CAPACITY RESULTS FOR WIRELESS COOPERATIVE COMMUNICATIONS WITH RELAY CONFERENCING

A Dissertation

by

CHUAN HUANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2012

Major Subject: Electrical Engineering

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ABSTRACT

Capacity Results for Wireless Cooperative Communications

with Relay Conferencing. (August 2012)

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Chair of Advisory Committee: Dr. Shuguang Cui

In this dissertation we consider cooperative communication systems with relay conferencing, where the relays own the capabilities to talk to their counterparts via either wired or wireless out-of-band links. In particular, we focus on the design of conferencing protocols incorporating the half-duplex relaying operations, and study the corresponding capacity upper and lower bounds for some typical channels and networks models, including the diamond relay channels (one source-destination pairs and two relays), large relay networks (one source-destination pairs and N relays), and interference relay channels (two source-destination pairs and two relays).

First, for the diamond relay channels, we consider two different relaying schemes, i.e., simultaneous relaying (for which the two relays transmit and receive in the same time slot) and alternative relaying (for which the two relays exchange their transmit and receive modes alternatively over time), for which we obtain the respective achievable rates by using the decode-and-forward (DF), compress-and-forward (CF), and amplify-and-forward (AF) relaying schemes with DF and AF adopted the conferencing schemes. Moreover, we prove some capacity results under some special conditions.

Second, we consider the large relay networks, and propose a "p-portion" conferencing scheme, where each relay can talk to the other "p-portion" of the relays. We

obtain the DF and AF achievable rates by using the AF conferencing scheme. It is proved that relay conferencing increases the throughput scaling order of the DF relaying scheme from $\mathcal{O}(\log(\log(N)))$ for the case without conferencing to $\mathcal{O}(\log(N))$; for the AF relaying scheme, it achieves the capacity upper bound under some conditions.

Finally, we consider the two-hop interference relay channels, and obtain the AF achievable rates by adopting the AF conferencing scheme and two different decoding schemes at the destination, i.e., single-user decoding and joint decoding. For the derived joint source power allocation and relay combining problem, we develop some efficient iterative algorithms to compute the AF achievable rate regions. Moreover, we compare the achievable degree-of-freedom (DoF) performance of these two decoding schemes, and show that single-user decoding with interference cancelation at the relays is optimal.

To my Dad

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CHAPTER I

INTRODUCTION

A. Overview of Relay Networks

In most recent wireless access standards, such as Long Term Evolution-Advanced (LTE-A) by the 3rd Generation Partnership Project (3GPP) and IEEE 802.16m for WiMAX, relaying has been proposed as one of the main performance enhancement technologies [1]. In these standards, relays are deployed to help the wireless access systems increase the system capacity and enlarge the coverage.

From the information-theoretical viewpoint, the capacity bounds of the traditional three-node full-duplex relay channel have been well studied [2–5], and various coding schemes, such as decode-and-forward (DF) and compress-and-forward (CF), have been proposed. For the half-duplex relay channel, in [5] and the references therein the authors have studied achievable rates and the power allocation problems.

For the case with two relay nodes and no direct link between the source and the destination, termed as the diamond relay channel, various achievable rates were derived in [6–13]. In particular, the authors in [6, 7] discussed the capacity upper bound and achievable rates using the DF and amplify-and-forward (AF) schemes under the full-duplex relaying mode. For the case with N relays, the authors in [8] used the bursty AF scheme to achieve the channel capacity within 1.8 bits with arbitrary channel gains and N values. In [9], the authors considered a different problem, where the relay-destination links are orthogonal, and derived achievable rates by assuming that each relay either cannot or can decode. Under the half-duplex mode, the authors in [10] discussed achievable rates using two time-sharing schemes,

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i.e., the simultaneous relaying and alternative relaying schemes. In [11], the authors further discussed the same problem and bounded the gap between the achievable rates and the upper bound to at most some constant bits. By further exploring partial collaboration between the two relays, the authors in [12, 13] developed some DF schemes based on dirty paper coding (DPC) and block Markov encoding (BME), where the DF scheme is shown to be optimal in some special cases [13].

For the large relay networks with N relay nodes, the asymptotic capacity bounds were studied in [14–17]. Considering the joint source channel coding problem for a special class of Gaussian relay networks [14], the capacity upper bound is asymptotically achieved by the AF relaying scheme as the number of relays tends to infinity. For general Gaussian relay networks, the authors in [15] obtained the achievable rate scaling law for the multiple-input and multiple-output (MIMO) relay networks with AF: For the coherent relaying case, with full forward-link channel state information (CSI) at the relays, the AF achievable rate scales as $\mathcal{O}(\log(N))$; for the noncoherent relaying case with zero forward-link CSI at the relays, it scales as $\mathcal{O}(\log(1))$. In [16], the authors studied the scaling laws of the DF, CF, and linear relaying schemes, and proved that the DF rate scales at most as $\mathcal{O}(\log(\log(N)))$ for the coherent relaying scheme. The authors in [17] mainly focused on the noncoherent case, and proved that the DF relaying scheme asymptotically achieves the capacity upper bound.

In practical communication systems, some nodes might have extra out-of-band connections with the others, e.g., through Bluetooth, WiFi, optical fiber, etc., to exchange certain information and improve the overall system performance. From the information-theoretical viewpoint, such kinds of interactions can be modeled as nodes conferencing [18–22]. Specifically, for multiple access channel (MAC) [18], encoder conferencing was used to exchange part of the source messages, and it is proved that one-round conferencing scheme is optimal, for which the two receivers exchange information only once. For the broadcast channel (BC) in [19], each decoder was designed to first compress the received signal, and then transmit the corresponding binning index number to its counterpart through the conferencing link. In [19, 20], it was shown that the one-round scheme is optimal for the physically degraded BC channel, while the two-round and three-round schemes can outperform the one-round one for general channel cases. Moreover, in [21] and [22], by adopting transmitter and receiver conferencing, achievable rates of the compound MAC channel were discussed, for which both the two receivers are required to fully decode the two source messages, and some capacity results for the degraded cases were provided.

In this dissertation, we introduce the idea of node conferencing into the cooperative communication networks, i.e., the diamond relay channels with the simultaneous and alternative relaying schemes, and single- or multi-user relay networks, by allowing the relays to talk to each other via out-of-band conferencing links. Moreover, we would like to address the following questions:

1. How to design efficient protocols to incorporate the relay conferencing among the relays and to excute relaying operations from the relays to the destinations?

2. Whether and when relay conferencing can (strictly) help the transmissions (in the sense of increasing the achievable rates) compared to the case without it?

3. How much gain can relay conferencing enable? Can we obtain more reward than the cost that we pay for relay conferencing?

4. When can the proposed relaying and conferencing schemes achieve the capacity upper bound?

B. Overview of Contributions

The main contribution and the structure of this dissertation are summarized as follows.

1. We first consider the half-duplex diamond relay channel with the simultaneous relaying scheme in Chapter 2, which consists of one source-destination pair and two relay nodes connected with two-way rate-limited out-of-band conferencing links. Three basic coding schemes are studied: For the DF scheme, we obtain an achievable rate by letting the source send a common message and two private messages; for the CF scheme, we exploit the conferencing links to help with the compression of the received signals, or to exchange messages intended for the second hop to introduce different levels of cooperations; for the AF scheme, we study the optimal combining strategy between the received signals from the source and the conferencing link. Moreover, we show that these schemes could achieve the rate upper bound under certain conditions. Finally, we evaluate various achievable rates for the Gaussian case with numerical results.

2. Next, the diamond relay channel with alternative relaying scheme is considered in Chapter 3, which consists of one source-destination pair and two relay nodes connected with rate-limited out-of-band conferencing links. In particular, we focus on the half-duplex alternative relaying strategy, in which the two relays operate alternatively over time. With different amounts of delay, two conferencing strategies are proposed, each of which can be implemented by either a general two-side conferencing scheme (for which both of the two conferencing links are used) or a special-case one-side conferencing scheme (for which only one of the two conferencing links is used). Based on the most general two-side conferencing scheme, we derive the achievable rates by using the DF and AF

relaying schemes, and show that these rate maximization problems are convex. By further exploiting the properties of the optimal solutions, the simpler one-side conferencing is shown to be equally good as the two-side conferencing in term of the achievable rates under arbitrary channel conditions. Based on this, the DF rate in closed-form is obtained, and the principle to use which one of the two conferencing links for one-side conferencing is also established. Moreover, the DF scheme is shown to be upper-bound-achieving under certain conditions with even one-side conferencing. For the AF relaying scheme, one-side conferencing is shown to be sub-optimal in general. Finally, numerical results are provided to validate our analysis.

3. Then, we extend the idea of relay conferencing to a half-duplex large relay network in Chapter 4, consisting of one source-destination pair and N relay nodes, each of which is connected with a subset of the other relays via signal-to-noise ratio (SNR)-limited out-of-band conferencing links. The asymptotic achievable rates of two basic relaying schemes with the "p-portion" conferencing strategy are studied: For the DF scheme, we prove that the DF rate scales as $\mathcal{O}(\log(N))$; for the AF scheme, we prove that it asymptotically achieves the capacity upper bound in some interesting scenarios as N goes to infinity.

4. Finally, in Chapter 5, we consider a two-hop interference network, which consists of two source-destination pairs and two relay nodes connected with SNR limited out-of-band conferencing links. Assuming that the AF relaying scheme is adopted, this network is shown to be equivalent to a two-user IC. By deploying two IC decoding schemes, i.e., single-user decoding and joint decoding, respectively, we characterize the achievable rate regions with a two-stage iterative optimization method: First, we fix the source power pair and maximize

the sum rate over the relay combining vector; second, we fix the relay combining vector and optimize the source power pair. Specifically, we design a new routine to compute the optimal relay combining vector, which is more efficient than the existing scheme. Furthermore, it is revealed that the AF scheme with relay conferencing achieves the full DoF, which outperforms the case without relay conferencing. Finally, simulation results show that relay conferencing can significantly improve the system performance under certain channel conditions.

C. Notations

Here, we briefly summarize the notations adopted in this dissertation in Table I.

Table I.: Notations

$\log(\cdot)$	base-2 logarithm
$\ln(\cdot)$	natural logarithm
x	scalar
$\boldsymbol{x}$	vector
$\boldsymbol{X}$	matrix
$\Re(x)$	real part of a complex number x
$\lvert x\rvert$	the amplitude of a complex number x
$\max\{x, y\}$	the maximum between two real numbers x and y
$\min\{x, y\}$	the minimum between two real numbers x and y
$\mathcal{C}(x) = \log(1 + x)$	the AWGN channel capacity
$\langle \boldsymbol{x}, \boldsymbol{y} \rangle$	the inner product of two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$
$\lvert\boldsymbol{x}\rvert$	the norm of vector $\boldsymbol{x}$
$\lvert\boldsymbol{A}\rvert$	the determinant of a matrix $\boldsymbol{A}$
$\mathrm{Rank}(\boldsymbol{X})$	the rank of a matrix $\boldsymbol{X}$
$\mathbb{E}(X)$	the expectation of a random variable X
$\mathrm{Tr}(\boldsymbol{X})$	the trace of a matrix $\boldsymbol{X}$
$\boldsymbol{X} \succeq 0$	$\boldsymbol{X}$ is a positive semidefinite matrix
$\mathrm{Diag}(\boldsymbol{x})$	a diagonal matrix with $\boldsymbol{x}$ as the diagonal elements
$X_N \xrightarrow{w.p.1} a$	$X_N \to a$ with probability 1, as $N \to +\infty$
$A_N \sim B_N$	$\lim_{N\to+\infty} \lvert A_N - B_N\rvert = 0$
$y_N \sim \mathcal{O}(\log(x_N))$	$\lim_{N\to+\infty} \frac{x_N}{y_N} = c$, where c is a positive constant

CHAPTER II

SIMULTANEOUS RELAYING DIAMOND CHANNEL

In this chapter, we consider the half-duplex diamond relay channel, which consists of one source-destination pair and two relay nodes. We assume that the relays can conduct conferencing with each other via some orthogonal out-of-band links [23]. In general, the conferencing links can be used to exchange compressed versions of the received signals at the relays [19], part of the messages intended to the destination between the two relays [18], or just to forward the received signal to the other relay [10]. With these ideas, we develop relaying schemes based on the conventional DF, CF, and AF schemes by exploiting the inter-relay conferencing, for both the cases of discrete memoryless channel (DMC) and Gaussian channel. Moreover, in stead of considering multi-round conferencing schemes [19, 20], we just concentrate on the simple one-round conferencing scheme, which means that the relays simultaneously process their received signal and conduct conferencing with the other in the same time slot.

Three basic coding schemes are studied: For the DF scheme, we obtain an achievable rate by letting the source send a common message and two private messages; for the CF scheme, we exploit the conferencing links to help with the compression of the received signals, or to exchange messages intended for the second hop to introduce different levels of cooperations; for the AF scheme, we study the optimal combining strategy between the received signals from the source and the conferencing link. Moreover, we show that these schemes could achieve the rate upper bound under certain conditions. Finally, we evaluate various achievable rates for the Gaussian case with numerical results.

The remainder of the chapter is organized as follows. In Section A, we introduce all assumptions and channel models. In Section B, we derive a rate upper bound and achievable rates for the DF, CF, and AF schemes. Moreover, we discuss some upper-bound-achieving cases. Section C shows some simulation and numerical results. Finally, this chapter is summarized in Section D.

A. Assumptions and System Model

In this chapter, we consider the diamond relay channel with out-of-band conferencing links between the relays, as shown in Fig. 1, which contains one source node, one destination node, and two relays. It is assumed that there is no direct link between the source and the destination. Furthermore, these two conferencing links are orthogonal to each other and outside the bandwidth used by the source-to-relay and relay-to-destination links. In this chapter, we only consider the simultaneous relaying scheme [10] with conferencing, leaving the alternative relaying scheme [10] for future works. To be concise, in each relaying scheme we generically describe the coding scheme for the i-th relay ($i = 1, 2$), where we use $(3 - i)$ to refer to the other relay index for the convenience of description.

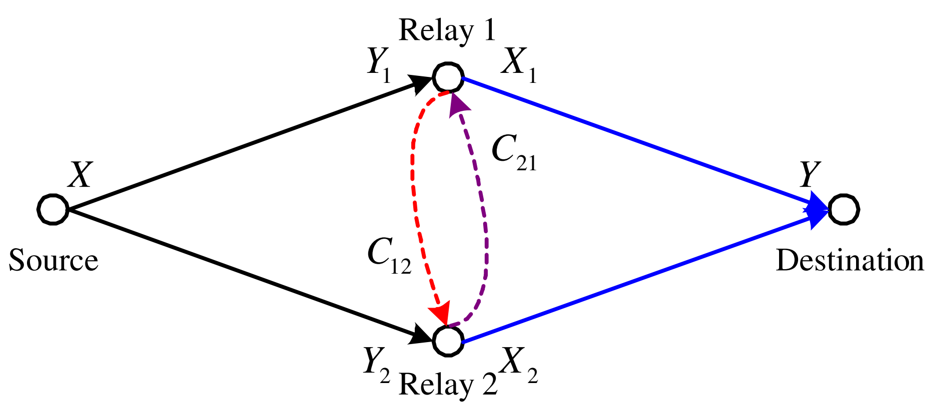
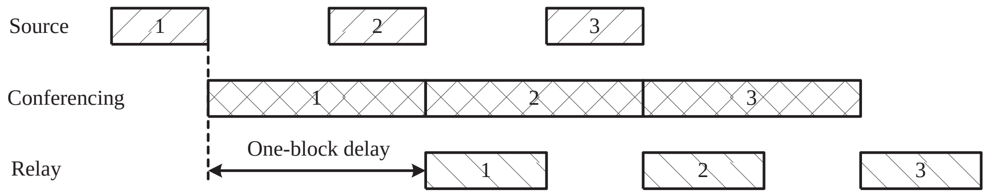


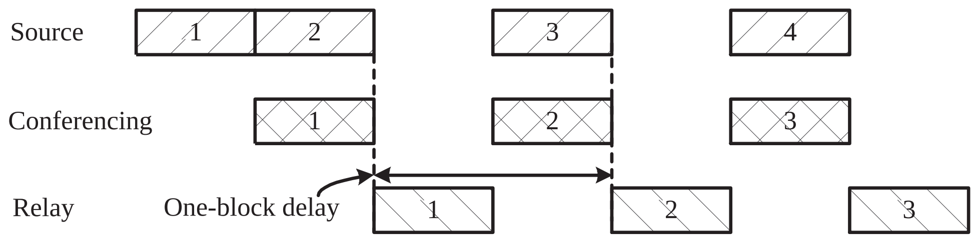
Fig. 1.: Diamond relay channel with conferencing links.

The time scheduling schemes of the source-relay, relay-destination, and conferencing transmissions are shown in Fig. 2(a) and Fig. 2(b) for different relaying schemes, respectively. The relay nodes work in a half-duplex mode: During the i-th transmission block, the source transmits the i-th message and the two relays listen in the first time slot; the relays simultaneously transmit the $(i-1)$-th source message and the destination listens in the second time slot. For the DF and CF relaying schemes, denote the time fraction allocated to the first slot as λ, with $\lambda \in (0,1)$, and that for the second slot as $\overline{\lambda} = 1-\lambda$; for the AF relaying scheme, let $\lambda = \frac{1}{2}$. After receiving the source signal, each relay temporarily stores this information for one-block, and forward this information to its counterpart via the conferencing link; then after obtained the information from the other relay, each relay generates a message and transmits it to the destination in the successive slot. Moreover, for both the DF and CF relaying schemes, we adopt the CF scheme as the conferencing strategy; and for the AF relaying scheme, we adopt the AF scheme for conferencing. Due to this assumption, the conferencing scheduling schemes for DF, CF, and AF are different: For the DF and CF relaying schemes, the block length of the conferencing link codewords is equal to the sum of those for the source and relay transmission codewords, as shown in Fig. 2(a); on the other hand, for the AF relaying scheme, the block lengths of these three codewords should be the same, as shown in Fig. 2(b). Furthermore, due to relay conferencing, there will be a one-block delay between the transmissions at the source and the relays, as shown in Fig. 2(a) and Fig. 2(b). Assume that during each block, the communication rate is R, and we need to transmit B blocks in total. Thus, the average information rate is $R\frac{B}{B+1} \to R$, as B goes to infinity. In this chapter, we focus on the one-block transmission and the associated coding scheme without specifying the delay in the sequel.

In this chapter, we consider both the DMC and Gaussian channel cases for the DF relaying scheme, while considering only the Gaussian case for the AF relaying scheme.



(a) For the DF and CF schemes.



(b) For the AF scheme.

Fig. 2.: Transmission scheduling scheme for the diamond relay channel with conferencing links.

These channel models are described as follows. For the DMC case, the diamond relay channel is defined as $(\mathcal{X}, \mathcal{X}_1, \mathcal{X}_2, \mathcal{P}, \mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y})$, where $\mathcal{X}, \mathcal{X}_1$, and $\mathcal{X}_2$ are the finite channel inputs at the source, relay 1, and relay 2, respectively; $\mathcal{Y}_1, \mathcal{Y}_2$, and $\mathcal{Y}$ are the finite channel outputs at relay 1, relay 2, and the destination, respectively; $\mathcal{P}$ denotes the collection of the conditional probabilities $p\left(y_1, y_2, y|x, x_1, x_2\right) = p\left(y_1, y_2|x\right) p\left(y|x_1, x_2\right)$ on $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ given $x \in \mathcal{X}$ and $y \in \mathcal{Y}$ given $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$, respectively. The channel is memoryless in the sense that for n channel uses, we have

$$p\left(\boldsymbol{y}_1, \boldsymbol{y}_2|\boldsymbol{x}\right) = \prod_{i=1}^{\lambda n} p\left(y_{1i}, y_{2i}|x_i\right),$$
$$p\left(\boldsymbol{y}|\boldsymbol{x}_1, \boldsymbol{x}_2\right) = \prod_{i=1}^{\bar{\lambda} n} p\left(y_i|x_{1i}, x_{2i}\right),$$

where the respective signal vectors are defined as follows: $\boldsymbol{x} = (x_1, \cdots, x_{\lambda n})$, $\boldsymbol{x}_1 = (x_{11}, \cdots, x_{1,\bar{\lambda}n})$, $\boldsymbol{x}_2 = (x_{21}, \cdots, x_{2,\bar{\lambda}n})$, $\boldsymbol{y}_1 = (y_{11}, \cdots, y_{1,\lambda n})$, $\boldsymbol{y}_2 = (y_{21}, \cdots, y_{2,\lambda n})$, and $\boldsymbol{y} = (y_1, \cdots, y_{\bar{\lambda}n})$.

Denote the capacity of the conferencing link from relay 1 to relay 2 as C_{12}, with C_{21} defined similarly. The inputs of the two conferencing links are within two integer sets $\mathcal{W}_1 = \left\{1, \cdots, 2^{nC_{12}}\right\}$ and $\mathcal{W}_2 = \left\{1, \cdots, 2^{nC_{21}}\right\}$, respectively. We assume that for the conferencing links, the receivers can perfectly decode the source messages without incurring any errors if the transmission rate is under the conferencing link rate. Note that the definition of the conferencing links is not the most general one [18], but enough to describe our proposed coding schemes in the next section.

A nonnegative rate R is achievable for the diamond relay channel with conferencing links, if there exists a codebook $\{\boldsymbol{x}(w)\}$, $w \in \left[1, \cdots, 2^{nR}\right]$, for the source node, a relaying mapping $\boldsymbol{x}_i(v_i) = \phi_i(\boldsymbol{y}_i, \boldsymbol{y}_{3-i,i}(w_{3-i}))$ for the i-th relay, $i = 1, 2$, where $v_i \in \left[1, \cdots, 2^{nR_i}\right]$ and $\boldsymbol{y}_{3-i,i}(w_{3-i})$ with $w_{3-i} \in \mathcal{W}_{3-i}$ is of length n and generated based on $\boldsymbol{y}_{3-i}$, and a decoding function $\widehat{w} = W(\boldsymbol{y})$, $\widehat{w} \in [1, \cdots, 2^{nR}]$, for the destination, such that the average error probability at the destination

$$P_e^{(n)} = \frac{1}{2^{nR}} \sum_{w=1}^{2^{nR}} \Pr\left\{\widehat{w} \neq w | w \text{ is sent}\right\} \to 0,$$

as n goes to infinity, where w is assumed to be uniformly distributed over $\left[1, \cdots, 2^{nR}\right]$. Note that the source code $\{\boldsymbol{x}(w)\}$ is with length λn and size 2^{nR}, and the relay code $\{\boldsymbol{x}_i(v_i)\}$ is with length $\overline{\lambda} n$ and size 2^{nR_i}, $i = 1, 2$. The capacity of the considered channel is defined as the maximum value over all achievable rates.

For the Gaussian channel case, we further define the following channel input-output relationship. The received signal y_i from the source at the i-th relay ($i = 1, 2$) is given as

$$y_i = h_i x + n_i, \ i = 1, 2, \tag{2.1}$$

where x is the signal transmitted by the source with average power P_S, h_i is the complex channel gain of the i-th source-to-relay link, and n_i's are the independently and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) noise with distribution $\mathcal{CN}(0,1)$.

In the second hop, signal x_i with average power P_R, is transmitted from the i-th relay to the destination; and the received signal y at the destination is given as

$$y = \sum_{i=1}^{2} g_i x_i + n, \tag{2.2}$$

where g_i is the complex channel gain of the i-th relay-to-destination link, and n is the CSCG noise with distribution $\mathcal{CN}(0,1)$. For convenience, we define the link SNRs as

$$\gamma_i = |h_i|^2 P_S,\ \widetilde{\gamma}_i = |g_i|^2 P_R,\ i = 1, 2. \tag{2.3}$$

B. Rate Upper Bound and Achievable Rates

In this section, we examine the rate upper bound and achievable rates of the considered channel with the following three relaying schemes: DF, CF, and AF. Moreover, we prove some upper-bound-achieving results under special channel conditions.

1. Rate Upper Bound

In this subsection, we first study the rate upper bound for the considered channel. Since the simultaneous relaying scheme described in Section A is adopted, we only consider the BC cut-set and MAC cut-set. Note that this upper bound is only for the simultaneous relaying protocol, not for the half-duplex diamond relay channel [10,13]. In this chapter, without introducing any confusions, we still call this bound as the rate upper bound for simplicity.

Theorem B.1 *The rate upper bound for the discrete memoryless diamond relay* *channel with conferencing links is given as*

$$C_{upper} = \max \frac{I\left(X;Y_1,Y_2\right) I\left(X_1,X_2;Y\right)}{I\left(X;Y_1,Y_2\right) + I\left(X_1,X_2;Y\right)}, \tag{2.4}$$

over distribution $p(x)p(y_1,y_2|x)p(x_1,x_2)p(y|x_1,x_2)$.

Proof: For any fixed distribution $p(x)p(y_1,y_2|x)p(x_1,x_2)\ p(y|x_1,x_2)$, by the cut-set bound, we have

$$C_{\text{upper}} = \max_{\lambda\in(0,1)} \min\left\{\lambda I\left(X;Y_1,Y_2\right), \overline{\lambda} I\left(X_1,X_2;Y\right)\right\},$$

which comes from the BC cut-set and MAC cut-set [10, 24]. We then optimize over λ to obtain the rate upper bound, and it is easy to see that the minimum value is achieved iff the two terms are equal, which means $\lambda^* = \frac{I(X_1,X_2;Y)}{I(X;Y_1,Y_2)+I(X_1,X_2;Y)}$. With this optimal λ, we obtain the upper bound in (2.4). ■

For the Gaussian case, we choose X and $X_1 = X_2$ to be independent CSCG with distributions $\mathcal{CN}\left(0,P_S\right)$ and $\mathcal{CN}\left(0,P_R\right)$, respectively, and these input distributions maximize both $I\left(X;Y_1,Y_2\right)$ and $I\left(X_1,X_2;Y\right)$ simultaneously, which means that they maximize C_{upper}; and the corresponding rate upper bound is given by the following corollary.

Corollary B.1 *For the Gaussian case, we have the following rate upper bound*

$$C_{upper} = \frac{\log\left(1+\gamma_1+\gamma_2\right)\log\left(1+\widetilde{\gamma}_1+\widetilde{\gamma}_2+2\sqrt{\widetilde{\gamma}_1\widetilde{\gamma}_2}\right)}{\log\left(1+\gamma_1+\gamma_2\right)+\log\left(1+\widetilde{\gamma}_1+\widetilde{\gamma}_2+2\sqrt{\widetilde{\gamma}_1\widetilde{\gamma}_2}\right)}. \tag{2.5}$$

2. DF Achievable Rate

Main idea: For the DF scheme, the source transmits three messages: one common message w_0 to both of the relays, and one private message to each of the relays, denoted as w_1 and w_2, respectively. In the i-th relay, it compresses the received signal from the source, and sends the corresponding binning index through the conferencing link to the other relay, which helps with decoding the desired common message. In the second hop, the channel is indeed a MAC with common information. In the next, we first consider the DMC case and then the Gaussian case.

a. DF Rate for the DMC Case

We first focus on the first hop that is a BC channel with receiver one-round conferencing, for which the authors in [19] investigated the two cases: send one independent message to each receiver and send one common message to both receivers. In this subsection, we extend their results with a more general coding scheme, and have the following lemma.

Lemma B.1 *An achievable rate region of the general discrete memoryless BC with common message and decoder conferencing is given as*

$$\boldsymbol{R}_{BC} = \bigcup_{p(u_0)p(u_1|u_0)p(u_2|u_0)x(u_0,u_1,u_2)p(y_1,y_2|x)p(\hat{y}_1|y_1)p(\hat{y}_2|y_2)} \left\{ \begin{array}{l} (R_0, R_1, R_2): \; R_0, R_1, R_2 \geq 0, \\ R_0 + R_i \leq \lambda I\left(U_0, U_i; \hat{Y}_{3-i}, Y_i\right), \\ R_0 + R_1 + R_2 \leq \lambda I\left(U_i; \hat{Y}_{3-i}, Y_i | U_0\right) \\ \qquad + \lambda I\left(U_0, U_{3-i}; \hat{Y}_i, Y_{3-i}\right) - \lambda I\left(U_1; U_2 | U_0\right), \\ 2R_0 + R_1 + R_2 \leq \lambda I\left(U_0, U_1; \hat{Y}_2, Y_1\right) \\ \qquad + \lambda I\left(U_0, U_2; \hat{Y}_1, Y_2\right) - \lambda I\left(U_1; U_2 | U_0\right), \end{array} \right\}. \tag{2.6}$$

subject to the following constraints

$$C_{i,3-i} \geq \lambda I\left(\hat{Y}_i; Y_i\right) - \lambda I\left(\hat{Y}_i; Y_{3-i}\right), \; i = 1, 2, \tag{2.7}$$

where R_0, R_1, and R_2 are the rates of the common message, the private messages *for the first relay and the second relay, respectively, and* U_0*,* U_1*,* U_2*,* $\hat{Y}_1$*, and* $\hat{Y}_2$ *are auxiliary random variables defined on arbitrary finite sets with the distribution given in (2.6).*

Proof: See Appendix 1. ■

For the second hop, i.e., the MAC with common message, its achievable rate region is well studied, which is presented in the following lemma.

Lemma B.2 *An achievable rate region for the discrete memoryless MAC with common message is given as [25]*

$$\boldsymbol{R}_{MAC} = \bigcup_{p(u)p(x_1|u)p(x_2|u)p(y|x_1,x_2)} \left\{ \begin{array}{l} (R_0, R_1, R_2): \;\; R_0, R_1, R_2 \geq 0, \\ R_1 \leq \overline{\lambda} I\left(X_1; Y|U, X_2\right), \\ R_2 \leq \overline{\lambda} I\left(X_2; Y|U, X_1\right), \\ R_1 + R_2 \leq \overline{\lambda} I\left(X_1, X_2; Y|U\right), \\ R_0 + R_1 + R_2 \leq \overline{\lambda} I\left(U, X_1, X_2; Y\right). \end{array} \right\}, \tag{2.8}$$

where U *is an auxiliary random variable defined on arbitrary finite set with the distribution given in (2.8).*

From the Lemmas B.1 and B.2, we have the following theorem for an achievable rate of the considered diamond relay channel.

Theorem B.2 *An achievable rate by using the DF scheme for the DMC diamond relay channel with conferencing links is given as*

$$R_{DF} = \max_{\lambda, (R_0, R_1, R_2) \in R_{BC} \bigcap R_{MAC}} R_0 + R_1 + R_2. \tag{2.9}$$

Corollary B.2 *For the DMC case, there exist finite* C_{12} *and* C_{21} *satisfying*

$$\begin{cases} C_{12} \leq \lambda^* H\left(Y_1|Y_2\right) \\ C_{21} \leq \lambda^* H\left(Y_2|Y_1\right) \end{cases} \tag{2.10}$$

with λ^* *defined in Theorem B.1, such that the rate upper bound given in (2.4) can be achieved by the DF relaying scheme.*

Proof: We consider the scheme that only one common message is transmitted at the source, which will provide an upper bound on the conferencing link rates to achieve the rate upper bound, since other smarter coding schemes might require smaller conferencing link rates. By choosing U_1 and U_2 as constants (also by Theorem 3 in [19]), (2.6) can be rewritten as

$$R \leq \lambda^* \min\left\{I\left(X; Y_1, \hat{Y}_2\right), I\left(X; Y_2, \hat{Y}_1\right)\right\}, \tag{2.11}$$

subject to $C_{12} \geq \lambda^* I\left(\hat{Y}_1; Y_1\right) - \lambda^* I\left(\hat{Y}_1; Y_2\right)$ and $C_{21} \geq \lambda^* I\left(\hat{Y}_2; Y_2\right) - \lambda^* I\left(\hat{Y}_2; Y_1\right)$. By choosing $\hat{Y}_1 = Y_1$ and $\hat{Y}_2 = Y_2$, it is observed that (2.11) equals to the rate upper bound given in (2.4), and the conferencing link rates should satisfy $C_{12} \geq \lambda^* H\left(Y_1|Y_2\right)$ and $C_{21} \geq \lambda^* H\left(Y_2|Y_1\right)$, which means that when $C_{12} = \lambda^* H\left(Y_1|Y_2\right)$ and $C_{21} = \lambda^* H\left(Y_2|Y_1\right)$, the rate upper bound is achieved. As mentioned in the beginning of this proof, these values are only upper bounds for C_{12} and C_{21}, and thus, this corollary is proved. ■

The above conditions imply that Y_i can be reliably transmitted to relay $(3-i)$ via the conferencing links, or it can be reliably estimated by relay $(3-i)$ (e.g., when $Y_1 = Y_2$). Therefore, both relays have access to Y_1 and Y_2. In this case, the two relays are effectively one node with two antennas, and the network behaves like a three-node two-hop relay channel without direct link, whose rate is already known and is achieved by the DF scheme [2].

Remark B.1 *It is worth noting that for any conferencing link rates larger than the right-hand sides of (2.10), respectively, the rate upper bound given in (2.4) can also be achieved. In Corollary B.2, it only provides an upper bound for $C_{i,3-i}$ to achieve the rate upper bound of the considered channel, which is the conditional entropy of the received signal at the i-th relay given by another relay's received signal. In the proof, we point out that this result is only a sufficient condition, which is due to the fact that the cut-set upper bound is relatively loose under general channel conditions [26]. For some special cases, the DF scheme can achieve with rate upper bound without conferencing. For example, when the BC channel part is deterministic, i.e., $Y_1 = f_1(X)$ and $Y_2 = f_2(X)$, where f_1 and f_2 are some deterministic functions, the BC cut-set bound is achieved by sending one private message to each relay [27], and this means that conferencing will not introduce any improvement.*

Remark B.2 *With $C_{i,3-i} = 0$, we claim that our proposed scheme is equivalent to the conventional DF scheme without conferencing. The reason is given as follows: For such a case, we choose $\hat{Y}_1$ and $\hat{Y}_2$ as constants, and $\boldsymbol{R}_{BC}$ defined in (2.6) will degrade to the rate region of a BC channel with one common message and two private messages [28]. Moreover, for the Gaussian BC channel without relay conferencing, we only need to transmit one common message to both relays and one private message to the better relay [10]. Thus, our scheme is a generalization of the conventional DF scheme, and our DF rate will be the same as or higher than that without conferencing.*

b. DF Rate for the Gaussian Case

For the BC part, it contains one transmitter and two receivers, and there are two received signals at each receivers (equivalent to that each receiver is equipped with two antennas). As such, it is equivalent to a vector BC [28]. Furthermore, since one of the received signals at each relay is just a degraded version of that from the other relay, and the noise term in $\hat{Y}_i$ is correlated with that in Y_i. Thus, the first hop is indeed a vector BC with correlated noises, which is not physically degraded in general. Therefore, it is possible to transmit a unique private message to each relay. For the compression at the relays, we choose $\hat{Y}_i = Y_i + N_{i,3-i}$, where $N_{i,3-i}$ is a CSCG random variable distributed as $\mathcal{CN}\left(0, \sigma^2_{i,3-i}\right)$. It is easy to check that the Pareto boundary [44] of the rate region over (R_0, R_1, R_2) is achieved when the variances of the compression noises are minimized, which means that the equality in (2.7) is achieved, i.e., the compression noise is set to have

$$\sigma^2_{i,3-i} = \frac{1 + \gamma_1 + \gamma_2}{(\gamma_{3-i} + 1)\left(2^{C_{i,3-i}/\lambda} - 1\right)}. \tag{2.12}$$

We now discuss the coding scheme for the Gaussian BC, which combines DPC and superposition coding [45]. We choose the transmitting signal $X = X_0 + X_1 + X_2$, where X_0, X_1, and X_2 denote the common message and the private messages intended to relay 1 and relay 2, respectively, and they are independent zero mean CSCG random variables with variances $\overline{\mu}P_S$, $\mu_1 P_S$, and $\mu_2 P_S$, respectively, where the positive parameters $\overline{\mu}$, μ_1, and μ_2 are power allocation factors for X_0, X_1, and X_2, respectively, with $\overline{\mu} + \mu_1 + \mu_2 = 1$.

At the relays, the common message is first decoded by both of them, and then each relay decodes its intended private message. Private messages are encoded using DPC [45]: If we first encode X_2, we use X_2 as a state information to help with encoding X_1; and in the decoding process, relay 1 can decode X_1 without interference from X_2; on the other hand, we can exchange the encoding and decoding orders to possibly obtain a better rate region. Therefore, an achievable rate region of the first hop is given as

$$\boldsymbol{R}_{\mathrm{BC}}\left(R_0, R_1, R_2\right) = \mathrm{Conv}\left(\bigcup_{\pi,\mu_1,\mu_2} R\left(\pi, \mu_1, \mu_2\right)\right), \tag{2.13}$$

where Conv$(\cdot)$ is the convex hull operator, and $R\left(\pi, \mu_1, \mu_2\right)$ is an achievable rate region under a given power allocation scheme (μ_1, μ_2) and encoding order $\pi \in \{\pi_{12}, \pi_{21}\}$ with $\pi_{i,3-i}$ meaning that the i-th relay's private message is encoded first. Specially, if X_2 is encoded first, we have

$$\boldsymbol{R}\left(\pi_{21}, \mu_1, \mu_2\right) = \left\{\begin{array}{l} \left(R_0, R_1, R_2\right): \\ R_0 \leq \min_{i=1,2} \lambda \log\left(1 + \frac{\overline{\mu}\gamma_i\left(1+\sigma_{3-i,i}^2\right)+\overline{\mu}\gamma_{3-i}}{\left(\left(\mu_1+\mu_2\right)\gamma_i+1\right)\left(1+\sigma_{3-i,i}^2\right)+\left(\mu_1+\mu_2\right)\gamma_{3-i}}\right) \\ R_1 \leq \lambda \log\left(1 + \mu_1\gamma_1 + \frac{\mu_1\gamma_2}{1+\sigma_{21}^2}\right) \\ R_2 \leq \lambda \log\left(1 + \frac{\mu_2\gamma_2\left(1+\sigma_{12}^2\right)+\mu_2\gamma_1}{\left(\mu_1\gamma_2+1\right)\left(1+\sigma_{12}^2\right)+\mu_2\gamma_1}\right) \end{array}\right\}, \tag{2.14}$$

and $R\left(\pi_{12}, \mu_1, \mu_2\right)$ can be computed similarly.

