QUEUEING BEHAVIOR OVER A GILBERT-ELLIOTT

PACKET ERASURE CHANNEL

A Thesis

by

YI CAI

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2011

Major Subject: Electrical Engineering

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ABSTRACT

Queueing Behavior over a Gilbert-Elliott Packet Erasure Channel. (December 2011)

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This thesis explores the queueing performance of a wireless communication system that transmits packets over a correlated erasure channel using the IEEE 802.11 protocol suit. The channel states and the queue length together form a Markov chain. Exploiting this mathematical structure, the probability of the queue exceeding a certain threshold can be obtained. Most previous contributions in this area treat code-rate selection, channel erasure probability and network congestion separately. In this thesis, a simple integrated approach, which jointly considers these factors, is introduced. This approach becomes especially valuable for capturing the performance of delay-sensitive communication systems over time-varying channels.

This thesis starts with a review of related work about correlated bit-erasure wireless channel models. A numerical study is then conducted to demonstrate the importance of optimizing overall system performance, and how this process impacts error-control coding at the physical layer. Following this exercise, a packet-erasure channel model with a Poisson arrival process is analyzed. The Baum-Welch algorithm is subsequently presented as a means to estimate the parameters of wireless communication systems. Furthermore, a matrix geometric method for obtaining the stationary

distribution of the ensuing Markov chain is discussed. This offers a new perspective on wireless communication in the context of delay-sensitive applications.

To complement the analysis platform put forth in this work, illustrative numerical results are contained in the last section of the thesis. From these results, design guidelines for improving the performance of delay-sensitive wireless communication systems are established. Although these results are obtained under simplifying assumptions, the overall methodology applies to more general situations, especially for wide-band delay-sensitive wireless communication applications.

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1. INTRODUCTION: THE IMPORTANCE OF RESEARCH

Contemporary wireless communication systems must be designed to accommodate the various applications that compose today's digital landscape. In particular, mobile devices must meet the needs of miscellaneous data flows in terms of delay tolerance and bandwidth requirements. As a result, we have witnessed a rapid proliferation of Wi-Fi access points, both in residential and commercial settings. These access points can serve as a primary digital bridge to the Internet for mobile devices, or they can be used to offload excess data traffic from a cellular infrastructure. Restricted by a narrow spectral bandwidth, interference and the broadcast nature of wireless environments, the effective capacity of wireless access networks can be very limited in certain circumstances. Worse still, the uncoordinated mode of operation typical of mobile Wi-Fi devices can further degrade performance and hinder the efficient management of available resources. In this thesis, we introduce a new method of measuring the instantaneous service quality of a Wi-Fi link. The proposed active estimation technique can be implemented on mobile devices, thereby enabling enhanced performance. Our measurement scheme can potentially be employed as a building block for traffic management, rate control and routing in wireless environments.

This thesis follows the style of *IEEE Transaction on Information Theory*.

Most systems that support a version of the IEEE 802.11 standard for wireless communication can operate at different data rates. An operating point for a particular device is selected according to local conditions, which including instantaneous signal power and noise levels. On Wi-Fi devices, claimed rates can vary from 1 Mbits/s to 54 Mbits/s, with each admissible value corresponding to a specific coding strategy and modulation scheme. The aforementioned factors can collectively influence the optimal code-rate, channel erasure probability and network congestion profile of a wireless communication system. As such, they can also significantly affect the quality of service perceived by the user of a mobile device, especially when dealing with high-throughput or delay-sensitive applications.

An important objective then is to better understand the tradeoff between code-rate, reliability and overall performance in wireless communication systems. This is key in designing efficient resource management schemes. In the past, much consideration has been given to understanding code-rate selection and queueing performance separately. When optimizing physical layer parameters, the goal is often set to maximizing throughput; as such, insights are drawn from information theory. With system parameters fixed, the operation of a wireless link is subsequently optimized following a strong hierarchical perspective. To further re-enforce this point, we stress that code-rate selection is impervious to the needs of active applications on most mobile devices. No explicit feedback path is in place between the corresponding layers.

One possible hurdle that can prevent mobile devices from implementing highly integrated decision mechanisms is model complexity. Indeed, when taking a multitude of possible factors into consideration, it can become difficult to develop accurate system models with reasonable complexity. Indeed, due to channel variability, modeling can be a daunting task for wireless communication systems based on the IEEE 802.11 standard. Although, it is possible to offer a theoretical perspective on algorithms that may work well in certain situations, the complexity of these algorithms is a major factor that can prevent them from being implemented on mobile platforms.

In this thesis, we keep the aforementioned difficulties in mind and offer a new perspective. We review previous results about transmitting random codes over a Gilbert-Elliott bit-erasure channel, and extend this framework to a packet-level abstraction. Rather than focus solely on popular bit-erasure channel models, a packet-erasure model is developed to limit the complexity of our algorithm for evaluating channel performance. A comparison of queueing performance based on video over IP (VoIP) for different Wi-Fi access points is presented. The proposed technique can be extended to various other contemporary applications. Most critically, this research introduces a methodology to assist mobile users in selecting the most suitable access point with respect to their current usage profile. This, in turn, leads to a significant improvement in resource allocation efficiency.

The remainder of this thesis is organized as follows. In Section 2, we introduce our research results on the Gilbert-Elliott bit-erasure channel and the importance to optimize code-rate for delay-sensitive systems. The packet-erasure channel model and the Baum-Welch algorithm used to estimate system parameters are introduced in Section 3. The M/G/1-like Markov system and its matrix geometric solution are studied in Section 4. In Section 5, we develop a more advanced system model with a Poisson arrival process and three different channel states. Numerical results showing the performance of our system are presented in Section 6. Finally, we offer some conclusions and discuss possible future work in Section 7.

2. RELATED PREVIOUS WORK

2.1 Introduction: Bit-Erasure Channel

Error correcting codes have played an instrumental role over the past decades in digital communication systems [1]. For instance, it is well known that one can improve the transmission reliability of a wireless link by increasing the block-length of the underlying coding scheme [2]. However, the stringent delay restrictions imposed on modern wireless communication systems can force an engineer to resort to short block codes. In such scenarios, the probability of decoding failure for individual codewords is not negligible. Moreover, packet retransmissions can lead to queue buildups at the transmitter, a phenomenon that results in longer latency [3]. Decoding attempts can become highly correlated over time for channels with memory. This further perturbs queueing behavior and end-to-end delay. Thus, it is essential to study the queueing behavior of communication systems subject to very stringent delay requirements.

Suppose our signal bits are transmitted over a Gilbert-Elliott erasure channel, which can either be in a $good$ state or a bad state as interpreted in Fig. 1. The transition probability matrix $\boldsymbol{P}$ for this channel can be expressed as

$$\boldsymbol{P} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}.$$

The corresponding conditional bit-erasure probabilities are ϵ_g and ϵ_b.

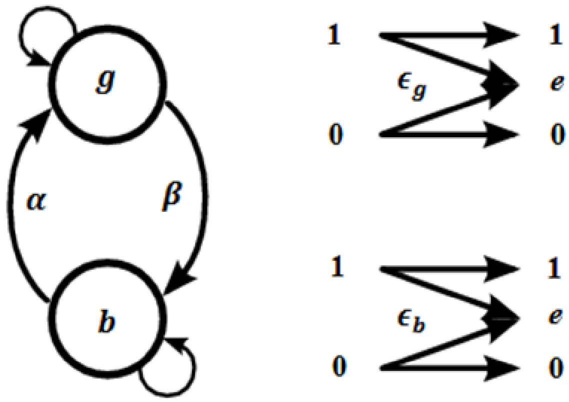

Fig. 1. A Gilbert-Elliott bit-erasure channel is employed to demonstrate the characteristics of a correlated wireless communication link. When the channel is in its good state, erasures occur with probability ϵ_g. Whereas when the channel is in its bad state, erasures have probability ϵ_b.

Leveraging previously published research [4], [5], [6], we wish to analyze the queueing behavior of our system as a function of length N and code-rate r. To complete our channel model and conduct this analysis, we have to calculate the conditional probability of decoding failure at the receiver end. Let E denote the number of erasures occurring in one codeword transmission. We can use the following matrix $\boldsymbol{P}_x$ to derive the distribution of E,

$$\boldsymbol{P}_x = \begin{bmatrix} (1-\alpha)(1-\epsilon_b+\epsilon_b x) & \alpha(1-\epsilon_b+\epsilon_b x) \\ \beta(1-\epsilon_g+\epsilon_g x) & (1-\beta)(1-\epsilon_g+\epsilon_g x) \end{bmatrix}.$$

Let $[\![x^j]\!]$ be the operator that maps a polynomial in x to the coefficient of x^j. Then, we can obtain the conditional probability for the number of erasures in terms of the N^{th} power of $\boldsymbol{P}_x$ as

$$Pr(E = e, C_{N+1} = d | C_1 = c) = [\![x^e]\!][\boldsymbol{P}_x^N]_{c,d}.$$

In this way, we are able to combine the physical channel model with the number of erasures occurring in one block, which provides a method to calculate the probability of decoding failure.

