PRESERVICE TEACHERS' CONTENT KNOWLEDGE OF FUNCTION CONCEPT WITHIN A CONTEXTUAL ENVIRONMENT

A Dissertation

by

IRVING ANTHONY BROWN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

Major Subject: Curriculum and Instruction

Preservice Teachers' Content Knowledge of Function

Concept within a Contextual Environment

PRESERVICE TEACHERS' CONTENT KNOWLEDGE OF FUNCTION CONCEPT WITHIN A CONTEXTUAL ENVIRONMENT

A Dissertation

by

IRVING ANTHONY BROWN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Approved by:

Co-Chairs of Committee,	Gerald O. Kulm
	Dennie L. Smith
Committee Members,	Jane F. Schielack
	Myeongsun Yoon
Head of Department,	Dennie L. Smith

August 2011

Major Subject: Curriculum and Instruction

ABSTRACT

Preservice Teachers' Content Knowledge of Function Concept

within a Contextual Environment. (August 2011)

Irving Anthony Brown, B.S., Lamar University; M.S., Tarleton State University

Co-Chairs of Advisory Committee: Dr. Gerald O. Kulm
Dr. Dennie L. Smith

The overarching goal of this dissertation research was to develop and measure the psychometric properties of an instrument to assess preservice teachers' content knowledge of the function concept embedded in contextual problems. This goal was accomplished through two research projects described in two studies. The first study reports on the collective case study that was used to pilot test the instrument and the second study details the rationale used in item selection and the psychometric properties of the new instrument. Unlike existing research studies that examine a broad range of function related topics using various forms of symbolic, tabular, and graphical representations as the basis for questions and problems, this study focused solely on function problems immersed in various real world contexts. Since this is not a common approach to measuring content knowledge of the function concept, the existing instruments in published studies were not found to be suitable for this specialized purpose. The psychometric measurements of the instrument did not suggest that the instrument was valid or reliable so more research will be required to validate the instrument. However, based on the preliminary results from testing, several potential suggestions can be made to teacher educations programs. Inferences drawn from the mathematical problem-solving cognition will aid in the development and validation of future instruments to assess the content knowledge of the mathematical function concept of preservice teachers as they complete contextualized problem-solving tasks.

DEDICATION

"For by him all things were created…"
Colossians 1:16

This work is dedicated to the elements of my being; God and my family. In all of my works I give the honor to God who is the head of my life and his son, my lord and savior, Jesus Christ. I am nothing without him in my life and my work would be meaningless if I did not work to serve him. I, as with every good thing I do, am created by him and for him.

I am truly blessed to have a wonderful, supportive and loving family that has been my anchor my entire life. Without their guidance and encouragement I would have not had the fortitude to pursue my dreams. Though my grandmother, Pricilla Nunnally, my aunt Dorothy Nunnally and my cousin Cofer L. McIntosh are no longer with us on earth, I carry them with me in my heart and their spirits will be with us forever.

I would be but half a man without my wife; Valerie is the center of me and my partner in every good thing I do. If anyone would ever look upon my accomplishments with favor, they should know that I am only the lesser half of the story. This was, is, and always will be an IrVal idea. These first 29 years have only marked the beginning of the wonders that God has in store for us.

"Forever Lovers,
Forever Friends.
A lifetime's a Short Time,
When Love Never Ends" (Davis, 1976)

ACKNOWLEDGEMENTS

I would like to thank my committee co-chairs, Dr. Dennie Smith and Dr. Gerald Kulm, and my committee members, Dr. Jane Schielack, and Dr. Myeongsun Yoon, for their guidance and support throughout the course of this research. The support from Drs. Kulm and Smith extend far beyond the reach of this study for they were instrumental in bringing me into the doctoral program and providing a strong start to my research career.

Thanks also go to my friends and colleagues and the department faculty and staff for making my time in the Department of Teaching, Learning, and Culture at Texas A&M University a wonderful experience. I also want to extend my gratitude to the faculty members in Texas and North Carolina, who served on my Subject Matter Expert panel, and to all the Texas and North Carolina professors and their preservice teachers who were willing to participate in the study.

I extend my heartfelt thanks to my parents, Vernon and Julia Brown, my sister, Melissa Brown and our extended family of Browns, Nunnallys and Picketts for their encouragement and support over these many years of my educational and professional journey. I include in my family my mentor of the last two decades, Dr. Richard L. Price or “Doc” as he is known on the campus of Lamar University. Dr. Price has been my steadfast supporter and a beacon of light on my academic path.

I owe a special debt of gratitude to my wonderful daughter Chasity. In addition to her love and support, she has been my data transcriber and proof-reader for many parts of this study and her help has been invaluable to my work. Last in this list, but first in my heart, I wish to thank my beautiful, gift-from-God wife for her love, patience and understanding. Valerie, I thank you for your sacrifice; few people know what the spouse of a Ph.D. student must endure and you are a shining exemplar of a supportive wife.

NOMENCLATURE

ACP	Alternative Certification Program - an alternate route to state teacher licensure from the traditional university based undergraduate teacher education programs.
ETS	Educational Testing Service
Middle School	The state of Texas had traditionally held certification for middle school mathematics teachers to include teachers in fourth through eighth grade classrooms. In 2008, that scope was narrowed to only include sixth through eighth grade teachers. It should also be noted that the term “middle” school is sometimes used synonymously used with “junior high” school and can only encompass the seventh and eighth grades. For the purposes of this study, the term middle school will mean grades four through eight.
Praxis™	The series of examinations were created by ETS to provide several states a means of assessment of preservice teachers’ general academic knowledge, specific subject knowledge and pedagogical knowledge for the various teacher certification programs across the U.S.
SACS	The Commission on Colleges of the Southern Association of Colleges and Schools is the regional body for the accreditation of degree-granting higher education institutions in the Southern states. These states include: Alabama, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Texas, and Virginia.
SBEC	State Board for Educator Certification
TEA	Texas Education Agency
TExES™	Texas Examinations of Educator Standards™ () - is the product of a collaborative effort between the State of Texas’ State Board for Educator Certification (SBEC), the Texas Education Agency (TEA) and the Educational Testing Service (ETS) to create a series of examinations to be used by state agencies as a partial requirement of teacher certification in the state of Texas.

TABLE OF CONTENTS

		Page
ABSTRACT		iii
DEDICATION		v
ACKNOWLEDGEMENTS		vi
NOMENCLATURE		viii
TABLE OF CONTENTS		ix
LIST OF FIGURES		xii
LIST OF TABLES		xiii
CHAPTER		
I	INTRODUCTION	1
	Statement of the Problem	9
	Statement of Purpose	10
	Research Questions	11
II	OBSERVING PRESERVICE TEACHERS' CONTEXTUALIZED FUNCTION PROBLEM SOLVING THROUGH TASK-BASED INTERVIEWS	13
	Introduction	13
	Problem	14
	Research Questions	15
	Theoretical Framework	16
	Method	18
	Participants	18
	Instrument	19
	Data Collection	21
	Results	21
	Data Analysis	21
	Interpretation of Individual Participants' Responses	27
	Teacher1	27

CHAPTER Page

Teacher2 and Teacher5 29
Teacher3 31
Teacher4 33
Teacher6 34
Discussion 36
Decoding Strategies 36
Procedural Approaches 37
Conceptual Approaches 38
Implications and Conclusions 39

III THE DEVELOPMENT AND PSYCHOMETRIC MEASUREMENT OF AN INSTRUMENT TO ASSESS PRESERVICE MATHEMATICS TEACHERS' CONTENT KNOWLEDGE IN CONTEXTUAL FUNCTION PROBLEM SOLVING 42

Background 42
Teachers' Knowledge of the Concept of Function 43
Test Development 45
Problem 47
Research Questions 49
Theoretical Framework 50
Item Selection and Review 50
Measurement Issues 52
Factor Analysis 53
Method 55
Development of the Instrument/Item Creation 55
Item Review 61
Item Review Participants – Subject Matter Experts 62
Item Review Data 62
Instrument Validation Participants 63
Instrument Validation Data 64
Results 65
Item Review 65
Instrument Validation 69
A Priori Assumptions about Links 73
Construct Validity 75
Factor Analysis 76
Discussion 79

CHAPTER	Page
IV CONCLUSIONS	84
Implications for Preservice Teacher Education	85
Limitations	92
Delimitations	93
Assumptions	94
REFERENCES	95
APPENDIX A	108
APPENDIX B	122
APPENDIX C	131
APPENDIX D	132
APPENDIX E	136
APPENDIX F	137
APPENDIX G	147
APPENDIX H	153
APPENDIX I	159
APPENDIX J	170
APPENDIX K	175
VITA	185

LIST OF FIGURES

FIGURE Page

1 Kulm's Problem Solving Model 8

2 Scores in Rank Order 71

3 Scores in Problem Order 71

4 Level of Demand of Mathematical Tasks with Score Ranks and Links 72

LIST OF TABLES

TABLE		Page
1	Frequency of Preservice Teachers' Strategies to Decode the Imbedded Function Concept from Contextualized Problems	23
2	Frequencies of Procedural Approaches Used by Preservice Teachers Use in Contextual Problem-Solving	23
3	Frequencies of Preservice Teachers Demonstration of Conceptual Knowledge of Functions in Contextual Problem-Solving	24
4	Webb's Depth of Knowledge Levels	59
5	Test Items in Webb's DoK Levels	61
6	Modification of Instrument Items Based on Item Review Results	66
7	Test Items in Smith & Stein's Framework	68
8	Problems, Scores and Ranks	70
9	Cross Tabulations on Linked Items	74
10	CFA Output Summary for Factors Based on Levels of Demands	77
11	CFA Output Summary for Factors Based Mathematical Topic	78

CHAPTER I

INTRODUCTION

The genesis of this research project occurred after I conducted several mathematics content knowledge review sessions for two groups of preservice teachers enrolled in alternative certification programs (ACP). In working with preservice middle school mathematics and secondary school mathematics/science teachers, I found that many of them struggled to understand the requirements and context of the problems more than the specific mathematical operations required for a solution. After I explained the problem context and requirements, most of the preservice teachers were able to perform the correct mathematical operations to find a solution. These experiences led me to form a basic hypothesis: middle and secondary school teachers do not have sufficient content knowledge of the function concept in contextual environments to perform at the high expectations of today’s STEM curriculum.

Generally speaking, teachers lack depth in mathematics content knowledge and the concept of function (Even, 1993; Kulm, 2008; Leinhardt, Zaslavsky, & Stein, 1990; Sherin, 2002; Stein, Baxter, & Leinhardt, 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979). Learning complex concepts, such as the concept of function, is

---

This dissertation follows the style of the *Journal for Research in Mathematics Education*.

much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995). Mathematics education researchers need a specialized instrument to ascertain the level of teachers' content knowledge of the function concept in a contextualized environment but such an instrument was not found in the literature. Having such an instrument would allow researchers to determine what steps were needed to improve teachers' content knowledge.

This study details the development of such an instrument through two empirical research projects. The first project examines preservice teachers' contextualized function problem-solving cognition by analyzing qualitative data collected in task-based interviews and serves as the pilot test of the instrument. The second project describes the item selection, development, and the psychometric properties of the instrument. The two projects are highly connected through an iterative process of design and testing.

In 1975, the National Advisory Committee on Mathematics Education (NACOME) paved the way for the public school reform efforts that restructured the teaching of mathematics in general and algebraic concepts in particular (O'Callaghan, 1998). Algebra, misunderstood by some to be merely a study of variables and symbolic manipulation (Driscoll, 1999), plays a central role in students' mathematical development. Algebra can be thought of as the mathematical "bridge" across which secondary students must pass to reach advanced mathematical concepts in high school (Dooren, Verschaffel, & Onghena, 2002) as well as post secondary studies in the science, technology, engineering, and mathematics (STEM) subject areas.

Robert Moses, the noted leader of the Algebra Project spoke of math literacy, and more specifically algebra, as the new focal point in civil rights:

> Today, I want to argue, the most urgent social issue affecting poor people and people of color is economic access. In today's world, economic access and full citizenship depend crucially on math and science literacy. I believe that the absence of math literacy in urban and rural communities throughout this country is an issue as urgent as the lack of registered Black voters in Mississippi was in 1961 (Moses & Cobb, 2001, p. 5).

Even though our nation's schools have made math literacy a priority in education, a persistent gap in algebraic achievement between students from minority groups and White students exists (Ladson-Billings, 1997; Moses & Cobb, 2001; Richardson, 2009).

Of the algebraic topics covered in middle and secondary schools, researchers recognize the concept of function as the single most important tool in algebra regarding a student's ability to apply mathematical concepts in sciences, engineering and other related contexts (Hollar & Norwood, 1999; Knuth, 2000; Lloyd & Wilson, 1998; O'Callaghan, 1998; Zbiek, 1998). Underdeveloped knowledge of the function concept hinders the mathematical development of students. The National Council of Teachers of Mathematics (NCTM) expects all students, beginning in grades six, to be able to "model and solve contextualized problems using various representations such as graphs, tables, and equations" (National Council of Teachers of Mathematics, 2000b), which requires students to possess a working knowledge of functions in contextualized environments.

With the focus of mathematics education research on students' achievement, it is important to note that teacher knowledge is the most important factor influencing student learning (Dooren et al., 2002; Lappan & Ferrini-Mundy, 1993; National Council of Teachers of Mathematics, 1991). Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (N. Webb, 1979) and earlier research found that teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). The lack of full understanding of the function concept impacts the preparation of teachers by bringing a marginalized view of functions to their classrooms (Leinhardt et al., 1990; Lloyd & Wilson, 1998; Stein et al., 1990).

Teachers' knowledge is not a static core of ideas but rather an ever changing pool of ideas that are supplemented, refined, and at times diminished during the instructional life of the teacher (Sherin, 2002). It is important that the initial pool of preservice teachers mathematical knowledge is as *deep and clear* as possible (Hill, Rowan, & Ball, 2005; Kulm, 1982; Li & Kulm, 2008). A rich source of knowledge for the teacher reduces the number of modifications to existing mathematical misconceptions and misrepresentations teachers must correct as they gain experience in the classroom. This pool of knowledge forms long before the preservice teacher enters a college classroom, but it is the college experience that is expected to deepen and widen the pool.

Some research suggests that the best way to learn the concept of function is over an extended period (years in some cases) (Yerushalmy, 2000). Through these years of mathematics instruction, are the preservice teachers actually absorbing the knowledge in such a way that it will provide an initial foundation that can be used in the classroom? Finding teachers that have acquired and maintained a strong concept of function can be particularly challenging in an environment where it has been reported that 65% of the middle school teachers neither have a mathematics degree nor have certification in mathematics (Li & Kulm, 2008).

Traditional algebraic instruction stresses the memorization of algebraic facts and symbolic manipulation at the expense of problem-solving skills and conceptualization (Hollar & Norwood, 1999; Karsenty, 2002; O'Callaghan, 1998). Research has uncovered the following disturbing corollary to this shortsighted view of algebraic instruction: students have been found to internally characterize word problems as artificially contrived classroom based scenarios that have little or no relation to problems of the real-world (Greer, Verschaffel, & De Corte, 2002; Verschaffel, De Corte, & Borghart, 1997). This suggests students feel free to suspend common sense approaches and the benefit of their personal real-world experiences and attack the word problem with simple mathematical facts and algorithms they have previously learned.

In their *Connections* standard, NCTM prescribes "Instructional programs from prekindergarten through grade 12 should enable all students to recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (National Council of Teachers of Mathematics, 2000a, p. 63). NCTM takes the position that mathematical lessons learned by students in connected curricula are richer and are better retained than a lesson of isolated mathematical thoughts. They also contend that through contextual lessons, students gain a better feel for both the utility of mathematics in other subject areas as well as a deeper understanding of the related content area of which the mathematics lesson is connected. Since research reflects more than one definition of contextual lessons (Roth, 1996), it is sensible to examine definitions and descriptions of contextual teaching and learning.

Williams (2007) describes contextual teaching as “a methodology of teaching that connects academic concepts to real-world conditions and encourages students to see how what they learn relates to their lives” (p. 572). Contextual teaching is also described as an integration of social constructivism, situated cognition, multiple intelligences and brain based learning theories (Lynch & Harnish, 2003; Williams, 2007). Roth (1996) cautions that contextualized problems are not just word problems with added verbiage, but rather “an expression is contextualized as part of meaningful practices rather than through an increase in (sign-based) situation descriptions” (p. 489). Roth’s minority opinion of contextual teaching follows a perspective held by John Dewey (Menand, 2001) that it is necessary for students to actively (kinesthetically) engage in the learning process. Current mathematics education research does not show a strong link between Dewey’s kinesthetic learning perspective and contextual teaching.

A significant aspect of the notion of contextual learning and problem-solving lies beyond the definition or description and entails how students are impacted by it. On the surface, research suggests the importance of “finding meaning by connecting academic work with daily lives” (Lynch & Harnish, 2003, p. 6) , and the idea that contextual lessons add meaning to both the mathematical topic as well as justify increased interest in academics (Lynch & Harnish; Wiseley, 2009). However, a deeper rationale for this study lies in the way contextual lessons help students convert seemingly correct mathematical answers into solutions based on “real-world” thinking. The following two problems best illustrate this point (Verschaffel et al., 1997, p. 340):

> Steve has bought 4 ropes of 2.5 metres each. How many ropes of 0.5 metre can he cut out of these 4 ropes? Steve has bought 4 planks of 2.5 metres each. How many planks of 1 metre can he saw out of these 4 planks?

Students can compute a correct response to the first problem without considering real-world consequences; 4 ropes * 2.5 meters/rope * 1 rope section/0.5 meters = 20 rope sections. But the same straightforward application of mathematics applied to the second problem leaves the student with a seemingly correct mathematical solution of 10 planks, which of course is impossible based on the context of the problem.

Research has shown empirically that students have a strong tendency not to use their common sense in solving word problems but would rather rely on absent-minded repetition of the drills practiced in school mathematics (Greer et al., 2002; Verschaffel et al., 1997). The following diagram (Figure 1) of the problem-solving process shows the dependence on interpreting the mathematical solution in light of contextual constraints to arrive at a “real” solution.

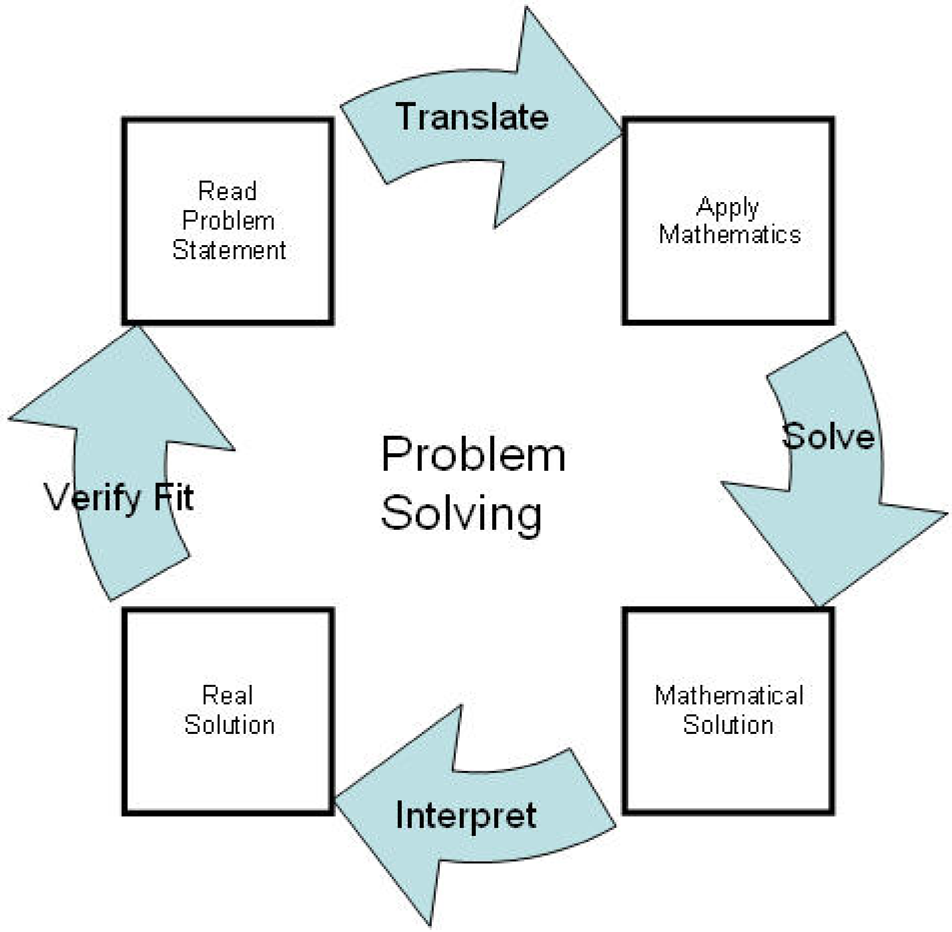


Figure 1 *Kulm's Problem-solving Model* (Kulm, 1982)

Since mathematics teachers have the greatest influence in students' mathematical development (Dooren et al., 2002), it is imperative that teachers possess sufficient depth of mathematical content knowledge and experience in contextual problem-solving to effectively lead student learning. Researchers believe one of the contributing factors to the disconnect between real-world insights in students' problem-solving abilities is "the way in which these problems are considered and used in current instructional practice and culture, and more specifically the lack of systematic attention to the modeling perspective by the teacher"(Verschaffel et al., 1997, p. 340). Likely reasons teachers avoid modeling highly contextualized problems in their classrooms are their inexperience and lack of confidence in solving this type of problem.

A teacher's ability to solve problems in a contextual (using real-world insights) environment needs to be assessed to give teacher educators insight into possible modifications to preservice programs and to provide school administrators a clearer perspective into the professional development needs of mathematics teachers. This need, and the fact that a contextual function instrument was not found in the literature, drives the rationale for creating and validating such an instrument.

Statement of the Problem

Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (N. Webb, 1979). More specifically, teachers lack depth in their conception of the mathematical function (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979). Learning complex concepts, such as the concept of function, is much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995) but research suggests that the primary use of contextual, real-world instruction is relegated to developmental mathematics courses in post secondary education (Grubb & Kraskouskas, 1992; Stone, Alfeld, Pearson, & Lewis, 2006; Wiseley, 2009) even though state teacher certification examinations are rich in contextualized problems (Educational Testing Service, 2009a, 2009b; Texas Education Agency, 2009).

If we, as mathematics educators, expect to improve preservice teachers' content knowledge of the function concept in a contextualized environment, then the first step will be to assess their current knowledge. Only after we have assessed their current level of knowledge can we plan modifications to preservice teacher education and perhaps teacher professional development programs. In order to make a sound assessment, we must have a reliable and valid instrument at our disposal. There are many extant survey instruments to measure a wide range of mathematical attitudes, affective behaviors, and content knowledge dimensions, including general function concepts, but an instrument that specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published.

Statement of Purpose

The overarching goal of this dissertation research was to develop and validate an instrument to assess preservice teachers' content knowledge of the function concept embedded in contextual problems. This goal was accomplished through two research projects described in two central chapters. Chapter II reports on the collective case study that was used to pilot test the instrument, and Chapter III details the rationale used in item selection and the psychometric properties of the new instrument. Unlike existing research studies that examine a broad range of function-related topics using various forms of non-contextual symbolic, tabular, and graphical representations as the basis for questions and problems, this study focuses solely on function assessment problems immersed in various real-world contexts.

Since this is not a common approach to measuring content knowledge of the function concept, the existing instruments in published studies were not found to be suitable for this specialized purpose. An instrument dedicated to measurement of content knowledge of functions using problems that are immersed in real-world contexts was developed and tested for validity and reliability measures.

Research Questions

The overarching question that defined and guided both research projects described in this study was, "how can preservice mathematics teachers' content knowledge of the function concept be assessed?" The following questions form the basis of this investigation:

1. How do preservice teachers demonstrate their knowledge of conceptual and procedural problem-solving skills related to contextualized mathematical function problems?
   a. How do preservice teachers decode the imbedded function concept from a contextualized problem?
   b. Which procedural approaches do preservice teachers use in problem-solving?
   c. How do preservice teachers demonstrate their conceptual knowledge of functions in problem-solving?

2. What are the key items in assessing contextual function concepts that should be included in an instrument to assess preservice mathematics teachers' knowledge?
3. What are the psychometric properties of an instrument developed to assess preservice middle and secondary mathematics teachers' knowledge of the mathematical concept of function within a contextual environment?

CHAPTER II
OBSERVING PRESERVICE TEACHERS' CONTEXTUALIZED FUNCTION PROBLEM-SOLVING THROUGH TASK-BASED INTERVIEWS

Introduction

In 1975, the National Advisory Committee on Mathematics Education (NACOME) paved the way for the public school reform efforts that restructured the teaching of mathematics in general and algebraic concepts in particular (O'Callaghan, 1998). Algebra, misunderstood by some to be merely a study of variables and symbolic manipulation (Driscoll, 1999), plays a central role in students' mathematical development. Algebra can be thought of as the mathematical "bridge" across which secondary students must pass to reach advanced mathematical concepts in high school (Dooren et al., 2002) as well as post secondary studies in the science, technology, engineering, and mathematics (STEM) subject areas.

Of the algebraic topics covered in middle and secondary schools, researchers recognize the concept of function as the single most important tool in algebra regarding a student's ability to apply mathematical concepts in sciences, engineering and other related contexts (Hollar & Norwood, 1999; Knuth, 2000; Lloyd & Wilson, 1998; O'Callaghan, 1998; Zbiek, 1998). Underdeveloped knowledge of the function concept hinders the mathematical development of students. The National Council of Teachers of Mathematics (NCTM) expects all students, beginning in grades six, to be able to "model and solve contextualized problems using various representations such as graphs, tables, and equations" (National Council of Teachers of Mathematics, 2000b), which requires students to possess a working knowledge of functions in contextualized environments.

Problem

Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (N. Webb, 1979). More specifically, teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979).

Learning complex concepts, such as the concept of function, is much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995) but research suggests that the primary use of contextual, real-world instruction is relegated to developmental mathematics courses in post secondary education (Grubb & Kraskouskas, 1992; Stone et al., 2006; Wiseley, 2009) even though state teacher certification examinations are rich in contextualized problems (Educational Testing Service, 2009a, 2009b; Texas Education Agency, 2009).

There are many extant instruments to measure a wide range of mathematical content topics and attitudes, but an instrument that specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published.

The purpose of this study was to describe the mathematical problem-solving processes related to contextualized mathematical function problems for six preservice middle and secondary school mathematics teachers at a large Southwestern, research intensive university. The results of the study will contribute to the validation of a new instrument to assess the content knowledge of the mathematical function concept of preservice teachers as they complete contextualized problem-solving tasks. Conclusions about the participants' mathematical problem-solving processes were derived from the verbal responses to interview questions, the written responses to the assessment instrument, and the interpretation of the researcher's observation of the preservice teachers' responses during task-based interviews. Inferences drawn from the mathematical problem-solving cognition will aid in the development and validation of an instrument to assess preservice mathematics teachers' knowledge of how to connect their knowledge mathematical concept of function to a contextual setting.

Research Questions

The research questions were driven by a desire to gain a richer understanding of the internal cognitions and procedures used by preservice teachers as they attempt to understand and apply the concept of function in a richly contextualized environment. The central question that guided this study was: How do preservice teachers demonstrate their knowledge of conceptual and procedural problem-solving skills related to contextualized mathematical function problems?

The following sub questions served to sharpen the focus of the study:

1. How do preservice teachers decode the imbedded function concept from a contextualized problem?

2. Which procedural approaches do preservice teachers use in problem-solving?

3. How do preservice teachers demonstrate their conceptual knowledge of functions in problem-solving?

Theoretical Framework

This study proposes to discover aspects of the participants' knowledge related to the task of solving contextualized mathematical function problems. Since it is not possible to directly access a person's thoughts, Piaget's clinical examination forms the theoretical basis for this study (Piaget, 1965). As Goldin (1997, 2000) builds on Piaget's base and focuses the work on mathematics education research through task-based interviews, he cautions that the outcomes from clinical interviews cannot be considered to be thoughts or cognitions. Goldin's perspective is guided by, "First, it is crucial to maintain carefully the scientific distinction between that which is observed and inferences that are drawn from observations" (1997, p. 52). Therefore a foundational theoretical postulation for this research is based on the notion that it is not possible to observe a mathematical construct within a student's mind, but rather the observations allow us to infer something about the student's internal mathematical constructs.

In addition to allowing us to infer qualities of the participant's internal mathematical cognitions, the theoretical framework should also describe how the characteristics of the mathematical tasks under study interact with those internal cognitions of the participant (Goldin, 1997). A task's language, mathematical content and structure, mathematical appropriateness, and the interview context are examples of the characteristics of the task. Kulm and others describe these as "task variables" in problem-solving research (Kulm, 1979; N. Webb, 1979). In this research project, the task's mathematical content and structure and the mathematical appropriateness of the task are principally based on the TExES™ and Praxis™ teacher certification examinations, upon which the primary research instrument is based (I. A. Brown, 2011).

The task's language and the interview context are key characteristics under study in this research project. This research assumes the theoretical position that the language of the contextualized function problems of the primary instrument will require the participant to work at a much higher level of cognition than when solving straightforward, "given these conditions, find the value of x" type of problems. By moving beyond the simple recall of definitions and basic algorithms, the participant will have the opportunity to demonstrate their deeper understanding of the central concepts of mathematical functions and their ability to use this knowledge to aid in their problem-solving approaches. The interview context is based on the design principle that holds the participant's free expression of mathematical ideas is of the utmost importance in the interview. Answers are to be accepted as "good" by the interviewer (researcher) whether the answers are correct or not. By using only non-directive follow-up questions and heuristic suggestions, the participant's responses are expected to yield rich data that will allow the researcher to infer qualities of the participant's internal mathematical constructs.