Next, we consider the MAC part. We choose $X_1 = \sqrt{\overline{\alpha}P}U + \sqrt{\alpha P}V_1$ and $X_2 = \sqrt{\overline{\beta}P}U + \sqrt{\beta P}V_2$, where U, V_1, and V_2 are independent CSCG variables with distribution $\mathcal{CN}\left(0,1\right)$. Thus, an achievable rate region of the MAC channel with common message is given as

$$\boldsymbol{R}_{\mathrm{MAC}}\left(R_0, R_1, R_2\right) = \left\{\begin{array}{l} R_1 \leq \overline{\lambda}\log\left(1+\alpha\widetilde{\gamma}_1\right) \\ R_2 \leq \overline{\lambda}\log\left(1+\beta\widetilde{\gamma}_2\right) \\ R_1 + R_2 \leq \overline{\lambda}\log\left(1+\alpha\widetilde{\gamma}_1+\beta\widetilde{\gamma}_2\right) \\ R_0 + R_1 + R_2 \leq \overline{\lambda}\log\left(1+\widetilde{\gamma}_1+\widetilde{\gamma}_2+2\sqrt{\overline{\alpha}\overline{\beta}\widetilde{\gamma}_1\widetilde{\gamma}_2}\right) \end{array}\right\}. \tag{2.15}$$

Therefore, as stated in Theorem B.2, an achievable rate by using the DF scheme is the maximum sum rate over the intersection of the regions given in (2.14) and (2.15).

Remark B.3 *From (2.12), we observe that when $\sigma^2_{i,3-i}$ goes to zero, $C_{i,3-i}$ goes to infinity. In other words, for the Gaussian case, only when C_{12} and C_{21} go to infinity, the DF scheme can achieve the rate upper bound, which is different from the DMC case. Intuitively, for Gaussian channels, the alphabet size of $\mathcal{X}$ is infinite, and each relay cannot reliably decode its counterpart's received signal with the limited help from the other relay.*

Remark B.4 *When γ_i goes to infinity, the optimal λ goes to 0, and the rate upper bound becomes the same as the MAC cut-set bound. In this case, the source only needs to transmit a common message, and both relays can successfully decode it. Therefore, for finite $C_{i,3-i}$ and $\widetilde{\gamma}_i$, the DF scheme can asymptotically achieve the cut-set bound as γ_i goes to infinity. On the other hand, when γ_i and $C_{i,3-i}$ are fixed, and $\widetilde{\gamma}_i$ goes to infinity, the upper bound cannot be asymptotically achieved. This is due to the fact that the BC cut-set bound cannot be achieved with finite-rate relay conferencing.*

3. CF Achievable Rates

In this subsection, we discuss three different coding schemes based on the conventional CF relaying scheme. The first two schemes exploit the conferencing links to partially or completely exchange the binning index of the compressed receiver signals at the relays, and we call them the partial cooperation CF scheme (PCF) and the full cooperation CF scheme (FCF), respectively, which implies how much cooperation is introduced in the MAC part; the third scheme uses the conferencing links to help compression, called as the conferencing assited CF scheme (CCF) scheme.

a. PCF Achievable Rate

Here each relay first compresses its received signal as $\hat{Y}_i$ independently and obtains the corresponding binning index. Then, each relay splits the binning index into two sub-messages, and transmit one of them to the other relay by conferencing. In the second hop, the active part of the system is nothing but a MAC channel with a common message. Since we only introduce partially cooperative transmission in the MAC channel, we call it as the partial cooperation CF scheme, i.e., PCF, as defined earlier.

DMC Case: We have the following theorem for a PCF achievable rate.

Theorem B.3 *An achievable rate by using the PCF scheme for the DMC case is given as*

$$R_{PCF} \leq \max \lambda I\left(X; \hat{Y}_1, \hat{Y}_2\right) \tag{2.16}$$

$$s.t.\ \lambda I\left(\hat{Y}_1; Y_1|\hat{Y}_2\right) \leq \overline{\lambda} I\left(X_1; Y|U, X_2\right) + C_{12} \tag{2.17}$$

$$\lambda I\left(\hat{Y}_2; Y_2|\hat{Y}_1\right) \leq \overline{\lambda} I\left(X_2; Y|U, X_1\right) + C_{21} \tag{2.18}$$

$$\begin{aligned} \lambda I\left(\hat{Y}_1, \hat{Y}_2; Y_1, Y_2\right) \leq \min\{&\overline{\lambda} I\left(X_1, X_2; Y|U\right) \\ &+ C_{12} + C_{21}, \overline{\lambda} I\left(X_1, X_2; Y\right)\}, \end{aligned} \tag{2.19}$$

over the distribution $p(x)p(y_1, y_2|x)p(\hat{y}_1|y_1)p(\hat{y}_2|y_2)p(u)\ p(x_1|u)p(x_2|u)p(y|x_1, x_2)$*, and* U *is an auxiliary random variable similarly defined as before.*

The proof of this theorem is straightforward: The coding scheme in the first hop is the same as that for the conventional CF scheme in [10]; the second hop with conferencing links is a MAC channel with conferencing encoders and its rate region is given in [18]. By a similar argument to that in [10], we can obtain the PCF rate as shown in this theorem.

Remark B.5 *For the case* $C_{i,3-i} = 0,\ i = 1,2$*, the PCF scheme is the same as the conventional CF scheme without conferencing [10]; for the case* $C_{i,3-i} > 0$*, the PCF scheme is not worse than the conventional CF scheme. Note that even when the MAC region is strictly enlarged compared to the case without conferencing, we still cannot claim that the PCF scheme is strictly better than the case without conferencing, since the right-hand side of (2.19) may not be strictly improved, and when (2.19) is dominant among these constraints, the PCF rate will be equal to the case without conferencing.*

Gaussian Case: We define the compression at the relays as $\hat{Y}_i = Y_i + \hat{N}_i,\ i = 1,2$, where $\hat{N}_i$ is the compression noise with distribution $\mathcal{CN}(0, \sigma_i^2)$.

Corollary B.3 *An achievable rate by using the PCF scheme for the Gaussian case is given as*

$$R_{PCF} = \max_{\lambda,\alpha,\beta,\sigma_1^2,\sigma_2^2} \lambda \log\left(1 + \frac{\gamma_1}{1+\sigma_1^2} + \frac{\gamma_2}{1+\sigma_2^2}\right) \tag{2.20}$$

$$s.t.\quad \lambda \log\left(1 + \frac{1}{\sigma_1^2}\left(1 + \frac{\gamma_1(1+\sigma_2^2)}{1+\sigma_2^2+\gamma_2}\right)\right) \leq \overline{\lambda} \log(1 + \alpha\widetilde{\gamma}_1) + C_{12}, \tag{2.21}$$

$$\lambda \log\left(1 + \frac{1}{\sigma_2^2}\left(1 + \frac{\gamma_2(1+\sigma_1^2)}{1+\sigma_1^2+\gamma_1}\right)\right) \leq \overline{\lambda} \log(1 + \beta\widetilde{\gamma}_2) + C_{21}, \tag{2.22}$$

$$\begin{aligned} &\lambda \log\left(1 + \frac{1+\gamma_1}{\sigma_1^2} + \frac{1+\gamma_2}{\sigma_2^2} + \frac{1+\gamma_1+\gamma_2}{\sigma_1^2\sigma_2^2}\right) \\ &\quad \leq \min\Big\{\overline{\lambda}\log(1 + \alpha\widetilde{\gamma}_1 + \beta\widetilde{\gamma}_2) + C_{12} + C_{21}, \\ &\qquad\qquad \overline{\lambda}\log\left(1 + \widetilde{\gamma}_1 + \widetilde{\gamma}_2 + 2\sqrt{\overline{\alpha}\overline{\beta}\widetilde{\gamma}_1\widetilde{\gamma}_2}\right)\Big\}. \end{aligned} \tag{2.23}$$

Remark B.6 *For given* γ_i*,* $C_{i,3-i}$*,* α*, and* β*, when* $\widetilde{\gamma}_i \to \infty$*, which means that the optimal* $\lambda \to 1$*, from (2.21), (2.22), and (2.23), we see that both* σ_1^2 *and* σ_2^2 *scale to 0, and (2.20) asymptotically achieves the rate upper bound* $\log(1 + \gamma_1 + \gamma_2)$*. Therefore, when* $\widetilde{\gamma}_i \to \infty$*, the PCF scheme asymptotically achieve the rate upper bound.*

Remark B.7 *We consider the case with fixed* γ_i *and* $\widetilde{\gamma}_i$ *and infinite large* $C_{i,3-i}$*. In order to achieve the rate upper bound* $\lambda^* \log(1+\gamma_1+\gamma_2)$*, where* λ^* *is given by Theorem B.1, it is required for the destination to perfectly recover* $\hat{Y}_i = Y_i$*. However, for the Gaussian channel case, the alphabet sizes of* Y_i*'s are infinity, and the MAC part of the considered channel, i.e., the second hop, is with only finite capacity. Hence, the MAC part cannot support the perfect reconstruction of* Y_i *at the destination. Thus,* with finite channel gains, the PCF scheme cannot achieve the rate upper bound even with infinite conferencing rates.

Remark B.8 *For the case that* $C_{i,3-i}$ *and* $\widetilde{\gamma}_i$ *are fixed, and* $\gamma_i \to \infty$*, only if the following condition*

$$C_{12} + C_{21} \geq \log\left(1 + \widetilde{\gamma}_1 + \widetilde{\gamma}_2 + 2\sqrt{\widetilde{\gamma}_1\widetilde{\gamma}_2}\right), \tag{2.24}$$

is satisfied, the rate upper bound can be approached. This is due to the following fact: If we fix σ_1^2 *and* σ_2^2*, and choose* $\lambda = \frac{\log\left(\widetilde{\gamma}_1+\widetilde{\gamma}_2+2\sqrt{\widetilde{\gamma}_1\widetilde{\gamma}_2}\right)}{\log(1+\gamma_1+\gamma_2)}$*, it is easy to check that (2.20) asymptotically achieves the upper bound, the constraints (2.21) and (2.22) become redundant, and (2.23) asymptotically holds when* $\gamma_i \to \infty$ *and (2.24) satisfied.*

b. FCF Achievable Rate

With FCF, after obtaining the compression of the received signal $\hat{Y}_i$, each relay finds the binning index (the number of bins is determined by the corresponding conferencing link rate), and send this binning index to the other relay. Based on its own received signal and the binning index from the other relay, each relay tries to decode the compressed signal of the other relay. Then, partition the two compressions again into some other bins and transmit the new binning indices to the destination. In this case, each relay has a full knowledge of these two binning indices, and transmits a common message X_r through the MAC channel to the destination. Since we introduce full cooperation over such a MAC channel, we call this scheme as the full cooperation CF scheme, i.e., FCF, as defined earlier.

DMC Case: We have the following theorem for an FCF achievable rate.

Theorem B.4 *An achievable rate by using the FCF scheme for the DMC case is given as*

$$R_{FCF} \leq \max \lambda I\left(X; \hat{Y}_1, \hat{Y}_2\right) \tag{2.25}$$

$$s.t. \quad C_{i,3-i} \geq \lambda I\left(\hat{Y}_i; Y_i\right) - \lambda I\left(\hat{Y}_i; Y_{3-i}\right), \; i = 1, 2, \tag{2.26}$$

$$\lambda I\left(\hat{Y}_1, \hat{Y}_2; Y_1, Y_2\right) \leq \overline{\lambda} I\left(X_r; Y\right), \tag{2.27}$$

over the distribution $p(x)p(y_1, y_2|x)p(\hat{y}_1|y_1)p(\hat{y}_2|y_2)p(x_r)\ p(y|x_r)$*. Here, we define* $p(y|x_r) = p(y|x_r, x_r)$ *according to* $p(y|x_1, x_2)$ *when* $x_1 = x_2 = x_r$*.*

Proof: See Appendix 2. ■

Gaussian Case: We choose the distributions of X and X_r as $\mathcal{CN}(0, P_S)$ and $\mathcal{CN}(0, P_r)$, respectively. Furthermore, the compressions at the relays are according to $\hat{Y}_i = Y_i + \hat{N}_i$, $i = 1, 2$.

Corollary B.4 *An achievable rate by using the FCF scheme for the Gaussian case is given as*

$$R_{FCF} \leq \max_{\lambda, \sigma_1^2, \sigma_2^2} \lambda \log\left(1 + \frac{\gamma_1}{1+\sigma_1^2} + \frac{\gamma_2}{1+\sigma_2^2}\right) \tag{2.28}$$

$$s.t. \quad \sigma_i^2 \geq \frac{1+\gamma_1+\gamma_2}{\left(\gamma_{3-i}+1\right)\left(2^{C_{i,3-i}/\lambda}-1\right)}, \; i = 1, 2, \tag{2.29}$$

$$\lambda \log\left(1 + \frac{1+\gamma_1}{\sigma_1^2} + \frac{1+\gamma_2}{\sigma_2^2} + \frac{1+\gamma_1+\gamma_2}{\sigma_1^2\sigma_2^2}\right) \leq \overline{\lambda} \log\left(1 + \widetilde{\gamma}_1 + \widetilde{\gamma}_2 + 2\sqrt{\widetilde{\gamma}_1\widetilde{\gamma}_2}\right). \tag{2.30}$$

Remark B.9 *It can be checked that when* $C_{i,3-i} = 0$*,* $R_{FCF} = 0$ *for any channel parameters. This suggests that the FCF scheme is worse than the conventional CF scheme when* $C_{i,3-i}$ *is relatively small. In this case, we should not use conferencing to obtain full cooperation in the second hop, and the PCF scheme should be adopted instead. Denote the optimal solution for the CF rate (by Theorem 5.8 in [10]) as* $\left(\overline{\sigma}_1^2, \overline{\sigma}_2^2, \overline{\lambda}\right)$*, and it is easy to check that this solution also satisfies the constraint in (2.30). Thus, the threshold* $\overline{C}_{i,3-i}$*, below which the FCF scheme performs worse than the CF scheme, is obtained when the equality in (2.29) is achieved, i.e.,*

$$\overline{C}_{i,3-i} = \overline{\lambda} \log\left(1 + \frac{1+\gamma_1+\gamma_2}{\overline{\sigma_i}^2\left(\gamma_{3-i}+1\right)}\right). \tag{2.31}$$

Remark B.10 *For any given finite* γ_i *and* $C_{i,3-i}$*, when* $\widetilde{\gamma}_i$ *goes to infinity, the optimal* λ *goes to 1. However, the compression noise power* σ_i^2 *cannot scale to 0 due to the constraints in (2.29), which means that the asymptotic rate upper bound cannot be achieved.*

Remark B.11 *For any given finite* $\widetilde{\gamma}_i$ *and* $C_{i,3-i}$*, when* $\gamma_i \to \infty$ *(assuming that* γ_1 *and* γ_2 *are on the same order), we choose* $\lambda = \frac{\log\left(\widetilde{\gamma}_1+\widetilde{\gamma}_2+2\sqrt{\widetilde{\gamma}_1\widetilde{\gamma}_2}\right)}{\log(1+\gamma_1+\gamma_2)} \to 0$*, while* σ_i^2 *scales on the order of* $\frac{1}{\gamma_i}$ *according to (2.29). For (2.30), it is easy to check that the left-hand side of the inequality is equal to the right-hand side asymptotically. Therefore, we conclude that the FCF scheme asymptotically achieves the rate upper bound as* $\gamma_i \to \infty$*.*

Remark B.12 *For any given finite* γ_i *and* $\widetilde{\gamma}_i$*, by the same argument as in Remark B.7, we conclude that as the conferencing link rates go to infinity, the FCF scheme cannot achieve the rate upper bound even with infinite conferencing link rates.*

Remark B.13 *When the conferencing link rates go to infinity, we observe that the PCF and FCF schemes achieve the same asymptotic performance. In particular,* *since the constraints (2.21), (2.22), and (2.29) become redundant as the conferencing link rates go to infinity, problem (2.20)-(2.23) and problem (2.28)-(2.30) can both be rewritten as*

$$\begin{aligned} R \leq \max_{\lambda,\sigma_1^2,\sigma_2^2} \lambda \log\left(1+\frac{\gamma_1}{1+\sigma_1^2}+\frac{\gamma_2}{1+\sigma_2^2}\right) \\ s.t.\;\; \lambda \log\left(1+\frac{1+\gamma_1}{\sigma_1^2}+\frac{1+\gamma_2}{\sigma_2^2}+\frac{1+\gamma_1+\gamma_2}{\sigma_1^2\sigma_2^2}\right) \\ \leq \overline{\lambda} \log\left(1+\widetilde{\gamma}_1+\widetilde{\gamma}_2+2\sqrt{\widetilde{\gamma}_1\widetilde{\gamma}_2}\right), \end{aligned}$$

which means that these two schemes achieve the same performance for the considered case.

c. CCF Achievable Rate

In this scheme, each relay generates its own compression intended for the second hop based on two signals: the received signal from the source, and the compressed signal from the other relay through the conferencing link.

DMC Case: We have the following theorem regarding a CCF achievable rate.

Theorem B.5 *As we use the conferencing links to help with compressing the received signal at the relays, an achievable rate by using the CCF scheme for the DMC case is given by*

$$\begin{aligned} R_{CCF} = \max \lambda I\left(X;\hat{Y}_1,\hat{Y}_2\right) \\ s.t.\; (2.7),\;\; \lambda I\left(\hat{Y}_1;Y_1,\hat{Y}_{21}|\hat{Y}_2\right) \leq \overline{\lambda} I\left(X_1;Y|X_2\right) \\ \lambda I\left(\hat{Y}_2;Y_2,\hat{Y}_{12}|\hat{Y}_1\right) \leq \overline{\lambda} I\left(X_2;Y|X_1\right) \\ \lambda I\left(\hat{Y}_1,\hat{Y}_2;Y_1,Y_2,\hat{Y}_{12},\hat{Y}_{21}\right) \leq \overline{\lambda} I\left(X_1,X_2;Y\right), \end{aligned} \tag{2.32}$$

over the distribution

$$p(x)p(y_1,y_2|x)p\left(\hat{y}_{12}|y_1\right)p\left(\hat{y}_{21}|y_2\right)p\left(\hat{y}_1|y_1,\hat{y}_{21}\right)p\left(\hat{y}_2|y_2,\hat{y}_{12}\right)p(x_1,x_2)p(y|x_1,x_2).$$

Proof: See Appendix 3. ■

Gaussian Case: We choose the distributions of transmit signals over the conferencing links as $\hat{Y}_{12}=Y_1+N_{12}$ and $\hat{Y}_{21}=Y_2+N_{21}$, respectively, where N_{12} and N_{21} are independent zero mean CSCG random variable, with variances defined the same as in (2.12). For the relay signals to the destination, we choose $\hat{Y}_1=aY_1+b\hat{Y}_{21}+V_1$ and $\hat{Y}_2=cY_2+d\hat{Y}_{12}+V_2$, where a, b, c, and d are some parameters, V_1 and V_2 are independent zero mean CSCG random variables with variances σ_1^2 and σ_1^2, respectively. Then, a CCF achievable rate for the Gaussian case is given as

$$\begin{aligned} R_{\text{CCF}} = & \max_{\lambda,a,b,c,d,\sigma_1^2,\sigma_2^2} \lambda\log\left(\frac{P_{\hat{Y}_1\hat{Y}_2}}{\hat{\sigma}_1^2\hat{\sigma}_2^2-|ad^*+bc^*|^2}\right) \\ \text{s.t.}\quad & \lambda\log\left(\frac{P_{\hat{Y}_1\hat{Y}_2}}{\sigma_1^2\left(|dh_1+ch_2|^2P+\hat{\sigma}_2^2\right)}\right)\le\overline{\lambda}\log\left(1+\widetilde{\gamma}_1\right) \\ & \lambda\log\left(\frac{P_{\hat{Y}_1\hat{Y}_2}}{\sigma_2^2\left(|ah_1+bh_2|^2P+\hat{\sigma}_1^2\right)}\right)\le\overline{\lambda}\log\left(1+\widetilde{\gamma}_2\right) \\ & \lambda\log\left(\frac{P_{\hat{Y}_1\hat{Y}_2}}{\sigma_1^2\sigma_2^2}\right)\le\overline{\lambda}\log\left(1+\widetilde{\gamma}_1+\widetilde{\gamma}_2\right), \end{aligned} \tag{2.33}$$

where

$$\begin{aligned} P_{\hat{Y}_1\hat{Y}_2} = & |ah_1+bh_2|^2P_S\hat{\sigma}_2^2+|dh_1+ch_2|^2P_S\hat{\sigma}_1^2+\hat{\sigma}_1^2\hat{\sigma}_2^2-|ad^*+bc^*|^2 \\ & -2\Re\left[\left(ah_1+bh_2\right)\left(dh_1+ch_2\right)^*\left(ad^*+bc^*\right)P_S\right], \end{aligned} \tag{2.34}$$

with $\hat{\sigma}_1^2=|a|^2+|b|^2\left(1+\sigma_{21}^2\right)+\sigma_1^2$, and $\hat{\sigma}_2^2=|c|^2+|d|^2\left(1+\sigma_{12}^2\right)+\sigma_2^2$. It is easy to check that the above objective function is not convex over a, b, c, and d jointly. Since it is difficult to compute the maximum rate, we try to find a sub-optimal but much simpler solution, i.e., letting $a=d=h_1^*$ and $b=c=h_2^*$, which will be used for the simulations in Section C.

Remark B.14 *Since the conventional CF scheme is just a special case of our scheme, by letting $\hat{Y}_{i,3-i}$ be a constant, a CCF achievable rate for the DMC case is the same as the case without conferencing [10]. Hence, with our setup, we conclude that the CCF rate is the same as or higher than the conventional CF rate. However, since only the sub-optimal solution for the combining problem at the relay is adopted, the CCF scheme may not perform better than the conventional CF scheme for the Gaussian case, and this will be shown in Section C.*

Remark B.15 *Considering another case when C_{12} and C_{21} go to infinity, by the same argument as Remark B.7, we conclude that the CF scheme cannot achieve the rate upper bound.*

4. AF Achievable Rate

In this subsection, to make the AF relaying scheme meaningful, we further assume that the conferencing links are Gaussian channels, which also use AF as the conferencing scheme. Without loss of generality, we assume that the input of the conferencing link is $x_{i,3-i} = y_i = h_i x + n_i$. Furthermore, we assume that the link gain of each conferencing link equals to 1, and the conferencing link output in the i-th relay is given as

$$y_{3-i,i} = x_{3-i,i} + n_{3-i,i}, \tag{2.35}$$

where $n_{3-i,i}$ is CSCG noise with distribution $\mathcal{CN}\left(0, \sigma^2_{3-i,i}\right)$. Based on the conferencing link rate constraints, the variance of $n_{3-i,i}$ is given as $\sigma^2_{3-i,i} \geq \frac{\gamma_{3-i}+1}{2^{C_{3-i,i}/2}-1}$, where the conferencing link rate is subject to a $\frac{1}{2}$ pre-log penalty due to the conferencing scheduling scheme introduced in Section A. Obviously, when the equality holds, the

AF scheme performs the best. Thus, we let

$$\sigma_{3-i,i}^2 = \frac{\gamma_{3-i} + 1}{2^{C_{3-i,i}/2} - 1}. \tag{2.36}$$

After the conferencing sessions, the relays combine the two received signals from the source node and the other relay, which leads to

$$x_i = a_{ii} y_i + a_{3-i,i} y_{3-i,i}, \tag{2.37}$$

where a_{ii} and $a_{3-i,i}$ are some complex parameters, and satisfy the following power constraints

$$\mathbb{E}\left(x_i^2\right) = |a_{ii}|^2 \left(|h_i|^2 P_S + 1\right) + |a_{3-i,i}|^2 \left(|h_{3-i}|^2 P_S + 1 + \sigma_{3-i,i}^2\right) \le P_R. \tag{2.38}$$

Therefore, the received signal at the destination is given as

$$\begin{aligned} y &= g_1 x_1 + g_2 x_2 + n \\ &= \left(a_{11} h_1 g_1 + a_{12} h_1 g_2 + a_{21} h_2 g_1 + a_{22} h_2 g_2\right) x \\ &\quad + \left(a_{11} g_1 + a_{12} g_2\right) n_1 + \left(a_{21} g_1 + a_{22} g_2\right) n_2 \\ &\quad + a_{21} g_1 n_{21} + a_{12} g_2 n_{12} + n, \end{aligned}$$

and an achievable rate by using the AF scheme is given as

$$R_{\mathrm{AF}} = \frac{1}{2} \log\left(1 + \gamma_{\mathrm{AF}}\right), \tag{2.39}$$

where γ_{AF} is the received SNR at the destination, given as

$$\gamma_{\mathrm{AF}} = \frac{|a_{11} h_1 g_1 + a_{12} h_1 g_2 + a_{21} h_2 g_1 + a_{22} h_2 g_2|^2 P_S}{|a_{11} g_1 + a_{12} g_2|^2 + |a_{21} g_1 + a_{22} g_2|^2 + |a_{21} g_1|^2 \sigma_{21}^2 + |a_{12} g_2|^2 \sigma_{12}^2 + 1}. \tag{2.40}$$

We now rewrite (2.40) to a matrix form, and maximize it to obtain the maximum

AF rate defined in (2.39). Thus, we have the following optimization problem

$$\begin{aligned} &\max \quad \frac{\mathbf{a}^H \mathbf{R} \mathbf{a}}{\mathbf{a}^H \mathbf{Q} \mathbf{a} + 1} \\ &\text{s.t.} \quad (2.38), \end{aligned} \tag{2.41}$$

where $\mathbf{a} = [a_{11}, a_{12}, a_{21}, a_{22}]^T$, $\mathbf{b} = [h_1^* g_1^*, h_1^* g_2^*, h_2^* g_1^*, h_2^* g_2^*]^T$, and the matrices $\mathbf{R} = \mathbf{b}\mathbf{b}^H$,

$$\mathbf{Q} = \begin{bmatrix} |g_1|^2 & g_1^* g_2 & 0 & 0 \\ g_1 g_2^* & |g_2|^2 \left(1 + \sigma_{12}^2\right) & 0 & 0 \\ 0 & 0 & |g_1|^2 \left(1 + \sigma_{21}^2\right) & g_1^* g_2 \\ 0 & 0 & g_1 g_2^* & |g_2|^2 \end{bmatrix}. \tag{2.42}$$

From (2.40), we know that $\mathbf{R}$ and $\mathbf{Q}$ are positive semidefinite. By a similar argument as in [46], this problem can be shown equivalent to

$$\begin{aligned} \max_{\mathbf{A}, t} \quad & t \\ \text{s.t.} \quad & \operatorname{Tr}\left(\mathbf{A}\left(\mathbf{R} - t\mathbf{Q}\right)\right) \geq t, \ (2.38), \\ & \operatorname{Rank}(\mathbf{A}) = 1, \mathbf{A} \succeq 0, \end{aligned} \tag{2.43}$$

where $\mathbf{A} = \mathbf{a}\mathbf{a}^H$. Using semidefinite relaxation [46], we aim to solve the following optimization problem:

$$\begin{aligned} \max_{\mathbf{A}, t} \quad & t \\ \text{s.t.} \quad & \operatorname{Tr}\left(\mathbf{A}\left(\mathbf{R} - t\mathbf{Q}\right)\right) \geq t, \\ & (2.38), \ \mathbf{A} \succeq 0. \end{aligned} \tag{2.44}$$

Remark B.16 *This optimization problem can be efficiently solved by bisection search over t; and for each t, the remaining problem is a convex feasibility problem, which* *can be efficiently solved using existing numerical tools, e.g., CVX [47]. However, the final solution may not be rank-1 to satisfy the constraint in (2.43); so we use the following randomization technique [46] to provide an approximate solution to the original rank-1 problem in (2.43): Denote the solution of problem (2.44) as* $\mathbf{A}^*$*, with its eigenvalue decomposition* $\mathbf{A}^* = \mathbf{U}\mathbf{D}\mathbf{U}^H$*; we choose* $\mathbf{a} = \mathbf{U}\mathbf{D}^{1/2}\mathbf{v}$*, where* $\mathbf{v}$ *is a vector of zero-mean unit-variance i.i.d. Gaussian random variables. We then scale* $\mathbf{a}$ *to make the power constraints (2.38) satisfied [48].*

Remark B.17 *If a rank-1 optimal solution for (2.44) can be found, our AF rate will be higher than the AF rate without conferencing, i.e., the case* $C_{i,3-i} = 0$*. This is due to the facts that the conventional AF relaying optimization problem is a special case of (2.41) with* $a_{12} = a_{21} = 0$*. However, sometimes we may not obtain the exact optimal solution of rank-1 for (2.44), such that there is a gap to the optimal value with the solution from the randomization method [48]. For these cases, our proposed AF scheme may not be better than the case without conferencing. It will be shown in Section C that for small conferencing link rates, our scheme performs worse than the conventional AF scheme without conferencing; but the reverse is true for large* $C_{i,3-i}$ *cases.*

Remark B.18 *It is easy to check that when* γ_i *goes to infinity, the AF scheme can achieve one-half of the rate upper bound, which is due to the half-duplex constraint. On the other hand, if both* γ_i *and* $\widetilde{\gamma}_i$ *are finite, the upper bound is not achievable even with infinite conferencing link capacity.*

C. Numerical Results

In this section, we present some numerical results to compare the performance among the proposed coding schemes. We consider both the symmetric and asymmetric cases.

1. Symmetric Case

For this case, we let $|h_1| = |h_2|$, $|g_1| = |g_2|$, and $C_{12} = C_{21} = C$. To further discuss the effects of the relative nodes location to achievable rates, we set the locations of the source node, the destination node, and the relays as $s_0 = (-1, 0)$, $s_3 = (0, 1)$, $s_1 = (d, -\sqrt{1-d^2})$, and $s_2 = (d, +\sqrt{1-d^2})$, respectively, where $d \in (-1, 1)$. Furthermore, we assume that the link gains satisfy $|h_i| = \frac{1}{|s_0 - s_i|}$ and $|g_i| = \frac{1}{|s_3 - s_i|}$, $i = 1, 2$. For the phases of h_i and g_i, we assume that they are uniform random variables over $[0, 2\pi]$. Moreover, we choose $P_S = P_R = 1$.

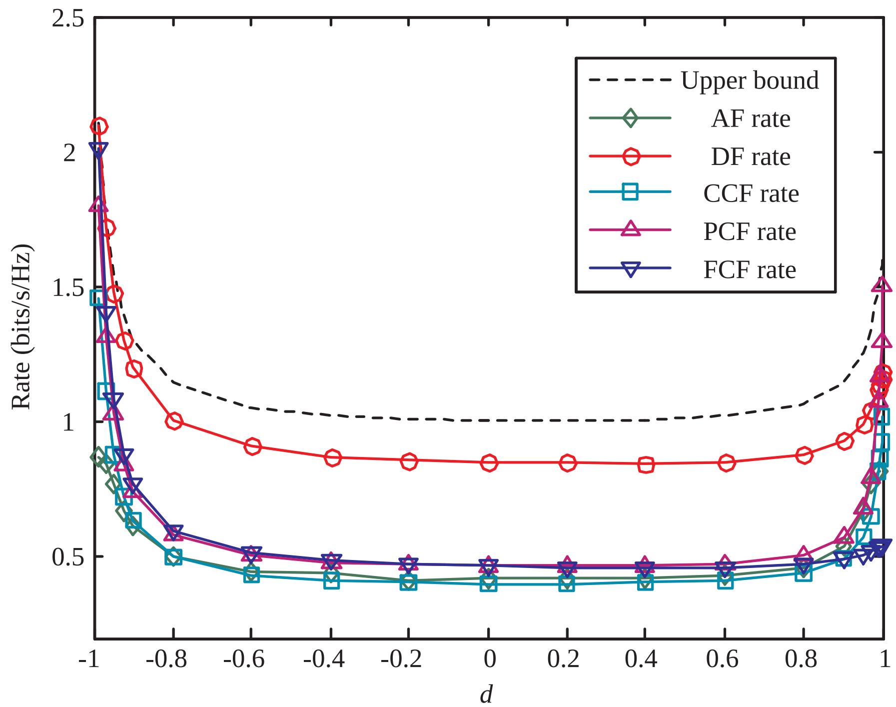


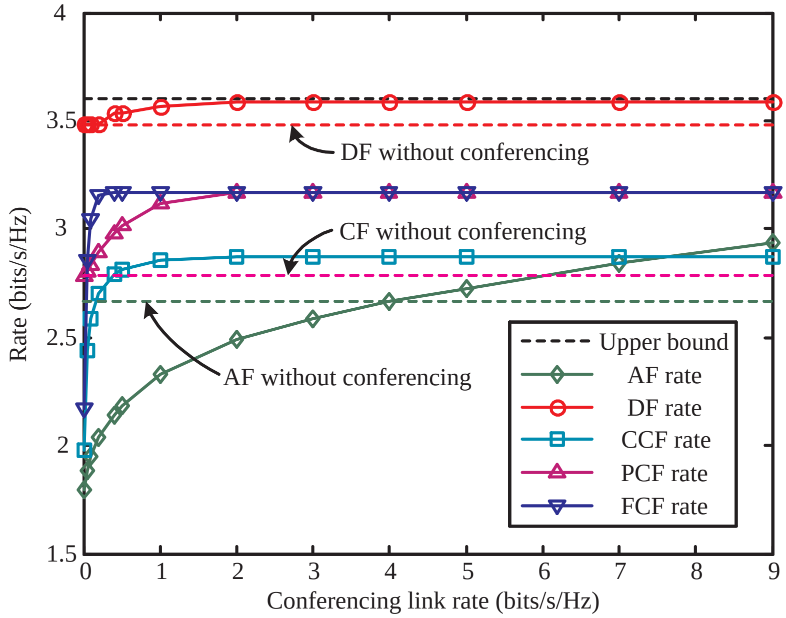
Fig. 3.: Achievable rates and cut-set upper bound for symmetric link gain case, with $C = 0.5$.

In Fig. 3, we compare the performance of the proposed schemes under the symmetric channel gain assumption for different relay locations. Here, we fix the conferencing link rate $C = 0.5$ bit/s/Hz. We observe that when d goes to -1, i.e., when the relays get close to the source node, the DF scheme asymptotically achieves the rate upper bound, so do the FCF and PCF schemes. Moreover, all three CF schemes outperform the AF scheme, but they are worse than the DF scheme. As d goes to 1, i.e., when the relays get close to the destination, we observe that the PCF scheme achieve the rate upper bound asymptotically, while the DF, AF, and FCF schemes are strictly suboptimal. For the case when d is around 0, the DF scheme performs the best among all the coding schemes, and the performances of the others are almost the same.

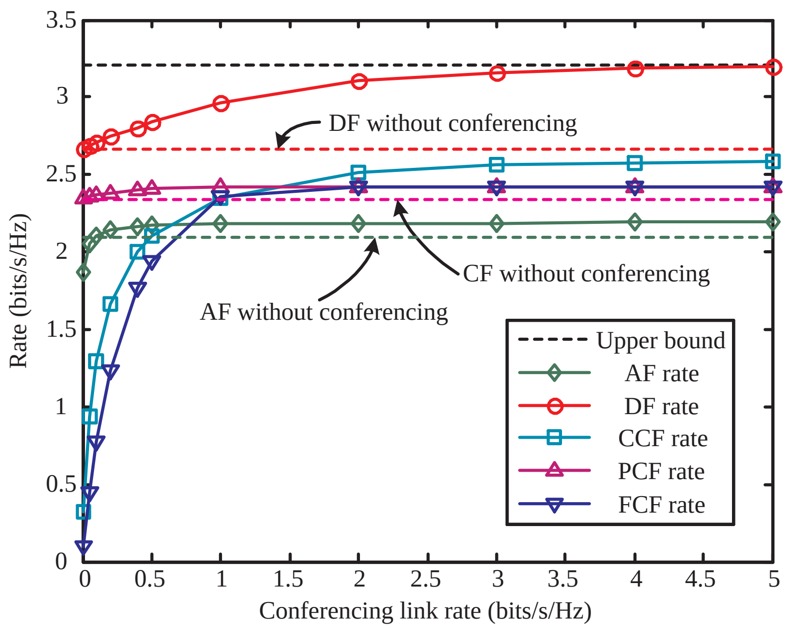
In Fig. 4(a) and Fig. 4(b), with different channel gains, we compare the performances of the coding schemes as the conferencing link rate increases. We consider two typical setups: the BC channel gains are larger than those of the MAC channel for Fig. 4(a), and the reverse case for Fig. Fig. 4(b). Overall, we observe that for each relaying scheme, there is an asymptotic performance limitation as the conferencing link rate increases.

Note that when $C = 0$, the proposed DF and PCF schemes are equivalent to the conventional DF and CF schemes. From these two subfigures, we observe that conferencing can strictly increase the DF and CF achievable rates using the proposed DF and PCF schemes, respectively. However, for the AF, CCF, and FCF schemes, they cannot guarantee to increase the AF and CF rates as we discussed before, respectively, especially when C is small.

For both cases shown in Fig. 4(a) and Fig. 4(b), the DF scheme gets close to the rate upper bound when C is large enough: For the good BC channel case, we need $C \geq 2$ bits/s/Hz, and for the good MAC channel case, we need $C \geq 4$ bits/s/Hz. For the PCF and FCF schemes, we observe that as C becomes large, they have the same performance; when C is very close to 0, the PCF scheme always performs better; for small C but not close to 0, the FCF scheme performs better in the good BC channel



(a) Good BC vs. bad MAC, $\gamma_i = 30$ dB, $\widetilde{\gamma}_i = 10$ dB



(b) Bad BC vs. good MAC, $\gamma_i = 10$ dB, $\widetilde{\gamma}_i = 30$ dB

Fig. 4.: Achievable rates over different conferencing link rates, $P_S = P_R = 1$.

case, and the reverse is true for the good MAC channel case. In the high conferencing rate regime, the CCF scheme performs better than the other two CF schemes for the good MAC channel case, and the reverse is true for the good BC channel case.

2. Asymmetric Case

In this subsection, we consider the cases that these links are with different qualities, which can model the scenarios that the two relays are located with different distances to the source node.

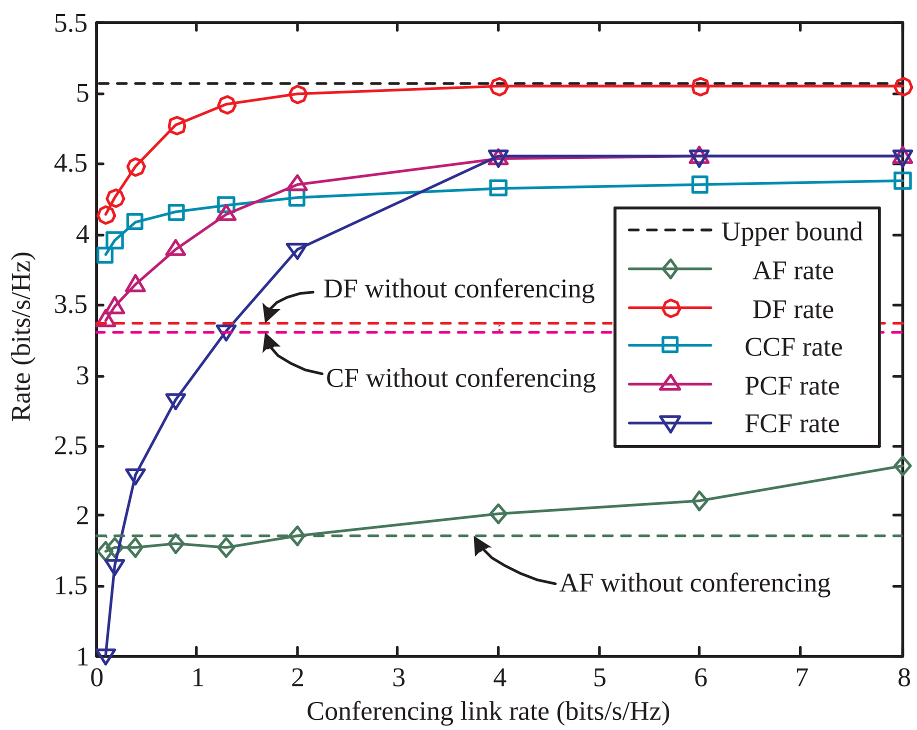


Fig. 5.: Achievable rates and cut-set upper bound for asymmetric link gain case, with $|\gamma_1|^2 = |\widetilde{\gamma}_2|^2 = 30$ dB and $|\widetilde{\gamma}_1|^2 = |\gamma_2|^2 = 10$ dB.

