# GENERALIZED CORRELATIONS TO ESTIMATE OIL RECOVERY AND PORE VOLUMES INJECTED IN WATERFLOODING PROJECTS

A Dissertation

by

ARNALDO LEOPOLDO ESPINEL DIAZ

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2010

Major Subject: Petroleum Engineering

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December 2010

Major Subject: Petroleum Engineering

# ABSTRACT


Generalized Correlations to Estimate Oil Recovery and Pore Volumes Injected in Waterflooding Projects.

(December 2010)

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When estimating a waterflood performance and ultimate recovery, practitioners usually prepare a plot of log of water-oil ratio vs. cumulative production or recovery factor and extrapolate the linear section of the curve to a pre-established economic limit of water production. Following this practice, engineers take the risk of overestimating oil production and/or underestimating water production if the economic limit is optimistic. Engineers would be able to avoid that risk if they knew where the linear portion of the curve finishes. We called this linear portion the “straight-line zone” of simply SLZ.

In this research, we studied that “straight-line zone” and determined its boundaries (beginning and end) numerically using mathematics rules. We developed a new procedure and empirical correlations to predict oil recovery factor at any water/oil ratio.

The approach uses the fundamental concepts of fluid displacement under Buckley-Leverett fractional flow theory, reservoir simulation, and statistical analysis from multivariate linear regression.

We used commercial spreadsheet software, the Statistical Analysis Software, a commercial numerical reservoir simulator, and Visual Basic Application software.

We determined generalized correlations to determine the beginning, end, slope, and intercept of this line as a function of rock and fluid properties, such as endpoints of relative permeability curves, connate water saturation, residual oil saturation, mobility ratio, and the Dykstra-Parsons coefficient. Characterizing the SLZ allows us to estimate the corresponding recovery factor and pore volumes injected at any water-oil ratio through the length of the SLZ .

The SLZ is always present in the plot of log of water-oil ratio vs. cumulative production or recovery factor, and its properties can be predicted. Results were correlated in terms of the Dykstra-Parsons coefficient and mobility ratio. Using our correlations, practitioners can estimate the end of the SLZ without the risk of overestimating reserves and underestimating water production. Our procedure is also a helpful tool for forecasting and diagnosing waterfloods when a detailed reservoir simulation model is not available.

# DEDICATION

To God, for giving me the opportunity and the privilege of living, studying, and working in Aggieland for a second time.

To my wife and kids, for their dedication, support, and sacrifice giving me always a reason to work out this degree.

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# CHAPTER I

# INTRODUCTION

For many years petroleum engineers plotted data on graph paper, and with a straightedge or a curve, drew average lines through the points or extrapolated those linear trends to estimate future performance. Now they use the same technique electronically.

A straight-line correlation is a linear correlation of several variables. Extrapolating a straight line in plots can be useful, especially when dealing with oil and water production data and recovery factors, but also may be dangerous, because recoverable oil may be overestimated.

Observing field data or simulation results from waterflooding projects, we can see that when no more movable oil is left in the reservoir, the curve of the widely used log of water-oil ratio (WOR) vs. oil recovery factor (RF) plot abruptly bends up. Therefore, extrapolating the zone before the curve bends up to a pre-established economic limit may drive the practitioner to overestimate recoverable oil and underestimate water production.

The log WOR vs. RF plot is an important, powerful and reliable diagnostic and forecasting tool because its shape is affected by very important factors that determine a waterflooding project performance, such as areal ($E_A$) and vertical ($E_i$) sweep efficiencies, displacement efficiency ($E_D$), rock properties, reservoir heterogeneity, fluid properties, and fluid saturations.

---

This dissertation follows the style of the *SPE Journal.*

In this chapter we discuss briefly these important factors, the most relevant previous works related to the WOR vs. oil production and RF plots, our objectives, and our contributions with this research. In the following chapters we will discuss our methodology, results, conclusions and recommendation.

$E_A$ is the fraction of the reservoir area contacted by injected water. This efficiency will depend upon the oil and water relative permeabilities ($k_{ro}$ and $k_{rw}$), Mobility Ratio (*M)*, injection pattern, pressure distribution between the injector and the producer, and areal heterogeneity (including fractures and directional permeability). Experimental correlations have been published to estimate $E_A$ for different mobility ratios (Craig, 1971).

$E_i$ is the fraction of the reservoir's vertical plane contacted by water. Vertical heterogeneity will determine this efficiency. Published correlations are available to estimate $E_i$ for different mobility ratio values (Dykstra and Parsons, 1950). $E_v$, or volumetric sweep efficiency, is the product of $E_A$ and $E_i$. $E_i$ can be expressed in terms of the Dykstra-Parsons coefficient (VDP). This coefficient indicates the degree of heterogeneity of a reservoir. VDP equal to 0 would correspond to an ideal, completely homogeneous reservoir, and 1 to a completely heterogeneous reservoir. Most reservoirs have VDPs between 0.8 and 0.95.

The Dykstra-Parsons coefficient is determined following these steps:

1. Divide permeability samples into layers of equal thickness.
2. Arrange the permeability data in decreasing order.

3. Determine for each value the percent of values with greater permeability and express each number as cumulative percentage, or “percent greater than.”
4. Plot the data on log-probability scale, with permeability in the log scale and percent in the probability scale.
5. Estimate the best fit for a straight line and determine permeability values at 84.1% and at 50%.
6. Determine the Dykstra-Persons coefficient using the expression:

$$VDP = \frac{k_{50} - k_{84.1}}{k_{50}} \; . \qquad (1)$$

$E_D$ is the ratio of displaced oil to the total flooded portion of the reservoir and can be estimated as function of water and oil saturation:

$$E_D = \frac{\overline{S_w} - S_{wc} - S_g}{1.0 - S_{wc} - S_g} \; , \qquad (2)$$

where

$\overline{S_w}$ = average water saturation behind the water front, fraction

$S_{wc}$ = connate water saturation, fraction

$S_g$ = gas saturation, fraction

and recovery efficiency ($E_R$) will be defined as:

$$E_R = E_A \times E_i \times E_D \; . \qquad (3)$$

Relative permeability is the ratio of effective permeability (permeability to water, oil, or gas when more than one phase is present in the reservoir) to some reference permeability. The most used reference permeability is the effective permeability to oil measured at the irreducible water saturation $(k_o)_{Swi}$. In this case, the value of the relative permeability of oil at $S_{wi}$ will always be 1.0. Sometimes we may find that the base permeability used is the permeability to air ($k_{\mathrm{air}}$). This procedure is not incorrect, but makes comparison of different relative permeability curves more difficult.

Other very important concepts to take into account when planning or analyzing a waterflooding project follow. These concepts are basic knowledge for our research.

Based on the reservoir continuity, especially considering continuity of the floodable pore volume, the injection scheme may be peripheral or have a specific injection pattern. If the reservoir is continuous and no permeability barrier is present between injectors and producers, peripheral water injectors can begin in the reservoir flanks, which can be an excellent way of displacing oil with water while saving investments by minimizing the number of injection wells.

The injected water will move from the edges of the reservoir to the center, and oil will be displaced toward the producer wells. Initial peripheral injection usually is expanded to interior reservoir injection to either reach symmetry or not. If the existing well density is high and/or sealing faults or permeability barriers are present, the peripheral scheme may not work efficiently, so an injection pattern should be applied to achieve a better sweep efficiency.

Large rock permeability is desired to ensure adequate injectivity and productivity and, if feasible, to accelerate the production response with a higher injection rate. However, large permeability variation or large contrast between layers will increase the risk of water channeling through large permeability intervals, leaving behind important oil volumes within zones of lesser permeability.

Mobility ratio (M) is defined as the ratio of the mobility of the displacing phase to the mobility of the displaced phase. $M$ may be estimated using $k_{rw}$ evaluated at the maximum water saturation (1-$S_{or}$), and the oil relative permeability evaluated at the connate water saturation ($S_{wc}$) (unflooded zone). This $M$ is called the "endpoint" mobility ratio (see Eq. 4).

$$M = \frac{\lambda_{\text{Displacing}}}{\lambda_{\text{Displaced}}} = \frac{\dfrac{(k_{rw})_{S_{w\max}}}{\mu_w}}{\dfrac{(k_{ro})_{S_{wc}}}{\mu_o}}, \quad \text{(4)}$$

where

$\lambda_{\text{displacing}}$ = mobility of the displacing phase (water)

$\lambda_{\text{displaced}}$ = mobility of the displaced phase (oil)

$(k_{rw})_{Sw\max}$ = relative permeability to water at the maximum water saturation, fraction

$(k_{ro})_{S_{wc}}$ = relative permeability to oil at the connate water saturation, fraction

$\mu_w$ = water viscosity, cp

$\mu_o$= oil viscosity, cp

This "endpoint" definition is used for simplicity in this work. Another definition for $M$ uses $k_{rw}$ evaluated at the average water saturation in the flooded zone at the water breakthrough, instead of the $k_{rw}$ evaluated at the maximum water saturation (1-$S_{or}$). This definition better explains the fluid behavior in the porous media (Craig, 1971) but is less practical to use in developing our correlations.

*M* will remain constant before water breakthrough but will begin to increase after that point with the increase of average water saturation in the water-swept zone.

Unfavorable *M* (M >> 1) may create some problems that may be handled with peripheral injection schemes. Large *M* values mean that water will move faster than oil; therefore, water fingering (water bypassing oil due to the higher mobility of the displacing phase) can occur and may cause early water breakthrough in producing wells. When the mobility ratio is much larger than one, injectivity will be larger than productivity, because water will flow through the porous medium easier than oil. To balance voidage and maintain pressure, more producers may be required than injectors. For very favorable mobility ratios (M<<1), productivity will be larger than injectivity because oil will be more mobile than water, and more injectors will be required than producers (Craig, 1971). Assuming steady-state, incompressible fluid behavior, and after gas fill-up is completed, we may neglect this effect in our forecasting since we will have one unit of produced liquid for each unit of injected water.

Initial free gas saturation may delay injection response due to the gas fill-up where injected water and displaced oil will fill the spaces previously occupied by free gas. To estimate oil and gas saturations at the beginning of the waterflood, the following expressions can be used (Craft and Hawkins, 1991):

$$S_o = \left(1.0 - \frac{N_{pp}}{N_{ob}}\right)\left(\frac{B_o}{B_{ob}}\right)\left(1.0 - S_{wc}\right) \quad (5)$$

and

$$S_g = 1.0 - S_o - S_{wc}, \quad (6)$$

where

$S_o$ = oil saturation at the beginning of the waterflood, fraction

$N_{pp}$ = primary oil production from the bubblepoint pressure to the current reservoir pressure, STB

$N_{ob}$ = original oil in place at the bubblepoint pressure, STB

$B_o$ = oil formation volume factor at current average reservoir pressure, RB/STB

$B_{ob}$ = oil formation volume factor at the bubblepoint pressure, RB/STB

$S_{wc}$ = connate water saturation, fraction

$S_g$ = gas saturation at the beginning of the waterflood, fraction

During the last 50 years, several attempts have been made to forecast waterflood performance and ultimate oil recovery by modeling the sweeping process of water displacing oil through the porous medium.

The accuracy of the prediction is mostly affected by the knowledge about effects of reservoir heterogeneity, fluid saturations, and mobility ratio. These factors affect displacement and areal and vertical sweep efficiencies. Water channeling that bypasses mobile oil remaining within the rock, causing low displacement and early breakthrough in producing wells, will reduce ultimate recovery.

Reasonable forecasts of waterflood performance can improve decisions regarding a waterflooding candidate and project feasibility. Waterflood performance can be estimated by various analytical and empirical methods based upon several assumptions that many times are ignored or violated.

Reservoir simulation is now one of the most comprehensive and widely used waterflood prediction tools. When used properly, it can be a very useful tool for waterflood design, planning, and surveillance. Simulation is especially useful when used to develop forecasts of a complex reservoir, varying fluid properties, and a no uniform well pattern.

However, while simulation can yield results that are superior to and more detailed than other methods, it also generally requires a lot more data and time. A range of uncertainty in the input data leads to a resulting band of uncertainty in the output. A good understanding of the uncertainties, multiple realizations, and sensitivity analysis will significantly increase the value and usefulness of the simulation results.

Since reservoir simulation requires more data and time to produce good forecasts, we focused our research in analytical and empirical methods used to develop more general but reasonable waterfloods forecasts.

Baker et al. (2003) showed that in the plot of log WOR vs. cumulative production, a linear extrapolation can be made of the linear region of the curve obtained up to a value of log WOR = 2 (WOR = 100). The authors highlighted the possibility of errors (underestimation of ultimate recovery) if exponential decline is used. However, they did not offer a procedure to extrapolate the curve in terms of the linear portion boundaries, giving only rules of thumb based on observations made from simulation results.

Ershaghi and Omoregie (1978) developed a procedure to extrapolate watercut curves to estimate recovery after injected water breakthrough, using the "*X*-function" (defined in Eq. 7) in an approach similar to the one used for the plot of log WOR vs. cumulative production:

$$X = -\left[\ln(\text{WOR}) - \frac{1}{\text{WOR} - 1}\right] \quad \text{.......} \quad (7)$$

In Ershaghi and Omoregie's (1978) approach, the straight-line relationship between the "$X$" factor and cumulative production ($N_p$) on a Cartesian plot may be extrapolated after WOR is equal to 1. This plot assumes a 1D Buckley-Leverett model, does not consider stratification, and assumes $E_A$ constant after water breakthrough. For values of WOR between 0.5 and unity, a linear relationship may be expected, but the abrupt increase of the WOR when oil saturation approaches $S_{or}$ is not considered, so extrapolations to the economic limit should be done with extreme caution.

Lo (1990) presented a procedure to estimate recovery for mature fields using relative permeability curves and production data. The method is used to calculate the slope of the log WOR vs. $N_p$ curve from the relative permeability curve data and determine ultimate recovery by extrapolating that curve. However, the end of the SLZ and the capability to predict reservoir behavior before breakthrough is not defined. This method assumes homogenous reservoirs.

Lo (1990) demonstrated that for 1D, linear, immiscible, and incompressible displacement, log WOR is linearly proportional to the cumulative oil production; therefore, a straight line will appear in a semilog plot of these variables. Lo used the 1D Buckley-Leverett analytical method and derived the slope of the curve from the relative permeability data, $S_{wc}$, $B_o$, and hydrocarbon pore volume (HCPV), using the expression:

$$Slope = \frac{(1-S_{wc})bB_O}{\text{HCPV}}, \quad (8)$$

where:

$S_{wc}$ = connate water saturation

$b$ = slope of the log of the best fit of the curve of water/oil relative permeability ratio vs. water saturation

HCPV = hydrocarbon pore volume ($V_P(1\text{-}S_{wc})/ V_P$)

Lo (1990) did not consider reservoir stratification (VDP), $M$, or sweep efficiencies in his study.

Despite all of the progress made by later researchers, Lo's work seemed to offer the best basis.

For our project, we incorporated the best contributions of other authors, as we modified Lo's approach to account for different reservoir and fluids properties and reservoir heterogeneity.

After studying the different authors and reviewing published information and case studies, we wanted to answer the following questions: How accurate is the extrapolated procedure using the log WOR vs. RF curve to estimate RF and WOR? Is the SLZ always present? How long it is? Is it always correct to extrapolate it to find ultimate recovery at an assumed economic limit? What are the SLZ boundaries (where does the SLZ begin and where does it end)?

The specific objectives of this research are:

1. To prove or disprove the existence, formation, and boundaries of the linear (SLZ) portion of the log WOR vs. RF plot as a function of reservoir

parameters, and to use this SLZ to estimate oil recoveries and water injection needs.

2. To develop generalized correlations using fractional flow theory, reservoir simulation, and field data to predict expected recovery factors for any WOR value.
3. To create a unique analytical and statistical tool that helps the user develop plans to maximize oil recovery and minimize water production and operating costs.
4. To validate our correlations and the tool developed in this project using field data from more than 80 reservoirs, including rock and fluid properties, production and injection rates, and recovery factors. That way we can ensure that our correlations and results are data driven. In this research we show two examples of forecasts using field cases results.

In summary, the purpose of our research is to characterize the linear portion (SLZ) of the plot of log WOR vs. RF in terms of its beginning and end so that practitioners can predict oil recovery at any WOR with a reasonable degree of accuracy.

Using fractional flow theory, we determined the slopes and intercepts of the SLZ when linear behavior is maintained in the plots of log WOR vs. RF for reasonable combinations of $M$, VDP , $S_{wc}$, $S_{orw}$, and $k_{roe}$, $k_{rwe}$ for oil- and water-wet rocks. We applied reservoir simulation to create a database of results for different heterogeneous cases, we developed multiple linear correlations to predict waterflood performance using basic production data, and we built a computer code to apply our newly developed correlations.

Our correlations estimate the beginning and the end of the SLZ to obtain recovery forecasts while avoiding the risk of extrapolating the WOR vs. RF line and overestimating reserves.

Our methodology includes recovery correlations that account for reservoir heterogeneity based on the VDP in a range of values from 0.5 to 0.99.

All results were compared with field data and results obtained from the reservoir simulator to perform a control check with all estimates.

# CHAPTER II

# METHODOLOGY

A waterflood oil production profile considering only production due to water injection—that is, injecting from the first day of operation—will present four typical basic stages that we can identify using the plot of log WOR vs. RF. These stages can be defined as (see Fig. 1):

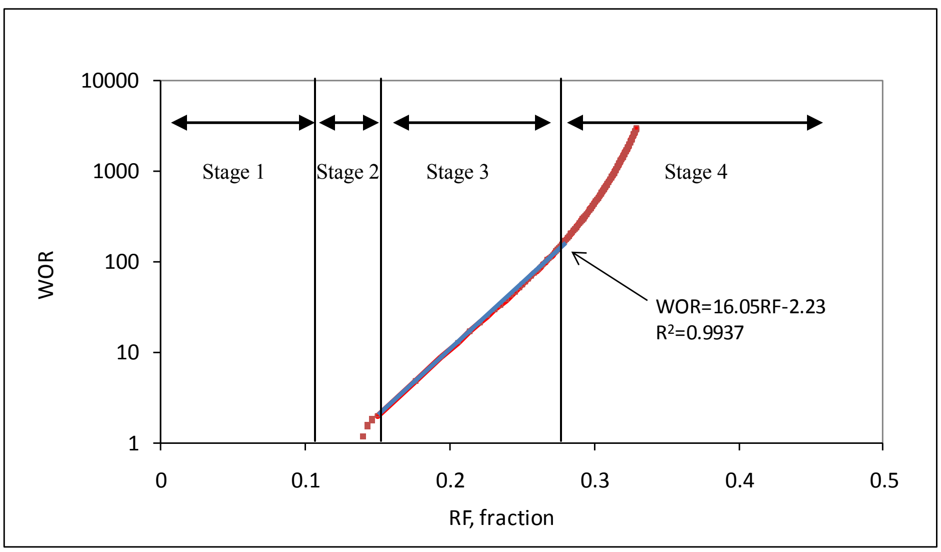


Fig. 1—Data from simulation. The plot shows a log WOR vs. RF plot, the curve stages, the SLZ zone, and the calculated slopes and intercepts for an example taken from the simulation runs with VDP = 0.9, $M = 3$, $S_{oi} = 0.6$, $S_{or} = 0.35$ and $S_{wi} = 0.4$.

1. Beginning of injection to water breakthrough: During this stage, oil banks are formed around the injection well. A water front is formed and advances through the reservoir layers.

2. Breakthrough to SLZ: This stage is the period of time from when injected water is first produced to when the SLZ begins to form. After a short period of stabilization, that according to our observations usually occurs between WOR = 0 and WOR = 1.5, the SLZ appears and the water production increases slowly, creating the SLZ in a semilog plot.

3. SLZ: Period when the change in the log of WOR with respect of the RF is constant. Water cut is increasing relatively slowly.

4. Incline WOR: The SLZ ends and WOR begins to increase above the SLZ trend.

By applying linear regression, we can determine that the SLZ in the example has a slope of 16.05 and an intercept of -2.23, with a regression coefficient (R2) higher than 0.99.

From the SLZ analysis, we can see that when producing the well after a value of WOR equal to 110, corresponding to a RF of approximately 28%, the water production will abruptly. The slope and the intercept of the SLZ will depend upon the reservoir and fluid characteristics.

From the example discussed above, we can see that an increase of more than one order of magnitude in WOR would be needed to obtain only 4% of additional recovery (32%), with a large amount of additional water injected and produced, which may increase costs and decrease economics of the project if this water production is not expected or planned.

Fig. 2 shows real data from an actual well in a field where waterflooding was conducted from the beginning of the field development. A sharp increase in WOR begins after the SLZ ends, just before reaching an RF of about 42%, as can be seen in the plot. This well is an example of the SLZ application.

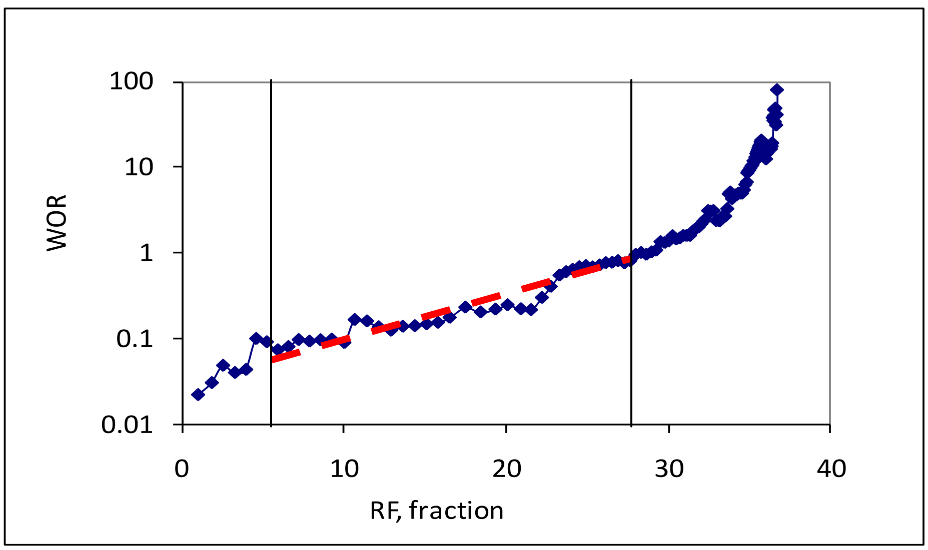


Fig. 2—Data from a real well. The SLZ is highlighted in red. This example shows the beginning of the SLZ between log WOR -1.5 and -1 and the end of the SLZ close to 0.1, with a RF = 42%. WOR increases abruptly after this point.

Table 1 presents a summary of the characteristics for each stage in a typical waterflooding project.

Table 1—Stages in the waterflooding life cycle and their characteristics.

| STAGES | CHARACTERISTICS |
|---|---|
| Beginning of injection to water breakthrough (Stage 1). | Oil banks formation.<br>Gas fill-up occurs.<br>Pre-breakthrough stage.<br>Secondary production begins. |
| Water breakthrough to SLZ(Stage 2). | Injection reaches the producer.<br>Water breakthrough occurs. |
| SLZ (Stage 3). | Oil rate peak. Water production increases constantly. |
| incline WOR (Stage 4). | Oil production decreases and water production increases fast.<br>High water cut is obtained. |

The steps during this research can be summarized as follows:

1. Perform literature review of worldwide oil industry experiences and lessons learned from real waterflooding projects including waterflooding performance estimation.
2. Build a database using a computer spreadsheet, including proprietary data available from existing fields consisting on permeability, porosity, fluid viscosity, *M*, voidage ratio, PVI, relative permeability curves, and all other

available parameters that can contribute to evaluate waterflooding performance.

3. Identify waterflood performance estimation methods that can be used for heterogeneous reservoirs with unfavorable mobility ratios.
4. Develop a set of correlations based on fractional flow theory to accurately evaluate reservoir performance under waterflood including the effect of initial fluid saturations, wettability, heterogeneity, and different viscosity ratios.
5. Build up this automated procedure in the Statistical Analysis Software (SAS*)* to create correlations to estimate the beginning and the end of the SLZ for homogeneous reservoirs, using reservoir and fluid properties, including recovery factors and pore volume injected. Prove the theoretical existence of the SLZ in all cases.
6. Build simple reservoir simulation models using commercial reservoir simulation systems to create multiple heterogeneous cases and to determine the existence of the SLZ in these runs for several combinations of M and VDP.
7. Develop an easy-to-use, user-friendly tool in VBA that includes the new methodology.

We used field data to test our correlations. Reservoir and fluids properties of more than 100 reservoirs and fields were used. Among those properties, we collected oil rate ($q_o$), water production rate ($q_w$), $N$, $N_p$ primary, $N_p$ secondary, PV, processing rates

(PVI/year), $\mu_o$, $\mu_w$, $S_{or}$, $S_{wc}$, relative permeability curves, cumulative water injection, wettability, PVT data and VDP.

Data quality check was performed. We tabulated all data collected and built log WOR vs. RF plots for each reservoir. To avoid the effects of errors in production measurements and specific operational problems in the data, we used a correlation coefficient ($R^2$) cutoff of 0.95 for the SLZs obtained from actual field data to select the curves to be used to calibrate our procedure. 84 reservoirs were used.

Data used is proprietary and restricted by Intellectual Property policies, therefore we did not obtain permission to publish all the results. However, we obtained permission to show two sets of data of two fields that we called Field A and Field B, both described in the Results section.

**The Module for Homogeneous Reservoirs**

We programmed and applied the conventional Buckley-Leverett (Buckley and Leverett, 1942) procedure using the Statistical Analysis Software (SAS) to calculate and differentiate the fractional flow ($df_w/dS_w$). Corey's relative permeability functions were used to estimate reasonable combinations of relative permeability curves using oil and water exponents (Molina, 1980). We called that code the "homogeneous module."

Corey's type functions (Molina, 1980) are expressed as:

$$k_{rw} = k_{rwe}\left[\frac{S_w - S_{wc}}{(1 - S_{wc} - S_{or})}\right]^{n_w} \qquad (9)$$

and

$$k_{ro} = k_{roe}\left[\frac{1 - S_w - S_{or}}{\left(1 - S_{wc} - S_{or}\right)}\right]^{n_o} \qquad (10)$$

where $k_{rwe}$ and $k_{roe}$ are the water and oil relative permeability curves endpoints and $n_w$ and $n_o$ are the water and oil Corey exponents (Molina, 1980). The base relative permeability used to develop the calculations to define a range for the oil relative permeability endpoint was the air permeability.

For normalized relative permeability curves, the program uses the following expression for dimensionless water saturation:

$$S_{wD} = \frac{\left(S_w - S_{wc}\right)}{\left(S_{w\max} - S_{wc}\right)}. \qquad (11)$$

and the following expressions for relative permeability values:

$$k_{rw} = k_{rwe}\left[S_{wD}\right]^{n_w} \qquad (12)$$

$$k_{ro} = k_{roe}\left[1 - S_{wD}\right]^{n_o}. \qquad (13)$$

Calculations at and after water breakthrough are computed to estimate the conditions at the flood front for all possible combinations of viscosity ratios (VR), $M$, and Corey's exponents for oil and water relative permeability endpoints, and $S_{wc}$ and $S_{orw}$ (see the homogeneous module's code in Appendix A).

For homogeneous cases, we differentiated fractional curves to calculate slopes for each saturation step:

$$f_w' = \left(\frac{df_w}{dS_w}\right). \qquad (14)$$

We calculated the water/oil ratio as the ratio of water rate to oil rate. Using Darcy's law for 1D, linear flow, we can express water and oil rate as follows:

$$q_w = -0.001127 \frac{k_w A}{\mu_w} \left[ \frac{\partial P_w}{\partial s} + 0.00694 \rho_w \sin\alpha \right] \quad (15)$$

and

$$q_o = -0.001127 \frac{k_o A}{\mu_o} \left[ \frac{\partial P_o}{\partial s} + 0.00694 \rho_o \sin\alpha \right], \quad (16)$$

so an expression for WOR will be:

$$\mathrm{WOR} = \frac{q_w}{q_o}, \quad (17)$$

or substituting Eq. 15 and 16 into Eq. 17, ignoring capillary pressure, assuming a horizontal reservoir, and solving, we have:

$$\mathrm{WOR} = \frac{k_{rw}}{k_{ro}} \frac{\mu_o}{\mu_w}. \quad (18)$$

Recovery factor and cumulative production can be expressed as dimensionless functions of average oil and connate water saturations. Using these values, we can generate the plot of log WOR vs. RF and analyze the SLZ.

$$\mathrm{RF} = \frac{N_P}{N} \quad (19)$$

where

$N$ = original oil in place, STB

or

$$\mathrm{RF} = \frac{1 - \overline{S_o} - S_{wc}}{1 - S_{wc}} \frac{B_o}{B_{oi}} \quad (20)$$

where $\overline{S_o}$ is average oil saturation, determined as:

$$\overline{S_o} = 1 - \overline{S_w} \; . \qquad (21)$$

Dimensionless PVI in fractional flow theory is calculated as the inverse of the slopes for each saturation step as shown in Eq. 14.

This PVI is related to the well's floodable pore volume and is equivalent to the ratio of the quantity of water injected to the total floodable pore volume associated to the well when no gas is present in the reservoir:

$$\mathrm{PV} = \frac{NB_o}{(1 - S_{wi})} \qquad (22)$$

where

$N$ = original oil in place, STB

$B_o$ = oil formation volume factor, RB/STB

$S_{wi}$ = irreducible water saturation, fraction

Since PVI is related to the well floodable pore volume, we assume an analysis on a well-by-well basis. As long as the producer is receiving the complete effect of the injector, regardless the injection scheme, the method will produce a good forecast since it works according to the fractional flow equation for homogeneous reservoirs only.

Since our basic calculations are dimensionless, we can use the processing rate (PVI/year) and the following equations to calculate reservoir performance in terms of production rates and cumulative production in barrels:

$$t = \frac{\mathrm{PVI}}{\mathrm{PVI} / y} \qquad (23)$$

where

$t$ = time, years

PVI = pore volume injected, fraction

PVI/$y$ = Pore Volume Injected per year

$$i_w = \frac{W_i - W_{i-1}}{\Delta t} \quad \text{(24)}$$

where

$i_w$ = water injection rate, STB/day

$W_i$ = total water injected at the evaluated time step, STB

$W_{i\text{-}1}$ = total water injected at previous time step, STB

$\Delta t$ = time difference, days

$$W_i = \frac{\mathrm{PVI} \times \mathrm{PV}}{B_w} \quad \text{(25)}$$

where

PV = reservoir pore volume, RB

$B_w$ = water formation volume factor, RB/STB

$$q_o = \frac{i_w \; B_o}{\mathrm{WOR} \times B_w + B_o} \quad \text{(26)}$$

where

$q_o$ = oil rate, STB/D

$B_o$ = oil formation volume factor, RB/STB

and

$$q_w = \frac{(i_w B_w) - (q_o B_o)}{B_o} \quad (27)$$

where

$q_w$= water rate, STB/D

The assumptions and limitations of this module are the following:

1. Buckley-Leverett displacement, 1D linear flow
2. Horizontal, homogeneous, isotropic reservoir
3. Incompressible fluids
4. Initial gas saturation is zero
5. Neglect capillary pressure (*Pc*)

Flow performance at and after breakthrough for recovery factors and water/oil ratio for each saturation step were calculated. All ranges and steps used in the homogeneous module to calculate each fractional flow case and each straight line are shown in Table 2.

Table 2—Parameters and ranges used in SAS to generate SLZs for homogeneous reservoirs in the homogeneous module.

| RANGES – FRACTIONAL FLOW | | | RUNS |
|---|---|---|---|
| Input data | Range | Steps | Combinations |
| Viscosity ratio | 0.1 to 1 | 0.2 | 5 |
| Oil exponent | 1 to 5 | 1.0 | 5 |
| Water exponent | 1 to 5 | 1.0 | 5 |
| Residual oil sat | 0.1 to 0.5 | 0.1 | 5 |
| Connate water sat | 0.1 to 0.5 | 0.1 | 5 |
| Oil relative permeability endpoint | 0.3 to 1.0 | 0.1 | 8 |
| Water relative permeability endpoint | 0.3 to 1.0 | 0.1 | 8 |
| Total runs (SLZ ) | | | 200,000 |

Our homogeneous module performs all the fractional flow calculations. A total of 200,000 different runs were made with SAS to generate the same number of SLZs for homogeneous cases.

All the SLZs were correlated, and general correlations for all the possible combinations of reservoir and fluid properties shown in Table 2 were obtained.

After all SLZs from the statistical model and all correlations for the homogeneous module were obtained, we needed to validate these results with reservoir simulation results and ensure that the SLZ was also present in simulation results and that its characteristics were reasonably similar to those estimated with the Buckley-Leverett 1D procedure. If results were consistent and similar in terms of RF at the end of the SLZ, and in terms of slopes and intercepts, we would be able to continue using reservoir simulation to estimate SLZs for different VDPs and build our heterogeneous module.

Fractional flow is a 1D model, where areal and vertical sweep efficiencies are assumed to be unity. In a reservoir simulator, we built a 3D model with an injection scheme of a 5-spot pattern. Comparing 1D model results with 3D model results would not be consistent, and 1D model results will overestimate recovery. However, we analyzed SLZ results from simulation for homogeneous cases (VDP = 0), and checked that areal sweep efficiency at the end of the SLZ was already unity in all cases. From simulation results, EA at the breakthrough was between 0.70 and 0.72.

Since our objective in this comparison (1D vs. 3D) was only to test the existence of the SLZ and not to produce RF estimates including VDP (we developed correlations for heterogeneous reservoirs later) we recognized our results as consistent, concluding that the SLZ existence was theoretically proved.

We compared fractional flow and reservoir simulation results in terms only of ultimate recoveries (recovery at the end of the SLZ), and slopes and intercepts, just to ensure that SLZs obtained from simulation were consistent with those obtained from fractional flow theory—in other words, just to be sure that the SLZ s was present in all cases and that the SLZs finished at the same points (same WOR and same RF), and they did. Consistency in results is shown later in Chapter III.

Thus, reservoir simulation models were built to validate the SAS results, for *M* of 0.6 to 5, for water- and oil-wet reservoirs, and for combinations of other variables. We made 720 runs for the homogeneous reservoir cases using reservoir simulation. These runs are summarized in Table 3.

Table 3—Variables and ranges used to generate reservoir simulation cases for homogeneous reservoirs and validate results from our new correlations.

| RANGES – SIMULATION RUNS | | | RUNS |
|---|---|---|---|
| Input Data | Range | Steps | Combinations |
| Mobility ratio | 0.6 to 5 | 1 | 5 |
| Oil exponent | 2 to 5 | 3 | 2 |
| Water exponent | 2 to 5 | 3 | 2 |
| Residual oil sat | 0.0 to 0.5 | 0.25 | 3 |
| Connate water saturation | 0.25 to 0.5 | 0.25 | 2 |
| Oil relative permeability endpoint | 0.3 to 1.0 | 0.7 | 2 |
| Water relative permeability endpoint | 0.3 to 0.5 | 0.1 | 3 |
| Total runs (SLZ ) | | | 720 |

All straight lines were correlated together to obtain the final expressions for the slope- and-intercept generalized correlations. Water saturation steps for all calculations are taken as $\Delta S_w = 0.001$.

**Analysis of the log WOR vs. RF Plot**

The application of the fractional flow equation and the frontal advance theory permits us to estimate waterflood performance, including the construction of the log WOR vs. RF plot.

We propose that calculating slopes and intercepts for all possible log WOR vs. RF plots from our correlations and generating a statistical solution for slopes and intercepts as a function of mobility and viscosity ratios, endpoints of oil and water relative permeability curve, Corey's function exponents for oil and water, and oil and water saturations, we can predict ultimate recovery and PVI for any combination of reservoir and fluid properties with a single equation for homogenous reservoirs, and we can use simulation runs to generate correlations to calculate RF and PVI for heterogeneous reservoirs as well, using different VDP and $M$ ranges.

Prior to determining the correlations, we estimated the extent of the SLZ in a general form. For that purpose we used the definition of a straight line knowing that the slope (first derivative) of a line is constant and its second derivative is zero. We computed the first and second derivatives numerically and excluded points that had second derivatives greater than |0.01| using an automated instruction on the computer code. Our set of correlations provides the RF at the end of the SLZ, when the second derivative is greater than |0.01| and advises the user to avoid extrapolations of the SLZ and overestimation of reserves. Fig.3 shows an example plot of the first derivative vs. RF. The first derivative is called DWOR in the plot, which is expressed as:

$$\mathrm{DWOR} = \frac{d\log(\mathrm{WOR})}{d(\mathrm{RF})} \quad \text{.......} \quad (28)$$

Using this set of correlations, we can forecast the expected behavior of the waterflood performance for a homogeneous reservoir in terms of the SLZ by predicting the curves' slopes and intercepts as functions of the variables defined earlier. Once each slope and intercept is calculated, recovery factors may be determined using the straight-line properties as shown in the next section.

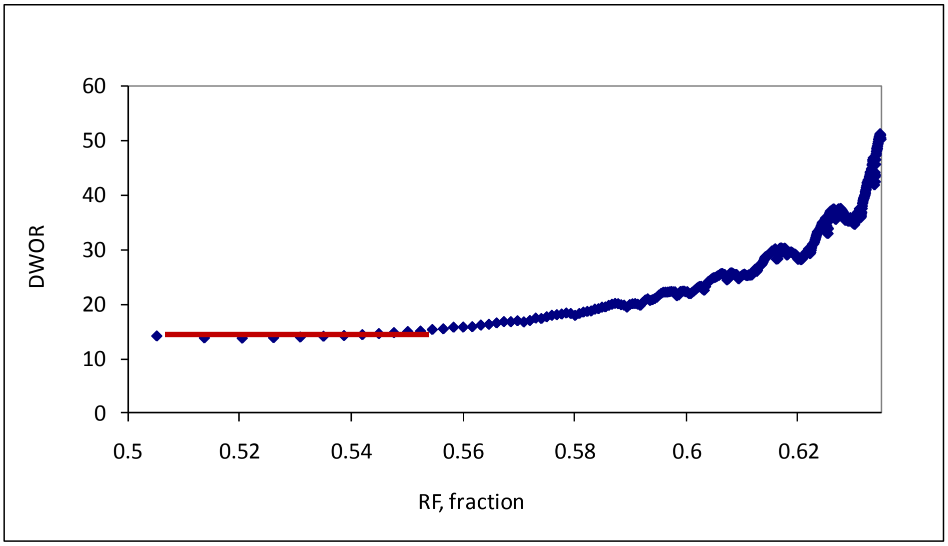


Fig.3—Plot of the first derivative of WOR with respect to RF showing the constant slope zone. RF range from 0.5 to 0.57 indicates the boundaries for the SLZ.

Applying the fractional flow theory and the frontal advance equation, we can forecast the expected behavior of the log WOR vs. RF plot for a homogeneous reservoir in terms of the SLZ by predicting the curves' slopes and intercepts and using only the same five rock and fluid parameters. In the next section, we present the statistical principles used to develop the correlations for any homogeneous reservoir using simple linear and multiple regressions. We also calculate water saturation at the water breakthrough ($S_{wBT}$), water cut at the breakthrough ($f_{wBT}$), and PVI correlations and represent them in tables and graphically. We can determine PVI for any RF provided that both variables are within the SLZ.

**Multiple Linear Regression Application**

Regression analysis establishes a mathematical relation between two or more variables that can determine a response, or set of dependent variables, from a given set of independent (or control variables). This statistical technique is useful to construct correlations for predicting trends of physical phenomena.

Using multiple linear regressions, we can analyze linear relationships among several variables, determining the slope and the intercept (numbers also called *regression coefficients*) of a straight line. The mean value of the data will be a function of the predictor (independent variable, usually plotted in the $x$ axis in a graph and called $x$). We need to describe the expected value of the mean of that independent variable, called $Y$, as the mean value of the function plus a random error term, as:

$$Y = \beta_o + \beta_1 x + \varepsilon \text{ , } \qquad (29)$$

where

$\beta_o$ = intercept (unknown regression coefficient)

$\beta_1$ = slope (unknown regression coefficient)

$x$ = independent variable, or "regressor"

$\varepsilon$ = random error term

When the set of correlations has only one regressor, it is called a "simple linear regression model." In this research, since we are using several regressors to understand the SLZ behavior, we will be using a "multiple linear regression model."

Technically, if we assume normal distribution of errors and that the mean and the variance of the random error term ε, are 0 and $\sigma^2$ respectively, we can say:

$$E(Y, x) = \beta_o + \beta_1 x\,, \quad \text{.......} \quad (30)$$

and

$$V(Y, x) = \sigma^2\,, \quad \text{.......} \quad (31)$$

with $E$ being the expected value of the mean of $Y$ for each $x$, and $V$ the variance of $Y$ with respect to $x$.

So the regression model can be defined as:

$$\mu_{Y,x} = \beta_o + \beta_1 x\,. \quad \text{.......} \quad (32)$$

The model is a line of mean values where the height of the regression line at any value of $x$ is the expected value of $Y$ for that $x$. The slope $\beta_1$ is the change in the mean of $Y$ for a unit change of $x$, and the variability of $Y$ at a particular value of $x$ is defined by the error variance $\sigma^2$. So there is a distribution of values of $Y$ at each $x$ value, and the variance of this distribution is the same at each $x$. If $\sigma^2$ is small, the observed values of $Y$ will fall close to the straight line. Is $\sigma^2$ is large, the observed values may deviate too much from the straight line. Since $\sigma^2$ is constant, the variability of $Y$ at any value of $x$ is the same (Montgomery and Runger, 2007).

In real-world problems, the values of slopes, intercepts, and error variance must be estimated from data, and future observations of $Y$ from possible values of $x$ are estimated using the statistical model.

Regression relationships are only valid for values of the regressor within the range of the original data, so large extrapolations are usually not accurate, or at least, more uncertainty is introduced in the process. For that reason, we developed a set of correlations using the SLZ boundaries to avoid extrapolations.

To estimate the unknown regression coefficients, or the slope and the intercepts for our log WOR vs. RF curves, we need to obtain the "best fit" of the data available, obtained from the application of the fractional flow equation and frontal advance theory, which are the first results from our SAS correlations.

Estimating these parameters is easy if we minimize the sum of the squares of the vertical deviations of the variance for each data set. This is the method of the "least squares."

If we use Eq. 29 to express $n$ observations in the sample, we may define the following equations to determine the least-squares estimates as:

$$\hat{\beta}_o = \bar{y} - \hat{\beta}_1 \bar{x} \quad \text{.......... (33)}$$

and

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} y_i x_i - \frac{\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} x_i\right)}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}, \quad \text{.......... (34)}$$

where

$$\bar{y} = \left(\frac{1}{n}\right)\sum_{i=1}^{n} y_i \quad \text{.......... (35)}$$

and

$$\bar{x} = \left(\frac{1}{n}\right)\sum_{i=1}^{n} x_i \text{ .......... } (36)$$

So using these equations, we can determine the fitted or estimated regression line as:

$$\bar{y} = \hat{\beta}_o + \hat{\beta}_1 x \text{ .......... } (37)$$

and the residual, which describes the error in the fit of the correlations to the $i^{th}$ observation $y_i$ and helps to determine the adequacy of the fitted model is described by

$$e_i = y_i - \hat{y}_i \text{ , .......... } (38)$$

which is used to estimate $\sigma^2$ as

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}{n-2} \text{ . .......... } (39)$$

Application of these formulas is really tedious, particularly for multiple regressions, and the possibilities for mistakes are really high, but they are all built into the SAS system and are the basis of our approach.

Using SAS we not only determined the best fit but also the estimated standard error of the slope and the intercept, which provides good guidance to examine the accuracy of the correlations.

Another important parameter to be taken into account is the $P_{-value}$, which is the lowest value that would lead to rejection of the null hypothesis $H_o$ of a given data, if $H_1$

is the hypothesis to be tested. In other words, the $P_{\text{-value}}$ is the probability that the test statistic will take on a value that is at least as extreme as the observed value of the statistic when the null hypothesis is true. The $P_{\text{-value}}$ gives information about the weight of evidence against $H_o$ and it allows us to make conclusions about the level of significance. In summary, the data used will be significant if the null hypothesis is rejected. Based on that, we may give a clearer definition of $P_{\text{-value}}$, saying that this is the smallest level at which the data is significant. The $P_{\text{-value}}$ must be as low as possible so we can ensure a good fit of the model.

The $P$-$_{\text{value}}$ is computed with the expression:

$$P_{-\text{value}} = 1 - P(x_1 < \overline{X} < x_2), \quad (40)$$

where

$P$ = probability

$x_1$ = lower value of the range analyzed

$x_2$ = higher value of the range analyzed

$\overline{X}$ = mean of the distribution

If the probability that the mean value within the observed values is high, 1-$P$ will be small; therefore, our correlation is making good predictions.

The $P_{\text{-value}}$ is calculated for each regressor, and a high $P_{\text{-value}}$ will mean that that specific regressor is not necessary to improve the accuracy of the correlation. Good regressors will show $P_{\text{-values}}$ less than 0.0001.

We used the $P_{\text{-value}}$ as our main indicator for significance for each parameter estimator, since a low value ($P$<0.0001) tells us that the slope and the intercept are good values. We rejected any parameter estimator with a $P$-value over the degree of confidence used (95% or $\alpha$=0.05).

Finally, we used the correlation coefficient, or coefficient of determination, $R^2$, which is the square of the correlation coefficient between $x$ and $Y$. This value may be dangerous, since it will increase artificially if more terms are added to the correlations, although some of these terms may exhibit low standard errors but may not be significant or necessary. In this case, their inclusion will increase the error. We used the adjusted $R^2$, which increases only if a new variable reduces the error mean square. This indicator penalizes the user for adding unnecessary terms to the correlations, trying to over-fit the model.

Further applications and details regarding these estimators can be found in Montgomery and Runger (2007) and Bain and Engelhardt (1992).

In this research we wanted to measure the effects of several parameters on the determination of the slope and intercept of the general correlations, so we now needed to consider the multiple linear correlation procedure.

For a multiple regression model, we built our correlation using a relationship with the following form:

$$Y = \beta_o + \beta_1 x_1 + \beta_2 x_2 + .....\beta_n x_n + \varepsilon \text{ , } \quad (41)$$

where the subscripts 1 to $n$ represent the number of variables used as regressors. This model describes a hyperplane in the $n$-dimensional space of the regressor variables. The multiple regression model developed here is used as an approximating function since the true functional relationship between Y and $x_1$, $x_2$, $x_3$, …, $x_n$ will remain unknown, but over an adequate range of independent variables, the linear regression model is an adequate approximation.

Our correlations include interaction effects, represented as cross-products of two or more variables, where applying the linear regression principles requires a variable transformation.

The regression coefficients can also be estimated from the method of least squares, where the parameter solution vector is obtained from a matrix of equal numbers of rows and columns.

The next step was to determine which variables were good for the correlations. We first selected those variables that were used to perform the fractional flow calculations. Based on the initial correlation coefficients (around 0.86), we tested some transformed variables, using those that intervene in the calculation of other variables such as mobility ratio and relative permeability curves, (for example, oil viscosity divided by Corey's oil exponent), until we obtained good initial $R^2$ results (above 0.9). However, we needed to improve the correlations methodically, so we used some built-in SAS procedures.

Four procedures are available: the stepwise regression, the forward selection, the backward elimination, and the *R*-squared selection. Use of the four methods is recommended so the analyst can compare results and make the best choice regarding which variables better fit the new model (Montgomery and Runger, 2007).

We used step-wise regression to build a sequence of regression models by adding and removing variables one at a time, finding the highest $R^2$ and the smallest $P$-value that define the strongest model.

The forward selection method will add variables like in the previous method, trying to maximize $R^2$, but will not eliminate added variables.

The backward elimination method begins with all variables included and will delete the worst regressors one at a time.

The $R$-squared method selects the combination of variables that reaches the highest correlation coefficient ($R^2$).

We ran all four methods and also checked our correlations for the lowest $P$-value for each statistic and with the covariance matrix (see outputs in Appendix G).

Correlation coefficients measure the extent of the association between two variables. Each such coefficient must lie between -1 and +1, inclusive. A positive coefficient indicates a positive association: a greater-than-expected outcome for one variable is likely to be associated with a greater-than-expected outcome for the other, while a smaller-than-expected outcome for one is likely to be associated with a smaller-than-expected outcome for the other. A negative coefficient indicates a negative association: a greater-than-expected outcome for one variable is likely to be associated with a smaller-than-expected outcome for the other, while a smaller-than-expected outcome for one is likely to be associated with a greater-than-expected outcome for the other. A coefficient of zero indicates no correlation at all.

The covariance matrix in our analysis indicated that all the variables could be used to develop the different correlations, but the variable selection procedures helped us to define the improvement of $R^2$ we can obtain with each variable. Both procedures were complementary.

The determination of influential points (outlier observations that do not improve the model and a few values too far from the mean) and possible variable transformations were studied.

All possible combinations for the input variables used were run and analyzed using variable selection procedures in SAS. Each SLZ boundary was defined, and for each SLZ, a slope and an intercept were obtained. All correlations presented a $R^2$ value around 0.99 with a maximum difference of about 2%.

Once each slope and intercept is calculated, recovery factors may be determined using the straight line properties as:

$$\log \mathrm{WOR} = RF \times m + I \quad \text{.......................} \quad (42)$$

where $m$ is the slope and $I$ is the intercept.

Also, we can define these slopes and intercept as $m = f_1(M, n_o, n_w, S_{wc}, S_{or}, k_{roe}, k_{rwe})$ and $I = f_2(M, n_o, n_w, S_{wc}, S_{or}, k_{roe}, k_{rwe})$. Eq. 42 can be used to determine RF, and solving for RF we have:

$$RF = \frac{Log\mathrm{WOR} - I}{m} \quad \text{.......................} \quad (43)$$

Therefore, to obtain the calculated recovery factor, only the WOR value was necessary as an input, since the correlations will give the boundaries of the SLZ from the second derivative analysis described before and the maximum WOR value to keep the RF results within the SLZ boundaries.

However, our objective was to obtain a generalized correlation to calculate *unique* slope and intercept expressions as functions of the previously determined variables to be used in all cases, so all obtained slopes and intercepts were correlated again to determine a general set of correlations to be used in any kind of reservoirs and conditions. Regressors used for correlations were viscosity ratio (VR), endpoint mobility ratio ($M$), Corey's oil and water exponents ($n_o$ and $n_w$), connate water saturation ($S_{wc}$), and residual oil saturation ($S_{or}$).

Many combinations were tested with the different correlation variables to optimize the correlation coefficient $R^2$ and obtain a value over 0.99 for the general correlation.

After analyzing and trying several variables that might affect the correlation coefficients, we realized that for combinations of $n_o$, $S_{or,}$ and $S_{wc}$ parameters, the correlation coefficients we obtained were not acceptable values ($R^2 < 0.80$). Therefore, we decided to separate the calculations and build two different correlations for water- and oil-wet rocks, suspecting that the oil-wet rocks would not be able to produce a good correlation because of the wettability effects in sweep efficiency discussed above. We analyzed the wettability impact based on the effects of each calculated variable.

Using the rules of thumb published by Craig (1971) that consider rocks water-wet when relative permeability curves cross each other after water saturation values of 0.5, and oil-wet when the curves cross before that value, our correlations separate and correlate SLZs based on wettability. To consider each type of rock, we created two codes: One code performs calculations for water-wet rocks and the other for oil-wet rocks, based on the values of $S_{wc}$, $S_{or}$, $k_{rwe}$, $k_{roe}$, $n_o$, and $n_w$ that affect the point where the relative permeability curves cross each other. If they crossed after $S_w$ = 0.5, we considered the rock to be water-wet. If the crossing point was before $S_w$ = 0.5, we considered the rock to be oil-wet.

The correlation variable $x_1$ considering *VR* and $n_o$ is defined as:

$$x_1 = \frac{\mathrm{VR}}{n_o}, \quad (44)$$

where

VR = viscosity ratio ($\mu_{\mathrm{water}}/\mu_{\mathrm{oil}}$)

$n_o$ = oil exponent from Corey's function

We used the variables described earlier and the log of mobility ratio to generate the correlations for homogeneous, water- and oil-wet reservoirs, and to calculate $f_{wBT}$ and $S_{wBT}$, slopes and intercepts, for the general log WOR vs. RF plot. Tables in Chapter III present the SAS output for the obtained correlations.