2.2 Coding Strategy and Queueing Behavior

In the previous section, we have discussed the relationship between channel state, channel correlation and the number of erasures. To evaluate the performance of the system, we need to specify a coding strategy. For simplicity, we employ random coding [7]. Given e erasures with a codeword of length N and with K information bits, the probability of decoding failure can be written as

$$P_f(N-K, e) \triangleq 1 - \prod_{i=0}^{e-1}(1 - 2^{i-(N-K)}).$$

Having introduced a channel model, we turn to the arrival and departure processes at the transmitter. In our system, we select block length N and code-rate r to be variables. Each arriving data packet is broken into length-K data segments, where $K = N \times r$, and the segments are encoded separately.

We assume that the arrival process is Poisson with rate λ and the number of information bits per data packet forms a sequence of independent and identically distributed random variables whose marginal distribution is geometric with parameter $\rho \in (0,1)$. Thus, the distribution for the length of each data packet becomes

$$Pr(L = l) = (1 - \rho)^{l-1}\rho \quad l = 1,2, \ldots$$

The profile of the service process is determined by the parameters of the Gilbert-Elliott channel and code-rate r. Generally, a lower code-rate will lead to a smaller probability of decoding failure, but it also requires more successfully decoded codewords to complete the transmission of one data packet. On the other hand, a higher code-rate behaves in the opposite fashion; fewer successful decoding events are needed to complete a packet transmission, yet the probability of successfully decoding a codeword is smaller. Thus, for a given channel model, we can vary the block length and code-rate to find the optimal value by exploring this natural tradeoff.

Once we have selected block length N and code-rate r, for a data packet with exactly L information bits, we need to successfully decode $M = \lceil L/rN \rceil$ consecutive codewords to complete the entire transmission of the data packet. The probability distribution for M is

$$Pr(M = m) = (1 - \rho_r)^{m-1}\rho_r \quad m = 1,2, \ldots$$

where parameter $\rho_r = 1 - (1 - \rho)^{rN}$ [4].

In our system, a very important feature is that the queue at the transmitter will discard a data segment if and only if it has been acknowledged that the receiver has already successfully decoded the relevant information. In other words, no packets are loss or dropped.

We use Q_s to denote the number of data packets waiting in the queue at the transmitter, and the corresponding channel state at the same instant is represented by C_{sN+1}. One can show that the aggregate process $U_s = (C_{sN+1}, Q_s)$ forms a Markov chain. More precisely, the evolution of our discrete-time system is akin to an M/G/1 queue. There are many established techniques to evaluate the characteristics of this system.

Using our previous characterization of the number of erasures occurring in a block conditioned on the initial channel state, we can calculate the transition probability from U_s to U_{s+1},

$$Pr\big(U_{s+1} = (d, q_{s+1})\big|U_s = (c, q_s)\big)$$

$$= \sum_{e \in \mathbb{N}_0} Pr(Q_{s+1} = q_{s+1}|E = e, Q_s = q_s) \times$$

$$Pr(E = e, C_{(s+1)N+1} = d|C_{sN+1} = c).$$

To find an analytical expression for this equation, we have to derive an expression for $Pr(Q_{s+1} = q_{s+1}|E = e, Q_s = q_s)$.

Suppose the number of packets in the queue is$Q_s = q_s$.Then, the possible values for Q_{s+1} are restricted to the countable set $\{q_s - 1, q_s, q_s + 1, q_s + 2, \dots\}$. The corresponding probabilities for these transitions are

$$Pr(Q_{s+1} = q_s + i|E = e, Q_s = q_s)$$

$$= P(i)(P_f(N - K, e) + (1 - P_f(N - K, e))(1 - \rho_r))$$

$$+P(i + 1)(1 - P_f(N - K, e))(1 - \rho_r).$$

$$Pr(Q_{s+1} = q_s - 1|E = e, Q_s = q_s)$$

$$= \big(1 - P(0)\big)\Big(1 - P_f(N - K, e)\Big)\rho_r$$

where $P(i) = \frac{e^{-\lambda N}(\lambda N)^i}{i!},\ i \geq 0$, represents the probability of i data packets arriving during the transmission of one codeword. When the queue is empty, i.e.. $Q_s = 0$, these probabilities reduce to

$$Pr(Q_{s+1} = i | E = e, Q_s = 0) = P(i)$$

$$= \frac{e^{-\lambda N}(\lambda N)^i}{i!} \qquad i = 1,2, \ldots$$

With these findings, we can get the probability transition matrix of the discrete-time M/G/1-type Markov process $\{U_s\}$.

2.3 Performance Evaluation

Based on these findings, we can leverage the analysis methodology introduced in previous work [1],[2],[3] to conduct a numerical study and find the optimal operating point for a given wireless communication system. There are many different characteristics that can be evaluated for our system. Herein, we focus on evaluating the probability that the number of packets waiting at the transmitter exceeds a prescribed threshold.

To loosely match the operation of a wireless GSM relay link, our simulation is based on the following conditions. The parameters related to the Gilbert-Elliott erasure channel are $\alpha = 0.02, \beta = 0.005, \epsilon_b = 0.49\ and\ \epsilon_g = 0.0025$. Consequently, the average erasure probability is 0.1. The block length varies between 1 and 80, and the number of information bits contained in every codeword lies between 1 and 60. The expected Poisson arrival rate is set to $\lambda = 1/300$ packets per channel use, and the average data packet length is taken to be 145 bits. When the channel bandwidth is $24.7\ KHz$, the system arrival rate is roughly $11.9\ Kb/s$. System performance as a function of block length N and information bits K is shown in Fig. 2 and Fig. 3.

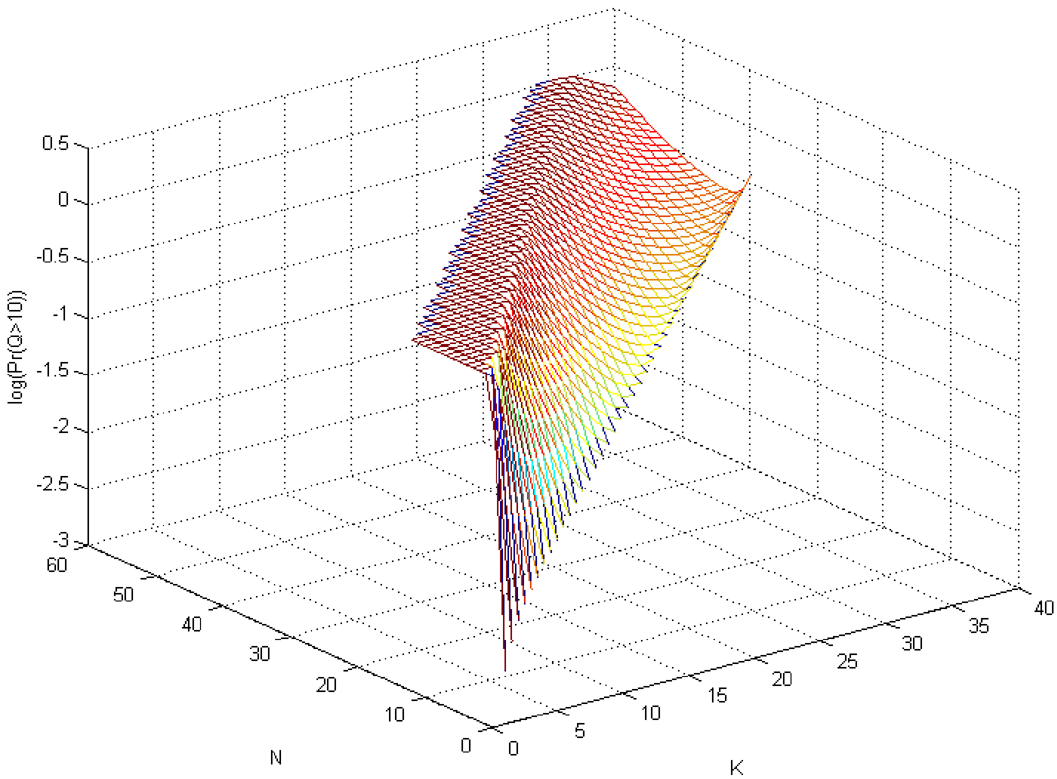

Fig. 2. The probability of the queue-length exceeding 10 is evaluated by varying block-length N and the number of information bits K. From the figure, we can see that the fastest working point requires both N and Kequal to 1. Also, for any specific block-length N, there is a corresponding optimal number of information bits K.

