PRODUCTION ECONOMICS MODELING AND ANALYSIS OF POLLUTING FIRMS: THE PRODUCTION FRONTIER APPROACH

A Dissertation

by

MAETHEE MEKAROONREUNG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2012

Major Subject: Industrial Engineering

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The Production Frontier Approach

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ABSTRACT

Production Economics Modeling and Analysis of Polluting Firms:

The Production Frontier Approach. (August 2012)

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Chair of Advisory Committee: Dr. Andrew Johnson

As concern grows about energy and environment issues, energy and environmental modeling and related policy analysis are critical issues for today's society. Polluting firms such as coal power plants play an important role in providing electricity to drive the U.S. economy as well as producing pollution that damages the environment and human health. This dissertation is intended to model and estimate polluting firms' production using nonparametric methods. First, frontier production function of polluting firms is characterized by weak disposability between outputs and pollutants to reflecting the opportunity cost to reduce pollutants. The StoNED method is extended to estimate a weak disposability frontier production function accounting for random noise in the data. The method is applied to the U.S. coal power plants under the Acid Rain Program to find the average technical inefficiency and shadow price of SO_2 and NO_x. Second, polluting firms' production processes are modeled characterizing both the output production process and the pollution abatement process. Using the law of conservation of mass applied to the pollution abatement process, this dissertation develops a new frontier pollutant function which then is used to find corresponding marginal abatement cost of pollutants. The StoNEZD method is applied to estimate a frontier pollutant function considering the vintage of capital owned by the polluting firms. The method is applied to estimate the average NO_x marginal abatement cost for the U.S. coal power plants under the current Clean Air Interstate Rule NO_x program. Last, the effect of a technical change on marginal abatement costs are investigated using an index decomposition technique. The StoNEZD method is extended to estimate sequential frontier pollutant functions reflecting the innovation in pollution reduction. The method is then applied to estimate a technical change effect on a marginal abatement cost of the U.S. coal power plants under the current Clean Air Interstate Rule NO_x program.

To Somchai and Piengpen Mekaroonreung and my family

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CHAPTER I

INTRODUCTION

1.1 Background and motivation

Production economics is a subject that investigates aspects of economic issues involving producers of goods and services in an economy. The producers are a number of production units called firms whose activities are to transform resources into goods and services.[1] Topics of current interest include its production functions, cost functions, profit functions, marginal productivity, technical efficiency, and technical change, which have been described in Frisch (1964), Shephard (1970), Varian (1992) and Mas-Colell et al. (1995).

Negative externalities, which cause inefficient resource allocation, occur when a firm producing goods or services does not bear the full cost of its activities, i.e. the firm's marginal cost is less than society's marginal cost. The imposition of an emissions tax (or Pigovian tax, Pigou, 1932) is one regulatory mechanism employed by society to address negative externalities. Other policy tools developed in recent decades include direct command and control actions, and/or cap and trade programs for pollution emission rights.

Efficient emission tax and subsidy policies can be implemented if the regulator knows the firm specific *shadow price* of pollutants. A pollutant's shadow price can be interpreted as the net loss for the firm to reduce pollution by one unit less; thus, the

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This dissertation follows the style of Energy Economics.

[1] In this dissertation, "firm" refers to any production unit at any scale that transforms inputs to outputs.

shadow price reflects the real value of marginally decreasing pollution by the firm. If there is no market for pollutant allowances[2], regulators will be unable to use market prices as a measure of a marginal abatement cost. However, shadow price estimates generally can provide enough information to support efforts to determine optimal taxes or subsidies.

An alternative to tax and subsidy policies is to take a cap and trade approach to control emissions (Hanley et al. 1997). Compared to more rigid command and control alternatives, these programs allows polluting firms flexibility to determine their own method of compliance to limit emissions.

In the U.S., the EPA's Acid Rain Program and the nitrogen oxide (NO_x) Budget Trading Program, both of which primarily affect coal power plants, are important examples of cap and trade programs. As an abundant fossil fuel coal offers a reliable domestic supply at very cheap cost. In fact, since 1989 it has generated about 47-56% of the electricity consumed in the U.S. (EIA, 2010). In coal power plants, fine coal powder is fed into boilers where it is burned, and the heat from the burning coal is then used to generate the high pressure and temperature steam that rotates the turbines that generate the end-product, electricity.

However, burning coal produces several harmful byproduct pollutants, notably sulfur oxide (SO_2), the major cause of acid rain, and NO_x, the major cause of both acid rain and ground level ozone. To mitigate these environmental problems, the Acid Rain Program and the NO_x Budget Trading Program implement an allowance that requires

[2] An allowance is a right to emit a fixed amount of a pollutant.

coal generators to hold allowances for each unit of SO_2 and NO_x released. Plants that reduce emissions below the number of SO_2 and NO_x allowances they hold are allowed to trade their unused allowances to others, sell them in the marketplace, or bank them for future use. The programs have resulted in dramatic reductions in emissions and are claimed to be more cost effective compared to command and control approaches (Stavins, 1998).

Yet, a pollutant allowance price that trades in the marketplace does not necessarily equal a pollutant shadow price (Drèze and Stern, 1990). For example, factors such as the price uncertainty of other fuels, rapid innovation of abatement technology and regulatory uncertainty can cause volatility of SO_2 and NO_x market prices (Burtraw and Szambelan 2009). In this regard, pollutant shadow price estimates could be used to compare with pollutant market prices in identifying the influence of these other factors. Moreover, regulators could combine both pollutant shadow and market prices information to understand polluting firms' production characteristics in order to improve future environmental regulatory policies.

The objective of this research is to use production economics theory to model the production of polluting firms and apply nonparametric estimation methods to perform *ex post* analysis, in particular, to estimate firm level shadow prices for pollutants. The U.S. domestic coal power plant sector is selected for empirical study for two reasons. First, coal power plants are important polluters in the U.S. economy Second, SO_2 and NO_x markets already exist in the U.S.; thus, SO_2 and NO_x shadow price estimates from coal power plants in this research can be directly compared to the SO_2 and NO_x market prices in the U.S.

1.2 Problem statement

A popular method to measure technical inefficiencies and to estimate the shadow prices of pollutants from polluting firms is the frontier production function approach imposing the weak disposability axiom between outputs and pollutants. Currently, this method constructs a frontier production function by either applying deterministic parametric approach or data envelopment analysis (DEA) without considering the presence of random noise in the data. However, since such methods can suffer from an outlier problem, Chapter III introduces a nonparametric estimation method for a frontier production function satisfying the weak disposability axiom with random noise in the data.

When considering negative externalities such as pollutants, regulators usually model the production of outputs as a function of inputs and pollutants. An aggregate production function in the current approaches is typically considered as a "black box" and does not consider an abatement process. Ignoring the types of abatement processes that some firms rely on to reduce their pollutants is a significant drawback of the current approaches. Moreover, existing models that consider pollutants as inputs in the production function have been subject to debate for not satisfying the conservation of mass law. Chapter IV, which discusses problems in existing models, proposes to model the overall production of a polluting firm in a network by first modeling the outputs production process and then modeling the pollution abatement process by imposing the law of conservation of mass.

The relationship between marginal abatement cost and technical change has been studied in a theoretical framework, but whether technical change increases or decreases the marginal abatement cost remains inconclusive. Moreover, methods to empirically measure the effect of a technical change on a marginal abatement cost are lacking. Therefore, Chapter V proposes a new nonparametric estimation method to investigate the effect of a technical change on a marginal abatement cost.

1.3 Organization of the dissertation

Chapter II describes the main methods for estimating a frontier production function. Chapter III introduces a new estimation approach that incorporates the weak disposability axiom into the convex nonparametric least squares and the stochastic non-smooth envelopment of data methods. Chapter IV proposes a production model of polluting firms considering a pollution abatement process and the law of conservation of mass. The results are estimates of pollutant functions and marginal abatement costs. Chapter V proposes a new frontier estimation method in multiple periods and develops a multiplicative decomposition technique to investigate the effect of a technical change on a marginal abatement cost. Chapter VI summarizes the findings and offers suggestions to extend the research.

CHAPTER II

METHODS TO BE USED

Estimating a production function is an important topic in empirical economics and productivity analysis. According to the neoclassical theory of production, a production function commonly satisfies properties such as monotonic increases in inputs, decreasing marginal productivity of inputs, and quasi-concavity. Several functional forms are used to represent production functions, such as Cobb-Douglas (Cobb and Douglas, 1928), Constant Elasticity of Substitution (Solow, 1956; Arrow et al., 1961) or translog (Christensen et al., 1973). In econometrics, a common method to estimate a production function is to assume a specific functional form for a production function and then apply regression techniques to estimate the production function's parameters.

A neoclassical production function is typically interpreted as a function which specifies the maximum output that can be produced for given input levels assuming that firms operate efficiently. However, a neoclassical production function does not consider that firms could operate inefficiently below the production function as a result of engineering or managerial problems. Thus, Koopman (1951) and Debreu (1951) proposed the concept of a frontier production function and Farrell (1957) proposed a measure of technical efficiency which was later extended by Boles (1967) and Afriat (1972).

DEA and stochastic frontier analysis (SFA) are the most common methods to estimate a frontier production function. DEA, Charnes et al. (1978), has been extensively used in the operations research field to measure firms' technical inefficiencies. DEA is a nonparametric method that estimates a frontier production function by solving a set of linear programming problems to construct a piecewise linear function that envelops all the observed data. DEA has the advantage that it can model a multiple output production frontier without assuming a functional form; however, it has been criticized because of its deterministic nature and lack of a statistical model of random noise. SFA, Aigner et al, (1977); Meeusen and van den Broeck, (1977), has been extensively used in the field of econometrics. SFA is a parametric regression method that estimates a frontier production function and firms' technical inefficiencies by using statistical methods such as the method of moments or maximum likelihood estimation. While SFA is attractive because it is a statistical method which allows random noise, it requires strong assumptions regarding the functional form for a frontier production function.

The main advantage of nonparametric methods over parametric methods is that they avoid the risk of misspecification. As stated in Li and Racine (2007), "nonparametric" refers to statistical techniques that do not require a specification of a functional form for the objects being estimated. In practice, functional forms of a frontier production function are rarely if ever known; thus, applying parametric methods to estimate frontier production functions can lead to well-known parametric misspecification problems. Several previous studies have conducted Monte Carlo simulations to show how nonparametric methods outperform parametric methods in estimating frontier production functions when the functional forms were misspecified (see Banker et al., 1993; Kittlesen, 1999; Kuosmanen, 2008; Kuosmanen and Johnson, 2010; and Kuosmanen and Kortelainen, 2011). The results of these studies support the use of nonparametric methods to estimate frontier production function.

2.1 Convex nonparametric least squares (CNLS)

A potential nonparametric regression method to estimate frontier production functions is CNLS, which was proposed by Hildreth (1954) and then extended by Handson and Pledger (1976). CNLS estimates a function satisfying continuity, monotonicity and globally concavity – the standard regularity conditions for a frontier production function in microeconomic theory. The primary disadvantages of previous CNLS methods, i.e., computational complexity and the absence of a closed form regression function, led Kuosmanen (2008) to develop an explicit formulation where the resulting function is a continuous, piecewise linear approximation that can be solved by a quadratic program.[3]

2.1.1 The CNLS problem

Consider a data set of n firms and let $\boldsymbol{x}_i \in R_+^M$ be an input vector, $y_i \in R_+$ be an output and f be an unknown frontier production function satisfying continuity, monotonicity and concavity. The regression model is written as

$$y_i = f(\boldsymbol{x}_i) + \varepsilon_i \qquad \forall i = 1, \dots, n \tag{2.1}$$

[3] From this point, CNLS refers to the convex nonparametric least squares method developed by Kuosmanen (2008).

where ε_i is a disturbance term with $E(\varepsilon_i) = 0\ \forall i$, $\text{Var}(\varepsilon_i) = \sigma^2 < \infty\ \forall i$ and $\text{Cov}(\varepsilon_i \varepsilon_j) = 0\ \forall i \neq j$. The CNLS problem can be formulated as the quadratic program

$$\min_{\alpha,\boldsymbol{\beta},\varepsilon} \sum_{i=1}^{n} \varepsilon_i^2 \tag{2.2.1}$$

$$\text{s.t.} \quad \varepsilon_i = y_i - (\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i) \qquad \forall i = 1, \dots, n \tag{2.2.2}$$

$$\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i \leq \alpha_h + \boldsymbol{\beta}_h{}'\boldsymbol{x}_i \qquad \forall i, h = 1, \dots, n \tag{2.2.3}$$

$$\boldsymbol{\beta}_i \geq 0 \qquad \forall i = 1, \dots, n \tag{2.2.4}$$

where α_i and $\boldsymbol{\beta}_i$ are the coefficients characterizing the hyperplanes of the frontier production function f. Note that α_i and $\boldsymbol{\beta}_i$ are specific to each firm i. The objective function (2.2.1) minimizes the sum of squared disturbance terms. The equality constraint (2.2.2) defines the disturbance term as the different between an observed output and an estimated output level. The inequality constraint (2.2.3) comprises a system of Afriat inequalities (Afriat, 1972), imposing the underlying frontier production function to be continuous and concave. The constraint (2.2.4) enforces monotonicity. Unlike DEA that only uses a few data points to construct a frontier production function, CNLS uses all of the data to estimate a production function. Thus, CNLS is more robust to outliers than DEA.

2.1.2 The explicit representor function

Given the coefficient estimates $(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i)$ from the CNLS problem (2.2), the CNLS estimator of the production function f, $\hat{f}$, can be constructed as the explicit representor function

$$\hat{f}(\boldsymbol{x}) = \min_i\{\hat{\alpha}_i + \widehat{\boldsymbol{\beta}}_i'\boldsymbol{x}\}. \tag{2.3}$$

In theory, $\hat{f}$ consists of n hyperplanes characterized by $(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i)$ $\forall i$; however, in practice, the number of estimated hyperplanes is usually lower than n. Both the coefficient estimates $(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i)$ and the estimator $\hat{f}$ are particularly useful because researchers are usually interested in the marginal properties of a frontier production function and the predicted output values. $\widehat{\boldsymbol{\beta}}$ can be interpreted as a set of partial derivative estimates on the production function. Thus, it is easy to calculate marginal properties such as marginal productivity, marginal rate of substitution, and elasticity of substitutions. Moreover, predicted output values at different input levels can be computed using equation (2.3), i.e., $\hat{y} = \min_i\{\hat{\alpha}_i + \widehat{\boldsymbol{\beta}}_i'\boldsymbol{x}\}$.

2.1.3 Uniqueness of the CNLS estimator

There is generally no unique optimal solution to the CNLS problem (2.2); rather, the optimal solution with different hyperplanes is characterized by other coefficient estimates $(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i)$. In other words, the CNLS estimator of the production function, $\hat{f}(\boldsymbol{x})$, is generally not unique, but the fitted output values at observed inputs, $\hat{f}(\boldsymbol{x}_i)$, are unique (Kuosmanen, 2008). In fact, given the fitted output values, it is possible to derive the tightest lower bound of the frontier production function as the explicit lower bound representor function

$$\hat{f}_{\min}(\boldsymbol{x}) = \min_{\alpha,\boldsymbol{\beta}}\{\alpha + \boldsymbol{\beta}'\boldsymbol{x} | \alpha + \boldsymbol{\beta}'\boldsymbol{x}_i \geq \hat{y}_i \quad \forall i = 1, \dots, n\} \tag{2.4}$$

where $\hat{y}_i = \hat{f}(\boldsymbol{x}_i)$ is the fitted output value. The tightest lower bound $\hat{f}_{\min}$ is a piecewise linear function satisfying continuity, monotonicity and concavity; thus, it can be used as the unique CNLS estimator of the frontier production function f.

2.2 Frontier production function based CNLS: additive disturbance model

To estimate the frontier production function, the CNLS method can be used to minimize the sum of the square of disturbances, which can be assumed as

1) composite (mixture of inefficiency and random noise) or

2) random (all deviations are random noise).

This chapter will elaborate on these composite and random disturbance term assumptions.

2.2.1 Frontier production function: applying stochastic non-smooth envelopment of data (StoNED)

To include both random noise and technical inefficiency in a CNLS style frontier production function, Kuosmanen and Kortelainen (2011) developed a two-stage method, StoNED, that combines the CNLS piecewise linear production function with the composite disturbance term from SFA. Consider the composite disturbance term

$$\varepsilon_i = v_i - u_i \qquad \forall i = 1, \dots, n \tag{2.5}$$

where v_i is a random noise, u_i is a technical inefficiency and v_i and u_i are assumed to be independent of each other. The random noise v_i is assumed to be the same as in section 2.1.1. The technical inefficiency, u_i, has a asymmetric distribution with $E(u_i) = \mu > 0 \ \forall i$, $\mathrm{Var}(u_i) = \sigma_u^2 < \infty \ \forall i$ and $\mathrm{Cov}(u_i u_j) = 0 \ \forall i \neq j$. The typical specific distribution assumption for v_i is the normal distribution, and the assumption for u_i is the half-normal, exponential or gamma distributions.

The composite disturbance term in (2.5) violates the Gauss-Markov property that $E(\varepsilon_i) = E(-u_i) = -\mu < 0$; thus, the composite disturbance term is modified as

$$y_i = [f(\boldsymbol{x}_i) - \mu] + [\varepsilon_i + \mu] = g(\boldsymbol{x}_i) + \vartheta_i \quad \forall i = 1, \dots, n \tag{2.6}$$

where $\vartheta_i = \varepsilon_i + \mu$ is a modified composite disturbance with $E(\vartheta_i) = E(\varepsilon_i + \mu) = 0$ and $g(\boldsymbol{x}_i) = f(\boldsymbol{x}_i) - \mu$ is an average production function. Since g inherits the continuity, monotonicity and concavity, the CNLS method can be used to find the estimator of the average production function g. The composite disturbance CNLS problem can then be formulated as:

$$\min_{\alpha,\boldsymbol{\beta},\vartheta} \sum_{i=1}^{n} \vartheta_i^2 \tag{2.7.1}$$

$$\text{s.t.} \quad \vartheta_i = y_i - (\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i) \qquad \forall i = 1, \dots, n \tag{2.7.2}$$

$$\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i \leq \alpha_h + \boldsymbol{\beta}_h{}'\boldsymbol{x}_i \qquad \forall i, h = 1, \dots, n \tag{2.7.3}$$

$$\boldsymbol{\beta}_i \geq 0 \qquad \forall i = 1, \dots, n. \tag{2.7.4}$$

where α_i and $\boldsymbol{\beta}_i$ are the coefficients that characterize the hyperplanes of the average frontier production function g. The composite disturbance CNLS problem (2.7) is the same as the CNLS problem (2.2) except that the composite disturbance CNLS problem minimizes the sum of squared modified composite disturbances.

The second stage of the StoNED method uses the modified composite residuals, $\hat{\vartheta}_i \; \forall i$, from (2.7) to separate the technical inefficiencies and random noises by applying the method of moments (Aigner et al., 1977; Kuosmanen and Kortelainen, 2011). Assuming that technical inefficiency has a half normal distribution, $u_i \sim |N(0, \sigma_u^2)|$, and that random noise has a normal distribution, $v_i \sim N(0, \sigma_v^2)$, the estimated standard deviation of technical inefficiency and random noise is written as

$$\hat{\sigma}_u = \sqrt[3]{\frac{\widehat{M}_3}{\left(\frac{2}{\pi}\right)\left(1 - \frac{4}{\pi}\right)}} \tag{2.8}$$

$$\hat{\sigma}_v = \sqrt{\widehat{M}_2 - \left(\frac{\pi - 2}{\pi}\right)\hat{\sigma}_u^2} \tag{2.9}$$

where $\widehat{M}_2 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\vartheta}_i - \hat{E}(\vartheta_i)\right)^2$ and $\widehat{M}_3 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\vartheta}_i - \hat{E}(\vartheta_i)\right)^3$ are the second and third sample central moment of the modified composite residuals. Moreover, $\widehat{M}_3$ should be negative so that $\hat{\sigma}_u$ is positive. Intuitively, the composite residuals should have negative skewness reflecting the presence of the technical inefficiency. The expected technical inefficiency is then calculated by

$$\hat{\mu} = \hat{\sigma}_u\sqrt{2/\pi}. \tag{2.10}$$

Given $(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i)$ from the CNLS problem (2.9), the unique StoNED estimator of the frontier production function is written as

$$\hat{f}_{\min}(\boldsymbol{x}) = \min_{\alpha,\boldsymbol{\beta}}\{\alpha + \boldsymbol{\beta}'\boldsymbol{x} | \alpha + \boldsymbol{\beta}'\boldsymbol{x}_i \geq \hat{y}_i \quad \forall i = 1, \ldots, n\} + \hat{\mu} \tag{2.11}$$

where $\hat{y}_i = \min_{h\epsilon\{1,\ldots,n\}}\{\hat{\alpha}_h + \widehat{\boldsymbol{\beta}}_h'\boldsymbol{x}_i\}$. First, the unique CNLS estimator of the average frontier production function, $\hat{g}_{\min}$, is obtained by using the tightest lower bound representor function (2.4) with the fitted output values, $\hat{y}_i$. Recall that $\hat{y}_i$ is calculated from the representor function (2.3) and $(\hat{\alpha}_i, \hat{\boldsymbol{\beta}}_i)$. Second, the frontier production function is obtained by additively shifting the unique CNLS estimator of the average frontier production function upward by the expected value of technical inefficiency.

2.2.2 Neoclassical production function

A neoclassical production function model assumes that a disturbance term contains only random noise

$$\varepsilon_i = v_i \qquad \forall i = 1, \dots, n \tag{2.12}$$

where v_i is a random noise with $E(v_i) = 0\ \forall i$, $\mathrm{Var}(v_i) = \sigma_v^2 < \infty\ \forall i$, $\mathrm{Cov}(v_i v_j) = 0\ \forall\ i \neq j$, and normally distributed. Since $\varepsilon_i = v_i$, the disturbance term does not contain the technical inefficiency term, u_i, which implies that $\mu = 0$ and $\sigma_u = 0$. Thus, the average frontier production function, g, is equivalent to the neoclassical production function, f. The solution of the CNLS problem (2.7) gives the hyperplane coefficient estimates, $(\hat{\alpha}_i, \hat{\boldsymbol{\beta}}_i)$, of the neoclassical production function.

With $(\hat{\alpha}_i, \hat{\boldsymbol{\beta}}_i)$ from the CNLS problem (2.7), the unique CNLS estimator of the neoclassical production function is written as

$$\hat{f}_{\min}(\boldsymbol{x}) = \min_{\alpha,\boldsymbol{\beta}}\{\alpha + \boldsymbol{\beta}'\boldsymbol{x} | \alpha + \boldsymbol{\beta}'\boldsymbol{x}_i \geq \hat{y}_i \quad \forall i = 1, \dots, n\} \tag{2.13}$$

where $\hat{y}_i = \min_{h\epsilon\{1,\dots,n\}}\{\hat{\alpha}_h + \hat{\boldsymbol{\beta}}_h{}'\boldsymbol{x}_i\}$. Note that when $\hat{\mu} = 0$, the unique estimator of the neoclassical production function (2.13) is equivalent to the unique estimator of the frontier production function (2.11).

2.3 Frontier production function based CNLS: multiplicative disturbance model

Since the additive disturbance model may not be suitable for all applications, inputs can appear in a logarithmic form to capture a nonlinear relationship between inputs and outputs. In short, a multiplicative disturbance model helps to control for heteroskedasticity resulting from more variability in output levels for firms operating at larger scale sizes (Caudil and Ford, 1993; Kuosmanen and Kortelainen, 2011). Moreover, a multiplicative disturbance model is usually applied in SFA studies when the frontier production functions are assumed to be Cobb-Douglas or translog function forms.

Consider a multiplicative disturbance model

$$y_i = f(\boldsymbol{x}_i)e^{\varepsilon_i} \qquad \forall i = 1, \dots, n \qquad (2.14)$$

where ε_i is a disturbance term with $E(\varepsilon_i) = 0 \forall i$, $\mathrm{Var}(\varepsilon_i) = \sigma^2 < \infty \ \forall i$ and $\mathrm{Cov}(\varepsilon_i \varepsilon_j) = 0 \ \forall i \neq j$. Similar to an additive disturbance model, the frontier production function can be either stochastic or neoclassical based on the assumption of the multiplicative disturbance.

2.3.1 Frontier production function

The StoNED method described in section 2.2.1 can be extended to estimate a frontier production function with a multiplicative disturbance. Consider a frontier production function model with multiplicative composite disturbance term

$$y_i = f(\boldsymbol{x}_i)e^{\varepsilon_i} = f(\boldsymbol{x}_i)e^{(v_i - u_i)} \qquad \forall i = 1, \dots, n. \qquad (2.15)$$

where assumptions regarding a random noise v_i and a technical inefficiency u_i are maintained as in section 2.2.1. Applying the log transformation to (2.15), the regression model is written as

$$\ln y_i = \ln f(\boldsymbol{x}_i) + \varepsilon_i = \ln f(\boldsymbol{x}_i) + v_i - u_i \qquad \forall i = 1, \dots, n. \tag{2.16}$$

The composite disturbance term in (2.16) violates the Gauss-Markov property that $E(\varepsilon_i) = E(v_i - u_i) = -\mu < 0$; thus, the composite disturbance term is modified as

$$\ln y_i = [\ln f(\boldsymbol{x}_i) - \mu] + [\varepsilon_i + \mu] = \ln g(\boldsymbol{x}_i) + \vartheta_i \qquad \forall i = 1, \dots, n \tag{2.17}$$

where $\vartheta_i = \varepsilon_i + \mu$ is a modified multiplicative composite disturbance term with $E(\vartheta_i) = E(\varepsilon_i + \mu) = 0$, and $g(\boldsymbol{x}_i) = f(\boldsymbol{x}_i)e^{-\mu}$ is an average production function. The multiplicative composite disturbance CNLS problem is then formulated as

$$\min_{\alpha,\boldsymbol{\beta},\vartheta} \sum_{i=1}^{n} \vartheta_i^2 \tag{2.18.1}$$

$$\text{s.t.} \quad \vartheta_i = \ln y_i - \ln(\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i) \qquad \forall i = 1, \dots, n \tag{2.18.2}$$

$$\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i \leq \alpha_h + \boldsymbol{\beta}_h{}'\boldsymbol{x}_i \qquad \forall i, h = 1, \dots, n \tag{2.18.3}$$

$$\boldsymbol{\beta}_i \geq 0 \qquad \forall i = 1, \dots, n. \tag{2.18.4}$$

where α_i and $\boldsymbol{\beta}_i$ are the coefficients characterizing the hyperplanes of the average frontier production function g.

Given$\left(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i, \hat{\vartheta}_i\right) \forall i$ from the CNLS problem (2.18), the unique StoNED estimator of the frontier production function is written as

$$\hat{f}_{\min}(\boldsymbol{x}) = \left[\min_{\alpha,\boldsymbol{\beta}}\{\alpha + \boldsymbol{\beta}'\boldsymbol{x} | \alpha + \boldsymbol{\beta}'\boldsymbol{x}_i \geq \hat{y}_i \quad \forall i = 1, \dots, n\}\right] e^{\hat{\mu}} \tag{2.19}$$

where $\hat{y}_i = \min_{h\epsilon\{1,\ldots,n\}}\{\hat{\alpha}_h + \widehat{\boldsymbol{\beta}}_h{}'\boldsymbol{x}_i\}$ and $\hat{\mu} = \hat{\sigma}_u\sqrt{2/\pi}$ obtained from the method of moments described in (2.8) and (2.9).Intuitively, the frontier production function is obtained by multiplicatively shifting the average production function upward by the expected value of technical inefficiency.

2.3.2 Neoclassical production function

Similar to section 2.2.2, when the multiplicative disturbance contains only the random noise, it implies $\mu = 0$ and $\sigma_u = 0$. Thus, the average frontier production function g is equivalent to the neoclassical production function, f. The solution of the CNLS problem (2.18) gives the hyperplane coefficient estimates$(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i)$ of the neoclassical production function.

Given $(\hat{\alpha}_i, \widehat{\boldsymbol{\beta}}_i)$ $\forall i$ from the CNLS problem (2.18), the unique CNLS estimator of the neoclassical production function is written as

$$\hat{f}_{\min}(\boldsymbol{x}) = \min_{\alpha,\boldsymbol{\beta}}\{\alpha + \boldsymbol{\beta}'\boldsymbol{x} \mid \alpha + \boldsymbol{\beta}'\boldsymbol{x}_i \geq \hat{y}_i \quad \forall i = 1, \ldots, n\} \tag{2.20}$$

where $\hat{y}_i = \min_{h\epsilon\{1,\ldots,n\}}\{\hat{\alpha}_h + \widehat{\boldsymbol{\beta}}_h{}'\boldsymbol{x}_i\}$. Note that when $\hat{\mu} = 0$, the unique estimator of the neoclassical production function (2.20) equals the unique estimator of the frontier production function (2.19).

2.4 Frontier production function with contextual variables

Contextual variables are variables that characterize firms' operational conditions and practices, such as the vintage of technology, managerial practices, etc. These contextual variables can be categorical, ordinal, interval or ratio scale (Banker and

Natarajan, 2008; Johnson and Kuosmanen, 2011; 2012) and can affect the level of firms' performance. Thus, investigating the effect of contextual variables on technical inefficiency can provide additional information about firms' production.

To estimate this effect, Johnson and Kuosmanen (2011) propose the method, stochastic semi-nonparametric envelopment of z variable data (StoNEZD). It extends StoNED such that the contextual variables are included in a composite disturbance term. A composite disturbance term considering contextual variables is written as

$$\varepsilon_i = v_i - u_i - \boldsymbol{\delta}'\boldsymbol{z}_i \tag{2.21}$$

where a random noise v_i and a technical inefficiency u_i are identically and independently distributed (i.i.d), and $\boldsymbol{z}_i \in R^r$ are the contextual variables and $\boldsymbol{\delta} \in R^r$ are the coefficients that capture the average effect of contextual variables on deviation from the frontier production function. Assuming that the contextual variables $\boldsymbol{z}$ only influence technical inefficiency, $u_i + \boldsymbol{\delta}'\boldsymbol{z}_i$ can be interpreted as the overall technical inefficiency of firm i where $\boldsymbol{\delta}'\boldsymbol{z}_i$ is the part of technical inefficiency captured by the contextual variables. From the disturbance term (2.21), the contextual variables CNLS problem is formulated as the nonlinear programming problem

$$\min_{\alpha,\boldsymbol{\beta},\vartheta} \sum_{i=1}^{n} \vartheta_i^2 \tag{2.22.1}$$

$$\text{s.t.} \quad \vartheta_i = \ln y_i - \ln(\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i) + \boldsymbol{\delta}'\boldsymbol{z}_i \qquad \forall i = 1, \dots, n \tag{2.22.2}$$

$$\alpha_i + \boldsymbol{\beta}_i{}'\boldsymbol{x}_i \le \alpha_h + \boldsymbol{\beta}_h{}'\boldsymbol{x}_i \qquad \forall i, h = 1, \dots, n \tag{2.22.3}$$

$$\boldsymbol{\beta}_i \ge 0 \qquad \forall i = 1, \dots, n. \tag{2.22.4}$$

where α_i and $\boldsymbol{\beta}_i$ are the coefficients characterizing the hyperplanes of the average frontier production function g. Note that $\boldsymbol{\delta}$ is unrestricted in sign and that a positive sign on δ implies that the contextual variables have positive effects on the output. The unique StoNEZD estimator of the frontier production function is obtained by using the two-step procedure described in section 2.3.1.

Based on Theorems 1 and 2 in Johnson and Kuosmanen (2011), the StoNEZD estimator for $\boldsymbol{\delta}$ isstatistically unbiased, consistent and asymptotically normally distributed as $\widehat{\boldsymbol{\delta}} \sim_a N(\boldsymbol{\delta}, (\sigma_u^2 + \sigma_v^2)(\boldsymbol{Z}'\boldsymbol{Z})^{-1})$ where $\boldsymbol{Z} = (\boldsymbol{z}_1, \ldots, \boldsymbol{z}_n)'$; thus, a standard t-test can be used to test the statistically significant of $\boldsymbol{\delta}$ effect on the production levels.

2.5 Estimation of the marginal product on the piecewise linear function

The marginal product of an input is equivalent to a partial derivative of the frontier production function with respect to an input, which is approximated as the slope on the estimated frontier production function. While it is possible to use $\widehat{\boldsymbol{\beta}}$ as an estimate of the marginal product of inputs, computational difficulty arises when the estimated frontier production function is non-smooth everywhere. The CNLS or StoNED estimator constructs piecewise linear functions that are non-differentiable at edge points. Specifically, the discontinuity of slopes at edge points causes non-unique partial derivatives (Charnes et al., 1985) as shown in figure 2.1. A partial derivative is different when taken from the left or from the right. To estimate an input's marginal products, this dissertation uses the method in Rosen et al. (1998) to find the partial derivatives of a piecewise linear function.

