

**DATA BIAS IN RATE TRANSIENT ANALYSIS OF SHALE GAS
WELLS**

A Thesis

by

AMMAR KHALIFA MOHAMMED AGNIA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2012

Major Subject: Petroleum Engineering

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Approved by:

Co-Chairs of Committee,	Robert A. Wattenbarger
	Bryan Maggard
Committee Members,	Yuefeng Sun
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ABSTRACT

Data Bias in Rate Transient Analysis of Shale Gas Wells.

(May 2012)

Ammar Khalifa Mohammed Agnia, B.Sc., Al-Tahadi University

Co-Chairs of Advisory Committee: Dr. Robert Wattenbarger
Dr. Bryan Maggard

Superposition time functions offer one of the effective ways of handling variable-rate data. However, they can also be biased and misleading the engineer to the wrong diagnosis and eventually to the wrong analysis. Since the superposition time functions involve rate as essential constituent, the superposition time is affected greatly with rate issues. Production data of shale gas wells are usually subjected to operating issues that yield noise and outliers. Whenever the rate data is noisy or contains outliers, it will be hard to distinguish their effects from common regime if the superposition time functions are used as plotting time function on log-log plots. Such deceiving presence of these flow regimes will define erroneous well and reservoir parameters. Based on these results and with the upsurge of energy needs there might be some costly decisions will be taken such as refracting or re-stimulating the well especially in tight formations.

In this work, a simple technique is presented in order to rapidly check whether there is data bias on the superposition-time specialized plots or not. The technique is based on evaluating the kernel of the superposition time function of each flow regime for the maximum production time. Whatever beyond the Kernel-Equivalent Maximum

Production Time (KEMPT) it is considered as biased data. The hypothesis of this technique is that there is no way to see in the reservoir more than what has been seen. A workflow involving different diagnostic and filtering techniques has been proposed to verify proposed notion. Different synthetic and field examples were used in this study.

Once the all problematic issues have been detected and filtered out, it was clear that whatever went beyond the KEMPT is a consequence of these issues. Thus, the proposed KEMPT technique can be relied on in order to detect and filter out the biased data points on superposition-time log-log plots. Both raw and filtered data were analyzed using type-curve matching of linear flow type-curves for calculating the original gas in-place (OGIP). It has been found that biased data yield noticeable reduced OGIP. Such reduction is attributed to the early fictitious onset of boundary dominated flow, where early false detection of the drainage boundaries defines less gas in-place occupied in these boundaries.

DEDICATION

With sincerity and devotion, my work is dedicated to Almighty Allah for His mercies and blessings.

With love and affection, my work is dedicated to those whose hearts are with me across distances, sharing my happiness and sadness, my parents and family.

With gratefulness and indebtedness, my work is dedicated to the one who gave me more than what I gave it, my country.

With respect and honor, my work is dedicated to the one who gave me his knowledge and patience to see the fruit of my work, my teacher.

With gratitude and appreciation, my work is dedicated to those who guided and encouraged me, my friends.

ACKNOWLEDGEMENTS

All praises and thanks to Allah Almighty for His infinite mercies and unlimited blessings.

I want to acknowledge and express my deep gratitude and appreciation to my advisor Dr. Robert A. Wattenbarger for his guidance and valuable contributions to this work. I am indebted to him for his patience with me and the unlimited support he gave me throughout my hard times. I am thankful to him for the knowledge that I gained from him within these two years, which he accumulated in more than my age in the industry.

I want to acknowledge my committee members Dr. Bryan Maggard and Dr. Yuefeng Sun for being in my advisory committee and accepting this work.

I also want to acknowledge all professors and instructors for valuable courses they taught at Texas A&M University.

Finally, I want to acknowledge my colleagues in Reservoir Modeling Consortium; Ahmed Alkoush, Anas Almarzooq, Haider Abdulal, Hasan Al-Ahmadi, Hassan Hamam, Orkhan Samandarli, Pahala Sinurat, Salman Mengal, Tan Tran, Vartit Tivayanonda, and last but not least Wahaj Khan for their intellectual discussions and friendship.

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CHAPTER I

INTRODUCTION

Recently, shale gas reservoirs appeared as a vital source of fuel to satisfy huge market demands for energy. According to Energy Information Administration (EIA) shale gas reservoirs production amounted to more than 8 Bcf per day in 2009. It is about 14 % of the total volume of dry natural gas produced in the United States and about 12% of the natural gas consumed in the United States. The EIA also projects that the shale gas share of U.S. natural gas production will continue to grow, reaching 45% of the total volume of gas produced in the United States by 2035.

Rate transient analysis is one of the valuable tools in evaluating reservoir performance. It provides valuable information on the reservoir such as original gas in place, drainage area, and the expected ultimate recovery. Although rate transient analysis is approaching the popularity as one of reservoir characterization tools, the diagnosis part of the production data is still challenging. One reason behind that is that the production data in nature is surveillance and monitoring data with little control considerable variance occurring during the acquisition of the data. Another reason is the limited diagnostic techniques in practice for the production data analysis. Moreover, within the techniques that are applied many are either observation-based approaches or rules of thumb.

This thesis follows the style of *SPE Journal*.

One of the challenging issues is the use of superposition time as part of diagnostic plots. Superposition time functions offered one of the effective ways of handling variable-rate data. Despite of that fact, superposition time functions can also be biased and misleading the engineer to the wrong diagnosis and eventually to the wrong analysis.

The superposition time functions behavior has been a subject of skepticism for long period of time. Some observations have been made on that subject by different authors such as Jargon and Van Poolen (1965), Dake (1978), Streltsova (1988), Anderson et al. (2006), Cvetkovic (2008), Nobakht and Mattar (2009), Anderson et al. (2010), and Nobakht et al. (2010). Those observations were uniquely tied to the rate fluctuations that make the superposition time function erratic. Recently the subject has received some attentions by Moghadam and Mattar (2011) and Bachman et al. (2011), though, the work shown in the literature served as big warns of the problem other than resorts.

1.1 Problem description

Since the superposition time functions involve rate as essential constituent, the superposition time is affected greatly with rate issues. Production data of shale gas wells are usually subjected to operating issues that yield noise and outliers. Whenever the rate data is noisy or contain outliers, it will be hard to distinguish their effects from common regime if the superposition time functions are used as plotting time function on log-log plots. Such deceiving presence of these flow regimes will define erroneous well and reservoir parameters. For example, rate outliers or liquid loading phenomena will hard to

be distinguished from Boundary-Dominated-Flow (BDF) regime if the superposition or material balance time functions are used as plotting function. In such cases, if such deceiving presence of BDF is believed, the Expected-Ultimate- Recovery (EUR) will be cut way down. Furthermore, based on these results and with the upsurge of energy needs there might be some other costly decisions will be taken such as refracing or re-stimulating the well especially in tight formations.

1.2 Motivation

Motivation for this work was derived by huge interest in modern production data analysis (rate transient analysis) especially for shale gas reservoirs where the pressure transient analysis is ineffective. It is also observed that the methodology employed by some production data analysts is blind application of production data analysis methods without consideration of data quality issues. A consequence of that methodology is to see what you need to see or to see what is not logical to be seen. Accordingly, it will be easy to fall in the trap of misinterpreting data issues as a reservoir signal rather than the operating problem. One of the challenging issues is the data bias caused by the use of superposition time as part of diagnostic plots. In order to cope with this challenge, this study leads to a proper use of superposition-time to help the analyst of production data to take the advantage of this valuable tool.

1.3 Research objectives

The objectives of this thesis are as follows.

- To develop a simple method, employing conventional techniques, in order to detect and filter out the biased data points on superposition-time plots.
- To validate the proposed method with a workflow involving different diagnostic and filtering techniques in order to recognize and filter out data issues of different synthetic and field examples were used in this study.
- To check the effect of the biased data on the OGIP.
- To demonstrate that production data diagnosis is a critical step before RTA, whereas blind application of production data analysis methods without consideration of data quality issues can lead to misinterpretation of the reservoir characteristics.
- To develop a work flow that guide the production data analyst to the proper use of superposition diagnostic plots
- To program a VBA program that performs proper use of superposition time functions according to the proposed work flow.

1.4 Organization of the thesis

This report is divided into six chapters. Organization of the report is developed in following way:

Chapter I is an introduction of the thesis which consists of problem description, motivation, objectives, and description of the thesis chapters.

Chapter II gives an overview of literature review done. This chapter will be viewing a literature overview on the history and the evolution of production data analysis, rate transient analysis, and the diagnostic procedures. Finally, it will be discussing the then will be discussing the literature regarding the superposition time function bias.

Chapter III gives an overview of superposition principle and its applications. A brief discussion on the nature of the production data will be also given.

Chapter IV will be discussing the main reasons that cause the bias in superposition-time plots. Synthetic data will be used with random noise and fluctuations. The behavior of the induced points on the superposition-time diagnostic plots will be observed. Finally, field cases will be analyzed accordingly.

Chapter V discusses the proper use of superposition time in rate transient analysis. In this chapter, a simple technique will be presented for detecting the data bias on the superposition-time specialized plots. To validate the proposed technique, a supplementary workflow of different diagnostic and filtering techniques will be presented.

Chapter VI presents the conclusions drawn from this work.

CHAPTER II

LITERATURE REVIEW

Production data analysis has witnessed tremendous changes since it has started in 1920's. It had made huge leap from the pure empirical relationships of decline curves to the firm bases of analytical solutions of the rate transient analysis. Nowadays, production data analysis is one of the valuable tools in evaluating reservoir performance. It provides valuable information on the reservoir such as original gas in place, drainage area, and the expected ultimate recovery. This chapter will be viewing a literature overview on the history and the evolution of production data analysis, rate transient analysis, and the diagnostic procedures. Finally, it will be discussing the then will be discussing the literature regarding the superposition time function bias.

2.1 The historical milestones in production data analysis

The production data analysis started in 1920's on purely empirical basis. It has started as empirical relationships with no technical background linked to the actual reservoir engineering. The main objective of it was to construct a decline function that fits the past history in order to be used in extrapolation to the future and assessing the revenue. Therefore, production data analysis was only used at that time as financial tool.

The earliest reported effort that studied the production drop over the time was done by Arnold and Anderson (1908). They stated that the production rate drops with constant fraction and can be modeled by a geometric series over equal time intervals.

They were the first who called that fraction “the decline” and observed the straight-line relationship of the rate versus time on semi-log paper.

Arnold and Darnell (1920) implemented the first empirical relations for economic purposes. Their approach for declining production curve was to assume that the production rate at any time is a constant fraction of the rate at a preceding date. That implies that the production drop over a given constant interval is a fixed fraction or percentage of the preceding production rate. Therefore, a decline curve showing this characteristic was easy to extrapolate since the rate-cumulative curve will exhibit a straight line on coordinate paper. Fig.1 shows an example of the proposed empirical approach.

Cutler (1924) investigated a large number of oilfield decline curves and concluded that the assumption of the constant-percentage decline gives too conservative results. He stated that a hyperbolic relationship on log-log paper would better describe the production decline.

The marginal improvement in this area came 20 years later when Arps (1945) expressed mathematically the constant-pressure exponential, hyperbolic and harmonic decline responses as shown in Fig.2 and 3 . This work was still partially empirical, but some parameters could be quantified. Nowadays, the method still has its continued popularity. One of the main reasons behind that is its simplicity and requires no knowledge of the reservoir or well parameters. Although it gives reasonable answers in many situations, it still lacks to the firm analytical background and is considered as an empirical approach.

Ramsay (1968) pointed out in a comprehensive literature review that in the period from 1964 to 1968 several papers were published and helped in understanding the decline curve analysis. However, a little new technology has been added.

In the 1970s the production data analysis has observed a new trend in the procedure of the analysis by the introduction of the type-curve matching. Slider (1968) developed an overlay method, as shown in Fig.4, to analyze rate-time data. The method is similar to the log-log type-curve matching procedure employed to analyze constant-rate data.

Gentry (1972) developed a graphical method displaying Arps' solutions which can be used as an overlay method to match the decline data and calculate the decline curve coefficients. Fig.5 shows relationship between production rate and cumulative production, whereas Fig.6 shows relationship between production rate and time.

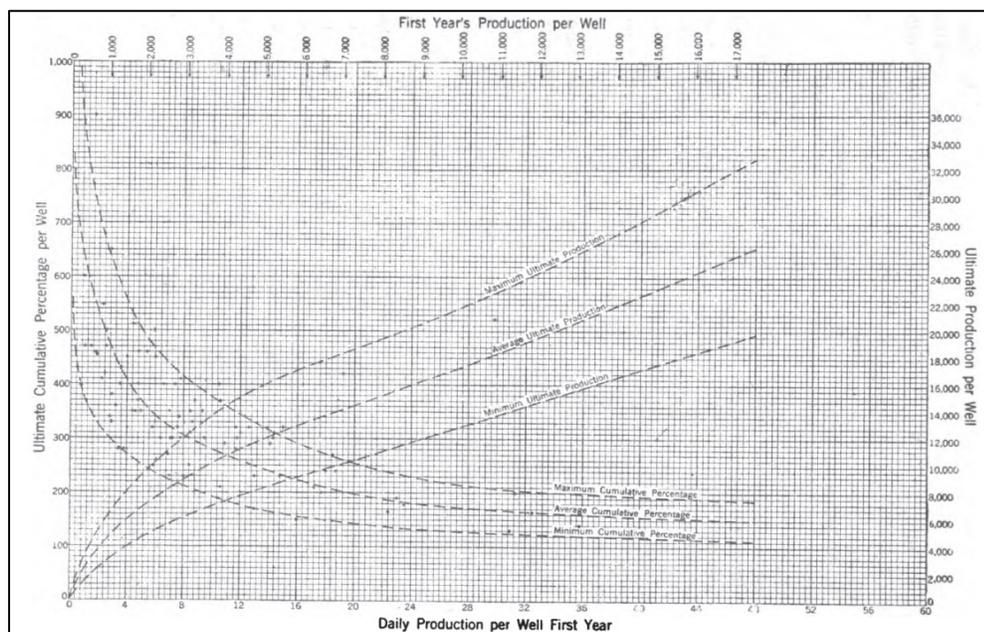


Fig. 1 The constant percentage decline proposed by Arnold and Darnell (1920)

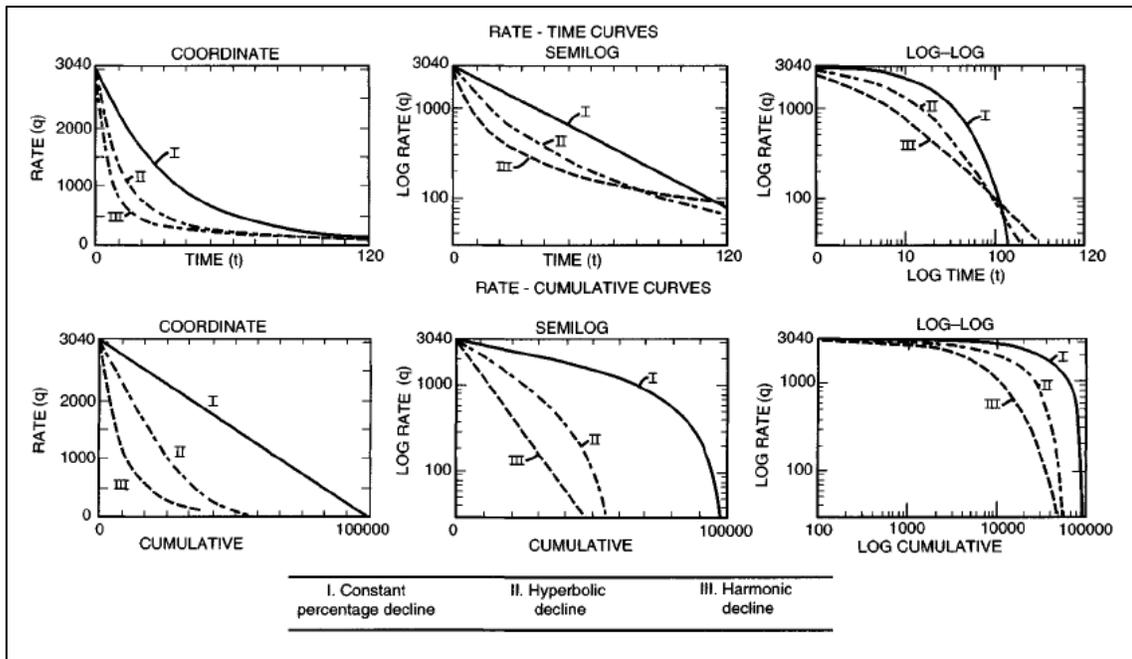


Fig. 2 Arp's decline curves

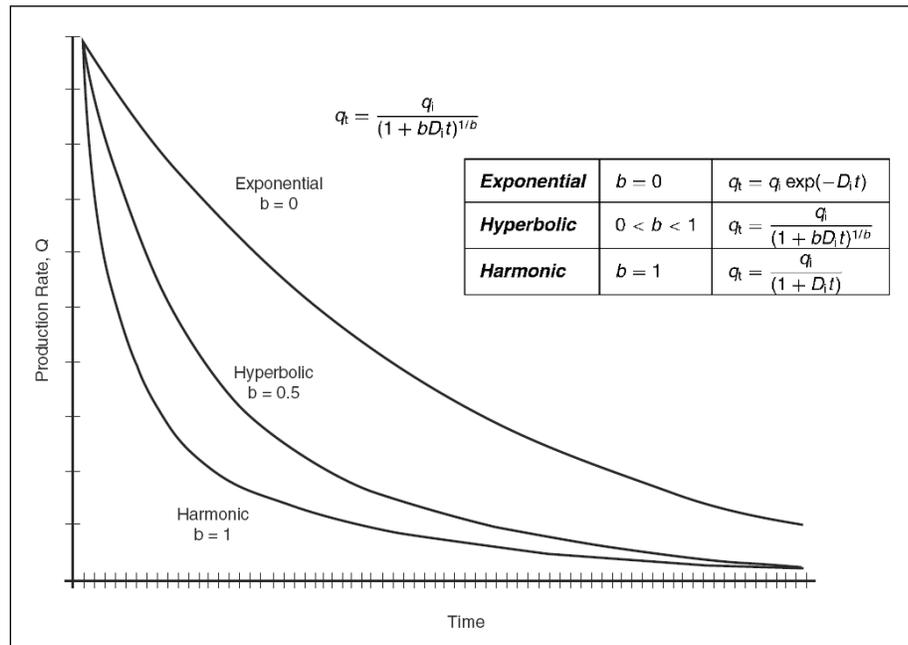


Fig. 3 Arp's empirical relationships proposed for decline curves

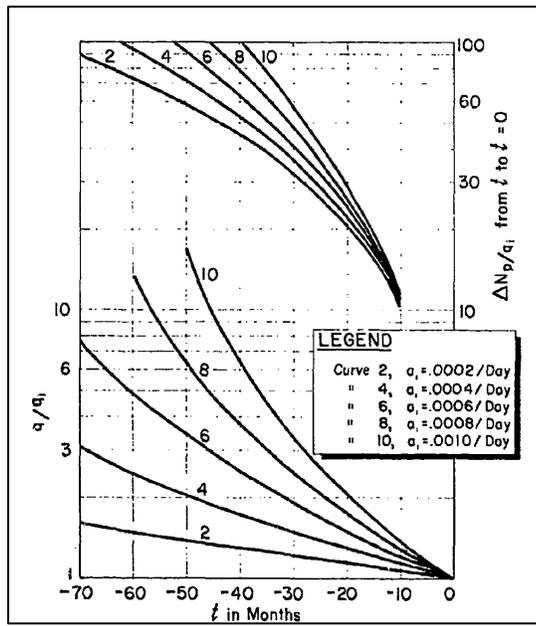


Fig. 4 Overlay method developed by Slider (1968)

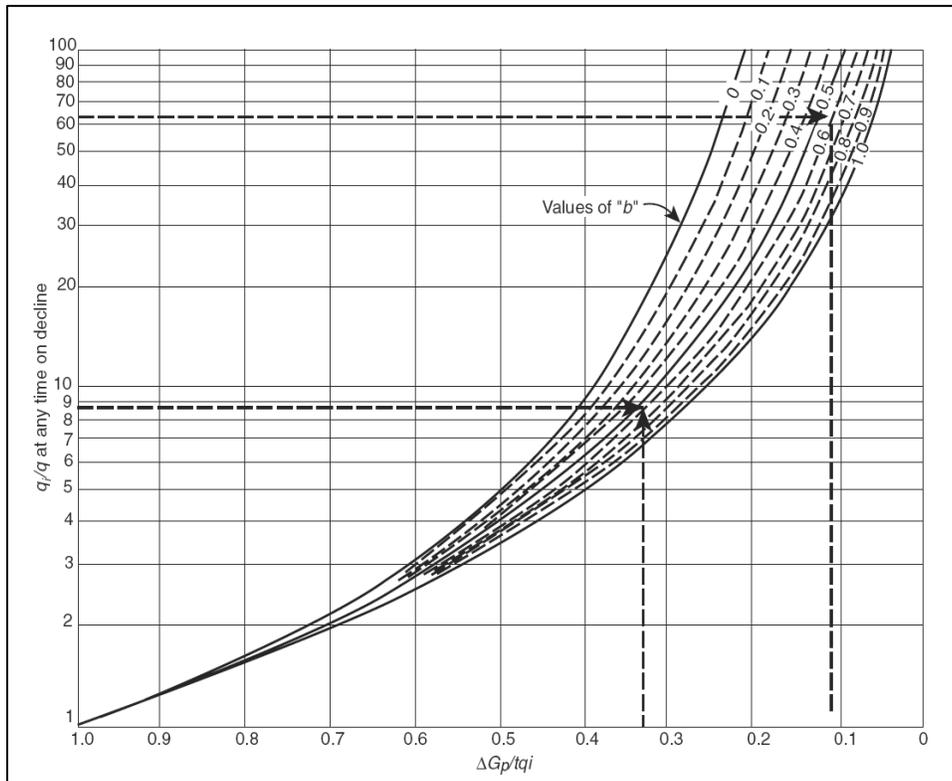


Fig. 5 Rate and cumulative production relationships developed by Gentry (1972)

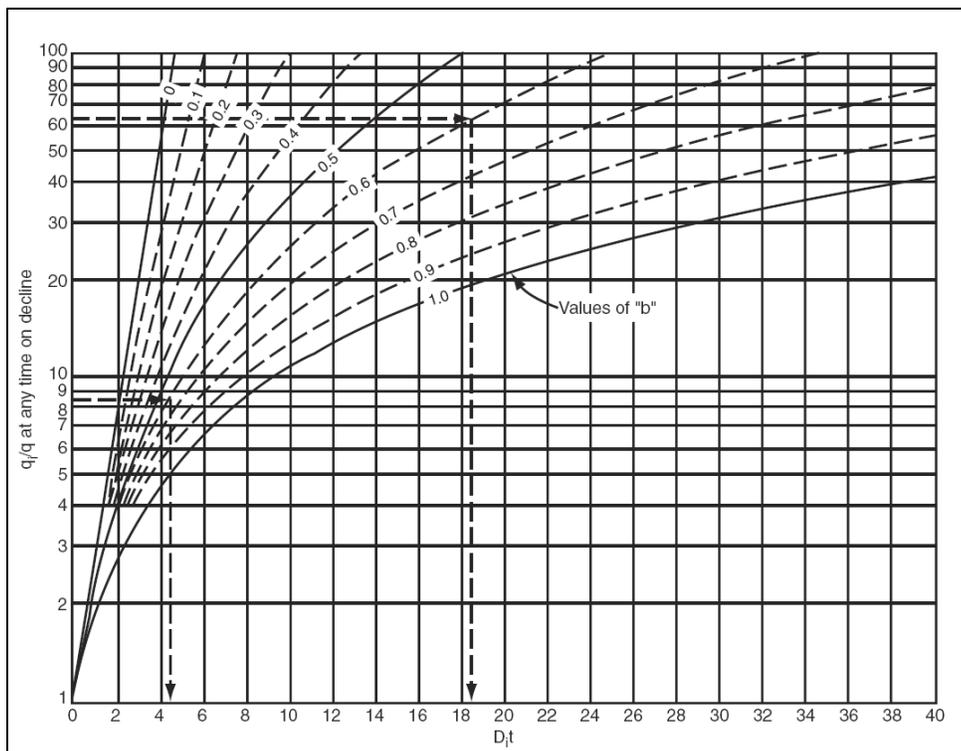


Fig. 6 Rate and time relationship developed by Gentry (1972)

The historical milestone came when Fetkovich (1973) combined the two families; empirical and analytical curves in one type-curves group as shown in Fig.7. Although, Agarwal et al. (1970) introduced the concept of type-curve analysis to the industry, Fetkovich (1973) was the first to extend the concept of type-curves to the analysis of production data. His type-curves are formed of the same Arps' depletion stems to analyze boundary dominated flow, and the constant-pressure type-curves developed by Everdingen and Hurst (1949) for transient production. He also introduced interpretation techniques of the providing more physical understanding of the drive mechanism causing the rate curvature. Moreover, he made the first attempt to define the initial rate decline, decline exponent, and initial decline by physical means. Later,

Raghavan (1993) discussed the conditions under which the decline curvature can be derived empirically.

Ehlig-Economides and Ramey (1981) presented an overview of the transient rate decline for a well production at constant bottom-hole pressure. They derived the analytical solution for exponential depletion for a closed boundary system. Carter (1985) extended Fetkovich's approach exclusively to gas case as shown in Fig.8. By the time advancing forward other type-curves were published later to address complex configurations such as layered and fractured reservoirs.

Fraim and Wattenbarger (1987) developed a normalized time for gas reservoirs producing at a constant wellbore pressure at boundary dominated flow. Their method matches the exponential decline to type-curve to evaluate different reservoir parameters. The normalized time does not have a significant effect on the transient flow and can be used for any reservoir shape.

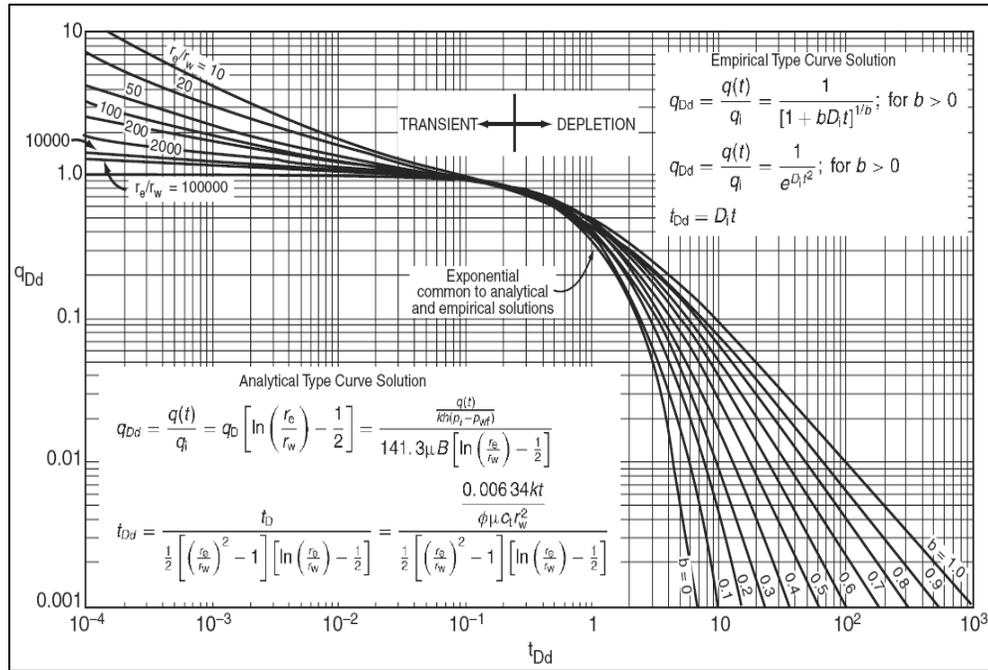


Fig. 7 Fetkovich type-curves

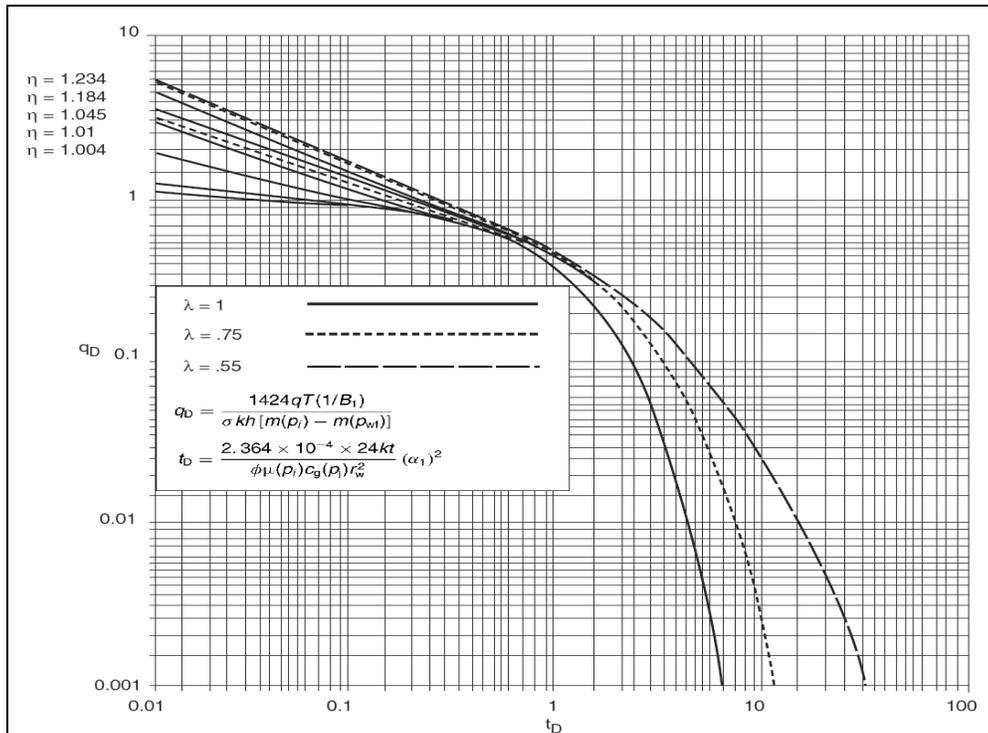


Fig. 8 Gas type-curves developed by Carter (1985)

2.2 The evolution of rate transient analysis

Although the production data analysis had made a huge leap forward in the 1980s, it is still assuming constant flowing pressure. The main failing of this assumption is that it completely ignores the flowing pressure data which can result in underestimating or overestimating the reserves.

At this stage the production data analysis started gaining the acceptance as an equivalent methodology in the theory and procedure to the pressure transient analysis. The constant-pressure type-curves were the counterpart of the well test constant-rate curves. Although both constant-pressure and constant-rate type-curves were valuable tools for the analysis of production and well test data, neither of their assumptions is the case of the typical oil and gas wells. The production conditions through the life of the well may, including testing, contain several periods of variable rate and shut-ins.

Since the rate transient analysis and the pressure transient analysis will be sharing a large technical kernel from this point and so on, it is better to change the gear to have a quick look on the development of the variable-rate, variable pressure treatment. Such development will be contributing later in the evolution of the rate transient analysis.

The analysis of the variable rate data has been studied by several authors. Cooper and Jacob (1946) developed a method for analyzing pressure data of aquifers pumped at variable discharge rate based on solution of Theis (1935). Everdingen and Hurst (1949) in their fundamental paper applied the principle of superposition to solve variable terminal pressure and variable terminal rate cases. The same principle of superposition

was used to calculate pressure function by Hutchinson and Sikora (1959) that they called the resistance function. Mueller (1962) used the same approach and called the pressure function the response function.

Coats et al. (1964) presented a method of determining the influence function in gas storage reservoirs. They derived a general model for the pressure response to unit-rate withdrawal from a reservoir.

Aron and Scott (1965) examined the rate variation effects on water well test analysis. Later, Ramey (1965) and then Winestock and Colpitts (1965) introduced the method of normalized drawdown method. The method involves graphing the $\Delta p/q$ vs. the logarithm of the producing time. The authors noticed that the reciprocal of the productivity index $\Delta p/q$ is often a linear function of the logarithm of producing time. The method theoretically exhibits a straight line whenever the radial flow regime dominates and it provides useful results if the pressure data of the producing well are to be analyzed.

Odeh and Jones (1965) presented a method of analyzing variable flow rate data. The method applies the superposition principle with the logarithmic approximation and approximating the variable rate data by step changes. The method involves graphing the $\Delta p/q$ vs. the superposition logarithmic time.

Jargon and Van Poolen (1965) proposed a superposition-based method in order to analyze variable-rate, variable-pressure well tests. Their approach also was suggested by Everdigen and Hurst (1949). The approach provided a mean by which varying flow rate data can be converted to constant-rate pressure response called the unit response

function. The authors noticed that if the pressure changes and/or rate changes are very drastic, the unit response function becomes erratic and oscillating and no answer can be obtained under these circumstances.

Sternberg (1968) applied the same approach for variable rate pumping test in the groundwater aquifers. Ridley (1975) improved the method of Odeh and Jones (1965) by suggesting a unified approach using superposition principle to analyze production and shut-in periods simultaneously. The method also uses the logarithmic approximation.

Everdingen and Meyer (1971) applied the principle of superposition to analyze low-permeability vertical fractured wells. They used the dimensionless vertical fracture constant rate solution where the relationship between real and dimensionless time is established through trial and error with the initial estimates provided by conventional Horner (1951) analysis.

Ramey (1976) revived the application of Gladfelter et al. (1955) in handling production rate change effects. The method is based on graphing the pressure difference divided by the instantaneous flow rate vs. the logarithm of the producing time.

Bostic et al. (1980) proposed a method of combining the production and buildup test data. They applied the superposition principle described by Everdingen and Hurst (1949) in the combination of the data. This combination is aimed to generate field data curves of sufficient length that the shape of the data curve may be sufficiently definitive.

Fetkovich and Vienot (1984) modified the approach of Odeh and Jones (1965) by including p_D instead of the logarithmic approximation. Their improvement was useful in

the analysis of low-permeability stimulated wells. They concluded that normalization analysis techniques work better for gradually changing rates than drastic changes.

Whitson and Sognesand (1990) demonstrated the application of Everdingen and Meyer (1971) by using full superposition to check the consistency of standard well-test analysis based on the constant-rate or constant-pressure assumptions.

Taking the advantage of the previously mentioned work, the breakthrough of the production data analysis improvements were made in 1990's with the methods that include the application of superposition and normalization. Such improvements contributed in the evolution of the rate transient analysis where in the theory production data analysis caught up with the pressure transient analysis. The rate transient analysis started as an independent methodology for the production data analysis based on analytical solutions. At this point the production data analysis started to split into two categories; rate-time relationship analysis and rate transient analysis.

Palacio and Blasingame (1993) developed a type-curves, shown in Fig.9, to give the performance of constant-rate and constant-pressure gas flow solutions. They also presented new modified time function and algorithm that can lead to harmonic declines and used to analyze gas production data of either gas or oil using type-curves analysis. These type-curves can be utilized to evaluate gas in place from variable rate or variable pressure production data. Their solution was based on a material balance time function which enabled the modeling of actual variable rate/variable pressure drop production conditions. The authors introduced also the integral function and its derivative as complementary plotting functions.

Agarwal et al. (1998) presented new production decline curves published specifically for hydraulically fracture wells of infinite and finite conductivity to analyze oil and gas production data from radial and vertically fractured wells, shown in Fig.10. These type-curves can be utilized to evaluation gas in place and different formation and completion properties. They combined the type-curve and decline curve analysis concepts to present new declines curves that can be used for gas production to estimate the OGIP in addition to formation properties. Their set of type-curves includes rate time, rate-cumulative and cumulative-time production. The two models for which the type-curves can be used are radial model and well with vertically fractures.

Wattenbarger et al. (1998) presented an analytical method of analysis for tight gas fractured wells in which the linear flow is dominant with late boundary effect, as shown in Fig.11. The authors developed equations for transient linear flow for both constant-pressure and constant-rate cases. Once the outer boundary is reached then the OGIP, drainage area, and the value of $(\sqrt{k} x_f)$ can be calculated. These calculations can be done without the need to know porosity, thickness or formation's permeability.

Helmy and Wattenbarger (1999) presented different approach for analyzing production data of gas wells subjected to shut-in intervals and dominated by linear flow at constant-pressure production. First they showed the application of the influence function which enables calculating the reservoir parameters and OGIP. Then construct a production trend without the effects of the shut-in periods. Their aim is to reduce the noise of production data due to multiple well shut-in periods of gas wells, as shown in Fig.12.

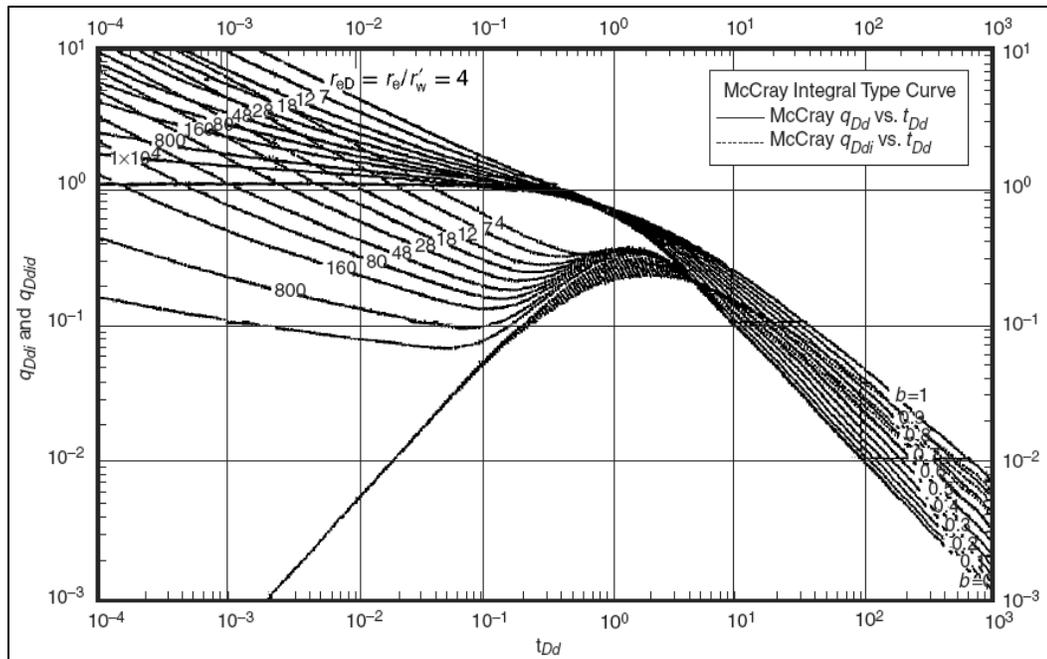


Fig. 9 Palacio and Blasingame type-curves

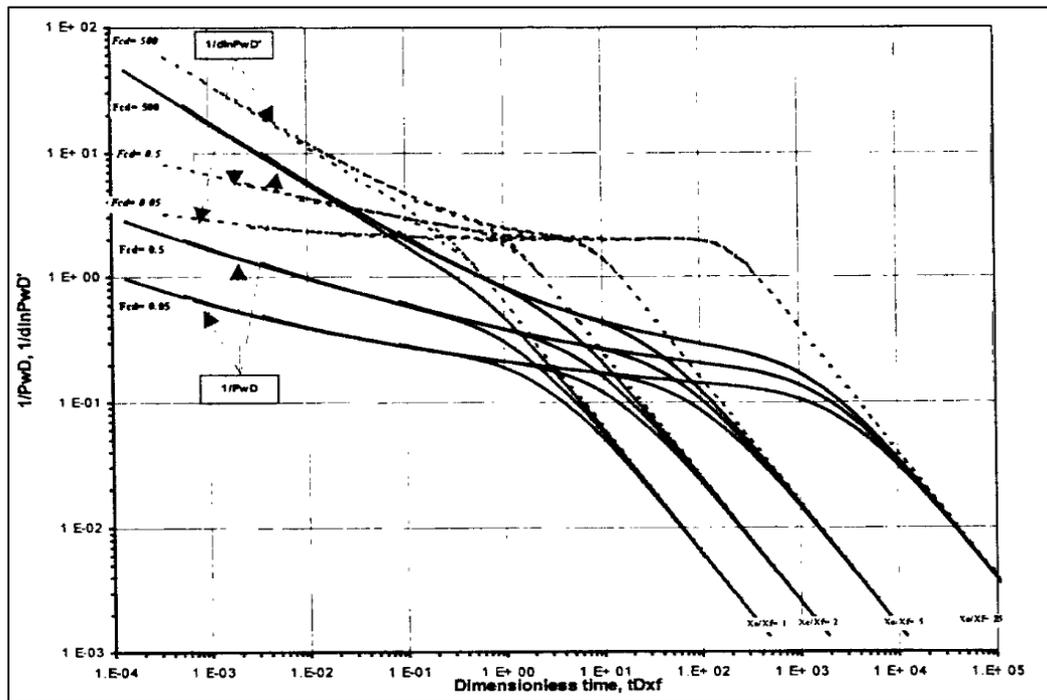


Fig. 10 Agarwal et al. type-curves

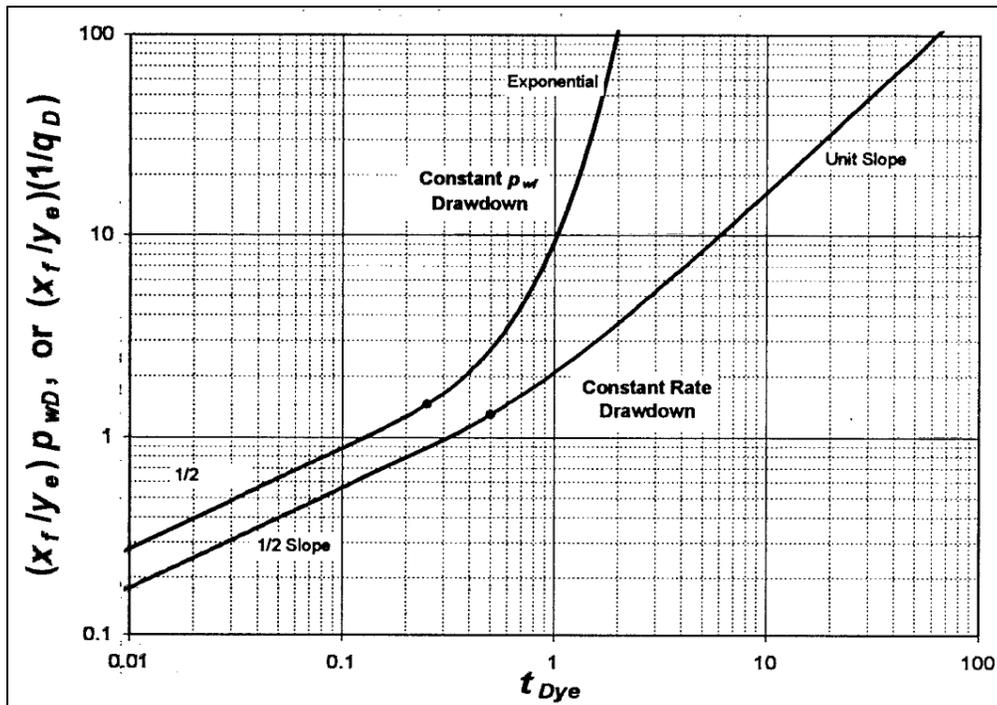


Fig. 11 Linear flow type-curves in closed reservoirs, from Wattenbarger et al. (1998)

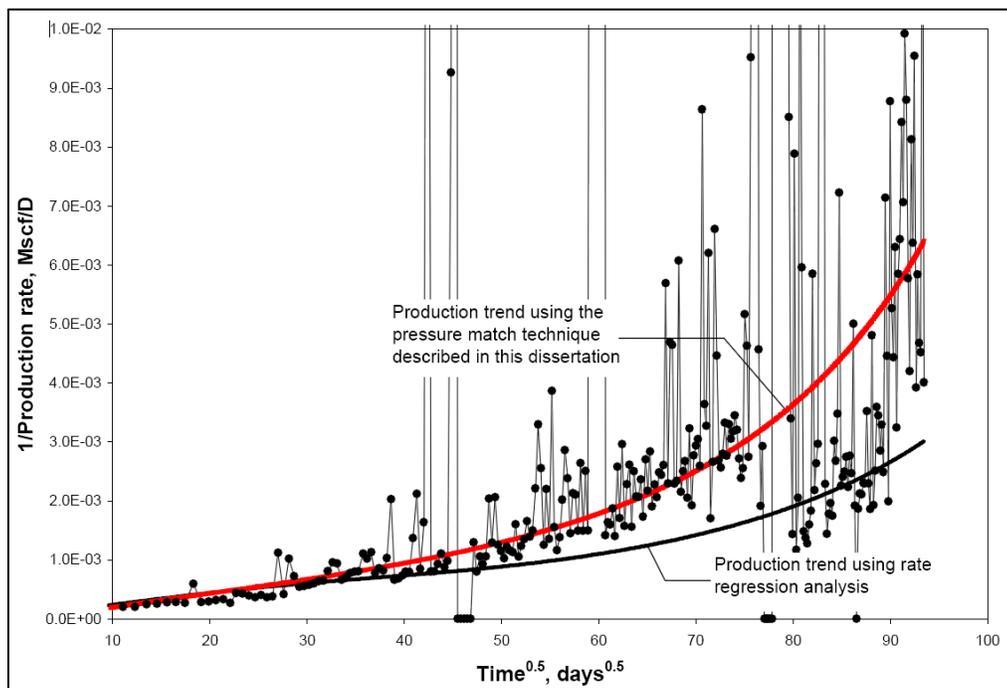


Fig. 12 Analysis of well performance with multiple shut-in periods, from Helmy and Wattenbarger (1999)

Ibrahim et al. (2003) introduced a new normalized pseudo time plotting function that can be used in the superposition method to provide more accurate estimation of the OGIP. The method has ability to analyze the fluctuating field data of both variable-pressure and variable-rate. The presented method is more accurate especially in highly depleted reservoirs. Fig.13 and 14 show the application of this method.

Bello and Wattenbarger (2008) studied the gas transient rate and identified five flow regions, shown in Fig.15, for multi-stage hydraulically fractured wells in shale gas formation. They presented equation for each of these regions and described an early skin effect for the linear flow region. A suggested procedure to analyze field data is also presented.

Al-Ahmadi and Wattenbarger (2011) proposed a fully transient triple porosity model. They showed that six different regions can be observed in the dimensionless rate versus time plot of constant-pressure solution. Those regions are alternating as linear and bilinear flows ending with boundary dominated flow regime of the system as shown in Fig.16.

As the time goes forward more solutions, type-curves, and improvements are introduced in the field of rate transient analysis to address more complex configurations of reservoirs and completions. The main two features that the field of rate transient had added in production data analysis are; first, the incorporation of the flowing pressure data along with production rates, second, the use of analytical solutions to calculate hydrocarbon-in-place and the expected recoverable reserves independently of production constraints.

