DEVELOPMENT AND TESTING OF ACHIEVEMENT FROM MULTIPLE MODES OF MATHEMATICAL REPRESENTATION: AUDIO, AUDIO-VISUAL, AND KINESTHETIC

A Dissertation

by

SERKAN ÖZEL

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2009

Major Subject: Educational Psychology

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ABSTRACT

Development and Testing of Achievement from Multiple Modes of Mathematical Representation: Audio, Audio-Visual, and Kinesthetic. (August 2009)

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This dissertation is comprised of three articles that build on and support each other. The first article is an extensive literature review, and the other two are empirical studies. In this literature review, the author discussed major theories about human learning processes to guide instructional designers about effective integration of multiple modes in interactive learning environments and explored the knowledge base on representations and manipulatives in mathematics education.

The first empirical study's purpose was to investigate effects of affordances provided with virtual learning environments at different treatment durations. Students from multiple sixth-grade classes were randomly assigned to one of three treatment groups differed by allocated session time (10-, 20-, and 30-minute). The online manipulative tool (OMT), which was designed to scaffold learning in operations with rational numbers, allowed students to use the following three components in any order: (a) audio, (b) audio-visual, and (c) manipulatives. Analyses showed that students who used manipulatives most achieved highest; whereas, students who used audio-visual most achieved the second highest. Additionally, the 30-minute group used each component of OMT the least. A meaningful increase in standard algorithm use over manipulatives suggested a transition from concrete to abstract thinking.

The second empirical study's purpose was to compare OMT's different representational aspects and to estimate OMT's effects on achievement and technology acceptance when compared to those of traditional classroom activities. Elementary- and middle-grade students were randomly assigned to the control group or one of three treatment groups: (a) audio-visual, (b) virtual-kinesthetic, and (c) dual-mode (virtual-kinesthetic and audio-visual combined).

When the control group was compared with experimental groups, pre- and post-test results suggested that OMT was more effective than traditional classroom activities in improving students' understanding of operations with rational numbers. When the students' achievement on pre- and post-tests among experimental groups was compared, no substantial difference was found. However, students in the dual-mode group scored the highest on the technology acceptance survey. Students' technology acceptances also differed among different SES levels but not genders. The results suggested that virtual manipulatives provided additional affordances for conceptual understanding. However, students' acceptances of technology should be considered when implementing new technologies.

DEDICATION

This dissertation is dedicated to my parents

and

my siblings and their families

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NOMENCLATURE

ASQ	After-Software Questionnaire
OMT	Online Manipulative Tool
CS	Component Selection
TAM-M	Technology Acceptance Model - Modified
TRA	Time Ratio for Audio
TRM	Time Ratio for Manipulatives
TRV	Time Ratio for Video

TABLE OF CONTENTS

		Page
ABSTRACT		iii
DEDICATION		v
ACKNOWLEDGEMENTS		vi
NOMENCLATURE		vii
TABLE OF CONTENTS		viii
LIST OF FIGURES		xii
LIST OF TABLES		xiv
CHAPTER		
I	INTRODUCTION: THE IMPORTANCE OF MULTIPLE REPRESENTATIONS AND VIRTUAL MANIPULATIVES	1
II	LITERATURE REVIEW	5
	Overview	5
	Virtual Manipulatives to Support Multiple Modes of Representations in Mathematics Learning	6
	Designing Learning Environments Using Multiple Modes of Representations	7
	Representations	12
	Multiple Modes of Representations	15
	Translational Skills Among Different Modes of Representation	16
	Manipulatives	17
	Physical Manipulatives	18
	Virtual Manipulatives	20
	Time Needed for Learning with Virtual Manipulatives	23
	Research on Virtual Manipulatives	24
	Students’ Use of Virtual Manipulatives	25
	Teachers’ Perceptions of Virtual Manipulatives	27
	Conclusion	30

CHAPTER Page

III LEARNING RATIONAL NUMBERS: WHAT AFFORDANCES DO VIRTUAL LEARNING ENVIRONMENTS PROVIDE? 31

Overview 31
Introduction 32
Affordances 32
Technology and Manipulatives 33
Affordances of Virtual Manipulatives 34
Learning Rational Numbers with Virtual Manipulatives 36
Time Needed for Learning with Virtual Manipulatives 36
Empirical Research on Virtual Manipulatives 38
Research Questions 39
Method 40
Participants 40
Intervention 41
Online manipulative tool 41
Procedures 44
Measures 46
Time 46
Component selection 46
Student achievement 46
Treatment duration 47
Results 47
Research Question 1: What Are the Relative Contributions of Audio, Audio-Visual, and Manipulative in OMT on Student Achievement? 47
Research Question 2: What Is the Relationship Between Treatment Duration (i.e., 10-, 20-, or 30-Minute) and the Time Spent on Each Component (i.e., TRA, TRV, and TRM)? 48
Research Question 3: How Does Student Achievement with OMT Change Between Sessions in Each Group (i.e., 10-, 20-, and 30-Minute Groups)? 51
Research Question 4: How Does Student Achievement with OMT Change During a Session in Each Group (i.e., 10-, 20-, and 30-Minute Groups)? 55
Discussion and Conclusion 56

CHAPTER Page

IV AN ONLINE MANIPULATIVE TOOL: EFFECTIVENESS OF THE TOOL AND STUDENTS' TECHNOLOGY ACCEPTANCE 61

Overview 61
Introduction 62
Same Concept, Different Representations 62
Beliefs and Attitudes 64
Assessment: The Crossroad Between Teaching and Learning 68
Research Questions 69
Method 71
Design 71
Participants.. 71
Procedure 72
Online Manipulative Tool.. 73
Measures 74
Data Analysis 76
Research question 1: What is the impact of the OMT on students' understandings of operations with rational numbers? How does the impact of the OMT on achievement differ by the representational mode (i.e., audio-visual, virtual-kinesthetic, and dual-mode)? 77
Research question 2: Do students' beliefs and attitudes toward usefulness of OMT change over time as they experience OMT? 77
Research question 3: Is there a difference in technology acceptances of (a) boys and girls and (b) students with different SES as measured by TAM-M? 77
Research question 4: Is there a difference in technology acceptances of students by the representational mode (i.e., audio-visual, virtual-kinesthetic, and dual mode) as measured by TAM-M? 78
Research question 5: How do students perform on OMT as compared to traditional paper-and-pencil methods? 78
Results 79

CHAPTER Page

Research Question 1: What Is the Impact of the OMT on Students' Understandings of Operations with Rational Numbers? How Does the Impact of the OMT on Achievement Differ by the Representational Mode (i.e., Audio-visual, Virtual-kinesthetic, and Dual-mode)? 79
Research Question 2: Do Students' Beliefs and Attitudes Toward Usefulness of OMT Change over Time as They Experience OMT? 80
Research Question 3: Is There a Difference in Technology Acceptances of (a) Boys and Girls and (b) Students with Different SES as Measured by TAM-M? ... 82
Research Question 4: Is There a Difference in Technology Acceptances of Students by the Representational Mode (i.e., Audio-visual, Virtual-kinesthetic, and Dual mode) as Measured by TAM-M? 84
Research Question 5: How do Students Perform on OMT as Compared to Traditional Paper-and-pencil Methods? 86
Discussion and Conclusion 87

V CONCLUSIONS 93

REFERENCES 97

APPENDIX A 115

APPENDIX B 117

APPENDIX C 119

APPENDIX D 121

APPENDIX E 123

APPENDIX F 124

VITA 126

LIST OF FIGURES

FIGURE		Page
1	A working example of Kaput's taxonomy of representation related to slope	14
2	Lesh's translation model for classroom instruction	15
3	(a) Screenshot of virtual fraction strips. (b) Screenshot of virtual area board	42
4	An example of clickable screen elements	43
5	Online manipulative tool time allotment of each group by session and week	45
6	Percentages of correct answers in each session for 10-minute group	51
7	Percentages of correct answers in each session for 20-minute group	52
8	Percentages of correct answers in each session for 30-minute group	53
9	Mean cumulative percentage of correct answers over nine sessions taken at 2-minute intervals in 10-, 20-, and 30-minute groups	56
10	95% CIs around gain scores for each group	80
11	CIs around means for each administration of ASQ	81
12	95% CIs around means for each administration of ASQ by group	82
13	Total scores on TAM-M by gender displayed with box plots and 95% CIs	83
14	Total scores on TAM-M by SES levels displayed with box plots and 95% CIs	84
15	Total scores on TAM-M by group displayed with box plots and 95% CIs	85

FIGURE Page

16 (a) Comparison of low and high performers on the posttest using 95% CIs.
(b) Comparison of low and high performers on algebraic notation scores on OMT using 95% CIs.
(c) Comparison of low and high performers on kinesthetic representation scores on OMT using 95% CIs 86

LIST OF TABLES

TABLE		Page
1	Participants' Demographic Information by Group	41
2	Summary of Regression Analysis for Student Component Selection Variables Predicting Student Achievement	48
3	Non-Parametric Correlations (Spearman ρ) (N = 60)	49
4	Descriptive Statistics by Group Membership	50
5	Frequencies of Answers by Method And Session in 30-Minute Group	54
6	Demographics of Grade 4, 5, 7, and 8 Sample	72

CHAPTER I

INTRODUCTION: THE IMPORTANCE OF MULTIPLE REPRESENTATIONS AND VIRTUAL MANIPULATIVES

In mathematics education, there has been a shift from classic to nontraditional teaching and learning practices with multiple representations (e.g. National Council of Teachers of Mathematics [NCTM], 2000; Rider, 2007). A substantial amount of research has demonstrated the effectiveness of multiple representations in enhancing students' conceptual understanding of mathematical concepts (e.g., Ainsworth, 2006; Amato, 2008; Fennell & Rowan, 2001; Gagatsis & Elia, 2004; Perry & Atkins, 2002; Suh, Moyer, & Heo, 2005). Different modes of representations can be integrated in teaching and learning environments using various instructional techniques and tools. One such tool that has been used in mathematics education is virtual manipulatives. Virtual manipulatives are interactive learning tools that can combine multiple representations and provide support for constructing mathematical knowledge (Moyer, Bolyard, & Spikell, 2002). Virtual manipulatives enhance students' attitudes toward mathematics as well as help students improve their problem-solving skills by scaffolding translation between different modes of representation (Crawford & Brown, 2003).

This dissertation follows the style of *Educational Technology Research and Development*.

Despite the research support for development of higher order thinking skills afforded by different representational forms presented via virtual manipulatives, little is understood about how students interact with multiple representations in virtual learning environments. Even though each representation provides similar information, the load that each representation puts on students' cognitive resources may differ (Larkin & Simon, 1987). Not only do individual representations have different impact on students' conceptual understanding but also integrating multiple representations may have interaction effects among different modes presented. Therefore, integration of multiple representations becomes an important consideration in instructional design. Consequently, to answer questions such as which representations students use and which representations are correlated with success in online mathematics learning tools, research on effects of web-based instructional tools such as virtual manipulatives on students' mathematical learning is needed (cf. Martin & Schwartz, 2005).

An important factor in students' benefiting from the aforementioned advantageous features of virtual manipulatives depends on their acceptance of this technology. Technology acceptance is paramount for actually using the technology (Lee, Kozar, & Larsen, 2003). Previous research suggested that the differential uses of technology in the schools are related to user backgrounds (e.g., gender). Information about inequities in students' beliefs and attitudes toward a technology tool due to student characteristics can provide insights about the differential use of technology in classrooms.

In the implementation of virtual manipulatives, time is an important consideration for both teachers and students because teachers have limited time to cover the curriculum and students need to develop not only conceptual understanding but also understanding of how manipulatives work (Rupe, 1986). Given that allocated time is fixed in a middle-grade mathematics class, time spent learning software means there will be less time allocated for learning the content. Thus, interactive learning environments should be transparent enough to increase the time on the content. In the literature no standard time has been established for using interactive learning environments (Bass, Ries, & Sharpe, 1986; Salerno, 1995). However, there is research arguing that students' achievement can be increased by providing more time on computer-assisted instruction (Louw, Muller, & Tredoux , 2008). Per contra, Morrison (2008) discusses an optimal time when the learning reaches a peak. Gain in achievement beyond this optimal time, if there is any, is virtual (Morrison; Son, & Sethi, 2006).

Multiple representations can help students improve conceptual understanding of mathematics. However, providing students with multiple modes of a concept may interfere with students' learning if information about their cognitive resources is ignored. Thus, effective integration of various representations in interactive learning tools is paramount. In this study, the guidelines for integrating multiple modes of representations in virtual manipulatives and the knowledge base on the effectiveness of virtual manipulatives were explored. Furthermore, given the importance of utilization of various modes of representations in interactive learning tools, an online tool in which students manipulate multiple representations of fractions was developed and its effectiveness on student achievement was investigated. Effects of students' beliefs about and attitudes toward learning tools led this study to analyze the relationship between beliefs and attitudes and students' achievement as well as the change in students' beliefs about and attitudes toward online manipulative tools (OMTs). Effect of different representational components in student learning at different treatment durations were also investigated in this study because time is an important consideration for teaching and learning with interactive learning tools.

CHAPTER II

LITERATURE REVIEW

Overview

Research has suggested the mode in which information is presented has impact on learners' understanding. Thus, the selection and combination of modes to present information becomes an important issue in educational technology. Various modes in which information is presented to learners form representations. This literature review lays the theoretical foundation for integrating multiple representations in interactive learning environments, in particular virtual manipulatives, to positively affect student learning. Virtual manipulatives are instructional tools that provide the opportunity to combine multiple representations in electronic environments as compared to physical manipulatives. Virtual manipulatives' capability to connect different modes of representation simultaneously has been demonstrated to improve conceptual understanding as well as positive attitudes toward mathematics. This article is intended to discuss major theories about human learning processes to guide instructional designers about effective integration of multiple modes in interactive learning environments and to explore the knowledge base on representations and manipulatives in mathematics education.

Virtual Manipulatives to Support Multiple Modes of Representation in Mathematics Learning

The importance of representation in mathematics education has been highlighted by numerous researchers (e.g. Goldin, 2003; Kaput, 1987; Lesh, 1979; National Council of Teachers of Mathematics [NCTM], 2000). Research studies indicate that providing accurate representations increases student understanding in mathematics. Moreover, presentation of concepts in multiple modes improves student acquisition (e.g., Capraro, Ding, Matteson, Li, & Capraro, 2007; Goldin & Shteingold, 2001; Kulm, Capraro, & Capraro, 2007). In k-12 education research, there is support for the use of physical and virtual manipulatives as hands-on, concrete, and kinesthetic representations of concepts to increase student motivation and achievement (e.g., Clements, 1999; Green, Piel, & Flowers, 2008; Moyer et al., 2002).

When presenting information in multiple modes of representation, utilizing students' cognitive resources effectively is paramount (Baddeley & Hitch, 1974; Clark & Mayer, 2008; Sweller, 1988). Thus, an important issue in instructional design is to develop effective presentation models for students. Representation of a concept in multiple modes may interfere with learning if information about students' cognitive resources is ignored by instructional designers. The development of instructional tools, such as virtual manipulatives, that effectively integrate multiple modes of representations needs to be informed by research.

The purpose of this review of literature is to discuss major theories about human learning processes to guide instructional designers about effective integration of multiple modes in interactive learning environments. In addition, research in mathematics education about representations and manipulatives is summarized. Thus, this article is intended to explore the guidelines for integrating multiple modes of representations in virtual manipulatives and the knowledge base on the effectiveness of virtual manipulatives.

Designing Learning Environments Using Multiple Modes of Representations

Research has suggested the mode in which information is presented has impact on learners' understanding (e.g., Leahy, Chandler, & Sweller, 2003; Marois, 2005; Marois & Ivanoff, 2005; Sweller, van Merrienboer, & Paas, 1998). Thus, the selection and combination of modes to present information becomes an important issue in educational technology. In this section, information processing theory is discussed to provide critical information about human learning processes. Moreover, Baddeley and Hitch's (1974) working memory model employed by information processing theory of learning and memory and an implication of working memory model, Sweller's (1988) cognitive load theory, are discussed to guide instructional designers about effective presentation of information in interactive learning environments.

"During our lifetime, our brain will have amassed 10^9 to 10^{20} bits of information, which is more than fifty-thousand times the amount of text contained in the U.S. Library of Congress, or more than five times the amount of the total printed material in the world!" (Marois, 2005, p. 30). This example shows the limits of human brain in storing information throughout one’s life. Even though the human brain has almost limitless capacity, the amount of information processed at a time is limited.

The processes of learning (i.e., processes of information) form the basic structure of information-processing theories of learning (Gagne, 1985). Models employed by information-processing theories of learning and memory posit internal structures for human brain: (a) sensory registers, (b) short-term memory (i.e., working memory), and (c) long-term memory. All the information received through senses (i.e., seeing, hearing, and touching) is send to sensory registers (Ellis & Hunt, 1983). The information selected (attentive selection) in sensory registers by human brain, then, transfers to short-term memory, which is a temporary storage and has a limited capacity in terms of the number of items can be held. The information in short-term memory is lost unless it is processed or practiced within a short period of time (i.e., 5 to 20 seconds). If the information is processed in short-term memory, then it is stored in long-term memory.

A human brain receives information by seeing, hearing, or touching (Ellis & Hunt, 1983). Any information received is directly sent to sensory registers. Only information that catches the human’s attention is transformed into patterns and sent to working memory. This process is called selective perception (Gagne, 1985; Gagne, Briggs, & Wager, 1992). For example, a visual mark “a” on a paper becomes the letter “a” when it is recognized in sensory registers and is transmitted to working memory. Working memory has not only limited time to keep information but also limited number objects to handle at a time (Gagne et al.; Marois, 2005). The information in working memory is lost if the information is not rehearsed within the given limited time or space.

If the information in working memory is transformed into meaningful form (i.e., semantic encoding), then it can enter long-term memory to be kept for long periods of time (Gagne et al.). Baddeley and Hitch (1974) proposed a model (i.e., working memory model) to explain working memory's functioning and transmitting information into long-term memory.

Working memory refers to "a brain system that provides temporary storage and manipulation of the information necessary for such complex cognitive tasks as language comprehension, learning, and reasoning" (p. 556) and stands at "the crossroads between memory, attention, and perception" (Baddeley, 1992, p. 559). Baddeley and Hitch (1974) originally described the working memory as consisting of one main component called *central executive* and two subcomponents: (a) *visuospatial sketch pad* and (b) *the phonological loop.* More recently, Baddeley (2000) proposed to add the third subcomponent called *episodic buffer*. The central executive is in charge of coordination of the subcomponents and integration of information coming from these subcomponents. The visuospatial sketch pad is responsible for maintenance and manipulation of visual representations whereas the phonological loop is carrying the load for storing and rehearsing verbal information. The last subcomponent, episodic buffer, which is controlled by the central executive, has a role of temporary interface between other subcomponents and long-term memory.

The two subcomponents, the phonological loop and the visuospatial sketch pad, can work independently to process information simultaneously. Each system has limited capacity to process information and can process one piece of information at a time.

Thus, for more efficient information acquisition these two systems should be utilized simultaneously instead of loading the information to one of the systems. Cognitive load theory emerges as an implication of the working memory model to take advantage of these two subcomponents and provide guidelines for instructional designers.

Sweller et al. (1998) stated “Cognitive load theory [CLT] has been designed to provide guidelines intended to assist in the presentation of information in a manner that encourages learner activities that optimize intellectual performance” (p. 251). Sweller et al. drew attention to the limited capacity of working memory and to the importance of selective use of learners’ cognitive resources for effective instruction. Sweller (1994) discussed ineffective instructional designs may interfere with learning by increasing the cognitive load.