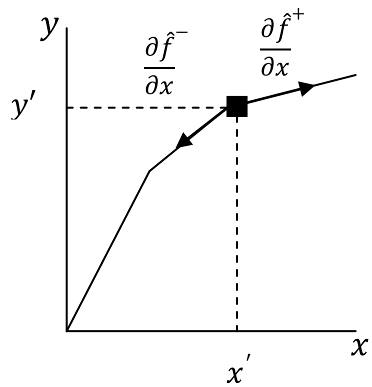

Figure 2.1 The marginal product of an input is non-unique at the edge point

The left and right partial derivatives of the frontier production function with respect to an input m at $\boldsymbol{x}$ are defined respectively as

$$\frac{\partial f(\boldsymbol{x})^-}{\partial x_m} = \lim_{h\to 0} \frac{f(x_1, \dots, x_m - h, \dots, x_M) - f(x_1, \dots, x_m, \dots, x_M)}{h} \tag{2.23}$$

$$\frac{\partial f(\boldsymbol{x})^+}{\partial x_m} = \lim_{h\to 0} \frac{f(x_1, \dots, x_m + h, \dots, x_M) - f(x_1, \dots, x_m, \dots, x_M)}{h} \tag{2.24}$$

From (2.23) and (2.24), the left and right partial derivatives of the frontier production function with respect to an input q at $\boldsymbol{x}$ are estimated respectively using

$$\frac{\partial f(\boldsymbol{x})^-}{\partial x_m} \approx -\frac{\hat{f}(\boldsymbol{x}^-) - \hat{f}(\boldsymbol{x})}{\epsilon} \tag{2.25}$$

$$\frac{\partial f(\boldsymbol{x})^+}{\partial x_m} \approx \frac{\hat{f}(\boldsymbol{x}^+) - \hat{f}(\boldsymbol{x})}{\epsilon} \tag{2.26}$$

where $\epsilon > 0$ is a small positive number, $\boldsymbol{x}^- = (x_1, x_2, .., x_m - \epsilon, \dots, x_M)$, $\boldsymbol{x}^+ = (x_1, x_2, .., x_m + \epsilon, \dots, x_M)$ and $\hat{f}(\boldsymbol{x})$ is the CNLS or StoNED estimator of the frontier production function.

While it is possible to use either $\frac{\partial f(\boldsymbol{x})^{-}}{\partial x_m}$ or $\frac{\partial f(\boldsymbol{x})^{+}}{\partial x_m}$ as an estimate for a marginal product of an input, $\frac{\partial f(\boldsymbol{x}_a, \boldsymbol{x})}{\partial x_m}$, this dissertation uses $\frac{\partial f(\boldsymbol{x})^{+}}{\partial x_m}$ because it is consistent with the definition of a marginal product of an input, i.e., an additional output produced by using an incremental amount of an input. Note that if $\boldsymbol{x}$ is not the edge point, $\frac{\partial f(\boldsymbol{x})^{-}}{\partial x_m} = \frac{\partial f(\boldsymbol{x})^{+}}{\partial x_m}$.

2.6 Testing normality against skewness of disturbances

This section reviews the Kuosmanen and Fosgerau (2009) test for skewness of a disturbance term. One difference between a neoclassical and a frontier production function is that a disturbance in a neoclassical production function has a symmetric distribution due to the normality of the random noise, while a disturbance in a frontier production function has an asymmetric distribution due to technical inefficiency. Therefore, the specification of a frontier production function as either neoclassical or frontier is based on the result of the hypothesis test considering the skewness of the disturbance term. Specifically, the hypothesis test is formulated as

H$_0$: disturbances, ε, are normally distributed.

H$_1$: disturbances, ε, are negatively skewed.

Rejection of the null hypothesis is interpreted as the evidence in the presence of negative skewness from technical inefficiency; thus, the frontier production function is supported. On the other hand, failing to reject the null hypothesis is interpreted as the lack of evidence in the presence of technical inefficiency; thus, the neoclassical production function is favorable.

Two test statistics are derived for testing the hypothesis. Let $m_j = \sum_{i=1}^{n}(\varepsilon_i - \bar{\varepsilon})^j/n$ be the sample jth central moment of the disturbance where $\bar{\varepsilon} = \sum_{i=1}^{n}\varepsilon_i/n$ is the sample mean of the disturbance. The first test statistic is the estimator of the skewness, $\sqrt{b_1}$, and is written as

$$\sqrt{b_1} = \frac{m_3}{{m_2}^{3/2}}. \tag{2.27}$$

While $\sqrt{b_1}$ can be used to test skewness of the disturbance, it may reject the null hypothesis even if the distribution is symmetric but has non-normal kurtosis (Poitras, 2006).

The second test statistic is the estimator of kurtosis, b_2, and is written as

$$b_2 = \frac{m_4}{{m_2}^2}. \tag{2.28}$$

The non-normal kurtosis can be interpreted as a sign of data problems, such as outliers or model misspecification. Specifically, the additional hypothesis test is formulated as

H_0: disturbances have normal kurtosis.

H_1: disturbances have non-normal kurtosis.

Therefore, b_2 can be used to test non-normal kurtosis of the disturbance and it provides additional information for the skewness hypothesis test.

A Monte Carlo simulation is applied to compute the critical values of the test statistics under the null hypothesis. Specifically, for a chosen large number M, M random pseudo-samples of n observations are drawn independently from $N(0,1)$ and then the $\sqrt{b_1}$ and b_2 test statistics are computed for each pseudo-sample. The critical value of the $\sqrt{b_1}$ statistic at the significant level α is the α percentile of the simulated distribution of the $\sqrt{b_1}$ statistic. Similarly, the critical value of the b_2 test statistic at the significant level α is the $\frac{\alpha}{2}$ percentile[4] of the simulated distribution of the $\sqrt{b_1}$ statistic.

Four possible outcomes results from the $\sqrt{b_1}$ and b_2 tests: 1) if the null hypothesis is not rejected by both $\sqrt{b_1}$ and b_2 tests, the result supports the neoclassic frontier production function; 2) if the null hypothesis is rejected by only $\sqrt{b_1}$ test, the result supports the frontier production function; 3) if the null hypothesis is rejected by only b_2 test, the neoclassical production function is plausible, but there may be data or model misspecification problems; 4) if the null hypothesis is rejected by both $\sqrt{b_1}$ and b_2 tests, no conclusion can be drawn due to data problems or model misspecification.

[4] The kurtosis test is the two-tailed test and the skewness test is the one-tailed test.

CHAPTER III

ESTIMATING THE SHADOW PRICES OF SO_2 AND NO_x FOR U.S. COAL POWER PLANTS: A WEAK DISPOSABILITY CNLS APPROACH*

3.1 Introduction

The Clean Air Act (CAA) which was last amended in 1990 requires the U.S. Environmental Protection Agency (EPA) to set National Ambient Air Quality Standards (NAAQS) for six of the principle pollutants considered harmful to public health and the environment. Of the six, SO_2 and NO_x are particularly egregious not only because they indirectly effect global warming (EPA, 2012b) but also increase acid rain. Title IV of the Clean Air Act Amendments (CAAA) of 1990 implemented through the Acid Rain Program and set goals to reduce annual SO_2 emissions by 10 million tons and NO_x by 2 million tons from 1980 levels via a two-phase tightening of the restrictions placed primarily on coal plants (EPA, 2007b). For SO_2, phase I (1995-1999) regulated 445 boiler units at mostly coal plants and Phase II (2000-present) regulated over 2000 boiler units with a capacity greater than 25 megawatts at all fossil fuel plants. NO_x reduction under Phase I tended to reduce annual NO_x emissions by over 400,000 tons per year, limiting the emissions of group 1 boilers (dry bottom wall fired and tangential boilers). Phase II further reduce annual NO_x emission to 2.1 million tons per year from both group 1 and group 2 boilers (wet bottom boilers, cyclones, cell burner boilers, and

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* Reprinted from Energy Economics, 34, Maethee Mekaroonreung and Andrew L. Johnson, Estimating the shadow prices of SO_2 and NO_x for U.S. coal power plants: A convex nonparametric least squares approach, 723-732., Copyright (2012), with permission from Elsevier.

vertically fired boilers), is still underway. An analysis of the effect of the Acid Rain Program is helpful in understanding the impacts in terms of reductions in pollution and the associated costs for continued reductions.

DEA has been extensively used to characterize firms' inputs usage to produce outputs as well as to measure firms' technical efficiency because DEA can estimate a multiple inputs/outputs frontier production function without assuming a functional form; however, the original DEA model constructed a frontier production function without considering undesirable outputs such as pollutants. Consequently, Färe et al. (1986) extended DEA by applying Shephard's (1970) concept of weak disposability between desirable outputs and pollutants to estimate a production frontier and evaluate the impact of environmental regulations on technical efficiency. Today, a DEA weak disposability production frontier is applied to measure firms' environmental performance. Färe et al. (1989) introduced a hyperbolic orientation to measure efficiency relative to a DEA weak disposability frontier and applied the method to measure U.S. pulp and paper mills' technical efficiency and output losses due to environmental regulations. Yaisawarng and Klein (1994) measured productivity change of U.S. coal power plants by computing Malmquist input-based productivity assuming a DEA weak disposability frontier. Tyteca (1997) measured environment performance indicators of U.S. fossil fuel power plants based on a DEA weak disposability frontier. Pasurka (2006) calculated changes in SO_2 and NO_x associated with technical change, technical efficiency change and changes in input and output levels of U.S. coal power plants using an output distance function relative to a DEA weak disposability frontier. Mekaroonreung and Johnson (2010) used

DEA and compared three approaches (hyperbolic efficiency measure; directional output distance function; linear transformation of pollutants) to estimate the technical efficiency of U.S. oil refineries. See Zhou et al. (2008) for a summary of other DEA weak disposability applications in energy and environmental studies.

Recently, Sueyoshi and Goto (2011) proposed the concept of natural and managerial disposability and applied the concepts to a DEA frontier. A non-radial efficiency measure compared environmental performances and computed the returns to scale and damages to scale of national oil companies in several countries and international oil companies. This paper will focus on the more standard weak disposability assumption as the frontier for undesirable outputs implied by managerial disposability violates free disposability of inputs.

The implementation of the weak disposability assumption relative to a variable returns to scale (VRS) frontier production function has been subject to considerable debate. For instance, Färe and Grosskopf (2003) proposed a new model to construct a VRS weakly disposable production possibility set by introducing a single abatement factor across all firms whereas Kuosmanen (2005) used a non-uniform abatement factor across firms. Kuosmanen and Podinovski (2008) demonstrated that a production possibility set constructed by a single abatement factor model does not satisfy convexity. Moreover, they proved that using non-uniform abatement factors allows the estimation of a VRS weakly disposable production possibility set to satisfy standard production axioms and the minimum extrapolation principle.

Many previous studies have estimated the shadow prices of undesirable outputs using distance functions. A ratio of the derivative of the distance function with respect to desirable output and the derivative of the distance function with respect to undesirable output characterizes the relative shadow price of the undesirable output, and parametric or nonparametric approaches can be used to estimate the distance function. The parametric approach is more widely used, because functions are everywhere differentiable. Färe et al. (1993) used an output distance function with the translog production function to estimate a shadow price of four undesirable pollutants for 1976 data describing pulp and paper mills in Michigan and Wisconsin. Coggins and Swinton (1996) took the same approach to estimate the shadow price of SO_2 for Wisconsin coal plants in 1990–1992. Färe et al. (2005) used a quadratic directional output distance function to estimate both technical efficiency and a shadow price of SO_2 for U.S. electric utilities in 1993 and 1997.

Despite its common usage, the parametric approach can be biased if the functional form is misspecified. Alternatively, a nonparametric approach, specifically DEA, can estimate a production frontier and the shadow prices of pollutants. Boyd et al. (1996) used a DEA frontier production function to estimate the shadow price of SO_2 for coal plants. Lee et al. (2002) used DEA when accounting for technical inefficiency to derive the shadow prices of SO_2, NO_x and total suspend particulates (TSP) for Korean coal- and oil-burning plants in 1990-1995. Researchers also acknowledge some major limitations of the alternative approach: greater sensitivity to outliers, the use of only a few observations to construct the frontier production function and lack of incorporating random noise in the data.

The advantages of CNLS and StoNED over DEA, i.e. they can consider random noise in the data and they are more robust to outliers than DEA, motivated us to apply them to estimate a weak disposability frontier production function. While DEA with weak disposability is well studied, there is lack of research that incorporates random noise to estimate the weak disposability frontier production function. This chapter describes the new methodology and applies it to measure the technical efficiency and to jointly estimate the shadow prices of SO_2 and NO_x for 336 boilers of 196 U.S. bituminous coal power plants under the Acid Rain Program during Phase II of CAAA. Moreover, there are no studies on the productive performance and shadow prices of SO_2 and NO_x using coal power plants during Phase II of CAAA. This chapter is organized as follows: the next section describes a method of estimating a frontier production function under weak disposability and the associated technical efficiency and shadow prices of SO_2 and NO_x. Section 3.3 describes the data set of 336 boilers of U.S. bituminous coal power plants in operation under the Acid Rain Program from 2000 to 2008. Section 3.4 presents the analysis and discusses the results and Section 3.5 summarizes the conclusions.

3.2 Model

3.2.1 A production possibility set assuming weak disposability and DEA weak disposability frontier production function

Let $\boldsymbol{x} \in R_+^M$ be a vector of inputs, $\boldsymbol{y} \in R_+^S$ be a vector of outputs and $\boldsymbol{b} \in R_+^J$ be a vector of pollutants. The production possibility set is defined as $T = \{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}): \boldsymbol{x} \text{ can produce } (\boldsymbol{y}, \boldsymbol{b})\}$. The assumptions defining the production possibility set are:

1. T is convex
2. There are variable returns to scale

The following axioms regarding production are restated when undesirable outputs are also produced:

3. Free disposability of inputs

   If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\boldsymbol{x}' \geq \boldsymbol{x}$, then $(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{b}) \in T$

4. Free disposability of outputs

   If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\boldsymbol{y}' \leq \boldsymbol{y}$, then $(\boldsymbol{x}, \boldsymbol{y}', \boldsymbol{b}) \in T$

5. Weak disposability between outputs and pollutants

   If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $0 \leq \varphi \leq 1$, then $(\boldsymbol{x}, \varphi\boldsymbol{y}, \varphi\boldsymbol{b}) \in T$

Based on the production possibilities axioms stated above, the variable returns to scale weakly disposable production possibility set T can be written as:

$$T = \{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in R_{+}^{M+S+J} | \boldsymbol{x} \geq \sum_{i=1}^{n} (\lambda_i + \mu_i)\boldsymbol{x}_i \; ; \; \boldsymbol{y} \leq \sum_{i=1}^{n} \lambda_i \, \boldsymbol{y}_i \; ;$$

$$\boldsymbol{b} \geq \sum_{i=1}^{n} \lambda_i \boldsymbol{b}_i \; ; \sum_{i=1}^{n} (\lambda_i + \mu_i) = 1, \qquad \lambda_i , \mu_i \geq 0 \, \} \tag{3.1}$$

where $\lambda_i s$ allows the convex combination of observed firms and $\mu_i s$ allows firms to scale down both outputs and pollutants while maintaining the same level of inputs. Formulation (3.1) differs from the Kuosmanen (2005) formulation in that the inequality sign in the pollutant constraints implies a negative shadow price on additional pollution and satisfies the economic intuition that pollutants incur costs to firms.

Using the weak disposable production possibility T in (3.1), the variable returns to scale output-oriented weak disposability DEA problem can be written as:

$$\max_{\theta, \lambda, \mu} \theta_o \tag{3.2.1}$$

s.t.

$$\sum_{i=1}^{n} \lambda_i \boldsymbol{y}_i \;\geq\; \theta_o \boldsymbol{y}_o \tag{3.2.2}$$

$$\sum_{i=1}^{n} \lambda_i \boldsymbol{b}_i \;\leq\; \boldsymbol{b}_o \tag{3.2.3}$$

$$\sum_{i=1}^{n} (\lambda_i + \mu_i)\boldsymbol{x}_i \;\leq\; \boldsymbol{x}_o \tag{3.2.4}$$

$$\sum_{i=1}^{n} (\lambda_i + \mu_i) = 1 \tag{3.2.5}$$

$$\lambda_i, \mu_i \;\geq 0 \qquad\qquad \forall i = 1, \ldots, n \tag{3.2.6}$$

where θ_o $\boldsymbol{y}_o$, $\boldsymbol{b}_o$ and $\boldsymbol{x}_o$ are the technical inefficiency, outputs, pollutants and inputs for observed firm o. The DEA problem (3.2) constructs the weak disposability frontier production function and estimates technical efficiency as the radial expansion of outputs.

3.2.2 Weak disposability frontier production function estimation

Consider a single output frontier production function model with a multiplicative disturbance term

$$y_i = f^w(\boldsymbol{x}_i, \boldsymbol{b}_i)\exp(\varepsilon_i) \qquad \forall i = 1,..,n \qquad (3.3)$$

where f^w is the frontier production function satisfying continuity, concavity, monotonicity and weak disposability and ε_i is the disturbance term for firm i. Note that the frontier production function in (3.3) treats pollutants as independent variables following Cropper and Oates (1992), who define this treatment as the standard approach to including pollutants within the environmental economics literature. Treating pollutants as independent variables has been used in several papers such as Pittman (1981), Hailu and Veeman (2000) and Considine and Larson (2006).

The motivations to employ a multiplicative disturbance model are twofold. First, as suggested in Kuosmanen and Kortelainen (2011), the multiplicative model allows the direct imposition of the assumptions of Constant Returns to Scale (CRS), Non-Increasing Returns to Scale (NIRS) or Non-decreasing Returns to Scale (NDRS). Specifically, CRS, NIRS and NDRS do not hold after an additive shift of the estimated frontier production function. Since the assumption of weak disposability between an output and pollutants requires the origin to be part of the convex production possibility set similar to the NIRS, the multiplicative disturbance term model is appealing. Second, the multiplicative disturbance term model helps to control for heteroskedasticity in power plants data.

3.2.2.1 Deterministic weak disposability frontier production function

Assuming that there is no statistical noise in the data; thus, any deviations from the estimated frontier are due to technical efficiency. Specifically the deterministic disturbance is written as

$$\varepsilon_i = -u_i \quad \forall i = 1, \dots, n \tag{3.4}$$

where $u_i \geq 0$ is the technical inefficiency. While noting that the CNLS objective function is to minimize the sum of square disturbances, when all of the disturbances are less than or equal to zero in the deterministic case, we can replace the sum of square disturbances by the sum of disturbances, see Kuosmanen and Johnson (2010). The multiplicative sign-constrained weak disposability CNLS problem is then formulated as

$$\min_{\alpha,\boldsymbol{w},\boldsymbol{c},\varepsilon} - \sum_{i=1}^{n} \varepsilon_i \tag{3.5.1}$$

$$\text{s.t.} \quad \varepsilon_i = \ln(y_i) - \ln(\alpha_i + \boldsymbol{w}_i^{'}\boldsymbol{x}_i + \boldsymbol{c}_i^{'}\boldsymbol{b}_i) \qquad \forall i = 1, \dots, n \tag{3.5.2}$$

$$\alpha_i + \boldsymbol{w}_i^{'}\boldsymbol{x}_i + \boldsymbol{c}_i^{'}\boldsymbol{b}_i \leq \alpha_h + \boldsymbol{w}_h^{'}\boldsymbol{x}_i + \boldsymbol{c}_h^{'}\boldsymbol{b}_i \qquad \forall i, h = 1, \dots, n \tag{3.5.3}$$

$$\alpha_i + \boldsymbol{w}_i^{'}\boldsymbol{x}_h \geq 0 \qquad \forall i, h = 1, \dots, n \tag{3.5.4}$$

$$\boldsymbol{w}_i, \boldsymbol{c}_i \geq 0, \varepsilon_i \leq 0 \qquad \forall i = 1, \dots, n. \tag{3.5.5}$$

where α_i, $\boldsymbol{w}_i$ and $\boldsymbol{c}_i$ are the coefficients characterizing the hyperplanes of the deterministic weak disposability frontier production function. The objective function maximizes the sum of the deterministic disturbance terms. Intuitively, the sign-constrained CNLS problem (3.5) estimates the hyperplanes characterizing the frontier production function that makes all firms look as efficient as possible using the minimum extrapolation principle of Banker et al. (1984), also referred to as the benefit of the doubt principle by Moesen and Cherchye (1998). The CNLS problem (3.5) has the additional inequality constraints (3.5.4) to impose the weak disposability between the output and pollutants. In line with Kuosmanen and Johnson (2010) which shows that the additive sign-constrained CNLS problem is equivalent to variable return to scale DEA problem, the multiplicative sign-constrained weak disposability CNLS problem can be shown to be equivalent to the weak disposability DEA problem (3.2) in the sense that both measure technical efficiency relative to the same estimated frontier production function

Proposition 3.1 In a single output case, the CNLS problem (3.5) is equivalent to the output-oriented weak disposability DEA problem (3.2).

Proof: See Appendix A

Moreover, the technical efficiency for firm i, TE_i, is obtained from the CNLS residual, $\hat{\varepsilon}_i, \forall i$

$$TE_i = e^{\hat{\varepsilon}_i} \qquad \forall i = 1, \dots, n. \tag{3.6}$$

Proposition 3.2 The technical efficiency estimates from the multiplicative sign-constrained weak disposability CNLS problem (3.4) equal the reciprocal of the technical efficiency estimates from the variable returns to scale output-oriented weak disposability DEA problem (3.2).

Proof: See Appendix A

Proposition 1 implies that the estimated deterministic weak disposability frontier production function from the CNLS problem (3.5) is unique since it is equivalent to the estimated DEA weak disposability frontier production function. However, it is more convenient to write the unique estimator of the frontier production function as an explicit function. Given $(\hat{\alpha}_i, \hat{\boldsymbol{w}}_i, \hat{\boldsymbol{c}}_i)$ from the sign-constrained CNLS problem (3.5), the unique CNLS estimator of the deterministic weak disposability frontier production function is written as

$$\hat{f}^{w}_{\min}(\boldsymbol{x}, \boldsymbol{b}) = \min_{\alpha, \boldsymbol{w}, \boldsymbol{c}} \{\alpha + \boldsymbol{w}'\boldsymbol{x} + \boldsymbol{c}'\boldsymbol{b} | \alpha + \boldsymbol{w}'\boldsymbol{x}_i + \boldsymbol{c}'\boldsymbol{b}_i \geq \hat{y}_i \quad \forall i = 1, \ldots, n\} \quad (3.7)$$

where $\hat{y}_i = \min_{h \epsilon \{1,\ldots,n\}} \{\hat{\alpha}_h + \hat{\boldsymbol{w}}_h{}'\boldsymbol{x}_i + \hat{\boldsymbol{c}}_h{}'\boldsymbol{b}_i\}$.

3.2.2.2 Stochastic weak disposability frontier production function

To estimate the weak disposability frontier production function assuming that there are both the random noise and the technical inefficiency in the data, the StoNED method in section 2.3.2.1 is extended. At the first stage, the multiplicative composite disturbance weak disposability CNLS problem is formulated as the following nonlinear program

$$\min_{\alpha, \boldsymbol{w}, \boldsymbol{c}, \vartheta} \sum_{i=1}^{n} \vartheta_i^2 \quad (3.8.1)$$

s.t.

$$\vartheta_i = \ln(y_i) - \ln(\alpha_i + \boldsymbol{w}_i'\boldsymbol{x}_i + \boldsymbol{c}_i'\boldsymbol{b}_i) \quad \forall i = 1, \ldots, n \quad (3.8.2)$$

$$\alpha_i + \boldsymbol{w}_i'\boldsymbol{x}_i + \boldsymbol{c}_i'\boldsymbol{b}_i \leq \alpha_h + \boldsymbol{w}_h'\boldsymbol{x}_i + \boldsymbol{c}_h'\boldsymbol{b}_i \quad \forall i, h = 1, \ldots, n \quad (3.8.3)$$

$$\alpha_i + \boldsymbol{w}_i'\boldsymbol{x}_h \geq 0 \quad \forall i, h = 1, \ldots, n \quad (3.8.4)$$

$$\boldsymbol{w}_i, \boldsymbol{c}_i \geq 0 \quad \forall i = 1, \ldots, n \quad (3.8.5)$$

where α_i, $\boldsymbol{w}_i$ and $\boldsymbol{c}_i$ are the coefficients characterizing the hyperplanes of the average weak disposability frontier production function.

Because CNLS identifies a production function that minimizes the sum of squared disturbances among all production functions that are continuous, monotonic increasing, concave and satisfy the weak disposability assumptions, it is important to check the following condition for the objective function.

Proposition 3.3 The objective function in the CNLS problem (3.8) is a convex function if and only if $\frac{y_i}{\alpha_i+\boldsymbol{w}_i'\boldsymbol{x}_i+\boldsymbol{c}_i'\boldsymbol{b}_i} \geq \frac{1}{e} \quad \forall i = 1, \ldots, n.$

Proof: See Appendix A

If the CNLS problem (3.8) has a convex objective function, then a local optimum to (3.8) is also a global optimum simplifying the optimization algorithms needed to find the global optimal solution to (3.8).

Given $\left(\hat{\alpha}_i, \widehat{\boldsymbol{w}}_i, \hat{\boldsymbol{c}}_i, \hat{\vartheta}_i\right) \forall i$ from the CNLS problem (3.8), the unique StoNED estimator of the stochastic weak disposability frontier production function is written as

$$\hat{f}^{w}_{\min}(\boldsymbol{x}, \boldsymbol{b}) = \left[\min_{\alpha, \boldsymbol{w}, \boldsymbol{c}}\{\alpha + \boldsymbol{w}'\boldsymbol{x} + \boldsymbol{c}'\boldsymbol{b} | \alpha + \boldsymbol{w}'\boldsymbol{x}_i + \boldsymbol{c}'\boldsymbol{b}_i \geq \hat{y}_i \quad \forall i = 1, \ldots, n\}\right] e^{\hat{\mu}} \tag{3.9}$$

where $\hat{y}_i = \min_{h \epsilon \{1,\ldots,n\}}\{\hat{\alpha}_h + \widehat{\boldsymbol{w}}_h{}'\boldsymbol{x}_i + \hat{\boldsymbol{c}}_h{}'\boldsymbol{b}_i\}$ and $\hat{\mu} = \hat{\sigma}_u\sqrt{2/\pi}$ obtained from the method of moment described in (2.8) and (2.9).

Given $\hat{\sigma}_u$ and $\hat{\sigma}_v$, the method introduced in Jondrow et al. (1982) can be used to estimate firm-specific inefficiency. Specifically, the conditional expectation of the technical inefficiency given the CNLS residual is written as

$$\hat{E}(u_i|\hat{\varepsilon}_i) = -\frac{\hat{\varepsilon}_i\hat{\sigma}_u{}^2}{\hat{\sigma}_u{}^2 + \hat{\sigma}_v{}^2} + \frac{\hat{\sigma}_u{}^2\hat{\sigma}_v{}^2}{\hat{\sigma}_u{}^2 + \hat{\sigma}_v{}^2}\left[\frac{\phi(\hat{\varepsilon}_i/\hat{\sigma}_v{}^2)}{1 - \Phi(\hat{\varepsilon}_i/\hat{\sigma}_v{}^2)}\right] \tag{3.10}$$

where $\hat{\varepsilon}_i = \hat{\vartheta}_i - \hat{\mu}$, ϕ is the standard normal density function and Φ is the standard normal cumulative distribution.

3.2.2.3 Neoclassical weak disposability frontier production function

In line with section 2.3.2.2, the average weak disposability frontier production function, g, is equivalent to the neoclassical weak disposability frontier production function, f. Given $(\hat{\alpha}_i, \widehat{\boldsymbol{w}}_i, \hat{\boldsymbol{c}}_i)\ \forall i$ from the CNLS problem (3.8), the unique CNLS estimator of the neoclassical weak disposability frontier production function is written as

$$\hat{f}^{w}_{\min}(\boldsymbol{x}, \boldsymbol{b}) = \min_{\alpha, \boldsymbol{w}, \boldsymbol{c}} \{\alpha + \boldsymbol{w}'\boldsymbol{x} + \boldsymbol{c}'\boldsymbol{b} | \alpha + \boldsymbol{w}'\boldsymbol{x}_i + \boldsymbol{c}'\boldsymbol{b}_i \geq \hat{y}_i \quad \forall i = 1, \dots, n\} \quad (3.11)$$

where $\hat{y}_i = \min_{h \epsilon \{1,\dots,n\}} \{\hat{\alpha}_h + \widehat{\boldsymbol{w}}_h{}'\boldsymbol{x}_i + \hat{\boldsymbol{c}}_h{}'\boldsymbol{b}_i\}$.

In section 3.4, the skewness and kurtosis of CNLS residuals tests are applied to select between the neoclassical or the frontier production function model. The results provide evidence for the presence of inefficiency.

3.2.3 Estimating shadow prices of pollutants

Assuming profit-maximizing behavior for each firm, the profit maximization problem for a production process with outputs and pollutants is

$$\pi(\boldsymbol{p}_y, \boldsymbol{p}_b, \boldsymbol{p}_x) = \max_{\boldsymbol{y}, \boldsymbol{b}, \boldsymbol{x}} \boldsymbol{p}_y{}'\boldsymbol{y} - \boldsymbol{p}_b{}'\boldsymbol{b} - \boldsymbol{p}_x{}'\boldsymbol{x} \quad (3.12.1)$$

s.t. $\quad F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y}) = 0 \quad$ (3.12.2)

where $\boldsymbol{p}_y = (p_{y_1}, \dots, p_{y_S})$, $\boldsymbol{p}_b = (p_{b_1}, \dots, p_{b_J})$and $\boldsymbol{p}_x = (p_{x_1}, \dots, p_{x_M})$ represent the price vectors of outputs, pollutants and inputs, respectively. $F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y})$ is the weak disposability transformation function corresponding to a multi-output production function. Since this study is interested in the shadow prices of pollutants, it imposes the constraint $F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y}) = 0$ so that only the frontier of the production possibility set is considered. Problem (3.13) applies the method of Lagrangian multipliers to (3.12)

$$\max_{\boldsymbol{y},\boldsymbol{b},\boldsymbol{x}} \boldsymbol{p}_y{}'\boldsymbol{y} - \boldsymbol{p}_b{}'\boldsymbol{b} - \boldsymbol{p}_x{}'\boldsymbol{x} + \zeta F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y}) \tag{3.13}$$

where ζ is the Lagrangian multiplier of the constraint. The first-order conditions (FOCs) of the problem (3.14) are

$$p_{y_s} + \zeta \frac{\partial F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y})}{\partial y_s} = 0 \tag{3.14.1}$$

$$-p_{b_j} + \zeta \frac{\partial F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y})}{\partial b_j} = 0 \tag{3.14.2}$$

$$-p_{x_m} + \zeta \frac{\partial F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y})}{\partial x_m} = 0 \tag{3.14.3}$$

$$0 = F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y}). \tag{3.14.4}$$

The shadow prices of pollutants are written as

$$p_{b_j} = p_{y_s} \left(\frac{\partial F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y})}{\partial b_j} \Big/ \frac{\partial F^w(\boldsymbol{x}, \boldsymbol{b}, \boldsymbol{y})}{\partial y_s} \right). \tag{3.15}$$

In the case of a single output production function (S=1), the first equality of the FOC (3.14.1) can be written as $p_y - \zeta = 0$, that is, the Lagrangian multiplier is equal to the price of an output. Thus, if the price of an output is known, the shadow prices of each pollutant can be estimated using the second equality in (3.14.2). The relative shadow prices of pollutants j for firm i are estimated as:

$$p_{b_{j_i}} = p_{y_i} \frac{\partial f^w(\boldsymbol{x}_i, \boldsymbol{b}_i)}{\partial b_{j_i}} \tag{3.16}$$

where p_{y_i} is the price of an output for firm i.

The partial derivative of the weak disposability frontier production function with respect to a pollutant j at observed data is needed as an input in calculating the relative shadow prices of pollutants (3.16). This study uses the estimator of the right partial derivative (2.26). Specifically, the partial derivative of the weak disposability frontier production function with respect to a pollutant j at $(\boldsymbol{x}_i, \boldsymbol{b}_i)$ for firm i is numerically estimated as

$$\frac{\partial f^w(\boldsymbol{x}_i, \boldsymbol{b}_i)}{\partial b_{j_i}} \approx \frac{\hat{f}^w_{\min}\left(\boldsymbol{x}_i, \boldsymbol{b}_i^{\,+}\right) - \hat{f}^w_{\min}(\boldsymbol{x}_i, \boldsymbol{b}_i)}{\epsilon} \tag{3.18}$$

where $\epsilon > 0$ is a small positive number and $\left(\boldsymbol{x}_i, \boldsymbol{b}_i^{\,+}\right) = (\boldsymbol{x}_i, b_{1_i}, b_{2_i}, \dots, b_{j_i} + \epsilon, \dots, b_{J_i})$. The choices of equation for $\hat{f}^w_{\min}$ depend on the frontier types; $\hat{f}^w_{\min}$ in (3.7), (3.9) and (3.11) are used for the deterministic, stochastic and neoclassical weak disposability frontier production function respectively.

To summarize, the method to estimate the technical efficiency and the shadow prices of pollutants is as follows:

1. If assume the deterministic disturbances (there is no random noise in the data)

   1.1 Solve the multiplicative sign-constrained weak disposability CNLS problem (3.5) to obtain $(\hat{\alpha}_i, \widehat{\boldsymbol{w}}_i, \hat{\boldsymbol{c}}_i, \hat{\varepsilon}_i)\ \forall i$.

   1.2 For each firm i, estimate the technical efficiency by using (3.6).

1.3 For each firm i, using (3.18) and $\hat{f}^{w}_{\min}$ in (3.7) to calculate $\frac{\partial f^{w}(\boldsymbol{x}_i,\boldsymbol{b}_i)}{\partial b_{j_i}}$. Then, estimate the shadow price of pollutant j by using (3.16).

2. If assume the composite disturbances (there is random noise in the data)

2.1 Solve the multiplicative composite disturbance weak disposability CNLS problem (3.8) to obtain $\left(\hat{\alpha}_i, \hat{\boldsymbol{w}}_i, \hat{\boldsymbol{c}}_i, \hat{\vartheta}_i\right) \forall i$.

