

ESSAYS IN INTERNATIONAL MACROECONOMICS AND FORECASTING

A Dissertation

by

JESUS ANTONIO BEJARANO ROJAS

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

Major Subject: Economics

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Approved by:

Chair of Committee,	Dennis W. Jansen
Committee Members,	Ryo Jinnai
	Anastasia Zervou
	Kishore S. Gawande
Head of Department,	Timothy J. Gronberg

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## ABSTRACT

Essays in International Macroeconomics and Forecasting. (August 2011)

Jesus Antonio Bejarano Rojas, B.A., Universidad del Rosario;

M.S., Universidad del Rosario

Chair of Advisory Committee: Dr. Dennis W. Jansen

This dissertation contains three essays in international macroeconomics and financial time series forecasting. In the first essay, I show, numerically, that a two-country New-Keynesian Sticky Prices model, driven by monetary and productivity shocks, is capable of explaining the highly positive correlation across the industrialized countries' inflation even though their cross-country correlation in money growth rate is negligible. The structure of this model generates cross-country correlations of inflation, output and consumption that appear to closely correspond to the data. Additionally, this model can explain the internal correlation between inflation and output observed in the data.

The second essay presents two important results. First, gains from monetary policy cooperation are different from zero when the elasticity of substitution between domestic and imported goods consumption is different from one. Second, when monetary policy is endogenous in a two-country model, the only Nash equilibria supported by this model are those that are symmetrical. That is, all exporting firms in both countries choose to price in their own currency, or all exporting firms in both countries choose to price in the importer's currency.

The last essay provides both conditional and unconditional predictive ability evaluations of the aluminum futures contracts prices, by using five different econometric models, in forecasting the aluminum spot price monthly return 3, 15, and 27-months ahead for the sample period 1989.01-2010.10. From these evaluations, the

best model in forecasting the aluminum spot price monthly return 3 and 15 months ahead is followed by a (VAR) model whose variables are aluminum futures contracts price, aluminum spot price and risk free interest rate, whereas for the aluminum spot price monthly return 27 months ahead is a single equation model in which the aluminum spot price today is explained by the aluminum futures price 27 months earlier. Finally, it shows that iterated multiperiod-ahead time series forecasts have a better conditional out-of-sample forecasting performance of the aluminum spot price monthly return when an estimated (VAR) model is used as a forecasting tool.

To God and my family

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## CHAPTER I

## INTRODUCTION

This dissertation contains three essays in international macroeconomics and forecasting. In the next chapter, I update the cross-country correlations in inflation, output and money growth rate calculated by Wang and Wen (2007) until 2008 Q1. As we will see these updated empirical findings do not differ substantially from the calculated by Wang and Wen (2007). Afterwards, I set up a two-country new Keynesian sticky price model, which has the same modeling framework presented by Wang and Wen (2007), but with four different features which are imperfect substitution between home and foreign goods, home bias consumption, inflation's inertia and both monetary and productivity shocks as an uncertainty sources. With the solution of this model, I can generate, first, a highly positive cross-country correlation in inflation across developed countries, even though when zero cross-country correlation in the money growth rate process across these countries is assumed <sup>1</sup>, second a positive cross-country correlation in output with values that do not differ substantially from those observed in the data from the industrialized countries, and third a positive inner-correlation between output and inflation. All these three results agree with the observed data between 1977Q1 and 2008Q1.

In Chapter III, I propose a SDGE model which has endogenous price-setting decision rule for exporting firms similar to that proposed by Devereux et al. (2004), the same preferences and consumption CES aggregator as Bhattarai (2009), and a

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<sup>1</sup>This dissertation follows the style of Journal of Monetary Economics.

<sup>1</sup>These countries are Australia, Canada, Japan, United Kingdom and United States. However, the results obtained in this paper do not differ so much from shorter data samples, which include more countries

central bank which precommits to a monetary policy rule and which may respond to domestic and foreign productivity shocks. The parameters of this policy rule, which determine the degree of response to each of these shocks, are derived optimally as the solution of an intertemporal maximization problem.<sup>2</sup>

This model has a microfounded invoicing decision rule to determine whether exported goods should be invoiced in domestic or foreign currency, the same as proposed by Devereux et al. (2004). As such, this model is comparable only with the perfect pass-through and zero pass-through versions of Corsetti and Pesenti (2004)'s and Corsetti and Pesenti (2005).

Unlike Bhattarai (2009) this model has a monetary policy rule which is endogenous to the aggregated exporting firms' invoicing currency. This feature allows a strategic interaction between the expectations of the firms and the monetary authority.

By solving this model through the second-order approximation method to the policy function, which was developed by Collard and Juillard (2001), I show two new important results in the international macroeconomics literature.

First, the theoretical result, obtained by Corsetti and Pesenti (2005) for a two country model with identical countries, of non-positive gains from international monetary cooperation is sensitive to the elasticity of substitution between consumption of domestic and foreign goods. Particularly, I show for different elasticity of substitution values, calibrated in international economics literature, that gains from international monetary cooperation are positive when two countries are identical and the elasticity of substitution between domestic and foreign goods consumption is different from one. In addition, I show for a different country size model that gains from interna-

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<sup>2</sup>The welfare function is defined as the expected discounted sum of the stream of future households' utility function.

tional monetary cooperation can be positive, negative or zero when the elasticity of substitution between domestic and foreign goods consumption is different from one.

Second, the theoretical result obtained by Bhattarai (2009) of strategic complementarities as a sufficient condition to support the existence of the asymmetric Nash Equilibrium (LCP,PCP) is not valid when monetary policy is determined optimally by the central bank. For different families of preferences and elasticity of substitution values, I show that the only Nash equilibria supported by a two-country New Keynesian SDGE model with optimal monetary policy are those which are symmetric. However, this model only supports the asymmetric Cooperative equilibria under certain preferences and elasticity of substitution values.<sup>3</sup>

Additionally, I can replicate and extend theoretical results derived in international macroeconomics literature by Corsetti and Pesenti (2004) and Corsetti and Pesenti (2005).

First, I extend for different families of preferences and elasticity of substitution values one of the theoretical results found by Corsetti and Pesenti (2005). That is, I show that (PCP,PCP) Nash Equilibrium always Pareto dominates (LCP,LCP) equilibrium.

Second, when the preferences are logarithmic in consumption, linear in labor, and there is unitary elasticity of substitution between domestic and foreign consumption, there exist two Nash Equilibria. In the first equilibrium (PCP,PCP), all the exporting firms, in both countries, choose to price in their own currency (PCP) and the optimal monetary policy is characterized by a model-derived reaction function which responds only to domestic productivity shocks. This monetary policy rule implies a flexible

---

<sup>3</sup>Since Bhattarai (2009) assumes exogenous processes for monetary policy in both countries, it is difficult to categorize if the asymmetric equilibria that they obtained are Nash or Cooperative equilibria.

exchange rate. In the second equilibrium (LCP,LCP), all the exporting firms, in both countries, choose to price in the importer's currency (LCP) and the optimal monetary policy is characterized by a model-derived reaction function, which responds symmetrically to both domestic and foreign productivity shocks. This monetary policy rule implies a pegged exchange rate.<sup>4</sup>

In Chapter IV, I propose six econometric models to predict the aluminum spot price monthly return and test statistically their prediction performance at 3, 15 and 27 steps-ahead-forecasts. The first model is a No-drift Random Walk, which has been broadly used in the forecasting evaluation literature, see Alquist and Kilian (2010), Swanson and Zeng (1998), Fama and French (1987) and so on. The second model is based on the Speculative Efficiency Hypothesis of futures contracts' prices to predict spot prices, provided by Bilson (1981). This model implies a single equation that relates the spot price at period  $t+h$  with the futures contract price at period  $t$ , which matures at period  $t+h$ . That is, if we have a 3-months future contract, the Speculative Efficiency Hypothesis model is represented by the spot price as a function of the 3 month futures price three months ago as long as I am using monthly data. The third model, based on the Financial Cost of Carry Theory, is a VAR in differences which has three endogenous variables: spot price, futures price and risk free rate. Since I found that aluminum futures prices, spot prices and treasury bills interest rates are cointegrated (see this paper appendix for more details), the fourth model is a VECM, which has these three latest variables.

The fifth model, based on the Storage Cost of Carry Theory, assumes that rather than having a commodity for hedging risk or speculation, there is an unobservable

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<sup>4</sup>This result is robust to country size. Also, we can see that when the two countries have the same size, optimal monetary policy are identical to those derived by Corsetti and Pesenti (2004).



benefit of having the commodity, which can be represented as a function of the inventories and the spot price volatility. For more details see Pindyck (2001).<sup>5</sup> Here, I represent this storage cost model through a VAR in differences, which has five endogenous variables: spot price, futures price, risk-free asset interest rate, inventories and spot price volatility. Since I found that aluminum futures prices, spot prices, inventories, spot price volatility and treasury bills interest rates are cointegrated (see this paper appendix for more details), the sixth model is a VECM, which has these five latest variables.

In contrast to most of the commodity forecasting evaluation literature, the predictive ability (or forecasting performance) evaluations, here, are not mainly based on ordinal comparisons between two models' expected loss functions. Instead, our forecasting performance evaluations are based on both unconditional and conditional prediction ability tests proposed by Giacomini and White (2006). The use of these tests have a great advantage over traditional ability prediction tests such as Diebold and Mariano (1995) tests because they allow to test predictive ability not only for non-nested models but also for nested models. As an example of nested models' forecasting performance evaluation, in this paper, is the comparison in conditional and unconditional forecasting performance between the Financial Cost of Carry vs. the Storage Cost models.

By using Giacomini and White (2006) predictive ability tests, I find that, for both 3 and 15 months ahead forecast, Storage Cost models does not improve the

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<sup>5</sup>This theory assumes that there is a benefit of having physical ownership of the commodity [see Brennan (1958) and Working (1949) for more details]. This benefit is called the convenience yield. Also, this theory assumes that there is a variable storage cost associated with storing the commodity until the expiration of the futures contract. As a result, the net storage cost is, defined as the difference between the total storage cost and the convenience yield, is a function of inventories and spot price volatility.

aluminum spot price monthly return's forecast with respect to the Financial Cost of Carry Models. However, these two models have a better spot price monthly return forecast performance than the Speculative Efficiency Hypothesis Model, which in turn has better conditional and unconditional forecast performance than the Random Walk model. Unlike the predictive ability test for 3 and 15 months ahead forecasts, I find that, for 27 months ahead forecasts, neither the Financial Cost of Carry nor Storage Cost models outperform the Speculative Efficiency model, however the Speculative Efficiency model has better conditional and unconditional forecasting performance than the Random Walk model.

Given the versatility of Giacomini and White (2006) conditional and unconditional tests to compare not only predictive ability between models but also forecasting methodologies, as an additional contribution to the forecasting evaluation literature, I find that both conditional and unconditional forecasting performance by iterating autoregressive models (this forecasting methodology is called iterated multistep forecast) is better than using horizon-specific estimated models (this forecasting methodology is called direct multistep forecast). Unlike Stock et al. (2006), I make a univariate forecast evaluation for the aluminum spot price and I use the asymptotic predictive ability tests provided by Giacomini and White (2006) that converge in distribution to conventional probability functions rather than using complicated bootstraps procedures as those provided by Stock et al. (2006).

Finally, I present a summary of this dissertation in Chapter V.

## CHAPTER II

A STRUCTURAL EXPLANATION OF COMOVEMENTS IN INFLATION  
ACROSS DEVELOPED COUNTRIES

## A. Introduction

During the last years, economists have expressed a deep interest in studying the international comovements in inflation across countries, particularly industrialized countries. In particular, Ciccarelli and Mojon (2008) and Neely and Rapach (2008) are interested in the role that global inflation movements play as an attractor of domestic inflation. Furthermore, Wang and Wen (2007) are interested in finding the sources and mechanisms explaining the observed international comovements in inflation. In the cross-country investigation of inflation dynamics prepared by Wang and Wen (2007), they find for the period covered between 1977 Q1 and 1998 Q4 that the average cross-country correlation in inflation for developed countries is high and positive, although the cross-country correlation in money growth rate is near zero. This finding is a puzzle, at least for people who believe in the quantity theory of money. In addition, they conclude that standard new Keynesian sticky-information and sticky-price models driven only by monetary shocks are not able to explain the highly positive cross-country correlation in inflation when the monetary shocks are uncorrelated across developed countries. In this paper, I update the cross-country correlations in inflation, output and money growth rate calculated by Wang and Wen (2007) until 2008 Q1. As we will see these updated empirical findings do not differ substantially from the calculated by Wang and Wen (2007). Afterwards, I set up a two-country new Keynesian sticky price model, which has the same modeling framework presented by Wang and Wen (2007), but with four different features which

are imperfect substitution between home and foreign goods, home bias consumption, inflation's inertia and both monetary and productivity shocks as an uncertainty sources. With the solution of this model, I can generate, first, a highly positive cross-country correlation in inflation across developed countries, even though when zero cross-country correlation in the money growth rate process across these countries is assumed <sup>6</sup>, second a positive cross-country correlation in output with values that do not differ substantially from those observed in the data from the industrialized countries, and third a positive inner-correlation between output and inflation. All these three results agree with the observed data between 1977Q1 and 2008Q1.

The remainder of the paper is organized as follows. I present the stylized facts that describe the puzzle found by Wang and Wen (2007). Then, I set up and calibrate the model with the four features mentioned above. Next, I explain how the assumptions of imperfect substitution and home bias consumption can generate a positive cross-country correlation in inflation between two symmetric countries when productivity shocks are assumed as a sole uncertainty's source in this model. Since productivity shocks, as a unique uncertainty's source, cannot generate a positive inner-correlation between output and inflation, I describe the mechanism of how this model can generate this inner-correlation by assuming monetary shocks as a unique uncertainty's source. As I mentioned above, having monetary shocks as unique uncertainty's source cannot generate a highly positive cross-country correlation in inflation, I explain how the assumption of inflation's inertia and both productivity and monetary shocks can generate jointly a positive cross-country correlation in inflation, con-

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<sup>6</sup>These countries are Australia, Canada, Japan, United Kingdom and United States. However, the results obtained in this paper do not differ so much from shorter data samples, which include more countries.

sumption and output, and a positive inner-correlation in inflation and output. Finally, I present the unconditional cross-country correlations in inflation, output, and consumption generated by this model's solution under the following three different scenarios. When productivity is the sole uncertainty's source, when money growth rate is the only uncertainty's source and when both productivity and money growth rate are the uncertainty's sources in this model.

## B. Stylized Facts

Wang and Wen (2007) calculated the cross-country correlation in inflation from 1977 Q1 until 1998 Q1. I updated this through 2008 Q1. As we can see, Table I, which displays data from 1977Q1 until 2008Q1, shows that the cross-country correlation in inflation between the industrialized countries is still very high and positive. In the same way, Table II shows that some cross-country correlations in M1 growth rate are near zero and some of them are negative.<sup>7</sup>

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<sup>7</sup>I do not include the European Union Countries, since many of them adopted the Euro from 1999.

Table I.

Cross-country Correlation in Inflation (mean = 0.6084)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Australia	1.0000	0.6320	0.4500	0.4576	0.5248
Canada	0.6320	1.0000	0.6020	0.6024	0.7756
Japan	0.4500	0.6020	1.0000	0.6543	0.6705
UK	0.4576	0.6024	0.6543	1.0000	0.7150
USA	0.5248	0.7756	0.6705	0.7150	1.0000

Table II.

Cross-country Correlation in Money Growth Rate (mean = 0.015)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Australia	1.0000	-0.0860	-0.1593	0.1019	0.0845
Canada	-0.0860	1.0000	0.0281	0.0399	0.0387
Japan	-0.1593	0.0281	1.0000	0.1380	0.0966
UK	0.1019	0.0399	0.1380	1.0000	-0.1289
USA	0.0845	0.0387	0.0966	-0.1289	1.0000

Table III shows that the cross-country correlation in output is positive but not as high as the cross-country correlation in inflation across these countries. Moreover, Table IV shows a very low domestic correlation between inflation and output.

Table III.

Cross-country Correlation in Output (mean = 0.3472)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Australia	1.0000	0.0469	0.6252	0.3270	0.4914
Canada	0.0469	1.0000	0.0981	0.0791	0.1260
Japan	0.6252	0.0981	1.0000	0.5314	0.5773
UK	0.3270	0.0791	0.5314	1.0000	0.5700
USA	0.4914	0.1260	0.5773	0.5700	1.0000

Table IV.

Domestic Correlation between Output and Inflation (mean = 0.2147)

Sample: 1977 Q1 - 2008 Q1

	Australia	Canada	Japan	UK	USA
Correlation	0.1961	0.1780	0.0702	0.2595	0.3698

From Tables I and II, it can be inferred that the average cross-country correlation in inflation across these countries is equal to 0.6 while the average cross-country correlation in money growth rate is equal to 0.0154. So the puzzle of highly positive cross-country correlation in inflation accompanied with a negligible cross-country correlation in money growth rate still holds. For the purpose of this research, this puzzle will be called the Wang and Wen's puzzle.

### C. The Model

#### 1. Environment

- Households live infinite number of periods; they consume a basket of final goods, which can be domestic or imported.
- There exists imperfect substitution in the consumption of domestic and foreign goods.
- Households are endowed with  $\bar{l}$  units of time, which they can spend on leisure or labor.
- Households are the owner of all firms.
- Only final goods are tradable.
- Intermediate goods firms are producing in a monopolistically competitive market.
- Final goods prices are sticky.
- Intermediate goods' factors are produced in a perfectly competitive market.
- There is a regime of floating exchange rate.



- Money supply is determined by an exogenous stochastic process for the money growth rate.
- Total factor productivity is determined by an exogenous stochastic process.

## 2. Households

The representative household chooses  $\{C_t, N_t, B(s^{t+1}), M_{t+1}\}$  which maximizes its lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\sigma} / (1-\sigma) - \psi N_t^{1+\eta} / (1+\eta) \right] \quad (2.1)$$

subject to:

$$T_t + B_t + M_t + \Pi_t + P_t W_t N_t \geq P_t C_t + \sum_{s^{t+1}}^{\infty} Q(s^{t+1}|s^t) B(s^{t+1}) + M_{t+1} \quad (2.2)$$

$$M_t + T_t + B_t \geq P_t C_t + \sum_{s^{t+1}}^{\infty} Q(s^{t+1}|s^t) B(s^{t+1}) \quad (2.3)$$

taking:  $M_t, T_t, P_t, \Pi_t, M_t, W_t, Q(s^{t+1}|s^t)$  as given, where  $C_t$  is the composed consumption at date t,  $N_t$  represents the worked hours,  $T_t$  is a lump sum transfer of flow of money that the representative household receives from the Government,  $M_t$  is the representative household's money holdings in home currency carried over from the last period,  $B_t$  are the nominal bonds expressed in domestic currency,  $P_t$  denotes the consumer's price index,  $W_t$  is the real wage (deflated by using the consumer price index) and  $Q(s^{t+1}|s^t)$  is the price at t of a bond that next period would yield  $B(s^{t+1})$  and  $\Pi_t$  is the intermediate goods producer's profit. The variables for the foreign

country are denoted with star.

*F.O.C*

The first order conditions from the representative household's maximization problem are as follows:

$$C_t : C_t^{-\sigma} - \lambda_t - \mu_t = 0 \quad (2.4)$$

$$N_t : -\psi N_t^\eta + \lambda_t W_t = 0 \quad (2.5)$$

$$M_{t+1} : -\lambda_t/P_t + \beta E_t \left[ \lambda_{t+1} + \mu_{t+1}/P_{t+1} \right] = 0 \quad (2.6)$$

$$\begin{aligned} B(s^{t+1}) : & \left[ \lambda_t + \mu_t/P_t \right] Q(s^{t+1}|s^t) - \\ & \beta \left[ \lambda(s^{t+1}|s^t) + \mu(s^{t+1}|s^t)/P(s^{t+1}|s^t) \right] = 0 \end{aligned} \quad (2.7)$$

where, at each period  $t$ , the representative household chooses  $C_{H,t}$  and  $C_{F,t}$  which minimizes its total expenditure:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \quad (2.8)$$

given, that they have chosen  $\{C_t\}_{t=0}^\infty$  previously and subject to:

$$C_t = C_{H,t}^\gamma C_{F,t}^{1-\gamma} \quad (2.9)$$

where  $C_{H,t}$  denotes the home consumption of the domestic final good,  $C_{F,t}$  denotes the home consumption of the foreign final good,  $P_{H,t}$  and  $P_{F,t}$  are their prices respectively.

As we can see (2.9) is an implication of assuming imperfect substitution in consumption between domestic and foreign goods.

The main result of this intratemporal minimization problem is that the economy's price level can be expressed as a function of the prices of home and foreign goods. That is,

$$P_t = \phi P_{H,t}^\gamma P_{F,t}^{1-\gamma} \quad (2.10)$$

where  $\phi = \left[ (\gamma/1 - \gamma)^{1-\gamma} + (1 - \gamma/\gamma)^\gamma \right]$

### 3. Firms

#### a. Final Good

Each country produces a single final good through the following production function:

$$Y_{H,t} = \left( \int_0^1 (Y_{H,t}(i))^{\zeta-1/\zeta} di \right)^{\zeta/\zeta-1} \quad (2.11)$$

where  $Y_{H,t}(i)$  is the intermediate  $i$  good which is non tradable and  $\zeta$  measures the elasticity of substitution among the intermediate goods,  $Y_{H,t}(i)$ .

The optimization problem of the final good producer is to find the optimal set of inputs  $Y_{H,t}(i)$  maximizing:

$$P_{H,t}Y_{H,t} - \int_0^1 P_{H,t}(i)Y_{H,t}(i)di \quad (2.12)$$

subject to (2.11)

*F.O.C*

$$Y_{H,t}(i) : P_{H,t}Y_{H,t}^{1/\zeta}Y_{H,t}^{-1/\zeta}(i) - P_{H,t}(i) = 0 \quad (2.13)$$

#### b. Intermediate Good Firms

Each intermediate good  $i$  is produced by a single monopolistically competitive firm according to the following technology:

$$Y_{H,t}(i) = A_{H,t}N_t(i) \quad (2.14)$$

$$A_{H,t} = A_{H,t}^o \exp^{\epsilon_t^A} \quad (2.15)$$

where  $A_{H,t}$  is the Total Factor Productivity (TFP) , which is the same for every  $i$  firm and  $\epsilon_t^A$  is a stochastic process which follows a normal distribution with zero mean and constant variance,  $\sigma_A^2$ .

