PROSPECTIVE MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA IN CHINA AND THE U.S.

A Dissertation

by

RONGJIN HUANG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2010

Major Subject: Curriculum and Instruction

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Approved by:

Co-Chairs of Committee,	Yeping Li
	Gerald Kulm
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	Victor Willson
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ABSTRACT

Prospective Mathematics Teachers' Knowledge for Teaching Algebra

in China and the U.S.

(December 2010)

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This study examined teachers' knowledge for teaching algebra, with a particular focus on teaching the concept of function and quadratic relations in China and the United States. An embedded mixed methods design was adapted, a design in which the main data set consists of written answers to a questionnaire, while the supportive data set is comprised of the written answers to open-ended questions and follow-up interviews. A structural equation model was adopted to analyze the status and structure of teacher knowledge for teaching algebra in China and the U.S. A qualitative analysis of the answers to the open-ended questions and follow-up interviews is aimed to further illustrate and interpret the quantitative findings.

Three hundred and seventy six Chinese and 115 U.S. prospective middle and high school mathematics teachers participated in this survey. Based on an extensively quantitative and qualitative data analysis, the following conclusions were made. First, the Chinese participants demonstrated a stronger knowledge for teaching algebra when compared with their U.S. counterparts. Second, the structure of knowledge for teaching algebra of the Chinese participants is much more interconnected than that of their U.S. counterparts. Third, the Chinese participants showed flexibility in choosing appropriate perspectives of function concept and in selecting multiple representations in contrast to their U.S. counterparts. Fourth, this flexibility is found to be closely related to school math and teaching math. Finally, the number of college math and math education courses taken impacts teachers' knowledge for teaching algebra.

The findings of this study hold several implications for mathematics teacher preparation in general and studies on mathematics teachers' knowledge in particular. Theoretically, the complexity of understanding and measuring mathematics teachers' knowledge for teaching was examined and discussed. This study also enriches the understanding of mathematics teachers' knowledge for teaching at middle and high schools in China and the United States. Specifically, the Chinese practice of developing teachers' basic knowledge, skills, and flexibility provides an alternative for U.S. mathematics teacher educators to reflect on their practice. Practically, what we can learn from this study to improve mathematics teacher preparation in China and the U.S. is discussed. Finally, the limitations of this study are discussed and further studies are suggested.

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NOMENCLATURE

MKT	Mathematics Knowledge for Teaching
KTA	Knowledge for Teaching Algebra
KTCF	Knowledge for Teaching the Concept of Function

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CHAPTER I

INTRODUCTION

Preparing future mathematics teachers with the appropriate mathematics knowledge needed for teaching is crucial for high quality teaching, eventually resulting in student learning (National Mathematics Advisory Panel [NMAP], 2008; RAND Mathematics Study Panel, 2003). Researchers have focused on understanding and measuring *mathematics knowledge for teaching* (denoted as MKT) in the past few decades (e.g., Ball, Hill, & Bass, 2005; Ferrini-Mundy, McCrory, & Senk, 2006; Hill, Ball, & Schilling, 2008; Kulm, 2008). For example, drawing on Shulman's (1986) seminal work on teacher knowledge, Ball and her colleagues have developed a refined framework and relevant instruments for measuring elementary mathematics knowledge for teaching (Ball et al., 2005; Ball, Thames, & Phelps, 2008). Moreover, researchers found that mathematics knowledge for teaching has a close relationship with classroom instruction (Hill, Blunk et al., 2008) and student achievement (Hill, Rowan, & Ball, 2005).

As mathematics literacy - particularly algebra - became an extension of the civil rights movement (Moses, 1995; Moses & Cobb, 2001), teaching algebra to all students became an important and challenging issue in the United States (Blume & Heckman, 2000; Creenes & Rubenstein, 2008; National Council of Teachers of Mathematics [NCTM], 2009; NMAP, 2008). Although many studies have focused on

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students' learning of algebra (e.g., Kaput, Blanton,& Moreno, 2008; Katz, 2007; Kieran, 2004, 2007), less attention was paid to studying teachers' knowledge for teaching algebra (e.g., Doerr, 2004; Even, 1993; Even& Tirosh, 1995, 2008). In order to understand and develop teacher *Knowledge for Teaching Algebra* (denoted as KTA), a research team at Michigan State University (Ferrini-Mundy et al., 2006) has worked on developing an instrument measuring KTA. They proposed a framework of KTA, including *School Algebra Knowledge* (i.e., algebra in secondary school, denoted as SA), *Advanced Mathematical Knowledge* (i.e., related college math such as calculus and abstract algebra, denoted as AM), and *Teaching Mathematics Knowledge* (i.e., knowledge of typical errors, canonical uses of school math, curriculum trajectories, etc., denoted as TM). An instrument grounded in this model has been developed and tested in the U.S. with an internal consistency Cronbach's alpha .8 (Floden & McCrory, 2007; Floden, McCrory, Reckase, & Senk, 2009).

Efforts to pursue high quality classroom teaching and student learning in mathematics have led researchers to explore the practices in high-achieving countries, such as China. Quite a number of comparative studies of mathematics education between China and the United States have covered a broad range of topics in mathematics education. These studies include student learning (i.e., Cai,1995, 2000, 2004), classroom teaching (i.e., Huang & Cai, 2010; Huang & Li, 2011; Stevenson, Chen, & Lee, 1993; Stevenson & Lee, 1995; Stigler & Hiebert, 1999), teachers' knowledge (i.e., An, Kulm, & Wu, 2004; Ma, 1999) and beliefs (i.e., An, Kulm, Ma, & Wang , 2006; Cai, 2000, 2006; Cai, Perry, Wong, &Wang, 2009), and curriculum (i.e., Fan & Zhu, 2007; Kulm &

Li, 2009; Li, Chen, & An, 2009).

With regard to teachers' knowledge and teacher preparation, Ma (1999) found that Chinese elementary mathematics teachers demonstrated a profound understanding of fundamental knowledge for teaching in contrast to their U.S. counterparts. A recent study on mathematics teacher preparation at the middle school level in Chinese Taiwan, South Korea, Bulgaria, Germany, Mexico and the United States, found that "in Chinese Taiwan and Korea, the level of mathematics preparation was very strong and in both countries, the amount of emphasis given to the practical issues of mathematics pedagogy was also extensive" (Schmidt et al., 2007, p. 1). Moreover, Li, Huang, and Shin (2008) revealed that the secondary teacher (including middle and high school levels) preparation programs in the Chinese Mainland and Korea emphasize teachers' learning of mathematics subject matter knowledge. In addition, a study comparing pedagogical content knowledge of middle school mathematics teachers between the U.S. and China (An et al., 2004) has found that the Chinese mathematics teachers emphasized gaining correct conceptual knowledge by relying on more rigid development of procedures. The U.S. teachers emphasized a variety of activities designed to promote creativity and inquiry to develop concept mastery, with a lack of connection between manipulative and abstract thinking, and between understanding and procedural development. Although these studies described some features of mathematics teacher knowledge for teaching in China and the U.S. in general, the characteristics of mathematics teachers' knowledge for teaching algebra in these two nations have not been explored empirically.

The practice in China attempts to prepare secondary mathematics teachers (including middle and high school levels) with a solid mathematics foundation and broad mathematics background, with less attention paid towards pedagogical knowledge preparation (Li et al., 2008). Compared to middle school mathematics teacher preparation in East Asia, the practice in the U.S. seems to place less emphasis on mathematics content knowledge and pedagogical content knowledge, but spend more time on learning pedagogical knowledge in general (Babcock et al., 2010; Schmidt et al., 2007). The differences between the teacher preparation systems in the U.S. and China may result in differences of teachers' knowledge for teaching. In this study, I aim to examine the status and characteristics of mathematics teachers' knowledge by focusing on their knowledge for teaching algebra and further examine the relationship between teachers' knowledge and relevant factors such as course taking.

Adapting an instrument cross-culturally is a challenging and important issue in comparative studies (Delaney, Ball, Hill, Schilling, & Zopf, 2008). In this study, I developed a questionnaire measuring mathematics knowledge for teaching algebra (KTA) based on an existing instrument developed by Michigan State University (Floden et al., 2009), and used it to collect prospective teachers' data in China and the U.S. Comparing the features of KTA between the United States and China could broaden and deepen our understanding of mathematics knowledge for teaching algebra. In addition, this study could contribute to developing and validating a survey instrument of KTA cross-culturally. Thus, this study holds implications for deepening the understanding of mathematic knowledge for teaching algebra, and improving mathematics teacher preparation in China and the U.S.

Although teaching algebra for all has been a slogan of mathematics education reformers (Edwards, 1990) for two decades, it is still a very challenging task for the current mathematics education reform (Katz, 2007; Kieran, 2004, 2007; NCTM, 2000, 2009). As described previously, there are salient differences between the U.S. and Chinese mathematics teacher preparation programs at secondary schools in terms of their emphasis of subject matter knowledge and pedagogical content knowledge (Li et al., 2008; Schmidt et al., 2007). It is expected that there are differences in the status and characteristics of teachers' mathematics knowledge for teaching particular contents in the U.S. and China. However, we do not know to what extent teachers are equipped with mathematics knowledge for teaching the core content, algebra. Thus, the current study is aimed to address the following questions: "What are the differences and similarities of secondary (i.e., grades 6-12) prospective teachers' knowledge for teaching algebra (KTA) between the U.S. and China? What are the relationships among different components of KTA within each country? With regard to the specific content of function, what are the differences and similarities of pre-service teachers' knowledge for teaching the concept of function? How do the courses that the pre-service teachers have taken relate to their performance in KTA?" In order to better describe this study, we use the following definitions and abbreviations.

Knowledge for teaching algebra (KTA) includes three types of knowledge for teaching, *i.e., school algebra knowledge, advanced mathematical knowledge and teaching algebra knowledge. School algebra knowledge* (SA) refers to the algebra covered in the curriculum from K-12. *Advanced mathematics knowledge* (AM) includes calculus, and abstract algebra which is related to the school algebra; and *teaching algebra knowledge* (TA) means typical errors, canonical uses of school math, and curriculum trajectories and so on. *Knowledge for teaching the concept of function* (KTCF) refers to the particular knowledge needed for teaching the concept, including the definition, representation, translation, operation of function and so on.

In addition, *Secondary School* in this study includes middle and high schools (i.e., grades 6 to 12). A more detailed explanation of the above definitions can be found in Chapter II.

Statement of Purpose

The purpose of this study is to examine pre-service teachers' *Knowledge for Teaching Algebra* (KTA) in China and the United States by using a mixed research method. In particular, I will compare teachers' KTA between these two countries at an item level and a structure level. At the item level, I mainly focused on mean differences. At the structure level, a SEM model (Structural Equation Model) was used to conduct a path model and measurement model analysis cross-culturally. In addition, teachers' *Knowledge for Teaching the Concept of Function* (KTCF) was investigated qualitatively.

Research Questions

The main purpose is to explore the characteristics of pre-service teacher knowledge for teaching algebra in China and the U.S. However, I also realize that the features of programs participants attended should have an impact on their performance of KTA.

Thus, the number of courses taken is selected as a key factor which may have direct effect on teachers' KTA. In particular, the study is aimed to answer the following research questions:

1. What are the differences and similarities of KTA between Chinese and U.S. pre-service teachers?
2. What are the relationships among different components of KTA within and between China and the U.S.?
3. What are the differences and similarities between Chinese and U.S. pre-service teachers' KTCF?
4. What are the relationships between pre-service teachers' status of KTA and their course- taking?

Delimitation

This study only examines pre-service teachers' knowledge for teaching algebra. Since algebra topics are mainly included in secondary (i.e., middle and high school) mathematics, I only focus on the population of pre-service secondary school teachers, not on elementary teachers.

CHAPTER II

LITERATURE REVIEW

It is a recent effort to measure mathematics teachers' knowledge needed for teaching, although there is more than 20 years of history in studying teachers' knowledge. Shulman's (1986) classification of subject matter knowledge, pedagogical knowledge, and curriculum knowledge laid the foundation for the study of teacher knowledge. Drawing on Shulman's framework, researchers have further refined and developed models to better describe and measure teacher knowledge needed for teaching (e.g., Ball et al., 2005; Krauss et al., 2008; Schmidt et al., 2007). This literature review consists of two sessions: teacher knowledge needed for teaching, and mathematics preparation of teachers in China and the U.S. In the first session, I reviewed the conceptualization of teacher knowledge for teaching in general and discussed the models for describing teacher knowledge for teaching mathematics. Then, I discussed relevant studies on teachers' knowledge needed for teaching algebra. In the second session, I analyzed the mathematics education systems and mathematics teacher preparation in China and the U.S., and summarized relevant studies on teachers' knowledge in mathematics in China and the U.S. Finally, grounded in the literature review, a framework for this study was proposed.

Knowledge Needed for Teaching

Great efforts have been made to seek what kind of knowledge a teacher needs to know in order to teach students effectively. In Shulman's (1986) seminal work on teacher's knowledge, he identified three categories, namely, *content knowledge, curriculum knowledge and pedagogical knowledge*. The first, *content knowledge* includes knowledge of the subject and its organizing structures. The teacher needs not only to understand *that* something is so; the teacher must further understand *why* it is so. The second category, *curricular knowledge*, is "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10). The third, *pedagogical content knowledge* (PCK) is described as follows:

> The most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the most useful ways of representing and formulating the subject that makes it comprehensible to others. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9)

Since Shulman(1986) coined the term PCK, many researchers have attempted to illustrate and clarify the nature of PCK and its implications for teacher education (e.g., Gess-Newsome, 1999). However, pedagogical content knowledge is often not clearly distinguished from other forms of teacher knowledge. For example, pedagogical content knowledge has been defined as "the intersection of knowledge of the subject with knowledge of teaching and learning" (Niess, 2005, p. 510) or as "that domain of teachers' knowledge that combines subject matter knowledge and knowledge of pedagogy" (Lowery, 2002, p. 69). Even a more careful description of PCK is still unclear as follows:

> Pedagogical content knowledge is a teacher's understandings of how to help students understand specific subject matter. It includes knowledge of how particular subject matter topics, problems, and issues can be organized, represented and adapted to the diverse interests and abilities of learners, and then presented for instruction. . The defining feature of pedagogical content knowledge is its conceptualization as the result of a *transformation* of knowledge from other domains (Magnusson, Krajcik, & Borko, 1999, p. 96).

According to the above definitions, PCK includes everything a teacher may need to know when teaching a particular topic, obscuring the differences between teacher action, belief, reasoning and knowledge.

Mathematics Teachers' Knowledge for Teaching

There is a widespread agreement that mathematics teachers need to have a deep understanding of mathematics (Ball, 1993; Grossman, Wilson, & Shulman, 1989; Ma, 1999). However, teachers' knowledge of mathematics alone is insufficient to support their attempts to teach mathematics effectively. In addition, in mathematics education, many studies defined PCK from different aspects. From example, Ball (1990) differentiated two dimensions of teachers' content knowledge: teachers' ability to execute an operation (division by a fraction) and their ability to represent that operation accurately for students. More recently, Ma (1999) described "profound understanding of fundamental mathematics" in terms of the connectedness, multiple perspectives, fundamental ideas, and longitudinal coherence. In addition, National Research Council [NRC] suggested that mathematics teachers need specialized knowledge that "includes an integrated knowledge of mathematics, knowledge of the development of students' mathematical understanding, and a repertoire of pedagogical practices that take into account the mathematics being taught and the students learning it."(Kilpatrick, Swafford, & Findell, 2001, p.428).

However, only in recent years, have researchers made efforts to conceptualize and measure particular mathematical knowledge that was considered pertinent and important for teaching (Ball & Bass, 2000; Ball et al., 2005). Furthermore, Hill and her colleagues further explored the relationship between mathematics knowledge needed for teaching and students' achievement (Hill et al., 2004), and classroom instruction (Hill et al., 2008). Growing attention was given to capture characteristics of teacher's knowledge needed for teaching specific content areas (e.g., Ball et al., 2005; Even, 1990, 1993; Ferrini-Mundy et al., 2006; Ma, 1999) which will be discussed in the following sections.

Researchers have attempted to understand *what* mathematical knowledge is entailed in teaching, how to assess it (Ball & Bass, 2000; Ball et al., 2005; Hill, Schilling, & Ball, 2004), and how to develop and refine ways to effectively promote mathematical knowledge for teaching (MKT) in teacher education and teacher professional development programs (NMAP, 2008; Stylianides & Stylianides, 2006). Ball and her colleagues have developed a specific framework describing mathematics knowledge for teaching (Ball et al., 2005). According to this model, subject matter knowledge is divided into two categories: *Common Content Knowledge* (CCK), which can be developed in anyone who has had school mathematics education, and *Specialized Content Knowledge* (SCK), which is used mainly by teachers. Meanwhile, the model makes a distinction between two main categories in pedagogical content knowledge: *Knowledge of Content and Students* (KCS) and *Knowledge of Content and Teaching* (KCT). This model highlights the kind of mathematical content knowledge that is the specialty of teachers, and recognizes that knowledge of mathematics for teaching is partially the product of content knowledge interacting with students in their learning processes and with teachers in their teaching practices.

Grounded in the concept of mathematics proficiency (Kilpatrick et al., 2001), Kilpatrick, Blume, and Allen (2006) proposed a framework for Mathematical Proficiency for Teaching. It suggests that *mathematical proficiency with content* (MPC) and *mathematical proficiency in teaching* (MPT) should be the main components for teachers to teach for mathematics proficiency. *The mathematical proficiency with content* (MPC) includes conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition, cultural and historical knowledge, knowledge of structure and conventions, and knowledge of connections within and outside the subject. The *mathematical proficiency in teaching* (MPT) consists of knowing students as learners, assessing one's teaching, selecting or constructing examples and tasks, understanding and translating across representations, understanding and using classroom discourse, knowing and using the curriculum, and knowing and using instructional tools and materials. This model basically illustrates Shulman's (1986) subject matter knowledge and pedagogical knowledge with a focus on mathematics proficiency.

Simon (2006) adopted the idea of a *Key Developmental Understanding* (KDU) in mathematics, namely, understanding a topic from multiple perspectives, building a well-structured knowledge web surrounding the topic as a way to think about understandings. KDUs are regarded as powerful springboards for learning and useful goals of mathematics instruction. Silverman and Thompson (2008) argued that developing MKT involves transforming these personal KDUs of a particular mathematical concept to an understanding of: (1) how this KDU could empower their students' learning of related ideas; (2) actions a teacher might take to support students' development of KDU and reasons why those actions might work. They further suggested a framework of mathematical knowledge for teaching as follows: A teacher has developed knowledge that supports conceptual understanding of a particular mathematical topic when he or she (1) has developed a KDU within which that topic exists, (2) has constructed models of the variety of ways students may understand the content, (3) has an image of how someone else might come to think of the mathematical idea in a similar way, (4) has an image of the kinds of activities and conversations about those activities that might support another person's development of a similar understanding of the mathematical idea, and (5) has an image of how students who have come to think about the mathematical idea in the specified way are empowered to learn related mathematical ideas. This framework opens up the possibility for the goal of mathematics teacher education to shift from positioning prospective teachers to develop particular MKT to developing professional practices that would support teachers' ability to continually develop MKT.

The previously described models enrich and/or extend Shulman's taxonomy with a focus on the mathematics subject. The third model even extends to include mathematics teachers' knowledge for professional development. In the next section, some specific research on teacher's knowledge needed for teaching algebra will be analyzed.

Teachers' Knowledge for Teaching Algebra

Algebra is an important part of school mathematics and is challenging for students to learn (NCTM, 2000; NMAP, 2008). Several researchers have proposed models of teachers' knowledge for teaching algebra (Artigue, Assude, Grugeon, & Lenfant, 2001; Even, 1990, 1993; Ferrini-Mundy et al., 2006; Li, 2007). Artigue and colleagues differentiated three dimensions of knowledge for teaching algebra as follows: (1) epistemological dimension; (2) cognitive dimension; and (3) didactic dimension. The *epistemological dimension* includes: (a) the complexity of the algebraic symbolic system and the difficulties of its historical development, and (b) how to flexibly use algebraic tools in solving different kinds of problems that are internal or external to the field of mathematics. The *cognitive dimension* deals with knowledge about learning processes in algebra, which includes knowing (a) the development of the student's algebraic thinking, and (b) students' interpretations of algebraic concepts and notations. The *didactic dimension* involves knowledge of (a) the algebra curriculum, and (b) the specific goals of algebraic teaching at a given grade, and so on.

Even (1990) identified and illustrated seven dimensions of subject matter knowledge based on an in-depth examination of function concept: (1) essential features, (2) different representations, (3) alternative ways of approaching, (4) the strength of the concept, (5) basic repertoire, (6) knowledge and understanding of a concept, and (7) knowledge about mathematics. *Essential features* refer to concept image by Vinner (1983) as the mental pictures of this concept, together with the set of properties associated with the concept (in the person's mind). It is crucial for teachers to judge if an instance belongs to a concept family by using an analytical judgment as opposed to a mere use of a prototypical judgment. It is necessary that teachers are able to correctly distinguish between concept examples and non-examples. *Different representations* give different insights which allow a better, deeper, more powerful and more complete understanding of a concept. When dealing with a mathematical concept in different representations, one may abstract the concept by grasping the common properties of the concept while ignoring the irrelevant characteristics that are imposed by the specific representation at hand. *Alternative ways of approaching the same concept* are used to deal with complex concepts in various forms, representations, labels and notations. *The strength of the concept* means the importance or power to open new possibilities, understand new concepts and capture the essence of the definition, as well as a more sophisticated formally mathematical knowledge. *Basic repertoire* includes powerful examples that illustrate important principles, properties, and theorems. The basic repertoire should be well known and familiar in order to be readily available for use. *Knowledge and understanding of a concept* means to achieve procedural proficiency and conceptual knowledge. The learning of a new concept or relationship implies the addition of a node or link to the existing cognition structure; thus making the whole more stable than before. *Knowledge about mathematics* includes knowledge about the nature of mathematics. This is a more general knowledge about a discipline which guides the construction and use of conceptual and procedural knowledge.

Comparing Artigue et al. (2001) and Evens' (1990) models, categories (1), (4), (6) and (7) of Even's category belong to the epistemological dimension while the others belong to the didactic dimension.

Ferrini-Mundy and her colleagues (2006) have developed a two-dimensional framework that describes mathematics knowledge for teaching algebra. In their model, the horizontal dimension indicates the fundamental *categories of knowledge* involved in teaching algebra, and the vertical dimension identifies several *tasks of teaching* in which teachers may apply their mathematical knowledge. The three overarching categories, *decompressing*, *trimming*, and *bridging*, are more sophisticated mathematical practices that utilize multiple elements of knowledge for teaching algebra and involve multiple tasks of teaching. *Categories of knowledge* include core content knowledge, representation, content trajectory, application and context, language and convention, and mathematical reasoning and proof. *Tasks of teaching* consist of analyzing students' work and thinking, designing, modifying and selecting mathematical tasks; establishing and revising mathematical goals for students; accessing and using tools and resources for teaching; explaining mathematical ideas and solving mathematical problems; building and supporting mathematical community and discourse. The framework illustrates the overall landscape of knowledge for teaching algebra: the major types of knowledge that may be used and contexts in which they may be used.

Flodden and McCrery (2007) have created a three dimensional construct, as illustrated in Figure 2.1 below, to guide the development of a measure of teachers' knowledge for teaching algebra.

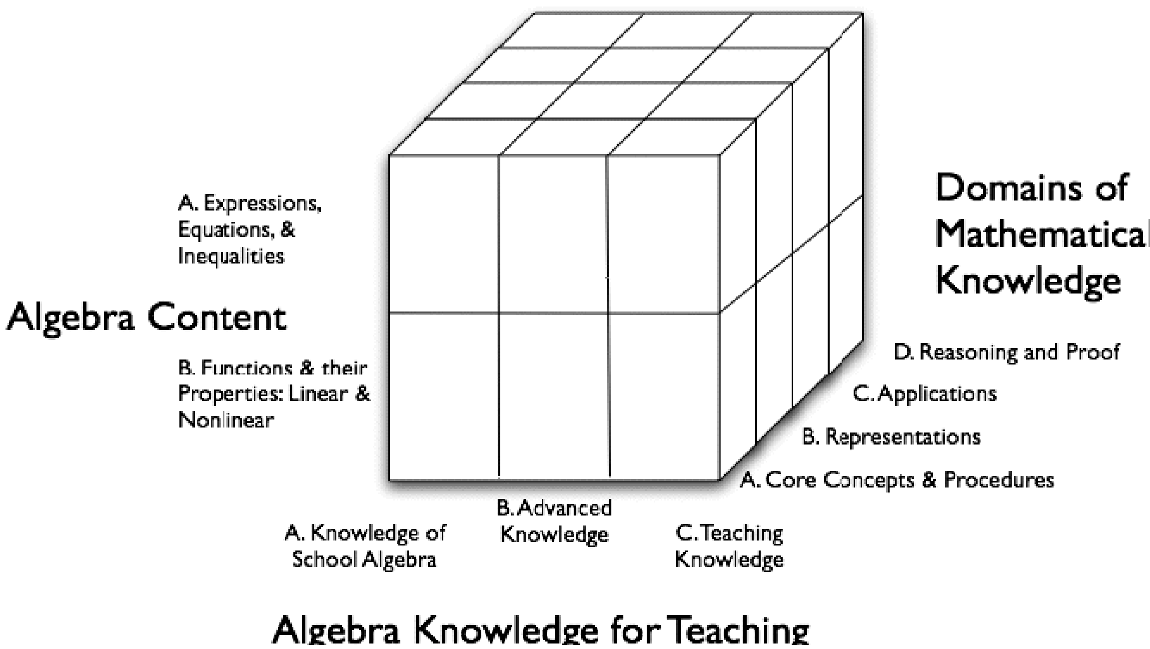

Figure 2.1. A framework for assessing knowledge for teaching algebra.

In this framework, the base of the matrix consists of three types of algebra knowledge for teaching including knowledge of school algebra, advanced algebra knowledge and teaching knowledge. *School Algebra Knowledge refers* to the algebra covered in the curriculum from K-12. *Advanced Algebra Knowledge* includes calculus, abstract algebra which is related to the school algebra. *Teaching Knowledge refers to* typical errors, canonical uses of school math, and curriculum trajectories and so on. There are four *domains of mathematical knowledge* or aspects of algebra teaching and learning (core concepts and procedures, representations, applications and contexts, and reasoning and proof on the Y-axis). The Z-axis contains two major themes in school *algebra content*: expressions, equations and inequalities, and functions and their properties. Assessment items can be specifically written for each cell in the matrix; for instance, knowledge of school algebra that is related to a core procedure for solving equations. Each assessment item would be uniquely located in Figure 2.1. as a coordinated system.

Based on the existing frameworks, Li (2007) reported a refined framework for investigating teachers' mathematical knowledge for teaching algebraic equation solving. His framework consists of three domains of knowledge: knowledge of the subject matter, knowledge of learners' conceptions and knowledge of didactic representations. *Knowledge of the subject matter* refers to mathematics subject matter as systems of established definitions, properties, facts, relations and connections; use of notations and representations; and methods for reasoning and problem solving. *Knowledge of learners' conceptions* includes the subject matter as understood by learners, including typical pre-conceptions, misconceptions, mistakes, questions, difficulties, strategies, reasoning, and factors that make a particular concept or procedure easy or hard.

Knowledge of didactic representations means that the subject matter is unpacked, linked, organized, and tailored through purposeful sequencing of topics and choices of examples, models, explanations, tasks, metaphors, and technological presentations.

Knowledge for Teaching Some Key Concepts in Algebra

Recent research has examined teachers' knowledge needed for teaching several important concepts in school algebra, such as function, expressions, and equations.

Teaching and Learning the Concept of Function

There have been different approaches to developing meaningful algebra (i.e., Bednarz, Kieran, & Lee, 1996; Hart, 1981; Usiskin, 1988). Usiskin summarized the following four approaches: (1) algebra as generalized arithmetic, (2) algebra as a study of procedures for solving certain kinds of problems, (3) algebra as the study of relationships among quantities, and (4) algebra as the study of structures. Learning algebra should include the following three core activities: generational, transformational and global/meta-level (Kieran, 2004). Function concept is one of the most important but difficult concepts across middle and high school levels (NCTM, 2000, 2006). Based on the theory of process-object duality of mathematics concept development (Sfard, 1991, 1992), a model of developing function concept is described as four stages: pre-function, action, process and object (Briedenbach, Dubinsky, Hawks, & Nichols, 1992). According to this mode, for *pre-function*, it means that the subject really does not display very much of a function concept. An *action*, is repeatable mental or physical manipulation of object, such a conception of function would involve, for example, the ability to plug numbers into an algebraic expression and calculate. A *process* involves a dynamic transformation of quantities according to some repeatable means that, given the same original quantity, will always produce the same transformed quantity. A function is conceived as of an *object*, if it is possible to perform action on it, in general actions that transform it.

In general, three representations are used for presenting functions: (1) geometrical representations including chart, graph, and histogram and so on; (2) numerical representations including numbers, table, and ordered number pairs and so on; and (3) algebraic representations including letter, formula, and mapping and so on (Verstappen, 1982). However, different representations play different roles in helping students understand the concept of function (Schwartz & Yerushalmy, 1992). For example, the algebraic representations benefit the understanding of function as a *process*; while the graphical representations help understand a function as an *object*. Moreover, some manipulations such as composition function performed on algebraic representations are easy to understand, while other manipulations transformations performed on graphical representations will be much easy to understand. Thus, it is critical to select appropriate representations with regard to different contexts.