Sweller et al. (1998) proposed instructional design principles to reduce cognitive load. *Split-attention effect*, one of the instructional design principles, helps to reduce cognitive load by physically integrating different sources of information in the instructional design in order to lower learners’ needs of mental integration (Sweller et al.). For example, let us think of a learner who is trying to learn how to use software by reading a manual. This learner needs to read the manual first and then apply his or her reading to the software. Thus, the process causes the learner to split the attention between reading the manual and then applying it to the software. In order to reduce cognitive load, instructions could be read to the learner while the learner practices with the software. In this latter case, the information in the manual is integrated in the software as audio. This integration reduces the cognitive load by letting the learner focus on the software while listening to the instructions.

Another instructional design principle suggested by Sweller et al. (1998) was the *modality effect.* This principle suggests incorporating visual and auditory components together to increase the capacity of working memory and decrease the cognitive load (Sorden, 2005). In his recent review of research on the modality effect principle, Mayer (2005) presented an example of a modality effect: Students who received instruction as oral-narration and graphics performed better than students who received instruction as on-screen text and graphics. The oral-narration-and-graphics group could use both auditory and visual channels; whereas, the on-screen-text-and-graphics group's visual channel suffered from being overloaded with two types of visual information.

Various modes in which information is presented form representations. Thus, the theories presented in this section lay the foundation for integrating multiple representations in interactive learning environments to positively affect student learning. Interactive learning environments such as virtual manipulatives have powerful features to combine different modes together on a computer screen. However, such environments should be developed by following instructional design principles of working memory, split-attention effect, and modality effect as described by Baddeley and Hitch (1974), and Sweller (1988).

Representations

Instructional designers use various representations to effectively present information. Goldin (2003) broadly defines representation as any configuration of characters, images, or concrete objects that can symbolize or *represent* something else. Representational systems are both internal and external in nature and can be created by forming individual representations such as letters, numbers, words, and real-life objects (Goldin). Kaput (1991) referred to internal representations as "mental structures" and defined them as "means by which an individual organizes and manages the flow of experience." Internal representation systems exist within the mind of the individual and consist of constructs to assist in describing the processes of human learning and problem solving in mathematics (Goldin, 1998). On the other hand, external representations are defined as “externalizations of internal systems of thought” and, in particular, mathematical representations as “simplifications of external systems” (Lesh, 1999, p. 331). Learners use external representations, such as marks on paper, sounds, or graphics on a computer screen, to organize the creation and elaboration of their own mental structures (Cifuentes & Hsieh, 2001). Unlike internal representation systems, external representation systems can easily be shared with and seen by others.

One of the essential goals of mathematics education is to develop internal representation systems that interact well with external representation systems (Goldin & Shteingold, 2001). Kaput (1987) identified five interacting types of internal and external representations: (a) mental representations (i.e., internal representations) that learners construct by reflecting on their experiences, (b) computer representations that model the mental representations through computer programs which allow for arrangement and manipulation of information, (c) explanatory representations consisting of models or analogies that create the interaction between mental and computer representations, (d) mathematical representations where one mathematical structure is represented by another mathematical structure, and (e) symbolic representations such as formal mathematical notations.

To understand Kaput's (1987) taxonomy of representation, a working example of different types of representation related to *slope* is presented as follows (see Figure 1). When learning about positive slopes, a student might internally imagine a hill. This mental representation can be replicated on a computer screen. The student can create his or her own model that incorporates his or her mental representation through a computer representation. If the model is a viable model, then this model can be an explanatory representation for the concept of slope. The student, then, can sketch a similar mathematical graph of the hill and can name it with the mathematical notation, slope. This graphical representation of slope can, then, be represented as $y = ax + b$ which is the symbolic representation of slope.

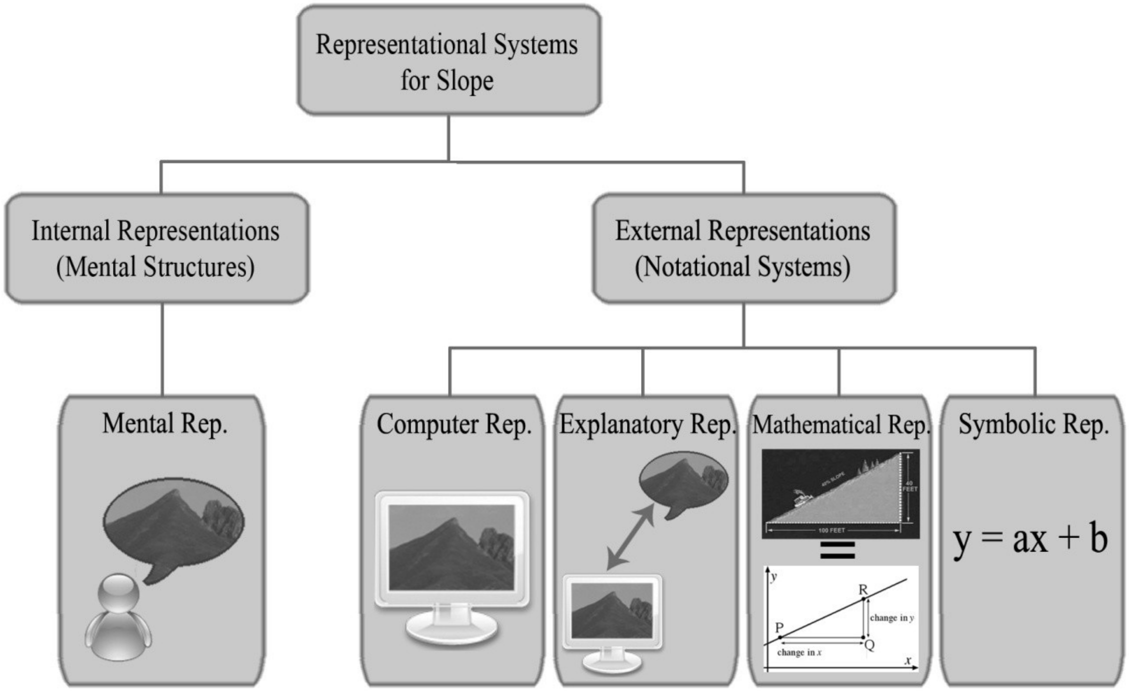


Figure 1. A working example of Kaput's (1987) taxonomy of representation related to slope.

In addition to the importance of the effective interactions between internal and external representations in the acquisition and use of mathematical knowledge, it is essential that students develop fluency among different external representations. Lesh (1979) enumerated multiple modes through which representations can be constructed: (a) manipulatives, (b) pictures, (c) real-life context, (d) verbal symbols, and (e) written symbols. Lesh also provided a translational model to depict the fluency among various representations (see Figure 2). To demonstrate deep understanding, students need to represent their mathematical ideas with different modes of representation and smoothly translate within and between those modes (Lesh, 1999; NCTM, 2000).

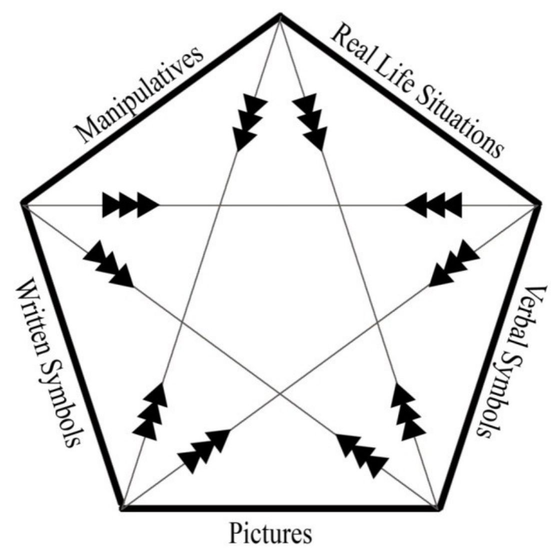


Figure 2. Lesh's (1979) translation model for classroom instruction.

Multiple Modes of Representation

Multiple modes of representation can be used by teachers and students to enhance understanding of mathematics. Most research has shown that providing students with accurate representations improves student learning (e.g., Capraro et al., 2007; Fennell & Rowan, 2001; Goldin & Shteingold, 2001; Kulm et al., 2007; Perry & Atkins, 2002). However, different representation modes might have differential impact on student understanding. One mode might be more relevant or effective than another one for teaching a specific concept (Ball, 1990). Not only accurate information but also appropriate presentation of information is crucial in teaching and learning.

Representations that let students actively involve the subject are more effective in student learning rather than the representations which do not support student active involvement. Providing multiple modes of representation goes beyond simply using a single mode in teaching and learning practices (e.g., Gagatsis & Elia, 2004; Suh, Moyer, & Heo, 2005). However, it is important to be cautious about integrating different modes. Providing redundant information with different modes might interfere with learning (Sweller, 1988).

Translational Skills Among Different Modes of Representation

To deepen students' understandings, teachers should provide students with multiple representations of a single concept and focus on students' transition ability from one representation to another. Teachers need to be able to present one concept in multiple modes without relying on a single mode and provide students with appropriate transitions among these representations (Ball, 1990). If teachers fail to implement the transitioning among different representations they present students with, students might build misconceptions (Bay, 2001; Cramer, Behr, Post, & Lesh, 1997).

Along with the teacher use of representation, a student's ability to represent a concept in multiple ways constitutes deep understanding of that concept. In mathematics education research, there is strong evidence that students can grasp the meaning of mathematical concepts by experiencing different mathematical representations (e.g., Amato, 2008; Fennell & Rowan, 2001; Gagatsis & Elia, 2004; Goldin & Shteingold, 2001; Perry & Atkins, 2002; Suh et al., 2005) and making connections and translations between these modes of representations (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). For example, asking a student to restate a problem in his or her own words, to draw diagrams to illustrate the concept, or to act out the problem are some ways of translating among representations. This translational skill among different modes of representation can support students' conceptual understanding (Suh & Moyer, 2007). Among various instructional tools that are used to provide students with different representations, manipulatives, particularly virtual manipulatives, occupy a big role in mathematics classrooms. In the next section, research on both physical and virtual manipulatives will be explored from multiple representations lens.

Manipulatives

In k-12 education, manipulatives are used as hands-on, concrete, and kinesthetic representations of concepts. The idea of using manipulatives in teaching and learning has been a well-known and accepted educational practice for a very long time. In his book, *Some Thoughts on Education,* the English philosopher and educator, John Locke, provided some early notes on the use of physical manipulatives for teaching the alphabet (Locke, 1836). Using concrete models prior to using abstract forms has been implemented as a strategy in teaching mathematics for almost two centuries (Brownell, 1928). During the late 1960s and early 1970s, a large number of researchers studied the effectiveness of physical manipulatives for mathematics instruction (e.g., Dienes, 1967; Fitzgerald, 1972; Kieren, 1969). With the emergence of personal computers, physical manipulatives have started to be converted into virtual manipulatives on computer screens. Thus, research on manipulatives have evolved to include studies about the effectiveness of not only physical (e.g. Green et al., 2008; Karshmer & Farsi, 2008; Moyer, 2001) but also virtual manipulatives (e.g., Moyer et al., 2002) as well as the comparison studies of both types (e.g., Clements, 1999; Crawford & Brown 2003; Forster 2006; Reimer & Moyer, 2005).

Physical Manipulatives

Physical manipulatives are physical objects that are specifically designed to promote learning by representing abstract mathematical ideas explicitly and concretely (Moyer, 2001). In order to develop robust mathematical understanding and to increase mathematics achievement, teachers in different grade levels use manipulatives in their instruction (Cauley & Seyfarth, 1995; McKinney, 1992; Suh & Moyer, 2007). Manipulatives provide students with kinesthetic representations of abstract concepts before they are presented with symbolic representations (Gardella, 2000). Students can build connections between concrete and abstract levels of mathematics as they see, touch, move, and rearrange manipulatives (Kanter, Dorfman, & Guillot, 1992).

Research supports the effectiveness of physical manipulatives in teaching mathematics (Dienes, 1967; Green et al., 2008; Sowell, 1989; Suydam & Higgins, 1977). Suydam and Higgins in their comprehensive review and synthesis of k-8 mathematics education research on activity-based learning concluded that regardless of students' prior achievements, abilities, and socioeconomic levels manipulatives with pictorial representations were more effective in student learning than symbolic procedures alone. In a later meta-analysis of sixty studies that analyzed the effectiveness of mathematics instruction with the use of manipulatives, Sowell concluded that mathematics achievement of students from kindergarten to post-secondary school improved through the use of manipulatives provided that teachers were knowledgeable about the manipulatives used.

To enhance students' experience with manipulatives, besides the aforementioned advantages of physical manipulatives, it is also important to explore disadvantages of physical manipulatives. Even though there is evidence in mathematics education that supports manipulatives as being helpful for teaching and learning, manipulatives cannot solve all problems associated with students' understanding of mathematical concepts (Ball, 1992; Baroody, 1989; Fenema, 1972). Although kinesthetic experience of manipulatives enhances student thinking and understanding, students are not always capable of drawing proper conclusions from their experiences with manipulatives (Ball).

Physical manipulatives can only provide students with one mode (i.e., kinesthetic representation) to acquire mathematics concepts. However, students are not necessarily capable of making the connection between kinesthetic and symbolic representations by themselves, and physical manipulatives are missing features of providing instructions, guidance, and feedback based on students' interaction with manipulatives to scaffold the transition from concrete to abstract concepts (Ball, 1992). For example, if a student needs guidance, he or she should ask his or her teacher at the teacher's convenience because the teacher is not only responsible for one students' learning but all the students in the classroom. In addition, students may not find opportunities to ask for the same instruction that their teacher provided with as many times as they need. Another drawback of physical manipulatives is the lack of immediate and specific feedback. Any feedback based on students' progress is given by their teacher if the teacher is at the right place at the right time. In other words, because each student does not have his or her own teacher, the teacher cannot monitor each student's progress continuously (Kim, 1993). Thus, the teacher may not catch each student's misconceptions or incorrect paths. All in all, despite the advantages of physical manipulatives in students' mathematical understanding, physical manipulatives lack some features to foster students' understanding.

Virtual Manipulatives

With the emergence of high performance Web technologies such as Java® and Flash®, virtual manipulatives are becoming capable of effectively addressing instructional design guidelines to facilitate mathematical representation. Virtual manipulatives are computationally enhanced online versions of physical manipulatives and can address the aforementioned drawbacks of physical manipulatives. Moyer et al. (2002) defined virtual manipulatives as "…interactive, Web-based visual representation of dynamic object[s] that present opportunities for constructing mathematical knowledge" (p. 373). Moyer et al. emphasized the importance of engaging nature of virtual manipulatives and the opportunity they provide students to control computer objects in similar ways as physical manipulatives do.

The term, virtual manipulative, is widely used to describe interactive kinesthetic representations of dynamic objects. However, it is possible to find different terminologies used in the literature such as mathlet (DiGiana, n.d.), widget (Miller, Brown, & Robinson, 2002), gizmo (Cholmsky, 2003), or computer manipulative (Clements & McMillen, 1996) in place of virtual manipulatives. Throughout this paper, the term, *virtual manipulative,* is consistently used as the descriptor for such tools.

Virtual manipulatives provide additional features that cannot be provided by physical manipulatives (Reimer & Moyer, 2005; Suh et al., 2005). One such feature is their capability to connect different modes of representation, such as dynamic visual and symbolic representations, for a single concept. On a computer screen, it is possible to show the relationship between iconic and symbolic representations simultaneously (Kaput, 1992). For example, a student can work on an area board and create area models to solve rational number problems that are presented in symbolic mode. As the student interacts with virtual area board, the symbolic representation can dynamically change to show the relationship between the concrete and abstract representations. This feature provides students with the opportunity to make connections between representations (Reimer & Moyer). However, the combinations of different representations and their connection need to follow instructional design principles for effective learning. For example, utilizing the modality effect principle some textual information can be incorporated in audio mode to increase the capacity of working memory and to reduce cognitive load.

Another feature of virtual manipulatives is they can be programmed to provide immediate and specific feedback to students regarding the correctness or incorrectness of their processes or solutions. (Clements & McMillen, 1996; Durmus & Karakirik, 2006). This feature provides students the opportunity to try possible solutions and also to learn from their own mistakes (Suh & Moyer, 2007). If their answer is incorrect, the immediate and specific feedback allows them to make corrections and prevent incorrect or faulty practice. In addition, this type of feedback reduces students' cognitive load by providing them with specific information to locate the error and work on it instead of splitting their attention to different parts of the problem to find the error. Moreover, this feature motivates students to continue when their answer is correct (Eggen & Kauchak, 2006).

In addition to the aforementioned beneficial features of virtual manipulatives, their availability and accessibility ease the integration of virtual manipulatives into teaching and learning (e.g. Reimer & Moyer, 2005; Moyer et al., 2002). For example, the National Library of Virtual Manipulatives (NLVM) provides numerous interactive Web-based virtual manipulatives free of charge on their Web site (available at http://nlvm.usu.edu). Another example would be NCTM's Illuminations Web site (available at http://illuminations.nctm.org) where teachers can find lesson plans with activities including virtual manipulatives. Moyer et al. pointed out the importance of such resources for teachers who have limited time and for students who need more interactive and engaging environments.

Despite the additional features of virtual manipulatives that can address the disadvantages of physical manipulatives, the favorable features of virtual manipulatives discussed above bring out a separate argument. Unlike physical manipulatives, virtual manipulatives provide students with flexibility of choosing among several alternative modes of representation, watching or listening to the instruction presented in the online tool as many times as they wish, and using a help screen. That is, students have the control of allocating their time when they are using online manipulative tools. Even though students enhance their conceptual understanding by interacting with different representations, will they be able to allocate their time to learn the content? Thus, time becomes an important consideration in virtual environments (Rupe, 1986).

Time Needed for Learning with Virtual Manipulatives

Time needed for learning is an important consideration for teachers as they implement virtual manipulatives in their lessons because students need enough time to both master the software and understand the concepts (Rupe, 1986). Spending more time on learning the software means there will be limited space to learn the content if the allocated time for instruction is fixed, which is the case for a middle-grade mathematics class. Thus, software with a user-friendly interface should be provided to students to allow them to spend more time on learning the content rather than having to focus on learning the software (Gadanidis, Gadanidis, & Schindler, 2003).

More time in using computer assisted instruction is associated with greater achievement (Louw et al., 2008) although there is no foundation in the literature establishing a standard time for using computer assisted instruction for teaching mathematics (Bass et al., 1986; Salerno, 1995). On the contrary, greater achievement is not always accomplished by providing more time. Moreover, there is research promoting optimal time for a student to reach a peak in his or her learning (Morrison, 2008; Son & Sethi, 2006).

Research on Virtual Manipulatives

Even though there are several individuals and groups who are developing virtual manipulatives, the research on the effectiveness of virtual manipulatives in mathematics education is limited (Bolyard & Moyer-Packenham, 2006; Triona & Klahr, 2003). For the current paper, a review of the literature on the effectiveness of virtual manipulatives in teaching and learning of mathematics was conducted. To locate the relevant published research studies, a two-step approach was used. First, a search was conducted on five databases, namely Academic Search Complete, Education Resources Information Center (ERIC), JSTOR, PsycINFO, and Wilson OmniFile FT Mega, using the keywords such as virtual manipulatives, mathlet, widgets, and gizmo combined with mathematics. As the second step, additional articles were obtained from the references of the articles found in the first step. This literature search resulted in nine studies that were published either in peer-reviewed journals or books from 2000 to 2009. These studies concentrated on either students' manipulative use in classrooms or teachers' perceptions about manipulatives.

