

**BAYESIAN PREDICTION OF MODULUS OF ELASTICITY OF  
SELF CONSOLIDATED CONCRETE**

A Thesis

by

CHANDAN BHATTACHARJEE

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2007

Major Subject: Civil Engineering

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Approved by:

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## ABSTRACT

Bayesian Prediction of Modulus of Elasticity of Self Consolidated Concrete.

(December 2007)

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Current models of the modulus of elasticity,  $E$ , of concrete recommended by the American Concrete Institute (ACI) and the American Association of State Highway and Transportation Officials (AASHTO) are derived only for normally vibrated concrete (NVC). Because self consolidated concrete (SCC) mixtures used today differ from NVC in the quantities and types of constituent materials, mineral additives, and chemical admixtures, the current models may not take into consideration the complexity of SCC, and thus they may predict the  $E$  of SCC inaccurately. Although some authors recommend specific models to predict the  $E$  of SCC, they include only a single variable of assumed importance, namely the compressive strength of concrete,  $f'_c$ . However there are other parameters that may need to be accounted for while developing a prediction model for the  $E$  of SCC. In this research, a Bayesian variable selection method is implemented to identify the significant parameters in predicting the  $E$  of SCC and more accurate models for the  $E$  are generated using these variables. The models have a parsimonious parameterization for ease of use in practice and properly account for the prevailing uncertainties.

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## 1 INTRODUCTION

Self consolidated concrete (SCC) is a unique concrete proportioned to flow into formwork corners and between reinforcement under its self weight without the need for external vibration and consolidation during concrete placement. The ease of placement results in reduced labor, the adequate compaction ensures quality and durability of the concrete, and the speed of placement results in increased production efficiency. Although the material costs of SCC are higher as much as 7% (Ouchi et al. 2003) due to costs of admixtures and quality control, labor costs can be reduced by 33% (Ouchi et al. 2003), thus reducing the total cost. Because of its desirable characteristics, many countries like Japan, the European Union and United States of America have begun adopting SCC in concrete construction. Due to this increased use of SCC, it is important to better estimate the critical properties of this material for a safe analysis and design. One of the critical parameters is the modulus of elasticity,  $E$ , of concrete. For example, this parameter is used to find deflections of structures for serviceability requirements and in seismic analysis and design for deformation and drift calculations. The  $E$  is also used in prestressed concrete design for calculating elastic shortening of concrete, creep loss, increase in prestressing steel strain and prestressing force in members of indeterminate frames.



A literature review reveals that the models for predicting  $E$  recommended by ACI, AASHTO and 7 other authors (see Table 1) have been developed based on tests from normally vibrated concrete (NVC). Because SCC mixtures used today differ from NVC in the quantities and types of constituent materials, mineral additives, and chemical admixtures, the current models do not take into consideration the complexity of SCC and are not accurate in assessing  $E$  of SCC.

**Table 1.** Current models for predicting  $E$  of NVC and SCC

Author	Model (SI units, MPa)	Model (US units, psi)
Models for NVC		
ACI 318 (2005)	$E = 4730 (f'_c)^{0.5}$	$E = 57,000 (f'_c)^{0.5}$
AASHTO (2006)	$E = 0.043K_1\lambda^{1.5}\sqrt{f'_c}$	$E = 33K_1\lambda^{1.5}\sqrt{f'_c}$
Ahmad and Shah (1985)	$E = 3.38 \times 10^{-5} \times \lambda^{2.5} (\sqrt{f'_c})^{0.65}$	$E = \lambda^{2.5} (\sqrt{f'_c})^{0.65}$
Jobse and Moustafa (1984)	$E = 0.103\lambda^{1.5} (\sqrt{f'_c})^{0.5}$	$E = 275.5\lambda^{1.5} (\sqrt{f'_c})^{0.5}$
ACI 363R-1992	$E = 3320 (f'_c)^{0.5} + 6890$	$E = 40,000 (f'_c)^{0.5} + 10^6$
CEB-1990	$E = 10,000 (f'_c + 8)^{1/3}$	$E = 276,000 (f'_c + 1160)^{1/3}$
Jensen (1943)	$E = \frac{41380}{1 + 13.8/f'_c}$	$E = \frac{6 \times 10^6}{1 + 2000/f'_c}$
Guitierrez and Canovas (1995)	$E = 8340 \sqrt[3]{f'_c}$	$E = 230,185 \sqrt[3]{f'_c}$
Cook (1989)	$E = 3.22 \times 10^{-5} \times \lambda^{2.5} (f'_c)^{0.315}$	$E = \lambda^{2.5} (f'_c)^{0.315}$
Models for SCC		
Persson (2001)	$E = 3700(f'_c)^{0.5}$	$E = 44,554(f'_c)^{0.5}$
Felekoglu et al. (2007)	$E = 1570(f'_c)^{0.8}$	$E = 4248(f'_c)^{0.8}$

A new model is needed for predicting the  $E$  of SCC that is accurate and properly accounts for the prevailing uncertainties. Although some authors (Persson 2001; Felekoglu et al. 2007) recommend models for predicting  $E$  of SCC (see Table 1), they do not attempt to identify any potential variable other than  $f'_c$ . A literature review indicates that there are other potential parameters such as water-cement ratio, cement to aggregate ratio, aggregate type, curing time and temperature, quantity and amount of fly ash, amount of silica fume, blast furnace slag, limestone filler, superplasticizers, air-entraining agents and water reducing agents that may need to be accounted for in developing an accurate prediction model for  $E$  of SCC. Therefore, there is a need to identify these variables using a standard selection procedure, and then develop an accurate model for predicting the  $E$  of SCC considering these critical variables. Also, the current models for predicting  $E$  are not dimensionless. Therefore, there is a need to derive a model that is dimensionless in its parameters and variables so that it can be used in any system of units.

In this study, Bayesian variable selection method (Brown et al. 1998; Chipman et al. 2001) is used to select the informative variables from a large set of candidate variables by comparing the posterior probabilities of competing models containing different subsets of variables. In addition to the more systematic Bayesian variable selection method, a second more intuitive method of analysis, called the method of stepwise addition of variables, is also considered in this research. This method explores the relationship of a particular independent variable to a dependent variable based on the potential trend in the scatter plot between the two variables. After identifying a model

that is both accurate and parsimonious (with as few parameters as possible), a regression analysis is carried out to assess the values of the unknown regression parameters. Finally, the accuracy of the developed predictive model is validated using a subset of data not used for the parameter estimation.