Putting it all together, the partial set of correlations for a homogeneous reservoir has the form:

$$\log WOR = RF \times m(M, \mathrm{VR}, n_o, n_w, S_{or}, S_{wc}, k_{ro,} k_{rw}) + I(M, \mathrm{VR}, n_o, n_w, S_{or}, S_{wc}, k_{ro}, k_{rw}), \quad (45)$$

**Reservoir Simulation Model for Homogeneous Reservoirs**

A simple, synthetic reservoir simulation model was built to control and calibrate our correlations. A general description of the simulation model can be seen in Table 4 (see Appendix C).

Our simulation two-phase model (oil and water) has grid dimensions of 19 x 19 x 10 with 3,610 cells. The lengths are 20 ft in the $x$ and $y$ directions, and 10 ft in the $z$ direction. It is ¼ of a 5-spot pattern with one producer and one injector, completed in all layers. Production and injection rates are constant at 2,000 STB/D. The production control method is by reservoir voidage, and the waterflood strategy is pressure maintenance. The solution method is fully implicit. OOIP, PV, and production and injection in barrels need to be multiplied by four to obtain the total pattern area values.

Reservoir and fluid properties were changed and tested according to the ranges shown before in Table 2, corresponding to each run evaluated in our new correlations. Fig. 4 shows a 3D screen example for the homogeneous reservoir model.

Table 4—Generalized model description for the reservoir (homogeneous and heterogeneous cases).

| PROPERTY | VALUE/DESCRIPTION |
|---|---|
| Grid dimensions, number (x,y,z) | 19, 19, 10-50 |
| Grid size, cells | 3610 |
| *Dx, Dy, Dz*, feet | 20, 20, 10 |
| Layers | 10 |
| Number of wells (*i/p*) | 1/1 |
| Producer completions | 19 19 1-10 (all layers) |
| Injector completions | 1 1 1-10 (all layers) |
| Production rate, BBL/D (not critical) | 2000 |
| Injection rate, BBL/D (not critical) | 2000 |
| Production control methods | Reservoir voidage (RESV) |
| Waterflood strategy | Pressure maintenance |
| Relative permeability curves | Variable – Corey's functions |
| Solution method | Implicit (AIM) |
| Porosity, fraction | 0.3 |
| Ave layer permeability, md | 200 |
| Phases(oil and water) | 2 |
| Water viscosity, cp | Variable |
| Oil viscosity, cp | Variable |
| *Sor,* fraction | Variable |
| *Swi,* fraction | Variable |
| *kv/kh* | 0.1-1 |
| Pattern | 5-spot |
| Reservoir pressure, psi (constant) | 3665 |

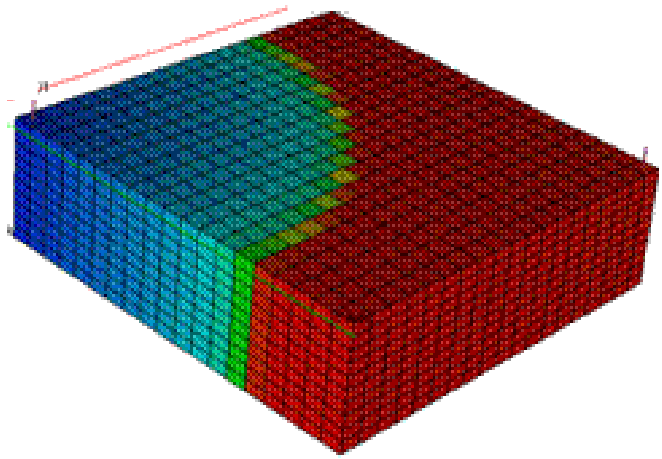

Fig. 4—Example of an oil saturation profile for a homogeneous reservoir at time 500 days, showing effective sweeping.

Once our correlations from the homogeneous module were determined, we applied those correlations to the same input we used in the simulation runs for each corresponding SLZ case to compare ultimate recovery results using both approaches for homogeneous reservoirs. Comparisons are shown in Chapter III.

To determine if a linear relationship existed between the two recovery factor calculations, we also compared ultimate recovery factor results (at the end of the SLZ) from the homogeneous module with the ultimate recovery factors estimated from the end of the SLZ obtained from the simulation results.

As we can see in the example shown in Fig. 5, we obtained straight lines in all cases. These lines however, had different slopes, intercepts, and lengths than those of corresponding homogeneous cases.


Fig. 5—Comparison of ultimate recovery factors for a homogeneous case vs. the same reservoir including VDP = 0.9 (including heterogeneity). A linear behavior is present. The case for VDP = 0.9 presents lower RF.

### Reservoir Simulation Model for Heterogeneous Reservoirs

Since the fractional flow equation applies for homogeneous reservoirs only, we used the reservoir simulation model to generate results with different VDP values and correlated them linearly with their correspondent model's results for the homogeneous reservoir case for input data set. The observations included the whole length of each SLZ.

Permeability values were changed to provide different VDP cases and a new data file was built with recovery factors, PVI, and log WOR values calculated for different *M*.

Fig. 6 is a 3D capture of the new model. See details of data files in Appendix D.

Heterogeneous module results will be discussed in Chapter III and in the module for heterogeneous reservoir section, later in this chapter. Runs for heterogeneous cases are shown in Fig. 7 and Fig. 8.

Injection expressed as PVI has an economic significance, since maximized recovery with a minimum of water injected will maximize net present value (NPV).

In this research, even when the log WOR vs. RF curve is independent of the injection rate, we wanted to understand the effect of different water injection rates on the ultimate recovery, so several exercises were run, in different scenarios of homogeneous and heterogeneous systems, with favorable and unfavorable mobility ratios.

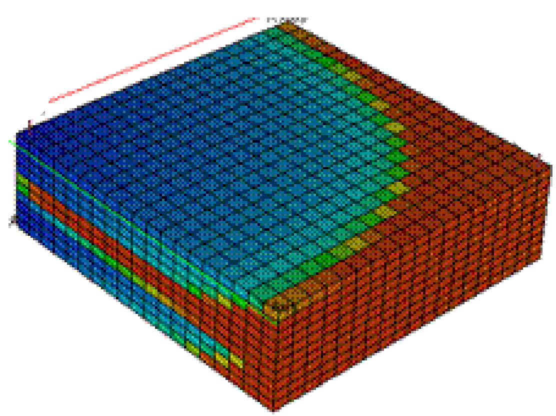

Fig. 6—Oil saturation profile for a heterogeneous reservoir with a VDP = 0.9. This figure shows irregular displacement at 500 days.

In Appendix H we include simulation run results where total fluid production and injection rates for each case were kept constant during the simulation runs to maintain reservoir pressure. Different scenarios were analyzed to determine if injecting more water per unit of time would affect the ultimate recovery and the correlations to be determined for heterogeneous reservoirs. Injection rates do not affect results in terms of plots of RF vs. PVI or log WOR vs. RF. The only important effect we found is the acceleration of recovery when higher rates are used.

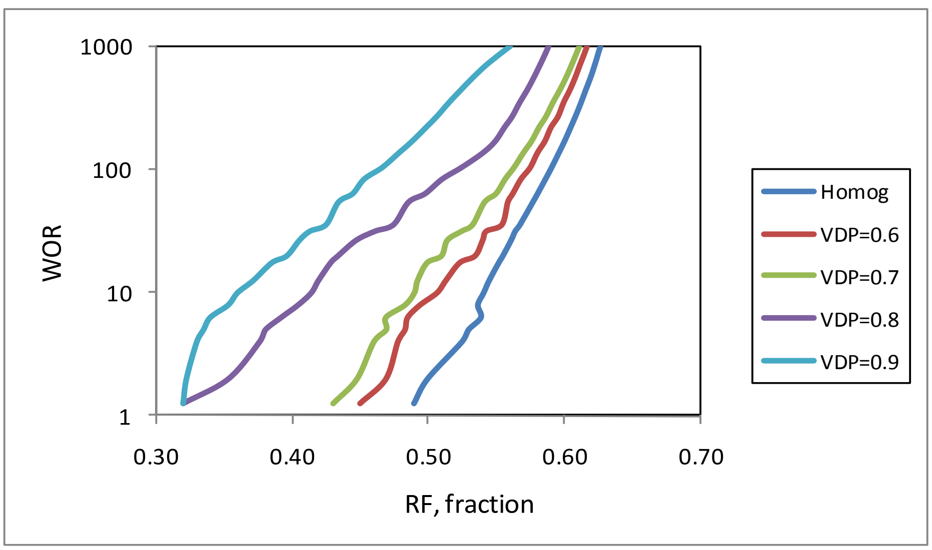


Fig. 7—Example of SLZs observed from reservoir simulation for different degrees of heterogeneity expressed by the Dykstra-Parsons coefficient. These results were correlated with our correlations for homogeneous reservoirs to generate the heterogeneous correlations for RF.

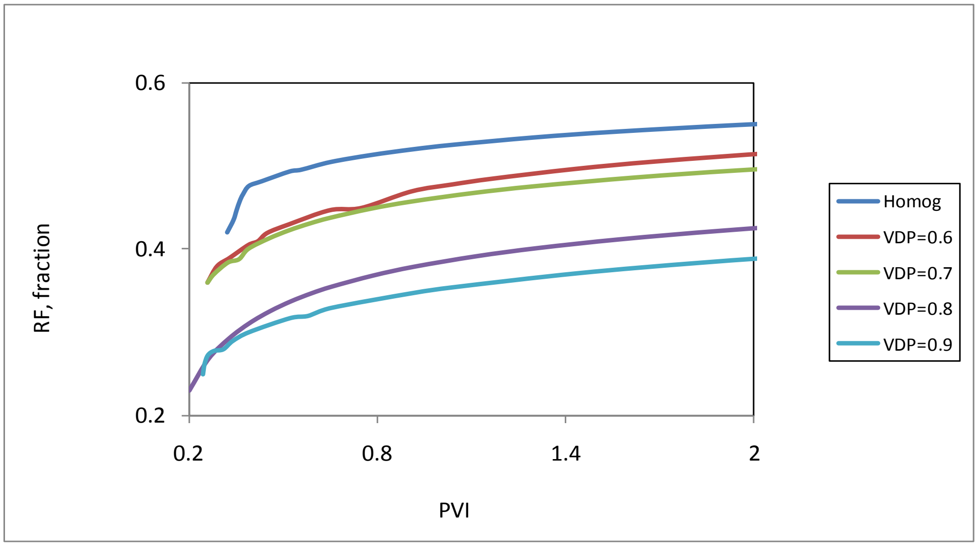


Fig. 8—Example of PVI observed from reservoir simulation for different degrees of heterogeneity expressed by the Dykstra-Parsons coefficient. These results were also correlated with our correlations for homogeneous reservoirs to generate the heterogeneous correlations for PVI.

For a homogenous reservoir, in the presence of a favorable *M*, we expected that increasing the injection and production rate [using a higher processing rate, but maintaining a voidage replacement ratio (VRR*)* close to one] should not affect the ultimate recovery, so an acceleration of production instead of an increase of reserves would take place in the field. However, we wanted to investigate if the higher probability of water fingering and oil left behind by high injection rates in the presence of a high VDP and/or an unfavorable M would lead to different results.

The same reservoir model used for each of the heterogeneous cases was used to determine the effect of different injection rates in the waterflood performance. We used a base injection rate of 200 STB/D as the lower rate, and 2,000 STB/D (one order of magnitude higher) as the higher rate.

Responses for all analyzed cases were similar in terms of the recovery factors and PVI obtained. Injection and production rates, VDP, and $M$ were changed, one at a time, and compared to determine possible impacts. The base case has the following combination of properties: $n_o$=3, $n_w$=5, $S_{or}$=0.2, $S_{wc}$=0.4, $k_{roe}$=0.9, $k_{rwe}$=0.55, VR=0.06 (for $M$=10) and VR=0.6 (for $M$=1). Cases with the same values of VDP and mobility ratio were compared to each other to isolate the effect of injection rate change. We needed to avoid including the already known heterogeneity and mobility ratio effects in this analysis. Keeping all other parameters unchanged, we changed the injection rates and compared recovery factors with PVI and log WOR values.

We can conclude that the most important effect of a higher injection rate, assuming no injectivity problems, is the acceleration or delay in production (see higher recovery for the same period of time for injection rate of 2,000 STB/D), which is not always favorable, because the impact of water injection costs must be taken into account.

However, at $M$ = 10 and VDP = 0.9, a higher recovery can be seen in the 200-B/D injection rate curve, a difference of less than 2%, if enough time is provided for that case. In this run, we allowed 135,000 days, which is an extremely long period of time for this kind of project.

After approximately 10 PVI, we can see recoveries of 0.414 for water injection rates ($i_w$) of 2,000 STB/D and 0.430 for water injection rates ($i_w$) of 200 STB/D. Also, for a recovery factor of 0.48 using $i_w$ of 200 STB/D, a log WOR of 3.0 is reached, while for the higher injection rate, a similar log WOR is obtained faster. (See Appendix H.)

In practice, at unfavorable *M* and high VDP, low injection rates may help to increase recovery and avoid the risk of water fingering, applying a sound reservoir management strategy. PVI would be smaller in time, so economics can also be improved.

The difference in recoveries using different injection rates may not be considered a limitation of the new procedure presented here, since the deviation is small and the impact is reached after very large amounts of water have been injected and when the WOR is away from the SLZ.

For areal heterogeneity, lower injection rates may be beneficial, but we will consider this issue as a design matter, due to the high degree of uncertainty and complexity involved.

The effect of high heterogeneity and mobility ratio are manifested in the lower recovery and earlier breakthrough times. Rate dependency of water coning was not studied.

Another variable we wanted to evaluate is the effect of the number of layers on the new correlation's results.

We used the same approach of comparing simulation results for different layer models, and we determined that no effect was important enough to consider. Ultimate recovery factors (URF) and slopes and intercepts of SLZ had no change. We used VDP

= 0.9 and $M$ = 10 to account for unfavorable effects and ran cases for 10 and 50 layers (see Appendix H for more details).

Another important variable we wanted to assess was the effect of crossflow in the recoveries of waterflood performance.

Craig (1971) discussed some experimental approaches regarding incremental recovery due to crossflow between layers. For homogeneous systems and at favorable mobility ratios ($M$), the results show that only a very small amount of additional recovery is obtained in those experiments. At unfavorable $M$, the same crossflow will produce less recovery than the one obtained when no or lower crossflow is present. As expected, high $M$ will produce fingering and bypassed oil in the rock that will reduce recovery. Under this situation, crossflow will hold back the advance of the front in the low-permeability layers.

At favorable $M$, crossflow values will tend to increase recovery efficiency, but at unfavorable $M$, efficiency will be reduced.

Thakur and Satter (1998) analyzed several crossflow levels in terms of vertical to horizontal permeability ratios. Simulation shows that ultimate recovery increases slightly with crossflow, about 0.5%; PVI may be slightly lower and breakthrough may occur later than in no-crossflow scenarios.

Even when these conclusions seem to be enough to understand the phenomenon, we wanted to use our reservoir simulation model to evaluate the effect of heterogeneity and the presence of "thief zones" in the system and determine if those situations could limit the applicability of our correlations. We performed runs for several scenarios to model crossflow effects. We used $M$ = 1.0 since the effects of fluctuations in that parameter had been discussed in previous works.

We used the same data file presented in Table 4 but we changed values for $k_v$ and $k_h$. Crossflow values ranged from 0 to 1. The value selected for $M$ was 1.0. Again, no important effect was found when considering crossflow in the reservoir. Results and some published references are shown in Appendix H.

For a homogeneous reservoir with a favorable $M$, crossflow will not affect performance or ultimate recovery. For the case of high VDP but low- to moderate-permeability values in all layers, the difference in oil and water rates, breakthrough time, and ultimate recovery will not affect estimates from our correlations with the log WOR vs. RF plot. However, when high-permeability layers are present, even when the VDP may be not too high, the system with no crossflow will present an earlier breakthrough in "thief zones," a higher watercut in early times, delayed production peak, and possibly a negative effect on the net present value due to the delayed production and water cycling, together with some oil left behind in the rock, especially in low-permeability zones.

Thief zones will reduce vertical sweep efficiency, so gel slugs may be used in the wellbore to divert flow and reduce permeability on thief zones, and/or to isolate thief zones. Water will bypass oil in high-permeability layers with high $M$.

**The Module for Heterogeneous Reservoirs**

The new correlations for heterogeneous reservoirs were built using multiple linear regression to create specific correlations for heterogeneous cases. Correlations were developed for VDP, *M*, RF, and WOR obtained from the simulation models for heterogeneous reservoir cases, for two scenarios of water-wet rocks (extreme and medium), two scenarios of oil-wet rocks, and their corresponding recovery factors obtained from the homogeneous reservoir model. Table 5 shows the variables and ranges used to estimate a total of 144 SLZs.

Table 5—Variables and ranges used to generate simulation cases for heterogeneous reservoirs using 2 scenarios for water-wet rocks and 2 scenarios for oil-wet rocks.

| RANGES | | | RUNS |
|---|---|---|---|
| Variables | Range | Steps | Number |
| M (water-wet) | 0.6 to 5 | 1 | 12 |
| VDP (water-wet) | 0.5 to 1 | 0.1 | 6 |
| Total cases (SLZ ) water-wet | | | 72 |
| M (oil-wet) | 0.6 to 5 | 1 | 12 |
| VDP (oil-wet) | 0.5 to 1 | 0.1 | 6 |
| Total cases (SLZ ) oil-wet | | | 72 |
| Total runs (SLZ ) | | | 144 |

We used transformed variables to improve correlation coefficients, and determined the best correlation variables using “Stepwise”, “Backward”, and “Forward” variable analysis procedures.

Two multiple linear correlations were developed, one for water-wet systems and other for oil-wet rocks. These correlations do not use fractional flow theory for the heterogeneous cases, but calculate recovery factors and PVI for any value of VDP and $M$ using correlated results from the simulator model and fractional flow results from the homogeneous cases (see Appendix E).

The program generated the correlations using two data files, one for water-wet and another for oil-wet systems, which contains all data to be correlated for both sets.

Each data file had the following fields: mobility ratio, log WOR, recovery factor for homogeneous reservoir (output from the homogeneous correlations), PVI correspondent to the correlated homogeneous reservoir recovery factor, and recovery factors for VDPs of 0.5, 0.6, 0.7, 0.8, 0.9 and 0.99 with their corresponding PVI, all for each value of log WOR. (See Appendixes B and C for the programs and result tables and examples of data files, respectively.)

Mobility ratio and WOR are independent variables, or predictors, because they are usually known (measured or calculated); and we set recovery factor and PVI as dependent variables. The regressors we used are $M$ and log WOR, but we needed transformed variables to better fit the correlations, since the log WOR curve has an exponential behavior when plotted against PVI, and a linear behavior (in the SLZ) when compared to RF for each VDP case. To determine the heterogeneous cases' recovery factors, we defined the new transformed variables (regressors) as:

$$x_{11} = RF^2{}_{\text{hom}}, \quad \text{(46)}$$

where

$RF^2_{\text{hom}}$ = square of the correlated recovery factor for homogeneous reservoirs,

$$x_{12} = e^{LogWOR}, \quad (47)$$

and

$$x_{13} = 10^{\log\text{WOR}} \quad (48)$$

The correlations we obtained (Appendix B) all show $R^2$ higher than 0.99. Results and all variables are shown and discussed in the next chapter.

Fig. 9 shows the workflow we used to generate our correlations using SAS, reservoir simulation, and VBA. Fig.10 shows how the codes work for each module.

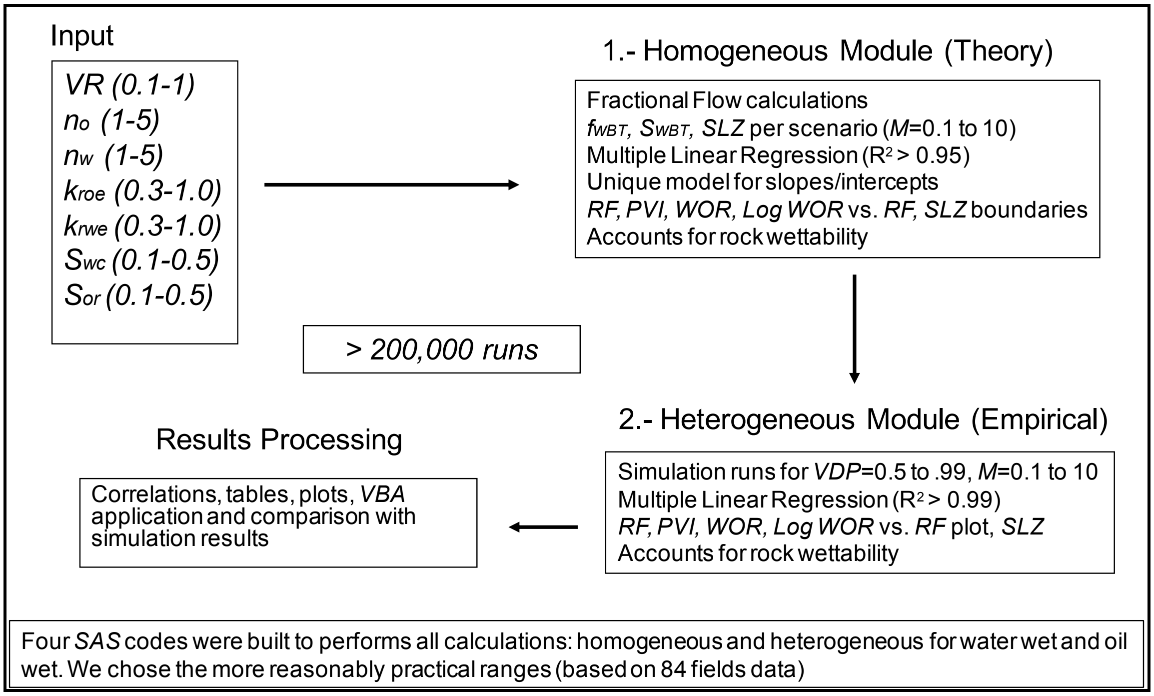


Fig. 9—Workflow used to generate our more than 200,000 runs for different cases, including correlations to estimate boundaries of the SLZs.

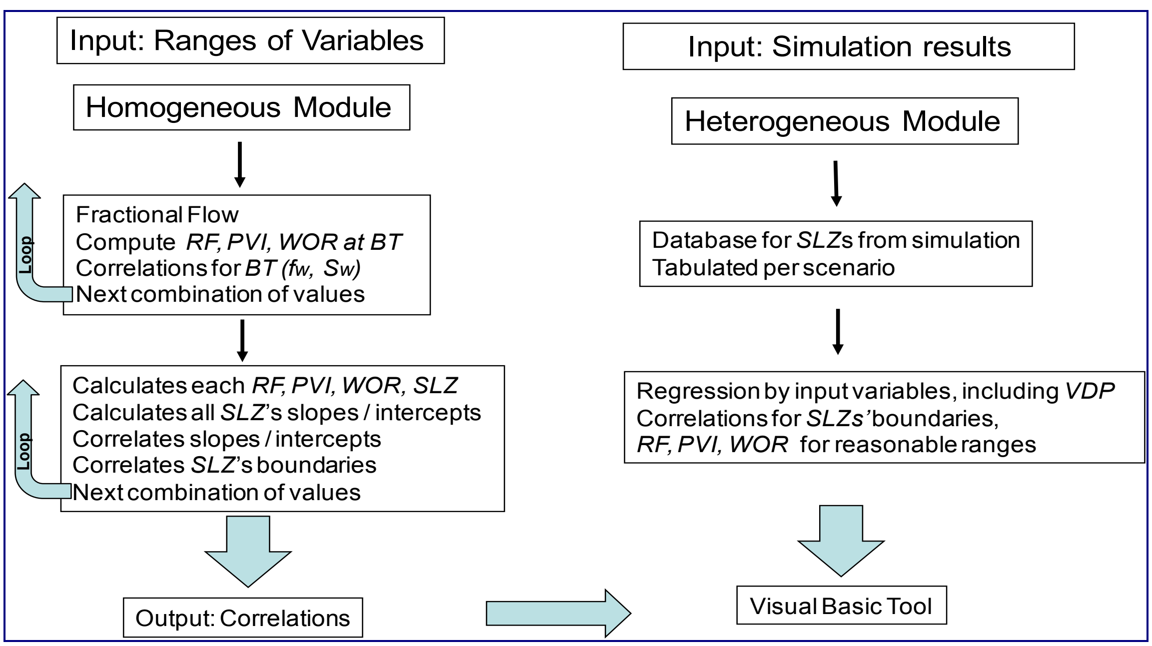


Fig.10—Structure and workflow of the regression codes.

Assumptions and limitations for the heterogeneous module are:

- Confined ¼ of a 5-spot pattern
- Multilayered reservoir
- No crossflow between layers
- Uniform permeability per layer
- Permeability $k$ changes with each layer
- Relative permeability curves are the same for all layers and are estimated using Corey-type equations
- Constant injection and production volumetric rates, $q_i = q_p$
- Initial $S_g = 0$

# CHAPTER III

# RESULTS

A new procedure and statistical correlations have been developed to predict oil recovery at any WOR and ultimate recovery for reservoirs under water injection, using the fractional flow equation and multiple linear regressions. We developed correlations to estimate the slopes and intercepts of the SLZs in the log WOR vs. RF plots for all reasonable combinations of $S_{or}$, $S_{wc}$, $n_o$, $n_w$, $M$, and VR, including the SLZ boundaries for homogenous reservoirs, and also correlations to estimate recovery for heterogeneous reservoirs within a range of VDP from 0.5 to 0.99 and $M$ from 0.1 to 10.

Limitations include that no dip angle, no capillary pressure, no initial gas saturation, and no gravity effects were considered. However, this methodology can be extended to include these cases and other special cases such as extra-heavy oil reservoirs and naturally fractured reservoirs.

After applying our correlations to cases with initial gas saturations from 1% to 16%, we observed that for waterfloods with small initial gas saturations ($S_{gi} < 0.04$), RF results are reasonably acceptable, with error less than 2% compared with recovery factors at the end of the SLZ found in simulation runs.

**SLZ Characteristics and Correlations**

The log WOR vs. RF plot shows an SLZ that is present in all studied cases. The correlations produced parameter estimates that multiply rock and fluid properties values used for regressions to calculate RF, WOR, and PVI for any value of VDP.

Typical behaviors relative to the effect of different values of rock and fluid properties in the SLZ are shown in Table 6.

Compensation can be seen if crossed effects. Descriptions of each variable and parameter estimates of correlations for homogeneous (ideal) cases are included in Table 7, Table 8 and Table 9. These correlations calculate water cut ($f_{wBT}$) and water saturation ($S_{wBT}$) at the water breakthrough and the slope and the intercept of each SLZ for homogeneous reservoirs. Parameters are slightly different for oil-wet rocks but having different correlations improves the correlation coefficients for both types of rock.

**Correlation for Homogeneous Reservoirs**

Four correlations were obtained for the homogeneous module to estimate the beginning and the end of the SLZ in terms of the minimum and maximum WOR corresponding to each SLZ boundary (WOR at the beginning and at the end of the SLZ), both estimated as functions of the input data used.

Table 6—Qualitative and relative comparison of the expected shapes and characteristics of the SLZ for different reservoir and fluid parameters combinations.

| PROPERTY CHARACTERISTIC | WATER BREAKTHROUGH | SLZ LENGTH | SLOPE | RF | WOR @ GIVEN RF |
|---|---|---|---|---|---|
| Water-wet rock | late | short | steep | high | low |
| Oil-wet rock | early | long | moderate | low | high |
| High viscosity ratio (μw/μo) | late | short | steep | high | low |
| Low viscosity ratio (μw/μo) | early | long | moderate | low | high |
| High VDP | early | long | moderate | low | high |
| Low VDP | late | short | steep | high | low |
| High mobility ratio | early | long | moderate | low | high |
| Low Mobility ratio | late | short | steep | high | low |
| High $S_{or}$ | early | long | moderate | low | high |
| Low $S_{or}$ | late | short | steep | high | low |
| High $S_{wc}$ | early | long | moderate | low | high |
| Low $S_{wc}$ | late | short | steep | high | low |

Table 7—Parameter estimates for homogenous reservoirs (water-wet rocks).

| GUIDE | | WATER-WET ROCKS | | | |
|---|---|---|---|---|---|
| Regressor | Variable | $f_{wBT}$ | $S_{wBT}$ | Slope | Intercept |
| Independent | Intercept | 0.71084 | 0.64301 | 1.29383 | -1.56680 |
| $x_1$ | VR/$n_o$ | 0.20212 | 0.23901 | 27.81540 | -30.39900 |
| $x_2$ | VR | -0.08234 | -0.10030 | -6.79110 | 8.83008 |
| $x_3$ | $n_o$ | 0.01779 | -0.0302 | 0 | 0 |
| $x_4$ | $n_w$ | 0.03932 | 0.03722 | 2.08358 | -1.50080 |
| $x_5$ | $S_{wc}$ | 0 | 0.34619 | 4.76521 | -0.10520 |
| $x_6$ | $S_{or}$ | 0 | -0.65380 | 22.31420 | -0.58340 |
| $x_7$ | log M | -0.09479 | -0.15480 | -4.7940 | 4.03107 |

Table 8—Parameter estimates for homogenous reservoirs (oil-wet rocks).

| GUIDE | | OIL-WET ROCKS | | | |
|---|---|---|---|---|---|
| Regressor | Variable | $f_wBT$ | $S_wBT$ | Slope | Intercept |
| Independent | Intercept | 0.65980 | 0.56187 | 0.77519 | -3.42540 |
| $x_1$ | VR/$n_o$ | 0.11888 | 0.16777 | 8.93439 | -11.94000 |
| $x_2$ | VR | -0.04844 | -0.06870 | -3.36470 | 4.78633 |
| $x_3$ | $n_o$ | 0.02099 | -0.02200 | 0 | 0 |
| $x_4$ | $n_w$ | 0.05127 | 0.04494 | 2.67364 | -1.41670 |
| $x_5$ | $S_{wc}$ | 0 | 0.41472 | 0 | -0.33000 |
| $x_6$ | $S_{or}$ | 0 | -0.58530 | 28.23280 | -0.06060 |
| $x_7$ | log M | -0.09342 | -0.14540 | -8.40360 | 5.30721 |

Table 9—Parameter estimates for PVI in homogeneous reservoirs. This correlations work for water and oil-wet rocks.

| REGRESSOR | VARIABLE | WATER-WET | OIL-WET |
|---|---|---|---|
| Independent | Intercept | -0.81246 | -0.86240 |
| $x_8$ | $M$ | 0.35196 | 0.41381 |
| $x_{12}$ | $e^{\log WOR}$ | 0.21624 | 0.24178 |
| $x_{13}$ | $10^{\log WOR}$ | 0.00800 | 0.00194 |

The correlations for maximum and minimum WOR to estimate the beginning and the end of the SLZ, for water-wet and oil-wet rocks in homogeneous reservoirs are the following.

Water-wet

$$\log WOR_{min} = 0.10547 + 0.31449(VR) + 0.08504(n_o) + 0.18375(n_w) + 0.56262(S_{or}) + 0.15521(S_{wc}) + 0.10024(k_{roe}) + 0.11893(k_{rwe}) - 0.01843(M) \quad (49)$$

$R^2 = 0.96$

$$\log WOR_{max} = 1.31556 - 0.24884(VR) + 0.60999(n_o) + 0.06739(n_w) - 0.66761(S_{or}) - 1.25442(S_{wc}) - 0.0454(k_{roe}) - 0.24884(k_{rwe}) + 0.06416(M) \quad (50)$$

$R^2 = 0.98$

Oil-wet

$$\log WOR_{min} = -0.10923 + 0.36546(VR) + 0.06644(n_o) + 0.18869(n_w) + 0.41633(S_{or}) + 0.17015(S_{wc}) + 0.23567(k_{roe}) - 0.04279(k_{rwe}) + 0.01746(M) \quad (51)$$

R2 = 0.96

$$\log WOR_{max} = 2.15118 - 0.85163(VR) + 0.68975(n_o) + 0.0229(n_w) - 1.14131(S_{or}) - 0.70038(S_{wc}) - 1.29377(k_{roe}) + 0.74267(k_{rwe}) - 0.12195(M) \quad (52)$$

R2 = 0.99

Different runs were made to test the correlation results and compare against simulator results to ensure that the fractional flow theory supported the procedure. Applying our correlations to the simulation results would support the quality of predictions and the results from the heterogeneous module. Correlations to estimate the end of the SLZ were always more accurate, presenting an error less than 2% compared to errors up to 4% in the beginning of the SLZ. However, determining the end of the SLZ was the main objective because we wanted to avoid overestimation of reserves and underestimation of the WOR through the extrapolation of the plot's curve.

Table 10 shows examples of input variables used to run this test. Table 11 shows the matches obtained for ultimate recovery, since at the end of the SLZ, $E_A$ is one in all results from the simulation runs.

Table 10—Reservoir data used for comparison.

| RESERVOIR | VR | $n_o$ | $n_w$ | $S_{or}$ | $S_{wc}$ | $k_{roe}$ | $k_{rwe}$ | M |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.0 | 3 | 5 | 0.20 | 0.40 | 0.90 | 0.55 | 0.6 |
| 2 | 1.0 | 2 | 2 | 0.50 | 0.25 | 0.90 | 0.90 | 1.0 |
| 3 | 0.06 | 3 | 5 | 0.20 | 0.40 | 0.90 | 0.55 | 10.2 |
| 4 | 0.2 | 2 | 2 | 0.50 | 0.25 | 0.90 | 0.90 | 5.0 |

Table 11—Comparison of results using correlations and simulation for the homogeneous module.

| RESULTS COMPARISON | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Water-wet, *M*=0.6 | | Oil-wet, *M*=1.0 | | Water-wet, *M*=10.0 | | Oil-wet, *M*=5 | |
| | Correlations | Simulator | Correlations | Simulator | Correlations | Simulator | Correlations | Simulator |
| RF | 0.62 | 0.63 | 0.41 | 0.40 | 0.57 | 0.57 | 0.31 | 0.30 |
| Log WOR | 2.3 | 2.3 | 1.7 | 1.7 | 1.7 | 1.7 | 2.0 | 2.0 |
| Slope | 22.5 | 22.0 | 21.0 | 23.0 | 15.4 | 15.4 | 15.0 | 15.3 |
| Intercept | -11.8 | -11.9 | -6.8 | -6.0 | -6.3 | -6.5 | -2.6 | -2.6 |
| PVI. | 2 | 2 | 1 | 1 | 4 | 4 | 3 | 3 |

Bigger differences in RF are obtained for oil-wet reservoirs but only are about 1% of ultimate recovery. Different wettabilities and *M* were run for two different homogeneous, synthetic reservoirs. The correlations matched the simulator, and trends for less recovery for unfavorable *M* are present. Fig.11 presents one of the examples in the form of an SLZ plot.

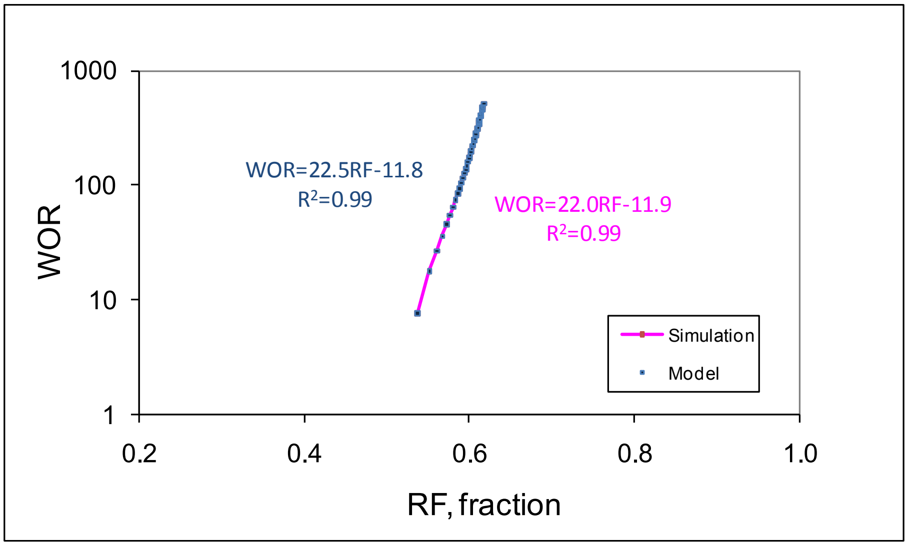


Fig.11—Comparison of SLZ estimated with our correlations almost perfectly matched one obtained from simulation results for a hypothetical water-wet, homogeneous reservoir (VDP = 0) and *M*=1.

Comparisons of results for homogeneous reservoirs were made for RF, slopes, and intercepts for four different reservoirs—with favorable and unfavorable mobility ratios and water-wet rocks, and with favorable and unfavorable mobility ratios but oil-wet rocks.

When comparing our correlations results with the reservoir simulation results, excellent matches were obtained with our new model results, as can be seen in Fig.12. In that figure, we compare only the ultimate recovery (RF at the end of the SLZ) obtained with our correlations with the one obtained from the SLZ in the corresponding simulation.

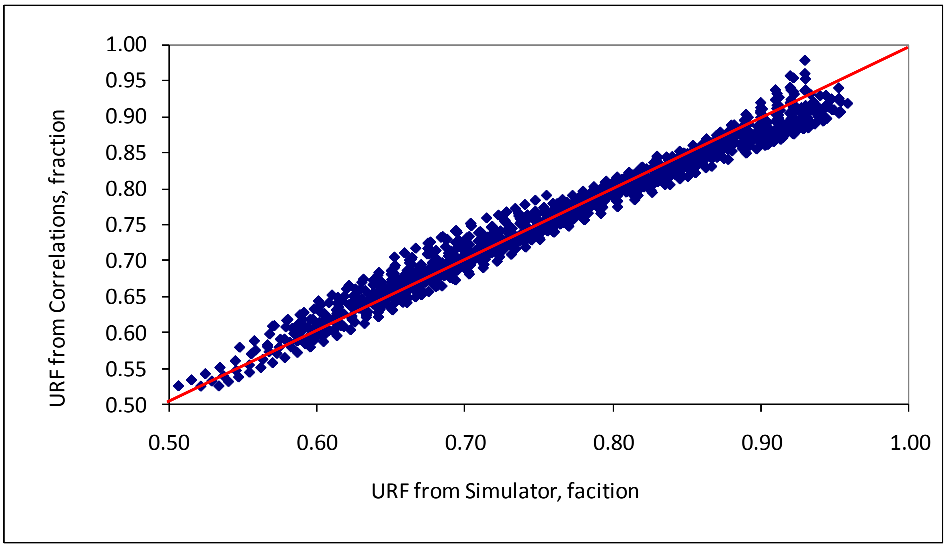


Fig.12—Estimated ultimate recovery at the end of 720 SLZ obtained from our correlations and from the simulation runs. Results matched very well.

The objective of this comparison is to verify that the SLZ obtained using fractional flow, the theoretical basis behind the SLZ, applies to the 3D simulation results. Since results matched, we can feel confident applying the correlations developed for heterogeneous reservoirs using only simulation.

We can see in Fig.12 that results from correlations developed for homogeneous reservoirs using fractional flow are reasonably close to those obtained for the same cases with simulation for homogeneous (SLZ = 0) reservoirs. Some outlier observations are located a little far from the Slope-1 trend line (in red) because of their oil-wet nature. The maximum error was 3% for water-wet systems and 4% for oil-wet systems. The same trend was found for the corresponding PVI.

The same comparison was made between the correlated slopes and intercepts of all SLZs calculated with our correlations for all combinations of cases and those calculated with the simulator. Fig.13 and Fig.14 show those results. Slope trends close to 1 can be seen in both plots.

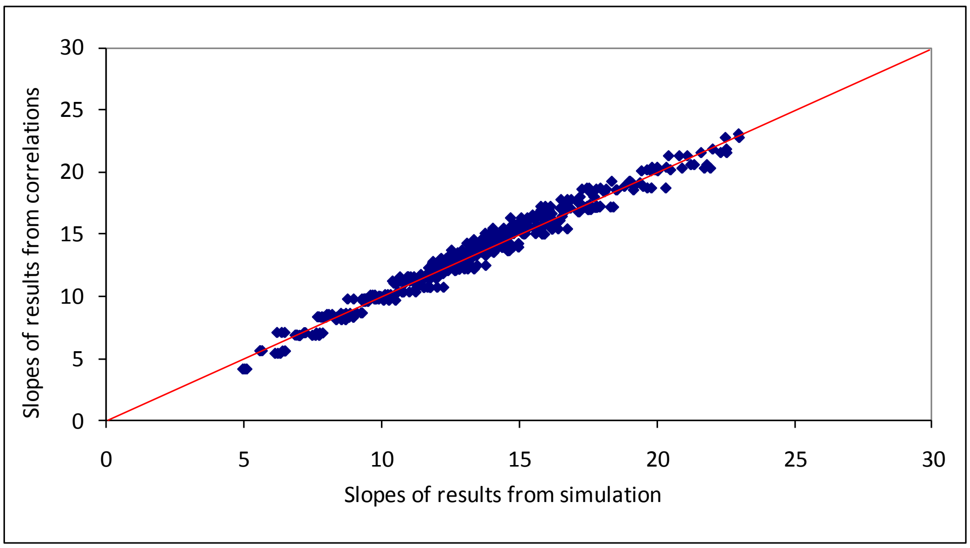


Fig.13—Our correlations reproduced the slopes determined from reservoir simulation (720 SLZs).

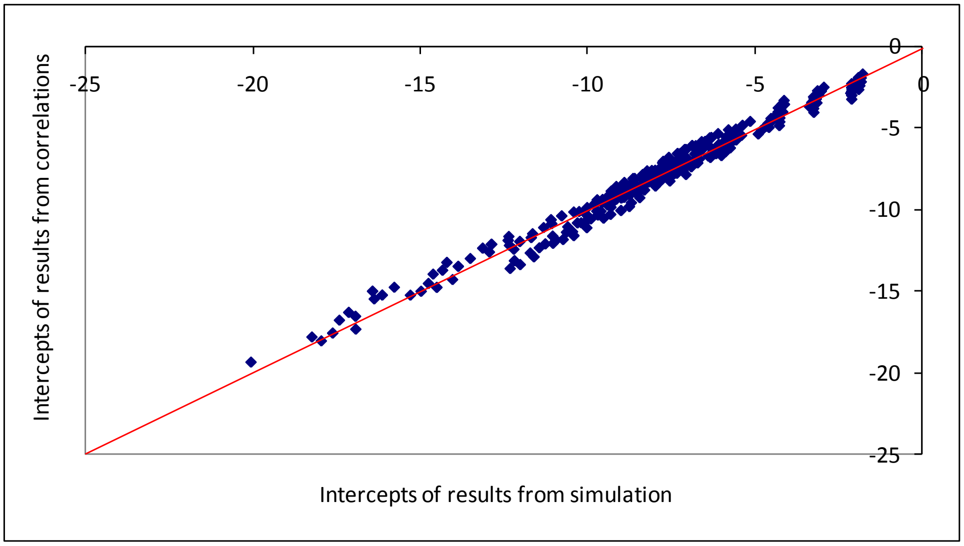


Fig.14—Our correlations captured intercepts determined from reservoir simulation (720 SLZs).

More results are shown in Fig.15, Fig.16, and Fig.17. As can be seen, the correlation matches the simulator runs results.

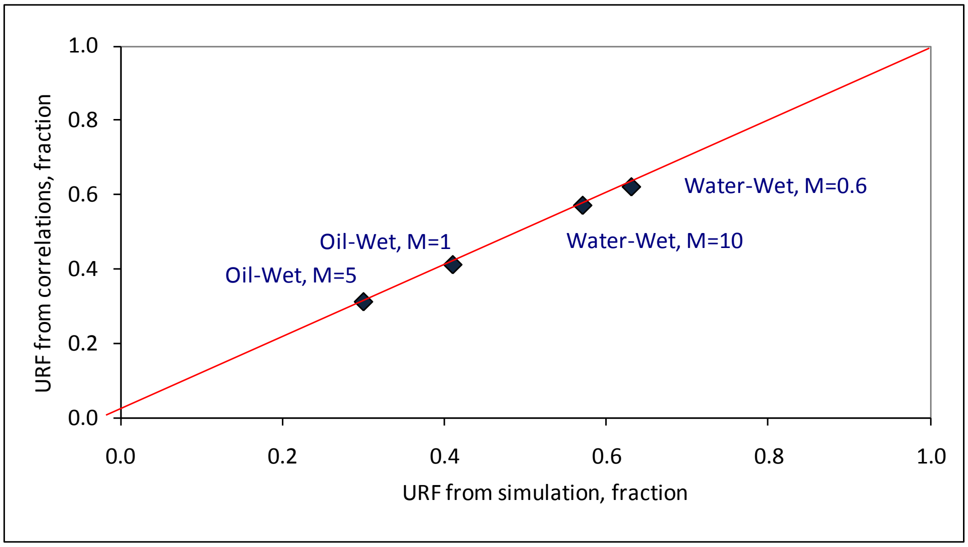


Fig.15—Final RF correlation from correlations and simulator for specific cases.

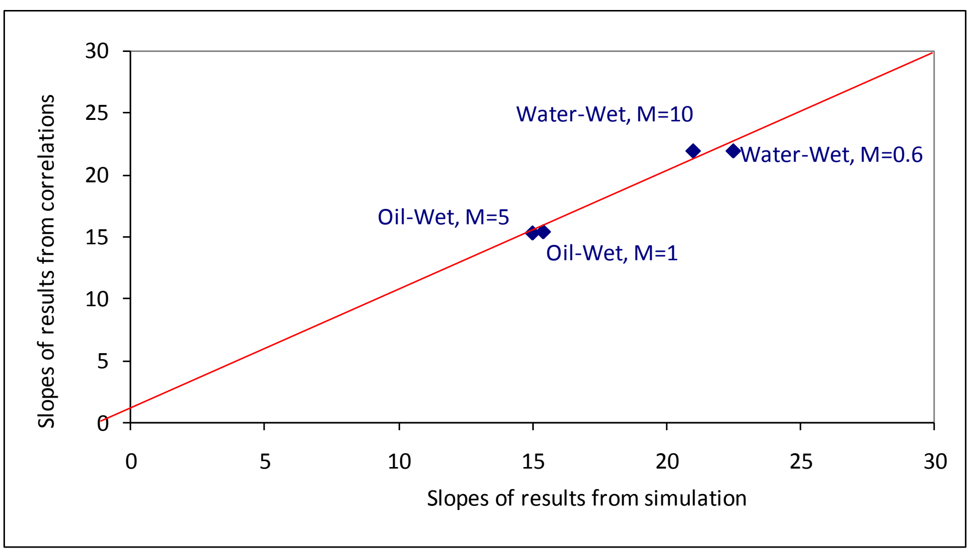


Fig.16—Slope correlations from correlations and simulator for specific cases.

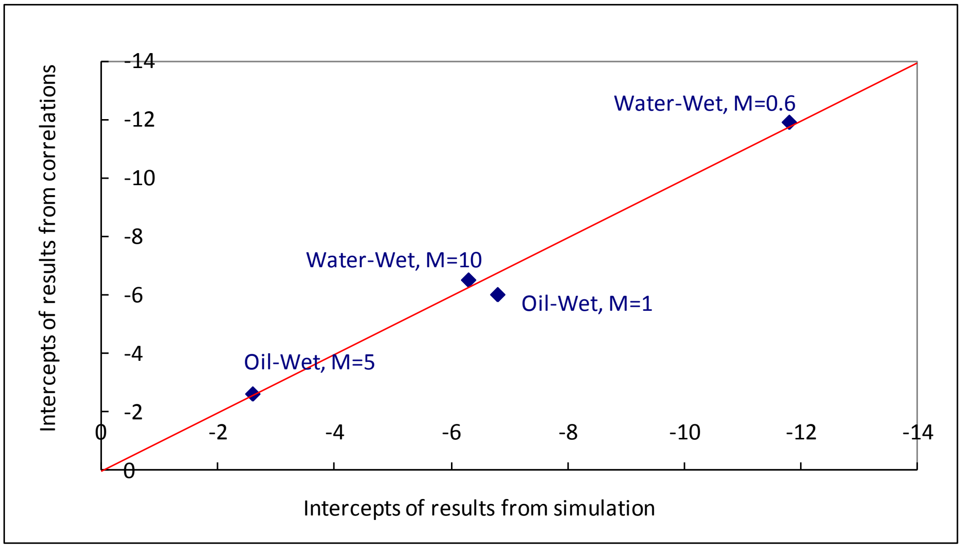


Fig.17—Intercept correlation from correlations and simulator for specific cases.

**Correlation for Heterogeneous Reservoirs**

Regressors and parameter estimates for heterogeneous reservoirs are shown in Table 12 and Table 13.

Table 12—Parameter estimates for heterogeneous reservoirs for water-wet rocks.

| GUIDE | | WATER-WET ROCKS | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Regressor | Variable | VDP = 0.6 | | VDP = 0.7 | | VDP = 0.8 | | VDP = 0.9 | |
| | | RF | PVI. | RF | PVI. | RF | PVI. | RF | PVI. |
| Independent | | -0.1802 | -1.05462 | 0.19068 | -1.2156 | 0.09829 | -2.0651 | 0.1718 | -0.4536 |
| $x_8$ | $M$ | 0 | 0.46384 | -0.0037 | 0.47344 | -0.0075 | 0.58703 | -0.0144 | 0.08084 |
| $x_9$ | $RF$ hom | 1.27511 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $x_{10}$ | log WOR | 0 | 0 | 0.02336 | 0 | 0.05354 | 0 | 0.07227 | 0 |
| $x_{11}$ | $RF^2{}_{\mathrm{hom}}$ | 0 | 0 | 0.91971 | 0 | 0.88719 | 0 | 0.44117 | 0 |
| $x_{12}$ | $e^{\log WOR}$ | 0 | 0.28413 | 0 | 0.31097 | 0 | 0.57103 | 0 | 0.19269 |
| $x_{13}$ | $10^{\log WOR}$ | 0 | 0.00912 | 0 | 0.01136 | 0 | 0.01217 | 0 | 0.02353 |

Table 13—Parameter estimates for heterogeneous reservoirs for oil-wet rocks.

| GUIDE | | OIL-WET ROCKS | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Regressor | Variable | VDP = 0.6 | | VDP = 0.7 | | VDP = 0.8 | | VDP = 0.9 | |
| | | RF | PVI. | RF | PVI. | RF | PVI. | RF | PVI. |
| Independent | | -0.0988 | -1.26805 | 0.09036 | -1.1147 | 0.06727 | -1.2674 | 0.0748 | -0.6497 |
| $x_8$ | $M$ | 0 | 0.49769 | -0.0023 | 0.43417 | -0.0049 | 0.45081 | -0.0054 | 0.18524 |
| $x_9$ | $RF$ hom | 1.27817 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $x_{10}$ | log WOR | 0 | 0 | 0.01459 | 0 | 0.03839 | 0 | 0.04112 | 0 |
| $x_{11}$ | $RF^2{}_{\text{hom}}$ | 0 | 0 | 1.78812 | 0 | 1.27549 | 0 | 0.88482 | 0 |
| $x_{12}$ | $e^{\log WOR}$ | 0 | 0.37376 | 0 | 0.30144 | 0 | 0.32235 | 0 | 0.22142 |
| $x_{13}$ | $10^{\log WOR}$ | 0 | 0.00238 | 0 | 0.00691 | 0 | 0.01279 | 0 | 0.0137 |

We developed different correlations for RF and PVI to simplify application of our correlation.

Parameter estimates for water-wet and oil-wet rocks were correlated independently; therefore, a better $R^2$ (higher than 0.9) is obtained using the two different correlations. Bigger differences are observed in values corresponding to higher SLZ, because oil-wet rocks show lower RF and higher PVI with higher heterogeneity than water-wet rocks. Using the tables above, application of the correlations can be easily programmed in a spreadsheet. However, a basic VBA program is provided in Appendix G for immediate application of the correlation.

Correlations to estimate RF and PVI for different VDPs in water-wet and oil-wet, heterogeneous reservoirs are the following.

