ESSAYS ON INTEREST RATE ANALYSIS WITH GOVPX DATA

A Dissertation

by

BONG JU SONG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2009

Major Subject: Economics

ESSAYS ON INTEREST RATE ANALYSIS WITH GOVPX DATA

A Dissertation

by

BONG JU SONG

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Approved by:

Chair of Committee,	Joon Y. Park
Committee Members,	Yoosoon Chang
	Hwagyun Kim
	Ximing Wu
Head of Department,	Larry Oliver

August 2009

Major Subject: Economics

ABSTRACT

Essays on Interest Rate Analysis with GovPX Data. (August 2009)

Bong Ju Song, B.A., Seoul National University;

M.A., Seoul National University; M.S., The University of Texas at Austin

Chair of Advisory Committee: Dr. Joon Y. Park

U.S. Treasury Securities are crucially important in many areas of finance. However, zero-coupon yields are not observable in the market. Even though published zero-coupon yields exist, they are sometimes not available for certain research topics or for high frequency. Recently, high frequency data analysis has become popular, and the GovPX database is a good source of tick data for U.S. Treasury securities from which we can construct zero-coupon yield curves. Therefore, we try to fit zero-coupon yield curves from low frequency and high frequency data from GovPX by three different methods: the Nelson-Siegel method, the Svensson method, and the cubic spline method.

Then, we try to retest the expectations hypothesis (EH) with new zero-coupon yields that are made from GovPX data by three methods using the Campbell and Shiller regression, the Fama and Bliss regression, and the Cochrane and Piazzesi regression. Regardless of the method used (the Nelson-Siegel method, the Svensson method, or the cubic spline method), the expectations hypothesis cannot be rejected in the period from June 1991 to December 2006 for most maturities in many cases. We suggest the possible explanation for the test result of the EH. Based on the overreaction hypothesis, the degree of the overreaction of spread falls over time. Thus, our result supports that the evidence of rejection of the EH has weaken over time.

Also, we introduce a new estimation method for the stochastic volatility model of the short-term interest rates. Then, we compare our method with the existing method. The results suggest that our new method works well for the stochastic volatility model of short-term interest rates.

To my family

ACKNOWLEDGMENTS

I would like to sincerely thank my advisor Joon Y. Park for his encouragement, understanding, patience, and guidance. I could never have completed this PhD program without him. I would also like to thank my committee members, Hwagyun Kim, Yoosoon Chang, and Ximing Wu, for their valuable suggestions. Joon Y. Park and Hwagyun Kim especially have been heavily involved in helping me progress towards the completion of my dissertation. I wish to thank and acknowledge the Department of Economics and the Program for Time Series with Applications in Financial Economics (PTSAFE) in which I have discussed and developed my dissertation.

I sincerely thank my colleagues Changwoo Nam, Minsoo Jeong, Joon-Young Hur, Yongok Choi, and Hyoung-Il Lee. They helped me a lot in running these programs and discussing my dissertation topics. Courtney Collins helped in writing the dissertation.

I would like to thank my parents, Soonho Song and Jungsoon Park. They have always supported me. My parents-in-law, Sangtae Kim and Jeumho Min have encouraged me with their best wishes.

Finally, I want to thank my wife, Kyounghee Kim and my son, Joonseo Song. They have always been there to cheer me up and to stand by me through the good times and the bad. I love you all.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

FIGURE Page

CHAPTER I

INTRODUCTION

The U.S. Treasury bond markets are crucially important in many areas of finance. The yield curves from these markets are instruments that give us information on asset pricing. Even though there are many issues of significance in interest rate analysis, we, in this research, specifically focus on the following three topics.

First, we study the modeling of a zero-coupon yield curve. Since zero-coupon yields are not observable in the market, it is necessary to construct zero-coupon yield curves through statistical methods. Even though many economists generally use the suggested zero-coupon yields such as CRSP, they are sometimes not available for certain research topics or for high frequency. Although there is a tendency for many economists to use their own zero-coupon yields, there are few papers that deal in detail with the modeling of zero-coupon yield curves. Recently, high-frequency data analysis has become popular, and many papers use the GovPX database, composed of tick data, in order to analyze term structure models. The GovPX database includes real-time quotes and trade data from most interdealer Treasury security brokers. Thus, the GovPX database is a good source of tick data from which we can construct zero-coupon yield curves. Therefore, considering parsimoniousness and popularity, we will plot the zero-coupon yield curve covering low and high frequencies from the GovPX database by the different methods. We will apply these new zero-coupon yield curves to the expectations hypothesis and a new estimation method of stochastic volatility for interest rates.

This dissertation follows the style of *Econometrica*.

Second, we hope to retest the expectations hypothesis (EH) with our previously constructed zero-coupon yield curves. The EH is one of the main issues in interest rate analysis. Fama and Bliss (1987), Campbell and Shiller (1991), and Cochrane and Piazzesi (2005) make important contributions to EH tests using different approaches. We contribute to the literature by retesting the EH with new zero-coupon yield curves which is constructed from GovPX data using the Fama-Bliss regression, the Campbell-Shiller regression, and the Cochrane-Piazzesi regression. Then, we discuss possible explanations in order to interpret our results.

Third, short-term interest rates are one of the most fundamental assets in the financial market. Many papers are developed in continuous time and assume that short-term interest rates follow a diffusion process. Ball and Torous (1999) introduce a stochastic volatility model of short-term interest rates that assumes the volatility itself is stochastic. There are different types of estimation methods of stochastic volatility models. Here we focus on a two stage estimation method. We introduce a new estimation method for a stochastic volatility model that uses the Martingale method and density-based filtering, and we compare our method with the existing method.

CHAPTER II

MODELING THE YIELD CURVE WITH GOVPX DATA

A. Introduction

The U.S. Treasury bond markets are crucially important in many areas of finance. The yield curves from these markets are instruments that give us information on asset pricing. However, zero-coupon yields are not observable in the market for a wide range of maturities. Therefore, we need to construct zero-coupon yield curves through statistical methods. Many methodologies are developed to derive the zero-coupon yield curve from observed data. However, many economists have little interest in the methodologies, and they generally use the suggested zero-coupon yields such as CRSP data. However, zero-coupon yield curves are sometimes not available for some research topics. It is necessary for economists to create their own zero-coupon yield data for their own research. Even though many methodologies are well known, it is not easy to get zero-coupon interest rates with all different maturities because of many technical issues. Also, for some models we cannot use a general data set. Therefore, there is a tendency for many economists to use their own zero-coupon yields.

There exist many methodologies to fit the yield curve. When we choose the method to construct the yield curve, we should consider both goodness of fit and smoothness of the curve since each method can supply different shapes of yield curve. There is a trade-off between these two factors. There exist two mainstream approaches of fitting the yield curve: a parsimonious representation and a spline representation. Nelson and Siegel (1987) suggest a parsimonious representation model that uses a parametric representation to capture many of the typical shapes of yield curves. The

Nelson-Siegel method has been extended by Svensson (1994) which incorporates additional flexibility. McCulloch (1971) introduces a spline method which uses a cubic spline. Also, many spline methods are suggested such as Waggoner (1997) and Anderson and Sleath (2001). Despite the existence of more flexible methods, the Bank of International Settlements (BIS) argues that one third of all central banks make use of the Nelson-Siegel method or Svensson method to generating zero-coupon yield curves. The spline method is especially used in cases involving the US. The Nelson-Siegel model is also popular among practitioners, and Diebold and Li (2006) argue that the Nelson-Siegel method can work well for term structure forecasts. Considering parsimoniousness and popularity, we focus on the Nelson-Siegel method, the Svensson method, and the cubic spline method.

Recently, high-frequency data analysis has become popular, and many papers, currently use the GovPX database, composed of tick data, in order to deal with term structure models. Therefore, if we use the GovPX database, we can fit zero-coupon yield curves for our research objectives. In this paper, we need to generate many types of zero-coupon yield curves since we hope to analyze many types of term structure models. Therefore, in this chapter, we study the method to fit the yield curve with the GovPX data set, and we are going to use these zero-coupon yield curves for the next chapters' topics.

We want to fit yield curves from GovPX data. Since GovPX data are tick data from the U.S. Treasury securities market, it is necessary to know the bond instrument, the structure the U.S. Treasury securities market, and the properties of the GovPX data set. Therefore, we will, at first, summarize the U.S. Treasury securities market, the GovPX database, and the zero-coupon yield curve methods. Then, we will construct our zero-coupon yield curves from the GovPX data with different frequencies.

The outline of this chapter is as follows. In section B, we study U.S. Treasury securities market. In section C, we study a description of the GovPX data set. Since the GovPX data set is a tick data set and is not organized well, it is indispensable to analyze the raw data set before we generate zero-coupon yield curve. Then, we have to study the zero-coupon yield curve with the GovPX data set. Section D explains how to do data cleaning and processing. In section E, we discuss the zero-coupon yield curve methods. In section F, we show our results, and we analyze them. Section G concludes this chapter.

B. U.S. Treasury Securities Market

Since U.S. Treasury Securities are issued by U.S. Department of Treasury, they are considered to be risk-free assets. Therefore, we can use them for pricing many other financial instruments. There are different types of Treasury securities. They are categorized as either discount or coupon securities with respect to the existence of coupon. Discount securities pay only face value at maturity without any interest, but coupon securities pay interest every six months as well as face value at maturity. Also, Treasury securities are categorized by maturities as Treasury bills, notes, bonds, and inflation-indexed securities. Table 2-1 explains the types of Treasury securities in detail.

Cash-management bills are issued irregularly if Treasury cash balances become too low. Their maturity is not regular and often less than 21 days. Treasury inflation indexed securities are bonds whose principal and coupon payment are adjusted for inflation using the consumer price index in order to maintain the purchasing power of their original investment. Therefore, cash-management bills and Treasury inflation-indexed securities should be treated differently from the other typical Trea-

Table 2-1.: Marketable U.S. Treasury Securities

Issue Type	Security Type	Issues
Treasury bills	discount	Cash-management, Three-month, Six-month
Treasury notes	coupon	Two-year, Five-year, Ten-year
Treasury bonds	coupon	Thirty-year
Treasury Inflation-Indexed Securities	coupon	Ten-year, Thirty-year

sury securities.

1. The Primary Market

The U.S. Treasury sells Treasury securities in the primary market by single-price auctions. The Treasury announces auction information that includes the amount and type of security, auction rules, and procedures several days in advance. Bids are typically submitted in multiples of $1,000 on auction day. Although the primary market is open to anyone, the primary dealers cover a large portion of trading volume.

The primary dealer system was organized by the Federal Reserve Bank of New York. As of September 2008, there was 19 primary dealers. The primary dealers are mainly firms that interact with the Federal Reserve Bank of New York for open market operations. The Treasury has a regular schedule to issue the Treasury securities as shown in Table 2-2 which is reported as of August 1999.

2. The Secondary Market

In the primary market, the Department of the Treasury issues Treasury securities through single price auctions. In the secondary market, trading takes place not

Table 2-2.: U.S. Treasury Auction Schedule

Issue Type	Issue Frequency
Three-month bill	weekly
Six-month bill	weekly
One-year bill	every 4 weeks
Two-year note	monthly
Five-year note	quarterly
Ten-year note	quarterly
Thirty-year bond	semi-annually

in over-the-counter markets. Primary dealers trade with customers such as banks, insurance companies, and non-primary dealers. Also, primary dealers trade with each other directly, or interdealer brokers execute trades between primary dealers and receive a fee.

Table 2-3 summarizes the volume and portion of daily trading of U.S. Treasury securities between April and August of 1994 according to trading agents based on Fleming (1997).

Table 2-3.: Daily Trading Volume of U.S. Treasury Securities

Trading Agents	Volume	Portion
Primary Dealer-Primary Dealer		
Interdealer Broker	$58.5 billion	42.6%
No intermediary	$ 4.9 billion	4.0%
Primary Dealer-Customer	$67.0 billion	53.4%

C. GovPX Data Set

GovPX is a consortium of several primary dealers and interdealer brokers, and it is organized in 1991. Since GovPX supplies the trading data of U.S. Treasury securities, it facilitates public access to the trading information of U.S. Treasury securities. GovPX data covers two-thirds of the interdealer broker market, and they contain security type, trading time, bid price, ask price, bid size, and ask size.

In Appendix A, Table A-26 explains the bulk of the information that is provided from GovPX data. Since GovPX does not include Cantor Fitzgerald Inc. which is prominent at the long-term maturity market, we should be careful of analyzing the long-term maturity securities from GovPX data

D. Data Cleaning and Processing

Since GovPX data is raw data, we have must clean and process it. Also, we must consider some issues which are important in the construct of yield curves. Thus, here we explain some important points for data cleaning and processing.

1. Time to Maturity

As we see above, there are many types of Treasury securities by maturity, and there exist significant differences among the volume of trading by maturity. Table 2-4 shows trading volume of the Treasury securities for on-the-run securities by maturity based on Fleming (1997) which gets the volume information from GovPX, Inc. between April and August of 1994.

Based on Table 4 the two-year note, the five-year note, and ten-year note are the most popular securities which cover three-fourths of the volume. The thirty-year Treasury bill was not issued from February 18, 2002 to February 8, 2006, and the

Table 2-4.: Trading Volume of U.S. Treasury Securities by Maturity

The Treasury Security Type	Portion
Three-month bill	7.4 %
Six-month bill	6.4 %
One-year bill	10.1 %
Cash-management bill	1.0 %
Two-year note	21.3 %
Three-year note	7.7 %
Five-year note	26.0 %
Ten-year note	21.3 %
Thirty-year bond	2.7 %

twenty-year bill was discontinued from January 1, 1887 to September 30 1993. Also, since GovPX does not include Cantor Fitzgerald Inc. which is prominent in the long-maturity segment of the market, it is advisable to exclude long-maturity Treasury securities for fitting yield curves with GovPX data. Therefore, we only consider maturities up to ten-year.

2. Security Type Issue

As we noted, cash-management bills and Treasury inflation-indexed securities are special. Therefore, we only use 3-month bills, 6-month bills, 1-year bills, 2-year notes, 5-year notes, and 10-year notes.

3. Trading Activity Issue

Trading activity is one of the key factors in determining the price of the security. There are three categories: when-issued securities, on-the-run securities, and off-the-run securities. A when-issued security is a security that has been notified for auction by the U.S. Department of Treasury but has not been issued yet. Even though when-issued securities have not been issued, they are allowed to be sold by a dealer to a customer in advance of the auctions in order to facilitate price discovery for auction and to reduce uncertainty about auction. An on-the-run security is a security that has most contemporarily been issued at a given maturity. An off-the-run security is a security that been issued before an on-the-run security at a given maturity. According to Fabozzi (2004), in 1998, on-the-run securities covered 71% of trading activity, off-the-run securities explain 23% of trading activity, and when-issued securities account for 6% of trading activity.

If a security is recently issued and more active, its price is higher than the securities that issued before. Therefore we should consider liquidity effects to construct a zero-coupon yield curve. Andersen and Benzoni (2006) rely on on-the-run securities, Gurkaynak, Sack, and Wright (2006) only use off-the-run securities, and Daily CRSP US Government Bonds files includes when-issued securities as well as on-the-run securities. We consider on-the-run securities and when -issued securities in our construction of zero-coupon yield curves.

4. Trading Time

Our period of data begins on Jan 1, 1991 and ends on Dec. 31 2006. Since the U.S. Treasury market is over-the-count, it is active for 24 hours, but it is most active during business days in the early morning through the late afternoon. Hence, we use the intra-day transaction record from 8:00 AM ET to 5:00 PM ET. This time window is appropriate because it includes the regular macroeconomic and monetary policy announcements, and the majority of the trading is done during these hours.

5. Cleaning Pricing

The GovPX dataset includes price information for all types of securities. However, zero-coupon bonds have only yield information. Thus, we have to compute bond prices from zero-coupon yields. Traditionally, given the zero-coupon yield, the price can be calculated as

$$Price = Face\ Value - (Face\ Value \times Yield \times \frac{Days\ Time\ Maturity}{360}) \tag{2.1}$$

E. Zero-coupon Yield Curve Methods

There are various methods to construct zero-coupon yield curves from Treasury securities that include discount or coupon securities. The methods can be categorized into the function-based approaches and the spline-based approaches. The function-based approaches use a single function over the entire maturity domain. The spline-based approaches use a piecewise polynomial where the individual segments are joined at the knot point.

Many central banks choose the Nelson-Siegel method or the Svensson method, the United States and the United Kingdom apply variants of the smoothed spline method. Table 2-5 summarizes the yield curve methods by central banks based on the Bank of International Settlements. We briefly discuss about the most popular methods below.

Table 2-5.: The Yield Curve Methods by Central Banks

Central Bank	Method	Relevant maturity
Belgium	Svensson or Nelson-Siegel	Couple days to 16 years
Canada	Exponetial Spline	3 month 30 years
France	Nelson-Siegel	Upto 10 years
Germany	Svensson	1 to 10 years
Italy	Nelson-Siegel	Upto 30 years
Japan	Smoothed Spline	1 to 10 years
Spain	Svensson	Upto 10 years
United Kingdom	Smoothed Spline	Upto 30 years
United States	Smoothed Spline	1 to 10 years

1. The Nelson-Siegel Method

Nelson and Siegel (1987) suggest a model which is flexible enough to catch the shapes generally associated with yield curves as follows:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau}) + \beta_{3t}(\frac{1-e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}) + u_t \tag{2.2}$$

where $y_t(\tau)$ is the zero-coupon yield.

There are some reasons why the Nelson-Siegal model is very popular. First, it is a parsimonious model which uses only four parameters, but it captures the typical yield curve shapes such as monotonic, humped and S -type shapes. Second, it has the desirable property that an instantaneous short rate value can be easily computed as follows:

$$y_t(0) = \beta_{1t} + \beta_{2t}; \quad y_t(\infty) = \beta_{1t} \tag{2.3}$$

Finally, the three parameters can be interpreted as as short, medium, and long. The parameter β_{1t} can be interpreted as a long-term factor since it is already confirmed that $y_t(\infty) = \beta_{1t}$. The parameter β_{2t} can be considered to be a short-term factor since $((1 - e^{-\lambda_t \tau})/\lambda_t \tau$ begins at 1 but decreases rapidly to 0. The parameter β_{3t} can be considered to be a medium-term factor since $((1 - e^{-\lambda_t \tau})/\lambda_t \tau) - e^{-\lambda_t \tau}$ begins at 0, and increases, and decreases to zero again.

Also, based on Diebold and Li (2006), β_{1t}, β_{2t}, and β_{3t} can be considered three latent factors. Since the three factors may be viewed as the yield curve level, the yield curve slope, and the yield curve curvature, and because long-term, short-term, and medium-term can also be considered yo be level, slope, and curvature, we can argue that β_{1t} is connected to the yield curve level, β_{2t} is connected to the yield curve slope, and β_{3t} is also connected to the yield curve curvature.

2. The Svensson Method

Svensson (1994) extends the Nelson-Siegel model as follows:

(2.4)

$$y_t(\tau) = \beta_{1t} + \beta_{2t}(\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}) + \beta_{3t}(\frac{1 - e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau}) + \beta_{4t}(\frac{1 - e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}) + u_t$$

The Svensson model adds a fourth component which mostly has an effect on medium-term maturities to improve the flexibility of the fitting curve. Therefore, the fitting curve is more flexible at the cost of two more parameters, β_{4t},λ_{2t}.

3. Cubic Spline Method

Spline methods use a piecewise polynomial rather than a single functional form over the entire maturity. Specifically, the cubic spline method uses cubic spline that is a piecewise cubic polynomial, twice differentiable everywhere. Since the individual segments are connected at the knot points, the number of knot points is critical in determining the goodness-of-fit and the smoothness. If we use the cubic spline method without smoothing technic, the interest curve tends to oscillate too much.

There are some methods to reduce the oscillation and increase the smoothness of a cubic spline. McCulloch (1971) uses a regression spline and Waggoner (1997) use smoothed splines. Also, Waggoner (1997) applies the variable roughness penalty (VRP) model for smoothness that allows more curvature for the short maturity. We briefly explain the smoothed spline method. At first, we need to choose node points, $\tau_0 < \tau_1 < ... < \tau_k$ for the cubic spline method and the smoothed cubic spline method. Let P_i be the observed price of i-th Treasury security. If we apply the cubic spline method, we minimize the objective function

$$\sum_{i=1}^{N} (P_i - \hat{P}_i^*(\psi))^2 \tag{2.5}$$

where ψ is the cubic spline. If we apply the smoothed cubic spline, we impose a penalty in the objective function in order to achieve the oscillations. Then the new objective function is as follows:

$$\sum_{i=1}^{N} (P_i - \hat{P}_i^*(\psi))^2 + \lambda(t) \int_0^{\tau_k} [\psi''(t)]^2 \mathrm{d}t \tag{2.6}$$

where $\lambda(t)$ is called a penalty function, which determines the tradeoff between fit and smoothness. As $\lambda(t)$ increases, smoothness will increase. We use smoothed cubic spline that Anderson and Sleath (2001) introduces. In Appendix A, Figure A-1 and Figure A-2 show zero-coupon yield curves that are generated by three different methods, Nelson-Siegel method, the Svensson method, and the cubic spline method using only active securities. The day representing each year is the last day of July, and the relevant maturity interval is from three-month to ten-year.

