INVESTIGATION OF THE EFFECT OF NON-DARCY FLOW AND MULTI-PHASE FLOW ON THE PRODUCTIVITY OF HYDRAULICALLY FRACTURED GAS WELLS

A Thesis

by

NASRALDIN ABDULSLAM A. ALARBI

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2011

Major Subject: Petroleum Engineering

Investigation of the Effect of Non-Darcy Flow and Multi-Phase Flow on the Productivity of Hydraulically Fractured Gas Wells

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Approved by:

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Committee Members,	Gioia Falcone
	Yuefeng Sun
Head of Department,	Stephen Holditch

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ABSTRACT

Investigation of the Effect of Non-Darcy Flow and Multi-Phase Flow on the Productivity of Hydraulically Fractured Gas Wells. (August 2011)

Nasraldin Abdulslam A. Alarbi, B.Eng., Al Tahadi University

Chair of Advisory Committee: Dr. Peter Valko

Hydraulic fracturing has recently been the completion of choice for most tight gas bearing formations. It has proven successful to produce these formations in a commercial manner. However, some considerations have to be taken into account to design an optimum stimulation treatment that leads to the maximum possible productivity. These considerations include, but not limited to, non-Darcy flow and multiphase flow effects inside the fracture. These effects reduce the fracture conductivity significantly. Failing to account for that results in overestimating the deliverability of the well and, consequently, to designing a fracture treatment that is not optimum.

In this work a thorough investigation of non-Darcy flow and multi-phase flow effects on the productivity of hydraulically fractured wells is conducted and an optimum fracture design is proposed for a tight gas formation in south Texas using the Unified Fracture Design (UFD) Technique to compensate for the mentioned effects by calculating the effective fracture permeability in an iterative way. Incorporating non-Darcy effects results in an optimum fracture that is shorter and wider than the fracture when only Darcy calculations are considered. That leads to a loss of production of 5, 18% due to dry and multiphase non-Darcy flow effects respectively. A comparison

between the UFD and 3D simulators is also done to point out the differences in terms of methodology and results. Since UFD incorporated the maximum dimensionless productivity index in the fracture dimensions design, unlike 3D simulators, it can be concluded that using UFD to design the fracture treatment and then use the most important fracture parameters outputs (half length and C_{fDopt}) as inputs in the simulators is a recommended approach.

DEDICATION

I dedicate this work, firstly, to almighty Allah for his guidance and help to me throughout my life. I also dedicate it to all my family and friends for all the support and encouragement they have shown.

ACKNOWLEDGEMENTS

First I would like Almighty Allah for his Grace and guidance. Then I thank my committee chair, Dr. Peter Valko, and my committee members, Dr. Gioia Falcone and Dr. Yuefeng Sun, for their guidance and support throughout the course of this research.

Thanks also go to my friends and colleagues and the department faculty and staff for making my time at Texas A&M University a great experience. I also want to thank the Ministry of High Education in Libya for providing me with the scholarship for my master's program.

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CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

Introduction

The huge increasing demand for power has made it essential to look for additional resources besides the easy to produce oil and gas reservoirs. These recourses have always been thought of as challenging and not worth producing, either for complexity of the development process, lack of the needed technology or most importantly for being not financially convenient. Low permeability gas reservoirs and heavy oil reservoirs are typical examples for these unconventional resources.

Unconventional reservoirs are reservoirs that cannot produce in high enough rates or economic volumes of hydrocarbons without the employment of one or more of stimulation techniques or enhanced oil recovery processes. The natural petrophysical characteristics and fluid properties of these reservoirs are not of a good quality to produce the oil and gas to the surface in an economical manner. That is attributed to either the too low permeability in the case of tight gas or the too high viscosity of the fluids in the case of heavy oils. Therefore the remedy should include ways to either cure permeability in the case of low permeability reservoirs or decrease viscosity for heavy oils. Unconventional reservoirs typically include tight gas sands, gas shales, coal bed methane, heavy oil, tar sands, and gas hydrates.

The importance of Producing low permeability gas reservoirs has become more

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This thesis follows the style of *SPE Journal*.

magnified with the increasing gas price and the improvements in the existing relatively new stimulation techniques to commercially develop such reserves. Hydraulic fracturing, namely, is the most successful and widely used treatment to stimulate low permeability rocks.

Hydraulic fracturing was first introduced to the oil industry in the early 1940s. Since then it has been the most successful and reliable means to commercialize low permeability reservoirs by stimulating the productivity of wells. That is achieved by creating relatively easy paths for the hydrocarbons to flow from the formation into the wellbore. These paths are filled with a propping agent to give the fracture sufficient permeability after the surface pressure has been released (closure).

Hydraulic fracturing is carried out by pumping big volumes of predesigned fluids down hole into the pay zone with high enough rates and pressures to overcome the fracture gradient of the formation and cause it to crack. The process starts with pumping a clean fluid called the pad to initiate the fracture and make it grow or “propagate”. Then, the designed fracturing fluid is pumped mixed with a propping agent and sometimes a fluid breaker. This fracturing fluid continues causing the fracture to propagate and transfers the proppant into the fracture. When the pressure is released and the well is put to production, the fracturing fluid breaks to a lower viscosity fluid, either by the effect of high temperature or by the help of the fluid breaker, and flows back out of the fracture in a process called “clean up”. The time for cleanup varies from a formation to another depending on many factors including: how well the fluid breaker was designed, leak off volume into the formation, formation permeability, damage around the fracture, proppant crushing, non-Darcy flow and others (Holditch, 1979). Clean up takes longer in tight gas wells compared to other formations due to the too low permeability and the significant effect of non-Darcy flow. After clean up, the propping agent is left inside the fracture forming a very conductive pathway for the flow of hydrocarbons from the formation to the wellbore.

Employing hydraulic fracturing as a stimulation technique involves a thorough knowledge of the formation permeability as well as the mechanical properties of the different layers in the reservoir. That is a key factor helps to predict the fracture growth and orientation. The fracture is believed to grow perpendicular to the minimum horizontal stress of the formation. A complete data set is a vital element for a successful fracture treatment design.

There are some phenomena associated with high production rate hydraulically fractured gas wells which have a negative effect on the fracture conductivity, and thus, on the deliverability of the wells. These phenomena include: Non-Darcy flow, the presence of immobile liquid along with the gas, multiphase flow, proppant crushing, proppant embedment and fines migration (Lopeze, Valko & Pham, 2004). Not accounting for these factors will lead to overestimation of the fracture conductivity and hence over evaluating the well production and – even more importantly – missing to realize some of the potential productivity from the given amount of resources spent on the stimulation treatment.

Non-Darcy flow is caused by the high rates and always associated with most of gas wells. The gas flowing with high velocity causes the flow to become turbulent and depart from Darcy's low. The high gas velocity also increases the inertial resistance of the porous medium to the flow generating an extra pressure drop that should be added to Darcy's equation.

Another form of Non-Darcy flow is caused by the presence of a liquid phase along with the flowing gas phase. That would result in reducing the cross sectional area for gas flow, inducing an additional pressure drop. Whether that liquid is immobile or flowing governs the magnitude by which the gas production is decreased. In particular, when the liquid phase is mobile, a great portion of the mechanical energy (pressure) loss is spent on acceleration the liquid bubbles, that periodically slowdown by hitting the solid matrix regions. Therefore, non-Darcy flow effects are especially severe in gas-liquid two phase flow.

One way to take into consideration of the non-Darcy effects within the formalism of Darcy flow is to use the concept of effective permeability that is considerably different from the nominal proppant permeability. Not being able to account for that leads to too optimistic predictions of the well capacity and, more importantly, to placing the proppant where it does not contribute optimally to the productivity of the well.

Literature Review

Unified Fracture Design

The main goal of the employment of a stimulation technique is to enhance the deliverability of the well so that the most possible hydrocarbon volumes in place can be produced. Unified fracture design UFD introduced by Economides, Oligney and Valko

(2002) suggests that the best design can be achieved by calculating optimum fracture dimensions which correspond to an optimum dimensionless fracture conductivity that leads to the highest possible productivity index.

Two famous approximations for the drainage area are the circular and rectangular shapes **Fig. 1**. The parameters of these shapes are related as follows:

$$A = r_e^2 \pi = x_e^2 \quad (1)$$

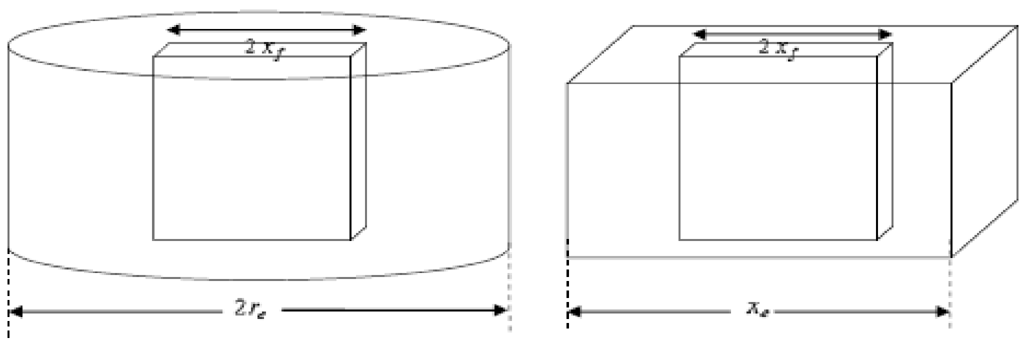

Fig. 1-Reservoir geometry comparison

In the rectangular drainage volume, the successfulness of a fracturing treatment depends on two dimensionless quantities, the penetration ratio and the dimensionless fracture conductivity (Economides, Oligney & Valko, 2002).

The penetration ratio I_x connects the fracture length to the reservoir length in the X direction as follows:

$$I_x = \frac{2\,x_f}{x_e} \quad (2)$$

where, I_x is the penetration ratio x_e is the drainage length. I_x ranges from 0 for not fractured reservoir to 1 for fully penetrating fracture.

The dimensionless fracture conductivity is defined as the ratio of the fracture conductivity to the ability of the formation to conduct fluids into the fracture.

$$C_{fD} = \frac{k_f\, w_f}{k\, x_f} \quad \text{(3)}$$

where, k_f is the proppant pack permeability, w_f is the propped fracture width, k is the reservoir permeability and x_f is the fracture half length.

A dimensionless proppant number relating the two mentioned dimensionless quantities can be introduced as:

$$N_{prop} = I_x^2\, C_{fD} = \frac{4\, k_f\, x_f\, w_f}{k\, x_e^2} = \frac{4\, k_f\, x_f\, w_f\, hp}{k\, x_e^2\, hp} \quad \text{(4)}$$

where N_{prop} is the dimensionless proppant number and h_p is the net pay thickness.

Treatment size (proppant volume in the pay zone) is the primary decision variable in UFD. For a fixed proppant mass, the proppant number can be calculated by:

$$N_{prop} = \frac{2\, k_f\, Volume\ of\ proppant\ reaching\ the\ net\ pay\ zone}{k\ Reservoir\ Drainage\ volume} \quad \text{(5)}$$

For each proppant number there is a unique optimum dimensionless fracture conductivity that corresponds to a maximum dimensionless productivity index. Fan et al. (2000) proposed correlations to calculate the optimum C_{fD} and J_D as a function of proppant number.

For proppant numbers less that 0.1 the optimum C_{fD} is 1.6.

And the max productivity index is given by

$$J_{Dmax} = \frac{1}{0.99 - 0.5 \ln N_{prop}} \quad (6)$$

For $0.1 \leq N_{prop} \leq 10$:

$$C_{fDopt} = 1.6 + \exp\left(\frac{-0.583 + 1.48 \ln N_{prop}}{1 + 0.142 \ln N_{prop}}\right) \quad (7)$$

And the corresponding maximum productivity index is as follows:

$$J_{Dmax} = \frac{\pi}{6} - \exp\left(\frac{0.423 - 0.311\, N_{prop} - 0.089 (N_{prop})^2}{1 + 0.667\, N_{prop} + 0.015 (N_{prop})^2}\right) \quad (8)$$

Fig. 2 and Fig. 3 constructed by Romero, Valko and Economides (2002) show the relation between the dimensionless fracture conductivity and the dimensionless productivity index as a function of the proppant number. The graphs were constructed using a direct boundary element method to calculate the performance of fractured wells. A subroutine written in *Mathematica* by Romero, Valko and Economides (2002) to describe these graphs is used in this work to calculate the optimum dimensionless fracture conductivity and the corresponding dimensionless productivity index for a given proppant number. For low proppant numbers (0.1 or less) it is clear that the maximum desired productivity index corresponds to a C_{fD} of 1.6 and it varies for proppant numbers bigger than 0.1.

