

OPTIMAL METER PLACEMENT AND TRANSACTION-BASED LOSS
ALLOCATION IN DEREGULATED POWER SYSTEM OPERATION

A Dissertation

by

QIFENG DING

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2004

Major Subject: Electrical Engineering

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ABSTRACT

Optimal Meter Placement and Transaction-based Loss Allocation in Deregulated Power System Operation. (December 2004)

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In this dissertation topics of optimal meter placement and transaction-based loss allocation in deregulated power system operation are investigated.

Firstly, Chapter II introduces the basic idea of candidate measurement identification, which is the selection of candidate measurement sets, each of which will make the system observable under a given contingency (loss of measurements and network topology changes). A new method is then developed for optimal meter placement, which is the choice of the optimal combination out of the selected candidate measurement sets in order to ensure the entire system observability under any one of the contingencies.

A new method, which allows a natural separation of losses among individual transactions in a multiple-transaction setting is proposed in Chapter III. The proposed method does not use any approximations such as a D.C. power flow, avoiding method induced inaccuracies. The power network losses are expressed in terms of individual power transactions. A transaction-loss matrix, which illustrates the breakdown of losses introduced by each individual transaction and interactions between any two transactions, is created. The network losses can then be allocated to each transaction based on the transaction-loss matrix entries.

The conventional power flow analysis is extended in Chapter IV to combine with the transaction loss allocation. A systematic solution procedure is formed in order to adjust generation while simultaneously allocating losses to the generators designated by individual transactions.

Furthermore, Chapter V presents an Optimal Power Flow (OPF) algorithm to optimize the loss compensation if some transactions elect to purchase the loss service from the Independent System Operator (ISO) and accordingly the incurred losses are fairly allocated back to individual transactions. IEEE test systems have been used to verify the effectiveness of the proposed method.

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CHAPTER I

INTRODUCTION

1.1 Electric Power Industry Deregulation

The electric utility industry is undergoing significant and irreversible changes that are reshaping an industry that has been remarkably stable for a long time. Where power generation was once dominated by vertically integrated investor-owned utilities (IOUs) that owned most of the generation capacity, transmission, and distribution facilities, the electric power industry now has many companies that produce and market wholesale and retail electric power [1]. The significant feature of these changes is to allow for competition, which is seen as necessary to increase the efficiency of electric energy production and distribution, and to offer a lower-price, high-quality and more secure product.

Several factors [1][2] have motivated the changes occurring in the electric power industry. First, technological advances have altered the economics of power production. Some new technologies like Flexible AC Systems (FACTS) have made possible the economic transmission of power over long distances so that customers can now be more selective in choosing an electricity supplier. Second, between 1975 and 1985, residential electricity prices and industrial electricity prices rose 13 percent and 28 percent. These rate increases, caused primarily by increases in utility construction and fuel costs, caused government officials to call into question the existing regulatory environment. Third, legislative and regulatory mandates like the Federal Energy Regulatory Commission (FERC) orders 888, 889 and other federal and state regulations have been causing the power industry to move rapidly toward the open transmission access, the unbundling of previously integrated services and the creation of regional transmission organizations (RTOs), Independent System Operators (ISOs) and so on.

This dissertation follows the style and format of *IEEE Transactions on Power Delivery*.

With the deregulation of the electric power industry, market participants such as generation companies, transmission and distribution companies, and retailer companies need to adjust the strategies of system operation and control in order to reduce costs, enhance asset utilization, improve planning and market management, better the system reliability and security, and nurture customer retention. There are many issues which need to be addressed. In this dissertation two main topics will be investigated. One is optimal meter placement, which is to optimize the cost of placing new meters against loss of measurements, branch outages and topology changes. Meters are required to be installed throughout the system so that the operating states can be observed. Moreover, redundant measurements are necessary to filter out measurement errors. Under deregulation, the cost of installing meters is a great concern to the utility companies while the goal of better monitoring and controlling the power systems can not be compromised. At the same time to ensure the electricity transactions there are more and more switching actions and topology changes throughout the network. As a result, how to reduce the cost of installing any new meters in the existing power network while maintaining the observability of the network against loss of measurements and topology changes, becomes an important topic. The other concern is about transmission loss allocation in a multi-transaction framework. Here the challenge is to fairly allocate the transmission losses to individual electricity transactions. In real-time operation, usually consumer meters measure their actual consumption while generators measure their actual electricity production which include the load consumption and the network losses. It brings about the problem of “who should pay for losses”. We all know losses in fact are the result of energy transactions through the transmission network where generators and consumers are engaged. Since the cost of losses can be substantial, how to reasonably, and relatively fairly allocate the transmission losses to individual transactions remains a difficult yet important issue to be resolved.

1.2 State Estimation, Observability Analysis and Meter Placement

Electric power system state estimation [3-8] was introduced by Fred Schweppe of MIT in 1969. The operating state of a power system is determined by the state

estimation (SE) function using a redundant set of real-time measurement data. The SE function is the basis for all advanced applications of an Energy Management System (EMS). The results of state estimation are used to compute various estimates for the line flows, losses and net bus injections. The deregulation of the electric power industry has transformed state estimation from an important application into a critical one [5]. Many critical commercial issues in the power market, such as congestion management, need to be formulated and addressed based on a precise model of the power system, which is derived from the state estimation process. Failure to obtain these quantities in real-time or miscalculating them, should be avoided in order to ensure proper accounting of power transactions as well as for system security. This implies that the state estimator should be made very robust against the topology changes (including branch outages and bus splitting) and temporary loss of measurements or remote terminal units (RTU). Observability analysis is another important procedure closely related to state estimation. State estimation will not be possible if there are not enough measurements. A system is determined to be observable if all the state variables (bus voltage magnitudes and relative phase angles) can be estimated using the available measurements. Various methods proposed for network observability analysis have been well documented in the literature [9-14]. If the power system is unobservable there will be a need to install new meters to make the system observable. Meter placement requires making decisions as to where and what types of meters should be placed. Many approaches are based on Integer Programming (IP) and heuristic solution techniques [15-19].

When installing a new state estimator or upgrading an existing one, measurement configuration will have to be considered in order to ensure that the system will be observable. The paper [20] investigates the meter placement problem with an objective of ensuring network observability against single branch outages. It presented a topological method to install new meters around the system for single branch outages. The papers [21][22] present a systematic procedure by which measurement systems can be optimally upgraded. The proposed method yields a measurement configuration that can withstand any single branch outage or loss of single measurement, without losing

network observability. It is a numerical method based on the measurement Jacobian and sparse triangular factorization, making its implementation easy in existing state estimators. However, that method is valid only for loss of single measurements and single branch outages, and also assumes that the original measurement system is observable. In a given practical power system there will be some specified contingencies which will include simultaneous loss of several measurements and/or outages of several branches. A new method [23] will be proposed in Chapter II, which greatly improves the unified approach presented in [21][22] to account for cases involving such contingencies. This is accomplished by extending the IP problem to consider more than one candidate for a given contingency.

1.3 Loss Allocation

The electric power industry is experiencing important changes brought about by the deregulation. Electric power generators and users engage in power transactions which take place over the transmission system and create losses. Transmission losses represent up to 5—10% of the total generation, and are worth millions of dollars per year. Consequently, the problem of “who should pay for losses” arises and the satisfactory sharing of the transmission system utilization costs among all market participants has become a key issue. Unfortunately, losses are expressed as a nonlinear function of line flows, and it becomes almost impossible to calculate exactly the losses that are incurred by each generator, load or transaction in the system.

A number of loss allocation schemes have been proposed in the literature to allocate the system losses to generators / loads in a pool market or to individual transactions in a bi-lateral contracts market. Based on different assumptions and approximations there are mainly four families of schemes: Pro rata methods [24], incremental transmission loss (ITL) methods [25-30], proportional sharing procedures [31-41], and loss formula methods [42-51]. Different loss allocation methods have been compared in [52-55].

Pro rata methods assign the transmission system losses to the generators and/or loads proportional to their active generation or load consumption. The electricity markets of England, Spain and Brazil are currently using Pro rata schemes to allocate the losses to

generators and/or consumers. This type of methods is simple to understand and implement. However, the network topology is never taken into account. Obviously it is not fair for two identical loads, which locate near generators and far away from generators respectively, to be allocated with the same amount of losses.

ITL methodologies use the sensitivities of losses to bus injections to allocate the losses to generators and loads. The paper [28] provides analyses and test results from a practical implementation of an incremental allocation procedure in the Norwegian electric system. The paper [29] solves a system of differential equations by using numerical integration where a distributed slack bus concept is used. The ITL methods depend on the selection of the slack bus and also the slack bus is allocated with no losses.

Proportional sharing methods, sometimes called flow-tracing schemes, assume that the power injections are proportionally shared among the outflows of each bus and trace the electricity down from the generation sources or up from the load sinks. The assumption here “the power flow reaching a bus from any power line splits among the lines evacuating power from the bus proportionally to their corresponding power flows” is neither provable nor disprovable. Also, it is not possible to allocate losses to generators and loads at the same time.

Recently, some loss-formula based methods have been presented. A quadratic loss formula is proposed in [42] to allocate transmission losses among trades. A "physical-power-flow-based" approach expresses the quadratic loss approximation with individual transactions in a multiple-transaction framework in [43]. Another loss allocation method is based on the bus impedance Z -bus matrix [44] and allocates transmission losses among loads and generators assuming a pool dispatch. A natural separation of the system losses among the network buses is derived using the loss formula. It does not however account for the interaction between different injections.

A new method [48], which allows a natural separation of losses among individual transactions in a multiple-transaction setting, will be discussed in Chapter III. The proposed method does not use any approximations such as a D.C. power flow, avoiding method induced inaccuracies. The power network losses are expressed in terms of

individual power transactions. A transaction-loss matrix, which illustrates the breakdown of losses introduced by each individual transaction and interactions between any two transactions, is created. The network losses can then be allocated to each transaction based on the transaction-loss matrix entries.

The use of a single slack bus to compensate for all the losses for all transactions has been widely discussed in many papers. However, it is also possible to have each transaction assign its own chosen buses for the loss compensation. It will certainly modify the system state, and will require further discussion of loss allocation along with loss compensation. In [47] and [49], the losses allocated to specific transactions are considered to be supplied by the generators corresponding to that transaction during the power flow calculation. This way, the power flow results indicate not only the system state but the transaction loss adjustment. Distributed slack buses have been used in [50] to compensate the system losses for bilateral transactions. Loss compensation in multiple transaction networks is discussed in [51]. The transactions have choices to elect self-acquisition of loss compensation at designated buses or to purchase the loss compensation service from the ISO. A linear optimization method is used to obtain the least-price loss compensation service. In [56], the transaction-based power flow analysis (TBPF) for transmission utilization allocation is proposed, which utilizes distributed purchase-sale pairs to replace the role of a single slack bus on energy imbalance during power flow calculation iterations. If transaction pairs compensate all losses for themselves, the single slack bus is not needed. The proposed method also is used for congestion management.

In Chapter IV, the conventional power flow analysis is extended to justify the transaction active generation after taking into account the loss allocation and transaction designated compensation generation. Unlike previous work, transactions can freely choose generators for loss compensation. Furthermore, in Chapter V an Optimal Power Flow (OPF) program is used to optimize the loss compensation if some transactions elect to purchase the loss service from the ISO and accordingly the incurred losses are fairly allocated back to individual transactions.

1.4 Contributions of the Dissertation

The main contributions of this dissertation include the following

1. An improved systematic optimal meter placement method is developed. The method allows the cost optimal selection of meters so that the system remains observable under any single or multiple measurement losses, branch outages, bus splitting and any other pre-defined contingency.
2. A new method which allocates losses to individual power transactions in a multiple transaction setting is developed. The derivations for the allocated losses directly follow the system loss formula and do not make any simplifying assumptions. It explicitly expresses the losses in terms of individual transactions, and this leads to a natural separation of system losses among all transactions in the system. A transaction-loss matrix, which illustrates coupling effects between any pair of transactions taking place in the system, is introduced to aid the implementation of the developed method.
3. The transaction framework is extended to include loss compensation selection. Transactions are allowed to freely designate generators for loss compensation. The conventional power flow analysis is combined with the transaction loss allocation and transaction designated loss compensation. Since the loss allocation itself depends on the solution, the two problems are combined and solved together. This combined formulation leads to a systematic solution procedure in order to adjust generation while simultaneously allocating losses to the generators designated by individual transactions.
4. An optimal power flow formulation in which the generation is dispatched in order to compensate for losses allocated to different individual transactions is proposed. The case where some transactions prefer instead to let the ISO to provide the loss compensation service is also considered. An optimization procedure, which yields the least-cost compensation from participating generators, is developed for this purpose by using an OPF model.

CHAPTER II

OPTIMAL METER PLACEMENT AGAINST CONTINGENCIES

2.1 Introduction

The operating state of a power system is determined by the State Estimation (SE) function using a redundant set of real-time data. With the assumption of knowing the parameters and the topology of the system, SE problem is commonly formulated as follows:

$$\begin{aligned} & \text{Minimize } f(z - h(x)) \\ & \text{Subject to } g(x) = 0 \\ & \quad \quad \quad c(x) \leq 0 \end{aligned} \tag{1}$$

where z is a vector of measurements usually including voltage magnitudes, real and reactive power injections and line flows, line current magnitudes, transformer taps, etc.; $h(x)$ is a vector function expressing measurements in terms of state variables, which usually are bus voltage magnitudes V and phase angles θ ; $f(z - h(x))$ is an objective function; and $g(x)$ and $c(x)$ are vector functions representing power flow quantities such as zero injections and voltage limits. Depending on the choice of the objective function, SE problem can be solved by different methods. The method most commonly used by commercial packages is, the Weighted Least Squares (WLS) method, having the following objective function:

$$f(z - h(x)) = (z - h(x))^T W (z - h(x)) \tag{2}$$

where the super-script “ T ” denotes vector/matrix transposition, and W is a diagonal matrix of measurement weights. These weights are assigned as the reciprocals of corresponding measurement variances. The solution of (1) yields the estimated operating states of the system based on the real-time measurements. Several other alternative methods [4-8] such as the WLS method, the Weighted Least Absolute Value (WLAV) method, the Weighted Least Median (WLM) method, etc., have been presented in the last two decades.

The ability to perform state estimation depends on whether sufficient measurements are well distributed throughout the power network. The system is considered as an observable network if enough measurements are available to estimate the voltage magnitudes and phase angles of all buses. There are numerous methodologies to determine the network observability by the real-time measurements, which can be classified into three categories: topological, numerical, and hybrid methods [9-14]. If the system is determined to be unobservable, new meters will have to be installed. This is referred to as the meter placement problem.

Whether a new state estimator is put into service or an existing one is being upgraded, placing new meters for improving or maintaining reliability of the measurement system is of great concern. Determination of the best possible combination of meters for monitoring a given power system is referred to as the optimal meter placement problem. In choosing the types and locations of new measurements, there may be several different concerns, such as:

- Maintaining an observable network when one or more measurements are lost.
- Maintaining an observable network when one or more network branches are disconnected.
- Maintaining an observable network when one or more network buses are split.
- Minimizing the cost of new metering.

Our goal is to present a systematic procedure which can yield a measurement configuration that can withstand any one or more branch outages, loss of one or more measurements and bus splitting without losing network observability.

2.2 Previous Work

The paper [20] presented a topological method for single branch outages. A linear programming based solution is proposed for choosing a measurement configuration that will make the system fully observable. The paper [21-22] proposed a unified approach, which generalized the meter placement problem formulation to simultaneously take into account both types of contingencies, namely loss of a branch or a measurement. The method is a numerical approach and can be implemented easily by modifying existing

state estimation program. Furthermore, the total cost of adding measurement as well as the number of additional measurements are simultaneously minimized by an integer programming (IP) program. The method, however, is valid only for loss of single measurements and single branch outages. In a given power system there are some specified contingencies which include losses of multi-measurements, outages of multi-branches, as well as bus splitting. Moreover, the unified method proposed in [21-22] introduces candidates for a contingency, which only needs one additional candidate measurements in order to make the system fully observable. If more than one candidate measurements simultaneously are necessary for a contingency to make the system observable, the candidate identification and IP problem need to be extended.

This chapter greatly improves the unified approach presented in [21-22] to against loss of multi-measurements, outages of multi-branches and bus splitting, extends the IP problem to consider more than one candidate for a contingency, and demonstrates method's application to several systems.

2.3 Problem Statement

The performance of a state estimator includes considerations of accuracy, as well as reliability. A reliable state estimator should continue operating even under contingencies such as branch outages or temporary loss of measurements. On the other hand, budgetary constraints prohibit expansion of measurement systems for the sake of redundancy. Hence, we should look at an optimization problem where the number of meters should be kept at a minimum while ensuring network observability for a predetermined set of contingencies.

One indicator of observability is the column rank of the measurement Jacobian matrix, H , whose column rank is not affected by the operating point, but essentially depends on the measurement configuration. Therefore, it is sufficient to evaluate H at flat start in order to study the effects of branch outages, measurement losses, and other contingencies, on its rank. Let the rows of Jacobian matrix H , be ordered so that the first m_e measurements are existing measurements. If the system is originally observable, the column rank of H should be full, i.e. equal to n . For each contingency, for example, loss

of a single measurement, a single branch outage, losses of multi-measurements, outages of multi-branches, or bus splitting, the column rank of H should then remain full. If not, then proper extra measurements should be added to make the H rank full. The choice of these additional measurements must be optimal so that the overall cost of adding these measurements should be a minimum.

The solution for this problem can be divided into two separate stages:

- One is “candidate measurements identification”, which is the selection of candidate measurement sets, each of which will make the system observable under a given contingency.
- The other is “optimal meter placement”, which is the choice of the optimal combination out of the selected candidate measurement sets in order to ensure the entire system observability under any one of the contingencies. Solution of this problem requires an integer programming approach.

2.4 Measurement Jacobian H Matrix

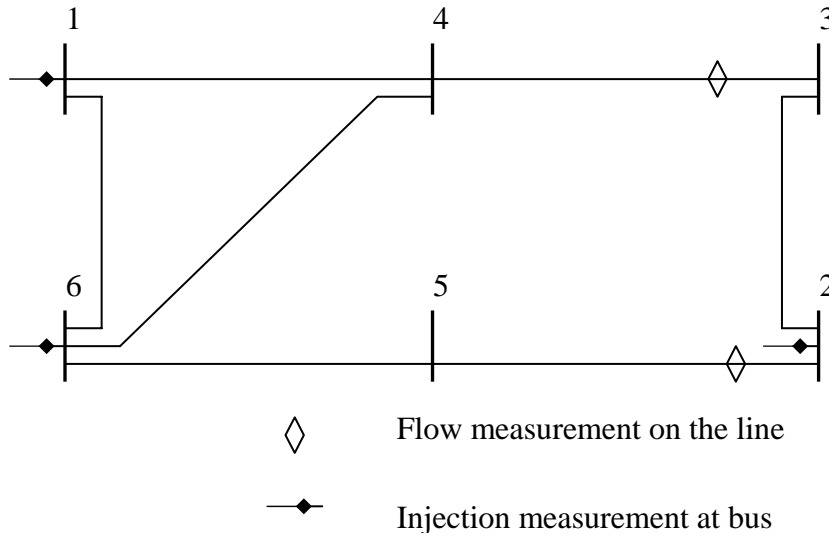


Fig. 1. 6-bus system example with measurements

Measurement Jacobian H matrix is the sub-matrix representing the gradient of the real power measurements with respect to all bus phase angles, in the decoupled model.

Let the rows of the H measurement Jacobian matrix be ordered such that the existing measurements are listed first as shown below:

$$H = \begin{bmatrix} H_{existing} \\ \dots \\ H_c \end{bmatrix} \begin{matrix} m_e \text{ existing measurements} \\ \\ m_c \text{ candidate measurements} \end{matrix} \quad (3)$$

where, $m = m_e + m_c$ is the total number of measurements that are either already existing (m_e existing measurements) or likely to be installed (m_c candidate measurements).

Let's take a 6-bus test system [21] as an example to build the measurement Jacobian H matrix assuming all the line impedance values are $j1$ and θ_1 is the reference angle.

All the measurements shown in the above Fig. 1 are considered as existing measurements, and all the injection measurements and flow measurements which are not shown in Fig. 1 are consider as candidate measurements.

Existing Measurements = [Injections: 1, 2, 6; Flows: 2-5, 3-4].

Candidate Measurements = [Injections: 3, 4, 5; Flows: 1-4, 1-6, 2-3, 4-6, 5-6].

The measurement Jacobian matrix H can be easily built as follows,

$$\begin{matrix} & \begin{bmatrix} \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \end{bmatrix} \\ \text{Inj.1} & \begin{bmatrix} 0 & 0 & -1 & 0 & -1 \end{bmatrix} \\ \text{Inj.2} & \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \end{bmatrix} \\ \text{Inj.6} & \begin{bmatrix} 0 & 0 & -1 & -1 & 3 \end{bmatrix} \\ \text{Flow 2-5} & \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \end{bmatrix} \\ \text{Flow 3-4} & \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \end{bmatrix} \\ \dots\dots\dots & \dots\dots\dots \\ \text{Inj.3} & \begin{bmatrix} -1 & 2 & -1 & 0 & 0 \end{bmatrix} \\ \text{Inj.4} & \begin{bmatrix} 0 & -1 & 3 & 0 & -1 \end{bmatrix} \\ \text{Inj.5} & \begin{bmatrix} -1 & 0 & 0 & 2 & -1 \end{bmatrix} \\ \text{Flow 1-4} & \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \end{bmatrix} \\ \text{Flow 1-6} & \begin{bmatrix} 0 & 0 & 0 & 0 & -1 \end{bmatrix} \\ \text{Flow 2-3} & \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \end{bmatrix} \\ \text{Flow 4-6} & \begin{bmatrix} 0 & 0 & 1 & 0 & -1 \end{bmatrix} \\ \text{Flow 5-6} & \begin{bmatrix} 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{matrix} \quad (4)$$

The upper part of the matrix is for the existing measurements while the lower part is for the candidate measurements.

2.4.1 Loss of Measurements

For the loss of one existing measurement k , we can set all entries of the k th row of the Jacobian matrix H equal to zeros or remove the k th row. If a contingency includes several measurement losses, then we set all entries of corresponding rows equal to zeros or remove all corresponding rows and have modified H matrix like:

$$H = \begin{bmatrix} H_{existing}^{mod} \\ \dots \\ H_c \end{bmatrix} \begin{matrix} m_e \text{ existing measurements} \\ \\ m_c \text{ candidate measurements} \end{matrix} \quad (5)$$

For example, if we lose the first measurement “Injection measurement at bus 1” in the above 6-bus test system, the modified H matrix will become

$$H = \begin{bmatrix} H_{existing}^{mod} \\ \dots \\ H_c \end{bmatrix} = \begin{matrix} & \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ \text{Inj.2} & 2 & -1 & 0 & -1 & 0 \\ \text{Inj.6} & 0 & 0 & -1 & -1 & 3 \\ \text{Flow 2-5} & 1 & 0 & 0 & -1 & 0 \\ \text{Flow 3-4} & 0 & 1 & -1 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \text{Inj.3} & -1 & 2 & -1 & 0 & 0 \\ \text{Inj.4} & 0 & -1 & 3 & 0 & -1 \\ \text{Inj.5} & -1 & 0 & 0 & 2 & -1 \\ \text{Flow 1-4} & 0 & 0 & -1 & 0 & 0 \\ \text{Flow 1-6} & 0 & 0 & 0 & 0 & -1 \\ \text{Flow 2-3} & 1 & -1 & 0 & 0 & 0 \\ \text{Flow 4-6} & 0 & 0 & 1 & 0 & -1 \\ \text{Flow 5-6} & 0 & 0 & 0 & 1 & -1 \end{matrix} \quad (6)$$

2.4.2 Loss of Branches

It is known that, network observability will be drastically affected by topology changes. Assuming that one contingency includes one or more branch outages, for each branch outage, say k - j branch is outaged, some related elements of H are modified like:

- $H_{ik}^{\text{mod}} = H_{ij}^{\text{mod}} = 0$ if measurement i is a line flow;
- $H_{ij}^{\text{mod}} = 0$, $H_{ik}^{\text{mod}} = H_{ik} + H_{ij}$, if measurements i is an injection at bus k .
- $H_{ik}^{\text{mod}} = 0$, $H_{ij}^{\text{mod}} = H_{ik} + H_{ij}$, if measurements i is an injection at bus j .

After modifying the related elements of Jacobian for all branch outages in that contingency, we have the measurement Jacobian matrix modified as:

$$H = \begin{bmatrix} H_{\text{existing}}^{\text{mod}} \\ \dots \\ H_c^{\text{mod}} \end{bmatrix} \begin{matrix} m_e \text{ existing measurements} \\ \\ m_c \text{ candidate measurements} \end{matrix} \quad (7)$$

For example, if the line 4-6 is outaged in the 6-bus system, the measurement Jacobian H matrix will be modified as

$$H = \begin{bmatrix} H_{\text{existing}}^{\text{mod}} \\ \dots \\ H_c^{\text{mod}} \end{bmatrix} = \begin{matrix} \text{Inj.1} \\ \text{Inj.2} \\ \text{Inj.6} \\ \text{Flow 2-5} \\ \text{Flow 3-4} \\ \dots\dots\dots \\ \text{Inj.3} \\ \text{Inj.4} \\ \text{Inj.5} \\ \text{Flow 1-4} \\ \text{Flow 1-6} \\ \text{Flow 2-3} \\ \text{Flow 4-6} \\ \text{Flow 5-6} \end{matrix} \begin{bmatrix} \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\ 0 & 0 & -1 & 0 & -1 \\ 2 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ \dots\dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (8)$$

2.4.3 Bus Splitting

In some substations, we often have operation modes shown in Fig. 2. Bus A and bus B are originally connected in the system, and in power system analysis they will often be treated as one bus. However, sometimes bus A and bus B will operate separately like shown in Fig. 2.