The theoretical foundation for the task-based interviews, as described by Goldin (1997) is based on three fundamental assumptions. First, it is assumed that the student's internal mathematical cognitions or "competencies and structures of such competencies" (p.55) are able to be inferred from external behaviors. Specifically, these internal structures are able to be inferred from the data gathered from task-based interviews when the task and the interview elicit certain cognitive and behavioral responses from the interviewee. The second assumption “is the idea that competencies are encoded in several different kinds of internal representations and that these interact with one another and with observable, external representations during problem-solving” (Goldin, p. 55). The third assumption is that a student’s representational actions are based on internal and/or external configurations which “symbolize” other representational configurations.

Method

Participants

The participants in this study were preservice middle school mathematics teachers enrolled in a preservice teachers’ mathematics education problem-solving course at a large Southwestern, research intensive university. Purposeful sampling was used to choose six students from the class based on their willingness to participate in this pilot study. To effect a *maximum variation* type of sampling (Creswell, 2007, p. 127), two of the highest scoring students and two of the lowest scoring students were chosen from the group that volunteered to participate along with two students that scored near the mean of the class scores.

Five of the six preservice teachers were preparing to teach middle school mathematics (education majors) while the sixth participant, Teacher4, was a mathematics major preparing to teach secondary school mathematics. One of the participants, Teacher6, was previously a mathematics major before becoming an education major. All six of the participants were classified as university juniors; four of which were White females; there was one Black female and one White male.

Instrument

There were two instruments, both created by the researcher, employed in this study. The primary instrument, the “Contextual Function Instrument”, was an initial draft of an instrument which consisted of fifteen multiple choice questions designed to assess middle and secondary school preservice mathematics teachers’ knowledge of the mathematical function concept in a contextually rich environment. This primary instrument was the focus of the pilot study and is the instrument that will ultimately be validated in a separate research study.

Prior to creating a new instrument, research literature was searched in an attempt to find an instrument that would satisfy the requirements of being highly contextualized as well as focusing on pre-calculus level function concepts. Included in the review of research literature were the following databases available on the Texas A&M University Library website: Academic Search Complete (EBSCO), ERIC, ScienceDirect, and the Web of Science (Texas A&M University, 2010). Additionally, function instruments were sought using GoogleScholar, and the test collection at Educational Testing Service (ETS).

The questions on the primary instrument were created by the researcher and are based on common competencies described in the TExES™ and Praxis™ teacher certification examinations for middle and secondary school preservice mathematics teachers (Educational Testing Service, 2009c; Texas Education Agency, 2009). Texas middle school mathematics teachers are required to demonstrate successful performance on the “115 Mathematics 4 – 8” examination and Texas secondary school mathematics teachers are required to demonstrate successful performance on the “135 Mathematics 8 – 12” examination (Texas Education Agency, 2008). North Carolina middle school mathematics teachers are required to demonstrate successful performance on the “Middle School Mathematics (0069)” examination and secondary school mathematics teachers are required to demonstrate successful performance on the “Mathematics: Content Knowledge (0061)” examination (Educational Testing Service, 2009a, 2009b, 2009c).

The second instrument was a task-based interview protocol script of five primary questions asked of the participants during the interview. In the instances when the five primary questions did not consume the entire 45 minute interview time slot, a sixth auxiliary question was used. The development of the interview protocol, which is shown in Appendix A, is based on the work of Goldin’s (1997) task based interview protocol creation guidelines:

> The script is written so that for each main question, exploration proceeds in four stages: (a) posing the question ("free" problem-solving) with sufficient time for the child to respond and only nondirective follow-up questions (e.g., "Can you tell me more about that?”); (b) heuristic suggestions if the response is not spontaneous (e.g., "Can you show me by using some of these materials?"); (c) guided use of heuristic suggestions, again to the extent that the requested description or behavior

> does not occur spontaneously (e.g., "Do you see a pattern in the cards?"); and (d) exploratory (metacognitive) questions (e.g., "Do you think you could explain how you thought about the problem?") (p. 45).

The goal of the instrument's design was to guide the interviewer in eliciting complete, coherent mathematical thoughts from the participant for each of the questions.

Data Collection

Qualitative data for this collective case study were collected face to face by the researcher (interviewer) in a conference room on campus, using the aforementioned protocol in 45 minute, semi structured, individual interviews. The interviews were audio recorded and those recordings were transcribed for data analysis. The researcher also made notes during the interview to supplement the transcription and the participants' written responses to the tasks presented.

Results

Data Analysis

The data consisted of the audio recorded interviews, the written problem-solving solutions from the participants, as well as the interviewer's notes. The audio tapes were transcribed and read several times by the researcher to help form initial open coding. Direct interpolation was also used to analyze data within each individual case. An in-depth depiction of the cases using narrative analysis was created to give a clearer image of the individual participants.

Each of the participants' responses (transcribed from audio recordings) was subdivided into "logical segments" of information that allowed the researcher to infer something of the participants thinking relating to the assigned task. The interview transcripts yielded a total of 209 logical segments for all six participants. Using a constant comparison approach (Boeije, 2002; Glaser, 1965; Maxwell, 2010), 18 researcher-defined codes were developed and used to interpret these 209 logical segments. For example, when asked about the method used to arrive at a particular answer, Teacher3's response included the statement, "I'm trying to remember, like, the equation for surface area and I'm thinking that since we really don't need to know how much it costs that would eliminate answer A and answer C." That logical segment was assigned the code, "eliminate answer choices." To validate the primary researcher's coding process four other researchers reviewed, supplemented, and modified the coding scheme.

The three research sub-questions were used as themes and categorical aggregation was employed to classify data across the six cases. The following tables show the codes (Table 1), the frequency of their occurrence (Table 2) and their rank within the specific groups (Table 3).

Table 1. *Frequency of Preservice Teachers' Strategies to Decode the Imbedded Function Concept from Contextualized Problems.*

Rank	Decoding Approach	Frequency
1	Decode the context and eliminate the extraneous text without getting overwhelmed by the amount of text.	61.4%
2	Use function notation and proper mathematical language to re-state and understand the problem context.	18.2%
3	Convert problem into their own words to better understand what the problem is asking.	11.4%
4	Relate the embedded function in familiar "x" and "y" terms even when the Cartesian coordinate system has nothing to do with the problem at hand.	4.5%
5	Assume the given mathematics problem is actually a "physics" problem and therefore that it is outside of their knowledge base.	4.5%

Table 2. *Frequencies of Procedural Approaches Used by Preservice Teachers Use in Contextual Problem-Solving.*

Rank	Procedural Approach	Frequency
1	Eliminate answer choices to reduce the probability of making a wrong answer selection.	35.4%
2	Use to mathematical definitions (from memory) to serve as foundations in their problem-solving strategies.	25.7%
3	Use graphing calculators.	10.6%
4	Use dimensional analysis to resolve disparities between the units given in the problems statement and the solution choices.	9.7%
5	Used conceptual understanding of mathematics and functions to guide their solution formulation.	8.8%
6	Decide on a potential formula for a solution, then "plugged in" one of the given numerical values and checked to see if the solution matches an answer choice.	4.4%
7	Apply proportional reasoning as an analytic method of solution.	2.7%
8	Use knowledge from prior problems and filtering of given information to find solution.	2.7%

Table 3. *Frequencies of Preservice Teachers Demonstration of Conceptual Knowledge of Functions in Contextual Problem-Solving.*

Rank	Demonstrated Approach	Frequency
1	Demonstrate the ability to apply real-world knowledge as constraints within their mathematical problem-solving.	38.5%
2	Use the definitions of a mathematical function as well as function notation.	30.8%
3	Use multiple representations to conceptualize the problem and its solution.	23.1%
4	Struggle with *unknown* constants in the problem even when the value of the problem is not necessary to find a solution.	3.8%
5	*Filter* decoded functional concepts to find the most appropriate use.	3.8%

The logical segments that included the five codes in Table 1 accounted for about 21% of the total segments across the six cases (44 out of 209). An analysis of the original questions asked, the participants' statements that contained the logical segments and the correct answers to the questions revealed that the participants were 100% successful when they converted the problem into their own words (rank #3) and when they related the problems to the Cartesian coordinate system (rank #4). The participants were found to be totally unsuccessful (0%) when the assumed the problem was actually a "physics" problem (rank #5). They were successful in about 63% of the cases when they tried to use function notation and proper mathematical language (rank #2), but only successful at a rate of about 26% when they attempted to decode the context of the problem and eliminate extraneous text (rank #1).

The logical segments related to the eight codes in Table 2 accounted for about 54% of the total segments across the six cases (113 out of 209). An analysis of the original questions asked, the participants' statements that contained the logical segments and the correct answers to the questions revealed that the participants were 93% successful when they eliminated answer choices (rank #1). The participants were found to be successful in 83% of the cases when they used graphing calculators (rank #3) and 80% successful when they used their conceptual understanding of mathematics and functions to guide their solution formulation (rank #5). They were successful in about 67% of the cases both when they applied proportional reasoning (rank #7) and when they used knowledge from prior problems and filtering of given information to find the solution (rank #8), but only successful at a rate of about 40% when after deciding on a potential formula for a solution, they "plugged in" one of the given numerical values and checked to see if the solution matched an answer choice (rank #6). Finally, the participants were found to be successful about 64% of the time when they used dimensional analysis to resolve disparities between the units given in the problem statement and the solution choices (rank #4).

The logical segments related to the five codes in Table 3 accounted for about 25% of the total segments across the six cases (52 out of 209). An analysis of the original questions asked, the participants' statements that contained the logical segments and the correct answers to the questions revealed that the participants were 100% successful when filtering decoded functional concepts to find the most appropriate (rank #5) and 95% successful when they demonstrated their ability to apply real-world knowledge as constraints (rank #1). The participants were found to be totally unsuccessful (0%) when they struggled with "unknown" constants in the problem (rank #4). They were successful in about 83% of the cases when they used multiple representations to conceptualize the problem and its solution (rank #3), and successful at a rate of about 75% when they attempted to use their definitions of a mathematical function as well as function notation (rank #2).

Even though the teachers' correct answers were not the primary focus of this study, it is still worth noting that the individual scores ranged from a low of 10% correct to a high of 80% correct. Overall, the six participants answered a total of 26 task related questions correctly out of the 59 task questions posed (44%). Due to the time constraint of the interview and the fact that some participants answered much more quickly than others, each of them were not asked the same number of questions. Participants were asked as few as eight questions and as many as 12 questions with the average being about 9.8 task questions.

It is interesting to note that none of the preservice teachers interviewed had a strong definition of a mathematical function in memory. When asked, "What is a mathematical function", the responses varied significantly. One participant responded:

> A mathematical function is a lot of things hmm, in my opinion, it's where (pause) I don't know how to explain, it's something I'm so used to doing, when I have to explain what it actually is, it's been so long since I've been told a definition that I just go through it. I wouldn't know how to explain it; it's something that comes with experience.

Another participant offered a specific definition that characterized a one-to-one correspondence between variables, and another said a mathematical function is something that shows a relationship between two or three things. Yet, even with a weak demonstrated function definition, all of the participants were successful about three-quarters of the time when they attempted to use their definitions of a mathematical function as well as functional notation. Rizzuti (1991), using 12^{th} grade mathematics students in her study, also found that her participants' weak function definitions did not hinder their function-related problem-solving ability.

Interpretation of Individual Participants' Responses

Teacher1

My initial impression of Teacher1 was that she was uncomfortable with the mathematics involved. Of the six participants, she displayed the least positive body language and she made the most negative statements about the questions on the instrument and her ability to answer the questions. My experience with students of mathematics suggests to me that her affective display was more likely related to her lack of confidence rather than lack of knowledge or skill. She answered two of the nine questions correctly, but more importantly she had a very diverse (and at times conflicting) range of responses to the questions.

She struggled with the definition of a function as well as the concept of direct and inverse proportionality, but during a discussion of a different problem she was able to demonstrate her knowledge of the function concept by implicitly expanding the functional relationship between heat and evaporation loss.

> …but a lot of things in the real-world isn't just linear, it has a lot of variability, variables throughout the day. I mean it could be sun shining one day so you are going to lose… with heat you are going to lose more

> water with evaporation whereas some days it's not sun shining so it's not going be; whereas [solution] D [she refers to a linear graph of evaporation losses versus pool radius] makes it seems like it will be sunshine every single day and in life it's really not. Some days are cloudy some days are not, so that it's not going to be perfect, staggering and that's why I say [solution] B [she refers to a quadratic graph of evaporation losses versus pool radius] because it has that curve so it kind of leaves more leeway to the days.

This problem explicitly states, "…that water evaporation is directly proportional to the surface area of the pool…", yet she was able to extend this idea based on prior knowledge that the evaporation rate of water is a function of heat. Unfortunately, this added knowledge may have played a role in the confounding of the answer. She chose the correct answer, but the line of thinking she used to arrive at that conclusion was flawed.

In her discussion of problem solutions, she demonstrated the ability to apply proper contextual meaning to the mathematics with statements like, "I would eliminate C because I know the radius wouldn't be negative; [there is] really not such a thing as a negative radius". But she was not consistent in this demonstrated ability, as shown by her notion in the passage above that the graph of a linear function suggests something "perfect" whereas the smooth curve of a quadratic function implies a more natural and "variable" response.

On several occasions Teacher1 made statements like, "… there's a lot of information in the problem and I'm trying to separate it in my mind so that I don't get anything confused", which served as examples of her demonstrated difficulties in decoding the imbedded function concept from contextualized problems. When she was able to fully grasp the concept within the contextual problem, she was able to demonstrate adequate mathematical procedural problem-solving skills. But these procedural skills were still overshadowed by statements like, "A mathematical function is a lot of things hmm, in my opinion, it's where (pause). I don't know how to explain, it's something I'm so used to doing, when I have to explain what it actually is (pause), it's been so long since I've been told a definition that I just go through it. I wouldn't know how to explain it; it's something that comes with experience." Teacher1 demonstrated knowledge of problem-solving procedures, like immediately eliminating as many answer choices as possible, but her ability to clearly demonstrate her conceptual knowledge of functions was not apparent.

Teacher2 and Teacher5

Teacher2 and Teacher5 are grouped together because they were quite similar in more than just demographics (White, female, preservice middle school teachers), they showed similar problem-solving approaches and responses. They had similar ratios of correct response to interview questions (2/10 and 3/12, respectively) and they both answered the same questions correctly and used similar reasoning in arriving at answer choices. In their solution to a problem that defined a function such that "the water evaporation is directly proportional to the surface area of the pool", both focused on a self-defined concept of change to explain their conceptual knowledge of functions. Teacher2 correctly chose answer choice B and explained, “I’m thinking that because it’s directly proportional, that means that the proportion is not changing or wouldn’t be changing and so I feel like it should be a straight line and not a curve or maybe I don’t know.” (It should be noted that independent of her self defined concept of change in graphs and its relationship to the term *directly proportional*, her answer choice contradicts her statement; choices B and C show the graphs of quadratic functions and choices A and D show the graphs of linear functions.) Teacher5 also correctly chose answer choice B and explained, “I would go with B just for the fact that that function is changing and I think it would change.” When asked why she would eliminate choice A (the graph of the linear function) she continued to explain, “Because of the changing like that one is constant…” Teacher5, like Teacher2, also used good real-world knowledge to constrain her choice to those with positive values on the independent axis knowing that the pool radius could not take on negative values.

Teacher2 and Teacher5 both struggled to decode the imbedded function concept from the contextualized problems. “Difficult, … like when they throw in the extra stuff that’s not needed” and “I think that the length of the problems and this has been the case with most of them, it’s just so overwhelming when you’re trying to think about everything at the same time” were characteristic responses by both teachers. Both teachers employed the “eliminate answer choices” problem-solving strategy as their primary procedural approach to problem-solving without giving sufficient cause for the elimination.

When they demonstrated their conceptual knowledge of functions in problem-solving, Teacher2 and Teacher5 both showed strengths in their ability to use real-world knowledge to properly constrain their problem solutions. However, they both struggled with foundational function concepts; they were unable to remember proper function definitions or were unable to apply some that they did recall. An exception to this difficulty was noted in their approach to find the zeros of a quadratic function. In this case, both teachers clearly demonstrated their understanding of the need to set the given function equal to zero and solve for the independent variable.

Teacher3

Teacher3 offered quite a contrast in style and demonstrated knowledge from Teacher1, Teacher2 and Teacher5. Teacher3 approached the problems with a calm, confident manner that allowed her to work through the most difficult problems without showing signs of frustration. She correctly answered 7 of the 10 questions, and one of the three incorrect responses was due to a simple calculation error rather than a conceptual shortcoming. Most interesting was the fact that she was the first interviewee that actually "spoke aloud her thoughts" as she solved the problems with very little prompting from the interviewer. Though it was not explicitly part of the task-based interview, Teacher3 displayed strong pedagogical skills in her explanation of her thoughts and problem-solving techniques.

She was able to consistently decode the imbedded function concept by carefully translating the text of the stated problem into her own words. She was then able to use these translated ideas to explicitly state what she believed the problem was asking and to link to definitions and concepts that she was able to recall. Additionally, Teacher3 was able to determine which parts of the text were clearly extraneous and eliminate them from future consideration as she continued her problem solution.

The primary procedural approach to problem-solving used by Teacher3 was to apply her knowledge of mathematical definitions and function concepts to properly set the problems up for an analytic solution. When posed with a problem that involved inverse proportional function, Teacher3 explained her thought in the following way, “I kind of saw that inversely proportional to the quantity so I knew that the proportion, the quantity wouldn’t be on top, it would be on bottom and I knew I would divide because when you find the inverse it’s one over something.” On several occasions she also relied on the graphing calculator as part of her problem-solving approach, but she also made it clear that, “if I can, I’ll solve it without using the calculator to better understand why it's doing what it's doing.” While she also used the technique of the elimination of answers, she was more likely to actually work through the problem and compare her answer with the answer choices available.

Teacher3’s primary means of demonstrating her conceptual knowledge of functions involved the way she interpreted and set-up her problem solution strategy. For example she stated, “We know that they’re looking for how much evaporation loss is lost per day with the gallons of water and I know that the water evaporation is directly proportional to the surface area.” She continued by describing the mathematical operations that will be necessary to complete the calculations and properly used dimensional analysis to convert a given quantity into a quantity more suitable for problem solution. Teacher3 also showed strength in her ability to use real-world knowledge to properly constrain her problem solutions.

Teacher4

Teacher4 offered the greatest contrast between his demonstrated knowledge as evidenced by the number of correct answers to the posed questions (4/8) and the quality of his responses during the interview. Like Teacher3, Teacher4 was quite impressive as he “spoke aloud his thoughts” as he solved the problems. He did this with minimal prompting from the interviewer. Also like Teacher3, he displayed strong pedagogical skills in his explanation of his thoughts and problem-solving techniques and he was very confident and relaxed during the interview. As a preservice secondary school mathematics teacher, Teacher4’s additional mathematics training was evident in his discussions. In three of the four incorrect answers to questions posed, Teacher4 knew exactly how to set up and solve the problem. It seems that he rushed the solutions (in two cases, he did the problems in his head) and therefore gave incorrect answers to concepts and procedures that he clearly understood. There was only one problem for which Teacher4 did not understand how to arrive at a correct solution.

Like Teacher3, he was able to consistently decode the imbedded function concept by carefully translating the text of the stated problem into his own words. He was then able to use these translated ideas to explicitly state what he believed the problem was asking and to link to the wealth of mathematical definitions and concepts that he easily recalled. Furthermore, Teacher4 was also able to determine which parts of the text were clearly extraneous and eliminate them from future consideration as he continued his problem solution.

Teacher4 also mirrored Teacher3's proclivity for using an analytical approach to problem-solving, but unlike Teacher3 he demonstrated a mild aversion to the use of technology in his solutions. He openly confessed, "Then my lack of technology skills, umm, made it difficult to figure out the exact point and I'm very slow at using a calculator." His secondary procedural approach to problem-solving was the proper use of elimination of answers as he worked through the problems.

Teacher4 demonstrated his conceptual knowledge of functions in statements like, "It says it's a modulating activity therefore I was thinking that the graph will have a series of ups and downs…" and "and then the fact that the formula has a square we knew that the graph should increase exponentially not linearly." Like Teacher2, Teacher3 and Teacher5, he also showed as a strong point his ability to use real-world knowledge to properly constrain his problem solutions. Teacher4 was particularly adept at using dimensional analysis to verify his problem solution strategy.

Teacher6

Teacher6 was a preservice middle school mathematics teacher, but unlike the others, she was a mathematics major before switching to education. It seemed that this additional training in mathematics allowed her to perform at a higher level of competence than all of the other preservice teachers that were interviewed in this study. Teacher6 also approached the problems with a calm, confident manner that allowed her to work through the most difficult problems without showing signs of frustration. She correctly answered eight of the ten primary questions, and since she showed signs of having a deeper understanding of the concepts, she was asked several conceptual "follow-up" questions which she also answered correctly. Teacher6 also "spoke aloud her thoughts" as she solved the problems with very little prompting from the interviewer and like Teacher3 and Teacher4, she displayed strong pedagogical skills in her explanation of her thoughts and problem-solving techniques.

Unlike Teacher3 and Teacher4, she was unable to initially decode the imbedded function concept as she translated the text of the stated problem into her own words. She freely confessed, "If I would have been paying more attention and read the problem more carefully, it might have taken a lot less time." It seemed that she merely needed time to become acclimated to the dense text in these contextualized problems, because after reading the first few problems, Teacher6 was also able to determine which parts of the text were extraneous and eliminate them from future consideration.

Teacher6 also mirrored Teacher3's proclivity for using an analytical approach to problem-solving, but unlike Teacher4 she demonstrated a higher level of affinity for the use of technology (the graphing calculator) in her solutions than all of the other teachers interviewed. After describing how to solve the problem conceptually she added, "Then, since it gave you the equation, you can pretty much just plug it into the calculator and find out where it's equal to zero." Her secondary procedural approach to problem-solving was the proper use of elimination of answers as she worked through the problems.

Teacher6 demonstrated her conceptual knowledge of functions in statements like, “Ok so it gives you a function of the activity of the thing which is t^2+3 times t^2- 5t + 4 and it says you only can remove the capsule when the activity is at zero, so we need to find the roots of that function” and “the dependent variable would be the evaporation loss per gallon per day, and [the independent variable is] the radius of the circle of the pool.” Like most of the other teachers, she also showed her ability to use real-world knowledge to properly constrain her problem solutions. Teacher6 also paid attention the units of the problem and was adept at using dimensional analysis to verify her problem solution strategy.

Discussion

Decoding Strategies

One of the questions under consideration in this study was to better understand how preservice teachers decode the embedded function concept from a contextualized problem. The most challenging aspect of this question is, “How much context makes a good contextual problem and how much does it take to simply muddy the waters?” It was not a goal of this study to answer that question directly, but that question remains a part of the conversation as we consider the results. A priori assumptions concerning this research question included the preservice teachers being either confounded by the language and sheer volume of text in the problems and/or being unable to recall or apply enough function concepts and definitions to be effective problem solvers.

The results showed that the most frequently used approach to decode the embedded function concept was to “eliminate the extraneous text” but the participants who used that approach were only successful about a quarter of the time. On the other hand, participants were always successful when they "converted problem into their own words for better understanding" or "related the embedded function in familiar 'x' and 'y' terms even when the Cartesian coordinate system has nothing to do with the problem at hand." However, these more successful approaches were rarely used. These results show that the methods used by the preservice teachers most often were not the methods that they were most successful with and the most successful methods were among the least used by the preservice teachers. Though the studies are quite different, these results are contrary to finding by Nathan and Koedinger where beginning algebra students employed the methods "that gave the highest likelihood of success (around 70%) and led to the greatest number of correct solutions when they were applied" (2000, p. 176). A similar study using preservice teachers as the participants has not been discovered.

Procedural Approaches

The preservice teachers showed a wider range of procedural approaches than they did in either decoding or demonstrating function concept knowledge. Their procedural approaches accounted for about half of the total observed strategies. The most frequently used procedure, "eliminating answer choices", was also the method that produced success on almost every item. Eliminating answer choices, along with using dimensional analysis, "top-down" analytic solution methods and multiple representations were the strategies hypothesized the preservice teachers would use, but it was not anticipated that eliminating answer choices would be used to that extent. Nor was it anticipated that the eliminating answer choices procedure would be used in such a casual manner as using the statement “it doesn’t seem right” as justification.

Eliminating answer choices is a popular test-taking strategy that is taught throughout most of a student’s K – 12 academic experiences, but it is best applied after careful consideration of the correct answer to the problem (Hong, Sas, & Sas, 2006; Texas Education Agency, 2009). Students should not eliminate an answer choice simply because “it doesn’t seem right”, but rather they should have a logical basis for answer elimination. “Students should NOT eliminate answer choices that they are not sure about, only those that they can logically show are wrong using either information in the question or facts that they know” (Noel, 2010).

Prior to data collection, the researcher hypothesized a strong demonstration of their knowledge of conceptual and procedural problem-solving skills related to contextualized mathematical function problems would have included more “top-down” analytic solutions (which would include the use of correct mathematical definitions in their problem-solving strategies), more dimensional analysis used in problem-solving as well as more discussions in multiple representations. Only the top two teachers displayed these types of problem-solving skills. These skills add to the richness of problem-solving methods as mathematics students are tasked with complex STEM problems (Connally et al., 2004).

A promising result in this study was observed in the preservice teachers’ ability to apply real-world knowledge as constraints within their mathematical problem-solving. This strategy was their most frequently demonstrated approach as well as a very successful approach for them. The ability to properly apply constraints to a novel mathematical problem to create a useful real-world solution is a key attribute in project-based learning environments, which are central to STEM curricula (Prince & Felder, 2006). Another promising result was found in their ability to correctly use multiple representations to find or discuss problem solutions. Unfortunately, it was only used about a quarter of the time.

Implications and Conclusions

Since today's middle and high school mathematics teachers provide the academic foundation for future STEM college graduates, it is important for mathematics educators to be able to assess just how well prepared the mathematics teachers are in integrating contextual function problem-solving into the 6 – 12 curriculum. In the setting of a university mathematics course, a freshman or sophomore student can recite function definitions, examples and counter-examples of mathematical functions. The path to academic achievement can seem quite linear; the instructor lectures on the function concept, the students reinforce the lecture with homework assignments and within a relatively short time frame the students demonstrate their acquisition of this new knowledge by passing an exam. But what happens a year or two later when that same preservice teacher is asked to demonstrate their knowledge of the function concept? As these preservice teachers make ready to transition to in-service mathematics teachers, how much conceptual knowledge of functions do they maintain? How do they apply this knowledge in problem-solving? How well can preservice teachers adapt classroom function concepts in contextual applications that tend to be buried in extraneous details and text?

The ability to retain problem-solving skills is not a matter of memory retention related to specific problems or problem types, but rather a profound knowledge of the foundational concepts that form the backbone of the mathematical problems (L. Ma, 1999). Researchers need a way to measure teachers' knowledge to develop better preservice programs and summative assessments prior to professional licensure. If a reliable and valid instrument is employed to establish preservice teachers' content knowledge of mathematical function, then that instrument can be used with induction year in-service middle and secondary school mathematics teachers to establish the quality and quantity of problems solving skills retained in the transition from student to teacher.

The ability of a teacher to adapt content knowledge into pedagogical content knowledge and to employ it in the classroom in such a way as to increase student learning of mathematics is a central theme in mathematics education research (Darling-Hammond & Youngs, 2002; Hill, 2007; Hill & Ball, 2004; Hill et al., 2005; Kulm, 2008; Lappan & Ferrini-Mundy, 1993). Results from this study have provided a view into how preservice teacher decode function concepts embedded in context similar to traditional problems found in other STEM courses. The use of this instrument, and its future generations, can provide mathematics education researchers quantifiable evidence of the interaction between preservice teachers' knowledge and the methods they use to convert that knowledge into useful problem-solving skills.

There are many extant instruments to measure a wide range of mathematical content topics and attitudes, but an instrument which specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published. In the effort to validate a new content knowledge of mathematical function instrument, a pilot test was required to gain further insight into the mathematical thinking of preservice teachers while they complete contextualized mathematical function problem-solving tasks. The purpose of this case study was to describe the mathematical problem-solving cognition related to contextualized mathematical function problems for preservice middle and secondary school mathematics teachers. Inferences drawn from the mathematical problem-solving cognition will aid in the development and validation of an instrument to assess preservice mathematics teachers' knowledge of the mathematical concept of function.

Results, in general, will help establish the mathematical content and style of the next generation of the contextual function instrument. Results from the decoding strategies employed by the preservice teachers in this study will impact the complexity of the context in future problems and the finding from the procedural approaches used will likely push future instrument development towards a free-response instrument rather than a multiple-choice instrument.

CHAPTER III

THE DEVELOPMENT AND PSYCHOMETRIC MEASUREMENT OF AN INSTRUMENT TO ASSESS PRESERVICE MATHEMATICS TEACHERS' CONTENT KNOWLEDGE IN CONTEXTUAL FUNCTION PROBLEM-SOLVING

Background

Of the algebraic topics covered in middle and secondary schools, researchers recognize the concept of function as the single most important tool in algebra regarding a student's ability to apply mathematical concepts in sciences, engineering and other related contexts (Hollar & Norwood, 1999; Knuth, 2000; Lloyd & Wilson, 1998; O'Callaghan, 1998; Zbiek, 1998). The National Council of Teachers of Mathematics (NCTM) expects all students, beginning in grade six, to be able to "model and solve contextualized problems using various representations such as graphs, tables, and equations" (National Council of Teachers of Mathematics, 2000a), which require students to possess a working knowledge of functions in contextualized environments.

