METHODS FOR COMPOSING TRADEOFF STUDIES UNDER UNCERTAINTY

A Thesis

by

CHRISTOPHER STEPHEN BILY

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2012

Major Subject: Mechanical Engineering

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Approved by:

Chair of Committee,	Richard J. Malak
Committee Members,	Daniel A. McAdams
	Martin A. Wortman
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ABSTRACT

Methods for Composing Tradeoff Studies under Uncertainty. (August 2012)

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Chair of Advisory Committee: Dr. Richard J. Malak

Tradeoff studies are a common part of engineering practice. Designers conduct tradeoff studies in order to improve their understanding of how various design considerations relate to one another. Generally a tradeoff study involves a systematic multi-criteria evaluation of various alternatives for a particular system or subsystem. After evaluating these alternatives, designers eliminate those that perform poorly under the given criteria and explore more carefully those that remain.

The capability to compose preexisting tradeoff studies is advantageous to the designers of engineered systems, such as aircraft, military equipment, and automobiles. Such systems are comprised of many subsystems for which prior tradeoff studies may exist. System designers conceivably could explore system-level tradeoffs more quickly by leveraging this knowledge. For example, automotive systems engineers could combine tradeoff studies from the engine and transmission subsystems quickly to produce a comprehensive tradeoff study for the power train. This level of knowledge reuse is in keeping with good systems engineering practice. However, existing procedures for generating tradeoff studies under uncertainty involve assumptions that preclude engineers from composing them in a mathematically rigorous way. In uncertain problems, designers can eliminate inferior alternatives using stochastic dominance, which compares the probability distributions defined in the design criteria space. Although this is well-founded mathematically, the procedure can be computationally expensive because it typically entails a sampling-based uncertainty propagation method for each alternative being considered.

This thesis describes two novel extensions that permit engineers to compose preexisting subsystem-level tradeoff studies under uncertainty into mathematically valid system-level tradeoff studies and efficiently eliminate inferior alternatives through intelligent sampling. The approaches are based on three key ideas: the use of stochastic dominance methods to enable the tradeoff evaluation when the design criteria are uncertain, the use of parameterized efficient sets to enable reuse and composition of subsystem-level tradeoff studies, and the use of statistical tests in dominance testing to reduce the number of behavioral model evaluations. The approaches are demonstrated in the context of a tradeoff study for a motor vehicle.

DEDICATION

This work is dedicated to my cousin Theresa, who lovingly, comfortingly, and bravely showed us all how to count our blessings, trust in God, and leave an everlasting impact on everyone we know.

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I would like to thank my committee chair, Dr. Richard Malak, for his support and guidance through this research process. Thanks also to my committee members, Dr. Daniel McAdams and Dr. Martin Wortman, for their support and willingness to participate in this process. Also a special thanks to Dr. Guy Curry for serving as a substitute on the oral defense.

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Finally thanks to my parents, sisters, and brother-in-law for their love and encouragement, and especially my soon-to-be wife, Erika, for her love, patience, support, and willingness to make the numerous weekend trips between Houston and College Station.

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1. INTRODUCTION

1.1 Problem Background

Engineered systems are becoming ever more prevalent in our world. To manage complexity on large systems engineering projects, design problems are commonly decomposed into multiple subsystem design problems. Multiple designers with appropriate technical expertise design the subsystems which they later integrate to form the system. In designing the subsystems, the system designers must determine which combination of components maximizes the overall performance of the system. Problems as such necessitate a means to model system-level performance metrics in terms of subsystem performance metrics to enable designers to understand the implications of their subsystem design decisions on the overall system performance.

For example, consider an automobile company who decides to develop a new fuel efficient vehicle to attract budget and environmentally conscious consumers. Following a typical systems engineering approach, system level requirements would be generated for the vehicle drivetrain and the design would be separated into multiple teams, each tasked with designing a specific component. Each team would consider multiple concepts and decide which will maximize system performance, as illustrated in Figure 1. If the teams individually consider the component tradeoffs in choosing a concept, they

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This thesis follows the style of Journal of Mechanical Design.

may not select the best system design. They must consider the effects of each component concept on the system performance. Enabling designers to compose the component tradeoffs to form system-level tradeoffs allows them to consider the implications of their subsystems design decisions on the overall system performance.

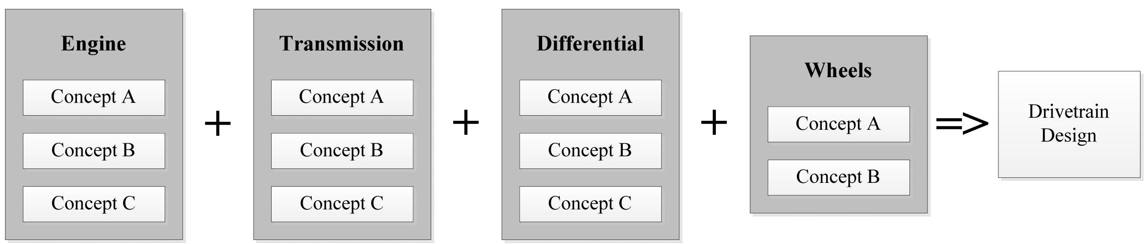

Figure 1. Automobile drivetrain design.

A tradeoff study is the activity of identifying proposed solutions to a design problem defined by their performance criteria, called *attributes*, which designers can use to support decision-making in choosing a design. Various authors have explored the use of tradeoff studies in decision making under deterministic conditions [1-8]. The general procedure for generating a tradeoff study can be summarized in three main steps:

1. Gather attribute data about feasible design implementations by sampling the behavioral model of the design.
2. Eliminate implementations a rational designer is guaranteed not to select, known as *dominated* implementations, using Pareto dominance.

3. Use remaining non-dominated implementations, called *efficient* implementations, to support decision making through visualization or fitting a model for computational use.

The ability to compose system-level tradeoff studies from subsystem tradeoff models is advantageous in enabling knowledge reuse, design effort coordination, and information linkage between system designers and component producers. Regarding knowledge reuse, designers often compose unique designs out of common types of components or subsystems. Frequently components are employed in previous designs where designers already evaluated the possible tradeoffs. Enabling designers to leverage this previous knowledge about component concepts is advantageous in design and especially applicable in composing system-level concepts from reusable tradeoff-space models of common components. Regarding design effort coordination, the organization of a typical engineering project is broken down into teams where each are assigned to develop each major subsystem. If the teams individually design the subsystems without considering the interactions with other subsystems, they may not select the best system design. They must consider the effects of each subsystem on the system performance. A means to compose the subsystem tradeoffs to form system-level tradeoffs would enable designers to consider fully the interactions between subsystems on the system performance when designing the subsystems. Regarding information linkage, systems engineering involves many designers with different technical expertise and backgrounds. Some designers will have specialized knowledge in specific components or subsystems, while others will have expertise in the overall system and system integration. Enabling designers to compose tradeoff studies allows them to incorporate detailed tradeoff considerations into their decision-making without requiring specialized domain knowledge about every component concept. Additionally situations arise where proprietary technology owned by an external company is used in a subsystem or component. Companies may be unwilling to provide detailed models of their designs in order to protect their proprietary technology. As such these companies can provide higher-level tradeoff space models of their designs which characterize the capabilities of their design to enable designers to consider them in engineered systems design without disclosing sensitive information. As a whole, the designers must make design decisions in order to maximize the overall performance of the system while dealing with the differences and availability of technical knowledge, which a means to compose subsystem tradeoffs into system level tradeoffs enables.

Consider again the previous automobile drivetrain example. Suppose the engine team is considering a concept by another company and do not have a detailed behavioral model, but instead a tradeoff space model for the concept. The transmission team is considering concepts which have been previously evaluated for other designs and want to leverage that knowledge to choose between concepts. The differential team is comparing a traditional open differential to a new hydraulic design developed by the internal R&D department, who provide a tradeoff space model of the concept due to the computationally expensive fluid dynamics simulations used in the behavioral model.

Even though the concepts have different design spaces, as the open differential is defined by material strength, number of gear teeth, gear modules, etc., and the hydraulic differential is defined by the hydraulic fluid density, fluid resistance, hydraulic pump specifications, etc., both share the same tradeoff space of cost, reliability, mass, and overall gear ratio. Each of these concept design situations necessitates the capability to compose subsystem tradeoffs in order to determine the drivetrain design tradeoffs and make design decisions.

Researchers have demonstrated the composition of tradeoff studies under deterministic conditions [9]. However designers are often faced with making decisions with uncertain information. Uncertainty originates from a wide variety of sources, including environmental factors, operating factors, manufacturing tolerances, and simplifying modeling assumptions [10-15]. In some design problems the uncertainty is assumed negligible or disregarded and decisions are evaluated as deterministic problems. However in many cases the uncertainty cannot be disregarded. Researchers have demonstrated techniques for non-compositional tradeoff studies under uncertainty [16]. In this thesis I present a combination of these two techniques to enable compositional tradeoff modeling under uncertainty.

Accounting for uncertainty when generating reusable tradeoff-space models results in a significant increase in computational expense. When uncertainty is considered in tradeoff studies, it is typically propagated through the behavioral models using a sampling-based method (Monte Carlo or quasi-Monte Carlo methods) which repeatedly samples the uncertain input variables and evaluates a deterministic behavioral model to produce estimators of the output variables' distributions for each alternative being considered [17]. The numerous behavioral model evaluations for each design implementation can become prohibitively expensive, especially in the case of complex models (computational fluid dynamics, finite element analysis, etc.). Well-established statistical hypothesis tests exist for comparing distribution parameter estimators. In this thesis I also present an extension to generating a tradeoff study under uncertainty that reduces the total number of behavioral model samples by incorporating in statistical hypothesis testing when eliminating inferior alternatives which a rational designer is guaranteed not to choose.

1.2 Prior Investigations into Composing Tradeoff Studies & Tradeoff Studies under Uncertainty

Malak et al. [9] extended the tradeoff study procedure to enable composition of reusable tradeoff studies under deterministic conditions. The key innovation of their approach is a parameterization technique as an extension to Pareto dominance. When generating reusable tradeoff-space models, the exact preferences of the designer are unknown. The designer's preferences will depend on the specific application they are employing the design in. For example, when designing a gear box for a specific application a designer has a preferred target value for the gear ratio. However when generating a reusable tradeoff-space model for the gear box concept, the exact application and target gear ratio are unknown to the designer. The reusable tradeoff-space model needs to be independent of problem specific knowledge. The parameterization technique identifies and eliminates alternatives a rational designer is guaranteed not to choose as a function of the attributes with unknown preferences. In the case of the gear box design, the parameterization technique eliminates an alternative which is guaranteed not to be chosen over another alternative that has the same gear ratio value.

In a later study, Malak et. al. [16] demonstrated a technique for non-compositional tradeoff studies under uncertainty. Their approach is significantly different than their approach in [9] due to the additional complexity from considering uncertainty. Design alternatives are characterized by distributions in the attribute space instead of deterministic points. Similar to their previous approach they eliminate alternatives a rational designer would not select; however the presence of uncertainty complicates the process. Pareto dominance is inappropriate for decisions under uncertainty as it involves the direct comparison of attributes, which becomes unclear when comparing uncertain attributes represented by random variables. They utilize stochastic dominance to eliminate dominated alternatives, which compares attribute distributions and is consistent with utility theory. They then demonstrate their approach in the context of a gear box example.

Various authors have proposed methods for eliminating dominated alternatives in problems considering uncertainty. However, these tend to be ad-hoc approaches that are not grounded in a rigorous decision theory. Mattson et al. [18] account for uncertainty into Pareto dominance by worsening the attributes' means based on their corresponding standard deviations, creating a conservative shifted efficient set. However this approach can be over-conservative in identifying the non-dominated alternatives. Iyer [19] employs toleranced dominance rules which evaluate Pareto dominance with a tolerance parameter in the dominance evaluations. However no method defines how to assign the tolerance parameter value. Furthermore using this approach it is only possible to identify a set of alternatives guaranteed to contain all non-dominated points (and possibly some dominated points) or a set of alternatives guaranteed to contain only non-dominated points (but possibly missing some non-dominated points). This inability to identify only the non-dominated set makes this approach undesirable for generating reusable tradeoff studies under uncertainty. Teich et al. [20] modify the Pareto dominance criterion by representing the attributes as random variables defined by uniform distributions and calculating the probabilities of an alternative dominating another. An alternative dominates another when, for each attribute, the worst-case interval bound of the dominating alternative is better than the best-case interval bound of the dominated alternative. This approach is limited in the assumption of a uniform distribution. Hughes et al. [21] represent the attributes as random variables defined by normal distributions and calculate the probability of one dominating another to determine an alternative's rank. Generally all of these approaches are similar to each other by they blur the non-dominated frontier and apply Pareto dominance rules. When considering uncertainty, Pareto dominance is insufficient because of its inability to consider the risk attitude of the designer. Thus these approaches are inadequate in eliminating dominated alternatives.