There are quite a number of studies on teachers' knowledge for teaching the concept of function (e.g., Even, 1990, 1992, 1993, 1998; Norman, 1992, 1993). For example, based on a study on 10 secondary teachers' knowledge of the function concept, Norman (1992) found that the secondary teachers tended to have *inflexible images* of the concept of function that restricted their abilities to identify functions in unusual contexts and to shift among representations of functions. The sampled teachers were able to give formal definitions of function, distinguish functions from relations, and correctly identify whether or not a given situation was functional. However, the teachers did not show strong connections between their informal notions of function and formal definitions, and were not comfortable with generating contexts for functions. Hitt (1994) investigated 117 mathematics teachers' ideas on function and found that the teachers had difficulty in constructing functions that were not continuous or were defined by different algebraic rules on different parts of the domain. Consistent with Norman's findings, Chinnappan and Thomas (2001) found that all four pre-service secondary teachers in their study had a preference of thinking about function graphically, had a weak understanding of representational connections, and a limited ability to describe applications of functions.

Based on a survey of teachers' knowledge about function with 152 prospective secondary teachers, Even (1993) found that many prospective secondary teachers did not hold a modern conception of a function as univalent correspondence between two sets. These teachers tended to believe that functions are always represented by equations and that their graphs are well-behaved. None of the teachers had a reasonable explanation of the need for functions to be univalent and over-emphasized the procedure of the "vertical line test" without concern for understanding. Given the often weak and fragile understanding of secondary mathematics teachers about the concept of function, it is not surprising to find that the knowledge of an experienced 5th grade teacher was missing several key ideas (such as univalence and unclear notions of dependency) and lacked a notion of the connectivity among representations (Leinhardt, Zaslavsky, &Stein, 1990; Stein, Baxter, & Leinhardt, 1990).

In sum, teachers have difficulties in understanding the concept of functions as univalent correspondence between two sets (sometimes, it is not presented by a formula, or it is not of a discontinuous graph), (2) shifting different representations flexibly, and (3) relating formal function notion to contextual situation which produce the function.

Teaching and Learning Expressions and Equations

Expressions. An algebraic expression can be seen as a string of symbols, a computational process, or as a representation of a number. An expression can also become a function representing change if the context changes (Sfard & Linchevski, 1994). Sfard and Linchevski also clarify a potential issue in understanding algebraic expressions when they note "…the difficulty lies … in the necessity to imbue the symbolic formulae with the double meanings: that of computational procedures and that of the objects produced…. To those who are well versed in algebraic manipulation (teachers among them), it may soon become totally imperceptible" (pp.198-199). The duality (procedure vs. structure) of algebraic expression results in students' learning difficulty. For example, when knowing x=3, y=2， find out the value of 3x+y, a procedure perspective is adopted. While simplifying the expression of 3x+y+8x, it is necessary to adopt an object (structure) perspective. In addition, it was found that extrapolating some manipulation rules to some contexts inappropriately is a common mistake (Matz, 1982). For example, applying distribution law o(AnB)=oAnoB to an inappropriate situation: $\sin(\alpha+\beta)=\sin(\alpha)+\sin(\beta)$ or applying "$\frac{AX}{A}=X$" to "$\frac{AX+BY}{X+Y}=A+B$" are common mistakes. For another example, applying an known rule (such as if ab=0 then a=0 or b=0) to an unfamiliar situation: (X-A)(X-B)=K $\Rightarrow$(X-A)=K or (X-B)=K is also a common mistake.

Equation. An *equation* is a combination of letters, operations and an equal sign such that if numbers are substituted for the letters, either a true or false proposition results. Aspects of student difficulty within this topic are well documented in the literature on students' algebra knowledge (Booth, 1984; Kieran, 1992; Wagner, Rachlin, & Jensen, 1984; Wagner & Kieran, 1989).

First of all, it is not easy to understand the meaning of the sign "=". In algebra, equal sign "=" means equivalence of two algebraic expressions (equation), or presents one expression by another one (computation). Alibali, Knuth, Hattikudur, Mcneil and Stephens (2007) investigated 81 students at grades 6, 7 and 8 about their understanding of equal sign and equation for three years. They found that overall the students increased their understanding of the two concepts, but some students at grade 8 still did not understand the equal sign deeply.

Second, when solving equations, students face two challenges: the meaning of equal sign and the reverse computation relationship between addition and subtraction (Booth, 1984; Sfard & Linchevski, 1994). For example, Sfard and Linchevski (1994) found that students at the age of 14 and 15 were able to solve the equation of 7x+157=248, but failed to solve equation of 112＝12x+247. They attribute the students' difficulty to two issues: the position and meaning of equal sign "=", and the subtraction of a larger number from a smaller number. Moreover, even though students understood the reverse computation relationship between addition and subtraction, they still did not understand that the order of computation cannot be changed arbitrarily. For them, it is still a big challenge to solve equations including combination computation (Piaget & Moreau, 2001; Ronbing, Ninowski, & Gray, 2006).

Two Perspectives about the Concept of Function: A Case Study of Quadratic Function

Process-product dichotomy is a widely accepted theory of mathematics concept development (Briedenbach et al., 1992; Schwartz &Yerushalmy, 1992; Sfard, 1992). With regard to the development of the function concept, according to the process perspective, a function is perceived of as linking x and y values: for each value of x, the function has only one corresponding y value. On the other hand, the object perspective regards functions or relations and any of its representation as entities. For example, functions could be regarded algebraically as members of parameterized classes, or in the plane graphs could be thought of as being rotated or translated (Moschkovich, Schoenfeld, & Arcavi, 1993).

Researchers further gave more detailed descriptions and illustrations of these two perspectives (Breidenback et al., 1992; Even, 1990; Moschkovick et al., 1993; Schwartz

&Yerushalmy, 1992; Sfard, 1992). For example, Schwartz and Yerushalmy (1992) illustrated the process-product distinction as follows:

> Consider the two functions: x+3 and 4+x-1.
>
> From the point of view of the process that is carried out with the recipe, these are two different recipes. If, however, one was to plot the output of each of these recipes again its input on a Cartesian plane then the two recipes would be indistinguishable. We see that the symbolic representation of function makes its process nature salient, while the graphical representation suppresses the process nature of the function and thus helps to make the function more entity-like. A proper understanding of algebra requires that students be comfortable with both of these aspects of function (p.265).

Furthermore Moschkovick et al. (1993) not only extensively illustrated the distinction between the process and object perspectives, but also emphasized the importance of connection between these two perspectives, and flexibility in switching from different perspectives. They discussed multiple methods of solving the following question:

> Why is the graph of y=3x steeper than the graph of y=2x? What about y=4x, y=5x, y=10x? (p.83).

In one method, by considering the equation of line L: $y = mx + b$ and two points P (x_1, y_1) and Q (x_2, y_2), an algebraic formula was resulted in: $m = \frac{y_2 - y_2}{x_1 - x_2}$. If taking any two points on L whose x coordinates differ by 1, then $y_2 = y_1 + m$. If m is positive, the graph of L raises m units for each unit change in x. Thus, large (positive) m corresponds to steeper slope. This method basically adopted a process perspective.

In another method, letting L_1: $y = m_1 x$, and L_2: $y = m_2 x$ pass respectively through the points (1, m_1), and (1, m_2), hence $m_2 > m_1$, L_2 rise more steeply. In this solution, two aspects of both the process and object perspective were adopted in the algebraic and graphical representations. From the object perspective, the individual equations and lines are considered as members of the parametric family $\{y = mx : m \in R\}$. But using points on the graphs and determining their coordinates using the equations of the lines, employs the process perspective.

Flexibility in Learning the Concept of Function: A Case Study of Quadratic Function.

In this part, I review the meanings of flexibility in using representations, and summarized the studies on teachers' knowledge for teaching algebra with regard to the flexibility in using representations.

Flexibility in using representations. Learning algebra with understanding require students to "understand the meaning of equivalent forms of expressions, equations, inequalities, and relations' (NCTM, 2000, p. 296). In order to make that understanding occur, teachers have to organize a classroom discussion to open questions about the equivalence. For example, with regard to quadratic equations or function, students need opportunities to discuss questions across equations, expression and functions as follows:

When solving $3x^2 + 3x + 3 = 0$, I can think of the task as finding the zeros of the function y= $3x^2 + 3x + 3$ .In the context of finding zeros, I can divide 3 in the equation. However, when working with the function of y= $3x^2 + 3x + 3$, we cannot divide all coefficients by 3. Why is that? (Chazan & Yerushalemy, 2003, p.124)

In the *Focal Points from pre-K to Grade 8* (NCTM, 2006), students (grade 8) are suggested to use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems. Students are expected to

1. Recognize a proportion ($y/x = k$, or $y = kx$) as a special case of a linear equation of the form $y = mx + b$, understanding that the constant of proportionality (m) is the slope and the resulting graph is a line through the origin.
2. Understand that the slope (m) of a line is a constant rate of change, so if the input, or x-coordinate, changes by a specific amount, a, the output, or y-coordinate, changes by the amount ma.
3. Translate among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations).
4. Describe how such aspects of a function as slope and y-intercept appear in different representations. (p.20).

In the *Focus of High School Mathematics* (NCTM, 2009), making sense and reasoning is the core value of learning mathematics in general, and algebra in particular. It was suggested that key elements of reasoning and sense making with algebraic symbols should include the following:

1. *Meaningful use of symbols.* Choosing variables and constructing expressions and equations in context; interpreting the form of expressions and equations; manipulating expression so that interesting interpretations can be made.
2. *Mindful manipulation.* Connecting manipulation with the laws of arithmetic; anticipating the results of manipulations; choosing procedures purposefully in context; picturing calculations mentally.
3. *Reasoned solving.* Seeing solution steps as logical deductions about equality; interpreting solutions in context.
4. *Connecting algebra with geometry.* Representing geometric situations algebraically and algebraic situations geometrically; using connections in solving problems.
5. *Linking expressions and functions.* Using multiple algebraic representations to understand functions; working with function notation.

In order to develop algebra fluency, attention should be paid to interpret expressions both at formal level and as statements about real-world situations. At the outset, the reasons and justifications for forming and manipulating expressions should be major emphasis of instruction (Kaput et al., 2008). As comfort with expressions grows, constructing and interpreting them require less and less effort and gradually become almost subconscious. For example, students should know which is most useful for finding the maximum value of the quadratic function:

$$1\frac{3}{16}+18t-16t^2, (t-\frac{19}{16})(t+\frac{1}{16}), (t-\frac{9}{16})^2+\frac{100}{16}$$

Although multiple representations of functions—symbolic, graphical, numerical, and verbal—are commonly seen, the idea of multiple *algebraic* representations of functions is less commonly made explicit. Different but equivalent ways of writing the same function may reveal different properties of the function (as illustrated by the above example). Building fluency in working with algebraic notation that is grounded in reasoning and sense making will ensure “that students can flexibly apply the powerful tools of algebra in a variety of contexts both within and outside mathematics” (NCTM, 2009, p.37).

Function is one of the most important tools for helping students make sense of the world around them and prepare them for further study in mathematics as well. Students’ continuing development of the concept of function must be rooted in reasoning, and likewise functions are an important tool for reasoning. Key elements of reasoning and sense making with functions include the following (NCTM, 2009, p.41):

1. *Using multiple representations of functions.* Representing functions in various ways, including tabular, graphic, symbolic (explicit and recursive), visual, and verbal; making decisions about which representations are most helpful in problem-solving circumstances; and moving flexibly among those representations.
2. *Modeling by using families of functions.* Working to develop a reasonable mathematical model for a particular contextual situation by applying knowledge of the characteristic behaviors of different families of functions.

3. *Analyzing the effects of parameters.* Using a general representation of a function in a given family (e.g., the vertex form of a quadratic, $f(x) = a(x-h)^2 + k$ to analyze the effects of varying coefficients or other parameters; converting between different forms of functions (e.g., the standard form of a quadratic and its factored form) according to the requirements of the problem-solving situation (e.g., finding the vertex of a quadratic or finding its zeros).

In summary, these documents suggested that the following aspects are important in algebra learning, particular with the learning of function: (1) building the connection among expressions, equations/inequality, and functions; (2) flexible use of multiple representations of a function and shift among different representations, and (3) flexible use of multiple expressions of a function. As Star and Rittle-Johnson (2009) argued, "understanding in algebra can be considered to consist of two complementary capacities, which we refer to as between and within representation fluency. The first concerns the ability to operate fluently between and cross multiple representations, while the second is about facility within each individual representation" (p.11). Thus, it is critical to have a flexible and adaptive use of representations and expressions.

The representation flexibility should include the following abilities: (1) Having the necessary diagrammatic knowledge to interact with the representations (de Jong et al., 1998; Roth & Bowen, 2001); (2) Being able to coordinate the translation and switching between representations within the same domain (de Jong et al., 1998; Gagatsis & Shiakalli, 2004; Lesh, Post, & Behr, 1987); and (3) Having the necessary strategic knowledge and skills to choose the most appropriate representation for each occasion (Uesaka & Manalo, 2006).

With regard to the specification of the concept of function, it is necessary to consider flexibility in two aspects. One is the flexibility in selecting perspectives of function: process and object, and shifting between these two perspectives (Breidenbach et al., 1992; Dubinsky & Harel, 1992; Sfard, 1991). Another is the flexibility in using appropriate representations of functions: tabular, graphic, symbolic, and verbal representations, and shifts between them (Even, 1998; Moschkovick et al., 1993).

Teachers' knowledge of representational flexibility. Some studies found that teachers do not have the appropriate knowledge of using representation flexibly. To investigate prospective mathematics teachers' subject knowledge, Even (1998) reported her finding on teachers' knowledge of using representations.   For example, she presented the following questions:

> If you substitute 1 for x in expression $ax^2 + bx + c$ (a, b and c are real numbers), you get a positive number, while substituting 6 gives a negative number.   How many real solutions does the equation $ax^2 + bx + c = 0$ have?   Explain.

Only 14% of the 152 subject correctly solved the problem. These subjects considered the function corresponding to y=$ax^2 + bx + c$, switched representations, and either referred to a graph mentally or actually sketched a graph. Most of the subjects (about 80%) did not show any attempt to look at another representation of the problem, and did not solve the problem. A large number of subjects were stuck with manipulation of inequalities: $a+b+c>0 \quad 36a+6b+c<0$. They were not able to find correct answers.

In addition, she also found that the subjects who used a point-wise approach (e.g., process perspective) were more successful in solving problems that involved different representations of function than subjects who used a global approach (e.g., object). For example, in the following question:

This is the graph (Figure 2.2) of the function f(x)=ax^2+bx+c. State whether a, b, and c are positive, negative or zero. Explain your decision.

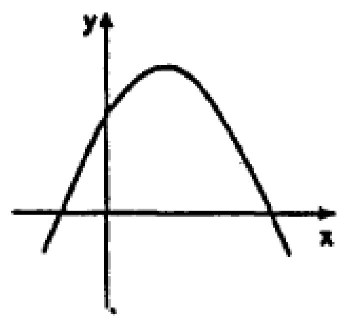

Figure 2.2. Graph of quadratic function

Only subjects who used a point-wise approach and looked at the y-intercept found correctly the sign of "c". They explained as follows: c->positive (when x=0, f(x)=c is positive).

In Black's (2007) study, he asked in-service high school mathematics teachers to explain their choice to following question to students. 22% of 67 participants got correct answers.

Mr. Seng's algebra class is studying the graph of $y = ax^2 + bx + c$ and how changing the parameters a, b, and c will cause different translations of the original graph (Figure 2.3).

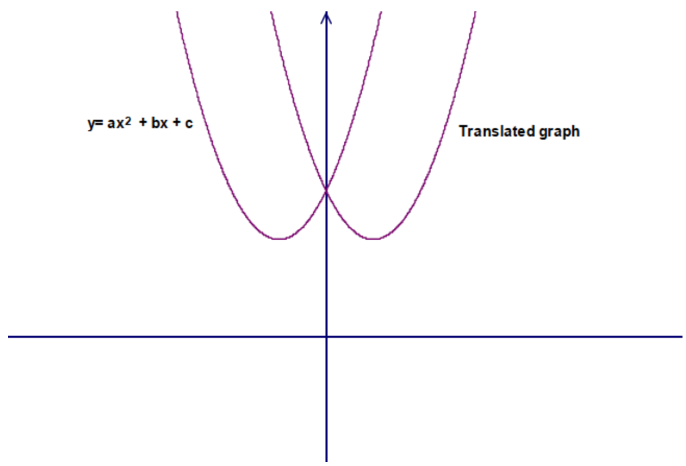

Figure 2.3. Graphs of original and translated quadratic functions

Which of the following is an appropriate explanation of the translation of the original graph $y = ax^2 + bx + c$ to the translated graph?

A. Only the **a** value changed. B. Only the **c** value changed.

C. Only the **b** value changed. D. At least two of the parameters hanged.

E. You cannot generate the translated graph by changing any of the parameters.

He concluded that the participant had "a lot of difficulty in both their answer selection, as well as in the explanations provided for those answers" (p.134). 19% the participants did not even attempt to answer the problem, and many more did not explain their answers. The finding seemed to suggest those mathematics teachers had difficulty in performing on function as an *object* (i.e., translation of graph) and shift between different representations (Using algebraic representation to explain graphic changes).

As argued by Moschkovick et al. (1993), "competence in the domain (i.e., line equation) consists of being able to move flexibly across representations and perspectives, where warranted: to be able to 'see' lines in the plane, in their algebraic form, or in tabular form, as objects when any of those perspective is useful, but also to switch to the process perspective, where that perspective is appropriate". (p.97)

Given the importance of developing students' flexibility in learning the concept of function, and the weakness of teachers' knowledge for teaching function promoting this flexibility, I focus on teachers' flexibility in shifting different perspectives (process vs. object), and selecting multiple representations (verbal, tabular, symbolic, and graphic). I will further illustrate the concept of flexibility in solving problems related to quadratic function in methodology part.

Mathematics Teacher Education in China and the U.S.

In this section, I briefly introduce teacher preparation systems in China and the U.S. providing a background for understanding teacher knowledge growth.

Mathematics Teacher Education in China

Since adopting the "nine-year compulsory education system" in 1986, teacher education has become a daunting task. Through approximately 20 years of effort, a three-staged process of “normal” education has been established and has made significant contribution to educating teachers from elementary to secondary levels in China. It means: (1) primary school teachers are trained in secondary normal schools; (2) junior high school teachers are trained in three-year teacher colleges; and (3) senior high school teachers are trained in four-year teacher colleges and normal universities. However, with the rapid development of the economy and technology in China, it has been an urgent agenda to upgrade and foster teachers’ quality. In order to meet this challenge, the Ministry of Education (1998) documented an action plan for revitalizing education in the 21st century. Two projects were launched; one is referred to as the “Gardener Project” and aims to establish continuing teacher education systems for practicing teachers. Meanwhile, the Ministry of Education (1999) enacted *a decision on deepening education reform and whole advancing quality education* in which comprehensive universities and Non-normal Universities were encouraged to engage in educating elementary and secondary teachers. This meant that the privilege of normal universities for teacher education changed. The Ministry of Education (2001a) put forward a process to improve an open teacher education system based on the existing Normal universities and supported by other universities, and to integrate prospective teacher preparation and practicing teachers’ professional development.

Through five years of development and research, teacher education has shown some changes:

1. Integration of education of prospective and practicing teachers;

2. Opening of teacher education in all qualified universities rather than just Normal universities;
3. Forming a new three-staged teacher education: where primary school teachers are trained in the three-year teacher colleges or four-year teacher colleges; the junior and senior high school teachers are trained in four-year teacher colleges and Normal universities, and some of the senior high school teachers are required to attain postgraduate level studies(Gu, 2006).

In 2004, there were more than 400 institutes conducting teacher education program and about 280 of them were teacher education universities or colleges. It was also found that one third of graduates who became teachers were from non-teacher education institutes (Yuan, 2004). This proportion has steadily increased in recent years. According to the *Educational Statistics Yearbook of China 2008* (Ministry of Education, 2009), there are only 188 normal institutions where preparing teachers at different levels is main purpose. In addition, there is now a flexible and encouraging accreditation system for teacher recruitment in China. University degree holders who wish to become school teachers and can pass some related examinations, usually pedagogy, psychology and subject didactics, in order to be a secondary school teacher.

Middle and high school mathematics typical were educated in mathematics department. Through analysis of the course design of mathematics department at a normal university; Li, Huang and Shin (2008) concluded that the secondary mathematics preparation program exhibits the following characteristics:

1. Providing prospective teachers with a foundation in profound mathematics knowledge and high advanced mathematics literacy
2. Emphasizing review and study of primary mathematics. It was believed that a profound understanding of primary mathematics and strong ability of solving problems in primary mathematics were crucial to being a qualified mathematics teacher at secondary schools. Due to the tradition of examination oriented teaching, a high level of problem solving ability is necessary for a qualified teacher;
3. Teaching practicum is limited. A six-week teaching practicum can only provide prospective teachers with a preliminary experience of teaching in secondary schools.

This reflects a belief that a solid mathematics base is vital for teacher preparation. Furthermore, higher mathematics courses are taken as a priority and privilege since prospective teachers will have less chance to learn them in their career lives. It is a main aim to foster prospective teachers with a bird's eye view of understanding elementary mathematics deeply rather than immediately connecting to what they will teach in schools; though there are special courses such as Modern Mathematics and School Mathematics, and Elementary Mathematics in Depth which connect higher mathematics to elementary mathematics. In contrast with the rigid requirement of mathematics, it is hoped that graduates learn teaching skills from their own practical teaching when they become teachers.

Mathematics Teacher Education in the U.S.

In the United States, there are three levels of pre-college education: elementary school(Grades K-5 or K-6); middle school or junior high school (grades 6-8 or 7-8); and high school (Grades 9-12). In 2007, forty-states had either a middle school or junior high school certification or endorsement requirement (National Middle School Association, 2007). Many of these states have special mathematics requirements for that certification or endorsements by the teachers' selected area of content expertise. In mathematics, these special requirements range from passing a test to completing the equivalent of an undergraduate minor in mathematics. The Conference Board of Mathematical Science [CBMS] (2001) recommendations call for the teaching of mathematics in middle school (grades 5-8) to be conducted by mathematics specialists; teachers specially educated to teach mathematic to the students of the grade levels they instruct. These teachers should have at least twenty-one semester hours in mathematics, including at least twelve semesters hours of fundamental ideas of mathematics appropriate for middles school teachers.

Middle school math teachers. Based on National NAEP survey (Smith, Arbaugh, & Fi, 2007), 85% of the nation's eight graders are taught by teachers who were certified by their state. When examined by teachers' degrees, 30% of the nation's eighth graders had teachers with an undergraduate degree in mathematics; 26 % had teachers with an undergraduate degree in mathematics education; and the remaining students were taught by a teacher with a degree in some other discipline. Thus, at least one-third of the nation's eighth-grade students were being taught mathematics by teachers without substantial mathematics training. According to National Report Card 2009 (National Assessment of Education Progress [NAEP], 2009), the situation gets worse: 27% of the nation's eighth graders had teachers with an undergraduate degree in mathematics; 30 % had teachers with an undergraduate degree in mathematics education. This is a major concern of the U.S. mathematics teacher education.

High school mathematics teachers. For high school mathematics teacher certification, states require from eighteen (in South Dakota) to forty-five (in California) semester hours of mathematics, equivalent to six to fifteen semester courses, or they require a major in the subject. When a number of credit hours of mathematics are specified for the certificate, almost half require thirty credit hours. The specific courses mentioned include three courses in calculus (two single-variable and on multivariable), linear algebra, geometry, and abstract algebra, plus a host of various electives.

The 2000 National Survey of Science and Mathematics Education (Whittington, 2002) was designed to identify trends in the areas of teacher background and experience, curriculum and instruction, and the availability and use of instructional resources. A total of 5,728 science and mathematics teachers in schools (1,367 of them were high school mathematics teachers) across the United States participated in this survey. According to this survey, 58% of mathematics teachers in grades 9-12 in their sample had an undergraduate major in mathematics, 22% had a degree in mathematics education, 10% had a degree in some other education field, and 10% had a degree in a field other than education or mathematics. In this sample, 96% of teachers had completed a course in calculus, 86 in probability and statistics, 83% in geometry, 82 % in linear algebra, 70 in advanced calculus, and 65% in differential equations and so on. The details are shown in Table 2.1.

Table 2.1
High School Mathematics Teachers Completing Various College Courses

Course	Percent of teachers
General methods of teaching	90
Methods of teaching mathematics	77
Supervised student teaching in mathematics	70
Instructional uses of computers/other technologies	43
Mathematics for middle school teachers	26
Geometry for elementary/middle school teachers	17
Calculus	96
Probability and statistics	86
Geometry	83
Linear algebra	82
College algebra/trigonometry/ elementary functions	80
Advanced calculus	70
Computer science course	68
Differential equations	65
Abstract algebra	65
Computer programming	62
Other upper division mathematics	60
Number theory	56
History of mathematics	41
Real analysis	38
Discrete mathematics	38
Applications of mathematics/ problem solving	37

The CBMS report (2001) recommends that high school teachers of mathematics have a major in mathematics; that includes a six-hour capstone course connecting their college mathematics course with high school mathematics. This recommendation stems from the view that teachers need to know the subject they will teach, they need to understand the broad range of the mathematical sciences their students will encounter in their careers, and they need to develop the habits of mind and dispositions towards doing mathematics that characterize effective workers in the field.

Studies on Teachers' Knowledge for Teaching in China and the U.S.

Thanks to Ma's (1999) work that revealed Chinese elementary teachers had a profound understanding of fundamental mathematics concepts in four areas: subtraction with regrouping, multi-digit multiplication, division by fractions, and the relationship between perimeter and area in contrast to U.S. counterparts, several studies have focused on mathematics teacher knowledge in China and the U.S.(An et al., 2004; Cai, 2005; Cai &Wang, 2006; Zhou, Peverly, & Xin, 2006). By comparing pedagogical content knowledge of middle mathematics teachers between the U.S. and China, An et al. (2004) found that the Chinese mathematics teachers emphasized gaining correct conceptual knowledge by relying on a more rigid development of procedures, while the United States teachers emphasized a variety of activities designed to promote creativity and inquiry in order to develop a concept mastery. In addition, a study comparing 162 U.S. and Chinese third grade mathematics teachers' expertise in teaching fractions (Zhou et al., 2006) found that Chinese teachers significantly outperformed their U.S. counterparts in subject matter knowledge, but they performed poorly in comparison to their U.S. counterparts on a test designed to measure general pedagogical knowledge. However, in pedagogical content knowledge there are no determinative patterns found.

Cai and his colleagues (Cai, 2000, 2005; Cai & Wang, 2006) have conducted a series of comparative studies on students' problem solving and teachers' construction of representations between China and the U.S. It was found that Chinese students preferred using symbolic representations while U.S. students tended to use pictorial representations (Cai, 1995, 2000). In addition, both Chinese and U.S. teachers used concrete representations for developing the concepts of ratio and average, but Chinese teachers tended to use symbolic representations for solving problem while U.S. teachers still preferred to use concrete representations when solving problems (Cai, 2005; Cai & Wang, 2006). Furthermore, Huang and Cai (2007) found that the U.S teachers tended to develop multiple representations simultaneously while the Chinese teachers tend to selectively use representations hierarchically.

These studies seem to suggest Chinese elementary mathematics teacher have a stronger subject matter knowledge, probably pedagogical content knowledge, compared with their U.S. counterparts. Meanwhile, Chinese mathematics teachers value symbolic representation more than U.S. mathematics teachers when solving problem.

However, there are no due comparative studies on teachers' knowledge needed for teaching special areas at middle and high school levels between China and the U.S. Also, teachers' representational flexibility which is closely related to teachers' beliefs and teaching has not been explored appropriately. Thus, in this study, we examined pre-service secondary school teachers' knowledge for teaching algebra with a focus on the concept of function and representation flexibility in China and the U.S. Therefore, the current study will contribute to our understanding of mathematics teacher's knowledge for teaching algebra in China and U.S. at middle and high schools and shed light on improvement of teacher preparations in China and the U.S.

Conclusion

This chapter provided a review of relevant literature, laying a theoretical foundation for this study. First of all, the development of the notion of teacher knowledge in general, and ways of defining and measuring mathematics teacher knowledge needed for teaching in particular were summarized. Second, the specific frameworks for studying mathematics knowledge for teaching algebra were analyzed and compared. Third, relevant studies on algebra teaching and learning, and teacher knowledge needed for teaching algebra were summarized. Fourth, the literature review focused on teacher knowledge for teaching the concept of function promoting flexibility in adapting appropriate perspectives and representations of function. Fifth, a brief summary of mathematics teacher preparation in China and the U.S. was presented. Finally, some comparative studies on teachers' knowledge for teaching between China and the U.S. are summarized.

CHAPTER III

METHODOLOGY

This study compared the characteristics of mathematics knowledge needed for teaching algebra between China and the United States. An embedded mixed methods design was adapted (Creswell & Clark, 2007). A design in which the main data set consists of written answers to a questionnaire which includes multiple choice items and open-ended items, while the supportive data set is comprised of the written answers to the open-ended items and follow up interviews. Its primary purpose was to compare the status and structure of teacher knowledge for teaching algebra through quantitatively analyzing the participants' performance in the KTA survey between the two countries. The second purpose was to further illustrate the similarities and differences in KTA through qualitatively analyzing the answers to the open-ended questions and follow up interviews which focus on the core concept of function. Based on the questionnaire of KTA by Floden and McCrory (2007), I developed an instrument for measuring teachers' knowledge needed for teaching algebra, with a focus on the concept of function. Then, completed questionnaires were collected from 376 pre-service Chinese mathematics teachers from five purposefully selected teachers' training institutions. At the same time, I also collected data from 115 U.S. pre-service teachers who were preparing to be mathematics and science teachers at the middle school level from a well respected university in the south of the United States. All the Chinese and U.S. data were scored and quantified as SPPS data set, and then the data set was analyzed through techniques including multiple comparisons and SEM model to answer the research questions. A qualitative analysis of the answers to the open-ended items was conducted to identify the strategies used and the flexibility in solving these problems. This chapter is organized into three parts. First, the process of developing a Chinese instrument based on the existing U.S. questionnaire is described in detail. Next, the ways of data collection are described. Finally, the methods of data analysis are depicted.