This thesis has four sections. After this introduction, the parameters that can affect the modulus of elasticity are discussed. Next, the theory of the Bayesian variable selection method is summarized. The following section describes the proposed model, the analysis procedure and the results of this study. Finally, the last section draws some conclusions.

## 2 PARAMETERS THAT MIGHT AFFECT THE MODULUS OF ELASTICITY

A thorough literature review shows that there are many parameters other than  $f'_c$  that might be important in predicting  $E$ . These parameters are broadly classified as constituent materials, SCMs and chemical admixtures.

### 2.1 Constituent materials

Several researchers have found that water cement ratio ( $w/c$ ) is an important parameter in predicting  $E$ . Based on data on NVC, Abram (1918), Asselanis and Tajirian (2005) and Felekoglu et al. (2007) showed that  $w/c$  is inversely proportional to  $E$ . Thus  $w/c$  may be an important parameter for predicting  $E$  of SCC. While for NVC a prediction of  $E$  only based on  $f'_{cr}$  (also inversely proportional to  $w/c$ ) is adequate, considering the complexity of SCC with respect to NVC,  $f'_{cr}$  alone may not be sufficient to predict  $E$  with the same accuracy as for NVC.

The cement to aggregate ratio is another variable that may influence  $E$ . Suwanvitaya et al. (2006) found that in NVC,  $E$  increased by over 100% with the increase in aggregate volume, keeping the  $w/c$  constant. Because cement to aggregate ratio is an important parameter in predicting  $E$  for NVC, it might also be an important parameter for SCC. Although its inclusion in the model for NVC was not essential, it might be necessary to include it in the model for SCC, considering the complexity of SCC with respect to NVC. Thus the cement to aggregate ratio should also be considered as a candidate variable in predicting  $E$ .

Other studies have shown that the type of aggregate used also affects the  $E$ . Turkmen and Kantarci (2006) found that when normal aggregate is replaced in SCC by expanded perlite aggregate (EPA) by 5, 10 and 15 percent, the  $f'_{cr}$  is reduced by 1, 3 and 7 percent respectively. Aitcin and Mehta (1990) found that using diabase and limestone aggregates produced concretes with significantly higher  $E$  and  $f'_{cr}$  than those using granite and river gravel. Sengul et al. (2002) prepared concrete mixtures with four different types of aggregates (Triassic crushed limestone, Devonian limestone, sandstone and basalt) and six classes of concrete (ranging from 2610 psi to 13,053 psi). Sengul et al. found that for low strength concretes Triassic limestone provides the highest  $f'_{cr}$ , and for high strength concretes basalt aggregate leads to the highest  $f'_{cr}$ . Thus specific types of aggregates may have a significant effect on  $f'_{cr}$  and, consequently, on  $E$ .

The work of other researchers suggests that curing temperature may also be a significant parameter in predicting  $E$ , but not curing time. Brunner (2005) studied SCC mixtures manufactured with varying  $w/c$  (0.45, 0.55, and 0.65) and air void contents, and subjected to different curing times (none, 24 hours, 48 hours, 6 days and 90 days) at 100% relative humidity and 23 °C. The specimens were then stored at 20 °C and 65% relative humidity. Brunner found that the influence of curing times on  $E$  is practically negligible. Stegmaier (2005) investigated the influence of curing temperatures on the mechanical properties of SCC using a hot air method of curing for 3 days. Stegmaier found that curing temperature hardly affects the  $E$  of SCC with low  $w/c$ . However, for

SCC with high  $w/c$ ,  $E$  decreases with increasing curing temperatures. Thus curing temperature may be an important parameter in predicting  $E$ .

## 2.2 SCMs and fillers

A literature review shows that fly ash quantity may have significant effect on  $E$ . Naik and Ramme (1989) found that concrete with 20% and 40% replacement of the cement with Class C fly ash gained 11% and 26% more  $f'_{cr}$  at 28 days compared to the mixtures without fly ash replacement. Similar effects can be expected on  $E$ . Similarly, Hammons and Smith (1990) found that the early age (14 days)  $f'_{cr}$  and  $E$  increased with increasing proportions of Class C fly ash. Lane and Best (1982) reported that the  $f'_{cr}$  and  $E$  of concrete with Class F fly ash is lower at early ages and slightly higher at later ages compared with similar concretes without fly ash. These studies show that fly ash has a direct relationship with  $E$ .

Other experimental data on SCC suggest that silica fume may be another important parameter in predicting  $E$ . Bentur and Goldman (1989) found that there is up to 25% increase in the 28-day  $f'_{cr}$  when 10% silica fume is added to the concrete mixture. Larrard (1989) experimented with four types of mineral additives (limestone filler, limestone ultrafine filler, siliceous ultrafine fillers and silica fume), and found that the use of silica fume is responsible for the highest increase in  $f'_{cr}$ . Thus silica fume is expected to have an effect also on  $E$ .

Research also shows that blast furnace slag could be another variable of significance in predicting  $E$ . Kaufmann et al. (2004) found that when 30% ultrafine cement (50%

OPC + 50% blast furnace slag) is added to 70 % OPC, there is an increase of 47% in the  $f'_{cr}$  compared to the  $f'_{cr}$  of the concrete with 100% OPC. Swamy and Bouikni (1990) found that 50 to 65% replacement of cement with slag produces similar  $f'_{cr}$  as OPC concrete at 3 and 7 days. This shows that the use of blast furnace slag has an effect on  $f'_{cr}$  and likely on  $E$ .

Other studies also show that limestone fillers may also have an important effect on  $E$ . Zhu and Gibbs (2005) found that the  $f'_{cr}$  of SCC mixes containing limestone and chalk powders were 60 to 80% higher at 7 days and 30 to 40% higher at 28 days, compared with the corresponding values for NVC. Bosiljkov (2003) found that the addition of filler improves the 28-day  $f'_{cr}$  of concrete mixes due to improved fine-particle packing. Thus limestone fillers may affect  $f'_{cr}$  and possibly  $E$ .

