RELIABILITY ANALYSIS OF SETTLEMENT USING AN UPDATED PROBABILISTIC UNIFIED SOIL COMPRESSION MODEL

A Thesis

by

AVERY CHRISTOPHER AMBROSE

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2011

Major Subject: Civil Engineering

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Approved by:

Chair of Committee,	Paolo Gardoni
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ABSTRACT

Reliability Analysis of Settlement Using an Updated Probabilistic Unified Soil Compression Model. (December 2011)

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Settlement of a structure is a matter of great concern. Both excessive and differential settlement can cause expensive damage to buildings and must be avoided. Most methods used to estimate settlement are both deterministic in nature and are based on elastic analysis of soils. To better estimate settlement, a probabilistic estimate that uses a more in depth analysis of the behavior of soil is required. This thesis develops a new probabilistic model for estimating settlement based on a probabilistic unified soil compression model. The model is then used to estimate the settlement of an embankment. Lastly, a reliability analysis of settlement is carried out on the settlement estimate of the embankment.

The new probabilistic unified soil compression model used in this thesis was developed based on a previously developed probabilistic unified soil compression model, accounting for further uncertainties into the model and correcting for errors in the model calibration. This model was calibrated using data from a site on the Venice Lagoon using a Bayesian approach. The model to estimate settlement was developed based on this probabilistic soil compression model and is unbiased in nature. Using this model, unbiased settlement estimates were obtained for an embankment also located in the Venice Lagoon.

Using the developed probabilistic model for settlement, reliability analysis was carried out. This reliability analysis involved assessing the conditional probability that, for a specific load and given soil properties, a specified settlement threshold would be reached or passed. Sensitivity and importance analysis were carried out, determining which parameters and random variables have the largest impact on the fragility estimates. Lastly, a closed-formed approximation based on the Central Limit Theorem was developed to allow for easier fragility estimation.

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1. INTRODUCTION

Chapter IV of the Ph.D. dissertation by Jung (2009), titled “Probabilistic Analysis of the Compressibility of Soils”, builds the framework through which the fragility estimates of settlement can be determined. That chapter starts by developing a new probabilistic coil compression model, then using this model to develop a probabilistic model for settlement. Using this probabilistic model for settlement, the fragility is then estimated.

Although the work in Chapter IV by Jung (2009) is thorough and complete, further inspection of that chapter reveals that there are errors in some of the analysis conducted. These errors cause the results in the chapter to also be erroneous. This thesis aims to correct the errors present in Chapter IV of Jung (2009) and also compare the corrected results to the erroneous results.

The work in Chapter IV of the Ph.D. dissertation by Jung (2009) is prefaced by the work done in Chapter III of the same Ph.D. dissertation. The work done in Chapter III is published in a journal article titled ‘Bayesian updating of a unified soil compression model’. In that article, Jung et al. (2009) developed a probabilistic model for soil compression that is based on a deterministic unified compression model developed by Biscontin et al. (2007). The model developed by Biscontin et al. (2007) is based on the compressibility of soils and works for a varying range of grade size distributions as long as the general mineralogy of the soil remains fairly constant. The

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This thesis follows the style of *Journal of Structural Engineering.*

model developed by Jung et al. (2009) does not take into account all of the uncertainties present, including those associated with the variability of loading steps in one-dimensional laboratory compression testing, and must be expanded to account for the additional uncertainties.

Following Chapter IV of Jung (2009), this thesis develops a new probabilistic model based on the model developed by Jung et al. (2009), taking into account the uncertainties arising from the variability of the loading steps in the one-dimensional laboratory testing. To account for these problems, the newly developed probabilistic model is formulated as an auto-regressive model. A Bayesian approach is used to calibrate this new probabilistic model based, ensuring that the model is unbiased and all uncertainties are taken into account. Calibration of this model is done using a database that includes a series of laboratory one-dimensional compression test. The database also includes the soil properties for the samples that are being tested. This database is from a testing site in the Venice Lagoon, in Italy.

A probabilistic model to estimate settlement is developed using the proposed probabilistic soil compression model. Using this model, unbiased settlement estimates can be obtained as long as the appropriate soil properties are ascertained and the compression model is calibrated in soils similar to the soil at the site whose settlement is being estimated. A settlement estimate is compared to the actual settlement at another site in the Venice Lagoon. The soil properties at this site and the actual settlement data have been exhaustively collected, making this site appropriate for the proposed comparison.

Using the developed probabilistic model for settlement, a reliability analysis is carried out. This reliability analysis involves assessing the conditional probability that, for a specific load and given soil properties, a specified settlement threshold will be reached or passed. These probabilities are ascertained taking into account all of the uncertainties present in the model and soil properties. Also, the sensitivities and importance measures of the model parameters are calculated. This helps determine which random variables and parameters have the most impact on the fragility estimates. Lastly, an approximate closed-form estimate of the fragility is developed, based on the Central Limit Theorem (CLT). This approximation allows the fragility to be estimated without the use of specialized reliability software and is relatively simple to calculate.

2. PROBABILISTIC SOIL COMPRESSION MODEL

2.1 Background

Biscontin et al. (2007) combines a one-dimensional compression model for cohesionless soils developed by Pestana and Whittle (1995) and evolved by Pestana (2002) with a conceptual approach that idealizes the soil as a two-phase combination of incompressible material (sand and silt particles) and a compressible matrix (clay-water phase). This model assumes that, at high stress levels, the one-dimensional compression response of soil converges to a curve that is linear in a double logarithmic space. This curve is referred to as the linear limiting compression curve (K_0-LCC) and is linear in the double logarithm space of void ratio and vertical effective stress ($\log e - \log p_v^{'}$), where e is the void ratio and $p_v^{'}$ is the effective vertical stress.

One advantage of this model is that the compression response can be characterized using only a few model parameters. One model parameter ρ_c is the slope of the K_0-LCC in double logarithm space. The location of the K_0-LCC is defined by a reference void ratio e_{1v} at atmospheric pressure p_{at}. The curvature of the compression curve in the region of lower pressure is described by the parameter α. Biscontin et al. (2007) finds that the reference void ratio e_{1v} is a function of the fines fraction FF of the soil, the reference void ratio for the granular phase e_{g1v} and the reference void ratio for the clay-water phase e_{c1v}. This relationship is expressed as

$$e_{1v} = \begin{cases} e_{g1v}(1-FF) - FF, & FF \leq 0.2 \\ e_{g1v}\exp(0.25-4.76FF) + (e_{c1v}-0.12)FF, & 0.2 < FF < 0.7 \\ e_{c1v}FF, & FF \geq 0.7 \end{cases} \quad (1)$$

These four model parameters (ρ_c, α, e_{g1v} and e_{c1v}) are dependent on soil mineralogy and can be used to predict the compression response for soils with similar mineralogy.

Jung et al. (2009) uses the same model parameters as Biscontin et al. (2007) and a similar model form to develop a probabilistic soil compression model. The purpose of creating the probabilistic model was to eliminate any bias associated with the input parameters from the deterministic model and to account for the uncertainties, epistemic and aleatory, present in the prediction process. To account for the uncertainty in the deterministic model, Jung et al (2009) adds an error term $\sigma\varepsilon_i$. This error term combines the standard deviation of the model error σ and a random variable with zero mean and unit variance ε_i.

