OPTIMAL CONTROL OF PROJECTS BASED ON KALMAN FILTER APPROACH FOR TRACKING & FORECASTING THE PROJECT PERFORMANCE

A Thesis

by

SRIKANT BONDUGULA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2009

Major Subject: Civil Engineering

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Approved by:

Chair of Committee,	Kenneth F. Reinschmidt
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ABSTRACT

Optimal Control of Projects Based on Kalman Filter Approach for Tracking & Forecasting the Project Performance.

(May 2009)

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Chair of Advisory Committee: Dr. Kenneth F. Reinschmidt

Traditional scheduling tools like Gantt Charts and CPM while useful in planning and execution of complex construction projects with multiple interdependent activities haven't been of much help in implementing effective control systems for the same projects in case of deviation from their desired or assumed behavior. Further, in case of such deviations project managers in most cases make decisions which might be guided either by the prospects of short term gains or the intension of forcing the project to follow the original schedule or plan, inadvertently increasing the overall project cost.

Many deterministic project control methods have been proposed by various researchers for calculating optimal resource schedules considering the time-cost as well as the time-cost-quality trade-off analysis. But the need is for a project control system which optimizes the effort or cost required for controlling the project by incorporating the stochastic dynamic nature of the construction-production process. Further, such a system must include a method for updating and revising the beliefs or models used for representing the dynamics of the project using the actual progress data of the project.

This research develops such an optimal project control method using Kalman Filter forecasting method for updating and using the assumed project dynamics model for forecasting the Estimated Cost at Completion (EAC) and the Estimated Duration at Completion (EDAC) taking into account the inherent uncertainties in the project progress and progress measurements. The controller is then formulated for iteratively calculating the optimal resource allocation schedule that minimizes either the EAC or both the EAC and EDAC together using the evolutionary optimization algorithm Covariance Matrix Adaption Evolution Strategy (CMA-ES). The implementation of the developed framework is used with a hypothetical project and tested for its robustness in updating the assumed initial project dynamics model and yielding the optimal control policy considering some hypothetical cases of uncertainties in the project progress and progress measurements.

Based on the tests and demonstrations firstly it is concluded that a project dynamics model based on the project Gantt chart for spatial interdependencies of sub-tasks with triangular progress rates is a good representation of a typical construction project; and secondly, it is shown that the use of CMA-ES in conjunction with the Kalman Filter estimation and forecasting method provides a robust framework that can be implemented for any kind of complex construction process for yielding the optimal control policies.

DEDICATION

DEDICATED TO MY FAMILY AND FRIENDS

ACKNOWLEDGEMENTS

I take this opportunity to express my sincere thanks to my thesis advisor Dr. Kenneth F. Reinschmidt for his guidance, encouragement and support throughout the duration of my graduate studies and research. This thesis would not have been possible without his vision and direction. I would also like to thank members of my committee Dr. Daren B.H. Cline and Dr. Ivan Damnjanovic for their timely advice and feedback. Finally, I take this opportunity to thank my parents, for their continued support and encouragement throughout the course of my studies, and my friends, who made my stay at Texas A&M University a wonderful experience.

NOMENCLATURE

BAC	Budget at Completion
BCWS	Budgeted Cost of Work Scheduled
BCWP	Budgeted Cost of Work Performed
CMA-ES	Covariance Matrix Adaption Evolution Strategy
COV	Coefficient of Variation
DAC	Duration at Completion
EAC	Estimated Cost at Completion
EDAC	Estimated Duration at Completion
EKF	Extended Kalman Filter
UKF	Unscented Kalman Filter

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1. INTRODUCTION

1.1 Importance of the Research

It has been shown by Bellman (Kirk 2004) that in case of deviations the optimal system or project trajectory for the remaining duration is different from the original optimal trajectory, and hence needs to be recalculated after every such deviation from the expected trajectory. The optimal project control policy will be a resource allocation schedule that minimizes this overall cost of completion of the project over the remaining duration. Several dynamic project control methods have been proposed for achieving this objective yielding the optimal resource allocation schedule for the rest of the duration of the project by minimization of the cost to complete.

All these methods are based on the assumption that the cost at completion is deterministic in nature. But the need is for a project control system which optimizes the expected value of the effort or cost required for controlling the project by incorporating the stochastic dynamical nature of the construction-production process. Further, even after the observation of apparent delays and deviation in the construction process the managers still rely on the initially assumed project dynamics model for making future control decisions. Hence we need a method for effectively updating the assumed progress model and using it with the project control methods to derive the control decisions.

This thesis follows the style of *Journal of Computing in Civil Engineering.*

This research aims to develop such an optimal project control method using Kalman Filtering algorithm for updating the assumed project progress model and using it for forecasting the progress of the project taking into account the inherent uncertainties in the project progress and progress measurements. The optimal resource allocation schedule can then be calculated by optimizing the future progress estimates using a suitable optimization algorithm.

1.2 Problem Statement

Project control of a typical project is a recursive process involving a) measurement and monitoring of the actual progress performance, b) revision of the assumed progress models to reflect the actual progress performance, c) forecasting future progress performance based on the revised or updated progress model, and finally, d) identification, quantification and optimization of the project controls that will steer the project towards the desired performance. The basic structure of such a controller is shown in Figure 1.

Firstly, an appropriate project progress model needs to be selected to represent the behavior of the project by explaining all the observable progress measurements in relation to all the project controls or resources. Using feedback from the actual project progress any approximate system or project model can be refined using an appropriate filtering technique for filtering out the process and measurement noises, disturbances and uncertainties.

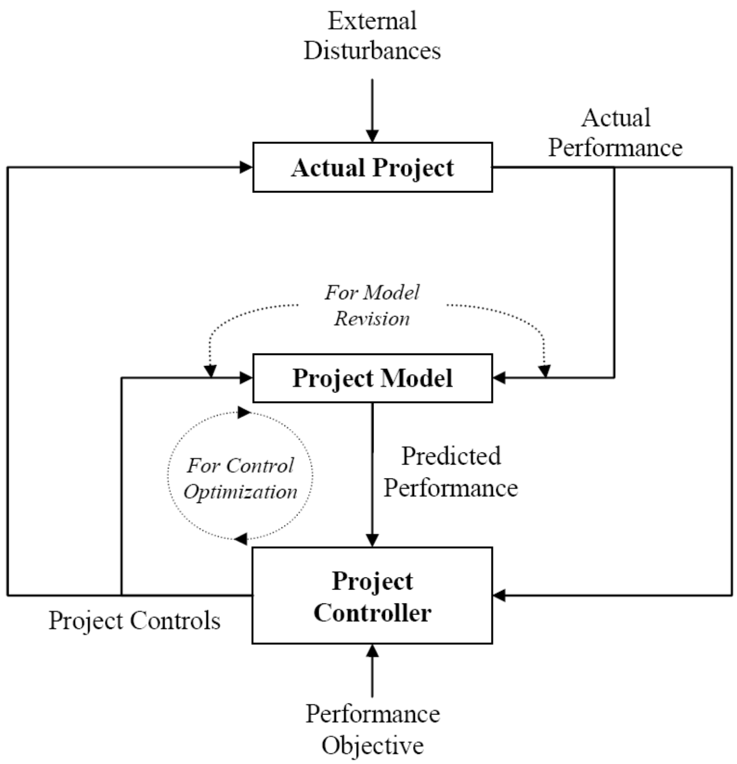


Figure 1 Basic structure of a project controller.

By recursively using the filtering technique the assumed model can be refined by updating the model parameters so that the outputs of the assumed model can be made to track the outputs of the actual project. The refined project model is then used for forecasting the future progress performance of the project. The purpose of a project controller is to determine the resource inputs for the project that produce the desired progress performance. The refined project model can be used for iteratively calculating these resource inputs.