### 2.3 Chemical admixtures

Several researchers found that superplasticizers may be a significant variable in predicting  $E$ . Sahmaran et al. (2006) concluded that the use of polycarboxylic ether, modified polycarboxylate and melamine formaldehyde superplasticizers increase the 28-day  $f'_{cr}$  by 15%, 25% and 21% compared to control concrete without any superplasticizer. Malhotra (1981) conducted similar experiments and found that the 28-day  $f'_{cr}$  of a concrete mixture with melamine formaldehyde condensate and naphthalene condensate superplasticizers are 11% and 18% more than the 28-day  $f'_{cr}$  of a control mixture with no superplasticizer. The corresponding values are 18% for naphthalene condensate and 6.5% for sulphonated polymer. Swamy (1989) found that when 2%

superplasticizers (by weight of binder) is added to a concrete with 50% fly ash replacement, the concrete achieves 50% of the 28-day  $f'_{cr}$  in 3 days and 70% of the 28-day  $f'_{cr}$  in 7 days. Siebel (1989) performed experiments with superplasticizers based on melamine and naphthalene sulphonates. He found that the  $f'_{cr}$  of the concretes with superplasticizers are 12% higher compared to the concretes without superplasticizers. These studies show that the addition of superplasticizers have an effect on  $f'_{cr}$ . Similar effects can be expected for  $E$ .

Other studies suggest that water reducing agents may also have an effect on  $E$ . Agarwal (2003) performed experiments with a water reducing agent made from creosote oil. He found that when this reducing agent is added at a proportion of 0.6% by weight of cement, the 28-day  $f'_{cr}$  of the concrete is increased by 11%. Artigues et al. (1990) studied the influence of carboxylic polymer based water reducing agent and found that when this admixture is added at 0.7% of the cement weight, there is an average increase in 28-day  $f'_{cr}$  by 17%. These results show that water reducing agents may be another important parameter in predicting  $f'_{cr}$  and  $E$ .

Some studies also showed that air entrainment may be an important parameter in predicting  $E$ . Ernzen and Carrasquillo (1992) found that the addition of 3% air entrainment can be associated with a decrease of 1% in  $f'_{cr}$ . Ansari et al. (2002) concluded that, due to the creation of larger air bubbles, concretes produced with the synthetic air entraining admixtures, in general, exhibit lower values of  $f'_{cr}$  than those



produced with vinsol resin agents. These studies show that air entrainment is a parameter that might be important in predicting  $f'_{cr}$  and similarly  $E$ .

### 3 BAYESIAN VARIABLE SELECTION

Bayesian variable selection (Brown et al. 1998; George and McCulloch 1997) is a statistical approach for selecting a subset of important variables from a larger set of candidate ones by comparing the posterior probabilities of competing models containing different choices of the variables. The advantages of this method compared to other selection methods are:

- 1) Accountability of uncertainty related to models and parameter values;
- 2) Ability to incorporate prior information in the form of expert opinion or previously collected data;
- 3) Ability to assess the joint effect of the predictors on the response variable and
- 4) Flexibility of allowing the sample size to be smaller than the number of available variables.

An excellent review monograph on Bayesian variable selection methods has been prepared by Chipman et al. (2001). Let  $\mathbf{Y}$  be the  $1 \times n$  vector of response variables, where  $n$  is the number of observations, and let  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_p$  be the corresponding  $1 \times n$  vectors of the independent variables (potential variables of significance in predicting  $\mathbf{Y}$ ). The present analysis aims at modeling the relationship between  $\mathbf{Y}$  and a subset of  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_p$ , where there is uncertainty about which  $\mathbf{X}$ 's to include in the model. A normal linear model relates  $\mathbf{Y}$  to  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_p$  as:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \quad (1)$$

where  $\varepsilon$  is a Gaussian random variable with zero mean and variance  $\sigma^2$  and where  $\Theta = (\boldsymbol{\theta}, \sigma)$  denotes the unknown model parameters.

In formulating the model, it is convenient to use a suitable transformation of the physical quantity of interest to justify the following assumptions: (a) the model variance  $\sigma^2$  is independent of  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_p$  (homoskedasticity assumption), and (b)  $\varepsilon$  follows a normal distribution (normality assumption). It may be explored which transformation is most appropriate by checking diagnostic plots of the data or the residuals against model predictors or individual regresses (Rao and Toutenburg 1997).

The variable selection problem arises when there is some unknown subset of the predictors with regression coefficients so small that it would be preferable to ignore them. Let each of the  $2^p$  possible subset choices among the  $\mathbf{X}$ 's be indexed by the vector  $\boldsymbol{\gamma}$  defined as

$$\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \dots, \gamma_p)' \quad (2)$$

where  $\gamma_i = 0$  or 1 according to whether  $\theta_i$  is small or large, respectively. Let  $q_\boldsymbol{\gamma} = \boldsymbol{\gamma}'\mathbf{1}$  denote the size of  $\boldsymbol{\gamma}$ , that is the number of components equal to 1. The distribution of interest in the Bayesian formulation can be written as

$$p(\boldsymbol{\gamma} | \mathbf{Y}) = \frac{p(\mathbf{Y} | \boldsymbol{\gamma})p(\boldsymbol{\gamma})}{\sum p(\mathbf{Y} | \boldsymbol{\gamma})p(\boldsymbol{\gamma})} \quad (3)$$

where  $p(\mathbf{Y} | \boldsymbol{\gamma}) = \int p(\mathbf{Y} | \Theta, \boldsymbol{\gamma})p(\Theta | \boldsymbol{\gamma})d\Theta$ . The term  $p(\boldsymbol{\gamma})$  is called the model space prior,  $p(\Theta | \boldsymbol{\gamma})$  is called the parameter prior,  $p(\mathbf{Y} | \boldsymbol{\gamma})$  is the marginal likelihood, and  $p(\boldsymbol{\gamma} | \mathbf{Y})$  is the posterior probability. An obvious choice for model selection is to choose

the most probable  $\gamma$ , that is the model  $\gamma$  that has the largest  $p(\gamma | \mathbf{Y})$ . Another option is to consider a set of models with high posterior probabilities. In this context, marginal models can also be used for variable selection. A marginal model identifies variables that appear most often in a set of models with high posterior probabilities. Prior specification and posterior calculations are discussed next.