With the focus of mathematics education research on students' achievement, it is important to note that teacher knowledge is the most important factor influencing student learning (Dooren et al., 2002; Lappan & Ferrini-Mundy, 1993; National Council of Teachers of Mathematics, 1991; The Education Alliance, 2006). Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (Kulm, 2008) and earlier research found that teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990).

Finding highly qualified teachers can be particularly challenging in an environment where it has been reported that 68.5% of the middle school teachers in the US have neither a major nor have certification in mathematic (Li & Kulm, 2008). Do middle and secondary school mathematics teachers have sufficient content knowledge of the function concept in contextual environments to perform at the high expectations of today's STEM curriculum?

Since mathematics teachers have the greatest influence in students' mathematical development (Dooren et al., 2002), it is imperative that teachers possess the depth of mathematical content knowledge and the experience of contextual problem-solving to effectively lead students. Researchers believe one of the contributing factors to the disconnect between real-world insights in students' problem-solving abilities is "the way in which these problems are considered and used in current instructional practice and culture, and more specifically the lack of systematic attention to the modeling perspective by the teacher"(Verschaffel et al., 1997, p. 340).

Teachers' Knowledge of the Concept of Function

A question that might serve to focus our attention could be phrased simply as, "What do mathematics teachers need to know to be effective in teaching functions in middle and secondary mathematics classrooms?" Generally speaking, middle and secondary school mathematics teachers are responsible for diverse range of subjects ranging from pre-algebra through AP® calculus and the concept of function is a common thread woven through all of these courses (Educational Testing Service, 2009c; W. Ma & Freedson, 2002; National Council of Teachers of Mathematics, 2000a; Texas

Education Agency, 2008, 2009). To be successful in any of these courses, a mathematics teacher must have a firm grasp of both the pedagogical knowledge necessary for the classroom instruction as well as mathematical content knowledge of the material they are teaching (Kulm, 2008; Mohr, 2008). Shulman (1986) cautions against viewing these notions as being mutually exclusive or even independent thoughts, but rather he introduces the notion of "pedagogical content knowledge" to provide a paradigm bridge between these complementary ideas. But there are still unanswered questions concerning teacher knowledge.

Shulman regards the following as "the central questions for disciplined inquiry into teacher education":

> What are the domains and categories of content knowledge in the minds of teachers? How, for example, are content knowledge and general pedagogical knowledge related? In which forms are the domains and categories of knowledge represented in the minds of teachers? What are promising ways of enhancing acquisition and develop? (p.9)

These questions are used to form the basis of his theoretical framework of teacher knowledge, which he divides into three categories; subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Of these three, this study will focus on subject matter content knowledge.

If content knowledge is viewed as an independent construct that only served the individual teacher, it could be discounted as irrelevant to the goals of classroom instruction. Research has shown that teacher's knowledge is not an independent construct within only the teacher, but rather that a teacher's mathematical content knowledge is directly related to pedagogical content knowledge and therefore has a direct effect on student learning (N. Webb, 1979). Just knowing that a teacher has specific content knowledge is not enough, we need to know that they have the appropriate knowledge for the content domain being taught and we need to know that they have a sufficient level of that knowledge (Sherin, 2002). It was the aim of this study to develop and validate an instrument to measure middle and secondary school preservice mathematics teachers' content knowledge of the function concept in a contextual environment.

Test Development

Testing, which has been in existence for about 3000 years (Allen & Yen, 2002), is used to measure a characteristic, trait or quality in a person or object. A test can also be defined as "an evaluative device or procedure in which a sample of an examinee's behavior in a specified domain is obtained and subsequently evaluated and scored using a standardized process" (American Educational Research Association, American Psychological Association, National Council on Measurement in Education, & Joint Committee on Standards for Educational and Psychological Testing (U.S.), 1999, p. 3). Carmines and Zeller (1979) describe measurement as a "process" of quantifying empirical data with abstract concepts. The creation and subsequent validation of a "test" or assessment instrument is firmly grounded in measurement theory. "Measurement theory is a branch of applied statistics that attempts to describe, categorize and evaluate the quality of measurements, improve the usefulness, accuracy and the meaningfulness of measurements, and propose methods for developing new and better measurement instruments" (Allen & Yen, 2002, p. 2). In the quest improve the usefulness, accuracy and meaningfulness of quantitative instruments, two common metrics are often used: reliability and validity.

When developing new instrumentation for testing, great care should be taken to show that the newly created instrument is both reliable and valid. Without verification of these two key qualities, the proposed instrumentation cannot be trusted to provide a useful link between observable phenomenon and abstract ideas (Carmines & Zeller, 1979). Generally speaking, reliability is a measure of how consistent an instrument is in measuring the same quality repeatedly. "Fundamentally, reliability concerns the extent to which an experiment, test, or any measure procedure yields the same results on repeated trials" (Carmines & Zeller, 1979, p. 11).

Common methods of assessing reliability in social science research are internal-consistency estimates of reliability and include the split halves method which can yield the Spearman-Brown coefficient for tests where the halves can be classified as parallel, or Cronbach's alpha (α) when the test halves are considered essentially τ-equivalent (Allen & Yen, 2002; Carmines & Zeller, 1979). The Kuder-Richardson #20 (K-R 20) coefficient, like the Spearman-Brown coefficient and Cronbach's alpha, is also a measure of an instrument's internal consistency reliability. These forms of internal consistency reliability testing procedure display a contrast to procedures that aim to measure reliability over a period of time, such as the test-retest reliability procedure (Huck, 2008).

Reliability is a measure of an instrument's consistency whereas validity quantifies how well an instrument is able to measure its intended dimension. This could commonly be phrased, "how well a test measures what it is supposed to measure" (Orcher, 2005). Sireci (2007, p. 478) summarizes key aspects of validity in the following:

- Validity is not a property of a test. Rather, it refers to the use of a test for a particular purpose.
- To evaluate the utility and appropriateness of a test for a particular purpose requires multiple sources of evidence.
- If the use of a test is to be defensible for a particular purpose, sufficient evidence must be put forward to defend the use of the test for that purpose.
- Evaluating test validity is not a static, one-time event; it is a continuous process.

The three major types of validity used in research in the social science are content validity, criterion-related validity, and construct validity (Allen & Yen, 2002; Carmines & Zeller, 1979; Orcher, 2005). This study focused on content and construct validity for the new instrument.

Problem

Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (Kulm, 2008). More specifically, teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979). Learning complex concepts, such as the concept of function, is much more effective when carried out within a contextual learning environment (de la Harpe & Wyber, 2001; Lankard, 1995) but research suggests that the primary use of contextual, real-world instruction is relegated to developmental mathematics courses in post secondary education (Grubb & Kraskouskas, 1992; Stone et al., 2006; Wiseley, 2009) even though state teacher certification examinations are rich in contextualized problems (Educational Testing Service, 2009a, 2009b; Texas Education Agency, 2009). There are many extant instruments to measure a wide range of mathematical content topics and attitudes, but an instrument which specifically measures knowledge of the function concept in a contextually rich environment for preservice and in-service mathematics teachers has not been published (I. A. Brown, 2010).

The purpose of this study was to create an instrument to assess mathematics teachers' content knowledge of the function concept in a contextual environment and measure the psychometric properties of the instrument for the purpose of measuring middle and secondary school preservice mathematics teachers' content knowledge of the function concept in a contextual environment. The results from this study will aid mathematics education researchers in developing future assessments of a teacher's ability to solve problems in a contextual environment. This may offer teacher educators insight into possible modifications to preservice programs and provide school administrators a clearer perspective into the professional development needs of mathematics teachers.

Research Questions

The research questions were driven by a desire to gain a deeper understanding of the complex nature of instrument development and the validation of an instrument for a particular mathematical assessment. The questions reflect the iterative *design-test-redesign* method used in this study. The initial set of problems used in the instrument was developed based on research findings and items used in the TExES™ and Praxis™ examinations (Brown, 2010). The questions were pilot tested with a sub-sample of middle school preservice mathematics teachers which prompted a redesign of the problem set. This redesigned problem set was then evaluated by the subject matter experts which again led to further redesign of the instrument. The goal of this design-test-redesign method was to determine which items should be included in the instrument and how those individual items chosen performed together as an instrument. The questions that formed the basis of this study were:

1. What are the key items in assessing contextual function concepts that should be included in an instrument to assess preservice mathematics teachers' knowledge?
2. What are the psychometric properties of an instrument developed to assess preservice middle and secondary mathematics teachers' knowledge of the mathematical concept of function within a contextual environment?

These two questions served as foundations to guide the research project reported in this paper.

Theoretical Framework

"Validity refers to the degree to which evidence and theory support the interpretations of test scores entailed by proposed uses of tests. Validity is, therefore, the most fundamental consideration in testing and evaluating tests" (American Educational Research Association et al., 1999, p. 9). Validity is not a characteristic of a test but rather it is a metric related to how the test is used nor is it a singular measurement but rather validity is established by an amalgamation of validity evidence (American Educational Research Association et al., 1999; Lissitz, 2009; Sireci, 2007). The *Standards for educational and psychological testing* "outline various sources of evidence that might be used in evaluating a proposed interpretation of test scores for particular purposes" (American Educational Research Association et al., 1999, p. 11) which are evidence based on test content, response processes, internal structure, relations to other variables, and consequences to testing. This research attempts to establish validity for the proposed use of the instrument through the processes of item review and factor analysis.

Item Selection and Review

"Evidence of content validity is needed whenever performance on a sample of items (the test) is used to make inferences about the broader domain of which the test items are a sample" (F. G. Brown, 1983, p. 132). This is particularly true of cognitive achievement tests like the instrument developed in this study. In defining content validity, the *Standards* define test content in following manner, "Test content refers to the themes, wording, and format of the items, tasks, or procedures regarding administration and scoring" (American Educational Research Association et al., 1999, p.

11). The goal of establishing content validity for this study is to show that the instrument represents a subset of the skills or competencies of the broader domain, namely preservice teachers' knowledge of the function concept in a contextual environment. The primary method for establishing content validity is by conducting a review of test items by an expert panel (Carmines & Zeller, 1979; Huck, 2008).

Standard 1.6 and Standard 1.7 serve to guide the item selection and review processes:

> Standard 1.6: When the validation rests in part on the appropriateness of test content, the procedures followed in specifying and generating test content should be described and justified in reference to the construct the test is intended to represent.
>
> Standard 1.7: When a validation rests in part on the opinions or decisions of expert judges, observers, or raters, procedures for selecting such experts and for eliciting judgments or ratings should be fully described. The qualifications, and experience, of the judges should be presented. The description of procedures should include any training and instructions provided, should indicate whether participants reached their decisions independently, and should report the level of agreement reached. If participants interacted with one another or exchanged information, the procedures through which they may have influenced one another should be set forth. (American Educational Research Association et al., 1999)

A pathway to provide evidence of validity can be developed by providing a logical structure that demonstrates the relationships between the test items and the construct under examination and providing the method for properly identifying and qualifying the experts who will ultimately judge the validity claims.

Measurement Issues

It is important to recognize that even if the sample chosen is truly representative of the population under study, there can still be errors associated with the collected data. One of the most obvious sources for error is due to survey non-response (Ott & Longnecker, 2001). To reduce the potential effects due to non-responsive participants, the researcher worked closely with most of the faculty at the universities under study to improve faculty *buy-in* on the research. The faculty members have also been offered the opportunity to join the researcher in future potential journal publications that use the collected data from their respective programs.

One area of bias that the instrument has very little protection against is respondent bias (Abbasi, 2000). Since the survey is anonymous, and consists of non-personal type questions, the natural instinct to protect ones self should not be as strong as with other types of surveys. Errors due to processing errors can be guarded against by using structure checks, duplication checks, and omission checks and by having the checks verified by an independent, third party (graduate student) to the research (Abbasi, 2000; Ott & Longnecker, 2001).

Due to the highly contextual nature of the instrument, there is also the potential for leading questions as well as unclear or poorly worded questions. The initial item review, discussed in a following section, should allow most of these type errors to be identified and corrected or eliminated. Care has been taken in the design of the items to make them both clear and neutral in language, but the language itself is another area of concern. It is assumed that preservice teachers are able to read at or above the level of the language used in the creation of the instrument, but as mentioned in the next section, the expert panel will provide feedback concerning any potential language level concerns they may perceive.

Factor Analysis

Factor analysis is a statistical method of data analysis that is used to help determine construct validity. “A prime use of factor analysis has been in the development of both the operational constructs for an area and the operational representatives for the theoretical constructs” (Gorsuch, 1983, p. 350). A construct, for the purposes of this study, can be thought of as a theoretical concept that logically binds test items. In developing an instrument to measure mathematical cognitive skills or knowledge, it is reasonable to question whether all of the test items are based on one particular mathematical concept or are several concepts represented in the items. Each of these constructs that bind groups of items, of a particular mathematical concept for example, will be a factor in the factor analysis of the instrument.

Since the problems were developed from three specific function content areas in the TExES™ and Praxis™ examinations, it is assumed that these three areas will also constitute the three factors in the instrument. It is also true that the test items were developed based on three of the four *Levels of Demands of Mathematical Tasks* as set forth by Smith and Stein (Smith & Stein, 1998). We hypothesize that a factor analysis will show that the items selected for use in the instrument are related based on the three mathematical function concepts under examination, namely linear functions, quadratic functions, and exponential functions. We further hypothesize that a factor analysis, based on different factors, will show that the items are correlated based on three of the levels of demand of mathematical tasks, namely Lower-Level Demands (procedures without connections), Higher-Level Demands (procedures with connections), and Higher-Level Demands (doing mathematics). Therefore, confirmatory factor analysis (CFA) will be used to verify the a priori hypotheses concerning the model structure, factors, and factor loading of the items in the instrument.

When qualifying model structures, several indices are used to guide the model refinement process. Brown (2006) describes the overall goodness of by saying:

> Goodness-of-fit indices provide a global descriptive summary of the ability of the model to reproduce the input covariance matrix, but the other two aspects of fit evaluation (localized strain, parameter estimates) provide more specific information about the acceptability and utility of the solution (p. 133).

In addition to the goodness-of-fit index, the Root Mean Square Error of Approximation (RMSEA), the Weighted Root Mean Square of Residual (WRMR), and the Comparative Fit Index / Tucker-Lewis Index (CFI/TLI) should be used to determine the model's psychometric properties (Albright & Park, 2009; T. A. Brown, 2006).

Method

Development of the Instrument/Item Creation

The initial draft of the instrument consisted of fifteen multiple choice questions to assess middle and secondary school preservice mathematics teachers' knowledge of the mathematical function concept in a contextually rich environment. A sample question follows:

> Pricilla's newly formed company, Aggie Pools, Inc., is in the business of installing round, in ground swimming pools. She offer pools from the economy sized (20 feet in diameter) to the "Hummer" of backyard pools (80 feet in diameter). A common concern of her customers is the amount of water lost each day due to evaporation. To maintain a constant level in the swimming pool, each gallon of water lost to evaporation must be made up with fresh water from the city water department. The local fresh water supply rate is $4.80 for the first 2000 gallons of water and $1.96 for each additional thousand gallons. Armed with the knowledge that water evaporation is directly proportional to the surface area of the pool and that evaporation loss rates in her service area average about 1/120 gallons per hour per ft^2 of pool surface area, she decides to create a formula that relates evaporation loss per day as a function of the radius of the pool.
>
> Of the four graphs below, which of the graphs best represents the real-world use of the above function?

The question above was followed by four graphical representations of functions. One of the four graphs will best represent the function described by the problem statement while the remaining three graphs will be incorrect or less desirable solutions to the problem.

The question above and fifteen similar questions were created by the authors and are based on common competencies described in the TExES™ and Praxis™ teacher certification examinations for middle and secondary school preservice mathematics teachers (Educational Testing Service, 2009c; Texas Education Agency, 2009). Texas middle school mathematics teachers are required to demonstrate successful performance on the "115 Mathematics 4 – 8" examination and Texas secondary school mathematics teachers are required to demonstrate successful performance on the "135 Mathematics 8 – 12" examination(Texas Education Agency, 2008). North Carolina middle school mathematics teachers are required to demonstrate successful performance on the "Middle School Mathematics (0069)" examination and North Carolina secondary school mathematics teachers are required to demonstrate successful performance on the "Mathematics: Content Knowledge (0061)" examination(Educational Testing Service, 2009a, 2009b, 2009c).

For the middle school function content, common requirements between the TExES™ (domains II & V) and the Praxis™ (content categories I & III) examinations included models, representations, and transformations, linear and non-linear functions, properties of functions and their graphs, mathematical models of real-world situations, analysis and evaluation models, use of multiple representations of a concept, and use mathematical models in other academic disciplines. The secondary school function content maintains all of the middle school function content with the additional requirement to be able to use the properties of polynomial, exponential, and trigonometric functions to analyze, graph, model, and solve problems based on TExES™ (domain II) and the Praxis™ (content categories IV-Trigonometry & V-Functions). The initial content in the instrument was based on four specific domain areas; linear, quadratic, trigonometric, and exponential functions.

A fundamental goal in item selection was to ensure alignment between the standards, in this case the TExES™ and Praxis™ examinations, and the instrument. The items in this version of the instrument were created to be consistent between (1) a common category of content between the standards, (2) Bloom's Taxonomy for the cognitive domain (Krathwohl, 2002), and (3) cognitive demand as defined by Webb's (2002, p. 4) "depth-of-knowledge". Webb (2007, p. 2) defines the following four levels of depth of knowledge:

> Level 1 (recall and reproduction) is the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple science process of procedure. A student answering a Level 1 item either knows the answer or does not.
>
> Level 2 (skills and concepts) includes the engagement of some mental processing beyond recalling or reproducing a response. The content knowledge or process involved is more complex than in Level 1. Keywords that generally distinguish a Level 2 item include 'classify,'

‘organize,’ ‘estimate,’ ‘make observations,’ ‘collect and display data,’ and ‘compare data.’

Level 3 (strategic thinking) requires reasoning, planning, using evidence, and higher level of thinking than the previous two levels. The complexity results because the multistep task requires more demanding reasoning.

Level 4 (extended thinking) Tasks at this level have high cognitive demands and are very complex. Students are required to make several connections, to related ideas within the content area or among content areas—and have to select or devise one approach among many solution alternatives. This level requires complex reasoning, experimental design and planning, and probably will require an extended period of time, either for the science investigation required by an objective, or for carrying out the multiple steps of an assessment item.

A careful review of TExES™ and Praxis™ examinations reveals there are no “Level 4” depth-of-knowledge competencies required, therefore there are no “Level 4” items in this instrument.

Table 4, adapted from Hess’ work on “cognitive complexity” (Hess, 2006, p. 6), shows the association between Bloom’s Taxonomy and Webb’s Depth of Knowledge (DoK) levels. The areas highlighted in bold print emphasize the intersection of the cognitive complexity items and the common competencies between the TExES™ and Praxis™ mathematics examinations relating to linear, quadratic, and exponential function problems that are of interest in this study.

Table 4. *Webb's Depth of Knowledge Levels.*

Bloom's Taxonomy	**Level 1 Recall & Reproduction**	**Level 2 Skills & Concepts**	**Level 3 Strategic Thinking/Reasoning**
Knowledge Define, list, memorize, name, order, recognize, relate, recall, state	- Recall or identify conversions between and among representations.		
Comprehension Classify, describe, explain, express, identify, locate, recognize, select, translate	- Make conversions between and among representations. - Evaluate an expression. - Solve a one step problem.	- Make basic inferences or logical predictions from data.	- Use concepts to solve non-routine problems. - Explain, generalize, or connect ideas using supporting evidence.
Application Apply, choose, demonstrate, illustrate, interpret, sketch, solve	**- Follow a simple procedure, algorithm, or formula. - Perform calculation. - Apply an algorithm or formula.**	**- Solve routine problem applying multiple concepts. - Retrieve information from a graph or a table and use it to solve a problem requiring multiple steps.**	**- Use concepts to solve non-routine problems. - Use reasoning, planning, and evidence.**
Analysis Analyze, calculate, compare, discriminate, examine	- Retrieve information from a graph or a table.	**Compare/contrast figures or data. - Select appropriate graph. - Interpret data from a simple graph.**	**- Analyze and draw conclusions. - Generalize a pattern - Interpret data from complex graph.**
Synthesis Rearrange, create, design, develop, formulate, organize, propose	- Brainstorm ideas, concepts, or perspectives related to a topic.	**- Use models to represent mathematical concepts.**	**- Develop a mathematical model for a complex situation.**
Evaluation Appraise, assess, defend, estimate, predict, select, evaluate			**- Cite evidence and develop a logical argument for concepts. - Verify reasonableness of results.**

Note: It should be noted again that "Level 4 – Extended Thinking" type items are beyond the scope of this study.

It can be seen from the above table that the inverted triangular shape of the bold items indicates a pattern. Items that will best fit these criteria will not be based on

Bloom’s lowest two levels of skills, “knowledge” or “comprehension”, but rather will be based on “application”, “analysis”, “synthesis”, and “evaluation”. These skills are consistent with contextualized function problems. It can also be seen that as the skill level progresses from “application” to “evaluation”, Webb’s DoK levels increase as well. Therefore, the item(s) with the lowest cognitive demand will be developed such that they will be associated with Bloom’s “application” level and Webb’s DoK “Level 1”, whereas the item(s) with the highest cognitive demand will be developed based on Bloom’s “synthesis” skill level and Web’s DoK “Level 3”.

From these criteria, fifteen items were created by the researcher for the initial version, version 1.1, of the instrument (see Appendix B). Table 5 illustrates the relationship between the created items and the alignment to the standards, Bloom’s skills, and Webb’s DoK. (#1 refers to item #1 in the instrument).

Table 5. *Test Items in Webb's DoK Levels.*

TExES & Praxis Function Content	Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
Level 1: Recall and Reproduction						
Linear			#9, #10	#1		
Quadratic			#3			
Exponential						
Trigonometric						
Level 2: Skills and Concepts						
Linear					#2	
Quadratic			#4	#11		
Exponential						
Trigonometric			#13, #14			
Level 3: Strategic Thinking / Reasoning						
Linear			#11	#12		
Quadratic					#5, #6,	
Exponential			#7, #8,	#15, #16		
Trigonometric						

Item Review

This initial draft of the instrument was reviewed by a panel of mathematics education experts and the results from the analysis of their comments were used to produce the second draft of the instrument. The subject matter experts (SME) were asked to verify the content areas represented in the problems, level of the language used in the instrument is consistent with the language level of the preservice teachers in their programs, and to provide commentary as to the level of difficulty of the problems.

Please see Appendix C for a copy of the letter sent to the SME panel and Appendix D for a copy of the "Contextual Function Survey - Subject Matter Expert Response Sheet."

Item Review Participants - Subject Matter Experts

12 faculty members from nine universities in the states of North Carolina and Texas were contacted via email and phone calls and asked to serve as an SME for this project. Nine faculty members from seven universities agreed to serve on the panel. Two of the faculty members held the rank of Assistant Professors, three were Associate Professors, three were Full Professors, and one member of the SME panel held the rank of Distinguished Professor. All of the SME were responsible for middle and/or secondary school mathematics teacher education and held positions in the mathematics department as mathematics educators, the mathematics education department or joint appointments in both departments. Two of the universities hold the Carnegie Classification (The Carnegie Foundation, 2010) designation of Master's Colleges and Universities-Larger Programs (Master's/L), two are Doctoral Research Universities (DRU), two are Research Universities-high research activity (RU/H), and one holds the designation of Research Universities-very high research activity (RU/VH).

Item Review Data

Data was collected from the SME in two modes 1) individual, personal interviews were completed at the researchers' home institution by the primary author, and 2) SME at remote institutions completed the SME Response Sheet on their own without collaboration between SME. Of the data collected remotely via the SME Response Sheets, there were a total of 5060 words written by SME. 39.5% of the responses were provided by the Full and Distinguished Professors and 57.8% provided by the Associate Professors. When the distribution is viewed from a per-person perspective, the Full and Distinguished Professors provided 19.8% per person and the Associate Professors contributed 19.3% per person. The sole Assistant Professor contributed less that 3% of the total responses.

The data from the SME response sheets were divided into two categories, 1) free response data, and 2) data specific to each item. Through substantive analyses of the free response data in the SME sheets, the primary researcher developed a set of response categories and key themes were identified. Review of the data specific to each item led to item revision or if the negative responses were significant, the item was replaced.

Instrument Validation Participants

The population under consideration for this study was middle and secondary grades preservice mathematics teachers in full-time, university teacher education programs within the states accredited by SACS. Since the states of Florida and Georgia currently use neither the Praxis™ nor TExES™ exams, students from those states are not part of the population. The sample for this study was comprised of 191 preservice teachers in 10 university teacher education programs in the states of Texas and North Carolina. This preservice teacher sample was chosen to represent the diverse preservice teacher population of the nine states accredited by SACS that use either the TExES™ or Praxis™ teacher certification examinations.

There were 148 (77.5%) females and 42 (22.0%), males with one preservice teacher not reporting, of the 191 preservice teachers in the sample used for this study. 24

(12.6%) of the participants classified themselves as sophomores, 65 (34.0%) were juniors, 93 (48.7%) were seniors, and 7 (3.7%) were Post-Bachelors with 2 not reporting their class. 123 (64.4%) of the preservice teachers are seeking middle school teaching certification and 62 (32.5%) are seeking secondary certification with 6 not reporting. 31 (16.2%) of the preservice teachers reported their academic major as being “Math and/or Science Education”, 31 (16.2%) were mathematics majors, 2 (1.0%) were physics majors, 16 (8.4%) reported “Multi-Disciplinary” majors, 7 (3.7%) reported “Other” with 11 not reporting an academic major.

Instrument Validation Data

A unique identifier was assigned to each of the completed Contextual Function Instruments (CFI) so that score transcription could be verified by an independent researcher. The multiple choice answers from the CFI were transcribed from the individual completed instruments into an Excel spreadsheet as raw data. A coding system was created to convert the checked class, certification, and college major demographic entries on the CFI into numeric codes for entry into the data sheet. A second worksheet ply was added to convert raw scores into dichotomous data for use in two statistical analysis software packages; SPSS (version 16.0) and Mplus® 6 (version 6.1) (Muthen & Muthen, 2010). An Excel formula was used to convert the raw answer (A, B, C, or D) into a “0” if the answer was incorrect or a “1” if the answer was correct.

The dichotomous data was then used in SPSS to calculate the descriptive statistics for the demographic information (sex, class, certification, and college major) and to also compute the descriptive information for the scores based on sex and class (see Appendix E). The SPSS data file was saved as an ASCII file to be used as an input data file to Mplus.

Results

Item Review

Version 1.1 of the Contextual Function Instrument was modified based on the results of analysis of the item review data from the Subject Matter Experts. The most significant concern from the SME panel was related to the framework used to classify the problems. The original framework was based on an integration of Bloom's taxonomy and Webb's Depth of Knowledge adapted from Hess' work on "cognitive complexity" (Hess, 2006). One of the SME responded in the following way:

> My general concern is the framework that you are using to classify / categorize the cognitive demand of an item is not sufficient. The framework by Hess (2006) which combines Bloom's Taxonomy and Webb's "depth of knowledge" framework is quite confusing to me; in my opinion it contradicts how Bloom's Taxonomy is supposed to be interpreted.

Based on responses like these and other related issues, several items were modified or replaced. Table 6 summarizes the changes from version 1.1 to version 4.1.

Table 6. *Modification Of Instrument Items Based on Item Review Results.*

Problem #	**Version 1.1**	**Version 4.1**
1	Replaced	New-linked to #8
2	Replaced	New-linked to #9
3	Modified; moved to #9	Modified old #5 -linked to #11
4	Modified; moved to #10	New-linked to #13 & #14
5	Modified; moved to #3	New-linked to #15
6	Modified; moved to #3	New-linked to #15
7	Replaced	New Problem
8	Replaced	New Problem
9	Replaced	Modified; from old #3
10	Replaced	Modified; from old #4
11	Replaced	New Problem
12	Replaced	New Problem
13	Replaced	Slightly modified; from old #7
14	Replaced	Slightly modified; from old #8
15	Replaced	Slightly modified; from old #16
16	Moved to #15	N/A

Five of the original problems were maintained in the final version after minor modifications. The remaining problems were replaced to allow the researchers to better infer the participants' knowledge of the function concept in contextualized problems and to bring all of the items under a tighter framework.

The entire original set of items asked the participants to demonstrate their ability to solve highly contextualized function problems, but did not allow the following basic question to be answered. "Did the participant fail to solve the contextualized problem because they were unable to understand the function concept because it was immersed in the given context, or did they simply not understand the function concept to begin with?" To offer more insight into this area, six new problems were created to test the participants' knowledge of linear, quadratic, and exponential functions without much problem context. These first six problems serve to establish the participant's basic function knowledge prior to asking the participant to demonstrate similar function knowledge in solving six linked, highly contextualized problems. For example, problem #4 gives the participant the function $P(t) = P_0 \cdot e^{rt}$; and explicitly given values for P_0 and r, the participant is asked to find P(11). Later in the instrument, problem # 13 asks the participant to solve a similar exponential problem using the same formula, but the values for t, P_0 and r are described or implied in the context of the problem. If the participant can correctly answer #4 but fails to correctly answer #13 our belief is that the context of the problem, and not the computation of the exponential function itself, caused the participant to fail to provide a correct solution.