Instrumentation

The translation equivalence and cultural adaptation of instruments in international comparative studies is an issue which has to be dealt with carefully and appropriately. For example, the TIMSS technical report (Chrostowski & Malak, 2004) suggested that translators should consider the following aspects when translating instruments: (1) identifying and minimizing cultural differences, (2) finding equivalent words and phrases, (3) ensuring that the reading level was the same in the target language as in the original international version, (4) ensuring that the essential meaning of the text did not change, (5) ensuring that the difficulty level of achievement items did not change, and (6) making changes in the instrument layout required due to translation. Both TIMSS and PISA (Organization for Economic Co-operation and Development [OECD], 2006) suggested adopting a double translation procedure (*i.e.* two independent translations from the source language, with reconciliation by a third person). This strategy offers two significant advantages when compared with the back translation procedure: (1) Equivalence of the source and target languages is obtained by using three different people (two translators and a reconciler) who all work on the source and the target versions; and (2) Discrepancies are recorded directly in the target language (OECD, 2006).

With regard to the adaptation of instruments of mathematics knowledge for teaching to different cultures, Delaney et al. (2008) highlighted several critical issues. These points include: (1) what teachers do during mathematics lessons, (2) teachers' conceptions about mathematics and about mathematics teaching, (3) the classroom contexts in which the knowledge is used, (4) differences in the types and sophistication of the explanations of students mistakes, (5) responding to student errors, (6) the presence and prevalence of specific mathematical topics, (7) the mathematical language used in the school, and (8) the content of the textbooks. They further suggested making relevant changes in the following four categories: the general cultural context related, the school cultural context related, mathematical substance related, and others.

In this study, I dealt with these issues using the following strategies: (1) content appropriateness in different cultures, and (2) equivalence of the instrument translation.

Content Appropriateness

A research team at Michigan State University has developed an instrument for measuring mathematics knowledge for teaching algebra for several years. The validity and reliability of the instrument has been tested in the United States context (Floden et al., 2009). In order to adapt this U.S. instrument into Chinese context, I first translated it into Chinese and invited three Chinese mathematicians and mathematics educators who are bilingual scholars to scrutinize the instrument and provide supplementary items if needed. Since the function concept is one of core concepts in algebra and is difficult for students to learn and for teachers to teach (Kieran, 2007; Even, 1990, 1993), I decided to include some open-ended items related to teacher knowledge needed for teaching the concept. Based on the study of the mathematics curriculum standards in China and the U.S., an extensive literature review on mathematics knowledge for teaching the concept of function, and consideration of the Chinese mathematicians and mathematics educators' suggestions, five open-ended questions were adapted or designed. These open-ended items were reviewed by two mathematics educators and one mathematician at the sample U.S. University. One of the mathematics educators has had extensive teaching experience at secondary schools (including middle and high schools) and universities in the U.S. While the other has had secondary mathematics teaching experience in China and several years of experience as a faculty member in math education programs in China and the United States. These items were judged appropriately for secondary (i.e., middle and high) mathematics teachers. Both of them examined these items carefully and improved the wording of some items. The mathematician also gave very detailed comments and corrections. Based on the feedback from them, I made a final English version.

Translation Equivalence

The English instrument was translated into Chinese by the researcher and another Ph.D. candidate in mathematics education from China. A third bilingual mathematics educator at the sample university compared these two Chinese versions and made a final version through discussions with the researcher.

Appropriateness of the Survey from Teachers' Perspectives

In order to understand teachers' perceptions of the questionnaire, we invited ten pre-service teachers in the last year of their four year bachelor degree program in secondary mathematics education from an eastern city of China and four in-service teachers from a secondary school in a northern city of China (two middle school teachers, and two high school teachers) to complete this survey within 45 minutes. Based on their written answer sheets, I identified several items which were answered incorrectly by some teachers for further interview inquiry.

I interviewed three of the four in-service teachers who completed this survey through a video conference system. One of them is a high school teacher with a senior position (Equivalent to an associate professor at universities in the U.S.). The others are middle school mathematics teachers. Both of them had more than six years of teaching experience. The interview was aimed at understanding participants' opinions about the overall difficulty, the appropriateness of items, and their interpretation of the identified mistakes.

These teachers expressed the following opinions in the interviews. First, overall the questionnaire is relatively easy, but the items were flexible, contextual, and covered broad topics. Second, they felt that the questions that were closely related to the advanced mathematics topics were the most difficult ones. Third, in general, the expression and background of problems were clear and easily understandable, but sometimes the problems were so novel that they had to be cautious in order to ensure that they correctly understood them. Fourth, the questionnaire was a little long to complete within 45 minutes. In addition, they believed the questionnaire could measure a teachers' knowledge for teaching algebra. Finally, they provided some suggestions for improving the questionnaire, including adding an introduction to the purpose of the questionnaire and refining the format of the questionnaire.

Based on the interview results, I have made a relevant improvement in the Chinese questionnaire which includes 25 items. 20 items, including 17 multiple choice items and 3 open ended items, were translated from the original English questionnaire, and an extra 5 open-ended items were created by the researcher. The format of the questionnaire was compacted from the original 14 pages to 8 pages. This includes the introductory information and 25 items. In addition, some specific terms which are easily misunderstood were highlighted with bold font. For instance, in item 10, large and bold words were used to emphasize consider "All lines in the Coordinate Plane". In Item 11, large and bold words were used to highlight to select the "NOT appropriate" one.

Finally, a questionnaire booklet was developed in this study. It includes two parts. The first part includes 17 multiple choice items and 8 open-ended items (items 18 to 25) with a focus on teachers' knowledge for teaching the concept of function. The second part is an answer booklet, including participants' backgrounds such as current grade and courses taking, a multiple choice answer table, and open-ended question answer sheets.

Three U.S. students who were studying in a Ph.D program in math education were invited to complete this survey within 45 minutes. This small pilot showed that the time for completing the survey was appropriate and the participants were able to understand and answer these questions. Thus, no further revision of the questionnaire appeared necessary. In the next section, justification of adaptation of the open-end questions will be given.

Measuring Knowledge for Teaching the Concept of Function

Based on an extensive literature review of teachers' knowledge for teaching the concept of function, we focused on two aspects of the concept of function: fluency and flexibility of knowledge for teaching function in general and for teaching quadratic functions in particular. With regard to the first aspect, I focused on the understanding of the concept of function in terms of shifting between two perspectives (*process vs. object*) appropriately. In addition to an original item 18 that required a process perspective to effectively answer the questions, I created two items. One item (item 24) that required using an object perspective in order to effectively complete the proof, and another (item 25), in which it is necessary to adopt the connection of the two perspectives in order to appropriately answer the item.

In order to measure teachers' knowledge for teaching quadratic functions, I focused on the fluency and flexibility of using different representations. Four items are used to gauge teacher knowledge for teaching the particular content field. Item 19 from the original questionnaire is used to measure knowledge of solving quadratic inequality by using algebraic and graphic representations. Item 21 is used to measure teacher knowledge in flexible use of algebraic and graphic representations and the connection of function, equation and inequality. Item 22 is used to measure the flexible use of multiple representations quadratic function (graphic and algebraic) and translation of graphs. Item 23 is designed to use multiple forms of algebraic representations and translations between different representations. These items are shown as follows:

18. a) On a test a student marked both of the following as non-functions

(i) f: $\boldsymbol{R} \to \boldsymbol{R}$, $f(x) = 4$, where $\boldsymbol{R}$ is the set of all the real numbers.

(ii) $g(x) = x$ if x is a rational number, and $g(x) = 0$ if x is an irrational number.

For each of (i) and (ii) above, decide whether the relation is a function, and write your answer in the Answer Booklet.

b) If you think the student was wrong to mark (i) or (ii) as a non-function, decide what he or she might have been thinking that could cause the mistake(s). Write your answer in the Answer Booklet.

(Adapted from Even (1993))

19. Solve the inequality $(x - 3)(x + 4) > 0$ in **two essentially** different ways. Show your work in the Answer Booklet.

21. If you substitute 1 for x in expression $ax^2 + bx + c$ (a, b and c are real numbers), you get a positive number, while substituting 6 gives a negative number. How many real solutions does the equation $ax^2 + bx + c = 0$ have?

One student gives the following answer:

According to the given conditions, we can obtain the following in-equations:

$a + b + c > 0$, and $36a + 6b + c < 0$.

Since it is impossible to find fixed values of a, b and c based on previous inequality, the original question is not solvable.

What do you think may be the reason for the students' answers? What are your suggestions to the student?

Write down your answers in as much detail as possible on your Answer Booklet.

(Adapted from Even (1998))

22. This item is adapted from Black (2007)) (See p. 33)

23. Given quadratic function $y = ax^2 + bx + c$ intersects x-axis at (-1, 0) and (3, 0), and its y-intercept is 6. Find the maximum of the quadratic function.

Show your work in as much detail as possible in the Answer Booklet.

(Adapted from NCTM (2009)

24. Prove the following statement:

If the graphs of linear functions $f(x) = ax + b$ and $g(x) = cx + d$ intersect at a point P on the x-axis, the graph of their sum function $(f + g)(x)$ must also go through P.

Show your work in the Answer Booklet.

25. When introducing the functions and the graphs in a middle school class (14-15 year-olds), tasks were used which consist of drawing graphs based on a set of pairs of numbers contextualized in a situation and from equations? One day, when starting the class, the following graph (Figure 3.1) was drawn on the blackboard and the pupils were asked to find a situation to which it might possibly correspond.

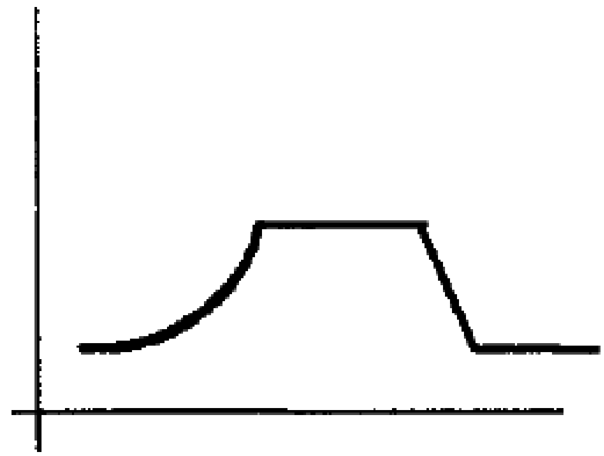

Figure 3.1. A graph presenting a daily life situation.

One student answered: ‘it may be the path of an excursion during which we had to climb up a hillside, the walk along a flat stretch and then climb down a slope and finally go across another flat stretch before finishing.’

How could you answer this student’s comments? What do you think may be the cause of this comment? Can you give any other explanations of this graph?

Write down your answers in as much detail as possible on your Answer Booklet.

(Adapted from Llinares (2000))

In sum, all the items and corresponding content areas, and types of knowledge are listed in Table 3.1.

Table 3.1
Items and Corresponding Content Areas and Knowledge Types

Item	Content area	Knowledge type
1	Expression	School mathematics
2	Equation	Teaching
3	Function	School mathematics
4	Function	Advanced mathematics
5	Equation	Teaching
6	Equation and expression	School Math
7	Function	Teaching
8	Function	Advanced math
9	Equation and expression	Advanced math
10	Function	Teaching
11	Function	Teaching
12	Equation and expression	Advanced math
13	Equation and expression	Advanced math
14	Equation	School math
15	Equation	Teaching
16	Function	Advanced math
17	Function	School math
18	Function	Teaching
19	Function	School math
20	Function	Advanced math
21	Equation and expression	Teaching
22	Function	Teaching
23	Function and equation	School math
24	Function and equation	Advanced math
25	Function	Teaching

Data Collection

In the following sections, I describe the process of the subject recruitment and data collection.

Chinese Data Collection

In China, there is only one type of secondary mathematics teacher preparation programs (including middle and high school teachers). There is no specific program for preparing middle school math teachers. These programs either are provided by normal universities or comprehensive universities. Usually, these programs are housed in a mathematics department (Li et al., 2008).

First, I discussed the selection of representative universities in China with a professor in a leading teacher education university in China. This professor was the former president of National Higher Teacher Education University Association. I raised two criteria: (1) the score of university entrance examination, and (2) the representativeness of different existing programs. With the help of the professor (and my personal connections with teacher education universities), I contacted math educators from seven Universities (Based on the educational institution list league in China in 2009, two belong Rank 1 (top 10), two belong Rank 2 (top 20), three belong Rank 3 (after 30)) through e-mails and phone contact. I explained the research purposes and the requirement of administrating the survey to them. First, participating students were required to have 45 minutes to complete a questionnaire. Second, ideally, about 60 junior and 60 senior students needed to be recruited to complete the survey. All target coordinators promised to help collect the data from their respective universities. I sent the questionnaire to respective coordinators from the seven universities (with written instructions of conducting the survey) in early spring 2010. I asked them to explain the survey to their students as part of their course work and request that the students complete the survey seriously and honestly within 45 minutes.

However, due to some physical difficulties (for example, in some universities, senior students were in the process of teaching practice, while in other universities, junior students were in the process of teaching practice), not all of them met the deadline and requirement. Two universities were not able to collect their data in time (one is rank 1 and the other is rank 3). One university only collected junior students' data while another university only collected senior students' data. All the completed questionnaires were sent to a professor in Shanghai, and the professor helped to scan the completed questionnaires into PDF files and e-mail them to me. I printed the questionnaires in March, 2010. Finally, 376 completed questionnaires were used for data analysis after excluding 8 copies from universities 3 and 4, and copies from university 5 due to lack of background information. The distribution of the competed questions is shown in Table 3.2.

Table 3. 2.

Demographic Information of the Chinese Participants

University	Code	Junior	Senior
Rank 1 University	1	59	50
Rank 2 University	2	71	
Rank 3 University	3	33	15
Rank 3 University	4	48	52
Rank 2 University	5		48

Note. According to the teacher education institution in China in 2009, teacher education institutions are ranked into different status; rank 1 (top 10) is the highest, followed by rank 2 (around top 20), and rank 3 (after top 30).

U.S. Data Collection

In the U.S., usually, high school mathematics teachers need to earn a bachelor's degree in math, with certain required credits in math education. However, there are different routes for training middle school math teachers. The first one is preparing middle school teachers as a part of the preparation of secondary school teachers. The second one is specifically preparing middle school teachers (i.e., math and science inter-discipline approach). The third is preparing middle school teachers as an extension of the preparation of elementary teacher (Dossey, Halvorsen, & McCrone, 2008; Schmidt et al., 2007).

I contacted three instructors who taught the mathematics education courses for junior and senior students at a large public school in the Southern United States. I explained the research project and requested their assistance to administrate my survey during part of their class duration (around 45 minutes). All of them allowed me to conduct the survey using their class.

Students were told that their participation in the survey is fully voluntary. Instructors introduced me to their students and allowed me to briefly introduce my research project. After briefing my research purpose and appreciating students' participation in this survey, I delivered the questionnaires to students and then collected the completed questionnaires after 45 minutes. All of the students who attended those classes completed the questionnaires. Finally, I collected 115 copies of questionnaires from the three classes. The demographic information of the U.S. sample is displayed in Table 3.3

Table 3.3
Demographic Information of the U.S. Participants

Program	Soph.	Junior	Senior	Total
Gr.4-8	11	48	31	90
Gr.6-12	0	5	4	9
Other	3	11	2	16
Total	14	64	37	115

Note. Gr. 4-8 presents the program prepared for science and mathematics teachers from grade 4 to 8. Gr.6-12 presents the program prepared for science and mathematics teachers from grade 6 to 12. Other refers certificate for teaching math and science at middle school.

The Table showed that the majority (79%) of the participants registered in the interdisciplinary program of math and science teachers at grades 4 to 8, while very few (7%) studied for programs of math and science teachers at grades 6 to 12. The remaining small part (14%) just took some courses for a certificate in teaching math or science at middle school. Among the participants, the majority (87%) was junior and senior students; only small proportions were sophomore students.

Interview of the Selected U.S. Participants

Originally, I intended to interview some participants from China and the U.S. to clarify participants' answers and probe their thoughts. In addition to their free explanations, the interviewees were intended to be also probed uniformly and non-uniformly. The uniform probes were presented to all subjects and were based on the analysis of pilot study and corresponding survey with in-service teachers. These probes represented themes that appeared in many of the written answers (such as mistakes in solving inequality, mistakes in explaining graphs). The non-uniform probing was based on the specific answers each subject gave to the questionnaire and was meant to clarify ambiguous answers and discover specific dimensions that seemed important.

However, based on a preliminary analysis of completed questionnaires from the U.S. and China, I found that Chinese participants provided very detailed and rich responses to the open-ended questions for us to analyze their thoughts and strategies. On the other hand, the U.S. participants provided relatively short and simple responses to the open-ended questions. So, I decided only conduct an interview with purposely selected U.S. participants.

Based on a detailed analysis of the answers of the participants from a class, I identified eight potential interviewees in terms of their performance such as typical correct answers and mistakes. Five of them agreed to attend an interview. The interview was conducted individually during the week after completing the survey. Each interview lasted about 20 minutes, and was audio recorded.

These interviewees were studying in the interdisciplinary program of middle school mathematics and science teacher preparation. We also collected information about the high school mathematics course taking (5 courses include: (i) Algebra I, (ii)Algebra II, (iii) Geometry, (iv)Pre-Calculus, and (v) Calculus) and college mathematics and mathematics education course taking. Except for one (Kerri, all names in the dissertation are fictitious) who took four high school mathematics courses, the others took five courses. The college courses included the following 17 courses:

(1)Structure of mathematics I , (2) Structure of mathematics II, (3)Basic concept of geometry, (4) Introduction to abstract mathematics, (5)Integration of mathematics and technology, (6)Problem solving in mathematics , (7)Integrated math , (8)Mathematics methods in middle, (9)Student teaching, (10)Freshman mathematics laboratory, (11) Analytic geometry and calculus, (12) Calculus, (13)Foundation of discrete mathematics, (14)Several variable calculus, (15)Liner algebra I , (16)Differential equations , and (17) Advanced calculus I. The courses taken are summarized in Table 3.4.

Table 3.4
Courses Taken by the U.S. Interviewees

Name	High school course taking	College courses taken	College courses being taking in the semester
Larry	(i)-(v)	(1)-(3),(6),(11),(12)	(4)-(7)
Jenny	(i)-(v)	(1)-(3),(6)	(4),(5),(7)
Kerri	(i)-(iv)	(1)-(3),(6),(10)-(12)	(4),(5),(7)
Alisa	(i)-(v)	(1)-(6)	(7)
Stacy	(i)-(v)	(1)-(4),(6),(10)-(12)	(5),(7)

Thus, all of the interviewees took at least four mathematics courses at high school and averaged 9 courses taken in college.

I designed some particular questions for each of the open-ended items. For example, in Item 18, I designed the following prompt questions: (1) How do you judge whether a relationship is a function or not? (2) What is the vertical line test? (3) What would you teach to your students? Can you give me an example?

For Item 19, I found some common mistakes, then I not only asked some general questions such as (1) When reading solving the inequality, what knowledge, skills and methods come to your mind? (2) What's your understanding about two essentially different ways? (Algebraic or Geometric methods?) (3) What do you think about the following operations?

A. (x-3)(x+4)>0-→ x-3>0, x+4>0, then x>3 and x>-4.

B. $x^2+x-12>0$; $x(x+1)>12$; x>12, x+1>12;

C. $x^2+x>-12$; $x^2>x-12$; $x>\sqrt{x-12}$;

In addition, I also probed whether participants can recall graphing method in solving quadratic equation or inequality.

For Item 20, I asked the following questions: Someone answers "Yes" and gives proof as follow:

$$\text{When } A=\begin{bmatrix} p & q \\ r & s \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ B=\begin{bmatrix} t & u \\ v & w \end{bmatrix} \text{then, } A\Delta B=\begin{bmatrix} 0t & 0u \\ 0v & 0w \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

What do you think about this "proof?"

For Item 21, I asked the following questions: (1) What are the reasons for the student to make his/her judgment? ; (2) To find solutions, are there other methods you can suggest to the student?

The prompt questions for Item 22 included: (1) What are the effects of change of parameters of a, b, c on the graph? (2) What algebraic manipulations may help you identify the key parameter(s)?

The prompt questions for Item 23 included (1) Which formula of quadratic function did you choose for finding the function? ;(2) How can you find the maximum of a given quadratic function?

The prompt questions for question 24 included: (1) What does it mean by intersecting at a point on x-axis? (2) What is the meaning of (f+g)(x)?

The probing questions for Item 25 were: (1) What are the missing parts of students' comment (two variables, X vs. Y); (2) How can you explain other real life situations by using this graph?

Data Analysis

The data analysis includes three phases: (1) quantifying the data: developing five level rubrics for quantifying the open-ended items; (2) analyzing KTA at item and structure levels; and (3) analyzing open-ended item qualitatively: focusing on problem solving methods or mistakes, and flexibility of using representations.

Quantifying the Data

For each multiple choice item (items 1 to 17), the correct choice was scored as 1, while wrong choice was scored as 0. For each open-ended item, I developed a five level rubric for scoring the answers from 0 to 4. For items 18, 19, 20 and 24, I adapted the rubrics from the original rubrics developed by Michigan State University with some modifications (treating blank and missing answer as 0) and specifications (adding some details). For example, for item 18, I developed the follow rubric:

Table 3. 5

Rubric for Coding Item 18

Score	Description
0	Blank or total wrong answers in (i) and (ii)
1	(I): (a) answer (i) is function, (ii) is not or inverse (b) explanation is missing or wrong OR (II): (a) answer (i) and (ii) are not function, but (b) give some relevant explanations.
2	(I): (i) (a) is correct: (i) and (ii) are function. (b) without explanation or giving wrong explanation Or (II): (a) one of (i) and (ii) is function, (b) give an correct explanation
3	To give the answers with the following elements: (a) Point out (i) and (ii) are functions ;(b) The explanations do not relate to the key element (multiple to one or one to one), rather some superficial features such as: the function (i) with constant value, and the function (ii) is not continuous or expressed by two expressions or there are many holes.
4	To give answers with the following elements: (a)Point out (i) and (ii) are functions ; (b)Point out that there is only one unique value corresponding to each value from domain value (such as one x value corresponds one y value, multiple x values correspond to one y value, but does not include one x value corresponds multiple y values). Or point out the use of the vertical line test.

Based on the study of existing rubrics, I developed a general criterion for scoring all open ended items as follows:

0-Blank or providing useless statements;

1-Providing several useful statements without a chain of reasoning for the correct answers;

2-Giving a correct answer but the explanations or procedures with major conceptual mistakes;

3-Giving a correct answer and appropriate explanations or procedures, with some minor mistakes;

4- Giving a correct answer with appropriate explanations and procedures.

Furthermore, based on the specification of each open-ended item, I further developed different rubrics for all of the open-ended items (See Appendix **A**). For example, the rubric for item 22 is described in Table 3. 6.

Table 3.6

Rubric for Coding Item 22

Score	Examples
0	Blank or useless statements
1	Gives partly features of graph when changing a, b, or c.
2	Selects C or D and gives some explanations, with some serious mistakes, such as if *a* is changed then the graph is moved up or down.
3	Gives answer C. However, reasons is not appropriately explained such as only mentioning the invariance of a or c.
4	Selects C and provides correct explanations such as: • Since change of *a* leads change of the openness, thus *a* is not changed; since y-intercept is not changed, so c is not changed. Thus, it is only possible to change b. • The translated graph is the symmetrical graph of original graph with regard to y-axis. So, symmetrical line $x = -\frac{b}{2a}$ should be changed. However, the openness of the graph is not changed, so *a* should be invariant. Thus, only b is changed to –b. • If f(x) and g(x) are symmetrical with regard to y-axis, then g(x)=f(-x), thus b is changed to –b.

Inter-Rater Reliability

Based on a preliminary examination of the open-items of 20 copies of U.S. questionnaires and 20 copies of Chinese questionnaires, I developed and tested the appropriateness of the rubrics. Then, I fully applied the finalized rubrics to the coding of the U.S. questionnaire.

After that, I and another secondary mathematics teacher scored the open-ended items of 109 copies of Chinese questionnaires from a high reputation institution separately. The inter-reliabilities of the items are 97% for item 18, 94% for item 19, 97% for item 20, 95% for item 21, 93% for item 22, 98 % for item 23, 86% for item 24, and 93 % for item 25. The disagreements were solved through discussing between raters and specifying the rubrics. The second mathematics teacher scored all of the remaining questionnaires. I double checked the codes of U.S. questionnaires, and 100 copies from the remaining Chinese questionnaires. The agreement was higher than 95%, and I made relevant corrections.

Developing Categories of Different Strategies of Solving Open-ended Items

There are a total of eight open-ended items. One of them is related to metric and logical inference (Item 20); Three of the items are related to function concepts in general (Items 18, 24, 25), while four of them are related to quadratic functions /equations /inequalities (Items 19, 21, 22, 23) in particular. For the metric item, the analysis was focused on the logical equivalence and metric operations.

A two dimensional framework was developed (see Table 3.7) for analyzing knowledge for teaching function concept. One aspect presents the perspectives of function concept (process vs. object), and the other presents different representations (verbal, tabular, algebraic, and graphical).

Table 3.7
A Framework for Investigating Alternative Perspectives of Function in Typical Representations

Perspective	Verbal	Tabular	Algebraic	Graphical
Process				
Object				

At the beginning, I tried to apply this two dimension framework to analyze all of the open-ended items. However, that attempt was found to be too complicated to implement. Then, I applied dimension of perspective of function to analyze items 18, 24, and 25 and the dimension of representations to analyze items 19, 21, 22, and 23.

With regard to Items 18, 24, and 25, the analysis centered on the perspectives adopted. The categories and relevant explanations are shown in Table 3.8.

Table 3. 8

Categories and Explanations with Regard to Function Concept in General

Item	Process	Object
18	Pointing out corresponding relationship between domain and range (one-to-one; multiple-to-one) [18P]	Point out the features of function expressions and graphs (a constant value or two expressions or one line, many holes /un-continuous curve) [18O]
24	Let f(x) and g(x) intersect at x-axis (p, 0), then, the following statements are true: (1) f(p) = 0 → ap + b = 0 → p = -b/a; (2) g(p) = 0 → cp + d = 0 → p = -d/c; (3) f(p) = g(p) → b/a = d/c → ad = bc; (4) f(p) = g(p) → ap + b = cp + d → p = -(b + d)/(a + c); According to (f+g)(p) = f(p) + g(p), and above statements, the student shows (f+g) (p) = 0.[24 P]	Let f(x) and g(x) intersect at x-axis (p, 0), then, f(p) = 0, g(p) = 0. So, (f+g)(p) = f(p) + g(p) = 0 + 0 = 0. Thus, (f+g) (p)=0. [24O]
25	It is necessary for students have a connection between two perspectives and a shift between graphical representation and verbal representation. The diagram could be interpreted as the following relations: (1) Height/distance vs. time [25C1]) (2) Velocity vs. time [25C2] (3) Housing/stock price vs. time [25C3] (4) Temperature vs. time [25C4]	

However, with regard to the items related to the quadratic functions /equations /inequalities (Items 19, 21, 22, & 23), the analysis was focused on the representations used and the shift between different presentations (which will be further illustrated in results).

Moreover, we defined the concept of flexibility in adopting different perspectives and different representations. Each shift between representations is coded as an event of flexibility if the participant is successful in solving the problem through this shift (e.g., score 3 or 4). For example, in item 22, the participants' responses can be categorized as three types, and each type presents a flexible event (See Table 3.9).

Table 3.9.
Categories and Flexibility with Regard to Item 22

Solutions /Mistakes Solution	Flexibility
The effects of changing of a, b and c on the changes of the graphs of quadratic function.	Yes (graph vs. algebra)
Symmetrical line $x=-\frac{b}{2a}$,*a* is not changed, then only *b* need to changed.	Yes (graph vs. algebra)
Based on the algebraic relationship g(x)=f(-x), finding the coefficients of g(x) (a1=a, b1=-b, and c1=c).	Yes (graph vs. algebra)
Mistakes	
Based on g(x)= $a(x-h)^2+b(x-h)+c$, make a statement that at least two of three (a, b, and c) need to be changed	No
According to $x=-\frac{b}{2a}$, $y=\frac{4ac-b^2}{4a}$, make a statement that at least two of three (a, b, and c) need to be changed.	No

For another example, the problem in item 23 could be solved in two steps: finding out the quadratic function and finding out the maximum. First of all, three forms of quadratic formula methods: $y=ax^2+bx+c$ (FM1); $y=a(x-x_1)(x-x_2)$ (FM2); and $y=a(x-h)^2+k$ (FM3) can be used for finding a quadratic function expression. Then, three methods were used for finding out the maximum value: (1) transforming into $y=a(x-h)^2+k$, then finding the maximum (MM1); (2) using formula $x=-\frac{b}{2a}$, $y_{maximum}=\frac{4ac-b^2}{4a}$ (MM2); and (3) taking the derivative: y'=0, x=1, then, $y_{maximum}=f(1)$ (MM3).

All the methods of solving the question are the combinations of above methods as follows:

1. FM1(step 1) and MM1 (step 2);
2. FM1 (step 1) and MM2(step 2);
3. FM2/FM3(step 1) and MM1(step 2);
4. M2/FM3(step 1) and MM2(step 2);
5. FM1/FM2/FM3 (step 1) and MM3 (step 2).