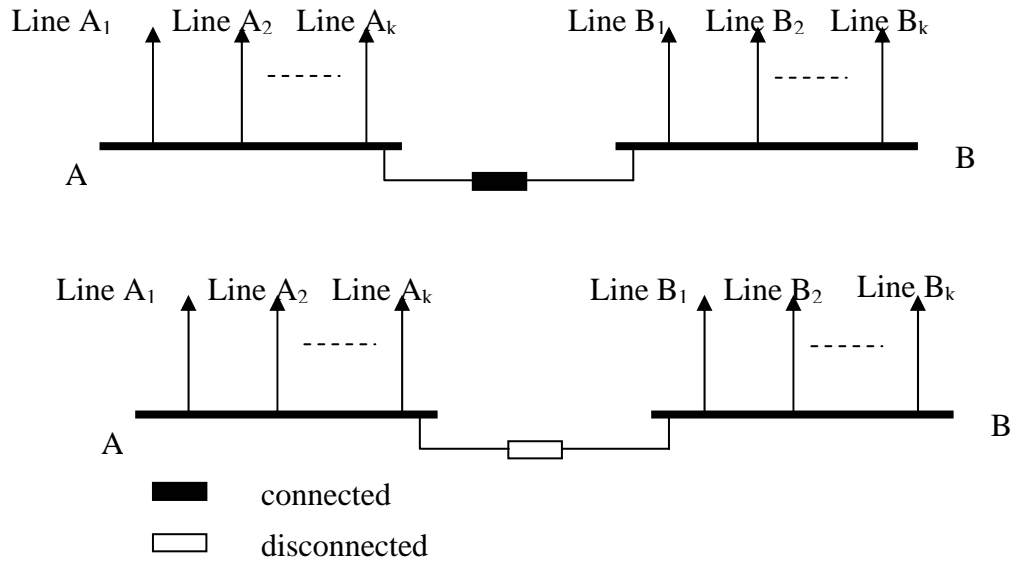


Fig. 2 Bus splitting

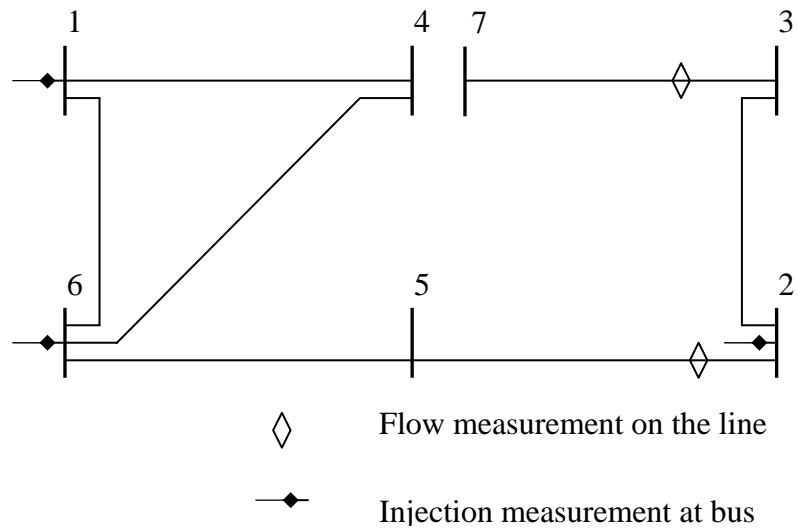


Fig. 3. 6-bus system example with bus splitting at bus 4

In order to consider such contingency in the optimal meter placement, firstly we should modify the H matrix like:

- The column number of H matrix will increase to $n+1$ since we split one bus to two buses. Assume that after splitting bus A will still use the previous bus number, for example i , then bus B will become bus $n+1$.
- $H_{k,n+1} = H_{k,i}$, $H_{k,i}^{\text{mod}} = 0$, (for the injection power measurement k on the bus j which is connected to bus B, and i is the bus number for bus A).
- $H_{l,n+1} = H_{l,i}$, $H_{l,i}^{\text{mod}} = 0$, (for the flow measurement l on the line which is connected with bus B, and i is the bus number for bus A).
- $H_{k,i}^{\text{mod}} = H_{k,i} + H_{k,j}$, $H_{k,j}^{\text{mod}} = 0$, (for the injection power measurement k on the splitting bus i , j is the bus connected to bus B.)

Remark: For the bus-splitting contingency, the column rank of H matrix will be $n+1$.

For example, if bus splitting occurs at bus 4 like shown in Fig. 3, the H matrix will be modified as

$$H = \begin{bmatrix} H_{existing}^{mod} \\ \dots \\ H_c^{mod} \end{bmatrix} = \begin{matrix} \text{Inj.1} \\ \text{Inj.2} \\ \text{Inj.6} \\ \text{Flow 2 - 5} \\ \text{Flow 3 - 7} \\ \dots\dots\dots \\ \text{Inj.3} \\ \text{Inj.4} \\ \text{Inj.5} \\ \text{Flow 1 - 4} \\ \text{Flow 1 - 6} \\ \text{Flow 2 - 3} \\ \text{Flow 4 - 6} \\ \text{Flow 5 - 6} \end{matrix} \begin{bmatrix} \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 \\ 0 & 0 & -1 & 0 & -1 & 0 \\ 2 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 3 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ \dots\dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ -1 & 2 & 0 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \quad (9)$$

2.5 Candidate Measurements Identification

2.5.1 Triangular Factorization of H matrix

For each pre-determined contingency, we can obtain the modified Jacobian H matrix by the method mentioned above for losses of measurements, outages of branches as well as bus splitting.

Triangular factorization on the modified H matrix with pivoting within existing m_e measurements, which occurs when zero pivoting is met and then the zero-pivoting row is moved to the end of the existing measurements, until we cannot select non-zero pivoting will yield the following:

$$H = \begin{bmatrix} L_e \\ \dots \\ M_r \\ \dots \\ M_c \end{bmatrix} [U_e] \quad (10)$$

where, the sparse lower triangular matrix L_e and sparse rectangular matrix M_r are corresponding to the existing measurements, and sparse rectangular matrix M_c is corresponding the candidate measurements. U_e is sparse upper triangular matrices. In carrying out the factorization procedure, pivoting is restricted to the existing m_e measurements. If the rank of the sparse lower triangular matrix L_e is full, i.e. n for $n+1$ bus system, then the system is observable. If not, we have to select candidate rows from M_c to make the matrix rank full. Those selected candidate rows are corresponding to candidate measurements which can be chosen to make the given system observable.

2.5.2 Candidate Measurement Selection

If the triangular factorization for the modified H matrix corresponding to one contingency still can be proceeded to n th row, which means that the rank of H matrix still is full after the contingency, we will say the contingency does not affect the observability of the system so we do not need to search for any candidate measurement.

If the result of triangular factorization on the modified H matrix says that the rank of the matrix is $n-1$, which means the contingency results in making the system unobservable, we can select candidates from the lower rectangular factor which looks like below:

$$\begin{array}{c}
 \cdot \\
 \cdot \\
 \vdots \\
 \cdot \\
 \cdot \\
 j_1 \\
 \cdot \\
 j_2 \\
 \cdot
 \end{array}
 \begin{bmatrix}
 \times & 0 & \cdots & \cdot & 0 \\
 & \times & \cdots & \cdot & 0 \\
 & & \ddots & \vdots & \vdots \\
 & & & \times & 0 \\
 & & M_r & \cdot & 0 \\
 & & & \cdot & \times \\
 & & & \cdot & \times \\
 & & & \cdot & \cdot
 \end{bmatrix}
 \begin{array}{c}
 \cdots \\
 L_e \\
 \vdots \\
 \cdots \\
 M_r \\
 \cdots \\
 M_c \\
 \vdots \\
 \cdots
 \end{array}
 \quad (11)$$

The measurements $j_1, j_2 \dots$ having non-zeros in the n th column of the lower rectangular factor in the M_c will be selected as candidates for that contingency.

More generally, if the factorization of the modified H matrix for one contingency shows the rank is $n-k$, we will have the lower rectangular factor which looks like below:

$$\begin{array}{c} \cdot \\ \cdot \\ \vdots \\ \cdot \\ \cdot \\ j_1 \\ \cdot \\ j_2 \\ \cdot \end{array} \begin{bmatrix} \times & 0 & \cdots & \cdot & 0 \\ & \times & \cdots & \cdot & 0 \\ & & \times & \vdots & \vdots \\ & & & \cdots & 0 \\ & & M_r & \cdot & 0 \\ & & & \times & \times \\ & & & \cdot & \\ & & & \times & \times \\ & & & \cdot & \end{bmatrix} \begin{array}{c} \cdots \\ L_e(rank = n - k) \\ \vdots \\ \cdots \\ M_r \\ \cdots \\ M_c \\ \vdots \\ \cdots \end{array} \quad (12)$$

For this case, additional k measurements are needed, and we have to select candidates from M_c to increase the rank.

As we know, for a matrix A , if we have a nonsingular matrix P , which makes $PA = L$, we will have $rank(A) = rank(L)$ and for a matrix like $A = \begin{bmatrix} A_1 & 0 \\ A_2 & A_3 \end{bmatrix}$, we have

$$rank(A) = rank(A_1) + rank(A_3) \quad (13)$$

As a result for the H matrix, after triangular factorization we have the above matrix (13). Obviously the rows, which contain nonzero elements, will be possible candidates to make the matrix rank full. So firstly we mark these nonzero rows as C (candidates). For each combination with k rows among these C nonzero rows, we will have the matrix like

$$\begin{bmatrix} L_e & 0 \\ M_{cl} & M_{cr} \end{bmatrix} \quad (14)$$

where M_{cl} and M_{cr} have k rows. If $rank(M_{cr}) = k$, then the whole matrix will be full; if not, this set of possible candidates will not be selected since to include those additional measurements will not make the system observable.

If we have l nonzero rows in M_c ($l \geq k$), then we will have C_l^k sets of possible candidates. By the rank analysis of the submatrix M_{cr} , we will know the number of

candidates among C_l^k sets of possible candidates, which can be selected to make the system observable.

Obviously the way described above possibly is time-consuming if many additional candidate measurements are needed to make the system observable since we have to do search for all candidate measurement combinations. Clearly the fewer additional candidate measurements needed, the less complicated to find all sets of candidates. In reality, there are very few cases that need more than five or more candidate measurements combined to make the system observable after a contingency, so it should not be very time-consuming to find all sets of candidate measurements.

2.5.3 Examples

For the 6-bus system if we assume we lose the first measurement “injection measurement at bus 1”, we will have the modified H matrix as

$$H = \begin{matrix} & \begin{matrix} \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \end{matrix} \\ \begin{matrix} \text{Inj.2} \\ \text{Inj.6} \\ \text{Flow 2-5} \\ \text{Flow 3-4} \\ \dots\dots\dots \\ \text{Inj.3} \\ \text{Inj.4} \\ \text{Inj.5} \\ \text{Flow 1-4} \\ \text{Flow 1-6} \\ \text{Flow 2-3} \\ \text{Flow 4-6} \\ \text{Flow 5-6} \end{matrix} & \begin{bmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 & 3 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ \dots\dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 3 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{matrix} \quad (15)$$

If we do triangular factorization on the modified H matrix (15) with pivoting within existing m_e measurements we will finally have

$$\begin{array}{l}
 \text{Inj.2} \\
 \text{Inj.6} \\
 \text{Flow 2-5} \\
 \text{Flow 3-4} \\
 \text{.....} \\
 \text{Inj.3} \\
 \text{Inj.4} \\
 \text{Inj.5} \\
 \text{Flow 1-4} \\
 \text{Flow 1-6} \\
 \text{Flow 2-3} \\
 \text{Flow 4-6} \\
 \text{Flow 5-6}
 \end{array}
 \begin{bmatrix}
 \theta_2 & \theta_6 & \theta_3 & \theta_5 & \theta_4 \\
 2 & 0 & 0 & 0 & 0 \\
 0 & 3 & 0 & 0 & 0 \\
 1 & 0 & 0.5 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 \\
 \text{.....} & \text{.....} & \text{.....} & \text{.....} & \text{.....} \\
 -1 & 0 & 1.5 & 1 & 0 \\
 0 & -1 & -1 & -1.3333 & 1.3333 \\
 -1 & -1 & -0.5 & 0.6667 & 0.3333 \\
 0 & 0 & 0 & 0 & -1 \\
 0 & -1 & 0 & -0.3333 & -0.6667 \\
 1 & 0 & -0.5 & 0 & 0 \\
 0 & -1 & 0 & -0.3333 & 0.3333 \\
 0 & -1 & 0 & 0.6667 & 0.3333
 \end{bmatrix}
 \tag{16}$$

The result of triangular factorization on the modified H matrix says that the rank of the matrix is 4, which means the loss of injection measurement at bus 1 results in making the system unobservable. Also, obviously from the lower candidate measurement part of (16) we see the following candidate measurements which can make system observable: injection measurement at bus 4, injection measurement at bus 5, flow measurement on line 1-4, flow measurement on line 1-6, flow measurement on line 4-6, and flow measurement on line 5-6.

2.6 Optimal Meter Placement — IP Problem Formulation

From the candidate selection procedure above, candidate measurements for each contingency can be obtained. The objective of the optimal selection procedure is to minimize the overall cost of this measurement system upgrade while making sure that all contingencies are properly taken into account. Each candidate measurement will be assigned the installation cost.

In order to obtain the optimal meter placement for those pre-determined contingencies (including a single measurement loss, a single branch outage, loss of

multi-measurements, multi-branch outages and bus splitting) we can construct an Integer Optimization (IP) Problem like:

$$\text{to minimize } C^T \cdot X \quad (17)$$

where, C is a cost vector, and X is a binary candidate measurement status vector like:

$$X(i) = \begin{cases} 1 \\ 0 \end{cases} \quad (18)$$

if the measurement i is a candidate $X(i)$ is 1; otherwise 0.

The constraints of this IP problem will be:

- For the case that only one additional candidate measurement is necessary for the contingency,

$$\sum_i x_i \geq 1 \text{ (} i\text{th measurement is the candidate for the contingency)}$$

- For the case that additional two candidate measurements are necessary for the contingency,

$$\sum_i x_{i_1} x_{i_2} \geq 1 \text{ (} i_1 \text{ \& } i_2 \text{ measurements are the candidates for the contingency)}$$

- For the case that additional k candidate measurements are necessary for the contingency,

$$\sum_i \prod_k x_{i_k} \geq 1 \text{ (Set } i_1 \dots i_k \text{ measurements are the candidates for the contingency)}$$

The constraints in the IP problem ensures that each contingency is assigned candidate measurements whereas the objective function penalizes with respect to both the total cost as well as the number of selected candidates. Solution of the IP problem described above yields measurements as the optimal choice that will ensure network observability under any pre-determined contingency at minimum cost.

We still take the above 6-bus system as the example to illustrate how to construct IP constraints. From chapter 2.5.3 we know injection measurement at bus 4, or injection measurement at bus 5, or flow measurement on line 1-4, or flow measurement on line 1-6, or flow measurement on line 4-6, and or flow measurement on line 5-6 make the

system observable if we lose the injection measurement at bus 1. If we denote that x_i is for the each candidate measurement, we will have x_1 for the injection measurement at bus 3, x_2 for the injection measurement at bus 4, x_3 for the injection measurement at bus 5, so on and so for. Then for the contingency of losing injection measurement at bus 1 we will have the IP constraint like:

$$\text{Inj.1 Loss} : x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \quad (19)$$

2.7 Algorithm

The following algorithm is proposed for selecting candidates and determining optimal meter placement based on the above analysis:

- Step 1. Form the measurements H matrix, which includes not only the existing measurements but the non-existing measurements as the candidates.
- Step 2. For a contingency modify the measurements H matrix, then perform the triangular factorization with pivoting and matrix manipulation within existing measurement rows, and obtain the column rank of the matrix.
- Step 3. Check if the column rank of the modified H matrix is full. If yes, go to Step 4. If not, select the candidate measurements that can make the H matrix full.
- Step 4. Check if all the contingencies have been done. If not, go to Step 2. If yes, the IP problem is constructed based on all selected candidates.
- Step 5. Yield the optimal meter placement which ensures the entire system will remain observable under any one of the contingencies.
- Step 6. Stop.

2.8 Numerical Tests

2.8.1 6-bus System

The simple 6-bus system with its measurement configuration shown in Fig. 1 is considered to illustrate the proposed algorithm. All the measurements shown in the Fig.

1 are considered as existing measurements, and all the injection measurements and flow measurements not shown in Fig. 1 are consider as candidate measurements.

Existing Measurements = [Injections: 1, 2, 6; Flows: 2-5, 3-4].

Candidate Measurements = [Injections: 3, 4, 5; Flows: 1-4, 1-6, 2-3, 4-6, 5-6].

The chosen installation cost vector C^T corresponding to the candidate measurements is:

$$[1, 0.2, 0.4, 0.4, 0.5, 1, 1, 1]$$

Assuming that x_i is for the each candidate measurement, we will have x_1 for the injection measurement at bus 3, x_2 for the injection measurement at bus 4, x_3 for the injection measurement at bus 5, so on and so for.

The optimal meter placement algorithm should provide a set of additional candidate measurements which will ensure network observability after any of contingencies in the list including single measurement losses, single branch outages, multi-measurement losses, multi-branch outages as well as bus splitting.

Firstly we consider the optimal meter placement only for loss of single measurements and single branch outages like stated in [21], not consider the other contingencies. For each contingency (either loss of single measurement or single branch outage), at most one additional candidate measurement is needed to make the system observable since the original network is observable. By the method introduced in Chapter 2.4 and 2.5, we can construct the original measurement Jacobian matrix H , modify H matrix for each contingency and choose the candidate measurements, which eventually are expressed as IP problem constraints. In Chapter 2.4 the original measurement Jacobian matrix H has been built as:

$$\begin{array}{l}
 \text{Inj.1} \\
 \text{Inj.2} \\
 \text{Inj.6} \\
 \text{Flow 2 - 5} \\
 \text{Flow 3 - 4} \\
 \text{.....} \\
 x_1 : \text{Inj. 3} \\
 x_2 : \text{Inj. 4} \\
 x_3 : \text{Inj. 5} \\
 x_4 : \text{Flow 1 - 4} \\
 x_5 : \text{Flow 1 - 6} \\
 x_6 : \text{Flow 2 - 3} \\
 x_7 : \text{Flow 4 - 6} \\
 x_8 : \text{Flow 5 - 6}
 \end{array}
 \begin{bmatrix}
 \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\
 0 & 0 & -1 & 0 & -1 \\
 2 & -1 & 0 & -1 & 0 \\
 0 & 0 & -1 & -1 & 3 \\
 1 & 0 & 0 & -1 & 0 \\
 0 & 1 & -1 & 0 & 0 \\
 \text{.....} & \text{.....} & \text{.....} & \text{.....} & \text{.....} \\
 -1 & 2 & -1 & 0 & 0 \\
 0 & -1 & 3 & 0 & -1 \\
 -1 & 0 & 0 & 2 & -1 \\
 0 & 0 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & -1 \\
 1 & -1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & -1 \\
 0 & 0 & 0 & 1 & -1
 \end{bmatrix}
 \tag{20}$$

If we lose the first existing measurement, the first row in the matrix (20) will be removed and the H matrix will be:

$$\begin{array}{l}
 \text{Inj.2} \\
 \text{Inj.6} \\
 \text{Flow 2 - 5} \\
 \text{Flow 3 - 4} \\
 \text{.....} \\
 x_1 : \text{Inj. 3} \\
 x_2 : \text{Inj. 4} \\
 x_3 : \text{Inj. 5} \\
 x_4 : \text{Flow 1 - 4} \\
 x_5 : \text{Flow 1 - 6} \\
 x_6 : \text{Flow 2 - 3} \\
 x_7 : \text{Flow 4 - 6} \\
 x_8 : \text{Flow 5 - 6}
 \end{array}
 \begin{bmatrix}
 \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\
 2 & -1 & 0 & -1 & 0 \\
 0 & 0 & -1 & -1 & 3 \\
 1 & 0 & 0 & -1 & 0 \\
 0 & 1 & -1 & 0 & 0 \\
 \text{.....} & \text{.....} & \text{.....} & \text{.....} & \text{.....} \\
 -1 & 2 & -1 & 0 & 0 \\
 0 & -1 & 3 & 0 & -1 \\
 -1 & 0 & 0 & 2 & -1 \\
 0 & 0 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & -1 \\
 1 & -1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & -1 \\
 0 & 0 & 0 & 1 & -1
 \end{bmatrix}
 \tag{21}$$

Like shown in Chapter 2.5.3, if we do triangular factorization on the modified H

matrix (21) with pivoting within existing m_e measurements we will finally have

$$\begin{array}{l}
 \text{Inj.2} \\
 \text{Inj.6} \\
 \text{Flow 2-5} \\
 \text{Flow 3-4} \\
 \dots\dots\dots \\
 x_1 : \text{Inj. 3} \\
 x_2 : \text{Inj. 4} \\
 x_3 : \text{Inj. 5} \\
 x_4 : \text{Flow 1-4} \\
 x_5 : \text{Flow 1-6} \\
 x_6 : \text{Flow 2-3} \\
 x_7 : \text{Flow 4-6} \\
 x_8 : \text{Flow 5-6}
 \end{array}
 \begin{bmatrix}
 \theta_2 & \theta_6 & \theta_3 & \theta_5 & \theta_4 \\
 2 & 0 & 0 & 0 & 0 \\
 0 & 3 & 0 & 0 & 0 \\
 1 & 0 & 0.5 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 \\
 \dots\dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\
 -1 & 0 & 1.5 & 1 & 0 \\
 0 & -1 & -1 & -1.3333 & 1.3333 \\
 -1 & -1 & -0.5 & 0.6667 & 0.3333 \\
 0 & 0 & 0 & 0 & -1 \\
 0 & -1 & 0 & -0.3333 & -0.6667 \\
 1 & 0 & -0.5 & 0 & 0 \\
 0 & -1 & 0 & -0.3333 & 0.3333 \\
 0 & -1 & 0 & 0.6667 & 0.3333
 \end{bmatrix}
 \quad (22)$$

Obviously from (22) we can see that measurements x_2 , x_3 , x_4 , x_5 , x_7 and x_8 are candidate measurements if we lose the injection measurement at bus 1. The candidate measurements can be expressed as the IP constraint (23).

$$\text{Inj.1 Loss} : x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \quad (23)$$

Similarly, we can have candidate measurements chosen for each single contingency and all of them will be expressed as the IP constraints:

to minimize $C^T \cdot X$

$$\text{Inj.1 Loss} : x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Inj.2 Loss} : x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$$

$$\text{Inj.6 Loss} : x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Flow2-5 Loss : } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$$

$$\text{Flow3-4 Loss : } x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 2-3 Outage : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 2-5 Outage : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 3-4 Outage : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

The outages of single branches 1-4, 1-6, 4-6 and 5-6 will not affect the network observability so that no IP constraint will correspond to them. The solution of the integer programming problem yields the injection measurement at bus-4 as the optimal choice that will ensure network observability under any single branch outage or loss of single measurement at minimum cost 0.2. This result is same as the result from the method in [21].

Next, besides the list of losses of each single existing measurement and outages of each single branch we add two extra contingencies:

Contingency 1: branch 4-6 outage and loss of injection measurements at buses 1 and 6;

Contingency 2: Contingency 2: branch 2-3 outage and loss of injection measurement at bus 2.