For construction projects the progress performance measures are the Estimated Cost at Completion (EAC) and the Estimated Duration at Completion (EDAC). The desired project performance is a) conformance to the project schedule requirements, and

b) minimum possible EAC. It is not uncommon for construction projects to overshoot the project deadline. Hence the conformance to schedule requirements is forced by imposing a penalty for schedule overruns. The estimated cost at completion (EAC) along with the cost of schedule overrun together constitutes the overall cost at completion of the project. At the start of the project the project activity schedule and resource allocation schedule are optimized by minimizing this overall cost at completion. But more often than not the actual project performance doesn't conform to the originally planned performance and necessary control measures need to be implemented so that the predicted future performance will be within the acceptable limits of the desired performance. Usually the main control measure in such cases is the change in current resource allocation schedule in the form of a revised resource acquisition and de-acquisition schedule.

But considering the stochastic nature of actual construction projects, the need is for a project control system which optimizes the effort or cost required for controlling the project by incorporating the stochastic dynamical nature of the construction-production process. The main component of such a stochastic optimal controller (Goodwin and Sin 1984) is the objective based forecasting algorithm that can forecast or predict the EAC and EDAC of the project given a resource allocation schedule. An offline controller can be devised that can iteratively and adaptively vary the resource allocation schedule for optimizing these forecasts. Any of the evolutionary algorithms can be used in conjunction with a robust forecasting algorithm to devise such an optimal project controller.

1.3 Research Objectives

The main objective of this research is to develop an optimal project control method for application to construction projects taking into account the inherent uncertainties in the project progress and progress measurements. As mentioned in previous sections, this includes the recursive processes of progress measurement, revision of the progress models, forecasting future progress, and optimizing the necessary project controls.

These sub-objectives that need to be addressed for achieving the main research objective are summarized below:

1. *Investigation of Project Progress Models*: The first and foremost objective is to investigate the usability of the three kinds of project models – the polynomial progress model, the sigmoid or logistic progress model, and the comprehensive model based on baseline plan and individual triangular progress rates – with the tracking and filtering framework. The evaluation of the model usability will be based on the tracking and forecasting performance observed by comparing the actual project performance with the tracked and forecasted performance in conjunction with the tracking and forecasting errors.
2. *Development of Filtering, Tracking and Forecasting Framework*: The next step in achieving the research objective is to develop a robust tracking and forecasting

framework capable of handling multiple project states and project model parameters while efficiently incorporating the model, process and the measurement uncertainties. The tracking system should be able to track both the project states – the progress of individual tasks – as well as the project values – the budgeted cost of work performed (BCWP) and the actual cost of work performed (ACWP). To handle nonlinear project progress models the Extended Kalman Filter as well as the Unscented Kalman Filter will need to be implemented.

3. *Development of Optimal Project Controller*: The final objective is to implement a framework for iterative optimization of the future progress performance estimates using a suitable optimization algorithm to yield the optimal resource allocation schedule. The performance estimate in this case is the overall cost at completion of the project including the resource costs as well as the cost of schedule overrun.

2. BACKGROUND AND LITERATURE REVIEW

Depending on the theoretical nature and background the overall process of project control can be separated into three sub-processes: a) tracking the actual progress performance and revision of the assumed progress models to reflect the actual progress performance, b) forecasting future progress performance based on the revised or updated progress model, and finally, c) identification, quantification and optimization of the project controls that will steer the project towards the desired performance. The following sections discuss in brief the theoretical background and literature review for each of these aspects.

2.1 Project Progress Models

The main objective of any general project progress model is to represent the complete behavior of the project by explaining all the observable progress measurements in relation to all the project resources. These models representing the project transition and project outputs are usually complex nonlinear functions. Due to their complexity it might be cumbersome to use these models with the tracking, forecasting and optimization system. To overcome this problem simpler models are chosen to represent the general dynamics of the system.

The first and the simplest of the models is based on the assumption that at any discrete time k the rate of progress $\dot{x}(k)$ of a project can be represented using a higher order polynomial (2.1) of the present project state $x(k)$ with a, b and c as the model parameters. This assumption is justified by the fact that any continuous function can be approximated to arbitrary precision on a finite interval using a suitable polynomial.

$$\dot{x}(k) = a + b[x(k)] + c[x(k)]^2 + . \tag{2.1}$$

The second model (2.2) is based on the conjecture that the rate of progress is directly proportional to the cumulative work completed and the amount of work to-be-completed (Reinschmidt 2007) (Barraza et al. 2000) (Barraza et al. 2004). In these models a particular Sigmoidal, Logistic or Triangular rate function (Reinschmidt 2007) is used to represent the progress of the whole project.

$$\dot{x}(k) = a[100 - x(k)] + b[x(k)][100 - x(k)] \tag{2.2}$$

In the above mentioned generalized models it is assumed that the whole construction project can be represented as a single continuous process. But in reality the construction projects have multiple subtasks each with nonlinear spatial interdependencies. A typical representation of such a model is the project Gantt chart or the baseline project plan. In a typical Gantt chart all the spatial interdependencies between the sub-tasks are accurately modeled. Such models have been extensively used in related works for the purpose of optimization and control of projects (Moselhi and Hassanein 2003) (Lee and Jong Min 2006) (Eldin and Senouci 1994). But the Gantt chart lacks the information about the progress dynamics of individual tasks. The rate of progress of the tasks is assumed to be linear and constant throughout the duration of the task.

2.2 Tracking, Filtering and Estimation

Filtering is the process of estimating the true value of the model parameter by removing the disturbances taking into account the inherent system and measurement noise (Goodwin and Sin 1984). The criterion for filtering depends on the intended purpose of the model and this criterion influences the choice of the filtering method. The simplest filtering techniques involve Least Square Estimates (Zarchan and Musoff 2005) and Recursive Least Square Estimates (Zarchan and Musoff 2005) which can be used to quickly estimate the actual model parameters as well as the variance of the error in their estimation by minimizing the square of the deviation between the model output and actual system outputs for a certain extent of time. Comprehensive filtering techniques like Maximum Likelihood Estimation and Bayesian Filtering (Goodwin and Sin 1984) can be used to give the best description of the nature of the project progress as well as the inherent disturbances and noise along with the model parameters.

Kalman Filter (Zarchan and Musoff 2005) (Goodwin and Sin 1984) is another such filtering algorithm used for estimation of the true state of a dynamic system with process and measurement uncertainties. It is usually used for tracking, prediction and control of complex dynamic systems such as spacecraft, satellites or missiles (Zarchan and Musoff 2005) and econometric modeling (Harvey 1987) . Kalman Filter can be used with any kind of models with accurate or subjective information about the uncertainties in the model, process and the measurement. Kalman Filter is a recursive algorithm that can be efficiently used with a large number of system states and model parameters.

Further it is an optimal filtering technique minimizing the variance of error in estimates. Thus it is better suited for use with simple as well as complex project progress models where the aim is to estimate mean and error variance of the model parameters. Though the actual Kalman Filter was designed for linear dynamic systems, other extensions of the Kalman Filter such as the Extended Kalman Filter (Zarchan and Musoff 2005) (Goodwin and Sin 1984) and Unscented Kalman Filter (Julier et al. 1995) have been developed to deal with nonlinear continuous and nonlinear non-differentiable dynamic systems respectively.