As far as the shifts between representations are concerned, I coded one event of demonstrating flexibility for each methods 1 to 4, but not method 5. It is because in the first four methods, it is necessary to shift from a different quadratic formula.

Quantitative Analysis

I analyzed the quantitative data from three aspects. First, I analyzed the item mean and performed a t-test detecting mean differences between China and the U.S. Then, I analyzed the relationships between different variables (including latent variables) by path model analysis and the fitness of the theoretical model of KTA by estimating instrument models. Third, I analyzed the correlation between the flexibility and other variables.

Interview Data Analysis

The U.S. interview data was analyzed to further illustrate pre-service teachers' responses (their thoughts) to open-ended items. It is aimed at providing a more detailed interpretation of participants' answers.

Framework for Data Analysis

The quantitative data analysis results were further illustrated and interpreted by the qualitative findings. The whole process of data analysis is described by Figure 3.2. According to this diagram, first, the items were quantified into quantitative data for item and construct analysis. With regard to item analysis, the item mean was analyzed and compared by using SPSS 16.0. and the path analysis and instrument model estimation were conducted by AMOS 16. In addition, a correlation analysis was used to investigate the relationship between flexibility and other variables, such as different knowledge components.

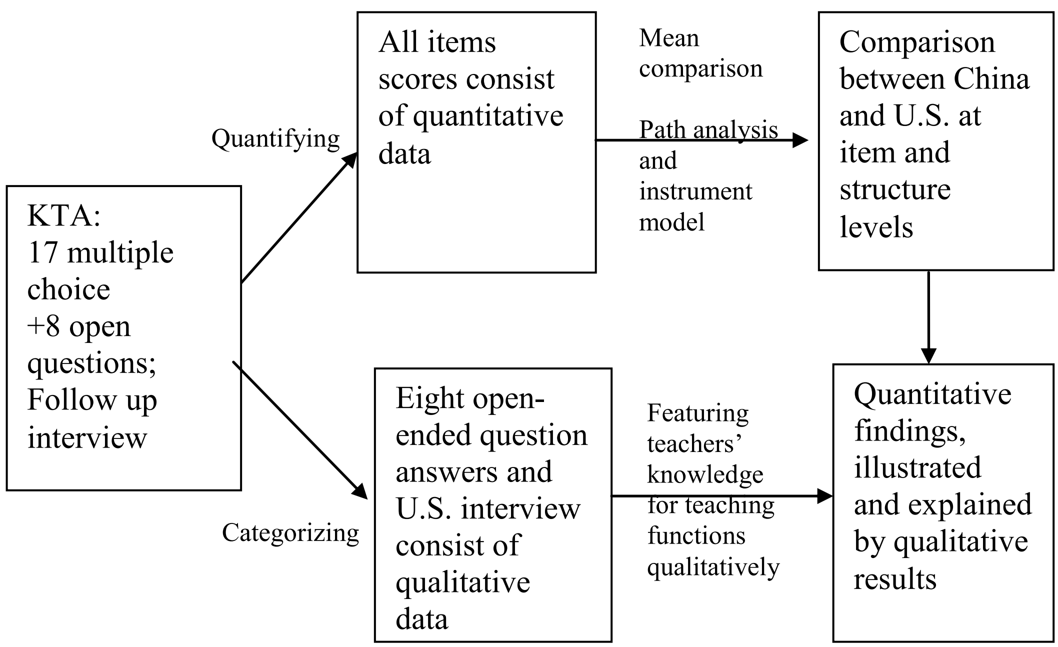

Figure 3.2. Process of data analysis.

With regard to the eight open-ended questions on teacher's knowledge for teaching algebra, a qualitative analysis was performed. The purpose was to identify the characteristics of teachers' knowledge for teaching the concept of function cross-culturally. A particular focus was put on the strategies used and flexibility in adapting perspectives of function concept and selecting representations. The follow up interview data analysis was used for further clarifying participants' knowledge for teaching the concept of function. Finally, the qualitative results were further used for interpreting quantitative findings and the conclusions of the study were made.

Conclusion

In this chapter, I describe the design of study which is a mixed method by using a questionnaire. The process of development of the instrument for this study was described and discussed. Next, the data collection procedures in China and the U.S. were described. After that, the methods of data analysis were illustrated in detail. Finally, I summarized the strategies to integrate the findings based on quantitative and qualitative analyses to make conclusion of the study.

CHAPTER IV

RESULTS

The findings of this study are organized into four sections. First, I report comparative results of KTA at item and structure levels between China and the U.S. Second, I present the relationship among background variables and components of KTA in China and the U.S. Third, I compare the similarities and differences of KTCF between China and the U.S. Fourth, I present an analysis of correlation between flexibility and other variables. Finally, I summarize the findings of the study with regard to the four research questions.

Comparison of KTA between China and the U.S.

Reliability of the Instrument

The questionnaire is designed to measure three types of knowledge: school mathematics, advanced mathematics and teaching mathematics. Each item belongs to one of the three categories. The distribution of items to different categories is shown in Table 4.1.

Thus, there are 7 items (1, 3, 6, 14, 17, 19, & 23) in *school mathematics*, 8 items (4, 8, 9, 12, 13, 16, 20, & 24) in *advanced mathematics*, and 10 items (2, 4, 7, 10, 11, 15, 18, 21, 22, & 25) in *teaching mathematics*. The reliabilities (Cronbach's Alpha) of the instrument are .877 (N=491, the whole sample), 0.613 (N=115, the U.S. sample), and .73 (N=376, the Chinese sample).

Table 4. 1.

Category of Knowledge for Teaching Algebra

Items	Types of knowledge	Items	Types of knowledge
1	1	14	1
2	3	15	3
3	1	16	2
4	2	17	1
5	3	18	3
6	1	19	1
7	3	20	2
8	2	21	3
9	2	22	3
10	3	23	1
11	3	24	2
12	2	25	3
13	2		

Note. 1-school mathematics; 2-advanced mathematics; and 3- teaching mathematics.

The Mean Differences of Items and Components between China and the U.S.

First, I compared the mean differences of multiple choice items. The mean and t-test values are listed in Table 4.2.

Table 4. 2.
Mean Differences of Multiple Choices between China and the U.S.

	Mean		T-Test
Item	China	U.S.	
1	**.90**	**.85**	**1.42**
2	.96	.57	8.38**
3	.96	.89	2.43**
4	.78	.15	16.06**
5	.95	.18	20.70**
6	**.38**	**.30**	**1.66**
7	.83	.57	5.20**
8	.90	.16	19.95**
9	.47	.20	5.90**
10	**.68**	**.66**	**.45**
11	.62	.78	-3.43**
12	.47	.27	4.01**
13	.64	.23	8.56**
14	.87	.40	9.64**
15	.66	.22	9.73**
16	.92	.29	14.19**
17	.96	.52	9.23**

This Table showed that Chinese participants performed better than the U.S. counterparts, except for on four items (1, 6, 10, and 11). On one item (11), the U.S. participants achieved a significant higher mean score than Chinese counterparts (Mean difference=0.16, t=3.43, p<0.001). On items 1, 6, & 10, although the Chinese participants scored higher than their U.S. counterparts, there was no significant difference. In all remaining items, Chinese participants' mean scores were significantly higher than the U.S. counterparts ($p<.001$). In addition to above mentioned items 1, 6, 10, and 11, other five items 4, 5, 8, 15, and 16, with significant differences between China and the U.S. are further discussed in detail in the next section.

With regard to the open-ended items and components of KTA, the means and tests of significance are displayed in Table 4.3.

Table 4.3.
Mean Differences of Open-ended Items and Components of KTA between China and the U.S.

	Mean		t-Test
Item	China	U.S.	
18	2.91	1.51	10.21**
19	3.66	.76	34.62**
20	3.47	.79	21.29**
21	2.97	.18	31.60**
22	2.64	1.40	8.44**
23	3.29	.29	33.01**
24	3.25	.02	46.63**
25	2.23	1.43	5.53**
SM	11.03	4.00	39.19**
AM	10.89	2.1	42.36**
TM	15.45	7.50	21.04**
KTA	37.37	13.60	39.40**

Note. SM-School mathematics; AM -Advanced mathematics; TM -Teaching mathematics; and KTA - Knowledge for teaching algebra.

In all the open-ended items, there are significant mean differences between China and the U.S. Chinese participants achieved significantly higher means in school mathematics (mean difference=7.03, t=39.19, p<.001), advanced mathematics (MD=8.78, t=42.36, p<.001), and teaching mathematics (MD=7.95, t=21.04, p<.001) than U.S. counterparts. Consequently, Chinese participants achieved significantly higher mean of KTA (MD=24.77, t=39.40, p<.001) than their U.S. counterparts.

Since the Chinese sample is relatively large, I did a multiple comparison of KTA with regard to different institutions. All the participating universities were classified into three ranks based on 2009 university league list in China. One high achieving university (rank 1) , two intermediate achieving universities (rank 2), and two low achieving universities (rank 3) (See Table 3.2 in Chapter III for details). It was found that there was no significant mean difference between rank 1 and rank 2 university but rank 1 and rank 2 universities had significantly higher mean score of KTA than rank 3 universities. Moreover, given the fact that only juniors or seniors in-service teachers came from rank 2 universities, I excluded participants from rank 2 universities for qualitative analysis. Thus, in this study, the high-achieving group consists of participants from rank 1 (N=109) and the low-achieving group consists of participants from rank 3 universities (N=147).

Analysis of Selected Multiple Choice Items

In this part, I examined several special multiple choice items in detail. These items include: item 11, in which U.S. participants outperformed Chinese counterparts; three items 1, 6, & 10, in which there are no significant mean difference between China and the U.S., and five items 4,5,8, 15 & 16, in which the means of participants from China were significantly higher than the U.S. I made a comparison between China high-achieving group (*N*=109), China low-achieving group (*N*=147), and the U.S. (*N*=115) in order to better understand how the participants answered these questions.

Item 11, in a first year algebra class, which of the following is **NOT** an appropriate way to introduce the concept of slope of a line?

A. Talk about the rate of change of a graph of a line on an interval.

B. Talk about speed as distance divided by time.

C. Toss a ball in the air and use a motion detector to graph its trajectory.

D. Apply the formula $slope = \frac{rise}{run}$ to several points in the plane.

E. Discuss the meaning of *m* in the graphs of several equations of the form $y = mx + b$.

The distribution of different choices of this item is displayed for China (high achieving vs. low achieving groups) and the U.S. sample in Table 4. 4. (C is the correct choice).

Table 4.4

The Choice Distribution of Item 11 in China and the U.S

	China		U.S.
	High (%)	Low (%)	(%)
A	11	8	8
B	11	11	5
C	64	61	78
D	12	20	5
E	2	1	6

More than 60% of Chinese participants can identify the correct choice and the Chinese participants also made a variety of wrong choices. These results may imply the Chinese participants are not familiar with different learning situations (particular contextual situations) for introducing the concept of slope. It may reflect that the Chinese participants' learning experience was limited in mathematical context. It may also reflect that they may memorize the formula of slope but do not understand the geometrical meaning of the formula (12% from high achieving group and 20 % for low-achieving group were not able to use the formula in China). On the other hand, the U.S. participants had a high rate of correct choice (78%). This may imply that the U.S. participants had a better understanding of the concept and were exposed to multiple application situations.

Item 1, at a storewide sale, shirts cost $8 each and pants cost $12 each. If S is the number of shirts and P is the number of pants bought, which of the following describes the expression 8S + 12P?

A. The number of shirts and pants bought   B. The cost of 8 shirts and 12 pants
C. The cost of P shirts and S pants   D. The cost of S shirts and P pants

The different choices of the item 1 are displayed in Table 4.5 (Correct choice is C)

Table 4.5

The Choice Distribution of Item 1 in China and the U.S

	China		U.S.
	High (%)	Low (%)	(%)
A	0	2	1
B	4	6	9
C	3	3	4
D	92	89	86

The Table showed both Chinese and U.S. participants made a high rate of correct choice D (greater than 86%). Only a small part of them made a wrong choice of B or C. The result may imply both Chinese and U.S. participants are familiar with presenting quantitative relationship by using algebra expressions.

Item 6, which of the following can be represented by areas of rectangles?

i. The equivalence of fractions and percents, e.g. $\frac{3}{5} = 60\%$

ii. The distributive property of multiplication over addition:

For all real numbers a, b, and c, we have $a(b + c) = ab + ac$

iii. The expansion of the square of a binomial: $(a + b)^2 = a^2 + 2ab + b^2$

A. ii only **B**. i and ii only **C**. i and iii only

D. ii and iii only **E**. i, ii, and iii

The different choices of the item 6 are displayed in Table 4.6 (Correct choice is E)

Table 4.6

The Choice Distribution of Item 6 in China and the U.S.

	China		US
	High (%)	Low (%)	(%)
A	3	3	7
B	3	15	16
C	4	14	26
D	56	35	22
E	35	34	30

This Table showed that both Chinese and U.S. participants achieved very low correct rate (between 30% to 35%), and no significant mean difference between China and the U.S. Interestingly, more than half (56%) Chinese participants from high-achieving group made choice D. That means they believed that the equation $\frac{3}{5} = 60\%$ can't be represented by the area of rectangle (Choice D). More than one third of participants from low achieving group and about one fourth of U.S. participants made the same choice (Choice D). In addition, about one fourth U.S. participants believed $a(b+c) = ab + ac$ can't represented by the area of a rectangle (Choice C). In summary, both Chinese and U.S. participants were low-scoring in using geometrical representation to present fraction/percentage and algebraic formula. That means the participants from both countries are not well prepared to link algebraic (arithmetic) and geometrical representations.

Item 10. A textbook includes the following theorem:

If line l_1 *has slope* m_1 *and line* l_2 *has slope* m_2 *then* $l_1 \perp l_2$ *if and only if*

$m_1 \cdot m_2 = -1$ *(i.e. "slopes are negative reciprocals").*

(McDougal Littell, Algebra 2)

Three teachers were discussing whether or not this statement generalizes to all lines in the Cartesian plane.

Mrs. Allen: The statement of the theorem is incomplete: it doesn't provide for the pair of lines where one is horizontal and one is vertical. Such lines are perpendicular.

Mr. Brown: The statement is fine: a horizontal line has slope 0 and a vertical line has slope ∞ and it's okay to think of 0 times ∞ as –1.

Ms. Corelli: The statement is fine; horizontal and vertical lines are not perpendicular.

Whose comments are correct?

A. Mrs. Allen only **B.** Mr. Brown only **C.** Ms. Corelli only

D. Mr. Brown and Ms. Corelli **E.** None are correct.

The different choices of the item 6 are displayed in Table 4.7 (Correct choice is A)

Table 4.7

The Choice Distribution of Item 10 in China and the U.S.

	China		U.S.
	High (%)	Low (%)	(%)
A	70	63	66
B	7	4	10
C	1	3	4
D	1	3	4
E	21	28	16

Both Chinese and U.S. participants had a similar rate of correct choice (about 67%). It is interesting that about one quarter of participants in the two countries did not agree with that given explanation (Choice E). Also, there was a small part of participants agreed “0 times -∞ as –1” by choosing B. This result alerts that it is in need to introduce a theorem more rigorously.

Since items 4, 5, & 16 are related to irrational function, irrational equation and derivative of polynomial, Chinese participants outperformed U.S. counterparts significantly. That means Chinese participants achieved high-scoring in advanced algebra computation. I further examined two other items 8 & 15, on which Chinese participants performed very well.

Item 8. The given graph represents speed vs. time for two cars (Figure 4.1). (Assume the cars start from the same position and are traveling in the same direction.) Use this information and the graph below to answer.

What is the relationship between the ***position*** of car A and car B at $t = 1$ hour?

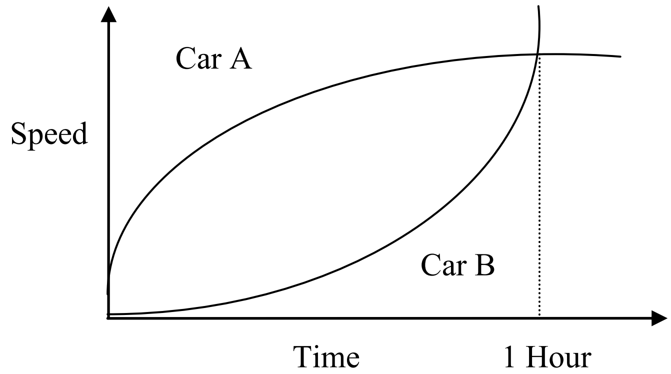

Figure 4.1. Diagram of the relationship between speed and time.

A. The cars are at the same position **B.** Car A is ahead of car B

C. Car B is passing car A **D.** Car A and car B are colliding

E. The cars are at the same position and car B is passing car A.

The different choices of the item 8 are displayed in Table 4.8 (Correct one is B)

Table 4.8

The Choice Distribution of Item 8 in China and the U.S.

	China		U.S.
	High (%)	Low (%)	(%)
A	0	2	19
B	96	88	16
C	1	2	13
D	1	2	4
E	2	5	48

The Chinese participants did extremely well in this item. The average correct rate is 90%. Even participants from the low-achieving group have about 88% correct rate. In order to get a correct answer, it is required to have a logical reasoning based on speed and time relationship and the graph. Only small part of the participants from the low-achieving group made their judgment based on the visual information without having an appropriate understanding of the meaning of speed vs. time graph (Choice A & E).

In the U.S. participants, only 16 % of them made a correct choice based on logical reasoning and graphical representation. About half of them (48%) made a wrong choice based on visual information only: intersection point and high over (Choice E) or partially using the visual information: intersection point (Choice A) or high over (Choice B). It may be that many U.S. participants used visual judgment rather than logical reasoning.

Item 15. Which of the following (taken by itself) would give **substantial** help to a student who wants to expand $(x+y+z)^2$?

i. See what happens in an example, such as $(3 + 4 + 5)^{2.}$

ii. Use $(x + y + z)^2 = ((x+ y) + z)^2$ and the expansion of $(a + b)^2$.

iii. Use the geometric model shown below (Figure 4.2).

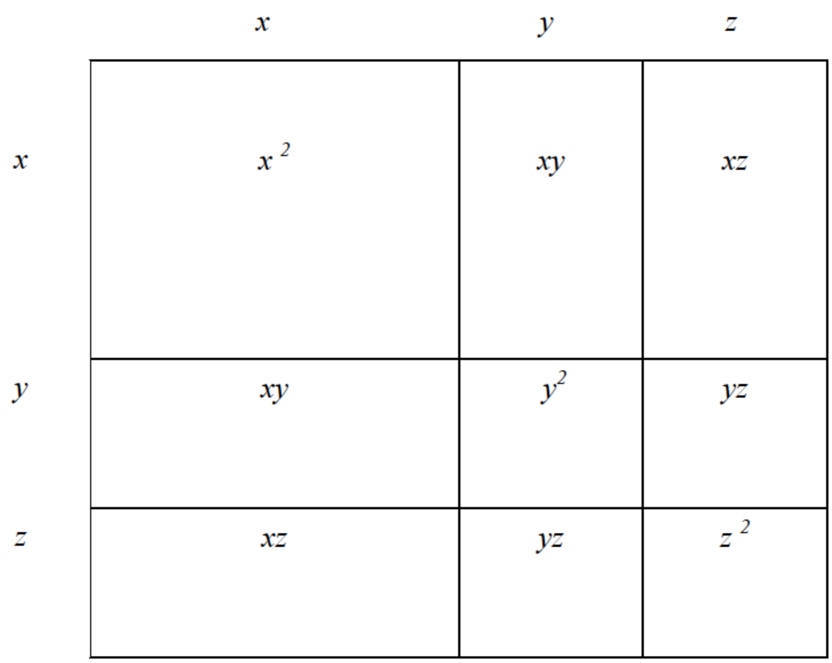

Figure 4. 2. Diagram of expansion of $(x+ y + z)^2$

A. ii only **B.** iii only **C.** i and ii only **D.** ii and iii only **E.** i, ii and iii

The different choices of the item 15 are displayed in Table 4.9 (Correct choice is D)

Table 4.9
The Choice Distribution of Item 15 in China and the U.S.

	China		U.S.
	High (%)	Low (%)	(%)
A	6	11	2
B	6	5	57
C	3	3	1
D	73	72	22
E	13	19	15

The Table showed that about 72 % Chinese participants made the correct choice. They realized that using algebraic computation and geometrical model can help students to understand the algebraic expansion. About 15% of the Chinese participants believed the numerical computation can also be helpful. However, more than half of U.S. participants believed only the geometry mode is helpful, while only 35 % of the participants recognized the usefulness of exploring algebraic expression. About 16% of U.S. participants believed the usefulness of numerical computation. Thus, the U.S. participants relied on the geometrical model to reason while the Chinese counterparts make their reason based on the geometrical model and algebraic computation.

In summary, the analysis of these purposely selected items show that both Chinese and U.S. participants scored high in expressing contextual situations using algebraic expressions (item 1) , and revealed a weakness in linking multiple representations such as numerical, algebraic and geometrical ones (item 6). Compared with the Chinese participants, the U.S. counterparts demonstrated strengths in understanding a concept from different aspects (item 11). However, when making reasoning or judgment, the U.S. participants often preferred to rely on visual or geometrical information, while the Chinese counterparts tended to make logical reasoning with the support of algebraic and geometrical representation and computation (item 8 & 15).

Relationship among Different Components of KTA

This session, I report path models in China and the U.S., and a measurement model in China. The relationships among background variables and different components of KTA are analyzed based on these models.

Path Model Analysis

In this part, I examined the relationship between background variables (course taking and grade) and components of KTA, and relationships between components of KTA. Researchers (e.g., Monk, 1994) suggested the number of courses taken by teachers is positively related to how much their students learn in mathematics at the secondary level. Thus, in this study, I created a conceptual frame for examining the relationship between background variables, including number of mathematics and mathematics education courses taken, and the components of KTA as shown in Figure. 4.3.

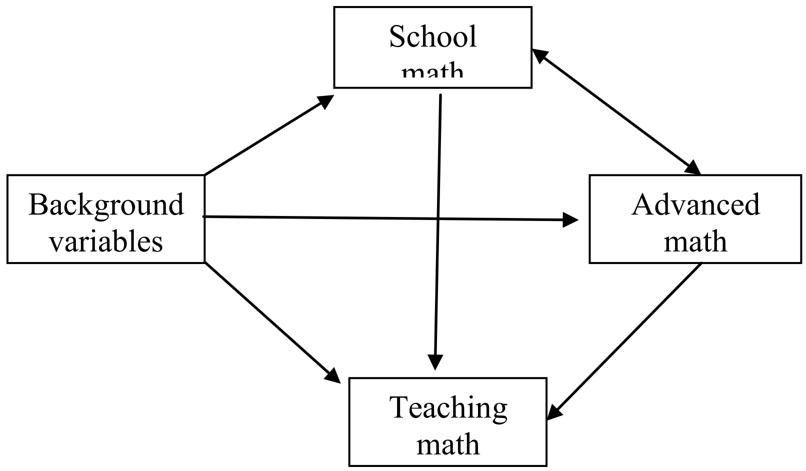

Figure 4. 3. Theoretical model relating background variables and components of KTA.

As outlined in Figure 4.3., the relationship between background variables (the number of math courses taken in high school, the number of courses taken in college, and the grade), and components of KTA (SM, AM and TM) were modeled using a series of path models (See Figure 4.4). It was assumed that SM and AM interact with each other and both of them impact TM. Furthermore, the KTA (including SM, AM and TM) is hypothesized to be a function of background variables. SM, AM and TM are considered as endogenous variables in the model. The aim of the path analysis is to include the entire variables which may contribute to the explanation of the variance in the endogenous variables. As it is impossible to include everything that may impact these variables, an error term is included in the model to be estimated for each endogenous variable (e.g., res, rea, and ret are the error terms for SM, AM, and TM respectively, see Figure 4.4). The errors reflect all those unobserved predictors that were not measured in this study nor included in this model.

In addition, this system of variables was hypothesized to be influenced by participants' background characteristics; including the number of high math courses taken, the number of college math education courses taken and grade level. These variables are considered to be exogenous (i.e., independent variables that are not dependent on or predicted by any other variables in the model). The variables are covaried to influence on KTA.

I estimated this mode by using AMOS 16. Initially, all the parameters for the background variables on the system of variables were estimated. Consequently, to achieve a good fit, some paths were deleted that were not significant. The final mode is presented in Figure 4.4. In this diagram, the bold arrow lines represent a significant effect while the dashed lines represent a non-significant effect. Estimates are in raw score form.

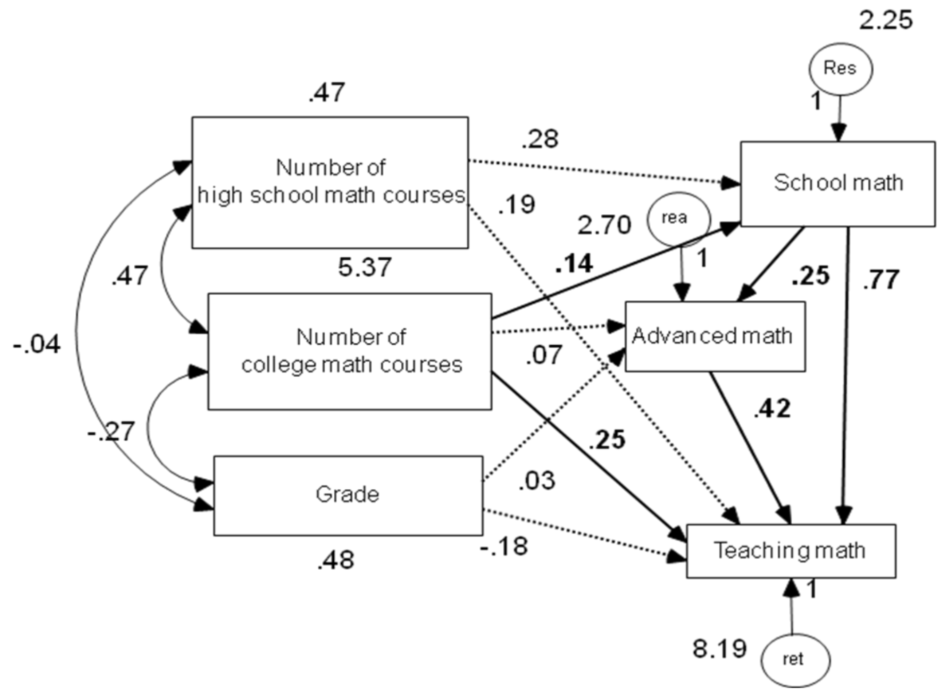

Figure. 4.4 . Final path model of the course taking, grade level, and KTA in the U.S.

In this model, the chi-square test for lack of fit was not significant, $\chi^2(2) = 0.08$, p=0.96. This means the data has a good fit (Hu & Bentler, 1999). Moreover, other fit indices showed there was a good fit. The comparative fit index CFI equals 1. This index can take on a value from 0 to 1 with values closer to 1 showing a better fit and value greater than .90 usually indicating a relatively good fit (Kline, 2005). The root mean square error of approximation (RMSEA) was 0.00. This index takes into account the complexity of the model, and it can range from 0 to 1 with less than 0.05 of presenting a good fit.

The parameters shown in Figure 4.4, school mathematics was found to have direct and significant effects on teaching mathematics (β=0.77, p<0.05) and advanced math (β=0.25, p<0.05). Number of college math courses was found to have a significant effect on school math (β=0.14, p<0.05) and teaching math (β=0.25, p<0.05) while advanced math was found to have a direct and significant effect on teaching math (β=0.42, p<0.05). However, the number of high school math courses and grade level were not found to have significant effects on school math (β=0.28) and teaching math (β=0.19), and number of college math courses was not found to have significant effect on advanced math (β=0.07).

Similarly, a final path model of the course taking, grade level, and KTA in China was created as Figure 4.5. The dashed lines represent non-significant effects while the bold lines represent significant effects. Since there was no number of courses taken in high school in China, we just focused on the number of courses taken in college.

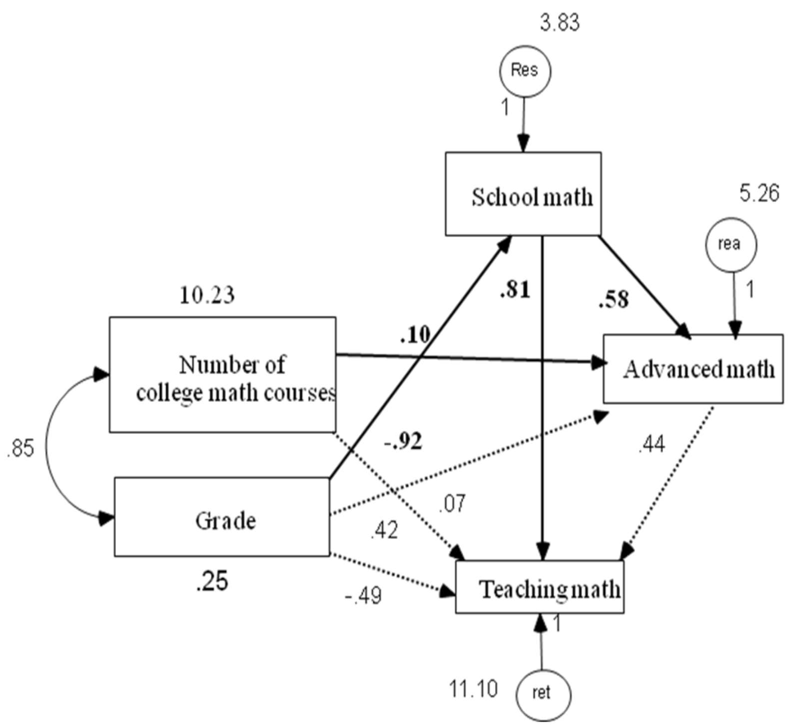

Figure 4.5 .Final path model of the course taking, grade level, and KTA in the China.