#### c. Price Setting

Following Calvo (1983), I assume that each individual firm resets its price with probability  $(1 - \theta)$  each period independently of the time elapsed since its last price adjustment. Thus, each period a measure  $(1 - \theta)$  of (randomly selected) firms reset their prices, while a fraction  $\theta$  keep their prices unchanged.

Let  $P_{H,t}(i)$  denotes the price set by a firm  $i$  adjusting its price in period  $t$ . Let

$\bar{P}_{H,t}(i)$  denotes the price set by a firm  $i$  adjusting its price in period  $t$ . Under the Calvo price setting structure,  $P_{H,t+k}(i) = \bar{P}_{H,t}(i)$  with probability  $\theta^k$  for  $k = 0, 1, 2, 3, 4, 5, 6, \dots$

Then, the firm's optimal price setting model is written as follows:

$$\bar{P}_{H,t}(i) = \arg \max \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} E_t [\bar{P}_{H,t}(i) - MC_{h,t}(i)] \quad (2.16)$$

subject to (2.13) and taking  $Y_{H,t+k}$  as given.

#### d. Price Index Dynamics

Under the assumed price-setting structure, the dynamics of the domestic price index is described by the following equation:

$$P_{H,t} = \left[ \theta P_{H,t-1}^{1-\zeta} + (1-\theta) \check{P}_{H,t}^{1-\zeta} \right]^{1/1-\zeta} \quad (2.17)$$

In order to introduce persistence in the New-Keynesian Phillips Curve, I use the same approach as Galí and Gertler (1999). Consider a fraction of firms,  $(1-\chi)$ , that follow the optimal updating price rule,  $\bar{P}_{H,t}$  and a fraction of firms  $\chi$  that follow a backward looking adjustment process,  $P_{h,t}^b$ . That is,

$$\check{P}_{H,t} = \left[ \chi (P_{H,t}^b)^{1-\zeta} + (1-\chi) (\bar{P}_{H,t})^{1-\zeta} \right]^{1/1-\zeta} \quad (2.18)$$

$$P_{H,t}^b = \bar{P}_{H,t-1} (1 + \pi_{t-1}) \quad (2.19)$$

#### 4. Government

The government transfers to individuals a lump sum transfer of flow of money.

$$T_t = M_t U_{t+1} \quad (2.20)$$

where the gross money growth rate follow the subsequent stochastic process:

$$U_t = (U_t)^\varsigma (\bar{U})^{1-\varsigma} \exp^{\epsilon_t^U} \quad (2.21)$$

where  $\bar{U}$  is the steady state money growth rate,  $\epsilon_t^U$  denotes the stochastic process which follows a normal distribution with zero mean and constant variance,  $\sigma_U^2$ .

Since both countries are identical, then the mathematical expressions and parameterization that are described above are the same for the other country.

#### 5. Equilibrium Conditions

$$Y_{H,t} = C_{H,t} + C_{F,t}^* \quad (2.22)$$

$$Y_{H,t} = C_{H,t}^* + C_{F,t} \quad (2.23)$$

$$\tau_t = (1 + \pi_{t+1}) \tilde{M}_{t+1} - \tilde{M}_t \quad (2.24)$$

$$\tau_t^* = (1 + \pi_{t+1}^*) \tilde{M}_{t+1}^* - \tilde{M}_t^* \quad (2.25)$$

where  $\tau_t = T_t/P_t$  and  $\tilde{M}_t = M_t/P_t$

## D. Model Predictions and Results

### 1. Calibration

Table V presents the calibration of this model which largely follows Wang and Wen (2007) with the addition of two parameters. The first one is the expenditure share on domestic goods  $\gamma$  which comes from allowing imperfect substitution in consumption between domestic and foreign goods, and assuming a Cobb-Douglas consumption aggregator. This value is between 0.7 and 0.94, which is a range under most of the calibrations and estimations of this parameter, in international macroeconomics, fall within. See Chari et al. (2002), Adolfson et al. (2005) and Lubik and Schorfheide (2005) for more details about this parameter estimation. The second one is the probability of adjusting the price based in the past period optimal reset price  $\chi$ . Also in this model, I use estimated and calibrated parameters from important papers in monetary economics and international macroeconomics such as Galí and Gertler (1999) and Backus et al. (1992) respectively.

Table V.

## Baseline Calibration

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<b>Parameter</b>	<b>Source</b>
$\sigma = 0.05$	Wang and Wen (2007)
$\eta = 0.05$	Wang and Wen (2007)
$\beta = 0.99$	Wang and Wen (2007)
$\varsigma = 0.60$	Wang and Wen (2007)
$\gamma = 0.81$	Calibrated by the author
$\theta = 0.80$	Gali and Gertler (1999)
$\chi = 0.85$	Gali and Gertler (1999)
$\sigma_A = \sigma_{A^*} = 0.085$	Backus et al. (1992)
$\sigma_U = \sigma_{U^*} = 0.85$	Backus et al. (1992)
$\rho(\epsilon^A, \epsilon^{A^*}) = 0.258$	Backus et al. (1992)
$\rho(\epsilon^U, \epsilon^{U^*}) = 0.00$	Wang and Wen (2007)

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Note: The symbol  $\rho(x, y)$  represents the unconditional correlation between two stochastic processes.



## 2. Predicted Unconditional Correlations

In this subsection, I present the results implied by this model when it is driven by a) only productivity shocks, b) only monetary shocks and c) both monetary and productivity shocks.

### a. Productivity Shocks Only

One important result provided by this model is its capability of generating cross-country correlation in inflation and output close to the observed data when productivity shocks in each country are the sole uncertainty's source, see Table VI. This interesting result is a consequence of the following three assumptions: imperfect substitution in consumption between foreign and home goods, home bias consumption and the presence of only productivity shocks processes in this model. To clarify the role of these assumptions behind this result, consider a world economy with only two countries USA and UK, then suppose that there is a one-time transitory but persistent shock to USA's productivity process. In response to this shock, USA's output jumps up causing that USA's final goods will be cheaper than UK's final goods. This change in the relative prices is reflected in a jump up of USA's terms of trade (a jump down in UK's terms of trade). Since I assume home bias consumption, USA's total consumption basket is cheaper than UK's total consumption basket, which is a jump up in USA's real exchange rate (obviously a jump down in UK's real exchange rate). Because of the risk sharing condition, this change in USA's real exchange rate implies a jump up in USA's total consumption and a jump down in UK's total consumption. On the other hand, this fall in UK's consumption shifts UK's labor supply curve to the right, which in turn generates a fall in UK's real wage. Inasmuch as the real marginal cost in both countries is a positive function of the domestic real wage

and the terms of trade, and a negative function of the domestic productivity process, UK's real marginal cost jumps down since both UK's terms of trade and UK's real wage have jumped down. This jump in UK's real marginal cost causes a fall in UK's domestic goods inflation (see equation (C.19), in the context of UK).

Moreover, since the effect of USA's terms of trade is lower than the effect of both USA's productivity and USA's real wage on USA's real marginal cost, then this latter variable jumps down and in turn USA's domestic goods inflation falls.

Since I have assumed imperfect substitution in consumption between home and foreign goods, then total inflation is a function not only of domestic goods inflation but also a function of foreign goods inflation. Therefore, it is necessary to consider the dynamics of foreign goods inflation of each country. As long as foreign goods inflation is a positive function of terms of trade growth rate, it is expected a fall in UK's foreign goods inflation and hence in the UK's total inflation. However, in USA the story is different because in response of this USA's productivity shock, USA's terms of trade jumps up and therefore USA's foreign goods inflation does increase.

Inasmuch as, USA's productivity shock has a stronger impact than USA's terms of trade on USA's real marginal cost, then domestic goods inflation falls. Since, I assume home bias consumption, then USA's total goods inflation also falls.

Therefore, we should expect a highly positive cross-country correlation in inflation between these two countries.

Furthermore, to put this model in perspective with the standard international RBC models, I assume a moderate positive cross-country correlation in productivity shocks between the two countries, assumed in this model, to generate a positive cross-country correlation in consumption. The cross-country correlation in consumption generated by this model is reported in Table VI which is similar to the reported by Backus et al. (1992).

Table VI.

Predicted Correlations - Source of Uncertainty: Productivity Shocks

$\gamma$	$\chi$	$\rho(\pi_t, \pi_t^*)$	$\rho(y_t, y_t^*)$	$\rho(c_t, c_t^*)$	$\rho(\pi_t, y_t)$
0.66	0.00	0.7746	0.7690	0.2373	-0.0511
0.66	0.50	0.8057	0.7879	0.2820	-0.0488
0.66	0.85	0.9351	0.9262	0.6936	-0.0401
0.76	0.00	0.5356	0.5720	0.1225	-0.0510
0.76	0.50	0.5666	0.5888	0.1474	-0.0489
0.76	0.85	0.7806	0.7873	0.4906	-0.0403
0.81	0.00	0.4463	0.4826	0.1247	-0.0508
0.81	0.50	0.4707	0.4958	0.1418	-0.0487
<b>0.81</b>	<b>0.85</b>	<b>0.6779</b>	<b>0.6881</b>	<b>0.4164</b>	<b>-0.0403</b>
<b>Data</b>		0.6000	0.3472	0.3200 <sup>c</sup>	0.2147

Note: a) The symbol  $\rho(x, y)$  represents the unconditional correlation between two stochastic processes. b) The bold numbers are the results obtained by using the Baseline calibration. c) This value was calculated by Kehoe and Perri (2002).

## b. Monetary Shocks Only

When I assume productivity shocks alone, the model does not capture the inner correlation of inflation and output observed in the data. That is, the model does not generate a Phillips curve. To understand how this model generates the positive relationship between inflation and output, I assume that money growth rate shocks are the only uncertainty sources in this model. To be consistent with the observed data, I assume that this shock is uncorrelated between the two countries, assumed in this model. As we can see in Table VII, with this shock the model is able to generate a positive inner-correlation between inflation and output but it is not able to generate a positive cross-country correlation in inflation between the two countries. The mechanism, which explains this result, is the following. Consider a world economy with only two countries USA and UK, then suppose that there is a one-time transitory but persistent shock to USA's money growth rate. This cause a jump up in USA's total consumption in view of the households have more money to spend in consumption goods. Since home bias consumption is assumed in this model, the raise in USA's domestic consumption is higher than the raise in USA's foreign consumption, USA's terms of trade jump up, this in turn causes a real depreciation of the dollar with respect to the sterling pound. This real depreciation of the dollar (or real appreciation of the sterling pound) causes a jump down in UK's consumption since the risk sharing condition holds.

Inasmuch as UK's consumption fall is larger than UK's real wage fall, UK's agents need to work more hours to increase their output and therefore improve their exports value. Since the real wages and the terms of trade in UK jumped off, UK's real marginal cost also jumps down and in turn UK's inflation decreases.

In USA the story is different. The jump up in USA's consumption shift backward the labor supply curve, which leads a raise in the real wage. This jump in the USA's real wage causes a jump up in USA's worked hours and therefore in USA's output. Since there is a real depreciation of the dollar with respect to the sterling pound and a higher real wage, the USA's real marginal cost jumps up also. This jump in USA's real marginal cost causes an increment in USA's inflation.

Therefore, the model captures an inner positive correlation between USA's inflation and USA's output.<sup>8</sup> However, with this monetary shock alone, the model is not able to generate the positive cross-country correlation in inflation between USA and UK.

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<sup>8</sup>Also we can see that this experiment shows a negative inner correlation between UK's inflation and UK's output. However, if I generate a money supply shock in UK, I will obtain the Phillips curve for UK. In the stochastic simulation's outcomes reported in Table VII, I assume uncorrelated monetary shocks for each country.

Table VII.

Predicted Correlations - Source of Uncertainty: Money Growth Rate Shocks

$\gamma$	$\chi$	$\rho(\pi_t, \pi_t^*)$	$\rho(y_t, y_t^*)$	$\rho(c_t, c_t^*)$	$\rho(\pi_t, y_t)$
0.66	0.00	-0.4779	0.2991	-0.4361	0.8519
0.66	0.50	-0.3650	0.2341	-0.4911	0.9030
0.66	0.85	0.1530	0.4568	-0.2754	0.4086
0.76	0.00	-0.2147	0.1610	-0.3496	0.9620
0.76	0.50	-0.1348	0.1253	-0.3812	0.9673
0.76	0.85	0.2151	0.3189	-0.1944	0.3752
0.81	0.00	-0.1385	0.1116	-0.2811	0.9812
0.81	0.50	-0.0842	0.0868	-0.3041	0.9703
<b>0.81</b>	<b>0.85</b>	<b>0.1801</b>	<b>0.2508</b>	<b>-0.1437</b>	<b>0.3543</b>
<b>Data</b>		0.6000	0.3472	0.3200 <sup>c</sup>	0.2147

Note: a) The symbol  $\rho(x, y)$  represents the unconditional correlation between two stochastic processes. b) The bold numbers are the results obtained by using the Baseline calibration. c) This value was calculated by Kehoe and Perri (2002).

### c. Productivity and Monetary Shocks

In order to generate a jointly high and positive cross-country correlation in inflation across countries, and a positive inner correlation between inflation and output, I assume the presence of both monetary and productivity shocks in this model. In addition, I show the role playing by the assumption of inflation's inertia in this model to generate these two important results.

As we saw above, the productivity shocks in this model generates a highly positive cross-country correlation in inflation but the monetary shocks in this model generates a negative cross-country correlation in inflation and a highly positive inner-correlation between inflation and output.

In addition to have both productivity and monetary shocks, it is important to see how the degree of inflation's inertia can affect the model's results. For example, when the degree of inflation's inertia,  $\chi$ , takes lower values, the effect of monetary policy in inflation is very high.<sup>9</sup> Therefore, when I include both productivity and the monetary shocks in this model, the cross-country correlation in inflation is negative. In addition, it is expected that the model generate a high inner correlation between inflation and output because the effects of monetary policy on inflation outweigh that of productivity shocks on inflation.

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<sup>9</sup>Inflation's inertia is the result of having a fraction of firms which adjust their prices following the backward looking rule represented by equations 2.18 and 2.19.

However, when the degree of inflation's inertia,  $\chi$ , takes high values, the effect of monetary policy in inflation is very low. Therefore, in Table VIII, I show that if I include both the productivity and the monetary shocks in this model, the positive cross-country correlation in inflation is still high and positive but lower than the case in which the model has only productivity shocks. In addition, with these two shocks the model still presents a positive inner correlation in inflation and output but lower than the case in which the model has only monetary shocks.

One remaining issue is the relative size of the cross-country correlation in inflation and the cross-country correlation in output. In the data the cross-country correlation in inflation is higher than the cross-country correlation in output, but in my model the opposite is true. Future work, like adding habit persistence in households consumption, sticky wages, or sticky imports prices, might usefully focus on addressing this issue.



Table VIII.

Predicted Correlations - Source of Uncertainty: Productivity and Money Growth Rate Shocks

$\gamma$	$\chi$	$\rho(\pi_t, \pi_t^*)$	$\rho(y_t, y_t^*)$	$\rho(c_t, c_t^*)$	$\rho(\pi_t, y_t)$
0.66	0.00	-0.0115	0.5883	-0.1006	0.3970
0.66	0.50	0.1365	0.5949	-0.0906	0.3800
0.66	0.85	0.6513	0.8031	0.3196	0.0992
0.76	0.00	0.1681	0.4160	-0.0844	0.3928
0.76	0.50	0.2464	0.4317	-0.0653	0.3578
0.76	0.85	0.5944	0.6636	0.2653	0.0843
0.81	0.00	0.1834	0.3439	-0.0425	0.3816
0.81	0.50	0.2350	0.3605	-0.0236	0.3426
<b>0.81</b>	<b>0.85</b>	<b>0.5184</b>	<b>0.5739</b>	<b>0.2471</b>	<b>0.0767</b>
<b>Data</b>		0.6000	0.3472	0.3200 <sup>c</sup>	0.2147

Note: a) The symbol  $\rho(x, y)$  represents the unconditional correlation between two stochastic processes. b) The bold numbers are the results obtained by using the Baseline calibration. c) This value was calculated by Kehoe and Perri (2002).

## E. Conclusion

One of the new challenges for the central banks is to identify what kind of domestic shocks affect the world economy and how they are transmitted to the rest of the world. In particular, we are interested in how the inflationary shocks are transmitted between countries.

In this paper I have presented a very simple two-country new-Keynesian model in which the inclusion of imperfect substitution between home and foreign consumption, home bias consumption, inflation's inertia and the existence of productivity shocks as well as monetary shocks are key for solving the Wang and Wen (2007)'s puzzle of the joint occurrence of positive cross-country correlation in inflation and a near-zero cross-country correlation in money growth.

Although this model adequately captures the signs and magnitude of the cross country-correlations in inflation, output, and consumption, this model tends to generate a stronger positive cross country correlation in output than inflation. Future work is needed to investigate whether this model can generate a stronger positive cross country correlation in inflation than in output.

## CHAPTER III

OPTIMAL MONETARY POLICY UNDER ENDOGENOUS CURRENCY  
INVOICING

## A. Introduction

Producers of export goods have to decide in which currency they set their prices, in their own currency or that of the importer. It is well known that firms in most countries around the world set prices of export goods in US dollars; however in developed countries like Germany, Switzerland, Italy and the United Kingdom, a considerable share of exports are invoiced in their own respective currencies.<sup>10</sup>

It turns out that the choice of invoicing currency is non-trivial. This decision can affect not only exporting firms profits but also macroeconomics variables such as nominal exchange rate, output and consumption. Devereux and Engel (2003), in a one-period two-country Dynamic Stochastic General Equilibrium (DSGE) model with sticky prices, found that the optimal monetary policy, particularly the degree of exchange rate flexibility, depends on the currency invoicing choice made by exporting firms. If all exporting firms invoice in the importer currency (Local Currency Pricing, LCP), the optimal monetary policy is a fixed nominal exchange rate, while if all exporting firms set their prices in their own currency (Producer Currency Pricing, PCP), the optimal monetary policy is a flexible exchange rate.<sup>11</sup> Note, however, that Obstfeld and Rogoff (1995) and Devereux and Engel (2003) take as given the price-setting decision of the exporting firm, so their model does not predict whether an

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<sup>10</sup>According to Gopinath and Rigobon (2008), the fraction of exports to the USA not priced in dollars is 40% from Germany, 38% from Switzerland, 22% from Italy, 21% from Japan, 20% from the UK.

<sup>11</sup>The last part of this result was obtained earlier by Obstfeld and Rogoff (1995).

economy will end up in a flexible exchange rate equilibrium with (PCP) or in a fixed exchange rate equilibrium with (LCP).

To address this problem Corsetti and Pesenti (2004) propose a model similar to Devereux and Engel (2003) but with an endogenous mechanism of price-setting for exporting firms. Their model yields two Nash Equilibria. In the first equilibrium (PCP,PCP), all exporting firms in both countries choose to price in their own currency (PCP) and the optimal monetary policy is characterized by a model-derived reaction function in which nominal money balances respond only to domestic productivity shocks. This monetary policy rule implies a flexible exchange rate.

In the second equilibrium (LCP,LCP), all exporting firms in both countries choose to price in the importer's currency (LCP) and the optimal monetary policy is characterized by a model-derived reaction function in which nominal money balances respond symmetrically to both domestic and foreign productivity shocks. This monetary policy rule implies a pegged exchange rate.

One issue with Corsetti and Pesenti (2004) is that the price setting objective function lacks of clear microfoundations, and this causes some problems in interpreting their derived *pass through* parameter, especially when it takes intermediate values between zero and one.<sup>12</sup> Devereux et al. (2004)'s price setting rule comes with clearer microfoundations but they assume an exogenous monetary policy.

An additional issue with Corsetti and Pesenti (2004) and Corsetti and Pesenti (2005) is that they assume a unitary intratemporal elasticity of substitution between

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<sup>12</sup>Corsetti and Pesenti (2004) define a mechanism in which firms choose an optimal level of pass-through, which takes values between 0 and 1. When this endogenous pass-through takes a value equal to 1, it implies that exporting firms are invoicing in their own currency while if it takes a value equal to 0, it implies that these same firms are invoicing in the importer currency. However, it is not clear what is the price-setting decision made by exporting firms when the pass-through is equal to an intermediate value, e.g. 0.5 or 0.7.

domestic and foreign goods consumption (Armington elasticity of substitution), but what is observed in the data is that the Armington elasticity of substitution is not necessarily equal to one. In fact, Reinert and Roland-Holst (1992) estimated for USA significant Armington elasticities of substitution for 163 goods whose values range between 0.14 and 3.49 and mean is around 1.5. Furthermore, many authors in international economics literature like Backus et al. (1995), Chari et al. (2002), Heathcote and Perri (2002), and Kydland et al. (2009) assume in their models Armington elasticity of substitution values different from one.

In order to carry out an empirical validation of the existing theory of currency invoices, Bhattarai (2009) set up a multiperiod sticky-prices two-country DSGE model to derive the endogenous price-setting conditions that allow exporting firms to determine whether to invoice their exports in PCP or LCP. By using more general consumer preferences or round about production,<sup>13</sup> he derived sufficient conditions that support the existence of asymmetric Nash equilibria. These equilibria are characterized by exporting firms in the Home country that invoice their exports in their own currency (PCP) and exporting firms in the Foreign country that invoice their exports in the importers currency (LCP). Bhattarai (2009) attributes the existence of this asymmetric equilibrium to the presence of strategic complementarities in his model.<sup>14</sup>

One issue with Bhattarai (2009) is that he assumes exogenous processes for monetary policy, for both countries. This assumption implies that monetary authority's decisions cannot be affected by foreign exporting firms' invoicing and this causes that his model predictions will be subject to the Lucas Critique.

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<sup>13</sup>This concept was originally proposed by Basu (1995).

<sup>14</sup>The concept of strategic complementarities in two-country models was defined by Steinsson (2008).

In this paper, I propose a two-country DSGE model which has endogenous price-setting decision rule for exporting firms similar to that proposed by Devereux et al. (2004), constant relative risk aversion preferences, CES consumption aggregator, and a central bank which precommits to a monetary policy rule and which may respond to domestic and foreign productivity shocks. The parameters of this policy rule, which determine the degree of response to each of these shocks, are derived optimally as the solution of the representative household's intertemporal welfare maximization problem.<sup>15</sup>

Since this model has the same exporting firms invoicing decision rule as the proposed by Devereux et al. (2004), then this model is comparable only with the perfect pass-through and zero pass-through versions of Corsetti and Pesenti (2004)'s and Corsetti and Pesenti (2005).

Unlike Bhattarai (2009), this model has a monetary policy rule which is endogenous to the aggregated exporting firms' currency invoicing choice. This feature allows a strategic interaction between the firm expectations and the monetary authority's reaction function.