In this thesis I adopted a probabilistic approach to handling uncertainty. A completely different approach to handling decision problems under uncertainty is fuzzy logic. Various research has been conducted in this area [22-25]. The justification for taking the probabilistic approach is based on the Dutch book argument (DBA) [26]. A Dutch book, defined in terms of a gamble, is a situation where it is possible structure wagers such that it will result in certain loss. It has been shown the axioms of probability set forth by Kolmogorov [27] are necessary and sufficient conditions to avoid the ability to construct a Dutch book.

1.3 Contributions

To address the problem described in the previous sections, this thesis presents and demonstrates a methodology for composing reusable component tradeoff-space models into system-level tradeoff studies and a method to reduce the computational expense associated with accounting for uncertainty, which I term *efficient Monte Carlo sampling*. The proposed methodology is built on the framework of composing tradeoff-space models under deterministic conditions, stochastic dominance, the dominance parameterization technique, and statistical hypothesis tests presented in literature. Stochastic dominance compares alternatives under uncertainty, the parameterization technique allows for reusability, and the hypothesis tests enable intelligent uncertainty propagation to reduce computational expense. The demonstration of the methodology and comparison of the results to a traditional fully-integrated design approach provides support for the methodology and it's effectiveness.

1.4 Thesis Contents

The remainder of this thesis is organized as follows. In the first part of Chapter 2 a more in-depth background on the dominance criterion under deterministic conditions and uncertainty is presented. This includes background on Pareto dominance for tradeoff studies under deterministic conditions, stochastic dominance for tradeoff studies under uncertainty, and the considerations taken when generating reusable tradeoff studies. Then in the second part, the statistical hypothesis tests used in the efficient Monte Carlo sampling method are presented.

In Chapter 3 the composition of system-level tradeoffs is presented in detail, including the generation of reusable tradeoff studies and the composition of them into system-level tradeoff studies. Then the efficient Monte Carlo sampling method is defined.

In Chapter 4 the composition of system-level tradeoffs is demonstrated in the context of an automobile drivetrain example. Reusable component tradeoff-space models are generated for transmission and differential concepts and composed into a system-level tradeoff study. A fully-integrated design approach to the drivetrain system tradeoff study is generated for comparison. Then the efficient Monte Carlo sampling method is demonstrated in the context of the automobile transmission. The method is compared to an approach where the number of Monte Carlo samples is fixed.

Chapter 5 presents and discusses the results of the demonstrations. Following the discussion, future work is presented.

2. TECHNICAL BACKGROUND

This section describes the technical background and tools used in the composing system-level tradeoff studies under uncertainty methodology and efficient Monte Carlo sampling method developed in this thesis. First the background on tradeoff studies is presented, and then the background on the statistical tests used in this thesis is presented.

2.1 Tradeoff Studies

The term tradeoff refers to a decision-making situation that involves worsening one quality or aspect of a system in return for improving another quality or aspect [28]. For example, in designing a structure, an engineer may be unable to reduce the weight of the structure without sacrificing the structure's strength. In this thesis, I refer to the qualities or aspects by which designers evaluate designs (figures of merit, criteria, performance metrics, etc.) as *attributes*. Mapping component concepts to a tradeoff space, where under deterministic conditions each dimension represents a design attribute, allows one to visualize various alternative design approaches. Designers then use the tradeoff information to support decision making. They may use a utility function to make a decision, or visualize the tradeoff information then select the final design, or may decide to develop additional concepts after viewing a variety of feasible alternatives. Some authors refer to visualizing the tradeoff information to make a decision as "design by shopping" [1].

A basic premise for this thesis is that designers should make decisions systematically and using methods that are sound with respect to the accepted norms of decisions theory. In particular it is built on the framework of multi-attribute utility theory (MAUT) [29], which is an extension of utility theory by von Neumann and Morgenstern [30]. Expected utility theory defines six axioms of "rationality" such that when they are satisfied there exists a utility function which reflects one's preferences between alternatives. The utility is a function of the design attributes, where an alternative with a greater utility value is preferred over another with a lesser utility value. Under uncertainty, the attributes are uncertain and a rational decision-maker prefers the alternative with maximum expected utility.

When visualizing or generating a mathematical model of the tradeoff space, it is beneficial to eliminate alternatives a rational designer would never select, known as dominated alternatives. The remaining alternatives are called efficient alternatives. Considering only the efficient set of alternatives eliminates clutter and allows designers to focus only on the alternatives of interest [31].

2.1.1 Dominance Analysis under Certainty

In deterministic problems designers compare alternatives solely on their attribute values since there is no associated uncertainty. The tradeoff space is a vector space defined by attributes for all alternatives. Figure 2 illustrates the deterministic generation of a tradeoff space. Two unique design spaces, which represent heterogeneous alternatives with unique design variables, for a design are mapped into the tradeoff space. Each design space represents a different concept with a unique design configuration. Since the tradeoff criteria are related to the designer's objectives, each concept for a component has the same tradeoff space. Each design implementation maps to a single point in the attribute space. Thus in this case the attribute space is referred to as the tradeoff space because attributes are all that is needed to test dominance.

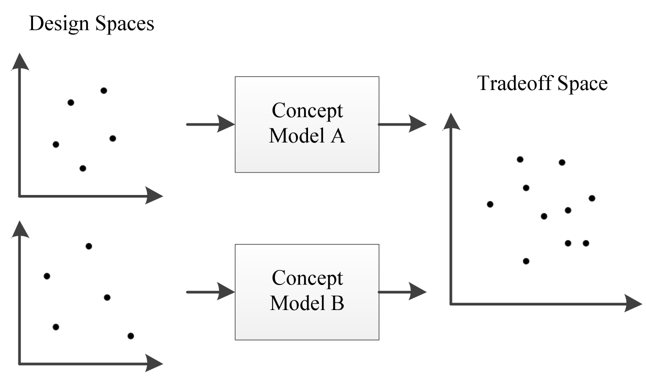

Figure 2. Deterministic generation of tradeoff space.

After defining the tradeoff space, dominated alternatives are eliminated using Pareto dominance, which is well documented [1, 29, 32]. The Pareto dominance criterion eliminates an alternative when there exists another alternative that is better in at least one attribute and at least as good in all other attributes.

The mathematical definition of Pareto dominance criteria requires defining some notation. Suppose a designer's preferences are monotonically increasing[1], that is the utility function is increasing, in each decision attribute x_i for $i = 1,2, \dots, N$, associated with a particular alternative. If $x = [x_1, x_2, \dots, x_N]$ denotes an attribute vector and X is the set of all attribute vectors, then Pareto dominance can be expressed as

> Pareto dominance [1]: an alternative with attribute vector $x'' \in X$ is said to be Pareto dominated by one with attribute vector $x' \in X$ if and only if $x_i' \geq x_i'' \forall i = 1,2, \dots, N$ and $x_i' > x_i'' \exists i = 1,2, \dots, N$.

The alternative with attribute vector x'' is dominated by alternative with attribute vector x' because Pareto dominance guarantees $u(x') > u(u'')$, where $u(\cdot)$ represents the utility function of the designer.

2.1.2 Dominance Analysis under Uncertainty

In uncertain problems, the attributes are random variables with associated probability distributions. The first part of Figure 3 illustrates the mapping of the design space to the attribute space containing alternatives' distributions. As such, the comparisons of alternatives are more involved. One cannot perform the deterministic comparisons required by the Pareto dominance criteria. The direct comparison of random variable attributes becomes meaningless, or at least requires additional qualification to have

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[1] Although stating monotonically increasing preferences, this is without loss of generality

meaning. Employing stochastic dominance to enables the comparison alternatives under uncertainty.

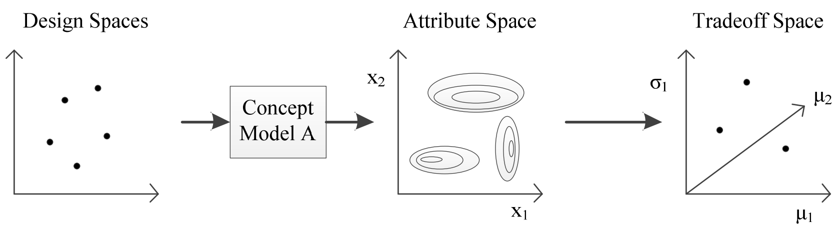

Figure 3. Mapping of design space to attribute space to tradeoff space under uncertainty. The tradeoff space is a 3D visualization of the tradeoff space that is higher dimensional.

Stochastic dominance has been primarily used in the areas of economics, finance, and statistics [33, 34]. Multiple stochastic dominance rules exist, each of which is appropriate for different types of decision making preferences (e.g. risk seeking vs. risk averting), which depend on the mathematical structure of the utility function employed. With the exception of Malak et al. [16, 35] , stochastic dominance has not been applied to engineering design problems.

Using stochastic dominance, designers compare two cumulative distribution functions and establish an order of preference between the two. Univariate stochastic dominance compares single-attribute distribution functions and identifies conditions in which $E[u(x')] > E[u(x'')]$, where $E[u(\cdot)]$ is the expected utility of an alternative and x' and

x'' are values of a single attribute for different alternatives. Univariate stochastic dominance has been well documented [33]. Three common univariate stochastic dominances classes have been defined: first-degree stochastic dominance (FSD), second-degree stochastic dominance (SSD), and third-degree stochastic dominance (TSD). Table 1 is a summary of the three univariate stochastic dominance classes with their assumptions and dominance criteria. Classes are defined by the type of utility function employed, which in the case of reusable component tradeoff space models is based on the assumption of what type of utility function a designer may have when composing system-level tradeoff studies. Each successive class is a subset of the previous; that is each successive class inherits the previous class's restrictions in addition to new ones. For example, an efficient set in FSD will have alternatives which are considered dominated and eliminated under SSD. It is best to evaluate alternatives with the most restrictive class justifiable by the intended use of the solutions in order to reduce problems with identifying the efficient set with less restrictive assumptions [36].

Table 1. Summary of three common univariate stochastic dominance classes.

Class	Utility Function Assumptions	Interpretation	Dominance Criterion
U_0	None	All utility functions	None
U_1	$= \left\{ u \in U_0 \,\middle\|\, \frac{du(x)}{dx} \geq 0 \right\}$	Monotonic utility functions	Option a with CDF F_a dominates option b with CDF F_b if and only if for all x: $F_a(x) \leq F_b(x)$
U_2	$= \left\{ u \in U_1 \,\middle\|\, \frac{d^2u(x)}{dx^2} \geq 0 \right\}$	Monotonic and non-risk taking utility functions	Option a with CDF F_a dominates option b with CDF F_b if and only if for all x: $\int_{-\infty}^{x} [F_b(t) - F_a(t)]dt \geq 0$
U_3	$= \left\{ u \in U_2 \,\middle\|\, \frac{d^3u(x)}{dx^3} \geq 0 \right\}$	Monotonic, non-risk taking, and decreasing absolute risk aversion utility functions	Option a with CDF F_a dominates option b with CDF F_b if and only if for all x: $\int_{-\infty}^{x} \int_{-\infty}^{v} [F_b(t) - F_a(t)]dtdv \geq 0$

The stochastic dominance classes described in Table 1 are for single-attribute decisions. This is rarely the case in engineering design where designers often must make decisions based on multiple attributes. Research has expanded univariate stochastic dominance to multivariate stochastic dominance to apply to multi-attribute problems. However multivariate stochastic dominance is a multiplex set of rules which can be difficult to apply [37-42]. In this case, I use the assumption of attribute marginal independence to simplify the stochastic dominance rules. Assuming the alternatives' attributes are independent of each other is a mathematically well-defined assumption which allows one to compare the marginal distributions for each attribute individually using the appropriate univariate stochastic dominance rules. This is not a perfect assumption for all engineering problems; however it is a good approximation for many. This assumption is used in this thesis and a more robust investigation is left for future work.

Mathematically expressing the multivariate stochastic dominance criterion with independence assumption requires defining some additional notation. Let $\geq_n^d$ denote nth degree stochastic dominance for d-dimensional distribution functions. Let A and B represent two decision alternative whose attributes are represented by multivariate distribution function $F_A(x)$ and $F_B(x)$, respectively. Let $F_{A,i}(x)$ and $F_{B,i}(x)$ represent the marginal distributions for the ith dimension where $i = 1,2, \ldots, d$ for $F_A(x)$ and $F_B(x)$, respectively.

> Multivariate stochastic dominance [43]: if $F_A(x)$ and $F_B(x)$ are marginally independent, then $F_A(x) \geq_n^d F_B(x)$ if and only if $F_{A,i}(x) \geq_n^1 F_{B,i}(x) \forall i = 1,2, \ldots, d$.

One can define several common random variable distributions with a few parameters (e.g. mean and variance for normal distributions, interval boundaries for uniform distributions). If any of these common probability distributions are a good model for the attribute distribution, designers can map the attribute space to a space of distribution parameters. In this case we define the tradeoff space as the space of distribution parameters because we can evaluate dominance using SSD by the alternatives' distribution parameters. The second part of Figure 3 shows the mapping of the attribute distributions to the tradeoff space in terms of the distribution parameters.