F. Results

As we noted before, we use only active securities whose maturities are less than ten-year. In Appendix A, Figure A-1 and Figure A-2 are the zero-coupon yield curves for the last day of June every year. They show that if we use active securities up to ten-year maturity zero-coupon yield curves by three methods (the Nelson-Siegel method, the Svensson method, and the cubic spline method) are not different. From Figure A-3 to Figure A-17 in Appendix A, we shows the zero coupon yield curve by methods and by frequencies.

The results from the Nelson-Sielgel method and the Svensson method are similar. However, as the frequency become higher and higher, the number of observation data will decrease and the fitted yield curve will not be stable. Specifically, as frequency increases, the zero-coupon yield curve by the cubic spline methods tend to oscillate. Since the cubic spline method is too sensitive to observed data, we use only the Nelson-Siegel method and the Svensson method for higher frequencies higher than hourly. Also, if we compare our yield curves with CRSP yield curves, we can see that they are similar. Figure A-18 through Figure A-22 in Appendix B support this argument.

G. Conclusion

U.S. treasury bond markets are crucially important in many areas of finance. Zero-coupon yields are, however, not observable in the market for a wide range of maturities. Therefore, we need either to use the suggested zero-coupon yields such as CRSP or to derive the zero-coupon yield curve from observed U.S. Treasury securities data. We, in this paper, deal with many types of term structure models such as the expectations hypothesis and stochastic volatility models with different frequencies. Therefore, it is necessary to fit the yield curve. Recently, the necessity of high-frequency data has increased.

In the next chapters, we will study many types of term structure models through the generated yield curves with low-frequency and high-frequency from GovPX data, which is a huge data set of tick data. Even though there are many methods used to fit yield curves, we use three parsimonious and popular models: the Nelson-Siegel method and the Svensson method, and the cubic spline method. Since we have zero-coupon yield curves at different frequency by different methods, we will be able to use these yield curves for our analysis in next chapters.

CHAPTER III

REVISITING THE EXPECTATION HYPOTHESIS OF INTEREST RATES WITH GOVPX DATA

A. Introduction

The expectations hypothesis (EH) is one of main topics of significance in term structure of interest rates. It is reasonable to assume that interest rates at different maturities move together since they are related to each other. The expectation hypothesis is a tool to explains this relationship. The expectations hypothesis implies that long-term bond yields are the average of future expected short-term bond yields. Many papers try to test the EH in financial economics and investigate whether the EH holds in different settings.

Among these papers, Fama and Bliss (1987), Campbell and Shiller (1991), and Cochrane and Piazzesi (2005) are the most distinguished. Fama and Bliss (1987) test whether or not the current forward rate can explain the expected return. Also, Campbell and Shiller (1991) test whether or not the expected change in the long-term bond yield can be predicted by the yield spread. The EH corresponds to the proposition that excess return can not be predicted. Cochrane and Piazzesi (2005) test whether one-year excess returns can be explained by five different maturity forward rates. Many papers, including the above mentioned tests, present evidence against the EH. There are many papers which try to explain the rejection of the EH. Campbell and Shiller (1991) suggest the two possible justifications against the EH, a time-varying risk premium and the overreaction hypothesis.

The EH is tested by using zero-coupon yields. Since zero-coupon yield curves should be extracted from coupon bonds as well as discount bonds, the construction of a zero coupon yield curve is an important factor to test the EH. In the previous chapter, we generate yield curves in different ways. Campbell and Shiller (1991) employ the zero coupon bond yield of McCulloch (1990) which uses the cubic spline method and covers the period December 1946 to Feburary 1987. Fama and Bliss (1987) employ CRSP data from January 1964 to December 1985. Cochrane and Piazzesi (2005) use CRSP data from 1964 to 2003.

There are some ways to produce yield curve bonds such as the Nelson-Siegel method, the Svennson method, and the cubic spline method. There are many papers which compare the methods of yield curve fitting. The figure of zero-coupon yield curves depends on the method used. However, few papers test the EH according to different zero-coupon yields curves which are constructed by several methods.

Also, there is an argument that the Treasury market has been changed after September 11, 2001. Mankiw and Miron (1986) examine the expectation theory with different periods in order to check if the rejection of the expectation hypothesis depends on the period. Bulkely, Harris, and Nawosah (2008) note that statistical evidence for the rejection of the EH has been weakened in the 1992-2004 period relative to the period of Campbell and Shiller (1991). Therefore, it is interesting to investigate with the zero-coupon yields from GovPX data whether or not the evidence against the EH has changed over time.

We contribute the literature by retesting the EH over time with different zero-coupon yields which are constructed from GovPx data. Thus it is valuable to revisit the EH by yield curve methods with the data which covers current period. In this chapter, we will test the EH with the yield curves based on the Nelson-Siegel method, the Svennson method, and the cubic spline method. Also, we try to suggest a possible explanation of the result that the evidence against the EH has been declined over time by using the overreaction hypothesis.

B. The Models of the Expectation Hypothesis

In the previous chapter, we explained how to generate yield curve from GovPx data. Here, we will introduce how to test the expectation hypothesis. There are many methods to test the EH. However, since we are interested in testing the EH with different types of zero coupon bond yield based on different methods, we hope to focus on straightforward test methods.

Campbell and Shiller (1991) test whether or not the expected change in the long-term bond yield can be predicted by the yield spread between the long-term and the short-term bond yield. We employ the notation based on Cochrane and Piazzesi (2005). Let us consider the price of an n- period bond as $P_t^{(n)}$. Then, we define the log yield of maturity n- period zero coupon bond at time t. Then, the log yield of zero coupon bond, $y_t^{(n)}$, is given as

$$y_t^{(n)} = -\frac{1}{n} \log P_t^{(n)} \tag{3.1}$$

Then, in order to test the EH, Campbell and Shiller (1991) run the following regression;

$$y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha + \beta \frac{1}{n-1}(y_t^{(n)} - y_t^{(1)}) + \epsilon_t. \tag{3.2}$$

If the EH holds then β should be equal to unity.

Fama and Bliss (1987) test the predictability of the forward rates on the expected excess return. We can define the log forward rate at time t for zero coupon bonds from time $t+n-1$ to $t+n$ as

$$f_t^{(n)} \equiv \log P_t^{(n-1)} - \log P_t^{(n)}, \tag{3.3}$$

and the log holding period return by price is defined as

$$r_{t+1}^{(n)} \equiv \log P_{t+1}^{(n-1)} - \log P_t^{(n)}. \tag{3.4}$$

Also, the excess log return is described as

$$rx_{t+1}^{(n)} \equiv r_{t+1}^{(n)} - y_t^{(1)}. \tag{3.5}$$

Then, the EH can be tested with the Fama and Bliss regression:

$$rx_{t+1}^{(n)} = \alpha + \beta(f_t^{(n)} - y_t^{(1)}) + \varepsilon_{t+1}^{(n)}. \tag{3.6}$$

If the EH holds, then β should be equal to zero.

Based on the statement that if the EH holds excess returns should not be predicted, Cochrane and Piazzesi (2005) test whether one-year excess return can be explained by five different maturity forward rates. Cochrane and Piazzesi (2005) test the predictability using the regression as

$$rx_{t+1}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} y_t^{(1)} + \beta_2^{(n)} f_t^{(2)} + \beta_3^{(n)} f_t^{(3)} + \beta_4^{(n)} f_t^{(4)} + \beta_5^{(n)} f_t^{(5)} + \epsilon_{t+1}^{(n)}. \tag{3.7}$$

Cochrane and Piazzesi (2005) also develop the following regression:

$$rx_{t+1}^{(n)} = b_n(\gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)}) + \epsilon_{t+1}^{(n)}. \tag{3.8}$$

This regression is suggested in order to characterize expected excess returns through one factor. They also suggest a two step estimation method to estimate the above regression.First, we regress the average excess return on forward rates,

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{(n)} = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)}) + \bar{\epsilon}_{t+1} \tag{3.9}$$

$$\overline{rx}_{t+1} = \gamma_1^T f_t + \bar{\epsilon}_{t+1}. \tag{3.10}$$

Secondly, we run the four regressions to estimate b_n

$$rx_{t+1}^{(n)} = b_n(\gamma^T f_t) + \epsilon_{t+1}^{(n)}, \quad n = 2, 3, 4, 5. \tag{3.11}$$

C. Data

We use the zero coupon yield curves that are derived from GovPx data with maturities from 1 to 5 years. As we mention, we use three different types of yield curves: the Nelson-Siegel Method, the Svensson method, and the cubic spline method.

At first, we hope to compare the results between the period from June 1964 to December 1979 and the period from June 1991 to December 2006 in order to know whether or not the empirical evidence against the expectations hypothesis has changed over time using CRSP data. In Appendix B, B-27 describes the summary statistics of two time series data sets in two distinct periods.

Then, we want to compare the results among three different methods in same period from June 1991 to December 2006 in order to know whether or not the empirical evidence of the expectations hypothesis changes according to the methods by which we derive the different zero-coupon yield curves. In Appendix B, Table B-28 and Table B-29shows the summary statistics of three time series data sets by different methods.

In Appendix B, Figure B-18 through Figure B-22 shows the four time series data sets, CRSP Data, the data from the Nelson-Siegel method, the data from the Svensson method, and the data from the cubic spline method in the period from June 1991 to December 2006. These figures support that the four time series data sets are very similar regardless of the yield curve method.

D. Test Results

We, here, analyze the results of the expectation hypothesis by different regression types, by different periods, and by different methods that we use for zero-coupon yield.

1. The Campbell Shiller Regression

First, we test the EH by the Campbell Shiller regression as

$$y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha + \beta \frac{1}{n-1}(y_t^{(n)} - y_t^{(1)}) + \epsilon_t \tag{3.12}$$

Under the EH, it should hold that $\beta = 1$. That is, the expected change in the long rate should be equal to the yield spread proportionally. Table 3-6 shows the results of the Campbell and Shiller regression among different periods. In case of the original period

Table 3-6.: Campbell Shiller Regression 1

$y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha + \beta\frac{1}{n-1}(y_t^{(n)} - y_t^{(1)}) + \epsilon_t$						
	Campbell Shiller		CRSP 1		CRSP 2	
Period	Jan1952-Feb1987		Jun1964-Dec1979		Jun1991-Dec2006	
	α	β	α	β	α	β
2 year		-1.0340	0.0039	-0.4823	-0.0032	-0.1390
		(0.6200)	(0.0037)	(0.6335)	(0.0067)	(1.2948)
3 year		-1.3960	0.0037	-0.7650	-0.0015	-0.6159
		(0.8830)	(0.0029)	(0.6732)	(0.0060)	(1.3920)
4 year		-1.7360	0.0039	-1.0258	-0.0008	-0.8655
		(1.0270)	(0.0023)	(0.7521)	(0.0050)	(1.3554)
5 year		-2.0220	0.0039	-1.1088	-0.0005	-0.8904
		(1.2050)	(0.0021)	(0.7816)	(0.0042)	(1.3181)

of Campbell and Shiller (1991) and CRSP 1 (Jun 1964-Dec 1979), β is significantly less than unity, its sign is negative, and the value of estimation falls monotonically with maturity. However, in the case of CRSP 2 (Jun 1991-Dec 2006), we cannot statistically reject the evidence of the EH based on the result. This result support that the evidence against the EH has changed over time.

Table 3-7 shows the result of the Campbell and Shiller regression with the data sets by different methods in the period from June 1991 to December 2006. As we show above, the results do not depend on the methods, and we cannot reject the EH statistically even though the value of β is negative.

Table 3-7.: Campbell Shiller Regression 2

$y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha + \beta \frac{1}{n-1}(y_t^{(n)} - y_t^{(1)}) + \epsilon_t$								
	CRSP		Nelson-Siegel		Svensson		Cubic Spline	
Period	Jun1991-Dec2006		Jun1991-Dec2006		Jun1991-Dec2006		Jun1991-Dec2006	
	α	β	α	β	α	β	α	β
2 year	-0.0032	-0.1390	-0.0036	-0.0372	-0.0022	-0.4219	-0.0017	-0.5124
	(0.0067)	(1.2948)	(0.0070)	(1.4118)	(0.0070)	(1.3331)	(0.0064)	(1.3292)
3 year	-0.0015	-0.6159	-0.0020	-0.4560	-0.0010	-0.7396	-0.0017	-0.7204
	(0.0060)	(1.3920)	(0.0062)	(1.4707)	(0.0061)	(1.4150)	(0.0060)	(1.3407)
4 year	-0.0008	-0.8655	-0.0013	-0.6494	-0.0007	-0.8634	-0.0011	-0.6369
	(0.0050)	(1.3554)	(0.0053)	(1.4494)	(0.0052)	(1.4130)	(0.0049)	(1.3501)
5 year	-0.0005	-0.8904	-0.0011	-0.7406	-0.0006	-0.9006	-0.0009	-0.6545
	(0.0042)	(1.3181)	(0.0046)	(1.4125)	(0.0044)	(1.3760)	(0.0044)	(1.4449)

2. The Fama and Bliss Regression

We test the EH by Fama and Bliss regression as

$$rx_{t+1}^{(n)} = \alpha + \beta(f_t^{(n)} - y_t^{(1)}) + \varepsilon_{t+1}^{(n)}. \tag{3.13}$$

If the EH holds then β should be equal to zero. Table 3-8 summarizes the results of the regression. In case of the original period of Fama and Bliss (1987) and CRSP 1 (Jun 1964-Dec 1979), since β is significantly greater than zero with respect to statistics, the EH is strongly rejected. However, in the period of CRSP 2 (Jun 1991-Dec 2006), we cannot strongly reject the evidence of the EH based on the result in the sense of statistics. This result also support that the evidence against the EH has changed over time.

Table 3-9 shows the results of the Fama and Bliss regression in the period from

Table 3-8.: Fama-Bliss Regression 1

$r_{t+1}^{(n)} - y_t^{(1)} = \alpha + \beta(f_t^{(n)} - y_t^{(1)}) + \epsilon_{t+1}$						
	Fama-Bliss		CRSP 1		CRSP 2	
Period	Jan1964-Dec1984		Jun1964-Dec1979		Jun1991-Dec2006	
	α	β	α	β	α	β
2 year	-0.2100	0.9100	-0.3882	0.7411	0.3168	0.5695
	(0.4100)	(0.2800)	(0.3730)	(0.3168)	(0.6721)	(0.6474)
3 year	-0.5100	1.1300	-0.8287	1.0437	0.3280	0.8369
	(0.6800)	(0.3700)	(0.5958)	(0.3837)	(1.1529)	(0.7227)
4 year	-0.9100	1.4200	-1.2351	1.2743	0.5227	0.8638
	(0.9200)	(0.4500)	(0.6766)	(0.4944)	(1.4509)	(0.7442)
5 year	-1.0600	0.9300	-1.4060	0.9261	0.8370	0.8198
	(1.3100)	(0.5300)	(0.9377)	(0.5098)	(1.5124)	(0.7516)

Table 3-9.: Fama-Bliss Regression 2

$r^{(n)}_{t+1} - y^{(1)}_t = \alpha + \beta(f^{(n)}_t - y^{(1)}_t) + \epsilon_{t+1}$								
	CRSP		Nelson-Siegel		Svensson		Cubic Spline	
Period	Jun1991-Dec2006		Jun1991-Dec2006		Jun1991-Dec2006		Jun1991-Dec2006	
	α	β	α	β	α	β	α	β
2 year	0.3168	0.5695	0.3235	0.7630	0.2163	0.7109	0.1696	0.7562
	(0.6721)	(0.6474)	(0.6874)	(0.3898)	(0.6962)	(0.6666)	(0.6412)	(0.6646)
3 year	0.3280	0.8369	0.2698	0.8981	0.3464	0.8150	0.5210	0.7167
	(1.1529)	(0.7227)	(1.0050)	(1.2065)	(1.1764)	(0.7502)	(1.1062)	(0.6083)
4 year	0.5227	0.8638	0.6952	0.7483	0.5981	0.8086	0.5728	0.8385
	(1.4509)	(0.7442)	(1.4943)	(0.7872)	(1.4749)	(0.7808)	(1.4179)	(0.8193)
5 year	0.8370	0.8198	0.9296	0.7695	0.8686	0.7990	0.7047	0.8959
	(1.5124)	(0.7516)	(1.7012)	(0.8060)	(1.6614)	(0.7824)	(1.6794)	(0.9470)

June 1991 to December 2006. The results of our yield data by the three methods are not different from CRSP, and we cannot strongly reject the EH based on the statistical estimation results.

3. The Cochrane and Piazzesi Regression

Cochrane and Piazzesi (2005) test the predictability of five forward rates in many ways. They test the predictability using the regression as

$$rx^{(n)}_{t+1} = \beta^{(n)}_0 + \beta^{(n)}_1 y^{(1)}_t + \beta^{(n)}_2 f^{(2)}_t + \beta^{(n)}_3 f^{(3)}_t + \beta^{(n)}_4 f^{(4)}_t + \beta^{(n)}_5 f^{(5)}_t + \epsilon^{(n)}_{t+1}. \tag{3.14}$$

Figure B-23 through Figure B-32 support the argument that the tent-shaped coefficients of Cochrane and Piazzesi (2005) depend on the period of as well as the method of generating zero coupon yield.

Also, they try to characterize expected excess returns through one factor as they suggest.

$$rx_{t+1}^{(n)} = b_n(\gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)}) + \epsilon_{t+1}^{(n)} \tag{3.15}$$

Therefore, we follow a two step approach. First, we estimate the γ as follows:

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{(n)} = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \gamma_3 f_t^{(3)} + \gamma_4 f_t^{(4)} + \gamma_5 f_t^{(5)}) + \bar{\epsilon}_{t+1} \tag{3.16}$$

Second,we run the four regressions to estimate b_n

$$rx_{t+1}^{(n)} = b_n(\gamma^T f_t) + \epsilon_{t+1}^{(n)}, \quad n = 2, 3, 4, 5. \tag{3.17}$$

Table 3-10 shows the results of the first step regression. As we see, in case of the original period of Cochrane and Piazzesi (2005) and CRSP 1 (Jun 1964-Dec 1979), some parameters are statistically strongly significant, and others are not significantly. In the period of CRSP 2 (Jun 1991-Dec 2006), some parameter results are statistically significant, and others are not significant. The evidence against the EH in the period of CRSP 1 (Jun 1964-Dec 1979) is stronger than in the period of CRSP 2 (Jun 1991-Dec 2006).

Table 3-11 summarizes the results of the first step regression in the period from June 1991 to December 2006. The results of our yield data by the three methods are not different from CRSP some parameter results are statistically significant, and others are not significant.

Table 3-12 shows the results of the second step regression. As we see, all coefficients are strongly significant regardless of the periods.

