ESSAYS ON ECONOMETRIC MODELING OF SUBJECTIVE PERCEPTIONS OF RISKS IN ENVIRONMENT AND HUMAN HEALTH

A Dissertation

by

TO NGOC NGUYEN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2008

Major Subject: Agricultural Economics

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Co-Chairs of Committee,	Richard T. Woodward
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ABSTRACT

Essays on Econometric Modeling of Subjective Perceptions of Risks
in Environment and Human Health. (May 2008)

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A large body of literature studies the issues of the option price and other ex-ante welfare measures under the microeconomic theory to valuate reductions of risks inherent in environment and human health. However, it does not offer a careful discussion of how to estimate risk reduction values using data, especially the modeling and estimating individual perceptions of risks present in the econometric models. The central theme of my dissertation is the approaches taken for the empirical estimation of probabilistic risks under alternative assumptions about individual perceptions of risk involved: the objective probability, the Savage subjective probability, and the subjective distributions of probability. Each of these three types of risk specifications is covered in one of the three essays.

The first essay addresses the problem of empirical estimation of individual willingness to pay for recreation access to public land under uncertainty. In this essay I developed an econometric model and applied it to the case of lottery-rationed hunting permits. The empirical result finds that the model correctly predicts the responses of 84% of the respondents in the Maine moose hunting survey.

The second essay addresses the estimation of a logit model for individual binary choices that involve heterogeneity in subjective probabilities. For this problem, I introduce the use of the hierarchical Bayes to estimate, among others, the parameters of distribution of subjective probabilities. The Monte Carlo study finds the estimator asymptotically unbiased and efficient.

The third essay addresses the problem of modeling perceived mortality risks from arsenic concentrations in drinking water. I estimated a formal model that allows for ambiguity about risk. The empirical findings revealed that perceived risk was positively associated with exposure levels and also related individuating factors, in particular smoking habits and one's current health status. Further evidence was found that the variance of the perceived risk distribution is non-zero.

In all, the three essays contribute methodological approaches and provide empirical examples for developing empirical models and estimating value of risk reductions in environment and human health, given the assumption about the individual's perceptions of risk, and accordingly, the reasonable specifications of risks involved in the models.

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I give special thanks to my wife for her patience and love. Finally, my dissertation is dedicated to my parents and my teachers who built the foundations for my primary education and made possible my further education.

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CHAPTER I

INTRODUCTION: RISK AND VALUATION

In the area of nonmarket valuation under uncertainty, the option price has been argued in (Graham, 1981) to be the most typically appropriate measure of ex-ante welfare effect of a change in risk involved in an event such as environmental quality or human health. However there are few empirical applications of the option price in the literature of environmental and health economics (Shaw, Riddel, and Jakus, 2005). As recognized in Smith (1992), a problem in an empirical study of the option price is the specification of probabilistic risk present in the econometric models. This problem continues to be a concern in the non-market valuation literature, for instance the research studies relating to lottery-rationed recreational activities such as hunting and fishing (Boxall, 1995; Akabua et al, 1999; Scrogin and Berrens, 2003).

Conventional approaches to the specification of probability include use of an objective probability or the scientific experts' estimates and elicited subjective probability. However, the objective probability approach is not always appropriate when individuals' perceptions of probabilistic risks are shown to be distinct from the objective probability or the experts' technical estimates (Slovic, 1987). If these are important, they must be elicited from subjects. But, in using these elicited probabilities, the approach might be problematic because of the risk communication issues. In contrast to those two approaches, the use of risk perceptions modeled and estimated from survey data has been recently suggested (Boxall, 1995; Riddel, 2007).

My dissertation focuses on approaches for empirical estimation of probabilistic risks under alternative assumptions about individual perceptions of risk involved. The cases that provide data typically involve individual discrete choices under risks inherent in environment and/or human health. I present my dissertation in the form of three essays, each of which is based on a specific assumption about individual perceptions of

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This dissertation follows the style of the *American Journal of Agricultural Economics*.

risks. The first essay, presented in the next chapter, follows the *objective probability* approach and develops an econometric model for estimating a theoretically-based option price from dichotomous response data. The model is applied to the case of Maine moose hunting permit to estimate the ex-ante benefit of a guarantee for participation in hunting. In this case the individual perceptions of the nonparticipation risk are assumed to be homogenous and aligned to the probability explicitly informed to them.

The second essay introduces the use of the hierarchical Bayes approach to model subjective probabilities inherent in a typical setting of individual choices under risk. An application using pseudo-data is presented to show the working of this approach in recovering predetermined parameters that generate the data. The first two essays are within the framework of the expected utility theory and without individual ambiguity about risks.

The third essay presents an empirical model of ambiguity about risk, a concept that is experimentally studied by Ellsberg (1961). In this essay I develop an econometric model for estimating individual subjective distributions of health risks using data from a survey involving arsenic contamination conducted in 2006 and 2007 (Shaw et al., 2006). In that survey, a risk ladder was used to communicate risk with the individuals and elicit the perceptions of risks associated with arsenic concentration in their drinking water. The model for perceived risks developed in this essay is based on an augmented probit function introduced by Lillard and Willis (2001).

Altogether, my dissertation is expected to make contributions in terms of methodology and empirical studies to the literature of nonmarket valuation of reductions of risks in environment and human health. The special focus is on modeling risk and uncertainty.

CHAPTER II

AN EMPIRICAL MODEL OF OPTION PRICE

WITH OBJECTIVE PROBABILITIES

Introduction

In this chapter I develop an empirical model within the expected utility framework to value a change in risks using discrete choice data. I also present an application of this model to the case of Maine moose hunting to estimate the benefit of eliminating the risk of not being drawn in an annual hunting lottery. The lottery scheme is one that randomly allocates hunting permits when supply is scarce relative to demand. In a hunting survey in 1992 that provides the data[1], hunters were asked whether or not they would be willing to pay a certain amount to guarantee themselves a hunting permit in the next year. If they chose not to pay any sum of money for this program, they could still participate in the annual lottery for the hunting permit with the usual number of granted permits.

This chapter provides estimates of the option price (OP) for Maine moose hunting permits using referendum price data from survey. The OP is Graham's (1981) measure of ex ante welfare based on the expected utility framework. The estimated OP indicates the individual's valuation of the program that effectively increases the probability of obtaining a permit to a value of one. The measurement of recreation values can be critical for the economically efficient management of hunting activities, especially when federal funding for wildlife management has diminished while at the same time many states face an expansion of urban residential areas and other human activities. The study of hunters' behaviors under the risks involved with permit lotteries

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[1] I thank Mr. Robert Paterson at Industrial Economics and Dr. Kevin Boyle at Virginia Tech for providing the data.

produces additional useful inputs for the management over the standard valuation models that assume there is no such risk.

As clarified later in this chapter, the specification of risk in this model follows the objective probability approach based on the assumption that the individuals' perceptions of risk are homogenous and aligned to the objective probability. In the case study of Main moose hunting option price, this assumption is reasonable since the risk of not being drawn in the lottery is a simple probability concept and especially this probability was clearly put in the survey question.

The organization of the remainder of this chapter is as follows. Next section provides a brief review of the literature on valuing hunting permits and on valuing environmental changes that involve uncertainty with the focus on the relevant econometric estimation methods. The following section constructs the theoretical and econometric models of the OP for the Maine moose hunting permit. Then one section summarizes the survey and questionnaires followed by the report and discussion of estimation results. The final section is devoted for concluding this chapter and transiting to more advanced topics of risks in the next two chapters.

Literature Review

In this section, I briefly review the travel cost method (TCM) literature on the valuation of hunting permits under a lottery-rationed system. Next, I discuss the referendum contingent valuation method and the generic model for estimating OP.

Lottery-Rationed Hunting Valuation with TCM

Within the non-market valuation literature, the estimation of the value of hunting and other recreational activities under a lottery-rationed system has been studied using various approaches. In such studies the hunting value is different than the usual values for resources or recreational activities because the supply of permits is constrained through a lottery. Loomis (1982), Boxall (1995) and Scrogin, Berrens, and Bohara (2000) propose variants on the travel cost framework to model the demand at aggregate or individual level. As an alternative to the standard travel cost method, a hedonic regression model is presented in Buschena, Anderson, and Leonard (2001) for obtaining the marginal value of a hunting permit.

Traditionally, the estimation of expected Marshallian consumer surplus for a hunting activity follows the standard travel cost method (TCM). The TCM utilizes the total number of trips actually taken as the dependent variable, with no risk or uncertainty prevalent in the model. It is implicitly assumed that the individual hunter knows everything with certainty, including how many trips he or she will take, environmental and stock conditions at the hunting areas, etc. However, this certainty approach is inappropriate in the context of a lottery-based hunting system because the lottery introduces an element of risk in participating in the activity. For example, Loomis (1982) showed that the standard TCM would result in biased estimation when a lottery system for hunting permits pertains, and suggested a modified version of the TCM that specifies per capita hunting permit applications in zones of origin as the dependent variable. This modified model follows the zonal TCM structure, which refers to the use of zonal level of data as against individual level. Scrogin, Berrens, and Bohara (2000) also essentially apply a zonal TCM, in which total zonal hunting permit applications for each site were treated as counts within a count-data model. They use their data to estimate expected consumer surplus associated with lottery-rationed hunting permits.

As an alternative to the zonal structure, Boxall (1995) presented the discrete choice TCM using data on individual choices of alternative lottery-rationed hunts for estimation of compensating surplus for a permit and for changes in site attributes. At the individual level, applications for hunting permits at specific hunting sites (destinations) were appropriately modeled as a discrete choice among a limited set of sites. Boxall’s model estimation follows the multinomial logit approach. Further, in realizing the effect of uncertainty in getting a permit, Boxall’s model specified permit applicants’ site choices based on their expected utilities. In addition, hunters were assumed homogeneous in their perception about the chance of being drawn. The chances were based on the probabilities of obtaining permits in the previous year.

More recently Scrogin and Berrens (2003) investigated a discrete choice model estimated in two stages. In the first stage of their model, individual expected access probabilities were estimated for the alternative lotteries by modeling the observed binary outcomes of being drawn or not drawn. Explanatory variables for the model of expected access probabilities include the probability of being drawn in the previous season and participant characteristics. In the second stage, the lottery choice model was developed by following the multinomial logit framework, conditioned on the first stage estimates of the access probabilities.

With the prevalence of using individual level of data, the discrete choice travel cost models seem to have emerged as the preferred approach to derive the value of lottery-rationed hunting and other similar recreational activities. However, as recognized in Boxall (1995), Scrogin and Berrens (2003), and Akabua et al. (1999) the key and challenging task in the analysis of these models is the specification of the hunters' individual perceived probability. This problem continues to be a concern in the literature.

In the next section I briefly review the option price concept and discuss the referendum-style contingent valuation method (CVM) to set the stage for the econometric model for option prices.

Option Price and Referendum Contingent Valuation Method

Option Price

The OP instead of other measures of ex ante welfare, such as the option value or expected surplus, has been shown to be the appropriate measure for valuing environmental changes under conditions involving risk (Graham, 1981). To clarify the meaning of the OP, first consider the example of a public project or policy that will improve on the quality (or level) of environmental service. Assume the quality of environmental service (X) takes a value of X_0 or X_1 contingent on state of nature ω (e.g.: weather), either good (ω=1) or bad (ω=0) respectively. The benefit of the project is generated from increasing the quality from X_1 to X_1' in the good state of nature and from

X_0 to X_0' in the bad state. In case $X_0' = X_1$ and $X_1' = X_1$ the project has the effect just as eliminating the risk of bad weather.

Assume further that the probability of the good state is π and that of the bad state is (1-π). These probabilities are also assumed to be well-known to individuals. We thus far have:

(2.1) $$X(\omega) = \begin{cases} X_1 & \text{if } \omega = 1 \text{ (good state)}, prob = \pi \\ X_0 & \text{if } \omega = 0 \text{ (bad state)} \quad , prob = 1 - \pi \end{cases}$$

Next, let $U(X_j, M)$ where $j = 0, 1$ be the ex-post indirect utility function that is common to the individuals and M be monetary income.

The expected surplus E(S) measure associated with this utility function is defined as the probability weighted sum of the compensating surpluses in the cases that the state of nature is good or bad. Let the surplus for an individual be S_1 in the good state and S_0 in the bad state. Then, the expected surplus is calculated as: $\pi S_1 + (1-\pi) S_0$. The values of S_1 and S_0 for an individual can be obtained by asking for the sure payment he or she is willing to pay for the project when the state of nature is observed. Formally, they are solutions of the equations:

(2.2) $$U(X_1, M) = U(X_1', M - S_1) \quad \text{for good state,}$$

and:

(2.3) $$U(X_0, M) = U(X_0', M - S_0) \quad \text{for bad state.}$$

In theory, the individual's OP is defined as the maximum amount that the individual is willing to pay for the project regardless of the state of nature tomorrow. For a formal definition of OP, let the expected utility of the individual at the status quo (without the project being undertaken) be V^*, then we have:

(2.4) $$V^* = \pi U(X_1, M) + (1-\pi) U(X_0, M)$$

For an individual who is assumed to be expected-utility maximizing, the amount of payment is chosen such that his or her new expected utility is not less than in the status quo. The values of OP as defined will solve the equation:

$$(2.5) \qquad \pi\, U(X_1', M - OP) + (1-\pi)\, U(X_0', M - OP) = \mathrm{V}^*$$

where V* is defined in (2.4).

If the OP is obtained via a survey question, the question must make it clear to the individual that the state of nature that will hold cannot be determined, and that the individual must pay his or her OP prior to, and in whatever the state of nature will occur. In general, the values of E(S) and OP are different. For a more detailed discussion about OP and expected surplus, see Graham (1981), Smith (1992), and Cameron (1997).

The Discrete-Choice Contingent Valuation Method

In order to empirically estimate OP as well as in other CVM practices, the use of referendum-style CVM has become very popular. In a typical referendum CVM application to hunting (no lottery involved), respondents might be asked if they are willing to pay to secure an improvement in the species population. Strictly speaking, a referendum format means that individuals are told that there will be a vote, and that the program will not be undertaken unless the majority (or some decision rule) votes for the referendum to support the program. However, the discrete choice style of asking the question (i.e. would you pay $*X* or not?) is often referred to as the referendum-style CVM even when there is no test of the vote.

Any errors or randomness in the conventional discrete choice or referendum CVM model (one without risk or uncertainty) are assumed to be attributable to the investigator’s failure to observe all the dimensions of the problem. These errors are typically introduced in a fashion that leads to estimation using the logit or probit models of discrete choice. Such errors are the conventional "investigator's" error and they are not synonymous with the randomness introduced as part of a known risk.