After the optimum dimensionless fracture conductivity has been determined, the optimum fracture dimensions can be calculated as follows:

$$x_{fopt} = \left(\frac{\mathrm{k}\, V_f}{\mathrm{C}_{fDopt}\, k\, h_p}\right)^{0.5} \quad (9)$$

$$w_{fopt} = (\frac{C_{fDopt} k\, V_f}{k_f\, h_p})^{0.5} \quad (10)$$

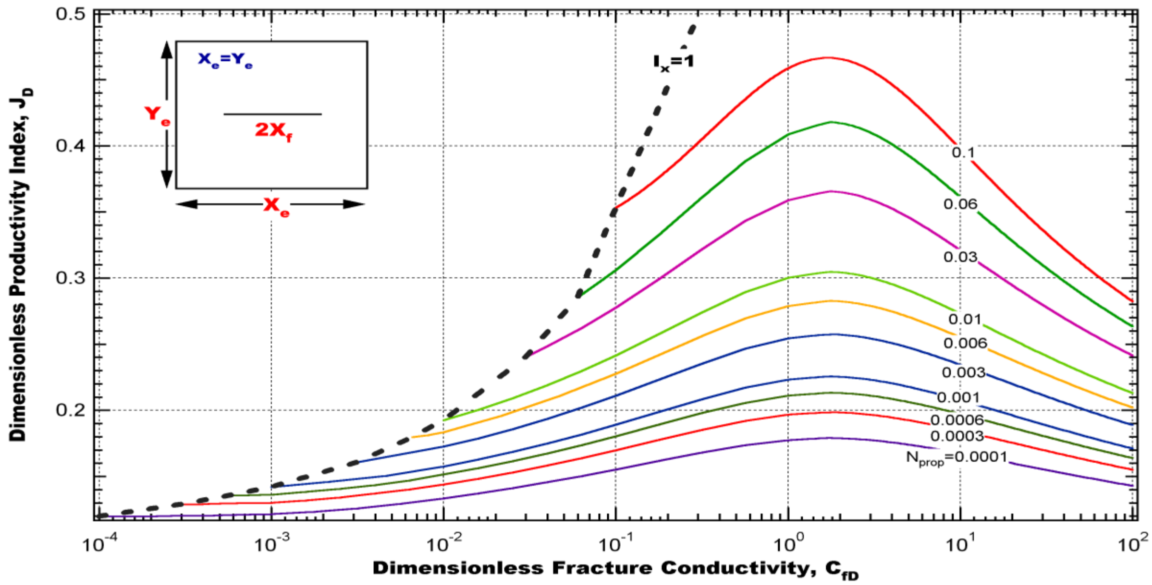

Fig. 2 – C_{fDopt} vs. J_D for $N_{prop} \leq 0.1$ (after Romero, Valko and Economides, 2002)

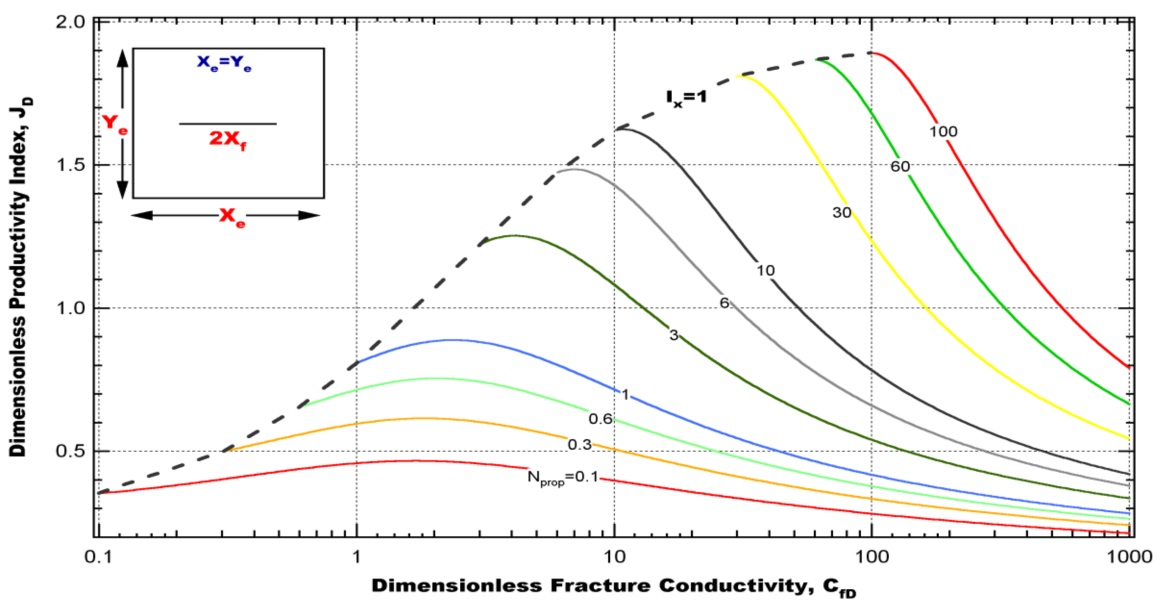

Fig. 3 – C_{fDopt} vs. J_D for $N_{prop} \geq 0.1$ (after Romero, Valko and Economides, 2002)

Having calculated the optimum fracture dimensions, a way to carry out the theoretical design has to be found according to the operational and financial constraints such as the pumping equipment pressure limitations and the worthiness of the stimulation treatment.

The injection process has to be analyzed to know whether or not the given proppant volume can be placed in the predesigned optimum fracture volume taking into consideration the fracture fluid efficiency and the concentration limits. If that is not possible, we should try changing the fracturing fluid, type of proppant or the pumping equipment. However, this can, most of the time, be difficult to implement on the well site. Therefore, a departure from the optimum design has to be adopted. This departure can be by keeping the length fixed and try to pump as much proppant mass as allowed by the pumping constraint. Although that will result in a smaller productivity index, it will lessen the treatment cost as well. Another scenario is to extend the fracture length to accommodate the available proppant mass. That will also result in a smaller J_D. However, the obtained J_D is the best possible considering the existing constraints. In high permeability reservoirs, a technique called the Tip Screen Out (TSO) is employed to depart from the primary optimum design (Economides, Oligney & Valko, 2002).

Unified Fracture Design vs. 3D Fracture Simulators

Hydraulic fractures are performed to stimulate the productivity of wells that are unable to deliver sufficient volumes of hydrocarbons in a commercial fashion. Therefore, the goal behind the employment of stimulation processes should be to maximize the well production not only to increase it. This goal can only be achieved by designing an optimum treatment using the available resources.

There is a profound difference between optimally designing a fracture treatment dimensions that lead to the maximum possible well productivity and a one that still gives reasonably good revenue, however, we can do better.

Commercial softwares used in the fracturing industry today use injection variable to predict fracture geometry (Economides and Demarchos 2008). They use the procedures constraints of the process to try to simulate the most important parameters in the design, namely the fracture length and width. These simulators do not emphasize the importance of estimating the optimum fracture dimensions that optimize the well performance first. They rather numerically solve the equations governing the fracture propagation to come up with a prospected treatment size that seems to be the optimum.

This is totally different in methodology from the fracture design models that try to first determine the fracture optimum dimensions to maximize the well productivity and then figure out the best way to achieve these dimensions accounting for the technical and practical procedures constraints.

The Unified Fracture Design approach (UFD) introduced by Economides and Valko predicts the optimum fracture length and width that corresponds to the maximum well productivity index. It is recommended that these dimensions are then used to design the other fracture treatment variables (i.e. injection variables) (Economides and Demarchos 2008). In UFD the fracture height is used as an input to predict the sought for optimum fracture geometry that are finally believed to correspond to the highest productivity index and thus the best possible production. In 3D simulators, however, height is simulated.

In contradiction to commercial fracture programs, the UFD does not emphasize net pressure match (Economides and Demarchos 2008); it rather focuses on finding the optimum fracture dimensions for the available proppant mass taking into account the operational constraints.

Knowing the optimum design, UFD suggests that the other variables are then estimated. Using the optimum dimensions, the fracture volume is calculated. And with a given maximum and minimum slurry concentration limits we can know whether or not we can place the given amount of proppant in that fracture volume. Consequently, the injection time, proppant schedule and fluid efficiency of the pumping process can be determined.

If the option to decide the proppant mass is feasible, different treatment sizes should be investigated to optimally design and choose the dimensions that result in the best J_D or decide whether or not the increase in J_D induced by a bigger treatment is justified from an economical point of view.

Since most other design programs are varying the proppant mass, the resultant design parameters are not expected to match with the ones obtained from the UFD optimum values. Therefore, it is hard to optimize the available mass to be pumped when the fracture dimensions are simulated (Economides and Demarchos 2008).

The above simplified design procedure (UFD) assumes that the permeability of the proppant pack is known at the start of the design. This is indeed the case if only Darcy effects are responsible for the pressure loss both in the formation and in the fracture. However, as Holditch and Morse (1976) and Gidley (1991) have showed, in the presence of non-Darcy effects the effective permeability cannot be known without considering the reservoir parameters and taking into account the actual inflow into the fracture. Therefore, some of the ideas from the UFD design approach can be used only in an iterative manner, updating the effective permeability of the proppant pack.

Non-Darcy flow

The pressure drop generated by fluid flow through porous media has always been described by Darcy's low Equation 11.

$$-\frac{dp}{dl} = \frac{\mu}{k}\,u \quad\text{.........................}(11)$$

where u is the superficial velocity of the fluid, k is the permeability, p is pressure, μ is viscosity, and l is the distance. This equation shows that the pressure drop in porous media is a result of the viscous forces.

However, Darcy conducted his test with low flow rates and using water as the flowing fluid. Therefore, the previous equation falls short to estimate pressure losses when high rates are involved due to the turbulence nature of the flowing regime which increases the inertial resistance (Holditch & Morse, 1976) to the flow resulting from increasing the velocity of the fluid.

Forcheimer in 1901 was the first to investigate the effect of turbulence on the pressure drop induced by fluids flowing with high rates. He introduced a new term to Darcy's low to account for the extra pressure drop, which is now called the non-Darcy term. The modified Darcy's low after accounting for the inertial forces contribution to pressure loss becomes as follows:

$$-\frac{dp}{dl} = \frac{\mu}{k}\,u + \beta\rho u^2 \quad (12)$$

where ρ is the fluid density and β is the non-Darcy coefficient.

Equation 12 implies that the pressure drop is a function of both the viscous energy losses and kinetic energy losses (Ergun, 1952). The kinetic energy loss or what is known among petroleum industry researchers as either the inertial loss term or non-Darcy term is the product of the fluid density, the second power of fluid velocity and the beta factor. This term is negligible in low flow rates situations. However, in high flow rates the non-Darcy term becomes bigger and can even dominate the Darcy term.

Researchers have used different terminology to describe β including: The coefficient of inertial resistance by Geertsma (1974), the non-Darcy flow coefficient by Evans, Hudson and Greenlee (1987), and Frederick and Graves (1994), The inertial flow coefficient by Pursel and Blakeley (1988) and Coles and Hartman (1998), the Forchheimer coefficient by Jin and Penny (1998), the coefficient of velocity by Firoozabadi and Katz (1979). In many of these terminologies, the name comes from what the researcher believes the cause of the nonlinearity introduced to Darcy's low.

Although Forchheimer himself attributed the pressure gradient increase to inertial resistance (Coles & Hartman, 1998), some early work ascribed the non-Darcy effect to turbulence only. However, more recent investigations have led to an agreement among most of researchers that the additional pressure drop is due to a combination of both inertial resistance and turbulence (Holditch & Morse, 1976). Some, however, attribute it to the acceleration and deceleration of the fluid through the porous media (Geertsma, 1974) regardless of the flow regime. Some others went even farther in invalidating the contribution of turbulence. Ma and Ruth (1997), for instance, stated that the non-Darcy flow effect starts much earlier than the criterion of turbulence. They used a bent tube model **Fig. 4**, which is analogous to the tortuous manner of the porous media, and found out that the non-Darcy effect starts at a Reynolds number that is, by many orders of magnitude, smaller than the value of transition from laminar to turbulent flow regimes in straight pipes. That led them to exclude turbulence as a reason for the nonlinearity introduced to Darcy's law in high rates and attribute it more to inertial effects. Miskimins, Lopeze and Barree (2005), also, reported that a reduction of 5-30 % in flow capacity can be a result of non-Darcy flow in low rate wells where turbulence is not likely to occur. The beta factor has also been described as the measure of the tortuosity in the flow path media (Geertsma, 1974), deemphasizing the importance of turbulence.