Table 3-10.: Cochrane-Piazessi Regression 1-1

$\overline{rx}_{t+1} = \gamma^T f_t + \bar{\epsilon}_{t+1}$			
	Cochrane-Piazessi	CRSP 1	CRSP 2
Period	Jan 1964-Dec 2003	Jun1964-Dec1979	Jun1991-Dec2006
γ_0	-3.2438	-5.2413	-6.4299
	(1.4546)	(2.2978)	(2.5584)
γ_1	-2.1353	-1.3840	-1.0571
	(0.3610)	(0.4264)	(1.2579)
γ_2	0.8083	0.7182	-1.9163
	(0.7351)	(1.0927)	(1.3832)
γ_3	3.0006	2.5316	6.1347
	(0.5016)	(0.7147)	(2.1339)
γ_4	0.8013	0.1905	-1.3530
	(0.4531)	(0.7809)	(1.4573)
γ_5	-2.0757	-1.3780	-0.6264
	(0.3358)	(0.6838)	(1.7116)

Table 3-11.: Cochrane-Piazessi Regression 1-2

	$\overline{rx}_{t+1} = \gamma^T f_t + \bar{\epsilon}_{t+1}$			
	CRSP	Nelson-Siegel	Svensson	Cubic Spline
Period	Jun1991-Dec2006	Jun1991-Dec2006	Jun1991-Dec2006	Jun1991-Dec2006
γ_0	-6.4299	-4.7881	-4.8956	-11.0540
	(2.5584)	(2.3745)	(3.4582)	(1.5316)
γ_1	-1.0571	1.7858	-0.2326	1.7950
	(1.2579)	(1.5589)	(1.8167)	(1.6475)
γ_2	-1.9163	-1.7214	-3.5076	-3.1826
	(1.3832)	(3.3533)	(5.6844)	(1.7418)
γ_3	6.1347	-6.2653	6.7790	6.4112
	(2.1339)	(2.9502)	(13.6710)	(4.9417)
γ_4	-1.3530	20.4080	-0.9048	-9.9142
	(1.4573)	(10.4630)	(19.1350)	(10.8940)
γ_5	-0.6264	-13.2400	-1.1503	7.1190
	(1.7116)	(6.9898)	(9.6473)	(5.9084)

Table 3-12.: Cochrane-Piazessi Regression 2-1

$rx_{t+1}^{(n)} = b_n(\gamma^T f_t) + \epsilon_{t+1}^{(n)}$			
	Cochrane-Piazessi	CRSP 1	CRSP 2
Period	Jan 1964-Dec 2003	Jun1964-Dec1979	Jun1991-Dec2006
	b_n	b_n	b_n
2 year	0.4652	0.4880	0.4492
	(0.0317)	(0.0502)	(0.0274)
3 year	0.8664	0.8459	0.8670
	(0.0217)	(0.0271)	(0.0284)
4 year	1.2353	1.1993	1.2283
	(0.0146)	(0.0317)	(0.0138)
5 year	1.4331	1.4668	1.4555
	(0.0403)	(0.0587)	(0.0435)

Table 3-13.: Cochrane-Piazessi Regression 2-2

	$rx_{t+1}^{(n)} = b_n(\gamma^T f_t) + \epsilon_{t+1}^{(n)}$			
	CRSP	Nelson-Siegel	Svensson	Cubic Spline
Period	Jun1991-Dec2006	Jun1991-Dec2006	Jun1991-Dec2006	Jun1991-Dec2006
	b_n	b_n	b_n	b_n
2 year	0.4492	0.6568	0.4493	0.4568
	(0.0274)	(0.0294)	(0.0343)	(0.0360)
3 year	0.8670	0.6761	0.8560	0.9356
	(0.0284)	(0.0308)	(0.0257)	(0.0352)
4 year	1.2283	1.1934	1.2006	1.1867
	(0.0138)	(0.0105)	(0.0107)	(0.0126)
5 year	1.4555	1.4737	1.4942	1.4208
	(0.0435)	(0.0311)	(0.0507)	(0.0373)

Table 3-13 shows the results of the second step regression. As we see, all coefficients are strongly significant regardless of methods. Also, we try to draw the regression coefficients of 1 year excess returns on forward rates at time t. The figures are well known as the tent-shaped regression coefficient. In the period from June 1964 to December 1979, the shape of coefficient value from CRSP data is not different from that of Cochrane and Piazzesi (2005). However, in the period from June 1964 to December 1979, the shapes of the coefficient value are different from the tent shape from our data regardless of the methods.

E. The Possible Explanation against the Evidence of the EH

We have shown that although the EH can be rejected significantly in the period June 1964 to December 1979, the EH can not be rejected significantly in the period June 1991 to December 2006. We, here, try to explain the situation the result that the statistical rejection of the EH has weakened over time. There are many papers to try to account for the rejection of the EH.

One issue to consider is a time-varying risk premium. When we test the EH, we assume that the risk premium is constant. However, if there exists time-varying risk premium, tests of the EH are biased downward from the theoretical value. Numerous researchers have studied this possibility. However, the results depends on the model specification for the risk premium, and Duffee (2002) argue that the time-varying risk premium may not be able to account for the scale of the rejection.

The other issue to consider is the overreaction hypothesis which is suggested by Campbell and Shiller (1991). According to the overreaction hypothesis, long rates overreact to the expectation of changes in future short rates. Thornton (2006) shows the following example. Let us assume that some event happens and it makes the future short-term rate increase. According to the EH, the long-term rate would change. However, the long-term rate overreacts. Then, the long-term rate falls over time. Hardouvelis (1994) argue that the overreaction hypothesis is the more likely explanation. We adopt the overreaction hypotheses in order to analyze our results which are summarized above.

1. The Measure of Overreaction

Campbell and Shiller (1991) introduce how to measure the overreaction of spread. We adopt their measure of overreaction of spread. The reader is referred to Campbell and Shiller (1991) for the detail. We can define the actual spread between the long rate, the n-period rate, and the short rate, the m-period rate, as

$$S_t^{(n,m)} = y_t^{(n)} - y_t^{(m)}. \tag{3.18}$$

Based on the EH, since long-term bond yields are the average of future expected short-term bond yields, we can specify the theoretical spread as

$$S_t^{(n,m)^*} \equiv y_t^{(n)^*} - y_t^{(m)} = \sum_{i=1}^{h-1}(1-\frac{i}{h})\Delta^m y_{t+im}^{(m)}. \tag{3.19}$$

If we assume that the model is

$$S_t^{(n,m)} = \kappa E_t S_t^{(n,m)^*} + c, \tag{3.20}$$

then the coefficient κ can be considered to be the degree of overreaction of spread. κ is greater than one under the assumption of the overreaction of spread. Also, if we run the regression of $S_t^{(n,m)^*}$ on $S_t^{(n,m)}$, then the coefficient will be $1/\kappa$. Therefore, we can check whether or not κ may change over time using the regression of $S_t^{(n,m)^*}$ on $S_t^{(n,m)}$.

We, here, run the above regression for two periods, June 1964 to December 1979 and June 1991 to December 2006. Table 3-14 summarizes the regression results.

We use the Chow test in order to check whether or not the coefficients for two periods are equal. Table 3-15 shows the results of Chow test. Based on the results of Table 10, we can reject at the 5% significance level the null hypothesis that the coefficients for two periods are equal. Also, since the coefficient can be interpreted as $1/\kappa$ we can compute the value of κ

Table 3-16 shows the results of the degree of overreaction. If the long-term periods are less than 3 years, the degrees of overreaction κ for every cases is greater one. Also, for almost every cases, κ is decreases over time if we compare the results for two periods. These results support that the statistical evidence against the EH has weakened. If κ is greater than one, it means that the long-term rate overreacts

Table 3-14.: Slope Coefficient on Regression of $S_t^{(n,m)^*}$ *on* $S_t^{(n,m)}$

	m							
n	1Month		3 Month		6Month		1Year	
	Period1	Period2	Period1	Period2	Period1	Period2	Period1	Period2
3Month	0.4001	0.5061						
	(0.0601)	(0.0665)						
6Month	0.4856	0.7524	0.3064	0.7825				
	(0.1622)	(0.0627)	(0.1904)	(0.1617)				
1 Year	0.5140	0.8310	0.3469	0.9529	0.0659	0.6206		
	(0.2148)	(0.1269)	(0.2654)	(0.3178)	(0.1702)	(0.4556)		
2 Year	0.5729	0.7851	0.6147	0.8324	0.6374	0.6969	0.2871	0.4305
	(0.3068)	(0.2507)	(0.3618)	(0.4813)	(0.3450)	(0.6117)	(0.3229)	(0.6474)
3 Year	0.7836	0.8305	0.9449	0.8843	1.0376	0.9543	0.8340	0.8783
	(0.2810)	(0.2906)	(0.2988)	(0.4747)	(0.2786)	(0.5579)	(0.3106)	(0.6058)
4 Year	0.8392	0.9698	1.0215	1.0352	1.1225	1.0632	1.0368	1.1272
	(0.2729)	(0.2573)	(0.2505)	(0.3760)	(0.2064)	(0.4284)	(0.2088)	(0.4614)
5 Year	0.9207	1.0291	1.0623	1.1071	1.1303	1.1388	1.0499	1.1767
	(0.2133)	(0.2056)	(0.1629)	(0.2872)	(0.1234)	(0.3164)	(0.1655)	(0.3433)

Table 3-15.: The Results of Chow Test

	m			
n	1Month	3 Month	6Month	1Year
	F(2,370)	F(2,370)	F(2,370)	F(2,380)
3Month	0.7967			
6Month	4.8052	5.3841		
1 Year	3.886	6.9302	5.9098	
2 Year	6.5934	8.7323	11.3101	7.5044
3 Year	15.3280	19.9924	28.3984	21.6497
4 Year	45.5869	55.9331	66.7247	56.6839
5 Year	101.5286	126.5526	155.1366	126.1031

and this overreaction can be interpreted as one of a pricing anomaly.

F. Conclusion

In this chapter, we retest the EH with many different regressions (the Campbell and Shiller regression, the Fama and Bliss regression, and the Cochrane and Piazessi regression) from different periods (June 1964 to December 1979 and June 1991 to December 2006), and by different data sets (the Nelson-Siegel method, the Svensson method, and the cubic spline method). As we see above, the results of the EH are different from before.

We argue that the evidence against the EH has weakened over time. Based on our results, in many cases, the EH can be statistically rejected in the period from June 1964 to December 1979, but the EH cannot be rejected in the period from June 1991

Table 3-16.: The Degree of Overreaction

	m							
n	1Month		3 Month		6Month		1Year	
	κ		κ		κ		κ	
	Period1	Period2	Period1	Period2	Period1	Period2	Period1	Period2
3Month	2.4993	1.9761						
6Month	2.0593	1.3291	3.2633	1.2779				
1 Year	1.9456	1.2034	2.8829	1.0495	15.1713	1.6114		
2 Year	1.7455	1.2737	1.6269	1.2014	1.5690	1.4349	3.4832	2.3229
3 Year	1.2762	1.2041	1.0583	1.1309	0.9638	1.0479	1.1990	1.1385
4 Year	1.1917	1.0311	0.9790	0.9660	0.8909	0.9405	0.9645	0.8872
5 Year	1.0862	0.9717	0.9413	0.9033	0.8847	0.8781	0.9524	0.8499

to December 2006. Although we try to test the EH using data sets by result of the Nelson-Siegel method, the Svensson method and the cubic spline method, the results are not different. Therefore, the yield curve fitting method does not affect the result of the test of the EH. Also, we can support the statement that the statistical evidence against the EH has weakened over time when we use the overreaction hypothesis in order to analyze our results.

CHAPTER IV

A NEW ESTIMATION METHOD FOR THE STOCHASTIC VOLATILITY MODEL OF SHORT-TERM INTEREST RATES

A. Introduction

Short-term interest rates are one of the most fundamental assets in the financial market. If we consider the one-factor affine term structure model, all interest rates can be characterized by an instantaneous interest rate which will be replaced with short-term interest rate. Therefore, the short-term interest rates are crucially important in financial market analysis. Many papers are developed in continuous time and assume that short-term interest rates follow a diffusion process:

$$dr_t = \mu_t dt + \sigma_t dW_t \tag{4.1}$$

where μ_t is the drift term which represents instantaneous mean and σ_t is the diffusion term which represents instantaneous variance.

Chan, Karoly, Longstaff, and Sanders (1992) (CKLS) suggest the following specific model:

$$dr_t = (\alpha + \beta r_{t-1})dt + \psi r_{t-1}^{\gamma} dW_t \tag{4.2}$$

where r_t is the short-term interest rate, and W_t is a standard Brownian motion. In this model, the drift function is characterized by a linear drift, and the diffusion function is characterized as the volatility of interest rate, which depends on the interest rate level. Many papers extend this CKLS model for the short-term interest rate. Cox, Ingersoll, and Ross (1985) (CIR) specify a square root model, $\gamma = 1/2$.

However, some papers argue that this CKLS model is inappropriate. For example, Ait-Sahalia (1996) provides strong evidence of a nonlinear drift function with deterministic volatility case.

Also, Ball and Torous (1999) assert that the volatility itself is stochastic. Ball and Torous (1999), Smith (2002), and Sun (2005) incorporate stochastic volatility into short-term interest rate models. It is valuable to review the existing volatility model of short-term interest rates before we introduce new estimation method for stochastic volatility of short-term interest rates.

Many papers use a discrete time approximation to analyze the volatility model of short-term interest rates. First of all, we can consider the Euler discrete time approximation with $\Delta = 1$ for the CKLS model as

$$r_t - r_{t-1} = (\alpha + \beta r_{t-1}) + \epsilon_t \tag{4.3}$$

$$\mathbb{E}(\varepsilon_t|\mathcal{F}_{t-1}) = 0,\ \mathbb{E}(\varepsilon_t^2|\mathcal{F}_{t-1}) \equiv \sigma_t^2 = \psi^2 r_{t-1}^{2\gamma}. \tag{4.4}$$

As we mentioned before, the volatility of the interest rate depends on the interest rate level in the CKLS model. Brenner, Harjes, and Kroner (1996) introduce the LEVELS-GARCH model as an extension of the CKLS given by

$$\mathbb{E}(\varepsilon_t^2|\mathcal{F}_{t-1}) \equiv \psi_t^2 r_t^{2\gamma} \tag{4.5}$$

$$\psi_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + b\psi_{t-1}^2. \tag{4.6}$$

In the above model, the volatility of the interest rate relies on previous volatility as well as the rate level. Ball and Torous (1999) incorporate stochastic interest rate volatility into the CKLS;

$$r_t - r_{t-1} = (\alpha + \beta r_{t-1}) + \sigma_t r_{t-1}^{\gamma}\epsilon_t \tag{4.7}$$

$$\ln\sigma_t^2 = \omega + \phi\ln\sigma_{t-1}^2 + \eta_t. \tag{4.8}$$

Ball and Torous (1999) assume that the volatility depends on the interest rate level, and that it is stochastic. This model is parsimonious and successfully flexible for the short-term interest rate model. In this paper, we focus on a two stage estimation method for this kind of stochastic volatility model. Andersen and Lund (1997) also extend the CKLS model like Ball and Torous (1999). Many papers use two stage estimation methods including Ball and Torous (1999), Smith (2002) and Sun (2005). Especially, if stochastic volatility is more specified as in the regime switch model and the logistic function volatility, the two stage estimation method is uniquely developed method for interest rate model.

Many papers based on two stage estimation method use ordinary least square (OLS) to estimate the parameters of drift function in interest process. At first we can apply OLS because OLS is consistent. If we apply OLS and we define realized residual, e_t, as $e_t = (r_t - r_{t-1}) - \alpha - \beta r_{t-1}$, then we can consider the estimation method of state space models. After taking the log of the squared residual, we can obtain

$$\ln e_t^2 = \ln \sigma_t^2 + 2\gamma \log r_{t-1} + \ln \epsilon_t^2 \tag{4.9}$$

since $e_t = \sigma_t r_{t-1}^{\gamma} \epsilon_t$. If we introduce new notation as $y = \ln e_t^2$, y is observable given the observed interest rate and parameter values. Also, $x = \ln \sigma_t^2$ can be interpreted as a state variable. Then, we can express the equations in the state-space form as

$$y_t = x_t + 2\gamma \log r_{t-1} + \ln \epsilon_t^2 \tag{4.10}$$

$$x_t = \omega + \phi x_{t-1} + \eta_t \tag{4.11}$$

.

Since $\ln \epsilon_t^2$ is log-chi-squared random variable, we cannot use the Kalman filter method. There are different methods to attack this problem. Ball and Torous (1999) use a non-Gaussian estimation method. Smith (2002) and Sun (2005) use quasi-maximum likelihood (QML) which is suggested by Harvey, Ruiz, and Shephard (1994). Let us examine the quasi-maximum likelihood in detail. After taking the log of the squared residual like Ball and Torous (1999), we can obtain

$$\ln e_t^2 = \ln \sigma_t^2 + 2\gamma \log r_{t-1} + \ln \epsilon_t^2. \tag{4.12}$$

We need to modify the system equation in order to apply the quasi-maximum likelihood. The mean of $\ln \epsilon_t^2$ is $\mathbb{E}(\ln \epsilon_t^2) = -1.2704$ and the variance of $\ln \epsilon_t^2$ is $Var(\ln \epsilon_t^2) = \frac{\pi^2}{2}$.

The quasi-maximum likelihood (QML) uses the likelihood function of a normal random variable as if $\ln \epsilon_t^2$ is a normal random variable with the mean, $\mathbb{E}(\ln \epsilon_t^2) =$

-1.2704, and the variance,$Var(\ln \epsilon_t^2) = \frac{\pi^2}{2}$. Then, the system equations are as follows:

$$y_t = x_t + 2\gamma \log r_{t-1} - 1.2704 + \xi_t \tag{4.13}$$

$$x_t = \omega + \phi x_{t-1} + \eta_t. \tag{4.14}$$

Smith (2002) suggests a Markov-switching model of short-term interest rates as

$$r_t - r_{t-1} = (\alpha + \beta r_{t-1}) + \sigma_i r_{t-1}^{\gamma} \epsilon_t \tag{4.15}$$

$$y_t = \omega_i + 2\gamma \log r_{t-1} + \ln \epsilon_t^2 \tag{4.16}$$

where $y = \ln e_t^2$ is observable given interest rate and parameter values as previously and $\omega_i = \ln \sigma_i$. Also Smith (2002) introduces a Markov-switching stochastic volatility model:

$$y_t = x_t + 2\gamma \log r_{t-1} + \ln \epsilon_t^2 \tag{4.17}$$

$$x_t = \omega_i + \phi x_{t-1} + \eta_t \tag{4.18}$$

Kalimipalli and Susmel (2004) makes use of the Monte Carlo Markov Chain (MCMC). However, there are some issues in previous methods which have been developed. At first, many stochastic volatility models used ordinary least squares to estimate the drift term of interest rate. Since the stochastic process of interest rates is close to a unit root which is a non-stationary process, the least square has the upward-biased problem for the speed of mean reversion. As a result, many papers report the speed of excessive mean reversion. Also, many papers use QML which has the advantages of computation work and adaptability to many cases. However, as Jacquier, Polson, and Rossi (1994) mention, the performance of QML depends on the parameter value. Therefore, we should be careful when using QML. If we use Bayesian analysis of stochastic volatility, the estimation will be computationally burdensome.

Park (2008) introduces a Martingale method to estimate the drift term without any specific assumption about diffusion term. If we use Martingale method, the upward bias problem will be mitigated. Therefore, we can adopt the Martingale method to estimate the drift term for the stochastic volatility model. Also, Tanizaki (1996) introduces nonlinear filters which include density-based filtering for the state space model. However, there is no paper which uses the density-based filtering for the short-term interest stochastic volatility model. Therefore, it is valuable to adopt the density-based filtering and study the empirical application for the stochastic volatility model of short-term interest rates.

Therefore, we introduce a new estimation method for the stochastic volatility model which uses Martingale estimation and the density-based filtering, we compare our new method with the existing method, and we apply our method to estimate the stochastic volatility model of short-term interest rates with the three-month interest rates which are constructed from GovPX data.

B. Model

We, in this chapter, assume a stochastic volatility model which Ball and Torous (1999) use.

$$dr_t = (\alpha + \beta r_t)dt + \sigma_t r_t^{\gamma} dW_{1,t} \tag{4.19}$$

$$d\log\sigma_t^2 = \kappa_2(\mu_2 - \log\sigma_t^2)dt + \xi dW_{2,t}. \tag{4.20}$$

As many papers use Euler approximation, we apply it as

$$r_{t+\Delta} - r_t = (\alpha + \beta r_t)\Delta + \sigma_t r_t^{\gamma} \sqrt{\Delta}\varepsilon_{1,t} \tag{4.21}$$

$$\log \sigma_{t+\Delta}^2 - \log \sigma_t^2 = \kappa_2(\mu_2 - \log \sigma_t^2)\Delta + \xi\sqrt{\Delta}\varepsilon_{2,t} \tag{4.22}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are independent. We adopt a two stage estimation method as Ball and Torous (1999), Smith (2005), and Sun(2005). We use Martingale estimation to estimate parameters for the drift term of the interes rate instead of ordinary least squares and apply density-based filtering to estimate stochastic volatility.

1. Martingale Method

Park (2008) introduces the Martingale method for the statistical inference in a conditional mean model given in continuous time. We can use the Martingale method to estimate the parameters of the drift term of the short-term interest rate without a specific assumption of functional form for the diffusion term. The idea of the Martingale Method is clear. The Dambis-Dubins-Schwartz Theorem (DDS Theorem) shows that any Martingale will be a time changed Brownian Motion with the time change derived by using of quadratic variation process. After the time-changed martingale, the increments follow standard normal distribution by DDS Theorem. The Martingale estimator (MGE) will minimize the distributional distance between the empirical distribution from the real data and the standard normal distribution. The Martingale method shows that the upward bias problem for mean reversion speed is much less than that of OLS with realistic parameter values.