Students' Use of Virtual Manipulatives

Five research studies reported in this section investigated effectiveness of virtual manipulatives on student achievement and motivation and learning characteristics that are afforded with virtual manipulatives. Research on physical manipulatives established that their use is associated with higher mathematics achievement and enhanced positive attitudes toward mathematics (Sowell, 1989). Because virtual manipulatives include additional features to advance teaching and learning, one would expect not only that virtual manipulatives improve student achievement and attitudes but also improve further than do physical manipulatives. Aligned with the expectations, research, which is limited to kindergarten and elementary school students, showed gains both in mathematics achievement and attitudes when virtual manipulatives were used (Moyer, Niezgoda, & Stanley, 2005; Reimer & Moyer, 2005; Steen, Brooks, & Lyon, 2006; Suh & Moyer, 2007; Suh et al., 2005).

Research on the comparison of virtual and physical manipulatives at elementary and middle grade levels showed that virtual manipulatives were generally more effective in improving students' conceptual and procedural understanding than physical manipulatives (Reimer & Moyer, 2005; Steen et al., 2006; Suh & Moyer, 2007). Reimer and Moyer conducted a study where they provided students with virtual manipulatives after students studied a subject with physical manipulatives to investigate additional improvement in achievement afforded by virtual manipulatives. Reimer and Moyer found a recognizable increase in students' conceptual knowledge (Cohen's $d = 0.35$) from pre- ($M = 9.58$, $SD = 4.53$) to post-test ($M = 11.00$, $SD = 3.61$) when students used virtual manipulatives but the increase in students' procedural knowledge from pre- (M = 12.63, SD = 1.34) to post-test (M = 12.74, SD = 1.10) was incremental (Cohen's d = 0.09). The small improvement in the procedural knowledge was due to the fact that students' procedural knowledge was already robust at the end of the instruction with physical manipulatives leaving less room for improvement with virtual manipulatives. Steen et al. found students who used virtual manipulatives achieved substantial improvement from pre- (M = 22.2, SD = 4.80) to post-test (M = 30, SD = 1) (Cohen's d = 2.25), but the increase in students' achievement from pre- (M = 27.7, SD = 1.80) to post-test (M = 29.9, SD = 1.20) who used physical manipulatives was not as high (Cohen's d = 1.44). Similarly, Suh and Moyer found students who were taught with virtual ($M_{difference}$ = 53.33, SD_{pool} = 17.32) and physical ($M_{difference}$ = 58.88, SD_{pool} = 21.32) manipulatives had considerable improvements in their algebraic relationships and representational fluencies (Cohen's d = 3.08, d = 2.76, respectively) although the gain with the virtual manipulatives was higher.

Research established some key learning characteristics of students that were afforded by the use of virtual manipulatives (Reimer & Moyer, 2005; Suh et al. 2005). Reimer and Moyer found that students were in favor of using virtual manipulatives over traditional activities and benefited from immediate and specific feedback feature of virtual manipulatives. Feedback scaffolded students' conceptual understanding and provided a safe learning environment where students could recognize and correct their mistakes and misconceptions (Reimer & Moyer; Suh et al.; Suh & Moyer, 2007). Students also capitalized on the interactive features of virtual manipulatives. Interactive features include capabilities of linking different modes of representations such as symbolic, iconic, verbal, and kinesthetic modes (Moyer et al., 2005; Reimer & Moyer; Suh et al.; Suh & Moyer). Suh et al. found that this linking capability emphasized the mathematical relationships. For example, presenting a symbolic representation with its verbal mode simultaneously helped students to develop their mathematical terminology (Reimer & Moyer). Moyer et al. found that students were more creative when they were building patterns on virtual manipulatives than on paper-and-pencil. Moyer et al. explained the reason for students' creativity on virtual manipulatives was that students could flexibly create patterns and test their ideas with virtual manipulatives. Moreover, students could revise their ideas based on their experiments with virtual manipulatives and communicate their mathematical thinking with others.

In summary, research studies reviewed in this paper provided evidence that using virtual manipulatives in classrooms increased student achievement, attitude, and creativity. It was also reported that students found the virtual manipulatives to be helpful. However, the evidence cannot be generalized to the whole population because sample sizes used in the studies reviewed were small, ranging from 19 to 46, and researchers did not provide their sample selection or assignment procedures.

Teachers' Perceptions of Virtual Manipulatives

There is substantial research support on the effectiveness of using manipulatives in classrooms (Sowell, 1989; Suydam & Higgins, 1977). Yet without teacher support mere occurrence of manipulatives does not promise achievement and conceptual understanding (Ball, 1992; Baroody, 1989). Thus, teachers' decisive roles in creating learning environments and beliefs about using strategies such as physical and virtual manipulatives have considerable impact on the effectiveness of such strategies. Four research studies reviewed in this section focused on teachers' perceptions of virtual manipulatives.

Research on teachers' beliefs about using virtual manipulatives across grades k-8 showed that teachers associated virtual manipulatives with higher student motivation and attitude toward mathematics. Teachers reported that virtual manipulatives are likely to improve students' motivation including those who have poor attitude toward mathematics and engage students in activities (Crawford & Brown, 2003; Dorward, 2002). Teachers also noted that virtual manipulatives allowed students to use and develop their creativity (Dorward).

In addition to teachers' positive views about the effects of virtual manipulatives on students' motivation and beliefs, teachers testified the distinct capability of virtual manipulatives to link information sources and to help students with visualizing problems through multiple representations (Crawford & Brown, 2003). Teachers also viewed virtual manipulatives as providing scaffolds for higher-order thinking and problem solving skills (Crawford & Brown). Another aspect of virtual manipulatives that teachers favored was virtual manipulatives facilitated the tracking of students' learning progresses (Crawford & Brown).

Research on teachers' pedagogical beliefs established some mediating factors as teachers' implement virtual manipulatives into their lesson plans. Gadanidis et al. (2003)

found teachers' beliefs about virtual manipulatives had impact on how they integrate virtual manipulatives into their teaching. Teachers who emphasized the importance of discovery learning tended to use virtual manipulatives to explore concepts, whereas teachers who valued teacher exposition tended to use virtual manipulatives only for demonstration or did not use virtual manipulatives at all. Moyer-Packenham, Salkind, and Bolyard (2008) found teachers mainly integrated virtual manipulatives for students' investigation of concepts and to strengthen students' skills. However, the use of virtual manipulatives for introduction or as games was rare.

In addition to teachers' pedagogical beliefs, another factor that influences teachers' selection of virtual manipulatives as an instructional strategy was their self-confidence levels with technology (Gadanidis et al., 2003). Teachers who were comfortable using technology were more likely to use virtual manipulatives. However, teachers who were not as self-confident about technology were more likely either not to use virtual manipulatives at all or to use them only for demonstration.

In summary, teachers' pedagogical beliefs and confidence levels were mediating factors for them to use virtual manipulatives in their classrooms. Teachers from different grade levels who had experiences with virtual manipulatives in their classrooms reported that students had higher motivation when they were engaged with virtual manipulatives. Studies included in this review of virtual manipulatives had limitations for making generalizations because the authors did not employ or explicitly state random selection or assignment procedures.

Conclusion

The use of multiple representations in mathematics teaching and learning has been promoted by various researchers (e.g. Goldin, 2003; Kaput, 1987; Lesh, 1979; NCTM, 2000). Presenting students with multiple modes of a concept improves student understanding (e.g., Capraro et al., 2007; Goldin & Shteingold, 2001; Kulm et al., 2007). However, exposing students to multiple modes of a concept may interfere with students' learning if information about their cognitive resources is ignored. Baddeley and Hitch (1974), Clark and Mayer (2008), and Sweller (1988) discussed ways of presenting information in order to use students' cognitive resources in the most efficient way.

Integration of multiple representations becomes an important consideration in instructional design. Application of instructional design principles related to multiple representations is paramount when a designer determines in what modes information should be presented. Instructional designers should carefully design virtual manipulatives in such a way that learners can process presented information efficiently without overloading students' working memories.

Limited research on virtual manipulatives concluded that virtual manipulatives help students develop conceptual understanding and improve their creativity. There is a need for further rigorous design and development research regarding use of virtual manipulatives and their effect on mathematical learning. In addition, virtual manipulatives should be carefully designed to address instructional design principles such as split-attention and modality effects.

CHAPTER III

LEARNING RATIONAL NUMBERS: WHAT AFFORDANCES DO VIRTUAL LEARNING ENVIRONMENTS PROVIDE?

Overview

The use of virtual learning environments for teaching and learning rational number concepts is common in today's middle-grades mathematics classrooms. Therefore, the purpose of this paper is to investigate effects of affordances provided with virtual learning environments at different treatment durations. Students from multiple sixth-grade classes were randomly assigned to one of three treatment groups that differed by allocated session time (10, 20, and 30 minutes). The online manipulative tool (OMT), which was designed to scaffold learning in operations with rational numbers, allowed students to use in any order or sequence three different components of OMT: (a) audio, (b) audio-visual, and (c) manipulatives. Participating students used OMT during their regular mathematics class (55 min.) so all the students received the same total amount of instruction. The regression analysis showed students who used manipulatives most achieved the highest as compared to audio and audio-visual ($\beta = .437$, $p < .001$). Additionally, as the treatment time increased, student spent less time on each component of OMT and spent more time on symbolic mode. There was a meaningful increase in the use of standard algorithm over manipulatives suggesting a transition from concrete to abstract thinking.

Introduction

This study reports effects of web-based instructional interventions on students' rational number learning and affordances provided by web-based interventions in student learning at different treatment durations. Previous research emphasizes the importance of affordances in instructional design (Norman, 1999) and effective integration of computer activities (Kaput, 1998) to achieve meaningful mathematical learning (Ball, 1988).

Affordances

Affordances are defined as the actionable properties between real world contexts and people (Gibson, 1977). That is, affordances are relationships between objects and a person. This relationship can change from one person to another. These types of affordances are called as perceived affordances (Norman, 1999). The difference stems from people's perceptions of affordances. For example, a chair may afford the action for sitting for one person who wants to sit; whereas, another person perceive the same chair as a step to reach a high point on a shelf. In the latter example, the action afforded by the chair is for using the chair as a ladder.

Interactive learning environments, such as virtual manipulatives, provide students with additional affordances for improved conceptual understanding and transitioning from guidance and instruction to actual learning activities. Although there is a substantial amount of research on the use of physical manipulatives (e.g., Green et al., 2008; Moyer, 2001), little is understood about how students interact with manipulatives and their affordances in virtual worlds for computer-facilitated mathematics learning.

Technology and Manipulatives

Manipulatives are “physical objects specifically designed to foster learning” (Zuckerman, Arida, & Resnick, 2005, p. 859), and virtual manipulatives are “computationally enhanced versions of physical objects, created in an effort to expand the range of concepts that children can explore through direct manipulation” (Zuckerman et al., p. 860). The replication of physical manipulatives in the form of computer applications provides additional features and advantages over traditional manipulatives (Reimer & Moyer, 2005; Suh et al., 2005; Zuckerman et al.).

Virtual manipulatives can be used in the same ways as concrete manipulatives. A computer mouse is the most commonly used interface for interacting with virtual manipulatives. With the mouse, students can flip, slide, and turn a virtual manipulative similar to the ways they can interact with a concrete manipulative. Moreover, virtual manipulatives can include additional features that make them more useful than concrete manipulatives for self-directed learning (Moyer et al., 2002). Instructional-support strategies incorporated into virtual manipulatives such as immediate feedback and help screens improve comprehension (Huang, Chern, & Lin, 2009; Yaman, Nerdel, & Bayrhuber, 2008) and self-efficacy (Wang & Wu, 2008). Some other affordances of virtual manipulatives include the safe environment they offer students to learn by guessing or trial-and-error (Suh et al., 2005). In fact, virtual manipulatives are identified as (a) helping students learn more about mathematical concepts by providing immediate and specific feedback, (b) reducing the amount of time it takes to learn to work with the manipulatives, and (c) enhancing students' enjoyment, attitude, and interest in learning mathematics (Reimer & Moyer, 2005).

Additionally, virtual manipulatives can provide a complete record of user interaction with the tool. For example, cursor movements and screen captures across time can be recorded so the teacher or the researcher can review students' processes as they attempt to answer each question. These archived data (screen captures) afford the teacher or the researcher the ability to examine, at length, the processes that may have led to errors or correct solutions even if the student has moved on to another question.

Affordances of Virtual Manipulatives

Virtual manipulatives are advantageous in their capability to provide additional affordances such as move-ability, draw-ability, highlight-ability, focus-ability, and record-ability. Combining multiple affordances helps students to increase their conceptual understanding (Norman, 1999). For example, integrating audio component into visual information provides additional affordances of listen-ability and playback-ability and helps to reduce cognitive load for learners which makes information acquisition easier (Ertl, Kopp, & Mandl, 2008; Kablan & Erden, 2008; Sweller et al., 1998). Connecting dynamic visual images with abstract symbols is another beneficial feature of virtual manipulatives. Unlike physical manipulatives, virtual manipulatives make use of graphics, numbers, and words on the computer screen to connect the pictorial with the symbolic mode (Suh et al., 2005).

A virtual manipulative can include auditory, audio-visual (i.e., dynamic), and kinesthetic (i.e., interactive) components, and each component puts in additional affordances. For example, auditory component can provide listen-ability affordance; whereas audio-visual component can provide watch-ability and guidance affordance, and virtual manipulatives can provide draw-ability, move-ability, feedback, highlight-ability, and focus-ability affordances. Sweller et al. (1998) suggested utilizing audio in designing online manipulative tools to decrease the cognitive load which led to easier processing of information. In his recent review of research, Mayer (2005) concluded that students who received instruction supplemented with audio performed better than students who received instruction supplemented with on-screen text. Auditory representation complements the information presented in visual format. The audio-visual component in a virtual manipulative has a dynamic nature (Kaput, 2006). In other words, objects provided in audio-visual component change with time. The dynamic feature of audio-visual component facilitates transitioning between representations via linking them together as a function of time. With the change in time, representations on the screen change simultaneously to make connections (Bolyard & Moyer-Packenham, 2006). For example, when a fraction with a denominator seven is created algorithmically, the fraction strip associated with the fraction can be divided into seven pieces simultaneously. In addition to the dynamic feature, virtual manipulatives have interactive aspect that allows students manipulate objects to observe the change in different components and make connections (Kaput). Unlike audio-visual mode, manipulatives will change according to students' inputs and interaction with the manipulatives.

Learning Rational Numbers with Virtual Manipulatives

The NCTM (2000) stated that middle-grade students should have a deep understanding of fractions. However, rational numbers are one of the most difficult concepts for students to master (Perie, Moran, & Lutkus, 2005). There are various strategies for teaching rational numbers (Naiser, Wright, & Capraro, 2004), and research has shown virtual manipulatives can be one of those (e.g., Reimer & Moyer, 2005; Suh et al., 2005; Witzel & Allsopp, 2007). However, teachers may avoid choosing virtual manipulatives as a strategy to teach rational numbers due to the lack of quality in the currently available virtual manipulatives (Donovan, 2008) or teachers' lack of training on the use of virtual manipulatives (Naiser et al.).

Time Needed for Learning with Virtual Manipulatives

Time needed for learning is an important consideration for teachers as they integrate virtual manipulatives both because students need enough time to master the software in addition to the concepts and because teachers have limited time to cover the curriculum. The effect of total amount of time devoted to instruction on student achievement has been investigated for more than 30 years. Research showed that the total amount of time allotted for instruction was a predictor of student success (Louw et al., 2008; Wiley & Harnischfeger, 1974). That is, the more time allocated, the higher student achievement will be. This theory was later refined because simply allocating more and more time was inefficient. This gave rise to the importance of engaged time with the learning task versus the allocated time. Learners were not necessarily engaged in the instruction for the duration of allocated time. For optimal learning, students need not only be provided with necessary allocated time but also spend adequate amount of engaged time with the learning task. Cognitive scientists differentiate between allocated or engaged time and time needed for learning (Son & Sethi, 2006). The time needed for learning is related to individual differences as well as learning environment. Allocating or spending less time than needed for learning has a negative effect on student achievement (Gettinger, 1985).

The time needed for learning is also an important factor for improved achievement with virtual manipulatives (Rupe, 1986). Students need enough time to become proficient about the concepts they are being taught. In computer assisted instruction students need to master not only content knowledge but also the software used. Given that allocated time is fixed in middle-grade mathematics class, time spent learning software means that there will be less time allocated for learning the content. Thus, the design of software should be as transparent as it could be to avoid students struggling with the software but spending more time on the content. There is no standard time established in the literature for using computer assisted instruction for teaching mathematics (Bass et al., 1986; Salerno, 1995). However, Louw et al. (2008) found that more time on computer assisted instruction results in greater achievement. Per contra, more time does not always equate to greater achievement. Morrison (2008) and Son and Sethi (2006) suggest that there is an optimal time when learning reaches a peak and that design of instruction should consider this time frame.

Empirical Research on Virtual Manipulatives

Even though there are several individuals and groups who are developing virtual manipulatives, there is limited research on virtual manipulatives' effectiveness. In general, of the available research some studies are on students' manipulative use in classrooms, and some others investigate teachers' perceptions of manipulatives. However, none of the studies have considered the time needed for learning with virtual manipulatives.

Classroom studies have mainly focused on the effectiveness of virtual manipulatives on mathematics achievement and student motivation. The findings from research on virtual manipulatives are somewhat mixed. Reimer and Moyer (2005), Suh and Moyer (2007), and Suh et al. (2005) showed statistically significant increases in students' achievement when the students used virtual manipulatives as compared to the students who used physical manipulatives or no manipulatives at all. However, in other studies no significant differences were found between students who used virtual or physical manipulatives (e.g., Dorward, 2002).

Research that focused on exploring teachers' perceptions of virtual manipulatives showed that to improve students' understandings of mathematics, teachers preferred using virtual manipulatives as cognitive technological tools (Moyer-Packenham et al., 2008). However, the frequency and allotted time for using virtual manipulatives differed among teachers (Mueller, Wood, Willoughby, Ross, & Specht, 2008). Teachers' pedagogical beliefs and confidence levels were mediating factors in their use of virtual manipulatives (Gadanidis et al., 2003; Hermans, Tondeur, van Braak, & Valcke, 2008; Mueller et al.). Regardless of the grade level, teachers reported their students had higher motivation and more engagement when using virtual manipulatives (Crawford & Brown, 2003; Dorward, 2002; Hermans et al.).

In response to the available research discussed above, I developed an online manipulative tool that incorporated various components of media for rational number concepts and assessed the effectiveness of their affordances in students' transition from concrete to abstract thinking. In addition, we investigated the optimal amount of engaged time with this tool for students to reach a peak in their learning.

Research Questions

Given the strong research support for the improvement of mathematical understanding with concrete manipulatives (e.g., Capraro et al., 2007; Fennell & Rowan, 2001; Gagatsis & Elia, 2004; Goldin & Shteingold, 2001; Kulm et al., 2007; Perry & Atkins, 2002; Suh et al., 2005), it is important to understand the effect of virtual manipulatives on student achievement. Using multiple components of media to present rational number concepts can provide evidence for understanding the relative contributions of each component and the correlation between affordances of each component and success (cf. Martin & Schwartz, 2005). In addition, the amount of time allotted with the online tool may have an effect on students' utilization of components based on their affordances. Investigating this relationship between the time and the use of different components can provide insight about students' transitioning from guidance and instruction to real instructional activity. An important consideration in learning with virtual manipulatives is students need to develop not only conceptual understanding but also understanding of how the manipulatives work. Thus, it is essential to investigate the optimal amount of time per session and number of sessions students need with virtual manipulatives to understand mathematical concepts.