In this model, the chi-square test of lack of fit was not significant, $\chi^2(1) = 0.583$, p=0.45. This means the data has a good fit. Moreover, other fit indices shown the model fit is good (CFI=1, RMSEA=0.000). The parameter estimates are shown for the Chinese model (see Figure 4.5) , the number of college math education courses was found to have a direct and significant positive effect on advanced math (β=0.10, p<0.05) while the grade level was found to have a direct and significant negative effect on school mathematics and advanced math (β=-0.92, p<0.05). School mathematics was found to have significant positive effects on advanced math (β=0.58, p<0.05) and teaching math (β=0.81, p<0.05). However, grade levels were not found to have an effect on advanced math (β=0.42) or teaching math (β=-.49). Number of college math courses was not found to have an effect on school math (β=0.07), nor was advanced math found to have an effect on teaching math (β=0.44).

Comparing the Chinese model and the U.S. model, the number of college math courses was not found to have an effect on advanced math (β=0.07) in the U.S., while in the effect was significant (β=0.10, p<0.05) in China. In addition, in the U.S. sample, number of college math courses taken was found to have a significant effect on teaching mathematics (β=0.25, p<0.05), while in the Chinese sample, the effect was not significant (β=0.07). This result may indicate that the Chinese teacher preparation emphasizes content knowledge while the U.S. teacher preparation emphasizes pedagogical knowledge. When considering the size of effect between different components in China and the U.S., it was found that there were stronger correlations in China than those in the U.S. (For example, the effect of school mathematics on teaching math is 0.81 in China while the corresponding effect is 0.77 in the U.S.; the effect school mathematics on advanced math is 0.58 in China while the corresponding effect is 0.25 in the U.S.). However, the differences of correlations (See Table 4.10) are not significant between China and the U.S. based on the Fisher's *Z* test.

Table 4.10

The Correlations of Different Components of KTA

	U.S. (N=115)			China (N=376)		
	SM	AM	TM	SM	AM	TM
SM	1	.242**	.449**	1	0.423**	.522**
AM	.242**	1	.336**	.423**	1	.449**
TM	.449**	.336**	1	.522**	.449**	1

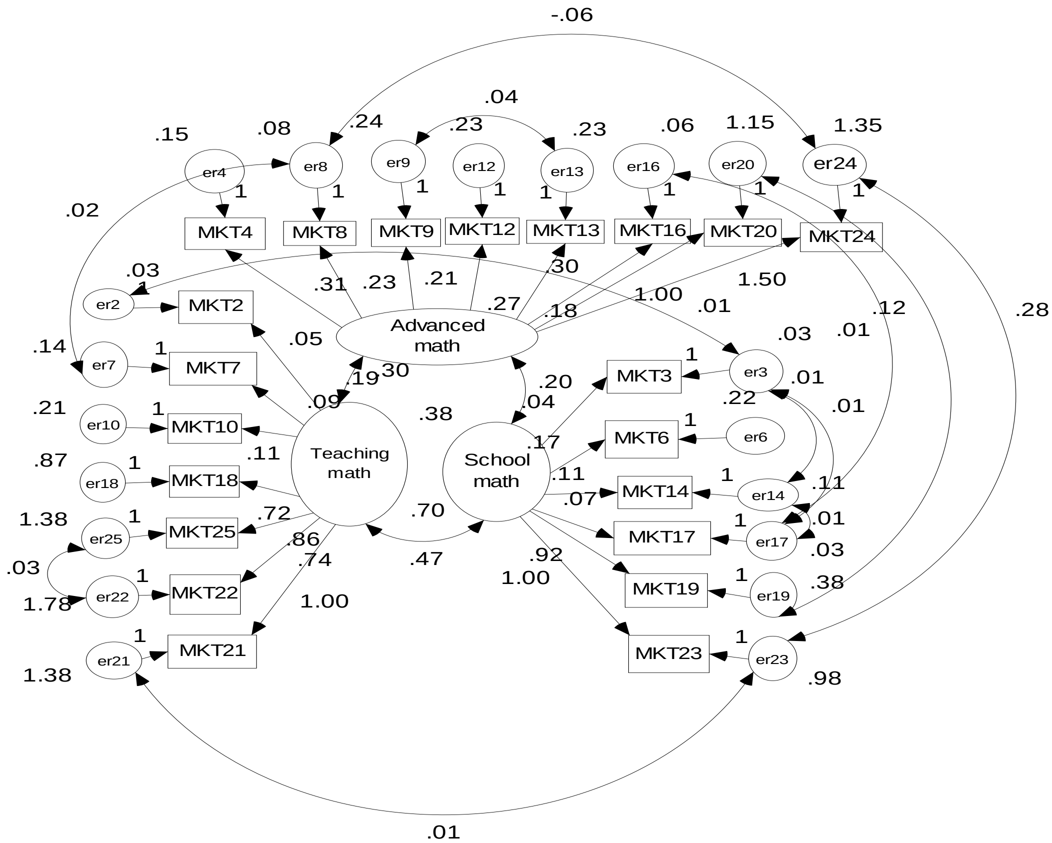

Figure 4.6. Final measurement model of KTA in China.

The bidirectional relationships between different observed variables are modeled (Figure 4.6) by allowing the error terms for each of these variables to covary (for example, labeled er 21 and er 23). Corvariance of error terms essentially reflects the correlation between two variables.

Estimates of measurement model parameters were obtained using AMOS 16 (Byrne, 2010). Initially, all parameters of the theoretical model were estimated. Consequently, based on the inspection of weight load (larger than .02) and model fit indexes, the model was re-estimated. The final measurement model with all parameter estimates is presented in Figure 4.6.

In this model, the chi-square test of lack of fit was significant, $\chi^2(174) = 245.34$, p<0.001. This showed the data (p<0.05) did not have a good fit to the model. However, other fit indices shown a relatively good fit. The Comparative Fit Indices (CFI) equaled 0.914 (>0.90) and the Root Mean Square Error of Approximation (RMSEA) was 0.03 (0.02-0.04) (<0.05). These two indexes indicate the Chinese data fit the model relatively well.

The standardized regression weights (larger than 0.10) of all the observed variables in the final model were displayed in Table 4.11.

This Table showed that except for four items (MKT13, MKT9, MKT7, MKT3), the others had weights greater than 0.20. Among these items, the open-ended items had relatively greater weights ranging from 0.420 to 0.677.

Table 4.11.
Standardized Regression Weights Estimate

Standardized regression weight			Estimate
MKT3	<---	SM	.147
MKT6	<---	SM	.215
MKT14	<---	SM	.211
MKT19	<---	SM	.677
MKT23	<---	SM	.530
MKT17	<---	SM	.243
MKT10	<---	TM	.206
MKT7	<---	TM	.196
MKT25	<---	TM	.522
MKT18	<---	TM	.544
MKT21	<---	TM	.579
MKT22	<---	TM	.420
MKT2	<---	TM	.211
MKT9	<---	AM	.185
MKT12	<---	AM	.237
MKT13	<---	AM	.163
MKT16	<---	AM	.474
MKT20	<---	AM	.374
MKT4	<---	AM	.323
MKT24	<---	AM	.488

In addition, the correlations between latent variables (components of KTA) are shown in the following Table 4.12. The correlations of these latent variables were relatively high (.75 to .91).

Table 4.12

The Correlation and Co-variance of Different Variables

Correlations			Estimates
SM	<-->	TM	.91
SM	<-->	AM	.75
TM	<-->	AM	.83

This Table shows that there was a high correlation between school mathematics and teaching mathematics (r=0.91). Compared with the correlation between school mathematics and advanced mathematics (r=0.74), the correlation between teaching mathematics and advanced mathematics (r=0.83) was relatively higher.

The final Chinese model of KTA showed that there are many links between errors, within the same component or across components. These links imply that these observed variables are not able to be used to measure different components of KTA exclusively, rather they are used to jointly measure an interconnected structure of knowledge for teaching algebra. When examining the linked items themselves, some of them are essentially related. For example, item 7 is about linear function and its graph while Item 8 is about using a graph of speed vs. time to interpret a daily life situation. For another example, item 3 is about polynomial function computation while the item 14 is about irrational equation solution. However, in some cases, two items are not obviously related. For example, item 19 is about solving quadratic inequality while item 20 is about matrix computation. For another example, item 23 is about finding quadratic function and its maximum while item 24 is about proving a proposition of sum of two linear functions.

The model seems to suggest that the theoretically and artificially exclusive components of KTA are essentially interconnected. That means that knowledge for teaching should be treated as a comprehensive and interconnected entity and construct.

However, with the U.S. data, the model is not admissible. It may be due to the small sample size (N=115 does not meet the minimum requirement of 10 × 25=250 cases required for a full estimation).

Comparisons of KTCF between China and the U.S.

In this part, I presented a detailed analysis on the participants' responses to the open-ended items in terms of their overall performance, typical strategies/methods, and misconceptions/mistakes. With the U.S. case, I complemented relevant analysis with the interview data. Rather than analyzing items one by one in order, I grouped these items into three categories: One item is related to matrix and logical inference (Item 20); Three items are related to function concept in general (Items 18, 24,& 25), and the remaining four open-ended items are related to quadratic functions/ equations/ inequalities (Items 19, 21, 22, & 23) in particular.

For the matrix item, the analysis was focused on the logical equivalence, and matrix computations. For the items related to the concept of function, the analysis was focused on the perspectives of function concept (process vs. object); for the items related to quadratic functions/equations/inequalities, a two dimension framework consists of the perspective of function, and the flexibility in using representations guides the data analysis. The framework is aimed to capture the understanding of the function concept and the flexibility in using representation (using verbal, tabular, algebraic, and graphical representations, translation between different representations and transformation within a representation) (see Table 3.8).

In the sessions that follow, I will present relevant findings in these three areas.

Logical Reasoning in Matrix System

Item 20 is an open-ended item used for measuring advanced knowledge. As Table 4.3 showed that there was a significant mean difference between China and the U.S. (Mean difference=2.68, t=21.29, p<0.001). Moreover, the score distribution is displayed in Figure 4.7.

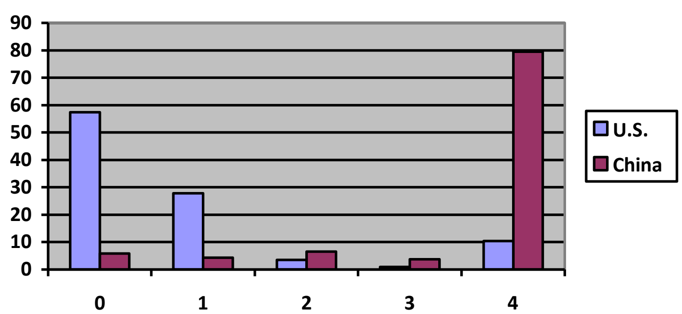

Figure 4.7. Score distribution of item 20.

This Figure shows that more than 80% of Chinese participants provided correct proofs while only about 10% of their U.S. counterparts did the same. Less than 6 % of

Chinese participants did not provide any useful information while more than half (57.4%) of the U.S. participants did the same.

The item 20 is as follows:

Let A= $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $B = \begin{bmatrix} t & u \\ v & w \end{bmatrix}$, Then $A \Delta B$ is defined to be $\begin{bmatrix} pt & qu \\ rv & sw \end{bmatrix}$. Is it true that if $A \Delta B = O$, then either A = **O** or B = **O** (where **O** represents the zero matrix) ? Justify your answer and show your work in the Answer Booklet.

In fact, the participants are required to provide a counterexample to disprove the statement. However, common misconception the U.S. participants made is to use a wrong logical reasoning as follows: $p \to q \Leftrightarrow q \to p$, namely, using the following logic "if A=0, then AΔB=0 or if B=0, then AΔB=0" to prove: "if AΔB=0, then A=0, or B=0" .More than a quarter of the U.S. participants made this mistake, while only a few of the Chinese counterparts made the same mistakes.

A few of the U.S. participants inappropriately generalized the same proposition from real number systems, namely $x.y = 0; x = 0$, $y = 0$. (*x*, *y* are real numbers) to matrix system. No Chinese participants made this overgeneralization.

U.S. participants' interpretation. The interviews with U.S. participants further confirmed their difficulties in providing a correct proof. Two of the five interviewees (Jenny and Stacy) gave the correct answer with an appropriate counterexample. For example, Stacy explained why she tried to disprove the statement as follows:" if someone wants to prove a proposition, s/he has to provide the whole process of proving.

However, if someone just wants to disprove a proposition, s/he only provides a counterexample, so, I considered to find a counterexample". The others gave a wrong judgment by providing examples such as "A=0, then $A\Delta B = O$" or "B=0, then $A\Delta B = O$" by "guess and check". However, when the researcher asked them "whether it is possible that if A $\neq$ 0, B $\neq$ 0, but $A\Delta B = O$", two of them (Larry and Alisa) took a second thought, and found a counterexample to disprove the statement. For example, Alisa gave a counterexample, A= $\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$. However, Kerri was still struggling with finding a counterexample by saying "it is a trick".

In summary, more than three-fourths (85%) of the U.S. participants were not able to provide any relevant information, and about one fourth were confused with the logical proposition relationship between "$p \rightarrow q$" and "$q \rightarrow p$".

Flexibility in Adopting Perspectives of Function Concept

The items 18, 24 and 25 are particularly used for measuring the knowledge of understanding and applying function concept from different perspectives (*process* and *object*). It is crucial for participants to provide correct answers and explanations if they select an appropriate perspective. Item 18 is in favor of using process perspective; item 24 is easily proved if adopting an object perspective. It is necessary to connect those two perspectives when solving item 25.

Responses to Item 18. There was a significant mean difference of item 18 between China and the U.S. (mean difference=1.4, t=10.21, p<0.001, see Table 4.3). The score distribution of the item is shown in Figure 4.8.

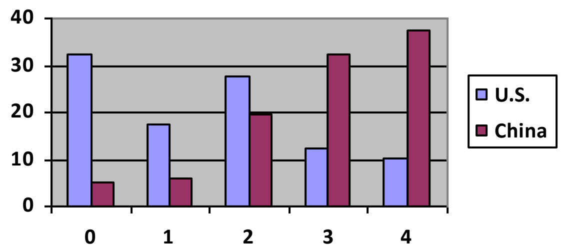

Figure 4.8. Score distribution of item 18.

The Table showed that about 23 % of U.S. participants got a correct answer (10.4 %) or almost correct answer with minor mistakes (12.2%), while there are about 70 % of the Chinese counterparts did the same. The following are correct examples in China and the U.S. (Figure 4. 9)

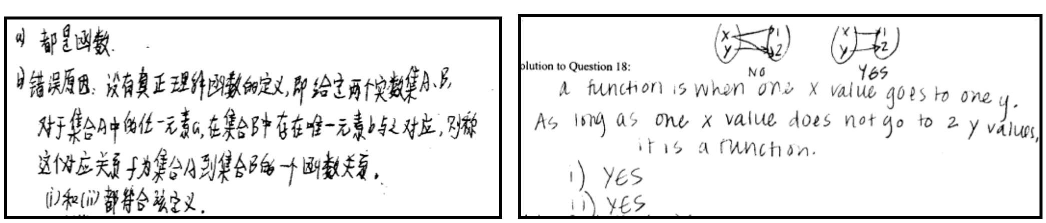

Figure 4.9. Examples of answers to item 18 in China and the U.S.

The Chinese participant directly used the definition of function: Let two real number sets A, B, if for any *a* x belongs to set A, there is only one *b* in the set B corresponding to *a*, then this corresponding relationship *f* from A to B is a function. According to this definition, (i) and (ii) are functions. However, the U.S. pre-service teacher used a diagram to visualize the function relationship and then make a judgment of these two given relations

Conversely, 32% of the U.S. participants' provided nothing or meaningless information about the solution of the item 18; only about 5% Chinese did the same. About 28% of U.S. participants and 19% of Chinese participants just gave correct answers without any interpretations or give one correct answer and relevant explanations.

Perspectives adopted in Item 18. In addition, the perspectives used in participants' interpretation are listed in Table 4.13.

Table 4.13
Perspectives Adopted in Item 18

Perspective	Description	Frequency		
		China		U.S. (%)
		High (%)	Low (%)	
Process	Corresponding relationship between domain and range (one-to-one/multiple-to-one)	51	31	6
Object	Algebraic expressions (constant value; two expressions) Graphic features (one line, many holes/un-continuous)	12	10	9

With regard to this item, it is more appropriate to adopt a process perspective. In China, in the high achieving group, more than half (51%) adopted the process perspective and in the low achieving group, more than one-third of the participants (31%) adopted this perspective. However, the U.S. participants preferred using object perspective (9%), namely, basing on function expressions and graphic features to using essentially corresponding relationship features (6%).

U.S. participants' interpretation in item 18. In responding to how they made their judgments, except for Jenny, the others reported they used the vertical line test (Larry, Alisa, and Stacy) or diagrams presenting corresponding relationship between two sets (Kerri). Jenny made her wrong judgment based on visual graphical images. Since she had a difficulty in drawing the graph of the second relation, she believed it is not a function. However, when asked whether she heard of the vertical line test, she clearly stated that "one x value can only have one corresponding y value; one x value cannot be corresponded to two y-values." Kerri said she "is a visual learner, and likes using diagrams representing the relationship between two sets (one-to-one or multiple-to-one, but not one-to-multiple)". Larry not only explained the vertical line test rule, but also showed an example ($x=y^2$) which cannot pass the vertical line test. Alisa and Stacy explained the rule by emphasizing "each input [value] should have only one [corresponding] value, but that does not mean that different [input] values cannot have same [corresponding] value"

With regard to students' mistakes, they attributed them to students' superficial understanding of the vertical line test rule (missing multiple x values correspond one y-value) or the confusion with "many holes", or the repeating output.

Four of them showed an accurate understanding of how to judge whether a relationship is a function or not based on corresponding relationship by using either vertical line test or diagrams to present the features of function relationship: one to one or multiple to one. They also realized that the "unusual" graphs of the function such as including constant value, many holes or in-continuality may confuse students' judgment.

Summary of item 18. More Chinese participants than U.S. counterparts adopted process perspective which protected them from the distraction of the unusual "appearances" of the function expressions or function graphs. These participants, who adopted a process perspective, can not only make a correct judgment, but also explain the reasons of making those mistakes.

Response to item 24. There was a significant mean difference of item 24 between China and the U.S. (MD=3.23, t=46.63, p<0.001). The score distribution of the item is shown in Figure 4.10.

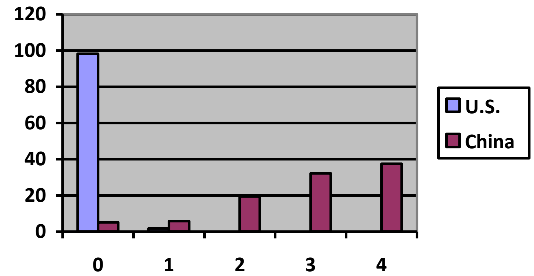

Figure 4.10. Score distribution of item 24.

Essentially, there were two proofs as follows:

Method 1: Let $f(x)$ and $g(x)$ intersect at x-axis (*p*, 0), then, the following statements are true:

(1) $f(p)=0 \rightarrow ap+b=0 \rightarrow p=-\frac{b}{a}$;

(2) $g(p)=0 \rightarrow cp+d=0 \rightarrow p=-\frac{d}{c}$;

(3) $f(p)=g(p) \rightarrow \frac{b}{a}=\frac{d}{c} \rightarrow ad=bc$;

(4) $f(p)=g(p) \rightarrow ap+b=cp+d \rightarrow p=-\frac{b+d}{a+c}$

According to the equation $(f+g)(p)=f(p)+g(p)$, and above statements, the participants could deduce $(f+g)(p)=0$. Thus, $(f+g)(x)$ passes at point (p, 0).

Method 2. Let $f(x)$ and $g(x)$ intersect at x-axis (p, 0), then, $f(p)=0$, $g(p)=0$

So, $(f+g)(p)=f(p)+g(p)=0+0$ Thus, $(f+g)(p)=0$.

In method 1, the underlying thinking method is to find the coordinate of the intersection point p and check whether $(f+g)(p)=0$, while the strategy in method 2 is based on the definition of equation root and the definition of the sum of functions. Thus, the method 1 is mainly guided by the process perspective, while the method 2 is essentially guided by the object perspective.

The Figure showed that almost all U.S. participants gave up the effort to find a proof or provided some irrelevant statement. Only two of them gave some statements which were useful for developing a proof. On the other hand, in China, more than one-third of the participants provided a correct proof and other one-third provided a rough correct proof with minor mistakes. About 5 % of the Chinese participants left it blank, and another 6% just gave some related statements but failed to create a proof.

Perspective adopted in item 24. When looking at the perspectives or the strategies used in attempting to find a proof, the distribution of using different perspectives is shown in Table 4. 14. The Table showed that more than half of the participants (80% in high achieving group and 60% in low-achieving group) in China adopted the object perspective so that they can perform with functions themselves and provide a proof effectively.

Table 4. 14
Perspectives Adopted in Item 24

Perspective	Description	Frequency		
		China		US(%)
		High (%)	Low (%)	
Process	Method 1	11	8	2
Process	Method 1	80	62	0

U.S. participants' explanations to item 24. Larry is the one who gave two concrete examples to explore the intersection points. However, in the interview, she used a general form of linear function, $f(x) = a_1 x + b_1$ and $g(x) = a_2 x + b_2$, and got a correct proof. Jenny just gave two concrete examples to explore, and then got stuck. The other three gave up their efforts to explain because they do not like proving.

Summary of item 24. More than 60% Chinese participants could prove roughly correct proofs (half of them with some minor mistakes). More importantly, they could adopt an appropriate perspective of function, namely, object perspective. Thus, they can operate function as an object, so that they avoid the difficulty in finding the function expression itself. However, the U.S. participants simply gave up their attempts to find a proof.

Response to item 25. There was a significant mean difference of item 25 between China and the U.S. (MD=0.80, t=5.53, p<0.001; See Table 4.3). The score distribution of the item is shown in Figure 4.11.

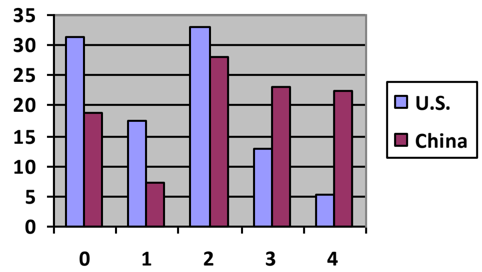

Figure 4.11. Score distribution of item 25.

This Figure showed that 45% of Chinese participants and about one-fifth (18%) of U.S. counterparts gave roughly correct answer and interpretation (3 or 4 scores). And about one-third (28.4%) of Chinese participants and more than one-third (33%) of U.S. counterparts either gave correct explanations or gave an appropriate interpretation. One-fifth (19%) of Chinese participants and one-third (31%) of U.S. counterparts gave useless information. For example, the Chinese participants gave correct answers as follows:

> "The student's explanation is correct. He/she links the life situation with the graph, if the x-axis represents time and the y-axis represents the vertical height. The graph can also be explained as a car driving. At the beginning, the driver speeds up, then drives it at a constant velocity, after that slows down, and finally drives at a constant velocity."

> "If the graph is seen as a height above sea level (h) and the time (t), then the students' opinion is right. They just understand function at a visualization level;

> This graph can also be used to present the changes of the stock market. In the morning, the price of the stock is increasing, and keeps the same during the recession at noon. In the afternoon, the price of the stock goes down, and finally stops at the price the same as the price at the beginning of the market."

16 U.S. participants (14%) pointed out that the student's interpretation could be improved by denoting x-axis as time while the y-axis as height above sea level. For example, one participant described it as follows:

> "That is a very creative answer, but [he/she] was not looking at the graph as a physical representation; we need to utilize it as a representation of data. The x-axis represents time while the y- axis represents height."

> Thirty one U.S. participants (31%) gave the situation of speed over time to illustrate the same diagram. For example, one participant gave "The graph could be showing speed vs. time where somebody is accelerating at an exponential rate, then goes a steady for period of time, and then slows at a constant rate, then stops."

Perspective adopted in item 25. It is necessary to have a connection of these two perspectives of function in order to get roughly correct answers. The different ways of explaining or interpreting the graph are displayed in Table 4.15

Table 4.15
Different Ways of Interpreting the Graph

Types of interpretation	Frequency		
	China		U.S. (%)
	High (%)	Low (%)	
Height vs. Time	20	12	4
Velocity vs. Time	51	39	31
Housing/Stock Price vs. Time	2	0	0
Temperature vs. time	2	3	0
Distance vs. Time	6	5	4

The Table showed that the majority of the participants explain, or interpret the graph as the graph of the relationship between velocity and time. It seemed that the participants who achieved a high score of KTA were in favor of a graph of velocity and time (51% in high achieving group in China, 39% in low achieving group in China and 31% in U.S. participants). The second frequent interpretation is the relationship of height and time. Again, the high score of KTA indicates the high frequency of interpretation with the relation of height and time (20 % in high achieving group in China, 12 % in low achieving group in China, and 4 % in U.S. participants).

U.S. participants' interpretation to item 25. Three of five interviewees (Larry, Alisa, and Stacy) realized that the original interpretation should be improved by pointing out the x-axis presenting time while y-axis presenting position (or height). All of them gave other examples of describing the diagram as the graph of speed over time, and two of them (Larry and Stacy) also mentioned about the graph of temperature over time. Two of them also mentioned they leaned the graph of distance over time in one course called math and technology using CBR. (The CBR 2 is TI's answer to an easy, affordable data collection device! Designed for teachers who want their students to collect and analyze real-world motion data, such as distance, velocity and acceleration).

Summary of item 25. The above analysis has shown that compared with the U.S. participants, the Chinese participants were more likely to give correct interpretations, as well as give more diverse interpretations. For those who gave correct interpretations, it is necessary to have flexibility in shifting ideas between process perspective and object perspective. Interview information further confirmed that U.S. participants generally demonstrated the appropriate knowledge about how to interpret the graph by using certain daily-life situations.

Summary of flexibility in selecting perspectives. The analysis of participants' responses to the three items suggests that compared with the U.S. participants, the Chinese participants demonstrated a flexibility in selecting appropriate perspectives of function concept, namely, *process and object.* Moreover, Chinese participants provided more diverse interpretations than the U.S. interpretations.

Flexibility in Using and Shifting Different Representations

The items 19, 21, 22, and 23 are deliberately designed and used for measuring knowledge for understanding and applying quadratic functions/equations/inequalities through flexibly using multiple representations. It is crucial for participants to flexibly use appropriate presentations and shift between different representations in order to solve them effectively. Regarding item 19, it is expected to have algebraic and graphic representations of equation and inequality. With regard to item 21, it is necessary to shift between algebraic and graphic representations in order to solve the problem. To solve the problem of item 22, it is necessary to have ability in translating graphic representations to algebraic representations. To solve the problem of item 23, it is required to use appropriate forms of algebraic expressions and transformations of different algebraic expressions, and translation between graphic and algebraic representations.

Response to Item 19. There was significant mean difference of this item between China and the U.S. (MD=2.90, t=34.62, p<.001). The score distribution of the item is displayed in Figure 4.12

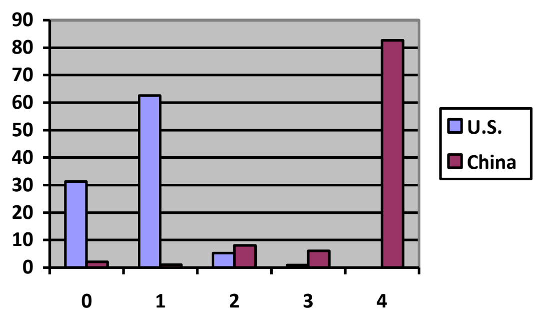

Figure 4.12. Score distribution of item 19.

The Figure showed that 82 % Chinese participants gave two essentially different solutions to the inequality, while only one U.S. participant gave two essentially different solutions. Moreover, about 6% of Chinese participants gave two correct algebraic solutions while only one U.S. participant did the same. In addition, one-third (31.3%) of

U.S. participants left it blank or gave some useless statements, while only 2.1% of Chinese participants did the same.

Strategies used in item 19. The different strategies or methods used to solve the inequality were displayed in Table 4.16. The Chinese participants demonstrated a high fluency and flexibility in solving the inequality. About 80% of participants from the high-achieving group and 51% of participants from low-achieving group gave two essentially different methods of solving the inequality. In addition, about 5% of the participants from each group gave two algebraic methods to solve the inequality. In contrast, the U.S. participants were struggling with solving the quadratic inequality (only two participants gave two methods correctly).

Table 4.16.
Different Methods of Solving Inequality in Item 19

Types of explanations	Frequency		
	China		U.S.
	High (%)	Low (%)	(%)
Two algebraic methods	5	6	0
Algebraic and interval sign	4	1	0
Algebraic and linear equation	1	1	1
Algebraic and quadratic function	80	51	1

Misconception Made by the U.S. Participants in item 19. The U.S. participants revealed numerous misconceptions and mistakes as displayed in Table 4. 17.