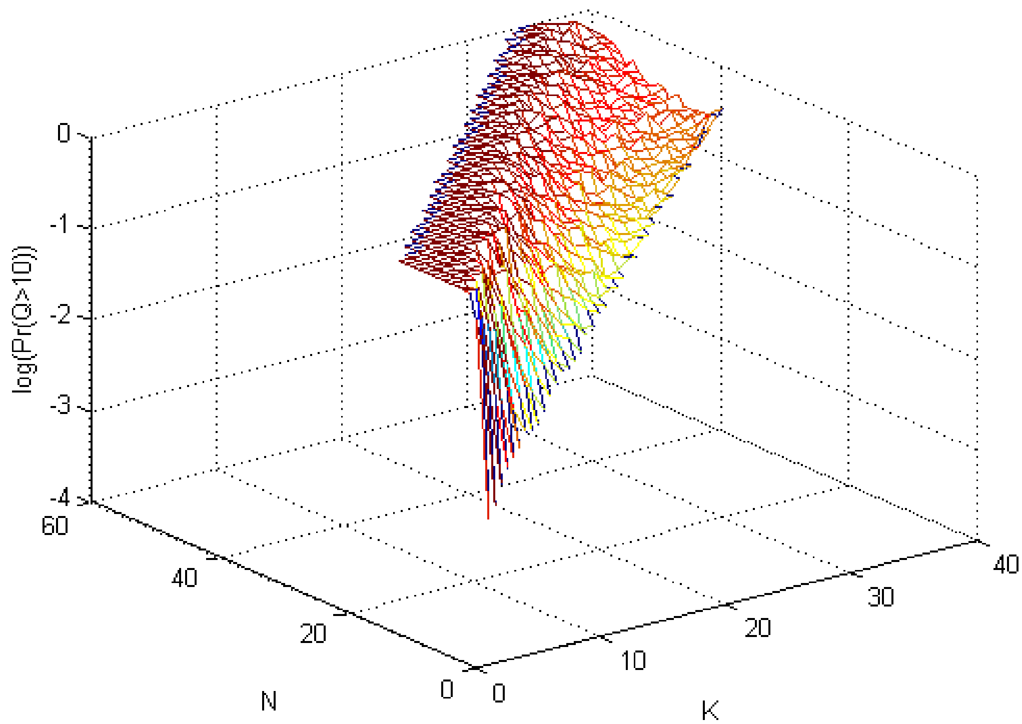

Fig. 3. Monte-Carlo simulation results for the tail probability of the queue-length exceeding 10. Not so surprisingly, these results closely match the theoretical values.

In our previous analysis, we have assumed that the transmitter has perfect knowledge about whether or not the receiver correctly decoded a sent codeword. In practice, we need to provide some feedback information to implement this control loop [8]. Below, we conduct an analysis about the influence of feedback on system performance.

Suppose that we are using a symmetric communication system, which means both sides of our system transmit and receive the same kind of codewords in a similar manner. By reciprocity, we can then account for feedback information and the arriving data segments within one block. Obviously, this process will only affect the probability of decoding failure. Suppose, for instance, that we need two bits of feedback information in every transmission. Now, instead of the previous encoding case, we have a $(N, K+2)$ random codeword. The probability of decoding failure thus becomes

$$P_f(N-K-2, e) \triangleq 1 - \prod_{i=0}^{e-1} \left(1 - 2^{i-(N-K-2)}\right).$$

Using these new parameters, we can reevaluate system performance. Ensuing results are shown in Fig. 4. Feedback takes care of the degenerate case where sending one information bit at a time is optimal.

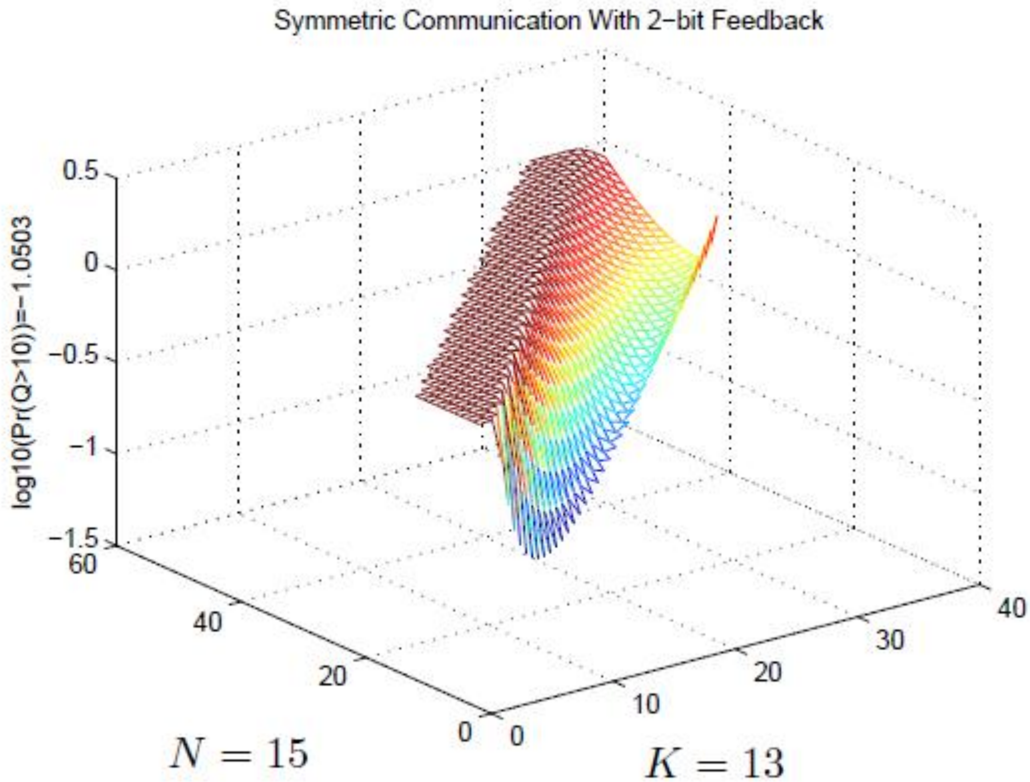

Fig. 4. The probability that the queue length exceeds 10 is evaluated with variable block-length N, information bits K, and 2-bit feedback information in every codeword.

From this figure, we can see that the optimized operating point has shifted from $(N = 1, K = 1)$ to $(N = 15, K = 13)$, which implies that feedback information bits can have significant influence on system performance. Hence, when designing a delay-sensitive wireless communication system, one should account for block-length, code-rate and also the feedback information.

3. PACKET-ERASURE CHANNEL

In the previous section, a bit-erasure channel model and random codes were jointly introduced to capture the evolution of a Markov communication system [9]. The matrix geometric method was instrumental in being able to evaluate the packet delay performance of a wireless communication system where the channel is defined at the bit level. We can employ the results obtained thus far to better design delay-sensitive wireless communication systems. However, we note that the channel parameters were specifically chosen to fit certain wireless communication environments. To accommodate a wide range of channels with frequently varying characteristics, we would like to introduce a new erasure channel model defined at the packet level. This model can essentially allow the mobile to rapidly and efficiently analyze the characteristics of the time-varying channel. Another advantage is that computational complexity at the packet level is much lower than that at the bit level, which can be a critical consideration whenever these algorithms are implemented on a mobile platform with limited computational power and finite battery life.

3.1 Hidden Markov Model

In wireless environments, we can use a Rayleigh fading model together with additive white Gaussian noise to model the characteristics of our communication link. In this case, the transmitted signal $x(t)$ is subject to multipath fading $h(t)$ and thermal noise $w(t)$, which is introduced by the receiving circuit. We can express the received signal $y(t)$ explicitly in the mathematic form $y(t) = h(t) * x(t) + w(t)$. Nevertheless, it can be very difficult to accurately describe the real-time characteristics of the fading process. And it is even harder to specify the time statistics of the function $h(t)$. For a terminal moving at constant velocity through a zero-mean proper complex Gaussian field, the time autocorrelation function of the envelope process $h(t)$ can be modeled as a zeroth-order Bessel function of the first kind. This latter characterization is still too complicated for us to conduct a proper queueing behavior analysis.

To circumvent these issues, we use a finite-state Markov chain to express the packet-based, time-evolution process of the wireless link. Suppose that data packets are transmitted over a Gilbert-Elliott erasure channel; and this channel can either be in a *good* state g with packet erasure probability ϵ_g or in a *bad* state b with packet erasure probability ϵ_b. We can construct a Markov process to capture the transitions between these two states. We denote the probability of transiting from state g to b by α and the transition probability in the reverse direction by β. According to this notation, we can write the transition probability matrix as follow,

$$\boldsymbol{P} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}.$$

A graphic interpretation of this channel is illustrated in Fig. 5.