If one assumes the distributions are normal, the SSD criterion simplifies to comparing the corresponding means (μ_A and μ_B) and variances (σ_A^2 and σ_B^2) [16]. This assumption largely simplifies the mathematics in applying SSD. When using this assumption, it is important to collect enough data to produce a reliable estimator of the true mean and variance, which is a topic discussed in the following chapter. Univariate stochastic dominance criterion for normal distributions eliminates alternatives in which another alternative has a better or equal mean and lesser or equal variance where at least one comparison is strict.

> Univariate normally-distributed second-degree stochastic dominance [44-46]: if $F_A(x)$ and $F_B(x)$ are normally-distributed then $F_A(x) \geq_2^1 F_B(x)$ if and only if $\mu_A \geq \mu_B$ and $\sigma_A^2 \leq \sigma_B^2$ where at least one of the inequalities is strict.

The case of the uniform distributions is a direct analog of the normal distribution case. If one assumes the distributions are uniform, the SSD criterion simplifies to comparing the lower bounds (a_A and a_B) and upper bounds (b_A and b_B). Univariate stochastic dominance criterion for uniform distributions eliminates alternatives in which another alternative has a better or equal mean and lesser or equal range where at least one comparison is strict.

> Univariate uniformly-distributed second-degree stochastic dominance: if $F_A(t)$ and $F_B(t)$ are uniformly-distributed, then $F_A(t) \geq_2^1 F_B(t)$ if and only if $\frac{a_A+b_A}{2} \geq \frac{a_B+b_B}{2}$ and $b_A - a_A \leq b_B - a_B$ where at least one of the inequalities is strict.

The proof of the univariate uniformly-distributed second-degree stochastic dominance is given in Appendix A.

When determining dominance conditions of normal or uniform distributions, one can use the normally-distributed SSD criterion for both cases, since the uniform distribution with the greater range will also have the greater variance. Mathematically proving this relationship can be shown starting with the range comparison from the uniformly-distributed SSD definition and transforming it into the variance comparison from the normally-distributed SSD definition, as shown below.

$$b_1 - a_1 \leq b_2 - a_2$$

$$(b_1 - a_1)^2 \leq (b_2 - a_2)^2$$

$$\frac{1}{12}(b_1 - a_1)^2 \leq \frac{1}{12}(b_2 - a_2)^2$$

$$Var(F_1) \leq Var(F_2)$$

In this thesis, SSD criterion is used to identify the efficient set. The simplified SSD criteria make intuitive sense. Often an engineer is willing to tradeoff the mean performance in order to reduce variability or range in an attribute. For example, in the design of a mechanical device, an engineer may be willing to tradeoff the mean lifetime performance in order to reduce lifetime variability. This type of tradeoff would be useful in determining warranty periods, maintenance schedules, etc., and is common in robust design [47, 48]. Thus representing alternatives in the tradeoff space by distribution parameters can be a good representation of the criteria which designers base decisions.

Figure 4 illustrates the differences in data flow when generating the efficient set of data for deterministic and uncertain problems, where the tradeoff exploration method under uncertainty utilizes second-order stochastic dominance with the statistical independence and distribution parameterization assumptions. Boxes with dashed lines represent the set of alternatives in the different spaces, and boxes with solid lines represent operations on the set.

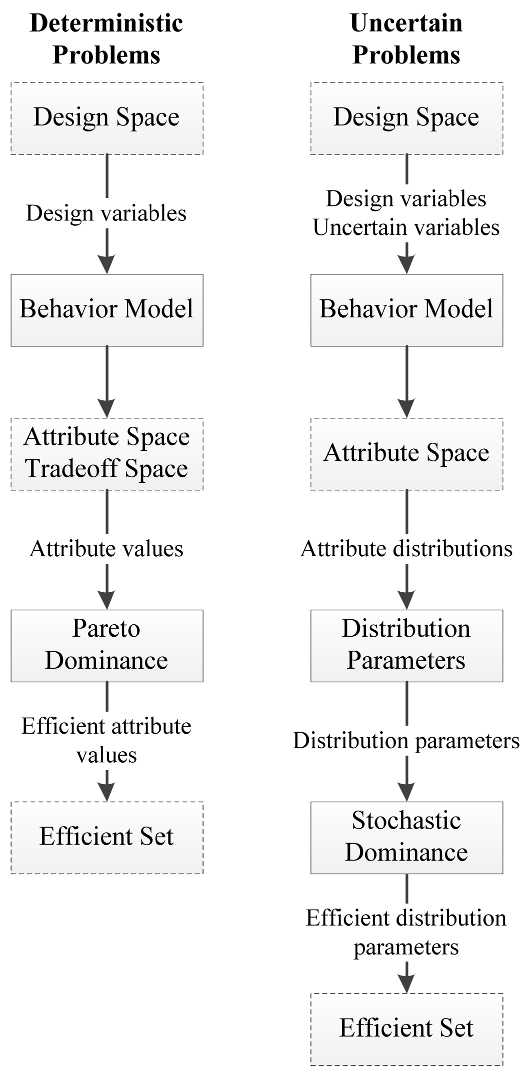

Figure 4. Efficient set generation flow chart for deterministic and uncertain decisions.

2.1.3 Composing Tradeoff Studies with Parameterized Efficient Sets

A principal assumption in the dominance comparisons presented thus far is one's preference in each tradeoff criterion is monotonic. That is to say one prefers to minimize or maximize every criterion, such as minimizing cost and maximizing reliability.

Problems arise in generating reusable component tradeoff studies where one has non-monotonic preference in some tradeoff criteria. Often these preferences arise from interfaces and when system-level criteria are mapped into subsystem-level criteria. Non-monotonic preferences can be “target seeking” or “target avoiding” and are specific to a design problem. Examples of “target seeking” preference is the gear ratio of a gear box. The target gear ratio comes from the system level of which the gear box is a part. Some system-level objectives will imply increasing the gear ratio, such as maximizing torque, while others imply decreasing the gear ratio, such as maximizing speed. Other examples of target seeking preferences include cylinder bore diameter [49], suspension spring constant [50], and heat exchange pipe diameter [51]. Examples of “target avoiding” preferences would be to avoid natural frequencies that lead to adverse vibrational and resonance effects.

Since the target values of non-monotonic preferences are problem specific, there is no method to identify the efficient set without that information. This limits the ability to generate reusable component tradeoff studies. The solution is to identify parameterized efficient sets. A parameterized efficient set is a collection of efficient sets each identified by their non-monotonically preferred attributes, which are called *parameters*. Monotonically preferred attributes are called *dominators*. Parameterized efficient sets under deterministic conditions, called parameterized Pareto sets, have been successfully utilized [32].

Building upon the classical Pareto dominance definition, let D denote the nonempty set of indices for the dominator attributes and P denote the set of indices for the parameter attributes. Parameterized Pareto dominance can be expressed as:

> Parameterized Pareto dominance [32]: an alternative with attribute vector $x'' \in X$ is parametrically dominated by one with attribute vector $x' \in X$ if and only if $x_i' = x_i'' \forall i \in P$,$x_i' \geq x_i'' \forall i \in D$, and $x_i' > x_i'' \exists i \in D$.

Once a parameterized Pareto set is generated, a designer can utilize it in a specific design by using the specific problem information to reduce it to the appropriate efficient set. However parameterized efficient sets have not been extended to the case of tradeoffs under uncertainty. In the next chapter, this thesis combines the stochastic dominance principles and parameterized efficient sets structure introduced here in a novel way in order to enable generating reusable component tradeoff space models for composing unique system-level tradeoff studies.

2.2 Statistical Tests

In engineering problems, the true attribute distribution parameters are generally unknown. Analytically propagating uncertainty from input design and environmental variables to the design attributes is difficult to apply in complex models. Typically propagation methods entail a sampling-based method (Monte Carlo, quasi-Monte Carlo) [14, 52], which are used to produce estimators of the true distribution parameters. The

Monte Carlo sampling procedure is illustrated in Figure 5. As such, it is important to collect enough samples to produce a reliable estimator of the true mean and variance for the stochastic dominance criterion.

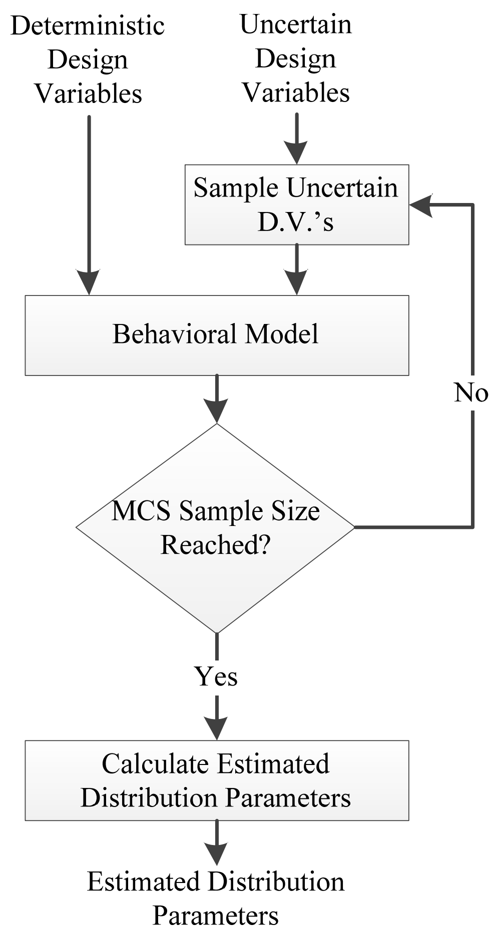

Figure 5. Random Monte Carlo sampling procedure.

When using distribution parameter estimators to evaluate second-order stochastic dominance criterion, it is necessary to consider the sampling distribution of the estimators. The SSD criterion consists of comparing differences in distribution parameters. In the case of normal distribution, these comparison tests are well documented [53]. Using these statistical tests, this thesis implements an efficient sampling method to determine dominance conditions using only the necessary number of samples.

In this thesis I use pseudorandom Monte Carlo sampling when applying these statistical hypothesis tests to reduce the total number of behavioral model samples. Other quasi-Monte Carlo sampling methods of variance reduction techniques, or methods to reduce the sample variance in estimating true distribution parameters, exist as another way to reduce sampling requirements. The statistical hypothesis tests can be incorporated quasi-Monte Carlo sampling methods in which the number of samples does not need to be known before initiating the sampling, such common random numbers [17]. Methods which require knowing the sample size before initiating the sampling, such as Latin hypercube, are difficult to integrate with these hypothesis tests because they require testing the distributions after each sample. The incorporation of these statistical hypothesis tests into other quasi-Monte Carlo sampling methods is left for future work.

The background on the statistical tests used in the efficient Monte Carlo sampling method presented in this thesis are described below. First the appropriate hypothesis tests for determining dominance using SSD are described. Then tools to allow for exceptions where the difference between distribution parameters are zero or negligible are described.

2.2.1 Difference in Sampling Distribution Parameters Hypothesis Testing

The normally distributed univariate SSD criterion consists of comparing the distributions' means and variances. When sampling to produce estimators for the mean and variance of a distribution, it can be shown the two estimators are independent [54]. This allows one to test the univariate stochastic dominance criterion as two separate tests for the difference in mean and the difference in variance. This section presents the hypothesis tests utilized in this thesis to compare estimators.

2.2.1.1 Difference of Means Test

Let population $A \sim N(\mu_A, \sigma_A^2)$ and population $B \sim N(\mu_B, \sigma_B^2)$. Let n_A and n_B samples be taken from population A and B, respectively. Let $\bar{x}_A$ and s_A^2 denote the sample mean and variance, respectively, for population A. Let $\bar{x}_B$ and s_B^2 denote the sample mean and variance, respectively, for population B. The appropriate hypothesis test is stated:

$$H_0: \mu_A = \mu_B$$

$$H_1: \mu_A > \mu_B$$

$$H_2: \mu_A < \mu_B$$

There is not an exact test statistic for these hypotheses, however, the statistic T_0^* is distributed approximately as t with degrees of freedom v if the null hypothesis is true, where

$$T_0^* = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

$$v = \frac{\left(\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}\right)^2}{\frac{(s_A^2/n_A)^2}{n_A + 1} + \frac{(s_B^2/n_B)^2}{n_B + 1}}$$

The hypotheses are tested as one-sided with significance level α. One rejects the null hypothesis and accepts the mean of population A is greater than population B (H_1) if

$$T_0^* > t_{\alpha,v}$$

Or one rejects the null hypothesis and accepts the mean of population B is greater than population A (H_2) if

$$T_0^* < -t_{\alpha,v}$$

Otherwise one fails to reject the null hypothesis. In this case more data is necessary to detect a difference between the sample means [53].