In Fig. 5, we plot achievable rates and the rate upper bound as functions of the conferencing link rates $C_{12} = C_{21} = C$ for the case that the source-relay and relay-destination links are not symmetrical, i.e., we set $|\gamma_1|^2 = |\widetilde{\gamma}_2|^2 = 30$ dB and $|\widetilde{\gamma}_1|^2 = |\gamma_2|^2 = 10$ dB. It is observed that the DF scheme can still achieve the rate upper bound as the conferencing link rates go to infinity asymptotically; for the PCF and FCF relaying schemes, the relay conferencing introduces about 1.2 bit gain when the conferencing link rates are large than 4 bits. for the AF relaying scheme, the relay conferencing provides gains only when the conferencing link rates are relatively large.

In Fig. 6(a) and Fig. 6(b), we plot achievable rates and the rate upper bound for the case with symmetric source-relay and relay-destination channel gains, while the conferencing link rates are different. Specifically, we set one conferencing link rate always equal to zero, i.e., $C_{12} = C$ and $C_{21} = 0$. We observe that the DF scheme cannot approach the rate upper bound even when the conferencing link rate is large for both the good BC (in Fig. 6(a)) and good MAC (in Fig. 6(b)) cases, which is due to the fact that relay 1 cannot decode all source messages. For the AF rate, it only changes slightly as the conferencing link rate increases; Moreover, for the good MAC case in Fig. 6(b), the PCF scheme performs better than the FCF scheme in the high conferencing link rate regime, which is different from the symmetric cases (shown in Fig. 4(a) and 4(b))and the good BC case with asymmetric conferencing link rates (shown in Fig. 6(a)).

D. Summary

Table II.: Upper-bound-achieving cases for the diamond relay channel with conferencing links

	DMC	Gaussian channel	
		$\gamma_i \to \infty$	$\widetilde{\gamma}_i \to \infty$
Schemes	DF with finite conferencing link rates	DF, FCF	PCF

In this chapter, we discussed the rate upper bound and achievable rates of the diamond relay channel with conferencing links. For the DF scheme, we derived an achievable rate by sending a common message and two private messages. We developed three new coding schemes based on CF and used the conferencing links to exchange certain compressed information between the relays, whose achievable rates were computed for both the DMC and Gaussian cases. For the AF scheme, we discussed the optimal combining problem between the signals from the source and the conferencing link at the relays, and use semidefinite relaxation and bisection search to efficiently obtain a sub-optimal solution. All the upper-bound-achieving cases are also summarized in Table II.

E. Appendix

1. Proof of Lemma B.1

Fix the distribution $p(u_0)p(u_1|u_0)p(u_2|u_0)p(y_1, y_2|x)\ p(\hat{y}_1|y_1)p(\hat{y}_2|y_2)$ and the function $x(u_0, u_1, u_2)$.

Codebook Generation: In the source, generate 2^{nR_0} i.i.d. sequences $\boldsymbol{u}_0(w_0)$, $w_0 \in \left[1 : 2^{nR_0}\right]$, according to the distribution $\prod_{j=1}^{\lambda n} p\left(u_{0,j}\right)$. For each $\boldsymbol{u}_0(w_0)$, generate 2^{nR_i} i.i.d. sub-codebooks $\boldsymbol{Q}_i(w_0, w_i)$, $w_i \in \left[1 : 2^{nR_i}\right]$, where each sub-codebook contains $2^{n\left(\widetilde{R}_i - R_i\right)}$ i.i.d. sequences $\boldsymbol{u}_i(w_0, l_i)$, $l_i \in \left[(w_i - 1)\, 2^{n\left(\widetilde{R}_i - R_i\right)} + 1 : w_i 2^{n\left(\widetilde{R}_i - R_i\right)}\right]$, according to $\prod_{j=1}^{\lambda n} p\left(u_{i,j}|u_{0,j}(w_0)\right)$. For each triple (w_0, w_1, w_2), define the set

$$\boldsymbol{Q}(w_0, w_1, w_2) = \{(u_1(w_0, l_1), u_2(w_0, l_2)) \in \boldsymbol{Q}_1(w_0, w_1) \times \boldsymbol{Q}_2(w_0, w_2) : \\ (u_0(w_0), u_1(w_0, l_1), u_2(w_0, l_2)) \in A_{\epsilon'}^n\}.$$

Conferencing function generation: Generate $2^{nR_i'}$ i.i.d. sequences $\hat{\boldsymbol{y}}_i(k_i)$,

$k_i \in \left[1 : 2^{nR'_i}\right]$, according to $\prod_{j=1}^{\lambda n} p\left(\hat{y}_{i,j}\right)$, where

$$p_{\hat{Y}_i}\left(\hat{y}_{i,j}\right) = \sum_{\mathcal{X},\mathcal{Y}_1,\mathcal{Y}_2} p\left(\hat{y}_i|y_i\right) p\left(y_1, y_2|x\right) p(x)$$

and $p(x) = \sum_{\mathcal{U}_1,\mathcal{U}_2} p(u_1, u_2, x)$. Randomly and uniformly partition the index set $\left[1 : 2^{nR'_i}\right]$ into $2^{nC_{i,3-i}}$ binnings $\boldsymbol{S}_i\left(s_i\right)$, $s_i \in \left[1 : 2^{nC_{i,3-i}}\right]$.

Encoding and Decoding: In the source, for each triple (w_0, w_1, w_2), pick one sequence pair $\left(\boldsymbol{u}_1\left(w_0, l_1\right), \boldsymbol{u}_2\left(w_0, l_2\right)\right) \in \boldsymbol{Q}\left(w_0, w_1, w_2\right)$, and generate a codeword $\boldsymbol{x}\left(w_0, w_1, w_2\right)$ according to $\prod_{i=1}^{\lambda n} p\left(x_i|u_1\left(w_0, l_1\right), u_2\left(w_0, l_2\right)\right)$; if no such pair exists, declare an error. This operation can be done reliably if [28]

$$\left(\widetilde{R}_1 - R_1\right) + \left(\widetilde{R}_2 - R_2\right) \geq \lambda I\left(U_1; U_2|U_0\right). \tag{2.45}$$

In the i-th relay, upon receiving $\boldsymbol{y}_i$, it tries to find a $\hat{\boldsymbol{y}}_i(k_i)$ such that $(\boldsymbol{y}_i, \hat{\boldsymbol{y}}_i(k_i)) \in A_\epsilon^n$, and this can be done reliably as n goes to infinity, if

$$R'_i \geq \lambda I\left(\hat{Y}_i; Y_i\right). \tag{2.46}$$

Then, the i-th relay finds the corresponding binning index number s_i, where $k_i \in \boldsymbol{S}_i(s_i)$, and sends it to the other relay through the conferencing link.

After receiving the conferencing message from its counterpart, the i-th relay first tries to find the unique $\hat{k}_{3-i}$ such that $\left(\hat{\boldsymbol{y}}_{3-i}(\hat{k}_{3-i}), \boldsymbol{y}_i\right) \in A_\epsilon^n$ with $\hat{k}_{3-i} \in \boldsymbol{S}_{3-i}(s_{3-i})$. This can be done reliably if

$$R'_{3-i} \leq \lambda I\left(\hat{Y}_{3-i}; Y_i\right) + C_{3-i,i}. \tag{2.47}$$

From (2.46) and (2.47), we obtain

$$C_{i,3-i} \geq \lambda I\left(\hat{Y}_i; Y_i\right) - \lambda I\left(\hat{Y}_i; Y_{3-i}\right). \tag{2.48}$$

Then, the i-th relay finds a unique pair $(\hat{w}_0, \hat{w}_i)$ satisfying

$$\left(\boldsymbol{u}_0(\hat{w}_0), \boldsymbol{u}_i(\hat{w}_0, \hat{l}_i), \hat{\boldsymbol{y}}_{3-i}(\hat{k}_{3-i}), \boldsymbol{y}_i\right) \in A_\epsilon^n,$$

and this can be done reliably if

$$\begin{cases} \widetilde{R}_i \leq \lambda I\left(U_i; \hat{Y}_{3-i}, Y_i | U_0\right) \\ R_0 + \widetilde{R}_i \leq \lambda I\left(U_0, U_i; \hat{Y}_{3-i}, Y_i\right). \end{cases} \tag{2.49}$$

From (2.45), (2.48), and (2.49), we obtain the rate region of the general broadcast channel with common message and conferencing as follows:

$$\boldsymbol{R}'_{\mathrm{BC}} = \bigcup_{p(u_0)p(u_1|u_0)p(u_2|u_0)p(x|u_1,u_2)p(y_1,y_2|x)p(\hat{y}_1|y_1)p(\hat{y}_2|y_2)} \left\{ \begin{array}{l} (R_0, R_1, R_2): \\ 0 \leq R_0, 0 \leq R_1 \leq \widetilde{R}_1, 0 \leq R_2 \leq \widetilde{R}_2, \\ \widetilde{R}_1 \leq \lambda I\left(U_1; \hat{Y}_2, Y_1 | U_0\right), \\ R_0 + \widetilde{R}_1 \leq \lambda I\left(U_0, U_1; \hat{Y}_2, Y_1\right), \\ \widetilde{R}_2 \leq \lambda I\left(U_2; \hat{Y}_1, Y_2 | U_0\right), \\ R_0 + \widetilde{R}_2 \leq \lambda I\left(U_0, U_2; \hat{Y}_1, Y_2\right), \\ \left(\widetilde{R}_1 - R_1\right) + \left(\widetilde{R}_2 - R_2\right) \geq \lambda I\left(U_1; U_2 | U_0\right), \\ \text{subject to: } (2.48). \end{array} \right\}. \tag{2.50}$$

Thus, the rate region R_{BC} is obtained from R'_{BC} using the Fourier-Motzkin elimination [50] to eliminate $\widetilde{R}_i$, $i = 1, 2$.

2. Proof of Theorem B.4

Fix the distribution as given in the theorem.

Codebook generation: Generate 2^{nR} i.i.d. sequences $\boldsymbol{x}(w)$, $w \in \left[1 : 2^{nR}\right]$, according to $\prod_{i=1}^{n} p(x_i)$. Generate $2^{n\hat{R}_i}$, $i = 1, 2$, i.i.d. sequences $\hat{\boldsymbol{y}}_i(w_i)$, $w_i \in \left[1 : 2^{n\hat{R}_i}\right]$, according to the distribution $p(\hat{y}_i) = \int p(x)p(y_i|x)p(\hat{y}_i|y_i)dxdy_i$. Randomly and uniformly partition the set $\left[1 : 2^{n\hat{R}_i}\right]$ into 2^{nR_i} binnings $\boldsymbol{S}_i(s_i)$, $s_i \in \left[1 : 2^{nR_i}\right]$. Randomly and uniformly partition the set $\left[1 : 2^{nR_i}\right]$ into $2^{nC_{i,3-i}}$ binnings $\boldsymbol{M}_i(m_i)$. Generate $2^{n(R_1+R_2)}$ i.i.d. sequences $\boldsymbol{x}_r\left(s_1, s_2\right)$, according to $p\left(x_r\right)$.

Encoding and decoding: At the source, it transmits $\boldsymbol{x}(w)$. At the i-th relay, $i = 1, 2$, it finds a $\hat{\boldsymbol{y}}_i(w_i)$ such that $(\hat{\boldsymbol{y}}_i(w_i), \boldsymbol{y}_i) \in A_\epsilon^n$, and this can be done reliably if $\hat{R}_i \geq \lambda I\left(\hat{Y}_i; Y_i\right)$. Then, at the i-th relay, it finds the conferencing binning index m_i, and sends it to the other relay through the conferencing link. Upon receiving m_{3-i}, the i-th relay decodes $\hat{\boldsymbol{y}}_{3-i}(w_{3-i})$ such that $\left(\hat{\boldsymbol{y}}_{3-i}(\hat{k}_{3-i}), \boldsymbol{y}_i\right) \in A_\epsilon^n$ with $\hat{k}_{3-i} \in \boldsymbol{M}_{3-i}(m_{3-i})$. This can be done reliably if $R'_{3-i} \leq \lambda I\left(\hat{Y}_{3-i}; Y_i\right) + C_{3-i,i}$. Thus, we satisfy the constraints in (2.26). Then, the i-th relay knows the binning index pair (s_1, s_2), and transmits $\boldsymbol{x}_r(s_1, s_2)$.

At the destination, it first decodes (s_1, s_2), and we obtain $R_1 + R_2 \leq \overline{\lambda} I\left(X_r; Y\right)$. Then, the destination decodes $(\hat{\boldsymbol{y}}_1, \hat{\boldsymbol{y}}_2)$ and the original message w. By a similar argument as in Section VC of [10], we obtain (2.27).

3. Proof of Theorem B.5

First fix the distribution as shown in the theorem.

Codebook Generation: Generate $\boldsymbol{x}(w)$ the same as those in Appendix 2. Generate $2^{nR_{i,3-i}}$ i.i.d. sequences $\hat{\boldsymbol{y}}_{i,3-i}(k_i)$, according to $\prod_{j=1}^{\lambda n} p(\hat{y}_{i,3-i}^j)$ with $p\left(\hat{y}_{i,3-i}\right) = \int p\left(y_i\right) p\left(\hat{y}_{i,3-i}|y_i\right) dy_i$. Randomly and uniformly partition the set $\left[1 : 2^{nR_{i,3-i}}\right]$ into $2^{nC_{i,3-i}}$ bins $\boldsymbol{S}_{i,3-i}(s_{i,3-i})$; generate $2^{nR_{i0}}$ i.i.d. sequences $\hat{\boldsymbol{y}}_i(w_i)$, according to the distribution $\prod_{j=1}^{\lambda n} p\left(\hat{y}_{i,j}\right)$ with $p\left(\hat{y}_i\right) = \int p\left(\hat{y}_i|y_i, \hat{y}_{3-i,i}\right) p\left(y_i, \hat{y}_{3-i,i}\right) dy_i d\hat{y}_{3-i,i}$. Randomly and uniformly partition the set $\left[1 : 2^{nR_{i0}}\right]$ into 2^{nR_i} bins $\widetilde{\boldsymbol{S}}_i(\widetilde{s}_i)$; and generate 2^{nR_i} i.i.d. sequences $\boldsymbol{x}_i(\widetilde{s}_i)$, according to $p_{X_i}(x_i)$.

Encoding and Decoding: At the source, it transmits $\boldsymbol{x}(w)$; in the i-th relay, the conferencing scheme is the same as the DF scheme, which is omitted here; and we obtain (2.7). Based on $\boldsymbol{y}_i$ and $\hat{\boldsymbol{y}}_{3-i,i}$, the i-th relay find a $\hat{\boldsymbol{y}}_i(k_i)$ such that $(\hat{\boldsymbol{y}}_i(k_i), \hat{\boldsymbol{y}}_{3-i,i}(w_{3-i,i}), \boldsymbol{y}_i) \in A_\epsilon^n$, and this can be done reliably if $R_{i0} \geq I\left(\hat{Y}_i; \hat{Y}_{3-i,i}, Y_i\right)$. Then, the i-th relay obtains the binning index $\widetilde{s}_i$ and sends $\boldsymbol{x}_i(\widetilde{s}_i)$ to the destination.

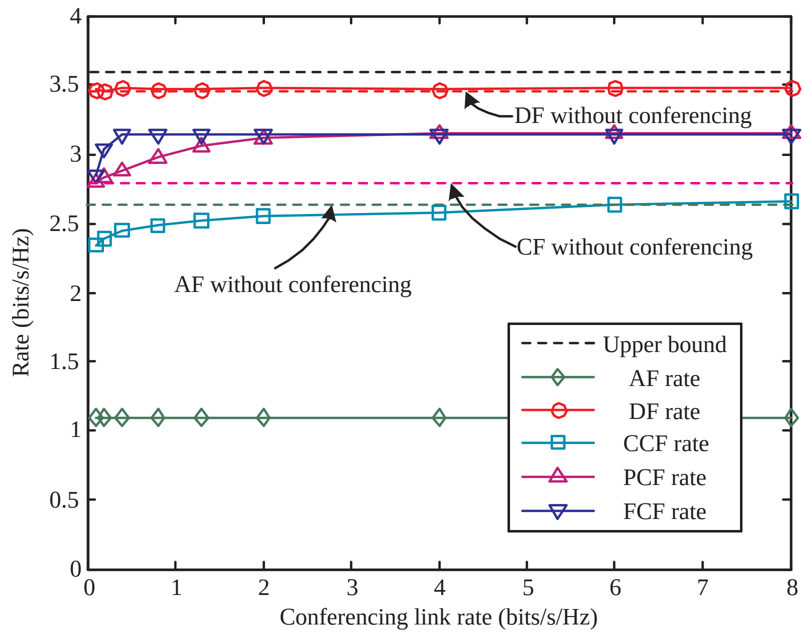
In the destination, upon receiving $\boldsymbol{y}$, it first decodes the pair $(\widetilde{s}_1, \widetilde{s}_2)$, and the rate region (R_1, R_2) is given by the MAC rate region as in [24, 28]. Then, the destination tries to decode $(\hat{\boldsymbol{y}}_1, \hat{\boldsymbol{y}}_2)$. Following a similar argument as in [10, 49], we have

$$\begin{cases} R_1 \geq \lambda I\left(\hat{Y}_1; Y_1, \hat{Y}_{21} | \hat{Y}_2\right) \\ R_2 \geq \lambda I\left(\hat{Y}_2; Y_2, \hat{Y}_{12} | \hat{Y}_1\right) \\ R_1 + R_2 \geq \lambda I\left(\hat{Y}_1, \hat{Y}_2; Y_1, Y_2, \hat{Y}_{12}, \hat{Y}_{21}\right) \end{cases}. \tag{2.51}$$

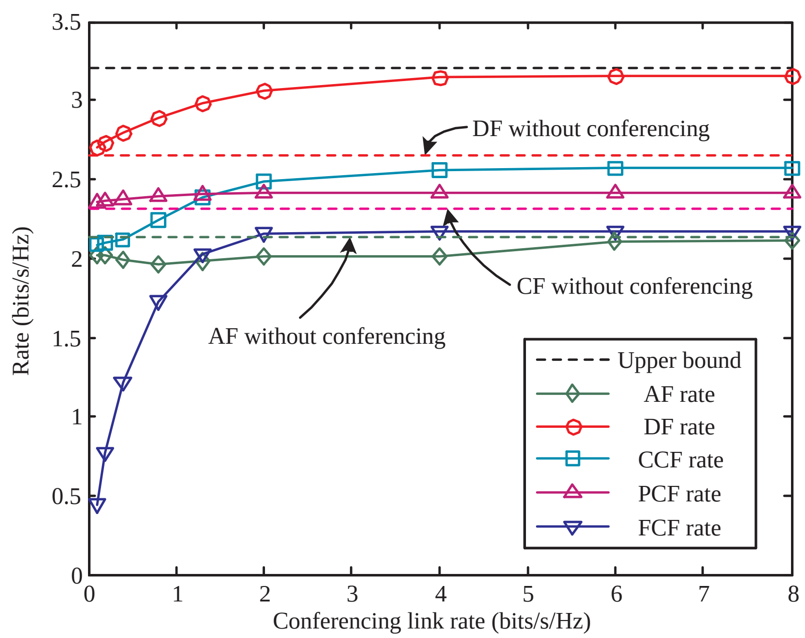
Finally, by finding a unique $\hat{w}$ such that $(\boldsymbol{x}(\hat{w}), \hat{\boldsymbol{y}}_1, \hat{\boldsymbol{y}}_2) \in A_\epsilon^n$, we obtain $R_{\text{CF}} = \lambda I\left(X; \hat{Y}_1, \hat{Y}_2\right)$. With the Fourier-Motzkin elimination [50], and the facts that

$$I\left(\hat{Y}_1, \hat{Y}_2; Y_1, Y_2, \hat{Y}_{12}, \hat{Y}_{21}\right) \geq I\left(\hat{Y}_1; Y_1, \hat{Y}_{21} | \hat{Y}_2\right) + I\left(\hat{Y}_2; Y_2, \hat{Y}_{12} | \hat{Y}_1\right),$$
$$I(X_1, X_2; Y) \leq I(X_1; Y | X_2) + I(X_2; Y | X_1),$$

the theorem is proved.



(a) Good BC vs. bad MAC, $\gamma_i = 30$ dB, $\widetilde{\gamma}_i = 10$ dB



(b) Bad BC vs. good MAC, $\gamma_i = 10$ dB, $\widetilde{\gamma}_i = 30$ dB

Fig. 6.: Achievable rates over different conferencing link rates with $C_{21} = 0$.

CHAPTER III

ALTERNATIVE RELAYING DIAMOND CHANNEL

In the previous chapter, we considered the diamond relay channel with the simultaneous relaying scheme. In this chapter, we consider another type of relaying scheme, i.e., alternative relaying, for which the two half-duplex relays are assumed to transmit and receive in different time slot and exchange their working modes alternatively over time. In practice, the alternative relaying strategy is more appealing than the simultaneous relaying strategy, due to the fact that it can achieve full multiplexing gain in the high SNR regime.

In this chapter, we consider the following two different conferencing strategies:

1. Conferencing strategy I: Relay conferencing for each source message is executed within the subsequent time slot after the relay receives the source signal by partially utilizing the conferencing links, and thus the decoding delay at the destination for each source message will be at most two time slots. By letting both or one of the two relays adopt the above conferencing strategy, we obtain the following two schemes:

   (a) Two-side conferencing: Use both of the two conferencing links, and both of the two relays are required to conference with each other;

   (b) One-side conferencing: Use one of the conferencing links, and one of the relays sends message to the its counterpart, while the other one keeps silent.

2. Conferencing strategy II: Relay conferencing for each source message is operated in the subsequent two time slots by fully utilizing the conferencing links, and thus the decoding delay for each source message at the destination will be more

than two time slots. Similar to the previous case, we can also introduce both of the two-side and one-side conferencing schemes with this conferencing strategy.

Intuitively, strategy II may lead to larger achievable rates compared to strategy I, since it allows higher conferencing rates between the two relays. Moreover, it is worth noting that one-side conferencing is just a special case of two-side conferencing, by letting one of the relays keep silent, and thus two-side conferencing in general will outperform one-side conferencing for both of the two conferencing strategies. Somewhat surprisingly, it will be shown that under certain conditions one-side conferencing is enough to achieve the same rate as the two-side conferencing, while it is much simpler to be implemented.

The remainder of the chapter is organized as follows. Section A introduces all assumptions and channel models. The rate upper bound and achievable rates obtained by using the DF and AF relaying schemes are discussed for conferencing strategy I in Section B, and the DF scheme under conferencing strategy II is investigated in Section C. Section D presents the numerical results. Finally, the paper is summarized in Section E.

A. System Model

We consider the Gaussian diamond relay channel, as shown in Fig. 7, which consists of one source node, one destination node, and two relays. It is assumed that there are no direct wireless links between either the source-destination pair or the two relays. However, between the two relays, there are two wired conferencing links, which are both rate-limited Gaussian. Due to the wired conferencing link assumptions, these two conferencing links can be considered orthogonal to each other and also orthogonal to the source-to-relay and relay-to-destination links.

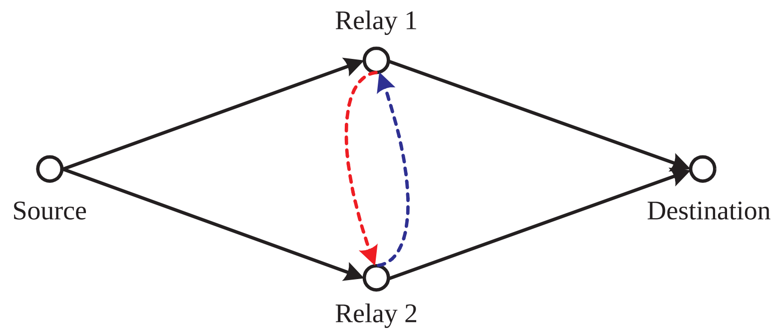


Fig. 7.: The diamond relay channel with out-of-band conferencing links.

In this chapter, it is assumed that the transmissions of the source and relays are slotted, and the half-duplex alternative relaying scheme is adopted, as shown in Fig. 8. Specifically, in the odd-numbered time slots, the source sends a message to relay 1, and relay 2 forwards a signal to the destination; in the even-numbered time slots, the roles of the two relays are exchanged. At the relays, the DF and AF relaying schemes are adopted for the transmissions to the destination. For the DF relaying scheme, we may allocate different time fractions to the odd and even time slots: Denote the time fraction allocated to the odd time slots as λ_1, and that to the even time slots as λ_2, with $\lambda_1 + \lambda_2 = 1$, $\lambda_1 \geq 0$, and $\lambda_2 \geq 0$. Note that among odd time slots or among over time slots, they are of equal length. For the AF relaying scheme, we set $\lambda_1 = \lambda_2 = \frac{1}{2}$.

The conferencing strategies shown in Fig. 9 are described as follows. For conferencing strategy I, the source transmits independent messages $\{w_k\}$ slot by slot; in the k-th time slot, when k is odd, relay 1 listens to the source, and relay 2 sends two signals, one to the destination about the source messages w_{k-2} and w_{k-1}, and the other to relay 1 about message w_{k-1} via the wired conferencing links; when k is even, the roles of these two relays are exchanged. For conferencing strategy II, in the k-th time slot, when k is odd, relay 1 listens and relay 2 sends two signals, one to the destination about messages w_{k-4} and w_{k-1} within the k-th time slot, and the other to

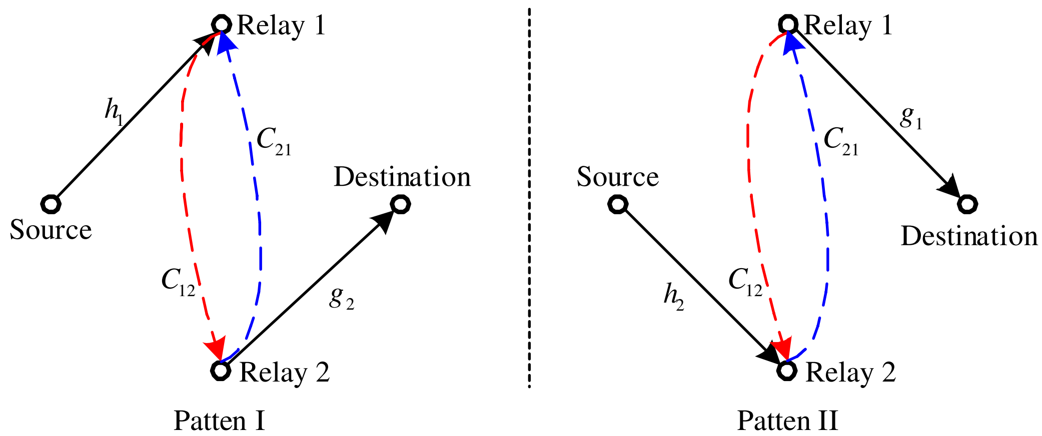
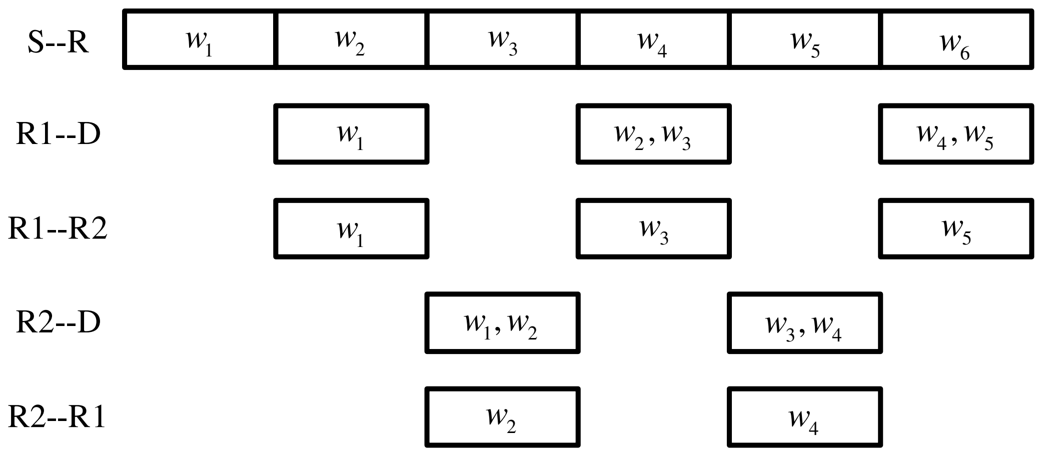


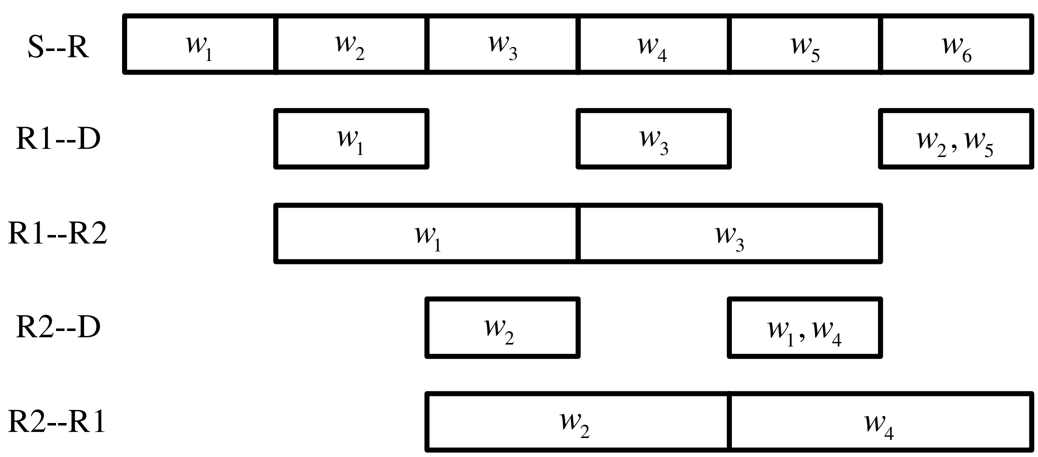
Fig. 8.: Two transmitting-receiving states for the alternative relaying diamond channel with conferencing links.

relay 1 about message w_{k-1} spreading over both of the k-th and the $(k+1)$-th time slots; when k is even, the roles of the two relays are exchanged. From Fig. 9, it is easy to see that strategy II fully utilizes the conferencing links, i.e., there are no idle time slots over the conferencing links, while strategy I only partially utilizes them. In Fig. 9, these two conferencing strategies for the most general two-side conferencing case are described. As stated in the introduction part, we are also interested in a special case of the two-side conferencing scheme, i.e., one-side conferencing. Taking conferencing strategy I as an example, we could just let relay 1 talk to relay 2 as in the two-side conferencing case, while relay 2 keeps silent; or only let relay 2 talk to relay 1 while relay 1 keeps silent. Similarly, the one-side conferencing scheme could as be defined for strategy II.

Note that for strategy II, the only way to deploy the conferencing strategy for the AF relaying scheme is to transmit the received signal at each relay to its counterpart repeatedly over the two subsequent conferencing time slots, since we can only forward the same received signal to the other relay via the conferencing links. As such, the two conferencing strategies for the AF relaying scheme are almost the same, only with



(a) Conferencing strategy I.



(b) Conferencing strategy II.

Fig. 9.: Relay conferencing strategies for the diamond relay channel with conferencing links, with "S", "R1", "R2", and "D" denoting the source, relay 1, relay 2, and destination, respectively.

different conferencing link SNRs (i.e., for conferencing strategy I, it suffers a $\frac{1}{2}$ penalty, with no penalty for strategy II.), and the results are quite similar (interestingly, we find that for the DF relaying scheme, it is not). Thus, in this chapter, we only consider the AF relaying scheme under conferencing strategy I, and omit the analysis for the other strategy.

Due to relay conferencing, there may be extra decoding delay. To be concise, we describe the coding schemes by using i and $(3-i)$, $i = 1, 2$, as the relay indices in the sequel. Take the k-th source message (to the i-th relay) for example: With conferencing strategy I, the destination needs to wait another λ_i-block[1] to obtain the signal from the $(3-i)$-th relay, which means that the decoding delay at the destination will be λ_i-block more compared with the case without relay conferencing[2]. However, when transmitting N messages in total, with N going to infinity, the effect of decoding delay to the average achievable rate can be neglected. Accordingly, in this chapter, we only consider the achievable rates over two successive time slots, since all coding schemes are operated periodically over time.

For the Gaussian channel case, the channel input-output relationships are given as follows. The received signal y_i at the i-th relay from the source, $i = 1, 2$, is given as

$$y_i = \sqrt{P_S} h_i x_i + n_i, \ i = 1, 2, \tag{3.1}$$

where x_i is the transmit signal from the source with unit average power, P_S is the source transmit power, h_i is the complex channel coefficient of the link from the source to the i-th relay, and n_i is the i.i.d. CSCG noise with distribution $\mathcal{CN}(0, 1)$.

[1]Here, one "block" consists of two successive time slots.

[2]It is worth noting that even without relay conferencing, there is still an λ_i-block decoding delay due to the relaying operation.

It is worth noting that in general we could allocate different power levels to the source messages at the odd and even time slots to maximize the overall system performance, which makes the achievable rate maximization problem hard to be tracked. In this chapter, we focus more on the relay operations, i.e., conferencing and relaying; and thus we assume uniform power allocation at all the source messages for simplicity.

For the Gaussian conferencing links, denote the SNR of the link from relay 1 to relay 2 as $\gamma_{1,2}$, while $\gamma_{2,1}$ is defined similarly. As such, the conferencing link rates $C_{i,3-i} = \mathcal{C}\left(\gamma_{i,3-i}\right)$, $i = 1, 2$, are achievable when rate-achieving codes are applied. Then, if the DF relaying scheme is adopted, relay i can send information to relay $3-i$ with the maximum rate $C_{i,3-i}$ reliably; for the AF relaying scheme, we assume that the AF scheme is also adopted as the conferencing scheme over the SNR-limited (equivalent to the notion of rate-limited in the DF case) conferencing links, which will be discussed with more details later in Section B.

After relay conferecing, each relay generates a signal t_i with unit average power, based upon the received signals from the source and the other relay. Then, the received signal z_i at the destination from the i-th relay is given as

$$z_i = \sqrt{P_R} g_i t_i + \widetilde{n}_i, \; i = 1, 2, \tag{3.2}$$

where P_R is the relay transmit power, g_i is the complex channel coefficient of the link from the i-th relay to the destination, and $\widetilde{n}_i$ is the i.i.d. CSCG noise with distribution $\mathcal{CN}(0, 1)$. For notation convenience, we denote the link SNRs as

$$\gamma_i = |h_i|^2 P_S, \; \widetilde{\gamma}_i = |g_i|^2 P_R, \; i = 1, 2. \tag{3.3}$$

B. Conferencing Strategy I

In this section, we examine the rate upper bound along with the DF and AF achievable rates for the considered channel with conferencing strategy I. Moreover, we prove some upper-bound-achieving results under special channel conditions.

1. Rate Upper Bound

In this subsection, we derive the rate upper bound for the considered channel. Note that the following rate upper bound is only applicable for the diamond relay channel with the alternative relaying strategy, not for the diamond relay channel with other strategies. To simplify notations, we call this bound as the rate upper bound in this chapter.

Theorem B.1 *Under conferencing strategy I, the rate upper bound for the alternative relaying diamond channel with conferencing links is given as*

$$\begin{aligned} C_{upper} = \max_{\lambda_1+\lambda_2=1} & \min \{ \lambda_1 \mathcal{C}(\gamma_1) + \lambda_2 \mathcal{C}(\gamma_2), \lambda_2 \mathcal{C}(\widetilde{\gamma}_1) + \lambda_1 \mathcal{C}(\widetilde{\gamma}_2), \\ & \lambda_1 C_{21} + \lambda_2 \left(\mathcal{C}(\gamma_2) + \mathcal{C}(\widetilde{\gamma}_1) + C_{12} \right), \lambda_1 \left(\mathcal{C}(\gamma_1) + \mathcal{C}(\widetilde{\gamma}_2) + C_{21} \right) + \lambda_2 C_{12} \} . \quad (3.4) \end{aligned}$$

Proof: This bound is derived by the cut-set bound [2] considering the alternative relaying scheme with conferencing links as given in Section A. Similar analysis can be found in [10] and thus skipped. ■

2. DF Achievable Rate

In this subsection, we first derive the DF rate for the general two-side conferencing, i.e., assuming that both of the two relays send information to each other via the conferencing links. After obtaining the most general expression of the DF rate in terms of a linear programming (LP) problem, we further exploit the properties of the optimal solution to simplify the coding scheme without sacrificing the DF rate, and show that one-side conferencing can also achieve the same DF rate as the two-side scheme. Then, we derive the DF rate in closed-form by solving the LP problem under different channel conditions.

a. Rate Formulation

First, we describe the main idea of the DF relaying scheme as follows: During the k-th time slot, the source transmits two messages w_k^i and w_k^{3-i} to the i-th relay by using superposition coding, with w_k^i targeted at the destination via the relay-to-destination link and w_k^{3-i} targeted at the $(3-i)$-th relay via the conferencing link; at the $(3-i)$-th relay, it transmits the messages w_{k-1}^{3-i} and w_{k-2}^{3-i} to the destination by using superposition coding, and w_{k-1}^i to the i-th relay, respectively; at the end of the k-th time slot, the i-th relay decodes messages w_k^i, w_k^{3-i}, and w_{k-1}^i, and the destination decodes w_{k-1}^{3-i} and w_{k-2}^{3-i}. Here, since all links in this channel are scheduled orthogonally over time or frequency, it is unnecessary to introduce any cooperation between the two relays and we only need to send independent messages cross the two time slots. Then, the DF rate is given in the following theorem.