Tracking involves the revision of the system or project model - specifically the model parameters - using feedback from the actual project progress. The process of parameter estimation is repeated whenever there is an update from the actual progress of the system. By recursively using the filtering technique the assumed model can be refined by updating the model parameters so that the outputs of the assumed model can be made to track the outputs of the actual project. The project model is then used for forecasting the future performance of the project, but only after it has been recursively updated by using all the available data about the actual progress of the project.

2.3 Forecasting Project Performance

Forecasting is the process of extrapolation of the present performance of the project or system into the future (Goodwin and Sin 1984). The reliability of the forecasts is dependent on the accuracy to which the project model is able to represent the actual project. Hence before using an approximate project dynamics model for extrapolation of the future forecasts, it has to be updated periodically by using a recursive filtering algorithm on all the available data about the actual progress of the project.

The forecast can be either description oriented or application oriented (Goodwin and Sin 1984) depending on the objective and context of the forecast. In descriptive forecasting the emphasis is on the general behavior of the system. Hence all the descriptive properties of the future system parameters are estimated by extrapolating the current properties of the same parameters. But, it is not always necessary to forecast the complete behavior of the system. In application oriented forecasting the emphasis is on estimating a particular prominent system parameter and the properties of the estimates of the other parameters are not important.

The forecast objective influences the choice of project model and the filtering-tracking algorithm. For description oriented forecasting comprehensive filtering techniques such as Maximum Likelihood Estimation and Bayesian Filtering will need to be used to give the best description of the behavior of the system along with the estimates of the system parameters. For objective oriented forecasting the much simpler filtering techniques such as Least Square Estimation, Recursive Least Square Estimation and Kalman Filtering will be sufficient.

Several forecasting methods and approaches have been proposed for predicting the estimated cost at completion and the estimated duration at completion. Most of these methods are based on the assumption that a) the estimates of the cost and duration at completion of the project can be modeled using a single continuous model for the entire project, and b) the model parameters can be estimated by analysis the past progress data (Barraza et al. 2004; Gardoni et al. 2007; Kim 2007; Teicholz 1993; Touran 1993).

2.4 Optimal Project Control

The purpose of a project controller is to determine the resource inputs for the project that produce the desired progress performance. For construction projects the progress performance measures are the estimated cost at completion (EAC) and the estimated duration at completion (EDAC). The desired project performance is a) conformance to the project schedule requirements, and b) minimum possible EAC. It is not uncommon for construction projects to overshoot the project deadline. Hence the conformance to schedule requirements is forced by imposing a penalty for schedule overruns. The estimated cost at completion (EAC) along with the cost of schedule overrun together constitutes the overall cost at completion of the project.

At the start of the project the project activity schedule and resource allocation schedule are optimized by minimizing this overall cost at completion. But more often than not the actual project performance doesn't conform to the originally planned performance and necessary control measures need to be implemented so that the predicted future performance will be within the acceptable limits of the desired performance. Usually the main control measure in such cases is the change in the current resource allocation schedule in the form of a revised resource acquisition and de-acquisition schedule.

It has been shown by Bellman (Kirk 2004) that in case of deviations the optimal system or project trajectory for the remaining duration is different from the original optimal trajectory, and hence needs to be recalculated after every such deviation from the expected trajectory. The optimal project control policy will be a resource allocation schedule that minimizes this overall cost of completion of the project over the remaining duration. Several dynamic project control methods have been proposed for achieving this objective (Handa and Barcia 1986) (Eldin and Senouci 1994) (Hegazy and Petzold 2003) (Lee and Jong Min 2006) yielding the optimal resource allocation schedule for the rest of the duration of the project by minimization of the cost to complete. All these methods are based on the assumption that the cost at completion is deterministic in nature. Further, in spite of apparent deviations and delays these project control methods use the initially assumed project progress models for deriving the optimal control policy leading to erroneous control.

But considering the stochastic nature of actual construction projects, the need is for a project control system which updates the assumed project progress model before using it for optimizing the expected value of the effort or cost required for controlling the project while incorporating the stochastic dynamical nature of the construction-production process. The main component of such a stochastic optimal controller (Goodwin and Sin 1984) is the objective based forecasting algorithm that can forecast or predict the EAC and EDAC of the project given a resource allocation schedule. An offline controller can be devised that can iteratively and adaptively varying the resource allocation schedule for optimizing these forecasts. Any of the evolutionary algorithms can be used in conjunction with a robust forecasting algorithm to devise such an optimal project controller (Hegazy and Petzold 2003; Zheng et al. 2004).

3. CONSTRUCTION AS A STOCHASTIC DYNAMIC PROCESS

The main objective of any general system model is to represent the complete behavior of a system by explaining all the observable outputs in relation to all the influencing factors. These models representing the project transition and project outputs are usually complex nonlinear functions. Due to their complexity it might be cumbersome to use these models with the tracking, forecasting and optimization system. To overcome this problem simpler models are chosen to represent the general dynamics of the system. If necessary these general dynamics of the system are further approximated. Further, even though the model used depicts the general dynamics of the system, the parameters within the model might not be deterministic (Maybeck 1979). The models of the dynamic systems are also influenced by many external disturbances which are nondeterministic or chaotic. The measurement of the outputs of simplified stochastic dynamic system model is equivalent to the random sampling of a stochastic process. Hence the measured system outputs don't show the whole response of the system.

Most of the construction processes with multiple spatially interdependent components involving nonlinear feedbacks need to be considered as stochastic dynamic systems. Like any typical stochastic dynamic system a construction project can be considered as a system with a particular state vector $x(t)$ indicating the state of the project at time t. The rate of change of state $\dot{x}(t)$ will be a function of the current state and applied controls $u(t)$ as represented using the continuous time equation (3.1), where

$v(t)$ represents the process noise, usually a zero mean Gaussian noise (Zarchan and Musoff 2005) attributed to simplification of the system mechanical models and model parameters (Goodwin and Sin 1984).

$$\dot{x}(t) = f[x(t), u(t), v(t), t] \tag{3.1}$$

For convenience and practicality it is appropriate to consider the construction process as a discrete dynamic system with each of the state and control variables taking discrete values and varying at discrete time intervals along the course of the project. By dividing the project duration into N time periods with each period of length Δt – usually the sampling or the measurement interval – this discrete dynamic system can be represented at any time period k using (3.2). The observation or measurements $z(k)$ from the projects are also functions of the state and control vectors as represented in (3.3) where $w(t)$ represents the measurement noise – usually zero mean Gaussian noise – attributed to either the actual error in measuring the outputs or the inherent error in the outputs of the system.

$$x(k+1) = f[x(k), u(k), v(k), k] \tag{3.2}$$

$$z(k) = h[x(k), u(k), k] + w(k) \tag{3.3}$$

3.1 Project Progress Models

For construction projects these models can either be comprehensive representing the complete dynamics of the project with provisions for all the degrees of freedom, or can be arbitrary with just the description of the basic and prominent dynamics of the project.

A complex project model might be successful in incorporating all the controllable inputs and subsequently accounting for all the measureable outputs provided the dynamics of the project are perfectly known. But the practical limitations of human visualization and intuition restrict the accuracy of estimation of the system dynamics. Due to this reason the actual response of the project might be much different from that of the project model. The actual progress trajectory of the project too often deviates from the planned or modeled progress trajectory. Hence even an approximate project dynamics model might be enough depending on the objective of the application using the model. But a comprehensive model will always provide better estimates than an approximated model. Hence a more complete model should be used where ever possible or necessary.