### 3.1 Selection of priors

The problem is selecting a submodel of Eq. (1) as indexed by  $\gamma$ . In the Bayesian formulation, the vector  $\gamma$  induces a prior on the regression coefficients of the form

$$p(\boldsymbol{\theta} | \sigma^2, \boldsymbol{\gamma}) = N_p(\mathbf{0}, \sigma^2 D_\gamma R D_\gamma) \quad (4)$$

where  $R$  is the correlation matrix and  $D_\gamma$  is a diagonal matrix with diagonal elements:

$$\begin{aligned} (D_\gamma)_{ii} &= \sqrt{v_{0i}} \quad \text{when } \gamma_i = 0 \\ &= \sqrt{v_{1i}} \quad \text{when } \gamma_i = 1 \end{aligned} \quad (5)$$

Under prior (4), the marginal prior distribution of each component of  $\boldsymbol{\theta}$  is given by

$$p(\theta_i) = (1 - p(\gamma_i))N(0, v_{0i}) + p(\gamma_i)N(0, \sigma^2 v_{1i}) \quad (6)$$

that is a scale mixture of two normal distributions, and  $v_{0i}$  and  $v_{1i}$  are the two hyperparameters of the distribution. The hyperparameters  $v_{0i}$  and  $v_{1i}$  are set small and large respectively so that when the data supports  $\gamma_i = 0$  over  $\gamma_i = 1$ , then  $\theta_i$  is small enough so that  $X_i$  will not be needed in the model. Eq. (4) is then coupled with the inverse gamma prior as shown here

$$p(\sigma^2 | \gamma) = p(\sigma^2) = IG(\nu/2, \nu\lambda/2) \quad (7)$$

where  $\nu$  and  $\lambda$  are the parameters of the inverse gamma prior. This coupling makes the conditional distribution of  $\boldsymbol{\theta}$  and  $\sigma^2$  given  $\gamma$  conjugate. This allows for the analytical marginalization of  $\boldsymbol{\theta}$  and  $\sigma^2$  from  $p(\mathbf{Y}, \boldsymbol{\theta}, \sigma^2 | \gamma) = p(\mathbf{Y} | \boldsymbol{\theta}, \sigma^2) p(\boldsymbol{\theta} | \sigma^2, \gamma) p(\sigma^2 | \gamma)$  to yield the following relationship:

$$p(\mathbf{Y} | \gamma) \propto |\mathbf{X}'\mathbf{X} + (D_\gamma R D_\gamma)^{-1}|^{-1/2} |D_\gamma R D_\gamma|^{-1/2} (\nu\lambda + S_\gamma^2)^{-(n+\nu)/2} \quad (8)$$

$$\text{where } S_\gamma^2 = Y'Y - Y'X(X'X + (D_\gamma R D_\gamma)^{-1})^{-1} X'Y \quad (9)$$

The prior used for  $\gamma$  is the Bernoulli prior, given by the expression:

$$p(\gamma) = w^{q_\gamma} (1-w)^{p-q_\gamma} \quad (10)$$

where  $w$  is the priori expected proportion of  $\mathbf{X}$ 's to be included in the model.

### 3.2 Calculation of posterior probability

Once the priors have been chosen, all the needed information for a Bayesian inference and decision is provided by the posterior probability. Eq. (8) is used to compute:

$$g(\gamma) \propto p(\mathbf{Y} | \gamma) p(\gamma) \propto p(\gamma | \mathbf{Y}) \quad (11)$$

The availability of such  $g(\gamma)$  can facilitate posterior calculation and estimation. In general, this simply entails calculating  $g(\gamma)$  for every  $\gamma$  and then summing over all  $\gamma$  values to obtain the normalizing constant.

In problems where  $p$  is large and where the exact calculation of  $p(\boldsymbol{\gamma} | \mathbf{Y})$  is not feasible, a stochastic search method is used for posterior inference. In this method, the following sequence is simulated:

$$\boldsymbol{\gamma}^{(1)}, \boldsymbol{\gamma}^{(2)}, \boldsymbol{\gamma}^{(3)}, \dots \quad (12)$$

The sequence starts from an initial value  $\boldsymbol{\gamma}^{(1)}$  and proceeds by successive simulation from a probability kernel  $p(\boldsymbol{\gamma} | \boldsymbol{\gamma}^{(j)})$ , and after sufficient iterations, converges to the posterior distribution  $p(\boldsymbol{\gamma} | \mathbf{Y})$ . The posterior probability helps to identify those high probability  $\boldsymbol{\gamma}$ , which are expected to appear more frequently. Thus MCMC methods can be used to stochastically search for high probability models. Metropolis Algorithm (Metropolis et al. 1953) is an important algorithm for constructing a kernel to simulate the sequence of Eq. (12). The corresponding algorithm is:

1. Simulate a value of  $\boldsymbol{\gamma} = \boldsymbol{\gamma}^*$  by randomly changing one component of  $\boldsymbol{\gamma}^{(j)}$ ;
2. Set  $\boldsymbol{\gamma}^{(j+1)} = \boldsymbol{\gamma}^*$  with probability  $\alpha^M(\boldsymbol{\gamma}^* | \boldsymbol{\gamma}^{(j)}) = \min\left\{\frac{g(\boldsymbol{\gamma}^*)}{g(\boldsymbol{\gamma}^{(j)})}, 1\right\}$ ;
3. Else  $\boldsymbol{\gamma}^{(j+1)} = \boldsymbol{\gamma}^{(j)}$ .

In this paper, we have used the implementation of the methods given in Brown et. al. (1998) that exploits fast updating schemes that use QR decompositions. The related Matlab codes can be found at [www.stat.tamu.edu/~mvannucci/webpages/codes.html](http://www.stat.tamu.edu/~mvannucci/webpages/codes.html).

## 4 APPLICATION OF BAYESIAN VARIABLE SELECTION

In this section the Bayesian variable selection method described above is used to construct an accurate and parsimonious probabilistic model to predict  $E$  of SCC.

### 4.1 Database used for analysis

Data for  $E$  of SCC is collected from the following sources: Khayat et al. (1997), Khayat et al. (2001), Kim et al. (1998), Khayat et al. (2000), Khayat (2000), Markus (2000), Asselanis and Tajirian (2005) and experimental results from Texas A&M University. A total of 275 data points are obtained. Each data point gives the value of  $E$  and of the independent predictor variables. Tables 2 and 3 show the ranges of the variables in the collected data.