Four major questions guided this study: (a) What are the relative contributions of audio, audio-visual, and manipulatives in OMT on d student achievement?, (b) What is the relationship between treatment duration (i.e., 10-, 20-, or 30-minute) and the time spent on each component of OMT (i.e., TRA, TRV, and TRM)?, (c) How does student achievement with OMT change between sessions in each group (i.e., 10-, 20-, and 30-minute groups)?, and (d) How does student achievement with OMT change during a session in each group (i.e., 10-, 20-, and 30-minute groups)?

Method

Participants

Sixth-grade students (32 female and 28 male) participated in this study. Participants were from three classrooms of a middle school located in the state of Texas. Eighteen students were Hispanic, 21 were African American, 11 were White, and 10 were other. This ethnic composition was similar to the district's which had 55% female, 33% Hispanic, 34% African American, 20% White, and 13% other. Table 1 presents the demographic information of the participants by group.

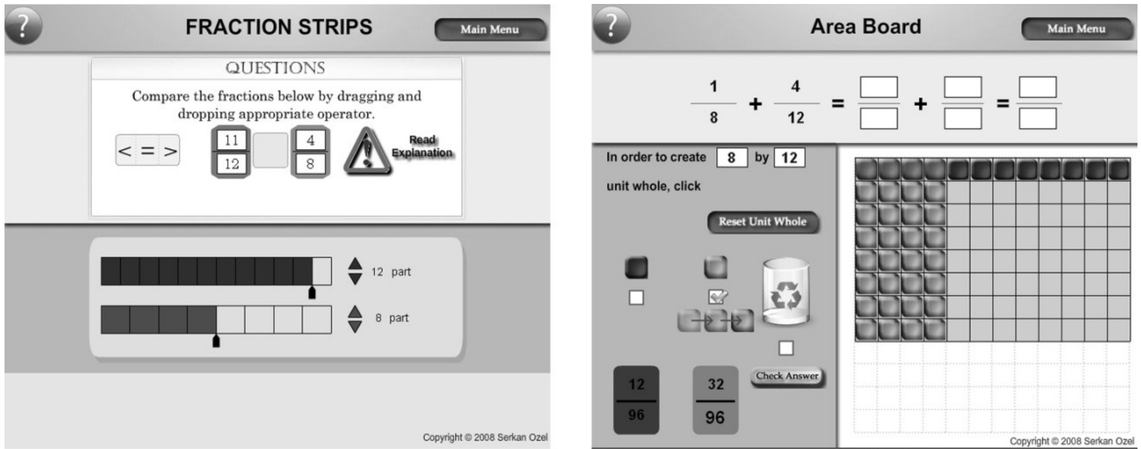
Table 1

Participants' Demographic Information by Group

Group	Gender		*N*
	Female	Male	
10-minute	13	7	20
20-minute	11	9	20
30-minute	8	12	20
Overall	32	28	60

Intervention

Online manipulative tool. The online manipulative tool was an interactive internet-based computer software program (available at http://coe.tamu.edu/~sozel/vm/) designed to present randomly generated addition, subtraction, multiplication, and comparison of rational number problems where each fraction was less than one, and the sums and products were all equal to or less than one.



3.a 3.b

Figure 3. (a) Screenshot of virtual fraction strips. (b) Screenshot of virtual area board.

The OMT contained two virtual manipulatives: (a) virtual area board and (b) virtual fraction strips (see Figures 3a and 3b, respectively). The problems were provided on the same screen with virtual manipulatives and in algorithmic form. The OMT also included audio and audio-visual components that students used if they preferred to do so. The use of any component in solution of problems provided in OMT was optional. That is, students had the control over components they prefer using in their solutions. The audio component consisted of instructions in the help menu and feedback for problems. In the help menu students could click any of the numbers, words, or symbols, and the tool read them aloud (See Figure 4). The feedback, which was provided in text as well as in audio format, included completeness of the algorithmic steps, each step's correctness, and if the answer is in the simplest form. The audio-visual component of OMT contained an instructional video on how to use the manipulatives. The video was provided to students to orient themselves to the online tool, and students could watch the videos as many times as they needed during their log in period.

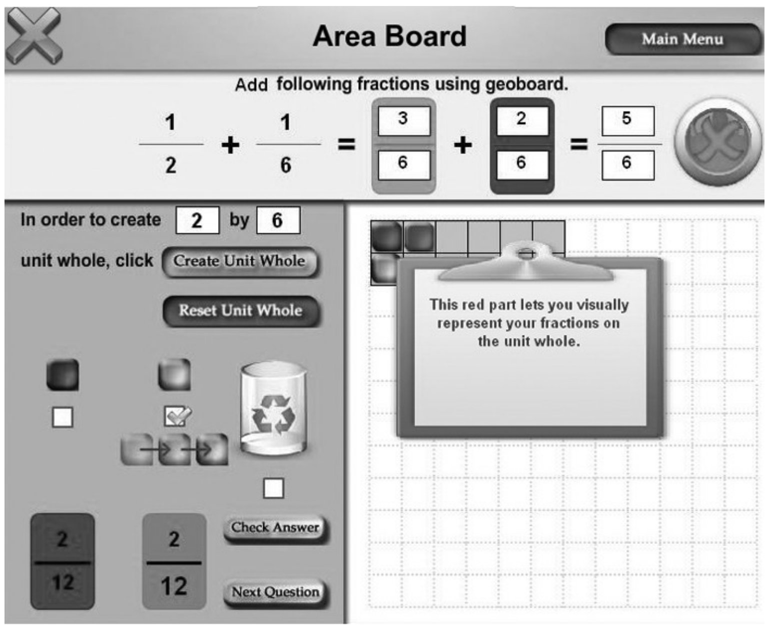


Figure 4. An example of clickable screen elements.

The OMT employed several research protocols to improve the quality of the data collected and to ensure that data were not the result of computer or internet-based resources. The OMT coded the content on the screen every 10 seconds. The coded information on the screen included every click, the question being solved, the current progress on manipulatives, the final solution in both symbolic and pictorial form, and the feedback provided for students' answers. The purpose of screen coding was to provide precise information about students' progress to ensure a complete accounting of each attempt. Additional protocols to ensure data dependability and reliability were collection of data regarding the total time spent on each item and component (i.e., audio, audio-visual, and manipulatives) and the internet protocol (IP) address. The time spent on each component was recorded based on the activation and de-activation times of the components.

Procedures

This study used experimental design where the intervention was OMT to scaffold learning of operations with rational numbers (i.e., addition, subtraction, multiplication, and comparison). This study is conducted in a spring semester. Students from three sixth-grade classes participated in the study. The students in each class were randomly assigned to one of three treatment groups that differed by the allocated time per session: (a) 10-minute group, (b) 20-minute group, and (c) 30-minute group. Each group used OMT three sessions per week over 3 weeks with the only difference among the groups being the time per session (see Figure 5). There were 20 randomly- assigned students in each group. This random assignment of students in each class to different treatment groups avoided the nesting issue of students within teacher (e.g. Raudenbush & Bryk, 2002) because in each group (i.e., 10-, 20-, and 30-minute groups) there were students from all of the teachers.

All groups used OMT without teacher or researcher assistance. The OMT allowed researchers to limit access for each student to a specific amount of time per day and three sessions per week. For example, a student assigned to the first group would be allowed to log in three times per week for 10 minutes per session. All students used their full allotment of time per week.

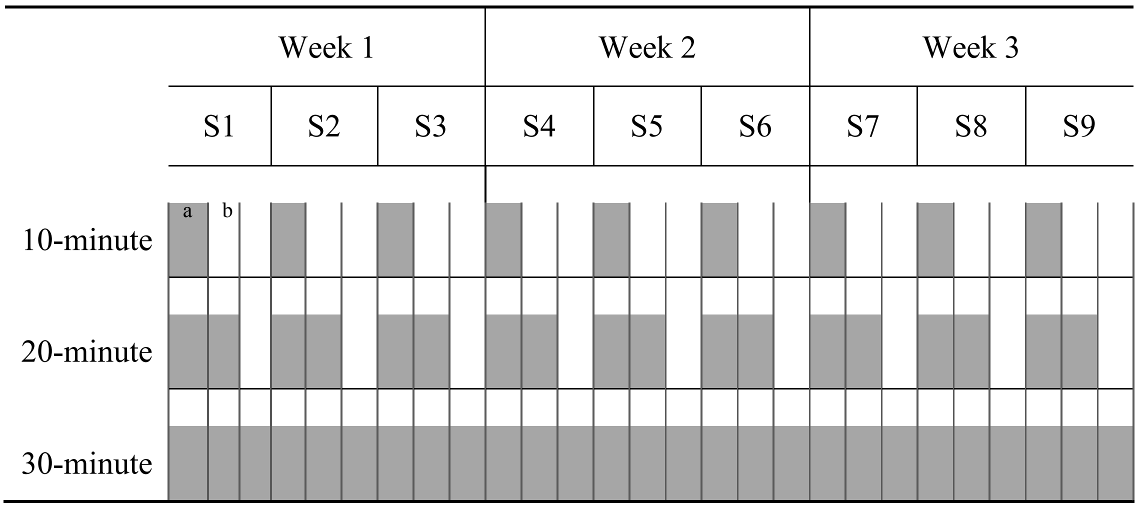


Figure 5. Online manipulative tool time allotment of each group by session and week.

Note. [a] Each shaded cell represents a 10-minute period use of OMT. [b] Each white cell represents a 10-minute period of teacher-assigned activity.

Participants used OMT for three weeks, which was the total time the teachers had allocated to learning rational number concepts covered by OMT. The mathematics class period was 55 minutes, and the intervention took place during this time only. All the students participated in direct instruction delivered by their teachers, but when the teachers assigned seatwork, participants logged onto the system. During the direct instruction the teacher followed the district curriculum and a textbook. The seatwork mainly consisted of teacher-prepared worksheets. All participants received their regular teacher instruction but were not held accountable for all assigned seatwork while they were using OMT. When students completed their computer time allotment, they returned to their class assignments. Participants in the same class were not all necessarily engaged with OMT for the same duration because students in the same class were randomly assigned to one of three conditions.

Measures

Time. The OMT provided students with three components, namely audio, audio-visual, and manipulatives. Students were free to use any of these components. Three time variables were created for each component to determine the time spent on each component: time ratio for audio (TRA), time ratio for video (TRV), and time ratio for manipulatives (TRM). TRA, TRV, and TRM were obtained by getting the ratio of time spent using audio, video, and manipulatives to the total group time assigned to OMT.

Component selection. Component selection (CS) variable to determine students' component preferences was created using the time spent on each component. The component that was used for the longest period of determined the students' component selections. For example, if a student used the audio component the longest, the CS for this student was coded as audio. The CS variable was coded as "1" for audio, "2" for audio-visual, and "3" for manipulatives.

Student achievement. Student achievement was assessed using the answers to problems presented by OMT. Students' answers to the problems were coded as either correct "1" or wrong "0." The total score for each student was calculated by getting the ratio of the number of correct answers to the total number of problems answered. It was reasonable to expect students who had more time would be able to attempt more items. Thus, the ratio provided a method for equitable comparisons across groups with different treatment durations. However, one caveat of calculating students' achievement scores by getting the ratio of the number of correct answers to the total number of problems answered is having extreme situations such as when a student who answers only one question which turns out to be correct may outperform another student who answers 20 questions correctly and 1 question incorrectly. However, the data were scrutinized for such cases, and none was observed.

Treatment duration. Treatment duration was a grouping variable. In other words, three different treatment durations (i.e., 10-, 20-, and 30-minute per session) determined three different groups.

Results

Research Question 1: What Are the Relative Contributions of Audio, Audio-Visual, and Manipulative in OMT on Student Achievement?

To determine the relative contributions of each component (i.e., audio, audio-visual, and manipulative) and student achievement, a multiple regression analysis was run. Students' CS variable, which had three levels, was coded into two dummy variables to compare (a) manipulative with audio-visual and audio and (b) audio-visual with audio on student achievement. The dummy variables were created to be independent from each other (r = 0). Overall the model was important, accounting for just over 37% of the variance in student achievement (see Table 2). Students who used manipulatives most achieved highest; whereas, students who used audio-visual most achieved the second highest (β = .437, $p < .001$). When achievement of students who used audio-visual most was compared to the achievement of students who used audio most, the audio-visual component was associated with higher achievement (β = .464, $p < .001$).

Table 2

Summary of Regression Analysis for Student Component Selection Variables Predicting Student Achievement

Predictors	*B*	*SE*	β	*p*	r_s
Manipulative vs. Audio-Visual & Audio	.086	.020	.437	< .001	.672
Audio-Visual vs. Audio	3.104	.691	.464	< .001	.741

Note. $R^2 = .392$ ($p < .001$; Adjusted $R^2 = .371$).

Research Question 2: What Is the Relationship Between Treatment Duration (i.e., 10-, 20-, or 30-Minute) and the Time Spent on Each Component (i.e., TRA, TRV, and TRM)?

Spearman rho correlation was run to investigate the relationship between treatment duration and the time spent using audio, audio-visual, and manipulatives (i.e., TRA, TRV, and TRM, respectively). As the treatment duration increased (i.e., 10-minute to 30-minute), TRA, TRV, and TRM decreased (see Table 3). Therefore, more time on OMT was associated with less time with each of the components. Because time spent on each of the three components across groups was coded as a ratio of the time each component was used to the total time, different treatment durations would not account for the obtained relationships.

Table 3

Non-Parametric Correlations (Spearman ρ) (N = 60)

	Treatment Duration	TRA	TRV	TRM
Treatment Duration	—			
TRA	-.801*	—		
TRV	-.612*	.873*	—	
TRM	-.342*	.761*	.829*	—

Note. TRA = time using audio, TRV = time using video, TRM = time using manipulatives.

*p < .001.

Cohen's *d* effect size estimates were computed to determine the relative magnitude of difference in TRA, TRV, and TRM across different treatment durations (see Table 4). 10-minute group was used as the baseline, and all effect size estimates were calculated from that baseline. When comparing treatment duration by TRA, the use of audio (i.e., TRA) decreased in the 20-minute group with the obtained effect of d =-2.718 and in the 30-minute group with the effect of d = -2.764. Thus, more time to engage with OMT was associated with less use of the auditory component. The TRV representation showed a similar pattern with the obtained effects of *d* = -1.291 for the 20-minute group and *d* = -1.840 for the 30-minute group. While not as dramatic, students used audio-visual component less, as they gained experience with OMT. When examining TRM by treatment duration, the interest here was if students moved away from using manipulatives and went directly to the algorithm. The TRM followed a similar decreasing pattern as TRA and TRV, and as the treatment duration increased the TRM decreased with the obtained effects of *d* =-.574 for the 20-minute group and *d* = -.951 for the 30-minute group.

Table 4

Descriptive Statistics by Group Membership

	10-Minute		20-Minute			30-Minute		
	M	*SD*	*M*	*SD*	Effect[a]	*M*	*SD*	Effect[b]
TRA	.320	.158	.016	.008	-2.718	.011	.005	-2.764
TRV	.160	.078	.080	.040	-1.291	.053	.026	-1.840
TRM	.576	.284	.432	.213	-.574	.352	.174	-.951

Note. CSR w/ = cumulative success ratio with using manipulatives, TUA = time using audio, TUV = time using video, TUM = time using manipulatives.

[a]Cohen's *d* effect between groups 1 and 2. [b]Cohen's *d* effect between groups 1 and 3.

Research Question 3: How Does Student Achievement with OMT Change Between Sessions in Each Group (i.e., 10-, 20-, and 30-Minute Groups)?

To examine the change in students' success with OMT in each group the average percentages of correct answers in each session for each group were calculated and presented in Figures 6, 7, and 8. As seen in Figure 6, in the 10-minute group the average percentage of correct answers was almost stable around 28% from the first to the fourth session and started to increase from the fifth session on with a peak, 35%, achieved in the last session. In the 20-minute group, a slow increase from 32% to 35% was observed between sessions 1 and 7. However, the average percentage of correct answers increased at a higher rate between sessions 7 and 9 with a peak, 43%, achieved in the last session (see Figure 7).

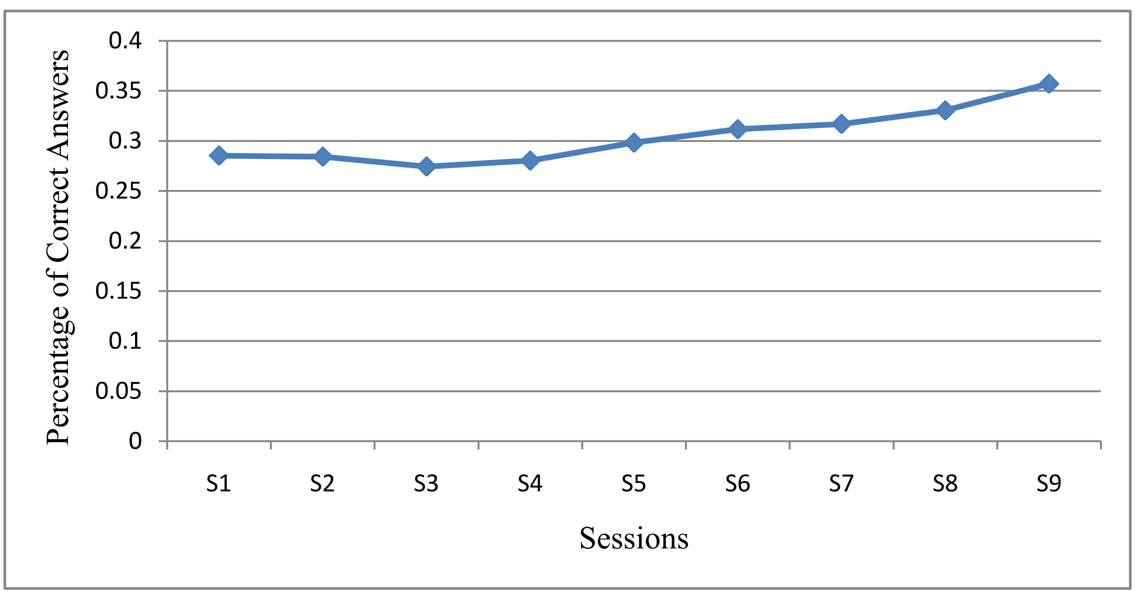


Figure 6. Percentages of correct answers in each session for 10-minute group.