Theorem B.2 *Under conferencing strategy I, the DF achievable rate for the alternative relaying diamond channel with conferencing links is given as*

$$\begin{aligned} P1: \quad & R_{DF} = \max \ R_{11} + R_{12} + R_{21} + R_{22} \qquad (3.5) \\ s.\ t. \quad & R_{3-i,i} \leq \lambda_i C_{3-i,i}, i = 1, 2, \\ & R_{i,i} + R_{i,3-i} \leq \lambda_i \mathcal{C}(\gamma_i), i = 1, 2, \\ & R_{3-i,3-i} + R_{i,3-i} \leq \lambda_i \mathcal{C}(\widetilde{\gamma}_{3-i}), i = 1, 2, \\ & \lambda_1 + \lambda_2 = 1, R_{i,j} \geq 0, \ i, j = 1, 2, \end{aligned}$$

where the design variables are $\{R_{11}, R_{12}, R_{21}, R_{22}, \lambda_1, \lambda_2\}$, R_{11} and R_{12} are the rates of messages w_k^1 and w_k^2 decoded by relay 1, respectively, when k is odd, and R_{21} and R_{22} are defined similarly when k is odd.

Proof: See Appendix 1. ■

Note that if we do not send w_k^2 at odd k and w_k^1 at even k, i.e., $R_{12} = R_{21} = 0$, it is observed that the DF rate with conferencing given in (3.5) is the same as that for the case without relay conferencing [10], which implies that our coding scheme is a natural extension of that in [10]. Next, we show that one-side conferencing is enough to achieve the same maximum DF rate.

Proposition B.1 *There exists one optimal point for Problem (P1) such that at least one of R_{12}^* and R_{21}^* is zero.*

Proof: We prove this proposition by construction. Without loss of generality, assume that $R_{12}^* \geq R_{21}^* > 0$, and $R_{i,j}^*,\ i,j \in \{1,2\}$ is the optimal point of Problem (P1). Then, construct a new point as $\widehat{R}_{12} = R_{12}^* - R_{21}^*$, $\widehat{R}_{21} = 0$, and $\widehat{R}_{i,i} = R_{i,i}^* + R_{21}^*$, $i = 1,2$. It is easy to check that $\widehat{R}_{i,j}$'s also satisfy the constraints of Problem (P1) and achieve the same optimal value as $R_{i,j}^*$'s, $i,j \in \{1,2\}$. Thus, the proposition is proved. ■

In practical communication system design, one-side conferencing simplifies the system requirements and thus is much easier to be implemented. In the next subsection, we will obtain the DF rate in closed-form, and show how to choose one of the two conferencing link for one-side conferencing under different channel conditions.

b. Closed-Form Expressions for the DF Rate

With Proposition B.1, it is easy to check that the optimal value of Problem (P1) is equal to the maximum between those of the following two problems, which are recast from Problem (P1) by letting $R_{21} = 0$ and $R_{12} = 0$, respectively.

$$\text{P1.1}: \quad \max \quad R_{11} + R_{12} + R_{22} \tag{3.6}$$

$$\text{s. t.} \quad R_{12} \leq \lambda_2 C_{12}, \tag{3.7}$$

$$R_{11} + R_{12} \leq \lambda_1 \mathcal{C}(\gamma_1), \;\; R_{11} \leq \lambda_2 \mathcal{C}(\widetilde{\gamma}_1), \tag{3.8}$$

$$R_{22} + R_{12} \leq \lambda_1 \mathcal{C}(\widetilde{\gamma}_2), \;\; R_{22} \leq \lambda_2 \mathcal{C}(\gamma_2), \tag{3.9}$$

$$R_{11} \geq 0, R_{12} \geq 0, R_{22} \geq 0, \lambda_1 + \lambda_2 = 1, \lambda_i \geq 0, i = 1, 2, \tag{3.10}$$

and

$$\text{P1.2}: \quad \max \quad R_{11} + R_{21} + R_{22} \tag{3.11}$$

$$\text{s. t.} \quad R_{21} \leq \lambda_1 C_{21}, \tag{3.12}$$

$$R_{11} \leq \lambda_1 \mathcal{C}(\gamma_1), \;\; R_{11} + R_{21} \leq \lambda_2 \mathcal{C}(\widetilde{\gamma}_1), \tag{3.13}$$

$$R_{22} \leq \lambda_1 \mathcal{C}(\widetilde{\gamma}_2), \;\; R_{22} + R_{21} \leq \lambda_2 \mathcal{C}(\gamma_2), \tag{3.14}$$

$$R_{11} \geq 0, R_{12} \geq 0, R_{22} \geq 0, \lambda_1 + \lambda_2 = 1, \lambda_i \geq 0, i = 1, 2. \tag{3.15}$$

Note that if Problems (P1) and (P1.1) achieve the same optimal value, the optimal point $(R_{11}^*, R_{12}^*, R_{22}^*, \lambda_1^*, \lambda_2^*)$ of Problem (P1.1) is also feasible to Problem (P1) with $R_{21}^* = 0$, and such an optimal point is also the solution for Problem (P1). Similar argument holds for the case that Problems (P1) and (P1.2) achieve the same optimal value. Therefore, solving Problems (P1.1) and (P1.2) is equivalent to solving Problem (P1). Before deriving the optimal solution for these two subproblems, we first introduce the following lemma.

Lemma B.1 *There exists one optimal solution of Problem (P1.1), which makes either both of the two constraints in (3.8) or those in (3.9) satisfied with equality, i.e.,* $\lambda_1^* \mathcal{C}(\gamma_1) - R_{12}^* = \lambda_2^* \mathcal{C}(\widetilde{\gamma}_1)$ *or* $\lambda_1^* \mathcal{C}(\widetilde{\gamma}_2) - R_{12}^* = \lambda_2^* \mathcal{C}(\gamma_2)$. *Similar result is true for*

Problem (P1.2).

Proof: We only prove the result for Problem (P1.1). If the optimal point of Problem (P1.1) satisfies that $R_{12}^* < \lambda_2^* C_{12}$, we can prove this lemma by contradiction using the same argument as that for Theorem 4.1 in [10]. Hence, we only need to consider the case with $R_{12}^* = \lambda_2^* C_{12}$. The main idea is shown as follows: By changing λ_1^* to $\lambda_1^* + \epsilon$, where ϵ is a small real value, i.e., $|\epsilon| \ll 1$, it can be shown that all other cases cannot be optimal, except for the case shown in this lemma. There are four possible other cases.

1. $\lambda_1^* \mathcal{C}(\gamma_1) - R_{12}^* < \lambda_2^* \mathcal{C}(\widetilde{\gamma}_1)$ and $\lambda_1^* \mathcal{C}(\widetilde{\gamma}_2) - R_{12}^* < \lambda_2^* \mathcal{C}(\widetilde{\gamma}_2)$: Choose $\epsilon > 0$, it is observed that the new solution satisfies $\widetilde{R}_{12} - R_{12}^* = -\epsilon C_{12}$, $\widetilde{R}_{11} - R_{11}^* = \epsilon C_{12} + \epsilon \mathcal{C}(\gamma_1)$, and $\widetilde{R}_{22} - R_{22}^* = \epsilon C_{12} + \epsilon \mathcal{C}(\widetilde{\gamma}_2)$. It is easy to check that the sum rate is improved, and thus this case cannot happen.

2. $\lambda_1^* \mathcal{C}(\gamma_1) - R_{12}^* < \lambda_2^* \mathcal{C}(\widetilde{\gamma}_1)$ and $\lambda_1^* \mathcal{C}(\widetilde{\gamma}_2) - R_{12}^* > \lambda_2^* \mathcal{C}(\widetilde{\gamma}_2)$: Change λ_1^* to $\lambda_1^* + \epsilon$, and it follows that the new solution satisfies $\widetilde{R}_{12} - R_{12}^* = -\epsilon C_{12}$, $\widetilde{R}_{11} - R_{11}^* = \epsilon C_{12} + \epsilon \mathcal{C}(\gamma_1)$, and $\widetilde{R}_{22} - R_{22}^* = -\epsilon \mathcal{C}(\gamma_2)$. If $\gamma_1 \geq \gamma_2$, choose ϵ as a positive value; otherwise, choose ϵ as a negative value. It is easy to check that the sum rate is improved, and thus this case cannot happen.

3. $\lambda_1^* \mathcal{C}(\gamma_1) - R_{12}^* > \lambda_2^* \mathcal{C}(\widetilde{\gamma}_1)$ and $\lambda_1^* \mathcal{C}(\widetilde{\gamma}_2) - R_{12}^* < \lambda_2^* \mathcal{C}(\widetilde{\gamma}_2)$: This case is similar to case 2).

4. $\lambda_1^* \mathcal{C}(\gamma_1) - R_{12}^* > \lambda_2^* \mathcal{C}(\widetilde{\gamma}_1)$ and $\lambda_1^* \mathcal{C}(\widetilde{\gamma}_2) - R_{12}^* > \lambda_2^* \mathcal{C}(\widetilde{\gamma}_2)$: This case is similar to case 1).

In conclusion, the proposition is proved. ■

Next, we show how to obtain the optimal point $(R_{11}^*, R_{12}^*, R_{22}^*, \lambda_1^*, \lambda_2^*)$ of Problem (P1.1), where the solution of Problem (P1.2) can be obtained similarly.

(i) If $\lambda_1^*\mathcal{C}(\gamma_1) - R_{12}^* = \lambda_2^*\mathcal{C}(\widetilde{\gamma}_1)$, it follows that $\lambda_1^* = \frac{\mathcal{C}(\widetilde{\gamma}_1)+R_{12}^*}{\mathcal{C}(\widetilde{\gamma}_1)+\mathcal{C}(\gamma_1)}$. Then, by (3.7) and noticing that $R_{12}^* \leq \lambda_1^*\mathcal{C}(\gamma_1)$ and $R_{12}^* \leq \lambda_1^*\mathcal{C}(\widetilde{\gamma}_2)$ (due to constraints (3.8) and (3.9)), R_{12}^* should satisfy the following conditions

$$\begin{cases} R_{12}^* \leq \frac{\mathcal{C}(\gamma_1)C_{12}}{\mathcal{C}(\gamma_1)+\mathcal{C}(\widetilde{\gamma}_1)+C_{12}} \\ R_{12}^* \leq \mathcal{C}\left(\gamma_1\right) \\ R_{12}^* \leq \frac{\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)}{\mathcal{C}(\widetilde{\gamma}_1)+\mathcal{C}(\gamma_1)-\mathcal{C}(\widetilde{\gamma}_2)} \end{cases}. \tag{3.16}$$

Since the right-hand side of the second constraint is an upper bound of the right-hand side of the first one, it follows that the second one is redundant. Thus, we obtain

$$0 \leq R_{12}^* \leq \min\left\{\frac{\mathcal{C}(\gamma_1)C_{12}}{\mathcal{C}(\gamma_1)+\mathcal{C}(\widetilde{\gamma}_1)+C_{12}}, \frac{\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)}{\mathcal{C}(\widetilde{\gamma}_1)+\mathcal{C}(\gamma_1)-\mathcal{C}(\widetilde{\gamma}_2)}\right\} \doteq k_1, \tag{3.17}$$

$R_{11}^* = \lambda_2^*\mathcal{C}(\widetilde{\gamma}_1)$, and $R_{22}^* = \min\left\{\lambda_1^*\mathcal{C}(\widetilde{\gamma}_2) - R_{12}^*, \lambda_2^*\mathcal{C}(\gamma_2)\right\}$. Thus, the optimal value of Problem (P1.1) is given as $R_1 = \max_{(3.17)}\left(R_{11}^* + R_{22}^* + R_{12}^*\right) = \max_{(3.17)} r_1(R_{12}^*)$, where

$$\begin{aligned} &r_1(R_{12}^*) \\ &= \frac{1}{\mathcal{C}(\gamma_1)+\mathcal{C}(\widetilde{\gamma}_1)} \min\left\{\begin{array}{c} \mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1) + \mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) + \left(\mathcal{C}(\gamma_1) - \mathcal{C}(\gamma_2)\right)R_{12}^* \\ \mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1) + \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2) + \left(\mathcal{C}(\widetilde{\gamma}_2) - \mathcal{C}(\widetilde{\gamma}_1)\right)R_{12}^* \end{array}\right\}. \end{aligned} \tag{3.18}$$

(ii) If $\lambda_1^*\mathcal{C}(\widetilde{\gamma}_2) - R_{12}^* = \lambda_2^*\mathcal{C}(\gamma_2)$, it follows that $\lambda_1^* = \frac{\mathcal{C}(\gamma_2)+R_{12}^*}{\mathcal{C}(\widetilde{\gamma}_2)+\mathcal{C}(\gamma_2)}$, and it is easy to check that

$$0 \leq R_{12}^* \leq \min\left\{\frac{\mathcal{C}(\widetilde{\gamma}_2)C_{12}}{\mathcal{C}(\gamma_2)+\mathcal{C}(\widetilde{\gamma}_2)+C_{12}}, \frac{\mathcal{C}(\widetilde{\gamma}_2)\mathcal{C}(\gamma_1)}{\mathcal{C}(\gamma_2)+\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\gamma_1)}, \frac{\left(\mathcal{C}(\widetilde{\gamma}_2)\right)^2}{\mathcal{C}(\gamma_2)}\right\} \doteq k_2. \tag{3.19}$$

Thus, the optimal value of Problem (P1.1) is given as $R_2 = \max_{(3.17)} r_2(R_{12}^*)$, where

$$r_2(R_{12}^*)$$
$$= \frac{1}{\mathcal{C}(\gamma_2)+\mathcal{C}(\widetilde{\gamma}_2)} \min \left\{ \begin{array}{c} \mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2)+\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)+\left(\mathcal{C}(\gamma_1)-\mathcal{C}(\gamma_2)\right) R_{12}^* \\ \mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2)+\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)+\left(\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\widetilde{\gamma}_1)\right) R_{12}^* \end{array} \right\}. \tag{3.20}$$

It is worth noting that the two terms in the min operation of (3.18) and (3.20) are all linear functions of R_{12}^*, and thus the optimal value of Problem (P1.1) is given by the max/min over two linear functions. Then, the optimal value of Problem (P1.1) can be obtained in the following cases, which are also shown in Fig. 10.

1. $\gamma_1 > \gamma_2$ and $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$: As provable and shown in Fig. 10(a), both of the two functions in (3.18) or (3.20) are strictly increasing over R_{12}^*, and thus the maximum values of (3.18) and (3.20) are achieved at $R_{12}^* = k_1$ or $R_{12}^* = k_2$, respectively. Then, the optimal value of Problem (P1.1) is given as

$$\max\left\{r_1(k_1), r_2(k_2)\right\}, \tag{3.21}$$

which implies that relay conferencing can strictly increase the DF rate in this case.

2. $\gamma_1 \leq \gamma_2$ and $\widetilde{\gamma}_2 \leq \widetilde{\gamma}_1$: As provable and shown in Fig. 10(b), both of the two functions in (3.18) or (3.20) are non-increasing over R_{12}^*, and thus the maximum values of (3.18) and (3.20) are achieved at $R_{12}^* = 0$. Thus, the optimal value of Problem (P1.1) is given as

$$\max\left\{r_1(0), r_2(0)\right\}, \tag{3.22}$$

which implies that using the conferencing link from relay 1 to relay 2 cannot improve the DF rate for this case.

3. $\gamma_1 > \gamma_2$ and $\widetilde{\gamma}_2 \leq \widetilde{\gamma}_1$: For either (3.18) or (3.20), one function in the min operation is increasing over R_{12}^*, while the other one is non-increasing. As such, we need to further compare the constant terms in them, and there are two subcases:

   (a) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) < \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: As provable and shown in Fig. 10(c), for both (3.18) and (3.20), the two functions in the min operation may have one intersection point for $R_{12}^* \geq 0$, which is given by $k_3 \doteq \frac{\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)-\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)}{\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}$. However, note that k_3 may not be within the region defined by (3.17) and (3.19). As such, for (3.18) and (3.20), their maximum values are achieved at $k_{01} = \min(k_1, k_3)$ or $k_{02} = \min(k_2, k_3)$, respectively. Thus, the optimal value of Problem (P1.1) is given as

$$\max \{r_1(k_{01}), r_2(k_{02})\}, \tag{3.23}$$

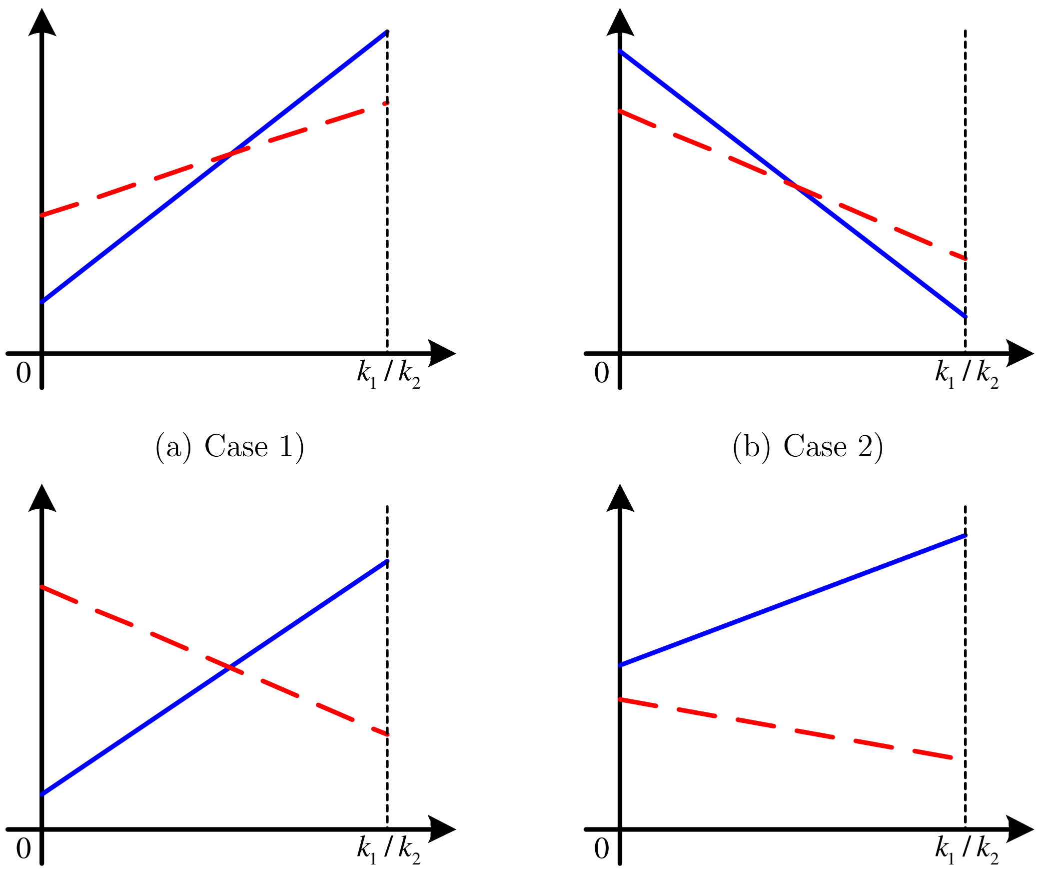
   which means that relay conferencing can strictly increase the DF rate in this case.

   (b) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) \geq \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: As provable and shown in Fig. 10(d), for both (3.18) and (3.20), the two functions in the min operation have no intersection points in the region defined by (3.17) and (3.19), respectively. Thus, the optimal value of Problem (P1.1) is given the same as (3.22), and it is concluded that by using the conferencing link from relay 1 to relay 2, the DF rate cannot be improved compared to the case without relay conferencing.

4. $\gamma_1 \leq \gamma_2$ and $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$: This case is similar to case 3), and the DF rate is given as

(a) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) > \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: The optimal value of Problem (P1.1) is given by (3.23);

(b) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) \leq \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: The optimal value of Problem (P1.1) is given by (3.22).



(a) Case 1)

(b) Case 2)

(c) Case 3)

(d) Case 4)

Fig. 10.: Four possible cases for the max/min solutions in (3.18) and (3.20), where k_1 and k_2 are given in (3.17) and (3.19), respectively.

Similar to Problem (P1.1), the optimal solution of Problem (P1.2) is summarized as follows.

1. $\gamma_1 > \gamma_2$ and $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$: The optimal value of Problem (P1.2) is given as

$$\max\left\{\widetilde{r}_1(0), \widetilde{r}_2(0)\right\}, \tag{3.24}$$

where $\widetilde{r}_1(R_{21}^*)$ and $\widetilde{r}_2(R_{21}^*)$ are defined as

$$\begin{aligned}&\widetilde{r}_1(R_{21}^*)\\ &= \frac{1}{\mathcal{C}(\gamma_1)+\mathcal{C}(\widetilde{\gamma}_1)} \min\left\{\begin{array}{c} \mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1)+\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)+\left(\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\widetilde{\gamma}_2)\right)R_{21}^* \\ \mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1)+\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)+\left(\mathcal{C}(\gamma_2)-\mathcal{C}(\gamma_1)\right)R_{21}^* \end{array}\right\},\end{aligned} \tag{3.25}$$

with

$$\begin{aligned}&0 \le R_{21}^* \le \\ &\quad \min\left\{\frac{\mathcal{C}(\widetilde{\gamma}_1)C_{21}}{\mathcal{C}(\gamma_1)+\mathcal{C}(\widetilde{\gamma}_1)+C_{21}}, \frac{\mathcal{C}(\widetilde{\gamma}_1^2)}{\mathcal{C}(2\widetilde{\gamma}_1)+\mathcal{C}(\gamma_1)}, \frac{\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\gamma_2)}{\mathcal{C}(\gamma_1)+\mathcal{C}(\widetilde{\gamma}_1)+\mathcal{C}(\gamma_2)}\right\} \doteq \widetilde{k}_1,\end{aligned}$$

and

$$\begin{aligned}&\widetilde{r}_2(R_{21}^*)\\ &= \frac{1}{\mathcal{C}(\gamma_2)+\mathcal{C}(\widetilde{\gamma}_2)} \min\left\{\begin{array}{c} \mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2)+\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)+\left(\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\widetilde{\gamma}_2)\right)R_{21}^* \\ \mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2)+\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)+\left(\mathcal{C}(\gamma_2)-\mathcal{C}(\gamma_1)\right)R_{21}^* \end{array}\right\},\end{aligned} \tag{3.26}$$

with

$$0 \le R_{21}^* \le \min\left\{\frac{\mathcal{C}(\widetilde{\gamma}_2)C_{12}}{\mathcal{C}(\gamma_2)+\mathcal{C}(\widetilde{\gamma}_2)+C_{12}}, \frac{\widetilde{\gamma}_1\widetilde{\gamma}_2}{\widetilde{\gamma}_2+\gamma_2-\widetilde{\gamma}_1}\right\} \doteq \widetilde{k}_2.$$

2. $\gamma_1 \le \gamma_2$ and $\widetilde{\gamma}_2 \le \widetilde{\gamma}_1$: The optimal value of Problem (P1.2) is given as

$$\max\left\{\widetilde{r}_1(\widetilde{k}_1), \widetilde{r}_2(\widetilde{k}_2)\right\}. \tag{3.27}$$

3. $\gamma_1 > \gamma_2$ and $\widetilde{\gamma}_2 \leq \widetilde{\gamma}_1$: There are two possible subcases:

   (a) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) < \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: The optimal value of Problem (P1.2) is given as (3.24).

   (b) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) \geq \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: The optimal value of Problem (P1.2) is given as

$$\max\left\{\widetilde{r}_1(\widetilde{k}_{01}), \widetilde{r}_2(\widetilde{k}_{02})\right\}, \tag{3.28}$$

   where $\widetilde{k}_{01}$ and $\widetilde{k}_{02}$ are defined as follows: $\widetilde{k}_{01} = \min\left(\widetilde{k}_1, \widetilde{k}_3\right)$ and $\widetilde{k}_{02} = \min\left(\widetilde{k}_2, \widetilde{k}_3\right)$, with $\widetilde{k}_3 \doteq \frac{\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2)-\mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)}{\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}$.

4. $\gamma_1 \leq \gamma_2$ and $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$: This is similar to case 3).

   (a) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) > \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: The optimal value of Problem (P1.2) is given by (3.24);

   (b) $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) \leq \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$: The optimal value of Problem (P1.2) is given by (3.27).

From the above analysis, it is observed that under the same channel conditions, at most one between R_{12}^* in Problem (P1.1) and R_{21}^* in Problem (P1.2) can be non-zero. Thus, the optimal value of Problem (P1) is achieved by one of the Problems (P1.1) and (P1.2) with non-zero $R_{i,3-i}^*$ (if there is), since these constant terms in (3.18) and (3.25) (same for (3.20) and (3.26)) are identical; for the case that both of $R_{i,3-i}^*$'s are zero, Problems (P1.1) and (P1.2) render the same optimal value, which is the same as that of Problem (P1). Therefore, we could obtain the DF rate in closed-form under different channel conditions, which is summarized in Table III. Moreover, it is worth noting that under arbitrary channel conditions, at most one between R_{12}^* and R_{21}^* is positive, which is coherent with the result in Proposition B.1 and indicates which one of the conferencing links should be used; furthermore, for some cases, both

Table III.: The DF rate under the conferencing strategy I and the corresponding one-side conferencing scheme.

Channel conditions	DF rate	Conferencing scheme
$\gamma_1 > \gamma_2$ and $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$	(3.21)	
$\gamma_1 > \gamma_2$, $\widetilde{\gamma}_2 \leq \widetilde{\gamma}_1$ and $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) < \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$	(3.23)	Relay 1 $\rightarrow$ Relay 2[1]
$\gamma_1 \leq \gamma_2$, $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$ and $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) > \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$	(3.23)	
$\gamma_1 < \gamma_2$ and $\widetilde{\gamma}_2 < \widetilde{\gamma}_1$	(3.27)	
$\gamma_1 > \gamma_2$, $\widetilde{\gamma}_2 \leq \widetilde{\gamma}_1$ and $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) > \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$	(3.28)	Relay 2 $\rightarrow$ Relay 1
$\gamma_1 \leq \gamma_2$, $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$ and $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) < \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$	(3.28)	
$\gamma_1 = \gamma_2$ and $\widetilde{\gamma}_2 = \widetilde{\gamma}_1$	(3.22)	
$\gamma_1 > \gamma_2$, $\widetilde{\gamma}_2 \leq \widetilde{\gamma}_1$ and $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) = \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$	(3.22)	No relay conferencing
$\gamma_1 \leq \gamma_2$, $\widetilde{\gamma}_2 > \widetilde{\gamma}_1$ and $\mathcal{C}(\gamma_1)\mathcal{C}(\gamma_2) = \mathcal{C}(\widetilde{\gamma}_1)\mathcal{C}(\widetilde{\gamma}_2)$	(3.22)	

1. This means that only the conferencing link from relay 1 to relay 2 is used, and the other one is not.

R^*_{12} and R^*_{21} are zero, which means under these channel conditions, relay conferencing is useless. In Table III, we also summarize which conferencing link should be used to deploy one-side relay conferencing and when relay conferencing cannot improve the DF rate.

Remark B.1 *From the above analysis, we observe that for the symmetric channel case, i.e.,* $\gamma_1 = \gamma_2$ *and* $\widetilde{\gamma}_1 = \widetilde{\gamma}_2$*, relay conferencing cannot improve the DF rate with the alternative relaying scheme, which is not true for the simultaneous relaying scheme [23].*

c. Asymptotic Performance

From [10], we know that for the diamond relay channel without relay conferencing, the DF scheme achieves the rate upper bound under arbitrary channel conditions. In the following, we show an asymptotic upper-bound-achieving result for the considered channel in this chapter.

Proposition B.2 *With conferencing strategy I and arbitrary channel coefficients, the DF relaying scheme achieves the rate upper bound given in (3.4) asymptotically as the conferencing link rates go to infinity.*

Proof: When $C_{i,3-i}$'s go to infinity, it is easy to see that the rate upper bound given in (3.4) is asymptotically equal to

$$C_{\text{upper}}^{\infty} = \max_{\lambda_i} \min \left\{ \lambda_1 \mathcal{C}(\gamma_1) + \lambda_2 \mathcal{C}(\gamma_2), \lambda_2 \mathcal{C}(\widetilde{\gamma}_1) + \lambda_1 \mathcal{C}(\widetilde{\gamma}_2) \right\}. \tag{3.29}$$

On the other hand, we set $R_{12} = R_{21} = 0$ in (3.5), and we observe that $R_{\text{DF}} \geq C_{\text{upper}}^{\infty}$. Thus, the proposition is proved. ■

Remark B.2 *For finite and positive $C_{i,3-i}$'s, it can be shown that the third and the fourth terms in (3.4) are generally larger than the DF rate define in (3.5), which implies why the DF relaying scheme cannot achieve this rate upper bound under general channel conditions. To see this point, we fix λ_i, $i = 1, 2$, and sum the following three constraints in (3.5) together: $R_{12} \leq \lambda_2 C_{12}$, $R_{22} + R_{21} \leq \lambda_2 \mathcal{C}(\gamma_2)$, and $R_{11} + R_{21} \leq \lambda_2 \mathcal{C}(\widetilde{\gamma}_1)$, which leads to*

$$\begin{aligned} R_{DF} &\leq R_{11} + R_{12} + 2R_{21} + R_{22} && (3.30)\\ &= \lambda_2 \left(\mathcal{C}(\gamma_2) + \mathcal{C}(\widetilde{\gamma}_1) + C_{12} \right) && (3.31)\\ &\leq \lambda_1 C_{21} + \lambda_2 \left(\mathcal{C}(\gamma_2) + \mathcal{C}(\widetilde{\gamma}_1) + C_{12} \right), && (3.32) \end{aligned}$$

where these two equalities in (3.30) and (3.32) are achieved only when $R_{21} = C_{21} = 0$. In general, since $C_{i,3-i} > 0$, we conclude that the rate upper bound given in (3.4) cannot be achieved by the DF scheme.

3. AF Achievable Rate

For the AF relaying scheme, each relay first linearly combines the received signals from the source and the other relay, and then transmits the combination to the destination under an individual relay power constraint.

We assume that the conferencing links are Gaussian. For simplicity, let the input of the conferencing link at the $(3-i)$-th relay be $x_{3-i,i} = y_{3-i}$ and the link gain of each conferencing link equal to 1. Thus, the conferencing link output at the i-th relay is given as

$$y_{3-i,i} = x_{3-i,i} + n_{3-i,i}, \tag{3.33}$$

where $n_{3-i,i}$ is the i.i.d. CSCG noise at the i-th relay with a distribution $\mathcal{CN}\left(0, \sigma^2_{3-i,i}\right)$. With the conferencing link rate constraint, $\sigma^2_{3-i,i}$ is given as

$$\sigma^2_{3-i,i} = \frac{\gamma_{3-i} + 1}{2^{C_{3-i,i}/2} - 1}, i = 1, 2. \tag{3.34}$$

After the relay conferencing, each relay combines the two received signals from the source and the other relay as $t_i = a_{ii} y_i + a_{3-i,i} y_{3-i,i}$, where a_{ii} and $a_{3-i,i}$ are the complex combining parameters, satisfying the following average transmit power constraint

$$\mathbb{E}\left(|t_i|^2\right) = |a_{ii}|^2 \left(\gamma_i + 1\right) + |a_{3-i,i}|^2 \left(\gamma_{3-i} + 1 + \sigma^2_{3-i,i}\right) \leq 1, \ i = 1, 2. \tag{3.35}$$

At the destination, we apply a sequential decoding process: Assume that at the k-th time slot, the previous $k-1$ source messages have already been successfully decoded; decode the k-th source message based on the received signals at the k-th and the $(k+1)$-th time slots, by treating the $(k+1)$-th message at the $(k+1)$-th time slot as noise. As such, the AF rate for each source message is given as

$$R_i = \frac{1}{2}\mathcal{C}\left(\frac{|a_{ii}|^2\gamma_i\widetilde{\gamma}_i}{1+|a_{ii}|^2\widetilde{\gamma}_i} + \frac{|a_{i,3-i}|^2\gamma_i\widetilde{\gamma}_{3-i}}{|a_{i,3-i}|^2\widetilde{\gamma}_{3-i}\left(1+\sigma_{i,3-i}^2\right)+|a_{3-i,3-i}|^2\widetilde{\gamma}_{3-i}(\gamma_{3-i}+1)+1}\right), \tag{3.36}$$

where R_1 and R_2 denote the rates for the source messages in odd and even time slots, respectively. Thus, the AF rate is given as

$$R_{\mathrm{AF}} = \max_{(3.35)} R_1 + R_2. \tag{3.37}$$

Then, we have the following result for the convexity of Problem (3.37).

Proposition B.3 *The AF rate maximization problem in (3.37) is concave over the combining parameters* $|a_{i,j}|^2$, $i,j \in \{1,2\}$.

Proof: See Appendix 2. ■

Even though the AF rate maximization problem in (3.37) can be solved by numerical algorithms, e.g., the interior point method [51], we know little about whether two-side conferencing is necessary with general channel coefficients. To obtain some insights for the proposed conferencing scheme, we further investigate the performance of the AF scheme for the cases when the second-hop link SNR $\widetilde{\gamma}_i$ goes to infinity and zero, respectively.

a. High SNR Regime

For the case with $\widetilde{\gamma}_i \to \infty$, (3.36) can be approximated as

$$R_i \approx \frac{1}{2}\mathcal{C}\left(\gamma_i + \frac{|a_{i,3-i}|^2\gamma_i}{|a_{i,3-i}|^2\left(1+\sigma_{i,3-i}^2\right)+|a_{3-i,3-i}|^2\gamma_{3-i}}\right). \tag{3.38}$$

Remark B.3 *In general, it is still difficult to derive the closed-form solution for Problem (3.37) with (3.38). However, it is worth noting that the term* γ_i *in* $\mathcal{C}(\cdot)$ *of (3.38) equals the received SNR for the case without relay conferencing when* $\widetilde{\gamma}_i \to \infty$*; moreover, the second term in* $\mathcal{C}(\cdot)$ *of (3.38) is the gain from relay conferencing. For some special cases, i.e., where* $\gamma_1 = \gamma_2$ *and we choose* $C_{i,3-i}$ *such that* $1 + \sigma^2_{i,3-i} = \gamma_i$*, it can be checked that the maximum AF rate is achieved at* $|a_{i,j}|^2 = \frac{1}{2\gamma_1}$*,* $i, j \in \{1, 2\}$ *(note that* R_i *is concave over* $|a_{i,j}|^2$*'s). Thus, for both* R_1 *and* R_2*, the conferencing gains are non-zero, which implies that two-side conferencing is necessary.*

b. Low SNR Regime

For the case with $\widetilde{\gamma}_i \to 0$, (3.36) can be approximated as

$$R_i \approx \frac{1}{2}\left(|a_{ii}|^2 \gamma_i \widetilde{\gamma}_i + |a_{i,3-i}|^2 \gamma_i \widetilde{\gamma}_{3-i}\right). \tag{3.39}$$

Then, the AF rate maximization problem can be recast as

$$\text{(P2)} \ \max_{(3.35)} \ \frac{1}{2}\left(|a_{11}|^2 \gamma_1 \widetilde{\gamma}_1 + |a_{12}|^2 \gamma_1 \widetilde{\gamma}_2 + |a_{22}|^2 \gamma_2 \widetilde{\gamma}_2 + |a_{2,1}|^2 \gamma_2 \widetilde{\gamma}_1\right), \tag{3.40}$$

which is a LP problem. It can be shown that Problem (P2) can be decomposed into two subproblems, and its optimal point can be constructed from those of the following two problems.

$$\text{(P2.1)} \ \max \ \frac{1}{2}\left(|a_{11}|^2 \gamma_1 \widetilde{\gamma}_1 + |a_{2,1}|^2 \gamma_2 \widetilde{\gamma}_1\right) \tag{3.41}$$

$$\text{s. t.} \quad |a_{11}|^2 \left(\gamma_1 + 1\right) + |a_{21}|^2 \left(\gamma_2 + 1 + \sigma^2_{21}\right) \le 1, \tag{3.42}$$

and

$$(\text{P2.2})\ \max\ \ \frac{1}{2}\left(|a_{12}|^2\gamma_1\widetilde{\gamma}_2+|a_{22}|^2\gamma_2\widetilde{\gamma}_2\right) \tag{3.43}$$

$$\text{s. t.}\quad |a_{22}|^2\left(\gamma_2+1\right)+|a_{12}|^2\left(\gamma_1+1+\sigma_{12}^2\right)\leq 1. \tag{3.44}$$

It is easy to check that one of the following two points is optimal for Problem (P2.1): $(|a_{11}|^2,|a_{21}|^2)=\left(\frac{1}{\gamma_1+1},0\right)$ and $(|a_{11}|^2,|a_{21}|^2)=\left(0,\frac{1}{\gamma_2+1+\sigma_{21}^2}\right)$, and thus its optimal value is $\widetilde{\gamma}_1\cdot\max\left\{\frac{\gamma_1}{\gamma_1+1},\frac{\gamma_2}{\gamma_2+1+\sigma_{21}^2}\right\}$. Similarly, for Problem (P2.2), its optimal value $\widetilde{\gamma}_2\cdot\max\left\{\frac{\gamma_2}{\gamma_2+1},\frac{\gamma_1}{\gamma_1+1+\sigma_{12}^2}\right\}$ is achieved by either $(|a_{22}|^2,|a_{12}|^2)=\left(\frac{1}{\gamma_2+1},0\right)$ or $(|a_{22}|^2,|a_{12}|^2)=\left(0,\frac{1}{\gamma_1+1+\sigma_{12}^2}\right)$. By considering different combinations of the possible optimal points of Problems (P2.1) and (P2.2), we obtain the optimal solution $|a_{i,j}^*|^2$, $i,j\in\{1,2\}$, of Problem (P2) as follows.

1. For the case with $\frac{\gamma_1}{\gamma_1+1}\geq\frac{\gamma_2}{\gamma_2+1+\sigma_{21}^2}$ and $\frac{\gamma_2}{\gamma_2+1}\geq\frac{\gamma_1}{\gamma_1+1+\sigma_{12}^2}$, we have $|a_{11}^*|^2=\frac{1}{\gamma_1+1}$, $|a_{22}^*|^2=\frac{1}{\gamma_2+1}$, and $|a_{12}^*|^2=|a_{21}^*|^2=0$; for this case, relay conferencing cannot improve the AF rate.

2. For the case with $\frac{\gamma_1}{\gamma_1+1}<\frac{\gamma_2}{\gamma_2+1+\sigma_{21}^2}$ and $\frac{\gamma_2}{\gamma_2+1}<\frac{\gamma_1}{\gamma_1+1+\sigma_{12}^2}$, we claim that it cannot happen. To see this point, consider the case with $C_{12}\to\infty$, i.e., $\sigma_{12}^2\to 0$, and we obtain that $\frac{\gamma_1}{\gamma_1+1}<\frac{\gamma_2}{\gamma_2+1+\sigma_{21}^2}<\frac{\gamma_2}{\gamma_2+1}$. Applying a similar argument for the other inequality, it follows that $\frac{\gamma_2}{\gamma_2+1}<\frac{\gamma_1}{\gamma_1+1}$, which contradicts with the previous inequality. As such, it is concluded that this case cannot happen.