2.2 Use $\hat{\vartheta}_i \; \forall i$ to estimate the $\sqrt{b_1}$ test statistic in (2.27) and the b_2 test statistic in (2.28). Then, test the presence of technical inefficiency by applying the skewness and kurtosis of disturbances tests described in section 2.6.

2.3 If the test support the stochastic weak disposability frontier production function

2.3.1 Use $\hat{\vartheta}_i \; \forall i$ to estimate $\hat{\sigma}_u$ in (2.8), $\hat{\sigma}_v$ in (2.9) and expected technical inefficiency, $\hat{\mu}$, in (2.10).

2.3.2 For each firm i, estimate the conditional expectation of the technical inefficiency, $\hat{E}(u_i|\hat{\varepsilon}_i)$, using (3.10).

2.3.3 For each firm i, using (3.18) and $\hat{f}^{w}_{\min}$ in (3.9) to calculate $\frac{\partial f^{w}(\boldsymbol{x}_i,\boldsymbol{b}_i)}{\partial b_{ij}}$. Then, estimate the shadow price of pollutant j by using (3.16).

2.4 If the test support the neoclassical weak disposability frontier production function

2.4.1 For each firm i, using (3.18) and $\hat{f}^{w}_{\min}$ in (3.11) to calculate $\frac{\partial f^{w}(\boldsymbol{x}_i,\boldsymbol{b}_i)}{\partial b_{ij}}$. Then, estimate the shadow price of pollutant j by using (3.16).

3.3 Data set

The balanced panel boiler-level data characterizes 336 units of U.S. bituminous coal-burning electricity plants in operation from 2000 to 2008. Bituminous coal power plants are mostly located in the eastern states and these power plants produce about 50% of the total electricity generated from coal. The location of the bituminous coal power plants in the data set are shown in figure 3.1. This form of coal has very high sulfur content. All boilers in the sample are either wall or tangential fired boilers, sub-groups of pulverized coal-fired boilers, which are regulated by the Acid Rain Program.

While it is possible that coal power plants maybe use other types of coal such as subbituminous and lignite, this chapter assumes that all coal power plants in the data set use bituminous coal as a main fuel. First, all coal power plants in the data set are reported in EPA (2012a) as power plants that use bituminous coal as primary fuel. Second, the eastern states coal power plants are located close to the bituminous coal fields (figure 3.2) in the Appalachian Mountains and parts of the Midwest while the western states coal power plants are located close to bituminous fields in Colorado and Utah. Due to the proximity to the bituminous fields, it is likely that these coal power plants should use bituminous coal as a primary source of fuel.

Figure 3.1 Location of the U.S. bituminous coal power plants under the Acid Rain Program in the data set

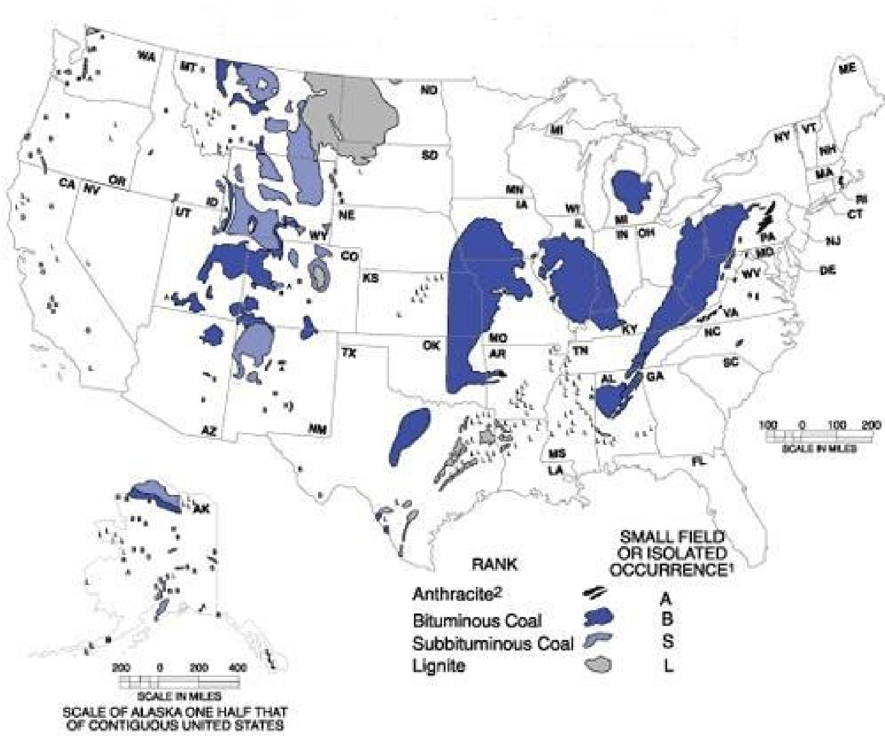

Figure 3.2 The U.S. coal reserves (EIA, 1999).

The output is the annual amount of electricity generated (in Megawatt-hours, MWh). The pollutants are the annual amount of SO_2 (tons) and NO_x (tons). The two inputs are capital and heat. The heat input (mmBtu), calculated by multiplying the quantity of fuel with the fuel's heat content, is the measure of fuel utilization. Information on electricity generated, amount of pollutants and heat input quantities are reported by the EPA database (EPA, 2011).

The boiler size (MW), the maximum rated output of a generator under specific conditions, is used as an instrumental variable for capital. The EPA's database reported the maximum heat input capacity (mmBtu/hr), a unit's maximum designed hourly heat input rate observed in the past five years, for each boiler unit. This study converts the maximum heat input capacity to estimate the boilers' sizes. The boilers' sizes in the sample range between 100 and 1426 MW.

Electricity prices ($/MWh) of each utility are reported in EIA861 (EIA, 2011b). Some of the utilities do not generate electricity; thus, this study matches the power plants in the sample to those utilities in which they have electricity production data and assume that electricity price in those utilities are the same as in power plants. Following Färe et al. (2005) approach, this study assumes that all generating boilers in the same power plants have the same electricity prices. This study derived electricity prices for each boiler by the average price of electricity sales for customers and for resale of each corresponding utility. For some utilities without electric price information, this study uses the state average retail electricity price reported in EIA (2011a).

From the original 491 bituminous coal power plant boilers data, this study constructs a 9 year balances panel data set based on the input output information described above. This study dropped 97 boilers for which their size are less than 100 MW, 55 boilers for which they are not pulverized coal-fired boilers and 3 boilers for which there are missing data on electricity and pollutants, leaving 336 boilers units in the sample. There was no entry or exit of coal power plants observed in the data gathered over this time horizon. The summary statistics are presented in Table 3.1.

Table 3.1 Statistics for the boiler units in the coal power plants (n=336)

Year	Variable[a]	Mean	Std. dev.	Min.	Max.
2000	Electricity	2141	1640	257	8315
	SO_2	12.57	11.45	0.30	76.28
	NO_x	4.69	3.55	0.78	18.68
	Heat input	20855	15672	3201	79135
	Price	52.26	14.51	17.26	113.80
2001	Electricity	2029	1618	236	10378
	SO_2	11.64	10.79	0.25	63.57
	NO_x	4.31	3.34	0.52	20.89
	Heat input	19741	15306	2283	86749
	Price	54.13	17.45	20.71	115.50
2002	Electricity	2062	1689	261	10474
	SO_2	11.33	10.73	0.24	87.59
	NO_x	4.22	3.42	0.36	20.97
	Heat input	19945	15762	2737	88046
	Price	50.44	17.06	21.44	111.60
2003	Electricity	2127	1721	242	10210
	SO_2	12.02	12.07	0.26	83.56
	NO_x	3.99	3.19	0.69	20.17
	Heat input	20412	15854	2291	92378
	Price	52.83	16.60	21.54	124.40
2004	Electricity	2097	1700	251	9940
	SO_2	11.67	11.38	0.22	75.75
	NO_x	3.57	2.80	0.40	15.17
	Heat input	20027	15674	2526	83167
	Price	55.13	17.46	22.42	125.50

Table 3.1 Continued

Year	Variable[a]	Mean	Std. dev.	Min.	Max.
2005	Electricity	2166	1785	266	11155
	SO_2	11.81	11.85	0.19	80.98
	NO_x	3.48	2.73	0.42	15.15
	Heat input	20605	16299	2803	92853
	Price	59.65	18.67	24.71	139.50
2006	Electricity	2160	1749	200	10363
	SO_2	11.25	11.40	0.20	71.92
	NO_x	3.42	2.74	0.38	16.59
	Heat input	20401	15939	2191	83026
	Price	65.43	19.36	28.58	154.50
2007	Electricity	2187	1765	83	10094
	SO_2	10.34	11.40	0.13	92.63
	NO_x	3.30	2.75	0.31	14.78
	Heat input	20789	16388	886	95973
	Price	67.53	20.85	23.57	152.20
2008	Electricity	2180	1763	83	10094
	SO_2	10.31	11.41	0.13	92.63
	NO_x	3.30	2.75	0.31	14.78
	Heat input	20740	16388	886	95973
	Price	73.98	22.04	36.06	165.70
	Boiler size	336	240	100	1426

[a] Unit of electricity, SO_2, NO_x, heat input, electricity price and boiler size are 10^3 MWh, 10^3 ton, 10^3 ton, 10^3 mmBTU, \$/MWh and MW, respectively.

3.4 Empirical results

To test whether the assumption of the frontier production function is more appropriate than the neoclassical production function, the skewness test is applied. The null hypothesis H_0 is: disturbances that are normally distributed is tested against an alternative hypothesis H_1: disturbances are negative skewed.[5] To provide additional information for the skewness hypothesis test, an additional hypothesis H_0, disturbances

[5] The simulated distribution of the skewness test statistic, $\sqrt{b_1}$, and the kurtosis test statistic, b_2, are constructed by a simple Monte Carlo simulation using M=10,000 Pseudo-samples of n=336 observations from $N(0,1)$.

are normal kurtosis, is tested against an alternative hypothesis H_1, disturbances are non-normal kurtosis. Table 3.2 reports the $\sqrt{b_1}$ and b_2 test statistics and the relevant p-values of the normality tests. As expected, the $\sqrt{b_1}$ statistics are negatively signed. At the 5% significance level, normality is rejected in favor of skewness in 2001–2004 and 2006-2008, which supports the frontier model, and cannot rejected in 2000 and 2005, which supports the neoclassical assumption. Thus, in these two years, this study uses the neoclassical production function model in which the disturbances contain only noise.

Table 3.2 Results of the skewness and kurtosis tests

Year	Test Statistics		P-values	
	$\sqrt{b_1}$	b_2	$\sqrt{b_1}$	b_2
2000	-0.084	3.987	0.263	0.003
2001	-0.271	4.022	0.020	0.003
2002	-0.382	4.069	0.002	0.002
2003	-0.546	4.004	0.000	0.003
2004	-0.496	4.738	0.000	0.000
2005	-0.168	3.334	0.102	0.095
2006	-0.618	4.785	0.000	0.000
2007	-0.560	5.334	0.000	0.000
2008	-0.488	5.318	0.000	0.000

Table 3.3 reports the estimated average shadow prices of SO_2 and NO_x, technical inefficiencies and related statistics, assuming a deterministic frontier production function. The estimated average shadow prices of SO_2 over the 9-year time horizon, range between 509 and 2,020 \$/ton and the estimated average prices of NO_x are between

3,671 and 11,679 \$/ton. The estimated average technical inefficiencies range between 0.883 and 0.902.

Table 3.4 reports the estimated average shadow prices of SO_2 and NO_x, technical inefficiencies and related statistics, using the production function model (2001-2004, 2006-2008) and the neoclassical production function model (2000, 2005). The estimated average shadow prices of SO_2 range between 201 and 343 \$/ton and the estimated average shadow prices of NO_x between 409 and 1,352 \$/ton. The estimated average technical inefficiencies range between 0.927 and 0.943. For all data sets, the estimated second and third moments of the residual, $\widehat{M}_2$ and $\widehat{M}_3$, have the correct signs; thus, the expected inefficiency terms can be calculated and used to estimate the shadow prices of both pollutants. The convexity condition, $\frac{y_i}{\hat{\alpha}_i+\widehat{\boldsymbol{w}}_i{}' \boldsymbol{x}_i+\hat{\boldsymbol{c}}_i{}' \boldsymbol{b}_i} \geq \frac{1}{e} \approx 0.368 \ \forall i,$ in Proposition 3.3 is satisfied for each year of data, which indicates that the objective function is globally convex. Therefore, a global optimal solution can always be found using standard nonlinear programming methods.

This study finds that applying the deterministic method results in higher estimated shadow prices than when assuming a composite or random disturbance term. Moreover, the estimated shadow prices in the deterministic case have a wider range. The deterministic weak disposability frontier production function estimates are more sensitive to variation. If outliers are present in the data set, the estimated frontier tends to have larger steep regions, thus $\frac{\partial f^w(\boldsymbol{x}_i,\boldsymbol{b}_i)}{\partial b_{j_i}}$ is large and the estimated shadow prices are higher. In general, when only a few extreme observations are used to construct a frontier, the result is more variation in the estimated shadow price.

Figure 3.3 shows the estimated average shadow prices compared with previous studies; note that every study uses different data sets and estimation methods as summarized in Table 3.5. From Figure 3.3, two conclusions can be drawn. First, the estimated shadow prices of SO_2 in this study using the stochastic or neoclassical production function models, ranging from 201-343 \$/ton, contain the estimates of Coggins and Swinton (1996) and are close to Färe et al. (2005). More importantly, they are in the range of EPA's SO_2 allowance auction prices. Second, the results also confirm that the estimated shadow prices from the stochastic or neoclassical production function models are generally lower than those from deterministic frontier models, and are likely more reasonable estimates of the prices from the EPA's allowance markets. Excluding Coggins and Swinton (1996),[6] the estimated average shadow price of SO_2 from deterministic frontier models (including DEA) are 509-3,107 \$/ton compared to 76-343\$/ton from the stochastic or neoclassical models. Table 3.6 shows that the SO_2 market prices are 130-1,550 \$/ton and the allowance auction prices are 126-860 \$/ton.[7] This study concludes that using the stochastic or neoclassical weak disposability frontier production function models provide more consistent estimates of market prices compared to using deterministic weak disposability frontier production function model.

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[6] Compared to other studies, this paper considers a limited number of boilers in Wisconsin all facing similar state regulations. The boilers in this study tend to have similar production characteristics; thus, if the data was collected carefully, the assumption a deterministic disturbance is more appropriate than in other studies with more heterogeneity and noise.

[7] Allowance auction price is the price for which the allowance is sold to the highest bidder in the annual EPA auction until no allowances remain. Market price is the price for which the allowance is traded on the open market.

Third, the estimated average shadow prices of NO_x are higher than SO_2. Using the stochastic or neoclassical weak disposability frontier production function models, the estimated average NO_x shadow prices of 409-1352 \$/ton are higher than the estimated average SO_2 shadow prices of 201-343\$/ton. This conclusion coincides with the observed prices in the SO_2 and NO_x allowance markets.

Table 3.6 shows the comparison of the estimated average shadow prices in this study to the pollutants' market prices. The SO_2 markets prices are obtained from EPA (2001, 2002a, 2003a, 2004a, 2005a 2006a, 2007a, 2008a, 2009a) and the NO_x markets prices are obtained from EPA (2003b, 2004b, 2005b, 2006b, 2007c, 2008b, 2009b). The estimated average SO_2 shadow prices in this study are slightly over-estimated between 2000 to 2003, within the range for 2004 and 2008 and under-estimated between 2005 and 2007, because the estimated average SO_2 shadow prices in this study are relatively stable year to year while the SO_2 market prices starts to increase in 2003, spikes during 2004-2005 and declines after 2005. The estimated average NO_x shadow prices in this study are within the range or close to NO_x market prices except in 2003, 2004 and 2005. During this time, the estimated average NO_x shadow prices in this study are lower than the market price because the NO_x market prices increase sharply. However, the estimated average NO_x shadow prices in this study have a similar trend of rising prices in 2004, 2005 and 2006 and dropping prices in 2007 and 2008. Figure 3.4 and 3.5 illustrate the estimated average shadow prices and the market prices.

Table 3.3 Statistics of the estimated shadow prices of SO_2 and NO_x ($/ton) and technical inefficiency in the deterministic weak disposability frontier production function model

Variable	Mean[a]	Std. dev.[a]	Min	Max
2000				
$Price_{SO2}$	743	11140	0	368805
$Price_{NOx}$	7536	9053	0	130271
TE	0.884	0.077	0.674	1.000
2001				
$Price_{SO2}$	1291	16816	0	391923
$Price_{NOx}$	9234	11713	0	58875
TE	0.879	0.077	0.665	1.000
2002				
$Price_{SO2}$	507	1041	0	18176
$Price_{NOx}$	4597	6425	0	52715
TE	0.888	0.072	0.685	1.000
2003				
$Price_{SO2}$	691	2712	0	79618
$Price_{NOx}$	6187	11262	0	80503
TE	0.886	0.073	0.637	1.000
2004				
$Price_{SO2}$	734	4296	0	125749
$Price_{NOx}$	7799	11272	0	75287
TE	0.883	0.073	0.612	1.000
2005				
$Price_{SO2}$	1033	11274	0	358869
$Price_{NOx}$	11679	13692	0	75962
TE	0.892	0.066	0.697	1.000
2006				
$Price_{SO2}$	954	9457	0	331175
$Price_{NOx}$	9994	13356	0	81886
TE	0.902	0.063	0.700	1.000
2007				
$Price_{SO2}$	780	8074	0	229204
$Price_{NOx}$	4044	7367	0	83184
TE	0.875	0.075	0.684	1.000
2008				
$Price_{SO2}$	2020	20215	0	458105
$Price_{NOx}$	3671	5583	0	73031
TE	0.869	0.077	0.618	1.000

[a] Weighted average by the amount of pollutants

Table 3.4 Statistics of the estimated shadow prices of SO_2 and NO_x ($/ton) and technical inefficiency in the stochastic and neoclassical weak disposability frontier production function model

Variable	Mean[a]	Std. dev.[a]	Min	Max
2000				
$Price_{SO2}$	201	255	0	2573
$Price_{NOx}$	1354	1281	0	8035
TE	N/A	N/A	N/A	N/A
2001				
$Price_{SO2}$	293	388	0	3218
$Price_{NOx}$	848	1649	0	11955
TE	0.943	0.036	0.818	1
2002				
$Price_{SO2}$	318	453	0	4899
$Price_{NOx}$	811	1542	0	16542
TE	0.938	0.042	0.801	1
2003				
$Price_{SO2}$	230	418	0	3338
$Price_{NOx}$	691	2047	0	17648
TE	0.927	0.049	0.743	1
2004				
$Price_{SO2}$	219	410	0	3215
$Price_{NOx}$	1211	3344	0	30712
TE	0.934	0.044	0.740	1
2005				
$Price_{SO2}$	246	1467	0	37648
$Price_{NOx}$	1352	3703	0	19071
TE	N/A	N/A	N/A	N/A
2006				
$Price_{SO2}$	343	3097	0	108436
$Price_{NOx}$	1301	3687	0	32640
TE	0.935	0.043	0.757	1
2007				
$Price_{SO2}$	237	421	0	4907
$Price_{NOx}$	409	1456	0	16775
TE	0.937	0.042	0.777	1
2008				
$Price_{SO2}$	239	666	0	11572
$Price_{NOx}$	609	2082	0	28770
TE	0.931	0.046	0.745	1

[a] Weighted average by the amount of pollutants

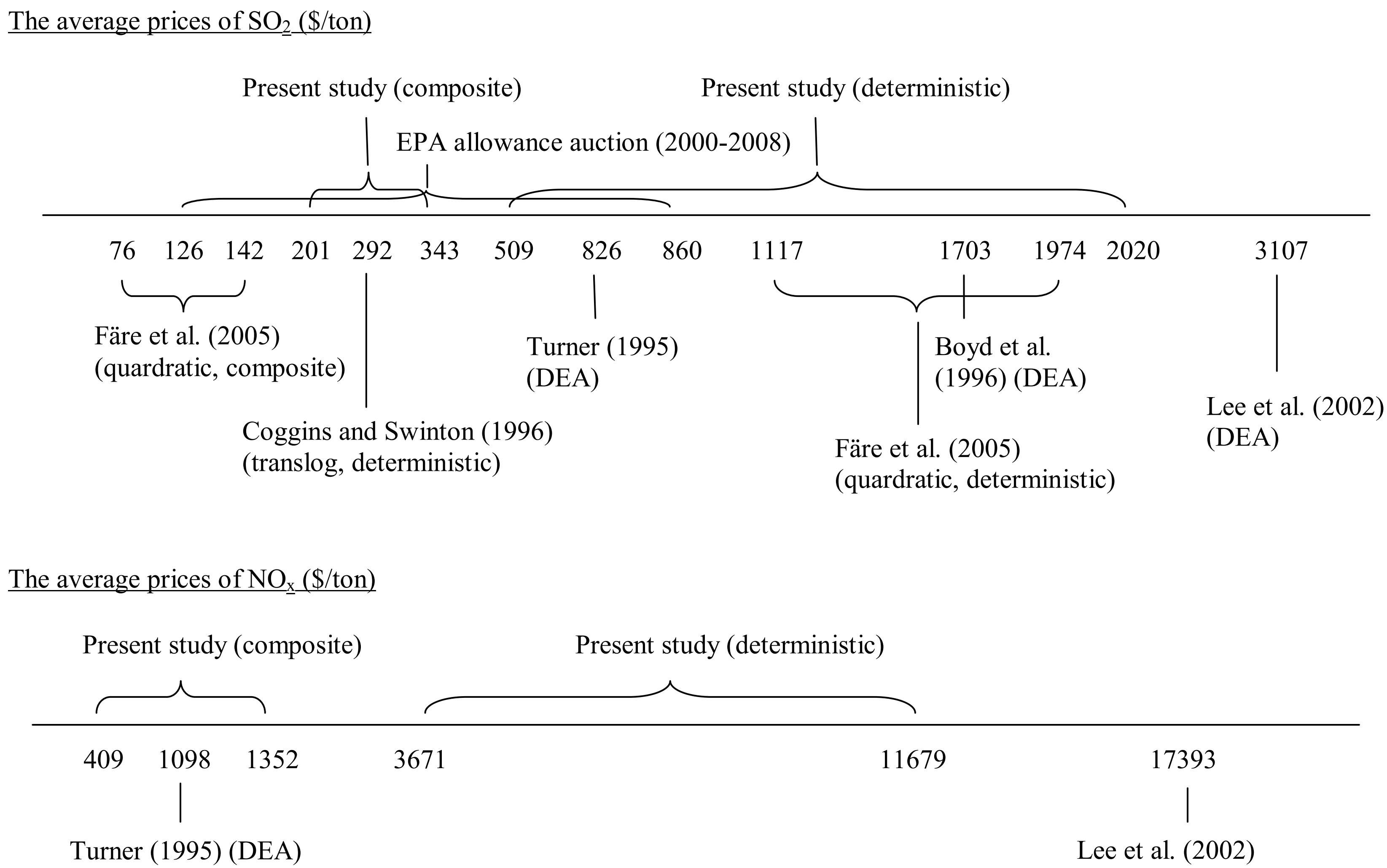

Figure 3.3 Comparison of the estimated average shadow prices of SO_2 and NO_x

Table 3.5 Data set comparisons of the electricity price used to estimate shadow prices of the pollutants

Study	Country	Year	Sample size	Price of electricity ($/MWh)
Boyd et al. (1996)	U.S.	1989	62	50.00
Coggin and Swinton (1996)	U.S.	1990-1992	42	36.38-65.87
Färe et al. (2005)	U.S.	1993, 1997	209	10.39-100.42
Lee et al. (2002)	Korea	1990-1995	43	66.67
Present study	U.S.	2000-2008	336	17.26-165.70

Table 3.6 Comparison between the pollutants market prices ($/ton) and the average shadow price estimates from the present study using composite disturbance case ($/ton)

Year	SO_2 prices		NO_x prices[a]	
	Market	Present study	Market	Present study
2000	130-155	201		1354
2001	135-210	293	600-1700[b]	848
2002	130-170	318		811
2003	150-220	230	2500-8000	691
2004	215-700	219	2100-3700	1211
2005	700-1550	246	2000-3500	1352
2006	430-740	343	900-2725	1301
2007	500-600	237	500-1000	409
2008	179-509	239	592-1400	609

[a]For 2003, 2004 and 2005, the range of NO_x market prices are approximated from graphs; for the other years, the range of NO_x market prices are explicitly stated in the EPA reports.
[b]For 2000, 2001 and 2002, EPA published three years of progress in a single OTC NO_x budget program report.

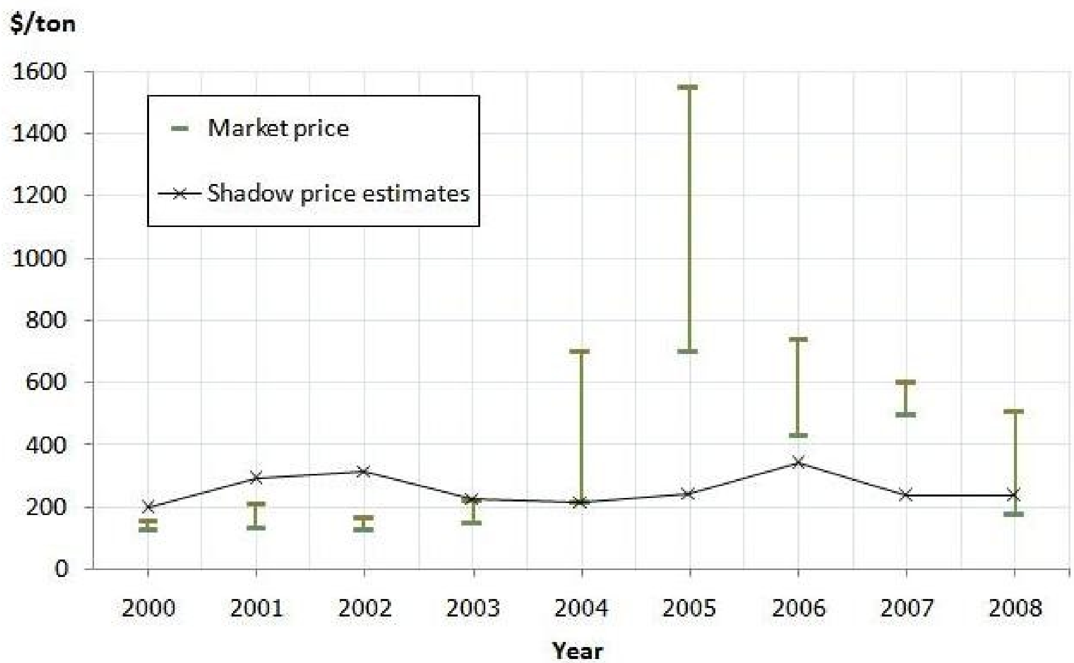

Figure 3.4 SO_2 market prices and the estimated average shadow prices ($/ton)

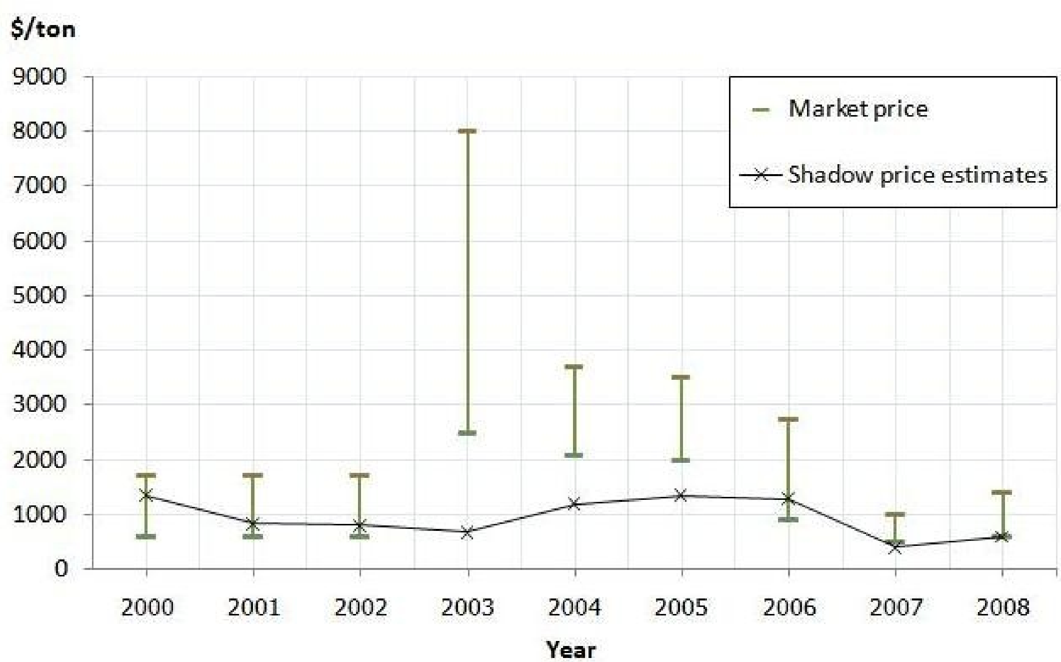

Figure 3.5 NO_x market prices and the estimated average shadow prices ($/ton)

Recall that the shadow prices of pollutants are estimates of the marginal abatement costs which should reflect the market prices for EPA's pollutant allowances. The estimated shadow prices in this study are derived based solely on the plants' production data; however, several other factors can affect the market allowance price. By allowing plants to buy, sell and bank allowances, the allowance prices reflect the cost of compliance with future regulation. The sharp increase in SO_2 and NO_x prices resulting from the EPA's Clean Air Interstate Rule (CAIR) programs which require further SO_2 and NO_x reduction from coal boilers beginning in 2010, caused an increase in the expected pollutant control costs in the future and provided incentives for plants to buy allowances and bank them for future use. Thus, allowance prices rose due to increased demand for allowances. After 2005, emission levels fell due to the increased use of gas-fired boilers and pollution control equipment. Thus, a sufficient supply of allowances in the market caused allowance market prices to fall.

3.5 Conclusions

This chapter proposes and implements a nonparametric methodology to estimate a frontier production function when pollutants are a result of the production process. It assumes that the traditional production axioms such as continuity, monotonicity and concavity with weak disposability between an output and the pollutants characterize the shape of an underlying frontier production function. In deterministic disturbance cases assuming no random noises in the data and an exact model specification, this chapter modifies the CNLS problem to minimize the sum of firms' one-sided deviations. In composite disturbance cases assuming there exists the random noise and technical inefficiency in the data, this chapter extends the StoNED method to include the weak disposability axiom. Similarly, in random disturbance cases assuming no technical inefficiency, this chapter extends the CNLS problem to include the weak disposability constraints. For each case, this chapter shows how to derive the unique estimator of the weak disposability frontier production functions which are later used to estimate shadow prices of pollutants. The proposed methodology was applied to derive the technical efficiencies and the SO_2 and NO_x shadow prices of 336 boilers of U.S. bituminous coal power plants under the Acid Rain Program during 2000-2008.

The main finding of this study is that, applying the StoNED method to estimate the stochastic weak disposability frontier production function, the estimated average shadow prices of SO_2 are between 201 and 343 $/ton and the estimated average shadow prices of NO_x are between 409 and 1,352 $/ton. Both estimated average shadow prices of SO_2 and NO_x are in reasonable ranges compared to the allowance market prices in the U.S. The proposed method can be applied to estimate shadow prices of other pollutants which can be used as references for marginal abatement costs for the industry. This marginal abatement cost is solely derived from production data so that it is not affected by market complexity.

Using the method to estimate the shadow prices of other pollutants will provide information to regulators and the power industry to establish references for marginal abatement costs. From the results in this chapter, it recommends the use of weak disposability StoNED method over weak disposability DEA which is likely to overestimate shadow prices due to extreme observations. Further cost analysis tools, such as the ones proposed in this chapter, will provide information to the EPA that maybe helpful in investigating the outcomes of their on-going pollution control policies.

CHAPTER IV

IMPOSING CONSERVATION OF MASS IN POLLUTANT FUNCTION ESTIMATES: NO_x GENERATION IN COAL-FIRED POWER PLANTS

4.1 Introduction

Pollution abatement is an important process for polluting firms because it allows firms to reduce their pollutants levels without having to reduce the production levels so that they can still produce outputs to meet customer demands and meet pollution regulations. In this case, the weak disposability between outputs and pollutants does not properly characterize the production of polluting firms with abatement processes. The new model of production that considers abatement process is necessary. Therefore, this chapter introduces a two-stage model of production and abatement. In the first stage, inputs are used to produce outputs resulting in byproduct pollutants. In the second stage, the pollutants are abated using abatement inputs where the law of conservation of mass is imposed. A pollutant function and the corresponding abatement cost and marginal abatement cost functions are developed.

While generally ignored in neoclassical economics modeling, the concept of conservation of mass has drawn attention in environmental economics modeling because it provides a physical link between outputs and pollutants. Ayres and Kneese (1969) modeled an entire economy as a material balance problem consisting of raw materials, goods, recycled and waste flows. Georgescu-Roegen (1986), Daly (1997) Baumgärter (2004) and Ebert and Welsch (2007) stated that production follows the law of thermodynamics, thus conservation of material inputs and pollutants in production processes must hold. Murty and Russell (2002) argued that the weak disposability assumption violates the law of mass conservation. Pethig (2006) applied the law of conversation of mass to link production and the abatement process. Other environmental economics models applying the law of conversation of mass include Nijkamp and van den Bergh (1997), van den Bergh (1999), Krysiak and Krysiak (2003), Coelli et al. (2007) and Førsund (2009).

A two-stage model of production and abatement can be used to model a production of U.S. coal power plants especially those in the eastern states because abatement processes are mainly used in these coal power plants to reduce NO_x emissions due to more restrict NO_x regulations than other areas.