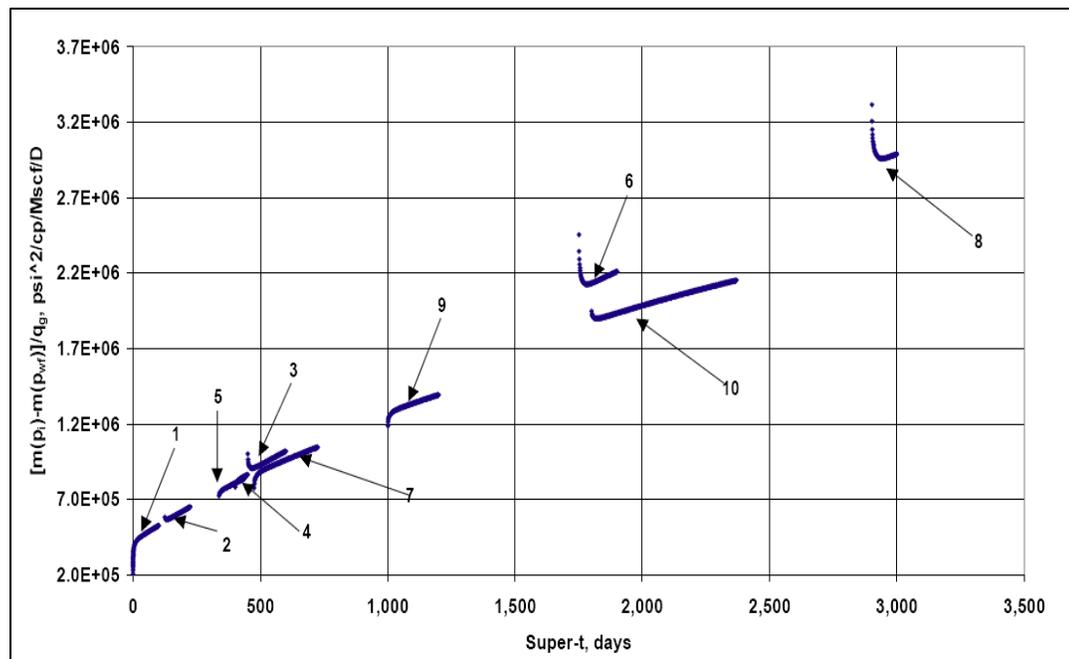


Fig. 13 Superposition pseudo-time plotting function, from Ibrahim et al. (2003)

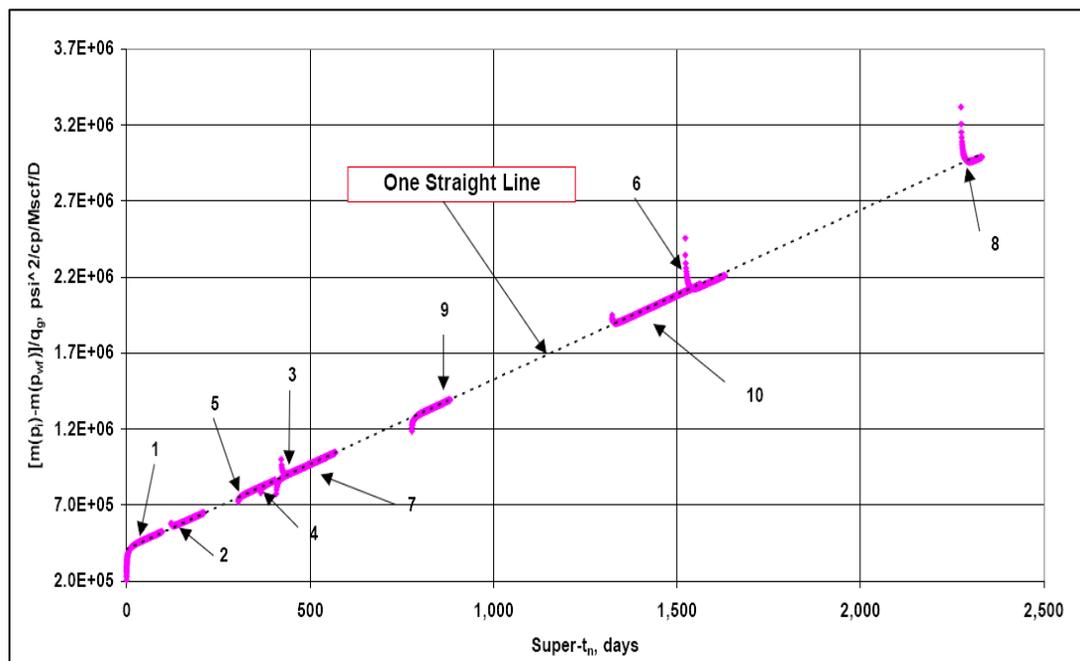


Fig. 14 Normalized superposition pseudo-time plotting function developed by Ibrahim et al. (2003)

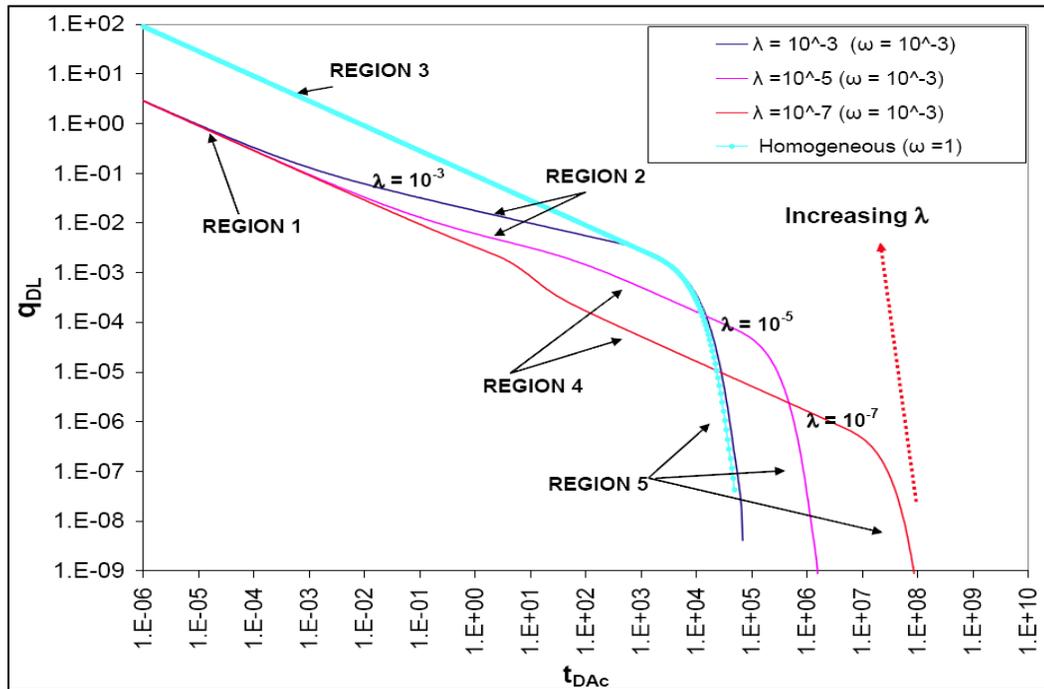


Fig. 15 Dual porosity type-curves developed by Bello and Wattenbarger (2008)

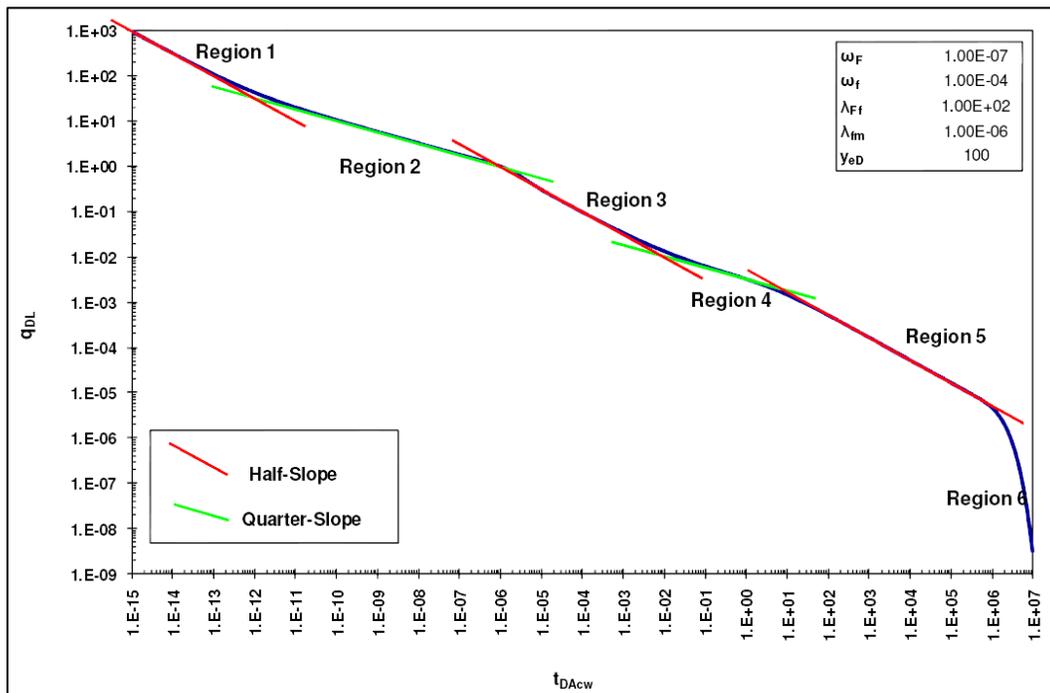


Fig. 16 Triple porosity type-curves developed by Al-Ahmadi and Wattenbarger (2011)

2.3 Diagnostic methods for production data analysis

By late 1990's production data analysis witnessed dramatic changes in the area of the analysis with the wide variety of the solutions proposed. That made it approaching the popularity as one of reservoir characterization tools. However, the diagnosis of production data analysis is still challenging. One reason behind that is that the production data in nature is surveillance and monitoring data with little control and considerable variance occurring during the acquisition of the data. Another reason is the limited diagnostic techniques in practice for the production data analysis. Moreover, within the techniques that are applied, many are either observation-based approaches or rules of thumb. Although rate transient type-curves offered the firm basis of the production data analysis, their virtue in the diagnostics is much superior to the analysis. However, there was no detailed work on the diagnostic of data quality for the transient rate analysis and it took almost ten years later to question the quality of the production data and the analysis methods.

Laustsen (1996) discussed the uses and misuses of the of production decline analysis. He presented an overview of the traditional rate-time methods and their misinterpretations. In addition, he discussed the rules to avoid these interpretations and how to identify production problems.

Mattar and Anderson (2003) presented a comprehensive presentation of all available method's for analyzing production data and highlighted the strengths and limitations of each method.

Anderson and Mattar (2004) presented a set of procedures that can be applied to analyze production data. Some of those procedures were specific and others are general, both on simple production charts and on type-curves. Their work offered diagnostic clues about detecting liquid loading, changing wellbore skin, detecting shift in productivity, and diagnosing external pressure support and interference. They demonstrated their work with six field practical examples and it was the first compilation of practical diagnostics for advanced production data analysis.

Kuchuk et al. (2005) proposed a deconvolution-based method for diagnostics in decline-curve analysis. They proposed a methodology for monitoring well productivity index (PI), forecasting production, and estimating reserves using pressure and flow rate data. Their identification technique start by computing constant-pressure rate (deconvolved rate) behavior based on deconvolution of pressure and rate measurements. Then, the deconvolved rate is compared with the exponential decline of the system under depletion as a base. Any deviation from the exponential decline may indicate nonlinearities about the system.

Kabir and Izgec (2006) provide guidance on the diagnosis of pressure-rate data, with an emphasis on characterizing the reservoir production mechanism. Their work offered a simple methodology to diagnose long term well performance especially when the reservoir response is boundary dominated. They presented mathematical proofs in support of the contentions in the study. However, their method had to rely on other independent methods in order to eliminate potential reservoir, wellbore and surface flow-line network issues before reaching reasonable conclusions.

Anderson et al. (2006) provided a comprehensive review of the techniques and tools that are used to diagnose the reservoir model and assess the reservoir conditions. They categorized the diagnostic techniques under three main categories; history and data correlation, reservoir diagnostics, and auxiliary diagnostics. The history and data correlation plots are limited to certain types of theories (e.g., constant p_{wf} , boundary-dominated flow, etc.). The primary value of these plots is the simplicity of the base theory upon which they are based. Moreover, they tend to be error-tolerant and useful for establishing correlation of the rate and pressure data. The reservoir diagnostics plots consist of a series of log-log plots which have strong theoretical ties to boundary-dominated flow and material balance. The fundamental purpose of these plots is the determination of the reservoir model. Auxiliary diagnostics plots are designed to be complimentary and to provide additional diagnostic value. These plots are used to assess reserves and the relative accuracy of the pressure data. Table-1 shows the proposed list of diagnostic plots.

Nobakht and Mattar (2009) reviewed many of the operating problems observed in practice, and discuss ways and patterns for recognizing them as operating issues rather than reservoir phenomena. They have presented a series of diagnostic plots to determine consistency of data for the proper analysis of production data. Their investigation in production data diagnostics was on four main areas; outlier removal, liquid loading in the wellbore, single phase flow in the reservoir consistency between rate and pressure.

Almarzooq (2010) studied the implications and flow behavior of the hydraulically fractured wells in shale gas formations. His extensive study included data

preparation and diagnosis of production data for 378 shale gas wells from the Barnett shale and 18 wells from Fayetteville shale. His systematic data evaluation was done based on Anderson et al. (2006) suggested procedure.

Ilk et al. (2011) present an integrated workflow including the use of diagnostic plots for reserves assessment in unconventional reservoirs. They provided a review of the existing production analysis and diagnostic techniques as well as the challenges associated with production analysis in unconventional reservoirs. They presented an extensive evaluation of the diagnostic tools for assessing data viability, checking data correlation along with flow regime identification. Based on the diagnostics and analysis results, they demonstrate with field examples the use of forward modeling (simulation) to predict future performance of single/multiple well(s) for various production and field development scenarios.

Table 1 – Proposed diagnostic plots , from Anderson et al. (2006)			
Proposed Diagnostic Plots	[gas variables]	Plot Code	Value in Practice
● <i>History and Data Correlation:</i> ^{1,14,15}			
—log(q) and p_{wf} vs. t	$[q_g]$	1	Good
—log(q) and log(N_p) vs. log(t)	$[q_g, G_p]$	2	Good
— p_{wf} vs. q	$[q_g]$	3	Moderate
— q and p_{wf} vs. N_p	$[q_g, G_p]$	4	Moderate
● <i>Reservoir Diagnostics:</i> ⁷⁻⁹			
—log($\Delta p/q$) vs. log(N_p/q)	$[\Delta m(p), q_g, G_p]$	5	Very Good
—log($q/\Delta p$) vs. log(N_p/q)	$[\Delta m(p), q_g, G_p]$	6	Very Good
● <i>Auxiliary Diagnostics:</i> ^{2,15,20-21}			
—log($q/\Delta p$) vs. log($N_p/\Delta p$)	$[\Delta m(p), q_g, G_p]$	7	Good
—log($1/q$) vs. log(N_p/q)	$[q_g, G_p]$	8	Very Good
—*(p_{wf}) _{meas} and (p_{wf}) _{decon} vs. t		9	Good
* Requires pseudopressure and pseudotime transformations for deconvolution.			

2.4 Remarks made in the literature regarding the superposition time function bias

The superposition time functions have been used as diagnosis tool for the presence of different flow regimes in the reservoir as well as for their analysis. Despite of that virtue that these time functions offer, they can also be a cause of misinterpretation for that data and eventually results in wrong results. For example, rate outliers or liquid loading phenomena will hardly to be distinguished from pseudo-steady state flow regime will if the superposition and material balance time functions are used as plotting function.

The first notice was made implicitly by Jargon and Van Poolen (1965) when they proposed a superposition-based method in order to analyze variable-rate, variable-pressure well test data. The authors noticed that if the pressure changes and/or rate changes are very drastic, the unit response function becomes erratic and oscillating and no answer can be obtained under these circumstances.

Dake (1978) discussed the use of the superposition for multi-rate flow tests and its requirement of presupposition of the flow regime. He stated, “ *The statements made in Chapter 7, sec. 8, about the possibility of incorrectly interpreting multi-rate test data through making an a priori judgment concerning the prevailing flow conditions are equally, if not more, valid in gas well test analysis.*”. Finally, he concluded the discussion with the statement “*ad-hoc assumption of transient flow can be particularly dangerous in the interpretation of multi-rate flow tests.*” which was a direct warning of the blind use of the superposition in time principle.

Streltsova (1988) investigate the influence of different variable production rate patterns on the subsequent buildup pressure behavior. She used superposition principle to investigate whether the variable flow history distorts the subsequent buildups or not. She concluded that a decreasing flow rate to a minimum value causes much more distortion in the subsequent buildup than a flow rate that increases to a maximum value prior to shut-in for all cases considered. Her recommendation was to avoid the flow rate decreasing to minimum prior to shut-in whenever possible.

Anderson et al. (2006) characterized the behavior of the liquid loading as an erratic behavior in the rate and pressure which can be confirmed by a set of diagnostic plots. One of these plots is the reciprocal of the gas rate versus the material balance superposition time function on log-log bases. However, they did not mention about the bias that the superposition time function has.

Cvetkovic (2008) presented an extensive review of analytically and empirically derived depletion rate-time solutions. He pointed out the limitations of the superposition time as he stated *“One such limitation, related to the use of the Horner approximation of the pseudo-producing time (e.g., material balance time), concerns the assumption that the flow rate is only permitted to be smoothly varying. Therefore, an erratic well-flow history shortly before a production data time level of interest can result in a significant error in the estimation of the equivalent superposition time function value for that time level. This limitation is also true for other production analysis methods that use the material balance time function as a substitute for the superposition time function”*.

Nobakht and Mattar (2009) investigated the diagnostics of production data. One of the main areas that they investigated is the outliers' removal. They emphasized on outlier removal as a first step in production data diagnostics in order to eliminate any incorrect interpretations. They noted that often outliers show a unit slope on the type-curves as shown in Fig.17. Nonetheless, there was no direct accusation to the role of superposition time function in that observation.

Anderson et al. (2010) they endorsed the use of actual time on the log-log plot for wells producing at constant-pressure conditions. They favored the use of the actual time over superposition time as they mentioned, "*Offers an advantage in wells that produce at close to constant pressure conditions, in the log-log plot is not susceptible to the superposition bias that is sometimes present*". That was the first explicit note on the bias of the superposition time use.

Nobakht et al. (2010) presented a method of production forecasting for tight /shale reservoirs exhibiting linear flow. While they are listing the advantages of their method they mentioned, "*Here are the advantages of the forecasting method proposed in this study: (1) it is not biased towards any flow regimes, as no superposition time functions are used*".

Moghadam and Mattar (2011) discussed the pitfalls of superposition when analyzing production data. They investigated the superposition behavior under the same rate history with different flow regimes, effect of speed of rate change, portion of the data affected, effect of different pressure profiles, and the effect of the outliers.

For the effect of rate history with different flow regimes, they investigated three cases; all data in transient radial flow, all data in transient linear flow, and all data in boundary dominated flow. They concluded that the choice of the superposition time function does not affect the early time data, but the late part of the data looks like the particular time flow regime.

They observed that the speed of rate decline has no effect on the behavior of the data regardless of the dominant flow regime. Furthermore, the choice of superposition time functions does not affect the early time data whereas the late time points are affected. They noted also that the portion of the data affected by the superposition is always the late time part when the rate drops below half of the initial rate.

The authors investigated the effect of three different pressure profiles; constant, increasing, and decreasing pressure profiles. They concluded that for the case decreasing pressure profile with increasing rate and transient flow the superposition time function choice has no effect on the shape of the data. However, for both cases of increasing pressure profile in transient flow and of constant-pressure production in boundary-dominated flow the choice of the superposition time function significantly affect the late time data. Finally, the case of constant-pressure production in transient flow has minimal effect of the choice of superposition time function.

They pointed out that the low flow with a steeper decline and exponential decline trends are affected by the choice of superposition time function. In addition, the outliers have been found of a great effect on the shape of the data with the use superposition time functions, and the effect is often magnified on the specialized plots. Finally the authors

recommended approach of Winestock and Colpitts (1965), which uses the instantaneous values of time instead of the superposition time.

Their work serves as a big warn to the analyst of production data. Although it was the first dedicated work on the susceptibility the superposition time functions behavior, their work was focused on illustration of the problem other than demonstrating the way of the proper use of superposition time functions and utilizing its advantages.

Bachman et al. (2011) used the rate transient analysis in the investigation of the effects of stress dependent reservoir permeability on the performance of stimulated horizontal gas wells. They pointed out that false boundary dominated flow signature is evident when rapid rate changes occur. This signature is characterized with slope of -1 on the log-log plot. In order to eliminate this defect they proposed a filtering algorithm. Their filtering algorithm is based on the use the weighted average of the $q_g/\Delta m$ term over specified number of neighboring points and comparing it with the value at the time of each point. If the difference is less than a specified tolerance, the value of the point will be plotted. They proposed also a method to verify that the late material balance time is the late in the time sequence. That was by splitting the production plot into different flow periods and checking their sequence as shown in Fig.18. However, the authors did not offer a detailed and clear treatment to the problem of superposition time bias.

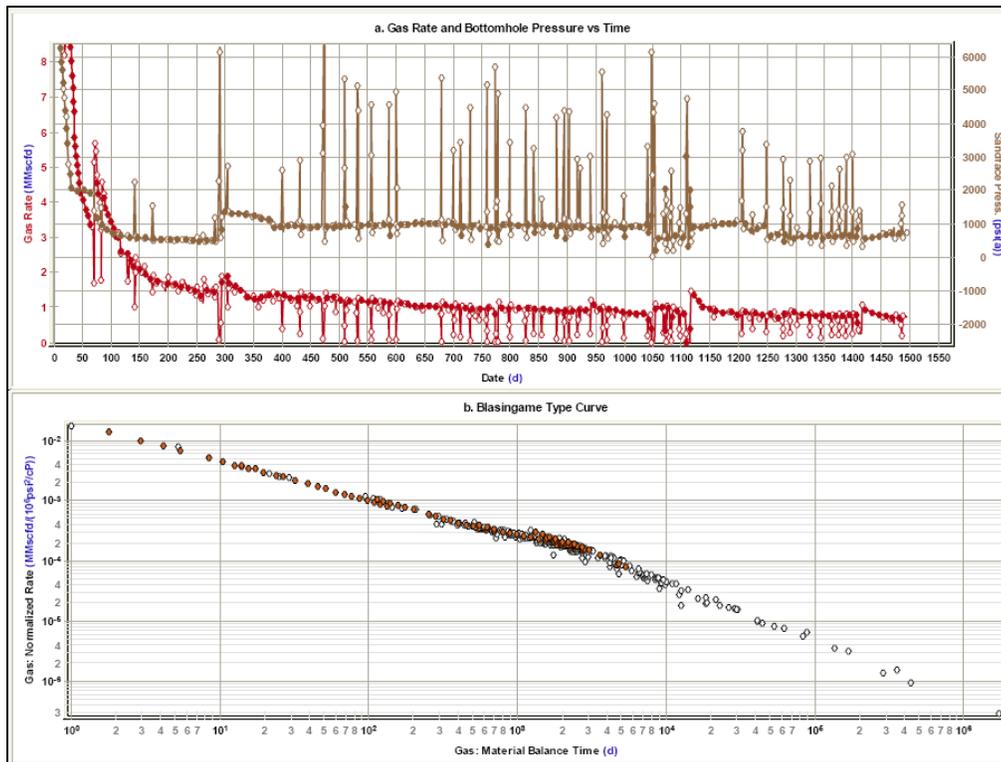


Fig. 17 The effect of outliers on the log-log of rate transient analysis, from Nobakht and Mattar (2009)

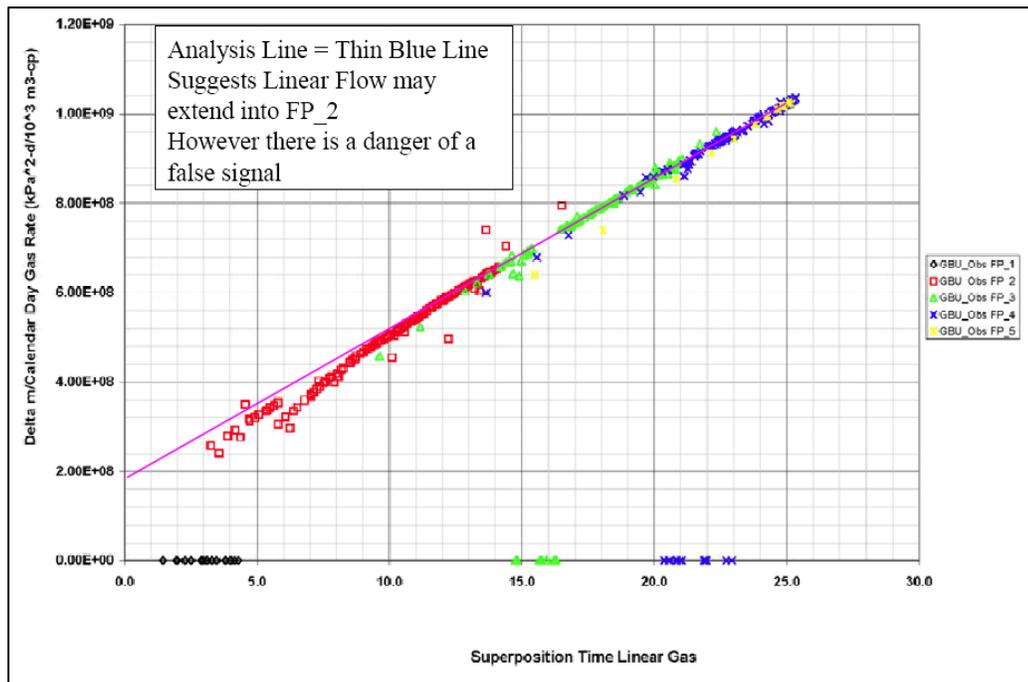


Fig. 18 Superposition time effect, from Bachman et al. (2011)

2.5 Summary

Production data analysis has witnessed tremendous changes since it has started in 1920's. It had made huge leap from the pure empirical relationships of decline curves to the firm bases of analytical solutions of the rate transient analysis. Nowadays, it reached the popularity and gained the acceptance as an equivalent methodology in the theory and practice to the pressure transient analysis. The constant-pressure type-curves are the counterpart of the well test constant-rate curves while they share the same variable-rate, variable-pressure treatment. Eventually, the rate transient analysis started sharing a large technical kernel with the pressure transient analysis. The diagnostic procedures came later to give clues about the consistency of the data and its practical use. However, there was large skepticism about the use of the superposition time functions and its behavior. Recently the subject has received some attentions, though, the work shown in the literature served as big warns of the problem other than resorts. The next chapter will be reviewing variable-rate, variable-pressure analysis techniques from the prospective of their relation to the superposition principle and application.

CHAPTER III

SUPERPOSITION PRINCIPLE AND APPLICATIONS

Production data analysis has huge leap in late 1990s where the theme has changed from the traditional to modern production data analysis. That is attributed to the introduction of four major developments. The first is the introduction of the analytical solutions that are exclusively developed for the approximating conditions for the production data. The second is the introduction of the permanent gauges that provide a continuous pressure measurement, the missing part in the traditional production data analysis. The third is the use of the superposition time functions which enabled the use of the already developed pressure transient solutions and techniques. The fourth is the introduction of the rate normalization and deconvolution. Consequently, the production data started having the theme of long-term continuous pressure and rate data. That theme dictated the change of the analysis and interpretation toward variable-rate, variable-pressure approach. At this stage, production data analysis crossed with large part of the pressure transient analysis techniques, principles, and solutions. Among of which are the principle of type-curve analysis, influence function, history matching, and straight-line approach. Therefore, this chapter will be reviewing those techniques from the prospective of their relation to the superposition principle and application. Finally, a brief discussion on the nature of the production data will be also given. The purpose of this chapter is to prepare the reader for the next chapter which will be discussing how the superposition can cause the bias in the rate transient analysis.

3.1 The principle of superposition

Simply, superposition principle is based on generating the solution of a complex problem with linear combination of simpler solutions. Therefore, it has been utilized in petroleum engineering into two main applications. The first is the superposition in time, which is used to simulate production histories by linear combinations of simple drawdown solutions of different starting times, as shown in Fig.19. The second is the superposition in space or image theory, which is used to simulate flow boundaries spatially by linear combinations of infinite responses of imaginary wells with the actual well.

As any other mathematical principle, superposition works under certain conditions and fails at others. The superposition principle in nature is a property of the linear differential equation. Therefore, the principle is not valid whenever nonlinearity is present such as multiphase flow. Moreover, the superposition is only valid for the region of its governing flow regime. The following Rules of superposition:

1. It is a property of linear differential equation, consequently, its use and validity only and strictly guaranteed to the solutions of linear differential equations.
2. Any linear combination of individual solutions honoring the linear differential equation is also a solution of the equation and honors its properties.
3. Different solutions of different differential equations cannot be superposed.

Accordingly, the principle of superposition in practice is limited to the linearity of the system. Therefore, the principle is not mathematically applicable to gas flow (without pseudo-corrections), multiphase flow, pressure dependent case, and reservoir model changes. Further the principle is limited to the systems in which one well perturbation appears (i.e., no nearby wells that cause an interference). Moreover, the superposition principle resides in the consistency of the measured pressure and rate data.

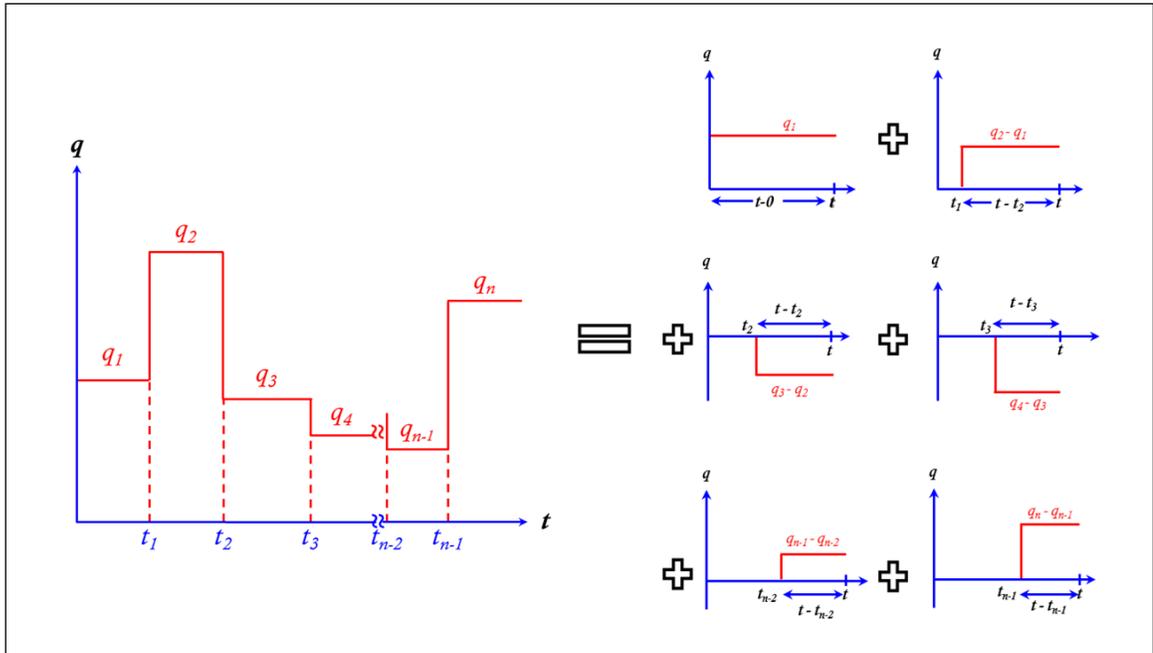


Fig. 19 Superposition principle

3.2 Superposition applications

The principle of superposition has been used in many engineering disciplines to solve transient problems such as in eclectic circuits and heat diffusion. Everdingen and Hurst (1949) were the first who demonstrated the application of the principle to petroleum engineering in order to calculate the bottom-hole pressure response resulting from variable production rate history. Ezekwe (2011) summarized the applications of the superposition principle in petroleum industry. He pointed that the principle of superposition is utilized in petroleum engineering to solve the following three cases:

1. Single well, multi-production rates case.
2. Multi-well reservoir system case.
3. Single well near a boundary (Image well) case.

Since the main concern of this work is the bias that superposition time introduce in the analysis, the second and third cases are out of the scope of this work. Therefore, the first case will be the main case for the discussion in this work. The application of the superposition principle has been demonstrated by Van Everdingen and Hurst (1949) in their fundamental paper in order to solve variable terminal pressure and variable terminal rate cases as shown in Fig.20. They used the principle in order to calculate the bottom-hole pressure response resulting from variable rate production according to Eq.(1).

$$p_{wf}(t) = p_i - \sum_{i=1}^n (q_i - q_{i-1}) \Delta p_{unit}(t - t_{i-1}) \dots \dots \dots (1)$$

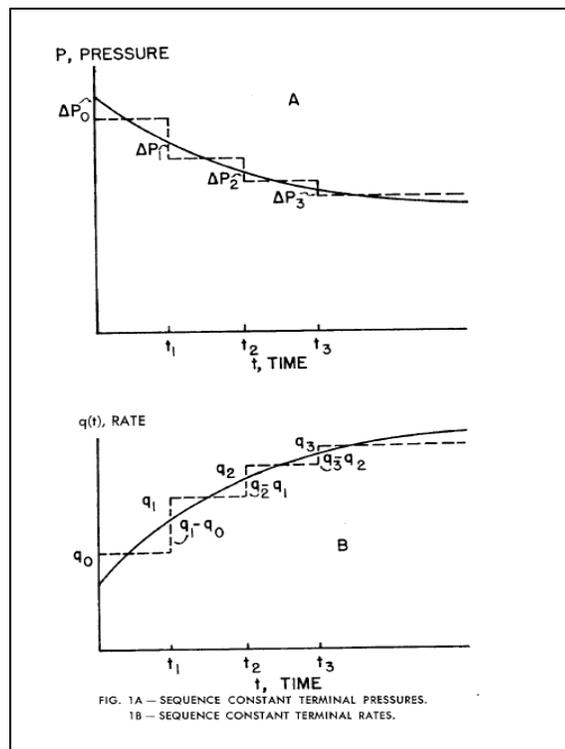


Fig. 20 Constant-rate and constant rate approximations used by Everdingen and Hurst (1949)

Generally, the principle of superposition is used to correct the time or pressure scales. The method which is applied for pressure scale correction is usually referred to as deconvolution. On other side, the method used for time correction assumes predefined reservoir flow regime model and referred to as convolutions.

The simplest form of the superposition time is the Horner time which is used to take into account the producing history time. It is convolution method as it presumes the radial flow regime in the reservoir. For a well-produced at constant rate q for time t_p then shut in for time Δt , the pressure change due to a combination of production and shut-in periods is equal to the combination of individual pressure changes due to each period. Mathematically, the build-up response at the shut-in time is equivalent to a continuation

of the drawdown with the same magnitude and negative sign (i.e., injection at rate $-q$) from time of shut-in as shown in Fig.21, and can be expressed by Eq.(2).

$$p_{shut-in}(t_p + \Delta t) = p_i - \Delta p_{prod}(t_p + \Delta t) + \Delta p_{inj}(\Delta t) \dots \dots \dots (2)$$

For the radial flow solution Eq.(2) the simplest form can be expressed as follows:

$$p_{ws} = p_i - m \left[\log \left(\frac{t_p + \Delta t}{\Delta t} \right) \right] \dots \dots \dots (3)$$

The term $\log \left(\frac{t_p + \Delta t}{\Delta t} \right)$ in Eq.(3) is the superposition time using the logarithmic

approximation of the radial flow solution. Horner plot is a simplified way to present the pressure versus the superposition time for radial flow as shown in Fig.22. However, the same principle can be applied to other flow regimes and eventually yield the appropriate specialized plot.

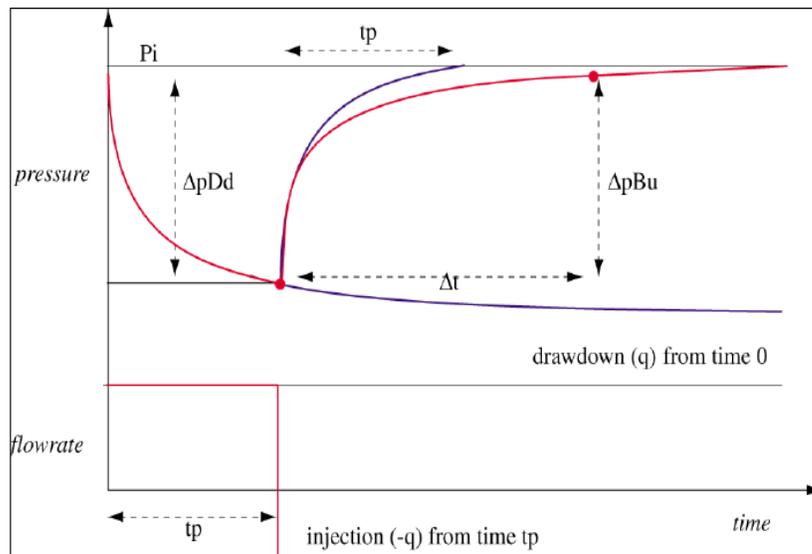


Fig. 21 Build-up superposition, form Houzé et al. (2011)

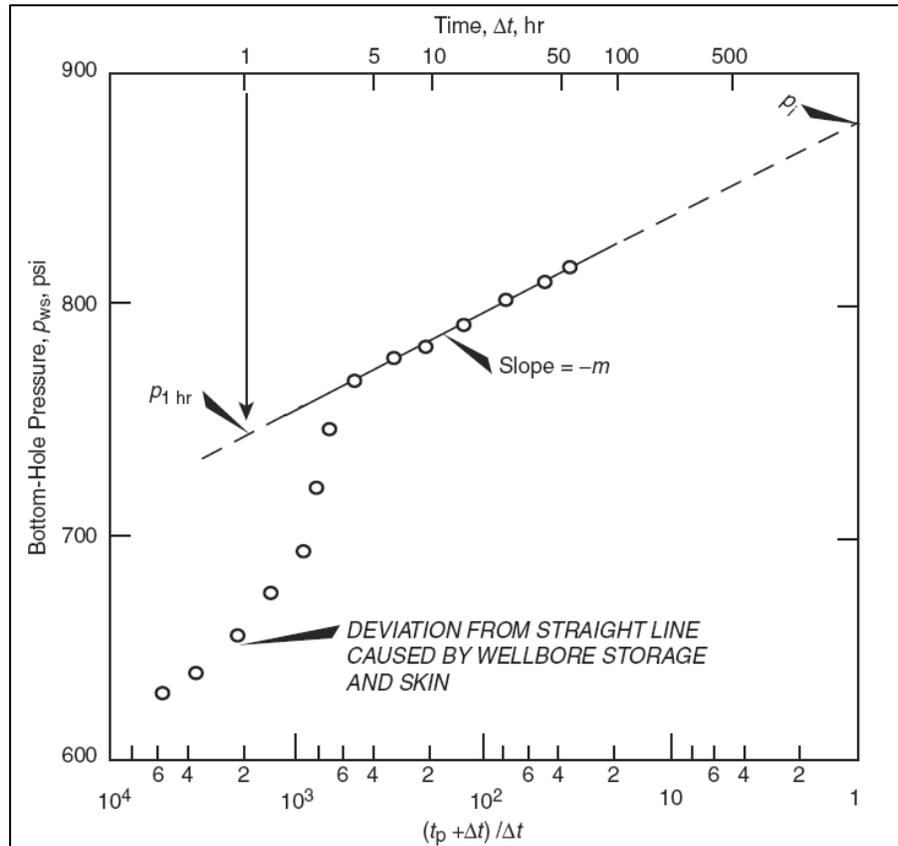


Fig. 22 Superposition time (Horner) plot for Bulid-ups, from Ahmed and McKinney (2005)

3.3 Influence function

Another concept which has been used in relation with the superposition principle is the influence function. The concept of influence function is used in many areas of science and engineering under various names. It is called unit response function, Green's function, resistance function, and memory function. It is used to relate the response of a system to an input. In reservoir engineering the influence function is used to relate the pressure to production rate. Everdingen and Hurst (1949) developed an influence

function using Laplace transforms for water-drive reservoirs. Hutchinson and Sikora (1959) then later Jargon and Van Poolen (1965) developed it in an algebraic forms. However, the field data inaccuracies is magnified during the desuperposition process, which results in influence function that is physically meaningless and cannot be extrapolated for future performance.

Coats et al. (1964) made the major improvement over the previously mentioned methods by imposing smoothness constraints. They found that exact solution to the diffusivity equation can be expressed by Eq.(4). It is the general model for the pressure response to unit-rate withdrawal from a reservoir. The transient flow is represented by the infinite sum of the exponential terms, whereas the stabilized flow is represented with the linear term. The smoothness constrains are imposed to ensure that the derived function is physically meaningful and can be correctly extrapolated. The first constraint Eq.(5) ensures that the function is monotonically increasing while the second Eq.(6) is to ensure that it does not bend down. Finally, the third constraint Eq.(7) forces the function to stabilize with time. The physical interpretation is that a constant rate of fluid into/out of a porous medium of arbitrary geometry and heterogeneity will result in an injection/production face pressure change which is always increasing but at a steadily decreasing rate. Given that Darcy flow and an initial state of equilibrium in the porous medium are assumed. The authors satisfied these constraints by the use of linear programming. The most important advantage of this approach is that it does not require a prior assumption of the reservoir geometry and properties. Therefore, the authors utilized this method to determine the influence function in gas storage reservoirs then used it

later to predict the future performance. Fig.23 shows the influence function derived from field data by Coats et al. (1964) and Fig.24 shows its application to match the field data.

$$F(t) = a_0 t + \sum_{i=1}^{\infty} a_i (1 - e^{-b_i t}) \dots\dots\dots (4)$$

$$F(t) > 0 \dots\dots\dots (5)$$

$$\frac{dF(t)}{dt} \geq 0 \dots\dots\dots (6)$$

$$\frac{d^2 F(t)}{dt^2} \leq 0 \dots\dots\dots (7)$$

Zais (1980) showed the application of the influence functions to describe geothermal reservoirs production behavior. In geothermal reservoir studies the influence functions is calculated to relate reservoir pressure to production rate, then later used to predict the reservoir behavior. He investigated the usefulness of Coats et al. (1964) constraints. He analyzed field data both with and without these constraints. He concluded that these constraints are necessary to ensure that the influence function derived is physically meaningful. Fig.25 shows derived influence functions both with and without constraints. Although the irregular curve is the best fit to data, it is physically meaningless and cannot be extrapolated. That curve was calculated by hand, while the others are using Coats et al. (1964) procedure. Other field data are also shown in Fig.26 as demonstration of the effectiveness of the procedure.

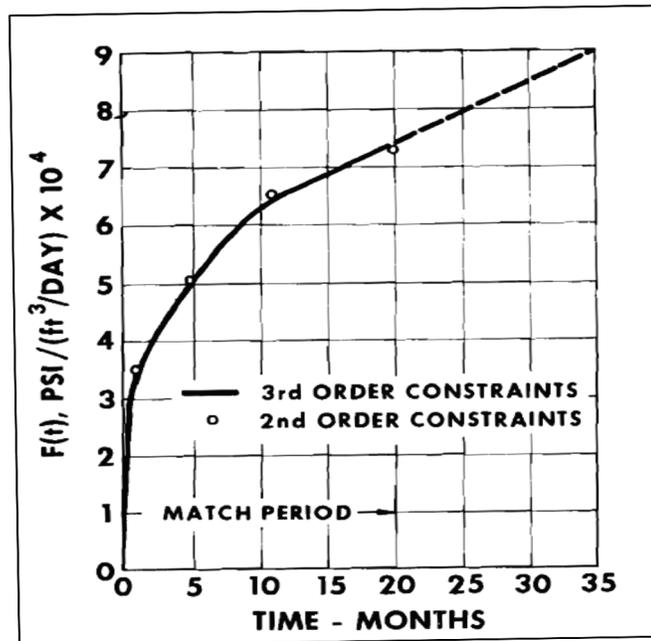


Fig. 23 Influence function derived from field data by Coats et al. (1964)

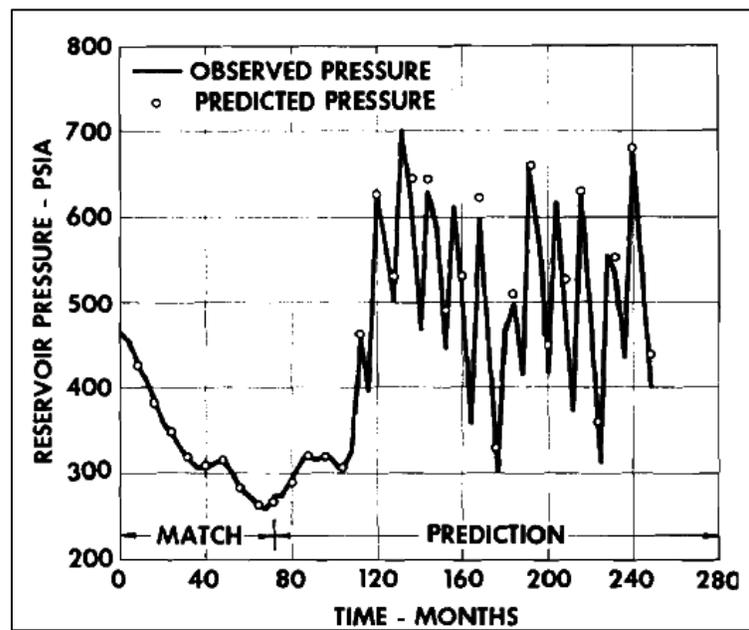


Fig. 24 Field data matched with the use of influence function, from Coats et al. (1964)

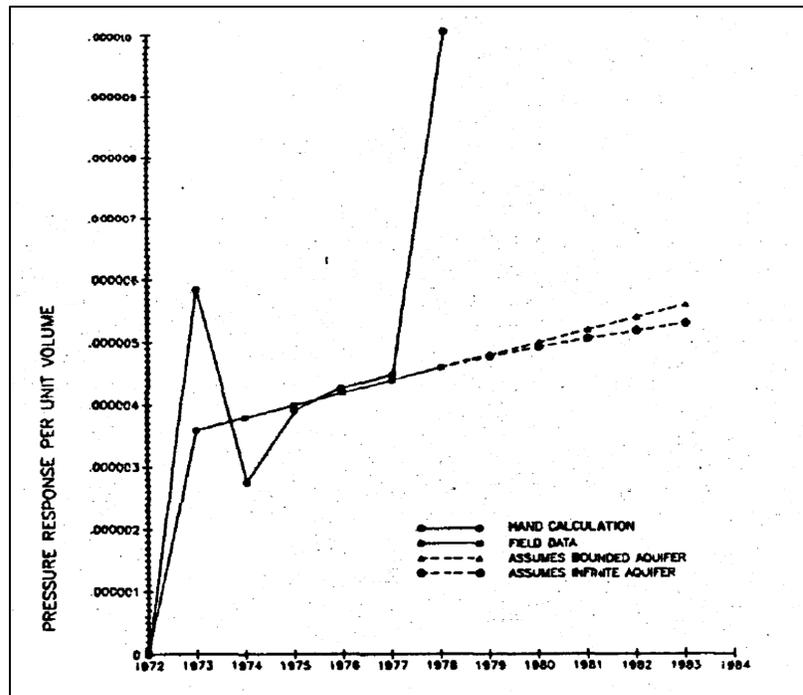


Fig. 25 Influence functions derived both with and without constraints, from Zais (1980)

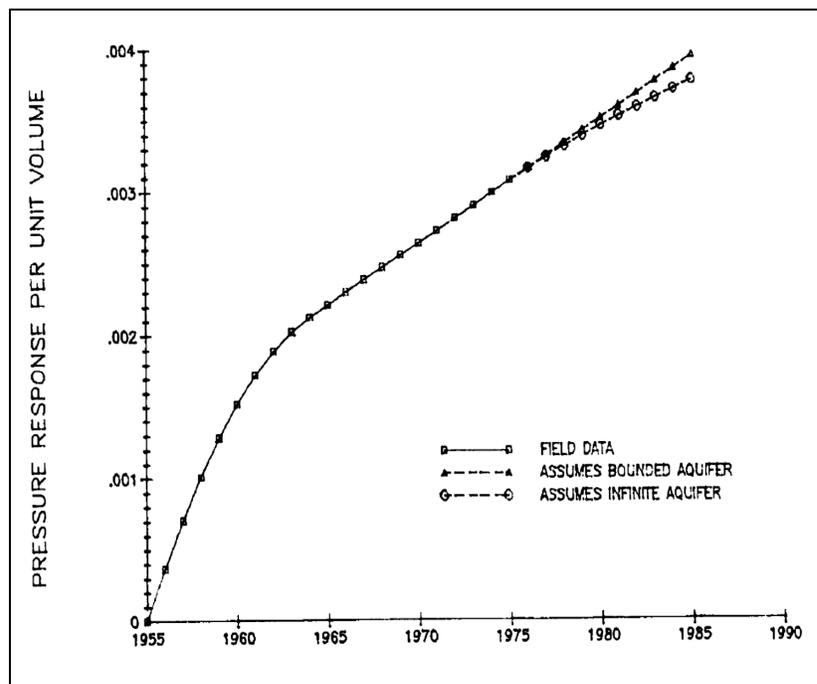


Fig. 26 Influence functions derived with constraints, from Zais (1980)

3.4 History matching

The other way of superposition application in variable rate production histories is the history matching procedure. History matching of variable rate and /or variable pressure data has strong relation to the principle of superposition. Furthermore, most of the work done with the help of the influence function have strong bond with the theory of history matching. Jargon and Van Poolen (1965), Odeh and Jones (1965), Ridley (1975), and Bostic et al. (1980) all used the principle of superposition to analyze variable-rate cases. However, the rigorous method was developed by Everdingen and Meyer (1971). The authors proposed that the time constant is varied until a linear fit of the pressure variable is made vs. superposition time. This dimensionless time constant is used in superposition calculations on the basis of rate change and dimensionless pressure and reservoir properties such as permeability, skin, and in some cases, drainage area. That requires the relationship between the real and dimensionless time. The relationship is established through trial and error with initial estimate obtained from conventional specialized plots. Although the method was developed for well testing purposes, it offered a simple way in which the drawdown and build-up data are analyzed simultaneously.