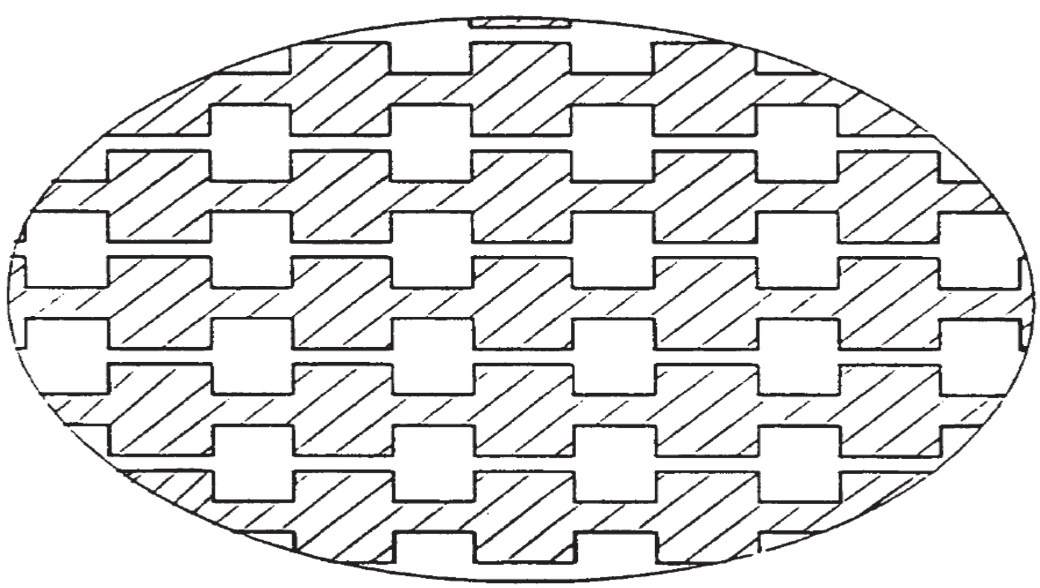

Fig. 4 - The bent pipe model by Ma and Ruth (1997)

Some Beta Correlations

Many equations have been developed to estimate the Beta factor. Some are theoretical using mathematical models to describe the flow through porous media and some are empirical developed by conducting laboratory tests and come up with a correlation relating beta to the properties of the porous media. The empirical correlations can be grouped into two main categories: the ones developed by testing proppants and the others developed by experimenting on cores or pack beds (Lopez, Valko & Pham, 2004). Choosing the right beta correlation is a vital factor to estimate optimum design parameters that compensates for specific non-Darcy flow conditions. Some examples of the two experimental categories are explained.

Cook (1973) conducted tests on many Brady sand mesh sizes at different closure stresses. He used the Forchheimer equation and plotted dp/l µ u versus $\rho u/$µ as in **Fig. 5**. The slop of that plot is the beta factor and the intercept is the reciprocal of the permeability. He computed the beta coefficient and the permeability for five different sand sizes and three different fluids: brine, gas and oil. He made a plot of beta and permeability. The plot follows the following general formula:

$$\beta = \frac{b}{k^a} \qquad (13)$$

where, a and b are empirical constants dependent on the type and mesh size of the examined proppant and the used system of units. Therefore, Cook's equation is applicable for most types of proppants with varying the constants.

Pursell and Blakeley (1988) claimed that the constants in cook's correlations do not apply for all situations such as high strength high permeability proppants as well as crushed proppants. He measured permeability and beta factor for Brady sand, interprop and Carbolite proppants and suggested different values for a and b. The two correlations were developed using the same units. Permeability is in darcies and beta is in ($atm.sec^2/gm$). **Table 1** shows a comparison between Cook's and Pursell's work.

Table 1. Cook's and Pursell and Blakeley's beta constants comparison.

Proppant size	Cook		Pursell and Blakely	
	a	b	a	b
12/20	1.34	2.63	1.144	0.635
20/40	1.54	2.65	1.123	0.326

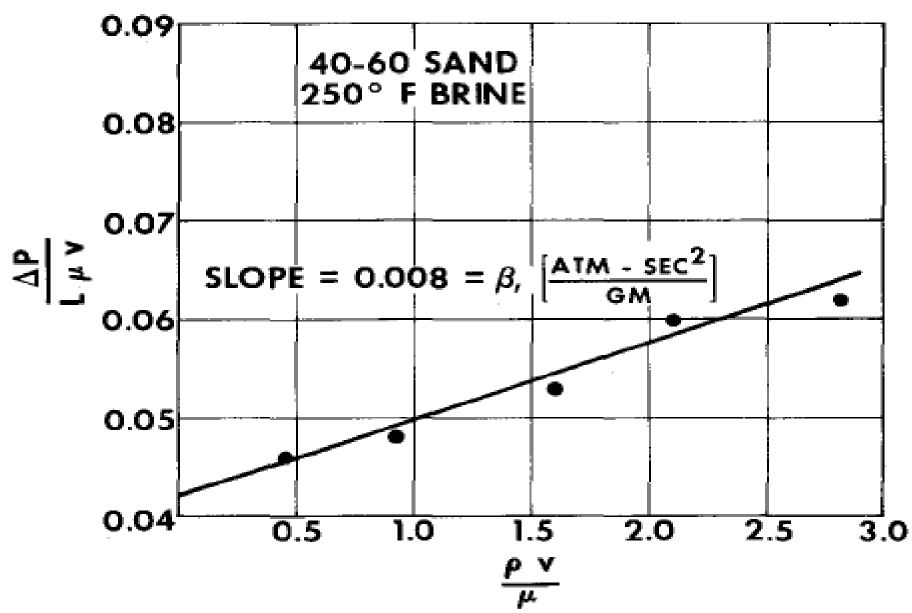

Fig. 5 - Cook's plot to determine beta

Maloney et al. (1989) tested sand packs under stresses from 1000 to 10000 psi. He used nitrogen with high rates as the flowing fluid to simulate the flowing of gas through propped fractures. He tested different sizes of sand, Ottawa sand and sintered bauxite. He combined the results to obtain generalized values of the constants considering beta in cm^{-1} and k_g in cm^2. He presented the following general form.

$$\beta = \frac{1.2\,E{-}03}{{k_f}^{1.7}\,{\varphi_p}^{0.5}} \quad (14)$$

Martins, Tayler & Leung (1990) tested various proppants and mesh sizes under different confining stress using Nitrogen as the flowing phase. Nitrogen was flowed with high rates to simulate field conditions. The same plot as cooks was used but for higher values for the X axis named "X" group as it is more relevant to field conditions. They proposed the following equation for beta using the same units as Cook's:

$$\beta = \frac{0.21}{k^{1.036}} \quad (15)$$

Penny and Jin (1995) studied non-Darcy flow and multiphase flow effects on fracture conductivity. They tested sands, resin coated sands, ceramics and bauxite. They combined the results for all the tested proppants and introduced a general Beta correlation that is identical to Cooks equation with varying the constants.

$$\beta = \frac{b}{K^{a}} \quad (16)$$

where, Permeability is in darcies and beta is in ($atm.sec^2/gm$) a and b are shown in **Table 2**.

Table 2. Penny and Jin's beta constants comparison.

Proppant Type	a	b
Jordan Sand	1.45	0.75
Resin Coated Sand	1.35	1
Light weight Ceramic	1.25	0.7
Bauxite	0.98	0.1

All of the previously mentioned correlations were developed by testing proppants. Some researchers, however, conducted their tests on cores or packed beds. Coles and Hartman (1998), for instance, estimated the beta factor for Limestone and sandstone cores from three different reservoirs and one outcrop by passing gas through them. The tests were done for both dry and saturated cores to investigate the effect of immobile liquid saturation on the flow process. They used paraffin wax as the liquid face to simulate gas condensate. Therefore, the authors claim that Equation 17 is valid for both one and two-phase flow for the tested cores exclusively. However, additional data for a wider variety of rocks is needed to generalize this relationship[2].

$$\beta = \frac{2.49\,E{+}11\,\varphi^{0.357}}{k^{1.79}} \quad (17)$$

where k is the effective permeability in md beta is in 1/ft and φ is effective porosity.

Ergun (1952) developed his beta correlation by testing gas flow through packed spheres, which resembles to a considerable extent gas flow thorough propped fractures. He examined the effect of flow rate, properties of the flowing fluid, the fractional void volume, orientation, size and shape and the surface of the granular surface. He considered one parameter effect at a time. Although his original correlation is presented using particle diameter, the particle diameter can be replaced by laminar permeability if the Carman-Kozny equation is utilized to give the form of Equation 18.

$$\beta = \frac{4.24\,E{+}04}{k_f^{0.5}\,\varphi^{1.5}} \qquad (18)$$

where, beta is in (1/m) and k_f is in md.

The Effect of Multiphase Flow on the Beta Factor

It has been proven by laboratory experiments that the presence of two phases simultaneously in a porous medium has a significant contribution in decreasing the ease with which the gas phase would flow and, therefore, increasing the value of the beta factor. That can be attributed to the decrease in the relative permeability to gas by the increase in the saturation of liquid. Whether the liquid phase is mobile or immobile governs the magnitude by which the permeability to gas flow is diminished. Two phase flow is likely to be present in the case of fluid flow in proppant packs in hydraulically fractured gas wells due to either to the presence of gas condensate, residual water saturation or the remains of fracturing fluids.

The very early work done to estimate the non-Darcy coefficient has not addressed the effect of multiphase flow. However, in hydraulic fractures, the presence of liquid phase along with the gas inside the fracture after the flow back of the fracturing fluids and putting the well on production will decrease the cross-sectional area available for the gas flow, which will obviously lead to a considerable reduction in the gas effective permeability. Failing to account for that, results in an overestimation of the gas flow rate and, consequently, over evaluating the well productivity. Geertsma (1974) was one of the first to investigate that effect. He found out that the previous work done was underestimating the beta factor if a two phase flow system is present. He reported that this underestimation can be by as much as a factor of 8 at liquid saturation of 30 % compared to the one phase flow system. The same 8 fold of increase occurs as the immobile liquid saturation increases from 40 to 70% as per Wong (1970). Evans, Hudson and Greenlee (1987) predicted an increase of three times in the beta factor with 20 % immobile liquid saturation above the dry case. They also pointed out that the presence of a small mobile liquid saturation could increase the beta factor by an order of magnitude. (Martins, Tayler & Leung, 1990) compared the flowing of dry gas to water saturated gas flow and concluded that the effect of 7.2 % water saturation was to increase the pressure drop by approximately 45 % relative to the dry gas case. They also tested mobile water saturation with low gas flow rate and found out that it may increase the pressure losses to 12 times compared to the dry case. They, however, noted that the effect of mobile water saturation decreases with higher gas rates.

Geertsma (1974) proposed an equation to estimate the beta factor considering the immobile water saturation effect.

$$\beta = \frac{0.005}{\varphi^{5.5}\ k^{0.5}}\left[\frac{1}{(1-S_w)^{5.5}\ k_r^{0.5}}\right] \qquad (19)$$

where s_w is the liquid saturation, φ is the porosity, k is the absolute permeability in md and k_r is the relative permeability of the gas phase and Beta is in 1/ft. This equation was generated from the researcher's dry case equation by simply placing the gas effective permeability instead of the absolute permeability and reducing the porosity by subtracting the saturation of the immobile liquid phase.

Martins, Tayler & Leung, (1990) defined the term gamma to evaluate the effect of water saturation on the non-Darcy coefficient. Gamma increase with increasing the water saturation according to the following equation.

$$\gamma = \frac{1.1}{(1-S_w)^{4.9}} \qquad (20)$$

Gamma represents the magnitude of increase in the one phase flow beta due to the presence of two-phase flow.

Equations 19 and 20 are valid only in the case of immobile water saturation. Mobile water saturation, however, was studied by some researchers, yet no direct equation was developed to estimate the magnitude by which the beta factor changes. However, Frederick and Graves (1994) developed a correlation and claimed its validity in the case of mobile liquid saturation. Although the correlation was developed using an immobile liquid saturation two phase system, the researchers plotted some mobile liquid saturation data from the literature and concluded that it follows the same trend of the immobile system. Therefore, Frederick Equation 21 is believed to be valid for mobile two-phase flow.

$$\beta = \frac{2.11\ \ 10^{10}}{{k_e}^{1.55}\,(\varphi(1-Sw))} \qquad (21)$$

where k_e the gas effective permeability in md and beta is in 1/ft.

Non-Darcy Flow Reduces Well Productivity

The effect of non-Darcy flow on hydraulically fractured gas wells deliverability has been studied intensively by a lot of researchers. It has been found that well productivity is significantly reduced by the non-Darcy flow inside the fracture. That is attributed to the big reduction in the proppant nominal permeability. Proppant suppliers conduct their permeability tests using low flow rates and stresses. Therefore, due to the non-Darcy flow conditions and the higher expected stresses, the nominal proppant permeability the manufacturers provide will be as much as an order of magnitude or more bigger than the actual packed permeability inside the fracture after the end of the stimulation treatment. That results in a smaller fracture conductivity than expected and thus less well productivity.

Miskimins, Lopez and Barree (2005) examined some cases and came up with the conclusion that non-Darcy flow effects could reduce flow capacity of low rate wells by 5-30%. That corresponded to a reduction in the cumulative production of 18% for the studied case over a 10 year period of production. Holditch and Morse (1976) investigated the effect of Non-Darcy flow on the productivity of hydraulically fracture wells and concluded that the fracture conductivity can be reduced by a factor of 20 or more leading to decreasing the gas well productivity index by 50 %. Handren *et al.* (2001) have studied two fields with fracture treated wells in south Texas. The old fracture designs were made based on Darcy flow consideration only. The new development wells, however, were treated with a design that accounts for non-Darcy and multiphase flow effects. An average increase in productivity of 20-30 % for the new wells compared to the offset wells treated with designs with no account for non-Darcy effects was noticed. (Martins, Tayler & Leung, 1990) conducted laboratory tests on proppant packs and reported a decrease in effective fracture conductivity by a factor of 10 as a result of non-Darcy flow.

