MODELING INFORMATION PROPAGATION ALONG TRAFFIC ON TWO PARALLEL ROADS

A Thesis

by

KAI YIN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2010

Major Subject: Civil Engineering

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Approved by:

Chair of Committee,	Xiu Wang
Committee Members,	Yunlong Zhang
	Darbha Swaroop
Head of Department,	John Niedzwecki

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ABSTRACT

Modeling Information Propagation Along Traffic

on Two Parallel Roads. (August 2010)

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Chair of Advisory Committee: Dr. Xiu Wang

IntelliDrive systems, including inter-vehicle communication and vehicle infrastructure integration, aim to improve safety, mobility and efficiency of transportation. They build on the wireless ad hoc network technologies, enabling vehicles to communicate with roadside infrastructure and with each other. The process of information propagation in a multi-hop network underlies the system design and efficiency. As of now, the research has been restricted to a single road of traffic. This work expands the study of information propagation to two parallel roads, a step further towards the discrete network case.

This thesis presents two methodologies to model the process of information propagation. By identifying an approximate Bernoulli process, we are able to derive the expectation and variance of propagation distance. A road separation distance of $\frac{\sqrt{3}}{2}$ times the transmission range distinguishes two cases for approximating the success probability in the Bernoulli process. In addition, our results take the single road as a special case. The numerical test shows that the developed approximation works well.

This work further identifies a Markov property for instantaneous information propagation along two parallel roads based on two types of transmission regions. Communication capable vehicles are assumed to follow two homogeneous Poisson processes on both roads. The Markov property enables us to derive exact expectation and variance of the propagation distance and further, obtain a recursive formula

for the probability distribution of successful propagation distance. The developed formulas enable numerical calculation of the characteristics of propagation process. We hope this research will shed light on studies of vehicular ad hoc networks on more general discrete roadway networks.

To my parents for their love, support and encouragement

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CHAPTER I

INTRODUCTION

Recently, a fascinating idea that allows vehicles to exchange traffic information has emerged in a new traffic system called IntelliDrive. It aims to significantly enhance the safety, mobility and quality of life [1], [2]. By integrating fast mobile computing and advanced wireless communication technologies into vehicles, the IntelliDrive system can connect vehicles with each other (via on-board equipment) and with roadside infrastructure (via roadside equipment). Drivers could be aware of potential hazards through in-vehicle display by receiving the information, such as speed, acceleration and deceleration rate, road conditions, etc., from their neighboring vehicles and roadside infrastructure. The transportation monitors could also use the traffic information transmitted from vehicles on the road networks to adjust transportation system operations. Figure 1 illustrates this scheme of communication. Compared with the traditional systems, which distribute information from a centered control station, this system provides more flexibility to individual vehicles and has well capability of strengthening a driver's awareness of surrounding traffic. It is anticipated that this system will remarkably improve the efficiency of the transportation operations. In order to design and deploy the intelliDrive system, one of the important issues is to know the property of information propagating in a vehicular network, which is the topic of this thesis.

This thesis follows the style and format of *IEEE Transactions on Wireless Communication.*

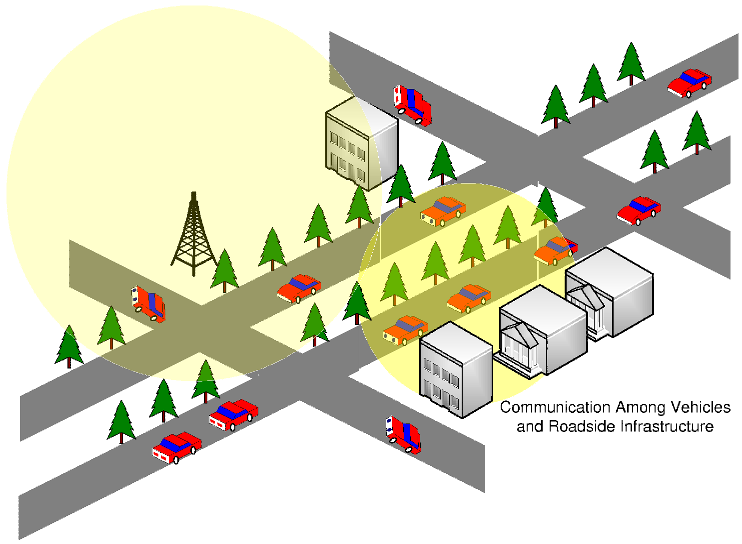


Fig. 1. Vehicles Communicate with Vehicles and Roadside Infrastructure

A. Background and Motivation

In 1970s, a project that enabled vehicles to communicate by radio was initiated in Japan [3], in order to reduce road congestion and accidents. Although the technology at that time could not provide high quality communication, it explored the potential benefits of sharing information among individual vehicles. Nowadays, the development of technologies in wireless local area network (WLAN) enables automobile industries to develop and equip reliable wireless devices on vehicles at low cost [4]. In 2009, the U.S. Department of Transportation (USDOT) and its public and private partners developed strategic plan for IntelliDrive program. This USDOT five years (2010 – 2014) plan mainly focuses on technical, non-technical, and safety, mobility, and environmental applications research [1]. In this system, vehicles are equipped with wireless communication devices, enabling Inter-Vehicle Communication (IVC) (IVC has also been called Vehicle-to-Vehicle (V2V) communication in the literature), Vehicle-Infrastructure Integration (VII), and connectivity among infrastructure, vehicles and other wireless stations. Similar programs, for examples, eSafety program in Europe and Advanced Safety Vehicles (ASV) program in Japan, are also under way.

There are mainly two objectives of IntelliDrive system. The first is to improve safety. By exchanging driving information such as acceleration and deceleration, IVC and VII techniques can monitor the traffic situations and alert the drivers as soon as the potential hazards take place, and even automatically navigate the vehicles. It also can potentially reduce a large portion of accidents caused by careless driving or drivers' misperception. For instance, the application of blind spot warning could alert a driver who tends to change lanes if another vehicle takes place in the blind spot, and the application of forward collision warning could notify a driver if they be not able to brake when there is a stopped vehicle ahead [1]. Moreover, the relative analysis shows that the estimated total benefit of safety is 41.8 billion dollars over the program horizon [5]. The second objective is to increase mobility. It is recognized that ineffective traffic control and inefficient use of road system capacity are due to limited traffic information available to planners and due to the performance limitations of drivers. The IntelliDrive system is intended to provide the information of vehicles in the traffic network such as the location and route of each vehicle so that the traffic manager is able to adjust traffic signals to improve the overall traffic flow. By the application such as adaptive cruise control, it can also reduce the headway between two vehicles, resulting in an increase of the road capacity [6]. The resulting benefits are to lighten the traffic congestion and could save 1.2 million gallons of fuel a year [5].

The networks like IntelliDrive systems are called vehicular ad hoc networks (VANETs) [7], referring to the self-organizing wireless communication networks among vehicles without the aid of any established infrastructure. In these systems, the information of one vehicle and/or its surroundings (for instance, speed and relative distance) is coded into a piece of so called *beacon message* for wireless transmission. For each wireless device, the power of a signal attenuates as the distance from this device increases. When the power is below a certain value, the message cannot be decoded. That means two vehicles equipped with the same kind of wireless communication device can exchange information only when their distance headway is less than the a certain distance, so called *communication range*. If two vehicles are not in the range of each other, their communication depends on *multi-hop* transmission. Since vehicles constantly leave and enter a segment of road, the communication connectivity among vehicles may vary all the time, which leads to concerns of the performance and challenges the design of VANETs.

It is recognized that the *information propagation distance*, determined from the source of vehicle delivering one piece of message to the furthest receiver, is a fundamental measure for the performance of IntelliDrive system, as it can help estimate the connectivity in a vehicular network and thereby guide the design of wireless communication devices and communication protocols. However, previous studies on this issue has been limited to the case of one (lane) straight road. Interactions among vehicles from different roads on a discrete traffic network have significant impact on effective information propagation along traffic streams. A basic situation is parallel roads, which might be found as two divided highway lanes or elevated road and its frontage road. Our rationale for studying this case is based on a question: if one cannot handle this case, how can one expect to address the case of discrete networks. This research begins with a study of the statistic property of information propagation on two parallel roads.

B. Problem Statement

Information propagation in VANETs depends on various parameters such as communication range, bandwidth of the device and number of hops. To avoid the complexity which prohibits development of useful results, in this study, we make three major assumptions:

- Message is transmitted and delivered instantaneously with respect to vehicle movement.
- The information propagation distance is measured based on instantaneous connectivity.
- The communication range is deterministic and no channel interference is considered.

The first assumption gives reasonable representation of the current wireless communication technologies, according to which the data transmission interval is less than 100 ms [8], [9]. This is also true for most applications of vehicle-to-vehicle communications [10], [4]. During such short period, the effect of vehicle movement can be omitted. Admittedly, this requirement can only be realized in practice with a large communication bandwidth device and short messages. The second assumption disregards the connectivity through vehicle mobility. If two neighboring vehicles, which travels in opposite directions, have a distance headway larger than the communication range of a wireless device, the message could not be delivered between them instantaneously. Although such message can be possibly sent to one vehicle in opposite direction after a period during which the transmitter keeps it, this kind of information propagation cannot be effective when applications relate to, for example, urgent safety issues. Hence, we only consider the vehicles with instantaneous connectivity. The third assumption states that the fading effect of the wireless device is not considered. Otherwise, it would lead to the highly complex expression results without better understanding of the basic network property. In fact, it is not difficult to extend the proposed method to the fading case.

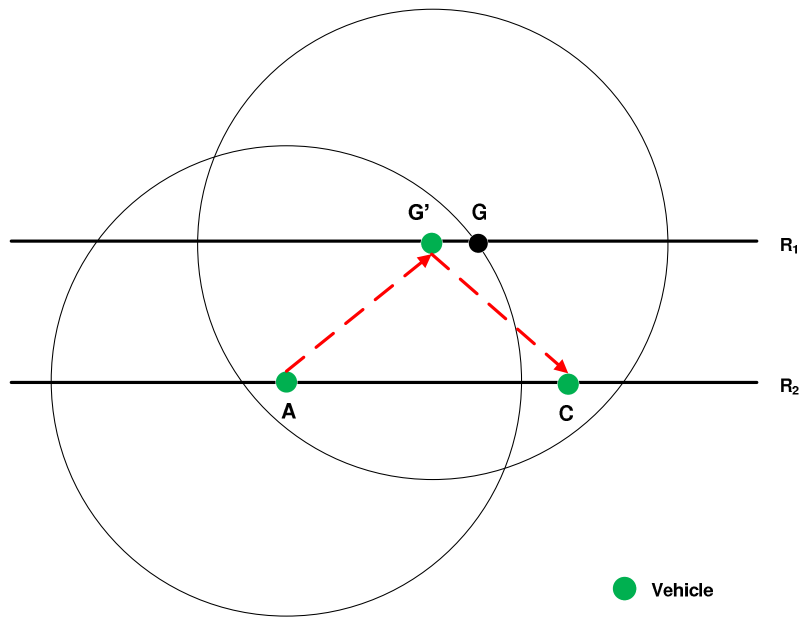


Fig. 2. An Illustrative Process of Information Propagation

We consider two parallel straight roadways denoted by R_1 and R_2, respectively. R_1 and R_2 separated by a distance d. And the traffic density on each road is λ_1 and λ_2, respectively. According to the assumptions, the direction of movement of traffic is not considered explicitly. Since usually the communication range L is significantly larger than the width of lanes, each road of multilanes can be considered a single lane. As illustrated in Figure 2, starting from a vehicle A on road R_2, information is propagated forward in one direction of interest, say, rightward. Vehicles on both roads within the communication range of A are able to receive and instantly further transmit the information forward. If we only consider the information transmission on road R_2 and if there is no vehicle present on road R_2 within the communication range L, vehicle A is not able to directly reach vehicle C. However, with help from vehicle G' on road R_1 within range L, information is able to propagate to vehicle C. Clearly, compared with the case of one road, information propagation is enhanced by vehicles on the second road [11].

The objective of this study is to find out the probabilistic property of the information propagation distance in terms of its expectation, variance and probability distribution. The propagation distance measures from the initiating vehicle to the last receiving one in the same direction as illustrated in Figure 3. According to such definition and assumptions, one can also treat the information propagation distance as the diameter of a random graph [12]. At the beginning of the Chapters III and IV, we will also provide mathematical description of the problem.

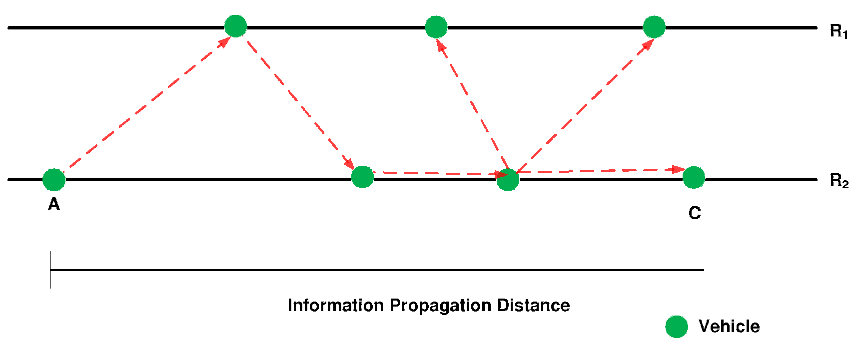


Fig. 3. An Illustration of Information Propagation Distance

C. Overview of the Proposed Methodology

To deal with the complexity of information propagation on two parallel roads, we first develop an approximate method. By identifying the conditions that the information fail to propagate further, the propagation process is approximated by a Bernoulli process. This method can also be applied to the case of one road [11]. In this model, the distance headway can be generalized to any distribution. The approximation generally works well but suffers significant errors in some cases.

Second, in order to get an exact solution, an analytical model is obtained, by defining the transmission region related to each transmitter. For maneuverability, we assume that the traffic along the roadway follows independent homogeneous Poisson distribution, which is often a standard assumption in traffic engineering studies. Although Poisson distribution might be violated as indicated in some studies [13], any lack of this assumption would render the model intractable.

Considering the particular vehicular network studied here, the information process can be viewed as an instance of a stochastic geometry process. Due to the inherent important relationship between geometry process and wireless communication network, the proposed analytical method has a potential to be generalized for other research on ad hoc network [14].

D. Research Objectives

The objective of this research is to model the information propagation process along two parallel roads. The focus is to develop models to estimate the expectation, variance and probability distribution of information propagation along two parallel roads under certain distributions of vehicle headway. The objectives are detailed as follows.

- Review and assessment of the current research on IntelliDrive system.
- Development of an approximate model to explicitly addresses the information propagation distance by assuming a general distribution for distance headway.

- Development of an exact probabilistic model based on a Markov process that captures probability distribution of information propagation along two parallel roads.

- Implement of numerical tests of the proposed models, including numerical calculation and simulation.

E. Thesis Overview

This thesis consists of five chapters. In Chapter II, the technique issues related to IntelliDrive, especially on IVC and VII systems, are reviewed. The concept of vehicular ad hoc networks and network routing are introduced. After reviewing the applications on safety, information service and motion control, the capacity and connectivity issues are described. In Chapter III, the information propagation is approximated by a Bernoulli process. The large number simulations under different situations are compared with the analytical results from the proposed model. Chapter IV starts with identifying two types of transmission regions associated with transmitting vehicles, and then proposes an exact model for the probability distribution of information propagation as well as its expectation and variance. A numerical method is introduced to obtain the solution from the model. Chapter V concludes the thesis by summarizing the contributions of this research and highlighting future work.

CHAPTER II

LITERATURE REVIEW

A. Introduction

The conventional traffic information systems connect vehicles to remote service centers, which collect information of vehicles and broadcast the traffic conditions or service information through radio. In a large scale transportation network, the response of such type of traffic information service system may have a large time delay and thereby be inefficient. Several characteristics of IntelliDrive or IVC and VII systems distinguish themselves from these traditional systems. First of all, the connectivity of vehicles is ad hoc based, i.e., decentralized. With wireless communication technologies, the networks undertake the short-range communication among vehicles without the aid of other infrastructure. In addition, due to high mobility of vehicles, the topology of wireless networks is time varying. These features challenge the wireless communication technology and traffic operations. In this chapter, we first introduce the technical issues of ad hoc wireless network. Then the possible applications of VII and IVC systems are described. Finally we discuss the simulation framework, capacity and connectivity issues.

B. Technical Issues of Vehicular Ad Hoc Networks

The architecture of vehicular ad hoc networks may follow the Open System Interconnection (OSI) seven layer model. The design of a network mainly focuses on each layer design and its protocol, which is a set of rules to ensure the possibility of communication among the network [15]. This section we only focus on the routing problem in the network layer. Before introducing this issue, we begin with the introduction of communication standard for IntelliDrive system. The main goal of standards is enabling communication systems developed by different companies to interoperate.

1. Standards

Recently, a novel communication standard called as Dedicated Short Range Communication (DSRC) has been proposed within the 5.850 to 5.925 GHz licensed band allocated by US Federal Communications Commission (FCC) and has been approved by ASTM International [16]. A kind of multi-channel wireless radio, including one control channel for safety messages and service channels for non-priority messages, is defined in this standard [7]. The message can be periodically sent in order to ensure the reception of other vehicles. Since DSRC is developed from IEEE 802.11a, naturally the Medium Access Control (MAC) layer of DSRC is similar to IEEE 802.11a [17]. Although the most widely used WLAN standard IEEE 802.11 works well in some small scale testbeds [18], which aim to protect the safety service for IVC and VII systems, the DSRC standard is different from IEEE 802.11a mainly in three aspects [1]:

- Design objective: while the design of IEEE 802.11a is optimized for local area networks with low mobility (3 mph), DSRC is aimed at high mobility vehicle applications.
- Medium Access Control (MAC) Layer: the control channel can support vehicular safety applications which is not a main objective of IEEE 802.11a.
- Physical Layer: comparing with 20 MHz channel of IEEE 802.11a, a tight spectrum of DSRC brings reduction of channel congestion and multi-path delay [19].