Whitson and Sognesand (1990) applied the method for field cases, shown in Fig.27 and Fig.28, and showed the use of full superposition to check the consistency of standard well-test analysis based on the constant-rate or constant-pressure assumption. Eq.(8) and Eq.(9) show the general formulas used in history matching for both variable-

pressure drop and variable-rate cases respectively.(should be modified for the gas case using the pseudo-pressure and pseudo-time).

$$q(t) = \frac{kh}{141.2B\mu} \sum_{i=1}^n (p_i - p_{wf,i}) q_D(t_D - t_{D,i-1}) \dots\dots\dots (8)$$

$$p(t) = p_i - \frac{141.2B\mu}{kh} \sum_{i=1}^n (q_i - q_{i-1}) p_D(t_D - t_{D,i-1}) \dots\dots\dots (9)$$

Orkhan et al. (2011) extended the approach of Everdingen and Meyer (1971) using a semi-analytic method for history matching the production of shale gas wells and future forecasting. They estimate reservoir parameters by history matching the production data of hydraulically fractured shale gas wells. Their method is based on the analytical solutions to dual-porosity reservoir models and the algebraic equations developed by Bello and Wattenbarger (2010). The general equations for rate and pressure drawdown superposition are given by Eq.(10) and Eq.(11). Where $[F_q(t)]$ is the rate response function and $[F_p(t)]$ is the pressure response function

$$\Delta p(t) = \sum_{i=1}^n (q_i - q_{i-1}) F_p(t_n - t_{i-1}) \dots\dots\dots (10)$$

$$q(t) = \sum_{i=1}^n (\Delta p_i - \Delta p_{i-1}) F_q(t_n - t_{i-1}) \dots\dots\dots (11)$$

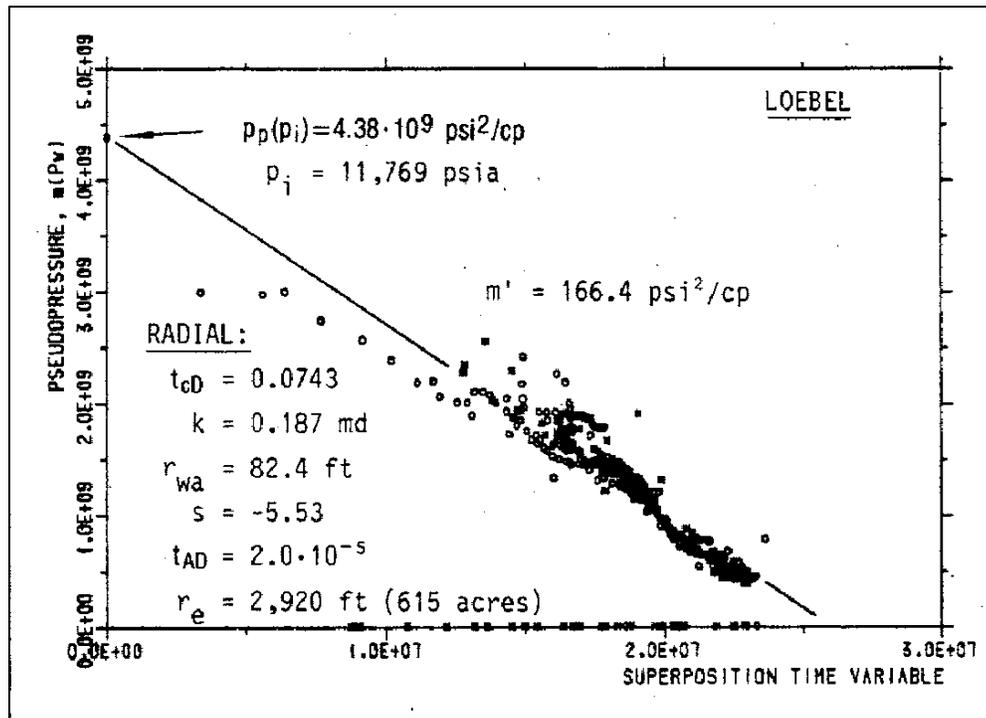


Fig. 27 Superposition plot used by Whitson and Sognesand (1990)

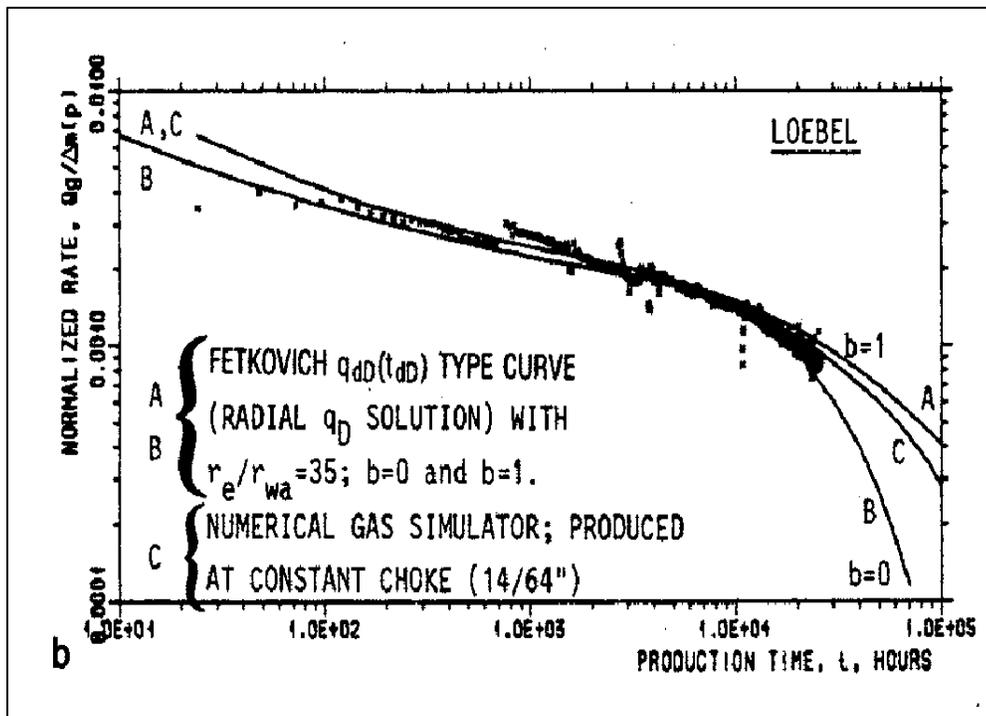


Fig. 28 Production history matched by Whitson and Sognesand (1990)

3.5 Type-curve analysis

The method of type-curve analysis was first developed for groundwater hydrology by Theis (1935). It was introduced to the petroleum industry earlier for constant-rate solutions by Agarwal et al. (1970). Authors such as Fetkovich (1973), Uraiet and Raghavan (1980), Ehlig-Economides and Ramey (1981), Carter (1985), and Wattenbarger et al. (1998) have considered the case of constant wellbore pressure. However, the pressure transient analysis predates the rate transient analysis and the area of pressure transient analysis is much mature compared to rate transient analysis. Accordingly, more constant-rate solutions are already developed. Providentially, still the constant-rate solutions can be used interchangeably for both pressure transient analysis and rate transient analysis with the help of superposition principle. The superposition principle says that the response of the system to a number of rate changes is equal to the sum of responses to each of the rate changes as if they were present by themselves. As a result, the superposition principle offered a resort to the analyst to use constant-rate solutions for production data analysis. For example, material balance time, depicted in Fig.29, is the superposition time function associated with boundary-dominated flow. It can be used effectively to convert constant pressure solution, shown in Fig.30, to the corresponding constant rate solution. As a result, the exponential curve plotted using material balance time becomes harmonic as shown in Fig.31. Base on this principle Palacio and Blasingame (1993) and Agarwal et al. (1998) developed their type-curves.

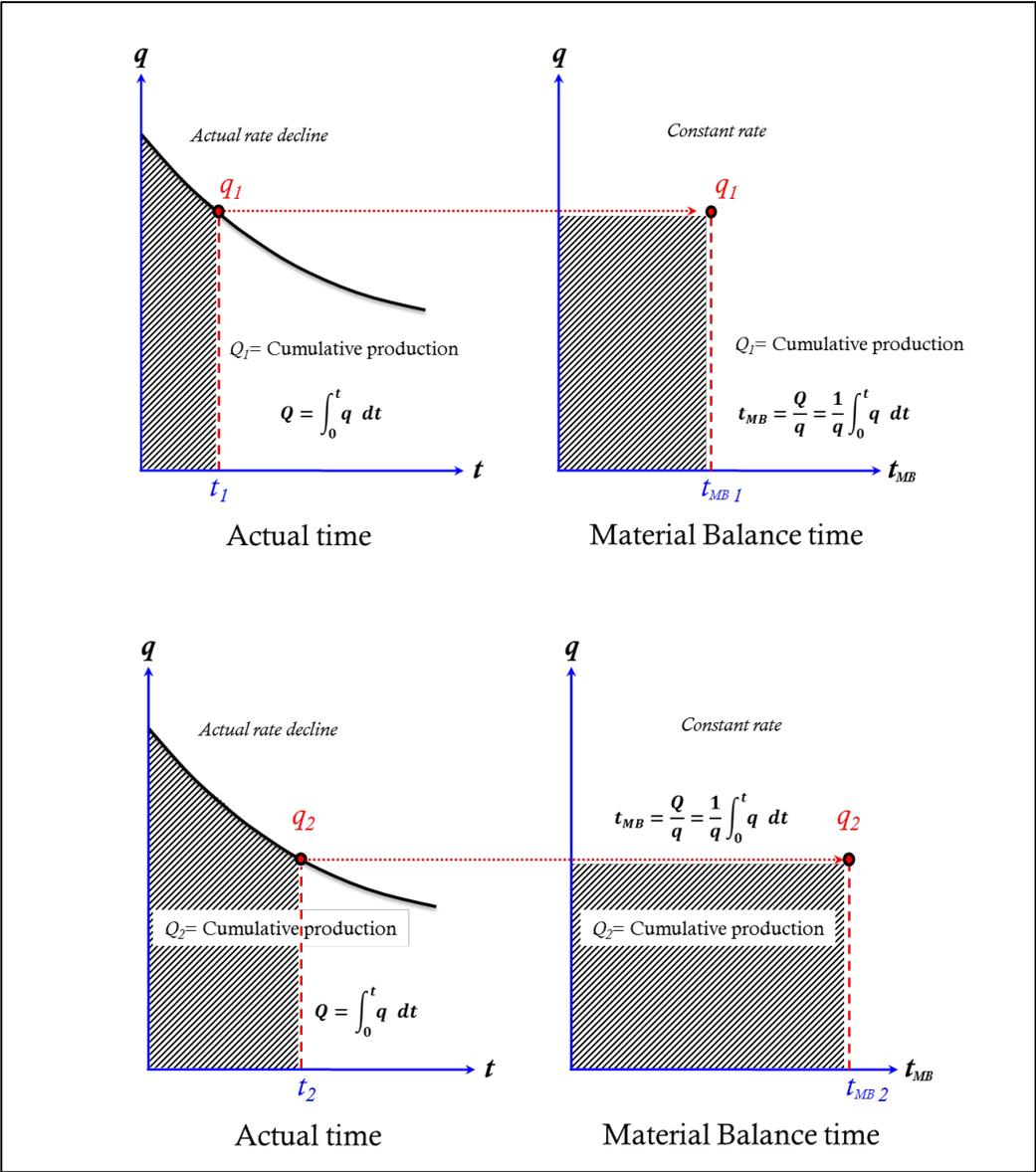


Fig. 29 The concept of material balance time, redrawn from FAST RTA (2010)

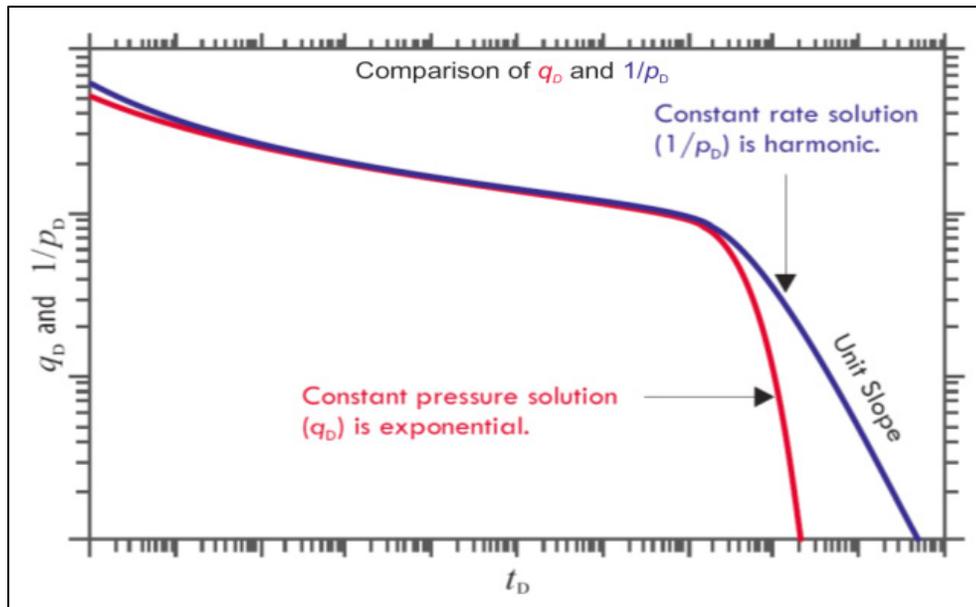


Fig. 30 Comparison of constant-rate and constant-pressure solutions, from FAST RTA (2010)

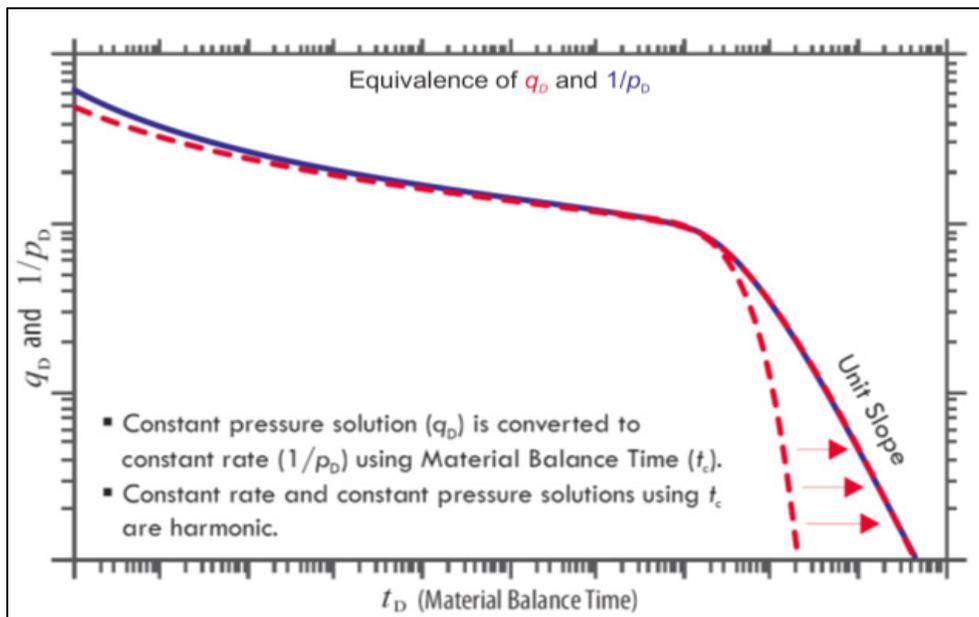


Fig. 31 Conversion of constant-pressure to constant-rate solution, from FAST RTA (2010)

3.6 Straight line analysis

The principle of the straight-line analysis is simply expressed by a plot of straight line equation with slope and intercept. The independent variable is a function of the elapsed time, whereas the transient function is the dependent variable. Whenever plotted, a straight line may be formed on part of the plot. The existence of the straight line relies on the occurrence and domination of the transient flow regime of interest. The slope and the intercept provide the controlling parameters of the flow regime. The advantages of the straight-line approach are its easiness in implementation and its ability for flow regime identification. Duong (1990) presented the advantages of the approach.

Straight-line techniques had been used in the groundwater hydrology before the petroleum industry. Theis (1935) introduced the straight-line semi-log analysis to groundwater hydrology. Later, Cooper and Jacob (1946) based on that solution developed a method for analyzing pressure data of aquifers pumped at variable discharge rate. Muskat (1937) presented to a trial and error method for finding static pressure from a build-up data using a straight line approach. He showed graphically that a plot of the pressure drop on the log axis versus time on the linear axis exhibits a straight line for the later portion of the plot if the guessed the static pressure is right. Nevertheless, the concept of straight-line analysis developed by Theis (1935) was brought to petroleum engineering in 1950s by Miller et al. (1950). They were the first who used the straight-line analysis for pressure transient data. They proposed plotting the pressure drop vs. production or shut-in time on a semi-log paper. Later, this method was known as Miller-Dyes-Hutchinson or MDH method. Subsequently, Horner (1951) took account of the

production history before the build-up by using the principle of superposition. He approximated the production history time by the total cumulative production divided by the last stabilized rate and superposed it to the shut-in time. Both MDH and Horner methods were developed for constant-rate cases. The technique was the dominant pressure transient interpretation tool in the 1960s and early 1970s.

Odeh and Jones (1965) developed the straight-line analysis method for the variable-rate, variable-pressure conditions. They used the superposition of the logarithmic time approximation of the radial flow as shown in Fig.32. However, the superposition time function can be modified for any other flow regime by changing its kernel. The kernel of the superposition time function for the radial flow is the log-time (Miller et al., 1950), it is the square root time for linear flow (Clark, 1968; Miller, 1962; Millheim and Cichowicz, 1968), fourth root of time for the bilinear flow (Cinco-Ley and Samaniego, 1981), the time for the pseudosteady-state (Jones, 1962) and wellbore storage (Ramey, 1970), and time to the power of minus half for the spherical flow (Moran and Finklea, 1962). In addition, the material balance time is used for boundary-dominated flow (Doublet et al., 1994; Palacio and Blasingame, 1993). Table-2 summarizes the superposition time functions for each flow regime, whereas the plots are shown in Fig.33 that are modified from Gringarten (2006). As in the case of type-curve analysis, the straight-line analysis took the advantage of the superposition time ability to convert the variable rate conditions to constant rate ones. That utilized the analyses that had been already developed constant-rate solutions. Table-3 lists the equations used for straight-line analysis adapted for gas flow.

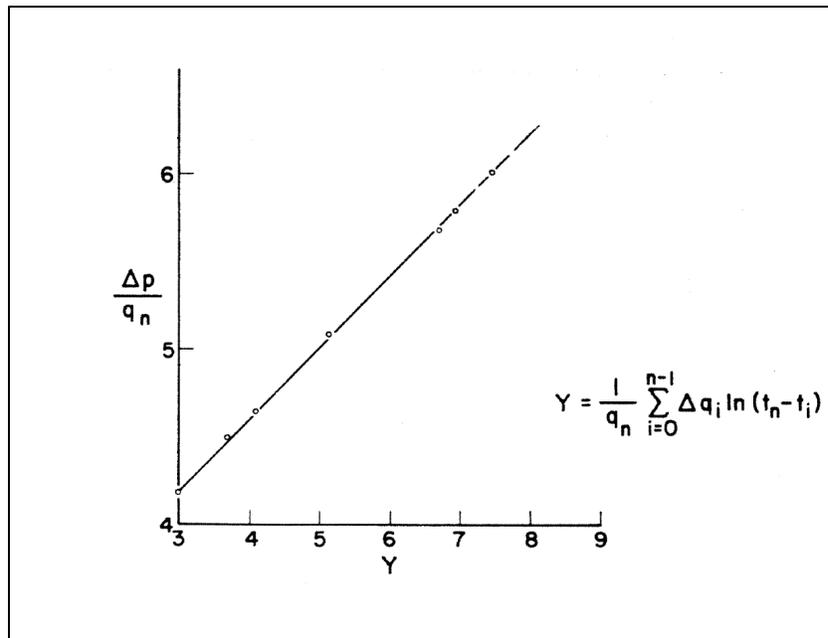


Fig. 32 Odeh and Jones (1965) approach for variable-rate, variable-pressure data

Table 2 Superposition time functions for each flow regime		
Flow regime	Nom	Superposition Time Functions
Radial	Super-log(t)	$\sum_{i=1}^n \frac{q_i - q_{i-1}}{q_n} \log(t_i - t_{i-1})$
Linear	Super-sqrt(t)	$\sum_{i=1}^n \frac{q_i - q_{i-1}}{q_n} \sqrt{t_i - t_{i-1}}$
Bilinear	Super-(t)	$\sum_{i=1}^n \frac{q_i - q_{i-1}}{q_n} \sqrt[4]{t_i - t_{i-1}}$
Spherical	Super(t) ^{1/4}	$\sum_{i=1}^n \frac{q_i - q_{i-1}}{q_n} \frac{1}{\sqrt{t_i - t_{i-1}}}$
Pseudo-Steady State	Super(t)	$\sum_{i=1}^n \frac{q_i - q_{i-1}}{q_n} (t_i - t_{i-1})$
Boundary dominated	t_{MB}	$\frac{G_{pn}}{q_n}$

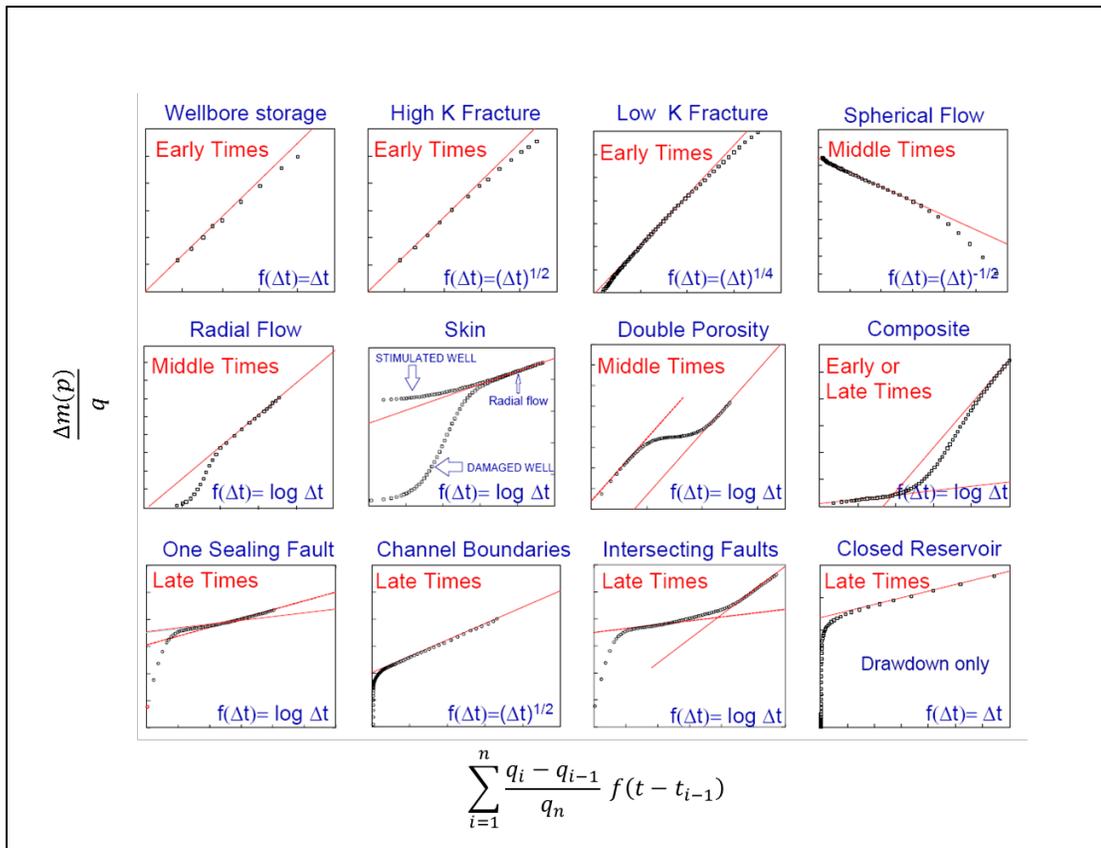


Fig. 33 Superposition time diagnostic plots used in Straight-line approach, from Gringarten (2006)

Table 3 Equations used in Straight-line approach				
Flow regime	y	slope	x	intercept
Radial	$\frac{\Delta m(p)}{q_g}$	$\frac{712 T}{k h}$	$super(\log(t))$	Const
Linear	$\frac{\Delta m(p)}{q_g}$	$\frac{200.81 T}{x_f \sqrt{k} h \sqrt{(\phi \mu c_t)_i}}$	$super(\sqrt{t})$	Const
Bilinear	$\frac{\Delta m(p)}{q_g}$	$\frac{982.69 T}{F_c \sqrt[4]{k} h \sqrt[4]{(\phi \mu c_t)_i}}$	$super(\sqrt[4]{t})$	Const
PSS	$\frac{\Delta m(p)}{q_g}$	$\frac{56.6 T}{V(\phi \mu c_t)_i}$	$super(t)$	Const

3.7 The nature of production data

The traditional production data is simply long-term data of continuous production history which might and might not be accompanied with surface pressure measurements, though in many cases it is assumed constant pressure production. The modern production data is a combination of the long term production rate data with continuous pressure measurements, provided by downhole gauges, in considerable variance in duration, quality and frequency, and accompanied with a multitude of shut-ins.

At a glance, production data may look like well-testing data as both of them are composed of rate and pressure data. Furthermore, the pressure transient and rate transient analyses share large part of the technical kernel. They share the same diffusion equations, assumptions, and limiting rules. In terms of flow regimes, both have an early-time flow that occurs while the transient wave is moving out into an infinite or semi-infinite acting reservoir which contains information about the reservoir. Early-time flow is then followed with late-time flow behavior dominates long-term data in which the reservoir reaches a state of pseudo-equilibrium governed by the mass balance.

Nevertheless, Anderson et al. (2006) defined the production data as "low-frequency low-resolution data", whereas pressure transient data as "high-frequency high resolution data". In comparison with pressure transient data, rate transient data analysis is poor in quality data with reduced quantity. One reason behind that is that the production data in nature is surveillance and monitoring data with little control and considerable variance occurring during the acquisition of the data. Another reason is that

the pressure measurements in well-testing are taken at extraordinary frequencies and accuracies, while the pressure data in the traditional production data analysis are either ignored or converted back to the formation from the surface measurements. Therefore, the methodology of the analysis and interpretation of the transient data depends on the operating conditions, duration, frequency and accuracy of the data. Table-4 lists the main aspects of the pressure transient and production data analyses.

In late 1990s and early 2000s the expense of Permanent Downhole Gauges (PDGs) started to be justified. Since then, PDGs have been introduced to the industry as a tool for monitoring and evaluating the reservoir performance. Beyond that objective, PDGs still provide considerable amount of data which if properly processed will help in the reservoir characterization. The combination of the long term continuous pressure with the production rate data, provided by PDGs, offered cost-free multiple pressure transient analysis and enabled for conducting the modern production data analysis. For the pressure transient analysis, the PDGs provide a continuous pressure history that includes large number of free-cost shut-ins. This multitude of shut-ins can be analyzed with pressure transient analysis to provide a global interpretation. Such global interpretation takes into account the change of the well conditions throughout its production life that may be caused by the natural damage, stimulation, evolution of flow conditions, multiphase production, and other reasons (Kamal, 2009). In addition to the free-cost shut-ins that the PDGs provide, the risk of the damaging the well usually involved in the conventional well testing operations is minimized. For production data analysis, the PDGs has taken the production data analysis a step further from the

classical methodology which was based on empirical and semi empirical use of the production rate data only to the modern production data analysis which has a solid theoretical basis.

Notwithstanding how much valuable are the data that PDGs offer, there are some hindering problems that prevent the fruitful use of such valuable data. The main two problems are concerned with the quantity and the quality of the data. With the high data acquisition rate of PDGs, there will be enormous amount of data points that cannot be analyzed in practical time frame. In addition, the acquired data are prone to errors such as outliers and noise which if not removed could result in inaccurate analyses. Moreover, the pressure transient analysis requires high frequency data for more descriptive diagnostic plots while in production data analysis lower frequency data is preferred to maintain the main picture and ignore the small fluctuations that make the data scatter on the diagnostic plots. Consequently, a tool is required to reduce the number of data, filter the outliers, identify the transients, and denoise the data while preserving the main features of the data.

Athichanagorn et al. (2002) developed a method processing and interpretation of long-term data acquired from permanent pressure gauges using wavelet transform. Their multistep procedure included outlier removal, denoising, transient identification, data reduction, and behavioral filtering. With this step the above mentioned problem has been sorted out, though the sampling is still an issue but not hindering hurdle.

Table 4 Common and different aspects of PTA and PDA , from Houzé et al. (2011)		
Aspect	Pressure Transient Analysis PTA	Production Data Analysis PDA
Theoretical bases	Same equations, superposition, analytical and numerical model. Same assumptions, limiting rules	
Major flow assumption	Constant rate	Constant pressure
Time range	Hours, days, sometimes weeks	Weeks, months, years
Periods of interest	Mostly shut-ins, clean productions possible	Production phases, but build-ups may be included
Data source	Well test measurements, formation tests, permanent downhole gages(PDGs)	Measured/allocated production surface pressure readings, permanent downhole gauges(PDGs)
Data frequency	High to very high	Average to low
Data quality	High	Average to low
Reservoir areas of interest	Whatever volume of investigation during the test and /or the shut-in	Well or group drainage area
The good old plots	MDH, Horner	Arps
The good old type-curves	Mckinley, Gringarten	Fetkovitch
Modern diagnostic plots	Log log with Bourdet derivative	Log log & superposition specialized plots, Blasingame, Wattenbarger, Agarwal type-curves with Integral and Bourdet derivatives
Main flow regime of interest	Infinite Acting Radial Flow	Pseudo Steady State(PSS) / Boundary-dominated flow
Main deliverables	kh & skin	Drainage area, original oil inplace (OOIP), original gas inplace (OGIP)
Diagnostic capability	High to very high	Average to low
Long term validity	Average to low	High to very high

At this stage the production data started having the theme of long-term continuous pressure and rate data. Consequently, the theme of the analysis and interpretation changed toward variable-rate, variable-pressure approach. Ilk (2005) categorized the methods dealing with variable-rate data into three main categories:

- Superposition and Convolution
- Rate Normalization and Material Balance Deconvolution
- Deconvolution

With the principle of superposition in time and rate normalization most the analytical models developed in pressure transient analysis can be used. That enabled the advanced use of production data. Since then, the production data analysis received appreciation and favor. However, the diagnosis part of production data analysis is still challenging. There is no established full checklist of all pitfalls and challenges in production data analysis. The main reason is nature of the production data is surveillance and monitoring data with little control and considerable variance occurring during the acquisition. That yields a lot of inconsistencies makes the data hard to diagnose. Those inconsistencies differ from cases to case in the effect and severity, whereas in many cases that depends on the expertise of the analyst. Anderson et al. (2006) provided a comprehensive review of the production data challenges and the techniques that can be used to diagnose the reservoir model and assess the reservoir conditions. Table-5 shows their list of the common production data problems and challenges.

Issue	Influence/ Severity
● <i>Pressure:</i>	
—No pressure measurement(s)	High
—Incorrect initial pressure estimate	High
—Poor $p_{tf} \rightarrow p_{wf}$ conversion (models)	Moderate
—Liquid loading: effect on $p_{tf} \rightarrow p_{wf}$ conversion	Moderate
—Incorrect location of pressure measurement	Very High
● <i>Flowrate:</i>	
—Rate allocations (potential for errors)	Moderate
—Liquid loading: effect on gas flowrate	Moderate
● <i>Well Completion:</i>	
—Zone changes: new/old perforations	Very High
—Changes in the wellbore tubulars	High
—Changes in surface equipment	Mod./High
—Stimulation: hydraulic fracturing	High
—Stimulation: acidizing, etc.	Moderate
● <i>General:</i>	
— Reservoir properties (ϕ, h, r_w, c_f, c_t , etc.)	Moderate
— Oil properties: B_o, R_{so}, μ_o, c_o , etc.	Moderate
— Gas properties: γ_g, T, z (or B_g), μ_g, c_g , etc.	Moderate
— Poor time-pressure-rate synchronization	Mod./High
— Poor time-pressure-rate correlation	Very High

Production data as acquired are usually prone to errors and inconsistencies starting from outliers and noise up to the problems with rate and pressure synchronization and consistencies. Nobakht and Mattar (2009) reviewed many of the operating problems observed in practice, and discuss ways and patterns for recognizing them as operating issues rather than reservoir phenomena. They specified four areas of investigation in production data diagnostics that are:

- Outlier removal
- Liquid loading in the wellbore
- Single phase flow in the reservoir
- Consistency (correlation) between rate and pressure

They listed some of the commonly encountered issues in production data that are worse than others and cause inconsistencies. Some of those issues are the missing flowing pressures, missing flow rates, rate or pressure averaging, wrong initial pressure, liquid loading, wrong pressure source and flow path, wrong production rate allocations, wrong water production rate data, significant increase in water-gas ratio, operational changes, wrong location of pressure gauge, and wrong rate and pressure synchronization.

Beside the production data issues, another problem arises which is the limited diagnostic techniques in practice for the production data analysis. Moreover, within these techniques that are applied many are either observation-based approaches or rules of thumb. Although the type-curves are considered as analysis method, their virtue in the diagnostics is much superior to the analysis. There some other tools that are used in production data diagnosis such as rate integral, derivative, and integral derivative. The rate integral is useful in removing the scatter in noisy data, but in many cases it dilutes the reservoir signal. The derivative it is good in amplifying the reservoir signal but it amplifies the noise as well. The rate integral derivative is in between, however, it still suffers with noisy data. The technique of the diagnosis is always dependent on the operating conditions, duration, frequency and accuracy of the data. Nonetheless, the nature of production data is noisy and scattered with fluctuations in rate and pressure.

Part of the diagnostic techniques used for production data analysis is the straight-line approach using superposition-time specialized plots. The superposition-time specialized plots are good in diagnosing the different flow regimes that might present in

the reservoir. Despite of that fact, superposition and material balance time functions can also be biased and misleading the engineer to the wrong diagnosis and eventually to the wrong results. For example, rate outliers or liquid loading phenomena will hard to be distinguished from pseudo-steady state flow regime if the superposition and material balance time functions are used as time plotting function. This observation is uniquely tied with the rate fluctuations which make the superposition time function erratic.

Although production data is subjected to many problematic issues that affect the data behavior, the diagnosis stage in this work will be only focusing on the expected reasons of biasing the superposition time specialized plots. This topic will be discussed thoroughly in the next two chapters. For other production data issues Anderson et al. (2006), Nobakht and Mattar (2009), and Almarzooq (2010) should be reviewed.

3.8 Summary

In this chapter the principle of superposition as well as it is applications have been briefly reviewed. In addition, the principle of type-curve analysis, influence function, history matching, and straight-line approach are uniquely tied to the principle of superposition. Those techniques have been also reviewed from the prospective of their relation to the superposition principle. Finally, a brief discussion on the nature of the production data was given. The purpose of this chapter is to prepare the reader for the next chapter, which will be discussing how the superposition can cause the bias in the rate transient analysis.

CHAPTER IV

SUPERPOSITION TIME BIAS IN THE RTA

Superposition time functions with the help of rate normalization established the firm ground of analyzing variable-rate, variable-pressure production data. One of the tools which are based on that ground is the straight-line approach using superposition-time specialized plots. The superposition-time specialized plots are good in diagnosing and analyzing the different flow regimes that might present in the reservoir. However, superposition time functions can be also biased and misleading in many cases. For example, rate outliers or liquid loading phenomena will hard to be distinguished from Boundary-Dominated-Flow (BDF) regime if the superposition or material balance time functions are used as plotting function. In such cases, if such deceiving presence of BDF is believed, the Expected-Ultimate- Recovery (EUR) will be cut way down. Furthermore, based on these results and with the upsurge of energy needs there might be some other costly decisions will be taken such as refracing or re-stimulating the well especially in tight formations.

The superposition time functions behavior has been a subject of skepticism for long period of time. Some observations have been made on that subject by different authors such as Jargon and Van Poolen (1965), Dake (1978), Streltsova (1988), Anderson et al. (2006), Cvetkovic (2008), Nobakht and Mattar (2009), Anderson et al. (2010), and Nobakht et al. (2010). Those observations were uniquely tied to the rate fluctuations that make the superposition time function erratic. Recently the subject has received some attentions by Moghadam and Mattar (2011) and Bachman et al. (2011),

though, the work shown in the literature served as big warns of the problem other than resorts.

Since the superposition time functions involve rate as essential constituent, the superposition time is affected greatly with rate issues. Calculated data points tend not to be sequential with superposition time but do tend to fall on a straight line represented by the superposition function chosen. Production data of shale gas wells are usually subjected to operating issues that yield noise and outliers. Whenever the rate data is noisy or contain outliers, it will be hard to distinguish their effects from common regime if the superposition time functions are used as plotting time function on log-log plots. Such deceiving presence of these flow regimes will define erroneous well and reservoir parameters. Based on these results and with the upsurge of energy needs there might be some costly decisions will be taken such as refracting or re-stimulating the well especially in tight formations.

This chapter will be discussing the main reasons that cause the bias in superposition-time specialized plots. Synthetic constant-pressure gas production depletion cases will be constructed using numerical Laplace inversion algorithm developed by Stehfest (1970) . Random noise and fluctuations will be then introduced to the previously generated cases and the behavior of the induced points on the superposition-time diagnostic plots will be observed. Finally, field cases will be analyzed accordingly.

4.1 The main problems

As any other method, the straight-line approach has its own limitations. The main limitation, as most of transient diagnosis and analysis tools, is the non-uniqueness. The non-uniqueness in straight-line approach is the inability to identify with confidence the real straight line which represents the flow regime of interest. In many cases an apparent straight-line does not prove the existence of a specific flow regime. That is mainly because of the nature of the production data. Production data in nature is surveillance and monitoring data with little control and considerable variance occurring during the acquisition of the data. Furthermore, the shale gas wells usually are subjected to liquid loading, multiphase flow, interference, or severe flow rate fluctuations. Consequently, the production data are much noisier than the pressure transient data.

Another aspect is the non-linearity. As any other mathematical principle, superposition works under certain conditions and fails at others. Most of the analytical solutions developed for transient analysis are derived from linear differential equations. The superposition principle in nature is a property of the linear differential equation. Thus, it is not valid whenever nonlinearity is introduced to the system. Consequently, the principle is not applicable mathematically to gas flow (without pseudo-corrections), multiphase flow, pressure dependent cases, and reservoir model changes.

For decades the superposition-time specialized plots are used for flow regimes identification. In practice, it is implemented by presuming the dominance of a certain flow regime and using its superposition time function for constructing the diagnostic plot. If the flow regime of interest is in reality dominant, then a straight line will be

shown during the time of its dominance. For instance, the BDF superposition-time function (material balance time) has been commonly used as default option on the diagnostic plots of production data. That is because it is believed that most of the production data is under of the influence of depletion. Nevertheless, for complete interpretation model, diagnostic plots must be constructed for all expected flow regimes.

In this work the straight-line technique will be modified and implemented on the log-log plots other than the Cartesian. Such modification was preferred for two reasons. The first is to have a unified theme since the log-log plot has been already used with the help of material balance time (superposition time function for BDF) as a diagnostic and analysis plot. The second reason is that the log-log plot is unique in exhibiting the linear relationships as straight line with unit slope whenever the relationship is satisfied by the occurrence of the flow regime of interest.

The implementation of superposition requires a priori knowledge of the flow model. Therefore, the way in which it is implemented may be of a serious risk. That is because the results obtained from the analysis may be affected by the analyst choice of the model. In other word, the analyst will plot what he needs to see. Fig.34 shows the production rate of well W1-1, a field case from Woodford shale. The superposition-time specialized plots of this well are shown in Fig.35 through Fig.40.