To bring all of the items under a tighter framework, we turn to a body of work compiled by Smith and Stein referred to as the "Levels of Demands of Mathematical Tasks". Smith and Stein (1998, p. 6) refer to the following four level of demands:

1. Lower-level demands (memorization) involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
2. Lower-level demands (procedures without connections) are algorithmic and use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task and are focused on producing correct answers instead of on developing mathematical understanding.

3. Higher-level demands (procedures with connections) focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas and are usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations.
4. Higher-level demands (doing mathematics) require complex and non-algorithmic thinking--a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example and require students to explore and understand the nature of mathematical concepts, processes, or relationships.

The redefined items are shown to fit into Smith and Stein's framework (Table 7).

Table 7: *Test Items in Smith & Stein's Framework. P1 Means Problem #1.*

	Linear	**Quadratic**	**Exponential**
Lower-Level Demands (Memorization)			
Lower-Level Demands (Procedures Without Connections)	P1, P2	P3	P4
Higher-Level Demands (Procedures With Connections)	P8	P5, P6, P10	P13
Higher-Level Demands (Doing Mathematics)	P7	P9, P11, P12, P15	P14

It can be seen that more than half of the problems are quadratic in nature and seven of the eight quadratic problems fall into the two Higher-Level Demand categories. The table also shows that none of the problems are classified as Lower-Level Demand (Memorization).

Instrument Validation

As stated earlier, the Contextual Function Instrument comprised of these 15 revised items, was distributed to the 191 preservice teachers that made up the instrument validation sample pool. Each multiple choice test was graded by the researcher and the resulting score data (represented as percent correct) was included with the demographic and item data. The overall average for all participants was 45.3% and the participant scores ranged from a high of 87% to a low of 7% (with a standard deviation of 16.6). The overall average for all items was 45.3% and the item scores ranged from a high of 90.6% for problem #1 (P1) to a low of 16.8% for P13. Table 8, and Figure 2 and Figure 3 summarize the scores and ranks of the problems.

Table 8. *Problems, Scores and Ranks.*

Problems in rank order			Problems in problem order		
Rank	**Problem**	**Score**	**Problem**	**Rank**	**Score**
1	P1	90.58%	P1	1	90.58%
2	P5	80.10%	P2	4	57.59%
3	P4	78.01%	P3	8	37.70%
4	P2	57.59%	P4	3	78.01%
5	P10	56.02%	P5	2	80.10%
6	P6	53.40%	P6	6	53.40%
7	P8	47.64%	P7	9	36.13%
8	P3	37.70%	P8	7	47.64%
9	P7	36.13%	P9	13	20.42%
10	P11	31.94%	P10	5	56.02%
11	P15	31.94%	P11	10	31.94%
12	P14	22.51%	P12	14	19.37%
13	P9	20.42%	P13	15	16.75%
14	P12	19.37%	P14	12	22.51%
15	P13	16.75%	P15	11	31.94%

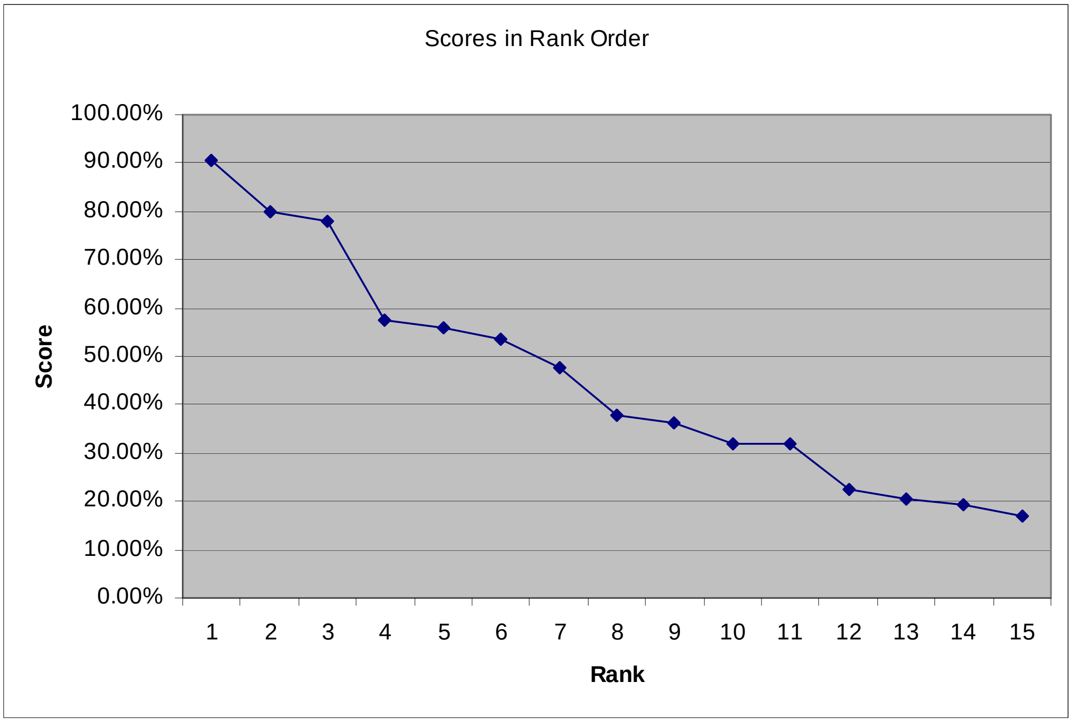


Figure 2. *Scores in Rank Order*.

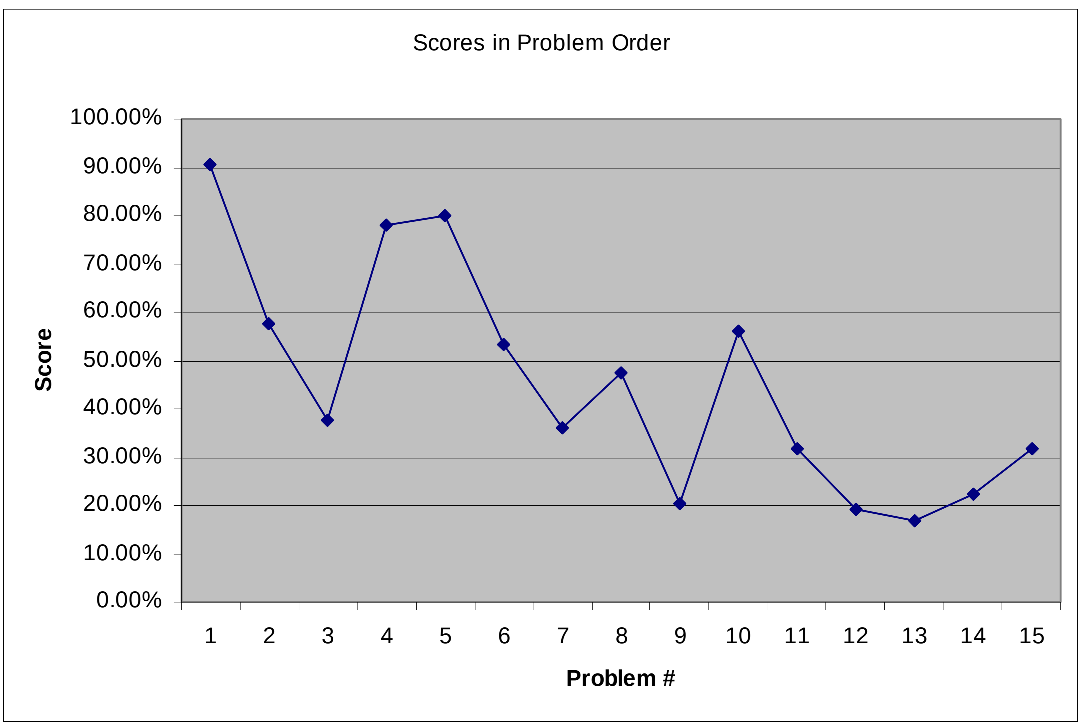


Figure 3. *Scores in Problem Order*.

The Levels of Demand of Mathematical Tasks framework can be revisited with ranks of the scores and the links between problems included in Figure 4, which shows, for example, that problem #1 (P1) has a score rank of 1 and is linked to problem #8 (P8) which has a rank of 7. The links shown above are based on a priori assumptions relating the items during creation, but we will now show the relative strength of the links based on analysis of the data.

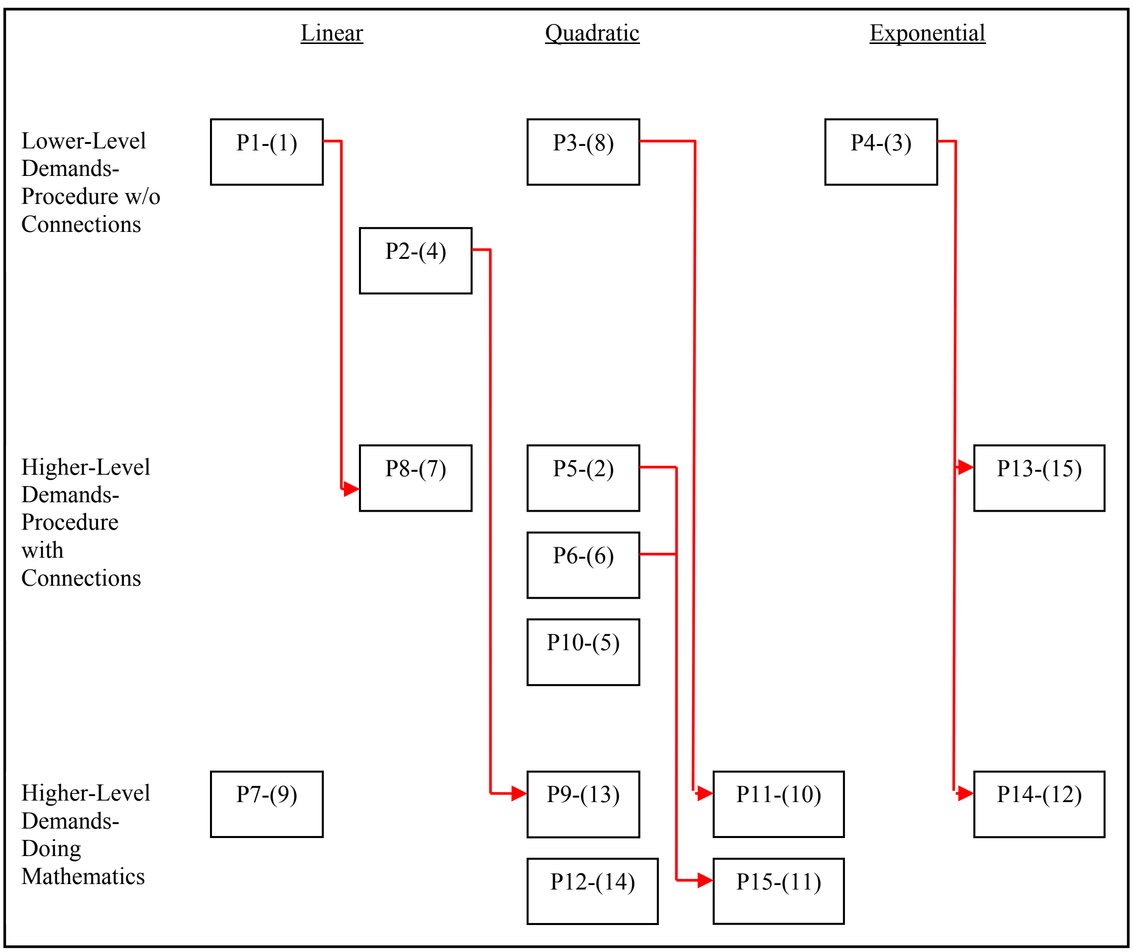


Figure 4. *Level of Demand of Mathematical Tasks with Score Ranks and Links.*

A Priori Assumptions about Links

To test a priori assumptions about the links between items, SPSS software was used to create cross tabulation data to demonstrate potential relationships between test items. As stated earlier, one of the primary purposes for creating new (redefined) test items was to be able to address the question, “Did the participant fail to solve the contextualized problem because they were unable to understand the function concept because it was immersed in the given context, or did they simply not understand the function concept to begin with?” The basic assumption is if the participant can correctly answer the lower demand linked problem but fails to correctly answer its higher demand linked complement then the context of the problem, and not the concept of the function itself, caused the participant to fail to provide a correct solution. Table 9 summarizes the cross tabulation of the responses for linked problems.

Table 9. *Cross Tabulation on Linked Items.*

		Incorrect	Correct	Total
			P8	
P1	Incorrect	14	4	18
	Correct	86	87	173
	Total	100	91	191
			P9	
P2	Incorrect	69	12	81
	Correct	83	27	110
	Total	152	39	191
			P11	
P3	Incorrect	79	40	119
	Correct	51	21	72
	Total	130	61	191
			P13	
P4	Incorrect	31	11	42
	Correct	128	21	149
	Total	159	32	191
			P14	
P4	Incorrect	31	11	42
	Correct	117	32	149
	Total	148	43	191
			P15	
P5	Incorrect	31	7	38
	Correct	99	54	153
	Total	130	61	191
			P15	
P6	Incorrect	71	18	89
	Correct	59	43	102
	Total	130	61	191

It can be seen in the table above that when participants correctly answered problem #1 (P1), then they had about an equal chance of correctly answering P8, but when they incorrectly answered P1 they were more than three times more likely to incorrectly answer P8. The link between P2 and P9 has a similar outcome, but demonstrates other information about the linkage. When participants correctly answered P2, they were about three times more likely to incorrectly answering P9. Unlike the link between P1 and P8 which simply moves from the Lower-Level Demands category to the next higher category and the link remains within the linear function domain, the link between P2 and P9 is more complex. We can see that P2 is a linear function within the Lower-Level Demands category, but its linked complement, P9, is two levels higher in the Higher-Level Demand (Doing Mathematics) category and it is also a quadratic function. The difference in ranks, 4 for P2 and 13 for P9, also demonstrates the increased difficulty between the problems. Nonetheless, when participants incorrectly answered P2 they were nearly six times more likely to incorrectly answer P9. The remainder of the linked problems shows similar trends and relationships.

Construct Validity

SPSS™ was also used to compute the value of Cronbach's Alpha for the instrument. All 191 cases were reported to be valid with no cases excluded; the value of Cronbach's Alpha for the 15 test items was 0.559. The Cronbach's Alpha was also computed for a modified case in which P4, P13, and P14 were removed from the data set due to negative correlations. The resultant value of Cronbach's Alpha was 0.546.

Factor Analysis

A key aspect of this project was the fact that several important a priori assumptions were made concerning relationships between test items. These assumptions effectively define the structure of the instrument by defining the items based on the categories of the Levels of Demands of Mathematical Tasks as well as the three mathematical topic areas chosen. Since this is an exploratory study, designed to better understand the factor structure of the instrument, confirmatory factor analysis (CFA) was run using two different factor structures.

The first CFA was completed using the Levels of Demands of Mathematical Tasks to define the factor structure (please see Appendix F for the complete Mplus CFA output). As defined in Figure 4 above, problems 1 – 4 were associated with factor 1 (Lower-Level Demands-Procedures Without Connections), problems 5, 6, 8, 10, & 13 were associated with factor 2 (Higher-Level Demands-Procedures With Connections), and problems 7, 9, 11, 12, 14, & 15 were associated with factor 3 (Higher-Level Demands-Doing Mathematics). A brief summary of the CFA output is in Table 10. The Chi-Square Test of Model Fit value was 123.4 with 87 degrees of freedom and a P-value of 0.0063. The other key parameters are as follows: the RMSEA was estimated at 0.047, the CFI was 0.814, the TFI was 0.775, and the WRMR was 0.964. It can be seen from the factor loadings (Estimates) that problems 13 and 14 show a negative loading. This finding is consistent with the fact that the score rank for P13 was 15 (16.8% success rate) and the rank for P4 was 12 (22.5% success rate).

Table 10. *CFA Output Summary for Factors Based on Levels of Demands.*

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1	**BY**				
	P1	1.000	0.000	999.000	999.000
	P2	0.880	0.313	2.806	0.005
	P3	1.449	0.478	3.035	0.002
	P4	1.668	0.496	3.359	0.001
F2	**BY**				
	P5	1.000	0.000	999.000	999.000
	P6	0.830	0.190	4.363	0.000
	P8	0.512	0.191	2.675	0.007
	P10	0.615	0.183	3.356	0.001
	P13	-0.274	0.212	-1.292	0.196
F3	**BY**				
	P7	1.000	0.000	999.000	999.000
	P9	0.353	0.432	0.818	0.413
	P11	0.295	0.348	0.847	0.397
	P12	0.997	0.502	1.987	0.047
	P14	-0.108	0.375	-0.287	0.774
	P15	2.15	0.816	2.634	0.008

The second CFA was completed using the mathematical topic areas to define the factor structure (please see Appendix G for the complete Mplus CFA output). As defined in Figure 2 above, problems 1, 2, 7, & 8 were associated with factor 1 (linear functions), problems 5, 6, 9, 10, 11, 12, & 15 were associated with factor 2 (quadratic functions), and problems 4, 13, & 14 were associated with factor 3 (exponential functions). A brief summary of the CFA output is in Table 11. The Chi-Square Test of Model Fit value was 124.8 with 87 degrees of freedom and a P-value of 0.0050. The other key parameters are as follows: the RMSEA was estimated at 0.048, the CFI was 0.807, the TFI was 0.767, and the WRMR was 0.966. Again, it can be seen from the factor loadings (Estimates) that problems 13 and 14 show a negative loading.

Table 11. *CFA Output Summary for Factors Based Mathematical Topic.*

		Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1	**BY**				
	P1	1.000	0.000	999.000	999.000
	P2	0.856	0.310	2.765	0.006
	P7	0.698	0.274	2.545	0.011
	P8	0.695	0.290	2.396	0.017
F2	**BY**				
	P3	1.000	0.000	999.000	999.000
	P5	0.946	0.210	4.512	0.000
	P6	0.786	0.165	4.765	0.000
	P9	0.135	0.209	0.646	0.519
	P10	0.569	0.156	3.649	0.000
	P11	0.111	0.175	0.635	0.525
	P12	0.477	0.213	2.235	0.025
	P15	0.971	0.185	5.259	0.000
F3	**BY**				
	P4	1.000	0.000	999.000	999.000
	P13	-0.176	0.179	-0.980	0.327
	P14	-0.087	0.128	-0.685	0.494

SPSS was used to test the correlation between P4 & P13 and between P4 & P14. This confirmed the suspected negative correlation between P4, P13, & P14 as shown by the results of the CFA. To improve the overall goodness of fit values for the model, P13 and P14 were removed. Since P13 and P14 are linked to

P4, and it would be nonsensical to create a model that has a factor with only one variable, P4 was also removed and the CFA were re-run on the modified data set. The full results of the revised CFA using the Levels of Demands of Mathematical Tasks to define the factor structure (please see Appendix H) and using the mathematical topic areas to define the factor structure (please see Appendix I) showed improved values for overall goodness of fit. The Mplus Chi-Square Test of Model Fit value was 69.6 with 51 degrees of freedom and a P-value of 0.043 (The other key parameters are as follows: the RMSEA was estimated at 0.044, the CFI was 0.857, the TFI was 0.814, and the WRMR was 0.878) for the Levels of Demand model and 79.9 with 53 degrees of freedom and a P-value of 0.042 (The other key parameters are as follows: the RMSEA was estimated at 0.043, the CFI was 0.854, the TFI was 0.818, and the WRMR was 0.886) for the mathematical topic areas model.

Discussion

With the focus of mathematics education research on students' achievement, it is important to note that teacher knowledge is the most important factor influencing student learning (Dooren et al., 2002; Lappan & Ferrini-Mundy, 1993; National Council of Teachers of Mathematics, 1991; The Education Alliance, 2006). Recent research reveals that, generally speaking, teachers lack depth in mathematics content knowledge (Kulm, 2008) and earlier research found that teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). A preliminary examination of the data used to develop and validate the Contextual

Function Instrument supports these claims. The overall average for all participants was 45.3% and the participant scores ranged from a high of 87% to a low of 7%. It should be noted that the data collected and reported in this study was not meant to be able to report on the actual function knowledge of preservice teachers, but rather to analyze the properties of a new instrument. Nonetheless, it’s difficult not to notice the low success rate on this unproven instrument.

Sherin (2002) notes that just knowing that a teacher has specific content knowledge is not enough, we need to know that they have the appropriate knowledge for the content domain being taught and we need to know that they have a sufficient level of that knowledge. For this reason, care was taken in the selection of appropriate function topics for this instrument. The items on the original instrument were created based on the mathematical topics of linear, quadratic, trigonometric, and exponential functions. These topics are most prominent in both the TExES™ and Praxis™ educator certification examinations (Educational Testing Service, 2009a, 2009b, 2009c; Texas Education Agency, 2009) and therefore should be represented on any instrument used to ascertain preservice teachers knowledge of function concepts.

It was discovered in this research that it is difficult to span four function areas on a test with less than 20 questions, and 20 question of a contextual nature require too much time for reasonable, volunteer testing. Therefore, it was decided to reduce the subject area in the revised instrument to only three function topics; linear, quadratic, and exponential functions. The study was frustrated by the fact that even though the contextual exponential problems were linked to a straightforward application problem (score of 78% and a rank of 3), the participants performed so poorly on the problems that they had to be eliminated as well. Based on the results of the cross tabulation, the most prudent action to take would be to create clearer contextual exponential problems as replacement rather than have an instrument that only tested linear and quadratic functions.

When developing new instrumentation for testing, great care should be taken to show that the newly created instrument is both reliable and valid. Without verification of these two key qualities, the proposed instrumentation cannot be trusted to provide a useful link between observable phenomenon and abstract ideas (Carmines & Zeller, 1979). The computed value for Cronbach's Alpha of 0.56 confirms the fact that several items in this instrument must be revised to increase this instrument's internal consistency reliability. For those items that otherwise fit well into the factor structure of the instrument but posses weak R^2 values, it would be wise to revise or replace them to be more consistent with items with similar items that posses large R^2 values.

The focus of this project was to create an instrument to assess mathematics teachers' content knowledge of the function concept in a contextual environment and measure the psychometric properties of the instrument for the purpose of measuring middle and secondary school preservice mathematics teachers' content knowledge of the function concept in a contextual environment. Knowing the logistical limitations in collecting data, the plan was not to bring the instrument into full reliable and valid form, but rather to examine key psychometric properties of the instrument to lay the groundwork for future research and continued development of this instrument. In addition to the factors and the overall goodness-of-fit index, the Root Mean Square Error of Approximation (RMSEA), the Weighted Root Mean Square of Residual (WRMR), and the Comparative Fit Index / Tucker-Lewis Index (CFI/TLI) were measured as recommended by the literature (Albright & Park, 2009; T. A. Brown, 2006).

One of the strongest aspects of this project was the procedure to determine content validity. The goal of establishing content validity was to show that the instrument represents a subset of the skills or competencies of the broader domain, namely preservice teachers' knowledge of the function concept in a contextual environment. The method for establishing content validity, by conducting a review of test items by an expert panel, is consistent with recommended practices in the research literature (Carmines & Zeller, 1979; Huck, 2008). Care was taken to follow Standard 1.6 and Standard 1.7 of the *Standards for educational and psychological testing* (American Educational Research Association et al., 1999) in specifying and generating test content. Also, since content validity rested on the opinions a Subject Matter Expert panel, the procedures for qualifying the experts were well defined.

It is clear from the psychometric properties of the instrument that the items on the instrument should have a tighter scope. While it is desired to have an instrument that can examine participants' knowledge in the areas of linear and quadratic functions, exponential functions and trigonometric functions, we recognize that those areas span too large of an area for a reliable and valid test that can be taken within a reasonable amount of time. Perhaps separate instruments can be created that can examine linear and quadratic functions while exponential and trigonometric functions can be examined in another instrument.

The relationships shown in the results suggest that highly contextualized problems should be linked to non-contextualized foundational function problems (similar to problems used in common instruments examining the knowledge of the function concept). While the results from link relationships were quite encouraging, the authors recognize that the current instrument only provides a good base for future instrument development. Future research should also seek to discover the potential relationship between the relative strengths of links and the instrument's psychometric properties.

CHAPTER IV

CONCLUSIONS

This chapter will conclude this project by describing how the results and conclusions Chapters II & III for a cohesive discussion that answers the overall question, "How should preservice mathematics teachers' content knowledge of the function concept be assessed?" this chapter will also describe how the sum of the results and conclusions from Chapters II and III add knowledge to the field of mathematics education. It is worthwhile to recount the origins of the project with the hope that a more complete picture of this complex and important landscape is drawn.

As explained in the introductory chapter, the genesis of this research project occurred after I conducted several mathematics content knowledge review sessions for groups of preservice teachers enrolled in alternative certification programs (ACP) within the state of Texas. In working with preservice middle school mathematics and secondary school mathematics and science teachers, I found that many of them struggled to understand the context and requirements of the problems on the TExES examination more than the specific mathematical operations required for a solution. Generally, after I explained what the problem requirements were, they were able to recall the appropriate mathematical content and perform the correct mathematical operations to find a solution.

The ACP enrolled preservice teachers from various academic backgrounds provided the preservice teacher had the requisite number of appropriate college mathematics courses. The fact that the preservice teachers with applied mathematics backgrounds (primarily science and engineering) seemed to have better command of the contextualized function problems on the TExES™ practice problems caused me to wonder if preservice teachers in traditional mathematics teacher education programs were better prepared for the rigors of the certification examinations. These thoughts led to the overarching question that defined and guided both research projects described in this study, "how should preservice mathematics teachers' content knowledge of the function concept be assessed?"

To answers this and the subsequent related questions, I reviewed existing mathematics education and teacher education literature to determine how content knowledge of the function concept has been accessed. This led me to also investigate if and how contextual teaching and learning promote effective methods in the mathematics classroom and whether or not a reliable and valid instrument existed that would be useful in assessing teachers' content knowledge of the function concept. Finally, I performed a review of literature in the field of assessment and instrument development to discover key psychometric properties that must be considered in the development of an effective instrument.

Implications for Preservice Teacher Education

In a 2009 speech at Teachers College, Secretary of Education Arne Duncan provided an less than admirable view of university based teacher education programs (Duncan, 2009, p. 1):

> Yet, by almost any standard, many if not most of the nation's 1,450 schools, colleges, and departments of education are doing a mediocre job

> of preparing teachers for the realities of the 21st century classroom. America's university based teacher preparation programs need revolutionary change—not evolutionary tinkering. But I am optimistic that, despite the obstacles to reform, the seeds of real change have been planted. America faces three great educational challenges that make the need to improve teacher preparation programs all the more urgent.

Not all of Secretary Duncan's comments were as damaging as the "mediocre" comment; he goes on in that speech to offer conciliatory statements to educational leadership for the monumental gains that have been made in the field of teacher education. Nonetheless, university education programs need to continually find new research-based ways improve teacher education.

The abundance of research literature in the area of mathematics teacher education demonstrates that the preparation of mathematics teachers is not a new area of concern, but some readers might be surprised to know how long this concern has been voiced in the research literature. As described in a 1907 article in *School Science and Mathematics*, Professor H. L. Coar of Marietta College presented of a paper entitled "The Teacher of Mathematics" where:

> He had found that conditions similar to those that Professor Skinner reported from Wisconsin held generally elsewhere. The preparation of the teacher of mathematics is very often quite inadequate. Universities, colleges, normal schools, and local school authorities could do much to better conditions (Whitney, Denton, & Jone, 1907, p. 70).

Even though the interest for improving the quality of mathematics teachers is not new, the past twenty-five years have been particularly fruitful in terms of producing high quality theoretical and empirical research (Hill, 2007; Hill & Ball, 2004).

The findings in this task-based interview research project (Chapter II) confirm research findings that generally speaking, teachers lack depth in mathematics content knowledge (N. Webb, 1979). More specifically, teachers find the concept of function particularly challenging (Even, 1993; Leinhardt et al., 1990; Sherin, 2002; Stein et al., 1990). A rich and practical understanding of the concept of function has been found to be at insufficient levels in college students as well as some middle and secondary school mathematics teachers (Sierpinska, 1992; N. Webb, 1979). This project revealed weaknesses and strengths of preservice teachers' contextual problem-solving ability.

Contrary to finding by Nathan and Koedinger, where beginning algebra students employed the methods "that gave the highest likelihood of success (around 70%) and led to the greatest number of correct solutions when they were applied" (2000, p. 176), the participants described in Chapter II did not use the decoding strategies that they found most successful. By far, the strategy used the most was the elimination of an answer choice. This suggests that the participants have worked themselves into a problem-solving rut. Rather than search their minds, though their problems solving experiences, to find the method most appropriate for the problem at hand they tend to try to apply a most familiar method to every problem.

Knowing that preservice teachers have this problem-solving liability, teacher educators could improve student outcomes by modifying problem choices sufficiently to give the students a wider view of problem-solving. Another strategy that could be employed by teacher educators is to require students to solve a problem using more than a single method and by specifying a method that must be used. An alternative to this last idea could be to also specify in the problem directions which problem-solving methods that cannot be used for this particular problem.

The most unproductive preservice teacher strategy (or way of classifying problems) was their insistence that some problems "were physics" problems. Upon making that statement, the students always failed to correctly solve the problem. Teacher educators could potentially improve problem-solving courses by borrowing problems from other STEM disciplines. The researcher hypothesized that students who recognized a math problem as a math problem, held themselves open to more mathematical problem-solving strategies than the student that discounted the problem as being outside of their knowledge domain, which may allow the student to mentally give up trying further.