Apart from the acid rain problem, NO_x from coal power plants contributes to the formation of ground-level ozone and thus has been the focus of both federal and regional regulations. There are several regional programs to address the problem of ground-level ozone. Under the 1990 CAA, the Ozone Transport Commission (OTC) cooperated with the EPA on developing and implementing regional solutions in the Northeast and Mid-Atlantic regions. OTC NO_x Reduction Programs were executed in 1995-2002. In 2003, the EPA replaced the OTC NO_x Reduction Programs with the NO_x State Implementation Plan (SIP) and called for further NO_x reductions across 22 eastern states. In 2009, the EPA developed the more stringent the CAIR NO_x program to cover 28 eastern states. To provide flexibility for power plants to meet NO_x restrictions cost-effectively, the EPA also developed a regional NO_x cap and trade program as part of phase II of its OTC NO_x Reduction Programs.

Although the EPA's proposed Cross-State Air Pollution Rule (CSAPR) is suspended pending legal review, U.S. power plants are under pressure to reduce emissions or close down. For example, in January 2012 Duke Carolinas announced the retirement of "1,667 MW of unscrubbed coal-fired capacity under a prescribed, enforceable schedule under an air permit-related settlement the utility reached with several environmental groups". The utility "had previously committed to retiring 200 MW of older, unscrubbed capacity at one of its older plants" as well as "retiring another 800 MW of unscrubbed capacity elsewhere by 2018", Carr (2012).

Effective abatement requires both technology for abatement and analysis to guide the implementation of abatement activities to maximize effectiveness. Common NO_x abatement technologies found in coal power plants can be categorized into two types: combustion controls and post-combustion controls. Combustion controls, especially low NO_x burner (LNB) and Overfire Air (OFA), are the most common NO_x abatement technologies used in coal plant boilers. In addition, many boilers require post-combustion control techniques such as Selective Non-Catalytic Reduction (SNCR) and Selective Catalytic Reduction (SCR).

Abatement analysis and implementation requires improved electricity production models which characterize the underlying production process and abatement process of byproduct pollutants. The remainder of this chapter is organized as follows. Section 4.2 develops the theoretical model of production considering abatement processes and introduces an underlying abatement function, a pollutant function and corresponding abatement cost, and a marginal abatement cost function. Section 4.2 also explains the method of estimating a frontier pollutant function using the StoNEZD method and associate marginal abatement cost of NO_x. Section 4.3 describes the data set of 325 boilers units in the U.S. coal power plants currently under the CAIR NO_x program. Section 4.4 presents and discusses the empirical results. Section 4.5 presents the conclusions.

4.2 Model

4.2.1 An abatement function and a pollutant function

Figure 4.1 shows the first stage of production where $\boldsymbol{x} \in R_+^M$ is a vector of production inputs used to produce a vector of outputs, $\boldsymbol{y} \in R_+^S$, which results in a vector of unabated byproduct pollutants, $\boldsymbol{b}_{int} \in R_+^P$.

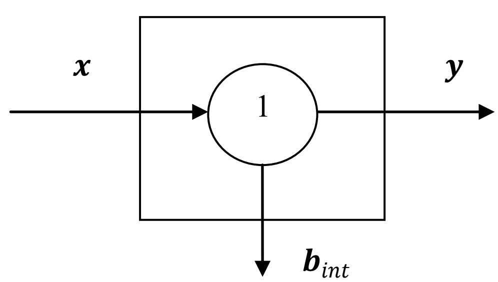

Figure 4.1 First stage of the production-abatement model: production

Definition 4.1 An output b_{int} is called a byproduct of production inputs $\boldsymbol{x}$ if the use of $\boldsymbol{x}$ in the production process generates b_{int}, and b_{int} is costly to dispose.

The same pollutant can be generated by multiple production inputs; for example, burning coal, natural gas or gasoline generates NO_x. Based on this fact, this study imposes that a byproduct can be written as a linear function of $\boldsymbol{x}$. Specifically,

$$b_{int} = h(\boldsymbol{x};\boldsymbol{o}) = \sum_{m=1}^{M} r_m(\boldsymbol{o})x_m \tag{4.1}$$

where $\boldsymbol{o} \in R_+^W$ is a vector of other variables describing production conditions, such as equipment technology or types. The amount of byproduct is determined by the amount of production input used multiplied by factors $\boldsymbol{r} \in R_+^W$ which is a function of the production conditions and correspond to each byproduct output. Thus, this study assumes that *unabated pollutants* $\boldsymbol{b}_{int}$ *are byproducts of production inputs* $\boldsymbol{x}$. This seems natural for chemical processes such as the burning of coal, yet the alternative assumption of weak disposability may be appropriate for situations in which undesirable outputs are the result of human error, for example, the production of medical services in hospitals.

Figure 4.2 illustrates the integrated production and abatement process with multiple unabated pollutants[8]. For simplicity, one abatement process for an unabated byproduct pollutant b_{int} is shown in Figure 4.3; however, the approach easily extends to multiple intermediate pollutants. Let $\boldsymbol{x}_a \in R_+^Q$ be a vector of abatement inputs. The portion abated is a and the final pollutant is b. Inspired by Ebert and Welsch (2007) who imposed law of the conservation of mass on material inputs, outputs and pollutants, this study imposes that the abatement process satisfies the law as

[8] An alternative abatement method for polluting firms is to switch to cleaner production inputs. For example, coal power plants could switch from high-sulfur coal to low-sulfur coal to reduce SO_2 levels. However, this chapter considers an abatement process as the main method to reduce pollutant levels.

$$b_{int} = a + b \tag{4.2}$$

and

$$0 \leq \frac{\partial b}{\partial b_{int}} \leq 1 \tag{4.3}$$

which imply that any marginal increase of byproduct pollutant must not lower the amount of final pollutant; however, the marginal increase is bounded by 1 due to the law.

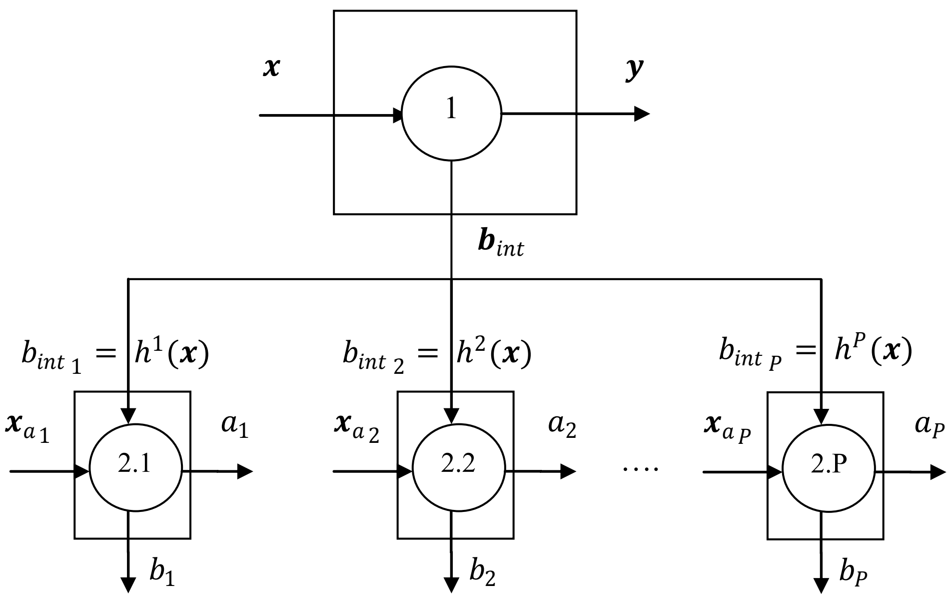

Figure 4.2 Integrated production-abatement model

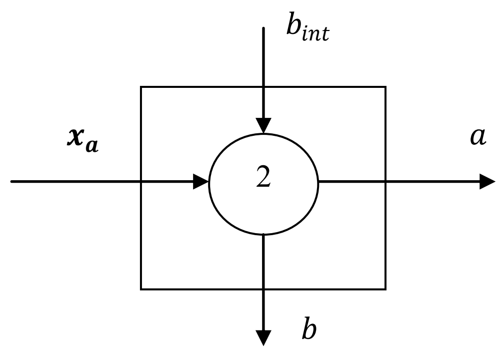

Figure 4.3 Second stage of the production-abatement model: abatement process

Pethig (2006) assumed an abatement activity to be a function of abating service labors and abating material inputs and describes the process by a function that satisfies some classical production function properties such as concavity and diminishing marginal productivity. Following Pethig (2006), the abatement function is defined as:

Definition 4.2 The function $A: R_+^{Q+1} \to R_+$ is an abatement function creating an abatement output a and satisfying the three properties:

1. $A(\boldsymbol{x}_a, b_{int}) = a$ and A is concave in $\boldsymbol{x}_a$ and b_{int}
2. $\frac{\partial A}{\partial x_{a_q}} \geq 0$ and $\frac{\partial^2 A}{\partial {x_{a_q}}^2} \leq 0$
3. $0 \leq \frac{\partial A}{\partial b_{int}} \leq 1$ and $\frac{\partial^2 A}{\partial {b_{int}}^2} \leq 0$

The first property states that abated output is a concave function of abatement inputs $\boldsymbol{x}_a$, and an unabated pollutant b_{int}. The second property implies that abated output is increasing in abatement inputs with decreasing rate. The third property implies that abated output is increasing in unabated pollutant with decreasing rate. However, the marginal product of unabated pollutant is bounded by 1 because $b_{int} = a + b$ and $0 \leq \frac{\partial b}{\partial b_{int}} \leq 1$; thus, $\frac{\partial b_{int}}{\partial b_{int}} = 1 = \frac{\partial A}{\partial b_{int}} + \frac{\partial b}{\partial b_{int}}$ implies that $0 \leq \frac{\partial A}{\partial b_{int}} \leq 1$.

It is possible to write an abatement output as a function of abatement inputs and production inputs to the production process given the production conditions. This leads to Proposition 1.[9]

Proposition 4.1 An abatement function $G(\boldsymbol{x}_a, \boldsymbol{x}; \boldsymbol{o}) = a$ satisfies the three properties:

1. $\frac{\partial G}{\partial x_{a_q}} \geq 0$ and $\frac{\partial^2 G}{\partial {x_{a_q}}^2} \leq 0$
2. $\frac{\partial G}{\partial x_m} \geq 0$ and $\frac{\partial^2 G}{\partial {x_m}^2} \leq 0$
3. G is concave in $\boldsymbol{x}_a$ and $\boldsymbol{x}$

Typically, measuring the level of final pollutant is easier than measuring the amount of abated output. Thus, the equations (4.2) and (4.3) are used to define a pollutant function and its properties in Proposition 2.

Proposition 4.2 A pollutant function $B(\boldsymbol{x}_a, \boldsymbol{x}; \boldsymbol{o}) = b$ satisfies the three properties:

1. $\frac{\partial B}{\partial x_{a_q}} \leq 0$ and $\frac{\partial^2 B}{\partial {x_{a_q}}^2} \geq 0$
2. $\frac{\partial B}{\partial x_m} \geq 0$ and $\frac{\partial^2 B}{\partial {x_m}^2} \geq 0$
3. B is convex in $\boldsymbol{x}_a$ and $\boldsymbol{x}$

Having defined the pollutant function, it is used to define an abatement cost function and to estimate the marginal abatement costs.

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[9] See Appendix B for all proofs.

4.2.2. An abatement cost minimization

First, this study defines the incurred cost when a firm uses abatement inputs to reduce pollutant. The abatement cost function can be derived by solving a constrained cost minimization problem. Based on this abatement cost function, we develop the marginal cost of abatement function and derive several properties of the abatement cost function and the marginal abatement cost function.

4.2.2.1. An abatement cost function

Consider a firm that optimizes the use of abatement inputs not to exceed a given level of a pollutant b and while consuming production inputs $\boldsymbol{x}$. The Abatement Cost Minimization Problem (ACMP) can be written as:

$$C\left(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b\right) = min_{\boldsymbol{x}_a}\left\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a; \boldsymbol{x}, \boldsymbol{o}) \leq b\right\} \tag{4.4}$$

where $\boldsymbol{w}_{x_a}$ is the unit cost of abatement inputs. The constraint imposes that a firm can choose the level of abatement to achieve the level of outputs and the pollutant level is not greater than b, a regulated absolute level of pollution. The properties of the abatement cost function are described in Preposition 4.3.

Proposition 4.3 The abatement cost function satisfies six properties:

1. Homogeneous of degree 1 in $\boldsymbol{w}_{x_a}$
2. Non-decreasing in $\boldsymbol{w}_{x_a}$
3. Concave function in $\boldsymbol{w}_{x_a}$
4. Non-decreasing in $\boldsymbol{x}$
5. Non-increasing in b
6. Convex function in b

In Figure 4.4, the first graph shows that at a given level of both production inputs $\boldsymbol{x}$ and other abatement inputs excluding x_{a_q}, a firm can reduce the amount of b by using more of an abatement input x_{a_q} with a decreasing rate. The second graph shows that at a fixed level of abatement inputs $\boldsymbol{x_a}$ and other production inputs excluding x_m, the amount of a pollutant b increases by using more of an arbitrary production input x_m with an increasing rate. The third graph shows the isoquant between two arbitrary abatement inputs at a given level of production inputs $\boldsymbol{x}$ and a pollutant b.

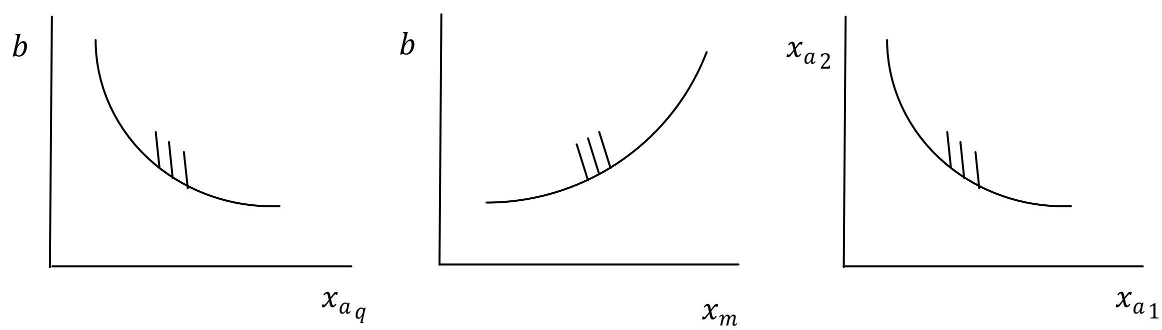

Figure 4.4 Two-dimensional isoquants of the pollutant function

4.2.2.2 A marginal abatement cost (MAC)

The MAC curve is a standard tool in environmental economics, climate change policy and emission trading appearing in Ellerman and Decaux (1998), Criqui et al. (1999) and Klepper and Peterson (2006) for example. Many studies use the MAC curve at the region/country level, but the actual concept comes from the firm level (Klepper and Peterson, 2006). Underlying the derivation of any MAC curve is the concept of the marginal cost of abatement; an incremental cost when firms reduce one more unit of a pollutant. Thus, the pollutant function, the abatement cost function and the MAC are all related. Figure 4.5 shows two graphs describing the standard shape. The right graph shows how MAC increases when more pollutant is abated and the left graph shows how it decreases when less pollutant is abated. Next, define the MAC is defined and and its properties are stated.

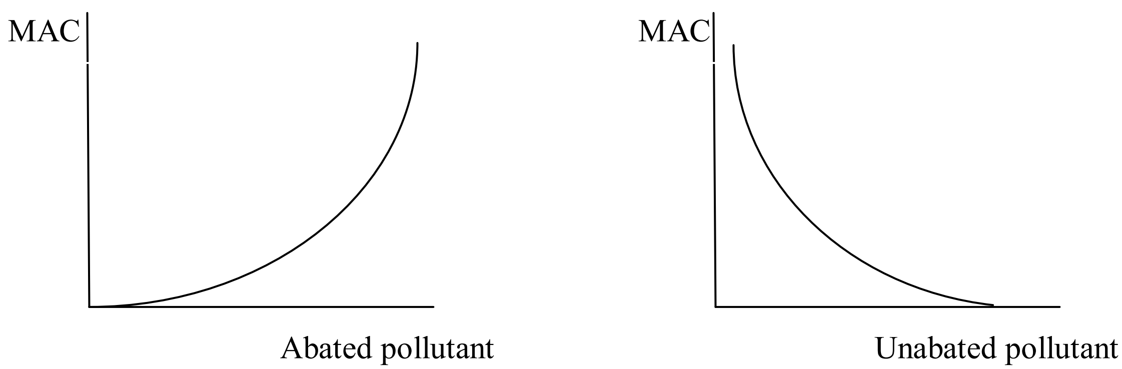

Figure 4.5 The MAC curves

Definition 4.3 MAC is an increase in an abatement cost when one more unit of a pollutant is abated. Mathematically, MAC equals $-\frac{\partial C}{\partial b}$.

Proposition 4.4 MAC satisfies two properties:

1. MAC is nonnegative, specifically, MAC equals the Lagrange multiplier of the ACMP problem.
2. MAC is non-increasing in b.

4.2.3 Frontier pollutant function estimation

From Proposition 4.2, a pollutant function B is continuous, monotonically increasing in production inputs x, monotonically decreasing in abatement inputs x_a and convex in both x and x_a; thus, the CNLS method can be used to find the estimator of the pollutant frontier function B. The frontier pollutant function is interpreted as a minimum amount of pollutant generated by given amounts of abatement inputs $\boldsymbol{x}_a$, production inputs $\boldsymbol{x}$ to the production process and the production conditions $\boldsymbol{o}$. For simplicity, this section assumes that the production conditions $\boldsymbol{o}$ are similar among firms i.e. power plants use the same types of boilers.

Consider a pollutant function model with a multiplicative disturbance term

$$b_i = B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)e^{\varepsilon_i} \qquad \forall i = 1, .., n \qquad (4.5)$$

where ε_i is the disturbance term for firm i with $E(\varepsilon_i) = 0\ \forall i$, $\mathrm{Var}(\varepsilon_i) = \sigma^2 < \infty\ \forall i$ and $\mathrm{Cov}(\varepsilon_i \varepsilon_j) = 0\ \forall i \neq j$.

Similar to the analysis of the frontier production function, ε_i can be deterministic, composite or random. However, deviations in firms' pollutant are in part due to the effects of contextual variables such as vintages of technology and managerial practices. This study extends the StoNEZD method in section 2.4 to measure the effect of the contextual variables on the level of a pollutant.

4.2.3.1 Frontier pollutant function with contextual variables

The multiplicative disturbance term considering contextual variables is written as

$$\varepsilon_i = v_i + u_i = v_i + u_i^l + \boldsymbol{\delta}'\boldsymbol{z}_i \qquad \forall i = 1, \dots, n \qquad (4.6)$$

where v_i is a random noise, $u_i = u_i^l + \boldsymbol{\delta}'\boldsymbol{z}_i \geq 0$ is the firm i overall technical inefficiency and $\boldsymbol{v}$ and $\boldsymbol{u}$ are i.i.d. u_i^l is the technical inefficiency of firm i that is not explained by the contextual variable, $\boldsymbol{\delta}'\boldsymbol{z}_i$ is the technical inefficiency of firm i that is explained by the contextual variables, $\boldsymbol{z}_i \in R^r$ are contextual variables of firm i and $\boldsymbol{\delta} \in R^r$ are coefficients that capture the average effect of contextual variables on deviation from the frontier pollutant function.

Consider the frontier pollutant function model with multiplicative composite disturbance (4.6)

$$b_i = B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)e^{\varepsilon_i} = B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)e^{\left(v_i + u_i^l + \boldsymbol{\delta}'\boldsymbol{z}_i\right)} \qquad \forall i = 1, \ldots, n. \qquad (4.7)$$

Applying the log transformation to (4.7), the regression model is written as

$$\begin{aligned} \ln b_i &= \ln B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i) + \varepsilon_i \\ &= \ln B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i) + \boldsymbol{\delta}'\boldsymbol{z}_i + v_i + u_i^l \\ &= \ln B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i) + \boldsymbol{\delta}'\boldsymbol{z}_i + \varepsilon_i^l \qquad \forall i = 1, \ldots, n. \qquad (4.8) \end{aligned}$$

The composite disturbance term in (4.8), ε_i^l, violates the Gauss-Markov property that $E(\varepsilon_i^l) = E(v_i + u_i^l) = \mu^l > 0$; thus, the composite disturbance term is modified as

$$\begin{aligned} \ln b_i &= \left[\ln B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i) + \mu^l\right] + \boldsymbol{\delta}'\boldsymbol{z}_i + \left[\varepsilon_i^l - \mu^l\right] \\ &= \ln K(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i) + \boldsymbol{\delta}'\boldsymbol{z}_i + \zeta_i \qquad \forall i = 1, \ldots, n \qquad (4.9) \end{aligned}$$

where $\zeta_i = \varepsilon_i^l - \mu^l$ is the modified multiplicative composite disturbance term with $E(\zeta_i) = E(\varepsilon_i^l - \mu^l) = 0$ and $K(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i) = B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)e^{\mu^l}$ is an average pollutant function. The contextual variables CNLS problem is then formulated as:

$$\min_{\alpha,\boldsymbol{\gamma},\boldsymbol{\rho},\boldsymbol{\delta},\zeta} \sum_{i=1}^{n} \zeta_i^2 \tag{4.10.1}$$

s.t. $$\zeta_i = \ln b_i - \ln\left(\alpha_i + \boldsymbol{\gamma}_i' \boldsymbol{x}_i + \boldsymbol{\rho}_i' \boldsymbol{x}_{a_i}\right) - \boldsymbol{\delta}' \boldsymbol{z}_i \qquad \forall i = 1, \dots, n \tag{4.10.2}$$

$$\alpha_i + \boldsymbol{\gamma}_i' \boldsymbol{x}_i + \boldsymbol{\rho}_i' \boldsymbol{x}_{a_i} \geq \alpha_h + \boldsymbol{\gamma}_h' \boldsymbol{x}_i + \boldsymbol{\rho}_h' \boldsymbol{x}_{a_i} \qquad \forall i, h = 1, \dots, n \tag{4.10.3}$$

$$\boldsymbol{\gamma}_i \geq 0, \boldsymbol{\rho}_i \leq 0 \qquad \forall i = 1, \dots, n \tag{4.10.4}$$

where α_i, $\boldsymbol{\gamma}_i$ and $\boldsymbol{\rho}_i$ are the coefficients characterizing hyperplanes of the average pollutant function K. The objective function (4.10.1) minimizes the sum of the squared value of the modified multiplicative composite disturbance terms. The equality constraints (4.10.2) define the modified disturbance term as the different between log-levels of an observed pollutant and an estimated pollutant minus the effect of the contextual variables. The inequality constraints (4.10.3) are a system of Afriat inequalities, imposing the underlying pollutant function to be continuous and convex. The constraints (4.10.4) enforce that the function is monotonic increasing in $\boldsymbol{x}_i$ and monotonic decreasing in $\boldsymbol{x}_{a_i}$. Note that $\boldsymbol{\delta}$ is unrestricted in sign and that a positive sign on δ implies that the contextual variable increases the observed level of pollutant.

Given the modified composite residuals, $\hat{\zeta}_i \; \forall i$, from (4.10), the method of moments is applied to separate the random noise and the technical inefficiency. Assuming that technical efficiency has a half normal distribution, $u_i^l \sim |N(0, \sigma_{u^l}^2)|$, and that the random noise has a normal distribution, $v_i \sim N(0, \sigma_v^2)$, the estimated standard deviation of the technical inefficiency and the random are written as

$$\hat{\sigma}_{u^l} = \sqrt[3]{\frac{\widehat{M}_3}{\left(\frac{2}{\pi}\right)\left(\frac{4}{\pi} - 1\right)}} \tag{4.11}$$

$$\hat{\sigma}_v = \sqrt{\widehat{M}_2 - \left(\frac{\pi - 2}{\pi}\right)\hat{\sigma}_u^2} \tag{4.12}$$

where $\widehat{M}_2 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\zeta}_i - \hat{E}(\zeta_i)\right)^2$ and $\widehat{M}_3 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\zeta}_i - \hat{E}(\zeta_i)\right)^3$ are the second and third sample central moment of the modified composite residuals. Unlike $\widehat{M}_3$ in the frontier production function model (2.8), $\widehat{M}_3$ in (4.11) should be positive so that the estimated $\hat{\sigma}_{u^l}$ is positive. The composite residuals will distribute with a positive skew if technical inefficiency is present.

Given $\left(\hat{\alpha}_i, \hat{\boldsymbol{\gamma}}_i, \hat{\boldsymbol{\rho}}_i, \hat{\zeta}_i\right) \forall i$ from the CNLS problem (4.10), the unique StoNEZD estimator of the frontier pollutant function is written as

$$\hat{B}_{\min}(\boldsymbol{x}_a, \boldsymbol{x}) = \left[\max_{\alpha,\boldsymbol{\gamma},\boldsymbol{\rho}}\left\{\alpha + \boldsymbol{\gamma}'\boldsymbol{x} + \boldsymbol{\rho}'\boldsymbol{x}_a \middle| \alpha + \boldsymbol{\gamma}'\boldsymbol{x}_i + \boldsymbol{\rho}'\boldsymbol{x}_{a_i} \leq \hat{b}_i \ \ \forall i\right\}\right] e^{-\widehat{\mu^l}} \tag{4.13}$$

where $\hat{b}_i = \max_{h \epsilon \{1,\dots,n\}}\left\{\hat{\alpha}_h + \hat{\boldsymbol{\gamma}}_h{}'\boldsymbol{x}_i + \hat{\boldsymbol{\rho}}_h{}'\boldsymbol{x}_{a_i}\right\}$ and $\widehat{\mu^l} = \hat{\sigma}_{u^l}\sqrt{2/\pi}$ obtained from the method of moments described equations (4.11) and (4.12). Intuitively, the frontier pollutant function is obtained by multiplicative shifting the average production function downward by the expected technical inefficiency.

Given the CNLS residuals, $\hat{\zeta}_i \ \forall i$, it is important to apply the skewness and kurtosis of the disturbance tests. First, the StoNEZD estimator for $\boldsymbol{\delta}$ from the problem (4.10) is statistically unbiased, consistent and asymptotically normally distributed as $\hat{\boldsymbol{\delta}} \sim_a N\left(\boldsymbol{\delta}, \left(\sigma_{u^l}^2 + \sigma_v^2\right)(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\right)$ where $\boldsymbol{Z} = (\boldsymbol{z}_1, \dots, \boldsymbol{z}_n)'$, thus a standard t-test can be used to test statically significant of $\boldsymbol{\delta}$ effect on the pollutant level; however, because the value of standard error depends on $\hat{\sigma}_{u^l}$ and $\hat{\sigma}_v$, it is important to test whether the stochastic or neoclassical pollutant function assumption[10] is more appropriate. Second, the unique StoNEZD estimator of the frontier pollutant function (4.13) also depends on $\hat{\sigma}_{u^l}$. If the test indicates that a frontier pollutant function model is appropriate, $\hat{\sigma}_{u^l}$ and $\hat{\sigma}_v$ are then estimated using (4.11) and (4.12) respectively. If the test indicates that a neoclassical pollutant function is appropriate, $\hat{\sigma}_{u^l} = 0$.

4.2.4 Estimating MACs

The firm specific MAC can be estimated by the use of Proposition 4. For an abatement cost minimization firm, Proposition 4 states that the MAC equals the Lagrange multiplier of the ACMP problem, λ and it has shown in Appendix B that $\lambda = -\frac{w_{x_{a_q}}}{\frac{\partial B\,(\boldsymbol{x}_a^*, \boldsymbol{x}; \boldsymbol{o})}{\partial x_{a_q}}}$ $\forall\, q$. Assuming that firm i minimizes the abatement cost using the production and abatement inputs at $(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)$, the MAC for firm i is estimated as

$$\mathrm{MAC}_i = -\frac{w_{x_{a_{q_i}}}}{\frac{\partial B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)}{\partial x_{a_{q_i}}}} \tag{4.14}$$

where $w_{x_{a_{q_i}}}$ is the unit cost of abatement input q for firm i and

[10] Or alternatively stated, the composite or random disturbance assumption is more appropriate.

$$\frac{\partial B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)}{\partial x_{a_{q_i}}} \approx \frac{\hat{B}_{\min}(\boldsymbol{x}_{a_i}^{+}, \boldsymbol{x}_i) - \hat{B}_{\min}(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)}{\epsilon} \tag{4.15}$$

where $\epsilon > 0$ is a small positive number and $(\boldsymbol{x}_{a_i}^{+}, \boldsymbol{x}_i) = (x_{a_{1_i}}, x_{a_{2_i}}, \dots, x_{a_{q_i}} + \epsilon, \dots, x_{a_{Q_i}}, \boldsymbol{x}_i)$ and $\hat{B}_{\min}$ is the unique StoNEZD estimator of the frontier pollutant function (4.13).

To summarize, the method to estimate MAC is as follows:

1. Solve the contextual variables CNLS problem (4.10) to obtain $(\hat{\alpha}_i, \hat{\boldsymbol{\gamma}}_i, \hat{\boldsymbol{\rho}}_i, \hat{\zeta}_i)\ \forall i$.
2. Use $\hat{\zeta}_i\ \forall i$ to estimate the $\sqrt{b_1}$ test statistic in (2.27) and b_2 test statistic in (2.28). Then, test for the presence of technical inefficiency by applying the skewness and kurtosis of disturbances tests described in section 2.6.
3. If the test supports the frontier pollutant function:

   3.1 Use $\hat{\zeta}_i\ \forall i$ to estimate $\hat{\sigma}_{u^l}$ in (4.11), $\hat{\sigma}_v$ in (4.12) and the expected technical inefficiency, $\widehat{\mu^l} = \hat{\sigma}_{u^l}\sqrt{2/\pi}$.

   3.2 For each firm i, using (4.15) and $\hat{B}_{\min}$ in (4.13) to calculate $\frac{\partial B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)}{\partial x_{a_{q_i}}}$. Then, estimate the MAC by using (3.16).
4. If the test supports the neoclassical pollutant function:

   4.1 $\hat{\sigma}_{u^l} = 0$; thus, $\widehat{\mu^l} = 0$. For each firm i, using (4.15) and $\hat{B}_{\min}$ in (4.13) to calculate $\frac{\partial B(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)}{\partial x_{a_{q_i}}}$. Then, estimate the MAC by using (3.16).

4.3 Data set

The boiler-level data characterizes 325 units of U.S. bituminous coal power plants operating in 2000-2008 and located in eastern states (Figure 4.6). All boilers are enrolled in the Acid Rain Program and are regulated by the CAIR NO_x program in 2009. Most are also in the NO_x Budget Trading Program with the exception of 43 boilers in Florida, Georgia, Missouri, Mississippi and parts of Alabama.

Coal power plants use different types of boilers to generate heat input, thus there are different levels of unabated pollutants per unit of heat input. The two most popular types of as pulverized coal (PC) fired boilers, wall and tangential, are selected in this study so that power plants have similar output production conditions. The sizes range between 65 and 1,426 MW. The heat input (mmBTU) as a proxy for the amount of coal burned is a production input to the electricity production process. The pollutant is NO_x (tons). Data on heat input and NO_x is taken from EPA (2011). Table 4.1 reports summary statistics.

Figure 4.6 Location of the U.S. bituminous coal power plants under the CAIR NO_x program in the data set

Abatement inputs are the resources the firm uses to reduce NO_x. Primary abatement systems for NO_x are low NO_x burner (LNB), Overfire air (OFA), selective non-catalytic reduction (SNCR) and selective catalytic reduction (SCR). Each system has different NO_x reduction performance and cost. A firm can choose to employ one of these systems or a mixture to minimize total NO_x abatement cost. Determining which system and size to use, involves several factors. For example, a bigger boiler with a higher maximum heat input rate requires a higher NO_x reduction performance system such as SCR or LNB along with SCR to meet a specified level of NO_x reduction. This study assumes that the abatement effort is based on the amount of pollutants generated and the abatement system used for each boiler. This study uses a proxy, measured in mmBtu, to quantify the abatement effort that is a function of the boiler's maximum heat input rate, the boiler's operating hours in a year (hr), and an abatement factor derived from the percentage reduction conditioned on the type of abatement system. This study estimates the unit cost of an abatement input for each type of abatement system (\$/mmBtu) based on the EPA Integrated Planning Model document (EPA, 2010)[11]. Table 4.1 shows the summary statistics for the abatement input data.

Table 4.1 shows that the average heat input levels fluctuate between 17747×10^3 and 18829×10^3 mmBtu for each year. However, NO_x levels continue to decrease because the coal plants employ more abatement inputs. The abatement input price also increases at approximately the rate of inflation throughout the time period.

Boiler vintage, defined as the time a boiler is put into operation, is important because different vintages are likely to have different NO_x emission levels. Typically, older boilers produce less electricity with more fuel due to technical limitations of the equipment or degradation. This study separates the boiler vintages into three groups, 1940-1959, 1960-1979 and 1980-, based on installation date as reported in Table 4.2.

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[11] See Appendix B for details of how the abatement input data and its unit cost are constructed.