2.2.1.2 Difference of Variances Test

Using the same notation form the difference of mean test, the hypothesis test is stated:

$$H_0: \sigma_A^2 = \sigma_B^2$$

$$H_1: \sigma_A^2 < \sigma_B^2$$

$$H_2: \sigma_A^2 > \sigma_B^2$$

For these hypotheses, the test statistic F_0 is distributed as F with $n_A - 1$ numerator and $n_B - 1$ denominator degrees of freedom if the null hypothesis is true, where

$$F_0 = \frac{s_A^2}{s_B^2}$$

The hypotheses are tested as one-sided with significance level α. One rejects the null hypothesis and accepts the variance of population A is less than population B (H_1) if

$$F_0 < f_{1-\frac{\alpha}{2}, n_A-1, n_B-1}$$

Or one rejects the null hypothesis and accepts the variance of population B is less than population A (H_2) if

$$F_0 > f_{\frac{\alpha}{2}, n_A-1, n_B-1}$$

Otherwise one fails to reject the null hypothesis. In this case more data is necessary to detect a difference between the sample variances. Note f_{α,d_1,d_2} is evaluated or extrapolated as the upper-tail points [53].

2.2.2 Maximum Sample Size

When evaluating hypothesis tests it is good practice to not only consider the statistical significance, α, but also the statistical power, $(1-\beta)$. Statistical power defines the probability of accepting a false null hypothesis. Operating characteristic curves relate sample size with the statistical power of the hypothesis test at a given significance. This section presents the operating characteristic curves for the hypothesis tests presented in this thesis which are used to limit the total number of samples taken when comparing identical or similar distributions.

2.2.2.1 Operating Characteristic Curves for Mean Comparison

Operating characteristic curves have been developed for the t-test used in the means hypothesis testing. The curves are determined for the case when the true variances of each population are equal for various $n = n_A = n_B$. Unfortunately if the true variances are different, the distribution of the test statistic is unknown when the null hypothesis is false, and no operating characteristic curves are available for that case.

However in some engineering problems, alternatives have similar variances and using the operating characteristic curves assuming equal true variances will present a rough estimate of the number of samples necessary. The operating characteristic curves for the one-sided t-test for significance levels 0.05 and 0.01 are shown in Figures 6 and 7, respectively. When using the curves, they must be entered with the sample size

$$n^* = 2n - 1$$

Or when determining the number of samples necessary, n, they must be calculated using the n^* value found on the operating characteristic curve, where

$$n = \frac{n^* + 1}{2}$$

The operating characteristic curves use the standardized distance d to measure the difference in mean where

$$d = \frac{\mu_1 - \mu_2}{2\sigma}$$

It is noted d is a parameter of σ, which is unknown. In this case, one may have to rely on a prior estimate or subjective estimate of σ to determine d, or define the difference in terms relative to σ [53].

For example, suppose a designer is performing the hypothesis tests with $\alpha = 0.01$. The designer wishes to detect differences between alternative means with a statistical power of 0.99 (probability of accepting false null hypothesis is $\beta = 0.01$). Additionally the minimum difference between the true means a designer wants to detect is one true standard deviation. Then $d = 0.5$, and evaluating at $\beta = 0.01$ gives approximately $n^* = 100$. Thus the designer will need to take at least $n = \frac{100+1}{2} = 50.5 \approx 51$, samples before terminating additional sampling if no difference between means is found.

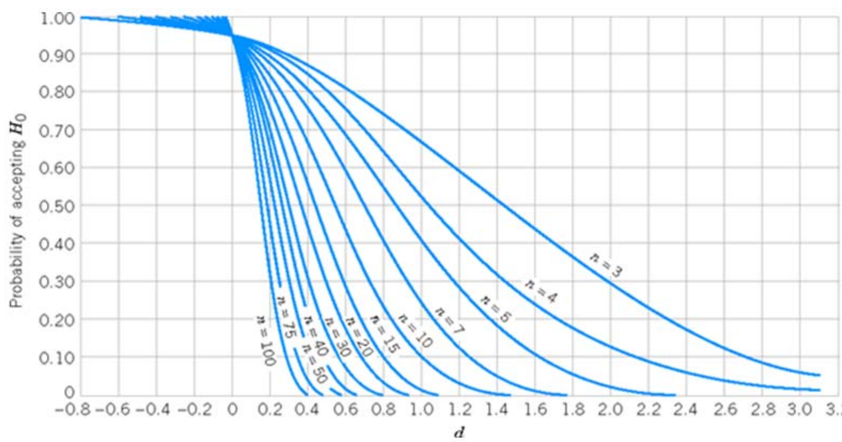

Figure 6. Operating characteristic curve for the one-sided t-test with significance level $\alpha = 0.05$ [53].

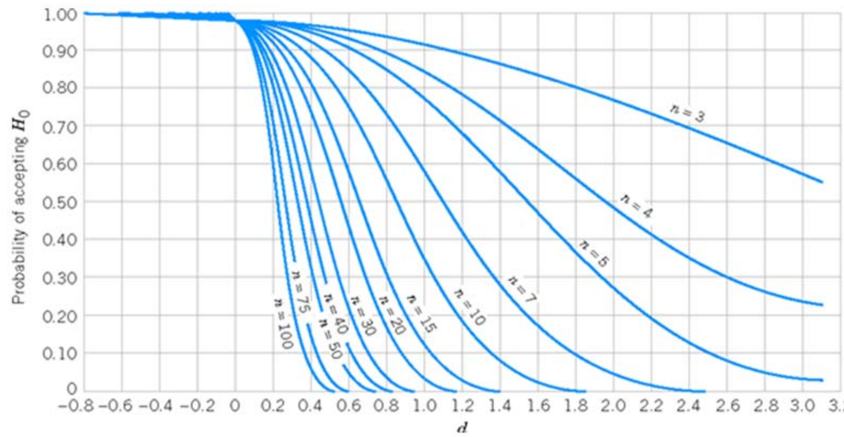

Figure 7. Operating characteristic curve for the one-sided t-test with significance level $\alpha = 0.01$ [53].

2.2.2.2 Operating Characteristic Curves for Variance Comparison

Operating characteristic curves have been developed for the F-test used in the variances hypothesis testing for various $n = n_A = n_B$. The operating curves for the one-sided F-test for significance level 0.05 and 0.01 are shown in Figures 8 and 9. The operating curves use the variable λ to measure the difference in variances where [53]

$$\lambda = \frac{\sigma_1}{\sigma_2}$$

For example, suppose a designer wishes to detect differences between alternative variances with a statistical power of 0.90 (probability of accepting false null hypothesis is $\beta = 0.10$). Additionally the minimum difference between the true variances a designer wants to detect is when one population standard deviation is double the other population standard deviation. Then $d = 2.00$, and evaluating at $\beta = 0.1$ gives approximately $n = 31$. Thus the designer will need to take at least 31 samples before terminating additional sampling if no difference between variances is found.

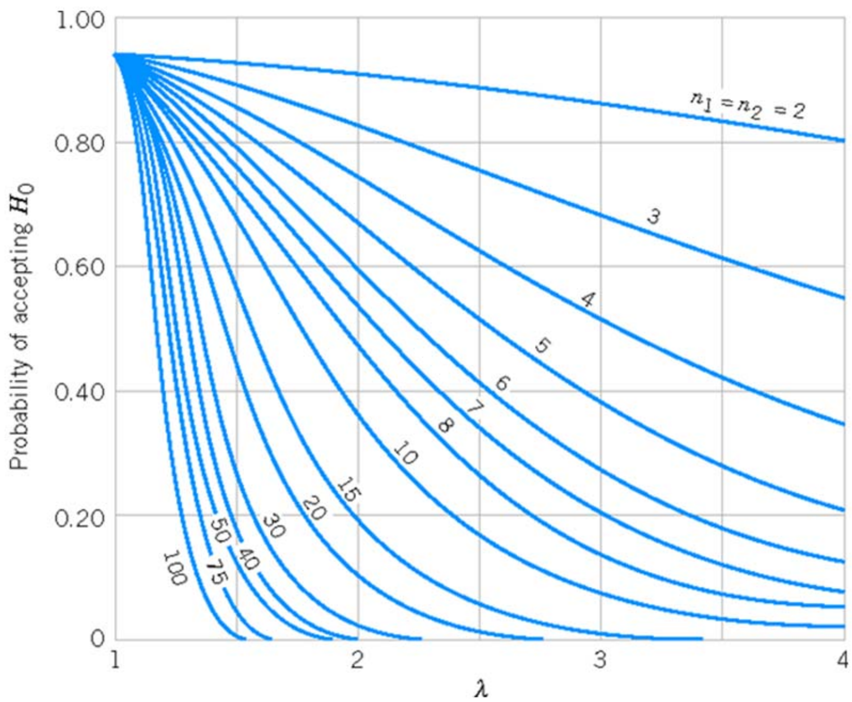

Figure 8. Operating characteristic curve for the one-sided F-test with significance level $\alpha = 0.05$ [53].

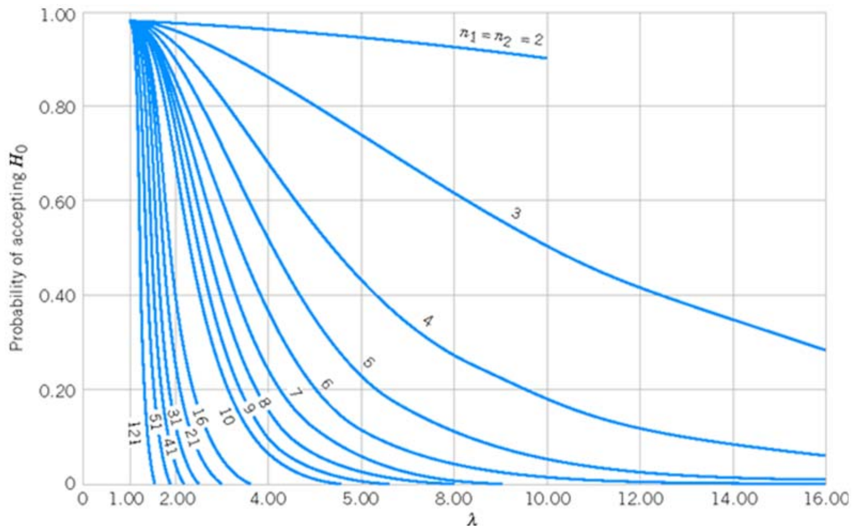

Figure 9. Operating characteristic curve for the one-sided F-test with significance level $\alpha = 0.01$ [53].

2.2.3 Hypothesis Testing Examples

Simple demonstrations illustrate the efficient hypothesis dominance testing procedures. Below the means and variances hypothesis tests are demonstrated for simple defined alternatives.

2.2.3.1 Difference of Means Example

Consider two alternatives A and B such that

$$A \sim N(0,1)$$

$$B \sim N(1,1)$$

Suppose one wants to determine which distribution has the higher mean without knowing the true distribution parameters. In this case, the difference of means hypotheses tests should indicate B has a greater mean.

Using the efficient sampling approach was simulated 100,000 times to analyze the number of samples necessary to determine orderings using significance level $\alpha = 0.01$. For each trial the initial sample size was two samples from both populations, which was tested using the difference in means hypotheses and incrementally increasing the number of samples for both populations until a difference in means was found. The resulting distribution of samples sizes is shown in Figure 10. Table 2 lists various statistics about the distribution.

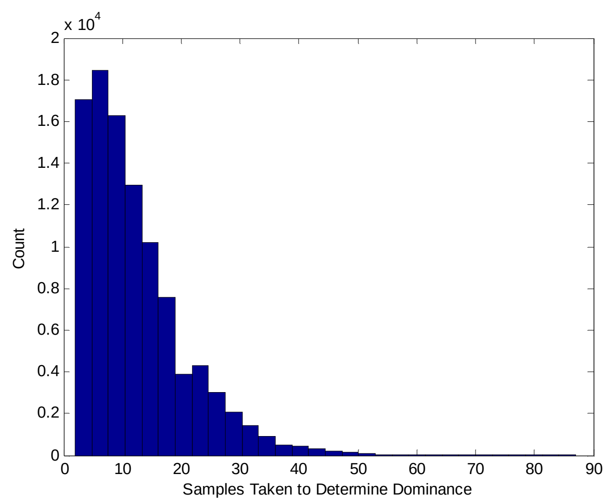

Figure 10. Histogram of sample size necessary to determine $\bar{x}_B > \bar{x}_A$.

Table 2. Number of samples necessary to determine mean ordering distribution statistics.

	Number of Samples
Mean	12.3
Min	2
Max	87
$n > 51$	0.18% (178)

Consider if we are willing to accept a Type II error rate of $\beta = 0.01$. Using the operating characteristic curves (Figure 7), the minimum number of samples necessary to detect the difference between the two populations is approximately 51 ($d = 0.05$; $n^* = 100$; $n = \frac{100+1}{2} \approx 51$). If a fixed number of Monte Carlo samples were used, 51 model evaluations would be necessary to ensure the statistical power. This value turns out to be quite conservative, as only 0.18% of the trials required more than 51 samples. This was caused by inexactness in reading values off the graph, where the correct n^* is between 100 and 75 (n is between 38 and 51). The benefit of the efficient approach is by limiting the sample size to 51, the average sample size is 12.3 samples per trial, which is a significant reduction in the number of model evaluations compared to a fixed sample size of 51.