The Effect of Proppant Type Choice on the Severity of Non-Darcy Flow

The non-Darcy coefficient is majorly affected by the structure of the pore throat (Noman & Archer, 1987), the grain size distribution (Pursell & Blakeley, 1988) as well as the permeability and the porosity of the porous media. Therefore, the choice of Proppant type is a vital factor to be considered when designing a fracture treatment to wells where non-Darcy effect is likely to be encountered. Picking a proppant with high conductivity and low beta factor will mitigate the seriousness of the mentioned effects significantly and increase production.

Different proppants have a range of conductivities under stress. The more resistant the proppant to crushing the best choice it is for fracturing in wells with high confining stress. Penny and Jin (1995) investigated the conductivity of various proppants as a function of stress. Thy noticed that Bauxite gives 10 times more conductivity that Jordan sand at closure stresses of 6000 psi. **Fig. 6** shows some proppants conductivity under stress comparison.

Handren *et al.* (2001) examined the effect of choosing different types of proppants on wells productivity in the Frio and Vicksburg reservoirs is south Texas. They used a production model that accounts for non-Darcy's effects. The use of Light Weight Ceramic (LWC) proppant provided an increase of productivity of 13 % over the resin coated sand (RCS) and of 100% over sand. **Fig. 7** shows the value of the beta factor for the most common used proppant in that field study.

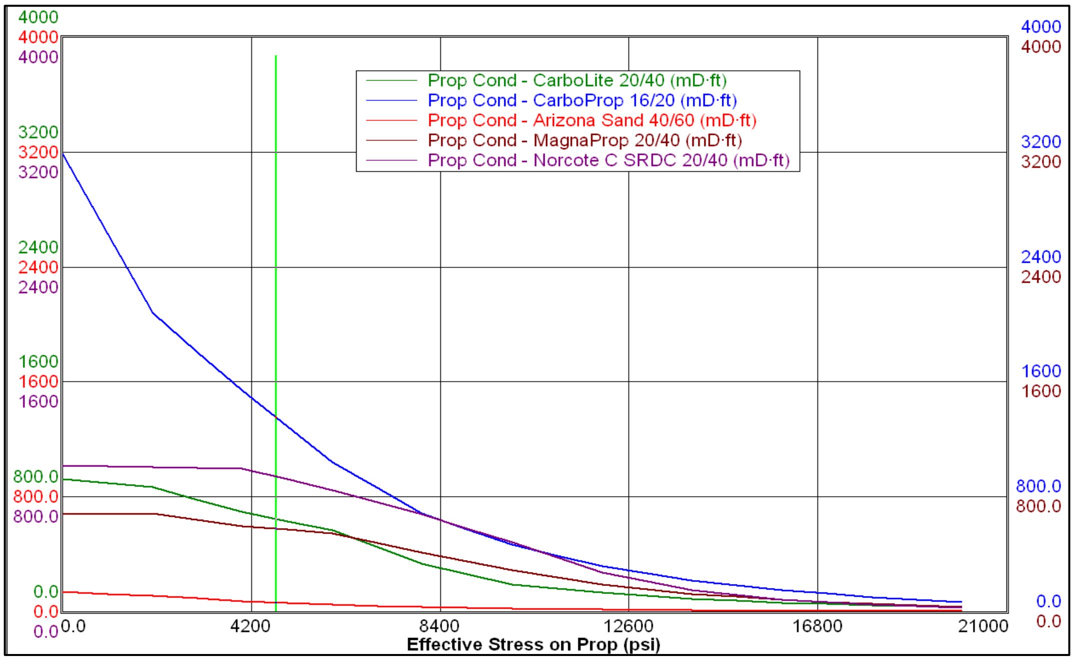

Fig. 6 - Conductivity of some proppants vs. closure stress (Fracpro)

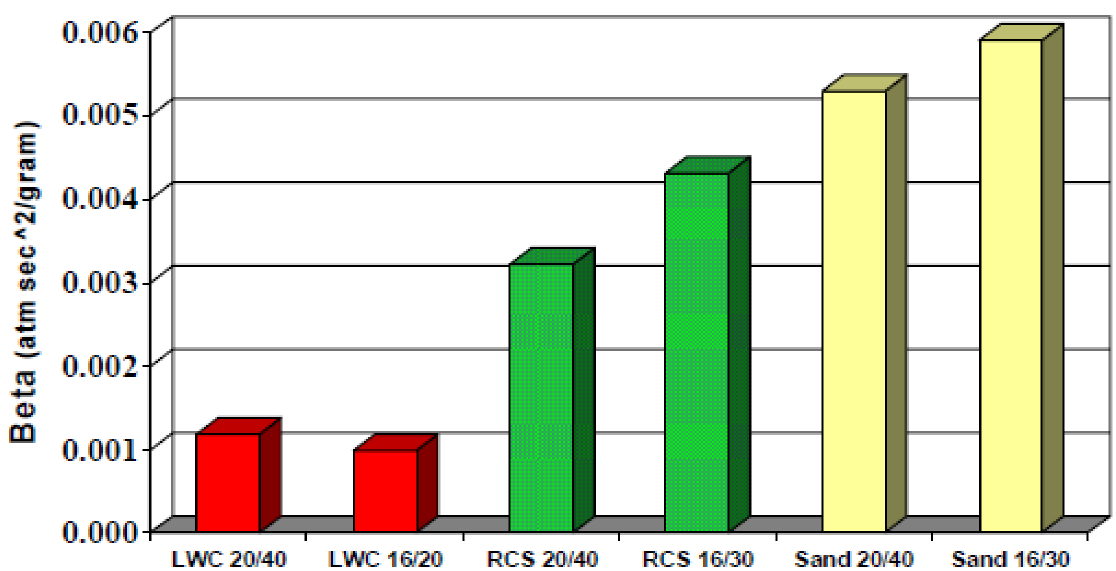

Fig. 7 - Common proppant beta factor comparison (after Handren, 2001)

CHAPTER II

OBJECTIVES OF THE STUDY

This work is intended to investigate the effect of non-Darcy flow in its different forms on the productivity of hydraulically fractured gas wells. An optimum fracture design using UFD technique to compensate for the mentioned effect is also included in this study. Moreover, a comparison is made between the UFD results and the results from three different 3D commercial fracture simulators. In details the objectives of this work are following:

- A thorough and comprehensive literature review is done to understand the problem from the point of view of different researchers.
- Design an optimum fracture treatment using a fixed proppant mass using UFD methodology for a tight gas formation developed by a vertically fractured vertical well with considering only Darcy flow conditions.
- Describe different correlations for the non-Darcy coefficient as far as their source and range of applicability.
- Use some of these correlations to calculate the non-Darcy coefficient for dry porous medium as well as liquid saturated media.
- Write a program using *Mathematica* to calculate the effective fracture permeability in an iterative way taking into account non-Darcy flow and multiphase flow effects and then design an optimum fracture treatment using UFD considering the mentioned effective fracture permeability.

- Compare the results for Darcy and non-Darcy fracture designs as far as the fracture parameters and the well productivity.
- Introduce some engineering choices that help mitigate the effect of non-Darcy flow and multi-phase flow.
- Use three different 3D fracture simulators (Fracpro, M-Frac, and FraCADE) to design the same treatment and compare the results with the UFD design.

CHAPTER III

APPROACH, PROCEDURES AND RESULTS

Non-Darcy flow and multiphase flow reduces the conductivity of hydraulic fractures. And as a result of that, the well productivity is negatively affected. In this work the phenomenon of non-Darcy flow is studied and its effect on the productivity of hydraulically fractured gas wells is evaluated. Some remedial procedures are suggested to lessen severity of that effect including the proposal of an optimum fracture treatment design using UFD technique for a tight gas reservoir is south Texas developed by a vertical well to compensate for Non-Darcy flow. That is accomplished by calculating the effective fracture permeability in an iterative process to account for the significant reduction imposed on the proppant nominal permeability provided by the proppant manufacturers as a result of non-Darcy flow. This method was presented by Lopez and Valko (2004). However, multiphase flow effect was not incorporated in their work and will be accounted for separately in this study. A dry as well as an immobile liquid saturated beta correlation will be used in the loop to calculate the effective fracture permeability in each case.

Fracture Design Using UFD

Unified fracture Design (UFD) is a methodology that aims to design optimum fracture dimensions that lead to a maximum well productivity. It starts with fixing the desired proppant mass to be pumped into the net pay. The most important design parameter is a dimensionless number called the proppant number, which connects the volume of the proppant into the net pay zone to the reservoir drainage volume. For each proppant number there is an optimum dimensionless fracture conductivity that corresponds to the highest value of dimensionless productivity index. A compromise between the fracture length and the width growth should be found. The resultant fracture dimensions are optimum since they correspond to the maximum well productivity. The equations and the procedures have been thoroughly explained in Chapter I.

Input Data

The reservoir input data used in the study are for a tight formation in the Frio reservoir in south Texas which has been used in non-Darcy effects field study by Handren *et al.* (2001). **Table 3** summarizes both the reservoir and the treatment input data.

Table 3. Input data.

Reservoir Data	
Gas gravity	0.66
Permeability (md)	0.15
Drainage area (acre)	80
Reservoir depth middle perforation (ft)	9000
Net pay thickness (ft)	66
Gross thickness (ft)	150
Closure stress (psi/ft)	0.9
Plain strain modulus (psi)	2×10^6
Temperature (F°)	180

Table 3. Continued.

Reservoir Data	
Initial pressure (psi)	6500
Porosity (%)	14
Water saturation (%)	0.40
Bottom hole flowing pressure (psi)	1500
Treatment Data	
Fracture height (ft)	150
Injection rate (bpm)	30
Leak off coefficient in net pay (ft/min$^{0.5}$)	0.003
Fluid loss multiplier in gross pay	0.5
Frac fluid rheology flow behavior index n	0.45
Rheology consistency index K (lbf/ft^{2})	0.9
Proppant mass (Lb)	250,000
Proopant packed porosity (%)	33
Proppant specific gravity	3.3
Proppant type	20/40 Jordan sand
Proppant permeability vs. stress (md)	27,000
Minimum and Maximum proppant final concentration(ppga)	4,15

Darcy Flow Calculation

An optimum fracture design is done to stimulate a vertical well developing a tight rectangular gas reservoir. As a first step, the calculation considers only Darcy flow conditions.

The design starts with calculating the volume of proppant pumped into the net pay using Equation 22.

$$Prop\ volume\ in\ net\ pay = \frac{mass\ of\ proppant\ \frac{h_n}{h_p}}{proppant\ density\ (1 - proppant\ porosity)} \quad \text{...... (22)}$$

The proppant number is then calculated using Equation 5. Having done that, the optimum dimensionless fracture conductivity and the maximum dimensionless fracture productivity index are calculated using subroutine functions written in *Mathematica* describing **Fig. 2** and **Fig. 3** developed by Romero, Valko and Economides (2002) which relate C_{fDopt} and J_{Dmax} to the proppant number. In this case **Fig. 3** is used since N_{prop} is bigger than 0.1 which is always the case in low permeability reservoirs.

The optimum fracture half length and width are then calculated using Equations 9 and 10. Knowing the maximum dimensionless productivity index, the maximum pseudo steady state gas production rate is determined by Equation 23.

$$q = \frac{\pi\ k\ h\ T_{sc}\ [m(p) - m(p_{wf})]}{T\ p_{sc}}\ J_D \quad \text{...... (23)}$$

Net Pressure, Injection Time, Efficiency and Pumping Schedule

After the optimum fracture parameters are obtained, a way to carry out the desired design has to be found according to the practical limitations on the job site. Job procedures including the time of injection, the volumes injected and the proppant schedule are considered.

The process starts with calculating the PKN average hydraulic fracture width using the following equation:

$$w_{w,0} = 9.15^{\frac{1}{2+2n}}\, 3.98^{\frac{n}{2+2n}}\, [\frac{1+2.14n}{n}]^{\frac{n}{2+2n}}\, K^{\frac{1}{2n+2}}\, (\frac{{h_f}^{1-n}\, \mathrm{qi}^{n}\, x_f}{E'})^{\frac{1}{2+2n}} \quad (24)$$

The width is then averaged out by multiplying the previous equation by $\pi/5$

$$w_{ave} = \frac{\pi}{5}\, w_{w,0} \quad (25)$$

The net pressure inside the fracture can be calculated by:

$$P_{net} = \frac{E}{2\, h_f}\, w_{w,0} \quad (26)$$

We now can calculate the injection time by solving the quadratic material balance equation for time.

$$\frac{q_i}{h_f\, x_f}\, t_e - (2\, \varkappa\ C_L)\sqrt{t_e} - \left(w_{ave} + 2\, S_p\right) = 0 \quad (27)$$

The injected volume is then given by:

$$V_i = \ q_i\, t \quad (28)$$

And the efficiency is

$$\eta = \frac{Fracture\ volume}{injected\ volume} = \frac{h_f\, x_f\, w_{ave}}{V_i} \quad (29)$$

Next step is to calculate the proppant schedule, which represents the proppant concentration as a function of time. Both the chemical engineering concentration and the mass of proppant added to a unit volume of clean fluid are calculated. Nolte analysis is used for this purpose. The process can be concluded in the following steps:

- From fluid efficiency calculate the exponent of proppant concentration, epsilon.

$$\varepsilon = \frac{1-\eta}{1+\eta} \quad (30)$$

- Calculate the pad volume and the time needed to pump it.

$$V_{pad} = \varepsilon\, V_i \quad (31)$$

$$t_{pad} = \varepsilon\, t_i \quad (32)$$

- Calculate the final proppant concentration.

$$c_e = \frac{proppant\ mass}{\eta\, V_i} \quad (33)$$

- Calculate proppant concentration schedule.

$$c = c_e \left(\frac{t-t_{pad}}{t_e-t_{pad}}\right)^{\varepsilon} \quad (34)$$

- Change the conventional chemical engineering proppant concentration to the added proppant concentration.

$$c_{added} = \frac{c}{1-\frac{c}{\rho_p}} \quad (35)$$

- Calculate the added proppant concentration schedule

$$c = c_{added} \left(\frac{t-t_{pad}}{t_e-t_{pad}}\right)^{\varepsilon} \quad (36)$$

The results for the fracture design considering only Darcy flow are summarized in **Table 4.**

Table 4. Darcy calculation summary.