Water-wet and heterogeneous calculations:

VDP > 0.55 and VDP <= 0.65

$$RF = -0.18015 + 1.27511(x_9) \quad (53)$$

$$PVI = -1.05462 + 0.46384(x_8) + 0.28413(x_{12}) + 0.00912(x_{13}) \quad (54)$$

VDP > 0.65 and VDP <= 0.75

$$RF = 0.19068 - 0.00368(x_8) + 0.02336(x_{10}) + 0.91971(x_{11}) \quad (55)$$

$$PVI=-1.2156 + 0.47344(x_8) + 0.31097(x_{12}) + 0.01136(x_{13}) \quad (56)$$

VDP > 0.75 and VDP <= 0.85

$$RF = 0.09829 - 0.00751(x_8) + 0.05354(x_{10}) + 0.88719(x_{11}) \quad (57)$$

$$PVI = -2.06505 + 0.58703(x_8) + 0.57103(x_{12}) + 0.01217(x_{13}) \quad (58)$$

VDP > 0.85 and VDP <= 0.99

$$RF = 0.1718 - 0.0144(x_8) + 0.07227(x_{10}) + 0.44117(x_{11}) \quad (59)$$

$$PVI = -0.45357 - 0.08084(x_8) + 0.19269(x_{12}) + 0.02353(x_{13}) \quad (60)$$

Oil-wet and heterogeneous calculations:

VDP > 0.55 and VDP <= 0.65

$$RF = -0.0988 + 1.27817 (x_9) \quad (61)$$

$$PVI = -1.26805 + 0.49769(x_8) + 0.37376(x_{12}) + 0.00238(x_{13}) \quad (62)$$

VDP > 0.65 and VDP <= 0.75

$$RF = 0.09036 – 0.00226(x_8) + 0.01459(x_{10}) + 1.78812(x_{11}) \quad (63)$$

$$PVI = -1.11471 + 0.43417(x_8) + 0.30144 (x_{12}) + 0.00691(x_{13}) \quad (64)$$

VDP > 0.75 and VDP <=0.85

$$RF = 0.06727 – 0.00489(x_8) + 0.03839(x_{10}) + 1.27549(x_{11}) \quad (65)$$

$$PVI = -1.26744 + 0.45081(x_8) + 0.32235(x_{12}) + 0.01279(x_{13}) \quad (66)$$

VDP > 0.85 and VDP <=0.99

$$RF = 0.0748 - 0.00535(x_8) + 0.04112(x_{10}) + 0.88482(x_{11}) \quad (67)$$

$$PVI = -0.64968 + 0.18524(x_8) + 0.22142(x_{12}) + 0.0137(x_{13}) \quad (68)$$

**ANOVA Tables for Correlations**

Analysis of variance (ANOVA) is a powerful technique to check for adequacy of correlations, which is determined by the information provided from four statistical indicators: correlation coefficient ($R^2$), mean square error ($\sigma 2$), $F$-statistic, and $P_{\text{-value}}$. A high value for $R^2$ (higher than 0.9) means that an accurate explanation of the behavior of the dependent variable will be obtained by studying the behavior of the independent variable. Our goal was to obtain $R^2$ as close as possible to 0.99. The mean square error is a measure of the error between the predicted value and the mean of the dependent variable, so a small value of the error means that the calculated value is closer to the mean. The $P_{\text{-value}}$ is related to the adequacy of the correlation to be used to predict values of the dependent variable within the range of investigation (sample). When the $P_{\text{-value}}$ is small, the correlation is also adequate (see explanations in Chapter II). Analysis of variances (ANOVA) tables follow.

Table 14—ANOVA for RF calculation for homogeneous, water-wet rocks.

| ANOVA TABLE | | | | |
|---|---|---|---|---|
| RF variable | $f_{wBT}$ | $S_{wBT}$ | Slope | Intercept |
| $R^2$ | 0.8537 | 0.9408 | 0.9223 | 0.9444 |
| $\sigma^2$ | 0.00053 | 0.00045 | 1.19005 | 0.54696 |
| $P_{\text{-Value}}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |

For homogeneous, water- and oil-wet reservoirs, Table 14 and Table 15 show $R^2$ numbers for $f_{wBT}$ and $S_{wBT}$, the slope and intercept correlations. These correlations are strong and can be used to calculate water cuts and saturations at breakthrough and slopes and intercepts to determine recoveries at any WOR, including ultimate recovery.

Table 15—ANOVA for RF calculation for homogeneous, oil-wet rocks.

| ANOVA TABLE | | | | |
|---|---|---|---|---|
| RF variable | $f_{wBT}$ | $S_{wBT}$ | Slope | Intercept |
| $R^2$ | 0.8751 | 0.9516 | 0.8627 | 0.9352 |
| $\sigma^2$ | 0.00057 | 0.00051 | 1.07414 | 0.20607 |
| $P_{\text{-Value}}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |

For heterogeneous reservoirs, statistics are shown in Table 16 and Table 17.

Table 16—ANOVA for RF calculation for heterogeneous, water-wet rocks.

| ANOVA TABLE | | | | |
|---|---|---|---|---|
| RF variable | VDP = 0.6 | VDP = 0.7 | VDP = 0.8 | VDP = 0.9 |
| $R^2$ | 0.9836 | 0.9966 | 0.9953 | 0.9987 |
| $\sigma^2$ | 6.6E-05 | 0.000017 | 4E-06 | 0.000001 |
| $P_{\text{-Value}}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |

Table 17—ANOVA for RF calculation for heterogeneous, oil-wet rocks.

| ANOVA TABLE | | | | |
|---|---|---|---|---|
| RF variable | VDP = 0.6 | VDP = 0.7 | VDP = 0.8 | VDP = 0.9 |
| $R^2$ | 0.971 | 0.9881 | 0.9926 | 0.9938 |
| $\sigma^2$ | 0.00008 | 0.00003 | 0.00003 | 0.00002 |
| $P_{\text{-Value}}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 |

Table 18—ANOVA for PVI correlation statistics, water-wet rock.

| ANOVA TABLE | HETEROGENEOUS-WATER-WET ROCK | | | | |
|---|---|---|---|---|---|
| PVI variable | Homog | VDP = 0.6 | VDP = 0.7 | VDP = 0.8 | VDP = 0.9 |
| $R^2$ | 0.9727 | 0.9691 | 0.965 | 0.9503 | 0.9963 |
| $\sigma^2$ | 0.30927 | 0.50905 | 0.81395 | 2.11876 | 0.19934 |
| $P_{\text{-Value}}$ | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 |

Table 19—ANOVA for PVI correlation statistics, oil-wet rock.

| ANOVA TABLE | HETEROGENEOUS-OIL-WET ROCK | | | | |
|---|---|---|---|---|---|
| PVI variable | Homog | VDP = 0.6 | VDP = 0.7 | VDP = 0.8 | VDP = 0.9 |
| $R^2$ | 0.9149 | 0.9239 | 0.9486 | 0.9511 | 0.9919 |
| $\sigma^2$ | 0.33022 | 0.61156 | 0.65109 | 1.33752 | 0.18589 |
| $P_{\text{-Value}}$ | <0.0001 | <0.0001 | <0.0001 | <0.0001 | <0.0001 |

Table 18 and Table 19 present very good correlation coefficients for the PVI correlations.

Consistently with previous results, four correlations were obtained to estimate the beginning and the end of the SLZ for the heterogeneous module. The expressions shown below are called "$WOR_{min}$" and "$WOR_{max}$" variables, referring to the minimum and maximum log WOR acceptable to ensure that the results are still within the SLZ area for heterogeneous reservoirs. These expressions are functions of *M* and SLZ:

Water-Wet – SLZ beginning:

$$\begin{aligned} \mathrm{WOR}_{\min} &= 4.91690 + 0.01738(M) - 10.83657(\mathrm{VDP}) \\ &+ 9.73473(\mathrm{VDP}^2) - 0.33856(10^{\mathrm{VDP}}) \end{aligned} \quad \text{..................................} \quad (69)$$

Oil-Wet – SLZ beginning

$$\begin{aligned} \mathrm{WOR}_{\min} &= 2.37473 + 0.02337(M) - 2.99912(VDP) \\ &+ 2.88892(VDP^2) - 0.22364(10^{VDP}) \end{aligned} \quad \text{..................................} \quad (70)$$

Water-Wet – SLZ end

$$\mathrm{WOR}_{\max} = 5.15914 + 0.13366(M) - 9.6964(\mathrm{VDP}) + 0.62073(10^{\mathrm{VDP}}) \quad \text{......} \ (71)$$

Oil-Wet – SLZ end

$$\mathrm{WOR}_{\max} = 1.93449 + 0.02621(M) - 0.45239(\mathrm{VDP}) - 0.00594(10^{\mathrm{VDP}}) \quad \text{.......} \ (72)$$

Minimum and maximum WOR value ranges within the different SLZs were determined with our correlations for the homogeneous and heterogeneous modules. We obtained a range of minimum WOR values (beginnings of the SLZ) for water-wet rocks from WOR = 1 to WOR = 15, and maximum WOR values (ends of the SLZ) from WOR = 30 to 489, all for SLZ values between 0.5 and 0.99. The mean values were WOR = 5 and WOR = 72, respectively. We defined ranges for oil-wet zones between WOR = 1 and WOR = 251 for heterogeneous cases.

For homogeneous cases, WOR ranges between 4 and 59 (min WOR) and 87 and 3090 (max WOR). Such extreme, unrealistic values for max WOR were obtained because the homogeneous module only considers the ideal case of SLZ = 0.

Fig. 18 presents ranges for each minimum and maximum WOR for homogeneous and heterogeneous cases for water-wet rocks. Fig. 19 shows ranges for oil-wet rocks. Table 20 shows minimum, maximum, and mean values for beginnings and ends of all SLZs for water- and oil-wet rocks, for both homogenous and heterogeneous reservoirs.

As can be seen, WOR ranges tend to be shorter for oil-wet rocks, as expected. Also, SLZs will finish earlier for heterogeneous rocks since they show shorter max WOR than the homogeneous cases.

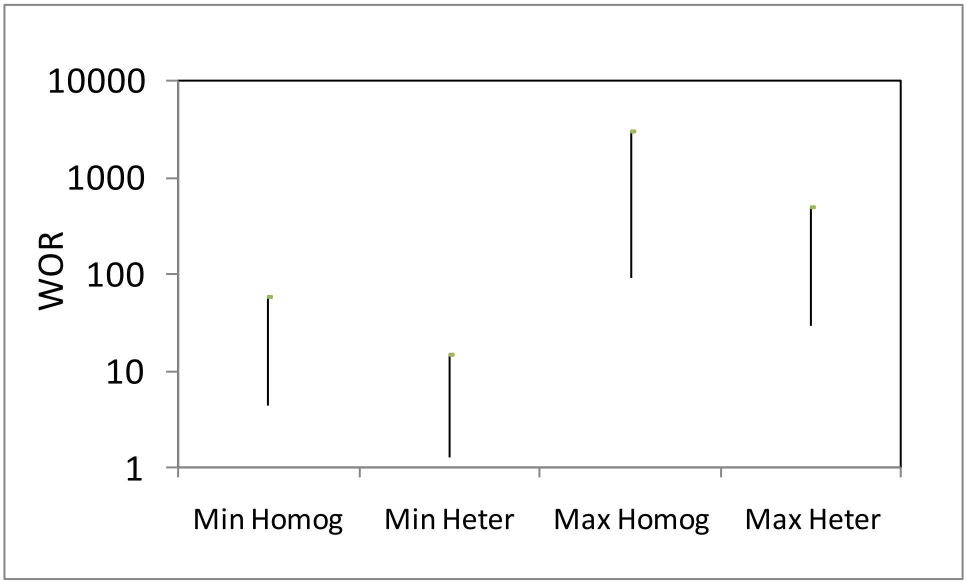


Fig. 18—Ranges for WOR determined with our correlations for both the homogeneous and the heterogeneous modules for water-wet rocks.

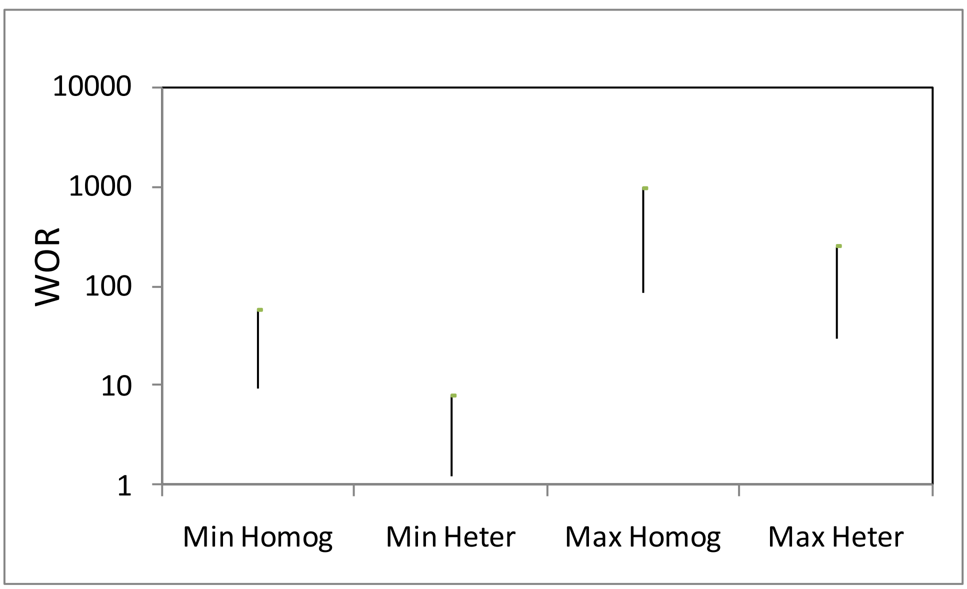


Fig. 19—Ranges for WOR determined with our correlations for both the homogeneous and the heterogeneous modules for oil-wet rocks.

Table 20—Minimum, maximum and mean values of WOR for different cases for water-wet and oil-wet reservoirs.

| WOR | WATER-WET | | | OIL-WET | | |
|---|---|---|---|---|---|---|
| | Min | Mean | Max | Min | Mean | Max |
| Min Homogeneous | 4 | 17 | 58 | 9 | 22 | 59 |
| Min Heterogeneous | 1 | 5 | 15 | 1 | 3 | 8 |
| Max Homogeneous | 93 | 308 | 3090 | 87 | 352 | 977 |
| Max Heterogeneous | 30 | 72 | 489 | 29 | 58 | 251 |

Comparisons of results for heterogeneous reservoirs are shown for ultimate RF (at the end of the SLZ) and its corresponding PVI for two different reservoirs—with favorable and unfavorable mobility ratios and water-wet rocks, and with favorable and unfavorable mobility ratios but oil-wet rocks. Results are shown in Fig. 20 and Fig. 21. As can be seen, using these extreme reservoir property values, and for all SLZs used, the correlations again match the simulator runs results for URF (at the end of the SLZ) and PVI.

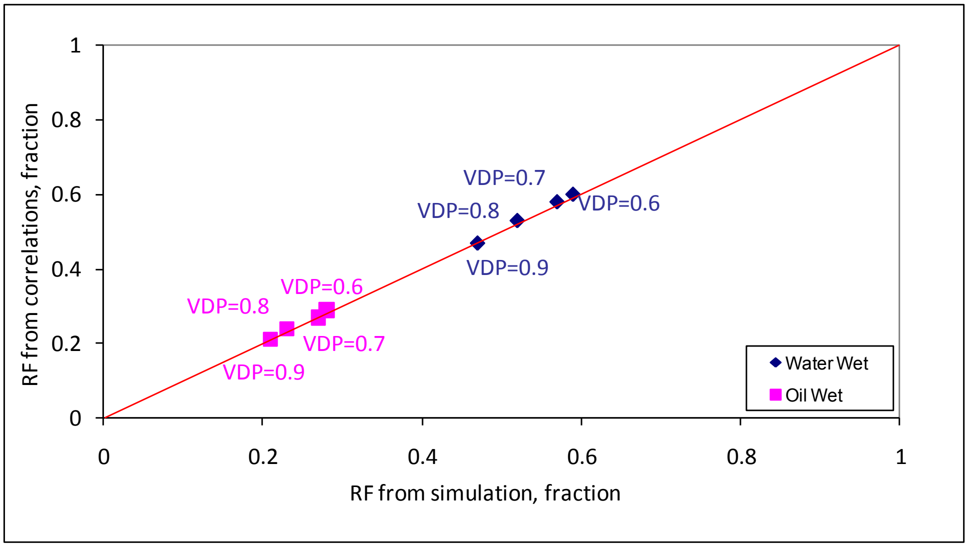


Fig. 20—URF for different wettabilities and SLZ from our correlations and from reservoir simulation.

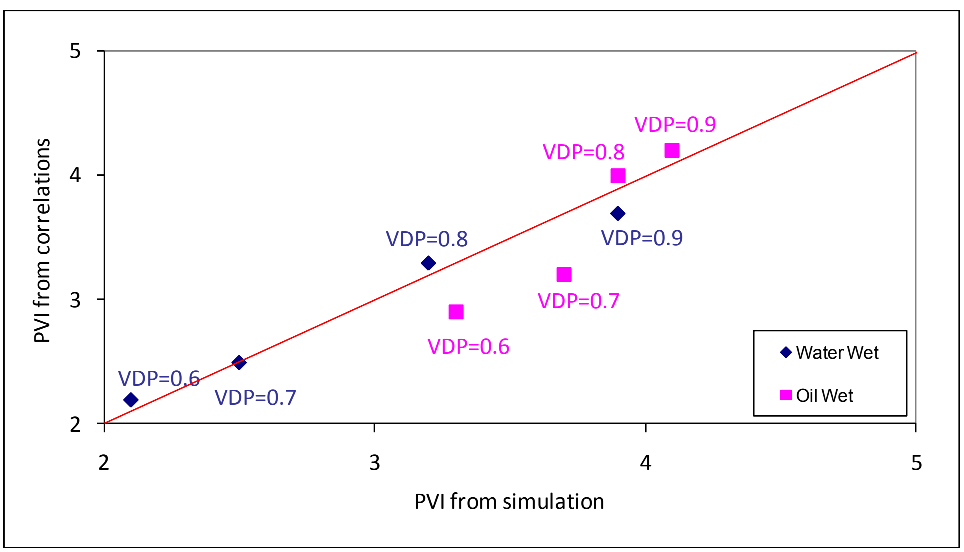


Fig. 21—Ultimate PVI for different wettabilities and SLZ from our correlations and from reservoir simulation.

Since comparing the correlations results with the simulator results was successful, we wanted to validate the correlations using field data to ensure the applicability of the correlations in real-life situations.

Two extreme cases were run to ensure that the correlations for the heterogeneous module were consistent with the reservoir simulation results in terms of predicting the SLZ behavior.

The first case presented is for an oil-wet rock reservoir with $M = 5$ and variations of SLZ from 0.7 to 0.9. These results permitted us to corroborate the accuracy of the correlations, as we can see in Fig. 22, Fig. 23 and Fig. 24.

The second case is for another oil-wet rock reservoir with $M = 30$ and variations of SLZ from 0.7 to 0.9. Results can be seen in Fig. 25, Fig. 26, and Fig. 27.

Results show that our correlations reproduced the SLZ behavior; therefore, the beginning and end of the SLZ can be predicted reasonably accurately.

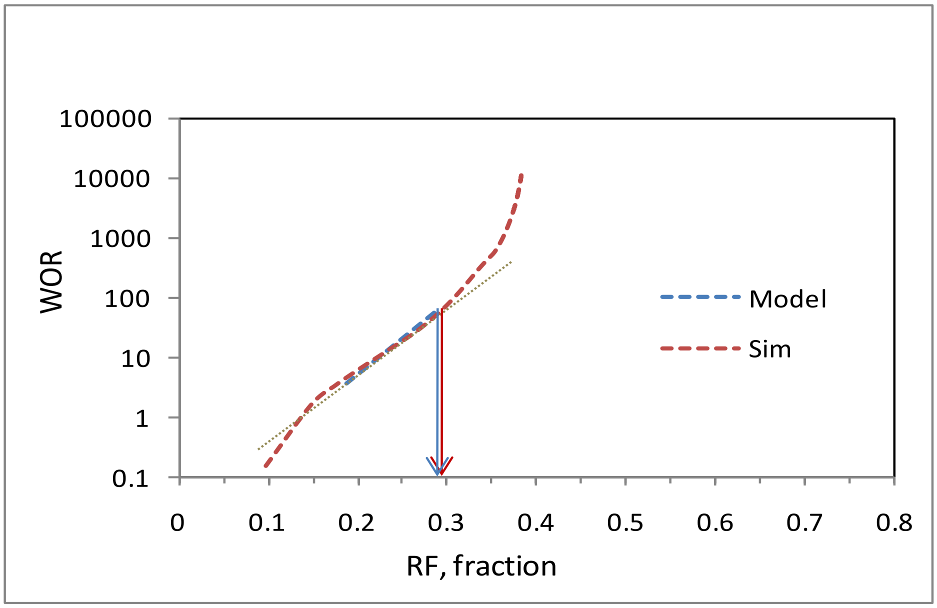


Fig. 22—Comparison of results from simulation and our correlations for an oil-wet rock with $M$ = 5 and SLZ = 0.7.

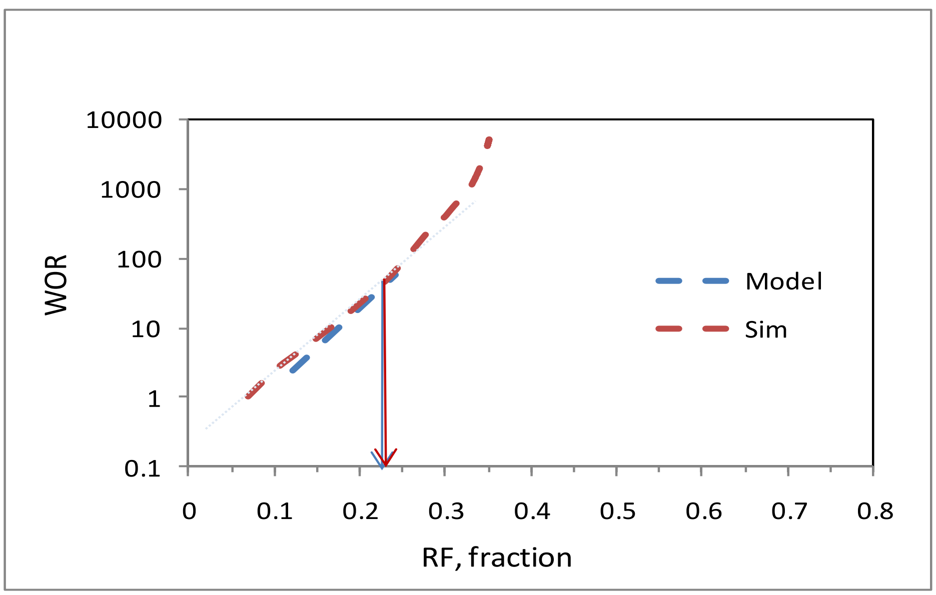


Fig. 23—Comparison of results from simulation to our correlations for an oil-wet rock with $M$ = 5 and VDP = 0.8.

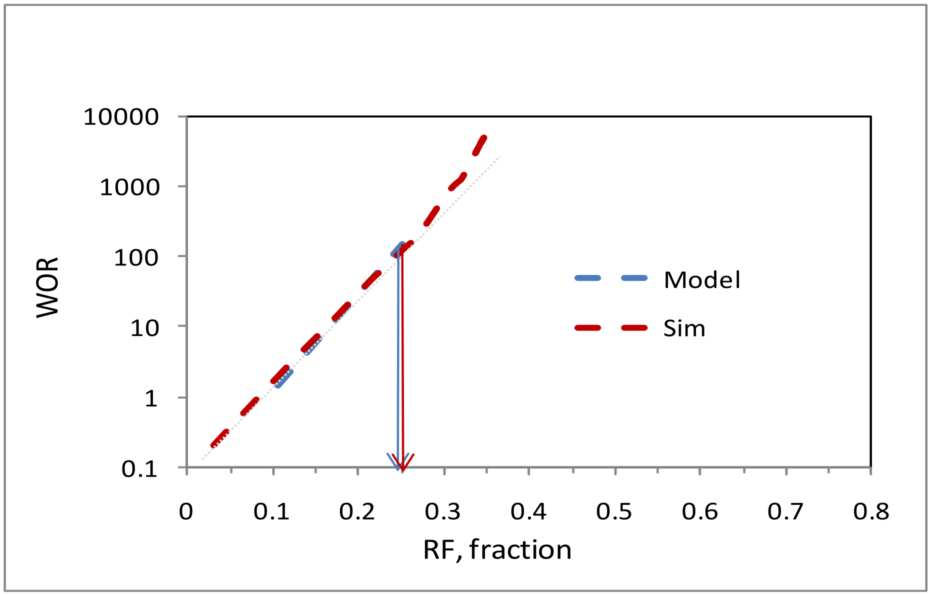


Fig. 24—Comparison of results from simulation and our correlations for an oil-wet rock with $M = 5$ and VDP = 0.9.

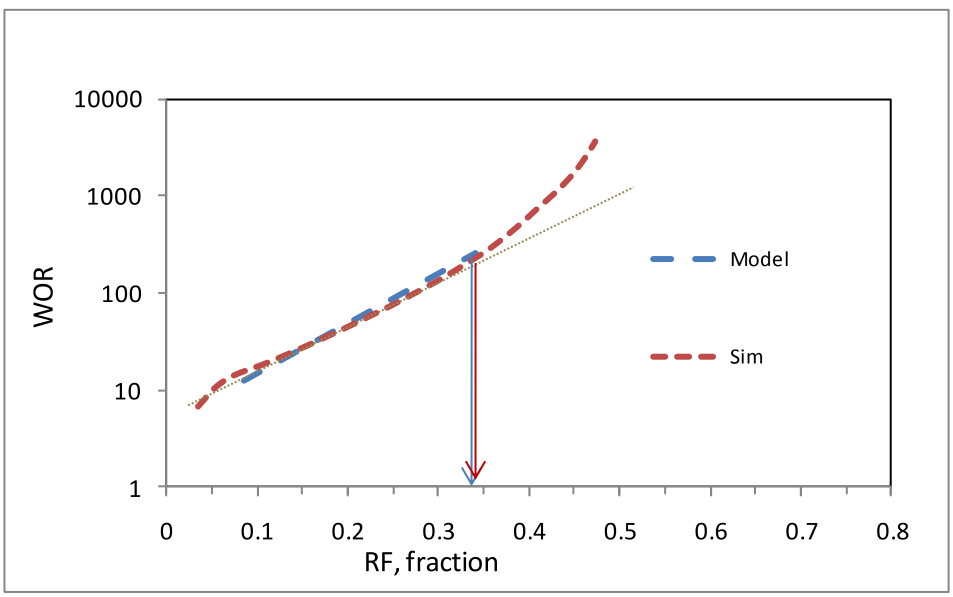


Fig. 25—Comparison of results from simulation to our correlations for an oil-wet rock with $M = 30$ and VDP = 0.7.

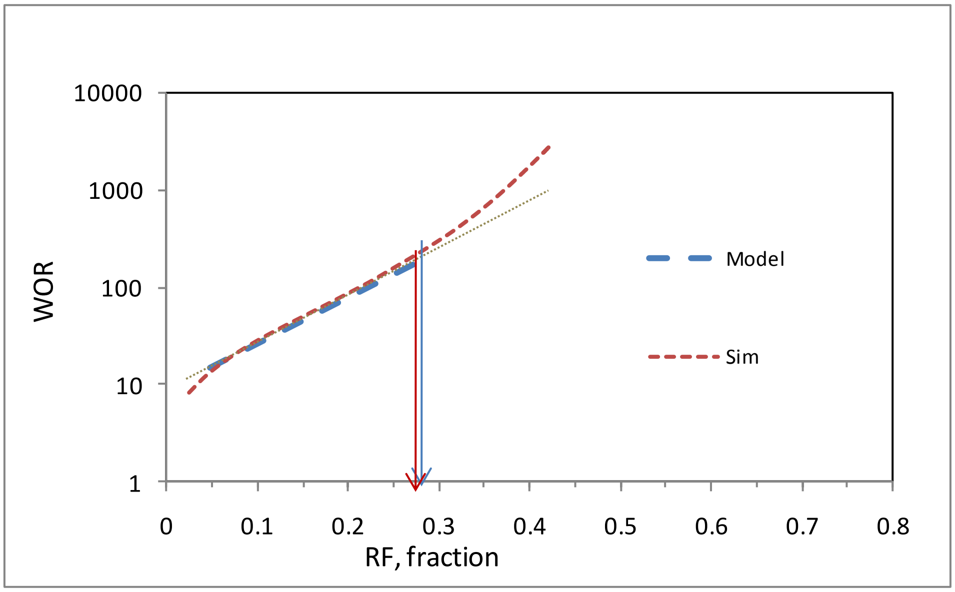


Fig. 26—Comparison of results from simulation with our correlations for an oil-wet rock with *M* = 30 and VDP = 0.8.

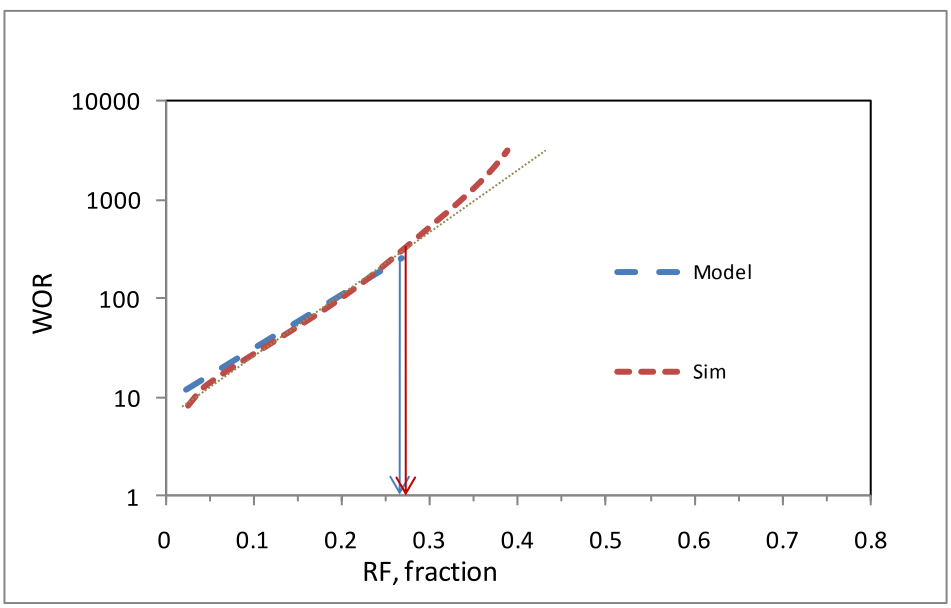


Fig. 27—Comparison of results from simulation with our correlations for an oil-wet rock with *M* =30 and VDP = 0.9.

## Field Cases

Eighty-four field cases were analyzed with our correlations. Two different field cases (called Field A and Field B) and several well performances are presented. Reservoir properties for the Field A case are shown in Table 21 and Table 22.

These reservoir data correspond to infill wells with no primary production. Runs were made with the original reservoir simulator and our new correlations. This example has a favorable M value of 0.76.

To apply our methodology, we developed a computer program in VBA that simplifies the correlation application (see Appendix F). A screen shot of the program illustrates the interface used by our code with input and output for all variables in Fig. 28.

Table 21—Field A case. Real and simulation model data.

| DATA FOR FIELD A | VALUE | UNITS |
|---|---|---|
| Connate water saturation, $S_{wc}$ | 0.38 | |
| Residual oil saturation, $S_{or}$ | 0.23 | |
| Residual gas saturation, $S_{gr}$ | 0.01 | |
| Initial gas saturation, $S_{gi}$ | 0.01 | |
| Water viscosity, $\mu_w$ | 0.9 | cp |
| Oil viscosity, $\mu_o$ | 1.2 | cp |
| Oil formation factor, $B_o$ | 1.15 | RB/STB |
| Water formation factor, $B_w$ | 1 | RB/STB |
| Area | 35 | Acres |
| Injection pressure | 2500 | psi |
| Reservoir pressure | 1200 | psi |
| Wellbore radius, $r_w$ | 0.3 | ft |
| End-point oil relative perm, $k_{roe}$ | 0.96865 | |
| End-point water relative perm, $k_{rwe}$ | 0.551 | |
| Oil Corey's function exponent, $n_o$ | 3.017 | |
| Water Corey's function exponent, $n_w$ | 1.8045 | |
| Oil in place (from simulation) OOIP | 4,633,054 | STB |
| $N_p$ (from simulation) | 1,861,397 | STB |
| URF – end of the SLZ (from simulation) | 40.18 | fraction |
| VDP | 0.8 | |

Table 22—Field A case. Average data for each layer from the simulation model.

| LAYERS DATA | | |
|---|---|---|
| Net h, ft | k,md | Porosity, % |
| 85 | 175 | 0.19 |
| 41 | 44 | 0.14 |
| 10.5 | 190 | 0.15 |
| 15 | 43 | 0.17 |
| 9.5 | 31 | 0.13 |
| 12 | 13 | 0.12 |
| 48 | 15 | 0.12 |
| Totals: | | |
| 221 | 73 | 0.15 |

Fig. 28 presents the input module where the user inputs the fluid and rock properties, the SLZ, and the WOR. The user must input the first 12 items requested (cells colored in white in Fig. 28) and run the case. When the program runs, it calculates the slope, the intercept, $M$, $f_{wBT}$, $S_{wBT}$, PVI, and RF for homogeneous cases, and the max WOR recommended, RF, and PVI for heterogeneous cases. A warning will be given if the user approaches the recommended max WOR or obtains a PVI higher than 2.5 for homogeneous cases, but the calculation will still be allowed so the user can adapt the program for any case and sensitivity.

A final RF of 39.6% is calculated at the end of the SLZ, based on the given log WOR. If a higher log WOR is used, PVI will increase rapidly while the RF will increase slowly. Since the correlations will calculate the end of the SLZ, the program will warn the user when too-high WOR values are used and linear extrapolation is are being done or when high PVI is required to obtain RF values.

| Input your data before running: | Enter Data here: |
|---|---|
| Reservoir Name | Field A |
| Oil Viscosity, cP | 1.2 |
| Water Viscosity, cP | 0.9 |
| Corey exponent for oil(no) | 3.017 |
| Corey exponent for water(nw) | 1.8045 |
| Oil rel perm curve end-point (kroe) | 0.96865 |
| Water rel perm curve end-point (krwe) | 0.551 |
| Residual oil saturation (Sor) | 0.23 |
| Connate water saturation (Swc) | 0.38 |
| Dykstra-Parsons coeff. (VDP) | 0.8 |
| Water wet=1 or Oil wet=2 | 1 |
| Estimated max operational WOR | 26.3 |
| Results: | |
| Slope | 14.39 |
| Intercept | -5.87 |
| fwBT | 0.835 |
| SwBT | 0.603 |
| Max WOR | 26.3 |
| Min WOR | 0.360 |
| Mobility Ratio | 0.758 |
| RF | 0.396 |
| PVI | 1.062 |

Fig. 28—Screen shot of the VBA program to apply the new correlations. Run for Field A well case.

One of the most important observations we can make at this point is that other analytical methods and the new methodology will offer reasonably similar results, with the difference that the traditional methods need more data and perform more calculations than the new correlations.

To compare our estimates with other analytical methods, we calculated and present results from all the methods, including our new correlations and field data (Fig. 29). We included Craig-Geffen-Morse (CGM), modified Buckley-Leverett (BL) to account for $E_A$ at the breakthrough, Dykstra-Parsons (DP), and Stiles (see Appendix J for definitions, assumptions and limitations of these methods).

Our new methodology shows an end of the SLZ similar to the field data's. Our ultimate recovery factor (at the end of the SLZ) is 39.6% (less than 1% different from the field data).

In Fig. 29 we can see that the two wells have an abrupt water production increase at the end of our SLZ and water production for both wells are far from the other analytical methods in terms of WOR or RF. Also, the SLZs found in the field data have slopes and beginnings similar to the one estimated with our correlations. Results from other analytical methods are too optimistic.

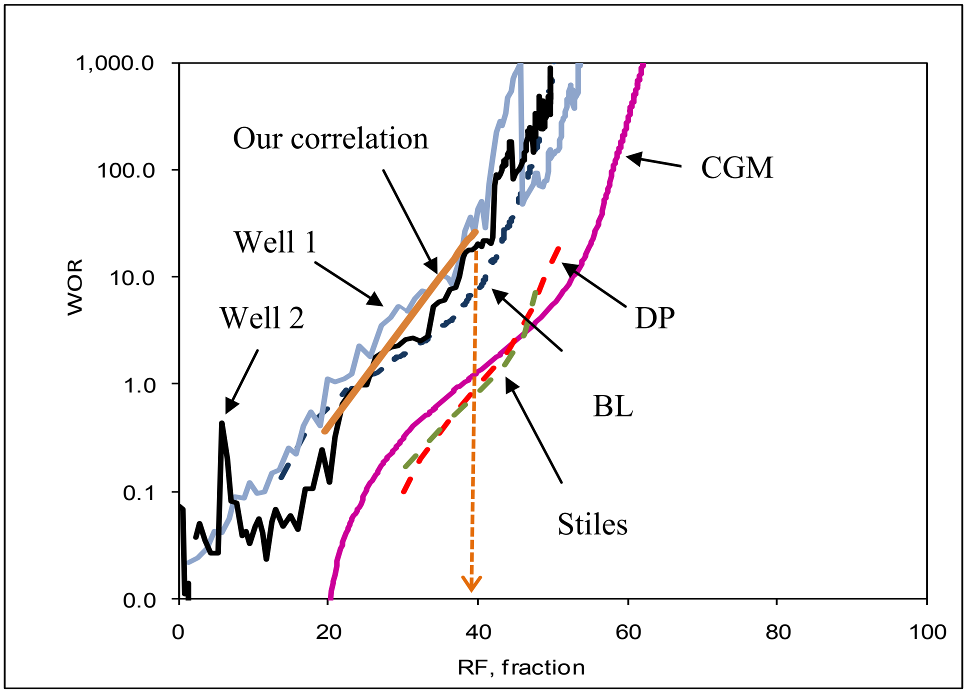


Fig. 29—Comparison of performance of two different wells from Field A with our new correlations and with four different analytical methods: Craig-Geffen and Morse (CGM), Dykstra-Parsons (DP), Modified Buckley-Leverett (BL), and Stiles.

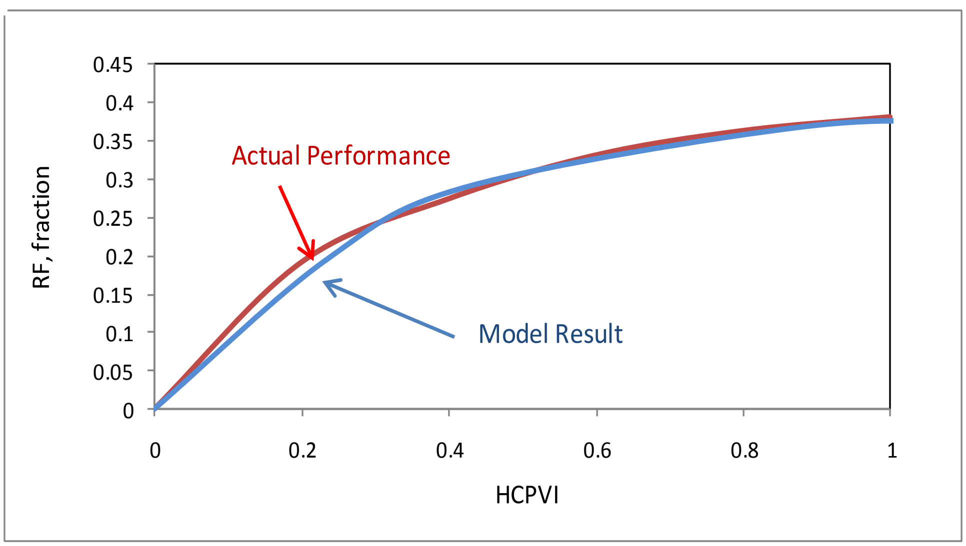


Fig. 30—RF calculated using our correlations and compared with the actual performance of Field A.

Comparing actual field performance with our correlations results, we see that our predictions match the actual behavior in terms of RF vs. hydrocarbon pore volumes injected (HCPVI), as seen in Fig. 30.

Fig. 31 shows a comparison of calculated and actual HCPVI and PVI estimated with the correlations, and Fig. 32 presents calculations of oil and water production rates using our correlations.

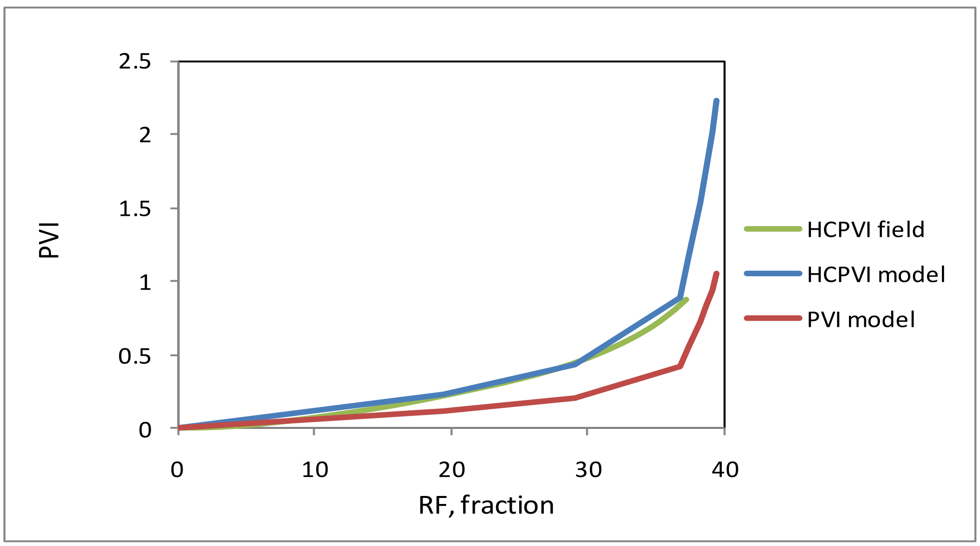


Fig. 31—Comparison of hydrocarbon pore volumes injected and total pore volume injected, estimated with the correlations and actual performance in Field A.

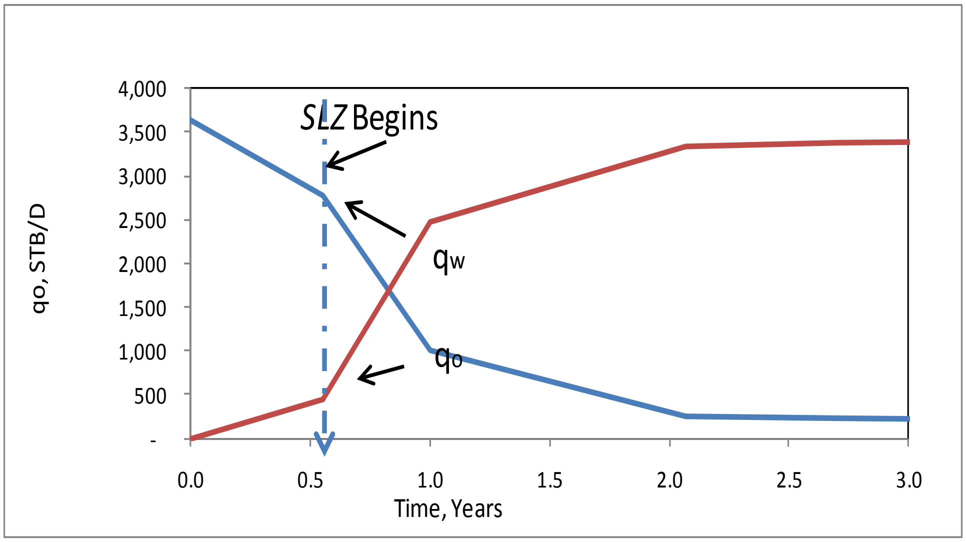


Fig. 32—Oil and water production rates per well estimated with the correlations. Field A.

Case 2, called Field B, presents a reservoir with unfavorable *M* and medium heterogeneity or SLZ (see Table 23 and Table 24). RF from simulation is 41.5% with high water cut, since the operator can handle water cuts above 98%. Layer data are average values.

Table 23—Field B Case. Real and original simulation model data.

| DATA FOR FIELD A | VALUE | UNITS |
|---|---|---|
| Connate water saturation, $S_{wc}$ | 0.17 | |
| Residual oil saturation, $S_{or}$ | 0.25 | |
| Residual gas saturation, $S_{gr}$ | 0 | |
| Initial gas saturation, $S_{gi}$ | 0 | |
| Water viscosity, $\mu_w$ | 0.25 | cp |
| Oil viscosity, $\mu_o$ | 2.54 | cp |
| Oil formation factor, $B_o$ | 1.108 | RB/STB |
| Water formation factor, $B_w$ | 1 | RB/STB |
| Area | 340 | Acres |
| Injection pressure | 1540 | psi |
| Reservoir pressure | 500 | psi |
| Wellbore radius, $r_w$ | 0.583 | ft |
| End-point oil relative perm, $k_{roe}$ | 1.0 | |
| End-point water relative perm, $k_{rwe}$ | 0.25 | |
| Oil Corey's function exponent, $n_o$ | 3 | |
| Water Corey's function exponent, $n_w$ | 2 | |
| Oil in place (from simulation) OOIP | 45,000,000 | Bbls |
| $N_p$ (from simulation) | 18,660,768 | Bbls |
| URF - end of the SLZ (from simulation) | 41.47 | fraction |
| VDP | 0.8 | |

Table 24—Field B Case. Average data for the only layer where the well is completed, taken from the original simulation model.

| LAYERS DATA | | |
|---|---|---|
| Net h, ft | k,md | Porosity, % |
| 55.5 | 340 | 0.17 |

Fig. 33 shows the tool's input and output for this case. High initial oil saturation and medium SLZ help to obtain good recovery at high production rates.

Using our methodology, recovery at 2.98 PVI is 37%. Fig. 34 shows data from two different wells, and even when the curves may be slightly different, the reservoir and fluid properties are similar and the SLZ estimated from our correlations matched the real data observations reasonably well. Results from other analytical methods are too optimistic.

Fig. 35 and Fig. 36 show performance estimations and oil and water production rates per well estimated from our correlation results.

In general, our methodology presents more realistic results than other methods, and can be used as an additional tool to support more detailed reservoir studies and simulations in later stages of planning and development.

| Input your data before running: | Enter Data here: |
|---|---|
| Reservoir Name | Field B |
| Oil Viscosity, cP | 2.54 |
| Water Viscosity, cP | 0.25 |
| Corey exponent for oil(no) | 3 |
| Corey exponent for water(nw) | 2 |
| Oil rel perm curve end-point (kroe) | 1 |
| Water rel perm curve end-point (krwe) | 0.25 |
| Residual oil saturation (Sor) | 0.25 |
| Connate water saturation (Swc) | 0.17 |
| Dykstra-Parsons coeff. (VDP) | 0.8 |
| Water wet=1 or Oil wet=2 | 1 |
| Estimated max operational WOR | 45.5 |
| Results: | |
| Slope | 10.15 |
| Intercept | -3.23 |
| fwBT | 0.803 |
| SwBT | 0.458 |
| Max WOR | 45.50 |
| Min WOR | 0.360 |
| Mobility Ratio | 2.540 |
| RF | 0.373 |
| PVI | 2.977 |

Fig. 33—Screen shot of the VBA program applying the new correlations to Field B well.

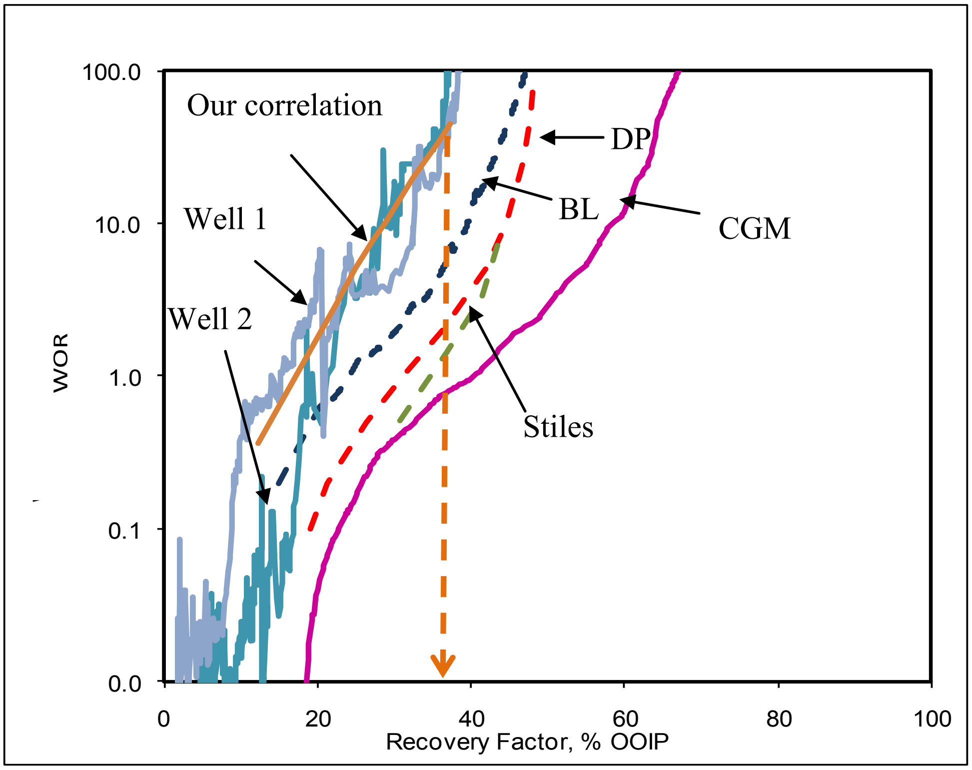


Fig. 34— Comparison of performance of two different wells from Field B with our new correlations and with four different analytical methods: Craig-Geffen and Morse (CGM), Dykstra-Parsons (DP), Modified Buckley-Leverett (BL), and Stiles.

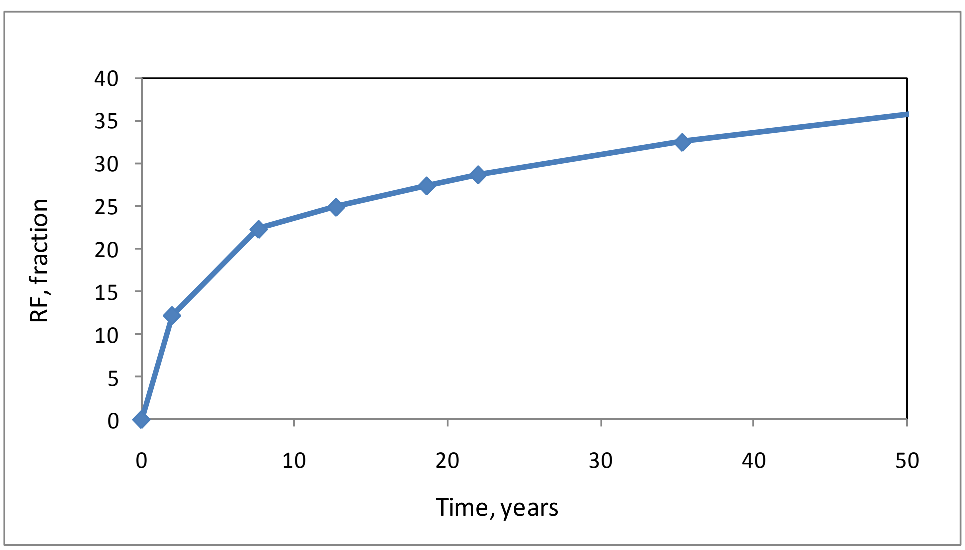


Fig. 35—RF vs. time calculated with our correlations for Field B.

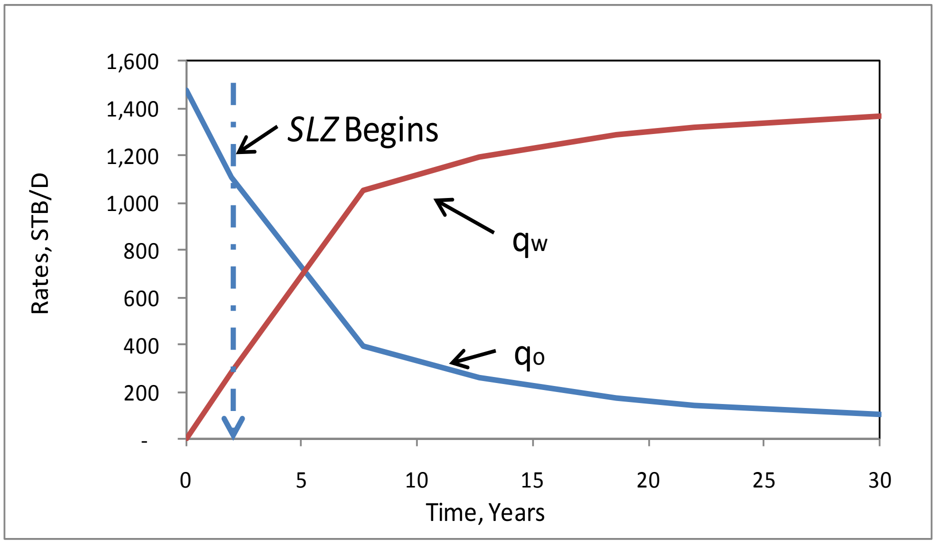


Fig. 36—Oil and water production rates per well estimated with the correlations. Field B.