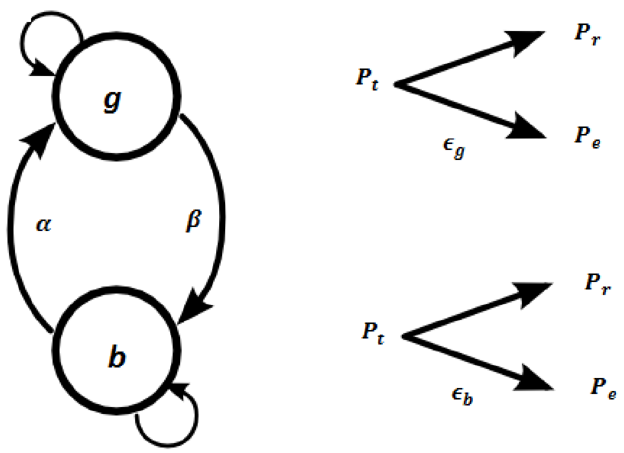

Fig. 5. A Gilbert-Elliott packet-erasure channel is employed to model the operation of a communication link with memory. This model captures both the uncertainty associated with transmitting packets over a noisy channel and correlation over time typical of several communication channels. $\boldsymbol{P_t}$, $\boldsymbol{P_r}$ and $\boldsymbol{P_e}$ correspondingly represent a transmitted packet, a received packet and an erased packet.

We employ random variable C_n to denote the state of the wireless channel at any given time n. In words, the entry $\boldsymbol{P}_{c,d}$ of the probability matrix $\boldsymbol{P}$ represents the probability of the channel jumping to state d from a previous state c, which is denoted as $Pr(C_{n+1} = d|C_n = c)$.

In this thesis, we want to analyze the queueing behavior of our system over a wireless channel with memory. To complete our channel model and simplify our analysis, we assume the packet arriving process to be a Bernoulli process with parameter $Pr(Arr)$. The departure process will be determined by the underlying channel state C_n, which must be inferred based on the packet erasure observations. Let O_n denote the channel observation at time n. Two different events, denoted o_b and o_g, can occur on the packet-erasure channel. Either the transmitted information is received properly (o_g) or it is lost due to a packet erasure (o_b).

As mentioned before, this research focuses on wireless channels with memory. Throughout, we assume that a buffer is present at the transmitter. At any given time n, the probability of observing a packet erasure, denoted by o_b, is equal to

$$Pr(O_n = o_b) = \big(Pr(C_{n-1} = b)\,\epsilon_b + Pr(C_{n-1} = g)\,\epsilon_g\big)\,Pr(Arr),$$

Correspondingly, the probability of a successful packet transmission, represented by o_g, can be written as

$$Pr\big(O_g = o_g\big) = 1 - (Pr(C_{n-1} = b)\,\epsilon_b + Pr(C_{n-1} = g)\,\epsilon_g)Pr(Arr).$$

The entire queued system forms a hidden Markov model. Given an observation sequence over time, standard algorithms can be employed to estimate the stationary distribution of the channel states, $Pr(C_n = c)$, and the state transition probabilities, $Pr(C_{n+1} = d|C_n = c)$. In the next section, the Baum-Welch algorithm is introduced to estimate system parameters based on the information gathered at the destination.

3.2 Baum-Welch Algorithm

Suppose the initial probabilities for channel states c_b and c_g are $\gamma = \{\gamma_b, \gamma_g\}$, respectively, the transition probability matrix is denoted by $\boldsymbol{P}$, and the conditional erasure probabilities are governed by $B = \big\{Pr(O_t|C_t)\,, O_t \in \{o_g, o_b\}, C_t \in \{c_g, c_b\}\big\}$. Altogether, this abstraction leads to a hidden Markov model with parameters $\Lambda =$

$\{\gamma, B, P\}$. Based on this model and after obtaining a series of observations $O = \{O_1, O_2, \ldots, O_n\}$, the Baum-Welch algorithm can be applied to estimate the parameters of this system [10], [11].

In order to estimate the given system parameters, we have to start from an assumed model with parameters $\Lambda' = \{\gamma', B', P'\}$. With this assumption, we are able to calculate the forward variable

$$\phi_t = Pr\{O_1, O_2, \ldots, O_t, C_t | \Lambda\}, \ 1 \leq t \leq n,$$

and the backward variable

$$\beta_t = Pr\{O_{t+1}, \ldots, O_n | C_t, \Lambda\}, \ 1 \leq t \leq n.$$

Once this is achieved, the next step is to update the parameters of the hidden Markov model to maximize the conditional probability $Pr(O|\lambda)$ according to the well-known *re-estimation formula*, which is not included in this thesis [12]. Notice the guaranteed convergence of the Baum-Welch algorithm. This estimation process yields a set of well-defined hidden Markov parameters, $\Lambda = \{\gamma, B, P\}$. With these specific values, we can further look into the queueing behavior of a wireless communication system and conduct numerical analysis.

4. QUEUEING MODEL

4.1 State Space Transition

To be consistent with our original notation for the bit-erasure channel model, we continue to use the same expressions U_s and Q_s to describe the state of the system. However, we emphasize that we are now analyzing system performance at the packet level, so the corresponding channel state for Q_s becomes C_{s+1}, which implies $U_s = (C_{s+1}, Q_s)$. Since arrivals are assumed to be Bernoulli, the evolution of the system is an instance of a quasi-birth-death process [12].

Based on the previous study of erasures occurring in a block conditioned on the channel state, we can calculate the transition probability from U_s to U_{s+1}

$$\begin{aligned} &Pr\big(U_{s+1} = (d, q_{s+1})\big|U_s = (c, q_s)\big) \\ &= Pr\big(C_{(s+1)+1} = d\big|C_{s+1} = c\big) \times \\ &Pr\big(Q_{s+1} = q_{s+1}\big|U_s = (c, q_s)\big). \end{aligned}$$

The first part of this equation is equivalent to our previous expressions with different values corresponding to entries in the channel transition probability matrix $\boldsymbol{P}$. It remains to derive a mathematical expression for $Pr\big(Q_{s+1} = q_{s+1}\big|U_s = (c, q_s)\big)$.

Suppose the current number of data packets in the queue is $Q_s = q_s$, and $q_s > 0$. According to the Bernoulli arrival process, the possible values for Q_{s+1} are restricted to the finite set $\{q_s - 1, q_s, q_s + 1\}$. For simplicity, we assume that a data packet is erased if and only if the channel is in its *bad* state, which necessarily means $\epsilon_g = 0$ and $\epsilon_b = 1$.

The corresponding probabilities for these transitions are

$$Pr\big(Q_{s+1} = q_s + 1\big|U_s = (C_{s+1}, q_s)\big) = Pr(Arr)\,Pr(C_{s+1} = b),$$

$$Pr\big(Q_{s+1} = q_s\big|U_s = (C_{s+1}, q_s)\big)$$

$$= (1 - Pr(Arr))\,Pr(C_{s+1} = b) + Pr(Arr)\,Pr(C_{s+1} = g),$$

$$Pr\big(Q_{s+1} = q_s - 1\big|U_s = (C_{s+1}, q_s)\big) = (1 - Pr(Arr))\,Pr(C_{s+1} = g).$$

When the queue is empty, $\{Q_s = 0\}$, the probabilities become

$$Pr\big(Q_{s+1} = 1\big|U_s = (c_s, 0)\big) = Pr(Arr),$$

$$Pr\big(Q_{s+1} = 0\big|U_s = (c_s, 0)\big) = 1 - Pr(Arr).$$

Collecting these findings, we can get the transition probability matrix of the Markov process $\{U_s\}$.

For convenience, we introduce the following mathematical notation,

$$\lambda_{cd} = Pr(U_{s+1} = (d, q + 1)|U_s = (c, q)),$$

$$\kappa_{cd} = Pr(U_{s+1} = (d, q)|U_s = (c, q)),$$

$$\mu_{cd} = Pr(U_{s+1} = (d, q - 1)|U_s = (c, q));$$

similarly, when the queue is empty, we get

$$\lambda_{cd}^0 = Pr(U_{s+1} = (d, 1)|U_s = (c, 0)),$$

$$\kappa_{cd}^0 = Pr(U_{s+1} = (d, 0)|U_s = (c, 0))$$

where $q \in \mathbb{N}_0$ and $c, d \in \{b, g\}$. A graphical representation for the state transitions is shown in Fig. 6.

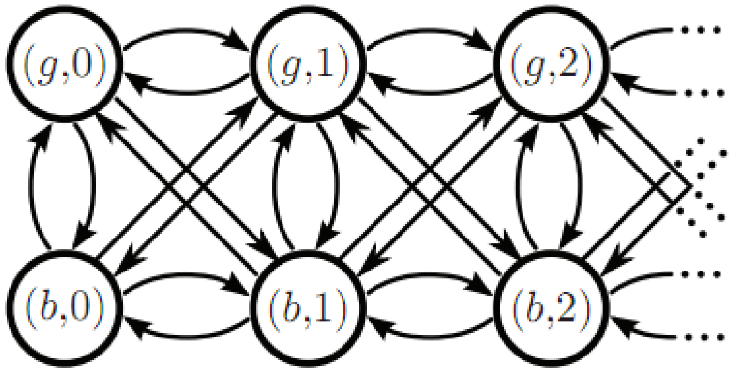

Fig. 6. State space and transition diagram for the aggregate queued process $\{U_s\}$ with a Bernoulli arrival process. Self-transitions are intentionally omitted.