In the model developed by Jung et al. (2009) the void ratio e_i at a given vertical stress p'_{vi} is dependent on the mean void ratio and vertical stress at the previous loading step, $\mathrm{E}(e_{i-1})$ and p'_{vi-1} respectively. In this case the mean void ratio at the previous loading step is represented by the expectation of the void ratio at the previous loading step. The model developed in Jung et al. (2009) is expressed as

$$e_i(\boldsymbol{\theta},\sigma) = \mathrm{E}[e_{i-1}(\boldsymbol{\theta},\sigma)]\left\{1-\rho_c\left\{1-\left[1-\frac{p'_{vi}}{p_{at}}\left(\frac{\mathrm{E}[e_{i-1}(\boldsymbol{\theta},\sigma)]}{e_{1v}(e_{g1v},e_{c1v},FF)}\right)^{1/\rho_c}\right]^{\alpha}\right\}\frac{p'_{vi-1}-p'_{vi}}{p'_{vi}}\right\} + \sigma\varepsilon_i, \quad (2)$$

$$i = 1, \dots, n_k$$

where $\mathbf{\theta} = \left(e_{g1v}, e_{c1v}, \rho_c, \alpha\right)$ is the vector of unknown model parameters, n_k represents the number of data points in each compression curve and where the subscript k is used to indicate a specific sample.

Since each point on the compression curve depends on the previous point, Jung et al. (2009) assumes that there is a correlation coefficient ρ between all the points on the same curve. Jung et al. (2009) also assumes that each compression curve is uncorrelated. Therefore the unknown parameters present in this probabilistic model were $\mathbf{\Theta} = (\mathbf{\theta}, \sigma, \rho)$. Although this correlation coefficient is not present in the model form, it is important in model calibration.

Two assumptions were made when this model was being developed. The first assumption was the homoskedasticity assumption and the second assumption was the normality assumption. These assumptions are very important when forming a probabilistic model. The homoskedasticity assumption assumes that the model error σ remains constant over the range of variables. The normality assumption is that the random variable ε_i is normally distributed.

Although, the probabilistic model developed by Jung et al. (2009) is unbiased and properly takes into account the epistemic and aleatory uncertainties, the model does not take into account the uncertainties that are brought about from the variability of the amplitude of the loading steps in the one-dimensional laboratory test and field conditions.

2.2 Proposed Probabilistic Model

To expand the model developed by Jung et al. (2009), taking into account the additional uncertainties, an auto-regressive model that accounts for the variability of the amplitude of the loading step in the laboratory one-dimensional compression test is proposed. The proposed model, which also utilizes a logarithmic variance stabilizing transformation, is expressed as

$$\ln[e_i(\boldsymbol{\theta},\sigma_i)] = \ln\left\{\mathrm{E}[e_{i-1}(\boldsymbol{\theta},\sigma_i)]\left\{1-\rho_c\left\{1-\left[1-\frac{p'_{vi}}{p_{at}}\left(\frac{\mathrm{E}[e_{i-1}(\boldsymbol{\theta},\sigma_i)]}{e_{iv}(e_{g1v},e_{civ},FF)}\right)^{1/\rho_c}\right]^{\alpha}\right\}\frac{p'_{vi-1}-p'_{vi}}{p'_{vi}}\right\}\right\}+\sigma_i\varepsilon_i \tag{3}$$
$$i = 1,\ldots,n_k$$

where all the model parameters are the same as those in Jung et al. (2009) except for the model error. The model error σ_i in this revised model is a function of the amplitude of the loading step and is expressed as

$$\sigma_i = \sigma(p'_{vi} - p'_{vi-1})^{\xi} \tag{4}$$

where σ and ξ are unknown model parameters and ε_i is a random variable with unit variance and zero mean. The logarithmic variance stabilizing transformation is used in this proposed model to ensure that the homoskedasticity assumption and the normality assumption are met.

As with the model developed by Jung et al. (2009), the data points on each compression curve are assumed to be correlated with a correlation coefficient ρ. Also, the individual compression curves are assumed to be uncorrelated. The covariance matrix, for all the data points, is

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_1 & & & \text{sym.} \\ 0 & \boldsymbol{\Sigma}_2 & & \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & \boldsymbol{\Sigma}_\mathrm{k} \end{bmatrix}_{N\times N} \tag{5}$$

where $\boldsymbol{\Sigma}_k$ is the covariance matrix for the k^{th} one-dimensional compression curve and N is the total number of data points. $\boldsymbol{\Sigma}_k$ can be expressed as

$$\boldsymbol{\Sigma}_\mathrm{k} = \begin{bmatrix} \sigma_1^2 & & & \text{sym.} \\ \rho\sigma_2\sigma_1 & \sigma_2^2 & & \\ \vdots & \vdots & \ddots & \\ \rho\sigma_{\mathrm{n_k}}\sigma_1 & \rho\sigma_{\mathrm{n_k}}\sigma_2 & \cdots & \sigma_{\mathrm{n_k}}^2 \end{bmatrix}_{\mathrm{n_k}\times\mathrm{n_k}} \tag{6}$$

Based on this newly formulated probabilistic model, the unknown model parameters are now $\boldsymbol{\Theta} = (\boldsymbol{\theta}, \sigma, \rho, \xi)$.

3. PROBABILISTIC MODEL CALIBRATION

3.1 Bayesian Updating

A Bayesian approach is used to calibrate this proposed probabilistic model. This approach uses the Bayesian updating rule to estimate the unknown parameters $\boldsymbol{\Theta}$ (Box and Tiao 1992). This approach is very useful for model calibration because it is capable of using many different forms of available data in its analysis. In general terms, the updating rule is expressed as

$$p(\boldsymbol{\Theta}|\mathbf{e}) \propto \kappa L(\boldsymbol{\Theta}|\mathbf{e}) p(\boldsymbol{\Theta}) \tag{7}$$

where $p(\boldsymbol{\Theta}|\mathbf{e})$ is the posterior distribution representing an updated knowledge about the model parameters $\boldsymbol{\Theta}$, based on the information provided by $\mathbf{e}$, which in our case would be experimental data. Also, $L(\boldsymbol{\Theta}|\mathbf{e})$ is the likelihood function that uses data contained in $\mathbf{e}$ to give objective information on $\boldsymbol{\Theta}$, $p(\boldsymbol{\Theta})$ is the prior distribution that represents all the available knowledge about $\boldsymbol{\Theta}$ prior to obtaining $\mathbf{e}$ and lastly, κ usually expressed as

$$\kappa = \left[\int L(\boldsymbol{\Theta}|\mathbf{e}) p(\boldsymbol{\Theta}) d\boldsymbol{\Theta} \right]^{-1} \tag{8}$$

is a normalizing factor. In general, this updating process can be repeated every time new, relevant data becomes available.

The prior distribution represents all the previously available information about the model parameters. In our case there is no prior distribution and a non-informative prior must be used. A non-informative prior is a distribution used when little or no previous information is available about the model parameters, when a non-informative prior is used the new information has a very large impact on the updating process and the prior has relatively little impact. Usually, a uniform distribution acts as an appropriate non-informative prior distribution (Box and Tiao 1992) but in cases where the probabilistic model is not a linear function on the model parameters $\mathbf{\Theta}$ a uniform distribution does not act as a non-informative prior. This is the case with the probabilistic model being proposed and a different non-informative prior must be used.