Other than the candidate measurements for the loss of single measurements and single branches, which are stated above, we also can obtain the candidate measurement sets for these two contingencies by the method in Chapter 2.4 and 2.5. Contingency 1 will be taken as the example. After we take into account the pre-defined contingency 1 we will have the updated Jacobian matrix as

$$\begin{array}{l}
\text{Inj.2} \\
\text{Flow 2 - 5} \\
\text{Flow 3 - 4} \\
\text{.....} \\
x_1 : \text{Inj.3} \\
x_2 : \text{Inj.4} \\
x_3 : \text{Inj.5} \\
x_4 : \text{Flow 1 - 4} \\
x_5 : \text{Flow 1 - 6} \\
x_6 : \text{Flow 2 - 3} \\
x_8 : \text{Flow 5 - 6}
\end{array}
\begin{bmatrix}
\theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\
2 & -1 & 0 & -1 & 0 \\
1 & 0 & 0 & -1 & 0 \\
0 & 1 & -1 & 0 & 0 \\
\text{.....} & \text{.....} & \text{.....} & \text{.....} & \text{.....} \\
-1 & 2 & -1 & 0 & 0 \\
0 & -1 & 2 & 0 & 0 \\
-1 & 0 & 0 & 2 & -1 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 \\
1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}
\tag{24}$$

If we do triangular factorization on the modified H matrix (24) with row pivoting within existing m_e measurements we will finally have

$$\begin{array}{l}
\text{Inj.2} \\
\text{Flow 2 - 5} \\
\text{Flow 3 - 4} \\
\text{.....} \\
x_1 : \text{Inj.3} \\
x_2 : \text{Inj.4} \\
x_3 : \text{Inj.5} \\
x_4 : \text{Flow 1 - 4} \\
x_5 : \text{Flow 1 - 6} \\
x_6 : \text{Flow 2 - 3} \\
x_8 : \text{Flow 5 - 6}
\end{array}
\begin{bmatrix}
\theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 \\
2 & 0 & 0 & 0 & 0 \\
1 & 0.5 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 \\
\text{.....} & \text{.....} & \text{.....} & \text{.....} & \text{.....} \\
-1 & 1.5 & -1 & 0 & 0 \\
0 & -1 & 2 & 1 & 0 \\
-1 & -0.5 & 0 & 1 & -1 \\
0 & 0 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 \\
1 & -0.5 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}
\tag{25}$$

The upper sub-matrix of existing matrix has rank order 3. At least two extra measurements from the lower sub-matrix are needed to make the matrix full rank. Since the combinations of measurements x_2 and x_3 , x_2 and x_5 , x_2 and x_8 , x_3 and x_4 , x_3 and

x_5 , x_4 and x_5 , x_4 and x_8 , as well as x_5 and x_8 make the matrix full rank, they can be included into IP constraints:

$$\begin{aligned} \text{Contingency 1: } & x_2 * x_3 + x_2 * x_5 + x_2 * x_8 + x_3 * x_4 + x_3 * x_5 \\ & + x_4 * x_5 + x_4 * x_8 + x_5 * x_8 \geq 1 \end{aligned}$$

Similarly we can select the candidate measurements for the contingency 2. The whole IP problem will be written as:

$$\text{to minimize } C^T \cdot X$$

$$\text{Inj.1 Loss : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Inj.2 Loss : } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$$

$$\text{Inj.6 Loss : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Flow2-5 Loss : } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$$

$$\text{Flow3-4 Loss : } x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 2-3 Outage : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 2-5 Outage : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 3-4 Outage : } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\begin{aligned} \text{Contingency 1: } & x_2 * x_3 + x_2 * x_5 + x_2 * x_8 + x_3 * x_4 + x_3 * x_5 \\ & + x_4 * x_5 + x_4 * x_8 + x_5 * x_8 \geq 1 \end{aligned}$$

$$\text{Contingency 2: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

After considering these two contingencies, the IP solver shows that the optimal measurement set for this system is the injection measurements at bus 4 and 5 with the

minimum installation cost 0.6. Hence, inclusion of these two additional measurements will maintain the system observable during any single line outage or loss of any single measurement and these two contingencies in the 6 bus system.

Now we include other two bus splitting contingencies

Contingency 3: Bus 4 is split to two buses (Fig. 3).

Contingency 4: Bus 5 is split to two buses (Fig. 4).

After we consider the pre-defined contingency 3 we will have the updated Jacobian matrix as

$$\begin{array}{l}
 \text{Inj.1} \\
 \text{Inj.2} \\
 \text{Inj.6} \\
 \text{Flow 2-5} \\
 \text{Flow 3-7} \\
 \text{.....} \\
 x_1 : \text{Inj.3} \\
 x_2 : \text{Inj.4} \\
 x_3 : \text{Inj.5} \\
 x_4 : \text{Flow 1-4} \\
 x_5 : \text{Flow 1-6} \\
 x_6 : \text{Flow 2-3} \\
 x_7 : \text{Flow 4-6} \\
 x_8 : \text{Flow 5-6}
 \end{array}
 \begin{bmatrix}
 \theta_2 & \theta_3 & \theta_4 & \theta_5 & \theta_6 & \theta_7 \\
 0 & 0 & -1 & 0 & -1 & 0 \\
 2 & -1 & 0 & -1 & 0 & 0 \\
 0 & 0 & -1 & -1 & 3 & 0 \\
 1 & 0 & 0 & -1 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & -1 \\
 \text{.....} & \text{.....} & \text{.....} & \text{.....} & \text{.....} & \text{.....} \\
 -1 & 2 & 0 & 0 & 0 & -1 \\
 0 & 0 & 2 & 0 & -1 & 0 \\
 -1 & 0 & 0 & 2 & -1 & 0 \\
 0 & 0 & -1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & -1 & 0 \\
 1 & -1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & -1 & 0 \\
 0 & 0 & 1 & -1 & 0 & 0
 \end{bmatrix}
 \tag{26}$$

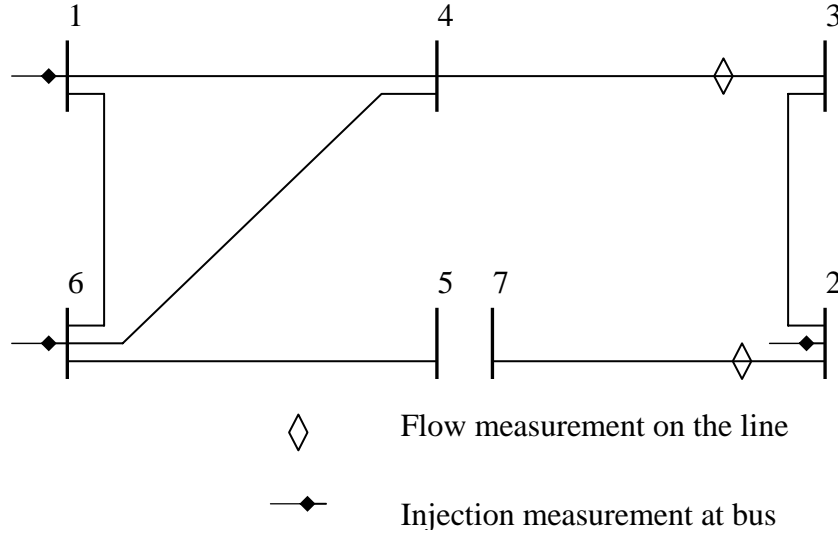


Fig. 4. 6-bus system with contingency 4

Since bus 4 is split to two buses, we have to add one bus into the Jacobian matrix (26). If we do triangular factorization on the modified H matrix (26) with row pivoting within existing m_e measurements we will finally have

$$\begin{array}{l}
 \text{Inj.1} \\
 \text{Inj.2} \\
 \text{Inj.6} \\
 \text{Flow 2-5} \\
 \text{Flow 3-7} \\
 \dots\dots\dots \\
 x_1 : \text{Inj.3} \\
 x_2 : \text{Inj.4} \\
 x_3 : \text{Inj.5} \\
 x_4 : \text{Flow 1-4} \\
 x_5 : \text{Flow 1-6} \\
 x_6 : \text{Flow 2-3} \\
 x_7 : \text{Flow 4-6} \\
 x_8 : \text{Flow 5-6}
 \end{array}
 \begin{bmatrix}
 \theta_4 & \theta_3 & \theta_6 & \theta_5 & \theta_2 & \theta_7 \\
 -1 & 0 & 0 & 0 & 0 & 0 \\
 0 & -1 & 0 & 0 & 0 & 0 \\
 -1 & 0 & 4 & 0 & 0 & 0 \\
 0 & 0 & 0 & -4 & 0 & 0 \\
 0 & 1 & 0 & -4 & 4 & 0 \\
 \dots\dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots & \dots\dots \\
 0 & 2 & 0 & -8 & 4 & 0 \\
 2 & 0 & -3 & -3 & -3 & -3 \\
 0 & 0 & -1 & 7 & 3 & 3 \\
 -1 & 0 & 1 & 1 & 1 & 1 \\
 0 & 0 & -1 & -1 & -1 & -1 \\
 0 & -1 & 0 & 4 & 0 & 0 \\
 1 & 0 & -2 & -2 & -2 & -2 \\
 1 & 0 & -1 & -5 & -5 & -5
 \end{bmatrix}
 \tag{27}$$

Obviously from (27) we can see that measurements x_2, x_3, x_4, x_5, x_7 and x_8 are candidate measurements if we split the bus 4. The candidate measurements can be expressed as the IP constraint (28).

$$\text{Contingency 3: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1 \quad (28)$$

The whole IP problem including all contingencies will be written as:

to minimize $C^T \cdot X$

$$\text{Inj.1 Loss: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Inj.2 Loss: } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$$

$$\text{Inj.6 Loss: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Flow2-5 Loss: } x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$$

$$\text{Flow3-4 Loss: } x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 2-3 Outage: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 2-5 Outage: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Branch 3-4 Outage: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\begin{aligned} \text{Contingency 1: } & x_2 * x_3 + x_2 * x_5 + x_2 * x_8 + x_3 * x_4 + x_3 * x_5 \\ & + x_4 * x_5 + x_4 * x_8 + x_5 * x_8 \geq 1 \end{aligned}$$

$$\text{Contingency 2: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Contingency 3: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

$$\text{Contingency 4: } x_2 + x_3 + x_4 + x_5 + x_7 + x_8 \geq 1$$

The optimal meter placement algorithm should provide a set of additional candidate measurements which will ensure network observability after any of contingencies in the list above. The IP solver shows that the optimal measurement set for this system is the injection measurements at bus 4 and 5 with the minimum installation cost 0.6. Hence, inclusion of these two additional measurements will maintain the system observable during any single line outage or loss of any single measurement and those pre-defined contingencies in the 6-bus system.

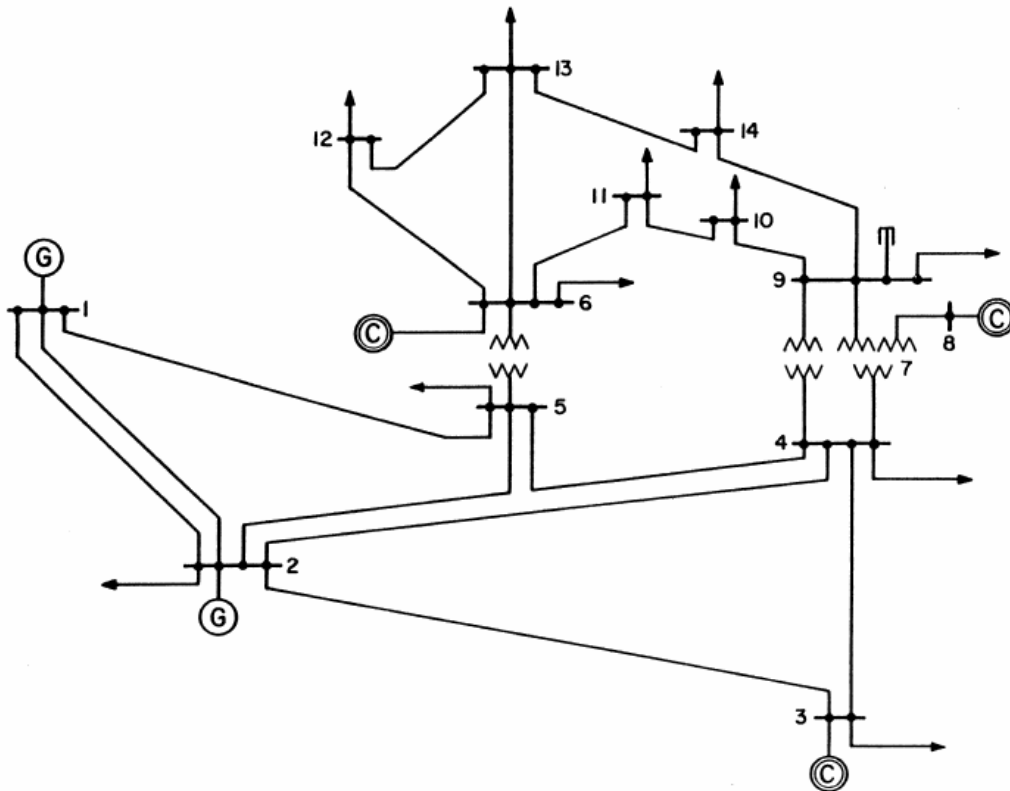


Fig. 5. IEEE-14 system

2.8.2 14-bus System

The IEEE-14 bus system shown in Fig. 5 with its measurement configuration is also used to demonstrate the proposed method.

Existing Measurements = [Injections: 12, 13, 6, 11, 7, 8, 5, 9, 10; Flows: 9-14, 7-9, 4-7, 7-8, 1-2, 2-3].

Candidate Measurements x_i = [Flows: 1-5, 2-4, 2-5, 3-4, 4-5, 4-9, 5-6, 6-11, 6-12, 6-13, 9-10, 10-11, 12-13, 13-14; Injections: 1, 2, 3, 4, 14].

The chosen installation cost vector C^T corresponding to the candidate measurements is:

[0.2, 1, 1, 1, 1, 1, 0.5, 0.5, 1, 0.4, 1, 0.6, 1, 0.5, 1, 1, 0.3, 0.6, 0.9]

Firstly we consider the optimal meter placement for loss of single measurements and single branch outages.

As a result, the contingency list consists of loss of each existing measurement and outage of each branch. For each contingency, at most one additional candidate measurement is needed to make the system observable. By the method introduced in Chapter 2.4 and 2.5, the candidate measurements, which are expressed as IP problem constraints, can be obtained for each existing measurement losses and single branch outages:

to minimize $C^T \cdot X$

$$Inj.12: x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1$$

$$Inj.13: x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1$$

$$Inj.6: x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1$$

$$Inj.11: x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1$$

$$\text{Inj.5: } x_1 + x_2 + x_3 + x_4 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1$$

$$\begin{aligned} \text{Inj.9: } & x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} \\ & + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\begin{aligned} \text{Inj.10: } & x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{12} + x_{13} + x_{14} \\ & + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\begin{aligned} \text{Flow9-14: } & x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} + x_{13} + x_{14} \\ & + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\text{Flow1-2: } x_1 + x_2 + x_3 + x_4 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1$$

$$\text{Flow2-3: } x_4 + x_{16} + x_{17} + x_{18} \geq 1$$

$$\text{Branch 4-7 Outage: } x_1 + x_2 + x_3 + x_4 + x_5 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1$$

$$\begin{aligned} \text{Branch 4-9 Outage: } & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} \\ & + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\text{Branch 2-3 Outage: } x_1 + x_2 + x_3 + x_4 + x_{15} + x_{16} + x_{17} + x_{18} \geq 1$$

$$\text{Branch 2-4 Outage: } x_4 + x_{17} + x_{18} \geq 1$$

$$\begin{aligned} \text{Branch 9-10 Outage: } & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} \\ & + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\begin{aligned} \text{Branch 6-11 Outage: } & x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} \\ & + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\begin{aligned} \text{Branch 6-12 Outage: } & x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} \\ & + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\begin{aligned} \text{Branch 10-11 Outage : } & x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} \\ & + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

$$\begin{aligned} \text{Branch 12-13 Outage : } & x_1 + x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_{10} + x_{13} \\ & + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \geq 1 \end{aligned}$$

Some branch outages such as branch 5-6 do not affect the system observability so there are no IP constraints for them. The solution of the integer programming problem yields the injection measurement at bus-3 as the optimal choice that will ensure network observability under any single branch outage or loss of single measurement at minimum cost 0.3.

Next, we consider the contingencies including loss of several measurements and outage of several branches.

Contingency 1: losses of injection measurements at bus 12 and bus 13;

Contingency 2: loss of the injection measurement at bus 9, and loss of branch 9-14;

Contingency 3: losses of injection measurements at bus 7 and 8, and loss of branch 7-9.

Besides the candidate measurements for the losses of single measurements and single branches, we also can obtain the candidate measurement sets for these three pre-determined contingencies, which also are expressed as IP constraints:

$$\text{Contingency 1: } x_9 * x_{10} + x_9 * x_{13} + x_9 * x_{14} + \dots \geq 1$$

$$\text{Contingency 2: } x_{11} * x_{14} + x_{11} * x_{18} + x_{11} * x_{19} + \dots \geq 1$$

$$\text{Contingency 3: } x_1 + x_2 + x_3 + x_4 + \dots \geq 1$$

In each contingency, for space limitation not all sets of candidate measurements are listed above. After considering these three pre-determined contingencies, the IP solver

shows that the optimal measurement set for this system is to include the injection measurement at bus 3 and the flow measurement in branch 6-13. Hence, inclusion of these additional measurements will maintain the system observable during any single line outage or loss of any single measurement and these three pre-determined contingencies in the IEEE-14 bus system.

2.8.3 30-bus System

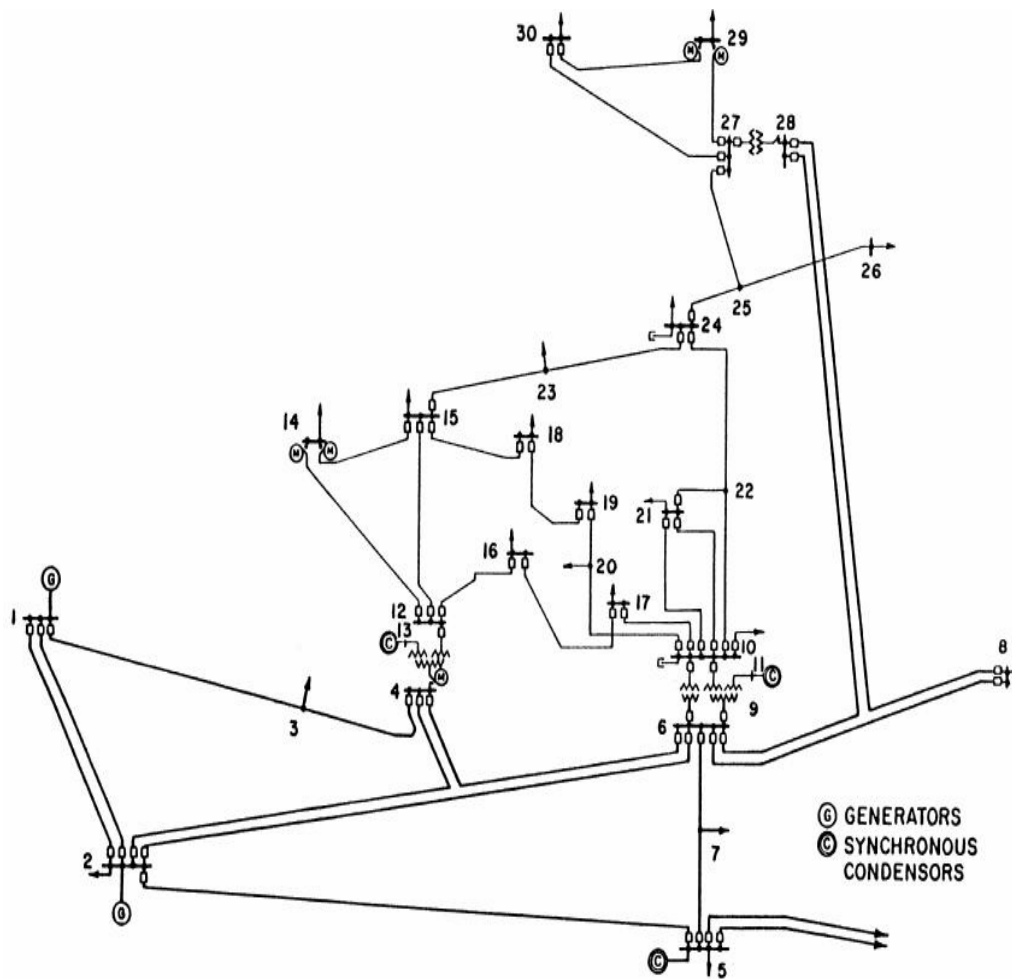


Fig. 6. IEEE-30 system

The IEEE-30 bus system shown in Fig. 6 with its measurement configuration is also used to demonstrate the proposed method.

Existing Measurements = [Injections: 1, 2, 3, 5, 8, 9, 10, 12, 13, 15, 21, 24, 26, 27; Flows: 1-2, 1-3, 2-5, 2-6, 9-11, 12-13, 12-16, 14-15, 16-17, 15-18, 18-19, 10-21, 15-23, 22-24, 25-26, 25-27, 28-27, 29-30, 6-28].

Candidate Measurements x_i = [Injections: 4, 6, 7, 11, 14, 16, 17, 18, 19, 20, 22, 23, 25, 28, 29, 30; Flows: 2-4, 3-4, 4-6, 5-7, 6-7, 6-8, 6-9, 6-10, 9-10, 4-12, 12-14, 12-15, 19-20, 10-17, 10-20, 10-22, 21-22, 23-24, 24-25, 27-29, 27-30, 8-28].

The chosen installation cost vector C^T corresponding to the candidate measurements is:

[0.4, 0.4, 0.6, 0.8, 0.8, 0.4, 0.4, 0.4, 0.6, 1, 2, 1, 0.6, 0.6, 0.6, 0.6, 0.2, 0.2, 2, 0.2, 0.2, 2, 0.4, 1, 0.4, 1, 0.4, 0.4, 1, 0.8, 0.8, 0.6, 2, 0.4, 0.4, 0, 0.2, 1, 0.2]

Similar to the previous two example systems, we consider loss of any single measurement and outage of any single branch at first. Due to space limits, only the corresponding IP constraints to loss of any single injection measurement are listed as follows:

to minimize $C^T \cdot X$

$$\text{Inj.5 Loss : } x_2 + x_3 + x_{20} + x_{21} \geq 1$$

$$\text{Inj.8 Loss : } x_2 + x_{14} + x_{22} + x_{38} \geq 1$$

$$\text{Inj.9 Loss : } x_2 + x_9 + x_{10} + x_{23} + x_{25} + x_{29} + x_{31} \geq 1$$

$$\text{Inj.10 Loss : } x_9 + x_{10} + x_{29} + x_{31} \geq 1$$

$$\begin{aligned} \text{Inj.12 Loss : } & x_1 + x_2 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} \\ & + x_{26} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \end{aligned}$$

$$\begin{aligned} \text{Inj.15 Loss : } & x_1 + x_2 + x_5 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} + x_{25} \\ & + x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \end{aligned}$$

$$\text{Inj.21 Loss : } x_2 + x_7 + x_9 + x_{10} + x_{11} + x_{23} + x_{24} + x_{25} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} \geq 1$$

$$\begin{aligned} \text{Inj.24 Loss : } & x_2 + x_7 + x_9 + x_{10} + x_{12} + x_{13} + x_{23} + x_{24} \\ & + x_{25} + x_{26} + x_{29} + x_{30} + x_{31} + x_{34} + x_{35} \geq 1 \end{aligned}$$

$$\text{Inj.27 Loss : } x_{15} + x_{16} + x_{36} + x_{37} \geq 1$$

We also have the IP constraints for loss of any single flow measurement and outage of any single branch. Compared with the A matrix in [21], obviously some injection measurement losses are not listed here, such as loss of injection measurements at bus 1, 2, 3, 13, 26. It is because loss of any of above five measurements will not affect the network observability and no extra measurement is needed. Solving the IP problem as proposed, the optimal measurement set will be the injection measurements at buses 6, 19, and the flow measurement in branch 27-29 with minimum installation cost 1.2.

Next we include other 5 contingencies into the contingency list besides losses of any single measurement and outages of any single branch.

Contingency 1: losses of injection measurements at buses 1, 3, and flow measurements in branches 1-2, 1-3; branch 4-12 is outaged;

Contingency 2: losses of injection measurements at buses 2, 5, and flow measurements in branches 2-5, 2-6; branches 1-2 and 2-4 are outaged;

Contingency 3: losses of injection measurement at bus 12, and flow measurements in branches 12-13, 12-16;

Contingency 4: loss of injection measurement at bus 26, and flow measurement in branch 25-26;

Contingency 5: loss of flow measurement in branch 29-30; branch 27-30 is outaged;

As a result, for the given contingency list, besides the candidate measurements for the loss of single measurements and single branches, we also can obtain the candidate measurement sets for these five contingencies, which also are expressed as IP constraints:

$$\text{Contingency 1: } x_1 * x_2 * x_9 + x_1 * x_2 * x_{10} + x_1 * x_2 * x_{17} + x_1 * x_2 * x_{18} + \dots \geq 1$$

$$\text{Contingency 2: } x_2 * x_3 * x_7 * x_{20} + x_2 * x_3 * x_{10} * x_{20} + x_2 * x_3 * x_{19} * x_{20} \\ + x_2 * x_3 * x_{20} * x_{23} + \dots \geq 1$$

$$\text{Contingency 3: } x_6 * x_7 + x_6 * x_9 + x_6 * x_{10} + x_6 * x_{29} + x_6 * x_{30} + \dots \geq 1$$

$$\text{Contingency 4: } x_{13} \geq 1$$

$$\text{Contingency 3: } x_{15} + x_{16} \geq 1$$

The IP solver shows that the optimal measurement set for this system is to include the injection measurements at buses 6, 19, 25, and 29, and the flow measurements in branches 3-4, 5-7, and 6-7. Hence, inclusion of these additional measurements will maintain the system observable during any single line outage or loss of any single measurement and those five given contingencies in the IEEE-30 bus system.

2.8.4 57-bus System

IEEE 57-bus system also is taken as an example. The network data and diagram can be found in [57-59].

Existing Measurements = [Injections: 1, 3, 4, 5, 7, 8, 10, 11, 12, 13, 16, 20, 21, 24, 29, 30, 31, 33, 34, 37, 38, 39, 44, 46, 48, 49, 52, 54, 55, 56, 57; Flows: 2-3, 4-5, 6-7, 6-8, 9-11, 3-15, 21-20, 24-26, 25-30, 22-38, 41-43, 15-45, 48-49, 50-51, 11-13, 14-15, 18-19, 23-24, 26-27, 27-28, 32-33, 35-36, 37-38, 42-41, 40-56, 55-9].