2.2.3.2 Difference of Variances Example

Consider two alternatives A and B such that

$$A \sim N(0,1)$$

$$B \sim N(0,2)$$

Suppose one wants to determine which distribution has the lower variance without knowing the true distribution parameters. In this case, the difference of variances hypotheses tests should indicate A has a lesser variance.

As in the difference of means example, the efficient sampling approach was simulated 100,000 times to analyze the number of samples necessary to determine orderings using significance level $\alpha = 0.01$. The testing approach is analogous to the differences of means example. The resulting distribution of samples sizes is shown in Figure 11. Table 3 lists the various statistics about the distribution.

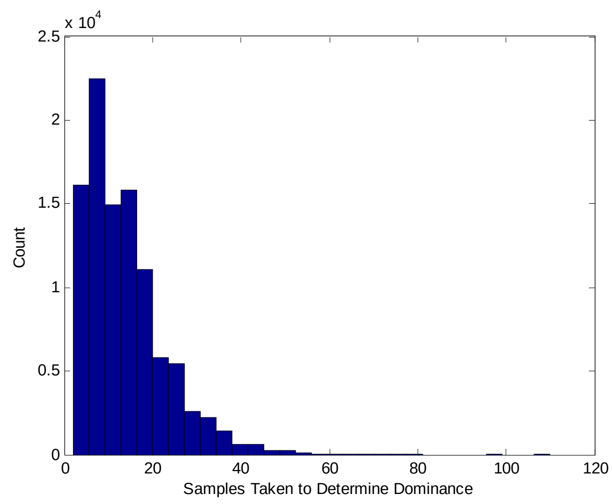

Figure 11. Histogram of sample size necessary to determine $\sigma_A^2 < \sigma_B^2$.

Table 3. Number of samples necessary to determine variances ordering distribution statistics.

	Number of Samples
Mean	13.7
Min	2
Max	110
$n > 45$	0.74% (738)

Consider if one is willing to accept a Type II error rate of $\beta = 0.01$. Using the operating characteristic curves (Figure 9), the minimum number of samples necessary to detect the difference between the two populations is approximately 45 ($\lambda = 2.00$). If a fixed number of Monte Carlo samples were used, 45 model evaluations would be necessary to ensure the statistical power. This value is reinforced by the example where 0.74% of trails required more than 45 samples to detect the difference. The benefit of the efficient approach is by limiting the sample size to 45, the average sample size is 13.7, which is a significant reduction in the number of model evaluations compared to a fixed sample size of 45.

3. PROPOSED METHODOLOGY

Two extensions to existing tradeoff methodology are presented in this thesis. First is the methodology for composing tradeoff studies under uncertainty. Second is the method for efficient identification of the efficient set under uncertainty which can be utilized in the methodology for composing tradeoff studies under uncertainty.

3.1 Methodology for Composing Tradeoff Studies under Uncertainty

Composing tradeoff studies under uncertainty is separated into two distinct sections: construction the subsystem-level tradeoff studies in a reusable manner, and composing the system-level tradeoff study. An overview of generating the subsystem-level tradeoff studies follows the steps:

1. Explore component design space
2. Map the attribute space to the tradeoff space using common distribution parameters to represent the probability distributions
3. Identify the parameterized efficient set using second-degree stochastic dominance rules
4. Fit predictive model to parameterized efficient set
5. Define domain in which predictive model is valid

An overview of composing the system-level tradeoff study follows the step

1. Develop system-level behavioral models

2. Sample the component predictive models within their valid domain
3. Transform the subsystem tradeoff criteria values to the system tradeoff criteria values
4. Identify system-level efficient set using second-degree stochastic dominance rules

3.1.1 Generating Reusable Component Tradeoff Models

Step 1: Explore component design spaces to generate representative samples of possible alternatives. When exploring the subsystem-level design spaces, the designer must sample the entire domain in which they are considering. Various sampling approaches can be used, including but not limited to random, quasi random, uniform, Latin hypercube, Monte Carlo, or via a genetic search. Identify attributes and categorize them as dominator or parameters for evaluating parameterized stochastic dominance. The attributes are calculated using standard engineering analysis and uncertainty propagation methods. Any resulting infeasible designs are eliminated at this step.

Step 2: Map the attribute space to the tradeoff space using common distribution parameters to enable dominance reasoning. It is important to verify each attribute's marginal distribution. This can be evaluated using a formal statistical goodness of fit test [55] or informal visual inspection. Sampling a few alternatives to verify attribute distributions is sufficient as each go through the same transformations in the concept behavioral models. Incorrectly describing an attribute's distribution will propagate error into the system-level attribute space. For example, representing a log-normal distribution as a normal distribution will not give accurate results.

Step 3: Identify the parameterized efficient sets using second-degree stochastic dominance to eliminate inferior alternatives. Designers must verify the assumption associated with the dominance rules. The non-risk taking assumption is reasonable in many engineering design problems, however one must verify this by considering their specific utility function and verifying the first and second derivatives are great than or equal to zero for all values. In this thesis, it is assumed the attribute distributions can be described by mean and variance parameters to satisfy the normality assumption. Designers should confirm these assumptions for any design.

Multivariate parameterized second-degree stochastic dominance is used to identify the parameterized efficient set. This is a key contribution of this thesis and is the extension of multivariate second-degree stochastic dominance rules based on the parameterized Pareto dominance rule. The multivariate parameterized second-degree stochastic dominance criterion with the attribute marginal independence and normality or uniformity assumptions used in this thesis can be expressed as

> <u>Multivariate (multi-attribute) parameterized second-degree stochastic dominance:</u> if $F_A(x)$ and $F_B(x)$ are marginally independent and normally distributed, then $F_A(x)$ parametrically dominates $F_B(x)$ by multivariate

SSD if and only if $\mu_{A,i} \geq \mu_{B,i} \forall i \in D$, $\sigma_{A,i}^2 \leq \sigma_{B,i}^2 \forall i \in D \cup P$, $\mu_{A,i} = \mu_{B,i} \forall i \in P$ where at least one of inequalities is strict.

Under parameterized second-degree stochastic dominance an alternative is dominated by another alternative which has equal parameter attribute means, is equal or better in every dominator attribute mean and is equal or lesser in all attribute variances, where at least one of the inequality comparisons is strict. The reason the parameter attribute variances are not parameterized is because a risk-averse decision-maker always prefers to minimize variance in all attributes even if the attribute preference is non-monotonic.

Step 4: Fit predictive models to the subsystem-level parameterized efficient sets for reusability. Fitting a predictive model is a process by which a model is created or chosen to generalize the finite set of collect data to a continuous model. This provides helpful inferences in cases in which all effects have not been adequately quantified or understood [56]. Predictive modeling is also helpful in efficiently exploring large design space [57]. The model can be fit using regression analysis or interpolation methods. A corresponding validation is necessary to ensure an accurate model fit. When fitting a predictive model to the parameterized efficient set, one must define the parameter attributes as independent variables. Various approaches to fitting a predictive model to a set of data are possible, including regression and Kriging [58]. The designer must make some insight to what will provide the best results.

Step 5: Define the domain in which the predictive models are valid to ensure they are used to quantify data the model was fit to predict. A predictive model will generate values for any combination of inputs, thus it is important to constraint the range of inputs to the domain in which the model was generated. In this thesis, the support vector domain description (SVDD) method is used to define the domain in which our data is contained. See [59, 60] for information on the support vector domain description method.

3.1.2 Composing System Tradeoff Study

Step 1: Develop system-level behavioral models that compute the system-level attributes as a function of the component attributes.

Step 2: Designers must sample the component predictive models within their valid domains. In this thesis, the models are sampled within the SVDD boundaries fit around the component tradeoff study data. When sampling the model, it is left to the designer to decide the adequate number of sample sufficient to compose the system tradeoff study. In general, more complex systems with additional components will require more samples than systems with fewer components.

Step 3: Transforming subsystem to system tradeoffs is accomplished using the problem specific relationships between the subsystem and system-level attributes. Each system design implementation must be analyzed using standard engineering analysis and uncertainty propagation methods. Uncertainty may be propagated analytically when possible, or through the use of Monte Carlo simulations [17]. After propagating the uncertainty to the system-level, the designer must again verify each attributes' distributions and represent them with distribution parameters.

Step 4: Identifying the system-level efficient set is identical to the process we used in Step 3 for generating reusable component tradeoff studies.

3.2 Method for Efficient Uncertainty Propagation Sampling

Step 1 of generating reusable tradeoff models and Step 3 of composing tradeoff studies involves transforming uncertain design variables to uncertain attributes. In the basic approach to propagating this uncertainty, Monte Carlo sampling is used where the number of samples is fixed. After taking the specified number of MCS samples, the distribution parameters are calculated. This process is repeated for each design configuration tested. Dominance conditions are determined after design configurations are evaluated. This approach is illustrated on the left side of Figure 12.

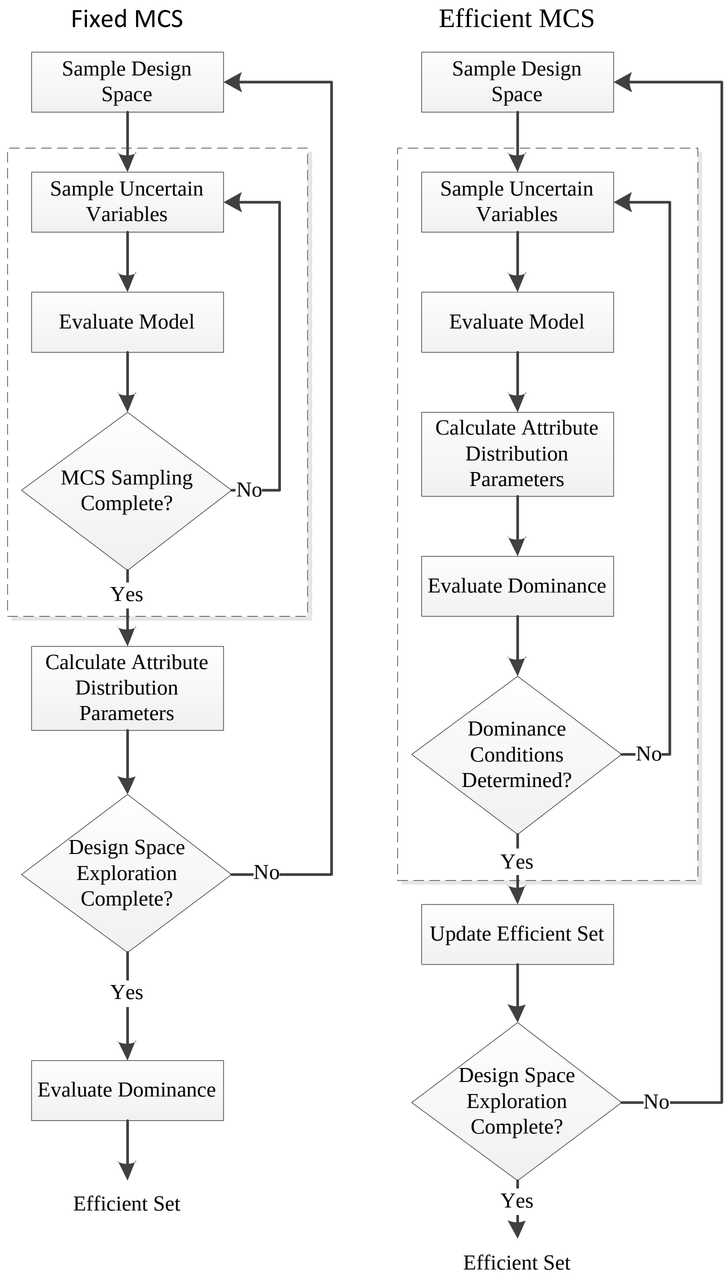

Figure 12. Comparison of generating tradeoff studies with fixed MCS sample sizes and efficient MCS sampling.

In the efficient sampling approach to generating a tradeoff study presented in this thesis, the dominance evaluation is incorporated inside the MCS, illustrated on the right side of Figure 12. After taking each MCS sample, the distribution parameters are calculated. It is important to verify each attribute's marginal distribution. Dominance is tested for the design configuration against the current efficient set using the hypothesis testing. Depending on the dominance conditions found, one of the following case procedures is followed.