Hanemann (1984) introduced the use of the referendum or discrete choice CVM and the random utility model (RUM) approach to build logit model for estimation of the Hicksian compensating and equivalent surplus for a hunting permit. Recently, Cameron (2005) used a modified version of the referendum CVM approach, allowing for risk ambiguity to estimate individual OP's for global climate change mitigation programs.

The Empirical Model for OP

The objective of this section is to develop a specific econometric model for the OP and derive the equation that allows the calculation of the OP for increasing the probability of obtaining a hunting permit to a value of one, i.e. a guarantee for permit. This elimination of risk inherent in the lottery is what is presented to hunters in the survey questionnaire.

Again, let M be income and the states be specified with the j index (j = 1 if awarded a permit, and j = 0 if not awarded a permit). Suppose that the individual derives his or her utility from income and other non-income activities such as hunting and that the individual utility function is linear in the logarithm of income (Hanemann, 1984):

$$U(j, M) = \alpha_j + \beta \log(M) \tag{2.6}$$

where β denotes marginal utility of a one-percent increase in income M; α_1 is all non-income utility including the utility obtained from hunting and α_0 is all non-income utility without hunting taking place. Non-income utility differs whether one hunts or not because of the value of this constant term. The difference $(\alpha_1 - \alpha_0)$ reflects the utility purely derived from hunting, should be positive. Note that the functional form in (2.6) allows for income effects, as the marginal utility of income is not assumed to be constant.

The discrete-choice CVM question offers the individual the option of buying a permit with certainty at a bid price *B*. Hence, the individual chooses between the expected utility if they answer "Yes" and pay the bid price *B*, and that obtained if they answer "No", V_y and V_n respectively:

$$V_y = \alpha_1 + \beta \log(M - B - C) + \varepsilon_y \tag{2.7}$$

and:

$$V_n = \pi [\alpha_1 + \beta \log(M - C)] + (1 - \pi) [\alpha_0 + \beta \log(M)] + \varepsilon_n \tag{2.8}$$

where *C* is the hunter's travel cost for a trip to the hunting site, π is the probability of being drawn in the lottery and the ε terms reflect components of the utility that are unobserved by the researcher.

What is different here from the usual (no risk) model is the expected utility derivation above. When the hunter says yes, he or she is guaranteed a permit, so the probability of obtaining a permit is increased to one. In (2.7) the hunter receives a permit with certainty; implicitly $\pi = 1$. In (2.8) the hunter declines the purchase of the guarantee and thus must take his or her chances of obtaining a permit. The first term on the right-hand-side of (2.8) represents the expected utility associated with being drawn. The second term represents the expected utility associated with not being drawn in the lottery. In this case the hunter keeps all his or her income, paying neither the option price, nor the travel costs for a trip. As in Graham's application of the expected utility framework, the risk model is state dependent: utility functions differ in their constant term specification in the two states (hunting vs. not hunting).

When offered an option to purchase the hunting permit, a respondent will accept the offer if the expected utility difference $\Delta V = (V_y - V_n) > 0$ and refuse it if otherwise. By subtracting (2.8) from (2.7) and rearranging, we reach the binary choice model with allowance for the risk associated with the lottery:

$$\Delta V = V_y - V_n = \alpha - \beta Q + \varepsilon \tag{2.9}$$

where: $\varepsilon = (\varepsilon_y - \varepsilon_n)$,

$\alpha = (\alpha_1 - \alpha_0)(1 - \pi)$,

and: $Q = [\pi \log(M - C) + (1 - \pi) \log(M)] - \log(M - B - C)$.

The term Q is the expected reduction in the logarithm of net income associated with buying the offer instead of participating in the lottery. In other words, Q measures, in logarithm term, the expected increase in expenditure for hunting by buying the offer. In the sample under study travel costs are relatively small to incomes so that Q can be approximated as $Q \approx \log(M) - \log(M - B - C)$. On the benefit side, the constant term α in (2.9) reflects the gain in expected hunting utility if buying the offer. On the cost side, the product term $\beta * Q$ reflects the loss in expected utility caused by the bid price and the destination travel cost if buying the offer.

Assuming ε follows a logistic distribution, we can estimate the parameters α and β in (2.9) by using a logit model with the observed Yes / No responses to the option offer being the dependent variable. Given the estimated values of α and β, the individual OP can be obtained by setting ΔV in (2.9) equal to zero and solving for bid B. First, solve for Q from the equation $\Delta V = 0$:

$$(2.10) \qquad Q = \log\left(\frac{M^{1-\pi}(M-C)^{\pi}}{M-C-B}\right) = (\alpha / \beta) + (\varepsilon / \beta) .$$

Then take exponents of both sides and solve for bid B to have:

$$(2.11) \qquad OP = (M - C) - M^{1-\pi}(M - C)^{\pi} \exp[-(\alpha / \beta)] \exp[-\varepsilon / \beta]$$

Note that the OP is a function of ε and so it is a random variable. Let EOP denote the expected value of OP with respect to ε. Take expectation for both sides of (2.11) to derive EOP, noting that $E_{\varepsilon}\{\exp[-\varepsilon/\beta]\}$ is moment generating function at (-1/β) of logistic distribution and equal to $\mathrm{Beta}(1-\frac{1}{\beta}\,;1+\frac{1}{\beta})$ where Beta( ) is the beta function:

$$(2.12) \qquad EOP = \left(M - C\right) - M^{1-\pi}(M-C)^{\pi} * \exp\left(-\frac{\alpha}{\beta}\right) * \mathrm{Beta}\left(1-\frac{1}{\beta}\,;1+\frac{1}{\beta}\right)$$

As mentioned previously, C is small relative to M and so the equation (2.12) can be approximated by (2.13), in which EOP is presented as a portion of income given appropriate values of α and β:

$$(2.13) \qquad EOP \approx M * \left\{1-\exp\left(-\frac{\alpha}{\beta}\right) * \mathrm{Beta}\left(1-\frac{1}{\beta};1+\frac{1}{\beta}\right)\right\}$$

It is shown from the EOP equation (2.12) that the effects of probabilistic risk (1−π) on EOP are indirectly through income as well as through hunting utility ($\alpha_1 - \alpha_0$). This is an ex-ante measure of welfare.

For the remainder of this chapter, I apply the empirical model derived in this section to estimate the OP for the case of the Maine moose hunting lottery.

Maine Moose Hunting and the Survey

Moose hunting in Maine is regulated much like in other states in the US and in Canada. One must apply for a permit in each year to be able to hunt in one of nineteen Wildlife Management Districts, which cover over 21,000 square miles and include six zones: NW, NE, C, SW, SC, and SE. The applicants take a chance in a public lottery conducted in mid-June of each year. Successful applicants will have a hunting season that is 6 days long. The success rate of hunters (those that killed or “bagged” a moose) in

1992 was 91%. For virtually all moose hunters then, winning the lottery leads to a high chance of bagging a moose.

In 1992 the 900 permits were to be awarded to hunt moose and as a result 69,237 individuals applied to participate in the permit lottery. Thus, the probability of being selected in the lottery (π) was 1.3 percent. This probability is similar to that of preceding years. In that year a random sample of 900 residents who applied for but did not receive a permit were sent a survey asking about a proposal to allow a small group of hunters the right to buy a permit with certainty outside of the lottery.[2] This sample of individuals was drawn using the same procedures as was used to allocate the 900 hunting permits and the response rate for this survey was 78 percent.

Two main sections in this survey were of my interest. First, there were a number of questions regarding the travel costs the hunter may incur, such as travel distance and time as well. Second, there was an OP question. The respondent was informed that the probability of winning the lottery in the previous year was 1.3%. They were also informed that the Maine Legislature had increased the number of moose hunting permits issued to Maine residents from 900 to 1000. The extra 100 permits were to be sold to resident hunters under the program to cover the current costs of managing Maine's moose herd. Then he or she was offered an amount to guarantee a permit in the following year. They were asked to response "yes" or "no" to purchase this guarantee. If they did not want to buy the guarantee, they could still participate in the annual lottery.

The last section of the survey elicits the socio-economic characteristics (age, gender, education, and income) of the individual. Income is categorized into 16 interval ranges and the respondents' income varies from less than \$5,000 to more than \$100,000. Shown in table 2.1 is a profile for the resident respondents. The data shows that there is only a small portion of respondents, 46 out of the 704 respondents, who have ever hunted in Maine as a permit holder, and 70 other people hunted as a subpermittee, a guest of the permittee without a right to an additional moose. The data also shows that respondents have expended a great deal of effort to obtain a permit. On average,

[2] The 900 residents who received a permit were also sent a survey. These responses are not relevant to this study about option price.

respondents had applied 7.3 times in the annual lotteries during the 1980-1991 period. Within the sample, there are 265 respondents who applied every year during this period. These permits are clearly highly prized, at least based on the effort exerted to get them.

Table 2.1. A Profile of the Resident Respondents

Description		Frequency
Number of respondents		704
	males	565
Average income		$32,662
Average age		41 years
Hunting experience:		
Ever hunted moose in Maine prior to 1992		116 people
	as a permit holder	46
	as a subpermittee	70
Hunting as a subpermittee in Maine in 1992		7
Past attempts to get a permit:		
Average years of having applications during 1980-91		7.30 years
Have applied every year during 1980-91		265 people

Data and Model Estimation

Data Description

Table 2.2 shows the summary statistics of data used for estimation of the logit model (2.9). In this table, the response variable (ANSWER) and Q are the two key variables to estimate the logit model (2.9) while bid price (BID), travel cost (TRAVEL), and income (INC) data are included in the value of Q. The other variables used for the variant models include socio-demographic characteristics (AGE, MALE, and EDUC) and hunting related factors (EVER for hunting experience and APPS for past effort to obtain a permit).

Table 2.2. Summary Statistics of Data

Variable	Description	Mean	Std.	Min.	Max.
ANSWER	Response (1: Yes, 0: No)	0.36	0.48	0	1
BID	Referendum price	1341	1571	9	4320
TRAVEL	Travel cost	66.38	51.81	0	320
INC	Income	32662	21468	5000	105000
Q	Expected cost of hunting (in log term)	0.09	0.22	0.0002	2.0938
AGE	Age in years	40.86	15.40	10	86
MALE	Dummy (1: male, 0: female)	0.81	0.39	0	1
EDUC	Ordinal categories (degrees) from 1 to 8	3.41	1.42	1	8
EVER	Dummy for hunting experience (1: ever hunted before 1992 and 0: never)	0.16	0.37	0	1
APPS	Number of applications from 1980-91	7.30	3.81	1	11

Five levels of bids were used, ranging from \$9 to \$4320. Table 2.3 shows how the percentage willing to pay a particular bid tends to decline as the bid level increases.

Table 2.3. Bids and Percentages of YES

Bid	Total Cases	Yes Responses	Percentage of Yes of Total (%)
9	146	124	84.9
128	126	80	63.5
780	147	13	8.8
1433	144	10	6.9
4320	140	16	11.4
	703	*243*	*34.6*

The mean travel cost is about $66 per individual. The travel cost is calculated as product of round-trip distance and estimated per-mile cost $0.32, to the nearest hunting site to the hunter. The average travel cost is much lower than the average referendum price offered to the hunter. Note that in the binary choice model, in a case where travel cost is far below the offered referendum payment, then the payment amount will likely dominate the travel cost in determining the outcome (Yes/No).

Model Estimation

I estimate the model (2.9) with three variant specifications denoted by M-1, M-2, and M-3 and the results of each specification are reported in table 2.4. Model M-1 includes a constant term and the key variable, Q, as defined in (2.9). M-1 is considered as the basic model, while other models are variants. Model M-2 augments M-1 with the two variables of gender and education. Model M-3 augments M-2 with additional explanatory variables: age, hunting experience, and past effort to obtain a permit. The socio-demographic variables (age, gender, and education) are introduced into the variant models as interaction terms with Q, as a result of assuming the β coefficient (marginal utility of a one-percentage increase in income) to be a linear function of these variables. On the other hand, the hunting related variables are introduced into M-3 as interaction terms with (1-π) as a result of assuming these factors linearly affecting hunting utility ($\alpha_1 - \alpha_0$)[3].

Table 2.4. Model Estimation Results

Variable	M-1 Coef. (p-val)	M-2 Coef. (p-val)	M-3 Coef. (p-val)
Constant	2.04 (0.000)	2.08 (0.000)	2.09 (0.000)
(1-π) ∗ EVER			-0.21 (0.646)
(1-π) ∗ APPS			0.0044 (0.924)
(- Q)	154.58 (0.000)	162.97 (0.000)	189.95 (0.003)
(- Q) ∗ AGE			-0.68 (0.265)
(- Q) ∗ MALE		-54.79 (0.076)	-54.05 (0.095)
(- Q) ∗ EDUC		10.91 (0.078)	11.09 (0.098)
Log likelihood	-247	-221	-220
D.F.	1	3	6
McFadden's R^2	0.357	0.373	0.375

Note: The variables shown in this table are defined in table 2.2

The estimation results show that α and β, the coefficients of constant term and (-*Q*) respectively, are consistently significant in all three models, with p-values near zero. They take positive signs, as expected according to underlying theory and assumptions.

[3] Together, we assume the function form of utility augmented with individual characteristics *Z* to be: $U(j,\ M;\ Z)\ =\ \alpha_j^0 + \alpha_j^z Z\ +\ (\beta^0 + \beta^z Z)\ M$. Note that α_j^z , marginal hunting utility of *Z*, is assumed state-dependent, otherwise it will be canceled when taking the utility difference Δ*V*.

Gender and education interacted with *Q*, are significant at the 10% level. The negative sign on the interaction term with gender predicts that there is a greater chance for a male respondent to accept the offer than a female, assuming the same values for other characteristics. Higher education is expected to have negative effect on the chance to accept the offer for the permit guarantee. Age, hunting experience, and past effort for a permit are statistically insignificant, as shown in M-3.

Further, the likelihood ratio (LR) test statistic for M-3 and M-2 is computed to be 1.572 and we fail to reject the null hypothesis that all three additional variables in M-3 are zero simultaneously. The LR-stat for M-2 and M-1 is 51.479 leading to rejecting the null. In terms of goodness of fit to data, the R-square of M-2 is a bit better than that of M-1, but the R-square of M-3 is not improved considerably, as compared to M-2. In addition, the log-likelihood of M-3 is not much different from that of M-2. In all, I prefer to use the model M-2 for estimating OP in the next section.

