

**DECISION SUPPORT SYSTEM (DSS) FOR MACHINE SELECTION:  
A COST MINIMIZATION MODEL**

A Dissertation

by

MAYRA I. MENDEZ PIÑERO

Submitted to the Office of Graduate Studies of  
Texas A&M University  
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2009

Major Subject: Industrial Engineering

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**ABSTRACT**

Decision Support System (DSS) for Machine Selection:

A Cost Minimization Model. (May 2009)

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Within any manufacturing environment, the selection of the production or assembly machines is part of the day to day responsibilities of management. This is especially true when there are multiple types of machines that can be used to perform each assembly or manufacturing process. As a result, it is critical to find the optimal way to select machines when there are multiple related assembly machines available. The objective of this research is to develop and present a model that can provide guidance to management when making machine selection decisions of parallel, non-identical, related electronics assembly machines. A model driven Decision Support System (DSS) is used to solve the problem with the emphasis in optimizing available resources, minimizing production disruption, thus minimizing cost. The variables that affect electronics product costs are considered in detail. The first part of the Decision Support System was developed using Microsoft Excel as an interactive tool. The second part was developed through mathematical modeling with AMPL9 mathematical programming language and the solver CPLEX90 as the optimization tools.

The mathematical model minimizes total cost of all products using a similar logic as the shortest processing time (SPT) scheduling rule. This model balances machine workload up to an allowed imbalance factor. The model also considers the impact on the product cost when expediting production. Different scenarios were studied during the sensitivity analysis, including varying the amount of assembled products, the quantity of machines at each assembly process, the imbalance factor, and the coefficient of variation (CV) of the assembly processes.

The results show that the higher the CV, the total cost of all products assembled increased due to the complexity of balancing machine workload for a large number of products. Also, when the number of machines increased, given a constant number of products, the total cost of all products assembled increased because it is more difficult to keep the machines balanced. Similar results were obtained when a tighter imbalance factor was used.

## DEDICATION

To Robert, my husband, your love means everything to me...

To Susan Janice, Roberto Alejandro, and Daniel Alejandro, my children,  
you are my inspiration...

To mom and dad for always being proud of me...

## ACKNOWLEDGEMENTS

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Finally, thanks to my friends from church, my family away from home. You were my moral support during these important years for me and my family.

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## **CHAPTER I**

### **INTRODUCTION**

#### **1.1. Motivation**

Every manufacturing firm needs to have reliable estimates of the cost of each product they make. A “good product cost estimate” is the basic information that management use together with sales prices when calculating their earnings.

Understanding the differences in each of their products and the processes or machines used to manufacture or assembly them, the company is in a better position to precisely calculate their profitability. Considering the lack of detailed cost estimates and cost models within the manufacturing assembly companies, this research examines detailed cost estimates of the products assembled with the purpose of minimizing total cost of assemblies. This cost model represents an excellent tool in helping the companies to understand where their earnings are coming from (or how the products’ costs are being generated). The cost model considers all aspects of product assembly and the resultant cost impacts.

The main contribution of this research is related with the level of detail considered when estimating the costs of the products because it includes each of the components of a product’s variable costs. This cost model can be a useful tool during budget preparation, short-term production planning, and in a continuous basis focusing

on cost improvement. Currently, many manufacturing firms do not delve into sufficient detail when calculating cost (i.e. indirect costs) because it is very time consuming, thus expensive. As many researchers agree:

“Traditional accounting systems allocate overhead as a percentage of direct labor hours and/or machine hours. As the volume of the product increases by 10%, the manufacturing overhead increases by 10%; this is assuming that the direct labor hours and machine hours increase proportionally. While this method of allocation is simple and fast, it does not reflect accurately on the actual product cost” (Ong, 1995; Dhavale, 1990; Miller et al., 1985; Cooper et al., 1988).

The model developed in this research directly helps in this kind of situations. This research is an extension of Méndez’(2001) previous research, but now focusing on using a model-driven decision support system tool on the assembly processes of printed circuit boards (PCB) with the ultimate objective of minimizing total cost.

## **1.2. Problem Description**

Considering the cost of the available alternatives is the way most commonly used to make a decision when assembly companies are dealing with machine selection within each assembly process. To facilitate this repetitive task, a cost model is developed following some basic assumptions: (1) an electronic product consists essentially of a printed circuit board (PCB) with electronics components soldered to it following a series of sequential steps, and (2) the cost of the electronic products is calculated assuming typical and generic assembly sequence and processes (refer to Figure 1.1).

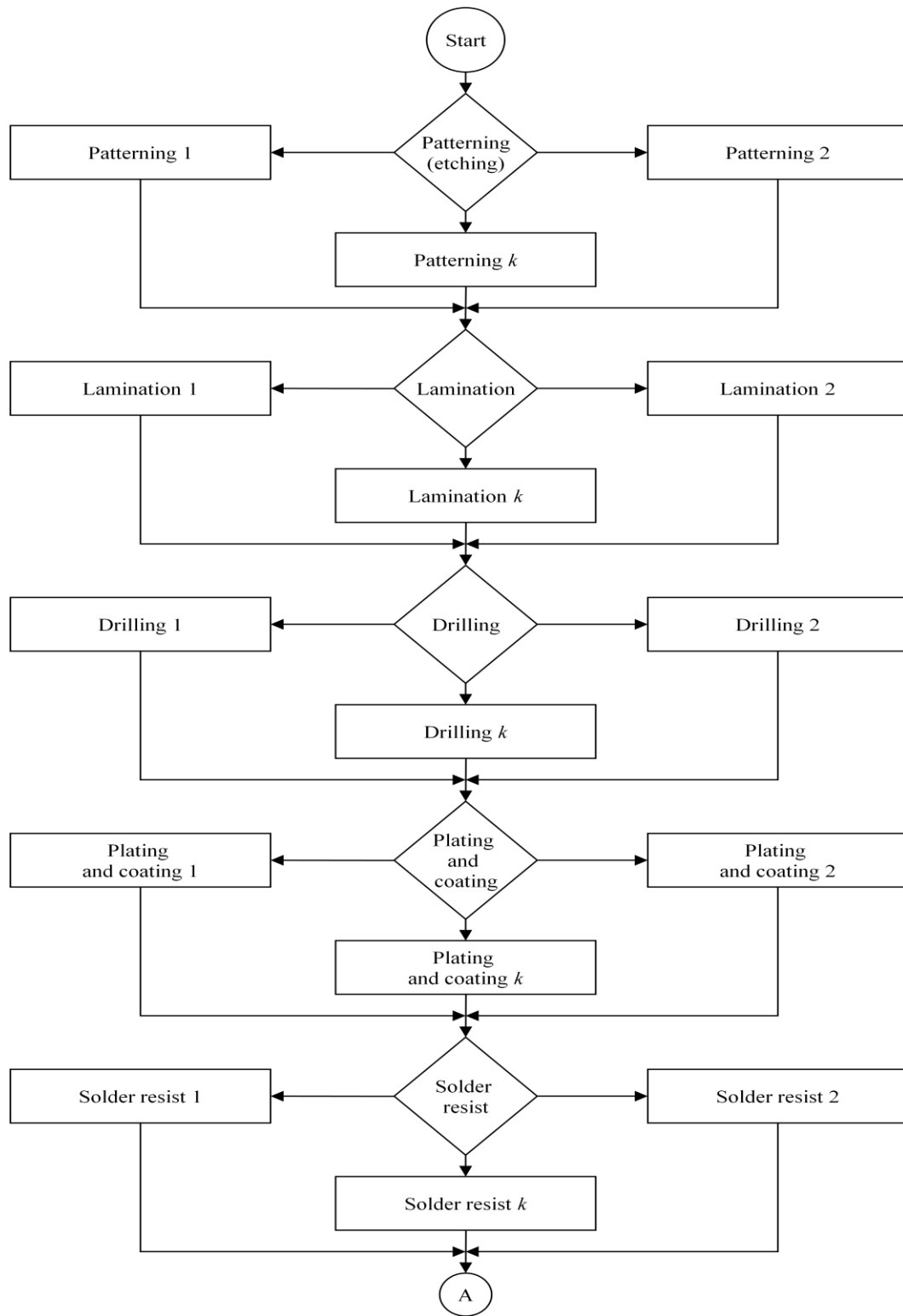


Figure 1.1. Assembly Processes Flowchart (Mendez, 2001; Clark, 1985)

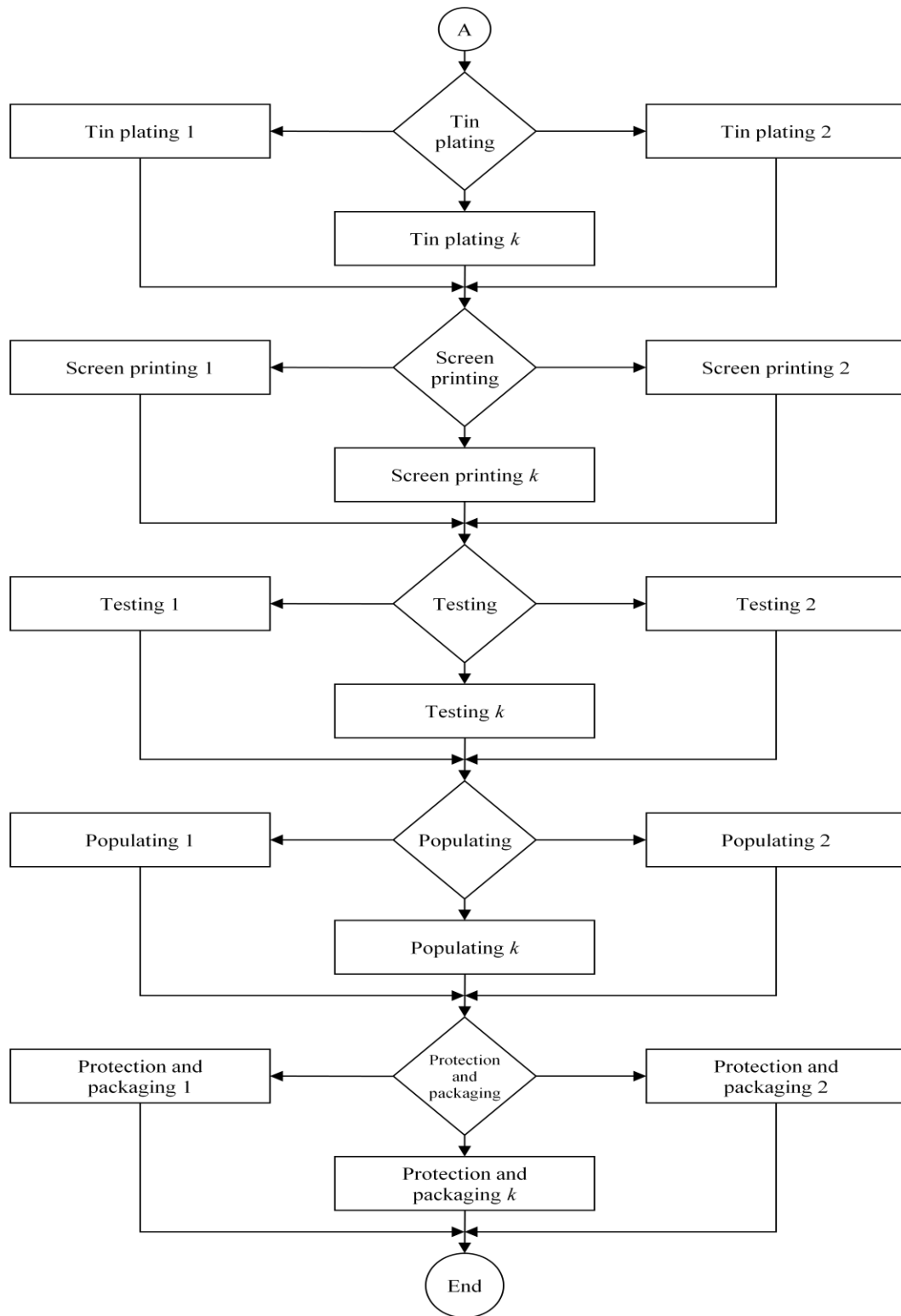


Figure 1.1 Continued



Crama et al. (2002) considered a generic assembly process which “consists in placing (inserting, mounting) a number of electronic components of prespecified types at prespecified locations on a bare board. Several hundred components of a few distinct types (resistors, capacitors, transistors, integrated circuits, etc.) may be placed on each board”. As pointed out by Ong (1995) on his cost estimate using an activity-based approach, “there will, however, be variations among different manufacturers and it is not appropriate here to describe all the processing steps but to have a general process flow which can then be used for our cost estimation”.

In each step of the assembly sequence, resources are consumed, hence cost is incurred. The variables that affect each assembly process are identified and considered in the machines’ selection problem based on how they influence product cost. It was critical during the progress of this research to understand how to estimate the cost and how resources are consumed during the assembly processes. Figures 1.2 and 1.3 present diagrams showing the cost model general concept and the consumption of resources, respectively.

The problem considered in this research includes the coefficient of variation (CV) as the measurement of the expected process variability because it is inherent in all manufacturing or assembly processes or systems. Being able to manage process variability may have a big impact on the effectiveness of the manufacturing process. As explained by Hopp and Spearman (2001), variability (or “the quality of nonuniformity of a class of entities”) is associated with randomness (related to probability), but not identical. The process time (and setup times) variability is considered in this research

and is measured based on the processes relative variability, for which the CV is used considering the standard deviation and the mean of these randomly generated times;  $CV = \frac{\sigma}{\mu}$ . In this research, process variability is classified based on its CV value as low variability ( $CV < 0.75$ ), moderate variability ( $0.75 \leq CV < 1.33$ ), and high variability ( $CV \geq 1.33$ ) (Hopp et al., 2001).

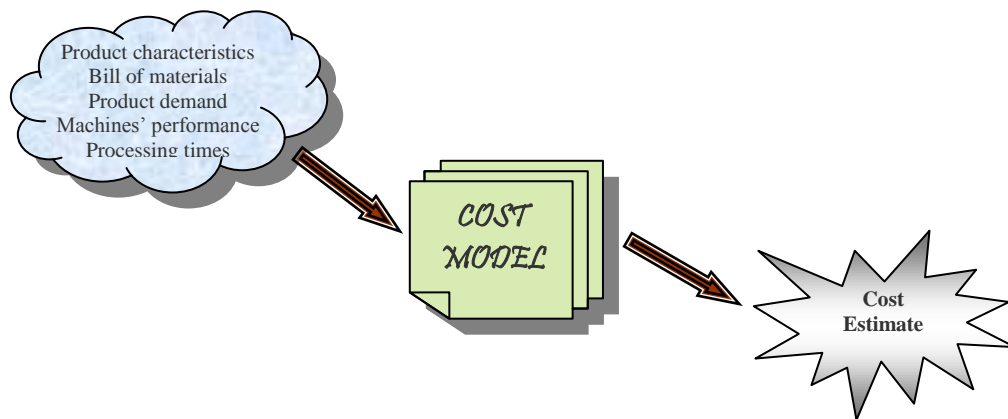


Figure 1.2. Cost Model Diagram (Mendez, 2001)

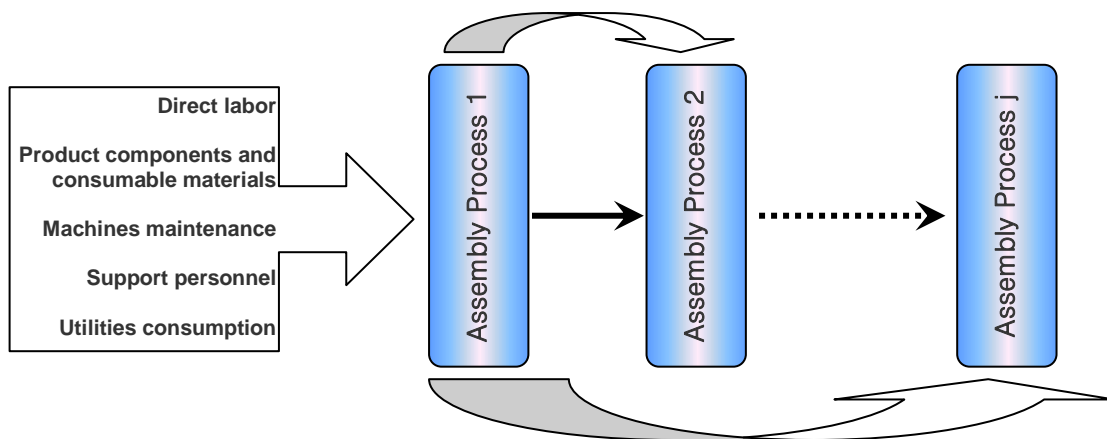


Figure 1.3. Resources Consumption Diagram (Mendez, 2001)

### **1.3. Organization of Dissertation**

This dissertation is divided in chapters pertaining to each part of the research. This chapter (CHAPTER I) includes the motivation for the research, the description of the problem and the review of the relevant literature. The remaining of the dissertation is organized as follows:

- CHAPTER II – Presents the development of the basic cost model using Microsoft Excel to estimate the cost of one product at a time selecting the machine with the minimum cost at each assembly process.
- CHAPTER III – Presents the development of the expanded models to optimize the total cost of all products assembled. The mathematical model was programmed with AMPL mathematical programming language and CPLEX was used as the solver.
- CHAPTER IV – Discusses the analysis of results and the sensitivity analyses. Different scenarios were generated by changing the amount of products, quantity of machines, imbalance factors allowed in machine workload balance, and coefficient of variation of the assembly processes.
- CHAPTER V – The last chapter summarizes this research, presents the conclusions and the recommendations for future research.

### **1.4. Literature Review**

The scope of this research includes model driven Decision Support Systems, mathematical modeling as an optimization tool, and cost modeling of electronics

products. As a result, it was important to perform an extensive review of literature concerning these areas. This section is divided based on relevant literature of the following major areas: (1) use and application of Decision Support Systems (DSS) in general; (2) use and application of mathematical modeling on assignment problems; (3) cost modeling and time reduction models for electronics products.

A Decision Support System can be defined as an interactive information system that supports business and organizational decision-making activities by compiling useful information from raw data, documents, personal knowledge, and/or business models to identify and solve problems and make decisions. Power and Sharda (2007) explained that “a model-driven Decision Support System includes computerized systems that use accounting and financial models, representational models, and/or optimization models to assist in decision-making”. Model-driven DSS has been used since the late 1960s (Power, 2003) as a management decision system and the first dissertation research using this methodology was presented by Scott Morton in 1967.

Some of the Decision Support Systems reviewed were models where minimizing costs was not necessarily the main objective. A DSS related to electronics products was proposed by Sandborn (2005) with the purpose of generating a model to determine the predictability of the reliability of electronics for scheduled maintenance concepts. Other authors, such as Sundararajan et al. (1998) developed a model to determine optimum production scenario based on the tradeoffs between service levels, costs, inventories, changeovers, and capacity. This model presented an application for a food processing industry where they mentioned minimizing cost as part of the challenge but the specifics

on how the cost was calculated were not presented. A Decision Support System that has been created to be used as a concurrent decision making tool was developed by Forgionne and Kohli (1996) with hospitals as their domain environment. This Decision Support System was created with the purpose of comparing it with a Management Support System where cost was not considered. Pillai (1990) developed another Decision Support System to identify and select alternatives that provide the highest manufacturing improvements and cost effectiveness. This work was performed for Intel and one of the areas it focused was in providing a baseline for a unit cost analysis. A cost model was developed using Intel's format due to their familiarity with it. This model did not include any additional detail on how to calculate the cost of the products and was used mainly to calculate the ROI (return on investment) of the alternatives considered.

Mathematical modeling or mathematical functions which present the end result of an operations research model where alternatives, restrictions, and an objective criterion are considered (Taha, 1997), has been used to solve problems in decision making as well. Pentico (2007) presented a survey of assignment problems where two or more sets are to be optimally matched. Within the models for multi-dimensional problems Gilbert and Hofstra (1988) mentioned the axial three-dimensional assignment problem considering jobs, workers, and machines as the three dimensions to match. Another authors, Gavish and Pirkul (1991) developed a mathematical model and heuristics procedures for the generalized assignment problem with multi-resources. These authors included cost on their mathematical models by being part of the objective functions, but it was presented as a parameter without any detail on its calculation.

LeBlanc et al. (1999) presented an extension of the multi-resource generalized assignment problem by splitting batches of products, while considering the effect of setup times and costs. LeBlanc's research differentiated from the others because the setup costs were considered independently of the total cost. Besides that, the other components of the total cost were not analyzed on detail.

There is another kind of generalized assignment problem that has been studied considering bottlenecks where capacity limitations have been included as part of the constraints. Some authors such as Mazzola and Neebe (1988) and Garfinkel (1971) included cost as a parameter on the objective functions without further details, while Geetha and Vartak (1994) did not consider cost at all on the models they proposed. Mathematical modeling considering workload balancing of parallel identical machines was presented by Rajakumar et al. (2004) where the researchers were looking to maximize production output optimizing overall performance. Ammons et al. (1997) presented a mathematical model to solve the problem of balancing workload in printed circuit cards assembly but the focus was to balance component placement times and setup times across the machines.

While reviewing literature on cost modeling for electronics products, Hillier and Brandeau (2001) can be mentioned as the ones who considered an operation assignment problem for a printed circuit board assembly process with the primary objective of minimizing total manufacturing cost, and a secondary objective of balancing machine workloads. They developed a binary integer program and a heuristic to solve the problem. These scholars considered the manufacturing cost "to be the expected total

amount of time required to produce all of the boards during the planning horizon". The main difference of Hillier and Brandeau work and the problem presented in this research is that they used identical assembly machines which implies that machine assignment at each assembly process was not required. It also means that the machine workload balance was done for all processes and not at each assembly process. Rajkumar and Narendran (1998) developed a heuristic for sequencing printed circuit board assembly to minimize setup times. Ong (1995) used Activity-Based Costing (ABC) in the early concept stage of design to estimate manufacturing costs of a printed circuit board assembly. This model provided details of how the product cost was calculated during the design of the product. On a previous research effort, Méndez (2001) developed an extremely detailed cost model for power electronics assemblies. Specifically the cost model was used for the fabrication of the boards and for new electronics' assemblies. Méndez cost model was designed for the manufacturing companies to thoroughly understand their new products' cost. It did not consider assigning machines and the objective of minimizing costs; it was a tool to estimate the cost of new products.