The equilibrium distribution for our system can be denoted by semi-infinite vector $\boldsymbol{\pi}$, where

$$\boldsymbol{\pi}(2q+i) = \begin{cases} Pr(C=b, Q=q) & if\ i=1 \\ Pr(C=g, Q=q) & if\ i=2, \end{cases}$$

and $\boldsymbol{\pi} = [\boldsymbol{\pi_0}\ \boldsymbol{\pi_1}\boldsymbol{\pi_2}\ \ldots]$. Vector $\boldsymbol{\pi_q}$ is known as the q^{th} level of the Markov chain and $\boldsymbol{\pi_q} = [\pi(2q+1)\ \pi(2q+2)]$.

Under proper stability conditions, we can apply the main theorem as $\boldsymbol{\pi T} = \boldsymbol{\pi}$ to solve for the equilibrium distribution of the queue length, where $\boldsymbol{T}$ is the transition probability matrix. As a quasi-birth-death process, there are a lot of methods to solve for the stationary distribution of our model. In the next section, we will introduce a matrix geometric method [12], [13].

4.2 Matrix Geometric Solution

In order to solve the main equilibrium equation $\boldsymbol{\pi T} = \boldsymbol{\pi}$, we can represent the probability transition matrix $\boldsymbol{T}$ as

$$\boldsymbol{T} = \begin{pmatrix} \boldsymbol{C_1} & \boldsymbol{C_0} & \boldsymbol{0} & \boldsymbol{0} & \cdots \\ \boldsymbol{A_2} & \boldsymbol{A_1} & \boldsymbol{A_0} & \boldsymbol{0} & \cdots \\ \boldsymbol{0} & \boldsymbol{A_2} & \boldsymbol{A_1} & \boldsymbol{A_0} & \cdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A_2} & \boldsymbol{A_1} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

where the sub-matrices $\boldsymbol{C_1}, \boldsymbol{C_0}, \boldsymbol{A_2}, \boldsymbol{A_1}$ and $\boldsymbol{A_0}$ are 2×2 real matrices. More specifically, we have

$$\boldsymbol{A_0} = \begin{bmatrix} \lambda_{bb} & \lambda_{bg} \\ \lambda_{gb} & \lambda_{gg} \end{bmatrix} \boldsymbol{A_1} = \begin{bmatrix} \kappa_{bb} & \kappa_{bg} \\ \kappa_{gb} & \kappa_{gg} \end{bmatrix} \boldsymbol{A_2} = \begin{bmatrix} \mu_{bb} & \mu_{bg} \\ \mu_{gb} & \mu_{gg} \end{bmatrix}.$$

When the queue is empty, the relevant sub-matrices can be written as

$$\boldsymbol{C_0} = \begin{bmatrix} \lambda_{bb}^0 & \lambda_{bg}^0 \\ \lambda_{gb}^0 & \lambda_{gg}^0 \end{bmatrix} \boldsymbol{C_1} = \begin{bmatrix} \kappa_{bb}^0 & \kappa_{bg}^0 \\ \kappa_{gb}^0 & \kappa_{gg}^0 \end{bmatrix}.$$

To derive the equilibrium distribution for a positive recurrent Markov chain with transition matrix $\boldsymbol{T}$, the crucial part is solving for the recursive matrix R, which is the limit of the matrix recursion

$$\boldsymbol{R}_{j+1} = \left(\boldsymbol{A}_0 + \boldsymbol{R}_j^2 \boldsymbol{A}_2\right)(\boldsymbol{I} - \boldsymbol{A}_1)^{-1}$$

starting from $\boldsymbol{R}_0 = \boldsymbol{0}$.

Then, the q^{th} level stationary distribution π_q satisfies $\boldsymbol{\pi_{q+1}} = \boldsymbol{\pi_q R}$ for $q \geq 1$ with $\boldsymbol{\pi_1} = \boldsymbol{\pi_0 Z}$ and

$$\boldsymbol{Z} = (\boldsymbol{I} - \boldsymbol{C}_1)\boldsymbol{A}_2^{-1}$$

$$\boldsymbol{\pi_0} = \begin{bmatrix} \frac{\beta}{\alpha+\beta} & \frac{\alpha}{\alpha+\beta} \end{bmatrix} (\boldsymbol{I} + \boldsymbol{Z}(\boldsymbol{I} - \boldsymbol{R})^{-1})^{-1}.$$

Given the above recursive relationship and the normalization condition, we can find the unique vector π_0, and π_j can then be computed iteratively.

5. MORE ADVANCED MODELS

5.1 Model with Poisson Arrival Process

In the previous part, we have obtained a solution for a correlated time-varying wireless communication system with a Bernoulli arrival process. Yet, this always remains an approximation because, in reality, there can be multiple packets arriving at the source within one time unit. A more sophisticated system model with a Poisson arrival process, which admits multiple arrivals per codeword transmission cycle, is presented below.

Suppose a Poisson arrival process with arrival rate ρ is introduced into our system model and, as a consequence, state transitions are no longer confined to adjacent states. The resulting state-transition diagram is illustrated in Fig. 7.

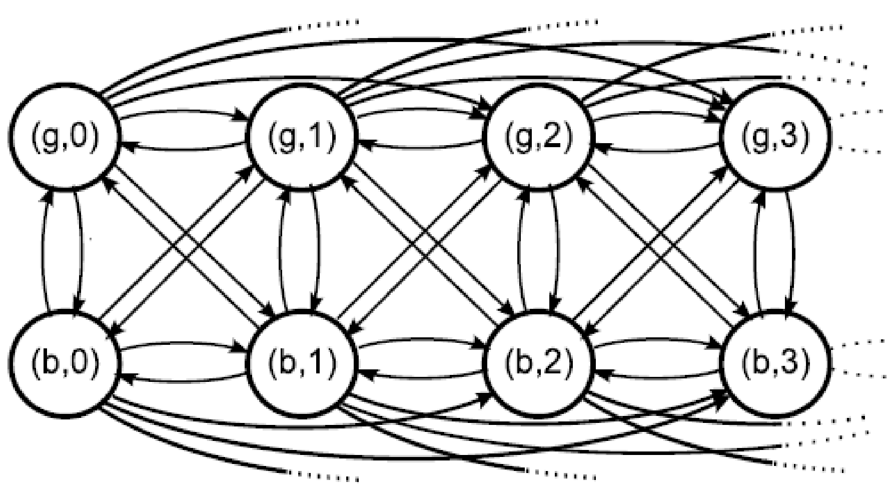

Fig. 7. State space and transition diagram for the aggregate queued process $\{U_s\}$ with Poisson arrivals. Self-transitions are intentionally omitted.

Much like in the preceding section, our first concern is to derive an expression for equation $Pr\left(Q_{s+1} = q_{s+1} \middle| U_s = (C_{s+1}, q_s)\right)$. To be consistent with previous sections, we continue to denote the current number of data packets in the queue by $Q_s = q_s$ ($q_s > 0$). However, the possible values for Q_{s+1}now becomes restricted to the countable set $\{q_s - 1, q_s, q_s + 1, q_s + 2, \ldots\}$. The corresponding transition probabilities are

$$Pr\left(Q_{s+1} = q_s + i \middle| U_s = (C_{s+1}, q_s)\right) = \xi_i \, Pr(C_{s+1} = b) + \xi_{i+1} Pr(C_{s+1} = g),$$

$$Pr\left(Q_{s+1} = q_s - 1 \middle| U_s = (C_{s+1}, q_s)\right) = \xi_0 \, Pr(C_{s+1} = g),$$

where $\xi_i = e^{-\rho} \frac{\rho^i}{i!}$ is the probability that exactly$i$ data packets arrive during the transmission of one codeword. When the queue is empty, $\{Q_s = 0\}$, the transition probabilities simply reduce to

$$Pr(Q_{s+1} = i | Q_s = 0) = \xi_i \, .$$

Again, we can introduce a convenient mathematical notation

$$\lambda_{cd}^{i} = Pr(U_{s+1} = (d, q + i) | U_s = (c, q)), \;\; i \geq 1,$$

$$\kappa_{cd} = Pr(U_{s+1} = (d, q) | U_s = (c, q)),$$

$$\mu_{cd} = Pr(U_{s+1} = (d, q - 1) | U_s = (c, q)).$$

When the queue is empty, we take

$$\lambda_{cd}^{i0} = Pr(U_{s+1} = (d, i) | U_s = (c, 0)),$$

$$\kappa_{cd}^{0} = Pr(U_{s+1} = (d, 0) | U_s = (c, 0)).$$

In this way, the probability transition matrix $\boldsymbol{T}$ can be expressed as