Candidate Measurements $x_i =$ [Injections: 2, 9, 14, 15, 18, 19, 22, 23, 25, 26, 27, 28, 32, 35, 36, 37, 40, 41, 42, 43, 45, 47, 50, 51, 53; Flows: 1-2, 3-4, 4-6, 8-9, 9-10, 9-12, 9-13, 13-14, 13-15, 1-15, 1-16, 1-17, 4-18, 5-6, 7-8, 10-12, 12-13, 12-16, 12-17, 19-20, 21-22, 22-23, 24-25, 28-29, 7-29, 30-31, 31-32, 32-34, 34-35, 36-37, 37-39, 36-40, 11-41, 38-44, 14-46, 46-47, 47-48, 49-50, 10-51, 13-49, 29-52, 52-53, 53-54, 54-55, 11-43, 44-45, 41-56, 42-56, 39-57, 56-57, 38-49, 38-48].

The chosen installation cost vector C^T corresponding to the candidate measurements is set to 0.1.

Firstly we consider loss of any single measurement and outage of any single branch. After obtaining all the IP constraints to loss of any single injection measurement and outage of any single branch, the optimal measurement set will be the injection measurements at buses 14, 22, 32, 36 with minimum installation cost 0.4.

Next we include other 5 contingencies into the contingency list besides losses of any single measurement and outages of any single branch.

Contingency 1: losses of injection measurements at bus 4, and flow measurements in branch 4-5;

Contingency 2: losses of injection measurements at bus 24, and flow measurements in branches 23-24, 24-26 and 26-28;

Contingency 3: losses of injection measurement at bus 48, and flow measurements in branch 48-49; branch 38-48 is outaged;

Contingency 4: losses of injection measurement at bus 37, and flow measurement in branch 37-38; branches 36-37 and 37-40 are outaged;

Contingency 5: loss of injection measurement at bus 3, losses of flow measurement in branch 2-3, 3-15; branch 3-4 is outaged;

As a result, for the given contingency list, besides the candidate measurements for the loss of single measurements and single branches, we also can obtain the candidate

measurement sets for these five contingencies, which also are expressed as IP constraints:

$$\text{Contingency 1: } x_1 + x_2 + x_3 + x_5 + x_6 + x_8 + x_9 + x_{14} + x_{16} + \dots \geq 1$$

$$\text{Contingency 2: } x_8 * x_9 * x_{10} + x_8 * x_9 * x_{14} + x_8 * x_9 * x_{15} + x_8 * x_{10} * x_{11} + \dots \geq 1$$

$$\text{Contingency 3: } x_4 + x_{22} + x_{60} + x_{61} + x_{62} \geq 1$$

$$\text{Contingency 4: } x_{14} + x_{15} + x_{16} + x_{17} + x_{53} + x_{54} + x_{57} \geq 1$$

$$\text{Contingency 5: } x_1 * x_3 + x_1 * x_4 + x_1 * x_5 + x_1 * x_{21} + x_1 * x_{22} + \dots \geq 1$$

The IP solver shows that the optimal measurement set for this system is to include the injection measurements at buses 2, 22, 23, and 35, and the flow measurements in branches 40-36, and 14-46. Hence, inclusion of these additional measurements will maintain the system observable during any single line outage or loss of any single measurement and those five given contingencies in the IEEE-57 bus system.

2.9 Conclusions

This chapter improves the unified method of [21-22] by accounting for the complex contingencies involving the loss of multiple measurements and branches, including bus splitting cases. Based on the modified measurement Jacobian H matrix for each contingency, one general candidate measurements selection method is introduced so that all candidates can be selected against any combination and number of possible contingencies. Furthermore, the IP problem is extended to consider the cases where two or more candidates should be combined for some contingency. Several numerical examples are given to verify the proposed method.

CHAPTER III
TRANSMISSION LOSS ALLOCATION IN A MULTIPLE-TRANSACTION
FRAMEWORK

3.1 Introduction

The electric power industry is experiencing important changes brought about by the deregulation. Electric power generators and users engage in power transactions which take place over the transmission system and create losses. Transmission losses represent up to 5—10% of the total generation, and are worth millions of dollars per year. In the past, losses were usually treated as an extra load in the system under the vertically integrated structure. In the current competitive model, however, its cost must be shared in a transparent and nondiscriminatory way. "Fair" allocation among different market participants has an important impact on the competitive operation of electric power markets.

As we know the total losses incurred in the power network usually is given by

$$P_L = \frac{1}{2} \sum_{i=0}^N \sum_{j \in H_i} \left\{ \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} [V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_i - \theta_j)] \right\} \quad (1)$$

where, P_L represents the total network losses in a N -bus system, H_i is the set of buses that are directly connected to bus $i = 1, \dots, N$, $R_{ij} + jX_{ij}$ is the line impedance value between bus i and bus j , and V_i, θ_i are the voltage magnitude and phase angle of bus i . Obviously the loss formula itself does indicate any transaction information.

It is recognized early on that the non-linear form of the loss expression would not lend itself to an exact loss allocation scheme. Any proposed loss allocation scheme can therefore only be judged on the basis of how reasonable it is, and this will be decided by the reaction of all the market participants. Hence, the main motivation here is to present a new method, which allows a natural separation of losses among individual transactions in a multiple-transaction setting.

3.2 Previous Work

A number of allocation schemes have been proposed to allocate the system losses to generators / loads in a pool market or to individual transactions in a bi-lateral contracts market. A quadratic loss formula is proposed in [42] to allocate transmission losses among trades. An integration scheme is presented to allocate losses for bilateral transactions in [29]. Several flow-tracing approaches have been used in [31], [33] and [35], which topologically determine contributions of individual generators to loads based on the proportional sharing principle. With a solved power flow case, a "physical-power-flow-based" approach that uses the quadratic loss approximation to distribute the losses among the transactions in a multiple-transaction framework is presented in [43]. The method provides loss allocations that are appropriate and behave in a physically reasonable manner despite having up to 16% error in some cases due to the approximations used in its derivation. Another loss allocation method is based on the bus impedance Z-bus matrix [44] and allocates transmission losses among loads and generators assuming a pool dispatch. A natural separation of the system losses among the network buses is derived using the loss formula. It does not however account for the interaction between different injections. In [45], branch flows are unbundled and then the branch loss formula for each transaction is derived. A more general transaction-based power flow analysis scheme is proposed in [47][56] to decompose the branch active power flows into individual transactions and also interactions between them with the consideration of the reactive power support. In [54], a cooperative game theory is utilized to allocate the loss cost to loads. Own-losses and cross-losses have been discussed in [55] and a CLP method is proposed to allocate cross-losses among multiple flow contributors.

In this chapter a new loss allocation method for the solved power flow case is presented. The proposed method does not use any approximations such as a D.C. power flow, avoiding method induced inaccuracies. The power network losses are expressed in terms of individual power transactions. A transaction-loss matrix, which illustrates the breakdown of losses introduced by each individual transaction and interactions between any two transactions, is created. The network losses can then be allocated to each transaction based on the transaction-loss matrix entries.

The chapter is organized in such a way that, the proposed formulation is presented first, followed by its implementation algorithm. Numerical examples are included at the end to illustrate the application of the proposed method to typical power systems.

3.3 Transaction Framework Formulation

3.3.1 Transaction Framework Formulation

In this section, we extend the multi-transaction framework definition in [43] for the system with n buses and M transactions considered in the system operation. Each load acts as a buyer to get its demand met through transactions with one or more sellers. Similarly, each generator acts as a seller and undertakes transactions with one or more buyers. For $m = 1, \dots, M$, the transaction $T^{(m)}$ involving the set of selling entities $S^{(m)}$, the set of buying entities $B^{(m)}$, the loss compensation portion $l^{(m)}$ and the MW amount $t^{(m)}$ is defined by the quadruplet

$$T^{(m)} = \{t^{(m)}, S^{(m)}, B^{(m)}, l^{(m)}\} \quad (2)$$

where

$$S^{(m)} = \{(s_i^{(m)}, \alpha_i^{(m)}) | i = 1, 2, \dots, N_s^{(m)}\}$$

$$B^{(m)} = \{(b_i^{(m)}, \beta_i^{(m)}) | i = 1, 2, \dots, N_b^{(m)}\}$$

For each transaction m , the selling bus $s_i^{(m)}$ provides the fraction $\alpha_i^{(m)}$ of the total MW amount $t^{(m)}$ while the buying bus $b_i^{(m)}$ receives the fraction $\beta_i^{(m)}$ where $\alpha_i^{(m)} \in [0,1]$ and $\beta_i^{(m)} \in [0,1]$ exist. $N_s^{(m)}$ is the number of the selling buses and $N_b^{(m)}$ is the number of the buying buses. The fraction $l^{(m)}$ of the transaction MW amount $t^{(m)}$ is the portion of the system losses at the system slack bus allocated to that transaction. Since $l^{(m)}$ is unknown before the loss allocation solution has been reached, it has to be computed iteratively.

3.3.2 Nodal Injection Expression

Since the nodal power injection can be written as a sum of individual transactions, eventually we can express the amount of the power injection P_h at any bus $h = 1, 2, \dots, n$ (n is designated as the slack bus) as follows:

$$P_h = \sum_{m=1}^M \delta_h^{(m)} t^{(m)} \quad (3)$$

where the components of the vector $\delta^{(m)}$ are

$$\delta_h^{(m)} = \begin{cases} \alpha_i^{(m)} & \text{if } s_i^{(m)} = h, i = 1, 2, \dots, N_s^{(m)} \\ -\beta_j^{(m)} & \text{if } b_j^{(m)} = h, j = 1, 2, \dots, N_b^{(m)} \\ \alpha_i^{(m)} - \beta_j^{(m)} & \text{if } s_i^{(m)} = b_j^{(m)} = h, \\ & i = 1, 2, \dots, N_s^{(m)}, j = 1, 2, \dots, N_b^{(m)} \\ 0 & \text{otherwise} \end{cases} \quad h \neq n \quad (4)$$

and for the slack bus n

$$\delta_n^{(m)} = \begin{cases} \alpha_i^{(m)} + l^{(m)} & \text{if } s_i^{(m)} = n, i = 1, 2, \dots, N_s^{(m)} \\ l^{(m)} - \beta_j^{(m)} & \text{if } b_j^{(m)} = n, j = 1, 2, \dots, N_b^{(m)} \\ \alpha_i^{(m)} - \beta_j^{(m)} + l^{(m)} & \text{if } s_i^{(m)} = b_j^{(m)} = n, \\ & i = 1, 2, \dots, N_s^{(m)}, j = 1, 2, \dots, N_b^{(m)} \\ l^{(m)} & \text{otherwise} \end{cases} \quad (5)$$

3.3.3 Examples

A three-bus test system shown in Fig. 7 with its transaction information is presented here for illustration.

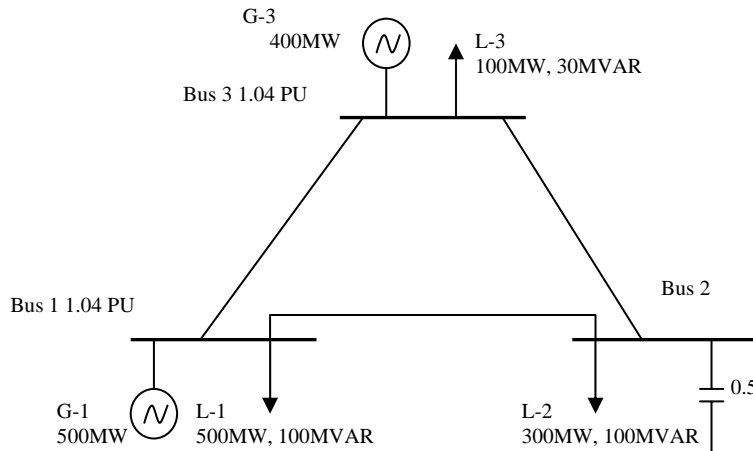


Fig. 7. 3-bus test system with transaction data

The test system has two generators scheduling to output 500MW and 400MW respectively to three loads that are 500MW, 300MW and 100MW respectively. We have two transactions occurring in the system buying from two generators and selling to three loads, and they are listed as

$$T^{(1)} = \{500, \{(1,100\%), \{(1,80\%), (2,20\%), I^{(1)}\}$$

$$T^{(2)} = \{400, \{(3,100\%), \{(1,25\%), (2,50\%), (3,25\%), I^{(2)}\}$$

Transaction $T^{(1)}$ has transaction amount of 500MW and the selling bus 1 provides all 500MW while the buying bus 1 and buying bus 2 take 80% and 20% of the full transaction amount respectively. Transaction $T^{(2)}$ has transaction amount 400MW and the selling bus 3 generates all 400MW while the buses 1, 2 and 3 buy 25%, 50% and 25% of the full transaction amount respectively.

The nodal injection then can be expressed by transactions. The bus 1 is taken as the example.

$$P_1 = \sum_{m=1}^2 \delta_1^{(m)} t^{(m)} = (100\% - 80\%)t^{(1)} + (-25\%)t^{(2)} \quad (6)$$

$$= 0.2t^{(1)} - 0.25t^{(2)}$$

where $t^{(1)}$ is the transaction amount of transaction $T^{(1)}$ and $t^{(2)}$ is the transaction amount of transaction $T^{(2)}$.

3.4 Proposed Allocation Scheme

3.4.1 Power System Losses

Since the transmission lines have resistance the power system will always incur losses while the electricity is transferred from the generating plants to consumers. Suppose that a solved power flow exists for a n -bus system, besides the system loss formula (1) expressed by the sum of individual line losses, the system losses also can be expressed by the vector of complex bus current injections, I , and the impedance Z -bus matrix as below:

$$P_L = \Re \left\{ \sum_{i=1}^n I_i^* \left(\sum_{j=1}^n R_{ij} I_j \right) \right\} \quad (7)$$

$$Z = R + jX = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1n} \\ R_{21} & R_{22} & \cdots & R_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nn} \end{bmatrix} + j \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & X_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & X_{nn} \end{bmatrix} \quad (8)$$

where R is the resistance part and X is the reactance part of the impedance matrix $Z = R + jX$.

Expressing the complex bus current injection I in rectangular coordinates as (9),

$$I = I_x + jI_y = \begin{bmatrix} I_{x1} \\ I_{x2} \\ \vdots \\ I_{xn} \end{bmatrix} + j \begin{bmatrix} I_{y1} \\ I_{y2} \\ \vdots \\ I_{yn} \end{bmatrix} \quad (9)$$

then, we will have (7) transformed to

$$P_L = \sum_{i=1}^n \sum_{j=1}^n R_{ij} (I_{xi} I_{xj} + I_{yi} I_{yj}) \quad (10)$$

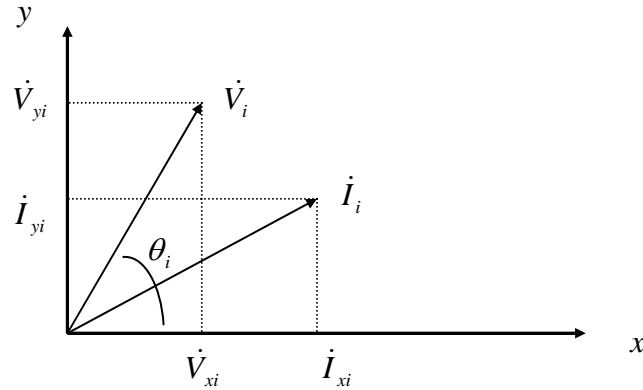


Fig. 8. Phasor projections of \dot{V}_i, \dot{I}_i

From Fig. 8 we know the nodal power injections P_i, Q_i of bus i can be expressed as:

$$\begin{cases} P_i = V_i I_{xi} \cos \theta_i + V_i I_{yi} \sin \theta_i \\ Q_i = V_i I_{xi} \sin \theta_i - V_i I_{yi} \cos \theta_i \end{cases} \quad (11)$$

The bus current injection then can be derived as:

$$\begin{cases} I_{xi} = (P_i \cos \theta_i + Q_i \sin \theta_i) / V_i \\ I_{yi} = (P_i \sin \theta_i - Q_i \cos \theta_i) / V_i \end{cases} \quad (12)$$

P_L can then be rewritten as:

$$\begin{aligned} P_L &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} (P_i \cos \theta_i + Q_i \sin \theta_i)(P_j \cos \theta_j + Q_j \sin \theta_j) \\ &+ \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} (P_i \sin \theta_i - Q_i \cos \theta_i)(P_j \sin \theta_j - Q_j \cos \theta_j) \end{aligned} \quad (13)$$

After some manipulations, it can be shown that the loss expression can be written as a sum of three different terms:

$$P_L = P_{LPP} + P_{LPQ} + P_{LQQ} \quad (14)$$

where

$$P_{LPP} = \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} (P_i \cos \theta_i \cdot P_j \cos \theta_j + P_i \sin \theta_i \cdot P_j \sin \theta_j) \quad (15)$$

$$\begin{aligned} P_{LPQ} &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} (P_i \cos \theta_i \cdot Q_j \sin \theta_j + Q_i \sin \theta_i \cdot P_j \cos \theta_j) \\ &- \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} (P_i \sin \theta_i \cdot Q_j \cos \theta_j + Q_i \cos \theta_i \cdot P_j \sin \theta_j) \end{aligned} \quad (16)$$

$$\begin{aligned} P_{LQQ} &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} (Q_i \sin \theta_i \cdot Q_j \sin \theta_j + Q_i \cos \theta_i \cdot Q_j \cos \theta_j) \\ &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} Q_i Q_j \cos \theta_{ij} \end{aligned} \quad (17)$$

The terms P_{LPP} and P_{LQQ} can be viewed as the loss components solely due to the active and the reactive power injections respectively, while P_{LPQ} may be considered to represent the losses induced by the interactions between the active and the reactive power injections.

3.4.2 Derivation of Transaction-Loss Matrix

Since the network loss expression P_L is still a function of the active power injection P_i and reactive power injection Q_i , equations (15)-(17) need to be reformulated in terms of the transaction-based injection equation (3) in order to express the system losses explicitly in terms of multiple transactions:

$$\begin{aligned}
P_{LPP} &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} (P_i \cos \theta_i \cdot P_j \cos \theta_j + P_i \sin \theta_i \cdot P_j \sin \theta_j) \\
&= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} P_i P_j \cos \theta_{ij} \\
&= \sum_{i=1}^n \left[P_i \left(\sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \cdot P_j \right) \right]
\end{aligned} \tag{18}$$

P_i and P_j in (18) can now be substituted by $P_i = \sum_{m1=1}^M \delta_h^{(m1)} t^{(m1)}$ and $P_j = \sum_{m2=1}^M \delta_h^{(m2)} t^{(m2)}$ as given in the transaction-based injection equation (3). These substitutions will allow the expression of the three components of the overall system loss in terms of the individual power transactions, as derived below:

$$\begin{aligned}
P_{LPP} &= \sum_{i=1}^n \left[\sum_{m1=1}^M \delta_i^{(m1)} t^{(m1)} \left(\sum_{j=1}^n \frac{R_{ij} \cos \theta_{ij}}{V_i V_j} \sum_{m2=1}^M \delta_j^{(m2)} t^{(m2)} \right) \right] \\
&= \sum_{i=1}^n \left\{ \sum_{m1=1}^M \delta_i^{(m1)} t^{(m1)} \left[\sum_{m2=1}^M \left(\sum_{j=1}^n \frac{R_{ij} \cos \theta_{ij}}{V_i V_j} \delta_j^{(m2)} \right) t^{(m2)} \right] \right\} \\
&= \sum_{m1=1}^M \left\{ \sum_{i=1}^n \delta_i^{(m1)} \left[\sum_{m2=1}^M \left(\sum_{j=1}^n \frac{R_{ij} \cos \theta_{ij}}{V_i V_j} \delta_j^{(m2)} \right) t^{(m2)} \right] \right\} t^{(m1)} \\
&= \sum_{m1=1}^M \left[\sum_{m2=1}^M \left(\sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \cdot \delta_i^{(m1)} \delta_j^{(m2)} \right) t^{(m2)} \right] t^{(m1)}
\end{aligned} \tag{19}$$

Similarly:

$$\begin{aligned}
P_{LPQ} &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \sin \theta_{ij} (-P_i Q_j + Q_i P_j) \\
&= \sum_{m=1}^M \left[\sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \sin \theta_{ij} (-\delta_i^{(m)} Q_j + Q_i \delta_j^{(m)}) \right] t^{(m)}
\end{aligned} \tag{20}$$

In the case of P_{LQQ} , a similar expression as (19) could easily be derived if the reactive power amount of each transaction were known. However, in the commercial marketplace the transactions are typically specified in terms of the real power traded without explicitly specifying the reactive power. Even though the reactive power support is an integral part of the transmission service, its induced real loss share should be further distributed to energy transactions. It is found that the transmission loss generated from reactive power support is less than 10% of the overall system losses under normal operating conditions. Hence, a simple solution to reallocate the losses due to the reactive power support can be suggested as:

$$P_{LQQ}^{(m)} = \frac{t^{(m)}}{\sum_{i=1}^M t^{(i)}} P_{LQQ} \tag{21}$$

Hence, based on (19)-(21) a transaction-loss matrix TL can be easily constructed as follows:

$$TL = \begin{matrix} & \begin{matrix} 1 & 2 & \dots & \dots & M \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ \vdots \\ M \end{matrix} & \begin{bmatrix} P_L^{(1,1)} & P_L^{(1,2)} & \dots & \dots & P_L^{(1,m)} \\ P_L^{(2,1)} & P_L^{(2,2)} & \dots & \dots & P_L^{(2,m)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_L^{(m,1)} & P_L^{(m,2)} & \dots & \dots & P_L^{(m,m)} \end{bmatrix} \end{matrix} \tag{22}$$

where

$$\begin{aligned}
P_L^{(m,m)} &= \frac{t^{(m)}}{\sum_{i=1}^M t^{(i)}} P_{LQQ} + \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \delta_i^{(m)} \delta_j^{(m)} t^{(m)} t^{(m)} \\
&\quad + \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \sin \theta_{ij} (-\delta_i^{(m)} Q_j + Q_i \delta_j^{(m)}) \cdot t^{(m)}
\end{aligned} \tag{23}$$

$$\begin{aligned}
P_L^{(m,k)} &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \cdot \delta_i^{(m)} \delta_j^{(k)} t^{(m)} t^{(k)} \\
&+ \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \cdot \delta_i^{(k)} \delta_j^{(m)} t^{(m)} t^{(k)} \quad k \neq m
\end{aligned} \tag{24}$$

The self-induced term $P_L^{(m,m)}$ is solely due to the individual transaction m , and the cross term $P_L^{(m,k)}$, which should further be evenly reallocated to the corresponding two single transactions, represents the losses accrued due to the interactions between transactions k and m .

3.4.3 Fair Loss Allocation

From the transaction-loss matrix indicating transaction self-induced losses and interaction losses between every two transactions, for each single transaction the allocated loss can be expressed as:

$$P_L^{(m)} = P_L^{(m,m)} + \sum_{k=1, k \neq m}^M \frac{1}{2} P_L^{(m,k)} \tag{25}$$

Thus, the total system loss expression (14) will be reformulated as:

$$P_L = \sum_{m=1}^M P_L^{(m)} \tag{26}$$

This formulation, as is the case in [25] and [43], can yield negative entries in the transmission-loss matrix TL. This is not inconsistent with the expectations [25] where the existence of a bilateral transaction can lead to counter flows reducing line losses. This information can be used different ways depending upon the agreed compensation policies by the involved parties of the transactions.

3.4.4 Flowchart

Since the portion of the system losses at the system slack bus for each single transaction is unknown before the solution is reached, an iterative solution whose flowchart is shown in Fig. 9, needs to be implemented.

The main computational burden associated with the iterative solution algorithm of Fig. 9 is in the evaluation of TL using (23) and (24). However, all calculations are made using the same single power flow solution. Considering the fact that the number of

transactions M is usually much less than the number of system buses, complexity of the algorithm will be in the order of the square of n .

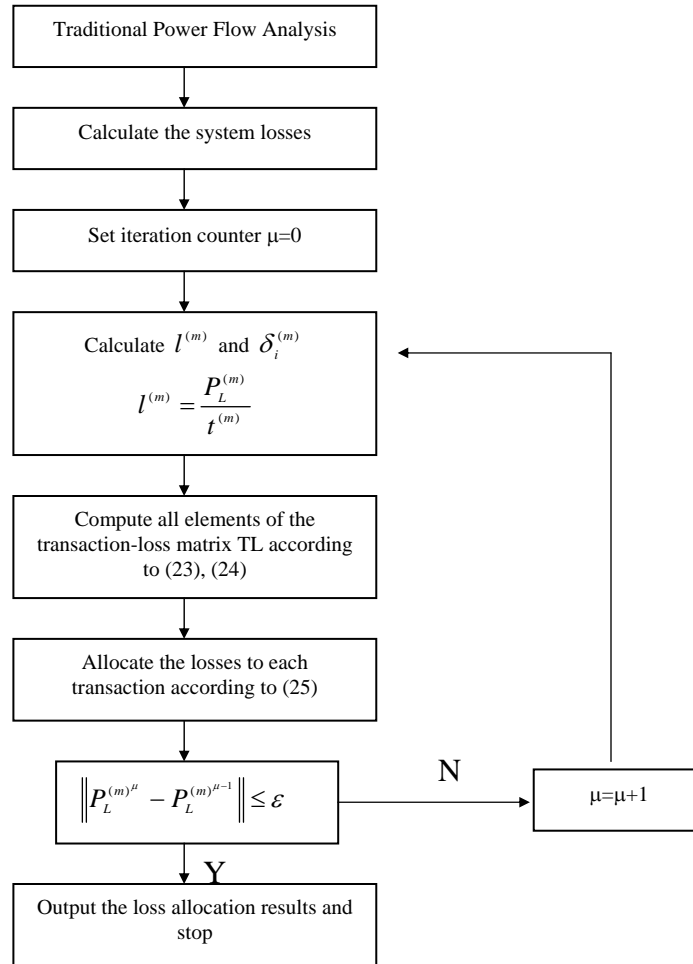


Fig. 9. Flowchart of the proposed approach

3.5 Numerical Results

A number of test systems have been used to test the effectiveness of the new proposed transaction-based Z-bus loss allocation scheme. In this paper, a three-bus system and the IEEE 30-bus system are presented as examples.