Table 4. 17

Misconceptions or Mistakes in Solving Inequality in Item 19

Types mistakes	Explanation	Examples	Frequency (%)
1	Misconception: if ab>0, then a>0, b>0	(x-3)(x+4)>0→ x-3>0, x+4>0, then x>3, x>-4.	37
2	Only Transforming into standard form	$x^2+x-12>0$ or $x^2+x>12$	21
3	Transforming into standard form and getting stuck	$x^2+x-12>0$ or x(x+1)>12 or x(x+1)=12	7.6
4	Working on the standard form with guess and check	$x^2+x-12>0$ → $x(x+1)>12$; →x>12, x+1>12→x>12, x>11	12
		$x^2+x>12$ → $x^2>x-12$ → $x>\sqrt{x-12}$	2.5
		$x^2+x-12>0$, or (x-3)(x+4)>0 →x_1=3, x_2=-4.	15
5	Drawing number line	Find partial answer: x>3 or x<-4	4
6	Using a table	x>3 (x=1, 2, 3, 4... or 0, -1, -2, -3,…).	4
7	ab>0→a>0/b or b>0/a	(x-3)(x+4)>0→ x-3>0/(x+4), then x>3	1.6

The Table showed that 44% of the U.S. participants adopted the inference: if *ab*>0, then *a*>0, *b*>0. None of them realized that *a* and *b* are possibly negative. In addition, none of them cared about the logical operations “or” or “and” between two logical propositions (such as *a*>0 and *b*>0 or *a*>0 or *b*>0). They also were satisfied with the solution “ $x>3, x>-4$ ” without any intention to further intersect or combine.

In order to find another method of solving the in-equation, an automatic alternative is to transform the factor form into standard form: $x^2 + x - 12 > 0$. 21% of them stopped with the standard form. 7.6 % of them were stuck with further algebraic operation: $x(x+1) > 12$ or $x(x+1) = 12$. Some of the participants went further with "guess and check strategies":

Mistake 1 (12%): $x^2 + x - 12 > 0 \rightarrow x(x+1) > 12$; →x>12, x+1>12→x>12, x>11.

Mistake 2 (15%): $x^2 + x - 12 > 0$,or (x-3)(x+4)>0 →x_1=3, x_2=-4.

Mistake 3 (2.5%): $x^2 + x > 12 \rightarrow x^2 > x - 12 \rightarrow x > \sqrt{x-12}$

In addition, there were some unexpected mistakes as follows:

"(x-3)(x+4)>0→ x-3>0, X+4>0, then X>3 & X>-4: -4<x<3";

"$x^2 + x > 12 \rightarrow x^2 > 12, x > 12 \rightarrow x > \sqrt{12}$";

"Solve by guess and check, x>3 because x=3 makes it zero, (x-3) =(x+4), $3 \neq 4$, so x >3";

"(x-3)(x+4)>0, if x=3, then "(x-3)(x+4)=0, not greater than 0, so (3, ∞)";

"$x^2 + x - 12 > 0$, $x^2 + x > 12$, x(x+1)>12, x>12 or x>11";

"$x^2 + x - 12 > 0$, $\sqrt{x^2} + x > \sqrt{12}$, 2x>$\sqrt{12}$ .x>$\sqrt{12}$ /2 "; and

"$x^2 + x - 12 > 0$, x^3>12."

Even though they were not able to find the correct answers, they took the risk of using different representations such as number line, or tabulation to explore the solution as follows(Figure 4.13):

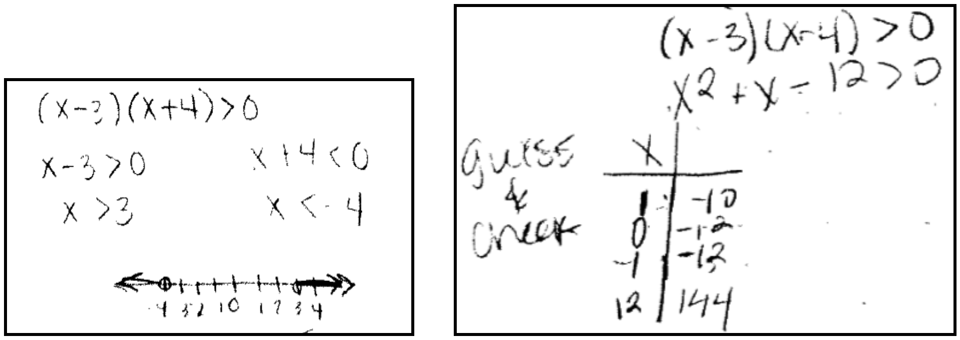

Figure 4.13. Different representations in solving inequality in the U.S.

In summary, it is encouraging that the U.S. participants tried to use different representations to explore different ways of solving the inequality, and take the risk of "guess and check". However, it is disappointing that nobody had attempted to use the graphic method, and almost all (except for one) were not able to provide a correct solution. Moreover, numerous misconceptions and mistakes were revealed when using the strategy of guess and check.

U.S. participants' explanations to item 19. One of the interviewees (Larry) just simplified the factored form into standard form ($x^2+x-12>0$) and then got stuck on handling the solution. She moved forward by "guess and check" such as "$x^2>x-12$, and then $x>\sqrt{x-12}$". She just square rooted them, even though it did not work (she knows that dividing something in inequality, the in-equal sign may be changed, but she did not memorize the exact rules).

By analogizing the property of equation: (x-3)(x+4)=0 $\rightarrow$x-3=0, or x+4=0, the remaining four interviewees made an inference as follows:

$(x-3)(x+4)>0 \rightarrow (x-3)>0,\ (x+4)>0 \rightarrow x>3, x>-4$.

In order to find a second method, Kerri took a risk by guess and check: $(x-3)(x+4)>0 \rightarrow x^2+x-12>0 \rightarrow x(x+1)>12 \rightarrow x>3$, while Stacy used root formula $x_{1,2}=\frac{-1+\sqrt{1-4(1)(-12)}}{2}$ =3 or -4, and got the same solution x>3, x>-4.

Nobody intended to work on "x>3, x>-4" further, such as the logical operations "and" or "or" and the operations of intersection and combination of sets. They seemed to be satisfied with the "solution".

When asked "if *ab*>0, what result can you deduce?" they realized that "if *ab*>0, then *a,b* both are positive, a, b both are negative". Then they realized that there are other solutions of the inequality. Kerri used a number line to find a correct solution x>3 and x<-4. Other two (Alisa and Stacy) guessed that x<-4 should be part of solutions. However, they still worked with algebraic representation to solve this inequality.

When asked if they can use a graphic method to solve equation or inequality, they recalled the graphs of quadratic equation. Aside from Jenny (she drew one without intersection with x-axis), the others drew a correct sketch and found the correct solutions with the support of the researcher. Moreover, Kerri not only presented the solutions by number line, but also drew two lines y=x-3 and y=x+4 to show how to use the graph of a linear equation to find the solutions of (x-3)(x+4)>0.

All the interviewees explained that they did not know how to use the quadratic function graph to solve inequality, although they knew the graphing method of solving linear equation. They learned quadratic function first (probably later at middle school or early at high school) and then inequality later at high school. These contents were taught separately. They were not taught how to use graphic representation to solve algebraic problems. They appreciated the method of integration of algebraic and graphic representations. Stacy said "it will be better to teach students with two methods, because some students are visual learners while others are algebraic learners."

In summary, in the U.S. sample, only one participant provided two correct algebraic solutions. Almost all of the U.S. participants were struggling with algebraic computation of inequality, with numerous mistakes when guessing and checking. However, the interviews showed that participants may have relevant content knowledge (such as quadratic function and its graph, quadratic equation, and inequality), but they did not have an interconnected knowledge network; they do not have problem solving experience in flexibly using different kinds of knowledge and relevant representations. However, when appropriately enlightened, they were able to build the connection between different types of relevant knowledge and find an appropriate solution. In addition, as pointed by the interviewees, the placement and presentation of the contents in textbooks and the ways of teaching the contents in classroom did not support them to build the connection between algebraic and graphic representations. This raises an important issues related to teachers' knowledge development.

Chinese participants' strategies used in item 19. On the other hand, the Chinese participants provided multiple methods to solve the inequality. Four- fifths of the participants provided two essentially different solutions (one is using algebraic manipulation and the other is using a graphing method). They provided correct procedural steps. There are no basic mistakes such as those made by the U.S. participants. For example, one participant gave the following two typical solutions:

Method 1: since (x-3)(x+4)>0, them (x-3)>0 and (x+4)>0$\Rightarrow$,x>3 and x>-4 or x-3<0 and x+4<0, $\Rightarrow$ x<3 and x<-4. So the solution is $\{x \mid x > 3 \text{ or } x < -4\}$;

Method 2 (graphing method): According to the graph of function f(x)=$(x-3)(x+4) = x^2 + x - 12$, when $x \in (-4,3), f(x) < 0.$ when $x \in (-\infty,-4) \cup (3,\infty), f(x) > 0.$ Thus, the solution of the inequality is $(-\infty,-4) \cup (3,\infty).$

One student gave a graphing method as follows: Sketch two lines: y=x-3 and y=x+4, then find the common regions where both lines are positive (above the x-axis) or negative (below the x-axis). The x-coordinate ranges of those regions are the solutions.

Summary of item 19. The above analysis showed that Chinese participants had sound knowledge in solving the inequality both algebraically and graphically. However, the U.S. participants were lack of this knowledge and skills in solving inequalities by using graphic methods, and made a lot of basic mistakes when trying to finding solutions. They did not realize that they can solve the inequality by quadratic function graphs.

Response to Item 21. Item 21 is used to measure teaching and students knowledge of solving quadratic equation. It is necessary to link algebraic and graphic representations. There was a significant mean difference of this item between Chinese and the U.S. participants (mean difference=2.79, t=31.60, p<.001). The score distribution of this item is displayed in Figure 4.14.

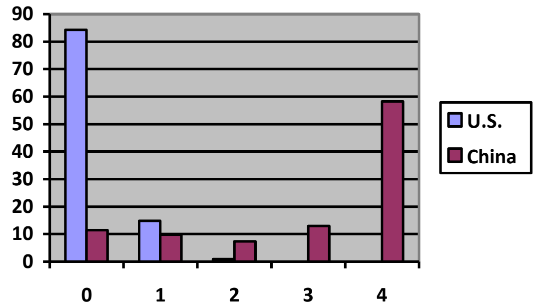

Figure 4.14. Score distribution of item 21

In the U.S. participants, only one gave correct explanations and useful suggestions as showed below.

(a) The student believes that he needs to know the values of *a,b,c*. He can't find these values because there are three variables in two equations.

(b) The student needs to think graphically. Since at x=1, the value is positive, then the graph is above the x-axis. The opposite is true when x=6. Therefore, the graph has to cross the x-axis and since it has degree two. It must have 2 solutions.

About 84% of them agreed with this student's explanation (actually, it is wrong), and they were stuck with the algebraic operation to find *a,b* and *c*, and had no idea about how to help the student get out of their difficulties. 15 % of the participants suggested plugging different values of *a, b*, and *c* (such as *a*=-10, *b*=-9, and *c*= 20) to see whether some patterns can be found. It is disappointing that when facing difficulty in using algebraic representation, they have no idea on how to think with graphic or geometrical representations.

In the Chinese participants, about three-fifths (58.2%) of the participants gave appropriate explanations of students' mistakes and provided a correct answer. For example, one participant gave detailed explanations of the student's reasons for his/her solution:

"Reasons: First, although the student masters some methods of solving problems, she/he directly applies that knowledge without considering the specific conditions of the problem. He/she was constrained by routine thinking methods; second, the student did not recall the method of judging zero points of function. The method can be used flexibly.

Suggestions:

(1) if you are not able to find *a,b*, and *c*, by using the given conditions (when x=1, the value is positive while x=6, the value is negative), why not try other methods ? Are there any ways which do not require to find a, b and c?

(2) Although we are not able to find accurate values (of roots) by using algebraic methods, then, why not use graphing method to make estimations? It should be helpful to guide students to draw a figure (similar to the Figure 4.13)

(3) Through observing this Figure, are there any intersection points of the function at x-axis?

(4) How many intersection points are there? According to the features of quadratic function, students could solve this question?

(5) Connecting zero points of a function to the roots of an equation lets students understand that learned knowledge can be flexibly applied to solve this problem. ”

For another example, one participant gave a solution (See the Figure 4.15).

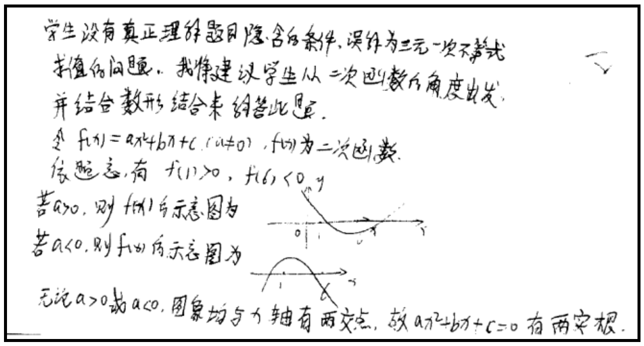

Figure 4. 15. An example of Chinese answers to item 22.

The participant believed that “the student did not fully understand the hidden condition of the problem and mistreated it as a problem of finding solution of a quadratic inequality”. S/he further suggested the student is to consider the problem by integration of algebra and geometry from a perspective of quadratic function. Then s/he drafted two sketches of the quadratic function ($f(x) = ax^2 + bx + c$, a>0 or a<0) according to the given conditions $f(1) > 0, f(6) < 0$, and concluded that there are two roots of the quadratic equation.

Another 13% of the participants identified students’ problems and gave correct answers with minor computational or notational errors. Other 7 % of the participants realized students mistakes and suggested using graphing method, but without any details.

About 10 % of them made more efforts to find *a,b* and *c* or judge the sign of discriminates. The remaining 10% left the item blank or wrote something not useful for solving the problem.

However, some participants tried to judge the numbers of roots based on the sign of $\Delta = b^2 - 4ac$, then they got stuck to some inappropriate algebraic manipulations. For example, one participant gave the following explanation:

Solution: if x=1, a+b+c>0; if x=6, 36a+6b+c<0, so 35a+5b<0; 7a+b<0, b<-7a.

(1) If b>0, c>0, then, a<0, then $b^2 - 4ac > 0$, then the equation has two roots.

(2) If b>0, c<0, then, a<0…

It is important to learn how to discuss and solve a problem according to different parameters, based on the sign of $\Delta \geq 0$ (two different roots, no real roots, and two equal roots) .Since there are several parameters, usually, we fix the values of some parameters, then adjust other parameters. Thus, the discussion will be very clear. (SC04-28).

Strategies used in item 21. The interpretations used in solving item 21 are displayed in Table 4.18

Table 4.18
Interpretations Used in Item 21

Interpretations	Frequency		
	China		U.S.
	High (%)	Low (%)	(%)
Correct explanations and correct graphing solutions	76	65	1
Using graphing method in general without providing solutions in detail	4	8	0

As analyzed above, the Chinese participants demonstrated an ability to use graphic methods to solve the algebraic equation. Moreover, compared with the low-achieving group, the high-achieving group seems to provide more complete and detailed solutions using graphing method.

In Even's (1998) study, it was found that only 14% of the 152 pre-service secondary mathematics teachers in the U.S. correctly solved this problem, and about 80% of them did not show any attempt to look at another representation of the problem. In the current study, only 1% of the 115 U. S. gave a correct answer while around 58% of 376 Chinese counterparts gave fully correct answers. The U. S. subjects in this study performed very poorly, and it may be due to the 80% of the U. S. participants prepared to be middle school math and science teachers. The correct rate of the Chinese pre-service teachers is higher than that (14%) of pre-service mathematics teachers in Even's (1998) study. Thus, the Chinese participants demonstrate strong knowledge and skills in shifting between symbolic and graphic representations.

U.S. participants' explanations to item 21. Three of the participants (Larry, Kerri, and Alisa) fully agreed with the student's statement. Namely, "Since it is impossible to find out fixed values of *a, b* and *c* based on the previously given inequalities, the original question is not solvable". They tried to find out *a,b*, and *c* through algebraic transformation but it did not work. They had no idea on how to help the student find a solution.

The other two interviewees (Jenny and Stacy) felt the problem may be solved, but they did not have any concrete ideas on how to solve it. What they could suggest to the student is to "try different ways, such as plugging more numbers between 1 and 6."(Jenny) or "explore in different ways such as plugging more numbers to see whether they can find certain patterns, rather than being stuck" (Stacy).

When asked whether they can try other methods such as graphical methods to solve, they tried to sketch the graphs and find the possible roots. Except for Larry, others were successful in finding the number of roots by examining the intersection points of the quadratic function. All of them said they had not thought in this way, they had not gotten these kinds of experiences in solving problems, but they realized the usefulness of graphing method in algebra.

Summary of item 21. The Chinese participants (more than 60%) demonstrated flexibility in using graphic representations to solve this problem, and only a small part of them (20%) were stuck with algebraic operations. However, in the U.S. counterparts, the majority of them (85%) struggled with algebraic manipulations, and failed to find correct answers and explain the students' mistakes. Less flexibility in using graphic representations was revealed when finding the number of roots of the quadratic equations.

Response to Item 22. This item is used to measure knowledge for understanding of the effects of changes of parameters of quadratic function on the changes of quadratic graphs. There was a significant mean difference between China and the U.S. (MD=1.42, t=-8.44, p<0.01). The score distribution of the item is displayed in Figure 4.16.

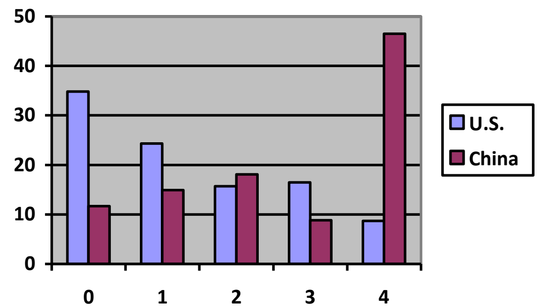

Figure 4.16. Score distribution of item 22.

About 10% of U.S. participants gave correct answers and appropriate explanations and 17% of them gave correct answers but failed to explain. Fifteen percent of the participants gave partially correct answers and explanations. Twenty three percent of them gave sporadic information about the effect of *a, b* and *c* changes. About 35% of them got lost, either leaving it blank or providing some wrong statements. In summary, one-third of U.S. participants had no ideas on solving and explaining this problem while about one-fourth of them gave roughly correct answers.

The graph also showed that 46.5% of Chinese participants gave the correct choice and appropriate explanations. They either explained by analyzing the effects of changes of *a*,*b*, and *d* on the changes of the graphs or analyzing the symmetrical features. For example, the following are some excerpts:

> "Because the change *a* results in changes of graph in the openness and wideness, however, the translated graph does not change the shape, so *a* is not changed. Since c represents the *y*-intercept, because the two graphs intersect y- axis at the same point. So *c* is not changed. So, only *b* can be changed." (Method 1, SC04019)

> "According to the graphs, a>0, the two graphs are symmetrical with regard to y-axis, thus, the symmetry line is also symmetrical with regard to y-axis. The symmetrical line of the left graph is $x=-\frac{b}{2a}$, so the symmetrical line of the right graph should be $x=\frac{b}{2a}$. Thus, only the b changed as –b" (Method 2, SC409).

> "Let original form: $y=ax^2+bx+c$, and the translated one: $y_1=a_1x^2+b_1x+c_1$. Sine y-intercepts are the same, namely, x=0, y=c= $y_1=c_1$, so, c=c_1. Since the graph y1 and y are symmetrical with regard to y- axis, so :
>
> $y=ax^2+bx+c=y_1=a_1(-x)^2+b_1(-x)+c_1$
>
> $\Rightarrow ax^2+bx=a_1x^2-b_1x \Rightarrow a=a_1, b=-b_1$. Thus, it is only needed to change *b* value." [Method 3, SC411]

Another 9 % of the participants gave a correct choice, but their explanations had minor errors. About 18% of the participants made correct choices and their explanations had serious mistakes.

Another 15% gave wrong choices but provide some useful explanations, and the remaining 11.7% just gave up any efforts to answer the item.

The strategies used or mistakes made in item 22. The following Table (Table 4.19) showed the different interpretations used.

Table 4.19.
Interpretations Used in Item 22

Interpretations	Frequency		
	China		U.S.
	High (%)	Low (%)	(%)
The effects of changes of *a, b* and *c* on the changes of graphs of quadratic functions. (Method 1)	7	14	11
Symmetrical line is $x = -\frac{b}{2a}$, *a* is invariant, then *b* can be changed only (Method 2).	31	27	2
According to g(x) =f(-x), find the coefficients of g(x) ($a_1 = a,\ b_1 = -b, c_1 = c$). (Method 3)	11	14	0

The Table showed that the Chinese participants used more ways (see previous explanation for details of the three methods) to interpret their answers than their U.S. participants did (two methods). Chinese participants not only used the general results of the effects of change of parameters on the changes of graphs (method 1), but also use the properties of symmetry both geometrically (method 2) and algebraically (method 3).

However, the U.S. participants mainly used the method 1 which was taught in typical texts.

Meanwhile, since the Chinese participants used more sophisticated algebraic multiplication, they made some errors. For example, one participant tried to transform the function expression as follows:

Because the translated function should be:

$g(x) = f(x-k) = a(x-k)^2 + b(x-k) + c = ax^2 - (bk+2ak)x + (ak^2 - bk + c)$.

Thus $ak^2 - bk + c = c$, $ak^2 - bk = 0$, k=0 or $k = \frac{b}{a}$. So at least two of these parameters of *a,b, c* should be changed.(SC04-22)

Similarly, another participant tried to find the vertex point: $x_1 = \frac{b}{2a}$, $y_1 = \frac{4ac - b^2}{4a}$, and explained that " since the y1 is the same[at the two vertex points of the two graphs], so only x1 could be changed. Thus, at least two parameters need to be changed. " (SC-04-24).

In Black's (2008) study, 20% of 76 U. S. high school mathematics teachers gave correct answers and relevant explanations to this problem. In the current study, 25% of 115 U. S. participants gave correct answers and explanations while 55% of 376 Chinese counterparts did the same. In this item, the U. S. participants in this study performed better than the subjects in Black's study. The Chinese participants in this study outperformed the U. S. counterparts remarkably.

U.S. interview participants' explanations to item 22. In the interview, two participants (Larry, and Kerri) clearly explained the effects of changing *a, b*, and *c* on the graphs of quadratic function, even though Kerri made a wrong choice in the survey.

Alisa and Stacy were able to explain the effect of changing *a* and *c* on the graph of quadratic function, but they were not sure about the effect of changes in b. Alisa made mistakes in drawing graph $f(x+h)$. Stacy knew how changes of *a* and *c* impact the changes of the graph but she was not clear about how changing *b* can impact the changes of graphs although she got a correct choice.

Jenny found the correct answer by explaining that changing b to negative b, the graph of quadratic function would reflect it over the y-axis because she "did a lot of exercises of translation of graphs in high school". However, she could not remember the details of the effect of changing *a*,*b* and *c* on the graph.

In summary, two of the participants were quite clear about how the changes of a, b impact on the changes of the graphs of quadratic function. Others were not quite sure how changes of these parameters impact on the changes of the graphs of quadratic function. One participant got the correct answer by relating the symmetric property, although she was not clear about the details of the effects of changing *a, b* and *c* on the graph.

Summary of item 22. More than half of the Chinese participants provided the correct choice and roughly appropriate explanations (46.5%+8.8%=55.3%) while only about one fourth (10%+17%=27%) of the U.S. counterparts did the same. In contrast, about one tenth (11%) of Chinese participants gave up any attempts to solve the problem, while more than one-third (35%) of the U.S. participants gave up. In addition, Chinese participants adopted more diverse ways to interpret by using the connections between geometrical symmetry property and algebraic function features, while their U.S. counterparts mainly used a routine way to interpret.

Response to item 23. There was a significant mean difference of item 23 between China and the U.S. (MD=3.00, t=33.01, p<.001). The score distribution of the item is displayed in Figure 4.17.

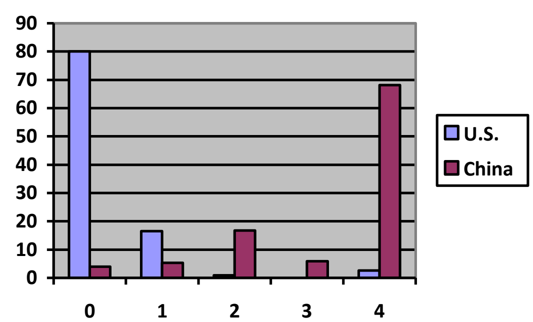

Figure 4.17. Score Distribution of Item 23.

Only three U.S. participants found the quadratic equation by solving a system of linear equations and found the maximum correctly. One participant found the quadratic equation but failed to find the maximum. About 20 % of the participants just drew a graph or list equations based on the given three points. Four-fifths of them left the item blank or wrote some useless statement.

The Figure showed that 67.8% of the Chinese participants correctly solved this problem. Usually, they used standard quadratic formula (i.e., $y = ax^2 + bx + c$) to find the function expressions, and then transformed it into a form of the vertex point (i.e., $y = a(x-h)^2 + k$) to find out the maximum. However, multiple strategies were used to find the solutions (see Figure 4.18). In one method, the special quadratic formula: $y = a(x - x_1)(x - x_2)$ was used, while in the other methods the standard quadratic formula $y = ax^2 + bx + c$ and Vièta theorem: $x_1.x_2 = \frac{c}{a}$, $x_1 + x_2 = -\frac{b}{a}$ (a=-2) were used.

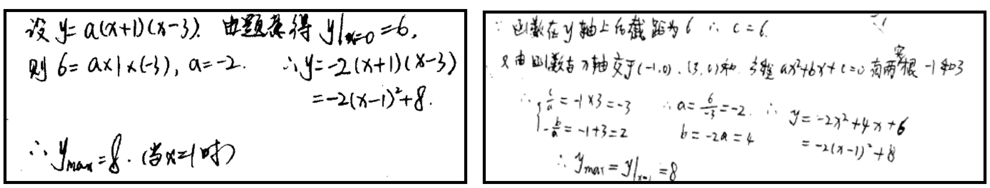

Figure 4. 18 .Two methods used in item 23 in China.

About 6 % of the Chinese participants found the correct quadratic expressions and tried to find the maximum by forming a perfect square format, however, they made mistakes in computation. Another 16.8% of the Chinese participants correctly found the quadratic expression, without further action to find maximum. About 5% of the Chinese participants can only find part of *a,b* and *c,* but failed to find the expression ,and the remaining 4% just left the item blank.

Strategies used in item 23. With regard to the strategies used to solve this problem, there are several methods. First of all, three forms of quadratic formula methods:

$y = ax^2 + bx + c$ (FM1); $y = a(x - x_1)(x - x_2)$ (FM2); and $y = a(x - h)^2 + k$ (FM3) could be used for finding the quadratic function expression. Then, three methods could be used for finding the maximum value: (1) transforming into $y = a(x - h)^2 + k$, then finding the maximum (MM1); (2) using formula $x = -\frac{b}{2a}$, $y_{maximum} = \frac{4ac - b^2}{4a}$ (MM2); and (3) taking derivative: y'=0, x=1, then, $y_{maximum} = f(1)$ (MM3). The strategies used are shown in Table 4.20.

Table 4.20
Different Strategies in Using Representations in Item 23

Strategies	Frequency		
	China		U.S.
	High (%)	Low (%)	(%)
1.FM1+MM1	22	18	3
2.FM1+MM2	28	32	0
3.FM2/FM3+MM1	12	1	0
4.FM2/FM3+MM2	8	10	0
5.FM1/FM2/FM3+MM3	4	1	0

The Table showed that the Chinese participants provided various strategies to solve the problems. The high achieving group showed more variability in adapting strategies (five methods) than did the low achieving group (essentially including three methods). Moreover, some participants (20% in high achieving group and 11% in low achieving group) demonstrate an ability to select the most appropriate formulas (strategies 3 or 4).

The following are some examples of flexibly using formula and properties of quadratic functions. For example, a Chinese participant from the high achieving group gave two methods:

> Method 1: using standard formula to find the quadratic function and make a completing the square formula and find the maximum (i.e. strategy 1).
>
> Method 2: according to the given condition, the symmetrical line is x=1.
>
> Let $f(x) = a(x-1)^2 + h$.
>
> Because f (-1)=0, f(0)=6.
>
> So, 4a+h=0 and a+h=6 $\begin{cases} 4a+h=0 \\ a+h=6 \end{cases}; \begin{cases} a=-2 \\ h=8 \end{cases}$, thus, $f(x) = -2(x-1)^2 + 8$. Thus the maximum is 8.(i.e. strategy 3) (SC04-15)

Another participant from the low-achieving group also gave two methods as follows:

> "Solution: c=6.
>
> Method 1: let $ax^2 + bx + c = 0$. $x_1 = -1, x_2 = 3$ are two roots, then,
>
> $x_1.x_2 = \frac{c}{a}$, a=-2; $x_1 + x_2 = -\frac{b}{a}$, b=4
>
> Thus, f(x)=-2 $x^2 + 4x + 6$, and symmetrical line is x=$\frac{x_1 + x_2}{2} = 1$,
>
> F(x)$_{max}$= $f(1) = 8$.
>
> Method 2: let y=a $(x-3)(x+4)$, plugging x=0, y=6, find a =-2, plugging the symmetrical line x-coordinate (x=1) and find the maximum." (LN4-02)

It is impressive that the participants flexibly used the Vièta theorem in method 1 and wisely chose an appropriate formula of the quadratic function in method 2.

The U.S. participants revealed a lack of basic skills of algebraic computation. In the following section, we try to get a better understanding of U.S. participants' thoughts about solving this problem.

U.S. participants' explanations of their solution. Two of the participants (Larry and Stacy) knew the process of solving the problem: finding the expression of quadratic equations by plugging given points, and then finding the maximum by taking derivative. Two of them (Larry and Alisa) realized that the maximum should be at x=1 due to its symmetry although they were not able to find correct expression of quadratic equations. Two of them (Kerri and Jenny) supposed that y-intercept is the maximum. They had difficulties finding the expression by plugging in the given points.

When asked whether they can use other formulas of quadratic equations to find the expression more effectively, they had no ideas about these formulas. Even when I showed them some formulas (such as $y = a(x - x_1)(x - x_2)$ or $y = a(x - h)^2 + k$), they did not know about them.