All of the plots are showing straight lines which can be an indication of the presence of all expected flow regimes (radial, linear, bilinear, and boundary dominated flow). Nonetheless, that all the plots show a straight line at late times, where is impossible for more than one flow regime to occur at the same period of time. Another

contradiction, is that the well is hydraulically fractured horizontal well where the radial flow is not expected or at least not at early production life. Moreover, hydraulically fractured tight gas wells are known with the domination of transient linear flow that may last for over 10 or 20 years in some cases (Wattenbarger et al., 1998). Such contradictions pose a very important question which is “*Are these plots reflecting the real reservoir behavior or it is a bias?*”

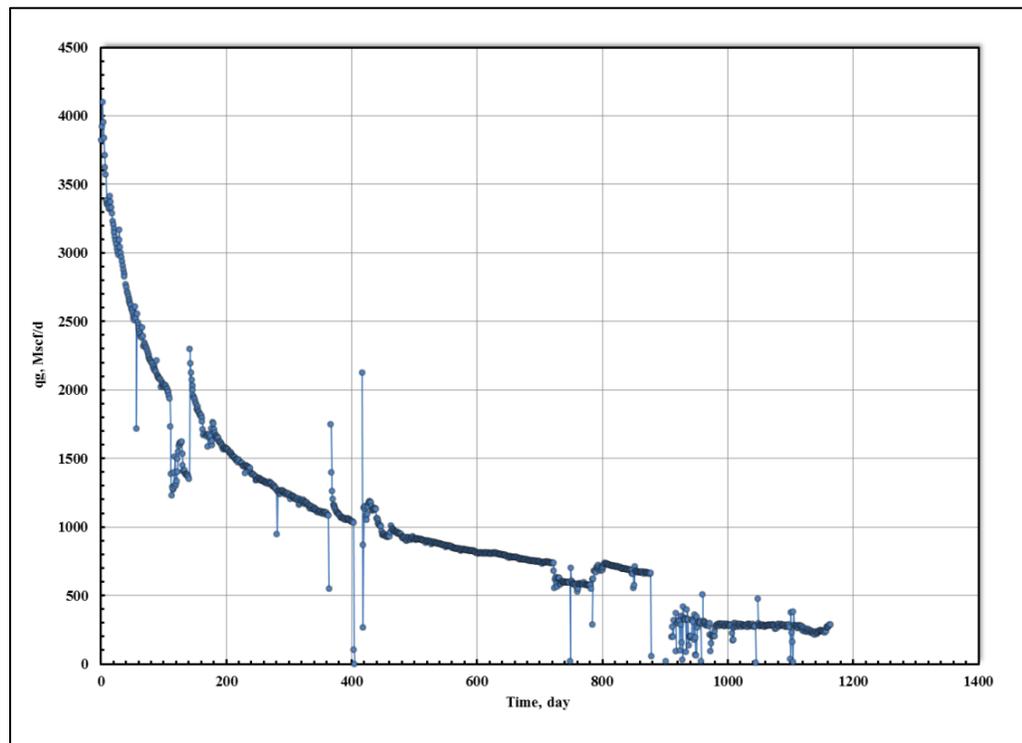


Fig. 34 Well W1-1 production data

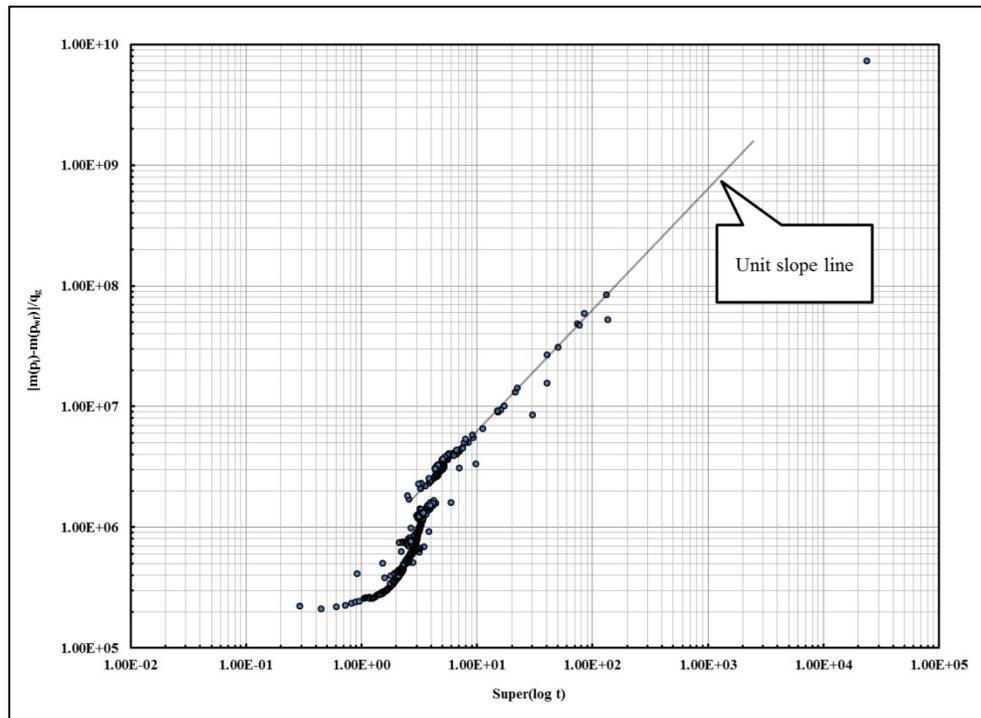


Fig. 35 Superposition-time specialized plot for radial flow

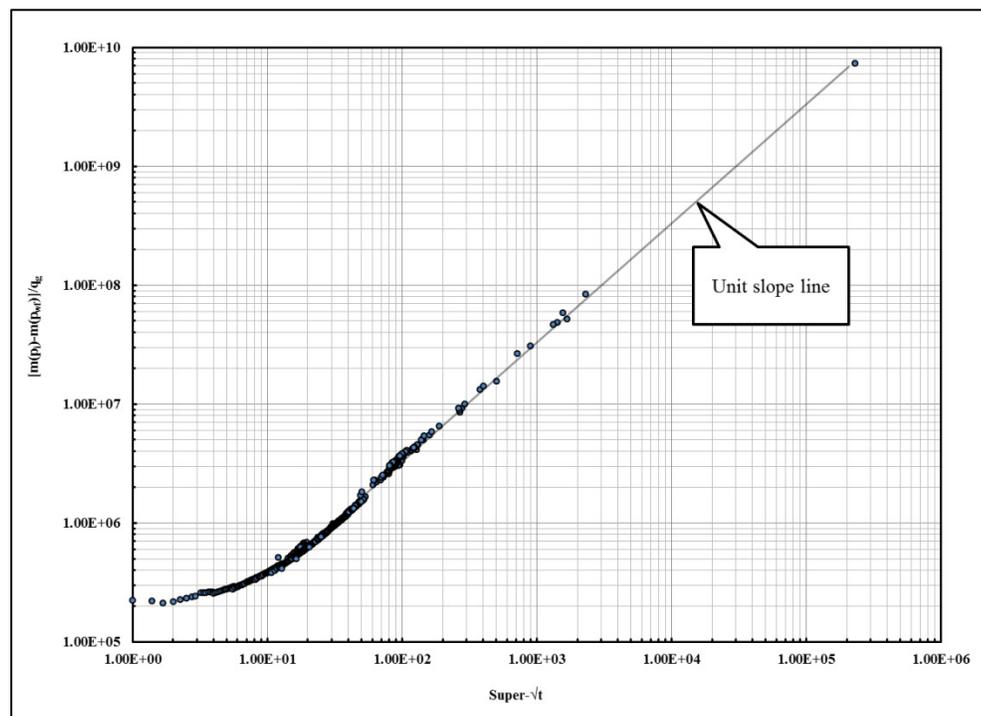


Fig. 36 Superposition-time specialized plot for linear flow

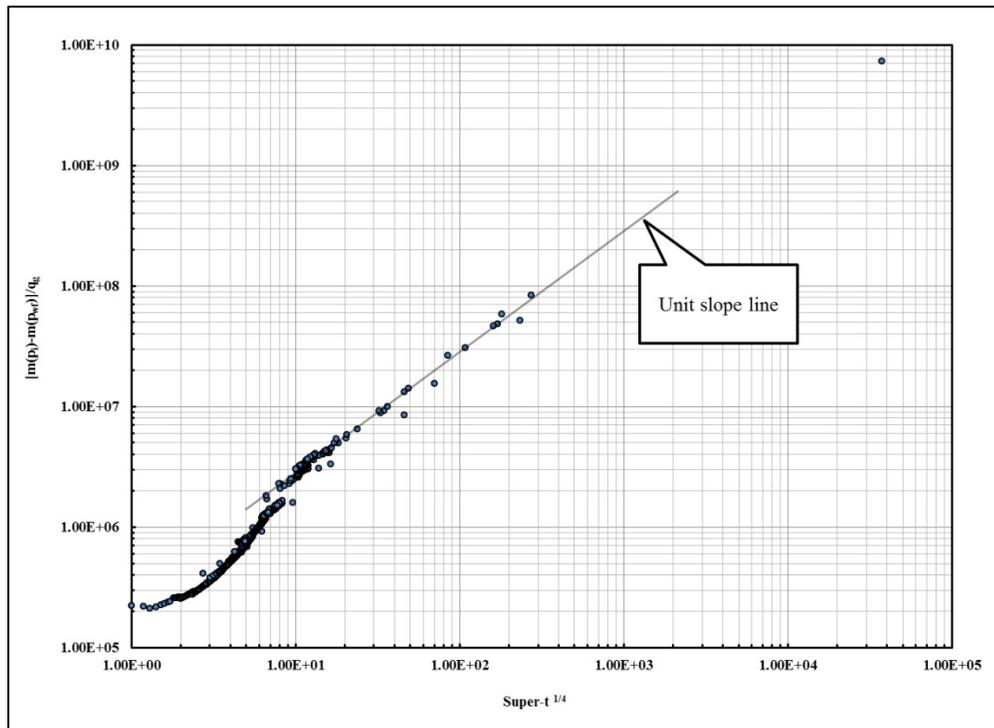


Fig. 37 Superposition-time specialized plot for bilinear flow

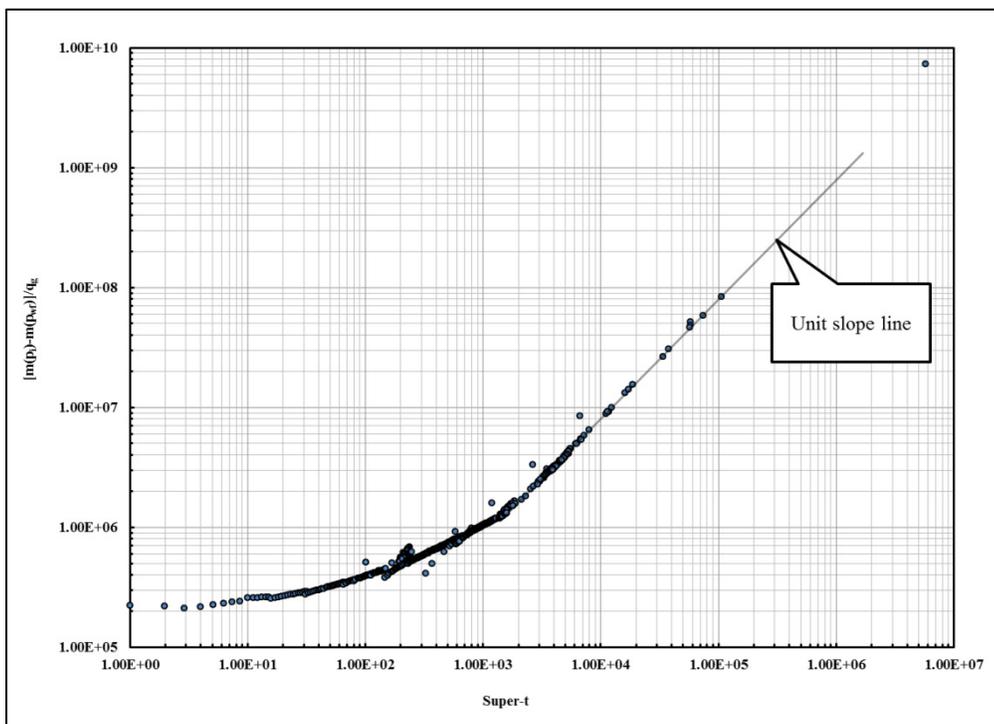


Fig. 38 Superposition-time specialized plot for Boundary dominated flow

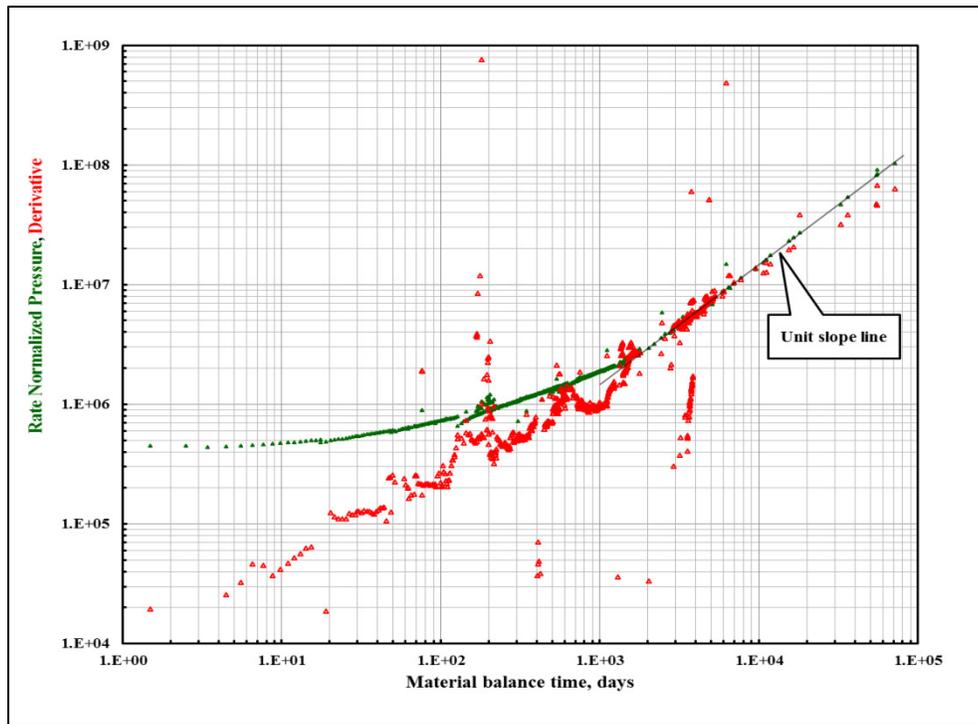


Fig. 39 Rate normalized pressure log-log diagnostic plot

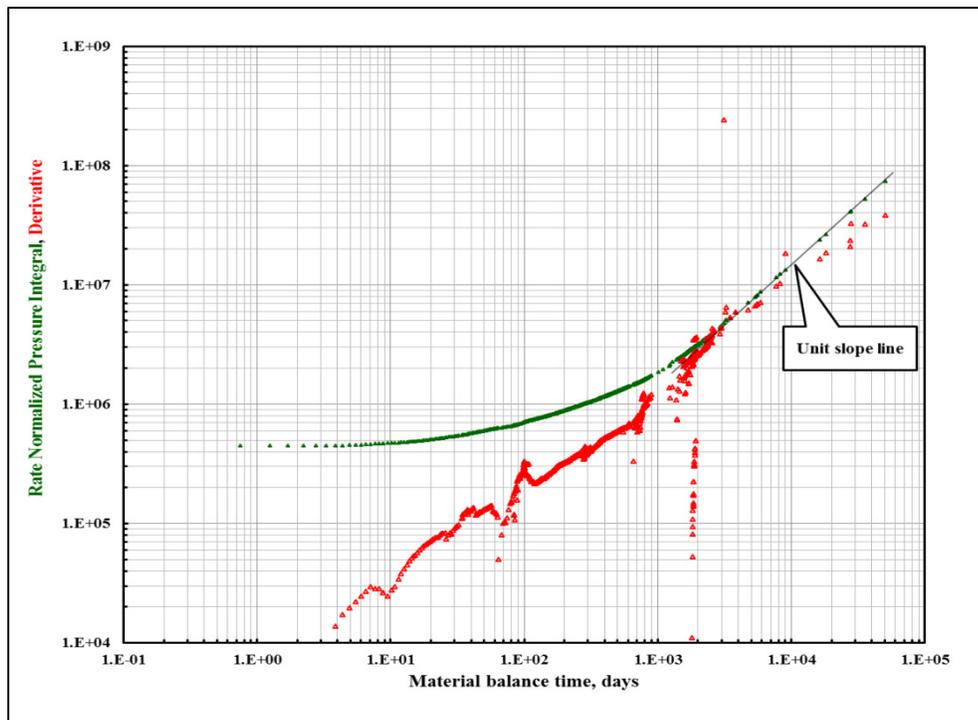


Fig. 40 Rate normalized pressure integral log-log diagnostic plot

From the case of W1-1 it is obvious that superposition-time specialized plots have a questionable behavior. In such case, production data analysis can be easily misinterpreted if only the specialized plots with superposition time functions are relied on as diagnostic tools. If these plots are really having the tendency to bias the analysis, the engineer will be deluded to the wrong diagnosis and eventually to the wrong results.

The following is a summary of the main reasons due to which the superposition time can be a source of bias in rate transient analysis:

- **Plotting nature:**
 - Nature of the time function (e.g. material balance time)
 - Nature of the plot (e.g. log-log plot)
- **Flow rate:**
 - Outliers
 - High scatter / fluctuations
 - Liquid loading / well clean-up
 - Rate allocation / averaging / prorating
- **Superposition principle**
 - Non-linearity (multiphase)
- **Change in the model**
 - Recompletion
 - Stimulation / Refracting
 - Interference

Each of these problems will be discussed and demonstrated with the help of the straight-line approach as diagnosing tool.

4.1.1 Plotting nature

For long time the drastic rate changes were thought as the only cause of the suspicious behavior in the analysis and diagnostic plots. Nonetheless, the nature of both the plot and the superposition time function are crucial in representing the data. For example, the plot of $\log(q)$ vs. $\log(t_{MB})$ makes any drop in the flow rate stretched on a negative unit slope line as shown in Fig.41 and Fig.42.

For any point $\log(q)$ vs. $\log(t_{MB} = Q/q)$ and any divisor (ϵ) due to drastic rate drop, the new point will be $\log(q/\epsilon)$ vs. $\log(Q/(q/\epsilon))$ and the slope between these two points will be:

$$slope = \frac{(\log(q) - \log(\epsilon)) - \log(q)}{\left(\log\left(\frac{Q}{q}\right) + \log(\epsilon)\right) - \log\left(\frac{Q}{q}\right)} = \frac{-\log(\epsilon)}{\log(\epsilon)} = -1 \dots\dots\dots (12)$$

Despite of the reason that makes the drastic rate drop, the nature of the plot and the time function are also of great importance for such spurious representation of the data. The log-log plot has the feature of expanding the low end while contracting the high end of the scale. Furthermore, it has the feature representing the power-law relationships with straight line. The log-log plot used for production data analysis has the rate in the vertical abscissa and the superposition time function on the horizontal axis. All the superposition time functions have the reciprocal of the instantaneous rate at each time step. The presence of the rate on both axes makes any drop in the rate stretched on a line of negative unit slope. When the rate drop dominates in both axes, such as in the case of constant or nearly constant-pressure production, the spurious unit slope points become observable on the log-log diagnostic plot.

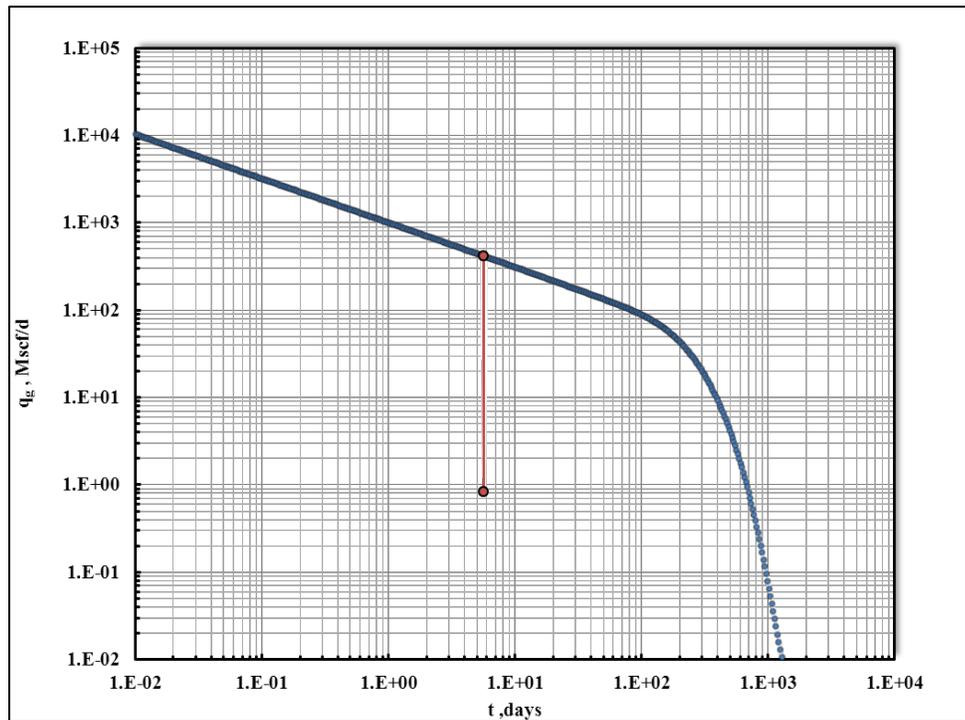


Fig. 41 Synthetic production data with the of drastic rate drop

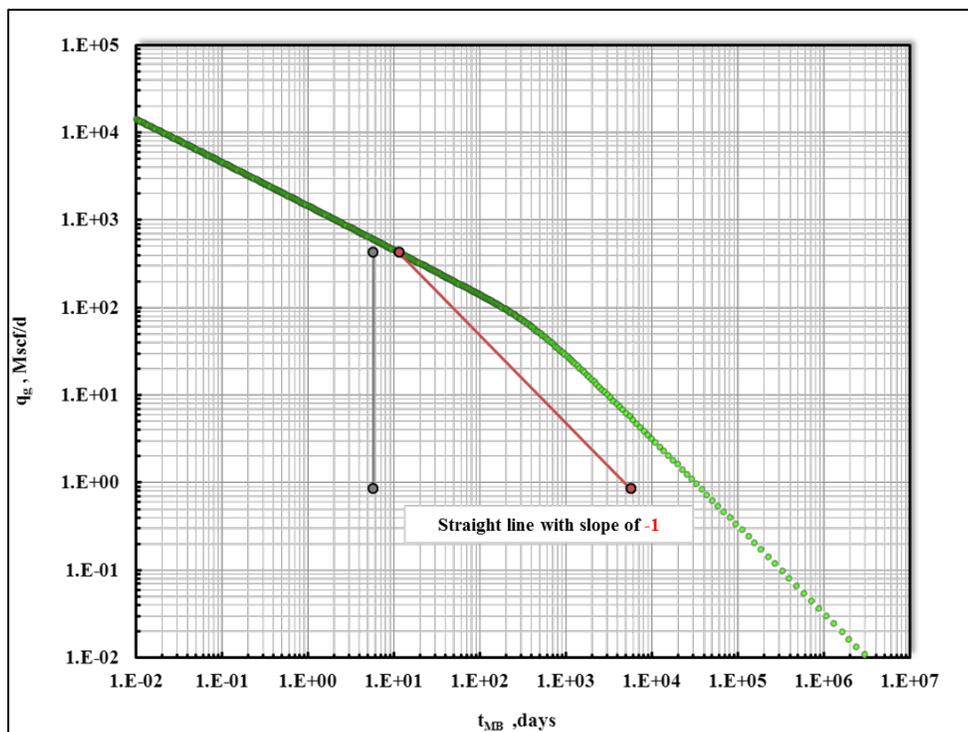


Fig. 42 The effect of drastic rate drop on the log-log plot

4.1.2 Flow rate issues

Flow rate data is commonly subjected to issues that cause inconsistencies. All of the rate issues related to the previously mentioned bias are centered on two main resulting effects; the noise and outliers. Noise is usually scattered data points around the main trend of the data, whereas outliers are points of data located far away from the main data trend. Some of the rate issues that cause the noise and outliers are the rate averaging, unstable operating conditions, liquid loading, flushing after shut-ins, sudden decline due to interference, production rate allocations, and missing flow rates. Out of importance are the following issues:

- *Rate averaging*

In some cases the rate is measured at high frequency. For reporting purposes, data storage capacity, calculation requirements, and consistency the rate data may require averaging to lower frequency quantities. For example, the well may produce for few hours a day and then the rate is averaged to daily basis. On daily scale, that day will be shown as a drastic drop compared to its neighboring points. On the log-log diagnostic plots, the point will be stretched on a negative unit slope line as shown in Fig.43 and Fig.44. Depending on the variance in rate data during that period, rate averaging may result in smooth, erratic, or meaningless numbers.

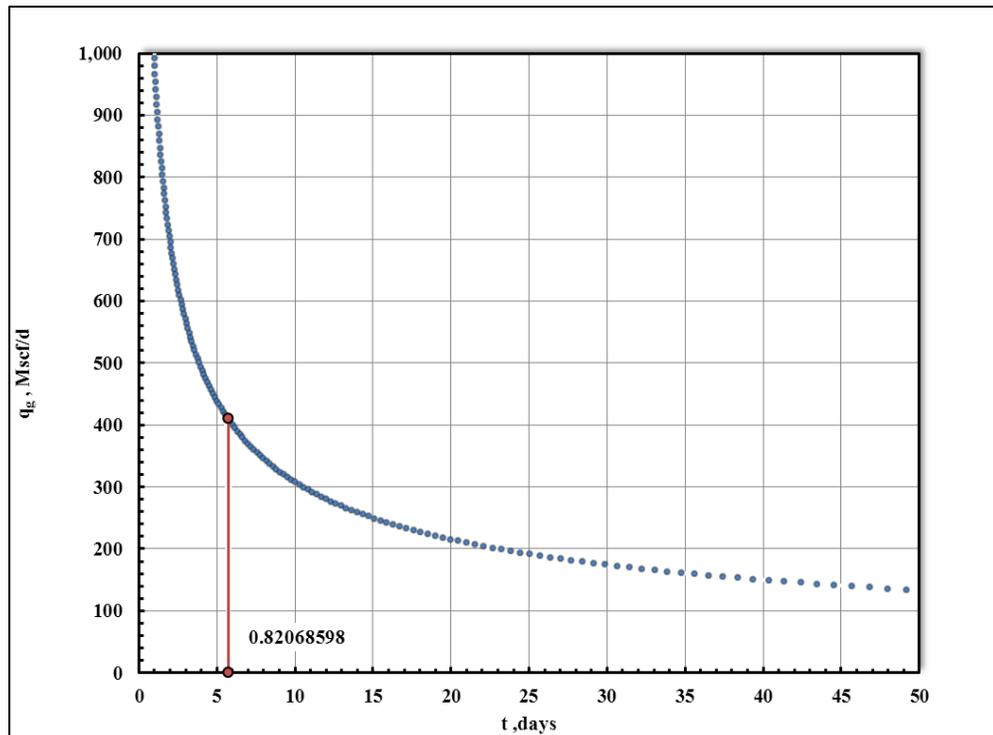


Fig. 43 Synthetic data with the effect of averaging the production of 236 Mscf during 12 hrs into daily rate

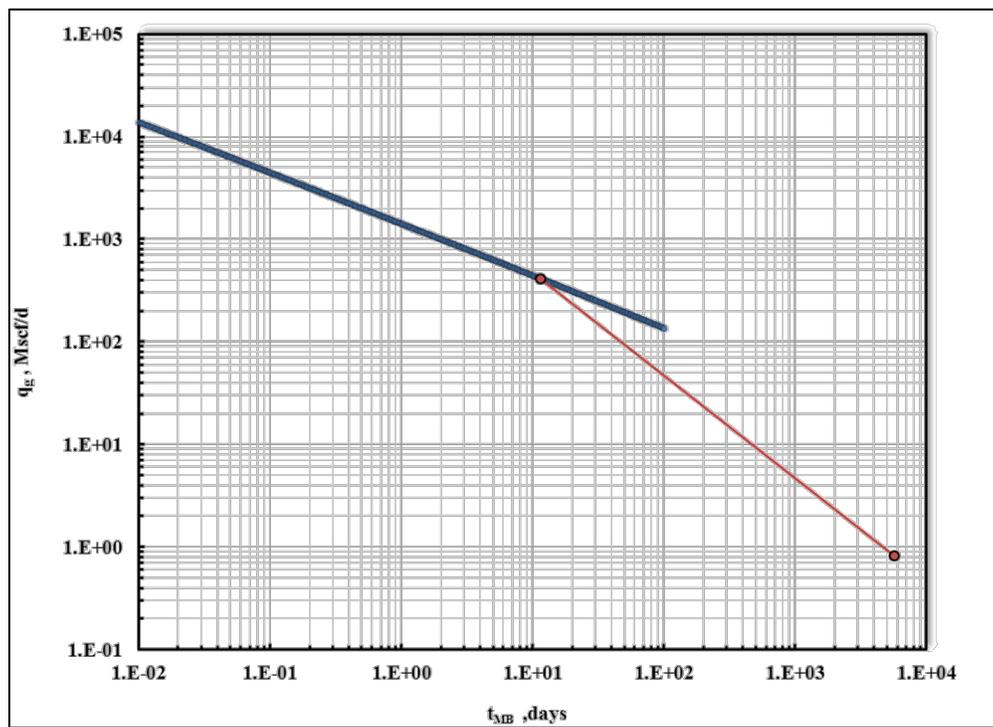


Fig. 44 The effect of rate averaging on the log-log plot

- *Unstable operating conditions*

Instability in the flow in the wellbore or any other instability in the operating conditions may result in considerable variance in rate data. For example, liquid level rises and falls in the wellbore and liquid slugs moving upward at fairly regular intervals and cause severe rate fluctuation and pressure variations. Another example is the market demand or any other technical reason that dictates producing at high or low production rates at different time intervals. As a result, the rate and pressure data will be highly scattered.

- *Liquid loading*

Liquid loading is a common problem in shale gas wells. Usually, it is attributed to liquids accumulating in the wellbore when the well has insufficient energy to carry the standing liquid column to the surface. This could be due to dropping reservoir pressure, oversized production tubing or an increase in the well's surface pressure. The problem causes a sudden and severe drop in the rate usually at late production times when the reservoir energy experiences significant depletion. Because it is characterized with severe rate drop at later time, liquid loading is at the top of the candidates that potentially deceive the analyst as spurious points forming unit slope line log-log diagnostic plots as shown in Fig.45 and Fig.46.

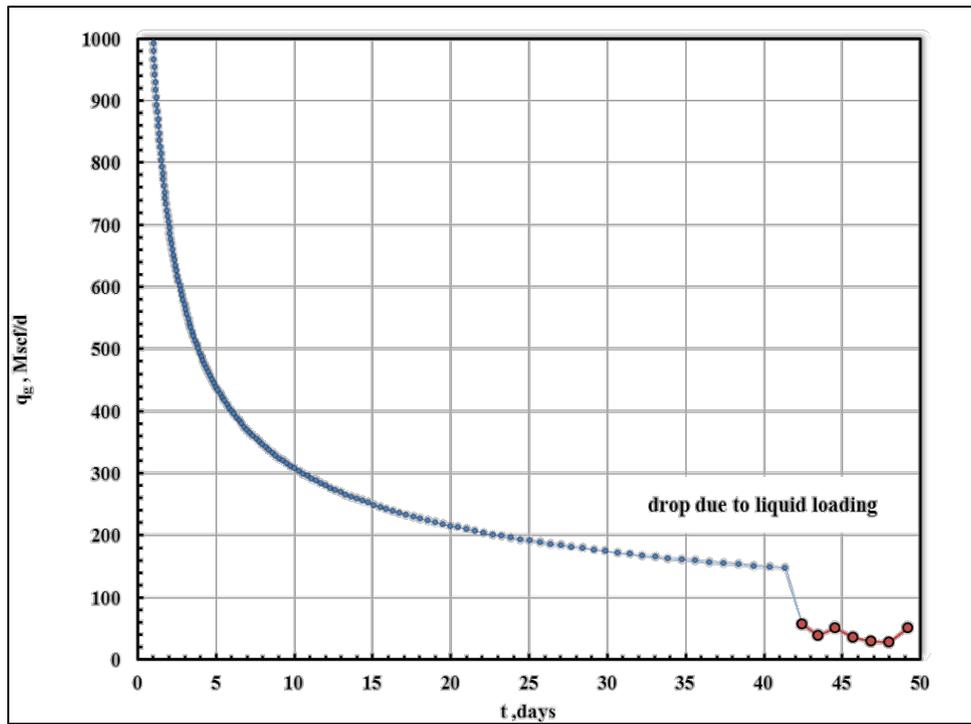


Fig. 45 Synthetic production data with the effect of liquid loading

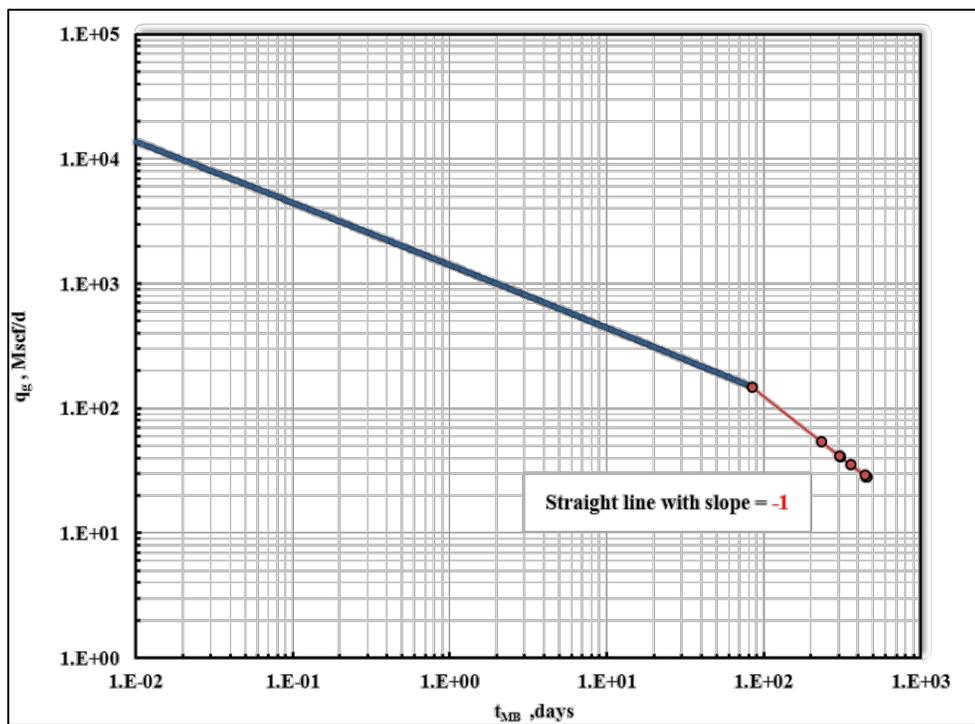


Fig. 46 The effect of the liquid loading on the log-log plot

- *Production rate allocations*

Rate allocation may satisfy the manifold volume calculations for flow rate proration, but it may not represent the actual performance of each well individually. Moreover, the rate allocation is always based on infrequent productivity tests that are assumed representative over a period of time. If the rate is allocated on a certain day at very low value compared to its neighboring points, it will be stretched on a negative unit slope line on the log-log diagnostic plots.

- *Shut-in effects*

It is commonly noticed in tight gas wells that frequent shut-ins cause severe scatter in the rate data. Following to each shut-in there is always production flush behavior. The magnitude and the drastic change in the rate are function in the reservoir energy, fluid compressibility, the durations of the shut-in, and the flowing period after it. The characteristics of this phenomenon are deeply discussed by Hale et al. (1989). However, Helmy and Wattenbarger (1999) noted that shut-ins effects hinder the analysis using conventional methods such as specialized plots and deconvolution. That is because of the drastic rate changes that accompany the flushing behavior.

- *Interference effects*

The interference effects would either cause the production rate to increase or decrease in adjacent wells whenever a new well is being hydraulically fractured. Although the low permeability makes it unlikely to observe interference in the shale

gas formation, different evidences indicated the occurrence of interference in tight gas wells. Those evidences are such as micro-seismic, chemical tracer, and the production rates of all the wells within the area on same calendar time cases. The interference between the wells can be attributed to many reasons such as short distance between the wells, interference of propagating hydraulic fractures, communication channels due to natural fractures, and maximum stress direction in the field. However, it is noticed that the interference is characterized with sudden drop in the rate and change in the rate of the decline. If the interference occurs at late time in the well production life, it will be potentially deceiving the analyst as spurious points forming unit slope line. Khan (2011) covered the interference effects and diagnosis in more detailed. Fig.47 shows a field case of six adjacent horizontal shale gas wells. The first producing well is W3-1 while the other came to production subsequently. Fig.48 shows that whenever a new well start production, W3-1 production drops suddenly to a new trend. The shut-ins and interference effect on log-log plot of well W3-1 are shown in Fig.49.

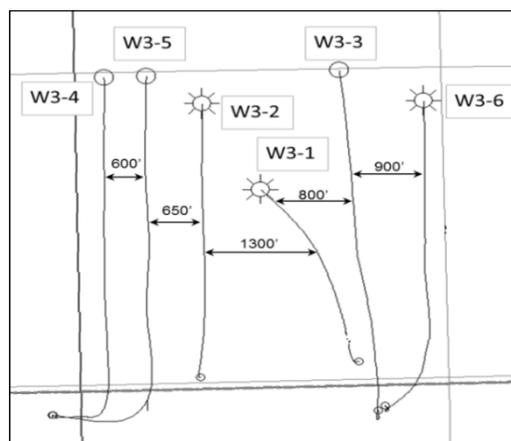


Fig. 47 W3 section with six adjacent wells

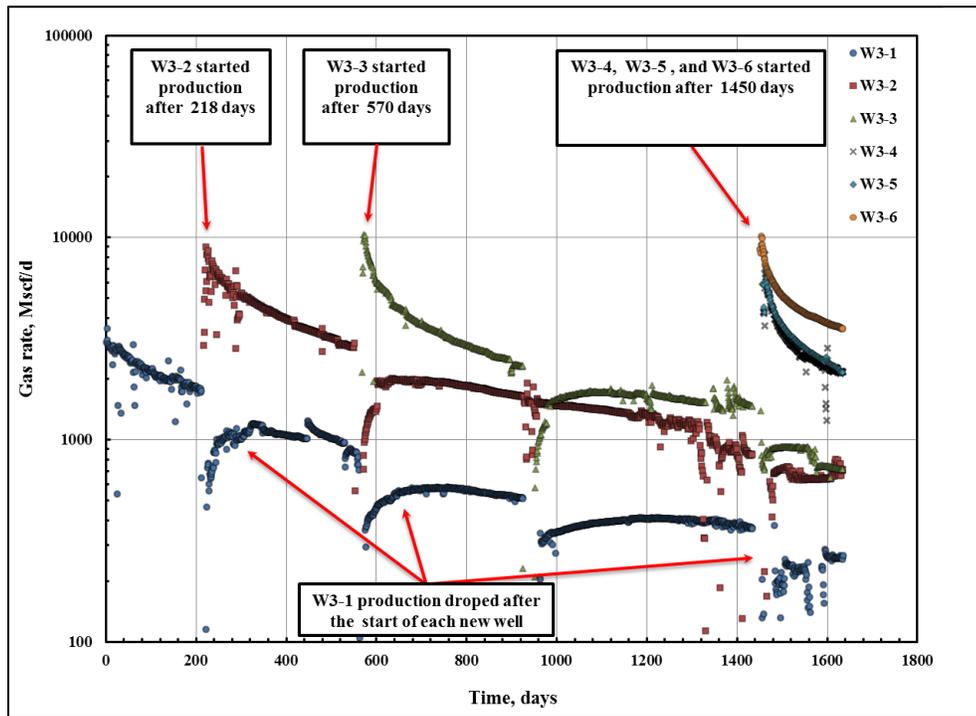


Fig. 48 The effect of interference on the production of well W3-1

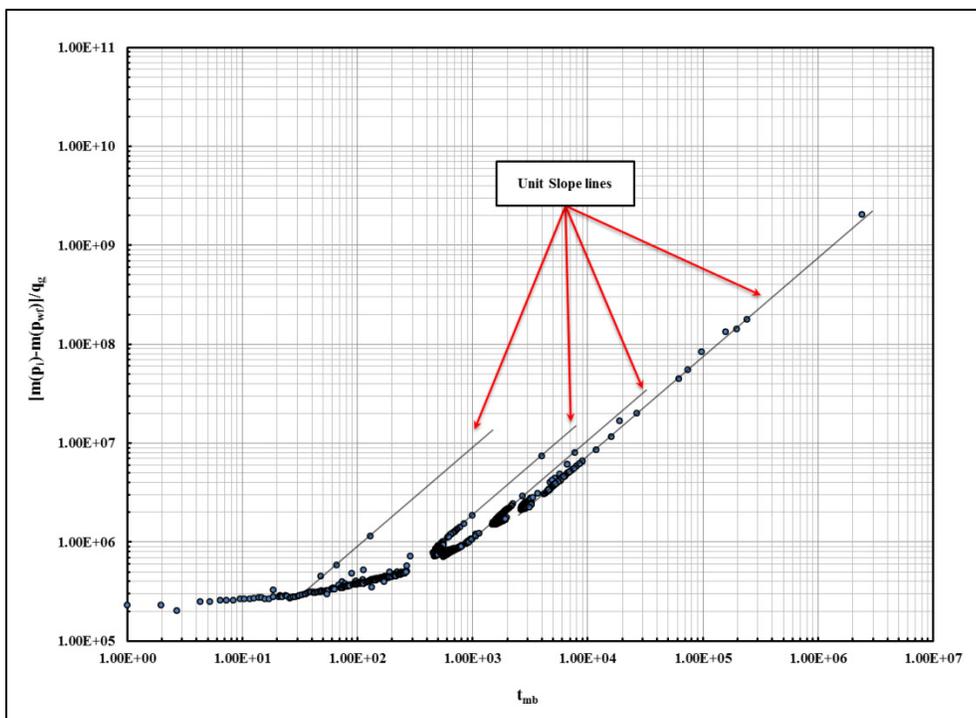


Fig. 49 The effect of the shut-ins and interference on the log-log plot

4.1.3 Superposition principle

Another aspect is the non-linearity. As any other mathematical principle, superposition works under certain conditions and fails at others. Most of the analytical solutions developed for transient analysis are derived from linear differential equations. The superposition principle in nature is a property of the linear differential equation. Thus, it is not applicable mathematically whenever nonlinearity is introduced to the system such as gas flow (without pseudo-corrections), multiphase flow, pressure dependent cases, and reservoir model changes.

4.1.4 Change in the model

Superposition, convolution, and deconvolution principles are mathematically attached to the Duhamel's theorem. Anderson et al. (2006) pointed out that Duhamel's integral becomes invalid in the physical sense if the reservoir model changes during the production sequence. They have mentioned examples of the scenarios where that invalidate the application of Duhamel's integral that can be concluded in:

- Well completions changes such as tubular or changes in the wellbore flow path
- Zonal changes such as flow path or reservoir model changes
- Liquid loading and water production due to multiphase flow.

Furthermore, the principle is limited to the systems in which one well perturbation appears (i.e., no nearby wells that cause an interference). Basically, that is because different solutions of different differential equations cannot be superimposed. Moreover, it resides in the consistency of the measured pressure and rate data.

4.2 Material balance time inadequacy in converting linear flow solutions

The other limitation of the superposition time application is in the conversion of the constant-pressure linear flow solution to its counterpart constant-rate solution. Although the material balance time provides the conversion of the constant-pressure production case to an equivalent constant-rate case, the conversion is incomplete in the case of linear flow regime as shown in Fig.50 and Fig.51.

Unlike the radial flow regime, the material balance time application to linear flow regime shows a pronounced difference and can be quantified. In terms of horizontal shift the material balance time should be multiplied by 1.233. On the other hand, the vertical shift of the normalized rate is about 1.41. Fig.52 shows both vertical and horizontal differences. However, these shifts apply only during pure transient linear flow. Anderson and Mattar (2003) investigated the applicability of the material balance time during the linear flow regime in rate transient analysis. They have noted that unless the production data being analyzed has very high resolution, the difference between the two solutions would certainly be considered insignificant.

Peter et al. (2011) reviewed the formulation, type-curve, specialized plots and superposition time that are used to analyze transient linear flow. They compared the conversion using both material balance time and linear superposition time. Their conclusion was that the material balance time preserves the characteristic shapes of the both transient linear flow and BDF, whereas the linear superposition time shows the BDF as transient linear flow as shown in Fig.53. Therefore, they preferred the use of material balance time when analyzing variable-rate data.

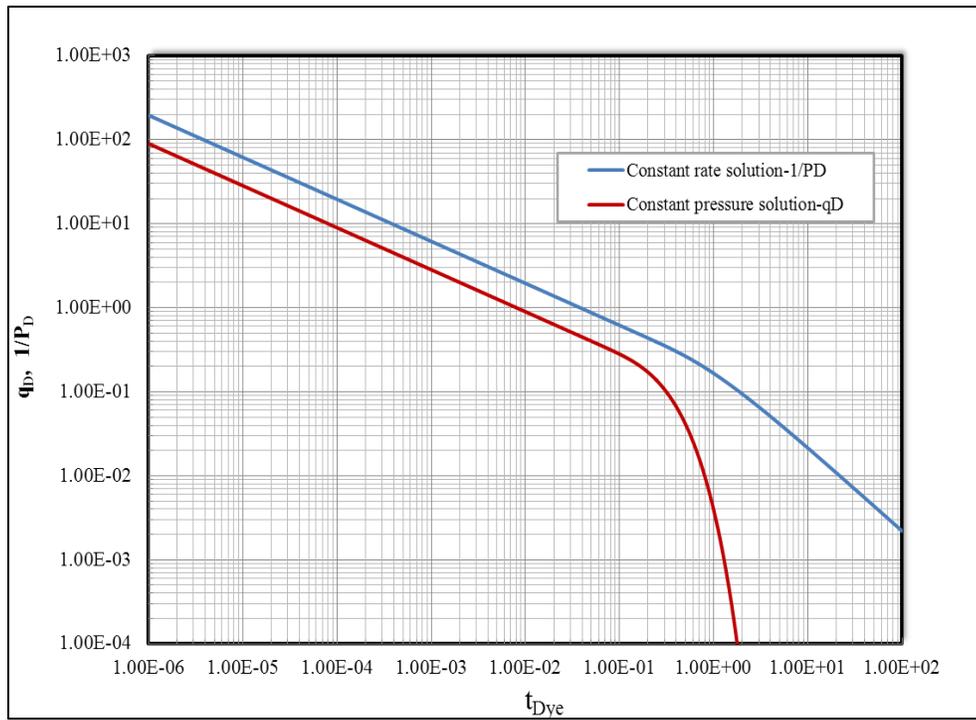


Fig. 50 Linear flow solutions, form Wattenbarger et al. (1998)

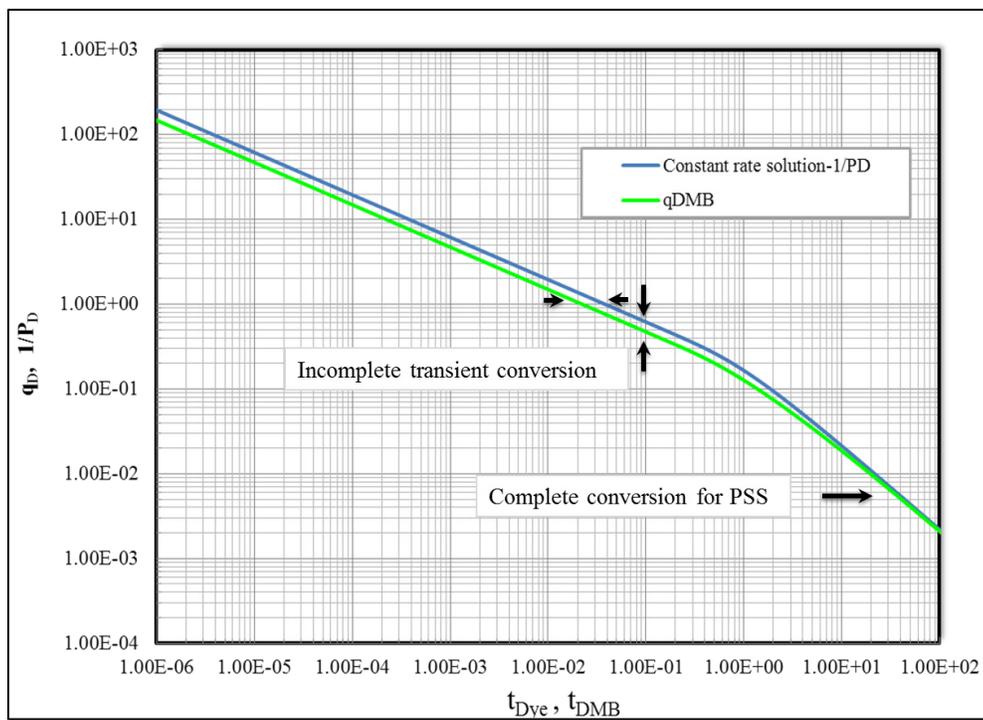


Fig. 51 Material balance time inadequacy in converting linear flow solutions

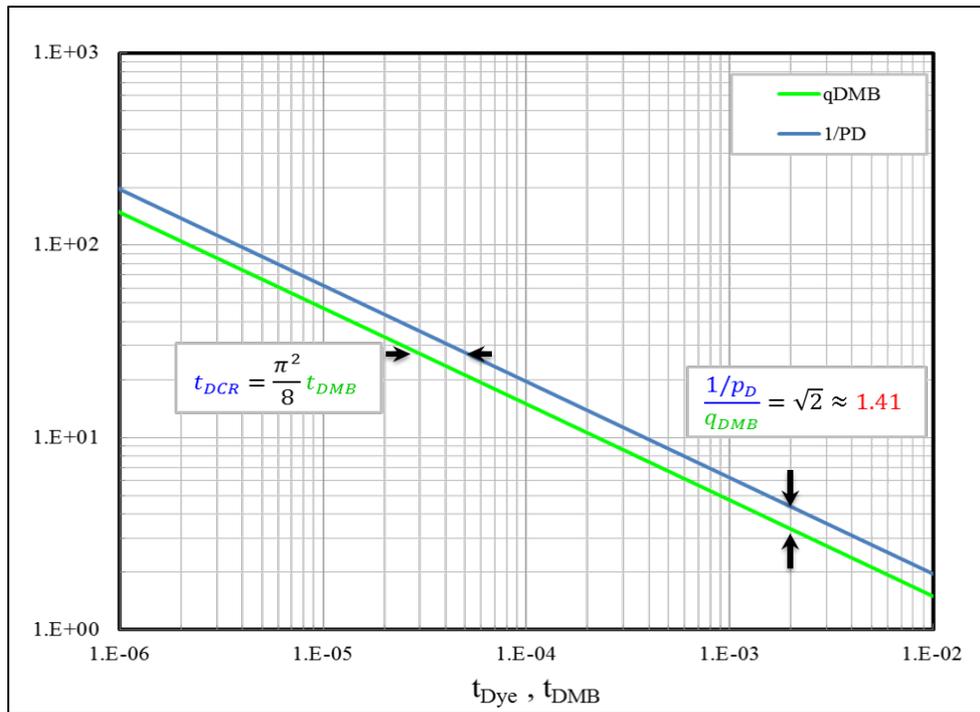


Fig. 52 Vertical and horizontal differences after using material balance time for linear flow

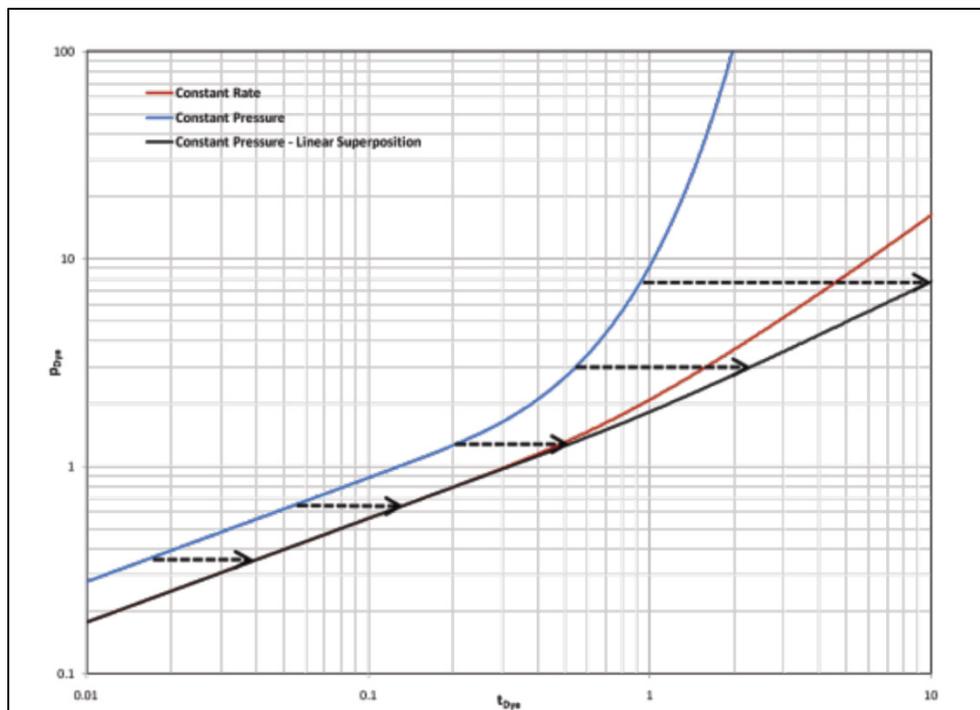


Fig. 53 The misrepresentation of BDF after using linear superposition time

4.3 Summary

The superposition time functions behavior has been a subject of skepticism for long period of time. The superposition-time specialized plots offered one of the effective ways of handling variable-rate, variable-pressure data. The approach has the virtue of diagnosing and analyzing the different flow regimes that might present in the reservoir. However, superposition time functions can be also biased and misleading in many cases. Furthermore, the shale gas wells usually are subjected to liquid loading, multiphase flow, interference, or severe flow rate fluctuations. For these cases, it will be hard to distinguish their effects from BDF regime if the superposition or material balance time functions are used as plotting function. In such cases, if such deceiving presence of BDF is believed, the Expected-Ultimate- Recovery (EUR) will be cut way down. Moreover, based on these results and with the upsurge of energy needs there might be some other costly decisions will be taken such as refracing or re-stimulating the well especially in tight formations.