By solving this model through the second-order approximation method to the policy function, which was developed by Collard and Juillard (2001), I find that after implementing the currency invoice rule developed by Devereux et al. (2004), this model can replicate the theoretical results obtained by Corsetti and Pesenti (2004) and Corsetti and Pesenti (2005) when the Armington elasticity of substitution is equal to one, and preferences are logarithmic in consumption and linear in labor. However, when I assume in this model more general preferences, such as constant relative risk aversion utility function, and Armington elasticity of substitution different from one,

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<sup>15</sup>The preferences and consumption aggregator assumed in this paper encompass those assumed by Corsetti and Pesenti (2005).

I find three new important results in the international macroeconomics literature.

First, the theoretical result, obtained by Corsetti and Pesenti (2005) for a two country model with identical countries, of non-positive gains in welfare from international monetary cooperation does not hold when the Armington elasticity of substitution is different from one. In fact, I find with this model that those gains in welfare are positive and significant under some particular Armington elasticity of substitution values. Furthermore, even though gains from international monetary cooperation are positive when the Armington elasticity of substitution is different from one, I show that if this elasticity is different from one but equal to the intertemporal elasticity of substitution in consumption and exporting firms in both countries set their prices in their own currency, gains from international monetary cooperation are equal to zero.

Second, for different risk aversion coefficients, Frisch elasticities and Armington elasticity of substitution values, this model predicts a unique and symmetric Nash equilibrium (PCP,PCP) when the Armington elasticity of substitution is less than one without matter whether there are or not strategic complementarities. In this equilibrium, all exporting firms in both countries choose to price in their own currency and the central bank's optimal monetary policy implies a flexible exchange rate. In addition, this model predicts multiple Nash equilibria when the Armington elasticity of substitution is larger and equal than one.

In the first equilibrium (PCP,PCP), all exporting firms, in both countries, choose to price in their own currency (PCP) and the optimal monetary policy is characterized by a model-derived reaction function which responds more aggressively to domestic than to foreign productivity shocks. This monetary policy rule implies a flexible exchange rate.

In the second equilibrium (LCP,LCP), all exporting firms, in both countries, choose to price in the importer's currency (LCP) and the optimal monetary pol-

icy is characterized by a model-derived reaction function, which responds in larger magnitude to foreign productivity shocks than domestic productivity shocks. This monetary policy rule implies a flexible exchange rate (with lower volatility than in the first equilibrium) or a pegged exchange rate depending on the Armington elasticity of substitution values.

In the third equilibrium (LCP,PCP), all exporting firms, in one country, choose to price in their own currency (PCP) and all the exporting firms, in the other country, choose to price in the importer's currency (LCP), the optimal monetary policy is characterized by a model-derived reaction function, which responds to both domestic and productivity shocks. This monetary policy rule implies a flexible exchange rate.

Third, unlike Bhattarai (2009), strategic complementarities is not a sufficient condition to support the existence of the asymmetric Nash Equilibrium (LCP,PCP) rather strategic substitution can be a sufficient condition to support this asymmetric Nash equilibrium.

The remainder of the paper is organized as follows. I present the model. Then, I introduce the methodology to determine the existence of the different Nash and Cooperative Equilibria in this model. This is important because here we can see an important application of the second order approximation method of the policy function for solving complex DSGE models, such as the presented in this paper, which do not have a closed form solution. Next, I present in detail the numerical model's results that support the theoretical results mentioned above. Finally, I conclude with a summary of the main paper results and suggestions for further research.



## B. The Model

### 1. Environment

There are two identical countries, each one with the following features:

- Households live an infinite number of periods; they consume a basket of final goods consisting of domestic and foreign goods.
- Households must use real balances to purchase final goods. (i.e. CIA constraint)
- There exists imperfect substitution in the consumption of domestic and foreign goods.
- Households own all firms.
- There exists a continuum of firms. Each firm produces a final good by using labor as input in a constant returns production function.
- The final goods market is characterized by monopolistic competition environment.
- Each firm sells the final good in both the domestic market and the foreign market.
- Each firm sets the price one period prior to the period in which it sells the final good. This means that firms have to set prices prior to observing certain stochastic shocks.
- Firms can choose to set the price of its exported final product either in domestic currency (PCP) or in foreign currency (LCP) through an endogenous price-setting rule, which it will be described later.

- The labor market is characterized by perfect competition. Labor is not tradable across countries.
- There is a central bank which seeks to maximize, at time  $t$ , the present value of the domestic representative household's indirect utility function. In terms of timing, firms set prices at time  $t-1$  and the central bank reacts at time  $t$ .
- The money supply is determined by the central bank's preannounced rule.
- The nominal exchange is determined by the balance of payments equilibrium.
- The source of uncertainty for each country is random labor productivity, determined by an exogenous stochastic process.

## 2. Households

Household  $j$ 's preferences are represented by the following lifetime utility function:

$$E_{t-1} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \frac{C_{\tau}(j)^{1-\sigma}}{(1-\sigma)} - \kappa \frac{N_{\tau}(j)^{1+\psi}}{(1+\psi)} \right] \quad (3.1)$$

where:

$$C_t(j) = \left[ (\chi)^{\frac{1}{\alpha}} C_{H,t}(j)^{\frac{\alpha-1}{\alpha}} + (1-\chi)^{\frac{1}{\alpha}} C_{F,t}(j)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}} \quad (3.2)$$

The parameter  $\chi$  determines the consumer's relative preference towards the home good and the parameter  $\alpha$  denotes the elasticity of substitution between domestic and foreign goods consumption.

Also,

$$C_{H,t}(j) = \left( \int_0^1 C_t(h, j)^{1-\frac{1}{\theta}} dh \right)^{\frac{\theta}{\theta-1}} \quad (3.3)$$

$$C_{F,t}(j) = \left( \int_0^1 C_t(f, j)^{1-\frac{1}{\theta}} df \right)^{\frac{\theta}{\theta-1}} \quad (3.4)$$

$C_t(h, j)$  denotes household  $j$ 's consumption of home variety  $\mathbf{h}$ ,  $C_t(f, j)$  denotes household  $j$ 's consumption of foreign variety  $\mathbf{f}$  and the parameter  $\theta$  denotes the elasticity of substitution across varieties.

$C_{H,t}(j)$  denotes household  $j$ 's aggregated consumption of domestic good, and  $C_{F,t}(j)$  denotes household  $j$ 's aggregated consumption of foreign good,  $C_t(j)$  is household  $j$ 's optimal consumption basket at date  $t$ , and  $N_t(h)$  represents the household  $j$ 's worked hours.

Household  $j$  chooses  $\{C_\tau(j), N_\tau(j), M_\tau(j), B_\tau(j), B_\tau^*(j)\}_{\tau=t}^\infty$ , which maximizes its lifetime utility (3.1) subject to the following budget constraint:

$$\begin{aligned} B_t(j) + S_t B_t^*(j) + M_t(j) &\leq (1 + i_{t-1})B_{t-1}(j) + (1 + i_{t-1}^*)S_t E_t B_{t-1}^*(j) + \\ &M_{t-1}(j) + W_t N_t(j) + T_t(j) + \int_0^1 \Pi_t(h) dh \\ &- P_t C_t(j) \end{aligned} \quad (3.5)$$

$$P_t C_t(j) \leq M_{t-1}(j) + T_t \quad (3.6)$$

where  $M_{t-1}(j), T_t(j), P_t, \Pi_t(h), W_t$ , are taken as given,  $T_t(j)$  is a lump sum transfer of money that household  $j$  receives from the monetary authority,  $M_t(j)$  is household  $j$ 's demand for money holdings in home currency at time  $t$  for spending at  $t+1$ ,  $B_t(j)$  is household  $j$ 's demand for nominal bonds issued in domestic currency at time  $t$ ,

$P_t$  denotes the home's consumer price index,  $W_t$  is the real wage (deflated using the consumer price index),  $i_t$  and  $i_t^*$  are the domestic and foreign bonds nominal yields paid at time  $t+1$ , and  $\Pi_t(h)$  is the final goods producer  $h$ 's profit. The variables for the foreign country are denoted with a star.

### *F.O.C*

The first order conditions from the representative household's maximization problem are:

$$C_t : C_t(j)^{-\sigma} - P_t\lambda_t(j) - P_t\mu_t(j) = 0 \quad (3.7)$$

$$N_t(j) : -\kappa N_t(j)^\eta + \lambda_t(j)W_t(j) = 0 \quad (3.8)$$

$$M_t(j) : -\lambda_t(j) + \beta E_t [\lambda_{t+1}(j) + \mu_{t+1}(j)] = 0 \quad (3.9)$$

$$B_t(j) : -\lambda_t(j) + (1 + i_t)\beta E_t [\lambda_{t+1}(j)] = 0 \quad (3.10)$$

$$B_t^*(j) : -\lambda_t(j)S_t + (1 + i_t^*)\beta E_t [S_{t+1}\lambda_{t+1}(j)] = 0 \quad (3.11)$$

#### a. International Risk Sharing

By the assumption of complete securities markets, similar countries, the first order conditions (3.7),(3.10),(3.11) and the foreign analogous of (3.7), it follows that

$$\frac{P_t^* S_t}{P_t} = \left( \frac{C_t}{C_t^*} \right)^\sigma \quad (3.12)$$

This is the efficient risk sharing condition used in international economy.

b. CPI Inflation

Given that household  $j$  has chosen  $\{C_t(j)\}_{t=0}^\infty$  previously, household  $j$  chooses at each period  $t$   $C_t(h, j)$  and  $C_t(f, j)$ , which minimize its total expenditure<sup>16</sup>:

$$P_{H,t} C_{H,t}(j) + P_{F,t} C_{F,t}(j) \quad (3.13)$$

subject to (3.2).

The main results from this intratemporal minimization problem are the following:

- Household  $j$ 's demand for varieties  $h$  and  $f$  as a function of the relative price and total consumption of domestic and foreign goods are given by:

$$C_t(h, j) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}(j) \quad (3.14)$$

$$C_t(f, j) = \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\theta} C_{F,t}(j) \quad (3.15)$$

- Household  $j$ 's total demands for the domestic and foreign goods are given by:

$$C_{H,t}(j) = \chi \left( \frac{P_{H,t}}{P_t} \right)^{-\alpha} C_t(j) \quad (3.16)$$

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<sup>16</sup>Since the representative household preferences are separable on every period and markets are complete, the intratemporal minimization problem can be solved independently from the intertemporal maximization problem described previously.

$$C_{F,t}(j) = (1 - \chi) \left( \frac{P_{F,t}}{P_t} \right)^{-\alpha} C_t(j) \quad (3.17)$$

- Using (3.16), (3.17) and (3.13), the home country's consumer price index is equal to:

$$P_t = [\chi P_{H,t}^{1-\alpha} + (1 - \chi) P_{F,t}^{1-\alpha}]^{\frac{1}{1-\alpha}} \quad (3.18)$$

where  $P_{H,t} = \left[ \int_0^1 p_t(h)^{1-\theta} dh \right]^{\frac{1}{1-\theta}}$  and  $P_{F,t} = \left[ \int_0^1 p_t(f)^{1-\theta} df \right]^{\frac{1}{1-\theta}}$ <sup>17</sup>

### 3. Firms

Each variety  $\mathbf{h}$  is produced by a single domestic firm and sold in both countries.

Producer  $\mathbf{h}$ 's output is produced by using the following technology:

$$Y_t(h) = A_t N_t(h) \quad (3.19)$$

$Y_t(h)$  denotes the final good output produced by Home's firm  $\mathbf{h}$ ,  $A_t$  is the Home's country-specific productivity process, where  $\hat{A}_t = \rho \hat{A}_{t-1} + u_t$ ,  $\hat{A}_t$  denotes Home's productivity deviations from its non stochastic steady state value and  $u_t \sim N(0, \Sigma)$ .<sup>18</sup>

Since the final goods market is characterized by a monopolistic competition's environment, firms are able to set the price of their final goods, in both domestic and foreign markets, to maximize their discounted expected profits.

- For the domestic market, we have:

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<sup>17</sup>These expressions are obtained from: minimize  $\int_0^1 p_t(h) C_t(h, j) dh$  subject to (3.3) and minimize  $\int_0^1 p_t(f) C_t(f, j) df$  subject to (3.4).

<sup>18</sup> $\Sigma$  is a 2x2 variance-covariance matrix of two stochastic processes  $u_t$  and  $u_t^*$ , where  $u_t^*$  is the random component of the foreign country-specific productivity process.

$$\bar{p}_t(h) = \arg \max E_{t-1} Q_{t-1,t} [p_t(h) - MC_t(h)] \int_0^1 C_t(h, j) dj \quad (3.20)$$

- For the foreign market, we have:

$$\bar{p}_t^f(h) = \arg \max E_{t-1} Q_{t-1,t} [S_t p_t^*(h) - MC_t(h)] \int_0^1 C_t^*(h, j^*) dj^* \quad (3.21)$$

Firms take the nominal price of labor,  $W_t$ , and the country-specific productivity shock,  $A_t$ , as given.

Since all domestic firms are homogeneous, in equilibrium we have:  $p_t(h) = P_{H,t}$  and  $p_t^*(h) = P_{H,t}^*$ . The same reasoning applies for the foreign firms. That is, in equilibrium we have  $p_t(f) = P_{F,t}$  and  $p_t^*(f) = P_{F,t}^*$ .

Here,  $\bar{p}_t^f(h)$  is the price of exported good set by the producer. This price can be set either in domestic money or in foreign currency. Here,  $p_t^*(h)$  is the price that firm h receives at the moment of selling its final good and it is the price that foreign consumers pay for the domestic good produced by firm h.  $MC_t(h)$  is the nominal marginal cost of firm h, which can be represented by the following expression:

$$MC_t(h) = W_t/A_t \quad (3.22)$$

In order to understand the currency-invoicing rule, we need the following definitions.

- If  $\bar{p}_t^f(h)$  is set in domestic currency,  $p_t^*(h) \equiv \bar{p}_t^f(h)/S_t$ . That is, if a US exporting firm sets its price in USD, the foreign consumer (say France) will pay  $p_t^*(h)$  which is denominated in EURO.
- If  $\bar{p}_t^f(h)$  is set in foreign currency,  $p_t^*(h) \equiv \bar{p}_t^f(h)$ . That is, if a US exporting firm sets its price in EURO, the foreign consumer, say France, will pay  $p_t^*(h)$

which is also denominated in EURO.

a. Currency Choice - PCP vs. LCP

The decision rule that firm  $h$  uses for invoicing its exports in either domestic currency or foreign currency is the same as proposed by Devereux et al. (2004).

**Definition 1.** Let  $\eta$  be a function from  $\mathfrak{R} \rightarrow \{0, 1\}$  such that:

$$\eta = 1 \text{ if } \{E_{t-1}Q_{t-1,t}\Pi_t^{PCP} > E_{t-1}Q_{t-1,t}\Pi_t^{LCP}\} \quad (3.23)$$

**Definition 2.**

$$p_t^*(h) \equiv p_t^f(h)/S_t^\eta \quad (3.24)$$

Unlike Corsetti and Pesenti (2004), expression (3.23) can take only two values, one or zero. Corsetti and Pesenti (2004) allowed intermediate values such as  $\eta = 0.5$  and in these cases it is no clear to determine whether a exporting firm sets its good price in domestic or in foreign currency.

Let me to cite the following proposition showed by Devereux et al. (2004)

**Proposition 1.** *The domestic firm  $h$  sets its price for the foreign market in domestic (foreign) currency if*

$$\left[ \frac{Var(S_t)}{2} - cov(MC_t, S_t) \right] > 0, (< 0) \quad (3.25)$$

*Proof.* See Appendix □

As Devereux et al. (2004) showed, this proposition is the result of comparing up to second order the expected profit that the exporting firm has when it sets its price in domestic (PCP) or in foreign currency (LCP). An important assumption in this proposition is that every firm  $h$  takes the stochastic discount factor as given. The idea is that actions by individual firms have a negligible impact on the market.



**Corollary 1.**

$$\eta = 1 \left\{ \left[ \frac{Var(S_t)}{2} - cov(MC_t, S_t) \right] > 0 \right\} \quad (3.26)$$

**Corollary 2.** *The foreign firm  $f$  sets its price for the domestic market in foreign (domestic) currency if*

$$\left[ \frac{Var(S_t)}{2} + cov(MC_t^*, S_t) \right] > 0, (< 0) \quad (3.27)$$

**Corollary 3.**

$$\eta^* = 1 \left\{ \left[ \frac{Var(S_t)}{2} + cov(MC_t^*, S_t) \right] > 0 \right\} \quad (3.28)$$

#### 4. Monetary Authority

The monetary authority transfers to households a lump sum transfer of money.

$$T_t = M_t - M_{t-1} \quad (3.29)$$

In contrast with Devereux et al. (2004) and Bhattarai (2009), the money supply in this model is determined optimally by the monetary authority at time  $t$ , one period after the exporting firms have set their prices. The monetary authorities are able to commit to the preannounced rule:

$$M_t = M(A_t, A_t^*, \nu_d, \nu_f) \quad (3.30)$$

$$M_t^* = M^*(A_t, A_t^*, \nu_d^*, \nu_f^*) \quad (3.31)$$

where  $A_t$  is the country-specific productivity process for the Home country and  $A_t^*$  is the country-specific productivity process for the Foreign country.  $A$  and  $A^*$  denotes the non-stochastic steady state value for the productivity of Home and Foreign country respectively, the parameter  $\nu_d$  denotes the monetary authority's degree of re-

sponse to domestic productivity shocks, while the parameter  $\nu_f$  denotes the monetary authority's degree of response to foreign productivity shocks. The  $\nu$ 's are determined optimally by a benevolent monetary authority who seeks to maximize the representative household's indirect welfare function. That is:

$$\{\bar{\nu}_d, \bar{\nu}_f\} = \arg \max E_{t-1} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \frac{\tilde{C}_{\tau}^{1-\sigma}}{(1-\sigma)} - \kappa \frac{\tilde{N}_{\tau}^{1+\psi}}{(1+\psi)} \right] \quad (3.32)$$

where  $\tilde{C}_t$  and  $\tilde{N}_t$  are the model's policy functions.

## 5. Equilibrium Conditions

The physical constraint for variety h is:

$$Y_t(h) \geq \int_0^1 C_t(h, j) dj + \int_0^1 C_t^*(h, j^*) dj^* \quad (3.33)$$

The physical constraint for variety f is:

$$Y_t(f) \geq \int_0^1 C_t(f, j) dj + \int_0^1 C_t^*(f, j^*) dj^* \quad (3.34)$$

The domestic labor market constraint is:

$$\int_0^1 N_t(j) dj \geq \int_0^1 N_t(h) dh \quad (3.35)$$

The foreign labor market constraint is:

$$\int_0^1 N_t^*(j) dj^* \geq \int_0^1 N_t^*(h) dh^* \quad (3.36)$$

The international bonds constraints are:

$$\int_0^1 B_t(j) dj + \int_0^1 B_t(j^*) dj^* = 0 \quad (3.37)$$

$$\int_0^1 B_t^*(j) dj + \int_0^1 B_t^*(j^*) dj^* = 0 \quad (3.38)$$

### C. Methodology to Find the Nash and the Cooperative Equilibria

#### 1. Nash Equilibria

This strategic situation can be described as a dynamic game in which firms in the two countries set their prices at time  $t-1$ , each country's central bank precommits at time  $t-1$  with an optimal monetary policy rule, which responds to both domestic and foreign productivity shocks at time  $t$  by taking the firms' pricing decision and the other country's central bank's reaction function as given.

The first step is to make a second order expansion of the Central Bank's objective function, (3.32), since it is computational burdensome to solve (3.32) by methodologies such as iterating on the Bellman equation.<sup>19</sup>

Therefore,

$$W_{t-1} \approx \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ C^{-\sigma} E_{t-1}(\tilde{C}_t - C) - \frac{\sigma}{2} C^{-\sigma-1} E_{t-1}(\tilde{C}_t - C)^2 \dots \right. \\ \left. - N^{\psi} E_{t-1}(\tilde{N}_t - N) - \frac{\psi}{2} N^{\psi-1} E_{t-1}(\tilde{N}_t - N)^2 \right] \quad (3.39)$$

Since markets are complete and  $\tilde{C}_t$  and  $\tilde{N}_t$  policy functions depend on  $\nu'$ s and productivity shocks, by applying the iterated expectations law to (3.39), the Central Bank's optimization problem turns into:

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<sup>19</sup>In chapter 6, Woodford (2003) argues why the expansion of (3.32) should be up to second order.

$$\{\bar{\nu}_H, \bar{\nu}_F\} = \arg \max \left[ C^{-\sigma} E_0(\tilde{C}_t - C) - \frac{\sigma}{2} C^{-\sigma-1} E_0(\tilde{C}_t - C)^2 \dots \right. \\ \left. - N^\psi E_0(\tilde{N}_t - N) - \frac{\psi}{2} N^{\psi-1} E_0(\tilde{N}_t - N)^2 \right] \quad (3.40)$$

where  $X$  denotes the non-stochastic steady state value of  $X_t$ ,  $\tilde{X}_t$  denotes  $X_t$ 's policy function, and  $X_t = \{C_t, N_t\}$ .

The second step is to solve the model described in the previous section by using the second order approximation method of the policy function.<sup>20 21 22</sup>

The third step is to solve simultaneously the optimization problem (3.40) for each country's Central Bank providing that all firms in each country have decided to set their prices either in domestic currency or in foreign currency (i.e. we impose an initial exporting firms' currency-invoicing decision). The solution algorithm is the following:

- Pick some initial values for the parameters  $\nu_H$ ,  $\nu_F$ ,  $\nu_H^*$  and  $\nu_F^*$ .
- Use the calibrated parameters, shown in the appendix, to solve the model.
- Solve the model by the method described in the second step.

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<sup>20</sup>Instead of using the standard linearization method, I use the second order approximation method of the policy function to avoid the spurious results described by Kim and Kim (2003). These authors show that if they evaluate the second order approximation of the welfare function by using only first order approximation of the policy function in a two-country SDGE model, then the economics welfare is higher under autarky than under free world trade. This spurious result contradicts the first welfare theorem.

<sup>21</sup>The method for computing the unconditional moments from the second order approximation of this model is the perturbations method implemented in DYNARE by Collard and Juillard (2001). DYNARE is matlab's toolbox developed by Maurice Juillard and Fabrice Collard. To get more information about this software, please follow the link: <http://www.dynare.org/>.

<sup>22</sup>The model's calibration is provided in the appendix.