Teacher educators can also build on the strengths displayed by the participants. The participants were extremely successful (95%) when they demonstrated their ability to apply real-world knowledge as constraints in problem-solving. This suggests an improvement in the earlier work by Vershaffel, et al. (1997) where participants failed to use real-world thinking in their problem-solving solutions. This student strength can be leveraged with a wider variety of problems (STEM based problems) to expand the preservice teachers' breadth of problem-solving knowledge and critical thinking skills.

The National Council for Accreditation of Teacher Education (NCATE) published a list of “Research Supporting the Effectiveness of Teacher Preparation” on its website (National Council for Accredition of Teacher Education, 2005, p. 1). In that list, it offers the following quote from (Wenglinksy, 2002):

> Wenglinsky looked at how math and science achievement levels of more than 7,000 eighth graders on the 1996 National Assessment of Educational Progress were related to measures of teaching quality and student social class background. He found that student achievement was influenced by both (1) teacher content background and (2) teacher education /professional development coursework, especially in how to work with diverse student populations and students with special needs. In addition, teaching practices, which had strong effects on achievement, were related to teacher training. Students performed better when teachers provided hands-on learning opportunities and helped student develop higher order thinking skills. These practices were related to the training they had received in developing critical thinking skills and related pedagogy.

Deep real-world problem-solving can help develop these critical thinking skills in preservice mathematics teachers (Greer et al., 2002; Verschaffel et al., 1997).

Another area that could be strengthened in teacher education programs is the preservice teachers’ definition of the concept of function. It is interesting to note that none of the preservice teachers interviewed had a strong definition of a mathematical function in memory. Yet, even with a weak demonstrated function definition, the participants were successful about three-quarters of the time when they attempted to use their definitions of a mathematical function as well as functional notation. Rizzuti (1991), using 12^{th} grade mathematics students in her study, also found that her participants' weak function definitions did not hinder their function related problem-solving ability. It can easily be argued that it may be permissible for a student to not have a proper definition for a mathematical function in mind if they can still use the concepts to properly set-up and solve problems. Teachers, on the other hand, are not afforded that luxury. Mathematics students have little hope of forming useful function concepts if their teachers exhibit weak function definitions.

The ability to retain problem-solving skills is not a matter of memory retention related to specific problems or problem types, but rather a profound knowledge of the foundational concepts that form the backbone of the mathematical problems (L. Ma, 1999). Researchers need a way to measure teachers' knowledge to develop better preservice programs and summative assessments prior to professional licensure. If a reliable and valid instrument is employed to establish preservice teachers' content knowledge of mathematical function, then that instrument can be used with induction year in-service middle and secondary school mathematics teachers to establish the quality and quantity of problems solving skills retained in the transition from student to teacher.

As noted previously, this study only examined preservice teachers' problem-solving cognitions related to linear, quadratic, and exponential functions. While these three topics offer a good start, much more work is needed to expand the body of knowledge to span the full pre-calculus function range. The majority of the existing research literature on the function concept is restricted to algebraic level discussions (1996; DeMarois & Tall, 2009; Gray & Tall, 1994; Hollar & Norwood, 1999; Kieran, 2008; Lambertus, 2007; Leinhardt et al., 1990; O'Callaghan, 1998; Sfard, 1991). A potential paradox to overcome is the fact that as the range of problem types increases, the internal consistency coefficient can tend to suffer leaving the instrument with lower reliability scores. This will require very careful problem selection.

The item creation and validation procedures outlined in this project should provide a good foundation to future projects that endeavor to create expanded versions of this instrument. Specifically, the importance of creating linked mathematical problems to give researchers the ability to distinguish between foundational function misconceptions and contextual shortcomings will hopefully be a component of future development in this area.

A related area of future research will involve converting the format of the instrument from a multiple choice test to an open response test. This researcher, as well as several members of the Subject Matter Expert panel, expressed interest in seeing the open response version of the instrument so that more of the student thinking would be exposed. The issue to overcome is the time constraint; most students used an elimination strategy to assist in the problem-solving process which also shortens the time for test completion. Presenting the test as an open response test would lengthen the time required beyond the 50 minutes time requirement of the multiple-choice format. Since many university based teacher education program classes are 50 minutes in duration, it would be difficult to administer a test of this length to a typical class. The alternative, of course, would be to shorten (reduce the number of problems) the test, but again there would be difficulties due to either too few functional topics covered or a degradation of psychometric properties due to greater variability in participant responses.

Limitations

This research study endeavored to develop and measure the psychometric properties of an instrument that can be used to quantify the level of contextual function concept knowledge of preservice middle and secondary school teachers in the southern region of the U.S. More specifically, the population is comprised of students enrolled in university teacher education programs within the eleven states whose colleges and universities are accredited by The Commission on Colleges of the Southern Association of Colleges and Schools (SACS) (Southern Assocation of Colleges and Schools, 2009) that use either the TExES™ or the Praxis™ teacher certification examinations. The ability to generalize the results will be limited due to the fact that the sample represents 14 universities accredited by SACS in two states.

Another limiting factor in this study is the sheer breadth of the notion of content knowledge of the function concept. The need for this study is justified by the fact that the function concept is both ubiquitous and powerful in the STEM fields. It is very difficult to encompass the entire function concept in one short, multiple choice instrument, therefore I concede that we are only able to measure certain important aspects of the concept, as determined by the mathematics education experts that participated in the review of the instrument.

Delimitations

An important aspect of the methodology was the selection of the participants that represent the sample of the population under study. This study did not try to understand the function concept knowledge of preservice teachers all over the U.S., but rather the scope is narrowed to students in two Southern states. A common element in nine of the eleven southern states under SACS is that either the Texas Examinations of Educator Standards™ (TExES™) or Praxis™ teacher certification examination is used by the state certification boards (Educational Testing Service, 2009c; Texas Education Agency, 2009). Therefore, to improve the validity of the instrument, items selected on the instrument were based on the competency areas as defined by the TExES™ and Praxis™ mathematics teacher certification examinations for middle and secondary school teachers.

This study also concerns only itself with preservice middle and secondary school mathematics teachers in the nine Southern states that use TExES™ and Praxis™ mathematics teacher certification examinations. Logistical constraints require further delimitation of the sample pool to participants within the states of Texas and North Carolina. These states were chosen because they span the teacher certification exams under consideration and both states have an extended system of teacher education programs.

Assumptions

This study assumed that all participants surveyed gave honest and thoughtful responses to all questions on the instrument. It was further assumed that all participants had a sufficient English language reading proficiency to fully comprehend the meaning of the questions.

REFERENCES

Abbasi, Z. (2000). Reducing measurement error in informal sector surveys. Retrieved January 10, 2010, from http://mospi.nic.in/informal_paper_17.htm

Albright, J. J., & Park, H. M. (2009). Confirmatory factor analysis using Amos, LISREL, Mplus, and SAS/STAT CALIS. Retrieved Feburary 16, 2011, from http://www.indiana.edu/~statmath/stat/all/cfa/index.html

Allen, M. J., & Yen, W. M. (2002). *Introduction to measurement theory*. Long Grove, IL: Waveland Press, Inc.

American Educational Research Association, American Psychological Association, National Council on Measurement in Education, & Joint Committee on Standards for Educational and Psychological Testing (U.S.). (1999). *Standards for educational and psychological testing*. Washington, D.C.: American Educational Research Association.

Boeije, H. (2002). A purposeful approach to the constant comparative method in the analysis of qualitative interviews. *Quality & Quantity, 36*(4), 391-409.

Brown, F. G. (1983). *Principles of educational and psychological testing* (3rd ed.). New York: Holt, Rinehart and Winston.

Brown, I. A. (2010). Observing preservice teachers' contextualized function problem solving through task-based interviews. Unpublished manuscript. Texas A&M University.

Brown, I. A. (2011). The development and psychometric measurement of an instrument to assess preservice mathematics teachers' content knowledge in contextual function problem solving. Unpublished manuscript. Texas A&M University.

Brown, T. A. (2006). *Confirmatory factor analysis for applied research*. New York: The Guilford Press.

Carmines, E. G., & Zeller, R. A. (1979). *Reliability and validity assessment* (Vol. 17). Newbury Park, CA: Sage Publications.

Connally, E., Hughes-Hallett, D., Gleason, A. M., Avenoso, F., Cheifetz, P., Davidian, A., et al. (2004). *Functions modeling change: A preparation for calculus*. Danvers, MA: John Wiley & Sons.

Creswell, J. W. (2007). *Qualitative inquiry & research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage Publications.

Darling-Hammond, L., & Youngs, P. (2002). Defining "highly qualified teachers": What does "scientifically-based research" actually tell us? . *Educational Researcher, 31*(9), 13-25.

Davis, M. (1976). Forever lovers. On *Forever lovers* [vinyl]. New York: Columbia Music.

de la Harpe, B., & Wyber, J. (2001). *Teaching and assessing more than content - a focus on team working skills*. Paper presented at the Expanding Horizons in Teaching and Learning. Retrieved from http://skillcity.iaaf.uwa.edu.au

DeMarois, P. (1996). *Beginning algebra students' images of the function concept.* Paper presented at the 22nd AMATYC Annual Conference, Long Beach, CA.

DeMarois, P., & Tall, D. (2009). Facets and layers of the function concept. Retrieved March, 30, 2009, from http://www2.dm.unito.it/paginepersonali/arzarello/fond_mat_sis/2_maggio/tall%20demarois-pme1996.pdf

Dooren, W. V., Verschaffel, L., & Onghena, P. (2002). The impact of preservice teachers' content knowledge on their evaluation of students' strategies for solving arithmetic and algebra word problems. *Journal for Research in Mathematics Education, 33*(5), 319-351.

Driscoll, M. (1999). *Fostering algebraic thinking: a guide for teachers, grades 6 - 10*. Portsmouth, NH: Heinemann.

Duncan, A. (2009). Teacher preparation: Reforming the uncertain profession—Remarks of Secretary Arne Duncan at Teachers College, Columbia University [press release]. Retrieved October 23, 2009, from http://www.ed.gov/print/news/speeches/2009/10/10222009.html

Educational Testing Service. (2009a). Mathematics: Content Knowledge (0061). *The Praxis Series.* Retrieved August 18, 2009, from http://www.ets.org/Media/Tests/PRAXIS/pdf/0061.pdf

Educational Testing Service. (2009b). Middle School Mathematics (0069). *The Praxis Series* Retrieved August 18, 2009, from http://www.ets.org/Media/Tests/PRAXIS/taag/0069/glance.htm

Educational Testing Service. (2009c). Teacher Licensure and Certification. Retrieved March 3, 2009, from http://www.ets.org/portal/site/ets/menuitem.fab2360b1645a1de9b3a0779f1751509/?vgnextoid=48c05ee3d74f4010VgnVCM10000022f95190RCRD&WT.ac=Praxis+Brochure+and+Front+Door

Even, R. (1993). Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education, 24*(2), 94-116.

Glaser, B. G. (1965). The constant comparative method of qualitative analysis. *Social Problems, 12*(4), 436-445.

Goldin, G. A. (1997). Chapter 4: Observing mathematical problem solving through task-based interviews. *Journal for Research in Mathematics Education. Monograph, 9*, 40-177.

Goldin, G. A. (2000). A scientific perspective on structured, task-based interviews in mathematics education research. In K. E. & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517-545). Mahwah, NJ: Erlbaum.

Gorsuch, R. L. (1983). *Factor analysis* (2nd ed.). Hillsdale, NJ: L. Erlbaum Associates.

Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A "Proceptual" View of Simple Arithmetic. *Journal for Research in Mathematics Education, 25*(2), 116-140.

Greer, B., Verschaffel, L., & De Corte, E. (2002). "The answer really is 4.5": Beliefs about word problems. In G. C. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 271-292). Netherlands: Springer.

Grubb, W. N., & Kraskouskas, E. (1992). *A Time to every purpose: Integrating occupational and academic education in community colleges and technical institutes*. Macomb, IL: University of California, Berkeley.

Hess, K. K. (2006). Exploring cognitive demand in instruction and assessment. Retrieved Februrary 7, 2010, from http://www.nciea.org/publications/DOK_ApplyingWebb_KH08.pdf

Hill, H. C. (2007). Mathematical knowledge of middle school teachers: Implications for the No Child Left Behind policy initiative. *Educational Evaluation and Policy Analysis, 29*(2), 95-114.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hollar, J. C., & Norwood, K. (1999). The effects of a graphing-approach intermediate algebra curriculum on students' understanding of function. *Journal for Research in Mathematics Education, 30*(2), 220-226.

Hong, E., Sas, M., & Sas, J. C. (2006). Test-taking strategies of high and low mathematics achievers. *Journal of Educational Research, 99*(3), 144-155.

Huck, S. W. (2008). *Reading statistics and research* (5th ed.). Boston, MA: Pearson.

Karsenty, R. (2002). What do adults remember from their high school mathematics? The Case of Linear Functions. *Educational Studies in Mathematics, 51*(1/2), 117-144.

Kieran, C. (2008). *Conceptualizing the learning of algebraic technique: Role of Tasks and Technology.* Paper presented at the 11th International Congress on Mathematical Education, Monterrey, Mexico.

Knuth, E. J. (2000). Student understanding of the cartesian connection: An exploratory study. *Journal for Research in Mathematics Education, 31*(4), 500-507.

Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. *Theory into Practice, 41*(4), 212-218.

Kulm, G. (1979). The classification of problem-solving research variables. In G. A. Goldin & C. E. McClintock (Eds.), *Task variables in mathematical problem solving* (pp. 1-22). Columbus, OH: Information Reference Center (ERIC/IRC).

Kulm, G. (1982). The development of mathematics problem solving ability in early adolescence. *School Science and Mathematics, 82*(8), 666-672.

Kulm, G. (Ed.). (2008). *Teacher knowledge and practice in middle grades mathematics*. Rotterdam, The Netherlands: Sense Publishers.

Ladson-Billings, G. (1997). It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education, 28*(6), 697-708.

Lambertus, A. J. (2007). *Students' understanding of the function concept: Concept images and concept definitions.* North Carolina State University, Raleigh, NC.

Lankard, B. A. (1995). New ways of learning in the workplace. ERIC digest No. 161. Retrieved November 12, 2009, from http://www.eric.ed.gov/ERICWebPortal/contentdelivery/servlet/ERICServlet?accno=ED385778

Lappan, G., & Ferrini-Mundy, J. (1993). Knowing and doing mathematics: A new vision for middle grades students. *The Elementary School Journal, 93*(5), 625-641.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1-64.

Li, Y., & Kulm, G. (2008). Knowledge and confidence of pre-service mathematics teachers: The case of fraction division. *ZDM Mathematics Education, 40*(5), 833-843.

Lissitz, R. W. (Ed.). (2009). *The Concept of Validity: Revisions, new directions, and applications*. Charlotte, NC: Information Age Publishing, Inc.

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29*(3), 248-274.

Lynch, R. L., & Harnish, D. (2003). *Implementing contextual teaching and learning by novice teachers* (No. ED-98-CO-0085, 1998–2003). Athens, GA: University of Georgia.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Ma, W., & Freedson, G. (2002). *5 steps to a 5: AP calculus AB*. New York, NY: McGraw-Hill.

Maxwell, J. A. (2010). Using numbers in qualitative research. *Qualitative Inquiry, 16*(6), 475-482.

Menand, L. (2001). *The metaphysical club*. New York, NY: Farrar, Straus and Giroux.

Mohr, M. J. (2008). Mathematics knowledge for teaching: The case of preservice teachers. In G. Kulm (Ed.), *Teacher knowledge and practice in middle grades mathematics* (pp. 19-43). Rotterdam: Sense Publishers.

Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Math literacy and civil rights*. Boston, MA: Beacon Press.

Muthen, L., & Muthen, B. (2010). *Mplus user's guide* (6th ed.). Los Angles.

Nathan, M. J., & Koedinger, K. R. (2000). Teachers' and researchers' beliefs about the development of algebraic reasoning. *Journal for Research in Mathematics Education, 31*(2), 168-190.

National Council for Accredition of Teacher Education. (2005). *Research supporting the effectiveness of teacher preparation*. Retrieved January 5, 2010, from http://www.ncate.org/public/conc1.asp?ch=48

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Retrieved February 10, 2011, from http://www.nctm.org/standards/content.aspx?id=26628

National Council of Teachers of Mathematics. (2000a). *Principles and standards for school mathematics*. Retrieved February 10, 2011, 2011, from http://www.nctm.org/standards/content.aspx?id=26792

National Council of Teachers of Mathematics. (2000b). Principles and standards for school mathematics. Reston, VA: Author.

Noel, D. (2010). *Eliminate the wrong answers*. Retrieved August 4, 2010, from http://www.rubies-scientific.com/company_info/rubies_info/eliminate_wrong_answers.htm

O'Callaghan, B. R. (1998). Computer-intensive algebra and students' conceptual knowledge of functions. *Journal for Research in Mathematics Education, 29*(1), 21-40.

Orcher, L. T. (2005). *Conducting research: Social and behavioral science methods.* Glendale, CA: Pyrczak.

Ott, R. L., & Longnecker, M. (2001). *An introduction to statistical methods and data analysis* (5th ed.). Pacific Grove, CA: Duxbury.

Piaget, J. (1965). *The child's conception of the world* (J. Tomlinson & A. Tomlinson, Trans.). Totowa, NJ: Littlefield, Adams & Co.

Prince, M. J., & Felder, R. M. (2006). Inductive teaching and learning methods: Definitions, comparisons, and research bases. *Journal of Engineering Education, 95*(2), 123-138.

Richardson, J. (2009). Equity and mathematics: An interview with Deborah Ball and Bob Moses. *Phi Delta Kappa, 91*(2), 54-59.

Rizzuti, J. M. (1991). *Students' conceptualizations of mathematical functions: The effects of a pedagogical approach involving multiple representations.* Unpublished Dissertation, Cornell University, Ithaca, NY.

Roth, W.-M. (1996). Where is the context in contextual word problems?: Mathematical practices and products in grade 8 students' answers to story problems. *Cognition and Instruction, 14*(4), 487-527.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin. *Educational Studies in Mathematics, 22*(1), 1-36.

Sherin, M. G. (2002). When teaching becomes learning. *Cognition and Instruction, 20*(2), 119-150.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Sierpinska, A. (1992). On understanding the notion of function. In G. Harel & E. Dubinsky (Eds.), *The concept of function: Aspects of epistemology and pedagogy* (Vol. 25, pp. 25-58). Washington, DC: MAA Notes.

Sireci, S. G. (2007). On validity theory and test validation. *Educational Researcher, 36*(8), 477-481.

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344-350.

Southern Assocation of Colleges and Schools. (2009). *Commission on colleges*. Retrieved October 5, 2009, from http://www.sacscoc.org/index.asp

Stein, M. K., Baxter, J. A., & Leinhardt, G. (1990). Subject-matter knowledge and elementary instruction: A Case from Functions and Graphing. *American Educational Research Journal, 27*(4), 639-663.

Stone, J. R., Alfeld, C., Pearson, D., & Lewis, M. V. (2006). *Building academic skills in context: Testing the value of enhanced math learning in CTE*. Columbus, OH: National Dissemination Center for Career and Technical Education The Ohio State University.

Texas A&M University. (2010). Library Homepage. Retrieved March 3, 2010, from http://library.tamu.edu/

Texas Education Agency. (2008). *Texas Essential Knowledge and Skills*. Retrieved March 11, 2009, from http://ritter.tea.state.tx.us/teks/

Texas Education Agency. (2009). *Texas Educator Certification*. Retrieved January 14, 2009, from http://www.texes.ets.org/texes/

The Carnegie Foundation. (2010). Standard Listings. Retrieved March 12, 2010, from http://classifications.carnegiefoundation.org/lookup_listings/standard.php

The Education Alliance. (2006). *Closing the Achievement Gap: Best Practices in Teaching Mathematics* Charleston, WV: Author.

Verschaffel, L., De Corte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the role of real-world knowledge in mathematical modeling of school word problems. *Learning and Instruction, 7*(4), 339-359.

Webb, N. (1979). Content and context variables in problem tasks. In G. A. Goldin & C. E. McClintock (Eds.), *Task variables in mathematical problem solving* (pp. 69-102). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Enviornmental Education.

Webb, N. L. (2002). *An analysis of the alignment between mathematics standards and assessments for three states*. Paper presented at the American Educational Research Association Annual Meeting. Retrieved from http://addingvalue.wceruw.org/Related%20Bibliography/Articles/Webb%20three%20states.pdf

Webb, N. L. (2007). *Aligning assessments and standards*. Retrieved February 7, 2010, from http://www.wceruw.org/news/coverStories/aligning_assessments_and_standards.php

Wenglinksy, H. (2002). How schools matter. The link between teacher classroom practices and student academic performance. *Education Policy Analysis Archives, 10*(12). Retrieved from http://epaa.asu.edu/epaa/v10n12/

Whitney, W., Denton, M., & Jone, F. (1907). Report of the meeting of the central association of science and mathematics teachers. *School Science and Mathematics, 7*(1), 68-79.

Williams, D. L. (2007). The what, why, and how of contextual teaching in a mathematics classroom. *The Mathematics Teacher, 100*(8), 572-575.

Wiseley, W. C. (2009). *Effectiveness of contextual approaches to developmental math in California community colleges.* Unpublished Dissertation, University of the Pacific, Stockton, CA.

Yerushalmy, M. (2000). Problem solving strategies and mathematical resources: A longitudinal view on problem solving in a function based approach to algebra. *Educational Studies in Mathematics, 43*(2), 125-147.

Zbiek, R. M. (1998). Prospective teachers' use of computing tools to develop and validate functions as mathematical models. *Journal for Research in Mathematics Education, 29*(2), 184-201.

APPENDIX A

OBSERVING PRESERVICE TEACHERS' CONTEXTUALIZED FUNCTION PROBLEM-SOLVING THROUGH TASK-BASED INTERVIEWS: **INTERVIEW PROTOCOL**

By
Irving A. Brown
Texas A&M University
Spring 2010

Date: ___/___/2010 Time:___:___ am/pm Interviewee # _______

Audio cassette # _______

Introduction:
Good (morning/afternoon/evening), my name is Irving Brown. I am a PhD candidate in Mathematics Education here at Texas A&M and this interview is part of a study that will be part of my dissertation research. Thank you for your participation. Before we begin, do you have any questions about the information sheet that you have been given concerning this study? Do you have any questions about your rights as a research participant? Our interview will be audio recorded and later transcribed for research purposes and I will also take notes during our interview. You will only be referred to in both my notes and the interview transcripts by a random "interviewee" number.

First, let me share some information about this study with you. The purpose of this study is to learn more about how preservice teachers solve contextualized function problems. I will be asking you some questions from the "Contextual Function Survey" you completed earlier and also asking you to complete some related tasks. It is important to note that you may or may not know how to correctly solve the problems; the focus of our discussion is not on a "correct" answer but rather, I am interested in your thoughts and problem-solving approaches to the various problems.

I am not interested in you giving me a single answer, but rather I am interested in finding out about your thinking. It won't be helpful to me if you think silently and just give me an answer. It will be more helpful if you tell me what you are thinking. I will be asking for clarifications and explanations about what you have done or said. This doesn't mean that you are right or wrong. I am just trying to make sure that I understand how you are thinking about the mathematics.

To make sure you have the tools you may need to solve the problems, I have a calculator, paper, and a pencil available if you would like to use any of them. I also have a laptop available with specialized graphing software installed for our use to enhance our mathematical discussions. Do you have any questions before we get started?

Question 1: ***[This is primarily a "warm-up" problem to prepare the participant for the mathematical thinking to come in questions 2 – 5. The response to the question "what is a function?" should provide valuable insight to the participants' basic understanding of the function concept.]***

Please refer to problem 2 on the Contextual Function Survey. To the best of your ability, please solve the problem and speak aloud your thoughts as you solve the problem. (*Give the interviewee sufficient time to work the problem with only non-directive follow-up questions. e.g.,* "Can you tell me more about that approach?")

(*If response is not spontaneous, give heuristic suggestion. e.g.,* "Can you solve the problem using the calculator or laptop?")

Question 1: (problem 2 continued)
(*If interviewee is "stuck" or non-responsive, offer guided heuristic suggestion; e.g.,*
"How does the given information relate to the terms in the formula? What does 'directly' proportional mean; what does 'inversely' proportional mean?")

Does this problem represent a mathematical function? What is a mathematical function?

Question 2:
Please refer to problem 9 on the Contextual Function Survey. To the best of your ability, please solve the problem and speak aloud your thoughts as you solve the problem.
(*Give the interviewee sufficient time to work the problem with only non-directive follow-up questions. e.g.,* "Can you tell me more about that approach?")

(*If response is not spontaneous, give heuristic suggestion. e.g.,* "Can you solve the problem using the calculator or laptop?")

Question 2: (problem 9 continued)
(*If interviewee is "stuck" or non-responsive, offer guided heuristic suggestion; e.g.,* "What formula would you use to set this problem up? How does the given information relate to the terms in the formula?")

Please explain how you approached this problem in your mind; which elements of the problem made it seem easy or difficult to you? How did you decide which elements of the problem statement to use and which to discard?

Question 3:
Please refer to problem 10 on the Contextual Function Survey. To the best of your ability, please solve the problem and speak aloud your thoughts as you solve the problem.
(*Give the interviewee sufficient time to work the problem with only non-directive follow-up questions. e.g.,* "Can you tell me more about that approach?")

(*If response is not spontaneous, give heuristic suggestion. e.g.,* "Can you solve the problem using the calculator or laptop?")

Question 3: (problem 10 continued)
(*If interviewee is "stuck" or non-responsive, offer guided heuristic suggestion; e.g.,* "What significant differences do you see in the four graphs? How does the given information relate to the significant differences in the graph?")

Please explain how you approached this problem in your mind; which elements of the problem made it seem easy or difficult to you?

Question 4:
Please refer to problem 12 on the Contextual Function Survey. To the best of your ability, please solve the problem and speak aloud your thoughts as you solve the problem.
(*Give the interviewee sufficient time to work the problem with only non-directive follow-up questions. e.g.,* "Can you tell me more about that approach?")

(*If response is not spontaneous, give heuristic suggestion. e.g.,* "Can you solve the problem using the calculator or laptop?")

Question 4: (problem 12 continued)
(*If interviewee is "stuck" or non-responsive, offer guided heuristic suggestion; e.g.,* "Let's use the specialized graphing software on the laptop to create a parabola. Generally speaking what should the parabola look like? Should it open up, down, to the left or to the right? Now, use the software tool to change the value of the v_y coefficient. How does that software visualization help you?")

Please explain how you approached this problem in your mind; which elements of the problem made it seem easy or difficult to you?

Question 5:
Please refer to problem 15 on the Contextual Function Survey. To the best of your ability, please solve the problem and speak aloud your thoughts as you solve the problem.
(*Give the interviewee sufficient time to work the problem with only non-directive follow-up questions. e.g.,* "Can you tell me more about that approach?")

(*If response is not spontaneous, give heuristic suggestion. e.g.,* "Can you solve the problem using the calculator or laptop?")

Question 5: (problem 15 continued)
(*If interviewee is "stuck" or non-responsive, offer guided heuristic suggestion; e.g.,* "What formula would you use to set this problem up? How does the given information relate to the terms in the formula?")

Please explain how you approached this problem in your mind; which elements of the problem made it seem easy or difficult to you? How did you decide which elements of the problem statement to use and which to discard?

Question 6*: [**_*Only if time permits the use of this additional question_**]
Please refer to problem 8 on the Contextual Function Survey. To the best of your ability, please solve the problem and speak aloud your thoughts as you solve the problem.
(*Give the interviewee sufficient time to work the problem with only non-directive follow-up questions. e.g.,* "Can you tell me more about that approach?")

(*If response is not spontaneous, give heuristic suggestion. e.g.,* "Can you solve the problem using the calculator or laptop?")

Question 6*: (problem 8 continued)
(*If interviewee is "stuck" or non-responsive, offer guided heuristic suggestion; e.g.,* "What formula would you use to set this problem up? How does the given information relate to the terms in the formula?")

Please explain how you approached this problem in your mind; which elements of the problem made it seem easy or difficult to you? (*If the interviewee solved the problem algebraically, ask them if they can solve it graphically, or visa-versa.*)

This concludes our interview. Thank you very much for your time and attention in sharing your problem-solving approaches with me. Your responses will be of significant value to my dissertation research.

Please remember that if you have any questions about this research in the future, please feel free to contact me using the email address or phone number on the information sheet.

Thank you again.

Appendix B

CONTEXTUAL FUNCTION INSTRUMENT (VER. 1.1)

TExES (4 – 8): Domain II, Comp 005 & 007; Domain V, Comp 015
Praxis (0069): Content Category I, Items 8 & 9; Category III, Items 4, 5 & 7

After many years of hard work at Elberton National Bank, Vernon has finally worked himself into his dream job. As the newest Corporate Loan Manager, Vernon was faced with making a loan decision on his first corporate account. Business was booming at Nunnally's Family Farming Implements so James and Ned decided to expand their "Garden Gopher" line of rakes, hoes, and mattocks which had been active since 1977. Since they were not ready to take on new partners for capitalization, they found themselves at Vernon's desk in the corporate loan department of the bank hoping for a $75,000.00 loan at either 4 ¼ % for 36 months or at 5 ½ % for 24 months. As part of the loan qualification process, Ned and James provided the following profit and sales data to show that the profit grew at a constant rate with sales:

Profit	**-471.00**	**216.50**	**1,726.25**	**4,888.75**	**8,329.00**	**10,529.00**	**12,729.00**
Units sold	**500**	**750**	**1299**	**2449**	**3700**	**4500**	**5300**

1) Before the first unit was sold, what was the cost (negative profit) of doing business?