Concluding the review of the literature, models were not found that while optimizing (minimizing) total cost, designing a decision support system, and/or developing a cost model were able to calculate the cost of the products with significant detail. The models where cost was thoroughly analyzed used Activity-Based Costing (ABC) as the accounting system to allocate their costs (Ong, 1995) or were applicable to assembly processes of new products and not necessarily applicable to existing products with the purpose of minimizing products' costs (Mendez, 2001).

## **1.5. Summary**

In this introductory chapter the motivation of this research was explained as well as the description of the problem studied. Review of literature on DSS, mathematical modeling for assignment problems, and cost modeling for electronics products followed confirming the need for further research. The explanation of the basic cost model is presented on the next chapter.



## **CHAPTER II**

### **DEVELOPMENT OF THE BASIC MODEL**

#### **2.1. Introduction**

This chapter describes the basic model for cost minimization of one product at a time for the machine selection problem. In this basic model, the total cost of only one product is considered to allow the researcher to delve into the details of cost estimating before complicating the model. This scenario considers the ten assembly processes for electronics products presented on Figure 1 and three parallel, non-identical, related machines per assembly process. The purpose of this simplified scenario is to show how the total cost of an assembled product can be calculated while being minimized at the same time.

The assumptions of the model are mentioned below. The details of the model development with the required equations to calculate the cost are explained. Interactive displays from Microsoft Excel that are used to solve the cost model are shown.

#### **2.2. Assumptions**

- The number and names of the assembly processes, and the machine quantities are given.
- The capacity in machine hours per period of each machine is given.

- Average direct labor employees' hourly salary is given. Direct labor employees quantity and direct labor employees average percent in each assembly process are generated by the uniform distribution.
- Average support personnel yearly salary is given. Support personnel quantity and the average percent of time in each assembly process are generated by the uniform distribution.
- Product demand per year is known and generated using the uniform distribution.
- Units per batch are generated by the uniform distribution.
- Cycle times (including handling times, processing times, setup times, and waiting times) are generated using the normal distribution.
- Product components quantities and costs, and consumable materials quantities and costs are generated with the uniform distribution.
- Mean-time-to failure and mean-time-to-repair are generated by the uniform distribution.
- Units of utilities consumed and utilities costs are generated by the uniform distribution.

The exponential and beta distributions are more commonly used to generate random times for parameters such as mean-time-to-failure and mean-time-to-repair, but for the purpose of this research the uniform distribution will suffice. The reason to use

the uniform distribution is that these parameters have minimum impact on the total cost of the product.

### **2.3. Model Development**

The different costs that affect the assembly of the electronics products are analyzed when developing the cost model. The necessary details (especially with the indirect costs) are considered to ensure the cost presented is accurate.

The cost model essentially calculates direct labor cost, materials cost and overhead or indirect costs. The sum of these main cost classifications give us the total cost of the product (Horngren et al., 2003; Castillo, 1998). When calculating product cost, it is important to clarify which accounting system this research supports. There are two main accounting systems that can be used to calculate product cost for a given period: absorption (or full) costing and variable (or direct) costing (Hilton, 1999). The basic difference between them lies in the treatment of the fixed manufacturing overhead costs. With absorption costing, these costs are included in the product cost that flow through the manufacturing accounts, treating them as inventoriable costs (costs incurred to purchase or manufacture goods). With variable costing, the fixed manufacturing overhead costs are not included as a product cost on the manufacturing accounts since they are treated as period costs (costs that are expensed during the time period in which they are incurred) (Hilton, 1999).

Variable costing accounting system is used when developing this cost model because the Decision Support System (DSS) developed during this research with the cost

model is to be used as a short-term decision making tool. Absorption costing considers capital investments; therefore it is more related to the capacity to produce than the actual production of specific units (Horngren et al., 2003). Another advantage of using variable costing is that it dovetails much more closely than absorption costing with any operational analyses that require a separation between fixed and variable costs (Hilton, 1999). One of these tools used by managers to plan and control business operations is Cost-Volume-Profit (CVP) analysis (Horngren et al., 2003; Hilton, 1999). When performing a CVP analysis, changes in costs and volume level are examined as well as their resulting effects on net income (Kinney et al., 2006).

### 2.3.1. Mathematical Equations for the Microsoft Excel Model

Using Microsoft Excel, an easy-to-use interactive model is created to calculate the cost of a given product. Required data is identified and mathematical equations are formulated for each one of the cost classifications identified on the previous chapter (direct labor, product components and consumable materials, machines maintenance, support personnel, and utilities consumption). The following notation presented in alphabetical order is used throughout the Microsoft Excel model:

$a$	index for product components, $a = \{1, 2, 3, 4, 5\}$
$b$	index for consumable materials, $b = \{1, 2\}$
$x$	index for product
$j$	index for processes, $j = \{PE, LM, DR, PC, SR, TP, SP, TT, PO, PP\}$
$k$	index for machines, $k = \{1, 2, 3\}$

- $s$  index for support personnel,  $s = \{ENG, QUAL, OTHER\}$
- $u$  index for utilities,  $u = \{WATER, ELEC, GAS\}$

### 2.3.1.1. Direct Labor Cost per Product

To calculate direct labor cost per product, processing times and setup times of each product at each process and machines are used. Hopp and Spearman (2001) define process time and setup time as follows: “process time is the time jobs are actually being worked on at the station” and “setup time is the time a job spends waiting for the station to be setup” (setup refers to the preparation of a machine for the product to be processed or assembled). The quantity of direct labor employees, the percent of time direct labor employees are assigned to each process and machine and the average direct labor employee salary per hour are considered.

The following notation is used for the data required to calculate the direct labor cost per unit for each assembled product:

- $\#DL_{jk}$  quantity of direct labor employees assigned to machine  $k$  on process  $j$
- $\%DL_{jk}$  average percent of time dedicated to machine  $k$  in process  $j$
- $\$DL$  average salary per hour for direct labor employees
- $PT_{xjk}$  process time of product  $x$  in process  $j$  and machine  $k$
- $SU_{xjk}$  setup time of product  $x$  in process  $j$  and machine  $k$

The mathematical equation for the direct labor cost per unit of each product ( $LC_x$ ) is shown in equation 2.1.

$$LC_x = [(SU_{xjk} + PT_{xjk}) \times \#DL_{jk} \times \%DL_{jk}] \times \$DL \quad \forall j, k \quad (2.1)$$

### 2.3.1.2. Material Cost per Product

Product components required and their costs, and consumable materials required and their costs are used to calculate the material cost per unit of each product. Product components are the parts that need to be inserted on the PCBs. Consumable materials are materials that are used at workstations but do not become part of the product sold (Hopp et al., 2001). Some examples presented by Méndez (2001) are: components tape, solder paste, adhesive glue, protection tape, flux, solder, alcohol, additives, etc.

The following notation is used for the data required to calculate the material cost per unit for each assembled product:

$CM_x$	consumable materials cost per unit for product $x$
$CM_{xjb}$	quantity of each consumable material $b$ required at process $j$
$\$CM_b$	cost of each consumable material $b$
$CP_x$	product components cost per unit for product $x$
$CP_{xja}$	quantity of each product component $a$ required at process $j$
$\$CP_a$	cost of each product component $a$

The mathematical equation for the material cost per unit of each product ( $MC_x$ ) is shown in equation 2.2.

$$MC_x = CP_x + CM_x = \sum_a (CP_{xja} \times \$CP_a) + \sum_b (CM_{xjb} \times \$CM_b) \quad \forall a, b, j \quad (2.2)$$

### 2.3.1.3. Overhead Cost per Product

To calculate the overhead cost per product, all indirect variable costs are considered in detail. Indirect costs are the ones that cannot be traced to a cost object in an economically feasible or cost-effective way (Horngren et al., 2003; Kinney et al., 2006). Variable cost is a cost that varies in total in direct proportion to changes in activity; it is a constant amount per unit (Kinney et al., 2006). For management to have a useful tool to use during their decision making process, overhead cost is divided into support personnel cost, utilities consumption cost, and machine maintenance cost. Support personnel cost per product is an allocation of the salaries paid to the support personnel assigned to the assembly processes. The allocation of the utilities consumption cost takes into consideration the utilities consumed at each assembly process based on the level of production assembled at each process. Machine maintenance cost is an allocation of the total machine maintenance cost during the accounting period to each machine within each assembly process based on the production level.

To calculate the three components of the overhead cost, the following data is required: total time (for this research, total time represents cycle time considering variability) that the products are on the assembly line and process time per product at each process and machine; quantity of support personnel assign to the assembly processes, the average percent of time they dedicate to each process and their average

yearly salary; units of each utility consumed and their cost per unit; total machine maintenance cost incurred and machine utilization based on machine hours run.

The following notation is used when calculating each component of the overhead cost per product:

$MM_x$	machine maintenance cost allocated to product $x$
$SP_x$	support personnel cost allocated to product $x$
$UC_x$	utilities consumption cost allocated to product $x$

The following notation is used for the data required to calculate the overhead cost per unit for each assembled product:

$HRS$	worked hours per worked week
$\$MM$	total machine maintenance cost incurred
$PT_{xjk}$	process time of product $x$ in process $j$ and machine $k$
$SP_s$	quantity of support personnel $s$ assigned to the assembly processes
$\%SP_x$	average percent of time of support personnel in product $x$
$\$SP$	average yearly salary of support personnel
$SUbh$	setup time per batch of product
$SU_{xjk}$	setup time of product $x$ in process $j$ and machine $k$
$TT_x$	total time of product $x$
$UC_u$	units of each utility $u$ consumed
$\$UC_u$	average cost per unit of each utility $u$
$UNbh_x$	units per batch of product



$WKS$  worked weeks per year

The mathematical equation for the overhead cost per unit of each product ( $OH_x$ ) is shown in equation 2.3.

$$\begin{aligned}
 OH_x &= SP_x + UC_x + MM_x & (2.3) \\
 &= \sum_s (SP_s \times \%SP_x) \times TT_x \times \frac{\$SP}{WKS \times HRS} \\
 &+ \sum_u (UC_u \times \$UC_u) \times PT_{xjk} & \forall j, k \\
 &+ \frac{\$MM}{(2 \times HRS)} \times (PT_{xjk} + SU_{xjk}) & \forall j, k
 \end{aligned}$$

$$SU_{xjk} = \frac{SUBh}{UNbh_x} \quad (2.4)$$

To calculate the setup time per product used in equation 2.3, setup time per batch and units per batch are used in equation 2.4. In order to make the calculations included in equation 2.3, additional mathematical equations are needed to calculate total time per product. Total time per product is calculated using cycle time per product and availability of the machines. Cycle time per product as defined by Hopp and Spearman (2001) (and applicable to this research) is calculated adding handling time, waiting time, setup time, and process time of each product. Handling time refers to the time it takes to move the in process product between assembly processes; waiting time is the time a

product has to wait to start an assembly process; setup time and process time were defined in section 2.3.1.1. Availability of a machine reflects a proportion between mean-time-to failure and mean-time-to repair (Hopp et al., 2001).

The following notation is required for the equations for total time per product, cycle time per product, and availability of each machine:

$HT_{xj}$	handling time of product $x$ to process $j$
$MTTF_{jk}$	mean-time-to failure of machine $k$ from process $j$
$MTTR_{jk}$	mean-time-to repair machine $k$ from process $j$
$PT_{xjk}$	process time of product $x$ in process $j$ and machine $k$
$SU_{xjk}$	setup time of product $x$ in process $j$ and machine $k$
$WT_{xjk}$	waiting time of product $x$ to start process $j$ in machine $k$

The mathematical equations for total time per product ( $TT_x$ ), cycle time per product per unit of each product ( $CT_x$ ), and availability of machines ( $AV_{jk}$ ) are shown in equations 2.5, 2.6, and 2.7.

$$TT_x = \frac{CT_x}{AV_{jk}} \quad (2.5)$$

$$CT_x = SU_{xjk} + HT_{xj} + WT_{xjk} + PT_{xjk} \quad (2.6)$$

$$AV_{jk} = \frac{MTTF_{jk}}{MTTF_{jk} + MTTR_{jk}} \quad (2.7)$$

#### 2.3.1.4. Total Cost per Product

The total cost per product ( $TC_x$ ) is calculated by adding the direct labor cost per product ( $LC_x$ ), the material cost per product ( $MC_x$ ), and the overhead cost per product ( $OH_x$ ), as shown in equation 2.8. The Microsoft Excel model uses the total cost per product equation as the objective to be minimized when selecting machines per assembly process for the given product.

$$TC_x = LC_x + MC_x + OH_x \quad (2.8)$$

#### 2.3.2. Microsoft Excel Model

The Microsoft Excel model is to provide an interactive easy-to-use tool for management during the short-term decision making process. To facilitate its use, this model gives the user different options (refer to Appendix A for details). The first screen on the Microsoft Excel model presents the main menu with the options to go to the data sheet to enter or revise data, or to go to the calculations and results sheet to see results (refer to Figure 2.1). When the user chooses to go to the data sheet, the Excel macro directs the worksheet to the data sheet (see Figure 2.2). If the user wants to go over the calculations and results, the Excel macro directs the worksheet to the calculations and results sheet (see Figures 2.3 and 2.4).

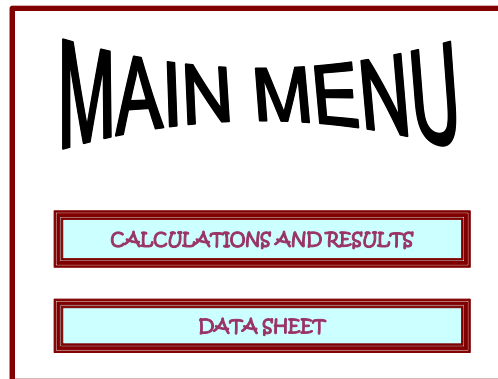


Figure 2.1. Microsoft Excel Model Main Menu

**Data required**[Back to Main Menu](#)

<b>Variable</b>	<b>Description</b>	<b>Units</b>	<b>Value</b>
$CM_{xjb}$	Consumable material $b$ required in process $j$	Number	$CM_{xjb}$
$\$CM_b$	Cost of consumable material $b$	\$/unit	$\$CM_b$
$CP_{xja}$	Component $a$ required in process $j$	Number	$CP_{xja}$
$\$CP_a$	Cost of component $a$	\$/unit	$\$CP_a$
$D_{yr}$	Total annual demand	Units/year	11,575
$\$DL$	Average direct labor cost per hour	\$/hour	\$ 8.50
$\%DL_{jk}$	% of time direct employee work on machine $k$ in process $j$	%	$\%DL_{jk}$
$\#DL_{jk}$	Quantity of direct employees working on machine $k$ in process $j$	Number	$\#DL_{jk}$
HRS	Worked hours per week	Hours/week	40
$HT_{xj}$	Handling or moving time of product $x$ from previous process to process $j$	Hours/unit	$HT_{xj}$
MH	Machine hours available per week	Hours/week	80
$\$MM$	Total machines maintenance cost	\$/month	\$ 1,000
$MTTF_{jk}$	Mean-time-to-failure of machine $k$ in process $j$	Hours	$MTTF_{jk}$
$MTTR_{jk}$	Mean-time-to-repair of machine $k$ in process $j$	Hours	$MTTR_{jk}$
$PT_{xjk}$	Process time on product $x$ of machine $k$ in process $j$	Hours/unit	$PT_{xjk}$
$SP_s$	Support personnel $s$	Number	$SP_s$
$\%SP_x$	% of time on product $x$ by support personnel $s$	%	$\%SP_x$
$\$SP$	Cost of support personnel per year	\$/year	\$ 60,000
Subh	Setup time per batch of product $x$	Hours/batch	Subh
$UC_u$	Utilities consumption $u$ per product	Units	$UC_u$
$\$UC_u$	Cost of utilities consumption $u$	\$/unit	$\$UC_u$
$UNb_x$	Total units of product $x$ per batch	Units/batch	1,000
WKS	Worked weeks per year	Weeks/year	50

Figure 2.2. Microsoft Excel Model Data Sheet

Using the calculations and results sheet, the total cost for product  $x$  can be obtained with all the details related to the cost components. Direct labor cost, material cost, and overhead costs per product are calculated independently and their summation represents the total cost per product for a single product. On the summarized results (refer to Figure 2.4), the total cost of the product is easily identified.

<b>To calculate cost of product x</b>											
Process	Machine ID	Setup time ( $SU_{xjk}$ )	Handling time ( $HT_{xj}$ )	Waiting time ( $WT_{xjk}$ )	Process time ( $PT_{xjk}$ )	Labor cost ( $LC_x$ )	Material cost ( $MC_x$ )	Overhead cost ( $OH_x$ )	Product cost ( $PC_x$ )	<i>Min (<math>PC_x</math>)</i>	
Patterning (etching) (PE)	PE1	0.0008	0.4978	0.3916	0.0001	\$ 0.0026	\$ 58.76	\$ 21.33	\$ 80.09	\$ 65.61	
	PE2	0.00088		0.3971	0.0001	\$ 0.0083		\$ 9.63	\$ 68.40		
	PE3	0.00082		0.2836	0.0001	\$ 0.0052		\$ 6.84	\$ 65.61		
Lamination (LM)	LM1	0.00077	0.7268	0.2800	0.0001	\$ 0.0049	\$ 79.65	\$ 23.79	\$ 103.45	\$ 88.87	
	LM2	0.00082		0.3952	0.0001	\$ 0.0026		\$ 9.61	\$ 89.26		
	LM3	0.00071		0.3547	0.0019	\$ 0.0074		\$ 9.22	\$ 88.87		

Figure 2.3. Microsoft Excel Model Calculations and Results Worksheet

Populating (PO)	Through-hole (THT)	PO1	0.00085	0.7976	0.3669	0.0001	\$ 0.0081	\$ 81.91	\$ 27.46	\$ 109.38	\$ 91.59
	Surface-mount (SMT)	PO2	0.00087		0.3713	0.2642	\$ 1.5021		\$ 92.81	\$ 176.22	
		PO3	0.00081		0.3989	0.0001	\$ 0.0077		\$ 9.67	\$ 91.59	
Protection & packaging (PP)		PP1	0.00089	0.7979	0.3772	2.6608	\$ 7.5414	\$ 70.54	\$ 871.36	\$ 949.44	\$ 80.06
		PP2	0.00089		0.3864	0.0123	\$ 0.0374		\$ 13.23	\$ 83.81	
		PP3	0.00083		0.3952	0.0001	\$ 0.0053		\$ 9.52	\$ 80.06	

**\$ 781.31**

Figure 2.4. Microsoft Excel Model Calculations and Results Worksheet (Summarized)

## **2.4. Summary**

This chapter presents the basic model to calculate the total cost per product for a single electronic product. The model presented minimizes the total cost by assigning the product to the minimum cost machine at each assembly process. The direct labor, material and overhead costs per product are also calculated and shown on the results.

If the user of the model presented needs to minimize the total cost of more than one product, a mathematical model will apply to optimize the total cost of all products. The next chapter presents a mathematical model to optimize the total cost of all products assembled in a given period of time.

## **CHAPTER III**

### **DEVELOPMENT OF THE EXPANDED MODELS**

#### **3.1. Introduction**

This chapter explains the expanded models for cost minimization of products assembled for the machine selection problem. Integer linear programming is used to model the problem with the objective of minimizing total cost. The mathematical model has multiple equations for each cost component of the total cost of all products assembled. An optimization model is capable of assigning multiple products to the minimum cost machine at each assembly process given some specific constraints.

The assumptions for these models are mentioned on the next section of this chapter. The details of the integer linear program follow the assumptions section. Production planning rules such as expediting production, which refers to moving a due date for a product (or customer) to an earlier date or time (Hopp et al., 2001) and machine workload balance, which refers to maximize or increase machine utilization (Hopp et al., 2001) are considered creating a more realistic environment while allowing the user to keep the same ultimate objective of assigning machines to products while minimizing total cost.

#### **3.2. Assumptions**

Most of the assumptions considered for the basic model also apply to the expanded models. These assumptions are mentioned below and are revised if applicable.

Additional assumptions needed for these optimization models are added. Assumptions are divided into deterministic or probabilistic depending on the nature of the data required.

- Assumptions for deterministic parameters:
  - The number of the assembly processes, the machine quantities, and the number of products assembled are given.
  - The capacity in machine hours per period of each machine is given.
  - Average direct labor employees' hourly salary is given.
  - Average support personnel yearly salary is given.
  
- Assumptions for probabilistic parameters:
  - Direct labor employees quantity and direct labor employees average percent in each assembly process are generated by the uniform distribution.
  - Support personnel quantity and the average percent of time in each assembly process are generated by the uniform distribution.
  - Products demand is known and generated by the uniform distribution.
  - Units per batch for each product are generated by the uniform distribution.
  - Cycle times (including handling times, processing times, setup times, and waiting times) are generated using the normal distribution. The left-



truncated normal distribution is used with a minimum allowed time of zero.

- Product components quantities and costs, and consumable materials quantities and costs are generated by the uniform distribution.
  - Mean-time-to failure and mean-time-to-repair are generated by the uniform distribution.
  - Units of utilities consumed and utilities costs are generated by the uniform distribution.
- General assumptions:
    - All times are used or calculated in hours.
    - Calculations are done considering a week as the period of time for production; the objective value represents a week worth of production.

The exponential and beta distributions are more commonly used to generate random times for parameters such as mean-time-to-failure and mean-time-to-repair, but for the purpose of this research the uniform distribution will suffice. The reason to use the uniform distribution is that these parameters have minimum impact on the total cost of all assembled products.