The other limitation of the superposition time application is in the conversion of the constant-pressure linear flow solution to its counterpart constant-rate solution. Material balance time has inadequacy in converting constant-pressure transient linear flow solution. Although the difference and can be quantified, it can be considered insignificant unless the production data being analyzed has very high resolution. Therefore, material balance time is found to be acceptable for all practical purposes. In the next chapter, a methodology for the proper use of superposition time will be presented.

CHAPTER V

THE PROPER USE OF SUPERPOSITION TIME IN RTA

In this chapter, a simple technique will be presented for detecting the data bias on the superposition-time specialized plots. The technique is based on evaluating the kernel of the superposition time function of each flow regime for the maximum production time. Whatever beyond the Kernel-Equivalent Maximum Production Time (KEMPT), it will be considered as biased data.

It has been found that rate issues are the main causes for the bias in superposition-time specialized plots. All of the troublesome rate issues are centered on two main resulting effects; the noise and outliers. The main rate issues that cause the noise and outliers are the rate averaging, unstable operating conditions, liquid loading, flushing after shut-ins, sudden decline due to interference, production rate allocations, and missing flow rates. These issues are all commonly encountered in the production data of shale gas wells. The effect of these issues is potently deceiving as the spurious points form a fictitious unit slope line log-log diagnostic plots. The severity of the problem increases when the rate drops drastically at late times.

Accordingly, different plotting and filtering techniques will be applied in order to identify and filter out the bias causing issues. Once the effects of all problematic issues are filtered out, it will be clear that whatever beyond the KEMPT is a consequence of these issues. Thus, the proposed KEMPT technique can be relied on in order to detect and filter out the biased data points on superposition-time specialized plots.

Finally, a workflow will be proposed for the proper use of superposition-time specialized plots in order to help the analyst of production data to take the advantage of this valuable tool. Both synthetic and field data will be used in demonstrating the use that workflow.

5.1 Kernel-equivalent maximum production time

A simple technique will be followed in order to rapidly check whether there is data bias on the superposition-time specialized plots or not. The technique is based on evaluating the kernel of the superposition time function of each flow regime for the maximum production time. Whatever beyond the KEMPT it is considered as biased data. The hypothesis of this technique is that there is no way to see in the reservoir more than what you have seen. The inspiring hypothesis which has a side of similarity was already discussed by Dake (1978) as he mentioned:

“It is frequently stated in the literature that the separate flow periods should be of short duration so that transient flow conditions will prevail at each rate. While this condition is necessary, it is insufficient for the valid application of transient analysis to the test. Instead, the entire test, from start to finish, should be sufficiently short so that transience is assured throughout the whole test period. The reason for this restriction is that the largest value of the dimensionless time argument, for which the p_D functions in equ. (7.69) must be evaluated, is equal to the total duration of the test.”

The author’s work was in the area of pressure transient analysis of multi-rate tests. He pointed out that a significant error can be made by automatically assuming that

a multi-rate flow test can be interpreted using transient analysis techniques. His observation was that the multi-rate test analysis can yield a different plot for each assumed boundary condition and the reason of such disparity is the nature of the analysis technique itself. Therefore, he proposed to evaluate the kernel of the superposition time function to the maximum production time in order to ensure that the whole test period is in transient flow.

In the case of rate transient analysis, it has been observed that all the specious points are beyond the KEMPT. For the synthetic data cases shown in the previous chapter, the superposition time of the induced points (due to the rate drop or liquid loading) is exceeding the KEMPT as shown in Fig.54 and Fig.55. The cases are developed for 50 days of constant-pressure pure linear flow. The relation between the actual time and the material balance time during linear flow has been derived based on the solution of Wattenbarger et al. (1998). The material balance time during linear flow has been found to be twice the actual time as shown in Appendix-B. Accordingly, the maximum material balance time for these cases is 100 days. However, all the specious points are beyond the 100 days on the log-log diagnostic plot.

For the field case of well W1-1 which has been previously presented all the fictitious unit slope lines are beyond the KEMPT on the log-log plots as shown in Fig.56 through Fig.61. Since it is a tight gas well, it is expected to have production data issues. Therefore, a workflow should be followed to confirm that the fictitious unit slope points are a consequence of production data issues.

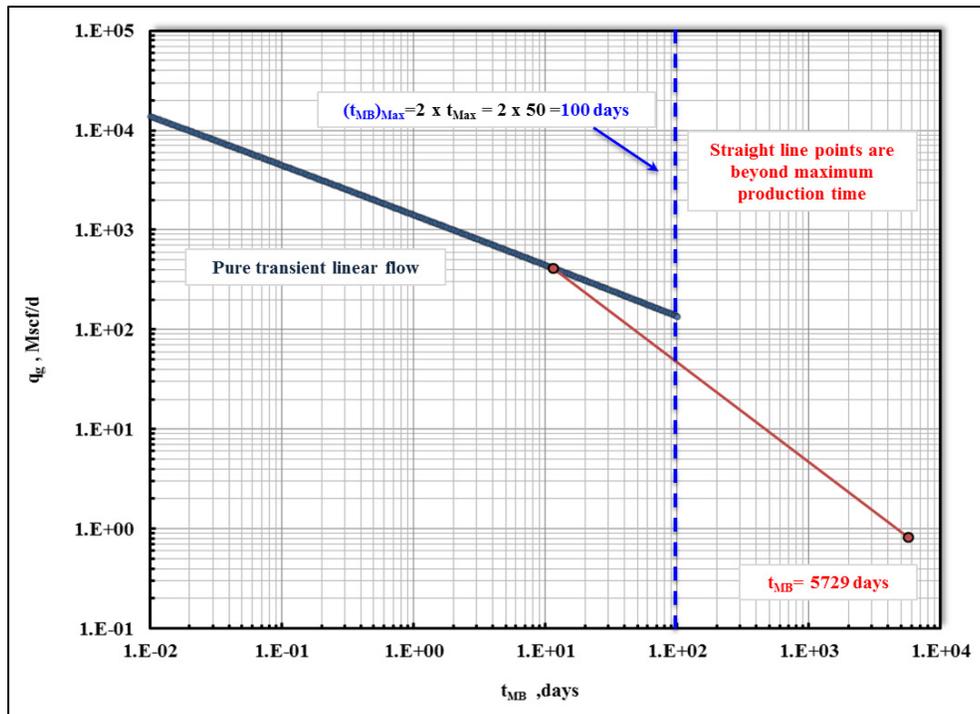


Fig. 54 Synthetic data with the effect of averaging making material balance time exceeds KEMPT

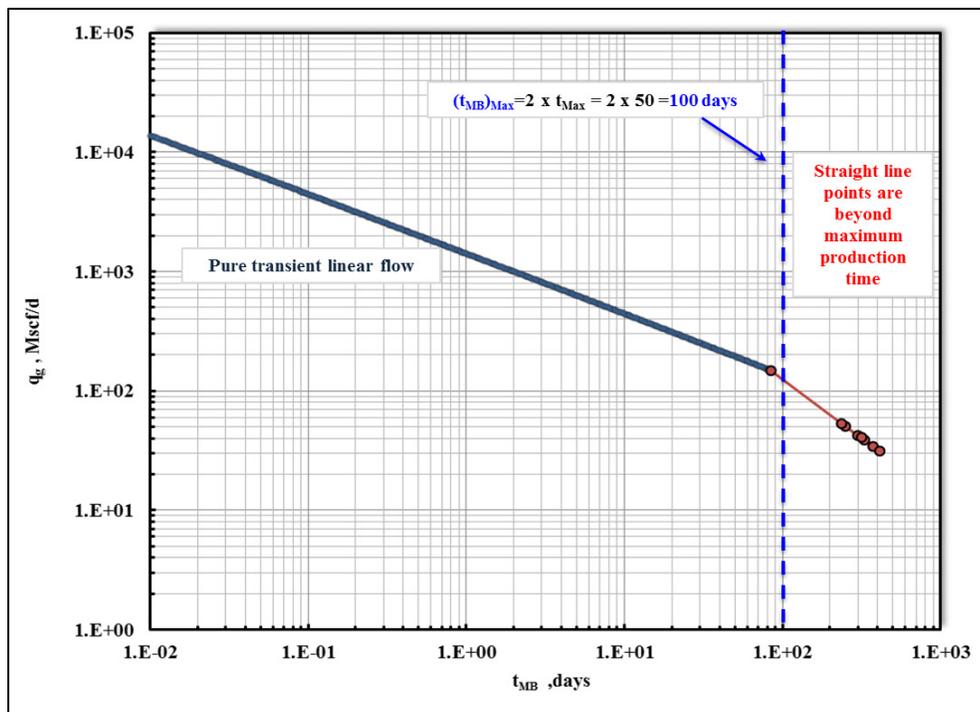


Fig. 55 Synthetic data with the effect of liquid loading making material balance time exceeds KEMPT

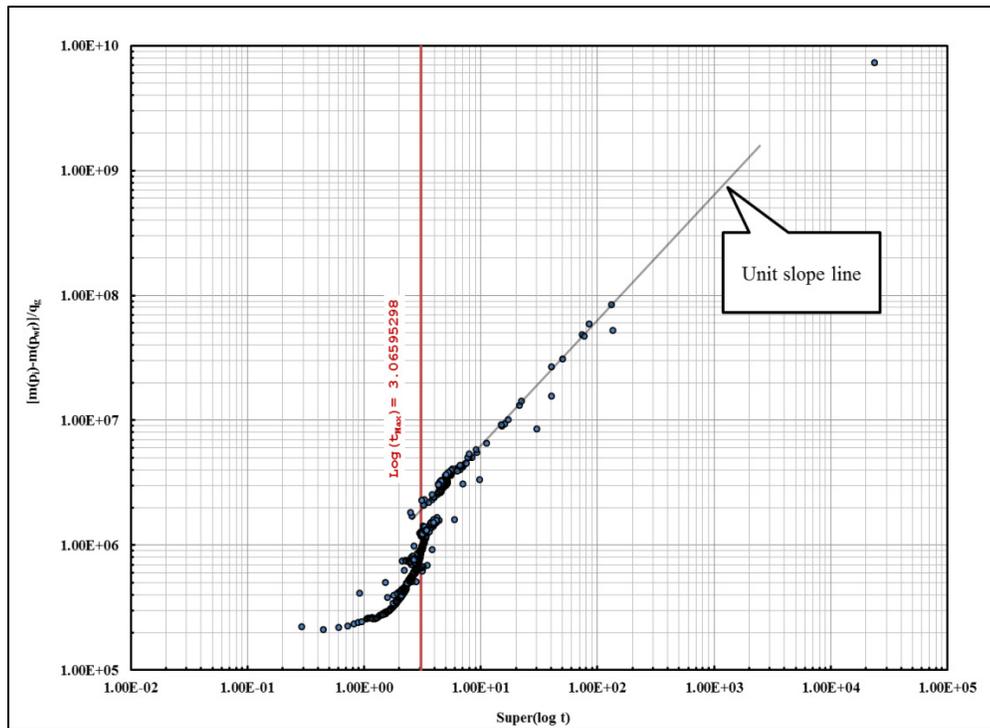


Fig. 56 Log-log Superposition-time plot for radial flow where straight lines are beyond KEMPT

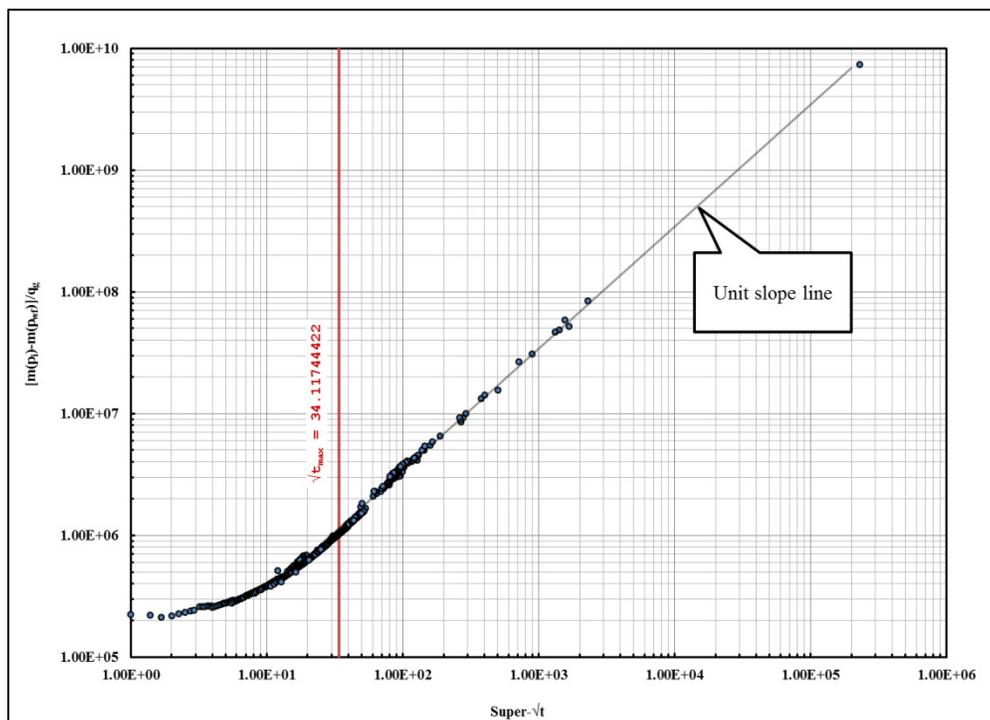


Fig. 57 Log-log Superposition-time plot for linear flow where straight lines are beyond KEMPT

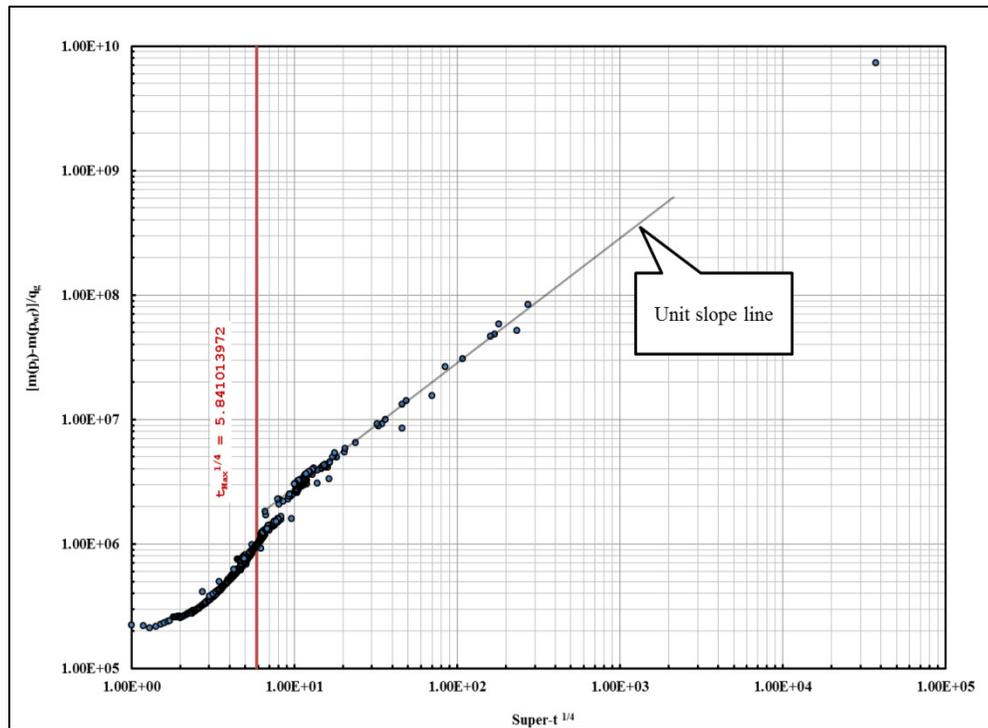


Fig. 58 Log-log Superposition-time plot for bilinear flow where straight lines are beyond KEMPT

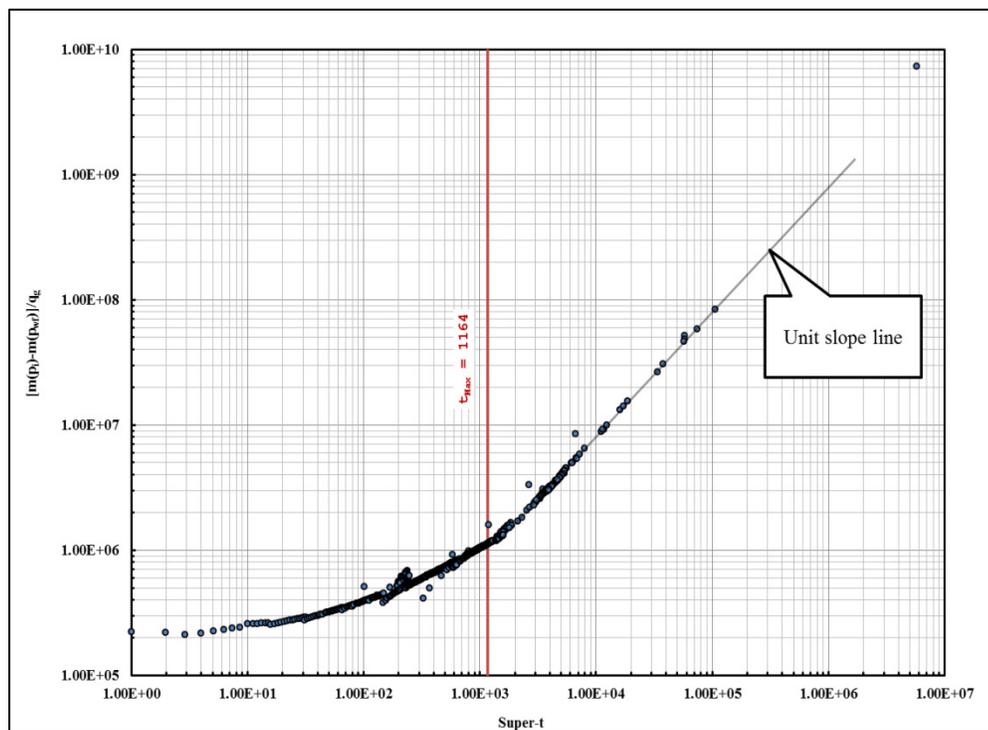


Fig. 59 Log-log Superposition-time plot for PSS flow where straight lines are beyond KEMPT

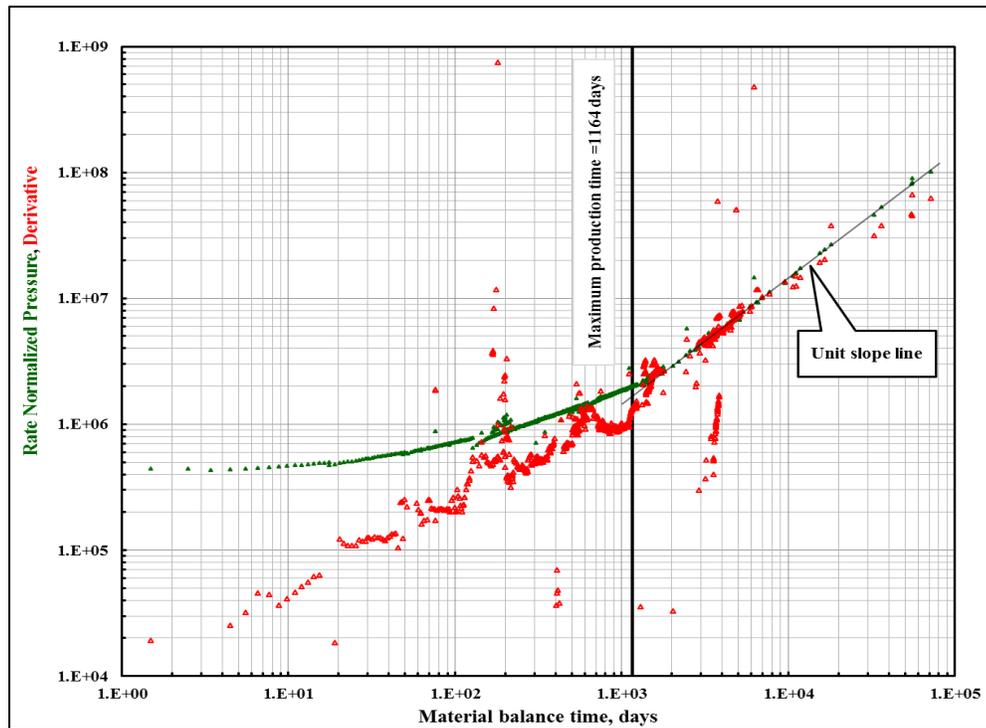


Fig. 60 RNP plot with straight lines are beyond KEMPT

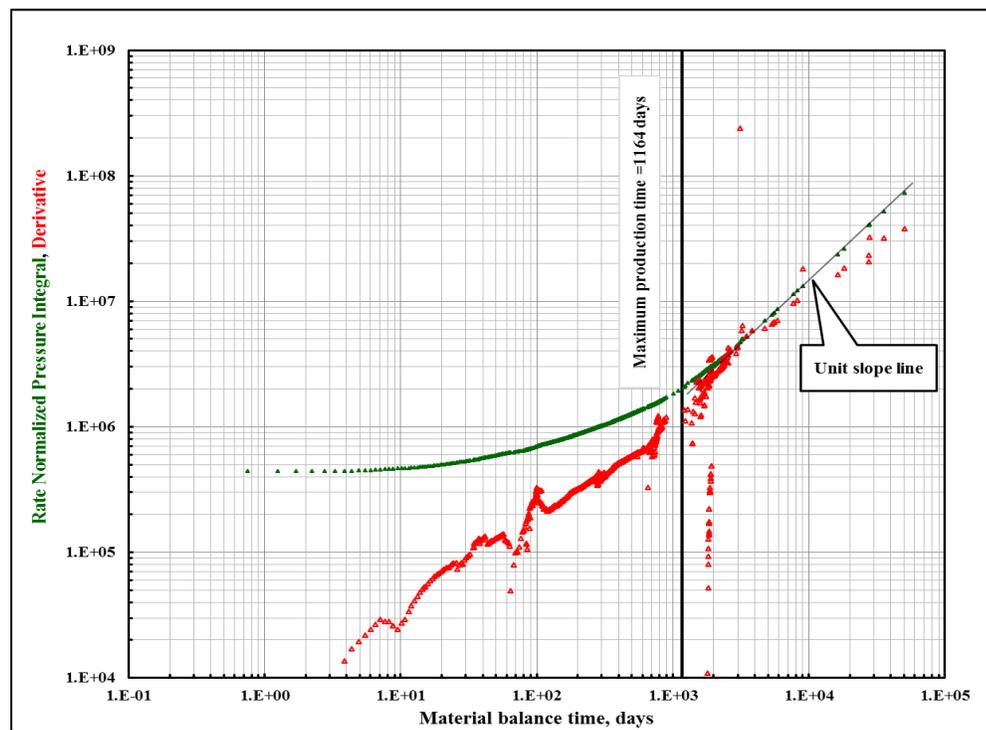


Fig. 61 RNPI plot with straight lines are beyond KEMPT

5.2 Proposed workflow

In order to check the cause of the unit slope points that are formed beyond the KEMPT, diagnostic and filtering techniques for production data issues should be followed. In this work the proposed workflow will be composed of three stages:

- 1- Diagnosing the production data for the suspected issues
- 2- Filtering out and editing the production data
- 3- Analyzing the filtered production data

As mentioned earlier, the production data is subjected to many problematic issues that affect the data behavior on the diagnostic and analysis plots. Production data can be of poor quality because of operational problems or changes in operating conditions, liquid loading, rate averaging, etc. If this poor quality data or inconsistent data is not recognized then it can easily be misinterpreted as a reservoir signal rather than the operating problem. In this work, the focus will be only on the expected reasons of biasing the superposition time specialized plots.

The workflow presented in this work is compiled from techniques presented in the literature and others based on observation. The diagnosis and filtering stages will be utilizing some of the techniques presented in the work of Anderson et al. (2006), Nobakht and Mattar (2009), and Almarzooq (2010). The analysis part will be using the linear flow type-curves developed by Wattenbarger et al. (1998) as an aid for type-curve matching. Type-curves matching will be used for calculating the OGIP and comparing the results with the unfiltered data.

5.2.1 Diagnosing production data for the suspected issues

Although production data is subjected to many problematic issues that affect the data behavior, the diagnosis stage in this work will be only focusing on the expected reasons of biasing the superposition time specialized plots. Of great importance, the issues that cause a sudden and severe rate drop. The effect of these issues is potentially deceiving as the spurious points form a fictitious unit slope line log-log diagnostic plots. The severity of the problem increases when the rate drops drastically at late times.

Some of the plotting techniques used for data diagnosis are based on observation and others are from the work of Anderson et al. (2006), Nobakht and Mattar (2009), and Almarzooq (2010). However, these plotting techniques will be used in this work exclusively for diagnosing the biasing issues of the superposition time specialized plots. The used diagnosing plots are:

1. q_g vs. t : it is good in showing changes in decline rate such as spikes and sudden drops in the trend.
2. *Turner plot*: This plot is used to check whether the flow rate is below the critical liquid lift velocity or not. It uses the ratio of critical-Turner-rate to gas-rate versus time. If the ratio of critical-Turner-rate to gas-rate is greater than one, then there is a likelihood of liquid loading according to the correlation presented by Turner et al. (1969).

3. *WGR plot*: it is a Cartesian plot of the water gas ratio versus time. It is used to check the presence of multiphase regions in the data. Nobakht and Mattar (2009) suggested that single phase flow assumption is approximately valid below a threshold value of 100bbl/MMscf. The gas wells exceeding this threshold limit may not be considered as single phase gas well. Since all the analytical models and type-curves use the assumption of single phase flow, data exceeding this limit should therefore be filtered out.
4. *Log (G_p) vs. log (t_{MB})* : this plot has the feature of distinguishing the transient flow as straight line while the sudden rate drops and fluctuations will be shown as horizontal swings.
5. *Log(q_g) vs. log(t)* : this plot is usually used for constant pressure cases as a diagnostic plot for the flow regimes. However, it is still usable for variable pressure cases as it has the feature of excluding the completion effects. Moreover, since no superposition rate function is used in this plot, it mitigates most of the influence of rate drops and fluctuations as deceiving straight lines.
6. *Log ($(m(p_i) - m(p_{wf})) / q_g$) vs. log(t)* : this plot is the counterpart of the pressure transient plots that uses pressure normalization. The normalization in this plot is to take account of the variable pressures. It still preserve the transient signal as straight lines, but rate drops and fluctuations will swing vertically instead of stretching on a unit slope line.

7. *Square root time specialized plot* : it is a Cartesian plot of the normalized pressure versus the square root of the production time. It is the specialized plot for diagnosing and analyzing the linear flow regime. It has the feature of distinguishing the linear flow as straight line, while other signals will be curved or clouded a distance from the trend. It is used because all tight gas wells are hydraulically fractured where the transient linear flow is expected to dominate for long period of time (Wattenbarger et al., 1998).
8. *Superposition time specialized plots with KEMPT* : once all the problematic rate issues are diagnosed, the superposition time specialized plots with KEMPT will be used to check whether these issues will be beyond the KEMPT or not. Thus, it will be confirmed if the unit slope lines formed beyond the KEMPT are fictitious or real lines.

When using these plots for diagnosing the production data of well W1-1 it has been found that the data include multiphase flow regions, liquid loading, and minor points of where the rate drops drastically. Fig.62 shows plot of WGR. It is clear that at the beginning of the well production the WGR is above the threshold value of 100bbl/MMscf (red horizontal line) which indicates multiphase (red points). That is expected since tight gas wells are undergone massive hydraulically fracturing which uses huge amounts of frac-water. Therefore, considerable amounts of that frac-water will flow back once the well production is commenced.

The well is also subjected to liquid loading at late production times as shown in the Fig.63. The ratio of ratio of critical-Turner-rate to gas-rate is greater than one (red line) which indicates the likelihood of liquid loading (green points) problem in the well. When these points are highlighted on the Cartesian plot of gas rate versus time, Fig.64, they are located at late production time with a sudden drop in the trend and some rate fluctuations.

The log-log plot of the gas rate versus the time is good in identifying the transient flow regimes while representing the rate fluctuations due to liquid loading in vertical direction other than stretching on a unit slope line. That is because it does not involve any superposition of the rate for the time scale. Fig.65 shows the log-log plot of the gas rate versus the time of well W1-1 production data. The log-log plot of the normalized pseudo-pressure versus time has an extra feature of including the wellbore and completion effects. However, this plot is shown in Fig.66 and dose not differ from Fig.65 in any aspect which can be attributed to the near constant-pressure operating conditions of well W1-1. The other figure which has similar characteristics is the Cartesian plot of the normalized pressure versus the square root of the production time, shown in Fig.67.

On Fig.68 the cumulative gas production is plotted versus material balance time in log-log plot. Transient flow is clearly distinguishable as straight line while the sudden rate drop and fluctuations (due to liquid loading) are shown as horizontal swings.

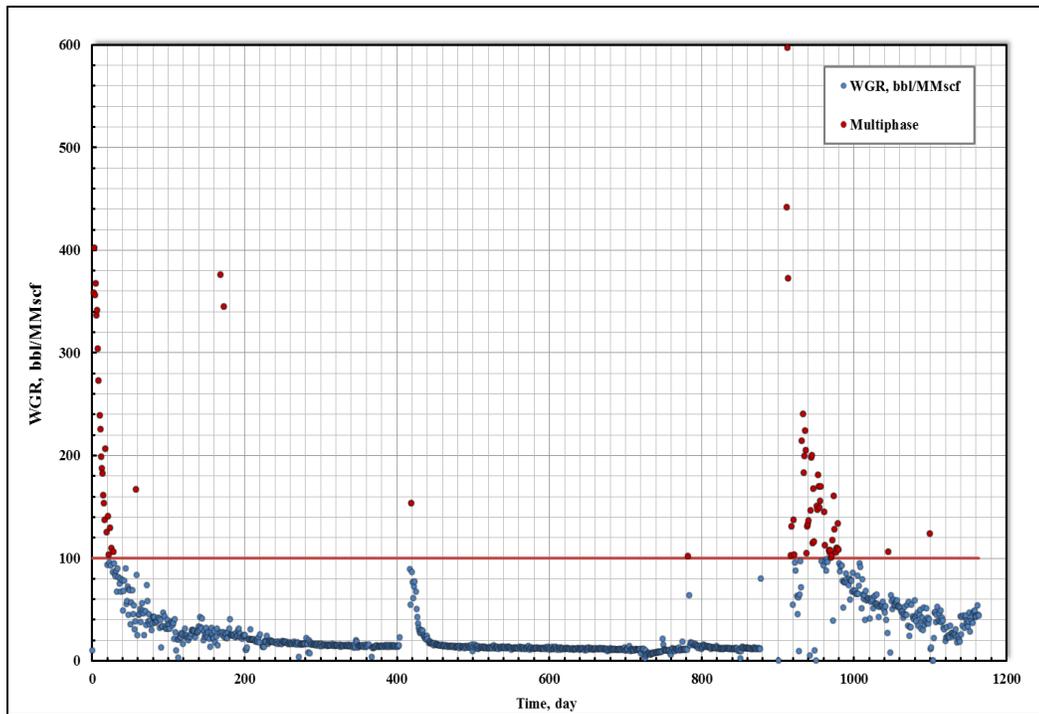


Fig. 62 Water gas ratio plot of well W1-1

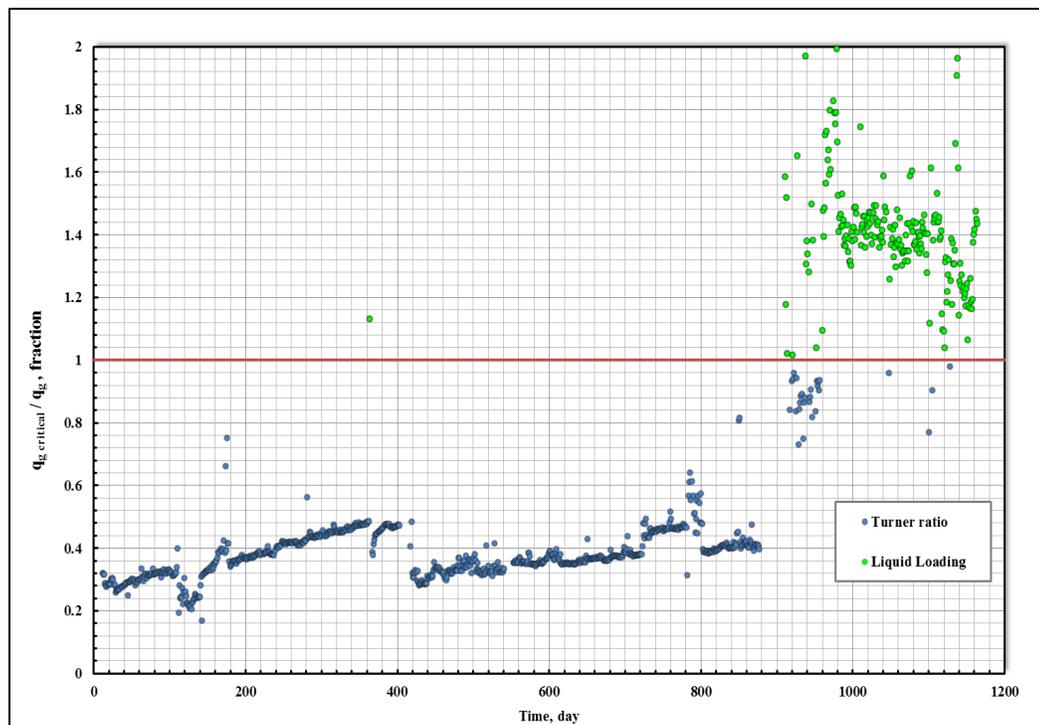


Fig. 63 Turner plot of well W1-1

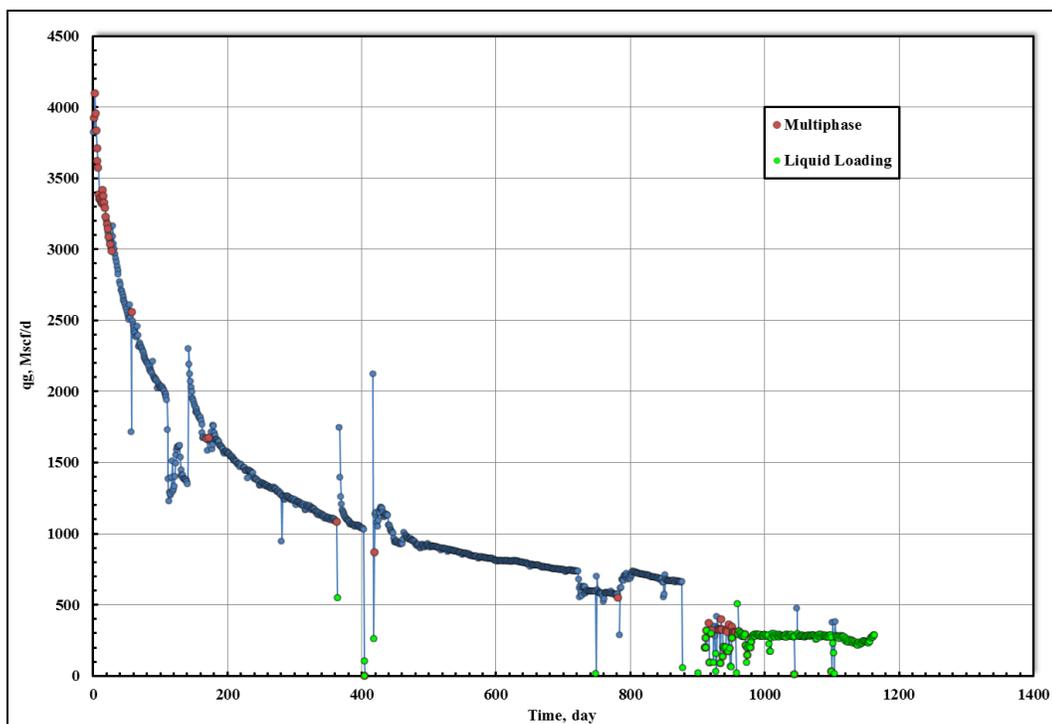


Fig. 64 Production rate data of well W1-1 with production issues highlighted

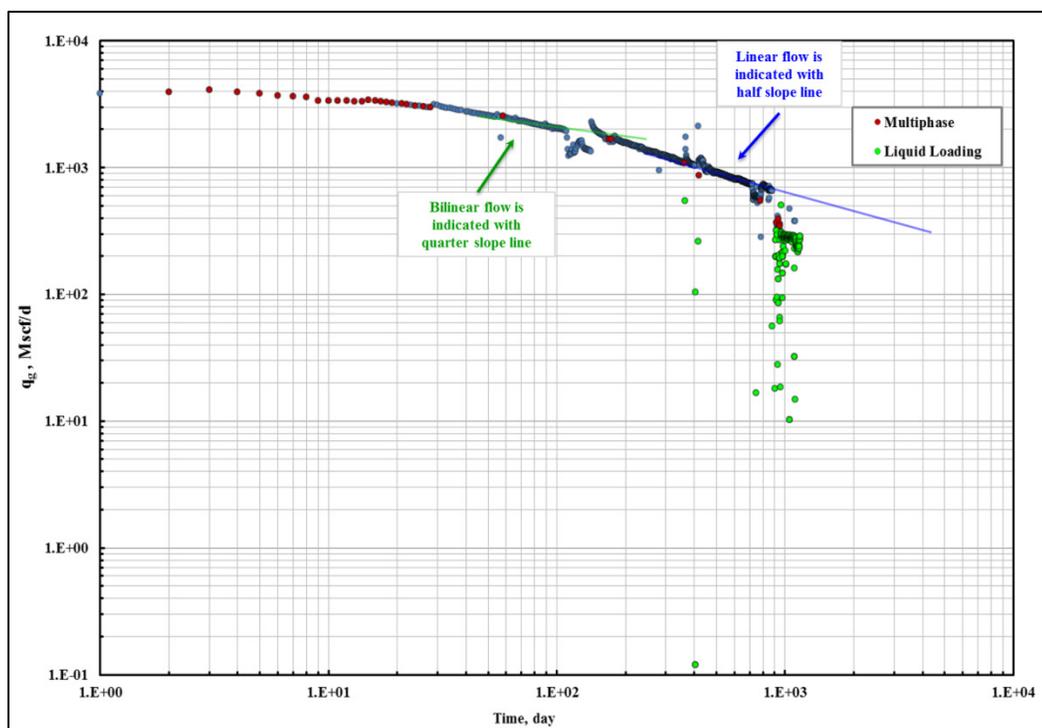


Fig. 65 Log-log plot of rate versus actual time of well W1-1 with production issues highlighted

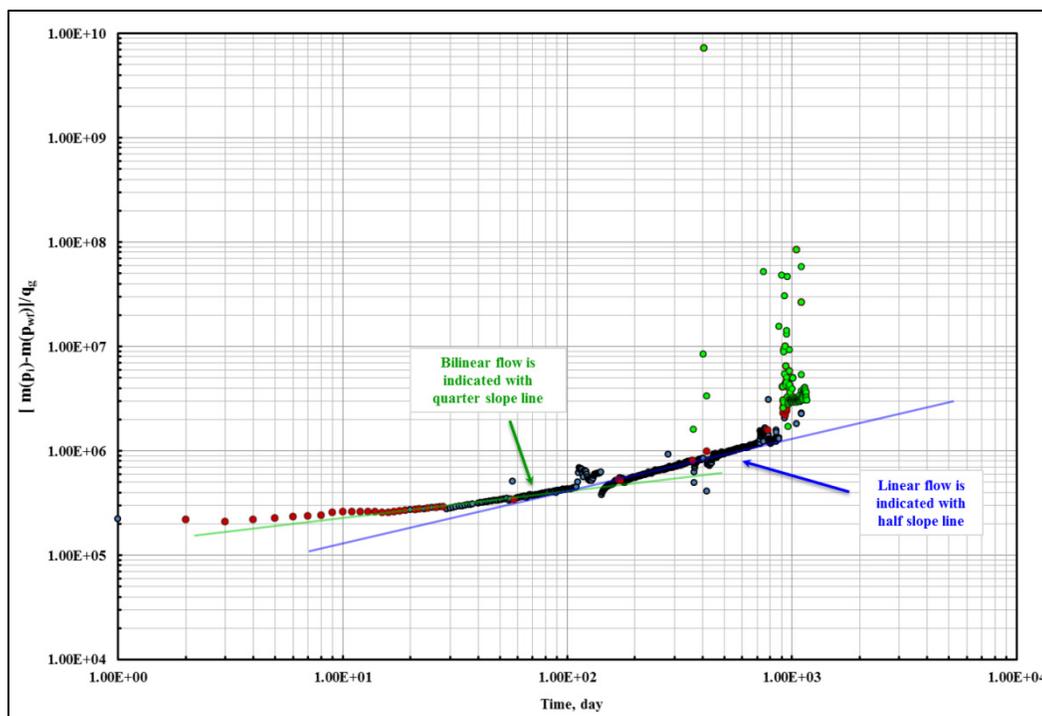


Fig. 66 Well W1-1 log-log plot of Normalized pressure vs. actual time with production issues highlighted

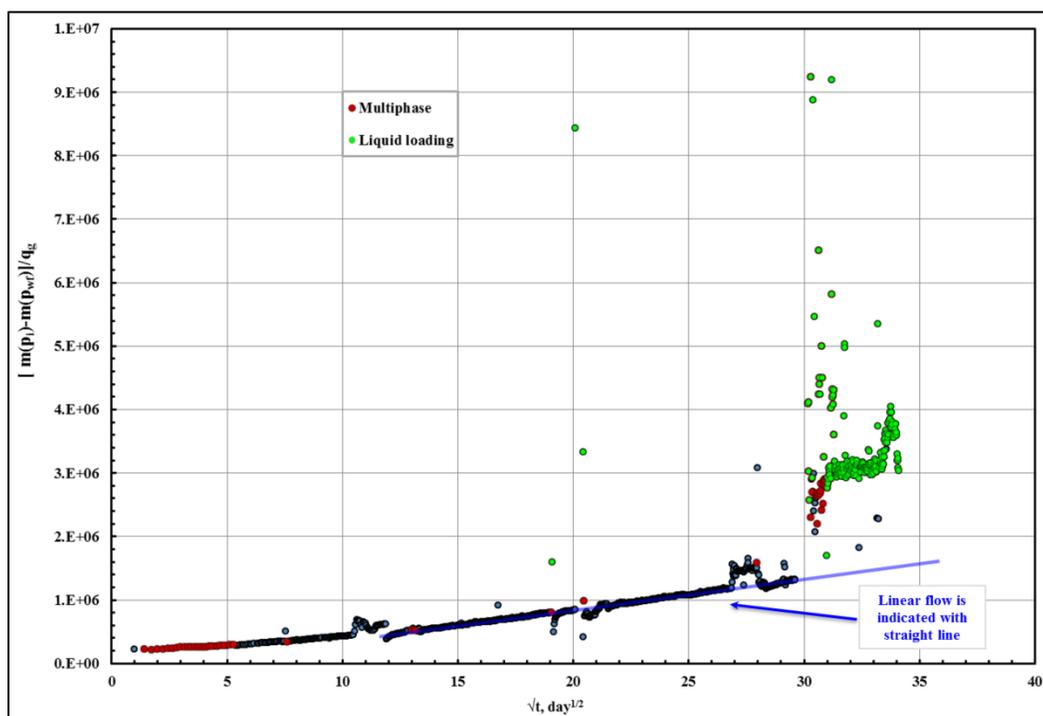


Fig. 67 Square root time plot of well W1-1 with production issues highlighted

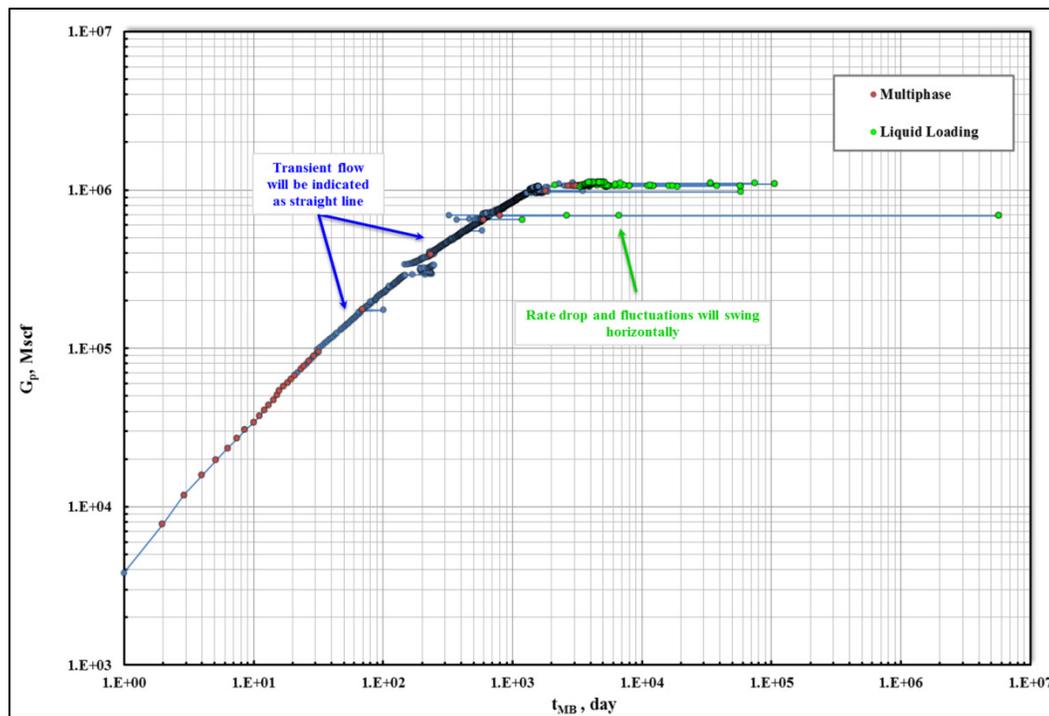


Fig. 68 Cummulative versus material balance time of well W1-1 with production issues highlighted

At this stage, there is a level of confidence to believe that the production data of well W1-1 exhibit the Bilinear and linear flow regimes with multiphase flow at early times and liquid loading at late times. In addition, there are few points scattered on the trend of the data.