We briefly summarize the Martingale method for our model. See Park (2008) for more detail. For our analysis, we define $U_t(\theta) = (r_t - r_0) - \int_0^t (\alpha + \beta r_s) ds$. Then, $dU_t(\theta)$ is considered to be an error after we handle the conditional mean process. We also define time change using the quadratic variation as

$$T_t = \inf_{s>0}\{\langle U\rangle_S > t\} \tag{4.23}$$

where $(\langle U\rangle_t)$ is the quadratic variation of (U_t). The quadratic variation is defined by

$$\langle U\rangle_t = p\lim_{\pi_n\to 0}\sum_{i=1}^{n}(U_{t_i} - U_{t_i})^2 \tag{4.24}$$

where $0 \equiv t_0 < ... < t_n \equiv t$ and $\pi_n = \max_{1\le i\le n}|t_i - t_{i-1}|$ By the DDS Theorem, the increments follow standard normal distribution after the time change,

$$Z_i(\theta) = \Delta^{-1/2}(r_{T_i} - r_{T_{i-1}} - \int_{T_{i-1}}^{T_i}(\alpha + \beta r_t)dt) \tag{4.25}$$

where Z_i is i.i.d and (T_i) is defined using the quadratic variation as before. Finally, the Martingale estimator (MGE) will minimize the Cramer-von-Mises (CvM) distance between the empirical distribution and the standard normal distribution as

$$\widehat{\theta} = \arg\min_{\theta\in\Theta}\int_{-\infty}^{\infty}[\Phi_N(r,\theta) - \Phi_N(r,\theta)]^2\varphi(r)dr. \tag{4.26}$$

If we use the Martingale method for our model, we can estimate the parameters,α and β. We can obtain the residual after the Martingale method as

$$y_t = \sigma_t r_t^{\gamma}\sqrt{\Delta}\epsilon_{1,t} \tag{4.27}$$

$$\log\sigma_{t+\Delta}^2 - \log\sigma_t^2 = \kappa_2(\mu_2 - \log\sigma_t^2)\Delta + \xi\sqrt{\Delta}\varepsilon_{2,t}. \tag{4.28}$$

This model is a state space model of nonlinear filter. Ball and Torous (1999), Smith (2002), and Sun (2005) take squared the log-linearization of the observation equation. Lee (2008) shows that the above stochastic volatility model can be transformed as the exponential form of the volatility function. We follow the transformation which Lee (2008) suggests in order to check whether or not the volatility factor is unit-root or near-unit root.Let us define a transformed latent factor x_t as

$$x_t = [\frac{\log \sigma_t^2}{\xi\sqrt{\Delta}} - \frac{\mu_2}{\xi\sqrt{\Delta}}]. \tag{4.29}$$

Then, our state space model is transformed as

$$y_t = \sqrt{\exp(\mu_2)\exp(\xi\sqrt{\Delta}x_t)} r_t^{\gamma} \Delta\varepsilon_{1,t} \tag{4.30}$$

$$x_{t+1} = (1-\kappa_2\Delta)x_t + \varepsilon_{2,t}. \tag{4.31}$$

Then, for estimation, we can write the observation equation and the transition equation as

$$y_t = \sqrt{\nu\exp(\lambda x_t)} r_t^{\gamma} \Delta\varepsilon_{1,t} \tag{4.32}$$

$$x_{t+1} = \alpha x_t + \varepsilon_{2,t}. \tag{4.33}$$

where $\nu = \exp(\mu_2)$, $\lambda = \xi\sqrt{\Delta}$, and $\alpha = (1-\kappa_2\Delta)$. From the transformed state space model, we will estimate the parameters, ν, λ, γ, and α. Then, we can get estimate values for original state space model's parameters, μ_2, ξ, κ_2, and γ. This is a nonlinear state space model. Therefore, we cannot apply the Kalman filter. There are several methods for a nonlinear state space model. The density-based nonlinear filtering is straightforward. Generally speaking, the conditional density cannot be obtained. Tanizaki (1996) suggests some methods to solve the computational burden. Among many methods, we apply the numerical integration because it is intuitive and straightforward.

We briefly summarize the density-based filtering for our model. See Lee (2008) for more detail. Since $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$are independent, our algorithm for the density-based filter is much simpler. For the prediction step, we utilize the relationship

$$
\begin{aligned}
p(x_t|\mathcal{F}_{t-1}) &= \int p(x_t|x_{t-1}, y_{t-1})p(x_{t-1}|\mathcal{F}_{t-1})dx_{t-1} \\
&\simeq \int_{-c+x_{t-1|t-2}}^{c+x_{t-1|t-2}} p(x_t|x_{t-1}, y_{t-1})p(x_{t-1}|\mathcal{F}_{t-1})dx_{t-1} \\
&= \int_{-c}^{c} p(x_t|z + x_{t-1|t-2}, y_{t-1})p(z + z + x_{t-1|t-2}|\mathcal{F}_{t-1})dz \\
&= \frac{h}{\sqrt{2\pi}} \sum_{j=1}^{m} \exp(-\frac{[z_j + x_{t-1|t-2} - (\alpha(z_j + x_{t-1|t-2})]^2}{2} \\
&\quad p(z + z + x_{t-1|t-2}|\mathcal{F}_{t-1})
\end{aligned}
$$

where $x_{t-1} = z + x_{t-1|t-2} and z = [-c, -c + h, ..., c - h]$. Therefore, in the numerical integration, $[-c, c]$ is the interval and h is the length of a partition. Let us analyze the updating step. Since $p(y_t|x_t)$ is given as a normal density function,we can express the likelihood function as

$$
\begin{aligned}
p(y_t|\mathcal{F}_{t-1}) &= \int p(y_t|x_t)p(x_t|\mathcal{F}_{t-1},\theta)dx_t \\
&\simeq \int_{-c+x_{t-1|t-2}}^{c+x_{t-1|t-2}} p(y_t|x_t)p(x_t|\mathcal{F}_{t-1},\theta)dx_t \\
&= \int_{-c}^{c} p(y_t|z+x_{t|t-1})p(z+x_{t|t-1}|\mathcal{F}_{t-1})dz \\
&= h\sum_{j=1}^{m} \frac{1}{\sqrt{2\pi\exp(\mu_2)\exp(\xi\sqrt{\Delta}x_{t|t-1})r_{t-1}^{2\gamma}\Delta}} \\
&\quad \exp(-\frac{y_t^2}{2\pi\exp(\mu_2)\exp(\xi\sqrt{\Delta}x_{t|t-1})r_{t-1}^{2\gamma}\Delta})p(z_j+x_{t|t-1}|\mathcal{F}_{t-1}).
\end{aligned}
$$

We can write the updating step by using the prediction step and the loglikelihood function as

$$
\begin{aligned}
p(y_t|\mathcal{F}_{t-1}) &= \int p(y_t|x_t)p(x_t|\mathcal{F}_{t-1})dx_t \\
&\simeq \frac{1}{\sqrt{2\pi\exp(\mu_2)\exp(\xi\sqrt{\Delta}x_{t|t-1})r_{t-1}^{2\gamma}\Delta}} \\
&\quad \exp(-\frac{y_t^2}{2\pi\exp(\mu_2)\exp(\xi\sqrt{\Delta}x_{t|t-1})r_{t-1}^{2\gamma}\Delta})\frac{p(z_j+x_{t|t-1}|\mathcal{F}_{t-1})}{p(y_t|\mathcal{F}_{t-1})}
\end{aligned}
$$

With the prediction step and the updating step, we can set the log likelihood function to be

$$
\widehat{\theta} = agr\max_{\theta\in\Theta_0}\sum_{t=1}^{n}\log p(y_t|\mathcal{F}_{t-1},\theta) \tag{4.34}
$$

As Lee (2008) emphasizes, choosing c and h is the most important issue. Larger c and smaller h will give us better result. However, it takes a long time to compute the the numerical integration. In our case, after we plot the density of the smoothed latent factors to check the value of the latent factors again and again, we choose c and h.

C. Simulation Study

We perform the Monte-Carlo simulation study. Our model is given as Ball and Torous (1999) and Andersen and Lund (1997) given by

$$
\begin{aligned}
dr_t &= (\alpha + \beta r_t)dt + \sigma_t r_t^{\gamma} dW_{1,t} && (4.35)\\
d\log\sigma_t^2 &= \kappa_2(\mu_2 - \log\sigma_t^2)dt + \xi dW_{2,t}. && (4.36)
\end{aligned}
$$

We generate the simulation data by the Euler approximation as many stochastic volatility models:

$$
\begin{aligned}
r_{t+\Delta} - r_t &= (\alpha + \beta r_t)\Delta + \sigma_t r_t^{\gamma}\sqrt{\Delta}\epsilon_{1,t} && (4.37)\\
\log\sigma_{t+\Delta}^2 - \log\sigma_t^2 &= \kappa_2(\mu_2 - \log\sigma_t^2)\Delta + \xi\sqrt{\Delta}\varepsilon_{2,t}. && (4.38)
\end{aligned}
$$

At first we generate the data with 5 minute frequency. Then, we select the generated data with weekly frequency, $\Delta = 1/52$ for fifty years because many papers use weekly frequency data to do simulations and estimations. The value of the parameters are followed from Andersen and Lund (1997) as

$$
\alpha = 0.96 \; \beta = -0.16 \; \mu_2 = -0.28 \; \kappa_2 = 1.04 \; \gamma = 0.54 \; \xi = 1.27 \qquad (4.39)
$$

We run each iteration 1000 times iteration with different random variables. We use Matlab, and employ the fminsearch optimization procedure. We also set the initial value as the true parameter value for numerical optimization. First, we estimate the parameters using ordinary least square (OLS) for the drift function in short-term interest rate and Kalman filter (KF) to estimate the rest of parameters. Second, we estimate the parameters for the drift function using Martingale estimator (MGE) and density-based filtering (DBF).

For the Martingale estimator (MGE), the number of sample after time change (TCN) is important. We follow Park (2008) and choose TCN=50. For density-based filtering, the values of c and h are critical for the simulation study. We choose c=20 and h=0.5 based on the density of smoothed latent factor. Then, we compare the performances between these two methods.

Tables 4-17 through Table 4-20 shows the summary of statistics of results for the OLS-Kalman filter, the MGE-Kalman filter, the OLS-DBF, and the MGE-DBF. Based on Table 1 and Table 2, we argue that OLS is biased for parameters α and β and MGE is better than OLS to estimate α and β. However, MGE does not affect the estimation performance of stochastic volatility part. Based on Table 4-17 through Table 4-20, density-based filtering (DBF) works well for the stochastic volatility model of short-term interest rates. Especially, DBF is better than Kalman filter for κ_2 and ξ.

Also, we show the results of the simulation graphically in Appendix C. Figures C-33 through C-36 make it easy to compare the methods. These results summarized by tables and figures support that our new estimation method of stochastic volatility model works well for a short-term interest rate model.

Table 4-17.: Simulation Result of OLS & Kalman Filter

	Alpha	Beta	Mu2	Kapp2	Gamma	Xi
True	0.9600	-0.1600	-0.2800	1.0400	0.5400	1.2700
Mean	1.6834	-0.2986	-0.1769	1.2069	0.5018	1.2770
Bias	0.7234	-0.1386	0.1031	0.1669	-0.0382	0.0070
Rbias	75.3540	-86.6037	36.8162	16.0444	-7.0752	0.5519
Std	0.8370	0.1478	0.3693	0.4170	0.0931	0.2093
Rmse	1.1063	0.2026	0.3834	0.4492	0.1007	0.2094

Table 4-18.: Simulation Result of MGE & Kalman Filter

	Alpha	Beta	Mu2	Kapp2	Gamma	Xi
True	0.9600	-0.1600	-0.2800	1.0400	0.5400	1.2700
Mean	0.9867	-0.1687	-0.1766	1.2111	0.5016	1.2773
Bias	0.0267	-0.0086	0.1034	0.1711	-0.0384	0.0073
Rbias	2.7778	-5.4276	36.9304	16.4498	-7.1086	0.5742
Std	1.0553	0.1552	0.3581	0.4241	0.0900	0.2133
Rmse	1.0556	0.1555	0.3728	0.4573	0.0978	0.2134

Table 4-19.: Simulation Result of OLS & DBF

	Alpha	Beta	Mu2	Kapp2	Gamma	Xi
True	0.9600	-0.1600	-0.2800	1.0400	0.5400	1.2700
Mean	1.6834	-0.2986	-0.1211	1.1660	0.4924	1.2619
Bias	0.7234	-0.1386	0.1588	0.1260	-0.0477	-0.0080
Rbias	75.3540	-86.6037	56.7355	12.1195	-8.8241	-0.6347
Std	0.8370	0.1478	0.3547	0.3258	0.1627	0.1401
Rmse	1.1063	0.2026	0.4669	0.3886	0.1696	0.1403

Table 4-20.: Simulation Result of MGE & DBF

True	0.9600	-0.1600	-0.2800	1.0400	0.5400	1.2700
Mean	0.9867	-0.1687	-0.0882	1.1726	0.4816	1.2667
Bias	0.0267	-0.0086	0.1918	0.1326	-0.0583	-0.0033
Rbias	2.7778	-5.4276	68.5010	12.7531	-10.8071	-0.2627
Std	1.0553	0.1552	0.3483	0.3303	0.1396	0.1468
Rmse	1.0556	0.1555	0.3976	0.3559	0.1513	0.1469

Table 4-21.: Summary Statistics for Weekly Three Month Treasury Bill

	Interest Rate Level		First Order Difference
Mean	3.8939	Mean	-0.0008
Std. dev	1.5734	Std. dev	0.1017
Minimum	0.7632	Minimum	-0.9270
Maximum	6.3517	Maximum	0.5894
Skewness	-0.5614	Skewness	-1.7285
Kurtosis	2.0538	Kurtosis	19.2082

D. Empirical Application

1. Data Description

For the empirical study, we use the three-month zero-coupon yields that are obtained in chapter 1 for weekly frequency. We use the period from the June 1991 to December 2006. Also, we use the hourly frequency three-month zero-coupon yield to obtain the realized volatility for weekly frequency yields.

Table 4-21 and Table 4-22 report the summary statistics of the level of the three-month interest rates and their first-order difference for weekly frequency and hourly frequency. Figure C-37 and Figure C-38 in Appendix C show the two time series data sets.

2. Result of Estimation

We estimate the parameters in the model in the way which we use for simulation study. First, we use ordinary least square (OLS) and Martingale estimator (MGE)to estimate the parameters of drift. Second, we use Kalman filter (KF) and density-

Table 4-22.: Summary Statistics for Hourly Frequency Treasury Bill

	Interest Rate Level		First Order Difference
Mean	3.9091	Mean	-0.00002
Std. dev	1.5690	Std. dev	0.0332
Minimum	0.7134	Minimum	-0.6121
Maximum	6.4139	Maximum	0.5729
Skewness	-0.5716	Skewness	-0.1079
Kurtosis	2.0741	Kurtosis	29.3067

based filering (DBF) to estimate the rest of parameters.

Tables 4-23 through Table 4-25 shows the result of estimation. Based on Mean absolute error(MAE)As we see below in the table and in the picture, the performance of MGE and DBF dominates the other results. MGE and DBF is the best to estimate parameters of both drift and stochastic volatility since MGE corrects the upward bias of OLS and density-based filtering corrects the approximation error of Kalman filter.

E. Conclusion

We have introduced a new estimation method for the stochastic volatility model which uses a Martingale method and the density-based filtering. The Martingale method improves the upward bias of OLS for the parameters in the drift function of interest rates. However, the Martingale method does not affect the estimation of stochastic volatility part with relevant parameter values which is based on real interest rates data. Density based filtering works well to estimate the stochastic volatility model of short-term interests. Specially, if we use density based filtering, we obtain better estimation result for the parameters which include mean reversion

Table 4-23.: Estimation Results for Weekly 3 Month Bill

Parameter	OLS & KF	MGE & KF	OLS & DBF	MGE & DBF
α	0.4804	1.8861	0.4804	1.8861
	(0.3709)	(0.0263)	(0.3709)	(0.0263)
β	-0.1346	-0.4260	-0.1346	-0.4260
	(0.0980)	(0.0039)	(0.0980)	(0.0039)
μ_2	-2.4883	-2.0470	-2.5020	-2.1568
	(0.9056)	(0.2544)	(0.2809)	(0.2660)
κ_2	16.9199	29.3966	13.4468	13.2803
	(116.5924)	(7.1330)	(3.3568)	(3.7056)
γ	0.4132	0.2525	0.4264	0.3218
	(0.4162)	(0.0940)	(0.1040)	(0.0992)
ξ	5.7768	7.9519	5.3069	4.8936
	(20.5096)	(1.1517)	(0.7502)	(0.7636)

Table 4-24.: Model Evaluation by Regression

Parameter	OLS & KF	MGE & KF	OLS & DBF	MGE & DBF
a	0.0187	0.0172	0.0322	0.0307
	(0.0043)	(0.0045)	(0.0030)	(0.0031)
b	4.3599	4.5294	1.5722	1.7903
	(0.6358)	(0.6654)	(0.2483)	(0.2709)
R^2	0.0546	0.0539	0.0469	0.0509

Table 4-25.: Model Evaluation by MAE

	OLS & KF	MGE & KF	OLS & DBF	MGE & DBF
MAE	0.03782	0.03779	0.03749	0.0373

speed of log-volatility,$\mu 2$, and diffusion term of log volatility,ξ. Therefore, if we use the Martingale method and density-based filtering for the stochastic volatility model of short-term interest rates, we can obtain better performance.

CHAPTER V

CONCLUSION

U.S. Treasury Securities are crucially important in many areas of finance. However, zero-coupon yields are not observable in the market. Even though published zero-coupon yields exist, they are sometimes not available for certain research topics or for high frequency. Recently, high frequency data analysis has become popular, and the GovPX database is a good source of tick data for U.S. Treasury securities from which we can construct zero-coupon yield curves. Therefore, we try to fit zero-coupon yield curves from low frequency and high frequency data from GovPX by three different methods: the Nelson-Siegel method, the Svensson method, and the cubic spline method.

Then, we try to retest the expectations hypothesis (EH) with new zero-coupon yields that are made from GovPX data by three methods using the Campbell and Shiller regression, the Fama and Bliss regression, and the Cochrane and Piazzesi regression. Regardless of the method used (the Nelson-Siegel method, the Svensson method, or the cubic spline method), the expectations hypothesis cannot be rejected in the period from June 1991 to December 2006 for most maturities in many cases.

Also, we introduce a new estimation method for the stochastic volatility model of short-term interest rates. We apply a Martingale method and density based filtering to estimate a stochastic volatility model of short-term interest rates, and we compare our method with the existing method. The result supports that our new method works well for the stochastic volatility model of short-term interest rates.

REFERENCES

AIT-SAHALIA, Y. (1996): "Testing Continuous-Time Models of the Spot Interest Rate," *Review of Financial Studies*, 9, 385–426.

ANDERSEN, T. G., AND L. BENZONI (2006): "Do Bonds Span Volatility Risk in the U.S. Treasury Market? Aspecification Test for Affin Term Structure Models," Working paper No. 2006-15, FRB of Chicago.

ANDERSEN, T. G., AND J. LUND (1997): "Estimating Continuous-Time Stochastic Volatility Models of the Short-Term Interest Rate," *Journal of Econometrics*, 77, 343–377.

ANDERSON, N., AND J. SLEATH (2001): "New Estimates of the UK Real and Nominal Yield Curve," Working paper, Bank of England.

BALL, C. A., AND W. N. TOROUS (1999): "The Stochastic Volatility of Short-Term Interest Rates: Some International Evidence," *The Journal of Finance*, 54, 2339–2359.

BRENNER, R. J., R. H. HARJES, AND K. F. KRONER (1996): "Another Look at Models of the Short-Term Interest Rate," *Journal of Financial and Quantitative Analysis*, 31(1), 85–107.

BULKELY, G., R. D. F. HARRIS, AND V. NAWOSAH (2008): "Revisiting the Expectations Hypothesis of the Term Structure of Interest Rates," Working paper No. 126, University of Exeter.

CAMPBELL, J. Y., AND R. J. SHILLER (1991): "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Syudies*, 58(3), 495–514.

Chan, K. C., A. Karoly, F. A. Longstaff, and A. B. Sanders (1992): "An Empirical Comparison of Alternative Models of the Short-Term Interest Rate," *The Journal of Finance*, 47(3), 1209–1227.

Cochrane, J. H., and M. Piazzesi (2005): "Bond Risk Premia," *American Economic Review*, 95(1), 138–160.

Cox, J., J. Ingersoll, and S. Ross (1985): "A Theory of the Term Structure of Interest Rates," *Econometrica*, 53, 385–407.

Diebold, F. X., and C. Li (2006): "Forecating the Term Structure of Government Bond Yileds," *Journal of Econometrics*, 130, 337–364.

Duffee, G. R. (2002): "Term Premia and Interest Rate Forecasts in Affine Models," *The Journal of Finance*, 57, 405–443.

Fabozzi, F. J. (2004): *The Handbook of Fixed Income Securities*. New York: McGraw-Hill.