Case A: If it is determined the current design configuration is dominated by *any* design in the efficient set

1. Discard current design configuration
2. Return to design space exploration

Case B: If it is determined the current design configuration dominates *any* design in the efficient set

1. Remove dominated design from efficient set
2. Add current design configuration to efficient set
3. Determine whether any other designs in the efficient set are dominated by the current design configuration using additional testing and sampling. Remove dominated designs from efficient set.
4. Return to design space exploration

Case C: If it is determined the current design configuration neither dominates nor is dominated by *all* designs in the efficient set

1. Add current design configuration to efficient set
2. Return to design space exploration

Case D: If dominance conditions cannot be determined for the current design configuration against a specific design in the efficient set

1. Take additional MCS samples, unless the maximum sample size is reached indicating negligible differences
    a. In the case of negligible differences, assume the current design configuration neither dominates nor is dominated by the specific design from the efficient set
2. After additional sampling, evaluate dominance conditions against efficient set again

The efficient sampling method is based on the assumption no two alternatives will have the same true distribution parameters, and if the hypothesis tests cannot determine a difference with the current sample size, additional sampling is needed. The hypotheses test are first tested using a small sample size from each alternative, and then the sample size is incrementally increased and retested until a difference can be found with a specified statistical significance. Once a difference is determined for both the means and variances tests, the univariate stochastic dominance rules are applied.

When comparing two alternatives and dominance cannot be determined, take additional MCS samples for the alternative with the least number of samples. If both designs have the same number of samples, sample both. This balances the number of samples taken for each alternative.

Incrementally increasing the sample size until the hypotheses testing detects a difference is based on the assumption that no two alternatives will have the same true distribution parameters. However situations will arise in which alternatives will have identical or negligible difference between distribution parameters. It is necessary for the designer to decide the acceptable probability of Type II error (β); the probability one accepts a false null hypothesis.

To account for these situations, one can limit the total number of samples taken based on the computation and time constraints, engineering intuition, or through the use of operating characteristic curves. Additionally designers can limit the number of these situations by increasing the significance level of the hypotheses tests, however this must be considered carefully as this indicates the confidence interval in which one tests the hypotheses. Statistical significance parameter α indicates the probability of rejecting a true null hypothesis, which in this case is the probability of incorrectly finding a difference between distribution parameters. This error can cause what would be an efficient implementation to be considered dominated and eliminated. This is a critical error since it eliminates an implementation a rational designer may choose. Statistical power parameter β indicates the probability of accepting a false null hypothesis, which in this case is the probability of incorrectly finding no difference between distribution parameters. This error can cause what would be a dominated implementation to be considered efficient and stored in the efficient set. This error is less critical since it preserves all implementations a rational designer may choose. As such it is better to maintain a better statistical significance and sacrifice statistical power when try to limit the number of samples. Operating characteristic curves give the general notion of how many samples are necessary to detect a generalized difference between distribution parameters at a stated statistical power $(1-\beta)$.

4. DEMONSTRATION OF METHODS

The methods for composing tradeoff studies under uncertainty and efficient uncertainty propagation sampling are demonstrated in separate case studies in order to focus on the specific approaches independently.

4.1 Demonstration of Composing Reusable Tradeoff Studies

The approach to composing tradeoff models in a multi-component system under uncertainty is demonstrated with an automobile drivetrain example. The object of the example is to illustrate the approach and support it with results from a traditional fully-integrated analysis approach.

4.1.1 System & Environment

The example is a multi-component automotive system. The components under consideration are its engine, transmission, differential, and wheels, illustrated in Figure 13. The design task is to explore design implementations of the transmission and differential concepts assuming the engine and wheels components have already been designed.

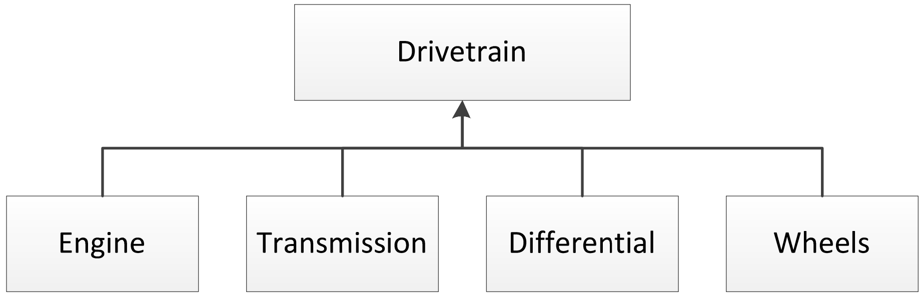

Figure 13. Automobile drivetrain components configuration.

The engine is modeled using a polynomial torque curve. Since the focus is on demonstrating the tradeoff study methodology rather than design results themselves, a generic curve representative of typical six-cylinder gasoline engine is used. The curve is shown in Figure 14.

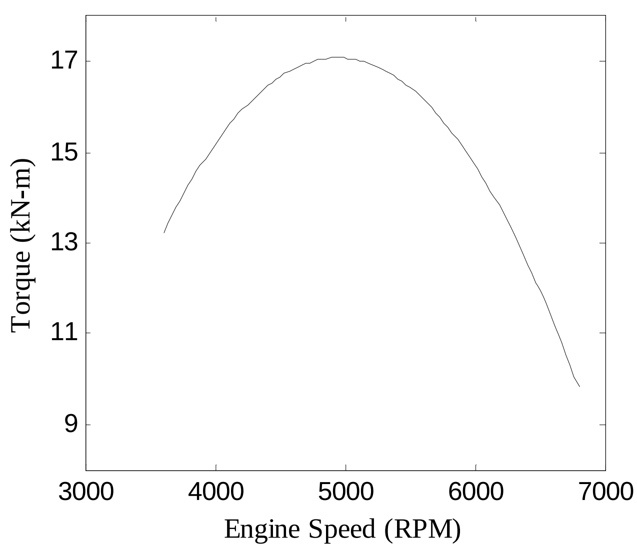

Figure 14. Automobile engine torque curve used in demonstration.

Table 4 enumerates the system, components, and environmental variables modeled with uncertainty. Normal distributions are notated by their mean and standard deviations, and uniform distributions are notated by their lower and upper bounds. The variables are assumed statistically independent.

Table 4. System, components, and environmental variables modeled with uncertainty.

Variable	Distribution	Units
Wheel diameter	N(0.9144,0.00635)	m
Drag reference area	N(0.790,0.05)	m^2
Drag coefficient	N(0.32,0.01)	
Rolling resistance coefficient	N(0.015,0.005)	
Mass density of air	U(1.1455,1.4224)	kg/m^3
Automobile mass	U(1360,1450)	kg/m^3
Gear material density	N(7850,50)	kg/mm^3
Gear material price	N(1.10,0.25)	$/kg
Gear material allowable stress	N(450,15)	N/mm^2
Max transmission gear ratio	U(2.1,2.9)	

4.1.2 Component Tradeoff Studies

4.1.2.1 Component Concepts

A single concept is developed for the transmission component. A simple four-speed manual transmission is modeled for the transmission component, illustrated in Figure 15. The transmission design variables considered are the number of teeth on each gear, effective face widths of meshing gears, and offset distance between the input shaft and layshaft. Many other design parameters, such as gear material, quality factor, etc., could be considered as well, but would add little to the example. Theses parameters are assumed the same for all concepts.

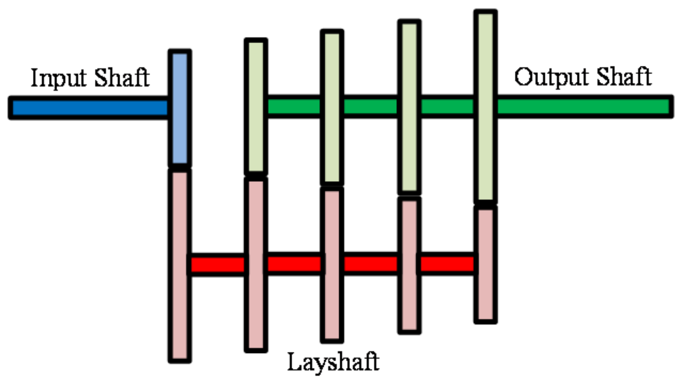

Figure 15. Simple four-speed manual transmission physical configuration.

A single concept is developed for the differential component. An open differential design is used for the differential component, shown in Figure 16. The differential design variables are restricted to the input gear and ring gear. The pinion and ring gears do not affect the gear ratio and are statically loaded when both wheels are rotating at the same speed. It is assumed they are sized to a higher reliability than the input and ring gears. The differential design variables considered are the number of teeth on each gear, effective face widths of meshing gears, and gear modules.

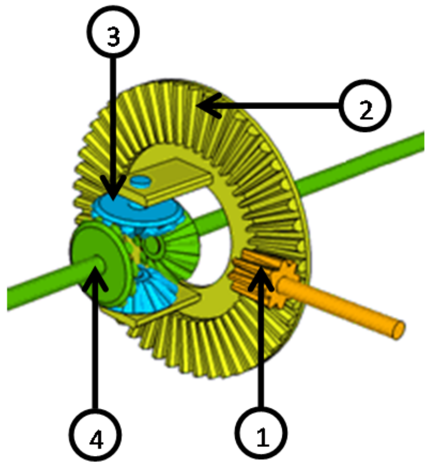

Figure 16. Open differential physical configuration (1: input gear, 2: ring gear, 3: pinion gears, 4: side gears)

The selected design variables are constrained to a wide domain in the design space. The number of teeth on a gear is allowed to vary from 5 to 50. The face widths of each gear mesh is allowed to vary from 1mm to 100mm. The transmission offset distance between the input shaft and layshaft is allowed to vary from 1mm to 1m. The differential gear modules are allowed to vary from 0.1 to 5.0.

4.1.2.2 Component Tradeoff Spaces, Stochastic Dominance, and Predictive Models

There exist many specific embodiments of the transmission and differential concepts, each with different properties depending on the values of design variables chosen by engineers. An important part of the proposed methodology is to determine the rational tradeoffs that engineers might make for each component—i.e., to identify their parameterized efficient sets.

Following Step 1 of the methodology, the tradeoff space for the transmission and differential components are explored through a systematic sampling of their design variables. The transmission space is explored such that the gear ratios are always decreasing for each successive speed and restricted to the ranges given in Table 5 in increments of 0.2. The differential space is explored such that the gear ratio ranges from 1.0 to 10.0 in increments of 0.1. Each design implementation is analyzed using standard engineering analysis and uncertainty propagation methods.

Table 5. Transmission gear ratio constraints.

Gear	Ratio Range
1st	2.1 - 2.9
2nd	1.5 - 2.3
3rd	0.9 - 1.7
4th	0.3 - 1.1

The transmission has seven decision attributes:

- Cost: manufacturing cost of materials and processing component. Prefer to minimize cost.
- Rotations to failure: number of output rotations with gear reliability of 0.99. Gear reliability assumed in series, where one gear failure causes component failure. Prefer to maximize rotations to failure.
- Mass: mass of materials used. Prefer to minimize mass.

- Gear ratio of 1st gear: the transformation ratio of the transmission when in 1st gear. Problem specific target seeking preference.
- Gear ratio of 2nd gear: the transformation ratio of the transmission when in 2nd gear. Problem specific target seeking preference.
- Gear ratio of 3rd gear: the transformation ratio of the transmission when in 3rd gear. Problem specific target seeking preference.
- Gear ratio of 4th gear: the transformation ratio of the transmission when in 4th gear. Problem specific target seeking preference.

The differential has four decision attributes: cost, rotations to failure, mass, and gear ratio. The attribute descriptions are identical to their respective transmission attribute counterparts.

Rotations to failure and gears ratios are calculated from the design variables using standard gear reliability analysis [61] with a gear reliability of 0.99. Mass is calculated using the volume of all gears and material density. Cost is a relationship using material mass and max gear diameter.

Following Step 3 of the methodology, the resulting attributes are mapped to tradeoff criteria by representing the distributions with common distribution parameters. Uncertainty propagates into the cost, rotations to failure, and mass attributes in both the transmission and differential components. The cost and mass attributes are normally distributed, which are represented in the tradeoff space with a mean and variance. The rotations to failure attribute is log-normally distributed, which is represented in the tradeoff space with a mean and variance of its corresponding normal distribution. The normal behavior of the attributes is the result of only linear transformation in the component behavior models on the many normally distributed inputs. Figures 17 and 18 show the corresponding normal attribute distributions for the transmission and differential components, respectively. Uncertainty does not propagate into the gear ratios. The resulting tradeoff space for the transmission component contains ten dimensions (two each for the cost, rotations to failure, and mass means and variances; one each for the four gear ratios). The resulting tradeoff space for the differential component contains seven dimensions (two each for the cost, rotations to failure, and mass means and variances; one for the gear ratio).