Jeffreys (1961) introduced Jeffreys' rule which can be used to construct an approximate non-informative prior. Jeffreys' rule assumes independence between the distribution of the model parameters $\boldsymbol{\theta}$ and the distribution of the model error, this independence can be expressed as

$$p(\mathbf{\Theta}) \propto p(\boldsymbol{\theta})p(\mathbf{\Sigma}) \tag{9}$$

where $\mathbf{\Sigma} = (\sigma, \beta, \rho)$ is the vector of model error parameters. Jeffreys' rule says that the prior distribution of $\boldsymbol{\theta}$ is proportional to the square root of the determinant of Fisher's information matrix. Fisher's information matrix is defined as the negative of the expectation of the second partial derivative of the natural logarithm of the likelihood function. Jeffrey's prior can be expressed as

$$p(\boldsymbol{\theta}) \propto \sqrt{|\mathcal{J}(\boldsymbol{\theta})|} = \sqrt{\left|-E_{\boldsymbol{\theta}}\left[\frac{\partial^2 lnL}{\partial\theta_i\partial\theta_j}\right]\right|} \tag{10}$$

where $|\cdot|$ is the determinant. The non-informative prior for the model error can be written as

$$p(\mathbf{\Sigma}) \propto |\Sigma|^{-(N+1)/2} \tag{11}$$

where Σ is the covariance matrix defined earlier (Gardoni et al. 2002).

The likelihood function $L(\boldsymbol{\Theta}|\mathbf{e})$ is proportional to the conditional probability of observing the experimental data for a set of model parameters. Since we are assuming that the data from each compression curve is uncorrelated, the likelihood function can be express as the product of the likelihood from each compression curve. The likelihood function is written as

$$L(\boldsymbol{\Theta}|\mathbf{e}) = \prod_{k=1}^{m}\left[(2\pi)^{-n_k/2}\left\{\prod_{i=1}^{n_k}(\sigma_i)\right\}^{-1} exp\left(-\frac{1}{2}\mathbf{r}_k^T(\boldsymbol{\theta})\mathbf{R}_k^{-1}\mathbf{r}_k(\boldsymbol{\theta})\right)\right] \tag{12}$$

where $\mathbf{r}_k$ is the normalized argument vector, defined as the vector of error between the actual data and the estimated data divided by the model error σ_i at that data point, and $\mathbf{R}_k$ is the correlation matrix. Maximizing the likelihood function gives the maximum likelihood estimate (MLE). The MLE is an estimate of the unknown model parameters based only on the experimental data.

The likelihood function was the first error in Chapter IV of Jung (2009). The likelihood function, used erroneously, is expressed as

$$L(\boldsymbol{\Theta}|\mathbf{e}) = \prod_{k=1}^{m}\left[(2\pi)^{-n_k/2}|\boldsymbol{\Sigma}_\mathrm{k}|^{-1} exp\left(-\frac{1}{2}\boldsymbol{\varepsilon}_k^T(\boldsymbol{\theta})\boldsymbol{\Sigma}_k^{-1}\boldsymbol{\varepsilon}_k(\boldsymbol{\theta})\right)\right] \tag{13}$$

where $\boldsymbol{\varepsilon}_k$ is the argument vector, defined as the difference between the actual data and the estimated data. This likelihood function does not properly account for the transformation in to the standard normal space and as a result causes incorrect parameter estimation.

3.2 Laboratory Data

The data used to calibrate this model comes from a series of laboratory one-dimensional soil compression test. The samples for these compression tests are from the Malamocco Test Site (MTS) in the Venice Lagoon in Italy. There are three distinct sets of samples that were tested and are being used for this calibration. The first set includes natural, undisturbed samples that belong to three soil classes (SM-SP, ML and CL). The second set includes reconstituted samples that are obtained using SP-SM and ML samples from the natural samples. The third set is made up of reconstructed mixtures of sand and clay, with varying clay to sand ratios. Table 1 shows the range of the initial void ratio e_0 and the Fines Fraction FF for each of the three data sets.

Table 1. Ranges of the material properties from the laboratory data

Sample	Material property	Range
Natural samples	Soil class	CL, ML, SP-SM
	e_0	0.600 – 1.237
	FF	0.01 – 0.78
Artificially reconstructed mixtures	Soil class	-
	e_0	0.680 – 1.160
	FF	0.01 – 0.67
Reconstituted samples	Soil class	ML, SP-SM
	e_0	0.606 – 1.454
	FF	0.01 – 0.11

3.3 Calibration Results

The probabilistic soil compression model is calibrated using the MTS laboratory data following the Bayesian updating rule. The statistics for the unknown model parameters **Θ** are presented in Table 2, including the parameter mean, standard deviation and correlation coefficients. The standard deviation of the model parameters represent the uncertainty associated with this parameter estimate. This uncertainty can be reduced by collecting more, appropriate data.

Table 2. Posterior statistics of the unknown parameters

Parameter	Mean	Standard deviation	Correlation coefficient						
			e_{g1v}	e_{c1v}	ρ_c	α	σ	ξ	ρ
e_{g1v}	2.76	0.135	1						
e_{c1v}	3.19	0.158	0.91	1					
ρ_c	0.229	0.058	0.63	0.60	1				
α	10.15	1.683	0.79	0.78	0.09	1			
σ	0.007	0.001	0.13	0.01	0.11	0.02	1		
ξ	0.322	0.017	–0.13	–0.01	–0.11	–0.02	–0.94	1	
ρ	0.222	0.023	–0.12	–0.06	–0.14	–0.01	0.06	–0.06	1

The error between the model prediction and the measured values using the results of the model calibration are shown in Figure 1. A prefect model fit would result in all the points along the zero error line, but of course this is not the case for this model. The dotted lines represent the standard deviation of the model error σ_i of the calibrated model on either side of the zero error line. Figure 1 shows that the majority of the data points lay within this dotted region.

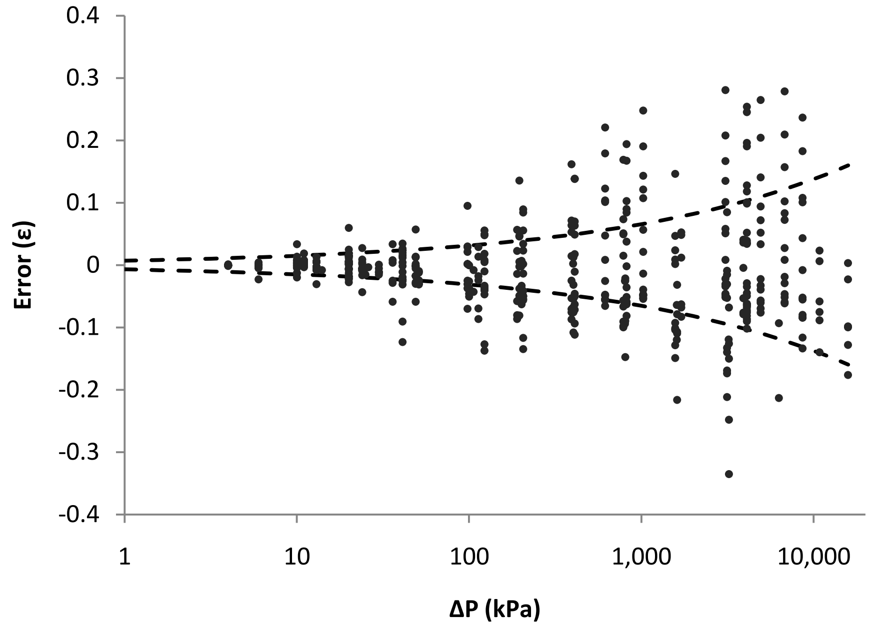

Figure 1. Comparison between the measured and predicted values based on the proposed probabilistic model

The statistics for the unknown model parameters presented by Jung (2009) are shown in Table 3. Comparing Table 2 to Table 3 shows that the mean value of e_{g1v} and e_{c1v} are slightly larger in the corrected model, with smaller standard deviations. The parameters ρ_c, σ and ξ have statistics that are almost the same in both calibrations, but ρ_c has a larger standard deviation. The parameter α has a mean value in the corrected model that is twice as large as the mean value in the previous calibration, the standard deviation for the parameter is smaller in the corrected calibration. Lastly, ρ has both a smaller mean and standard deviation in the corrected calibration.