**Table 2.** Ranges of variables used in this research

Name of the variable	Range
Modulus of elasticity	3,031,287 – 7,112,795 psi
Unit weight	2,102 – 2,425 kg/m <sup>3</sup>
Total cementitious (binder) materials	300 – 650 kg/m <sup>3</sup>
Fine aggregate	600 – 950 kg/m <sup>3</sup>
Coarse aggregate	700 – 1,032 kg/m <sup>3</sup>
High range water reducing agent	0.96 – 12.95 kg/m <sup>3</sup>
Viscous modifying agent	0 – 0.075 % of cement
Slump flow	389 – 750 mm
Silica fumes	0 – 18 kg/m <sup>3</sup>
Cement content	150 – 589 kg/m <sup>3</sup>
Fly ash quantity	0 – 253 kg/m <sup>3</sup>
Limestone powder quantity	0 – 200 kg/m <sup>3</sup>
Blast furnace slag	0 – 236 kg/m <sup>3</sup>
Set retarding agent	0 – 1.06 L/m <sup>3</sup>
Compressive strength	4,177.1 – 16,918.2 psi
Total aggregate content	1,300 – 1,982 kg/m <sup>3</sup>

#### 4.2 Analysis with collected data

The collected data points are randomly divided in two groups, 2/3 of the data points are used in the variable selection process and to assess the resultant model. The remaining 1/3 of the data are used to quantify the accuracy of the developed model. Considering the non-negative nature of the  $E$ , the logarithmic variance-stabilizing transformation is selected among other possible transformations to generate a homoskedastic model.



**Table 3.** Ranges of normalized variables used in this research

Normalized variable	Range
Water / binder ratio	0.23 – 0.50
Viscous modifying agent	0 – 0.075 % of cement
Binder / aggregate ratio	0.18 – 0.39
High range water reducing agent / cement	0.0059 – 0.0423
Fly ash / cement	0 – 0.67
Blast furnace slag / cement	0 – 1
Limestone powder / cement	0 – 0.54
Silica fumes / cement	0 – 0.058
Fine aggregate / coarse aggregate	0.656 – 1.11
Binder / coarse aggregate	0.36 – 0.76
Binder / fine aggregate	0.36 – 0.86
Slump flow × unit weight / compressive strength	0.000144 – 0.000497
Set retarding agent / cement quantity	0 – 0.00234

Thus, the model is formulated as follows:

$$\ln(Y_r) = \theta_0 + \theta_1 \ln(X_{1r}) + \dots + \theta_p \ln(X_{pr}) + \sigma_r \varepsilon_r \quad r = 1, 2, \dots, n \quad (13)$$

In formulating the model, the  $\mathbf{Y}$  and the  $\mathbf{X}$  are grouped in two alternate ways. In the first group, a suitable combination of the variables are defined so that  $\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_p$  and

$\mathbf{Y}$  are dimensionless and thus each model parameter  $\theta_1, \theta_2, \dots, \theta_p$  is also dimensionless.

This group is henceforth called the dimensionless group, and has the advantage that the finalized model can be used irrespective of the system of units. In the second group, some of the elements of  $\mathbf{X}$  are dimensionless and some are not normalized and have

dimensions. This group is henceforth called the mixed group. The mixed group has the advantage of being more flexible and what can be selected for the predictive model, as the Bayesian variable selection process can select any important variable without limiting the choice on the dimensionless variables.

In each of these groups, two classes of models are formulated. One class includes only the independent variables without any square or cross term of the independent variables. The second class includes the squares and cross terms. After the selection of the significant variables is carried out for each class, a linear regression analysis is used to assess the reduced model for each class.

### 4.3 Parameter choices

The selection of parameters is important in this analysis as the values of the parameters justify the assumptions made. The hyperparameter  $\nu_{0i}$  is set to 0 and  $\nu_{1i}$  is set to 5. Recommended choices for the parameter  $\nu_{1i}$  are in the range 0.1 to 10. The parameter  $w$  in the Bernoulli prior is set to 2, as the current models for  $E$  generally contain 2 variables. For the inverse gamma prior, the parameter  $\lambda$  can be thought of as a prior estimate of  $\sigma^2$ , and has been chosen as  $0.1\sigma_Y^2$ . A vague specification of this inverse gamma prior is then obtained by setting  $\nu$  to 2. The choice  $R = I$  is also made as this works well in practice.

### 4.4 Results of analysis

Summary of the finalized models from all groups is presented here. The model selected for the dimensionless group without square terms or cross terms is:

$$E = 59874(f'_{cr})^{0.389} \left(\frac{w}{b}\right)^{-0.105} (sf \times \lambda)^{0.611} \left(\frac{b}{a}\right)^{-0.307} \quad (14)$$

where  $w/b$  is the water to binder ratio by weight, where binder is a term collectively used for cement, silica fume, fly ash and blast furnace slag,  $sf$  is the slump flow,  $\lambda$  is the unit weight of the concrete and  $b/a$  is the binder to aggregate ratio by weight.

Model (14) was selected with the following procedure. First, the models selected by the Bayesian variable selection method are listed in descending order according to their posterior probability. Then, the best 10 of these models are compared according to their prediction performances, as described in the validation section below. Once the variables are selected, least squares estimates are used to compute the coefficients in Eq. (14).

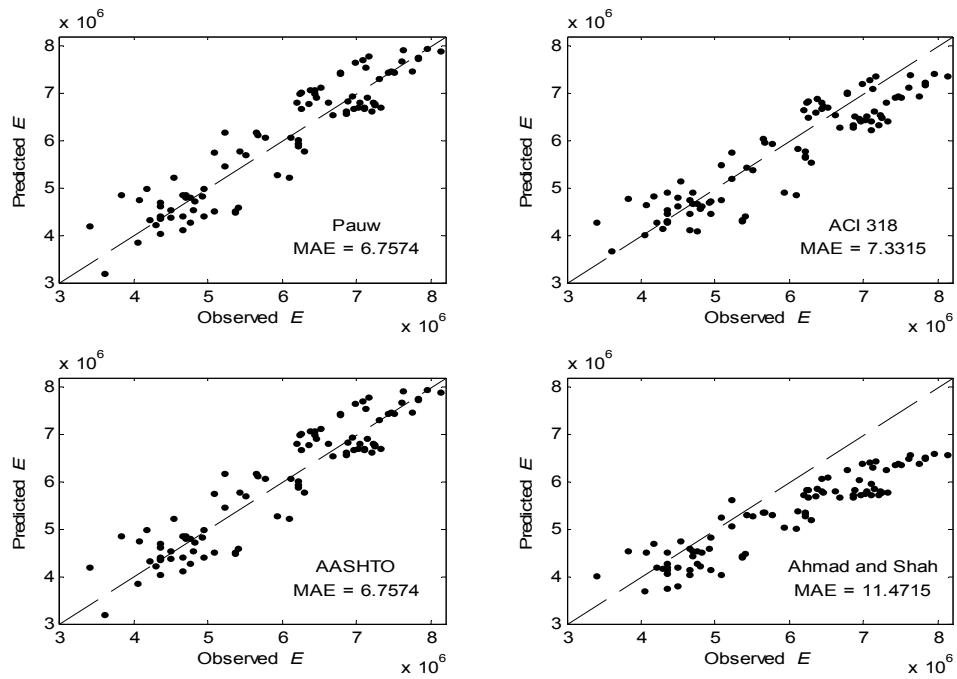
The proposed model relates the  $f'_{cr}$  to  $E$  in contrast to the existing models (as in Table 1) which relate  $f'_c$  to  $E$ . However, as the  $f'_{cr}$  values are generally higher than  $f'_c$  values, the predicted values of  $E$  based on  $f'_{cr}$  would generally be non-conservative.