The next step is to see the behavior of the indicated issues on the superposition time specialized plots. If these issues are forming unit slope lines, then these lines are fictitious. Fig.69 through Fig.72 show the superposition time specialized plots with the major issues of the production data of well W1-1. It is clear that all the unit slope lines are fictitious as the constituting points of these lines are mainly formed of liquid loading. Furthermore, it is worth to mention that these lines are located beyond the KEMPT.

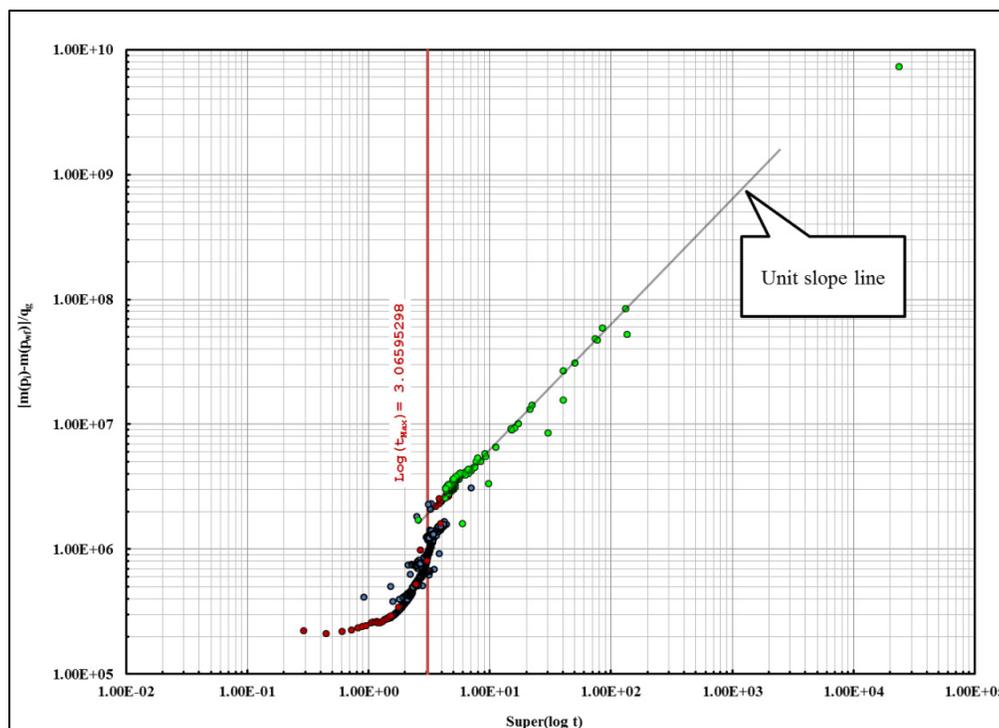


Fig. 69 Log-log Superposition-time plot for radial flow of well W1-1 with production issues highlighted

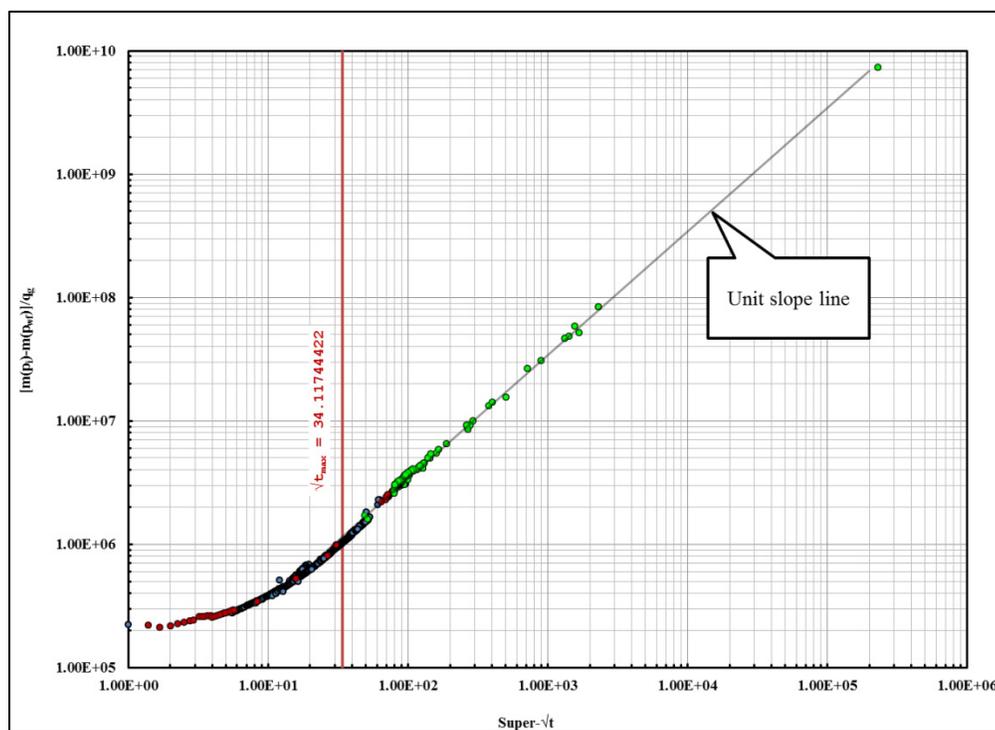


Fig. 70 Log-log Superposition-time plot for linear flow of well W1-1 with production issues highlighted

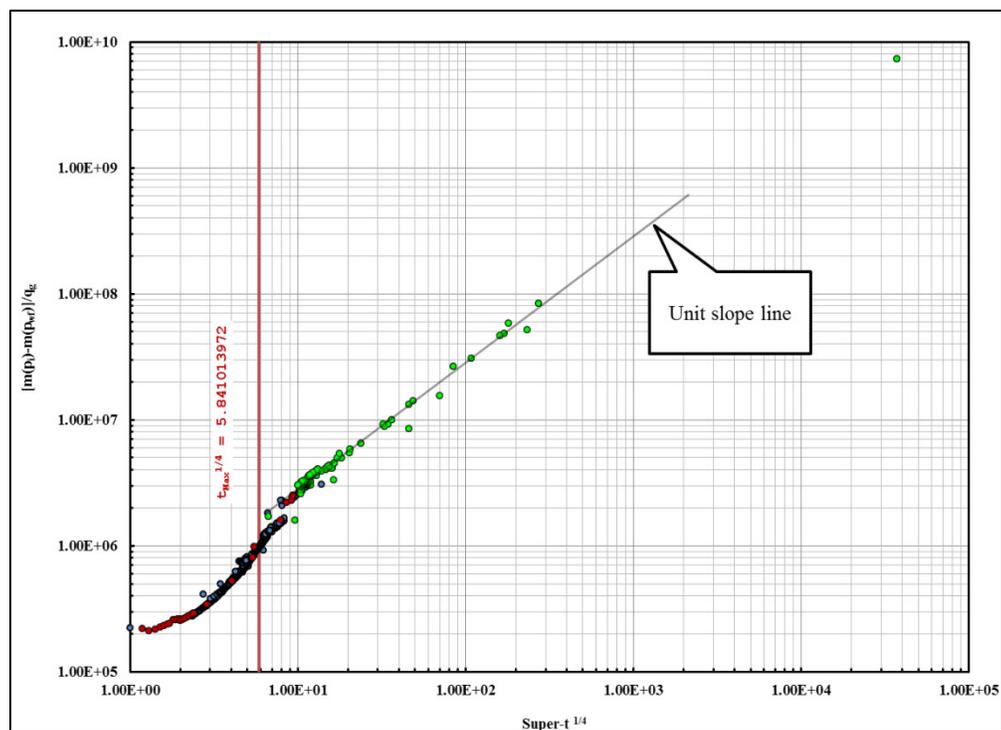


Fig. 71 Log-log Superposition-time plot for bilinear flow of well W1-1 with production issues highlighted

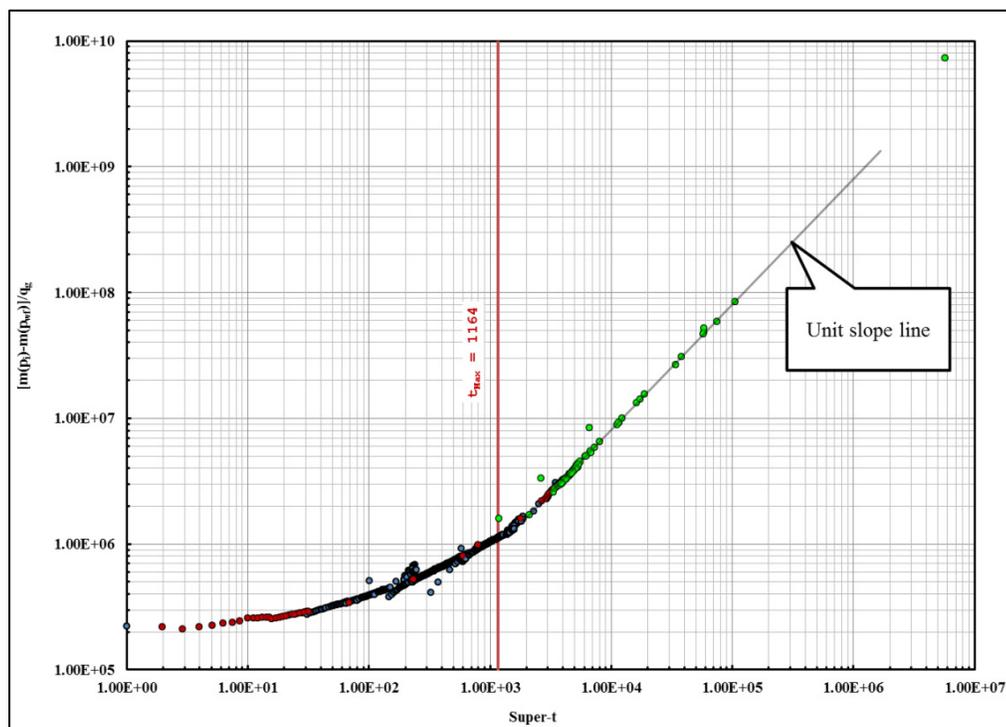


Fig. 72 Log-log Superposition-time plot for PSS flow of well W1-1 with production issues highlighted

5.2.2 Filtering out and editing the production data

In the previous section the production data of well W1-1 has been diagnosed for the biasing issues. It has been confirmed that all unit slope lines located beyond the KEMPT are fictitious where the main constituting points of these lines are formed of liquid loading points. Accordingly, it is clear that if these problematic issues are not recognized then they can easily be misinterpreted as a reservoir signal rather than operating problem. In this chapter the problematic issues will be filtered out. Thus, it should be clear whether there are any other real reservoir signals beyond the KEMPT or not.

Since the superposition time functions involve rate components, the superposition time is affected greatly with rate issues. Therefore, the main filtering techniques presented in this work will be concerned with filtering rate issues. If the production data is suffering liquid loading, then flow rate points below the critical-Turner-rate according to the correlation presented by Turner et al. (1969) should be filtered out.

Rate noise and outliers are best filtered with the median filter. The median filter is a nonlinear filter which is best known for filtering out impulse characteristics such as noise spikes in a signal. However, it still preserves the main trend of the data and edges.

Median filter was introduced as a tool for data analyses by Tukey (1977) as he first called it "running median" (Coyle et al., 1993). Later, it became a canonical signal and image processing operation. In signal and image processing, median filter is best known for its salt and pepper noise removal aptitude. Therefore, it is typically used in

removal of noise in scanned images, removal of cosmic ray spikes from image data, and cleaning pitches from noise in speech processing (Moore and Jorgenson, 1993).

In this work, the median filter will be utilized for rate noise and outliers removal. In the median filtering operation, a filtering window, usually of odd width, is stepped one sample at a time along the production data. The rate values in the neighborhood window are ranked according to their magnitude. The median value is calculated for the specified window points, then comparing it with the value of each point. Whenever the difference is within a certain tolerance, the point will be preserved, otherwise filtered out. Fig.73 shows daily production data with considerable amount of noise. When the median filter of 19 points width window was applied to this data, all the noise has been removed while the main data trend was preserved as shown in Fig.74.

(Muhammed, 2006) summarized the advantages of the median filters over smoothing filters. He pointed out that median filters have four advantageous features:

1. The output pixel value is one of the neighboring values. Since the output values are of only of those present in the neighborhood (no averaging), new “unrealistic” values are not created.
2. Median filtering does not shift boundaries, as can happen with conventional smoothing filters (a contrast dependent problem).
3. Since the median is less affected than the mean by extreme values (outliers), those extreme values are more effectively removed.
4. Since edges are minimally degraded, median filters can be applied repeatedly.

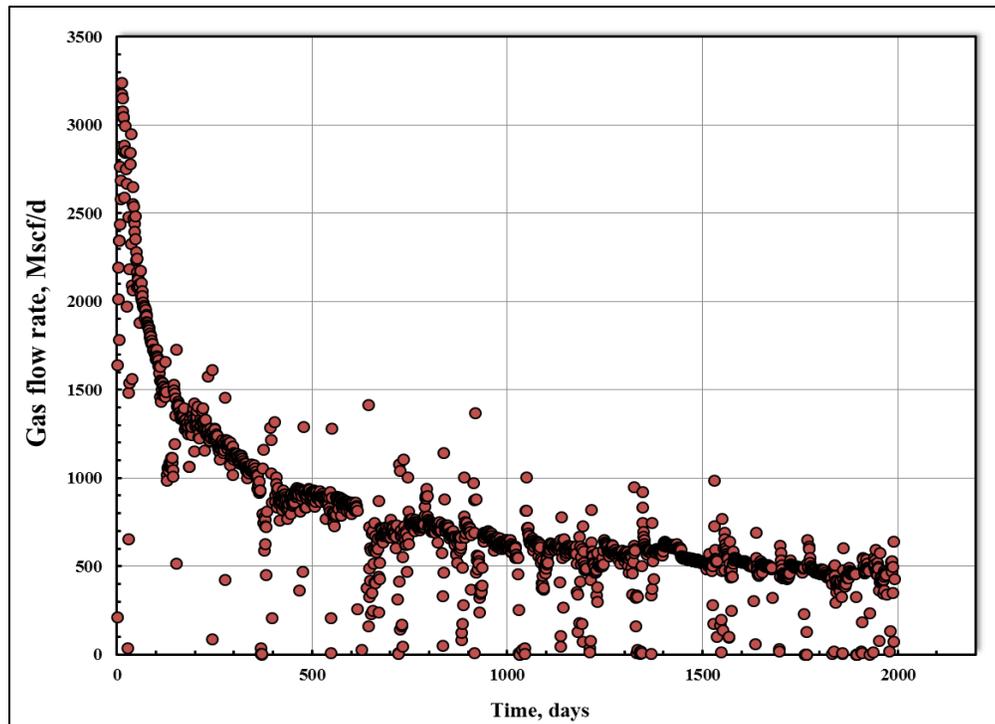


Fig. 73 Daily production data with considerable amount of noise

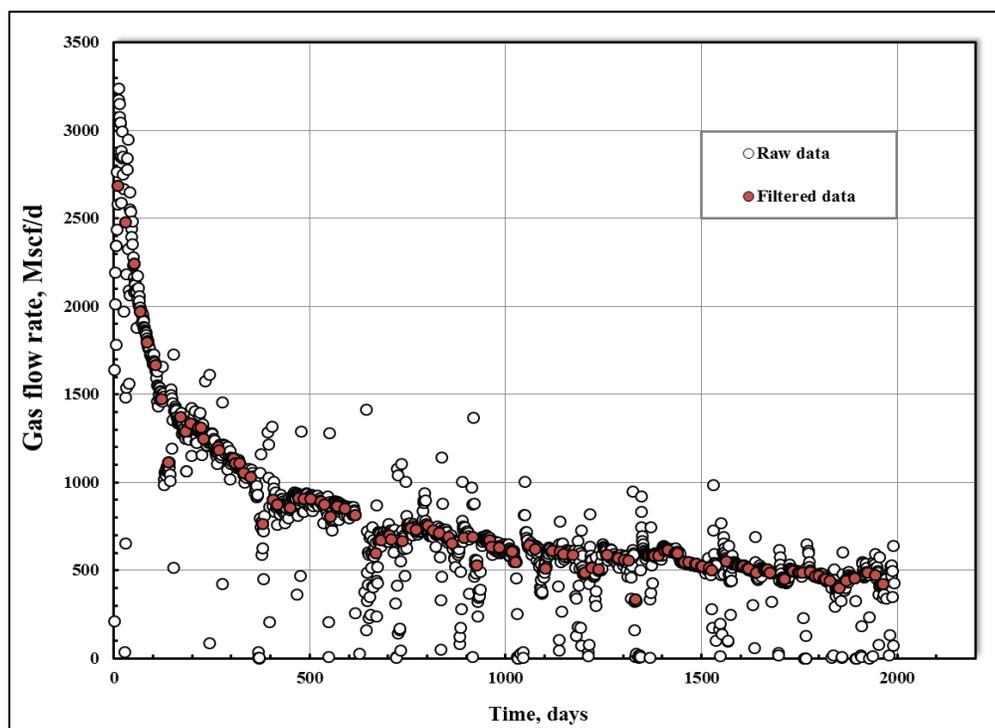


Fig. 74 Daily production data filtered with median filter of 19 points width window

It is worth to mention that production data should be checked whether it is allocated, averaged, or prorated. Data undergone these processes, should be edited and retained to its original daily state whenever it is possible. Additionally, partial day production data should not be included as they have a high potential to show as rate outliers (sudden and drastic drop).

The proposed filtering techniques was applied the production data well W1-1. The liquid loading data points as well as the outliers have been removed from all specialized superposition plots. Fig.75 shows that there is no any indication of the presence of the radial flow. In shale gas hydraulically fractured horizontal well, the radial flow is not expected or at least not at early production life since these wells are known with the domination of transient linear flow for long term production times. Fig.76 still exhibits linear flow regime but before the KEMPT, which is logically acceptable. Fig.77 also indicates the presence of the bilinear flow regime where it starts and ends before the linear flow, which is the expected behavior. Fig.78 shows no indication of the BDF, however it confirms clearly the presence of the linear and bilinear flow regime.

Thus, it became evident that whatever points beyond the KEMPT are deceiving spurious points even if they formed a unit slope line log-log diagnostic plots. Accordingly, the proposed diagnostic and filtering workflow stages confirmed that the fictitious unit slope lines are a consequence of production data issues. Accordingly, it is clear that if such problematic issues are not recognized then they would easily be misinterpreted as a reservoir signal rather than operating problem.

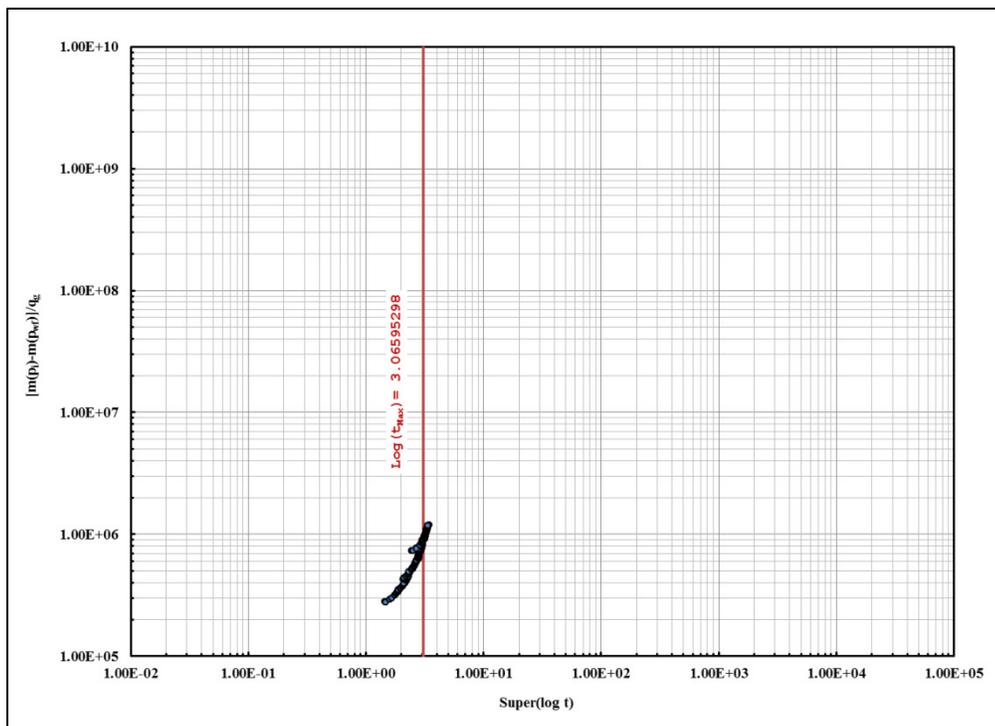


Fig. 75 Log-log Superposition-time plot for radial flow of well W1-1 with production issues filtered out

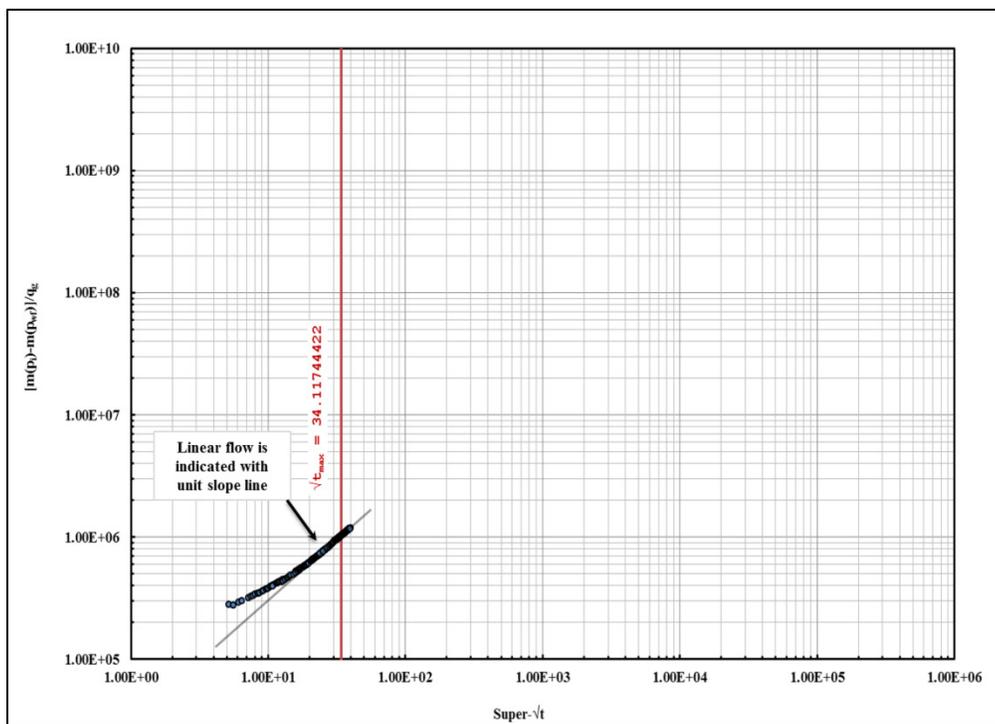


Fig. 76 Log-log Superposition-time plot for linear flow of well W1-1 with production issues filtered out

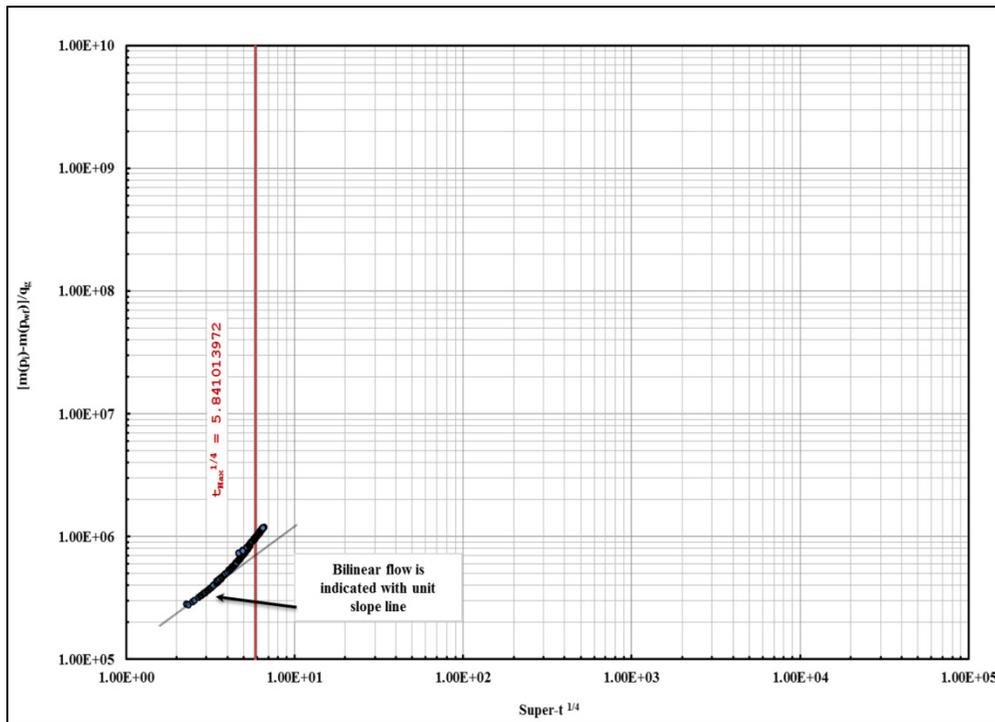


Fig. 77 Log-log Superposition-time plot for bilinear flow of well W1-1 with production issues filtered out

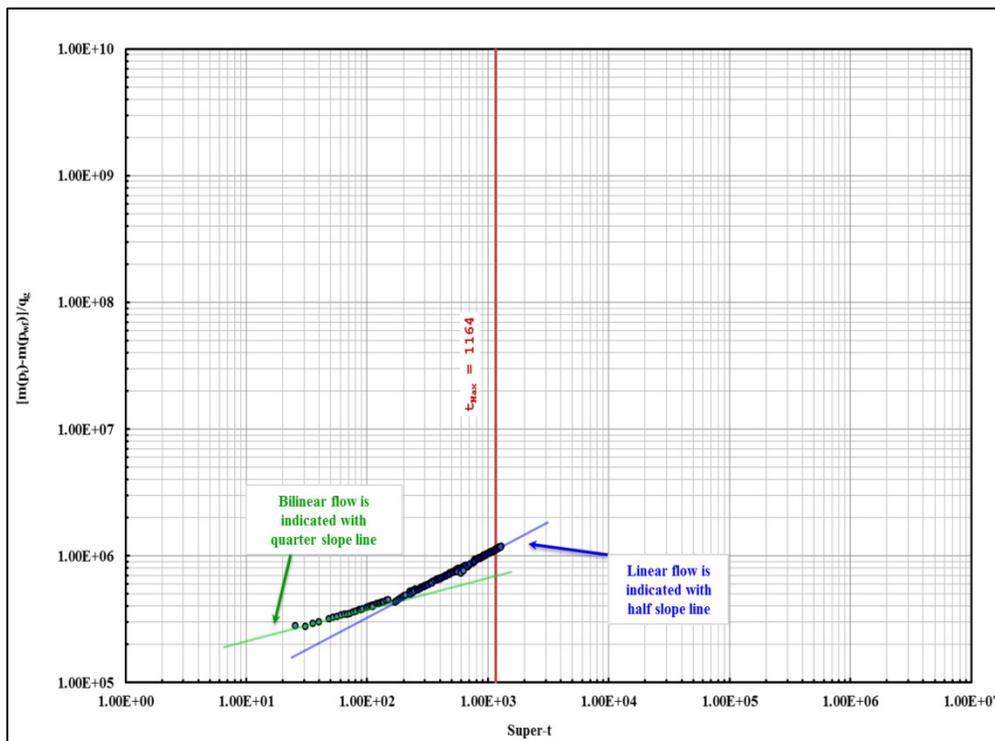


Fig. 78 Log-log Superposition-time plot for BDF flow of well W1-1 with production issues filtered out

5.2.3 Analyzing the filtered production data

After the proposed diagnostic and filtering workflow stages have confirmed that the fictitious unit slope lines are a consequence of production data issues. It is good idea to analyze both raw and filtered data and compare their results. This will enable recognizing the consequence of the data bias resulted from the superposition processes.

Shale gas hydraulically fractured horizontal wells are known with the domination of transient linear flow for long term production times. Therefore, the analysis part will be using the linear flow type-curves developed by Wattenbarger et al. (1998) as an aid for type-curve matching. The procedure of type-curve matching is described in Appendix-B. Type-curves matching will be used for calculating the OGIP and comparing the results with the unfiltered data.

It has been noticed in the previous chapter that the superposition time has limitation application is in the conversion of the constant-pressure linear flow solution to its counterpart constant-rate solution. Material balance time has inadequacy in converting constant-pressure transient linear flow solution. However, the difference can be considered insignificant unless the production data being analyzed has very high resolution, which is not the conventional case of the production data. Anyway, to be in the safe side the constant-pressure type-curves will be used along with the actual time for type-curves matching. Then the converted constant-pressure type-curves will be used along with the material balance time type-curves matching.

Fig.79 shows the unfiltered production data of the well W1-1 on the plot of $\log \left(\frac{m(p_i) - m(p_{wf})}{q_g} \right)$ vs. $\log(t)$. Although the data involves the liquid loading (green

points) as well as multiphase flow (red points), this plot is not affected with these issues. The type-curves matching yielded an OGIP of 3.5002 Bscf. This value represents the minimum OGIP that the transient wave has explored since the drainage boundaries has not been reached yet.

This value of the OGIP will be reduced to 2.3273 Bscf when using the material balance time with the converted constant-pressure type-curves for matching. Fig.80 shows that liquid loading points are the main constituents of the fictitious unit slope line shown on the plot. Such signature would be potentially deceiving as an indication of BDF. The reduction in the OGIP is about 33.51 %. Such reduction is attributed to the early false detection of the drainage boundaries, which defines the gas in-place occupied in these boundaries.

Once the production data is filtered as shown in Fig.81 and Fig.82, using the previously proposed filtering techniques, both actual time and superposition time plots yielded the same values of OGIP of 3.5002 Bscf.

In addition, more three field cases are presented in the Appendix-D. The cases are from different shale gas reservoirs; Woodford, Marcellus, and Barnett shale. All of these field cases are suffering production issues such as presented in this chapter. The workflow proposed in this chapter has been applied for these cases and the observations are pertinent to what have been presented in this chapter

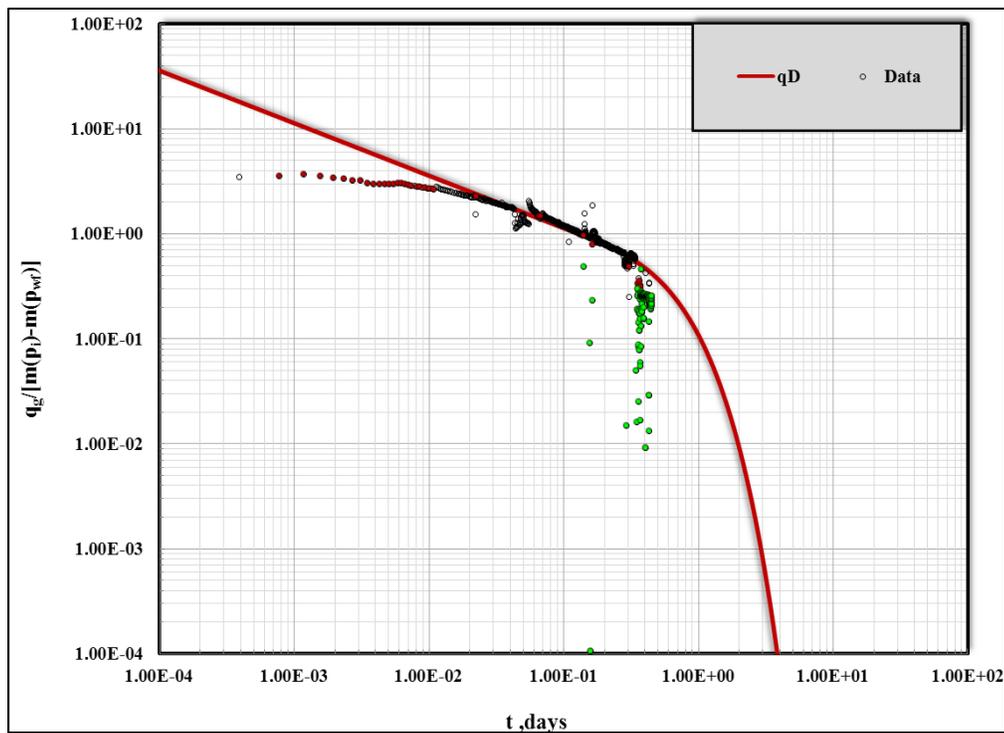


Fig. 79 Constant pressure type-curve matching of unfiltered data of well W1-1

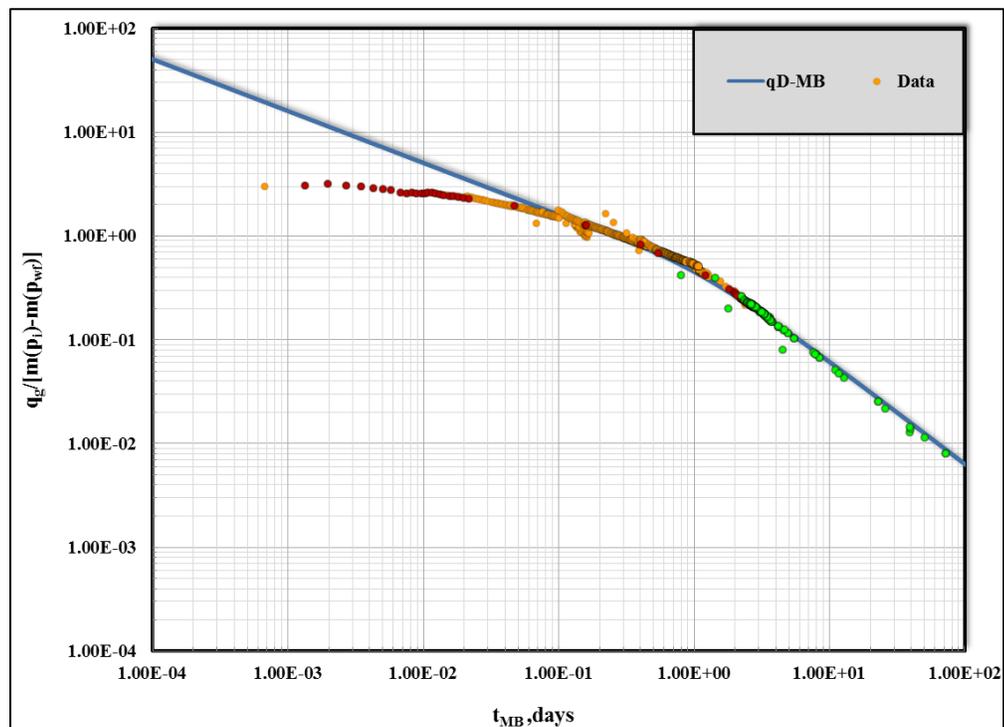


Fig. 80 Type-curve matching of constant-rate unfiltered data of well W1-1

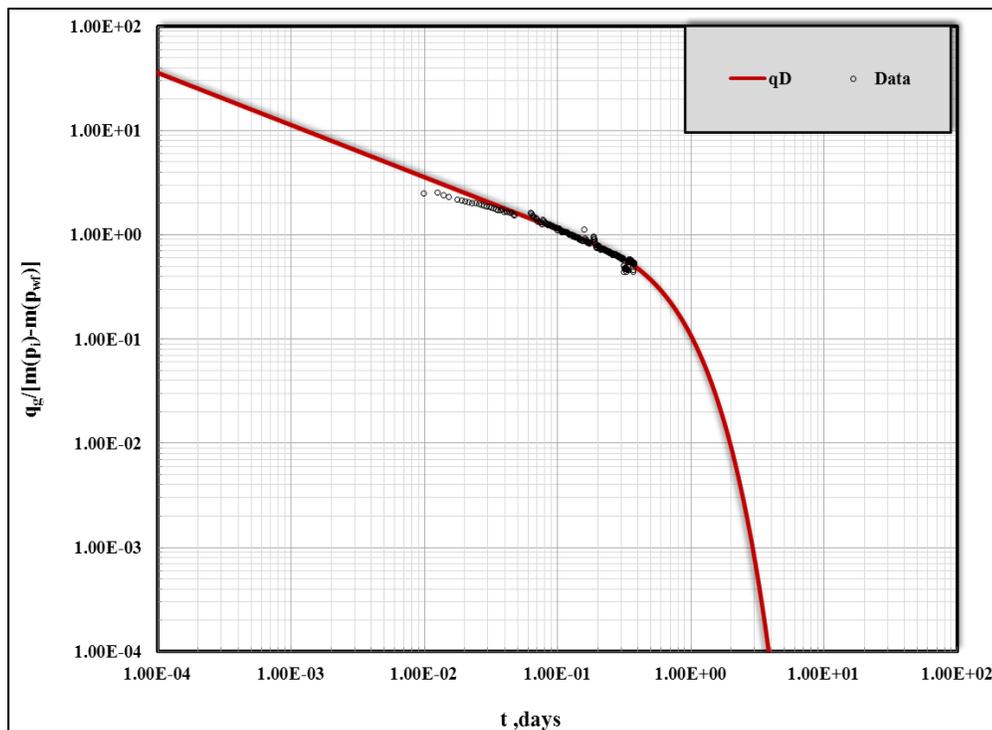


Fig. 81 Constant pressure type-curve matching of filtered data of well W1-1

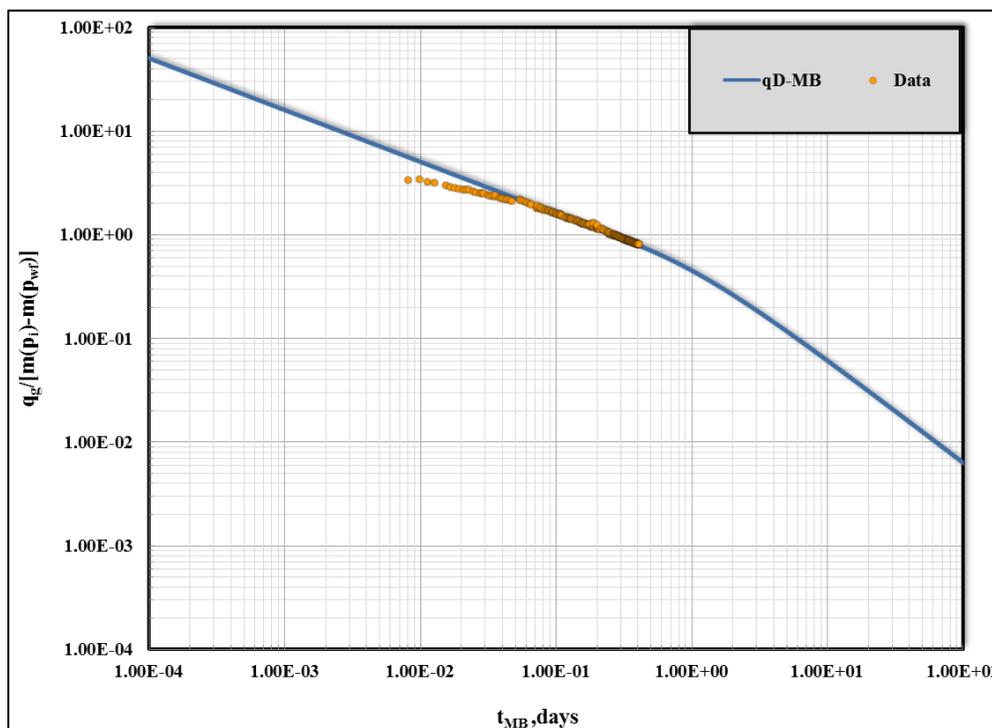


Fig. 82 Type-curve matching of constant-rate filtered data of well W1-1

5.3 Summary

A simple technique been presented in order to rapidly check whether there is data bias on the superposition-time specialized plots or not. The technique is based on evaluating the kernel of the superposition time function of each flow regime for the maximum production time. Whatever beyond the Kernel-Equivalent Maximum Production Time (KEMPT) it is considered as biased data. The hypothesis of this technique is that there is no way to see in the reservoir more than what you have seen.

In order to check the cause of the unit slope points that are formed beyond the KEMPT, diagnostic techniques for production data issues were proposed. Thus, it will be confirmed if the unit slope lines formed beyond the KEMPT are fictitious or real lines. It has been confirmed that all unit slope lines located beyond the KEMPT are fictitious where the main constituting points of these lines are a consequence of production data issues.

Different filtering techniques have been proposed in order to filter out the bias causing issues. Since the superposition time functions involve rate components, the superposition time is affected greatly with rate issues. Therefore, filtering techniques have been proposed for filtering rate issues. Of importance are the noise, outliers, and liquid loading problem. Rate noise and outliers are best filtered with the median filter, while liquid loading are best filtered using liquid loading filter based on the correlation presented by Turner et al. (1969).

Once the effects of all problematic issues are filtered out, it will be clear that whatever beyond the KEMPT is a consequence of these issues. Thus, the proposed

KEMPT technique can be relied on in order to detect and filter out the biased data points on superposition-time specialized plots.

Both raw and filtered data were analyzed using type-curve matching of linear flow type-curves developed by Wattenbarger et al. (1998). Type-curves matching were used for calculating the OGIP and comparing the results of raw with filtered data. It has been found that biased data yield noticeable reduced OGIP. Such reduction is attributed to the fictitious onset of boundary dominated flow. In other words, early false detection of the drainage boundaries defines less gas in-place occupied in these boundaries.

CHAPTER VI

CONCLUSIONS

From the presented work, following conclusions can be drawn:

- Production data diagnosis is a critical step before RTA, whereas blind application of production data analysis methods without consideration of data quality issues can lead to misinterpretation of the reservoir characteristics. Therefore, Analyst should be alert in using commercial software for RTA.
- The nature of $\log q$ vs. $\log \text{super-}f(t)$ plot makes any drop in the flow rate to be stretched on a unit slope line. Therefore, production data must be explored on actual time scale first.
- Outliers ,high scatter, liquid loading, well clean up, and interference can be misleading on superposition time scale
- All unit slope lines located beyond the KEMPT are fictitious where the main constituting points of these lines are a consequence of production data issues.
- Biased data yield noticeable reduced OGIP. Such reduction is attributed to the early fictitious onset of boundary dominated flow, where early false detection of the drainage boundaries defines less gas in-place occupied in these boundaries.
- The proposed KEMPT technique can be relied on in order to detect and filter out the biased data points on superposition-time plots.