The current research studies the use of three different types of project models for use with the filtering, tracking and forecasting implementation. The first model (3.4) assumes that the progress of a project can be represented using a higher order polynomial. This assumption is justified by the fact that any nonlinear function can be approximated to arbitrary precision using a suitable polynomial. The second model (3.5) is based on the basic dynamics of the construction project that the rate of progress is directly proportional to the amount of work completed and the amount of work to be completed (Reinschmidt 2007). In these models a particular Sigmoid or Triangular rate function (Reinschmidt 2007) is used to represent the progress of the whole project. The third project model is more comprehensive based on the spatial activity interdependencies from the Gantt chart or the baseline plan as well as the nonlinear progress dynamics of each sub-activity.

$$\dot{x}(k) = a + b[x(k)] + c[x(k)]^2 \tag{3.4}$$

$$\dot{x}(k) = a[100 - x(k)] + b[x(k)][100 - x(k)] \tag{3.5}$$

In all these models it is assumed that in spite of the model specification error or lack of it, there is some inherent process noise as well as measurement noise that needs to be taken into consideration while tracking and forecasting the future performance. Hence the model parameters representing the project progress are also considered a part of the state vector so that any change in these parameters can also be modeled. The outputs $z(k)$ of the system can represent either the states of the subtasks or the budgeted and actual costs of the project. The choice of the output depends on the application in which the model is used. In the present study both types of output models are used.

3.2 Project Control Models

The rate of progress of the work increases with increase in the quantity of resources. But more often than not this increase in the rate of progress of work decreases with the increase in the quantity of resources. This effect is model using a *Rate of Progress Factor* $r_i(k)$ which influences the rate of progress $\dot{x}_i(k)$ of the task i at time k according to (3.6), $\dot{x}_{ni}(k)$ being the nominal rate of progress of the task i governed by (3.8). The diminishing increase of this Rate of Progress Factor $r_i(k)$ with the increase in quantity of resources is modeled using (3.7), where $u_{ij}(k)$ is the quantity of resource j assigned to task i at time k.

$$\dot{x}_i(k) = r_i(k)\dot{x}_{ni}(k) \tag{3.6}$$

$$r_i(k) = c_i\left[\frac{e^{u_{ii}(k)} - 1}{e^{u_{ii}(k)}}\right] \qquad c_i = 1.6 \quad \text{for } i = 1,2,3 \tag{3.7}$$

3.3 Test Project

For the purpose of demonstrating and testing the optimal control methodology, a simple construction project is assembled with three interdependent sub-tasks – Task 1, Task 2 and Task 3 – reflecting the general nature of actual construction projects.

3.3.1 The Test Project – Project Schedule

Any subsequent task is allowed to start only after 90% of the previous task has been complete. Further it is assumed that the nominal rate of progress of each of these tasks is variable following the general conjecture about project dynamics that the rate of progress of a task is proportional to the quantity of the task completed as well as the quantity to be completed. A Sigmoid or Triangular function can be used to represent this rate of progress. In this research the Triangular function is used to demonstrate the usability of the proposed method with non-linear and non-differentiable project models. The triangular functions for the rate of progress of the sub-tasks are as shown in (3.8), where for each Task i, $x_i(k)$ represents its state at time k in terms of the percentage of task complete, $\dot{x}_{ni}(k)$ the nominal rate of progress in terms of rate of change in percentage of task complete, and a_i and b_i the rate parameters mentioned in Table 1.

$$\dot{x}_{ni}(k) = \begin{cases} 0.001 & x_i(k) \le 0 \\ a_i\sqrt{x_i(k)} & 0 < x_i(k) \le 5 \\ b_i\sqrt{100 - x_i(k)} & x_i(k) \le 100 \end{cases} \tag{3.8}$$

Table 1 The model parameters for the rate of progress of sub-tasks in the test project.

Parameter	*Task 1*	*Task 2*	*Task 3*
a:	0.30	0.10	0.25
b:	0.30	0.10	0.25

The Gantt chart for this test project based on the progress model (3.8) with corresponding model parameters a and b from Table 1 is shown in Figure 2. The nominal rates of progress and the progress trajectories of the tasks - simulated discretely with intervals of 10 time units - are shown in Figure 3 and in Figure 4 respectively.

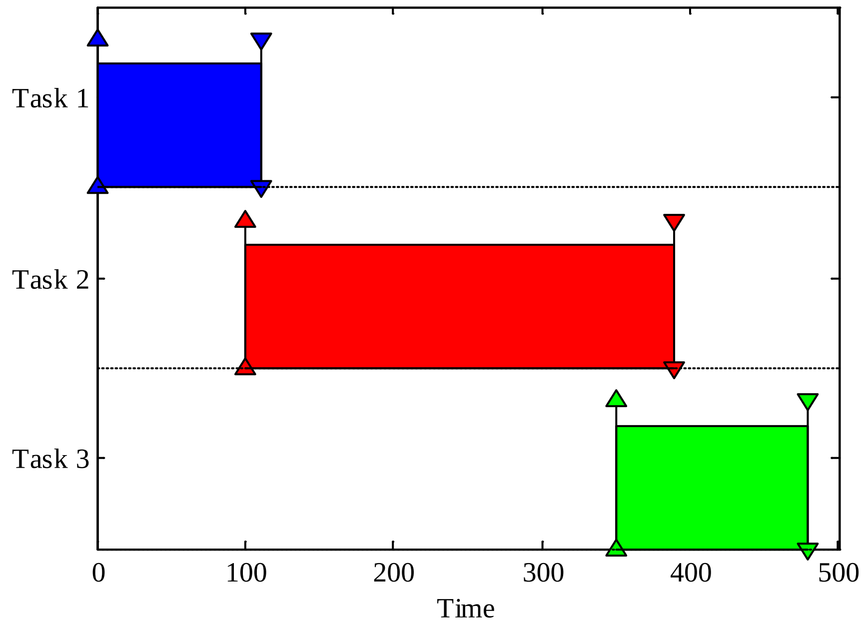


Figure 2 The Gantt chart for the test project.

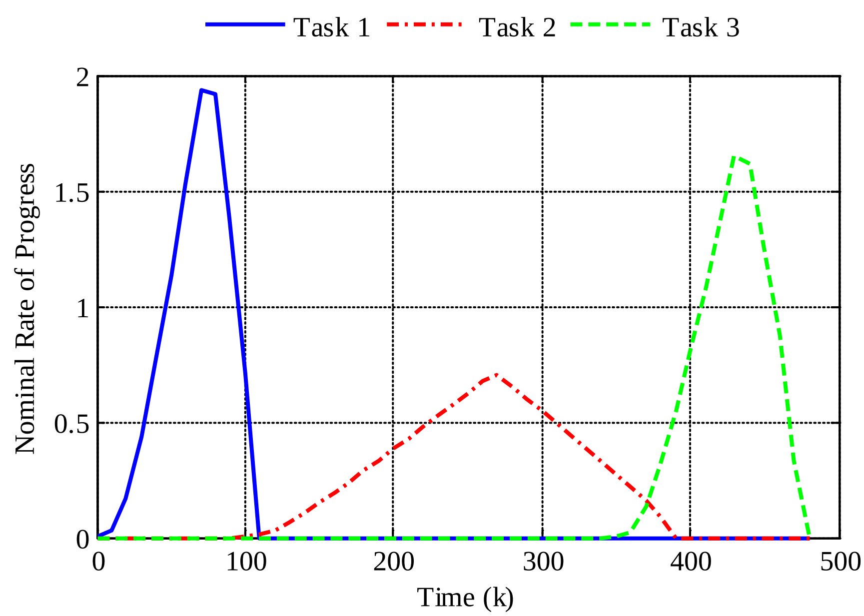


Figure 3 Triangular rates of progress of the sub-tasks in the test project.