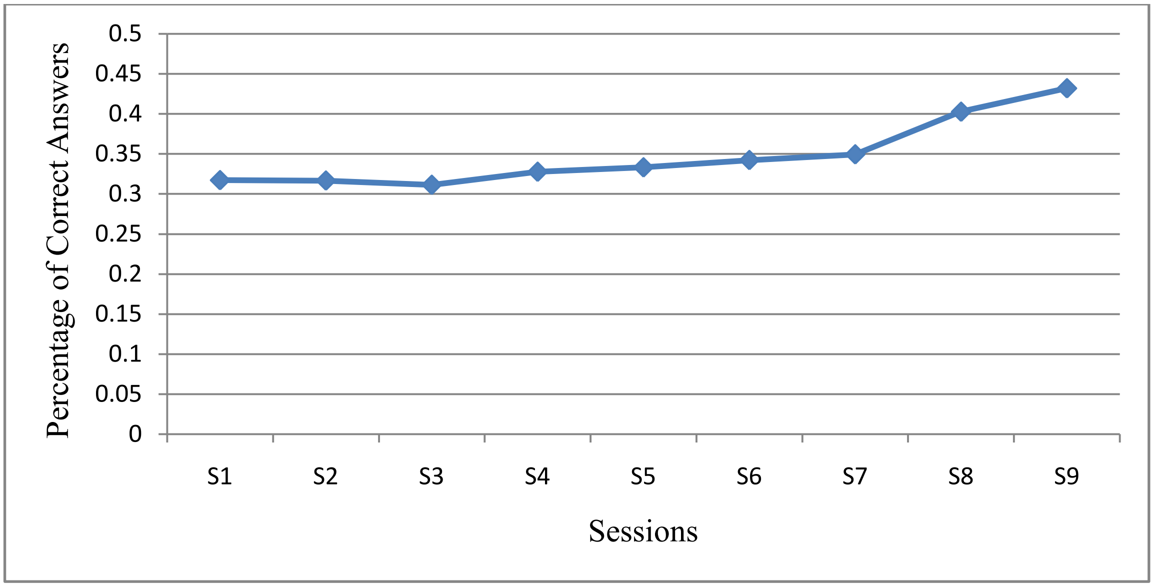


Figure 7. Percentages of correct answers in each session for 20-minute group.

The achievement pattern between sessions in the 30-minute group was substantially different than the other two groups. Students in the 30-minute group started with 74% correct response rate, which was almost twice as high as the other groups. However, this percentage decreased to as low as 69% in the fourth session (see Figure 8). From the fourth session on, the average percentage of correct answers started to increase at a decreasing rate till the last session, where a slight improvement was achieved compared to the initial correct answer rate.

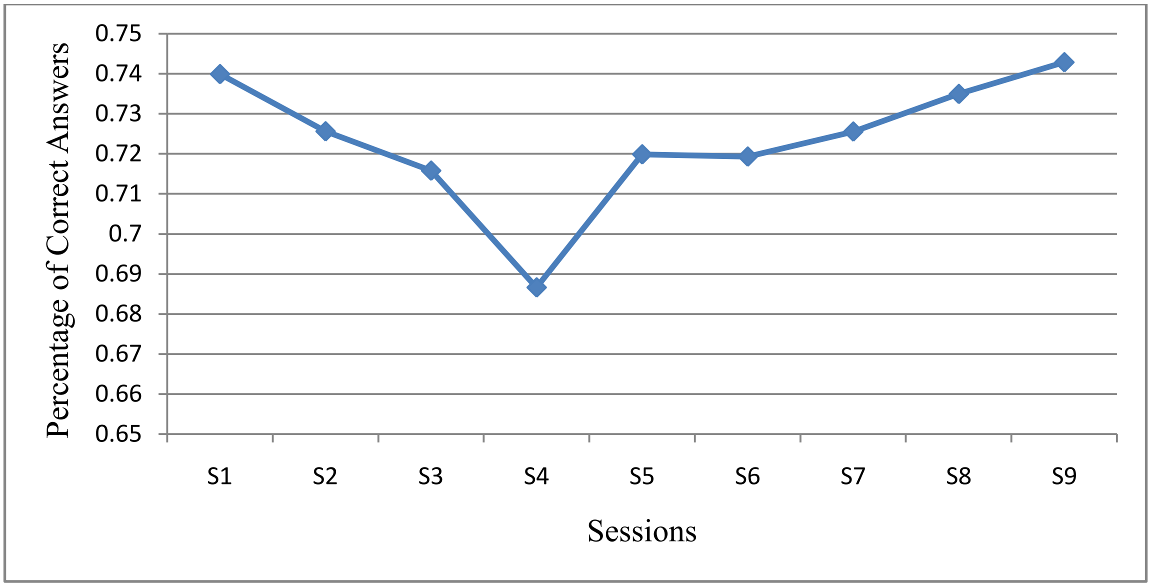


Figure 8. Percentages of correct answers in each session for 30-minute group.

Because the achievement pattern of the 30-minute group differed considerably from 10- and 20-minute groups, solution methods in the 30-minute group were examined closely (see Table 5). Students in the 30-minute group used the algorithm method increasingly more from the first to the last session with an effect size of $r = .69$ ($p < .001$). During the first four sessions, the incorrect response percentage when the algorithm was used was more than two times as much as the percentage of incorrect responses when the manipulatives were used. Starting from the fifth session the magnitude of the difference between the incorrect answer rate when the algorithm was used and when the manipulatives were used decreased till the eighth session when students became more likely to provide a correct answer when they used the algorithm.

Table 5

Frequencies of Answers by Method and Session in 30-Minute Group

Session			Answer 0	Answer 1	Total
1	Method	Algorithm	67.4%	21.5%	34.1%
		Manipulative	32.6%	78.5%	65.9%
2	Method	Algorithm	66.7%	19.0%	34.1%
		Manipulative	33.3%	81.0%	65.9%
3	Method	Algorithm	68.4%	19.5%	35.0%
		Manipulative	31.6%	80.5%	65.0%
4	Method	Algorithm	68.3%	25.2%	39.3%
		Manipulative	31.7%	74.8%	60.7%
5	Method	Algorithm	51.9%	34.1%	39.3%
		Manipulative	48.1%	65.9%	60.7%
6	Method	Algorithm	45.5%	42.7%	43.5%
		Manipulative	54.5%	57.3%	56.5%
7	Method	Algorithm	42.2%	48.2%	46.7%
		Manipulative	57.8%	51.8%	53.3%
8	Method	Algorithm	39.6%	57.2%	52.7%
		Manipulative	60.4%	42.8%	47.3%
9	Method	Algorithm	40.0%	65.3%	58.9%
		Manipulative	60.0%	34.7%	41.1%

Note. Correct answer = 1; incorrect answer = 0.

Research Question 4: How Does Student Achievement with OMT Change During a Session in Each Group (i.e., 10-, 20-, and 30-Minute Groups)?

Figures 9 present the mean cumulative percentage of correct answers over nine sessions taken at 2-minute intervals in 10-, 20-, and 30-minute groups. The data from nine sessions were averaged for each group using the following 2-step procedure: First, the cumulative correct answer percentages at 2-minute intervals were calculated for each session. Then, the averages of the percentages for each 2-minute interval over nine sessions were calculated. For example, percentages of cumulative correct answers at minute 2 from each session were averaged, and this procedure was repeated for each 2-minute interval in each group.

As seen in Figure 9, the correct response percentage in the 10-minute group increases during the sessions up to 30.4% with a decreasing rate in the second half of the sessions. In the 20-minute group the percentage of correct answers increases up to 37.5% till approximately 12th minute and then decreases till 16th minute to 34% where it virtually plateaus (see Figure 9). The percentage of correct responses during the first 20 minutes of the 30-minute group displayed a similar pattern as the 20-minute group (see Figure 9). The correct answers increased in the first half of the sessions, although to a larger percentage (i.e., 64%) than the 20-minute group, and then decreased and reached plateau till the 20th minute. After the 20th minute the correct response percent started increasing for approximately 6 minutes at a lower rate than it did during the first 10 minutes and then a plateau pattern appeared again.

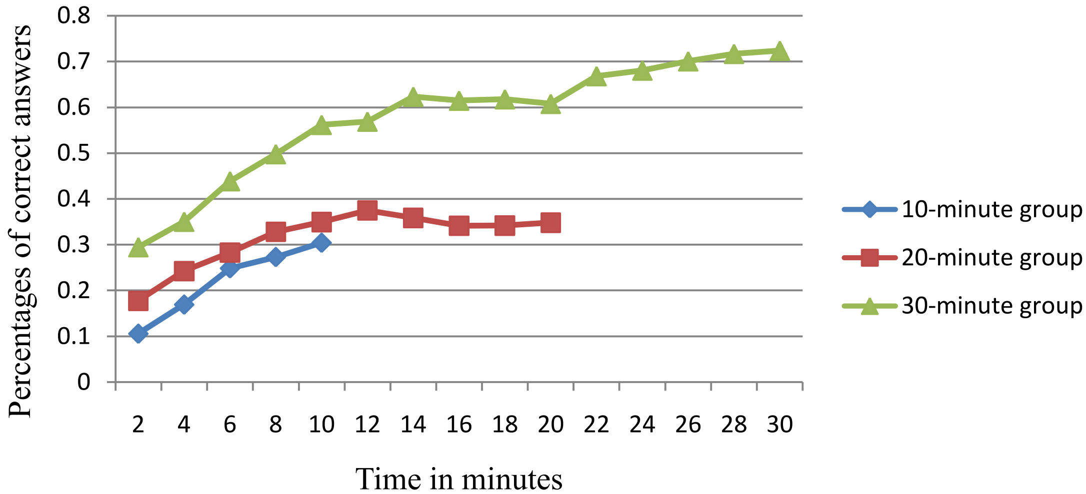


Figure 9. Mean cumulative percentage of correct answers over nine sessions taken at 2-minute intervals in 10-, 20-, and 30-minute groups.

Discussion and Conclusion

When the achievements of students who used different components were compared, it was found that students who used manipulatives most achieved highest. Those who used audio-visual most achieved the second highest. This finding supports the previous research that kinesthetic learning is more active than audio and audio-video and associated with both information perception and information processing (Felder & Silverman, 1988). Thus, students who used the manipulatives most might have not only perceived but also processed the concepts related to operations with fractions as they experimented with manipulatives. This finding implies audio received as stimuli and had the role of attention catcher. Because of the content of the audio component (i.e., providing instructions), audio was probably not enough by itself to transfer information into long-term memory. However, audio-visual component, which provided students with guidance and worked examples, could have afforded guidance for cognition. Moreover, this finding indicates that manipulatives support processes in information registry, attention, semantic encoding, and retrieval of information without overloading students' cognitive resources. One could assume the reason for the effect of manipulatives is that they provided spatial affordances (i.e., move-ability), media affordances (i.e., draw-ability), feedback affordances, and visual affordances.

The analyses showed that as the treatment duration increased, students used audio and audio-visual components of OMT less. Audio component in OMT provided students with guidance and instructions but the content. However, audio-visual is consisted of information related to instructions as well as the content. Moreover, the longer treatment duration was associated with less use of the manipulatives. In other words, students in 30- and 20-minute groups used the manipulatives less frequently than the students in 10-minute group. It is especially interesting that even though students' more frequent use of the manipulatives was associated with better success, there was a meaningful increase in effect for students showing a transition away from the manipulative and toward the standard algorithm. This result supports Ball's (1992) findings that students require guidance prior to solving problems symbolically. This finding indicates that audio and audio-video components of OMT afford guidance when enough time is provided (30-minutes) so that students were able to move forward toward the symbolic mode. Students could possibly encode the verbal and visual information afforded by audio and audio-video components meaningfully as they use this information with manipulatives. This meaningful encoding could have helped students to transfer this information into long-term memory.

In computer assisted instruction students need to master not only content knowledge but also the software used. Students in 30-minute group received substantially higher scores than students in 20- and 10-minute groups in the initial session. This finding suggest the initial session needs to be long enough so that students have enough time with a manipulative tool in order to learn the tool, thus, they can use the tool to learn content.

Technology integration facilitates multiple affordances in a way that improves transition from concrete manipulatives to abstract thinking and provides a foundation for continued learning (Reimer & Moyer, 2005). However, early transitions with insufficient guidance from the directions to the actual learning activity may result in a decrease in students' achievement (Ball, 1992). The finding that students' success initially dropped in the 30-minute group led to a closer investigation of this group's online tool use. One of the hypotheses for this finding was that students started with a method (i.e., algorithm or manipulative) that was familiar to them in the first session and changed to the other method, at which they were not as proficient, resulting in a decrease in their percentage of correct answers. As the students became experienced with the new method, their success started to increase till they became as proficient with the new method as they were with the initial mode. Further analyses supported this hypothesis as discussed in the third research question.

Time is one of the major problems that teachers encounter in their classrooms (Rupe, 1986). Teachers need to carefully allocate their instructional time for each lesson. To completely cover state curriculum, they have to address objectives within a limited window of time. Therefore, making optimal use of time for each activity is an important factor (Morrison, 2008; Son and Sethi, 2006). In the case of an online manipulative tool, this optimal time contains the time needed for learning the tool and the content. When the student achievement in the 10-minute group was explored, there was a steady increase and no plateau pattern. This suggests that the students in 10-minute group were still in the process of learning the content. An interesting finding when the student achievement was examined across time within each group was that the similarity between the 20-minute group and the first 20 minutes of the 30-minute group. Both graphs almost reached plateaus approximately between 14th and 20th minutes. Students in 20- and 30-minute groups reached their first plateau in their learning curve. When the 30-minute group's performance was analyzed from the 22nd to 30th minute, there was only 5-point increase observed. Students in this group started with an average score of 30 points and reached an average score of 68 points at their 22nd minute. In order to have 5-point increase in their average scores, students should have received almost perfect scores within the last eight minutes. In other words, as Morrison and Son and Sethi suggested, students reached their peak. Thus, it is suggested that such tools can be provided students with an optimal time of 21 to 25 minutes.

This study has useful implications for teaching and learning practice as well as for instructional designers. One of the results of this study was that students who spent the most time on manipulatives had the highest achievement scores. This finding implies for instructional designers that affordances provided via virtual manipulatives are crucial considerations for developments of such tools. These affordances include guidance, feedback, move-ability, draw-ability, highlight-ability, and focus-ability. Teachers should also be cognizant about the affordances of learning environments they prefer to use in their classrooms.

Another implication would be related to guidance. Students need guidance in their processes of learning. Nevertheless, guidance should be faded so that students can spend more time on the content. Teachers should monitor students' performance and decrease guidance after as students gain competence. Similarly instructional designers should develop learning tools that monitor students' progress to provide responsive guidance. An additional implication about guidance/instruction would be that students' early transitioning from guidance/instruction to actual learning activity hiders learning. Teachers should be aware of their students' performances, and based on their performances, teachers should provide additional guidance or feedback. Similarly, instructional designers can develop built-in assessment techniques that could monitor students' progress and provide guidance or feedback if there is a decrease.

CHAPTER IV

AN ONLINE MANIPULATIVE TOOL: EFFECTIVENESS OF THE TOOL AND STUDENTS' TECHNOLOGY ACCEPTANCE

Overview

Given the strong research support for improvement of students' conceptual understanding with multiple representations, it is important to understand effects of different representations on student achievement. In this study, an online manipulative tool (OMT) was introduced to students to support their study of rational number operations. The purpose of this experimental study is to compare different representational aspects of OMT and to estimate OMT's effects on achievement and technology acceptance when compared to those of traditional classroom activities. Elementary- and middle-grade students were randomly assigned to the control group ($N = 14$) or one of the following three treatment groups: (a) audio-visual ($N = 14$), (b) virtual-kinesthetic ($N = 15$), and (c) dual-mode (virtual-kinesthetic and audio-visual combined) groups ($N = 11$). When the control group was compared with experimental groups, pre- and post-test results suggested OMT was more effective than traditional classroom activities in improving students' understanding of operations with rational numbers. When the students' achievement on pre- and post-tests among experimental groups was compared, no substantial difference was found. However, students in the dual-mode group scored the highest on the technology acceptance survey. Students' technology acceptances also differed among different ethnicities and SES levels but not genders. The results suggest learning can be facilitated by virtual manipulatives. However, students' acceptances of technology should be considered when implementing new technologies.

Introduction

This study reports effects of an online manipulative tool (OMT) on students' understanding of operations with rational numbers, more specifically fraction comparisons and addition, subtraction, and multiplication of fractions when students are presented with different modes of representations. In addition, students' beliefs about the usefulness of OMT and their acceptance of the newly introduced OMT were investigated. The study also explored students' achievement on the questions presented by OMT versus the students' achievement on paper-and-pencil assessments. Previous research on OMTs establishes the importance of such tools on students' achievement in mathematics (e.g., Reimer & Moyer, 2005; Suh & Moyer, 2007; Suh et al., 2005). Furthermore, research showed students built positive attitudes toward mathematics when they used OMTs in their mathematics classes (Moyer et al., 2005; Reimer & Moyer; Steen et al., 2006; Suh & Moyer; Suh et al.).

Same Concept, Different Representations

In mathematics education there is substantial research demonstrating the effectiveness of multiple modes of representation in helping students develop conceptual understanding (e.g., Ainsworth, 2006; Amato, 2008; Fennell & Rowan, 2001; Gagatsis & Elia, 2004; Perry & Atkins, 2002; Suh et al., 2005). Multiple modes facilitate different perspectives on a particular concept thereby scaffolding deeper understanding (Ainsworth, Bibby, & Wood, 1997). For example, presenting a fraction as a linear model on a fraction strip helps students build the concept of fraction as a number; whereas, an area model emphasizes the numerator and denominator of a fraction in relation to partioning a whole. Dienes (1973) argued presenting the same concept with various representations helps students build abstract mathematical thinking. When students are exposed to symbolic or concrete representations of mathematical concepts prior to learning formal mathematical notations, they can link the concrete representations with abstract mathematical ideas. Thus, integrating various representations combines the strength and eliminates the weakness of any single representation (Elia, Gagatsis, & Demetriou, 2007).

Even though multiple representations help students understand mathematical concepts, different representations have varying degrees of effects on teaching and learning of constructs (Duval, 2002). Although some representations may provide similar information, each representation can have different loads on students' cognitive resources (Larkin & Simon, 1987). For example, the cognitive load of understanding a diagram can be more than the cognitive resources needed when the diagram is accompanied with textual information physically linked to related segments of the diagram, thereby helping students' with mental integration of information and making recognition and understanding easier. Thus, selecting appropriate representations is an important consideration for effective instruction.

Not only individual representations have different impact on students' conceptual understanding but also integrating multiple representations may have interaction effects. Even though some research indicates that interaction among representational modes helps students' conceptual understanding (Elia et al., 2007), this interaction may hinder learning if the representations are not chosen and integrated carefully (Mayer, 2005; Sweller et al., 1998). Elia et al. found that students had more difficulties solving problems presented with informational pictures as compared to problems presented in verbal mode. Students who were presented problems with informational pictures had to combine the information presented in text with the picture, thus, splitting their attention between both the pictorial and verbal representations. This allocation of cognitive resources into several processes was indicated to reduce effectiveness in information processing. Given that there were unfavorable results of ineffective integration of various representations into instruction, Sweller et al. proposed instructional design principles for presenting information in different modes. Further discussion about these instructional design principles can be found in Chapter II.

Beliefs and Attitudes

Along with the appropriate use of multiple representations, students' beliefs and attitudes toward mathematics and instructional strategies used in mathematics classrooms can affect students' achievement (DeBellis & Goldin, 1993; McLeod, 1992).

Goldin (2000) conjectured possible relationships between affective states of students' feelings and students' problem-solving heuristics. Moreover, Goldin inferred how these affective representations can improve or inhibit mathematical problem solving skills. For example, frustration during problem solving may prevent a student from pursuing the solution or, on the contrary, motivate the student to find the solution.