A. $471.00
B. $1846.00
C. $2750.00
D. $75,000.00

2) How much money more money is earned for each 1,000 unit increase in sales?
A. $471.00
B. $1846.00
C. $2750.00
D. $75,000.00

TExES (4 – 8): Domain II, Comp 006 & 007; Domain V, Comp 015 & 016
Praxis (0069): Content Category I, Items 9; Content Category III, Items 5, & 8

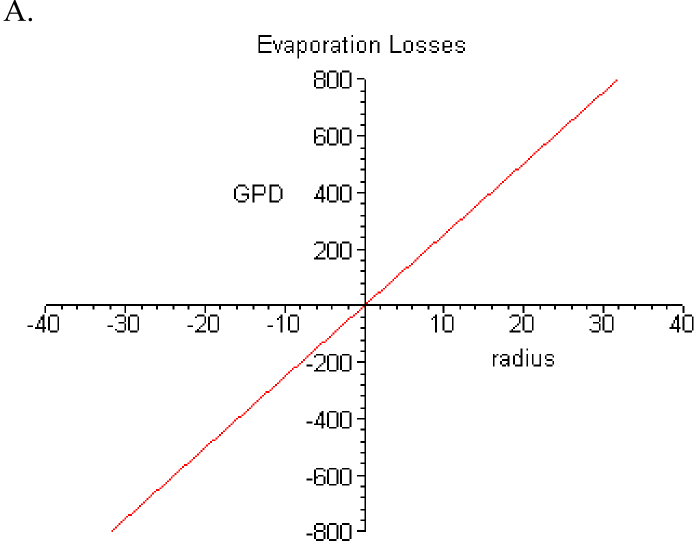
Pricilla's newly formed company, Aggie Pools, Inc., is in the business of installing round, in ground swimming pools. She offers pools from the economy sized (20 feet in diameter) to the "Hummer" of backyard pools (80 feet in diameter). A common concern of her customers is the amount of water lost each day due to evaporation. To maintain a constant level in the swimming pool, each gallon of water lost to evaporation must be made up with fresh water from the city water department. The local fresh water supply rate is $4.80 for the first 2000 gallons of water and $1.96 for each additional thousand gallons. Armed with the knowledge that water evaporation is directly proportional to the surface area of the pool and that evaporation loss rates in her service area average about 1/120 gallons per hour per ft^2 of pool surface area, she decides to create a formula that relates evaporation loss per day as a function of the radius of the pool.

3) Which of the 4 choices below, best represents the function described above?

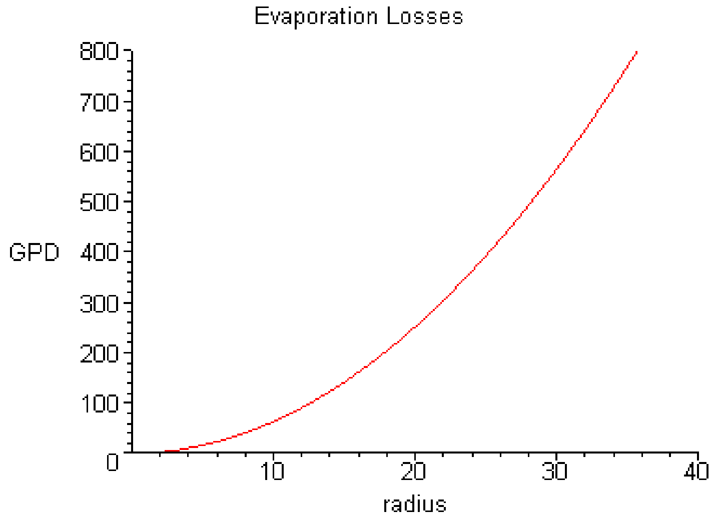
A. Evap. Loss = $1.96 * x + $4.80
B. Evap. Loss = $0.2 \pi r^2$
C. Evap. Loss = $(1/120)*(\pi r^2) + \$4.80$
D. Evap. Loss = (1/120)*(area)

4) Of the four graphs below, which of the graphs best represents the "real world" use of the above function?

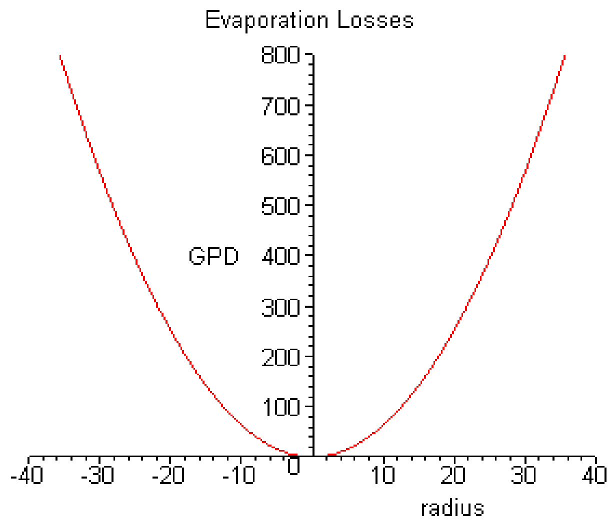
A.



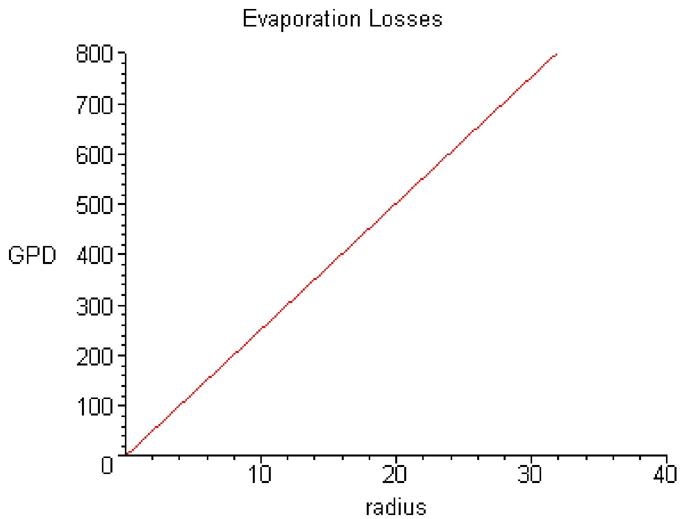
B. CORRECT ANSWER



C.



D.



TExES (4 – 8): Domain II, Comp 006; Domain V, Comp 015 & 016
Praxis (0069): Content Category I, Items 9 & 10; Content Category III, Items 5

It was Mr. I. Diot's lifelong aspiration to one day become an award winner. What he didn't realize was that his unique antics would one day win him the "Darwin Award" for his personal contribution to help cleanse the gene pool. I. Diot came up with the brilliant plan to test his new bullet proof helmet by firing a gun straight up into the air, run inside the house to get the helmet, run back out to the exact spot below the falling bullet, and count up ten seconds before putting on the helmet. Mr. Tightpockets, the insurance claims adjuster handling the death benefit for Mrs. I. Diot, calculated the time for the bullet to travel into the sky and become the last thing on I. Diot's mind by using the formula: $y = y_0 + v_{y0} * t + ½*a_y * t^2$. Additional facts used to calculate the bullet's travel time were as follows: it took I. Diot 40 seconds to run into the house to retrieve the helmet and run back to the exact test spot, the value of $v_{y0} = 750$ ft/sec, the value of $a_y = -32.2$ ft/sec^2, and he initially fired the gun from head level at $y_0 = 6$ ft.

5) What was the last number Mrs. I. Diot heard her husband say, before she whipped out the cell phone to call the insurance claims adjuster and the travel agent?

A. "6"
B. "7"
C. "8"
D. "9"

6) Besides not trying such a reckless stunt in the first place, what else could I. Diot have done to improve his chance of survival?

A. Purchased a higher quality helmet
B. Moved the spot for the stunt twice as far from the house
C. Clean and test fire the gun before the stunt
D. Held the gun 2 feet above his head before firing

TExES (8 – 12): Domain II, Comp 008; Domain V, Comp 019
Praxis (0061): Content Category: Functions, Items 1, & 4

The question really boiled down to whether or not Bugs Meany was really at the coffee shop at midnight as he claimed, or was he across town committing the cat burglary of the century? His friends claim that his alibi is valid because he drank coffee with them until midnight, but other seemingly reliable witnesses told police that Buzz left the coffee shop about 8pm. At his wits end, the police chief calls in Encyclopedia Brown, the boy detective, to crack the case. Encyclopedia Brown's first suggestion is to draw a sample of Bugs' blood to determine the remaining level of caffeine, the psycho-active substance that gives coffee its "kick." The lab determined that the blood sample drawn at 8:00am retained 33 mg of caffeine of the original 325 mg of caffeine consumed by Bugs in the two large espressos he drank the night before. Using the knowledge that caffeine leaves the average person (approx. 145 pounds) at a continuous rate of about 15% per hour, Encyclopedia Brown smiles and tells the chief to put the handcuffs on Bugs Meany and read him the Miranda warning.

7) Based on the average person's weight assumption, how much caffeine should have remained in Bugs' blood?

A. 97.89
B. 84.25
C. 72.52
D. 62.42

8) Bugs attempts to counter Encyclopedia Brown's findings by pointing out that he is, at 197 pounds, much larger than the average person and Encyclopedia Brown should have used the fact that caffeine leaves a person of his size at a continuous rate of 19%. Using this new information, about how many hours had passed since Bugs drank the two espressos to have retained the 33mg found in his blood at 8:00am?

A. 8 hours
B. 10 hours
C. 12 hours
D. 14 hours

TExES (4 – 8): Domain II, Comp 005; Domain V, Comp 015
Praxis (0069): Content Category I, Item 8; Category III, Items 5, & 8

Chasity and Denise, owners of "Pickett's Custom Accoutrements" and makers of the finest handbags in the South, want to be able to predict the production cost of their most popular handbag "Gretha's Grip". Business has been growing in the 18 months of handbag production, and their highest sales in a single month have been 147 handbags. After a careful analysis of the variable costs (costs based on the number of bags produced) and fixed costs (the cost of doing business before a single bag is produced), they are able to describe the relationship between the total monthly cost, C, (fixed cost + variable costs) and the number of bags produced in a month, n, as follows:

$$C = \$39.86 * n + \$1608.00$$

9) What is the monthly fixed cost associated with producing "Gretha's Grip"?
A. $1647.86
B. $39.86
C. $1608.00
D. $1568.14

10) What is the monthly variable cost associated with producing "Gretha's Grip"?
A. $1608.00*n/month
B. $39.86*n/month
C. $1608.00*n/bag
D. $39.86*n/bag

TExES (8 – 12): Domain II, Comp 005; Domain V, Comp 019
Praxis (0061): Content Category: Functions, Items 5

The goal of creating new and exciting machines has driven Valerie to excel in her studies since she was a little girl. Now, as a manufacturing engineer, she has created a most amazing pair of magical machines that contradict the law of conservation of mass. Her machines increase the mass of ice cream fed into them! The first machine, she calls the "Pi" machine, gives as an output 2.5 times the amount fed to it. For example, if she feeds 2 kilograms in, she will get 5 kilograms out; 4 kilograms in, gives 10 kilograms out. The second machine, Valerie refers to as the "Sigma" machine, gives an output 6.2 kilograms more than the amount fed to it. For example, if she feeds 2 kilograms in, she will get 8.2 kilograms out; 4 kilograms in, gives 10.2 kilograms out.

11) Assuming her machines work as described, how much ice cream should she feed into each machine to get an equal output from both machines?

A. 3.14 kilograms
B. 4.13 kilograms
C. 5.68 kilograms
D. 8.65 kilograms

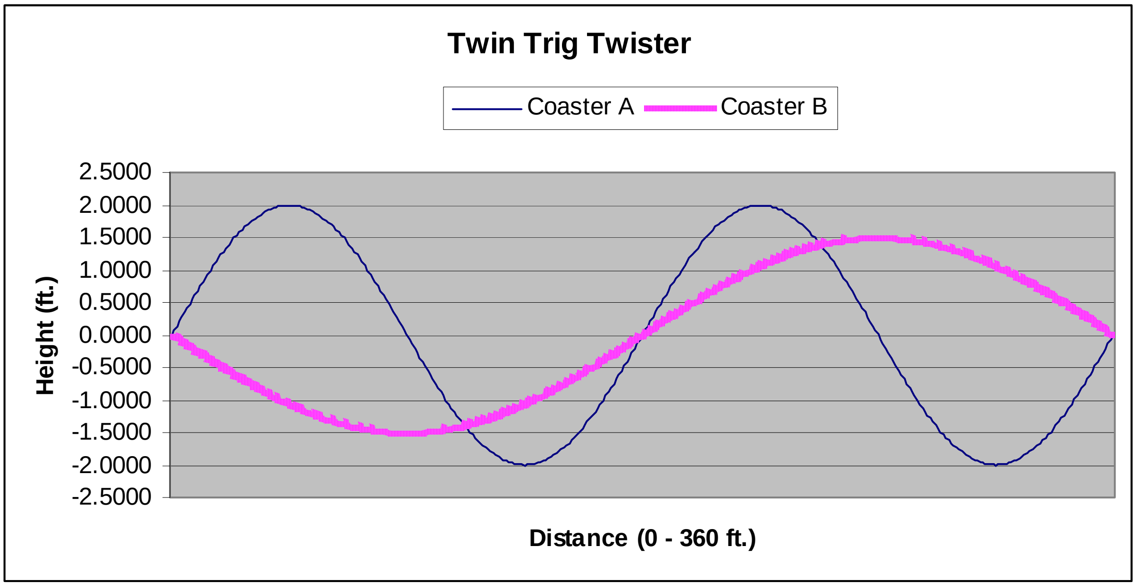
12) The output of the "Pi" machine can be fed to the input of the "Sigma" machine to make a composite machine or the output from the "Sigma" machine can be fed to the input of the "Pi" machine for a different composite machine. It should be noted that one of the composite configurations is more productive than the other. If Valerie feeds 5 kilograms of ice cream into the more productive of the two possible composite machines, how many kilograms will be produced?

A. 8.7 kilograms
B. 15.5 kilograms
C. 18.7 kilograms
D. 28.0 kilograms

TExES (8 – 12): Domain II, Comp 009; Domain V, Comp 019
Praxis (0061): Content Category: Trigonometry, Items 1, 3 & 4; Functions, Items 4

Just looking at the sparkle in Trinard's eyes is enough to know that the source of this beautiful light comes from daughters Kyndall and Kale. Today's task is to use his new camera, that his wife Karen purchased for their eleventh wedding anniversary, to take the perfect photo of his little girls as they ride the "Twin Trig Twister" kiddy coaster at "Fun-Land" amusement park. For him the perfect photo opportunity will occur when both girls, on their separate coasters, will be at the same height.

The height of coaster A (h_1) is defined by the formula $h_1 = 2 * \sin(2*x)$ and the height of coaster B (h_2) is defined by the formula $h_2 = -1.5 * \sin(x)$; where x is the linear distance traveled by the respective coaster. The figure below shows the graphical representation of the formulas for the two coasters. Both coasters start at the same x distance (zero feet) and height and end at the same height 360 feet away.



13) Approximately how many feet away from the start of the coaster ride should Trinard set up his camera for the first opportunity for the "perfect" shot?

A. 108 feet
B. 112 feet
C. 128 feet
D. 148 feet

14) Approximately how many feet away from the end of the coaster ride can Trinard set up his camera for the last opportunity for the "perfect" shot?

A. 108 feet
B. 112 feet
C. 128 feet
D. 148 feet

TExES (4 – 8): Domain II, Comp 006; Domain V, Comp 015 & 016
Praxis (0069): Content Category I, Items 9, & 10

An international spy, Agent 008, has just been rushed into the medical unit at headquarters. A full body radiographic scan has revealed that Agent 008 has been injected with a synthetic neurotoxin capsule that has modulating activity (toxicity) in the human body. The activity of natural neurotoxins tends to grow exponentially until they cause irreversible damage and become terminal for the patient. It was found that the activity for this particular synthetic neurotoxin is dependent on the time it has been in the human body by the following formula: Activity = $t^4 - 5t^3 + 7t^2 - 15t + 12$. Assisted by a particularly bright mathematics teacher, it was shown that the activity formula for this capsule could alternatively be expressed as: Activity = $(t^2 + 3)(t^2 – 5t + 4)$, where t represents the time in days the patient has been exposed to the neurotoxin. It has also been determined that if the capsule remains in the 185 pound body of Agent 008 for more than 8 days, the activity will have risen to sufficient levels to become terminal. The medical team has decided to operate on Agent 008 to remove the capsule, but they can only remove the capsule when the activity of the neurotoxin capsule is at zero otherwise he will suffer irreversible medical damage.

15) How many opportunities will the medical team have to safely remove the capsule from Agent 008?

A. 1
B. 2
C. 3
D. 4

16) Assuming it takes 36 hours to prep the equipment and staff for the safe removal of the capsule, on which day should the surgical staff perform the operation?

A. Day 1
B. Day 2
C. Day 3
D. Day 4

APPENDIX C

SUBJECT MATTER EXPERT SOLICITATION

Hello Dr. Xxxx,

I hope that you have had both a productive and regenerative summer. Last we spoke, back in early spring, I was preparing to send my “Contextual Function Survey” to you and several other faculty members in Texas and North Carolina for expert feedback. My sending the survey was delayed while we planned and implemented a modified course of action for my research project. My committee thought it prudent that I include a qualitative study with preservice mathematics teachers to strengthen the overall findings in my dissertation research. Now that this qualitative study is complete, I am ready to proceed with the validation study of the instrument.

I thank you again for agreeing to be a subject matter expert (SME) in my study. I have attached a copy of the instrument for your examination along with the answer key, and detailed answers for select problems. The purpose of the detailed answers is to give you further insight into my rationale for creating the problems. If you would, please provide any feedback you feel will aid in the refinement of this instrument such as content, language and wording. The primary focus of the SME feedback is to establish content validity. To further this goal, I have also included a (short) relevant “item creation” excerpt from my proposal. I have also attached a copy of the IRB exemption for this project.

Please respond on the “Subject Matter Expert – Response Sheet”, save it as a Word document and return it to me as an email attachment. I understand how busy the beginning of the semester can be, so I can’t set a “deadline” for your response. If you could return your response within the next two weeks, that would be extremely helpful in allowing me time to revise and send the survey out for testing early in the semester.

Thank you for your time and consideration,

Irving

Irving A. Brown
PhD Candidate
Mathematics Education
Teaching, Learning, and Culture
Texas A&M University
Office: Harrington Tower 205
Phone: (512) 694-4762 Wireless
Phone: (979) 845-0561 Office
Email: iab4phd@tamu.edu

APPENDIX D

CONTEXTUAL FUNCTION SURVEY
SUBJECT MATTER EXPERT – RESPONSE SHEET

Name:
Date:
University Affiliation:

Please indicate your approval or suggestions for Content and Wording/Language for each item on the survey.

Item 1
Content:

Wording/Language:

Item 2
Content:

Wording/Language:

Item 3
Content:

Wording/Language:

Item 4
Content:

Wording/Language:

Item 5
Content:

Wording/Language:

Item 6
Content:

Wording/Language:

Item 7
Content:

Wording/Language:

Item 8
Content:

Wording/Language:

Item 9
Content:

Wording/Language:

Item 10
Content:

Wording/Language:

Item 11
Content:

Wording/Language:

Item 12
Content:

Wording/Language:

Item 13
Content:

Wording/Language:

Item 14

Content:

Wording/Language:

Item 15
Content:

Wording/Language:

APPENDIX E

DESCRIPTIVE STATISTICS

Descriptive Statistics for Score by Sex

Sex = Female		Statistic	Standard Error
Mean		44.1441	1.31681
95% Confidence Interval	Lower	41.5418	
For Mean	Upper	46.7465	
Median		40.0000	
Variance		256.632	
Standard Deviation		16.0197	
Range		73.33	

Sex = Male		Statistic	Standard Error
Mean		49.2063	2.79143
95% Confidence Interval	Lower	43.5689	
For Mean	Upper	54.8438	
Median		40.0000	
Variance		327.268	
Standard Deviation		18.0905	
Range		66.67	

Descriptive Statistics for Score by Class

Class = Sophomore		Statistic	Standard Error
Mean		40.8333	2.78923
95% Confidence Interval	Lower	35.0634	
For Mean	Upper	46.6033	
Median		40.0000	
Variance		186.715	
Standard Deviation		13.66437	
Range		53.33	

Class = Junior		Statistic	Standard Error
Mean		45.1282	1.94348
95% Confidence Interval	Lower	41.2457	
For Mean	Upper	49.0108	
Median		40.0000	
Variance		245.513	
Standard Deviation		15.66885	
Range		73.33	

APPENDIX F

CONTEXTUAL FUNCTION INSTRUMENT (VER. 4.1)

INFORMATION SHEET

Pre-Service Teachers' Function Knowledge

Introduction

The purpose of this form is to provide you (as a prospective research study participant) information that may affect your decision as to whether or not to participate in this research.

You have been asked to participate in a research study to better understand preservice mathematics teachers' understanding of the function concept. The purpose of this study is to develop a survey instrument to examine preservice teachers' knowledge of the function concept. You were selected to be a possible participant because have been identified as a middle or secondary school preservice mathematics teacher.

.

What will I be asked to do?

If you agree to participate in this study, you will be asked to spend about 50 minutes to complete a 15 question survey to demonstrate your content knowledge of the function concept in a contextual problem-solving enviornment. You will be asked to respond to mathematics questions and problems. You will only be asked to complete the survey one time.

What are the risks involved in this study?

The risks associated with this study are minimal, and are not greater than risks ordinarily encountered in daily life.

What are the possible benefits of this study?

You will receive no direct benefit from participating in this study; however, Results from this study, including the newly developed instrument, may possibly help mathematics teacher educators better assess preservice mathematics teachers' content knowledge of the function concept.

Do I have to participate?

No. Your participation is voluntary. You may decide not to participate or to withdraw at any time without your current or future relations with Texas A&M University, any universities within the state of Texas, or university within the University of North Carolina system being affected.

Who will know about my participation in this research study?

This study is anonymous and the attached survey sheet will not ask for your name or any other identifiers.

No individual survey results will be used in the study; only grouped data will be included in any sort of report that might be published

Whom do I contact with questions about the research?

If you have questions regarding this study, you may contact Irving A. Brown, (512) 694-4762, iabrown@tamu.edu. or Dr. Gerald Kulm, (979) 862-4407, gkulm@tamu.edu.

Whom do I contact about my rights as a research participant?
This research study has been reviewed by the Human Subjects' Protection Program and/or the Institutional Review Board at Texas A&M University. For research-related problems or questions regarding your rights as a research participant, you can contact these offices at (979)458-4067 or irb@tamu.edu.

Participation
Please be sure you have read the above information, asked questions and received answers to your satisfaction. If you would like to be in the study, please follow the directions on the attached survey.

Contextual Function Instrument

Demographic Information

1) Sex: ____ Female ____ Male

2) Academic Classification:
____ Sophomore ____ Junior ____ Senior ____ Post Bac

3) Mathematics Teacher Certification Level:
____ Middle School ____ High School

4) Undergraduate Major:

____

Instructions

- Relax; this is not a test but simply a study.
- Circle your answer choices.
- Only spend about an hour on the questions.
- Please feel free to use calculator or graphing calculator.
- Please do not use friends, computers, or the internet.
- You can detach and keep the first page (Information Sheet)
- Relax and enjoy a little low stress problem-solving.

Thank you for your participation!

Contextual Function Instrument

1) Linear velocity (v) can be expressed as a function of time (t) by several different formulas. If time, and therefore velocity, are allowed to vary while holding the initial velocity (v_0) and acceleration (a) constant, the function takes the form $v(t) = v_0 + a \bullet t$.

Given $v(t) = v_0 + a \bullet t$; where $v(2) = 0$, $v_0 = 60$**;** find a.

A. -20
B. -30
C. -40
D. -50

2) Management at Elberton National Bank has been informed by the lead bookkeeper, Vernon, that the number N of Certificates of Deposit (CD) sold each month appears to be inversely proportional to the quantity (P + 22.5) where P is the price of one certificate in dollars.

If k is the constant of proportionality, how would Vernon express N as a function of P?

A. N(P) = k + (P + 22.5)
B. N(P) = k • (P + 22.5)
C. N(P) **=** k / (P + 22.5)
D. N(P) = k - (P + 22.5)

3) The vertical distance y, (in feet above the ground), a ball travels after being thrown directly upward from an initial height of y_0 (feet) with an initial velocity v_y (in feet/sec) can be expressed as a function of time (t) in the following formula:
$y(t) = y_0 + v_y \bullet t + ½ \bullet a_y \bullet t^2$, where a_y is the acceleration due to gravity in ft/sec^2. Given the value of v_y = 60 ft/sec, the value of a_y = -32.2 ft/sec^2, and the ball was initially thrown from a platform at y_0 = 20 feet above the ground, calculate the time (in seconds) the ball travels before striking ground.

A. (-0.308 & 4.03)
B. (4.03 & -0.308)
C. 3.3 seconds
D. 4 seconds

4) Given the function $P(t) = P_0 \bullet e^{rt}$; find P(11) if P_0 = 100 and r =1.05.

A. 10,377,703.68
B. 103,777.04
C. 5,987,414.17
D. 10,677,703.68

5) Given $f(x) = x^2 - x - 6$, find the x-intercepts.

A. x = 6 & x = 7
B. x = -3 & x = 2
C. x = -2 & x = 3
D. x = -6 & x = 0

6) Given the function $f(t) = t^4 - 169$, how many real roots and how many imaginary roots does the equation have?

A. 4 real roots & 0 imaginary roots
B. 3 real roots & 1 imaginary roots
C. 2 real roots & 2 imaginary roots
D. 0 real roots & 4 imaginary roots

Starting to Stop

Valerie wanted to know more about the braking potential of her new pickup truck. She found that from an initial velocity (v_0) of 60 miles/hr, she was able to bring her truck to a safe stop in a time of exactly 9 seconds. The velocity function, $v(t)$, can take the form: $v(t) = a \cdot t + v_0$. Assume a constant rate of deceleration, a.

7) Which equation below best represents her truck's velocity as a function of time?

E. $v(t)$ = 6.6667 miles/hr^2 • time - 60 miles/hr
F. $v(t)$ = -6.6667 miles/hr^2 • time + 60 miles/hr
G. $v(t)$ = 6.6667 miles/hr/seconds • time - 60 miles/hr
H. v(t) = -6.6667 miles/hr/seconds • time + 60 miles/hr

8) If Valerie starts at 30 miles/hr and she is able to double the rate of deceleration, how long will it take the truck to stop?

A. 1.125 seconds
B. 2.250 seconds
C. 3.750 seconds
D. 4.500 seconds

Evaporation Exasperation

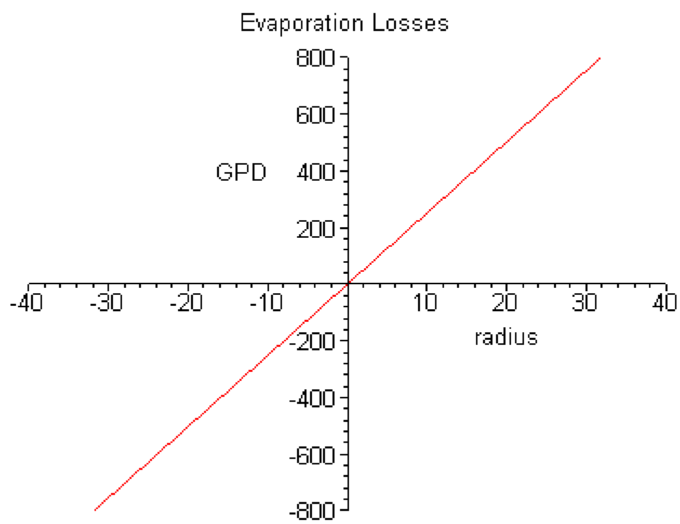
Priscilla's newly formed company, Aggie Pools, Inc., is in the business of installing circular, in-ground swimming pools. She offers pools from the economy sized (20 feet in diameter) to the "Hummer" of backyard pools (80 feet in diameter). A common concern of her customers is the amount of water lost each day due to evaporation. To maintain a constant water level in the swimming pool, each gallon of water lost to evaporation must be made up with fresh water from the city water department. The local fresh water supply rate is $4.80 for the first 2000 gallons of water and $1.96 for each additional thousand gallons. Armed with the knowledge that water evaporation is directly proportional to the surface area of the pool and that evaporation loss rates in her service area average about 1/120 gallons per hour per ft^2 of pool surface area, she decides to create a formula that relates evaporation loss (L) in gallons per day as a function of the radius of the pool.

9) Which of the 4 choices below, best represents the function described above (where r is the radius (in feet) of the pool)?

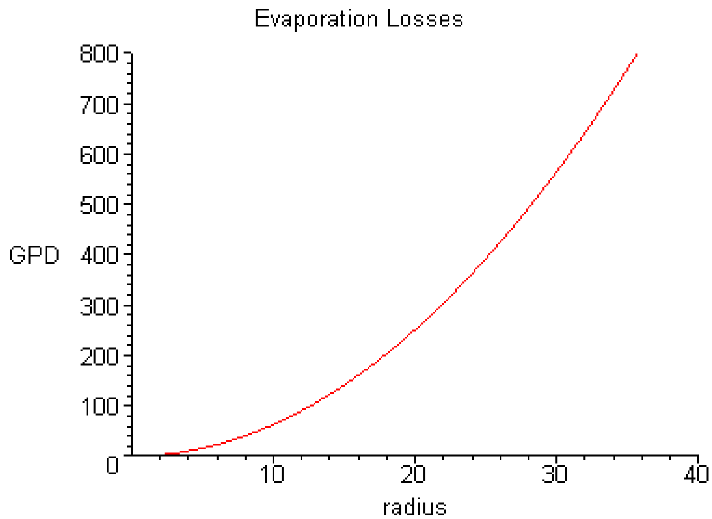
E. $L(r) = \$1.96 \bullet r + \$4.80$
F. $L(r) = 0.2\,\pi\, r^2$
G. $L(r) = (1/120)(\pi\, r^2) + \$4.80$
H. $L(r) = (1/120)(\pi\, r^2)$

10) Of the four graphs that follow, choose the one that best corresponds to the swimming-pool context of this function?