3. For the case with $\frac{\gamma_1}{\gamma_1+1}<\frac{\gamma_2}{\gamma_2+1+\sigma_{21}^2}$ and $\frac{\gamma_2}{\gamma_2+1}\geq\frac{\gamma_1}{\gamma_1+1+\sigma_{12}^2}$, we have $|a_{11}^*|^2=|a_{12}^*|^2=0$, $|a_{22}^*|^2=\frac{1}{\gamma_2+1}$, and $|a_{21}^*|^2=\frac{1}{\gamma_2+1+\sigma_{21}^2}$; for this case, the source does not need to send information to relay 1, and both of the two relays forward the signals received at relay 2 to the destination.

4. For the case with $\frac{\gamma_1}{\gamma_1+1}\geq\frac{\gamma_2}{\gamma_2+1+\sigma_{21}^2}$ and $\frac{\gamma_2}{\gamma_2+1}<\frac{\gamma_1}{\gamma_1+1+\sigma_{12}^2}$, we have $|a_{11}^*|^2=\frac{1}{\gamma_1+1}$,

$|a_{12}^*|^2 = \frac{1}{\gamma_1+1+\sigma_{12}^2}$, and $|a_{22}^*|^2 = |a_{21}^*|^2 = 0$; this case leads to results opposite to case 3).

Remark B.4 *From the above analysis, we conclude that for the case with* $\widetilde{\gamma}_i \to 0$, *after obtaining the two signals from the source and its counterpart relay, each relay should only forward the one with a higher SNR and discard the other one. Moreover, cases 3 and 4 suggest that in the low SNR regime, increasing power gain is more critical than increasing multiplex gain for the AF relaying scheme. This is opposite to the result in the high SNR regime, where the alternative relaying scheme is optimal in the sense of achieving the full multiplexing gain as stated in the introduction part.*

C. Conferencing Strategy II

In this section, we consider conferencing strategy II, and derive the rate upper bound and the DF achievable rate. We will show that the DF rate will be greatly improved compared with that of strategy I. As we discussed before, we omit the analysis for the AF relaying scheme, since its result is only different by some constant factors from that of strategy I.

1. Rate Upper Bound

Note that for this case, the conferencing links can be fully utilized, and thus there will be no penalty terms λ_i's over the conferencing link rates $C_{i,3-i}$'s. As such, we obtain the following result for the rate upper bound, which is similar to Theorem B.1.

Theorem C.1 *Under conferencing strategy II, the rate upper bound for the alterna-*

tive relaying diamond channel with conferencing links is given as

$$C_{upper} = \max_{\lambda_1+\lambda_2=1} \min\{\lambda_1\mathcal{C}(\gamma_1)+\lambda_2\mathcal{C}(\gamma_2), \lambda_2\mathcal{C}(\widetilde{\gamma}_1)+\lambda_1\mathcal{C}(\widetilde{\gamma}_2),$$
$$\lambda_2\left(\mathcal{C}(\gamma_2)+\mathcal{C}(\widetilde{\gamma}_1)\right)+C_{12}+C_{21}, \lambda_1\left(\mathcal{C}(\gamma_1)+\mathcal{C}(\widetilde{\gamma}_2)\right)+C_{12}+C_{21}\}. \quad (3.45)$$

2. DF Achievable Rate

The DF coding scheme under conferencing strategy II is similar to that for scheme I, and thus we only describe its differences from the previous scheme. For the k-th source message (sending to the i-th relay), it contains two sub-messages w_k^i and w_k^{3-i} via superposition coding. After relay i decodes them, it sends w_k^i to the destination in the $(k+1)$-th time slot, and w_k^{3-i} to the other relay in the $(k+1)$-th and the $(k+2)$-th time slots via the conferencing link. As such, the rate of message w_k^{3-i} is no longer subject to the conferencing link rates constraints, i.e., $R_{i,3-i} \le C_{i,3-i},\ i=1,2$ is not required. In the $(k+3)$-th time slot, relay $3-i$ sends w_k^{3-i} to the destination together with the message w_{k+3}^{3-i}. Accordingly, we have the following result for the DF rate.

Theorem C.2 *Under conferencing strategy II, the DF achievable rate for the alternative relaying diamond channel with conferencing links is given as*

$$P3: \ \max \quad R_{DF} = R_{11}+R_{12}+R_{21}+R_{22} \quad (3.46)$$

$$s.\ t.\ \ R_{3-i,i} \le C_{3-i,i},\ i=1,2, \quad (3.47)$$

$$R_{i,i}+R_{i,3-i} \le \lambda_i\mathcal{C}(\gamma_i),\ i=1,2, \quad (3.48)$$

$$R_{3-i,3-i}+R_{i,3-i} \le \lambda_i\mathcal{C}(\widetilde{\gamma}_{3-i}),\ i=1,2, \quad (3.49)$$

$$R_{i,j} \ge 0, i,j\in\{1,2\},\ \lambda_1+\lambda_2=1,\ i=1,2, \quad (3.50)$$

where R_{ii} and $R_{i,3-i}$, $i=1,2$, are defined the same as those in Problem (P1).

Table IV.: Lower bound on $C_{12}+C_{21}$ for the DF scheme to achieve the rate upper bound under conferencing strategy II.

Channel conditions	Minimum $C_{12}+C_{21}$
$\gamma_1 > \gamma_2, \widetilde{\gamma}_2 > \widetilde{\gamma}_1$	$\min\{\mathcal{C}(\gamma_1), \mathcal{C}(\widetilde{\gamma}_2)\}$
$\gamma_1 \le \gamma_2, \widetilde{\gamma}_2 \le \widetilde{\gamma}_1$	$\min\{\mathcal{C}(\gamma_2), \mathcal{C}(\widetilde{\gamma}_1)\}$
$\gamma_1 > \gamma_2, \widetilde{\gamma}_1 \le \widetilde{\gamma}_2, \widetilde{\gamma}_2 \ge \gamma_1$	$\mathcal{C}(\widetilde{\gamma}_2)$
$\gamma_1 \le \gamma_2, \widetilde{\gamma}_1 > \widetilde{\gamma}_2, \widetilde{\gamma}_2 \le \gamma_1$	$\mathcal{C}(\gamma_1)$
$\gamma_1 > \gamma_2, \widetilde{\gamma}_1 > \widetilde{\gamma}_2, \gamma_1 > \widetilde{\gamma}_2$	$\frac{\mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1)}{\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)} - g(\lambda_0)$[1]
$\gamma_1 \le \gamma_2, \widetilde{\gamma}_1 < \widetilde{\gamma}_2, \gamma_1 < \widetilde{\gamma}_2$	$\frac{\mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1)}{\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)} - g(\lambda_0)$

1. $g(\cdot)$ is defined in (3.59), and $\lambda_0 = \frac{\mathcal{C}(\gamma_2)-\mathcal{C}(\widetilde{\gamma}_1)}{\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}$.

Since Problem (P3) has a similar structure as Problem (P1), it can be shown that one-side conferencing is also optimal for the DF scheme under conferencing strategy II, and the optimal solution of Problem (P3) can be obtained by a similar routine as that in Section B, which is omitted for simplicity. Here, we first have the following proposition to show a rate result for the DF scheme.

Proposition C.1 *Under conferencing strategy II, the DF relaying scheme achieves the corresponding rate upper bound with finite conferencing link rates, i.e., with $C_{12}+C_{21}$ larger than or equal to the values summarized in Table IV.*

Proof: See Appendix 3. ■

Remark C.1 *Compared to the asymptotic upper-bound-achieving result for conferencing strategy I given in Proposition B.2, Proposition C.1 guarantees that conferencing strategy II is practically feasible, and only finite conferencing link rates are necessary to achieve the rate upper bound.*

Proposition C.2 *If the conferencing link rates are symmetric, i.e., $C_{12} = C_{21}$, the DF rate under conferencing strategy II is the same as the rate upper bound under conferencing strategy I with arbitrary channel coefficients.*

Proof: First, for the cases that the either one of the first two terms in the min operation of (3.4) is the smallest among these four terms, it is easy to check that it can be achieved by setting $R_{i,3-i} = 0$, $i = 1, 2$, in Problem (P3). On the other hand, if the third term in (3.4) is the smallest one, it is achievable for the DF scheme due to the following fact: As a similar argument of Remark B.2, it can be shown that

$$R_{\mathrm{DF}} \leq R_{11} + R_{12} + 2R_{21} + R_{22} \tag{3.51}$$

$$\leq C_{12} + \lambda_2 \left(\mathcal{C}(\gamma_2) + \mathcal{C}(\widetilde{\gamma}_1)\right), \tag{3.52}$$

where (3.52) equals the third term in (3.4), the equality in (3.51) is achieved only when $R_{21} = 0$, and the equality in (3.52) is achieved only when the constraints (3.47)-(3.49) achieves the equality for the case of $i = 2$. Thus, define $R_{12} = C_{12}$, $R_{21} = 0$, $R_{11} = \lambda_2\mathcal{C}(\widetilde{\gamma}_1)$, and $R_{22} = \lambda_2\mathcal{C}(\gamma_2)$. Since we assume that the third term in (3.4) is the smallest one, it is easy to check that this solution is also feasible for the constraints in (3.48) and (3.49) for the case of $i = 1$. As such, the third term in (3.4) is achievable for the DF scheme. For the fourth term in (3.4), it can be shown that it is achievable by applying a similar argument as in the previous case. Therefore, this proposition is proved. ■

For the case that the conferencing link rates are not the same, i.e., $C_{12} \neq C_{21}$, the DF rate under conferencing strategy II may be either larger or smaller than the rate upper bound under conferencing strategy I, which will be shown in the next section by numerical results.

D. Numerical Results

In this section, we present some numerical results to compare the performances of the proposed coding schemes. Here, we only consider the asymmetric channel case, i.e., $\gamma_1 = \widetilde{\gamma}_2$ and $\gamma_2 = \widetilde{\gamma}_1$, and also show the performance of the DF simultaneous relaying scheme given in [23] as a comparison, which usually performs the best among various coding schemes under the simultaneous relaying mode.

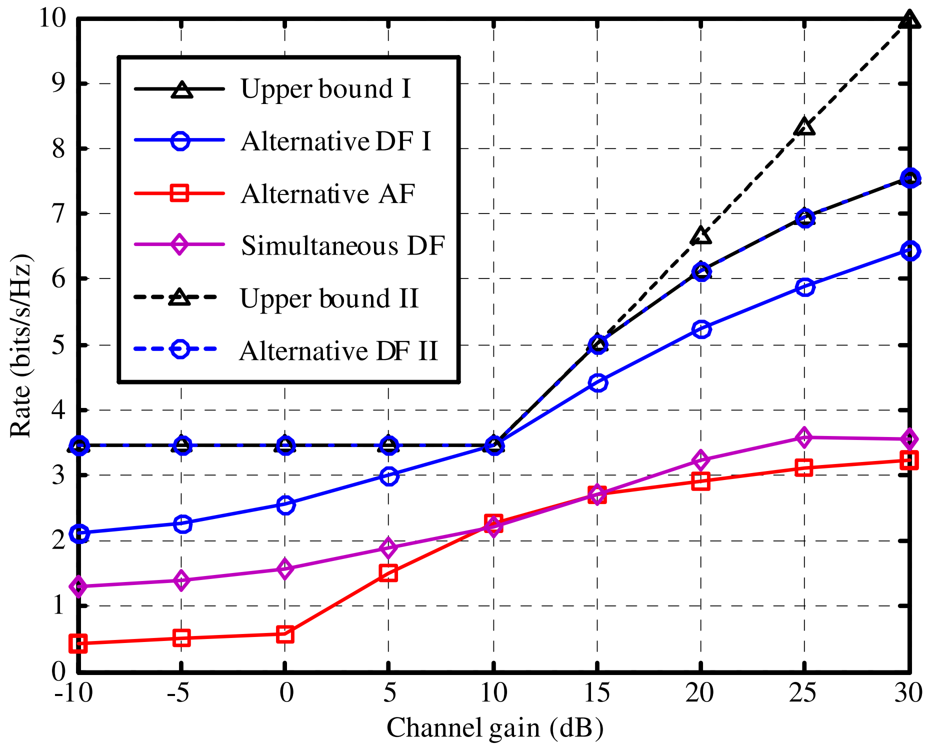


Fig. 11.: Comparison of the rate upper bounds and various achievable rates under different channel conditions, with $C_{12} = C_{21} = 5$ bits/s/Hz, $\gamma_1 = \widetilde{\gamma}_2 = 10$ dB, and different $\gamma_2 = \widetilde{\gamma}_1$.

In Fig. 11, for the two relay conferencing strategies, we plot the rate upper bounds and various achievable rates as functions of link gains. Here, we let $C_{12} = C_{21} = 5$ bits/s/Hz, $\gamma_1 = \widetilde{\gamma}_2 = 10$ dB, and $\gamma_2 = \widetilde{\gamma}_1$ change over $[-10, 30]$ dB. It is observed that the two upper bounds coincide when the channel gain is relatively small, i.e., when below 15 dB. For the DF relaying scheme, it achieves the rate upper bound when the channel gain is less than 10 dB under conferencing strategy II, while only at the point of 10 dB under conferencing strategy I.

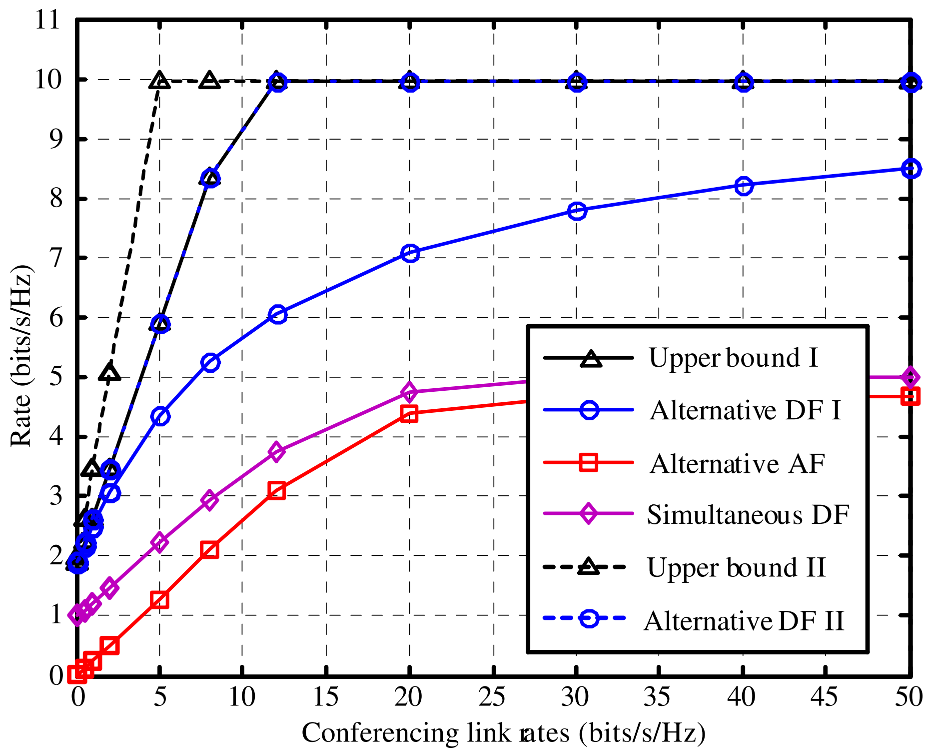


Fig. 12.: Comparison of the rate upper bounds and various achievable rates over different conferencing link rates, with $C_{12} = C_{21}$, $\gamma_1 = \widetilde{\gamma}_2 = 10$ dB, and $\gamma_2 = \widetilde{\gamma}_1 = 30$ dB.

In Fig. 12, we plot the rate upper bounds and various achievable rates as functions of the conferencing link rates for both of the two conferencing strategies. Here, we assume $C_{12} = C_{21}$, $\gamma_1 = \widetilde{\gamma}_2 = 10$ dB, and $\gamma_2 = \widetilde{\gamma}_1 = 30$ dB. It is observed that relay conferencing can significantly increase these achievable rates for both of the simultaneous and alternative relaying schemes. Moreover, although it is proved in Proposition B.2 that under conferencing strategy I, the alternative DF scheme can asymptotically achieve the rate upper bound as C_{12} goes to infinity, unfortunately it approaches the upper bound very slowly: Even when C_{12} are 50 bits/s/Hz, the gap between them is still about 1 bits/s/Hz; while under conferencing strategy II, the DF scheme achieves the corresponding upper bound with relative small conferencing link rates, i.e., about 12 bits/s/Hz.

E. Summary

In this chapter, we considered the alternative relaying diamond relay channel with conferencing links. We derived the DF and AF achievable rates for two conferencing strategies, and showed that these rate maximization problem are convex. For the DF relaying scheme, by further exploiting the properties of the optimal solution, one-side conferencing was shown to be optimal for the DF scheme with both of the two conferencing strategies. Then, we obtained the DF rate in closed-form, and explicitly showed the rules on which conferencing link should be used under given channel conditions for one-side conferencing. Interestingly, the DF scheme was shown to be upper-bound-achieving with the help of finite conferencing link rates under conferencing strategy II, whose lower bounds were also derived. For the AF relaying scheme, we studied the optimal combining strategy, and showed that one-side conferencing is not optimal in general. Furthermore, some asymptotic optimal combining strategies were obtained in both the high and low SNR regimes.

F. Appendix

1. Achievability Proof of the DF Rate

Codebook Generation: First, note that we only need to generate in total two sets of codebooks for the odd and even time slots, respectively; for simplicity, we use the subscript s, $s = 1, 2$, to distinguish these two sets of codebooks, i.e., $s = 1$ for that in odd time slots and $s = 2$ for that in even ones. The codebooks at the source are generated as follows: Generate $2^{nR_{s,s}}$ i.i.d. sequences $\boldsymbol{u}_s(q_s^1)$, where $q_s^1 \in \left[1 : 2^{nR_{s,s}}\right]$, according to the distribution $\prod_{j=1}^{\lambda_s n} p(u_{s,j})$; for each $\boldsymbol{u}_s(q_s^1)$, generate $2^{nR_{s,3-s}}$ i.i.d. sequences $\boldsymbol{x}_s(q_s^1, q_s^2)$, where $q_s^2 \in \left[1 : 2^{nR_{s,3-s}}\right]$, with the distribution $\prod_{j=1}^{\lambda_s n} p(x_{s,j}|u_{s,j})$. For the conferencing links, generate $2^{nR_{3-s,s}}$ i.i.d. sequences $\mathcal{M}_{3-s}(v_{3-s})$, where $v_{3-s} \in [1 : R_{3-s,s}]$. For the relay-destination transmissions, generate $2^{n(R_{3-s,3-s}+R_{s,3-s})}$ i.i.d. sequences $\boldsymbol{t}_{3-s}(s_s^1, s_s^2)$ by a similar superposition coding method as that of $\boldsymbol{x}_s$, where $s_s^1 \in [1 : R_{3-s,3-s}]$ and $s_s^2 \in [1 : R_{s,3-s}]$, according to the distribution $\prod_{j=1}^{\lambda_s n} p(t_j)$.

Encoding and decoding: At the beginning of the k-th time slot, $k = 1, 2, \cdots$, where the source sends message to the i-th relay with $i = 1$ for odd k and $i = 2$ for even k, the source splits the message w_k into two submessages w_k^1 and w_k^2, and transmits $\boldsymbol{x}_i(w_k^1, w_k^2)$; the $(3-i)$-th relay transmits $\mathcal{M}_i(w_{k-1}^{3-i})$ to the i-th relay via the conferencing link; the $(3-i)$-th relay transmits $\boldsymbol{t}_i(w_{k-1}^{3-i}, w_{k-2}^{i})$ to the destination.

At the end of the k-th time slot, the i-th relay obtains $\mathcal{M}_i(w_{k-1}^{3-i})$ from the $(3-i)$-th relay. Since we assume that the conferencing links are noiseless, the i-th relay can successfully decode message w_{k-1}^{3-i} if

$$R_{3-i,i} \leq \lambda_i C_{3-i,i}. \tag{3.53}$$

Simultaneously, the i-th relay obtains $\boldsymbol{y}_i$ from the source. Then, it decodes (w_k^1, w_k^2), and this can be done reliably if

$$R_{i,i} + R_{i,3-i} \leq \lambda_i I\left(X_i; Y_i\right) = \lambda_i \mathcal{C}\left(\gamma_i\right), \tag{3.54}$$

where (3.54) is obtained by choosing X_i as a Gaussian random variable with a distribution $\mathcal{CN}\left(0, P_S\right)$. At the destination, it decodes $(w_{k-1}^{3-i}, w_{k-2}^{i})$, and this can be done reliably if

$$R_{3-i,3-i} + R_{i,3-i} \leq \lambda_i I\left(T_{3-i}; Z_i\right) = \lambda_i \mathcal{C}\left(\widetilde{\gamma}_{3-i}\right), \tag{3.55}$$

where (3.55) is obtained by choosing T_{3-i} as a Gaussian random variable with a distribution $\mathcal{CN}(0, P_R)$. Based on the above analysis, we can obtain the DF achievable rate as shown in (3.5).

2. Proof of Proposition B.3

To prove this result, we only need to show that R_i defined in (3.36) is concave over $|a_{i,j}|^2$'s [51], due to the convexity of the constraints in (3.35). Then, since the function $y = \mathcal{C}(x)$ is concave and non-decreasing, we need to prove that the function within the $\mathcal{C}(\cdot)$ function in (3.36) is concave [51]. Moreover, noticing that $\frac{|a_{ii}|^2\gamma_i\widetilde{\gamma}_i}{1+|a_{ii}|^2\widetilde{\gamma}_i}$ is concave over $|a_{ii}|^2$, we only need to show that the second fraction in $\mathcal{C}(\cdot)$ is also concave. By letting $x = |a_{i,3-i}|^2$ and $y = |a_{3-i,3-i}|^2$, and normalizing the coefficients of x in the numerator and the denominator both to 1, it is equivalent to prove that $z = \frac{x}{x+ay+b}$, where a and b are some positive constants, is concave. Then, check the Hessian matrix of function z as

$$\boldsymbol{H} = \frac{1}{(x+ay+b)^3}\begin{bmatrix} -2ay-2b & ax-a^2y-ab \\ ax-a^2y-ab & 2a^2x \end{bmatrix}. \tag{3.56}$$

Noticing that $a > 0,\ b > 0,\ x \geq 0$, and $y \geq 0$, it is easy to show that $-2ay - 2b < 0$ and $|\boldsymbol{H}| = \frac{1}{(x+ay+b)^3}\left[-2a^2x(2ay+2b) - (ax-a^2y-ab)^2\right] < 0$, which implies that $\boldsymbol{H}$ is negative semidefinite and function z is concave. Therefore, the proposition is proved.

3. Proof of Proposition C.1

Similar to the proof of Proposition C.2, it is easy to check that the first and the second terms in (3.45) can be achieved by the DF rate given in Problem (P3). However, for the third and the fourth terms in (3.45), by a similar argument as Remark B.2, it can be shown that these two terms cannot be achieved by the DF relaying scheme. As such, the DF relaying scheme achieves the rate upper bound only for the case that the last two terms are redundant, i.e., for the optimal λ_i^* achieving the maximum value of the following optimization problem,

$$\widetilde{C}_{\text{upper}} = \max_{\lambda_1+\lambda_2=1} \min \left\{\lambda_1 \mathcal{C}(\gamma_1) + \lambda_2 \mathcal{C}(\gamma_2), \lambda_2 \mathcal{C}(\widetilde{\gamma}_1) + \lambda_1 \mathcal{C}(\widetilde{\gamma}_2)\right\}, \tag{3.57}$$

we always have $\lambda_2^* \left(\mathcal{C}(\gamma_2) + \mathcal{C}(\widetilde{\gamma}_1)\right) + C_{12} + C_{21} \geq \widetilde{C}_{\text{upper}}$ and $\lambda_1^* \left(\mathcal{C}(\gamma_1) + \mathcal{C}(\widetilde{\gamma}_2)\right) + C_{12} + C_{21} \geq \widetilde{C}_{\text{upper}}$. Therefore, the rate upper bound is achieved by the DF scheme only when the following relationship is satisfied

$$C_{12} + C_{21} \geq \widetilde{C}_{\text{upper}} - \min \left\{\lambda_2^* \left(\mathcal{C}(\gamma_2) + \mathcal{C}(\widetilde{\gamma}_1)\right), \lambda_1^* \left(\mathcal{C}(\gamma_1) + \mathcal{C}(\widetilde{\gamma}_2)\right)\right\}. \tag{3.58}$$

Denote

$$g(\lambda_1^*) = \min \left\{\lambda_2^* \left(\mathcal{C}(\gamma_2) + \mathcal{C}(\widetilde{\gamma}_1)\right), \lambda_1^* \left(\mathcal{C}(\gamma_1) + \mathcal{C}(\widetilde{\gamma}_2)\right)\right\}, \tag{3.59}$$

and it follows that $g(0) = g(1) = 0$. Then, in order to compute the lower bound on $C_{12} + C_{21}$ to achieve the rate upper bound, we only need to compute $\widetilde{C}_{\text{upper}}$ and the corresponding λ_i^*. For Problem (3.57), it follows that

1. $\gamma_1 > \gamma_2, \widetilde{\gamma}_2 > \widetilde{\gamma}_1$: It is obtained that $\lambda_1^* = 1$, and thus, $\widetilde{C}_{\text{upper}} = \min\left\{\mathcal{C}(\gamma_1), \mathcal{C}(\widetilde{\gamma}_2)\right\}$;

2. $\gamma_1 \leq \gamma_2, \widetilde{\gamma}_2 \leq \widetilde{\gamma}_1$: It is obtained that $\lambda_1^* = 0$, and thus $\widetilde{C}_{\text{upper}} = \min\left\{\mathcal{C}(\gamma_2), \mathcal{C}(\widetilde{\gamma}_1)\right\}$;

3. $\gamma_1 > \gamma_2, \widetilde{\gamma}_1 \leq \widetilde{\gamma}_2, \widetilde{\gamma}_2 \geq \gamma_1$: It is obtained that $\lambda_1^* = 1$, and thus $\widetilde{C}_{\text{upper}} = \mathcal{C}(\widetilde{\gamma}_2)$;

4. $\gamma_1 \leq \gamma_2, \widetilde{\gamma}_1 > \widetilde{\gamma}_2, \widetilde{\gamma}_2 \leq \gamma_1$: It is obtained that $\lambda_1^* = 1$, and thus $\widetilde{C}_{\text{upper}} = \mathcal{C}\left(\gamma_1\right)$;

5. $\gamma_1 > \gamma_2, \widetilde{\gamma}_1 > \widetilde{\gamma}_2, \gamma_1 > \widetilde{\gamma}_2$: It is obtained that $\lambda_1^* = \lambda_0$, and thus $\widetilde{C}_{\text{upper}} = \frac{\mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2) - \mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1)}{\mathcal{C}(\widetilde{\gamma}_2) - \mathcal{C}(\widetilde{\gamma}_1) - \mathcal{C}(\gamma_1) + \mathcal{C}(\gamma_2)}$;

6. $\gamma_1 \leq \gamma_2, \widetilde{\gamma}_1 < \widetilde{\gamma}_2, \gamma_1 < \widetilde{\gamma}_2$: It is obtained that $\lambda_1^* = \lambda_0$, and thus $\widetilde{C}_{\text{upper}} = \frac{\mathcal{C}(\gamma_2)\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\gamma_1)\mathcal{C}(\widetilde{\gamma}_1)}{\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}$;

where $\lambda_0 = \frac{\mathcal{C}(\gamma_2)-\mathcal{C}(\widetilde{\gamma}_1)}{\mathcal{C}(\widetilde{\gamma}_2)-\mathcal{C}(\widetilde{\gamma}_1)-\mathcal{C}(\gamma_1)+\mathcal{C}(\gamma_2)}$. Thus, the lower bound on $C_{12} + C_{21}$ to achieve the rate upper bound is given in Table IV.

CHAPTER IV

SINGLE-USER LARGE RELAY NETWORKS

In Chapters 2 and 3, we investigated the achievable rates for the four-node diamond relay channel with rate-limited out-of-band conferencing links between the two relays, and it was shown that the DF scheme could achieve the cut-set bound even with finite conferencing link rates in some special channel conditions. In this chapter, we extend these results to the large Gaussian relay networks with SNR-limited AF conferencing links among the relays, and focus on the asymptotic achievable rates of the DF and AF relaying schemes. It is shown that the relay conferencing can improve these achievable rates, and some asymptotic capacity results can be established under certain conditions.

The rest of this chapter is organized as follows. In Section A, we introduce the assumptions and channel models. In Section B, we discuss the DF and AF achievable rates. In Section C, we present some simulation and numerical results. Finally, the paper is concluded in Section D.

A. Assumptions and System Model

In this chapter, we consider a large relay network with out-of-band conferencing links among the relays, as shown in Fig. 13, which consists of one source-destination pair and N relays. We assume that there is no direct link between the source and destination.

The time scheduling of the transmissions at the source, relays, and conferencing links is shown in Fig. 14. The relay nodes work in a half-duplex mode: The source transmits and the relays listen in the first time slot; the relays simultaneously transmit and the destination listens in the second time slot. For simplicity, we allocate equal

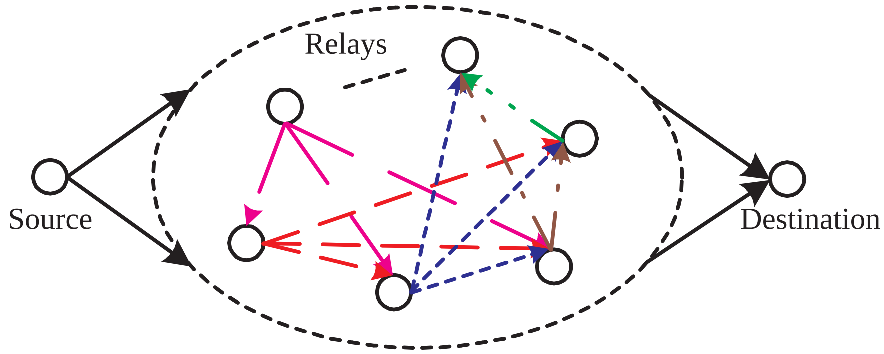


Fig. 13.: The large relay networks with conferencing links.

time durations to the two hops [15, 17]. Note that the conferencing links use out-of-band connections, which are orthogonal to other conferencing links, the source-to-relay links, and the relay-to-destination links. Based on these assumptions, the source-to-relay and the conferencing transmissions are scheduled during the same time slot.

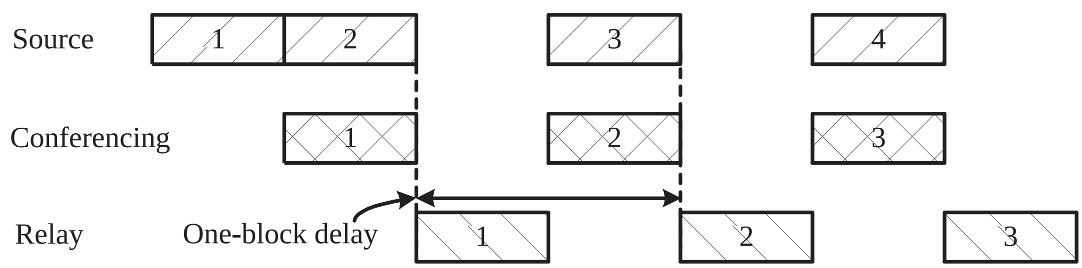


Fig. 14.: Transmission scheduling scheme for the large relay networks with conferencing links.

Due to the relay conferencing, each relay needs to wait the conferencing signals from the other relays before forwarding information to the destination. Thus, there is a one-block delay between the transmissions at the source and the relays, as shown in Fig. 14, which requires the relays to buffer one block of source signals for each relaying operation. Assume that during each data block, the communication rate is R, and we need to transmit B blocks in total. Thus, the average information rate is $R\frac{B}{B+1} \to R$, as B goes to infinity, such that the effect of the one-block delay is negligible. In this chapter, we focus on the one-block transmission to study the associated relaying and conferencing schemes without specifying the delay in the proof of the achievability.

We assume that each relay can conference with a subset of other relays via orthogonal wired links. In this chapter, we adopt a deterministic "p-portion conferencing" scheme: each relay can conference with other M relays, and

$$\lim_{N\to+\infty} \frac{M+1}{N} = p. \tag{4.1}$$

Without loss of generality, we assume that the i-th relay forwards its received signal to the relays with indices $i+k, i = 1, 2, \cdots, N$, and $k = 1, \cdots, M$, via the conferencing links. With a little abuse of notation, we use $i+k$ to denote the $(i+k)_N$-th relay, where $(\cdot)_N$ means the modula over N (and also $i-k$ is defined similarly). Particularly, when $N = M+1$, we call the scheme as "complete conferencing". Note that there exist many other conferencing schemes, i.e., random conferencing with any other M relays; for simplicity, the p-portion deterministic conferencing scheme is adopted here to provide a tractable achievable rate. In practical systems, it is costly to deploy MN conference links, which is exactly the reason why we propose a p-portion conferencing protocol to limit the percentage of conferencing connections. We will study the impact of p on the tradeoff between the system performance and the system installation cost.

We further define the following channel input-output relationship. In the first hop, the received signal y_i at the i-th relay, $i = 1, 2, \cdots, N$, is given as

$$y_i = \sqrt{P_s} h_i x + n_i, \tag{4.2}$$

where x is the signal transmitted by the source, P_s is the transmit power at the source node, h_i is the complex channel gain of the i-th source-to-relay link, which is assumed known to the source, and n_i's are the i.i.d. CSCG noise with distribution $\mathcal{CN}(0, N_0)$. Note that there are no particular assumptions on the distributions of h_i's, which are just assumed to be independent, of zero-mean, and with uniformly and positively bounded second-order and fourth-order statistics, i.e., $0 < b_1 \leq \mathbb{E}\left(|h_i|^2\right) \leq b_2 < +\infty$ and $0 < c_1 \leq \mathbb{E}\left(|h_i|^4\right) \leq c_2 < +\infty$ for arbitrary i.

Regardless of whether the relays work with the DF or AF relaying scheme, for the conferencing links, we assume that only AF is used as the conferencing scheme to forward the received signal of the i-th relay to the $(i+k)$-th relay, and the received signal at the $(i+k)$-th relay via the conferencing link is given as

$$y_{i,i+k} = \sqrt{\frac{P_c}{P_s\mathbb{E}\left(|h_i|^2\right) + N_0}} f_{i,i+k} y_i + n_{i,i+k}, \tag{4.3}$$

where $f_{i,i+k}$ is the complex link gain, $n_{i,i+k}$ is the CSCG noise with distribution $\mathcal{CN}(0, N_0)$, and P_c is the transmit power at the conferencing links. Here, the constant coefficient $\sqrt{\frac{P_c}{P_s\mathbb{E}(|h_i|^2)+N_0}}$ is used to satisfy the average transmit power constraint of the conferencing link. Due to the out-of-band and possible wired conferencing link assumptions, we assume that $f_{i,i+k}$ is a fixed positive constant and uniformly and positively bounded (similarly defined as that for $\mathbb{E}\left(|h_i|^2\right)$). Since the inputs of conferencing links may not be Gaussian, we adopt the transmit SNR $\frac{P_c}{N_0}$ as the quality metric of the conferencing links for convenience, instead of the rate constraints as in [23].

In the second hop, x_i with unit average power is transmitted from the i-th relay to the destination, and the received signal y at the destination is given as

$$y = \sum_{i=1}^{N} \sqrt{P_r} g_i x_i + n, \tag{4.4}$$

where g_i is the complex channel gain of the i-th relay-to-destination link, P_r is the transmit power at each relay, and n is the CSCG noise with distribution $\mathcal{CN}(0, N_0)$. We also assume that g_i's are independent, of zero mean, and with uniformly and positively bounded $\mathbb{E}\left(|g_i|^2\right)$ and $\mathbb{E}\left(|g_i|^4\right)$.

In this chapter, we assume that for the i-th relay, it knows h_i, h_j, $f_{j,i}$, and g_i, where $j \in \mathcal{A}_i \subset \{1, \cdots, N\}$ and $\mathcal{A}_i$ is the set of the indices corresponding to the relays connected to the i-th relay via the conferencing links. In practice, to obtain h_j's, one solution is to let the source send out one symbol pilot, and each relay then forward this pilot to the other relays via the conferencing links. After receiving such forwarded pilot signals, each relay can estimate h_j's, since the conferencing link gains $f_{j,i}$'s are assumed to be constant and known. To obtain g_i, we assume that the relay-to-destination links are reciprocal such that only one pilot signal from the destination is needed.

B. Capacity Upper Bound and Achievable Rates

In this section, we examine the capacity upper bound and the achievable rates of the considered networks with the DF and AF relaying schemes, respectively. Moreover, we prove some capacity-achieving results under special conditions.

1. Preliminary Results and Capacity Upper Bound

In this subsection, we first present some preliminary results and the capacity upper bound.

Lemma B.1 *Let $\{X_i \geq 0,\ i = 1, \cdots, N\}$ be independent random variables, whose means and variances are uniformly and positively bounded, respectively. Then, we* *have*

$$\log\left(1+\sum_{i=1}^{N}X_i\right)-\log\left(1+\sum_{i=1}^{N}\mathbb{E}\left(X_i\right)\right)\xrightarrow{w.p.1}0, \tag{4.5}$$

$$\log\left(\sum_{i=1}^{N}X_i\right)-\log\left(\sum_{i=1}^{N}\mathbb{E}\left(X_i\right)\right)\xrightarrow{w.p.1}0. \tag{4.6}$$

Proof: By the Corollary 2.3 in [52], we have (4.5); and we could obtain (4.6) similarly. ■

Using this lemma and the classic BC cut-set bound [2], we obtain the following capacity upper bound.

Theorem B.1 *(BC cut-set bound) The capacity upper bound for the two-hop large Gaussian relay network is given as*

$$C_{upper}\leq\frac{1}{2}\log\left(1+\frac{P_s}{N_0}\sum_{i=1}^{N}|h_i|^2\right) \tag{4.7}$$

$$\xrightarrow{w.p.1}\frac{1}{2}\log\left(1+\frac{P_s}{N_0}\sum_{i=1}^{N}\mathbb{E}\left(|h_i|^2\right)\right) \tag{4.8}$$

$$\sim\mathcal{O}\left(\log(N)\right) \tag{4.9}$$

Proof: (4.7) is by the result in [15], and (4.8) is by (4.5). Let $\mu=\frac{1}{N}\sum_{i=1}^{N}\mathbb{E}\left(|h_i|^2\right)$, which is positively bounded, and we obtain (4.9). ■

2. The DF Achievable Rate

In [16], the authors showed that the DF rate scales on the order of $\mathcal{O}\left(\log(\log(N))\right)$ without conferencing among the relays, where the source chooses an optimal a subset of relays to decode the source message and let the rest keep silent in the second hop transmission. In this subsection, we adopt a different scheme to require all the relays to decode the source message and transmit in the second hop. Obviously, compared to the previous scheme [16], our scheme is not optimal in term of relay subset selection, while it is enough to show the improvement of the achievable rate scaling behavior introduced by relay conferencing. Note that both the schemes in [16] and our proposed DF scheme require full channel CSI at the source node. Our main result for the DF relaying scheme is given as the following theorem.