Parameter	**Value**
N_{prop}	1.2
C_{fDopt}	2.6
J_{Dmax}	0.92
x_{fopt}(ft)	623
w_{fopt} (in)	0.11
Fracture conductivity (md-ft)	250
Net pressure (psi)	652
Average PKN width (in)	0.73
Injected volume (bbls)	2386
Injection time (min)	159
Fluid efficiency	0.42
V_{pad} (bbls)	953
t_{pad} (min)	63
Pad percentage (%)	39
Required final concentration (ppg)	5.8
Required added concentration (ppga)	7.3
Gas flow rate (Mscf/d)	18981

Fig. 8 and **Fig. 9** illustrate the proppant concentration and the added proppant concentration schedule.

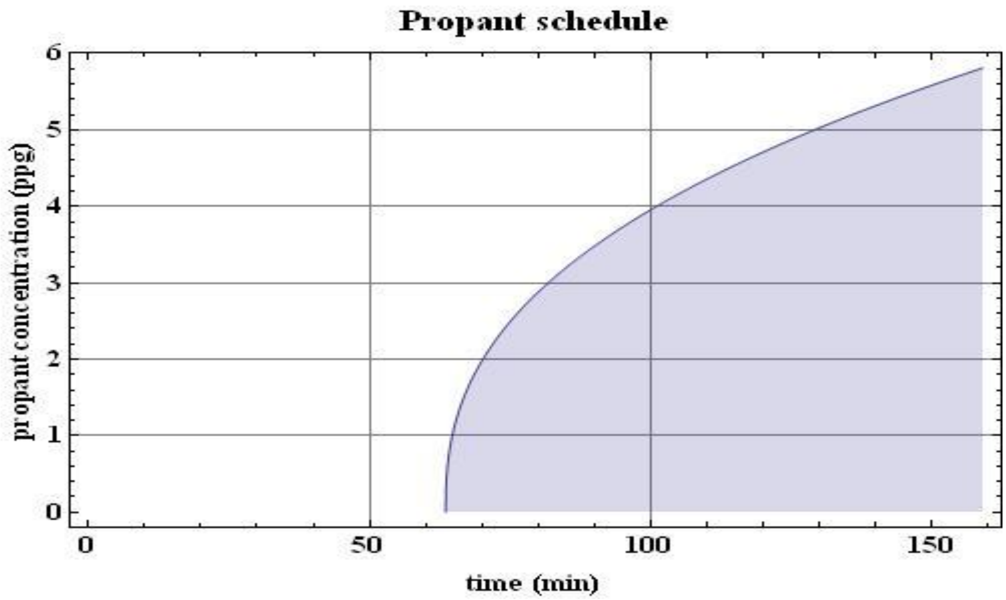

Fig. 8 - Proppant concentration schedule

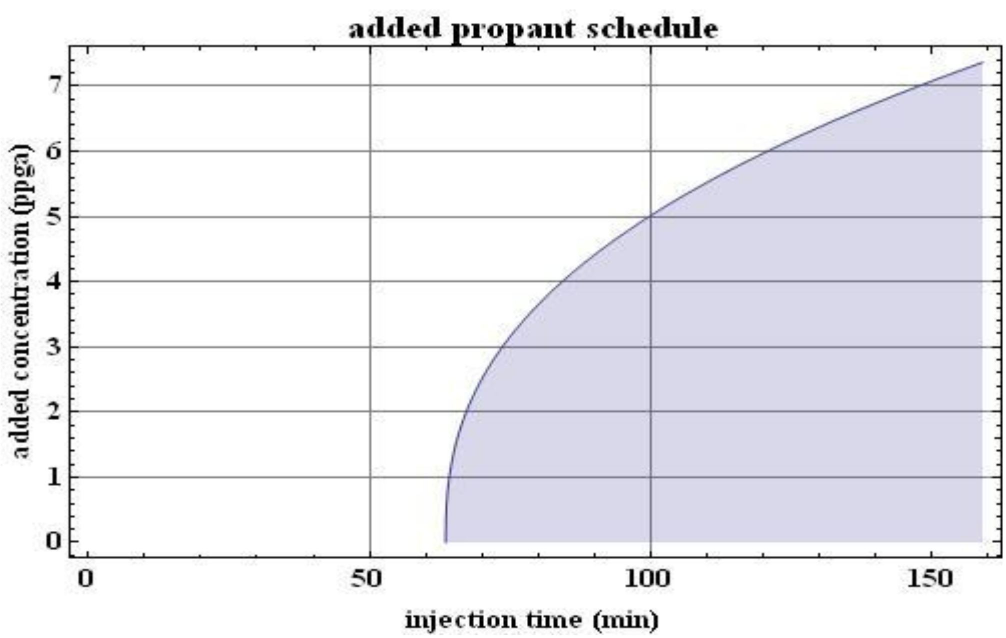

Fig. 9 - Added proppant concentration schedule

Non-Darcy Calculations

Due to many factors including the high inertial resistance to flow, high flow rates and turbulence, Gas flow in hydraulically fractured wells cannot be described by Darcy's low. Instead, Forchheimer's low is believed to be more considerate. Therefore, when designing a fracture treatment, corrected fracture permeability must be used instead of the nominal permeability of the fracture provided by the proppant manufacturers. That results in a shorter and wider fracture that compensates for the non-Darcy conditions. Gidley (1991) proposed a way to calculate the corrected permeability by dividing the Forchheimer's low for pressure drop by the Darcy's low.

Dividing Equation 12 by Equation 11 we obtain the following:

$$\frac{\left(-\frac{dp}{dl}\right)_F}{\left(-\frac{dp}{dl}\right)_D} = 1 + \frac{\beta\rho u\, k_D}{\mu} \quad (37)$$

where, the subscript F describes the Forchheimer or non-Darcy flow pressure drop and D indicates the Darcy flow condition.

By definition, the Reynolds number is:

$$N_{Re} = \frac{\beta\rho u\, k}{\mu} \quad (38)$$

$$\frac{\left(-\frac{dp}{dl}\right)_F}{\left(-\frac{dp}{dl}\right)_D} = 1 + N_{Re} \quad (39)$$

The Forchheimer (i.e. non-Darcy or effective) permeability can be expressed by:

$$\frac{1}{k_F} = \frac{1}{k_D} + \frac{\beta\rho u}{\mu} \quad (40)$$

$$\frac{k_F}{k_D} = \frac{1}{1 + N_{Re}} \quad (41)$$

And finally,

$$k_F = \frac{k_D}{1+N_{Re}} \quad (42)$$

where k_F is the effective fracture permeability after accounting for non-Darcy flow, and k_D is the fracture Darcy permeability.

In this work the corrected permeability is referred to as the fracture effective permeability or the non-Darcy permeability $K_{N.D}$ and the Darcy permeability is equivalent to the permeability of the proppant under stress provided by the manufacturers. As seen in the Reynolds number equation, there appears the factor Beta. Depending on the flow conditions in the fracture, different beta correlations can be used to evaluate the magnitude of non-Darcy effects. In this work two scenarios are considered.

- One phase non-Darcy flow (beta is calculated using Ergun's Equation 18).
- Immobile liquid saturation two phase flow (beta is calculated using Geertsma's Equation 19).

To optimally design a fracture that considers the fracture effective permeability and thus compensates for the non-Darcy effects, the fracture design calculations are done in an iterative manner. We start the calculation the same way as in the Darcy design until the gas flow rate is calculated. Then the loop is incorporated in the design as follows:

- Convert the surface gas production rate calculated from the Darcy fracture design to the insitu gas rate inside one wing of the fracture.

$$qinsitu = \frac{q}{2}\left(\frac{T}{\mathrm{T}_{sc}}\frac{\mathrm{P}_{sc}}{\mathrm{P}_{wf}}\mathrm{Z}_{pwf}\right), \quad (43)$$

- Calculate the gas velocity inside the fracture.

$$u = \frac{qinsitu}{h_f \, w_{fopt}} \quad \text{(44)}$$

- Calculate the non-Darcy coefficient, beta, using a suitable correlation for the flow condition in the porous media. That includes the use of Equation 18 by Ergun for the dry gas case and Equation 19 by Geertsma for the immobile liquid saturation two-phase flow case. The relative permeability curves for the two-phase flow are taken from Barree and Conway (2009).
- Calculate the porous media Reynolds number

$$N_{Re} = \frac{\beta \rho \, u \, k}{\mu} \quad \text{(45)}$$

- Calculate the effective non-Darcy permeability inside the fracture

$$K_{N.D} = \frac{k_f}{1 + N_{Re}} \quad \text{(46)}$$

- Calculate the new proppant number using $K_{N.D}$ instead of k_f

$$N_{prop} = \frac{2 \, K_{N.D} \ volume\ of\ proppant\ in\ the\ net\ pay}{k\ reservoir\ volume} \quad \text{(47)}$$

- Calculate C_{fDopt} and J_{Dmax} from **Fig. 2** and **Fig. 3** using the new N_{prop}
- Calculate the optimum fracture half length and width from Equations 9 and 10.
- Calculate the new gas flow rate using the new maximum dimensionless productivity index.
- Repeat the process until convergence is reached by gaining the same value for effective permeability in the last step as the previous one.

The iteration process is illustrated in the following **Fig. 10**.

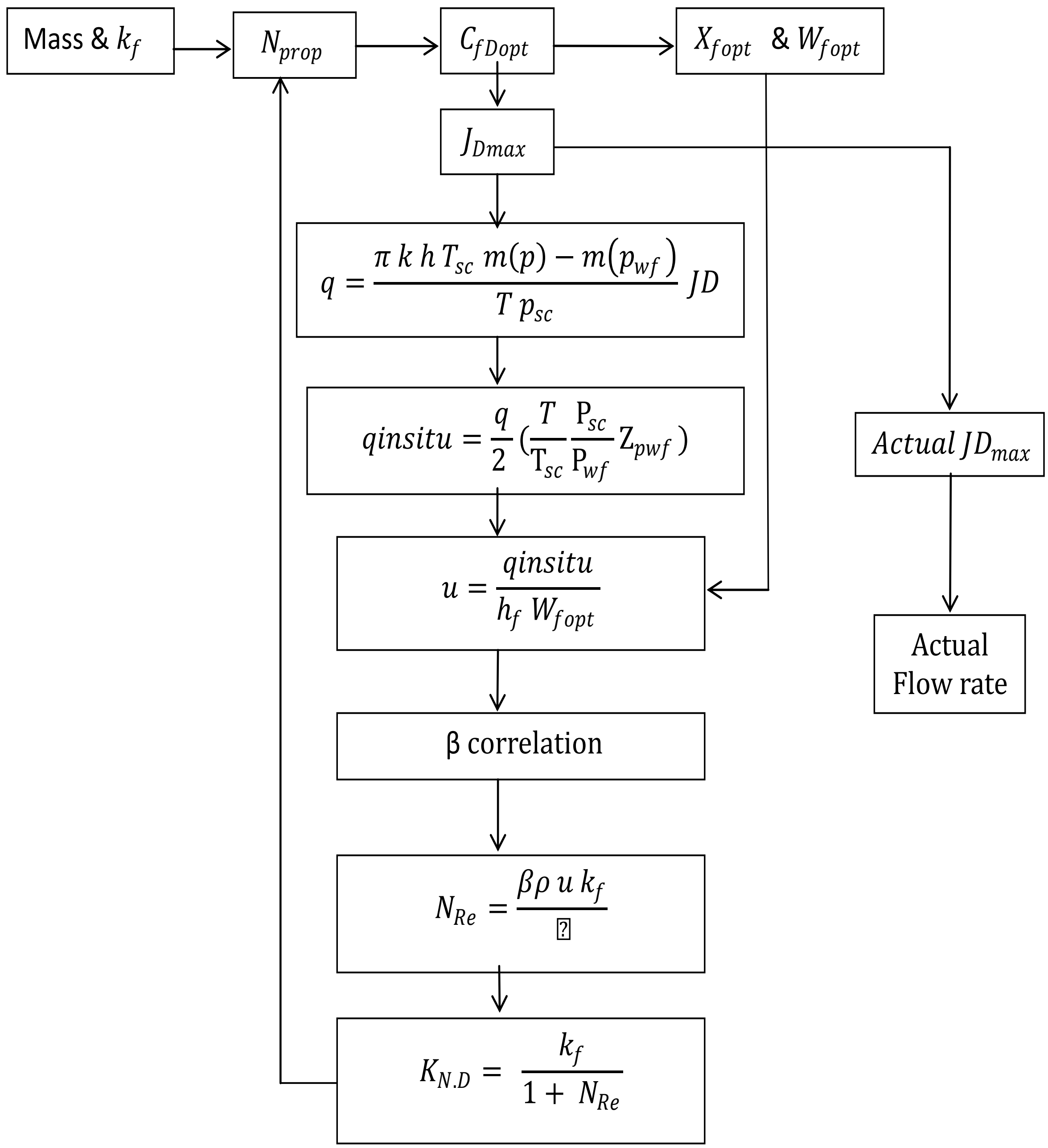

Fig. 10 – Summary of fracture design considering non-Darcy flow

In the case of using Ergun equation as the beta correlation in the design loop the effective permeability converges after 22 iterations at 7.5 darcy. **Fig. 11** shows the effective permeability versus iteration number. The behavior of effective fracture permeability in the other case of the use Geertsma as the beta correlation is similar with lower value of 543 md for permeability after convergence due to higher value of beta factor which increases with the increase of liquid saturation as illustrated in **Fig. 12.**