Because of virtual manipulatives' additional features to facilitate teaching and learning mathematics, one would expect virtual manipulatives to help students improve their achievement in mathematics as well as enhance their attitudes toward mathematics. Indeed, research showed higher student achievement in mathematics was associated with better student attitudes towards mathematics (Fennema & Sherman, 1976; Van Eck, 2006). In the case of physical manipulative use, students had higher achievement in mathematics and better attitudes toward mathematics when physical manipulatives were implemented into instruction (Sowell, 1989). Aligned with these expectations, research, which is limited to kindergarten and elementary school students, showed gains both in mathematics achievement and attitudes when virtual manipulatives were used (Moyer et al., 2005; Reimer & Moyer, 2005; Steen et al., 2006; Suh & Moyer, 2007; Suh et al., 2005).

Virtual manipulatives offer important learning characteristics favored by students (Reimer & Moyer, 2005; Suh et al. 2005). Reimer and Moyer found that students preferred virtual manipulatives over traditional activities because virtual manipulatives could provide immediate and specific feedback. This feature of virtual manipulatives enhanced students' attitude toward mathematics by providing a safe learning environment where students could learn from their own mistakes (Reimer & Moyer; Suh et al.; Suh & Moyer, 2007). Additionally, in this safe learning environment students could use their creativity more wisely than on paper-and-pencil activities (Moyer et al., 2005). Students could flexibly create patterns, freely test their ideas with virtual manipulatives, and share these ideas and their mathematical thinking with others.

Despite the aforementioned advantageous features of virtual manipulatives, students' perceptions of virtual manipulatives are an important mediating factor in the effect of these features on students' achievement. When students are presented with a new technology such as virtual manipulatives, students need to accept the new technology in order to derive its advantages (Ching, 1999). In other words, if students do not accept new technology, it is most likely the new technology will either not be used by students or will not be beneficial for students' learning. Therefore, it is important to emphasize technology acceptance in classrooms when implementing a new technology.

Davis (1989) developed the Technology Acceptance Model (TAM) to explain and predict acceptance behaviors and usage intentions of a new technology. The TAM includes three constructs: *perceived usefulness, perceived ease of use,* and *enjoyment*. A student's belief that the new technology will enhance his or her achievement is called perceived usefulness (Davis). In order for students to use a technological tool, first they should believe the tool will enhance their performance. Perceived ease of use, on the other hand, is the belief that the new technology will be free of effort (Davis). Perceived ease of use as reflected in the user friendliness of a new technology is also an important factor in technology's perceived usefulness (Yi & Hwang, 2003). If a user struggles with the tool, then the difficulties encountered can affect the perceived usefulness. According to Yi and Hwang enjoyment is an external factor that influences perceived ease of use. Davis defined enjoyment as the extent to which a student found the tool enjoyable. Vrielink (2008) argued that learning takes place when students accept a technology, thus, students' perceptions of the technological tool will affect students' performances.

The technology acceptance model has been used widely to explore how technology acceptance is related to user backgrounds (e.g., gender or ethnicity) as well as different technologies or tasks (King & He, 2006). However, studies on technology acceptance were mostly conducted with adult participants, sometimes with college students, and very rarely with k-12 students. Because technology acceptance is paramount for actually using the technology (Lee, 2003), it is important to understand the differences in k-12 students' technology acceptance. Previous research suggested that the differential use of technology in the schools is related to the achievement gap between different ethnicities (e.g., Kirby & Styron, 1994). In addition to ethnicity, some studies indicated that differences in the extent to which technology is used in classrooms were related to gender (Selby & Ryba, 1993). Although the difference related to gender is narrowing, male students reported using technology more frequently than their female peers (Miller, Schweingruber, & Brandenburg, 2001). Information about the inequities in technology acceptance due to student characteristics can provide insights about the aforementioned differential use of technology in classrooms.

Assessment: The Crossroad Between Teaching and Learning

Assessment is a continuing process that measures a learner's performance and progress toward establishing learning outcomes and that provides feedback to improve learning (Center for Teaching, Learning, and Assessment, n.d.). Wiliam (2008) described this ongoing process as a bridge between teaching and learning and suggested that assessment should therefore be learning oriented. Peltenburg, van den Heuvel-Panhuizen, and Doig (2009) argued learning-oriented assessments can be designed in interactive learning environments.

A learning-oriented assessment can be embedded into an instructional design (Wiliam, 2008), and this type of assessment is called *dynamic assessment* (Peltenburg et al., 2009). Dynamic assessment takes place during the learning process rather than at the end of the learning as in traditional assessment. Thus, dynamic assessment is more effective than a traditional assessment model in identifying the reasons for failure or learner's ability (Lidz, 1991). In traditional assessments students need to show their performance without any feedback and manipulatives. However, dynamic assessments evaluate students' performances on learning tasks as they interact with the learning environment (Peltenburg et al.). This assessment method is capable of revealing students' hidden capacities because students have the opportunity to interact with the learning environment and to learn the subject matter (Pletenburg et al., Allsop et al., 2008). In particular, low performing students can benefit the most from dynamic assessment (Allsop et al.).

Interactive environments with embedded dynamic assessment can evaluate students' progress and provide immediate and specific feedback with no apparent assessment. Virtual manipulatives, in particular, can be programmed to track students' progress and provide hints and feedback accordingly (Reimer & Moyer, 2005; Suh et al., 2005; Suh & Moyer, 2007). These hints and feedback provide support and engage students in problem-solving processes and help students adjust their mathematical thoughts (Wiliam, 2008). The freedom offered to students by virtual manipulatives to experiment their ideas can guide them to a successful performance. Thus, the embedded assessment within virtual manipulatives not only measures students' progress but also scaffolds their learning.

Research Questions

Given the strong research support for the improvement of students' conceptual understanding with multiple representations (e.g., Ainsworth, 2006; Amato, 2008; Fennell & Rowan, 2001; Gagatsis & Elia, 2004; Perry & Atkins, 2002; Suh et al., 2005), it is important to understand the effect of different modes of representation on student achievement. Using multiple representations via technology to present rational number concepts can provide evidence for understanding the relationship between the representational mode use and achievement (cf. Martin & Schwartz, 2005). In addition, students' beliefs and attitudes toward mathematics and instructional strategies used in classrooms can affect students' achievement (DeBellis & Goldin, 1993; McLeod, 1992). Therefore, it is important to know how students' beliefs and attitudes toward OMT change over time and how the beliefs and attitudes are related with students' characteristics and representational modes. Moreover, students perform better and reveal their hidden competences when assessment is embedded in instruction (i.e., invisible assessment) (Peltenburg et al., 2009). Thus, it is essential to investigate the relationship between scores received on OMT and paper-based test.

Five major questions guided this study:

1. What is the impact of the OMT on students' understandings of operations with rational numbers? How does the impact of the OMT on achievement differ by the representational mode (i.e., audio-visual, virtual-kinesthetic, and dual-mode)?
2. Do students' beliefs and attitudes toward usefulness of OMT change over time as they experience OMT?
3. Is there a difference in technology acceptances of (a) boys and girls and (b) students with different SES as measured by TAM-M?
4. Is there a difference in technology acceptances of students by the representational mode (i.e., audio-visual, virtual-kinesthetic, and dual mode) as measured by TAM-M?
5. How do students perform on OMT as compared to traditional paper-and-pencil methods?

Method

Design

The experimental design was intended to compare different aspects of OMT and to estimate the effects of OMT over traditional classroom activities. Elementary- and middle-grade students from fourth, fifth, seventh, and eighth grades participated in the study. The students were randomly assigned to the control group ($N = 14$) or one of the following three treatment groups: (a) audio-visual ($N = 14$), (b) virtual-kinesthetic ($N = 15$), and (c) dual-mode (virtual-kinesthetic and audio-visual components combined) group ($N = 11$). Random assignment was performed within each grade separately in order to avoid over assignment to any group in each grade. Excel was used to generate random numbers to assign students into groups. Even though students in each classroom were assigned to the control and experimental groups, the classes remained intact. That is, students in each classroom stayed together as they participated in the study.

Participants

Fifty-four elementary- and middle-grade students from five classes at a college preparatory charter school participated in the study. The charter school is located in the state of Texas. Only one classroom at each grade (i.e., fourth, seventh, and eighth) was included in the study except for fifth grade, which was represented with two classes in the sample. Table 5 presents the demographic information for the participants.

Table 6

Demographics of Grade 4, 5, 7, and 8 Sample

Grade	Gender		*N*
	Female	Male	
4	42%	58%	14
5	40%	60%	20
7	91%	9%	11
8	45%	55%	11
Overall	52%	48%	54

Note. Percentages may not add up 100% because of rounding.

Procedure

The students in the experimental groups used OMT for six sessions each of which was 30-minute long. The completion of six sessions for all groups took two weeks. The participating classes remained intact throughout the study. In other words, even though students were randomly assigned to different groups (i.e., three experimental and one control) in each classroom, students within a classroom participated in the study together and within their classroom periods. The study is conducted in a spring semester.

All students participated in direct instruction delivered by their teacher. After the instruction on the fraction concepts was over, students in experimental groups logged onto the system for additional activities with the OMT. The control group did not receive any treatment other than teacher-assigned activities. The teacher-assigned activities were from the textbooks, Holt, Rinehart and Winston or Pearson.

Online Manipulative Tool

The online manipulative tool (OMT) is interactive internet-based computer software designed to present students with problems on addition, subtraction, multiplication, and comparison of rational numbers. For each comparison problem the fractions were less than one, and for addition, subtraction, and multiplication problems the fractions and the results were equal to or less than one.

The software differed among experimental groups based on the mode used providing the same content. The audio-visual group watched an instructional video on operations with rational numbers using virtual manipulatives, whereas the virtual kinesthetic group actively used virtual manipulatives to solve rational number questions. The dual-mode group was provided with the opportunity to use virtual manipulatives as the virtual-kinesthetic group and to watch the same instructional videos given to audio-visual group as they wished.

The OMT consisted of two virtual manipulatives: (a) a virtual area board and (b) virtual fraction strips. The virtual area board provided an interactive environment where students could use area model to add, subtract, and multiply rational numbers. The virtual fraction strips presented students with comparison of rational numbers in a similar manner to physical fraction strips. Both of the manipulatives can be accessed at http://coe.tamu.edu/~sozel/vm/hm/.

The tool employed several research protocols to improve the quality of the data collected and to ensure that data were not the result of computer or internet-based resources. The OMT coded the content on the screen every 5 seconds. The coded information on the screen included every click, the question being solved, the current progress on manipulatives, the final solution in both algorithm and manipulative representations, and the feedback provided for students' answers. The purpose of screen coding was to provide precise information about students' progress to ensure a complete accounting of each attempt. Additional protocols to ensure data dependability and reliability were collection of data regarding the total time spent on each item and the internet protocol (IP) address.

Measures

The students were administered a pre- and a post- paper-and-pencil test on addition, subtraction, multiplication, and comparison of rational numbers. Students were not allowed to use the OMT when they took the paper-and-pencil tests. Two versions of both pre- and post-tests were administered: versions A of the pre- and post-tests (see Appendices A and B, respectively) to fourth and fifth grades and versions B of the pre- and post-tests (see Appendices C and D, respectively) to seventh and eighth grades. Although the content and structure were the same on both versions on pre- and post-administrations, the difficulty level was higher on versions B. Each version consisted of 10 items on rational number operations investigating students' algorithmic and representational skills.

Pre- and post-tests were evaluated using a rubric that had a score ranging from 0 to 19. The rubric used to evaluate each version of pre- and post-test was as follows. In each version, the first three questions were about comparison of two fractions, and the answers were scored 1 for correct and 0 for incorrect. Questions 4 through 7 required students to translate pictorially represented fractions and operations of fractions into formal mathematical notations. Students' answers were scored as 1 for correct and 0 for incorrect. Students were provided addition, subtraction, and multiplication problems in the last three questions and asked to represent the problems pictorially. Two different scores were calculated for this set of problems: algorithmic and pictorial scores. Algorithmic scores were either correct (i.e., 1) or incorrect (i.e., 0), whereas, pictorial scores were based on a 3-point scale: Each correctly represented fraction earned 1 point (adding up to 2 because there were 2 fractions in each question) and the resulting fractions were scored with another 1 point. Then these scores were added to create the pictorial score for that particular problem. For example, if a student could only represent one of the two fractions and the resulting fraction, the student received 2 points for his or her pictorial score.

To investigate students' perspectives on the usefulness of the OMT when learning operations with fractions, an after-software questionnaire (ASQ) (see Appendix E) was adapted from Vrielink (2006). The ASQ was a 4-item 5-point Likert-type questionnaire (1 = *strongly disagree* to 5 = *strongly agree*). The ASQ was administered to students in experimental groups at the end of every other session. That is, students completed ASQ three times throughout the study. The ASQ was not administered to students in the control group because they did not use the software. The Cronbach's alpha reliability estimates for first, second, and third administrations of ASQ were .95, .94, and .96, respectively.

At the end of the study, students in the experimental groups were administered the *Technology Acceptance Model Modified* (TAM-M) questionnaire (see Appendix F) adapted from Vrielink (2006). The original TAM was developed by Davis (1989) for adults. The TAM-M version was validated for students between ages 12 and 17 (Vrielink) and contained an 11-item 5-point Likert-type questionnaire (1 = *strongly disagree* to 5 = *strongly agree*). The questionnaire addresses three constructs: (a) enjoyment, (b) ease of use, and (c) usefulness. The Cronbach's alpha reliability estimate for the whole TAM-M questionnaire was .96. For each construct the Cronbach's alpha reliabilities were as follows: .90 for ease of use, .91 for usefulness, and .91 for enjoyment for the data in hand.

Data Analysis

A discussion of each analysis in relation to the research questions is presented below.

Research question 1: What is the impact of the OMT on students' understandings of operations with rational numbers? How does the impact of the OMT on achievement differ by the representational mode (i.e., audio-visual, virtual-kinesthetic, and dual mode)? To determine the impact of the OMT on students' understanding of operations with rational numbers, gain scores from pre to post paper-and-pencil tests were compared across groups. First, gain scores were calculated as the arithmetic difference between post- and pre-tests. Subsequently, 95% confidence intervals (CIs) were constructed around the gain scores for each group. CIs allowed for the comparison of experimental and control groups as well as the comparisons of the experimental groups within each other.

Research question 2: Do students' beliefs and attitudes toward usefulness of OMT change over time as they experience OMT? Students' beliefs and attitudes toward usefulness of OMT were measured by ASQ. To determine students' perspectives about usefulness of OMT over time, confidence intervals (CIs) around the means were calculated for each administration of ASQ. In addition, CIs around the means for each ASQ administration were investigated by group (audio-visual, virtual-kinesthetic, and dual mode). CIs allowed for the examination of the change in students' beliefs and attitudes toward the OMT within each group and across groups.

Research question 3: Is there a difference in technology acceptances of (a) boys and girls and (b) students with different SES as measured by TAM-M? The technology acceptance levels (as measured with TAM-M) of students with different sexes or SES levels are discussed using CIs around the means and box-plots, which allowed for comparisons of distributions side-by-side.

Research question 4: Is there a difference in technology acceptances of students by the representational mode (i.e., audio-visual, virtual-kinesthetic, and dual mode) as measured by TAM-M? The technology acceptance levels of students (as measured with TAM-M) of students by the representational mode are displayed in box-plots to allow for comparisons of distributions side-by-side. In addition, CIs for the mean in each group is presented.

Research question 5: How do students perform on OMT as compared to traditional paper-and-pencil methods? The students were divided into two groups based on their posttest scores: Students who scored below 9 out of the maximum possible score of 19 were grouped as low performers, and students who scored at or above 9 were grouped as high performers. Then, the performances of the low and high performers on OMT during the last session were compared using CIs. The performances on OMT were evaluated both on algebraic notation and on kinesthetic representation. The scores on algebraic notation and on kinesthetic representation were obtained as the ratio of the number of correct answers to the total number of items attempted.

Results

Research Question 1: What Is the Impact of the OMT on Students' Understandings Of Operations with Rational Numbers? How Does the Impact of the OMT on Achievement Differ by the Representational Mode (i.e., Audio-visual, Virtual-kinesthetic, and Dual mode)?

The CIs around the gain scores for each group are presented in Figure 10. There were recognizable differences between the control group and each of the experimental groups. The magnitude of the differences as investigated using Cohen's *d*s were .59 between the control (*M* = .79, *SD* = 5.0) and the audio-visual groups (*M* = 3.38, *SD* = 3.73), .47 between the control and the virtual-kinesthetic groups (*M* = 2.90, *SD* = 3.78), and .68 between the control and the dual-mode groups (*M* = 3.82, *SD* = 3.66). Thus, the results suggested that OMT were more effective than traditional classroom activities in improving students' understanding of operations with rational numbers.

The CIs in figure 10, also allow us to compare the gain scores across the experimental groups. As seen in the figure 10, the gain in each experimental group was similar to the gains in other experimental groups. In other words, there were not differences in the achievement of students in different experimental groups by the representational mode.

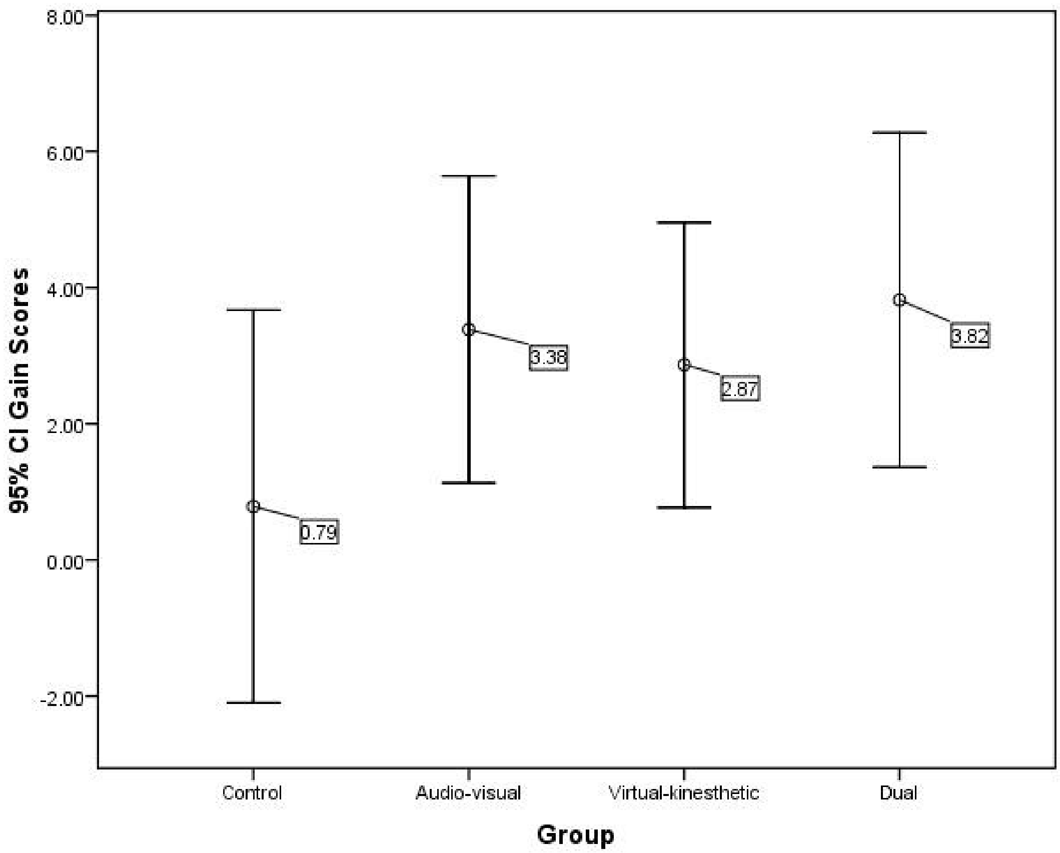


Figure 10. 95% CIs around gain scores for each group.