Table 4.1 Statistics for the boiler units in the coal power plants (n=325)

Year	Variable[a]	Mean	Std. dev.	Min.	Max.
2000	Heat input	18829	15238	866	79135
	NO_x	4.32	3.48	0.25	18.68
	Abatement input	10033	10398	0	52360
	Abatement input price	0.07	0.08	0	0.50
2001	Heat input	17747	14809	997	86749
	NO_x	3.97	3.28	0.25	20.89
	Abatement input	10212	11212	0	60428
	Abatement input price	0.08	0.09	0	0.50
2002	Heat input	17852	15262	568	88046
	NO_x	3.86	3.37	0.10	20.97
	Abatement input	11102	12295	0	74251
	Abatement input price	0.09	0.11	0	0.51
2003	Heat input	18441	15497	1052	92378
	NO_x	3.65	3.13	0.30	20.17
	Abatement input	13278	14941	0	90901
	Abatement input price	0.13	0.14	0	0.64
2004	Heat input	17881	15068	248	82628
	NO_x	3.15	2.63	0.05	14.69
	Abatement input	14917	16223	0	76999
	Abatement input price	0.18	0.18	0	0.64
2005	Heat input	18514	15903	885	92853
	NO_x	3.07	2.60	0.25	15.15
	Abatement input	16152	17700	0	91039
	Abatement input price	0.20	0.19	0	0.64
2006	Heat input	18253	15413	207	83026
	NO_x	2.97	2.51	0.07	13.85
	Abatement input	16395	17623	0	82334
	Abatement input price	0.21	0.20	0	0.64
2007	Heat input	18676	15891	134	95973
	NO_x	2.87	2.50	0.04	14.43
	Abatement input	16466	17200	0	79852
	Abatement input price	0.22	0.19	0	0.64
2008	Heat input	17801	15657	58	93785
	NO_x	2.60	2.37	0.02	16.53
	Abatement input	16714	17759	0	87371
	Abatement input price	0.23	0.20	0	0.65

[a] Unit of heat input, NO_x, abatement input and abatement input price are 10^3 mmBtu, 10^3 ton, 10^3 mmBtu and 10^3 \$/mmBtu respectively.

Table 4.2 Boiler vintages

Boilers installation date	Contextual variables	Number of boilers
1940-1959	z_1	136
1960-1979	z_2	154
1980-	NA	35

Table 4.3 reports the change in number of boilers in 2000-2008 corresponding with the three abatement types categorized by percentage of NO_x reduction reported in Table 8 in Appendix. The first type, which includes SNCR and all combustion control abatement systems without any post-combustion control abatement, has 25-45 percent NO_x reduction. The second type, which includes all pre-combustion control abatement systems with SNCR, has 66.3-75.4 percent NO_x reduction. The third type, which includes all abatement systems with SCR, has 80-91 percent NO_x reduction. Table 3 also shows that number of boilers without NO_x abatement systems declines and that an increasing number of boilers install post-combustion abatement systems throughout the years. For example, in 2000-2008 84 more boilers (26 percent) install SCR, and 32 more boilers (10 percent) with combustion control abatement install SNCR. During the same time period, boilers without SCR and SNCR (52-74 percent) still remain the primary NO_x reduction technique.

Table 4.3 Number (and percentage) of boilers corresponding with abatement types categorized by percentage of NO_x reduction

Year	No abatement	Percentage NO_x reduction		
		25 - 45	66.3 - 75.4	80 - 91
2000	80 (24.6)	230 (70.8)	11 (3.4)	4 (1.2)
2001	65 (20.0)	239 (73.5)	12 (3.7)	9 (2.8)
2002	56 (17.2)	234 (72.0)	13 (4.0)	22 (6.8)
2003	43 (13.2)	215 (66.2)	15 (4.6)	52 (16.0)
2004	34 (10.5)	198 (60.9)	19 (5.8)	74 (22.8)
2005	33 (10.2)	187 (57.5)	25 (7.7)	80 (24.8)
2006	27 (8.3)	179 (55.1)	36 (11.1)	83 (25.5)
2007	23 (7.1)	176 (54.2)	40 (12.3)	86 (26.4)
2008	25 (7.7)	169 (52.0)	43 (13.2)	88 (27.1)

The decline in NO_x emission levels is also due to the improved performance of NO_x emission reduction throughout the time periods. Table 4.4 shows the average and standard deviation of NO_x per heat input (lb/mmBtu) from boilers in different NO_x abatement categories. We observe that the average NO_x emission rates generally decline for every type of abatement and no abatement plants. Furthermore, NO_x emission rates for boilers with more extensive NO_x abatement systems (66.3-75.4 and 80-91 percent NO_x reduction) have less variation than those with less extensive NO_x abatement systems (25-45 percent NO_x reduction) and no abatement systems; however, variability is still generally increasing among these boilers over the time horizon as more plants adapt extensive NO_x abatement systems.

Table 4.4 Boilers' average (and standard deviation) of NO_x per heat input (lb/mmBtu) corresponding with abatement type categorized by percentage of NO_x reduction

Year	No abatement	Percentage NO_x reduction		
		25 - 45	66.3 - 75.4	80 - 91
2000	0.593	0.472	0.435	0.393
	(0.186)	(0.120)	(0.059)	(0.059)
2001	0.595	0.464	0.432	0.369
	(0.175)	(0.117)	(0.064)	(0.067)
2002	0.588	0.468	0.441	0.335
	(0.177)	(0.130)	(0.078)	(0.075)
2003	0.557	0.450	0.387	0.341
	(0.152)	(0.117)	(0.083)	(0.119)
2004	0.467	0.411	0.375	0.317
	(0.136)	(0.093)	(0.081)	(0.087)
2005	0.457	0.392	0.370	0.297
	(0.116)	(0.098)	(0.074)	(0.081)
2006	0.443	0.392	0.342	0.294
	(0.117)	(0.098)	(0.083)	(0.076)
2007	0.455	0.383	0.292	0.283
	(0.119)	(0.103)	(0.078)	(0.075)
2008	0.444	0.372	0.304	0.282
	(0.133)	(0.083)	(0.089)	(0.094)

4.4 Empirical results

To decide whether the assumption of a frontier pollutant function or an average pollutant function is more appropriate, the skewness and kurtosis of the CNLS residuals are tested as discussed in section 4.2.3.1. Table 4.4 reports the skewness ($\sqrt{b_1}$) test statistics in which the null hypothesis H_0, disturbances are normally distributed, is tested against an alternative hypothesis H_1, disturbances are *positive* skewed. Table 4.4 also reports the kurtosis (b_2) test statistics in which the null hypothesis H_0, disturbances are normal kurtosis, is tested against an alternative hypothesis H_1, disturbances are non-normal kurtosis[12]. The $\sqrt{b_1}$ statistics are significant and positively signed at the 5 percent level in 2001 and not significant in 2000, 2002 and 2003. For the 2001 data, the null hypothesis is rejected for the $\sqrt{b_1}$ test at 5 percent significant level, but cannot be rejected for the b_2 test at 10 percent significant level; thus, the frontier pollutant function model is the preferred for the 2001 data. For 2000, 2002, 2003, 2004 and 2005, the null hypothesis related to both $\sqrt{b_1}$ and b_2 tests cannot be rejected at the 10% significant level; thus, the neoclassical pollutant function model is supported. For 2006, 2007 and 2008, the null hypothesis cannot be rejected for the $\sqrt{b_1}$ test but is rejected for the b_2 test due to excess kurtosis. In this case, the frontier pollutant function model is rejected but the neoclassical pollutant function model assuming normality may be poorly specified[13]. Figure 4.7 shows the model specification for year 2000-2008 data.

[12] The simulated distribution of the $\sqrt{b_1}$ and b_2 are constructed via a Monte Carlo simulation using M=10,000 Pseudo-samples of n=325 observations from $N(0,1)$.

[13] This study chooses the model that failed the test with the smallest margin after testing alternative specifications that also failed the kurtosis test.

$\sqrt{b_1}$ test		b_2 test: Cannot reject H_0	b_2 test: Reject H_0
	Cannot reject H_0	The neoclassical pollutant function model 2000, 2002, 2003, 2004, 2005	The neoclassical pollutant function model but possible data problems or model specification 2006, 2007, 2008
	Reject H_0	The frontier pollutant function model 2001	The frontier pollutant function model but possible data problems or model specification

Figure 4.7 The model specification based on the skewness and kurtosis tests.

It is observed that for 2004-2008 the values of $\sqrt{b_1}$ has a negative sign (the sign opposite of what they would be expected based on the discussion in Kumbhakar and Lovell (2000). Some authors (Greene, 2008) argue that it indicates the model is misspecified, but the efficiency literature now recognizes that wrong skewness frequently arises and can result from the sampling issue (Simar and Wilson, 2009). Alternatively, this study argues that the wrong sign of $\sqrt{b_1}$ can result even with a correctly specified model if a few firms operate more efficiently than others. Table 4.5 reports the statistics of the positive CNLS residuals (boilers operate less environmentally productively than the average boilers) and negative CNLS residuals (boilers operate more environmentally productively than the average boilers). First note that after 2002 there are fewer observations with negative residuals indicating there is a larger spread of negative residual values (larger dispersion in performance). Table 4.5 shows that the average and the standard deviation of the size of the negative CNLS residuals increase whereas the average and the standard deviation of the positive CNLS residuals are more stable. While the frontier pollutant function characterizes the NO_x emission of a majority of boilers, a subset of shows significantly better environmental performance. Boilers in this smaller group typically use SCR abatement system.[14] Tables 4.3 and 4.4 show that the number of boilers with superior environmental performance in 2003-2004 makes up 27% of boilers operating in 2008.

This study suggests two reasons for these findings. Since the NO_x State Implementation Plan was being implemented in 2003-2004 and the EPA was in the process of issuing the more stringent CAIR NO_x program to take effect in 2009, each year more coal power plants responded to the new regulations by using more efficient post-combustion abatement systems such as SNCR and SCR. In addition, the CAIR NO_x program provides incentives for some plants to reduce NO_x levels below the cap level in order to trade unused allowances or bank them for future use.

[14] Recall that boilers in this group operate under the pollutant frontier; thus, an increase number of these boilers causes a negative value of $\sqrt{b_1}$ (negative skewness).

Table 4.5 Results of the skewness and kurtosis test and the statistics of estimated CNLS residuals

Year	Test Statistics		P-values		Positive distance			Negative distance		
	$\sqrt{b_1}$	b_2	$\sqrt{b_1}$	b_2	%	Mean	Std.Dev	%	Mean	Std.Dev
2000	0.127	3.295	0.177	0.114	49	0.175	0.141	51	-0.168	0.128
2001	0.283	3.325	0.020	0.098	50	0.176	0.147	50	-0.172	0.121
2002	0.146	3.157	0.142	0.223	52	0.175	0.151	48	-0.187	0.137
2003	0.056	3.212	0.347	0.176	53	0.173	0.150	47	-0.192	0.148
2004	-0.191	3.301	0.924	0.111	51	0.177	0.132	49	-0.182	0.153
2005	-0.112	2.871	0.798	0.367	51	0.177	0.129	49	-0.185	0.136
2006	-0.474	4.203	0.999	0.001	54	0.172	0.138	46	-0.201	0.170
2007	-1.241	6.246	1.000	0.000	58	0.197	0.147	42	-0.271	0.277
2008	-1.046	4.821	1.000	0.000	55	0.258	0.173	45	-0.320	0.320

Table 4.6 reports the δ_1 and δ_2 estimates that capture the effects of boiler vintage on pollutant level for the dummy variables z_1 and z_2 representing boiler put into operation between 1940-1959 and 1960-1979, respectively. Both δ_1 and δ_2 have positive and significant signs as expected, which implies that older vintages have higher NO_x emissions. On average, boilers installed between 1940-1959 and between 1960-1979 have 11.9%-29.4% and 3%-18.1% higher NO_x emissions than those installed after 1980. Interestingly, the vintage effect decrease over time is an indication that maintenance overhauls or upgrades might exist during the study periods in fact improving older boilers' performance.

Table 4.6 Contextual variable parameter estimates

Year	$\hat{\delta}_1$	t-statistic	$\hat{\delta}_2$	t-statistic
2000	0.288[a]	15.421	0.165[a]	9.419
2001	0.294[a]	11.783	0.178[a]	7.606
2002	0.287[a]	14.498	0.181[a]	9.728
2003	0.243[a]	12.046	0.150[a]	7.928
2004	0.221[a]	11.255	0.158[a]	8.560
2005	0.181[a]	9.410	0.150[a]	8.306
2006	0.189[a]	9.132	0.130[a]	6.678
2007	0.119[a]	4.440	0.030[c]	1.172
2008	0.217[a]	6.656	0.067[b]	2.189

[a] Significant at the 1% level or better
[b] Significant at the 5% level or better
[c] Significant at the 10% level or better

Table 4.7 reports that the estimated average NO_x MACs range between 589 and 4,426$/ton. The difference between the estimated average NO_x MACs for 2000-2003 and 2004-2008 is significant. Referring to the calculation of MAC from (4.14), the two factors causing its rise are an increase in unit cost of abatement inputs, w_{x_a}, and a change in the slope of the pollutant function with respect to abatement inputs, $\frac{\partial B}{\partial x_a}$. Referring again to Table 4.1 and 4.3, observe that w_{x_a} are increasing because more power plants use more costly abatement inputs in later years. Moreover, the less-negative estimates of $\frac{\partial \hat{B}}{\partial x_a}$ imply that the plants are cleaner. Simply stated, the findings indicate it is more costly for cleaner plants to abate an additional unit of NO_x.

The MACs of NO_x can provide an indicator of the NO_x shadow prices or the NO_x allowance prices. Table 4.7 compares the NO_x shadow price estimates in this study to the NO_x allowance prices in the market obtained from EPA (2003b, 2004b, 2005b, 2006b, 2007c, 2008b, 2009b). During the NO_x program of OTC (2000-2002), NO_x allowance prices ranged between 600 and 1,700 \$/ton. During the NO_x budget trading program (2003-2009), NO_x allowance prices dramatically fluctuated between 2,100 and 8,000 \$/ton in 2003, decreased and were less volatile between 2,000 and 3,700 \$/ton in 2004-2005 and dropped back to 500 and 2,725 \$/ton in 2006-2008. The reason for a significant fluctuation in market prices in early 2003 was that power plants were uncertain about the NO_x control costs which were affected by trend in abatement strategy, energy demand and uncertainty in natural gas prices (Burtraw and Szambelan 2009; EPA, 2004b). Moreover, NO_x control costs in the future are expected to increase due to the CAIR NO_x program which requires further NO_x reduction from coal boilers beginning in 2009. However, NO_x allowance prices fell and were more stable after 2003 and fell sharply after 2005 because of a decrease in demand for NO_x allowances. Power plants have adequately banked allowances prior to 2005 and have become cleaner by increasing the use of NO_x abatement equipment (EPA 2006b; 2007c; 2008b). Figure 4.8 illustrates the estimated average MACs and the market prices.

It is important to note that the estimated NO_x MACs may be overstated compared to NO_x allowance prices for several reasons. Similar to the argument described in Färe et al. (2005), plants make long-term capital investments in NO_x abatement equipment based on expected future regulations or expected NO_x allowance prices. The financial decision affects NO_x allowance prices in two ways. First, prices can fluctuate depending on the supply and demand of NO_x allowances. Second, having installed the abatement equipment, if the plant then operates the abatement equipment at full capacity, the result is additional NO_x reduction beyond the regulatory requirement which the plant can bank as future allowances to meet future regulations or sell in the marketplace. However, the NO_x allowance prices were below the expected NO_x control cost (EPA 2007c; 2008b) which is consistent with the results in this section.

Table 4.7 Comparison between the NO_x prices in the market (\$/ton) and the NO_x price estimates in the study (\$/ton)

Year	Market	Study[b]			
		Mean[a]	Std.dev.[a]	Min	Max
2000		1225	1500	58	9369
2001	600-1700[c]	943	875	43	6040
2002		724	988	56	9093
2003	2500-8000	1057	1103	203	5934
2004	2100-3700	2991	3015	117	10377
2005	2000-3500	5471	6484	270	22449
2006	900-2725	3836	3967	321	26523
2007	500-1000	2469	3191	153	27940
2008	592-1400	2948	4406	257	31454

[a] Weighted average by the amount of pollutants
[b] For 2003 to 2005, the range of NO_x market prices are approximated from graphs; for all other years, the market prices range are explicitly stated in the EPA reports.
[c] For year 2000 to 2002, three years of OTC NO_x budget program progress was reported in a single OTC NO_x budget program report.

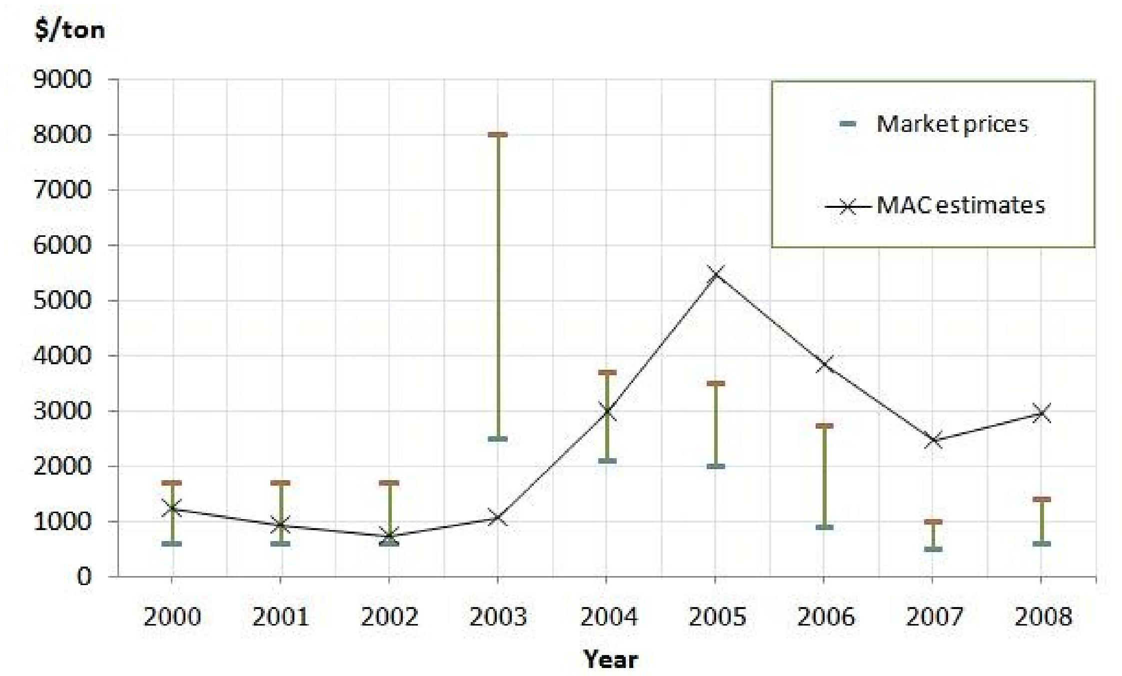

Figure 4.8 NO_x market prices and the estimated average MACs ($/ton)

4.5 Conclusions

This chapter describes a production model including the abatement processes. Applying the law of conservation of mass in the abatement process, a pollutant function is derived. The theoretical abatement cost function and MAC are derived using the pollutant function.

The pollutant function characterizing 325 boiler units in the eastern U.S. bituminous coal power plants under the CAIR NO_x program between 2000-2008 is estimated using StoNEZD. In most cases, a neoclassical average production function was preferred to a frontier pollutant function because the inefficiency term is found to be statistically insignificant. This chapter argues this is due to the NO_x abatement technical progress of a small subset of the plants. The estimated average NO_x MACs of coal power plants in the sample are between 724 and 5,471 $/ton. These numbers, which are in the range of projected NO_x prices, are likely to be higher than the NO_x allowances' prices in the market.

This chapter suggests and demonstrates that abatement processes are an essential part of electricity production in the U.S. As several NO_x programs have been implemented to address acid rain and ground level ozone problems, U.S. coal power plants responded to the more stringent regulations by installing LNB, OFA, SCR and SNCR equipments. The model developed allows the effects of environmental regulations to be quantified and provide the basis for prediction of future responses to environmental regulations.

CHAPTER V

A NONPARAMETRIC METHOD TO ESTIMATE A TECHNICAL CHANGE EFFECT ON THE MARGINAL ABATEMENT COSTS OF U.S. COAL POWER PLANTS

5.1 Introduction

The relationship between innovation and environmental policy has received considerable attention in recent years in part because the Porter Hypothesis (Porter and van der Linde, 1995) suggested that more stringent environmental policy could provide incentives for firms to develop new pollution controls that could also augment general productivity. The enactment of the 1990 CAA resulted in environmental programs and regulations that are designed to reduce NO_x, the key pollutant in ground level ozone and acid rain. Coal power plants are the primary generators of NO_x. From 2000 to 2008, eastern U.S. coal power plants operating under the OTC NO_x budget program and the NO_x budget trading program significantly lowered their NO_x emission to meet the regulated reduction targets. As reported in EPA (2009b), the average regional ozone season NO_x emission[15]from affected coal, oil, and gas power plants decreased from 1,256 thousand tons in 2000, to 849 thousand tons in 2003, and 481 thousand tons in 2008. For affected coal power plants, the average regional ozone season NO_x emission decreased from 800 thousand tons in 2003 to 456 thousand tons in 2008, while the average levels of heat input were relatively stable between 4.91 and 5.15 billion mmBtu.

[15] Regional ozone season NO_x emission is the level of NO_x emission between May 1 and September 30 in twenty affected eastern states.

Furthermore, the average emission rate was reduced from 0.32 lb/mmBtu in 2003 to 0.18 lb/mmBtu in 2008. One reason for the dramatic decrease is believed to be the adoption of NO_x abatement technologies such as LNB, OFA, SCR and SNCR (EPA, 2009b). However, the average NO_x emission rate from affected non-controlled units also decreased from about 0.55 lb/mmBtu in 2003 to about 0.32 lb/mmBtu in 2008.

From Chapter IV, the MACC is a standard analytical tool in environmental economics that links firms' emission levels to an additional cost of reducing a unit of pollution emission, or MAC. Firms' MAC provides valuable information for determining pollution taxes, setting the level of emission permits, and estimating prices of pollutants in allowance markets. As stated in EPA (2009b), NO_x allowance prices should reflect firms' specific NO_x MAC; thus, a variety of emission control decisions can be made based on the firms' NO_x MAC.

Technical change can result in either reduced or increased MAC. In general, a number of theoretical models simply assume that technical change directly lowers MAC at all abatement levels, Milliman and Prince (1989), Rosendahl (2004), Bramoulle and Olsen (2005) and Fischer et al. (2003). However, Baker et al. (2008) reviewed several theoretical models and concluded that different approaches to derive MAC and model technical change can produce different conclusions. One example considered a nested CES production function and technical change represented by acknowledge parameter. When knowledge can substitute for both fossil and non-fossil energy inputs, MAC must be lowered by technical change; however, when knowledge can substitute for only fossil energy, MAC increases with technical change at higher levels of abatement. Baker et al.

(2006) provided another example in which technical change was modeled by an R&D parameter. MAC is lower when R&D leads to a uniform quantity reduction in emissions and higher when R&D causes proportional emission reduction. In conclusion, technical change does not necessarily imply a reduction in MAC. However, several of the models explored in Baker et al. (2008) have strong parametric or substitution assumptions. Thus the motivation for this chapter, *to develop nonparametric tools for estimating the empirical impact of technical change on MAC*.

Technical change is viewed as a shift of production frontier over time. The most common method of representing technical change in existing economic studies is to assume Hicks neutral technical change (Hick, 1966), Solow neutral technical change (Solow, 1956; 1957) or biased technical change through coefficients within particular parametric production functions. In the nonparametric frontier literature, the production frontier is constructed contemporaneously or sequentially (Tulkens and Van den Eeckaut, 1995). Contemporaneous production frontiers are those in which each time period's production frontier is estimated independently using only corresponding time period observations. Using them allows technological regress, meaning that production frontiers can move inward from previous periods. In contrast, sequential production frontiers use all observations from past periods up to the current period to ensure that the estimated production frontier envelops all observations, meaning that only technological progress exists.

To measure the effect of technical change on economic factors, researchers apply index numbers and the decomposition method to derive meaningful components including technical change. One example of this approach is the construction of the Malmquist productivity index. Färe et al. (1994) constructed contemporaneous production frontiers and estimate a set of distance functions to derive the Malmquist productivity index and its components. The estimated Malmquist productivity index is decomposed into an efficiency change effect, an activity effect, and a technical change effect; Färe et al. (1994) applied this technique to the productivity growth in OECD countries. Alternative Malmquist productivity index decompositions include Ray and Desli (1997) and Balk (2001). Shestalova (2003), who argued that only technological progress likely exists in the manufacturing industry, decomposed the Malmquist productivity index based on a sequential production frontier to evaluate productivity change in manufacturing in OECD countries. Grifell-Tatje and Lovell (1999) decomposed the profit change of Spanish banks into productivity, activity, and price effects; a technical change effect is included in the productivity effect and each term in profit decomposition is computed by the distance functions calculated using the sequential frontiers method.

The objective of this chapter is to measure the effect of technical change on NO_x MAC of U.S. coal power plants in 2000-2008. During this period, coal power plants significantly reduced their NO_x emission levels. This study investigates if these results derive from normal replacement of equipment, or from innovation induced from more stringent NO_x regulation programs. To measure the innovation effect on NO_x MAC, this study develops a two-stage decomposition method for the MAC change index. The first stage decomposes the MAC change index into a technical change effect and a non-technical change effect. The second stage decomposes the non-technical change effect into a pollutant level effect, production input level effect, and abatement input cost effect. To empirically implement the MAC change index decomposition, this study develops a three-step estimation method. The first step estimates multiple-period sequential pollutant frontiers.[16] As reported in EPA (2009b), the NO_x emissions per heat inputs from U.S. coal power plants have decreased consistently since 2000. Therefore, technological progress in NO_x reduction exists and it is appropriate to use sequential pollutant frontiers to analyze the innovation effect. While the method to nonparametricly estimate sequential production frontiers in deterministic cases already exists by using sequential DEA (Tulkens and Van den Eeckaut, 1995), there is no method that can estimate sequential productions in stochastic cases. This paper introduces a modified version of the CNLS that can estimate multiple-period sequential production frontiers when noise is considered. The second step recovers unobserved abatement cost minimization points on the estimated pollutant frontiers by solving several linear programming problems. The third step calculates a technical change effect and a non-technical change effect of MAC decomposition.

This chapter is organized as follows. Section 5.2 describes the decomposition of the MAC change index. The three-step estimation procedure is described in sections 5.3.1, 5.3.2, and 5.3.3. The data set describing the electricity generating resources, emissions, and abatement inputs is described in section 5.4. Section 5.5 presents and discusses the empirical results and section 5.6 gives conclusions.

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[16] Pollutant frontier is a function that describes a minimum level of pollutants given levels of production inputs and abatement inputs. More details appear in section 2.1.

5.2 MAC decomposition

The strategy to decompose the MAC ratio $\frac{\text{MAC}^{t+1}}{\text{MAC}^{t}}$ in multiple stages is motivated by the method of Grifell-Tatje and Lovell (1999). In the first stage, the MAC ratio is decomposed into a technical change effect and a non-technical change effect. In the second stage, the non-technical change effect is decomposed into a pollutant level effect, a production input level effect, and an abatement input cost effect.

Figure 5.1 shows an example in which innovative activities have taken place. Technical progress is observed and the frontier pollutant function shifts down from period t to $t+1$. Assume at time t that a firm operates at an arbitrary point A with $(\boldsymbol{x}_a^A, \boldsymbol{x}^A, b^A)$ and at time $t+1$, it operates at point M with $(\boldsymbol{x}_a^M, \boldsymbol{x}^M, b^M)$.

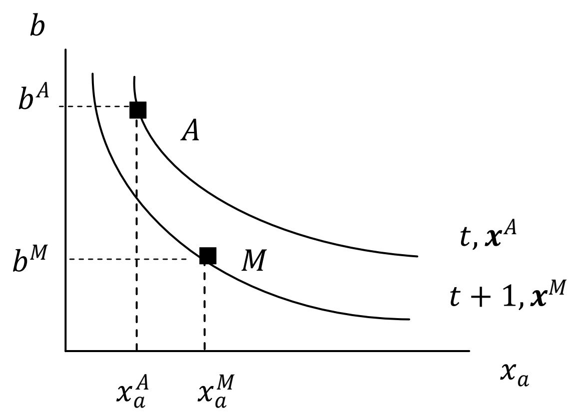

Figure 5.1 Technical progress exists from period t to period $t+1$

Two factors affect the choices of abatement inputs from period t to $t+1$. This study identifies 1) a technical change effect and 2) a non-technical change effect. A technical change effect changes the use of abatement inputs due to a shift in the frontier pollutant function holding other factors fixed. A non-technical change effect causes changes in the level of abatement inputs due to the firm's other activities in the same period.

To quantify the change in MAC between a period t frontier pollutant function and a period $t+1$ frontier pollutant function, the ratio $\frac{\text{MAC}^{t+1}}{\text{MAC}^{t}}$ is used as

$$\frac{\text{MAC}^{t+1}}{\text{MAC}^{t}} = \left(\frac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}}\right)\left(\frac{\frac{\partial B^{t}(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right) \tag{5.1}$$ [17]

where B^t, B^{t+1} $w_{x_{a_q}}^{t}$ and $w_{x_{a_q}}^{t+1}$ are the frontier pollutant function and the unit cost of abatement input q at period t and $t+1$ respectively. The MAC ratio (5.1) can be multiplicatively decomposed into a technical change effect and a non-technical change effect using period $t+1$ technology as [18]

$$\frac{\text{MAC}^{t+1}}{\text{MAC}^{t}} = \left(\frac{\frac{\partial B^{t}(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}\right)\left(\frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\frac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}}\right) \tag{5.2}$$

TC change   Non-TC change

Figure 5.2 illustrates the decomposition in (5.2). The left graph shows the technical change effect when the firm reduces the amount of an abatement input from x_a^A to x_a^G

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[17] The notation $B^t(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)$ has the same meaning as $B^t(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A)$ while the reason to put b^A in the bracket is to emphasize that $B^t(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A) = b^A$.

[18] A non-technical change effect can also be decomposed on the period t technology. Decomposing the non-technical change effect on both the period t and the period t+1 technologies allows a MAC ratio decomposition as described in the Appendix. However, components of the non-technical effect on the period t technology occasionally have infeasible solutions in practice as noted by Grifell-Tatje and Lovell (1999). This problem of infeasibility is similarly described in Ray and Mukhergee (1996).

while maintaining the same level of production inputs, $\boldsymbol{x}^A$, and emitting the same level of a pollutant, b^A. Note that if the technical progress does not exist, the firm will be unable to reduce the amount of an abatement input from x_a^A to x_a^G. The right graph shows a non-technical change effect within the period $t+1$, when the firm changes the amount of an abatement input from x_a^G to x_a^M reduce the amount of pollutant from b^A to b^M.

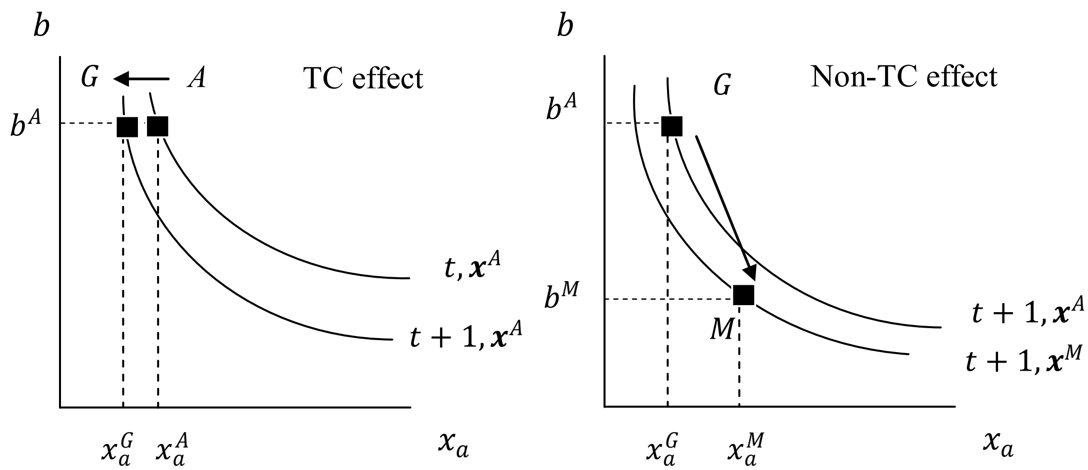


Figure 5.2 Technical change effect and a non-technical change effect at period $t+1$

The non-technical change effect is composed of three sub effects: 1) a pollutant level effect which changes the use of abatement inputs due to the change of pollution level, 2) a production input level effect which changes the use of abatement inputs due to the change of outputs level, and 3) an abatement inputs cost effect which changes the use of abatement inputs due to the change in the unit cost of abatement inputs. Figure 5.3 illustrates the decomposition of the non-technical change effect on the period $t+1$

pollutant frontier. Specifically, the figure shows the decomposition of the first term in the bracket of the non-technical change effect in Table 5.1 where a firm changes the amount of abatement input from x_a^G to x_a^M in period $t+1$. To capture the pollutant level effect and the production input level effect at $t+1$, assume that the firm uses information on abatement input costs from period $t+1$, $\boldsymbol{w}_{x_a}^{t+1}$, to decide the abatement input mix satisfying the pollutant and production input level constraints. The first graph shows the abatement input cost effect and the abatement input cost changes from $\boldsymbol{w}_{x_a}^t = \left(w_{x_{a1}}^t, w_{x_{a2}}^t\right)$ to $\boldsymbol{w}_{x_a}^{t+1} = \left(w_{x_{a1}}^{t+1}, w_{x_{a2}}^{t+1}\right)$ where $\frac{w_{x_{a1}}^t}{w_{x_{a2}}^t} \leq \frac{w_{x_{a1}}^{t+1}}{w_{x_{a2}}^{t+1}}$. To minimize the cost of abatement when the abatement input cost changes,[19] the firm adjusts the mix of abatement inputs from $\boldsymbol{x}_a^G$ to $\boldsymbol{x}_a^H$ in which abatement input 2 is used more than abatement input 1 due to the relative costs. The second graph shows the pollutant level effect when the firm increases the use of abatement input from x_a^H to x_a^K to reduce the pollutant level from b^A to b^M while maintaining production input levels, $\boldsymbol{x}^A$. The third graph shows the production input level effect when the firm reduces the use of abatement input from x_a^K to x_a^M and the production input level from $\boldsymbol{x}^A$ to $\boldsymbol{x}^M$ while still maintaining the pollutant level, b^M.