- Get the unconditional first two moments for the variables  $C_t$  and  $N_t$  and plug them into each country's central bank welfare function.
- Obtain the optimal values for  $\nu_H, \nu_F$ , which maximizes the welfare function (3.40) for the domestic country given the initial values of  $\nu_H^*$  and  $\nu_F^*$ .
- Obtain the optimal values for  $\nu_H^*, \nu_F^*$ , which maximizes the welfare function (3.40) for the foreign country given the values of  $\nu_H$  and  $\nu_F$  that were obtained in the previous item.
- Repeat the last two steps until the norm of  $\nu'$ s vector reach a convergence criterion defined by the researcher.<sup>23</sup>

The fourth step is to check if the solution obtained from the above steps is a subgame perfect Nash equilibrium. We need to check if the exporting firms currency-invoicing decisions, determined by (3.25) and (3.27), are the same as those taken as given by the Central Bank at the moment of choosing its own  $\nu'$ .<sup>24</sup>

## 2. Cooperative Equilibria

This strategic situation can be described as a dynamic game in which firms, in both countries, set their prices at time  $t-1$ , each country's central bank precommits at time  $t-1$  with an optimal monetary policy rule, which responds to both domestic and foreign productivity shocks at time  $t$  by taking as given the firms' pricing decision as given. Unlike the non-cooperative case, here both central banks want to choose their optimal monetary policy such that they maximize aggregate world welfare. That is:

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<sup>23</sup>In this paper, I set a convergence criterion of  $10e-5$ .

<sup>24</sup>To know this firm's currency-invoicing decision, we need to obtain from the model's solution the following unconditional moments  $cov(MC_t, S_t)$ ,  $cov(MC_t^*, S_t)$  and  $var(S_t)$  and plug them in (3.25) and (3.27).

$$\begin{aligned} \{\bar{\nu}_d, \bar{\nu}_f, \bar{\nu}_d^*, \bar{\nu}_f^*\} = \arg \max E_{t-1} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \zeta \left[ \frac{\tilde{C}_\tau^{1-\sigma}}{(1-\sigma)} - \kappa \frac{\tilde{N}_\tau^{1+\psi}}{(1+\psi)} \right] + \\ (1-\zeta) \left[ \frac{(\tilde{C}_\tau^*)^{1-\sigma}}{(1-\sigma)} - \kappa \frac{(\tilde{N}_\tau^*)^{1+\psi}}{(1+\psi)} \right] \end{aligned} \quad (3.41)$$

where  $\zeta$  denotes each country's bargaining power<sup>25</sup>,  $X$  denotes the non-stochastic steady state value of  $X_t$  and  $\tilde{X}_t$  denotes  $X_t$ 's policy function and  $Xt = \{C_t, Nt, C_t^*, N_t^*\}$

Since markets are complete and  $\tilde{C}_t, \tilde{N}_t, \tilde{C}_t^*, \tilde{N}_t^*$  policy functions depend on  $\nu$ 's and productivity shocks, by approximating (3.41) up to second order and by applying the iterated expectations law to (3.41), the Central Banks' jointly optimization problem turns into:

$$\begin{aligned} \{\bar{\nu}_d, \bar{\nu}_f, \bar{\nu}_d^*, \bar{\nu}_f^*\} = \arg \max \zeta \left[ C^{-\sigma} E_0(\tilde{C}_t - C) - \frac{\sigma}{2} C^{-\sigma-1} E_0(\tilde{C}_t - C)^2 \dots \right. \\ \left. - N^\psi E_0(\tilde{N}_t - N) - \frac{\psi}{2} N^{\psi-1} E_0(\tilde{N}_t - N)^2 \right] \\ + (1-\zeta) \left[ (C^*)^{-\sigma} E_0(\tilde{C}_t^* - C^*) - \frac{\sigma}{2} (C^*)^{-\sigma-1} E_0(\tilde{C}_t^* - C^*)^2 \dots \right. \\ \left. - (N^*)^\psi E_0(\tilde{N}_t^* - N^*) - \frac{\psi}{2} (N^*)^{\psi-1} E_0(\tilde{N}_t^* - N^*)^2 \right] \end{aligned} \quad (3.42)$$

The two first steps to find the cooperative equilibria are the same as those described in the previous subsection. However, the third step changes a little bit with respect to the described in the previous subsection. That is:

- Pick up some initial values for the parameters  $\nu_H, \nu_F, \nu_H^*$  and  $\nu_F^*$ .
- Use the calibrated parameters, shown in the appendix, to solve the model.

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<sup>25</sup>In this paper I assume that both countries has the same bargaining power even when they have different size. The reason of this assumption is to understand how the country size affects the monetary cooperation gains for each country.

- Solve the model by the method described in the second step.
- Get the unconditional first two moments for the variables  $C_t, N_t, C_t^*, N_t^*$  and plug them in each country's central bank welfare function.
- Obtain the optimal values for  $\nu_H, \nu_F, \nu_H^*, \nu_F^*$ , which maximizes the welfare function (3.42).

The fourth step is to evaluate if the solution obtained above is a cooperative equilibrium. To evaluate it, we need check that the exporting firms currency-invoicing decisions, determined by (3.25) and (3.27), are the same as the taken as given by the central banks at the moment of choosing their own  $v'$ .<sup>26</sup>

#### D. Theoretical Results

In this section, I present the theoretical results obtained by this model under different combination of preferences, Armington elasticities of substitution and international monetary policy strategies (uncoordinated and coordinated international monetary policy)

##### 1. Loglinear Preferences

When preferences are logarithmic in consumption, ( $\sigma = 1$ ), linear in labor, ( $\psi = 0$ ) and the Armington elasticity of substitution is equal to one, ( $\alpha = 1$ ), this model replicates the following theoretical results derived analytically by Corsetti and Pesenti (2004) and Corsetti and Pesenti (2005).

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<sup>26</sup>To know this firm's currency-invoicing decision, we need to obtain from the model's solution the following unconditional moments  $cov(MC_t, S_t)$ ,  $cov(MC_t^*, S_t)$  and  $var(S_t)$  and plug them in (3.25) and (3.27).

Table IX.

Nash Equilibria, when  $\sigma = 1$ , and  $\psi = 0$ 

$\alpha$	Invoice Invoice		$\nu_d$	$\nu_f$	$\nu_d^*$	$\nu_f^*$	sd( $S_t$ )	E(U)	E( $U^*$ )
	Home	Foreign							
0.25	PCP	PCP	1.6045	-0.6045	-0.6045	1.6045	6.17	-3.4316	-3.4316
0.50	PCP	PCP	1.1552	-0.1552	-0.1552	1.1552	3.66	-3.1287	-3.1287
0.70	PCP	PCP	1.0456	-0.0456	-0.0456	1.0456	3.05	-2.9256	-2.9256
0.85	PCP	PCP	1.0105	-0.0105	-0.0105	1.0105	2.85	-2.7781	-2.7781
1.00	PCP	PCP	1.0000	0.0000	0.0000	1.0000	2.79	-2.6316	-2.6316
1.00	LCP	LCP	0.5000	0.5000	0.5000	0.5000	0.00	-2.8171	-2.8171
1.30	PCP	PCP	1.0427	-0.0427	-0.0427	1.0427	3.03	-2.3404	-2.3404
1.30	LCP	LCP	0.4325	0.5675	0.5675	0.4325	0.38	-2.5832	-2.5832
1.50	PCP	PCP	1.1364	-0.1364	-0.1364	1.1364	3.56	-2.1641	-2.1641
1.50	LCP	LCP	0.3875	0.6125	0.6125	0.3875	0.63	-2.4311	-2.4311
2.00	LCP	LCP	0.2750	0.7250	0.7250	0.2750	1.26	-2.0638	-2.0638
2.00	PCP	LCP	-0.1136	1.1136	-1.3030	2.3030	3.32	-2.6571	-2.4006
2.20	PCP	LCP	-0.4975	1.4976	-2.9673	3.9673	6.90	-6.1724	-2.4735
3.00	LCP	LCP	0.0500	0.9500	0.9500	0.0500	2.52	-1.3857	-1.3857
4.00	LCP	LCP	-0.1750	1.1750	1.1750	-0.1750	3.77	-0.7827	-0.7827
5.00	LCP	LCP	-0.4000	1.4000	1.4000	-0.4000	5.03	-0.2548	-0.2548



Table X.

Cooperative Equilibria, when  $\sigma = 1$ , and  $\psi = 0$ 

$\alpha$	Invoice						Gains from			
	Home	Foreign	$\nu_d$	$\nu_f$	$\nu_d^*$	$\nu_f^*$	$sd(S_t)$	E(U)	E( $U^*$ )	Coop. (%)
0.25	PCP	PCP	1.0	0.0	0.0	1.0	2.79	-3.3638	-3.3638	1.9750
0.50	PCP	PCP	1.0	0.0	0.0	1.0	2.79	-3.1197	-3.1197	0.2850
0.70	PCP	PCP	1.0	0.0	0.0	1.0	2.79	-2.9245	-2.9245	0.0360
0.85	PCP	PCP	1.0	0.0	0.0	1.0	2.79	-2.7780	-2.7780	0.0020
1.00	PCP	PCP	1.0	0.0	0.0	1.0	2.79	-2.6316	-2.6316	0.0000
1.00	LCP	LCP	0.5	0.5	0.5	0.5	0.00	-2.8171	-2.8171	0.0000
1.30	PCP	PCP	1.0	0.0	0.0	1.0	2.79	-2.3387	-2.3387	0.0750
1.30	LCP	LCP	0.5	0.5	0.5	0.5	0.00	-2.5798	-2.5798	0.1309
1.50	PCP	PCP	1.0	0.0	0.0	1.0	2.79	-2.1434	-2.1434	0.9563
1.50	LCP	LCP	0.5	0.5	0.5	0.5	0.00	-2.4217	-2.4217	0.3863
2.00	LCP	LCP	0.5	0.5	0.5	0.5	0.00	-2.0263	-2.0263	1.8201
3.00	LCP	LCP	0.5	0.5	0.5	0.5	0.00	-1.2354	-1.2354	79.980
4.00	LCP	LCP	0.5	0.5	0.5	0.5	0.00	-0.4446	-0.4446	67.910
5.00	LCP	LCP	0.5	0.5	0.5	0.5	0.00	0.3462	0.3462	144.220

First, by comparing Tables IX and X we can see that there are not gains from international monetary cooperation when the Armington elasticity of substitution is equal to one. This result is independent of the invoicing currency chosen by exporting

firms.<sup>27</sup>

Second, in Table IX we can see that when the Armington elasticity of substitution is equal to one, there are two Nash Equilibria. The first equilibrium (PCP,PCP), is characterized by all exporting firms, in both countries, choosing to price in their own currency (PCP) and the optimal monetary policy is characterized by a model-derived reaction function which responds more aggressively to domestic than to foreign productivity shocks. This monetary policy rule implies a flexible exchange rate. The second equilibrium (LCP,LCP), is characterized by all the exporting firms, in both countries, choosing to price in the importer's currency (LCP) and the optimal monetary policy is characterized by a model-derived reaction function, which responds in similar magnitude to both foreign and domestic productivity shocks. This monetary policy rule implies a pegged exchange rate.

Nevertheless, when the Armington elasticity of substitution is not equal to one, the theoretical results derived analytically by Corsetti and Pesenti (2004) and Corsetti and Pesenti (2005) do not hold. First, by comparing Tables IX and (X) we can see that there are positive gains from international monetary cooperation when the Armington elasticity of substitution is different from one. This result is independent of the currency invoicing choice made by exporting firms. Furthermore, in Table IX we can see that when the Armington elasticity of substitution is quite far from one, positive gains from international monetary cooperation becomes more significant. For example, when the Armington elasticity of substitution is equal to 0.25 or 5, the representative household expected welfare increases in 2% or 144% respectively. However, when the Armington elasticity is equal to 0.85 or 1.3, the representative

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<sup>27</sup>These results are not demonstrated by a formal proof here but they can be obtained by solving numerically this model. This numerical solution is provided in this paper's matlab code, which is available upon request.

household expected welfare increases in 0% or 0.07% respectively.

Second, in Table IX we can see that when the Armington elasticity of substitution is less than one, there is a unique and symmetric Nash Equilibrium. This Nash equilibrium (PCP,PCP), is characterized by all exporting firms, in both countries, choosing to price in their own currency (PCP) and the optimal monetary policy is characterized by a model-derived reaction function which responds more aggressively to domestic than foreign productivity shocks. This monetary policy rule implies a flexible exchange rate.

Third, in Table IX we can see that when the Armington elasticity of substitution is larger and equal than one, there are multiple Nash Equilibria. The first Nash equilibrium (PCP,PCP) has the same features as the previously described. The second Nash equilibrium (LCP,LCP), is characterized by all exporting firms, in both countries, choosing to price in the importer's currency (LCP) and the optimal monetary policy is characterized by a model-derived reaction function, which responds in larger (except when  $\alpha = 1$ ) magnitude to foreign productivity shocks than domestic productivity shocks. This monetary policy rule implies a flexible exchange rate (with lower volatility than in the first equilibrium) or a pegged exchange rate depending on the Armington elasticity of substitution value. The third Nash equilibrium (LCP,PCP) is characterized by all exporting firms, in one country, choosing to price in their own currency (PCP) and all exporting firms, in the other country, choosing to price in the importer's currency (LCP), the optimal monetary policy is characterized by a model-derived reaction function, which responds to both domestic and productivity shocks. This monetary policy rule implies a flexible exchange rate.

Fourth, in Table IX, we can see, for different Armington elasticities of substitution values, that the (PCP,PCP) Nash equilibrium Pareto dominates the (LCP,LCP) Nash equilibrium.

## 2. CRRA Preferences and Same Country Size

When preferences are represented by a constant relative risk aversion (CRRA) utility function, most of the above theoretical results still hold. Furthermore, under these general preferences, I find two additional new results in international macroeconomics literature. First, when the Armington elasticity of substitution is different from one but equal to the intertemporal elasticity of substitution in total consumption and all exporting firms, in both countries, invoice their exports in their own currency (PCP,PCP), there are no gains from international monetary cooperation. Second, theoretical results obtained by Bhattarai (2009) do not hold in this model. For example, strategic complementarities  $\sigma + \psi < 1$ , as defined by Steinsson (2008) is not the only sufficient condition to support the asymmetric Nash equilibrium (PCP,LCP), in which all exporting firms, in one country, choose to price in their own currency (PCP) and all the exporting firms, in the other country, choose to price in the importer's currency (LCP). Instead, strategic substitution  $\sigma + \psi > 1$  can also support the asymmetric Nash equilibrium (PCP,LCP).

Table XI.

Nash Equilibria, when  $\sigma = 0.3$ , and  $\psi = 0.3$ 

$\alpha$	Invoice		$\nu_d$	$\nu_f$	$\nu_d^*$	$\nu_f^*$	$\text{sd}(S_t)$	$E(U)$	$E(U^*)$
	Home	Foreign							
0.50	PCP	PCP	1.8961	0.2706	0.2706	1.8961	4.54	-0.6903	-0.6903
0.70	PCP	PCP	1.7116	0.4550	0.4550	1.7116	3.51	-0.4355	-0.4355
0.85	PCP	PCP	1.6346	0.5321	0.5321	1.6346	3.08	-0.2703	-0.2703
1.00	PCP	PCP	1.5833	0.5833	0.5833	1.5833	2.79	-0.0418	-0.0418
1.00	LCP	LCP	1.0833	1.0833	1.0833	1.0833	0.00	-0.1250	-0.1250
1.30	PCP	PCP	1.5213	0.6454	0.6454	1.5213	2.45	0.1498	0.1498
1.30	LCP	LCP	1.0293	1.1374	1.1374	1.0293	0.27	-0.1393	-0.1393
1.50	PCP	PCP	1.4960	0.6706	0.6706	1.4960	2.31	0.3106	0.3106
1.50	LCP	LCP	0.9969	1.1697	1.1697	0.9969	0.44	-0.0093	-0.0093
2.00	LCP	LCP	0.9268	1.2399	1.2399	0.9268	0.80	0.2714	0.2714
2.00	PCP	LCP	0.5842	1.5825	0.5316	1.6350	0.14	0.4095	-0.0704
2.20	PCP	LCP	0.5105	1.6562	0.4718	1.6948	0.10	0.5521	-0.0805
3.00	LCP	LCP	0.8196	1.3471	1.3471	0.8196	1.34	0.6956	0.6956
3.00	PCP	LCP	0.1996	1.9670	0.1728	1.9939	0.07	1.0826	-0.3460
4.00	LCP	LCP	0.7417	1.4250	1.4250	0.7417	1.74	1.0010	1.0010
4.00	PCP	LCP	-0.2395	2.4061	-0.3299	2.4966	0.23	1.6301	-1.1205

Table XII.

Cooperative Equilibria, when  $\sigma = 0.3$ , and  $\psi = 0.3$ 

$\alpha$	Invoice								Gains from	
	Home	Foreign	$\nu_d$	$\nu_f$	$\nu_d^*$	$\nu_f^*$	$sd(S_t)$	E(U)	E( $U^*$ )	Coop. (%)
0.50	PCP	PCP	1.65	0.52	0.52	1.65	3.16	-0.6645	-0.6645	3.74
0.70	PCP	PCP	1.62	0.55	0.55	1.62	3.00	-0.4303	-0.4303	1.18
0.85	PCP	PCP	1.60	0.57	0.57	1.60	2.89	-0.2694	-0.2694	0.32
1.00	PCP	PCP	1.58	0.58	0.58	1.58	2.79	-0.0418	-0.0418	0.00
1.00	LCP	LCP	1.08	1.08	1.08	1.08	0.00	-0.1250	-0.1250	0.00
1.30	PCP	PCP	1.55	0.62	0.62	1.55	2.61	0.1509	0.1509	0.78
1.30	PCP	PCP	1.08	1.08	1.08	1.08	0.00	-0.1386	-0.1386	0.46
1.50	PCP	PCP	1.53	0.64	0.64	1.53	2.51	0.3126	0.3126	0.65
1.50	LCP	LCP	1.08	1.08	1.08	1.08	0.00	-0.0076	-0.0076	17.67
2.00	LCP	LCP	1.08	1.08	1.08	1.08	0.00	0.2768	0.2768	1.99
3.00	LCP	LCP	1.08	1.08	1.08	1.08	0.00	0.7109	0.7109	2.20
4.00	LCP	LCP	1.08	1.08	1.08	1.08	0.00	1.0267	1.0267	2.56

By comparing Tables XI and XII, we can see that for different Armington elasticity of substitution values and under strategic complementarities (e.g  $\sigma = 0.3$  and  $\psi = 0.3$ ), there are positive gains from international monetary cooperation.

In addition, in Table XI we can see that for different Armington elasticity of substitution values which are less than one, there exists only a unique Nash equilibrium, (PCP,PCP), while for Armington elasticity of substitution values larger and equal

than one, there exist multiple Nash equilibria. Furthermore, we can see also in Table XI that expected welfare associated to (PCP,PCP) Nash equilibrium is larger than that of the (LCP,LCP).

Table XIII.

Nash Equilibria, when  $\sigma = 2$ , and  $\psi = 1$

$\alpha$	Invoice Invoice		$\nu_d$	$\nu_f$	$\nu_d^*$	$\nu_f^*$	sd( $S_t$ )	E(U)	E( $U^*$ )
	Home	Foreign							
0.50	PCP	PCP	1.0000	-0.3333	-0.3333	1.0000	3.73	-4.3913	-4.3913
0.70	PCP	PCP	0.8964	-0.2297	-0.2297	0.8964	3.15	-4.0852	-4.0852
0.85	PCP	PCP	0.8577	-0.1910	-0.1910	0.8577	2.93	-3.8985	-3.8985
1.00	PCP	PCP	0.8333	-0.1667	-0.1667	0.8333	2.79	-5.9363	-5.9363
1.00	LCP	LCP	0.3333	0.3333	0.3333	0.3333	0.00	-6.5260	-6.5260
1.30	PCP	PCP	0.8017	-0.1350	-0.1350	0.8017	2.62	-3.4871	-3.4871
1.30	LCP	LCP	0.2991	0.3675	0.3675	0.2991	0.24	-3.9063	-3.9063
1.50	PCP	PCP	0.7845	-0.1178	-0.1178	0.7845	2.52	-3.3559	-3.3559
1.50	LCP	LCP	0.2809	0.3858	0.3858	0.2809	0.37	-3.7985	-3.7985
2.00	PCP	PCP	0.7417	-0.0751	-0.0750	0.7416	2.28	-3.1131	-3.1131
2.00	LCP	LCP	0.2460	0.4207	0.4207	0.2460	0.61	-3.5946	-3.5946
3.00	LCP	LCP	0.2023	0.4644	0.4644	0.2023	0.92	-3.3448	-3.3448
4.00	LCP	LCP	0.1761	0.4906	0.4906	0.1761	1.10	-3.1977	-3.1977

Table XIV.

Cooperative Equilibria, when  $\sigma = 2$ , and  $\psi = 1$ 

$\alpha$	Invoice						Gains from			
	Home	Foreign	$\nu_d$	$\nu_f$	$\nu_d^*$	$\nu_f^*$	$sd(S_t)$	E(U)	E( $U^*$ )	Coop. (%)
0.50	PCP	PCP	1.00	-0.33	-0.33	1.00	3.73	-4.3913	-4.3913	0.0000
0.70	PCP	PCP	0.92	-0.25	-0.25	0.92	3.29	-4.0846	-4.0846	0.0137
0.85	PCP	PCP	0.87	-0.21	-0.21	0.87	3.02	-3.8982	-3.8982	0.0079
1.00	PCP	PCP	0.83	-0.17	-0.17	0.83	2.79	-5.9363	-5.9363	0.0000
1.00	LCP	LCP	0.33	0.33	0.33	0.33	0.00	-6.5260	-6.5260	0.0000
1.30	PCP	PCP	0.77	-0.10	-0.10	0.77	2.43	-3.4846	-3.4846	0.0718
1.30	LCP	LCP	0.33	0.33	0.33	0.33	0.00	-3.9045	-3.9045	0.0445
1.50	PCP	PCP	0.73	-0.07	-0.07	0.73	2.24	-3.3486	-3.3486	0.2172
1.50	LCP	LCP	0.33	0.33	0.33	0.33	0.00	-3.7944	-3.7944	0.1075
2.00	PCP	PCP	0.67	0.00	0.00	0.67	1.86	-3.0880	-3.0880	0.8051
2.00	LCP	LCP	0.33	0.33	0.33	0.33	0.00	-3.5832	-3.5832	0.3156
3.00	LCP	LCP	0.33	0.33	0.33	0.33	0.00	-3.3193	-3.3193	0.7631
4.00	LCP	LCP	0.33	0.33	0.33	0.33	0.00	-3.1610	-3.1610	1.1494



By comparing Tables XIII and XIV, we can see that for different Armington elasticity of substitution values and under strategic substitution (e.g  $\sigma = 2$  and  $\psi = 1$ ), there are positive gains from international monetary cooperation. However, when  $\alpha = 0.5$  we can see that gains from international monetary cooperation are equal to zero.