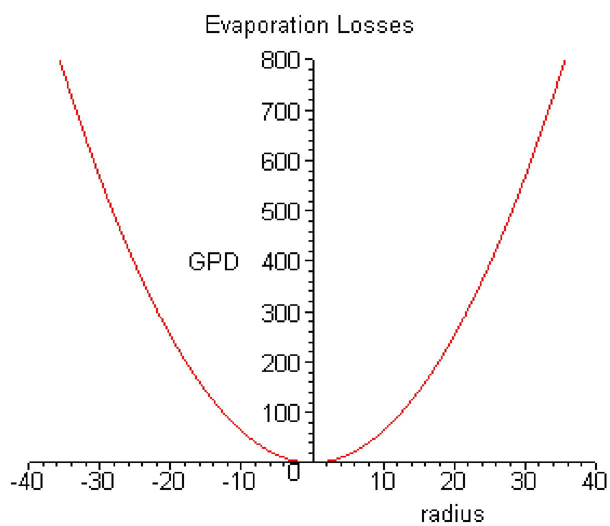
E.



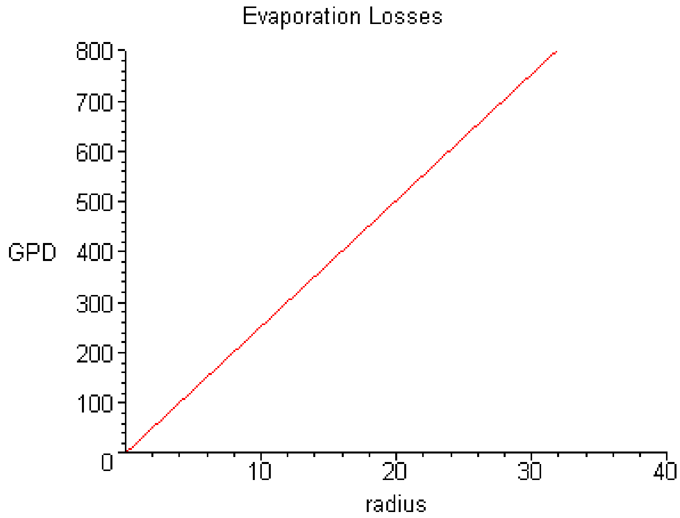
F.


Evaporation Losses
GPD
0
100
200
300
400
500
600
700
800
10
20
30
40
radius

G.

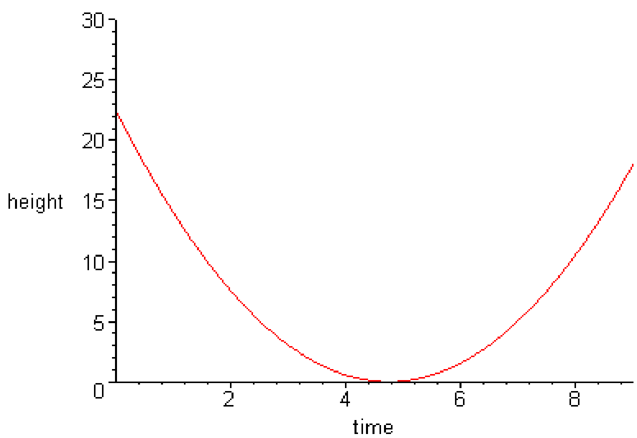

Evaporation Losses
GPD
0
100
200
300
400
500
600
700
800
-40
-30
-20
-10
10
20
30
40
radius

H.



Bungee Fun

Chasity, in her new job as safety engineer for an amusement park, is analyzing data for a new bungee jump attraction in the water park area. The special feature of this bungee attraction is that the bungee ride is customized for each individual such that the rider barely touches the surface of the pool at the lowest point in the first fall from the platform. While analyzing the height vs. time (t) data for a jumper tethered to a bungee cord over the pool (for a single fall and rebound cycle), she notices that the function fits the form, height(t) = $at^2 + bt + c$, where height is the height (in feet) above the surface of the pool and time is measured in seconds. She plots the data and observes the following graph.



11) Based on the information above, what can Chasity deduce about the coefficients of the function?

A) $b^2 - 4ac > 0$
B) $b^2 - 4ac < 0$
C) $b^2 - 4ac = 0$
D) $c^2 = a^2 + b^2$

For the next question, please consider how changes in the coefficients a_y, v_y, *and* y_0 *affect the shape of the curve defined by the following function:*

$$y(t) = y_0 + v_y \bullet t + ½\bullet a_y \bullet t^2$$

12) If the coefficients 'a_y' and 'y_0' are constants, what effect does changing the 'v_y' coefficient have on the graph of the function?

A. The vertex of the parabola shifts horizontally to the left or right.
B. The parabola opens wider or closes to become narrower.
C. The vertex of the parabola shifts vertically up or down.
D. The vertex of the parabola moves in a parabolic arc.

<u>Encyclopedia Brown and the Coffee House Caper</u>

The question really boiled down to whether or not Bugs Meany was really at the coffee shop at midnight as he claimed, or was he across town committing the burglary of the century? His friends claim that his alibi is valid because he drank coffee with them until midnight, but other seemingly reliable witnesses told police that Bugs left the coffee shop about 8pm. At his wit's end, the police chief calls in Encyclopedia Brown, the boy detective, to crack the case. Encyclopedia Brown's first suggestion is to draw a sample of Bugs' blood to determine the remaining level of caffeine, the psycho-active substance that gives coffee its "kick." The lab determined that the blood sample drawn at 8:00am retained 33 mg of caffeine of the original 325 mg of caffeine consumed by Bugs in the two large espressos he drank the night before. Using the knowledge that caffeine leaves the average person (approx. 145 pounds) at a continuous rate of about 15% per hour, Encyclopedia Brown smiles and tells the chief to put the handcuffs on Bugs Meany and read him the Miranda warning.

13) Based on the "average person's weight" assumption, how much caffeine should have remained in Bugs' blood if he indeed drank coffee until midnight?

E. 97.89 mg
F. 84.25 mg
G. 72.52 mg
H. 62.42 mg

14) Bugs attempts to counter Encyclopedia Brown's findings by pointing out that he is, at 197 pounds, much larger than the average person and Encyclopedia Brown should have used the fact that caffeine leaves a person of his size at a continuous rate of 19% per hour. Using this new information, about how many hours had passed since Bugs drank the two espressos to have retained the 33mg of caffeine found in his blood at 8:00am?

E. 8 hours
F. 10 hours
G. 12 hours
H. 14 hours

The Peril of Agent 008

An international spy, Agent 008, has just been rushed into the medical unit at headquarters. A full body radiographic scan has revealed that Agent 008 has been injected with a synthetic neurotoxin capsule that has modulating activity (toxicity) in the human body. It was found that the activity for this particular synthetic neurotoxin is dependent on the length of time it has been in the human body by the following function: $\text{Activity}(t) = (t^2 + 3)(t^2 - 5t + 4)$, where t represents the time in days the patient has been exposed to the neurotoxin. It has also been determined that if the capsule remains in Agent 008 for more than 7 days, the activity will have risen to sufficient levels to become terminal. The medical team has decided to operate on Agent 008 to remove the capsule, but they can only remove the capsule when the activity of the neurotoxin capsule is at zero otherwise he will suffer irreversible medical damage.

15) Assuming it takes 36 hours to prep the equipment and staff for the safe removal of the capsule, on which day should the surgical staff perform the operation?

E. Day 1
F. Day 2
G. Day 3
H. Day 4

APPENDIX G

CONFIRMATORY FACTOR ANALYSIS FOR FACTORS DEFINED BY LEVEL OF MATHEMATICAL DEMAND

```
Mplus VERSION 6.1
MUTHEN & MUTHEN
01/06/2011   2:18 PM

INPUT INSTRUCTIONS

  TITLE:      this is a CFA with all of the
              actual categorical data
              factor loadings are fixed at one
              factors are defined by levels of demand
  DATA:       FILE IS CFI_Data.dat;
  VARIABLE:   NAMES ARE p1-p15;
              CATEGORICAL ARE p1-p15;
  MODEL:      F1 BY p1-P4;
              F2 BY p5 p6 p8 p10 p13;
              F3 BY p7 p9 p11 p12 p14 p15;



INPUT READING TERMINATED NORMALLY



this is a CFA with all of the
actual categorical data
factor loadings are fixed at one
factors are defined by levels of demand

SUMMARY OF ANALYSIS

Number of groups                              1
Number of observations                      191

Number of dependent variables                15
Number of independent variables               0
Number of continuous latent variables         3
```

Observed dependent variables

Binary and ordered categorical (ordinal)

P1	P2	P3	P4	P5	P6
P7	P8	P9	P10	P11	P12
P13	P14	P15			

Continuous latent variables

F1	F2	F3

Estimator	WLSMV
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20
Parameterization	DELTA

Input data file(s)
CFI_Data.dat

Input data format FREE

UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES

P1		
Category 1	0.094	18.000
Category 2	0.906	173.000
P2		
Category 1	0.424	81.000
Category 2	0.576	110.000
P3		
Category 1	0.623	119.000
Category 2	0.377	72.000
P4		
Category 1	0.220	42.000
Category 2	0.780	149.000
P5		
Category 1	0.199	38.000
Category 2	0.801	153.000

P6		
Category 1	0.466	89.000
Category 2	0.534	102.000
P7		
Category 1	0.639	122.000
Category 2	0.361	69.000
P8		
Category 1	0.524	100.000
Category 2	0.476	91.000
P9		
Category 1	0.796	152.000
Category 2	0.204	39.000
P10		
Category 1	0.440	84.000
Category 2	0.560	107.000
P11		
Category 1	0.681	130.000
Category 2	0.319	61.000
P12		
Category 1	0.806	154.000
Category 2	0.194	37.000
P13		
Category 1	0.832	159.000
Category 2	0.168	32.000
P14		
Category 1	0.775	148.000
Category 2	0.225	43.000
P15		
Category 1	0.681	130.000
Category 2	0.319	61.000

THE MODEL ESTIMATION TERMINATED NORMALLY

WARNING: THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
DEFINITE. THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE FOR A
LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN TWO LATENT
VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO LATENT VARIABLES.
CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.

PROBLEM INVOLVING VARIABLE F2.

MODEL FIT INFORMATION

Number of Free Parameters 33

Chi-Square Test of Model Fit

Value	123.381*
Degrees of Freedom	87
P-Value	0.0063

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.047	
90 Percent C.I.	0.026	0.065
Probability RMSEA <= .05		0.594

CFI/TLI

CFI	0.814
TLI	0.775

Chi-Square Test of Model Fit for the Baseline Model

Value	300.466
Degrees of Freedom	105
P-Value	0.0000

WRMR (Weighted Root Mean Square Residual)

Value	0.964

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1 BY				
P1	1.000	0.000	999.000	999.000
P2	0.880	0.313	2.806	0.005
P3	1.449	0.478	3.035	0.002
P4	1.668	0.496	3.359	0.001
F2 BY				
P5	1.000	0.000	999.000	999.000
P6	0.830	0.190	4.363	0.000
P8	0.512	0.191	2.675	0.007
P10	0.615	0.183	3.356	0.001
P13	-0.274	0.212	-1.292	0.196
F3 BY				
P7	1.000	0.000	999.000	999.000
P9	0.353	0.432	0.818	0.413
P11	0.295	0.348	0.847	0.397
P12	0.997	0.502	1.987	0.047
P14	-0.108	0.375	-0.287	0.774
P15	2.150	0.816	2.634	0.008
F2 WITH				
F1	0.311	0.106	2.934	0.003
F3 WITH				
F1	0.138	0.067	2.066	0.039
F2	0.165	0.066	2.489	0.013
Thresholds				
P1$1	-1.315	0.126	-10.452	0.000
P2$1	-0.191	0.091	-2.097	0.036
P3$1	0.313	0.092	3.395	0.001
P4$1	-0.773	0.101	-7.631	0.000
P5$1	-0.845	0.104	-8.167	0.000
P6$1	-0.085	0.091	-0.941	0.347
P7$1	0.355	0.093	3.827	0.000
P8$1	0.059	0.091	0.651	0.515

P9$1	0.827	0.103	8.034	0.000
P10$1	-0.152	0.091	-1.664	0.096
P11$1	0.469	0.094	4.972	0.000
P12$1	0.864	0.104	8.299	0.000
P13$1	0.964	0.108	8.943	0.000
P14$1	0.755	0.101	7.495	0.000
P15$1	0.469	0.094	4.972	0.000

Variances

F1	0.190	0.112	1.698	0.089
F2	0.361	0.141	2.563	0.010
F3	0.142	0.089	1.608	0.108

R-SQUARE

Observed Variable	Estimate	Residual Variance
P1	0.190	0.810
P2	0.147	0.853
P3	0.400	0.600
P4	0.530	0.470
P5	0.361	0.639
P6	0.248	0.752
P7	0.142	0.858
P8	0.095	0.905
P9	0.018	0.982
P10	0.137	0.863
P11	0.012	0.988
P12	0.142	0.858
P13	0.027	0.973
P14	0.002	0.998
P15	0.658	0.342

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.829E-03
(ratio of smallest to largest eigenvalue)

Beginning Time: 14:18:19
Ending Time: 14:18:20
Elapsed Time: 00:00:01

APPENDIX H

CONFIRMATORY FACTOR ANALYSIS FOR FACTORS DEFINED BY MATHEMATICAL TOPIC

Mplus VERSION 6.1
MUTHEN & MUTHEN
03/12/2011 6:44 PM

INPUT INSTRUCTIONS

```
  TITLE:      this is a CFA with all of the
              actual categorical data
              factor loadings are fixed at one
              factors are defined by mathematical topic
  DATA:       FILE IS CFI_Data.dat;
  VARIABLE:   NAMES ARE p1-p15;
              CATEGORICAL ARE p1-p15;
  MODEL:      F1 BY p1 p2 p7 P8;
              F2 BY p3 p5 p6 p9 p10 p11 p12 p15;
              F3 BY p4 p13 p14;
```

INPUT READING TERMINATED NORMALLY

this is a CFA with all of the
actual categorical data
factor loadings are fixed at one
factors are defined by mathematical topic

SUMMARY OF ANALYSIS

Number of groups	1
Number of observations	191
Number of dependent variables	15
Number of independent variables	0
Number of continuous latent variables	3

Observed dependent variables

Binary and ordered categorical (ordinal)

P1	P2	P3	P4	P5	P6
P7	P8	P9	P10	P11	P12
P13	P14	P15			

Continuous latent variables

F1 F2 F3

Estimator	WLSMV
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20
Parameterization	DELTA

Input data file(s)

CFI_Data.dat

Input data format FREE

UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES

P1		
Category 1	0.094	18.000
Category 2	0.906	173.000
P2		
Category 1	0.424	81.000
Category 2	0.576	110.000
P3		
Category 1	0.623	119.000
Category 2	0.377	72.000
P4		
Category 1	0.220	42.000
Category 2	0.780	149.000
P5		
Category 1	0.199	38.000
Category 2	0.801	153.000
P6		
Category 1	0.466	89.000
Category 2	0.534	102.000
P7		

Category 1	0.639	122.000
Category 2	0.361	69.000
P8		
Category 1	0.524	100.000
Category 2	0.476	91.000
P9		
Category 1	0.796	152.000
Category 2	0.204	39.000
P10		
Category 1	0.440	84.000
Category 2	0.560	107.000
P11		
Category 1	0.681	130.000
Category 2	0.319	61.000
P12		
Category 1	0.806	154.000
Category 2	0.194	37.000
P13		
Category 1	0.832	159.000
Category 2	0.168	32.000
P14		
Category 1	0.775	148.000
Category 2	0.225	43.000
P15		
Category 1	0.681	130.000
Category 2	0.319	61.000

THE MODEL ESTIMATION TERMINATED NORMALLY

WARNING: THE RESIDUAL COVARIANCE MATRIX (THETA) IS NOT POSITIVE DEFINITE.
THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE FOR AN OBSERVED
VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN TWO OBSERVED
VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO OBSERVED VARIABLES.
CHECK THE RESULTS SECTION FOR MORE INFORMATION.
PROBLEM INVOLVING VARIABLE P4.

MODEL FIT INFORMATION

Number of Free Parameters 33

Chi-Square Test of Model Fit

Value	124.762*
Degrees of Freedom	87
P-Value	0.0050

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.048	
90 Percent C.I.	0.027	0.066
Probability RMSEA <= .05		0.564

CFI/TLI

CFI	0.807
TLI	0.767

Chi-Square Test of Model Fit for the Baseline Model

Value	300.466
Degrees of Freedom	105
P-Value	0.0000

WRMR (Weighted Root Mean Square Residual)

Value	0.966

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1 BY				
P1	1.000	0.000	999.000	999.000
P2	0.856	0.310	2.765	0.006
P7	0.698	0.274	2.545	0.011
P8	0.695	0.290	2.396	0.017
F2 BY				
P3	1.000	0.000	999.000	999.000
P5	0.946	0.210	4.512	0.000
P6	0.786	0.165	4.765	0.000
P9	0.135	0.209	0.646	0.519
P10	0.569	0.156	3.649	0.000
P11	0.111	0.175	0.635	0.525
P12	0.477	0.213	2.235	0.025
P15	0.971	0.185	5.259	0.000
F3 BY				
P4	1.000	0.000	999.000	999.000
P13	-0.176	0.179	-0.980	0.327
P14	-0.087	0.128	-0.685	0.494
F2 WITH				
F1	0.295	0.093	3.167	0.002
F3 WITH				
F1	0.336	0.124	2.707	0.007
F2	0.505	0.089	5.664	0.000
Thresholds				
P1$1	-1.315	0.126	-10.452	0.000
P2$1	-0.191	0.091	-2.097	0.036
P3$1	0.313	0.092	3.395	0.001
P4$1	-0.773	0.101	-7.631	0.000
P5$1	-0.845	0.104	-8.167	0.000
P6$1	-0.085	0.091	-0.941	0.347
P7$1	0.355	0.093	3.827	0.000
P8$1	0.059	0.091	0.651	0.515
P9$1	0.827	0.103	8.034	0.000
P10$1	-0.152	0.091	-1.664	0.096

P11$1	0.469	0.094	4.972	0.000
P12$1	0.864	0.104	8.299	0.000
P13$1	0.964	0.108	8.943	0.000
P14$1	0.755	0.101	7.495	0.000
P15$1	0.469	0.094	4.972	0.000

Variances

F1	0.280	0.163	1.716	0.086
F2	0.450	0.115	3.898	0.000
F3	1.642	1.528	1.074	0.283

R-SQUARE

Observed Variable	Estimate	Residual Variance	
P1	0.280	0.720	
P2	0.205	0.795	
P3	0.450	0.550	
P4	Undefined	-0.642	0.16415E+01
P5	0.403	0.597	
P6	0.278	0.722	
P7	0.136	0.864	
P8	0.135	0.865	
P9	0.008	0.992	
P10	0.145	0.855	
P11	0.006	0.994	
P12	0.102	0.898	
P13	0.051	0.949	
P14	0.013	0.987	
P15	0.424	0.576	

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.907E-03
(ratio of smallest to largest eigenvalue)

Beginning Time: 18:44:04
Ending Time: 18:44:05
Elapsed Time: 00:00:01

APPENDIX I

CONFIRMATORY FACTOR ANALYSIS FOR FACTORS DEFINED BY LEVEL OF MATHEMATICAL DEMAND WITH P4, P13, & P14 REMOVED FROM DATA SET

```
Mplus VERSION 6.1
MUTHEN & MUTHEN
03/13/2011  10:41 AM

INPUT INSTRUCTIONS

  TITLE:     this is a CFA with all of the
             actual categorical data
             factor loadings are fixed at one
             factors are defined by levels of demand
  DATA:      FILE IS CFI_Data2.dat;
  VARIABLE:  NAMES ARE p1-p3 p5-p12 p15;
             CATEGORICAL ARE p1-p3 p5-p12 p15;
  MODEL:     F1 BY p1-P3;
             F2 BY p5 p6 p8 p10;
             F3 BY p7 p9 p11 p12 p15;
  OUTPUT:    SAMPSTAT Standardized;


INPUT READING TERMINATED NORMALLY



this is a CFA with all of the
actual categorical data
factor loadings are fixed at one
factors are defined by levels of demand

SUMMARY OF ANALYSIS

Number of groups                                 1
Number of observations                         191

Number of dependent variables                   12
Number of independent variables                  0
```

Number of continuous latent variables 3

Observed dependent variables

Binary and ordered categorical (ordinal)

P1	P2	P3	P5	P6	P7
P8	P9	P10	P11	P12	P15

Continuous latent variables

F1 F2 F3

Estimator	WLSMV
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20
Parameterization	DELTA

Input data file(s)
CFI_Data2.dat

Input data format FREE

UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES

P1		
Category 1	0.094	18.000
Category 2	0.906	173.000
P2		
Category 1	0.424	81.000
Category 2	0.576	110.000
P3		
Category 1	0.623	119.000
Category 2	0.377	72.000
P5		
Category 1	0.199	38.000
Category 2	0.801	153.000
P6		
Category 1	0.466	89.000
Category 2	0.534	102.000
P7		
Category 1	0.639	122.000
Category 2	0.361	69.000

P8		
Category 1	0.524	100.000
Category 2	0.476	91.000
P9		
Category 1	0.796	152.000
Category 2	0.204	39.000
P10		
Category 1	0.440	84.000
Category 2	0.560	107.000
P11		
Category 1	0.681	130.000
Category 2	0.319	61.000
P12		
Category 1	0.806	154.000
Category 2	0.194	37.000
P15		
Category 1	0.681	130.000
Category 2	0.319	61.000

SAMPLE STATISTICS

ESTIMATED SAMPLE STATISTICS

SAMPLE THRESHOLDS

	P1$1	P2$1	P3$1	P5$1	P6$1
1	-1.315	-0.191	0.313	-0.845	-0.085

SAMPLE THRESHOLDS

	P7$1	P8$1	P9$1	P10$1	P11$1
1	0.355	0.059	0.827	-0.152	0.469

SAMPLE THRESHOLDS

	P12$1	P15$1
1	0.864	0.469

SAMPLE TETRACHORIC CORRELATIONS

	P1	P2	P3	P5	P6
P1					
P2	0.261				
P3	0.152	0.299			
P5	0.475	0.229	0.220		
P6	-0.030	0.143	0.360	0.338	
P7	0.221	0.009	0.109	0.246	0.250
P8	0.371	0.154	0.093	0.289	0.145
P9	-0.391	0.217	0.110	-0.016	0.008
P10	0.394	0.013	0.402	0.247	0.062
P11	-0.186	-0.005	-0.077	-0.045	0.200
P12	0.190	0.283	0.336	-0.108	0.156
P15	0.067	0.332	0.564	0.286	0.380

SAMPLE TETRACHORIC CORRELATIONS

	P7	P8	P9	P10	P11
P8	0.144				
P9	-0.004	0.019			
P10	0.119	0.134	0.007		
P11	0.076	-0.076	0.128	0.031	
P12	0.131	0.113	0.215	0.111	0.309
P15	0.300	0.144	0.224	0.180	0.142

SAMPLE TETRACHORIC CORRELATIONS

	P12	P15
P15	0.213	

THE MODEL ESTIMATION TERMINATED NORMALLY

WARNING: THE LATENT VARIABLE COVARIANCE MATRIX (PSI) IS NOT POSITIVE
DEFINITE. THIS COULD INDICATE A NEGATIVE VARIANCE/RESIDUAL VARIANCE FOR A
LATENT VARIABLE, A CORRELATION GREATER OR EQUAL TO ONE BETWEEN TWO LATENT
VARIABLES, OR A LINEAR DEPENDENCY AMONG MORE THAN TWO LATENT VARIABLES.

CHECK THE TECH4 OUTPUT FOR MORE INFORMATION.
PROBLEM INVOLVING VARIABLE F3.

MODEL FIT INFORMATION

Number of Free Parameters 27

Chi-Square Test of Model Fit

Value	69.624*
Degrees of Freedom	51
P-Value	0.0425

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.044	
90 Percent C.I.	0.009	0.068
Probability RMSEA <= .05		0.639

CFI/TLI

CFI	0.857
TLI	0.814

Chi-Square Test of Model Fit for the Baseline Model

Value	195.835
Degrees of Freedom	66
P-Value	0.0000

WRMR (Weighted Root Mean Square Residual)

Value	0.878

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1 BY				
P1	1.000	0.000	999.000	999.000
P2	0.991	0.378	2.622	0.009
P3	1.663	0.594	2.801	0.005
F2 BY				
P5	1.000	0.000	999.000	999.000
P6	0.916	0.287	3.189	0.001
P8	0.596	0.254	2.345	0.019
P10	0.742	0.257	2.884	0.004
F3 BY				
P7	1.000	0.000	999.000	999.000
P9	0.455	0.458	0.994	0.320
P11	0.341	0.358	0.953	0.341
P12	1.122	0.552	2.034	0.042
P15	2.304	0.899	2.564	0.010
F2 WITH				
F1	0.224	0.094	2.386	0.017
F3 WITH				
F1	0.132	0.068	1.942	0.052
F2	0.140	0.062	2.259	0.024
Thresholds				
P1$1	-1.315	0.126	-10.452	0.000
P2$1	-0.191	0.091	-2.097	0.036
P3$1	0.313	0.092	3.395	0.001
P5$1	-0.845	0.104	-8.167	0.000
P6$1	-0.085	0.091	-0.941	0.347
P7$1	0.355	0.093	3.827	0.000
P8$1	0.059	0.091	0.651	0.515
P9$1	0.827	0.103	8.034	0.000
P10$1	-0.152	0.091	-1.664	0.096
P11$1	0.469	0.094	4.972	0.000
P12$1	0.864	0.104	8.299	0.000
P15$1	0.469	0.094	4.972	0.000

Variances				
F1	0.165	0.113	1.466	0.143
F2	0.307	0.152	2.018	0.044
F3	0.129	0.084	1.532	0.125

STANDARDIZED MODEL RESULTS

STDYX Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1 BY				
P1	0.407	0.139	2.932	0.003
P2	0.403	0.102	3.967	0.000
P3	0.676	0.129	5.247	0.000
F2 BY				
P5	0.554	0.137	4.036	0.000
P6	0.507	0.106	4.790	0.000
P8	0.330	0.115	2.869	0.004
P10	0.411	0.109	3.774	0.000
F3 BY				
P7	0.359	0.117	3.065	0.002
P9	0.163	0.150	1.089	0.276
P11	0.122	0.125	0.978	0.328
P12	0.402	0.142	2.829	0.005
P15	0.826	0.148	5.591	0.000
F2 WITH				
F1	0.995	0.217	4.594	0.000
F3 WITH				
F1	0.909	0.193	4.704	0.000
F2	0.707	0.165	4.288	0.000
Thresholds				
P1$1	-1.315	0.126	-10.452	0.000
P2$1	-0.191	0.091	-2.097	0.036
P3$1	0.313	0.092	3.395	0.001

P5$1	-0.845	0.104	-8.167	0.000
P6$1	-0.085	0.091	-0.941	0.347
P7$1	0.355	0.093	3.827	0.000
P8$1	0.059	0.091	0.651	0.515
P9$1	0.827	0.103	8.034	0.000
P10$1	-0.152	0.091	-1.664	0.096
P11$1	0.469	0.094	4.972	0.000
P12$1	0.864	0.104	8.299	0.000
P15$1	0.469	0.094	4.972	0.000
Variances				
F1	1.000	0.000	999.000	999.000
F2	1.000	0.000	999.000	999.000
F3	1.000	0.000	999.000	999.000

STDY Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1 BY				
P1	0.407	0.139	2.932	0.003
P2	0.403	0.102	3.967	0.000
P3	0.676	0.129	5.247	0.000
F2 BY				
P5	0.554	0.137	4.036	0.000
P6	0.507	0.106	4.790	0.000
P8	0.330	0.115	2.869	0.004
P10	0.411	0.109	3.774	0.000
F3 BY				
P7	0.359	0.117	3.065	0.002
P9	0.163	0.150	1.089	0.276
P11	0.122	0.125	0.978	0.328
P12	0.402	0.142	2.829	0.005
P15	0.826	0.148	5.591	0.000
F2 WITH				
F1	0.995	0.217	4.594	0.000
F3 WITH				
F1	0.909	0.193	4.704	0.000

F2	0.707	0.165	4.288	0.000

Thresholds

P1$1	-1.315	0.126	-10.452	0.000
P2$1	-0.191	0.091	-2.097	0.036
P3$1	0.313	0.092	3.395	0.001
P5$1	-0.845	0.104	-8.167	0.000
P6$1	-0.085	0.091	-0.941	0.347
P7$1	0.355	0.093	3.827	0.000
P8$1	0.059	0.091	0.651	0.515
P9$1	0.827	0.103	8.034	0.000
P10$1	-0.152	0.091	-1.664	0.096
P11$1	0.469	0.094	4.972	0.000
P12$1	0.864	0.104	8.299	0.000
P15$1	0.469	0.094	4.972	0.000