2. Routing Problem

Because of the various applications and highly dynamic vehicular networks topology, the routing, i.e., finding good paths to deliver the packets between the sender and the receiver, becomes a problem to the network design. This problem relates to the address requirement in network layer. Many safety applications may require the network layer to provide geographical addresses [20]. The relative routing, called GeoCast or position-based routing [21], assumes that each equipped vehicle keeps a routing table including the geographical positions of others. Traditionally ad hoc wireless networks use fixed addresses and any node that receives a query broadcasted by a sender can reply uniquely to that node. Compared with using fixed addresses, using geographical addresses in vehicular networks may enable each vehicle to maintain a relative small routing table and thus to save the communication and storage overhead [22], [23]. Hence, some authors have studied how to map fixed addressed to geographical ones [24], [25]. The geographical routing protocols can also help the sender forward a message to a specific group of vehicles within some road section. This can be done by three scenarios: 1) some changed form of broadcast flooding scenario [26], which is a group of vehicles that include all one-hop peers of each member; 2) clustering scenario, which partitions a vehicular network into several hierarchical groups to reduce the amount of routing information [7]; 3) some advanced forwarding scheme, for instance, a distributed MAC scheme in [27] and a distributed relay selection scheme [28].

C. Applications

In this section we present an overview of several applications and related issues. Three main classes of applications are considered: general information support, safety warning services and vehicular motion control. The security issue is also discussed at the end of this section.

1. General Information Support

General information support applications provide updated road conditions, weather report, local information and general information limited in time or time (such as parking restrictions) [29], [4]. Usually it is not necessary to require real-time delivery in this kind of service. The information delivery is expected to have a large communication coverage in the entire traffic network. In addition, drivers are able to send queries to the traffic control center or the database of peers.

2. Safety Warning Based Application

The major objective of the VII and IVC application is about safety. These applications are aimed to send an alert to drivers to avoid the potential danger situations. The services may include two major classes: emergency and non-emergency warning. While the former may relate to road and travel condition warning, signal and stop sign violation warning, and speed limit warning, the latter relates to the event-driven messages such as incident warning and deceleration warning [1]. These applications have hard real-time constraints with short distance between the sender and receiver. In addition, each warning application may require a different communication range [10].

One problem for such application in highway system is that high speed vehicles may pass the wireless communication area of a roadside infrastructure quickly, resulting in the difficulty to set up the communication link between vehicle and roadside infrastructure during such a short period. Therefore, how to supply the accurate, robust, and reliable information to every vehicle is a technique problem in real application.

3. Vehicular Motion Control

This application aims to improve driving safety and enhance mobility service for individual or groups of vehicles. Through IVC and VII systems, vehicles can continuously exchange information such as position, velocity and acceleration with each other or with the service station. Since the evaluation of wireless communication technology in field tests can guarantee that the transmission latency varies no larger than one second, typically smaller than human reaction time [10], it is possible to utilize this information to automatically control vehicle velocity and acceleration or deceleration and ensure the safety of the driver. The application in cruise control [30] enables vehicles to follow each other closely and the traffic flow is expected to be improved. The relative theoretical work regarding platoon control is concerned with the string stability [31], [32].

IVC and VII techniques can help cooperative driving at intersections and highway ramp metering. The project CarTALK describes such application [33] and indicates that it may improve the traffic operations. Compared with maintaining platoon stability where information can be delivered backward, cooperative assistance is more complex since the information should be gathered from all direction, and the entire communication area should cover all vehicles near intersections or ramps. It requires quick computation to determine a safe and efficient strategy. The study in [34] represents all schedules as a spanning tree according to the proposed safety driving patterns. Since it is efficient to search a proper trajectory control plan within a schedule tree, the cooperative control through VII and IVC can be reliable. In [35], the cooperative system puts each vehicle's state into a group of coupled extended Kalman filters and then automatically control the trajectory for each car in order to minimize the delay as well as fuel consumption. The authors also show the good performance of the Kalman filtering to a nonlinear multi-state vehicle model, indicating a promising feature in real application. Described in [36], the Intelligent Speed Adaptation systems can use VII techniques to predict the driver's reaction and take control of vehicle if necessary.

4. Security Issues

With the rapid development of IVC and VII systems, privacy is becoming a public concern. While the activities of a driver on the public roads may be revealed to the public, privacy laws have not kept up with these issues resulting from uses of advanced information technology [37]. Such privacy concern is known as "the problem of privacy in public" [38]. However, until now only a few publications have been found related to security topic. The authors in [39] discuss the security issues when vehicles can exchange information with the local neighbors, though it is true to conclude that there is no need to consider confidentiality. The authors in [40] propose a method which aims to use the public-key infrastructure to secure communications and thereby to enhance the privacy of vehicles. They also examine the privacy limitations of sharing certificates among vehicles. A design for the security of geographical routing is presented in [41]. The key is to use the location and time information to find the secure neighbors.

D. Modeling Vehicular Ad Hoc Networks

An appropriate model of wireless ad hoc networks, particularly a probabilistic model simplifying physical world without losing basic characteristics, can give some insights into the performance and help design IVC and VII systems. Many ground breaking analytical results are obtained ranging from the connectivity issues to the network capacity. On the other hand, simulation is necessary to explore the performance, because very often the complexity of IVC and VII systems exceeds the capability of analytical methods due to a large number of factors, such as signal fading and interference, and because field tests are too costly to conduct [2]. This section is focused on the simulation and two basic topics, network capacity and connectivity.

1. Simulation Issues

Computer simulations serve as a basic tool to evaluate the functionality of IVC and VII systems. An appropriate simulator must be capable of modeling both wireless network communication and traffic flow. Unfortunately, currently no integrated simulator exists with a full range of components, though there are many advanced simulators in each individual area and a few number of efforts in progress [42], [43]. From the perspective of wireless communication, one challenge of simulation comes from simulating radio signal propagation with various environment factors. Besides the path loss effect, i.e., the power reduction as signal propagates in space, the radio wave signal may be diffracted and scatted by the environment, and the mobility of vehicle may also cause a time-variant distortion to the signal strength, known as shadowing and fading respectively [44]. The multi-path loss effect also plays a significant role in communication. It arises when the same messages transmitted through different internodes arrive at a receiver simultaneously. In vehicular networks, information propagation usually takes place in a multi-hop manner. The percentage of equipped vehicles, known as market penetration rate, may affect the multi-hop delivery since this value can vary significantly in different places in large scale networks. To deal with some of these issues in simulation, currently, there are several wireless communication simulators, such as Qualnet, NS-2, OPTNET, and NCTUns.

A traffic simulator is needed to describe the movement of vehicles according to the constrains such as road topology and speed limit. From the perspective of traffic engineering, a qualified simulator should employ at least microscopic and macroscopic mobility models. The former tracks individual vehicles and mimics individual driver's behaviors with response to surrounding traffic, such as car following and lane changing; the latter deals with the vehicular traffic flow at an aggregate level, describing the phenomena such as trip generation [45], [46]. Current studies tend to integrate the traffic and wireless communication simulators, for example, combing NS-2 and microscopic simulator VISSIM [47], and to explore the performance of vehicular networks, topics including transverse message delivery [48], packet-routing protocols [49], packet collision rate [50] and safety improvement [44].

There are many benefits for establishing a realistic simulation framework. One is to crucial to test current theoretical models and design of next generation networks [46]. Although it is argued that field tests provide much valuable information, field tests are often restricted to addressing small-scale issues. This is because large-scale deployment of IVC and VII systems are too expensive [51]. Hence, evaluation of IntelliDrive systems in simulation will remain crucial for a long time.

2. Capacity of Vehicular Networks

It is important to measure the "amount of information flow" in the wireless ad hoc network. This issue is studied by the capacity of networks from different perspectives. In a pioneered work, Gupta and Kumar [52] study the transport capacity which measures the distance-rate throughput that are transported per second. They investigate a model of fixed ad hoc networks where the source and destination are randomly located with a fixed communication range. The main result shows that the capacity reduces linearly as $\Theta(\frac{1}{\sqrt{n}})$ with the decreasing number of nodes n. Along this strategy, the work in [53] shows the expected transport capacity can be upper-bounded by the multiple of total transmission powers of nodes in some cases. The authors in [54] apply percolation theory to find a lower bound on the achievable bit rate in a wireless network. Although there may be some interference in the communication, their results show that in some cases the total amount of information sent by all sources can be transmitted by the nodes along the end-to-end connected paths. For the case of independent fading channel between any pair of nodes, the exact expression of per-node capacity can also be found if multi-hop connection only allows data forwarding [55].

The transmission capacity, defined as the throughput of successful transmitting nodes in the network per unit area, is first introduced in [56] with an outage constraint. Most interestingly, the transmission capacity can be tightly bounded in many situations. Until now, the transmission capacity has been studied from the perspectives of design and performance analysis, addressing issues such as interference cancelation [57], power control [58], and the relationship with outage probability and data rate [59]. However, most of these studies focus on a one-hop wireless ad hoc network. For the VANETs, several works have shown the mobility of nodes will increase the capacity of the network [60], [61]. The results show that the end-to-end throughput does not significantly lose with the growth of number of nodes in the entire network. In the transportation area, Du *et al.* in [62] consider the VANETs with a broadcast transmission protocol and study the broadcast capacity measured by the maximum number of successful concurrent transmissions. Due to the particular characteristics of traffic flow, various effects such as traffic density and vehicular distribution are investigated by two integer programming models.

3. Connectivity of Vehicular Networks

Estimating connectivity of wireless ad hoc networks like IVC and VII systems is important for understanding the effectiveness of system and designing routing protocols. The study in [63] investigates the probability distribution of the minimum number of hops, given fixed number of uniformly distributed nodes on a rectangular area. Although closed form expressions can be derived for the cases that the nodes are one or two hops connected, the cases of hops larger than two are only studied by simulation. Mullen [64] presents two approximate models for the distance distribution of connected nodes within a rectangular region, assuming that the static nodes are independently distributed in each dimension. The work by Orris and Barton [65] and its correction by Zanella *et al.* [66] provide the distribution of the number of nodes within one-hop area with log-normal fading effect. Although this work does not allow multi-hop connection, it provides some guideline and inspires further research. Mukherjee and Avidor [67] extend the study to the multi-hop network with homogeneous Poisson distributed nodes in a plane, considering the fading channel and power consumption with battery-operated nodes. Kuo and Liao [68] are motivated by multi-hop connection and obtain the path connectivity probability in a network.

Although there are many studies on the connectivity of ad hoc wireless networks, the work on IVC and VII systems from connectivity perspective is rare. The IVC and VII systems distinguish themselves from other ad hoc networks in several ways. In contrast to the previous studies where nodes are randomly distributed within a circle or a rectangular plane and are randomly assigned with a speed, groups of vehicles travel in a segment of road along a fixed direction whereby traffic density, volume and speed follow certain relationship. That means the mobility of nodes is predictable with a constrained road topology. Since vehicles can provide enough power to the wireless devices, power consumption is not an issue. Additionally, most previous work focus on one-hop broadcast communication but IVC and VII systems adopt multi-hop strategy in real application. Inspired by these particular features, many researchers from transportation and electronic engineering areas make great effort. Wang [11] studies the information propagation along one road. This work defines a relay process to obtain a closed formula for information propagation distance when the traffic follows homogeneous Poisson distribution. Wu *et al.* [69] considers the mobility effect on the information propagation. The vehicles are assumed to have the same communication range and a constant delay between receiving and transmitting at the same vehicle. In light of this strategy, the study in [48] considers the effects of two direction traffic flow, assuming the messages to be delivered within a tolerable delay. When no vehicle locates within the communication range, a vehicle can broadcast the information until it reaches another one in opposite direction. The investigation of such kind of connectivity is conducted by simulation. Jin and Recker [70] extend to study the inhomogeneous Poisson traffic where the positions of vehicles are known through simulation or field observation.

E. Summary

This chapter reviews the literature for vehicular ad hoc networks. Beginning with a short introduction of features of vehicular ad hoc networks, the DSRC standard and the routing problem in network layer are discussed. These problems are fundamental to development of any advanced models on IVC and VII systems. After introducing the applications to general information service, safety warning system and the motion control, the security issues are also noted. In simulation, the wireless communication model and mobility model are both introduced. Finally, important issues about capacity and connectivity are discussed.

CHAPTER III

METHODOLOGY I: BERNOULLI APPROXIMATION

A. Introduction

In this chapter, we study the process of instantaneous information propagation along two parallel lines of traffic as stated in Chapter I Section B. In the context of instantaneous propagation, the propagation distance is equivalent to the distance of the furthest connected vehicle on the same road. This problem can be also described by the random geometric graph [12] in the following.

1. Random Geometric Graphs Description

According to the assumptions in Chapter I Section B, we are able to use random geometric graphs to model the vehicle ad hoc networks. We assume the network nodes are randomly placed on two parallel lines, and a communication link connects two nodes if the distance between them is not greater than the communication range. A mathematical model for this case is as follows. Let $\|\cdot\|$ be the Euclidean norm, and R_1 and R_2 be two parallel lines on $\mathbb{R}^2$. The distance of these two lines is $d > 0$. Let $f_1(\cdot)$ and $f_2(\cdot)$ be some two probability density function (p.d.f.) on R_1 and R_2, respectively. Note that $f_1(\cdot)$ and $f_2(\cdot)$ are defined on $\mathbb{R}^1$. Also, let $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \ldots$ be independent and identically distributed (i.i.d.) random variables with the density $f_1(\cdot)$, where $\mathbf{X}_i$ denotes he random location of node i on the line R_1. Similarly, let $\mathbf{Y}_1, \mathbf{Y}_2, \mathbf{Y}_3, \ldots$ be i.i.d. random variables with the density $f_2(\cdot)$ on the line R_2. The ensemble of graphs with undirected links connecting all those pairs $\{\mathbf{z}_i, \mathbf{z}_j\}$ (where $\mathbf{z}_i \in \{\mathbf{X}_i\}$ or $\{\mathbf{Y}_i\}$) with $\|\mathbf{z}_i - \mathbf{z}_j\| \leq L$, $L > 0$, is called *random geometric graph* [12], which is denoted by $G(\mathcal{X}, L)$.

In this model, when we say 'one node can send the information to another one', we mean there exists a link connecting these two nodes. The problem of *information propagation* can be described as follows. Let W be any connected component of $G(\mathcal{X}, L)$ and $\{\mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_N\} \subset W$, where N is a random integer valued variable and it depends on W. The node $\mathbf{Y}_i$ should locate between $\mathbf{Y}_{i-1}$ and $\mathbf{Y}_{i+1}$ on R_2. We define the *information propagation distance* D_2 on R_2 in this chapter as follows:

$$D_2 \triangleq \sup\{\|\mathbf{y}_i - \mathbf{y}_j\| : i, j \in \{0, 1, \ldots, N\}\}.$$

In this definition, one can see that $D_2 = \|\mathbf{y}_0 - \mathbf{y}_N\|$. Let $H_i = \|\mathbf{y}_i - \mathbf{y}_{i-1}\|$. Then we can express D_2 as

$$D_2 = \sum_{i=1}^{N} H_i. \tag{3.1}$$

We are primarily concerned with the expectation $E[D_2]$ and variance $V(D_2)$ in this chapter.

2. Ideas About the Solution

In this chapter, the ideas for the solution of Equation (3.1) are straightforward: to find an approximation that H_i are independent. In details, we are going to do:

- to find some geometric region (called Bernoulli region) related to each pair $\{\mathbf{y}_i, \mathbf{y}_{i-1}\}$ (they may not necessarily connect with each other), where the length of one edge of this region is equal to H_i;
- to find the probability of H_i by conditioning on the same connected component;
- under some situation, one can approximately treat Bernoulli regions to be independent with each other;

- the probability for each Bernoulli region occurring can be seen as the same; so that the information propagation process can be approximated by Bernoulli process.

In the next sections, we will first identify two cases in terms of the distance d between R_1 and R_2, where in the first case the proposed approximation performs well.

This chapter is organized as follows. In Section B, we introduce a Bernoulli process to approximate the information propagation. With the help of a Bernoulli region, we derive the mean and variance of the propagation distance. In section C, numerical simulations are conducted to asses accuracy of the results from using the Bernoulli process, and to show how distance between the two parallel roads impacts the propagation distance. The paper concludes with Section D.

B. Approximation with a Bernoulli Model

We consider the information being relayed along one road rightward, say R_2. Vehicle A is the transmitting vehicle on R_2 as in Figure 4. Suppose G and H are the furthest points directly reachable by vehicle A on both roads. $ABGH$ is the according parallelogram with $BG = AH = L$. We identify two cases. Figure 4 left has point B within the communication range of vehicle A, representing Case I. In case I, it is clear that vehicles left of the parallelogram $ABGH$ do not matter in further propagating information rightward from vehicle A: any vehicle left of point B on road R_1 would have to resort to vehicles within BG for further propagation. By the language of random graph, it means that any node in BG can connect with A and only these nodes can connect the nodes (if any) at the right of point G and H. We further define $r = 2(L - \sqrt{L^2 - d^2})$ in Case I, representing two times the horizontal *shift* of the two horizontal sides of each parallelogram.

If point B of the parallelogram is outside vehicle A's transmission region, one gets Case II as in the right of Figure 4. In Case II, it is not possible to have a parallelogram $ABGH$ within the communication range of A, and point B will be out of reach by the vehicle at point A as in Figure 4. In the later part, we define for Case II the point B as the leftmost reachable point by A, in which case $ABGH$ will not be a parallelogram.