# CHAPTER IV

# CONCLUSIONS AND RECOMMENDATIONS

On the basis of these research results, we offer the following conclusions and recommendations.

1. The plot of log WOR vs. RF always presents an SLZ for all reasonable combinations of *M*, fluid saturations, and relative permeability curves developed using Corey-type functions.
2. We characterized the SLZ by developing generalized empirical correlations using fractional flow theory and numerical reservoir simulation. These correlations may be used to estimate reservoir performance, ultimate recovery factors, water/oil ratio, and pore volumes injected for reservoirs under a waterflood. This procedure is faster than reservoir simulation and other analytical methods.
3. The correlations were developed using multivariate linear regression analysis. Our correlations provide the boundaries, slopes, and intercepts of the SLZ. The software also produces results in plots and tables, all in one report and query.
4. Using the automated tool we developed in this research that calculates the SLZ for any combination of mobility ratio, fluid saturations, and oil and water Corey exponents, we can estimate RF and PVI up to the ultimate recovery, avoiding the risks of extrapolating the SLZ beyond its boundary, overestimating reserves, and underestimating WOR .

5. This analysis must be performed on a well-by-well basis since production data will be affected by different operational problems such as well stimulations, infill drilling, and pattern changes. These may all mask net injection effects in performance and recovery. However, by detecting these deviations of the estimated SLZ, we can determine the effects of interference from one well to another, so the correlations can be used as a diagnostic tool.
6. Our correlations can be used accurately with multilayered or heterogeneous reservoirs, with or without crossflow, and with any value of injection rates. Initial gas saturation should not be higher than 4% to avoid overestimation of reserves.
7. Our procedure estimates results that can be compared and benchmarked with other analytical methods and simulation results. Also, these results can be used in most companies to support reserves calculation processes, to benchmark simulation results, and to generate first estimates of water injection requirements and production rates, facilities, and costs.
8. This procedure simplifies early identification of opportunities for the oil industry and supports existing reservoir simulation models.
9. Using our correlations as a diagnostic tool, the user may compare actual performance with the expected performance for the parameters given (for example, too-early breakthrough may indicate higher heterogeneity, water channeling, fractures, or thief zones). Other problems such as water loss or completions problems may be inferred.

10. This procedure may be used to generate correlations for special cases: extra-heavy oil fields and dip-angled, naturally fractured reservoirs. Cases with high initial gas saturations may be analyzed with this procedure, when good quality, comprehensive databases are available. Using the software, the practitioner can develop adjusted correlations based on the specific databases and analogies the practitioner may want to analyze.
11. Data required to use the correlations are usually available production data. This makes the analysis easy and fast to achieve, and results will be accurate and reliable without the need of additional and more expensive and time-consuming processes, such as reservoir simulation, at early stages of planning, contributing to cost and time efficiency.

# NOMENCLATURE

A = area, acres

$B_o$ = oil formation volume factor, RB/STB

$B_{ob}$ = oil formation volume factor at the bubblepoint pressure, RB/STB

$B_w$ = water formation volume factor, RB/STB

$e$ = base of the natural logarithm (≈ 2.71828)

$E(Y,x)$ = expected value of the mean of variable $Y$ for each value of $x$

$E_A$ = *a*real sweep efficiency, percentage

$E_D$ = Displacement Efficiency, fraction

$e_i$ = residual, or error in the fit of the correlations

$E_i$ = Vertical Sweep Efficiency, fraction

$E_R$ = Recovery Efficiency, fraction

$E_v$ = volumetric sweep efficiency after waterflood, fraction

$f_w$ = water fraction on the fractional flow calculation, fraction

$f_{wBT}$ = water fraction at breakthrough, fragment

$H_1$ = hypothesis to be tested

HCPV = hydrocarbon pore volume, fraction of PV

HCPVI = hydrocarbon pore volume injected, fraction of PV

$H_o$ = null hypothesis

$I$ = intercept, dimensionless

$i_w$ = injection rate, STBW/D

$k_{air}$ = Permeability to air, mD

$k_i$ = permeability to oil in a percentile ($i$) in the log-probability plot of permeability to estimate VDP, cp (as in Eq. 1)

$k_o$ = effective permeability to oil, fraction

$k_{ro}$ = oil relative permeability

$k_{roe}$ = oil relative permeability endpoint

$k_{rw}$ = water relative permeability

$k_{rwe}$ = water relative permeability endpoint

$k_w$ = effective permeability to water, fraction

$M$ = mobility ratio

$m$ = slope of the SLZ, fraction

$N$ = original oil in place, RB

$n_o$ = Corey oil exponent

$N_{ob}$ = original oil in place at the bubblepoint pressure, STB

$N_p$ = cumulative production, STB

$N_{pp}$ = primary cumulative production, STB

$NPV$ = Net Present Value, $

$n_w$ = Corey water exponent

$P$ = probability

$P_o$ = pressure in the oil phase, psi

$PV$ = reservoir pore volume, fraction

$P_{-value}$ = smallest level of data significance

PVI = pore volume injected, dimensionless

$P_w$ = pressure in the water phase, psi

$q_o$ = oil production, bbl/day

$q_p$ = total fluid production rate, STB

$q_w$ = water production, bbl/day

$R^2$ = statistical correlation coefficient, fraction

$RF$ = recovery factor, dimensionless

$RF_{\text{heter}}$ = recovery factor for heterogeneous reservoirs, dimensionless

$RF_{\text{homog}}$ = recovery factor for homogeneous reservoirs, dimensionless

$s$ = fluids saturation, fraction

$S_g$ = gas saturation, fraction

$S_{gi}$ = initial gas saturation, fraction

SLZ = Straight-line zone in the logwor vs. RF plot

$S_{or}$ = residual oil saturation after waterflood, fraction

$S_w$ = water saturation, fraction

$S_{w\,max}$ = max water saturation, fraction

$S_{wBT}$ = water saturation at the breakthrough, fraction

$S_{wc}$ = connate water saturation at the moment of discovery, fraction

$S_{wD}$ = dimensionless water saturation

$S_{wi}$ = irreducible water saturation, fraction

$\overline{S_o}$ = average oil saturation, fraction

$\overline{S_w}$ = average water saturation in the flooded zone, fraction

$t$ = time, days

$URF$ = Ultimate Recovery Factor, fraction

$V(Y,x)$ = variance of $Y$ with respect to $x$

VDP = Dykstra-Parsons Coefficient

VR = viscosity ratio, $\mu_w / \mu_o$

VRR = Voidage Replacement Ratio, fraction

$W_i$ = total water injected, STB

WOR = water/oil ratio, ratio

$x_i$ = correlation regressor ($i$ =1 to 14, in tables in Chapter III)

$\varepsilon$ = random error term

$\Delta t$ = time difference, days

$\alpha$ = reservoir dip angle, degrees

$\beta_1$ = intercept as an unknown regression coefficient in statistics notation

$\beta_o$ = slope as an unknown regression coefficient in statistics notation

$\lambda_{displaced}$ = mobility of the displaced phase

$\lambda_{displacing}$ = mobility of the displacing phase

$\mu_o$ = oil viscosity, cp

$\mu_w$ = water viscosity, cp

$\mu_{Y,x}$ = mean of Y with respect to x

$\rho_o$ = oil density

$\rho_w$ = water density

$\sigma$ = standard error

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# APPENDIX A

## BASIC PROGRAMS FOR HOMOGENEOUS RESERVOIRS

SAS general program to estimate homogeneous reservoir waterflood performance. This example is for water-wet systems. For oil-wet systems, only change section 4. For the following lines:

```
data BT;
* for BT:
begin counter for saturation and evaluation of relative permeabilities
and fractional flow;
  dsw= 0.001;
   bo= 1;
   boi= 1;
       e=2.71828182845904523536O3;

*ranges estimated from the relative perm curves analized and the Craig
approximation for wettability;
   Do uwuo= 0.1 To 1 by 0.2;
     Do po= 2 To 3.0 by 1;
       Do pw= 2 To 5.0 by 1;
         Do swc= 0.15 To 0.4 by 0.1;
           Do sor= 0.1 To 0.2 by 0.1;
           Do kroe=0.7 to 1 by 0.1;
            Do krwe=0.55 to 0.8 by 0.1;
 swmax= 1 - sor;
 sw=swc;

 do sw=swmax To swc by -dsw;
   *normalized water saturation;
   swd = (sw - swc) / (swmax - swc);
  If swd = 1 Then
  kro = 0.000000001;
      else kro = kroe*(1 - swd) ** po;
 If swd = 1 Then
      krw = krwe*(swd) ** pw;
      else krw = krwe*(swd) ** pw;
If swd = 1 Then
      fw = 1;
      else fw = 1 / (1 + (kro / krw) * (uwuo));
If swd = 1 Then
      dfw_1 = 1;
      else dfw_1 = (fw - 0) / (sw - swc);
if swd=1 then
  dfw_2 = 0;

  krod = kroe*(1 - (swd + 0.0001)) ** po;
  krwd = krwe*(swd + 0.0001) ** pw;
```

```
  fwd = 1 / (1 + krod / krwd * uwuo);
  dfw_3 = (fwd - fw) / 0.0001;


*Compare two successive slopes;


  diff = dfw_1 - dfw_2;

  If diff <= 0 Then
  fwbt = fw;
      If diff <= 0 Then
  swbt = sw;
      If diff <= 0 Then
  p_inj1 = 1 / (dfw_3+00000.1);
      If diff <= 0 and p_inj1>0 or p_inj1<1 Then
  swav1 = (1 - fw) / dfw_1 + sw;
      If diff <= 0 and p_inj1>0 or p_inj1<1 Then
  soav1 = 1 - swav1;
      If diff <= 0 and p_inj1>0 or p_inj1<1 Then
  RF a = ((1 - soav1 - swc) / (1 - swc)) * bo / boi;
      WOR =(krw/kro)*(1/uwuo);
      LWOR =log10(WOR o+0.0000001);
      WOR o=WOR ;
      M=(krwe/kroe)/uwuo;
      lM=log10(M);
      output;
If diff <= 0 and p_inj1>0 Then
sw = swc;
*If diff <= 0 Then;
      dfw_2 = dfw_1;

      *output;
      end;
        end;
              end;
                        end;
                              end;
                                    end;
                                          end;
                                                end;
data Work.BT;
set Work.BT;
rename po=no pw=nw uwuo=VR;
if diff>=0 then delete;
if p_inj1<=0 then delete;
if p_inj1>1 then delete;
proc sort; by VR no nw swc sor kroe krwe M;
data Work.BT;
set Work.BT;
e=2.71828182845904523536028;
x1=(VR/no);
x2=VR;
x3=no;
```

```
x4=nw;
x5=swc;
x6=sor;
x7=log10(M);
proc reg;model fwbt= x1 x2 x3 x4 x7 ;
*proc reg;*model fwbt= no nw lM /vif;
output out=front P=Predicted R=Residual;
proc reg;model swbt= x1 x2 x3 x4 x5 x6 x7 ;
*proc reg;*model swbt= no nw swc sor lM /vif;
output out=satur P=Predicted R=Residual;
*proc gplot; *plot fwbt*lM;*by VR no nw swc sor ;
*plot swbt* lM;*by VR no nw swc sor ;
*plot fwbt * swbt;*by VR no nw swc sor;*run;

data recovery1;
*  This  program  performs  Fractional  Flow  Theory  calculations  to
determine recovery factors and
WOR values based upon the LogWOR vs. RF plot. The staight line zone is
determined fron the "nose" region to the
beginning  of  the  change  in  slope  more  than  1%,  using  the  second
derivative approach.
Following  module  (data  recovery1)  performs  the  calculation  described
above.;

* Loop for different water saturation values, relative perm curves, and
fractional flow curve:;

   dsw= 0.001;
   bo= 1;
   boi= 1;
       e=2.71828182845904523536O3;
*ranges estimated from the relative perm curves analized and the Craig
approximation for wettability;

   Do uwuo= 0.1 To 1 by 0.2;
     Do po= 2 To 3.0 by 1;
       Do pw= 2 To 5.0 by 1;
         Do swc= 0.15 To 0.4 by 0.1;
           Do sor= 0.1 To 0.2 by 0.1;
            Do kroe=0.7 to 1 by 0.1;
               Do krwe=0.55 to 0.8 by 0.1;
   swmax= 1 - sor;
   sw=swc;

   do sw=swc To swmax by dsw;
   *normalized water saturation;
   swd= (sw - swc) / (1 - sor - swc) + 0.0000001;
   kro= kroe*(1 - swd) ** po;
   krw= krwe*(swd) ** pw;

   *evaluates incremental for derivatives;


   krod= kroe*(1 - (swd + 0.0001)) ** po;
```

```
  krwd= krwe*(swd + 0.0001) ** pw;
  fwd= 1 / (1 + krod / krwd * uwuo);

 *evaluates fractional flow curve;
   fw= 1 / (1 + kro / krw * uwuo);
 *Evaluates derivative of the fractional flow curve as well as
derivative from Swc;

  If sw = swc Then
  dfw_1 = 100;
           Else dfw_1= (fwd - fw) / 0.0001;
     If sw = swc Then
  dfw_2 = 0.1;
           Else dfw_2= (fw - 0) / (sw - swc);
     If sw= swc Then
  swav1= 100;
           Else swav1= (1 - fw) / dfw_2 + sw;
     If sw= swc Then
  nose= 100;
           Else nose= (RF a - RF a1) / (WOR - WOR o+.0001);
     If sw= swc Then
      RF a= 0 ;
           Else RF a= ((1 - soav1 - swc) / (1 - swc)) * bo / boi;
     soav1= 1 - swav1;
     WOR = (krw / kro) * (1 / uwuo);
     RF a= ((1 - soav1 - swc) / (1 - swc)) * bo / boi;
     RF a2= RF a - RF a1;
     * PVI SLZ :;
     LWOR = log10(WOR o + 0.000001);
     LWOR 2 = LWOR - LWOR 1;
     dzero = LWOR 2 / (RF a2+.00001);
     SLZ = dzero - dzero1;
     SLZ 2 = SLZ / (dzero + 0.001);

     p_inj1= 1 / (dfw_1 + 0.000001);
  p_inj2= 1 / dfw_2;
  p_inj12 = p_inj1-PVI.;
     dzeroPVI. = p_inj12 /(RF a2 + 0.00001);
     SLZ PVI = dzeroPVI. - dzero1PVI. ;
     SLZ PVI.2 = SLZ PVI / (dzeroPVI. + 0.001);
  LWOR = log10(WOR o + 0.000001);
     nose= (RF a - RF a1) / (WOR - WOR o);
     M=(krwe/kroe)/uwuo;
     lM=log10(M);
   *stores old value of recovery factor to evaluate nose derivative and
increase counter
   *write all other values in generic results spreadsheet;

  RF a1= RF a;
  WOR o= WOR ;
  LWOR 1= LWOR ;
     PVI = p_inj1;
  dzero1= dzero;
```

```
        dzero1PVI. = dzeroPVI.;
              output ;

    end;
         end;
               end;
                         end;
                               end;
                                     end;
                                           end;
                                                 end;
 *proc print; *var nose RF RF 1 lWOR lWOR 1 lWOR 2 dzero dzero1 SLZ SLZ
2 p_inj1 RF 2; *by uwuo po pw sor swc;
* This module deletes negative noses and fixes the tolerance for the
SLZ region;
data Work.recovery1;
set Work.recovery1;
rename po=no pw=nw uwuo=VR;
if nose < 0 or nose > 50 then delete;
if RF a > 1 then delete;
IF SLZ 2<-0.01 or SLZ 2>0.01 then delete;

* Activate the following instruction to determine the optimim RF vs PVI
only. If not activated, the program will determine the critical RF and
PVI.;
*IF SLZ PVI.2<-0.01 or SLZ PVI.2>0.01 then delete;
proc sort; by VR no nw swc sor kroe krwe M;
data Work.bt;
set Work.bt;
proc sort; by VR no nw swc sor kroe krwe M;
data merged;
merge       Work.recovery1
            Work.Bt;
by VR no nw swc sor kroe krwe M;
* This module calculates and creates a table with the slopes and
intercepts for LWOR vs. RF plots for each VR, no, nw, sor, swc case;
proc reg data= merged outest=slopes; by VR no nw swc sor kroe krwe M;
model LWOR = RF a;
run;
data Work.slopes;
set Work.slopes;
*rename RF =Slope;
e=2.71828182845904523536030;
x1=(VR/no);
x2=VR;
x3=no;
x4=nw;
x5=swc;
x6=sor;
x7=log10(M);
*proc reg; *model Slope= x2 x3 no nw sor swc lM /vif;;* kroe krwe
M/vif;
proc reg; model Slope= x1 x2 x4 x5 x6 x7 ;* kroe krwe M/vif;
```

```
output out=slopesres P=predicted1 R=residual1;
*This procedure performs the regression for the intercepts and creates
a table for calculated values;
proc reg; model Intercept= x1 x2 x4 x5 x6 x7 ;* kroe krwe M/vif;
*proc reg; *model Intercept= x2 x3 no nw sor swc lM /vif;
output out=interres P=predicted2 R=residual2;
run;
data merged2;
merge       Work.merged
            Work.interres;
by VR no nw swc sor kroe krwe M;
data Work.merged2;
set Work.merged2;
x1=VR/no;
slopecalc=predicted1;
interceptcalc=predicted2;
logWOR calc=slopecalc*RF a1+interceptcalc;
RF acalc=(lWOR -interceptcalc)/slopecalc;
diflogWOR =(logWOR calc-lWOR )/lWOR ;
difRF =(RF acalc-RF a1)/RF a1;
output;
run;
data Work.merged2;
set Work.merged2;
x1=VR/no;
if difRF > 0.05 then delete;
proc reg; model Slope= x1 x2 x4 x5 x6 x7 ; * kroe krwe M/vif;
output out=slopesres P=predicted3 R=residual3;
*This procedure performs the regression for the intercepts and creates
a table for calculated values;
proc reg; model Intercept= x1 x2 x4 x5 x6 x7 ;* kroe krwe M/vif;
*proc reg; *model Intercept= x2 x3 no nw sor swc lM /vif;
output out=interres P=predicted4 R=residual4;
run;
data Work.merged2;
set Work.merged2;
proc reg;model Slope= x1 x2 x4 x5 x6 x7 /selection=forward slentry=0.05
details; run;
proc reg;model Slope= x1 x2 x4 x5 x6 x7 /selection=backward slstay=0.05
details; run;
proc reg;model Slope= x1 x2 x4 x5 x6 x7 /selection=stepwise
slentry=0.08 details; run;
proc reg;model Slope= x1 x2 x4 x5 x6 x7 /selection=rsquare cp mse; run;
proc corr data=merged2 cov out=outcov (type=cov)nocorr noprint;
var x1 x2 x4 x5 x6 x7;
run;
proc print data=merged2(obs=12);
run;
proc mianalyze data=merged2 edf=30 mult;
var x1 x2 x4 x5 x6 x7;
run;
```

**Results from the Water-wet model:**

The REG Procedure
Model: MODEL1
Dependent Variable: fwbt

| | |
|---|---|
| Number of Observations Read | 2880 |
| Number of Observations Used | 2880 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 8.84325 | 1.76865 | 3355.12 | <.0001 |
| Error | 2874 | 1.51503 | 0.00052715 | | |
| Corrected Total | 2879 | 10.35828 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.02296 | R-Square | 0.8537 |
| Dependent Mean | 0.86652 | Adj R-Sq | 0.8535 |
| Coeff Var | 2.64964 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.71084 | 0.00543 | 130.99 | <.0001 |
| x1 | 1 | 0.20212 | 0.01815 | 11.14 | <.0001 |
| x2 | 1 | -0.08234 | 0.00858 | -9.60 | <.0001 |
| x3 | 1 | 0.01779 | 0.00174 | 10.24 | <.0001 |
| x4 | 1 | 0.03932 | 0.00038266 | 102.76 | <.0001 |
| x7 | 1 | -0.09479 | 0.00330 | -28.71 | <.0001 |

The REG Procedure
Model: MODEL1
Dependent Variable: swbt

| | |
|---|---|
| Number of Observations Read | 2880 |
| Number of Observations Used | 2880 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 7 | 20.51846 | 2.93121 | 6518.51 | <.0001 |
| Error | 2872 | 1.29146 | 0.00044967 | | |
| Corrected Total | 2879 | 21.80993 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.02121 | R-Square | 0.9408 |
| Dependent Mean | 0.64132 | Adj R-Sq | 0.9406 |
| Coeff Var | 3.30654 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.64301 | 0.00529 | 121.54 | <.0001 |
| x1 | 1 | 0.23901 | 0.01676 | 14.26 | <.0001 |
| x2 | 1 | -0.10025 | 0.00792 | -12.65 | <.0001 |
| x3 | 1 | -0.03016 | 0.00161 | -18.79 | <.0001 |
| x4 | 1 | 0.03722 | 0.00035343 | 105.30 | <.0001 |
| x5 | 1 | 0.34619 | 0.00484 | 71.53 | <.0001 |
| x6 | 1 | -0.65383 | 0.00790 | -82.73 | <.0001 |
| x7 | 1 | -0.15479 | 0.00305 | -50.75 | <.0001 |

The REG Procedure
Model: MODEL1
Dependent Variable: slope

| | |
|---|---|
| Number of Observations Read | 226977 |
| Number of Observations Used | 226977 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 3205594 | 534266 | 448942 | <.0001 |
| Error | 226970 | 270106 | 1.19005 | | |
| Corrected Total | 226976 | 3475700 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 1.09090 | R-Square | 0.9223 |
| Dependent Mean | 12.30964 | Adj R-Sq | 0.9223 |
| Coeff Var | 8.86212 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 1.29383 | 0.02174 | 59.51 | <.0001 |
| x1 | 1 | 27.81540 | 0.05745 | 484.14 | <.0001 |
| x2 | 1 | -6.79106 | 0.03085 | -220.16 | <.0001 |
| x4 | 1 | 2.08358 | 0.00209 | 995.21 | <.0001 |
| x5 | 1 | 4.76521 | 0.02929 | 162.68 | <.0001 |
| x6 | 1 | 22.31418 | 0.04731 | 471.61 | <.0001 |
| x7 | 1 | -4.79404 | 0.01729 | -277.35 | <.0001 |

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

| | |
|---|---|
| Number of Observations Read | 226977 |
| Number of Observations Used | 226977 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 2109903 | 351651 | 642917 | <.0001 |
| Error | 226970 | 124144 | 0.54696 | | |
| Corrected Total | 226976 | 2234047 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.73957 | R-Square | 0.9444 |
| Dependent Mean | -6.21398 | Adj R-Sq | 0.9444 |
| Coeff Var | -11.90168 | | |

Parameter Estimates

| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Intercept | Intercept | 1 | -1.56684 | 0.01474 | -106.30 | <.0001 |
| x1 | | 1 | -30.39889 | 0.03895 | -780.46 | <.0001 |
| x2 | | 1 | 8.83008 | 0.02091 | 422.26 | <.0001 |
| x4 | | 1 | -1.50075 | 0.00142 | -1057.4 | <.0001 |
| x5 | | 1 | -0.10520 | 0.01986 | -5.30 | <.0001 |
| x6 | | 1 | -0.58337 | 0.03208 | -18.19 | <.0001 |
| x7 | | 1 | 4.03107 | 0.01172 | 344.00 | <.0001 |

---

## Results from the Oil-wet model:

The REG Procedure
Model: MODEL1
Dependent Variable: fwbt

| | |
|---|---|
| Number of Observations Read | 8441 |
| Number of Observations Used | 8441 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 33.95133 | 6.79027 | 11820.1 | <.0001 |
| Error | 8435 | 4.84563 | 0.00057447 | | |
| Corrected Total | 8440 | 38.79696 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.02397 | R-Square | 0.8751 |
| Dependent Mean | 0.84045 | Adj R-Sq | 0.8750 |
| Coeff Var | 2.85183 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.65980 | 0.00374 | 176.61 | <.0001 |
| x1 | 1 | 0.11888 | 0.01107 | 10.74 | <.0001 |
| x2 | 1 | -0.04844 | 0.00494 | -9.81 | <.0001 |
| x3 | 1 | 0.02099 | 0.00122 | 17.15 | <.0001 |
| x4 | 1 | 0.05127 | 0.00023332 | 219.74 | <.0001 |
| x7 | 1 | -0.09342 | 0.00177 | -52.75 | <.0001 |

The REG Procedure
Model: MODEL1
Dependent Variable: swbt

| | |
|---|---|
| Number of Observations Read | 8441 |
| Number of Observations Used | 8441 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 7 | 84.21711 | 12.03102 | 23696.8 | <.0001 |
| Error | 8433 | 4.28149 | 0.00050771 | | |
| Corrected Total | 8440 | 88.49860 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.02253 | R-Square | 0.9516 |
| Dependent Mean | 0.41177 | Adj R-Sq | 0.9516 |
| Coeff Var | 5.47210 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.56187 | 0.00360 | 156.00 | <.0001 |
| x1 | 1 | 0.16777 | 0.01040 | 16.12 | <.0001 |
| x2 | 1 | -0.06865 | 0.00464 | -14.79 | <.0001 |
| x3 | 1 | -0.02195 | 0.00115 | -19.08 | <.0001 |
| x4 | 1 | 0.04494 | 0.00021935 | 204.86 | <.0001 |
| x5 | 1 | 0.41472 | 0.00417 | 99.42 | <.0001 |
| x6 | 1 | -0.58525 | 0.00208 | -281.07 | <.0001 |
| x7 | 1 | -0.14536 | 0.00166 | -87.31 | <.0001 |

The REG Procedure
Model: MODEL1
Dependent Variable: slope

| | |
|---|---|
| Number of Observations Read | 9600 |
| Number of Observations Used | 9600 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 241628 | 48326 | 12058.3 | <.0001 |
| Error | 9594 | 38449 | 4.00765 | | |
| Corrected Total | 9599 | 280077 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 2.00191 | R-Square | 0.8627 |
| Dependent Mean | 15.56270 | Adj R-Sq | 0.8626 |
| Coeff Var | 12.86352 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.77519 | 0.17979 | 4.31 | <.0001 |
| x1 | 1 | 8.93439 | 0.36963 | 24.17 | <.0001 |
| x2 | 1 | -3.36474 | 0.20697 | -16.26 | <.0001 |
| x4 | 1 | 2.67364 | 0.01827 | 146.30 | <.0001 |
| x6 | 1 | 28.23280 | 0.18275 | 154.49 | <.0001 |
| x7 | 1 | -8.40355 | 0.13878 | -60.55 | <.0001 |

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Number of Observations Read 9600
Number of Observations Used 9600

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 51074 | 8512.33172 | 23092.0 | <.0001 |
| Error | 9593 | 3536.24062 | 0.36863 | | |
| Corrected Total | 9599 | 54610 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.60715 | R-Square | 0.9352 |
| Dependent Mean | -5.59580 | Adj R-Sq | 0.9352 |
| Coeff Var | -10.85005 | | |

Parameter Estimates

| Variable | Label | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|---|
| Intercept | Intercept | 1 | -3.42537 | 0.05516 | -62.10 | <.0001 |
| x1 | | 1 | -11.94031 | 0.11210 | -106.51 | <.0001 |
| x2 | | 1 | 4.78633 | 0.06277 | 76.25 | <.0001 |
| x4 | | 1 | -1.41665 | 0.00554 | -255.60 | <.0001 |
| x5 | | 1 | -0.33004 | 0.11085 | -2.98 | 0.0029 |
| x6 | | 1 | -0.06060 | 0.05542 | -1.09 | 0.2742 |
| x7 | | 1 | 5.30721 | 0.04209 | 126.09 | <.0001 |

# APPENDIX B

## EXAMPLE OF BASIC PROGRAMS FOR HETEROGENEOUS RESERVOIRS

**Water-wet system program:**

**Recovery:**

```
data allMandVDP;
set allMandVDP;;
x2=homo**2;
proc reg;model vdp06= homo /vif r clm cli alpha=.05;
run;
proc reg;model vdp07= M lwor x2 /vif r clm cli alpha=.05;
run;
proc reg;model vdp08= M lwor x2 /vif r clm cli alpha=.05;
output out=vdp08i P=predicted8i R=residual8i;
run;
proc reg;model vdp09= M lwor x2 /vif r clm cli alpha=.05;
run;
*proc gplot;
*plot vdp08 * predicted8i;
*plot lwor * predicted8i;by M;
*plot lwor * (vdp06 vdp07 vdp08 vdp09);
*run;
```

**PVI:**

```
option ls=120 ps=75 nocenter nodate;
title 'Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and
0.6';
data allMandVDPandPVI;
infile  "c:\0001  My  Research\Tesis  Total\Writings  and  Calculations\  Nov
19\heterwaterwetPVIdata" ;
input M lwor rfhomo PVIhomo rf06 PVI06 rf07 PVI07 rf08 PVI08 rf09 PVI09;
*cards;
data allMandVDPandPVI;
set allMandVDPandPVI;
e=2.71828182845904523536O3;
x9=M;
x13=e**(lwor);
x14=10**(lwor);
proc reg;model PVIhomo= x8 x12 x13 /r clm cli alpha=.05;
output out=homo P=predictedh R=residualh;
run;
proc reg;model PVI06= x8 x12 x13 /r clm cli alpha=.05;
output out=vdp06 P=predicted6 R=residual6;
run;
proc reg;model PVI07= x8 x12 x13 /r clm cli alpha=.05;
output out=vdp07 P=predicted7 R=residual7;
run;
proc reg;model PVI08= x8 x12 x13 /r clm cli alpha=.05;
output out=vdp08 P=predicted8 R=residual8;
run;
proc reg;model PVI09= x8 x12 x13 /r clm cli alpha=.05;
output out=vdp09 P=predicted9 R=residual9;run;
```

**Results from the Water-wet model, Recovery Factors:**

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
125

The REG Procedure
Model: MODEL1
Dependent Variable: vdp06

| | |
|---|---|
| Number of Observations Read | 82 |
| Number of Observations Used | 82 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 0.31629 | 0.31629 | 4801.35 | <.0001 |
| Error | 80 | 0.00527 | 0.00006587 | | |
| Corrected Total | 81 | 0.32156 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00812 | R-Square | 0.9836 |
| Dependent Mean | 0.52645 | Adj R-Sq | 0.9834 |
| Coeff Var | 1.54170 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -0.18015 | 0.01024 | -17.60 | <.0001 |
| homo | 1 | 1.27511 | 0.01840 | 69.29 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
129

The REG Procedure
Model: MODEL1
Dependent Variable: vdp07

| | |
|---|---|
| Number of Observations Read | 82 |
| Number of Observations Used | 82 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 0.37584 | 0.12528 | 7560.63 | <.0001 |
| Error | 78 | 0.00129 | 0.00001657 | | |
| Corrected Total | 81 | 0.37713 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00407 | R-Square | 0.9966 |
| Dependent Mean | 0.50843 | Adj R-Sq | 0.9964 |
| Coeff Var | 0.80063 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.19068 | 0.01494 | 12.76 | <.0001 |
| X8 | 1 | -0.00368 | 0.00127 | -2.89 | 0.0050 |
| X10 | 1 | 0.02336 | 0.00354 | 6.59 | <.0001 |
| X11 | 1 | 0.91971 | 0.06093 | 15.10 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
133

The REG Procedure
Model: MODEL1
Dependent Variable: vdp08

| | |
|---|---|
| Number of Observations Read | 82 |
| Number of Observations Used | 82 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 0.67749 | 0.22583 | 5539.75 | <.0001 |
| Error | 78 | 0.00318 | 0.00004077 | | |
| Corrected Total | 81 | 0.68067 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00638 | R-Square | 0.9953 |
| Dependent Mean | 0.45022 | Adj R-Sq | 0.9951 |
| Coeff Var | 1.41814 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.09829 | 0.02344 | 4.19 | <.0001 |
| X8 | 1 | -0.00751 | 0.00200 | -3.76 | 0.0003 |
| X10 | 1 | 0.05354 | 0.00556 | 9.64 | <.0001 |
| X11 | 1 | 0.88719 | 0.09556 | 9.28 | <.0001 |

| | |
|---|---|
| Sum of Residuals | 0 |
| Sum of Squared Residuals | 0.00318 |
| Predicted Residual SS (PRESS) | 0.00350 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
137
The REG Procedure
Model: MODEL1
Dependent Variable: vdp09

| | |
|---|---|
| Number of Observations Read | 82 |
| Number of Observations Used | 82 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 0.58779 | 0.19593 | 19763.9 | <.0001 |
| Error | 78 | 0.00077325 | 0.00000991 | | |
| Corrected Total | 81 | 0.58856 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00315 | R-Square | 0.9987 |
| Dependent Mean | 0.40614 | Adj R-Sq | 0.9986 |
| Coeff Var | 0.77525 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.17180 | 0.01156 | 14.86 | <.0001 |
| X8 | 1 | -0.01440 | 0.00098386 | -14.64 | <.0001 |
| X10 | 1 | 0.07227 | 0.00274 | 26.37 | <.0001 |
| X11 | 1 | 0.44117 | 0.04712 | 9.36 | <.0001 |

## Results from the Water-wet model, PVI:

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
161

The REG Procedure
Model: MODEL1
Dependent Variable: PVIhomo

| | |
|---|---|
| Number of Observations Read | 81 |
| Number of Observations Used | 81 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 849.00414 | 283.00138 | 915.07 | <.0001 |
| Error | 77 | 23.81351 | 0.30927 | | |
| Corrected Total | 80 | 872.81765 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.55612 | R-Square | 0.9727 |
| Dependent Mean | 2.73765 | Adj R-Sq | 0.9717 |
| Coeff Var | 20.31364 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -0.81246 | 0.18097 | -4.49 | <.0001 |
| X8 | 1 | 0.35196 | 0.05886 | 5.98 | <.0001 |
| X12 | 1 | 0.00800 | 0.00076159 | 10.50 | <.0001 |
| X13 | 1 | 0.21624 | 0.03685 | 5.87 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
165

The REG Procedure
Model: MODEL1
Dependent Variable: PVI06

| | |
|---|---|
| Number of Observations Read | 81 |
| Number of Observations Used | 81 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 1230.20736 | 410.06912 | 805.56 | <.0001 |
| Error | 77 | 39.19689 | 0.50905 | | |
| Corrected Total | 80 | 1269.40425 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.71348 | R-Square | 0.9691 |
| Dependent Mean | 3.36284 | Adj R-Sq | 0.9679 |
| Coeff Var | 21.21653 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -1.05462 | 0.23218 | -4.54 | <.0001 |
| X8 | 1 | 0.46384 | 0.07551 | 6.14 | <.0001 |
| X12 | 1 | 0.00912 | 0.00097710 | 9.34 | <.0001 |
| X13 | 1 | 0.28413 | 0.04727 | 6.01 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
169

The REG Procedure
Model: MODEL1
Dependent Variable: PVI07

| | |
|---|---|
| Number of Observations Read | 81 |
| Number of Observations Used | 81 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 1725.72439 | 575.24146 | 706.73 | <.0001 |
| Error | 77 | 62.67400 | 0.81395 | | |
| Corrected Total | 80 | 1788.39839 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.90219 | R-Square | 0.9650 |
| Dependent Mean | 3.81432 | Adj R-Sq | 0.9636 |
| Coeff Var | 23.65272 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -1.21560 | 0.29359 | -4.14 | <.0001 |
| X8 | 1 | 0.47344 | 0.09548 | 4.96 | <.0001 |
| X12 | 1 | 0.01136 | 0.00124 | 9.20 | <.0001 |
| X13 | 1 | 0.31097 | 0.05978 | 5.20 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
173

The REG Procedure
Model: MODEL1
Dependent Variable: PVI08

| | |
|---|---|
| Number of Observations Read | 81 |
| Number of Observations Used | 81 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 3120.38902 | 1040.12967 | 490.92 | <.0001 |
| Error | 77 | 163.14421 | 2.11876 | | |
| Corrected Total | 80 | 3283.53322 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 1.45559 | R-Square | 0.9503 |
| Dependent Mean | 5.17494 | Adj R-Sq | 0.9484 |
| Coeff Var | 28.12777 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -2.06505 | 0.47368 | -4.36 | <.0001 |
| X8 | 1 | 0.58703 | 0.15405 | 3.81 | 0.0003 |
| X12 | 1 | 0.01217 | 0.00199 | 6.10 | <.0001 |
| X13 | 1 | 0.57103 | 0.09645 | 5.92 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
177

The REG Procedure
Model: MODEL1
Dependent Variable: PVI09

| | |
|---|---|
| Number of Observations Read | 81 |
| Number of Observations Used | 81 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 4122.74491 | 1374.24830 | 6894.11 | <.0001 |
| Error | 77 | 15.34892 | 0.19934 | | |
| Corrected Total | 80 | 4138.09384 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.44647 | R-Square | 0.9963 |
| Dependent Mean | 5.29914 | Adj R-Sq | 0.9961 |
| Coeff Var | 8.42536 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -0.45357 | 0.14529 | -3.12 | 0.0025 |
| X8 | 1 | 0.08084 | 0.04725 | 1.71 | 0.0911 |
| X12 | 1 | 0.02353 | 0.00061143 | 38.48 | <.0001 |
| X13 | 1 | 0.19269 | 0.02958 | 6.51 | <.0001 |

## Oil-wet system program:

### Recovery:

```
data allMandVDP;
set allMandVDP;;
x2=homo**2;
proc reg;model vdp06= homo /r clm cli alpha=.05;
run;
proc reg;model vdp07= M lwor x2 /r clm cli alpha=.05;
run;
proc reg;model vdp08= M lwor x2 /r clm cli alpha=.05;
run;
proc reg;model vdp09= M lwor x2 /r clm cli alpha=.05;
run;
*proc gplot;
*plot homo * (vdp06 vdp07 vdp08 vdp09);
*run;
```

PVI:

```
option ls=120 ps=75 nocenter nodate;
title 'Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and
0.6';
data allMandVDPandPVI;
infile "c:\0001 My Research\Tesis Total\Writings and Calculations\Ultimos a Nov
19\heterwaterwetPVIdata" ;
input M lwor rfhomo PVIhomo rf06 PVI06 rf07 PVI07 rf08 PVI08 rf09 PVI09;
data allMandVDPandPVI;
set allMandVDPandPVI;
e=2.71828182845904523536O3;
x9=M;
x11=e**(lwor);
x12=10**(lwor);
proc reg;model PVIhomo= x8 x12 x13 /r clm cli alpha=.05;
run;
proc reg;model PVI06= x8 x12 x13 /r clm cli alpha=.05;
run;
proc reg;model PVI07= x8 x12 x13 /r clm cli alpha=.05;
run;
proc reg;model PVI08= x8 x12 x13 /r clm cli alpha=.05;
run;
proc reg;model PVI09= x8 x12 x13 /r clm cli alpha=.05;
run;
*proc gplot;
*plot homo * (vdp06 vdp07 vdp08 vdp09);
*run;
```

## Results from the Oil-wet model, Recovery Factor:

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
197

The REG Procedure
Model: MODEL1
Dependent Variable: vdp06

| | |
|---|---|
| Number of Observations Read | 84 |
| Number of Observations Used | 84 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 0.21885 | 0.21885 | 2746.83 | <.0001 |
| Error | 82 | 0.00653 | 0.00007967 | | |
| Corrected Total | 83 | 0.22538 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00893 | R-Square | 0.9710 |
| Dependent Mean | 0.26381 | Adj R-Sq | 0.9707 |
| Coeff Var | 3.38349 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -0.09880 | 0.00699 | -14.14 | <.0001 |
| homo | 1 | 1.27817 | 0.02439 | 52.41 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
201

The REG Procedure
Model: MODEL1
Dependent Variable: vdp07

| | |
|---|---|
| Number of Observations Read | 84 |
| Number of Observations Used | 84 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 0.21292 | 0.07097 | 2206.41 | <.0001 |
| Error | 80 | 0.00257 | 0.00003217 | | |
| Corrected Total | 83 | 0.21549 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00567 | R-Square | 0.9881 |
| Dependent Mean | 0.25536 | Adj R-Sq | 0.9876 |
| Coeff Var | 2.22102 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.09036 | 0.01160 | 1.06 | 0.2907 |
| X8 | 1 | -0.00226 | 0.00052347 | -6.90 | <.0001 |
| X10 | 1 | 0.01459 | 0.00217 | 10.92 | <.0001 |
| X11 | 1 | 1.78812 | 0.04889 | 15.37 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
205

The REG Procedure
Model: MODEL1
Dependent Variable: vdp08

| | |
|---|---|
| Number of Observations Read | 84 |
| Number of Observations Used | 84 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 0.30137 | 0.10046 | 3562.41 | <.0001 |
| Error | 80 | 0.00226 | 0.00002820 | | |
| Corrected Total | 83 | 0.30362 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00531 | R-Square | 0.9926 |
| Dependent Mean | 0.22262 | Adj R-Sq | 0.9923 |
| Coeff Var | 2.38536 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.06727 | 0.00627 | 10.73 | <.0001 |
| X8 | 1 | -0.00489 | 0.00055858 | -8.76 | <.0001 |
| X10 | 1 | 0.03839 | 0.00260 | 14.77 | <.0001 |
| X11 | 1 | 1.27549 | 0.11041 | 11.55 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6
209

The REG Procedure
Model: MODEL1
Dependent Variable: vdp09

| | |
|---|---|
| Number of Observations Read | 84 |
| Number of Observations Used | 84 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 0.24670 | 0.08223 | 4306.13 | <.0001 |
| Error | 80 | 0.00153 | 0.00001910 | | |
| Corrected Total | 83 | 0.24823 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.00437 | R-Square | 0.9938 |
| Dependent Mean | 0.20143 | Adj R-Sq | 0.9936 |
| Coeff Var | 2.16950 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 0.07480 | 0.00516 | 14.50 | <.0001 |
| X8 | 1 | -0.00535 | 0.00045967 | -11.63 | <.0001 |
| X10 | 1 | 0.04112 | 0.00214 | 19.23 | <.0001 |
| X11 | 1 | 0.88482 | 0.09086 | 9.74 | <.0001 |

## **Results from the OIL-wet model, PVI:**

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6 9609

The REG Procedure
Model: MODEL1
Dependent Variable: PVIhomo

Number of Observations Read 87
Number of Observations Used 87

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 294.48907 | 98.16302 | 297.26 | <.0001 |
| Error | 83 | 27.40850 | 0.33022 | | |
| Corrected Total | 86 | 321.89757 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.57465 | R-Square | 0.9149 |
| Dependent Mean | 1.75655 | Adj R-Sq | 0.9118 |
| Coeff Var | 32.71468 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -0.86243 | 0.17016 | -5.07 | <.0001 |
| X8 | 1 | 0.41381 | 0.05868 | 7.05 | <.0001 |
| X12 | 1 | 0.00194 | 0.00075435 | 2.57 | 0.0119 |
| X13 | 1 | 0.24178 | 0.03561 | 6.79 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6 9610

The REG Procedure
Model: MODEL1
Dependent Variable: PVI06

| | |
|---|---|
| Number of Observations Read | 87 |
| Number of Observations Used | 87 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 616.55169 | 205.51723 | 336.05 | <.0001 |
| Error | 83 | 50.75989 | 0.61156 | | |
| Corrected Total | 86 | 667.31158 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.78203 | R-Square | 0.9239 |
| Dependent Mean | 2.45954 | Adj R-Sq | 0.9212 |
| Coeff Var | 31.79562 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -1.26805 | 0.23157 | -5.48 | <.0001 |
| X8 | 1 | 0.49769 | 0.07986 | 6.23 | <.0001 |
| X12 | 1 | 0.00238 | 0.00103 | 2.32 | 0.0228 |
| X13 | 1 | 0.37376 | 0.04846 | 7.71 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6 9614

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6 9617

The REG Procedure
Model: MODEL1
Dependent Variable: PVI07

Number of Observations Read 87
Number of Observations Used 87

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 997.31606 | 332.43869 | 510.59 | <.0001 |
| Error | 83 | 54.04059 | 0.65109 | | |
| Corrected Total | 86 | 1051.35665 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.80690 | R-Square | 0.9486 |
| Dependent Mean | 2.77621 | Adj R-Sq | 0.9467 |
| Coeff Var | 29.06492 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -1.11471 | 0.23894 | -4.67 | <.0001 |
| X8 | 1 | 0.43417 | 0.08240 | 5.27 | <.0001 |
| X12 | 1 | 0.00691 | 0.00106 | 6.52 | <.0001 |
| X13 | 1 | 0.30144 | 0.05001 | 6.03 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6 9618

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6 9621

The REG Procedure
Model: MODEL1
Dependent Variable: PVI08

| | |
|---|---|
| Number of Observations Read | 87 |
| Number of Observations Used | 87 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 2157.88326 | 719.29442 | 537.78 | <.0001 |
| Error | 83 | 111.01378 | 1.33752 | | |
| Corrected Total | 86 | 2268.89704 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 1.15651 | R-Square | 0.9511 |
| Dependent Mean | 3.77747 | Adj R-Sq | 0.9493 |
| Coeff Var | 30.61598 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -1.26744 | 0.34246 | -3.70 | 0.0004 |
| X8 | 1 | 0.45081 | 0.11810 | 3.82 | 0.0003 |
| X12 | 1 | 0.01279 | 0.00152 | 8.42 | <.0001 |
| X13 | 1 | 0.32235 | 0.07167 | 4.50 | <.0001 |

Gral corr behavior, Homo, VDP 0.6, 0.7, 0.8, and 0.9 with M= 1, 3 and 0.6 9625

The REG Procedure
Model: MODEL1
Dependent Variable: PVI09

| | |
|---|---|
| Number of Observations Read | 87 |
| Number of Observations Used | 87 |

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 1888.85706 | 629.61902 | 3387.07 | <.0001 |
| Error | 83 | 15.42881 | 0.18589 | | |
| Corrected Total | 86 | 1904.28586 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.43115 | R-Square | 0.9919 |
| Dependent Mean | 3.44690 | Adj R-Sq | 0.9916 |
| Coeff Var | 12.50832 | | |

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | -0.64968 | 0.12767 | -5.09 | <.0001 |
| X8 | 1 | 0.18524 | 0.04403 | 4.21 | <.0001 |
| X12 | 1 | 0.01370 | 0.00056597 | 24.20 | <.0001 |
| X13 | 1 | 0.22142 | 0.02672 | 8.29 | <.0001 |

# APPENDIX C

## EXAMPLE OF BASIC DATA FILES FOR HETEROGENEOUS RESERVOIRS

Table 25—Data for heterogeneous reservoirs, example for water-wet systems.

| M | LogWOR | RF homo | PVI homo | RF 0.6 | PVI 0.6 | RF 0.7 | PVI 0.7 |
|---|---|---|---|---|---|---|---|
| 0.6 | 0.0 | 0.49 | 0.30 | 0.45 | 0.35 | 0.43 | 0.30 |
| 0.6 | 0.3 | 0.50 | 0.32 | 0.47 | 0.38 | 0.45 | 0.32 |
| 0.6 | 0.6 | 0.53 | 0.35 | 0.48 | 0.39 | 0.46 | 0.37 |
| 0.6 | 0.7 | 0.53 | 0.35 | 0.48 | 0.41 | 0.47 | 0.39 |
| 0.6 | 0.8 | 0.54 | 0.37 | 0.49 | 0.42 | 0.47 | 0.42 |
| 0.6 | 0.9 | 0.54 | 0.39 | 0.49 | 0.46 | 0.48 | 0.46 |
| 0.6 | 1.0 | 0.54 | 0.42 | 0.51 | 0.52 | 0.49 | 0.49 |
| 0.6 | 1.1 | 0.55 | 0.46 | 0.51 | 0.59 | 0.49 | 0.55 |
| 0.6 | 1.2 | 0.55 | 0.52 | 0.52 | 0.65 | 0.50 | 0.59 |
| 0.6 | 1.3 | 0.56 | 0.55 | 0.53 | 0.79 | 0.51 | 0.72 |
| 0.6 | 1.4 | 0.56 | 0.65 | 0.54 | 0.85 | 0.51 | 0.79 |
| 0.6 | 1.5 | 0.56 | 0.71 | 0.54 | 0.92 | 0.52 | 0.98 |
| 0.6 | 1.6 | 0.57 | 0.78 | 0.55 | 1.18 | 0.53 | 1.18 |
| 0.6 | 1.7 | 0.58 | 1.04 | 0.56 | 1.31 | 0.54 | 1.44 |
| 0.6 | 1.8 | 0.58 | 1.18 | 0.56 | 1.44 | 0.55 | 1.77 |
| 0.6 | 1.9 | 0.59 | 1.44 | 0.57 | 1.71 | 0.56 | 2.10 |
| 0.6 | 2.0 | 0.59 | 1.70 | 0.57 | 2.10 | 0.56 | 2.50 |
| 0.6 | 2.1 | 0.60 | 2.09 | 0.58 | 2.50 | 0.57 | 3.03 |
| 0.6 | 2.2 | 0.60 | 2.49 | 0.59 | 3.03 | 0.58 | 3.62 |
| 0.6 | 2.3 | 0.60 | 3.01 | 0.59 | 3.55 | 0.58 | 4.28 |
| 0.6 | 2.4 | 0.61 | 3.53 | 0.60 | 4.34 | 0.59 | 5.07 |
| 0.6 | 2.5 | 0.61 | 4.32 | 0.60 | 5.13 | 0.59 | 6.06 |
| 0.6 | 2.6 | 0.62 | 5.11 | 0.60 | 6.19 | 0.60 | 7.24 |
| 0.6 | 2.7 | 0.62 | 6.15 | 0.61 | 7.37 | 0.60 | 8.63 |
| 0.6 | 2.8 | 0.62 | 7.33 | 0.61 | 8.69 | 0.61 | 10.21 |
| 0.6 | 2.9 | 0.62 | 8.64 | 0.61 | 10.41 | 0.61 | 12.05 |
| 0.6 | 3.0 | 0.63 | 9.56 | 0.62 | 11.46 | 0.61 | 13.24 |

Table 26—Data for heterogeneous reservoirs, example for oil-wet systems.