### **3.3. Model Development**

An integer linear program (ILP) is developed with the objective of minimizing the total cost of all products assembled as an optimization tool. This total cost of all products includes different cost components such as direct labor cost, material cost, and overhead cost. AMPL9 (Fourer et al., 2003) is used for the mathematical programming and ILOG CPLEX90 as the solver of the model.

For the mathematical model, the cost equations developed for the basic model in Chapter II are used to calculate the costs, but they are extended to consider multiple subscripts (products, processes, and machines) for the parameters when applicable. The same logic used on Chapter II to develop a cost model based on variable costing accounting system is used for the mathematical model. This mathematical model is a powerful tool when compared to the Microsoft Excel model because it is able to show the user to which machine type at each assembly process should *each product* be assigned to minimize the total cost of *all products* assembled.

#### **3.3.1. Integer Linear Programming Model (ILP)**

The mathematical model used to solve the problem presented in this research is an integer linear program. An integer linear program can be defined as a linear program in which the variables are restricted to integer or discrete values (Taha, 1997). The nature of the problem researched, allowed the model to be solved based on binary values (i.e., 0 or 1) of the decision variables which make the program a binary linear program.

The following notation in alphabetical order is used throughout the mathematical models presented in this chapter for the indexes and the corresponding sets:

$a$	index for product components, $a \in A$
$b$	index for consumable materials, $b \in B$
$i$	index for products, $i \in I$
$j$	index for processes, $j \in J$
$k$	index for machines, $k \in K$
$s$	index for support personnel, $s \in S$
$u$	index for utilities, $u \in U$

The following notation is used throughout the mathematical models presented in this chapter for the parameters considered in the formulation:

- Parameters with a single data value:

$DLc$	average salary per hour of direct labor employees
$HRS$	worked hours per week
$MMc$	total machine maintenance cost per week
$N$	number of products assembled during a given period of time
$SPc$	average salary of support personnel per year
$WKS$	working weeks per year
$MH_{jk}$	machine hours available on machine $k$ in process $j$

- Parameters with random values generated using probability distributions:

$CM_{ijb}$	consumable material $b$ for product $i$ in process $j$
$CMc_b$	cost per unit of consumable material $b$
$CP_{ija}$	product component $a$ for product $i$ in process $j$
$CPc_a$	cost per unit of product component $a$
$DLp_j$	average percent of time direct labor employees work in process $j$
$DLq_j$	quantity of direct labor employees assigned to process $j$
$Dyr_i$	demand per year of product $i$
$HT_{ij}$	handling or moving time of product $i$ from previous process to process $j$
$MTTF_{jk}$	mean-time-to-failure of machine $k$ in process $j$
$MTTR_{jk}$	mean-time-to-repair of machine $k$ in process $j$
$PT_{ijk}$	process time of product $i$ in process $j$ and machine $k$
$SPp_j$	average percent of time of support personnel in process $j$
$SPq_s$	quantity of support personnel $s$
$SUB_{ijk}$	machine setup time per batch of product $i$ in process $j$ and machine $k$
$Ub_i$	units per batch of product $i$
$UCc_u$	cost per unit of utility $u$
$UCq_{jku}$	units of utility $u$ consumed by machine $k$ in process $j$
$WT_{ijk}$	waiting time of product $i$ in process $j$ and machine $k$

- Computed parameters:

$AV_{jk}$	availability of machine $k$ in process $j$
$C_{ijk}$	total cost of product $i$ in process $j$ and machine $k$

$CT_{ijk}$	cycle time of product $i$ in process $j$ and machine $k$
$L_{ijk}$	labor cost of product $i$ in process $j$ and machine $k$
$M_{ijk}$	material cost of product $i$ in process $j$ and machine $k$
$MM_{ijk}$	machine maintenance cost per product $i$ in process $j$ and machine $k$
$MU_{ijk}$	maximum machine utilization for product $i$ at process $j$ and machine $k$
$O_{ijk}$	overhead cost of product $i$ in process $j$ and machine $k$
$SC_{ijk}$	machine setup cost of product $i$ in process $j$ and machine $k$
$SP_{ijk}$	support personnel cost for product $i$ in process $j$ and machine $k$
$SU_{ijk}$	machine setup time of product $i$ in process $j$ and machine $k$
$TT_{ijk}$	total time on the assembly line of product $i$ in process $j$ and machine $k$
$UC_{ijk}$	utilities consumption cost of product $i$ in process $j$ and machine $k$

The decision variables are defined as follows:

$$X_{ijk} = \begin{cases} 1 & \text{if product } i \text{ is assigned to process } j \text{ and machine } k, \\ 0 & \text{otherwise.} \end{cases}$$

The ILP mathematical model is expressed with the objective function and constraints that follow. The mathematical equations for the parameters that need to be calculated are also shown below.

$$\text{Minimize } Z = \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} C_{ijk} X_{ijk} \quad \forall i, j, k \quad (3.1)$$

subject to

$$\sum_{i \in I} X_{ijk} \leq N \quad \forall j, k \quad (3.2)$$

$$\sum_{k \in K} X_{ijk} = 1 \quad \forall i, j \quad (3.3)$$

$$\sum_{i \in I} (MU_{ijk} X_{ijk}) \leq MH_{jk} \quad \forall j, k \quad (3.4)$$

$$X_{ijk} = \{0, 1\} \quad \forall i, j, k \quad (3.5)$$

In the formulation, equation (3.1) is the objective function to minimize the total cost of all assembled products. The first constraint (3.2) states that each product  $i$  can be assigned to each process  $j$  and machine  $k$  only once. In constraint (3.3), it is specified that each product  $i$  is assigned to only one machine  $k$  in each process  $j$ . Constraint (3.4) shows that available machine hours for product  $i$  in process  $j$  and machine  $k$  cannot be exceeded. The last constraint (3.5) defines the decision variables as binary.

When analyzing the capacity constraint (refer to equation 3.4), an additional mathematical expression is needed to calculate machine utilization; this is shown in equation 3.4a.

$$MU_{ijk} = (PT_{ijk} + SU_{ijk}) \times \frac{Dyr_i}{WKS} \quad \forall i, j, k \quad (3.4a)$$

Additional mathematical equations are required to go into the details of the cost components when calculating the total cost per product ( $C_{ijk}$ ). These equations are presented next grouped by the cost component they affect.

The following mathematical equations are used to calculate direct labor cost per product. They consider setup times (based on setup per batch of products) and processing times, direct labor employees and direct labor average wages per hour.

$$L_{ijk} = PT_{ijk} \times DLq_j \times DLp_j \times DLc + SC_{ijk} \quad \forall i,j,k \quad (3.6)$$

$$SC_{ijk} = SU_{ijk} \times DLq_j \times DLp_j \times DLc \quad \forall i,j,k \quad (3.7)$$

$$SU_{ijk} = \frac{Sub_{ijk}}{Ub_i} \quad \forall i,j,k \quad (3.8)$$

Equation (3.6) calculates the labor cost per product considering process time per product, quantity of direct labor employees, the percentage of time they worked at each process, the average rate per hour for direct labor employees, and the setup cost per product. On equation (3.7) the setup cost per product is calculated using setup time per product, quantity of direct labor employees, the percentage of time they worked at each process, and the average rate per hour for direct labor employees. The purpose of equation (3.8) is to convert the setup time per batch into setup time per product.

The following mathematical equation is used to calculate material cost per product (refer to equation 3.9). It includes the cost of the required product components by considering the units used of each product component and the components cost per unit. It also includes the cost per product of the consumable materials by considering the units required of each consumable material and the consumable materials cost per unit.

$$M_{ijk} = \sum_{a \in A} (CP_{ija} \times CPc_a) + \sum_{b \in B} (CM_{ijb} \times CMc_b) \quad \forall i,j,k \quad (3.9)$$

The following mathematical equations are used to calculate overhead cost per product. These equations calculate variable support personnel cost per product, utilities consumption cost per product, and machine maintenance cost per product.

$$O_{ijk} = SP_{ijk} + UC_{ijk} + MM_{ijk} \quad \forall i,j,k \quad (3.10)$$

$$SP_{ijk} = \sum_{s \in S} (SPq_s \times SPP_j) \times TT_{ijk} \times \frac{SPc}{WKS \times HRS} \quad \forall i,j,k \quad (3.11)$$

$$TT_{ijk} = \frac{CT_{ijk}}{AV_{jk}} \quad \forall i,j,k \quad (3.12)$$

$$CT_{ijk} = SU_{ijk} + HT_{ij} + WT_{ijk} + PT_{ijk} \quad \forall i,j,k \quad (3.13)$$

$$AV_{jk} = \frac{MTTF_{jk}}{MTTF_{jk} + MTTR_{jk}} \quad \forall j,k \quad (3.14)$$

$$UC_{ijk} = \sum_{u \in U} (UCq_{jku} \times UCc_u) \times PT_{ijk} \quad \forall i,j,k \quad (3.15)$$

$$MM_{ijk} = \frac{MMc}{(2 \times HRS)} \times (SU_{ijk} + PT_{ijk}) \quad \forall i,j,k \quad (3.16)$$

In the formulation for overhead cost per product, equation (3.10) summarizes the variable indirect costs by adding support personnel cost per product, utilities consumption cost per product, and machine maintenance cost per product. The support personnel cost per product equation (3.11) considers the quantity of support personnel



with the percentage of time worked at each process, the total time products are in the assembly line and the average weekly support personnel salary. Equation (3.12) calculates the total time products are in the assembly line by considering the cycle time per product and the availability of the machines. The cycle time is calculated on equation (3.13) by adding setup time, handling time, waiting time, and process time of each product at each process and machine. The equation for the availability of the machines (3.14) considers a ratio between the mean-time-to-failure and mean-time-to repair of the each machine. The utilities consumption cost per product in equation (3.15) is calculated based on units of utilities consumed and its cost per unit, and the process time per product. The total machine maintenance expenses are allocated to each product based on the process time per product and the setup time per product. These are used to calculate the machine maintenance cost per product in equation (3.16).

The mathematical equation to calculate total cost for all products based on direct labor cost ( $L_{ijk}$ ), material cost ( $M_{ijk}$ ), and overhead cost ( $O_{ijk}$ ) is next (refer to equation 3.17). It considers the required weekly demand of each product to be assembled:

$$C_{ijk} = (L_{ijk} + M_{ijk} + O_{ijk}) \times \frac{Dyr_i}{WKS} \quad \forall i, j, k \quad (3.17)$$

### 3.3.1.1. ILP Solving Methodology

AMPL9 mathematical programming language with CPLEX90 as the solver is used to solve the mathematical model explained in the previous section. AMPL9 uses the branch and bound (B&B) algorithm to solve ILP problems. The B&B algorithm is

one of the two most commonly used methods to solve ILP problems (the other one is the cutting plane method) and it is more successful (Taha, 1997) computationally speaking.

The logic behind the ILP algorithms as explained by Taha (1997) is to begin by relaxing the binary variables to the continuous range  $[0, 1]$ . This result in a regular linear programming (LP) problem, which is solved first to identify an optimum based on the continuous range for the decision variable. Then, constraints are added to iteratively modify the LP solution until an optimum extreme point that satisfies the integer requirements is found.

The optimum obtained when the LP is solved is equivalent to the bounding part of the B&B algorithm, and the sub-problems created when the optimum solution is not an integer is the branching part of the algorithm. The analogy to the problem presented in this research is that with the bounding part, an upper bound is found for the minimization problem, and the branching is done with values zero and one for the decision variables.

A series of different sets of data is used to solve the mathematical model with the purpose of verifying the applicability of the model as a decision tool under different scenarios. The scenarios are based on a moderate process variability using the coefficient of variation (CV), different number of assembly machines at each process, and different amounts of assembled products. When analyzing the scenarios, the objective of the ILP model (minimize total cost of all products assembled) can be compared. The scenarios considered are presented in Table 3.1. The combination of the selected scenarios makes a total of nine sets of runs of the mathematical model.

Table 3.1- Scenarios to Solve the ILP Model

<i>CV – Moderate = 0.9</i>			
<i>Machines</i>	<i>Products</i>		
4	10	100	1000
7	10	100	1000
10	10	100	1000

These mathematical models are increasing their size in terms of variables and constraints as the number of products and machines increase. Table 3.2 summarizes the size of these ILP models for each selected scenario mentioned in Table 3.1.

Table 3.2 - Sizes of the ILP Models

<b><i>MCV = 0.9</i></b>			
<i>Products</i>	<i>Machines per Process</i>	<i>Number of Variables</i>	<i>Number of Constraints</i>
10	4	400	180
	7	700	240
	10	1000	300
100	4	4000	1080
	7	7000	1140
	10	10000	1200
1000	4	40000	10080
	7	70000	10140
	10	100000	10200

Considering that some models are fairly large, a time limit of 3600 seconds is used with the solver. Since the solver checks the remaining time at only certain points while the logic of the model is being followed, it could run over the established time limit. For the purpose of this research, solutions obtained up to 25% over the time limit (up to 3900 seconds) are considered optimum if a feasible solution is found.

### **3.3.2. Integer Linear Programming Model with Expediting (ILPx)**

Considering the reality of a manufacturing environment, there are a lot of times when a priority may be assigned to some products. This could be for different reasons but in most cases the priority is determined by management based on the company's relationship with the customers. Among the reasons to prioritize production are the customer orders that were not finished during the previous period of time (i.e., week), commonly called backlogs. Backlogs create production disruption because the production planned for the current period of time has to be adjusted to accommodate and prioritize these products. Another common reason to prioritize production could be established by management based on customers that need their products to be shipped first and are willing to pay a premium on the price if applicable. A third reason to prioritize production is when a product is harder to be assembled (not necessarily related to time) and management can decide to finish them first because production can be run smoothly after that.

For any of the reasons mentioned above or any other reason management might have to give priorities to some products, the concept called expediting production is now

considered in the mathematical model presented in section 3.3.1. This new model is called ILPx. It is included in the model by considering the impact on the total cost of the products assembled. Specifically, the direct labor cost will increase when special treatment is given to a product. Material cost is not affected since demand is not changed, and overhead cost is not affected either because the same support is received from the support personnel, the machine maintenance cost does not change, and the products still consumed the same units of utilities.

The following parameters are added to the mathematical model to include expediting production:

$EXP_i$  expedite condition to determine products  $i$  to be expedited

$EXP\%$  percentage of increase in direct labor cost when expediting

Following the logic use during the development of the mathematical model,  $EXP_i$  is generated using a uniform (binary) distribution, where 1 means product needs to be expedited and 0 means otherwise. The  $EXP\%$  factor is given by management.

Equation (3.6), used to calculate direct labor cost per product ( $L_{ijk}$ ) is now replaced by equations (3.6) and (3.6a) to consider expediting cost. Equation 3.6 is used if  $EXP_i = 0$  (product  $i$  do not need to be expedited), while equation 3.6a is to be used when products need to be expedited,  $EXP_i = 1$ .

$$L_{ijk} = PT_{ijk} \times DLq_j \times DLP_j \times DLc + SC_{ijk} \quad \forall i, j, k \quad (3.6)$$

$$L_{ijk} = (PT_{ijk} \times DLq_j \times DLP_j \times DLc + SC_{ijk}) \times (1 + EXP\%) \quad \forall i, j, k \quad (3.6a)$$

The total cost of products assembled under the expediting scenarios is calculated the same way (with equation 3.17), but choosing the applicable equation to represent the direct labor cost per product from equations 3.6 and 3.6a.

The same scenarios selected to evaluate the ILP model (refer to Table 3.1) are going to be used to solve the ILPx model. Therefore, these ILPx models have the same sizes as the ILP models (refer to Table 3.2) for each scenario. The solving methodology explained in section 3.3.1.1 for the ILP model, also applies for the ILPx model.

### **3.3.3. Integer Linear Programming Model with Machine Workload Balance (ILPb)**

Another important situation in assembly environments when selecting machines to assemble each product at each assembly process, is maintaining machine workload balanced. The machine workload balance problem is studied with the idea in mind of creating additional realistic environments for the machine selection problem while keeping the objective of minimizing total cost.

Machine workload balance can be defined as distributing the available workload among the available machines as equally as possible (Rajakumar et al., 2004). It has the purpose of improving machine utilization by distributing the scheduled production as much as possible among the available machines. As a result, when machine workload is balanced, waiting times between processes might be minimized. Another impact of balancing machine workload is less work-in-process inventory, the goods or product between adjacent processes (Silver et al., 1998). If waiting times are lower, there will be

less accumulation of inventory between processes. Due to natural process variability and the variation in processing times, obtaining a perfect machine workload balance is not easily achievable.

Balancing machine workload within an assembly line can be also beneficial when we consider machine maintenance. There is a direct relationship between machine maintenance needed, the amount of time the machines are run, and the machine maintenance cost. This applies to both preventive and corrective maintenance. When machine workload balance is improved, preventive maintenance can be appropriately scheduled which can result in an improved machine performance and availability (as calculated in equation 3.14).

The machine workload balance is measured by using the hours that machines are run at each process to satisfy product demand. A factor to measure the required percentage of machine workload balance is now included in the ILP model developed in this research (now called ILPb). This factor is based on the maximum imbalance percentage allowed at each assembly process by management.

The following parameters are added to the ILP model presented in section 3.3.1 to include the machine workload balance problem to the problem previously describe:

$\delta$	machine workload imbalance factor
$AvgMU_{jk}$	average machine utilization per process $j$ and machine $k$
$K$	number of machines available per process

The following mathematical equation (3.18) is added to the ILP presented in section 3.3.1 to calculate the average machine utilization (in machine hours) of machine  $k$  in process  $j$ .

$$AvgMU_{jk} = \sum_{i \in I} MU_{ijk} / K \quad \forall j, k \quad (3.18)$$

The following constraint is added to the constraints set of the ILP model presented in section 3.3.1 to balance the machine workload based on the given imbalance factor  $\delta$ :

$$AvgMU_{jk}(1-\delta) \leq \sum_{i \in I} (MU_{ijk}X_{ijk}) \leq AvgMU_{jk}(1+\delta) \quad \forall j, k \quad (3.19)$$

The total cost of products assembled under the machine workload balance scenarios (ILPb model) is calculated the same way (with equation 3.17) as in the ILP presented in section 3.3.1, with the only difference been that constraint 3.19 is now included. The results given by the ILPb are based on minimizing total cost of products assembled with the basic constraints previously explained (equations 3.2 – 3.5) and with this additional constraint of machine workload balance (refer to equation 3.19).

After adding the machine workload balance constraint to the problem previously discussed, more scenarios are created to solve the mathematical model. To the scenarios presented in Table 3.1, additional scenarios are considered to run the mathematical



model under two different values of the imbalance factor  $\delta$  (40% and 25%). All the scenarios are presented in Table 3.3. The combination of the selected scenarios makes a total of eighteen sets of runs of the mathematical model.

The size of the ILPb mathematical models increases from the ILP models due to their additional constraints to balance the assembly machines workload. Table 3.4 summarizes the size of these ILPb models for each selected scenario mentioned in Table 3.3. Since the scenarios show two different machine workload imbalance factors, the ILPb models need to be run twice, one for each imbalance factor.

Table 3.3 - Scenarios to Solve the ILPb Model

<i>CV – Moderate = 0.9</i>						
<i>Machines</i>	<i><math>\delta = 0.40</math></i>			<i><math>\delta = 0.25</math></i>		
	<i>Products</i>			<i>Products</i>		
4	10	100	1000	10	100	1000
7	10	100	1000	10	100	1000
10	10	100	1000	10	100	1000

The solution methodology explained on section 3.3.1.1 for the ILP model, also used for the ILPx model, applies to the ILPb models as well. This solution methodology with the B&B algorithm is now used for the different scenarios presented in Table 3.3.

Table 3.4 - Sizes of the ILPb Models

<b><i>MCV = 0.9</i></b>			
<i>Products</i>	<i>Machines per Process</i>	<i>Number of Variables</i>	<i>Number of Constraints</i>
10	4	400	240
	7	700	280
	10	1000	340
100	4	4000	1120
	7	7000	1180
	10	10000	1240
1000	4	40000	10120
	7	70000	10180
	10	100000	10240

### 3.4. Summary

This chapter presents the integer linear program to minimize the total cost of all electronic products assembled during a given production period of a week. This is achieved by choosing the minimum cost available machine at each assembly process. The direct labor, material and overhead costs per product are also calculated and shown on detail.

In this chapter, two expanded ILP models are also explained; one to include expediting production (ILPx), and the other one to include machine workload balance (ILPb). The next chapter presents the analysis of results and sensitivity analyses.

## **CHAPTER IV**

### **ANALYSIS OF RESULTS**

#### **4.1. Introduction**

This chapter presents the results obtained after running the models explained in Chapter III. First, results are presented for the ILP explained in section 3.3.1. Then, the results for both expanded models are presented. The ILPx model is explained in section 3.3.2, and the ILPb model is explained in section 3.3.3.

The results of the ILP model are presented for the scenarios considering a moderate coefficient of variation, different machine quantities per assembly process, and different number of products assembled as shown in Table 3.1. The results for the ILPx model are also presented for the scenarios shown in Table 3.1. The scenarios presented in Table 3.2 are used to show the results of the ILPb model.

#### **4.2. Results for the ILP Model**

The results obtained on the runs of the ILP model are shown for the objective function - total cost, and for key parameters that help in the decision making process such as: average machine utilization and average machine workload imbalance. These results are presented by number of machines, number of products, and the process variability (using the CV).

The results related to how the AMPL9/CPLEX90 performed are presented based on the iterations required by each scenario and the run time. These are also presented by number of machines and by number of products.