Furthermore, for many combinations of  $\alpha$  and  $\sigma$  such that  $\alpha = \frac{1}{\sigma}$  and all exporting firms, in both countries, invoice their exports in their own currency (PCP,PCP), it is possible to find numerically that gains from international monetary cooperation are equal to zero.<sup>28</sup>

In addition, in Table XV we can see that unlike Bhattarai (2009) the asymmetric Nash equilibrium (LCP,PCP) can be also supported under strategic substitution (i.e. when  $\sigma + \psi > 1$ ) and when the Armington elasticity of substitution takes values equal to 2 or 2.2.

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<sup>28</sup>The matlab's code to check this result can be provided by the author upon request.

Table XV.

Nash Equilibria, when  $\sigma = 1$ , and  $\psi = 0.5$ 

$\alpha$	Invoice Invoice		$\nu_d$	$\nu_f$	$\nu_d^*$	$\nu_f^*$	sd( $S_t$ )	E(U)	E( $U^*$ )
	Home	Foreign							
0.50	PCP	PCP	1.2207	-0.2207	-0.2207	1.2207	4.03	-3.2241	-3.2241
0.70	PCP	PCP	1.0844	-0.0844	-0.0844	1.0844	3.27	-2.9576	-2.9576
0.85	PCP	PCP	1.0319	-0.0319	-0.0319	1.0319	2.97	-2.7858	-2.7858
1.00	PCP	PCP	1.0000	0.0000	0.0000	1.0000	2.79	-2.6316	-2.6316
1.00	LCP	LCP	0.5000	0.5000	0.5000	0.5000	0.00	-2.9098	-2.9098
1.30	PCP	PCP	0.9668	0.0332	0.0332	0.9668	2.61	-2.3655	-2.3655
1.30	LCP	LCP	0.4509	0.5491	0.5491	0.4509	0.27	-2.6959	-2.6959
1.50	PCP	PCP	0.9555	0.0445	0.0445	0.9555	2.55	-2.2146	-2.2146
1.50	LCP	LCP	0.4229	0.5771	0.5771	0.4229	0.43	-2.5753	-2.5753
2.00	PCP	PCP	0.9386	0.0607	0.0613	0.9382	2.45	-1.9113	-1.9115
2.00	LCP	LCP	0.3650	0.6350	0.6350	0.3650	0.75	-2.3302	-2.3302
2.00	PCP	LCP	0.4044	0.5956	-0.2636	1.2636	1.87	-2.2687	-2.0645
2.20	LCP	LCP	0.3457	0.6543	0.6543	0.3457	0.86	-2.2496	-2.2496
2.20	PCP	LCP	0.3956	0.6044	-0.3032	1.3032	1.95	-2.1923	-1.9438
3.00	LCP	LCP	0.2840	0.7160	0.7160	0.2840	1.21	-1.9955	-1.9955
4.00	LCP	LCP	0.2300	0.7700	0.7700	0.2300	1.51	-1.7777	-1.7777

## E. Conclusion

In a model of endogenous invoicing and endogenous monetary policy, I have found, first, that gains from international monetary cooperation between two countries are different from zero when the elasticity of substitution between domestic and foreign goods consumption (Armington elasticity of substitution) is different from one.

Second, I have found also that when the Armington elasticity of substitution is less than one, this model predicts a unique Nash equilibrium, in which all exporting firms in both countries choose to price in their own currency and the central bank's optimal monetary policy implies a flexible exchange rate. However, this model predicts multiple Nash equilibria, when the Armington elasticity of substitution is larger and equal than one.

Third, I have also found that strategic complementarities, defined by Steinsson (2008), is not the only sufficient condition to support the existence of asymmetric Nash Equilibria such as (LCP,PCP) or (PCP,LCP). Furthermore, these asymmetric Nash equilibria can be supported under strategic substitution. (i.e. when  $\sigma + \psi < 1$ )

Fourth, under different sets of preferences and elasticities of substitution values, this model confirms theoretical results obtained previously in the international macroeconomics literature, (see Devereux and Engel (2003), Corsetti and Pesenti (2004) and Corsetti and Pesenti (2005)) such as that (PCP,PCP) Nash Equilibrium Pareto dominates the (LCP,LCP) Nash Equilibrium.

This model can be extended for different country sizes. In particular, this requires to deal with the risk sharing condition constant and to model a Nash Bargaining solution to determine the optimal international monetary policy under coordination as long as the different-country size assumption implies different units of welfare measure for each country.

Finally, it is also important to incorporate segmented markets into this model to determine endogenously which fraction of exporting firms set their prices either in domestic or in foreign currency. This idea is motivated from Fukuda (1994) who says that firms are indifferent to price their exports in domestic or foreign currency when they have access to futures market.

## CHAPTER IV

SPOT PRICE RETURNS FORECAST EVALUATION FOR ALUMINUM  
TRADED AT LONDON METAL EXCHANGE

## A. Introduction

Commodity price forecasters frequently use commodity futures contracts' prices as commodity spot prices' predictor. For instance, Bopp and Sitzler (1987), Gurcan (1998), Swanson and Zeng (1998) and Abosedra and Baghestani (2004) conclude that crude oil futures prices are unbiased and efficient predictors of crude oil spot price. However, important works such as Fama and French (1987) and Alquist and Kilian (2010) conclude the opposite. In the case of metals and agricultural products, Chinn and Coibion (2010) found that futures prices of those products are good predictors of their respective spot prices while Bernard et al. (2008) found that aluminum futures contracts' prices yield most of the times out-of-sample forecasting errors, for aluminum future spot prices, than those made by a No-drift Random Walk Model.<sup>29</sup> During the last 20 years, aluminum has become a very popular input for producing cars, airplanes, buildings, beverages cans, and diverse appliances, see Sander and Slatter (2008). This aluminum popularity has been reflected in its high consumption growth rate of 3.1% and 5% during the 90's and the last five years respectively, see Luo and Soria (2008). Because of this aluminum's leaning, its market, now, is more exposed to diverse uncertainty shocks and, therefore, to have highly volatile prices. In order to hedge against aluminum spot price volatility risk, speculators and aluminum consumers (in this case final good firms) arrange into forward or futures contracts.

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<sup>29</sup>Chinn and Coibion (2010) included metals such as gold, silver, aluminum, copper, lead, nickel and tin.

Aluminum futures contracts are mainly traded either at the London Exchange Metals (LME) or at the New York Stock Exchange (NYMEX). Unlike other commodities markets such as oil and energy, there are few authors that have been studied aluminum futures prices role as a predictor of the aluminum spot price. Among those authors, Heaney (2002) shows, by using autoregressive models, that model testing ability of futures prices to forecast subsequent spot prices with models based on Carrying Costs Theory have lesser forecast mean squared error than those models that are not based on Carrying Cost Theory. By using other approach, Bernard et al. (2008) used stochastic models such as Random Walks, ARCHs, GARCHs and Factor models to evaluate the aluminum futures contracts' prices efficiency as an aluminum spot price predictor. They found that mean-reverting models with stochastic convenience yields have a better out of sample forecast performance than those competing models.

One caveat about the above conclusions is that they are based on ordinal comparisons among one-step ahead forecast expected loss functions (e.g. Mean Squared Error or Mean Absolute Error) rather than statistical testing comparisons between forecasts loss functions. In fact, these conclusions can dramatically change if not only consider the forecasting loss function's expected value but also its variance. That is, one can have a model producing the lowest mean squared forecasting error but with very volatile squared forecasting errors. To avoid these ordinal comparisons between forecasting loss functions' expected value, Otto (2011), by using an alternative approach, found evidences to not reject the null hypothesis of speculative efficiency that LME metals futures contract' prices does have to predict their respective spot price.<sup>30</sup>

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<sup>30</sup>Speculative efficiency hypothesis means that under the condition of risk neutrality and zero transaction costs, the future spot price at  $t+h$  is equal to the future contract price at  $t$ , whose maturity is at  $t+h$ , plus a zero mean i.i.d random variable.

In that paper Otto (2011) instead of using out of sample errors forecast evaluations to test futures prices performance in predicting spot prices, she used ARMA's family models to test the influence of the present and past forecast errors, for 3 and 15 months futures contracts, on the future forecast error for six base metals traded at LME.<sup>31</sup>

In this paper, we propose six econometric models to predict the aluminum spot price monthly return and test statistically their prediction performance at 3, 15 and 27 steps-ahead-forecasts. The first model is a No-drift Random Walk, which has been broadly used in the forecasting evaluation literature, see Alquist and Kilian (2010), Swanson and Zeng (1998), Fama and French (1987) and so on. The second model is based on the Speculative Efficiency Hypothesis of futures contracts' prices to predict spot prices, provided by Bilson (1981). This model implies a single equation that relates the spot price at period  $t+h$  with the futures contract price at period  $t$ , which matures at period  $t+h$ . That is, if we have a 3-months future contract, the Speculative Efficiency Hypothesis model is represented by the spot price as a function of the 3 month futures price three months ago as long as we are using monthly data. The third model, based on the Financial Cost of Carry Theory, is a VAR in differences which has three endogenous variables: spot price, futures price and risk free rate. Since we found that aluminum futures prices, spot prices and treasury bills interest rates are cointegrated (see this paper appendix for more details), the fourth model is a VECM, which has these three latest variables.

The fifth model, based on the Storage Cost of Carry Theory, assumes that rather than having a commodity for hedging risk or speculation, there is an unobservable benefit of having the commodity, which can be represented as a function of the in-

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<sup>31</sup>These six base metals are zinc, aluminum, cooper, tin, nickel and lead.

inventories and the spot price volatility. For more details see Pindyck (2001).<sup>32</sup> Here, we represent this storage cost model through a VAR in differences, which has five endogenous variables: spot price, futures price, risk-free asset interest rate, inventories and spot price volatility. Since we found that aluminum futures prices, spot prices, inventories, spot price volatility and treasury bills interest rates are cointegrated (see this paper appendix for more details), the sixth model is a VECM, which has these five latest variables.

In contrast to most of the commodity forecasting evaluation literature, our predictive ability (or forecasting performance) evaluations are not mainly based on ordinal comparisons between two models' expected loss functions. Instead, our forecasting performance evaluations are based on both unconditional and conditional prediction ability tests proposed by Giacomini and White (2006). The use of these tests have a great advantage over traditional ability prediction tests such as Diebold and Mariano (1995) tests because they allow to test predictive ability not only for non-nested models but also for nested models. As an example of nested models' forecasting performance evaluation, in this paper, is the comparison in conditional and unconditional forecasting performance between the Financial Cost of Carry vs. the Storage Cost models.

By using Giacomini and White (2006) predictive ability tests, we find that, for both 3 and 15 months ahead forecast, Storage Cost models does not improve the aluminum spot price monthly return's forecast with respect to the Financial Cost of

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<sup>32</sup>This theory assumes that there is a benefit of having physical ownership of the commodity [see Brennan (1958) and Working (1949) for more details]. This benefit is called the convenience yield. Also, this theory assumes that there is a variable storage cost associated with storing the commodity until the expiration of the futures contract. As a result, the net storage cost is, defined as the difference between the total storage cost and the convenience yield, is a function of inventories and spot price volatility.



Carry Models. However, these two models have a better spot price monthly return forecast performance than the Speculative Efficiency Hypothesis Model, which in turn has better conditional and unconditional forecast performance than the Random Walk model. Unlike the predictive ability test for 3 and 15 months ahead forecasts, we find that, for 27 months ahead forecasts, neither the Financial Cost of Carry nor Storage Cost models outperform the Speculative Efficiency model, however the Speculative Efficiency model has better conditional and unconditional forecasting performance than the Random Walk model.

Given the versatility of Giacomini and White (2006) conditional and unconditional tests to compare not only predictive ability between models but also forecasting methodologies, as an additional contribution to the forecasting evaluation literature, we find that both conditional and unconditional forecasting performance by iterating autoregressive models (this forecasting methodology is called iterated multistep forecast) is better than using horizon-specific estimated models (this forecasting methodology is called direct multistep forecast). Unlike Stock et al. (2006), we make a univariate forecast evaluation for the aluminum spot price and we use the asymptotic predictive ability tests provided by Giacomini and White (2006) that converge in distribution to conventional probability functions rather than using complicated bootstraps procedures as those provided by Stock et al. (2006).

This paper is organized as follows. Next, I present a more detailed description of the models to estimate. Then, I describe the data and forecasting evaluation methodology, and report the results. Next, I present the performance evaluation between iterated and direct multistep forecasts. Finally, I provide some conclusions.

## B. Forecasting Models

In this section, we describe in more detail the six models, mentioned above, to forecast the aluminum spot price. Some of them are estimated in this paper

### 1. Random Walk Model

As we mentioned in the introduction, the Random Walk model is the benchmark model in most of the forecasting evaluation literature. The Random walk model is an implication of assuming that the efficient markets hypothesis does hold. The efficient market hypothesis asserts that if markets are efficient, then there does not exist any arbitrage opportunities. Therefore, a particular stock's future return, in an efficient market, will be equal to zero. See Fama (1970) and Samuelson (1965).

Therefore, under the efficient markets hypothesis, we assume that aluminum spot prices are represented by the following expression:

$$S_{t+h} = S_t + e_{t+h} \quad (4.1)$$

where  $S_{t+h}$  denotes the aluminum spot price  $h$  periods ahead,  $e_{t+h}$  denotes a zero mean i.i.d random variable.

### 2. Speculative Efficiency Model

This model is also broadly used in the commodity forecasting evaluation literature because many authors have found, for many commodities, that a particular commodity future price is an unbiased predictor of its respective future spot price, see Bilson (1981), Alquist and Kilian (2010), Otto (2011) and so on. Therefore, this model can be represented by the following equation:

$$S_{t+h} = F_{t+h,t} + e_{t+h} \quad (4.2)$$

where  $F_{t+h,t}$  denotes the aluminum future price with maturity in  $t + h$  periods and quoted at period  $t$ ,  $e_{t+h}$  denotes a zero mean iid random variable.

### 3. Financial Cost of Carry Model

The future-spot price parity states that any investment strategy in a particular asset or commodity should give to the investor the same return. For example, consider the following strategy: suppose that buying a particular commodity costs  $S_t$  at period  $t$ , hence the investor invests an amount of money equal to  $S_t$ , and invest it in a safe asset that provide him a risk free interest rate equal to  $r_t$ . At the same time he opens a long position in this commodity's future contract within which he promises to buy the commodity at a price equal to  $F_{t+h,t}$  with the gross return of investing in the safe asset. Next, at period  $t + h$  he expects to receive  $S_{t+h}$  as a revenue for selling this commodity at period  $t + h$ . Therefore, this investor expects to obtain the following holding period return,  $R_{t+h,t}^1$ .

$$R_{t+h,t}^1 = \frac{S_{t+h} - F_{t+h,t} + (1 + r_t)S_t - S_t}{S_t} \quad (4.3)$$

Now, consider another investment strategy, suppose that the same investor buys, at period  $t$ , the same commodity at  $S_t$ , then he expects to sell this commodity at price  $S_{t+h}$  at period  $t + h$ . In addition, he has to face a constant net storage cost  $c$  for holding this commodity for  $h$  periods.<sup>33</sup> Therefore, he expects the following holding

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<sup>33</sup>If  $c$  is negative, it can be interpreted as net constant convenience yield otherwise it can be interpreted as net storage cost.

period return,  $R_{t+h,t}^2$ .

$$R_{t+h,t}^2 = \frac{S_{t+h} - S_t}{S_t} - c \quad (4.4)$$

by subtracting (4.4) from (4.3), we have:

$$R_{t+h,t}^2 - R_{t+h,t}^1 = \frac{F_{t+h,t} - S_t}{S_t} - c - r_t \quad (4.5)$$

If the future-spot price parity holds,  $R_{t+h,t}^2 - R_{t+h,t}^1$  will be equal to zero. Then, through substituting (4.5) left-hand side by zero, we have the well-known future-spot price parity theorem or cost of carry relationship:

$$S_t = \frac{F_{t+h,t}}{1 - c - r_t} \quad (4.6)$$

Since we have assumed, in this section a constant storage cost but variable risk free interest rate along time, as Damodaran (2011) did, we rename 4.6 as the financial cost of carry relationship.

In applied financial econometrics literature, many authors, see Brenner and Kroner (1995), Otto (2011), Fama and French (1987) and so on, parameterize theoretical results as the described above to test them statistically. Unlike these authors, instead of testing whether (4.6) does hold or not, we want to evaluate whether aluminum futures prices and risk-free interest rate have a better forecasting performance for the aluminum spot price than the Random Walk and the Speculative Efficiency models.

Since  $S_t$ ,  $F_{t+h,t}$  and  $r_t$  are non stationary stochastic processes, the parameterized version of (4.6) in differences can be represented by the following expression:

$$\Delta s_t = \beta_0 + \beta_1 \Delta f_{t+h,t} + \beta_2 \Delta r_t + e_t \quad (4.7)$$

where  $s_t$ ,  $f_{t+h,t}$  denote the natural logarithms of  $S_t$  and  $F_{t+h,t}$  respectively and  $e_{t+h}$  denotes a zero mean i.i.d random process.

As long as we are going to forecast  $h > 1$  periods ahead, we need to know also the processes that govern,  $f_{t+h,t}$  and  $r_t$ . Thus, a parsimonious way to represent these three stochastic processes dynamics is through the following reduced-form VAR model:

$$\Delta X_t = \gamma_0 + \sum_{k=1}^K \gamma_k \Delta X_{t-k} + V_t \quad (4.8)$$

where  $X_t = \{s_t, f_{t+h,t}, r_t\}'$ ,  $\gamma_0$ ,  $\gamma_k$  are 3x1 vectors of parameters, and  $V_t$  is a 3x1 errors vector.

Inasmuch as the above VAR model only captures the short-run relationship among these three variables, we made a cointegration test among aluminum 3-months futures price, spot price and Treasury-bills-90-days interest rate for three different periods (1/1989 to 2/1999, 1/1995 to 2/2005 and 6/2000 to 7/2010 ).<sup>34</sup>

From this cointegration test, we found that there is at least one cointegration vector among these three variables for each one of these periods. See Appendix for more details.<sup>35</sup>

As long as we found a long run relationship among these three variables, at least for the 3-months futures prices and 15-months futures prices, we propose the following VECM as a candidate model to forecast the aluminum spot price.

That is:

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<sup>34</sup>We made the same cointegration tests among 15-months futures price, spot price and 1 year treasury bonds interest rate, and 27-months futures price, spot price and 2 year treasury bonds interest rate.

<sup>35</sup>Unlike the cointegration tests for 3 and 15-months futures contracts' prices, for the 27-months futures, there is not any cointegration vector among 27-months futures price, spot price and 2 year treasury bonds interest rate.

$$\Delta X_t = \gamma_0 + \sum_{k=1}^K \gamma_k \Delta X_{t-k} + \eta X_{t-1} + V_t \quad (4.9)$$

#### 4. Storage Cost Theory Model

Unlike the latest model, we assume in this subsection that  $c$  is changing over time (let's from now to change its notation by  $c_t$ ). Since this variable is unobservable, authors such as Brennan (1958), Working (1949) and Pindyck (2001) assert that  $c_t$  is a function of inventories and spot price volatility. Besides, Heaney (2002) have found that including aluminum inventories and spot price volatility in a VECM model alike (4.9), improves the aluminum future spot price predictions by producing lower means squared errors than models that do not have included these two additional variables.

Then, by using the same logic as in the previous subsection, we represent the relationship among aluminum futures contract's price, spot price, inventories, spot price volatility and risk free interest rate through the following VAR model:

$$\Delta X_t = \gamma_0 + \sum_{k=1}^K \gamma_k \Delta X_{t-k} + V_t \quad (4.10)$$

where  $X_t = \left\{ s_t, f_{t+h,t}, r_t, z_t, \int_0^t \sigma_s ds \right\}'$ ,  $z_t$  is the logarithm of aluminum inventories,  $\sigma_t$  is the spot price conditional volatility,  $\gamma_0$  and  $\gamma_k$  are 5x1 vectors of parameters.

Inasmuch as the above VAR model only captures the short-run relationship among these five variables, we made a cointegration test among aluminum 3-months futures contracts' price, spot price, inventories, spot price volatility and treasury bills 90 days interest rate for three different periods (1/1989 to 2/1999, 1/1995 to 2/2005 and 6/2000 to 7/2010 ).<sup>36</sup>

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<sup>36</sup>We made the same cointegration tests among aluminum 15-months futures price,

From this cointegration test, we found that there is at least one cointegration vector among these five variables for each one of these periods. See Appendix for more details.

As long as we found a long run relationship among these five variables, at least for the 3-months, 15-months and 27-months futures prices, we propose the following VECM as a candidate model to forecast the aluminum spot price.

$$\Delta X_t = \gamma_0 + \sum_{k=1}^K \gamma_k \Delta X_{t-k} + \eta X_{t-1} + V_t \quad (4.11)$$

where  $X_t = \{s_t, f_{t+h,t}, r_t, z_t, \sigma_t\}'$ ,  $z_t$  is the logarithm of aluminum inventories,  $\sigma_t$  is the spot price conditional volatility,  $\gamma_0$  and  $\gamma_k$  are 5x1 vectors of parameters.

### C. Predictive Ability Tests between Models

#### 1. Data

Spot price, 3, 15 and 27 months official futures contracts' prices and inventories for aluminum, used for the above models estimation and prediction, were obtained from the London Metal Exchange (LME). The sample period, used in this paper, goes from January of 1989 to October 2010. Since 15 months and 27 months futures contracts' delivery dates are not daily (in fact, their delivery dates are on each month's third Wednesday), we decided to use monthly frequency data. Every monthly observation represents an average of the daily close prices and inventories traded at the LME. <sup>37</sup>

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spot price, inventories, spot price volatility and 1 year treasury bonds interest rate, and among aluminum 27-months futures price, spot price, inventories, spot price volatility and 2 year treasury bonds interest rate.