3.5.1 3-bus System

First, a three-bus system is used to illustrate and examine the proposed method. The parameters and topology are shown in Table 1 and Fig. 7. Five different cases are listed. Note that the power flow solution and loss allocation results in Table 2 and Table 3 and hence the total system losses are the same for all five cases, while the number and type of transactions vary, yielding different loss allocation solutions.

TABLE 1
LINE CHARACTERISTICS FOR A 3-BUS SYSTEM

From Bus	To Bus	$R (p.u.)$	$X (p.u.)$	$C (p.u.)$
1	2	0.02	0.08	0.00
1	3	0.03	0.12	0.00
2	3	0.02	0.06	0.00

Case 1:

$$T^{(1)} = \{500, \{(1,100\%)\}, \{(1,100\%)\}, l^{(1)}\}$$

$$T^{(2)} = \{400, \{(3,100\%)\}, \{(2,75\%), (3,25\%)\}, l^{(2)}\}$$

Case 2:

$$T^{(1)} = \{500, \{(1,100\%)\}, \{(1,80\%), (2,20\%)\}, l^{(1)}\}$$

$$T^{(2)} = \{400, \{(3,100\%)\}, \{(1,25\%), (2,50\%), (3,25\%)\}, l^{(2)}\}$$

Case 3:

$$T^{(1)} = \{500, \{(1,100\%)\}, \{(1,60\%), (2,40\%)\}, l^{(1)}\}$$

$$T^{(2)} = \{400, \{(3,100\%)\}, \{(1,50\%), (2,25\%), (3,25\%)\}, l^{(2)}\}$$

Case 4:

$$T^{(1)} = \{500, \{(1,100\%)\}, \{(1,40\%), (2,60\%)\}, l^{(1)}\}$$

$$T^{(2)} = \{400, \{(3,100\%)\}, \{(1,75\%), (3,25\%)\}, l^{(2)}\}$$

Case 5:

$$T^{(1)} = \{400, \{(1,50\%), (3,50\%)\}, \{(1,100\%)\}, l^{(1)}\}$$

$$T^{(2)} = \{400, \{(1,50\%), (3,50\%)\}, \{(1,25\%), (2,75\%)\}, l^{(2)}\}$$

$$T^{(3)} = \{100, \{(1,100\%)\}, \{(3,100\%)\}, l^{(3)}\}$$

TABLE 2
POWER FLOW RESULTS OF THE 3-BUS SYSTEM

Bus number	Voltage (p.u.)	Angle (degree)
1	1.0400	-5.0197
2	0.9834	-7.6768
3	1.0400	0.0000

TABLE 3
LOSS ALLOCATION RESULTS FOR THE 3-BUS SYSTEM ($P_L = 13.983MW$)

Alloc. Loss	Case 1	Case 2	Case 3	Case 4
Transaction 1 (MW)	0.587	2.352	4.118	5.883
Transaction 2 (MW)	13.396	11.630	9.865	8.100

In case 1, since the selling and buying buses of transaction 1 are the same, it is intuitively expected that no losses should be assigned to transaction 1 except for the loss due to the system reactive power support. Accordingly most of the system losses are allocated to transaction 2. The delivered amount to bus 1 by T-1 is gradually decreased while the amount delivered from bus 1 to bus 2 by T-1 is increased, going from case 1 to case 4. This requires that the losses for which T-1 is responsible should also increase, leading to an increase in its allocation of losses.

Case 5 yields the transaction-loss matrix (MW) as:

$$\begin{matrix} & T^{(1)} & T^{(2)} & T^{(3)} \\ \begin{matrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{matrix} & \begin{bmatrix} 6.830 & 4.006 & -6.554 \\ 4.006 & 9.839 & -1.979 \\ -6.554 & -1.979 & 1.841 \end{bmatrix} \end{matrix},$$

where, the loss allocations per transaction can be computed as:

Transaction 1: 5.556 MW

Transaction 2: 10.853 MW

Transaction 3: -2.426 MW

If different slack bus is chosen, the total system losses will be different so that the transaction-loss matrix will be not same. For instance, if bus 1 is chosen as the system slack bus, then the case 5 will yield the transaction-loss matrix (MW) as:

$$\begin{array}{l}
 T^{(1)} \\
 T^{(2)} \\
 T^{(3)}
 \end{array}
 \begin{bmatrix}
 T^{(1)} & T^{(2)} & T^{(3)} \\
 6.487 & 4.203 & -6.235 \\
 4.203 & 9.643 & -1.585 \\
 -6.235 & -1.585 & 1.745
 \end{bmatrix}$$

where, the total system losses will be 13.257MW and loss allocations per transaction can be computed as:

Transaction 1: 4.970 MW

Transaction 2: 10.452 MW

Transaction 3: -2.165 MW

TABLE 4
POWER FLOW RESULTS OF THE 3-BUS SYSTEM (BUS 1 AS THE SLACK BUS)

Bus number	Voltage (p.u.)	Angle (degree)
1	1.0400	0.0000
2	0.9838	-2.9406
3	1.0400	4.5120

Clearly the selection of the system slack bus will slightly change the power flow results shown in Table 4 and the loss allocation results.

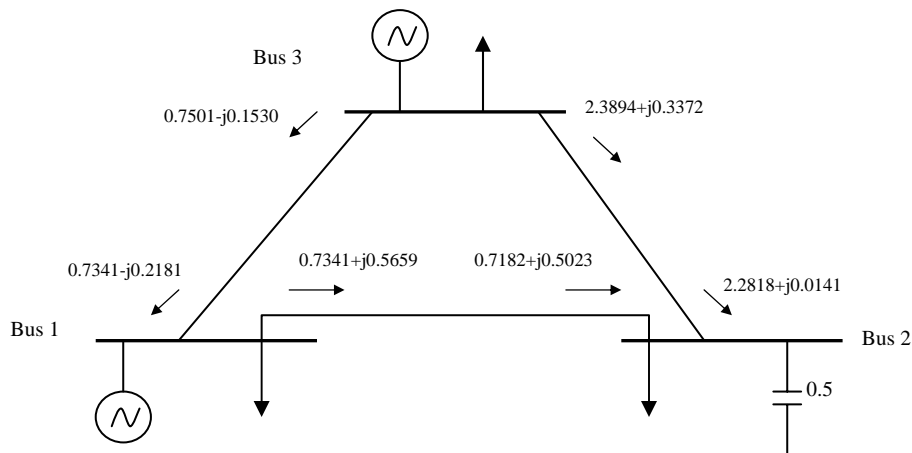


Fig. 10. Line flows (p.u.) of case 5 ($t^{(3)} = 100MW$)

Note that the cross terms between $T^{(3)}$ and $T^{(1)}$, and between $T^{(3)}$ and $T^{(2)}$ are negative. This implies that the losses to $T^{(3)}$ and $T^{(1)}$ decrease when $T^{(3)}$ and $T^{(1)}$ coexist. Transaction 1 buys 200MW from the selling bus 3, to supply the customer at bus 1 while transaction 3 transfers 100MW from bus 1 to bus 3. From Fig. 10 it is obvious that transaction 1 causes a flow in the same direction as the net flow while transaction 3 causes a flow in the opposite direction. Following the terminology of "dominant flow" and "counter flow" of [43], transaction 1 and 3 will follow the dominant and counter flow directions respectively. The benefit due to the counter flow is reflected by the cross term $P_L^{(1,3)} = -6.554MW$, which will be shared by both transactions.

Similar argument applies to $T^{(2)}$ and $T^{(3)}$. Note that the calculated loss allocation to transaction 3 is negative (-2.426 MW). This is due to the fact that transaction 3 causes only counter flows which help reduce the system losses. Hence, transaction 3 will receive a negative loss allocation. This illustrates that even when the network power flow solution is same, like in cases (1)-(5), loss allocation to individual transactions may be significantly different if the transactions are different. Furthermore, it is possible for a "well positioned" transaction to receive negative loss allocation.

As an illustration, consider the case where the amount of the transaction 3 is varied around the base case described above while keeping the other two transactions fixed. Fig. 11 shows the plots of the system losses and the losses allocated to each transaction, both of which decrease when the amount of transaction 3 is increased. This will be true until a turning point is reached, after which the system losses will increase when the amount of transaction 3 increases. This is due to the fact that transaction 3 initially is on the counter flow direction helping to reduce the system losses. At some point transaction 3 will become aligned with the dominant flow, causing the system losses to increase again.

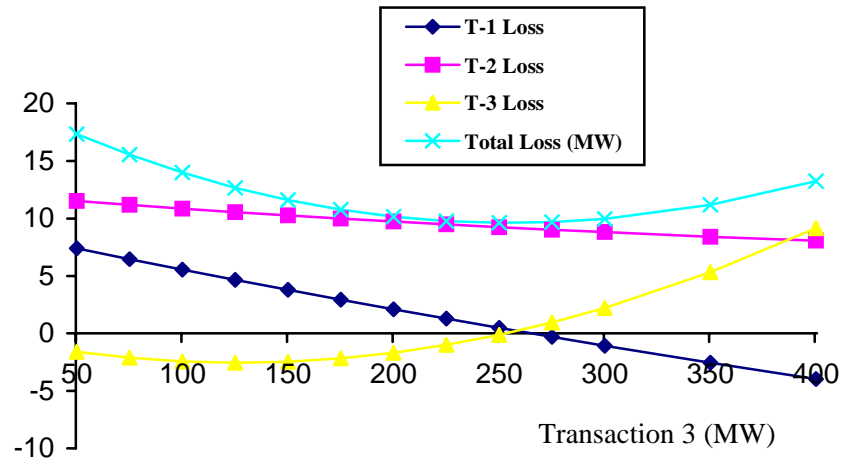


Fig. 11. Loss allocation curves when $t^{(3)}$ changes

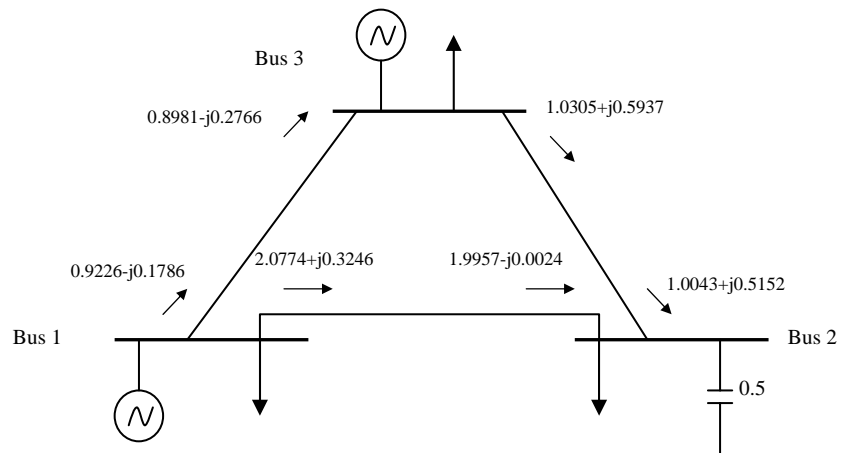


Fig. 12. Line Flows (p.u.) of Case 5 ($t^{(3)} = 400MW$)

Fig. 12 presents the network flow distribution when the amount of transaction $T^{(3)}$ is $t^{(3)} = 400\text{MW}$. The network loss value is 13.24 MW. The transaction-loss matrix (MW) is

$$\begin{array}{l} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{array} \begin{array}{ccc} T^{(1)} & T^{(2)} & T^{(3)} \\ \left[\begin{array}{ccc} 6.890 & 3.283 & -25.021 \\ 3.283 & 9.664 & -6.487 \\ -25.021 & -6.487 & 24.910 \end{array} \right] \end{array}$$

and the loss allocations are:

Transaction 1: -3.979 MW

Transaction 2: 8.062 MW

Transaction 3: 9.157 MW

In this case, transaction 3 tries to bring 400MW from bus 1 to bus 3 while transaction 1 buys 200MW from bus 3 to supply bus 1. It is clear from Fig. 12 that transaction 3 follows the dominant flow direction while transaction 1 is in the counter flow direction. Consequently the transaction 1 will receive a negative loss allocation like shown in the loss allocation results.

3.5.2 IEEE 30-bus System

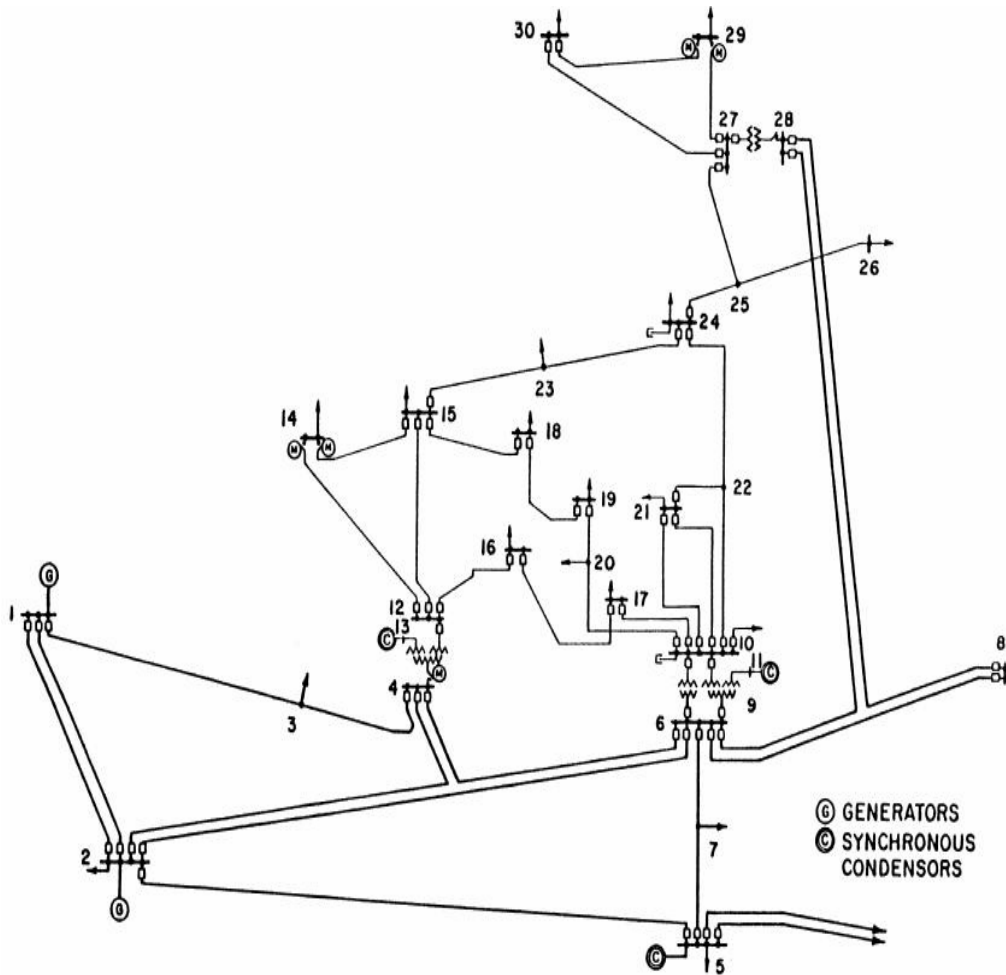


Fig. 13. IEEE 30-bus system for loss allocation

The loss allocation scheme is tested also on a larger IEEE 30-bus system shown in Fig. 13. The existing load and generation is assumed to reflect an existing transaction, T-1. Then, a generator is added at bus 5, and an additional bilateral transaction T-2 (buy 20MW from bus 5, sell to bus 2) is introduced into the system.

In the base case without transaction T-2, bus 5 appears as a power sink. Both lines 5-2 and 5-7 carry power into bus 5. 80.07MW power flows into bus 5 from bus 2 and 14.13MW power flows into bus 5 from bus 7. The base case has 17.633MW total system

loss. After we put the transaction T-2 into the system, the flows on lines 5-2 and 5-7 will become 68.06MW and 6.14MW. Obviously, the transaction T-2, which transfers power out of bus 5 to bus 2, does not follow the dominant flow direction. It will produce a counter flow and should therefore have negative loss allocation.

The network loss value is 15.933MW. The transaction-loss matrix (MW) is

$$\begin{matrix} & T^{(1)} & T^{(2)} \\ T^{(1)} & \begin{bmatrix} 17.461 & -1.710 \end{bmatrix} \\ T^{(2)} & \begin{bmatrix} -1.710 & 0.182 \end{bmatrix} \end{matrix}$$

and the loss allocations are:

Transaction 1: 16.606 MW

Transaction 2: -0.673 MW

As expected transaction T-2 receives a negative loss allocation of -0.673 MW. Also we see that the total system loss value has dropped from 17.633MW in the base case to 15.933MW although the system has transferred 20MW more power. The reason behind that is that transaction T-2 brings in 20MW in generation at bus 5 and has reduced flows on both lines 5-2 and 5-7 thus the losses on those two line significantly decreased. It can be expected that the total system loss will reduce along with the increase of the amount of transaction T-2 since T-2 will reduce the line flows on lines 5-2 and 5-7. By some point the amount of T-2 will exceed the load value at bus 5, then the flow direction of lines 5-2 and 5-7 will change and transaction T-2 will produce “dominant flow” instead of “counter flow” and receive positive loss allocation accordingly.

TABLE 5
LINE FLOW AND T-2 LOSS ALLOCATION RESULTS (MW)

T-2 amount	0	20	80	160
Line 5-2 flow	-80.07	-68.06	-32.01	16.03
Line 5-7 flow	-14.13	-6.14	17.81	49.77
T- 2 Loss Allocation	0	-0.673	-1.103	2.109

Fig. 14 shows the variation of losses as the transaction amount for T-2 is increased. From Fig. 14 we can see that initially the total system losses as well as the losses allocated to both T-1 and T-2 decrease with increasing T-2. Total loss and loss allocation for T-1 will continue to decrease even after the losses allocated to T-2 start to increase.

This is due to the fact that T-2 no longer follows the counter flow direction. Finally as T-2 transaction amount reaches 160MW, as shown in Table 5, both lines 5-2 and 5-7 will be transferring power out of bus 5. At this time, transaction T-2 follows the dominant flow direction, and as expected its loss allocation becomes positive. The network loss value is 12.741MW. The transaction-loss matrix (MW) is

$$\begin{matrix} & T^{(1)} & T^{(2)} \\ T^{(1)} & \begin{bmatrix} 17.033 & -12.804 \end{bmatrix} \\ T^{(2)} & \begin{bmatrix} -12.804 & 8.511 \end{bmatrix} \end{matrix}$$

and the loss allocations are:

Transaction 1: 10.632 MW

Transaction 2: 2.109 MW

Further increases in the T-2 transaction amount will cause an increase in the total system losses.

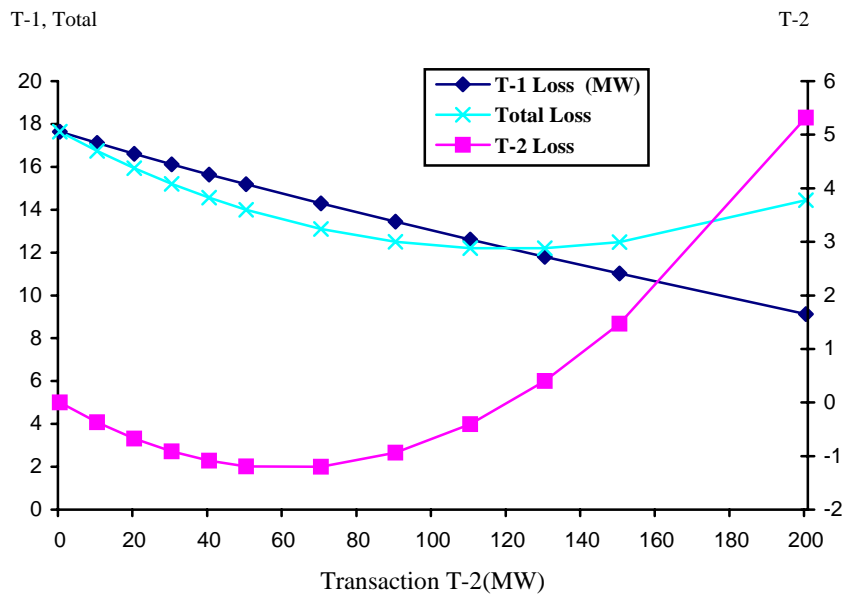


Fig. 14. Loss allocation curves when T-2 changes

These examples and the associated loss allocations calculated by the proposed approach appear consistent with expectations. Furthermore, the total system losses accurately match the sum of allocated losses for all existing transactions.

3.6 Conclusions

This chapter presents a loss allocation method which allocates system losses among multiple transactions. It is directly derived from the loss formula without making any approximations. A transaction-loss matrix is created in order to illustrate the effects of the transaction itself as well as its interaction with other existing transactions on the allocated losses to the individual transaction. Since the proposed method calculates the losses and allocations iteratively, the sum of the allocated losses to each transaction will match the total system loss exactly. Numerical examples are given to illustrate that the new method yields loss allocation results that are intuitively reasonable and consistent with expectations. It is further noted that, even though a single slack bus is used to compensate for all the losses for all transactions in this paper; it is possible to have each transaction assign its own chosen bus for the loss compensation. Formulation and implementation of such a scheme will be reported in the following chapters.

CHAPTER IV

POWER FLOW ANALYSIS WITH FAIR LOSS ALLOCATION

4.1 Introduction

Bilateral transactions in the power network generate transmission losses and a loss allocation method aiming to allocate the total network losses back to individual transactions has been proposed in the previous chapter. It is conceivable that sellers and buyers of transactions intend to self-balance their own incurred losses by designating loss compensation generators. Such a choice will provide market participants with some control over their contractual loss and overall transaction cost. However it will yield a different power flow solution, necessitating further discussion of loss allocation along with loss compensation.

Consider the power flow equations for bus i :

$$\begin{cases} P_i = P_{Gi} - P_{Di} = V_i \sum_{j \in i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ Q_i = Q_{Gi} - Q_{Di} = V_i \sum_{j \in i} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \end{cases} \quad (1)$$

where, P_i and Q_i are active and reactive bus injections; P_{Gi} and Q_{Gi} are active and reactive generation while P_{Di} and Q_{Di} are active and reactive loads respectively; and $G_{ij} + jB_{ij}$ is the element of the admittance matrix Y . In the conventional power flow analysis, except P_G of the slack bus P_G of all other buses will be pre-specified and the power flow calculation is executed based on those pre-determined P_G . However, in an open market environment all transactions are responsible for the transmission losses. It suggests that the generation of transactions may be adjusted in order to compensate the allocated transaction losses. If we have known the losses allocated back to individual transactions and transactions are willing to compensate the allocated losses by their designated compensation generators, P_G of those designated generators would have had to be updated before the transactions are committed. It means the power flow analysis with loss allocation and compensation will have to not only calculate the losses for each

individual transactions but also adjust the output of the loss compensation generators. Consequently the system operating state will not be same as we obtain from the conventional power flow analysis.

4.2 Previous Work

Among the schemes proposed so far for the allocation of losses [24-43], there are different approaches ranging from allocating losses to generators / loads in a pool market to allocating them to individual transactions in a bi-lateral contract market. A single slack bus is often used to compensate for all the losses incurred by transactions, but it is possible to have each transaction assign its own chosen buses for the loss compensation. One possible solution which uses distributed slack buses is described in [50] where losses are compensated for bilateral transactions. In [51], the same problem is formulated for the multiple transaction case where transactions get to designate generator buses for compensation of losses allocated to them or they may opt for purchasing loss compensation service from the ISO. While very comprehensive, this approach may lead to possible inaccuracies as shown in [43] due to its use of the DC power flow approximation and the LP optimization model. A very detailed analysis has been done in [56] on transaction-based power flow analysis, which utilizes transaction pairs to replace the single slack bus. It states that if all transactions compensate their own losses, the single slack bus is not required.

This chapter presents an alternative solution to the above problem. First, the multi-transaction framework definition is extended to include loss compensation entities for transactions so that each individual transaction is able to freely choose any generators instead of the system slack bus for loss compensation. Unlike some previous papers, the transactions are allowed to select a single generator or multiple generators for loss compensation and do not necessarily designate the system slack or their own generators for loss compensation. Next, the conventional power flow analysis is combined with the transaction loss allocation method [48] that is developed in the previous chapter. The method allows a natural separation of losses among individual transactions in a multiple-transaction setting. This combined formulation leads to a systematic solution procedure in order to adjust generation while simultaneously allocating losses to the generators designated by individual transactions.

The chapter is organized in such a way that, the proposed formulation is presented first, followed by its implementation algorithm. Numerical examples are included at the end to illustrate the application of the proposed method to typical power systems.