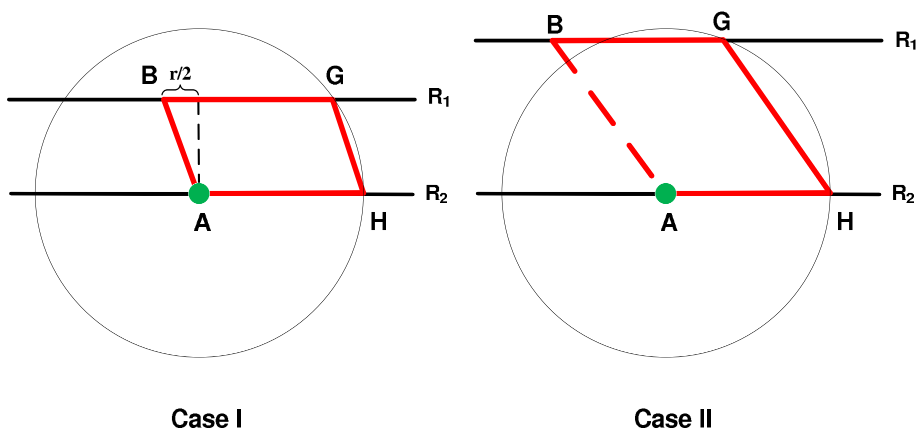


Fig. 4. Two Cases of Point B on Road R_1

Case I has the following relationship: $d \leq \frac{\sqrt{3}}{2}L$. Similarly, Case II corresponds to the situation in which $d > \frac{\sqrt{3}}{2}L$. Here $d = \frac{\sqrt{3}}{2}L$ is the critical point separating two cases, at which the parallelogram $ABGH$ has exactly three points BGH on the circle of radius L about point A. The increase of the separation distance d makes point B fall outside the circle, implying Case II, while the decrease draws point B to within the circle, implying Case I. We first discuss Case I.

1. Case I

Suppose vehicle A is the transmitting vehicle on R_2 and C is the immediate next vehicle to A on R_2. That is, there is no vehicle in the segment AC. As shown in

Figure 5, $ABGH$ is the according parallelogram defined earlier. We further assume that E is the leftmost point on road R_1 that can reach vehicle C on road R_2. Let $ED = EC = FC = L$, and $CDEF$ is the according parallelogram. The region $ABDC$, accords to a vehicle gap AC on road R_2, is called the *Bernoulli region* of vehicle A in Case I. It is worth noting that BD has the length of $AC + r$. In addition, one has to note that Figure 5 just illustrates an example for the Bernoulli region. The distance headway of vehicle A and C can be any value. The two parallelogram $ABGH$ and $CDEF$ could have some overlap. Hence, any distance headway on R_2 is associated with a Bernoulli region. Or, equivalently, *any* vehicle (as A in Figure 5) on road R_2 has a Bernoulli region with according points B and D defined above. Such region is critical to information propagation. In fact, we have

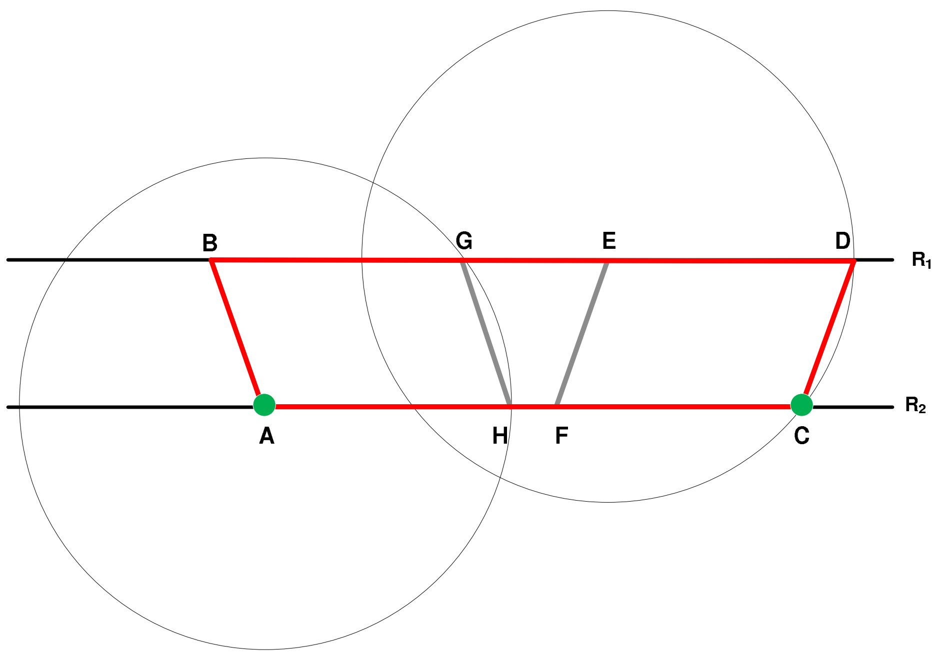


Fig. 5. A Bernoulli Region $ABDC$ During Propagation

Proposition 1 *Information propagation along road R_2 is gapped out at a vehicle* *location A if and only if the associated Bernoulli region has a vehicle gap larger than L on both roads.*

By the language of random graph, when we say 'the information propagation is gapped out', we mean that two neighboring vehicles on R_2 are not in the same connected component. The Proposition 1 is obvious: if there is no link to connect A and C, the only situation for them in the same connected component is that some nodes in the one connected component in BD connect with A and C, as shown in Figure 5.

It is clear that information propagation gaps out if and only if the process fails to get through a Bernoulli region. Each vehicle on road R_2 has an according Bernoulli region with a probability of success. Since the vehicle gaps on each road are *i.i.d.*, one may roughly treat each Bernoulli region as having an *identical and independent* probability of success. Propagating through vehicles on road R_2 can therefore be considered as a Bernoulli process whose 'trials' are the Bernoulli regions. The success means successful propagation through the Bernoulli region and failure means otherwise. The gap out probability associated with a Bernoulli region is denoted by p_r. In this paper, we assess the value of p_r by assuming independent Bernoulli regions. This assessment of p_r could be fairly accurate but not exact, as explained below. This is why we call the Bernoulli process here an *approximate* Bernoulli process.

This treatment using the Bernoulli process is not accurate because the probability of gap-out at a 'trial' could be slightly correlated to the probability of no gap out at its proceeding trial. The Bernoulli regions associated with two consecutive vehicles on road R_2, i.e., $ABDC$ and $CGEF$ as in Figure 6, have an overlap of length r on road R_1. When two consecutive gaps larger than L are present on road R_2, not gapping out in the first Bernoulli region might indicate, to a certain degree, vehicle presence in the overlap section on road R_1. From such perspective, r is an important parameter in this study as it indicates the extent to which vehicular interaction takes place between the two roads.

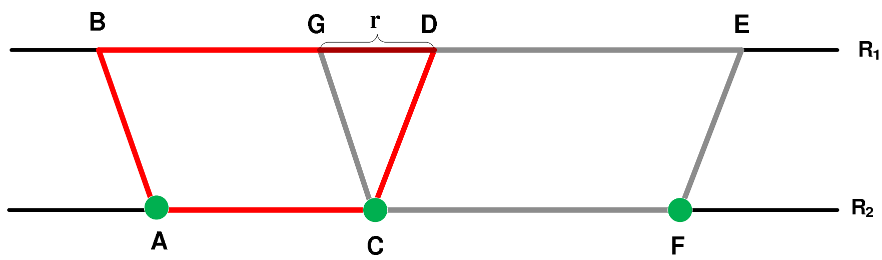


Fig. 6. Overlap between Two Consecutive Bernoulli Regions

According to the Bernoulli process, the random number of gaps on R_2 (or, Bernoulli regions) until the first gap out follows a Geometric distribution, whose mean is $\frac{1}{p_r}$. In other words, if the number of gaps before a gap out is denoted by N, then we have $E[N] = \frac{1}{p_r} - 1$. In order to get the expected propagation distance on road R_2, we need to calculate the expected length of the gap before a gap-out. By the language of random geometric graphs, such treatment is that we treat H_i as i.i.d. random variables (in Equation 3.1) under the condition that associated nodes in one connected component.

As vehicle presence on both roads are independent of each other, the probability of gap out, p_r, is calculated as the product of two probabilities: one for a gap g larger than L on road R_2 and the other for a gap larger than L on road R_1 within the range $g + r$, where g is the vehicle gap on road R_2. Since the distance headway (or gap) of vehicles is assumed homogeneous along the roads, the probability of a gap larger than L on road R_1 within a range y, denoted by $p(y)$, can be calculated recursively by using conditional probability. If we denote by g_{next} the distance headway between the starting point on R_1 and next vehicle along the direction, by noting that $g(y|\{g_{next} > L\}) = 1$, when $y > L$, we have

$$\begin{aligned} p(y) &= E(p(y|g_{next})) \\ &= \int_0^L f_1(t)p(y-t)dt + \int_L^\infty f_1(t) \cdot 1 \, dt. \end{aligned} \tag{3.2}$$

Obvious when $y > L$, $p(y) = 0$. Hence we have

$$p(y) = \begin{cases} \int_0^L f_1(t)p(y-t)dt + 1 - F_1(L), \text{when } \ y \geq L, \\ 0, \text{when } \ y < L. \end{cases} \tag{3.3}$$

Where $f_i(\cdot)$ is the probability density function of the distance headway on road i, whose cumulative function is $F_i(\cdot)$. For the gapout probability of a distance y on road R_1, Equation (3.3) means that if no vehicle is present in $[0, L]$, whose probability is $1 - F_1(L)$, there is a gapout; if a vehicle is present at distance t in $[0, L]$, the failure of detour on R_1 then depends on gapout over the remaining length $y - t$, the probability for which is therefore $f_1(t)p(y-t)dt$. Note here that t is the location of the first vehicle from the beginning of the Bernoulli region on road R_1. Figure 7 illustrates this recursive relation.

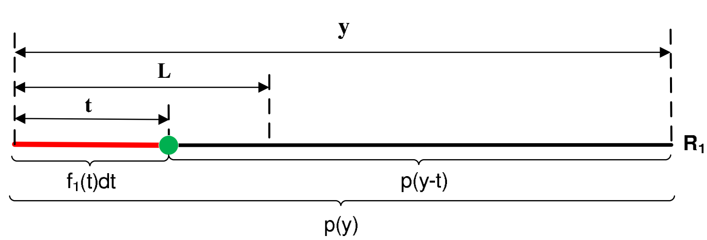


Fig. 7. Illustrate Equation 1: Conditional on Next Vehicle

The failure (gap out) probability of a Bernoulli region is $p_r = \int_L^\infty f_2(t)p(t+r)dt$, meaning that there is a vehicle distance headway larger than L on both roads. If we denote the event of success Bernoulli region by $\mathcal{S}$, the expected length of each *distance headway* or *gap* associated with one Bernoulli region, denoted by $E[g|\mathcal{S}]$, can be calculated as follows.

$$E[g|\mathcal{S}] = \frac{\int_0^L tf_2(t)dt + \int_L^\infty tf_2(t)(1-p(t+r))dt}{1-\int_L^\infty f_2(t)p(t+r)dt}. \tag{3.4}$$

The denominator is the success probability of a Bernoulli region. The probability density function $f_2(\cdot)$ in the numerator divided by the denominator leads to the conditional probability density function on no gap-out. The first term in the numerator corresponds to the case of a gap smaller than L on R_2; and the second term refers to the case of a larger than L gap on road R_2 but of a success of the according Bernoulli region.

If we denote $E[g]$ as the expected distance headway on road R_2, Equation (3.4) can be simplified as follows

$$E[g|\mathcal{S}] = \frac{E[g] - \int_L^\infty tf_2(t)p(t+r)dt}{1-\int_L^\infty f_2(t)p(t+r)dt}. \tag{3.5}$$

Therefore, the expected propagation distance can be approximated by the following.

$$\begin{aligned} E[D_2] &= E[g|\mathcal{S}] \times (\frac{1}{p_r} - 1) \\ &= \frac{E[g|\mathcal{S}](1-p_r)}{p_r} \\ &= \frac{E[g] - \int_L^\infty tf_2(t)p(t+r)dt}{p_r}. \end{aligned} \tag{3.6}$$

In addition, by using the Bernoulli process, we are able to find the variance of propagation distance D_2, $V(D_2)$. Recall the Equation 3.1, $D_2 = \sum_{i=1}^{N} H_i$, where H_i are distance headways indexed from the initial transmitting vehicle. According to previous analysis, H_i are i.i.d. with H whose distribution is the same with g under the event $\mathcal{S}$. By the law of total variance, the variance of propagation distance can be obtained. We then have

Proposition 2 *The mean and variance of information propagation distance on road R_2 may be expressed as follows.*

$$
\begin{aligned}
E[D_2] &= \frac{E[g] - \int_L^\infty t f_2(t) p(t+r) dt}{\int_L^\infty f_2(t) p(t+r) dt}, \\
\textit{and,} & \\
V(D_2) &= V[E[\sum_{i=1}^{N} H_i | N]] + E[V(\sum_{i=1}^{N} H_i | N)] \\
&= V(NE[H]) + E[NV(H)] \\
&= E^2(H)V(N) + E[N]V(H), \qquad (3.7)
\end{aligned}
$$

where H has a probability density function as follows.

$$
f(t) = \begin{cases} f_2(t)/(1-p_r), & for\ t \le L, \\ f_2(t)(1-p(t+r))/(1-p_r), & for\ t > L. \end{cases} \qquad (3.8)
$$

With formula (3.8), we are able to numerically get $E[H]$ and $V(H)$. Therefore, Equation (3.7) can be evaluated with numerical method easily.

The proposed model can also be applied to the special case when there is only one road. Setting $\lambda_1 = 0.0$, one can show that Equations (3.6) and (3.7) leads to the following results.

Proposition 3 *Information propagation along a single road has an expected propa-*

gation distance and variance as follows.

$$
\begin{aligned}
E[D] &= \frac{\int_0^L t f(t) dt}{1 - F(L)}, \quad \textit{and} \\
V(D) &= \frac{\int_0^L t^2 f(t) dt}{1 - F(L)} + (E[D])^2,
\end{aligned}
\tag{3.9}
$$

where $f(\cdot)$ and $F(\cdot)$ are the respective density and cumulative functions of distance headway with two roads combined, and D is the information propagation distance.

We provide a short explanation. Note that $p(t + r) = 1.0$ always holds when $\lambda_1 = 0$. Substitution of $D = D_2$ into Equation (3.6) gives $E[D_2] = \frac{\int_0^L t f_2(t) dt}{1 - F_2(L)}$. Since N has a Geometric distribution, then

$$E[N] = \frac{F(L)}{1 - F(L)}, \quad \text{and} \quad V(N) = \frac{F(L)}{(1 - F(L))^2}.$$

Hence, by Equation (3.7), we have the expression of $V(D)$.

Proposition 3 was developed in Wang *et al* [71]. Here by a Bernoulli process, we have provided an alternative proof.

2. Case II

As stated earlier, Case II satisfies such a condtion: $d > \frac{\sqrt{3}}{2} L$.

In Figure 8, the gap between two consecutive vehicles A and C is larger than L on road R_2 as we are interested in the gap out probability. B is the left most point on road R_1 that the vehicle at point A on road R_2 can reach within a communication range L, *i.e.*, $AB = L$. E is the furthest point on road R_1 horizontally left of point C that is able to directly reach point C, i.e., $EC = L$. D is the furthest reach to the right directly from point E on road R_1, *i.e.*, $ED = L$. In addition, G is the rightmost point on road R_1 directly reachable by vehicle A, and clearly $BG = 2\sqrt{L^2 - d^2} \neq r$.

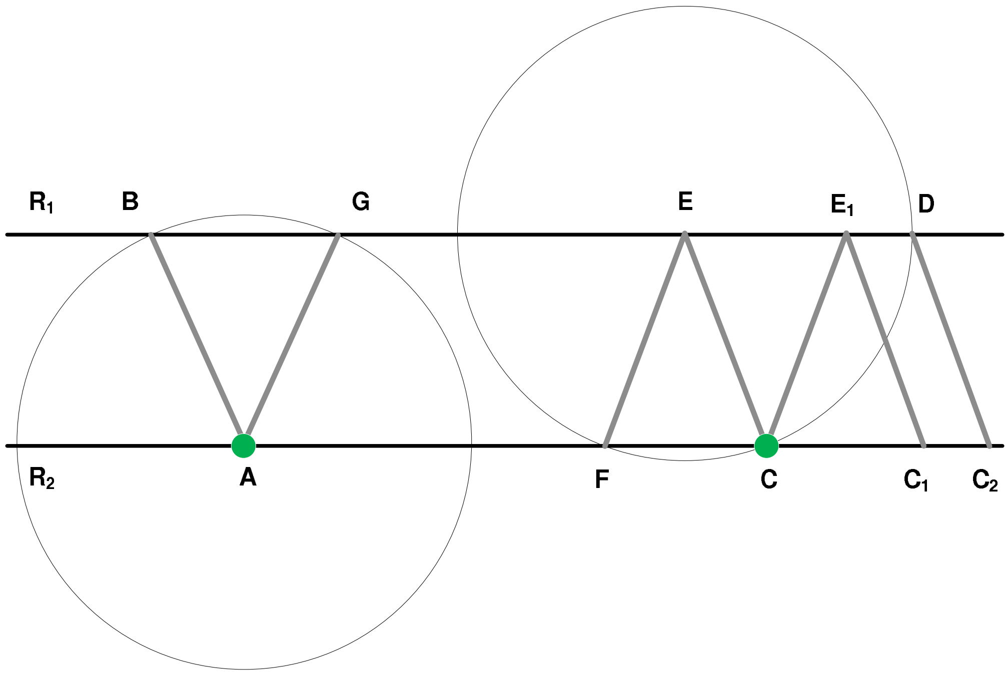


Fig. 8. An *Approximate* Bernoulli Region in Case II

F is the leftmost point reachable on road R_2 by a vehicle at point E with $CF = BG$. E_1 is the leftmost point to the right of vehicle C horizontally that can reach vehicle C. C_1 is the rightmost point reachable on R_2 by point E_1. C_2 on road R_2 corresponds to point D on road R_1. We have $CC_2 = L$. Obviously, $BE = AC$ and $BD = AC + L$.

We still refer to $ABDC$ as a Bernoulli region, although with slight notational abuse about points B and D defined earlier. Note that the Bernoulli region in Case I has $BD = AC + r$, which is different from Case II due to the different situations. In order to take a detour on road R_1 to C on R_2, there are two necessary conditions. First, there must be vehicle presence within the range $[B, G]$ on road R_1 if otherwise, no vehicle within the section of a length $L - BG$ left of B is capable of propagating further till beyond G. This first condition implies ignorance of information coming from left of point B on road R_1. Second, vehicles cannot gap out within $[G, D]$ in order to assist the propagation. Note that vehicles have probabilistic presence on road R_1, given the vehicle gap AC on road R_2.