Research Question 2: Do Students' Beliefs and Attitudes Toward Usefulness of OMT Change over Time as They Experience OMT?

The CIs around the means are given in Figure 11. The mean scores on ASQ steadily increased across administrations. The Cohen's *d* effect size for the mean difference between the third (M = 13.08, SD = 5.10) and the first administrations (M = 11.25, SD = 4.74) was 0.4. After investigating the overall trend in students' perspectives about usefulness of OMT as measured by ASQ, the CIs around the means for each administration of ASQ were examined within each experimental group and across groups. As seen in Figure 12, there is an increasing trend in the means in the audio-visual and dual-mode groups; whereas, the mean for the virtual-kinesthetic group was almost the same across administrations. A recognizable finding was that the mean score for the dual-mode group in the first administration was below the means of other groups. However, in the last administration the mean for the dual-mode group was above the means of all other groups. The Cohen's *d* effect size for the increase in the mean of the dual-mode group from the first (M = 9.54, SD = 5.22) to the last administration (M = 14.27, SD = 6.40) was 0.8. However, the precision of the mean estimate for the dual-mode group was lower than the mean estimates for other groups as reflected in the widths of the CIs. This was partly due to the relatively smaller sample size in the dual-mode group.

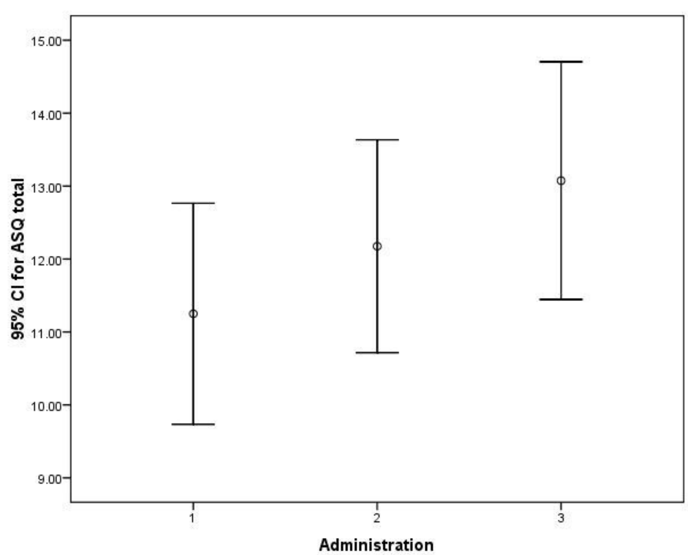


Figure 11. CIs around means for each administration of ASQ.

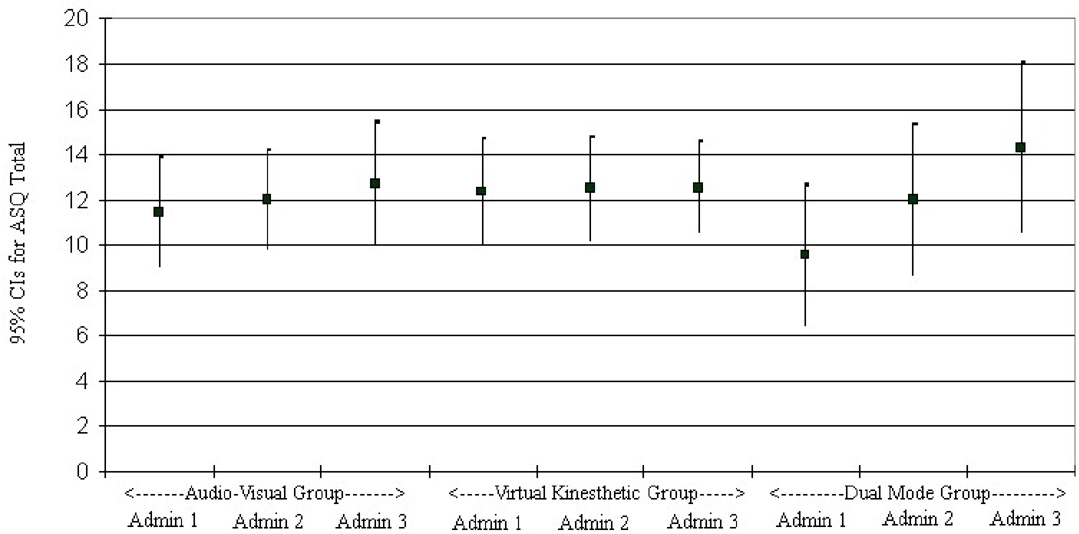


Figure 12. 95% CIs around means for each administration of ASQ by group.

Research Question 3: Is There a Difference in Technology Acceptances of (a) Boys and Girls and (b) Students with Different SES as Measured by TAM-M?

The distributions of scores for boys and girls on TAM-M are displayed in Figure 13. As conveyed by the box-plots, the distributions of scores for boys and girls were similar although the variation was a little higher for girls. Also, CIs suggested no difference between boys' and girls' technology acceptances.

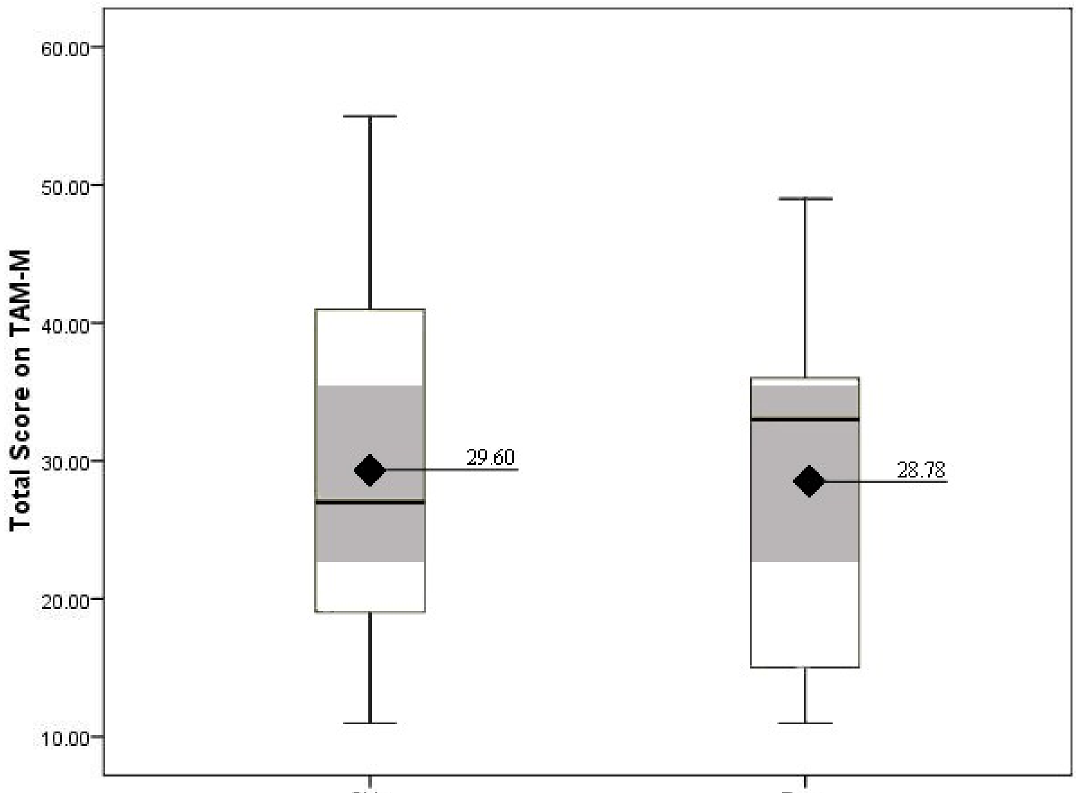


Figure 13. Total scores on TAM-M by gender displayed with box plots and 95% CIs. The straight horizontal lines in the boxes represent the median. The diamonds represent the means. The gray areas represent 95% CIs around means.

The distributions of total scores on TAM-M by SES are displayed in Figure 14. On average students with low SES scored higher than students with high SES. In fact, approximately 50% of students with low SES got scores comparable to high SES students in the upper quartile. The CIs also indicated statistically significant difference between students with low (M = 32.47, SD = 13.45) and high SES (M = 24.53, SD = 11.20) in their average score on TAM-M (Cohen's d = .62).

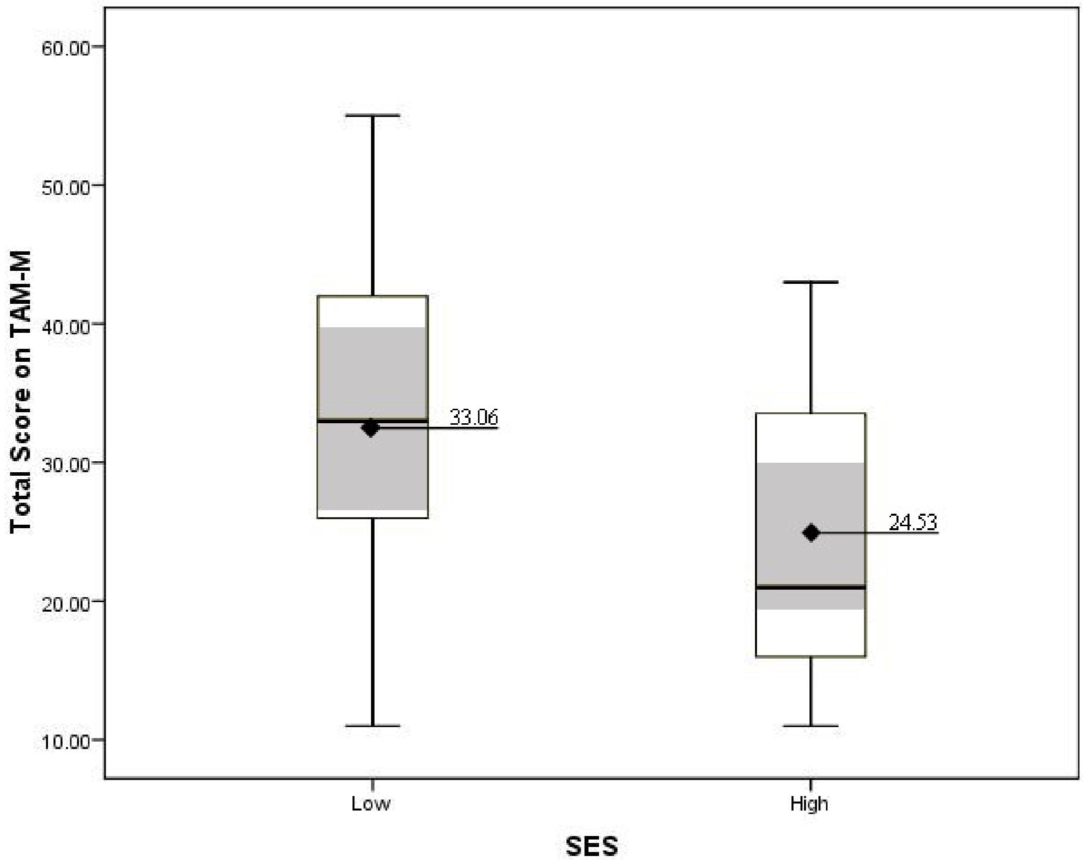


Figure 14. Total scores on TAM-M by SES levels displayed with box plots and 95% CIs. The straight horizontal lines in the boxes represent the median. The diamonds represent the means. The gray areas represent 95% CIs around means.

Research Question 4: Is There a Difference in Technology Acceptances of Students by the Representational Mode (i.e., Audio-visual, Virtual-kinesthetic, and Dual-mode) as Measured by TAM-M?

As seen in Figure 15, the average TAM-M scores of students in the audio-visual group were lower than that of students in virtual-kinesthetic and dual-mode groups.

Further, more than 50% of the students in the dual-mode group had scores comparable to the scores in the upper quartile of both the audio-visual and virtual-kinesthetic groups. Regarding the score variation, the variation in the dual-mode group was higher - especially for students who were below the median - than the variations in the other two groups.

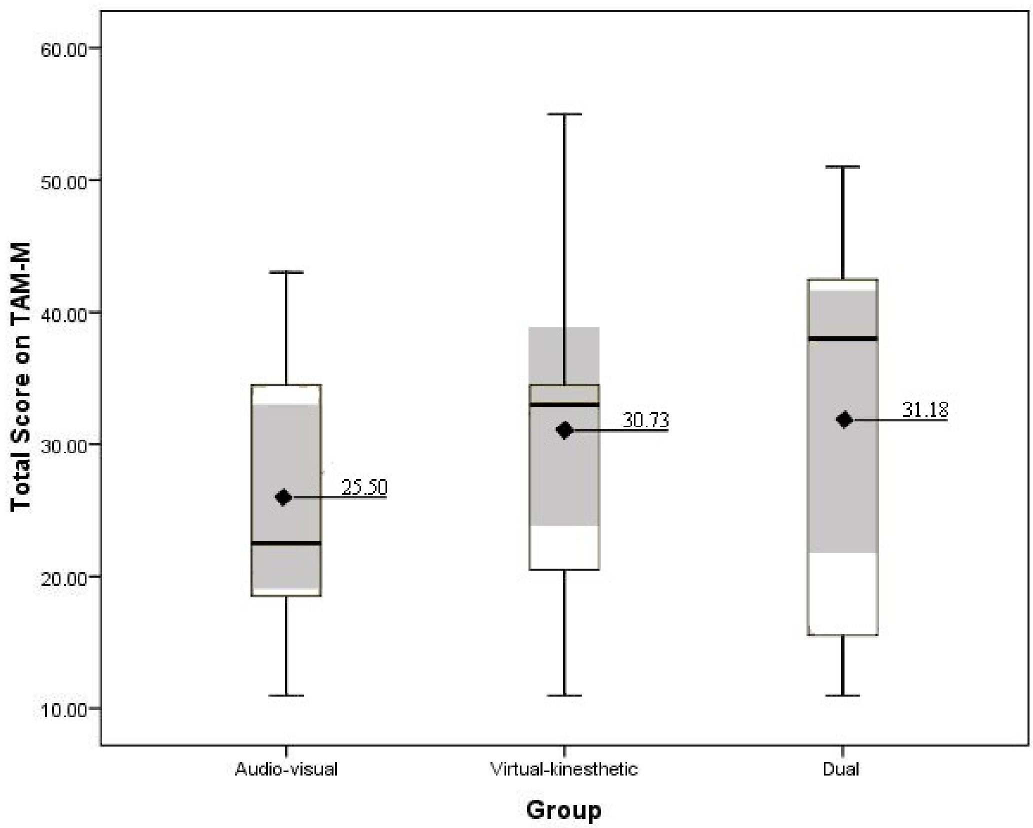


Figure 15. Total scores on TAM-M by group displayed with box plots and 95% CIs. The straight horizontal lines in the boxes represent the median. The diamonds represent the means. The gray areas represent 95% CIs around means.

Research Question 5: How do Students Perform on OMT as Compared to Traditional Paper-and-pencil Methods?

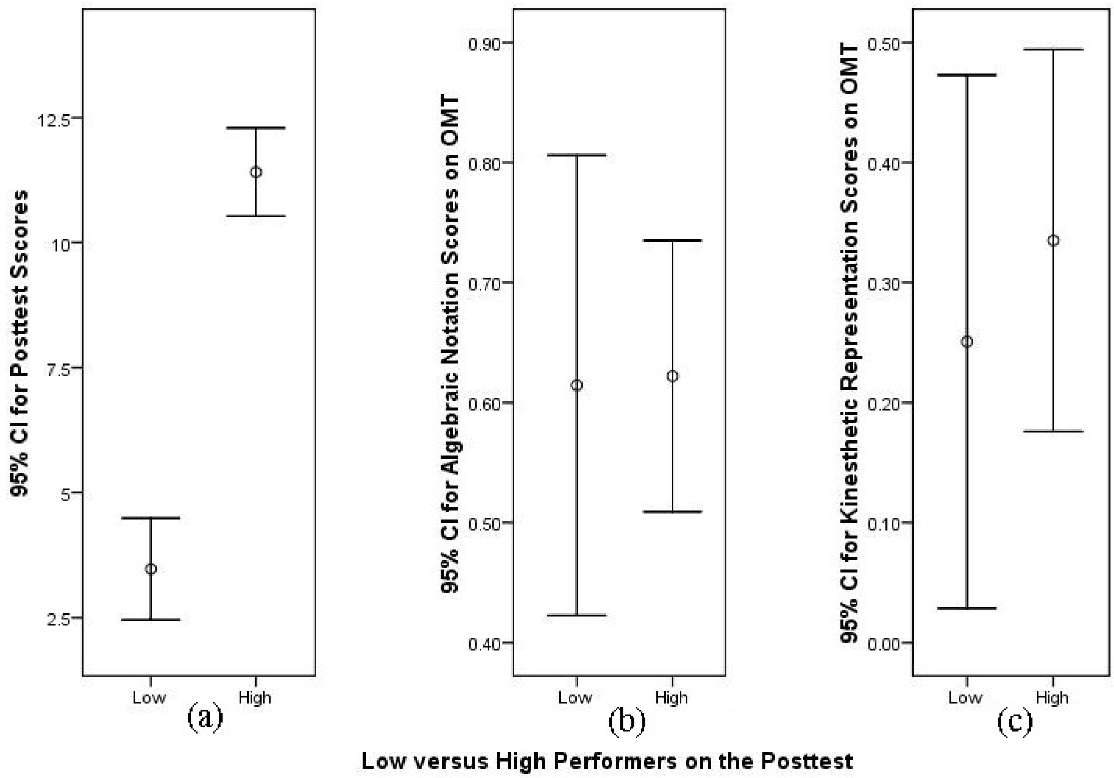


Figure 16. (a) Comparison of low and high performers on the posttest using 95% CIs. (b) Comparison of low and high performers on algebraic notation scores on OMT using 95% CIs. (c) Comparison of low and high performers on kinesthetic representation scores on OMT using 95% CIs.

As seen in Figure 16a, there is a statistically significant and large difference between the low and high performers on the paper-and-pencil posttest. However, on neither the algebraic notation nor the kinesthetic representation scores on OMT are there differences between the low and high performers (see Figure 16b and 16c). The average scores of low and high performers both on OMT algebraic notations and kinesthetic representations are similar although the variation in the low performers is higher.

Discussion and Conclusion

When the impact of the OMT on students' understanding of operations with rational numbers was analyzed, it was found that students who used the OMT achieved better than the students who attended teacher-assigned classroom activities. Teacher-assigned classroom activities included textbook activities related to operations with rational numbers. Students who used the OMT received activities in different representational modes including audio-visual, kinesthetic, or both audio-visual and kinesthetic. On the other hand, the students in the control group were provided with only textbook activities. This finding suggests that each representational mode presented provided different affordances that could not be found in traditional classroom activities such as move-ability, draw-ability, feedback, and focus-ability affordances in manipulatives and watch-ability, playback-ability, focus-ability, and highlight-ability in audio-video representation. Each affordance in video and manipulative provided students with opportunity to watch (video) or manipulate (manipulative) the transitioning from concrete representation to the symbolic mode. One could imply that students could use these affordances to process the information and give meaning to them to transfer into long-term memory. Moreover, they could retrieve this information to be able to successful on the paper-and-pencil test. This finding supports the current literature on the effectiveness of teaching and learning fractions with online learning tools (Reimer & Moyer, 2005; Suh & Moyer, 2007).