[19] While maintaining the level of pollutant at b^A and the level of production inputs at $\boldsymbol{x}^A$ in consistent with point A in period t+1

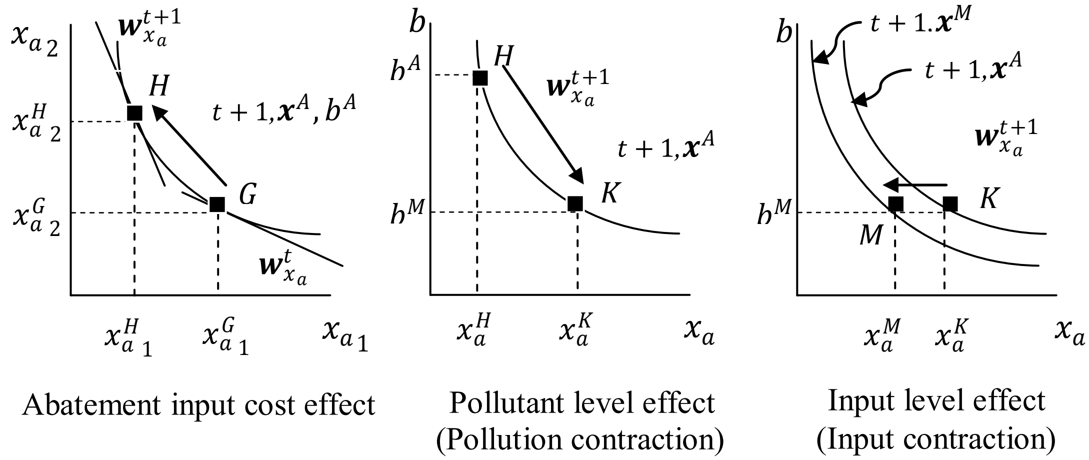
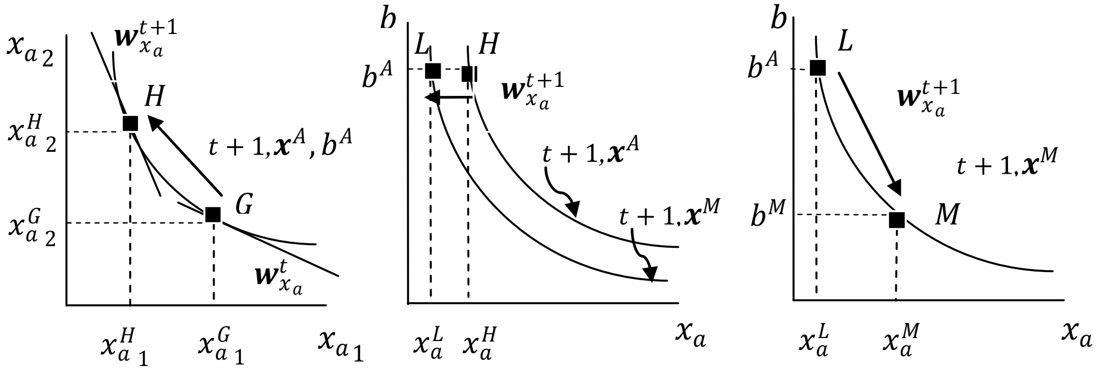


Figure 5.3 The first decomposition of a non-technical change effect at period $t + 1$

However, the sequence of non-technical change effect decomposition leads to different estimates of the pollutant level effect and the production input level effect. Figure 5.4 shows an alternative decomposition of the non-technical change effect on the period $t + 1$ pollutant frontier when interchanging the production input level effect term and the pollutant effect term. Table 1 summarizes the two alternative non-technical change effect decompositions on the period $t + 1$ pollutant frontier.



Abatement input cost effect

Input level effect
(Input contraction)

Pollutant level effect
(Pollution contraction)

Figure 5.4 The second decomposition of a non-technical change effect at period $t+1$

Table 5.1 The decomposition of a non-technical change effect at period $t+1$

Figure	Abatement input cost effect	Pollutant level effect	Production input level effect
5.3	$\dfrac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}} \dfrac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}}$	$\dfrac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^K;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}}$	$\dfrac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^K;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}$
5.4	$\dfrac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}} \dfrac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}}$	$\dfrac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^L;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}$	$\dfrac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^L;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}}$

The abatement input cost effect is consistent for both decompositions

$$\frac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}} \frac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}} \tag{5.3}$$

There are two different terms for the pollutant level effect; thus, following Färe et al. (1994), this study takes a geometric mean between these two terms to calculate the pollutant level effect. The pollutant level effect is written as

$$\left(\frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^K;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}} \times \frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^L;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right)^{\frac{1}{2}} \tag{5.4}$$

Finally, the production input level effect is written as

$$\left(\frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^L;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}} \times \frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^K;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right)^{\frac{1}{2}} \tag{5.5}$$

To compute the technical change effect and the non-technical change effect using the MAC ratio (5.2), the marginal products of the abatement input need to be estimated at the points A, G, H, K, L and M. Note that all points are shown in figures 5.3 and 5.4.

5.3 The estimation method

This section describes the estimation method to measure the technical change effect on MAC. Section 5.3.1 introduces contemporaneous frontier pollutant functions in which random noise is considered and each period frontier pollutant function is estimated by solving the CNLS problem. Based on a contemporaneous frontier pollutant function, this section develops a method to estimate the sequential frontier pollutant functions described in section 5.3.2. The sequential method consists of estimating fitted pollutant values by solving the modified CNLS problem. Then these fitted pollutant values are used to construct a series of unique pollutant frontiers function by applying the technique to construct unique CNLS production frontiers with the sequential DEA technique in Tulkens and Van den Eeckaut (1995). Based on the estimated sequential frontier pollutant functions, section 5.3.3 describes the method to find the abatement cost minimization points A, G, H, K, L and M by solving the set of linear programs.

5.3.1 Estimating a contemporaneous frontier pollutant function

Consider a frontier pollutant function at period t characterized by the pollutant equation with a multiplicative disturbance term

$$b_i^t = B^t(\boldsymbol{x}_{a_i}^{\ t}, \boldsymbol{x}_i^t)e^{\varepsilon_i^t} \qquad \forall i = 1, \dots, n \quad \forall t = 1, \dots, T \qquad (5.6)$$

where ε_i^t is the disturbance term at period t. Similar to the section 4.3.1, the multiplicative disturbance term considering contextual variables in an arbitrary period t can be written as

$$\varepsilon_i^t = v_i^t + {u_i^l}^t + \boldsymbol{\delta}^{t\prime}\boldsymbol{z}_i^t \qquad \forall i = 1, \dots, n \quad \forall t = 1, \dots, T \qquad (5.7)$$

where v_i^t is a random noise at period t, ${u_i^l}^t$ is the technical inefficiency of firm i at period t that is not explained by the contextual variable, $\boldsymbol{\delta}^{t\prime}\boldsymbol{z}_i^t$ is the technical inefficiency of firm i at period t that is explained by the contextual variables, $\boldsymbol{z}_i^t \in R^r$ are contextual variables at period t and $\boldsymbol{\delta}^t \in R^r$ are coefficients that capture the average effect of contextual variables on deviation from the frontier pollutant function at period t.

Based on the contextual variables CNLS problem (4.10), the contemporaneous CNLS problem with the disturbance term (5.7) at specific period s is then formulated as

$$\min_{\alpha^s,\boldsymbol{\gamma}^s,\boldsymbol{\rho}^s,\boldsymbol{\delta}^s,\zeta^s} \sum_{i=1}^{n} \zeta_i^{s\,2} \tag{5.8.1}$$

s.t. $$\zeta_i^s = \ln b_i^s - \ln(\alpha_i^s + \boldsymbol{\gamma}_i^{s\prime}\boldsymbol{x}_i^s + \boldsymbol{\rho}_i^{s\prime}\boldsymbol{x}_{a\,i}^{\;s}) - \boldsymbol{\delta}^{s\prime}\boldsymbol{z}_i^s \qquad \forall i = 1,\dots,n \tag{5.8.2}$$

$$\alpha_i^s + \boldsymbol{\gamma}_i^{s\prime}\boldsymbol{x}_i^s + \boldsymbol{\rho}_i^{s\prime}\boldsymbol{x}_{a\,i}^{\;s} \;\geq\; \alpha_h^s + \boldsymbol{\gamma}_h^{s\prime}\boldsymbol{x}_i^s + \boldsymbol{\rho}_h^{s\prime}\boldsymbol{x}_{a\,i}^{\;s} \qquad \forall i,h = 1,\dots,n \tag{5.8.1}$$

$$\boldsymbol{\gamma}_i^s \geq 0 \text{ and } \boldsymbol{\rho}_i^s \leq 0 \qquad \forall i = 1,\dots,n \tag{5.8.1}$$

where α_i^s, $\boldsymbol{\gamma}_i^s$ and $\boldsymbol{\rho}_i^s$ are the coefficients characterizing hyperplanes of the average pollutant function at period s.

Given $\left(\hat{\alpha}_i^s, \hat{\boldsymbol{\gamma}}_i^s, \hat{\boldsymbol{\rho}}_i^s, \hat{\zeta}_i^s\right) \forall i$ from the CNLS problem (5.8), the unique StoNEZD estimator of the frontier pollutant function at the specific period s is written as

$$\hat{B}_{\min}^s(\boldsymbol{x}_a, \boldsymbol{x}) = \left[\max_{\alpha,\boldsymbol{\gamma},\boldsymbol{\rho}}\{\alpha + \boldsymbol{\gamma}'\boldsymbol{x} + \boldsymbol{\rho}'\boldsymbol{x}_a \,|\, \alpha + \boldsymbol{\gamma}'\boldsymbol{x}_i^s + \boldsymbol{\rho}'\boldsymbol{x}_{a\,i}^{\;s} \leq \hat{b}_{\mathrm{i}}^s \;\; \forall i\}\right] e^{-\widehat{\mu^l}^s} \tag{5.9}$$

where $\hat{b}_{\mathrm{i}}^s = \max_{h\epsilon\{1,\dots,n\}}\{\hat{\alpha}_h^s + \hat{\boldsymbol{\gamma}}_h^{s\prime}\boldsymbol{x}_i^s + \hat{\boldsymbol{\rho}}_h^{s\prime}\boldsymbol{x}_{a\,i}^{\;s}\}$ and $\widehat{\mu^l}^S = \hat{\sigma}_{u^{l^s}}\sqrt{2/\pi}$ obtained from the method of moment described in (4.11) and (4.12). Given the CNLS residuals, $\hat{\zeta}_i^s$ $\forall i$, the skewness and kurtosis of disturbances tests are applied to indicate if the technical inefficiency exists i.e whether $\hat{\sigma}_{u^l} = 0$. The test results are used to formulate a standard t-test for $\hat{\boldsymbol{\delta}}^s$ and calculate the expect technical inefficiency $\widehat{\mu^l}^s$.

Note that the estimated contemporaneous pollutant frontiers from solving (5.8) and (5.9) might satisfy a sequential pollutant frontier condition if a technical progress is significant enough so that the estimated frontiers in each period do not cross. However, for an arbitrary data set, this method is not guaranteed to generate sequential pollutant frontiers. Therefore, next section proposes the solution method.

5.3.2 Estimating sequential frontier pollutant functions in multiple periods

Let $t+$ be the set of periods greater than t, $t+=\{s|s>t\}$. Using the concept of sequential production functions, meaning that technical regress is not possible (see figure 5.2), the condition between the frontier pollutant function at period t and $t+$ can be written as

$$B^{t+}(\boldsymbol{x}_a,\boldsymbol{x}) \leq B^{t}(\boldsymbol{x}_a,\boldsymbol{x}) \qquad \forall t=1,\dots,T-1 \qquad (5.10)$$

Condition (5.10) indicates that the pollution possibility set in $t+$ includes the pollution possibility set from period t. Consider a frontier pollutant function using a CNLS representator function, then $B^{t}(\boldsymbol{x}_a,\boldsymbol{x})=\max_h\{\alpha_h^t+\boldsymbol{\gamma}_h^t{}'\boldsymbol{x}+\boldsymbol{\rho}_h^t{}'\boldsymbol{x}_a\}$ and $B^{t+}(\boldsymbol{x}_a,\boldsymbol{x})=\max_h\{\alpha_h^{t+}+\boldsymbol{\gamma}_h^{t+}{}'\boldsymbol{x}+\boldsymbol{\rho}_h^{t+}{}'\boldsymbol{x}_a\}$; thus, condition (5.10) can be written as

$$\max_h\{\alpha_h^{t+}+\boldsymbol{\gamma}_h^{t+}{}'\boldsymbol{x}_i+\boldsymbol{\rho}_h^{t+}{}'\boldsymbol{x}_{a_i}\} \leq \max_h\{\alpha_h^{t}+\boldsymbol{\gamma}_h^{t}{}'\boldsymbol{x}_i+\boldsymbol{\rho}_h^{t}{}'\boldsymbol{x}_{a_i}\}$$

$$\forall i,h=1,\dots n \quad \forall t=1,\dots,T-1 \qquad (5.11)$$

For a production unit observed at two points in time, $(\boldsymbol{x}_i^t,\boldsymbol{x}_{a_i}^{\ t})$ and $(\boldsymbol{x}_i^{t+},\boldsymbol{x}_{a_i}^{\ t+})$, the CNLS problem (5.8) will assign a frontier pollutant function coefficients for each observation such that $B^{t}\left(\boldsymbol{x}_{a_i}^{\ t},\boldsymbol{x}_i^t\right)=\max_h\{\alpha_h^t+\boldsymbol{\gamma}_h^t{}'\boldsymbol{x}_i^t+\boldsymbol{\rho}_h^t{}'\boldsymbol{x}_{a_i}^{\ t}\}=\alpha_i^t+\boldsymbol{\gamma}_i^t{}'\boldsymbol{x}_i^t+\boldsymbol{\rho}_i^t{}'\boldsymbol{x}_{a_i}^{\ t}$ and $B^{t+}\left(\boldsymbol{x}_{a_i}^{\ t+},\boldsymbol{x}_i^{t+}\right)=\max_h\{\alpha_h^{t+}+\boldsymbol{\gamma}_h^{t+}{}'\boldsymbol{x}_i^{t+}+\boldsymbol{\rho}_h^{t+}{}'\boldsymbol{x}_{a_i}^{\ t+}\}=\alpha_i^{t+}+\boldsymbol{\gamma}_i^{t+}{}'\boldsymbol{x}_i^{t+}+\boldsymbol{\rho}_i^{t+}{}'\boldsymbol{x}_{a_i}^{\ t+}$. Thus, condition (5.11) can be written as

$$\max_h\{\alpha_h^{t+}+\boldsymbol{\gamma}_h^{t+}{}'\boldsymbol{x}_i^t+\boldsymbol{\rho}_h^{t+}{}'\boldsymbol{x}_{a_i}^{\ t}\} \leq \alpha_i^t+\boldsymbol{\gamma}_i^t{}'\boldsymbol{x}_i^t+\boldsymbol{\rho}_i^t{}'\boldsymbol{x}_{a_i}^{\ t}$$

and $$\alpha_i^{t+}+\boldsymbol{\gamma}_i^{t+}{}'\boldsymbol{x}_i^{t+}+\boldsymbol{\rho}_i^{t+}{}'\boldsymbol{x}_{a_i}^{\ t+} \leq \max_h\{\alpha_h^{t}+\boldsymbol{\gamma}_h^{t}{}'\boldsymbol{x}_i^{t+}+\boldsymbol{\rho}_h^{t}{}'\boldsymbol{x}_{a_i}^{\ t+}\}$$

$$\forall i, h = 1, \ldots n \quad \forall t = 1, \ldots, T-1 \tag{5.12}$$

Note that the sequential frontier condition (5.12) imposes the sequential relationship among pollutant functions; however, if unexplained technical inefficiency is significant, $\mu^s > 0$, then the CNLS problem should be solved adjusting the hyperplanes of the pollutant function for technical inefficiency. To formulate the CNLS problem satisfying the pollutant function properties and the sequential condition, the disturbance term is written as

$$\xi_i^t = v_i^t \qquad \forall i = 1, \ldots, n \quad \forall t = 1, \ldots, T \tag{5.13}$$

where v_i^t is a random noise at period t and the modified log pollutant level is written as

$$\ln b_i^t - \widehat{\mu^l}^t \qquad \forall i = 1, \ldots, n \quad \forall t = 1, \ldots, T \tag{5.14}$$

where $\widehat{\mu^l}^t$ is the the expected technical inefficiency at period t. Combing the contemporaneous CNLS problem (5.8) and the sequential condition (5.12) with the disturbance term (5.13) and the modified log pollutant level (5.14), the sequential CNLS problem is formulated as

$$\min_{\alpha^t, \boldsymbol{\gamma}^t, \boldsymbol{\rho}^t, \boldsymbol{\delta}^t, \xi^t} \sum_{t=1}^{T} \sum_{i=1}^{n} {\xi_i^t}^2 \tag{5.15.1}$$

s.t. $$\xi_i^t = \left(\ln b_i^t - \widehat{\mu^l}^t\right) - \ln(\alpha_i^t + \boldsymbol{\gamma}_i^{t\prime} \boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime} \boldsymbol{x}_{a\,i}^{\ t}) - \boldsymbol{\delta}^{t\prime} \boldsymbol{z}_i^t$$

$$\forall i = 1, \ldots, n \quad \forall t = 1, \ldots, T \tag{5.15.2}$$

$$\alpha_i^t + \boldsymbol{\gamma}_i^{t\prime} \boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime} \boldsymbol{x}_{a\,i}^{\ t} \geq \alpha_h^t + \boldsymbol{\gamma}_h^{t\prime} \boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t\prime} \boldsymbol{x}_{a\,i}^{\ t}$$

$$\forall i, h = 1, \ldots, n \quad \forall t = 1, \ldots, T \tag{5.15.3}$$

$$\max_h\{\alpha_h^{t+} + \boldsymbol{\gamma}_h^{t+\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t+\prime}\boldsymbol{x}_{a_i}^{\ t}\} \leq \alpha_i^t + \boldsymbol{\gamma}_i^{t\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime}\boldsymbol{x}_{a_i}^{\ t}$$

$$\forall i, h = 1, \ldots, n \quad \forall t = 1, \ldots, T-1 \qquad (5.15.4)$$

$$\alpha_i^{t+} + \boldsymbol{\gamma}_i^{t+\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_i^{t+\prime}\boldsymbol{x}_{a_i}^{\ t+} \leq \max_h\{\alpha_h^t + \boldsymbol{\gamma}_h^{t\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_h^{t\prime}\boldsymbol{x}_{a_i}^{\ t+}\}$$

$$\forall i, h = 1, \ldots, n \quad \forall t = 1, \ldots, T-1 \qquad (5.15.5)$$

$$\boldsymbol{\gamma}_i^t \geq 0 \text{ and } \boldsymbol{\rho}_i^t \leq 0 \qquad \forall i = 1, \ldots, n \quad \forall t = 1, \ldots, T \qquad (5.15.6)$$

where α_i^t, $\boldsymbol{\gamma}_i^t$ and $\boldsymbol{\rho}_i^t$ are the coefficients characterizing hyperplanes of the frontier pollutant function at period t, B^t. The objective function (5.15.1) minimizes the sum of squared of disturbances (5.13) summed over multiple periods. The equality constraints (5.15.2) define the disturbance using the modified log pollutant level (5.14). The constraints (5.15.3) and (5.15.6) are the same as the constraints (5.8.3) and (5.8.4) in the contemporaneous CNLS problem (5.8). The constraints (5.15.3) and (5.15.4) enforce the sequential frontier condition. An iterative procedure is used to solve the sequential CNLS problem (5.15). The proposed iterative procedure is the modified version of the algorithm proposed in Lee et al. (2011); see the Appendix for details.

Given $(\hat{\alpha}_i^t, \hat{\boldsymbol{\gamma}}_i^t, \hat{\boldsymbol{\rho}}_i^t)\ \forall i, \forall t$ from the sequential CNLS problem (5.15), the unique CNLS estimator of the frontier pollutant function at the specific period s is written as

$$\hat{B}_{\min}^s(\boldsymbol{x}_a, \boldsymbol{x}) = \max_{\alpha,\boldsymbol{\gamma},\boldsymbol{\rho}}\left\{\alpha + \boldsymbol{\gamma}'\boldsymbol{x} + \boldsymbol{\rho}'\boldsymbol{x}_a \mid \alpha + \boldsymbol{\gamma}'\boldsymbol{x}_i^t + \boldsymbol{\rho}'\boldsymbol{x}_{a_i}^{\ t} \leq \hat{b}_{\mathrm{i}}^t\ \forall i; \forall t = 1, \ldots, s\right\} \quad (5.16)$$

where $\hat{b}_{\mathrm{i}}^t = \max_{h\epsilon\{1,\ldots,n\}}\left\{\hat{\alpha}_h^t + \hat{\boldsymbol{\gamma}}_h^{t\prime}\boldsymbol{x}_i^t + \hat{\boldsymbol{\rho}}_h^{t\prime}\boldsymbol{x}_{a_i}^{\ t}\right\}\ \forall t = 1, \ldots, s$ and $\hat{\alpha}_i^{\ t}$, $\hat{\boldsymbol{\gamma}}_i^{\ t}$ and $\hat{\boldsymbol{\rho}}_i^{\ t}\ \forall i;\ \forall t$ are the coefficient estimates from the problem (5.15). Notice that the estimator of the frontier pollutant function (5.16) uses the concept in the sequential DEA. The estimator of the frontier pollutant function at period s uses the data from all previous periods

$\left(\boldsymbol{x_i}^t, \boldsymbol{x}_{a_i}^t, \hat{b}_i^{\,t}\right)$ $\forall i = 1, \dots, n$ and $\forall t = 1,\dots,s$ to ensure the sequential pollutant frontiers condition.

5.3.3 Finding abatement cost minimization points

This section describes the method to identify the abatement cost minimization points A, G, H, K, L and M, i.e. the solutions of the abatement cost minimization problem, $\min_{x_a}\left\{\boldsymbol{w}_{x_a}{}'\boldsymbol{x}_a : B^s(\boldsymbol{x}_a; \boldsymbol{x}) \leq b\right\}$. Given the unique CNLS estimator of the frontier pollutant function at period s (5.16), the cost minimization points can be found by solving the cost minimization problem, $\min_{x_a}\left\{\boldsymbol{w}_{x_a}{}'\boldsymbol{x}_a : \hat{B}^s_{\min}(\boldsymbol{x}_a; \boldsymbol{x}) \leq b\right\}$ where $\hat{B}^s_{\min}$ is characterized by the coefficient estimates $\left(\hat{\alpha}_i^{\,s}, \hat{\boldsymbol{\gamma}}_i^{\,s}, \hat{\boldsymbol{\rho}}_i^{\,s}\right)$ $\forall i$ obtained by solving the following linear program

$$\max_{\alpha^s, \boldsymbol{\gamma}^s, \boldsymbol{\rho}^s} \alpha^s + \boldsymbol{\gamma}^{s\prime}\boldsymbol{x_i}^s + \boldsymbol{\rho}^{s\prime}\boldsymbol{x}_{a_i}^s \tag{5.17.1}$$

$$\text{s.t.} \quad \alpha^s + \boldsymbol{\gamma}^{s\prime}\boldsymbol{x_i}^t + \boldsymbol{\rho}^{s\prime}\boldsymbol{x}_{a_i}^t \leq \hat{b}_i^{\,t} \qquad \forall i = 1, \dots, n \quad \forall t = 1, \dots, s \tag{5.17.2}$$

$$\boldsymbol{\gamma}^s \geq 0 \text{ and } \boldsymbol{\rho}^s \leq 0. \tag{5.17.3}$$

Given $\left(\hat{\alpha}_i^{\,s}, \hat{\boldsymbol{\gamma}}_i^{\,s}, \hat{\boldsymbol{\rho}}_i^{\,s}\right)$ $\forall i$ from the problem (5.17), the solution to the cost minimization problem is found by solving the following linear program

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x_a}{}'\boldsymbol{x}_a \tag{5.18.1}$$

$$\text{s.t.} \quad \hat{\alpha}_i^{\,s} + \hat{\boldsymbol{\gamma}}_i^{\,s\prime}\boldsymbol{x} + \hat{\boldsymbol{\rho}}_i^{\,s\prime}\boldsymbol{x}_a \leq b \qquad \forall i = 1, \dots n \tag{5.18.2}$$

$$\boldsymbol{x}_a \geq 0. \tag{5.18.3}$$

Given abatement input cost $\boldsymbol{w}_{x_a}$, the level of the production input $\boldsymbol{x}$, the level of the pollutant b and the estimated pollutant frontier parameters $\hat{\alpha}_i^{\,t}$, $\hat{\boldsymbol{\gamma}}_i^{\,t}$ and $\hat{\boldsymbol{\rho}}_i^{\,t}$ $\forall i = 1, \dots, n$

and $\forall t = 1, \ldots, T$, the abatement cost minimization points A, G, H, K, L and M for each firm i are found by solving the linear programming problem based on (5.17).[20]

5.3.4 Estimating the marginal product of an abatement input

The remaining task is to compute the marginal product of an abatement input at both observed and unobserved points. As shown in figure 5.5, the abatement cost minimization points on a piecewise linear frontier pollutant function are likely to exist at edge points, meaning that a partial derivative will differ when taken from the left or from the right.

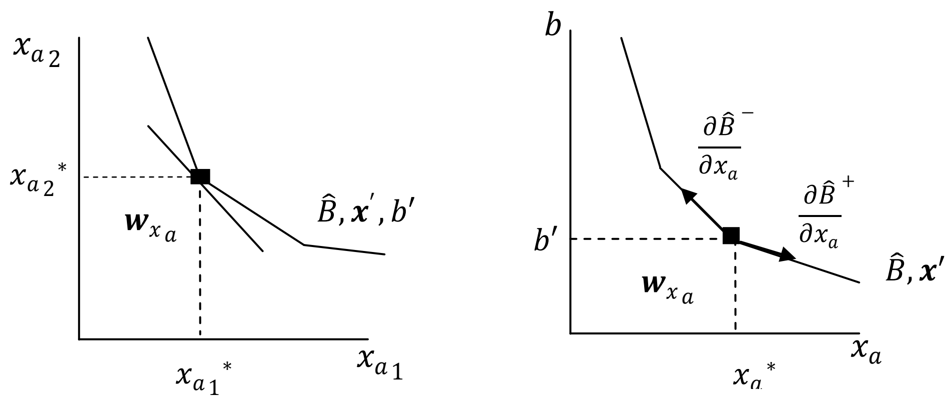


Figure 5.5 The abatement cost minimization occurs at edge points where the marginal product of an abatement input is non-unique.

Let $(\boldsymbol{x}_a, \boldsymbol{x})$ be the abatement cost minimizing production possibility on the pollutant frontier at period s obtained from solving (5.17). To estimate marginal products of an abatement input, the equation (4.15) is applied. Specifically

[20] This involves solving four sets of linear programs for each observation (see the Appendix).

$$\frac{\partial B^s(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)}{\partial x_{a_{q_i}}} \approx \frac{\hat{B}^s_{\min}(\boldsymbol{x}_{a_i}{}^+, \boldsymbol{x}_i) - \hat{B}^s_{\min}(\boldsymbol{x}_{a_i}, \boldsymbol{x}_i)}{\epsilon} \tag{5.19}$$

where $\epsilon > 0$ is a small positive number, $(\boldsymbol{x}_{a_i}{}^+, \boldsymbol{x}_i) = (x_{a_{1_i}}, x_{a_{2_i}}, .., x_{a_{q_i}} + \epsilon, \dots, x_{a_{Q_i}}, \boldsymbol{x}_i)$ and $\hat{B}^s_{\min}(\boldsymbol{x}_a, \boldsymbol{x})$ is the unique CNLS estimator of the sequential frontier pollutant function (5.16).

To summarize, the three-step estimation method to decompose the MAC ratio is as follows:

1. Estimate the sequential pollutant frontiers:

   1.1 Estimate the expected technical inefficiency $\widehat{\mu^l}^t\ \forall t = 1, \dots, T$ as described in section 5.3.1.

   1.2 Estimate the fitted pollutant values, $\hat{b}_i^{\,t}\ \forall i = 1, \dots, n;\ \forall t = 1, \dots, T$ by using the algorithm for solving the sequential CNLS problem (5.15) introduced in the Appendix. Given $\hat{b}_i^{\,t}\ \forall i; \forall t$, the unique CNLS estimator of the sequential frontier pollutant function (5.16) is obtained.

   1.3 For each period s, solve the linear programming problem (5.17) to obtain $(\hat{\alpha}_i^{\,s}, \hat{\gamma}_i^{\,s}, \hat{\rho}_i^{\,s})\ \forall i$. Solving for all periods gives $(\hat{\alpha}_i^{\,t}, \hat{\gamma}_i^{\,t}, \hat{\rho}_i^{\,t})\ \forall i = 1, \dots, n; \forall t = 1, \dots, T$.

2. For each firm i, find the abatement cost minimization points A, G, H, K, L and M by solving the linear programs based on (5.18).

3. Calculate a technical change effect and a non-technical change effect of MAC decomposition.

3.1 For each firm i, find the marginal products estimates at points A, G, H, K, L and M using (5.19).

3.2 Estimate the technical change effect and non-technical change effect using equation (5.2), the abatement input cost effect using equation (5.3), the pollutant level effect using equation (5.4), and the production input level effect using equation (5.5).

5.4 Data set

The same data set in chapter IV is analyzed. The panel boiler-level data consists of 325 units of U.S. bituminous coal power plants operating in 22 eastern states under the CAIR NO_x program. This chapter analyzes a technical change effect by investigating the data at three points in time: 2000, 2004, and 2008. In 2000, most of the plants were not regulated by the NO_x Budget Trading Program, 2004 was the first year that most were affected by the NO_x Budget Trading Program,[21] and 2008 was the last year of the NO_x Budget Trading Program before the transition to the more stringent CAIR NO_x program. See section 4.5 for data descriptions and Table 4.1 and 4.2 for summary statistics of the data.

5.5 Empirical results and analyses

The results of the skewness of CNLS residuals test are based on the result in section 4.4. For 2000 and 2004, the null hypothesis related to both $\sqrt{b_1}$ and b_2 tests

[21] The NO_x budget trading program was promulgated in 1998.

cannot be rejected at the 10% significant level; thus, the result does not support the present of technical inefficiency implying that $\sigma_u = 0$. For 2008, the null hypothesis cannot be rejected for the $\sqrt{b_1}$ test but is rejected for the b_2 test due to excess kurtosis. In this case, the present of technical inefficiency is rejected implying that $\sigma_u = 0$, but assuming that the disturbance contains only normal random noise may be poorly specified. In conclusion, there is not enough statistical evidence of the present of technical inefficiency in the sample implying that the expected technical inefficiency $\hat{\mu}^t = 0\ \forall t$.

The decomposition results for the change in MAC in 2000-2004 and 2004-2008 are reported in Table 5.2. The results are geometric averages over the number of power plants listed in the last column and the breakdowns of power plants excluded from the analysis are reported in the Appendix. Technical change accounted for 28.3 percent of NO_x MACs reduction in 2000-2004 and 26.5 percent in 2004-2008 for this sample. However, NO_x MACs increased about 30 percent in 2000-2004 and 14.8 percent in 2004-2008, mostly due to non-technical change which accounted for 81 percent of NO_x MACs increase in 2000-2004 and 56.1 percent in 2004-2008.

Table 5.2 MAC change decomposition for the coal power plants

Year	MAC change	Technical change effect	Non-technical change effect	Number
2000-2004	1.300	0.717	1.810	172
2004-2008	1.148	0.735	1.561	229

Table 5.3 reports the decomposition of the non-technical change effect into a pollutant level effect, a production input level effect, and an abatement input cost effect. On average, the abatement input cost effect accounted for 53.8 percent of NO_x MAC increase in 2000-2004 and 28 percent in 2004-2008, and is the largest contributor to the non-technical change effect. As plant operators began to install advanced abatement equipment, especially SCR and SNCR, the higher capital and operational costs (EPA, 2010) of such systems resulted in higher MAC.

The pollutant level effect accounted for 20.9 percent of NO_x MAC increase in 2000-2004 and 13.8 in 2004-2008. During these periods, power plants significantly lowered their NO_x emission levels for two reasons. Under the EPA's NO_x budget program, each state is required to reduce its NO_x emission cap every year; affected power plants were allowed fewer NO_x allowances and therefore reduced their NO_x emission levels. Second, the NO_x budget program allowed operators to bank their unused allowances for future use; thus operators began to further reduce NO_x emissions. Other factors such as uncertain regulatory conditions also contributed banking allowances.

Finally, the results related to the production input level effect are mixed; however, this effect has a limited contribution to changes in NO_x MAC when compares to the abatement input cost effect and the pollutant level effect. On average, the input level effect contributed only 2.6 percent of NO_x MACs decrease in 2000-2004 and 7.1 percent of NO_x MACs increase in 2004-2008, because heat input levels remained relatively stable over the observed years. In fact, on average, the amount of heat input decreased only 4.96 percent in 2000-2004 and 0.44 percent in 2004-2008. On the other hand, NO_x levels decreased 27.08 percent in 2000-2004 and 17.46 percent in 2004-2008. Coal power plants typically use coal-burning boilers to generate heat input, while gas-burning or oil-burning boilers are used for additional heat input generation during periods of increased demand for electricity. This is the primary reason heat input levels are stable in our sample.