$$\boldsymbol{T} = \begin{pmatrix} \widehat{\boldsymbol{A}} & \widehat{\boldsymbol{F}}^{(\mathbf{1})} & \widehat{\boldsymbol{F}}^{(\mathbf{2})} & \widehat{\boldsymbol{F}}^{(\mathbf{3})} & \cdots \\ \boldsymbol{B} & \boldsymbol{A} & \boldsymbol{F}^{(\mathbf{1})} & \boldsymbol{F}^{(\mathbf{2})} & \cdots \\ \mathbf{0} & \boldsymbol{B} & \boldsymbol{A} & \boldsymbol{F}^{(\mathbf{1})} & \cdots \\ \mathbf{0} & \mathbf{0} & \boldsymbol{B} & \boldsymbol{A} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Above, “$\boldsymbol{A}$”, “$\boldsymbol{F}$”, and “$\boldsymbol{B}$” represent local, forward, and backward transition blocks, respectively. These 2×2 matrices can be obtained by using the method introduced in Section 4,

$$\boldsymbol{F}^{(\boldsymbol{i})} = \begin{bmatrix} \lambda_{bb}^{i} & \lambda_{bg}^{i} \\ \lambda_{gb}^{i} & \lambda_{gg}^{i} \end{bmatrix} \qquad \boldsymbol{A} = \begin{bmatrix} \kappa_{bb} & \kappa_{bg} \\ \kappa_{gb} & \kappa_{gg} \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} \mu_{bb} & \mu_{bg} \\ \mu_{gb} & \mu_{gg} \end{bmatrix}.$$

When the queue is empty, the relevant sub-matrices become

$$\widehat{\boldsymbol{F}}^{(\boldsymbol{i})} = \begin{bmatrix} \lambda_{bb}^{i0} & \lambda_{bg}^{i0} \\ \lambda_{gb}^{i0} & \lambda_{gg}^{i0} \end{bmatrix} \; \widehat{\boldsymbol{A}} = \begin{bmatrix} \kappa_{bb}^{0} & \kappa_{bg}^{0} \\ \kappa_{gb}^{0} & \kappa_{gg}^{0} \end{bmatrix}.$$

This system is a discrete-time Markov chain of the M/G/1-type, and the following part presents another matrix geometric method which can be used for solving the equilibrium distribution of this system [14].

Let $\boldsymbol{G}$ be the limiting matrix of the recursion $\boldsymbol{G_{i+1}} = -\boldsymbol{L^{-1}}(\boldsymbol{B} + \sum_{\boldsymbol{j=1}}^{\infty} \boldsymbol{F}^{(\boldsymbol{j})} \boldsymbol{G_i^{j+1}})$ starting from $\boldsymbol{G_0} = \mathbf{0}$, where $\boldsymbol{L} = \boldsymbol{A} - \boldsymbol{I}$. Then, the stationary probability vectors $\boldsymbol{\pi_j}$ associated with $\boldsymbol{T}$ are given by

$$\boldsymbol{\pi_j} = -\left(\boldsymbol{\pi_0} \widehat{\boldsymbol{S}}^{(\boldsymbol{j})} + \sum_{\boldsymbol{k=1}}^{\boldsymbol{j-1}} \boldsymbol{\pi_k} \boldsymbol{S}^{(\boldsymbol{j-k})}\right) \boldsymbol{S}^{(\mathbf{0})^{-\mathbf{1}}} \quad j = 1,2, \dots,$$

where $\boldsymbol{F}^{(\mathbf{0})} \triangleq \boldsymbol{L},\ \widehat{\boldsymbol{S}}^{(\boldsymbol{j})} = \sum_{l=j}^{\infty} \widehat{\boldsymbol{F}}^{(\boldsymbol{l})} \boldsymbol{G}^{\boldsymbol{l-j}} (j \geq 1),\ \ and\ \boldsymbol{S}^{(\boldsymbol{j})} = \sum_{l=j}^{\infty} \boldsymbol{F}^{(\boldsymbol{l})} \boldsymbol{G}^{\boldsymbol{l-j}}$ $(j \geq 0)$. The initial distribution vector $\boldsymbol{\pi_0}$ is uniquely determined by normalization and can be found by solving

$$\boldsymbol{\pi_0}\left[\left(\widehat{\boldsymbol{L}} - \widehat{\boldsymbol{S}}^{(\mathbf{1})} \boldsymbol{S}^{(\mathbf{0})} \boldsymbol{B}\right)^{\diamond} \middle| \mathbf{1}^{\boldsymbol{T}} - \boldsymbol{H} \mathbf{1}^{\boldsymbol{T}}\right] = [\mathbf{0} | \mathbf{1}],$$

where $\boldsymbol{H} = \sum_{\boldsymbol{j=1}}^{\infty} \widehat{\boldsymbol{S}}^{(\boldsymbol{j})} \left(\sum_{\boldsymbol{j=0}}^{\infty} \boldsymbol{S}^{(\boldsymbol{j})} \right)^{-\boldsymbol{1}}$, $\widehat{\boldsymbol{L}} = \widehat{\boldsymbol{A}} - \boldsymbol{I}$, and the symbol "⋄" is an operator that discards the last column of the corresponding matrix.

With the provided iterative relationship and the normalization condition, we can numerically evaluate the stationary distribution vector $\boldsymbol{\pi}$ of a given wireless communication system.

5.2 Triple States Model

In the previous sections, we have introduced a model for a wireless communication link and a methodology for analyzing the ensuing queueing behavior. This model features two channel states, $good$ and bad. To make the model more precise, we can increase the state space to include three different channel states.

5.2.1 State Augmentation

In this section, we consider a slightly more complex channel model by introducing an additional channel state. We refer to this third scenario as the $moderate$ state, and we assume that the corresponding packet-erasure probability is given by ϵ_m. For simplicity and to be consistent with the previous system model characteristics, we keep the erasure probabilities of the extreme states fixed, $\epsilon_g = 0$ and $\epsilon_b = 1$. On the other hand, we assume that the erasure probability for the moderate state is $\epsilon_m \in (0,1)$. After introducing a third channel state, our system model should be updated. We postulate that the system can only transition between adjacent states. Admissible transition probabilities are illustrated in the Fig. 8. The transition probability matrix consequently becomes

$$P = \begin{bmatrix} 1-\alpha & \alpha & 0 \\ \beta & 1-\beta-\gamma & \gamma \\ 0 & \rho & 1-\rho \end{bmatrix}.$$

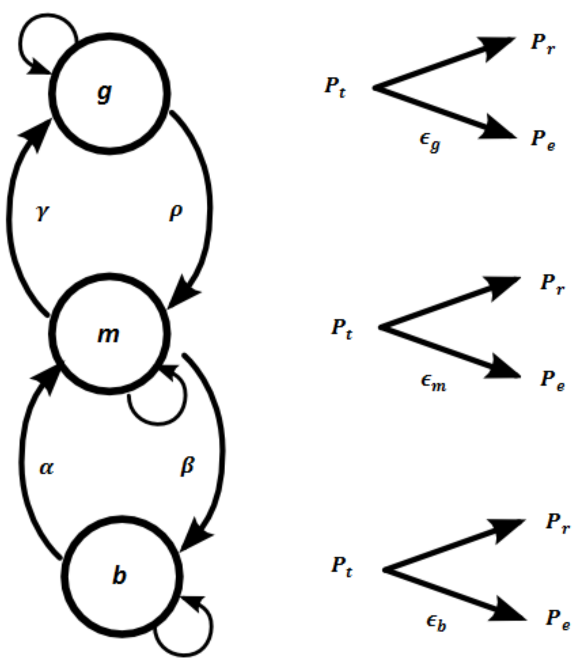

Fig. 8. Three states transition diagram. A new system state named m is added.

Again, we can employ the Baum-Welch algorithm to obtain estimate for this transition probability matrix.