Theorem B.2 *Using the p-portion conferencing strategy, the DF rate scales on the order of* $\mathcal{O}(\log(N))$.

Proof: Based on the principle of maximum ratio combining (MRC), the received SNR at the relay is the sum of the SNRs in (4.2) and (4.3). Thus, for the first hop, the maximum rate supported at the i-th relay is given as

$$R_i = \frac{1}{2}\log\left(1 + \frac{|h_i|^2 P_s}{N_0} + \frac{P_s}{N_0}\sum_{k=1}^{M}\frac{\frac{P_c}{P_s\mathbb{E}(|h_{i-k}|^2)+N_0}|f_{i-k,i}|^2|h_{i-k}|^2}{\frac{P_c}{P_s\mathbb{E}(|h_{i-k}|^2)+N_0}|f_{i-k,i}|^2+1}\right) \tag{4.10}$$

$$\xrightarrow{w.p.1} \frac{1}{2}\log\left(1 + \frac{P_s}{N_0}\left(\mathbb{E}\left(|h_i|^2\right) + \sum_{k=1}^{M}\frac{P_c|f_{i-k,i}|^2\mathbb{E}\left(|h_{i-k}|^2\right)}{P_c|f_{i-k,i}|^2 + P_s\mathbb{E}\left(|h_{i-k}|^2\right) + N_0}\right)\right) \tag{4.11}$$

$$= \frac{1}{2}\log\left(1 + (M+1)\frac{P_s}{N_0}\mu_{\mathrm{DF}}\right), \tag{4.12}$$

where (4.11) is by the Lemma B.1, and

$$\mu_{\mathrm{DF}} = \frac{1}{M+1}\left[\mathbb{E}\left(|h_i|^2\right) + \sum_{k=1}^{M}\frac{P_c|f_{i-k,i}|^2\mathbb{E}\left(|h_{i-k}|^2\right)}{P_c|f_{i-k,i}|^2 + P_s\mathbb{E}\left(|h_{i-k}|^2\right) + N_0}\right],$$

which is positively bounded. Thus, we have $R_i \sim \mathcal{O}(\log(N))$.

In the second hop, we assume that all relays transmit simultaneously, and the transmit signal at the i-th relay is $x_i = \sqrt{\frac{1}{\mathbb{E}(|g_i|^2)}}g_i^* x$. Thus, the received signal at the destination is given as

$$y = \underbrace{\sum_{i=1}^{N}\sqrt{\frac{P_r}{\mathbb{E}\left(|g_i|^2\right)}}|g_i|^2}_{Q_0} x + n, \tag{4.13}$$

and the maximum rate supported in the second hop is given as

$$R_{\mathrm{MAC}} = \frac{1}{2}\log\left(1+\frac{Q_0^2}{N_0}\right) \tag{4.14}$$

$$\sim \log\left(\frac{Q_0}{\sqrt{N_0}}\right) \tag{4.15}$$

$$\xrightarrow{w.p.1} \log\left(\frac{\mathbb{E}\left(Q_0\right)}{\sqrt{N_0}}\right) \tag{4.16}$$

$$= \frac{1}{2}\log\left(\frac{P_r}{N_0}N^2\mu^2\right), \tag{4.17}$$

where (4.15) is valid as $N \to \infty$, (4.16) is by (4.6), $\mathbb{E}\left(Q_0\right) = \sqrt{P_r}\sum_{i=1}^{N}\frac{\mathbb{E}\left(|g_i|^2\right)}{\sqrt{\mathbb{E}(|g_i|^2)}} = \sqrt{P_r}\sum_{i=1}^{N}\sqrt{\mathbb{E}\left(|g_i|^2\right)}$, and $\mu = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\mathbb{E}\left(|g_i|^2\right)}$.

Therefore, the DF achievable rate is given as

$$R_{\mathrm{DF}} = \min\left\{\min_i\{R_i\}, R_{\mathrm{MAC}}\right\}. \tag{4.18}$$

Since R_i and R_{MAC} scale as $\mathcal{O}\left(\log\left(N\right)\right)$ and $\mathcal{O}\left(\log\left(N^2\right)\right)$, respectively, R_{DF} scales with the order of $\mathcal{O}\left(\log\left(N\right)\right)$. ■

Remark B.1 *For the complete conferencing scheme, i.e., $M = N-1$, the DF scheme is not capacity-achieving, since the SNR penalty term $\frac{P_c|f_{i-k,i}|^2}{P_c|f_{i-k,i}|^2+P_s\mathbb{E}(|h_{i-k}|^2)+N_0}$ is uniformly and positively bounded and strictly less than 1. For the case $0 < p < 1$, obviously, the DF scheme is also not capacity-achieving, and suffers another $(1-p)$-portion power gain loss.*

3. AF Achievable Rate

In this subsection, we discuss the AF relaying scheme. Since we assume no global CSIs at the relays, the network-wide optimal combining at the relays as proposed in [23] cannot be deployed. Thus, with only local CSIs, MRC across conferencing signals is another good choice, which maximizes the received SNR at the relays. Unfortunately, MRC makes the rate expression too complicated to obtain any clean results. Instead, here we combine the received signals y_i and $y_{i-k,i}$'s at the i-th relay as

$$t_i = h_i^* y_i + \sum_{k=1}^{M} \sqrt{\frac{P_s \mathbb{E}\left(|h_{i-k}|^2\right) + N_0}{P_c}} \frac{1}{f_{i-k,i}} h_{i-k}^* y_{i-k,i}. \tag{4.19}$$

Then, the transmit signal at the i-th relay is given as

$$\begin{aligned} x_i &= a_i \sqrt{P_r} g_i^* t_i && (4.20)\\ &= a_i \sqrt{P_r} g_i^* \left(\sum_{k=0}^{M} \sqrt{P_s} |h_{i-k}|^2 x + \sum_{k=0}^{M} h_{i-k}^* n_{i-k} \right.\\ &\qquad \left. + \sum_{k=1}^{M} \sqrt{\frac{P_s \mathbb{E}\left(|h_{i-k}|^2\right) + N_0}{P_c}} \frac{h_{i-k}^* n_{i-k,i}}{f_{i-k,i}} \right), && (4.21) \end{aligned}$$

where a_i is the power control factor to satisfy $\mathbb{E}(x_i) \leq P_r$, and it is chosen as

$$\begin{aligned} a_i^2 = \mathbb{E}^{-1}\left(|g_i|^2\right) & \left[P_s \mathbb{E} \left(\sum_{k=0}^{M} |h_{i-k}|^2 \right)^2 \right.\\ & \left. + \sum_{k=0}^{M} \mathbb{E}\left(|h_{i-k}|^2\right) \left(1 + \frac{P_s \mathbb{E}\left(|h_{i-k}|^2\right) + N_0}{P_c |f_{i-k,i}|^2} \right) \right]^{-1}. && (4.22) \end{aligned}$$

Remark B.2 *This combining scheme is not valid for the case without relay conferencing, i.e., the conferencing link SNR* $\frac{P_c}{N_0} = 0$*. Moreover, if* $|f_{i,i+k}|$ *or* $\frac{P_c}{N_0}$ *is close to zero, it will boost the conferencing link noise* $n_{i,i+k}$*, which may make the performance even worse than the case without conferencing. However, our analysis will show that for uniformly and positively bounded* $|f_{i,i+k}|$*'s and arbitrary* $\frac{P_c}{N_0}$*, the AF scheme performs well as* $N \to \infty$*.*

Based on (4.4) and (4.21), the received signal at the destination is given as

$$y = \sum_{i=1}^{N} g_i x_i + n \tag{4.23}$$

$$= \sqrt{P_r P_s} \underbrace{\sum_{i=1}^{N} a_i |g_i|^2 \left(\sum_{k=0}^{M} |h_{i-k}|^2 \right)}_{Q_1} x + \sqrt{P_r} \sum_{i=1}^{N} \left(\sum_{k=0}^{M} a_{i+k} |g_{i+k}|^2 \right) h_i^* n_i$$
$$+ \sqrt{P_r} \sum_{i=1}^{N} \sum_{k=1}^{M} \sqrt{\frac{P_s \mathbb{E}\left(|h_{i-k}|^2\right) + N_0}{P_c}} \frac{1}{f_{i-k,i}} a_i |g_i|^2 h_{i-k}^* n_{i-k,i} + n. \tag{4.24}$$

Then, the AF achievable rate is given as

$$R_{\mathrm{AF}} = \frac{1}{2} \log \left(1 + \frac{P_s P_r Q_1^2}{\left(P_r Q_2 + P_r Q_3 + 1\right) N_0} \right), \tag{4.25}$$

where

$$Q_2 = \sum_{i=1}^{N} \left(\sum_{k=0}^{M} a_{i+k} |g_{i+k}|^2 \right)^2 |h_i|^2, \tag{4.26}$$

$$Q_3 = \sum_{i=1}^{N} \sum_{k=1}^{M} |a_i|^2 \frac{P_s \mathbb{E}\left(|h_{i-k}|^2\right) + N_0}{P_c |f_{i-k,i}|^2} |g_i|^4 |h_{i-k}|^2. \tag{4.27}$$

Now we have

$$\log \left(1 + \frac{P_s P_r Q_1^2}{\left(P_r Q_2 + P_r Q_3 + 1\right) N_0} \right) \tag{4.28}$$

$$\sim \log \left(\frac{P_s P_r Q_1^2}{\left(P_r Q_2 + P_r Q_3 + 1\right) N_0} \right) \tag{4.29}$$

$$= 2 \log \left(\sqrt{\frac{P_s P_r}{N_0}} Q_1 \right) - \log \left(P_r Q_2 + P_r Q_3 + 1 \right) \tag{4.30}$$

$$\xrightarrow{w.p.1} 2 \log \left(\sqrt{\frac{P_s P_r}{N_0}} \mathbb{E}\left(Q_1\right) \right) - \log \left(P_r \mathbb{E}\left(Q_2\right) + P_r \mathbb{E}\left(Q_3\right) + 1 \right), \tag{4.31}$$

$$\sim \log \left(1 + \frac{P_s P_r \mathbb{E}^2\left(Q_1\right)}{\left(P_r \mathbb{E}\left(Q_2\right) + P_r \mathbb{E}\left(Q_3\right) + 1\right) N_0} \right), \tag{4.32}$$

where (4.31) is by the Lemma B.1. Notice that (4.29) and (4.32) are valid since we only add or ignore a constant term, which can be neglected in the case of $N \to +\infty$.

As $N \to +\infty$, we have

$$\mathbb{E}\left(Q_1\right) = \sum_{i=1}^{N} a_i \mathbb{E}\left(|g_i|^2\right) \left(\sum_{k=0}^{M} \mathbb{E}\left(|h_{i-k}|^2\right) \right) = N(M+1)\mu_1, \tag{4.33}$$

$$\mathbb{E}\left(Q_2\right) = \sum_{i=1}^{N} \mathbb{E}\left(\left(\sum_{k=0}^{M} a_{i+k}|g_{i+k}|^2 \right)^2 \right) \mathbb{E}\left(|h_i|^2\right) = N(M+1)^2\mu_2, \tag{4.34}$$

$$\mathbb{E}\left(Q_3\right) = \sum_{i=1}^{N} \sum_{k=1}^{M} |a_i|^2 \frac{P_s \mathbb{E}\left(|h_{i-k}|^2\right) + N_0}{P_c |f_{i-k,i}|^2} \mathbb{E}\left(|g_i|^4\right) \mathbb{E}\left(|h_{i-k}|^2\right) = NM\mu_3, \tag{4.35}$$

where

$$\mu_1 = \frac{1}{N} \sum_{i=1}^{N} a_i \mathbb{E}\left(|g_i|^2\right) \left(\frac{1}{M+1} \sum_{k=0}^{M} \mathbb{E}\left(|h_{i-k}|^2\right) \right), \tag{4.36}$$

$$\mu_2 = \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}\left(\left(\frac{1}{M+1} \sum_{k=0}^{M} a_{i+k}|g_{i+k}|^2 \right)^2 \right) \mathbb{E}\left(|h_i|^2\right), \tag{4.37}$$

$$\mu_3 = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{M} \sum_{k=1}^{M} |a_i|^2 \frac{P_s \mathbb{E}\left(|h_{i-k}|^2\right) + N_0}{P_c |f_{i-k,i}|^2} \mathbb{E}\left(|g_i|^4\right) \mathbb{E}\left(|h_{i-k}|^2\right). \tag{4.38}$$

Since we assume that $\mathbb{E}\left(|h_i|^2\right)$, $\mathbb{E}\left(|g_i|^2\right)$, and $\mathbb{E}\left(|g_i|^4\right)$ are uniformly and positively bounded, $|a_i|$, μ_1, μ_2, and μ_3 are also bounded and positive. For the p-portion conferencing scheme, since $\mathbb{E}\left(Q_3\right)$ scales on a smaller order than $\mathbb{E}\left(Q_2\right)$ as N goes to infinity, we obtain the AF rate as

$$R_{\mathrm{AF}} \xrightarrow{w.p.1} \frac{1}{2} \log \left(1 + N \frac{\mu_1^2}{\mu_2} \frac{P_s}{N_0} \right). \tag{4.39}$$

Remark B.3 *The term Q_3 is the contribution of the conferencing link noises. Since $\frac{\mathbb{E}(Q_3)}{\mathbb{E}(Q_2)} \to 0$, we conclude that for the p-portion conferencing scheme, the conferencing link noises are asymptotically negligible as $N \to +\infty$. This suggests that for large relay networks with AF, we do not need high quality conferencing links, i.e., even with small $\frac{P_c}{N_0}$, and the performance of the AF scheme is reasonably good for large N.*

It is difficult to verify whether the AF scheme is capacity-achieving or not for the case with $0 < p < 1$ and generally distributed h_i's and g_i's. In the following, we prove two special capacity-achieving cases, which may be applied to many widely-used scenarios.

Theorem B.3 *If h_i's and g_i's are i.i.d., respectively, the AF scheme asymptotically achieves the capacity upper bound (4.8) as N goes to infinity for arbitrary $0 < p < 1$ and $\frac{P_c}{N_0} > 0$.*

Proof: Since h_i's and g_i's are i.i.d., $\mathbb{E}\left(|h_i|^2\right)$, $\mathbb{E}\left(|g_i|^2\right)$, and $\mathbb{E}\left(|g_i|^4\right)$ are identical over different i's, respectively. Let us examine the term $\frac{\mu_1^2}{\mu_2}$, and we have

$$\frac{\mu_1^2}{\mu_2} = \frac{1}{N}\sum_{i=1}^{N}\mathbb{E}\left(|h_i|^2\right)\frac{\sum_{j=1}^{N}\mathbb{E}\left(|h_j|^2\right)\left(\sum_{k=0}^{M}a_{i+k}\mathbb{E}\left(|g_{i+k}|^2\right)\right)\left(\sum_{t=0}^{M}a_{j+t}\mathbb{E}\left(|g_{j+t}|^2\right)\right)}{\sum_{j=1}^{N}\mathbb{E}\left(|h_j|^2\right)\mathbb{E}\left(\left(\sum_{s=0}^{M}a_{j+s}|g_{j+s}|^2\right)^2\right)} \tag{4.40}$$

$$= \frac{\mathbb{E}\left(|h_i|^2\right)}{N}\sum_{i=1}^{N}\underbrace{\frac{N\mathbb{E}^2\left(|g_i|^2\right)\left[\left(\sum_{k=0}^{M}a_{i+k}\right)\left(\sum_{t=0}^{M}a_t\right)\right]}{\mathbb{E}^2\left(|g_i|^2\right)\sum_{j=1}^{N}\sum_{s_1\neq s_2}a_{j+s_1}a_{j+s_2}+\mathbb{E}\left(|g_i|^4\right)M\sum_{j=1}^{N}a_j^2}}_{C_i}. \tag{4.41}$$

From (4.22), we have $a_i^2 \approx \frac{1}{\mathbb{E}(|h_i|^2)\mathbb{E}(|g_i|^2)P_sM^2}$ for large M, and we have

$$C_i \approx \frac{\mathbb{E}^2\left(|g_i|^2\right)N(M+1)^2}{\mathbb{E}^2\left(|g_i|^2\right)NM(M+1)+\mathbb{E}\left(|g_i|^4\right)MN} \to 1. \tag{4.42}$$

Hence, we have $\frac{\mu_1^2}{\mu_2} \to \mathbb{E}\left(|h_i|^2\right)$. Therefore, the theorem is proved. ■

Theorem B.4 *For independent but not necessarily identically distributed h_i's or g_i's, the full conferencing scheme, i.e., $N = M + 1$, asymptotically achieves the capacity upper bound as N goes to infinity for arbitrary $\frac{P_c}{N_0} > 0$.*

Proof: For the complete conferencing scheme, we obtain

$$Q_1 = \left(\sum_{i=1}^{N} a_i |g_i|^2\right) \sum_{k=1}^{N} |h_k|^2, \tag{4.43}$$

$$Q_2 = \left(\sum_{i=1}^{N} a_i |g_i|^2\right)^2 \sum_{k=1}^{N} |h_k|^2. \tag{4.44}$$

By a similar argument as in the previous theorem, we can show $\frac{P_r Q_3 + 1}{P_r Q_2} \xrightarrow{w.p.1} 0$ as N goes to infinity such that we obtain

$$R_{\mathrm{AF}} = \frac{1}{2} \log \left(1 + \frac{P_s \sum_{k=1}^{N} |h_k|^2}{\left(1 + \frac{P_r Q_3 + 1}{P_r Q_2}\right) N_0}\right) \tag{4.45}$$

$$\xrightarrow{w.p.1} \frac{1}{2} \log \left(1 + \frac{P_s}{N_0} \sum_{k=1}^{N} |h_k|^2\right). \tag{4.46}$$

Therefore, the capacity upper bound is asymptotically achieved. ■

C. Numerical Results

In this section, we present some simulation and numerical results to compare the performance among the proposed coding schemes. For simplicity, we assume that h_i's and g_i's are i.i.d. complex Gaussian random variable of $\mathcal{CN}(0,1)$, $|f_{i,i+k}| = 1$, $P_s = 1$, $P_r = 1$, and $N_0 = 1$. The rates in all the simulations, are averaged over 1000 fading realizations.

In Fig. 15, we show the capacity upper bound and the achievable rates for different p values, as the number of relays increases. For the AF relaying scheme, the gap between the upper bound and the achievable rate is very small for $p = 0.2$ and large N values. For the DF relaying scheme, when N is large, we observe that the DF rate and the capacity upper bound have the same scaling behavior.

In Fig. 16, we plot the achievable rates as functions of p. For the AF relaying

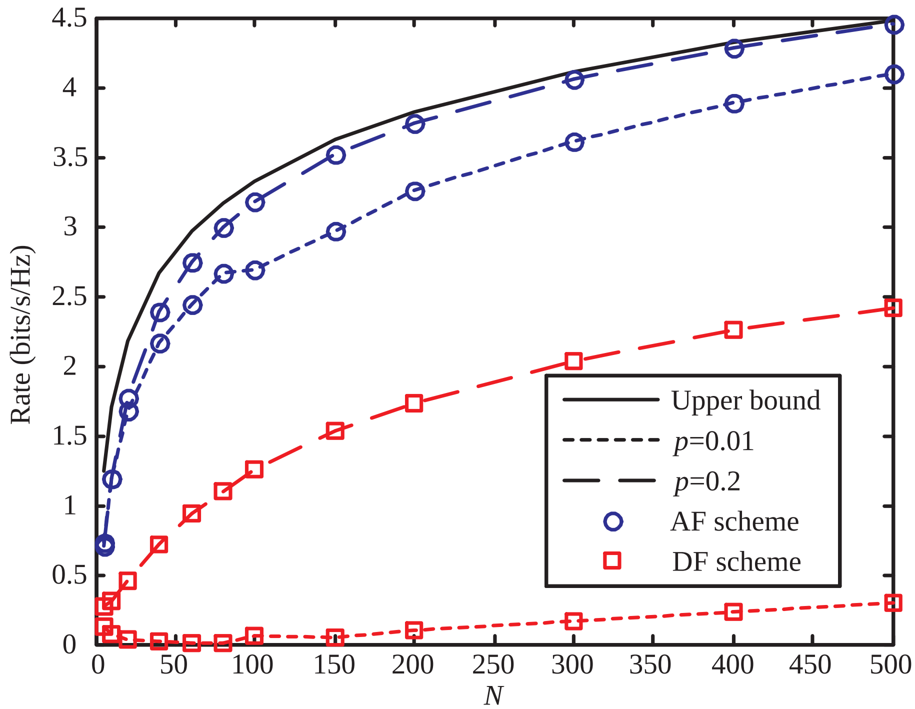


Fig. 15.: Achievable rates vs. the number of relays, $P_s = 1$, $P_r = 1$, $P_c = 1$, and $|f_{i,k}| = 1$.

scheme, the p value does not need to be large to achieve most of the gains, i.e., around $p = 0.3$; on the other hand, conferencing may not strictly improve the AF rate: When p is close to zero, the achievable rate is lower than the case without relay conferencing, which is due to the sub-optimality of the combining scheme at the relays. For the DF relaying scheme, relay conferencing always helps, and there is a significant rate improvement as p increases.

In Fig. 17, we plot the achievable rates as functions of the conferencing link SNR. It is observed that with medium-quality conferencing links (the SNRs of the conferencing links are around 5 dB), we achieve most of the gains introduced by relay conferencing for both the AF and DF relaying schemes.

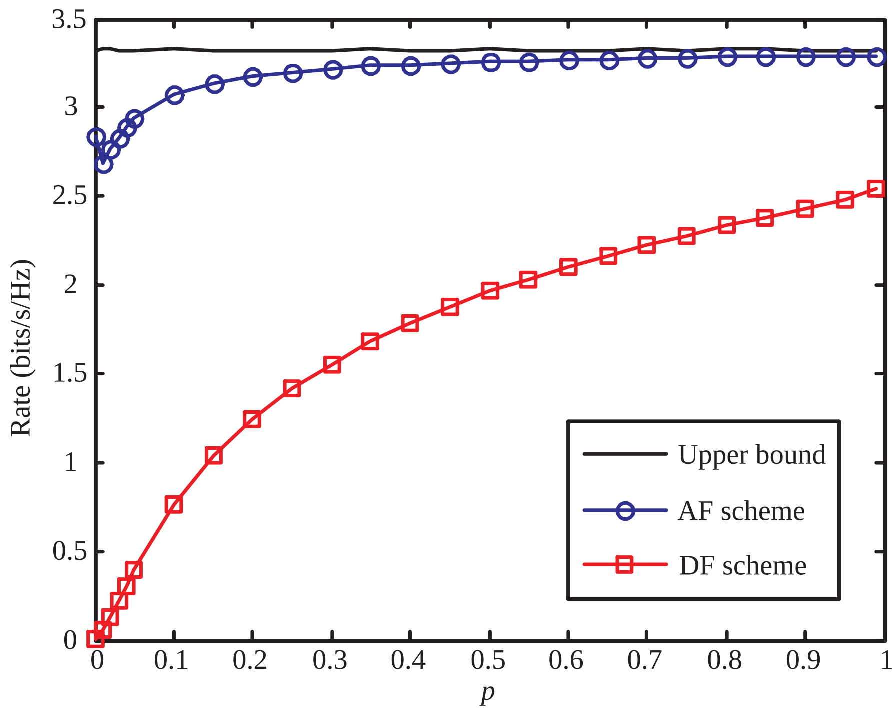


Fig. 16.: Achievable rates vs. the conferencing ratio, $P_s = 1$, $P_r = 1$, $P_c = 1$, $|f_{i,k}| = 1$, and $N = 100$.

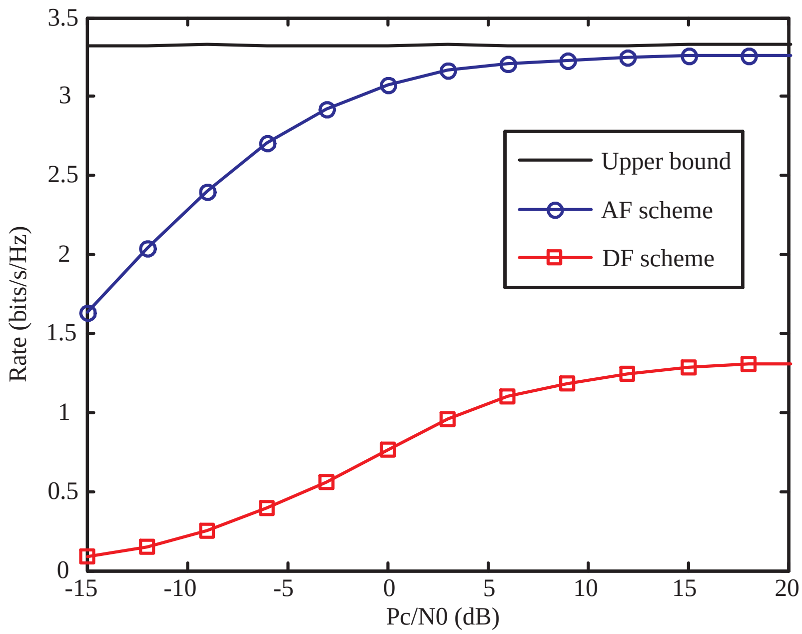


Fig. 17.: Achievable rates vs. the conferencing link SNR, $P_s = 1$, $P_r = 1$, $|f_{i,k}| = 1$, $N = 100$, and $p = 0.1$.

D. Summary

In this chapter, we investigated the achievable rate scaling laws of the DF and AF relaying schemes in a large Gaussian relay networks with conferencing links. We showed that for the DF relaying scheme, the rate scales as $\mathcal{O}(\log(N))$, compared to $\mathcal{O}(\log(\log(N)))$ for the case without conferencing; for the AF relaying scheme, we proved that if the channel fading coefficients h_i's and g_i's are i.i.d., respectively, or $N = M+1$, it asymptotically achieves the capacity upper bound as N goes to infinity.

CHAPTER V

TWO-USER INTERFERENCE RELAY NETWORKS

In this chapter, we focus on the multi-user wireless systems. In the literature, the one-hop IC has been extensively studied, where various upper bounds and achievable rates have been established [29–31]. Specifically, the best known achievable rate result was obtained by Han and Kobayashi [29], where each source splits its message into two parts and each destination decodes its own desired message and part of the interference. In general, the capacity of IC remains unknown, and the capacity results are established only for some special cases, e.g., the strong [30] and weak [31] interference cases.

An important extension of the one-hop IC is the so-called two-hop interference networks [32–36], where two sources send messages to their intended destinations via two separate relays. Essentially, such a network is a cascade of two ICs, which may arise in many practical scenarios, such as two neighboring base-stations communicating with two mobile users via two relays in an LTE-A cellular system. For such a two-hop interference network, the authors in [32–34] derived the achievable rates by using the DF and AF relaying schemes under the weak or strong interference assumptions, while no general capacity results were established. In [35], the authors investigated this network by using a deterministic model and bounded the gap between the achievable rate and the upper bound within a constant number of bits under arbitrary channel conditions. More recently, the authors in [36] proved that under the full-duplex relaying mode, i.e., the relay can transmit and receive at the same time and frequency, the maximum degree-of-freedom (DoF) of 2 can be achieved by a specifically designed DF scheme, while only $\frac{3}{2}$ DoF can be achieved by the AF scheme with constant channel coefficients.

Among these relaying schemes, the AF scheme is more attractive for practical implementation in cellular or other systems. When both the source and relays are equipped with multiple antennas, the optimal AF relaying matrix design problem was considered in [37–39] for point-to-point communication under different design principles: maximum achievable rate, minimum mean square error (MMSE), and optimal quality-of-service (QoS), respectively. In [40–43], the authors considered the optimal design for the multipoint-to-multipoint networks assisted with AF relays, where both the non-robust and robust design methods were considered in [43]. Note that all these aforementioned works are based on the single-user decoding scheme, which treats other users' signals as noise and may lead to certain suboptimality under general channel setups. In addition, the above works did not explore any information exchanges among the relays.

In this chapter, we consider a two-hop interference network, which contains two sources, two destinations, and two relays with out-of-band conferencing links. We assume that each source wants to transmit a message to its desired destination. As shown in this chapter, such a system with AF relaying and relay conferencing is equivalent to a two-user IC. Instead of characterizing the rate region via the complicated general Han-Kobayashi scheme [29], we turn to two more practical decoding schemes:

1. Single-user decoding scheme: Each destination tries to decode its own message and treat the other source's message as noise;

2. Joint decoding scheme: Each destination decodes both of the source messages.

Compared with the previous works [40–42], in this chapter, we further consider the conferencing between the two relays, and concentrate on how to characterize the rate region by optimizing over the source powers and the relay combining vector.

The rest of the chapter is organized as follows. Section A introduces the assumptions and channel models. In Sections B and C, we study the rate regions for both the single-user decoding and joint decoding schemes, respectively. In Section D, we compare the performances of the two schemes in the high SNR regime. Section E presents some numerical results. Finally, the paper is summarized in Section F.

A. System Model

In this chapter, we consider a two-hop interference network, as shown in Fig. 18, which contains two sources, two destinations, and two relays with out-of-band SNR-limited conferencing links. We assume that there are no direct links between any source-destination pairs. Denote the received SNR of the conferencing link from relay 1 to relay 2 as γ_{12}, with γ_{21} defined similarly. Furthermore, these two conferencing links are assumed to be orthogonal to each other and outside the frequency band used by the source-to-relay and relay-to-destination links, or even use wired connections.

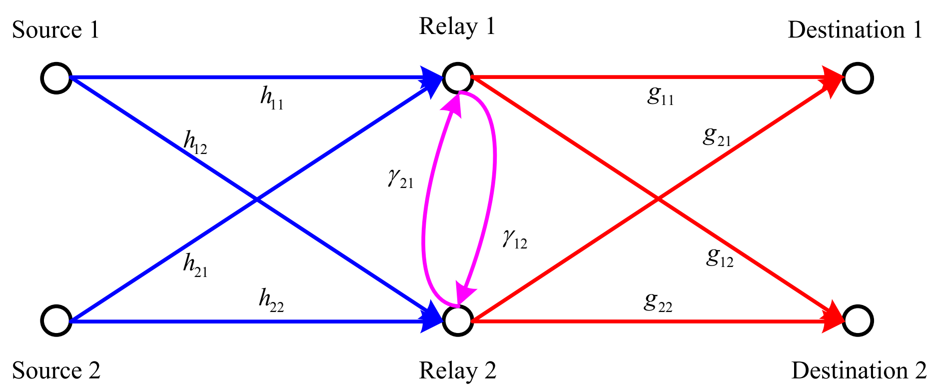


Fig. 18.: Two-hop interference networks with out-of-band SNR-limited conferencing links.

It is assumed that the relay nodes work in a half-duplex mode: The sources simultaneously transmit and the two relays listen in the first time slot; the relays simultaneously transmit and the destinations listen in the second time slot. Moreover, relay conferencing is scheduled at the same time slot as the source-to-relay transmissions. It is assumed that the AF scheme is adopted as the relaying and conferencing schemes, such that the lengthes of the above transmission blocks are the same. With the above assumptions, the time scheduling of the transmissions at the sources, relays, and conferencing links is described as follows, also shown in Fig. 19. The i-th source, $i = 1, 2$, sends independent messages $w_i(1)$ in the first time slot, and $w_i(t)$, $t = 2, 3, \cdots$, in the $(2t-2)$-th time slot sequentially; during the $(2t-2)$-th time slot, $t \geq 2$, the i-th relay forwards the received signal about the $w_i(t-1)$-th message to its counterpart relay via the conferencing link; after receiving the signals from the source and the other relay, each relay sends the combined signals about the $w_1(t)$-th and the $w_2(t)$-th source messages, $t = 1, 2, \cdots$, to the destination in the $(2t+1)$-th time slot. Due to relay conferencing, there is a one-block delay between the transmissions at the sources and the relays, which enables the relays to buffer one block of the source signal for each relaying operation. Assume that during each block, the communication rate is R, and we need to transmit B blocks in total. Thus, the average information rate is $R\frac{B}{B+1} \to R$ as B goes to infinity, such that the effect of the one-block delay is negligible. As such, here we only focus on one-block transmission and the associated relay operations without specifying the delay in the analysis.

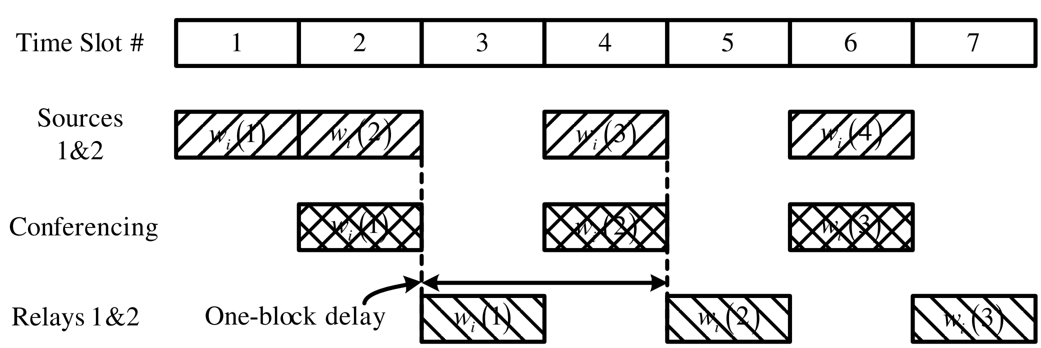


Fig. 19.: Transmission scheduling scheme for the two-hop interference network with conferencing links.

The channel input-output relationships of the discussed network are given as follows. To be concise, when we describe the signal relationship at the i-th relay

$(i = 1, 2)$, we use $(3-i)$ to index the other relay. The received signal at the i-th relay is given as

$$y_i = h_{1i}x_1 + h_{2i}x_2 + n_i, \tag{5.1}$$

where $x_j,\ j = 1, 2$, is the circularly symmetric complex Gaussian (CSCG) signal from the j-th source with power p_j, h_{ji} is the complex channel gain from the j-th source to the i-th relay, n_i is the independent and identically distributed (i.i.d.) CSCG noise with zero mean and unit variance, i.e., $n_i \sim \mathcal{CN}(0, 1)$. For the sources, we consider two different power constraints on the channel inputs: With the individual source power constraints, we define the source power region as

$$\mathcal{P} = \{(p_1, p_2) :\ 0 \le p_1 \le P_{S1},\ 0 \le p_2 \le P_{S2}\},$$

where P_{S1} and P_{S2} are the maximum power budgets at source 1 and source 2, respectively; with the sum power constraint, we have

$$\mathcal{P} = \{(p_1, p_2) :\ p_1 + p_2 \le P_S, p_1 \ge 0, p_2 \ge 0\},$$

where P_S is the maximum sum power budget over both sources.

For the conferencing links, the received signal at the $(3-i)$-th relay, $i = 1, 2$, is given as

$$r_{i,3-i} = f_{i,3-i} \cdot (h_{1i}x_1 + h_{2i}x_2 + n_i) + n_{i,3-i}, \tag{5.2}$$

where $f_{i,3-i} = \sqrt{\gamma_{i,3-i}}e^{j\theta_{i,3-i}}E_i^{-\frac{1}{2}}$ is a coefficient regulating the input power to make the conferencing link received SNR not bigger than $\gamma_{i,3-i}$, E_i is a constant to be defined later, $\theta_{i,3-i}$ is the phase of the link channel coefficient, and $n_{i,3-i}$ is the i.i.d. CSCG noise with distribution $\mathcal{CN}(0, 1)$. Without loss of generality, we assume wired connections for the conferencing links, such that $\theta_{i,3-i}$ is fixed. In fact, our following analysis is also valid for the time-varying conferencing link case as long as $\theta_{i,3-i}$ can be learned at the i-th relay. For the parameter E_i, the best choice is to set it equal to the power of the received signal y_i, i.e., $E_i = |h_{1i}|^2 p_1 + |h_{2i}|^2 p_2 + 1$. However, this will make the source power allocation problem intractable, since $f_{i,3-i}$ is a function of the source power values. To simplify the analysis, we adopt a normalizing method to make the worst-case scenario still satisfy the conferencing link SNR constraints: For the case with individual source power constraints, we let $E_i = |h_{1i}|^2 P_{S1} + |h_{2i}|^2 P_{S2} + 1$; for the case with a sum source power constraint, we let $E_i = \left(|h_{1i}|^2 + |h_{2i}|^2\right) P_S + 1$.

After receiving the signals from the sources and the other relay, each relay first linearly combines the two received signals as

$$t_i = c_{i,i} y_i + c_{3-i,i} r_{3-i,i}, \tag{5.3}$$

where the combining parameter $c_{i,i}$ and $c_{3-i,i}$ satisfy the following power constraints at the relays: With the individual relay power constraints under power budgets P_{R1} and P_{R2} respectively at the two relays, we have $|t_1|^2 \leq P_{R1}$ and $|t_2|^2 \leq P_{R2}$, i.e.,

$$\mathcal{C} = \left\{ \boldsymbol{c} : \ \boldsymbol{c}^H \boldsymbol{W}_{R1} \boldsymbol{c} \leq P_{R1}, \text{ and } \boldsymbol{c}^H \boldsymbol{W}_{R2} \boldsymbol{c} \leq P_{R2} \right\}, \tag{5.4}$$

where $\boldsymbol{c} = [c_{11}, c_{21}, c_{12}, c_{22}]^T$ and $\boldsymbol{W}_{Ri} = p_1 \boldsymbol{w}_{Ri1} \boldsymbol{w}_{Ri1}^H + p_2 \boldsymbol{w}_{Ri2} \boldsymbol{w}_{Ri2}^H + \boldsymbol{W}_{Rin}$, $i = 1, 2$. In particular, $\boldsymbol{w}_{Ri1}$, $\boldsymbol{w}_{Ri2}$, and $\boldsymbol{W}_{Rin}$ are contributed by source 1, source 2, and the noises at the relays, respectively, i.e., $\boldsymbol{w}_{R11} = [h_{11}, h_{12} f_{21}, 0, 0]^H$, $\boldsymbol{w}_{R12} = [h_{21}, h_{22} f_{21}, 0, 0]^H$, $\boldsymbol{W}_{R1n} = \mathrm{Diag}[1, |f_{21}|^2 + 1, 0, 0]$, $\boldsymbol{w}_{R21} = [0, 0, h_{11} f_{12}, h_{12}]^H$, $\boldsymbol{w}_{R22} = [0, 0, h_{21} f_{12}, h_{22}]^H$, and $\boldsymbol{W}_{R2n} = \mathrm{Diag}[0, 0, |f_{12}|^2 + 1, 1]$; with the sum relay power constraint under the total power budget P_R over the two relays, we have $|t_1|^2 + |t_2|^2 \leq P_R$, i.e.,

$$\mathcal{C} = \left\{ \boldsymbol{c} : \ \boldsymbol{c}^H \boldsymbol{W}_R \boldsymbol{c} \leq P_R \right\}, \tag{5.5}$$

where $\boldsymbol{W}_R = \boldsymbol{W}_{R1} + \boldsymbol{W}_{R2}$.