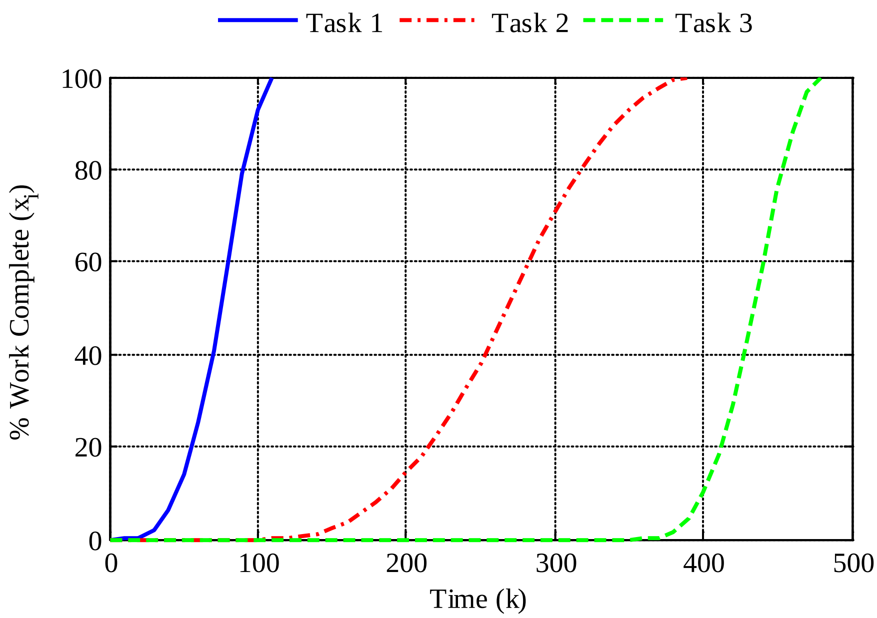


Figure 4 The nominal progress trajectories of sub-tasks in the test project.

3.3.2 Test Project – The Project Controls

For modeling the project controls it was assumed that each of the three sub-tasks of the test project needs one type of resource each for their progress. The assignment of the resources for each of the sub-tasks is as shown in Table 2.

Table 2 The resources required for progress of each of the sub-tasks in the test project.

	Task 1	Task 2	Task 3
Resource :	Resource 1	Resource 2	Resource 3

This relation between the quantity of resources and Rate of Progress Factor defined by (3.7) is depicted in Figure 5. This model has been specifically chosen so that the rate of progress Rate of Progress Factor r_i is equal to 1 for a unit quantity of resource for all the tasks at any point of time. In this numerical example the relation (3.7) is assumed to be of continuous nature for simplicity sake. But in actual projects it is discrete with the Rate of Progress factor discretely increasing with the continuous increase in the quantity of resources. Further, in reality the quantity of resources can only be increased in discrete quantities.

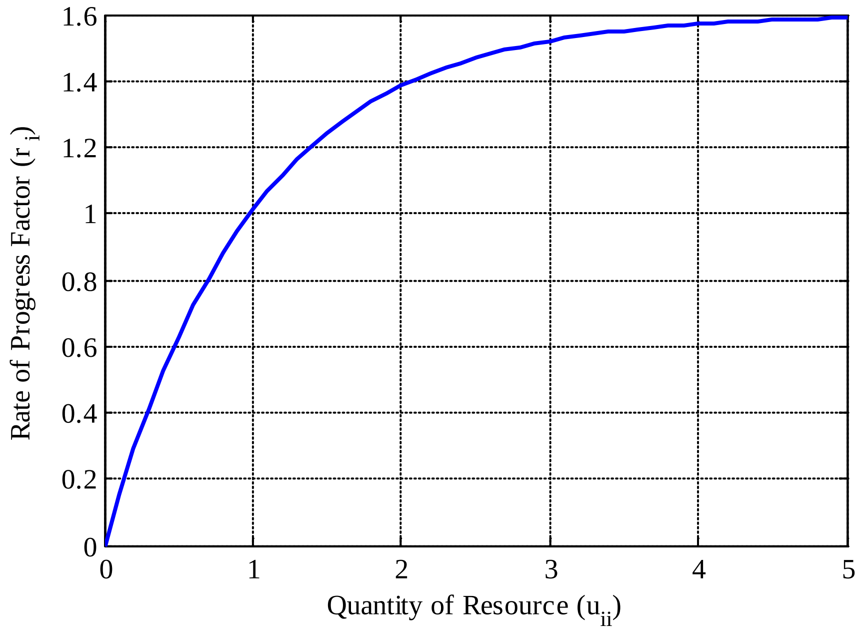


Figure 5 The rate of progress factor as a function of the quantity of resources.

For practicality and feasibility it would be appropriate to limit the project controls to the directly and independently controllable aspects of the project resources, such as the project man power or crew. In this research the project control is limited to the number of crews assigned to a particular sub-task at a particular time along the course of the project. The reasons for this choice are: a) for most of the construction activities the concerned crews are main operators and users of the other indirect resources such as material, tools & machinery and hence directly influence activity progress, b) acquisition and de-acquisition of crews is the most viable form of control vested with the construction managers, and c) data for the productivity and influence of construction crews on the rate of progress of activities is consistently documented for all projects and hence can be used for formulation and revision of the progress models like (3.7) & (3.6).

3.3.3 Test Project – The Project Cost

The overall cost of the project is divided into two parts a) the Resource Cost for including the cost of all the project controls and b) the Other Cost to include all the other costs during the course of the project. The Resource Cost is the cost of the project resources – the manpower and crew for all the sub-tasks. For this test project the unit costs of the three resources in terms of cost units per unit crew per time period are as shown in Table 3. The Resource Cost till a point of time T - $RC(T)$ - is given by (3.9), where $u_{ii}(k)$ is the quantity of resource i assigned to task i (as mentioned in the previous section, task i only uses resource i) at time k and rc_i the unit cost of the resource i as mentioned inTable 3.

$$RC(T) = \sum_{k=1}^{T} \sum_{i} u_{ii}(k).rc_i \tag{3.9}$$

Table 3 The unit costs of the direct resources for the sub-tasks in the test project.

	Resource 1	Resource 2	Resource 3
Unit Cost (rc_i) *:* */ crew / time period*	0.20	0.30	0.50

The Other Cost is defined to include all the remaining project costs such as a) the cost of materials for the sub-tasks, b) the cost of other indirect resources like tools and machinery, and c) the project overhead and other intangible costs. For simplicity it is assumed that these costs are all directly proportional to the quantity of work completed. This Other Cost till a point of time T - $OC(T)$ - is given by (3.10) where oc_i is the cumulative unit cost of the above mentioned indirect resources & costs in terms of cost units per unit progress of task i. The values of oc_i are mentioned in Table 4. At any time T along the course of the project, the Total Project Cost $C(T)$ is the sum of the Resource Cost $RC(T)$ and Other Cost $OC(T)$.

$$OC(T) = \sum_i x_i(T).oc_i \tag{3.10}$$

Table 4 The unit costs of the indirect resources for the sub-tasks in the test project.

	Task 1	*Task 2*	*Task 3*
Unit Cost (oc_i) *:* */ unit progress*	1.00	4.00	7.00

3.3.4 Test Project – The Original Project Plan

An original baseline project plan was created with the intent of forcing all the sub-tasks of the project to follow their nominal progress trajectories as represented by the progress model (3.8) and progress parameters in Table 1. With such a baseline project schedule the rates of progress and progress trajectories for the sub-tasks will be as shown in Figure 3 and Figure 4 respectively. The resources are assigned so that the progress rates $\dot{x}_i(k)$ of the tasks will be same as nominal progress rates $\dot{x}_{ni}(k)$. Using (3.6) it can be observed that such a project progress can be achieved when the Rate of Progress Factor $r_i(k)$ is equal to 1 for all the sub-tasks at every time k. As mentioned earlier the project control model has been specifically chosen so that the rate of progress Rate of Progress Factor r_i is equal to 1 for a unit quantity of resource for all the tasks at any point of time. Hence the resource allocation schedule for the baseline project plan will be as shown in Figure 6 with a uniform value of 1 for the corresponding resource during the corresponding periods along the course of the project. The planned start and finish times for the sub-task based the planned schedule are as mentioned in Table 5.