In summary, the teachers tried to find expressions of quadratic equations by using the standard formula, and then found the maximum by taking derivative and plugging x=1. However, they had difficulty in finding the correct expression due to the complexity of algebraic manipulation. Some of them tried to use the symmetry to find maximum. In addition, nobody was aware of using other appropriate formulas to find the expressions as Chinese counterparts did. It seems that the standard formula of quadratic equation is the only one they were familiar with.

Summary of item 23. The analysis of the responses to the item 23 showed that Chinese participants demonstrated a sound and flexible knowledge for solving this problem. They not only could shift from process perspective (point-wise) to object perspective (graphic and algebraic representations) but also and flexibly selected most appropriate formula (algebraic representations) to solve this problem. On the other hand, the U.S. participants revealed a shortage of using relevant knowledge to solve the problem.

Summary of the representational flexibility in China and the U.S. The analysis of the responses to the four items which focused on solving and interpreting quadratic functions/equations/inequalities provides a consistent picture of teacher knowledge for teaching the concept of quadratic relation. Overall, the Chinese participants not only demonstrated a sound knowledge needed for teaching the concept, but also showed the flexibility in using representations appropriately. In contrast, the U.S. counterparts revealed their shortage of basic knowledge for teaching the concept and flexibility in using representations.

An Analysis of Correlation between Flexibility and Other Variables

Due to the importance of developing teachers' flexibility in using appropriate representations, I developed an indicator of flexibility. Flexibility in this study is defined as a shift between different representations or a transformation between different forms within a representation.

For example, in item 22, if a participant explained as follows: "since the symmetrical line is $x = -\frac{b}{2a}$ while the openness of the graph is not changed so the parameter *a* will not be changed, thus, only b can be changed", I coded the response as flexibility (shift from graphic representation to algebraic representation). So, each of the three strategies used (see Table 4.19) can be coded as a flexibility.

In another example, in item 23, if a participant adopted standard form of quadratic function (i.e., $y = ax^2 + bx + c$) to find the expressions and then reorganize it as a form of $y = a(x-h)^2 + k$ in order to find out the maximum, I coded a flexibility (transformation within algebraic representation). Thus, each of the strategies 1-4 used for solving item 23 (see Table 4.20) can be coded as a flexibility.

The number of times that flexibility occurred in items 19, 21, 22, & 23 were counted as the value of a variable denoted as flexibility, ranging from 0 to 4. The indicator of flexibility was examined through two aspects: (1) multiple group comparison; and (2) regression analysis with regard to SM, AM, and TM as independent variables in different groups.

Mean difference across different groups. The means scores and standard deviation of flexibility are displayed in Table 4.21

Table 4. 21

Mean Scores and Standard Deviation of Flexibility

	Mean	St. Deviation
1.China High-achieving	3.09	1.050
2.China Low-achieving	2.76	1.190
3.U. S.	.29	.491

Multiple group comparison showed that the mean difference between China high-achieving and low-achieving group is significant (mean difference=.33, p=0.008). The mean difference between China low-achieving group and the U.S. group is also significant (mean difference=2.47, p=0.000). Thus, there are significant differences of flexibility between different groups of participants. The higher the KTA score the participants have, the more flexible they are in selecting representations.

Prediction of different components of KTA. Using flexibility as the dependent variable and three components of KTA as independent variables, I did a regression analysis in different groups. (let y=Flexibility, x_1=School mathematics, x_2=Teaching mathematics, x_3=Advanced mathematics)

In China high-achieving group, the regression equation is as follows:

$$y = -1.09 + 0.12x_1 + 0.17x_2$$

Knowledge of school mathematics and teaching mathematics explains 56% of the variance in flexibility ($R^2 = 0.56$, $F(2)=67.16$, $p<0.001$).

In China low-achieving group, the regression equation is as follows:

$$y = -1.69 + 0.26x_1 + 0.12x_2$$

Knowledge of school mathematics and teaching mathematics explain 74% of variance in flexibility ($R^2 = 0.74$, $F(2)$=200.9, p<0.001).

In the United States, the regression equation is as follows:

$$y = -.50 + 0.03x_1 + 0.09x_2$$

Knowledge of school mathematics and teaching mathematics explain 42% of variance in flexibility ($R^2 = 0.42$, $F(2)$=40.80, p.<0.001).

However, in the whole sample (including Chinese and the U.S. participants N=371), the regression equation is as follows:

$$y = -1.41 + 0.19x_1 + 0.12x_2 + 0.03x_3 .$$

Knowledge of school mathematics, teaching mathematics and advanced mathematics explain 84% of variance in flexibility ($R^2 = 0.84$, $F(3)$= 663.84, p.<0.001).

At the same time, flexibility is highly correlated to each of school mathematics(r=.87, p<.001), teaching mathematics (r=.86, p<.001) and advanced mathematics (r=.81, p<.001).

In summary, overall, the higher KTA scores the participants achieve, the higher flexibility they have. Meanwhile, the flexibility can be significantly predicted by KTA and it is highly correlated to all the components of KTA.

Summary of the Findings

The findings of this study can be summarized in line with four research questions: (1) the differences and similarities of KTA in Chinese and U.S. pre-service teachers, (2) the relationship between different components of KTA, (3) the difference and similarities of knowledge for teaching the concept of functions, and (4) the relationship between pre-service teachers' status of KTA and their courses taken.

The Differences and Similarities of KTA in Chinese and U.S. Pre-service Teachers

In all 17 multiple choice items, except for four items, the Chinese participants had significantly higher mean scores than their U.S. counterparts. In all 8 open-ended items, the Chinese participants significantly outperformed their U.S. counterparts. Based on detailed examination of individual items, I found that: (1) the U. S. participants showed a better understanding of introducing the slope concept from multiple perspectives than the Chinese counterparts; (2) both the U. S. and Chinese participants revealed weaknesses in presenting numerical relations and algebraic equations using geometrical representations; (3)the Chinese participants tended to make their judgments based on visual and graphical information and underlying conceptual understanding and logical reasoning while their U. S. counterparts tended to make their judgments mainly based on visual information without paying close attention to underlying concepts and logical reasoning; and (4) the Chinese participants demonstrated strong knowledge and skills in algebraic manipulation and quadratic functions/equations/inequalities.

The Relationship between Different Components of KTA

With regard to the U.S. sample, school mathematics was found to have significant effects on teaching mathematics and advanced math. Advanced math was found to have a direct and significant effect on teaching math. Regarding the Chinese sample, school mathematics was found to have direct and significant effects on advanced math and teaching math.

Difference and Similarities of Knowledge for Teaching the Concept of functions

The analysis on the items of measuring teachers' knowledge for teaching the concept of function revealed the following results: (1) the Chinese participants demonstrated sound knowledge and skills needed for solving the problems and interpreting their solutions, while their U.S. counterparts revealed their limitations in basic knowledge and skills; (2) the Chinese participants showed flexibility in selecting appropriate perspectives of function concept while the U.S. counterparts showed disadvantages in adopting appropriate perspectives; and (3) the Chinese participants demonstrated flexibility in using multiple representations while the U.S. counterparts revealed limited knowledge and ability in adopting multiple representations appropriately. In addition, the Chinese participants were willing to provide more diverse interpretations than their U.S. counterparts.

There are significant differences of flexibility between China and the U.S. The Chinese participants demonstrated greater flexibility in using representations and perspectives than their U.S. counterparts. Overall, the KTA scores are highly correlated to the flexibility and the flexibility can be significantly predicted by KTA.

The Relationship between KTA and Courses Taken

In China, the number of math courses taken was found to have a significant effect on advanced mathematics, but the effects of the number of courses taken on school mathematics and teaching mathematics were found not significant. In the U.S., the number of math courses taken was found to have significant effects on school mathematics and teaching mathematics, but it did not have significant effect on advanced mathematics. These findings imply that the Chinese teacher preparation programs may emphasize content knowledge while the U.S. teacher preparation programs may emphasize pedagogical knowledge.

CHAPTER IV

CONCLUSIONS AND DISCUSSIONS

Before discussing my findings, I would like to point out the disparity of sampling and courses taken between China and the U.S. First, the U.S. sample was a convenience sample taken from an interdisciplinary middle grade math and science program in a large public university. The U.S. participants came primarily from one of three routes of preparing middle grade mathematics teachers: a program designed exclusively for middle grade teachers' preparation. The Chinese participants were sampled from a pre-service preparation program (there is only one type of mathematics teacher preparation program for middle and high schools in China although there are variations regarding course design and arrangement) from purposely selected universities (high, normal, and low reputation universities). On average, the U.S. participants had taken seven mathematics content and mathematics education courses, while the Chinese participants had taken 14 such courses. So caution should be taken in interpreting the findings of this study in which comparisons are made between Chinese and U.S. participants due to the disparity of the sample.

In this chapter, I first summarized and discussed the main findings of this study in line with the research questions. Then, I discussed the limitation of this study and proposed some topics for further studies.

Conclusions

Knowledge for Teaching Algebra in China and the U.S.

Based on our survey, overall the Chinese participants had a much greater knowledge for teaching algebra than their U.S. counterparts. When looking at the items in detail, several interesting results were found as follows: (1) the U.S. participants showed a better understanding of introducing the concept of slope from multiple perspectives than the Chinese counterparts; (2) both the U.S. and Chinese participants revealed weaknesses in presenting numerical relations and algebraic equations using geometrical representations; (3) the Chinese participants tended to make their judgments based on visual information and underlying conceptual understanding and logical reasoning while the U. S. participants tended to make their judgments mainly based on visual information without paying close attention to underlying concepts and logical reasoning; and (4) Chinese participants demonstrated strong knowledge and skills in algebraic manipulation and quadratic functions/equations/inequalities.

These findings are parallel to Ma's (1999) findings that Chinese elementary mathematics teachers had a profound understanding of fundamental mathematics knowledge, and An et al.'s (2004) observation that Chinese middle grade mathematics teachers emphasized developing students' understanding and mastering knowledge through rigorous and traditional methods, when compared with their U.S. counterparts. Moreover, Li and his colleagues (2008) found that the secondary (including middle grade) mathematics preparation program in China put great efforts to develop students' sound and broad content knowledge and mathematics education knowledge. Thus, it is reasonable to expect that Chinese pre-service teachers may have sound mathematics content knowledge. On the other hand, it is a publicly acknowledged observation that many U.S. mathematics teachers do not have the adequate mathematics knowledge that they need to teach (e.g., Ball & Bass, 2000; CBMS, 2001). The study further alerted that the U.S. pre-service middle grade teachers need to make great efforts to meet the recommendations by influential documents (CBMS, 2001; Kilpatrick et al., 2001; NMAP, 2008). The weaknesses of the U.S. middle grade mathematics preparation programs were identified by international comparative studies (Babcock et al., 2010; Schmidt et al., 2007). The teacher preparation programs in East Asia including Korea, Chinese Taiwan demonstrated their strengths in mathematics content knowledge and pedagogical content knowledge compared with the U.S. ones. This study seems to suggest that the middle mathematics teacher preparation programs in China shares some features with other East Asian teacher preparation systems such as emphasizing mathematics content knowledge and pedagogical content knowledge, rather than pedagogical knowledge in general.

The weakness in presenting numerical relations and algebraic equations using geometrical representations in China and the U.S. calls for preparing teachers with connections of different brand of knowledge and flexibility in using different representations if we want to implement mathematics curriculum as recommended by NCTM (2000, 2009). It was recommended that students should understand the meaning of equivalent forms of expressions, equations, inequalities, and relations (NCTM, 2000), the connection between algebra and geometry, the link of expressions and function, and the flexible use of representations (NCTM, 2009). Thus pre-service teachers should be equipped with relevant knowledge and skills so that they may be able to organize their classroom instruction with the necessary learning opportunities.

The observation that Chinese participants tended to make their judgments based on underlying conceptual understanding and logical reasoning while the U. S. tended to make their judgments mainly based on visual information may partially echo Cai (2005) finding that Chinese teachers put more value on abstract representation than U.S. counterparts. Pre-service teachers may be able to bring their learning experience in pre-university into their reasoning and decision making (Ball, 1990). Some studies found that Chinese students preferred to use abstract representations (Cai, 1995), and performed better in tasks required no-visual representations (Brenner, Herman, Ho, & Zimmer, 1999), compared with U.S. counterparts. Moreover, comparing the ways to prove Pythagoras's theorem, Mainland Chinese teachers' preferred to use mathematical proofs with algebraic manipulation rather than using visual verification as Hong Kong teachers did (Huang & Leung, 2004). Furthermore, Chinese mathematics teaching is well known by its emphasis on mastering mathematics knowledge and skills and rigorous mathematics reasoning (Huang & Li, 2009; Leung, 1995, 2005). So, it is plausible that Chinese pre-service teachers preferred to make their judgments based on underlying concepts and logical reasoning, rather than visual information provided only. On the other hand, mathematics teaching in the U.S. classroom has always been described as emphasizing low-level, rather than high-level cognitive processes (i.e., memorizing and recalling facts and procedures rather than reasoning about and connecting ideas or solving complex problems) (Hiebert et al., 2005; Silver, Mesa, Moriss, Star, & Benken, 2009; Wood, Shin, & Doan, 2006). If taken the U.S. students' preference in using visual representations and lack of ability in mathematical reasoning together, then it may be understandable why U.S. pre-service teachers tended to make their justification based on visual information given, without paying close attention to underlying concepts.

The Relationship between Different Components of KTA

The path analysis revealed that components of KTA in the Chinese sample are much more highly correlated than those in U.S. sample. That means that Chinese participants have a more interconnected KTA structure than U. S. counterparts. Measurement model analysis also confirms that Chinese participants have a highly correlated KTA structure.

Pre-service teachers' knowledge structure mainly is impacted by their learning experience in pre-university and university. Cai and Wang (2010) found that Chinese expert teachers put more emphasis on coherence of delivering a good lesson than their U.S. counterparts. Moreover, Huang, Li, and He (2010) revealed that both novice and expert teachers in China viewed coherently developing a lesson as one salient feature of effective teaching. When examining classroom instruction, coherence and connection of lessons are essential features of teaching in China (Chen &Li, 2010; Wang & Murphy, 2004). Lesson coherence could be carried out through organizing systematic and varying classroom activities by sticking to mandatorily well-designed textbooks (Huang, Li, & Ma, 2010; Huang, Rowntree, Yetkiner, & Li, 2010). At the university level, lecture is the dominating teaching method (Li, Zhao et al., 2008), which may be conducive to transmitting knowledge systematically. Possibly, pre-service teachers develop their well-structured and interconnected knowledge bases through their learning experience in pre-university and university.

The Difference and Similarities of Knowledge for Teaching the Concept of Functions

The open-ended items were used to measure teachers' knowledge for teaching the concept of function in terms of the perspectives adopted and representations used. In general, a function concept should be developed from process to object perspective (Briedenbach et al., 1992; Sfard, 1990, 1993). A deep understanding a function concept could be partially reflected by taking an appropriate perspective or shifts between these two perspectives flexibly. Moreover, using multiple representations and shifting between different representations are the manifestations of understanding a function concept and relevant skills. The analysis of the open-ended items from multiple aspects showed that, the Chinese counterparts seemed to have:

(1) Strong algebraic and graphic transformational skills and procedural fluency;
(2) Multiple strategies of solving algebraic problems by integrating algebraic and geometrical representations;
(3) Appropriately taking perspectives and shifting between different perspectives of function; and

(4) Appropriate use of representations and flexible shifts between different representations.

On the other hand, the U.S. participants struggled with basic algebra manipulation and had limited knowledge in flexible use of perspectives and representations. Both qualitative and quantitative analyses showed that Chinese participants seemed to be more flexible than U. S. counterparts in terms of shifting between different perspectives and selecting appropriate representations of function.

It was found that the scores of KTA predict flexibility significantly, and the scores of KTA can explain more than four-fifths of the variance of flexibility. Meanwhile, the number of college math courses taken and grade level were not found to have significant prediction of the growth of flexibility. These results imply that developing flexibility is not a methodology and/or maturation issue, rather a comprehensive issue with the development of knowledge for teaching. That means we have to equip pre-service teachers with a well-structured knowledge base in order to develop their flexibility.

How can Chinese pre-service middle grade teachers develop their flexibility while developing their procedural fluency and conceptual understanding? Many studies explored how Chinese in-service teachers develop their professional knowledge and expertise (Huang & Bao, 2006; Huang & Li, 2009; Li, 2004; Li &Li, 2009; Li, Huang, & Yang, 2011; Ma, 1999; Yang, 2009). However, little research on pre-service teacher learning in China has been done. Based on the characteristics of secondary mathematics teacher preparation programs, Chinese pre-service teachers are exposed to broad advanced mathematics content knowledge, some math education theories, and an extensive study on school mathematics, with little chance of student teaching (only around 4-6 weeks) (Li et al., 2008). So, they mainly obtain mathematics knowledge for teaching from their learning experience at pre-university and university.

With regard to mathematics classroom (pre-university) teaching in China, based on an extensive literature review, Huang and Li (2009) summarized the following features: (1) setting and achieving comprehensive and feasible teaching objectives; (2) having a detailed and well designed lesson plan that not only covers sufficient content to teach but also offers alternatives to develop the content coherently; (3) emphasizing the formation and development of knowledge and mathematics reasoning; (4) emphasizing knowledge connection and instruction coherence; (5) practicing new knowledge with systematic variation problems; (6) making a balance between the teacher’s guidance and students’ self explorations; and (7) summarizing key points in due course and assigning homework (p.99).

Considering content coverage and presentation in high school (Li, Zhang, & Ma, 2009), university teacher preparation programs, the ways Chinese pre-service teachers taught as described above, the Chinese pre-service teachers are able to develop sound subject content knowledge and a well-structured knowledge base. Based on this assumption, we argued that it is possible for Chinese pre-service teachers to develop their fluency and flexibility simultaneously.

First, the teachers’ sound content knowledge reduce cognitive load and leave space for developing strategies in solving problems (Richland, Zur, & Holyoak, 2007). Second, the solid and interconnected knowledge base provides the foundations for developing flexibility. That means that pre-service teachers have a rich recipe of strategies for solving individual problems, and different representations for presenting mathematics concepts and mathematics problems. Third, it is a traditional and common practice to develop multiple approaches to solving a problem and developing multiple problems derived from the same problem or the same problem solving strategy in Chinese mathematics classroom (Cai & Nie, 2007; Huang, Mok, & Leung, 2006). Implementing this approach of teaching requires learners to compare different strategies and select the most appropriate one for a particular type of problems. This comparison is an effective way to develop learners' flexibility in solving problems (Star & Seifert, 2006; Star & Rittle-Johnson, 2009).

In summary, the teacher preparation practice in China seems to provide the opportunities for pre-service teachers to develop their sound knowledge for teaching, and flexibility in selecting appropriate strategies and using appropriate representations of function. However, more empirical studies need to be done to explore relevant factors and mechanisms.

The Relationship between Pre-service Teachers' KTA and Their Course Taking

This study revealed that the courses taken do have an effect on KTA although the patterns in China and the U.S. differ. Chinese teacher preparation programs seem to emphasize content knowledge while the U.S. teacher preparation programs put more emphasis on pedagogical knowledge. Meanwhile, the Chinese participants seem to have a more interconnected KTA structure than U.S. counterparts.

With regard to the number of courses taken, there was a big difference between China and the U.S. The number of courses taken in mathematics content and mathematics education, on average, the U.S. participants had 7 while the Chinese participants had 14. Thus, results of this study may be influenced by the difference of courses taken.

It is not surprising that there was such a big difference in the number of courses taken between China and the U.S.; in China, mathematics teachers for middle and high schools are required to major in mathematics. There was no distinction in preparing mathematics teachers for middle and high schools. Whether graduates work in high school or middle school depends on job market and reputation of university where they graduated. However, in the U.S., There was substantial difference between middle and high mathematics teachers in terms of programs attended. In this study, the U.S. participants were from an interdisciplinary program of middle grade mathematics and science teachers. It was suggested that it will be beneficial for middle and high school teachers to specialize in the subject field that they will be teaching (CBMS, 2001; National Commission of Mathematics and Science Teaching [NCMST], 2000). In particular, CBMS (2001) recommended middle grade mathematics teacher preparation program should include at least 21 semester-hours of mathematics including two types of courses (as the participants in this study did). One focuses on developing a deep understanding of the mathematics they will be teaching. The other aims at strengthening and broadening understanding of mathematical connections between one educational level and the next, connections between elementary and middle grades as well as between middle grades and high schools.

As far as the algebra is concerned, the CBMS (2001) recommended developing a deep understanding of variables and functions as follows: (1) relate tabular, symbolic, and graphical representations to functions; (2) relate proportional reasoning to linear functions; (3) recognize change patterns associated with linear, quadratic, and exponential functions and their inverses; and (4) draw and use "qualitative graphs" to explore meaning of graphs of functions. Meanwhile, students need to demonstrate the following skills: (1) represent physical situations symbolically; (2) graph linear, quadratic, exponential functions and their inverses and understand physical situations calling for each; (3) solve linear and quadratic equations and inequalities; and (4)exhibit fluency in working with symbols. (pp.108-109)

Comparing with these recommendations in the U.S. and the practice in China, in order to implement curriculum standards (NCTM, 2000, 2006), the middle grade mathematics teacher preparation programs in the U.S. not only need to add more mathematics and mathematics education course, but also need to improve the quality of courses and the quality of teaching.

Discussions

In the sections that follow, I discussed relevant issues needed to be further explored. These issues include measure knowledge for teaching mathematics cross-culturally, comparison of KTM in different content areas, developing basic knowledge and skills and flexibility simultaneously, and what we can learn from this study.

Can Knowledge for Teaching Mathematics be Measured across Cultures?

Different models have been developed to define and measure mathematics knowledge for teaching. Although great efforts have been made to develop reliable instruments for measuring MKT, There was not any a well-developed instrument available so far. Even the most popular one, developed by University of Michigan, the low reliability of KCS (knowledge for content and student) (Schilling, Blunk, & Hill, 2007) prevents it being used in study relating teacher knowledge to student achievement (Hill et al., 2004). With regard to KTA instrument, although Floden et al. (2009) reported a high inter reliability (Cronbach alpha (α) .80 for whole instrument), the present study showed a relatively low reliability (α= 0.613 for the U.S. sample (*N*=115) and α=.73 for the Chinese sample (*N*=376).

In addition, the Chinese measurement model of KTA indicates that three components of KTA are highly correlated, and there are many links between different observed variables (items). It seems to suggest the complexity of mathematics knowledge for teaching: multiple venues for success and multiple solutions drawn on various knowledge and skills (Floden et al., 2009) results in the difficulty in measuring teacher knowledge for teaching by several isolated items. Ball, Thames, and Phelps (2008) realized that "it is not always easy to discern where one of our categories divides from the next, and this affects the precision (or lack thereof) of our definitions" (p.403), and they also recognized that formulation of mathematical knowledge for teaching is culturally specific or dependent on teaching styles (Delaney et al., 2008). Based on specifying the relationship between Shulman's (1986, 1987) theorizations of "pedagogical content knowledge" and "the knowledge base for teaching," and Ball et al.'s (2008) notions of "specialized content knowledge" (SCK) and "mathematical knowledge for teaching," Lawrence (2010) concluded that Ball's model of "mathematical knowledge for teaching" would perhaps be made more useful for analyses of the kinds of knowledge-practice breakdown. However, if teacher knowledge is emphasized teachers' professional judgment as Shulman's, then the bridge-building, namely "the wisdom of practice" is crucial. Furthermore, a shift in focus from mathematics teachers' knowledge to their knowledgeable practices might facilitate Ball's efforts to bridge professional knowledge and teaching practice (Lawrence, 2010).

Moreover, when developing an instrument for cross-culturally comparative studies of mathematical knowledge for teaching, the equivalence of coverage of items becomes a challenging issue. As pointed by Delaney et al. (2008), factors such as the teaching strategies, teacher beliefs, classroom contexts, the presence and prevalence of specific mathematical topics and the content of the textbooks should be considered. It was found that teaching strategies and emphases across different grades should be important factors influencing teachers' knowledge for teaching. For example, in the current study, when U.S. participants were asked why they did not connect algebra and graphic representations when solving quadratic inequality, they said they were taught separately by different teachers in different grades and they did not realize that these different types of knowledge are interconnected. Thus, it is important to consider teaching strategies across different grades.

In addition, the content placement across pre-college and college should be considered. For example, some contents taught at college in the U.S. are the contents in high school in China. In other contents, a reverse situation occurs. In order to achieve common classifications of three components of KTA (*school math, advanced math and teaching math*), this content equivalence should be considered.

Do Chinese Teachers Have a Sound Knowledge for Teaching?

This study showed that Chinese middle and high school pre-service teachers demonstrated solid knowledge for teaching algebra, with certain flexibility in taking perspectives of function and selecting multiple representations of functions. Overall, Chinese pre-service secondary mathematics teachers have a deep understanding of mathematics knowledge for teaching algebra.

This study extends Ma's (1999) investigation on elementary mathematics teachers' knowledge in China and the United States in some ways. The findings suggest that Chinese secondary mathematics teachers have a profound understanding of mathematical knowledge for teaching algebra. In Ma's study, the focus was on the connection of relevant concepts, while this study focused on connection of different perspectives and representations of function, and also the flexibility in selection of representations. This study also enriched and extended An et al. (2004) observations of pedagogical content knowledge of middle school mathematics teachers in China. An et al. found that the Chinese mathematics teachers emphasized gaining the correct conceptual knowledge by reliance on traditional, more rigid development of procedures. This study provides some interpretations on why Chinese pre-service teachers could emphasize on gaining correct conceptual knowledge by reliance on "rigid development of procedures."

When considering the findings by Even (1998) and Black (2007), the strengths of Chinese mathematics teachers' knowledge for teaching algebra are even more prominent. For example, 20% of 76 U. S. high school mathematics teachers in Blacks' (2007) study got correct answers, while 55% of 376 Chinese participants in the current study got correct answers.

The finding that the Chinese secondary pre-service mathematics teachers have sound mathematical knowledge for teaching algebra echoes the observation that secondary mathematics teacher preparation programs in China emphasizes mathematics content courses and mathematics education courses (Li et al., 2008).

Basic Knowledge and Skills and Flexibility in Problem Solving

If the core value of algebra learning is to develop students flexibility in translations between different representations and transformations between different forms within a presentation (Star & Rittle-Johnson, 2009) and to make sense of algebra through building connections between different brands of knowledge and different representations (NCTM, 2000, 2009), then, it is the key to equip pre-service teachers with relevant knowledge for teaching promoting these values.

Although math education in China has a long tradition to pursue basic knowledge and basic skills of mathematics, mathematical thinking and rigorous logical reasoning (Zhang, Li, & Tan, 2004), great efforts have been made since 2001 to develop students' exploratory, collaborative learning, creative thinking, mathematical communication (Ministry of Education, 2001, 2003). A study on comparing 10 novice and 10 expert teachers' views of effective mathematics (Huang, Li, & He, 2010) found that all the participating teachers valued students' mastering of mathematical knowledge and skills, and their development in mathematical thinking methods and abilities. Compared with novice teachers, expert teachers emphasized more on the development of students' mathematical thinking and higher order thinking abilities. These findings seem to suggest that the mathematics teachers in the context of curriculum reform in China have been making a balance between mastering knowledge and skills and developing mathematics thinking and creative thinking.

The survey revealed that Chinese pre-service teachers demonstrated sound knowledge and skills and fluency in algebra computation. In particular, the analysis of the open-ended items showed the Chinese pre-service teachers not only have high fluency in algebraic computation, but also have flexibility in selecting appropriate function perspectives and using multiple representations. Moreover, the Chinese participants not only performed well, but also used various methods.

Thus, the Chinese participants demonstrated high procedural fluency and a deep understanding of the concepts. It seems that the Chinese teachers develop their procedural fluency, conceptual understanding, and flexibility in adopting appropriate perspectives and selecting appropriate representations simultaneously.

What Can We Learn from the Study?

In the United States, it is a publicly recognized problem that teachers do not have adequate knowledge of what they will be teaching. In this study, compared with Chinese counterparts, the weakness of the U.S. pre-service teacher's knowledge for teaching algebra is evident. Thus, I consider what U.S. mathematics educators may learn from Chinese practice in teacher preparation programs.

First, adding more mathematics content courses to the existing teacher preparation programs may be necessary. Since the number of courses taken impact on KTM and the U.S. participants took fewer courses than the Chinese participants, it is necessary to add more compulsory courses in mathematics and mathematics education in the U.S. middle grade mathematics teacher preparation program. Second, teaching approaches to mathematics education courses may also need to be improved. In China, the dominated teaching method is lecture with frequent probing questions which may be conducive to developing pre-service teachers' systematic and interconnected knowledge, but may constrain their opportunities to develop creativity and inquiry. On the other hand, in the U.S., there are a lot of individual/on-line learning, students' presentations, and projects in math education courses. These kinds of activities may be beneficial to developing individual exploration, team cooperation, presentation and communication skills.

However, pre-service teachers may not be able to acquire necessary knowledge and skills systematically.

It may be important to ensure our pre-service teachers to have a deep understanding of core concepts and build an interconnected knowledge base through multiple approaches including direct instruction, problem-based teaching and learning, cases studies and inquiry project. Moreover, it is also crucial to develop pre-service teachers' ability to transfer school math to be more easily accessible and meaningful to their students, and to learn from their lesson designing and teaching. Some programs focusing on designing, teaching and reflecting on lessons of teaching core concepts have demonstrated promising future in mathematics teacher preparation programs (e.g., Hiebert & Morris, 2009).