Table 3. Posterior statistics of the unknown parameters presented in Jung (2009)

Parameter	Mean	Standard deviation	Correlation coefficient						
			e_{g1v}	e_{c1v}	ρ_c	α	σ	ξ	ρ
e_{g1v}	2.63	0.239	1						
e_{c1v}	2.81	0.303	0.96	1					
ρ_c	0.233	0.021	0.66	0.57	1				
α	5.86	2.83	0.89	0.93	0.29	1			
σ	0.009	0.001	0.13	0.03	0.26	–0.01	1		
ξ	0.296	0.017	–0.11	–0.01	–0.31	0.05	–0.76	1	
ρ	0.709	0.053	0.06	0.04	–0.02	0.06	0.62	–0.02	1

4. SETTLEMENT ESTIMATES

Using the proposed probabilistic soil compression model a probabilistic settlement model is developed. To estimate the settlement, the compression model, along with the soil's material properties, the soil's geometric properties and the loading conditions, is required. The settlement S is calculated by dividing the soil into layers and summing the change in thickness ΔH_i of each of these layers. Each layer will have its own initial thickness H_{0i}, fines fraction FF, initial void ratio e_{0i} and change in void ratio $\Delta e_i(\mathbf{x}, \mathbf{\Theta})$. This model for settlement can be expressed as

$$S(\mathbf{x}, \mathbf{\Theta}) = \sum_{i=1}^{r} \Delta H_i = \sum_{i=1}^{r} \left[\frac{\Delta e_i(\mathbf{x}, \mathbf{\Theta})}{1 + e_{0i}} H_{0i} \right] \tag{14}$$

where $\mathbf{x} = (Q, H_{01}, e_{01}, \ldots, H_{0r}, e_{0r})$ is the vector of material and geometric properties for each layer and the applied pressure Q. The values found in the $\mathbf{x}$ vector are also random variables and must be assigned appropriate distributions, means and standard deviations.

For each layer, the loading conditions and soil properties are used to determine the initial effective vertical stress. Boussinesq's Method or another appropriate method can be used to determine the change of effective vertical stress and therefore the final effective stress. The initial and final effective vertical stress along with the soil properties and the compression model is used to determine the final void ratio which can be subtracted from the initial void ratio to determine the change in void ratio.

To test the probabilistic settlement estimates, the settlement model is used to estimate the settlement of an embankment constructed at the Treporti Test Site (TTS). Although our model was calibrated using samples from the MTS, the TTS is also in Venice Lagoon and believed to have soil mineralogy similar to that of the MTS. The embankment is a circular earth embankment that reaches 6.7 m high and has a diameter of 40 m. Data on the load being applied as a result of this embankment and the soil geometry and material properties are located in Simonini (2004) and Tonni and Gottardi (2011). The final vertical pressure applied on the soil as a result of the embankment is approximately 106.5 kPa. These articles also present the actual settlement data that will be compared to the estimates.

Soil material and geometric properties for the TTS are obtained using data provided in Simonini (2004). The initial stress state for each of the soil layers below the embankment is determined using the saturated unit weight of the soil. Next, the final stress state of each layer is determined by using the initial stress state, the applied pressure Q and Boussinesq's equation for stress below a center of a circular load. The change in void ratio $\Delta e_i(\mathbf{x}, \mathbf{\Theta})$ is then calculated using the soil properties found in Simonini (2004) and the final and initial stress condition in conjunction with the probabilistic soil compression model developed earlier. This change in void ratio $\Delta e_i(\mathbf{x}, \mathbf{\Theta})$, along with the initial void ratio e_{0i} and layer thickness H_{0i}, is used in Eq. (14) to estimate the settlement S at the center of the embankment. Since this is a probabilistic model, the parameters in vector $\mathbf{x}$ need to have, not only a mean value, but a standard deviation and distribution type. The mean values are assigned from data provided in Simonini (2004). Since these values must be non-negative the distribution type to all these parameters is a lognormal distribution. Also, following convention the standard deviations are based on an assumed coefficient of variation (COV) of 10%. This COV reflects the confidence of the values obtained from site investigation.

The settlement model yields a settlement estimate around 580 mm for the settlement below the center of the embankment. This estimate is larger than the actual settlement readings reported in both Simonini (2004) and Tonni and Gottardi (2011). Simonini (2004) reports a settlement around 460 mm recorded in December of 2003 while Tonni and Gottardi (2011) reports a settlement around 525 mm recorded in June of 2008. The increasing trend of the settlement data shows that the final settlement may be larger than those reported in literature, bringing the final settlement closer to the estimated settlement. Figure 2 shows a comparison of the accumulated settlement for the estimate and the reported settlement. Figure 2 also shows that the model has larger settlement in areas where the reported settlement is not very large and smaller settlement in areas where the reported settlement is substantial.

Although the predictions from this model are supposed to be unbiased, the model does overestimate the settlement of the embankment. One possible reason for this over estimation is that even though the TTS is also in the Venice Lagoon, the soil at the TTS may have mineralogy that is not the same as the mineralogy as the MTS, where the samples used to calibrate the soil compression model were obtained from. This possible difference in mineralogy could lead to parameter estimation that is not accurate for the TTS and lead to the overestimation of the settlement of the embankment.

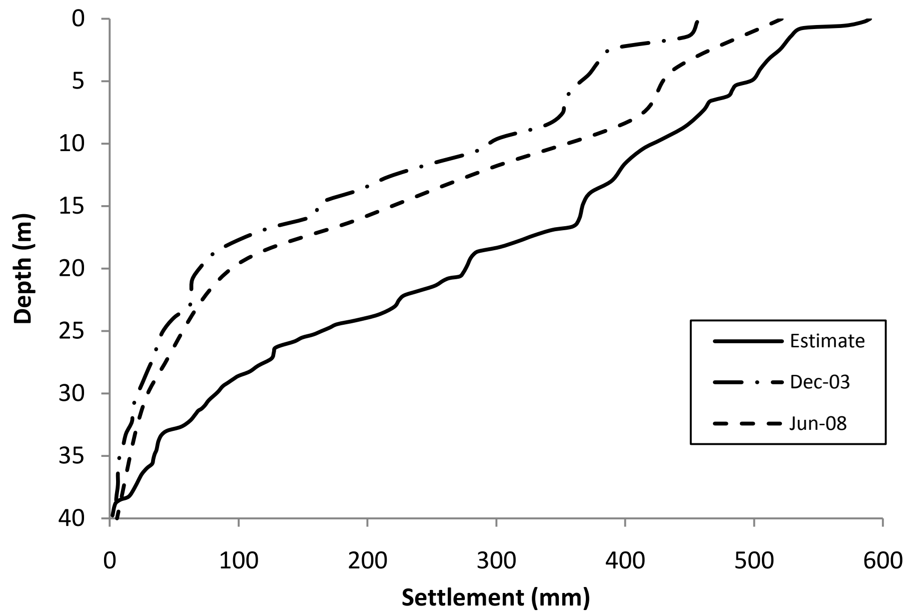

Figure 2. Comparison between estimated and actual accumulated settlement

Another possible cause for the overestimation of the settlement is that the range of initial void ratios used to calibrate the soil compression model (0.600 – 1.454) was a lot smaller than the initial void ratios of the soil below the embankment (0.75 – 3.74). It is possible that the model is over estimating the incremental settlement in layers whose void ratio is larger than those used for the soil compression model calibration. Further inspection shows that the void ratio of the layer of soil whose depth is 34 m is larger than those used to calibrate the model and the settlement estimate seems to spike at this point. This trend can also be seen between depths of 25-30 m, where the model appears to be severely over predicting the settlement of these layers and has very large initial void ratios. This also occurs at depth of 18 m.