For a wireless communication system with Poisson arrivals, the queue length evolution process still follows an M/G/1-type Markov chain. However, the transition probabilities of the queue length should be modified accordingly,

$$Pr\big(Q_{s+1} = q_s + i \big| U_s = (C_{s+1}, q_s)\big)$$

$$= \xi_i(Pr(c_s = b) + Pr(C_{s+1} = m)\,\epsilon_m)$$

$$+\xi_{i+1}\big(P\,r(C_{s+1} = g) + Pr(C_{s+1} = m)\,(1 - \epsilon_m)\big),$$

$$Pr\big(Q_{s+1} = q_s - 1 \big| U_s = (C_{s+1}, q_s)\big)$$

$$= \xi_0\,(Pr(C_{s+1} = g) + Pr(C_{s+1} = m)(1 - \epsilon_m)).$$

Despite this minor difference in the transition probability expressions, the matrix repetition structure for this system remains unchanged. Using the notations derived in the last section, these sub-blocks are 3×3 matrices that can be expressed as

$$\boldsymbol{F}^{(i)} = \begin{bmatrix} \lambda_{bb}^{i} & \lambda_{bm}^{i} & \lambda_{bg}^{i} \\ \lambda_{mb}^{i} & \lambda_{mm}^{i} & \lambda_{mg}^{i} \\ \lambda_{gb}^{i} & \lambda_{gm}^{i} & \lambda_{gg}^{i} \end{bmatrix} \quad \boldsymbol{A} = \begin{bmatrix} \kappa_{bb} & \kappa_{bm} & \kappa_{bg} \\ \kappa_{mb} & \kappa_{mm} & \kappa_{mg} \\ \kappa_{gb} & \kappa_{gm} & \kappa_{gg} \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} \mu_{bb} & \mu_{bm} & \mu_{bg} \\ \mu_{mb} & \mu_{mm} & \mu_{mg} \\ \mu_{gb} & \mu_{gm} & \mu_{gg} \end{bmatrix}.$$

When the queue is empty, the relevant sub-matrices become

$$\widehat{\boldsymbol{F}}^{(i)} = \begin{bmatrix} \lambda_{bb}^{i0} & \lambda_{bm}^{i0} & \lambda_{bg}^{i0} \\ \lambda_{mb}^{i0} & \lambda_{mm}^{i0} & \lambda_{mg}^{i0} \\ \lambda_{gb}^{i0} & \lambda_{gm}^{i0} & \lambda_{gg}^{i0} \end{bmatrix} \widehat{\boldsymbol{A}} = \begin{bmatrix} \kappa_{bb}^{0} & \kappa_{bm}^{0} & \kappa_{bg}^{0} \\ \kappa_{mb}^{0} & \kappa_{mm}^{0} & \kappa_{mg}^{0} \\ \kappa_{gb}^{0} & \kappa_{gm}^{0} & \kappa_{gg}^{0} \end{bmatrix}.$$

Using this notation for matrices together with the matrix geometric solution provided in the last section, we can numerically evaluate the packet queueing behavior in a wireless communication system.

5.2.2 Model Comparison

With the Baum-Welch algorithm introduced in the previous part, we can obtain either a two-states system model with parameters $\Lambda_0 = \{\gamma_0, B_0, P_0\}$ or a three-states model with parameters $\Lambda_1 = \{\gamma_1, B_1, P_1\}$. To compare the suitability of these two different models, we review some model-comparison method, see Mackay's "Information Theory, Inference, and Learning Algorithms" [15].

Suppose we have an observation sequence $O = \{O_1, O_2, \dots, O_n\}$, and we want to infer how probable Λ_1 is relative to Λ_0. To perform model comparison, we have to find the posterior probabilities for these two models $Pr(\Lambda_0|O)$ and $Pr(\Lambda_1|O)$.

Using Bayes' theorem, we can rewrite these arguments as

$$Pr(\Lambda_0|O) = Pr(O|\Lambda_0)\, Pr(\Lambda_0) \,/\, Pr(O),$$

$$Pr(\Lambda_1|O) = Pr(O|\Lambda_1)\, Pr(\Lambda_1) \,/\, Pr(O).$$

The normalizing constant in both cases is $Pr(O)$, which is the total probability of getting the observed data. If Λ_0 and Λ_1 are the only models under consideration, this probability is given by the sum rule:

$$Pr(O) = Pr(O|\Lambda_0)\, Pr(\Lambda_o) + Pr(O|\Lambda_1) Pr(\Lambda_1).$$

To evaluate the posterior probabilities of the hypotheses, we need to assign values to the prior probabilities $Pr(\Lambda_0)$ and $Pr(\Lambda_1)$; for instance, we may elect to set these to $1/2$ each.

Based on these statements, the posterior probability ratio of model Λ_1 to $\Lambda_0$is

$$\frac{Pr(\Lambda_1|O)}{Pr(\Lambda_0|O)} = \frac{Pr(O|\Lambda_1)}{Pr(O|\Lambda_0)}.$$

Still, we need to evaluate the observation-dependent terms $Pr(O|\Lambda_0)$ and $Pr(O|\Lambda_1)$. We can give names to these quantities. They are measures of how much the observation sequence favors either Λ_0 or Λ_1, and we can call them the *evidence*.

Furthermore, we can use the posterior probability ratio to represent which model is more favorable by the observations.

Again, we can compute the forward variable

$$\phi_t = Pr\{O_1, O_2, \dots, O_t, C_t | \Lambda\},\ 1 \le t \le n.$$

Suppose there are totally N possible channel states for C_t, we can easily see that the required probability is given by

$$Pr(O|\Lambda) = \sum_{i=1}^{N} \phi_n(i)$$

where $\phi_n(i) = Pr\{O_1, O_2, \dots, O_n, C_n = i | \Lambda\},\ 1 \le i \le N.$

We are now ready to perform our numerical simulation. Specifically, we obtain a sequence of observations of length 1000 from the TAMU Evans library and use the Baum-Welch algorithm to estimate the system parameters for both Λ_0 and Λ_1. The results are shown in the following Table 1.

Table 1 Estimated parameters for two competing models

	α	β	γ	ρ	ϵ_g	ϵ_m	ϵ_b
2-states	0.0072	0.0540	NA	NA	0.0126	NA	0.9799
3-states	0.0180	0.2098	0.0455	0.0324	0	0.5822	0.9944

Also, we can use an observation sequence to calculate the *evidence* for Λ_0 and Λ_1. For illustrative purpose, we can vary the length of the observation sequence and calculate the posterior probability ratio. The detailed results are shown in Table 2, where evi stands for evidence.

Table 2 Posterior probabilities for different sequence length

Sequence Length	$Log_{10}(evi)$ for 2-states Model	$Log_{10}(evi)$ for 3-states Model
100	-26.0286	-25.3383
200	-59.7470	-53.5295
300	-75.5893	-67.9250
400	-84.7725	-78.4764
500	-92.7509	-86.9748
600	-103.9010	-99.0341
700	-110.2615	-106.0579
800	-117.8970	-112.8002
900	-124.9018	-120.4000
1000	-127.7762	-123.2273

To explore this further, we plot these data points in Fig. 9 and compare the evidence for the two different models. An alternate interpretation is offered in Fig. 10, where the posterior probability ratio is plotted to show which model is favored by the observation sequence.

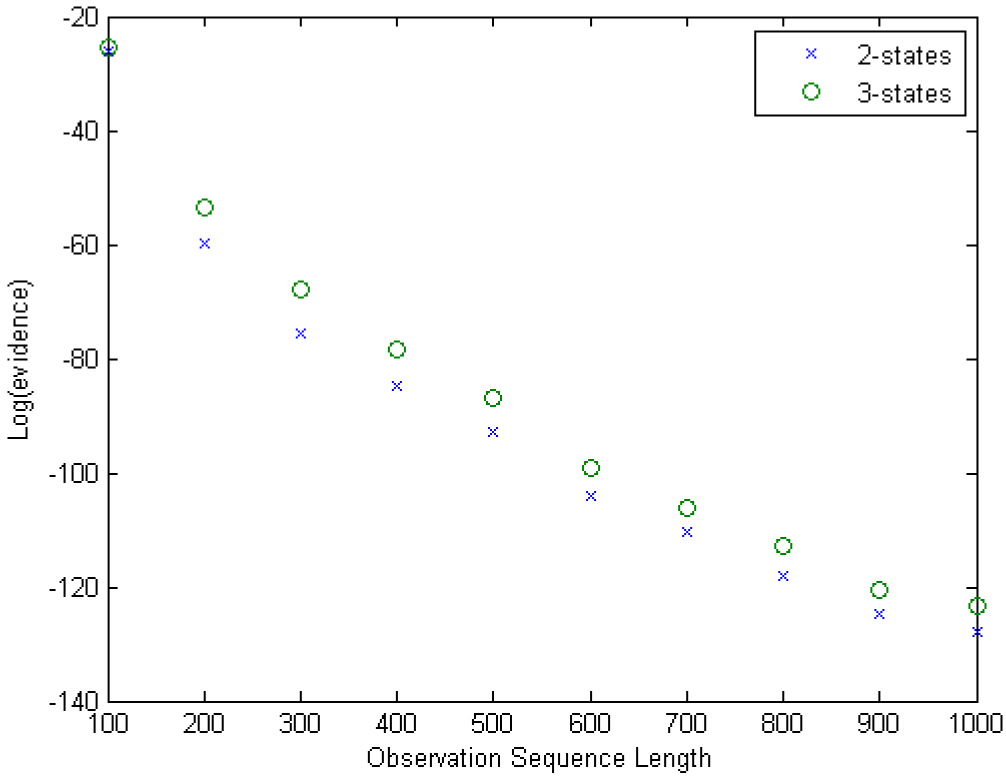

Fig. 9. Corresponding evidence for 2-states model and 3-states model. The evidences show that the 3-state model is generally more convincing than the 2-state model.