At the i-th destination, the received signal is given as

$$z_i = g_{1i}t_1 + g_{2i}t_2 + w_i, \tag{5.6}$$

where $g_{ji},\ j = 1,2$, is the complex channel gain from the j-th relay to the i-th destination, w_i is the i.i.d. CSCG noise with distribution $\mathcal{CN}(0,1)$. By (5.1), (5.2), (5.3), and (5.6), the overall input-output relationship of this network is given as

$$\begin{aligned} z_i = \boldsymbol{w}_{1,i}^H \boldsymbol{c} x_1 + \boldsymbol{w}_{2,i}^H \boldsymbol{c} x_2 + \left(g_{1,i}c_{11} + g_{2,i}f_{i,3-i}c_{12}\right) n_1 \\ + \left(g_{1,i}f_{i,3-i}c_{21} + g_{2,i}c_{22}\right) n_2 + g_{1,i}c_{21}n_{21} + g_{2,i}c_{12}n_{12} + w_i,\ i = 1,2, \end{aligned} \tag{5.7}$$

where with

$$\begin{aligned} \boldsymbol{w}_{1,i} &= [h_{11}g_{1,i}, h_{12}g_{1,i}f_{21}, h_{11}g_{2,i}f_{12}, h_{12}g_{2,i}]^H, \\ \boldsymbol{w}_{2,i} &= [h_{21}g_{1,i}, h_{22}g_{1,i}f_{21}, h_{21}g_{2,i}f_{12}, h_{22}g_{2,i}]^H. \end{aligned}$$

The power of the noise terms in (5.7) is $\boldsymbol{c}^H \boldsymbol{W}_{in} \boldsymbol{c} + 1$, where $\boldsymbol{W}_{in}$ is given as

$$\boldsymbol{W}_{in} = \begin{bmatrix} |g_{1i}|^2 & 0 & g_{1i}^* g_{2i} f_{12} & 0 \\ 0 & |g_{1i}|^2(f_{21}^2+1) & 0 & g_{1i}^* g_{2i} f_{21} \\ g_{1i} g_{2i}^* f_{12} & 0 & |g_{2i}|^2(f_{12}^2+1) & 0 \\ 0 & g_{1i} g_{2i}^* f_{21} & 0 & |g_{2i}|^2 \end{bmatrix}. \tag{5.8}$$

From (5.7), we know that the AF two-hop interference networks with relay conferencing is equivalent to an IC, but with possibly correlated noises.

Next, we define the rate region $C(\mathcal{P},\mathcal{C})$ of the considered two-hop interference network, subject to the source and relay power constraints, respectively, as

$$C(\mathcal{P},\mathcal{C}) \triangleq \bigcup_{(p_1,p_2)\in\mathcal{P},\ \boldsymbol{c}\in\mathcal{C}} \{(R_1,R_2)\}, \tag{5.9}$$

where (R_1, R_2) is the rate pair achieved with a certain coding and power allocation scheme.

In this chapter, we mainly focus on two special decoding schemes: single-user decoding, where each destination only decodes its desired message; and joint decoding, where each destination decodes both of the source messages. In the following two sections, we will discuss how to characterize the AF achievable rate regions for the single-user and joint decoding schemes, respectively. Note that due to the fundamental non-convexity of the joint source power and relay combining optimization problem, it is not guaranteed that all the Pareto boundary points [54] of the rate regions can be found. However, the proposed algorithms are still efficient and meaningful to provide some reasonable good achievable rate regions by fully exploring the hidden convexity of the decomposed subproblems.

B. Single-User Decoding

In this section, we assume that each destination tries to decode its own desired message, and treats the signal intended to the other destination as noise. With these assumptions, the achievable rate region is given as

$$C(\mathcal{P},\mathcal{C}) \triangleq \bigcup_{(p_1,p_2)\in\mathcal{P},\ \boldsymbol{c}\in\mathcal{C}} \left\{ (R_1,R_2) \;\middle|\; \begin{array}{l} R_1 \leq \frac{1}{2}\log\left(1+\frac{p_1\boldsymbol{c}^H\boldsymbol{W}_{11}\boldsymbol{c}}{p_2\boldsymbol{c}^H\boldsymbol{W}_{12}\boldsymbol{c}+\boldsymbol{c}^H\boldsymbol{W}_{1n}\boldsymbol{c}+1}\right) \\ R_2 \leq \frac{1}{2}\log\left(1+\frac{p_2\boldsymbol{c}^H\boldsymbol{W}_{22}\boldsymbol{c}}{p_1\boldsymbol{c}^H\boldsymbol{W}_{21}\boldsymbol{c}+\boldsymbol{c}^H\boldsymbol{W}_{2n}\boldsymbol{c}+1}\right) \end{array} \right\}, \tag{5.10}$$

where $\boldsymbol{W}_{ij} = \boldsymbol{w}_{ij}\boldsymbol{w}_{ij}^H,\ i,j \in \{1,2\}$.

Typically, the rate region can be characterized by the weighted-sum maximization method [54]. However, in our case, the weighted sum will involve coupled nonconvex functions, which is hard to solve. Instead, here we apply the rate profile method adopted in [54], and define the rate-profile vector as $\boldsymbol{\beta} = [\beta, 1-\beta]^T$, where $0 \leq \beta \leq 1$. Such a method could decouple the nonconvex weighted-sum objective function and transfer the nonconvex elements into individual constraints. Specifically, to characterize the whole rate region, we compute a sum rate maximization problem for all possible rate-profile vectors, i.e.,

$$\begin{aligned}
\max_{R_{\text{sum}}, p_1, p_2, \boldsymbol{c}} \quad & R_{\text{sum}} \\
\text{s. t.} \quad & \frac{1}{2}\log\left(1 + \frac{p_1 \boldsymbol{c}^H \boldsymbol{W}_{11} \boldsymbol{c}}{p_2 \boldsymbol{c}^H \boldsymbol{W}_{12} \boldsymbol{c} + \boldsymbol{c}^H \boldsymbol{W}_{1n} \boldsymbol{c} + 1}\right) \geq \beta R_{\text{sum}}, \\
& \frac{1}{2}\log\left(1 + \frac{p_2 \boldsymbol{c}^H \boldsymbol{W}_{22} \boldsymbol{c}}{p_1 \boldsymbol{c}^H \boldsymbol{W}_{21} \boldsymbol{c} + \boldsymbol{c}^H \boldsymbol{W}_{2n} \boldsymbol{c} + 1}\right) \geq (1-\beta) R_{\text{sum}}, \\
& (p_1, p_2) \in \mathcal{P}, \ \boldsymbol{c} \in \mathcal{C}.
\end{aligned} \tag{5.11}$$

Although this problem is still not convex, we observe that if we fix either (p_1, p_2) or $\boldsymbol{c}$, the remaining problem is efficiently solvable. Hence, we turn to a two-stage iterative method, and approximately solve Problem (5.11) via iterations between the following two sub-problems:

1. Fix (p_1, p_2), maximize R_{sum} over $\boldsymbol{c} \in \mathcal{C}$;

2. Fix $\boldsymbol{c}$, maximize R_{sum} over $(p_1, p_2) \in \mathcal{P}$.

Remark B.1 *Note that the above algorithm cannot guarantee to obtain all the boundary points of the rate region defined in (5.10), since it is difficult to find the global optimal solution of the non-convex Problem (5.11). However, it provides us an efficient way to obtain an meaningful achievable rate region. In addition, we here fix the optimization order to first optimize over* $\boldsymbol{c}$ *for a given* (p_1, p_2)*. Generally, we can start the optimization problem in an arbitrary order between these two sub-problems. The reason why we take such an order is that vector* $\boldsymbol{c}$ *is of a higher dimension than the source power pair* (p_1, p_2)*, such that it is relatively hard to find a feasible solution for* $\boldsymbol{c}$ *to initiate the algorithm.*

1. Optimization over $\boldsymbol{c}$ at the Relays

In this subsection, we consider the relay optimization problem with a fixed source power pair. Then, Problem (5.11) can be rewritten as

$$\begin{aligned} \max_{R_{\text{sum}}, \boldsymbol{c}} \quad & R_{\text{sum}} \\ \text{s. t.} \quad & \boldsymbol{c}^H \boldsymbol{F}_1 \boldsymbol{c} \geq 1,\ \boldsymbol{c}^H \boldsymbol{F}_2 \boldsymbol{c} \geq 1,\ \boldsymbol{c} \in \mathcal{C}, \end{aligned} \tag{5.12}$$

where $\boldsymbol{F}_1 = \frac{1}{\lambda_1} p_1 \boldsymbol{W}_{11} - p_2 \boldsymbol{W}_{12} - \boldsymbol{W}_{1n}$, $\boldsymbol{F}_2 = \frac{1}{\lambda_2} p_2 \boldsymbol{W}_{22} - p_1 \boldsymbol{W}_{21} - \boldsymbol{W}_{2n}$, $\lambda_1 = 2^{2\beta R_{\text{sum}}} - 1$, and $\lambda_2 = 2^{2(1-\beta) R_{\text{sum}}} - 1$. Furthermore, Problem (5.12) is equivalent to the following problem

$$\begin{aligned} \max_{R_{\text{sum}}, \boldsymbol{C}} \quad & R_{\text{sum}} \\ \text{s. t.} \quad & \operatorname{Tr}(\boldsymbol{F}_1 \boldsymbol{C}) \geq 1,\ \operatorname{Tr}(\boldsymbol{F}_2 \boldsymbol{C}) \geq 1,\ \boldsymbol{C} \in \mathcal{C}_0, \\ & \operatorname{Rank}(\boldsymbol{C}) = 1,\ \boldsymbol{C} \succeq 0, \end{aligned} \tag{5.13}$$

where $\boldsymbol{C} = \boldsymbol{c}\boldsymbol{c}^H$, and in particular, for the sum relay power constraint case, we have

$$\mathcal{C}_0 = \{\boldsymbol{C} : \operatorname{Tr}(\boldsymbol{W}_R \boldsymbol{C}) \leq P_R\}; \tag{5.14}$$

for the individual relay power constraint case, we have

$$\mathcal{C}_0 = \{\boldsymbol{C} : \operatorname{Tr}(\boldsymbol{W}_{R1} \boldsymbol{C}) \leq P_{R1},\ \operatorname{Tr}(\boldsymbol{W}_{R2} \boldsymbol{C}) \leq P_{R2}\}. \tag{5.15}$$

In general, due to the rank-1 constraint, Problem (5.13) is not convex. Next, we will show that this rate maximization problem can be exactly solved by a sequence of power minimization problems (as defined next) without considering the rank-1 constraint, which is the so-called semidefinite relaxation (SDR) approach [53].

1. *Sum Relay Power Constraint*: We consider the following semidefinite program

(SDP) power minimization problem for a given set of R_{sum} and $\boldsymbol{\beta}$,

$$\begin{aligned} \min_{\boldsymbol{C}} \quad & p_R = \operatorname{Tr}\left(\boldsymbol{W}_R \boldsymbol{C}\right) \\ \text{s. t.} \quad & \operatorname{Tr}\left(\boldsymbol{F}_1 \boldsymbol{C}\right) \geq 1,\ \operatorname{Tr}\left(\boldsymbol{F}_2 \boldsymbol{C}\right) \geq 1, \\ & \boldsymbol{C} \succeq 0. \end{aligned} \tag{5.16}$$

2. *Individual Relay Power Constraint*: Similarly to the previous case, we consider the following SDP power minimization problem.

$$\min_{\boldsymbol{C}} \quad p_{R1} = \operatorname{Tr}\left(\boldsymbol{W}_{R1} \boldsymbol{C}\right) \tag{5.17}$$

$$\text{s. t.} \quad \operatorname{Tr}\left(\boldsymbol{F}_1 \boldsymbol{C}\right) \geq 1,\ \operatorname{Tr}\left(\boldsymbol{F}_2 \boldsymbol{C}\right) \geq 1,\ \operatorname{Tr}\left(\boldsymbol{W}_{R2} \boldsymbol{C}\right) \leq P_{R2}, \tag{5.18}$$

$$\boldsymbol{C} \succeq 0. \tag{5.19}$$

First, we claim that if the rank-1 optimal solution of Problem (5.16) or (5.17)-(5.19) can be found, which will be probed later, the sum rate maximization problem in (5.12) can be efficiently solved via the following bi-section search algorithm up to an accuracy requirement ϵ:

<u>Algorithm 1</u>:

- Initialize $r_{\text{low}} = 0$ and $r_{\text{up}} = r_{\max}$;
- Repeat

  1. Set $\frac{1}{2}\left(r_{\text{low}} + r_{\text{up}}\right) \rightarrow r$;
  2. Solve Problem (5.16) with the given $R_{\text{sum}} = r$ and obtain the optimal point p_R^*; similarly, solve Problem (5.17)-(5.19) with the given $R_{\text{sum}} = r$ and obtain the optimal point p_{R1}^*.
  3. Update r with the bi-section search: For the sum relay power constraint

case, if $p_R^* \leq P_R$, set $r_{\text{low}} = r$; otherwise, set $r_{\text{up}} = r$. For the individual relay power constraint case, if $p_{R1}^* \leq P_{R1}$, set $r_{\text{low}} = r$; otherwise, set $r_{\text{up}} = r$.

- Until $r_{\text{up}} - r_{\text{low}} < \epsilon$.

Here, the upper limit $r_{\max}$ of the searching range can be determined by the cut-set upper bound shown in Section VI.

For the sum relay power constraint case, as shown in [54], if the rank of the optimal solution of Problem (5.16) is larger than 1, an equivalently optimal rank-1 solution can be efficiently constructed via finding a matrix decomposition as in [55] and solving a linear programming problem as in Appendix E of [54].

For the individual relay power constraint case, we have the following theorem to guarantee that the rank-1 solution of the power minimization Problem (5.17)-(5.19) can be efficiently constructed. [1] As shown in Remark G.1, the proposed algorithm for the construction of rank-1 solution is much more efficient than that given in [56].

Theorem B.1 *Assuming that an optimal solution $\boldsymbol{C}^*$ with rank $r > 1$ is found for Problem (5.17)-(5.19), a rank-1 solution $\boldsymbol{C}^{**}$ can be constructed based on $\boldsymbol{C}^*$ efficiently.*

Proof: See Appendix 1. ■

In conclusion, for both the sum and individual relay power constraint cases, Problem (5.13) can be efficiently solved.

[1]Note that the existence conditions of rank-1 solutions for general quadratic SDP relaxation problems are nicely summarized in [56]. Our result here can be considered as a special case, with a differently tailored rank-1 construction routine.

2. Optimizing (p_1, p_2) at the Sources

In this subsection, we fix $\boldsymbol{c}$ and optimize over p_1 and p_2. Problem (5.11) is rewritten as

$$\max_{R_{\text{sum}}, p_1, p_2} \quad R_{\text{sum}} \tag{5.20}$$

$$\text{s. t.} \quad p_1 \boldsymbol{c}^H \boldsymbol{W}_{11} \boldsymbol{c} - p_2 \lambda_1 \boldsymbol{c}^H \boldsymbol{W}_{12} \boldsymbol{c} \geq \lambda_1 \left(\boldsymbol{c}^H \boldsymbol{W}_{1n} \boldsymbol{c} + 1 \right), \tag{5.21}$$

$$p_2 \boldsymbol{c}^H \boldsymbol{W}_{22} \boldsymbol{c} - p_1 \lambda_2 \boldsymbol{c}^H \boldsymbol{W}_{21} \boldsymbol{c} \geq \lambda_2 \left(\boldsymbol{c}^H \boldsymbol{W}_{2n} \boldsymbol{c} + 1 \right), \tag{5.22}$$

$$(p_1, p_2) \in \mathcal{P}. \tag{5.23}$$

where λ_1 and λ_2 are defined the same as before. It is easy to check that this problem is not convex, due to the non-convex constraints (5.21) and (5.22). However, by fixing R_{sum} as a constant, consider the following feasibility problem:

$$\text{find}_{\{p_1, p_2\}} \quad 0 \tag{5.24}$$

$$\text{s. t.} \quad p_1 \boldsymbol{c}^H \boldsymbol{W}_{11} \boldsymbol{c} - p_2 \lambda_1 \boldsymbol{c}^H \boldsymbol{W}_{12} \boldsymbol{c} \geq \lambda_1 \left(\boldsymbol{c}^H \boldsymbol{W}_{1n} \boldsymbol{c} + 1 \right), \tag{5.25}$$

$$p_2 \boldsymbol{c}^H \boldsymbol{W}_{22} \boldsymbol{c} - p_1 \lambda_2 \boldsymbol{c}^H \boldsymbol{W}_{21} \boldsymbol{c} \geq \lambda_2 \left(\boldsymbol{c}^H \boldsymbol{W}_{2n} \boldsymbol{c} + 1 \right), \tag{5.26}$$

$$(p_1, p_2) \in \mathcal{P}. \tag{5.27}$$

Thus, Problem (5.20)-(5.23) can be efficiently solved via a bisection search method over R_{sum}, since for a given R_{sum} at each searching step, Problem (5.24)-(5.27) is convex over (p_1, p_2).

For Problem (5.24)-(5.27), we have the following result to efficiently check its feasibility. First, consider the two hyperplanes defined as follows:

$$\begin{cases} p_1 \boldsymbol{c}^H \boldsymbol{W}_{11} \boldsymbol{c} - p_2 \lambda_1 \boldsymbol{c}^H \boldsymbol{W}_{12} \boldsymbol{c} = \lambda_1 \left(\boldsymbol{c}^H \boldsymbol{W}_{1n} \boldsymbol{c} + 1 \right) \\ -p_1 \lambda_2 \boldsymbol{c}^H \boldsymbol{W}_{21} \boldsymbol{c} + p_2 \boldsymbol{c}^H \boldsymbol{W}_{22} \boldsymbol{c} = \lambda_2 \left(\boldsymbol{c}^H \boldsymbol{W}_{2n} \boldsymbol{c} + 1 \right) \end{cases}, \quad p_1, p_2 \in \mathcal{R}. \tag{5.28}$$

For each of the hyperplanes, the set of (p_1, p_2) actually forms a straight line. It is observed that there may be one intersection point $(\widetilde{p}_1, \widetilde{p}_2)$ of these two hyperplanes, where $\widetilde{p}_i$ is given as

$$\widetilde{p}_1 = \frac{\begin{vmatrix} \lambda_1\left(\boldsymbol{c}^H \boldsymbol{W}_{1n}\boldsymbol{c}+1\right) & -\lambda_1 \boldsymbol{c}^H \boldsymbol{W}_{12}\boldsymbol{c} \\ \lambda_2\left(\boldsymbol{c}^H \boldsymbol{W}_{2n}\boldsymbol{c}+1\right) & \boldsymbol{c}^H \boldsymbol{W}_{22}\boldsymbol{c} \end{vmatrix}}{\begin{vmatrix} \boldsymbol{c}^H \boldsymbol{W}_{11}\boldsymbol{c} & -\lambda_1 \boldsymbol{c}^H \boldsymbol{W}_{12}\boldsymbol{c} \\ -\lambda_2 \boldsymbol{c}^H \boldsymbol{W}_{21}\boldsymbol{c} & \boldsymbol{c}^H \boldsymbol{W}_{22}\boldsymbol{c} \end{vmatrix}}, \ \widetilde{p}_2 = \frac{\begin{vmatrix} \boldsymbol{c}^H \boldsymbol{W}_{11}\boldsymbol{c} & \lambda_1\left(\boldsymbol{c}^H \boldsymbol{W}_{1n}\boldsymbol{c}+1\right) \\ -\lambda_2 \boldsymbol{c}^H \boldsymbol{W}_{21}\boldsymbol{c} & \lambda_2\left(\boldsymbol{c}^H \boldsymbol{W}_{2n}\boldsymbol{c}+1\right) \end{vmatrix}}{\begin{vmatrix} \boldsymbol{c}^H \boldsymbol{W}_{11}\boldsymbol{c} & -\lambda_1 \boldsymbol{c}^H \boldsymbol{W}_{12}\boldsymbol{c} \\ -\lambda_2 \boldsymbol{c}^H \boldsymbol{W}_{21}\boldsymbol{c} & \boldsymbol{c}^H \boldsymbol{W}_{22}\boldsymbol{c} \end{vmatrix}}. \tag{5.29}$$

Then, we have the following result for the feasibility of Problem (5.24)-(5.27).

Proposition B.1 *Problem (5.24)-(5.27) is feasible if and only if* $(\widetilde{p}_1, \widetilde{p}_2)$ *defined in (5.29) exists and satisfies the source power constraint in (5.27).*

Proof: The "if" part is obvious, and thus its proof is omitted. For the "only if" part, since $\boldsymbol{c}^H \boldsymbol{W}_{i,j}\boldsymbol{c} \geq 0$, $i, j = 1, 2$, it is easy to check that only when $(\widetilde{p}_1, \widetilde{p}_2)$ exists and $\widetilde{p}_i \geq 0$, $i = 1, 2$, Problem (5.24)-(5.27) may be feasible, as shown in Fig. 20. Then, we only need to prove that if Problem (5.24)-(5.27) is feasible, $(\widetilde{p}_1, \widetilde{p}_2)$ always satisfies the relay power constraint (5.27). To see this point, it is noticed that the slopes of these two lines defined in (5.28) are both non-negative (also due to the fact that $\boldsymbol{c}^H \boldsymbol{W}_{i,j}\boldsymbol{c} \geq 0$, and see Fig. 20), such that any feasible point $(\widehat{p}_1, \widehat{p}_2)$ defined by constraint (5.25)-(5.26) must satisfy $\widetilde{p}_i \leq \widehat{p}_i$, $i = 1, 2$. With the individual source power constraint, if $(\widehat{p}_1, \widehat{p}_2)$ is feasible, it is easy to check that $(\widetilde{p}_1, \widetilde{p}_2)$ is also feasible; with the sum source power constraint, if $(\widehat{p}_1, \widehat{p}_2)$ is feasible, we conclude that $(\widetilde{p}_1, \widetilde{p}_2)$ is also feasible since $\widetilde{p}_1 + \widetilde{p}_2 \leq \widehat{p}_1 + \widehat{p}_2 \leq P_S$. In other words, if Problem (5.24)-(5.27) is feasible, $(\widetilde{p}_1, \widetilde{p}_2)$ is always a feasible point of Problem (5.24)-(5.27). Therefore, this proposition is proved. ■

Based on this proposition, the feasibility of Problem (5.24)-(5.27) can be checked

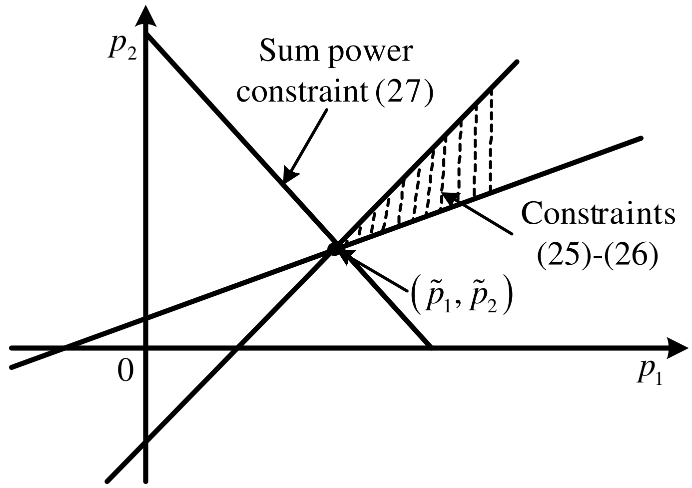


Fig. 20.: One example of the feasible set of Problem (5.24)-(5.27) with sum source power constraint.

as follows. First, compute $(\widetilde{p}_1, \widetilde{p}_2)$. We then check whether $(\widetilde{p}_1, \widetilde{p}_2)$ satisfies constraint (5.27) or not: If it is, Problem (5.24)-(5.27) is feasible; if not, Problem (5.24)-(5.27) is not. As such, we obtain the following algorithm to solve Problem (5.20)-(5.23).

Algorithm 2:

- Initialize $r_{\text{low}} = 0$ and $r_{\text{up}} = r_{\max}$;
- Repeat
  1. Set $\frac{1}{2}(r_{\text{low}} + r_{\text{up}}) \to r$;
  2. Set $R_{\text{sum}} = r$, and obtain $(\widetilde{p}_1, \widetilde{p}_2)$ by using (5.29);
  3. Update r with the bi-section search: If $(\widetilde{p}_1, \widetilde{p}_2)$ satisfies constraint (5.27), set $r_{\text{low}} = r$; otherwise, set $r_{\text{up}} = r$.
- Until $r_{\text{up}} - r_{\text{low}} < \epsilon$.

Here, $r_{\max}$ can also be chosen as the cut-set bound given in Section VI.

3. Iterative Algorithm

Based on the above analysis, we summarize the iterative algorithm for Problem (5.11) for a particular rate profile vector $\boldsymbol{\beta}$ as:

Algorithm 3:

- Set the initial values of (p_1, p_2);
- Repeat
  1. Solve Problem (5.13) by using Algorithm 1, and obtain a rank-1 solution $\boldsymbol{c}_i^*$ by using Appendix 1;
  2. Set $\boldsymbol{c} = \boldsymbol{c}_i^*$, and solve Problem (5.20)-(5.23) by using Algorithm 2. Denote (p_{1i}^*, p_{2i}^*) as the optimal point obtained in this stage, and set this pair as the initial value for the next round of iteration;
- Until an accuracy requirement is met.

The initial values of (p_1, p_2) are chosen based on the following observations: For the sum source power constraint case, it is a necessary condition for the optimal (p_1^*, p_2^*) that the equality of the sum source power constraint is achieved; otherwise, we can scale the pair simultaneously with a factor $\frac{P_S}{p_1+p_2}$, and then the new pair will give a rate pair larger than the previous one. As such, we choose the initial values as $p_1 = \alpha P_S$ and $p_2 = (1-\alpha)P_S$, where $\alpha \in [0, 1]$. For the individual source power constraint case, it is necessary at the optimal point that at least one of source powers should be equal to its maximum value; otherwise, with a similar argument of scaling, a higher rate can be achieved. Thus, we choose $p_1 = P_{S1}$ and $p_2 = \alpha P_{S2}$, or $p_1 = \alpha P_{S1}$ and $p_2 = P_{S2}$.

With the above two-stage iteration scheme for the source power allocation and relay combining problem as defined in (5.11), we have the following proposition addressing the convergence issue.

Proposition B.2 *The optimal value given by using Algorithm 3 is convergent for arbitrary initial values and channel realizations under both individual and sum power constraints at the sources and relays.*

Proof: Denote the sum rate in the i-th iteration of Algorithm 3 after the first and the second stages as $R_{\text{sum}}^{(1i)}$ and $R_{\text{sum}}^{(2i)}$, respectively. Since the optimal values of each stage satisfy $R_{\text{sum}}^{(11)} \leq R_{\text{sum}}^{(12)} \leq R_{\text{sum}}^{(21)} \leq R_{\text{sum}}^{(22)} \leq \cdots$, where $R_{\text{sum}}^{(ij)}$ is an upper-bounded sequence (e.g., bounded by the cut-set upper bound given in Section E), this proposition is proved. ■

It is worth noting that although the optimal value of Problem (5.11) obtained by using Algorithm 3 is convergent, its associated optimal point may not converge. This is due to the fact that during each iteration, the optimal points of Problem (5.16) and Problem (5.17)-(5.19) may not be unique. Moreover, even when the optimal point obtained by using Algorithm 3 converges, it may not be globally optimal, due to the non-convexity of the overall problem in (5.11). As such, Algorithm 3 may not obtain all the boundary points of the AF rate region defined in (5.10); however, it still provides a meaningful inner bound for the capacity region of the considered two-hop interference network.

Remark B.2 *By setting $c_{12} = c_{21} = 0$, Problem (5.11) degrades to the traditional two-hop AF interference networks without conferencing [34], where the proposed algorithms are still valid and also provide a relatively simple way to compute a suboptimal solution. As far as we know, there are no efficient algorithms to compute the global optimal solution for such a problem, except for the exhaustive search method. Thus, we may conclude that our AF scheme with the help of relay conferencing is more* *general than the traditional AF scheme, and the newly derived rate region is equal to or larger than that of the traditional case, if the same two-stage iteration algorithm is used. This is also true for the joint decoding scheme discussed in the next section.*

C. Two-User Joint Decoding

With the joint decoding scheme, each destination tries to decode both of the source messages independently. Thus, the overall channel given in (5.7) is indeed the compound MAC [21,22]. Accordingly, for each receiver, the rate pair is within the capacity region of a MAC, and the rate region for the two-hop interference network is given as the intersection of capacity regions of the two MACs, which is given as

$$\begin{cases} R_1 \leq \frac{1}{2}\log\left(1+\frac{p_1\boldsymbol{c}^H\boldsymbol{W}_{i1}\boldsymbol{c}}{\boldsymbol{c}^H\boldsymbol{W}_{in}\boldsymbol{c}+1}\right) \\ R_2 \leq \frac{1}{2}\log\left(1+\frac{p_2\boldsymbol{c}^H\boldsymbol{W}_{i2}\boldsymbol{c}}{\boldsymbol{c}^H\boldsymbol{W}_{in}\boldsymbol{c}+1}\right) \\ R_1+R_2 \leq \frac{1}{2}\log\left(1+\frac{p_1\boldsymbol{c}^H\boldsymbol{W}_{i1}\boldsymbol{c}+p_2\boldsymbol{c}^H\boldsymbol{W}_{i2}\boldsymbol{c}}{\boldsymbol{c}^H\boldsymbol{W}_{in}\boldsymbol{c}+1}\right) \end{cases}, \quad i=1,2, \tag{5.30}$$

where $\boldsymbol{W}_{ij}$'s and $\boldsymbol{W}_{in}$'s are defined the same as in the previous section. Similarly, using the rate profile method, we have the following sum rate maximization problem to characterize the rate region:

$$\begin{aligned} \max_{R_{\text{sum}},\boldsymbol{c},p_1,p_2} \quad & R_{\text{sum}} \qquad (5.31) \\ \text{s. t.} \quad & p_1\boldsymbol{c}^H\boldsymbol{W}_{i1}\boldsymbol{c} \geq \lambda_1\left(\boldsymbol{c}^H\boldsymbol{W}_{in}\boldsymbol{c}+1\right), \\ & p_2\boldsymbol{c}^H\boldsymbol{W}_{i2}\boldsymbol{c} \geq \lambda_2\left(\boldsymbol{c}^H\boldsymbol{W}_{in}\boldsymbol{c}+1\right), \\ & p_1\boldsymbol{c}^H\boldsymbol{W}_{i1}\boldsymbol{c}+p_2\boldsymbol{c}^H\boldsymbol{W}_{i2}\boldsymbol{c} \geq \lambda_0\left(\boldsymbol{c}^H\boldsymbol{W}_{in}\boldsymbol{c}+1\right), \\ & (p_1,p_2)\in\mathcal{P},\ \boldsymbol{c}\in\mathcal{C},\ i=1,2, \end{aligned}$$

where $\lambda_0 = 2^{2R_{\text{sum}}}-1$, λ_1 and λ_2 are defined the same as in the previous section. Since Problem (5.31) is non-convex, we adopt the same two-stage iterative method as

Algorithm 3. Specifically, we first fix (p_1, p_2) and optimize over $\boldsymbol{c}$; but unfortunately, this problem is not convex. Equivalently, we rewrite this problem as

$$\begin{aligned} \max_{R_{\text{sum}}, \boldsymbol{C}} \quad & R_{\text{sum}} \\ \text{s. t.} \quad & \text{Tr}\left(\left(p_1 \boldsymbol{W}_{i1} - \lambda_1 \boldsymbol{W}_{in}\right) \boldsymbol{C}\right) \geq \lambda_1, \\ & \text{Tr}\left(\left(p_2 \boldsymbol{W}_{i2} - \lambda_2 \boldsymbol{W}_{in}\right) \boldsymbol{C}\right) \geq \lambda_2, \\ & \text{Tr}\left(\left(p_1 \boldsymbol{W}_{i1} + p_2 \boldsymbol{W}_{i2} - \lambda_0 \boldsymbol{W}_{in}\right) \boldsymbol{C}\right) \geq \lambda_0, \\ & \text{Rank}\left(\boldsymbol{C}\right) = 1,\ \boldsymbol{C} \succeq 0,\ \boldsymbol{C} \in \mathcal{C}_0,\ i = 1, 2, \end{aligned} \tag{5.32}$$

where $\mathcal{C}_0$ is defined the same as the previous section. By using the SDR approach [53], we solve the following problem to obtain an approximate solution by ignoring the rank-1 constraint:

$$\begin{aligned} \max_{R_{\text{sum}}, \boldsymbol{C}} \quad & R_{\text{sum}} \\ \text{s. t.} \quad & \text{Tr}\left(\left(p_1 \boldsymbol{W}_{i1} - \lambda_1 \boldsymbol{W}_{in}\right) \boldsymbol{C}\right) \geq \lambda_1, \\ & \text{Tr}\left(\left(p_2 \boldsymbol{W}_{i2} - \lambda_2 \boldsymbol{W}_{in}\right) \boldsymbol{C}\right) \geq \lambda_2, \\ & \text{Tr}\left(\left(p_1 \boldsymbol{W}_{i1} + p_2 \boldsymbol{W}_{i2} - \lambda_0 \boldsymbol{W}_{in}\right) \boldsymbol{C}\right) \geq \lambda_0, \\ & \boldsymbol{C} \succeq 0,\ \boldsymbol{C} \in \mathcal{C}_0,\ i = 1, 2. \end{aligned} \tag{5.33}$$

This problem can be efficiently solved via the bi-section search over R_{sum} (similar to Algorithm 2), since for each given R_{sum}, the resulting feasibility problem is a convex SDP problem. Generally, the optimal point $\boldsymbol{C}^*$ of Problem (5.33) is not of rank-1, and the rank-1 reconstruction method in Section III cannot be applied here. Instead, we turn to the following approximate randomization method [57]: Calculate the eigen-decomposition for $\boldsymbol{C}^*$ as $\boldsymbol{C}^* = \boldsymbol{U}\boldsymbol{D}\boldsymbol{U}^H$, and choose $\boldsymbol{c}_l = c_l \boldsymbol{U}\boldsymbol{D}^{1/2}\boldsymbol{v}_l$, where the elements of $\boldsymbol{v}_l$ are independent complex random variables with zero-mean and unit-variance and c_l is a constant to make $\boldsymbol{c}_l$ satisfy the equality of the power constraints in Problem (5.32). In the simulations, we generate a set of $\boldsymbol{c}_l$, $l = 1, \cdots, L$, and choose the one which maximizes R_{sum} as the optimal point $\boldsymbol{c}^*$ for Problem (5.31).

Then, we fix $\boldsymbol{c} = \boldsymbol{c}^*$ and optimize over (p_1, p_2). Note that for the individual source power constraint case, each source should transmit with its maximum power. With the sum source power constraint, the source power allocation problem is then given as

$$\begin{aligned} \max_{R_{\text{sum}}, p_1, p_2} \quad & R_{\text{sum}} \\ \text{s. t.} \quad & p_1 \boldsymbol{c}^H \boldsymbol{W}_{i1} \boldsymbol{c} \geq \lambda_1 \left(\boldsymbol{c}^H \boldsymbol{W}_{in} \boldsymbol{c} + 1 \right), \\ & p_2 \boldsymbol{c}^H \boldsymbol{W}_{i2} \boldsymbol{c} \geq \lambda_2 \left(\boldsymbol{c}^H \boldsymbol{W}_{in} \boldsymbol{c} + 1 \right), \\ & p_1 \boldsymbol{c}^H \boldsymbol{W}_{i1} \boldsymbol{c} + p_2 \boldsymbol{c}^H \boldsymbol{W}_{i2} \boldsymbol{c} \geq \lambda_0 \left(\boldsymbol{c}^H \boldsymbol{W}_{in} \boldsymbol{c} + 1 \right), \\ & (p_1, p_2) \in \mathcal{P}, \;\; i = 1, 2. \end{aligned} \tag{5.34}$$

This problem is a convex problem over $(R_{\text{sum}}, p_1, p_2)$, which can be efficiently solved by some optimization tools, e.g., CVX [47].

As the above two steps iterate, since we cannot exactly solve Problem (5.32), the approximate solution of Problem (5.32) may not strictly improve the sum rate compared to the previous iteration. Hence, when we iteratively compute Problem (5.33) and Problem (5.34), the sum rate sequence may not be convergent. To minimize this deficiency, we take the following two measures: First, we choose a relatively large L for the randomization method used in rank-1 construction for Problem (5.33); second, we terminate the iteration mannually, when the iteration cannot improve the sum rate anymore. Accordingly, we summarize the algorithm as follows.

Algorithm 4:

- Set the initial values of (p_1, p_2);

- Repeat

  1. Solve Problem (5.33) via the bi-section search method over R_{sum}, and use the randomization method to construct a rank-1 solution $\boldsymbol{c}_i^*$;

  2. Set $\boldsymbol{c} = \boldsymbol{c}_i^*$, and solve Problem (5.34). Denote R_{sum}^* and (p_{1i}^*, p_{2i}^*) as the optimal sum rate and source power pair obtained in this stage, respectively. Set (p_{1i}^*, p_{2i}^*) as the initial value for the next round of iteration;

- Until the maximum iteration number is reached, or R_{sum}^* is less than or equal to that in the previous iteration.

The initial values of (p_1, p_2) can be set the same as those in the previous section for the sum source power constraint case. Note that Algorithm 4 is used only for the sum source power constraint case; for the individual source power constraint case, we only need to set $p_1 = P_{S1}$ and $p_2 = P_{S2}$, and directly solve Problem (5.33).

D. Asymptotic Behaviors of the Two Decoding Schemes

In the above two sections, we presented two different decoding schemes. In this section, we study the performance of these schemes in both the high and low SNR regimes.

1. High SNR Regime

In this subsection, we assume that both the source and relay powers go to infinity.

Proposition D.1 *When the source and relay powers scale with the same order, i.e., $a_1 P_S \leq P_R \leq a_2 P_S$, where a_1 and a_2 are positive constants, the two-hop interference network with relay conferencing achieves DoF of one by using the single-user decoding* *scheme if*

$$\boldsymbol{w}_{11}, \boldsymbol{w}_{22} \notin \mathcal{O}\left(\boldsymbol{w}_{12}, \boldsymbol{w}_{21}\right), \tag{5.35}$$

where $\mathcal{O}\left(\boldsymbol{w}_{12}, \boldsymbol{w}_{21}\right)$ *is the subspace spanned by* $\boldsymbol{w}_{12}$ *and* $\boldsymbol{w}_{21}$*; it achieves DoF of* $\frac{1}{2}$ *by the joint decoding scheme with arbitrary channel realizations.*

Proof: See Appendix 2. ■

Note that condition (5.35) is usually satisfied in practice. For example, if these channel coefficients are i.i.d., by the theory of random matrix, we know that condition (5.35) is satisfied almost surely. Moreover, from the proof, we know that in the high SNR regime, the relay combining vector can be chosen to be orthogonal to the "interference space", which is indeed the interference cancelation scheme [40].