There are two cases for one vehicle within the segment $[B, G]$ to propagate information to beyond point E.

- Information is propagated to a vehicle within the range $[E, E_1]$. In this case, information is able to reach vehicle C on road R_1.
- There is no vehicle within the range $[E, E_1]$. But information is propagated to a vehicle within the range $[E_1, D]$. we consider this case as not gapping out on road R_2 (approximately). Note that in this case, the vehicle at location C may actually be skipped.

The above discussions conclude that *there should be no gap of L or larger over the distance BD in order for information to detour to the vehicle at point C.* As a matter of fact, information getting through BD could have come from the vehicle at point A or from vehicles prior to location B on road R_1. For manoeuvrability, we simplify the process by ignoring the information that could have come from vehicles on road R_1 left of point B. The effect of this ignorance is minimized by choosing the larger vehicle density for road R_2.

The above discussions simplify the process by assuming that information getting through to a vehicle at point within E_1D successfully propagates to the vehicle at point C at a probability 1.00. This assumption could partially compensate for the underestimate of information getting through BG by ignoring that coming from left of point B, as in Figure 8.

Therefore, the probability of getting through R_1 from vehicle A to C is approximated as follows by slightly modifying Equation (3.3), using the fact that $BG = 2\sqrt{L^2 - d^2}$ and $BD = AC + L$.

$$p(y) = \begin{cases} \int_0^{2\sqrt{L^2-d^2}} f_1(t)p(y-t)dt + 1 - F_1(2\sqrt{L^2-d^2}), \text{for } \ y \geq L, \\ 0, \text{for } \ y < L. \end{cases} \tag{3.10}$$

In calculation for Case II, the according new formulas in Proposition 2 are as follows.

$$E[D_2] \quad = \quad \frac{\int_0^L tf_2(t)dt + \int_L^\infty tf_2(t)(1-p(t+L))dt}{\int_L^\infty f_2(t)p(t+L)dt}.$$

and H has a probability density function as follows.

$$f(t) = \begin{cases} f_2(t)/(1-p_r), \ \ for \ t \leq L, \\ f_2(t)(1-p(t+L))/(1-p_r), \ \ \ for \ t \geq L. \end{cases}$$

C. Numerical Tests

The numerical tests are designed to show the propagation distance in relation to the system parameters such as communication range, vehicle density and road separation distance.

A discrete numerical method was applied to solve the equations for the expectation and variance of successful propagation distance. A small step-length h is used for discretization to estimate the integrals of function $f(x)$, i.e., the integral $\int_0^a f(x)dx$ is approximated by $\sum_{i=0}^n f(ih)h$. When a is sufficiently large, for example, when a includes a vehicle distance headway at a probability of *almost* 1.0, one gets the integration $\int_0^\infty f(x)dx$. In our case, we take $a = 30L$. The $p(t)$, seen in Equations (1) though (8), can be estimated recursively by discretizing the integral and using the boundary value $p(0) = 0$.

The simulation only takes into account the statistical property of distance headway without the consideration of a real mobility. And the communication connectivity is also simulated to be instantaneous. The more sophisticated simulation will be studied in the future.

An observation is worthy of a note. When a test instance is constructed, it only reflects the relativity of distance and vehicle density. The test instances constructed should cover a range of the relative magnitude of parameters. We set the communication range to be a standard unit 1.0. All the other lengths are measured against this unit, including the vehicle density.

We have tested both the case of Poisson vehicle distribution and other independent vehicle distance headway distributions using the formulas developed. We conduct 4000 times simulation for each instance to benchmark results from the analytical formulas.

1. Poisson Distribution of Vehicles on the Roads

In testing the formulas for Case I, the density for road R_2 is set to be 0.2, 0.6, 1.0, 1.5 and 2.0 respectively, each corresponding to a set of lower densities on road R_1. Each pair of road densities for both roads has a set of varying road separation distances of 0.1, 0.5 and 0.8 respectively. Except for a particular case with a road R_2 density of 0.2, the expectation and variance of propagation distance are highly accurate in all instances of Case I.

The test result for Case II shows high accuracy, but slightly poorer than in Case I. For each case in Case I, by increasing the road separation distance to 0.9, 0.94 and 0.98, we get corresponding instances in Case II. All the results are provided in Table IV to Table VIII in the Appendix A.

The analytical approximation in case II is generally very close to the simulation results if the vehicle density on road 1 is smaller than that on road 2. However, we have a few cases in which both roads have high densities, which gives rise to large errors in the calculated numbers compared with simulation results, a distinct example of which is seen at $\lambda_1 = \lambda_2 = 2.0$ and $D = 0.98$. See Table IV for details. This is most likely attributed to the violation of our assumptions in the development of the formulas for Case II. In Case II, the derivation implies that road 2 has a longer propagation distance. Based on this assumption, no vehicle left of point B on road 1 in Figure 8 is able to transmit the information to vehicles beyond point B. Magnitude of errors in Case II may be interpreted as a result of the extent to which this assumption is violated. For example, when road separation distance gets closer to the critical value $\frac{\sqrt{3}}{2}$, the approximation errors in Case II become smaller because in this instance, the formulas for Case II depend less on the assumption. The reason for the less dependence is due to $ABDC$ being closer to the Bernoulli region defined in Figure 5.

2. Gamma Distance Headway Distribution

We also test a select number of cases in which the vehicle distance headway follows Gamma distributions on both roads. We set μ_2 to be 2.0, and μ_1 being 1.2, 1.0. and 0.8 respectively, where μ_i is the mean distance headway on road R_i, $i = 1, 2$. The road separation is 0.01, 0.1, 0.3, 0.5, 0.7, 0.8 respectively. In these cases, we set the variance of vehicle headway on road 2 is 0.5 times its mean,and road 1 has a variance of vehicle headway equal to its mean. Note that Gamma distribution becomes exponential when the mean and variance are equal. The test results are seen in Table IX and Table X in the Appendix B. It appears that formulas in Case I provide good estimates for the mean. Analytical variance tends to be larger than from simulation though, the difference between them increases with road separation.

3. Truncated Gaussian Distance Headway Distribution

We test on traffic with distance headway following the truncated Gaussian distribution, whose density function is as follows.

$$f(x;a) = \frac{\phi(\frac{x-\mu}{\sigma})}{1-\Phi(\frac{a-\mu}{\sigma})},\ x \geq a,$$

where $\phi(\cdot)$ is Gaussian density function, $\Phi(\cdot)$ Gaussian cumulative function, a the truncation point, μ the mean and σ the standard deviation of Gaussian distribution. Here a can be set up to a reasonable value to control the minimum headway allowed. Generally, the smaller the μ reaches, the higher the density becomes. As earlier, L is set to 1.0. The coefficient of variation, ratio between standard deviation and mean, of the distribution is set to 0.8. The results of each instance are provided with the analytical values followed by simulation values, as in Table XI of the Appendix C.

4. Information Propagation Along Two Zigzag Roads

In real application, two roads might not be suitable to be considered as the parallel lines. There is a need to test the proposed model in a different geometric structure of the roads. In this section, we apply our developed formulas to two parallel zigzag roads as illustrated in Figure 9.

The center lines of the two zigzag roads are apart from each other by a distance d. The actual zigzag roads have an angle α with the center line. We test a series of instances with α varying from 10 to 20 degrees. Each zigzag section is set to be $0.8L$, where L is the length of communication range and is standardized to be 1.0. The headway on both roads follow Poisson distributions and the successful information propagation distance is measured by two different kinds of distance: 1) horizontal distance, measured horizontally from the sender and the last receiver on the same road; 2) actual curve distance between the sender and the furthest receiver. From Table XII to Table XV in Appendix D, λ_1 and λ_2 denote the density on road R_1 and R_2, respectively; for the simulation results, 'Horiz' and 'Curve' denote the horizontal and curve distance, respectively. The theoretical results based on the proposed model, denoting by 'Theoretical (Condensed)', is calculated based on the curve density divided by $cos(\alpha)$. The motivation to do so is that we want to check in which manner the approximation works well.

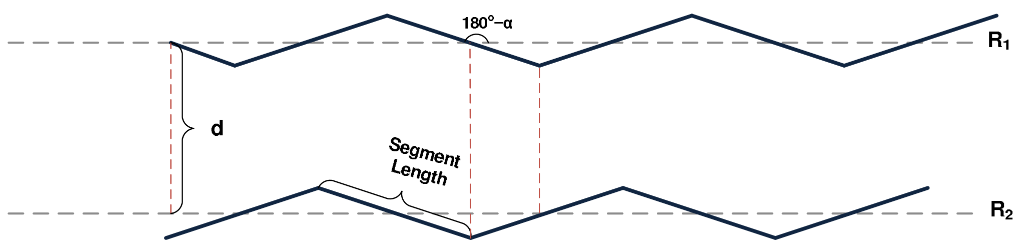


Fig. 9. Propagation Along Zigzag Roads

From the results one can figure out that if the horizontal successful propagation distance is measured, by using the condensed density, the model works very well in estimation of both expectation and variance for all instances in Case I. However, if the curve distance is measured, as the α becomes larger, say, up to 20 degree, the model would underestimate the successful propagation distance. If the curve density is used, the theoretical results tend to be smaller than those when α gets larger. For Case II it is also obvious that when the density on R_1 becomes relative smaller, the theoretical results get accurate.

5. Approximation to Probability Distribution of Successful Propagation

In application, people are concerned with the probability distribution of the propagation distance. In light of the result in Wang [11], it is suspected whether the probability distribution of propagation distance follows a Gamma type. Therefore, we construct for each case a Gamma distribution by setting the Gamma parameters in such a way that the resulting Gamma distribution has the same mean and variance as calculated with our formulas. We then graphically compare this approximate Gamma distribution with the simulated results. Although such approach is too heuristic, it helps understand the 'approximate property' of the information propagation.

The simulated frequency distribution and the constructed Gamma distribution show a good fit in general. Two cases as examples are presented. In the first case, the distance headway follows an exponential distribution; in the second, the distance headway follows a Gamma distribution.

The Case of Exponential Distance Headway

In this case, the headway on each road is assumed to follow an exponential distribution. All the parameters are shown in Table I. And the results are shown in Figure 10 where the approximate Gamma function is denoted by the blue line, and the simulation result frequency distribution by the red line.

Table I. Parameters for Gamma Approximation to Exponential Headway

Parameters for Poisson Headway Distribution	
$\lambda_1 = 1.2$	$d = 0.5$
$\lambda_2 = 1.5$	$L = 1.0$
Moments of Successful Propagation Distance	
Theoretical Mean = 3.1859	Simulation Mean = 3.2927
Theoretical Variance = 12.9437	Simulation Variance = 13.8804
Parameters for Gamma Approximation	
Mean of Gamma Distribution = Theoretical Mean of Propagation Distance	
Variance of Gamma Distribution = Theoretical Variance of Propagation Distance	

Case of Distance Headway with Gamma Distribution

The headway on each road is assumed to follow a Gamma distribution to make a more general case, and the variance of headway on road R_2 equals 0.5 times its mean, and the variance of headway on road R_1 equals its mean. The values of parameters can be found in Table II. The results are shown in Figure 11 where the Gamma distribution is denoted by blue line and the simulation by red line.

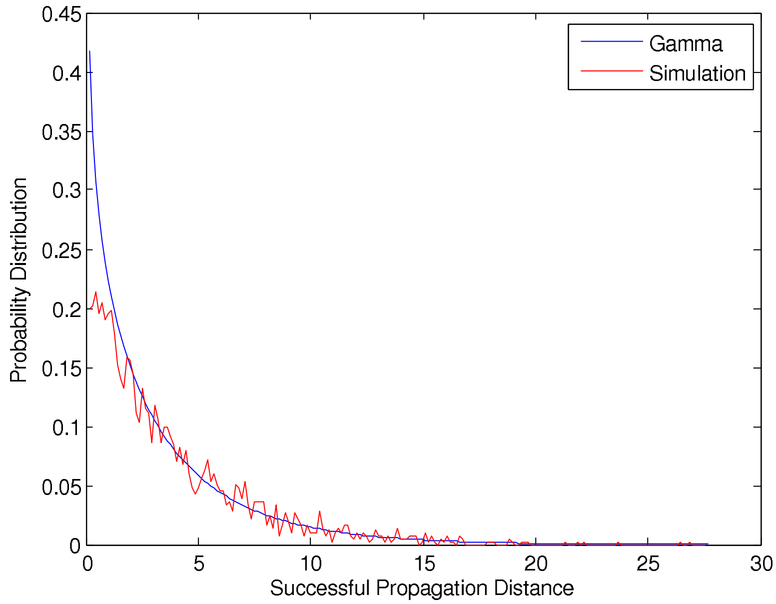


Fig. 10. Gamma Approximation with Exponential Distance Headway

D. Summary

In this chapter, we characterize the information propagation in terms of expectation and variance by establishing a Bernoulli approximation. We use the simulation to evaluate the quality of this approximation. According to the simulation, our developed formulas are accurate in most cases. Since the proposed model does not depend on any specific distribution, thus the approximation can be adapted to a class of general headway distributions. However, it is apparent that in some situations the Bernoulli regions have some correlation. Moreover, when the traffic density increases, such approximation may lead to significant inaccuracy. Hence an exact method is needed to address these issues. In next chapter, we aim at establishment of an exact model.

Table II. Parameters for Gamma Approximation

Parameters for Gamma Headway Distribution	
Mean Headway on $R_1 = 1.0$	$d = 1.0$
Mean Headway on $R_2 = 1.0$	$L = 1.0$
Moments of Successful Propagation Distance	
Theoretical Mean = 1.9108	Simulation Mean = 1.8816
Theoretical Variance = 5.5412	Simulation Variance = 5.3965
Parameters for Gamma Approximation	
Mean of Gamma Distribution = Theoretical Mean of Propagation Distance	
Variance of Gamma Distribution = Theoretical Variance of Propagation Distance	

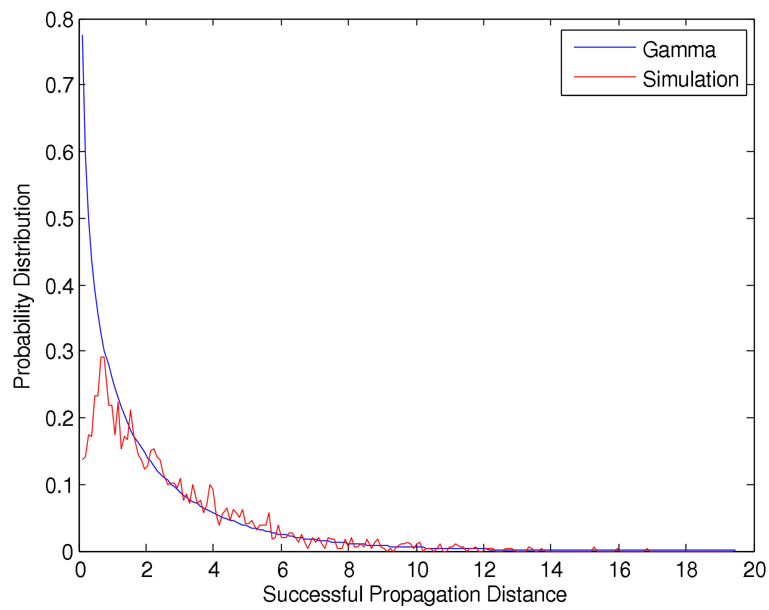


Fig. 11. Gamma Approximation when Headway Follows Gamma Distribution

CHAPTER IV

METHODOLOGY II: MARKOV MODEL

A. Introduction

Chapter III identifies a Bernoulli process to *approximate* this propagation process, which performs very well in most cases, but it suffers significant errors in a small number of other cases. In contrast, this chapter develops exact models to rigorously characterize this propagation process. Not only the expectation and variance of information propagation are given, also the probability distribution is provided based on the proposed model.

The problem studied in this chapter is almost same to the statement in Chapter III. However, the propagation distance measures horizontally from the initiating vehicle to the furthest receiving one in this chapter. The furthest receiver is not necessary on the same road with the initial sender. This definition is slightly different from Chapter III because it helps simplify the formula in this chapter. In fact, one can easily use the proposed methods here to find a solution for the definition of propagation distance in Chapter III. By the language of random geometric graphs, using the same notation in Chapter III section A, the information propagation distance is defined as:

$$D_2 \triangleq \sup\{\|\mathbf{y}_0 - \mathbf{z}_i\| : z_i \in \{\mathbf{X}_i\} \text{ or } \{\mathbf{Y}_i\} \subset W\}.$$

We highlight the ideas of modeling as follows:

- to identify a transmission region associated with each vehicle node such that the additional information propagation distance does not depend on what has happened prior to this region;

- to identify Markov property and define the states in terms of transmission regions and associated parameters;
- to carefully divide the whole probability space without overlap by the location of 'first node' defined in next section;
- to establish the recursive relationship for probability distribution, expectation and variance of information propagation.

In Section B, we introduce the Markov process equivalent of the propagation process. With the Markov process, we derive the mean and variance of the propagation distance. Section C describes the probability distribution of propagation distance with the same strategy in Section B. And then conclusion is drawn in Section E.

B. Modeling with a Markov Process

To establish the Markov relationship, we will firstly introduce the concept of transmission region and its types, and then describe the transitional relationship between the different regions.

1. Transmission Regions and Types

Consider a transmitter node N_1 on road R_2 as in Figure 12. We assume propagation goes *rightwards* and suppose N_4 and N_3 are the two right most nodes that N_1 can reach on both roads. We refer to a parallelogram $N_1N_2N_3N_4$, where the segments $N_1N_4 = N_1N_3 = N_2N_3 = L$, as the *transmission region* of type 2 associated with N_1. Similarly, a node N_1' present on road R_1 also has its transmission region, $N_1'N_2'N_3'N_4'$

referred to as transmission region of type 1, as in Figure 12. The following result is important to developing the Markov process later.