Individual EOPs and Discussion

The individual expected Option Prices (EOP) over the sample are computed by substituting the estimated coefficients of the model M-2 into the EOP equation (2.12). The summary statistics of EOP is reported in table 2.5 and the histogram in figure 2.1.[4] We find the average EOPs over the sample is $384.65. This approach finds that more than 80% of respondents have an implied EOP greater than $77 and less than $740.

Table 2.5. Expected Option Price (EOP) over the Sample

Mean	Std.Dev.	Minimum	Maximum
384.65	274.62	0.07	1613.64

[4] The sample of EOP is truncated at zero to take out 17 out of 531 EOPs that are negative. The average EOP of the sample without truncation is $370.79.

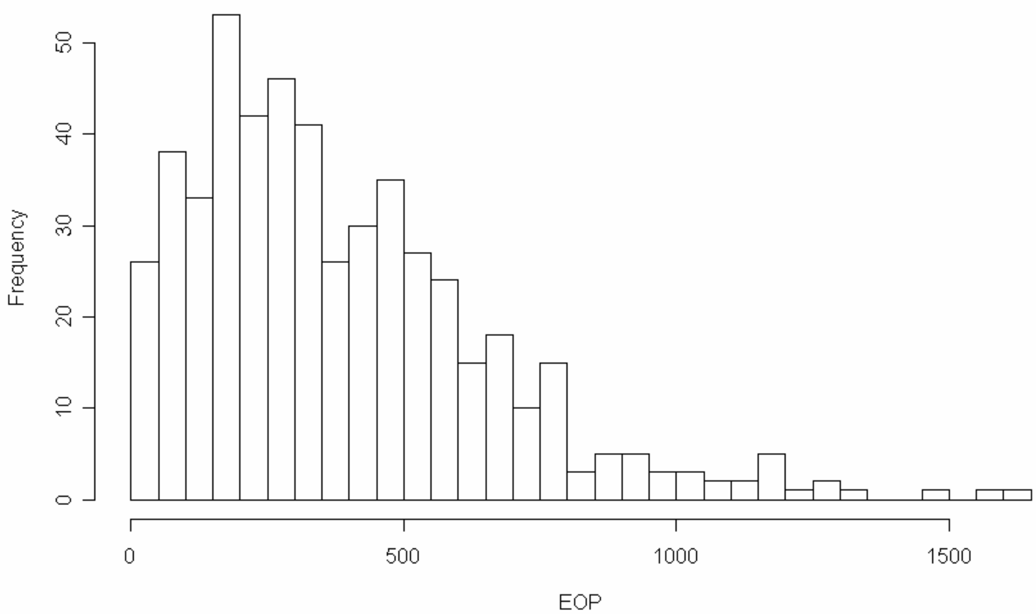


Figure 2.1. Histogram for Expected Option Price

In order to evaluate how well the estimated logit models predict the binary responses, I use the prediction rule based on the comparison between predicted probabilities from the logit models and the threshold which is share of actual Yes responses in the survey (table 2.3). The predicted response takes value of 1 (Yes to buy the option) if the predicted probability exceeds the threshold and of 0 otherwise. As shown in table 2.6, the three models perform quite well in prediction and the performance difference among them is not substantial. Model M-3 has the best performance in the prediction with a percentage of correct prediction to be 82.169 while model M-2 and M-1 obtain percentages of 81.926 and 81.926 respectively.

Table 2.6. Actual and Predicted Responses

Model M-1

		Predicted	
		"No"	"Yes"
Actual	"No"	313	64
	"Yes"	48	172
Correct prediction percentage (%): 81.240			

Model M-2

		Predicted	
		"No"	"Yes"
Actual	"No"	309	65
	"Yes"	42	176
Correct prediction percentage (%): 81.926			

Model M-3

		Predicted	
		"No"	"Yes"
Actual	"No"	285	58
	"Yes"	39	162
Correct prediction percentage %): 82.169			

Concluding Remarks

In this chapter I have presented an empirical model to value the elimination of the risk of not being drawn in a lottery that randomly allocates hunting permits. An estimate of the mean OP for the Maine moose hunting permit from the 1992 survey has been provided. The theoretical derivation from the expected utility framework shows that the individual OP reflects the increase in their expected net hunting benefit thanks to risk elimination. The estimated model specifies the significant determinants of the hunters' responses, including the informed probability of being successful in the annual lottery, referendum price and travel cost. The model correctly predicts the responses of 84% of the respondents in the Maine moose hunting survey.

In the case under study, the risk facing the individuals is a simple probability concept and especially it is clearly informed to the respondents. So in this situation it is likely quite reasonable to use the objective probability approach. However, the homogeneity assumed for the hunter's risk perceptions here seems not often to be the case. Slovic (1987) found that the individual conceptualization of risk is much richer than that of the expert and their perception of risk tend to be heterogeneous across individuals. In the next two chapters I present the modeling approaches addressing this issue, which is at the core of empirical researches in the area of valuation under uncertainty.

CHAPTER III

A HIERARCHICAL BAYES (HB) MODEL OF SUBJECTIVE PROBABILITIES

Introduction and Literature Review

The Subjective Probability (SP) Theory

In the von Neumann-Morgenstern theory, probabilities are assumed to be objective and typically considered being inherent in nature. However, the SP idea pioneered by Ramsey (1926) and De Finetti (1974) argued that by observing the bets people make on a horse race, one can presume that these reflect their personal beliefs on the outcome of the horse race. Thus, subjective probabilities, which are defined as personal beliefs in risks, can be inferred from observation of people's choices. The SP idea was axiomatized and developed into a full theory, the SP theory, by Leonard J. Savage in his Foundations of Statistics (1954). The SP theory assures that well-defined probabilistic beliefs are revealed by choice behavior.

In order to make the SP idea clear, consider an illustrative example adapted from Schmeidler (1989, p. 574). Supposes a bettor draws a ball from an urn that contains balls of either red or black color and the ratio between these two types is not known to him or her. Denote by R and B the event of drawing a red ball and a back ball respectively. Now consider a bet that offers \$100 if R happens and \$0 if otherwise. According to the SP theory, if the bettor is indifferent between betting on R for \$100 and betting on another risky event with an (objective) probability of 3/7 for \$100 then the subjective probability of the event R is equal to its risk equivalent, i.e. $prob(R) = 3/7$.

Empirical Estimation of SP

The empirical estimation of SP has been a concern in the literature of decision making and nonmarket valuation under risks. For example, Boxall (1995) suggests using estimates of hunters' perception of their chance from permit lotteries instead of using objective probabilities in modeling hunters' choice of participation in the lotteries as I have in the earlier chapter. This suggestion is in line with findings in Slovic (1987) that the individual conceptualization of risk is much richer than that of the expert and that perception of risk tends to be heterogeneous across individuals. In other words, there is no reason to expect that one individual's perception of risks is the same as someone else's.

Viscusi and Evans (1998) estimate individual's perceptions of risks associated with household chemical products by using survey data on how much the individual is willing to pay for the safer product. The estimation is based on the concept of prospective reference theory (Viscusi, 1989), which asserts that risk belief is in effect a weighted average between an individual's prior probability and some objective information about the risks given to the individual, which follows from a Bayesian learning framework.

In empirical research on lottery-rationed access to public resource and welfare, Scrogin and Berrens (2003) also use the logit/probit probabilities to be proxies of the individuals' perception of their chance in the current lottery. The logit/probit models take observed outcomes of being drawn or not being drawn in the lottery as the dependent variable. While seeking proxies for subjective perceptions, this approach in fact obtains 'objective' measure of expected chance of being drawn in the lotteries since the outcomes of being drawn or not drawn in a lottery are independently distributed from people's estimates of that chance.

Shaw, Walker, and Benson (2005) also use predicted probabilities from a logit model of observed choices or decisions to treat arsenic contaminated water, which act as indicators of the households' assessment of the risks of drinking the water. The assumption made in their approach is that there is a positive relationship between the predicted probability of treatment and individual assessment of arsenic risk from drinking water. This approach was initially taken in a similar study (but on toxic contamination of fish) by Jakus and Shaw (2003).

As distinct from the approaches presented above, this chapter explores for the first time the application of the hierarchical Bayes (HB) approach to model and estimate subjective probabilities using discrete choice data. While the HB approach can apply to various choice settings to estimate subjective probabilities, this chapter uses binary valuation responses (*yes*/*no* to an offer relating to buying a lottery) to illustrate the procedure of the HB modeling method. The basic framework of the expected utility model (EUM) is assumed throughout this chapter, as opposed to any alternatives to the EUM[5].

The remainder of this chapter consists of two major parts. The first part describes a typical binary decision setting involving risk and sets up the HB model. The second part shows an example using simulated data for the estimation of the HB model. The final section provides the conclusions and introduces to the subjective risks with ambiguity, the topic of next chapter.

The Hierarchical Bayes Model

Background on the HB Approach

Applications of the HB approach have become widespread in many areas such as biostatistics and marketing (Rossi, et al. 2005). In his book about discrete choice methods with simulation, Train (2002) devotes one chapter to the practice of the HB approach, applied specifically to mixed logit models. However, most typical applications of the HB approach assume linear forms in parameters, deterministic and random, even

---

[5] Many studies have shown that people behave in ways that systematically violate the expected utility maximization (see Starmer (2000) for an overview). Among evidences the two most well-known are Allais (1953) and Ellsberg (1961) paradoxes. The former leads to violating the independence axiom while the latter refutes the neutrality of uncertainty about subjective probability. As a result, theorists devise non-expected utility models that relax some assumptions underlying the expected utility framework. Among the most popular are the rank-dependent models such as Quiggin (1981) and Schmeidler (1989). The issue of uncertainty about probability is the topic to be explored in next chapter.

though in theory the approach can be applied to nonlinear models. As is made clear below, the special position of probabilities in relation with other parameters in a decision model makes the model under study here nonlinear.

The Binary Choice Setting and Probability Model

Consider a sample of N individuals who are offered the opportunity to buy a lottery ticket at a price of B. If an individual is drawn in the lottery s/he will have access to a service of which the benefit, denoted by α, is unknown to the researcher. For an example, the benefit obtained by winning the lottery is the utility from hunting as in the case of moose hunting permit explored in the previous chapter.

Following Hanemann (1984) the ex-post indirect utility of an individual i is supposedly derived from the unknown benefit α and the money income, denoted by M, that is:

$$(3.1) \qquad U(j, M_i) = \alpha_j + \beta * M_i \ ,$$

where $j \in \{1, 0\}$ represents the 2 states of being drawn or not being drawn in the lottery. Let π_i represent the individual i's subjective probability for his or her chance of being drawn in the lottery. The expected utilities associated with buying (accepting the offer) and not buying (declining the offer) a lottery ticket are determined respectively as:

$$(3.2) \qquad V_{iy} = \pi_i * U(1, M_i - B_i - C_i) + (1 - \pi_i) * U(0, M_i - B_i) + \varepsilon_{iy} ,$$

and:

$$(3.3) \qquad V_{in} = U(0, M_i) + \varepsilon_{in} ,$$

where the error terms above reflect the unobserved components of utility, M and B represents gross income and bid price respectively, and C represents total costs associated with consumption of the service, e.g. travel costs. Subtracting V_n from V_y

yields the increment in the individual expected utility when choosing to buy the prospect:

$$(3.4) \qquad \Delta V_i = V_{yi} - V_{ni} = \pi_i * \alpha - \beta * (B_i + \pi_i * C_i) + \varepsilon_i ,$$

where $\varepsilon_i = \varepsilon_{yi} - \varepsilon_{ni}$ and $\alpha = \alpha_1 - \alpha_0$.

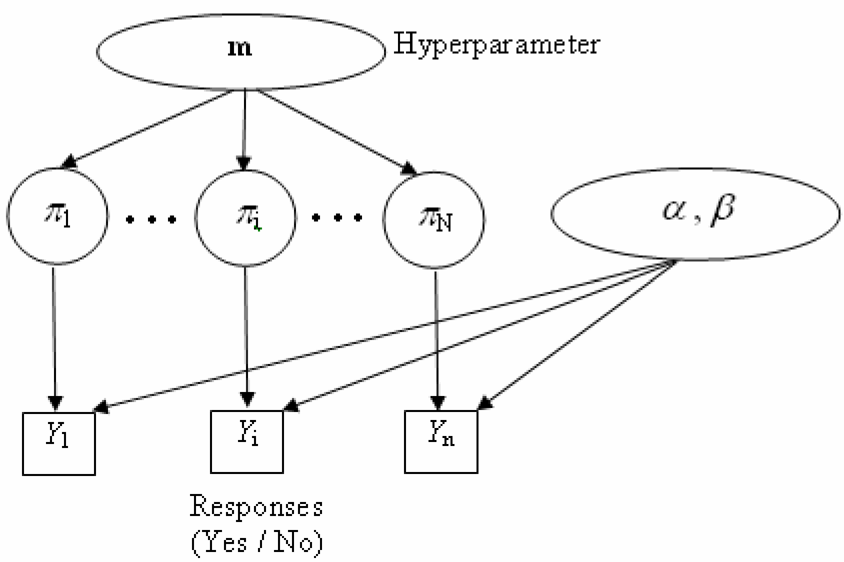
The individuals are assumed to be maximizing subjective expected utility (SEU), so they will accept the offer if ΔV_i is positive and decline the offer otherwise. Assume ε follows a standard logistic distribution. The probability that an individual *i* accepts the offer can be derived directly from (3.4) as:

$$(3.5) \qquad \varphi_i = \text{Prob}(Y_i = 1) = \text{Prob}(\Delta V_i > 0) = \Omega[\pi_i * \alpha - \beta * (B_i + \pi_i * C_i)] ,$$

where $Y_i \in \{1 \ (\textit{yes}), 0 \ (\textit{no})\}$ defines for the response for individual *i*, and Ω represents the cdf of the standard logistic distribution. It is clear that the probability model in (3.5) represents a logit model that involves risks, because of the presence of the probability term, π_i.

Now the question of concern is how to estimate α and β in (3.5) and the distribution of π_i over the sample given a dataset of individual characteristics, B_i, C_i, and observed responses Y_i. There are two approaches for the estimation of (3.5): the maximum simulated likelihood (MSL) and the HB approach. However, as shown in Train (2002), the HB approach has theoretical advantages from both a classical and Bayesian perspective and it works faster computationally. Further, while maximum likelihood estimation is susceptible to a flat or nearly flat likelihood function, often due to an insufficient number of observations, the Bayesian approach can still work in this case (Rossi, 2005, page 19). This research chooses the HB for the analysis and in the next section I will build the model based on this approach.