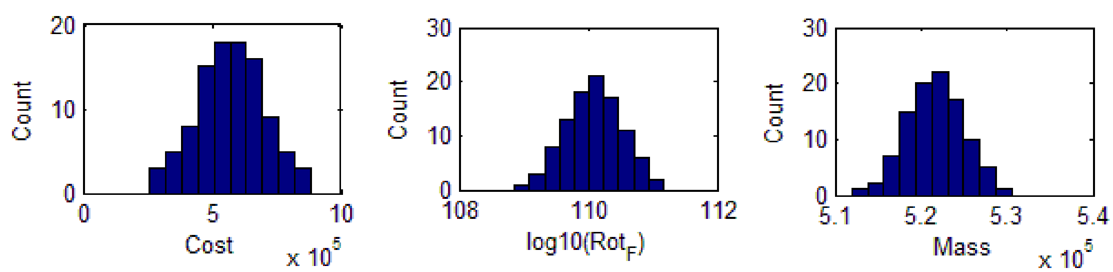

Figure 17. Transmission cost, rotations to failure, and mass attributes histograms showing normal distributions.

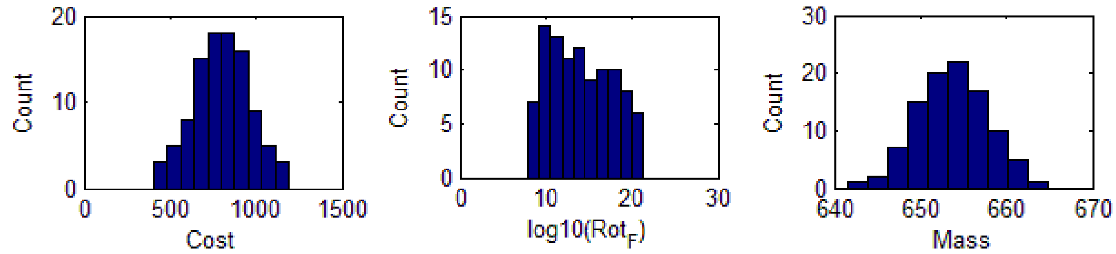

Figure 18. Differential cost, rotations to failure, and mass attribute histograms showing normal distributions.

Referring to Step 3 of the methodology, the resulting tradeoff criteria are evaluated to eliminate dominated alternatives using second-degree stochastic dominance. The cost, rotations to failure, and mass attributes are monotonically preferred, thus are used as dominator parameters in the second-order stochastic dominance evaluation. The gear ratio attributes are non-monotonically preferred, where the target values are derived from the system-level acceleration and top speed attributes. Thus the gear ratio attributes serve as the parameters in creating the parameterized efficient set.

Executing Step 4 of the methodology, a predictive tradeoff model is fitted to the parameterized efficient set for each component. A nonlinear curve-fitting algorithm in the least-squares sense is employed to find the coefficients of a user-defined function. For each component, the tradeoff criteria are used as variables in the function. A summation of a combination of linear terms, cross terms, and a constant term is used in the model. The transmission model uses ten linear terms, nine cross terms, and one constant. The differential model uses seven linear terms, six cross terms, and one constant. The models are validated using hold-back validation [62] to calculate the root mean squared error as a measure of fit. The model is fitted to 80% of the data, and the remaining 20% is used to validate the model. Following Step 5 of the methodology, a support vector domain description (SVDD) is fit around the data to define the domain in which the model is valid. The transmission and differential tradeoff space explorations are summarized in Table 6.

Table 6. Transmission and differential tradeoff space exploration summaries.

	Transmission	Differential
Tradeoff study		
Total implementations	1,000,000	240,000
Stochastic dominance		
Efficient implementations	228,558	99,824
Predictive model		
Sample implementations	182,846	79,859
Validation implementations	45,712	19,965
Validation RMSE	2.337×10^{-3}	6.665×10^{-12}

4.1.3 Composing System Tradeoff Study

The system-level tradeoff study is composed combining the produced component tradeoff studies. The results are supported by a traditional fully-integrated approach which explores the design space of the entire system.

Following Step 1 of the methodology, the tradeoff space for the drivetrain is composed by first sampling one-million implementation combinations from the transmission and differential component tradeoff study predictive models. The models are randomly sampled within their SVDD boundaries to ensure the predicted values are valid. Referring to Step 2 of the methodology, the component tradeoffs are mapped to the drivetrain tradeoffs. Each design combination is analyzed using standard engineering analysis and Monte Carlo simulation methods.

The automobile drivetrain has five decisions attributes:

- Cost: manufacturing cost of materials and processing drivetrain. Equal to the sum of the costs of components. Prefer to minimize cost.
- Distance to failure: distance traveled to system failure. Components assumed in series, where one component failure causes system failure. Prefer to maximize distance to failure.
- Mass: mass of materials used. Equal to the sum of the masses of components. Prefer to minimize mass.
- Top Speed: top speed of drivetrain when traveling on a 30° incline. Prefer to maximize top speed.
- Acceleration time: time taken to achieve 60MPH from rest when traveling on a 30° incline. Prefer to minimize acceleration time.

The top speed and acceleration time attributes are calculated using a system dynamics model of the vehicle. Ideal and instantaneous shifting, with partial clutch engagement modeled as a linear torque relationship between no torque at rest and first-speed idle torque at first-speed idle velocity.

The drivetrain attribute distributions are verified and distribution parameters are set to represent the possible tradeoffs. Uncertainty propagates into each attribute in the drivetrain system from uncertainties in the attributes of its components. The cost, mass, top speed, and acceleration time are normally distributed, which we represent in the tradeoff space with a mean and standard deviation. The distance to failure attribute is log-normally distributed, which we represent in the tradeoff space with a mean and standard deviation of its corresponding normal distribution. Figure 19 shows the corresponding normal attribute distributions for the drivetrain system. The resulting tradeoff space for drivetrain system contains ten dimensions (two each for the cost, distance to failure, mass, top speed, and acceleration time means and variances).

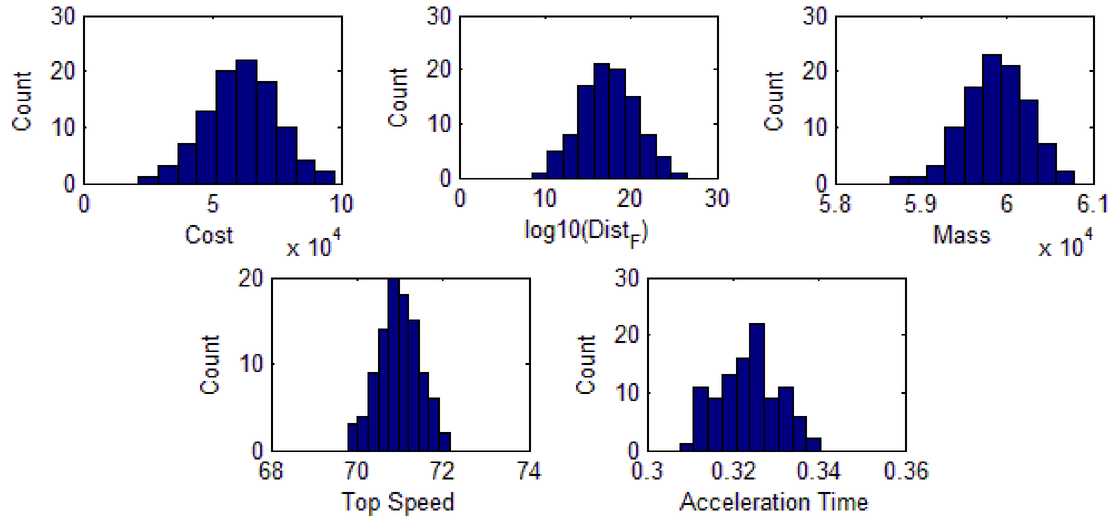

Figure 19. Drivetrain cost, distance to failure, mass, top speed, and acceleration time histograms showing normal distributions.

Using Step 3 of the methodology, the resulting tradeoff criteria are evaluated to eliminate dominated points using second-degree stochastic dominance producing the efficient set. All tradeoff criteria are monotonically preferred for the drivetrain system, thus are all used as dominators in the second-degree stochastic dominance evaluation.

4.1.4 Evaluation of Results

The composed system tradeoff study is compared against a traditional fully-integrated approach generated by exploring the design space from the system level. The design variable constraints are identical to the constraints used in generating the reusable transmission and differential component tradeoff studies. The key difference is the component attribute distributions are not represented by a predictive model of distribution parameters which are used to represent the distribution in composing the system tradeoff study. Instead the set of component attribute values obtained via Monte Caro simulation is used directly in calculating the system attributes.

The results of each approach are sets of ten-dimensional points which are difficult to compare directly. To aid in comparison, each set of points is generalized into a continuous model. Using support vector domain descriptions, the domain volumes the two studies occupy are modeled. If the composed tradeoff study and traditional tradeoff study have similar results, it is expect a large proportion of the data in each study will intersect both support vector domain descriptions, as illustrated in Figure 20. As the proportion of implementations that intersects both domain descriptions increases, one expects the domain description boundaries to approach each other. While this admittedly is not a compelling metric of comparison, it will produce meaningful results.

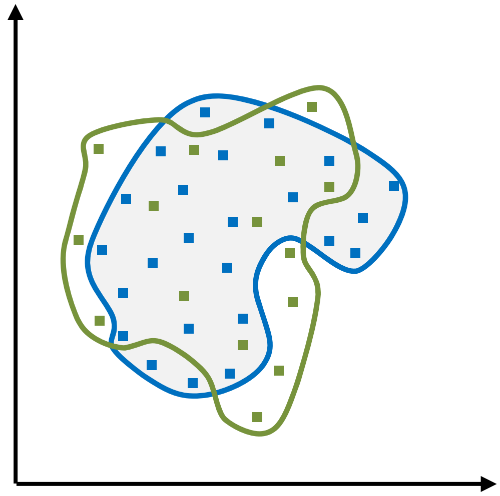

Figure 20. Illustrative representation of two-dimensional intersecting support vector domain descriptions.

Due to the large sizes of the two study data sets, each SVDD is calculated around a sample of the study data and additional data is systematically added to the sample until all implementations in the set are contained in the SVDD. The implementations from each study are compared against the other study's SVDD. Table 7 shows a summary of the two study approaches and comparison results. The results show 807,603 of the 866,756 (93.2%) alternatives from the composed tradeoff study and 634,204 of the 655,049 (96.8%) alternatives from the traditional tradeoff study are contained by both SVDD models. The large percentage of data overlap between the SVDD models is indicative of similar drivetrain tradeoff studies.

Table 7. Composed and traditional fully-integrated approaches tradeoff space comparisons.

	Composed	Traditional
Tradeoff study Total implementations	1,000,000	1,000,000
Stochastic dominance Efficient implementations	866,756	655,049
Generating SVDD Sample implementations	13,902	14,010
SVDD Overlap Overlapping implementations	807,603 (93.2%)	634,204 (96.8%)
Non-overlapping implementations	59,153 (6.8%)	20,845 (3.2%)

4.2 Demonstration of Efficient Uncertainty Propagation Sampling

The approach of efficient Monte Carlo sampling to identify the efficient set under uncertainty is demonstrated on the framework of the previous demonstration but since the focus is on uncertainty propagation and not composing system-level tradeoff studies only the transmission component is considered. The objective is to illustrate the approach and compare it with a basic approach with a set Monte Carlo sample size.

4.2.1 Problem Setup

The engine is modeled using the same polynomial torque curve used in the composition demonstration (Figure 14), which is used in calculating the stresses in the transmission gears. Table 8 enumerates the system and environmental variables modeled with uncertainty. Normal distributions are notated by their means and variances, and uniform distributions are notated by their lower and upper bounds. The variables are assumed statistically independent.

Table 8. System and environmental variables modeled with uncertainty.

Variable	Distribution	Units
Wheel diameter	N(0.75,0.01)	m
Gear material density	N(7850,250)	kg/m^3
Gear material price	U(0.85,1.35)	$/kg
Gear material allowable stress	N(450,225)	N/mm^2
Gear application factor	U(1.1,3.0)	

The transmission concept is identical to the one used in the composition demonstration. It is a simple four-speed manual transmission (Figure 15) defined by the number of teeth on each gear, effective face widths of meshing gears, and the offset distance between the input shaft and layshaft. The number of teeth on a gear is allowed to vary from 5 to 50. The face widths of each gear mesh is allowed to vary from 1mm to 100mm. The transmission offset distance between the input shaft and layshaft is allowed to vary from 1mm to 1m.

When exploring the transmission design space, again the gear ratios are constrained for each of the four gears to the ranges shown in Table 5 above. The gear ratios are constrained to be always decreasing for successive speeds. These constraints ensure the transmission design is sensible and useful. We define for this demonstration the designer has no preference with respect to gear ratios, which leaves three decision attributes in which dominance is evaluated.

- Cost: manufacturing cost of materials and processing component. Prefer to minimize cost.
- Rotations to failure: number of output rotations with gear reliability of 0.99. Gear reliability assumed in series, where one gear failure causes system failure. Prefer to maximize rotations to failure.
- Mass: mass of materials used. Prefer to minimize mass.

The decision attributes are calculated using the same behavior models developed in the composition demonstration. Uncertainty propagates into all three decision attributes. The cost is uniformly distributed, which we represent with a lower and upper bounds. The rotations to failure is log normally distributed, which we represent with a mean and standard deviation of its corresponding normal distribution. The mass is normally distributed, which we represent with a mean and standard deviation. Figure 21shows the corresponding distributions of the attributes.