4.3 Transaction Framework Formulation

4.3.1 Transaction Framework Formulation

In this section, we extend the multi-transaction framework definition in [43] and [48] for the system with n buses and M transactions. Each load acts as a buyer to get its demand met through transactions with one or more sellers. Similarly, each generator acts as a seller and undertakes transactions with one or more buyers. For $m = 1, \dots, M$, the transaction $T^{(m)}$ involving the set of selling entities $S^{(m)}$, the set of buying entities $B^{(m)}$, the set of loss compensating entities $C^{(m)}$, the loss compensation portion $l^{(m)}$ and the MW amount $t^{(m)}$ is defined by the quintuplet

$$T^{(m)} = \{t^{(m)}, S^{(m)}, B^{(m)}, C^{(m)}, l^{(m)}\} \quad (2)$$

where

$$S^{(m)} = \left\{ \left(s_i^{(m)}, \alpha_i^{(m)} \right), i = 1, 2, \dots, N_s^{(m)} \right\}$$

$$B^{(m)} = \left\{ \left(b_i^{(m)}, \beta_i^{(m)} \right), i = 1, 2, \dots, N_b^{(m)} \right\}$$

$$C^{(m)} = \left\{ \left(c_i^{(m)}, \gamma_i^{(m)} \right), i = 1, 2, \dots, N_c^{(m)} \right\}$$

For each transaction m , the selling bus $s_i^{(m)}$ provides the fraction $\alpha_i^{(m)}$ of the total MW amount $t^{(m)}$ while the buying bus $b_i^{(m)}$ receives the fraction $\beta_i^{(m)}$ where $\alpha_i^{(m)} \in [0,1]$ and $\beta_i^{(m)} \in [0,1]$ exist. The loss compensation bus $c_i^{(m)}$ ($c_i^{(m)}$ is not necessarily among the selling buses of the transaction) supplies the fraction $\gamma_i^{(m)}$ of the transaction allocated loss where $\gamma_i^{(m)} \in [0,1]$. $N_s^{(m)}$ is the number of the selling buses, $N_b^{(m)}$ is the number of the buying buses and $N_c^{(m)}$ is the number of the loss compensation buses. The fraction $l^{(m)}$ of the transaction MW amount $t^{(m)}$ is the portion of the system losses allocated to that transaction. Since $l^{(m)}$ will not be known before the loss allocation solution is reached, it must be computed iteratively.

4.3.2 Nodal Injection Expression

Since the nodal power injections can be written as a sum of individual transactions, eventually we can express the amount of the power injection at any bus as follows:

$$P_h = \sum_{m=1}^M \delta_h^{(m)} t^{(m)} \quad (3)$$

where the components of the vector $\delta_h^{(m)}$ are

$$\delta_h^{(m)} = \begin{cases} \alpha_i^{(m)} & \text{if } s_i^{(m)} = h, i = 1, 2, \dots, N_s^{(m)} \\ \alpha_i^{(m)} + \gamma_k^{(m)} l^{(m)} & \text{if } s_i^{(m)} = c_k^{(m)} = h, i = 1, 2, \dots, N_s^{(m)}, \\ & k = 1, 2, \dots, N_c^{(m)} \\ -\beta_j^{(m)} & \text{if } b_j^{(m)} = h, j = 1, 2, \dots, N_b^{(m)} \\ \gamma_k^{(m)} l^{(m)} - \beta_j^{(m)} & \text{if } b_j^{(m)} = c_k^{(m)} = h, j = 1, 2, \dots, N_b^{(m)}, \\ & k = 1, 2, \dots, N_c^{(m)} \\ \alpha_i^{(m)} - \beta_j^{(m)} & \text{if } s_i^{(m)} = b_j^{(m)} = h, i = 1, 2, \dots, N_s^{(m)}, \\ & j = 1, 2, \dots, N_b^{(m)} \\ \alpha_i^{(m)} - \beta_j^{(m)} + \gamma_k^{(m)} l^{(m)} & \text{if } s_i^{(m)} = b_j^{(m)} = c_k^{(m)} = h, \\ & i = 1, 2, \dots, N_s^{(m)}, j = 1, 2, \dots, N_b^{(m)}, \\ & k = 1, 2, \dots, N_c^{(m)} \\ \gamma_k^{(m)} l^{(m)} & \text{if } c_k^{(m)} = h, k = 1, 2, \dots, N_c^{(m)} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

4.3.3 Examples

A three-bus test system shown in Fig. 15 with its transaction information is presented here for illustration. The test system has two generators scheduling to output 500MW and 400MW respectively to three loads that are 500MW, 300MW and 100MW respectively. We have two transactions occurring in the system buying from two generators and selling to three loads, and they are listed as

$$T^{(1)} = \{500, \{(1, 100\%), \{(1, 80\%), (2, 20\%), \{(1, 100\%), l^{(1)}\}$$

$$T^{(2)} = \{400, \{(3, 100\%), \{(1, 25\%), (2, 50\%), (3, 25\%), \{(1, 50\%), (3, 50\%), l^{(2)}\}$$

Transaction $T^{(1)}$ has transaction amount of 500MW and the selling bus 1 provides all 500MW while the buying bus 1 and buying bus 2 take 80% and 20% of the full transaction amount respectively. Bus 1 is also selected by the transaction $T^{(1)}$ to fully

compensate the allocated losses. Transaction $T^{(2)}$ has transaction amount 400MW and the selling bus 3 generates all 400MW while the buses 1, 2 and 3 buy 25%, 50% and 25% of the full transaction amount respectively. Bus 1 and bus 3 are designated to compensate the allocated losses of the transaction $T^{(2)}$ with 50% each.

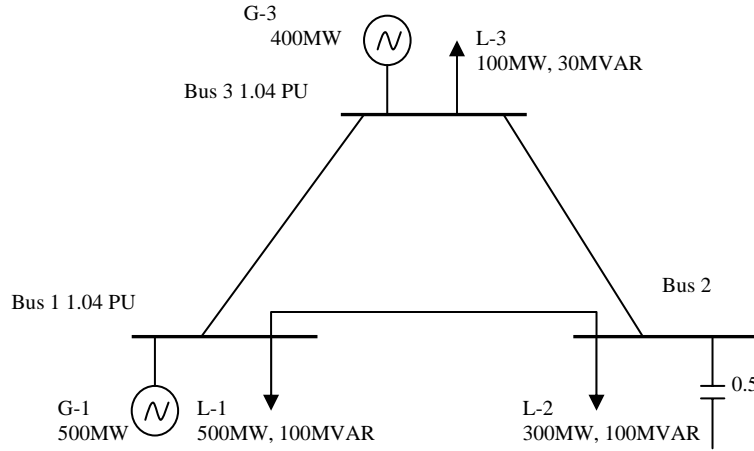


Fig. 15. 3-bus test system

The nodal injection then can be expressed by transactions. The bus 1 is taken as the example.

$$P_1 = \sum_{m=1}^2 \delta_1^{(m)} t^{(m)} = (100\% - 80\% + 100\% \cdot l^{(1)} + 50\% \cdot l^{(2)}) t^{(1)} + (-25\%) t^{(2)} \quad (5)$$

where $t^{(1)}$ is the transaction amount of transaction $T^{(1)}$ and $t^{(2)}$ is the transaction amount of transaction $T^{(2)}$.

4.4 Mathematical Formulation

4.4.1 Loss Allocation Scheme

A loss allocation method in a multiple transaction framework is described in the previous chapter. The approach is based on a symmetric transaction-loss matrix TL which can be constructed as follows:

$$TL = \begin{matrix} & 1 & 2 & \cdots & \cdots & M \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ \vdots \\ M \end{matrix} & \begin{bmatrix} P_L^{(1,1)} & P_L^{(1,2)} & \cdots & \cdots & P_L^{(1,m)} \\ P_L^{(2,1)} & P_L^{(2,2)} & \cdots & \cdots & P_L^{(2,m)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_L^{(m,1)} & P_L^{(m,2)} & \cdots & \cdots & P_L^{(m,m)} \end{bmatrix} \end{matrix} \quad (6)$$

where

$$\begin{aligned} P_L^{(m,m)} &= \frac{t^{(m)}}{\sum_{i=1}^M t^{(i)}} P_{LQQ} + \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \delta_i^{(m)} \delta_j^{(m)} t^{(m)} t^{(m)} \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \sin \theta_{ij} (-\delta_i^{(m)} Q_j + Q_i \delta_j^{(m)}) \cdot t^{(m)} \end{aligned} \quad (7)$$

$$\begin{aligned} P_L^{(m,k)} &= \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \delta_i^{(m)} \delta_j^{(k)} t^{(m)} t^{(k)} \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \frac{R_{ij}}{V_i V_j} \cos \theta_{ij} \delta_i^{(k)} \delta_j^{(m)} t^{(m)} t^{(k)} \quad k \neq m \end{aligned} \quad (8)$$

The self-induced loss term $P_L^{(m,m)}$ is solely due to the individual transaction m , and the cross term $P_L^{(m,k)}$, which should further be evenly reallocated to the corresponding two single transactions, represents the losses accrued due to the interactions between transactions k and m . As a result, for each transaction, the allocated loss can be calculated as:

$$P_L^{(m)} = P_L^{(m,m)} + \sum_{k=1, k \neq m}^M \frac{1}{2} P_L^{(m,k)} \quad (9)$$

Summing the losses allocated to each transaction, the total system loss can be calculated as:

$$P_L = \sum_{m=1}^M P_L^{(m)} \quad (10)$$

4.4.2 Power Flow Analysis with Loss Compensation

In this section, the loss allocation scheme outlined above will be incorporated into the power flow solution algorithm. Consider the power flow equations for bus i :

$$\begin{cases} P_i = P_{Gi} - P_{Di} = V_i \sum_{j \in i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ Q_i = Q_{Gi} - Q_{Di} = V_i \sum_{j \in i} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \end{cases} \quad (11)$$

where, P_i and Q_i are active and reactive bus injections; P_{Gi} and Q_{Gi} are active and reactive generation while P_{Di} and Q_{Di} are active and reactive loads respectively.

When using a single slack bus in power flow problem formulation, the real power injections of the generator buses will be predetermined except for the chosen slack bus. However, if transactions are self-compensating, i.e. for each individual transaction the loss compensation buses $c_i^{(m)}$ and their compensation fractions $\gamma_i^{(m)}$ are known, then the real power generation of the designated loss compensation buses will have to be updated iteratively during the power flow solution. Thus, the real power injections at these buses will be re-written as:

$$P_i = P_{Gi} + \Delta P_{Gi} - P_{Di} = V_i \sum_{j \in i} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (12)$$

where, ΔP_{Gi} is the loss allocated to generation bus i .

Hence, the power flow equations can be written in compact form as:

$$f(V, \theta, \Delta P_{Gi}) = 0 \quad (13)$$

where ΔP_{Gi} is calculated by:

$$\Delta P_{Gi} = \sum_{m=1}^M \gamma_{Gi}^{(m)} l^{(m)} t^{(m)} \quad (14)$$

$$l^{(m)} = \frac{P_L^{(m)}}{t^{(m)}} \quad (15)$$

In (15), the transaction allocated losses are calculated by (6)-(10).

4.4.3 Flowchart

Since the loss $P_L^{(m)}$ allocated to the transaction m will not be known before the solution is reached, the solution of (13) will be iterative as shown by the flowchart in Fig. 16. The solution is initialized by first solving a power flow without performing any loss allocation so that the combined solution can be found by performing few more iterations after this initialization. At each iteration, power flow equations are solved using fixed ΔP_{Gi} , followed by updates of $P_L^{(m)}$, $I^{(m)}$ and ΔP_{Gi} using the loss allocation scheme described in the previous chapter.

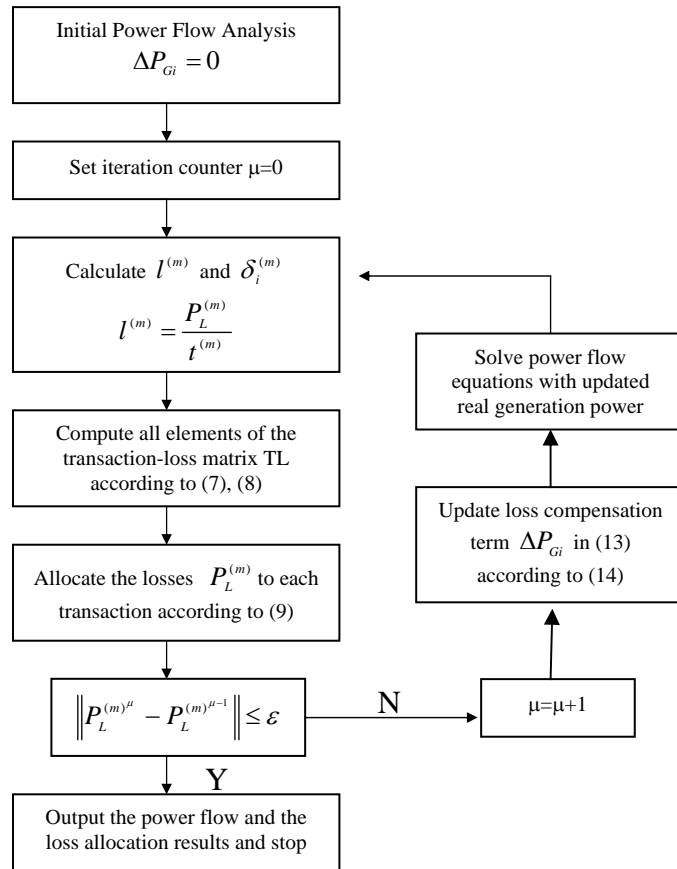


Fig. 16. Flowchart of the proposed power flow analysis with loss allocation approach

4.5 Numerical Results

A number of test systems have been used to test the effectiveness of the new proposed power flow analysis with the transaction-based loss allocation scheme. In this paper, a three-bus system, a 5-bus system [25], and the well-known IEEE Reliability Test System (RTS) are presented as examples.

4.5.1 3-bus System

First, a three-bus system is used to illustrate and examine the proposed method. The parameters and topology are shown in the previous chapter. Five different cases are listed in Table 6.

TABLE 6
TRANSACTION DATA FOR A 3-BUS SYSTEM (MW)

	Case 1		Case 2		Case 3		Case 4		Case 5		
	$T^{(1)}$	$T^{(2)}$	$T^{(1)}$	$T^{(2)}$	$T^{(1)}$	$T^{(2)}$	$T^{(1)}$	$T^{(2)}$	$T^{(1)}$	$T^{(2)}$	$T^{(3)}$
$t^{(m)}$	500	400	500	400	500	400	500	400	400	400	100
S	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%	1,50%	1,50%	1,100%
									3,50%	3,50%	
B	1,100%	2,75%	1,80%	1,25%	1,60%	1,50%	1,40%	1,75%	1,100%	1,25%	3,100%
		3,25%	2,20%	2,50%	2,40%	2,25%	2,60%	3,25%		2,75%	
				3,25%		3,25%					
C	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%	1,50%	1,50%	1,100%
									3,50%	3,50%	

If all the losses are compensated by the slack bus 3 (all C_s in Table 1 are set to (3,100%)), then the power flow solution and hence the total system losses will be the same for all five cases, while the number and type of transactions vary, yielding different loss allocation solutions reported in Table 7. The generation values are $P_{G1} = 500MW$ and $P_{G3} = 413.983MW$ and the system losses are 13.983MW.

TABLE 7
LOSS ALLOCATION RESULTS FOR THE 3-BUS SYSTEM WHEN LOSSES ARE COMPENSATED BY THE SLACK BUS 3

Alloc. Loss	Case 1	Case 2	Case 3	Case 4
Transaction 1 (MW)	0.587	2.352	4.118	5.883
Transaction 2 (MW)	13.396	11.630	9.865	8.100

Then the loss compensation sets in Table 6 for different transactions 1-4 are used for power flow computation and results are shown in Table 8.

TABLE 8
LOSS ALLOCATION RESULTS FOR THE 3-BUS SYSTEM WHEN LOSSES ARE COMPENSATED BY DIFFERENT BUSES (MW)

Alloc. Loss	Case 1	Case 2	Case 3	Case 4
Transaction 1	0.57	2.305	4.068	5.86
Transaction 2	13.38	11.547	9.685	7.80
Total Losses	13.95	13.852	13.753	13.66
G-1	500.57	502.31	504.07	505.86
G-3	413.38	411.55	409.68	407.80

Obviously compared with the results in Table 7, the total system losses are different in different cases because different generation buses compensate the system losses. Since we require each transaction in cases 1-4 to have its own generator to compensate losses, the generation values in Table 8 show they have been updated with the allocated losses. We also note the total losses and losses allocated to both transactions decrease when we have transaction own loss compensation generation buses, which means that each transaction is able to have options to choose its own loss compensation generation for less loss allocation. Furthermore, if all transactions select their own generation buses for loss compensation, then the system slack bus becomes not necessary. We take case 4 in Table 7 and Table 8 as the example.

In Table 7 we have the system slack bus to compensate all the transaction losses. The system losses 13.983MW will be taken by the system slack bus and then transactions 1 and 2 will be allocated with 5.833MW and 8.1MW respectively. During the calculation only one power flow analysis is needed since there is no need to update the generation with the allocated losses. Case 4 in Table 7 yields the transaction-loss matrix (MW) as:

$$\begin{matrix} T^{(1)} & T^{(2)} \\ T^{(1)} & \begin{bmatrix} 12.717 & -13.666 \\ -13.666 & 14.933 \end{bmatrix} \end{matrix} \quad (16)$$

In Table 8 transactions will use their own generators to compensate the allocated losses, so the generation will be updated during calculation. After convergence the transaction-loss matrix (MW) is:

$$\begin{array}{c} T^{(1)} \\ T^{(2)} \end{array} \begin{array}{cc} T^{(1)} & T^{(2)} \\ \left[\begin{array}{cc} 12.981 & -14.243 \\ -14.243 & 14.916 \end{array} \right] \end{array} \quad (17)$$

and the losses allocated to transaction 1 and transaction 2 are 5.86MW and 7.80MW respectively. Obviously we see that the generators G-1 and G-3 have extra output 5.86MW and 7.80MW respectively besides their transaction amount. Also we see that the values in the transaction-loss matrix are different with those in (16) and (17). It is because the transaction allocated losses also cause line losses in the system.

Case 5 has transactions which have more than one generator to compensate the transaction allocated losses. After calculation we have the transaction-loss matrix (MW) as:

$$\begin{array}{c} T^{(1)} \\ T^{(2)} \\ T^{(3)} \end{array} \begin{array}{ccc} T^{(1)} & T^{(2)} & T^{(3)} \\ \left[\begin{array}{ccc} 6.649 & 3.592 & -6.312 \\ 3.592 & 9.736 & -1.756 \\ -6.312 & -1.756 & 1.755 \end{array} \right] \end{array} \quad (18)$$

where the transaction loss allocation results are:

Transaction 1: 5.289MW

Transaction 2: 10.654MW

Transaction 3: -2.280MW

The half of transactions 1 and 2 allocated losses will go to generator G-1 and the other half will go to generator G-3. The losses allocated to transaction 3 will be compensated by generator G-1. The simple calculation can be done and the total losses taken by G-1 and G-3 will be 5.692MW and 7.971MW respectively. At the same time the power flow results tell us after we exclude the transaction amount the extra output values from G-1 and G-3 are 5.692MW and 7.971MW. As a result, the losses allocated to individual transactions have been updated into their designated compensation generators and the eventual power flow results reflect the generation adjustment with loss allocation and loss compensation.

4.5.2 5-bus System

We next test the method on a 5-bus system [25] shown in Fig. 17 and Table 9.

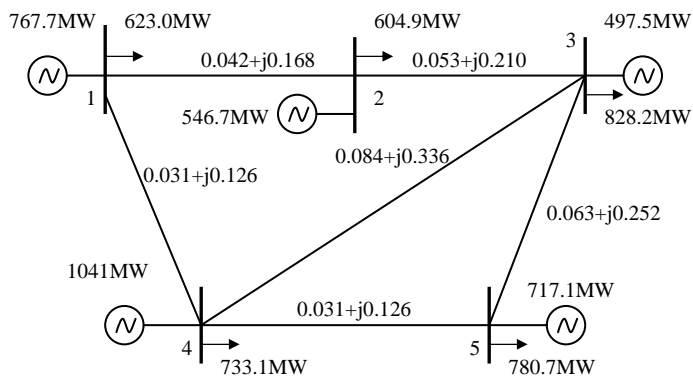


Fig. 17. 5-bus system

TABLE 9
LINE CHARACTERISTICS FOR A 5-BUS SYSTEM

From Bus	To Bus	$R (p.u.)$	$X (p.u.)$	$C (p.u.)$
1	2	0.042	0.168	0.00
1	4	0.031	0.126	0.00
2	3	0.053	0.210	0.00
3	4	0.084	0.336	0.00
3	5	0.063	0.252	0.00
4	5	0.031	0.126	0.00

Like [25], we have 25 bilateral transactions between each different two buses in the system shown in Fig. 17. We examine the loss allocation with different loss compensation and report the results in Table 10. Line flows are shown in Fig. 18.

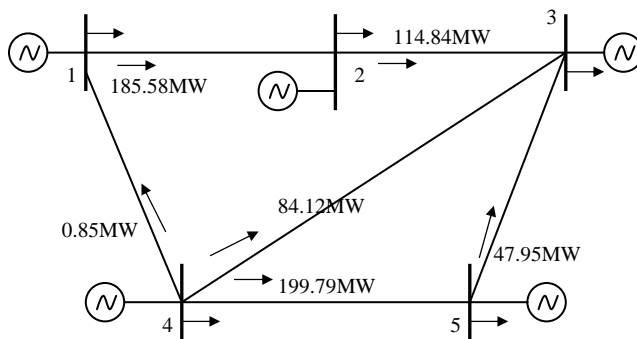


Fig. 18. Flows of the 5-bus system

All transactions have same signs and same trend of the allocated losses as in [25] and most of them have similar results though a few transactions have different loss allocation values due to different method. For instance, all self-transactions 1, 7, 13, 19 and 25 in [25] have 0 allocated losses. However, here those transactions have been allocated with a little bit of losses due to the reactive power support.

TABLE 10
LOSS ALLOCATION RESULTS OF THE 5-BUS SYSTEM FOR DIFFERENT LOSS COMPENSATION (MW)

Transaction	$t^{(m)}$	S	B	Loss Allocation ¹	Loss Allocation ²	Loss Allocation ³
$T^{(1)}$	65.7	1, 100%	1, 100%	0.26	0.33	0.0
$T^{(2)}$	14.1	1, 100%	2, 100%	0.93	1.03	1.2
$T^{(3)}$	203.7	1, 100%	3, 100%	19.73	21.67	32.3
$T^{(4)}$	203.8	1, 100%	4, 100%	-2.45	-1.45	-0.1
$T^{(5)}$	280.4	1, 100%	5, 100%	13.40	15.88	18.4
$T^{(6)}$	115.1	2, 100%	1, 100%	-6.68	-6.78	-8.6
$T^{(7)}$	155.8	2, 100%	2, 100%	0.62	0.74	0.0
$T^{(8)}$	249.3	2, 100%	3, 100%	8.69	8.96	16.7
$T^{(9)}$	10.4	2, 100%	4, 100%	-0.77	-0.73	-0.8
$T^{(10)}$	16.1	2, 100%	5, 100%	-0.23	-0.17	-0.3
$T^{(11)}$	158.9	3, 100%	1, 100%	-14.14	-13.89	-20.6
$T^{(12)}$	201.3	3, 100%	2, 100%	-5.43	-5.17	-12.3
$T^{(13)}$	2.3	3, 100%	3, 100%	0.01	0.01	0.0
$T^{(14)}$	115.0	3, 100%	4, 100%	-12.07	-11.32	-14.9
$T^{(15)}$	20.0	3, 100%	5, 100%	-0.90	-0.81	-1.5
$T^{(16)}$	125.3	4, 100%	1, 100%	2.50	2.19	0.0
$T^{(17)}$	206.0	4, 100%	2, 100%	16.88	17.77	17.3
$T^{(18)}$	176.7	4, 100%	3, 100%	19.94	21.21	28.1
$T^{(19)}$	279.1	4, 100%	4, 100%	1.10	1.43	0.0
$T^{(20)}$	253.9	4, 100%	5, 100%	16.20	17.69	16.7
$T^{(21)}$	158.1	5, 100%	1, 100%	-6.31	-6.99	-9.5
$T^{(22)}$	27.6	5, 100%	2, 100%	0.61	0.56	0.5
$T^{(23)}$	196.2	5, 100%	3, 100%	10.40	10.23	16.7
$T^{(24)}$	124.8	5, 100%	4, 100%	-6.98	-6.97	-7.5
$T^{(25)}$	210.4	5, 100%	5, 100%	0.83	1.01	0.0
Total System Losses				56.15	66.44	71.75

Loss Allocation¹: all system losses are compensated by the slack bus 4.

Loss Allocation²: each transaction uses its own generator to compensate the allocated losses.

Loss Allocation³: results from [10].

TABLE 11
GENERATION CHANGES OF THE 5-BUS SYSTEM FOR DIFFERENT LOSS COMPENSATION (MW)

	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5
Generation ¹	767.70	546.70	497.5	1097.05	717.1
Generation ²	805.16	548.72	466.32	1101.19	714.95
Generation ³	819.5	553.6	448.2	1103.2	717.2

Fig. 18 shows that bus 3 is a sink which has all connected lines transferring power into. It means that all transactions selling power from bus to other buses will produce “counter flows” so that those transactions will be allocated with negative losses. Also, the transactions selling power to bus 3 are possible to be allocated with positive losses since they follow “dominant flow” directions. Table 10 shows that our proposed method and the method in [25] both have negative losses allocated to the former transactions and positive losses to the latter.

Apparently in Table 10 the total system loss value when all losses are compensated by the slack bus 4 is less than that when each transaction compensates the allocated losses by its own supplying generator. It is possibly because all the transactions from bus 3 to other buses (transaction 11, 12, 14 and 15), which produce “counter flows”, are rewarded negative losses causing the generation at bus 3 to decrease. Other buses have to transfer more power to bus 3 so that the overall system losses increase. Table 11 shows the updated generation values with loss allocation and loss compensation. Since bus 3 is allocated with negative losses its generation reduces while others have to output extra for their own loss allocation portions.