The unknown parameters  $\Theta = (\theta, \sigma)$  of the selected model are also estimated using a Bayesian updating procedure. This makes use of the importance sampling method proposed by Ditlevsen and Madsen (1996) and Gardoni (2002). The sampling distribution is assumed Normal, centered in the MLE estimates and with covariance matrix given by the negative of the inverse of the Hessian of the log-likelihood of the data evaluated at the maximum-likelihood estimator. Posterior means and standard deviations of the model parameters are reported in Table 4. The accuracy of the model can be judged by the standard deviation of the model error, sigma, with lower values representing better accuracy.

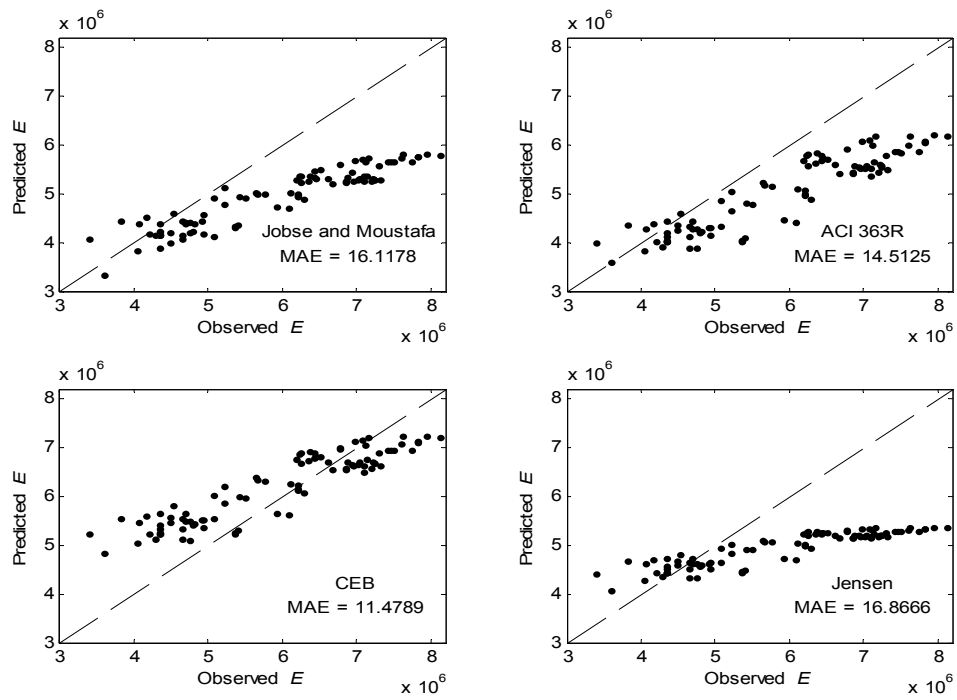
**Table 4.** Estimates of  $\Theta$  for dimensionless group without cross terms

Parameter	Mean	Standard deviation	Correlation coefficient						
			$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\sigma$	
$\theta_1$	11.32	0.14	1						
$\theta_2$	0.56	0.08	-0.39	1					
$\theta_3$	-0.25	0.04	-0.52	0.76	1				
$\theta_4$	0.18	0.08	-0.21	0.98	0.67	1			
$\theta_5$	-0.24	0.03	0.38	-0.24	-0.08	-0.24	1		
$\sigma$	0.076	0.003	0.03	0.005	-0.01	0.01	0.01	1	

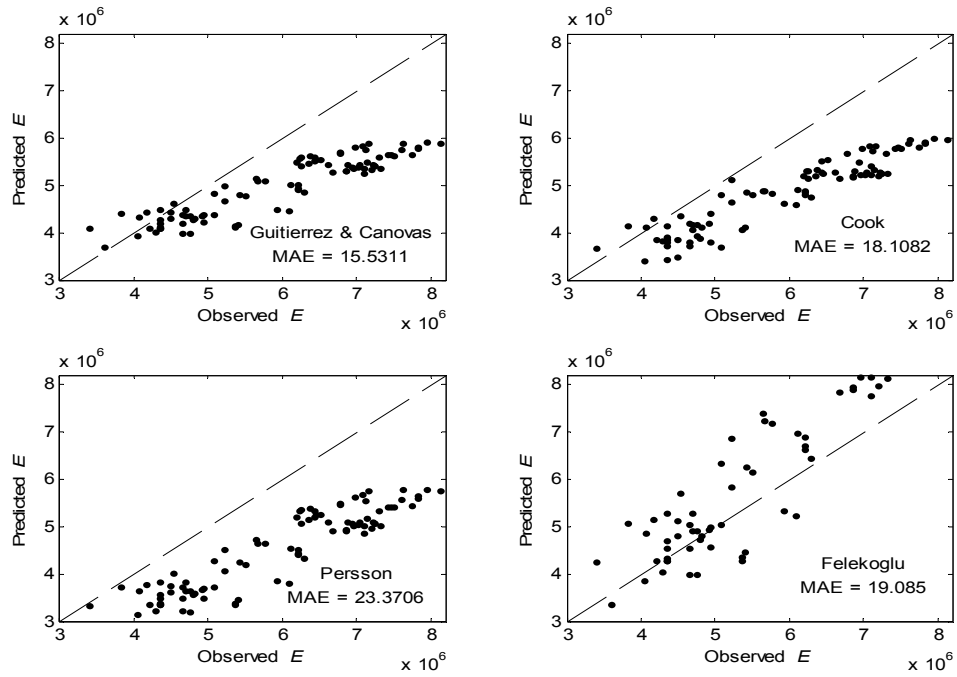
The accuracy of the model is also compared from the plot of predicted versus the observed values of  $E$ . Such graphs have been drawn for predicted values using the models by all the authors in Table 1. These graphs are shown in Figures 1, 2 and 3. It can be noted that the models by Ahmad and Shah, Jobse and Moustafa, ACI 363R, CEB, Jensen, Guitierrez and Canovas, Cook and Persson underestimate the  $E$  at higher values, whereas the model by Felekoglu overestimates the  $E$  at higher values. Also, the model by CEB overestimates the  $E$  at lower values. The corresponding graphs for the proposed model are shown in Figure 4. It can be noted that most of the points lie closer to the 1:1 line, and so the predictions by the proposed model are more accurate. The  $\pm\sigma$  bounds of the model are also drawn. It is found that most of the points in the graph lie between these two bounds, and thus the model is accurate. The model in Eq. (14) is concise and so it can be used for quick calculation with good accuracy. Because the terms in the model are dimensionless, the model can be used irrespective of system of units used.