The settlement estimates presented in Jung (2009) are significantly less that those estimated above. This happens partly as a result of Jung (2009) using a compression model that was not calibrated properly, but more importantly Jung (2009) does not take into account the pore pressure in the soil. This leads to an estimated settlement of around 450 mm, which is more than 130 mm less than the estimate made using the correct model parameters and taking into account the pore pressure. Figure 3 shows the comparison of the settlement estimates between the corrected estimate and the estimate from Jung (2009).

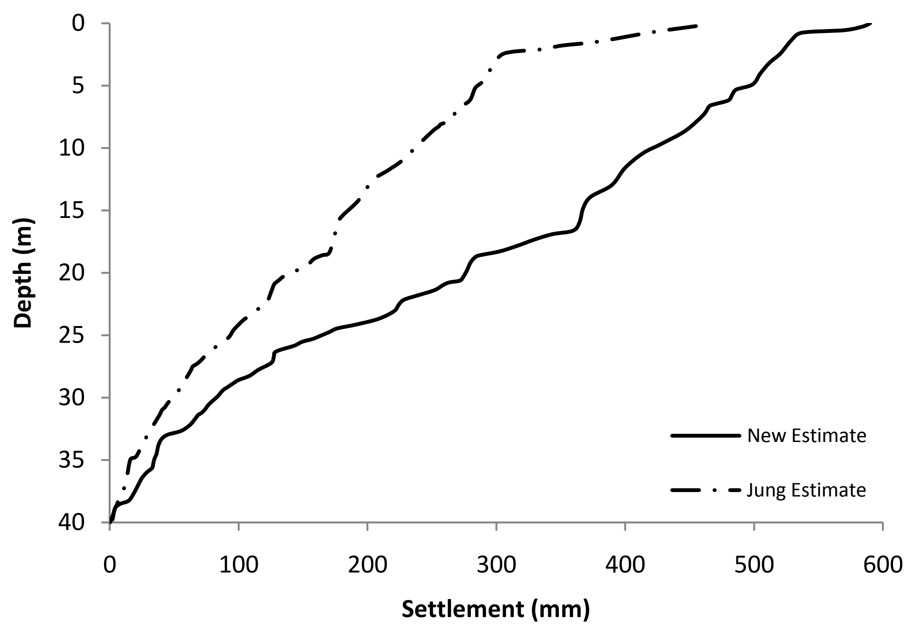

Figure 3. Comparison between corrected estimate and Jung (2009) estimate

5. RELIABILITY ANALYSIS OF SETTLEMENT

5.1 Fragility Estimates

Using the developed probabilistic model for estimating settlement, along with an applied pressure and a settlement threshold S_T, the conditional probability of attaining or exceeding the settlement threshold for the given applied pressure can be obtained. This conditional probability is often referred to as the fragility. A limit state function $g(\mathbf{x}, \boldsymbol{\Theta}, S_T)$ is defined such that the limit state function returns a value less than or equal to zero if the limit state is exceeded (Ditlevsen and Madsen 1996). In our case the limit state should return a value less than or equal to zero if the settlement estimated from the given load exceeds the defined settlement threshold. This limit state function is expressed as

$$g(\mathbf{x}, \boldsymbol{\Theta}, S_T) = S_T - S(\mathbf{x}, \boldsymbol{\Theta}) \tag{15}$$

Using this limit state function, the fragility can be expressed as

$$F(\mathbf{x}, \boldsymbol{\Theta}, S_T) = P[\{g(\mathbf{x}, \boldsymbol{\Theta}, S_T) \leq 0\} | S_T, Q] \tag{16}$$

where $P[A|\mathbf{b}]$ is the conditional probability that the event A occurs given that the values of $\mathbf{b}$ are known.

In order to take into account all of the uncertainties present in the limit state function a predictive estimate of fragility $F(\mathbf{x}, S_T)$ will be used. This has the advantage of incorporating the epistemic uncertainties present in the model parameters $\boldsymbol{\Theta}$, where as a point estimate of fragility will not incorporate these uncertainties. Since no closed-form solution is available for predictive estimates of fragility, the First Order Reliability

Method (FORM) and Monte Carlo simulation (MC) will be used to obtain the fragility estimates. FORM is a linearizing approximate method of estimating the fragility but is very fast, on the other hand MC is very time consuming but can ensure the accuracy of the answer.

Unfortunately, there is also an uncertainty involved with estimating fragility. Gardoni et al. (2002) proposes an approximation to calculate the bounds on the fragility estimates. The approach estimates the standard deviation of the reliability index $\beta(\mathbf{x}, S_T)$ which is defined as

$$\beta(\mathbf{x}, S_T) = \Phi^{-1}[1 - F(\mathbf{x}, S_T)] \tag{17}$$

where $\Phi^{-1}[\cdot]$ is the inverse of the standard normal cumulative probability. The approximate bounds are expressed as

$$\left\{\Phi\left[-\tilde{\beta}(\mathbf{x}, S_T) - \sigma_\beta(\mathbf{x}, S_T)\right], \Phi\left[-\tilde{\beta}(\mathbf{x}, S_T) + \sigma_\beta(\mathbf{x}, S_T)\right]\right\} \tag{18}$$

where Φ is the standard normal cumulative probability and σ_β is the standard deviation of the reliability index. Using a first order Taylor series expansion around the mean point, the variance of the reliability index is approximated by

$$\sigma_\beta^2(\mathbf{x}, S_T) \approx \nabla_{\mathbf{\Theta}}\beta(\mathbf{x}, S_T)\mathbf{\Sigma}_{\mathbf{\Theta\Theta}}\nabla_{\mathbf{\Theta}}\beta(\mathbf{x}, S_T)^{\mathrm{T}} \tag{19}$$

where $\mathbf{\Sigma}_{\mathbf{\Theta\Theta}}$ is the posterior covariance matrix of $\mathbf{\Theta}$ and $\nabla_{\mathbf{\Theta}}\beta(\mathbf{x}, S_T)$ is the gradient of $\beta(\mathbf{x}, S_T)$ at the mean point $\mathbf{M}_{\mathbf{\Theta}}$ which is computed by FORM.

Figure 4 and Figure 5 show the fragility estimates for the embankment with respect to the settlement threshold S_T given an applied pressure Q of 106.5 kPa and the applied pressure Q given the settlement threshold S_T of 580 mm. Both figures include fragility estimates obtained using MC (represented by circular points) and FORM (represented by a solid line), while the FORM fragility estimate also includes fragility bounds (represented by dashed lines). Figure 4 shows that as the settlement threshold S_T increases, the probability of exceedance decreases. Adversely, Figure 5 shows that as the applied pressure Q is increased, the probability of exceedance decreases. Both Figures 4 and 5 show that the fragility estimates obtained using MS are very similar to those obtained using FORM, this can be expected because of the relative simplicity of the limit state function and because there was only one limit state function.