| M | LogWOR | RF homo | PVI homo | RF 0.6 | PVI 0.6 | RF 0.7 | PVI 0.7 |
|---|---|---|---|---|---|---|---|
| 0.6 | 0.0 | 0.23 | 0.20 | 0.17 | 0.16 | 0.20 | 0.18 |
| 0.6 | 0.1 | 0.23 | 0.21 | 0.19 | 0.17 | 0.20 | 0.18 |
| 0.6 | 0.3 | 0.25 | 0.22 | 0.19 | 0.18 | 0.20 | 0.19 |
| 0.6 | 0.4 | 0.25 | 0.23 | 0.20 | 0.21 | 0.21 | 0.21 |
| 0.6 | 0.5 | 0.26 | 0.26 | 0.22 | 0.23 | 0.21 | 0.22 |
| 0.6 | 0.6 | 0.27 | 0.28 | 0.22 | 0.26 | 0.22 | 0.26 |
| 0.6 | 0.8 | 0.27 | 0.30 | 0.23 | 0.33 | 0.23 | 0.32 |
| 0.6 | 0.9 | 0.28 | 0.35 | 0.25 | 0.39 | 0.24 | 0.39 |
| 0.6 | 1.0 | 0.28 | 0.39 | 0.25 | 0.44 | 0.25 | 0.42 |
| 0.6 | 1.1 | 0.28 | 0.42 | 0.26 | 0.52 | 0.25 | 0.45 |
| 0.6 | 1.2 | 0.29 | 0.47 | 0.27 | 0.58 | 0.26 | 0.52 |
| 0.6 | 1.3 | 0.29 | 0.52 | 0.27 | 0.65 | 0.26 | 0.65 |
| 0.6 | 1.4 | 0.30 | 0.56 | 0.28 | 0.78 | 0.27 | 0.71 |
| 0.6 | 1.5 | 0.30 | 0.65 | 0.28 | 0.91 | 0.27 | 0.84 |
| 0.6 | 1.6 | 0.31 | 0.78 | 0.29 | 1.04 | 0.28 | 0.97 |
| 0.6 | 1.7 | 0.31 | 0.91 | 0.30 | 1.30 | 0.28 | 1.10 |
| 0.6 | 1.8 | 0.31 | 1.04 | 0.30 | 1.43 | 0.29 | 1.30 |
| 0.6 | 1.9 | 0.31 | 1.17 | 0.30 | 1.69 | 0.29 | 1.56 |
| 0.6 | 2.0 | 0.32 | 1.30 | 0.31 | 1.95 | 0.29 | 1.88 |
| 0.6 | 2.1 | 0.32 | 1.43 | 0.31 | 2.21 | 0.30 | 2.33 |
| 0.6 | 2.2 | 0.32 | 1.69 | 0.31 | 2.46 | 0.30 | 2.79 |
| 0.6 | 2.3 | 0.32 | 1.95 | 0.32 | 2.85 | 0.31 | 3.37 |
| 0.6 | 2.4 | 0.32 | 2.21 | 0.32 | 3.24 | 0.31 | 4.02 |
| 0.6 | 2.5 | 0.32 | 2.46 | 0.32 | 3.63 | 0.31 | 4.73 |
| 0.6 | 2.6 | 0.32 | 2.85 | 0.32 | 4.15 | 0.32 | 5.58 |
| 0.6 | 2.7 | 0.33 | 3.24 | 0.32 | 4.67 | 0.32 | 6.42 |
| 0.6 | 2.8 | 0.33 | 3.63 | 0.32 | 5.32 | 0.32 | 7.33 |
| 0.6 | 2.9 | 0.33 | 4.15 | 0.32 | 6.10 | 0.32 | 8.30 |
| 0.6 | 3.0 | 0.33 | 4.41 | 0.33 | 6.62 | 0.32 | 8.89 |

# APPENDIX D

## SIMULATION MODEL FOR HOMOGENEOUS RESERVOIR

```
------------------------------------------------------------------------
-- Black Oil model for waterflooding process.
-- This model is only for WATER INJECTION.
-- Grid 19x19x10 = 3610 cells
------------------------------------------------------------------------

RUNSPEC
TITLE
350204 homogeneo vs heterogeneo VR=0.1
DIMENS
-- NX  NY  NZ
  19  19  10 /

NONNC

-- NOSIM

OIL

WATER

FIELD

IMPLICIT

EQLDIMS
  1 100  50  1  50 /

TABDIMS
--# of sat #of PVT  max # sat max # press max # of   max # Rs
--tables  tables  nodes   nodes    FIP regions  nodes
  1    1     100     100      1       100  /

WELLDIMS
--max #  #of conn  max #  max # wells
--wells  per well  groups  per group
  5   50     2    5 /

START
 19 'JUL' 2006 /

-- Specifies the size of the stack for Newton iterations
NSTACK
10 /

UNIFOUT

UNIFIN
```

```
DEBUG
 2 0 0 0 0 0 1/


GRID
--============
-- PERMX,Y CONSTTE = 120

EQUALS
'DX'    20 /
'DY'    20 /
'DZ'    10 /
'PORO'  0.3 /
'TOPS'  8325    1 19 1 19  1  1  /
'PERMX'  200   1 19 1 19  1  1  /
'PERMX'  200   1 19 1 19  2  2  /
'PERMX'  200   1 19 1 19  3  3  /
'PERMX'  200   1 19 1 19  4  4  /
'PERMX'  200   1 19 1 19  5  5  /
'PERMX'  200   1 19 1 19  6  6  /
'PERMX'  200   1 19 1 19  7  7  /
'PERMX'  200   1 19 1 19  8  8  /
'PERMX'  200   1  19 1 19  9  9  /
'PERMX'  200   1 19 1 19  10 10  /
/

COPY
   'PERMX'  'PERMY' 1 19 1 19 1 10 /
   'PERMX'  'PERMZ' 1 19 1 19 1 10 /
/


MULTIPLY
--    Kv/Kh
 PERMZ  0.1   1 19 1 19 1 10 /
/

INIT


PROPS  =============

SWOF
-- Sw   Krw     Kro                         PCwo
-- Sat water int = 0.4
0.40000                    0.00000             0.90000  0.00000
0.40667                    0.00000             0.85575  0.00000
0.41333                    0.00000             0.81297  0.00000
0.42000                    0.00000             0.77164  0.00000
0.42667                    0.00000             0.73173  0.00000
0.43333                    0.00000             0.69323  0.00000
0.44000                    0.00001             0.65610  0.00000
```

| | | | |
|---|---|---|---|
| 0.44667 | 0.00001 | 0.62032 | 0.00000 |
| 0.45333 | 0.00002 | 0.58587 | 0.00000 |
| 0.46000 | 0.00004 | 0.55271 | 0.00000 |
| 0.46667 | 0.00007 | 0.52083 | 0.00000 |
| 0.47333 | 0.00011 | 0.49020 | 0.00000 |
| 0.48000 | 0.00018 | 0.46080 | 0.00000 |
| 0.48667 | 0.00026 | 0.43260 | 0.00000 |
| 0.49333 | 0.00038 | 0.40557 | 0.00000 |
| 0.50000 | 0.00054 | 0.37969 | 0.00000 |
| 0.50667 | 0.00074 | 0.35493 | 0.00000 |
| 0.51333 | 0.00100 | 0.33128 | 0.00000 |
| 0.52000 | 0.00134 | 0.30870 | 0.00000 |
| 0.52667 | 0.00175 | 0.28717 | 0.00000 |
| 0.53333 | 0.00226 | 0.26667 | 0.00000 |
| 0.54000 | 0.00289 | 0.24716 | 0.00000 |
| 0.54667 | 0.00365 | 0.22863 | 0.00000 |
| 0.55333 | 0.00455 | 0.21105 | 0.00000 |
| 0.56000 | 0.00563 | 0.19440 | 0.00000 |
| 0.56667 | 0.00691 | 0.17865 | 0.00000 |
| 0.57333 | 0.00840 | 0.16377 | 0.00000 |
| 0.58000 | 0.01015 | 0.14974 | 0.00000 |
| 0.58667 | 0.01217 | 0.13653 | 0.00000 |
| 0.59333 | 0.01451 | 0.12413 | 0.00000 |
| 0.60000 | 0.01719 | 0.11250 | 0.00000 |
| 0.60667 | 0.02025 | 0.10162 | 0.00000 |
| 0.61333 | 0.02373 | 0.09147 | 0.00000 |
| 0.62000 | 0.02768 | 0.08201 | 0.00000 |
| 0.62667 | 0.03214 | 0.07323 | 0.00000 |
| 0.63333 | 0.03715 | 0.06510 | 0.00000 |
| 0.64000 | 0.04277 | 0.05760 | 0.00000 |
| 0.64667 | 0.04905 | 0.05070 | 0.00000 |
| 0.65333 | 0.05604 | 0.04437 | 0.00000 |
| 0.66000 | 0.06382 | 0.03859 | 0.00000 |
| 0.66667 | 0.07243 | 0.03333 | 0.00000 |
| 0.67333 | 0.08195 | 0.02858 | 0.00000 |
| 0.68000 | 0.09244 | 0.02430 | 0.00000 |
| 0.68667 | 0.10398 | 0.02047 | 0.00000 |
| 0.69333 | 0.11665 | 0.01707 | 0.00000 |
| 0.70000 | 0.13052 | 0.01406 | 0.00000 |
| 0.70667 | 0.14568 | 0.01143 | 0.00000 |
| 0.71333 | 0.16222 | 0.00915 | 0.00000 |
| 0.72000 | 0.18022 | 0.00720 | 0.00000 |
| 0.72667 | 0.19980 | 0.00555 | 0.00000 |
| 0.73333 | 0.22103 | 0.00417 | 0.00000 |
| 0.74000 | 0.24404 | 0.00304 | 0.00000 |
| 0.74667 | 0.26892 | 0.00213 | 0.00000 |
| 0.75333 | 0.29579 | 0.00143 | 0.00000 |
| 0.76000 | 0.32477 | 0.00090 | 0.00000 |
| 0.76667 | 0.35598 | 0.00052 | 0.00000 |
| 0.77333 | 0.38954 | 0.00027 | 0.00000 |
| 0.78000 | 0.42558 | 0.00011 | 0.00000 |
| 0.78667 | 0.46424 | 0.00003 | 0.00000 |
| 0.79333 | 0.50567 | 0.00000 | 0.00000 |

```
0.80000                    0.55000           0.00000 0.00000
-- Sat oil res = 0.2

/

-- PVT PROPERTIES OF WATER
-- REF. PRES. REF. FVF COMPRESSIBILITY REF VISCOSITY VISCOSIBILITY
PVTW
   4014.7   1.0    3.13D-6      1.0      0 /

ROCK
--  REF. PRES  COMPRESSIBILITY
   14.7     3.0D-6     /

DENSITY
--    OIL  WATER  GAS
    49.1  62.428 0.06054 /


PVDO
--to be updated
--Press  Bo  Visc
 400 1.0004 5.160
 1200 1.0003 5.164
 2000 1.0002 5.167
 2800 1.0001 5.550
 3600 1.0000 5.000
 4400 0.9999 5.0001
 5200 0.9998 5.0002
 6000 0.9997 5.0003
 7000 0.9996 5.0004
 8000 0.9995 5.0005
 9000 0.9994 5.0006
  /


SOLUTION =====================

-- DATA FOR INITIALISING FLUIDS TO POTENTIAL EQUILIBRIUM
--
--  DATUM DATUM  OWC  OWC  GOC  GOC  RSVD  RVVD  SOLN
--  DEPTH   PRESS   DEPTH  PCOW DEPTH  PCOG TABLE TABLE  METH
EQUIL
   8400  3665.3518  15000  0  /

RPTRST
 BASIC=2 /

SUMMARY ============


--REQUEST PRINTED OUTPUT OF SUMMARY FILE DATA
```

```
RUNSUM

-- FIELD OIL PRODUCTION, Cumulative oil prod. for field and for every well
FOPR
FOPT
WOPT
/
WOPR
/

-- FIELD WATER INJ. RATE, Cumulative water inj. for field and for every well
FWIR
FWIT
WWIT
/
WWIR
/

--INSTANTANEOUS WATER CUTS FOR FIELD AND FOR EVERY WELL.
FWCT
WWCT
/
FWPR
WWPR
/

--OIL IN PLACE for field and for every FIP Region
FOIP
ROIP
/

--Water in place for field and for every FIP region
FWIP
RWIP
/

--Average PRESSURE for field and for every FIP region
FPR
RPR
/

-- WELL BOTTOM-HOLE PRESSURE
WBHP
/

TCPU
ELAPSED
SEPARATE

RPTONLY

SCHEDULE ===============================================================
```

```
-------- THE SCHEDULE SECTION DEFINES THE OPERATIONS TO BE SIMULATED
----------------------------------------------------------------------

-- WELL SPECIFICATION DATA
--
--WELL  GROUP LOCATION BHP  PI
--NAME  NAME  I   J  DEPTH DEFN
WELSPECS
'INJ1'    'G'  1   1  1* 'WATER'  1*  'STD'  'SHUT'  'NO'  /
'PROD1'     'G' 19 19  1* 'WATER'  1*  'STD'  'SHUT'  'NO'  /
/

-- COMPLETION SPECIFICATION DATA
--
--WELL  -LOCATION-  OPEN/ SAT CONN WELL
--NAME  I J K1 K2  SHUT TAB FACT DIAM
COMPDAT
'PROD1' 19 19 1 10 'OPEN'  2*  0.5  /
'INJ1'    1  1 1 10 'OPEN'  2*  0.5  /
 /
-- SELECCIONAR WCONPROD Y WCONINJE SEGUN LA SENSIBILIDAD QUE QUERA (PCTTE
O DECL)
WCONPROD
--WELL  OPEN/ CNTL  OIL  WATER GAS  LIQU RES  BHP
--NAME  SHUT  MODE  RATE RATE RATE RATE RATE
'PROD1' 'OPEN' 'RESV' 4* 2000 4* /
 /


WCONINJE
-- PARA RECORDAR: PARA MANTENER PRESION CONTROLAR RESV. SI CONTROLO LRAT O
RATE Y QUIERO MANTENER PRESION
-- DEBO AJUSTAR POR FACTORES VOLUMETRICOS BW Y BO.
--WELL   INJ   OPEN/  CNTL   SURF . FLOW  RESV. FLOW BHP
--NAME  TYPE  SHUT    MODE  RATE RATE   RATE       UPPER
'INJ1' 'WATER' 'OPEN' 'RESV' 1* 2000 4* /
 /

-- WCONPROD
-- 'PROD1' 'OPEN' 'LRAT' 3* 20 5* /
-- /
--
-- WCONINJE
-- 'INJ1' 'WATER' 'OPEN' 'RATE' 20 5* /
-- /

RPTSCHED
FIP=2 WELLS=4 /


TUNING
 1* 10 /
/
```

```
/

TSTEP
-- 25 years
900*50
 /

END   ==================================================================
```

# APPENDIX E

## EXAMPLE OF SIMULATION MODEL FOR HETEROGENEOUS RESERVOIR

```
-----------------------------------------------------------------------
-- Black Oil model for waterflooding process.
-- This model is only for WATER INJECTION.
-- Grid 19x19x10 = 3610 cells
-----------------------------------------------------------------------

RUNSPEC
TITLE
350204 homogeneous vs. heterogeneous VR=0.1
DIMENS
-- NX  NY  NZ
  19  19  10 /

NONNC

OIL

WATER

FIELD

IMPLICIT

EQLDIMS
  1 100  50  1  50 /

TABDIMS
--# of sat #of PVT  max # sat max # press max # of   max # Rs
--tables  tables  nodes   nodes    FIP regions  nodes
  1    1     100     100      1       100  /

WELLDIMS
--max #  #of conn  max #  max # wells
--wells  per well  groups  per group
  5   50     2    5 /

START
 19 'JUL' 2006 /

-- Specifies the size of the stack for Newton iterations
NSTACK
10 /

UNIFOUT

UNIFIN

DEBUG
```

```
 2 0 0 0 0 0 1/

GRID

EQUALS
'DX'    20 /
'DY'    20 /
'DZ'    10 /
'PORO'  0.3 /
'TOPS'  8325    1 19 1 19  1  1  /
'PERMX'  200   1 19 1 19  1  1  /
'PERMX'  140   1 19 1 19  2  2  /
'PERMX'  110   1 19 1 19  3  3  /
'PERMX'  80   1 19 1 19  4  4  /
'PERMX'  60   1 19 1 19  5  5  /
'PERMX'  40   1 19 1 19  6  6  /
'PERMX'  10   1 19 1 19  7  7  /
'PERMX'   5   1 19 1 19  8  8  /
'PERMX'   1   1  19 1 19  9  9  /
'PERMX'  0.2   1 19 1 19  10 10  /
/

COPY
   'PERMX'  'PERMY' 1 19 1 19 1 10 /
   'PERMX'  'PERMZ' 1 19 1 19 1 10 /
/


MULTIPLY
--    Kv/Kh
 PERMZ  0.1  1 19 1 19 1 10 /
/

INIT


PROPS  =============

SWOF
-- Sw   Krw     Kro                         PCwo
-- Sat water int = 0.4
0.25                        0           0.9         0
0.254166667                 0.00025                 0.87025  0
0.258333333                 0.001    0.841          0
0.2625                      0.00225                 0.81225  0
0.266666667                 0.004    0.784          0
0.270833333                 0.00625                 0.75625  0
0.275                       0.009    0.729          0
0.279166667                 0.01225                 0.70225  0
0.283333333                 0.016    0.676          0
0.2875                      0.02025                 0.65025  0
0.291666667                 0.025    0.625          0
```

| | | | |
|---|---|---|---|
| 0.295833333 | 0.03025 | 0.60025 | 0 |
| 0.3 | 0.036 | 0.576 | 0 |
| 0.304166667 | 0.04225 | 0.55225 | 0 |
| 0.308333333 | 0.049 | 0.529 | 0 |
| 0.3125 | 0.05625 | 0.50625 | 0 |
| 0.316666667 | 0.064 | 0.484 | 0 |
| 0.320833333 | 0.07225 | 0.46225 | 0 |
| 0.325 | 0.081 | 0.441 | 0 |
| 0.329166667 | 0.09025 | 0.42025 | 0 |
| 0.333333333 | 0.1 | 0.4 | 0 |
| 0.3375 | 0.11025 | 0.38025 | 0 |
| 0.341666667 | 0.121 | 0.361 | 0 |
| 0.345833333 | 0.13225 | 0.34225 | 0 |
| 0.35 | 0.144 | 0.324 | 0 |
| 0.354166667 | 0.15625 | 0.30625 | 0 |
| 0.358333333 | 0.169 | 0.289 | 0 |
| 0.3625 | 0.18225 | 0.27225 | 0 |
| 0.366666667 | 0.196 | 0.256 | 0 |
| 0.370833333 | 0.21025 | 0.24025 | 0 |
| 0.375 | 0.225 | 0.225 | 0 |
| 0.379166667 | 0.24025 | 0.21025 | 0 |
| 0.383333333 | 0.256 | 0.196 | 0 |
| 0.3875 | 0.27225 | 0.18225 | 0 |
| 0.391666667 | 0.289 | 0.169 | 0 |
| 0.395833333 | 0.30625 | 0.15625 | 0 |
| 0.4 | 0.324 | 0.144 | 0 |
| 0.404166667 | 0.34225 | 0.13225 | 0 |
| 0.408333333 | 0.361 | 0.121 | 0 |
| 0.4125 | 0.38025 | 0.11025 | 0 |
| 0.416666667 | 0.4 | 0.1 | 0 |
| 0.420833333 | 0.42025 | 0.09025 | 0 |
| 0.425 | 0.441 | 0.081 | 0 |
| 0.429166667 | 0.46225 | 0.07225 | 0 |
| 0.433333333 | 0.484 | 0.064 | 0 |
| 0.4375 | 0.50625 | 0.05625 | 0 |
| 0.441666667 | 0.529 | 0.049 | 0 |
| 0.445833333 | 0.55225 | 0.04225 | 0 |
| 0.45 | 0.576 | 0.036 | 0 |
| 0.454166667 | 0.60025 | 0.03025 | 0 |
| 0.458333333 | 0.625 | 0.025 | 0 |
| 0.4625 | 0.65025 | 0.02025 | 0 |
| 0.466666667 | 0.676 | 0.016 | 0 |
| 0.470833333 | 0.70225 | 0.01225 | 0 |
| 0.475 | 0.729 | 0.009 | 0 |
| 0.479166667 | 0.75625 | 0.00625 | 0 |
| 0.483333333 | 0.784 | 0.004 | 0 |
| 0.4875 | 0.81225 | 0.00225 | 0 |
| 0.491666667 | 0.841 | 0.001 | 0 |
| 0.495833333 | 0.87025 | 0.00025 | 0 |
| 0.5 | 0.9 | 0 | 0 |

-- Sat oil res = 0.2

/

```
-- PVT PROPERTIES OF WATER
-- REF. PRES. REF. FVF COMPRESSIBILITY REF VISCOSITY VISCOSIBILITY
PVTW
   4014.7   1.0    3.13D-6      3.0      0 /

ROCK
--  REF. PRES  COMPRESSIBILITY
   14.7     3.0D-6     /

DENSITY
--    OIL  WATER  GAS
    49.1  62.428 0.06054 /


PVDO
--to be updated
--Press  Bo  Visc
 400 1.0004 5.160
 1200 1.0003 5.164
 2000 1.0002 5.167
 2800 1.0001 5.550
 3600 1.0000 5.000
 4400 0.9999 5.0001
 5200 0.9998 5.0002
 6000 0.9997 5.0003
 7000 0.9996 5.0004
 8000 0.9995 5.0005
 9000 0.9994 5.0006
  /



SOLUTION ====================

-- DATA FOR INITIALIZING FLUIDS TO POTENTIAL EQUILIBRIUM
--
--  DATUM DATUM  OWC  OWC  GOC  GOC  RSVD  RVVD  SOLN
--  DEPTH   PRESS   DEPTH  PCOW DEPTH  PCOG TABLE TABLE  METH
EQUIL
   8400  3665.3518  15000  0  /

RPTRST
 BASIC=2 /

SUMMARY ===========


--REQUEST PRINTED OUTPUT OF SUMMARY FILE DATA

RUNSUM

-- FIELD OIL PRODUCTION, Cumulative oil prod. for field and for every well
FOPR
```

```
FOPT
WOPT
/
WOPR
/

-- FIELD WATER INJ. RATE, Cumulative water inj. for field and for every well
FWIR
FWIT
WWIT
/
WWIR
/

--INSTANTANEOUS WATER CUTS FOR FIELD AND FOR EVERY WELL.
FWCT
WWCT
/
FWPR
WWPR
/

--OIL IN PLACE for field and for every FIP Region
FOIP
ROIP
/

--Water in place for field and for every FIP region
FWIP
RWIP
/

--Average PRESSURE for field and for every FIP region
FPR
RPR
/

-- WELL BOTTOM-HOLE PRESSURE
WBHP
/

TCPU
ELAPSED
SEPARATE

RPTONLY

SCHEDULE ============================================================
-------- THE SCHEDULE SECTION DEFINES THE OPERATIONS TO BE SIMULATED
----------------------------------------------------------------------

-- WELL SPECIFICATION DATA
--
```

```
--WELL  GROUP LOCATION BHP  PI
--NAME  NAME  I   J  DEPTH DEFN
WELSPECS
'INJ1'    'G'  1   1  1* 'WATER'  1*  'STD'  'SHUT'  'NO'  /
'PROD1'     'G' 19 19  1* 'WATER'  1*  'STD'  'SHUT'  'NO'  /
/

-- COMPLETION SPECIFICATION DATA
--
--WELL  -LOCATION-  OPEN/ SAT CONN WELL
--NAME  I J K1 K2  SHUT TAB FACT DIAM
COMPDAT
'PROD1' 19 19 1 10 'OPEN'  2*  0.5  /
'INJ1'    1  1 1 10 'OPEN'  2*  0.5  /
 /
WCONPROD
--WELL  OPEN/ CNTL  OIL  WATER GAS  LIQU RES  BHP
--NAME  SHUT  MODE  RATE RATE RATE RATE RATE
'PROD1' 'OPEN' 'RESV' 4* 2000 4* /
 /


WCONINJE
--WELL   INJ   OPEN/  CNTL   SURF . FLOW  RESV. FLOW BHP
--NAME  TYPE  SHUT    MODE  RATE RATE   RATE       UPPER
'INJ1' 'WATER' 'OPEN' 'RESV' 1* 2000 4* /
 /

-- WCONPROD
-- 'PROD1' 'OPEN' 'LRAT' 3* 20 5* /
-- /
--
-- WCONINJE
-- 'INJ1' 'WATER' 'OPEN' 'RATE' 20 5* /
-- /
RPTSCHED
FIP=2 WELLS=4 /

TUNING
 1* 10 /
/
TSTEP
-- 25 years
900*50 / END
```

# APPENDIX F

## APPLICATION IN VBA FOR QUICK SLZ AND RECOVERY CALCULATIONS

Arnaldo Espinel

```
' This procedure calculates Waterflooding projects performance
' Part one:
' Declaration of Variables
Option Explicit


' Fractional Flow Calculations
Sub homogeneous()
    Dim i, j, moil, mwater, no, nw, kroe, krwe, sor, swc, M, x8, VDP, WOR, logM, lWOR, e, x1, x12, x13, VR, Slope, Intercept, RF, RFprev, RFheter, PVI, PVIhomo, PVIhomorev, PVIrev, fwBT, SwBT, maxlog, wet As Double
    Dim maxlog05, maxlog06, maxlog07, maxlog08, maxlog09, maxlog05a, maxlog06a, maxlog07a, maxlog08a, maxlog09a, minwor As Double
   Dim points, sw(10000), kro(10000), krw(10000), swd(10000), fw(10000), pointsVDP(10000), RFheter2, RFheterx, RFhetery, PVI2 As Double
   Dim x2, x3, x4, x5, x6, x7, x9, x10, x11, x120, x130, lwor2, RFprev2, RF2, worhomomin, worhomomina, worhomomax, worhomomaxa As Double

' Data Table construction

With ThisWorkbook.Worksheets("Calculations")
    Range("d1,d200:n1,n200").ClearContents
    Cells(1, 1) = "Input your data before running:"
     Cells(2, 1) = "Reservoir Name"
    'Name = .Cells(2, 2)
     Cells(3, 1) = "Oil Viscosity, cP"
    moil = Cells(3, 2)
    Cells(4, 1) = "Water Viscosity, cP"
    mwater = Cells(4, 2)
    Cells(5, 1) = "Corey exponent for oil(no)"
    no = Cells(5, 2)
    Cells(6, 1) = "Corey exponent for water(nw)"
    nw = Cells(6, 2)
    Cells(7, 1) = "Oil rel perm curve end-point (kroe)"
    kroe = Cells(7, 2)
    Cells(8, 1) = "Water rel perm curve end-point (krwe)"
    krwe = Cells(8, 2)
    Cells(9, 1) = "Residual oil saturation (Sor)"
    sor = Cells(9, 2)
    Cells(10, 1) = "Connate water saturation (Swc)"
    swc = Cells(10, 2)
    Cells(11, 1) = "Dykstra-Parsons coeff. (VDP)"
    VDP = Cells(11, 2)
    Cells(12, 1) = "Water-wet=1 or Oil-wet=2"
    wet = Cells(12, 2)
    Cells(13, 1) = "Estimated max operational WOR"
    WOR = Cells(13, 2)
```

```
'wettability according to kr's plot:
 points = 10
 For i = 1 To points + 1
   If i = 1 Then
       sw(i) = swc
       Else
       sw(i) = swc + (1 - swc - sor) * (i - 1) / points
   End If
   'normalizing
   swd(i) = (sw(i) - swc) / (1 - sor - swc)
   kro(i) = kroe * (1 - swd(i)) ^ no
   krw(i) = krwe * swd(i) ^ nw

   If krw(i) = 0 Then
       fw(i) = 0
       Else
       fw(i) = 1 / (1 + kro(i) * mwater / (krw(i) * moil))
   End If

   Cells(1, 10) = "Sw"
   'Cells(1, 11) = "SwD"
   Cells(1, 11) = "Kro"
   Cells(1, 12) = "Krw"
   Cells(1, 13) = "fw"
   Cells(i + 1, 10) = sw(i)
   'Cells(i + 1, 11) = swd(i)
   Cells(i + 1, 11) = kro(i)
   Cells(i + 1, 12) = krw(i)
   Cells(i + 1, 13) = fw(i)


   Next i

' Parameters calculations
M = (krwe / mwater) / (kroe / moil)
x8 = M
logM = Log(M) / Log(10)
e = 2.71828182845905
x1 = (mwater / moil) / no
x2 = (mwater / moil)
x3 = no
x4 = nw
x5 = swc
x6 = sor
x7 = logM

'Homogeneous oil-wet:
If wet = 2 Then
   Slope = 0.77519 + 8.93429 * x1 - 3.36474 * x2 + 2.67364 * x4 + 28.2328 * x6 - 8.40355 * x7
   Intercept = -3.4254 - 11.94 * x1 + 4.78633 * x2 - 1.4167 * x4 - 0.33004 * x5 - 0.0606 * x6 + 5.30721 *
x7
   fwBT = 0.6598 + 0.11888 * x1 - 0.04844 * x2 + 0.02099 * x3 + 0.05127 * x4 - 0.09342 * x7
```

```
   SwBT = 0.56187 + 0.16777 * x1 - 0.0687 * x2 - 0.022 * x3 + 0.04494 * x4 + 0.41472 * x5 - 0.5853 *
x6 - 0.1454 * x7

'Homogeneous water-wet
Else
   Slope = 1.29383 + 27.8154 * x1 - 6.7911 * x2 + 2.08358 * x4 + 4.76521 * x5 + 22.3142 * x6 - 4.794 *
x7
   Intercept = -1.5668 - 30.399 * x1 + 8.83008 * x2 - 1.5008 * x4 - 0.5834 * x6 - 0.1052 * x5 + 4.03107 *
x7
   fwBT = 0.71084 + 0.20212 * x1 - 0.08234 * x2 + 0.01779 * x3 + 0.03932 * x4 - 0.09479 * x7
   SwBT = 0.64301 + 0.23901 * x1 - 0.1003 * x2 - 0.0302 * x3 + 0.03722 * x4 + 0.34619 * x5 - 0.6538 *
x6 - 0.1548 * x7
End If

'*****************************************************************
'All max log general:

'If wet = 2 Then
  ' maxlog = 1.82775 + 0.0629 * x8 - 0.47157 * VDP
'Else
   'maxlog = 5.15914 + 0.13366 * x8 - 9.6964 * VDP + 0.62073 * (10 ^ VDP)

'End If

'**************************************************************

'All max log with constrains for case dependent cases

VR = mwater / moil


'maxwor and minwor homo:
'water-wet:
If wet = 1 Then
worhomomina = 10 ^ (0.10547 + 0.31449 * VR + 0.08504 * no + 0.18375 * nw + 0.56262 * sor +
0.15521 * swc + 0.10024 * kroe + 0.11893 * krwe - 0.01843 * M)
worhomomaxa = 10 ^ (1.31556 - 0.24884 * VR + 0.60999 * no + 0.06739 * nw - 0.66761 * sor - 1.25442
* swc - 0.0454 * kroe - 0.24884 * krwe + 0.06416 * M)
Else
'oil-wet:
worhomomina = 10 ^ (-0.10923 + 0.36546 * VR + 0.06644 * no + 0.18869 * nw + 0.41633 * sor +
0.17015 * swc + 0.23567 * kroe - 0.04279 * krwe + 0.01746 * M)
worhomomaxa = 10 ^ (2.15118 - 0.85163 * VR + 0.68975 * no + 0.0229 * nw - 1.14131 * sor - 0.70038 *
swc - 1.29377 * kroe + 0.74267 * krwe - 0.12195 * M)
End If



If wet = 2 Then
   maxlog05 = 1.93449 + 0.02621 * x8 - 0.45239 * (0.5) - 0.00594 * (10 ^ (0.5))
Else
   maxlog05 = 5.15914 + 0.13366 * x8 - 9.6964 * (0.5) + 0.62073 * (10 ^ (0.5))
End If
```

```
If wet = 2 Then
    maxlog06a = 1.93449 + 0.02621 * x8 - 0.45239 * (0.6) - 0.00594 * (10 ^ (0.6))
Else
    maxlog06a = 5.15914 + 0.13366 * x8 - 9.6964 * (0.6) + 0.62073 * (10 ^ (0.6))
End If
If wet = 2 Then
    maxlog07a = 1.93449 + 0.02621 * x8 - 0.45239 * (0.7) - 0.00594 * (10 ^ (0.7))
Else
    maxlog07a = 5.15914 + 0.13366 * x8 - 9.6964 * (0.7) + 0.62073 * (10 ^ (0.7))
End If

If wet = 2 Then
    maxlog08a = 1.93449 + 0.02621 * x8 - 0.45239 * (0.8) - 0.00594 * (10 ^ (0.8))
Else
    maxlog08a = 5.15914 + 0.13366 * x8 - 9.6964 * (0.8) + 0.62073 * (10 ^ (0.8))
End If

If wet = 2 Then
    maxlog09a = 1.93449 + 0.02621 * x8 - 0.45239 * (0.9) - 0.00594 * (10 ^ (0.9))
Else
    maxlog09a = 5.15914 + 0.13366 * x8 - 9.6964 * (0.9) + 0.62073 * (10 ^ (0.9))
End If

If maxlog06a >= maxlog05 Then
maxlog06 = maxlog05 - (maxlog05 * 0.1)
Else
maxlog06 = maxlog06a
End If
If maxlog07a >= maxlog06a Then
maxlog07 = maxlog06a - (maxlog06a * 0.1)
Else
maxlog07 = maxlog07a
End If
If maxlog08a >= maxlog07a Then
maxlog08 = maxlog07a - (maxlog07a * 0.1)
Else
maxlog08 = maxlog08a
End If
If maxlog09a >= maxlog08a Then
maxlog09 = maxlog08a - (maxlog08a * 0.1)
Else
maxlog09 = maxlog09a
End If


If VDP <= 0.5 Then
    maxlog = maxlog05
    Else
    If VDP > 0.5 And VDP <= 0.65 Then
    maxlog = maxlog06
    End If
    If VDP > 0.65 And VDP <= 0.75 Then
    maxlog = maxlog07
```

```
   End If
   If VDP > 0.75 And VDP <= 0.85 Then
   maxlog = maxlog08
   End If
   If VDP > 0.85 And VDP <= 1 Then
   maxlog = maxlog09
   End If
End If
If VDP > 0.5 Then
If wet = 1 Then
'minwor = 10 ^ (5.14786 * (VDP ^ 2) - 8.44063 * (VDP) + 3.81754)
minwor = 10 ^ (0.01738 * M + 9.73473 * (VDP ^ 2) - 10.83657 * (VDP) + 4.9169 - 0.33856 * (10 ^
VDP))
Else
'minwor = 10 ^ (1.27008 - 1.27427 * (VDP ^ 2))
minwor = 10 ^ (2.37473 + 0.02337 * M - 2.99912 * VDP - 0.22364 * (10 ^ VDP) + 2.88892 * VDP ^ 2)
End If
Else
If wet = 1 Then
minwor = 10 ^ (0.01738 * M + 9.73473 * (0.5 ^ 2) - 10.83657 * (0.5) + 4.9169 - 0.33856 * (10 ^ 0.5))
Else
minwor = 10 ^ (2.37473 + 0.02337 * M - 2.99912 * 0.5 - 0.22364 * (10 ^ 0.5) + 2.88892 * 0.5 ^ 2)
End If
End If

If VDP = 0.9 And wet = 2 Then
minwor = 5
End If



If worhomomina > (10 ^ (2.37473 + 0.02337 * M - 2.99912 * 0.5 - 0.22364 * (10 ^ 0.5) + 2.88892 * 0.5 ^
2)) Then
worhomomin = worhomomina
Else
worhomomin = (10 ^ (2.37473 + 0.02337 * M - 2.99912 * 0.5 - 0.22364 * (10 ^ 0.5) + 2.88892 * 0.5 ^ 2))
* 1.3
End If


If worhomomaxa > 10 ^ (maxlog05) Then
worhomomax = worhomomaxa
Else
worhomomax = 10 ^ (maxlog05) * 1.3
End If


Cells(26, 1) = "WOR homo min"
Cells(26, 2) = worhomomin
Cells(27, 1) = "WOR homo max"
Cells(27, 2) = worhomomax
```

```
'*****************************************************
'Recovery Calculations
lWOR = Log(WOR) / Log(10)
RFprev = (lWOR - Intercept) / Slope
' RF constrain
If RFprev > (1 - x6) Then
RF = (1 - x6)
Else
RF = RFprev
End If

x12 = e ^ (lWOR)
x13 = 10 ^ (lWOR)
lwor2 = Log(0.1) / Log(10)
x120 = e ^ ((Log(worhomomax) / Log(10)))
x130 = 10 ^ ((Log(worhomomax) / Log(10)))



Cells(14, 4) = "Message:"
Cells(14, 1) = "Results:"
Cells(15, 1) = "Slope"
Cells(15, 2) = Slope
Cells(16, 1) = "Intercept"
Cells(16, 2) = Intercept
Cells(17, 1) = "fwBT"
Cells(17, 2) = fwBT
Cells(18, 1) = "SwBT"
Cells(18, 2) = SwBT
Cells(19, 1) = "Max WOR"
Cells(19, 2) = 10 ^ maxlog
Cells(20, 1) = "Min WOR"
Cells(20, 2) = minwor
Cells(21, 1) = "Recovery Factor homogeneous"
Cells(21, 2) = RF
Cells(22, 1) = "PVI homogeneous"
Cells(23, 1) = "Mobility Ratio"
Cells(23, 2) = x8


' Calculation of SLZ end:

If VDP < 0.5 Then
   RFheter = RF
   Else
'water-wet and heterogeneous calculations
   If VDP >= 0.5 And VDP <= 0.65 And wet = 1 Then
   RFheter = -0.18015 + 1.27511 * RF
   PVI = -1.05462 + 0.46384 * x8 + 0.00912 * x13 + 0.28413 * x12
   End If
   If VDP > 0.65 And VDP <= 0.75 And wet = 1 Then
   RFheter = 0.19068 - 0.00368 * x8 + 0.02336 * lWOR + 0.91971 * RF ^ 2
```

```
    PVI = -1.2156 + 0.47344 * x8 + 0.01136 * x13 + 0.31097 * x12
    End If
    If VDP > 0.75 And VDP <= 0.85 And wet = 1 Then
    RFheter = 0.09829 - 0.00751 * x8 + 0.05354 * lWOR + 0.88719 * RF ^ 2
    PVI = -2.06505 + 0.58703 * x8 + 0.01217 * x13 + 0.57103 * x12
    End If
    If VDP > 0.85 And VDP <= 1 And wet = 1 Then
    RFheter = 0.1718 - 0.0144 * x8 + 0.07227 * lWOR + 0.44117 * RF ^ 2
    PVI = -0.45357 - 0.08084 * x8 + 0.02353 * x13 + 0.19269 * x12
    End If
End If

    'oil-wet and heterogeneous calculations
If VDP <= 0.5 Then
    RFheter = RF
    Else
    If VDP > 0.5 And VDP <= 0.65 And wet = 2 Then
    RFheter = -0.0988 + 1.27817 * RF
    PVI = -1.26805 + 0.49769 * x8 + 0.00238 * x13 + 0.37376 * x12
    End If
    If VDP > 0.65 And VDP <= 0.75 And wet = 2 Then
    RFheter = 0.09036 - 0.00226 * x8 + 0.01459 * lWOR + 1.78812 * RF ^ 2
    PVI = -1.11471 + 0.43417 * x8 + 0.00691 * x13 + 0.30144 * x12
    End If
    If VDP > 0.75 And VDP <= 0.85 And wet = 2 Then
    RFheter = 0.06727 - 0.00489 * x8 + 0.03839 * lWOR + 1.27549 * RF ^ 2
    PVI = -1.26744 + 0.45081 * x8 + 0.01279 * x13 + 0.32235 * x12
    End If
    If VDP > 0.85 And VDP <= 1 And wet = 2 Then
    RFheter = 0.0748 - 0.00535 * x8 + 0.04112 * lWOR + 0.88482 * RF ^ 2
    PVI = -0.64968 + 0.18524 * x8 + 0.0137 * x13 + 0.22142 * x12
    End If
End If


If wet = 1 Then
    PVIhomo = -0.81246 + 0.35196 * x8 + 0.008 * x13 + 0.21624 * x12
    Else
    PVIhomo = -0.86243 + 0.41381 * x8 + 0.00194 * x13 + 0.24178 * x12
    End If

    If PVIhomo < 0.2 Then
    PVIhomorev = 0.2
    Else
    PVIhomorev = PVIhomo
    End If

    Cells(22, 2) = PVIhomorev



If VDP <= 0.5 Then
    PVIrev = PVIhomorev
```

```
    Else
    PVIrev = PVI
End If

If PVI < 0.1 Then
    PVIrev = 0.1
    Else
    PVIrev = PVI

End If


If RFheter > RF Then
RFheterx = RF * 0.9
Else
RFheterx = RFheter
End If


If RFheter < 0 Then
RFhetery = RF * 0.9
Else
RFhetery = RFheter
End If



If RFheter < 0 Then
Cells(24, 1) = "RF heterogeneous"
Cells(24, 2) = RFhetery
Else
Cells(24, 1) = "RF heterogeneous"
Cells(24, 2) = RFheterx
End If




Cells(25, 1) = "PVI Heterogeneous"
If VDP <= 0.5 Then
Cells(25, 2) = PVIhomorev
Else
Cells(25, 2) = PVIrev
End If

If PVIhomo >= 3 Or lWOR > maxlog Then
Cells(15, 4) = "Warning: due to the high WOR used, your RF value may be close to or beyond he end of the SLZ"
Cells(16, 4) = "Correlations are defined only for the SLZ so higher values of max WOR must be ingnored"
Cells(17, 4) = "If you obtain recovery factors for heterogeneous reservoirs higher than the correspondent"
Cells(18, 4) = "homogeneous recovery factor, you are extrapolating the SLZ and overestimating reserves"
Cells(19, 4) = "if that is the case, please try a lower WOR value (please check suggested Max Log WOR)"
Else
```

```
Cells(15, 4) = "No problem has been detected. Please continue your evaluation"

End If


Cells(24, 1) = "RF heterogeneous"
Cells(24, 2) = RFheter



End With



End Sub
```

# APPENDIX G

## VARIABLES SELECTION PROCESSES AND COVARIANCE MATRIX RESULTS

### Variable Selection and Covariance Matrix for the water-wet homogeneous model (Slope)

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Number of Observations Read 226977
Number of Observations Used 226977

Forward Selection: Step 1

Statistics for Entry
DF = 1,226975

| Variable | Model Tolerance | R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 1.000000 | 0.4993 | 226317 | <.0001 |
| x2 | 1.000000 | 0.4417 | 179557 | <.0001 |
| x4 | 1.000000 | 0.3137 | 103754 | <.0001 |
| x5 | 1.000000 | 0.0042 | 964.74 | <.0001 |
| x6 | 1.000000 | 0.0127 | 2919.78 | <.0001 |
| x7 | 1.000000 | 0.4748 | 205174 | <.0001 |

Variable x1 Entered: R-Square = 0.4993 and C(p) = 1235459

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 1735328 | 1735328 | 226317 | <.0001 |
| Error | 226975 | 1740372 | 7.66768 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 8.32573 | 0.01019 | 5114987 | 667084 | <.0001 |
| x1 | 23.96600 | 0.05038 | 1735328 | 226317 | <.0001 |

Bounds on condition number: 1, 1

---

Forward Selection: Step 2
The SAS System 09:56 Saturday, October 3, 2009 2888

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 2

Statistics for Entry
DF = 1,226974

| Variable | Model Tolerance | R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.121152 | 0.4993 | 17.78 | <.0001 |
| x4 | 0.999971 | 0.8173 | 394957 | <.0001 |
| x5 | 0.990126 | 0.4993 | 12.16 | 0.0005 |
| x6 | 0.977372 | 0.5483 | 24657.2 | <.0001 |
| x7 | 0.222493 | 0.5188 | 9233.19 | <.0001 |

Variable x4 Entered: R-Square = 0.8173 and C(p) = 306744.1

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 2840550 | 1420275 | 507543 | <.0001 |
| Error | 226974 | 635150 | 2.79834 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.93459 | 0.01189 | 74098 | 26479.2 | <.0001 |
| x1 | 24.06865 | 0.03043 | 1750175 | 625434 | <.0001 |
| x4 | 2.00145 | 0.00318 | 1105223 | 394957 | <.0001 |

Bounds on condition number: 1, 4.0001

---

Forward Selection: Step 3

The SAS System 09:56 Saturday, October 3, 2009 2889

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 3

Statistics for Entry
DF = 1,226973

| Variable | Model Tolerance | R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.120862 | 0.8177 | 566.47 | <.0001 |
| x5 | 0.985080 | 0.8193 | 2606.96 | <.0001 |
| x6 | 0.972575 | 0.8857 | 135979 | <.0001 |
| x7 | 0.221813 | 0.8291 | 15755.4 | <.0001 |

Variable x6 Entered: R-Square = 0.8857 and C(p) = 106791.5

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| Model | 3 | 3078507 | 1026169 | 586397 | <.0001 |
| Error | 226973 | 397193 | 1.74996 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.63770 | 0.01350 | 25755 | 14717.4 | <.0001 |
| x1 | 25.42569 | 0.02435 | 1908473 | 1090583 | <.0001 |
| x4 | 2.06667 | 0.00252 | 1172641 | 670097 | <.0001 |
| x6 | 20.77006 | 0.05633 | 237957 | 135979 | <.0001 |

Bounds on condition number: 1.0282, 9.1697

---------------------------------------------------------------------------------------------

Forward Selection: Step 4

The SAS System 09:56 Saturday, October 3, 2009 2890

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 4

Statistics for Entry
DF = 1,226972

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.120806 | 0.8865 | 1458.51 | <.0001 |
| x5 | 0.950012 | 0.8951 | 20208.0 | <.0001 |
| x7 | 0.221782 | 0.8969 | 24655.3 | <.0001 |

Variable x7 Entered: R-Square = 0.8969 and C(p) = 74090.55

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 3117425 | 779356 | 493733 | <.0001 |
| Error | 226972 | 358275 | 1.57850 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 0.51508 | 0.01877 | 1188.54656 | 752.96 | <.0001 |
| x1 | 18.69810 | 0.04869 | 232816 | 147492 | <.0001 |
| x4 | 2.04557 | 0.00240 | 1145224 | 725515 | <.0001 |
| x6 | 20.67050 | 0.05350 | 235648 | 149286 | <.0001 |
| x7 | -2.35963 | 0.01503 | 38918 | 24655.3 | <.0001 |

Bounds on condition number: 4.537, 44.33

---------------------------------------------------------------------------------------------

Forward Selection: Step 5

The SAS System 09:56 Saturday, October 3, 2009 2891

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 5

Statistics for Entry
DF = 1,226971

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.068854 | 0.9132 | 42651.0 | <.0001 |
| x5 | 0.949226 | 0.9057 | 21107.9 | <.0001 |

Variable x2 Entered: R-Square = 0.9132 and C(p) = 26468.81

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 3174100 | 634820 | 477738 | <.0001 |
| Error | 226971 | 301600 | 1.32880 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.85479 | 0.02061 | 25483 | 19177.6 | <.0001 |
| x1 | 27.14027 | 0.06055 | 266957 | 200900 | <.0001 |
| x2 | -6.73090 | 0.03259 | 56675 | 42651.0 | <.0001 |
| x4 | 2.05376 | 0.00220 | 1154038 | 868479 | <.0001 |
| x6 | 20.85748 | 0.04909 | 239849 | 180500 | <.0001 |
| x7 | -4.83303 | 0.01826 | 93057 | 70030.7 | <.0001 |

Bounds on condition number: 14.523, 164.04

---

Forward Selection: Step 6

The SAS System 09:56 Saturday, October 3, 2009 2892

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 6

Statistics for Entry
DF = 1,226970

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.949089 | 0.9223 | 26463.8 | <.0001 |

Variable x5 Entered: R-Square = 0.9223 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 3205594 | 534266 | 448942 | <.0001 |
| Error | 226970 | 270106 | 1.19005 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.29383 | 0.02174 | 4214.70878 | 3541.61 | <.0001 |
| x1 | 27.81540 | 0.05745 | 278940 | 234393 | <.0001 |
| x2 | -6.79106 | 0.03085 | 57684 | 48472.0 | <.0001 |
| x4 | 2.08358 | 0.00209 | 1178686 | 990447 | <.0001 |
| x5 | 4.76521 | 0.02929 | 31493 | 26463.8 | <.0001 |
| x6 | 22.31418 | 0.04731 | 264688 | 222417 | <.0001 |
| x7 | -4.79404 | 0.01729 | 91544 | 76924.2 | <.0001 |

Bounds on condition number: 14.526, 203.73

---

All variables have been entered into the model.

The SAS System 09:56 Saturday, October 3, 2009 2893

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Summary of Forward Selection

| Step | Variable Entered | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|
| 1 | x1 | 1 | 0.4993 | 0.4993 | 1235459 | 226317 | <.0001 |
| 2 | x4 | 2 | 0.3180 | 0.8173 | 306744 | 394957 | <.0001 |
| 3 | x6 | 3 | 0.0685 | 0.8857 | 106792 | 135979 | <.0001 |
| 4 | x7 | 4 | 0.0112 | 0.8969 | 74090.5 | 24655.3 | <.0001 |
| 5 | x2 | 5 | 0.0163 | 0.9132 | 26468.8 | 42651.0 | <.0001 |
| 6 | x5 | 6 | 0.0091 | 0.9223 | 7.0000 | 26463.8 | <.0001 |

The SAS System 09:56 Saturday, October 3, 2009 2894

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Number of Observations Read 226977
Number of Observations Used 226977

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.9223 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 3205594 | 534266 | 448942 | <.0001 |
| Error | 226970 | 270106 | 1.19005 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.29383 | 0.02174 | 4214.70878 | 3541.61 | <.0001 |
| x1 | 27.81540 | 0.05745 | 278940 | 234393 | <.0001 |
| x2 | -6.79106 | 0.03085 | 57684 | 48472.0 | <.0001 |
| x4 | 2.08358 | 0.00209 | 1178686 | 990447 | <.0001 |
| x5 | 4.76521 | 0.02929 | 31493 | 26463.8 | <.0001 |
| x6 | 22.31418 | 0.04731 | 264688 | 222417 | <.0001 |
| x7 | -4.79404 | 0.01729 | 91544 | 76924.2 | <.0001 |

Bounds on condition number: 14.526, 203.73

------------------------------------------------------------------------------------------------

Backward Elimination: Step 1

The SAS System 09:56 Saturday, October 3, 2009 2895

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Backward Elimination: Step 1

Statistics for Removal
DF = 1,226970

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0803 | 0.8420 | 234393 | <.0001 |
| x2 | 0.0166 | 0.9057 | 48472.0 | <.0001 |
| x4 | 0.3391 | 0.5832 | 990447 | <.0001 |
| x5 | 0.0091 | 0.9132 | 26463.8 | <.0001 |
| x6 | 0.0762 | 0.8461 | 222417 | <.0001 |
| x7 | 0.0263 | 0.8959 | 76924.2 | <.0001 |

All variables left in the model are significant at the 0.0500 level.