Table 4.1 - Results for the ILP Model

<b>MCV = 0,9</b>						
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>	<i>Solver Iterations</i>	<i>Run time (sec.)</i>
10	4	1,850	35.26	50.62%	101	10.31
	7	3,843	33.10	55.59%	124	16.81
	10	4,590	24.60	68.09%	109	13.38
100	4	1,611	32.43	51.59%	1000	21.47
	7	2,946	26.47	64.19%	1000	36.57
	10	4,229	23.13	69.79%	1000	71.90
1000	4	1,642	35.01	48.31%	10000	83.68
	7	2,973	29.68	59.94%	10000	1430.17
	10	4,237	25.99	65.91%	10000	5102.06*

From the results on Table 4.1, it can be seen that when the amount of products or machines increase, the average machine workload imbalance also increases while the average machine utilization tends to decrease. This is because the required machine hours are distributed between more machines creating higher machine workload imbalances in average, and lowering the average machine utilization as well. The total cost of all products assembled increases when the number of machines per process

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\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

increases with the same amount of products assembled, but there is no relation if the number of products assembled changes. This is because the higher the amount of products to assemble, the lower the expected demand per product which may or may not result in an increase in total cost. Increasing the number of products creates a more complicated model because balancing machine workload will require that more iterations will be analyzed which can also result in more time to solve. When the number of products increases; it does not mean that the total cost has to increase because the model continues to look for an optimum (minimization) solution.

From the information on Table 4.1 it can be explained the impact on the total cost that varying the number of products and machines has. The total cost for 10 products under the four machines scenario is \$1,850k and for 1000 products under the same scenario is \$1,642k. At the same time, this cost for 10 products and ten machines is \$4,590k and for 1000 products and ten machines is \$4,237k. These results were not expected by the researcher since usually more products mean higher total cost. This is an important finding to understand that what determine how total cost is affected is the complexity of the model and how the constraints reflect it.

Results obtained from the runs of the ILP model also show the range of machine utilization among the machines used per process. These are shown in Table 4.2. It can be seen that these ranges varies from 16.71 to 47.37 for the 10 products scenarios, from 18.22 to 36.16 for the 100 products scenarios, and from 23.29 to 38.29 for the 1000 products scenarios. The ranges of the minimum and maximum machine hours vary from 16 to 24 hours on the 10 products scenarios to between 6 and 7 hours on the 1000

products scenarios; the higher the amount of products, the more consistent the machine utilization is. It is important to review the results considering these machine utilization ranges because when the minimum and maximum values are too distant, the average does not have necessarily reflect the reality of the process.

Table 4.2 - Machine Utilization Ranges for the ILP Model

<b>MCV = 0.9</b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	23.59	47.37
	7	25.96	41.31
	10	16.71	33.17
100	4	28.37	36.16
	7	22.55	28.84
	10	18.22	26.93
1000	4	31.56	38.29
	7	25.68	33.03
	10	23.29	29.55

#### 4.3. Results for the ILPx Model

The results presented for the ILPx model are in the same format as the results for the ILP model. For each combination of number of assembled products and number of machines per assembly process, the total cost, the average machine utilization and the average machine workload imbalance are shown. The results related to the performance

of the mathematical programming language and the solver (AMPL9 and CPLEX90) are included as well.

Analyzing the results found for the ILPx model, it can be seen that the trends on the results are very similar to the ones obtained with the ILP model. From Table 4.3, it can be seen that the total cost for 10 products under the four machines scenario is \$1,566k and for 1000 products under the same scenario is \$1,522k. At the same time, this cost for 10 products and ten machines is \$4,047k and for 1000 products and ten machines is \$3,926k. Table 4.4 shows that the ranges of the minimum and maximum average machine hours vary from 14 to 18 hours on the 10 products scenarios to between 5 and 6 hours on the 1000 products scenarios.

Table 4.3 - Results for the ILPx Model

<b>MCV = 0.9</b>						
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>	<i>Solver Iterations</i>	<i>Run time (sec.)</i>
10	4	1,566	35.95	48.50%	103	27.88
	7	2,771	26.45	64.45%	108	28.14
	10	4,047	19.27	75.13%	107	28.66
100	4	1,737	34.94	47.85%	1000	5.27
	7	3,162	29.52	59.90%	1000	45.66
	10	4,497	25.18	66.98%	1000	81.12
1000	4	1,522	32.06	52.66%	10000	93.98
	7	2,743	25.92	64.94%	10000	1640.98
	10	3,946	22.39	70.65%	10000	5758.20*

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\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

Table 4.4 - Machine Utilization Ranges for the ILPx Model

<b><i>MCV = 0.9</i></b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	27.76	45.46
	7	19.39	37.41
	10	12.58	26.56
100	4	31.32	39.94
	7	23.38	33.06
	10	18.67	29.53
1000	4	29.68	34.64
	7	22.93	28.58
	10	19.82	25.77

In general, the average machine utilization has a tendency to be lower on the ILPx model when compare to the ILP model due to the additional complexity added to the ILPx model to consider expediting production (refer to Figure 4.1). Meanwhile, the average machine workload imbalance has a tendency to be higher than the ILP model for the same reason (refer to Figure 4.2).



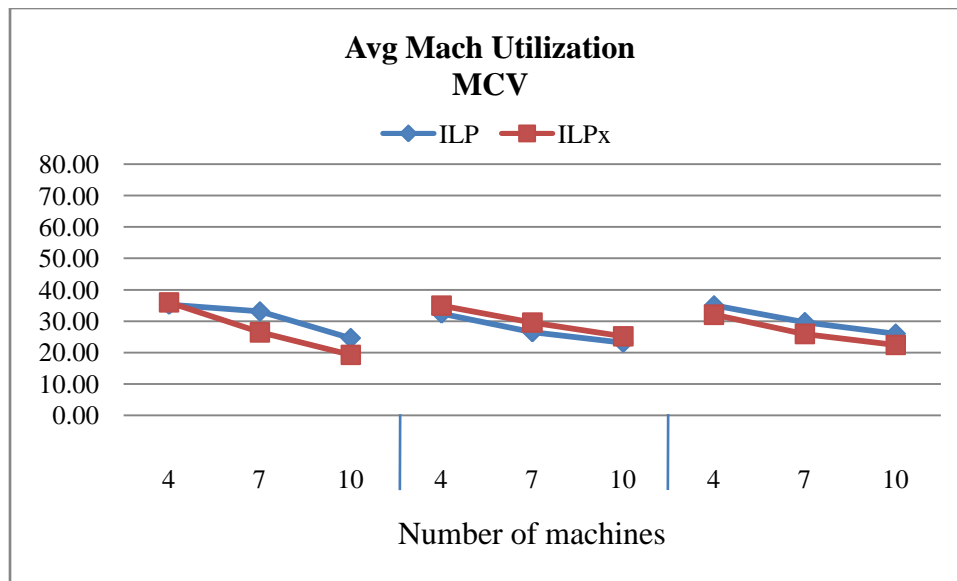


Figure 4.1. Average Machine Utilization - MCV (ILP & ILPx)

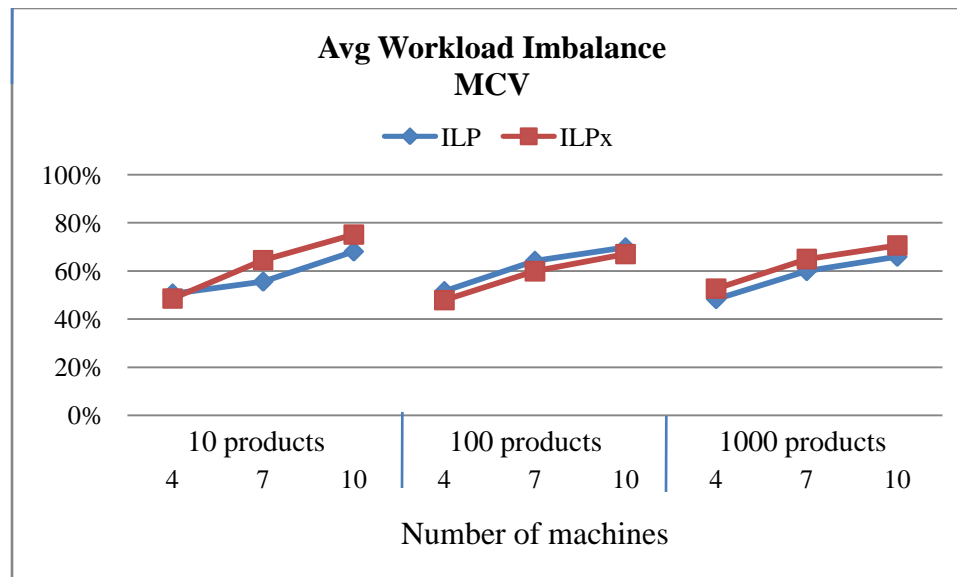


Figure 4.2. Average Machine Workload Imbalance - MCV (ILP & ILPx)

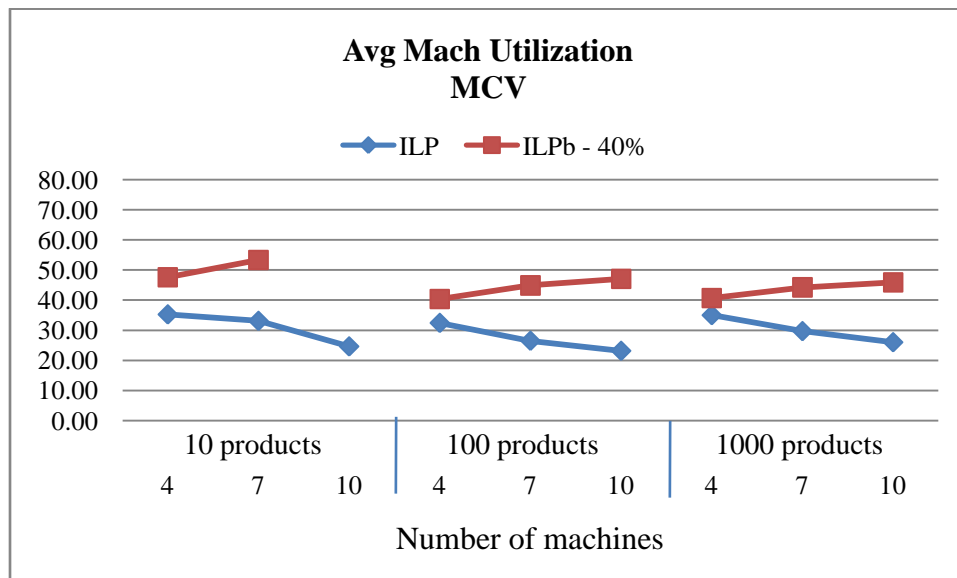
#### 4.4. Results for the ILPb Model

The results presented for the ILPb model follow the same format as the results for the ILP model, but considering the scenarios for the different values of the machine workload imbalance factor. The total cost, the average machine utilization and the average machine workload imbalance are shown for the different combinations of number of assembled products and number of machines per assembly process. The performance of AMPL9 with CPLEX90 is included by showing the iterations and the B&B nodes generated at each run.

The results included on Table 4.5 for the ILPb model with  $\delta = 40\%$  indicate that for the machine workload balance model, the higher the number of machines, the higher the machine utilization given the same number of products (refer to Figure 4.3). As expected, the average workload imbalance is almost constant given that the model has constraints to balance machine workload (refer to Figure 4.4). The total cost, when evaluated at each scenario, has the tendency to increase when balancing machine workload. This is because the machines chosen at each process for each product will not necessarily be the minimum cost ones to satisfy the machine workload balance constraint. As can be seen in Figure 4.5, this is not the case for the 10 products scenarios.

Table 4.5 - Results for the ILPb Model with  $\delta=40\%$ 

<b>MCV = 0.9</b>				
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>
10	4	1,598	47.63	30.73%
	7	3,050	53.36	27.02%
	10	Infeasible	-	-
100	4	1,637	40.39	39.68%
	7	3,058	44.91	39.30%
	10	4,479	47.12	38.78%
1000	4	1,655	40.67	39.93%
	7	3,052	44.26	39.91%
	10	5,257	45.90	39.84%

Figure 4.3. Average Machine Utilization - MCV (ILP & ILPb;  $\delta=40\%$ )

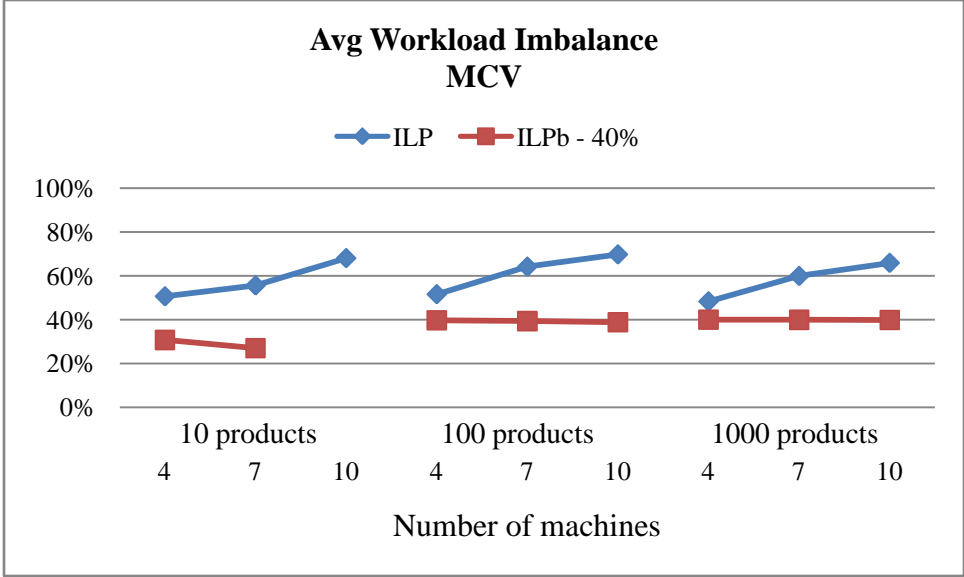


Figure 4.4. Average Machine Workload Imbalance - MCV (ILP & ILPb;  $\delta=40\%$ )

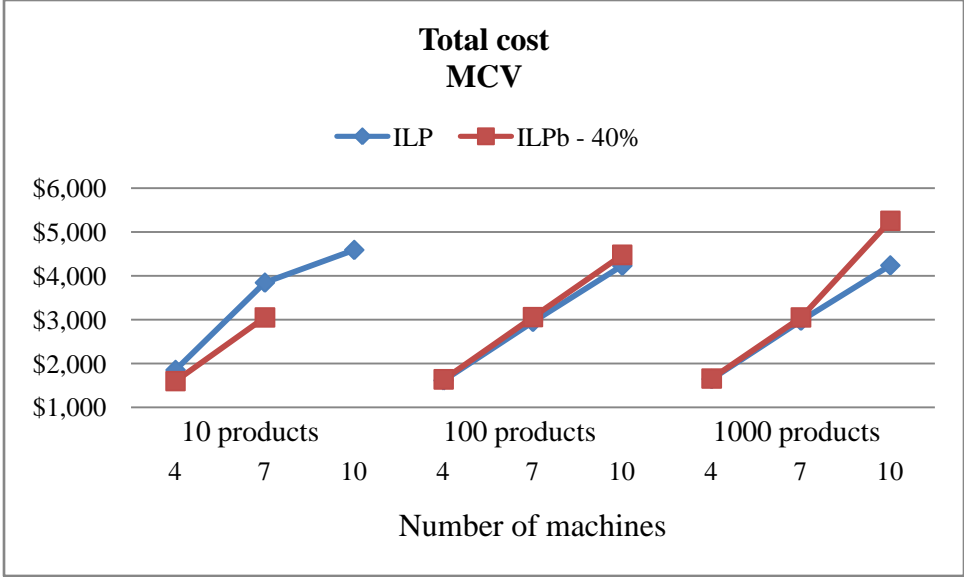


Figure 4.5. Total Cost - MCV (ILP & ILPb;  $\delta=40\%$ )

When analyzing the performance of the solver (refer to Table 4.6), the amount of iterations and B&B nodes increased significantly as the size of the model increased. The algorithm was not able to find an optimum solution for three of the scenarios (identified by \*) within the time limit of 3600 seconds. The scenario of 10 products with ten machines was determined to be infeasible after only 14 seconds. This is mainly due to the complexity of assigning products to machine while balancing within the given range with  $\delta = 40\%$ .

Table 4.6 - Solver Results for the ILPb Model with  $\delta=40\%$

<b>MCV = 0.9</b>				
<i>Products</i>	<i>Machines</i>	<i>Solver Iterations</i>	<i>B&amp;B Nodes</i>	<i>Run time (sec.)</i>
10	4	1326	299	7.56
	7	708	3	13.62
	10	-	-	14.45
100	4	2360392	279079	3605.30
	7	3101127	256375	10821.90*
	10	2812860	236941	18031.20*
1000	4	65721	11000	1480.18
	7	184764	19141	8733.54*
	10	224815	13049	3615.53

Table 4.7 shows the ranges of the minimum and maximum average machine hours when  $\delta = 40\%$ . These values vary from 12 to 14 hours for the 10 products scenarios, and from 1 to 2 hours for the 1000 products scenarios. As the results in Table

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\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

4.5, the higher the number of machines, given the same amount of products, the higher the machine utilization.

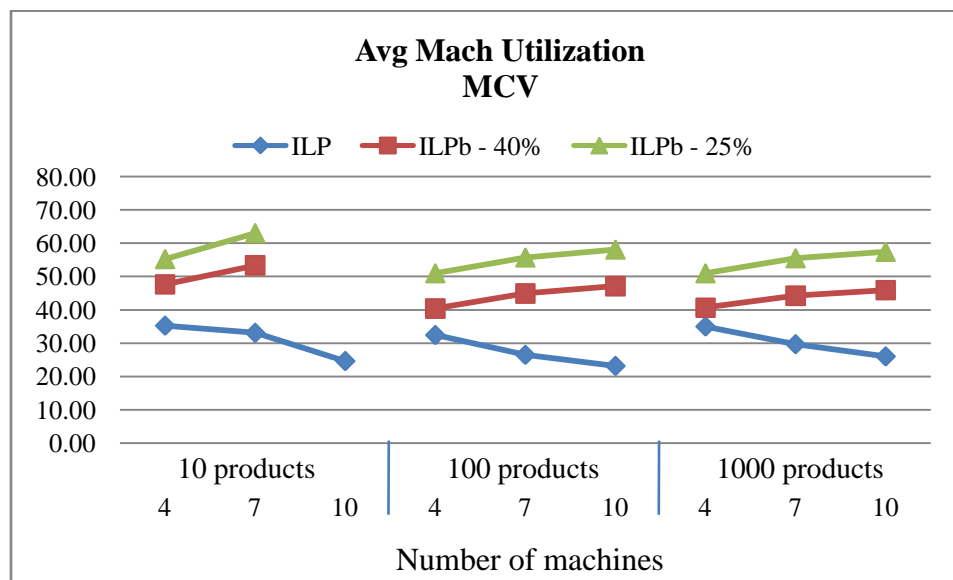
Table 4.7 - Machine Utilization Ranges for the ILPb Model with  $\delta=40\%$

<b><i>MCV = 0.9</i></b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	41.77	56.15
	7	46.54	58.88
	10	-	-
100	4	39.27	41.87
	7	43.87	47.08
	10	46.16	48.36
1000	4	39.78	41.38
	7	43.26	44.66
	10	45.50	46.26

The results for the ILPb model with an imbalance factor of 25% are included on Tables 4.8 – 4.10. The same effects in total cost, average machine utilization, and average machine workload imbalance are observed as in the ILPb model with 40% imbalance factor (refer to Figures 4.6, 4.7, and 4.8).

Table 4.8 - Results for the ILPb Model with  $\delta=25\%$ 

<b>MCV = 0.9</b>				
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>
10	4	1,637	55.23	19.46%
	7	3,218	63.06	16.48%
	10	Infeasible	-	-
100	4	1,701	50.99	24.63%
	7	3,165	55.74	24.28%
	10	4,662	58.12	23.84%
1000	4	1,701	50.98	24.94%
	7	2,719	55.50	24.86%
	10	5,414	57.44	24.88%

Figure 4.6. Average Machine Utilization - MCV (ILP & ILPb;  $\delta=40\%$  &  $25\%$ )

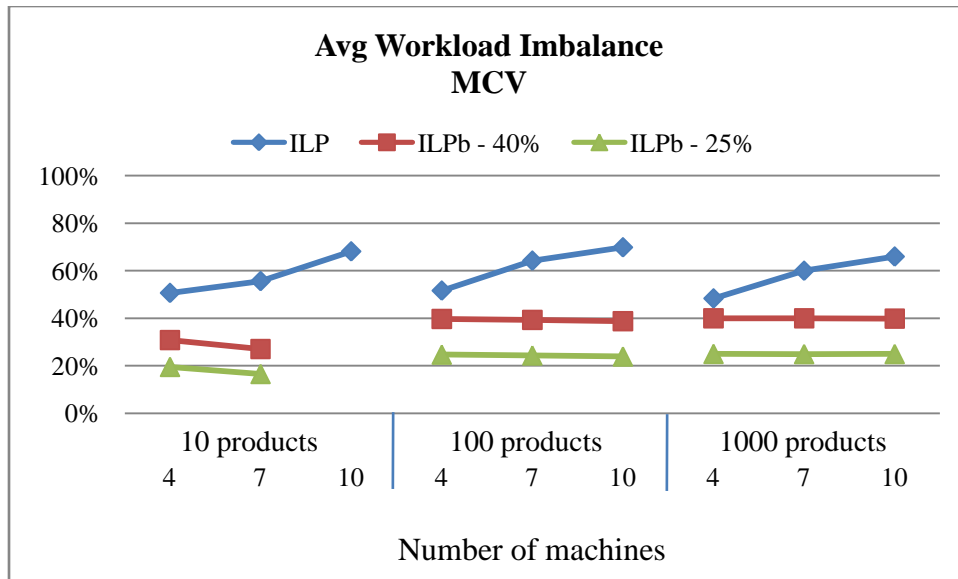


Figure 4.7. Average Machine Workload Imbalance – MCV(ILP & ILPb;  $\delta=40\%&25\%$ )

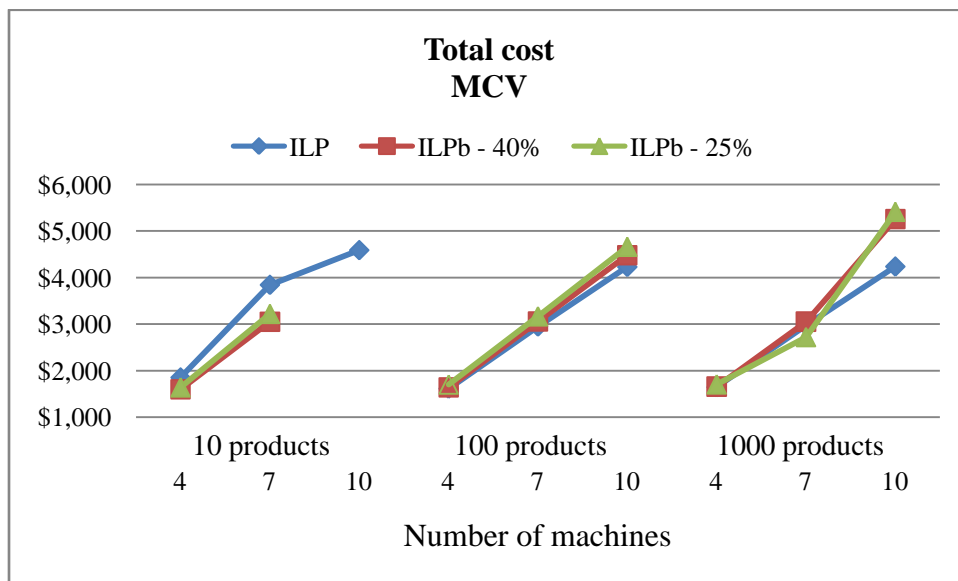


Figure 4.8. Total Cost - MCV (ILP & ILPb;  $\delta=40\% & 25\%$ )



The analysis of the performance of the solver (refer to Table 4.9) shows that only the scenarios with 10 products and four or seven machines and the one with 1000 products and seven machines can find an optimum solution within the time limit of 3600 seconds. Once again, the 10 products with ten machines scenario is infeasible. The solver was able to find feasible solutions for the scenarios with 100 and the others with 1000 products even when it went over the established time limit.