<sup>37</sup>If an investor opened a long position in aluminum 3-months futures contract on 3/31/2011, the LME associated warehouse has to deliver the aluminum's quantity, specified in the contract, by 6/30/2011 or this investor has to close his position on

For the risk free interest rates, we use the 90-days USA Treasury bills, the 1-year USA Treasury bonds and the 2-years USA Treasury bonds nominal interest rates from the period January of 1989 to October 2010, this interest rate data was taken from Federal Reserve Board (FRB) webpage. Alike the Aluminum futures prices, we use monthly frequency for the risk free interest rates data instead of daily or weekly data.<sup>38</sup> Finally, the aluminum spot price volatility is calculated as the monthly average of the daily standard deviation of the aluminum official spot prices return of the last 20 days, ending at day  $t$ .<sup>39</sup>

## 2. Methodology

Since we are going to use the Giacomini and White (2006) conditional and unconditional predictive ability tests to evaluate the above models forecasting performance, we need to use the "rolling window" estimation method to avoid that estimation uncertainty vanishes asymptotically, see Giacomini and White (2006) for more details about the importance of using this estimation method.<sup>40</sup>

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6/28/2011. However, if the same investor opened a long position in aluminum 15-months futures contract on 3/31/2011, the LME associated warehouse has to deliver on 6/20/2011, which is the third Wednesday of June 2012, or this investor has to close his position on 6/28/2012. In the same way, if this investor opened a long position in aluminum 27-months futures contract on 3/31/2011, the LME associated warehouse has to deliver on 6/20/2013, which is the third Wednesday of June 2013, or this investor has to close his position on 6/28/2013.

<sup>38</sup>Treasury bills and bonds interest rates monthly data are calculated by the FRB as the simple average of their respective weekly rates.

<sup>39</sup>According to LME, the official prices are, verified by the Quotations Committee, based on the last bids and offers at the close of the second trading session for each metal.

<sup>40</sup>The following steps, describe the rolling window estimation method. First, to estimate a model and to forecast the variable of interest until the desired horizon by using the first  $m$  observations as information set. Second, to estimate and to forecast again the same model with the same number of observations  $m$  but dropping the first observation and adding the  $(m+1)$ th observation. Then repeat the second step but dropping second observation and adding the  $(m+2)$ th observation to the second step



We set the rolling window length to 120 months, so we have an out-of-sample forecast evaluation period of 139 months, 127 months, and 113 months if our desired forecast horizon is 3, 15 and 27 months ahead respectively. For example, if we want to evaluate the forecasting performance for a particular model three months ahead, we need first to estimate it, from 1989:1 until 1999:1, and then forecast the aluminum spot price three months ahead. This implies that our out-of-forecast sample will start on 1999:4. As long as we are using the rolling window estimation method, we estimate the same model from 1989:2 until 1999:2 and, then forecast it three months ahead. This implies that our second out-of-sample forecast observation will be on 2000:1. Then, we continue with the same procedure until obtaining our last out-of-sample forecast observation, which is on 2010:10.

The use of the rolling window estimation method raises the following questions, first how many lags should the VAR and the VECM include as long as the information criteria that we use to set the number of lags might change as we change the window estimation sample set? Second, how many cointegration vectors should we include in VECM as long as the number of cointegrating vectors might change when we change the window estimation sample set?

In this paper, we solve the first question by programming a simple algorithm that chooses the optimal number of lags for each VAR and VECM at each different  $m$  size rolling window through using any of the standard lag information criteria; in this paper, we use the Schwartz information criterion. However, dealing with the second question can be computationally burdensome because in our forecast evaluation we need to use at least 112  $m$  size rolling windows.

Once we have our respective out-of-sample forecast, we compute their forecasting

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sample and so on so forth.

error and hence its respective loss functions. Here, we use the usual squared error loss function,

$$L_{t+h}(Y_{t+h}, M_{t+h,t}^j) = (Y_{t+h} - M_{t+h,t}^j)^2 \text{ for every } h > 0 \quad (4.12)$$

where  $Y_{t+h}$  denotes the observed variable of interest at period  $t+h$  and  $M_{t+h,t}^j$  denotes the variable of interest  $h$ -periods-ahead forecast, which is produced by the estimated  $j$ -th model with information observed until period  $t$ .

Inasmuch as we want to test statistically which model has a better forecasting performance, Giacomini and White (2006) propose the following hypothesis testing:

For a given loss function  $L$  and  $\sigma$ -field,  $\mathcal{G}_t$ , the null hypothesis of equal conditional predictive ability of  $M_{t+h,t}^j$  and  $M_{t+h,t}^l$  for a desired horizon  $t+h$  is represented by,

$$H_0 : E [L_{t+h}(Y_{t+h}, M_{t+h,t}^j) - L_{t+h}(Y_{t+h}, M_{t+h,t}^l) | \mathcal{G}_t, ] = 0 \text{ a.s } t=1,2, \dots \quad (4.13)$$

Depending on the  $\sigma$ -field's elements, Giacomini and White (2006) proposes the following two kinds of predictive ability tests: the conditional and the unconditional predictive ability test.

#### a. Conditional Predictive Ability Test

In the conditional predictive test, it is assumed that the  $\sigma$ -field  $\mathcal{G}_t = \mathcal{F}_t$ , where,  $\mathcal{F}_t = \sigma(W'_1, \dots, W'_t, X'_{t+1})'$ ,  $W_t \equiv (Y_t, X'_t)$ ,  $Y_t$  is the variable of interest and  $X_t$  is a vector of predictor variables. To carry out this test, Giacomini and White (2006) show that testing (4.13) is equivalent to test the following equation when  $\mathcal{G}_t = \mathcal{F}_t$ .

$$H_{0,d} : E [d_t \Delta L_{m,t+h}] = 0 \text{ a.s for all } t \geq 1 \quad (4.14)$$

where  $\Delta L_{m,t+h} = L_{t+h}(Y_{t+h}, M_{t+h,t}^j) - L_{t+h}(Y_{t+h}, M_{t+h,t}^l)$ ,  $d_t$  is a  $q \times 1$   $\mathcal{F}_t$ -measurable vector or also known as test function.

Then, to test (4.14), Giacomini and White (2006) proposed the following statistic tests:

for  $h = 1$

$$T_{m,n}^d = n \bar{Z}'_{m,t} \hat{\Omega}_n^{-1} \bar{Z}_{m,t} \xrightarrow{d} \chi_q^2 \quad (4.15)$$

where,

$$\bar{Z}_{m,t+1} = \left( n^{-1} \sum_{t=m}^{T-1} Z_{m,t+1} \right) \quad (4.16)$$

$$Z_{m,t+1} = d_t \Delta L_{m,t+1} \quad (4.17)$$

$$\hat{\Omega}_n \equiv n^{-1} \sum_{t=m}^{T-1} Z_{m,t+1} Z'_{m,t+1} \quad (4.18)$$

for  $h > 1$

$$T_{m,n,h}^d = n \bar{Z}'_{m,t,h} \tilde{\Omega}_n^{-1} \bar{Z}_{m,t,h} \xrightarrow{d} \chi_q^2 \quad (4.19)$$

where,

$$\bar{Z}_{m,t+h} = \left( n^{-1} \sum_{t=m}^{T-h} Z_{m,t+h} \right) \quad (4.20)$$

$$Z_{m,t+h} = d_t \Delta L_{m,t+h} \quad (4.21)$$

$$\tilde{\Omega}_n \equiv n^{-1} \sum_{t=m}^{T-h} Z_{m,t+h} Z'_{m,t+h} + n^{-1} \sum_{s=1}^{h-1} w_{n,s} \sum_{t=m}^{T-h} \left[ Z_{m,t+h} Z'_{m,t+h-s} + Z_{m,t+h-s} Z'_{m,t+h} \right] \quad (4.22)$$

$\tilde{\Omega}_n$  is, in simple words, the Newey and West (1987) estimator of  $\Omega_n$  and  $w_{n,s}$  is a weight function, which are defined in detail by Giacomini and White (2006).

If we fail to reject the null hypothesis, that is when  $T_{m,n,h}^d \leq \chi_q^2(\alpha)$ , Giacomini and White (2006) advise to use the most parsimonious model.<sup>41</sup>

However, if we reject  $H_0$ , that is when  $T_{m,n,h}^d > \chi_q^2(\alpha)$ , we cannot use the mean squared error to rank the models conditional predictive ability because this test is conditional to the information observed until period  $t$ . In that case, they suggested carrying out the following decision rule for forecasting performance ranking:

1. Regress  $\Delta L_{m,t+h}$  on  $d_t$  over the out-of-sample period  $t = m+1, \dots, T-h$  and let  $\hat{\delta}'_n$  denotes the regression coefficient.

2. Compute the following indicator function:  $I_{n,c} = n^{-1} \sum_{t=m}^{T-h} 1 \left\{ \hat{\delta}'_n > c \right\}$ , where  $1 \left\{ \hat{\delta}'_n > c \right\}$  equals 1 if  $\hat{\delta}'_n > c$  and 0 otherwise. Where  $c$  is user-specified threshold, Giacomini and White (2006) set  $c$  equal to zero.

3. If  $1 \left\{ \hat{\delta}'_n > c \right\} = 1$ , model  $M^l$ , has a better forecasting performance than  $M^j$  at period  $t+h$ .

4. Since  $I_{m,n}$  denotes the proportion of times that model  $M^l$  performs better than  $M^j$ , then if  $I_{m,n} > 0.5$ , we choose model  $M^l$ .

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<sup>41</sup> $\alpha$  denotes the test's significance level.

### b. Unconditional Predictive Ability Test

Unlike the conditional predictive test, in the unconditional predictive ability test is assumed that the  $\sigma$ -field  $\mathcal{G}_t = \{\phi, \Omega\}$ . For  $h \geq 1$ , Giacomini and White (2006) propose the following test statistic, which coincide with that proposed by Diebold and Mariano (1995) test except that the  $\Delta L_{m,n}$  assumed by Giacomini and White (2006) is different and more general than that assumed by Diebold and Mariano (1995), because the former avoid that models estimation uncertainty vanish asymptotically.

$$t_{m,n,h} = \frac{\Delta \bar{L}_{m,n}}{\hat{\sigma}_n / \sqrt{n}} \xrightarrow{d} N(0, 1) \quad (4.23)$$

where  $\hat{\sigma}_n^2$  is a HAC estimator of the asymptotic variance  $\sigma_n^2 = \text{var} [\sqrt{n} \Delta \bar{L}_{m,n}]$ .

If we fail to reject the null hypothesis, that is when  $T_{m,n,h}^d \leq z_{(\alpha)/2}$ , Giacomini and White (2006) proposes to use the most parsimonious model. However, if we reject  $H_0$ , that is when  $T_{m,n,h}^d > Z_{(\alpha)/2}$ , we choose the model that has the lowest mean squared error as long as this is a unconditional test.

## 3. Results

Tables XVI, XVII and XVIII summarize the conditional and unconditional 3, 15 and 27 months-ahead predictive ability tests between the models described in section 2. The first column reports the pairs of models to compare. Where (1) denotes the Random Walk model, (2) represents the Speculative Efficiency Hypothesis model, (3) denotes the VAR version of the Financial Cost of Carry model, (4) represents the VECM version of the Financial Cost of Carry Model, (5) denotes the VAR version of the Storage Cost model and (6) represents the VECM version of the Storage Cost model. The second column reports the conditional predictive ability test values for

each pair of models' comparison whereas the third column presents their respective p-values.<sup>42</sup>

The fourth column reports the indicator function  $I_{n,c}$ , which notes the proportion of times that the rightmost model, reported in the first column, outperforms the leftmost model, reported also in the first column. The fifth column reports either a number or a character,  $N$ . The former indicates which one of the two comparing models has the best aluminum spot price's conditional forecasting performance whereas the later indicates that both models have the same aluminum spot price's conditional forecasting performance.

The fifth column's value is determined, first, by contrasting if the p-value, reported in the third column, is either lower or higher than the 10% significance level. If this p-value is lower than 10%, we fail to reject (4.14) and, therefore, we conclude that both models have the same conditional forecasting performance. Nevertheless, If this p-value is higher than 10%, we reject (4.14), and, therefore, we conclude that both models have a different conditional forecasting performance. In order to determine which one of those two comparing models has the best conditional future spot price's forecasting performance, we use the model selection rule described in the fourth step of the last subsection.

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<sup>42</sup>The test values, reported in the second column, are obtained by evaluating  $\Delta L_{m,t+h}$  and  $d_t$ , obtained by comparing two particular models' out-of-sample spot price forecast, in equation (4.19). The p-values, reported in the third column, are obtained by evaluating the test values, reported in the second column, in a standard  $\chi$ -square distribution function.

From this model selection rule's value, which is reported in the fourth column, we conclude that the leftmost model, reported in the first column, have a better conditional future spot price's forecasting performance if  $I_{n,c}$  is less than 0.5.

The sixth column reports the unconditional predictive ability test values for each pair of models comparison whereas the seventh column presents their respective p-values.<sup>43</sup>

The eighth column reports either a number or a character,  $N$ . The former indicates which one of the two comparing models has a better spot price forecasting performance for any period whereas the later indicates that both models have the same unconditional forecasting performance.<sup>44</sup>

All the above model estimations, forecasts and predictive ability tests are programmed in Eviews 5.0 and their codes are available upon request. These predictive ability tests provided in my code produce the same results as those provided by Giacomini's webpage.

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<sup>43</sup>The test values, reported in the sixth column, result from evaluating  $\Delta L_{m,t+h}$ , previously obtained by comparing two particular models out-of-sample spot price forecast, in equation (4.23). The p-values, reported in the seventh column, results from evaluating the test values, reported in the sixth column, in a t-student distribution function.

<sup>44</sup>The seventh column's value is determined, first, by contrasting if the p-value, reported in the seventh column, is lower or higher than the 10% significance level. If this p-value is lower than 10%, we fail to reject (4.14) and, therefore, we conclude that both models have the same unconditional forecasting performance. Nevertheless, If this p-value is higher than 10%, we reject (4.14), and, therefore, we conclude that both models have a different unconditional forecasting performance. In order to determine which one of those two comparing models have a better unconditional future spot price forecasting performance, we choose the model having the lowest mean squared error.

Table XVI.

Forecasting Performance Evaluation Test: Three-months-ahead

Models	Conditional				Unconditional		
	Test-Value	P-value	$I_{n,c}$	Winner	Test-Value	P-value	Winner
(2) vs (1)	15.4859	0.0004	0.0074	(2)	-3.8227	0.0002	(2)
(3) vs (1)	12.4138	0.0020	0.1471	(3)	-3.4618	0.0007	(3)
(4) vs (1)	13.0279	0.0015	0.1397	(4)	-3.3556	0.0010	(4)
(5) vs (1)	12.5639	0.0019	0.1397	(5)	-3.4491	0.0007	(5)
(6) vs (1)	12.1586	0.0023	0.1397	(6)	-3.603	0.0004	(6)
(3) vs (2)	10.7410	0.0047	0.1618	(3)	-3.1410	0.0021	(3)
(4) vs (2)	11.3094	0.0035	0.1544	(4)	-3.0424	0.0028	(4)
(5) vs (2)	10.9097	0.0043	0.1618	(5)	-3.1266	0.0022	(5)
(6) vs (2)	10.4881	0.0053	0.1618	(6)	-3.2868	0.0013	(6)
(3) vs (5)	5.0912	0.0784	0.0221	(3)	-1.3908	0.1665	N
(4) vs (6)	5.8351	0.0541	0.8971	(6)	0.1680	0.8668	N
(3) vs (6)	4.0006	0.1353	NA	N	0.1838	0.8544	N

From Table XVI we conclude, first, that the Speculative Efficiency Hypothesis model (i.e. model 2) has a better both conditional and unconditional three-months-ahead forecasting performance than the Random Walk model (i.e. model 1). Therefore, the aluminum 3-months futures contracts' prices are useful to predict the aluminum spot price monthly return three months ahead.



Second, we can see that both the Financial Cost of Carry and the Storage Cost Theory models have the best both conditional and unconditional three-months-ahead forecasting performance than both the Random Walk model and the Speculative Efficiency Hypothesis model. Therefore, we bring to a close that even though the 3-months futures contracts' prices are helpful to predict the aluminum spot price monthly return three months ahead, variables such as the 90 days-Treasury-bills nominal interest rate, aluminum inventories and aluminum spot price volatility complement the aluminum 3-months futures contracts' prices in forecasting its spot price three months ahead.

Third, we can see that if we use the VAR approach for both the Financial Cost of Carry model and the Storage Cost model, we find that the former has a better forecasting performance than the later. In contrast, if we use the VECM approach for these two models, the result is the opposite.

Fourth, since from the VAR approach for the Financial Cost of Carry model has the best aluminum spot price's forecasting performance and since from the VECM approach for the Storage Cost model has the best aluminum spot price monthly return's forecasting performance model, we test the conditional and the unconditional predictive ability between these two models. From this test, we conclude that these two models have the same aluminum spot price monthly return forecasting performance. Therefore, as Giacomini and White (2006) suggest, we choose the VAR approach for the Financial Cost of Carry model has the best three-months-ahead forecasting performance for the aluminum spot price monthly return because the VAR approach is more parsimonious than the VECM approach.

Table XVII.

Forecasting Performance Evaluation Test: Fifteen-months-ahead

Models	Conditional				Unconditional		
	Test-Value	P-value	$I_{n,c}$	Winner	Test-Value	P-value	Winner
(2) vs (1)	42.7457	0.0000	0.0446	(2)	-2.4459	0.0158	(2)
(3) vs (1)	5.8728	0.0531	0.1607	(3)	-2.2813	0.0242	(3)
(4) vs (1)	5.5226	0.0632	0.1607	(4)	-2.2557	0.0258	(4)
(5) vs (1)	5.8744	0.0530	0.1607	(5)	-2.2835	0.0241	(5)
(6) vs (1)	6.0900	0.0476	0.1607	(6)	-2.2993	0.0231	(6)
(3) vs (2)	4.3079	0.1160	NA	N	-1.8748	0.0631	(3)
(4) vs (2)	3.9631	0.1379	NA	N	-1.8506	0.0666	(4)
(5) vs (2)	4.3076	0.1160	NA	N	-1.8762	0.0629	(5)
(6) vs (2)	4.5207	0.1043	NA	N	-1.8905	0.0610	(6)
(3) vs (5)	2.1463	0.3419	NA	N	-1.2368	0.2185	N
(4) vs (6)	3.0691	0.2156	NA	N	0.8144	0.4170	N

From Table XVII we conclude, first, that the Speculative Efficiency Hypothesis model has a better both conditional and unconditional fifteen-months-ahead forecasting performance for the aluminum spot price monthly return than the Random Walk model. Therefore, the aluminum 15-months futures contracts' prices are useful to predict the aluminum spot price monthly return fifteen months ahead.

Second, we can see that both the Financial Cost of Carry and the Storage Cost

Theory models have a better conditional and unconditional fifteen-months-ahead forecasting performance for the aluminum spot price than the Random Walk model.

In contrast, they have the same fifteen-months-ahead conditional forecasting performance as the Speculative Efficiency Hypothesis model. However, the Financial Cost of Carry and the Storage Cost Theory models have a better fifteen-months-ahead unconditional forecasting performance than the Speculative Efficiency Hypothesis model. From this result, it is difficult to provide a conclusion because it is expected that if both models had the same conditional forecasting performance for all periods, they will have the same unconditional forecasting performance. [ i.e equation (4.14) with  $\mathcal{G}_t = \mathcal{F}_t$  implies equation (4.14) with  $\mathcal{G}_t = \{\Omega_t, \phi_t\}$ ] However, these outcomes are not reflecting this implication. Thus, according to Giacomini and White (2006) this outcome happens because either the conditional forecasting performance test has a lower power or the unconditional test is oversized.

Therefore, we bring to a close that variables such as the 1-year-Treasury-bonds nominal interest rate, aluminum inventories and aluminum spot price volatility may or not complement the aluminum 15-months futures contracts' price to forecast the aluminum spot price monthly return fifteen months ahead.

Third, if we use either the VAR or the VECM approach for both the Financial Cost of Carry model and the Storage Cost model, we find that both models have the same both conditional and unconditional forecasting performance for the aluminum spot price monthly return.

From these three above results, we can conclude that if the conditional predictive ability test has a lower power, the 15-months aluminum futures contracts' prices will be the best predictor for the aluminum spot prices fifteen months ahead. However, if the unconditional predictive ability test is oversized either the Financial Cost of Carry model or the storage Cost model will be the best predictor for the aluminum

spot price fifteen months ahead.

Table XVIII.

Forecasting Performance Evaluation: Twenty-Seven-months-ahead

Models	Conditional				Unconditional		
	Test-Value	P-value	$I_{n,c}$	Winner	Test-Value	P-value	Winner
(2) vs (1)	7.9726	0.0186	0.1477	(2)	-3.3493	0.0011	(2)
(3) vs (1)	4.9968	0.0822	0.2386	(3)	-2.8492	0.0052	(3)
(4) vs (1)	3.4630	0.1770	NA	N	-2.1018	0.0378	(4)
(5) vs (1)	4.9966	0.0822	0.2386	(5)	-2.848	0.0052	(5)
(6) vs (1)	5.0066	0.0818	0.2386	(6)	-2.8488	0.0052	(6)
(3) vs (2)	4.3226	0.1152	NA	N	-1.0723	0.2859	N
(4) vs (2)	3.8955	0.1426	NA	N	-0.1657	0.8687	N
(5) vs (2)	4.3201	0.1153	NA	N	-1.0701	0.2868	N
(6) vs (2)	4.3125	0.1158	NA	N	-1.0677	0.2879	N
(3) vs (5)	5.4511	0.0655	0.1477	(3)	-1.0588	0.2919	N
(4) vs (6)	2.1231	0.3459	NA	N	1.0259	0.3071	N

From Table XVIII we conclude, first, that the Speculative Efficiency Hypothesis model has a better both conditional and unconditional twenty-seven-months-ahead forecast performance than the Random Walk model.

Therefore, the aluminum 27-months futures contracts' prices are useful to predict the aluminum spot price monthly return twenty-seven months ahead.

Second, unlike the three and fifteen months-ahead predictive ability tests, we can see that both the Financial Cost of Carry and the Storage Cost Theory models have the same conditional twenty-seven-months-ahead forecasting performance as the Speculative Efficiency model.

Therefore, we bring to a close that variables such as the 2-years-Treasury-bonds nominal interest rate, aluminum inventories and aluminum spot price volatility do not complement the aluminum 27-months futures price in forecasting the spot price monthly return twenty-seven months ahead.

Fourth, the VAR approach for the Financial Cost of Carry model has a better conditional forecasting performance than its respective VECM approach. However, the VAR approach for the Storage Cost Theory model has the same unconditional forecasting performance as its respective VECM.