To begin, it is necessary to have a distribution functional form assumed for the subjective probabilities π_i for the logit model (3.5) to be estimable. At this point, I denote by $P(\pi_i | \mathbf{m})$ the generic distribution of π_i characterized by vector of parameters **m** and I defer specifying any form for the distribution of π_i until the next section. Using this assumption, the probability model (3.5) possesses a hierarchical structure of parameters in which the first level of parameters includes π_i, α, and β and the second level includes **m**, which is termed the *hyperparameter*, as shown in figure 3.1. In addition, it is clear that the logit model (3.5) is nonlinear in parameters due to interactions between π_i and other parameters.



Note: The variables in this figure except **m** are defined in equation (3.5) and **m** represents vector of parameters that characterizes the sample distribution of π_i.

Figure 3.1. Structure of the hierarchical model for the lottery choice model

The HB Model

This section will build on (3.5) to construct a HB model, which is comprised of three components: the likelihood function $P(\mathbf{Y} \mid \boldsymbol{\theta})$, the prior distribution $P(\boldsymbol{\theta})$, and the posterior distribution $P(\boldsymbol{\theta} \mid \mathbf{Y})$ where $\boldsymbol{\theta}$ is a vector that represents all parameters in the model as specified below. Especially, the prior distribution is structured with first- and second-level priors corresponding to the levels of parameters (Gelman *et al*., 2004).

For convenience in notation, define vectors:

$$\mathbf{Y} = (Y_1, \ldots, Y_n)\ ,$$
$$\boldsymbol{\pi} = (\pi_1, \ldots, \pi_N)\ ,$$
and: $$\boldsymbol{\theta} = (\boldsymbol{\pi}, \alpha, \beta, \mathbf{m})\ ,$$

where $\boldsymbol{\theta}$ is vector of all parameters in (3.5).

Likelihood Function: $P(\mathbf{Y} \mid \boldsymbol{\theta})$

In order to derive the likelihood function from (3.5), suppose the individual *i*'s response Y_i follows a Bernoulli distribution with parameter φ_i determined in (3.5). Then the choice probability of the individual *i* conditional on (π_i, α, β) will be:

(3.6) $$P(Y_i \mid \pi_i, \alpha, \beta) = \varphi_i^{\,Y_i} * (1 - \varphi_i)^{(1-Y_i)}\ .$$

Assuming choices by the individuals are made independently, multiplication of (3.6) over *i* yields the likelihood function of observed responses:

(H-1) $$P(\mathbf{Y} \mid \boldsymbol{\pi}, \alpha, \beta, \mathbf{m}) = P(\mathbf{Y} \mid \boldsymbol{\pi}, \alpha, \beta) = \prod_{i=1}^{n} P(Y_i \mid \pi_i, \alpha, \beta)\ ,$$

where the first equality holds because the data distribution, $P(\mathbf{Y} \mid \boldsymbol{\pi}, \alpha, \beta, \mathbf{m})$, depends only on ($\boldsymbol{\pi}, \alpha, \beta$) while the hyperparameter **m** affects Y through $\boldsymbol{\pi}$.

Prior: $P(\boldsymbol{\theta})$

In specifying a joint prior distribution for $\boldsymbol{\theta}$, first assume independence between the risk-related parameters **($\boldsymbol{\pi}$, m)** and the other parameters, (α, β). Further assume that α and β are independently distributed and that perceptions of the probability are independent across all of the individuals. Together these assumptions imply:

$$
\text{(H-2)} \qquad \begin{aligned} P(\boldsymbol{\pi}, \alpha, \beta, \mathbf{m}) &= P(\boldsymbol{\pi}, \mathbf{m}) * P(\alpha, \beta) \\ &= P(\boldsymbol{\pi} \mid \mathbf{m}) * P(\mathbf{m}) * P(\alpha) * P(\beta) \\ &= \prod_{i=1}^{n} P(\pi_{\mathrm{i}} \mid \mathbf{m}) * P(\mathbf{m}) * P(\alpha) * P(\beta) \end{aligned}
$$

where the first-level priors include $P(\alpha)$, $P(\beta)$, and $P(\pi_{\mathrm{i}} \mid \mathbf{m})$ and the second-level is $P(\mathbf{m})$.

Posterior: $P(\boldsymbol{\theta} \mid \mathbf{Y})$

Following Bayes' rule, we obtain the un-normalized joint posterior distribution of all parameters by multiplying the likelihood (H-1) by the prior (H-2):

$$
\text{(H-3)} \qquad P(\boldsymbol{\theta} \mid \mathbf{Y}) \propto \prod_{i=1}^{n} P(Y_{\mathrm{i}} \mid \pi_{\mathrm{i}}, \alpha, \beta) * P(\pi_{\mathrm{i}} \mid \mathbf{m}) * P(\alpha) * P(\beta) * P(\mathbf{m})
$$

In theory, samples for each of the parameters can be drawn if the specific forms of all functions $P(.)$ in the posterior (H-3) are defined. Since the posterior distribution shown in (H-3) is not a standard distribution, the sampling requires using the Monte Carlo Markov Chain (MCMC) algorithms. The rest of this chapter presents a numerical example using pseudo-data to illustrate the computational algorithm and to evaluate on the performance of the application of HB framework to recover the parameters in (H-3).

An Example Using the Pseudo Data

This section presents a numerical example of the HB approach discussed above using pseudo data. The process includes generating a number of datasets for the model in (3.5) and then implementing the sampling algorithm described below to simulate the posterior distributions of the parameters. The sample sizes are chosen to be increasing to verify the approaching of the posterior distributions around the population parameters, i.e. these distributions will have variances smaller and means closer to true values of the parameters when the sample larger. Details of the process are described in the three main steps below.

Step 1: Generating the pseudo-data:

- Assume values for the population parameters (α, β, m);
- Generate ε_i, π_i, B_i and C_i (details given below);
- Derive choices Y_i using the binary choice rule in (3.4).

Step 2: Using the sampling algorithm described in the previous section and the data generated in Step 1 to simulate the posterior distribution, mean and variance, for each of the parameters (α, β, m).

Step 3: Evaluating how well posterior samples of the parameters approach the true parameters.

The results from implementing these three steps are presented next.

The Pseudo Data

The parameters α and β are in this example presumed to be 1 and 0.001 respectively. Since α represents the unobserved utility derived from a good or service and β is the marginal utility of money, it implies a monetary benefit of $1,000. The data for ε_i is drawn from standard logistic distribution, as presumed in the formulation of the model 3.4. For the case of π_i it is assumed following a standard triangular distribution, denoted by *Trig*(min=0,max=1,mode = m), with the value of m is 0.2. The true values and prior distributions of the parameters α, β, and m are shown in table 3.1 below.

Table 3.1. True Values vs. Priors of the Parameters

	α	β	m
True value	1	0.001	0.2
Prior distribution	$I(0, 50)$	$I(0, 1)$	$I(0, 1)$

Note: $I(a, b)$ represents the uniform distribution on the interval from a to b; α and β respectively represent non-income utility and marginal utility of income defined in equation (3.1); m is mode of a standard triangular distribution.

For prices of bid, B_i, they are assigned by random draws from a predetermined pool of ten values from 100 to 2000: {100, 150, 200, 250, 300, 400, 500, 800, 1000, 2000}. For data of cost associated with consumption, C_i, they are randomly drawn from the set of positive values from 0 to 3000. The choice of bids follows the usual approach used in the contingent valuation method to have the portion of Yes responses close to 50%. Such a bid vector can improve the efficiency of dichotomous choice parameter estimates. For a detailed discussion on bid design issues see Haab and McConnell (2002, page 128).

In this study I run simulations for the 60 samples of three different sizes (100, 800, and 1500) so we have 20 random samples of each size. For each sample, data is generated and fed into the posterior sampling process to draw posteriors for the parameters of concern.

Table 3.1 also presents the prior distributions assumed for the parameters. In practice, the specification of priors depends heavily on the researcher's knowledge about these parameters. For simplicity and illustration purpose, here I assume that there is no information about the distributions of the parameters except for the lower bound, which is 0, and the upper bound, which could be a quite large positive number. One way to capture this knowledge is to use locally uniform distributions, denoted by $I(.)$, with the support bounded on a certain range of non-negative values. In this particular example, prior distribution for α is assumed to be $I(0, 50)$, a uniform distribution over the range of (0, 50), while true value of α is 1. For β the prior is $I(0, 1)$ while its true value is 0.001. For m the prior is $I(0, 1)$ and its true value is 0.2. The priors chosen for the parameters are quite far from their true values so that it is easy to observe the approaching of posterior distributions of the parameters towards the true values when samples getting larger sizes.

Computation: The Gibbs Sampling Algorithm

In order for the computation to proceed, we need to assign distribution functional forms for the priors present in (H-3). Above I have assumed functional forms for the prior distribution. We still need a functional form for the prior distribution of π_i over individuals in the sample, $P(\pi_i \mid \mathbf{m})$.

For a distribution to be eligible for being prior distribution of π_i, that distribution needs to be one with the support bounded between 0 and 1. Examples of adequate representation of the population of probabilistic risks include beta, triangular, truncated normal, and locally uniform distributions. In practice, a true distribution of π_i might be known to the researcher or not known. It is ideal for the researcher to know and hence use the true distribution of π_i over the sample. Otherwise, she needs to make an assumption about the distribution. Since beta distribution is very flexible in capturing variability over a fixed range, it is potentially a good candidate for the assumption about unknown distribution of π_i. However, the beta distribution is complicated mathematically (Covello and Merkhofer, 1993, p. 61). For the reason of ease in computation and isolation of other sources of error or bias from the estimator itself, I assume the prior distribution of π_i to be the standard triangular distribution, $Trig(.\mid m)$ where m represents for the mode of this distribution.

Given all the specific functional forms in (H-3), the un-normalized posterior in (H-3) can be rewritten as:

(H-4) $$P(\boldsymbol{\theta} \mid \mathbf{Y}) \propto \prod_{i=1}^{n} [P(Y_i \mid \pi_i, \alpha, \beta) * Trig(\pi_i \mid m)] * I(\alpha) * I(\beta) * I(m) \ ,$$

where $P(Y_i \mid \pi_i, \alpha, \beta)$ is determined by (3.6) and (3.5).

In order to draw from (H-4) the samples for each of the parameters, we need to implement the Monte Carlo Markov Chain (MCMC) algorithms. Most applications of MCMC have used the Gibbs sampler (Gilks, 1996). This is especially true for HB models. The Gibbs sampler is to generate an instance from the distribution of each random variable in turn, conditional on the current values of the other variables (see for details Gelfand and Smith, 1990). So the Gibbs algorithm requires knowing all full conditional posteriors, i.e. the distribution of one parameter given all other quantities in the model. It is ideal if all conditional posteriors are standard, so that drawing from the conditionals becomes straightforward. Otherwise another MCMC algorithm should be used to draw from those conditional posteriors.

For the case under study, the nonlinearity in the model makes all conditional posteriors non-standard distribution, as shown below, hence combining Gibbs sampler with another MCMC algorithm is chosen for computation. The rest of this section is dedicated for the implementation of this algorithm, including deriving all full conditional posteriors from (H-4) and describing specific steps of the sampling procedure.

Full Conditional Posteriors

Given the joint posterior distribution in (H-4), the full conditional posteriors for each of the parameters (π_i, α, β, m) are obtained below, from (G-1) to (G-4). Note that all of these conditionals are non-standard distributions and this makes the sampling procedure complicated, as discussed next.

(G-1) $$P(\pi_i \mid \alpha, \beta, m, Y_i) \propto P(Y_i \mid \pi_i, \alpha, \beta) * Trig(\pi_i \mid m) \ , \quad \forall i = 1,\ldots, N$$

$$\text{(G-2)} \qquad P(\alpha \,|\, \boldsymbol{\pi}, \beta, m, \mathbf{Y}) \quad \propto \quad \prod_{i=1}^{n} [P(Y_i \,|\, \pi_i, \alpha, \beta)] * I(\alpha) \,,$$

$$\text{(G-3)} \qquad P(\beta \,|\, \boldsymbol{\pi}, \alpha, \mathrm{m}, \mathbf{Y}) \quad \propto \quad \prod_{i=1}^{n} [P(Y_i \,|\, \pi_i, \alpha, \beta)] * I(\beta) \,,$$

$$\text{(G-4)} \qquad P(m \,|\, \boldsymbol{\pi}, \alpha, \beta, \mathbf{Y}) \quad \propto \quad \prod_{i=1}^{n} [Trig(\pi_i \,|\, m)] * I(m) \,.$$

The Sampling Procedure

The implementation of the Gibbs sampling method includes a large number of iterations during which an instance of each member parameter of $\boldsymbol{\theta}$ is drawn from the conditional posteriors (G-1)-(G-4) based on the current values of the other parameters. The main steps within a single iteration are summarized below:

Step 0. At start, initial values are assigned to all parameters, $\boldsymbol{\theta}^0 = (\boldsymbol{\pi}^0, \alpha^0, \beta^0, m^0)$.

Step 1. Draw a value of π_i from (G-1) conditional on initial values of α^0, β^0, and m^0. This is done for every i = 1, N. As (G-1) consists of product of the likelihood for the ith individual and the prior of π_i, the draw from (G-1) can be done using a simple and computationally efficient algorithm, Smith and Gelfand's (1992) rejection sampling method applied in Bayesian settings.

Step 2. Draw a value of α from (G-2) conditional on current values of β and m and new value of $\boldsymbol{\pi}$ obtained from step 1.

Step3. Draw a value of β from (G-3) conditional on current value of m and new values of $\boldsymbol{\pi}$ and α obtained from steps 1 and 2.

Step4. Draw a value of m from (G-4) conditional on current value of m and new values $\boldsymbol{\pi}$ and α obtained from steps 1, 2, and 3.

The sampling of α, β, and m from (G-2), (G-3), and (G-4) respectively can be carried out following the Metropolis-Hasting algorithm (see for details Chib and Greenberg, 1995), which is a very general approach and can be used practically for any density. In this example, the proposal distribution used for generating candidate for α is chosen as $N(\alpha_1,1^2)$, normal distribution with mean to be the current value of α itself, and standard deviation to be 1. Similarly, the proposal distribution for β takes the form $N(\beta_1, 0.01^2)$. For m, the proposal chosen is a uniform distribution over [0, 1]. The proposal distributions and the acceptance probabilities are shown in table 3.2 below.