Table 4.9 - Solver Results for the ILPb Model with  $\delta=25\%$

<b><i>MCV = 0.9</i></b>				
<i>Products</i>	<i>Machines</i>	<i>Solver Iterations</i>	<i>B&amp;B Nodes</i>	<i>Run time (sec.)</i>
10	4	87344	10935	12.87
	7	1102	21	14.23
	10	-	-	14.65
100	4	2938354	390010	7220.37*
	7	3159529	334801	14429.40*
	10	2504534	256000	21634.30*
1000	4	165138	38231	5108.80*
	7	216681	19164	3611.50
	10	415624	10484	12092.60*

The ranges of the minimum and maximum average machine hours used when  $\delta = 25\%$  are shown on Table 4.10. These are 15 to 19 hours for the 10 products feasible scenarios, and 1 to 2 hours for the 1000 products scenarios showing once again, the machine utilization increasing when the number of machines increases.

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\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

Table 4.10 - Machine Utilization Ranges for the ILPb Model with  $\delta = 25\%$ 

<b>MCV = 0.9</b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	47.27	65.90
	7	56.00	70.60
	10	-	-
100	4	47.16	53.49
	7	54.68	56.91
	10	54.83	60.39
1000	4	50.35	51.55
	7	54.94	55.99
	10	57.17	57.89

#### 4.5. Sensitivity Analyses

To understand in detail the behavior of the different mathematical models studied during the course of this research, sensitivity analyses can be done. At the same time, these analyses create more realistic environment that can closely relate to real electronics assemblies companies. Due to the importance of controlling the process variability at any manufacturing plant, the sensitivity analyses should be done by varying the coefficient of variation (CV). In specific, additional scenarios can be created if low CV (LCV) and high CV (HCV) are considered. As explained by Hopp and Spearman, low variability is when  $CV < 0.75$ , moderate variability is when  $0.75 \leq CV < 1.33$ , and high variability is when  $CV \geq 1.33$  (Hopp et al., 2001).

The sensitivity analyses are going to be done adding scenarios with a LCV of 0.4, and a HCV of 1.4 to the ILP and ILPb models. The ILPx model will not be

compared as part of the sensitivity analyses because its behavior is similar to the ILP model. The additional scenarios increase the total runs of the mathematical model to twenty seven for the ILP model, and to fifty four for the ILPb model. Tables 4.11 and 4.12 summarize the scenarios for the sensitivity analyses.

Table 4.11 - Scenarios of the ILP Model for the Sensitivity Analyses

<i>CV</i>	<i>Machines</i>	<i>Products</i>		
0.4	4	10	100	1000
	7	10	100	1000
	10	10	100	1000
0.9	4	10	100	1000
	7	10	100	1000
	10	10	100	1000
1.4	4	10	100	1000
	7	10	100	1000
	10	10	100	1000

The results of the sensitivity analyses runs are shown on Tables 4.13 and 4.14 for the ILP model with LCV, and on Tables 4.15 and 4.16 for the ILP model with HCV. The scenario of 1000 products with ten machines was the only one of the ILP model that had to be run over the time limit to obtain a feasible solution when considering a LCV (refer to Table 4.13). The same scenario needed more time to find a feasible solution when using a HCV (refer to Table 4.15).

Table 4.12 - Scenarios of the ILPb Model for the Sensitivity Analyses

<i>CV</i>	<i>Machines</i>	$\delta = 0.40$			$\delta = 0.25$		
		<i>Products</i>			<i>Products</i>		
0.4	4	10	100	1000	10	100	1000
	7	10	100	1000	10	100	1000
	10	10	100	1000	10	100	1000
0.9	4	10	100	1000	10	100	1000
	7	10	100	1000	10	100	1000
	10	10	100	1000	10	100	1000
1.4	4	10	100	1000	10	100	1000
	7	10	100	1000	10	100	1000
	10	10	100	1000	10	100	1000

Table 4.13 - Results for the ILP Model with LCV

<b><i>LCV = 0.4</i></b>						
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>	<i>Solver Iterations</i>	<i>Run time (sec.)</i>
10	4	1,619	38.50	38.27%	101	23.63
	7	2,971	36.07	44.94%	121	23.82
	10	4,318	33.74	50.08%	144	24.21
100	4	1,514	39.78	30.82%	1000	6.38
	7	2,763	38.32	38.22%	1000	19.60
	10	3,986	35.88	44.45%	1001	52.50
1000	4	1,480	41.79	25.17%	10000	144.51
	7	2,693	39.50	35.06%	10000	1526.30
	10	3,856	37.36	40.59%	10000	4840.12*

---

\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

Table 4.14 - Machine Utilization Ranges for the ILP Model with LCV

<b>LCV = 0.4</b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	30.23	53.69
	7	27.67	44.97
	10	27.84	47.06
100	4	35.70	44.59
	7	34.12	44.65
	10	30.57	41.76
1000	4	38.87	44.08
	7	35.18	41.70
	10	33.88	40.17

Table 4.15 - Results for the ILP Model with HCV

<b>HCV = 1.4</b>						
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>	<i>Solver Iterations</i>	<i>Run time (sec.)</i>
10	4	1,631	39.29	54.95%	113	8.44
	7	2,869	27.04	70.84%	104	8.72
	10	4,114	26.30	72.33%	109	9.25
100	4	1,552	36.41	57.88%	1000	4.23
	7	2,766	27.61	70.55%	1000	21.33
	10	3,972	23.33	76.28%	1000	60.91
1000	4	1,622	38.17	56.26%	10000	169.69
	7	2,899	29.56	68.75%	10000	2082.19
	10	4,140	25.30	74.12%	10000	6599.96*

---

\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

Table 4.16 - Machine Utilization Ranges for the ILP Model with HCV

<b><i>HCV = 1.4</i></b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	16.69	52.51
	7	18.52	38.44
	10	16.23	34.03
100	4	32.13	41.79
	7	23.65	31.00
	10	20.80	26.95
1000	4	35.03	42.70
	7	26.03	34.98
	10	21.96	30.05

Tables 4.17, 4.18, and 4.19 include the results of the sensitivity analyses for the ILPb model with LCV and  $\delta = 40\%$ . Similar results are presented on Tables 4.20, 4.21, and 4.22 for the ILPb model with  $\delta = 25\%$ .

When running the ILPb model with  $\delta = 40\%$  and a LCV, there were two scenarios that ran over the time limit to find a feasible solution; 100 products with seven and ten machines (refer to Table 4.18). With a LCV and  $\delta = 25\%$ , there were three scenarios that ran over the time limit and were able to find feasible solutions; 100 products with seven machines, and 1000 products with seven and ten machines (refer to Table 4.21). For the scenario with 100 products and ten machines, the solver was not able to find an integer feasible solution even when running during 4528 seconds through over four millions of iterations and almost four hundred thousand B&B nodes analyzed (refer to Tables 4.20 and 4.21).

Table 4.17 - Results for the ILPb Model with LCV and  $\delta=40\%$ 

<b>LCV = 0.4</b>				
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>
10	4	1,642	42.31	26.19%
	7	3,153	50.06	22.70%
	10	4,557	52.84	18.91%
100	4	1,516	39.85	28.97%
	7	2,778	38.90	36.53%
	10	4,014	39.69	36.73%
1000	4	1,480	41.80	25.15%
	7	2,684	39.69	34.69%
	10	4,405	38.44	38.84%

Table 4.18 - Solver Results for the ILPb Model with LCV and  $\delta=40\%$ 

<b>LCV = 0.4</b>			
<i>Products</i>	<i>Solver Iterations</i>	<i>B&amp;B Nodes</i>	<i>Run time (sec.)</i>
10	343	22	30.44
	723	62	31.32
	601	0	38.45
100	1740	0	15.50
	4562486	162870	7442.89*
	3549392	145127	14722.90*
1000	16015	0	80.92
	22059	0	3031.31
	136920	7000	2304.27

---

\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

Table 4.19 - Machine Utilization Ranges for the ILPb Model with LCV and  $\delta=40\%$ 

<b>LCV = 0.4</b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	36.37	51.86
	7	43.98	55.31
	10	48.63	55.68
100	4	35.99	44.85
	7	37.21	42.52
	10	37.23	44.38
1000	4	38.87	44.08
	7	36.57	42.08
	10	37.59	40.76

Table 4.20 - Results for the ILPb Model with LCV and  $\delta=25\%$ 

<b>LCV = 0.4</b>				
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>
10	4	1,678	48.48	15.30%
	7	3,235	55.39	13.17%
	10	4,817	59.44	13.36%
100	4	1,524	42.79	23.90%
	7	2,840	46.57	24.20%
	10	No solution <sup>♦</sup>	-	-
1000	4	1,486	42.58	23.66%
	7	2,710	45.59	24.88%
	10	4,478	47.27	24.78%

---

<sup>♦</sup> No integer feasible solution was found by the solver within the established time limit.



Table 4.21 - Solver Results for the ILPb Model with LCV and  $\delta=25\%$ 

<b>LCV = 0.4</b>				
<i>Products</i>	<i>Machines</i>	<i>Solver Iterations</i>	<i>B&amp;B Nodes</i>	<i>Run time (sec.)</i>
10	4	1260	166	30.82
	7	3862	571	32.02
	10	788	0	41.44
100	4	3222683	223423	3607.78
	7	3390492	308321	11087.20*
	10	4733138	382381	4528.37 <sup>♦</sup>
1000	4	38617	2000	1302.38
	7	536531	40998	7654.19*
	10	602878	13009	4512.08*

Table 4.22 - Machine Utilization Ranges for the ILPb Model with LCV and  $\delta=25\%$ 

<b>LCV = 0.4</b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	42.88	54.00
	7	51.12	59.66
	10	51.72	62.89
100	4	41.23	44.35
	7	45.94	47.42
	10	-	-
1000	4	41.52	43.90
	7	45.24	45.96
	10	47.00	47.57

---

\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

<sup>♦</sup> No integer feasible solution was found by the solver within the established time limit.

The results for the sensitivity analyses for the ILPb model with HCV and  $\delta = 40\%$  are presented on Tables 4.23, 4.24, and 4.25. Using  $\delta = 25\%$ , generated results are shown on Tables 4.26, 4.27, and 4.28. It can be seen on Tables 4.23 and 4.24 that the scenarios with 10 products, a HCV, and  $\delta = 40\%$  were not feasible and the solver determined that in less than twenty-one seconds for each scenario. Table 4.24 shows that for four of the scenarios, optimum solutions were not found within the time limit, but feasible solutions were obtained instead.

Table 4.23 - Results for the ILPb Model with HCV and  $\delta=40\%$

<b><i>HCV = 1.4</i></b>				
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>
10	4	Infeasible	-	-
	7	Infeasible	-	-
	10	Infeasible	-	-
100	4	1,610	52.22	39.55%
	7	3,010	57.40	39.46%
	10	4,407	59.98	38.82%
1000	4	1,673	52.41	39.96%
	7	3,118	56.70	39.89%
	10	4,517	58.92	39.83%

Table 4.24 - Solver Results for the ILPb Model with HCV and  $\delta=40\%$ 

<b>HCV = 1.4</b>				
<i>Products</i>	<i>Machines</i>	<i>Solver Iterations</i>	<i>B&amp;B Nodes</i>	<i>Run time (sec.)</i>
10	4	177	0	1.33
	7	0	0	2.41
	10	0	0	20.11
100	4	2989214	397287	3628.74
	7	3050901	334436	10849.60*
	10	2681919	234221	18090.50*
1000	4	160295	26541	3609.13
	7	174827	15441	10828.70*
	10	200681	11088	18049.90*

Table 4.25 - Machine Utilization Ranges for the ILPb Model with HCV and  $\delta=40\%$ 

<b>HCV = 1.4</b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	-	-
	7	-	-
	10	-	-
100	4	50.61	53.94
	7	55.33	60.86
	10	58.27	61.76
1000	4	51.90	52.96
	7	56.46	57.24
	10	58.27	59.41

---

\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

Table 4.26 - Results for the ILPb Model with HCV and  $\delta=25\%$ 

<b>HCV = 1.4</b>				
<i>Products</i>	<i>Machines</i>	<i>Total Cost (\$M)</i>	<i>Avg. Machine Utilization</i>	<i>Avg. Workload Imbalance</i>
10	4	Infeasible	-	-
	7	Infeasible	-	-
	10	Infeasible	-	-
100	4	1,687	65.31	24.72%
	7	3,141	70.25	24.36%
	10	Infeasible	-	-
1000	4	1,745	65.06	24.95%
	7	3,275	71.00	24.88%
	10	4,748	73.31	24.80%

Table 4.26 shows that when considering a HCV and  $\delta = 25\%$ , the 10 products scenarios are infeasible (same as when  $\delta = 40\%$ ) and it was quickly determined by AMPL9/CPLEX90, in less than twenty-two seconds for each scenario (refer to Table 4.27). There is another scenario under the same considerations; 100 products and ten machines, that was found infeasible by the solver as well, but it took over 18000 seconds. In addition, for five scenarios the solver was able to find feasible solutions within the established time limit. These scenarios are for 100 products with four and seven machines, and all of the 1000 products scenarios.

Table 4.27 - Solver Results for the ILPb Model with HCV and  $\delta=25\%$ 

<b>HCV = 1.4</b>				
<i>Products</i>	<i>Machines</i>	<i>Solver Iterations</i>	<i>B&amp;B Nodes</i>	<i>Run time (sec.)</i>
10	4	-	-	1.77
	7	-	-	3.33
	10	-	-	21.56
100	4	3126767	382000	7231.43*
	7	2994235	336366	14482.30*
	10	2181	0	18092.00
1000	4	168787	32623	7222.81*
	7	309168	15461	14437.70*
	10	271590	11362	24672.50*

Table 4.28 - Machine Utilization Ranges for the ILPb Model with HCV and  $\delta=25\%$ 

<b>HCV = 1.4</b>			
<i>Products</i>	<i>Machines</i>	<i>Min Average Machine Utilization</i>	<i>Max Average Machine Utilization</i>
10	4	-	-
	7	-	-
	10	-	-
100	4	62.47	68.09
	7	66.92	73.41
	10	-	-
1000	4	63.20	66.49
	7	70.10	71.50
	10	72.65	73.88

---

\* This result is not optimum, it represents the best bound from the B&B algorithm at the time limit.

#### 4.5.1. Comparison of Results from the Sensitivity Analyses

Graphs are generated combining different scenarios to compare the results from the sensitivity analyses. This helps to understand the behavior of the models and how the different variables considered may affect the results. Considering the process variability, analyzed in this research based on the coefficient of variation (CV), as the main factor to do sensitivity analyses, the comparison of the results are being made by presenting them for each level of CV within each mathematical model developed.

The following graphs (refer to Figures 4.9, 4.10, and 4.11) show the average machine utilization under each level of CV for the ILP, ILPb with  $\delta = 40\%$ , and ILPb with  $\delta = 25\%$ , respectively.

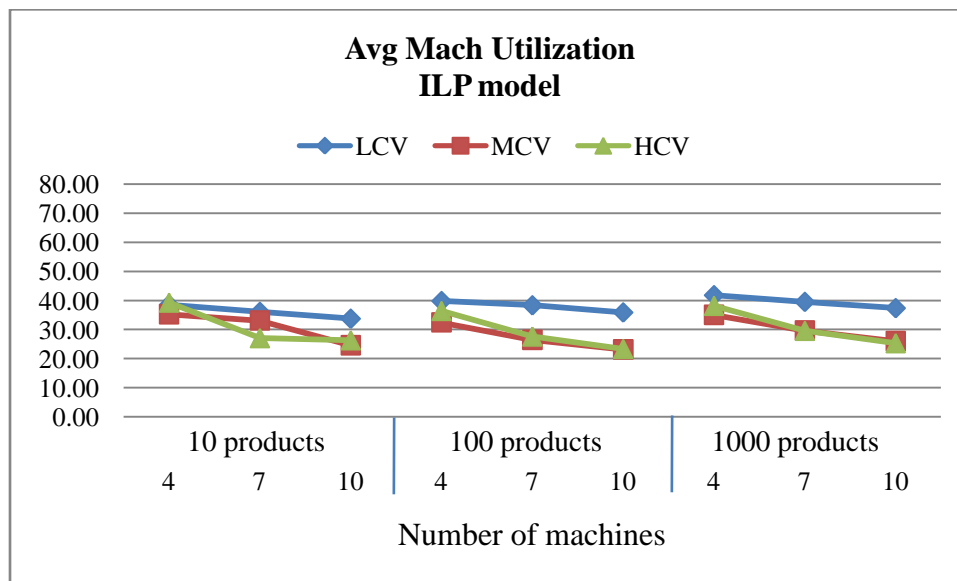


Figure 4.9. Average Machine Utilization – ILP Model

It can be seen on Figure 4.9 that the highest average machine utilizations for each combination of number of products and number of machines are obtained with the LCV. Also, the lower the number of machines, the higher the average machine utilization will be disregarding the number of products or the level of the CV. When comparing MCV and HCV, there are no significant differences in the average machine utilization given the same combination of number of products and machines.

Under the ILPb model scenarios with  $\delta = 40\%$  (refer to Figure 4.10), the higher the CV, the higher the average machine utilization for each combination of number of products and machines. Again, when comparing MCV and HCV, there are no significant differences in the average machine utilization given the same combination of number of products and machines. The missing points on the graphs represent infeasible scenarios.

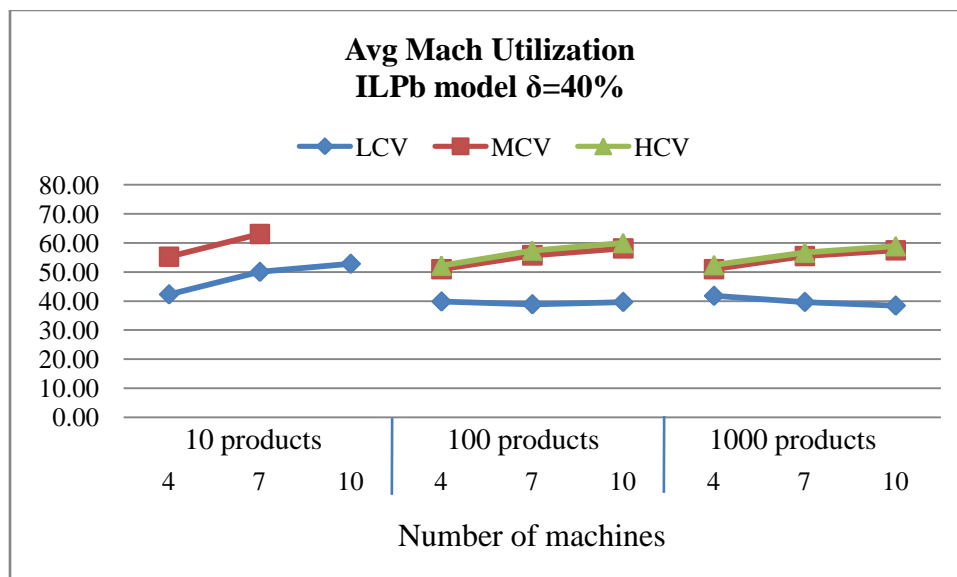


Figure 4.10. Average Machine Utilization – ILPb Model;  $\delta=40\%$

Analyzing the ILPb model with  $\delta = 25\%$  (refer to Figure 4.11), the higher the CV, the higher the average machine utilization for each combination of number of products and number of machines. On this model, due to the imbalance factor being more restrict, there is a noticeable difference on the average machine utilization between MCV and HCV resulting in higher numbers for the HCV. The missing points on the graphs also represent infeasible scenarios.