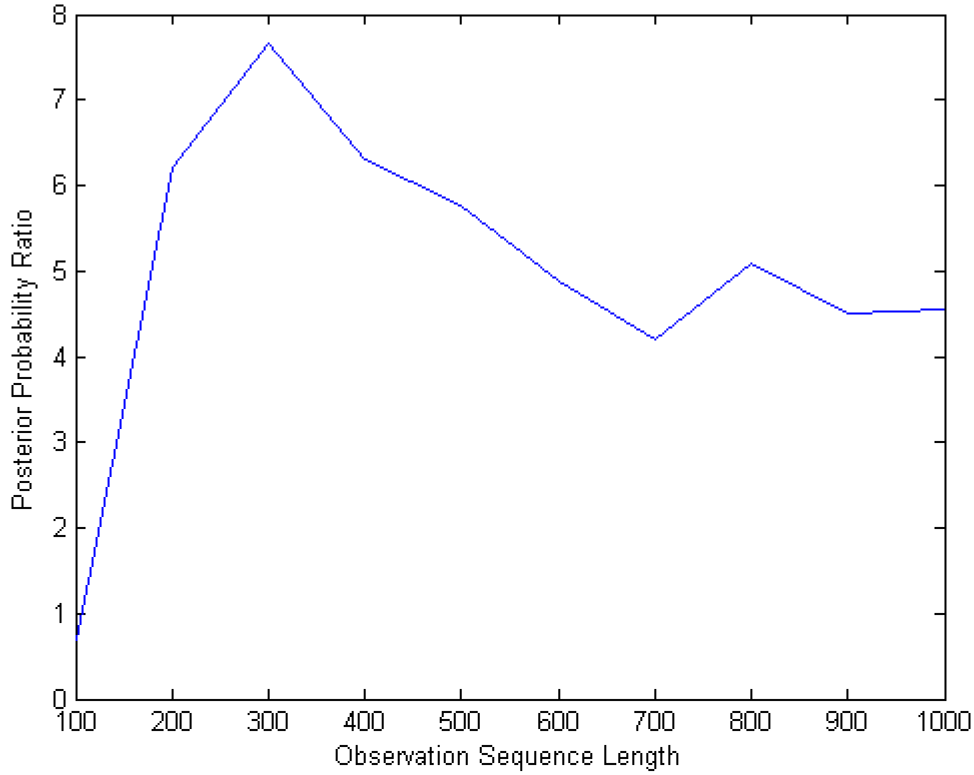

Fig. 10. Posterior probability ratio for 3-state model normalized by the evidence of 2-state model. The ratio is plotted in log scale and is always above 1. It means the more complicated 3-state model is always much more favorable than the 2-state model with very little dependence on the observation length.

6. NUMERICAL RESULTS

The mathematical structure of the algorithm introduced above makes it possible to compute the equilibrium distribution of the queue at the transmitter. Based on it, the potential performance of many wireless communication applications can be analyzed. As an illustrative example, we employ the contemporarily widely utilized video over IP applications and offer a comparison between three different Wi-Fi access points, labeled AP_1, AP_2 and AP_3. Moreover, the detected signal powers for these three access points are correspondingly $-62dBm$, $-85dBm$ and $-83dBm$.

The channel state model is divided into time interval of 1 millisecond, and a UDP socket program is created to simulate the video over IP application and obtain the observation sequence $O = \{O_1, O_2, \ldots, O_n\}$. The packet size for the UDP socket is set to 500 bytes and packets will arrive every 10 millisecond. These settings result in a corresponding data rate of $400\ Kb/s$, which loosely matches the medium quality video over IP applications with CIF technology. Now, we can apply our mathematic methodology by setting $Pr(Arr) = 0.1$ and retrieving the channel state model $\Lambda = \{\gamma, B, P\}$ with the analysis method introduced in the previous sections. The results are shown in Fig. 11.

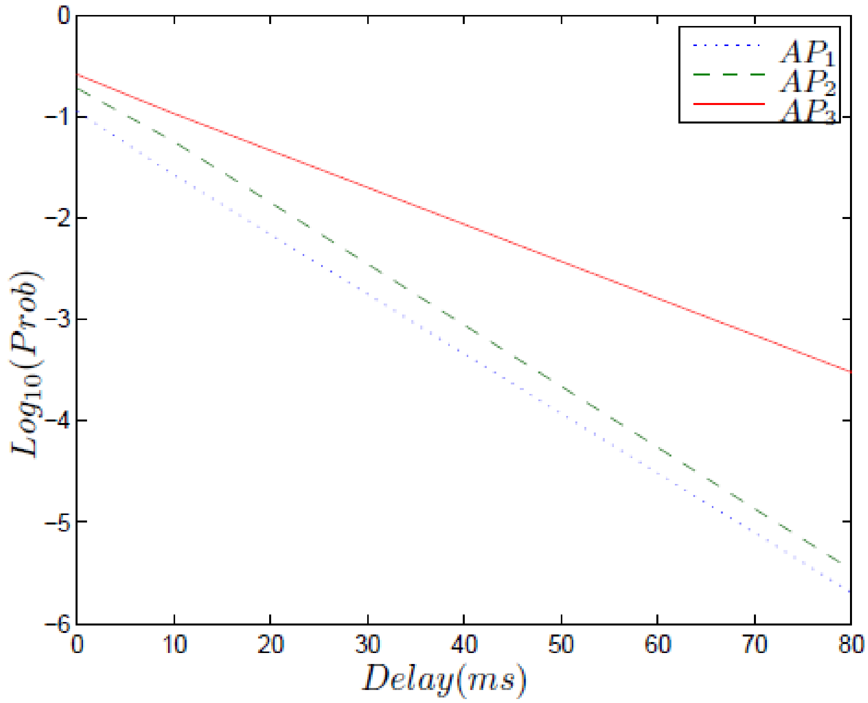

Fig. 11. Numerical results of the packet delay probability exceeding certain thresholds for different Wi-Fi access points. $Prob$ represents the tail probability that the packet delay exceeds the given threshold.

From the above figure, we can see that, although the signal power for AP_1 and AP_2 differs significantly, their quality of service in terms of packet delay are nearly the same. On the other hand, for AP_2 and AP_3, the services that they can support are quite different, despite having nearly identical signal power levels. Thus, the analysis method introduced above can make significant improvement in determining which Wi-Fi access points are capable of supporting a prescribed application, especially when the detected signal power levels for those available access points are very similar to one another.

7. SUMMARY AND CONCLUSIONS

With the fast development of contemporary wireless communication technologies and the significant improvement in the computation ability of mobile devices, advanced techniques to optimizing the performance of wireless communication systems can now be implemented on mobile devices. These algorithms should put great emphasis, not only on maximum throughput, but also on packet delay, especially for delay-sensitive applications. This thesis proposed a hidden Markov system model and provided two different prospective methodologies for analyzing packet delay, either at a bit-level or at a packet-level. The first analysis method takes the channel coding strategy into consideration and the underlying bit-erasure channel characteristics at the physical layer. The second approach introduces a highly integrated model by merging these factors into a simple characterization. This relatively simple method can more easily be implemented on modern mobile devices. In particular, the second method yields a tractable Baum-Welch algorithm for estimating the desired system parameters.

For illustrative purpose, the proposed algorithms were implemented on an Android mobile platform and results were obtained for three different Wi-Fi access points, which are attached to the same background router. The results of this thesis are encouraging since they show that, although the signal power for different access points may be a factor that influences the effective capacity of a link, the packet delay for an access point can have very little dependence on it. More specifically, in a wireless communication network, congestion is a factor that should not be neglected. The analysis methodology introduced in this thesis provides a possible point of view on how to avoid excessive bandwidth contending among different service receivers and efficiently allocate limited communication resources to different users.

Moreover, the algorithms provided in this thesis can be implemented on a distributed system. Mobile devices sharing a same wireless communication resource can independently make decisions by conducting the analysis with the methodology presented in this thesis. Generally speaking, conducting this analysis can cause some overhead for the entire communication system because the server and clients will have to estimate the required parameters by exchanging packets. However, since in this way every user is making a wise decision, this overhead hopefully remains negligible compared to the unnecessary network congestion resulting from bad management.

In the last part of this thesis, some more advanced models were introduced and comparisons were done between these and the previous models. Our results show that more complicated models can be more convincing and heavily favored by an observation sequence. However, as a model grows more complicated, the computational complexities for the Baum-Welch algorithm and the matrix geometric solution also increase dramatically. This may be an obstacle that prevents these more elaborate algorithms from being implemented on mobile devices.

To resolve the tension between computational complexity and high precision, a future research possibility is to perform the computations on the server, which may give us a centralized control system. The mobile devices and server can exchange information by sending small data packets. After adequate estimates are obtained, the server can send messages to control the entire communication system. This way, the efficiency of the system can be optimized in a global fashion.

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