Table 5.3 The non-technical change effect decomposition for the coal power plants

Year	Non-technical change effect	Pollutant level effect	Production input level effect	Abatement input cost effect
2000-2004	1.810	1.209	0.974	1.538
2004-2008	1.561	1.138	1.071	1.280

The effects of boiler vintages, δ_1 and δ_2, on pollutant level are reported in Table 5.4. The results are similar to the results in chapter IV (Table 4.6) which finds that both δ_1 and δ_2 are positively signed and significant. It implies that older vintages increase NO_x emissions. On average, boilers commissioned in 1940-1959 have 21.8%-27.4% and in 1960-1979 have 6.8%-16.3% higher NO_x emissions than those entering operation after 1980. The vintage effect decreased in 2000-2008, possibly due to increased maintenance, upgrades, and replacement.

Table 5.4 Contextual variable parameter estimates

Year	$\hat{\delta}_1$	t-statistic	$\hat{\delta}_2$	t-statistic
2000	0.274[a]	14.687	0.156[a]	8.887
2004	0.226[a]	11.532	0.163[a]	8.816
2008	0.218[a]	6.702	0.068[b]	2.235

[a] Significant at the 1% level or better

[b] Significant at the 5% level or better

To summarize, several factors resulted in the MAC change during the NO_x budget program. A non-technical change effect was caused by operators adjusting their abatement inputs to lower NO_x levels while maintaining a given level of heat input and abatement input cost. Increases in NO_x MACs primarily resulted from the higher capital and operational cost of the new abatement systems (the abatement input cost effect) and lower NO_x pollutant levels (the pollutant level effect) due to both programs. The boilers in this analysis consumed relatively constant amounts of heat input over the analysis period; thus, changes in MACs were not attributed to changes in the heat input level effect. In 2000-2008, on average, technical change lowered the NO_x MACs of coal power plants.

5.6 Conclusions

This chapter describes the effect of technical change on firms' MAC. This chapter develops a new decomposition of the MAC change ratio consisting of a technical change effect and a non-technical change effect. The non-technical change effect was further decomposed into three sub factors, an abatement input cost effect, a pollutant level effect, and a production input level effect. The decomposition allowed identification of the sources of MAC change. To measure each effect empirically, this chapter develops a methodology consisting of three steps: 1) a nonparametric estimation method of sequential frontier pollutant functions in a stochastic framework, 2) a calculation of unobserved abatement cost minimization points based on the estimated sequential frontier pollutant functions, and 3) a calculation of the MAC change decomposition based on marginal product of abatement inputs at abatement cost minimization points.

The methodology is applied to a data set of 325 boilers in 134 U.S. bituminous coal power plants operating under the current CAIR NO_x program. A technical change effect is analyzed by investigating the data at three points in time: 2000, 2004, and 2008. This chapter finds that the significant NO_x reduction was due to more stringent regulations and that the higher MAC was due to widespread installation of advanced post-combustion abatement system such as SCR and SNCR. This study concludes that even though technical change exists and lowers MAC, the technical change effect is overwhelmed by the effects of regulation and post-combustion equipment.

An important question in the cap and trade program is whether emission permits should be given to polluting firms for free or they should be auctioned. Free and auctioned permits instrument provide different incentives for firms to promote innovation and diffusion, especially when technical change has different effects on MAC. Milliman and Prince (1989) stated that if technical change decreases MAC, auction permits provide the most incentive for industry to develop pollution control innovations and to promote diffusion across firms. On the other hand, Baker et al. (2008)

concluded that if technical change increases MAC, emission subsidies provide the most incentive for innovation and diffusion and free permits are better instruments than auctioned permits in diffusion promotion. Because technical change lowered NO_x MAC for coal power plants under the CAIR NO_x program, auctioning permits would promote innovation and diffusion in coal power plants which may not be achieved currently because permits are given away.

CHAPTER VI

CONCLUSIONS

6.1 Summary

Chapter II reviewed the CNLS and the StoNED methods and summarized how to estimate a frontier production function. The estimation method's steps included solving a CNLS problem, testing the skewness of the CNLS residuals, finding the expected technical efficiency by the method of moments, estimating the fitted values of an output from the representor function and finally deriving the unique estimator of the frontier production function. The chapter also described a variant of the StoNEZD method to estimate a frontier production function when considering contextual variables as well as a method to estimate a partial derivative on the frontier production function.

Chapter III presented a new estimation method which incorporated the weak disposability axiom into the CNLS and the StoNED methods. Unlike prior deterministic or DEA weak disposability models, the proposed methods are less sensitive to outliers because they allow the presence of random noise in the data. The chapter described how to apply the weak disposability CNLS and StoNED methods to estimate technical inefficiency and shadow prices of SO_2 and NO_x from U.S. coal power plants regulated under the EPA's Acid Rain Program in 2000–2008. The major finding was that the StoNED method gave more reasonable estimated shadow prices, i.e. they were within the range of EPA allowance auction and market prices, than the estimated shadow prices obtained via the deterministic method. The estimated average shadow prices of SO_2 were between 201 and 343 \$/ton, and the estimated average shadow prices of NO_x were between 409 and 1,352 \$/ton.

One-stage frontier production function models assuming pollutants as production inputs have been criticized for violating the physical law of conservation of mass. Moreover, these models neglect abatement processes which are important for polluting firms. Chapter IV addressed these issues by proposing a production model of polluting firms considering both a model of the output production process and a model of the pollution abatement processes in which the law of conservation of mass was imposed. The result established a pollutant function, an abatement cost function and a MAC. The StoNEZD method was extended to estimate the frontier pollutant function of the bituminous coal power plants under the CAIR NO_x program during 2000–2008 and the effect of plant vintage on the pollutant level. The estimated average NO_x MACs for the plants between 724 and 5,471 \$/ton were in the range of the EPA's projected NO_x prices, yet were likely to be higher than the NO_x market prices.

Chapter V then proposed a new method to estimate an effect of technical change on a MAC. First, it developed a new decomposition of the MAC change ratio consisting of a technical change effect and a non-technical change effect. The non-technical change effect was further decomposed into an abatement input cost effect, a pollutant level effect and a production input level effect. Second, Chapter V developed the estimation method of sequential frontier pollutant functions in a stochastic framework; sequential frontier pollutant functions are used to find the components of the decomposition. The main finding was that technical change in 2000–2008 lowered the average NO_x MACs of bituminous coal power plants under the CAIR NO_x program, but that the average NO_x MACs increased due to the effects of regulation and the installation of post-combustion equipment.

6.2 Main contributions

The important contributions of this dissertation are outlined below.

Methodology contributions

1. Chapter III developed an estimation method which includes the weak disposability axiom in a frontier production function considering random noise in the data.
2. Chapter IV developed a modeling of polluting firms considering abatement processes and the law of conservation of mass.
3. Chapter IV developed a derivation of a pollutant function and corresponding MAC in a production economic framework.
4. Chapter V developed an estimation of the technical change effect on a MAC using an index decomposition technique.
5. Chapter V developed an estimation method of sequential frontier functions considering random noise in the data.

Empirical contributions

1. From Chapter III, the estimated average technical inefficiencies range between 0.927 and 0.943. The estimated average SO_2 shadow prices for bituminous coal power plants under the Acid Rain Program are between 201 and 343$/ton. The

estimated average NO_x shadow prices for bituminous coal power plants under the Acid Rain Program are between 409 and 1,352$/ton. Both the average estimated shadow prices of SO_2 and NO_x are in reasonable ranges and likely to be lower than the allowance market prices.

2. From Chapter IV, the estimated average NO_x MACs for bituminous coal power plants under the CAIR NO_x program are between 724 and 5,471$/ton. The estimated average NO_x MACs are in the range of projected NO_x prices, but are likely to be higher than the NO_x allowance market prices.
3. From Chapter V, in 2000–2008, technical change lowered average NO_x MACs, but non-technical change increased average NO_x MACs for bituminous coal power plants under the CAIR NO_x program. On average, technical change accounted for 28.3 percent of NO_x MAC reduction in 2000–2004 and 26.5 percent of NO_x MAC reduction in 2004–2008. The abatement input cost effect accounted for 53.8 percent of NO_x MAC increase in 2000–2004 and 28 percent of NO_x MAC increase in 2004–2008. The pollutant level effect accounted for 20.9 percent of NO_x MAC increase in 2000–2004 and 13.8 of NO_x MAC increase in 2004–2008. Finally, the input level effect contributed only 2.6 percent of NO_x MAC reduction in 2000–2004 and 7.1 percent of NO_x MAC increase in 2004–2008.

6.3 Further research

This dissertation uses a single output multiple input frontier production function model because the only output from coal power plants is electricity and a single pollutant multiple input frontier pollutant function model because each pollutant has different abatement processes. The research described should be extended by considering a multiple output (pollutant) multiple input frontier production function model in which outputs (pollutants) are joint products from the same production (abatement) process. The regression model in Collier et al. (2011) could be used to extend the weak disposability CNLS and StoNED methods allowing multiple outputs and inputs or multiple pollutants and inputs.

The pollutant function is derived using the underlying assumption that byproduct pollutants have a linear relationship with production inputs. This seems logical for chemical processes such as burning coal in which the amount of coal, SO_2 and NO_x can be written in chemical equations which are linear. However, a linear assumption might not hold for some processes, for example, a byproduct that is monotonic increasing in production inputs. Further research might derive a more general pollutant function relaxing the linear relationship between byproduct pollutants and production outputs.

A MAC is derived by the assumption that polluting firms have to produce a given level of outputs; thus, the firm must use a fixed level of production inputs at the first stage. This assumption is reasonable for a coal power plant industry and for the U.S. domestic coal plant industry which burns a stable amount of coal to produce baseload electricity. The model in this dissertation should be extended to consider a situation where polluting firms jointly determine the amount of different inputs and abate pollutants to minimize the overall cost of production. Thus, the profit maximization problem integrated with the abatement cost minimization problem yields an alternative formulation for MAC.

Chapter V can be extended in two directions. First, the decomposition method could include Johnson and Ruggiero's (2011) model of the effect of contextual variables on the MAC. Second, the decomposition method could consider how to distinguish between the effects of embodied and disembodied technical change.

Additionally, different mathematical tools for estimation could be applied. First, the partial derivative estimates are calculated to provide shadow prices, MAC and MAC decomposition. Thus, it would be more convenient to estimate a smooth frontier production function so that the partial derivative estimates is unique and not likely to equal zero. Second, the nonconvex objective function in the multiplicative disturbance model makes it difficult to find global optimal solutions. More advanced algorithms for solving nonconvex problems could be used. Alternatively, the nonconvex problems could be approximated by a linear transformation.

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APPENDIX A

SUPPORTING DISCUSSIONS FOR CHAPTER III

A.1 Proof of Proposition 3.1

In a single output case, we can transform the problem (3.2) into an additive form:

$$\max_{\phi,\lambda,\mu}\left\{\emptyset_o \left| \begin{array}{ll} \sum_{i=1}^{n}\lambda_i y_i \geq y_o + \emptyset_o & \\ \sum_{i=1}^{n}\lambda_i b_{ij} \leq b_{oj} & \forall j = 1,\dots,J \\ \sum_{i=1}^{n}(\lambda_i + \mu_i)x_{im} \leq x_{om} & \forall m = 1,\dots,M \\ \sum_{i=1}^{n}(\lambda_i + \mu_i) = 1 & \\ \lambda_i, \mu_i \geq 0 & \forall i = 1,\dots,n \end{array}\right.\right\} \tag{A.1}$$

where $\theta_o = 1 + \frac{\emptyset_o}{y_o}$. Applying duality theory of linear programming, the LP problem (A.1) has a dual problem

$$\min_{\alpha,\boldsymbol{w},\boldsymbol{c}}\left\{(\alpha + \boldsymbol{w}'\boldsymbol{x}_o + \boldsymbol{c}'\boldsymbol{b}_o) - y_o \left| \begin{array}{ll} \alpha + \boldsymbol{w}'\boldsymbol{x}_i + \boldsymbol{c}'\boldsymbol{b}_i \geq y_i & \forall i \\ \alpha + \boldsymbol{w}'\boldsymbol{x}_i \geq 0 & \forall i \\ \boldsymbol{w}, \boldsymbol{c} \geq 0 & \forall i \end{array}\right.\right\}. \tag{A.2}$$

y_o can be removed from the objective function since it is a constant. Taking the logarithm of the objective function and the first set of constraints, because the logarithm is a monotonic transformation for values greater than or equal to 1, and adding the negative of $\ln y_o$ to the objective function since it is a constant, problem (A.2) becomes

$$\min_{\alpha,\boldsymbol{w},\boldsymbol{c}}\left\{\ln(\alpha+\boldsymbol{w}'\boldsymbol{x}_o+\boldsymbol{c}'\boldsymbol{b}_o)-\ln y_o\left|\begin{array}{ll}\ln(\alpha+\boldsymbol{w}'\boldsymbol{x}_i+\boldsymbol{c}'\boldsymbol{b}_i)\geq \ln y_i & \forall i\\ \alpha+\boldsymbol{w}'\boldsymbol{x}_i\geq 0 & \forall i\\ \boldsymbol{w},\boldsymbol{c}\geq 0 & \forall i\end{array}\right.\right\} \qquad \text{(A.3)}$$

Introducing a new variable $\varepsilon_o=\ln y_o-\ln(\alpha+\boldsymbol{w}'\boldsymbol{x}_o+\boldsymbol{c}'\boldsymbol{b}_o)$ and adding an additional constraint, problem (A.3) can be equivalently written as

$$\min_{\alpha,\boldsymbol{w},\boldsymbol{c},\varepsilon}\left\{-\varepsilon_o\left|\begin{array}{ll}\varepsilon_o=\ln y_o-\ln(\alpha+\boldsymbol{w}'\boldsymbol{x}_o+\boldsymbol{c}'\boldsymbol{b}_o) & \\ \ln(\alpha+\boldsymbol{w}'\boldsymbol{x}_i+\boldsymbol{c}'\boldsymbol{b}_i)\geq \ln y_i & \forall i\\ \alpha+\boldsymbol{w}'\boldsymbol{x}_i\geq 0 & \forall i\\ \boldsymbol{w},\boldsymbol{c}\geq 0 & \forall i\end{array}\right.\right\}. \qquad \text{(A.4)}$$

Instead of solving (A.4) separately for each firm, it is possible to combine *n* optimization formulations and solve simultaneously for all firms. Since $\varepsilon_i,\alpha_i,\boldsymbol{w}_i$ and $\boldsymbol{c}_i$ are estimated independently for each firm, (A.5) minimize the sum of ε_i as

$$\min_{\alpha,\boldsymbol{w},\boldsymbol{c},\varepsilon}\left\{-\sum_{i=1}^{n}\varepsilon_i\left|\begin{array}{ll}\varepsilon_i=\ln y_i-\ln(\alpha_i+\boldsymbol{w}_i{}'\boldsymbol{x}_i+\boldsymbol{c}_i{}'\boldsymbol{b}_i) & \forall i\\ \ln(\alpha_h+\boldsymbol{w}_h{}'\boldsymbol{x}_i+\boldsymbol{c}_h{}'\boldsymbol{b}_i)\geq \ln y_i & \forall i,h\\ \alpha_h+\boldsymbol{w}_h{}'\boldsymbol{x}_i\geq 0 & \forall i,h\\ \boldsymbol{w}_i,\boldsymbol{c}_i\geq 0 & \forall i\end{array}\right.\right\}. \qquad \text{(A.5)}$$

By construction, $\varepsilon_i\leq 0$; thus, this constraint is added to the problem. Moreover, it can add the inefficiency term ε_i to the right side of the second set of constraints because of the monotonicity assumption. Note that the constraints are binding if $i=h$, and inequality otherwise

$$\min_{\alpha,\boldsymbol{w},\boldsymbol{c},\varepsilon}\left\{-\sum_{i=1}^{n}\varepsilon_i\left|\begin{array}{ll}\varepsilon_i=\ln y_i-\ln(\alpha_i+\boldsymbol{w}_i{}'\boldsymbol{x}_i+\boldsymbol{c}_i{}'\boldsymbol{b}_i) & \forall i\\ \ln(\alpha_h+\boldsymbol{w}_h{}'\boldsymbol{x}_i+\boldsymbol{c}_h{}'\boldsymbol{b}_i)+\epsilon_i\geq \ln y_i & \forall i,h\\ \alpha_h+\boldsymbol{w}_h{}'\boldsymbol{x}_i\geq 0 & \forall i,h\\ \boldsymbol{w}_i,\boldsymbol{c}_i\geq 0,\varepsilon_i\leq 0 & \forall i\end{array}\right.\right\}. \qquad \text{(A.6)}$$

Since $\ln y_i - \varepsilon_i = \ln(\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i)$, the second set of constraints can be written as $\ln(\alpha_h + \boldsymbol{w}_h{}'\boldsymbol{x}_i + \boldsymbol{c}_h{}'\boldsymbol{b}_i) \geq \ln(\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i)\ \forall i, h = 1, \ldots, n$. Removing the logarithm from this second set of constraints allows the problem (A.6) to be equivalently written as:

$$\min_{\alpha,\boldsymbol{w},\boldsymbol{c},\varepsilon}\left\{-\sum_{i=1}^{n}\varepsilon_i \left| \begin{array}{ll} \varepsilon_i = \ln y_i - \ln(\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i) & \forall i \\ \alpha_h + \boldsymbol{w}_h{}'\boldsymbol{x}_i + \boldsymbol{c}_h{}'\boldsymbol{b}_i \geq \alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i & \forall i, h \\ \alpha_h + \boldsymbol{w}_h{}'\boldsymbol{x}_i \geq 0 & \forall i, h \\ \boldsymbol{w}_i, \boldsymbol{c}_i \geq 0, \varepsilon_i \leq 0 & \forall i \end{array}\right.\right\} \tag{A.7}$$

which is the problem (3.5). □

A.2 Proof of proposition 3.2

By construction, $\theta_i y_i = y_i + \emptyset_i$, thus $\theta_i = 1 + \emptyset_i / y_i$. By duality between the problem (A.1) and (A.2), $\emptyset_i = (\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i) - y_i$. This gives $\theta_i = (\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i)/y_i$. By construction the variable $\varepsilon_i = \ln y_i - \ln(\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i)$, thus $e^{\varepsilon_i} = y_i / (\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i) = 1/\theta_i$. □

A.3 Proof of proposition 3.3

Let the function $\Omega(\phi_1, \ldots, \phi_n) = \sum_{i=1}^{n}(\ln y_i - \ln\phi_i)^2$. Since $\frac{\partial^2\Omega}{\partial\phi_i^2} = \frac{2}{\phi_i^2}(1 - \ln\phi_i + \ln y_i)\ \forall i$ and $\frac{\partial^2\Omega}{\partial\phi_i\partial\phi_j} = 0\ \forall i, j = 1, \ldots, n$, all non-diagonal elements in the Hessian matrix of the function Ω are equal to zero. Thus, the function Ω is convex if and only if

$\frac{\partial^2 \Omega}{\partial \phi_i^2} = \frac{2}{\phi_i^2}(1 - \ln\phi_i + \ln y_i) \geq 0\ \forall i$. This condition is equivalent to $0 < \phi_i \leq e y_i\ \forall i$. Since the objective function of the CNLS problem (3.8) is a composition with an affine function $\phi_i = \alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i\ \forall i$, it is convex if the function Ω is convex if and only if $\alpha_i + \boldsymbol{w}_i{}'\boldsymbol{x}_i + \boldsymbol{c}_i{}'\boldsymbol{b}_i \leq e y_i\ \forall i$. □

APPENDIX B

SUPPORTING DISCUSSIONS FOR CHAPTER IV

B.1 Proof of Proposition 4.1

1. $\frac{\partial G}{\partial x_{a_q}} = \frac{\partial A}{\partial x_{a_q}} \geq 0$. $\frac{\partial^2 G}{\partial {x_{a_q}}^2} = \frac{\partial^2 A}{\partial {x_{a_q}}^2} \leq 0$.

2. $\frac{\partial G}{\partial x_m} = \frac{\partial A}{\partial b_{int}} \frac{\partial b_{int}}{\partial x_m} \geq 0$ since $0 \leq \frac{\partial A}{\partial b_{int}} \leq 1$ and $\frac{\partial b_{int}}{\partial x_m} \geq 0$. $\frac{\partial^2 G}{\partial {x_m}^2} = \frac{\partial A}{\partial b_{int}} \frac{\partial b_{int}^2}{\partial {x_m}^2} + \left(\frac{\partial b_{int}}{\partial x_m}\right)^2 \frac{\partial^2 A}{\partial {b_{int}}^2} = 0 + \left(\frac{\partial b_{int}}{\partial x_m}\right)^2 \frac{\partial^2 A}{\partial {b_{int}}^2} \leq 0$ since $\frac{\partial b_{int}^2}{\partial {x_m}^2} = 0$, $\left(\frac{\partial b_{int}}{\partial x_m}\right)^2 \geq 0$ and $\frac{\partial^2 A}{\partial {b_{int}}^2} \leq 0$.

3. Since A is concave in $\boldsymbol{x}_a$ and b_{int} and $b_{int} = h(\boldsymbol{x}; \boldsymbol{o})$ where h is monotonic increasing and linear in $\boldsymbol{x}$. Thus, A is concave in $\boldsymbol{x}_a$ and $\boldsymbol{x}$.

B.2 Proof of Proposition 4.2

1. $\frac{\partial B}{\partial x_{a_q}} = \frac{\partial}{\partial x_{a_q}}(b_{int} - G) = 0 - \frac{\partial G}{\partial x_{a_q}} \leq 0$ since $\frac{\partial G}{\partial x_{a_q}} \geq 0$. $\frac{\partial^2 B}{\partial {x_{a_q}}^2} = \frac{\partial^2}{\partial {x_{a_q}}^2}(b_{int} - G) = 0 - \frac{\partial^2 G}{\partial {x_{a_q}}^2} \geq 0$ since $\frac{\partial^2 G}{\partial {x_{a_q}}^2} \leq 0$.

2. $\frac{\partial B}{\partial x_m} = \frac{\partial}{\partial x_m}(b_{int} - G) = \frac{\partial b_{int}}{\partial x_m} - \frac{\partial G}{\partial x_m} \geq 0$ since $\frac{\partial G}{\partial x_m} = \frac{\partial A}{\partial b_{int}} \frac{\partial b_{int}}{\partial x_m} \geq 0$ and $0 \leq \frac{\partial A}{\partial b_{int}} \leq 1$ thus $\frac{\partial b_{int}}{\partial x_m} \geq \frac{\partial G}{\partial x_m}$. $\frac{\partial^2 B}{\partial {x_m}^2} = \frac{\partial^2}{\partial {x_m}^2}(b_{int} - G) = \frac{\partial^2 b_{int}}{\partial {x_m}^2} - \frac{\partial^2 G}{\partial {x_m}^2} = 0 - \frac{\partial^2 G}{\partial {x_m}^2} \geq 0$ since $\frac{\partial^2 G}{\partial {x_m}^2} \leq 0$.

3. $b_{int} = h(\boldsymbol{x};\boldsymbol{o}) = \boldsymbol{0}'\boldsymbol{x}_a + \sum_{s=1}^{S} h_s(\boldsymbol{o})\, y_s$ is an affine function on R_+, thus, $h(\boldsymbol{x};\boldsymbol{o})$ is a convex function. From Proposition 4.1.3, G is a concave function, thus, $-G$ is a convex function. Since $B = b_{int} - G$, B is a convex function.

B.3 Proof of Proposition 4.3

1. Let $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b) = \min_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \le b\}$ and let $\boldsymbol{x}_a(\boldsymbol{w}_{x_a}, \boldsymbol{x})$ solve this ACMP, thus, $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b) = \boldsymbol{w}_{x_a}\boldsymbol{x}_a(\boldsymbol{w}_{x_a}, \boldsymbol{x})$. Consider a new ACMP with prices $\lambda\boldsymbol{w}_{x_a}, \lambda \ge 0$. The new abatement cost can be written as $C^*(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b) = \min_{\boldsymbol{x}_a}\{\lambda\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \le b\} = \lambda\min_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \le b\}$.Thus, $C^*(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b) = \lambda\boldsymbol{w}_{x_a}\boldsymbol{x}_a(\boldsymbol{w}_{x_a}, \boldsymbol{x}) = \lambda C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b)$.
2. Assume that $\boldsymbol{w}_{x_a}^1 \ge \boldsymbol{w}_{x_a}^2$, it needs to show that $C(\boldsymbol{w}_{x_a}^1, \boldsymbol{x}, b) \ge C(\boldsymbol{w}_{x_a}^2, \boldsymbol{x}, b)$. Let $\boldsymbol{x}_a^1 = \operatorname{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}^1\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}^1, \boldsymbol{o}) \le b\}$ and $\boldsymbol{x}_a^2 = \operatorname{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}^2\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}^2, \boldsymbol{o}) \le b\}$. This implies that $\boldsymbol{w}_{x_a}^2\boldsymbol{x}_a^1 \ge \boldsymbol{w}_{x_a}^2\boldsymbol{x}_a^2$ since $\boldsymbol{x}_a^2$ is a cost minimizer with $\boldsymbol{w}_{x_a}^2$ and $\boldsymbol{w}_{x_a}^1\boldsymbol{x}_a^1 \ge \boldsymbol{w}_{x_a}^2\boldsymbol{x}_a^1$ since $\boldsymbol{w}_{x_a}^1 \ge \boldsymbol{w}_{x_a}^2$. Thus, $C(\boldsymbol{w}_{x_a}^1, \boldsymbol{x}, b) = \boldsymbol{w}_{x_a}^1\boldsymbol{x}_a^1 \ge \boldsymbol{w}_{x_a}^2\boldsymbol{x}_a^1 \ge \boldsymbol{w}_{x_a}^2\boldsymbol{x}_a^2 = C(\boldsymbol{w}_{x_a}^2, \boldsymbol{x}, b)$.
3. Let $\boldsymbol{x}_a^1 = \operatorname{argmin}_{x_a}\{\boldsymbol{w}_{x_a}^1\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \le b\}$ and $\boldsymbol{x}_a^2 = \operatorname{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}^2\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \le b\}$. For $0 \le \lambda \le 1$, let $\boldsymbol{w}_{x_a}^3 = \lambda\boldsymbol{w}_{x_a}^1 + (1-\lambda)\boldsymbol{w}_{x_a}^2$. It needs to show that $C(\boldsymbol{w}_{x_a}^3, \boldsymbol{x}, b) \ge \lambda C(\boldsymbol{w}_{x_a}^1, \boldsymbol{x}, b) + (1-\lambda)C(\boldsymbol{w}_{x_a}^2, \boldsymbol{x}, b)$. Let $\boldsymbol{x}_a^3 = \operatorname{argmin}_{x_a}\{\boldsymbol{w}_{x_a}^3\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \le b\}$, thus $C(\boldsymbol{w}_{x_a}^3, \boldsymbol{x}, b) = \boldsymbol{w}_{x_a}^3\boldsymbol{x}_a^3$ Then, $C(\boldsymbol{w}_{x_a}^3, \boldsymbol{x}, b) = \boldsymbol{w}_{x_a}^3\boldsymbol{x}_a^3 = \lambda\boldsymbol{w}_{x_a}^1\boldsymbol{x}_a^3 + (1-\lambda)\boldsymbol{w}_{x_a}^2\boldsymbol{x}_a^3 \ge \lambda C(\boldsymbol{w}_{x_a}^1, \boldsymbol{x}, b) + (1-\lambda)C(\boldsymbol{w}_{x_a}^2, \boldsymbol{x}, b)$ since $C(\boldsymbol{w}_{x_a}^1, \boldsymbol{x}, b) \le \boldsymbol{w}_{x_a}^1\boldsymbol{x}_a^3$ and $C(\boldsymbol{w}_{x_a}^2, \boldsymbol{x}, b) \le \boldsymbol{w}_{x_a}^2\boldsymbol{x}_a^3$.

4. Assume that $\boldsymbol{x}^1 \geq \boldsymbol{x}^2$, it needs to show that $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}^1, b) \geq C(\boldsymbol{w}_{x_a}, \boldsymbol{x}^2, b)$. Let $\boldsymbol{x}_a^1 = \text{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}^1, \boldsymbol{o}) \leq b\}$ and $\boldsymbol{x}_a^2 = \text{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}^2, \boldsymbol{o}) \leq b\}$. Since $B(\boldsymbol{x}_a^1, \boldsymbol{x}^1, \boldsymbol{o}) = B(\boldsymbol{x}_a^2, \boldsymbol{x}^2, \boldsymbol{o}) = b$ and $\boldsymbol{x}^1 \geq \boldsymbol{x}^2$, it implies that $\boldsymbol{x}_a^1 \geq \boldsymbol{x}_a^2$ since B is non-decreasing in $\boldsymbol{x}$ by Proposition 4.2.2. Thus, $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}^1, b) = \boldsymbol{w}_{x_a}\boldsymbol{x}_a^1 \geq \boldsymbol{w}_{x_a}\boldsymbol{x}_a^2 = C(\boldsymbol{w}_{x_a}, \boldsymbol{x}^2, b)$.
5. Assume that $b^1 \geq b^2$, it needs to show that $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^1) \leq C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^2)$. Let $\boldsymbol{x}_a^1 = \text{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \leq b^1\}$ and $\boldsymbol{x}_a^2 = \text{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \leq b^2\}$. Since $B(\boldsymbol{x}_a^1, \boldsymbol{x}, \boldsymbol{o}) = b^1 \geq b^2 = B(\boldsymbol{x}_a^2, \boldsymbol{x}, \boldsymbol{o})$, it implies that $\boldsymbol{x}_a^1 \leq \boldsymbol{x}_a^2$ since B is non-increasing in $\boldsymbol{x}_a$ by Proposition 4.2.1. Thus, $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^1) = \boldsymbol{w}_{x_a}\boldsymbol{x}_a^1 \leq \boldsymbol{w}_{x_a}\boldsymbol{x}_a^2 = C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^2)$.
6. Let $\boldsymbol{x}_a^1 = \text{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \leq b^1\}$ and $\boldsymbol{x}_a^2 = \text{argmin}_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \leq b^2\}$. For $0 \leq \lambda \leq 1$, let $\boldsymbol{x}_a^3 = \lambda\boldsymbol{x}_a^1 + (1-\lambda)\boldsymbol{x}_a^2$ and $b^3 = \lambda b^1 + (1-\lambda)b^2$. It needs to show that $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^3) \leq \lambda C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^1) + (1-\lambda)C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^2)$. Since B is convex in $\boldsymbol{x}_a$, $B(\boldsymbol{x}_a^3, \boldsymbol{x}; \boldsymbol{o}) = B(\lambda\boldsymbol{x}_a^1 + (1-\lambda)\boldsymbol{x}_a^2, \boldsymbol{x}; \boldsymbol{o}) \leq \lambda B(\boldsymbol{x}_a^1, \boldsymbol{x}; \boldsymbol{o}) + (1-\lambda)B(\boldsymbol{x}_a^2, \boldsymbol{x}; \boldsymbol{o}) \leq \lambda b^1 + (1-\lambda)b^2 = b^3$. Thus, $\boldsymbol{x}_a^3$ is a candidate for $\min_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \leq b^3\}$. Since $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^1) = \boldsymbol{w}_{x_a}\boldsymbol{x}_a^1$ and $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^2) = \boldsymbol{w}_{x_a}\boldsymbol{x}_a^2$, it implies that $\lambda C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^1) + (1-\lambda)C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^2) = \boldsymbol{w}_{x_a}(\lambda\boldsymbol{x}_a^1 + (1-\lambda)\boldsymbol{x}_a^2) = \boldsymbol{w}_{x_a}\boldsymbol{x}_a^3$. Since $\boldsymbol{x}_a^3$ is a candidate for $\min_{\boldsymbol{x}_a}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a : B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) \leq b^3\}$, $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^3) \leq \boldsymbol{w}_{x_a}\boldsymbol{x}_a^3 = \lambda C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^1) + (1-\lambda)C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b^2)$.

B.4 Proof of Proposition 4.4

1. Let λ be the Lagrange multiplier for the constraint in the ACMP, the Langrangian problem of the ACMP can be written as $\min_{\boldsymbol{x}_a,\lambda}\{\boldsymbol{w}_{x_a}\boldsymbol{x}_a + \lambda(B(\boldsymbol{x}_a, \boldsymbol{x}, \boldsymbol{o}) - b)\}$. If $(\boldsymbol{x}_a^*)$ is optimal in the ACMP, then for some λ, two first order conditions must hold: 1.) $\lambda = -\frac{w_{x_{a_q}}}{\frac{\partial B(x_a^*, x; o)}{\partial x_{a_q}}}\ \forall\, q = 1, \dots, Q$ and 2.) $B(\boldsymbol{x}_a^*, \boldsymbol{x}; \boldsymbol{o}) - b = 0$. Consider if the abatement cost function is differentiated with respect to b, $\frac{\partial C}{\partial b} = \sum_{q=1}^{Q} w_{x_{a_q}} \frac{\partial x_{a_q}{}^*}{\partial b}$, and substituting $w_{x_{a_q}}$ by $-\lambda\frac{\partial B(\boldsymbol{x}_a^*, \boldsymbol{x}; \boldsymbol{o})}{\partial x_{a_q}}$ obtains $\frac{\partial C}{\partial b} = -\lambda \sum_{q=1}^{Q} \frac{\partial B}{\partial x_{a_q}} \frac{\partial x_{a_q}{}^*}{\partial b}$. Since $B(\boldsymbol{x}_a^*, \boldsymbol{x}; \boldsymbol{o}) - b = 0$, $\sum_{q=1}^{Q} \frac{\partial B}{\partial x_{a_q}} \frac{\partial x_{a_q}{}^*}{\partial b} - 1 = 0$. Thus, $\frac{\partial C}{\partial b} = -\lambda$. The Lagrange multiplier λ is nonnegative since $\frac{\partial B(\boldsymbol{x}_a^*, \boldsymbol{x}; \boldsymbol{o})}{\partial x_{a_q}} \geq 0$, then MAC = $-\frac{\partial C}{\partial b}$ is nonnegative and equals $\lambda = -\frac{w_{x_{a_q}}}{\frac{\partial B(\boldsymbol{x}_a^*, \boldsymbol{x}; \boldsymbol{o})}{\partial x_{a_q}}}\ \forall\, q = 1, \dots, Q$.