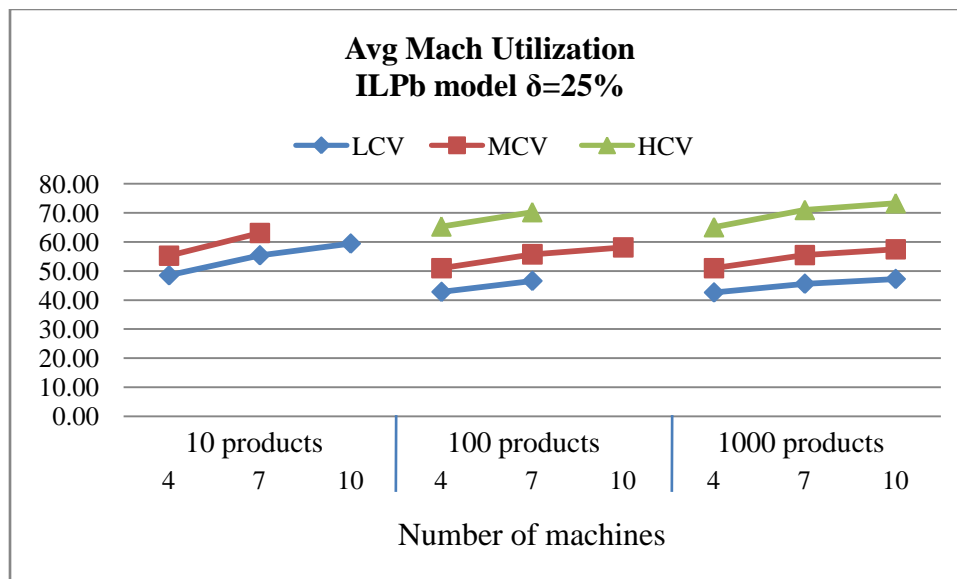


Figure 4.11. Average Machine Utilization – ILPb Model;  $\delta=25\%$

The next variable to compare is the average machine workload imbalance. Figures 4.12, 4.13, and 4.14 show these comparisons for each level of CV for the ILP, ILPb with  $\delta = 40\%$ , and ILPb with  $\delta = 25\%$ , respectively.



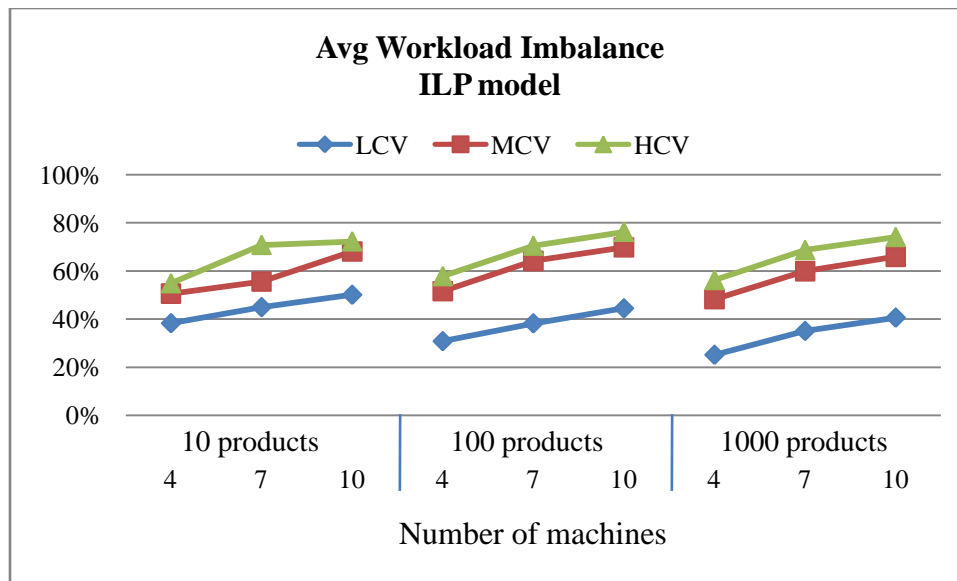


Figure 4.12. Average Machine Workload Imbalance - ILP Model

When analyzing the comparison of the average machine workload imbalance for the ILP model (refer to Figure 4.12), it shows that the higher the level of the CV, the higher the machine workload imbalance for each combination of number of products and number of machines. This result was expected since the ILP model does not consider machine workload balance at all; it minimizes assembly cost based on the assignment of products to machines considering machine capacity.

Figures 4.13 and 4.14 demonstrate that for the ILPb model, since machine workload balance is one of the constraints, the maximum workload imbalance observed will be close to 40% and 25% depending on the value of the imbalance factor. The level of the CV does not have a significant impact except for the 10 products scenarios, where

the machine workload imbalance is noticeable lower than the maximum allowed. The missing points are again related to infeasible scenarios.

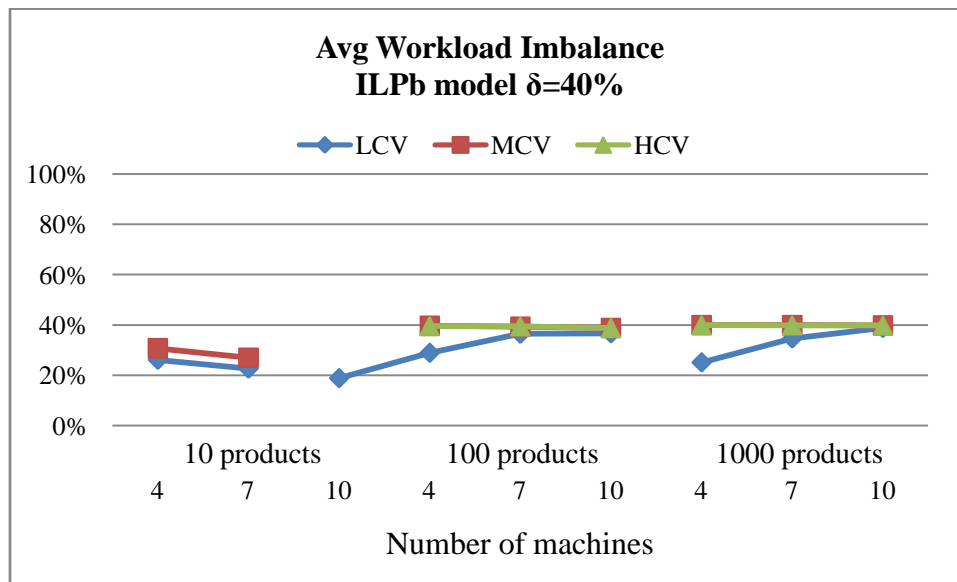


Figure 4.13. Average Machine Workload Imbalance - ILPb Model;  $\delta=40\%$

The last set of graphs on the comparisons from the sensitivity analyses show the impact on the total cost of all products assembled. Figures 4.15, 4.16, and 4.17 present the total cost for each level of CV for the ILP, ILPb with  $\delta = 40\%$ , and ILPb with  $\delta = 25\%$ , respectively.

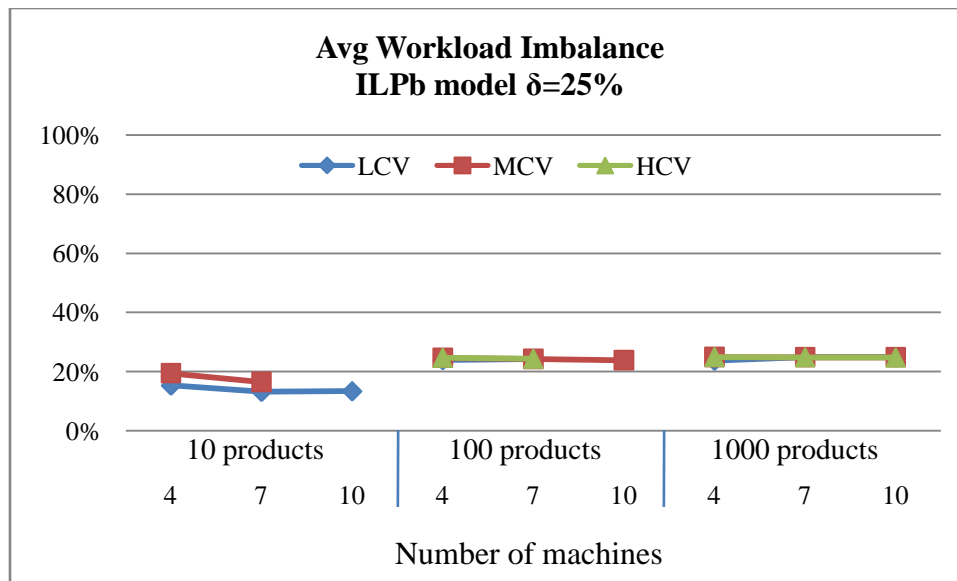


Figure 4.14. Average Machine Workload Imbalance - ILPb Model;  $\delta=25\%$

The ILP model graph for total cost (refer to Figure 4.15) show that the higher the level of the CV and the higher the number of machines within each number of products scenario, the higher the total cost of assembling all products. It can be seen that there is no significant variation on the total cost for the same number of machines when assembling different amounts of products. This means that when there are four machines, if the products assembled are 10, 100, or 1000, the total cost do not vary significantly because the expected demand per product is not the same to be able to assemble all products given the available capacity. Same reasoning applies when there are seven or ten machines.

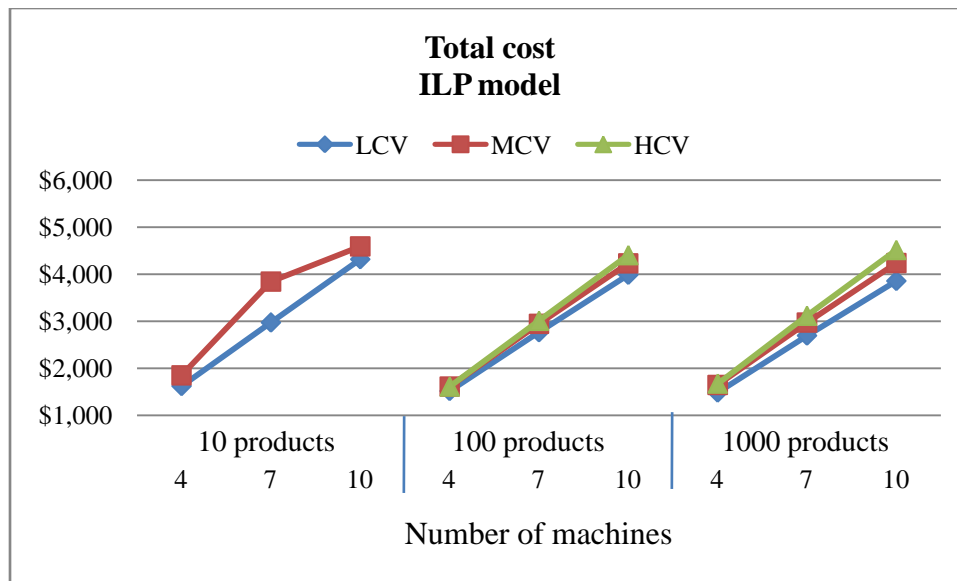
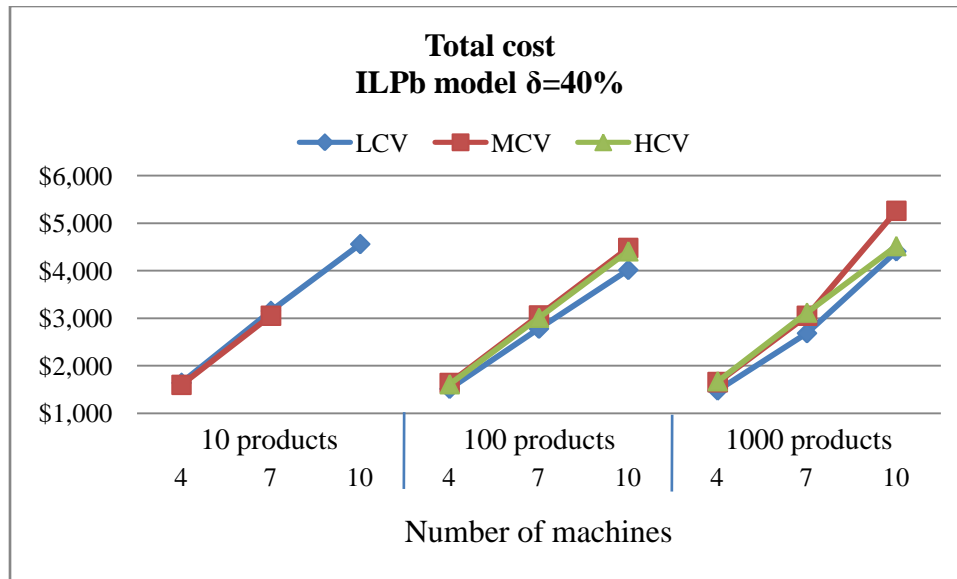
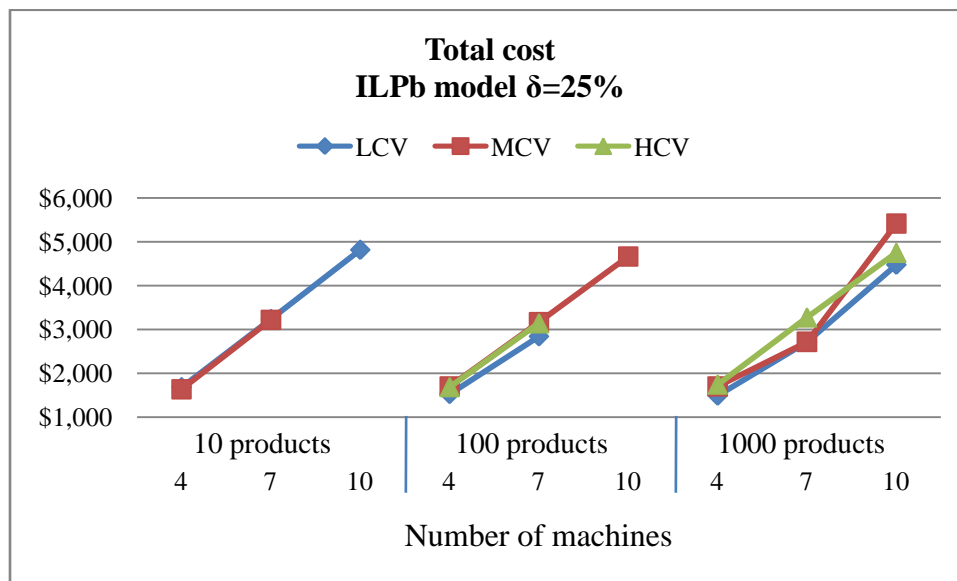


Figure 4.15. Total Cost – ILP Model

When analyzing the total cost for the ILPb models (refer to Figures 4.16 and 4.17), it also shows that the higher the level of the CV and the higher the number of machines for each scenario keeping the number of products constant, the higher the total cost of assembling all products. Again, the reasoning about having the same number of machines, but changing the amounts of products assembled also applies for these models. There is one obvious exception when there are 1000 products assembled with ten machines per assembly process because the total cost is significantly higher under the MCV scenarios for both imbalance factors ( $\delta = 40\%$  &  $25\%$ ).

Figure 4.16. Total Cost – ILPb Model;  $\delta=40\%$ Figure 4.17. Total Cost – ILPb Model;  $\delta=25\%$

#### **4.6. Summary**

This chapter includes the results of the runs of all models: ILP, ILP<sub>x</sub>, and ILP<sub>b</sub>. These results are presented on tables including total cost, average, minimum, and maximum machine utilization, and average machine workload balance for each identified scenario combining different number of assembly machines and different amounts of assembled products. Results presenting the performance of the mathematical programming language and the solver are also presented in tables for each identified scenario.

A series of sensitivity analyses were performed for the ILP and ILP<sub>b</sub> models where additional levels of process variability (using CV) were considered on the scenarios analyzed. Graphs were generated for the comparison of the additional scenarios and analyses of these results were discussed.

The next chapter is presents a summary of the research with its conclusions. It also includes recommendation for future research.

## CHAPTER V

### SUMMARY, CONCLUSIONS, AND FUTURE RESEARCH

#### 5.1. Introduction

This chapter summarizes the problem presented in this research of optimizing machine selection in an electronics assembly environment. The methodology used to solve the problem for each integer linear programming model is explained. The results obtained are discussed for the selected scenarios. Recommendations of the applicability of the developed models are also included while discussing the conclusions of this work. The opportunities for future research are mentioned and explained based on the researcher's knowledge and experience.

#### 5.2. Summary of Research

A model driven Decision Support System (DSS) was developed to solve the problem described in Chapter I with the emphasis in optimizing available resources, minimizing production disruption, thus minimizing cost. The variables that affect the costs of the assembly of electronics products were considered in detail and used while developing the cost model under the premises of variable costing accounting system.

The first model of the Decision Support System was developed using Microsoft Excel as an interactive tool to estimate the cost of a single electronic product assembly. This basic model for one product was created in Microsoft Excel due to its practical and

common use among most businesses (refer to Chapter II). It uses macros to facilitate entering the required data and to do the calculations needed (refer to Appendix A).

Mathematical modeling was used for the optimization model of all products assembled using AMPL9 as the mathematical programming language and CPLEX90 as the solver for the model. Branch and bound (B&B) algorithm is used when solving the models with CPLEX90. This model was created to provide an optimization tool for the assembly of many products during the same period of time. The mathematical model was then enhanced by adding constraints to balance machines workload up to an allowed imbalance factor. Another variation of the mathematical model calculates the direct labor cost per product in a different way to consider the impact on the product cost when expediting production. The mathematical models are discussed on detail in Chapter III.

Multiple scenarios were identified with different combinations of the amount of assembled products, the quantity of machines at each assembly process, and the allowed imbalance factor for the assembly machines workload. These scenarios were analyzed with the corresponding mathematical model and the results are presented in Chapter IV.

The analysis of the results obtained showed that the mathematical model considering expediting production (ILPx) performs in a similar way as the general model (ILP). The higher the number of machines per process, the higher the average machine workload imbalance due to having more options (in terms of machines) to assign the products at each assembly process. If the variable being changed is the amount of products, then the higher it is, the more consistent the machine utilization is among all assembly processes and machines. The ILPx model has slightly higher average machine



workload imbalance and slightly lower machine utilization in general than the ILP model because of its additional complexity.

Due to the nature of the ILPb models which have constraints to balance machine workload at each assembly process, the results were as expected in the sense that the average machine utilization increases when the amount of products assembled also increases while keeping the average machine workload imbalance almost constant. This is the opposite of the results obtained with the ILP and ILPx models.

Sensitivity analyses were performed to understand the behavior of the mathematical models (ILP and ILPb) under additional scenarios. These scenarios were created by changing the coefficient of variation (CV) of the assembly processes. Since the analysis of results was done using a moderate coefficient of variation (MCV), for the sensitivity analyses low and high coefficients of variation (LCV and HCV) were used. Results from the sensitivity analyses show that when evaluating the ILP model with LCV and HCV only for one scenario (1000 products with ten machines) the solver was not able to find an optimum solution within the established time limit, but feasible integer solutions were found in less than two hours. For all other scenarios, optimum solutions were found as fast as in four seconds and in no more than thirty-five minutes. No noticeable differences were found in the performance of the ILP model under the MCV and HCV considerations.

The results of the sensitivity analyses for the ILPb models show that when considering a LCV with  $\delta = 40\%$  or  $25\%$ , there was only one scenario (100 products with ten machines and  $\delta = 25\%$ ) where the solver was not able to find an integer feasible

solution within the time limit, but the scenario was not recognized as infeasible by AMPL9/CPLEX90. There were some scenarios for which the solver found feasible (not optimum) solutions within the established time limit. These scenarios are the ones with 100 products and seven or ten machines when  $\delta = 40\%$ ; and the ones with 100 products and seven machines or 1000 products and seven or ten machines when  $\delta = 25\%$ . The results for the HCV show that all scenarios with 10 products were found infeasible by the solver for both levels of the imbalance factor ( $\delta = 40\%$  and  $25\%$ ). The only other infeasible scenario was with 100 products and ten machines given a  $\delta = 25\%$ . There were many scenarios that found feasible (not optimum) solutions within the established time limit. These scenarios are the ones with 100 or 1000 products and seven or ten machines when  $\delta = 40\%$ , and the ones with 100 or 1000 products and any number of machines when  $\delta = 25\%$  (with the exception of 100 products and ten machines that is mentioned above).

### **5.3. Conclusions**

While looking for the best way to select machines when there are multiple related assembly machines available, minimizing the total cost (or using the SPT rule) of assembling all products proved to be an appropriate method. The main difference in the execution of the ILP and ILPb mathematical models lies on how they choose the assembly machines at each assembly process for the demanded products. The ILP model always assign the minimum cost (minimum time as well) available machine at each assembly process unless there is no capacity (machine hours) left for the given period of

time. The ILPb does not necessarily assign the minimum cost available machine to be able to balance the machines workload within each assembly process up to the selected imbalance factor. Therefore, the ILPb model requires more machine hours in average than the ILP model to assembly the same amount of product demand. This is how the average machine utilization increased when running the ILPb model while the average machine workload balance decreased. As a result, usually the total cost of assembling all products is higher with the ILPb model than with the ILP model given other variables are kept constant.

Analyzing the total cost of all products assembled, since there is not a significant difference, the ILPb model is recommended over the ILP model. The average difference in total cost fluctuates between 5 and 10% when comparing the ILPb versus the ILP model depending on the level of the CV. Between the two levels of the imbalance factors studied ( $\delta = 40\%$  and  $25\%$ ), the average difference in total cost goes from 2 to 5%. For this reason, the mathematical model considering  $\delta = 25\%$  is highly recommended because it provides for a better machine workload balance.