Table 3.2. The Proposed Distributions and the Probabilities of Acceptance

Parameter	Proposal distribution	Probability of acceptance
α	$\alpha_2 \mid \alpha_1 \sim N(\alpha_1,1^2)$	min(1, *aprob*) where $aprob = \dfrac{\prod_{i=1}^{n} P(Y_i \mid \pi_i, \alpha_2, \beta) * I(\alpha_2 \mid 0,50)}{\prod_{i=1}^{n} P(Y_i \mid \pi_i, \alpha_1, \beta) * I(\alpha_1 \mid 0,50)}$
β	$\beta_2 \mid \beta_1 \sim N(\beta_1,0.01^2)$	min(1, *aprob*) where $aprob = \dfrac{\prod_{i=1}^{n} P(Y_i \mid \pi_i, \alpha, \beta_2) * I(\beta_2 \mid 0,1)}{\prod_{i=1}^{n} P(Y_i \mid \pi_i, \alpha, \beta_1) * I(\beta_1 \mid 0,1)}$
m	$m_2 \mid m_1 \sim I(0,1)$	min(1, *aprob*) where $aprob = \dfrac{\prod_{i=1}^{n} Trig(\pi_i \mid m_2)}{\prod_{i=1}^{n} Trig(\pi_i \mid m_1)}$

Note: $I(x \mid a, b)$ represents density of the uniform distribution on (a, b); α and β respectively represent non-income utility and marginal utility of income defined in equation (3.1); m is mode of a standard triangular distribution; subscripts 1 and 2 respectively represent current value and proposed next value for the Monte Carlo Markov Chain simulation process.

Simulation Results and Analysis

As discussed in the previous section, the simulation process includes drawing samples of α, β, and m from their joint posterior distribution (H-4) so that their posterior distributions can be obtained. The simulation results for 60 datasets of three sizes (100, 800, and 1500) are reported in two tables in Appendix A. Table A-1 presented in Appendix A shows means and the Table A-2 shows standard deviations of the posterior distributions of the parameters α, β, and m.

Each row in those two tables corresponds to an instance of the computational process discussed above, i.e. drawing random samples of size N (=100, 800, 1500), inputting these data into the MCMC simulation to get posterior distributions of the parameters, and finally, calculating means and standard deviations of these posterior distributions. It is clear that means and standard deviations obtained from such a process have a degree of randomness. So we should not evaluate an estimator, in terms of unbiasedness and efficiency, based on just one instance of the process. Take an example, for the experiment corresponding to line “Sample 10”, the posterior mean of α estimated for N = 100 (=0.986) is closer to the true α than the estimate for N = 1500 (=1.448). This result seems to contradict the common-sense notion that a larger sample should provide better estimate. However, as seen in Table A-1, on average an estimate with N=1500 leads to a prediction closer to the true value and the standard deviation of these estimates decreases as N increase. The evaluation of the hierarchical Bayesian estimator used in this study will be made under this perspective as follows.

Summary statistics for sampling distribution of the posterior mean and standard deviation are shown in table 3.3 and table 3.4. In Bayesian statistics, the posterior estimate is an information weighted compromise between prior and data estimates. In the numerical example presented in this section, the prior belief is presumed to be quite far from the true parameters and it is expected from the HB procedure that the posterior means approach the true parameter values. The result presented in table 3.3 shows the actual working of the HB across three different sample sizes. In the analysis below, I first compare the posteriors derived from datasets of the smallest size (100) with the prior and then evaluate the converging to the true parameters as sample size grows.

For the 20 samples of N = 100, the average posterior mean for α is 2.145, much closer to its true value (= 1) rather than the prior mean for α (= 25). Similarly for β, the average posterior mean is about $1.344{*}10^{-3}$ while the actual value is $1{*}10^{-3}$ and the prior mean is $5{*}10^{-3}$. So the estimator improves substantially the posterior belief on the true parameters of α and β even with small samples. However, the result for m is not as good as for α or β. The posterior mean for m is 0.378 a value close to the middle between its true value (= 0.2) and the prior mean (= 0.5).

It is shown from table 3.3 that the HB procedure provides an estimator that is less biased and more efficient as the sample size grows. For example, at small size of N = 100 the average posterior mean of α is 2.145 and while it's true value is 1. However, it can be observed that the sampling distribution of posterior mean converges in probability to the true parameters when the datasets get larger, i.e. average values of the posterior means gets closer to the true values and their variances gets smaller.

Consider the posterior means of α across three sizes of sample. Its average posterior mean is 2.145 for N = 100, reduced to 1.058 for N = 800 and again reduced to 1.051 for N = 1500. So there's tending to the true value of α, which is 1. In terms of variance, the standard deviations are 1.251, 0.421, and 0.290 respectively. The similar pattern also occurs with the posterior means of β and the hyperparameter m. For β, its average posterior means change from 1.344 to 1.007 and to 0.998 respectively in increasing sample sizes. These values approaches the true value (= 1) at smaller standard deviations of 0.421, 0.177, and 0.100 respectively. For the hyperparameter m, the converging to its true value occurs at a slower rate compared to α and β. With N = 800, the mean error for α estimate is 0.058 (= 5.8% of the true value) and the mean error for β estimate is $1.007{*}10^{-3}$ (= 0.7%) respectively, while the mean error for m estimate is 0.022 (= 11%) even with greater sample size of N = 1500.

In this particular example, since the prior is biased from the true parameters while the data is generated from the true parameters, the bias in posterior estimates is associated with the prior, not because of the estimator. When sample size grows the estimator is able to get rid of this type of bias. These results allow us to believe that the HB procedure is capable of recovering the true parameters given sufficient data.

Summary statistics of posterior standard deviations is presented in table 3.4. The table shows that on average the standard deviation of posterior distribution is consistently reduced as sample size grows. This property is true for all three parameters α, β, and m. That means the posterior distributions are likely to be more focused for larger dataset. This result is consistent with Bayesian theory.

Table 3.3. Summary Statistics of Posterior Means for α, β, and m

α

	Size		
	100	800	1500
Mean	2.145	1.058	1.051
Std	1.251	0.421	0.290

β (in 10^{-3})

	Size		
	100	800	1500
Mean	1.344	1.007	0.998
Std	0.421	0.177	0.100

m

	Size		
	100	800	1500
Mean	0.378	0.257	0.222
Std	0.084	0.080	0.068

Note: α, β, and m are defined in table 3.1

Table 3.4. Summary Statistics of Posterior Standard Deviations for α, β, and m

α

	Size		
	100	800	1500
Mean	1.263	0.448	0.359
Std	0.546	0.057	0.034

β (in 10^{-3})

	Size		
	100	800	1500
Mean	0.480	0.173	0.134
Std	0.152	0.019	0.012

m

	Size		
	100	800	1500
Mean	0.263	0.191	0.161
Std	0.025	0.047	0.038

Note: α, β, and m are defined in table 3.1

Further Research Topics

The results from the above example shows that the posterior distributions of the population parameters α, β, and m approach their true values as the sample size sufficiently large. Further, as shown in (G-1) it is possible to draw samples of individual subjective probabilities. An interesting question is, under which situations in practice do we have sufficient information on the individuals to infer their subjective probabilities individually. There are at least two ways that inferences on individual SP can be improved. First, more relevant individual characteristics are introduced into the model. Second, the lottery is repeated so that multiple observations of an individual's decisions can be realized. A brief discussion of the latter case will be discussed next together with another topic for further research relating to the generalized triangular distribution for individual SP.

HB with Panel Data

Consider a model with multiple individuals' responses to different levels of bid. Let t subscript denote the index of bid and t = 1, T. In fact, T is not necessarily the same for all individuals but for convenience subscript i is suppressed in T. For an example, the double-bounded bid approach used in contingent valuation (Hanemann, Loomis, and Kanninen, 1991) will give the individual two levels of bid to get her two responses to the offer. Another case that multiple responses/choices can be observed is the repetition of lotteries over periods. In those cases, the probability of observed responses will be a joint probability over a series of t choices:

$$(3.7) \qquad P(Y_{i1,\ldots,}\, Y_{iT} |\, \pi_i\,,\, \alpha,\, \beta) = \prod_{t=1}^{T} \varphi_i^{Yit} * (1 - \varphi_i)^{(1-Yit)}$$

It is the same procedure as in this chapter to construct the HB model for the multiple participation setting. With more observations on revelation of individual SP, π_i, through (3.7), the likelihood will be sharper and thereby the inference on π_i will be closer to the true values.

General Triangular Distribution

In the example presented in the second part of this chapter, the standard triangle distribution with lower-bound and upper bound are fixed at 0 and 1 respectively is used as the actual distribution and the prior distribution of SPs over the sample. As discussed above it is usual that the researcher may not know the actual distribution of SPs, and hence beta distribution, a very flexible distribution, is more appealing to be the prior rather than the standard triangle distribution. Another adequate distribution is the general triangle distribution of which the parameters include min, max, and mode. While less complicated mathematically compared to beta distribution, the general triangular distribution can be used as a proxy for the beta distribution except when the latter takes irregular shapes Johnson (1997).

The use of general triangular distribution in this framework requires an adaptation from the algorithm used in the example by adding two more population parameters, *min* and *max* of the triangular distribution, into the full conditional distributions shown in (G-1, G-4). Further the algorithm for computing (*min*, *max*, *mode*) of the triangular can developed from the version for the standard triangular by transforming the density of the latter distribution (Johnson, 1997).

Concluding Remarks

In this chapter, I have shown an application of the HB approach to model subjective probabilities in a typical discrete choice setting under uncertainty. The model found is a HB nonlinear logit model with individual SP treated as individual-level parameters. I have also applied the MH within Gibbs algorithm to simulate the posteriors for the sixty different datasets generated from assumed distributions. It is found that given a sufficiently large sample size of data the HB framework can perform well in recovering parameters of the distribution of SPs over the sample. However, if dataset is generated from a one-shot lottery such as in the Maine moose survey presented in the previous chapter then it is less likely that we can estimate individual SPs precisely. In such a case we can only simulate sample values such as mean for option price rather than individual option prices.

As for the specification of individual risk perceptions in the model, the subjective probabilities are heterogeneous among the individuals. But the individuals are still assumed certain about the probabilistic risk, i.e. no ambiguity about risk. However, in several cases people might be uncertain about the risk they face such as the case of mortality risk. In fact, many researchers have noted discrepancies between the assessments of risk based on statistical information, by so-called experts, and the subjective risks of the public, who may or may not be informed. When there are substantial differences, an individual’s behavior is better explained by his subjective risks than by objective risks, but a problem arises when subjective risks are imprecise or individuals are in fact ambiguous about the risks. This is exacerbated when experts do not know risks, leaving all decision makers facing uncertainty.

In the next chapter I develop and estimate a formal model of perceived risk distributions that allows for such risk ambiguity. The data used for estimation was collected from a survey given to a sample of people living in areas with potential health risk problems from drinking water contaminated with naturally occurring arsenic.

CHAPTER IV

AN ECONOMETRIC MODEL OF

SUBJECTIVE DISTRIBUTIONS OF MORTALITY RISKS

Introduction

The purpose of this chapter is to model and estimate subjective or perceived risks that individuals have for mortality related to consuming drinking water with high arsenic concentrations. Arsenic has been demonstrated to increase the usual average risks of dying from lung and bladder cancers that exists for the U.S. population, and can cause other health problems in both adults and children. However, the causal relationships are imprecise. The model developed here allows for individuals to be uncertain or ambiguous about these risks.

The data used in the modeling below are obtained from a survey in a study on arsenic contamination in selected regions in the United States (see Shaw, et al. 2006 for a complete description). As will be enumerated below, respondents answer questions posed in an information brochure during a follow-up telephone call that allows interaction between the trained interviewer and the respondent. Each information brochure mailed to households in the survey contained a risk ladder, a common means of visually conveying and communicating risks to subjects in surveys and experiments. The survey questionnaire format allowed the respondents to mark either a distinct, single rung on the risk ladder, or a range of risks on the ladder if they chose to do so.

Methodologically, the perceived distribution of risk is modeled as a probit function of the sum of two terms representing factors of median and variance of the distribution. This basic modeling approach was first developed by Lillard and Willis (2001), and is applied by Hill, Perry and Willis (2005), and extended and applied by Riddel (2007). Such a modeling approach allows estimating the factors that influence the median and variance of the subject’s perceived arsenic risk distribution.

The organization of the remainder of this chapter is as follows. The next section briefly reviews some relevant literature to provide background on the topic of risk ambiguity. The following section summaries the arsenic survey questionnaire and describes the approach taken to obtain or elicit perceived risks from the respondents. Then I develop an empirical model for stated or indicated risks based on an assumption about the individuals' behaviors in marking on the risk ladder. Finally, the estimation result using the arsenic survey data is reported and discussed.

The research in this chapter makes two contributions to the literature. First, it provides an empirical study in the area of environmental economics that may guide arsenic risk reduction policy and programs in the United States. This is one of the first studies that focus on U.S. arsenic exposure from a socio-economic point of view. Second, it provides another, though yet still rare, empirical application of new approaches in modeling health risks that allow for, and include ambiguity from survey responses.

Background and Brief Literature Review

Arsenic and Health Risk

Arsenic, a semi-metal and an element in the periodic table, has long been known to be an acute toxin. It has been shown that consumption of water contaminated with arsenic at high levels may cause skin damage or problems with circulatory systems, and may increase the risk of getting lung or bladder cancer. Since January 2006 the new EPA regulatory standard for arsenic in public water systems has been reduced to 10 parts-per-billion (ppb) from 50 ppb. This new regulation was promulgated to provide more protection for about 13 million Americans in areas with naturally occurring arsenic in their water supplies (USEPA, 2006). Sources of arsenic contamination in drinking water are from natural deposits in the earth or from agricultural and industrial practices. Ground water sources tend to be contaminated with arsenic at higher levels as compared to surface water sources such as lakes and rivers.

Even though scientists generally agree that human exposure to arsenic can cause health effects, the exact dose-mortality relationship between arsenic levels and health risks is still controversial. The estimated increases in the health risks that accompany a level of 50 ppb vary (Shaw et al., 2006). The dose-response relationship is especially uncertain at low levels, below 10 ppb, so some scientists still believe the newly mandated 10 ppb threshold is not low enough to ensure safety. However, the 1999 report to the United States Environmental Protection Agency (USEPA) by the National Research Council (NRC) showed that evidence of risk at lower dose is very weak. Burnett and Hahn (2001) raised concerns about the data and the methodology employed by the USEPA to estimate the risks of low-level exposure. They also cast doubt on inferences from animal and epidemiological studies, and the way that the USEPA quantified low-dose risks from arsenic. In addition, they believe that the dose-response relationship from arsenic should be a nonlinear relationship and that the actual risk from low levels of arsenic concentration is much less than the results from EPA's linear dose-response model.