When gain scores on achievement among experimental groups were compared, there was no difference among experimental groups. Representational modes in experimental groups were dynamic and interactive in nature. Students in the audio-visual group were presented rational numbers concepts in a dynamic media; whereas, students in the virtual-kinesthetic group used interactive online manipulatives. On the other hand, students in the dual-mode group were provided with both the dynamic media and interactive online manipulatives to choose from. Hypothetically, one would expect students who use interactive online manipulatives achieve better than the students who watch a dynamic media because interactive online manipulatives provide students with the opportunity to manipulate objects, to dynamically see changes in different modes, and to receive immediate and specific feedback for their solution strategies. However, no difference based on representational mode was found in this study (see Figure 1). This finding suggests that different representational modes resulted similar performance via different affordances. That is, students could meaningfully encoded information into their long-term memory and retrieve it efficiently using their cognitive resources. This result is aligned with Kaput's (2006) and Bolyard and Moyer-Packenham's (2006) findings that dynamic video and interactive manipulative has similar effect on students conceptual understanding of mathematics.

The overall analyses showed that the mean ASQ scores of students who used the OMT increased from the first administration to the last. That is, students' perspectives on the usefulness of the OMT increased over time. One of the hypotheses for this finding was that because students had not used virtual manipulatives before, initially they might have had difficulties in understanding how the manipulatives worked. This hypothesis is supported by the previous research concluding that if users do not have difficulties when using a technology, they are more likely to perceive that particular technology more useful (Yi & Hwang, 2003). This hypothesis also supports the more positive change in students' perceptions of usefulness of the OMT in the dual-mode group as compared to the virtual-kinesthetic group. The dynamic video presented students in the dual-mode group scaffolded students' manipulative skills. When these students had difficulties using manipulatives, they could easily switch to the videos where they could learn how to use manipulatives as well as about operations with rational numbers. This finding could also imply that students could have mastered the tool and developed conceptual understanding (i.e., meaningfully encoded) as they spent more time on OMT so that they could have transferred information into long-term memory and retrieved from it easily.

The analyses on TAM-M survey showed no difference between boys' and girls' technology acceptance levels. This finding is in accordance with the current research indicating that the gap between male and female students' technology uses is narrowing (Hargittai & Shafer, 2006; Ono & Zavodny, 2003). However, the results indicated that students' technology acceptances differed among different SES groups. More specifically students from low SES families had higher acceptance levels as compared to students from high SES families. However, students from higher income families have been found to use computers in school and in their homes more frequently than students from lower-income families (Becker, 2001; Coley, Cradler, & Engel, 1997). One can conclude that the mechanism of SES (i.e., parent literacy, parental help, accessibility to resources/materials) might have influence on students' perceptions of technology. Because low SES group might have not experienced such tools before, they could be more open to the tool when they were first introduced. On the other hand, students from high SES could have access different technologies (iPods, computers at home, etc.) so that they were not very excited about the tool and had lower scores.

When the TAM-M scores of students in experimental groups were analyzed, it was found that almost 50% of the students in both the virtual-kinesthetic and dual-mode groups scored similar to the students in the upper quartile of the audio-visual group. Thus, most of the students who used virtual manipulatives (either in the virtual-kinesthetic or dual-mode groups) had better perceptions on OMT. Because the virtual manipulatives were common in both groups, the results suggest that students enjoyed using virtual manipulatives and found them more useful than the dynamic video component because they could create their own models and express their creativities on them. This finding supports previous research by Moyer et al. (2005) in that these researchers also concluded virtual manipulatives promote students' creativity. Moreover, even though students in the virtual-kinesthetic and dual-mode groups had similar mean TAM-M scores, more than half of the students in the dual-mode group scored higher on TAM-M than three quarters of students in the virtual-kinesthetic group. The dynamic video component in the dual-mode group could have an effect on the easiness of OMT and resulted in higher scores on TAM-M. The video could have supported students when they had difficulties using virtual manipulatives thereby helping them feel more comfortable using the newly introduced virtual manipulatives.

When students' scores on OMT were analyzed based on their paper-based post-test scores, there was no difference on either students' algebraic scores or students' kinesthetic representation scores between low and high performers. Low-performing students on the paper-based test showed their competence on not only kinesthetic representations but also algebraic notations on OMT. This result signifies the importance of the embedded dynamic assessment taking place during the learning process. It is apparent that lower-performing students achieved better in a learning oriented dynamic assessment than a paper-based test. The OMT scaffolded students to reveal their hidden competencies. This finding is in accordance with Allsop et al. (2008) who concluded that low-performing students could benefit the most from dynamic assessments.

This study has important implications for teaching and learning practice as well as for instructional design. Online learning tools such as dynamic video and interactive manipulatives improve students' conceptual understanding of mathematics. Thus, teachers should consider incorporating such tools in their classrooms. However, students' acceptance of such tools depends on how easy to use the tool so that instructional designers should develop user-friendly interfaces for online learning environments. When utilizing a learning tool in their classrooms, teachers need to provide students with enough time to spend on the learning tool in order for students to learn the tool and have fewer difficulties. In addition, teachers and instructional designers should pay special attention to affordances of media provided with online learning tools. Because, additional affordances incorporated in virtual manipulatives were more acceptable by students.

In this study, TAM-M was used to assess k-12 students' technology acceptances. However, surveys to investigate technology acceptances of k-12 students are not common. Thus, future research is needed on the development of such surveys for k-12 students. In addition, later studies can explore teachers' technology acceptances and its relation to the implementation of technology in instruction.

CHAPTER V

CONCLUSIONS

The major purposes of these studies were (a) to design virtual manipulatives that incorporate multiple components (i.e., audio, audio-video, and manipulative), (b) to investigate the virtual manipulatives' effectiveness in helping students develop conceptual mathematical understanding, and (c) to examine affordances provided with components of virtual manipulatives. Thus, an online manipulative tool (OMT) was developed based on the guidelines in the literature for presentation of information and affordances in virtual manipulatives. The first experimental study was, then, designed to investigate effects of affordances provided with virtual learning environments at different treatment durations.

The first study concluded that the longer treatment duration was associated with less use of the manipulatives. In longer treatment groups (i.e., 20- and 30-minute groups) students were transitioning away from the manipulative and toward the standard algorithm. This finding suggested that the students who spent more time on OMT could solve the problems with the algorithmic procedures instead of using on audio, audio-visual, and manipulative components of OMT. The results of the first study also showed that students who used virtual manipulatives more frequently than audio or audio-visual modes were more likely to achieve better on OMT. This finding might be due to the kinesthetic feature (i.e., virtual manipulatives), which provided students with an active environment where they could create and test their models. Moreover, feedback based on students' interaction with virtual manipulatives could have scaffolded students' understanding of operations with rational numbers concept.

Utilizing the findings from the first study, the second study was designed to investigate OMT's impact on students' understanding of operations with rational numbers over traditional classroom activities (e.g., textbook based assignments). Because in the first study it was found that more time with the OMT allowed for transition from concrete to abstract thinking, the second experimental study was designed with set time per session (i.e., 30-minute) over two weeks (6 sessions total). It was found that students who used OMT (i.e., any experimental group) achieved better on paper-based tests than the students who attended teacher-assigned classroom activities. The students in experimental groups were presented with dynamic and/or interactive representational forms in OMT. The symbolic notations and kinesthetic manipulatives appeared on the same screen so students had the opportunity to test their abstract thinking with manipulatives or watch how symbolic representations are translated into pictorial representations. Therefore, OMT provided a support and scaffold for these students to translate among representations and to develop conceptual understanding of operations with rational numbers.

In addition, the random assignment of students to groups in which they were provided with a specific representational mode or combination of two (i.e., audio-visual, virtual-kinesthetic, and dual-mode) allowed for the comparison of different affordances' effects on paper-based test performances. Students' gain scores from pre- to post- paper-and-pencil tests showed no difference among experimental groups. This finding suggests that different representational modes resulted similar performance via different affordances. That is, students could meaningfully encoded information into their long-term memory and retrieve it efficiently using their cognitive resources. This result is aligned with Kaput's (2006) and Bolyard and Moyer-Packenham's (2006) findings that dynamic video and interactive manipulative has similar effect on students conceptual understanding of mathematics.

The second study also explored students' achievement on the questions presented by OMT versus the students' achievement on paper-and-pencil assessments. Dynamic assessments embedded in virtual environments are known to be capable of revealing students' hidden capacities because they take place during the learning process and provide immediate and specific feedback. In particular, low performing students can benefit the most from dynamic assessments. Students' scores on OMT showed no difference between low- and high-performing students on paper-and-pencil tests. Students who performed low on paper-and-pencil tests showed their competence on not only kinesthetic representations but also algebraic notations on OMT.

In addition to the findings on students' achievement, students' beliefs about the usefulness of OMT and their acceptance of the newly introduced technology were investigated. Students' thoughts about the usefulness of OMT improved over time. Students' perception of usefulness is influenced by the extent to which the technology can be used free of effort. Thus, as students became familiar with OMT and understand how the manipulatives worked, they perceived OMT more useful. Also when students received an explanation via dynamic video on how the symbolic notations are translated into pictorial representations with virtual manipulatives, more positive change was observed in students' perceptions of OMT's usefulness.

The representational modes that students were exposed to also had influences on students' technology acceptances. Students who used the virtual manipulatives (i.e., virtual-kinesthetic and dual-mode groups) had better perceptions of OMT than students who did not interact with the manipulatives (i.e., audio-visual group). Further, who had the dynamic video option in addition to the opportunity to work with the manipulatives had better acceptances of OMT. The relationship between the student characteristics and technology acceptances were also explored. Aligned with the current research findings that the difference between genders in the technology use is narrowing, no difference was found between boys' and girls' technology acceptances. However, SES related differences were identified in technology acceptance. Students from low SES families were also more accepting OMT than students from high SES families. To inform instructional designers about what makes technology more appealing to different student groups further large scale studies are needed.

This study has some limitations that can be addressed in further studies. Currently there are not widely used surveys to explore k-12 students' technology acceptances. Consequently, survey developments regarding k-12 students' technology acceptances are needed. In addition, further research can explore teachers' technology acceptances and how this relates to their technology use in their classrooms and students' achievement.

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APPENDIX A

PRE-TEST FOR 4TH AND 5TH GRADES

Compare the following fractions. Write <, >, or =. Show your work.

1) $\frac{5}{9} \;\square\; \frac{4}{11}$

2) $\frac{3}{5} \;\square\; \frac{2}{3}$

3) $\frac{6}{9} \;\square\; \frac{8}{12}$

Add, subtract, and multiply the following fractions. Write your answer in the simplest form. Show your work.

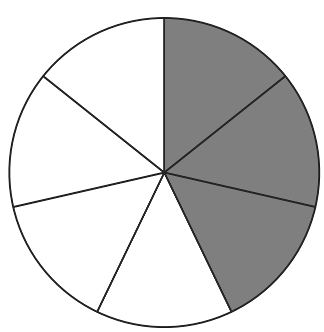
4) $\frac{3}{10} + \frac{1}{2} =$

5) $\frac{4}{5} - \frac{1}{3} =$

6) $\frac{3}{4} \times \frac{4}{9} =$

What is the fraction for the shaded part?

7)



a) $\frac{3}{5}$

b) $\frac{3}{4}$

c) $\frac{3}{7}$

d) $\frac{4}{7}$

Represent the following addition, subtraction, and multiplication in the space provided next to the questions and solve the problems. Do not forget to show your work.

8) $\frac{2}{5} + \frac{1}{6} =$

9) $\frac{2}{3} - \frac{1}{9} =$

10) $\frac{3}{5} \times \frac{2}{3} =$

APPENDIX B

POST-TEST FOR 4TH AND 5TH GRADES

Compare the following fractions. Write <, >, or =. Show your work.

1) $\frac{5}{6} \ \square \ \frac{7}{9}$

2) $\frac{3}{5} \ \square \ \frac{4}{7}$

3) $\frac{3}{9} \ \square \ \frac{2}{6}$

Add, subtract, and multiply the following fractions. Write your answer in the simplest form. Show your work.

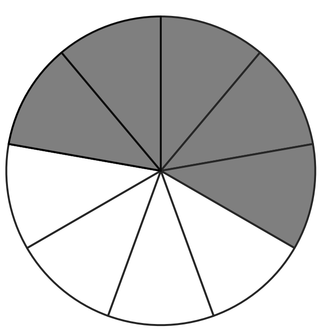
4) $\frac{4}{9} + \frac{2}{3} =$

5) $\frac{7}{8} - \frac{1}{2} =$

6) $\frac{2}{5} \times \frac{3}{4} =$

What is the fraction for the shaded part?

7)



e) $\frac{5}{9}$

f) $\frac{4}{5}$

g) $\frac{9}{5}$

h) $\frac{5}{4}$

Represent the following addition, subtraction, and multiplication in the space provided next to the questions by creating your own grids and shading the appropriate areas. You will also need to solve the problems. Do not forget to show your work.

8) $\frac{2}{7}+\frac{1}{3}=$

9) $\frac{7}{9}-\frac{2}{5}=$

10) $\frac{2}{5}\times\frac{3}{4}=$

APPENDIX C

PRE-TEST FOR 7TH AND 8TH GRADES

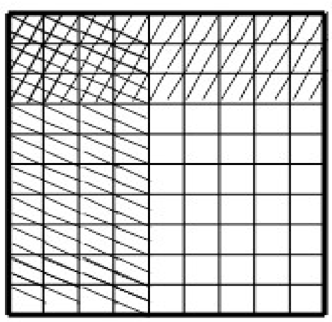
Compare the following fractions. Write <, >, or =. Show your work.

1) $62.5\% \quad \square \quad \frac{19}{32}$

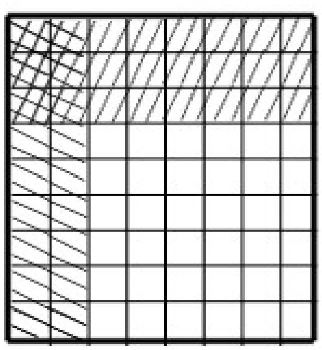
2) $\frac{9}{16} \quad \square \quad \frac{8}{17}$

3) $\frac{24}{32} \quad \square \quad \frac{25}{31}$

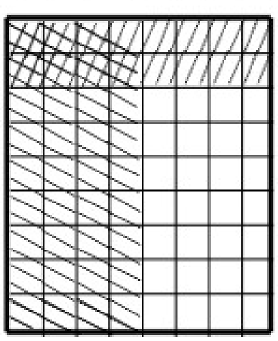
Add, subtract, and multiply the following fractions. Write your answer in the simplest form. Show your work.

4) 

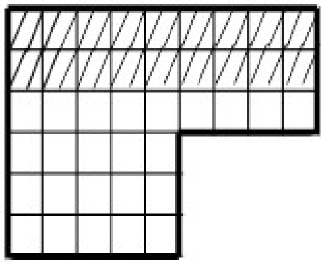
Find and solve the addition problem for the area model represented on the left. Show your work.

5) 

Find and solve the subtraction problem for the area model represented on the left. Show your work.

6) 

Find and solve the multiplication problem for the area model represented on the left. Show your work.

7) 

Find the fraction for the shaded part on the figure left. Please show your work.

Represent the following addition, subtraction, and multiplication in the space provided next to the questions and solve the problems. Do not forget to show your work.

8) $\frac{8}{9} + \frac{1}{11} =$

9) $\frac{8}{36} - \frac{1}{9} =$

10) $\frac{24}{36} \times \frac{12}{16} =$

APPENDIX D

POST-TEST FOR 7TH AND 8TH GRADES

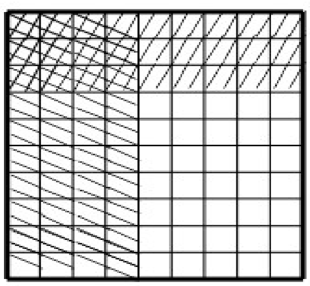
Compare the following fractions. Write <, >, or =. Show your work.

1) 52.5% $\square$ $\frac{14}{29}$

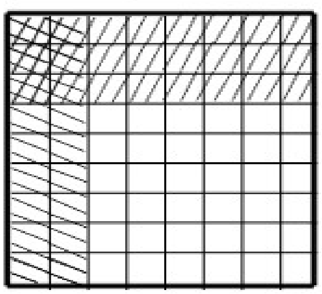
2) $\frac{9}{14}$ $\square$ $\frac{10}{15}$

3) $\frac{7}{21}$ $\square$ $\frac{14}{42}$

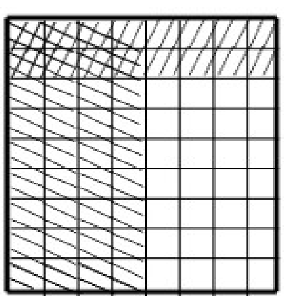
Add, subtract, and multiply the following fractions. Write your answer in the simplest form. Show your work.

4) 

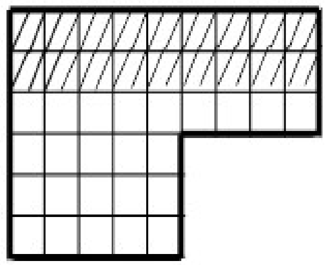
Find and solve the addition problem for the area model represented on the left. Show your work.

5) 

Find and solve the subtraction problem for the area model represented on the left. Show your work.

6) 

Find and solve the multiplication problem for the area model represented on the left. Show your work.

7) 

Find the fraction for the shaded part on the figure left. Please show your work.

Represent the following addition, subtraction, and multiplication in the space provided next to the questions and solve the problems. Do not forget to show your work.

8) $\frac{3}{10} + \frac{1}{11} =$

9) $\frac{3}{8} - \frac{8}{32} =$

10) $\frac{12}{17} \times \frac{5}{6} =$

APPENDIX E

AFTER-SOFTWARE QUESTIONNAIRE

Directions: Please answer the following questions by putting a check mark with the appropriate response.

		Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1.	Using the software would increase my productivity in this course.					
2.	Using the software would enhance my effectiveness in this course.					
3.	I found the software would be useful in this course.					
4.	Using the software would improve my performance in this course.					

APPENDIX F

TECHNOLOGY ACCEPTANCE MODEL MODIFIED QUESTIONNAIRE

Directions: This questionnaire gives you an opportunity to tell us your reactions to the software you used. Your responses will help us understand what aspects of the software you are particularly concerned about and the aspects that satisfy you. To as great a degree as possible, think about all the tasks that you have done with the software while you answer these questions.

Please read each statement and indicate how strongly you agree or disagree with the statement by putting a check mark with the appropriate response.

Please write comments to elaborate on your answers.

		Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
1.	I found the software easy to use. **(EU)**					
2.	Using the software would increase my productivity in this course. **(U)**					
3.	I found it easy to get the software to do what I want it to do. **(EU)**					
4.	I had fun using the software. **(E)**					
5.	Using the software would enhance my effectiveness in this course. **(U)**					

6.	I found the software would be useful in this course. **(U)**					
7.	My interaction with the software was clear and understandable. **(EU)**					
8.	Using the software was pleasant. **(E)**					
9.	Using the software would improve my performance in this course. **(U)**					
10.	Learning to use the software was easy for me. **(EU)**					
11.	I found using the software to be enjoyable. **(E)**					

Note. E = Enjoyment, EU = Ease of Use, and U = Usefulness.

VITA

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Özel, S., Yetkiner, Z. E., & Capraro, R. M. (2008). Technology in K-12 mathematics classrooms. *School Science and Mathematics, 108*, 80-85.

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