Fama, E. F., and R. R. Bliss (1987): "The Information in Long-Maturity Forward Rates," *American Economic Review*, 77(4), 680–692.

Fleming, M. J. (1997): "The Round-the-Clock Market of U.S. Treasury Securities," *Federal Reserve Bank of New York Economic Policy Review*, 3, 9–32.

Gurkaynak, R. S., B. P. Sack, and J. H. Wright (2006): "The U.S. Treasury Yield Curve: 1961 to the Present," Working Paper No. 2006-28, Board of Governors of the Federal Reserve System Research Paper Series.

Hardouvelis, G. A. (1994): "The Term Structure Spread and Future Changes in Long and Short Rates in the G7 Countries," *Journal of Monetary Economics*, 33, 255–283.

HARVEY, A. C., E. RUIZ, AND N. SHEPHARD (1994): "Multivariate Stochastic Variance Models," *The Review of Economic Studies*, 61, 247–264.

JACQUIER, E., N. G. POLSON, AND P. E. ROSSI (1994): "Bayesian Analysis of Stochastic Volatility Models," *Journal of Business and Economic Statistics*, 12(4), 69–87.

KALIMIPALLI, M., AND R. SUSMEL (2004): "Regime-Switching Stochastic Volatility and Short-Term Interest Rates," *Journal of Empirical Finance*, 11, 309–329.

LEE, H. I. (2008): "Stochastic Volatility Models with Persistent Latent Factors: Theory and Its Applications to Asset Prices," Dissertation.

MANKIW, N. G., AND J. A. MIRON (1986): "The Changing Behavior of the Term Structure of Interest Rates," *The Quarterly Journal of Economics*, 101(2), 211–228.

MCCULLOCH, J. H. (1971): "Measuring the Term Structure of Interest Rates," *The Journal of Business*, 44(1), 19–31.

——— (1990): *The Handbook of Monetary Economics*. Amsterdam: North Holland.

NELSON, C. R., AND A. F. SIEGEL (1987): "Parsimonious Modeling of Yield Curves," *The Journal of Business*, 60(4), 473–489.

PARK, J. Y. (2008): "Martigale Regression and Time Change," Working Paper.

SMITH, D. R. (2002): "Markov-Switching and Stochastic Volatility Diffusion Models of Short-Term Interest Rates," *Journal of Business and Economic Statistics*, 20(2), 183–197.

SUN, L. (2005): "Regime Shift in Interest Rate Volatility," *Journal of Empirical Finance*, 12, 418–434.

Svensson, L. E. (1994): "Estimating Forward Interest Rates with the Extended Nelson & Siegel Method," Working paper No. 114, International Monetary Fund.

Tanizaki, H. (1996): *Nonlinear Filters: Estimation and Application.* 2nd, New York: Springer.

Thornton, D. L. (2006): "Tests of the Expectations Hypothesis: Resolving the Campbell-Shiller Paradox," *Journal of Money, Credit, and Banking*, 38(2), 511–542.

Waggoner, D. F. (1997): "Spline Methods for Extracting Interesr Rate Curves from Coupon Bond Prices," Working paper 97-10, Federal Reserve Bank of Atlanta.

APPENDIX A

ZERO-COUPON YIELD CURVES

Table A-26.: Description of GovPX's Variables

Variable	Type	Description	Categories
CUSIP	Char	CUSIP	
DATE	Num	Date	
ACTIVE	Char	Active Code	A(Active),N(Non-Active)
			W(When-issued)
ALIAS	Char	Bond Type	3-month, 6-month, 1-year, etc
COUPON	Num	Coupon	0(discount), 1(coupon)
MATDATE	Num	Maturity Date	
TIME	Num	Time	
BIDPRC	Num	Bid Price	
BIDYLD	Num	Bid Yield	
BIDSIZE	Num	Bid Size	
ASKPRC	Num	Ask Price	
ASKYLD	Num	Ask Yield	
ASKSIZE	Num	Ask Size	
LTPRC	Num	Last Trade Price	
LTYLD	Num	Last Trade Yield	
LTSIZE	Num	Last Trade Size	
INDBID	Num	Indicative Bid Price	
INDASK	Num	Indicative Ask Price	

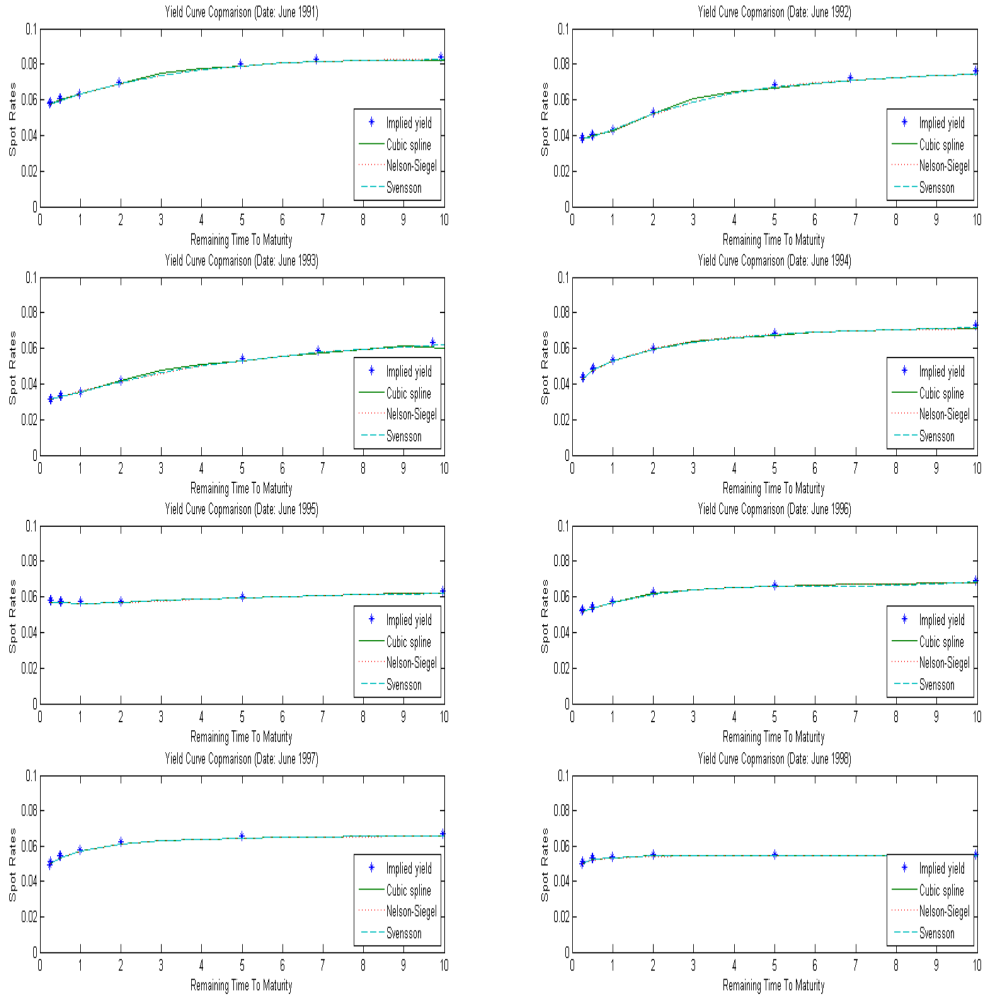

Fig. A-1.: The Comparison of Three Methods (period:1991-1998)

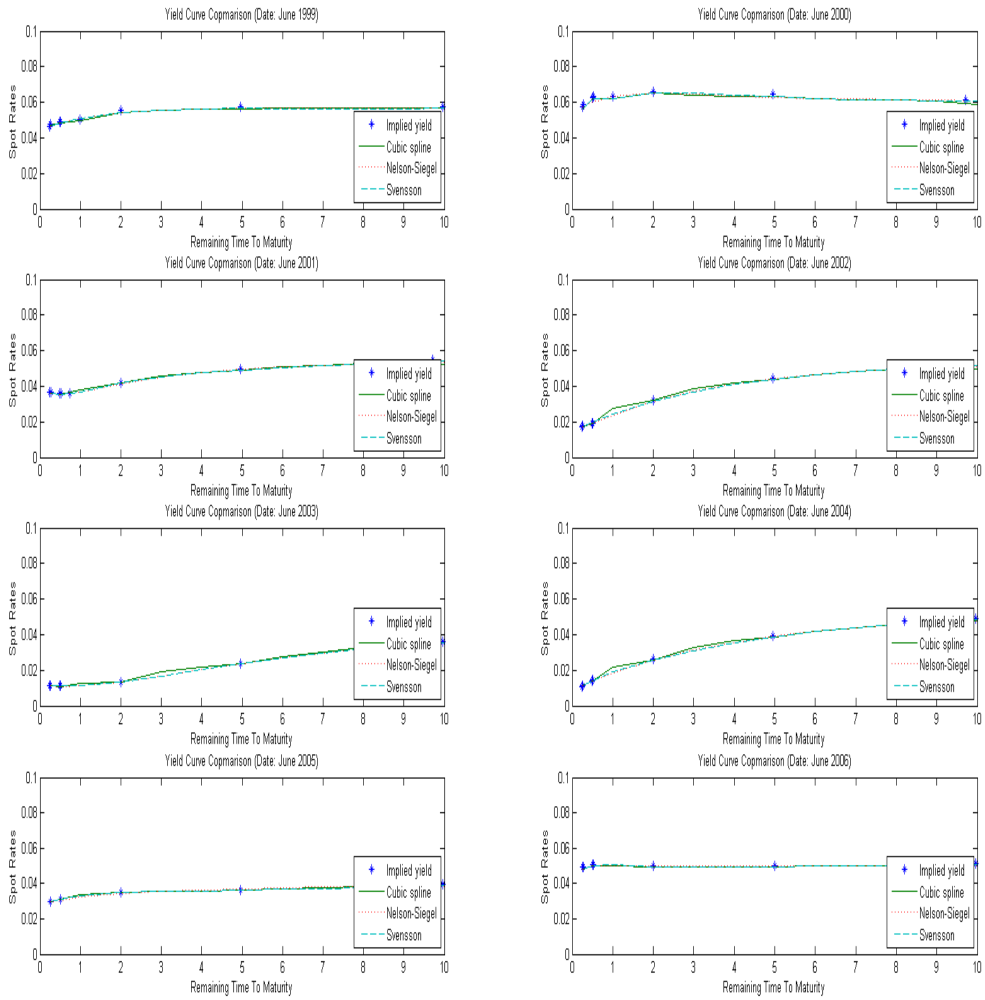

Fig. A-2.: The Comparison of Three Methods (period:1999-2006)