Table 5 The planned start and finish times for the sub-task of test project.

	Task 1	*Task 2*	*Task 3*
Start :	0	100	350
Finish :	110	390	480

According to this schedule - calculated at discrete times with a time period of 10 units - the Duration at Completion (DAC) is 480. Using the project cost equations (3.9) and (3.10) along with the unit costs in Table 3 and Table 4 the planned project cost is calculated along the course of the project. This overall planned project cost represents the Budgeted Cost of Work Scheduled (BCWS). The BCWS trajectory for this test project is a shown in Figure 7. The overall project cost at the end of the project - the Budget at Completion (BAC) – based on this schedule is 1691 cost units. The Other Cost $OC(T)$ can be used to represent the cumulative overall progress of the project. The planned trajectory of $OC(T)$ for the test project is as shown in Figure 8. The BAC in this case is 1201. The process is complete when BCWS reaches BAC.

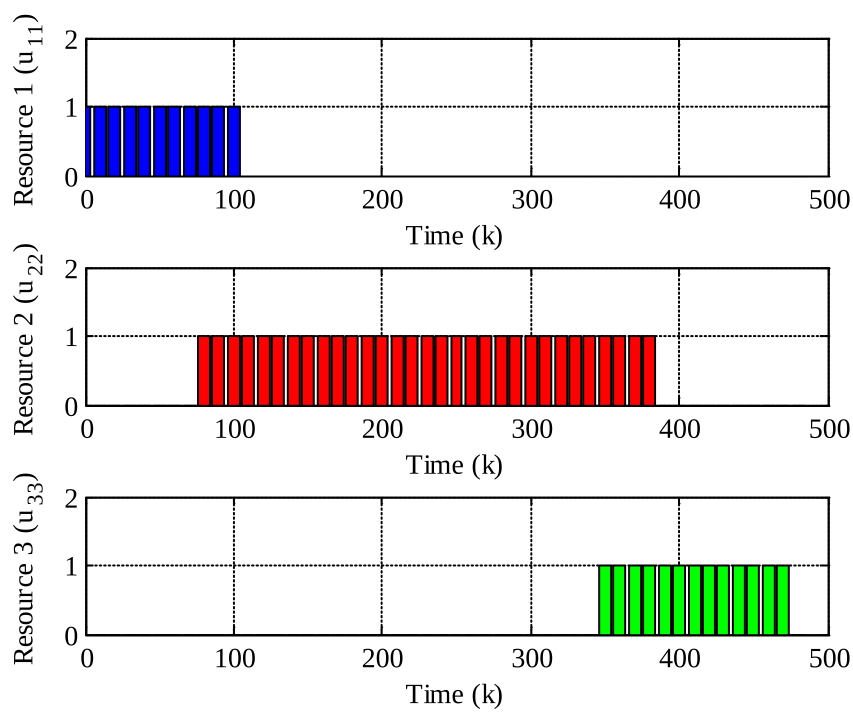


Figure 6 The planned resource allocation schedule for the test project.

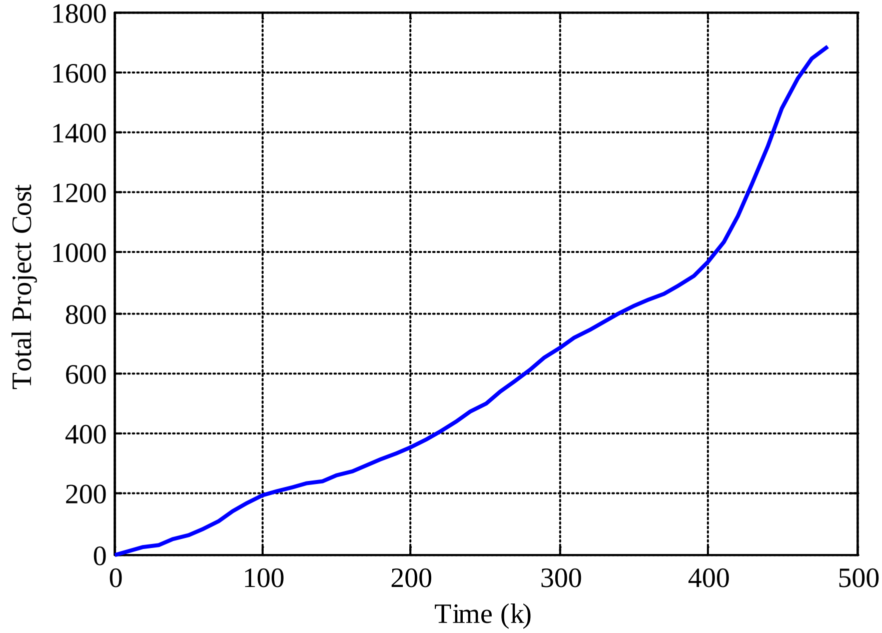


Figure 7 The trajectory of the overall planned project cost - BCWS - for the test project.

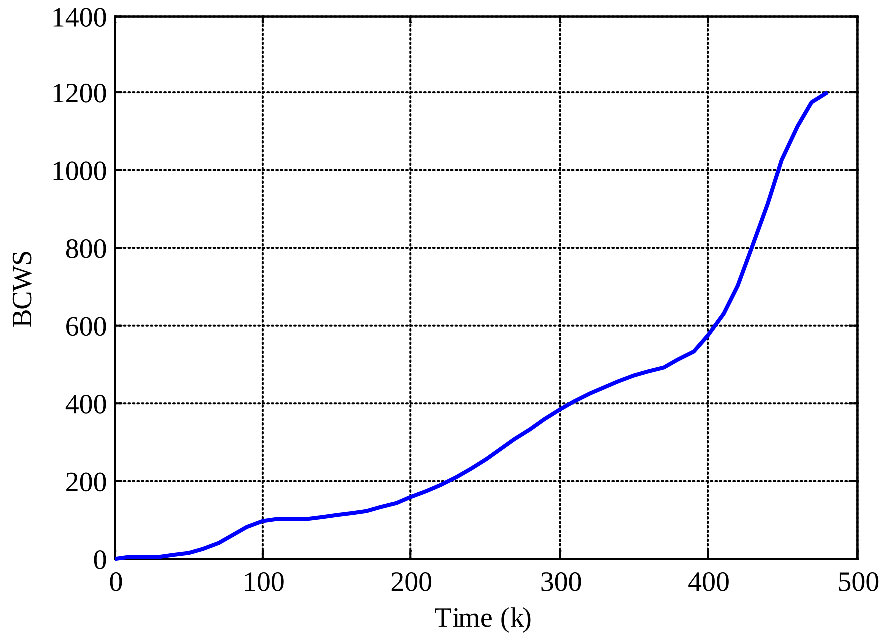


Figure 8 The trajectory of the indirect project cost representing the cumulative overall progress of the project.

4. TRACKING AND FORECASTING PROJECT PERFORMANCE

Every project model (4.1) in itself is an approximation, some more accurate than others. Further the parameters within the model might not be deterministic. To account for the stochastic nature of the project while using simplified models the following assumptions are made.