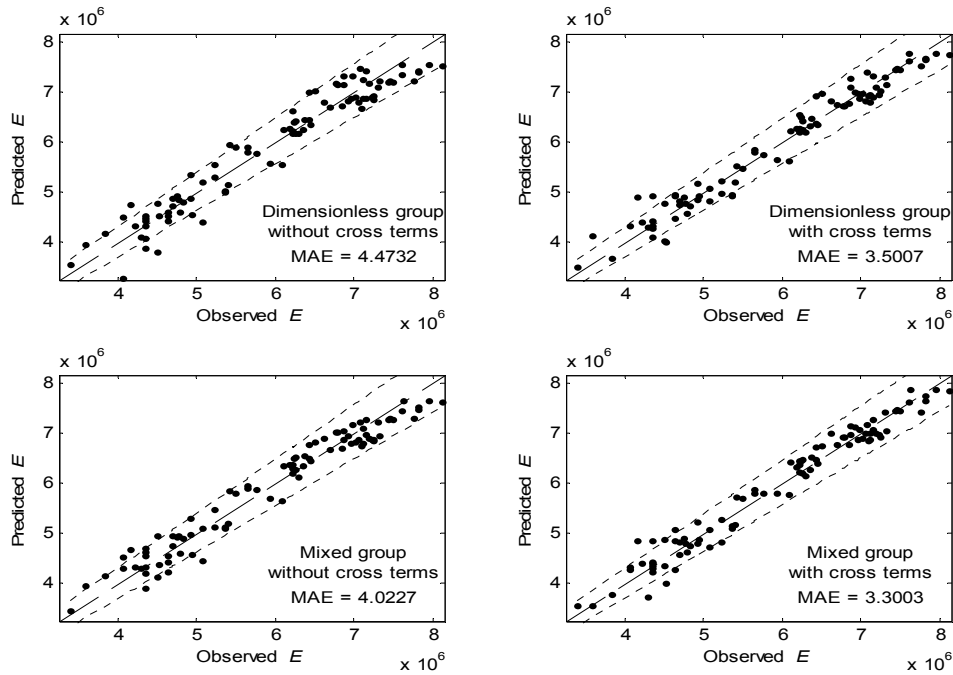
**Fig. 1.** Graph of predicted versus observed values of  $E$  from the models by Pauw, ACI 318, AASHTO and Ahmad and Shah



**Fig. 2.** Graph of predicted versus observed values of  $E$  from the models by Jobse and Moustafa, ACI 363R, CEB and Jensen



**Fig. 3.** Graph of predicted versus observed values of  $E$  from the models by Guitierrez and Canovas, Cook, Persson and Felekoglu



**Fig. 4.** Graph of predicted versus observed values of  $E$  from the models proposed in this research

The domain of search for the best model in case of the dimensionless group is restricted as the individual terms can be combined in only the patterns which make the combined term dimensionless. So, the term  $sf$  is always associated with  $f'_{cr}$ . Thus in the model in Eq. (14), the term  $sf$  is included, even though it is not an important parameter.

The model selected for the dimensionless group with square terms and cross terms is:

$$\log\left(\frac{E}{f'_{cr}}\right) = 11.5 + 0.565 \log\left(\frac{w}{b}\right) + 0.602 \log\left(\frac{sf \times \lambda}{f'_{cr}}\right) - 0.175 \log\left(\frac{b}{C_a}\right) + \quad (15)$$



$$\log\left(\frac{w}{b}\right)\left\{0.237\log\left(\frac{w}{b}\right)+0.109\log\left(\frac{b}{a}\right)\right\}$$

where  $b/C_a$  is the binder to coarse aggregate ratio by weight.

It is expected (and will be shown in the validation section) that the model selected for the dimensionless group with cross terms is more accurate than the model selected without cross terms. But the better accuracy is obtained as a compromise with the complexity of the model. So, the model in Eq. (15) can be used for detailed calculation with requirements of better accuracy, and can be used irrespective of the system of units followed. The estimates of the parameter  $\Theta = (\theta, \sigma)$  of the model are listed in Table 5. In the plot of the predicted versus the observed values as shown in Figure 4, the majority of the points lie within the  $\pm\sigma$  bounds, and closer to 1:1 line than the models in Table 1. So, the proposed model is accurate in predicting the values of  $E$ .

**Table 5.** Estimates of  $\Theta$  for dimensionless group with cross terms

Parameter	Mean	Standard deviation	Correlation coefficient								
			$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\theta_6$	$\theta_7$	$\sigma$	
$\theta_1$	11.7	0.14	1								
$\theta_2$	0.53	0.13	-0.03	1							
$\theta_3$	0.18	0.22	0.56	0.51	1						
$\theta_4$	0.15	0.12	0.07	0.98	0.47	1					
$\theta_5$	0.0001	1.72	0.09	-0.07	0.02	-0.08	1				
$\theta_6$	0.08	0.22	0.64	0.38	0.98	0.34	0.05	1			
$\theta_7$	0.22	0.14	-0.32	0.33	-0.12	0.33	-0.24	-0.26	1		
$\sigma$	0.075	0.14	-0.16	0.04	-0.10	0.03	-0.08	-0.23	0.16	1	

The model selected for the mixed group without square terms or cross terms in SI units is:

$$E = 4226 \left( \frac{w}{b} \right)^{-0.308} (f'_{cr})^{0.383} \left( \frac{b}{a} \right)^{-0.219} \quad (16)$$

The similar model in US customary units is:

$$E = 91126 \left( \frac{w}{b} \right)^{-0.308} (f'_{cr})^{0.383} \left( \frac{b}{a} \right)^{-0.219} \quad (17)$$

The estimates of the parameter  $\Theta = (\theta, \sigma)$  of the model are listed in Table 6. The plot of predicted versus observed values of  $E$  are shown in Figure 4. Most of the points lie within the  $\pm\sigma$  bounds, and closer to 1:1 line than the models in Table 1. So, the proposed model is accurate. The domain of search for the best model in case of the mixed group is flexible, as the Bayesian variable selection method chooses models without any limitations. It is noted that the term  $sf$  is not selected in the model in Eq. (16) and Eq. (17), confirming that it is not an important parameter as discussed earlier in case of the model in Eq. (14).