In China, the Chinese pre-service teachers in this study demonstrated their strengths in mathematical knowledge for teaching algebra and their flexibility in using appropriate representations. However, they need to learn more about how to develop students' creativity, discovery learning, and collaborative learning which are advocated in the new curriculum standards in China. Moreover, they may need to learn how to develop a concept from multiple perspectives, how to build the connections between arithmetic, algebra and geometry.

Limitation

There are several limitations in this study. Even though, the researcher considered the representativeness of the Chinese sample such as university entrance scores, programs and regions, the number of teacher education institutions in China is too large to be sampled by an individual research effort. In China, There was only one approach for preparing secondary school mathematics teachers (including middle school teachers) housed in mathematics department. In the U.S., there are three approaches for preparing middle mathematics teachers. The first prepares teachers to teach all secondary mathematics, including lower secondary/middle grades. The second focuses on specifically and exclusively on preparing teachers for lower secondary/middle school grades. The third approach prepares lower secondary/middle school teachers as an extension of elementary teacher preparation. In this study, the researcher mainly selected the participants from the second type of program in a respected university (only very small part of the participants from the first approach program). Because of this disparity, the sample at most reflects a low level of secondary math teacher preparation program in the U.S. Cautions should be taken when interpreting the differences between China and the U.S. In order to make an appropriate comparison, more wide samples from the different approaches in the U.S. should be included. Since the essential differences of programs between China and the U.S., the numbers of courses taken are different. The number of courses taken in mathematics content and mathematics education, on average, the U.S. participants had 7 while the Chinese participants had 14. So caution should be taken in interpreting the findings of this study in which comparisons are made between U.S. and Chinese participants due to the disparity of the sample.

Second, the small size of the U.S. sample prohibited building a measurement model. If the U.S. sample size can be increased large enough to build structure equation models, then, we can conduct more sophisticated and extensive comparisons.

Moreover, although the reliability (α=0.88) for the whole sample (N=491) is high, and the reliability (α=0 .73) for the Chinese sample (N=376) is acceptable, but the reliability (α=0.613) for the U.S. sample (*N*=115) is relatively low. So, the results based on this instrument should be interpreted cautiously.

Recommendation

With regard to pre-service teachers' mathematics knowledge for teaching, more questions need to be further explored. For example, what is meant by mathematics knowledge for teaching algebra and other secondary mathematics topics? How can it be measured? Do Chinese pre-service teachers have sound mathematics knowledge for teaching in all areas of school mathematics? What strategies are effective in developing pre-service teachers' basic knowledge and skills, and flexibility through teacher preparation program in the U.S.?

Study on the Meaning of Mathematics Knowledge Needed for Teaching

Concerning with the first question, many studies found the weaknesses of the existing instruments, such as the KTA in this study. This calls for researching into the nature of mathematical knowledge for teaching, and to what extent, it can be measured. Building bridges between knowledge and practice toward a knowledgeable practice or wisdom of practice (Lawrence, 2010) shed light on defining and measuring teachers' knowledge needed for teaching.

Teachers' Knowledge for Teaching Other Topics in China and the U.S.

The second research question concerns specialty of topics investigated. Mathematics education in China has a tradition of emphasizing basic knowledge and skills (Zhang et al., 2004). Due to the core position of algebra in school mathematics, the teaching of algebra both at secondary schools and universities are emphasized. A great deal of time was spent on learning and practicing algebraic computation in high school and university. So, pre-service teachers in China may have strength in KTA, but it does not mean they should have strong knowledge for teaching other content areas, particular in some newly added contents such as probability and statistics. So, it will be interesting and meaningful to compare MKT in other content areas.

Are There Some Teaching Strategies Effective for Developing Flexibility?

Studies suggest that some strategies in China are effective for students learning with procedural fluency and conceptual understanding, such as problem-based teaching, practicing with varying problems. Can these strategies be applied to teaching pre-service teacher preparation courses in the U.S.? What are the effective teaching strategies for equipping pre-service teachers with sound basic knowledge and skills, and flexibility in problem solving? It should be interesting to explore how U.S. mathematics educators can adopt some Chinese strategies to the teacher preparation programs in the U.S. to develop teachers' knowledge needed for teaching.

Coda

It is the desire to find the similarities and difference of KTM and provide implications for improving mathematics teacher preparation in China and the U.S. that led me to do this comparative study. Finally, the findings illustrated many more differences than similarities, and a sampling effect on Chinese participants' superiority in knowledge for teaching algebra. Although, the findings may not be generalized due to the limitation of the sample, the detailed description and analysis should provide referents for international mathematics educators, particularly the Chinese and U.S. mathematics educators to reflect what they can learn from this study. Moreover, it should be exciting to derive more meaningful research questions based on this study.

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APPENDIX A

RUBRICS

Item 18

Score	Description
4	Give answers with the following elements: (a)Point out (i) and (ii) are functions ; (b)Point out that there is only one unique value corresponding to each value from domain value (such as one to one, multiple to one, but not one to multiple).
3	Give the answers with the following elements: (a) Point out (i) and (ii) are functions ;(b) The explanations do not relate to the key element (multiple to one or one to one), rather some superficial features such as: the function (i) with constant value, and the function (ii) is not continuous or expressed by two expressions or there are many holes.
2	(I): (i) (a) is correct: (i) and (ii) are function. (b) without explanation or giving wrong explanation Or (II): (a) one of (i) and (ii) is function, (b) give an correct explanation
1	(I): (a) answer (i) is function, (ii) is not or inverse (b) explanation is missing or wrong OR (II): (a) answer (i) and (ii) are not function, but (b) give some relevant explanations.
0	Blank or total wrong answers in (i) and (ii)

Item 19

Score	Examples
4	1.S notes that for ab to be positive either both a & b are positive or both are negative. Solves and finds x > 3 or x < -4. 2.Solve x – 3 = 0 and x + 4 = 0; plot x = 3 and x = -4 on the number line. Identify whether (x-3)(x+4) is positive or negative on the three intervals determined by these 2 points. 3.Rewrite (x-3)(x+4) and solve $x^2 + x - 12 = 0$ using quadratic formula. Then use one of Method 1 or 2 above. 4.Graph y = (x-3)(x+4). Identify x-values where parabola is above the x-axis.
3	1. Give two algebraic correctly. 2. One method is correct while there are minor mistakes with the other method.
2	1. There is only one correct method and solution. 2.The two methods shown are essentially the same.
1	Based on the assumption that if ab is positive, then a & b are positive, namely,

	a>0, b>0. (Without & or between two inequality).
0	Blank or no mathematically useful statement

Item 20

Score	Explanation
4	Give correct answers with an counterexample
3	Give correct answer with a correct counterexample but minor calculation error or notational error or sloppy comment.
2	Give a correct judgment but not provide relevant explanations.
1	There is at least one correct useful statement. For example, States YES or True but has something that might be relevant to the situation. For example, give examples such as A=0, then AΔB=0 or B=0, then AΔB=0.
0	Blank or useless information

Item 21

Score	Examples
4	Reason: stick to finding algebra expression and discriminate, without realizing graphical representation. Solution: According the given conditions, sketch a graph of $y = f(x) = ax^2 + bx + c$ and finding one root in [1, 6]. Moreover, according to the symmetry of quadratic function. The quadratic function should have two intersection points at x-axis , namely, there are two roots of the $ax^2 + bx + c = 0$ （a$\neq$0）
3	For example, they just pointed one root.
2	Just say students should consider by integrating numerical and pictorial representations.
1	Providing at least one related and useful statement
0	Blank or some information not related to solving this problem

Item 22

Score	Examples
4	Give answer is C and provide different explanation such as: • Change of a leads change of the openness, thus a is not changed; the y-intercept is not changed, so c is not changed. Thus, it is only possible to change b. • The translated graph is the symmetrical graph of original graph with regard to y symmetrical axis. So b is changed into –b.
3	Answer C However, reasons is not explained appropriately such as only mentioning a or c the invariance.
2	Give C or D and gives some explanations, with some serious mistakes, such as if a is changed then the graph is moved up or down.
1	Or give partly the features of graph when changing a,b, and c.

Item 23

Score	Examples
4	Use different formula to find the quadratic function. Find the maximum by using formula, symmetrical feature.
3	Find correct quadratic function expression but make mistakes in finding maximum.
2	Only find a correct quadratic function, without further attempt to find maximum.
1	Find one of a, b, c.

Item 24

Score	Example
4	Method 1: let f(x) and g(x) intersect at x-axis (p, 0), then, f(p) = 0, g(p) = 0. So, (f+g)(p) = f(p) + g(p) = 0 + 0 = 0. Thus, f+g (p, 0) Method 2: let f(x) and g(x) intersect at x-axis (p, 0), then, the following statements are true: (1) f(p) = 0 → ap + b = 0 → p = -b/a; (2) g(p) = 0 → cp + d = 0 → p = -d/c; (3) f(p) = g(p) → b/a = d/c → ad = bc; (4) f(p) = g(p) → ap + b = cp + d → p = -(b + d)/(a + c);According to (f+g)(p) = f(p) + g(p), and above statements, to deduce (f+g) (p) = 0.Thus, (f+g)(x) pass at point (p, 0)
3	All major points are made but one small piece may be skipped: Based on the above propositions (1) – (4) , and deduce (f+g)(x) = f(x) + g(x) = (a + c)x + (b + d), and get (f+g)(p) = 0， but make some minor mistakes.
2	Understand f(p) = 0, g(p) = 0, and (f+g)(p) = f(p) + g(p), but they did not get (f+g)(p) = 0 or (f+g)(x) passes at point (p, 0). Although getting (f+g)(x) = f(x) + g(x) = (a + c)x + (b + d), but fail to deduce (f+g)(p) = 0 by using previous propositions.
1	Understand f and g pass P(p, 0), then, f(p)= g(p) =0, without further reasoning. Deduce (f+g)(x) = f(x) + g(x) = (a + c)x + (b + d) without further reasoning.

Item 25

Score	Examples
4	Point out, if the x-axis presents time, and the y-axis present the height above sea level, them origin explanation is correct. Other examples include: velocity vs. time, distance vs. time, or temperature vs. time and etc.
3	Point out it is not appropriate to describe the real situation without using mathematical relation (the meaning of x, and y). Gives a correct example, but not provide details.
2	Student provides appropriate improving suggestion. Or give an detailed example
1	The student points the inappropriate, such as direct description based on daily situation, or gives a piece of information about an example.

APPENDIX B

INTERVIEW TRANSCRIPTS OR KEY POINTS

18. a) On a test a student marked both of the following as non-functions
(i) f: $\boldsymbol{R} \rightarrow \boldsymbol{R}$, $f(x) = 4$, where $\boldsymbol{R}$ is the set of all the real numbers.
(ii) $g(x) = x$ if x is a rational number, and $g(x) = 0$ if x is an irrational number.
For each of (i) and (ii) above, decide whether the relation is a function, and write your answer in the Answer Booklet.
b) If you think the student was wrong to mark (i) or (ii) as a non-function, decide what he or she might have been thinking that could cause the mistake(s).
Write your answer in the Answer Booklet.

Questions: (1) How do you judge whether a relationship is a function or not?
(2) What is vertical line test?
(3) What would you teach to your students? Can you give me an example?)

Larry: Larry got correct judgment without explanation and analysis students' learning difficulties. When asking how she make her judgment, she said to use vertical line test and draw a diagram ($x=y^2$) and then judge it is not. She believed that students may be confused by may holes, but the vertical line test can be passed.

Jenny: Jenny made wrong answers without explanation. She had difficult in plotting the graph, such as how to calculate rational and irrational number separately. She made her choice based on visual image of function. When asking, she knew the vertical line test.

Kerri: Kerri clearly use diagrams to explain the concept of function (one-one or multiple to one, but not one-multiple) and used it to judge, as stated " a function is when one x value goes to one y, as long as one x value does not go to 2 y values, it is a function"

Alisa: Alisa clearly use vertical line test. Although there are many holes in (ii), but it still passes vertical line tests. Students may be confused by horizontal line or vertical line test?

Stacy: She made correct judgment. Stacy stated she used vertical line test line. "Each input [value] should have only one value, but that does not means different input could not have different values". Students may be confused by the repeating output as stated "he could have seen outputs repeating and said non-function, even though there is actually one possible output for every input".

21. If you substitute 1 for x in expression $ax^2 + bx + c$ (a, b and c are real numbers), you get a positive number, while substituting 6 gives a negative number. How many real solutions does the equation $ax^2 + bx + c = 0$ have?
One student gives the following answer:
According to the given conditions, we can obtain the following in-equations:

$a+b+c>0$, and $36a+6b+c<0$.

Since it is impossible to find fixed values of *a, b* and *c* based on the previous inequations, the original question is not solvable.

Write down your answers in as much detail as possible on your Answer Booklet.

Questions: (1) What do you think may be the reason for the student' answers?
(2) To find solution, what other mathematical objects do think may be related to the equations?

Larry: Larry totally agrees students' comments. There are three parameters unknown. How can find the solution o the equations? She tried to use algebraic transformation, but nothing can be done. She said she was stuck to algebraic operations, there is no idea to suggest student go ahead. Even when the interviewer suggested drawing a graph, she still thinks it is impossible because all the three coefficients are unknown. Even when the interviewer drew a sketch of the quadratic function, she is not able to build the relations between roots and intersection points.

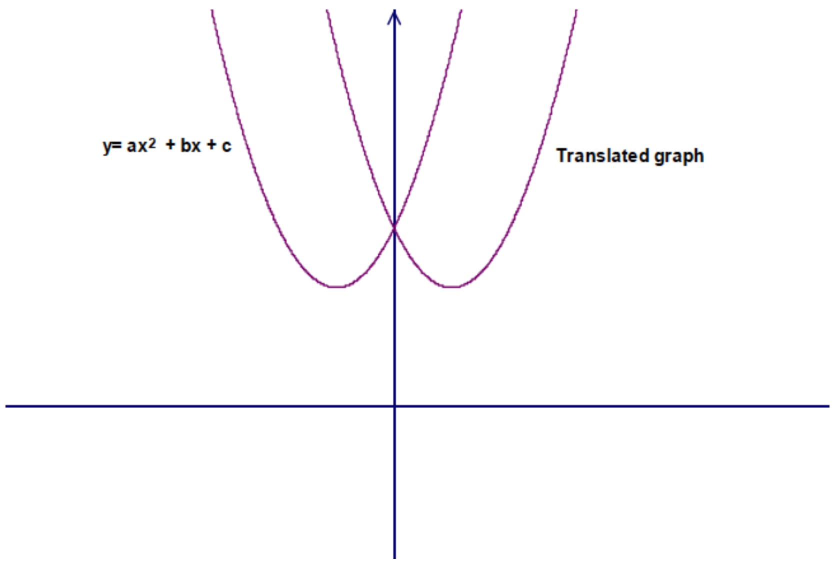
Jenny: Jenny is honestly to say she just jumped the conclusion. I actually do not how to solve this problem at all. What she could suggest is to ask students to try different ways, such as plugging more numbers between 1 and 6. She directly asked the interviewer if he can tell a method. When the interviewer asked to read the question carefully, to see the question is to find the number of root. And then if you have difficult in thinking algebraically, can you consider in other representations? Can you use graphical representations? Then the interviewer drew a sketch of a quadratic function based on the given conditions. Then the interviewee was enlightened to think roots and intersect points. She is not quite sure, but finally she found the roots. However, she said she did not have this experience in solving in-equality by graphing.

Kerri: Kerri agrees with the students' statement. She have not ideas how to find the results of the questions. They tried to find *a,b,* and *c* by adopting the ideas from algebra. But it does not work. Even when hinting to use the given conditions to draw a graph, she still get stuck because she do not know a,b and c. When the interviewer drew the sketch, she realized there are two roots.

Alisa: Honestly, she did not know how to solve the question. She agrees students. I do not know how to solve this problem. When asking whether she can fix out the number of roots by other methods. She guesses she can. She drew a sketch of a quadratic function, and found the number of roots (two). She realized the power of graphing method and will teach her students.

Stacy: She has not concreted idea about how to solve this problem. But she would like to suggest students to explore in different ways such as plugging more numbers to see whether they can find pattern, rather than being stuck. However, she still intended to find out a, b and c. when enlightening whether graphing method can be used, she drew a correct graph, and found the number of roots.

22. Mr. Seng's algebra class is studying the graph of $y = ax^2 + bx + c$ and how changing the parameters *a, b,* and *c* will cause different translations of the original graph.



Which of the following is an appropriate explanation of the translation of the original graph $y = ax^2 + bx + c$ to the translated graph?

A. Only the *a* value changed. **B**. Only the *c* value changed.
C. Only the *b* value changed. **D**. At least two of the parameters changed.
E. You cannot generate the translated graph by changing any of the parameters.

Explain your answer choice in as much detail as possible. Show your work in the Answer Booklet

Questions: (1) What are the effects of change of parameters of a,b,c on the change of graph?
(2) What do you find the changes and invariance after the translations ?
(3) What algebraic operations may help to identify the key parameter(s)

Larry: Change c, the graph is moved up or down; Change b, the graph is move from left t right or reverse; change a, the shapes of the graph is changed; so only b can be changed (correct answers should be C)

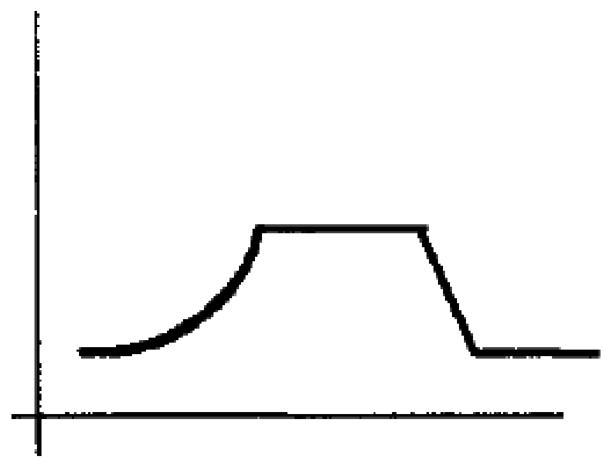
Jenny: She did a lot of translations of these graphs in high school: b->-b just influence on the left and right. But she is not clear about the effect of changing a, b, c. it is an good way to show students different example to explain the effect a,b, and c.

Kerri: c –up/ down, b-wide and narrow, b can only be changed. (just make wrong choice).

Alisa: C-change, up/down. B-change, make wider or narrower. Made mistakes in drawing graphs. So, the judge is wrong. She did not know, how to shift graph. (X+h);

Stacy: She knows how changes of a, and c impact on the changes of graph. But she is not clear about the change of b and its impact. Solution is correct. But she is not sure why! Learn in algebra I and II

23. When introducing the functions and the graphs in a class of middle school (14-15 year-old), tasks were used which consist of drawing graphs based on a set of pairs of numbers contextualized in a situation and from equations. One day, when starting the class, the following graph was drawn on the blackboard and the pupils were asked to find a situation to which it might possibly correspond.



One student answered: 'it may be the path of an excursion during which we had to climb up a hillside, the walk along a flat stretch and then climb down a slope and finally go across another flat stretch before finishing.'

How could you answer this student's comments? What do you think may be the cause of this comment? Can you give any other explanations of this graph?

Write down your answers in as much detail as possible on your Answer Booklet.

Questions: (1) What are the missing part of students' comment (two variables, X vs. Y);

(2) How can you explain other real life situation by using this graph?

Larry: She drew a diagram with x-axis (time) and y-axis (position), and explained that how this diagram makes sense of the situation. She also gave two examples of Speed Vs. time and Temperature Vs. time.

Jenny: Jenny said it could be Speed over time. In one course of math and technology, she learned this kind of graph through experiment.

Kerri: She also talks about speed over time. And gave a detailed description as increase, constant, decrease and constant.

Alisa: She also understood the graph as distance over time, or speed over time. The key is the relationship between two variables of x and y.

Stacy: she explained X-axis represents time, while y-axis represents distance, namely, height. She described the change trends such as flat, growing up, and going down. In general, it could represent the distance vs. time (CBR) or *temperature vs. time.*

24*. Given quadratic function $y = ax^2 + bx + c$ intersects x-axis at (-1, 0) and (3, 0), and its y-intercept is 6. Find the maximum of the quadratic function.
Show your work in as much detail as possible in the Answer Booklet.
Questions: (1) What kinds formula of the quadratic equation do you prefer to use?(how many different formulas have you learned) ?
(2) To find maximum, which formula do you prefer?

Larry: She knew the methods: plugging three points into quadratic equations to find a,b and c. and then taking derivative to find the point where the y-value is the maximum. She also pointed out that the graph should be symmetry on line x=1 and when x=1 the y value should be the maximum. But she did not know any other forms of quadratic equations.

Jenny: She has difficulty to find a,b,and c. she guessed that y-intercept is the maximum. However, when drawing a sketch of quadratic equation, she realized that the maximum should be x=1.

Kerri: She supposed that the y-intercept is the maximum. She just drew three points and try to see the maximum. She realized the method, but cannot remember the formula clearly.

Alisa: She found the expression of quadratic equation (in general form) by plugging three coordinates of points. And then using the symmetry, she realized that when x=1, y=8 is the maximum. However, she do not have any ideas about the other forms could be used for solving this problem.

Stacy: She made mistakes in plugging x values in order to find the a,b,and c. But she knows that she can find the maximum by taking derivates. She did realize there are other formulas

25*. Prove the following statement:

If the graphs of linear functions $f(x) = ax + b$ and $g(x) = cx + d$ intersect at a point P on the x-axis, the graph of their sum function $(f + g)(x)$ must also go through P.

Show your work in as much detail as possible in the Answer Booklet.

Q: (1) What does mean by intersect at a point on x-axis?

(2) What is the mean of (f+g)(x)?

(3) What do you want to prove?

Larry: In her answer to questionnaire, she used two concrete examples to computation. However, during interview, she used the general form and find correct proof. As following: ax1+b=0, ax2+b=0; (f+g)(x)=(ax1+b)+(ax1+b)=0+0=0;

Jenny: She just listed two concrete examples, and showed how to the intersection points. Then she failed to show how to prove.

Kerri: She gave up the question in questionnaire.

Alisa: She gave up the question in questionnaire.

Stacy: She realized how to use visual methods. She do not like proving, and has no idea to prove it.

APPENDIX C

SME MODEL PARAMETERS

Table C1. Selected AMOS Output for the Final Chinese Model: Unstandardized and Standardized Estimate

			Estimate	S.E.	C.R.	P
			Regression Weights			
MKT3	<---	SM	.045	.019	2.358	.018
MKT6	<---	SM	.168	.050	3.359	***
MKT14	<---	SM	.114	.034	3.309	***
MKT10	<---	TM	.115	.034	3.358	***
MKT25	<---	TM	.863	.119	7.280	***
MKT18	<---	TM	.723	.096	7.567	***
MKT8	<---	AM	.234	.058	4.066	***
MKT9	<---	AM	.213	.079	2.677	.007
MKT12	<---	AM	.273	.084	3.269	.001
MKT13	<---	AM	.181	.075	2.406	.016
MKT16	<---	AM	.296	.060	4.924	***
MKT20	<---	AM	1.000			
MKT4	<---	AM	.308	.076	4.035	***
MKT17	<---	SM	.074	.020	3.768	***
MKT24	<---	AM	1.498	.305	4.907	***
MKT22	<---	TM	.739	.120	6.151	***
MKT21	<---	TM	1.000			
MKT23	<---	SM	1.000			
MKT19	<---	SM	.916	.124	7.410	***
MKT2	<---	PCK1	.048	.014	3.433	***
MKT7	<---	TM	.089	.028	3.190	.001
			Standardized Regression weights			
MKT3	<---	SM	.147			
MKT6	<---	SM	.215			
MKT14	<---	SM	.211			
MKT10	<---	PCK1	.206			
MKT25	<---	PCK1	.522			
MKT18	<---	PCK1	.544			

			Estimate	S.E.	C.R.	P
MKT8	<---	AM	.341			
MKT9	<---	AM	.185			
MKT12	<---	AM	.237			
MKT13	<---	AM	.163			
MKT16	<---	AM	.474			
MKT20	<---	AM	.374			
MKT4	<---	AM	.323			
MKT17	<---	SM	.243			
MKT24	<---	AM	.488			
MKT22	<---	PCK1	.420			
MKT21	<---	PCK1	.579			
MKT23	<---	SM	.530			
MKT19	<---	SM	.677			
MKT2	<---	PCK1	.211			
MKT7	<---	PCK1	.196			
			Covariances			
SM	<-->	PCK1	.469	.078	6.020	***
PCK1	<-->	AM	.299	.062	4.851	***
SM	<-->	AM	.200	.045	4.432	***
			Correlations			
SM	<-->	PCK1	.908			
PCK1	<-->	AM	.827			
SM	<-->	AM	.747			

Table C2.
Selected AMOS Output for Final Chinese Model: Goodness-of-Fit Statistics

Model fit Summary

CMIN

Model	NPAR	CMIN	DF	P	CMIN/ DF
Default model	57	245.347	174	.000	1.410
Saturated model	231	.000	0		
Independence model	21	1036.888	210	.000	4.938

RMR, GFI

Model	RMR	GFI	AGFI	PGFI
Default model	.027	.943	.925	.711
Saturated model	.000	1.000		
Independence model	.146	.696	.666	.633

Baseline comparison

Model	NFI Delta1	RFI rho1	IFI Delta2	TLI rho2	CFI
Default model	.763	.714	.917	.896	.914
Saturated model	1.000		1.000		1.000
Independence model	.000	.000	.000	.000	.000

RMSEA

Model	RMSEA	LO 90	HI 90	PCLOSE
Default model	.033	.023	.042	.999
Independence model	.102	.096	.109	.000

AIC

Model	AIC	BCC	BIC	CAIC
Default model	359.347	366.452	583.334	640.334
Saturated model	462.000	490.793	1369.735	1600.735
Independence model	1078.888	1081.506	1161.409	1182.409

ECVI

Model	ECVI	LO 90	HI 90	MECVI
Default model	.958	.858	1.079	.977

Model fit Summary

CMIN

Model	NPAR	CMIN	DF	P	CMIN/ DF
Saturated model	1.232	1.232	1.232	1.309	
Independence model	2.877	2.619	3.155	2.884	

Table C3.

Selected AMOS Output for the Final American path analysis Model: Unstandardized

			Estimate	S.E.	C.R.	P
		Regression Weights				
SM	<---	N_Highmath	.284	.212	1.342	.180
SM	<---	N_Collemath	.143	.063	2.277	.023
AD	<---	N_Collemath	.070	.069	1.014	.311
AD	<---	Grade	.030	.221	.137	.891
AD	<---	SM	.253	.101	2.519	.012
TM	<---	N_Highmath	.195	.408	.478	.633
TM	<---	Grade	-.178	.386	-.461	.645
TM	<---	N_Collemath	.247	.124	1.991	.047
TM	<---	AD	.422	.161	2.620	.009
TM	<---	SM	.769	.181	4.244	***
SM	<---	N_Highmath	.284	.212	1.342	.180
SM	<---	N_Collemath	.143	.063	2.277	.023
AD	<---	N_Collemath	.070	.069	1.014	.311
AD	<---	Grade	.030	.221	.137	.891
AD	<---	SM	.253	.101	2.519	.012
TM	<---	N_Highmath	.195	.408	.478	.633
TM	<---	Grade	-.178	.386	-.461	.645
TM	<---	N_Collemath	.247	.124	1.991	.047
TM	<---	AD	.422	.161	2.620	.009
TM	<---	SM	.769	.181	4.244	***
SM	<---	N_Highmath	.284	.212	1.342	.180
		Covariances				
N_Highmath	<-->	N_Collemath	.472	.153	3.080	.002
N_Collemath	<-->	Grade	-.272	.151	-1.798	.072
N_Highmath	<-->	Grade	-.037	.044	-.839	.401

Table C4

Selected AMOS Output for Final American Path Analysis Model: Goodness-of-Fit Statistics

Model fit Summary

CMIN

Model	NPAR	CMIN	DF	P	CMIN/ DF
Default model	19	.079	2	.961	.040
Saturated model	21	.000	0		
Independence model	6	73.016	15	.000	4.868

RMR, GFI

Model	RMR	GFI	AGFI	PGFI
Default model	.007	1.000	.998	.095
Saturated model	.000	1.000		
Independence model	.915	.796	.714	.568

Baseline comparison

Model	NFI Delta1	RFI rho1	IFI Delta2	TLI rho2	CFI
Default model	.999	.992	1.027	1.248	1.000
Saturated model	1.000		1.000		1.000
Independence model	.000	.000	.000	.000	.000

RMSEA

Model	RMSEA	LO 90	HI 90	PCLOSE
Default model	.000	.000	.000	.971
Independence model	.182	.141	.225	.000

AIC

Model	AIC	BCC	BIC	CAIC
Default model	38.079	40.497	90.722	109.722
Saturated model	42.000	44.673	100.184	121.184
Independence model	85.016	85.779	101.640	107.640

ECVI

Model	ECVI	LO 90	HI 90	MECVI

Model fit Summary					
CMIN					
Model	NPAR	CMIN	DF	P	CMIN/ DF
Default model	.325	.342	.342	.346	
Saturated model	.359	.359	.359	.382	
Independence model	.727	.530	.987	.733	

Table C5

Selected AMOS Output for the Final Chinese path analysis Model: Unstandardized

			Estimate	S.E.	C.R.	P
		Regression Weights				
SM	<---	Grade	-.919	.204	-4.510	***
AD	<---	N_Collemath	.098	.044	2.236	.025
AD	<---	Grade	.423	.288	1.472	.141
AD	<---	SM	.583	.061	9.634	***
TM	<---	AD	.437	.075	5.823	***
TM	<---	SM	.807	.098	8.217	***
TM	<---	N_Collemath	.066	.064	1.027	.305
TM	<---	Grade	-.489	.419	-1.167	.243
		Covariances				
N_Collemath	<-->	Grade	.846	.093	9.112	***

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