#### D. Predictive Ability Tests between Iterated and Direct Multistep Forecasting Methods

As we reported in the last section, all of our forecast performance evaluations are based on multiperiod forecasts and our estimated models are autoregressive. In the forecasting literature, see Stock et al. (2006), there are two methods to produce multiperiod forecasts with autoregressive models. The first one is through estimating the model to forecast and then iterating upon that model to obtain its multiperiod forecast. The second one is through regressing model's dependent variables  $h$ -periods-hence against a constant and both present and past values of both dependents and independents variables.

In this section, we are going to apply the Giacomini and White (2006)'s conditional and unconditional predictive ability tests to see which one of these two forecasting methods has the best conditional and unconditional predictive ability for the aluminum spot price monthly return three, fifteen, and twenty-seven months ahead. Unlike Stock et al. (2006), we use, here, asymptotic forecasting performance tests rather than using complicated bootstrapping procedures which are computationally expensive, hardly precise and often hard to replicate, see Brooks (2008).

## 1. Methodology

To implement the Giacomini and White (2006) predictive ability tests, we need, first, to compute the forecasting error loss function,  $L_{t+h}(X_{t+h}, X_{t+h}^{\hat{j}})$  for each multiperiod forecasting method.

### a. Iterated Multiperiod Forecasting Method

Consider an estimated version of (4.8), represented by the following expression:

$$\Delta X_t = \hat{\gamma}_0 + \sum_{k=1}^K \hat{\gamma}_k \Delta X_{t-k} + V_t \quad (4.24)$$

Iterate (4.24)  $h - k$  times to obtain  $\Delta \hat{X}_{t+h}$  then, compute  $L_{t+h}(\Delta X_{t+h}, \Delta \hat{X}_{t+h}^i)$ .

### b. Direct Multiperiod Forecasting Method

Instead of estimating (4.8) and iterating upon it, direct forecasting method requires estimating the following VAR model:

$$\Delta X_t = \gamma_0 + \sum_{k=0}^K \gamma_k \Delta X_{t-k-h} + W_t \text{ for } h > 1 \quad (4.25)$$

where  $W_t$  is a vector of errors, which have the same number of rows as  $\Delta X_t$ . After estimating, (4.25), compute  $\Delta \hat{X}_{t+h}$  and, then,  $L_{t+h}(\Delta X_{t+h}, \Delta \hat{X}_{t+h}^d)$  which have the same sample size as  $L_{t+h}(\Delta X_{t+h}, \Delta \hat{X}_{t+h}^i)$ .

c. Conditional and Unconditional Predictive Ability Tests

Once  $L_{t+h}(\Delta X_{t+h}, \Delta \hat{X}_{t+h}^d)$  and  $L_{t+h}(\Delta X_{t+h}, \Delta \hat{X}_{t+h}^i)$  have been obtained, we need to substitute them into (4.19) to reject or not the null hypothesis of equal forecasting performance between the two methodologies. Here, we also assume  $d_t = [1, \Delta L_{t+h}(\Delta X_{t+h}, \Delta \hat{X}_{t+h}^j)]$  and  $\sigma$ -field  $\mathcal{G}_t = \mathcal{F}_t$  for the conditional predictive ability test and  $\sigma$ -field  $\mathcal{G}_t = \{\phi, \Omega\}$  for the unconditional predictive ability test.

In this paper, we do not make any direct multistep forecast by using VECM models [e.g. models (4) and (6)] because if we want to make direct multistep forecasts, the VECM represented by (4.9) will turn into the following heterodox VECM:

$$\Delta X_t = \gamma_0 + \sum_{k=0}^K \gamma_k \Delta X_{t-k-h} + \eta X_{t-1-h} \quad (4.26)$$

Estimating this heterodox VECM is out of this paper's scope and will be an interesting as a future work research related to develop direct multistep forecasting methods for VECM models.

## 2. Results

Tables XIX, XX and XXI summarize the conditional and unconditional 3, 15 and 27 months-ahead predictive ability tests, for the aluminum spot price, between these two multistep forecasting methodologies. The first column reports the two multi-

period forecasting methodologies to compare, direct,  $D$ , and iterated,  $I$ . The second column reports the model that we use for this exercise. The third column reports the conditional predictive ability test values whereas the fourth column presents their respective p-values.

The fifth column reports the function  $I_{n,c}$ , which indicates the proportion of times that the direct forecasting method outperforms the iterated multistep forecasting method. The sixth column reports either a number or a character,  $N$ . The former indicates which one of the two comparing forecasting methodologies has the best conditional forecasting performance whereas the later indicates that both forecasting methodologies have the same conditional forecasting performance.

The seventh column reports the unconditional predictive ability test values whereas the eighth column presents their respective p-values. The ninth column reports either a number or a character,  $N$ . The former indicates which one of the two comparing forecasting methodologies has the best spot price forecasting performance for any period whereas the later indicates that both forecasting methodologies have the same unconditional forecasting performance.



Table XIX.

Multistep Forecasting Performance Evaluation - Direct vs Iterated: Three-months-ahead

Methods	Model	Conditional				Unconditional		
		Test	P-value	$I_{n,c}$	Winner	Test	P-value	Winner
(D) vs (I)	(3)	12.1388	0.0023	0.8897	(I)	1.5198	0.1309	N
(D) vs (I)	(5)	10.8744	0.0044	0.9485	(I)	1.3665	0.1740	N

Table XX.

Multistep Forecasting Performance Evaluation - Direct vs. Iterated: Fifteen-months-ahead

Methods	Model	Conditional				Unconditional		
		Test	P-value	$I_{n,c}$	Winner	Test	P-value	Winner
(D) vs (I)	(3)	8.9531	0.0114	0.9286	(I)	1.1455	0.2542	N
(D) vs (I)	(5)	5.0684	0.0793	0.8661	(I)	1.4765	0.1423	N

Table XXI.

Multistep Forecasting Performance Evaluation - Direct vs Iterated: Twenty-seven-months-ahead

Methods	Model	Conditional				Unconditional		
		Test	P-value	$I_{n,c}$	Winner	Test	P-value	Winner
(D) vs (I)	(3)	3.6349	0.1624	NA	N	0.7048	0.4829	N
(D) vs (I)	(5)	3.4536	0.1779	NA	N	0.8287	0.4096	N

From Tables XIX, XX and XXI I conclude, first, that for both Financial Cost of Carry and Storage Cost Theory models, iterated multistep forecasting methodology has a better conditional predictive ability than the direct forecasting methodology for aluminum spot price monthly return three and fifteen months ahead. However, this is not true for the aluminum spot price monthly return forecast twenty-seven months ahead.

On the other hand, these two forecasting methodologies have the same unconditional predictive ability for aluminum spot price three, fifteen and twenty-seven months ahead.

Therefore, we conclude that the iterated multistep forecasting methodology is more efficient than the direct multistep forecasting methodology if we want to forecast, by using whichever of VAR models presented in this paper, the aluminum spot price monthly return for a specified period, in this paper those specified periods are three and fifteen months ahead.

## E. Conclusion

We have explored the ability of futures contracts' prices, inventories, risk-free-assets interest rate and spot price volatility to predict the spot price monthly return three, fifteen and twenty seven months ahead for the aluminum traded at the London Metal Exchange (LME). By applying the conditional and unconditional predictive ability tests developed by Giacomini and White (2006) and using the Speculative Efficiency model we have found that aluminum 3, 15, 27 futures contracts' prices have a better forecasting performance for the aluminum spot price three, fifteen and twenty-seven months ahead than a No-drift Random Walk.

In addition, we have found that risk-free-asset interest rates, aluminum inventories and spot price volatility complement the aluminum futures contracts' prices performance in forecasting the aluminum spot price three and fifteen months ahead. In particular, we have found that a VAR version of the Financial Cost of Carry model is has the best forecasting performance for the aluminum spot price monthly return three and fifteen months ahead among the models presented in this paper.

A new application of the Giacomini and White (2006), conditional and unconditional predictive ability tests, applied in this paper, is the forecasting performance comparison between the iterated multistep and the directed multistep forecasting methods, broadly discussed by Stock et al. (2006), for autoregressive models. From this application, we have found that the iterated multistep forecasting method of VAR models, used for aluminum spot price prediction, produces better three and fifteen months-ahead forecasts for the aluminum spot price monthly return than those produced by the direct multistep forecasting method. This result is robust as long as we used a long sample size and an asymptotic forecasting performance tests instead of using complicated and, sometimes, non- accurate, bootstrapping methods.

## CHAPTER V

## CONCLUSION

In Chapter II, I have presented a very simple two-country new-Keynesian model in which the inclusion of imperfect substitution between home and foreign consumption, home bias consumption, inflation's inertia and the existence of productivity shocks as well as monetary shocks are key for solving the Wang and Wen (2007)'s puzzle of the joint occurrence of positive cross-country correlation in inflation and a near-zero cross-country correlation in money growth.

Although this model adequately captures the signs and magnitude of the cross country-correlations in inflation, output, and consumption, this model tends to generate a stronger positive cross country correlation in output than inflation. Future work is needed to investigate whether this model can generate a stronger positive cross country correlation in inflation than in output.

In Chapter III, I have found, first, that gains from international monetary cooperation between two countries are different from zero when the elasticity of substitution between domestic and foreign goods consumption (Armington elasticity of substitution) is different from one.

Second, I have found also that when the Armington elasticity of substitution is less than one, this model predicts a unique Nash equilibrium, in which all exporting firms in both countries choose to price in their own currency and the central bank's optimal monetary policy implies a flexible exchange rate. However, this model predicts multiple Nash equilibria, when the Armington elasticity of substitution is larger and equal than one.

Third, I have also found that strategic complementarities, defined by Steinsson (2008), is not the only sufficient condition to support the existence of asymmetric

Nash Equilibria such as (LCP,PCP) or (PCP,LCP). Furthermore, these asymmetric Nash equilibria can be supported under strategic substitution. (i.e. when  $\sigma + \psi < 1$ )

Fourth, under different sets of preferences and elasticities of substitution values, this model confirms theoretical results obtained previously in the international macroeconomics literature, (see Devereux and Engel (2003), Corsetti and Pesenti (2004) and Corsetti and Pesenti (2005)) such as that (PCP,PCP) Nash Equilibrium Pareto dominates the (LCP,LCP) Nash Equilibrium.

This model can be extended for different country sizes. In particular, this requires to deal with the risk sharing condition's constant and to model a Nash Bargaining solution to determine the optimal international monetary policy under coordination as long as the different-country size assumption implies different units of welfare measure for each country.

Finally, it is also important to incorporate segmented markets into this model to determine endogenously which fraction of exporting firms set their prices either in domestic or in foreign currency. This idea is motivated from Fukuda (1994) who says that firms are indifferent to price their exports in domestic or foreign currency when they have access to futures market.

In Chapter IV, I have explored the ability of futures contracts' prices, inventories, risk-free-assets interest rate and spot price volatility to predict the spot price monthly return three, fifteen and twenty seven months ahead for the aluminum traded at the London Metal Exchange (LME). By applying the conditional and unconditional predictive ability tests developed by Giacomini and White (2006) and using the Speculative Efficiency model I have found that aluminum 3, 15, 27 futures contracts' prices have a better forecasting performance for the aluminum spot price monthly return three, fifteen and twenty-seven months ahead than a No-drift Random Walk.

In addition, I have found that risk-free-asset interest rates, aluminum inven-

ories and spot price volatility complement the aluminum futures contracts' prices performance in forecasting the aluminum spot price monthly return three and fifteen months ahead. In particular, I have found that a VAR version of the Financial Cost of Carry model is has the best forecasting performance for the aluminum spot price monthly return three and fifteen months ahead among the models presented in this paper.

A new application of the Giacomini and White (2006), conditional and unconditional predictive ability tests, applied in this paper, is the forecasting performance comparison between the iterated multistep and the directed multistep forecasting methods, broadly discussed by Stock et al. (2006), for autoregressive models. From this application, I have found that the iterated multistep forecasting method of VAR models, used for aluminum spot price monthly return prediction, produces better three and fifteen months-ahead forecasts for the aluminum spot price than those produced by the direct multistep forecasting method. This result is robust as long as I used a long sample size and an asymptotic forecasting performance tests instead of using complicated and, sometimes, non- accurate, bootstrapping methods.

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## APPENDIX

## A. Description of Data

This appendix describes the data source and range. From the (IFS) database, I obtained series of Consumer Price Index (CPI), Real Gross Domestic Output (GDP) and Money Supply, (M1). All these data are available for these five countries. The range goes from 1977 Q1 to 2008 Q1. The cross-country correlation in inflation is computed from the quarterly percent change in the CPI. The cross-country correlation in output is computed from the percent deviation of the GDP from its long run trend, which is obtained through the Hodrick and Prescott (1997) filter.

## B. Model's Structure

## Efficient Risk Sharing Condition

In this subsection, I present the derivation of the Efficient Risk Sharing Condition implied by the households' intertemporal maximization of this model:

by substituting (3.7) in (2.7), we get:

$$\frac{C_t^{-\sigma}}{P_t} Q(s^{t+1}|s^t) - \beta \left[ \frac{C^{-\sigma}(s^{t+1}|s^t)}{P(s^{t+1}|s^t)} \right] \quad (\text{B.1})$$

since these two countries are the same, then we have:

$$\frac{(C_t^*)^{-\sigma}}{P_t^*} Q^*(s^{t+1}|s^t) - \beta \left[ \frac{(C^*)^{-\sigma}(s^{t+1}|s^t)}{P^*(s^{t+1}|s^t)} \right] \quad (\text{B.2})$$

then by dividing (B.1) by (B.2), we get:

$$\frac{C_t^{-\sigma}}{(C_t^*)^{-\sigma}} \frac{RER_t}{E_t} = \frac{C^{-\sigma}(s^{t+1}|s^t)}{(C^*)^{-\sigma}(s^{t+1}|s^t)} RER(s^{t+1}|s^t) E(s^{t+1}|s^t) \quad (\text{B.3})$$

where:  $RER_t \equiv E_t P_t^*/P_t$  is the real exchange rate and  $E_t$  is the nominal exchange rate.

If we assume that initially the two economies are perfectly symmetric (i.e. in state  $s$  they have the same prices and marginal utility), then (B.3) implies:

$$RER_t = \frac{(C^*)^{-\sigma}(s^{t+1}|s^t)}{C^{-\sigma}(s^{t+1}|s^t)} \quad (\text{B.4})$$

which is nothing but the *Efficient Risk Sharing Condition*.

#### Intratemporal Optimization Problem

At each period  $t$ , the representative household chooses  $C_{H,t}$  and  $C_{F,t}$  which minimizes its total expenditure:

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \quad (\text{B.5})$$

given that they have chosen  $\{C_t\}_{t=0}^{\infty}$  previously.<sup>45</sup>

subject to:

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<sup>45</sup>Since the representative household preferences are separable on every period, I can solve this intratemporal minimization problem independently from the intertemporal maximization problem described previously.

$$C_t = C_{H,t}^\gamma C_{F,t}^{1-\gamma} \quad (\text{B.6})$$

as we can see (B.6) is an implication of assuming imperfect substitution in consumption between domestic and foreign goods.

where  $C_{H,t}$  denotes the home consumption of the domestic final good,  $C_{F,t}$  denotes the home consumption of the foreign final good,  $P_{H,t}$  and  $P_{F,t}$  are their prices respectively.

*F.O.C*

$$C_{H,t} : P_{H,t} - \nu_t \gamma C_{H,t}^{\gamma-1} C_{F,t}^{1-\gamma} = 0 \quad (\text{B.7})$$

$$C_{F,t} : P_{F,t} - \nu_t (1 - \gamma) C_{H,t}^\gamma C_{F,t}^{-\gamma} = 0 \quad (\text{B.8})$$

$$\nu_t : C_t - C_{H,t}^\gamma C_{F,t}^{1-\gamma} = 0 \quad (\text{B.9})$$

where  $\nu$  is the Lagrange multiplier associated to (3.2).

The main implication of this intratemporal minimization problem is:

$$P_t = \phi P_{H,t}^\gamma P_{F,t}^{1-\gamma} \quad (\text{B.10})$$

where  $\phi = \left[ (\gamma/1 - \gamma)^{1-\gamma} + (1 - \gamma/\gamma)^\gamma \right]$

### Price Setting

Following Calvo (1983) I assume that each individual firm resets its price with probability  $(1 - \theta)$  each period, independently of the time elapsed since its last price

adjustment. Thus, each period a measure  $(1 - \theta)$  of (randomly selected) firms reset their prices, while a fraction  $\theta$  keep their prices unchanged.

Let  $P_{H,t}(i)$  denotes the price set by a firm  $i$  adjusting its price in period  $t$ . Let  $\bar{P}_{H,t}(i)$  denotes the price set by a firm  $i$  adjusting its price in period  $t$ . Under the Calvo price setting structure,  $P_{H,t+k}(i) = \bar{P}_{H,t}(i)$  with probability  $\theta^k$  for  $k = 0, 1, 2, 3, 4, 5, 6, \dots$ . Then, the firm's optimal price setting model is written as follows:

$$\bar{P}_{H,t}(i) = \operatorname{argmax} \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} E_t [\bar{P}_{H,t}(i) - MC_{h,t}(i)] \quad (\text{B.11})$$

subject to (2.13) and taking  $Y_{H,t+k}$  as given.

*F.O.C*

$$\begin{aligned} & \sum_{k=0}^{\infty} Q_{t,t+k} E_t \left[ (1 - \zeta) (\bar{P}_{H,t}(i)/P_{H,t})^{-\zeta} Y_{H,t+k} \right] + \dots \\ & \sum_{k=0}^{\infty} Q_{t,t+k} E_t \left[ \zeta (\bar{P}_{H,t}(i)/P_{H,t})^{-\zeta-1} RMC_{H,t}(i) Y_{H,t+k} \right] = 0 \end{aligned} \quad (\text{B.12})$$

$$\sum_{k=0}^{\infty} Q_{t,t+k} E_t \left[ ((\bar{P}_{H,t}(i)/P_{H,t}) - (\zeta/\zeta - 1) RMC_{H,t}(i)) Y_{H,t+k} \right] = 0 \quad (\text{B.13})$$

where  $RMC_{H,t}(i) = MC_{h,t}(i)/P_{H,t}$ .

Since all firms resetting prices in any given period and having identical technology they will choose the same price, I henceforth drop the  $i$  subscript.

$$\sum_{k=0}^{\infty} Q_{t,t+k} E_t \left[ ((\bar{P}_{H,t}/P_{H,t}) - (\zeta/\zeta - 1) RMC_{H,t}) Y_{H,t+k} \right] = 0 \quad (\text{B.14})$$



where  $\zeta/1 - \zeta$  denotes the intermediate firms' mark-up.

### C. Equations implied by the model

This appendix presents the system of dynamic equations implied by this model under Flexible Prices and under Sticky Prices.

#### *Flexible Prices*

- From (3.7),(3.8) and (3.9) we have:

$$\frac{\psi}{\beta} N_t^\eta C_{H,t+1|t}^\sigma = W_t \quad (\text{C.1})$$

- Recalling (3.2) we have:

$$C_t = C_{H,t}^\gamma C_{F,t}^{1-\gamma} \quad (\text{C.2})$$

- Recalling (B.3), we have

$$RER_t = \frac{(C^*)_t^{-\sigma}}{C_t^{-\sigma}} \quad (\text{C.3})$$

- From (B.12) and (B.13) we have:

$$C_{H,t}/C_{F,t} = \left( \frac{\gamma}{1-\gamma} \right) TOT_t \quad (\text{C.4})$$

where  $TOT_t = \frac{P_{F,t}}{EP_{H,t}^*}$ . Note that  $TOT_t = \frac{P_{F,t}}{P_{H,t}}$  since LOOP holds.

- By dividing (3.18) from its first lagged values, we have:

$$(1 + \pi_t) = (1 + \pi_{H,t})^\gamma (1 + \pi_{F,t})^{1-\gamma} \quad (\text{C.5})$$

- Recalling (2.14) and assuming that all intermediate goods forms have the same technology, we have:

$$Y_{H,t} = A_{H,t} N_t \quad (\text{C.6})$$

- Recalling (2.15)

$$A_{H,t} = A_{H,t}^\rho \exp^{\epsilon_t^A} \quad (\text{C.7})$$

- The real marginal cost associated to the technology described in (2.14), is defined as:  $RMCH_{t} \equiv P_t W_t / P_{H,t} A_{H,t}$  and the terms of trade is defined as:  $TOT_t \equiv P_{F,t} / P_{H,t}$ .