- The model states $x(k)$ at any instant of time are Gaussian.
- The model contains some process noise $v(k)$ - usually zero mean Gaussian noise (Zarchan and Musoff 2005) attributed to simplification of the project dynamics model and model parameters (Goodwin and Sin 1984).
- The model contains some measurement noise $w(k)$ - zero mean Gaussian noise, attributed to either the actual error in measuring the outputs or the inherent stochastic nature of the outputs of the project.

$$x(k+1) = f[x(k), u(k), v(k), k] \tag{4.1}$$

$$z(k) = h[x(k), u(k), k] + w(k) \tag{4.2}$$

The reasons for choosing a Gaussian distribution for all the states, disturbances and noises are twofold (Julier et al. 2000). Firstly, the mean and covariance are the distribution parameters of interest. Secondly, given the mean and covariance, a Gaussian distribution is the least informative and can represent the maximum possible amount of randomness. To find the actual response of the system the process and measurement noise needs to be filtered. Filtering is the process of extracting the actual system parameter estimates by removing the disturbances taking into account the inherent system and measurement noise (Goodwin and Sin 1984). The present study uses the optimal filtering technique known as Kalman Filtering (Zarchan and Musoff 2005) to estimate the system parameters. A Kalman filter is a recursive optimal estimation algorithm. This filtering technique is optimal in the sense that it minimizes the variance of error in estimates of the system parameters (Zarchan and Musoff 2005).

4.1 Tracking, Filtering and Estimation Using Kalman Filter

A typical Kalman Filter is based on a predictor-corrector structure (Julier et al. 2000). The first step involves the propagation of the mean $x(k)$ and error covariance $P_{xx}(k)$ of the present state of the project through the project model (3.2) to predict the distribution parameters – estimate of mean state $\hat{x}(k+1|k)$, estimate of covariance of error in state $\hat{P}_{xx}(k+1|k)$, estimate of mean measurement $\hat{z}(k+1|k)$, estimate of covariance of the error in measurement $\hat{P}_{zz}(k+1|k)$ and estimate of cross-covariance of the error in state and measurement $\hat{P}_{xz}(k+1|k)$ – at the future time $k+1$. These predicted estimates at time $k+1$ are then corrected or updated using the data – usually the measurement $z(k+1)$ – from the actual progress of the project. The updating is based on the linear update rule (4.3).

$$\begin{aligned} \hat{x}(k+1|k+1) &= \hat{x}(k+1|k) + K(k+1)v(k+1) \\ P_{xx}(k+1|k+1) &= \hat{P}_{xx}(k+1|k) - K(k+1)P_{vv}(k+1|k)K^T(k+1) \end{aligned} \tag{4.3}$$

where $v(k+1)$ is the residual or error in output estimation, $P_{vv}(k+1|k)$ is the covariance of the output estimation error given the covariance of the measurement error $R(k+1)$, and $K(k+1)$ the Kalman Gain, all calculated using (4.4).

$$\begin{aligned} v(k+1) &= z(k+1) - \hat{z}(k+1|k) \\ P_{vv}(k+1|k) &= \hat{P}_{zz}(k+1|k) + R(k+1) \\ K(k+1) &= \hat{P}_{xz}(k+1|k) P_{vv}^{-1}(k+1|k) \end{aligned} \tag{4.4}$$

Though the actual Kalman Filter was designed for linear dynamic systems, other extensions of the Kalman Filter like the Extended Kalman Filter (Zarchan and Musoff 2005) (Goodwin and Sin 1984) and Unscented Kalman Filter (Julier et al. 1995) have been developed to deal with nonlinear continuous differentiable and nonlinear discontinuous non-differentiable dynamic systems respectively. The main difference in these two variants of Kalman Filter is the method of prediction of the distribution parameters – $\hat{x}(k+1|k)$, $\hat{P}_{xx}(k+1|k)$, $\hat{z}(k+1|k)$, $\hat{P}_{zz}(k+1|k)$ and $\hat{P}_{xz}(k+1|k)$ – of the future states and outputs of the project.

This process of parameter estimation by using Kalman Filters is repeated whenever there is an update in the actual progress of the system. By recursively using the filtering technique the assumed model can be refined by updating the model parameters so that the outputs of the assumed model can be made to track the outputs of the actual project. The tracking ability of the project model can be used as a main criterion for deciding its usability. The following sections describe in detail the implementations of the Extend and Unscented Kalman Filters along with their performance, limitations and implications when used with the different project models for tracking and forecasting the progress of the above mentioned project. For testing the performance of the first two models for tracking and forecasting it is assumed that the control vectors are redundant and have no effect on the dynamics of the system. The tasks progress at their nominal rates of progress.

4.1.1 The Extended Kalman Filter

The Extended Kalman Filter (EKF) uses the fact that the error in prediction of states and outputs of the project can be approximated by using a Taylor Series expansion (Zarchan and Musoff 2005) (Goodwin and Sin 1984) of the project model (4.1) and (4.2). The truncation of this expansion to the first order then yields a linear approximation for propagation of the project states. The linear approximation to the error in prediction of future state - $\tilde{x}(k+1|k)$ - obtained by expanding (4.1) about the present error $\tilde{x}(k|k)$ and truncating it to the first term is (4.5)

$$\tilde{x}(k+1|k) \simeq \nabla f_x \tilde{x}(k|k) + \nabla f_v v(k) \tag{4.5}$$

where ∇f_x is the Jacobian of (4.1) with respect to $x(k)$, and ∇f_v is the Jacobian of (4.1) with respect to $v(k)$. The predicted mean and error-covariance of the future project states are then calculated using (4.6) and (4.7) respectively (Julier et al. 2000).

$$\hat{x}(k+1|k) = f[\hat{x}(k), u(k)] + \tilde{x}(k+1|k) \tag{4.6}$$

$$\hat{P}_{xx}(k+1|k) = \nabla f_x P_{xx}(k) \nabla^T f_x + \nabla f_v Q(k+1) \nabla^T f_v \tag{4.7}$$

Here Q represents the covariance matrix of the process noise representing the model specification error form unaccounted factors. The mean and error-covariance of the future project outputs, and the cross covariance between the project states and outputs are calculated using a similar linear approximation (4.8).

$$\begin{aligned}
&\hat{z}(k+1|k) = h[\hat{x}(k+1|k), u(k+1)] \\
&\hat{P}_{zz}(k+1|k) = \nabla h_x \hat{P}_{xx}(k+1|k)\nabla^T h_x \\
&\hat{P}_{xz}(k+1|k) = \hat{P}_{xx}(k+1|k)\nabla^T h_x
\end{aligned} \tag{4.8}$$

Now that the distribution parameters – $\hat{x}(k+1|k)$, $\hat{P}_{xx}(k+1|k)$, $\hat{z}(k+1|k)$, $\hat{P}_{zz}(k+1|k)$ and $\hat{P}_{xz}(k+1|k)$ – of the future states and outputs of the project at time $k+1$ are predicted, the linear update equations (4.4) and (4.3) can be used to correct the estimates $\hat{x}(k+1|k+1)$ and $\hat{P}_{xx}(k+1|k+1)$ using the actual system output $z(k+1)$.

The Extended Kalman Filter can be used with any linear or nonlinear project model provided it is continuous and differentiable. In this study the EKF is used for recursive tracking and forecasting of the project states using two different families of general project models (a) a model with no information about the nature of the project; based on a second order polynomial rate equation, and (b) a model with the basic information about the dynamics of the project; based on a sigmoid rate equation.