**Table 6.** Estimates of  $\Theta$  for mixed group without cross terms

Parameter	Mean	Standard deviation	Correlation coefficient				
			$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\sigma$
$\theta_1$	11.41	0.04	1				
$\theta_2$	-0.31	-0.05	-0.43	1			
$\theta_3$	0.39	0.04	-1.02	0.70	1		
$\theta_4$	-0.21	-0.04	0.35	0.22	-0.05	1	
$\sigma$	0.076	0.04	-0.08	0.04	0.08	-0.06	1

The model selected for the mixed group with square terms and cross terms in US customary units is:

$$\log(E) = 11.57 + 0.106 \log(b) \log(f'_{cr}) + \log(w) \left\{ 0.16 \log\left(\frac{w}{b}\right) - 0.137 \log\left(\frac{b}{a}\right) \right\} + 0.493 \log\left(\frac{w}{b}\right) \log\left(\frac{b}{a}\right) \quad (18)$$

where  $b$  is the binder weight.

The estimates of the parameter  $\Theta = (\theta, \sigma)$  of the model are listed in Table 7. The plot of predicted versus observed values of  $E$  are shown in Figure 4. The majority of the points lie within the  $\pm\sigma$  bounds, and closer to 1:1 line than the models in Table 1. So, the proposed model is accurate. Similar to the observation in the dimensionless group, the model in Eq. (18) with cross terms is expected to be more accurate than the model in Eq. (16) and Eq. (17) without cross terms. But the accuracy is at the cost of complexity of the model. Thus the model in Eq. (18) can be used in detailed calculation with requirement for better accuracy.

**Table 7.** Estimates of  $\Theta$  for mixed group with cross terms

Parameter	Mean	Standard deviation	Correlation coefficient						
			$\theta_1$	$\theta_2$	$\theta_3$	$\theta_4$	$\theta_5$	$\sigma$	
$\theta_1$	11.46	0.04	1						
$\theta_2$	0.11	0.04	-0.34	1					
$\theta_3$	0.13	0.04	0.76	0.30	1				
$\theta_4$	-0.17	-0.05	0.95	-0.05	0.91	1			
$\theta_5$	0.48	0.04	0.68	0.19	0.89	0.83	1		
$\sigma$	0.075	0.04	0.001	0.02	0.01	0.005	-0.001	1	

#### 4.5 Validation of the models

To facilitate the use in practice of a predictive model, the model needs to have a parsimonious parameterization (i.e., as few parameters as possible). From a statistical point of view, a parsimonious model is also preferable in order to avoid loss of precision of the model due to inclusion of unimportant predictors and to avoid overfit of the data. The accuracy of a set of parsimonious candidate models can be assessed using the Mean Average Error (MAE) defined as:

$$\text{MAE} = \frac{\sum_{r=1}^n |\hat{E}_r - E_r| / E_r}{n_f} \quad (19)$$

where  $\hat{E}_r$  is the predicted values of  $E_r$  for each data point in the 1/3 of the data that were not used to assess the models, and  $n_f$  is the number of observations from 1/3 of the data. The MAE is a measure of the average relative error of the predicted values from

the observed values, and thus provides a measure of the accuracy level achieved by the model in predicting  $E$ . In order to achieve a parsimonious model with acceptable accuracy, the competing models are also compared by two set of coefficients: Akaike information criteria (AIC) (Akaike 1974) and Bayesian information criteria (BIC) (Schwarz 1978), which measures the relative goodness of fit of the models by trading off the complexity of the model with the goodness of fit of the models. Thus a model with the lowest values of AIC and BIC might be preferred. The accuracy of the models proposed in this research are compared to the accuracy of the models in Tables 1 and 2 using MAE, AIC and BIC values computed for the  $E$  of each of those models. The comparison for all the models is given in Table 8. It can be seen that the MAE, AIC and BIC values for the 4 proposed models are lower than those of the other models indicating that the proposed models are more accurate than the other models. The comparison of the values also indicate that the models with square and cross terms are more accurate than those without them as already noted in results of analysis section.

**Table 8.** MAE, AIC and BIC values of various models

Model	MAE	AIC	BIC
Jensen (1943)	16.86	3,014	3,017
Pauw (1960)	6.757	2,856	2,861
Jobse and Moustafa (1984)	16.11	3,027	3,032
Ahmad and Shah (1985)	11.47	2,996	3,001
Cook (1989)	18.10	3,046	3,051
CEB-1990	11.47	2,894	2,896
ACI 363R-1992	14.51	2,972	2,975
Gutierrez and Canovas (1995)	15.53	2,989	2,992
Persson (2001)	23.37	3,038	3,040
ACI 318 (2005)	7.331	2,838	2,840
AASHTO (2006)	6.757	2,856	2,861
Felekoglu et al. (2007)	19.08	3,030	3,033
Dimensionless model without cross terms	4.474	2,335	2,348
Dimensionless model with cross terms	3.500	2,606	2,329
Mixed model without cross terms	4.022	2,311	2,328
Mixed model with cross terms	3.300	2,282	2,292

## 5 CONCLUSIONS

In the present study, a Bayesian variable selection method is implemented on 275 data points to identify the variables of importance in predicting  $E$  of SCC. A linear regression analysis is done with the selected variables to derive models for predicting  $E$  of SCC. Two groups of models are derived: the dimensionless group with dimensionless variables, and the mixed group with both dimensionless variables and variables with dimensions. In each group, two classes of models are considered: without square or cross terms and with square and cross terms. The following observations are made:

- 1) The  $E$  of SCC is related to more constituent variables such as  $\frac{w}{b}$ ,  $\frac{b}{a}$ ,  $\frac{b}{C_a}$  than is the  $E$  of NVC.
- 2) The models obtained for the dimensionless group can be used irrespective of the system of units followed.
- 3) For each group of results, a model is proposed for quick calculation with good accuracy, and a detailed model is proposed with better accuracy.

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