NOMENCLATURE

c_f	= formation compressibility, 1/psi
c_g	= gas compressibility, 1/psi
c_t	= total compressibility, 1/psi
c_w	= water compressibility, 1/psi
G_p	= cumulative production, Mscf
k_m	= matrix permeability, md
$m(p_i)$	= initial pseudo-pressure, psi ² /cp
$m(p_{wf})$	= bottom hole flowing pseudo-pressure, psi ² /cp
$OGIP$	= original gas in place, Bscf
p	= pressure, psi
p_D	= dimensionless pressure
Δp_{unit}	= unit rate pressure function, psi
q_{DL}	= dimensionless rate
q_{DMB}	= dimensionless rate converted with dimensionless material balance time
q_g	= gas rate, Mscf/Day
S_{gi}	= initial gas saturation
t_D	= dimensionless time
t_{DCR}	= dimensionless time for constant rate solution
t_{DCP}	= dimensionless time for constant pressure solution
t_{DMB}	= dimensionless material balance time
T	= temperature, R
V	= volume, ft ³
x_f	= fracture half length, ft
x_e	= reservoir length, ft
y_e	= reservoir width, ft
y_{eD}	= dimensionless reservoir width
z	= compressibility factor

Greek Symbols

μ_g	= gas viscosity, cp
ϕ	= porosity

REFERENCES

- Agarwal, R.G., Al-Hussainy, R., and Ramey Jr., H.J. 1970. An Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow: I. Analytical Treatment. *SPE Journal* **10** (3): 279-290. doi: 10.2118/2466-pa
- Agarwal, R.G., Gardner, D.C., Kleinstieber, S.W. et al. 1998. Analyzing Well Production Data Using Combined Type Curve and Decline Curve Analysis Concepts. Paper Society of Petroleum Engineers 49222-MS presented at the SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, 09/27/1998. doi: 10.2118/49222-MS
- Ahmed, T.H. and McKinney, P.D. 2005. *Advanced Reservoir Engineering*. Burlington, MA: Elsevier/Gulf Professional Pub.
- Al-Ahmadi, H.A. and Wattenbarger, R.A. 2011. Triple-Porosity Models: One Further Step Towards Capturing Fractured Reservoirs Heterogeneity. Paper Society of Petroleum Engineers 149054-MS presented at the SPE/DGS Saudi Arabia Section Technical Symposium and Exhibition, Al-Khobar, Saudi Arabia, 05/15/2011. doi: 10.2118/149054-MS
- Almarzooq, A. 2010. The Implications and Flow Behavior of the Hydraulically Fractured Wells in Shale Gas Formation. Master of Science. Texas A&M, College Station, Texas.
- Anderson, D. and Mattar, L. 2004. Practical Diagnostics Using Production Data and Flowing Pressures. Paper Society of Petroleum Engineers 89939-MS presented at the SPE Annual Technical Conference and Exhibition, Houston, Texas, 26/09/2004. doi: 10.2118/89939-MS
- Anderson, D.M. and Mattar, L. 2003. Material-Balance-Time During Linear and Radial Flow. Paper Petroleum Society of Canada 2003-201 presented at the Canadian International Petroleum Conference, Calgary, Alberta, 07/10/2003. doi: 10.2118/2003-201
- Anderson, D.M., Nobakht, M., Moghadam, S. et al. 2010. Analysis of Production Data from Fractured Shale Gas Wells. Paper Society of Petroleum Engineers 131787-MS presented at the SPE Unconventional Gas Conference, Pittsburgh, Pennsylvania, USA, 02/23/2010. doi: 10.2118/131787-MS
- Anderson, D.M., Stotts, G.W.J., Mattar, L. et al. 2006. Production Data Analysis - Challenges, Pitfalls, Diagnostics. Paper 102048 presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, USA, 09/27/2006. doi: 10.2118/102048-MS

- Arnold, R. and Anderson, R. 1908. *Preliminary Report on the Coalinga Oil District, Fresno and Kings Counties, California*. Washington, DC: Govt. Print. Off.
- Arnold, R. and Darnell, J.L. 1920. *Manual for the Oil and Gas Industry under the Revenue Act of 1918*. New York: John Wiley & Sons, Inc.
- Aron, G. and Scott, V.H. 1965. Simplified Solutions for Decreasing Flow in Wells. *Proc. American Society of Civil Engineers* **91** (HY5): 1–12.
- Arps, J.J. 1945. Analysis of Decline Curves. *Petroleum Transactions, AIME* **160**: 228-247. doi: 10.2118/945228-G
- Athichanagorn, S., Horne, R.N., and Kikani, J. 2002. Processing and Interpretation of Long-Term Data Acquired from Permanent Pressure Gauges. *SPE Reservoir Evaluation & Engineering* **5** (5): 384-391. doi: 10.2118/80287-pa
- Bachman, R.C., Vikram, S., Dinara, K. et al. 2011. Examining the Effects of Stress Dependent Reservoir Permeability on Stimulated Horizontal Montney Gas Wells. Paper Society of Petroleum Engineers 149331 presented at the Canadian Unconventional Resources Conference, Alberta, Canada, 11/15/2011. doi: 10.2118/149331-MS
- Bello, R.O. and Wattenbarger, R.A. 2008. Rate Transient Analysis in Naturally Fractured Shale Gas Reservoirs. Paper Society of Petroleum Engineers 114591-MS presented at the CIPC/SPE Gas Technology Symposium 2008 Joint Conference, Calgary, Alberta, Canada, 06/16/2008. doi: 10.2118/114591-MS
- Bello, R.O. and Wattenbarger, R.A. 2010. Multi-Stage Hydraulically Fractured Horizontal Shale Gas Well Rate Transient Analysis. Paper Society of Petroleum Engineers 126754-MS presented at the North Africa Technical Conference and Exhibition, Cairo, Egypt, 02/14/2010. doi: 10.2118/126754-MS
- Bostic, J.N., Agarwal, R.G., and Carter, R.D. 1980. Combined Analysis of Postfracturing Performance and Pressure Buildup Data for Evaluating an Mhf Gas Well. *SPE Journal of Petroleum Technology* **32** (10): 1711-1719. doi: 10.2118/8280-pa
- Carter, R.D. 1985. Type Curves for Finite Radial and Linear Gas-Flow Systems: Constant-Terminal-Pressure Case. *SPE Journal* **25** (5): 719-728. doi: 10.2118/12917-pa
- Cinco-Ley, H. and Samaniego, F. 1981. Transient Pressure Analysis for Fractured Wells. *SPE Journal of Petroleum Technology* **33** (9): 1749-1766. doi: 10.2118/7490-pa

- Clark, K.K. 1968. Transient Pressure Testing of Fractured Water Injection Wells. *SPE Journal of Petroleum Technology* **20** (06): 639-643. doi: 10.2118/1821-pa
- Coats, K.H., Rapoport, L.A., Mccord, J.R. et al. 1964. Determination of Aquifer Influence Functions from Field Data. *SPE Journal of Petroleum Technology* **16** (12): 1417-1424. doi: 10.2118/897-pa
- Cooper, H.H. and Jacob, C.E. 1946. A Generalized Graphical Method of Evaluating Formation Constants and Summarizing Well-Field History. *Trans. Am. Geophys. Union.* **27** (4): 526-534.
- Coyle, E.J., Gabbouj, M., and Jisang, Y. 1993. Order Statistic-Based Nonlinear Filters: Stack Filters and Weighted Median Filters. In *Nonlinear Digital Signal Processing, 1993. IEEE Winter Workshop*:T_3.1-T_3.9.
- Cutler, W.W. 1924. *Estimation of Underground Oil Reserves by Oil-Well Production Curves*. Washington, DC: Govt. Print. Off.
- Cvetkovic, B. 2008. Well Production Decline. Doctor of Philosophy. Norwegian University of Science and Technology.
- Dake, L.P. 1978. *Fundamentals of Reservoir Engineering*. Developments in Petroleum Science, 8. Amsterdam; New York: Elsevier Scientific Pub. Co.
- Doublet, L.E., Pande, P.K., Mccollum, T.J. et al. 1994. Decline Curve Analysis Using Type Curves--Analysis of Oil Well Production Data Using Material Balance Time: Application to Field Cases. Paper Society of Petroleum Engineers 28688 presented at the International Petroleum Conference and Exhibition of Mexico, Veracruz, Mexico, 10/13/1994. doi: 10.2118/28688-ms
- Duong, A.N. 1990. A Straight-Line Approach to Determine the Distance to Barriers. *SPE Production Engineering* **5** (1): 65-69. doi: 10.2118/17917-pa
- Ehlig-Economides, C.A. and Ramey, J., H. J. 1981. Transient Rate Decline Analysis for Wells Produced at Constant Pressure. *SPE Journal* **21** (1): 98-104. doi: 10.2118/8387-pa
- Everdingen, A.F.V. and Hurst, W. 1949. The Application of the Laplace Transformation to Flow Problems in Reservoirs. *Petroleum Transactions, AIME* **186**: 305-324. doi: 10.2118/949305-G
- Everdingen, A.F.V. and Meyer, L.J. 1971. Analysis of Buildup Curves Obtained after Well Treatment. *SPE Journal of Petroleum Technology* **23** (04): 513-524. doi: 10.2118/2864-pa

- Ezekwe, N. 2011. *Petroleum Reservoir Engineering Practice*. Upper Saddle River, NJ: Prentice Hall.
- FAST RTA. 3.5. Fekete Engineering Software & Services.
- Fetkovich, M.J. 1973. Decline Curve Analysis Using Type Curves. Paper 4629 presented at the Annual Fall Meeting of the Society of Petroleum Engineers of AIME, Las Vegas, 09/30/1973. doi: 10.2118/4629-MS
- Fetkovich, M.J. and Vienot, M.E. 1984. Rate Normalization of Buildup Pressure by Using Afterflow Data. *SPE Journal of Petroleum Technology* **36** (12): 2211-2224. doi: 10.2118/12179-pa
- Fraim, M.L. and Wattenbarger, R.A. 1987. Gas Reservoir Decline-Curve Analysis Using Type Curves with Real Gas Pseudopressure and Normalized Time. *SPE Formation Evaluation* **2** (4): 671-682. doi: 10.2118/14238-pa
- Gentry, R.W. 1972. Decline-Curve Analysis. *SPE Journal of Petroleum Technology* **24** (01): 38-41. doi: 10.2118/3356-pa
- Gladfelter, R.E., Tracy, G.W., and Wilsey, L.E. 1955. Selecting Wells Which Will Respond to Production-Stimulation Treatment. Paper American Petroleum Institute 55-117 presented at the Drilling and Production Practice, Amarillo, USA, 03/01/1955. doi: 10.2118/55-117
- Gringarten, A.C. 2006. From Straight Lines to Deconvolution: The Evolution of the State of the Art in Well Test Analysis. Paper Society of Petroleum Engineers 102079 presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, USA, 09/24/2006. doi: 10.2118/102079-ms
- Hale, B.W., Firth, C.H., Hansen, R.C. et al. 1989. Postbuildup Drawdown Analysis of Tight Gas Wells in the Rocky Mountains. *SPE Reservoir Engineering* **4** (3): 288-296. doi: 10.2118/16804-pa
- Helmy, M.W. and Wattenbarger, R.A. 1999. Analysis of Well Performance with Multiple Shut-in Periods. Paper 53932 presented at the Latin American and Caribbean Petroleum Engineering Conference, Caracas, Venezuela, 04/21/1999. doi: 10.2118/53932-MS
- Horner, D.R. 1951. Pressure Build-up in Wells. Paper World Petroleum Congress 4135 presented at the 3rd World Petroleum Congress, The Hague, the Netherlands, 05/28/1951. doi: 10.2118/4135

- Houzé, O., Viturat, D., Fjaere, O. et al. 2011. Dynamic Data Analysis. In:557. Paris, France: KAPPA Engineering
- Hutchinson, T.S. and Sikora, V.J. 1959. A Generalized Water-Drive Analysis. *Petroleum Transactions, AIME* **216**: 169-178. doi: 10.2118/1123-G
- Ibrahim, M., Robert, A.W., and Helmy, W. 2003. Determination of Ogip for Wells in Pseudosteady-State-Old Techniques, New Approaches. Paper Society of Petroleum Engineers 84286 presented at the SPE Annual Technical Conference and Exhibition, Denver, Colorado, 10/05/2003. doi: 10.2118/84286-ms
- Ilk, D. 2005. Deconvolution of Variable Rate Reservoir Performance Data Using B-Splines. Master of Science. Texas A&M University, College Station, Texas.
- Ilk, D., Creties David, J., and Thomas Alwin, B. 2011. Production Analysis in Unconventional Reservoirs - Diagnostics, Challenges, and Methodologies. Paper Society of Petroleum Engineers 144376 presented at the North American Unconventional Gas Conference and Exhibition, The Woodlands, Texas, USA, 06/14/2011. doi: 10.2118/144376-ms
- Jargon, J.R. and Van Poolen, H.K. 1965. Unit Response Function from Varying-Rate Data. *SPE Journal of Petroleum Technology* **17** (08): 965-969. doi: 10.2118/981-pa
- Jones, P. 1962. Reservoir Limit Test on Gas Wells. *SPE Journal of Petroleum Technology* **14** (06): 613-619. doi: 10.2118/24-pa
- Kabir, C.S. and Izgec, B. 2006. Diagnosis of Reservoir Behavior from Measured Pressure/Rate Data. Paper Society of Petroleum Engineers 100384 presented at the SPE Gas Technology Symposium, Calgary, Alberta, Canada, 05/15/2006. doi: 10.2118/100384-ms
- Kamal, M.M. 2009. *Transient Well Testing*. Richardson, Texas.: Society of Petroleum Engineers.
- Khan, W. 2011. *Data Diagnostics - Interference Study of Field Production Data*. Department of Petroleum Engineering: Texas A&M University.
- Kuchuk, F.J., Hollaender, F., Gok, I.M. et al. 2005. Decline Curves from Deconvolution of Pressure and Flow-Rate Measurements for Production Optimization and Prediction. Paper Society of Petroleum Engineers 96002-MS presented at the SPE Annual Technical Conference and Exhibition, Dallas, Texas, 10/09/2011. doi: 10.2118/96002-MS

- Laustsen, D. 1996. Practical Decline Analysis Part 1-Uses and Misuses. *Journal of Canadian Petroleum Technology* **35** (9): 34-38. doi: 10.2118/96-09-das
- Mattar, L. and Anderson, D.M. 2003. A Systematic and Comprehensive Methodology for Advanced Analysis of Production Data. Paper Society of Petroleum Engineers 84472-MS presented at the SPE Annual Technical Conference and Exhibition, Denver, Colorado, 10/05/2003. doi: 10.2118/84472-MS
- Miller, C.C., Dyes, A.B., and C. A. Hutchinson, J. 1950. The Estimation of Permeability and Reservoir Pressure from Bottom Hole Pressure Build-up Characteristics. *Petroleum Transactions, AIME* **189**: 91-104. doi: 10.2118/950091-G
- Miller, F.G. 1962. Theory of Unsteady-State Influx of Water in Linear Reservoirs. *J. Inst. Pet* **48** (467): 365-379.
- Millheim, K.K. and Cichowicz, L. 1968. Testing and Analyzing Low-Permeability Fractured Gas Wells. *SPE Journal of Petroleum Technology* **20** (02): 193-198. doi: 10.2118/1768-pa
- Moghadam, S. and Mattar, L. 2011. Pitfalls of Superposition When Analyzing Production Data. Paper Society of Petroleum Engineers 146727 presented at the Canadian Unconventional Resources Conference, Alberta, Canada, 11/15/2011. doi: 10.2118/146727-ms
- Moore, A.W. and Jorgenson, J.W. 1993. Median Filtering for Removal of Low-Frequency Background Drift. *Analytical Chemistry* **65** (2): 188-191. doi: 10.1021/ac00050a018
- Moran, J.H. and Finklea, E.E. 1962. Theoretical Analysis of Pressure Phenomena Associated with the Wireline Formation Tester. *SPE Journal of Petroleum Technology* **14** (08): 899-908. doi: 10.2118/177-pa
- Mueller, T.D. 1962. Transient Response of Nonhomogeneous Aquifers. *SPE Journal* **2** (1): 33-43. doi: 10.2118/1604-g
- Muhammed, H.H. 2006. *Medical Image Analysis*. Huddinge, Sweden School of Technology and Health, KTH.
- Muskat, M. 1937. Use of Data Oil the Build-up of Bottom-Hole Pressures. *Petroleum Transactions, AIME* **123**: 44-48. doi: 10.2118/937044-G
- Nobakht, M. and Mattar, L. 2009. Diagnostics of Data Quality for Analysis of Production Data. Paper Petroleum Society of Canada 2009-137 presented at the

- Canadian International Petroleum Conference, Calgary, Alberta, 06/16/2009. doi: 10.2118/2009-137
- Nobakht, M., Mattar, L., Moghadam, S. et al. 2010. Simplified yet Rigorous Forecasting of Tight/Shale Gas Production in Linear Flow. Paper Society of Petroleum Engineers 133615-MS presented at the SPE Western Regional Meeting, Anaheim, California, USA, 05/27/2010. doi: 10.2118/133615-MS
- Odeh, A.S. and Jones, L.G. 1965. Pressure Drawdown Analysis, Variable-Rate Case. *SPE Journal of Petroleum Technology* **17** (08): 960-964. doi: 10.2118/1084-pa
- Orkhan, S., Hasan, A.a.A., and Robert, A.W. 2011. A Semi-Analytic Method for History Matching Fractured Shale Gas Reservoirs. Paper Society of Petroleum Engineers 144583 presented at the SPE Western North American Region Meeting, Anchorage, Alaska, USA, 07/05/2011. doi: 10.2118/144583-ms
- Palacio, J. and Blasingame, T. 1993. Decline Curve Analysis Using Type Curves: Analysis of Gas Well Production Data. Paper Society of Petroleum Engineers 25909 presented at the SPE Joint Rocky Mountain and Low Permeability Symposium, Denver, Colorado, 04/12/1993. doi: 10.2118/25909-MS
- Peter, L., Louis, M., and Samane, M. 2011. Analyzing Variable Rate/Pressure Data in Transient Linear Flow in Unconventional Gas Reservoirs. Paper Society of Petroleum Engineers 149472 presented at the Canadian Unconventional Resources Conference, Alberta, Canada, 11/15/2011. doi: 10.2118/149472-ms
- Raghavan, R. 1993. *Well Test Analysis*. Prentice Hall Petroleum Engineering Series. Englewood Cliffs, New Jersey.: PTR Prentice Hall.
- Ramey, J., H. J. 1965. Non-Darcy Flow and Wellbore Storage Effects in Pressure Build-up and Drawdown of Gas Wells. *SPE Journal of Petroleum Technology* **17** (02): 223-233. doi: 10.2118/1058-pa
- Ramey, J., H. J. 1970. Short-Time Well Test Data Interpretation in the Presence of Skin Effect and Wellbore Storage. *SPE Journal of Petroleum Technology* **22** (01): 97-104. doi: 10.2118/2336-pa
- Ramey, J., H. J. 1976. Verification of the Gladfelter-Tracy-Wilsey Concept for Wellbore Storage-Dominated Transient Pressures During Production. *Journal of Canadian Petroleum Technology* **15** (2): 84-85. doi: 10.2118/76-02-09
- Ramsay, H.J. 1968. The Ability of Rate-Time Decline Curves to Predict Future Production Rates. Master of Science. The University of Tulsa, Tulsa, Oklahoma.

- Ridley, T.P. 1975. The Unified Analysis of Well Tests. Paper Society of Petroleum Engineers 5587 presented at the Fall Meeting of the Society of Petroleum Engineers of AIME, Dallas, Texas, 28/09/1975. doi: 10.2118/5587-ms
- Slider, H.C. 1968. A Simplified Method of Hyperbolic Decline Curve Analysis. *SPE Journal of Petroleum Technology* (03). doi: 10.2118/1936-pa
- Stehfest, H. 1970. Algorithm 368: Numerical Inversion of Laplace Transforms [D5]. *Commun. ACM* **13** (1): 47-49. doi: 10.1145/361953.361969
- Sternberg, Y.M. 1968. Simplified Solution for Variable Rate Pumping Test. *American Society of Civil Engineers* **94** (HY1): 177-180.
- Streltsova, T.D. 1988. *Well Testing in Heterogeneous Formations*. New York: Wiley.
- Theis, C.V. 1935. The Relation between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage. *Trans. Am. Geophys. Union*. **16** (2): 519-524.
- Tukey, J.W. 1977. *Exploratory Data Analysis*. Reading, Massachusetts: Addison-Wesley Pub. Co.
- Turner, R.G., Hubbard, M.G., and Dukler, A.E. 1969. Analysis and Prediction of Minimum Flow Rate for the Continuous Removal of Liquids from Gas Wells. *SPE Journal of Petroleum Technology* **21** (11): 1475-1482. doi: 10.2118/2198-pa
- Uraiet, A.A. and Raghavan, R. 1980. Unsteady Flow to a Well Producing at a Constant Pressure. *SPE Journal of Petroleum Technology* **32** (10): 1803-1812. doi: 10.2118/8768-pa
- Wattenbarger, R.A., El-Banbi, A.H., Villegas, M.E. et al. 1998. Production Analysis of Linear Flow into Fractured Tight Gas Wells. Paper Society of Petroleum Engineers 39931-MS presented at the SPE Rocky Mountain Regional/Low-Permeability Reservoirs Symposium, Denver, Colorado, 04/05/1998. doi: 10.2118/39931-MS
- Whitson, C.H. and Sognesand, S. 1990. Application of the Van Everdingen-Meyer Method for Analyzing Variable-Rate Well Tests. *SPE Formation Evaluation* **5** (1): 67-75. doi: 10.2118/15482-pa
- Winestock, A.G. and Colpitts, G.P. 1965. Advances in Estimating Gas Well Deliverability. *Journal of Canadian Petroleum Technology* **4** (4): 111-119. doi: 10.2118/65-03-01

Zais, E.J. 1980. *Production Decline Analysis Using Influence Functions*. Corvallis, Oregon: Elliot Zais & Assoc., Inc.

APPENDIX A

FILTERING TECHNIQUES

The main filtering techniques presented in this work are concerned with filtering rate problematic issues. The main effective filtering techniques for rate problems are the liquid-loading filter and the median filter.

A.1 Liquid-loading filter

Liquid-loading filter is based on the correlation presented by Turner et al. (1969). The correlation was developed for a droplet model. The model assumes free flowing liquid in the wellbore which forms droplets suspended in the gas stream. The liquid droplets are subjected to two forces; gravity and drag forces. The gravity pulls the droplets down, while drag force pushes the droplets upward due to flowing gas as shown in Fig(A-1). If the velocity of the gas is sufficient, the drops are carried to the surface. Otherwise, they fall and accumulate in the wellbore. The model was tested for large number of real data. It is also verified to liquid-gas ratio up to 130bbl/MMscf, whereas the range of applicability might be for surface pressures as low as 5 to 800 psi.

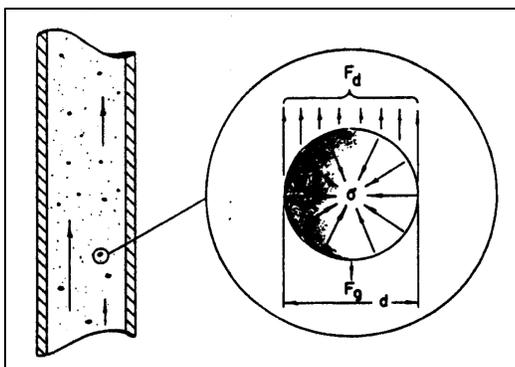


Fig.A- 1 Droplet model, form Turner et al. (1969)

Two variations of the correlation were developed, one for free water production and the other for free condensate production. The calculation of minimum gas velocity for each is:

$$v_{g (water)} = \frac{5.62(67 - 0.0031p)^{1/4}}{(0.0031p)^{1/2}} \dots\dots\dots(A-1)$$

$$v_{g (condensate)} = \frac{4.02(45 - 0.0031p)^{1/4}}{(0.0031p)^{1/2}} \dots\dots\dots(A-2)$$

From the minimum gas velocity, the minimum gas flow rate required for liquids removal can be calculated using:

$$q_{critical} = \frac{(3.06 p v_g A)}{ZT} \dots\dots\dots(A-3)$$

If the production data is suffering liquid loading, then gas flow rate will drop the critical-Turner-rate ($q_{critical} / q_g > 1$). Liquid loading is characterized with erratic production rate data, sudden increase in decline rate, and/or severe rate fluctuations and spikes. Since the superposition time functions involve rate components, the superposition time will be greatly affected with liquid loading. Accordingly, points below the critical-Turner-rate should be filtered out.

A.2 Median filter

Median filter was introduced as a tool for data analyses by Tukey (1977). He first called it "running median" (Coyle et al., 1993). Later, it became a canonical signal and image processing operation. In signal and image processing, median filter is best known for its salt and pepper noise removal aptitude. Therefore, it is typically used in removal of noise in scanned images, removal of cosmic ray spikes from image data, and cleaning pitches from noise in speech processing (Moore and Jorgenson, 1993).

Rate noise and outliers are best filtered with the median filter, which is best known for filtering out impulse characteristics such as noise spikes. However, it still preserves and passes broader features of the data.

In median filtering operation, a filtering window, usually of odd width, is stepped one window at a time along the production data. The rate values in the neighborhood window are ranked according to their magnitude. The median value is calculated for the specified window points, then comparing it with the value of each point. Whenever the difference is within a certain tolerance, the point will be preserved, otherwise filtered out. It is worth to mention that the filtration is applied on the rate data after calculating the superposition times. The median filtering process is depicted in Fig(A-2) and Fig(A-3), whereas Table A-1 lists a VBA code which explain the process of the median filtering.

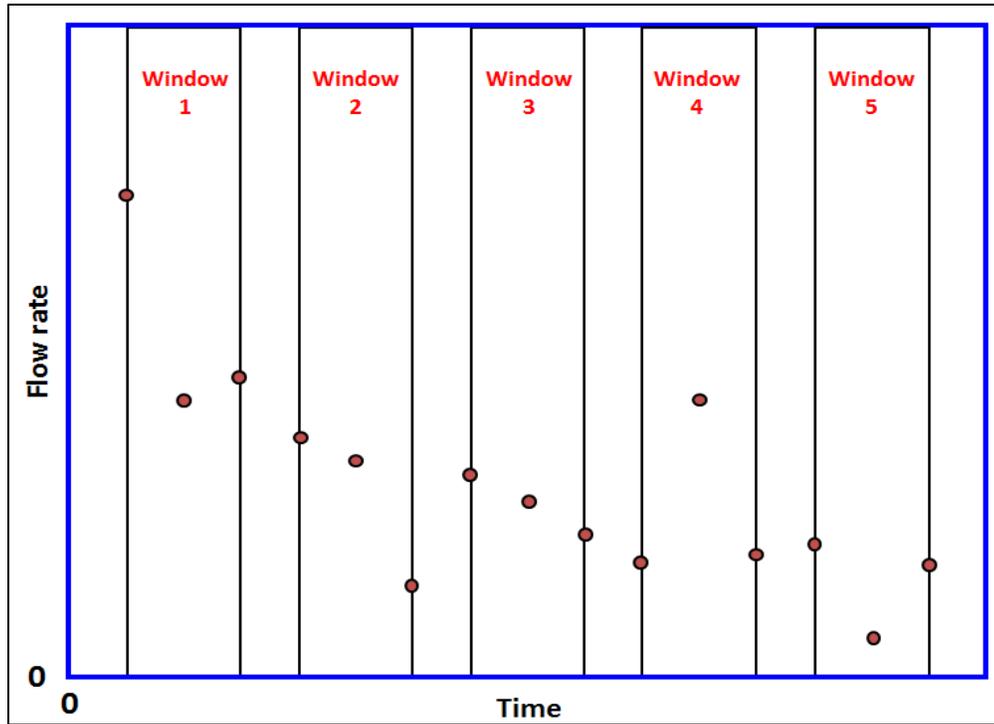


Fig.A- 2 Raw data to be filtered with median filter with 3 points window

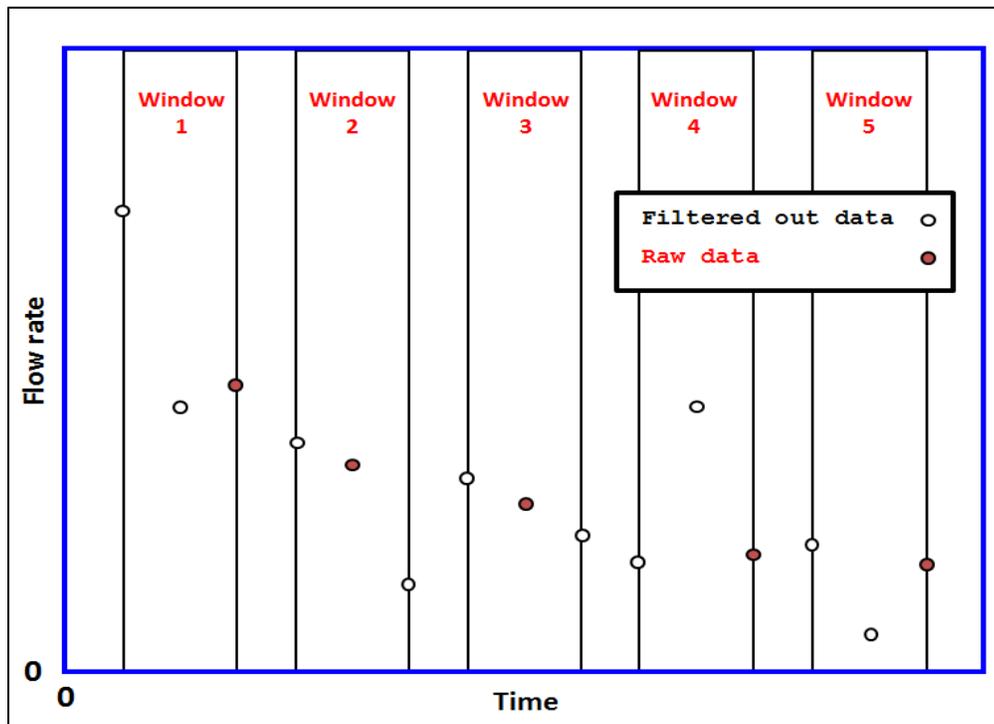


Fig.A- 3 Depiction of the median filtering process with 3 points window

Table A- 1 VBA code of Median filter

```
Sub Median()  
Dim n As Integer, i As Integer, j As Integer, P() As Double, Median As  
Double, k As Double  
  
With ThisWorkbook.Worksheets("Sheet1")  
    n = Application.CountA(Sheets("Sheet1").Range("D4:D10000"))  
    r = .Cells(2, 2) `filter window width  
    j = 0  
    ReDim P(1 To r) As Double  
  
    For i = 1 To n  
        j = j + 1  
        P(j) = .Cells(3 + i, 4)  
  
        If j = r Then  
            Median = Application.Median(P())  
  
            For k = 1 To r  
                If P(k) < Median Or P(k) > Median Then  
                    .Cells(3 + i - r + k, 5) = ""  
                Else  
                    .Cells(3 + i - r + k, 5) = .Cells(3 + i - r + k, 4)  
                End If  
            Next k  
  
            j = 0  
        End If  
  
    Next i  
End With  
End Sub
```

APPENDIX B

TYPE-CURVES MATCHING PROCEDURE

Wattenbarger et al. (1998) presented an analytical solution for linear flow. They presented analytical method of analysis for tight gas fractured wells in which the linear flow is dominant with late boundary effect. Linear flow is characterized by half slope line when production rate is plotted versus time on log-log plot. Once the outer boundary is detected, the OGIP can be calculated. The analytical solutions are represented by:

- Constant rate production:

$$\left(\frac{x_e}{y_e}\right) p_D = \frac{\pi}{2} \left[\frac{1}{3} + t_{Dye} \right] - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp[-n^2 \pi^2 t_{Dye}] \dots \dots \dots (B-1)$$

- Constant pressure production:

$$\left(\frac{x_f}{y_e}\right) \frac{1}{q_D} = \frac{\frac{\pi}{4}}{\sum_{n_{odd}}^{\infty} \exp\left[-\frac{n^2 \pi^2}{4} t_{Dye}\right]} \dots \dots \dots (B-2)$$

For calculating the OGIP, this work will be using the type-curves matching procedure proposed by Theis (1935). Using the dimensionless variables for constant pressure gas case:

- First using the dimensionless rate:

$$q_D = \frac{1422 T}{kh} \frac{q_g}{[m(p_i) - m(p_{wf})]} \dots \dots \dots (B-3)$$

$$q_D \left(\frac{y_e}{x_f} \right) = \frac{1422 T}{kh} \frac{q_g}{[m(p_i) - m(p_{wf})]} \left(\frac{y_e}{x_f} \right) \dots\dots\dots (B-4)$$

$$k \left(\frac{x_f}{y_e} \right) = \frac{1422 T}{h} \frac{q_g}{q_D \left(\frac{y_e}{x_f} \right)} \dots\dots\dots (B-5)$$

Let the term $\frac{q_D \left(\frac{y_e}{x_f} \right)}{q_g [m(p_i) - m(p_{wf})]}$ represented by R_{shift} , then;

$$k \left(\frac{x_f}{y_e} \right) = \frac{1422 T}{h} \frac{1}{R_{shift}} \dots\dots\dots (B-6)$$

- Then, using the dimensionless rate:

$$t_{Dye} = \frac{0.00633 kt}{\phi \mu c_t y_e^2} \dots\dots\dots (B-7)$$

$$\frac{y_e}{\sqrt{k}} = \sqrt{\frac{0.00633}{\phi \mu c_t} \left(\frac{1}{\frac{t_{Dye}}{t}} \right)} \dots\dots\dots (B-8)$$

Let the term $\left(\frac{1}{\frac{t_{Dye}}{t}} \right)$ represented by $\left(\frac{1}{t_{shift}} \right)$, and then simplifying the equations

more.

$$\frac{y_e}{\sqrt{k}} = \sqrt{\frac{0.00633 \left(\frac{1}{t_{shift}} \right)}{\phi \mu c_t}} \dots\dots\dots (B-9)$$

$$x_f \sqrt{k} = \sqrt{\frac{0.00633 \left(\frac{1}{t_{shift}} \right)}{\phi \mu c_t}} \frac{1422 T}{h} \frac{1}{R_{shift} \sqrt{t_{shift}}} \dots\dots\dots (B-10)$$

The drainage can be then calculated using the equation proposed by Wattenbarger et al. (1998):

$$A = 4 \times y_e \times x_e \dots\dots\dots (B-11)$$

$$A = \frac{4 \times 0.00633}{\phi \mu c_t} \frac{1422 T}{h} \frac{1}{R_{shift} t_{shift}} \dots\dots\dots (B-12)$$

Finally, the OGIP is calculated using:

$$OGIP = \frac{A \phi h S_{gi}}{B_{gi}} \dots\dots\dots (B-13)$$

APPENDIX C

INADEQUACY IN CONVERTING TRANSIENT LINEAR FLOW

The superposition time application shows limitation of in the conversion of the constant-pressure linear flow solution to its counterpart constant-rate solution. Material balance time has inadequacy in converting constant-pressure transient linear flow solution. Although the material balance time provides the conversion of the constant-pressure production case to an equivalent constant-rate case, the conversion is incomplete in the case of linear flow regime. Unlike the radial flow regime, the material balance time application to linear flow regime shows a pronounced difference and can be quantified. In terms of horizontal shift the difference is 1.233, whereas the vertical shift of the normalized rate is about 1.41. However, these shifts apply only during pure transient linear flow. Using the early time approximations of Wattenbarger et al. (1998) solutions:

- Constant pressure early time approximation :

$$q_D = \frac{2}{\pi \sqrt{\pi t_{DCP}}} \dots\dots\dots (C-1)$$

- Constant rate early time approximation :

$$\frac{1}{p_D} = \frac{1}{\sqrt{\pi t_{DCR}}} \dots\dots\dots (C-2)$$

As a reference point, it is good to have an idea about the difference between these two solutions at purely transient linear flow. The vertical difference is:

$$\frac{1}{\frac{p_D}{q_D}} = \frac{\pi}{2} \approx 1.5708 \dots\dots\dots (C-3)$$

Solve the early time approximation equations simultaneously for t_{DCR} assuming that the q_D will be shifted completely to $1/p_D$, as;

$$\frac{1}{\sqrt{\pi t_{DCR}}} = \frac{2}{\pi \sqrt{\pi t_{DCP}}} \dots\dots\dots (C-4)$$

With simplification then horizontal difference will be about 2.46 as follows:

$$t_{DCR} = \frac{\pi^2}{4} t_{DCP} \dots\dots\dots (C-5)$$

Using this relation the constant rate early time approximation can also be represented as:

$$\frac{1}{p_D} = \frac{1}{\sqrt{\pi \left(\frac{\pi^2}{4} t_{DCP} \right)}} \dots\dots\dots (C-6)$$

In order to quantify the difference as resulted from the application of material balance time, first the dimensionless material balance time should be calculated:

$$t_{DMB} = \frac{Q_D}{q_D} \dots\dots\dots (C-7)$$

The dimensionless cumulative production is calculated as:

$$Q_D = \int_0^{t_{DCP}} q_D dt_{DCP} \dots\dots\dots (C-8)$$

Substituting the dimensionless cumulative production and the dimensionless production rate gives:

$$t_{DMB} = \frac{\int_0^{t_{DCP}} \frac{2}{\pi\sqrt{\pi t_{DCP}}} dt_{DCP}}{\frac{2}{\pi\sqrt{\pi t_{DCP}}}} \dots\dots\dots(C-9)$$

Integrating and simplifying the equation further then the dimensionless material balance time will be:

$$t_{DMB} = 2 t_{DCP} \dots\dots\dots(C-10)$$

Using this relation, the horizontal difference between the actual constant-pressure time and the material balance time can be quantified as follows:

$$t_{DCP} = \frac{\pi^2}{8} t_{DMB} \dots\dots\dots(C-11)$$

Converting the constant-pressure solution to an equivalent constant-rate using the material balance time as follows:

$$q_{DMB} = \frac{2}{\pi\sqrt{\pi 2 t_{DCP}}} \dots\dots\dots(C-12)$$

$$\frac{p_D}{q_{DMB}} = \frac{\frac{1}{\sqrt{\pi \left(\frac{\pi^2}{4} t_{DCP} \right)}}}{\frac{2}{\pi\sqrt{\pi 2 t_{DCP}}}} \dots\dots\dots(C-13)$$

$$\frac{p_D}{q_{DMB}} = \sqrt{2} \approx 1.41 \dots\dots\dots(C-14)$$

APPENDIX D

ADDITIONAL FIELD CASES

D.1 Well W-84, Barnett shale

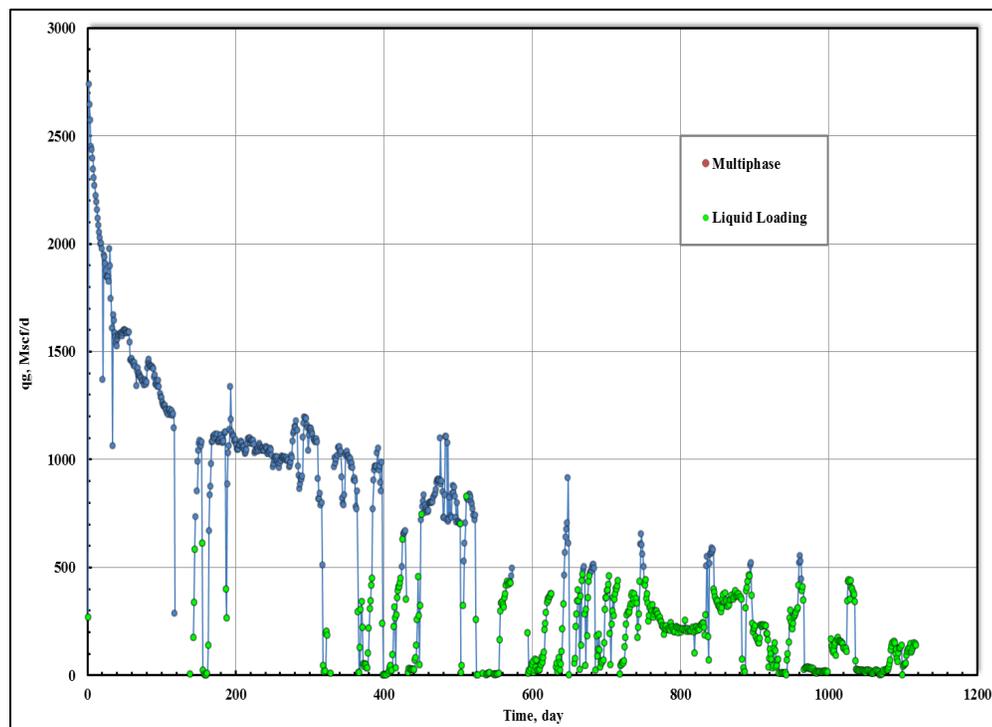


Fig.D- 1 Production rate data of well W-84 with production issues highlighted

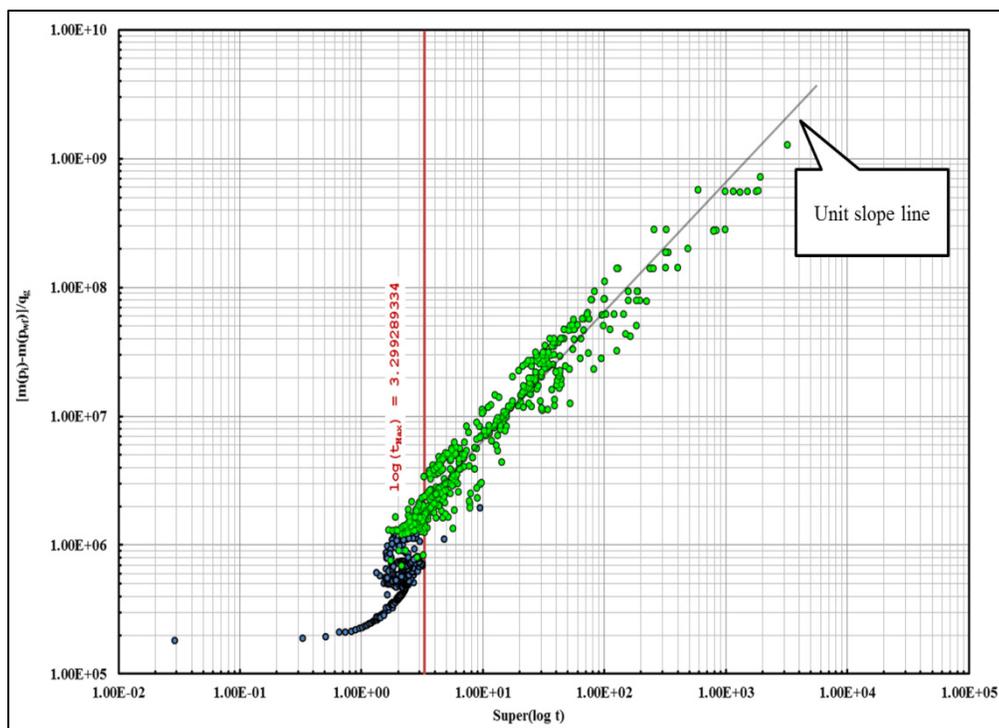


Fig.D- 2 Log-log Superposition-time plot for radial flow of well W-84 with rate issues highlighted

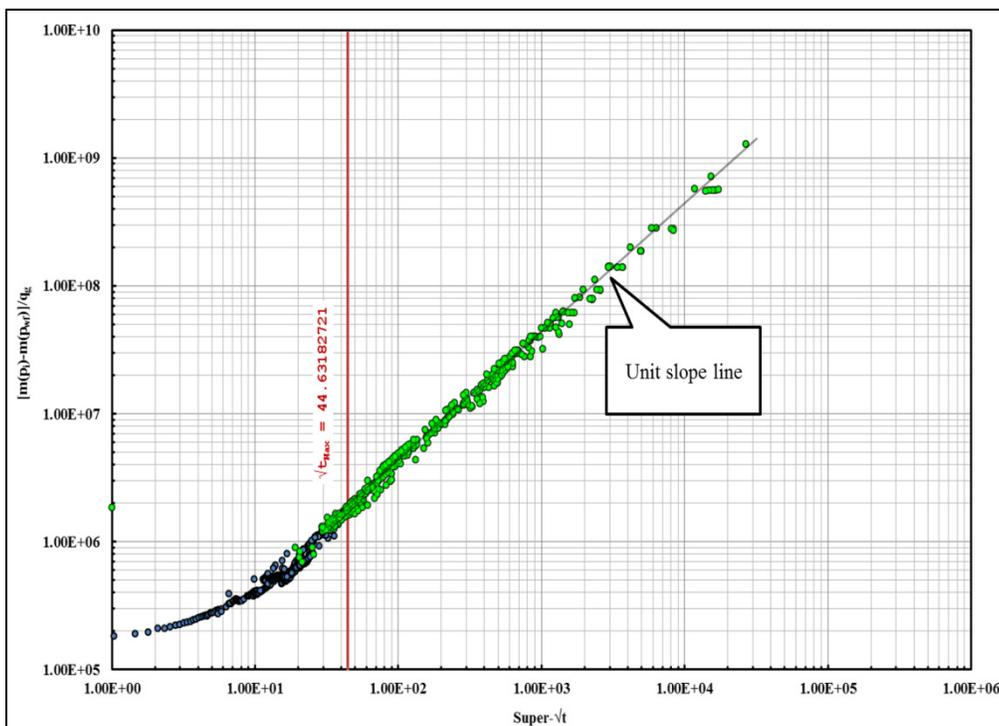


Fig.D- 3 Log-log Superposition-time plot for linear flow of well W-84 with rate issues highlighted

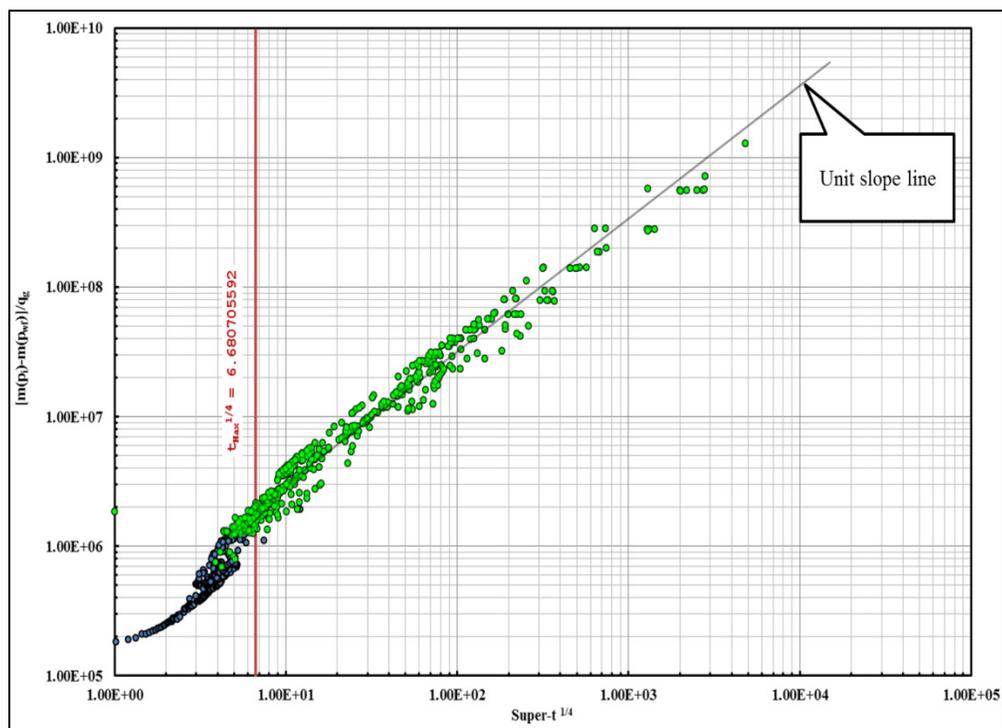


Fig.D- 4 Log-log Superposition-time plot for bilinear flow of well W-84 with rate issues highlighted

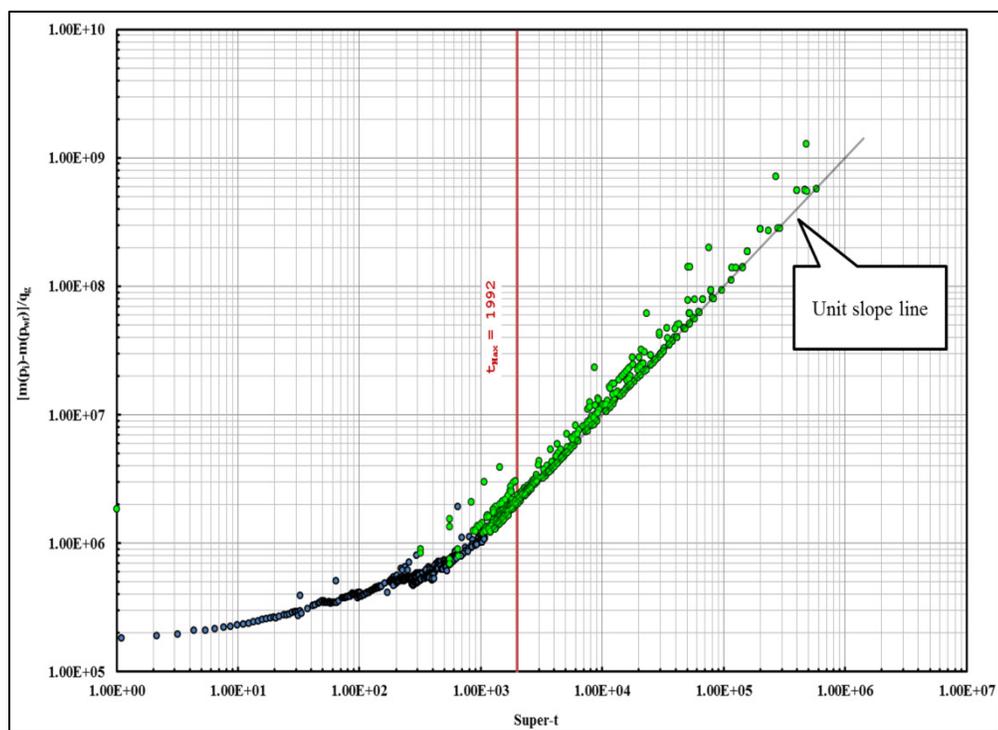


Fig.D- 5 Log-log Superposition-time plot for PSS flow of well W-84 with rate issues highlighted