Proposition 4 *Information propagates forward from a vehicle* only if *there is a vehicle present on the segments of the according transmission region.*

This necessity condition is self-evident. If no vehicle is present in the transmission region, a gap of L on both roads wil terminate the propagation. In a similar light, in considering further propagation, vehicles left of this parallelogram can be safely overlooked. The propagation process between the two roads is a process of transition within and between transmission regions of the two types.

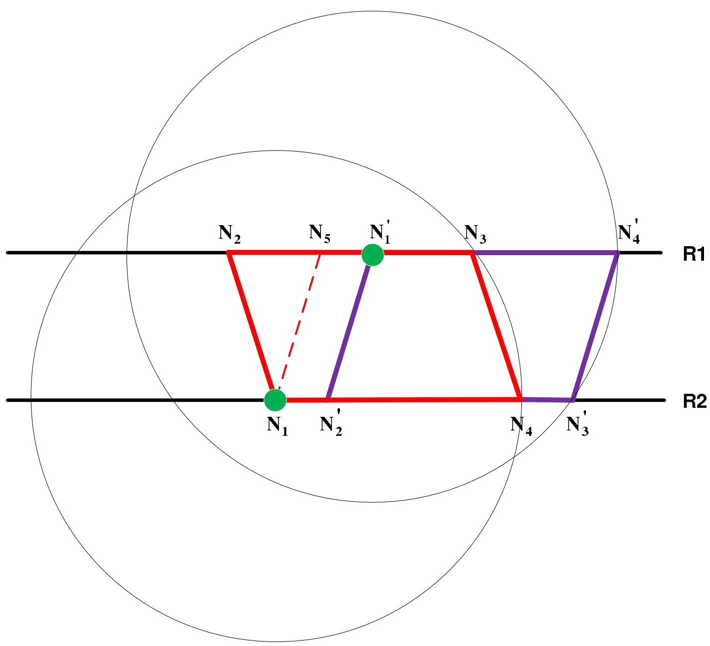


Fig. 12. Illustration of Transmission Regions of Type 1 and 2

We only study the case with $d \leq \frac{\sqrt{3}L}{2}$ because it guarantees an entire transmission region within a communication range. One may verify that the parallelogram

$N_1N_2N_3N_4$ cannot be held within the circle in Figure 12 if $d > \frac{\sqrt{3}L}{2}$. We leave this case to the future study.

An inherent characteristic indicative of the two road network is a segment N_2N_5 as in Figure 12. Its length is a function of road separation and communication range, and it indicates the degree to which vehicles on both roads interact with each other. We denote its length by r. Clearly, $\frac{r}{2} = L - \sqrt{L^2 - d^2}$.

2. Transition Between Regions and State Parameters

In this section, we delineate the process of transition within and between the transmission regions, and introduce two parameters associated with each transmission region: void distance and revisit distance. A transition state is characterized by a transmission region and its two parameters.

Consider a transmitting vehicle denoted by N_1 on road R_2. Vehicle N_1 has a transmission region of type 2, N_1DCA as in Figure 13 (a) and (b). A vehicle on either road in this region may further propagate the information forward. Again, we do not consider vehicles left of this transmission region as they would still resort to vehicles in this region to further propagate.

Two cases of transition from N_1 may take place. In the first case, the *first* node N_2 takes place on road R_2 as shown in figure 13(a). By the term 'first' on road R_2, we mean no node presence in the intervals AB and N_1N_2, and one node presence N_2 whereas $N_1N_2 = AB$. Throughout the paper later, the term 'first vehicle' has a similar meaning. In this case, the transmission region of type 2 ($ACDN_1$) associated with node N_1 transits into a transmission region of the same type ($BEFN_2$) associated with the immediate node N_2. It may be equivalently taken as moving the transmission region of N_1 rightwards and stopping whenever the line AN_1 hits a node on either road. In this particular case, the first node 'hit' is N_2 on road R_2. This is an example of transition between the same regions of type 2.

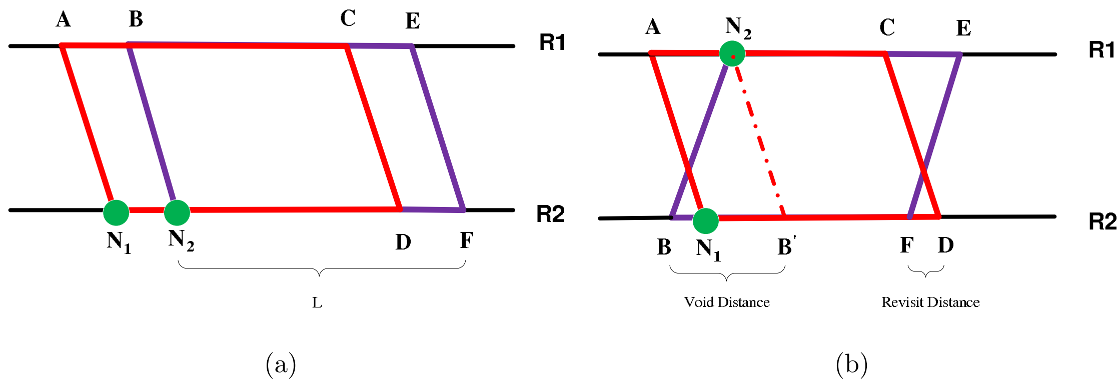


Fig. 13. Transition (a) Type 2 to Type 2 Regions, and (b) Type 2 to Type 1 Regions

The second case of transition is from a type 2 region of node N_1 into a transmission region of type 1. Refer to Figure 13(b). Still consider moving the line AN_1 rightwards and stopping as a new line $B'N_2$ when the first node hit takes place at N_2 on road R_1. In this case, there is no node presence in N_1B' and in AN_2, but there is one node at location N_2 on road R_1. Now the node N_2 becomes the new transmitting node. Associated with node N_2 is a new transmission region of type 1, N_2BFE.

Note that when transiting from a type 2 area into type 1 as shown in Figure 13(b), there is an associated change in the state parameters. In the new type 1 region associated with N_2, there is a segment BB' in which there is no vehicle to be considered again for future propagation: those nodes in it, if any at all, have been used prior to reaching node N_1. We call this segment BB' *void distance*, first of the two state parameters. Ignoring nodes in the void distance does not compromise the capability of further propagation. In addition, a segment DF was in the previous transmission region for N_1, but not in this new transmission region for N_2. In order not to compromise the capability of propagation due to this transition from type 2 into type 1, we shall not drop this segment DF from consideration. In doing so, we associate with the resulting type 1 region $BFEN_2$ a second parameter, *re-visit distance.*

This means that the propagating capability of the new region of type 1 comes from its effective coverage region $B'FEN_2$ plus the revisit distance DF, as seen in Figure 13(b). The capability of effective additional propagation can therefore be well defined by the new resulting transmission region and its two associate parameters.

It is obvious that the type 1 region transits in a similar fashion. One can figure out that a starting state, defined by a transmission region and two parameters, has a probability of transiting into a new state. We will explain the transition probability between states in more details later. Additionally, note that the void distance and revisit distance for a type 1 region always take place on road R_2, and those of type 2 regions always on road R_1.

3. State Parameters and Expected Propagation Distance

To further detail the above discussions, we define the notation below.

Notation

r	A constant, $\frac{r}{2} = L - \sqrt{L^2 - d^2}$.
$D_i(v, u)$	Random propagation distance starting from a node location with transmission region type i with void distance v and revisit distance u.
$d_i(v, u)$	Expectation of $D_i(v, u)$
$V_i(v, u)$	Variance of $D_i(v, u)$
$F_i(d, v, u)$	The probability of horizontal propagation beyond distance d on the two roads when the initial node takes place on road i with void distance v and revisit distance u. d is the horizontal distance starting from the initial node

Note that the void and re-visit distances are always on the different roads of the associated vehicles. r is the limit to the void distance. We call a state for a transmission region with two associated parameters. In addition, we have,

Proposition 5 *In the case* $d \leq \frac{\sqrt{3}}{2}L$*, there holds* $u \leq v \leq r$.

Proposition 5 is self explanatory from Figure 13 (b). Figure 13 (b) shows a transition from a type 2 region AN_1DC to a type 1 region $BFEN_2$. The associated re-visit distance $u = FD = BN_1 \leq BB'$ while the void distance $v = BB' \leq r$. Figure 13 (a) shows equal cuts to both u and v when transits into the same region of type 2, maintaining the same relationship as in Figure 13 (b). Proposition 5 is important to our later derivation.

Note that any re-visit distance has been caused by an advance node, L distance prior to the end of the re-visit distance, on the same road. As in Figure 13 (b), the re-visit distance FD associated with node N_2 is due to an earlier node N_1: N_1 has an associated region N_1DCA that covers FD.

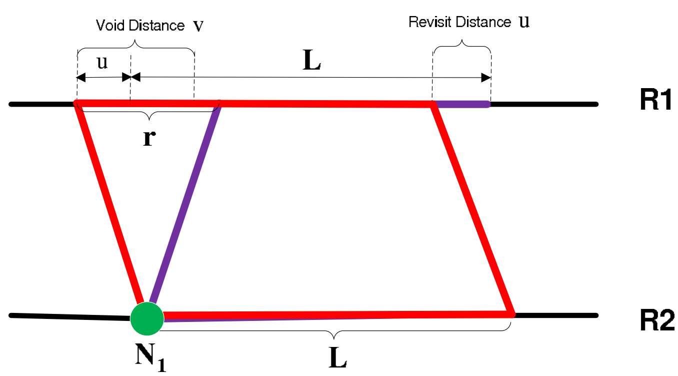


Fig. 14. Transmitting Node N_1 and its Void and Revisit Distances

Formulation

Suppose a transmitting node N_1 on road R_2 has a propagation distance $D_2(v,u)$. See Figure 14 for illustration. $D_2(v,u)$ can be recursively expressed depending on location of the next node in the transmission region of N_1. Several cases of first node location are explained as follows.

Case 1: First node on road R_2

If the *first* node is present on road R_2 denoted by N_2 as in Figure 15, the resulting transmission region will still be of type 2, with only differences in the void distance and the revisit distance, The resulting state is represented by a type 2 region with two associated parameters. The resulting new void distance and the new revisit distance depend on location of N_2 on road R_2. The word 'first' has the same meaning as explained earlier.

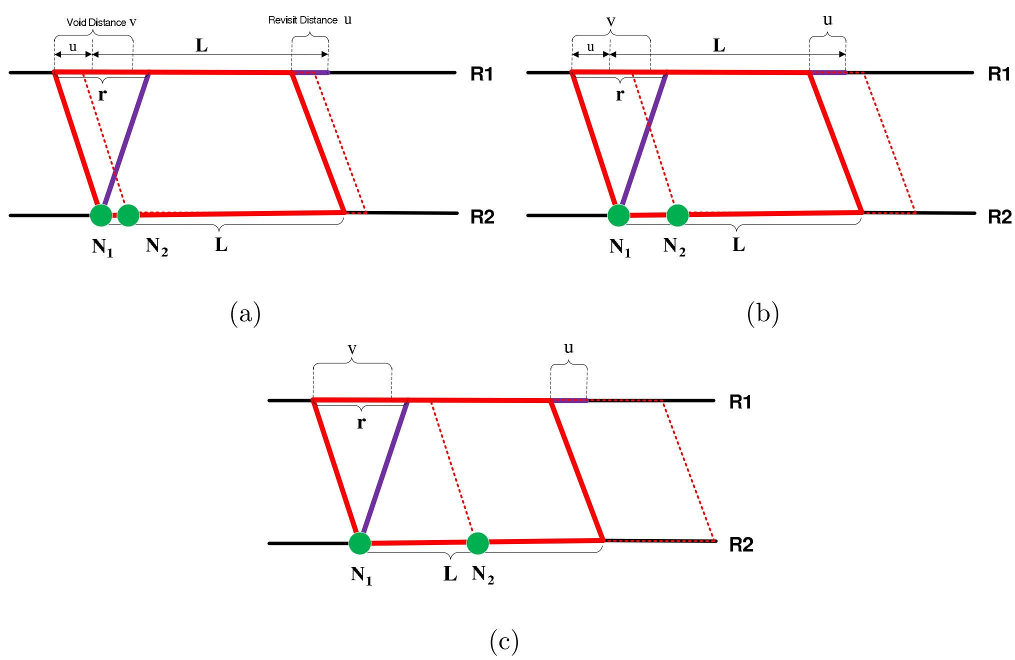


Fig. 15. Transition Between Regions of Type 2

In this case, there are three segments on road R_2 in which the first node could take place: (1) The first node N_2 has a distance to N_1 in the range $[0, u]$. This distance is short enough to keep both parameters positive in the resulting state, as illustrated in Figure 15(a). (2) N_2 has a distance to N_1 in the range $[u, v]$. This reduces the revisit distance to zero and keeps the void distance positive, as seen in Figure 15(b). (3) N_2 has a distance to N_1 in $[v, L]$. This reduces both parameters to zero as illustrated in Figure 15(c). Therefore, corresponding to the three situations, the total propagation distance in this case may be recursively expressed as follows.

$$\begin{aligned}\int_0^u \lambda_2 e^{-\lambda_2 t}(t + D_2'(v-t, u-t))dt \ &+ \ \int_u^v \lambda_2 e^{-t\lambda_2}(t + D_2'(v-t, 0))dt \\ &+ \ \int_v^L \lambda_2 e^{-(t-v)\lambda_1 - t\lambda_2}(t + D_2'(0,0))dt\end{aligned}$$

where $D_2'(v, u)$ is independent and identically-distributed $(i.i.d.)$ random variable with $D_2(v, u)$. D_2' represents additional distance to propagate from the new node. The term $\lambda_2 e^{-(t-v)\lambda_1 - t\lambda_2}dt$ represents the first node at location $t > v$ on road R_2.

Case 2: First node on road $R1$

In this case, the new transmission region will be of type 1 transitioned from type 2, as shown in Figure 16 where N_2 is the next node. Be aware that the first node presence on road R_1 cannot be in the void distance of the transmission region. Several cases arise, each giving rise to a new transmission region of different parameters (e.g. void distance and revisit distance). Measured by distance on R_1 to the left most end of the transmission region for N_1, N_2 may take place in three sections on road R_1: $[v, r)$, $[r, L)$, and $[L, L+u]$. When N_2 falls at t within $[v, r]$, the new void distance is r, and revisit distance of the resulting transmission region for N_2 is $r-t$, as seen in Figure 16(a). If N_2 takes place in $[r, L]$, the resulting void distance is r and revisit distance is zero, as illustrated in Figure 16(b). If N_2 falls at t in the revisit range $[L, L+u]$, the resulting void distance is $L+r-t$ and the revisit distance is zero, as seen in Figure 16(c). Therefore, the total propagation distance in this case may be recursively expressed as follows.

$$\int_v^r \lambda_1 e^{-(t-v)\lambda_1 - t\lambda_2}(t - \frac{r}{2} + D_1^{'}(r, r-t))dt + \int_r^L \lambda_1 e^{-(t-v)\lambda_1 - t\lambda_2}(t - \frac{r}{2} + D_1^{'}(r, 0))dt$$
$$+ \int_L^{L+u} \lambda_1 e^{-(t-v)\lambda_1 - L\lambda_2}(t - \frac{r}{2} + D_1^{'}(r-(t-L), 0))dt.$$

where $D_1^{'}(v, u)$ is *i.i.d.* random variable with $D_1(v, u)$, representing additional distance to propagate from N_2. The term $\lambda_1 e^{-(t-v)\lambda_1 - t\lambda_2}dt$ is the probability of having the first node at location t. $t - \frac{r}{2}$ is additional horizontal distance forward due to the transition.

The first term has a distance setback, which will be adjusted in Case 3 below.

Case 3: No node presence in the transmission region of N_1 and its Revisit Distance

In this case, the propagation terminates at N_1. The expected additional propagation distance is zero. Note that N_1 might not be as far horizontally as the previous node on R_1 that has caused the revisit distance u. If $u \geq \frac{r}{2}$, there is one previously analyzed node on R_1 farther than N_1 horizontally. This is because that a location at $\frac{r}{2}$ on road R_1 of the transmission region has the same horizontal distance as node N_1. In this case, $D_2(v, u)$ will be adjusted by the following quantity:

$$e^{-L\lambda_2 - (L+u-v)\lambda_1}\left[u - \frac{r}{2}\right]_+$$

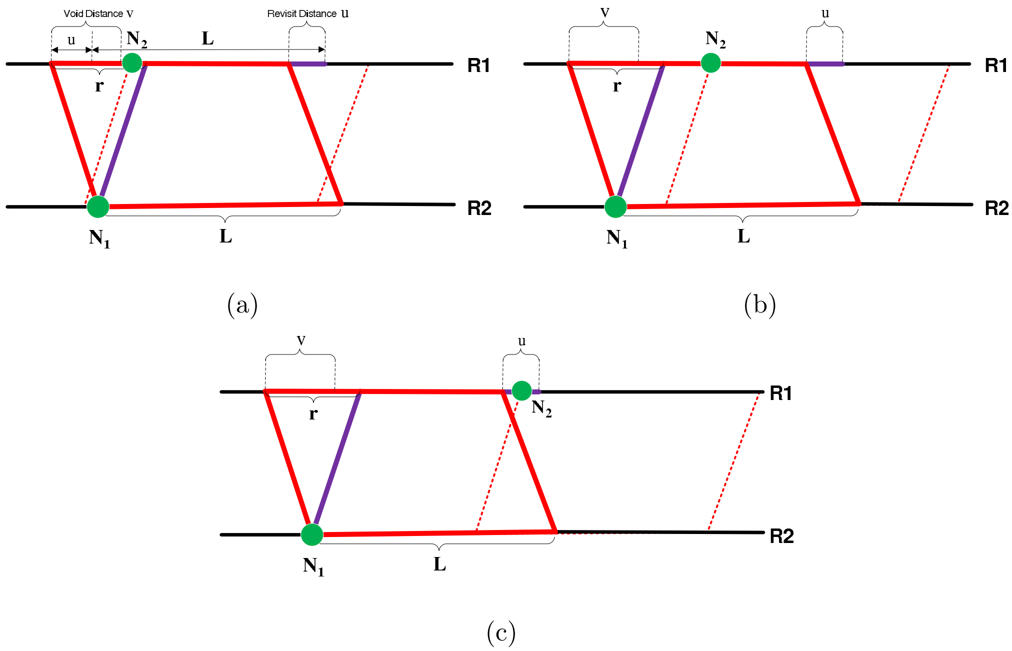


Fig. 16. Transition From Type 2 to Type 1 Regions

where $[x]_+$ equals x if $x \geq 0$ or 0 if $x < 0$. The exponential term above is for the no-node probability. Without this adjustment, there would be a distance setback from the previous node on road R_1 transiting to node N_1. In essence, the revisit distance of a region indicates location of the previous node that has reached so far. Therefore, it is used for adjustment to all the distance setback due to transition between regions.