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Number of Observations Read 226977
Number of Observations Used 226977

Stepwise Selection: Step 1

Statistics for Entry
DF = 1,226975

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 1.000000 | 0.4993 | 226317 | <.0001 |

| | | | | |
|---|---|---|---|---|
| x2 | 1.000000 | 0.4417 | 179557 | <.0001 |
| x4 | 1.000000 | 0.3137 | 103754 | <.0001 |
| x5 | 1.000000 | 0.0042 | 964.74 | <.0001 |
| x6 | 1.000000 | 0.0127 | 2919.78 | <.0001 |
| x7 | 1.000000 | 0.4748 | 205174 | <.0001 |

Variable x1 Entered: R-Square = 0.4993 and C(p) = 1235459

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 1735328 | 1735328 | 226317 | <.0001 |
| Error | 226975 | 1740372 | 7.66768 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 8.32573 | 0.01019 | 5114987 | 667084 | <.0001 |
| x1 | 23.96600 | 0.05038 | 1735328 | 226317 | <.0001 |

Bounds on condition number: 1, 1

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Stepwise Selection: Step 2

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The REG Procedure

Model: MODEL1

Dependent Variable: slope

Stepwise Selection: Step 2

Statistics for Entry

DF = 1,226974

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.121152 | 0.4993 | 17.78 | <.0001 |
| x4 | 0.999971 | 0.8173 | 394957 | <.0001 |
| x5 | 0.990126 | 0.4993 | 12.16 | 0.0005 |
| x6 | 0.977372 | 0.5483 | 24657.2 | <.0001 |
| x7 | 0.222493 | 0.5188 | 9233.19 | <.0001 |

Variable x4 Entered: R-Square = 0.8173 and C(p) = 306744.1

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 2840550 | 1420275 | 507543 | <.0001 |
| Error | 226974 | 635150 | 2.79834 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| Intercept | 1.93459 | 0.01189 | 74098 | 26479.2 | <.0001 |
| x1 | 24.06865 | 0.03043 | 1750175 | 625434 | <.0001 |
| x4 | 2.00145 | 0.00318 | 1105223 | 394957 | <.0001 |

Bounds on condition number: 1, 4.0001

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Stepwise Selection: Step 3

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 3

Statistics for Removal
DF = 1,226974

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.5035 | 0.3137 | 625434 | <.0001 |
| x4 | 0.3180 | 0.4993 | 394957 | <.0001 |

Statistics for Entry
DF = 1,226973

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.120862 | 0.8177 | 566.47 | <.0001 |
| x5 | 0.985080 | 0.8193 | 2606.96 | <.0001 |
| x6 | 0.972575 | 0.8857 | 135979 | <.0001 |
| x7 | 0.221813 | 0.8291 | 15755.4 | <.0001 |

Variable x6 Entered: R-Square = 0.8857 and C(p) = 106791.5

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 3078507 | 1026169 | 586397 | <.0001 |
| Error | 226973 | 397193 | 1.74996 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.63770 | 0.01350 | 25755 | 14717.4 | <.0001 |
| x1 | 25.42569 | 0.02435 | 1908473 | 1090583 | <.0001 |
| x4 | 2.06667 | 0.00252 | 1172641 | 670097 | <.0001 |
| x6 | 20.77006 | 0.05633 | 237957 | 135979 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 3

Bounds on condition number: 1.0282, 9.1697

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Stepwise Selection: Step 4

Statistics for Removal
DF = 1,226973

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.5491 | 0.3366 | 1090583 | <.0001 |
| x4 | 0.3374 | 0.5483 | 670097 | <.0001 |
| x6 | 0.0685 | 0.8173 | 135979 | <.0001 |

Statistics for Entry
DF = 1,226972

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.120806 | 0.8865 | 1458.51 | <.0001 |
| x5 | 0.950012 | 0.8951 | 20208.0 | <.0001 |
| x7 | 0.221782 | 0.8969 | 24655.3 | <.0001 |

Variable x7 Entered: R-Square = 0.8969 and C(p) = 74090.55

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 3117425 | 779356 | 493733 | <.0001 |
| Error | 226972 | 358275 | 1.57850 | | |
| Corrected Total | 226976 | 3475700 | | | |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 4

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 0.51508 | 0.01877 | 1188.54656 | 752.96 | <.0001 |
| x1 | 18.69810 | 0.04869 | 232816 | 147492 | <.0001 |
| x4 | 2.04557 | 0.00240 | 1145224 | 725515 | <.0001 |
| x6 | 20.67050 | 0.05350 | 235648 | 149286 | <.0001 |
| x7 | -2.35963 | 0.01503 | 38918 | 24655.3 | <.0001 |

Bounds on condition number: 4.537, 44.33

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Stepwise Selection: Step 5

Statistics for Removal

DF = 1,226972

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0670 | 0.8299 | 147492 | <.0001 |
| x4 | 0.3295 | 0.5674 | 725515 | <.0001 |
| x6 | 0.0678 | 0.8291 | 149286 | <.0001 |
| x7 | 0.0112 | 0.8857 | 24655.3 | <.0001 |

Statistics for Entry
DF = 1,226971

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.068854 | 0.9132 | 42651.0 | <.0001 |
| x5 | 0.949226 | 0.9057 | 21107.9 | <.0001 |

Variable x2 Entered: R-Square = 0.9132 and C(p) = 26468.81

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 5

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 3174100 | 634820 | 477738 | <.0001 |
| Error | 226971 | 301600 | 1.32880 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.85479 | 0.02061 | 25483 | 19177.6 | <.0001 |
| x1 | 27.14027 | 0.06055 | 266957 | 200900 | <.0001 |
| x2 | -6.73090 | 0.03259 | 56675 | 42651.0 | <.0001 |
| x4 | 2.05376 | 0.00220 | 1154038 | 868479 | <.0001 |
| x6 | 20.85748 | 0.04909 | 239849 | 180500 | <.0001 |
| x7 | -4.83303 | 0.01826 | 93057 | 70030.7 | <.0001 |

Bounds on condition number: 14.523, 164.04

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Stepwise Selection: Step 6

Statistics for Removal
DF = 1,226971

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0768 | 0.8364 | 200900 | <.0001 |
| x2 | 0.0163 | 0.8969 | 42651.0 | <.0001 |

| | | | | |
|---|---|---|---|---|
| x4 | 0.3320 | 0.5812 | 868479 | <.0001 |
| x6 | 0.0690 | 0.8442 | 180500 | <.0001 |
| x7 | 0.0268 | 0.8865 | 70030.7 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 6

Statistics for Entry
DF = 1,226970

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.949089 | 0.9223 | 26463.8 | <.0001 |

Variable x5 Entered: R-Square = 0.9223 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 3205594 | 534266 | 448942 | <.0001 |
| Error | 226970 | 270106 | 1.19005 | | |
| Corrected Total | 226976 | 3475700 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.29383 | 0.02174 | 4214.70878 | 3541.61 | <.0001 |
| x1 | 27.81540 | 0.05745 | 278940 | 234393 | <.0001 |
| x2 | -6.79106 | 0.03085 | 57684 | 48472.0 | <.0001 |
| x4 | 2.08358 | 0.00209 | 1178686 | 990447 | <.0001 |
| x5 | 4.76521 | 0.02929 | 31493 | 26463.8 | <.0001 |
| x6 | 22.31418 | 0.04731 | 264688 | 222417 | <.0001 |
| x7 | -4.79404 | 0.01729 | 91544 | 76924.2 | <.0001 |

Bounds on condition number: 14.526, 203.73

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Stepwise Selection: Step 7

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 7

Statistics for Removal
DF = 1,226970

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| x1 | 0.0803 | 0.8420 | 234393 | <.0001 |
| x2 | 0.0166 | 0.9057 | 48472.0 | <.0001 |
| x4 | 0.3391 | 0.5832 | 990447 | <.0001 |
| x5 | 0.0091 | 0.9132 | 26463.8 | <.0001 |
| x6 | 0.0762 | 0.8461 | 222417 | <.0001 |
| x7 | 0.0263 | 0.8959 | 76924.2 | <.0001 |

All variables left in the model are significant at the 0.1500 level.

All variables have been entered into the model.

Summary of Stepwise Selection

| Step | Variable Entered | Variable Removed | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|
| 1 | x1 | | 1 | 0.4993 | 0.4993 | 1235459 | 226317 | <.0001 |
| 2 | x4 | | 2 | 0.3180 | 0.8173 | 306744 | 394957 | <.0001 |
| 3 | x6 | | 3 | 0.0685 | 0.8857 | 106792 | 135979 | <.0001 |
| 4 | x7 | | 4 | 0.0112 | 0.8969 | 74090.5 | 24655.3 | <.0001 |
| 5 | x2 | | 5 | 0.0163 | 0.9132 | 26468.8 | 42651.0 | <.0001 |
| 6 | x5 | | 6 | 0.0091 | 0.9223 | 7.0000 | 26463.8 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

R-Square Selection Method

Number of Observations Read 226977
Number of Observations Used 226977

| Number in Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 1 | 0.4993 | 1235459 | 7.66768 | x1 |
| 1 | 0.4748 | 1307009 | 8.04283 | x7 |
| 1 | 0.4417 | 1403672 | 8.54964 | x2 |
| 1 | 0.3137 | 1777411 | 10.50920 | x4 |
| 1 | 0.0127 | 2656558 | 15.11866 | x6 |
| 1 | 0.0042 | 2681290 | 15.24833 | x5 |
| 2 | 0.8173 | 306744.1 | 2.79834 | x1 x4 |
| 2 | 0.7724 | 437832.0 | 3.48565 | x4 x7 |
| 2 | 0.7466 | 513178.3 | 3.88070 | x2 x4 |
| 2 | 0.5483 | 1092158 | 6.91636 | x1 x6 |
| 2 | 0.5188 | 1178295 | 7.36799 | x1 x7 |
| 2 | 0.5161 | 1186372 | 7.41034 | x6 x7 |
| 2 | 0.4993 | 1235346 | 7.66712 | x1 x2 |
| 2 | 0.4993 | 1235382 | 7.66731 | x1 x5 |
| 2 | 0.4833 | 1281987 | 7.91166 | x2 x6 |
| 2 | 0.4782 | 1297127 | 7.99104 | x2 x7 |
| 2 | 0.4749 | 1306603 | 8.04072 | x5 x7 |
| 2 | 0.4417 | 1403500 | 8.54877 | x2 x5 |
| 2 | 0.3366 | 1710476 | 10.15828 | x4 x6 |
| 2 | 0.3144 | 1775494 | 10.49918 | x4 x5 |
| 2 | 0.0149 | 2650073 | 15.08471 | x5 x6 |
| 3 | 0.8857 | 106791.5 | 1.74996 | x1 x4 x6 |
| 3 | 0.8299 | 269723.4 | 2.60423 | x4 x6 x7 |
| 3 | 0.8291 | 272103.0 | 2.61671 | x1 x4 x7 |

| | | | | |
|---|---|---|---|---|
| 3 | 0.8193 | 300685.6 | 2.76657 | x1 x4 x5 |
| 3 | 0.8177 | 305417.4 | 2.79138 | x1 x2 x4 |
| 3 | 0.8051 | 342254.6 | 2.98453 | x2 x4 x6 |
| 3 | 0.7773 | 423370.6 | 3.40983 | x2 x4 x7 |
| 3 | 0.7730 | 435871.8 | 3.47537 | x4 x5 x7 |
| 3 | 0.7475 | 510399.1 | 3.86613 | x2 x4 x5 |
| 3 | 0.5674 | 1036418 | 6.62413 | x1 x6 x7 |
| 3 | 0.5505 | 1085862 | 6.88337 | x1 x5 x6 |
| 3 | 0.5483 | 1092145 | 6.91632 | x1 x2 x6 |
| 3 | 0.5317 | 1140650 | 7.17063 | x1 x2 x7 |
| 3 | 0.5205 | 1173480 | 7.34277 | x2 x6 x7 |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

R-Square Selection Method

| Number in Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 3 | 0.5189 | 1178284 | 7.36795 | x1 x5 x7 |
| 3 | 0.5167 | 1184622 | 7.40119 | x5 x6 x7 |
| 3 | 0.4993 | 1235273 | 7.66676 | x1 x2 x5 |
| 3 | 0.4842 | 1279493 | 7.89861 | x2 x5 x6 |
| 3 | 0.4783 | 1296864 | 7.98969 | x2 x5 x7 |
| 3 | 0.3366 | 1710478 | 10.15833 | x4 x5 x6 |
| 4 | 0.8969 | 74090.55 | 1.57850 | x1 x4 x6 x7 |
| 4 | 0.8951 | 79507.26 | 1.60690 | x1 x4 x5 x6 |
| 4 | 0.8865 | 104662.5 | 1.73879 | x1 x2 x4 x6 |
| 4 | 0.8442 | 228011.3 | 2.38553 | x1 x2 x4 x7 |
| 4 | 0.8364 | 250790.2 | 2.50496 | x2 x4 x6 x7 |
| 4 | 0.8350 | 255078.8 | 2.52745 | x4 x5 x6 x7 |
| 4 | 0.8309 | 266771.3 | 2.58876 | x1 x4 x5 x7 |
| 4 | 0.8198 | 299222.5 | 2.75890 | x1 x2 x4 x5 |
| 4 | 0.8110 | 325047.6 | 2.89431 | x2 x4 x5 x6 |
| 4 | 0.7782 | 420941.5 | 3.39710 | x2 x4 x5 x7 |
| 4 | 0.5812 | 996202.7 | 6.41329 | x1 x2 x6 x7 |
| 4 | 0.5693 | 1030980 | 6.59563 | x1 x5 x6 x7 |
| 4 | 0.5505 | 1085860 | 6.88338 | x1 x2 x5 x6 |
| 4 | 0.5317 | 1140627 | 7.17053 | x1 x2 x5 x7 |
| 4 | 0.5213 | 1171244 | 7.33107 | x2 x5 x6 x7 |
| 5 | 0.9132 | 26468.81 | 1.32880 | x1 x2 x4 x6 x7 |
| 5 | 0.9057 | 48476.95 | 1.44420 | x1 x4 x5 x6 x7 |
| 5 | 0.8959 | 76929.19 | 1.59338 | x1 x2 x4 x5 x6 |
| 5 | 0.8461 | 222422.2 | 2.35623 | x1 x2 x4 x5 x7 |
| 5 | 0.8420 | 234398.2 | 2.41902 | x2 x4 x5 x6 x7 |
| 5 | 0.5832 | 990452.4 | 6.38316 | x1 x2 x5 x6 x7 |
| 6 | 0.9223 | 7.0000 | 1.19005 | x1 x2 x4 x5 x6 x7 |

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The CORR Procedure

6 Variables: x1 x2 x4 x5 x6 x7

Covariance Matrix, DF = 226976

x1 x2 x4 x5 x6 x7

| | | | | | | |
|---|---|---|---|---|---|---|
| x1 | 0.013310993 | 0.030600508 | -0.000682703 | -0.000919871 | -0.000867544 | -0.037908526 |
| x2 | 0.030600508 | 0.080044782 | 0.003738304 | -0.001959494 | -0.001905864 | -0.098515099 |
| x4 | -0.000682703 | 0.003738304 | 1.215603899 | -0.006237114 | -0.003772409 | -0.008763795 |
| x5 | -0.000919871 | -0.001959494 | -0.006237114 | 0.006438250 | -0.000661045 | 0.002311448 |
| x6 | -0.000867544 | -0.001905864 | -0.003772409 | -0.000661045 | 0.002498730 | 0.002401774 |
| x7 | -0.037908526 | -0.098515099 | -0.008763795 | 0.002311448 | 0.002401774 | 0.138854170 |

Simple Statistics

| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
|---|---|---|---|---|---|---|
| x1 | 226977 | 0.16623 | 0.11537 | 37731 | 0.03333 | 0.45000 |
| x2 | 226977 | 0.41589 | 0.28292 | 94397 | 0.10000 | 0.90000 |
| x4 | 226977 | 3.18472 | 1.10254 | 722859 | 2.00000 | 5.00000 |
| x5 | 226977 | 0.23984 | 0.08024 | 54439 | 0.15000 | 0.35000 |
| x6 | 226977 | 0.15113 | 0.04999 | 34303 | 0.10000 | 0.20000 |
| x7 | 226977 | 0.40354 | 0.37263 | 91594 | -0.21388 | 1.02996 |

Pearson Correlation Coefficients, N = 226977
Prob > |r| under H0: Rho=0

| | x1 | x2 | x4 | x5 | x6 | x7 |
|---|---|---|---|---|---|---|
| x1 | 1.00000 | 0.93747 | -0.00537 | -0.09937 | -0.15043 | -0.88176 |
| | | <.0001 | 0.0106 | <.0001 | <.0001 | <.0001 |
| x2 | 0.93747 | 1.00000 | 0.01198 | -0.08632 | -0.13476 | -0.93445 |
| | <.0001 | | <.0001 | <.0001 | <.0001 | <.0001 |
| x4 | -0.00537 | 0.01198 | 1.00000 | -0.07050 | -0.06845 | -0.02133 |
| | 0.0106 | <.0001 | | <.0001 | <.0001 | <.0001 |
| x5 | -0.09937 | -0.08632 | -0.07050 | 1.00000 | -0.16481 | 0.07731 |
| | <.0001 | <.0001 | <.0001 | | <.0001 | <.0001 |
| x6 | -0.15043 | -0.13476 | -0.06845 | -0.16481 | 1.00000 | 0.12894 |
| | <.0001 | <.0001 | <.0001 | <.0001 | | <.0001 |
| x7 | -0.88176 | -0.93445 | -0.02133 | 0.07731 | 0.12894 | 1.00000 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | |

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The CORR Procedure

6 Variables: x1 x2 x4 x5 x6 x7

Covariance Matrix, DF = 226976

| | x1 | x2 | x4 | x5 | x6 | x7 |
|---|---|---|---|---|---|---|
| x1 | 0.013310993 | 0.030600508 | -0.000682703 | -0.000919871 | -0.000867544 | -0.037908526 |
| x2 | 0.030600508 | 0.080044782 | 0.003738304 | -0.001959494 | -0.001905864 | -0.098515099 |
| x4 | -0.000682703 | 0.003738304 | 1.215603899 | -0.006237114 | -0.003772409 | -0.008763795 |
| x5 | -0.000919871 | -0.001959494 | -0.006237114 | 0.006438250 | -0.000661045 | 0.002311448 |
| x6 | -0.000867544 | -0.001905864 | -0.003772409 | -0.000661045 | 0.002498730 | 0.002401774 |
| x7 | -0.037908526 | -0.098515099 | -0.008763795 | 0.002311448 | 0.002401774 | 0.138854170 |

Simple Statistics

| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
|---|---|---|---|---|---|---|
| x1 | 226977 | 0.16623 | 0.11537 | 37731 | 0.03333 | 0.45000 |

| | | | | | | |
|---|---|---|---|---|---|---|
| x2 | 226977 | 0.41589 | 0.28292 | 94397 | 0.10000 | 0.90000 |
| x4 | 226977 | 3.18472 | 1.10254 | 722859 | 2.00000 | 5.00000 |
| x5 | 226977 | 0.23984 | 0.08024 | 54439 | 0.15000 | 0.35000 |
| x6 | 226977 | 0.15113 | 0.04999 | 34303 | 0.10000 | 0.20000 |
| x7 | 226977 | 0.40354 | 0.37263 | 91594 | -0.21388 | 1.02996 |

Pearson Correlation Coefficients, N = 226977
Prob > |r| under H0: Rho=0

| | x1 | x2 | x4 | x5 | x6 | x7 |
|---|---|---|---|---|---|---|
| x1 | 1.00000 | 0.93747 | -0.00537 | -0.09937 | -0.15043 | -0.88176 |
| | | <.0001 | 0.0106 | <.0001 | <.0001 | <.0001 |
| x2 | 0.93747 | 1.00000 | 0.01198 | -0.08632 | -0.13476 | -0.93445 |
| | <.0001 | | <.0001 | <.0001 | <.0001 | <.0001 |
| x4 | -0.00537 | 0.01198 | 1.00000 | -0.07050 | -0.06845 | -0.02133 |
| | 0.0106 | <.0001 | | <.0001 | <.0001 | <.0001 |
| x5 | -0.09937 | -0.08632 | -0.07050 | 1.00000 | -0.16481 | 0.07731 |
| | <.0001 | <.0001 | <.0001 | | <.0001 | <.0001 |
| x6 | -0.15043 | -0.13476 | -0.06845 | -0.16481 | 1.00000 | 0.12894 |
| | <.0001 | <.0001 | <.0001 | <.0001 | | <.0001 |
| x7 | -0.88176 | -0.93445 | -0.02133 | 0.07731 | 0.12894 | 1.00000 |
| | <.0001 | <.0001 | <.0001 | <.0001 | <.0001 | |

## Variable Selection and Covariance Matrix for the water-wet homogeneous model (Intercept)

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Number of Observations Read 226977
Number of Observations Used 226977

Forward Selection: Step 1

Statistics for Entry
DF = 1,226975

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 1.000000 | 0.6227 | 374538 | <.0001 |
| x2 | 1.000000 | 0.4955 | 222965 | <.0001 |
| x4 | 1.000000 | 0.2717 | 84678.4 | <.0001 |
| x5 | 1.000000 | 0.0133 | 3062.80 | <.0001 |
| x6 | 1.000000 | 0.0225 | 5212.99 | <.0001 |
| x7 | 1.000000 | 0.5333 | 259349 | <.0001 |

Variable x1 Entered: R-Square = 0.6227 and C(p) = 1314262

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 1391052 | 1391052 | 374538 | <.0001 |
| Error | 226975 | 842995 | 3.71404 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -2.64708 | 0.00709 | 517052 | 139215 | <.0001 |
| x1 | -21.45737 | 0.03506 | 1391052 | 374538 | <.0001 |

Bounds on condition number: 1, 1

---

Forward Selection: Step 2
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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 2

Statistics for Entry
DF = 1,226974

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.121152 | 0.6332 | 6545.48 | <.0001 |
| x4 | 0.999971 | 0.8988 | 619402 | <.0001 |
| x5 | 0.990126 | 0.6240 | 833.80 | <.0001 |
| x6 | 0.977372 | 0.6237 | 598.29 | <.0001 |
| x7 | 0.222493 | 0.6280 | 3259.17 | <.0001 |

Variable x4 Entered: R-Square = 0.8988 and C(p) = 186344.4

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 2007980 | 1003990 | 1008016 | <.0001 |
| Error | 226974 | 226067 | 0.99601 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.12789 | 0.00709 | 89644 | 90003.7 | <.0001 |
| x1 | -21.53406 | 0.01816 | 1400973 | 1406592 | <.0001 |
| x4 | -1.49533 | 0.00190 | 616928 | 619402 | <.0001 |

Bounds on condition number: 1, 4.0001

---

Forward Selection: Step 3

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 3

Statistics for Entry
DF = 1,226973

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.120862 | 0.9154 | 44413.1 | <.0001 |
| x5 | 0.985080 | 0.8988 | 0.28 | 0.5970 |
| x6 | 0.972575 | 0.8988 | 63.73 | <.0001 |
| x7 | 0.221813 | 0.9008 | 4451.07 | <.0001 |

Variable x2 Entered: R-Square = 0.9154 and C(p) = 118706.2

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 2044976 | 681659 | 818308 | <.0001 |
| Error | 226973 | 189071 | 0.83301 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.04664 | 0.00650 | 82637 | 99203.0 | <.0001 |
| x1 | -30.97117 | 0.04776 | 350305 | 420530 | <.0001 |
| x2 | 4.10467 | 0.01948 | 36997 | 44413.1 | <.0001 |
| x4 | -1.51325 | 0.00174 | 630295 | 756648 | <.0001 |

Bounds on condition number: 8.2739, 52.648

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Forward Selection: Step 4

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 4

Statistics for Entry
DF = 1,226972

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.984545 | 0.9154 | 30.18 | <.0001 |
| x6 | 0.972117 | 0.9154 | 177.11 | <.0001 |
| x7 | 0.126408 | 0.9443 | 118198 | <.0001 |

Variable x7 Entered: R-Square = 0.9443 and C(p) = 337.3204

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 2109721 | 527430 | 962883 | <.0001 |
| Error | 226972 | 124327 | 0.54776 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.69538 | 0.01209 | 10767 | 19655.7 | <.0001 |
| x1 | -30.33430 | 0.03877 | 335280 | 612090 | <.0001 |
| x2 | 8.82201 | 0.02092 | 97393 | 177801 | <.0001 |
| x4 | -1.49834 | 0.00141 | 617349 | 1127040 | <.0001 |
| x7 | 4.03128 | 0.01173 | 64744 | 118198 | <.0001 |

Bounds on condition number: 14.518, 126.9

---

Forward Selection: Step 5

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 5

Statistics for Entry
DF = 1,226971

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.984346 | 0.9444 | 3.57 | 0.0590 |
| x6 | 0.972108 | 0.9444 | 306.22 | <.0001 |

Variable x6 Entered: R-Square = 0.9444 and C(p) = 33.0606

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 2109888 | 421978 | 771403 | <.0001 |
| Error | 226971 | 124159 | 0.54703 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.60130 | 0.01323 | 8017.73125 | 14656.9 | <.0001 |
| x1 | -30.38399 | 0.03885 | 334582 | 611638 | <.0001 |
| x2 | 8.82875 | 0.02091 | 97509 | 178252 | <.0001 |

| | | | | | |
|---|---|---|---|---|---|
| x4 | -1.50009 | 0.00141 | 615684 | 1125512 | <.0001 |
| x6 | -0.55121 | 0.03150 | 167.51209 | 306.22 | <.0001 |
| x7 | 4.03193 | 0.01172 | 64764 | 118394 | <.0001 |

Bounds on condition number: 14.523, 164.04

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Forward Selection: Step 6

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 6

Statistics for Entry
DF = 1,226970

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.949089 | 0.9444 | 28.06 | <.0001 |

Variable x5 Entered: R-Square = 0.9444 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 2109903 | 351651 | 642917 | <.0001 |
| Error | 226970 | 124144 | 0.54696 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.56684 | 0.01474 | 6181.05193 | 11300.7 | <.0001 |
| x1 | -30.39889 | 0.03895 | 333163 | 609116 | <.0001 |
| x2 | 8.83008 | 0.02091 | 97524 | 178302 | <.0001 |
| x4 | -1.50075 | 0.00142 | 611502 | 1118000 | <.0001 |
| x5 | -0.10520 | 0.01986 | 15.34805 | 28.06 | <.0001 |
| x6 | -0.58337 | 0.03208 | 180.90739 | 330.75 | <.0001 |
| x7 | 4.03107 | 0.01172 | 64724 | 118334 | <.0001 |

Bounds on condition number: 14.526, 203.73

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All variables have been entered into the model.

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Summary of Forward Selection

| Step | Variable Entered | Label | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|
| 1 | x1 | | 1 | 0.6227 | 0.6227 | 1314262 | 374538 | <.0001 |
| 2 | x4 | | 2 | 0.2761 | 0.8988 | 186344 | 619402 | <.0001 |
| 3 | x2 | | 3 | 0.0166 | 0.9154 | 118706 | 44413.1 | <.0001 |
| 4 | x7 | | 4 | 0.0290 | 0.9443 | 337.320 | 118198 | <.0001 |
| 5 | x6 | | 5 | 0.0001 | 0.9444 | 33.0606 | 306.22 | <.0001 |
| 6 | x5 | | 6 | 0.0000 | 0.9444 | 7.0000 | 28.06 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Number of Observations Read 226977
Number of Observations Used 226977

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.9444 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 2109903 | 351651 | 642917 | <.0001 |
| Error | 226970 | 124144 | 0.54696 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.56684 | 0.01474 | 6181.05193 | 11300.7 | <.0001 |
| x1 | -30.39889 | 0.03895 | 333163 | 609116 | <.0001 |
| x2 | 8.83008 | 0.02091 | 97524 | 178302 | <.0001 |
| x4 | -1.50075 | 0.00142 | 611502 | 1118000 | <.0001 |
| x5 | -0.10520 | 0.01986 | 15.34805 | 28.06 | <.0001 |
| x6 | -0.58337 | 0.03208 | 180.90739 | 330.75 | <.0001 |
| x7 | 4.03107 | 0.01172 | 64724 | 118334 | <.0001 |

Bounds on condition number: 14.526, 203.73

---

Backward Elimination: Step 1

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Backward Elimination: Step 1

Statistics for Removal
DF = 1,226970

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| x1 | 0.1491 | 0.7953 | 609116 | <.0001 |
| x2 | 0.0437 | 0.9008 | 178302 | <.0001 |
| x4 | 0.2737 | 0.6707 | 1118000 | <.0001 |
| x5 | 0.0000 | 0.9444 | 28.06 | <.0001 |
| x6 | 0.0001 | 0.9444 | 330.75 | <.0001 |
| x7 | 0.0290 | 0.9155 | 118334 | <.0001 |

All variables left in the model are significant at the 0.0500 level.

The SAS System 09:56 Saturday, October 3, 2009 5803

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Number of Observations Read 226977
Number of Observations Used 226977

Stepwise Selection: Step 1

Statistics for Entry
DF = 1,226975

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 1.000000 | 0.6227 | 374538 | <.0001 |
| x2 | 1.000000 | 0.4955 | 222965 | <.0001 |
| x4 | 1.000000 | 0.2717 | 84678.4 | <.0001 |
| x5 | 1.000000 | 0.0133 | 3062.80 | <.0001 |
| x6 | 1.000000 | 0.0225 | 5212.99 | <.0001 |
| x7 | 1.000000 | 0.5333 | 259349 | <.0001 |

Variable x1 Entered: R-Square = 0.6227 and C(p) = 1314262

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 1391052 | 1391052 | 374538 | <.0001 |
| Error | 226975 | 842995 | 3.71404 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -2.64708 | 0.00709 | 517052 | 139215 | <.0001 |
| x1 | -21.45737 | 0.03506 | 1391052 | 374538 | <.0001 |

Bounds on condition number: 1, 1

---

Stepwise Selection: Step 2

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 2

Statistics for Entry
DF = 1,226974

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.121152 | 0.6332 | 6545.48 | <.0001 |
| x4 | 0.999971 | 0.8988 | 619402 | <.0001 |
| x5 | 0.990126 | 0.6240 | 833.80 | <.0001 |
| x6 | 0.977372 | 0.6237 | 598.29 | <.0001 |
| x7 | 0.222493 | 0.6280 | 3259.17 | <.0001 |

Variable x4 Entered: R-Square = 0.8988 and C(p) = 186344.4

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 2007980 | 1003990 | 1008016 | <.0001 |
| Error | 226974 | 226067 | 0.99601 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.12789 | 0.00709 | 89644 | 90003.7 | <.0001 |
| x1 | -21.53406 | 0.01816 | 1400973 | 1406592 | <.0001 |
| x4 | -1.49533 | 0.00190 | 616928 | 619402 | <.0001 |

Bounds on condition number: 1, 4.0001

---

Stepwise Selection: Step 3

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 3

Statistics for Removal
DF = 1,226974

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.6271 | 0.2717 | 1406592 | <.0001 |
| x4 | 0.2761 | 0.6227 | 619402 | <.0001 |

Statistics for Entry
DF = 1,226973

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.120862 | 0.9154 | 44413.1 | <.0001 |
| x5 | 0.985080 | 0.8988 | 0.28 | 0.5970 |
| x6 | 0.972575 | 0.8988 | 63.73 | <.0001 |

| | | | | |
|---|---|---|---|---|
| x7 | 0.221813 | 0.9008 | 4451.07 | <.0001 |

Variable x2 Entered: R-Square = 0.9154 and C(p) = 118706.2

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 2044976 | 681659 | 818308 | <.0001 |
| Error | 226973 | 189071 | 0.83301 | | |
| Corrected Total | 226976 | 2234047 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.04664 | 0.00650 | 82637 | 99203.0 | <.0001 |
| x1 | -30.97117 | 0.04776 | 350305 | 420530 | <.0001 |
| x2 | 4.10467 | 0.01948 | 36997 | 44413.1 | <.0001 |
| x4 | -1.51325 | 0.00174 | 630295 | 756648 | <.0001 |



The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 3

Bounds on condition number: 8.2739, 52.648

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Stepwise Selection: Step 4

Statistics for Removal
DF = 1,226973

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.1568 | 0.7586 | 420530 | <.0001 |
| x2 | 0.0166 | 0.8988 | 44413.1 | <.0001 |
| x4 | 0.2821 | 0.6332 | 756648 | <.0001 |

Statistics for Entry
DF = 1,226972

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.984545 | 0.9154 | 30.18 | <.0001 |
| x6 | 0.972117 | 0.9154 | 177.11 | <.0001 |
| x7 | 0.126408 | 0.9443 | 118198 | <.0001 |

Variable x7 Entered: R-Square = 0.9443 and C(p) = 337.3204

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| Model | 4 | 2109721 | 527430 | 962883 | <.0001 |
| Error | 226972 | 124327 | 0.54776 | | |
| Corrected Total | 226976 | 2234047 | | | |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 4

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.69538 | 0.01209 | 10767 | 19655.7 | <.0001 |
| x1 | -30.33430 | 0.03877 | 335280 | 612090 | <.0001 |
| x2 | 8.82201 | 0.02092 | 97393 | 177801 | <.0001 |
| x4 | -1.49834 | 0.00141 | 617349 | 1127040 | <.0001 |
| x7 | 4.03128 | 0.01173 | 64744 | 118198 | <.0001 |

Bounds on condition number: 14.518, 126.9

---

Stepwise Selection: Step 5

Statistics for Removal
DF = 1,226972

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.1501 | 0.7943 | 612090 | <.0001 |
| x2 | 0.0436 | 0.9008 | 177801 | <.0001 |
| x4 | 0.2763 | 0.6680 | 1127040 | <.0001 |
| x7 | 0.0290 | 0.9154 | 118198 | <.0001 |

Statistics for Entry
DF = 1,226971

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.984346 | 0.9444 | 3.57 | 0.0590 |
| x6 | 0.972108 | 0.9444 | 306.22 | <.0001 |

Variable x6 Entered: R-Square = 0.9444 and C(p) = 33.0606

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 5

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 2109888 | 421978 | 771403 | <.0001 |

```
Error        226971     124159    0.54703
Corrected Total   226976    2234047


       Parameter   Standard
  Variable   Estimate    Error  Type II SS F Value Pr > F

  Intercept   -1.60130   0.01323  8017.73125 14656.9 <.0001
  x1      -30.38399   0.03885    334582  611638 <.0001
  x2       8.82875   0.02091    97509  178252 <.0001
  x4      -1.50009   0.00141    615684 1125512 <.0001
  x6      -0.55121   0.03150  167.51209  306.22 <.0001
  x7       4.03193   0.01172    64764  118394 <.0001

       Bounds on condition number: 14.523, 164.04
-------------------------------------------------------------------------------------------

          Stepwise Selection: Step 6


           Statistics for Removal
             DF = 1,226971

          Partial     Model
   Variable    R-Square   R-Square  F Value  Pr > F

   x1       0.1498    0.7947   611638  <.0001
   x2       0.0436    0.9008   178252  <.0001
   x4       0.2756    0.6688  1125512  <.0001
   x6       0.0001    0.9443   306.22  <.0001
   x7       0.0290    0.9154   118394  <.0001


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            The REG Procedure
             Model: MODEL1
       Dependent Variable: Intercept Intercept

          Stepwise Selection: Step 6

           Statistics for Entry
             DF = 1,226970

               Model
  Variable    Tolerance   R-Square  F Value  Pr > F

  x5       0.949089    0.9444   28.06  <.0001


   Variable x5 Entered: R-Square = 0.9444 and C(p) = 7.0000


           Analysis of Variance

              Sum of      Mean
Source          DF    Squares     Square  F Value  Pr > F

Model          6   2109903    351651   642917  <.0001
Error        226970     124144    0.54696
Corrected Total   226976    2234047


       Parameter   Standard
  Variable   Estimate    Error  Type II SS F Value Pr > F
```

| | | | | | |
|---|---|---|---|---|---|
| Intercept | -1.56684 | 0.01474 | 6181.05193 | 11300.7 | <.0001 |
| x1 | -30.39889 | 0.03895 | 333163 | 609116 | <.0001 |
| x2 | 8.83008 | 0.02091 | 97524 | 178302 | <.0001 |
| x4 | -1.50075 | 0.00142 | 611502 | 1118000 | <.0001 |
| x5 | -0.10520 | 0.01986 | 15.34805 | 28.06 | <.0001 |
| x6 | -0.58337 | 0.03208 | 180.90739 | 330.75 | <.0001 |
| x7 | 4.03107 | 0.01172 | 64724 | 118334 | <.0001 |

Bounds on condition number: 14.526, 203.73

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Stepwise Selection: Step 7

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 7

Statistics for Removal
DF = 1,226970

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.1491 | 0.7953 | 609116 | <.0001 |
| x2 | 0.0437 | 0.9008 | 178302 | <.0001 |
| x4 | 0.2737 | 0.6707 | 1118000 | <.0001 |
| x5 | 0.0000 | 0.9444 | 28.06 | <.0001 |
| x6 | 0.0001 | 0.9444 | 330.75 | <.0001 |
| x7 | 0.0290 | 0.9155 | 118334 | <.0001 |

All variables left in the model are significant at the 0.1500 level.

All variables have been entered into the model.

Summary of Stepwise Selection

| Step | Variable Entered | Variable Removed | Label | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|---|
| 1 | x1 | | | 1 | 0.6227 | 0.6227 | 1314262 | 374538 | <.0001 |
| 2 | x4 | | | 2 | 0.2761 | 0.8988 | 186344 | 619402 | <.0001 |
| 3 | x2 | | | 3 | 0.0166 | 0.9154 | 118706 | 44413.1 | <.0001 |
| 4 | x7 | | | 4 | 0.0290 | 0.9443 | 337.320 | 118198 | <.0001 |
| 5 | x6 | | | 5 | 0.0001 | 0.9444 | 33.0606 | 306.22 | <.0001 |
| 6 | x5 | | | 6 | 0.0000 | 0.9444 | 7.0000 | 28.06 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept

R-Square Selection Method

Number of Observations Read 226977
Number of Observations Used 226977

Number in

| Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 1 | 0.6227 | 1314262 | 3.71404 | x1 |
| 1 | 0.5333 | 1679313 | 4.59374 | x7 |
| 1 | 0.4955 | 1833463 | 4.96521 | x2 |
| 1 | 0.2717 | 2747720 | 7.16837 | x4 |
| 1 | 0.0225 | 3765798 | 9.62172 | x6 |
| 1 | 0.0133 | 3803119 | 9.71165 | x5 |
| 2 | 0.8988 | 186344.4 | 0.99601 | x1 x4 |
| 2 | 0.7891 | 634398.7 | 2.07572 | x4 x7 |
| 2 | 0.7586 | 759162.2 | 2.37638 | x2 x4 |
| 2 | 0.6332 | 1271063 | 3.60996 | x1 x2 |
| 2 | 0.6280 | 1292446 | 3.66148 | x1 x7 |
| 2 | 0.6240 | 1308623 | 3.70047 | x1 x5 |
| 2 | 0.6237 | 1310212 | 3.70430 | x1 x6 |
| 2 | 0.5369 | 1664351 | 4.55770 | x2 x7 |
| 2 | 0.5368 | 1665044 | 4.55937 | x5 x7 |
| 2 | 0.5364 | 1666439 | 4.56273 | x6 x7 |
| 2 | 0.4986 | 1820893 | 4.93493 | x2 x6 |
| 2 | 0.4986 | 1821186 | 4.93564 | x2 x5 |
| 2 | 0.2848 | 2694241 | 7.03952 | x4 x6 |
| 2 | 0.2779 | 2722338 | 7.10723 | x4 x5 |
| 2 | 0.0426 | 3683411 | 9.42322 | x5 x6 |
| 3 | 0.9154 | 118706.2 | 0.83301 | x1 x2 x4 |
| 3 | 0.9008 | 178397.0 | 0.97685 | x1 x4 x7 |
| 3 | 0.8988 | 186230.4 | 0.99573 | x1 x4 x6 |
| 3 | 0.8988 | 186345.9 | 0.99601 | x1 x4 x5 |
| 3 | 0.7943 | 613321.7 | 2.02494 | x2 x4 x7 |
| 3 | 0.7897 | 632002.7 | 2.06996 | x4 x5 x7 |
| 3 | 0.7896 | 632299.9 | 2.07067 | x4 x6 x7 |
| 3 | 0.7590 | 757374.4 | 2.37208 | x2 x4 x6 |
| 3 | 0.7589 | 757672.1 | 2.37279 | x2 x4 x5 |
| 3 | 0.6680 | 1129025 | 3.26768 | x1 x2 x7 |
| 3 | 0.6345 | 1266022 | 3.59782 | x1 x2 x5 |
| 3 | 0.6341 | 1267479 | 3.60133 | x1 x2 x6 |
| 3 | 0.6295 | 1286306 | 3.64670 | x1 x5 x7 |
| 3 | 0.6290 | 1288245 | 3.65137 | x1 x6 x7 |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept

R-Square Selection Method

| Number in Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 3 | 0.6256 | 1302430 | 3.68556 | x1 x5 x6 |
| 3 | 0.5414 | 1646347 | 4.51433 | x5 x6 x7 |
| 3 | 0.5402 | 1651197 | 4.52602 | x2 x5 x7 |
| 3 | 0.5398 | 1652556 | 4.52929 | x2 x6 x7 |
| 3 | 0.5030 | 1803208 | 4.89233 | x2 x5 x6 |
| 3 | 0.2948 | 2653561 | 6.94152 | x4 x5 x6 |
| 4 | 0.9443 | 337.3204 | 0.54776 | x1 x2 x4 x7 |
| 4 | 0.9154 | 118438.7 | 0.83236 | x1 x2 x4 x6 |
| 4 | 0.9154 | 118662.3 | 0.83290 | x1 x2 x4 x5 |
| 4 | 0.9008 | 178304.6 | 0.97663 | x1 x4 x6 x7 |
| 4 | 0.9008 | 178396.4 | 0.97685 | x1 x4 x5 x7 |
| 4 | 0.8988 | 186224.6 | 0.99572 | x1 x4 x5 x6 |
| 4 | 0.7947 | 611473.9 | 2.02049 | x2 x4 x5 x7 |
| 4 | 0.7947 | 611741.9 | 2.02113 | x2 x4 x6 x7 |
| 4 | 0.7905 | 628903.6 | 2.06249 | x4 x5 x6 x7 |

| | | | | |
|---|---|---|---|---|
| 4 | 0.7595 | 755145.2 | 2.36671 | x2 x4 x5 x6 |
| 4 | 0.6694 | 1123323 | 3.25395 | x1 x2 x5 x7 |
| 4 | 0.6688 | 1125677 | 3.25962 | x1 x2 x6 x7 |
| 4 | 0.6358 | 1260526 | 3.58459 | x1 x2 x5 x6 |
| 4 | 0.6311 | 1279824 | 3.63109 | x1 x5 x6 x7 |
| 4 | 0.5444 | 1633980 | 4.48454 | x2 x5 x6 x7 |
| 5 | 0.9444 | 33.0606 | 0.54703 | x1 x2 x4 x6 x7 |
| 5 | 0.9444 | 335.7502 | 0.54776 | x1 x2 x4 x5 x7 |
| 5 | 0.9155 | 118339.3 | 0.83212 | x1 x2 x4 x5 x6 |
| 5 | 0.9008 | 178306.5 | 0.97663 | x1 x4 x5 x6 x7 |
| 5 | 0.7953 | 609121.1 | 2.01482 | x2 x4 x5 x6 x7 |
| 5 | 0.6707 | 1118005 | 3.24114 | x1 x2 x5 x6 x7 |
| 6 | 0.9444 | 7.0000 | 0.54696 | x1 x2 x4 x5 x6 x7 |

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The CORR Procedure

6 Variables: x1 x2 x4 x5 x6 x7

Covariance Matrix, DF = 226976

| | x1 | x2 | x4 | x5 | x6 | x7 |
|---|---|---|---|---|---|---|
| x1 | 0.013310993 | 0.030600508 | -0.000682703 | -0.000919871 | -0.000867544 | -0.037908526 |
| x2 | 0.030600508 | 0.080044782 | 0.003738304 | -0.001959494 | -0.001905864 | -0.098515099 |
| x4 | -0.000682703 | 0.003738304 | 1.215603899 | -0.006237114 | -0.003772409 | -0.008763795 |
| x5 | -0.000919871 | -0.001959494 | -0.006237114 | 0.006438250 | -0.000661045 | 0.002311448 |
| x6 | -0.000867544 | -0.001905864 | -0.003772409 | -0.000661045 | 0.002498730 | 0.002401774 |
| x7 | -0.037908526 | -0.098515099 | -0.008763795 | 0.002311448 | 0.002401774 | 0.138854170 |

Simple Statistics

| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
|---|---|---|---|---|---|---|
| x1 | 226977 | 0.16623 | 0.11537 | 37731 | 0.03333 | 0.45000 |
| x2 | 226977 | 0.41589 | 0.28292 | 94397 | 0.10000 | 0.90000 |
| x4 | 226977 | 3.18472 | 1.10254 | 722859 | 2.00000 | 5.00000 |
| x5 | 226977 | 0.23984 | 0.08024 | 54439 | 0.15000 | 0.35000 |
| x6 | 226977 | 0.15113 | 0.04999 | 34303 | 0.10000 | 0.20000 |
| x7 | 226977 | 0.40354 | 0.37263 | 91594 | -0.21388 | 1.02996 |

Pearson Correlation Coefficients, N = 226977
Prob > |r| under H0: Rho=0

| | x1 | x2 | x4 | x5 | x6 | x7 |
|---|---|---|---|---|---|---|
| x1 | 1.00000 | 0.93747 | -0.00537 | -0.09937 | -0.15043 | -0.88176 |
| | | <.0001 | 0.0106 | <.0001 | <.0001 | <.0001 |
| x2 | 0.93747 | 1.00000 | 0.01198 | -0.08632 | -0.13476 | -0.93445 |
| | <.0001 | | <.0001 | <.0001 | <.0001 | <.0001 |
| x4 | -0.00537 | 0.01198 | 1.00000 | -0.07050 | -0.06845 | -0.02133 |
| | 0.0106 | <.0001 | | <.0001 | <.0001 | <.0001 |
| x5 | -0.09937 | -0.08632 | -0.07050 | 1.00000 | -0.16481 | 0.07731 |
| | <.0001 | <.0001 | <.0001 | | <.0001 | <.0001 |
| x6 | -0.15043 | -0.13476 | -0.06845 | -0.16481 | 1.00000 | 0.12894 |

```
  <.0001   <.0001   <.0001   <.0001          <.0001

x7   -0.88176   -0.93445   -0.02133    0.07731    0.12894    1.00000
  <.0001   <.0001   <.0001   <.0001   <.0001
```

## Variable Selection and Covariance Matrix for the oil-wet homogeneous model (Slope)

```
           The REG Procedure
            Model: MODEL1
          Dependent Variable: slope

       Number of Observations Read   804144
       Number of Observations Used   804144

          Forward Selection: Step 1


           Statistics for Entry
            DF = 1,804142

                Model
 Variable    Tolerance   R-Square  F Value  Pr > F

 x1        1.000000    0.1396   130519  <.0001
 x2        1.000000    0.1680   162387  <.0001
 x4        1.000000    0.3387   411943  <.0001
 x5        1.000000    0.0025  1977.47  <.0001
 x6        1.000000    0.0880  77621.3  <.0001
 x7        1.000000    0.2200   226815  <.0001


  Variable x4 Entered: R-Square = 0.3387 and C(p) = 5475160


           Analysis of Variance

              Sum of      Mean
Source         DF    Squares     Square  F Value  Pr > F

Model          1    3455232    3455232   411943  <.0001
Error       804142    6744864    8.38765
Corrected Total  804143    10200095


      Parameter   Standard
 Variable   Estimate    Error  Type II SS F Value Pr > F

 Intercept   7.64402   0.01005   4855739  578915 <.0001
 x4       1.85608   0.00289   3455232  411943 <.0001

        Bounds on condition number: 1, 1
-------------------------------------------------------------------------------------------------

          Forward Selection: Step 2
            The SAS System   11:24 Saturday, October 3, 2009 9654

           The REG Procedure
            Model: MODEL1
          Dependent Variable: slope

          Forward Selection: Step 2
```

Statistics for Entry
DF = 1,804141

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.999919 | 0.4745 | 207755 | <.0001 |
| x2 | 0.999585 | 0.4972 | 253513 | <.0001 |
| x5 | 0.999993 | 0.3414 | 3180.42 | <.0001 |
| x6 | 0.915514 | 0.5758 | 449401 | <.0001 |
| x7 | 0.999529 | 0.5472 | 370089 | <.0001 |

Variable x6 Entered: R-Square = 0.5758 and C(p) = 3224002

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 5873298 | 2936649 | 545780 | <.0001 |
| Error | 804141 | 4326797 | 5.38064 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -0.37485 | 0.01442 | 3637.70377 | 676.07 | <.0001 |
| x4 | 2.32777 | 0.00242 | 4975388 | 924683 | <.0001 |
| x6 | 18.71166 | 0.02791 | 2418067 | 449401 | <.0001 |

Bounds on condition number: 1.0923, 4.3691

---

Forward Selection: Step 3

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 3

Statistics for Entry
DF = 1,804140

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.963891 | 0.7962 | 869279 | <.0001 |
| x2 | 0.964070 | 0.8246 | 1141135 | <.0001 |
| x5 | 0.995161 | 0.5830 | 13964.4 | <.0001 |
| x7 | 0.954563 | 0.9039 | 2746752 | <.0001 |

Variable x7 Entered: R-Square = 0.9039 and C(p) = 108082.3

Analysis of Variance

Sum of Mean

| Source | DF | Squares | Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 9220243 | 3073414 | 2522273 | <.0001 |
| Error | 804140 | 979853 | 1.21851 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.10594 | 0.00702 | 109601 | 89947.1 | <.0001 |
| x4 | 2.40763 | 0.00115 | 5313344 | 4360526 | <.0001 |
| x6 | 23.48963 | 0.01359 | 3639193 | 2986593 | <.0001 |
| x7 | -7.23629 | 0.00437 | 3346944 | 2746752 | <.0001 |

Bounds on condition number: 1.1437, 9.8566

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Forward Selection: Step 4

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 4

Statistics for Entry
DF = 1,804139

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.391421 | 0.9059 | 16448.8 | <.0001 |
| x2 | 0.265209 | 0.9044 | 3784.50 | <.0001 |
| x5 | 0.994818 | 0.9131 | 84836.7 | <.0001 |

Variable x5 Entered: R-Square = 0.9131 and C(p) = 21029.47

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 9313752 | 2328438 | 2112486 | <.0001 |
| Error | 804139 | 886343 | 1.10223 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.62404 | 0.00688 | 61412 | 55715.9 | <.0001 |
| x4 | 2.41514 | 0.00110 | 5343579 | 4847987 | <.0001 |
| x5 | 6.50617 | 0.02234 | 93509 | 84836.7 | <.0001 |
| x6 | 23.76092 | 0.01296 | 3704509 | 3360933 | <.0001 |
| x7 | -7.25876 | 0.00415 | 3366599 | 3054362 | <.0001 |

Bounds on condition number: 1.1497, 17.191

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Forward Selection: Step 5

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 5

Statistics for Entry
DF = 1,804138

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.391415 | 0.9150 | 17900.7 | <.0001 |
| x2 | 0.265179 | 0.9135 | 3789.78 | <.0001 |

Variable x1 Entered: R-Square = 0.9150 and C(p) = 3062.767

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 9333053 | 1866611 | 1731187 | <.0001 |
| Error | 804138 | 867042 | 1.07823 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 0.72599 | 0.00956 | 6220.21933 | 5768.94 | <.0001 |
| x1 | 2.03790 | 0.01523 | 19301 | 17900.7 | <.0001 |
| x4 | 2.41858 | 0.00109 | 5355805 | 4967239 | <.0001 |
| x5 | 6.49437 | 0.02209 | 93169 | 86409.5 | <.0001 |
| x6 | 23.82882 | 0.01283 | 3719881 | 3450002 | <.0001 |
| x7 | -6.59415 | 0.00645 | 1128375 | 1046511 | <.0001 |

Bounds on condition number: 2.5803, 41.936

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Forward Selection: Step 6

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Forward Selection: Step 6

Statistics for Entry
DF = 1,804137

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.119539 | 0.9153 | 3057.77 | <.0001 |

Variable x2 Entered: R-Square = 0.9153 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 9336338 | 1556056 | 1448650 | <.0001 |
| Error | 804137 | 863758 | 1.07414 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.01676 | 0.01089 | 9357.77363 | 8711.85 | <.0001 |
| x1 | 2.96582 | 0.02264 | 18428 | 17155.8 | <.0001 |
| x2 | -0.65198 | 0.01179 | 3284.47749 | 3057.77 | <.0001 |
| x4 | 2.42011 | 0.00108 | 5359086 | 4989175 | <.0001 |
| x5 | 6.50844 | 0.02205 | 93561 | 87102.5 | <.0001 |
| x6 | 23.84656 | 0.01281 | 3723084 | 3466099 | <.0001 |
| x7 | -6.83971 | 0.00782 | 822256 | 765500 | <.0001 |

Bounds on condition number: 8.3655, 126.58

------------------------------------------------------------------------------------------------

All variables have been entered into the model.

The SAS System 11:24 Saturday, October 3, 2009 9659

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Summary of Forward Selection

| Step | Variable Entered | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|
| 1 | x4 | 1 | 0.3387 | 0.3387 | 5475160 | 411943 | <.0001 |
| 2 | x6 | 2 | 0.2371 | 0.5758 | 3224002 | 449401 | <.0001 |
| 3 | x7 | 3 | 0.3281 | 0.9039 | 108082 | 2746752 | <.0001 |
| 4 | x5 | 4 | 0.0092 | 0.9131 | 21029.5 | 84836.7 | <.0001 |
| 5 | x1 | 5 | 0.0019 | 0.9150 | 3062.77 | 17900.7 | <.0001 |
| 6 | x2 | 6 | 0.0003 | 0.9153 | 7.0000 | 3057.77 | <.0001 |

The SAS System 11:24 Saturday, October 3, 2009 9660

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Number of Observations Read 804144
Number of Observations Used 804144

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.9153 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 9336338 | 1556056 | 1448650 | <.0001 |
| Error | 804137 | 863758 | 1.07414 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.01676 | 0.01089 | 9357.77363 | 8711.85 | <.0001 |
| x1 | 2.96582 | 0.02264 | 18428 | 17155.8 | <.0001 |
| x2 | -0.65198 | 0.01179 | 3284.47749 | 3057.77 | <.0001 |
| x4 | 2.42011 | 0.00108 | 5359086 | 4989175 | <.0001 |
| x5 | 6.50844 | 0.02205 | 93561 | 87102.5 | <.0001 |
| x6 | 23.84656 | 0.01281 | 3723084 | 3466099 | <.0001 |
| x7 | -6.83971 | 0.00782 | 822256 | 765500 | <.0001 |

Bounds on condition number: 8.3655, 126.58

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Backward Elimination: Step 1

The SAS System 11:24 Saturday, October 3, 2009 9661

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Backward Elimination: Step 1

Statistics for Removal
DF = 1,804137

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0018 | 0.9135 | 17155.8 | <.0001 |
| x2 | 0.0003 | 0.9150 | 3057.77 | <.0001 |
| x4 | 0.5254 | 0.3899 | 4989175 | <.0001 |
| x5 | 0.0092 | 0.9061 | 87102.5 | <.0001 |
| x6 | 0.3650 | 0.5503 | 3466099 | <.0001 |
| x7 | 0.0806 | 0.8347 | 765500 | <.0001 |

All variables left in the model are significant at the 0.0500 level.