The complex dose-response relationship is also due to many other issues and factors such as various consumption and exposure thresholds, confounding influences, and the latency period. Some biologists and toxicologists insist that there is a threshold below which arsenic concentration causes no effects. However, this conclusion was drawn from experiments on animals, and extrapolation to humans is questioned by a growing number of arsenic researchers (Wilson, 2001). In addition, there are many confounding factors affecting exposure and mortality rates of arsenic concentration. Smoking is an example of a strong confounding factor. The NRC indicates smoking as a factor that may substantially increase the risks associated with arsenic exposures. The effects of smoking on mortality risks from arsenic exposure need more research before having an exact estimate, but indications are a doubling of the risks for a smoker. Finally, the mapping from arsenic exposure to health risks is made even more problematic by the latency issue because there's no agreement in the estimates of the length of the latency period in contracting the cancers; an additional question relates to the length of the period to recovery when arsenic exposure ceases.

The controversy that exists among experts related to quantifying the dose - response relationship between arsenic levels and health risks no doubt contributes to the public's confusion about the risks of arsenic. A standard procedure in finding the public's sense of the risk is to inform them using the best available data and information. Thus, it is easy to see that in the case of arsenic this information might well contribute to, rather than reduce, confusion about the risks. This is clearly a case where people hold ambiguous and heterogeneous perceptions of risks, and this ambiguity is a compelling reason that any economic modeling related to arsenic risks must address this type of complexity, the perceived risks.

Ambiguity and Heterogeneity

The psychologist Paul Slovic (1987) found that the individual conceptualization of risk is much richer than that of the expert and their perception of risk tend to be heterogeneous across individuals. Slovic's work, as well as that done by other psychologists and risk scholars, has demonstrated why we need to look at perceived risks in stead of objective probabilities, especially if we wish to predict behavior in the face of risks. However, perceived risks are often fraught with ambiguity, making the risk, at one extreme, tantamount to total uncertainty. As in the case of arsenic where people's perceptions of risks are influenced by conflicting expert estimations, there is more likely to be ambiguity.

The notion of ambiguity was defined by the psychologist Daniel Ellsberg (1961) as the "quality depending on the amount, type, reliability, and 'unanimity' of information." More generally, Frisch and Baron (1988) define ambiguity as uncertainty about a probability, created by missing information that is relevant and could be known. Ellsberg (1961) was one of the first we know of to address such ambiguity, and the concept has been extended and analyzed both theoretically (e.g. Segal 1987), and in several laboratory experiments in economics and psychology. There are several possible psychological reasons for why people are ambiguous about some types of risks (e.g. Heath and Tversky, 1991 and Fox and Tversky, 1995). Most work in economics on ambiguity, and on risks in general, has focused on financial risks, but while mortality risks relate to many decisions people make, they have been studied much less.

Mortality risks might be calculated for average people in certain populations, from certain diseases, but these vary greatly by age. For example, the Center for Disease Control regularly updates age-specific estimates of mortality risk, for several diseases, providing one source of estimates that are supposedly "objective" measures of mortality risk. However, people asked to determine their own probability of dying from a particular cause, and at a particular point in their life, may face difficulty in doing so because of emotions involved in the task, or due to a lack of information at their disposal. There is good reason to think that ambiguity may be prevalent in subjective estimates of death or survival risks for any number of causes.

Recently Cameron (2005), and Riddel and Shaw (2006) introduce ambiguity about environmental and mortality risks in somewhat rare empirical models that do not rely on laboratory experiments; each of these relies on survey data, although Cameron's (2005) data is for classroom students. In each of these studies the introduction of an ambiguity term into an empirical model is somewhat ad hoc, at least from a theoretical point of view. From a statistical point of view, the models are sensible in that each relies on introduction of non-linear terms for probability, a break from the standard linear-in-probabilities expected utility model (EUM).

Somewhat more formally, Lillard and Willis (2001), hereafter L&W, introduce the probit function approach to model the relationship between the information that a respondent has about the probability of a given outcome and the shape of the density function of her probabilistic beliefs. This approach is then used in combination with the modal responses hypothesis to explain a link between an individual's degree of uncertainty and her propensity to give responses as focal point answers at zero, one-half, or one and as exact answers. The basic idea is that high degree of uncertainty results in clusters or "heaps" of responses around certain probabilities. This may indicate that people have an easier time understanding certain values for risks, thus clinging to these in their subjective estimates. The modal response hypothesis is tested empirically using data from a large set of subjective probability questions from the Health and Retirement Survey (HRS). Hill, Perry and Willis (2005), HPW hereafter, extend the L&W's work and develop an econometric model to estimate the determinants of individual-level uncertainty about personal longevity, and they also use HRS data. Riddel (2007) applies parts of HPW in her test of various specifications for subjective risk distributions, using a different data set on perceived mortality risks from nuclear waste storage and transportation.

Next, because the development of our risk model is tied to data from the arsenic survey, I discuss the sample and survey data used for my analysis. While the survey sample is small, an advantage of it over larger survey samples such as the HRS, is that the subjects are informed about risks using risk communication devices. Another advantage is that while the HRS forces an individual to make a point estimate about survival risks, the survey used here does not.

The Survey and Sample Statistics

The sample consists of households living in five communities exposed to arsenic levels in excess of the new EPA standard of 10 ppb. Table 4.1 provides information on the sources of drinking water and arsenic exposure, including the mean and range of contamination where appropriate, for the five communities. The public water supply systems of Albuquerque, Fernley and Oklahoma City were not in compliance with federal standard for arsenic. The Outagamie County/Appleton Wisconsin region was selected for the study because of the high arsenic levels in privately owned wells. Private drinking water wells are not regulated under the Safe Drinking Water Act, so any knowledge that well owners have about their well quality is obtained on their own or in conjunction with a state or local health agency.

Table 4.1. Profile of Arsenic Concentration in the Locations

Group	Area	Obs	Source	Mean (ppb)	Range (ppb)
1	Albuquerque, NM	54	Public	25	20 ÷ 30
2	Fernley, NV	108	Public	40	No range
3	Oklahoma City, OK	80	Public	17.5	14 ÷ 21
4	Outagamie County, WI	55	Private, tested	3.84	No range
4	Outagamie County, WI	43	Private, not tested	—	5 ÷ 100
5	Appleton, WI	5	Private, tested	6.9	No range
5	Appleton, WI	8	Private, not tested	—	5 ÷ 100
Total		353			

Survey materials were designed using three focus groups in Utah and Wisconsin communities with known arsenic problems. The survey followed a telephone-mail-telephone format. Respondent households were selected via random digit dialing and completed a first round survey asking about preferences for different public goods, concerns about environmental risks from atmospheric and water pollutants, how tap water was used in the household, and demographic information. At the conclusion of the first round survey a respondent was asked if he or she would be willing to participate in a second round survey focusing on contamination of drinking water by naturally occurring arsenic. Those willing to participate were sent a booklet about the risks of arsenic exposure. The booklet asked respondents to consider the risks of arsenic, which were elicited during the follow-up telephone survey. Survey activities were conducted during Fall 2006 and Spring 2007.

There were 748 respondents in the screener survey. Out of them 565 people (75.53%) agreed to participate in the follow-up survey and 161 people rejected to participate. Finally, there were 353 people (47.19% of the 748) who actually completed the follow-up survey. The response rates across the five communities are shown in table 4.2. While the selection bias issue is inevitable for any survey, the response rate of 47.19% is not so low that may cause serious problems for inferences. There is another reason that may support for this belief. As shown in table 4.2, the response rates across the five groups are not very different, ranging from around 35.06% to 54.82%. Such similar proportions are likely to maintain characteristics of every community such as arsenic concentration in the total sample. This is important because the arsenic concentration is a key variable in the model of perceived arsenic risks as discussed in the next section.

Table 4.2. Response Rates

Group	Number of Respondents		Response Rate (%)
	Screener Survey	Follow-up Survey	
1	154	54	35.06
2	197	108	54.82
3	187	80	42.78
4	182	98	53.85
5	28	13	46.43
	748	353	47.19

Explanation and Elicitation of Risk and Uncertainty

The information brochure mailed to each respondent following the initial telephone contact described the sources of arsenic contamination, the effects of long-term exposure, and the new 10ppb EPA standard for arsenic. Following the explanation of the standard, the booklet informed participants of the level of arsenic in the drinking water in their community. Respondents were then provided with detailed information regarding the specific risks of arsenic exposure, with emphasis on the confounding factors that affect risk, especially smoking.

The risk ladder included "rungs" indicating that baseline risks unrelated to arsenic exposure were about 60 per 100,000 people, the risk at 50 ppb was about 1000 per 100,000 people, and the risks of a smoker exposed at 50 ppb was about 2000 per 100,000 people. The ladder also included rungs associated with other common and not-so-common risks ranging from the risk of death by lightning strike to death from heart disease (see figure 4.1). The risk ladder expressed mortality risks as the number of deaths per 100,000 people in the population rather than as probabilities because several studies have shown that numerical probabilities are difficult for people to process, especially when risks are small**.** Risk ladders have been used for many years as a good device to enhance peoples' understanding of morbidity and mortality risks (see Loomis and DuVair, for example).

Respondents were then asked to think about the mortality risks from arsenic exposure for themselves as well as for other family members, and to express their best estimate of the mortality risk at current exposure levels. Each respondent was asked to put a single mark on the risk ladder if they are certain about the risks; if the respondent could not provide a point estimate of risk, they were asked to place two marks on the ladder, for lowest and highest values of risk. During the second-round telephone interview respondents were asked which rungs they had marked on the ladder (the right hand scale in figure 4.1).

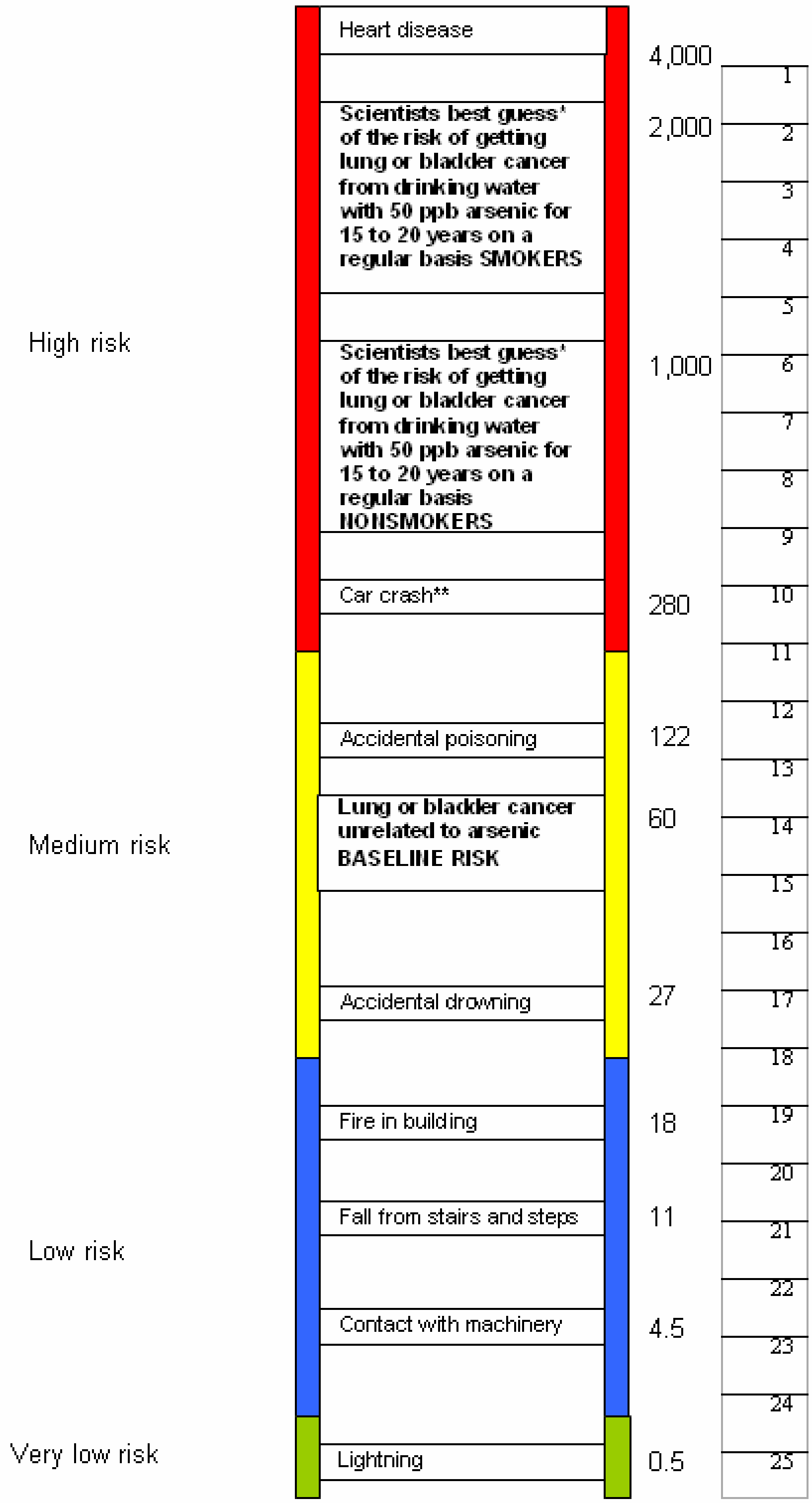


Figure 4.1. The risk ladder used in the arsenic survey

(Risk of deaths per 100,000 over 20 years)

The Sample and Basic Statistics

The full sample included 353 households, of which 69% obtain their water from a public water system and 31% from private wells. The risk elicitation process was quite successful (table 4.3). Of the 353 people who completed the second round survey, 198 (56%) provided a point estimate of risk, 99 (28%) provided an ambiguous risk estimate, and 56 (16%) could not or refused to provide an estimate of risk.