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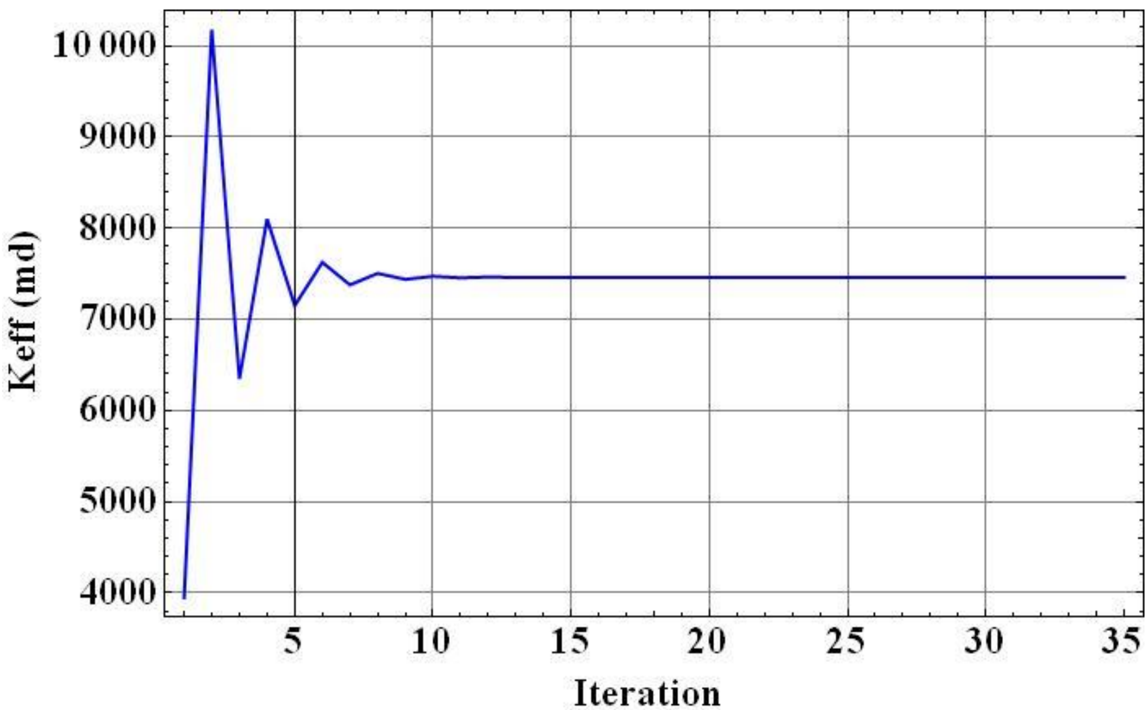

Fig. 11 - Effective permeability vs. iteration using Ergun equation

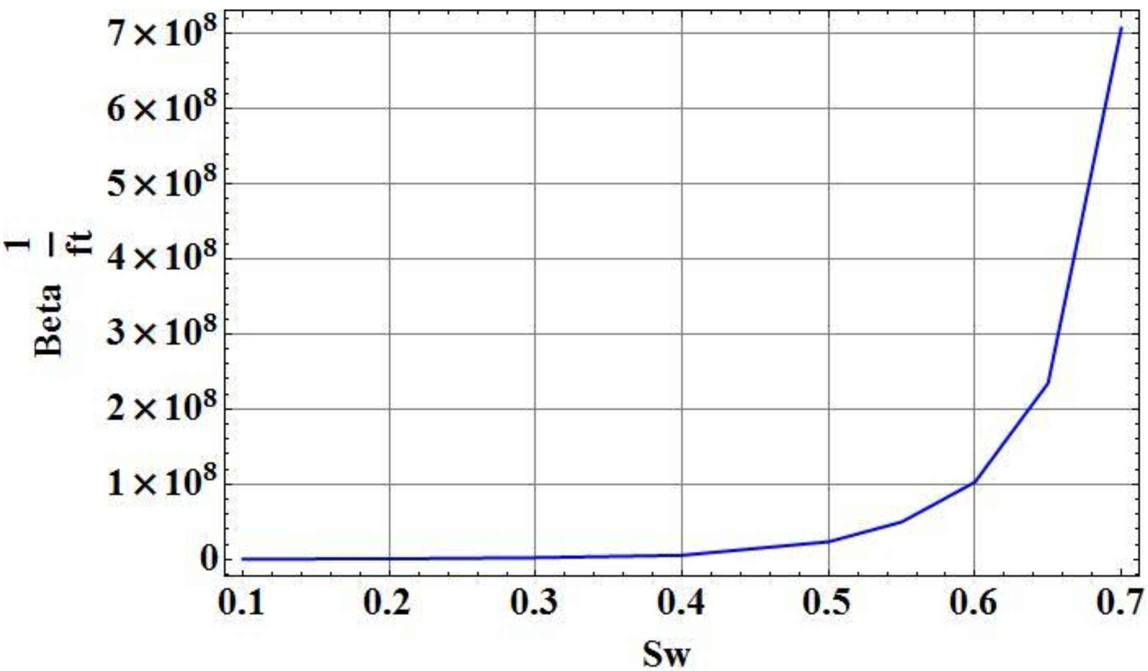

Fig. 12 - Beta factor vs. liquid saturation

Having designed the optimum fracture dimensions accounting for non-Darcy conditions, the same procedures used to obtain a complete pumping schedule for the fracture design in the Darcy case, explained previously, can then be used for the non-Darcy flow and multiphase flow designs. It is, however, possible that other operational constraints (e.g. limit on maximum pumpable proppant concentration) prohibit the placement of proppant in the optimum way. In such a case, several options are available for the engineer: In the case of soft formations, a TSO design may be adopted. In cases when TSO is not an option because of the high net pressures, the design should target the minimum departure from the optimum placement.

The results for the three cases are summarized and compared to the Darcy case in **Table 5**.

Table 5. Comparison between Darcy and non-Darcy designs.

Case	Darcy	Dry non-Darcy	Two-phase flow
N_{prop}	1.2	0.33	0.024
C_{fDopt}	2.6	1.76	1.63
J_{Dmax}	0.92	0.63	0.35
x_{fopt}(ft)	623	403	113
w_{fopt} (in)	0.11	0.17	0.61
β (1/ft)	-	5.1×10^{4}	6.2×10^{6}
N_{Re}	-	2.6	48
Fracture permeability (md)	27,000	7460	543
Fracture conductivity (md-ft)	249	106	27
Gas flow rate (Mscf/d)	18981	13175	7215
5 year Cum production (MMscf)	4263	4032	3512

It can be seen that accounting for the non-Darcy effects results in an optimum fracture that is shorter and wider. The permeability inside the fracture can be reduced by almost four folds as a result of the non-Darcy flow in a dry gas flow. However, the permeability impairment can be as much as more than an order of magnitude when liquid phase is present in the media. As a result of that, the conductivity of the fracture, and thus the gas production rate, is significantly reduced with the increase in the severity of the non-Darcy effects. It is true that we theoretically have more production (18,981 Mscf/d) when we account only for Darcy flow conditions. However, in reality these perfect flow conditions do not exist in hydraulically fractured gas wells. On the contrary, one or the two of the other mentioned non-Darcy flow scenarios is more likely to occur. Being not able to account for that will result in a fracture design that is not optimum even within the possibilities, and consequently produces less gas.

The effect of non-Darcy flow on cumulative production was evaluated by conducting a production forecast for five years for all the cases using Promat production analysis program. **Fig. 13** shows a comparison of the three scenarios. The production loss due to the non-Darcy flow conditions is 5, and 18 % for dry and immobile liquid saturation respectively. This loss in production occurs regardless of having an optimum fracture design that compensates for non-Darcy effects. However, if the mentioned effects are ignored, more production loss is expected.

By considering a bigger drainage area, (160 acres instead of 80 acres) the severity of the non-Darcy effects on the cumulative production becomes clearer. **Fig. 14** shows a production loss of 11 and 29 % for the dry and immobile two-phase non-Darcy flow effects respectively.

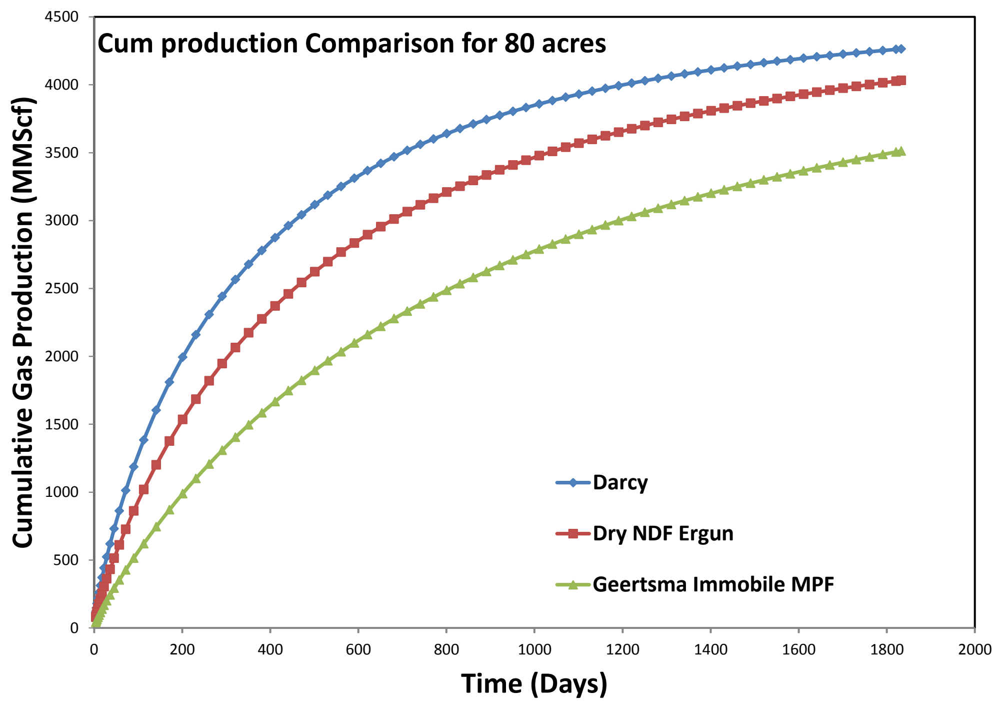

Fig. 13 - Darcy vs. non-Darcy cum production comparison (A=40 acres)

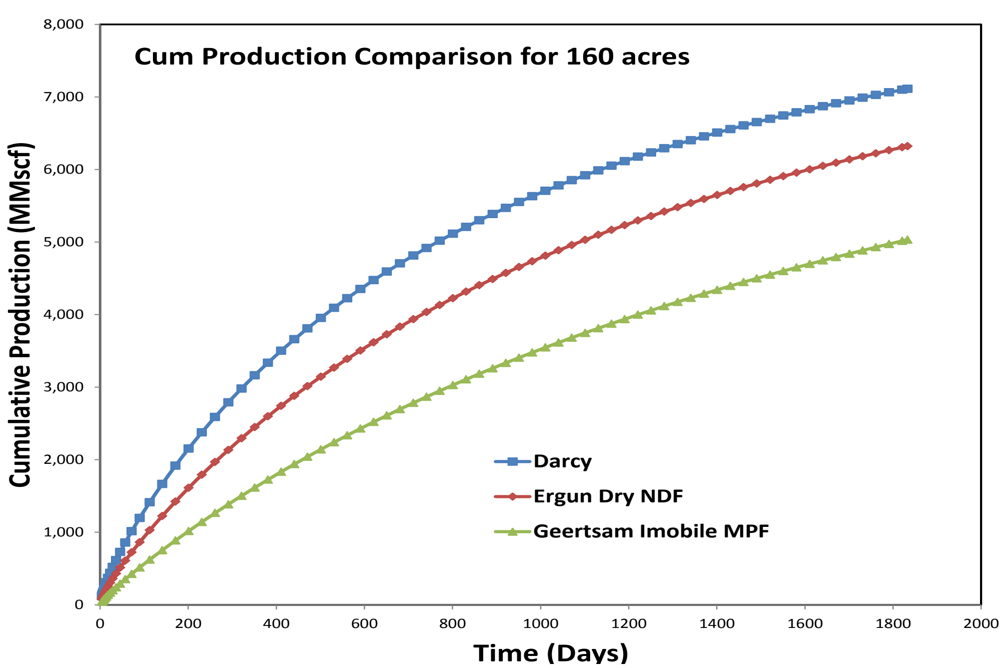

Fig. 14 - Darcy vs. non-Darcy cum production comparison (A=80 acres)