The SAS System 11:24 Saturday, October 3, 2009 9662

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Number of Observations Read 804144
Number of Observations Used 804144

Stepwise Selection: Step 1

Statistics for Entry
DF = 1,804142

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| x1 | 1.000000 | 0.1396 | 130519 | <.0001 |
| x2 | 1.000000 | 0.1680 | 162387 | <.0001 |
| x4 | 1.000000 | 0.3387 | 411943 | <.0001 |
| x5 | 1.000000 | 0.0025 | 1977.47 | <.0001 |
| x6 | 1.000000 | 0.0880 | 77621.3 | <.0001 |
| x7 | 1.000000 | 0.2200 | 226815 | <.0001 |

Variable x4 Entered: R-Square = 0.3387 and C(p) = 5475160

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 3455232 | 3455232 | 411943 | <.0001 |
| Error | 804142 | 6744864 | 8.38765 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 7.64402 | 0.01005 | 4855739 | 578915 | <.0001 |
| x4 | 1.85608 | 0.00289 | 3455232 | 411943 | <.0001 |

Bounds on condition number: 1, 1

---------------------------------------------------------------------------------------------

Stepwise Selection: Step 2

The SAS System 11:24 Saturday, October 3, 2009 9663

The REG Procedure

Model: MODEL1

Dependent Variable: slope

Stepwise Selection: Step 2

Statistics for Entry

DF = 1,804141

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.999919 | 0.4745 | 207755 | <.0001 |
| x2 | 0.999585 | 0.4972 | 253513 | <.0001 |
| x5 | 0.999993 | 0.3414 | 3180.42 | <.0001 |
| x6 | 0.915514 | 0.5758 | 449401 | <.0001 |
| x7 | 0.999529 | 0.5472 | 370089 | <.0001 |

Variable x6 Entered: R-Square = 0.5758 and C(p) = 3224002

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 5873298 | 2936649 | 545780 | <.0001 |
| Error | 804141 | 4326797 | 5.38064 | | |
| Corrected Total | 804143 | 10200095 | | | |

Parameter Standard

| Variable | Estimate | Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -0.37485 | 0.01442 | 3637.70377 | 676.07 | <.0001 |
| x4 | 2.32777 | 0.00242 | 4975388 | 924683 | <.0001 |
| x6 | 18.71166 | 0.02791 | 2418067 | 449401 | <.0001 |

Bounds on condition number: 1.0923, 4.3691

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Stepwise Selection: Step 3

The SAS System 11:24 Saturday, October 3, 2009 9664

The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 3

Statistics for Removal
DF = 1,804141

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x4 | 0.4878 | 0.0880 | 924683 | <.0001 |
| x6 | 0.2371 | 0.3387 | 449401 | <.0001 |

Statistics for Entry
DF = 1,804140

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.963891 | 0.7962 | 869279 | <.0001 |
| x2 | 0.964070 | 0.8246 | 1141135 | <.0001 |
| x5 | 0.995161 | 0.5830 | 13964.4 | <.0001 |
| x7 | 0.954563 | 0.9039 | 2746752 | <.0001 |

Variable x7 Entered: R-Square = 0.9039 and C(p) = 108082.3

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 9220243 | 3073414 | 2522273 | <.0001 |
| Error | 804140 | 979853 | 1.21851 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 2.10594 | 0.00702 | 109601 | 89947.1 | <.0001 |
| x4 | 2.40763 | 0.00115 | 5313344 | 4360526 | <.0001 |
| x6 | 23.48963 | 0.01359 | 3639193 | 2986593 | <.0001 |
| x7 | -7.23629 | 0.00437 | 3346944 | 2746752 | <.0001 |

The SAS System 11:24 Saturday, October 3, 2009 9665

The REG Procedure
Model: MODEL1

Dependent Variable: slope

Stepwise Selection: Step 3

Bounds on condition number: 1.1437, 9.8566

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Stepwise Selection: Step 4

Statistics for Removal
DF = 1,804140

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x4 | 0.5209 | 0.3830 | 4360526 | <.0001 |
| x6 | 0.3568 | 0.5472 | 2986593 | <.0001 |
| x7 | 0.3281 | 0.5758 | 2746752 | <.0001 |

Statistics for Entry
DF = 1,804139

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.391421 | 0.9059 | 16448.8 | <.0001 |
| x2 | 0.265209 | 0.9044 | 3784.50 | <.0001 |
| x5 | 0.994818 | 0.9131 | 84836.7 | <.0001 |

Variable x5 Entered: R-Square = 0.9131 and C(p) = 21029.47

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 9313752 | 2328438 | 2112486 | <.0001 |
| Error | 804139 | 886343 | 1.10223 | | |
| Corrected Total | 804143 | 10200095 | | | |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 4

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.62404 | 0.00688 | 61412 | 55715.9 | <.0001 |
| x4 | 2.41514 | 0.00110 | 5343579 | 4847987 | <.0001 |
| x5 | 6.50617 | 0.02234 | 93509 | 84836.7 | <.0001 |
| x6 | 23.76092 | 0.01296 | 3704509 | 3360933 | <.0001 |
| x7 | -7.25876 | 0.00415 | 3366599 | 3054362 | <.0001 |

Bounds on condition number: 1.1497, 17.191

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Stepwise Selection: Step 5

Statistics for Removal
DF = 1,804139

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x4 | 0.5239 | 0.3892 | 4847987 | <.0001 |
| x5 | 0.0092 | 0.9039 | 84836.7 | <.0001 |
| x6 | 0.3632 | 0.5499 | 3360933 | <.0001 |
| x7 | 0.3301 | 0.5830 | 3054362 | <.0001 |

Statistics for Entry
DF = 1,804138

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.391415 | 0.9150 | 17900.7 | <.0001 |
| x2 | 0.265179 | 0.9135 | 3789.78 | <.0001 |

Variable x1 Entered: R-Square = 0.9150 and C(p) = 3062.767

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 5

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 9333053 | 1866611 | 1731187 | <.0001 |
| Error | 804138 | 867042 | 1.07823 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 0.72599 | 0.00956 | 6220.21933 | 5768.94 | <.0001 |
| x1 | 2.03790 | 0.01523 | 19301 | 17900.7 | <.0001 |
| x4 | 2.41858 | 0.00109 | 5355805 | 4967239 | <.0001 |
| x5 | 6.49437 | 0.02209 | 93169 | 86409.5 | <.0001 |
| x6 | 23.82882 | 0.01283 | 3719881 | 3450002 | <.0001 |
| x7 | -6.59415 | 0.00645 | 1128375 | 1046511 | <.0001 |

Bounds on condition number: 2.5803, 41.936

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Stepwise Selection: Step 6

Statistics for Removal
DF = 1,804138

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0019 | 0.9131 | 17900.7 | <.0001 |

| | | | | |
|---|---|---|---|---|
| x4 | 0.5251 | 0.3899 | 4967239 | <.0001 |
| x5 | 0.0091 | 0.9059 | 86409.5 | <.0001 |
| x6 | 0.3647 | 0.5503 | 3450002 | <.0001 |
| x7 | 0.1106 | 0.8044 | 1046511 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 6

Statistics for Entry
DF = 1,804137

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.119539 | 0.9153 | 3057.77 | <.0001 |

Variable x2 Entered: R-Square = 0.9153 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 9336338 | 1556056 | 1448650 | <.0001 |
| Error | 804137 | 863758 | 1.07414 | | |
| Corrected Total | 804143 | 10200095 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 1.01676 | 0.01089 | 9357.77363 | 8711.85 | <.0001 |
| x1 | 2.96582 | 0.02264 | 18428 | 17155.8 | <.0001 |
| x2 | -0.65198 | 0.01179 | 3284.47749 | 3057.77 | <.0001 |
| x4 | 2.42011 | 0.00108 | 5359086 | 4989175 | <.0001 |
| x5 | 6.50844 | 0.02205 | 93561 | 87102.5 | <.0001 |
| x6 | 23.84656 | 0.01281 | 3723084 | 3466099 | <.0001 |
| x7 | -6.83971 | 0.00782 | 822256 | 765500 | <.0001 |

Bounds on condition number: 8.3655, 126.58

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Stepwise Selection: Step 7

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

Stepwise Selection: Step 7

Statistics for Removal
DF = 1,804137

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| x1 | 0.0018 | 0.9135 | 17155.8 | <.0001 |
| x2 | 0.0003 | 0.9150 | 3057.77 | <.0001 |
| x4 | 0.5254 | 0.3899 | 4989175 | <.0001 |
| x5 | 0.0092 | 0.9061 | 87102.5 | <.0001 |
| x6 | 0.3650 | 0.5503 | 3466099 | <.0001 |
| x7 | 0.0806 | 0.8347 | 765500 | <.0001 |

All variables left in the model are significant at the 0.1500 level.

All variables have been entered into the model.

Summary of Stepwise Selection

| Step | Variable Entered | Variable Removed | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|
| 1 | x4 | | 1 | 0.3387 | 0.3387 | 5475160 | 411943 | <.0001 |
| 2 | x6 | | 2 | 0.2371 | 0.5758 | 3224002 | 449401 | <.0001 |
| 3 | x7 | | 3 | 0.3281 | 0.9039 | 108082 | 2746752 | <.0001 |
| 4 | x5 | | 4 | 0.0092 | 0.9131 | 21029.5 | 84836.7 | <.0001 |
| 5 | x1 | | 5 | 0.0019 | 0.9150 | 3062.77 | 17900.7 | <.0001 |
| 6 | x2 | | 6 | 0.0003 | 0.9153 | 7.0000 | 3057.77 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

R-Square Selection Method

Number of Observations Read 804144
Number of Observations Used 804144

| Number in Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 1 | 0.3387 | 5475160 | 8.38765 | x4 |
| 1 | 0.2200 | 6602729 | 9.89382 | x7 |
| 1 | 0.1680 | 7096460 | 10.55332 | x2 |
| 1 | 0.1396 | 7365841 | 10.91315 | x1 |
| 1 | 0.0880 | 7855962 | 11.56784 | x6 |
| 1 | 0.0025 | 8668600 | 12.65333 | x5 |
| 2 | 0.5758 | 3224002 | 5.38064 | x4 x6 |
| 2 | 0.5472 | 3496078 | 5.74407 | x4 x7 |
| 2 | 0.4972 | 3970051 | 6.37719 | x2 x4 |
| 2 | 0.4745 | 4185944 | 6.66557 | x1 x4 |
| 2 | 0.3830 | 5054671 | 7.82599 | x6 x7 |
| 2 | 0.3414 | 5450425 | 8.35462 | x4 x5 |
| 2 | 0.3122 | 5727555 | 8.72480 | x2 x6 |
| 2 | 0.2780 | 6052382 | 9.15869 | x1 x6 |
| 2 | 0.2226 | 6577904 | 9.86067 | x5 x7 |
| 2 | 0.2202 | 6600531 | 9.89089 | x2 x7 |
| 2 | 0.2202 | 6601186 | 9.89177 | x1 x7 |
| 2 | 0.1703 | 7074330 | 10.52377 | x2 x5 |
| 2 | 0.1680 | 7096284 | 10.55310 | x1 x2 |
| 2 | 0.1420 | 7343123 | 10.88282 | x1 x5 |
| 2 | 0.0928 | 7810505 | 11.50713 | x5 x6 |
| 3 | 0.9039 | 108082.3 | 1.21851 | x4 x6 x7 |
| 3 | 0.8246 | 861021.0 | 2.22426 | x2 x4 x6 |

| | | | | |
|---|---|---|---|---|
| 3 | 0.7962 | 1131535 | 2.58560 | x1 x4 x6 |
| 3 | 0.5830 | 3155247 | 5.28881 | x4 x5 x6 |
| 3 | 0.5499 | 3469833 | 5.70902 | x4 x5 x7 |
| 3 | 0.5476 | 3492287 | 5.73901 | x1 x4 x7 |
| 3 | 0.5473 | 3494410 | 5.74185 | x2 x4 x7 |
| 3 | 0.4997 | 3946501 | 6.34574 | x2 x4 x5 |
| 3 | 0.4975 | 3967227 | 6.37342 | x1 x2 x4 |
| 3 | 0.4771 | 4161801 | 6.63333 | x1 x4 x5 |
| 3 | 0.3892 | 4995766 | 7.74731 | x5 x6 x7 |
| 3 | 0.3837 | 5047906 | 7.81696 | x1 x6 x7 |
| 3 | 0.3835 | 5050579 | 7.82053 | x2 x6 x7 |
| 3 | 0.3176 | 5676075 | 8.65604 | x2 x5 x6 |

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The REG Procedure
Model: MODEL1
Dependent Variable: slope

R-Square Selection Method

| Number in Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 3 | 0.3125 | 5724356 | 8.72054 | x1 x2 x6 |
| 3 | 0.2834 | 6000599 | 9.08953 | x1 x5 x6 |
| 3 | 0.2228 | 6575874 | 9.85796 | x2 x5 x7 |
| 3 | 0.2228 | 6576446 | 9.85873 | x1 x5 x7 |
| 3 | 0.2202 | 6600487 | 9.89084 | x1 x2 x7 |
| 3 | 0.1704 | 7074144 | 10.52354 | x1 x2 x5 |
| 4 | 0.9131 | 21029.47 | 1.10223 | x4 x5 x6 x7 |
| 4 | 0.9059 | 89798.73 | 1.19409 | x1 x4 x6 x7 |
| 4 | 0.9044 | 103811.2 | 1.21280 | x2 x4 x6 x7 |
| 4 | 0.8328 | 783788.8 | 2.12110 | x2 x4 x5 x6 |
| 4 | 0.8265 | 843183.3 | 2.20043 | x1 x2 x4 x6 |
| 4 | 0.8044 | 1053550 | 2.48143 | x1 x4 x5 x6 |
| 4 | 0.5503 | 3466177 | 5.70414 | x1 x4 x5 x7 |
| 4 | 0.5501 | 3468315 | 5.70700 | x2 x4 x5 x7 |
| 4 | 0.5476 | 3492239 | 5.73895 | x1 x2 x4 x7 |
| 4 | 0.5000 | 3943632 | 6.34191 | x1 x2 x4 x5 |
| 4 | 0.3899 | 4989181 | 7.73852 | x1 x5 x6 x7 |
| 4 | 0.3896 | 4991999 | 7.74229 | x2 x5 x6 x7 |
| 4 | 0.3837 | 5047888 | 7.81694 | x1 x2 x6 x7 |
| 4 | 0.3179 | 5672741 | 8.65160 | x1 x2 x5 x6 |
| 4 | 0.2228 | 6575825 | 9.85791 | x1 x2 x5 x7 |
| 5 | 0.9150 | 3062.767 | 1.07823 | x1 x4 x5 x6 x7 |
| 5 | 0.9135 | 17160.85 | 1.09706 | x2 x4 x5 x6 x7 |
| 5 | 0.9061 | 87107.50 | 1.19049 | x1 x2 x4 x6 x7 |
| 5 | 0.8347 | 765505.2 | 2.09667 | x1 x2 x4 x5 x6 |
| 5 | 0.5503 | 3466104 | 5.70405 | x1 x2 x4 x5 x7 |
| 5 | 0.3899 | 4989180 | 7.73853 | x1 x2 x5 x6 x7 |
| 6 | 0.9153 | 7.0000 | 1.07414 | x1 x2 x4 x5 x6 x7 |

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The CORR Procedure

6 Variables: x1 x2 x4 x5 x6 x7

Covariance Matrix, DF = 804143

x1 x2 x4 x5 x6 x7

| | | | | | | |
|---|---|---|---|---|---|---|
| x1 | 0.014765359 | 0.031255086 | 0.001222166 | 0.000010498 | -0.002168265 | -0.027338461 |
| x2 | 0.031255086 | 0.080380462 | 0.006449020 | 0.000045538 | -0.005114094 | -0.070126382 |
| x4 | 0.001222166 | 0.006449020 | 1.247236758 | -0.000152301 | -0.031440474 | -0.006994508 |
| x5 | 0.000010498 | 0.000045538 | -0.000152301 | 0.002761387 | -0.000334695 | 0.000051812 |
| x6 | -0.002168265 | -0.005114094 | -0.031440474 | -0.000334695 | 0.009380917 | 0.005847031 |
| x7 | -0.027338461 | -0.070126382 | -0.006994508 | 0.000051812 | 0.005847031 | 0.083268051 |

Simple Statistics

| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
|---|---|---|---|---|---|---|
| x1 | 804144 | 0.22587 | 0.12151 | 181635 | 0.06667 | 0.50000 |
| x2 | 804144 | 0.55709 | 0.28351 | 447984 | 0.20000 | 1.00000 |
| x4 | 804144 | 3.28964 | 1.11680 | 2645347 | 2.00000 | 5.00000 |
| x5 | 804144 | 0.05796 | 0.05255 | 46606 | 0 | 0.15000 |
| x6 | 804144 | 0.34562 | 0.09686 | 277931 | 0.20000 | 0.50000 |
| x7 | 804144 | 0.60734 | 0.28856 | 488388 | 0.05799 | 1.22185 |

Pearson Correlation Coefficients, N = 804144
Prob > |r| under H0: Rho=0

| | x1 | x2 | x4 | x5 | x6 | x7 |
|---|---|---|---|---|---|---|
| x1 | 1.00000 | 0.90724 | 0.00901 | 0.00164 | -0.18423 | -0.77967 |
| | | <.0001 | <.0001 | 0.1404 | <.0001 | <.0001 |
| x2 | 0.90724 | 1.00000 | 0.02037 | 0.00306 | -0.18624 | -0.85717 |
| | <.0001 | | <.0001 | 0.0061 | <.0001 | <.0001 |
| x4 | 0.00901 | 0.02037 | 1.00000 | -0.00260 | -0.29066 | -0.02170 |
| | <.0001 | <.0001 | | 0.0200 | <.0001 | <.0001 |
| x5 | 0.00164 | 0.00306 | -0.00260 | 1.00000 | -0.06576 | 0.00342 |
| | 0.1404 | 0.0061 | 0.0200 | | <.0001 | 0.0022 |
| x6 | -0.18423 | -0.18624 | -0.29066 | -0.06576 | 1.00000 | 0.20921 |
| | <.0001 | <.0001 | <.0001 | <.0001 | | <.0001 |
| x7 | -0.77967 | -0.85717 | -0.02170 | 0.00342 | 0.20921 | 1.00000 |
| | <.0001 | <.0001 | <.0001 | 0.0022 | <.0001 | |

## Variable Selection and Covariance Matrix for the oil-wet homogeneous model (Intercept)

The SAS System 11:24 Saturday, October 3, 2009 9673

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Number of Observations Read 804144
Number of Observations Used 804144

Forward Selection: Step 1

Statistics for Entry
DF = 1,804142

Model

| Variable | Tolerance | R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 1.000000 | 0.3622 | 456589 | <.0001 |
| x2 | 1.000000 | 0.3171 | 373441 | <.0001 |
| x4 | 1.000000 | 0.4968 | 793944 | <.0001 |
| x5 | 1.000000 | 0.0001 | 50.51 | <.0001 |
| x6 | 1.000000 | 0.1311 | 121300 | <.0001 |
| x7 | 1.000000 | 0.4244 | 592931 | <.0001 |

Variable x4 Entered: R-Square = 0.4968 and C(p) = 8087357

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 1809024 | 1809024 | 793944 | <.0001 |
| Error | 804142 | 1832260 | 2.27853 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -0.50232 | 0.00524 | 20969 | 9202.79 | <.0001 |
| x4 | -1.34302 | 0.00151 | 1809024 | 793944 | <.0001 |

Bounds on condition number: 1, 1

---

Forward Selection: Step 2
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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 2

Statistics for Entry
DF = 1,804141

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.999919 | 0.8514 | 1918851 | <.0001 |
| x2 | 0.999585 | 0.7981 | 1199963 | <.0001 |
| x5 | 0.999993 | 0.4968 | 59.39 | <.0001 |
| x6 | 0.915514 | 0.5238 | 45559.8 | <.0001 |
| x7 | 0.999529 | 0.9017 | 3312667 | <.0001 |

Variable x7 Entered: R-Square = 0.9017 and C(p) = 932648.9

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 3283387 | 1641693 | 3688633 | <.0001 |
| Error | 804141 | 357898 | 0.44507 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.43947 | 0.00282 | 661458 | 1486194 | <.0001 |
| x4 | -1.31669 | 0.00066631 | 1737991 | 3905000 | <.0001 |
| x7 | 4.69352 | 0.00258 | 1474362 | 3312667 | <.0001 |

Bounds on condition number: 1.0005, 4.0019

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Forward Selection: Step 3

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 3

Statistics for Entry
DF = 1,804140

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.392046 | 0.9269 | 276818 | <.0001 |
| x2 | 0.265256 | 0.9018 | 378.32 | <.0001 |
| x5 | 0.999982 | 0.9017 | 128.12 | <.0001 |
| x6 | 0.874328 | 0.9026 | 7418.51 | <.0001 |

Variable x1 Entered: R-Square = 0.9269 and C(p) = 487884.8

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 3375039 | 1125013 | 3397876 | <.0001 |
| Error | 804140 | 266245 | 0.33109 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.53964 | 0.00435 | 41395 | 125024 | <.0001 |
| x1 | -4.43726 | 0.00843 | 91652 | 276818 | <.0001 |
| x4 | -1.32052 | 0.00057474 | 1747821 | 5278941 | <.0001 |
| x7 | 3.23636 | 0.00355 | 274849 | 830126 | <.0001 |

Bounds on condition number: 2.5517, 18.309

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Forward Selection: Step 4

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 4

Statistics for Entry
DF = 1,804139

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x2 | 0.119625 | 0.9541 | 476347 | <.0001 |
| x5 | 0.999935 | 0.9269 | 279.86 | <.0001 |
| x6 | 0.872932 | 0.9274 | 6201.80 | <.0001 |

Variable x2 Entered: R-Square = 0.9541 and C(p) = 7248.815

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 3474084 | 868521 | 4177085 | <.0001 |
| Error | 804139 | 167201 | 0.20793 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.17965 | 0.00419 | 119759 | 575970 | <.0001 |
| x1 | -9.52673 | 0.00995 | 190529 | 916333 | <.0001 |
| x2 | 3.57900 | 0.00519 | 99044 | 476347 | <.0001 |
| x4 | -1.32651 | 0.00045554 | 1763070 | 8479352 | <.0001 |
| x7 | 4.57904 | 0.00342 | 372360 | 1790836 | <.0001 |

Bounds on condition number: 8.3594, 75.147

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Forward Selection: Step 5

The SAS System 11:24 Saturday, October 3, 2009 9677

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 5

Statistics for Entry
DF = 1,804138

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.999840 | 0.9541 | 206.53 | <.0001 |
| x6 | 0.872418 | 0.9545 | 6824.93 | <.0001 |

Variable x6 Entered: R-Square = 0.9545 and C(p) = 422.3553

Analysis of Variance

Sum of Mean

| Source | DF | Squares | Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 3475491 | 695098 | 3371391 | <.0001 |
| Error | 804138 | 165794 | 0.20618 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.36277 | 0.00472 | 104464 | 506674 | <.0001 |
| x1 | -9.49002 | 0.00992 | 188683 | 915159 | <.0001 |
| x2 | 3.56864 | 0.00517 | 98414 | 477331 | <.0001 |
| x4 | -1.31499 | 0.00047456 | 1583102 | 7678420 | <.0001 |
| x6 | 0.46240 | 0.00560 | 1407.13254 | 6824.93 | <.0001 |
| x7 | 4.55087 | 0.00342 | 364145 | 1766189 | <.0001 |

Bounds on condition number: 8.3644, 100.41

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Forward Selection: Step 6

The SAS System 11:24 Saturday, October 3, 2009 9678

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Forward Selection: Step 6

Statistics for Entry
DF = 1,804137

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.994669 | 0.9545 | 417.36 | <.0001 |

Variable x5 Entered: R-Square = 0.9545 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 3475577 | 579263 | 2811017 | <.0001 |
| Error | 804137 | 165708 | 0.20607 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.37660 | 0.00477 | 103204 | 500823 | <.0001 |
| x1 | -9.48883 | 0.00992 | 188630 | 915372 | <.0001 |
| x2 | 3.56742 | 0.00516 | 98334 | 477189 | <.0001 |
| x4 | -1.31476 | 0.00047457 | 1581662 | 7675406 | <.0001 |
| x5 | 0.19733 | 0.00966 | 86.00390 | 417.36 | <.0001 |
| x6 | 0.47064 | 0.00561 | 1450.20388 | 7037.47 | <.0001 |
| x7 | 4.54955 | 0.00342 | 363805 | 1765456 | <.0001 |

Bounds on condition number: 8.3655, 126.58

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All variables have been entered into the model.

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Summary of Forward Selection

| Step | Variable Entered | Label | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|
| 1 | x4 | | 1 | 0.4968 | 0.4968 | 8087357 | 793944 | <.0001 |
| 2 | x7 | | 2 | 0.4049 | 0.9017 | 932649 | 3312667 | <.0001 |
| 3 | x1 | | 3 | 0.0252 | 0.9269 | 487885 | 276818 | <.0001 |
| 4 | x2 | | 4 | 0.0272 | 0.9541 | 7248.82 | 476347 | <.0001 |
| 5 | x6 | | 5 | 0.0004 | 0.9545 | 422.355 | 6824.93 | <.0001 |
| 6 | x5 | | 6 | 0.0000 | 0.9545 | 7.0000 | 417.36 | <.0001 |

The SAS System 11:24 Saturday, October 3, 2009 9680

The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Number of Observations Read 804144
Number of Observations Used 804144

Backward Elimination: Step 0

All Variables Entered: R-Square = 0.9545 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 3475577 | 579263 | 2811017 | <.0001 |
| Error | 804137 | 165708 | 0.20607 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.37660 | 0.00477 | 103204 | 500823 | <.0001 |
| x1 | -9.48883 | 0.00992 | 188630 | 915372 | <.0001 |
| x2 | 3.56742 | 0.00516 | 98334 | 477189 | <.0001 |
| x4 | -1.31476 | 0.00047457 | 1581662 | 7675406 | <.0001 |
| x5 | 0.19733 | 0.00966 | 86.00390 | 417.36 | <.0001 |
| x6 | 0.47064 | 0.00561 | 1450.20388 | 7037.47 | <.0001 |
| x7 | 4.54955 | 0.00342 | 363805 | 1765456 | <.0001 |

Bounds on condition number: 8.3655, 126.58

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Backward Elimination: Step 1

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Backward Elimination: Step 1

Statistics for Removal
DF = 1,804137

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0518 | 0.9027 | 915372 | <.0001 |
| x2 | 0.0270 | 0.9275 | 477189 | <.0001 |
| x4 | 0.4344 | 0.5201 | 7675406 | <.0001 |
| x5 | 0.0000 | 0.9545 | 417.36 | <.0001 |
| x6 | 0.0004 | 0.9541 | 7037.47 | <.0001 |
| x7 | 0.0999 | 0.8546 | 1765456 | <.0001 |

All variables left in the model are significant at the 0.0500 level.

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Number of Observations Read 804144
Number of Observations Used 804144

Stepwise Selection: Step 1

Statistics for Entry
DF = 1,804142

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 1.000000 | 0.3622 | 456589 | <.0001 |
| x2 | 1.000000 | 0.3171 | 373441 | <.0001 |
| x4 | 1.000000 | 0.4968 | 793944 | <.0001 |
| x5 | 1.000000 | 0.0001 | 50.51 | <.0001 |
| x6 | 1.000000 | 0.1311 | 121300 | <.0001 |
| x7 | 1.000000 | 0.4244 | 592931 | <.0001 |

Variable x4 Entered: R-Square = 0.4968 and C(p) = 8087357

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 1 | 1809024 | 1809024 | 793944 | <.0001 |
| Error | 804142 | 1832260 | 2.27853 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -0.50232 | 0.00524 | 20969 | 9202.79 | <.0001 |
| x4 | -1.34302 | 0.00151 | 1809024 | 793944 | <.0001 |

Bounds on condition number: 1, 1

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Stepwise Selection: Step 2

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 2

Statistics for Entry
DF = 1,804141

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.999919 | 0.8514 | 1918851 | <.0001 |
| x2 | 0.999585 | 0.7981 | 1199963 | <.0001 |
| x5 | 0.999993 | 0.4968 | 59.39 | <.0001 |
| x6 | 0.915514 | 0.5238 | 45559.8 | <.0001 |
| x7 | 0.999529 | 0.9017 | 3312667 | <.0001 |

Variable x7 Entered: R-Square = 0.9017 and C(p) = 932648.9

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 2 | 3283387 | 1641693 | 3688633 | <.0001 |
| Error | 804141 | 357898 | 0.44507 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.43947 | 0.00282 | 661458 | 1486194 | <.0001 |
| x4 | -1.31669 | 0.00066631 | 1737991 | 3905000 | <.0001 |
| x7 | 4.69352 | 0.00258 | 1474362 | 3312667 | <.0001 |

Bounds on condition number: 1.0005, 4.0019

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Stepwise Selection: Step 3

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 3

Statistics for Removal
DF = 1,804141

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| x4 | 0.4773 | 0.4244 | 3905000 | <.0001 |
| x7 | 0.4049 | 0.4968 | 3312667 | <.0001 |

Statistics for Entry
DF = 1,804140

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.392046 | 0.9269 | 276818 | <.0001 |
| x2 | 0.265256 | 0.9018 | 378.32 | <.0001 |
| x5 | 0.999982 | 0.9017 | 128.12 | <.0001 |
| x6 | 0.874328 | 0.9026 | 7418.51 | <.0001 |

Variable x1 Entered: R-Square = 0.9269 and C(p) = 487884.8

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 3 | 3375039 | 1125013 | 3397876 | <.0001 |
| Error | 804140 | 266245 | 0.33109 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -1.53964 | 0.00435 | 41395 | 125024 | <.0001 |
| x1 | -4.43726 | 0.00843 | 91652 | 276818 | <.0001 |
| x4 | -1.32052 | 0.00057474 | 1747821 | 5278941 | <.0001 |
| x7 | 3.23636 | 0.00355 | 274849 | 830126 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 3

Bounds on condition number: 2.5517, 18.309

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Stepwise Selection: Step 4

Statistics for Removal
DF = 1,804140

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0252 | 0.9017 | 276818 | <.0001 |
| x4 | 0.4800 | 0.4469 | 5278941 | <.0001 |
| x7 | 0.0755 | 0.8514 | 830126 | <.0001 |

Statistics for Entry
DF = 1,804139

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| x2 | 0.119625 | 0.9541 | 476347 | <.0001 |
| x5 | 0.999935 | 0.9269 | 279.86 | <.0001 |
| x6 | 0.872932 | 0.9274 | 6201.80 | <.0001 |

Variable x2 Entered: R-Square = 0.9541 and C(p) = 7248.815

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 3474084 | 868521 | 4177085 | <.0001 |
| Error | 804139 | 167201 | 0.20793 | | |
| Corrected Total | 804143 | 3641284 | | | |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 4

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.17965 | 0.00419 | 119759 | 575970 | <.0001 |
| x1 | -9.52673 | 0.00995 | 190529 | 916333 | <.0001 |
| x2 | 3.57900 | 0.00519 | 99044 | 476347 | <.0001 |
| x4 | -1.32651 | 0.00045554 | 1763070 | 8479352 | <.0001 |
| x7 | 4.57904 | 0.00342 | 372360 | 1790836 | <.0001 |

Bounds on condition number: 8.3594, 75.147

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Stepwise Selection: Step 5

Statistics for Removal
DF = 1,804139

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0523 | 0.9018 | 916333 | <.0001 |
| x2 | 0.0272 | 0.9269 | 476347 | <.0001 |
| x4 | 0.4842 | 0.4699 | 8479352 | <.0001 |
| x7 | 0.1023 | 0.8518 | 1790836 | <.0001 |

Statistics for Entry
DF = 1,804138

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.999840 | 0.9541 | 206.53 | <.0001 |
| x6 | 0.872418 | 0.9545 | 6824.93 | <.0001 |

Variable x6 Entered: R-Square = 0.9545 and C(p) = 422.3553

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 5

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 5 | 3475491 | 695098 | 3371391 | <.0001 |
| Error | 804138 | 165794 | 0.20618 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.36277 | 0.00472 | 104464 | 506674 | <.0001 |
| x1 | -9.49002 | 0.00992 | 188683 | 915159 | <.0001 |
| x2 | 3.56864 | 0.00517 | 98414 | 477331 | <.0001 |
| x4 | -1.31499 | 0.00047456 | 1583102 | 7678420 | <.0001 |
| x6 | 0.46240 | 0.00560 | 1407.13254 | 6824.93 | <.0001 |
| x7 | 4.55087 | 0.00342 | 364145 | 1766189 | <.0001 |

Bounds on condition number: 8.3644, 100.41

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Stepwise Selection: Step 6

Statistics for Removal
DF = 1,804138

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0518 | 0.9027 | 915159 | <.0001 |
| x2 | 0.0270 | 0.9274 | 477331 | <.0001 |
| x4 | 0.4348 | 0.5197 | 7678420 | <.0001 |
| x6 | 0.0004 | 0.9541 | 6824.93 | <.0001 |
| x7 | 0.1000 | 0.8545 | 1766189 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 6

Statistics for Entry
DF = 1,804137

| Variable | Tolerance | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x5 | 0.994669 | 0.9545 | 417.36 | <.0001 |

Variable x5 Entered: R-Square = 0.9545 and C(p) = 7.0000

Analysis of Variance

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 6 | 3475577 | 579263 | 2811017 | <.0001 |
| Error | 804137 | 165708 | 0.20607 | | |
| Corrected Total | 804143 | 3641284 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | -3.37660 | 0.00477 | 103204 | 500823 | <.0001 |
| x1 | -9.48883 | 0.00992 | 188630 | 915372 | <.0001 |
| x2 | 3.56742 | 0.00516 | 98334 | 477189 | <.0001 |
| x4 | -1.31476 | 0.00047457 | 1581662 | 7675406 | <.0001 |
| x5 | 0.19733 | 0.00966 | 86.00390 | 417.36 | <.0001 |
| x6 | 0.47064 | 0.00561 | 1450.20388 | 7037.47 | <.0001 |
| x7 | 4.54955 | 0.00342 | 363805 | 1765456 | <.0001 |

Bounds on condition number: 8.3655, 126.58

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Stepwise Selection: Step 7

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept Intercept

Stepwise Selection: Step 7

Statistics for Removal
DF = 1,804137

| Variable | Partial R-Square | Model R-Square | F Value | Pr > F |
|---|---|---|---|---|
| x1 | 0.0518 | 0.9027 | 915372 | <.0001 |
| x2 | 0.0270 | 0.9275 | 477189 | <.0001 |
| x4 | 0.4344 | 0.5201 | 7675406 | <.0001 |
| x5 | 0.0000 | 0.9545 | 417.36 | <.0001 |
| x6 | 0.0004 | 0.9541 | 7037.47 | <.0001 |
| x7 | 0.0999 | 0.8546 | 1765456 | <.0001 |

All variables left in the model are significant at the 0.1500 level.

All variables have been entered into the model.

Summary of Stepwise Selection

| Step | Variable Entered | Variable Removed | Label | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
|---|---|---|---|---|---|---|---|---|---|
| 1 | x4 | | | 1 | 0.4968 | 0.4968 | 8087357 | 793944 | <.0001 |
| 2 | x7 | | | 2 | 0.4049 | 0.9017 | 932649 | 3312667 | <.0001 |
| 3 | x1 | | | 3 | 0.0252 | 0.9269 | 487885 | 276818 | <.0001 |
| 4 | x2 | | | 4 | 0.0272 | 0.9541 | 7248.82 | 476347 | <.0001 |
| 5 | x6 | | | 5 | 0.0004 | 0.9545 | 422.355 | 6824.93 | <.0001 |
| 6 | x5 | | | 6 | 0.0000 | 0.9545 | 7.0000 | 417.36 | <.0001 |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept

R-Square Selection Method

Number of Observations Read 804144
Number of Observations Used 804144

| Number in Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 1 | 0.4968 | 8087357 | 2.27853 | x4 |
| 1 | 0.4244 | 9366681 | 2.60637 | x7 |
| 1 | 0.3622 | 10466608 | 2.88823 | x1 |
| 1 | 0.3171 | 11262427 | 3.09217 | x2 |
| 1 | 0.1311 | 14550014 | 3.93464 | x6 |
| 1 | 0.0001 | 16864986 | 4.52788 | x5 |
| 2 | 0.9017 | 932648.9 | 0.44507 | x4 x7 |
| 2 | 0.8514 | 1821656 | 0.67288 | x1 x4 |
| 2 | 0.7981 | 2763550 | 0.91425 | x2 x4 |
| 2 | 0.5238 | 7610608 | 2.15636 | x4 x6 |
| 2 | 0.4968 | 8086702 | 2.27836 | x4 x5 |
| 2 | 0.4777 | 8424946 | 2.36504 | x6 x7 |
| 2 | 0.4469 | 8969616 | 2.50462 | x1 x7 |
| 2 | 0.4275 | 9312710 | 2.59254 | x1 x6 |
| 2 | 0.4245 | 9365198 | 2.60599 | x2 x7 |
| 2 | 0.4244 | 9366109 | 2.60622 | x5 x7 |
| 2 | 0.3856 | 10051877 | 2.78196 | x2 x6 |
| 2 | 0.3638 | 10437196 | 2.88070 | x1 x2 |
| 2 | 0.3622 | 10465206 | 2.88788 | x1 x5 |
| 2 | 0.3172 | 11260785 | 3.09175 | x2 x5 |
| 2 | 0.1321 | 14532145 | 3.93007 | x5 x6 |
| 3 | 0.9269 | 487884.8 | 0.33109 | x1 x4 x7 |
| 3 | 0.9026 | 916774.8 | 0.44100 | x4 x6 x7 |
| 3 | 0.9018 | 931834.1 | 0.44486 | x2 x4 x7 |
| 3 | 0.9017 | 932374.2 | 0.44500 | x4 x5 x7 |
| 3 | 0.8541 | 1773558 | 0.66056 | x1 x4 x6 |
| 3 | 0.8518 | 1814215 | 0.67098 | x1 x2 x4 |
| 3 | 0.8515 | 1820770 | 0.67266 | x1 x4 x5 |
| 3 | 0.8019 | 2695840 | 0.89690 | x2 x4 x6 |
| 3 | 0.7982 | 2762476 | 0.91398 | x2 x4 x5 |
| 3 | 0.5241 | 7605164 | 2.15497 | x4 x5 x6 |
| 3 | 0.4979 | 8068522 | 2.27371 | x1 x6 x7 |
| 3 | 0.4782 | 8416824 | 2.36296 | x5 x6 x7 |
| 3 | 0.4777 | 8424313 | 2.36488 | x2 x6 x7 |
| 3 | 0.4699 | 8562982 | 2.40042 | x1 x2 x7 |

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The REG Procedure
Model: MODEL1
Dependent Variable: Intercept

R-Square Selection Method

| Number in Model | R-Square | C(p) | MSE | Variables in Model |
|---|---|---|---|---|
| 3 | 0.4469 | 8968818 | 2.50442 | x1 x5 x7 |
| 3 | 0.4283 | 9297823 | 2.58873 | x1 x2 x6 |
| 3 | 0.4281 | 9300776 | 2.58948 | x1 x5 x6 |

| | | | | |
|---|---|---|---|---|
| 3 | 0.4245 | 9364605 | 2.60584 | x2 x5 x7 |
| 3 | 0.3864 | 10038927 | 2.77864 | x2 x5 x6 |
| 3 | 0.3639 | 10435745 | 2.88033 | x1 x2 x5 |
| 4 | 0.9541 | 7248.815 | 0.20793 | x1 x2 x4 x7 |
| 4 | 0.9274 | 477998.5 | 0.32856 | x1 x4 x6 x7 |
| 4 | 0.9269 | 487437.3 | 0.33098 | x1 x4 x5 x7 |
| 4 | 0.9027 | 916053.0 | 0.44082 | x2 x4 x6 x7 |
| 4 | 0.9026 | 916113.5 | 0.44083 | x4 x5 x6 x7 |
| 4 | 0.9018 | 931548.3 | 0.44479 | x2 x4 x5 x7 |
| 4 | 0.8545 | 1767524 | 0.65901 | x1 x2 x4 x6 |
| 4 | 0.8542 | 1771502 | 0.66003 | x1 x4 x5 x6 |
| 4 | 0.8519 | 1813309 | 0.67075 | x1 x2 x4 x5 |
| 4 | 0.8021 | 2693222 | 0.89623 | x2 x4 x5 x6 |
| 4 | 0.5197 | 7682816 | 2.17487 | x1 x2 x6 x7 |
| 4 | 0.4984 | 8059903 | 2.27150 | x1 x5 x6 x7 |
| 4 | 0.4782 | 8416141 | 2.36279 | x2 x5 x6 x7 |
| 4 | 0.4699 | 8562496 | 2.40029 | x1 x2 x5 x7 |
| 4 | 0.4290 | 9285872 | 2.58567 | x1 x2 x5 x6 |
| 5 | 0.9545 | 422.3553 | 0.20618 | x1 x2 x4 x6 x7 |
| 5 | 0.9541 | 7042.474 | 0.20787 | x1 x2 x4 x5 x7 |
| 5 | 0.9275 | 477194.0 | 0.32835 | x1 x4 x5 x6 x7 |
| 5 | 0.9027 | 915376.7 | 0.44064 | x2 x4 x5 x6 x7 |
| 5 | 0.8546 | 1765461 | 0.65849 | x1 x2 x4 x5 x6 |
| 5 | 0.5201 | 7675411 | 2.17297 | x1 x2 x5 x6 x7 |
| 6 | 0.9545 | 7.0000 | 0.20607 | x1 x2 x4 x5 x6 x7 |

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The CORR Procedure

6 Variables: x1 x2 x4 x5 x6 x7

Covariance Matrix, DF = 804143

| | x1 | x2 | x4 | x5 | x6 | x7 |
|---|---|---|---|---|---|---|
| x1 | 0.014765359 | 0.031255086 | 0.001222166 | 0.000010498 | -0.002168265 | -0.027338461 |
| x2 | 0.031255086 | 0.080380462 | 0.006449020 | 0.000045538 | -0.005114094 | -0.070126382 |
| x4 | 0.001222166 | 0.006449020 | 1.247236758 | -0.000152301 | -0.031440474 | -0.006994508 |
| x5 | 0.000010498 | 0.000045538 | -0.000152301 | 0.002761387 | -0.000334695 | 0.000051812 |
| x6 | -0.002168265 | -0.005114094 | -0.031440474 | -0.000334695 | 0.009380917 | 0.005847031 |
| x7 | -0.027338461 | -0.070126382 | -0.006994508 | 0.000051812 | 0.005847031 | 0.083268051 |

Simple Statistics

| Variable | N | Mean | Std Dev | Sum | Minimum | Maximum |
|---|---|---|---|---|---|---|
| x1 | 804144 | 0.22587 | 0.12151 | 181635 | 0.06667 | 0.50000 |
| x2 | 804144 | 0.55709 | 0.28351 | 447984 | 0.20000 | 1.00000 |
| x4 | 804144 | 3.28964 | 1.11680 | 2645347 | 2.00000 | 5.00000 |
| x5 | 804144 | 0.05796 | 0.05255 | 46606 | 0 | 0.15000 |
| x6 | 804144 | 0.34562 | 0.09686 | 277931 | 0.20000 | 0.50000 |
| x7 | 804144 | 0.60734 | 0.28856 | 488388 | 0.05799 | 1.22185 |

Pearson Correlation Coefficients, N = 804144
Prob > |r| under H0: Rho=0

x1 x2 x4 x5 x6 x7

| | | | | | | |
|---|---|---|---|---|---|---|
| x1 | 1.00000 | 0.90724 | 0.00901 | 0.00164 | -0.18423 | -0.77967 |
| | | <.0001 | <.0001 | 0.1404 | <.0001 | <.0001 |
| x2 | 0.90724 | 1.00000 | 0.02037 | 0.00306 | -0.18624 | -0.85717 |
| | <.0001 | | <.0001 | 0.0061 | <.0001 | <.0001 |
| x4 | 0.00901 | 0.02037 | 1.00000 | -0.00260 | -0.29066 | -0.02170 |
| | <.0001 | <.0001 | | 0.0200 | <.0001 | <.0001 |
| x5 | 0.00164 | 0.00306 | -0.00260 | 1.00000 | -0.06576 | 0.00342 |
| | 0.1404 | 0.0061 | 0.0200 | | <.0001 | 0.0022 |
| x6 | -0.18423 | -0.18624 | -0.29066 | -0.06576 | 1.00000 | 0.20921 |
| | <.0001 | <.0001 | <.0001 | <.0001 | | <.0001 |
| x7 | -0.77967 | -0.85717 | -0.02170 | 0.00342 | 0.20921 | 1.00000 |
| | <.0001 | <.0001 | <.0001 | 0.0022 | <.0001 | |

# APPENDIX H

# EVALUATION OF THE INJECTION RATES, CROSSFLOW AND LAYERS NUMBER IN THE RECOVERY FACTOR AND PVI RESULTS USING THE SIMULATION MODEL

## Injection Rates Effect

Table 27—Example cases to evaluate the effect of different injection rates using the reservoir simulator characteristics as described in Table 2.

| Case # | Heterogeneity (VDP ) | Mobility Ratio | Injection Rate (B/D) |
|---|---|---|---|
| 1 | 0.0 | 1 | 200 |
| 2 | 0.0 | 10 | 200 |
| 3 | 0.0 | 1 | 2000 |
| 4 | 0.0 | 10 | 2000 |
| 5 | 0.9 | 1 | 200 |
| 6 | 0.9 | 10 | 200 |
| 7 | 0.9 | 1 | 2000 |
| 8 | 0.9 | 10 | 2000 |

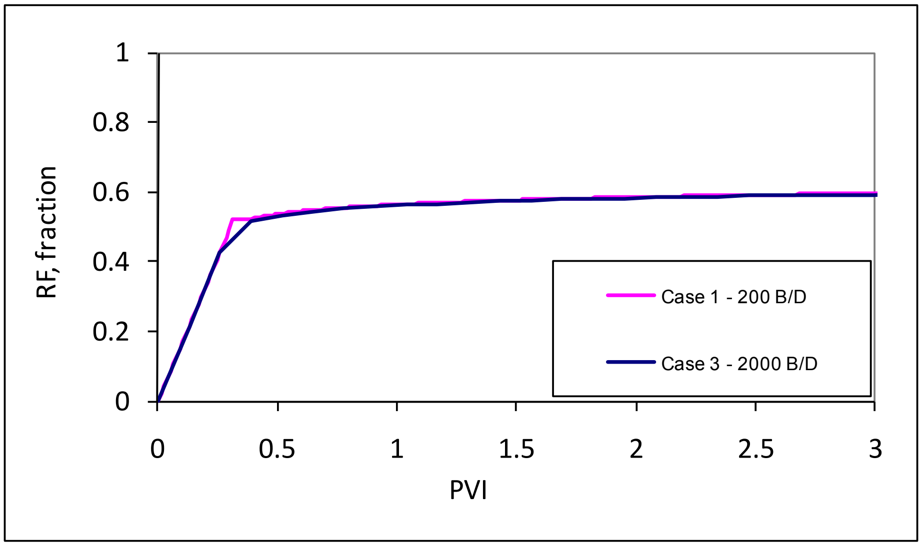


Fig. 37—Comparison of recovery factors obtained using different injection rates and $M$=1 for homogeneous reservoirs.

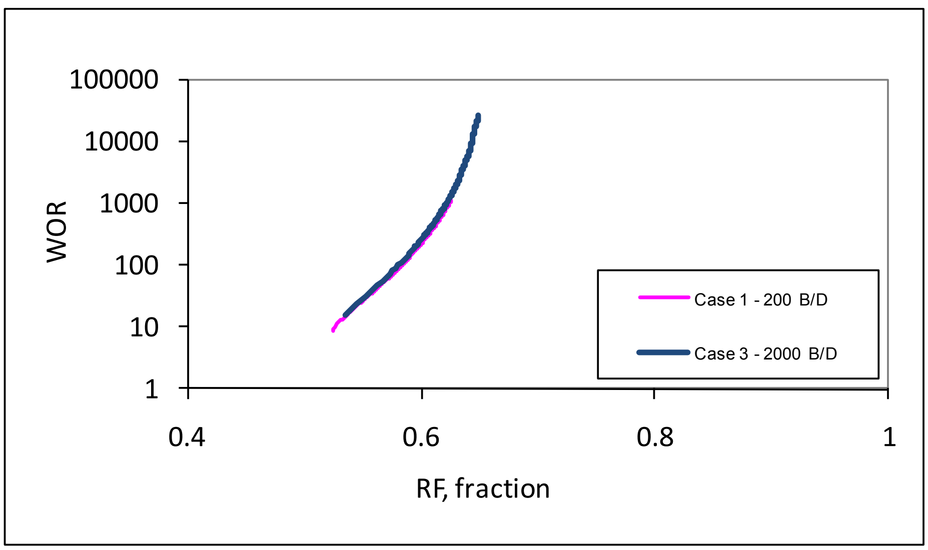


Fig. 38—Comparison of SLZs obtained using different injection rates and $M$=1 for homogeneous reservoirs.

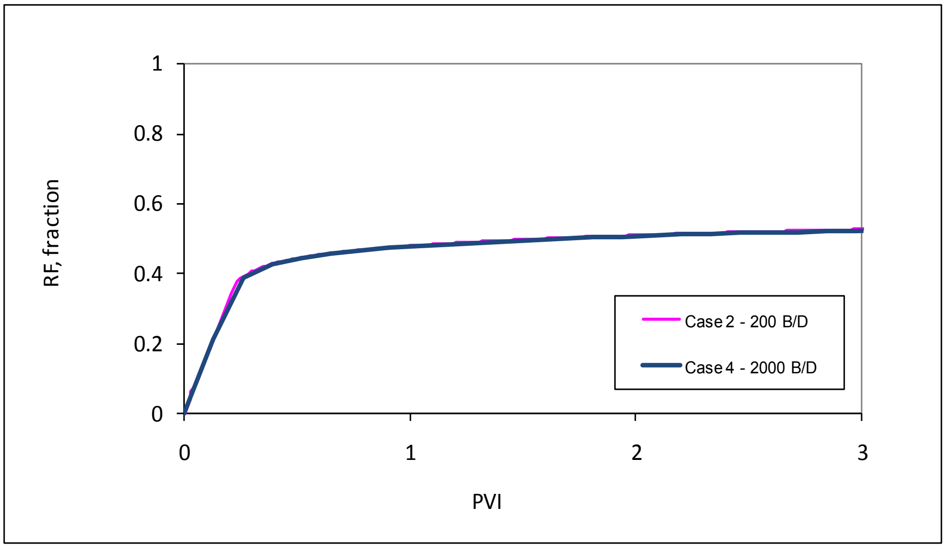


Fig. 39—Comparison of recovery factors obtained using different injection rates and $M$=10 for homogeneous reservoirs.

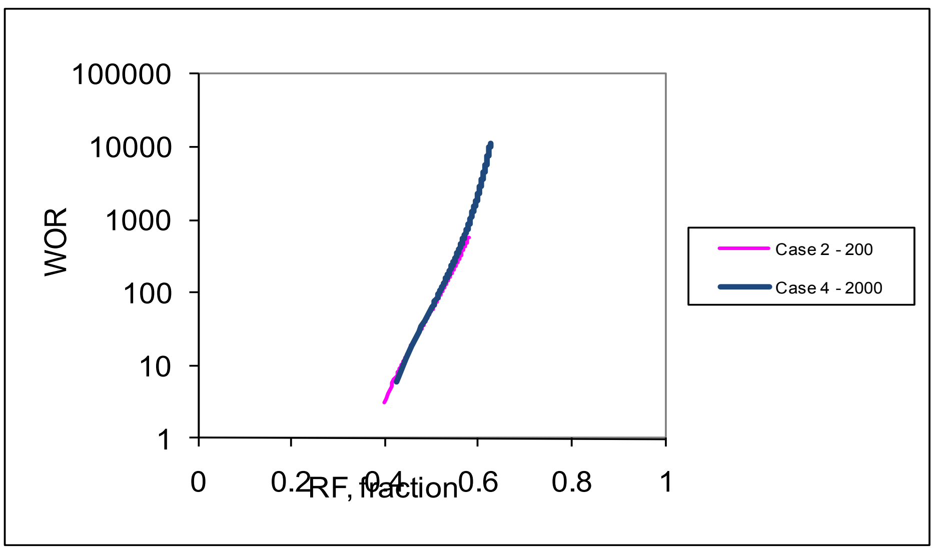


Fig. 40—Comparison of SLZs obtained using different injection rates and $M$=10 for homogeneous reservoirs.