4.5.3 The IEEE Reliability Test System (RTS)

The IEEE Reliability Test System (RTS) [59] is also used to illustrate the proposed procedure. The RTS system diagram is shown in Fig. 19, which has 24 buses, 38 circuits, and 14 generators. The generation and load data has been slightly changed for calculation convenience reasons. The total system load is 2800MW. We assume there are four transactions in the system, which are listed in Table 12.

TABLE 12
TRANSACTIONS IN THE IEEE RTS SYSTEM

Transaction m	$t^{(m)}$ (MW)	$S^{(m)}$	$B^{(m)}$	$C^{(m)}$
1	600	{(1,30%),(2,30%), (7,40%)}	{(1,16.67%),(2,16.67%), (3,30%),(6,16.67%), (7,20%)}	{(1,100%)}
2	900	{(22,33.33%), (23,66.67%)}	{(4,8.33%),(5,8.33%), (8,16.67%),(9,16.67%), (10,22.22%),(13,27.78%)}	{(22,50%),(23,50%)}
3	100	{(13,100%)}	{(16,100%)}	{(13,100%)}
4	1200	{(15,16.67%),(16,16.67%), (18,33.33%),(21,33.33%)}	{(14,16.67%),(15,25%), (18,29.17%),(19,16.67%), (20,12.5%)}	{(18,50%),(21,50%)}

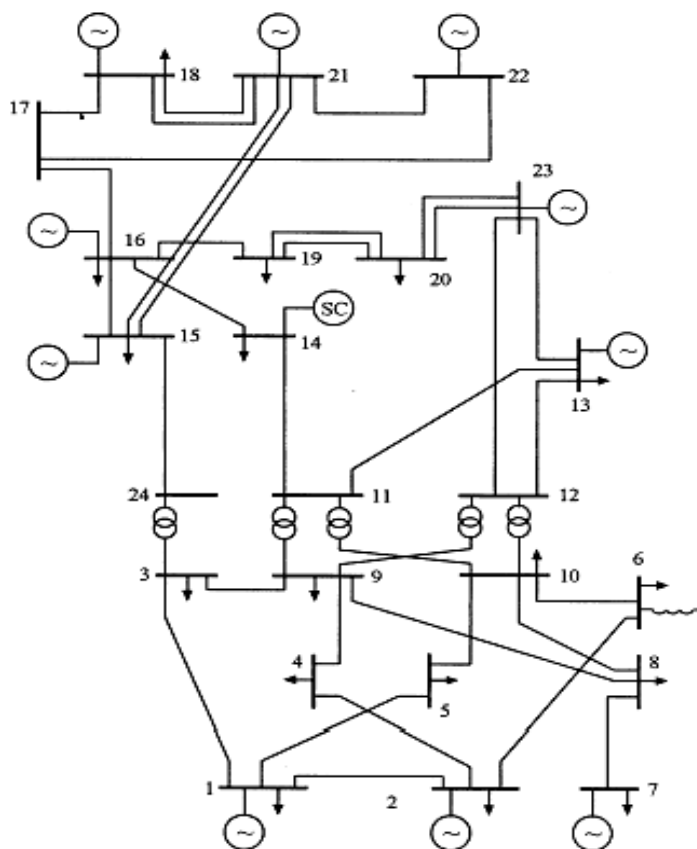


Fig. 19. IEEE RTS 24-bus system

Assuming bus 15 as the system slack bus the initial power flow results show that the total system losses are 49.73MW, which are fully compensated by the extra output from the generator of bus 15. However, since all individual transactions designate their own generators to compensate their allocated losses, the power flow solution will be slightly different from the initial power flow results with 50.78MW of the total system losses. Both the initial power flow results and power flow results with loss compensation are shown in Table 13.

After convergence the transaction-loss matrix (MW) is calculated as:

$$\begin{matrix}
 & T^{(1)} & T^{(2)} & T^{(3)} & T^{(4)} \\
 \begin{matrix} T^{(1)} \\ T^{(2)} \\ T^{(3)} \\ T^{(4)} \end{matrix} & \begin{bmatrix} 11.968 & -18.216 & 0.977 & -2.654 \\ -18.216 & 44.877 & -5.615 & 10.070 \\ 0.977 & -5.615 & 0.714 & -2.177 \\ -2.654 & 10.070 & -2.177 & 10.828 \end{bmatrix} & & &
 \end{matrix} \quad (19)$$

TABLE 13
POWER FLOW RESULTS OF IEEE RTS 24 BUS SYSTEM (BUS 15 AS THE SLACK BUS)

Bus	Initial Power Flow Results						Power Flow Results with Loss Compensation					
	P_g (MW)	Q_g (MVar)	P_d (MW)	Q_d (MVar)	V (p.u.)	θ (degree)	P_g (MW)	Q_g (MVar)	P_d (MW)	Q_d (MVar)	V (p.u.)	θ (degree)
1	180	13.88	100	22	1.0350	-18.9601	182.02	13.24	100	22	1.0350	-3.9806
2	180	-18.04	100	20	1.0350	-19.0682	180	-18.05	100	20	1.0350	-4.0923
3	0	0	180	37	0.9741	-17.4628	0	0	180	37	0.9744	-2.6616
4	0	0	75	15	0.9914	-21.9824	0	0	75	15	0.9915	-7.0320
5	0	0	75	14	1.0278	-22.2792	0	0	75	14	1.0278	-7.3097
6	0	0	100	28	1.0843	-24.5457	0	0	100	28	1.0842	-9.5827
7	240	43.94	120	25	1.0250	-18.5856	240	43.92	120	25	1.0250	-3.6408
8	0	0	150	35	0.9968	-22.5155	0	0	150	35	0.9968	-7.5710
9	0	0	150	36	0.9903	-20.1118	0	0	150	36	0.9904	-5.1836
10	0	0	200	40	1.0486	-22.3024	0	0	200	40	1.0486	-7.3431
11	0	0	0	0	1.0024	-16.5284	0	0	0	0	1.0025	-1.6067
12	0	0	0	0	1.0175	-15.5641	0	0	0	0	1.0173	-0.5752
13	100	84.71	250	54	1.0200	-14.9659	97.30	86.04	250	54	1.0200	0.0000
14	0	-31.88	200	39	0.9800	-12.9466	0	-32.61	200	39	0.9800	1.9116
15	249.73	-21.08	300	64	1.0140	0.0000	200	-14.72	300	64	1.0140	14.5421
16	200	35.19	100	20	1.0170	-1.3833	200	34.44	100	20	1.0170	13.3830
17	0	0	0	0	1.0381	3.1104	0	0	0	0	1.0380	17.9948
18	400	141.54	350	68	1.0500	4.4102	406.72	141.80	350	68	1.0500	19.3038
19	0	0	200	37	1.0221	-4.2449	0	0	200	37	1.0222	10.6830
20	0	0	150	26	1.0378	-4.5532	0	0	150	26	1.0379	10.5119
21	400	110.39	0	0	1.0500	5.3409	406.72	113.06	0	0	1.0500	20.1977
22	300	-30.37	0	0	1.0500	10.9782	319	-30.52	0	0	1.0500	26.2588
23	600	131.29	0	0	1.0500	-3.8659	619	130.28	0	0	1.0500	11.2729
24	0	0	0	0	0.9817	-6.4574	0	0	0	0	0.9823	8.1788

and then the transaction loss allocation results are: Transaction 1: 2.022MW; Transaction 2: 37.996MW; Transaction 3: -2.694MW; Transaction 4: 13.448MW.

Also we can easily observe that the allocated losses to individual transactions have been compensated by their designated generators. The generator of bus 1 output extra 2.022MW to compensate the allocated losses to transaction 1. Generators of bus 22 and bus 23 both output extra about 19MW to compensate the allocated losses 37.996MW to transaction 2. Generators of bus 18 and bus 21 both output extra about 6.72MW to compensate allocated losses 13.448MW to transaction 4. Since transaction 3 produces “counter flow” and is allocated with negative losses, the loss compensation generator will be awarded with reduced output of 2.694MW.

TABLE 14
POWER FLOW RESULTS OF IEEE RTS 24 BUS SYSTEM (BUS 23 AS THE SLACK BUS)

Bus	Initial Power Flow Results						Power Flow Results with Loss Compensation					
	P_g (MW)	Q_g (MVar)	P_d (MW)	Q_d (MVar)	V (p.u.)	θ (degree)	P_g (MW)	Q_g (MVar)	P_d (MW)	Q_d (MVar)	V (p.u.)	θ (degree)
1	180	13.43	100	22	1.0350	-15.6905	182.02	13.24	100	22	1.0350	-15.2536
2	180	-18.16	100	20	1.0350	-15.7912	180	-18.05	100	20	1.0350	-15.3653
3	0	0	180	37	0.9747	-14.4539	0	0	180	37	0.9744	-13.9346
4	0	0	75	15	0.9915	-18.6848	0	0	75	15	0.9915	-18.3050
5	0	0	75	14	1.0278	-18.9619	0	0	75	14	1.0278	-18.5827
6	0	0	100	28	1.0842	-21.2008	0	0	100	28	1.0842	-20.8557
7	240	43.88	120	25	1.0250	-15.2432	240	43.92	120	25	1.0250	-14.9139
8	0	0	150	35	0.9968	-19.1736	0	0	150	35	0.9968	-18.8440
9	0	0	150	36	0.9905	-16.7976	0	0	150	36	0.9904	-16.4566
10	0	0	200	40	1.0486	-18.9363	0	0	200	40	1.0486	-18.6161
11	0	0	0	0	1.0025	-13.2119	0	0	0	0	1.0025	-12.8798
12	0	0	0	0	1.0170	-12.0619	0	0	0	0	1.0173	-11.8482
13	100	87.24	250	54	1.0200	-11.4591	97.30	86.04	250	54	1.0200	-11.2730
14	0	-33.97	200	39	0.9800	-9.8283	0	-32.61	200	39	0.9800	-9.3616
15	200	-17.83	300	64	1.0140	2.4965	200	-14.72	300	64	1.0140	3.2691
16	200	29.38	100	20	1.0170	1.4500	200	34.44	100	20	1.0170	2.1100
17	0	0	0	0	1.0382	5.8236	0	0	0	0	1.0380	6.7218
18	400	140.74	350	68	1.0500	7.0693	406.72	141.80	350	68	1.0500	8.0308
19	0	0	200	37	1.0223	-0.9637	0	0	200	37	1.0222	-0.5900
20	0	0	150	26	1.0379	-0.8915	0	0	150	26	1.0379	-0.7610
21	400	111.14	0	0	1.0500	7.9498	406.72	113.06	0	0	1.0500	8.9247
22	300	-30.51	0	0	1.0500	13.6278	319	-30.52	0	0	1.0500	14.9858
23	649.18	128.49	0	0	1.0500	0.0000	619	130.28	0	0	1.0500	0.0000
24	0	0	0	0	0.9828	-3.7748	0	0	0	0	0.9823	-3.0942

We also do another test to verify the final power flow results will not be related to the selection of the system slack bus in the power flow calculation because all the losses are compensated by the transaction loss compensation generators. Table 14 lists the power flow results by using different slack bus in the initial power flow analysis. Obviously the initial power flow results are different from that in Table 13 since different slack buses are used. However the power flow results with loss compensation are exactly same except that the values of the bus voltage phase angles look different because different reference buses are used. It clearly shows that the power flow results are independent of the selection of the slack bus [56] because all transactions have chosen their own generators for loss compensation and the system slack bus is no longer needed to compensate the transmission losses.

4.6 Conclusions

This chapter firstly extends the transaction framework definition by adding transaction loss compensation bus set so that each transaction is able to choose any generator for loss compensation. Secondly, this paper analyzes the power flow computation with fair loss allocation, which allocates system losses among multiple transactions. The loss allocation scheme is directly derived from the loss formula without making any approximations. A power flow scheme with loss allocation and loss compensation on transaction chosen buses finally is presented. Since the proposed method calculates the power flow, losses and allocations iteratively, the sum of the allocated losses to each transaction will match the total system loss exactly. Numerical examples are given to illustrate that the new method yields loss allocation results that are intuitively reasonable and consistent with expectations.

CHAPTER V

POWER FLOW ANALYSIS WITH OPTIMAL LOSS ALLOCATION

5.1 Introduction

In the last chapter we discussed about the power flow analysis with loss allocation and loss compensation provided that all individual transactions designate compensation generators. Power flow solution algorithm should then incorporate the chosen loss allocation strategy so that the solution will yield a system state and generation dispatch consistent with this loss allocation strategy. However, some transactions may choose not to designate any specific loss compensation generators, then this will provide an opportunity for the ISO to implement a least-cost loss compensation solution by dispatching the loss compensation service to certain generators while the network constraints are taken into account. This option also may allow certain transactions to cover their allocated losses at a lower cost than otherwise would incur.

The paper [51] presents a method by using the LP formulation to determine the solution that gives the least-price at which the ISO can acquire the loss compensation service from participating generators. Because of its use of DC power flow approximation and the LP optimization model it may cause possible inaccurate solution. As we know, naturally it is an optimization problem to dispatch least-cost loss compensation service with loss allocation. So in this chapter a new scheme, in which an Optimal Power Flow (OPF) model is then utilized to optimize the loss compensation for those transactions electing to purchase the loss service from the ISO and accordingly the incurred losses are fairly allocated back to individual transactions, will be proposed. The proposed scheme provides all transactions the capability to either choose self-compensation or ISO-compensation for their allocated losses with less cost.

The chapter is organized in such a way that, the proposed formulation is presented first, followed by its implementation algorithm. Numerical examples are included at the end to illustrate the application of the proposed method to typical power systems.

5.2 Transaction Framework Formulation

The same transaction framework definition in the previous chapter will be used here. For $m = 1, \dots, M$, the transaction $T^{(m)}$ involving the set of selling entities $S^{(m)}$, the set of buying entities $B^{(m)}$, the set of loss compensating entities $C^{(m)}$, the loss compensation portion $l^{(m)}$ and the MW amount $t^{(m)}$ is defined by the quintuplet

$$T^{(m)} = \{t^{(m)}, S^{(m)}, B^{(m)}, C^{(m)}, l^{(m)}\} \quad (1)$$

where

$$S^{(m)} = \left\{ \left(s_i^{(m)}, \alpha_i^{(m)} \right), i = 1, 2, \dots, N_s^{(m)} \right\}$$

$$B^{(m)} = \left\{ \left(b_i^{(m)}, \beta_i^{(m)} \right), i = 1, 2, \dots, N_b^{(m)} \right\}$$

$$C^{(m)} = \left\{ \left(c_i^{(m)}, \gamma_i^{(m)} \right), i = 1, 2, \dots, N_c^{(m)} \right\}$$

In the previous chapter, all transactions designate the loss compensation generators, so all loss compensation buses $c_i^{(m)}$ and corresponding compensation fractions $\gamma_i^{(m)}$ are pre-defined. Here we will assume that transactions $m \in M_A$ choose to compensate the allocated losses by their own designated generators while transactions $m \in M_B$ ($M_A \cup M_B = M$) select to use ISO's loss compensation service. Then, since for each self-compensated transaction the set of loss compensating entities $C^{(m)}$ (loss compensation buses $c_i^{(m)}$ and corresponding compensation fractions $\gamma_i^{(m)}$) is known, the allocated losses can be obtained after each power flow solution is reached. However, for the ISO-compensated transactions the compensating buses $c_i^{(m)}$ and their compensation fractions $\gamma_i^{(m)}$ are unknown before each optimal power flow solution. The selection of the compensating buses $c_i^{(m)}$ and their compensation fractions $\gamma_i^{(m)}$ will be discussed in chapter 5.3. The nodal power injections also can be written by the same expressions in chapter 4.3.2.

5.3 Problem Statement

5.3.1 Optimal Loss Compensation

If some of the transactions choose to have the ISO designate independent units for their loss compensation while others are willing to compensate on their own [51], then a modified solution will have to be found. In this case, the ISO may acquire the loss compensation through having the generation price and determining the least-price distribution by an OPF program [60].

For the ISO-compensated transactions the compensating buses $c_i^{(m)}$ and their compensation fractions $\gamma_i^{(m)}$ are unknown before each optimal power flow solution, which is to minimize the cost or the price or the total loss while all the necessary network limits are satisfied. Assume that K generators are willing to participate in the ISO's system loss compensation service. Then, the OPF model for the optimal loss allocation will be:

$$\min \sum_k^K f_{Gk,B}(\Delta P_{Gk,B}) \quad (2)$$

subject to the constraints:

$$P_{Gi}^{sp} + \Delta P_{Gi,A} + \Delta P_{Gi,B} - P_{Di} - P_i(V, \theta) = 0 \quad (3)$$

$$Q_{Gi} - Q_{Di} - Q_i(V, \theta) = 0 \quad (4)$$

$$S_{ij} \leq S_{ij}^{\max} \quad (5)$$

$$V_i^{\min} \leq V_i \leq V_i^{\max} \quad (6)$$

$$0 \leq \Delta P_{Gk,B} \leq P_{Gk}^{\max} - P_{Gk}^{sp} - \Delta P_{Gk,A} \quad 1 \leq k \leq K \quad (7)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max} \quad (8)$$

where the objective function $f_{Gk,B}(\Delta P_{Gk,B})$ can be the least-price, least-cost or least system loss; (3) and (4) are active and reactive power flow equations; (5) and (6) are the line flow and bus voltage limits respectively; and (7) and (8) are the generator active and reactive power output limits. After the optimal solution is reached, the ISO-compensated

losses $\sum_k^K \Delta P_{Gk,B}$ will have to be allocated to each transaction. With the assumption that each participated unit evenly compensates each transaction losses, then the generator k will have loss compensation fraction $\gamma_k^{(m)}$ for transaction m given by:

$$\gamma_k^{(m)} = \frac{\Delta P_{Gk,B}}{\sum_{k=1}^K \Delta P_{Gk,B}} \quad (9)$$

5.3.2 Flowchart

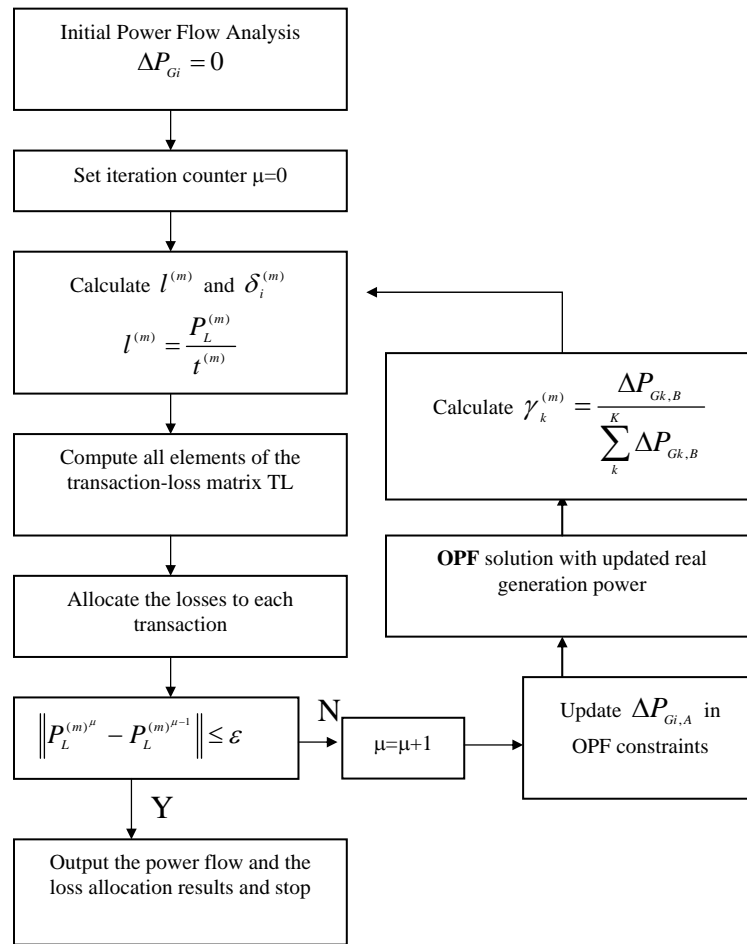


Fig. 20. Flowchart of the proposed OPF approach

In order to solve the OPF problem (2)-(8) with loss allocation described in chapter III, losses have to be computed, allocated to each transaction and assigned to loss compensation generation buses. Since the system losses are unknown before the solution is reached, an

iterative solution shown in Fig. 20 needs to be implemented. Here we can observe that if no self-compensating transactions exist, then no $\Delta P_{Gk,A}$ in the flowchart needs to be updated. It will make the problem same as OPF problem to optimize the system losses except the generation limits will be updated by transactions.

5.4 Numerical Results

Several test systems of varying sizes are used to test the effectiveness of the proposed power flow and optimization procedures with loss allocation. As also stated in [48], the computational burden is proportional to the number of transactions and is not significantly affected by the system size. Hence, only the results related to a three-bus system, a 5-bus system [25] and the IEEE RTS 24-bus system will be presented here in an attempt to keep the examples reproducible and easy to follow.

5.4.1 3-bus System

Consider the 3-bus system whose network data are given in chapter III. Four different cases are presented as listed in Table 15. In Table 15, S, B and C stands for the selling entity, buying entity and loss compensating entity respectively while $t^{(m)}$ represents the transaction MW amount in the multi-transaction framework definition.

TABLE 15
TRANSACTION DATA WITH COMPENSATION FOR A 3-BUS SYSTEM (MW)

	Case 1		Case 2		Case 3		Case 4	
	$T^{(1)}$	$T^{(2)}$	$T^{(1)}$	$T^{(2)}$	$T^{(1)}$	$T^{(2)}$	$T^{(1)}$	$T^{(2)}$
$t^{(m)}$	500	400	500	400	500	400	500	400
S	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%
B	1,100%	2,75%	1,80%	1,25%	1,60%	1,50%	1,40%	1,75%
		3,25%	2,20%	2,50%	2,40%	2,25%	2,60%	3,25%
				3,25%		3,25%		
C	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%	1,100%	3,100%

TABLE 16
LOSS ALLOCATION RESULTS FOR THE 3-BUS SYSTEM WHEN LOSSES ARE COMPENSATED BY DESIGNATED GENERATORS (MW)

Alloc. Loss	Case 1	Case 2	Case 3	Case 4
Transaction 1	0.57	2.305	4.068	5.86
Transaction 2	13.38	11.547	9.685	7.80
Total Losses	13.95	13.852	13.753	13.66
G-1	500.57	502.31	504.07	505.86
G-3	413.38	411.55	409.68	407.80

The power flow results for different cases with loss allocation and loss compensation are shown in Table 16. As expected, the total system losses are different in different cases because different generation buses compensate for the system losses. Since each transaction in cases 1-4 is required to have its own generator to compensate losses, the generation values in Table 16 show that they are updated with the allocated losses. It is also noted that the total losses and losses allocated to both transactions decrease when there are transactions electing loss compensation generation buses, i.e. when each transaction chooses its own loss compensation generation for less loss allocation.

Next, the same system is used to implement optimal loss allocation. Case 4 is taken as an example. Table 17 lists the test results of three scenarios when least system loss is used as the optimization objective function.

TABLE 17
OPTIMAL LOSS ALLOCATION RESULTS FOR CASE 4 (LEAST LOSS) (MW)

	1	2	3
Transaction 1	5.860	6.015	6.015
Transaction 2	7.800	7.242	7.242
Total Losses	13.660	13.257	13.257
G-1	505.860	513.257	513.257
G-3	407.800	400.000	400.000

1: Both transactions are self-compensated; 2: Transaction 1 is self loss compensated while transaction 2 is ISO compensated; 3: Both are compensated by ISO loss compensation service.

It is observed that after the optimization, the latter two scenarios have less total system loss compared to the base case. Also it is noticed that the latter two scenarios have the same results. This is due to the fact that if ISO has G-1 to compensate for the losses, then transaction 2 will incur less system losses. It is also interesting to see that in the latter two scenarios transaction 1 will have more allocated losses compared to that of scenario 1. It clearly shows the loss compensation generation selection will have an impact on the loss allocation results and the ISO-compensation will result in least system losses but may not have least loss allocation for all transactions.

If the MW prices of G-1 and G-3 are known, the least loss cost optimization also can be obtained. For instance, if the price of G-1 and G-3 are 12 \$/MWh and 11.38 \$/MWh respectively, then an optimal loss allocation will be obtained when both transactions are compensated by the ISO as shown in Table 18 below.

TABLE 18
OPTIMAL LOSS ALLOCATION RESULTS FOR CASE 4 (LEAST COST) (MW)

Transaction 1	Transaction 2	Total Losses	G-1	G-3	Total Cost (\$)
5.596	7.762	13.578	507.250	406.330	159.04

5.4.2 5-bus System

Next, consider the 5-bus system shown in Fig. 21 and used in [25]. As assumed in [25], 5 bilateral transactions within the same buses and 20 bilateral transactions between different pairs of buses are assumed in the system. Five different loss compensation scenarios are studied and the results are shown in Tables 19 and 20. These five scenarios are described below:

- Scenario 1: All system losses are compensated by the slack bus 4;
- Scenario 2: Each transaction uses its own generator to compensate for their allocated loss;
- Scenario 3: All transactions buy ISO-acquired loss compensation service with least system loss optimization;
- Scenario 4: All transactions buy ISO-acquired loss compensation service with least loss cost optimization;
- Scenario 5: All transactions use least cost ISO-compensation, except for 3, 17 and 23 which use their own generators for loss compensation.