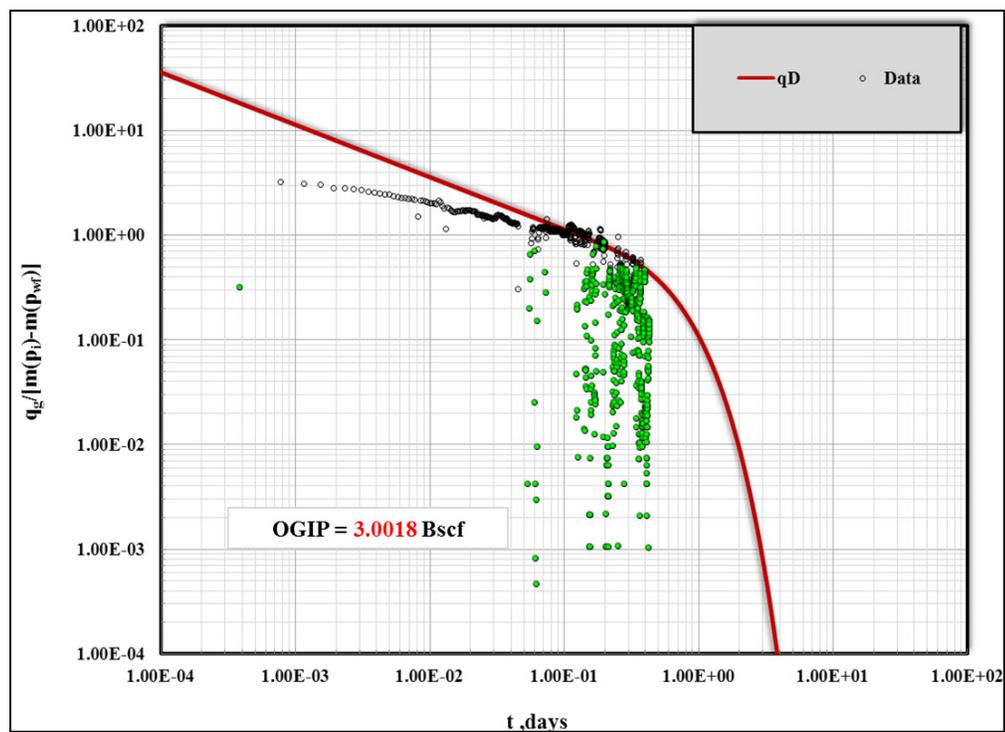


Fig.D- 6 Constant pressure type-curve matching of filtered data of well W-84

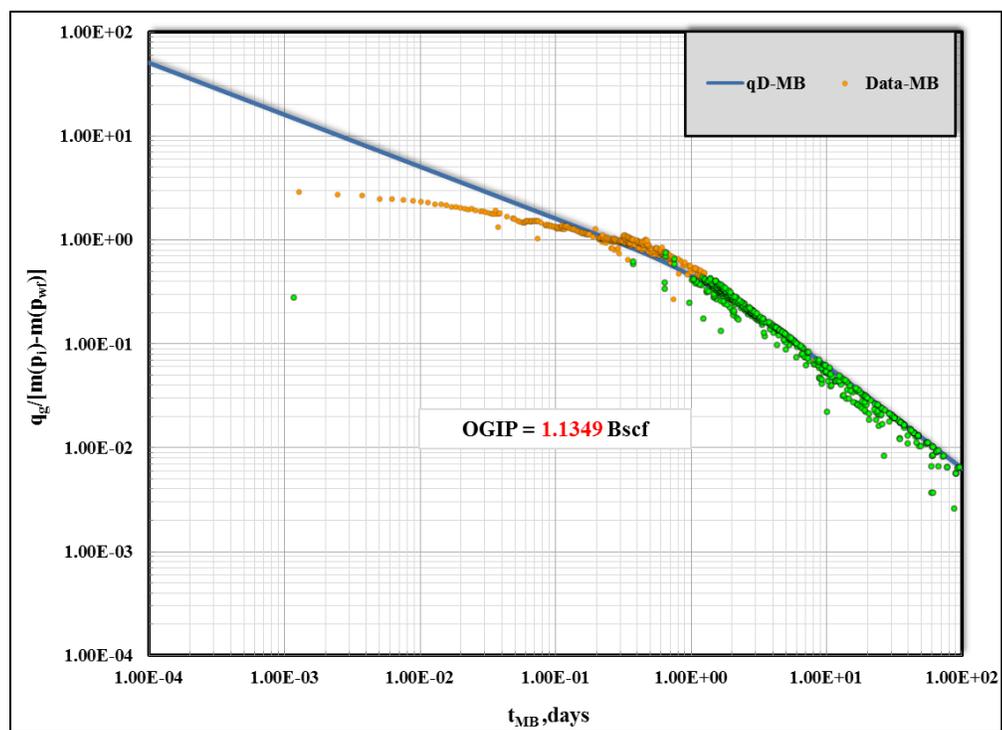


Fig.D- 7 Type-curve matching of constant-rate unfiltered data of well W-84

D.2 Well W-114, Barnett shale

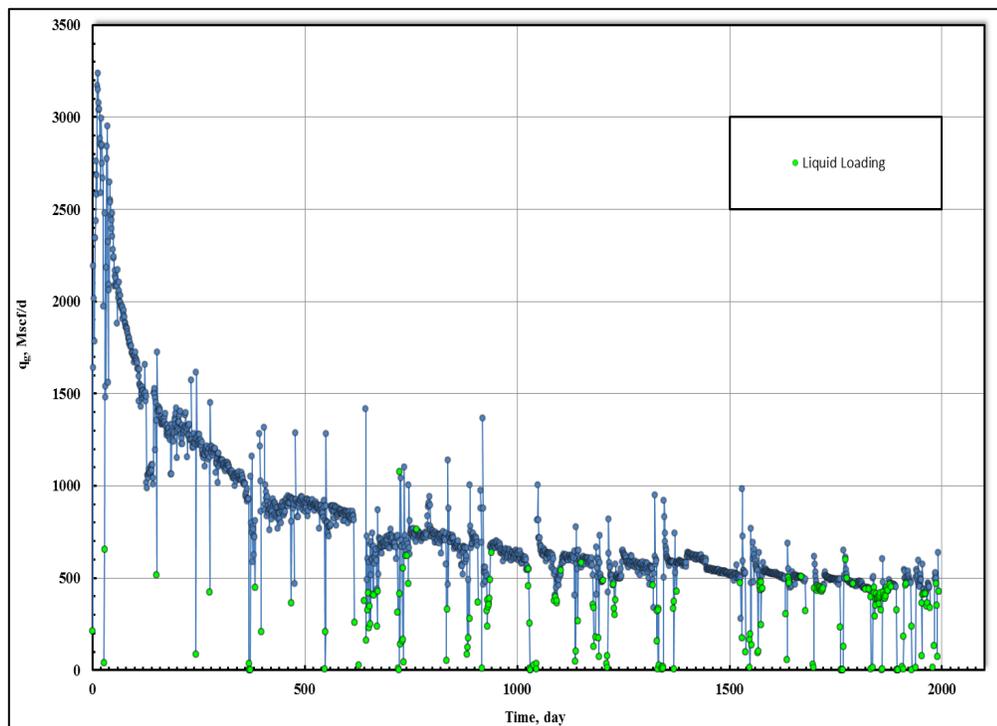


Fig.D- 8 Production rate data of well W-114 with production issues highlighted

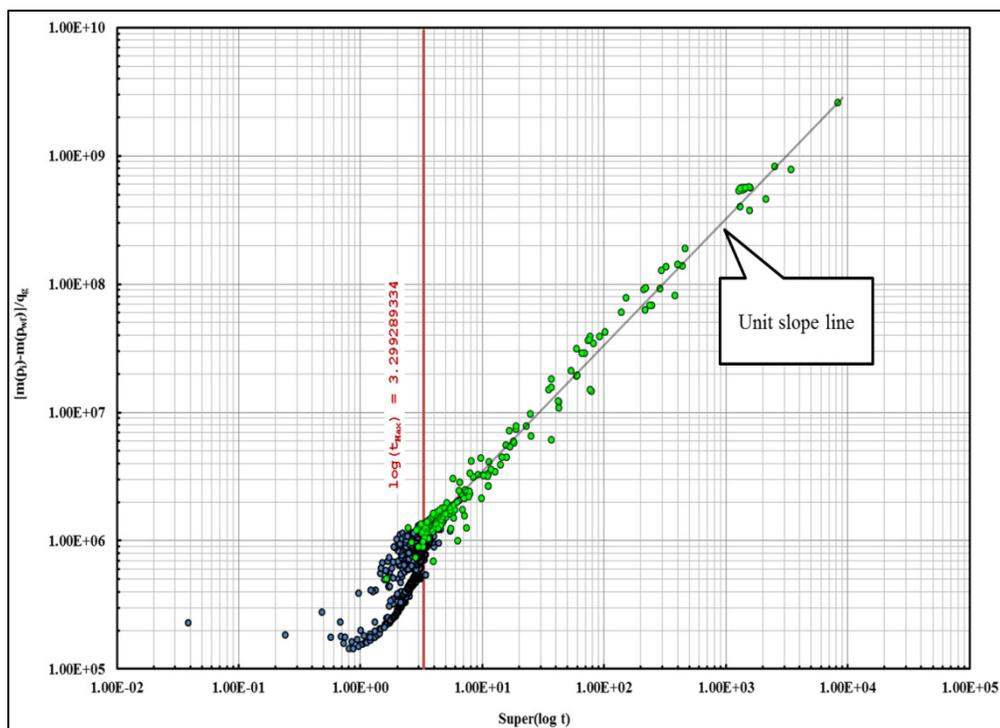


Fig.D- 9 Log-log Superposition-time plot for radial flow of well W-114 with rate issues highlighted

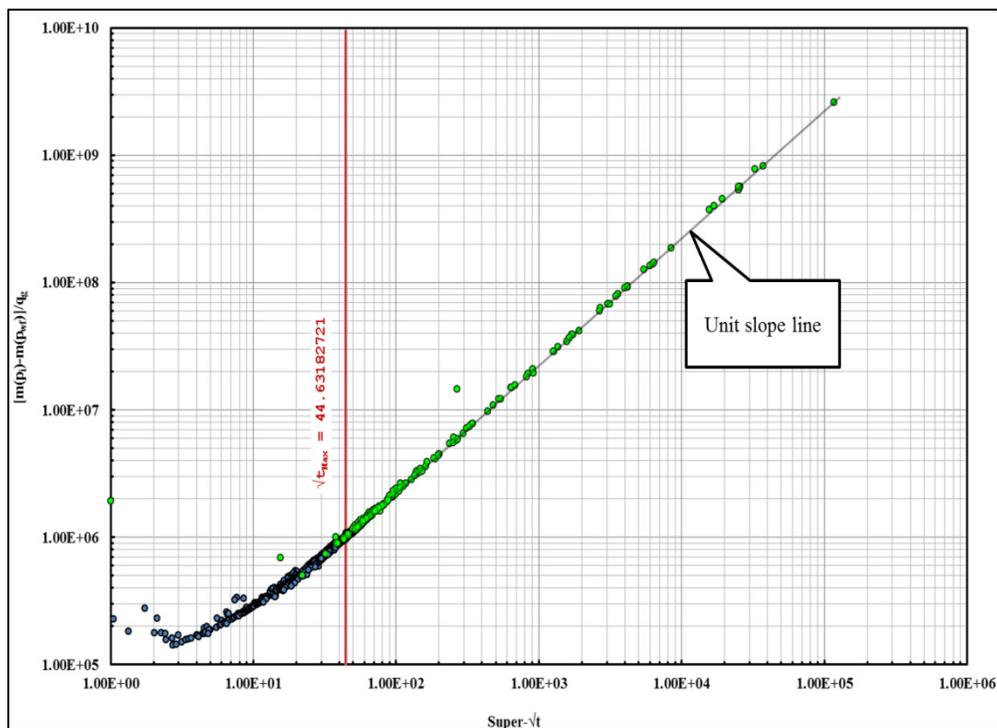


Fig.D- 10 Log-log Superposition-time plot for linear flow of well W-114 with rate issues highlighted

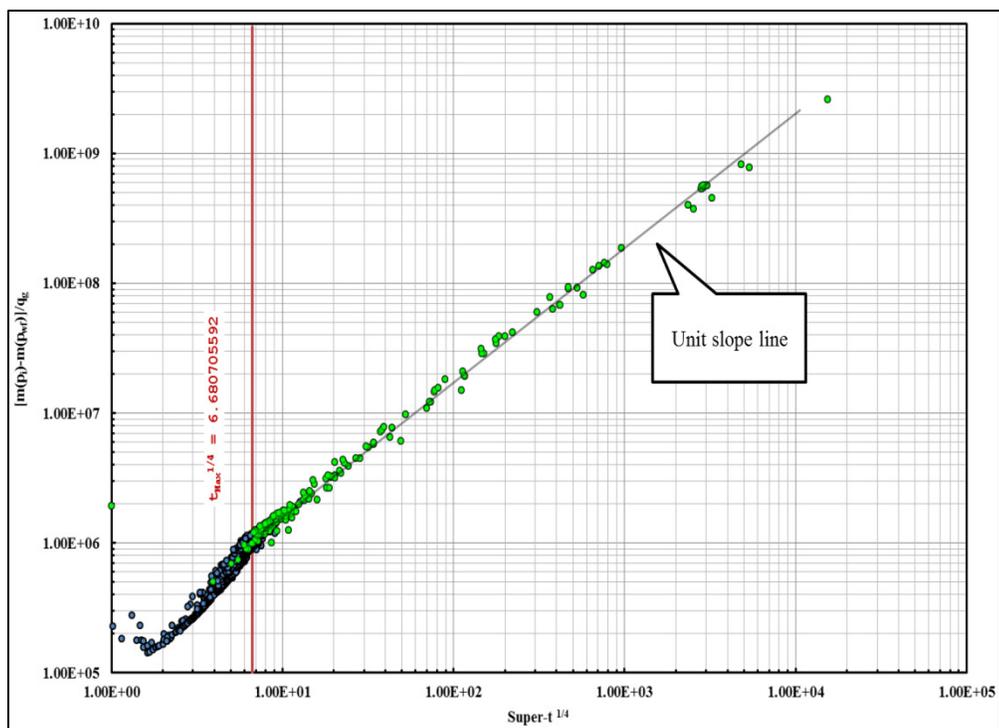


Fig.D- 11 Log-log Superposition-time plot for bilinear flow of well W-114 with rate issues highlighted

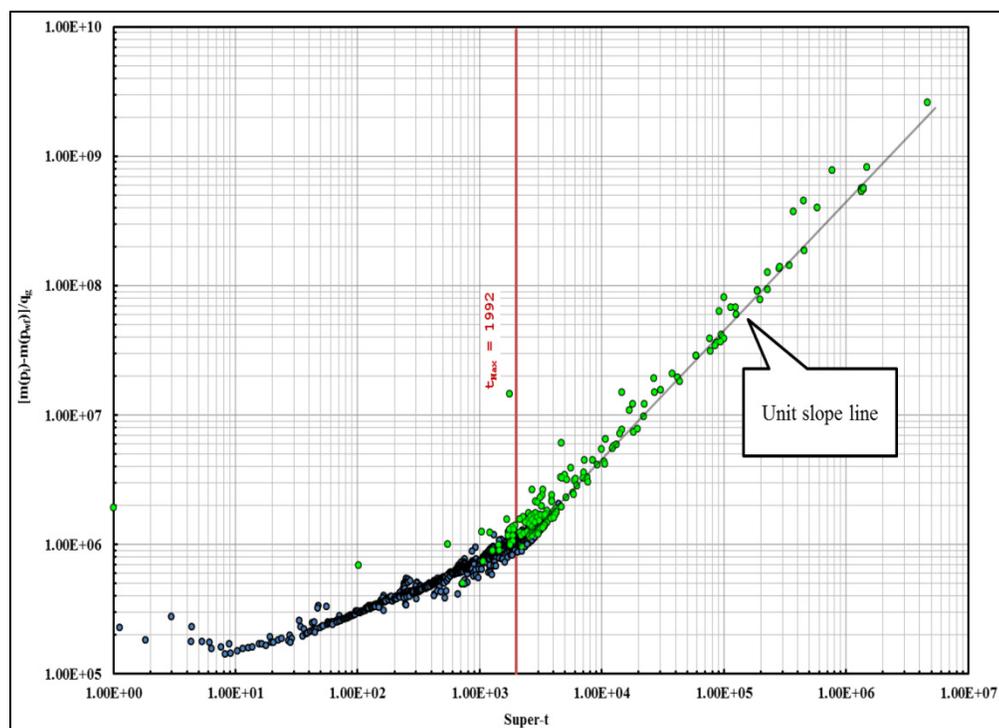


Fig.D- 12 Log-log Superposition-time plot for PSS flow of well W-114 with rate issues highlighted

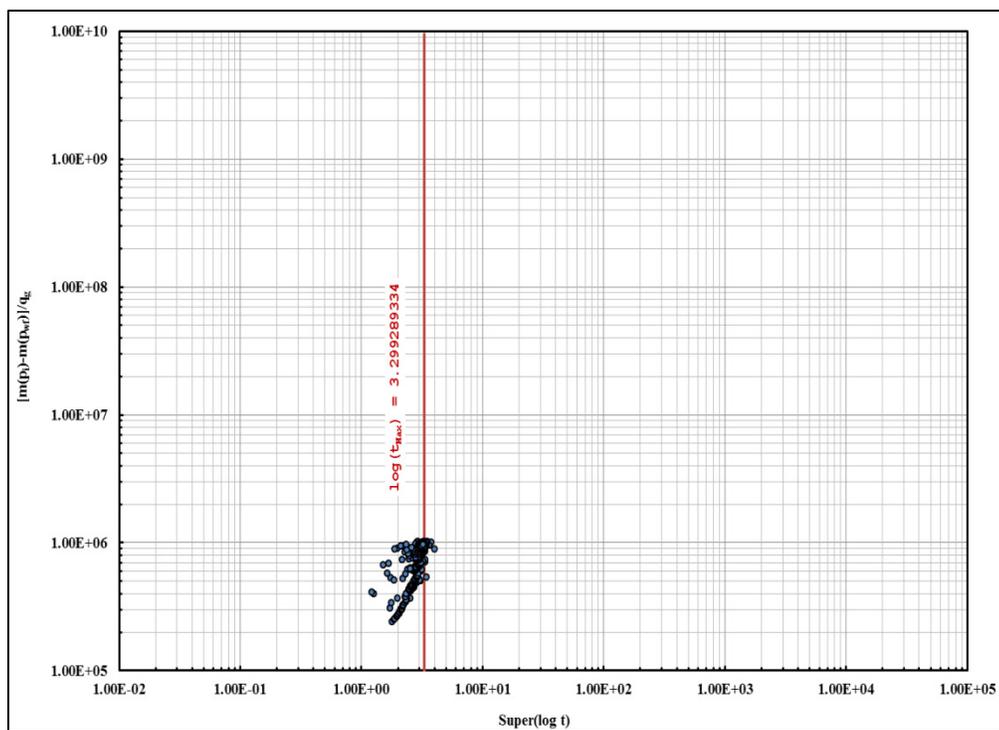


Fig.D- 13 Log-log Superposition-time plot for radial flow of well W-114 with rate issues filtered out

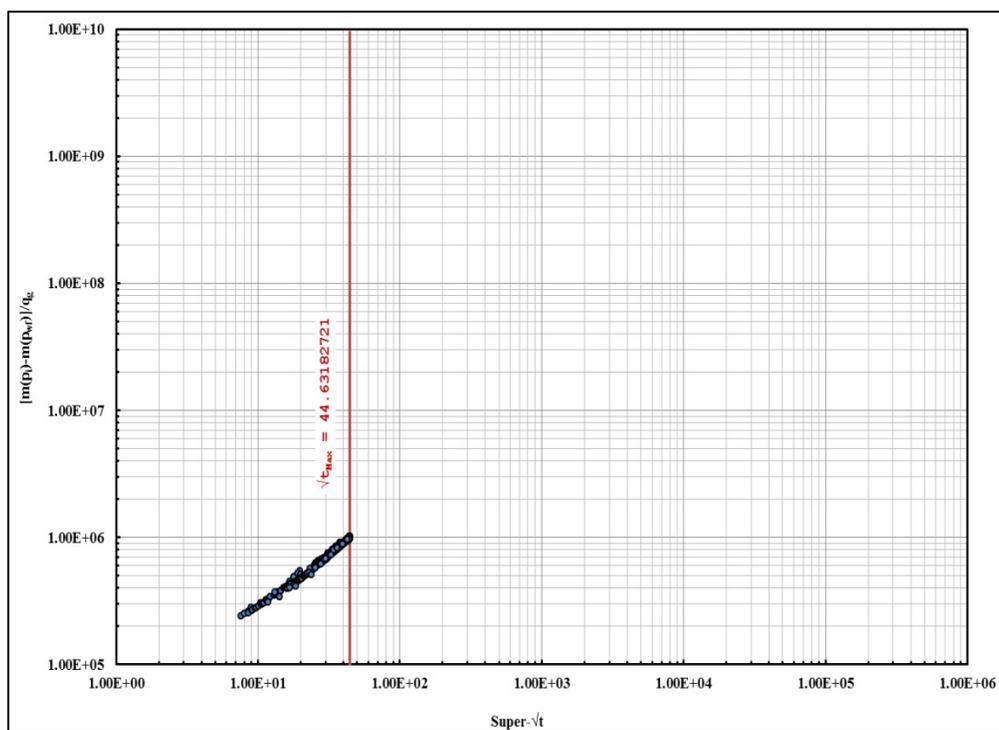


Fig.D- 14 Log-log Superposition-time plot for linear flow of well W-114 with rate issues filtered out

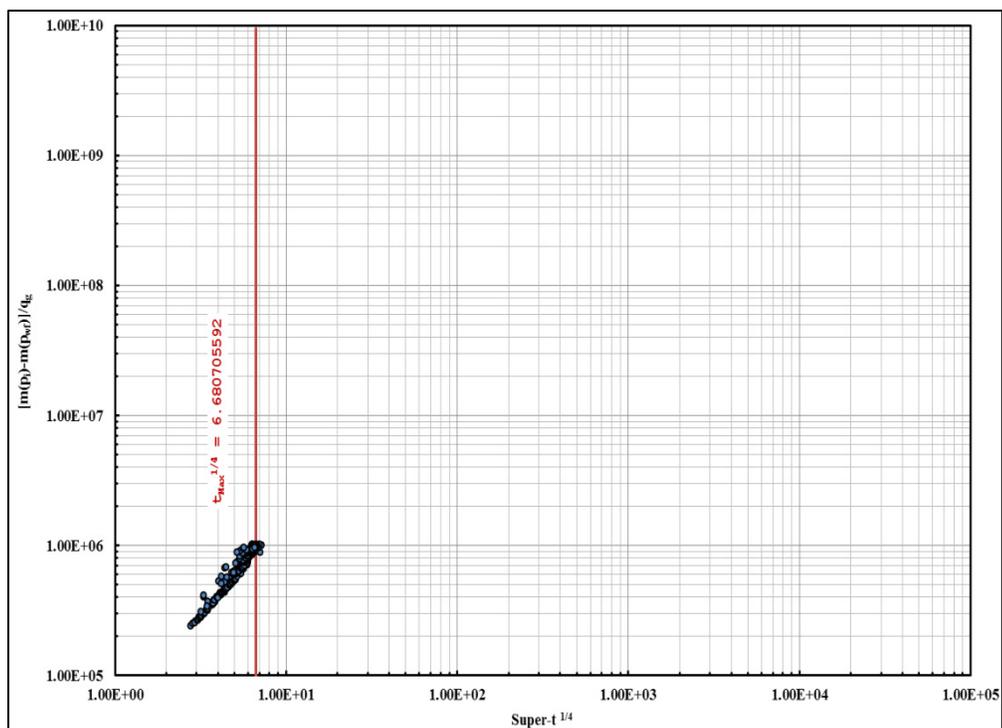


Fig.D- 15 Log-log Superposition-time plot for bilinear flow of well W-114 with rate issues filtered out

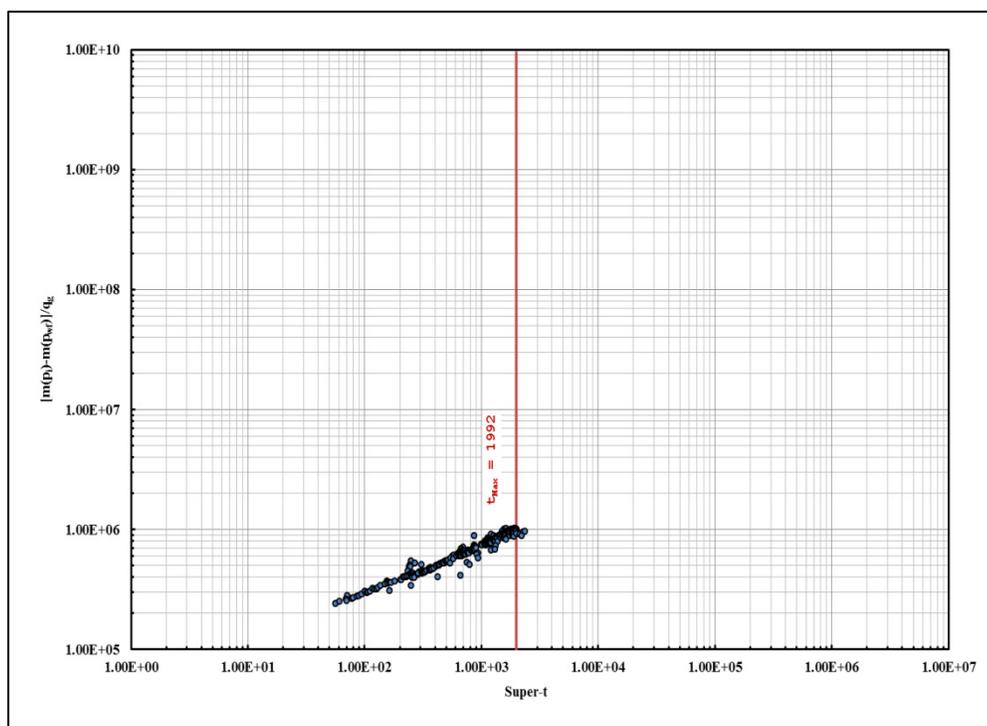


Fig.D- 16 Log-log Superposition-time plot for PSS flow of well W-114 with rate issues filtered out

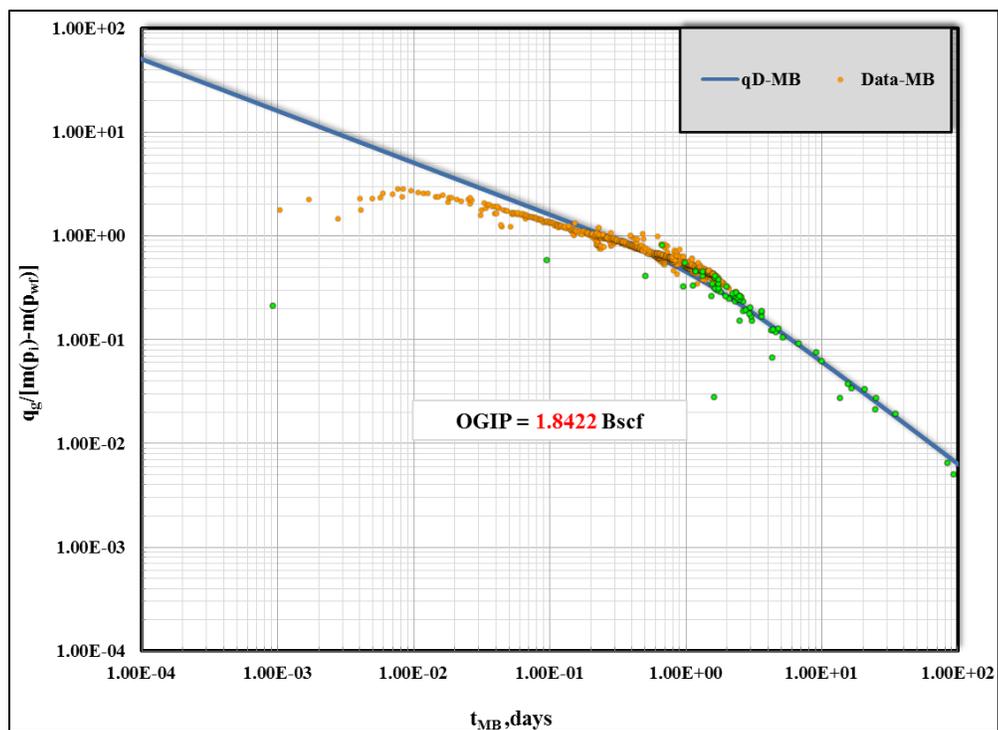


Fig.D- 17 Type-curve matching of constant-rate unfiltered data of well W-114

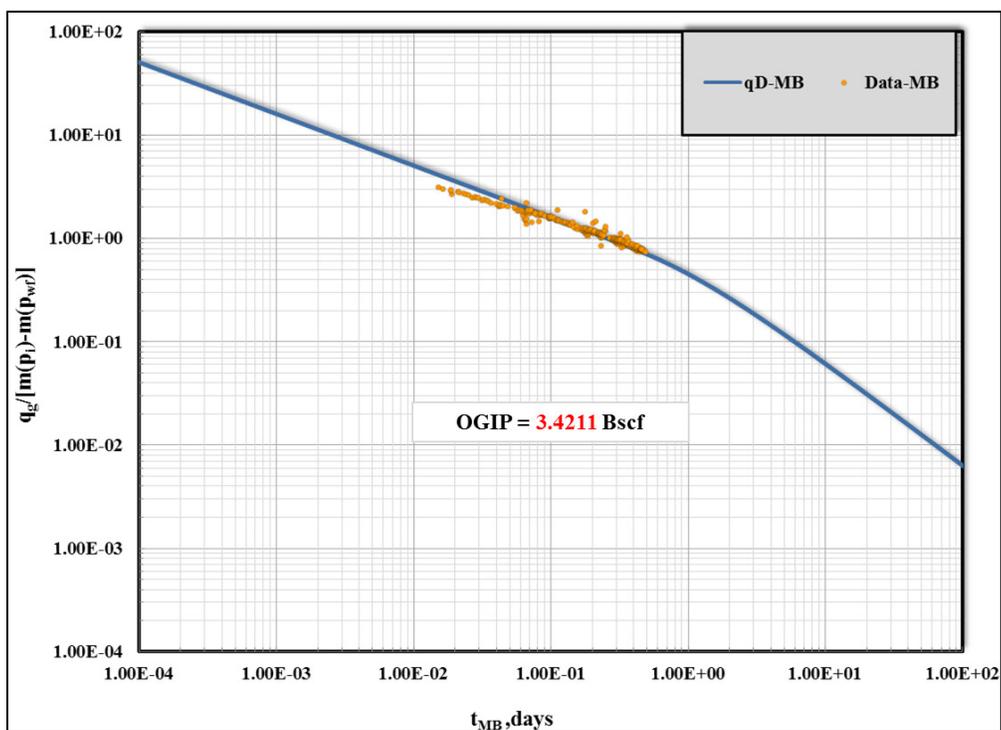


Fig.D- 18 Type-curve matching of constant-rate filtered data of well W-114

D.3 Well W-4, Marcellus shale

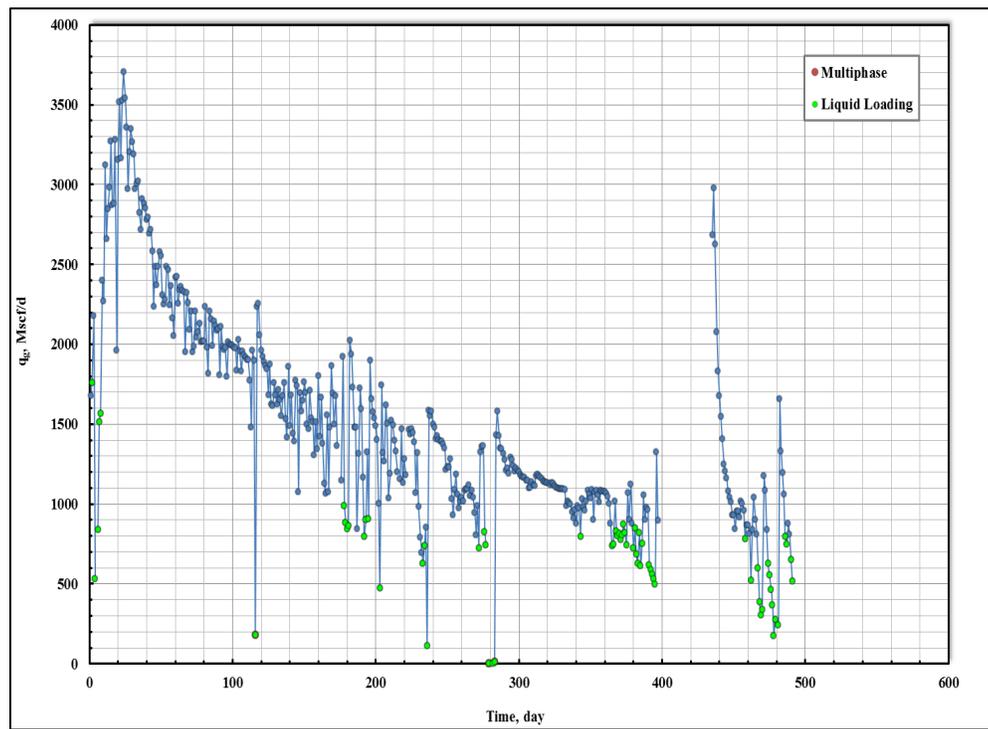


Fig.D- 19 Production rate data of well W-4 with production issues highlighted

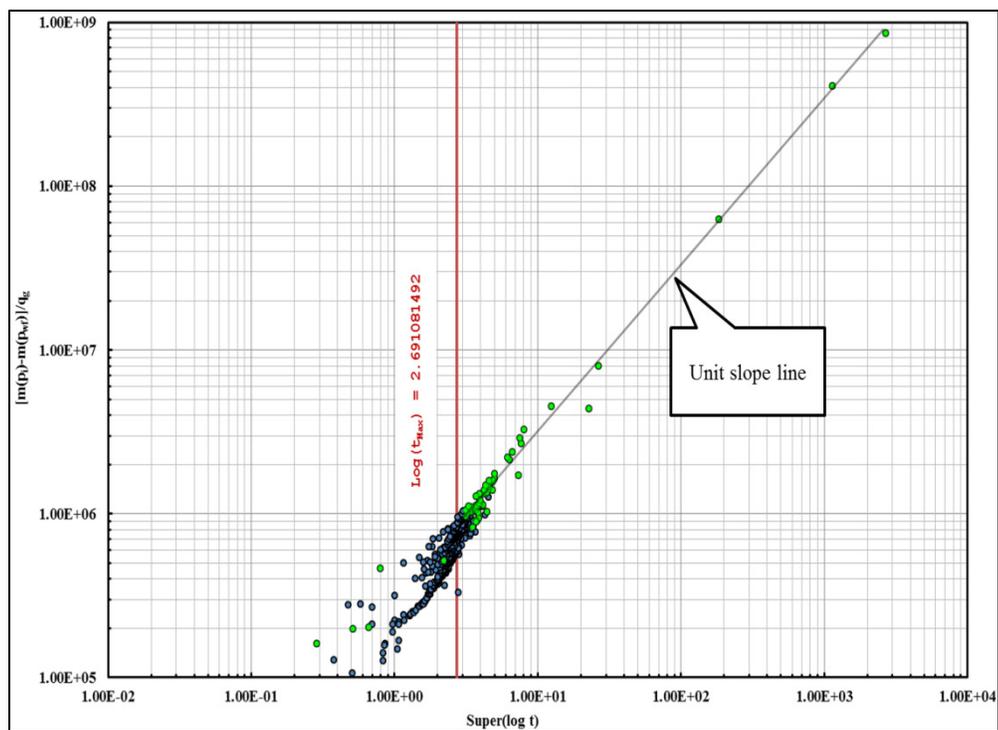


Fig.D- 20 Log-log Superposition-time plot for radial flow of well W-4 with rate issues highlighted

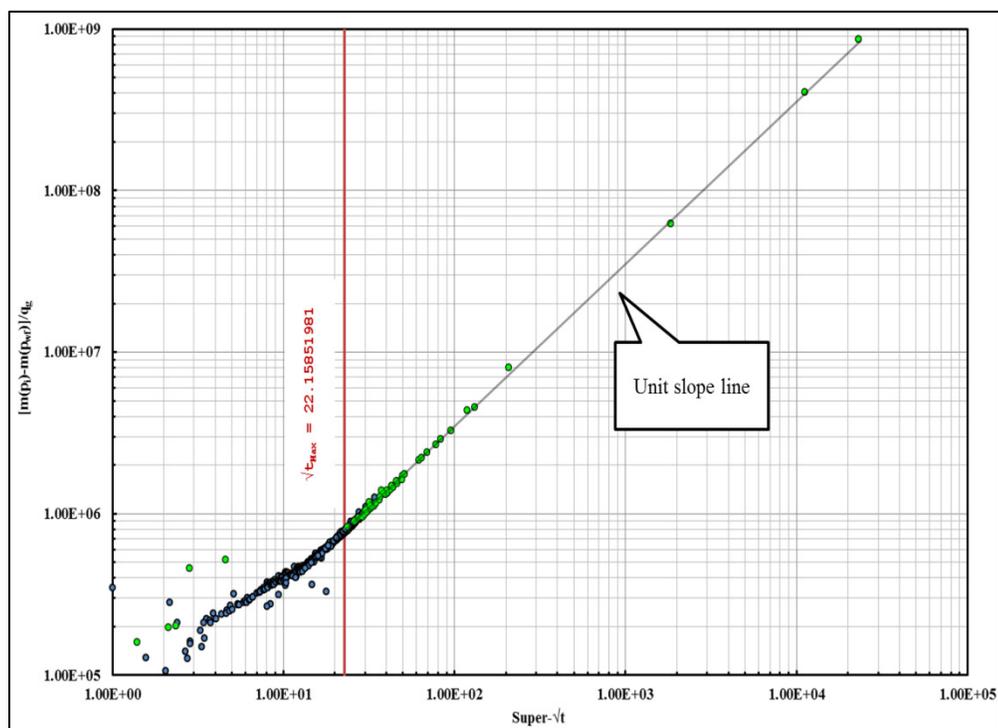


Fig.D- 21 Log-log Superposition-time plot for linear flow of well W-4 with rate issues highlighted

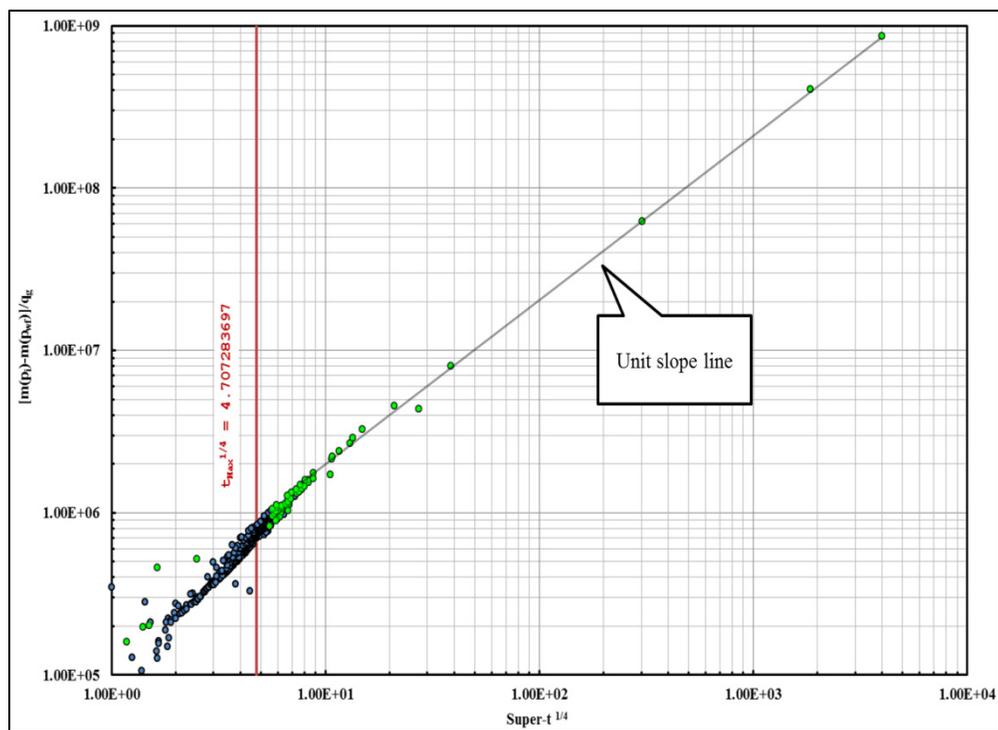


Fig.D- 22 Log-log Superposition-time plot for bilinear flow of well W-4 with rate issues highlighted

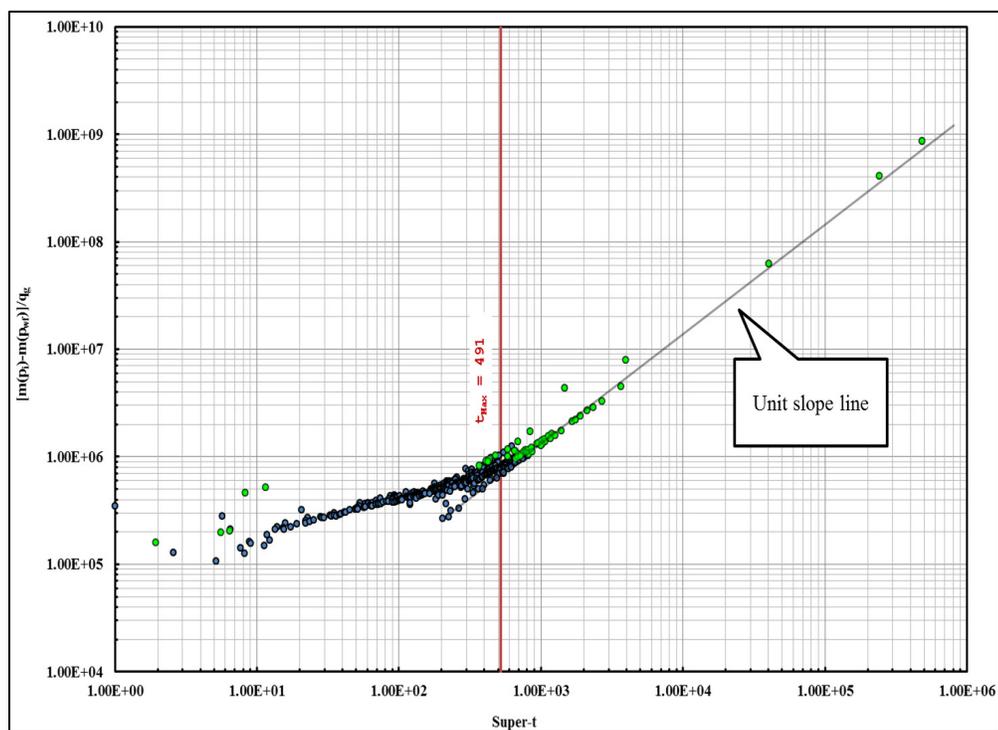


Fig.D- 23 Log-log Superposition-time plot for PSS flow of well W-4 with rate issues highlighted

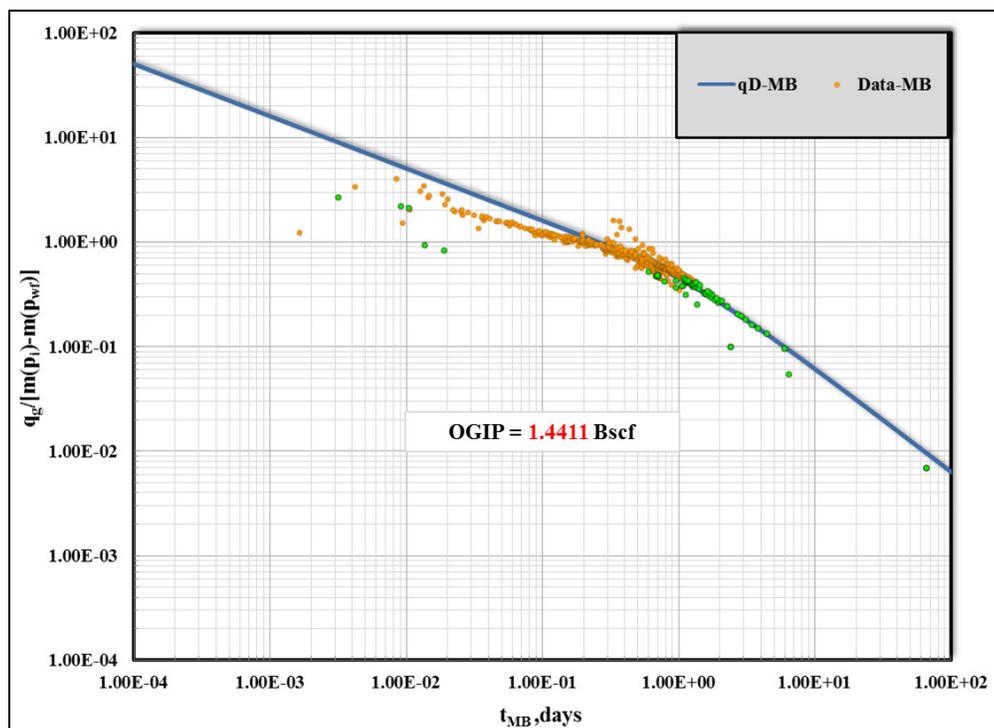


Fig.D- 24 Type-curve matching of constant-rate unfiltered data of well W-4

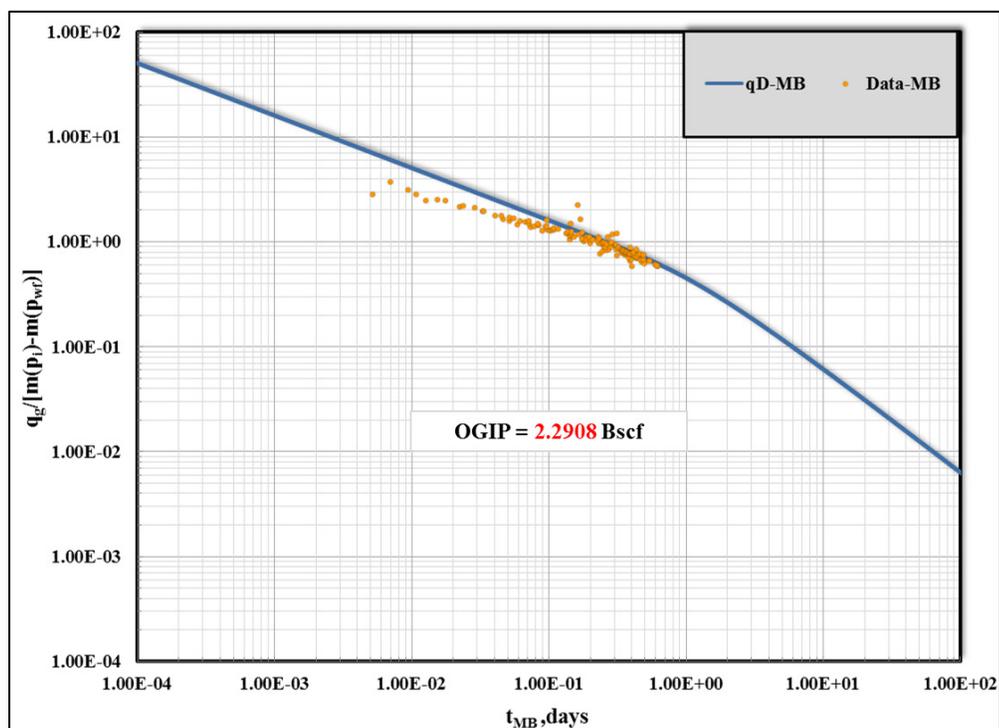


Fig.D- 25 Type-curve matching of constant-rate filtered data of well W-4

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