Remark D.1 *Considering the cut-set bound [36] and the half-duplex relaying constraint, the maximum DoF value of the two-hop interference networks is one. Thus, Proposition D.1 indeed proves that in the high SNR regime, single-user decoding with interference cancelation at the relay is asymptotically optimal for the AF relaying scheme in the sense of achieving the maximum DoF. As shown in [36], for the case without relay conferencing, only* $\frac{3}{4}$ *DoF is achieved (under the half-duplex constraint) with the AF scheme and constant channel coefficients. In conclusion, it is shown that relay conferencing closes the gap between the DoF achieved by the AF relaying scheme and the DoF upper bound in high SNR regime.*

2. Low SNR Case

In this subsection, we consider the case that the source and relay powers both go to zero.

Proposition D.2 *The achievable rate regions obtained by using the single-user and* *joint decoding schemes are asymptotically the same when the source and relay transmission powers both go to zero.*

Proof: First, we have $\log(1+x) \to \frac{x}{\ln 2}$ and $1+x \to 1$, as $x \to 0$. Using these results, it is easy to observe that both the rate regions given by (5.10) and (5.30) are asymptotically equal to

$$\begin{cases} R_1 \leq \frac{1}{2\ln 2} p_1 \boldsymbol{c}^H \boldsymbol{W}_{11} \boldsymbol{c} \\ R_2 \leq \frac{1}{2\ln 2} p_2 \boldsymbol{c}^H \boldsymbol{W}_{22} \boldsymbol{c} \end{cases}. \tag{5.36}$$

Thus, this proposition is proved. ■

E. Simulation Results

In this section, we present some simulation results to validate our analysis about the achievable rate regions for both the two decoding schemes. For convenience, we assume that for the conferencing links, $\theta_{i,3-i} = 0,\ i = 1, 2$; for other links, h_{ii} and g_{ii}, $i = 1, 2$, are i.i.d. CSCG with distribution $\mathcal{CN}(0, 1)$, and $h_{i,3-i}$ and $g_{i,3-i}$ are also i.i.d. CSCG with distributions $\mathcal{CN}(0, a)$ and $\mathcal{CN}(0, b)$, respectively, where a and b are parameters reflecting the power of the cross-link interference at each hop.

In the simulations, we only consider two typical scenarios: 1) The first hop is a weak IC, and the second hop is a strong interference channel, i.e., $a < 1$ and $b > 1$, respectively; and 2) both of the two hops are weak ICs, i.e., $a, b < 1$. Specifically, the two scenarios are set up as:

1. Scenario I: $a = 0.1$; $b = 10$;

2. Scenario II: $a = 0.1$; $b = 0.1$.

Moreover, we only present the results for the sum source/relay power constraints, and the cases with individual power constraints are omitted for conciseness. All the simulations are based on the average over 1000 channel realizations and computed with the core optimization tool CVX [47].

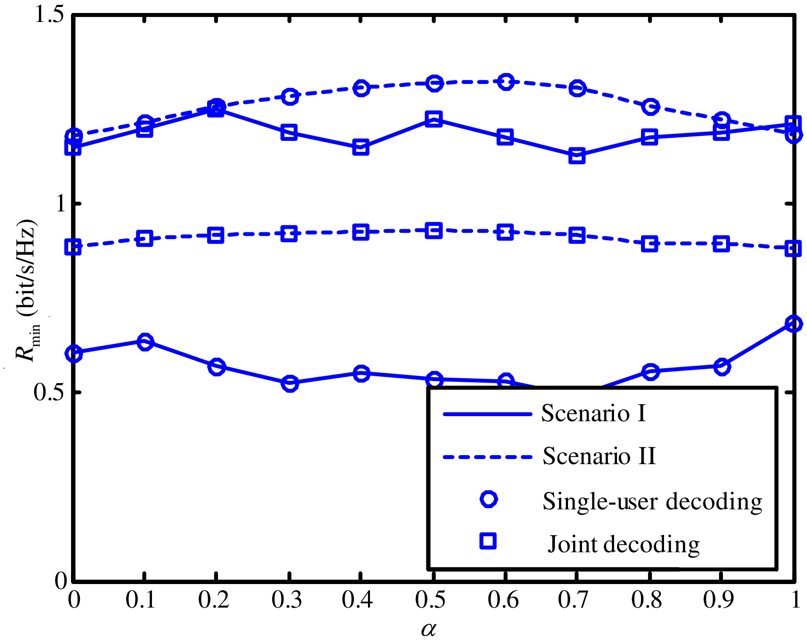
First, we study the effect of initial values of the source power pair (p_1, p_2), where we pick a particular boundary point that gives the maximum $R_{\min}$, where $R_{\min} = \min(R_1, R_2)$ and it is computed by setting the rate profile vector $\boldsymbol{\beta} = [0.5, 0.5]$. As shown in Fig. 21(a) and Fig. 21(b), we plot $R_{\min}$ over different initial power allocation factors $\alpha = \frac{p_1}{P_S}$ for both scenarios I and II with different SNR values. For the conferencing links, we set $\gamma_{12} = \gamma_{21} = \gamma_c = 0$ dB, and the maximum number of iterations for both algorithms as 5. It is observed that for the high SNR case, $R_{\min}$ is not sensitive to the initial values, except for the case when α is close to 0; for the low SNR case, the initial value of α plays a relatively important role compared to the high SNR case, and the peak $R_{\min}$ is usually achieved for $\alpha \in [0.4, 0.6]$. Moreover, it is also observed that for both high and low SNR cases, single-user decoding performs better than joint decoding in Scenario II, and vice versa for Scenario I. However, this may not be true for the case with general SNR values.

Next, we compare the AF achievable rate region with the capacity upper bound. The upper bound used in this chapter is derived by the MAC and BC cut-set bounds [28] and is given by

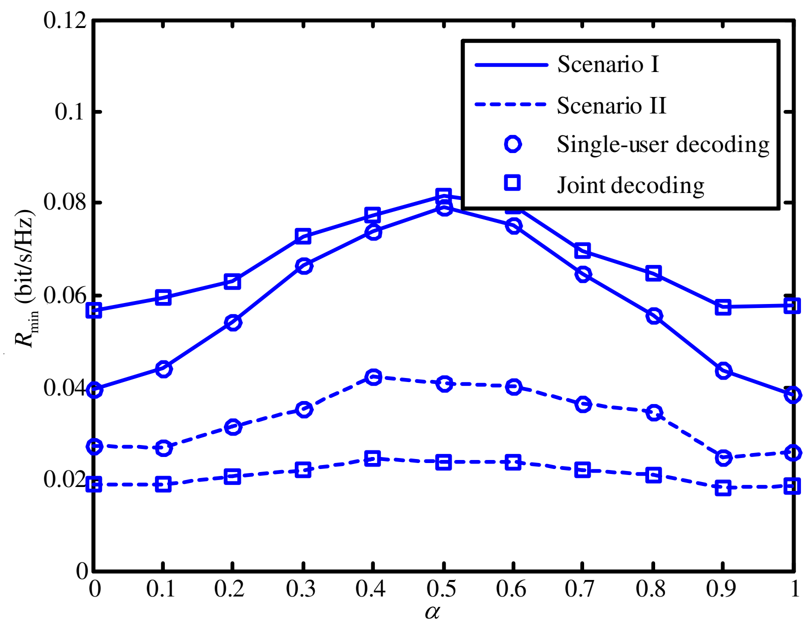
$$C_{\text{upper}}(R_1, R_2) = C_{\text{MAC}}(R_1, R_2) \bigcap C_{\text{BC}}(R_1, R_2), \tag{5.37}$$

where $C_{\text{MAC}}(R_1, R_2)$ is the MAC channel rate region [28] with a sum source power constraint, and $C_{\text{BC}}(R_1, R_2)$ is the BC channel rate region, which can be characterized by the MAC-BC duality [28, 58]. The cut-set bound corresponds to the case where the conferencing link SNR goes to infinity, for which these two relays can be regarded as two co-located antennas.

In Fig. 22 and Fig. 23, we plot the AF achievable rates without time sharing

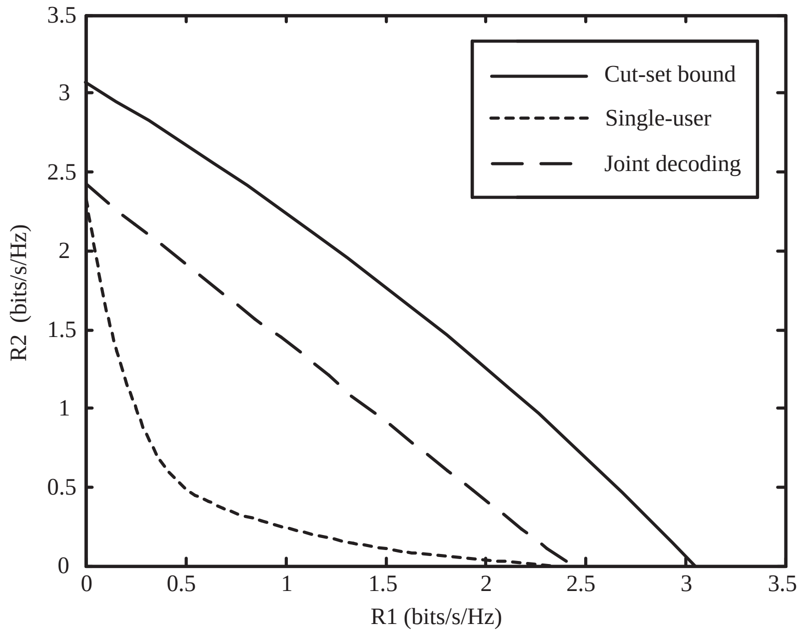


(a) High SNR case: $P_S = P_R = 20$ dB;

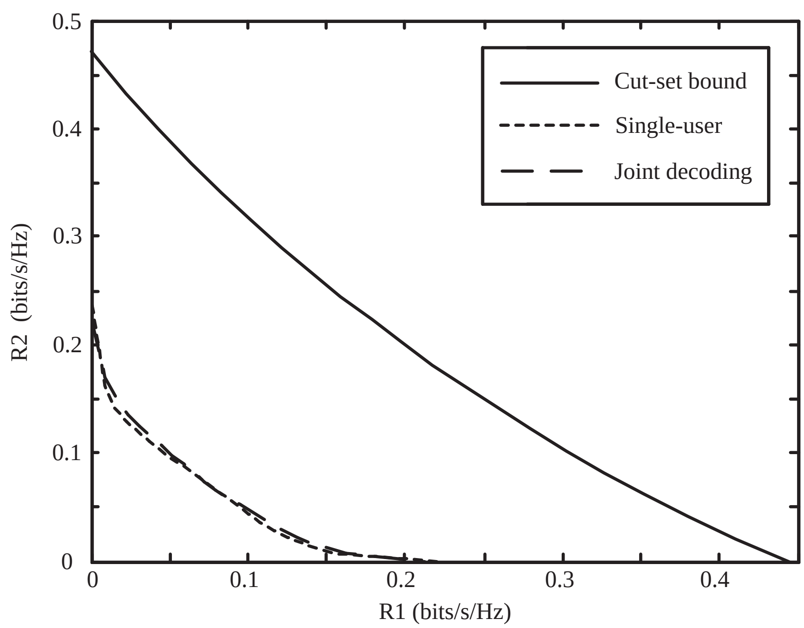


(b) Low SNR case: $P_S = P_R = 0$ dB.

Fig. 21.: The effect of initial value on the maximum minimum rate $R_{\min}$, $\gamma_c = 0$ dB, $\boldsymbol{\beta} = [0.5, 0.5]$.

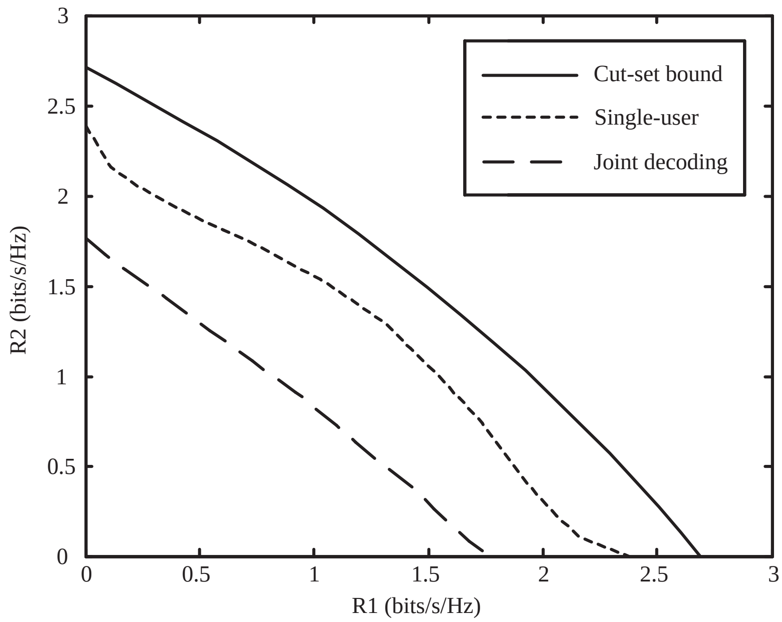


(a) High SNR case: $P_S = P_R = 20$ dB;

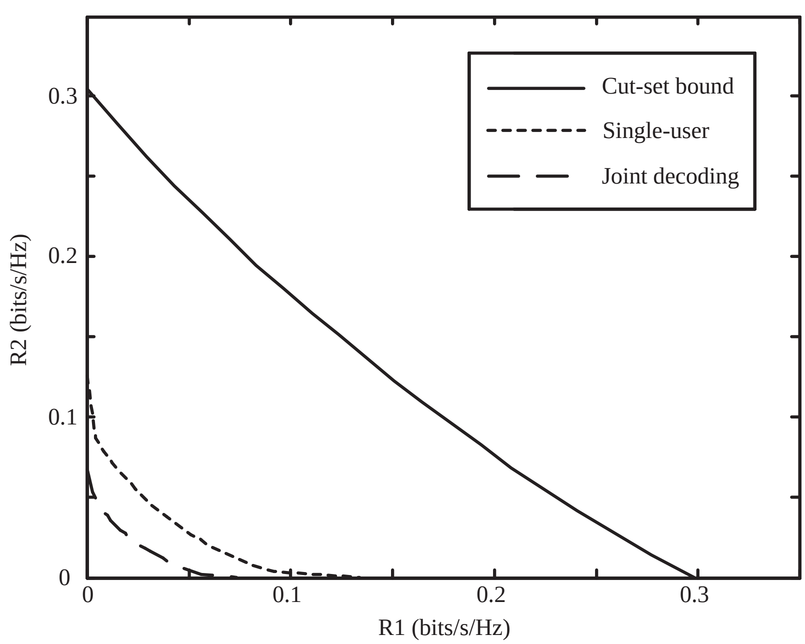


(b) Low SNR case: $P_S = P_R = 0$ dB.

Fig. 22.: The achievable rate region for Scenario I, $\gamma_c = 0$ dB.



(a) High SNR case: $P_S = P_R = 20$ dB;



(b) Low SNR case: $P_S = P_R = 0$ dB.

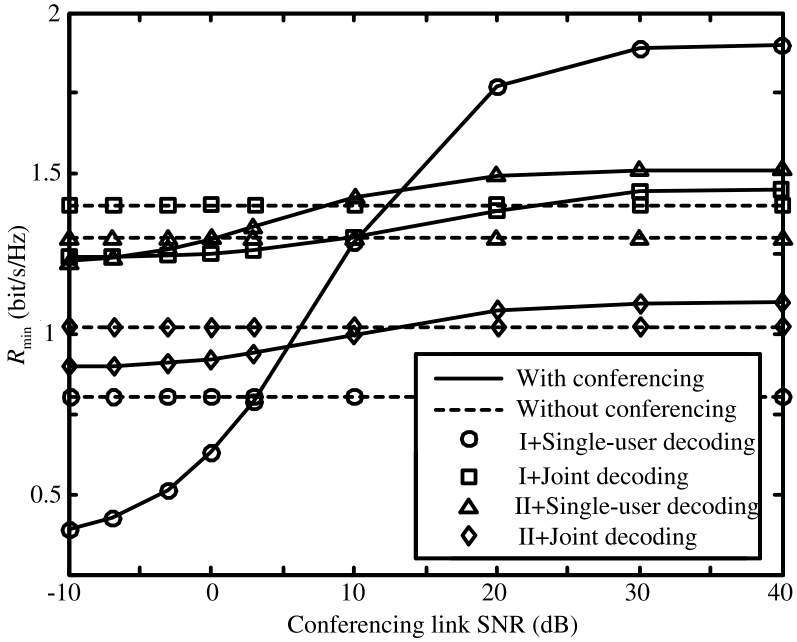
Fig. 23.: The achievable rate region for Scenario II, $\gamma_c = 0$ dB.

or convex hulling and the capacity upper bound under different channel conditions. It is observed that the AF relaying scheme performs better in the high SNR regime than in the low SNR regime. For Scenario I with conferencing link SNR $\gamma_0 = 0$ dB, the joint decoding scheme is always better than the single-user decoding scheme, and the performance difference is more significant in the high SNR regime; for Scenario II, single-user decoding scheme performs always better.

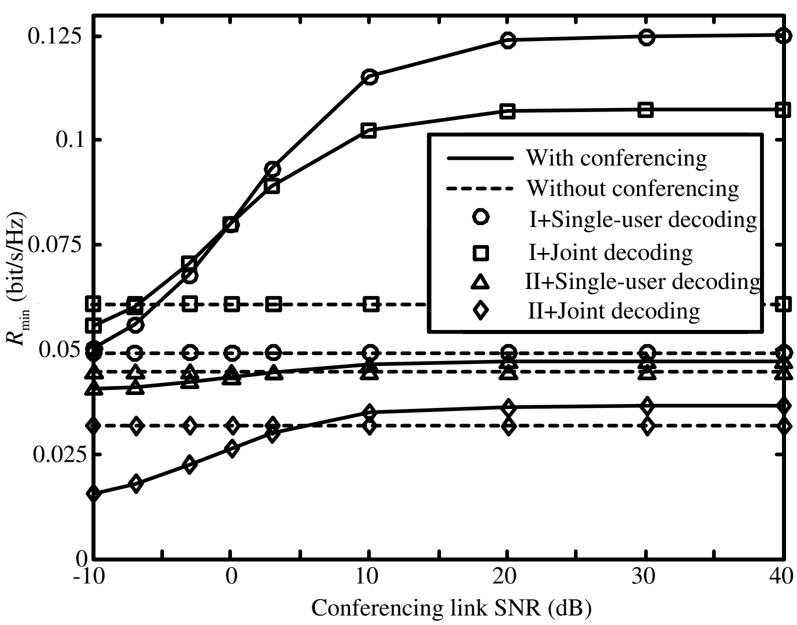
Finally, we examine the effect of the conferencing link quality on the minimum rate. In Fig. 24(a) and Fig. 24(b), we plot $R_{\min}$ over different conferencing link SNRs for both of the two scenarios with different source and relay power levels. Moreover, we show the results for the case without relay conferencing by using exhaustive search, which provides a benchmark to illustrate the improvement induced by relay conferencing. It is observed that for the high source/relay power case, relay conferencing brings a relatively large gain for single-user decoding at scenario I compared with those with other setups; for the low source/relay power case, relay conferencing significantly improves the rates for both the two decoding schemes at scenario I, while it introduces a little rate gain for scenario II. Furthermore, most of the gain introduced by relay conferencing can be achieved when $\gamma_c \approx 20$ dB.

F. Summary

In this chapter, we investigated the source power allocation and relay combining strategies for the two-hop AF interference network with conferencing relays. By using the rate profile method, we developed a two-stage iterative algorithm to efficiently characterize the achievable rate region for both the single-user decoding and joint decoding schemes. In particular, we proposed a more efficient routine to compute the optimal solution for the relay combining problem under the individual relay power



(a) High SNR case: $P_S = P_R = 20$ dB;



(b) Low SNR case: $P_S = P_R = 0$ dB.

Fig. 24.: The effect of conferencing link SNR on the minimum rate $R_{\min}$, $\boldsymbol{\beta} = [0.5, 0.5]$.

constraint. Moreover, we showed that relay conferencing can improve the DoF in the high SNR regime compared to the case without conferencing. In particular, the single-user decoding scheme is asymptotically optimal to achieve the DoF upper bound with simple linear beamforming schemes, while the joint decoding scheme is strictly suboptimal in term of DoF.

G. Appendix

1. Proof of Theorem B.1 and the Computation Complexity Analysis

First, we cite the following lemma, which is proved in [59].

Lemma G.1 *Given that Tr*$(\boldsymbol{A}\boldsymbol{C}^*) \geq 0$ *and Tr*$(\boldsymbol{B}\boldsymbol{C}^*) \geq 0$*, there exists a decomposition for* $\boldsymbol{C}^*$ *such that*

$$\boldsymbol{C}^* = \sum_{j=1}^{r} \boldsymbol{x}_j \boldsymbol{x}_j^H, \tag{5.38}$$

and $\boldsymbol{x}_j^H \boldsymbol{A} \boldsymbol{x}_j \geq 0,\ \boldsymbol{x}_j^H \boldsymbol{B} \boldsymbol{x}_j \geq 0,\ j = 1, \cdots, r.$

We consider the third constraint in (5.18), and denote $p_0 = \mathrm{Tr}(\boldsymbol{W}_{R2}\boldsymbol{C}^*)$. Since $\boldsymbol{W}_{R2}$ is positive semidefinite, we have $p_0 \geq 0$. First, let us consider the case $p_0 > 0$. By Lemma G.1, let $\boldsymbol{A} = \boldsymbol{F}_1 - \frac{1}{p_0}\boldsymbol{W}_{R2}$ and $\boldsymbol{B} = \boldsymbol{F}_2 - \frac{1}{p_0}\boldsymbol{W}_{R2}$, we obtain that there exists $\boldsymbol{C}^* = \sum_{j=1}^{r} \boldsymbol{x}_j \boldsymbol{x}_j^H$ with $\boldsymbol{x}_j^H \left(\boldsymbol{F}_1 - \frac{1}{p_0}\boldsymbol{W}_{R2}\right) \boldsymbol{x}_j \geq 0,\ \boldsymbol{x}_j^H \left(\boldsymbol{F}_2 - \frac{1}{p_0}\boldsymbol{W}_{R2}\right) \boldsymbol{x}_j \geq 0,\ j = 1, \cdots, r$. Define $y_{i,j} = \boldsymbol{x}_j^H \boldsymbol{W}_{Ri} \boldsymbol{x}_j$, and $z_{i,j} = \boldsymbol{x}_j^H \boldsymbol{F}_i \boldsymbol{x}_j,\ i = 1, 2$, and $j = 1, \cdots, r$. Thus, it is easy to check that Problem (5.17)-(5.19) has the same minimum objective value as the following linear programming (LP) problem

$$\begin{aligned}\min_{t_1,\cdots,t_r} \quad & \sum_{j=1}^{r} y_{1,j} t_j \\ \text{s. t.} \quad & \sum_{j=1}^{r} z_{1,j} t_j \geq 1, \ \sum_{j=1}^{r} z_{2,j} t_j \geq 1, \ \sum_{j=1}^{r} \frac{y_{2,j}}{p_0} t_j = 1, \\ & t_j \geq 0, \ j = 1, \cdots, r,\end{aligned} \tag{5.39}$$

which is based on the facts that for each feasible set of $t_j \geq 0$, $\sum_{j=1}^{r} t_j \boldsymbol{x}_j \boldsymbol{x}_j^H$ corresponds to a feasible solution for Problem (5.17)-(5.19), such that when $t_j = 1$, $\forall j$, the same minimum value can be achieved in both the above LP problem and the Problem (5.17)-(5.19). By Lemma G.1, we obtain $z_{i,j} \geq \frac{1}{p_0} y_{2,j}$ for any i and j. Thus, for any t_j's, we have $\sum_{j=1}^{r} z_{1,j} t_j \geq \frac{1}{p_0} \sum_{j=1}^{r} y_{2,j} t_j = 1$ and $\sum_{j=1}^{r} z_{2,j} t_j \geq \frac{1}{p_0} \sum_{j=1}^{r} y_{2,j} t_j = 1$. As such, Problem (5.39) is further equivalent to the following problem

$$\begin{aligned}\min_{t_1,\cdots,t_r} \quad & \sum_{j=1}^{r} y_{1,j} t_j \\ \text{s. t.} \quad & \sum_{j=1}^{r} \frac{y_{2,j}}{p_0} t_j = 1, \ t_j \geq 0, \ j = 1, \cdots, r.\end{aligned} \tag{5.40}$$

Therefore, by the property of basic feasible solution for LP, we can prove that there is at least one optimal solution of Problem (5.40) with only one $t_{j_0} > 0$ and all other t_j's equal to zero. Then, the optimal rank-1 solution is given as $\boldsymbol{C}^{**} = t_{j_0} \boldsymbol{x}_{j_0} \boldsymbol{x}_{j_0}^H$.

For the case $p_0 = 0$, without loss of generality, we assume $\mathrm{Tr}\left(\left(\boldsymbol{F}_1 - \boldsymbol{F}_2\right) \boldsymbol{C}^*\right) \geq 0$ (otherwise, we could reverse the following definitions), and let $\boldsymbol{A} = \boldsymbol{F}_1 - q_2 \boldsymbol{F}_2$, where $q_2 = \mathrm{Tr}\left(\boldsymbol{F}_2 \boldsymbol{C}^*\right)$. Then, by Lemma G.1, we obtain that there exists $\boldsymbol{C}^* = \sum_{j=1}^{r} \boldsymbol{x}_j \boldsymbol{x}_j^H$, and $\boldsymbol{x}_j^H \left(\boldsymbol{F}_1 - \boldsymbol{F}_2\right) \boldsymbol{x}_j \geq 0, \ j = 1, \cdots, r$. Define $y_{i,j}$ and $z_{i,j}$ the same as the previous case, and consider the following problem, which also has the same minimum optimal value as problem (5.17)-(5.19).

$$\begin{aligned} \min_{t_1,\cdots,t_r} \quad & \sum_{j=1}^{r} y_{1,j} t_j \\ \text{s. t.} \quad & \sum_{j=1}^{r} z_{1,j} t_j \geq 1, \ \sum_{j=1}^{r} \frac{z_{2,j}}{q_2} t_j = 1, \ \sum_{j=1}^{r} y_{2,j} t_j = 0, \\ & t_j \geq 0, \ j = 1, \cdots, r. \end{aligned} \tag{5.41}$$

By the same argument as the previous case, we obtain $\sum_{j=1}^{r} z_{1,j} t_j \geq \sum_{j=1}^{r} \frac{z_{2,j}}{q_2} t_j = 1$ for any t_j, which means that $\sum_{j=1}^{r} z_{1,j} t_j \geq 1$ is redundant. Moreover, since $\boldsymbol{W}_{R2}$ is positive semidefinite, we have $y_{2,j} = \boldsymbol{x}_j^H \boldsymbol{W}_{R2} \boldsymbol{x}_j \geq 0$; on the other hand, we have $0 = \mathrm{Tr}(\boldsymbol{W}_{R2} \boldsymbol{C}^*) = \sum_{j=1}^{r} y_{2,j}$. Based on these two observations, we conclude $y_{2,j} = 0$ for any j. Thus, for any t_j, $\sum_{j=1}^{r} y_{2,j} t_j = 0$ is always true, which means that this constraint is also redundant. Accordingly, Problem (5.41) is equivalent to

$$\begin{aligned} \min_{t_1,\cdots,t_r} \quad & \sum_{j=1}^{r} y_{1,j} t_j \\ \text{s. t.} \quad & \sum_{j=1}^{r} \frac{z_{2,j}}{q_2} t_j \geq 1, \ t_j \geq 0, \ j = 1, \cdots, r. \end{aligned} \tag{5.42}$$

By the same argument as the $p_0 > 0$ case, we know that the rank-1 solution can be efficiently constructed, which is given as $\boldsymbol{C}^{**} = t_{j_0} \boldsymbol{x}_{j_0} \boldsymbol{x}_{j_0}^H$ with j_0 indexing the single positive t_j. In conclusion, the theorem is proved.

Remark G.1 *It is worth noting that to solve the linear programming problem in (5.42), we only need to find the minimum value among $y_{1,j} t_j$, $j = 1, \cdots, r$, where $t_j = \frac{q_2}{z_{2,j}}$, i.e., solve*

$$\min_j \left\{ y_{1,j} \frac{q_2}{z_{2,j}} \right\}. \tag{5.43}$$

Thus, for the proposed algorithm for the construction of the rank-1 solution, the *computation burden is mainly on how to find the matrix decomposition in Lemma G.1. By [59, 60], this matrix decomposition can be obtained by using only one eigen-decomposition routine for matrix $\boldsymbol{C}^*$, whose computation complexity is on the order of $\mathcal{O}(N^3)$ [61], with N the dimension of $\boldsymbol{C}^*$, and some linear operations with the complexity on the order of $\mathcal{O}(N^2)$. Since the proposed algorithm for the construction of the rank-1 solution is a one-shot scheme, i.e., iterations are not needed, its computation complexity is on the order of $\mathcal{O}(N^3)$. On the other hand, for Algorithm 1 in [56], it is observed that for each iteration, we need to compute two eigen-decomposition routines for two matrices with dimensions of N and r, respectively, and all other linear operations are with the complexity on the order of $\mathcal{O}(N^2)$. Moreover, it is easy to check that this algorithm requires only one iteration for the best case (i.e., $r = 2$), while $N - 1$ iterations for the worst case (i.e., $r = N$). Thus, its worst-case complexity is on the order of $\mathcal{O}(N^4)$. Based on the above analysis, we conclude that the proposed algorithm for the construction of the rank-1 solution is more efficient than that in [56] especially when r is large.*

2. Proof of Theorem D.1

For the single-user decoding method, from (5.10), we observe that $\boldsymbol{W}_{12}$ and $\boldsymbol{W}_{12}$ are both of rank-1. By condition (5.35), there always exists a $\boldsymbol{c}_{01}$ such that $\boldsymbol{c}_{01}^H \boldsymbol{W}_{12} \boldsymbol{c}_{01} = |\boldsymbol{w}_{12}^H \boldsymbol{c}_{01}|^2 = 0$, $\boldsymbol{c}_{01}^H \boldsymbol{W}_{21} \boldsymbol{c}_{01} = |\boldsymbol{w}_{21}^H \boldsymbol{c}_{01}|^2 = 0$, and $\boldsymbol{w}_{11}^H \boldsymbol{c}_{01} \neq 0$. Similarly, there exists a $\boldsymbol{c}_{02}$ such that $\boldsymbol{c}_{02}^H \boldsymbol{W}_{12} \boldsymbol{c}_{02} = |\boldsymbol{w}_{12}^H \boldsymbol{c}_{02}|^2 = 0$, $\boldsymbol{c}_{02}^H \boldsymbol{W}_{21} \boldsymbol{c}_{02} = |\boldsymbol{w}_{21}^H \boldsymbol{c}_{02}|^2 = 0$, and $\boldsymbol{w}_{22}^H \boldsymbol{c}_{02} \neq 0$. Thus, we let $\boldsymbol{c}_0 = d\,(\boldsymbol{c}_{01} + \boldsymbol{c}_{02})$, where d is a constant making the relay power constraint satisfied. Here, we assume $\langle \boldsymbol{c}_{01}, \boldsymbol{c}_{02} \rangle \geq 0$, which can be satisfied by changing the direction of one of them without violating the previous conditions. Provided with this $\boldsymbol{c}_0$ and the rate pair given by (5.10), we have

$$R_i = \frac{1}{2}\log\left(1 + p_i\frac{\boldsymbol{c}_0^H\boldsymbol{W}_{ii}\boldsymbol{c}_0}{\boldsymbol{c}_0^H\boldsymbol{W}_{in}\boldsymbol{c}_0 + 1}\right), \ i = 1, 2. \tag{5.44}$$

Furthermore, we have

$$\frac{\boldsymbol{c}_0^H\boldsymbol{W}_{ii}\boldsymbol{c}_0}{\boldsymbol{c}_0^H\boldsymbol{W}_{in}\boldsymbol{c}_0 + 1} \geq \frac{\boldsymbol{c}_0^H\boldsymbol{W}_{ii}\boldsymbol{c}_0}{\sigma|\boldsymbol{c}_0|^2 + 1} \tag{5.45}$$

$$\geq \frac{d|\boldsymbol{c}_{0i}^H\boldsymbol{w}_{ii}|^2}{\sigma|\boldsymbol{c}_0|^2 + 1}, \tag{5.46}$$

where σ is the largest eigenvalue of $\boldsymbol{W}_{in}$. Since $\boldsymbol{W}_{in}$ is a positive semidefinite and non-zero matrix, we know $\sigma > 0$; and (5.46) is due to the assumption $\langle \boldsymbol{c}_{01}, \boldsymbol{c}_{02}\rangle \geq 0$. Moreover, when $a_1P_S \leq P_R \leq a_2P_S$, by (5.4), we know $d > 0$ when $P_S \to \infty$. Therefore, we conclude $\lim_{P_S\to\infty}\frac{R_i}{\log(P_S)} = \frac{1}{2}$, and the result for the single-user decoding case is proved.

For the joint decoding method, according to (5.30) and by as similar argument as the above analysis, it is easy to see $R_1 + R_2 \to \frac{1}{2}\log(d_0P_S)$, as P_S goes to infinity, where d_0 is a positive constant. Thus, the theorem is proved.

CHAPTER VI

CONCLUSIONS

A. Summary of Dissertation Contributions

This dissertation investigated the information-theoretical bounds for the cooperative communication systems with relay conferencing. By designing the protocols jointly with the relaying and conferencing schemes, various achievable rates were obtained, and some capacity results were derived under certain special channel conditions. Moreover, we specified when relay conferencing can outperform the case without conferencing under different channel and network models with different channel conditions by either analysis or simulations. Specifically, we summarize the main contributions of this dissertation as follows.

1. We started with the simultaneous relaying diamond channel in Chapter 2, where the two relays were assumed to transmit and receive in the same time slot. We obtained the rate upper bound via the cut-set bounds and various achievable rates by modifying the conventional DF, CF, and AF relaying schemes. In particular, we obtained the following results.

   (a) For the DF relaying scheme, we let the source transmit one common message to both relays and one private message to each relay. We proved that for the DMC case, the DF scheme achieves the rate upper bound with finite conferencing link rates; for the Gaussian channel case, the rate upper bound is asymptotically achieved when the source-to-relay link SNR go to infinity.

   (b) For the CF relaying scheme, we developed three schemes: one using conferencing links to help the compression, and the other two using them to partially or fully exchange the binning indices of the compressed receiver signals. We proved that for the Gaussian case, when the SNRs of the source-relay links or the relay-destination links go to infinity, the rate upper bound is asymptotically achievable.

(c) For the AF relaying scheme, we investigated the optimal combining problem between the received signals from the source (via the source-relay link) and the other relay (via the conferencing link). Generally, the resulting problem is not a convex problem, while semidefinite relaxation is applied to transform it to a quasi-convex problem.

2. We then considered the alternative relaying diamond channel in Chapter 3, where the two relays transmit and receive in different time slots and exchange their modes alternatively. Two different conferencing strategies were proposed by utilizing the conferencing links in different amount of time, which lead to different decoding delays at the destination. For both of the two conferencing strategies with the general two-side conferencing scheme, we derived the DF and AF achievable rates. For the DF relaying scheme, we formulated the rate maximization problem as a LP problem; for the AF relaying scheme, it was shown that the optimal linear combining problem is convex. By exploiting the properties of the optimal solutions for the above two problems, we further obtained the following results:

   (a) For the DF relaying scheme, it was proved that the one-side conferencing scheme is optimal to achieve the maximum DF rates achieved by the two-side conferencing scheme for both of the two conferencing strategies. Based on this property, we derived the DF rates in closed-form under different channel coefficients, and further determined: (i) when relay conferencing is unnecessary; (ii) when relay conferencing is necessary, and which one of the conferencing links should be used. Moreover, we proved that the DF scheme achieves the rate upper bound under conferencing strategy I asymptotically as the conferencing link rates go to infinity, while only finite conferencing rates are required under strategy II.

(b) For the AF relaying scheme, it was shown that: (i) When the second-hop relay-to-destination link SNRs become asymptotically large, two-side conferencing is necessary; (ii) when these link SNRs go to zero, one-side conferencing is asymptotically optimal, and each relay only needs to forward the signal with a higher SNR to the destination.

3. In Chapter 4, we applied relay conferencing to the large relay networks, which consists of one source-destination pair and N relays. In particular, the conferencing links were assumed to be SNR-limited and with the AF conferencing scheme. We obtained the DF and AF achievable rates, and further examined the asymptotic behaviors of these schemes as N goes to infinity. It was shown that the relay conferencing can improve the scaling order of the DF relaying scheme, and some asymptotic capacity results are established under certain conditions with the AF relaying scheme.

4. Finally, we considered the two-hop interference relay channels, which consist of two source-destination pair and two relays. Here, we only focused on two different decoding schemes at the destination: single-user decoding and joint decoding.

   (a) We derived the AF achievable rate for the two decoding schemes under both the sum and individual source/relay power constraints. To effectively quantify the rate region and solve the optimal source power allocation and relay combining problems, we adopted a two-stage iterative optimization method: First, we fixed the source power pair and maximized the sum rate over the relay combining vector under relay power constraints; second, we fixed the relay combining vector at the optimal point of the previous stage, and maximized the sum rate over the source power levels. Then, the iteration continued. We designed a new algorithm to compute the optimal solution for the relay combining problem under the individual relay power constraint, which is more efficient than the existing scheme especially for the worst-case scenario.

(b) We compared the single-user and joint decoding schemes under both the high and low SNR assumptions, and obtained the following results: As the SNR goes to infinity, the single-user decoding scheme is asymptotically optimal in the sense of achieving the maximum DoF of 1, while without relay conferencing, only 3/4 DoF can be achieved by the AF scheme; as the SNR goes to zero, both the decoding schemes achieve the same rate region asymptotically.

B. Future Work

We propose the following possible extensions to the work presented in this dissertation.

1. In Chapter 2 and 3, we separately considered two relay scheduling schemes, i.e., simultaneous and alternative relaying, for the same diamond relay channel. Thus, to obtain an unified strategy jointly considering these two relay scheduling schemes with relay conferencing will be more interesting and challenging.

2. In Chapter 4, we adopted the "p-portion" conferencing scheme, whose complexity is high for the large N case. One possible way to avoid this implementation issue is to choose a constant number of conferencing links for each relay to conference. However, how to choose which subset of the relays for each one and how to design the proper conferencing and relaying protocols still need to be carefully investigated.

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