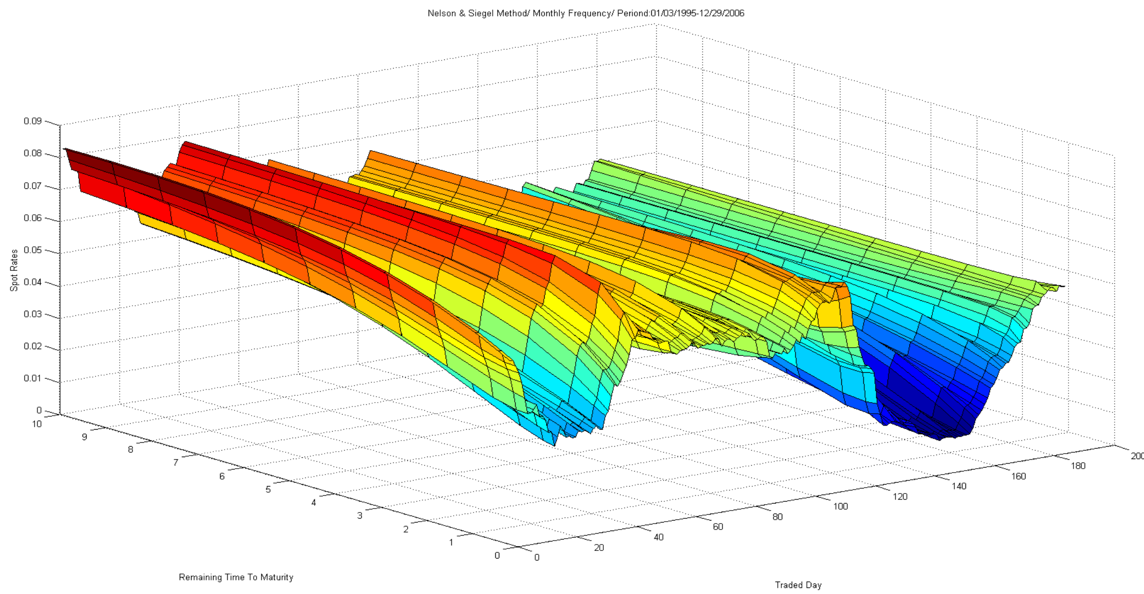

Fig. A-3.: The Nelson-Siegel Yield Curves with Monthly Frequency

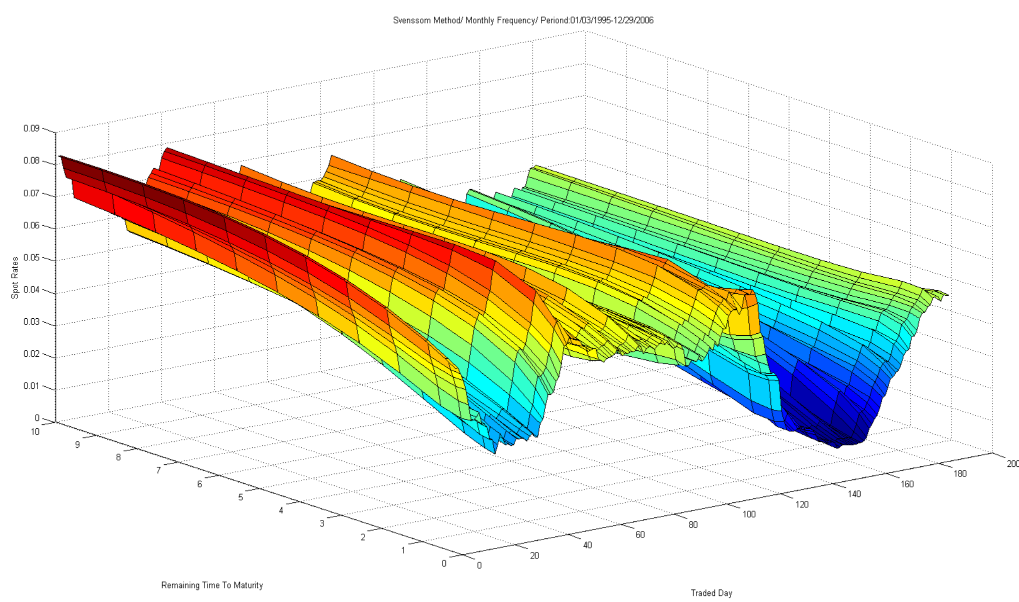

Fig. A-4.: The Svensson Yield Curves with Monthly Frequency

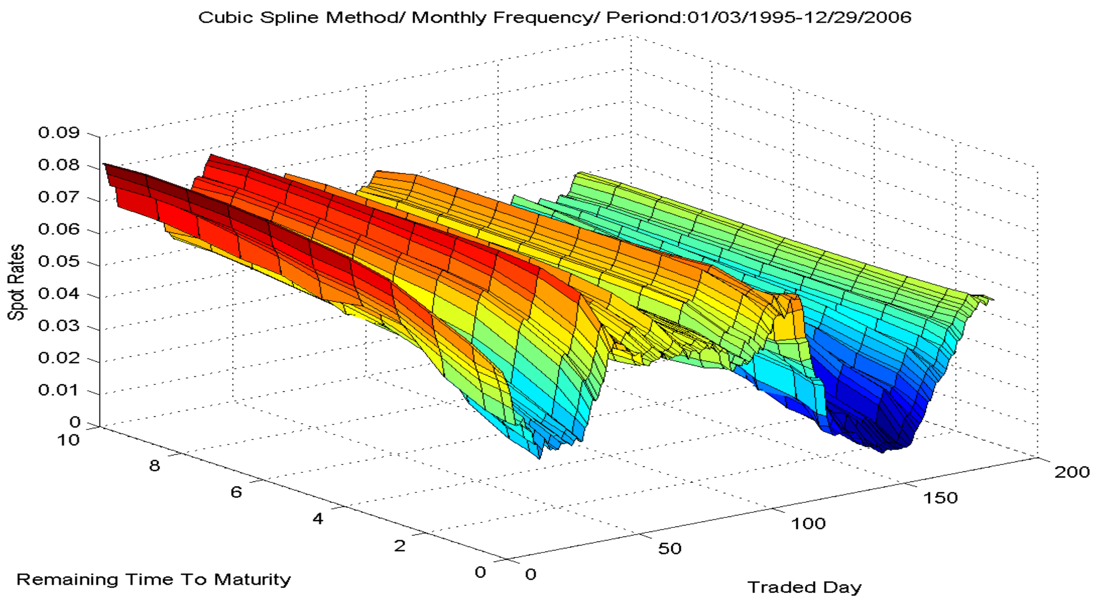

Fig. A-5.: The Cubic Spline Yield Curves with Monthly Frequency

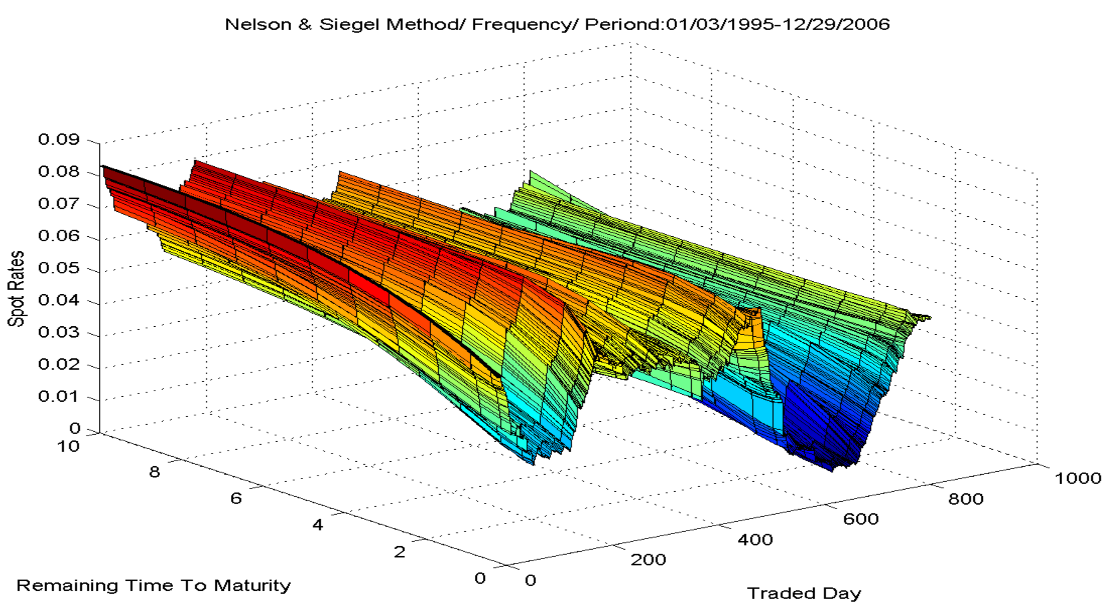

Fig. A-6.: The Nelson-Siegel Yield Curves with Weekly Frequency

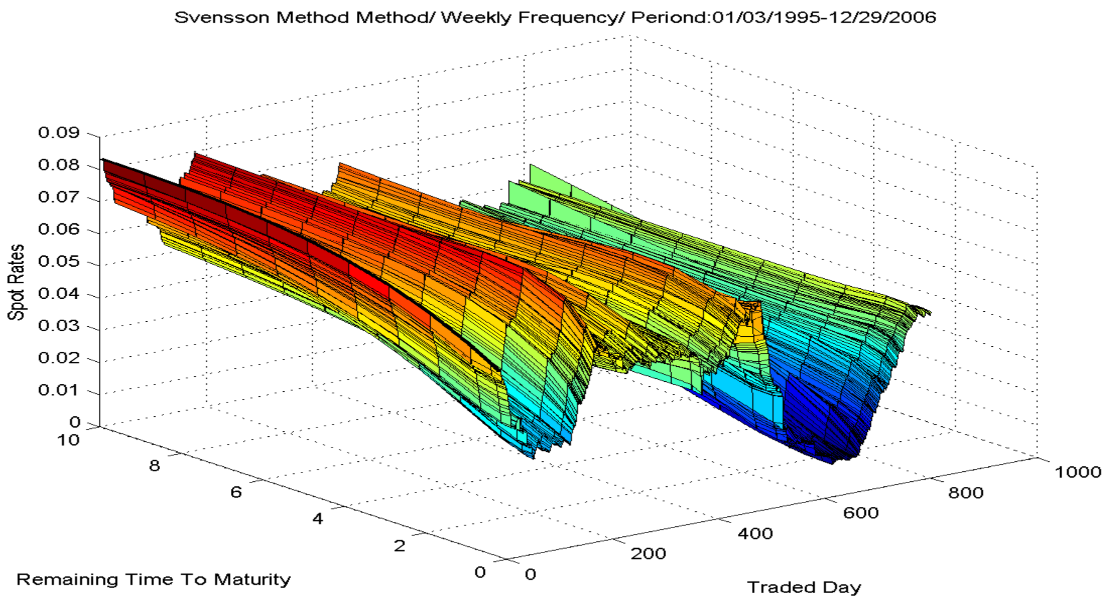

Fig. A-7.: The Svensson Yield Curves with Weekly Frequency

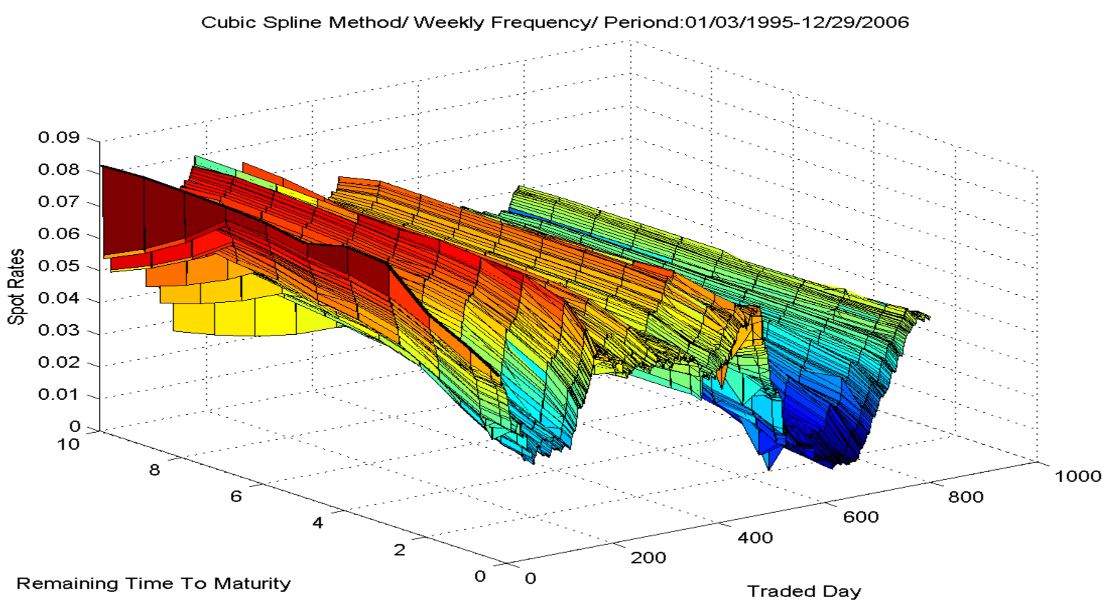

Fig. A-8.: The Cubic Spline Yield Curves with Weekly Frequency

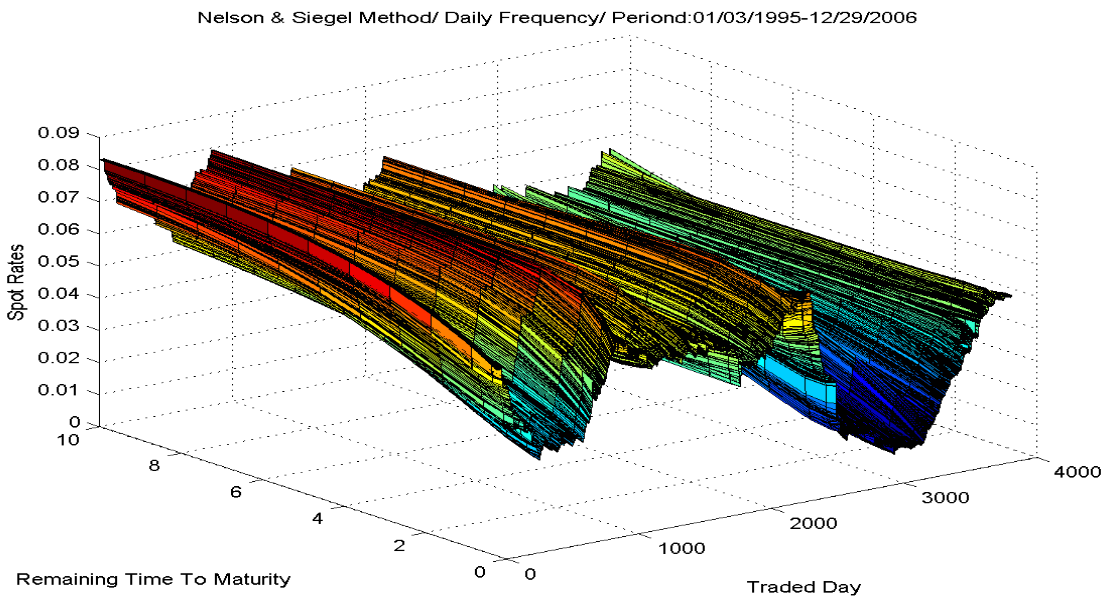

Fig. A-9.: The Nelson-Siegel Yield Curves with Daily Frequency

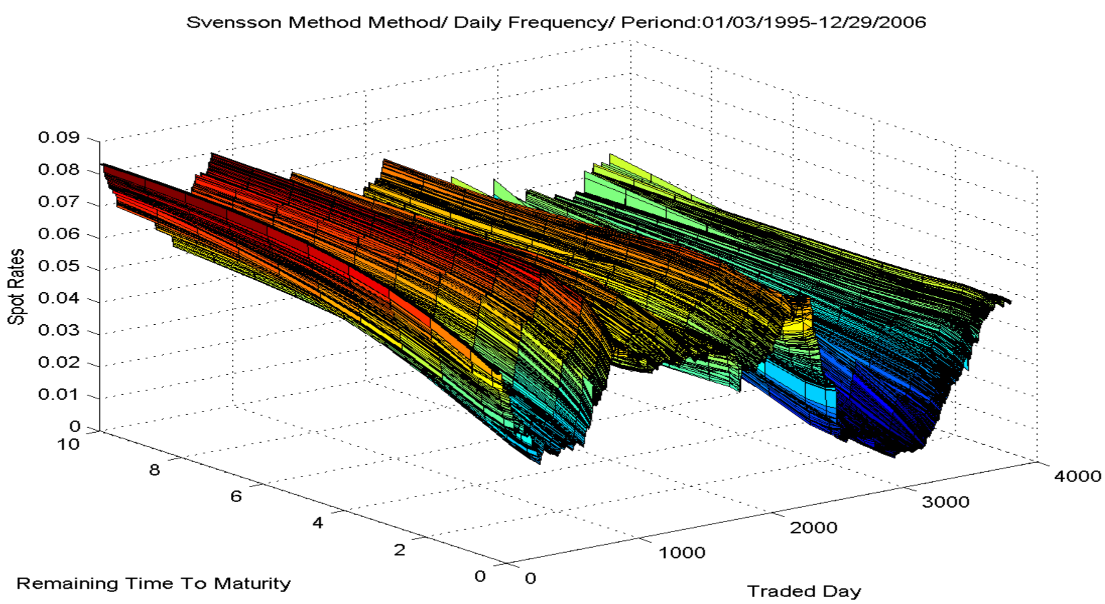

Fig. A-10.: The Svensson Yield Curves with Daily Frequency

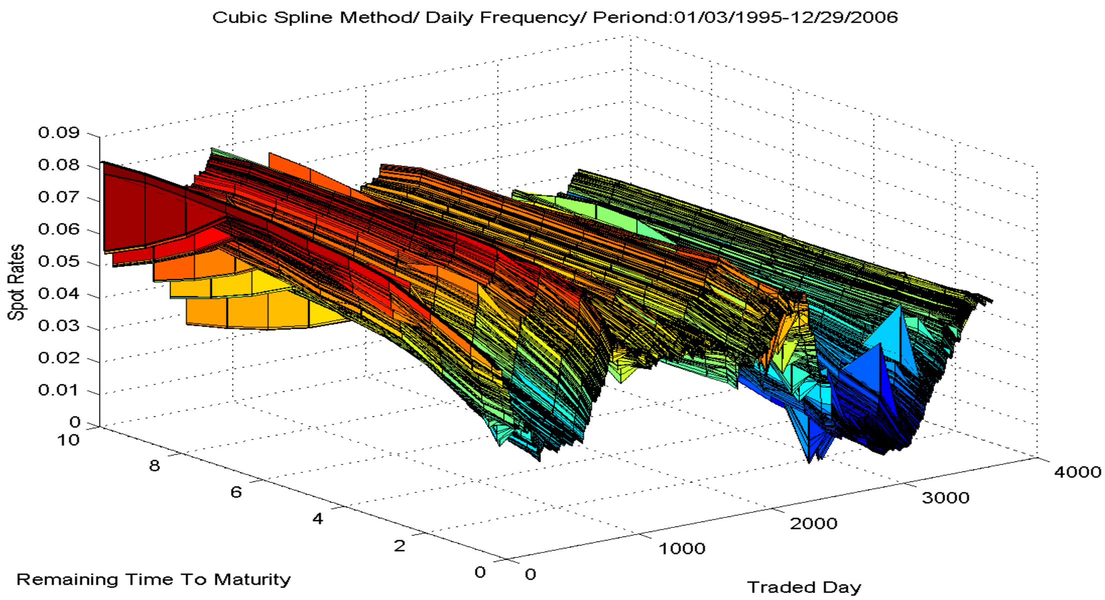

Fig. A-11.: The Cubic Spline Yield Curves with Daily Frequency

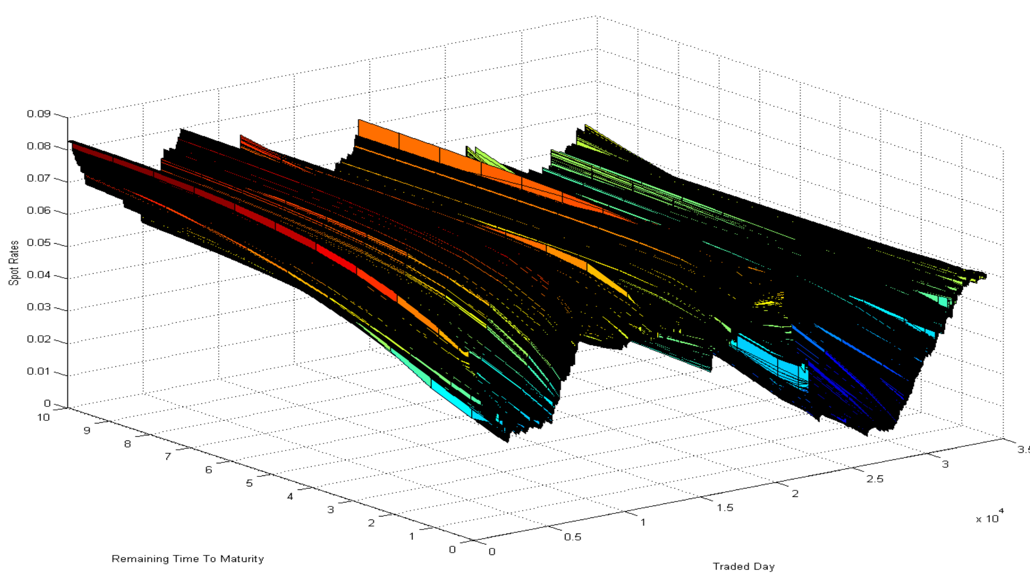

Fig. A-12.: The Nelson-Siegel Yield Curves with Hourly Frequency

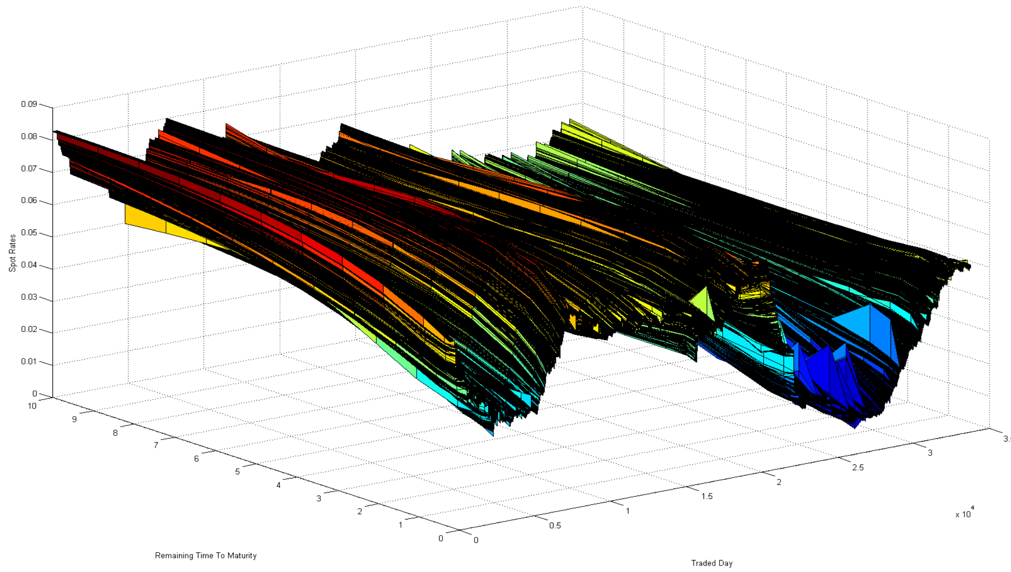

Fig. A-13.: The Svensson Yield Curves with Hourly Frequency

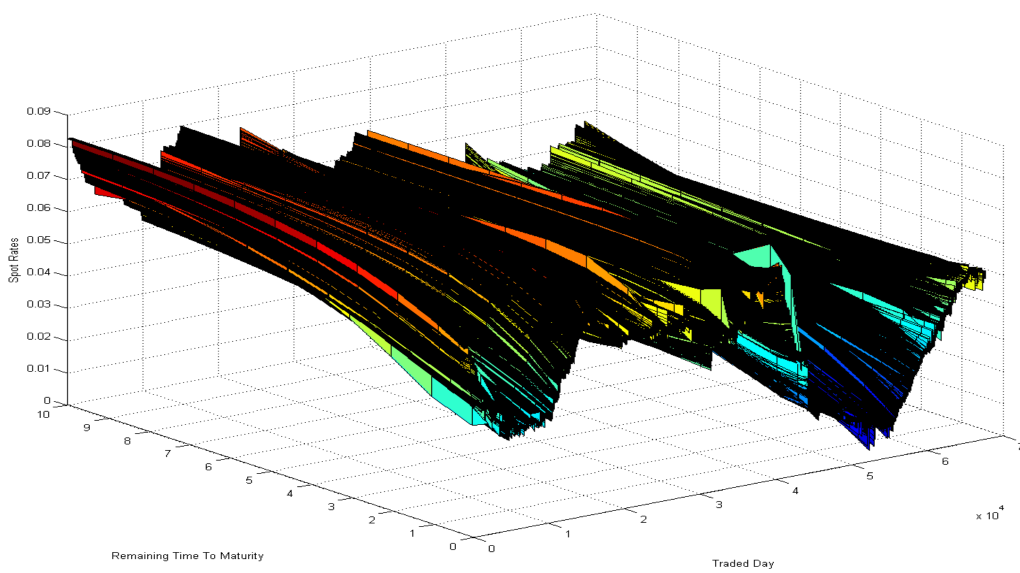

Fig. A-14.: The Nelson-Siegel Yield Curves with 30 Minutes Frequency

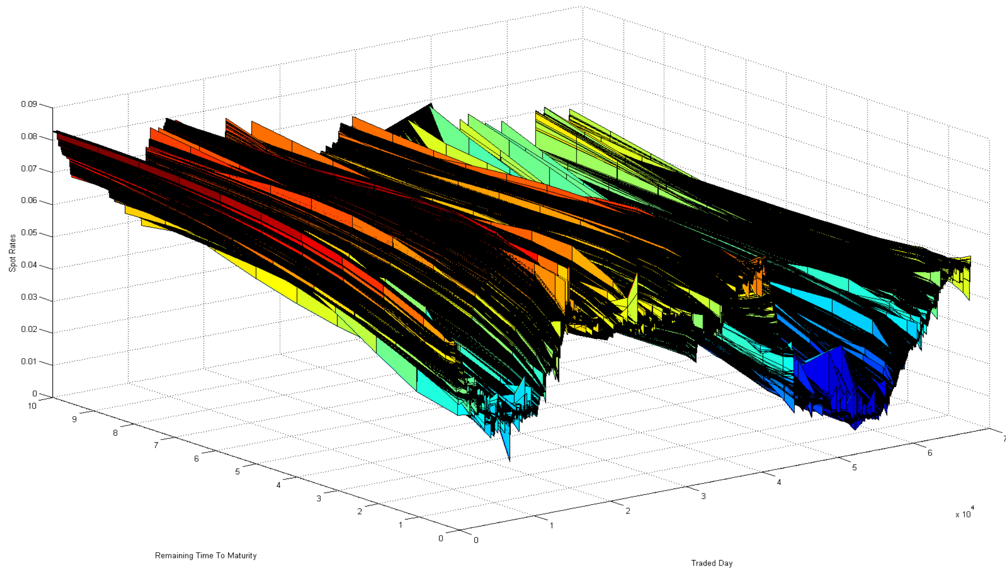

Fig. A-15.: The Svensson Yield Curves with 30 Minutes Frequency

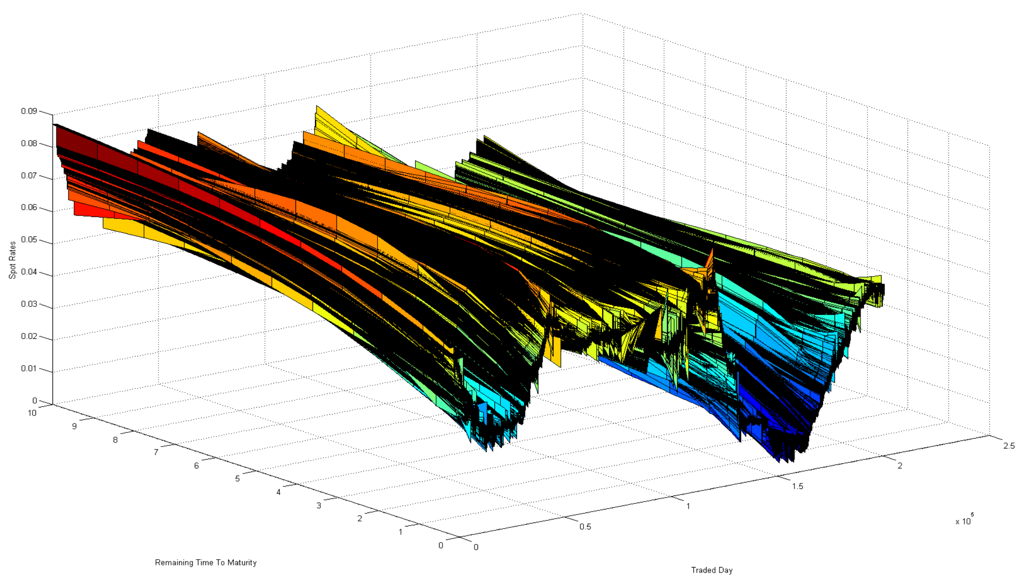

Fig. A-16.: The Nelson-Siegel Yield Curves with 10 Minutes Frequency

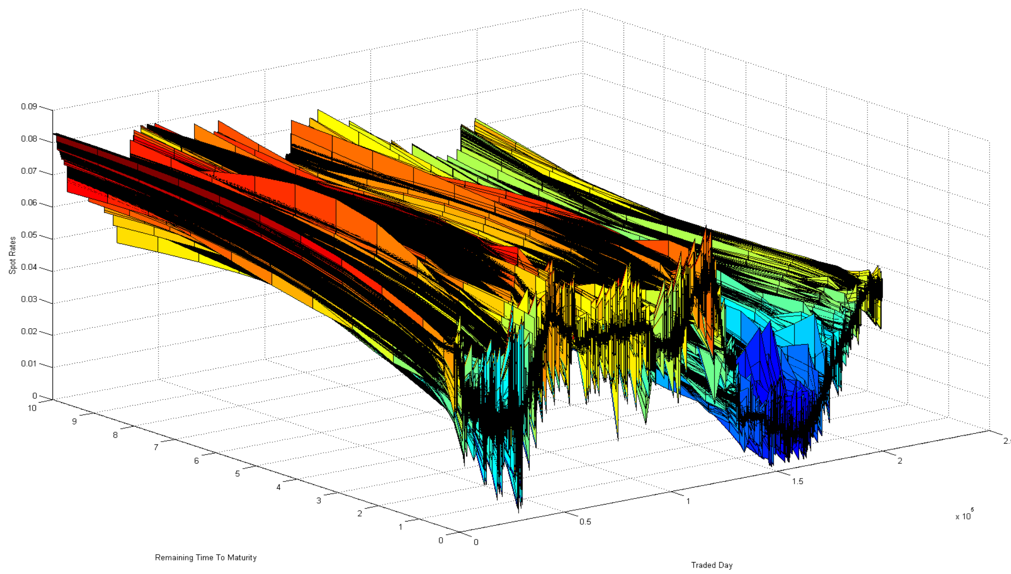

Fig. A-17.: The Svensson Yield Curves with 10 Minutes Frequency

APPENDIX B

REVISITING THE EH OF INTEREST RATES WITH GOVPX DATA

Table B-27.: Summary Statistics of CRSP Data

	CRSP (Jun1964-Dec1979)				CRSP(Jun1991-Dec2006)			
Maturity	Mean	Std	Min	Max	Mean	Std	Min	Max
1 year	4.0279	1.7150	1.0424	6.4837	4.4325	1.4437	1.7499	6.5927
2 year	4.2732	1.5639	1.3013	6.5932	4.5875	1.3240	2.1072	6.6044
3 year	4.4904	1.4063	1.6235	6.5622	4.7058	1.2347	2.3973	6.5864
4 year	4.6594	1.2760	2.0009	6.6343	4.8104	1.1591	2.6503	6.5955
5 year	4.7724	1.1610	2.3532	6.6636	4.9062	1.0939	2.8782	6.6039