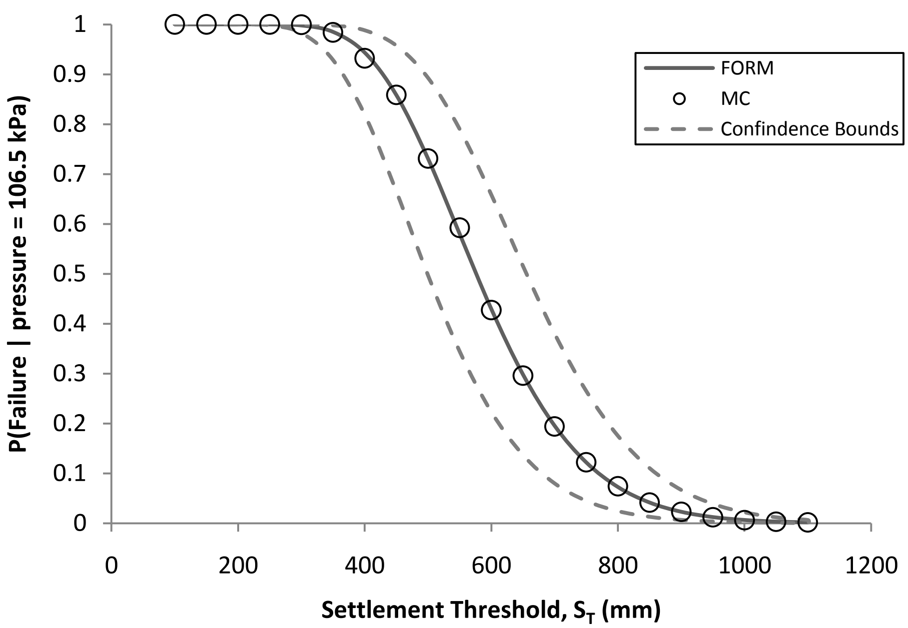

Figure 4. Fragility estimates with respect to settlement threshold

The fragility estimates for the embankment with respect to the settlement threshold S_T given an applied pressure Q of 106.5 kPa presented in Jung (2009) are shown in Figure 6. This figure has a similar shape to Figure 4, but appears to be shifted to the right and less steep than the fragility estimates in Figure 4.

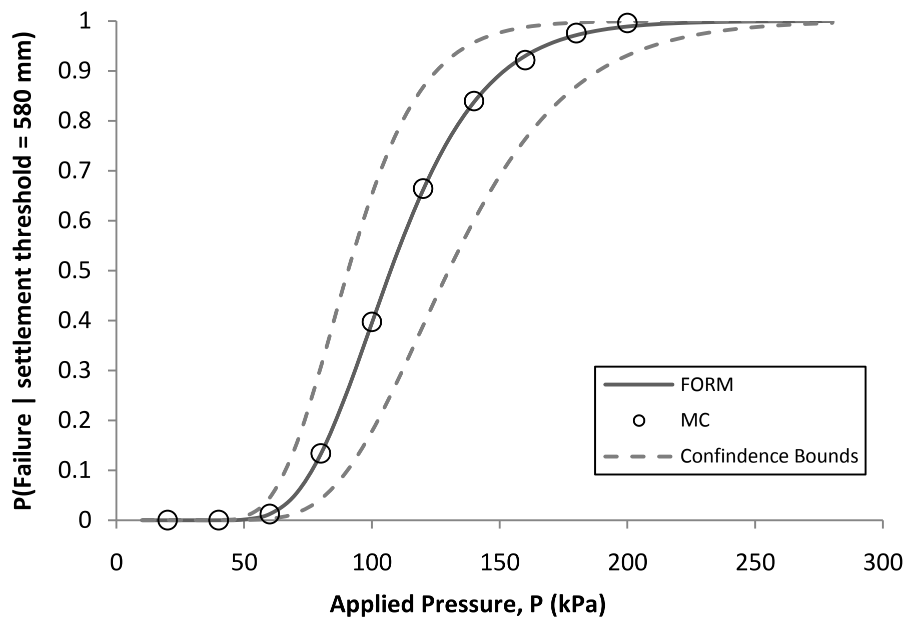

Figure 5. Fragility estimates with respect to applied pressure

5.2 Sensitivity and Importance Measures

Sensitivity and importance measures can be calculated using the results of the FORM analysis. Sensitivity measures are used to determine how a change in the parameters of the limit state function or a change in the distribution of the random variables will affect the fragility estimates. The importance gives a measure of the random variable whose associated uncertainty has the largest impact on the calculation of the fragility estimates.

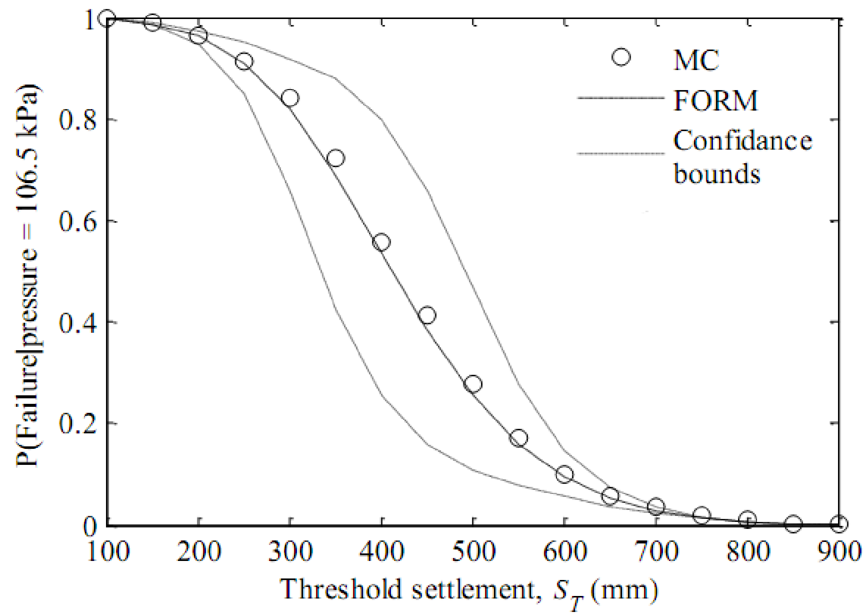

Figure 6. Fragility estimates with respect to threshold settlement from Jung (2009)

Sensitivity measures give the influence that the mean values of $\mathbf{\Theta}$ and $\mathbf{x}$ have on the probability of failure and the reliability index. The sensitivity measures are a function of the gradient of the reliability index (Hohenbichler and Rackwitz 1986), with respect to the random variables, estimated using FORM. For comparisons to take place, this gradient is scaled using the standard deviation of each parameter. This scaling allows for a comparison that takes into account the uncertainty of each parameter. This definition of sensitivity is express as

$$\boldsymbol{\delta} = \mathbf{D}\nabla_{\mathbf{\Theta}}\beta \tag{20}$$

where $\boldsymbol{\delta}$ is the vector of sensitivity measures, $\mathbf{D}$ is the diagonal matrix containing standard deviation of the random variables along the main diagonal and $\nabla_{\mathbf{\Theta}}\beta$ is the gradient of the reliability index with respect to the random variables.