For the least loss cost optimization, it is assumed that the prices of generators at buses 1, 2, 3, 4 and 5 are 12.5 \$/MWh; 15 \$/MWh; 17 \$/MWh; 19 \$/MWh and 22 \$/MWh respectively.

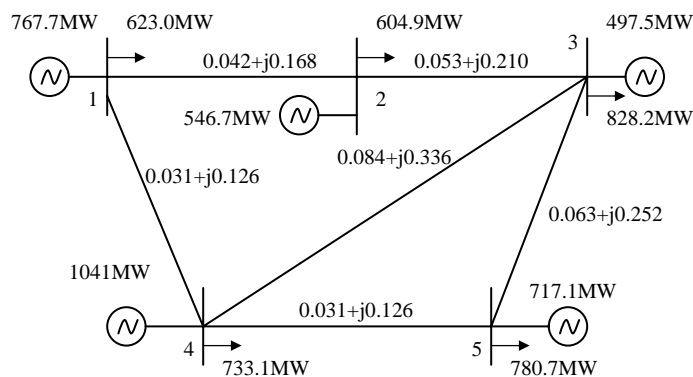


Fig. 21. 5-bus test system with transaction data

Apparently in Table 19 the total system loss value when all losses are compensated by the slack bus 4 is less than that when each transaction compensates the allocated losses by its own supplying generator. This is possibly due to all the transactions from bus 3 to other buses (transaction 11, 12, 14 and 15), which produce “counter flows”, and are rewarded negative losses causing the generation at bus 3 to decrease (Table 20). When all transactions use their own generators to compensate the allocated losses, other buses have to transfer more power to bus 3 so that the overall system losses increase. It is due to this same reason that when all the transactions acquire loss compensation from the ISO with least system loss, the generator at bus 3 is used for acquisition.

TABLE 19
LOSS ALLOCATION RESULTS OF THE 5-BUS SYSTEM FOR DIFFERENT LOSS COMPENSATION (MW)

Trans.	$t^{(m)}$	S	B	1	2	3	4	5
$T^{(1)}$	65.7	1, 100%	1, 100%	0.26	0.33	0.157	0.254	0.248
$T^{(2)}$	14.1	1, 100%	2, 100%	0.93	1.03	0.778	1.010	0.959
$T^{(3)}$	203.7	1, 100%	3, 100%	19.73	21.67	16.792	21.654	20.483
$T^{(4)}$	203.8	1, 100%	4, 100%	-2.45	-1.45	-1.215	0.198	-1.014
$T^{(5)}$	280.4	1, 100%	5, 100%	13.40	15.88	11.778	16.502	14.345
$T^{(6)}$	115.1	2, 100%	1, 100%	-6.68	-6.78	-5.798	-7.354	-6.959
$T^{(7)}$	155.8	2, 100%	2, 100%	0.62	0.74	0.373	0.603	0.588
$T^{(8)}$	249.3	2, 100%	3, 100%	8.69	8.96	7.398	9.609	9.058
$T^{(9)}$	10.4	2, 100%	4, 100%	-0.77	-0.73	-0.611	-0.695	-0.720
$T^{(10)}$	16.1	2, 100%	5, 100%	-0.23	-0.17	-0.173	-0.143	-0.210
$T^{(11)}$	158.9	3, 100%	1, 100%	-14.14	-13.89	-12.339	-15.663	-14.781
$T^{(12)}$	201.3	3, 100%	2, 100%	-5.43	-5.17	-5.010	-6.202	-5.796
$T^{(13)}$	2.3	3, 100%	3, 100%	0.00	0.00	0.00	0.00	0.00
$T^{(14)}$	115.0	3, 100%	4, 100%	-12.07	-11.32	-9.891	-11.668	-11.703
$T^{(15)}$	20.0	3, 100%	5, 100%	-0.90	-0.81	-0.761	-0.872	-0.913
$T^{(16)}$	125.3	4, 100%	1, 100%	2.50	2.19	1.347	0.847	1.569
$T^{(17)}$	206.0	4, 100%	2, 100%	16.88	17.77	13.083	15.352	15.948
$T^{(18)}$	176.7	4, 100%	3, 100%	19.94	21.21	16.043	19.296	19.316
$T^{(19)}$	279.1	4, 100%	4, 100%	1.10	1.43	0.668	1.080	1.053
$T^{(20)}$	253.9	4, 100%	5, 100%	16.20	17.69	12.786	15.677	15.211
$T^{(21)}$	158.1	5, 100%	1, 100%	-6.31	-6.99	-5.884	-8.081	-6.896
$T^{(22)}$	27.6	5, 100%	2, 100%	0.61	0.56	0.429	0.459	0.569
$T^{(23)}$	196.2	5, 100%	3, 100%	10.40	10.23	8.402	10.07	9.96
$T^{(24)}$	124.8	5, 100%	4, 100%	-6.98	-6.97	-5.688	-6.740	-6.535
$T^{(25)}$	210.4	5, 100%	5, 100%	0.83	1.01	0.503	0.814	0.794
Total System Losses				56.15	66.44	43.174	56.016	54.581

TABLE 20
GENERATION CHANGES OF THE 5-BUS SYSTEM FOR DIFFERENT LOSS COMPENSATION (MW)

	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Total Loss Cost (\$)
1	767.70	546.70	497.50	1097.05	717.10	1065.0
2	805.16	548.72	466.32	1101.19	714.95	1064.8
3	767.70	546.70	540.58	1041.00	717.10	734.0
4	823.62	546.70	497.50	1041.00	717.10	700.2
5	796.28	546.70	497.50	1056.94	727.06	865.1

It is also observed from Tables 19 and 20 that for scenario 5, the allocated losses to transactions 3, 17 and 23 are 20.48MW, 15.95MW and 9.96 MW respectively, which have been assigned to their designated generators at buses 1, 4 and 5. These results indicate that the transactions will have different loss allocations with different loss compensation choices. As a result, it gives the transactions flexibility to either self-compensate their allocated losses or buy the loss compensation service from the ISO depending on their motivation.

5.4.3 IEEE RTS 24-bus System

The IEEE RTS 24-bus system described in the previous chapter is also used to verify the effectiveness of the proposed optimal loss compensation procedure. The following three cases have been investigated:

- Case 1. All four transactions have the system slack bus 15 compensate the total system losses;
- Case 2. All four transactions designate their own loss compensation generators as shown in the previous chapter;
- Case 3. All four transactions have the system choose loss compensation generators to minimize the system losses.

The power flow results of three cases are shown in Table 21, which lists bus voltages, phase angles as well as generation output. Obviously the power flow solutions are slightly different for three cases because the generation output to compensate the transmission losses is different among cases. The final loss allocation results are reported in Table 22 for these three cases. After comparing the total transmission losses for these three cases, we find that case 3 has least total system losses and least losses allocated to each individual transactions. By designating loss compensation generators transactions

however are allocated with more losses since the total system losses increase. It may mean sometime transactions may have ISO figure out the best way to minimize the losses instead of designating their own loss compensation generators.

TABLE 21
POWER FLOW RESULTS OF THREE CASES IN THE IEEE RTS 24 BUS SYSTEM

Bus	Case 1				Case 2				Case 3			
	P_g (MW)	Q_g (MVar)	V (p.u.)	θ (degree)	P_g (MW)	Q_g (MVar)	V (p.u.)	θ (degree)	P_g (MW)	Q_g (MVar)	V (p.u.)	θ (degree)
1	180	13.88	1.0350	-18.9601	182.02	13.24	1.0350	-3.9806	180	12.98	1.0350	-17.8177
2	180	-18.04	1.0350	-19.0682	180	-18.05	1.0350	-4.0923	180	-18.50	1.0350	-17.9142
3	0	0	0.9741	-17.4628	0	0	0.9744	-2.6616	0	0	0.9751	-16.7063
4	0	0	0.9914	-21.9824	0	0	0.9915	-7.0320	0	0	0.9918	-20.7998
5	0	0	1.0278	-22.2792	0	0	1.0278	-7.3097	0	0	1.0279	-21.0672
6	0	0	1.0843	-24.5457	0	0	1.0842	-9.5827	0	0	1.0845	-23.2940
7	240	43.94	1.0250	-18.5856	240	43.92	1.0250	-3.6408	240	43.63	1.0250	-17.3351
8	0	0	0.9968	-22.5155	0	0	0.9968	-7.5710	0	0	0.9970	-21.2671
9	0	0	0.9903	-20.1118	0	0	0.9904	-5.1836	0	0	0.9909	-18.9080
10	0	0	1.0486	-22.3024	0	0	1.0486	-7.3431	0	0	1.0488	-21.0212
11	0	0	1.0024	-16.5284	0	0	1.0025	-1.6067	0	0	1.0026	-15.2438
12	0	0	1.0175	-15.5641	0	0	1.0173	-0.5752	0	0	1.0176	-14.1648
13	100	84.71	1.0200	-14.9659	97.30	86.04	1.0200	0.0000	146.98	75.86	1.0200	-13.1957
14	0	-31.88	0.9800	-12.9466	0	-32.61	0.9800	1.9116	0	-35.93	0.9800	-12.0608
15	249.73	-21.08	1.0140	0.0000	200	-14.72	1.0140	14.5421	200	-19.17	1.0140	0.0000
16	200	35.19	1.0170	-1.3833	200	34.44	1.0170	13.3830	200	28.20	1.0170	-1.0693
17	0	0	1.0381	3.1104	0	0	1.0380	17.9948	0	0	1.0382	3.3125
18	400	141.54	1.0500	4.4102	406.72	141.80	1.0500	19.3038	400	140.79	1.0500	4.5618
19	0	0	1.0221	-4.2449	0	0	1.0222	10.6830	0	0	1.0223	-3.6281
20	0	0	1.0378	-4.5532	0	0	1.0379	10.5119	0	0	1.0379	-3.6787
21	400	110.39	1.0500	5.3409	406.72	113.06	1.0500	20.1977	400	111.09	1.0500	5.4458
22	300	-30.37	1.0500	10.9782	319	-30.52	1.0500	26.2588	300	-30.50	1.0500	11.1210
23	600	131.29	1.0500	-3.8659	619	130.28	1.0500	11.2729	600	126.47	1.0500	-2.8532
24	0	0	0.9817	-6.4574	0	0	0.9823	8.1788	0	0	0.9834	-6.1831
Total losses: 49.73MW				Total losses: 50.78MW				Total losses: 46.99MW				

TABLE 22
LOSS ALLOCATION RESULTS OF THREE CASES (MW)

Transaction m	Case 1	Case 2	Case 3
1	1.950	2.022	2.179
2	37.229	37.996	34.875
3	-2.826	-2.694	-2.442
4	23.373	13.448	12.372
Total losses	49.74MW	50.78	46.99MW

From Table 21 we see that in case 3 in order to minimize the total system losses the generator of bus 13 needs to output extra 46.99MW beyond the specified transaction amount. Next if put a limit of maximum generation 120MW to the generator of bus 13, the power flow result with optimal loss compensation for case 3 will be different and shown in Table 23. Obviously the generation limit of bus 13 becomes active during OPF calculation and the total system losses will increase to 47.35MW compared with the results of case 3 in Table 21. It indicates that since the active generation limit, the compensation from bus 13 has to be replaced by the more expensive compensation at buses 2 and 7, resulting in about 1% increase in the total system losses. The above test results also illustrate the proposed procedure has the capability to effectively handle the physical constraints of the power systems.

TABLE 23
POWER FLOW RESULTS OF IEEE RTS 24 BUS SYSTEM

Bus	Initial Power Flow Results					
	P_g (MW)	Q_g (MVar)	P_d (MW)	Q_d (MVar)	V (p.u.)	θ (degree)
1	180	13.14	100	22	1.0350	-16.8665
2	204.69	-22.96	100	20	1.0350	-16.8567
3	0	0	180	37	0.9752	-16.4072
4	0	0	75	15	0.9916	-20.2154
5	0	0	75	14	1.0276	-20.4518
6	0	0	100	28	1.0840	-22.8430
7	242.67	43.37	120	25	1.0250	-16.8892
8	0	0	150	35	0.9968	-20.9157
9	0	0	150	36	0.9908	-18.7263
10	0	0	200	40	1.0485	-20.7563
11	0	0	0	0	1.0028	-15.2858
12	0	0	0	0	1.0181	-14.2262
13	120.00	78.09	250	54	1.0200	-13.5486
14	0	-36.52	200	39	0.9800	-12.0963
15	200	-20.57	300	64	1.0140	0.0000
16	200	28.58	100	20	1.0170	-1.0978
17	0	0	0	0	1.0382	3.2941
18	400	140.86	350	68	1.0500	4.5480
19	0	0	200	37	1.0222	-3.7007
20	0	0	150	26	1.0379	-3.7892
21	400	111.03	0	0	1.0500	5.4362
22	300	-30.49	0	0	1.0500	11.1080
23	600	126.62	0	0	1.0500	-2.9839
24	0	0	0	0	0.9839	-6.0732

and the transaction loss allocation results are:

Transaction 1: 2.719MW

Transaction 2: 34.638MW

Transaction 3: -2.514MW

Transaction 4: 12.509MW

5.5 Conclusions

In this chapter, an optimal power flow formulation with loss allocation and loss compensation is presented. In this formulation, transactions will have options to choose either self-loss-compensation or purchasing ISO loss compensation service. Since the proposed method calculates the optimal power flow, losses and allocations iteratively, the sum of the allocated losses to each transaction will match the total system loss exactly. The loss compensation on a self-loss-compensation basis and that provided by ISO can easily coexist. Numerical examples are given to illustrate that this method yields optimal loss allocation results that are intuitively reasonable.

CHAPTER VI

CONCLUSIONS

6.1 Summary of Contributions

The implementation of a two-stage optimal meter placement is discussed in chapter II. A systematic procedure to choose candidate measurements for the loss of multi-measurements, outages of multi-branches and bus splitting contingencies has been proposed. An IP formulation then is presented to obtain the optimal selection among the list of all candidate measurements. It is a numerical method based on the measurement Jacobian and sparse triangular factorization, making its implementation easy in existing state estimators. Several test systems with different contingencies have been used to demonstrate the proposed method.

Chapter III investigates the power system loss allocation problem. Without making any simplifying assumptions such as DC power flow approximation, a new method which allocates losses to individual power transactions in a multiple transaction setting is derived from the system loss formula. It explicitly expresses the losses in terms of individual transactions, and this leads to a natural separation of system losses among all transactions in the system. A transaction-loss matrix, which illustrates coupling effects between any pair of transactions taking place in the system, is also introduced. Different test systems with various transactions have been used to verify the effectiveness of the proposed method. The new loss allocation method will calculate the losses to each individual transactions after the power flow analysis if no transactions designate their own loss compensation generators.

However, if transactions have their own generators for loss compensation, the conventional power flow needs to be extended to adjust generation with transaction allocated losses. Since the loss allocation itself depends on the solution, the two problems have to be combined and solved together. The combined formulation is built in Chapter IV and leads to a systematic solution procedure adjusting generation while simultaneously allocating losses to the generators designated by individual transactions.

Since some transactions may choose not to designate any specific loss compensation generators, then this will provide an opportunity for the ISO to implement a least-cost loss compensation solution. This option also may allow certain transactions to cover their allocated losses at a lower cost than otherwise would incur. An optimal power flow formulation in which the generation is dispatched in order to compensate for losses allocated to different transactions is proposed in Chapter V. Since the proposed method calculates the optimal power flow, losses and allocations iteratively, the sum of the allocated losses to each transaction will match the total system loss exactly. The loss compensation on a self-loss-compensation basis and that provided by ISO can easily coexist. Several test systems have been used to evaluate the performance of the proposed approaches.

6.2 Future Work

We can never claim our work is finished. There is still a lot of room for further developments. In the future, our research work can be improved in the following directions:

1. Single measurement and combinations of measurements are discussed in the proposed optimal meter placement method. In reality a single RTU usually carries multiple measurements, how to extend the proposed method to losses of multiple RTUs need to be investigated. Moreover, the practical power network is very big, so how to implement the method to the real system with reasonable performance speed needs to be studied.
2. The presented loss allocation method is mainly to allocate losses to transactions under the multi-transaction environment. Under the pool context whether or not a similar loss allocation scheme which fairly allocated the system losses to individual generators can be studied in the future.
3. The system losses usually are about real power losses. However, different market participants are providing reactive power support for the real power transactions. How to allocate the reactive power support is also a great concern.

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APPENDIX

LOSS ALLOCATION PROGRAM SOURCE CODES

Loss allocation subroutines in C language are listed here.

Header Files:

- **"node.h"**

```
class Node
{
public:
    int number,n$nn[MXNODE],o$nn[MXNODE],type[MXNODE];
    char name[MXNODE][STRLENGTH];
    double vnn[MXNODE],ann[MXNODE];
    double vnnmax[MXNODE],vnnmin[MXNODE];
    double injp[MXNODE],injq[MXNODE];
    double BusPL[MXNODE][MXNODE],BusQL[MXNODE][MXNODE];
    double BusPloss[MXNODE],BusQloss[MXNODE];

    Node();
    virtual ~Node();

};
```

- **"transaction.h"**

```
class transaction
{
public:
    int number,sellnumber[MXT],buynumber[MXT];
    double volume[MXT],sportion[MXT][MXNODE]
        ,bportion[MXT][MXNODE];
    double delta[MXT][MXNODE];
    double ploss[MXT][MXT];
    double pl[MXT];
    int sellbus[MXT][MXNODE],buybus[MXT][MXNODE];

    transaction();
    virtual ~transaction();

};
```

- **"jac.h"**

```
class Jac
{
public:
    double bmatrix[MXNODE][MXNODE],gmatrix[MXNODE][MXNODE];
    double xmatrix[MXNODE][MXNODE],rmatrix[MXNODE][MXNODE];

    Jac();
    virtual ~Jac();

};
```

Source Code Files:

▪ "losscal.cpp"

```

////////////////////////////////////
// Power Flow Loss Cal Program          //
// Based on Fast Decoupled Method       //
// Version 1.0                           //
//                                       //
// Author: Qifeng Ding                   //
// Date  : Aug. 20, 2002                 //
// Copyrights                             //
//                                       //
// This subroutine is to calculate the system losses //
// and allocate them to individual transactions //
// Input: Solved power flow results      //
//   1. node magnitude vn and angle va   //
//   2. system impedance matrix Z=R+jX   //
// Output:                                //
//   1. Transaction-loss matrix          //
//   2. Transaction allocated losses      //
//                                       //
// Flowchart:                             //
//   1. Read transaction data             //
//   2. Calculate losses incurred by Q    //
//   3. Calculate coefficient Delta       //
//   4. Calculate Transaction-Loss Matrix //
//   5. Allocate losses to individual transactions //
//   6. Converged? If No, go to 3        //
//   7. Output results.                  //
//                                       //
////////////////////////////////////

```

```

#include "math.h"
#include "stdio.h"
#include <iostream>
#include <string>

#include "Node.h"
#include "Jac.h"

#include "Matrix.h"
#include "transaction.h"

extern class Node node;
extern class Jac jac;

class transaction trans;

void LossCal()
{
    double Plqq = 0.0;
    double Ploss1 = 0.0;
    double Ploss2 = 0.0;

```

```

double deltaP =0.0;

    ReadTransaction(); // Read transaction data
    Plqq = PLQQCal(); // Calculate losses incurred by Q

    CalDelta();

do{
    Ploss1 = CalPlmatrix(Plqq);
    deltaP = Ploss1 - Ploss2;
    Ploss2 = Ploss1;
    ChgDelta();
    } while (fabs(deltaP)>0.000001);
}

void ReadTransaction()
{
    int i,j;

    char tmps[1000];

    char title[STRLENGTH];
    char FileName[STRLENGTH];

    int notrans;
    FILE *fp;

    cout<<"To enter the transaction data File:";
    gets(FileName);
    cout<<"\n"<<endl;

    fp = fopen(FileName,"rt");
    if ( fp == NULL ) { cout<<" No FILE exists! "; exit(0); }

    else{

        if(!fgets(tmps,1000,fp))
        {
            printf("cannot open the file");
            return;
        }; //if

        {
            fscanf(fp,"%s",title);
            fscanf(fp,"%d",&trans.number);
            for ( i=1; i<=trans.number; i++ )
            {
                fscanf(fp,"%s",title);
                fscanf(fp,"%d",&notrans);
                fscanf(fp,"%lf",&trans.volume[notrans]);
                trans.volume[notrans] = trans.volume[notrans]/100.0;
                fscanf(fp,"%d",&trans.sellnumber[notrans]);
            }
        }
    }
}

```

```

        fscanf(fp,"%d",&trans.buynumber[notrans]);

        for ( j=1; j<=trans.sellnumber[notrans]; j++ )
        {
            fscanf(fp,"%d",&trans.sellbus[notrans][j]);
            fscanf(fp,"%lf",&trans.sportion[notrans]
                [node.n$nn[trans.sellbus[notrans][j]]]);
        }
        for ( j=1; j<=trans.buynumber[notrans]; j++ )
        {
            fscanf(fp,"%d",&trans.buybus[notrans][j]);
            fscanf(fp,"%lf",&trans.bportion[notrans]
                [node.n$nn[trans.buybus[notrans][j]]]);
        }
    }
}

fclose(fp);
}
}

double PLQQCal()
{
    int i,j;

    double Plqq = 0.0;

    for ( i=1; i<=node.number; i++ )
    {
        for ( j=1; j<=node.number; j++ )
        {
            Plqq += (jac.rmatrix[i][j]/(node.vnn[i]*node.vnn[j]))
                *node.injq[i]*node.injq[j]
                *cos(node.ann[i]-node.ann[j]);
        }
    }

    FILE *fp;

    fp = fopen ("lfresults.dat","a+");
    fprintf(fp,"\nPLQQ = %8.4f",Plqq);
    fclose(fp);
    return Plqq;
}

void CalDelta()
{
    int i,j;

    for ( i=1;i<=trans.number;i++ )
    {
        for ( j=1; j<=node.number; j++ )
        {
            trans.delta[i][j]=trans.sportion[i][j]-trans.bportion[i][j];
        }
    }
}

```

}

```

double CalPlmatrix(double Plqq)
{
    int i, j,k,m;
    double PTotal=0.0;
    FILE *fp;

    fp = fopen ("lfresults.dat","a+");
    fprintf(fp, "\n ----- \n");

    for ( i=1;i<=trans.number;i++ )
    {
        PTotal += trans.volume[i];
    }

    for ( m=1; m<=trans.number;m++)
    {
        trans.ploss[m][m]= trans.volume[m]*Plqq/PTotal;
        for ( i=1; i<=node.number; i++ )
        {
            for ( j=1; j<=node.number; j++ )
            {
                trans.ploss[m][m]+=(jac.rmatrix[i][j]
                    /(node.vmn[i]*node.vmn[j]))
                    *(cos(node.ann[i]-node.ann[j])
                    *trans.delta[m][i]*trans.delta[m][j]
                    *trans.volume[m]*trans.volume[m]
                    -sin(node.ann[i]-node.ann[j])
                    *trans.delta[m][i]*node.injq[j]
                    *trans.volume[m]
                    +sin(node.ann[i]-node.ann[j])
                    *trans.delta[m][j]*node.injq[i]
                    *trans.volume[m]);
            }
        }
    }

    for ( m=1; m<=trans.number; m++ )
    {
        trans.pl[m]=0.0;
    }
    for ( m=1;m<=trans.number;m++ )
    {
        for (k=1;k<=trans.number;k++ )
        {
            if (k==m) continue;
            else
            {
                trans.ploss[m][k] = 0.0;
                for ( i=1; i<=node.number; i++ )
                {
                    for ( j=1; j<=node.number; j++ )
                    {

```

```

double temp;
temp =(jac.rmatrix[i][j]/(node.vnn[i]*node.vnn[j]))
      *(cos(node.ann[i]-node.ann[j])
        *trans.delta[m][i]*trans.delta[k][j]
        *trans.volume[m]*trans.volume[k]);
trans.ploss[m][k]+=temp;
if(temp !=0.0 && k>m){
  trans.pl[m]+=0.5*temp;
  trans.pl[k]+=0.5*temp;
}
temp =(jac.rmatrix[i][j]/(node.vnn[i]*node.vnn[j]))
      *(cos(node.ann[i]-node.ann[j])
        *trans.delta[k][i]*trans.delta[m][j]
        *trans.volume[m]*trans.volume[k]);
trans.ploss[m][k] += temp;
if (temp != 0.0 && k>m) {
  trans.pl[m]+=0.5*temp;
  trans.pl[k]+=0.5*temp;
}
}
}
}
}
for ( m=1; m<=trans.number; m++ )
{
  for ( k=1; k<=trans.number; k++ )
  {
    fprintf(fp,"%8.5f  ",trans.ploss[m][k]);
  }
  fprintf(fp,"\n");
}

PTotal = 0.0;

for ( m=1; m<=trans.number; m++ )
{
  trans.pl[m] += trans.ploss[m][m];
}

for ( m=1; m<=trans.number; m++ )
{
  PTotal += trans.pl[m];
  fprintf (fp, "Transaction %d : %8.5f \n", m, trans.pl[m]);
}

fprintf (fp,"Calcalated Total Loss: %8.5f\n", PTotal);

return PTotal;
}

```

```
void ChgDelta()
{
    int i, ni;

    ni = node.number;

    for ( i=1;i<=trans.number;i++ )
    {
        trans.delta[i][ni]=trans.sportion[i][ni]-trans.bportion[i][ni]
            +trans.pl[i]/trans.volume[i];
    }
}
```


VITA

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