The recursive propagation distance expressed above starts from a transmitting node on road R_2. The propagation distance from a node on road R_1 may be expressed in a similar way. Summarizing over all the cases discussed above gives the following the results.

Proposition 6 *The propagation distance along both roads satisfies the following relationship. Replacing each* $D_i(\cdot,\cdot)$ *with the according* $d_i(\cdot,\cdot)$ *gives the expected distances,* $i = 1, 2$.

$$
\begin{aligned}
D_1(v,u) &= \int_0^u \lambda_1 e^{-\lambda_1 t}(t + D_1^{'}(v-t, u-t))dt \\
&+ \int_u^v \lambda_1 e^{-t\lambda_1}(t + D_1^{'}(v-t, 0))dt \\
&+ \int_v^L \lambda_1 e^{-t\lambda_1-(t-v)\lambda_2}(t + D_1^{'}(0, 0))dt \\
&+ \int_v^r \lambda_2 e^{-(t-v)\lambda_2-t\lambda_1}(t - \frac{r}{2} + D_2^{'}(r, r-t))dt \\
&+ \int_r^L \lambda_2 e^{-(t-v)\lambda_2-t\lambda_1}(t - \frac{r}{2} + D_2^{'}(r, 0))dt \\
&+ \int_L^{L+u} \lambda_2 e^{-(t-v)\lambda_2-L\lambda_1}(t - \frac{r}{2} + D_2^{'}(r-(t-L), 0))dt \\
&+ e^{-L\lambda_1-(L+u-v)\lambda_2}\left[u - \frac{r}{2}\right]_+, \qquad (4.1)
\end{aligned}
$$

$$
\begin{aligned}
D_2(v,u) &= \int_0^u \lambda_2 e^{-\lambda_2 t}(t + D_2^{'}(v-t, u-t))dt \\
&+ \int_u^v \lambda_2 e^{-t\lambda_2}(t + D_2^{'}(v-t, 0))dt \\
&+ \int_v^L \lambda_2 e^{-(t-v)\lambda_1-t\lambda_2}(t + D_2^{'}(0, 0))dt \\
&+ \int_v^r \lambda_1 e^{-(t-v)\lambda_1-t\lambda_2}(t - \frac{r}{2} + D_1^{'}(r, r-t))dt \\
&+ \int_r^L \lambda_1 e^{-(t-v)\lambda_1-t\lambda_2}(t - \frac{r}{2} + D_1^{'}(r, 0))dt \\
&+ \int_L^{L+u} \lambda_1 e^{-(t-v)\lambda_1-L\lambda_2}(t - \frac{r}{2} + D_1^{'}(r-(t-L), 0))dt \\
&+ e^{-L\lambda_2-(L+u-v)\lambda_1}\left[u - \frac{r}{2}\right]_+, \qquad (4.2)
\end{aligned}
$$

where $u \leq v \leq r$ as in Proposition 5.

Although it is difficult to obtain analytic solutions for these two linear integral equations, one can still find the characteristics of information propagation with the numerical solution. One way of calculation is to discretize the integration and then to obtain an array of equations. For practical purposes, one may consider the initial sender node being in a state $(0, 0)$.

Formulas 4.1 and 4.2 enables to analyze the effect of void distance on propagation distance as well as that of communication range, node density and road separation distance.

4. Variance of Propagation Distance

In this section, we will analyze the variance of propagation distance based on the Law of Total Variance. This will use the expected propagation distance calculated earlier.

$$\begin{aligned} V(D_i(v,u)) &= E(V(D_i(v,u)|\tau)) + V(E(D_i(v,u)|\tau)) \\ &= E(V(D_i(v,u)|\tau)) + E\left((E(D_i(v,u)|\tau) - E(D_i(v,u)))^2\right) \end{aligned}$$

where $i = 1, 2$ and τ is location of the first equipped node.

To illustrate how the Law of Total Variance works in our case, we take the variance of $D_1(v, u)$ for example, assuming a transmitting vehicle on R_1. If the first node on R_1 falls in the range $[0, u]$, the location of which is denoted by $\tau_{1u} \in [0, u]$. It does not eliminate the revisit distance in the resulting transmission region of the same type. The resulting parameters are $(v - t, u - t)$. The according terms of variance is expressed as follows. Conditional on $t \in [0, u]$, we have

$$
\begin{aligned}
&E(Var(D_i(v,u)|\tau_{1u})) + Var(E(D_i(v,u)|\tau_{1u})) \\
&= \int_0^u \lambda_1 e^{-\lambda_1 t} V_1(v-t, u-t)dt + \int_0^u \lambda_1 e^{-\lambda_1 t}(t + d_1(v-t, u-t) - d_1(v,u))^2 dt \\
&= \int_0^u \lambda_1 e^{-\lambda_1 t}[V_1(v-t, u-t) + (t + d_1(v-t, u-t) - d_1(v,u))^2]dt
\end{aligned}
$$

where the notations are as defined earlier. Similarly, the next node could be at locations in $[u, v]$ and $[v, L]$ on R_1, and in $[v, r]$, $[r, L]$ and $[L, L+u]$ on R_2. It is aware of the case with no node presence within the transmission region and the re-visit distance, for which the according terms in $E(Var(D_1(v,u)|\tau)) = 0$. However, the according terms in $Var(E(D_1(v,u)|\tau))$ are not zero. In fact we have $E(D_1(v,u)|\tau) = 0$ and $E(D_1(v,u))) = d_1(v,u)$. Thus the variance term according to the case of no node presence is given as follows.

$$
\begin{aligned}
E(V(D_1(v,u)|\tau)) &= 0, \\
V(E(D_1(v,u)|\tau)) &= e^{-\lambda_1 L - \lambda_2(L+u-v)} d_1^2(v,u).
\end{aligned}
$$

Summing over all the cases, we have the following results for R_1.

Proposition 7 *The variance of $D_1(v,u)$ can be calculated as follows.*

$$
\begin{aligned}
V_1(v,u) &= \int_0^u \lambda_1 e^{-\lambda_1 t}[V_1(v-t,u-t)+(t+d_1(v-t,u-t)-d_1(v,u))^2]dt \\
&+ \int_u^v \lambda_1 e^{-t\lambda_1}[(t+d_1(v-t,0)-d_1(v,u))^2+V_1(v-t,0)]dt \\
&+ \int_v^L \lambda_1 e^{-t\lambda_1-(t-v)\lambda_2}[(t+d_1(0,0)-d_1(v,u))^2+V_1(0,0)]dt \\
&+ \int_v^r \lambda_2 e^{-(t-v)\lambda_2-t\lambda_1}[(t-\frac{r}{2}+d_2(r,r-t)-d_1(v,u))^2+V_2(r,r-t)]dt \\
&+ \int_r^L \lambda_2 e^{-(t-v)\lambda_2-t\lambda_1}[(t-\frac{r}{2}+d_2(r,0)-d_1(v,u))^2+V_2(r,0)]dt \\
&+ \int_L^{L+u} \lambda_2 e^{-(t-v)\lambda_2-L\lambda_1}[(t-\frac{r}{2}+d_2(r-(t-L),0)-d_1(v,u))^2 \\
&\qquad +V_2(r-(t-L),0)]dt \\
&+ e^{-L\lambda_1-(L+u-v)\lambda_2}d_1(v,u)^2. \qquad (4.3)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
V_2(v,u) &= \int_0^u \lambda_2 e^{-\lambda_2 t}[V_2(v-t,u-t)+(t+d_2(v-t,u-t)-d_2(v,u))^2]dt \\
&+ \int_u^v \lambda_2 e^{-t\lambda_2}[(t+d_2(v-t,0)-d_2(v,u))^2+V_2(v-t,0)]dt \\
&+ \int_v^L \lambda_2 e^{-t\lambda_2-(t-v)\lambda_1}[(t+d_2(0,0)-d_2(v,u))^2+V_2(0,0)]dt \\
&+ \int_v^r \lambda_1 e^{-(t-v)\lambda_1-t\lambda_2}[(t-\frac{r}{2}+d_1(r,r-t)-d_2(v,u))^2+V_1(r,r-t)]dt \\
&+ \int_r^L \lambda_1 e^{-(t-v)\lambda_1-t\lambda_2}[(t-\frac{r}{2}+d_1(r,0)-d_2(v,u))^2+V_1(r,0)]dt \\
&+ \int_L^{L+u} \lambda_1 e^{-(t-v)\lambda_1-L\lambda_2}[(t-\frac{r}{2}+d_1(r-(t-L),0)-d_2(v,u))^2 \\
&\qquad +V_1(r-(t-L),0)]dt \\
&+ e^{-L\lambda_2-(L+u-v)\lambda_1}d_2(v,u)^2 \qquad (4.4)
\end{aligned}
$$

where $u \leq v \leq r$.

C. Probability Distribution of Successful Propagation

The probability distribution of successful propagation distance is probably the most important measure to characterize the information propagation process. It enables researchers to study point connectivity and other critical properties on a network.

We start with a node on road R_2. As defined earlier, $P_2(x, v, u)$ denotes the probability of propagation beyond a horizontal distance x starting from a state (u, v). Similar to the analysis of expected distance, we can calculate the probability distribution conditional on location of the 'first node' in a transmission region. For simplicity of presentation, we define a function as follows.

$$P_i(x, v, u) = 1, \text{if } x \leq max\{0, u - \tfrac{r}{2}\},\ i = 1, 2. \tag{4.5}$$

This explains the above definition of Equation 4.5: A negative x means a location left of the starting node. Presence of the initiating node itself proves a successful propagation beyond that negative point. In addition, location at $x = u - \frac{r}{2}$ was the previous node on the other of road R_i covering up to the farthest point of the revisit distance u. That previous node is an evidence of propagation beyond it. Therefore we have equations that follow.

$$
\begin{aligned}
P_1(x,v,u) &= \int_0^u \lambda_1 e^{-\lambda_1 t} P_1(x-t, v-t, u-t)dt \\
&+ \int_u^v \lambda_1 e^{-\lambda_1 t} P_1(x-t, v-t, 0)dt \\
&+ \int_v^L \lambda_1 e^{-(t-v)\lambda_2 - t\lambda_1} P_1(x-t, 0, 0)dt \\
&+ \int_v^r \lambda_2 e^{-(t-v)\lambda_2 - t\lambda_1} P_2(x-t+\frac{r}{2}, r, r-t)dt \\
&+ \int_r^L \lambda_2 e^{-(t-v)\lambda_2 - t\lambda_1} P_2(x-t+\frac{r}{2}, r, 0)dt \\
&+ \int_L^{L+u} \lambda_2 e^{-(t-v)\lambda_2 - L\lambda_1} P_2(x-t+\frac{r}{2}, r-t+L, 0)dt. \qquad (4.6) \\
P_2(x,v,u) &= \int_0^u \lambda_2 e^{-\lambda_2 t} P_2(x-t, v-t, u-t)dt \\
&+ \int_u^v \lambda_2 e^{-\lambda_2 t} P_2(x-t, v-t, 0)dt \\
&+ \int_v^L \lambda_2 e^{-(t-v)\lambda_1 - t\lambda_2} P_2(x-t, 0, 0)dt \\
&+ \int_v^r \lambda_1 e^{-(t-v)\lambda_1 - t\lambda_2} P_1(x-t+\frac{r}{2}, r, r-t)dt \\
&+ \int_r^L \lambda_1 e^{-(t-v)\lambda_1 - t\lambda_2} P_1(x-t+\frac{r}{2}, r, 0)dt \\
&+ \int_L^{L+u} \lambda_1 e^{-(t-v)\lambda_1 - L\lambda_2} P_1(x-t+\frac{r}{2}, r-t+L, 0)dt, \qquad (4.7)
\end{aligned}
$$

where t in the last integrals of each equation is the horizontal distance from the left vertex of each transmission region. The recursive relationship in the above equations is similar to that for the expectation earlier.

D. Numerical Results

1. Illustrative Numerical Results for Expectation and Variance

We numerically calculate the expected value and variance for a select number of instances for the purpose of illustration. We solve the integral equations of expectations and variances adopting a simple method —- discretizing the integrations and solving the resultant arrays of equations. Technically, we partition the interval [0, r] into n subintervals of size $h = r/n$ and the parameters u and v take values of integer times h. Just like for any function $f(t)$, $\int_a^b f(t)dt \approx \sum_{i=k}^m f(ih)h$. The resulting array of linear equations are easily solved with popular software such as Matlab. Worth mentioning is that r is smaller than the communication range L. Because r is not a large value, the number of states (u, v) in this discrete manner is not too large to efficiently compute given the current computational power for linear systems.

Recognizing that distance is relative, we scale the communication range to be a standard unit 1.0. All other measures including the node density are scaled accordingly. The density for road R_2 is set to be 0.6 and 1.0, each corresponding to a set of lower densities on road R_1.

The results are tabulated in Table III, where d denotes the distance between the two roads, E for $d_2(0, 0)$, and V for $V_2(0, 0)$. The numbers for E and V in each instance has an analytical value followed by a simulation one. The simulation has 4000 runs each time.

2. Gamma Approximation to the Probability Distribution

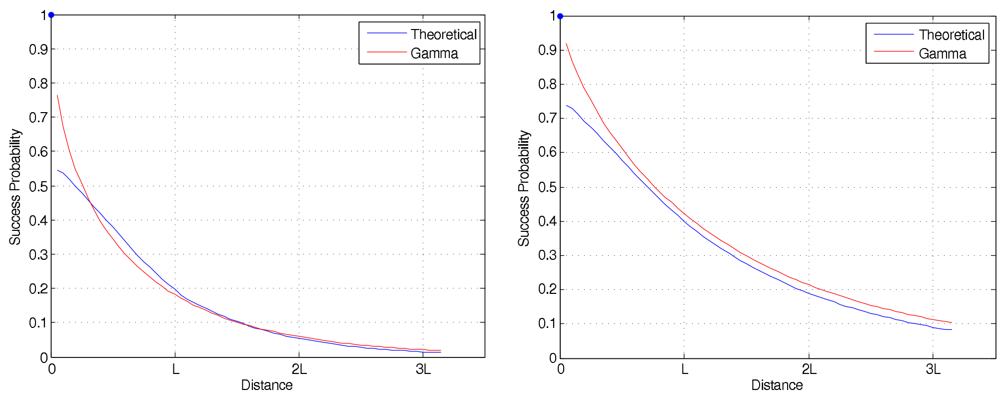
Equations (4.6) and (4.7) give a way to numerically calculate probability of propagation distance. Similar to numerical calculation of the expectation and variance, discretization of the integration leads to arrays of equations. Solving the equation

Table III. Numerical Results on Expectation and Variance of Propagation Distance

$\lambda_2 = 0.6$						
d		0.1	0.3	0.5	0.7	0.8
$\lambda_1 = 0.6$	E	0.9271	0.8784	0.7842	0.6588	0.5885
		0.9619	0.9048	0.8032	0.6526	0.6100
	V	1.4094	1.2803	1.0476	0.7905	0.6776
		1.5381	1.3677	1.0573	0.8159	0.7341
$\lambda_1 = 0.4$	E	0.7141	0.6818	0.6187	0.5336	0.4857
		0.7023	0.6740	0.6105	0.5564	0.5098
	V	0.9422	0.8666	0.7278	0.5719	0.5043
		0.9196	0.8259	0.7027	0.6366	0.5308
$\lambda_2 = 1.0$						
d		0.1	0.3	0.5	0.7	0.8
$\lambda_1 = 1.0$	E	2.1749	2.0295	1.7630	1.4387	1.2692
		2.1867	2.0468	1.8216	1.5004	1.2732
	V	5.9032	5.1527	3.9152	2.6938	2.1805
		5.9666	5.3441	4.0991	3.0565	2.2653
$\lambda_1 = 0.8$	E	1.7907	1.6815	1.4773	1.2225	1.0874
		1.7509	1.6634	1.4951	1.2869	1.1216
	V	4.1972	3.7140	2.8911	2.0471	1.6859
		3.9444	3.6357	3.0072	2.4059	1.9397

arrays gives the numerical results. There might be other methods for the numerical calculation, which is not discussed here.

As conjectured in the one road case by [11], Gamma distribution could serve as a good approximation to $P_i(x,0,0)$, especially when the coefficient of variation is near 1.0. The Gamma distribution is defined by setting its two parameters to yield the mean and variance as calculated from the analytical formulas. We illustrate by two instances as in Figure 17.



(a) $\lambda_1 = 0.4, \lambda_1 = 0.6, D = 0.7$ (b) $\lambda_1 = 0.8, \lambda_1 = 1.0, D = 0.7$

Fig. 17. Gamma Approximation to Success Probability

E. Summary

This chapter studies the information propagation between vehicles along two parallel roads as a special case of discrete network, assuming that vehicles follow two homogeneous Poisson processes. Interactions of vehicles between the two roads is clearly a significant factor in this process. Those interactions complicate modeling of the stochastic propagation process. We identify an inherent Markov model that determines the propagation behavior. Under our assumption, Case I can be addressed very well. With help of two types of transmission regions, one can find the probability distribution of information propagation as well as the expectation and variance.

CHAPTER V

SUMMARY AND CONCLUSIONS

A. Contribution

This thesis investigates the process of instantaneous information propagation along two parallel roads in a vehicular ad hoc network. Understanding this problem helps address the connectivity issues and design VANETs for the intelliDrive systems. Compared with the previous studies of IVC and VII systems on one line, the proposed models in this research extend to the case of two roads. To our knowledge, no study of connectivity on discrete networks has conducted in the past. Although in the wireless communication area, many studies have explored the mobile ad hoc networks in two dimensional plane, their results cannot be directly applied to discrete traffic networks. In short, there are mainly two contributions.