Table 4.3. Risk Responses

Variable	Frequency (N) or Mean indicated
Initial marks on risk ladder	One = 198
	Two = 99
	Could not decide = 40
	Refused = 16

Figure 4.2 shows histogram of the mortality risks from the dataset, combining information from those making single marks and the indicated midpoint of ranges that people state. The distribution seems fairly even along the range from the highest line on the risk ladder (number 1, which corresponds to 4,000 of 100,000 deaths) to the lowest line (line 25 on the risk ladder in figure 4.1 corresponds to only 0.5 deaths out of 100,000) even though there are two relatively high densities at the two extremes.

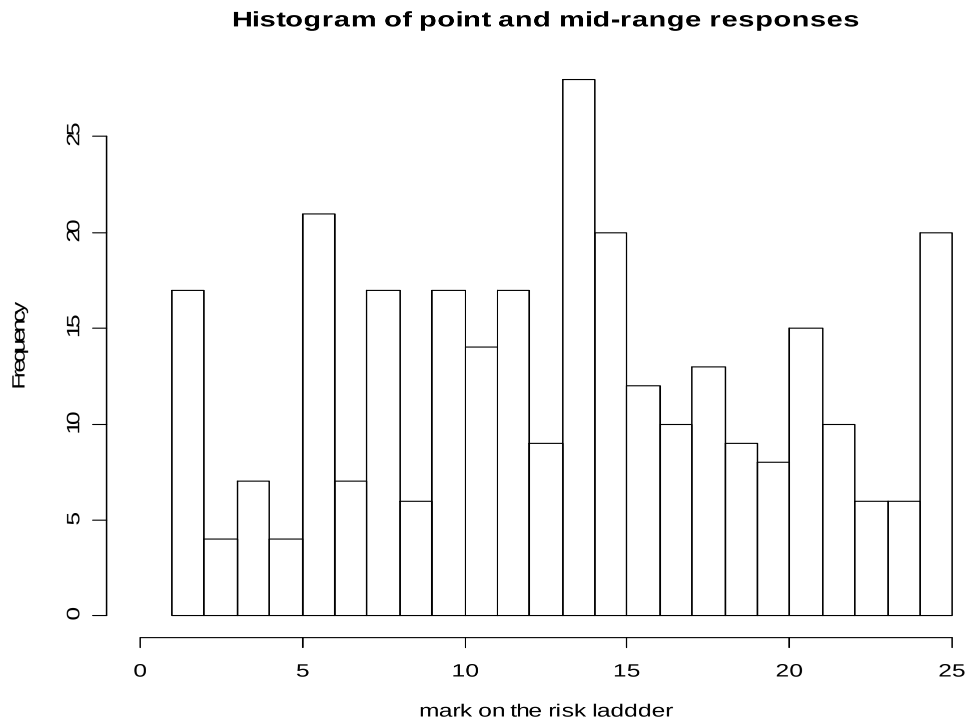


Figure 4.2. Distribution of risk responses

The survey questionnaire includes questions for the respondents about the households' current and historical health status, uses of tap water, choices of water treatment, perceptions of the health risks from arsenic in their drinking water, and their willingness to pay for safe water. In the survey, the respondents were also informed the level of arsenic concentration of their drinking water. In the analysis below PPB and PPB_RG are defined as the variables that respectively represent mid-range and range of the arsenic concentrations in their drinking water, of which the profile is shown in table 4.1 above.

Table 4.4 presents the basic statistics of the variables used in my analysis, the notation of these variables in the modeling below, and some additional variables related to those model variables, to give a feel for the characteristics of the sample.

The average age of the sample is around 51 years (ranging from 20 to 93 years): the survey protocol does not allow for respondents under the age of 18, as is often the case in such survey efforts. Education was thought to possibly influence risk responses, though the direction of influence is an empirical issue. While cognitive ability may increase with education and lead to a better understanding of information about risks, awareness of the complicated nature of the risk issues may also increase ambiguity. Education is categorized into 7 levels, from no high school as lowest educational level, to the highest, an advanced degree. The majority of the sample have an education from at least high school to an advanced degree with quite even distribution across those levels. There are only 26 people without a high school degree.

A person's health status may also influence his or her perceived risks. Again, the direction of influence is not obvious: people in poor health may feel that they have little additional mortality risk from any event or exposure, but people in good health may believe they can fight off or avoid extreme consequences of exposure. The respondents are asked to rate their own health status in one of the 5 levels from excellent (coded 1) to poor condition (coded 5). Most of the sample, 322 out of 353 people, rate their health good, very good, or excellent.

Most in the sample have some type of health insurance. Only 14 people in the sample do not have any type of health insurance. Health insurance purchases may reflect the individual's perceptions of many types of risk. Health insurance might be purchased because of the financial means to obtain health care, but some types of coverage also may reflect preferences for taking risks. That is, a risk-taker might choose to purchase no health insurance, while a person with extreme concerns about the possibility of contracting diseases may purchase a large amount of coverage. For a more detailed discussion about alternative reasons to why people buy health insurance see Nyman (2002).

Though relevant to individual perception of risks, the health insurance purchase will not be included as an independent variable in the model because of two reasons. First, health insurance is likely to be an effect from individual's perception of arsenic risk rather than a causal factor. That is, the higher risk perceived may cause higher probability that the individual to purchase health insurance, rather than the inverse. Since the model aims to explain the effects of the causal factors, the inclusion of this variable is not appropriate. Second, there are only 14 respondents out of 353 who did not have health insurance. So, it is not likely that there's sufficient variation in this variable to see its effect if any on the perceived risks.

When the respondents receive the information brochure in the mail, they are shown that smoking has an effect on increasing the mortality risks associated with arsenic in drinking water. Therefore smoking is expected to have a significant effect on people's thoughts about their own and others' mortality risks. In the empirical analysis, I use two dummy variables relevant to the smoking habit. One dummy variable takes a value of 1 if the individual has ever smoked in their life and is equal to 0 otherwise. The other dummy variable takes a value equal to 1 if the respondent has smoked in past but has now stopped, and equals 0 if they have smoked and still do at the time the survey questionnaire is administered to them. In the sample, there are many past smokers: 161 people of our 353 in the sample have smoked in the past. Among these past smokers, 116 people said they had stopped smoking and 45 people were still smoking.

Some occupations have been shown to increase mortality risks from arsenic exposure because the individual is simultaneously exposed to substances that may enhance lung and bladder cancer risks. The survey thus also asks the respondents if they have worked in the jobs that may increase basic lung and bladder cancer risks, such as manufacturing paint, manufacturing textiles, leather, or rubber products, beautician or hairstylist, packaging dues or other chemicals, printing, and aluminum worker. As shown in table 4.4 there are 93 respondents who have worked, or still work in those occupations. They are again told in the information brochure about the possibility that working in these types of jobs may increase risks.

Table 4.4. Basic Statistics of Key Variables

Variable	Notation	Frequency
Gender	MALE	Male = 200, female = 153
Age	AGE	Mean = 51 (of 350 reporting)
Education	EDU	No high school = 15
		Some high school = 11
		High school degree = 85
		Some college = 66
		2-year college = 61
		4-year college = 65
		Advanced degree = 49
Occupation	OCC	Risky jobs = 92 , None = 261
Self-rating health status	HEALTH	Excellent = 97
		Very good = 130
		Good = 95
		Fair = 27
		Poor = 4
Have private or public insurance?		Yes = 336, No = 14
Homeowner (=1 if yes, 0 = No)		1 = 320, 0 = 21 (12 missing)
Have smoked in the past?	SMOEVER	Yes = 161, No = 192
If smoked are you smoking now?	SMOQUIT	Yes = 45, No = 116
Is all water you drink from the tap?		Yes = 231, No = 122
Do you use a treatment device?	TREAT	Yes = 182, No = 171

Note: Total responses may not equal 353 because of item non-response or skipped questions.

Modeling the Perceived Risks

The Probit Function Approach

There are several density functional forms that one might use to model perceived risks such as the beta distribution. Any other probability distributions with the support on [0, 1] is a logical one to use in modeling the risk responses. Here I use a probit functional form introduced in L&W (2001) to model the subjective risk perceptions of the individuals. Below is a brief specification of this function.

Denote by p_i the individual *i*'s probabilistic belief about his or her own mortality risk of arsenic present in their drinking water. The probit approach treats p_i as a random variable that is defined as:

(4.1) $$p_i = \Phi(m_i + u_i) \text{, where } u_i \sim N(0, \sigma_i^2) .$$

In equation (4.1), $\Phi(.)$ is the probit function, m_i represents all of the information that forms the individual *i*'s belief about the probability, and u_i is a random variable with standard deviation, σ_i, which represents all information that determines the individual *i*'s uncertainty/ambiguity about the risk. As mentioned above, this ambiguity might relate to a lack of information that the individual understands that she has about the risks she faces. It is implied from (4.1) that $(m_i + u_i) \sim N(m_i, \sigma_i^2)$. For an individual who is certain about the risk, $\sigma_i = 0$ and his/her belief p_i is thus degenerate at a point probability equal to $\Phi(m_i)$.

Let $F(p_i)$ and $f(p_i)$ be the cumulative distribution function (c.d.f) and probability distribution function (p.d.f) of p_i , respectively. The equation for $F(p_i)$ is derived from (4.1) as:

(4.2) $$F(p_i) = \Phi\left[\frac{\Phi^{-1}(p_i) - m_i}{\sigma_i}\right] .$$

Taking the derivative of the right-hand-side (hereafter RHS) of (4.2) yields the equation for the p.d.f., or $f(p_i)$. It can be verified from (4.2) that $\Phi(m_i)$ is the median of $f(p_i)$. For convenience in notation, the subscript *i* will be suppressed in all that follows below.

Assumptions Regarding the Subjective Risk Responses

In order to formulate a likelihood function for the stated risk responses on the risk ladder, we need an assumption about the relationship between an individual's responses to the survey questions that elicit individual beliefs about the arsenic risks and the functional form of their subjective beliefs, $f(p)$. For the arsenic risks under this study, I assume that a probability range response $[p_1, p_2]$ reflects the individual's belief that probability mass of $f(p)$ outside this range approaches zero. This assumption is quite intuitive. It says that the range a respondent selected reflects a perceived distribution of risk that has the most heavy mass within that range. Further, I treat a probability point response as a special case of range response in that the range is bounded by the two midpoints from the rung chosen and the two rungs next to it. Hereafter, we refer to this assumption as the probability mass assumption.

Likelihood Function

The information contained in the two factors *m* and σ must be parameterized to derive an estimable model. To do so, let:

(4.3) $$m = X\alpha \ ,$$

and:

(4.4) $$\ln\sigma = Z\beta \ ,$$

where *X* and *Z* are two sets of variables that influences people's subjective thoughts about median and variance of risk respectively, where X and Z need not, but may have some identical elements; α and β reflect weights that the individual put on factors present in the equations. Note that the form in (4.4) as a semi-log ensures that σ will be positive.

Substituting (4.3) and (4.4) into equation (4.2) yields the distribution of the risk belief in terms of causal factors for which we have data:

$$(4.5) \qquad F(p) = \Phi\left[\frac{\Phi^{-1}(p) - (X\alpha)}{\exp(Z\beta)}\right]$$

The likelihood of range $p \in [p_1, p_2]$ is specified as:

$$(4.6) \qquad prob(p_1 \leq p \leq p_2) = F(p_2) - F(p_1)$$

The likelihood of point $p = \mathrm{p_o}$ can be calculated as a special case of (4.6) where two ladder rungs are degenerate at half way from the marked rung of the ladder and the two next closest rungs. Let p_{ou} denote the mid-range from $\mathrm{p_o}$ to the risk at the next upper rung and p_{ou} denote the mid-range from $\mathrm{p_o}$ to the risk at the next lower rung on the risk ladder. The likelihood of point $p = \mathrm{p_o}$ can be written as:

$$(4.7) \qquad prob(\ p = p_o) = F(p_{ou}) - F(p_{ol})$$

For consistence in treating data between point and range responses, a two-mark response $[p_1, p_2]$ in equation (4.6) is treated in computation as range $[\mathrm{p_{2u}}, \mathrm{p_{1l}}]$. Multiplication of (4.6) or (4.7) over the appropriate respondents, i.e. those who provide a point estimate versus a range estimate, yields the likelihood function for the entire sample under this first assumption. Maximizing this function will yield the maximum likelihood (ML) estimates of α and β. Note that the objective function to be maximized is a function of the parameters conditional on data. So, the procedure to compute standard errors and p-values is similar to the one for a normal likelihood function.

Estimation Results and Discussion

Table 4.5 reports the parameter estimates for the probit function, the standard errors, p-values, and significant levels of α and β from the estimation using the arsenic risk data. The coefficients of the median-shifting variables such as the arsenic parts per billion concentrations (PPB) represent the marginal effect of these variables on the Z-score, i.e. the inverse probit function Φ^{-1} of the median of perceived distribution of arsenic risks, $f(p_i)$, as mentioned above.

The choice of variables in the model is based on the expectation and intuition of relevance of these variables to location or variance of individual perceived risks. It is informed to the respondents that levels of arsenic concentration and smoking behaviors will critically affect mortality risks facing them. So the variables related to these factors are included in the model. In addition, since health status and health related factors such as risky occupation are likely to affect any health risks they are also included in the model. For the factors of variance, it is quite intuitive that range of arsenic concentration will have a positive impact on diffusion of perceived risks. Education, as discussed in above section, is factor of cognitive ability and it is expected to reduce uncertainty inherent in individual perceptions.