Figure 7 shows the sensitivity measurements for the fragility estimates represented in Figure 4. Although the sensitivities for the parameters contained in **x** are computed, they are left out because they are negligible. It can be seen from Figure 7 that the mean value of ρ_c is the most sensitive parameter and has the largest effect on the fragility estimates. Although not shown, the sensitivities corresponding to Figure 5 were also computed and also show that the fragility estimates are most sensitive to the mean value of ρ_c.

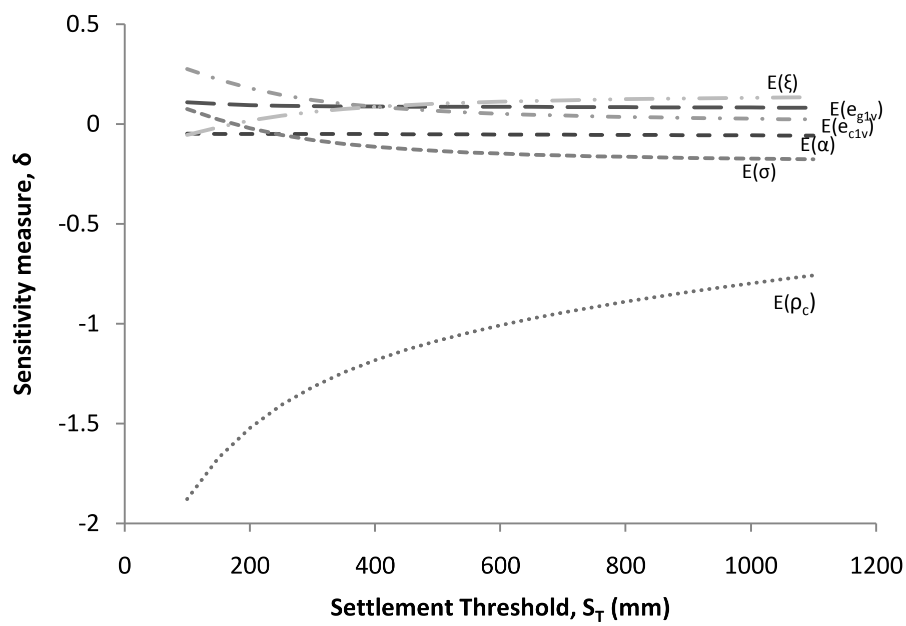

Figure 7. Sensitivity measures with respect to settlement threshold

The sensitivity measures presented in Jung (2009) can be seen in Figure 8. Figure 8 shows that the mean value of ρ_c has the largest impact on the fragility estimates, in agreement with Figure 7. Figure 8 also shows that the mean values of α and

e_{g1v} have a large impact on fragility estimates; this is not shown in the corrected sensitivities of Figure 7.

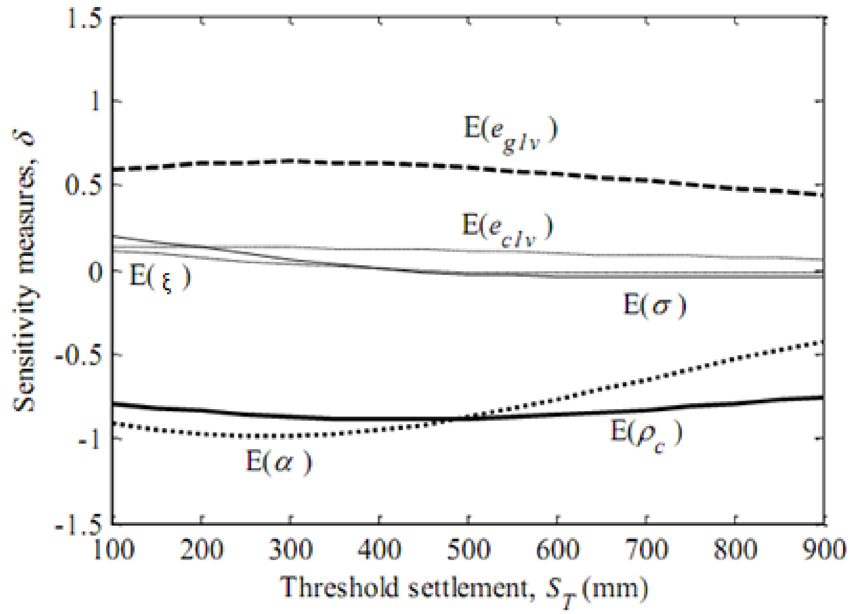

Figure 8. Sensitivity measures with respect to settlement threshold from Jung (2009)

The importance measure tells which of the random variable have the largest impact when estimating the fragility. This impact is associated with the uncertainty associated with that random variable. The normalized importance vector is defined by Der Kiureghian and Ke (1995) as

$$\gamma^{\mathrm{T}} = \frac{\hat{\boldsymbol{\alpha}}^{\mathrm{T}} \mathbf{J}_{\mathrm{u}^*,\mathrm{z}^*} \mathbf{D}'}{\left\| \hat{\boldsymbol{\alpha}}^{\mathrm{T}} \mathbf{J}_{\mathrm{u}^*,\mathrm{z}^*} \mathbf{D}' \right\|} \tag{21}$$

where z is the vector of all the random variables, $\mathbf{J}_{u^*,z^*}$ is the Jacobian transferring from the original space z to the standard normal space u, with respect to the design point and $\mathbf{D}'$ is the diagonal matrix of the standard deviation of the variables in the standard space.

Figure 9 shows the importance measures for the fragility estimates represented in Figure 4. Figure 9 shows that ρ_c is the most important variable when estimating fragility. This is expected since the fragility estimates were most sensitive to changes in ρ_c. Similar analysis of the fragility estimates shown in Figure 5 confirm that ρ_c is the most important variable when estimating fragility.

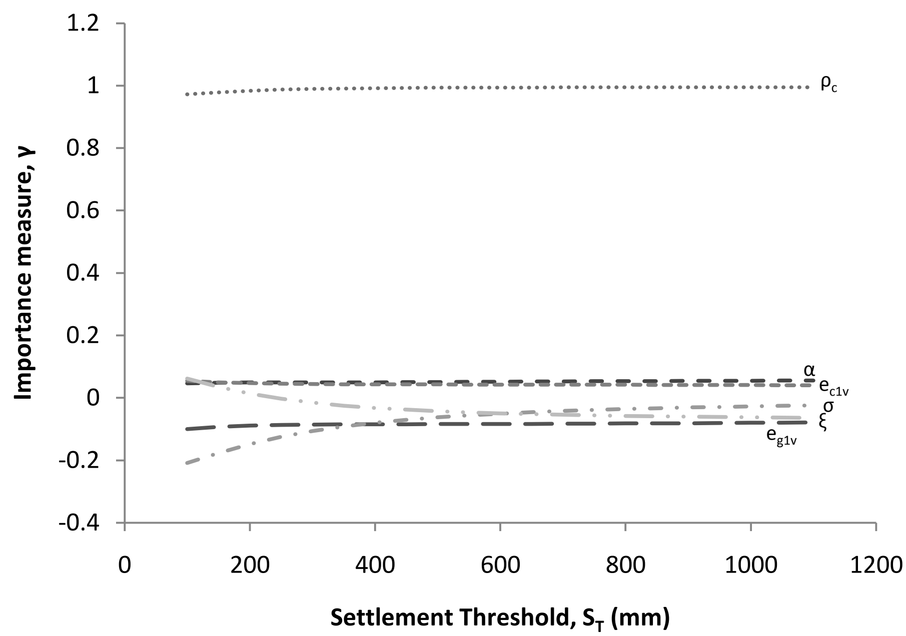

Figure 9. Importance measures with respect to settlement threshold

The importance measures presented in Jung (2009) can be seen in Figure 10. Figure 10 shows that ρ_c is the most important parameter when estimating fragility, in agreement with Figure 9. Figure 10 also shows that α and e_{g1v} are also important parameters when estimating fragility; this is not shown in the corrected importance measures of Figure 9.