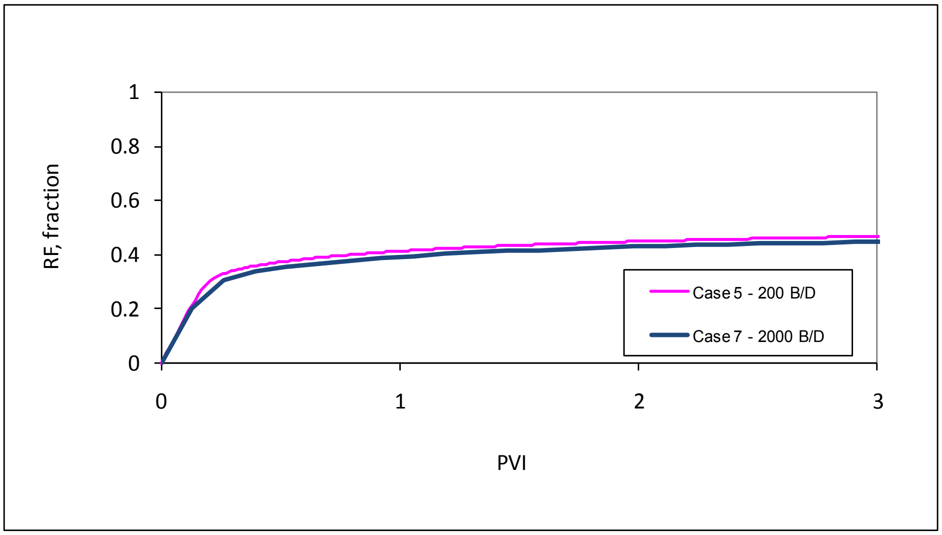


Fig. 41—Comparison of recovery factors obtained using different injection rates and *M*=1 and VDP=0.9.

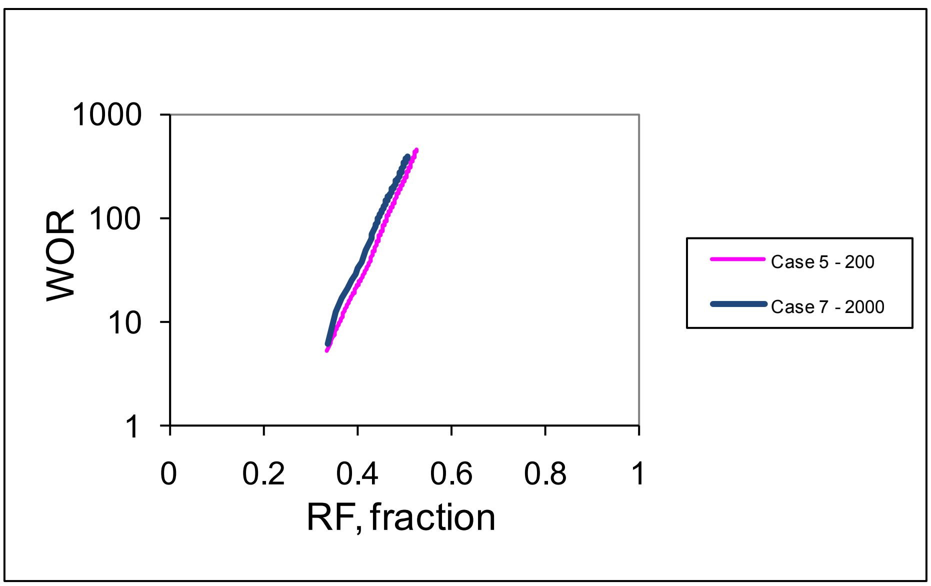


Fig. 42—Comparison of SLZs obtained using different injection rates and *M*=1 and VDP=0.9.

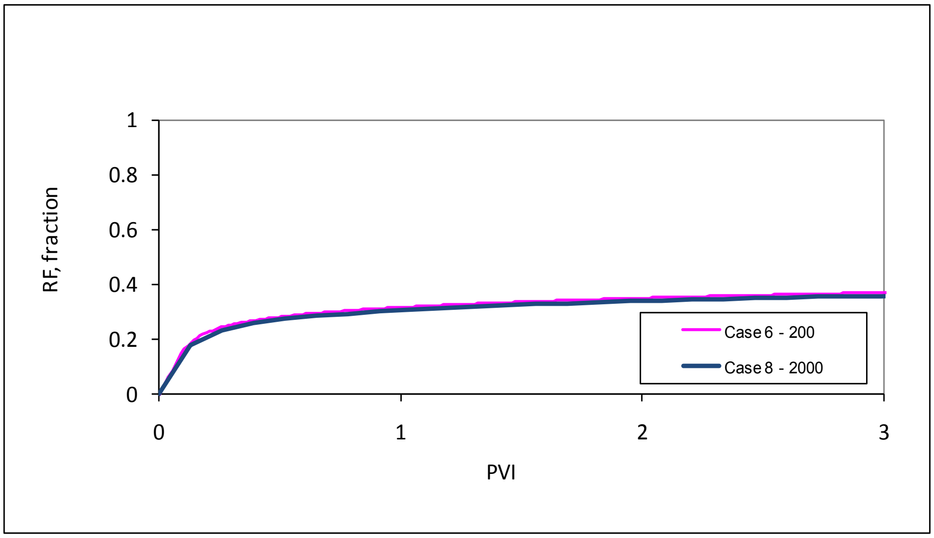


Fig. 43—Comparison of recovery factors obtained using different injection rates and *M*=10 and VDP=0.9.

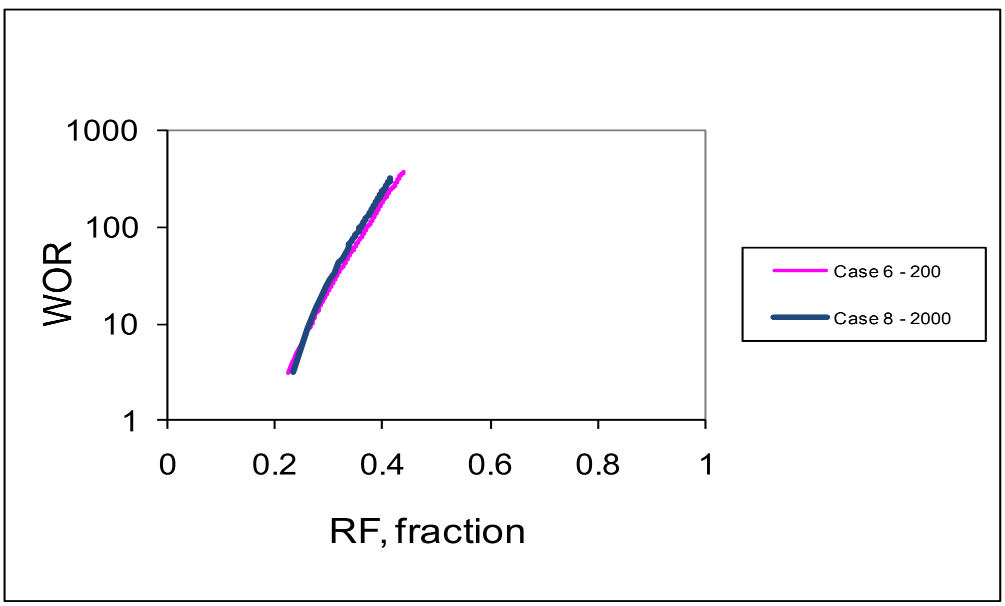


Fig. 44—Comparison of SLZs obtained using different injection rates and *M*=10 and VDP=0.9.

**Number of Layers Effect**

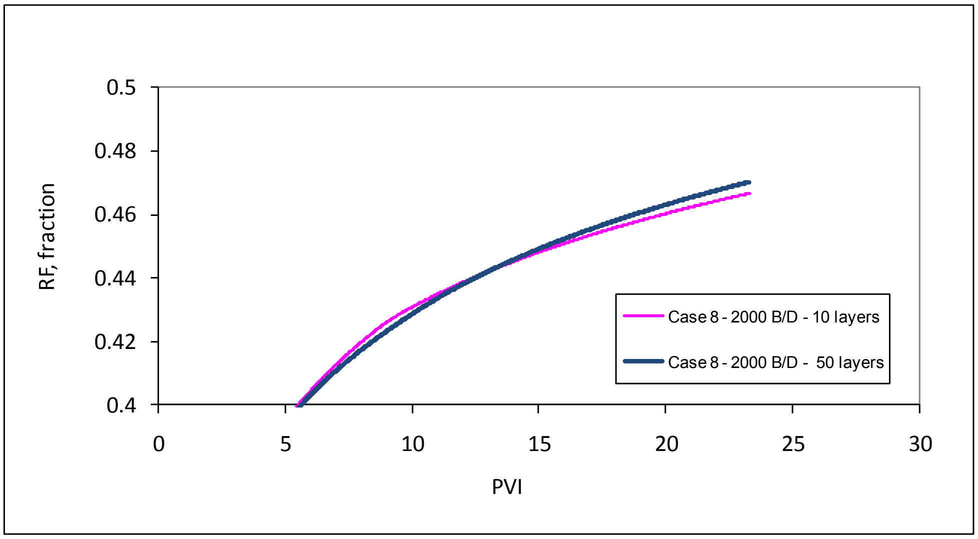


Fig. 45—Recovery comparison for different number of layers in the same reservoir.

Fig. 45 shows the same behavior for both cases, so we can infer that not an important effect is observed, considering that only at high PVI a difference will be obtained. Also, Fig. 46 shown the SLZ behavior and no important effect is noticed.

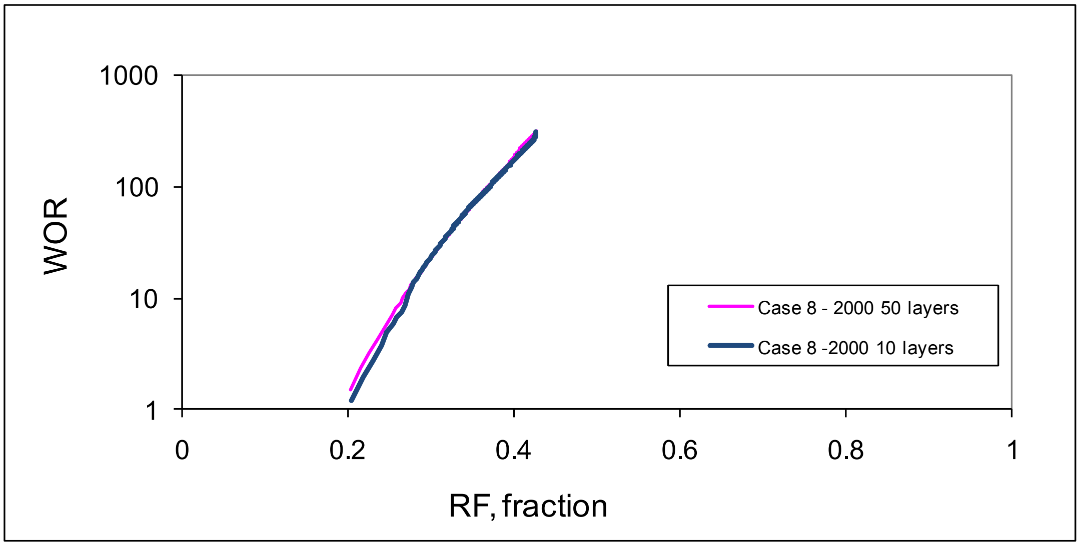


Fig. 46—SLZ plot for heterogeneous cases (VDP=0.9) with $M$=10. No difference in behavior is shown for the number of layers used.

**Crossflow Effect**

Three scenarios of the same 10-layers reservoirs in Table 28 were evaluated:

<u>*Scenario 1*</u>: Homogeneous reservoir (VDP=0), no crossflow ($k_v$=0) and crossflow ($k_v$=$k_h$).

<u>*Scenario 2*</u>: Heterogeneous reservoir (VDP=0.93), no extreme values of permeability ($k$ from 0.1 to 200 mD), no crossflow and crossflow.

<u>*Scenario 3*</u>: Heterogeneous reservoir (VDP=0.85), 3 theft zones ($k$ from 10 to 3000 mD), no crossflow and crossflow.

Table 28—Permeability values per layer used in the crossflow exercise.

| Layer | Scenario 1 | Scenario 2 | Scenario 3 |
|---|---|---|---|
| 1 | 200 | 200 | 2000 |
| 2 | 200 | 100 | 100 |
| 3 | 200 | 80 | 800 |
| 4 | 200 | 60 | 60 |
| 5 | 200 | 0.1 | 2000 |
| 6 | 200 | 0.2 | 100 |
| 7 | 200 | 10 | 10 |
| 8 | 200 | 150 | 150 |
| 9 | 200 | 100 | 3000 |
| 10 | 200 | 50 | 50 |
| VDP | 0 | 0.93 | 0.85 |

Table 28 shows the VDP input data and figures present the simulation runs results. In Fig. 47, RF  vs. PVI curve behavior for the homogenous reservoir is presented. We cannot see any difference in performances. Fig. 48 presents a plot of WOR  vs. RF . In any of them we can see a difference. We can conclude that crossflow will not affect homogeneous reservoir behavior.

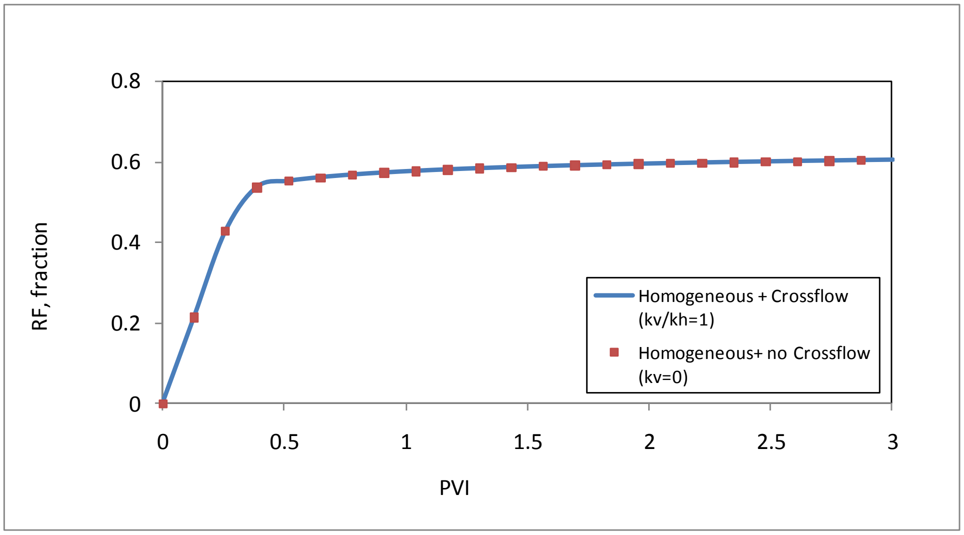


Fig. 47—Crossflow effect in homogeneous reservoir.

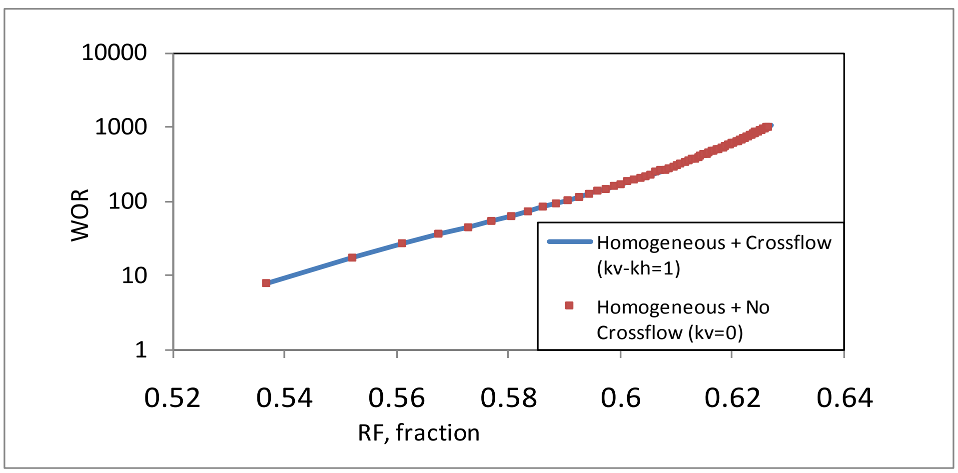


Fig. 48—Log WOR vs. RF plot for homogeneous reservoir with and without crossflow.

Same set of plots are presented for Scenario 2, with a VDP of 0.93 but with relatively low permeability values in all layers. No important difference can be detected in the plots due to crossflow effects.

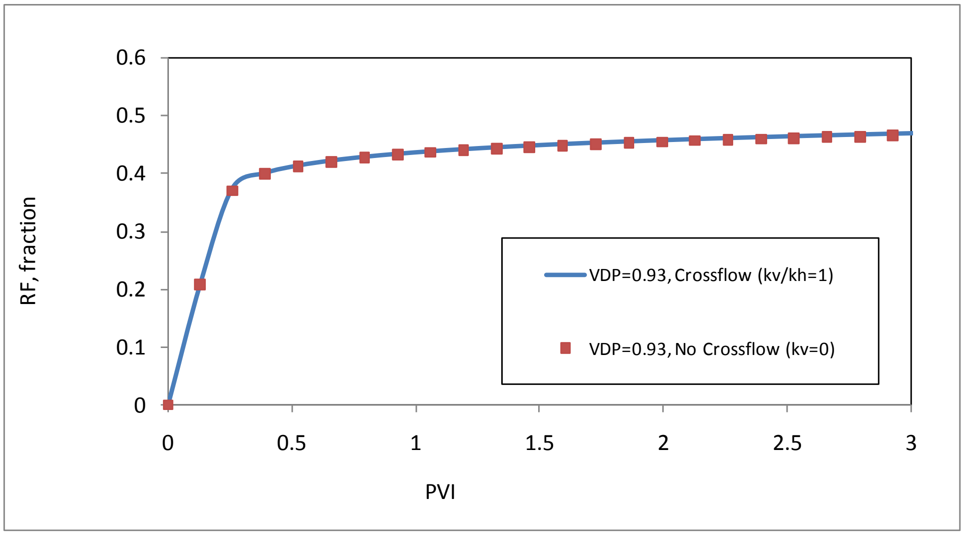


Fig. 49—Crossflow effect in heterogeneous reservoir (VDP=0.93).

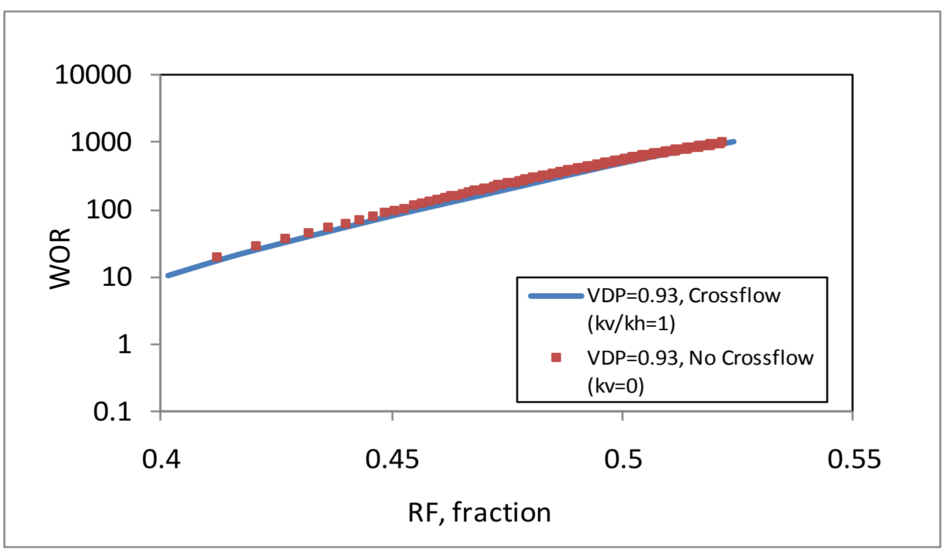


Fig. 50—WOR vs. RF plot for heterogeneous reservoir with and without crossflow (VDP=0.93).

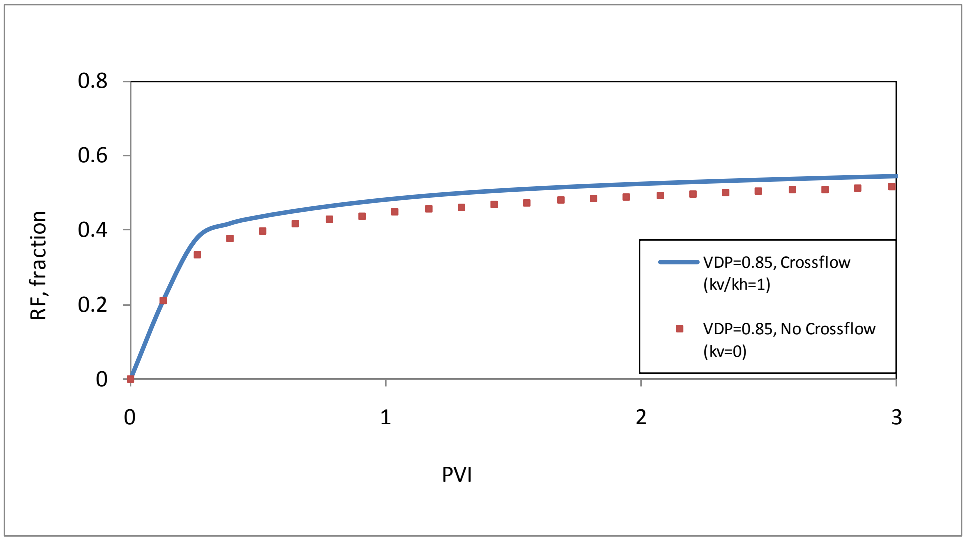


Fig. 51—Crossflow effect in heterogeneous reservoir (VDP = 0.85). For larger VDP, if there is a “thief zone”, the effect will be stronger.

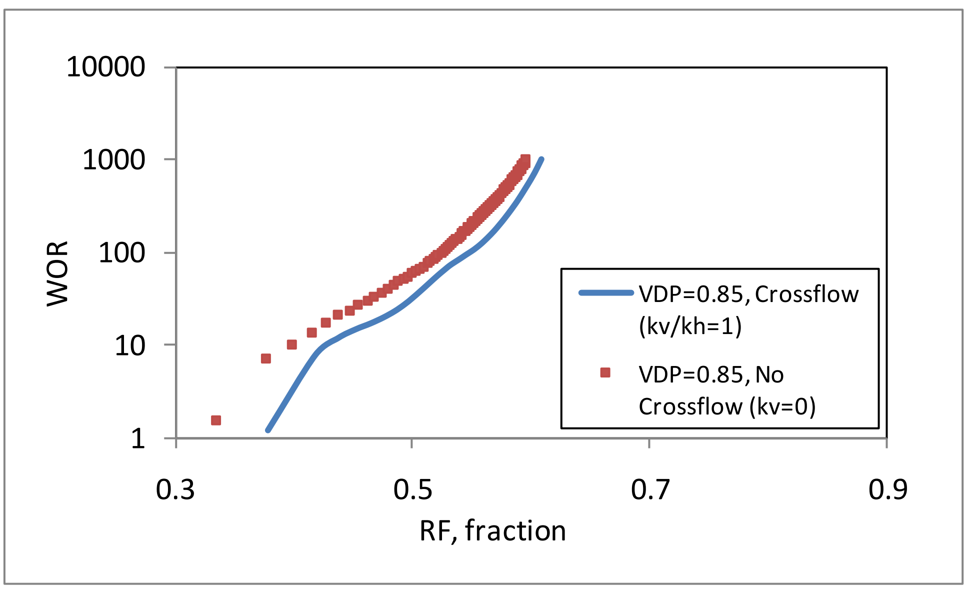


Fig. 52—WOR vs. RF plot for heterogeneous reservoir with and without crossflow (VDP = 0.85).

In Fig. 52 we can see a little difference in the two SLZs. However ultimate recovery does not show an important difference that may affect results in our statistical model. This case has "thief zones". Vertical permeability contrast is less than Scenario 2, but high permeability layers represent a sort of "channels" that are expected to take most of the water injected since horizontal permeability values are extremely high. This situation can be seen in many reservoirs. Thief zones with higher permeability will increase the difference in recovery, WOR and PVI.

In these cases, earlier breakthrough may happen for the no-crossflow case. High permeability layers are acting as theft zones, taken more water and bypassing oil in the reservoir, leaving low permeability zones with less sweep efficiency (red in Fig. 53 and Fig. 54) showing oil saturation profiles at a water cut of 99%.

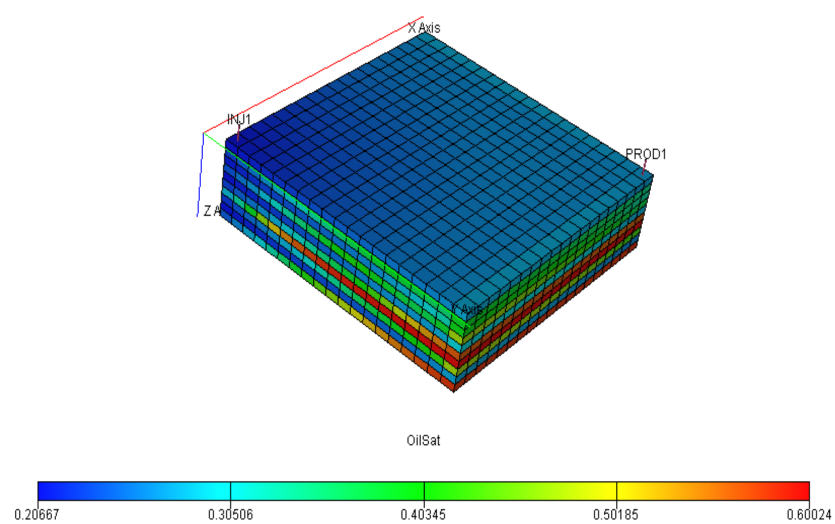


Fig. 53—Oil saturation profile for no crossflow reservoir, with VDP = 0.85.

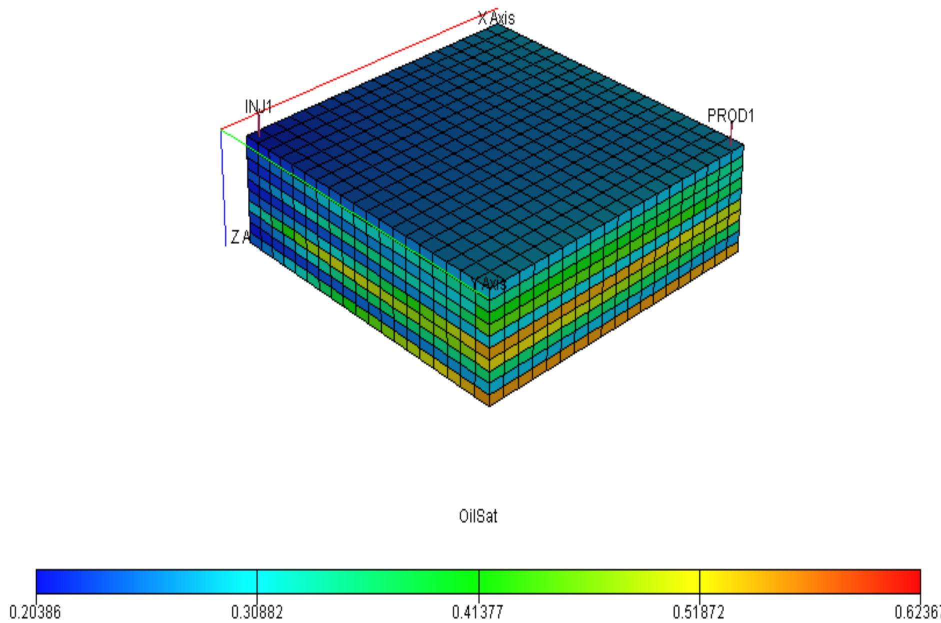


Fig. 54—Oil saturation profile for reservoir with crossflow and VDP = 0.85.

**Some Published Works Regarding Crossflow Effect**

According to Kumar (2005), factors controlling recovery and fingering are high average permeability and *M*. In these cases, depletion will cause important increments of *M* since at lower pressures the oil will be more viscous, so the presence of thief zones may affect recovery and water cycling. Heterogeneity affects recovery more for high *M* (less resistance to flow for water) so thief zones will contribute to recovery mostly during primary depletion. Accurate *M* and relative permeability measurements for each layer will be essential to optimize operations and maximize recovery. Highly correlated (continuous), thin, thief zones will reduce recovery for high *M*. Mechanical or chemical blocking of thief zones on the injection or production side may be a remedy needed in these cases (Yang and Ershaghi, 2005).

Willhite (1986) explains that oil and water can move between layers depending on relative permeability relationships and potential differences, but because permeability differences, the front in the high permeability zone (*HPZ*) will move faster than in the low permeability zone (*LPZ*). Viscous crossflow can occur from *LPZ* to *HPZ* because mobility ratio may be higher in the *HPZ*.

Also, capillary forces can cause crossflow: if the rock is water-wet, water from *HPZ* can imbibe into *LPZ* and *some* oil from *LPZ* can go to *HPZ* where is displaced.

Craig (1971) explains that when $k_v << k_h$: Reservoir can perform as one with very little or no crossflow (30 mD vs. 3000 mD = 0.01).

Ahmed (2002) says that "substantial reservoir uniformity is one of the major physical criterions for successful waterflooding. For example, if the formation contains a stratum of limited thickness with a very high permeability (i.e., thief zone), rapid channeling and bypassing will develop. Unless this zone can be located and shut off, the producing water–oil ratios will soon become too high for the flooding operation to be considered profitable. The lower depletion pressure that may exist in the highly permeable zones will also aggravate the water-channeling tendency due to the high permeability variations".

Ultimate recovery in previous plots in this section are very similar, including same slopes and intercepts in both trend lines, which may drive us to conclude that our model will still Work properly in those cases. Also, all these findings confirm conclusions from other authors commented above.

For cases with extremely high Mobility Ratios, the user must be careful applying any prediction method, including reservoir simulation, because even simulators may have problems accounting for high Mobility Ratio effects (Avery et al., 1987). Techniques such as simulators based on nine-point finite difference, or the use of a pre-selected grid orientation, will improve forecast when an unfavorable $M$ is present.

# APPENDIX I

## Fractional Flow and Frontal Advance Theory

In waterflooding projects, water displaces oil through the rock in an immiscible process. This process is desired to *be* efficient, meaning that we want the water to displace as much oil as possible. The term "Displacement Efficiency" ($E_D$) is then referred to the fraction of oil saturations swept by water from the reservoir that can be explained using the principles of the Frontal Advance Theory and the Fractional Flow (Buckley-Leverett) Equation as a 1-D process.

Beginning with the Darcy's law, and following the derivation process shown by Craig (1971), we can write this expression in consistent units, for oil and water separately as follows:

$$u_o = -\frac{k_o}{\mu_o}\left(\frac{\partial P_o}{\partial x} + g\rho_o \sin\alpha\right) \quad \text{(I.1)}$$

and

$$u_w = -\frac{k_w}{\mu_w}\left(\frac{\partial P_w}{\partial x} + g\rho_w \sin\alpha\right) \quad \text{(I.2)}$$

where:

$u_o$ = oil phase velocity

$u_w$ = water phase velocity

$k_o$ = effective permeability to oil

$k_w$ = effective permeability to water

$\mu_o$ = oil viscosity, cP

$\mu_w$ = water viscosity, cP

$P_o$ = pressure in oil phase

$P_w$ = pressure in water phase

$x$ = distance along direction of movement

$g$ = acceleration due to gravity

$\rho_o$ = oil density

$\rho_w$ = water density

$\alpha$ = angle of the reservoir dip with the horizontal

Rearranging Eq. I.1 and I.2 and substituting, we obtain:

$$u_o \frac{\mu_o}{k_o} = -\frac{\partial P_o}{\partial x} - g\rho_o \sin\alpha \quad \text{(I.3)}$$

and

$$u_w \frac{\mu_w}{k_w} = -\frac{\partial P_w}{\partial x} - g\rho_w \sin\alpha \quad \text{(I.4)}$$

Subtracting Eq. I.3 from Eq. I.4 we obtain:

$$u_w \frac{\mu_w}{k_w} - u_o \frac{\mu_o}{k_o} = -\left(\frac{\partial P_w}{\partial x} - \frac{\partial P_o}{\partial x}\right) - g(\rho_w - \rho_o)\sin\alpha \quad \text{(I.5)}$$

Since the difference of the pressure in the oil phase minus the pressure in the water phase is defined as Capillary Pressure ($P_c$), and the density difference is the difference between the water density and the oil density, we can write:

$$u_w \frac{\mu_w}{k_w} - u_o \frac{\mu_o}{k_o} = \frac{\partial P_C}{\partial x} - g\Delta\rho \sin\alpha \quad \text{(I.6)}$$

but we can express Eq. I.6 in terms of the total velocity *($u_t$)* as:

$$u_w \frac{\mu_w}{k_w} - (u_t - u_{w)}) \frac{\mu_o}{k_o} = \frac{\partial P_C}{\partial x} - g\Delta\rho \sin\alpha \quad \text{(I.7)}$$

and solving Eq. I.7 and dividing both members by $u_t$:

$$u_w \left( \frac{\mu_w}{k_w} + \frac{\mu_o}{k_o} \right) - u_t \frac{\mu_o}{k_o} = \frac{\partial P_C}{\partial x} - g\Delta\rho \sin\alpha \quad \text{(I.8)}$$

$$\frac{u_w}{u_t} \left( \frac{\mu_w}{k_w} + \frac{\mu_o}{k_o} \right) - \frac{\mu_o}{k_o} = \frac{1}{u_t} \left( \frac{\partial P_C}{\partial x} - g\Delta\rho \sin\alpha \right) \quad \text{(I.9)}$$

and solving for the fraction of water velocity of the total velocity (*fw*):

$$\frac{u_w}{u_t} = f_w = \frac{\dfrac{\mu_o}{k_o} + \dfrac{1}{u_t} \left( \dfrac{\partial P_C}{\partial x} - g\Delta\rho \sin\alpha \right)}{\dfrac{\mu_w}{k_w} + \dfrac{\mu_o}{k_o}} \quad \text{(I.10)}$$

or

$$f_w = \frac{1 + \dfrac{k_o}{u_t \mu_o} \left( \dfrac{\partial P_C}{\partial x} - g\Delta\rho \sin\alpha \right)}{1 + \dfrac{\mu_w}{\mu_o} \dfrac{k_o}{k_w}} \quad \text{(I.11)}$$

Which is the Fractional Flow Equation including Capillary Pressure, fluids density and dip angle of the reservoir.

We assumed Capillary Pressure negligible and a horizontal reservoir. We simplify our equation as:

$$f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o}\frac{k_o}{k_w}} \quad \text{(I.12)}$$

With the Fractional Flow Equation, we can determine the water cut ($f_w$) at any point in the reservoir where water saturation is known.

Frontal Advance Equation (Craig, 1971), makes two main assumptions: mass transfer between phases does not exist and fluids are incompressible. We also consider an infinitesimal element of rock with a constant porosity ($\phi$) and area ($A$) in the direction of flow with distance $x$. Applying the Mass Conservation and Material Balance principles we can define the water mass rate entering and leaving the element at point $x$, and the water accumulation in the element as:

$$(q_w \rho_w)_x \quad \text{(I.13)}$$

$$(q_w \rho_w)_{x+\Delta x} \quad \text{(I.14)}$$

$$A\phi\Delta x \frac{\partial}{\partial t}(S_w \rho_w) \quad \text{(I.15)}$$

and we can express water accumulation = water in – water out as:

$$(q_w \rho_w)_x - (q_w \rho_w)_{x+\Delta x} = A\phi\Delta x \frac{\partial}{\partial t}(S_w \rho_w) \quad \text{(I.16)}$$

or

$$\frac{\partial}{\partial x}(q_w \rho_w) + A\phi \frac{\partial}{\partial t}(S_w \rho_w) = 0 \quad \text{(I.17)}$$

eliminating density difference:

$$\frac{\partial}{\partial x}(q_w) + A\phi \frac{\partial}{\partial t}(S_w) = 0 \quad \text{(I.18)}$$

and solving for changes in water saturation with time:

$$\left(\frac{\partial S_w}{\partial t}\right)_x = -\frac{1}{A\phi}\left(\frac{\partial q_w}{\partial x}\right)_t \quad \text{(I.19)}$$

but $q_w$ is a function of both water saturation and time, so:

$$dq_w = \left(\frac{\partial q_w}{\partial S_w}\right)_t dS_w + \left(\frac{\partial q_w}{\partial t}\right)_{S_w} dt \quad \text{(I.20)}$$

Taking the derivative with respect to $x$ at a fixed time and solving for the change of water saturation with respect to length:

$$\left(\frac{\partial S_w}{\partial x}\right)_t = \frac{\left(\frac{\partial q_w}{\partial x}\right)_t}{\left(\frac{\partial q_w}{\partial S_w}\right)_t} \quad \text{(I.21)}$$

Water saturation is a function of $x$ and t, so giving the same treatment we gave to $q_w$ we have:

$$dS_w = \left(\frac{\partial S_w}{\partial x}\right)_t dx + \left(\frac{\partial S_w}{\partial t}\right)_x dt = 0 \quad \text{(I.22)}$$

We made $dS_W = 0$ because we are looking at a plane of constant $x$ where $S_W$ changes. Thus

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{-\left(\frac{\partial S_w}{\partial t}\right)_L}{\left(\frac{\partial S_w}{\partial x}\right)_t} \quad \text{(I.23)}$$

and substituting Eq. I.19 into Eq. I.21 we have:

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{1}{A\phi}\left(\frac{\partial q_w}{\partial S_w}\right)_t \quad \text{(I.24)}$$

but we also have that

$$q_w = f_w q_t \quad \text{(I.25)}$$

and differentiating with respect to $S_W$ *at a constant t:*

$$\left(\frac{\partial q_w}{\partial S_w}\right)_t = f_w\left(\frac{\partial q_t}{\partial S_w}\right)_t + q_t\left(\frac{\partial f_w}{\partial S_w}\right)_t \quad \text{(I.26)}$$

but the change of velocity with respect to $S_W$ at any time is zero because the fluids are incompressible, so we finally have, substituting Eq. I.26 into Eq. I.24 :

$$\left(\frac{\partial x}{\partial t}\right)_{S_w} = \frac{q_t}{A\phi}\left(\frac{\partial f_w}{\partial S_w}\right)_t \quad \text{(I.27)}$$

This is the linear Frontal Advance Equation for water, based upon conservation of mass and assuming incompressible fluids. It states that the rate of advance (velocity= distance /time) of the saturation front inside the reservoir will be equal to superficial velocity of the total fluid, times the change of the fractional flow with water saturation. We can also say that particular water saturation propagates through a porous rock at a constant velocity. To determine that velocity, we need to apply the Fractional Flow Equation.

Some limitations of the frontal advance solution are related to the basic assumptions, such as:

1. The two immiscible fluids are considered incompressible.
2. Linear or radial flow in only one direction is assumed.

3. Initial fluids saturations are uniform.
4. Applies only to stabilized displacement processes.
5. Layers are assumed to be homogeneous and isotropic. Constant rock properties except permeability per layer, layer thickness and porosity in the reservoir.
6. Layer-cake mode with no crossflow between layers.
7. Steady State flow.
8. Gravity segregation, dip angle and capillary pressure are neglected.
9. The vertical efficiency is unity within each layer.

**General Application**

In this section, we summarize a typical application of the fractional flow equation and the Frontal Advance Theory. In later sections we will show the approach developed in this research.

Rewriting Eq. I.12 in terms of the oil and water relative permeabilities ratios we have:

$$f_w = \frac{1}{1 + \frac{\mu_w}{\mu_o}\frac{k_{ro}}{k_{rw}}} \quad \text{(I.28)}$$

Since the relative permeabilities are functions of the water saturation ($S_w$), $f_w$ is also a function of the water saturation. It is used to construct a plot of water front ($f_w$) vs. water saturation ($S_w$) to determine displacement performance.

To construct the plot, we need a set of relative permeability curves generated from a model or determined by special core analysis and oil and water viscosity values.

For computation of performance at water breakthrough, the plot needed is shown in Fig. 55, where an example of a fractional flow performance with water saturation can be seen. According to Welge procedure, a tangent drawn from the initial water saturation ($S_{wc}$) touches the curve at $S_w$=0.69 and at $f_w$=0.93. These are the values of those variables at the water front, and will be the values at the water breakthrough when the injected water front reaches the producer well.

The stabilized zone includes all water saturations from $S_{wc}$ to $S_{WBT}$=0.69 and water saturations will increase after breakthrough up to the maximum water saturation (1-$S_{or}$). Reading the point where the tangent reaches $f_w$=1, we can determine the average water saturation behind the front. In Fig. 55, this value is 0.72.

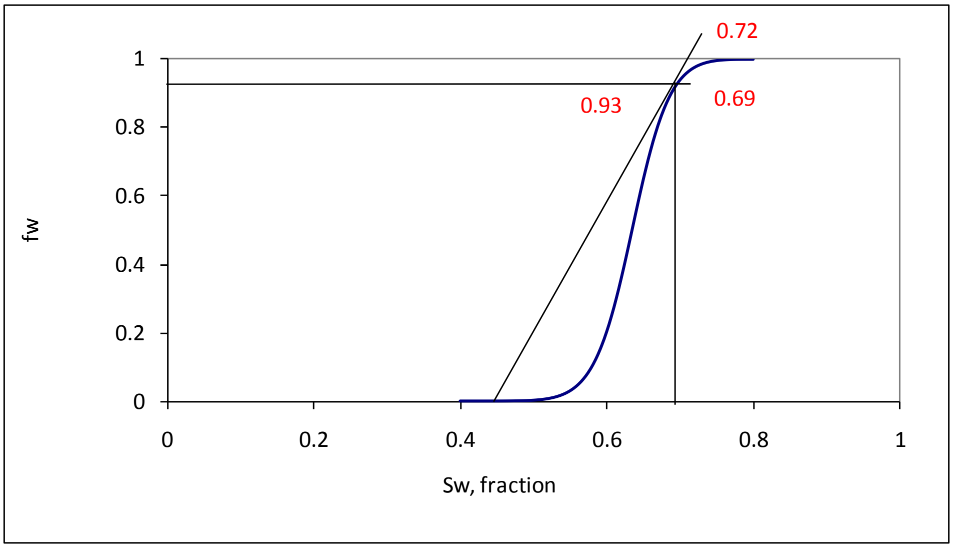


Fig. 55—Example of fractional flow curve showing the breakthrough at a $f_w$ of 0.93 and a $S_w$ of 0.69. The average water saturation at the breakthrough is 0.72.

To determine performance after breakthrough (the non-stabilized portion of fractional flow curve), several tangents to the portion of the fractional flow curve after breakthrough can be determined to identify the $f_w$ value for each water saturation value, until we reach the maximum water saturation (1-$S_{or}$). The extrapolation of these tangents to the value of $f_w$=1 provide the values of $S_{Wave}$.

Even when the Welge's graphical method is useful, in some cases it may be difficult to determine where the tangent intersects the curve. High viscous oils can present a stepped curve where the tangent cannot be identified properly. A numerical approach is better than the graphical method and using Fractional Flow Theory we obtain Eq. I.29:

$$\overline{S_w} = S_{Wf} + \frac{1 - f_{wf}}{\left(\frac{df_w}{dS_w}\right)_{S_{wf}}} \quad \text{……………………………………………… (I.29)}$$

Using this information (just the appropriate water-oil relative permeability curves and the oil and water viscosities), cumulative oil production and water injection, injection and production rates, and WOR can be computed.

Wettability is an important factor that control displacement efficiency in the waterflooding process. A decrease of water-wetness will make water permeability increase (more water flow) and oil permeability decrease (less oil flow). The fractional flow curve for an oil-wet rock is steeper than the water-wet one. That will make displacement of oil by water in an oil-wet rock less efficient than that in a water-wet rock, or more PVI will be needed to achieve an equivalent recovery factor.

The viscosity ratio will also affect displacement since the more viscous the oil, the stepper the slope of the fractional flow curve after breakthrough, so more PVI will be required to produce oil, because the expression for PVI for linear systems is:

$$PVI = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{S_{wf}}} \quad \text{…………………………………………………..(I.30)}$$

The same effect will be obtained for high or unfavorable mobility ratio, since water mobility will be higher than oil mobility and this parameter is affected by the oil viscosity (see Fig. 56).

Gravity forces will affect also when the dip angle is high. In that case, the dip angle component in the fractional flow curve must be considered. Usually oil density is less than water density, so water should move slower or faster than oil, depending on where the water is injected in the reservoir, down dip or up dip, respectively.

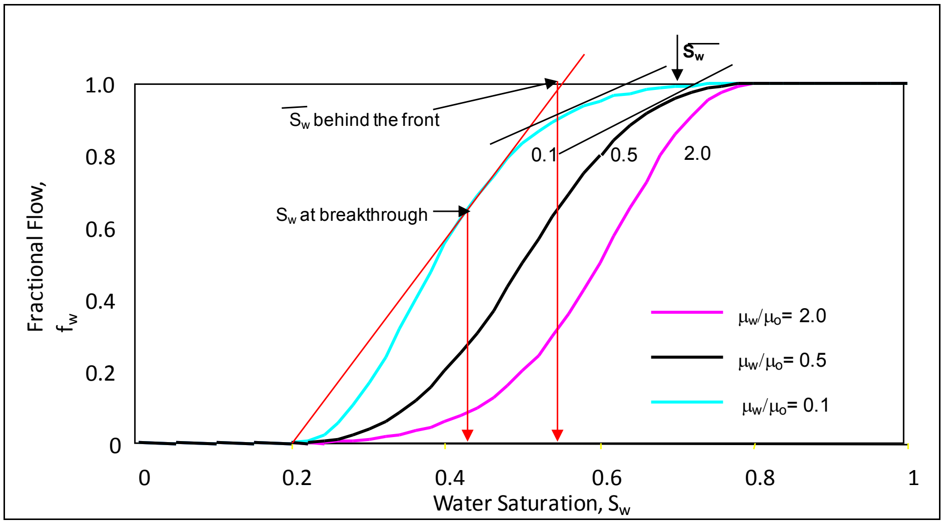


Fig. 56—Effects of viscosity ratio in fractional flow curves, maintaining the same relative permeability curves, $S_{wi}$ and $S_{or}$.

# APPENDIX J

# ANALYTICAL METHODS FOR ESTIMATING WATERFLOOD RECOVERY

## Analytical and Empirical Forecasting Procedures

During the last 50 years, several attempts have been made to forecast waterflood performance and ultimate oil recovery by modeling the sweeping process of water displacing oil through the porous medium.

The accuracy of the prediction is mostly affected by the knowledge about effects of reservoir heterogeneity, fluid saturations, and mobility ratio. These factors affect displacement and areal and vertical sweep efficiencies. Water channeling that bypasses mobile oil remaining within the rock, causing low displacement and early breakthrough in producing wells, will reduce ultimate recovery.

Reasonable forecasts of waterflood performance can improve decisions regarding a waterflooding candidate and project feasibility. Waterflood performance can be estimated by various analytical methods based upon several assumptions that many times are ignored or violated.

A brief description of the most used analytical and empirical forecasting methods follows:

## Craig-Geffen-Morse Method

The Craig-Geffen-Morse prediction method (CGM) can be used to estimate waterflood performance (Craig, 1971). It is based on the Buckley-Leverett theory that is concerned with displacement mechanisms and considers oil displacement by water in either a linear or a radial system. The method estimates oil recovery with the required volume of water injected for that recovery in a waterflood system as a function of time. CGM considers multilayered systems, variable injection rate, and areal sweep efficiency as main parameters.

In addition to displacement in the swept area, CGM uses experimental correlations that account for areal sweep efficiency at water breakthrough and relates the areal sweep efficiency after breakthrough to the cumulative injected water. The method accounts for a gradual improvement in areal sweep efficiency with continued water injection and for increasing displacement of oil behind the front in the contacted zone until areal sweep efficiency is equal to unity.

The method performs all calculations for a single layer. Results of injection and production rates for the other layers can be extrapolated in proportion to the flow capacity (*kh*) values and pore volumes of the other layers. The vertical sweep efficiency is considered unity. The summation process accounts for vertical sweep of the multilayered system.

The total oil production is the sum of the oil displaced from the initial swept region and the additional oil produced as a result of the increase in areal sweep. Oil production begins before water breakthrough, just after gas fill-up. The water production is then the water injected minus the oil produced.

The method discussed in the original paper (Craig, 1954) did not consider multilayer reservoirs, but a revised version was extended to stratified reservoirs (Craig 1971).

The main assumptions of the CGM method are:

1. Multilayered reservoirs with no crossflow between layers. Each layer is homogeneous but the method accounts for vertical heterogeneity.

2. Linear or radial flow can be explained using Darcy's law. Applies for , incompressible fluids and isothermal process.

3. Steady-state flow.

4. Only rock properties that change per layer are permeability, layer thickness, and porosity.

5. Gravity segregation between oil and water, dip angle, and capillary pressure are neglected.

6. Gas fill-up per zone is completed before production begins.

7. Areal sweep efficiency is calculated with experimental correlations and increases up to 100%. Also 100% vertical sweep efficiency is assumed for each layer.

**Dykstra-Parsons Method**

Dykstra-Parsons' method (Dykstra and Parsons, 1950) is concerned mostly with reservoir stratification. Used for predicting waterflood behavior in stratified systems, the method combines laboratory results with theoretical studies. This method requires the use of VDP, $M$, the initial or connate water saturation ($S_{wc}$), and fractional oil recovery at a specified water/oil ratio. Dykstra and Parsons introduced the vertical coverage ($C_v$) parameter, determined experimentally, to account for vertical sweep efficiency. This parameter is determined using VDP, *M,* and $C_v$ correlations, and later $C_v$ is multiplied times the areal sweep efficiency, determined using Craig's correlations (1971). The main assumptions of this method are:

1. Multilayered reservoirs with no crossflow between layers.

2. Linear flow; incompressible fluids; isothermal process; can be explained using Darcy's law.

3. Steady-state flow.

4. Only rock properties that change per layer are permeability, layer thickness, and porosity.

5. Gravity segregation between oil and water is ignored.

6. Piston-like displacement with no oil production from behind the front, ignoring relative permeability effects.

The results obtained from this method tend to be optimistic, related mainly to the assumption of piston-like displacement, but the method is accurate for highly heterogeneous reservoirs with any *M*.

**Stiles Method**

The Stiles method handles reservoir heterogeneity and is used in stratified reservoirs. The method is subject to the following assumptions (Stiles, 1949):

1. Multilayered reservoirs with no crossflow between layers.

2. Linear flow; incompressible fluids; isothermal process; can be explained using Darcy's law.

3. Steady-state flow.

4. Only rock properties that change per layer are permeability, layer thickness, and porosity.

5. Gravity segregation between oil and water is ignored.

6. Piston-like displacement with no oil production from behind the front.

7. Flood front penetration into each layer is proportional to the capacity of the layer (thickness × permeability). This is equivalent to assuming the mobility ratio is unity.

Stiles' method is more realistic for multilayered reservoirs. The method calculates permeability distribution from capacity distribution and considers the physical structure of the reservoir better than Dykstra-Parsons, which is based on statistical and experimental correlations. Running both methods and comparing results can be a good approach to estimate recovery when high VDP and $M$ are present. Input data and reservoir characteristics will dictate which method should be used in each case.

Other practical approaches include the work published by Craig (1971), which presents different graphic correlations for areal sweep efficiencies as a function of $M$. Graphic, experimental correlations are used to determine areal and vertical sweep efficiency and to determine displacement efficiency and cumulative production ($N_p$). This method provides accurate results for more favorable $M$ ($M$<1).

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