- We develop an approximation method, which builds on Bernoulli regions associated with each vehicle in the network. The approximation method can be applied to general distribution of distance headway of traffic.

- We also develop an exact Markov process to study information propagation. Two types of transmission regions are identified. Accordingly an exact model is developed to describe the probability distribution of information propagation in terms of recursive integral equations, capturing the core feature of information propagation in the vehicular ad hoc networks.

B. Future Research

There are some work left in this research that we wish to accomplish. First of all, the proposed model does not take the fading effect and transmission latency into account. In wireless communication, the fading effect is a significant issue for reliable communication and connectivity. Transmission latency is also important due to the high mobility of vehicles. Second, the real traffic flow pattern has not been explored in the proposed model, possibly causing the results inaccurate at least in some extreme traffic situations. Therefore, the future work includes the consideration of channel fading, latency and traffic patterns. In addition, since the simulation framework is critical to validation of theoretical models, the future work will also focus on developing a simulation framework by combing the simulators NS-2 and VISSIM.

REFERENCES

[1] "The website of IntelliDrive(SM)." http://www.intellidriveusa.org.

[2] F. Dion, J.-S. Oh, and R. Robinson, "Virtual testbed for assessing probe vehicle data in intellidrive systems," *IEEE Transactions on Intelligent Transportation Systems*, Accepted, 2010.

[3] H. Kawashima, "Japanese perspective of driver information system," *Transportation*, vol. 17, pp. 263–284, September 1990.

[4] P. Papadimitratos, A. La Fortelle, K. Evenssen, R. Brignolo, and S. Cosenza, "Vehicular communication systems: Enabling technologies, applications, and future outlook on intelligent transportation," *IEEE Communications Magazine*, vol. 47, pp. 84 –95, November 2009.

[5] "Vehicle-Infrastructure Integration (VII) Initiative benefit-cost analysis." http://www.intellidriveusa.org, May 2008.

[6] S. E. Shladover, X.-Y. Lu, and D. Cody, "Develpment and evaluation of selected modiblity applications for VII: concept of operations," working paper, California PATH, UCB-ITS-PWP-2009-3, March 2009.

[7] T. Willke, P. Tientrakool, and N. Maxemchuk, "A survey of inter-vehicle communication protocols and their applications," *IEEE Communications Surveys Tutorials*, vol. 11, no. 2, pp. 3–20, 2009.

[8] L. Briesemeister, L. Schafers, and G. Hommel, "Disseminating messages among highly mobile hosts based on inter-vehicle communication," in *Proc. of IEEE Intelligent Vehicles Symposium*, pp. 522 –527, Dearborn, MI, USA, 2000.

[9] T. Ohyama, S. Nakabayashi, Y. Shiraki, and K. Tokuda, "A study of real-time and autonomous decentralized DSRC system for inter-vehicle communications," in *Proc. of International IEEE Conference: Intelligent Transportation Systems*, pp. 190–195, Dearborn, MI, USA, October 2000.

[10] R. Chen, W.-L. Jin, and A. Regan, "Broadcasting safety information in vehicular networks: issues and approaches," *IEEE Network*, vol. 24, pp. 20 –25, January-February 2010.

[11] X. Wang, "Modeling the process of information relay through intervehicle communication," *Transportation Research Part B: Methodological*, vol. 41, pp. 684–700, July 2007.

[12] M. Penrose, *Random Geometric Graphs*, p. 13. New York: Oxford University Press, 2003.

[13] W.-L. Jin, *Kinematic Wave Models of Network Vehicular Traffic*. PhD dissertation, University of California, Davis, September 2003.

[14] W. S. Kendall and I. E. Molchanov, *New Perspectives in Stochastic Geometry*, ch. 16, pp. 520–554. New York: Oxford Univrsity Press, 2010.

[15] A. S. Tanenbaum, *Computer Networks*. New Jersey: Prentice Hall, fourth ed., 2002.

[16] "Standard specification for telecommunications and information exchange between roadside and vehicle systems – 5 GHz band Dedicated Short Range Communications (DSRC) Medium Access Control (MAC) and Physical Layer (PHY) specifications." ASTM E2213-03, 2003.

[17] M. Sichitiu and M. Kihl, "Inter-vehicle communication systems: a survey," *IEEE Communications Surveys Tutorials*, vol. 10, no. 2, pp. 88 –105, 2008.

[18] F. W. Hui and P. Mohapatra, "Experimental characterization of multi-hop communications in vehicular ad hoc network," in *Proc. of 2nd ACM International Workshop Vehicular Ad Hoc Networks*, pp. 85–86, Cologne, Germany, September 2005.

[19] D. Jiang, V. Taliwal, A. Meier, W. Holfelder, and R. Herrtwich, "Design of 5.9 GHZ DSRC-based vehicular safety communication," *IEEE Wireless Communications*, vol. 13, pp. 36 –43, October 2006.

[20] Y. Morgan, "Accurate positioning using short-range communications," in *Proc. of Ultra Modern Telecommunications Workshops*, pp. 1–7, Saint Petersburg, Russia, Octobor 2009.

[21] A. Bachir and A. Benslimane, "A multicast protocol in ad hoc networks inter-vehicle geocast," in *Proc. of 57th IEEE Semiannual Vehicular Technology Conference*, vol. 4, pp. 2456–2460, Jeju, Korea, April 2003.

[22] X. Ma, M.-T. Sun, G. Zhao, and X. Liu, "An efficient path pruning algorithm for geographical routing in wireless networks," *IEEE Transactions on Vehicular Technology*, vol. 57, pp. 2474–2488, July 2008.

[23] C. Maihofer, "A survey of geocast routing protocols," *IEEE Communications Surveys and Tutorials*, vol. 6, no. 2, pp. 32 –42, 2004.

[24] C. Lochert, M. Mauve, H. Fussler, and H. Hartenstein, "Geographic routing in city scenarios," *SIGMOBILE Mobile Computing and Communications Review*, vol. 9, no. 1, pp. 69–72, 2005.

[25] J. Bernsen and D. Manivannan, "Review: Unicast routing protocols for vehicular ad hoc networks: A critical comparison and classification," *Pervasive and Mobile Computing*, vol. 5, no. 1, pp. 1–18, 2009.

[26] H. Alshaer and E. Horlait, "An optimized adaptive broadcast scheme for inter-vehicle communication," in *Proc. of 61st Vehicular Technology Conference*, vol. 5, pp. 2840–2845, Stockholm, Sweden, May 2005.

[27] J. Peng and L. Cheng, "A distributed mac scheme for emergency message dissemination in vehicular ad hoc networks," *IEEE Transactions on Vehicular Technology*, vol. 56, pp. 3300 –3308, November 2007.

[28] Y. Bi, L. X. Cai, X. Shen, and H. Zhao, "Efficient and reliable broadcast in inter-vehicle communication networks: A cross layer approach," *IEEE Transactions on Vehicular Technology*, vol. 59, pp. 2404–2417, June 2010.

[29] A. N. Shirshanka, A. N, S. Das, B. Zhou, G. Pau, and M. Gerla, "Adtorrent: Digital billboards for vehicular networks," in *Proc. of IEEE/ACM International Workshop on Vehicleto-Vehicle Communications*, San Diego, California, USA, July 2005.

[30] Y. Zhang, B. Kosmatopoulos, P. Ioannou, and C. Chien, "Using front and back information for tight vehicle following maneuvers," *IEEE Transactions on Vehicular Technology*, vol. 48, pp. 319–328, January 1999.

[31] X. Liu, A. Goldsmith, S. Mahal, and J. Hedrick, "Effects of communication delay on string stability in vehicle platoons," in *Proc. of IEEE Intelligent Transportation Systems*, Oakland, CA, USA, August 2001.

[32] R. Middleton and J. Braslavsky, "String instability in classes of linear time invariant formation control with limited communication range," *IEEE Transactions on Automatic Control*, vol. 55, pp. 1519–1530, July 2010.

[33] D. Reichardt, M. Miglietta, L. Moretti, P. Morsink, and W. Schulz, "CarTALK 2000: safe and comfortable driving based upon inter-vehicle-communication," in *Proc. of IEEE Intelligent Vehicle Symposium*, vol. 2, pp. 545–550, Versailles, France, June 2002.

[34] L. Li and F.-Y. Wang, "Cooperative driving at blind crossings using intervehicle communication," *IEEE Transactions on Vehicular Technology*, vol. 55, pp. 1712 –1724, November 2006.

[35] A. Farahmand and L. Mili, "Cooperative decentralized intersection collision avoidance using Extended Kalman Filtering," in *Proc. of IEEE Intelligent Vehicles Symposium*, pp. 977–982, Xi'an, Shaanxi, China, June 2009.

[36] M. Bohm and A. Frotscher, "Bidirectional I2V communication applications for advanced invehicle ISA systems," in *Proc. of IEEE Vehicular Networking Conference (VNC)*, pp. 1–6, Tokyo, Japan, Octobor 2009.

[37] M. Zimmer, "Surveillance, privacy and the ethics of vehicle safety communication technologies," *Ethics and Information Technology*, vol. 7, pp. 201–210, December 2005.

[38] H. Nissenbaum, "Protecting privacy in an information age: The problem with privacy in public law and philosophy," *Law and Philosophy*, vol. 17, pp. 559–596, 1998.

[39] M. E. Zarki, S. Mehrotra, G. Tsudik, and N. Venkatasubramanian, "Security

issues in a future vehicular network," in *Proc. of European Wireless*, pp. 270–274, Florence, Italy, February 2002.

[40] E. van den Berg, T. Zhang, and S. Pietrowicz, "Blend-in: A privacy-enhancing certificate-selection method for vehicular communication," *IEEE Transactions on Vehicular Technology*, vol. 58, pp. 5190 –5199, November 2009.

[41] A. Festag, P. Papadimitratos, and T. Tielert, "Design and performance of secure geocast for vehicular communication," *IEEE Transactions on for Vehicular Technology*, vol. 59, pp. 2456–2471, June 2010.

[42] M. Piorkowski, M. Raya, A. L. Lugo, P. Papadimitratos, M. Grossglauser, and J.-P. Hubaux, "TraNS: Realistic Joint Traffic and Network Simulator for VANETs," *ACM SIGMOBILE Mobile Computing and Communications Review*, vol. 12, January 2008.

[43] Y. Balcioglu, F. Ozguner, U. Ozguner, and K. Redmill, "A multiple intersection integrated wireless and vehicular network simulator," in *Proc. of IEEE Vehicular Networking Conference*, pp. 1–5, Tokyo, Japan, October 2009.

[44] M. Killat, *The Impact of Inter-Vehicle Communication on Vehicular Traffic.* PhD dissertation, University of Karlsruhe (TH), Germany, July 2009.

[45] S. P.Hoogendoorn and P. L. Bovy, "State-of-the-art of vehicular traffc flow modelling," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 215, no. 4, pp. 283–303, 2001.

[46] J. Harri, F. Filali, and C. Bonnet, "Mobility models for vehicular ad hoc networks: A survey and taxonomy," research report RR-06-168, Institure Eurecom, France, March 2006.

[47] B. Park and J. Lee, "Assessing sustainability impacts of route guidance system under cooperative vehicle infrastructure environment," in *Proc. of IEEE International Symposium on Sustainable Systems and Technology*, pp. 1–6, Tempe, AZ, USA, May 2009.

[48] A. Kesting, M. Treiber, and D. Helbing, "Connectivity statistics of store-and-forward intervehicle communication," *IEEE Transactions on Intelligent Transportation Systems*, vol. 11, pp. 172 –181, March 2010.

[49] W. Wang, F. Xie, and M. Chatterjee, "Small-scale and large-scale routing in vehicular ad hoc networks," *IEEE Transactions on Vehicular Technology*, vol. 58, pp. 5200 –5213, November 2009.

[50] B. Jarupan, Y. Balcioglu, E. Ekici, F. Ozguner, and U. Ozguner, "An integrated wireless intersection simulator for collision warning systems in vehicular networks," in *Proc. of IEEE International Conference on Vehicular Electronics and Safety*, pp. 340–345, Columbus, OH, USA, September 2008.

[51] S. E. Shladover, "Develpment Path Analysis for Cooperative ITS Systems," working paper, UCB-ITS-PWP-2009-4, California PATH Program, University of California, Berkeley.

[52] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, March 2000.

[53] L.-L. Xie and P. Kumar, "On the path-loss attenuation regime for positive cost and linear scaling of transport capacity in wireless networks," *IEEE Transactions on Information Theory*, vol. 52, pp. 2313–2328, June 2006.

[54] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in

the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, pp. 1009–1018, March 2007.

[55] Y. Nebat, R. Cruz, and S. Bhardwaj, "The capacity of wireless networks in nonergodic random fading," *IEEE Transactions on Information Theory*, vol. 55, pp. 2478–2493, June 2009.

[56] S. Weber, X. Yang, J. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Transactions on Information Theory*, vol. 51, pp. 4091–4102, December 2005.

[57] S. Weber, J. Andrews, X. Yang, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with successive interference cancellation," *IEEE Transactions on Information Theory*, vol. 53, pp. 2799 –2814, August 2007.

[58] S. Weber, J. Andrews, and N. Jindal, "The effect of fading, channel inversion, and threshold scheduling on ad hoc networks," *IEEE Transactions on Information Theory*, vol. 53, pp. 4127 –4149, November 2007.

[59] M. Haenggi, "Outage, local throughput, and capacity of random wireless networks," *IEEE Transactions on Wireless Communications*, vol. 8, pp. 4350–4359, August 2009.

[60] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks," *IEEE/ACM Transactions on Networking*, vol. 10, pp. 477–486, August 2002.

[61] S. Diggavi, M. Grossglauser, and D. Tse, "Even one-dimensional mobility increases the capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 51, pp. 3947–3954, November 2005.

[62] L. Du, S. Ukkusuri, W. F. Y. D. Valle, and S. Kalyanaraman, “Optimization models to characterize the broadcast capacity of vehicular ad hoc networks,” *Transportation Research Part C: Emerging Technologies*, vol. 17, pp. 571–585, December 2009.

[63] C. Bettstetter and J. Eberspacher, “Hop distances in homogeneous ad hoc networks,” in *Proc. of 57th IEEE Semiannual Vehicular Technology Conference*, pp. 2286–2290, Jeju, Korea, April 2003.

[64] J. Mullen, “Robust approximations to the distribution of link distances in a wireless network occupying a rectangular region,” *Mobile Computing and Communications Review*, vol. 7, pp. 80–91, April 2003.

[65] J. Orriss and S. Barton, “Probability distributions for the number of radio transceivers which can communicate with one another,” *IEEE Transactions on Communications*, vol. 51, pp. 676–681, April 2003.

[66] A. Zanella, M. Stramazzotti, F. Fabbri, E. Salbaroli, D. Dardari, and R. Verdone, “Comments on ”probability distributions for the number of radio transceivers which can communicate with one another”,” *IEEE Transactions on Communications*, vol. 57, pp. 1287–1289, May 2009.

[67] S. Mukherjee and D. Avidor, “Connectivity and transmit-energy considerations between any pair of nodes in a wireless ad hoc network subject to fading,” *IEEE Transactions on Vehicular Technology*, vol. 57, pp. 1226 –1242, March 2008.

[68] J.-C. Kuo and W. Liao, “Hop count distribution of multihop paths in wireless networks with arbitrary node density: Modeling and its applications,” *IEEE Transactions on Vehicular Technology*, vol. 56, pp. 2321 –2331, July 2007.

[69] H. Wu, R. Fujimoto, G. Riley, and M. Hunter, "Spatial propagation of information in vehicular networks," *IEEE Transactions on Vehicular Technology*, vol. 58, pp. 420–431, January 2009.

[70] W.-L. Jin and W. Recker, "An analytical model of multihop connectivity of inter-vehicle communication systems," *IEEE Transactions on Wireless Communications*, vol. 9, pp. 106–112, January 2010.

[71] X. Wang, T. Adams, W. Jin, and Q. Meng, "The process of information propagation in a traffic stream with a general node headway: a revisit," *Transportation Research Part C.*, vol. 18, pp. 367–375, 2010.

APPENDIX A

POISSON DISTRIBUTION OF VEHICLES

The distance headway distribution is assumed exponential with parameters λ_1 and λ_2 on road R_1 and road R_2, respectively. Clearly, the two parameters reflect the traffic densities on the two roads, respectively. In the following tables, both analytical results based on the equations and simulation results are provided. In the tables that follow, d denotes the distance between the two roads, E the expected propagation distance, and V variance of the propagation distance. Furthermore, the numbers for E and V in each instance below start with the analytical value followed by its according simulation counterpart.