Table B-28.: Summary Statistics of Our Yield Data

	Nelson-Siegel(Jun1991-Dec2006)				Svensson (June1991-Dec2006)			
Maturity	Mean	Std	Min	Max	Mean	Std	Min	Max
1 year	4.0279	1.7150	1.0424	6.4837	4.4325	1.4437	1.7499	6.5927
2 year	4.2732	1.5639	1.3013	6.5932	4.5875	1.3240	2.1072	6.6044
3 year	4.4904	1.4063	1.6235	6.5622	4.7058	1.2347	2.3973	6.5864
4 year	4.6594	1.2760	2.0009	6.6343	4.8104	1.1591	2.6503	6.5955
5 year	4.7724	1.1610	2.3532	6.6636	4.9062	1.0939	2.8782	6.6039

Table B-29.: Summary Statistics of Our Yield Data

	Cubic Spline(Jun1991-Dec2006)			
Maturity	Mean	Std	Min	Max
1 year	4.0279	1.7150	1.0424	6.4837
2 year	4.2732	1.5639	1.3013	6.5932
3 year	4.4904	1.4063	1.6235	6.5622
4 year	4.6594	1.2760	2.0009	6.6343
5 year	4.7724	1.1610	2.3532	6.6636

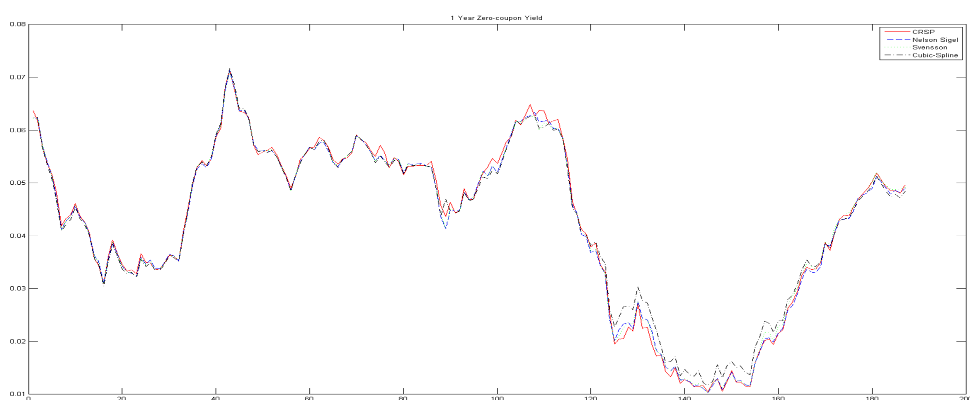

Fig. B-18.: One Year Zero-Coupon Yields for 06/1991 - 12/2006

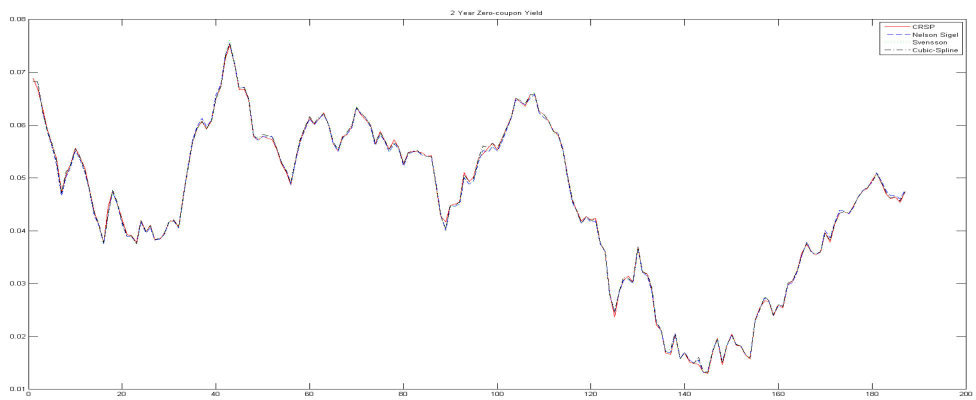

Fig. B-19.: Two Year Zero-Coupon Yields for 06/1991 - 12/2006

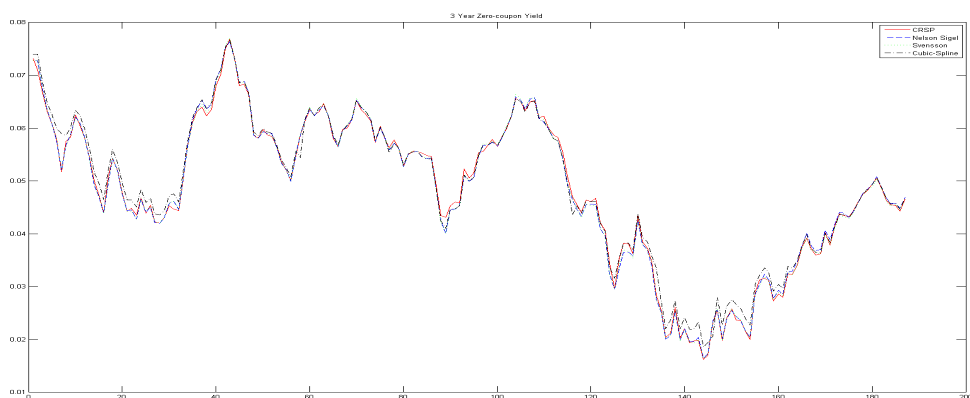

Fig. B-20.: Three Year Zero-Coupon Yields for 06/1991 - 12/2006

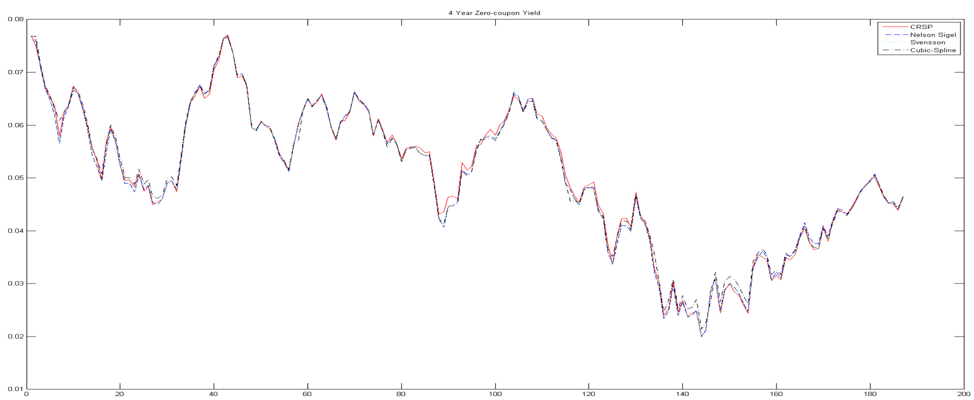

Fig. B-21.: Four Year Zero-Coupon Yields for 06/1991 - 12/2006

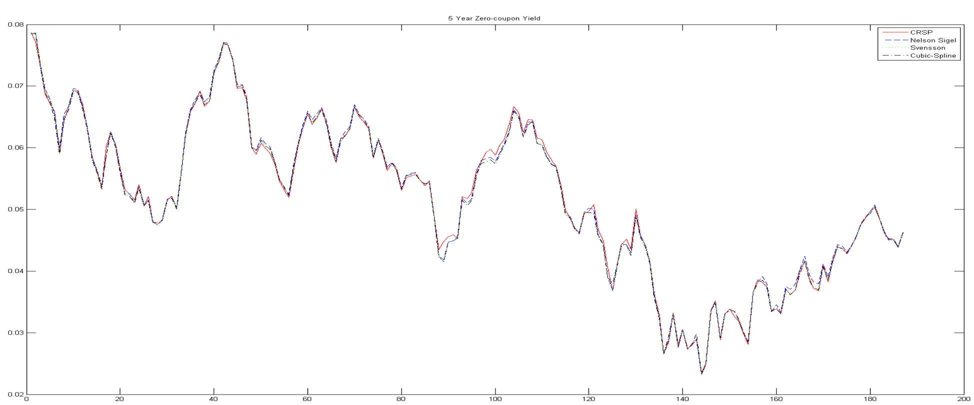

Fig. B-22.: Five Year Zero-Coupon Yields for 06/1991 - 12/2006

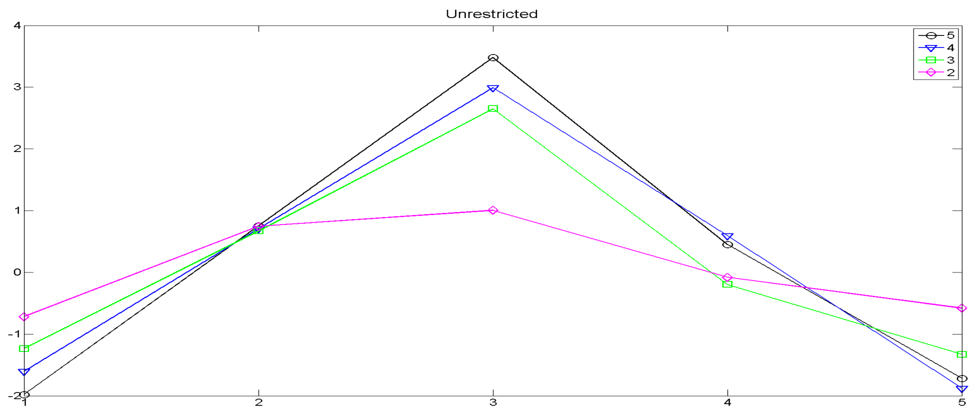

Fig. B-23.: Coefficients of Unrestricted Model of Cochrane and Piazzesi from CRSP (Jun1964-Dec1979)

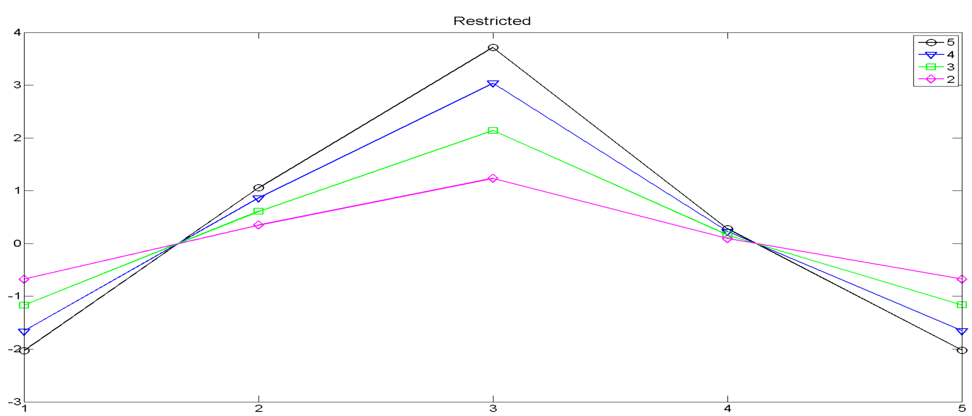

Fig. B-24.: Coefficients of Restricted Model of Cochrane and Piazzesi from CRSP (Jun1964-Dec1979)

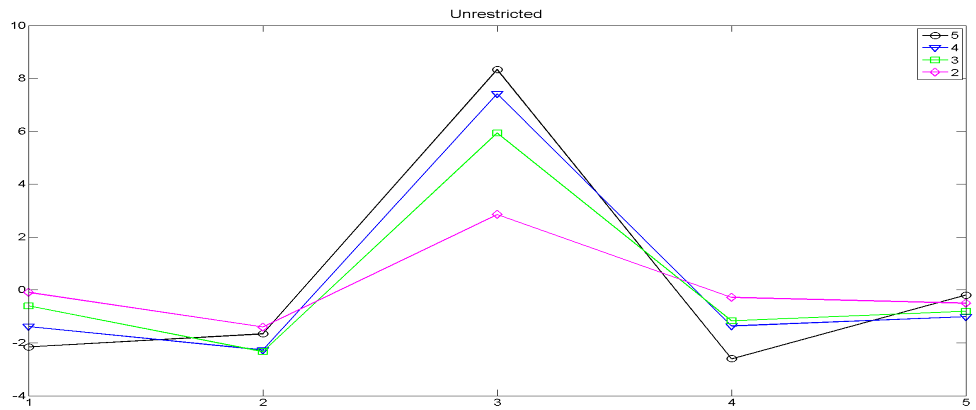

Fig. B-25.: Coefficients of Unrestricted Model of Cochrane and Piazzesi from CRSP (Jun1991-Dec2006)

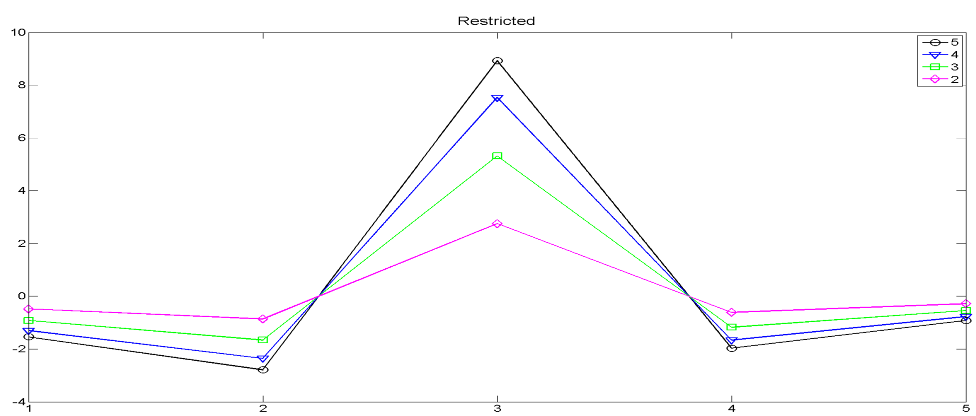

Fig. B-26.: Coefficients of Restricted Model of Cochrane and Piazzesi from CRSP (Jun1991-Dec2006)

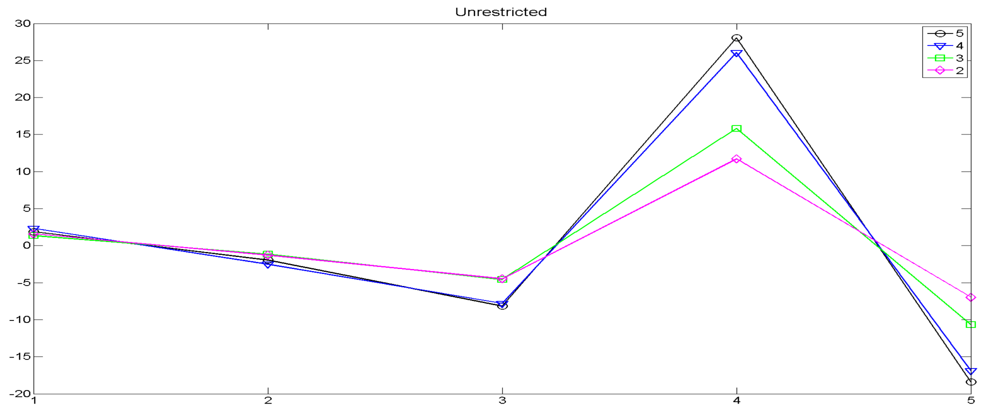

Fig. B-27.: Coefficients of Unrestricted Model of Cochrane and Piazzesi from Our Data by Nelson-Siegel Method (Jun1991-Dec2006)

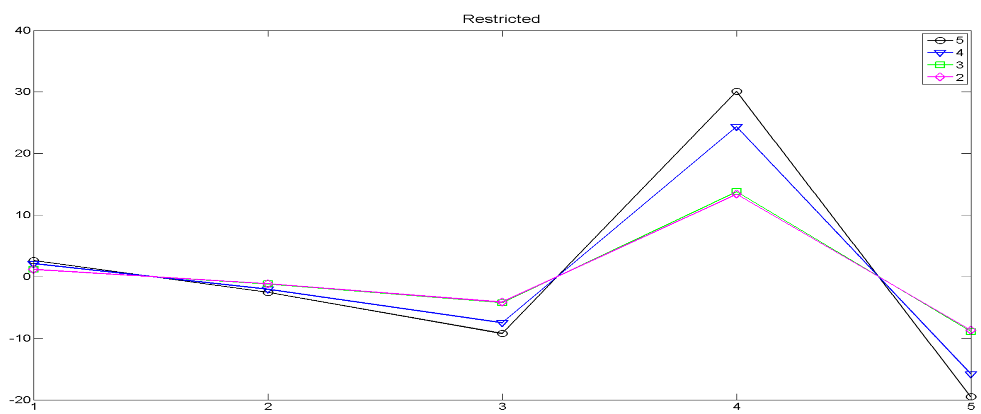

Fig. B-28.: Coefficients of Restricted Model of Cochrane and Piazzesi from Our Data by Nelson-Siegel Method (Jun1991-Dec2006)

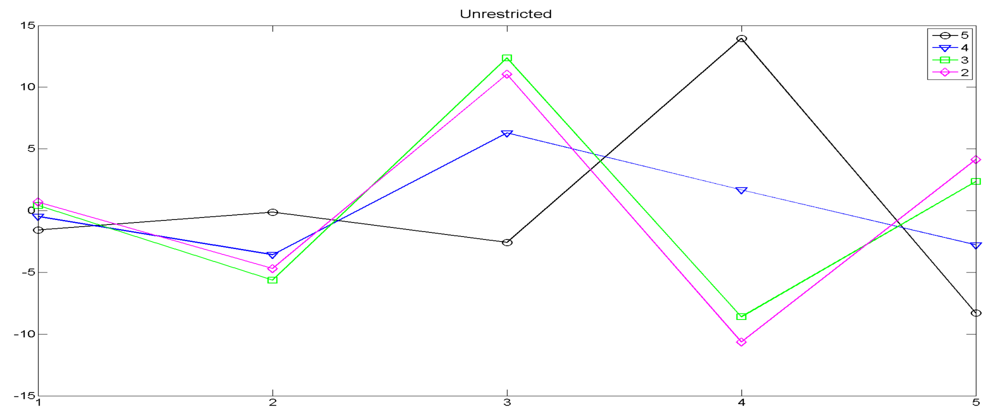

Fig. B-29.: Coefficients of Unrestricted Model of Cochrane and Piazzesi from Our Data by Svensson Method (Jun1991-Dec2006)

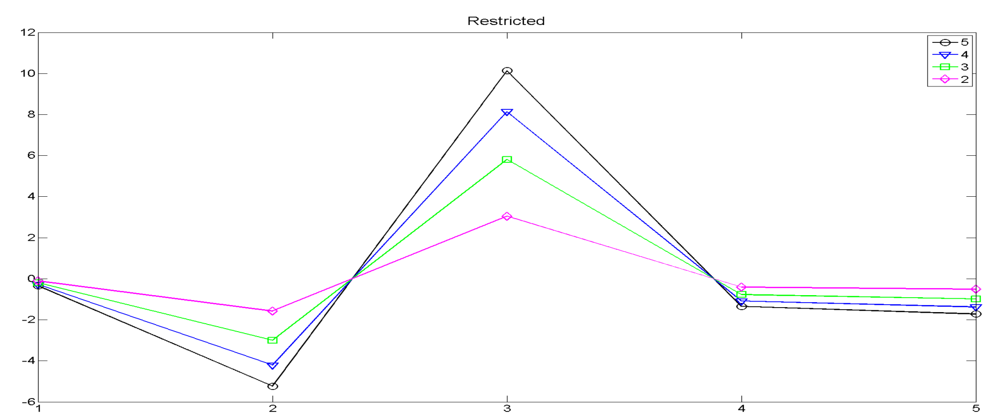

Fig. B-30.: Coefficients of Restricted Model of Cochrane and Piazzesi from Our Data by Svensson Method (Jun1991-Dec2006)