Even for the more restricted scenarios, the ILPb model when running with AMPL9/CPLEX90 is able to find feasible solutions. There can be scenarios that need over four millions iterations to be solved and over three hundred B&B nodes to be analyzed within a time range of five to seven hours. As expected, the higher the level of the CV, the higher the average total cost of all products assembled, and the higher the average machine utilization will be when using the ILPb model with any of the

imbalance factors identified. The scenarios varying the CV were presented for the purpose of relating to more realistic scenarios in an electronics assemblies environment.

#### **5.4. Future Research**

There are some options that have been considered by the researcher to expand the use of the integer linear programs developed during the course of this research. These options have two main purposes: to improve the mathematical models presented in this research considering the same domain, electronics products assemblies; and to extend the applicability of these integer linear programs to other manufacturing scenarios.

When focusing on the electronics assembly environment, the following are some opportunities to create optimization models with more applications:

- To apply additional production planning or production scheduling rules to the mathematical models presented in this research. One of these methodologies is to consider the bottleneck assembly process, the one who takes more time on the assembly line. The problem of emphasizing on the bottleneck process can be included in the mathematical model by adding additional constraints. It would be crucial to understand the applicability of considering the bottleneck process in a mathematical model like the ones presented in this research to guarantee a significant contribution to the existing literature. The problem of bottleneck process has been studied by many researchers and therefore, our contribution would be related to include it in a mathematical model where cost is calculated in a very detailed manner and the objective is to minimize it.

- As explained at the beginning of this dissertation, the variable costing accounting system was used to develop the cost models explained. An additional research focus can be to develop this kind of mathematical model but considering a full or absorption costing accounting system for long-term decision making. There could be a different group of users for this kind of minimization model; the ones looking into strategic planning and long term planning in an electronics assembly environment. If a full costing accounting system is used, the major changes to the mathematical model created during this research would be on the way the overhead costs are calculated. In this research, only the variable manufacturing overhead costs were considered when developing the cost model. Under a full costing accounting system, the fixed manufacturing overhead costs would have to be considered as well.
- Another alternative for future research is the development of heuristics procedures to solve the ILPb models discussed in this research. These procedures would help to solve the scenarios where too many constraints are included making the solving time to increase extraordinarily. In specific, these heuristics procedures could have the purpose to solve those scenarios where an integer feasible solution was found within five to seven hours, but to obtain an optimum solution may have taken an undetermined amount of time.

With the focus on extending the applicability of these integer linear programs to other manufacturing scenarios, the following are some opportunities to be considered:

- To study the differences on how are the costs (mainly overhead costs) generated under different manufacturing or assembly environments and apply these differences

to the mathematical models developed in this research. The differences identified could affect the way the parameters used to calculate the total cost of all products are computed, or they could have an effect on the required data and/or the way it is generated. Another possible impact would be on the constraints identified for the mathematical model since these can be the same, but most probably, it could imply that additional (or different) constraints may be needed.

- The heuristic procedures and the bottleneck considerations mentioned above can be analyzed for different manufacturing environments as well and determine its applicability.

## REFERENCES

- Ammons, J.C., Carlyle, M., Cranmer, L., DePuy, G., Ellis, K., McGinnis, L.F., Tovey, C.A., Xu, H., 1997. Component allocation to balance workload in printed circuit card assembly systems. *IIE Transactions*, 29, 265-275.
- Castillo, C., 1998. Cost modeling system for the IC packaging industry. Ph.D. Dissertation, Texas A&M University, College Station, TX.
- Clark, R.H., 1985. *Handbook of Printed Circuit Manufacturing*. 1st. ed., New York, Van Nostrand Reinhold.
- Cooper, R., Kaplan, R.S., 1988. How cost accounting distorts product costs. *Management Accounting*, 69(10), 20-27.
- Crama, Y., van de Klundert, J., Spieksma, F., 2002. Production planning problems in printed circuit board assembly. *Discrete Applied Mathematics*, 123, 339-361.
- Dhavale, D.G., 1990. A manufacturing cost model for computer-integrated manufacturing systems. *Production and Operations Management*, 10(8), 5-18.
- Forgionne, G.A., Kohli, R., 1996. HMSS: a management support system for concurrent hospital decision making. *Decision Support Systems*, 16(3), 209-230.
- Fourer, R., Gay, D.M., Kernighan, B.W., 2003. *AMPL - A Modeling Language for Mathematical Programming*. 2nd ed. 2003, Pacific Grove, California, Thomson Learning.
- Garfinkel, R.S., 1971. An improved algorithm for the bottleneck assignment problem. *Operations Research*, 19(7), 1747-1751.
- Gavish, B., Pirkul, H., 1991. Algorithms for the multi-resource generalized assignment problem. *Management Science*, 37(6), 695-713.

- Geetha, S., Vartak, M.N., 1994. The three-dimensional bottleneck assignment problem with capacity constraints. *European Journal of Operational Research*, 73, 562-568.
- Gilbert, K.C., Hofstra, R.B., 1988. Multidimensional assignment problems. *Decision Science*, 19(2), 302-317.
- Hillier, M.S., Brandeau, M.L., 2001. Cost minimization and workload balancing in printed circuit board assembly. *IIE Transactions*, 33(7), 547-557.
- Hilton, R.W., 1999. *Managerial Accounting*. Fourth ed., New York, Irwin McGraw-Hill.
- Hopp, W.J., Spearman, M.L., 2001. *Factory Physics*. 2nd ed., New York, McGraw-Hill Higher Education.
- Horngren, C.T., Datar, S.M., Foster, G., 2003. *Cost Accounting - A Managerial Emphasis*. 11th ed., Upper Saddle River, New Jersey: Prentice Hall.
- ILOG CPLEX. Available from: <http://www.ilog.com/products/cplex/>.
- Kinney, M.R., Prather-Kinsey, J., Raiborn, C.A., 2006. *Cost Accounting: Foundations and Evolutions*. 6e ed., Mason, Ohio: Thomson South-Western.
- LeBlanc, L.J., Shtub, A., Anandalingam, G., 1999. Formulating and solving production planning problems. *European Journal of Operational Research*, 112, 54-70.
- Mazzola, J.B., Neebe, A.W., 1988. Bottleneck generalized assignment problems. *Engineering Costs and Production Economics*, 14, 61-65.
- Méndez-Piñero, M.I., 2001. Development of cost models for power electronics assemblies. M.S. Thesis, University of Puerto Rico, Mayaguez, PR.
- Miller, T., Vollmann, T., 1985. The hidden factory. *Harvard Business Review*, 65(5), 142-150.



- Ong, N.S., 1995. Manufacturing cost estimation for PCB assembly: An activity-based approach. *International Journal of Production Economics*, 38(2-3), 159-172.
- Pentico, D.W., 2007. Assignment problems: A golden anniversary survey. *European Journal of Operation Research*, 176, 774-793.
- Pillai, D.D., 1990. Developing a decision support system for optimizing automated wafer fabrication. *IEEE Transactions on Components, Hybrids, and Manufacturing Technology*, 13(1), 94-102.
- Power, D.J., 2003. A brief history of decision support systems. Available from: <http://www.inta.gov.ar/bariloche/desarrollo/gesrural/trabajos/ssd/Bibliografia/SSD/Historia%20y%20Evoluci%C3%B3n/A%20Brief%20History%20of%20Decision%20Support%20Systems.pdf>.
- Power, D.J., Sharda, R., 2007. Model-driven decision support systems: Concepts and research directions. *Decision Support Systems*, 43(3), 1044-1068.
- Rajakumar, S., Arunachalam, V.P., Selladurai, V., 2004. Workflow balancing strategies in parallel machine scheduling. *The International Journal of Advanced Manufacturing Technology*, 23(23), 366-374.
- Rajkumar, K., Narendran, T.T., 2003. A heuristic for sequencing PCB assembly to minimize set-up times. *Production Planning and Control*, 9(5), 465-476.
- Sandborn, P., 2005. A decision support model for determining the applicability of prognostic health management (PHM) approaches to electronic systems. In: 2005 Proceedings of the Annual Reliability and Maintainability Symposium, pp. 422-427.
- Silver, E.A., Pyke, D.F., Peterson, R., 1998. *Inventory Management and Production Planning and Scheduling*. Third ed., New York: John Wiley and Sons.
- Sundararajan, S., Srinivasan, G., Staehle, W.O., Zimmers, E.W., 1998. Application of a decision support system for operational decisions. *Computers and Industrial Engineering*, 35(1-2), 141-144.

Taha, H.A., 1997. Operations Research: An Introduction. Sixth ed., Upper Saddle River, New Jersey: Prentice Hall.

## APPENDIX A

### MICROSOFT EXCEL MODEL

In this appendix, the logic used to create the Microsoft Excel Model is explained in detail and the different worksheets used are included. The purpose of this model is to estimate the cost of assembly a given electronic product and use that information when selecting a machine to assign the product at each assembly process based on the minimum cost of assembly. The model considers direct labor cost, material cost, and overhead cost to calculate the total cost per unit.

To use this model, the user can start by selecting on the main worksheet (refer to Figure A.1) to go to the data sheet or to the calculations and results worksheet. When using the model for the first time to estimate the cost of assembly a particular product, the user should select the data sheet to be able to identify which data is required. This cost model works with macros to facilitate the data entry.

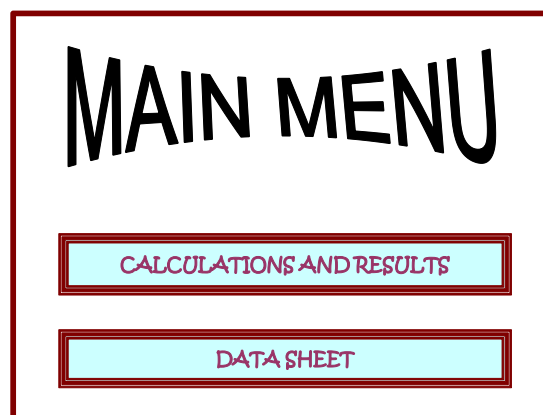


Figure A.1. Microsoft Excel model main menu

In the data sheet (refer to Figure A.2), the user will enter the required information for the parameters that require a single value (identified on light yellow). If the data needs to be entered in an array format, the user will click on each named button (identified on bright yellow) to be redirected to where the data can be entered (refer to Figure A.3). There are many of these data values that are generated using probability distributions. For these parameters, the user has the option to change the distribution values or enter different data values on the spaces available.

**Data required**

[Back to Main Menu](#)

<b>Variable</b>	<b>Description</b>	<b>Units</b>	<b>Value</b>
$CM_j$	Consumable material $j$	Number	$CM_j$
$\$CM_j$	Cost of consumable material $j$	\$/unit	$\$CM_j$
$CP_i$	Component $i$	Number	$CP_i$
$\$CP_i$	Cost of component $i$	\$/unit	$\$CP_i$
$D_{yr}$	Total annual demand	Units/year	10,258
$\$DL$	Average direct labor cost per hour	\$/hour	\$ 8.50
$\%DL_{jk}$	% of time direct employee work on machine $k$ in process $j$	%	$\%DL_{jk}$
$\#DL_{jk}$	Quantity of direct employees working on machine $k$ in process $j$	Number	$\#DL_{jk}$
HRS	Worked hours per year	Hours/year	2,000
$HT_{xp}$	Handling or moving time of product $x$ from previous process to process $p$	Hours/unit	$HT_{xp}$
MH	Machine hours available per week	Hours/week	80
$\$MM$	Total machines maintenance cost	\$/month	\$ 20,000
$MTTF_{pm}$	Mean-time-to-failure of machine $m$ in process $p$	Hours	$MTTF_{pm}$
$MTTR_{pm}$	Mean-time-to-repair of machine $m$ in process $p$	Hours	$MTTR_{pm}$
$PT_{xpm}$	Process time on product $x$ of machine $m$ in process $p$	Hours/unit	$PT_{xpm}$
$SP_k$	Support personnel $k$	Number	$SP_k$
$\%SP_x$	% of time on product $x$ by support personnel $k$	%	$\%SP_x$
$\$SP_{yr}$	Cost of support personnel per year	\$/year	\$ 60,000
$SU_{xpm}$	Setup time per batch of product $x$	Hours/batch	$SU_{xpm}$
$UC_u$	Utilities consumption $u$ per product	Units	$UC_u$
$\$UC_u$	Cost of utilities consumption $u$	\$/unit	$\$UC_u$
$UN(bh)_x$	Total units of product $x$ per batch	Units/batch	1,000
WKS	Worked weeks per year	Weeks/year	50

Figure A.2. Microsoft Excel model data sheet

<b>SU<sub>(bh)</sub></b>		Setup time per batch of product $x$ on process $j$ and machine $k$			
		<b>ROUNDUP(NORMDIST(RAND(),0.5,0.45,FALSE),4)</b>			
		<i>Machines</i>			
		1	2	3	
<i>Process</i>	1	0.6403	0.7287	0.8792	
	2	0.8149	0.4836	0.8727	
	3	0.8862	0.6725	0.5791	
	4	0.6631	0.5367	0.5772	
	5	0.7807	0.6706	0.5201	
	6	0.6602	0.6574	0.8687	
	7	0.6287	0.5437	0.4873	
	8	0.5046	0.7037	0.8824	<a href="#">Back to Data Sheet</a>
	9	0.8732	0.7173	0.8699	
	10	0.8862	0.5860	0.5271	<a href="#">Back to Main Menu</a>
<b>PT<sub>xjk</sub></b>		Process time of product $x$ on process $j$ and machine $k$			
		<b>ROUNDUP(NORMDIST(RAND(),0.1,0.09,FALSE),4)</b>			
		<i>Machines</i>			
		1	2	3	
<i>Process</i>	1	0.0587	2.4184	0.0001	
	2	0.0013	2.8468	0.0001	
	3	0.0315	0.0001	0.0001	
	4	0.0001	0.0001	0.9304	
	5	0.0498	0.0001	2.8049	
	6	0.0001	0.0006	0.0001	
	7	3.6259	0.7274	0.0001	
	8	2.5839	0.2390	0.0001	<a href="#">Back to Data Sheet</a>
	9	0.0001	0.0001	0.0001	
	10	2.7147	0.0005	0.0001	<a href="#">Back to Main Menu</a>
<b>WT<sub>xjk</sub></b>		Time that product $x$ has to wait to start a process $j$ on machine $k$			
		<b>ROUNDUP(NORMDIST(RAND(),1,1,FALSE),4)</b>			
		<i>Machines</i>			
		1	2	3	
<i>Process</i>	1	0.3887	0.3971	0.3283	
	2	0.2882	0.3637	0.2988	
	3	0.2629	0.3654	0.3730	
	4	0.2863	0.3647	0.3289	
	5	0.3957	0.3769	0.3903	
	6	0.3988	0.3844	0.2974	
	7	0.3984	0.3902	0.3112	
	8	0.3988	0.3920	0.3333	<a href="#">Back to Data Sheet</a>
	9	0.3253	0.3839	0.3831	
	10	0.3555	0.3746	0.3990	<a href="#">Back to Main Menu</a>

Figure A.3. Microsoft Excel model arrays data sheet

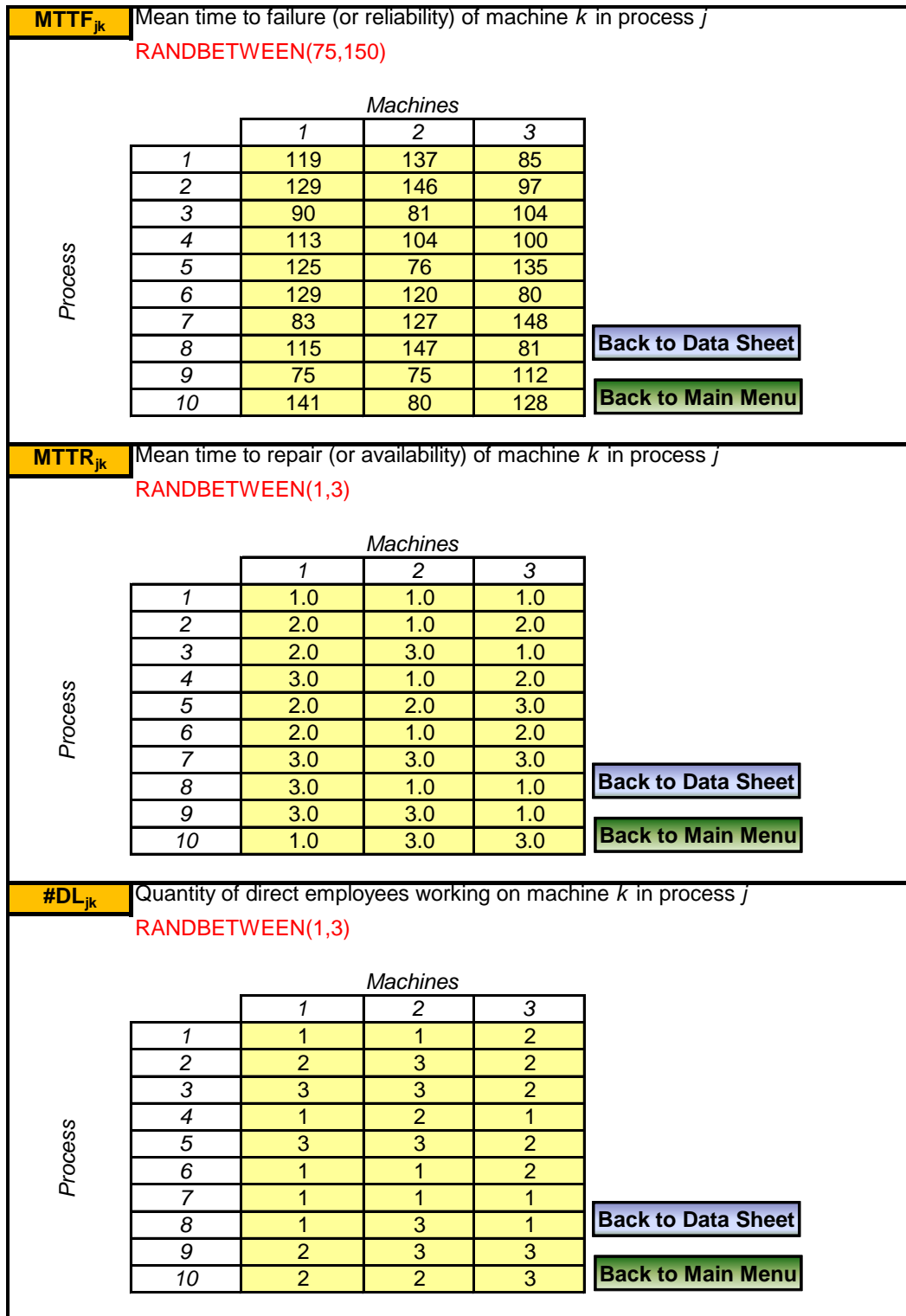


Figure A.3 Continued

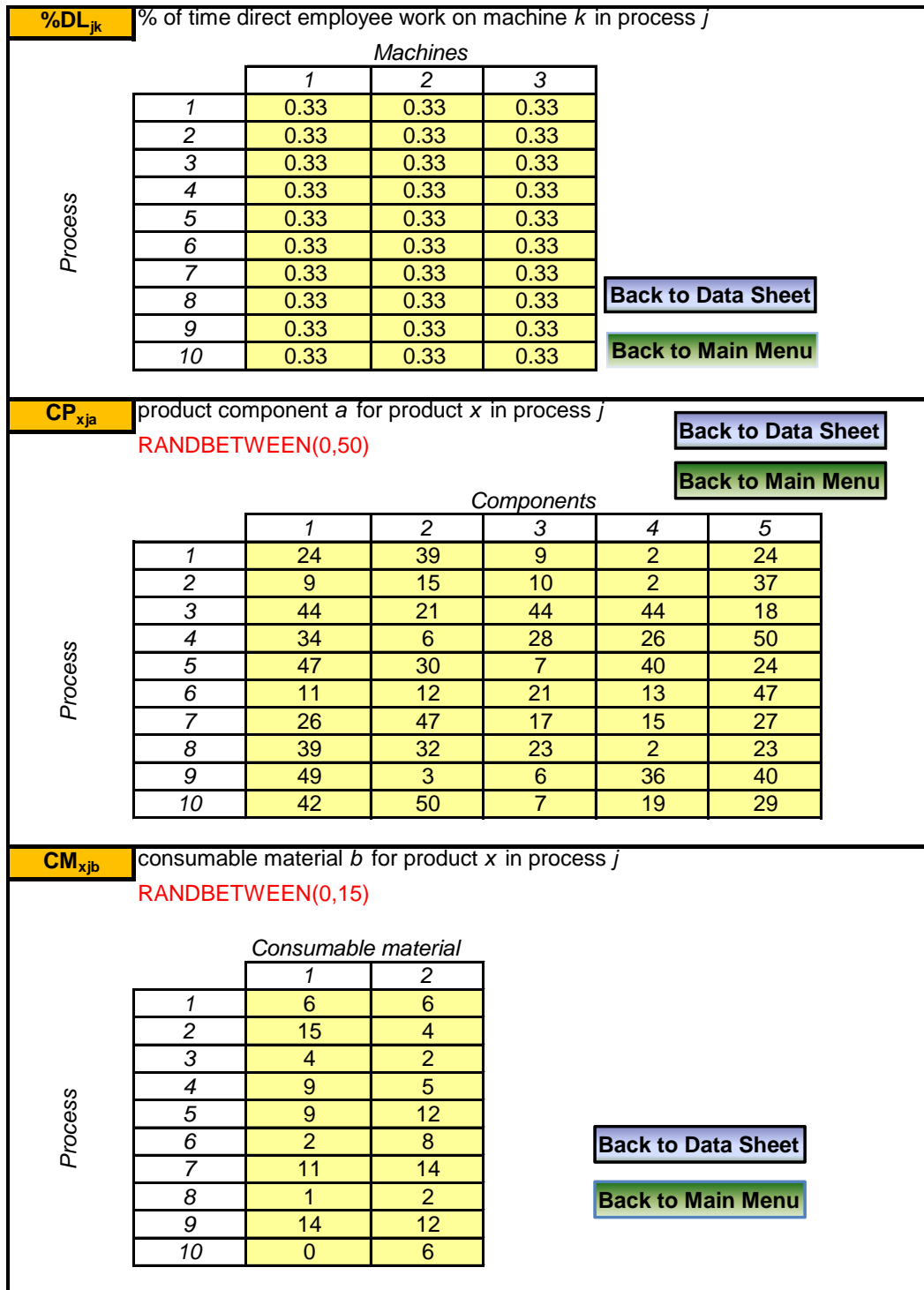


Figure A.3 Continued

<b>HT<sub>xj</sub></b>	Handling or moving time of product x from previous process to process j $\text{ROUNDUP}(\text{NORMDIST}(\text{RAND}(),0.5,0.5,\text{FALSE}),4)$																								
<i>Process</i>	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="text-align: center;">1</td><td style="text-align: center;">0.6379</td></tr> <tr><td style="text-align: center;">2</td><td style="text-align: center;">0.7979</td></tr> <tr><td style="text-align: center;">3</td><td style="text-align: center;">0.7563</td></tr> <tr><td style="text-align: center;">4</td><td style="text-align: center;">0.6853</td></tr> <tr><td style="text-align: center;">5</td><td style="text-align: center;">0.7920</td></tr> <tr><td style="text-align: center;">6</td><td style="text-align: center;">0.7446</td></tr> <tr><td style="text-align: center;">7</td><td style="text-align: center;">0.7976</td></tr> <tr><td style="text-align: center;">8</td><td style="text-align: center;">0.6255</td></tr> <tr><td style="text-align: center;">9</td><td style="text-align: center;">0.7961</td></tr> <tr><td style="text-align: center;">10</td><td style="text-align: center;">0.7918</td></tr> </table>	1	0.6379	2	0.7979	3	0.7563	4	0.6853	5	0.7920	6	0.7446	7	0.7976	8	0.6255	9	0.7961	10	0.7918				
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Figure A.3 Continued



Once all required data is entered, the user will choose the calculations and results worksheet (refer to Figure A.4) by going first to the main menu worksheet and selecting the option for calculations and results. On this worksheet the user is going to be able to review the estimate of the cost in detail for each one of the cost components mentioned above, and for the total cost as well. The user will also be able to see which machine was assigned at each assembly process which in a real manufacturing or assembly environment would mean the selected production sequence for the given product.