Since this model does not determine the optimal path for nominal variables such as price levels, we have to rewrite the real marginal cost as follows:

By using the definitions of  $RMCH_t$  and  $TOT_t$  described above and (3.18), we have:

$$RMCH_{H,t} = \phi \frac{W_t}{A_{H,t}} TOT_t^{1-\gamma} \quad (\text{C.8})$$

- Since prices are flexible, by (B.14) the optimality condition for intermediate goods firms reduces to:

$$RMCH_{H,t} = (1 - \zeta / \zeta) \quad (\text{C.9})$$

- Using the definition of  $TOT_t$  and dividing it by its first lagged value, we have:

$$(1 + \pi_{F,t}) = \frac{TOT_t}{TOT_{t-1}} \quad (\text{C.10})$$

- From (2.20) and (2.24) we have:

$$\frac{\tilde{M}_{t+1}}{\tilde{M}_t} = \frac{U_{t+1}}{1 + \pi_t + 1} \quad (\text{C.11})$$

- Recalling (2.21), we have:

$$U_t = (U_t)^\varsigma (\bar{U})^{1-\varsigma} \exp^{\epsilon_t^U} \quad (\text{C.12})$$

- Recalling (2.22), we have:

$$Y_{H,t} = C_{H,t} + C_{F,t}^* \quad (\text{C.13})$$

- By using the equilibrium condition (2.24), the household's budget constraint (3.5), the consumer price index (3.18), the definition of terms of trade and the fact that  $\Pi_t \equiv P_{H,t}Y_{H,t} - P_tW_{H,t}N_t$ , we obtain the equilibrium CIA constraint:

$$Y_{H,t} = M_{t+1}(1 + \pi_{t+1})\phi TOT_t^{1-\gamma} \quad (\text{C.14})$$

- By using the above definition of real exchange rate, the consumer price index of each country and the LOOP, we obtain the following relationship between the real exchange rate and the terms of trade as follows:

$$RER_t = \frac{\phi}{\phi^*} TOT_t^{2\gamma-1} \quad (\text{C.15})$$

*Sticky Prices*

By log-linearizing the above system of equations around the steady state and by including sticky prices, we have the following dynamic system of linear rational expectations equations:

$$\eta n_t + \sigma c_{t+1|t} = w_t \quad (\text{C.16})$$

$$c_t = \gamma c_{H,t} + (1 - \gamma) c_{F,t} \quad (\text{C.17})$$

$$c_t - c_t^* = \frac{1}{\sigma} r e r_t \quad (\text{C.18})$$

$$c_{H,t} = c_{F,t} + tot_t \quad (\text{C.19})$$

$$\pi = \gamma \pi_{H,t} + (1 - \gamma) \pi_{F,t} \quad (\text{C.20})$$

$$y_{H,t} = a_{H,t} + n_t \quad (\text{C.21})$$

$$a_t = \rho a_{t-1} + \epsilon_t^A \quad (\text{C.22})$$

$$rmc_t = w_t - a_{H,t} + (1 - \gamma) tot_t \quad (\text{C.23})$$

By log-linearizing (2.17)-(2.19) and (B.14) around zero inflation and by solving the system of equations implied by this log-linearization, which is described in Gali and Monacelli (2005) and Gali and Gertler (1999)), we obtain the NKPC:

$$\pi_{H,t} = \frac{(1-\chi)(1-\theta)(1-\beta\theta)}{\Delta} rmc_t + \frac{\beta\theta}{\Delta} \pi_{H,t+1|t} + \frac{\chi}{\Delta} \pi_{H,t-1} \quad (\text{C.24})$$

$$\pi_{F,t} = tot_t - tot_{t-1} \quad (\text{C.25})$$

$$m_{t+1} = m_t + u_{t+1} - \pi_{t+1} \quad (\text{C.26})$$

$$u_t = \varsigma u_{t-1} + \epsilon_t^U \quad (\text{C.27})$$

$$y_{H,t} = c_{H,t} + c_{F,t}^* \quad (\text{C.28})$$

$$y_{H,t} = m_{t+1} + \pi_{H,t+1} + (1-\gamma)tot_t \quad (\text{C.29})$$

$$rer_t = (2\gamma - 1)tot_t \quad (\text{C.30})$$

#### D. Currency Invoicing Rule

##### Proof of Proposition 1

*Proof.* Let

$$E_{t-1}Q_{t-1,t}\Pi_t^{PCP} = E_{t-1}Q_{t-1,t} [p_t^{PCP}(h) - MC_t(h)] \int_0^1 C_t^*(h, j^*) dj^* \quad (\text{D.1})$$

By plugging the analog, for the foreign country, of equations (3.14) and (3.16) in (D.1), we have:

$$E_{t-1}Q_{t-1,t}\Pi_t^{PCP} = E_{t-1}Q_{t-1,t} [p_t^{PCP}(h) - MC_t(h)] \int_0^1 \left( \frac{p_t^{PCP}(h)}{S_t P_{H,t}^*} \right)^{-\theta} \chi \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\alpha} C_t^*(j) dj^* \quad (D.2)$$

$$E_{t-1}\Pi_t^{PCP}(h) = E_{t-1}Q_{t-1,t} \{ [p_t^{PCP}(h)]^{1-\theta} S_t^\theta - MC_t(h) [p_t^{PCP}(h)]^{-\theta} S_t^\theta \} Z_t \quad (D.3)$$

where  $Z_t \equiv Q_{t-1,t} \chi [P_{H,t}^*]^{(\theta-\alpha)} [P_t^*]^\alpha \int_0^1 C_t^*(j) dj^*$ ,  $Q_{t-1,t}$  is the firm's exogenous discount factor.

The profit maximization price for the firm, under PCP, is equal to:

$$p_t^{PCP}(h) = \frac{\theta}{1-\theta} \frac{E_{t-1} (MC_t(h) S_t^\theta Z_t)}{E_{t-1} (S_t^\theta Z_t)} \quad (D.4)$$

By substituting (D.4) in (D.3), we have the final goods firms' expected discounted profit under PCP. That is,

$$E_{t-1}\Pi_t^{PCP}(h) = \tilde{\theta} [E_{t-1} (MC_t(h) S_t^\theta Z_t)]^{1-\theta} [E_{t-1} (S_t^\theta Z_t)]^{-\theta} \quad (D.5)$$

where  $\tilde{\theta} \equiv \left( \frac{1}{1-\theta} \right) \left( \frac{\theta}{1-\theta} \right)^{-\theta}$

Let

$$E_{t-1}Q_{t-1,t}\Pi_t^{LCP} = E_{t-1}Q_{t-1,t} [S_t p_t^{LCP}(h) - MC_t(h)] \int_0^1 C_t^*(h, j^*) dj^* \quad (D.6)$$

By plugging the analog, for the foreign country, of equations (3.14) and (3.16) in

(D.6), we have:

$$E_{t-1}Q_{t-1,t}\Pi_t^{LCP} = E_{t-1}Q_{t-1,t} [S_t p_t^{LCP}(h) - MC_t(h)] \int_0^1 \left( \frac{p_t^{LCP}(h)}{P_{H,t}^*} \right)^{-\theta} \chi \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\alpha} C_t^*(j) dj^* \quad (D.7)$$

$$E_{t-1}\Pi_t^{LCP}(h) = E_{t-1}Q_{t-1,t} \{ [S_t p_t^{LCP}(h)]^{1-\theta} - MC_t(h) [p_t^{LCP}(h)]^{-\theta} \} Z_t \quad (D.8)$$

where  $Z_t \equiv Q_{t-1,t} \chi [P_{H,t}^*]^{(\theta-\alpha)} [P_t^*]^\alpha \int_0^1 C_t^*(j) dj^*$ ,  $Q_{t-1,t}$  is the firm's exogenous discount factor.

The profit maximization price for the firm, under LCP, is equal to:

$$p_t^{LCP}(h) = \frac{\theta}{1-\theta} \frac{E_{t-1}(MC_t(h)Z_t)}{E_{t-1}(S_t Z_t)} \quad (D.9)$$

by substituting (D.4) in (D.3), we have the final goods firms' expected discounted profit under PCP. That is,

$$E_{t-1}\Pi_t^{LCP}(h) = \tilde{\theta} [E_{t-1}(MC_t(h)S_t Z_t)]^{1-\theta} [E_{t-1}(S_t^Z)]^{-\theta} \quad (D.10)$$

where  $\tilde{\theta} \equiv \left( \frac{1}{1-\theta} \right) \left( \frac{\theta}{1-\theta} \right)^{-\theta}$ .

Equations (D.5) and (D.10) are the same as Devereux et al. (2004), hence the rest of this proof is identical to the provided, in the appendix, by Devereux et al. (2004).

□

## E. Model's implicit solution

The following is the system of equations that characterize the equilibrium in this model:

$$M_t = 1 + \nu_d (A_t - A) + \nu_f (A_t^* - A^*) \quad (\text{E.1})$$

$$M_t^* = 1 + \nu_f^* (A_t^* - A^*) + \nu_d^* (A_t - A) \quad (\text{E.2})$$

$$\frac{P_t^* S_t}{P_t} = \left( \frac{C_t}{C_t^*} \right)^\sigma \quad (\text{E.3})$$

$$MC_t = \frac{\kappa P_t C_t^\sigma N_t^\psi}{A_t} \quad (\text{E.4})$$

$$MC_t^* = \frac{\kappa P_t^* (C_t^*)^\sigma (N_t^*)^\psi}{A_t^*} \quad (\text{E.5})$$

$$P_{H,t} = \frac{\theta}{\theta - 1} \frac{E_{t-1} (Q_{t-1,t} MC_t C_{H,t})}{E_{t-1} (Q_{t-1,t} C_{H,t})} \quad (\text{E.6})$$

$$P_{F,t} = \frac{\theta}{\theta - 1} S_t^{\eta^*} \frac{E_{t-1} (Q_{t-1,t}^* MC_t^* C_{F,t})}{E_{t-1} (Q_{t-1,t}^* C_{F,t} S_t^{\eta^* - 1})} \quad (\text{E.7})$$

$$P_{H,t}^* = \frac{\theta}{\theta - 1} S_t^{-\eta} \frac{E_{t-1} (Q_{t-1,t} MC_t C_{H,t}^*)}{E_{t-1} (Q_{t-1,t} C_{H,t}^* S_t^{1-\eta})} \quad (\text{E.8})$$

$$P_{F,t}^* = \frac{\theta}{\theta - 1} \frac{E_{t-1} (Q_{t-1,t}^* MC_t^* C_{F,t}^*)}{E_{t-1} (Q_{t-1,t}^* C_{F,t}^*)} \quad (\text{E.9})$$

$$M_t = P_t C_t; \quad (\text{E.10})$$



$$M_t^* = P_t^* C_t^*; \quad (\text{E.11})$$

$$P_t = \begin{cases} \frac{1}{\tilde{\chi}} [\chi P_{H,t}^{1-\alpha} + (1-\chi) P_{F,t}^{1-\alpha}]^{\frac{1}{1-\alpha}} & \text{if } \alpha \neq 1 \\ \frac{1}{\tilde{\chi}} P_{H,t}^\chi P_{F,t}^{1-\chi} & \text{if } \alpha = 1 \end{cases} \quad (\text{E.12})$$

where  $\tilde{\chi} = \chi^\chi (1-\chi)^{(1-\chi)}$

$$P_t^* = \begin{cases} \frac{1}{\tilde{\chi}} [\chi (P_{H,t}^*)^{1-\alpha} + (1-\chi) (P_{F,t}^*)^{1-\alpha}]^{\frac{1}{1-\alpha}} & \text{if } \alpha \neq 1 \\ \frac{1}{\tilde{\chi}} (P_{H,t}^*)^\chi (P_{F,t}^*)^{1-\chi} & \text{if } \alpha = 1 \end{cases} \quad (\text{E.13})$$

$$Y_t = A_t N_t \quad (\text{E.14})$$

$$Y_t^* = A_t^* N_t^* \quad (\text{E.15})$$

$$Y_t = C_{H,t} + C_{H,t}^*; \quad (\text{E.16})$$

$$Y_t^* = C_{F,t} + C_{F,t}^*; \quad (\text{E.17})$$

$$C_{H,t} = \chi \left( \frac{P_{H,t}}{P_t} \right)^{-\alpha} C_t \quad (\text{E.18})$$

$$C_{F,t} = (1-\chi) \left( \frac{P_{F,t}}{P_t} \right)^{-\alpha} C_t \quad (\text{E.19})$$

$$C_{H,t}^* = \chi \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\alpha} C_t^* \quad (\text{E.20})$$

$$C_{F,t}^* = (1 - \chi) \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\alpha} C_t^* \quad (\text{E.21})$$

$$(A_t - A) = \rho(A_{t-1} - A) + u_t \quad (\text{E.22})$$

$$(A_t^* - A^*) = \rho(A_{t-1}^* - A^*) + u_t^* \quad (\text{E.23})$$

Where:  $v_t = \{u_t, u_t^*\}$  is a vector, which follows a normal distribution with zero mean and covariance matrix,  $\Sigma$ .

$$Q_{t-1,t} = \beta * \left( \frac{P_{t-1}}{P_t} \right) \left( \frac{C_{t-1}}{C_t} \right)^\sigma \quad (\text{E.24})$$

$$Q_{t-1,t}^* = \beta * \left( \frac{P_{t-1}^*}{P_t^*} \right) \left( \frac{C_{t-1}^*}{C_t^*} \right)^\sigma \quad (\text{E.25})$$

## F. Calibration

Since this model's solution is numerical, we need to set some values for this model's parameters. These parameter values are reported in Table XXII, which are taken from well known papers in international macroeconomics.

Table XXII.

## Calibration

Parameter	Source
$\beta = 0.99$	Backus et al. (1992)
$\theta = 10$	Wang and Wen (2007)
$\rho = 0.9$	Backus et al. (1992)
$\Sigma_{1,1} = 1^a$	Calibrated by the author
$\Sigma_{1,2} = 0.5$	Backus et al. (1992)
$\Sigma_{2,2} = 1$	Calibrated by the author

Note: a) I assume that both countries have identical productivity shocks' distributions.

## G. Cointegration Tests

## Financial Cost of Carry Model

Tables XXIII and XXIV show the Johansen cointegration tests among aluminum's 3 months futures contracts price, aluminum's spot price and the 90-days USA Treasury bills. The first column denotes the number of cointegration vectors, the second column the Johansen's test value and the third column denotes the p-values, which are taken from MacKinnon-Haug-Michelis (1999) and computed by using Eviews 5.0. The cointegration model used in this paper assumes the existence of constant term in both the VAR structure and the Cointegration vector and by using the Schwartz criterium in the VAR model, the optimal number of lags is equal to 1.

Table XXIII.

Trace Test: Financial Cost of Carry Theory Using 3-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
	None	53.6451	0.0000
1989:1 - 1999:1	At most 1	15.5213	0.0496
	None	42.5812	0.0010
1995:1 - 2005:1	At most 1	9.4778	0.3230
	None	32.0677	0.0269
2000:7 - 2010:7	At most 1	5.7646	0.7231

Table XXIV.

Maximum Eigenvalue Test: Financial Cost of Carry Theory Using 3-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
	None	38.1382	0.0001
1989:1 - 1999:1	At most 1	10.4095	0.1864
	None	33.1034	0.0007
1995:1 - 2005:1	At most 1	6.8929	0.5019
	None	26.3031	0.0085
2000:7 - 2010:7	At most 1	3.3212	0.9231

Tables XXV and XXVI show the Johansen cointegration tests among aluminum's 15 months futures contracts price, aluminum's spot price and the 1-year USA Treasury bonds. The first column denotes the number of cointegration vectors, the second column the Johansen's test value and the third column denotes the p-values, which are taken from MacKinnon-Haug-Michelis (1999) and computed by using Eviews 5.0. The cointegration model used in this paper assumes the existence of constant term in both the VAR structure and the Cointegration vector and by using the Schwartz criterium in the VAR model, the optimal number of lags is equal to 1.

Table XXV.

Trace Test: Financial Cost of Carry Theory Using 15-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	30.0303	0.0470
	At most 1	11.1538	0.2022
1995:1 - 2005:1	None	24.7299	0.1713
	At most 1	8.3236	0.4315
2000:7 - 2010:7	None	30.5470	0.0409
	At most 1	7.7182	0.4960

Table XXVI.

Maximum Eigenvalue Test: Financial Cost of Carry Theory Using 15-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
	None	18.8765	0.1005
1989:1 - 1999:1	At most 1	6.3807	0.5649
	None	16.4064	0.2020
1995:1 - 2005:1	At most 1	5.9955	0.6138
	None	22.8288	0.0286
2000:7 - 2010:7	At most 1	5.7861	0.6408

Tables XXVII and XXVIII show the Johansen cointegration tests among aluminum's 15 months futures contracts price, aluminum's spot price and the 1-year USA Treasury bonds. The first column denotes the number of cointegration vectors, the second column the Johansen's test value and the third column denotes the p-values, which are taken from MacKinnon-Haug-Michelis (1999) and computed by using Eviews 5.0. The cointegration model used in this paper assumes the existence of constant term in both the VAR structure and the Cointegration vector and by using the Schwartz criterium in the VAR model, the optimal number of lags is equal to 1.

Table XXVII.

Trace Test: Financial Cost of Carry Theory Using 27-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	36.0322	0.0084
	At most 1	17.2978	0.0265
	At most 2	5.3593	0.0206
1995:1 - 2005:1	None	22.1015	0.2929
	At most 1	8.1601	0.4484
2000:7 - 2010:7	None	28.21603	0.0752
	At most 1	9.0425	0.3615

Table XXVIII.

Maximum Eigenvalue Test: Financial Cost of Carry Theory Using 27-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
	None	18.7344	0.1048
1989:1 - 1999:1	At most 1	11.9386	0.1130
	None	13.9413	0.3698
1995:1 - 2005:1	At most 1	5.5905	0.6661
	None	19.1735	0.0919
2000:7 - 2010:7	At most 1	7.1842	0.4676

#### Cointegration Tests: Storage Cost Theory Model

Tables XXIX and XXX show the Johansen cointegration tests among aluminum's 3 months futures contracts price, aluminum's spot price and the 90-days USA Treasury bills. The first column denotes the number of cointegration vectors, the second column the Johansen's test value and the third column denotes the p-values, which are taken from MacKinnon-Haug-Michelis (1999) and computed by using Eviews 5.0. The cointegration model used in this paper assumes the existence of constant term in both the VAR structure and the Cointegration vector and by using the Schwartz criterium in the VAR model, the optimal number of lags is equal to 1.



Table XXIX.

Trace Test: Storage Cost Theory Using 3-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	113.1957	0.0000
	At most 1	70.1952	0.0001
	At most 2	30.5780	0.0406
	At most 3	16.6187	0.0337
	At most 4	6.0057	0.0143
1995:1 - 2005:1	None	102.1955	0.0000
	At most 1	57.6308	0.0046
	At most 2	25.3661	0.1488
	At most 3	7.5544	0.5142
	At most 4	1.3999	0.2367
2000:7 - 2010:7	None	75.9171	0.0150
	At most 1	39.5220	0.2401
	At most 2	13.9377	0.8441
	At most 3	4.4076	0.8680
	At most 4	0.6137	0.4334

Table XXX.

Maximum Eigenvalue Test: Storage Cost Theory Using 3-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	43.0004	0.0031
	At most 1	39.6172	0.0009
	At most 2	13.9593	0.3684
	At most 3	10.6130	0.1747
	At most 4	6.0057	0.0143
1995:1 - 2005:1	None	44.5646	0.0019
	At most 1	32.2647	0.0116
	At most 2	17.8117	0.1370
	At most 3	6.1545	0.5935
	At most 4	1.3999	0.2367
2000:7 - 2010:7	None	36.3951	0.0245
	At most 1	25.5843	0.0882
	At most 2	9.5300	0.7875
	At most 3	3.7940	0.8804
	At most 4	0.6137	0.4334

Tables XXXI and XXXII show the Johansen cointegration tests among aluminum's 3 months futures contracts price, aluminum's spot price and the 90-days

USA Treasury bills. The first column denotes the number of cointegration vectors, the second column the Johansen's test value and the third column denotes the p-values, which are taken from MacKinnon-Haug-Michelis (1999) and computed by using Eviews 5.0. The cointegration model used in this paper assumes the existence of constant term in both the VAR structure and the Cointegration vector and by using the Schwartz criterium in the VAR model, the optimal number of lags is equal to 1.

Table XXXI.

Trace Test: Storage Cost Theory Using 15-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	102.9325	0.0000
	At most 1	52.4992	0.0172
	At most 2	30.0574	0.0467
	At most 3	13.1915	0.1080
	At most 4	5.7244	0.0167
1995:1 - 2005:1	None	94.6649	0.0002
	At most 1	55.5934	0.0079
	At most 2	29.1114	0.0598
	At most 3	7.6625	0.5022
	At most 4	2.3500	0.1253
2000:7 - 2010:7	None	84.8900	0.002
	At most 1	41.9359	0.1605
	At most 2	20.1830	0.4105
	At most 3	9.3136	0.3372
	At most 4	0.5135	0.4736

Table XXXII.

Maximum Eigenvalue Test: Storage Cost Theory Using 15-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	50.4333	0.0002
	At most 1	22.4417	0.1986
	At most 2	16.8659	0.1784
	At most 3	7.4671	0.4355
	At most 4	5.7244	0.0167
1995:1 - 2005:1	None	39.0716	0.0110
	At most 1	26.4820	0.0687
	At most 2	21.4489	0.0451
	At most 3	5.3125	0.7020
	At most 4	2.3500	0.1253
2000:7 - 2010:7	None	42.9542	0.0032
	At most 1	21.7529	0.2333
	At most 2	10.8693	0.6604
	At most 3	8.8001	0.3032
	At most 4	0.5135	0.4736

Tables XXXIII and XXXIV show the Johansen cointegration tests among aluminum's 3 months futures contracts price, aluminum's spot price and the 90-days

USA Treasury bills. The first column denotes the number of cointegration vectors, the second column the Johansen's test value and the third column denotes the p-values, which are taken from MacKinnon-Haug-Michelis (1999) and computed by using Eviews 5.0. The cointegration model used in this paper assumes the existence of constant term in both the VAR structure and the Cointegration vector and by using the Schwartz criterium in the VAR model, the optimal number of lags is equal to 1.

Table XXXIII.

Trace Test: Storage Cost Theory Using 27-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	120.9274	0
	At most 1	67.7809	0.0003
	At most 2	40.1094	0.0023
	At most 3	21.0192	0.0066
	At most 4	6.3761	0.0116
1995:1 - 2005:1	None	88.8593	0.0007
	At most 1	50.0714	0.0305
	At most 2	26.7679	0.1074
	At most 3	7.9311	0.4728
	At most 4	2.7598	0.0967
2000:7 - 2010:7	None	92.6748	0.0003
	At most 1	43.4766	0.1214
	At most 2	24.3260	0.1870
	At most 3	10.0388	0.2778
	At most 4	0.4577	0.4987

Table XXXIV.

Maximum Eigenvalue Test: Storage Cost Theory Using 27-Months Futures Contract's Price

Sample	Number of CE	Test Value	P-value
1989:1 - 1999:1	None	53.1466	0.0001
	At most 1	27.6714	0.0487
	At most 2	19.0901	0.0942
	At most 3	14.6432	0.0436
	At most 4	6.3761	0.0116
1995:1 - 2005:1	None	38.7879	0.0120
	At most 1	23.3035	0.1609
	At most 2	18.8368	0.1017
	At most 3	5.1713	0.7201
	At most 4	2.7598	0.0967
2000:7 - 2010:7	None	49.1983	0.0004
	At most 1	19.1505	0.4030
	At most 2	14.2872	0.3419
	At most 3	9.5812	0.2409
	At most 4	0.4577	0.4987



## VITA

Jesus Antonio Bejarano Rojas was born in Bogota DC, Colombia. He received his Bachelor of Arts degree in economics from Universidad del Rosario in 2002, and Master of Arts degree in economics from Universidad del Rosario in 2004. He has worked for Banco de La Republica (Colombia Central Bank) since 2001. He joined the graduate program in economics at Texas A&M University in August 2006, and received his Doctor of Philosophy in August 2011 under the supervision of Dr. Dennis Jansen. His research interests include international macroeconomics and applied financial econometrics. Dr. Bejarano-Rojas can be reached at the Economics Research Department, Banco de la Republica, Bogota DC, Colombia, South America. His email address is [jbejarro@banrep.gov.co](mailto:jbejarro@banrep.gov.co)

The typist for this dissertation was Jesus Antonio Bejarano Rojas.