Among the variables that are expected to be relevant to the location of the median of the subjective distribution of arsenic risk, PPB, which represents the mean of arsenic concentrations in parts per billion, and the two variables related to smoking habit, SMOEVER (ever smoking) and SMOQUIT (quit smoking) are found to be of highly significant: the p-values of PPB, SMOEVER, and SMOQUIT are 0.0002, 0.0088, and 0.0026 respectively. The influence of the concentration is consistent with our expectation and intuition that people live in the region with the higher levels of arsenic in their drinking water tend to have higher perceived risks from arsenic. The positive coefficient of SMOEVER shows that a respondent who has smoked at some point in

Table 4.5. ML Estimation of Median and Variance Factors

Variables	Description	Coef.		Std.Err.	p-val
1. *Factors of median*					
ONE_X	Constant	-3.2345	***	0.1651	0.0000
PPB	Mean parts per billion	0.0111	***	0.0030	0.0002
SMOEVER	1: Ever smoking; 0: Never	0.3481	***	0.1321	0.0088
SMOQUIT	1: Quit; 0: Still smoking	-0.4417	***	0.1454	0.0026
AGE	Age	-0.0044	*	0.0027	0.1074
HEALTH	1: Excellent, …, 5: Poor	0.0713	*	0.0438	0.1043
TREAT	1: Treatment; 0: No treatment	-0.0417		0.0809	0.6065
OCC	1: Risky occupation; 0: None	-0.0486		0.0971	0.6172
MALE	1: Male; 0: Female	0.0067		0.0839	0.9366
2. *Factors of variance*					
ONE_Z	Constant	-0.4708	***	0.1639	0.0043
SMOEVER	1: Ever smoking; 0: Never	0.1535	*	0.1038	0.1400
PPB_RG	Range of parts per billion	0.0592		0.0559	0.2907
PPB_RG2	Squared PPB_RG	-0.0077		0.0062	0.2115
EDU	Education (1 ÷ 7)	-0.0191		0.0306	0.5343
Log-likelihood	-637.371				

Note: Asterisk (*), double asterisk (**), and triple asterisk (***) denote variables significant at 15%, 5%, and 1% respectively.

their life has a higher median of the risk perception than those who have never smoked. Further, the negative coefficient of SMOQUIT reveals that a smoker who once smoked, but who quit smoking perceives risk at lower level than a smoker who continues smoking at the time of the survey. These results are aligned with the information in the mail brochure sent to the respondents, which clearly explained the serious effects of smoking in increasing risks of arsenic. This is one of the contributing factors to risks about which the scientists who study arsenic are most certain. Evidently, most in the sample processed this information appropriately.

In addition to the above influences on the median, HEALTH is also a factor that is significant at the 0.10 level. The positive sign of HEALTH coefficient shows that a person in a poorer health condition perceives higher risk than those who are in better health status. This evidence seems to be reasonable, and my interpretation of this is that a person with poor health is more likely to think that a poor health state increases risks of mortality, probably because of the perception that the person is more vulnerable to any health risks than a healthy person is.

The variable AGE is also found to be statistically significant at the 0.10 level. The negative sign of AGE shows that an older person tends to perceive less risk from arsenic than a younger person. This is likely to be relevant to the latency period for the arsenic-induced cancers: an older person might think that time left to live is relatively short compared to the latency period of 10 or 20 years.

The remaining median shifting variables are statistically insignificant. The p-value of the gender variable is around 0.93 and so there’s no significant difference between male and female in their perceived levels of risk from arsenic concentration. Those who treated their drinking water (TREAT = 1) perceive lower risks than those who do not treat (TREAT = 0) but the difference is not statistically significant. In addition, the risky occupations tend not to impact on the individual’s perceptions of risks.

Among the factors potentially affecting the distribution variance (σ^2), the smoking is the most significant factor, as compared to education and range of arsenic contamination. The signs of PPB_RG and PPB_RG2, positive and negative respectively, provide evidence that the individual with drinking water with a wider range of PPB is more likely to perceive a mortality risk distribution that is more diffuse, and that this effect is decreasing in the level of PPB. In addition, individuals who have sometime smoked are more uncertain, i.e. perceiving a more diffuse distribution of risk, while education tends to reduce the people’s uncertainty about their risk facing them. The smoking effect on the variance is significant at the 0.15 level. The education result would be consistent with the thought that education may reduce ambiguity; however, the coefficient of education is also not significantly different from zero.

Table 4.6 reports summary statistics for the mean, median and standard deviations of individual perceptions of risk over the sample. The average value of the median risk over the sample is found to be 155.3 (deaths per 100,000). This level of risk is between line 11 and line 12 on the risk ladder in figure 4.1. Again, note that the ladder has 25 lines coded from 1 to 25 in decreasing order. The smallest median value is about 21.4 (per 100,000 deaths), which is between line 18 and line 19 and the maximum for the sample is about 1015 (per 100,000), which is a bit above line 6.

Table 4.6. Summary Statistics of Estimated Risk Perceptions

(unit: per 100,000)

	Median	Mean	Standard deviation
Min.	21.4	129.8	223.9
Average	155.3	560.1	1234.0
Max.	1015.0	2797.0	4966.0

Table 4.7 below shows the estimation result for the restricted model that includes only factors of median (*X*'s) but not factors of variance (*Z*'s). Compared to the full model, the restricted model provides very similar coefficient estimates and p-values for almost all variables, except for AGE and HEALTH of which the p-values are reduced from around 0.10 to 0.03 and 0.02 respectively. However, the advantages of the full model over the restricted model are in greater log-likelihood and findings on the variance impacts of the factors, especially for smoking, a prominent subject of the literature of mortality risk perceptions. Two questions were central to the litigation and remain central to the smoking policy debate. Those questions - do smokers understand the risks of smoking, and does smoking impose net financial costs on the states? - are at

the heart of W. Kip Viscusi's research for an informed smoking policy (Viscusi, 2002). As discussed above, the empirical finding from the full model shows that while having smoked at all in the past raises perceived risk relative to a current non-smoker, someone who has quit smoking apparently believes that his or her perceived risk is not only below that of current smokers, but also below that of people who have never smoked. In addition, those who ever smoke are expected to have more diverse subjective distribution of mortality risk from arsenic than those who don’t. This finding contributes an empirical evidence for the research on the behavior of smokers with respect to mortality risk perceptions.

Table 4.7. Estimation of Model without Variance Factors (*Z*)

Variables	Description	Coef.		Std.Err.	p-val
1. *Factors of median*					
ONE_X	Constant	-3.2465	***	0.1664	0
PPB	Mean parts per billion	0.0114	***	0.0029	0.0001
SMOEVER	1: Ever smoking; 0: Never	0.3191	***	0.1245	0.0108
SMOQUIT	1: Quit; 0: Still smoking	-0.3847	***	0.1326	0.0039
AGE	Age	-0.0057	**	0.0027	0.0349
HEALTH	1: Excellent, …, 5: Poor	0.1005	**	0.0428	0.0193
TREAT	1: Treatment; 0: No treatment	-0.0439		0.0803	0.5847
OCC	1: Risky occupation; 0: None	-0.051		0.0982	0.6042
MALE	1: Male; 0: Female	0.0075		0.0831	0.9280
2. *Factors of variance*					
ONE_Z	Constant	-0.4946	***	0.0459	0

Note: Asterisk (*), double asterisk (**), and triple asterisk (***) denote variables significant at 15%, 5%, and 1% respectively.

Further Research Topic

In this chapter I have used the probability mass assumption consistent with the fact that a range marked on the risk ladder by our subjects implies the mass of the latent subjective distribution is heavily concentrated on that range. Alternatively, the modal response hypothesis in L&W (2001) assumes that the respondents report the *most likely* value of p, i.e. the mode of $f(p)$. This implies that for the individual who chose a point response at p, the mode of $f(p)$ is assumed to be p. In their original framework, the survey used by L&W (2001) asked respondent to choose a number between 0 and 100 to represent their probabilistic beliefs. Note that there's no elicitation of uncertainty in their survey. In contrast, the arsenic survey uses the risk ladder and allows single or double marks that the respondent makes on this ladder. So it is natural to extend the modal response hypothesis to cover probability range response $[p_1, p_2]$ in that it reflects the individual's belief that the mode of $f(p)$ lies within this range.

Compared to the probability mass assumption, the modal response hypothesis does not restrict the probability mass outside the range selected to be small. The latter assumption just suggests that the most likely (*mode*) value of the mortality rate will be within the stated range for the individual, while the chance for values outside that range might be considerable. It is also worth nothing that these two assumptions are not exclusive each other.

Concluding Remarks

Many, if not all environmental stressors on human health involve some uncertainty, or unknown risks. Most analysis of the effect of stressors on health either ignore risk and uncertainty altogether, or assume that risks are known. Few such studies use the individual's perceived mortality or health risks from a stressor. In this chapter I have presented an approach that uses the probit function to model individual's perceived distribution of mortality risks from arsenic in drinking water. Factors that influence the median and variance of the perceived risk distribution are included as arguments of the probit function. Further, the parametric estimation model here has been derived from the assumption relating to the markings on a risk ladder that people make, and the corresponding probability mass distribution.

This modeling approach is applied to the case of the arsenic survey to estimate perceptions of the respondents about risks associated with arsenic in their drinking water, and ambiguity or uncertainty about these risks is allowed in the approach. The findings show that the level of arsenic concentrations that households face, which is measured in parts per billions, and the respondents' smoking habits are of the strongest influences that affect the medians of the individuals' perceived distributions of risks. Health status and age are also significant factors influencing the median. I also explore influences on the variance of the perceived risk distribution, finding that again, the range of arsenic concentration is found having a positive and significant relationship with the variance. This variance represents an individual's ambiguity about mortality risk faced by the household.

CHAPTER V

SUMMARY AND CONCLUSION

In the three essays of my dissertation, I have presented the modeling approaches for empirical estimation of probabilistic risks, which is critical to valuation of risky goods such as risks inherent in environment and human health. The first essay briefly reviews the option price concept as a measure of ex-ante benefit and develops an empirical model to estimate option price from individual discrete choice data. The first essay also provides a case study in which I use an empirical measure of the option price to estimate the ex-ante benefit of a guarantee of participation for a Maine moose hunter. In this essay, I assume the hunters were homogeneous in their thought about the chance they had to face in the annual lottery allocating the hunting permits. This chance was presented to them as the actual statistical odds of winning the lottery.

In the second essay, I relax the homogeneity assumption and consider the individuals' heterogeneity in their thoughts about the probabilistic risk inherent in a risky event such as lottery. However no ambiguity about risks is assumed in this essay, as well as in the first one.

The third essay introduces ambiguity for the risks, a characteristic usually observed in individual perceptions of mortality risks. The modified probit approach has been shown capable in estimating factors of median and variance of subjective distributions of risks using the individuals' markings on risk ladder.

All three of the essays are set in the general area of nonmarket valuation under uncertainty with particular focus on econometric modeling of individual perceptions of risks in environmental quality and human health. They contribute empirical examples and methodological approaches to developing estimation models given the assumption about the individual's perceptions of risk, and accordingly, the reasonable specifications of risks involved in the models.

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APPENDIX A

Table A-1: Sampling Distribution of Posterior Means of α, β, and m

	α true value = 1			β (in 10^{-3}) true value = 1			m true value = 0.2		
	Size			Size			Size		
Sample	100	800	1500	100	800	1500	100	800	1500
1	3.771	1.433	0.875	1.550	1.138	0.923	0.420	0.218	0.307
2	2.560	0.976	0.846	1.272	0.982	0.937	0.290	0.243	0.309
3	0.815	2.077	0.894	0.750	1.515	0.974	0.359	0.212	0.274
4	2.770	1.446	0.994	1.426	1.227	0.950	0.327	0.303	0.246
5	2.609	1.596	1.447	1.687	1.262	1.214	0.238	0.245	0.276
6	1.464	0.581	0.662	1.032	0.922	0.855	0.562	0.256	0.316
7	3.635	1.678	0.928	1.420	1.056	0.919	0.400	0.288	0.214
8	3.484	1.282	1.130	1.954	1.082	0.978	0.502	0.210	0.180
9	1.269	0.725	1.322	1.245	0.882	1.023	0.237	0.301	0.173
10	0.986	1.177	1.448	1.089	0.993	1.189	0.370	0.281	0.174
11	3.732	1.179	1.268	1.749	0.938	1.035	0.392	0.202	0.314
12	2.002	0.646	1.114	0.914	0.997	1.002	0.354	0.111	0.184
13	0.821	0.882	1.477	1.022	1.037	1.144	0.402	0.102	0.290
14	0.731	0.685	0.775	1.060	0.832	0.878	0.461	0.283	0.226
15	4.901	0.957	1.322	2.586	0.991	1.012	0.390	0.422	0.170
16	1.791	0.895	1.151	1.277	0.945	1.045	0.301	0.283	0.153
17	0.787	0.487	0.773	1.021	0.760	0.901	0.382	0.221	0.222
18	1.493	0.664	1.306	1.113	0.853	1.100	0.295	0.217	0.067
19	2.415	0.765	0.456	1.515	0.796	0.915	0.489	0.327	0.148
20	0.872	1.023	0.824	1.189	0.931	0.961	0.380	0.416	0.201
Average	2.145	1.058	1.051	1.344	1.007	0.998	0.378	0.257	0.222
STD	1.251	0.421	0.290	0.421	0.177	0.100	0.084	0.080	0.068

Table A-2: Sampling Distribution of Posterior Standard Deviations of α, β, and m

	α true value = 1			β (in 10^{-3}) true value = 1			m true value = 0.2		
	Size			Size			Size		
Sample	100	800	1500	100	800	1500	100	800	1500
1	1.770	0.496	0.346	0.603	0.183	0.133	0.278	0.170	0.177
2	1.401	0.469	0.344	0.441	0.169	0.128	0.239	0.178	0.172
3	0.639	0.575	0.347	0.284	0.220	0.138	0.265	0.160	0.188
4	1.677	0.491	0.319	0.545	0.194	0.115	0.243	0.224	0.138
5	1.429	0.508	0.371	0.558	0.197	0.148	0.216	0.145	0.138
6	0.937	0.360	0.313	0.397	0.159	0.127	0.278	0.227	0.218
7	2.458	0.473	0.354	0.720	0.171	0.128	0.295	0.239	0.166
8	1.751	0.518	0.387	0.653	0.188	0.139	0.283	0.186	0.187
9	0.927	0.408	0.383	0.402	0.161	0.134	0.216	0.209	0.118
10	0.784	0.470	0.433	0.386	0.179	0.172	0.272	0.174	0.238
11	1.664	0.491	0.351	0.549	0.173	0.133	0.242	0.184	0.184
12	1.261	0.401	0.363	0.393	0.153	0.130	0.275	0.107	0.149
13	0.671	0.453	0.402	0.320	0.162	0.152	0.294	0.098	0.162
14	0.590	0.409	0.346	0.318	0.160	0.135	0.292	0.184	0.192
15	2.251	0.451	0.371	0.876	0.189	0.130	0.272	0.290	0.139
16	1.226	0.412	0.383	0.471	0.167	0.134	0.244	0.206	0.111
17	0.632	0.349	0.337	0.322	0.138	0.123	0.266	0.169	0.147
18	1.154	0.385	0.392	0.434	0.149	0.136	0.229	0.171	0.071
19	1.340	0.413	0.279	0.546	0.161	0.119	0.291	0.237	0.157
20	0.691	0.434	0.354	0.375	0.179	0.133	0.270	0.260	0.159
Average	1.263	0.448	0.359	0.480	0.173	0.134	0.263	0.191	0.161
STD	0.546	0.057	0.034	0.152	0.019	0.012	0.025	0.047	0.038

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