To calculate cost of product x										Back to Main Menu			Back to Data Sheet					
Process	Machine ID	Setup time (SU <sub>xjk</sub> )	Handling time (HT <sub>xj</sub> )	Waiting time (WT <sub>xjk</sub> )	Process time (PT <sub>xjk</sub> )	Cycle time (CT <sub>x</sub> )	Availability	Total time (TT <sub>x</sub> )	Labor cost (LC <sub>x</sub> )	Prod comp cost (CP <sub>x</sub> )	Cons. materials cost (CM <sub>x</sub> )	Material cost (MC <sub>x</sub> )	Support personnel cost (SP <sub>x</sub> )	Utilities consumption cost (UC <sub>x</sub> )	Machine maintenance cost (MM <sub>x</sub> )	Overhead cost (OH <sub>x</sub> )	Product cost (PC <sub>x</sub> )	Min (PC <sub>x</sub> ) (LC and OH)
Patterning (etching) (PE)	PE1	0.00049	0.6379	0.3738	0.0001	1.0123	97%	1.0480	\$ 0.0050	\$ 34.25	\$ 1.20	\$ 35.45	\$ 0.2882	\$ 0.0030	\$ 0.0059	\$ 0.30	\$ 35.75	\$ 35.54
	PE2	0.00083		0.3904	0.0028	0.3940	97%	0.4067	\$ 0.0308				\$ 0.1119	\$ 0.0840	\$ 0.0363	\$ 0.23	\$ 35.71	
	PE3	0.00088		0.2702	0.0001	0.2712	97%	0.2788	\$ 0.0028				\$ 0.0767	\$ 0.0030	\$ 0.0098	\$ 0.09	\$ 35.54	
Lamination (LM)	LM1	0.00088	0.7979	0.3987	2.4852	3.6827	98%	3.7407	\$ 14.0878	\$ 24.19	\$ 1.79	\$ 25.98	\$ 1.0288	\$ 74.5174	\$ 24.8608	\$ 100.41	\$ 140.47	\$ 26.10
	LM2	0.00083		0.3713	0.0001	0.3722	99%	0.3774	\$ 0.0053				\$ 0.1038	\$ 0.0030	\$ 0.0093	\$ 0.12	\$ 26.10	
	LM3	0.00076		0.3797	0.0001	0.3806	97%	0.3926	\$ 0.0049				\$ 0.1080	\$ 0.0030	\$ 0.0086	\$ 0.12	\$ 26.10	
Drilling (DR)	DR1	0.00066	0.7563	0.3959	2.7793	3.9322	97%	4.0550	\$ 7.8765	\$ 57.98	\$ 0.58	\$ 58.56	\$ 1.1153	\$ 83.3358	\$ 27.7996	\$ 112.25	\$ 178.69	\$ 58.65
	DR2	0.00073		0.2614	0.0001	0.2622	99%	0.2654	\$ 0.0024				\$ 0.0730	\$ 0.0030	\$ 0.0083	\$ 0.08	\$ 58.65	
	DR3	0.00078		0.3563	0.0001	0.3572	97%	0.3698	\$ 0.0075				\$ 0.1017	\$ 0.0030	\$ 0.0088	\$ 0.11	\$ 58.68	
Plating & coating (PC)	PC1	0.00081	0.6853	0.3099	0.0001	0.9961	98%	1.0184	\$ 0.0077	\$ 47.08	\$ 1.36	\$ 48.44	\$ 0.2801	\$ 0.0030	\$ 0.0091	\$ 0.29	\$ 48.74	\$ 48.56
	PC2	0.00084		0.3250	4.3957	4.7215	98%	4.8132	\$ 24.9137				\$ 1.3238	\$ 131.8027	\$ 43.9654	\$ 177.09	\$ 250.45	
	PC3	0.00059		0.3922	0.0001	0.3929	98%	0.4015	\$ 0.0019				\$ 0.1104	\$ 0.0030	\$ 0.0069	\$ 0.12	\$ 48.56	
Solder Resist (SR)	SR1	0.00081	0.7920	0.3986	1.5565	2.7479	98%	2.8040	\$ 13.2371	\$ 51.45	\$ 2.13	\$ 53.58	\$ 0.7712	\$ 46.6708	\$ 15.5731	\$ 63.02	\$ 129.83	\$ 53.70
	SR2	0.00073		0.3799	4.2466	4.6272	98%	4.7459	\$ 36.1023				\$ 1.3053	\$ 127.3320	\$ 42.4733	\$ 171.11	\$ 260.79	
	SR3	0.00088		0.3775	0.0001	0.3785	99%	0.3815	\$ 0.0028				\$ 0.1049	\$ 0.0030	\$ 0.0098	\$ 0.12	\$ 53.70	
Tin plating (TP)	TP1	0.00075	0.7446	0.3808	0.0001	1.1263	98%	1.1450	\$ 0.0072	\$ 34.21	\$ 1.06	\$ 35.27	\$ 0.3149	\$ 0.0030	\$ 0.0085	\$ 0.33	\$ 35.60	\$ 35.40
	TP2	0.00062		0.3983	0.0001	0.3990	98%	0.4058	\$ 0.0061				\$ 0.1116	\$ 0.0030	\$ 0.0072	\$ 0.12	\$ 35.40	
	TP3	0.00056		0.3709	2.6323	3.0038	98%	3.0744	\$ 7.4598				\$ 0.8456	\$ 78.9281	\$ 26.3286	\$ 106.10	\$ 148.83	
Screen printing (SP)	SP1	0.00089	0.7976	0.2505	0.2078	1.2568	99%	1.2664	\$ 1.7738	\$ 46.31	\$ 2.53	\$ 48.84	\$ 0.3483	\$ 6.2308	\$ 2.0869	\$ 8.67	\$ 59.28	\$ 48.95
	SP2	0.00072		0.3443	0.0001	0.3451	99%	0.3476	\$ 0.0070				\$ 0.0956	\$ 0.0030	\$ 0.0082	\$ 0.11	\$ 48.95	
	SP3	0.00058		0.3825	0.0004	0.3835	97%	0.3966	\$ 0.0028				\$ 0.1091	\$ 0.0120	\$ 0.0098	\$ 0.13	\$ 48.97	
Testing (TT)	TG1	0.00053	0.6255	0.3943	0.0010	1.0213	99%	1.0348	\$ 0.0087	\$ 40.28	\$ 0.31	\$ 40.59	\$ 0.2846	\$ 0.0300	\$ 0.0153	\$ 0.33	\$ 40.93	\$ 40.93
	TG2	0.00071		0.3964	4.2941	4.6912	98%	4.8029	\$ 36.5059				\$ 1.3210	\$ 128.7563	\$ 42.9481	\$ 173.03	\$ 250.12	
	TG3	0.00077		0.2512	4.2941	4.5461	98%	4.6543	\$ 12.1688				\$ 1.2801	\$ 128.7563	\$ 42.9487	\$ 172.99	\$ 225.74	
Populating (PO)	PO1	0.00084	0.7961	0.2872	0.0003	1.0844	98%	1.1078	\$ 0.0064	\$ 44.46	\$ 2.58	\$ 47.04	\$ 0.3047	\$ 0.0090	\$ 0.0114	\$ 0.33	\$ 47.37	\$ 47.37
	PO2	0.00049		0.3987	4.2047	4.6039	99%	4.6687	\$ 11.9147				\$ 1.2841	\$ 126.0756	\$ 42.0519	\$ 169.41	\$ 228.37	
	PO3	0.00052		0.3857	3.4099	3.7961	97%	3.9135	\$ 28.9886				\$ 1.0764	\$ 102.2440	\$ 34.1042	\$ 137.42	\$ 213.45	
Protection & packaging (PP)	PP1	0.00087	0.7918	0.2921	4.3219	5.4067	99%	5.4525	\$ 36.7435	\$ 51.63	\$ 0.66	\$ 52.29	\$ 1.4996	\$ 129.5898	\$ 43.2277	\$ 174.32	\$ 263.35	\$ 52.41
	PP2	0.00067		0.3990	0.0001	0.3998	98%	0.4089	\$ 0.0044				\$ 0.1125	\$ 0.0030	\$ 0.0077	\$ 0.12	\$ 52.42	
	PP3	0.00083		0.3761	0.0001	0.3770	97%	0.3868	\$ 0.0026				\$ 0.1064	\$ 0.0030	\$ 0.0093	\$ 0.12	\$ 52.41	
<b>\$ 447.62</b>																		

Figure A.4. Microsoft Excel model calculations and results worksheet

## APPENDIX B

### FILES FOR AMPL9

In this Appendix, the files used to solve the different mathematical models are included. The files required by AMPL9 with CPLEX90 are: the model (.mod) files, the data (.dat) files, and the script (.scs) files. This Appendix is divided for each integer linear program.

#### 1. The ILP Model

The model file used for the integer linear program is included next, followed by the data file, and the scripts used to include all the required commands needed to solve the ILP model. The script file shown is for the 10 products scenarios. If running the model with 100 or 1000 products, the script file needs the instruction “let lastprod := 10;” to be adjusted to reflect the corresponding number of products, 100 or 1000.

*Decision Support System - Mathematical Model (.mod file)*

```

set J;           # processes
set S;           # support personnel
set U;           # utilities
set A;           # product components
set B;           # consumable materials

# Parameters:

param first integer;      # first product/machine number
param lastprod >= first integer; # last product number
param lastmach >= first integer; # last machine number
set I := first..lastprod; # products
set K := first..lastmach; # machine types
param DLc >= 0.0;        # average direct labor employees salary per hour
param HRS >= 0.0;        # scheduled machine hours per week

```

```

param MMc >= 0.0;      # total machine maintenance cost per week
param SPc >= 0.0;      # average cost of support personnel per year
param WKS >= 0;        # scheduled weeks per year

```

# Parameters with random values generated in script file:

```

param Dyr {I};         # total annual demand for each product i
param HT {I,J};        # handling or moving time of product i from previous process to process j
param PT {I,J,K};      # process time of product i at process j and machine k
param SUb {I,J,K};     # machine setup time per batch for product i at process j and machine k
param WT {I,J,K};     # waiting time of product i at process j in machine k

```

# Parameters with random values:

```

param CM {i in I, j in J, b in B} := round(Uniform(0,15));
# consumable material b needed for product i in process j

```

```

param CMc {b in B} := Uniform(0.5,1.0);
# cost per unit of consumable material b

```

```

param CP {i in I, j in J, a in A} := round(Uniform(0,50));
# component a needed for product i in process j

```

```

param CPc {a in A} := Uniform(0.10,0.50);
# cost per unit of component a

```

```

param DLq {j in J} := round(Uniform(1,3));
# quantity of direct employees working in process j

```

```

param DLp {j in J} := Uniform(0.30,0.50);
# average percent of time direct employee work in process j

```

```

param MH {j in J, k in K} default 80;
# machine hours available per week for machine k in process j

```

```

param MTF {j in J, k in K} >= 0.0 := round(Uniform(75,150));
# mean-time-to-failure of machine k in process j

```

```

param MTTR {j in J, k in K} >= 0.0 := Uniform(0.50,1.50);
# mean-time-to-repair of machine k in process j

```

```

param SPp {j in J} := Uniform(0.05,0.15);
# average percent of time of support personnel in process j

```

```

param SPq {s in S} := round(Uniform(1,3));
#quantity of support personnel s

```

```

param Ub {i in I} := round(Uniform(500,1500));
# units per batch of product i

```

```

param UCc {u in U} := Uniform(0.05,0.10);
# cost of utility u per unit

```

```

param UCq {j in J, k in K, u in U} := Uniform(10,300);
# units of utility u consumed by machine k in process j

# Computed parameters:

# To calculate the machine setup time per unit for product i at process j and machine k
param SU {i in I, j in J, k in K} := SUB[i,j,k] / Ub[i];

# To calculate maximum machine utilization for product i at process j and machine k
param MUtil {i in I, j in J, k in K} := (PT[i,j,k] + SU[i,j,k]) * (Dyr[i]/WKS);

# To calculate average machine utilization per process for each product i and machine k
param AvgMUtil {j in J, k in K} := (sum {i in I} MUtil[i,j,k]) / card (K);

# To calculate average machine utilization per process
param TAvGMUtil {j in J} := (sum {k in K} AvgMUtil[j,k]) / card (K);

# To calculate the cycle time of product i at process j and machine k
param CT {i in I, j in J, k in K} := SU[i,j,k] + HT[i,j] + WT[i,j,k] + PT[i,j,k];

# To calculate the availability of machine k in process j
param AV {j in J, k in K} := MTTF[j,k] / (MTTF[j,k] + MTTR[j,k]);

# To calculate the total time of product i at process j and machine k
param TT {i in I, j in J, k in K} := CT[i,j,k] / AV[j,k];

# To calculate the machine setup cost for product i at process j and machine k
param SC {i in I, j in J, k in K} := SU[i,j,k] * DLq[j] * DLp[j] * DLc;

# To calculate the direct labor cost for product i at process j and machine k
param L {i in I, j in J, k in K} := PT[i,j,k] * DLq[j] * DLp[j] * DLc + SC[i,j,k];

# To calculate the materials cost for product i at process j and machine k
param M {i in I, j in J, k in K} := sum {a in A} (CP[i,j,a] * CPc[a]) + sum {b in B} (CM[i,j,b] * CMc[b]);

# To calculate the support personnel cost for process j
param SP {i in I, j in J, k in K} := sum {s in S} (SPq[s] * SPp[j]) * TT[i,j,k] * (SPc / (WKS * HRS));

# To calculate the utilities cost for product i at process j and machine k
param UC {i in I, j in J, k in K} := sum {u in U} (UCq[j,k,u] * UCc[u]) * PT[i,j,k];

# To calculate the machine maintenance cost for product i at process j and machine k
param MM {i in I, j in J, k in K} := MMc / HRS * (SU[i,j,k] + PT[i,j,k]);

# To calculate the overhead cost for product i at process j and machine k
param O {i in I, j in J, k in K} := SP[i,j,k] + UC[i,j,k] + MM[i,j,k];

# To calculate the cost of product i at process j and machine k
param C {i in I, j in J, k in K} := (L[i,j,k] + M[i,j,k] + O[i,j,k]) * (Dyr[i] / WKS) > 0.0;

# Decision variables:

var X {I,J,K} binary;          # to assign product i to process j and machine k

```

# Objective function - to minimize total cost of all products assembled

```
minimize Total_cost:
    sum {i in I, j in J, k in K} (C[i,j,k] * X[i,j,k]);
```

# Constraints:

# (1) Each product i can be assigned to each process j and machine k only once.

```
subject to AssignProduct {j in J, k in K}:
    sum {i in I} X[i,j,k] <= card(I);
```

# (2) Each product i is assigned to only one machine k at each process j.

```
subject to AssignMachine {i in I, j in J}:
    sum {k in K} X[i,j,k] = 1;
```

# (3) To make sure available machine hours for product i at process j and machine k are not exceeded.

```
subject to Capacity {j in J, k in K}:
    sum {i in I} (MUtil[i,j,k] * X[i,j,k]) <= MH[j,k];
```

*Decision Support System - Data file*

```
set J      :=      PE LM DR PC SR TP SP TT PO PP;      # processes
set S      :=      ENG QUAL OTHER ;                   # support personnel
set U      :=      WATER ELEC GAS ;                   # utilities
set A      :=      1 2 3 4 5 ;                         # product components
set B      :=      1 2 ;                               # consumable materials
```

# Parameters without subscripts:

```
param DLc := 8.50;
param HRS := 80;
param MMc := 10000;
param SPc := 60000;
param WKS := 50;
```

```
###      Script file for DSS.mod and DSS.dat
###
###      SCENARIO: PRODUCTS = 10, 100, 1000
###              MACHINES = 4, 7 & 10
###
###      Coefficient of Variation = 0.9 (MCV)
###
###      W/O Machine Workload Balance
```

```
option log_file 'DSS.tmp';
```

```

model DSS.mod;
data DSS.dat;

option presolve 0;
option solution_precision 4;
option solver cplexamp;

let first := 1;
let lastmach := 1;
let lastprod := 10;

for {1..3} {
    let lastmach := lastmach + 3;

    for {i in I} {
        if lastmach = 4 then let Dyr[i] := round(Uniform(100000,120000) / card(I));
        else
            if lastmach = 7 then let Dyr[i] := round(Uniform(200000,220000) / card(I));
            else
                let Dyr[i] := round(Uniform(300000,320000) / card(I));
            }
    }

    for {i in I, j in J} {
        repeat {
            let HT[i,j] := Normal(0.50,0.50);
            if HT[i,j] <= 0 then continue;
        } while HT[i,j] <= 0;
    }

    for {i in I, j in J, k in K} {
        repeat {
            let PT[i,j,k] := Normal(0.10,0.09);
            if PT[i,j,k] <= 0 then continue;
        } while PT[i,j,k] <= 0;
    }

    for {i in I, j in J, k in K} {
        repeat {
            let SUB[i,j,k] := Normal(0.50,0.70);
            if SUB[i,j,k] <= 0 then continue;
        } while SUB[i,j,k] <= 0;
    }

    for {i in I, j in J, k in K} {
        repeat {
            let WT[i,j,k] := Normal(1,1);
            if WT[i,j,k] <= 0 then continue;
        } while WT[i,j,k] <= 0;
    }

    option cplex_options 'bestbound timelimit 3600';
    solve;

```

```

display lastprod;
display lastmach;

display _ampl_elapsed_time;
display _total_solve_elapsed_time;
display Total_cost;

display {j in J} (sum {k in K, i in I} (MUtil[i,j,k] * X[i,j,k])) / card (K);

display sum {j in J} (((sum {k in K, i in I} (MUtil[i,j,k] * X[i,j,k])) / card (K))) / card (J);

display sum {j in J} (sum {k in K} (abs (sum {i in I} MUtil[i,j,k] * X[i,j,k] - TAvGMUtil[j]) /
TAvGMUtil[j]) / card (K)) / card (J);
}

```

## 2. The ILPx Model

The model and the data files used for the integer linear program with the expediting production option are the same as the ILP model with only a few differences. The script file to be used is the same, and as mentioned above, the number of products has to be changed depending on the amount of products assembled.

On the model file, the following parameters are added:

```

param EXPcost > 0.0; # cost of expediting production (in percentage)
param EXP {i in I} := round(Uniform01()); # to determine which products will be expedited

```

The direct labor cost per product is calculated in a different way considering expediting production. Therefore, on the ILP model file, the parameter used to calculate direct labor cost per product must be substituted by:

```

# To calculate the direct labor cost for product i at process j and machine k
param L {i in I, j in J, k in K} := if EXP[i] = 0 then PT[i,j,k] * DLq[j] * DLp[j] * DLc + SC

```

On the data file for the ILPx, a parameter value is added:

```

param EXPcost := 0.25;

```



### 3. The ILPb Model

The model and the data files used for the integer linear program with the expediting production option are the same as the ILP model with only two differences on the model file.

On the model file, the following parameter and constraint are added:

```
param delta >= 0.0;           # machine workload imbalance factor

# (4) To balance machine utilization based on a given imbalance factor.
subject to MachineUtilization {j in J, k in K}:
    AvgMUtil[j,k] * (1 - delta) <= sum {i in I} (MUtil[i,j,k] * X[i,j,k]) <= AvgMUtil[j,k] * (1 + delta);
```

The script file to be used is basically the same with the addition of the following “for loop” after the “for loop” of the machines. As mentioned above, the number of products has to be changed depending on the amount of products assembled.

```
let delta := 0.55;

for {1..2} {
let delta := delta - 0.15;
}.
```

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