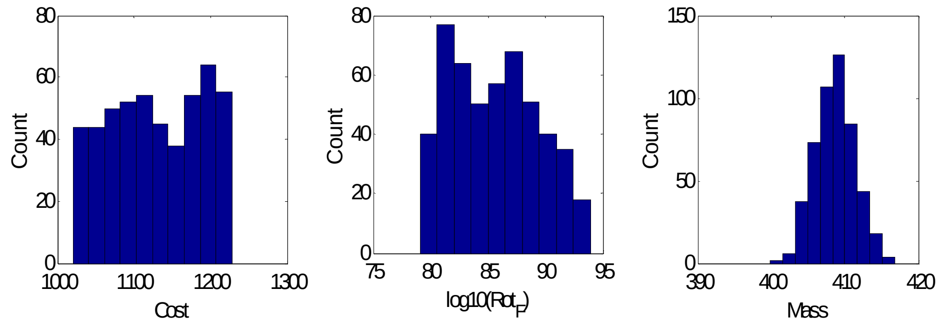

Figure 21. Transmission cost, rotations to failure, and mass distribution histograms showing distributions.

In exploring the transmission design space, 10,000 design implementations were generated. The hypothesis tests were evaluated using a significance level $\alpha = 0.01$, and the maximum number of MCS samples was capped at $N = 100$ (determined using generalized difference parameters $d = 0.5$ and $\lambda = 1.5$ at $\beta = 0.01$). For comparison, the efficient approach was run concurrently with a fixed MCS sample size approach with a sample size of $N = 100$.

4.2.2 Evaluation of Results

The efficient sampling approach and traditional fixed sample size approach produced 290 efficient implementations in the tradeoff space. The efficient approach had a significant reduction in transmission behavior model evaluations compared to the fixed MCS sample size approach. Overall, the efficient sampling approach had 60% fewer model evaluations compared to using a fixed MCS sample size of 100. The largest reduction in model evaluations was for dominated implementations, which on average required 38.4 model evaluations per implementation to determine dominance. The reduction for efficient implementations was not as drastic, which on average required 92.0 model evaluations per implementation to determine dominance. This is indicative that the efficient implementations have negligible differences between attributes, as 83.8% (243/290) of them were sampled the capped maximum of 100 samples and entered into the efficient set because the difference between at least one attribute distribution parameter could not be determined (Case D). The breakdown of the number of behavioral model evaluations for both approaches is shown in Table 9.

Table 9. Number of behavioral model evaluations compared between efficient and fixed MCS sampling size approaches.

	Efficient Approach	Fixed MCS Approach
Number of model evaluations		
Efficient implementations	26,401	29,000
Dominated implementations	372,595	971,000
Total	398,996	1,000,000

5. SUMMARY

The approach to composing tradeoff studies under uncertainty holds promise and warrants further investigation. The approach is based on stochastic dominance rules which handle the uncertainty in a mathematically rigorous manner conjointly with the parameterization technique to support composition and reuse. This allows designers to leverage previous knowledge in new designs and for a better way to coordinate efforts among teams of engineers, where teams can work concurrently and break tasks associated with the generation of the system tradeoff study down into tasks that can be completed by teams working on specific subsystem components. The results from the automobile drivetrain showed that the new approach generated similar data compared to a fully-integrated design space exploration approach.

In the demonstration, the large percentage of overlapping data between SVDD models is indicative of similar results but insufficient to provide compelling support for its actual feasibility. The results merit further study to verify success of the approach. A limitation of the approach is the assumption of normality or uniformity and independence in the second-order stochastic dominance evaluation. These assumptions are used to formalize the tradeoff space and simplify the dominance criterion. Further investigation is needed to analyze the sensitivity of violating these assumptions. Additionally the stochastic dominance criterion could be expanded for other distribution types, including the possibility of developing an empirical evaluation of stochastic dominance which would eliminate the distribution type assumption.

A potential complication with the generation of reusable parameterized efficient sets is changes in the uncertainty. In the demonstration the uncertainty values used in generating the reusable component tradeoff studies was identical to the uncertainty associated with the composed system. Problems may arise in using a parameterized efficient set generated using different uncertainty than associated in the system design. For example, using the parameterized efficient sets generated in our example for a drivetrain with a different gear material price distribution may affect the results. Further study is needed to assess the impact of this complication on the approach.

The key benefits of the approach are the ability to compose tradeoff studies and reuse of knowledge. The ability compose tradeoff studies enables the breakdown of tradeoff study generation tasks in a way that reflects the organization of typical systems engineering project, where teams are assigned to develop each major subsystem. The reuse of knowledge by fitting predictive models to parameterized efficient sets generated using stochastic dominance principles, enables designers to consider the various tradeoffs of different concepts under uncertainty. This reduces the need to duplicate previous efforts and allows a designer to consider the various tradeoffs without specialized domain-specific knowledge. Additionally the approach can potentially be used for abstracting together physically heterogeneous concepts. This has been demonstrated in the deterministic case [9, 32, 51] and for the non-compositional case under uncertainty [16]. Demonstrating it for composition under uncertainty is left for future research.

The approach to efficient Monte Carlo sampling is computationally effective in identifying the efficient set of alternatives. The approach is based on stochastic dominance rules which handle the uncertainty in a mathematically rigorous manner conjointly with the appropriate statistical hypothesis tests to identify the efficient set with specified confidence. This allows designers to identify the efficient set faster and with less model evaluations. The results from the automobile transmission demonstration showed that the new approach significantly reduced the number of model evaluations.

In the demonstration, the approach effectively identified dominated implementations but does not quantify the accuracy of the distribution estimators of the efficient implementations. Beyond identifying the efficient set, additional samples may be necessary to obtain more accurate values for the distribution parameters of the efficient implementations.

The key benefit of our approach is the ability to minimize model evaluations in identifying the efficient set. This prevents additional computation time for complex models. In our approach, random Monte Carlo sampling was used. Further investigation is warranted into applications of this approach, including incorporating the statistical tests into other sampling methods (i.e. Latin hypercube, stratified sampling, cluster sampling etc.).

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APPENDIX A

UNIVARIATE UNIFORMLY-DISTRIBUTED SECOND-DEGREE STOCHASTIC DOMINANCE PROOF

Theorem 1 [45]: Let F_1 and F_2 be two (cumulative) distributions, and $u(x)$ a non-decreasing function, with finite values for any finite x. A necessary and sufficient condition for F_1 dominates F_2 is: $F_1(x) \leq F_2(x)$ for every x, and $F_1(x_0) < F_2(x_0)$ for some x_0.

Theorem 2 [45]: Let F_1 and F_2 be two (cumulative) distributions. A necessary and sufficient conditions for F_1 dominates F_2, for every $u(x)$ which is non-decreasing and concave, is

$$\int_{-\infty}^{x} [F_2(t) - F_1(t)]dt \geq 0$$

for every x, and $F_2 \neq F_1$ for some x_0.

Theorem 3 [45]: Let F_1, F_2 be two distributions with mean values μ_1, μ_2, respectively, such that for some $x_0 < \infty$, $F_1 \leq F_2$ for $x < x_0$ (and $F_1 < F_2$ for some $x_1 < x_0$) and $F_1 \geq F_2$ for $x \geq x_0$; then F_1 dominates F_2 (for concave utility functions) if and only if $\mu_1 \geq \mu_2$.

Definition 1: For a uniform distribution defined by lower bound a and upper bound b, the probability distribution function is $f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & x < a \; and \; x > b \end{cases}$ and cumulative probability distribution function is $F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}$.

Theorem 4: Let $F_1(x)$ and $F_2(x)$ be two uniform distributions defined by bounds $[a_1, b_1]$ and $[a_2, b_2]$, respectively. Let $\frac{a_1+b_1}{2} \geq \frac{a_2+b_2}{2}$. Then F_1 dominates F_2 for all concave $u(x)$, if and only if $b_1 - a_1 \leq b_2 - a_2$.

Proof:

If $a_1 = a_2$ and $b_1 = b_2$, F_1 and F_2 are identical.

If $a_1 > a_2$ and $b_1 > b_2$, for $a_2 \leq x \leq b_1$ (cases A & B in Figure 22), $\frac{x-a_1}{b_1-a_1} < \frac{x-a_2}{b_2-a_2}$, thus $F_1(x) \leq F_2(x)$ for all x ($F_1(x) < F_2(x)$ for $a_2 < x < b_1$ and $F_1(x) = F_2(x)$ for $x \leq a_2$ and $x \geq b_1$). F_1 dominates F_2 by Theorem 1.

If $a_1 \leq a_2$ and $b_1 \geq b_2$ (cases C & D in Figure 22) where at least one of the inequalities is strict, or $a_1 \geq a_2$ and $b_1 \leq b_2$ where at least one of the inequalities is strict (either uniform distribution bounds is within the other's), we have an intersection point at x_0, where

$$x_0 = \frac{a_1 b_2 - a_2 b_1}{a_1 - b_1 - a_2 + b_2}$$

If $a_1 \leq a_2$ and $b_1 \geq b_2$ (case C in Figure 22) where at least one of the inequalities is strict, and $\frac{a_1+b_1}{2} > \frac{a_2+b_2}{2}$, for $x < x_0$, $\frac{x-a_1}{b_1-a_1} > \frac{x-a_2}{b_2-a_2}$, thus $F_1(x) \geq F_2(x)$ for $x < x_0$ ($F_1(x) > F_2(x)$ for $a_1 < x < x_0$ and $F_1(x) = F_2(x)$ for $x < a_1$); and for $x > x_0$, $\frac{x-a_1}{b_1-a_1} < \frac{x-a_2}{b_2-a_2}$, thus $F_1(x) \leq F_2(x)$ for $x > x_0$ ($F_1(x) < F_2(x)$ for $x_0 < x < b_1$ and $F_1(x) = F_2(x)$ for $x \geq b_1$). Thus, the conditions for Theorem 3 is not satisfied, and F_1 cannot dominate F_2.

If $a_1 > a_2$ and $b_1 < b_2$ (case D in Figure 22) where at least one of the inequalities is strict, and $\frac{a_1+b_1}{2} > \frac{a_2+b_2}{2}$, for $x < x_0$, $\frac{x-a_1}{b_1-a_1} < \frac{x-a_2}{b_2-a_2}$, thus $F_1(x) \leq F_2(x)$ for $x < x_0$ ($F_1(x) < F_2(x)$ for $a_2 < x < x_0$ and $F_1(x) = F_2(x)$ for $x < a_2$); and for $x > x_0$, $\frac{x-a_1}{b_1-a_1} > \frac{x-a_2}{b_2-a_2}$, thus $F_1(x) \geq F_2(x)$ for $x > x_0$ ($F_1(x) > F_2(x)$ for $x_0 < x < b_2$ and $F_1(x) = F_2(x)$ for $x \geq b_2$). Thus F_1 dominates F_2 by Theorem 3.

Q.E.D.

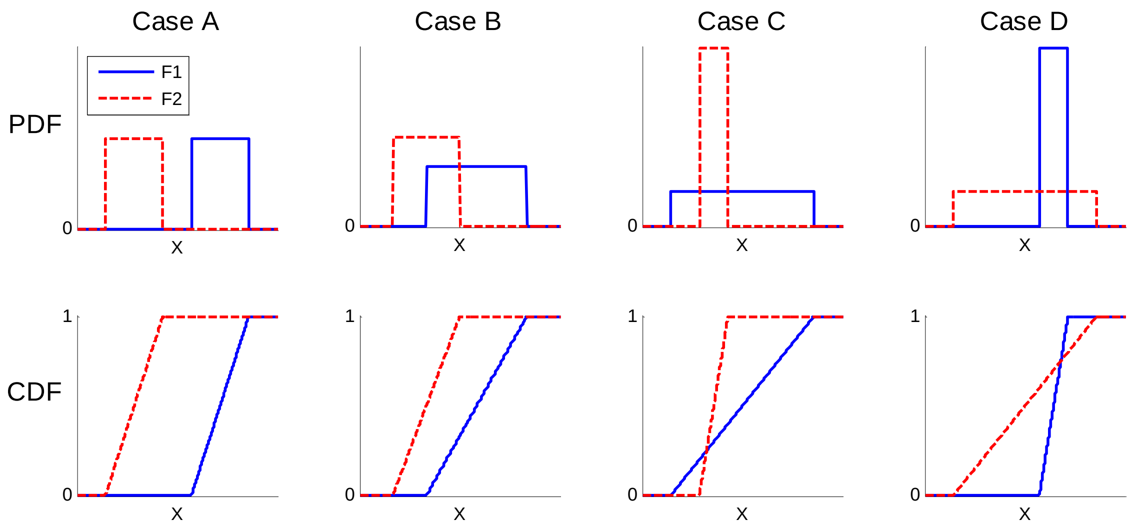

Figure 22. Uniform distribution possible cases when evaluating SSD.

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