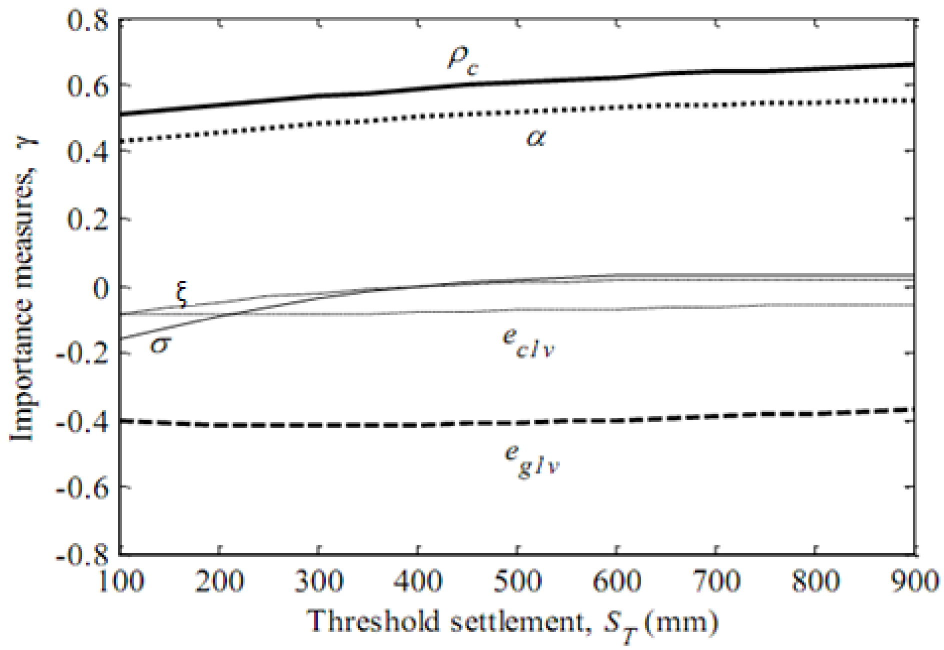

Figure 10. Importance measures with respect to settlement threshold from Jung (2009)

5.3 Closed-Form Approximation Based on CLT

An approximate, closed-form solution is developed to estimate the fragility. This approximation is based on the Central Limit Theorem (CLT). The use of the CLT is possible because of the large number of layers involved in calculating the total settlement. Following the CLT, the distribution of $S(\mathbf{x}, \mathbf{\Theta})$ is approximately normal with mean value $\mathrm{E}(S)$ and standard deviation σ_S. Therefore the approximate fragility can be written as

$$F(\mathbf{x}, \mathbf{\Theta}, S_T) = \Phi\left[-\frac{S_T - \mathrm{E}(S)}{\sigma_S}\right] = 1 - \Phi\left[\frac{S_T - \mathrm{E}(S)}{\sigma_S}\right] \tag{22}$$

$\mathrm{E}(S)$ is calculated using the mean value of all the random variables, σ_S is calculated using a first order Taylor series approximation. This approximation is written as

$$\sigma_S^2 \approx (\nabla_{\mathbf{\Theta}} S)' \mathbf{\Sigma}_{\mathbf{\Theta}} (\nabla_{\mathbf{\Theta}} S) \tag{23}$$

where $\nabla_{\mathbf{\Theta}} S$ is the gradient of the settlement with respect to $\mathbf{\Theta}$ and $\mathbf{\Sigma}_{\Theta}$ is the covariance matrix of the model parameters. A fourth order finite difference operator is used to estimate $\nabla_{\mathbf{\Theta}} S$.

Figure 11 shows the approximate fragility with respect to S_T compared to the FORM estimates and the MC estimates. The fragility estimates using this approximate method are very similar to those found using FORM and MC. The estimates vary the most from FORM and MC when the fragility is close to either zero or one.

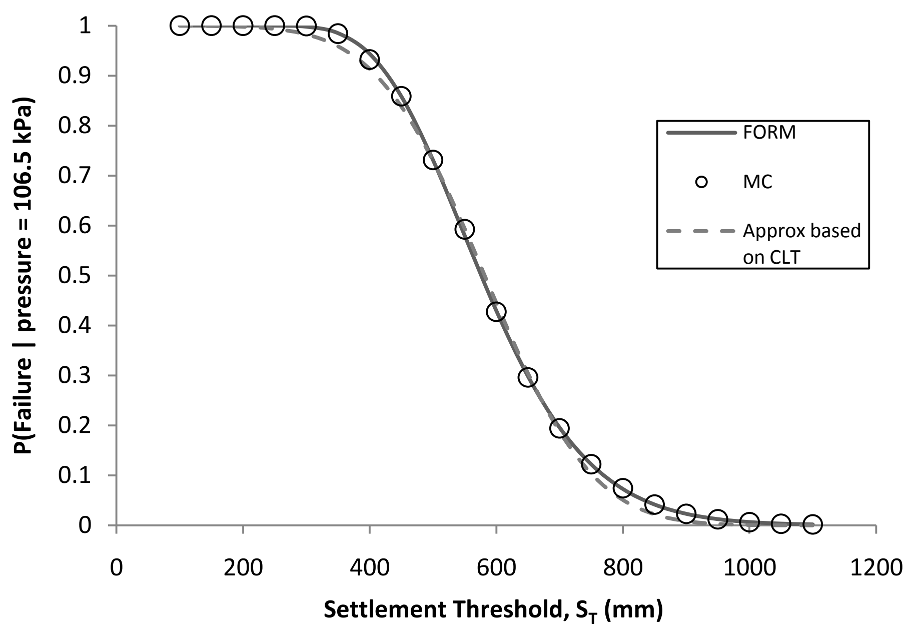

Figure 11. Fragility estimates using approx. based on CLT, FORM and MC

6. CONCLUSION

The probabilistic soil compression model developed by Jung et al. (2009) is modified to account for the uncertainties brought about by the variability of the loading steps in the laboratory one-dimensional loading test. This modification involves the use of an auto-regressive model form and a variance stabilizing transformation.

The updated soil compression model is used to formulate a settlement model. This settlement model is then used to estimate the settlement of an embankment and the estimated settlement compared to the actual recorded settlement and found to be slightly conservative.

Fragility estimates for the settlement of the embankment are obtained using both FORM and MC. These fragility estimates show that, for a specific applied pressure, the probability of exceedance diminishes as the settlement threshold increases. Also, the probability of exceedance increases, for a specific settlement threshold, as the applied pressure increases.

Sensitivity and importance measures are determined for the fragility estimates. Sensitivity measures show that $E(\rho_c)$ is the most sensitive model parameter and has the most impact on the fragility estimates. Similarly, the importance measures show that ρ_c is the random variable whose uncertainty has the highest impact on fragility estimates. The findings of the sensitive and importance analysis show that the slope ρ_c of the K_0-LCC is the most important parameter in the developed probabilistic soil compression model.

A method to approximate the fragility based on the CLT is developed. This method gives very accurate estimates compared to FORM and MC and appears to be a good alternative to specialized reliability software.

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