Variances

F1	1.000	0.000	999.000	999.000
F2	1.000	0.000	999.000	999.000
F3	1.000	0.000	999.000	999.000

STD Standardization

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1 BY				
P1	0.407	0.139	2.932	0.003
P2	0.403	0.102	3.967	0.000
P3	0.676	0.129	5.247	0.000
F2 BY				
P5	0.554	0.137	4.036	0.000
P6	0.507	0.106	4.790	0.000
P8	0.330	0.115	2.869	0.004
P10	0.411	0.109	3.774	0.000
F3 BY				
P7	0.359	0.117	3.065	0.002
P9	0.163	0.150	1.089	0.276
P11	0.122	0.125	0.978	0.328
P12	0.402	0.142	2.829	0.005
P15	0.826	0.148	5.591	0.000

F2 WITH				
F1	0.995	0.217	4.594	0.000
F3 WITH				
F1	0.909	0.193	4.704	0.000
F2	0.707	0.165	4.288	0.000
Thresholds				
P1$1	-1.315	0.126	-10.452	0.000
P2$1	-0.191	0.091	-2.097	0.036
P3$1	0.313	0.092	3.395	0.001
P5$1	-0.845	0.104	-8.167	0.000
P6$1	-0.085	0.091	-0.941	0.347
P7$1	0.355	0.093	3.827	0.000
P8$1	0.059	0.091	0.651	0.515
P9$1	0.827	0.103	8.034	0.000
P10$1	-0.152	0.091	-1.664	0.096
P11$1	0.469	0.094	4.972	0.000
P12$1	0.864	0.104	8.299	0.000
P15$1	0.469	0.094	4.972	0.000
Variances				
F1	1.000	0.000	999.000	999.000
F2	1.000	0.000	999.000	999.000
F3	1.000	0.000	999.000	999.000

R-SQUARE

Observed Variable	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value	Residual Variance
P1	0.165	0.113	1.466	0.143	0.835
P2	0.162	0.082	1.983	0.047	0.838
P3	0.457	0.174	2.624	0.009	0.543
P5	0.307	0.152	2.018	0.044	0.693
P6	0.257	0.107	2.395	0.017	0.743
P7	0.129	0.084	1.532	0.125	0.871
P8	0.109	0.076	1.435	0.151	0.891
P9	0.027	0.049	0.545	0.586	0.973
P10	0.169	0.090	1.887	0.059	0.831
P11	0.015	0.031	0.489	0.625	0.985
P12	0.162	0.114	1.414	0.157	0.838

P15	0.683	0.244	2.796	0.005	0.317

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.690E-03
(ratio of smallest to largest eigenvalue)

Beginning Time: 10:41:18
Ending Time: 10:41:18
Elapsed Time: 00:00:00

APPENDIX J

CONFIRMATORY FACTOR ANALYSIS FOR FACTORS DEFINED BY MATHEMATICAL TOPIC WITH P4, P13, & P14 REMOVED FROM DATA SET

```
Mplus VERSION 6.1
MUTHEN & MUTHEN
03/13/2011  10:20 AM

INPUT INSTRUCTIONS

  TITLE:      this is a CFA with all of the
              actual categorical data
              factor loadings are fixed at one
              factors are defined by mathematical topic
  DATA:       FILE IS CFI_Data2.dat;
  VARIABLE:   NAMES ARE p1-p3 p5-p12 p15;
              CATEGORICAL ARE p1-p3 p5-p12 p15;
  MODEL:      F1 BY p1 p2 p7 P8;
              F2 BY p3 p5 p6 p9 p10 p11 p12 p15;



INPUT READING TERMINATED NORMALLY



this is a CFA with all of the
actual categorical data
factor loadings are fixed at one
factors are defined by mathematical topic

SUMMARY OF ANALYSIS

Number of groups                                1
Number of observations                        191

Number of dependent variables                  12
Number of independent variables                 0
Number of continuous latent variables           2
```

Observed dependent variables

Binary and ordered categorical (ordinal)

P1	P2	P3	P5	P6	P7
P8	P9	P10	P11	P12	P15

Continuous latent variables

F1 F2

Estimator	WLSMV
Maximum number of iterations	1000
Convergence criterion	0.500D-04
Maximum number of steepest descent iterations	20
Parameterization	DELTA

Input data file(s)

CFI_Data2.dat

Input data format FREE

UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES

P1		
Category 1	0.094	18.000
Category 2	0.906	173.000
P2		
Category 1	0.424	81.000
Category 2	0.576	110.000
P3		
Category 1	0.623	119.000
Category 2	0.377	72.000
P5		
Category 1	0.199	38.000
Category 2	0.801	153.000
P6		
Category 1	0.466	89.000
Category 2	0.534	102.000
P7		
Category 1	0.639	122.000
Category 2	0.361	69.000
P8		
Category 1	0.524	100.000

Category 2	0.476	91.000
P9		
Category 1	0.796	152.000
Category 2	0.204	39.000
P10		
Category 1	0.440	84.000
Category 2	0.560	107.000
P11		
Category 1	0.681	130.000
Category 2	0.319	61.000
P12		
Category 1	0.806	154.000
Category 2	0.194	37.000
P15		
Category 1	0.681	130.000
Category 2	0.319	61.000

THE MODEL ESTIMATION TERMINATED NORMALLY

MODEL FIT INFORMATION

Number of Free Parameters 25

Chi-Square Test of Model Fit

Value	71.995*
Degrees of Freedom	53
P-Value	0.0423

* The chi-square value for MLM, MLMV, MLR, ULSMV, WLSM and WLSMV cannot be used
for chi-square difference testing in the regular way. MLM, MLR and WLSM chi-square difference testing is described on the Mplus website. MLMV, WLSMV, and ULSMV difference testing is done using the DIFFTEST option.

RMSEA (Root Mean Square Error Of Approximation)

Estimate	0.043	
90 Percent C.I.	0.009	0.067
Probability RMSEA <= .05		0.652

CFI/TLI

CFI	0.854
TLI	0.818

Chi-Square Test of Model Fit for the Baseline Model

Value	195.835
Degrees of Freedom	66
P-Value	0.0000

WRMR (Weighted Root Mean Square Residual)

Value	0.886

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
F1 BY				
P1	1.000	0.000	999.000	999.000
P2	0.949	0.367	2.586	0.010
P7	0.768	0.329	2.336	0.019
P8	0.730	0.342	2.135	0.033
F2 BY				
P3	1.000	0.000	999.000	999.000
P5	0.707	0.204	3.471	0.001
P6	0.682	0.164	4.157	0.000
P9	0.189	0.204	0.929	0.353
P10	0.545	0.154	3.545	0.000
P11	0.141	0.169	0.830	0.406
P12	0.518	0.209	2.479	0.013
P15	1.008	0.208	4.839	0.000
F2 WITH				
F1	0.286	0.100	2.853	0.004
Thresholds				
P1$1	-1.315	0.126	-10.452	0.000

P2$1	-0.191	0.091	-2.097	0.036
P3$1	0.313	0.092	3.395	0.001
P5$1	-0.845	0.104	-8.167	0.000
P6$1	-0.085	0.091	-0.941	0.347
P7$1	0.355	0.093	3.827	0.000
P8$1	0.059	0.091	0.651	0.515
P9$1	0.827	0.103	8.034	0.000
P10$1	-0.152	0.091	-1.664	0.096
P11$1	0.469	0.094	4.972	0.000
P12$1	0.864	0.104	8.299	0.000
P15$1	0.469	0.094	4.972	0.000

Variances

F1	0.240	0.158	1.517	0.129
F2	0.509	0.133	3.831	0.000

R-SQUARE

Observed Variable	Estimate	Residual Variance
P1	0.240	0.760
P2	0.216	0.784
P3	0.509	0.491
P5	0.255	0.745
P6	0.237	0.763
P7	0.141	0.859
P8	0.128	0.872
P9	0.018	0.982
P10	0.151	0.849
P11	0.010	0.990
P12	0.137	0.863
P15	0.517	0.483

QUALITY OF NUMERICAL RESULTS

Condition Number for the Information Matrix 0.749E-02
(ratio of smallest to largest eigenvalue)

Beginning Time: 10:20:41
Ending Time: 10:20:41
Elapsed Time: 00:00:00

APPENDIX K

THE FUNCTION CONCEPT AND ITS ROLE IN SCHOOL MATHEMATICS: A REVIEW OF THE CRITICAL LITERATURE

Introduction

In 1975, the National Advisory Committee on Mathematics Education (NACOME) paved the way for the public school reform efforts that restructured the teaching of mathematics in general and algebraic concepts in particular(O'Callaghan, 1998). Algebra, misunderstood by some to be merely a study of variables and symbolic manipulation (Driscoll, 1999), plays a central role in students' mathematical development. The importance of algebra in school mathematics is not a new finding. In a 1907 issue of School Science and Mathematics, Professor E. B. Skinner of the University of Wisconsin spoke on the teaching of mathematics in Wisconsin. He said, "in general pupils came to the university and especially to the scientific schools of the university inadequately prepared in algebra"(Whitney, Denton, & Jones, 1907, p. 70).

Algebra can be thought of as the mathematical "bridge" across which secondary students must pass to reach advanced mathematical concepts in high school (Dooren, Verschaffel, & Onghena, 2002) as well as post secondary studies in the science, technology, engineering, and mathematics (STEM) subject areas. But, we have found that simply requiring schools to teach courses in algebra is not enough. Traditional algebraic instruction stresses the memorization of algebraic facts and symbolic manipulation at the expense of problem solving skills and conceptualization (Hollar & Norwood, 1999; Karsenty, 2002; O'Callaghan, 1998). These research findings led to algebraic reform efforts in K – 12 programs.

Of the algebraic topics covered in middle and secondary schools, researchers recognize the concept of function as the single most important tool in algebra regarding a student's ability to apply mathematical concepts in sciences, engineering and other related contexts (Hollar & Norwood, 1999; Knuth, 2000; Lloyd & Wilson, 1998; O'Callaghan, 1998; Zbiek, 1998). Underdeveloped knowledge of the function concept hinders the mathematical development of students and efforts to give the function concept greater emphasis in school mathematics have been under way since 1883 (M. Carlson & Oehrtman, 2005). The National Council of Teachers of Mathematics (NCTM) expects all students, beginning in grades six, to be able to "model and solve contextualized problems using various representations such as graphs, tables, and equations" (National Council of Teachers of Mathematics, 2000), which requires students to possess a working knowledge of functions. One of the first challenges for students, as well as some teachers, is to find an adequate definition of the function concept.

In a post secondary education pre-calculus text, a function is defined as "a rule which takes certain numbers as inputs and assigns to each input number exactly one output number. The output is a function of the input" (Connally, et al., 2004, p. 2). This seemingly simple definition of the concept of function is actually just the tip of a mathematical iceberg that has offers quite a challenge to students from middle school through post secondary graduate studies.

This review of critical literature in the area of the mathematical function concept and its role in school mathematics will explore the various definition of the function concept after a brief examination of its history. This literature review will then examine the role and importance the function concept has in school mathematics before looking at common misconceptions students, and some teachers, hold concerning the function. Finally the review will examine integral characteristics students should possess to have a rich and flexible knowledge of the function concept.

A Brief History of the Function Concept

While Gottfried Wilhelm Leibniz (1694 – 1716) is generally credited with creating the mathematical term "function" in 1694 while defining his work in the development of calculus (Burton, 1999), the precursors to the concept go back as far as 4000 years (Kleiner, 1989). Kleiner and Burton both note the evolution of the function concept as a logical continuation of the joining of Descartes geometry and the mathematics of Asia and the Middle East called algebra. From this union came new mathematics referred to as analytic geometry and calculus, developed independently by Isaac Newton (1642 – 1727) and Gottfried Leibniz, was soon to follow (Burton, 1999).

Influences From the Developers of the Calculus

Even (1990, 1993) points to the evolution of the function concept from the operational perspective held by developers of calculus to more of a structural object view. While many mathematics educational researchers (DeMarois & Tall, 2009; Even, 1990; Gray & Tall, 1994; Haimes, 1996; Hollar & Norwood, 1999; Leinhardt, Zaslavsky, & Stein, 1990; O'Callaghan, 1998) agree on the importance of moving mathematics students from operational perspectives of the function to the structural object view, the evolution of the concept from its precalculus roots in the literature is worthy of closer examination. The Seldens illustrate the importance of Newton's and Leibniz's calculus problem solving methods in the formative period of the function concept (Selden & Selden, 1992).

> The calculus of Newton and Leibniz was not cast in the mold of functions, but was rather an ingenious collection of problem-solving methods applicable to curves, which were paths generated by moving points. In the 17^{th} century, the study of motion, from Kepler's work on

> the planets to Huygen's work on the pendulum, was central, and functional relationships were expressed in words and the language of proportion. It took time before calculus was recast in an algebraic and symbolic mold with curves specified by formulas or equations. Once this happened, attention was paid to the relationships holding between the symbols. Terminology was needed to represent quantities dependent on other quantities in formulas or equations (p. 4-5).

Kleiner echoes this perspective and adds two specific reasons why the function concept was not developed prior to the 18th century (Kleiner, 1989). First, mathematics prior to 18th century was still in the process of developing algebraic symbols (Burton, 1999), which was foundational to the symbolic nature of the function concept . Second, there was a lack of motivation among early mathematicians to express new abstract notions when so few mathematical examples existed that required abstraction. The Seldens and Kleiner (1989; 1992) agree that early mathematics had a propensity to focus on finite solutions to specific problems; abstractions and generalizations were to follow as mathematics continued to be refined from about 1650 – 1850. Kleiner (1989, p. 2) lists the refinements as follows:

- Extension of the concept of number to embrace real and (to some extent) even complex numbers (Bombelli, Stifel, et al.);
- The creation of a symbolic algebra (Viète, Descartes, et al.);
- The study of motion as a central problem of science (Kepler, Galileo, et al.);
- The wedding of algebra and geometry (Fermat, Descartes, et al.).

Shift in Emphasis from Geometry to Function

While Leibniz is credited with being the author of the term "function", Leonhard Euler (1707 – 1783) (Burton, 1999) introduced the *f(x)* notation and in his 1748 work "Introduction in Analysis Infinitorum" where he defined a function as "an analytical expression composed in any manner from the variable quantity and numbers or constant quantities" (Selden & Selden, 1992, p. 5). It was during this period that the function concept replaced geometry as the central theme in calculus. Euler's perspective on the function concept was tested and refined over several years of scholarly debate between himself, Daniel Bernoulli (1700 – 1782), and Jean le Rond d'Alembert (1717 – 1783) concerning the famous "Vibrating-String Problem" (Burton, 1999; Kleiner, 1989; Selden & Selden, 1992).

Even though there was no clear winner in the debate concerning the "Vibrating-String Problem" (Kleiner, 1989; Malik, 1980), the concept of function was advanced in two significant ways. First, for the first time, functions were described by piecewise expressions over varying intervals. Prior to this extension, functions were thought to be only described over continuous intervals. Second, functions were no longer bound to symbolic expressions. Mathematician of the late 18th century recognized curves that could be "drawn freehand" as functions (Kleiner, 1989). The "Vibrating-String Problem" did not end the scholarly debates concerning the function concept. On the contrary, it seems that it ushered in a new era of scholarly debate that attracted work by renowned mathematicians as Fourier, Cauchy, Riemann, Weierstrass, Lebesgue, and Borel which further developed the concept of function into the 20th century (Burton, 1999; Kleiner, 1989; Selden & Selden, 1992). Two notable mathematicians, Dirichlet and Bourbaki are excluded from the list above, will be examined in greater detail in the next section.

Current Working Definitions of Function

Readers might find it interesting that while the Principles and Standards for School Mathematics from the National Council of Teachers of Mathematics (NCTM) has placed great emphasis on the function concept, a definition of the function concept is lacking from the standard (National Council of Teachers of Mathematics, 2000). Smith points out the fact that consistent definitions can be found in many textbooks, "In many ways, the topic of functions should be better delineated than that of algebra. Unlike algebra, function has a definition that is essentially common to all textbooks and has been fairly consistent for some time" (2003, p. 141). Smith goes on to quote two definitions of the function concept that encompass the general ideas from many texts.

Smith (2003, p. 141) quotes a 1986 precalculus text by Swokowski, the following definition, "A function from a set D to a set E is a correspondence that assigns to each element x of D a unique element y of E." Closer inspection of this first definition reveals an emphasis on the relationship (correspondence) between two pre-existing sets. It may be noteworthy to some that the sets described here are not restricted to real numbers or any number system for that matter. From a 1997 college algebra text by Demarois, McGowen, and Whitkanack, Smith (p. 141) quotes the following function definition, "A function is a process that receives input… and returns a value called the output. …There is exactly one output for each input." This second definition demonstrates a strong process view of the function concept; a common example used in school mathematics to illustrate this perspective is the "function machine" which can be found in many textbooks. With these thoughts in mind, the modern definition of function will be examined in the next section.

The "Modern" Dirichlet-Bourbaki Definition of Function

The introduction of the modern definition of function is primarily credited to Johann Peter Gustav Lejeune Dirichlet (1805 – 1859) (Burton, 1999; Kleiner, 1989; Rizzuti, 1991; Selden & Selden, 1992). Dirichlet's definition of function, "y is a function of x if for any value of x there is a rule which gives a unique value of y corresponding to x" (Malik, 1980, p. 491; Rizzuti, 1991, p. 20) is primarily based on the idea of correspondence between sets. This notion is contrasted with the classical definition of function, based on Euler's work, which places the emphasis on the covariation between the variables in the function (Rizzuti).

The exemplar for the classical function definition could be a "well behaved" linear function such as, $f(x) = 2x - 3$. This linear function emphasizes the covariation of the variables, x and $f(x)$; as x increases, so does $f(x)$ and as x decreases, $f(x)$ decreases as well. Rizzuti (p. 20) uses the following as an exemplar of the Dirichlet definition of a function: $f(x) = 1$, for rational x and $f(x) = 0$ for all irrational x. It is important to note that the graphical representation of this function is discontinuous and it will pass the school student's "vertical line" test. This exemplar stresses correspondence and the lack of covariation between the variables (Rizzuti, 1991). The Dirichlet definition, which is based on the correspondence between real numbers, needed additional modification to become the modern definition (Eves, 1997).

The Dirichlet definition of function was limited to real numbers but the Bourbaki definition, sometimes referred to as "ordered pair" (Selden & Selden, 1992), extends beyond the grasp of numbers. "'Nicholas Bourbaki' is a pseudonym for a group of mathematicians which set out in 1936, and continues to this day, to demonstrate that all of mathematics can be built on axioms and set theory" (Rizzuti, 1991). Rizzuti uses the following two definitions of the function concept, taken from high school textbooks, to exemplify the Dirichlet-Bourbaki definition of function:

> A function consists of two sets, the domain and the range, together with a paring that assigns to each member of the domain exactly one member of the range. (p.23)
>
> A function is a relation with the property: if (a,b) and (a,c) belong to the function, then b = c. (p.23)

This Dirichlet-Bourbaki definition of function is also known as the "set-theoretic" definition which emphasizes it foundation in modern set theory (Rizzuti).

> The Dirichlet-Bourbaki approach defined as functions many correspondences that were not recognized as functions by previous generations of mathematicians. Among these correspondences are discontinuous functions, functions de- fined on split domains (i.e., by different rules on different subdomains), functions with a finite number of exceptional points, and functions defined by means of a graph. (Vinner & Dreyfus, 1989, p. 357)

Function Definition Categories

The working definitions of the function concept can be described in from two perspectives; first, we have examined the function from the viewpoint of mathematicians and educators. This section will examine student conceptions of functions, which in some cases fail to align effectively with what may be considered the “right” concepts (M. Carlson & Oehrtman, 2005; Clement, 1989; DeMarois, 1996; Lambertus, 2007; O'Callaghan, 1998; Schwarz & Hershkowitz, 1999; Sfard, 1991). It should be noted that the label “student” used here is quite flexible and range from middle school students in mathematics classes through preservice teachers in their final stages of university study and early career middle school mathematics teachers participating in a “in-service” mathematics training course (Vinner & Dreyfus, 1989).

The aforementioned students have been the focus in several research studies to explore and categorize students’ function concept definitions. Vinner & Dreyfus (1989, pp. 359-360) identified the following six categories for student function concept definitions:

I. Correspondence: A function is any correspondence between two sets that assigns to every element in the first set exactly one element in the second set (the Dirichlet-Bourbaki definition).
II. Dependence Relation: A function is a dependence relation between two variables (y depends on x).
III. Rule: A function is a rule. A rule is expected to have some regularity, whereas a correspondence may be "arbitrary." The domain and the codomain were usually not mentioned here, contrary to Category I, where they were.
IV. Operation: A function is an operation or a manipulation (one acts on a given number, generally by means of algebraic operations, in order to get its image).
V. Formula: A function is a formula, an algebraic expression, or an equation.
VI. Representation: The function is identified, in a possibly meaningless way, with one of its graphical or symbolic representations.

In this particular study, Vinner & Dreyfus reported in their findings “An examination of the data indicates that the percentage of students giving some version of [the Dirichlet-Bourbaki] definition increased with the level of the mathematics course the students were taking” (p. 360).

Function Concept Definition vs. Concept Images

When discussing students’ function concept definitions, the research literature is rich in the associated topic of concept images (Confrey & Smith, 1995; Dubinsky & Harel, 1992; Monk, 1992; Norman, 1992; Rizzuti, 1991; Selden & Selden, 1992; Sfard, 1992; Sierpinska, 1992; Smith, 2003; Tall & Vinner, 1981; Vinner, 1983, 1992; Vinner

& Dreyfus, 1989). Tall & Vinner (1981) introduce the notion of concept image with the following passage:

> Many concepts which we use happily are not formally defined at all, we learn to recognise them by experience and usage in appropriate contexts. Later these concepts may be refined in their meaning and interpreted with increasing subtlety with or without the luxury of a precise definition. Usually in this process the concept is given a symbol or name which enables it to be communicated and aids in its mental manipulation. But the total cognitive structure which colours the meaning of the concept is far greater than the evocation of a single symbol. It is more than any mental picture, be it pictorial, symbolic or otherwise. During the mental processes of recalling and manipulation a concept, many associated processes are brought into play, consciously and unconsciously affecting the meaning and usage. We shall use the term *concept image* to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. (pp. 151-152)

The notion of concept image is contrasted with concept definition in that a concept definition is a verbal definition, void of an individual's mental images, which is used to describe and specify a particular concept (Hartter, 2009; Sfard, 1992; Sierpinska, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Important distinctions for mathematics educators are that students access and use these two potentially disparate parcels of information in different ways (Lambertus, 2007; Vinner, 1983, 1992; Vinner & Dreyfus, 1989). Vinner found that when a selected group of high achieving mathematics students were asked to give the definition of the function concept, more than half were able to give an adequate definition. When the same group of students was asked to solve function related problems that required specific reasoning, only about a third of the students that were able to give an adequate definition were also able to use that definition in their mathematical reasoning (1992). Vinner theorizes that in many situations, students rely on their concept images and do not consult their concept definitions at all.

The Importance of the Function Concept in School Mathematics

The function concept plays a central and unifying role in mathematics (Selden & Selden, 1992). The NCTM standard (National Council of Teachers of Mathematics, 2000, p. 222) for grades 6-8 sets the following expectations concerning the function concept:

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;

- Relate and compare different forms of representation for a relationship;
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

The standard for grades 9-12 follows a similar tenor (p.296) in their expectations of student knowledge of the function:

- Generalize patterns using explicitly defined and recursively defined functions;
- Understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;
- Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- Interpret representations of functions of two variables.

Incongruent with these expectations, research has shown that teachers continue to struggle with the function concept (Brown & Slough, 2009; Kulm, 2008; Norman, 1992; Selden & Selden, 1992; Vinner, 1992). Since research has show teachers' knowledge to be a key factor in student learning (You, 2006), it is not surprising that school students also have difficulty with the function concept (DeMarois, 1996; DeMarois & Tall, 2009; Gray & Tall, 1994; Hollar & Norwood, 1999; Kieran, 2008; Lambertus, 2007; Leinhardt, et al., 1990; O'Callaghan, 1998; Sfard, 1991).

Studies have shown that learning the function concept is a complex task and even high performing students have weak function concepts (M. Carlson & Oehrtman, 2005; Kulm, 2008; Vinner & Dreyfus, 1989). "Students who think about functions only in terms of symbolic manipulations and procedural techniques are unable to comprehend a more general mapping of a set of input values to a set of output values; they also lack the conceptual structures for modeling function relationships in which the function value (output variable) changes continuously in tandem with continuous changes in the input variable" (M. Carlson & Oehrtman, 2005). These reasoning skills serve as a foundation to higher concepts in mathematics as well as other contents areas within the STEM disciplines (Brown & Slough, 2009; M. Carlson & Oehrtman, 2005; Kaput, 1992).

Common Misconceptions and Issues

One of the most common function misconceptions revealed in the research literature is students' belief that a function is always defined by a well-behaved algebraic equation (Dubinsky & Harel, 1992; Norman, 1992; Selden & Selden, 1992; Sfard, 1992; Sierpinska, 1992; Vinner, 1992). Many students don't recognize that functions can be defined over disjoint domains or be displayed in a discontinuous graph. The correspondence between sets, the basis for the Dirichlet definition of the function, is thought to be a rule that displays regularities over the entire domain. Rizzuti points out that the Dirichlet-Bourbaki definition of function is too abstract for many students, and that even after learning the definition, students are unable to effectively use the definition in mathematical reasoning (1991).

As a corollary to his discussion on concept images and concept definitions, Vinner (1992) adds the notion of compartmentalization which basically describes the situation which occurs when two incompatible items of knowledge exist in someone's mind without them being aware of it. When a person's concept image is sufficiently different and incompatible with their concept definition, they will tend to use one to answer one type of problem and the other for different problems not perceive the conflict (Lambertus, 2007; Sfard, 1992).

Sfard (1992) indicates a particularly problematic issue for students. If two symbolic different algebraic formulas produce the same output for a given input, students perceive the functions to be different. Norman (1992) study reveals many secondary teachers' dependence on a single form of representation for functions, which was most often the graph. Students were also found to be unsure about the use of the equal sign and many "view functions simply as two expressions separated by an equal sign" (M. P. Carlson & Bloom, 2005, p. 3).

Integral Student Characteristics of the Concept of Function

Freudenthal points to the notions of arbitrariness and equivalence as being the two essential features of the function concept (1983) but this simply addresses a full understanding and use of the modern function definition. A richer discussion is introduced by considering students' conceptual development. Students tend to begin their practical use of the function concept in what is referred to as an *action view of functions* (M. Carlson & Oehrtman, 2005, pp. 8-9; Dubinsky & Harel, 1992). As these students mature mathematically, it is recommended that they develop a *process view of function* (M. Carlson & Oehrtman, 2005, pp. 8-9; Dubinsky & Harel, 1992).

> An *action* conception of function would involve the ability to plug numbers into an algebraic expression and calculate. It is a static conception in that the subject will tend to think about it one step at a time (e.g., one evaluation of an expression). A student whose function conception is limited to actions might be able to form the composition of two functions, defined by algebraic expressions, by replacing each

> occurrence of the variable in one expression by the other expression and then simplifying; however, the students would probably be unable to compose two functions that are defined by tables or graphs.
>
> A *process* conception of function involves a dynamic transformation of quantities according to some repeatable means that, given the same original quantity, will always produce the same transformed quantity. The subject is able to think about the transformation as a complete activity beginning with objects of some kind, doing something to these objects, and obtaining new objects as a result of what was done. When the subject has a process conception, he or she will be able, for example, to combine it with other processes, or even reverse it. Notions such as 1-1 or onto become more accessible as the students' process conception strengthens.

This notion of the separate conceptions of action and process should not be confused with Gray and Tall's (1994) notion of the function "procept" which suggests a way of viewing a function as both a process and as a concept (Sajka, 2003). The function $f(x) = 5x - 1$ can be viewed as a process when the "operation" class of the function definition described above. In this sense, the student can calculate a value for $f(x)$ for each value of x given. But the function can also be viewed as a concept when the "correspondence" class of the function definition is used. The above notions of action and process might be best aligned with Sfard's analysis of the formation of the function concept. Sfard (1991) defined the three stages of concept development interiorization, condensation, and reification to "correspond to three degrees of structuralization" (p.18).

VITA

Irving Anthony Brown
4232 TAMU
College Station, TX 77843
(512) 694-4762
iabrown@tamu.edu

Education

Texas A&M University — College Station, TX
Ph.D. in Curriculum & Instruction — August 2011
with emphasis in Mathematics Education

Tarleton State University - Central Texas — Killeen, TX
M.S. in Mathematics — May 2006

Lamar University — Beaumont, TX
B.S. in Chemical Engineering — August 1991
.

Research Interests

- Middle and secondary school mathematics teacher preparation
- Problem-solving in Mathematics
- Computational technology in mathematics education
- Science, Technology, Engineering, and Mathematics (STEM) education

Honors

- National Science Foundation (NSF) CADRE Fellow
- Member of Kappa Delta Pi – International Honor Society in Education
- TAMU - Graduate Diversity Fellowship (3yrs. full funding)
- Pi Mu Epsilon National Math Honor Society Student Member
- Omega Chi Epsilon National Chemical Engineering Honor Society Student Vice President
- McMaster Honor Scholar
- National Society of Black Engineers Student Vice President

Appointments and Memberships

External Board Member, Advisory Board, Quality Enhancement Plan/The Academy
Huston-Tillotson University, 2010-Present