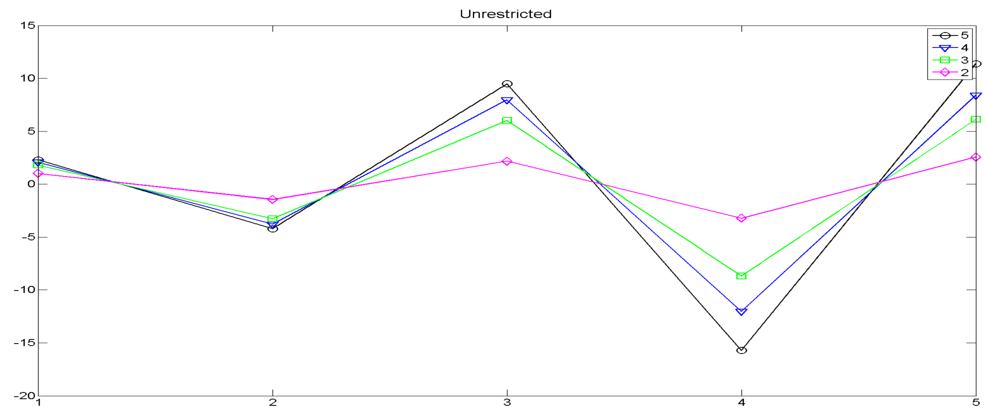

Fig. B-31.: Coefficients of Unrestricted Model of Cochrane and Piazzesi from Our Data by the Cubic Spline Method (Jun1991-Dec2006)

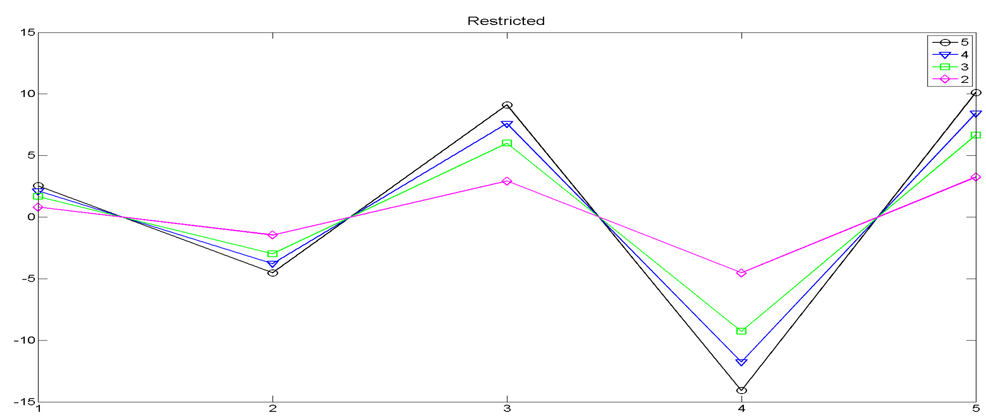

Fig. B-32.: Coefficients of Restricted Model of Cochrane and Piazzesi from Our Data by the Cubic Spline Method (Jun1991-Dec2006)

APPENDIX C

ESTIMATION OF STOCHASTIC VOLATILITY MODEL OF INTEREST RATES

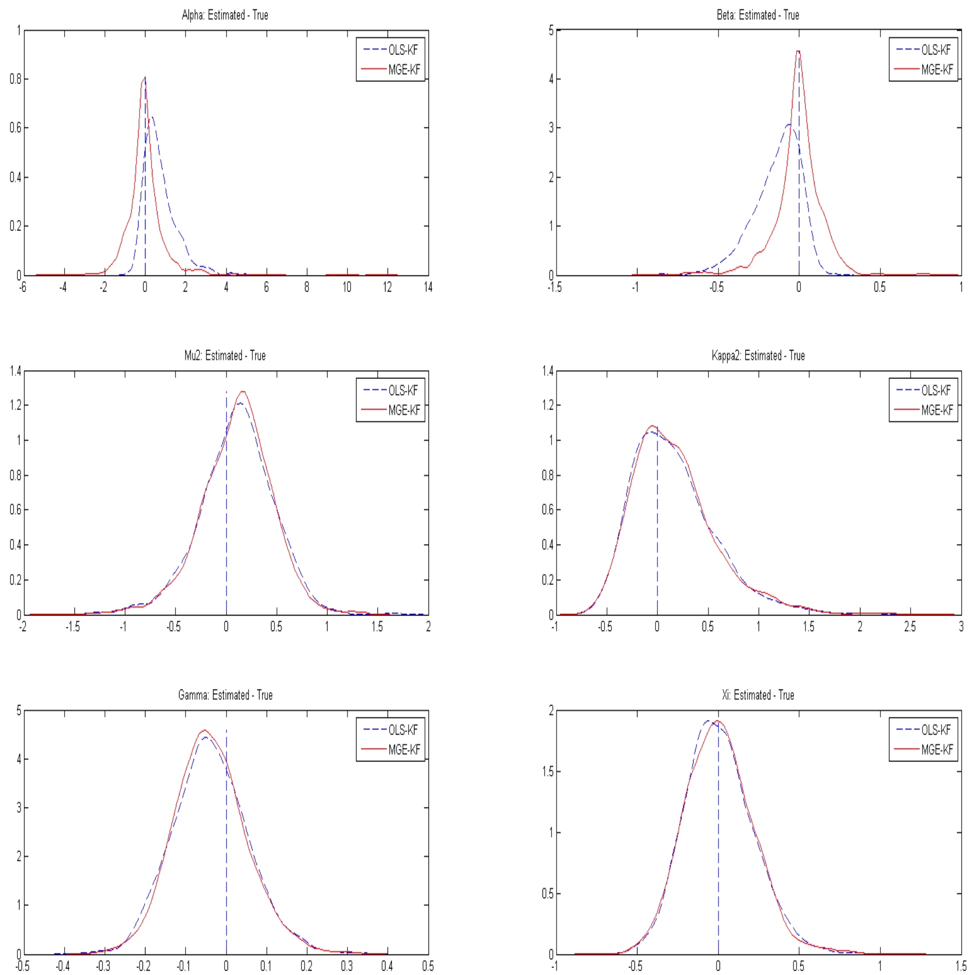

Fig. C-33.: The Comparison between OLS-KF and MGE-KF

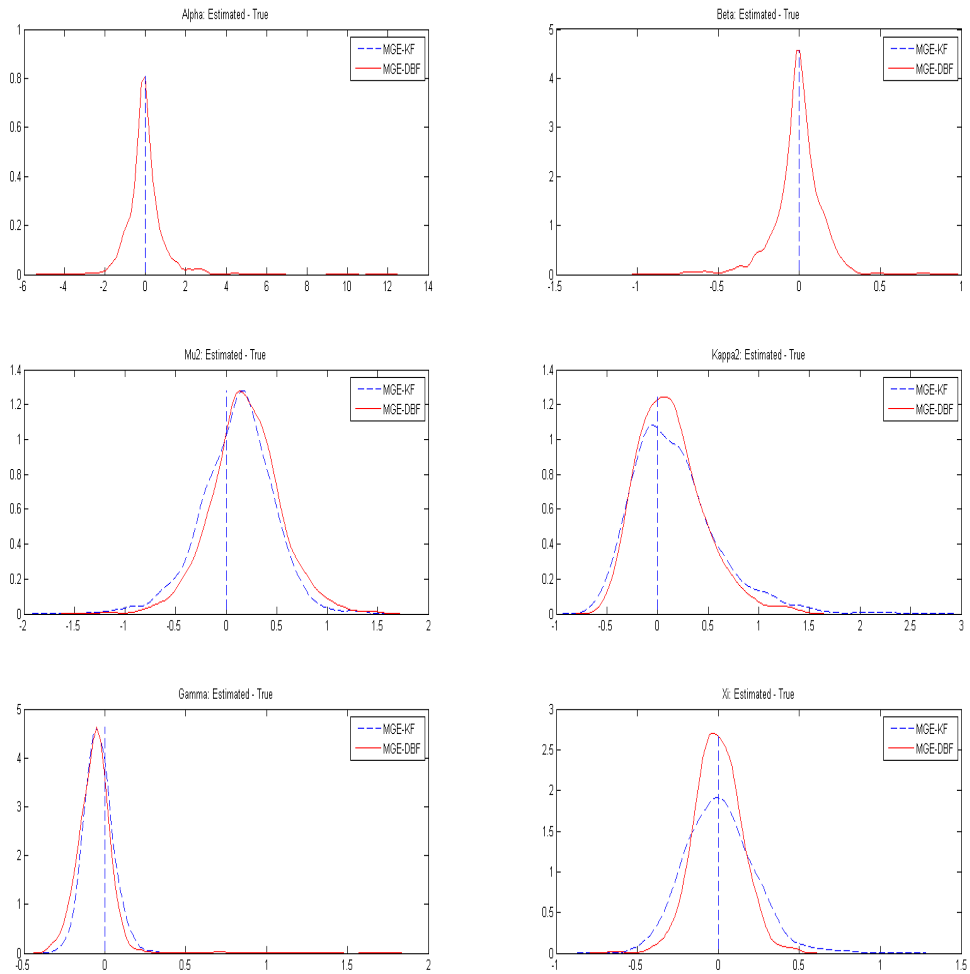

Fig. C-34.: The Comparison between MGE-KF and MGE-DBF

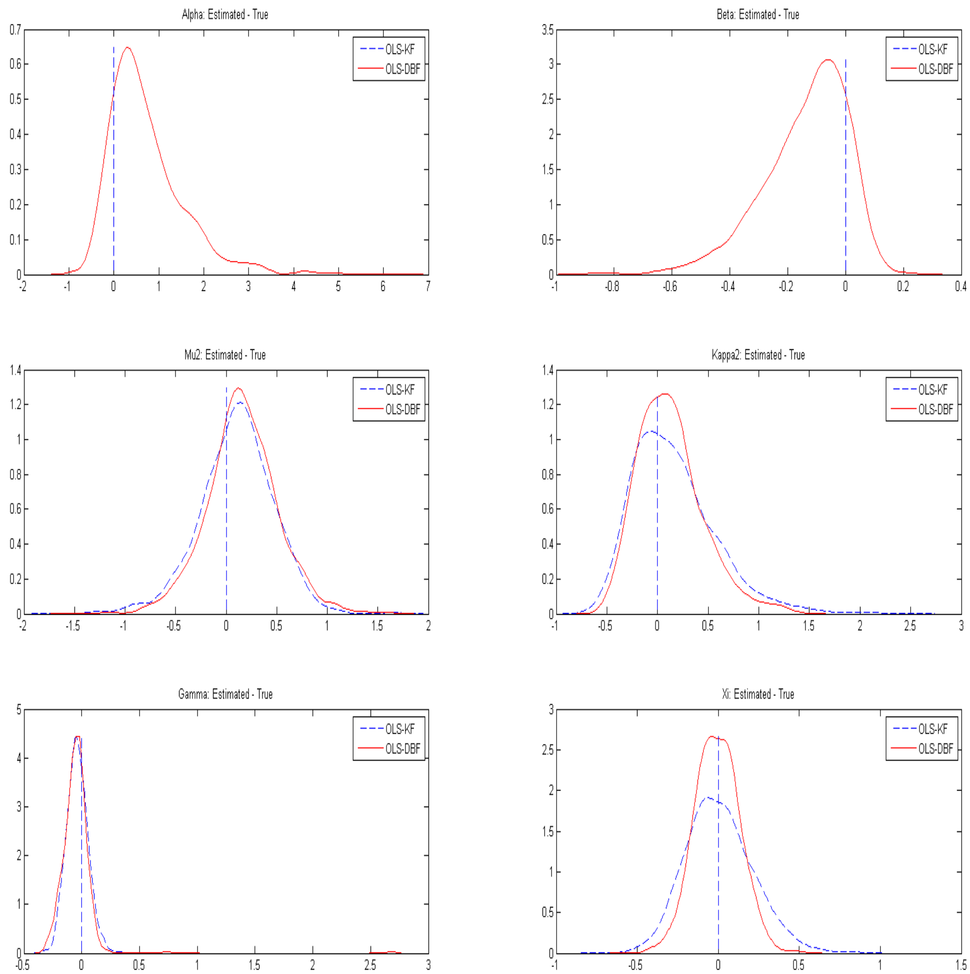

Fig. C-35.: The Comparison between OLS-KF and OLS-DBF

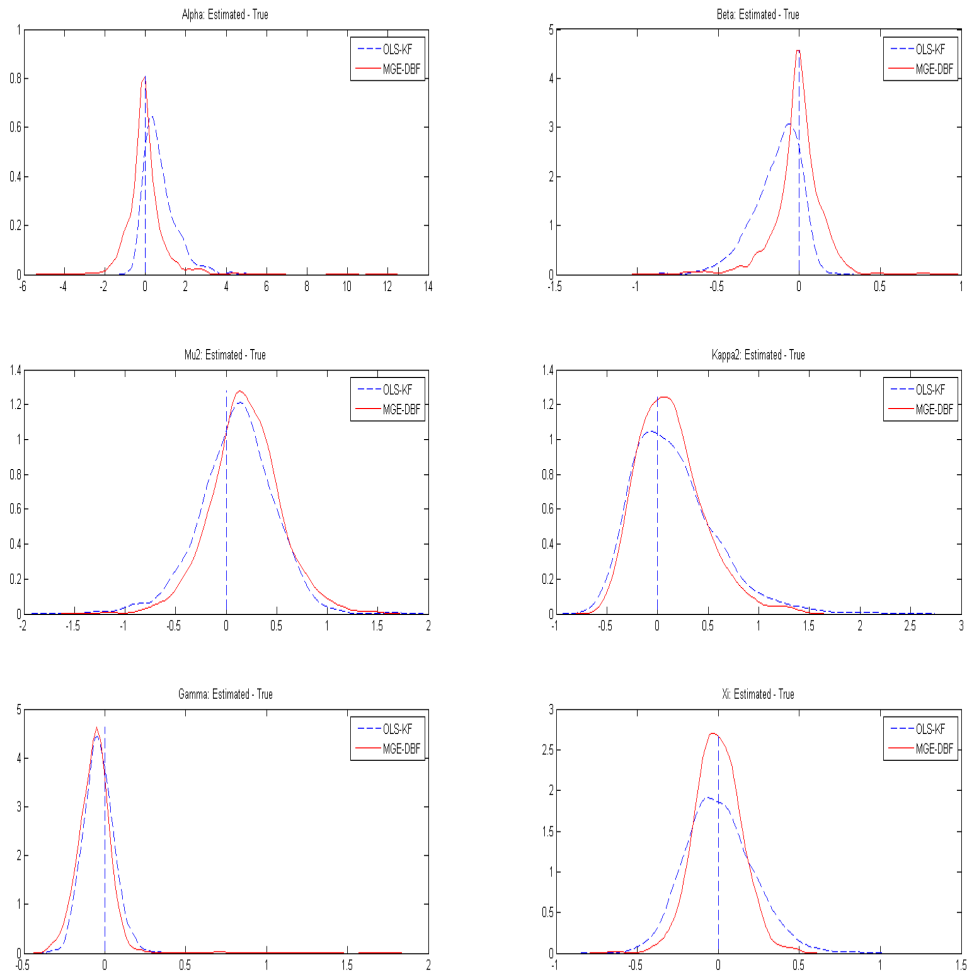

Fig. C-36.: The Comparison between OLS-KF and MGE-DBF

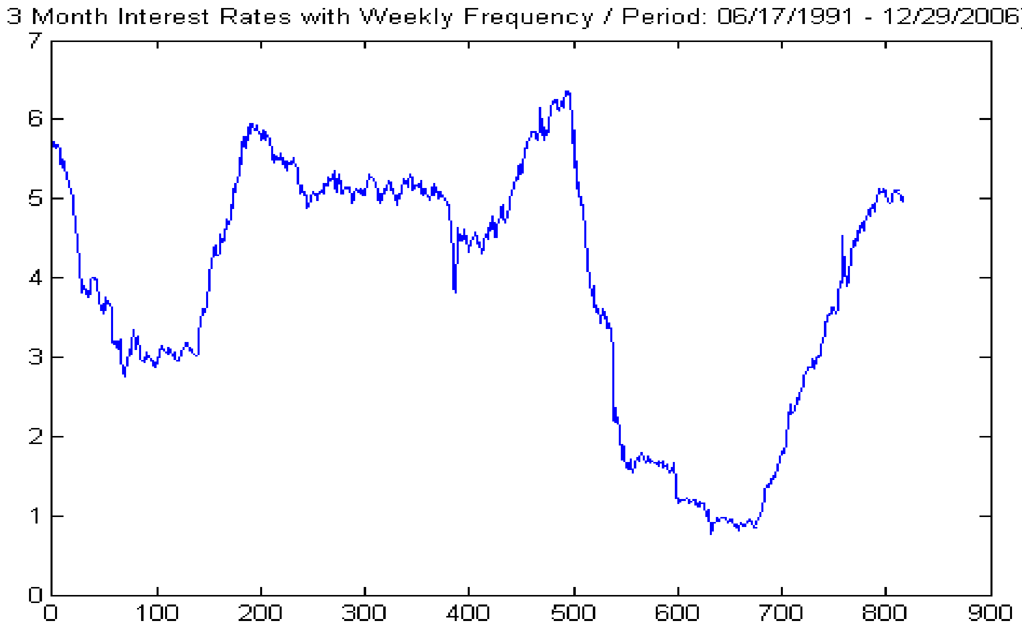

Fig. C-37.: Three Month Interest Rates with Weekly Frequency

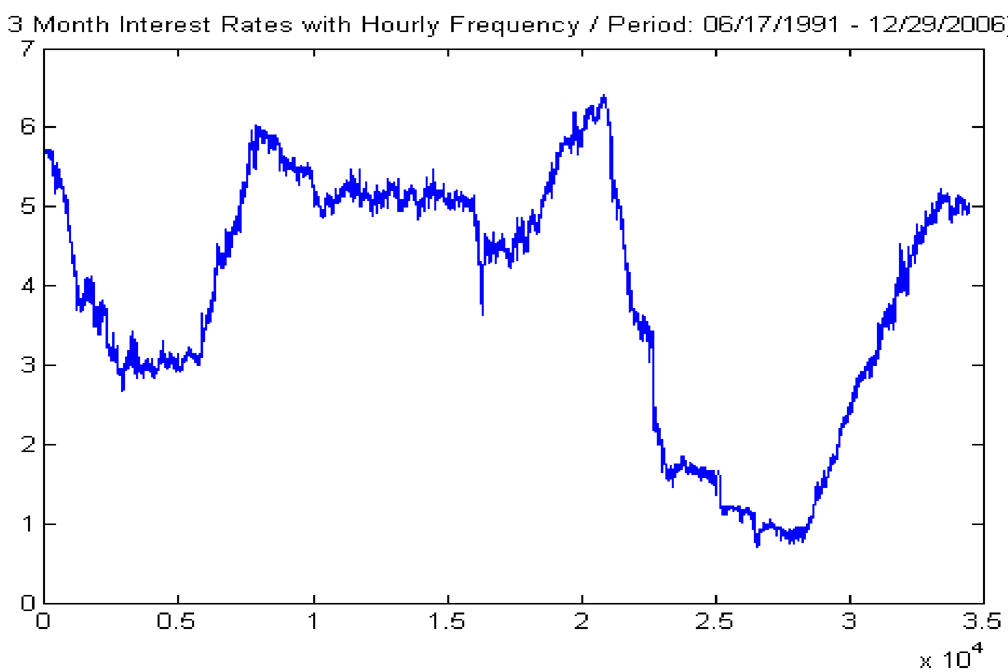

Fig. C-38.: Three Month Interest Rates with Hourly Frequency

VITA

Bong Ju Song received his Bachelor of Arts degree in Economics from Seoul National University in February of 1999. He received his Master of Arts degree in Economics from Seoul National University in February of 2002, and he received his Master of Science degree in Economics from the University of Texas at Austin. He received his Ph.D. in Economics from Texas A&M University in August of 2009. His fields of specialization are Econometrics and Financial Economics. Bongju Song can be reached at Korea Air Force Academy, P.O. Box 335-2 Ssangsu-Ri, Namil-Myun, Cheongwon-Gun, Chungbuk, 363-849, Republic of Korea, or by email at econosong@yahoo.co.kr