Ways to Lessen the Severity of Non-Darcy Flow Effect

Non-Darcy flow is a phenomenon that is of the nature of porous media and, therefore, cannot be eliminated or totally controlled. However, some engineering choices can be done to optimize the stimulation treatment and make the impact of non-Darcy flow as limited as possible. The most vital practice to limit the reduction in gas production due to non-Darcy flow is to design an optimum fracture that at least partly compensates for those effects by correcting the fracture conductivity as previously explained. However, some other choices can be made within the fracture design to help optimize it more. That includes, but is not limited to, choosing a proppant with better permeability and less beta factor and pumping more proppant mass. These choices, however, should be subject to an economical study to prove worthy. In this work, though, some scenarios are run changing design parameters, namely proppant permeability and mass, to theoretically examine their effect on the productivity regardless of their financial convenience. This sensitivity analysis was limited to the dry non-Darcy flow design. All the calculations were done for 160 acres drainage area to be able to see a clear sign of the importance of each parameter.

The Effect of Proppant Permeability

Choosing a proppant with better characteristics i.e. higher permeability and lower beta factor can be a very effective practice to mitigate the severity of non-Darcy flow. Some cases are shown in **Fig. 15**. It can be seen that using CarboLite proppant with a nominal permeability of 190 darcy instead of Jordan sand with 27 darcy may increase the cumulative production by 443 MMscf, which is equivalent by 5% of increase. With a price of 5$ per Mscf, this increase is worth 2.2 million Dollars. It can be concluded that the severity of non-Darcy flow can be lessened by almost 45% (from losing 11% production compared to the Darcy design to losing only 5%) by choosing Carbolite proppant instead of Jordan sand.

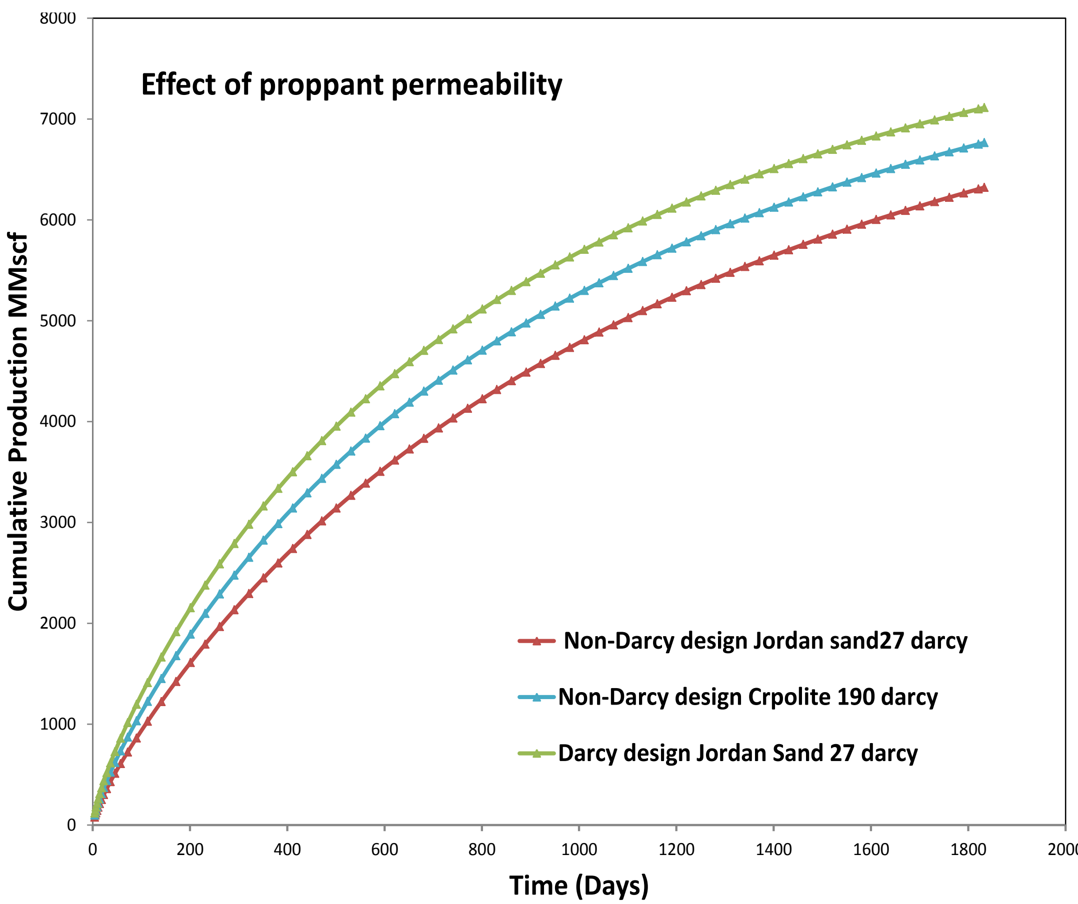

Fig. 15 – The effect of proppant permeability

The Effect of Proppant Mass

The effect of changing the proppant mass and keeping the same type of proppant was investigated. The base case of 250,000 lbs. of Jordan sand proppant was compared to 50,000 and 500,000 lbs. It has been found that increasing the mass by 2 times increases the production by 8 % in five years. That represents a reduction in the severity of non-darcy effect by 38 % (from losing 11% production compared to the Darcy case to losing only 4 %). However, reducing the mass by five times reduces the production by 15 %. **Fig.16** Shows the effect of the amount of mass pumped into the fracture on the well productivity.

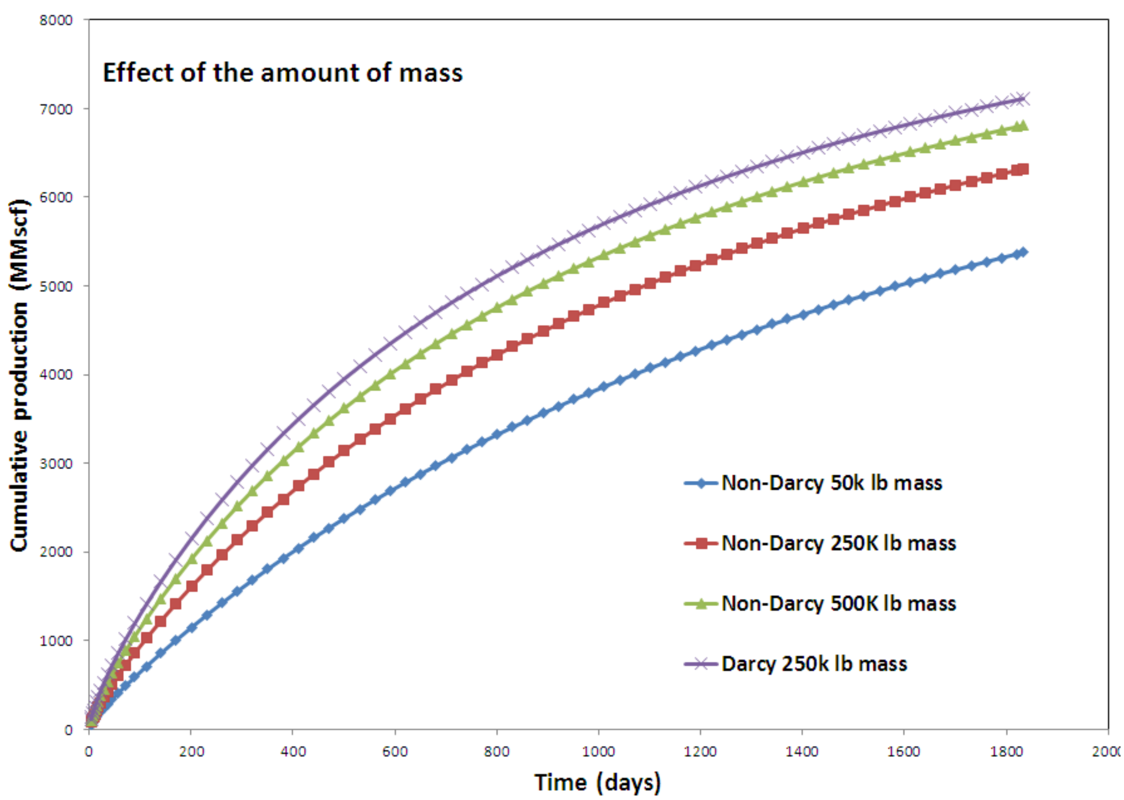

Fig. 16 – The effect of proppant mass

CHAPTER IV

UFD vs. 3D FRACTURE SIMULATORS

This chapter is dedicated to shed some light on the profound differences in philosophy between the UFD approach used to design the fracture treatment in this research and the 3D fracture simulators used in the industry.

The main goal from a stimulation technique should be to maximize the production of the candidate well, not only to increase it. As mentioned before, UFD serves this goal by designing optimum fracture dimensions that lead to a maximum well productivity index using a fixed amount of proppant.

The initial parameter in UFD methodology is the proppant mass. A fracture volume ready to accommodate the given mass is calculated, from which a dimensionless proppant number is obtained. Associated with that proppant number there is a maximum goal dimensionless fracture conductivity and a maximum dimensionless productivity index, to which a compromise between the fracture half-length and fracture width is linked. Fracture height is an input in UFD and the injection parameters and pressure match are not involved in the primary process in which optimum fracture dimensions are designed. They are rather calculated later on based on the pumping limitations.

On the contrary, 3D fracture simulators numerically solve all the flow governing equations to come up with the fracture dimensions. These equations include: mass conservation equation, the continuity equation, the momentum conversation equation (equation of motion), the width pressure relationship and the fracture propagation based on the concept of stress intensity factor and fracture toughness relationship. Unlike UFD, the pressure distribution inside the fracture as well as the leak off properties are vital factors affecting the fracture length and width. In addition, fracture height is also simulated rather than fixed.

Regardless of the mentioned differences in philosophy between the UFD and the fracture numerical simulators, in this work, the same base Darcy calculation fracture treatment done using UFD is also done using three known fracture programs (Fracpro, MFrac and FracCADE) only to have an idea about how far or close the results can be as far as the fracture most important parameters and what are the possible changes that can be done to design a fracture treatment using the simulators which can best match the treatment designed according to UFD.

A try is made to best mimic the UFD input by using the same proppant mass when possible (MFrac and FracCADE). The same proppant permeability is also maintained by either changing the proppant damage factor (Fracpro and MFrac) or permeability retained factor (FracCADE) or to edit the proppant permeability as a function of closure stress to obtain the 27,000 md Jordan sand permeability under stress used in the UFD case. The C_{fDopt} obtained from the UFD as an output is also used as goal conductivity in the simulators input data, when possible (Fracpro and MFrac), and couple it with the total mass.

MFrac

Regardless of the mentioned difference in philosophy, among the commercial fracture programs, MFrac developed by Meyer & associate, Inc., can be best compared to UFD since the proppant mass can be fixed and can be coupled with the dimensionless fracture conductivity as the input options **Fig. 17**. However, the input data needs to be carefully chosen and edited to best mimic the parameters in UFD. In this work, the mass is 250,000 lbs., the C_{fDopt} is 2.6, the final proppant concentration is 8 ppg, the permeability versus closure stress is set at 27,000 md and Leak off properties is assumed to be contained only in the reservoir gross height of 150 ft and the leak off multiplier in the shale layers is 0.5.