Table IV.: Instances of Exponential Headway at $\lambda_2 = 2.0$

$\lambda_2 = 2.0$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 0.2$	E	2.6163	2.4660	2.2576	2.2136	2.2009	2.2576
		2.6291	2.4815	2.1707	2.1509	2.3129	2.2393
	V	8.3862	7.4931	6.3406	6.1096	6.0429	6.0349
		8.3771	7.6324	6.2323	5.8723	6.9114	6.2274

Table IV.: (Continued)

$\lambda_2 = 2.0$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 0.4$	E	3.1113	2.8018	2.3877	2.2633	2.2134	2.2009
		3.0062	2.6694	2.4599	2.2434	2.2808	2.1916
	V	11.6432	9.5446	7.0530	6.3727	6.1081	6.0438
		10.8335	9.1231	7.2428	6.4460	6.1823	6.5348
$\lambda_1 = 0.6$	E	3.6960	3.2129	2.5862	2.3517	2.2398	2.2029
		3.7140	3.1669	2.6131	2.4177	2.3431	2.2929
	V	16.1372	12.3752	8.2113	6.8543	6.2467	6.0544
		17.1476	11.8683	8.1146	7.8916	6.4712	6.2594
$\lambda_1 = 0.8$	E	4.3863	3.7093	2.8551	2.4821	2.2834	2.2054
		4.3901	3.7136	2.8347	2.5968	2.4435	2.3348
	V	22.3380	16.2562	9.9120	7.5969	6.4797	6.0681
		22.4843	16.0576	9.7043	8.3828	7.4897	7.3776
$\lambda_1 = 1.0$	E	5.2010	4.3036	3.1991	2.6579	2.3468	2.2090
		5.1607	4.2211	3.1708	2.7975	2.6367	2.5097
	V	30.8972	21.5632	12.3064	8.6551	6.8265	6.0867
		30.9821	19.6625	12.2637	9.6766	8.8439	7.9201
$\lambda_1 = 1.2$	E	6.1623	5.0113	3.6258	2.8827	2.4323	2.2138
		6.1833	4.9362	3.6066	3.1974	3.0978	2.6989
	V	42.7192	28.8155	15.6130	10.1024	7.3078	6.1126
		39.9055	29.6488	15.7733	12.8741	11.9535	9.1578

Table IV.: (Continued)

$\lambda_2 = 2.0$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 1.4$	E	7.2966	5.8511	4.1455	3.1609	2.5418	2.2206
		7.0643	5.6454	4.1856	3.4504	3.2973	2.9280
	V	59.0621	38.7316	20.1396	12.0378	7.9472	6.1480
		58.6426	38.3815	20.6432	14.7272	14.1258	11.3427
$\lambda_1 = 1.6$	E	8.6356	6.8452	4.7712	3.4979	2.6772	2.2296
		8.6578	6.8334	4.6095	4.1060	3.7288	3.3272
	V	81.6785	52.3054	26.3137	14.5944	8.7735	6.1959
		86.7528	52.5455	26.1259	20.6484	18.1198	15.8284
$\lambda_1 = 1.8$	E	10.2167	8.0206	5.5190	3.9003	2.8407	2.2415
		10.0141	8.0166	5.5345	4.6526	4.3466	3.6814
	V	113.0139	70.9145	34.7279	17.9493	9.8216	6.2589
		105.0044	69.7134	34.2559	26.4321	22.4226	17.2950
$\lambda_1 = 2.0$	E	12.0844	9.4093	6.4087	4.3760	3.0344	2.2567
		11.9310	9.4989	6.4212	5.2115	4.9744	4.1341
	V	156.4857	96.4719	46.2018	22.3378	11.1355	6.3401
		148.8270	94.0844	48.9465	33.7764	30.0973	22.5572

Table V.: Instances of Exponential Headway at $\lambda_2 = 1.5$

$\lambda_2 = 1.5$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 0.2$	E	1.5909	1.4914	1.3601	1.3330	1.3248	1.3241
		1.5857	1.4734	1.3512	1.3444	1.2504	1.3641
	V	3.5498	3.1431	2.6463	2.5492	2.5197	2.5177
		3.5836	3.0704	2.6561	2.4956	2.3066	2.6697
$\lambda_1 = 0.4$	E	1.9102	1.7079	1.4448	1.3650	1.3326	1.3254
		1.8504	1.7163	1.4866	1.3631	1.3638	1.3455
	V	4.9905	4.0507	2.9668	2.6661	2.5475	2.5229
		4.9656	4.0167	3.0003	2.4637	2.7275	2.5846
$\lambda_1 = 0.6$	E	2.2898	1.9777	1.5776	1.4231	1.3493	1.3269
		2.2175	1.9910	1.6233	1.4202	1.4391	1.3648
	V	6.9873	5.3281	3.5051	2.8860	2.6080	2.5289
		6.7457	5.4012	3.7979	2.8366	2.9953	2.7851
$\lambda_1 = 0.8$	E	2.7401	2.3078	1.7609	1.5108	1.3775	1.3288
		2.6796	2.2873	1.7607	1.5999	1.5299	1.4206
	V	9.7504	7.1047	4.3145	3.2334	2.7119	2.5362
		9.6620	6.8648	4.5104	3.5000	3.3558	2.8838
$\lambda_1 = 1.0$	E	3.2738	2.7070	1.9990	1.6308	1.4192	1.3313
		3.3438	2.6597	2.0260	1.7561	1.6595	1.4942
	V	13.5699	9.8603	5.4757	3.7386	2.8696	2.5457
		14.6653	9.3438	5.8894	4.7211	4.0426	3.1665

Table V.: (Continued)

$\lambda_2 = 1.5$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 1.2$	E	3.9053	3.1859	2.2977	1.7864	1.4767	1.3347
		3.7922	3.2927	2.3335	2.0241	1.8102	1.6269
	V	18.8475	12.9437	7.1038	4.4418	3.0924	2.5583
		17.5766	13.8804	7.7589	5.7811	5.0603	3.9668
$\lambda_1 = 1.4$	E	4.6522	3.7593	2.6647	1.9812	1.5500	1.3392
		4.5355	3.6857	2.6114	2.3179	2.0486	1.8385
	V	26.1395	17.5985	9.3606	5.3964	3.3931	2.5752
		24.7830	16.5469	8.9771	7.9980	6.1810	5.4124

Table VI. Instances of Exponential Headway at $\lambda_2 = 1.0$

$\lambda_2 = 1.0$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 0.2$	E	0.8789	0.8174	0.7407	0.7253	0.7203	0.7203
		0.8809	0.8219	0.7308	0.7316	0.6976	0.7084
	V	1.3878	1.2109	1.0085	0.9707	0.9575	0.9583
		1.4658	1.2166	0.9456	0.9976	0.9541	0.9878
$\lambda_1 = 0.4$	E	1.0723	0.9488	0.7929	0.7448	0.7249	0.7214
		1.0714	0.9657	0.8019	0.7526	0.7396	0.7266
	V	2.0005	1.5995	1.1496	1.0219	0.9690	0.9623
		1.9719	1.7040	1.1348	1.0275	1.0360	1.0293
$\lambda_1 = 0.6$	E	1.3057	1.1175	0.8781	0.7814	0.7348	0.7227
		1.2590	1.1373	0.8873	0.8314	0.8244	0.7585
	V	2.8618	2.1661	1.3994	1.1218	0.9947	0.9667
		2.6795	2.2642	1.4475	1.3145	1.2957	1.0434
$\lambda_1 = 0.8$	E	1.5863	1.3288	0.9993	0.8380	0.7521	0.7243
		1.5416	1.3634	0.9877	0.9298	0.8625	0.7557
	V	4.0670	2.9756	1.7909	1.2856	1.0402	0.9718
		3.8586	3.1107	1.7473	1.6350	1.3956	1.1332
$\lambda_1 = 1.0$	E	1.9226	1.5890	1.1609	0.9177	0.7781	0.7262
		1.9858	1.5901	1.0878	0.9856	0.9231	0.8432
	V	5.7473	4.1179	2.3717	1.5320	1.1112	0.9778
		6.0198	4.1090	2.2898	1.8237	1.6483	1.3757

Table VII. Instances of Exponential Headway at $\lambda_2 = 0.6$

$\lambda_2 = 0.6$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 0.2$	E	0.4616	0.4256	0.3826	0.3746	0.3714	0.3718
		0.4625	0.4349	0.3895	0.3808	0.3999	0.3742
	V	0.5626	0.4822	0.3952	0.3817	0.3736	0.3768
		0.5507	0.5260	0.4185	0.3912	0.4303	0.3642
$\lambda_1 = 0.4$	E	0.5746	0.5027	0.4139	0.3864	0.3740	0.3732
		0.5752	0.4833	0.4115	0.3907	0.3795	0.3908
	V	0.8412	0.6615	0.4620	0.4084	0.3795	0.3821
		0.5266	0.6441	0.5065	0.4147	0.4103	0.3934
$\lambda_1 = 0.6$	E	0.7142	0.6056	0.4673	0.4091	0.3799	0.3746
		0.7300	0.5955	0.4662	0.4205	0.4057	0.3754
	V	1.2436	0.9366	0.5912	0.4605	0.3921	0.3877
		1.2706	0.8762	0.6109	0.5146	0.4454	0.3619

Table VIII. Instances of Exponential Headway at $\lambda_2 = 0.2$

$\lambda_2 = 0.2$							
		Case I			Case II		
d		0.1	0.5	0.8	0.9	0.94	0.98
$\lambda_1 = 0.2$	E	0.2462	0.2380	0.2338	0.2370	0.2344	0.2362
		0.1366	0.1201	0.1125	0.1097	0.1113	0.1041
	V	0.1337	0.1118	0.0894	0.1053	0.0872	0.1058
		0.1369	0.1007	0.0874	0.0827	0.0829	0.0789

APPENDIX B

GAMMA DISTRIBUTION OF THE VEHICLE HEADWAY

The headway on each road is assumed to follow a Gamma distribution with the parameters shown in Table IX. The theoretical and simulation results are shown in Table X.

Table IX. Configuration of Gamma Distribution

On Road 1	Mean of Gamma Distribution: μ_1
	Variance of Gamma Distribution: μ_1
On Road 2	Mean of Gamma Distribution: μ_2
	Variance of Gamma Distribution: $0.5\mu_2$

Table X. Analytical and Simulation Results

$\mu_2 = 1.0$							
d		0.01	0.1	0.3	0.5	0.7	0.8
$\mu_1 = 1.2$	E	2.0091	1.9950	1.8726	1.6380	1.3361	1.1807
		2.0073	1.9935	1.8924	1.6563	1.3195	1.1819
	V	6.0299	5.9556	5.3284	4.2145	2.9533	2.3791
		6.3135	5.7853	5.6661	4.1474	3.0711	2.3346
$\mu_1 = 1.5$	E	1.6163	1.6072	1.5260	1.3611	1.1380	1.0235
		1.7259	1.6868	1.6146	1.3781	1.1591	1.0294
	V	4.1071	4.0663	3.7099	3.0324	2.2139	1.8365
		4.8706	4.6104	4.4811	3.2178	2.3368	1.8945
$\mu_1 = 1.8$	E	1.3556	1.3496	1.2956	1.1803	1.0154	0.9303
		1.4862	1.4275	1.4163	1.2126	0.9954	0.9105
	V	3.0120	2.9883	2.7773	2.3519	1.8012	1.5426
		3.8272	3.4366	3.3335	2.5664	1.8260	1.6399

APPENDIX C

TRUNCATED GAUSSIAN DISTRIBUTION OF THE VEHICLE HEADWAY

Table XI.: Instances of Truncated Gaussian Headway

$\mu_2 = 0.5, a = 0.01$							
		Case I			Case II		
d		0.2	0.5	0.8	0.9	0.94	0.98
$\mu_1 = 4.0$	E	4.0664	3.9808	3.7870	3.5697	3.6107	3.6720
		4.3995	4.2121	3.8474	3.7832	3.7621	3.6763
	V	19.1593	18.4042	16.7506	14.9849	15.3117	15.8053
		23.8971	23.0550	17.1167	15.4219	16.3400	16.0258
$\mu_1 = 3.0$	E	4.2193	4.0975	3.8167	3.4963	3.5556	3.6446
		4.5921	4.4444	3.7313	3.6975	3.6791	3.7320
	V	20.5435	19.4358	16.9992	14.4097	14.8738	15.5836
		23.9911	23.0739	16.0244	15.9636	16.6468	16.4426

Table XI.: (Continued)

$\mu_2 = 1.0, a = 0.01$							
		Case I			Case II		
d		0.2	0.5	0.8	0.9	0.94	0.98
$\mu_1 = 3.0$	E	0.5395	0.5046	0.4535	0.3362	0.3638	0.4020
		0.5732	0.5185	0.4518	0.4178	0.4334	0.4471
	V	0.7169	0.6353	0.5268	0.2559	0.3186	0.4072
		0.8505	0.6933	0.5448	0.4534	0.4773	0.5318
$\mu_1 = 2.0$	E	0.6247	0.5634	0.4707	0.2595	0.3088	0.3763
		0.6858	0.5716	0.4660	0.4461	0.4360	0.4557
	V	0.9238	0.7701	0.5633	0.0896	0.1961	0.3473
		1.2108	0.8508	0.5561	0.5358	0.5116	0.5521

APPENDIX D

INFORMATION PROPAGATION FOR TWO ZIGZAG ROADS

From Table XII to Table XV, the simulation results of information propagation are measured in two ways: 1) horizontal distance, measured horizontally from the sender and the last receiver on the same road; and 2) curve distance, measured along the curve. They are denoted by 'Horiz' and 'Curve', respectively. Theoretical results are also provided in terms of projected density, i.e., the curve density divided by $cos(\alpha)$.

Table XII. Instances of Exponential Headway on Zig-Zag Roads for $\lambda_1 = 0.5$

$\lambda_1 = 0.5, \lambda_2 = 2.0$						
			Case I			Case II
d			0.3	0.5	0.7	0.9
$\alpha = 10$	Horiz	E	3.3664	3.0594	2.7132	2.3175
		V	13.7026	11.6453	9.1897	6.5161
	Curve	E	3.4183	3.1066	2.7551	2.3532
		V	14.1286	12.0073	9.4754	6.7087
Theoretical		E	3.3689	3.1007	2.7431	2.3743
(Condensed)		V	13.5001	11.5346	9.1449	6.9555
$\alpha = 15$	Horiz	E	3.3509	3.1932	2.8206	2.5264
		V	13.4040	12.1114	9.3242	7.8947
	Curve	E	3.4691	3.3058	2.9201	2.6155
		V	14.3663	12.9809	9.9936	8.4616
Theoretical		E	3.5213	3.2382	2.8608	2.4696
(Condensed)		V	14.6347	12.4809	9.8666	7.4638
$\alpha = 20$	Horiz	E	3.6461	3.3592	2.9912	2.6634
		V	16.3080	13.5451	10.5167	8.3137
	Curve	E	3.8801	3.5747	3.1831	2.8344
		V	18.4684	15.3395	11.9098	9.4150
Theoretical		E	3.7539	3.4479	3.0401	2.6141
(Condensed)		V	16.4545	13.9953	11.0177	8.2688

Table XIII. Instances of Exponential Headway on Zig-Zag Roads for $\lambda_1 = 0.7$

$\lambda_1 = 0.7, \lambda_2 = 2.0$						
			Case I			Case II
d			0.3	0.5	0.7	0.9
$\alpha = 10$	Horiz	E	3.9288	3.5479	3.0228	2.5199
		V	18.0274	15.2847	11.3196	8.5763
	Curve	E	3.9894	3.6026	3.0694	2.5588
		V	18.5879	15.7599	11.6716	8.8429
Theoretical (Condensed)		E	3.9664	3.5758	3.0673	2.4898
		V	18.4081	15.1225	11.3109	7.6142
$\alpha = 15$	Horiz	E	4.0655	3.5859	3.1493	2.5967
		V	19.5763	15.0090	12.5503	8.3987
	Curve	E	4.2089	3.7124	3.2604	2.6883
		V	20.9818	16.0866	13.4514	9.0017
Theoretical (Condensed)		E	4.1572	3.7440	3.2068	2.5938
		V	20.0693	16.4498	12.2629	8.1962
$\alpha = 20$	Horiz	E	4.3199	3.8764	3.2776	2.9147
		V	21.7084	17.3916	13.4926	9.7362
	Curve	E	4.5971	4.1257	3.4879	3.1017
		V	24.5842	19.6955	15.2800	11.0259
Theoretical (Condensed)		E	4.4496	4.0015	3.4199	2.7521
		V	22.7546	18.5891	13.7913	9.1223

Table XIV. Instances of Exponential Headway on Zig-Zag Roads for $\lambda_1 = 1.0$

$\lambda_1 = 1.0, \lambda_2 = 2.0$						
			Case I			Case II
d			0.3	0.5	0.7	0.9
$\alpha = 10$	Horiz	E	4.9275	4.4419	3.7132	2.9614
		V	26.7850	22.0993	16.4029	11.0786
	Curve	E	5.0035	4.5105	3.7705	3.0071
		V	27.6178	22.7864	16.9129	11.4231
Theoretical (Condensed)		E	5.0783	4.4749	3.7125	2.7508
		V	29.4708	23.1713	16.2638	9.2081
$\alpha = 15$	Horiz	E	5.2089	4.6258	3.7804	3.1169
		V	30.5780	24.7879	16.7966	11.8781
	Curve	E	5.3927	4.7890	3.9138	3.2268
		V	32.7734	26.5676	18.0025	12.7309
Theoretical (Condensed)		E	5.3451	4.7042	3.8966	2.8745
		V	32.4157	25.4158	17.7728	9.9713
$\alpha = 20$	Horiz	E	5.6613	4.9242	3.9747	3.1638
		V	35.5937	27.7942	17.8093	11.2393
	Curve	E	6.0246	5.2402	4.2297	3.3669
		V	40.3090	31.4762	20.1686	12.7282
Theoretical (Condensed)		E	5.7563	5.0573	4.1793	3.0636
		V	37.2323	29.0738	20.2211	11.1960

Table XV. Instances of Exponential Headway on Zig-Zag Roads for $\lambda_1 = 1.5$

$\lambda_1 = 1.5, \lambda_2 = 2.0$						
			Case I			Case II
d			0.3	0.5	0.7	0.9
$\alpha = 10$	Horiz	E	7.6225	6.3632	5.3606	3.8998
		V	63.4701	46.7651	33.3066	18.7436
	Curve	E	7.7401	6.4614	5.4433	3.9600
		V	65.4435	48.2191	34.3421	19.3263
Theoretical (Condensed)		E	7.6912	6.6145	5.3065	3.4539
		V	65.2747	48.8942	32.1199	14.2038
$\alpha = 15$	Horiz	E	7.9230	6.9048	5.5446	4.1859
		V	66.3784	52.8162	33.9887	20.7906
	Curve	E	8.2024	7.1484	5.7402	4.3336
		V	71.1441	56.6083	36.4290	22.2833
Theoretical (Condensed)		E	8.1539	7.0022	5.6079	3.6311
		V	72.9145	54.4267	35.6033	15.5652
$\alpha = 20$	Horiz	E	8.8628	7.3733	5.9583	4.4887
		V	88.2312	60.8332	42.0374	22.6167
	Curve	E	9.4316	7.8464	6.3407	4.7767
		V	99.9195	68.8921	47.6062	25.6128
Theoretical (Condensed)		E	8.8751	7.6049	6.0754	3.9042
		V	85.6677	63.6214	41.3629	17.7853

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