2. $C(\boldsymbol{w}_{x_a}, \boldsymbol{x}, b)$ is non-increasing and convex in b.

B.5 Construction of an abatement input and an abatement unit cost

Let Q_B be the maximum heat input rate and O_{hr} be the operating hours[22] in a year. Let η be a final abatement factor for the abatement system. The method to calculate η is described below. The abatement factor is derived from the percentage reduction

[22] Information on the maximum heat input rate and the operating hours are reported in the EPA database (EPA 2011).

efficiencies for each type of abatement system. Such information is reported in EPA (1997) and Srivastava et al. (2005). An abatement input x_a is:

$$x_a = Q_B O_{hr} \eta \tag{B.1}$$

Table B.1 shows an approximate percentage of NO_x reduction for each abatement system. However, plants may invest in a new abatement system during a year. Thus, the final abatement factors are derived by weighing the old and the new abatement system factor when the plants put these systems into operation. For example, if a plant currently using LNB changes to use LNB+SCR in August, the final abatement factor $\eta = \left(\frac{7}{12}\right) \times 0.394 + \left(\frac{5}{12}\right) \times 0.890 = 0.6$.

Table B.1 Abatement systems used in the U.S. plants during the study period

Abatement system	% reduction	Abatement system	% reduction
OFA	25.0	LNB+OFA+SNCR (dry bottom wall fired)	73.8
LNB	39.4	LNB+closed-coupled OFA+SNCR	67.7
LNB+OFA (dry bottom wall fired)	52.3	LNB+separated OFA+SNCR	70.9
LNB+closed-coupled OFA	41.3	LNB+closed-coupled/separated OFA+SNCR	75.4
LNB+separated OFA	47.1	OFA+SCR	85.0
LNB+closed-coupled/separated OFA	55.2	LNB+SCR	89.0
SNCR	45.0	LNB+OFA+SCR (dry bottom wall fired)	90.5
SCR	80.0	LNB+closed-coupled OFA+SCR	88.3
OFA+SNCR	66.3	LNB+separated OFA+SCR	89.4
LNB+SNCR	69.8	LNB+closed-coupled/separated OFA+SCR	91.0

The NO_x abatement cost is composed of a capital cost (\$/kW), fixed operation and maintenance (O&M) cost (\$/kW-yr) and variable O&M cost (mills/kWh) (EPA, 2010). To approximate a final abatement unit cost $\boldsymbol{w}_{x_a}$, convert the fixed capital and O&M costs into variable costs. The unit costs of each term in the abatement costs are reported in Table B.2. The coefficient of capital cost, C_a and the fixed O&M unit cost, FOM_a, are approximated as a function of the boiler capacity (MW). The coefficient of the variable cost, VOM_a, is fixed except for VOM_a of SCR system which is a function of boiler capacity. Estimate the abatement unit cost (\$/mmBtu) using the equation:

$$\text{abatement unit cost} = \frac{1}{3.413}\left[\frac{C_a}{(20 \times 365 \times 24)} + \frac{FOM_a}{(365 \times 24)} + VOM_a\right] \tag{B.2}$$

The first term is the unit cost related to capital. Assume that the abatement equipment lifespan is 20 years (EPA 2002b). The example below calculates an abatement unit cost for a 574MW boiler using LNB+closed-coupled OFA+SCR abatement system:

$$C_a = 15000 \times \left(\frac{300}{574}\right)^{0.359} + 169000 \times \left(\frac{242.72}{574}\right)^{0.27} = 145838.08$$

$$FOM_a = 170 \times \left(\frac{300}{574}\right)^{0.359} + 790 \times \left(\frac{242.72}{574}\right)^{0.27} = 760.85$$

$$VOM_a = 0 + 0.71 \times \left(\frac{242.72}{574}\right)^{0.27} = 0.646$$

$$\text{abatement unit cost} = \frac{1}{3.413}\left[\frac{145838.08}{(20 \times 365 \times 24)} + \frac{760.85}{(365 \times 24)} + 0.646\right] = 0.459$$

Similar to η, the final abatement unit cost, $\boldsymbol{w}_{x_a}$, is derived by weighing the old and the new abatement unit cost when a plant places a new system in operation.

Table B.2 Abatement cost coefficients of several abatement systems

Abatement system	Abatement cost coefficient		
	Capital ($\$\times10^3$/MW)	Fixed O&M ($\$\times10^3$/MW-yr)	Variable O&M (\$/MWh)
OFA	$10\times(300/MW)^{0.359}$	$0.12\times(300/MW)^{0.359}$	0.021
LNB	$29\times(300/MW)^{0.359}$	$0.31\times(300/MW)^{0.359}$	0.064
LNB+OFA (dry bottom wall fired)	$39\times(300/MW)^{0.359}$	$0.43\times(300/MW)^{0.359}$	0.085
LNB+closed-coupled OFA	$15\times(300/MW)^{0.359}$	$0.17\times(300/MW)^{0.359}$	0
LNB+separated OFA	$21\times(300/MW)^{0.359}$	$0.22\times(300/MW)^{0.359}$	0.029
LNB+closed-coupled/separated OFA	$24\times(300/MW)^{0.359}$	$0.27\times(300/MW)^{0.359}$	0.029
SNCR	$0.5\times[29\times(200/MW)^{0.577}+ 33\times(100/MW)^{0.681}]$	$0.5\times[0.3\times(200/MW)^{0.577}+ 0.35\times(100/MW)^{0.681}]$	0.79
SCR	$169\times(242.72/MW)^{0.27}$	$0.79\times(242.72/MW)^{0.27}$	$0.71\times(242.72/MW)^{0.11}$

Source: U.S. EPA (2011)

APPENDIX C

SUPPORTING DISCUSSIONS FOR CHAPTER V

C.1 A complete marginal abatement cost ratio decomposition

The MAC ratio (5.1) can be alternatively decomposed into a technical change effect and a non-technical change effect based on period t data:

$$\frac{\mathrm{MAC}^{t+1}}{\mathrm{MAC}^{t}}.= \underbrace{\left(\frac{\frac{\partial B^{t}(\boldsymbol{x}_{a}^{A};\ \boldsymbol{x}^{A}, b^{A})}{\partial x_{a_q}}}{\frac{\partial B^{t}(\boldsymbol{x}_{a}^{F};\ \boldsymbol{x}^{M}, b^{M})}{\partial x_{a_q}}} \frac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}} \right)}_{\text{Non-TC change}} \underbrace{\left(\frac{\frac{\partial B^{t}(\boldsymbol{x}_{a}^{F};\ \boldsymbol{x}^{M}, b^{M})}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_{a}^{M};\ \boldsymbol{x}^{M}, b^{M})}{\partial x_{a_q}}} \right)}_{\text{TC change}} \tag{C.1}$$

The first graph in figure C.1 illustrates the non-technical change effect at period t when a firm changes the amount of abatement input from x_a^A to x_a^F in order to reduce the amount of pollutant from b^A to b^M while holding production input use constant at $\boldsymbol{x}^M$. The second graph shows that the technical change effect enables the firm to reduce abatement input from x_a^F to x_a^M while maintaining the same input level, $\boldsymbol{x}^M$ and emitting the pollutant level, b^M.

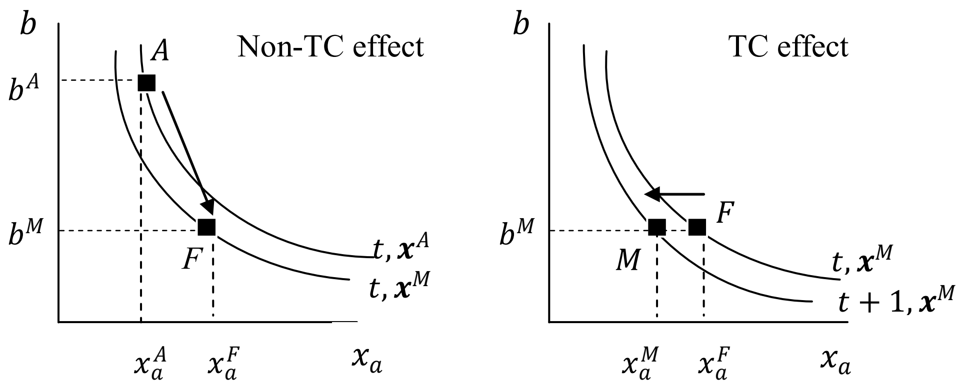


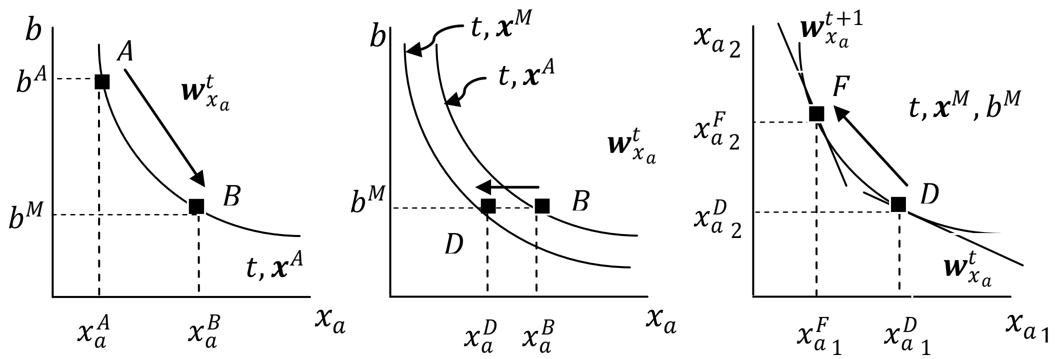
Figure C.1 A non-technical change effect at t and a technical change effect

Following Färe et al. (1994), the geometric mean of two MAC ratios in (5.2) and (C.1) is taken to avoid selecting an arbitrary base period in defining the technical and the non-technical change effects:

$$\frac{\mathrm{MAC}^{t+1}}{\mathrm{MAC}^{t}} = \left[\left(\frac{\dfrac{\partial B^{t}(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}\right)\left(\frac{\dfrac{\partial B^{t}(\boldsymbol{x}_a^F;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right)\right]^{\frac{1}{2}} \qquad \text{TC change}$$

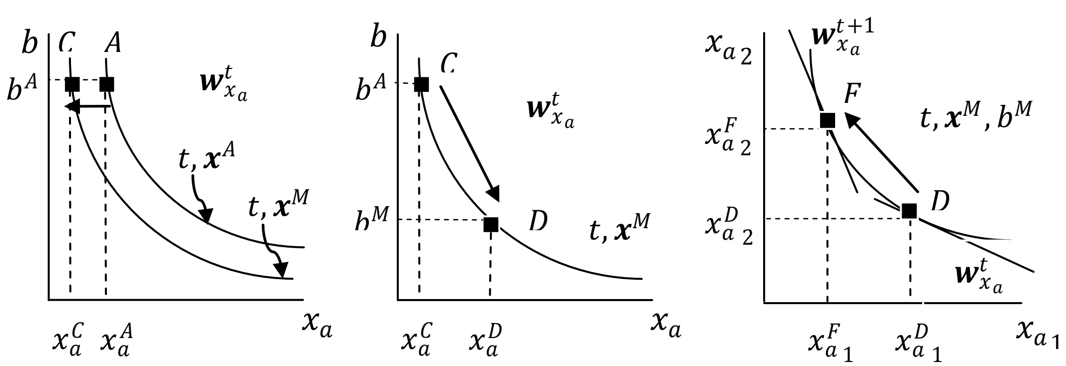
$$\times\left[\left(\frac{\dfrac{\partial B^{t}(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t}(\boldsymbol{x}_a^F;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\frac{w_{x_a}^{t+1}}{w_{x_a}^{t}}\right)\left(\frac{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\dfrac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\frac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}}\right)\right]^{\frac{1}{2}} \qquad \text{Non-TC change} \tag{C.2}$$

Similar to period $t+1$, it is also possible to decompose the non-technical change effect at period t in the same manner. Figure C.2 and C.3 illustrate the two possible decompositions of the non-technical change effect in period t. A firm minimizes the abatement cost by using period t abatement input costs, $\boldsymbol{w}_{x_a}^t$, to identify the abatement input mix.



Pollutant level effect (Pollution contraction) | Input level effect (Input contraction) | Abatement input cost effect

Figure C.2 The first decomposition of a non-technical change effect at period t



Input level effect (Input contraction) | Pollutant level effect (Pollution contraction) | Abatement input cost effect

Figure C.3 The second decomposition of a non-technical change effect at period t

Table C.1 The decomposition of a non-technical change effect at period t

Figure	Abatement input cost effect	Pollutant level effect	Production input level effect
C.2	$\dfrac{\frac{\partial B^t(\boldsymbol{x}_a^D;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^F;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}} \dfrac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}}$	$\dfrac{\frac{\partial B^t(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^B;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}}$	$\dfrac{\frac{\partial B^t(\boldsymbol{x}_a^B;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^D;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}$
C.3	$\dfrac{\frac{\partial B^t(\boldsymbol{x}_a^D;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^F;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}} \dfrac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}}$	$\dfrac{\frac{\partial B^t(\boldsymbol{x}_a^C;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^D;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}$	$\dfrac{\frac{\partial B^t(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^C;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}}$

The abatement input cost effect is written as the geometric mean between the cost effect in periods t and $t+1$:

$$\left(\frac{\frac{\partial B^t(\boldsymbol{x}_a^D;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^F;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}} \times \frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^G;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}} \right)^{\frac{1}{2}} \frac{w_{x_{a_q}}^{t+1}}{w_{x_{a_q}}^{t}} \tag{C.3}$$

There are two different terms for the pollutant level effect in each period; thus, a geometric mean is taken to calculate the pollutant level effect for each period

$$\left(\frac{\frac{\partial B^t(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^B;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}} \times \frac{\frac{\partial B^t(\boldsymbol{x}_a^C;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^D;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right)^{\frac{1}{4}}$$

$$\times \left(\frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^K;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}} \frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^L;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right)^{\frac{1}{4}} \tag{C.4}$$

The production input level effect is derived in the same manner as the pollutant level effect. The production input level effect is written as:

$$\left(\frac{\frac{\partial B^t(\boldsymbol{x}_a^A;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^C;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}} \frac{\frac{\partial B^t(\boldsymbol{x}_a^B;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}}{\frac{\partial B^t(\boldsymbol{x}_a^D;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right)^{\frac{1}{4}}$$

$$\times \left(\frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^H;\ \boldsymbol{x}^A, b^A)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^L;\ \boldsymbol{x}^M, b^A)}{\partial x_{a_q}}} \frac{\frac{\partial B^{t+1}(\boldsymbol{x}_a^K;\ \boldsymbol{x}^A, b^M)}{\partial x_{a_q}}}{\frac{\partial B^{t+1}(\boldsymbol{x}_a^M;\ \boldsymbol{x}^M, b^M)}{\partial x_{a_q}}}\right)^{\frac{1}{4}} \tag{C.5}$$

The full MAC ratio decomposition can be estimated following the methodology described in the summary at the end of section 5.3. The four points, *B* ,*C, D,* and *F*, are calculated in step 2 and the corresponding marginal products are estimated in step 3. Finally, technical and the non-technical change effects are calculated using (C.2), the abatement input cost effect using (C.3), the pollutant level effect using (C.4), and the production input level effect using (C.5).

C.2 The algorithm for solving the sequential CNLS problem (5.15)

When the number of observations and time periods are large, the number of constraints of the sequential CNLS problem (5.15) requires an algorithm to reduce the computational burden. Following Lee et al. (2011), the proposed algorithm solves the sequential CNLS problem (5.15) by solving the relaxed sequential CNLS problem composing of the set of constraints that are likely to be binding and then iteratively adds a set of violated constraints until the optimal solution is found. The relaxed sequential CNLS problem is formulated as

$$\min_{\alpha^t,\boldsymbol{\gamma}^t,\boldsymbol{\rho}^t,\boldsymbol{\delta}^t,\xi^t} \sum_{t=1}^{T}\sum_{i=1}^{n} {\xi_i^t}^2 \qquad \text{(C.6.1)}$$

s.t.

$$\xi_i^t = \left(\ln b_i^t - \widehat{\mu^l}^t\right) - \ln(\alpha_i^t + \boldsymbol{\gamma}_i^{t\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime}\boldsymbol{x}_{a_i}^{\ t}) - \boldsymbol{\delta}^{t\prime}\boldsymbol{z}_i^t \qquad \forall i = 1,\dots,n \quad \forall t = 1,\dots,T \qquad \text{(C.6.2)}$$

$$\alpha_i^t + \boldsymbol{\gamma}_i^{t\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime}\boldsymbol{x}_{a_i}^{\ t} \ \geq\ \alpha_h^t + \boldsymbol{\gamma}_h^{t\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t\prime}\boldsymbol{x}_{a_i}^{\ t} \qquad \forall (i,h) \in U \quad \forall t = 1,\dots,T \qquad \text{(C.6.3)}$$

$$\alpha_h^{t+} + \boldsymbol{\gamma}_h^{t+\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t+\prime}\boldsymbol{x}_{a_i}^{\ t} \leq \alpha_i^t + \boldsymbol{\gamma}_i^{t\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime}\boldsymbol{x}_{a_i}^{\ t} \qquad \forall (i,h) \in V \quad \forall t = 1,\dots,T-1 \qquad \text{(C.6.4)}$$

$$\alpha_i^{t+} + \boldsymbol{\gamma}_i^{t+\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_i^{t+\prime}\boldsymbol{x}_{a_i}^{\ t+} \leq \alpha_h^t + \boldsymbol{\gamma}_h^{t\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_h^{t\prime}\boldsymbol{x}_{a_i}^{\ t+} \qquad \forall (i,h) \in W \quad \forall t = 1,\dots,T-1 \qquad \text{(C.6.5)}$$

$$\boldsymbol{\gamma}_i^t \geq 0 \text{ and } \boldsymbol{\rho}_i^t \leq 0 \qquad \forall i = 1,\dots,n \quad \forall t = 1,\dots,T \qquad \text{(C.6.6)}$$

where U is the subset of all the convexity constraints and V and W are the subsets of the first and second sequential constraints.

The Algorithm

1. Let $iter = 0$ and let $U = V = W = \emptyset$.

2. Find an initial solution, $(\alpha_i^{t,iter}, \boldsymbol{\gamma}_i^{t,iter}, \boldsymbol{\rho}_i^{t,iter})$ $\forall i$, by solving

$$\min_{\alpha^t,\boldsymbol{\gamma}^t,\boldsymbol{\rho}^t,\boldsymbol{\delta}^t,\xi^t} \left\{ \sum_{t=1}^{T}\sum_{i=1}^{n} {\xi_i^t}^2 \left| \begin{array}{ll} \xi_i^t = (\ln b_i^t - \hat{\mu}^t) - \ln(\alpha_i^t + \boldsymbol{\gamma}_i^{t\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime}\boldsymbol{x}_{a_i}^{\ t}) - \boldsymbol{\delta}^{t\prime}\boldsymbol{z}_i^t & \forall i;\ \forall t \\ \alpha_i^t + \boldsymbol{\gamma}_i^{t\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t\prime}\boldsymbol{x}_{a_i}^{\ t} \ \geq \ \alpha_{i+1}^t + \boldsymbol{\gamma}_{i+1}^{t}{}'\boldsymbol{x}_i^t + \boldsymbol{\rho}_{i+1}^{t}{}'\boldsymbol{x}_{a_i}^{\ t} & \forall i;\ \forall t \\ \boldsymbol{\gamma}_i^t \geq 0 \text{ and } \boldsymbol{\rho}_i^t \leq 0 & \forall i;\ \forall t \end{array} \right. \right\}$$

3. While the convexity constraint is violated $\left(\max_{i,h}\left\{\left(\alpha_h^{t,iter} + \boldsymbol{\gamma}_h^{t,iter\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t,iter\prime}\boldsymbol{x}_{a_i}^{\ t}\right) - \left(\alpha_i^{t,iter} + \boldsymbol{\gamma}_i^{t,iter\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t,iter\prime}\boldsymbol{x}_{a_i}^{\ t}\right)\right\} > 0\right)$, the first sequential constraint is violated $\left(\max_{i,h}\left\{\left(\alpha_h^{t+,iter} + \boldsymbol{\gamma}_h^{t+,iter\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t+,iter\prime}\boldsymbol{x}_{a_i}^{\ t}\right) - \left(\alpha_i^{t,iter} + \boldsymbol{\gamma}_i^{t,iter\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t,iter\prime}\boldsymbol{x}_{a_i}^{\ t}\right)\right\} > 0\right)$, or the second sequential constraint is violated $\left(\max_i \min_h\left\{\left(\alpha_i^{t+,iter} + \boldsymbol{\gamma}_i^{t+,iter\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_i^{t+,iter\prime}\boldsymbol{x}_{a_i}^{\ t+}\right) - \left(\alpha_h^{t,iter} + \boldsymbol{\gamma}_h^{t,iter\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_h^{t,iter\prime}\boldsymbol{x}_{a_i}^{\ t+}\right)\right\} > 0\right)$, do

   3.1 For each period t, select 90th percentile and over violated convexity constraints (C.6.3), $\left(\alpha_h^{t,iter} + \boldsymbol{\gamma}_h^{t,iter\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t,iter\prime}\boldsymbol{x}_{a_i}^{\ t}\right) - \left(\alpha_i^{t,iter} + \boldsymbol{\gamma}_i^{t,iter\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t,iter\prime}\boldsymbol{x}_{a_i}^{\ t}\right) > 0$, and let U' be the set of observation pairs corresponding with the selected violated convexity constraints. Update $U = U \cup U'$.

   3.2 For each period t, select 90th percentile and over violated first sequential constraints (C.6.4), $\left(\alpha_h^{t+,iter} + \boldsymbol{\gamma}_h^{t+,iter\prime}\boldsymbol{x}_i^t + \boldsymbol{\rho}_h^{t+,iter\prime}\boldsymbol{x}_{a_i}^{\ t}\right) - \left(\alpha_i^{t,iter} +\right.$

$\boldsymbol{\gamma}_i^{t,iter}{}'\boldsymbol{x}_i^t + \boldsymbol{\rho}_i^{t,iter}{}'\boldsymbol{x}_{a}{}_i^t\big) > 0$, and let V' be the set of observation pairs corresponding with the selected violated convexity constraints. Update $V = V \cup V'$.

3.3 For each observation i at each period t, select the most violated second sequential constraints (C.6.5),

$\min_h\{\big(\alpha_h^{t+,iter} + \boldsymbol{\gamma}_h^{t+,iter}{}'\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_h^{t+,iter}{}'\boldsymbol{x}_{a}{}_i^{t+}\big) - \big(\alpha_i^{t,iter} + \boldsymbol{\gamma}_i^{t,iter}{}'\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_i^{t,iter}{}'\boldsymbol{x}_{a}{}_i^{t+}\big)\} > 0$, and let W' be the set of observation pairs corresponding with the selected violated convexity constraints. Update $W = W'$.

3.4 Solve the relaxed sequential CNLS problem (C.6) to obtain $(\alpha_i^{t,iter+1}, \boldsymbol{\gamma}_i^{t,iter+1}, \boldsymbol{\rho}_i^{t,iter+1})\ \forall i, \forall t$.

3.5 Update $iter = iter + 1$.

As with Lee at el. (2011), the algorithm finds a set of initial solutions using the Afriat approach. Since the obtained solutions are not feasible, the algorithm at steps 3.1 and 3.2 iteratively adds some of the most violated convexity constraints (C.6.3) to the set U and some of the most violated first sequential constraints (C.6.4) to the set V. For all violated constraints, the algorithm selects only the 90th percentile and over violated constraints.

Unlike the convexity and the first sequential constraint in which each observation at each time period might require several observation pairs of constraints, the second sequential constraints (C.6.5) require at most one observation pair constraint because the estimated pollutant at $t+$ period must be lower than the maximum estimated pollutant using pollutant frontier parameters at t period, or $\alpha_i^{t+} + \boldsymbol{\gamma}_i^{t+\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_i^{t+\prime}{\boldsymbol{x}_a}_i^{t+} \leq \max_h\{\alpha_h^t + \boldsymbol{\gamma}_h^{t\prime}\boldsymbol{x}_i^{t+} + \boldsymbol{\rho}_h^{t\prime}{\boldsymbol{x}_a}_i^{t+}\}\ \forall i = 1, \dots, n\ \forall t = 1, \dots, T$. At each iteration, if violated constraints exist, step 3.3 of the algorithm selects the most violated constraint and adds it to the set W. Moreover, the algorithm renews set W at each iteration by removing all the previous constraints in the set W so that it does not retain unnecessary constraints from last iteration.

C.3 Finding abatement cost minimization points on estimated pollutant frontiers

This section provides the specific linear programs used to find the cost minimization points at A, B, C, D, F, G, H, L, K and M. Note that all linear programs are based on the linear program (5.18) described in section 5.3.3. The required parameters are 1) the estimated coefficients ${\hat{\alpha}_i}^t$, ${\hat{\boldsymbol{\gamma}}_i}^t$ and ${\hat{\boldsymbol{\rho}}_i}^t$ $\forall i = 1, \dots, n;\ \forall t = 1, \dots, T$ from the linear programming problem (5.17), 2) the abatement input cost ${\boldsymbol{w}_{x_a}}_i^t$ $\forall i = 1, \dots, n;\ \forall t = 1, \dots, T$, 3) the points at period s, $(\boldsymbol{x}_i^A, \hat{b}_i^A)$ $\forall i = 1, \dots, n$ where $\hat{b}_i^A = {\hat{\alpha}_i}^s + {\hat{\boldsymbol{\gamma}}_i}^{s\prime}{\boldsymbol{x}_i}^A + {\hat{\boldsymbol{\rho}}_i}^{s\prime}{\boldsymbol{x}_a}^A$, and 4) the points at period $s+1$, $(\boldsymbol{x}_i^M, \hat{b}_i^M)$ $\forall i = 1, \dots, n$ where $\hat{b}_i^M = {\hat{\alpha}_i}^{s+1} + {\hat{\boldsymbol{\gamma}}_i}^{s+1\prime}{\boldsymbol{x}_i}^M + {\hat{\boldsymbol{\rho}}_i}^{s+1\prime}{\boldsymbol{x}_a}^M$.

For each firm o, the cost minimization point A can be recovered by solving the following linear program

$$\min_{\boldsymbol{x}_a} {\boldsymbol{w}_{x_a}}_o^{s\prime}\boldsymbol{x}_a$$

$$\text{s.t.} \quad {\hat{\alpha}_i}^s + {\hat{\boldsymbol{\gamma}}_i}^{s\prime}{\boldsymbol{x}_o}^A + {\hat{\boldsymbol{\rho}}_i}^{s\prime}\boldsymbol{x}_a \leq \hat{b}_o^A \qquad \forall i = 1, \dots n$$

$$\boldsymbol{x}_a \geq 0 \qquad \text{(C.7)}$$

For each firm *o*, the cost minimization point B can be recovered by solving the following linear program

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x\,a_o}^{\;s\,\prime}\boldsymbol{x}_a$$

s.t. $$\widehat{\alpha}_i^{\,s} + \widehat{\boldsymbol{\gamma}}_i^{\,s\prime}\boldsymbol{x}_o^{\,A} + \widehat{\boldsymbol{\rho}}_i^{\,s\prime}\boldsymbol{x}_a \leq \widehat{b}_o^M \qquad \forall i = 1, \ldots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.8}$$

For each firm *o*, the cost minimization point C can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x\,a_o}^{\;s\,\prime}\boldsymbol{x}_a$$

s.t. $$\widehat{\alpha}_i^{\,s} + \widehat{\boldsymbol{\gamma}}_i^{\,s\prime}\boldsymbol{x}_o^{\,M} + \widehat{\boldsymbol{\rho}}_i^{\,s\prime}\boldsymbol{x}_a \leq \widehat{b}_o^A \qquad \forall i = 1, \ldots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.9}$$

For each firm *o*, the cost minimization point D can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x\,a_o}^{\;s\,\prime}\boldsymbol{x}_a$$

s.t. $$\widehat{\alpha}_i^{\,s} + \widehat{\boldsymbol{\gamma}}_i^{\,s\prime}\boldsymbol{x}_o^{\,M} + \widehat{\boldsymbol{\rho}}_i^{\,s\prime}\boldsymbol{x}_a \leq \widehat{b}_o^M \qquad \forall i = 1, \ldots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.10}$$

For each firm *o*, the cost minimization point F can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x\,a_o}^{\;s+1\,\prime}\boldsymbol{x}_a$$

s.t. $$\widehat{\alpha}_i^{\,s} + \widehat{\boldsymbol{\gamma}}_i^{\,s\prime}\boldsymbol{x}_o^{\,M} + \widehat{\boldsymbol{\rho}}_i^{\,s\prime}\boldsymbol{x}_a \leq \widehat{b}_o^M \qquad \forall i = 1, \ldots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.11}$$

For each firm o, the cost minimization point G can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x_{a_o}}^{s\,\prime}\boldsymbol{x}_a$$

s.t. $$\hat{\alpha}_i^{s+1} + \hat{\boldsymbol{\gamma}}_i^{s+1\prime}\boldsymbol{x}_o^{A} + \hat{\boldsymbol{\rho}}_i^{s+1\prime}\boldsymbol{x}_a \leq \hat{b}_o^A \qquad \forall i = 1, \dots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.12}$$

For each firm o, the cost minimization point H can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x_{a_o}}^{s+1\prime}\boldsymbol{x}_a$$

s.t. $$\hat{\alpha}_i^{s+1} + \hat{\boldsymbol{\gamma}}_i^{s+1\prime}\boldsymbol{x}_o^{A} + \hat{\boldsymbol{\rho}}_i^{s+1\prime}\boldsymbol{x}_a \leq \hat{b}_o^A \qquad \forall i = 1, \dots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.13}$$

For each firm o, the cost minimization point K can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x_{a_o}}^{s+1\prime}\boldsymbol{x}_a$$

s.t. $$\hat{\alpha}_i^{s+1} + \hat{\boldsymbol{\gamma}}_i^{s+1\prime}\boldsymbol{x}_o^{A} + \hat{\boldsymbol{\rho}}_i^{s+1\prime}\boldsymbol{x}_a \leq \hat{b}_o^M \qquad \forall i = 1, \dots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.14}$$

For each firm o, the cost minimization point L can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x_{a_o}}^{s+1\prime}\boldsymbol{x}_a$$

s.t. $$\hat{\alpha}_i^{s+1} + \hat{\boldsymbol{\gamma}}_i^{s+1\prime}\boldsymbol{x}_o^{M} + \hat{\boldsymbol{\rho}}_i^{s+1\prime}\boldsymbol{x}_a \leq \hat{b}_o^A \qquad \forall i = 1, \dots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.15}$$

For each firm o, the cost minimization point M can be recovered by solving the following linear program:

$$\min_{\boldsymbol{x}_a} \boldsymbol{w}_{x_{a_o}}^{s+1\prime}\boldsymbol{x}_a$$

$$\text{s.t.} \qquad \hat{\alpha}_i^{s+1} + \hat{\boldsymbol{\gamma}}_i^{s+1\prime}\boldsymbol{x}_o{}^M + \hat{\boldsymbol{\rho}}_i^{s+1\prime}\boldsymbol{x}_a \leq \hat{b}_o^M \qquad \forall i = 1, \dots n$$

$$\boldsymbol{x}_a \geq 0 \tag{C.16}$$

C.4 The breakdown of power plants excluded in MAC decomposition results

Table 5.2 considers only power plants for which MAC change can be completely decomposed. In practice, it cannot be fully decomposed if the technical change effect term is zero or the non-technical change effect term is undefined. Specifically, if $\frac{\partial B^t(\boldsymbol{x}_a^A; \boldsymbol{x}^A, b^A)}{\partial x_a} = 0$, the technical change effect term is zero and if $\frac{\partial B^{t+1}(\boldsymbol{x}_a^K; \boldsymbol{x}^K, b^K)}{\partial x_a} = 0$, $\frac{\partial B^{t+1}(\boldsymbol{x}_a^L; \boldsymbol{x}^L, b^L)}{\partial x_a} = 0$ or $\frac{\partial B^{t+1}(\boldsymbol{x}_a^M; \boldsymbol{x}^M, b^M)}{\partial x_a} = 0$, the non-technical change effect is undefined, because either the pollutant level effect term, or the production input level effect term, or both are undefined. Moreover, since many power plants in the study do not use NO_x abatement equipment, there is no information on abatement input cost and MAC does not exist. Table C.2 reports the number of boilers with zero technical change effect terms, undefined non-technical change effect terms, or no MAC that are excluded from the analysis.

Table C.2 Number of boilers for which the MAC change cannot be decomposed

Year	Technical change effect is zero	Non-technical change effect is undefined	No. MAC
2000-2004	17	66	70
2004-2008	6	54	36

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