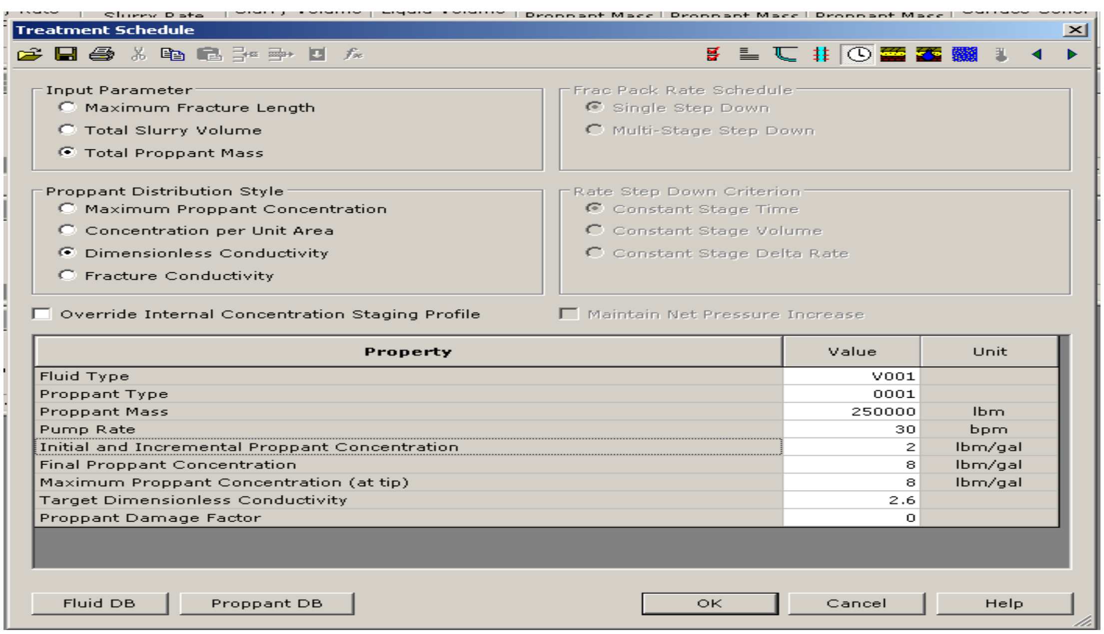

Fig. 17 - MFrac input options

The permeability of the fracture after considering the closure stress should be 27,000 md in the final report, which equals the input permeability in UFD. That can be done by either changing the proppant damage factor or editing the proppant permeability understress table. The latter was done in this work.

Fracpro

Fracpro is a stimulation software developed by CARBO. Unlike MFrac, the mass cannot be a fixed input in Fracpro. Instead, many treatment sizes are proposed for the goal dimensionless fracture conductivity which should be the output optimum dimensionless conductivity from UFD. The treatment with a proppant mass that is most comparable to the UFD case is then manually selected and its parameters i.e. half-length and dimensionless fracture conductivity are used as pre-selected design parameters in the first iteration to design the final treatment **Fig. 18**.

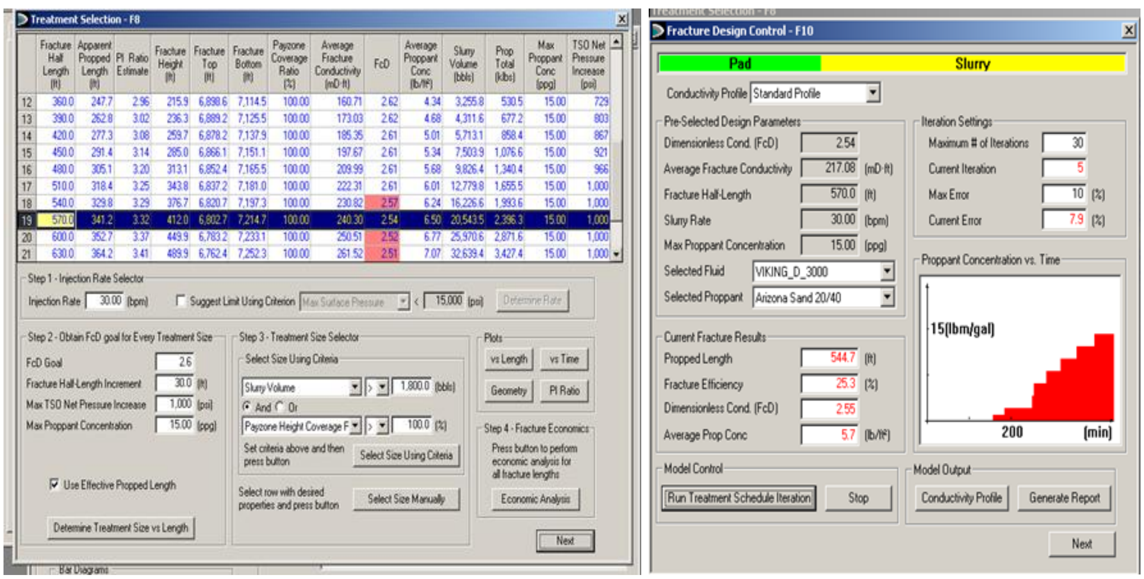

Fig. 18 - Fracpro design selection and design control

FraCADE

FracCADE is a commercial fracture design program developed by Schlumberger. Proppant mass is also not a preliminary input in FracCADE. It can, however, be fairly controlled by editing the proppant concentration in the pumping schedule. The proppant permeability can be fixed at 27,000 md to match the UFD input by using permeability retained factor or by simply giving a value to permeability under stress tables as shown in **Fig. 19**.

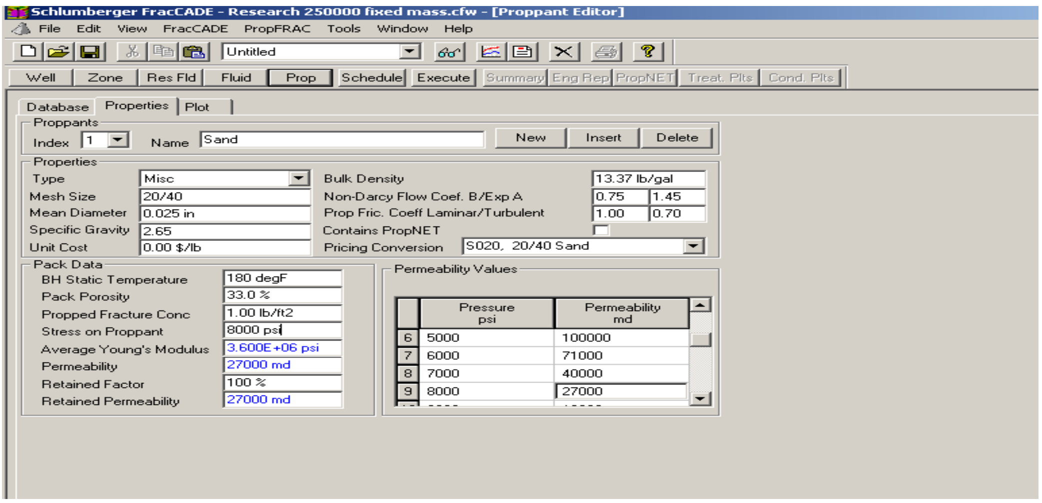

Fig. 19 - Editing proppant permeability in FracCADE

Table 6 summarizes the fracture simulators outputs and compares them to the UFD results for the Darcy case. Although the 3D fracture simulators use a completely different philosophy to solve for the fracture dimensions, it can be concluded that to some extent comparable results can be obtained from these simulators if the input data was edited carefully, as explained previously, to best mimic the UFD design.

Table 6. UFD vs. fracture 3D simulators.

Case	UFD	MFrac	Fracpro	FracCADE
x_f(ft)	623	514	544	393
w_f (in)	0.11	0.092	0.091	0.081
C_{fD}	2.6	2.6	2.6	7.5
Fracture propped height(ft)	150	334	330	600
Proppant mass (1000 Lbs.)	250	250	245	251
Fracture permeability (md)	27,000	27,000	27,000	27,000
Fracture conductivity (md-ft)	250	200	340	441
Net pressure (psi)	652	337	600	200
Average hydraulic width (in)	0.73	0.5	0.52	0.29
Injected volume (bbls)	2386	4539	4005	4670
Pad volume (bbl)	953	2464	1802	2241
Pad percentage (%)	39	54	45	48
Injection time (min)	159	151	200	164
Fluid efficiency	0.42	0.31	0.33	0.52

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Summary

Searching the literature and then using some of the already developed equations and correlation to investigate the effects of non-Darcy flow and multi-phase flow and design a fracture that compensates for them, we conclude the following:

- The flow in hydraulically fractured gas wells cannot be described by Darcy's low. It should rather be described by Forchheimer's equation to account for the extra pressure drop.
- The extra pressure drop is induced by a combination of the high gas flow velocity, turbulence, high inertial resistance to the flow and the tortuosity of the porous media.
- An additional pressure drop occurs when liquid saturation is present in the media due to the reduction in the cross-sectional area for the gas flow and thus the decrease in the gas relative permeability.
- The additional pressure drop due to non-Darcy flow is represented by the non-Darcy flow coefficient beta.
- The Beta factor is a property of the porous media and the stress state.
- The permeability of the fracture can be reduced by four folds due to non-Darcy flow in the dry gas flow case and can be reduced by more than an order of magnitude with the existence of liquid phase.

- In the studied case in this work, non-Darcy flow reduces the gas productivity by 5 and 18 % for dry and immobile liquid saturation cases respectively.
- By increasing the drainage area to 160 acres instead of 80 acres the decrease in productivity becomes 11 and 29% for the same two cases.
- The mentioned productivity losses take place regardless of designing an optimum fracture that takes into account non-Darcy effects. However, the losses are expected to be more significant if non-Darcy conditions are not considered.
- Accounting for non-Darcy conditions in the UFD results in a shorter and wider fracture.
- Non-Darcy conditions are of the nature of tight formations and, therefore, cannot be eliminated. However, some engineering procedures with in the optimum fracture design can mitigate their effects.
- Using CarboLite proppant, which has a higher permeability, instead of Jordan Sand can lessen the non-Darcy effects by as much as 45 %.
- Using two times more mass of the same proppant lessens the non-Darcy effect by 38 %.
- UFD uses a different philosophy than the available 3-D commercial fracture simulators. UFD starts with proppant mass to design fracture dimensions that result in a maximum dimensionless productivity index without the involvement of the injection constrains or the pressure distribution.

- Fracture simulators solve the mass conservation, the momentum conservation, the continuity and width vs. pressure equations to come up with fracture geometry without considering whether or not that geometry is optimum.
- By carefully editing the input in the fracture simulators to best mimic the UFD parameters, fairly close results can be obtained.

Recommendations

- When designing a fracture treatment in gas wells the permeability of the fracture must be corrected to account for non-Darcy condition inside the fracture.
- When calculating the fracture effective permeability a suitable beta correlation should be chosen for each different flow conditions.
- Proppant with low beta factor and high permeability should be used when non-Darcy flow is likely to be encountered.
- It is recommended to design an optimum fracture treatment using UFD and then use the outputs i.e. x_f and C_{fD} as an input, when possible, in a fracture simulator to obtain the other design aspects such as the pumping schedule.

NOMENCLATURE

A	=	Drainage area, acres.
a	=	Constant in beta correlation.
b	=	Constant in beta correlation.
c_{added}	=	Added proppant concentration, ppga.
c_e	=	Final proppant concentration, ppg.
C_{fD}	=	Dimensionless fracture conductivity.
C_L	=	Leak-off coefficient, ft/min$^{0.5}$.
$\frac{dp}{dl}$	=	Pressure drop in a porous medium, psi.
E'	=	Plain strain modulus, psi.
h_f	=	Fracture height, ft.
h_n	=	Height of net pay, ft.
h_p	=	Height of pay zone, ft.
I_x	=	Penetration ration.
J_D	=	Dimensionless productivity index.
K	=	Rheology consistency index, lbf/ft^2.
k	=	Reservoir permeability, md.
k_e	=	Effective permeability, md.
k_f	=	Fracture permeability, md.
$K_{N.D}$	=	Non-Darcy effective fracture permeability, md.
k_r	=	Relative permeability.
$m(p)$	=	Gas pseudo pressure, psi.

n	=	Rheology flow behavior index.
N_{prop}	=	Proppant number.
N_{Re}	=	Reynolds number.
P_{net}	=	Net Pressure, psi.
P_{sc}	=	Standard conditions pressure, psi.
p_{wf}	=	Down hole flowing pressure, psi.
q	=	Gas flow rate, Mscf/d.
qi	=	Injection rate, bpm.
q_{insitu}	=	Gas flow rate inside the fracture, Mscf/d.
r_e	=	Reservoir drainage radius, ft.
S_p	=	Spurt loss coefficient, ft.
s_w	=	Water saturation.
T	=	Reservoir temperature, F°.
t_e	=	Injection time, min.
t_{pad}	=	Pad pumping time, min.
T_{sc}	=	Standard condition temperature, F°.
u	=	Gas velocity inside the fracture, ft/s.
V_i	=	Injected volume, bbl.
V_{pad}	=	Pad volume, bbl.
w_{ave}	=	Average hydraulic fracture width, in.
w_f	=	Fracture width, in.
$w_{w,0}$	=	Average hydraulic fracture width, in.

x_e	=	Reservoir length, ft.
x_f	=	Fracture half length, ft.
Z	=	Gas compressibitly factor.
μ	=	Gas viscosity, cp.
β	=	Non-Darcy flow coefficient, 1/ft.
ε	=	The exponent of proppant concentration epsilon.
ρ	=	Gas density, lb/ft^3.
ρ_p	=	Proppant density, lb/ft^3.
φ	=	Porosity.
φ_p	=	Proppant porosity.
γ	=	Ratio of two-phase versus single-phase pressure loss.
η	=	Fluid efficiency.
$\varkappa$	=	Nolte's function.

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