4.1.2 The Unscented Kalman Filter

The first step of a Kalman Filter involves the propagation of the mean $x(k)$ and covariance $P_{xx}(k)$ of the present state of the project through the project model to predict the distribution parameters – $\hat{x}(k+1|k)$, $\hat{P}_{xx}(k+1|k)$, $\hat{z}(k+1|k)$, $\hat{P}_{zz}(k+1|k)$ and $\hat{P}_{xz}(k+1|k)$ – of the future states and outputs of the project. These predicted estimates at time $k+1$ are then corrected or updated using the data – usually the measurement $z(k+1)$ – from the actual progress of the project.

The Extended Kalman Filter (EKF) uses the fact that the error in prediction of states and outputs of the project can be approximated by using a Taylor Series expansion of the project model. The truncation of this expansion to the first order then yields a linear approximation for propagation of the distribution parameters of the project states using the Jacobian of the project model equations. But this approximation is valid only for differentiable functions. Further, the truncation of higher order terms in the expansion may not always be appropriate and can introduce significant errors.

As discussed in the previous section, a more complete and reliable model for the project usually involves the use of non-differentiable functions. And sometimes the model might be a black-box model with no information about the structure of the progress dynamics function. In such cases the EKF approach cannot be used for predicting the distribution parameters – $\hat{x}(k+1|k)$, $\hat{P}_{xx}(k+1|k)$, $\hat{z}(k+1|k)$, $\hat{P}_{zz}(k+1|k)$ and $\hat{P}_{xz}(k+1|k)$ – of the future states and outputs of the project. To overcome this problem a new approach for propagation of the means and covariances has been proposed by Julier and Uhlmann (1996). This approach - known as the Unscented Transformation (discussed in Section 4.1.3) – has been shown to work successfully for any kind of nonlinear transformation of Gaussian random numbers (Julier et al. 2000) (Lefebvre et al. 2002).

4.1.3 Unscented Transformations of Means and Covariances

In this method the approximations of Gaussian probability distributions are transformed using the process and measurement equations of the project model. These transformed approximations are then used to calculate the moments of the transformed variables. This method is based on statistical linear regression of probability distributions (Lefebvre et al. 2002). If *n* is the length of the state vector *2n+1* regression points X_i are chosen for approximating the distribution with weights W_i

$$\begin{aligned} X_i &= \hat{x} + \left(\sqrt{(n+k)P_{xx}}\right)_i & W_i &= 1/2(n+k) \\ X_{n+1} &= \hat{x} & W_i &= k/(n+k) \\ X_{i+n+1} &= \hat{x} - \left(\sqrt{(n+k)P_{xx}}\right)_i & W_i &= 1/2(n+k) \end{aligned} \tag{4.9}$$

where $\left(\sqrt{(n+k)P_{xx}}\right)_i$ is the *i*th column of $\sqrt{(n+k)P_{xx}}$, and k is the degree of freedom in the choice of the regression points and is taken as 3 in the present study. These regression points are then transformed using the process equation of the model to later calculate the moments of the transformed distribution.

$$\begin{aligned} X_i(k+1|k) &= f[X_i(k), u(k), k] \\ \hat{x}(k+1|k) &= \sum_{i=1}^{2n+1} W_i X_i(k+1|k) \\ P_{xx}(k+1|k) &= \sum_{i=1}^{2n+1} W_i [X_i(k+1|k) - \hat{x}(k+1|k)].[X_i(k+1|k) - \hat{x}(k+1|k)]^T \end{aligned} \tag{4.10}$$

The same regression points are passed through the measurement equation to find the moments of the outputs and the cross covariance between the states and outputs.

$$
\begin{aligned}
Z_i(k+1|k) &= h[X_i(k+1|k), u(k+1), k+1] \\
\hat{z}(k+1|k) &= \sum_{i=1}^{2n+1} W_i Z_i(k+1|k) \\
P_{zz}(k+1|k) &= \sum_{i=1}^{2n+1} W_i [Z_i(k+1|k) - \hat{z}(k+1|k)].[Z_i(k+1|k) - \hat{z}(k+1|k)]^T \\
P_{xz}(k+1|k) &= \sum_{i=1}^{2n+1} W_i [X_i(k+1|k) - \hat{x}(k+1|k)].[Z_i(k+1|k) - \hat{z}(k+1|k)]^T
\end{aligned}
\tag{4.11}
$$

Once these distribution parameters have been estimated using the model equation for predicting the future states and outputs, the original Kalman corrector equations (4.3) can be used to update the estimates. The Kalman Filter using this Unscented Transformation approach is known as the Unscented Kalman Filter (UKF) (Julier and Uhlmann 1997).

4.2 Forecasting Future Project Performance

Forecasting is the process of extrapolation of the present performance of the project or system into the future (Goodwin and Sin 1984). The reliability of the forecasts is dependent on the accuracy to which the project model is able to represent the actual project dynamics. Depending on the nature of the project model either of the above mentioned filtering methods can be used to revise the assumed model by tracking the actual progress of the project. The revised model can then used to calculate the estimates of interest at a specified future date. In the present study where the objective is to develop an optimal project controller, the main estimates of interest are the expected value of Cost Estimate at Completion (EAC) and Estimated Duration at Completion (EDAC) along with the Variances of their estimation errors.

For differentiable functions these means and variances at a future time can be estimated by using the linear approximation approach used for the Extended Kalman Filter as shown in section 4.1.1. For non-differentiable and other black box functions this can be achieved by using the unscented approach discussed in section 4.1.3.

4.3 Application and Discussions

In this study first the EKF is used for recursively tracking and forecasting of the project states using two different families of general project models: a) a model with no information about the nature of the project; based on a second order polynomial rate equation, and b) a model with the basic information about the dynamics of the project; based on a sigmoid rate equation. Next the UKF is used with a more comprehensive model with basic information about the dynamics of each subtask along with the spatial interdependencies between the various tasks of the project. Each of the models is used with the appropriate tracking and filtering method to revise the model parameters. After tracking the available project progress data, the updated model is used to extrapolate the future behavior or trajectory of the project which is then compared with the actual project progress data.

4.3.1 Using the Polynomial Project Progress Model

When there is no information about the general dynamics of the system it can be assumed that a general polynomial model can be used to represent the rate of progress of the project in terms of its BCWP. The rate of change of BCWP is represented by a second order polynomial (4.12) with three model parameters a, b and c.

$$\frac{d}{dt}BCWP(t) = a + b[BCWP(t)] + c[BCWP(t)]^2 \tag{4.12}$$

To account for any uncertainty in this project dynamics model it is assumed that these model parameters themselves are variable states of the project model and need to be updated to reflect the behavior of the actual project by tracking the actual BCWP trajectory. The project states and outputs for this system at any discrete time k are

$$x(k) = \begin{bmatrix} BCWP(k) \\ a(k) \\ b(k) \\ c(k) \end{bmatrix} \qquad z(k) = BCWP(k) \tag{4.13}$$

The initial values of the state vectors were all set to 0 assuming there is no information about the initial conditions. The initial variance of BCWP was set to 0. It was assumed that the output measurement has a variance of 2. With this project dynamics model the BCWP of the test project was tracked over the whole duration of the project using the Extended Kalman Filtering algorithm equations (4.5) - (4.8) with a sampling time of 5. The project is deemed complete when the BCWP is equal to the BAC derived from the project plan. The point in time where the BCWP reaches BAC is the Estimated Duration at Completion (EDAC) of the project. To observe the influence of the process noise five different trials were conducted using different values for the process noise, starting with a value of 0. The tracking performance of this model for five different values of process noise are as shown in Figure 9 to Figure 13. It was observed that the tracking performance is best when the process noise is at least 10^{-7}. The tracking error and its theoretical 95% $(\sim 2\sigma)$ estimation bounds for a process noise value of 10^{-7} are as shown in Figure 14.

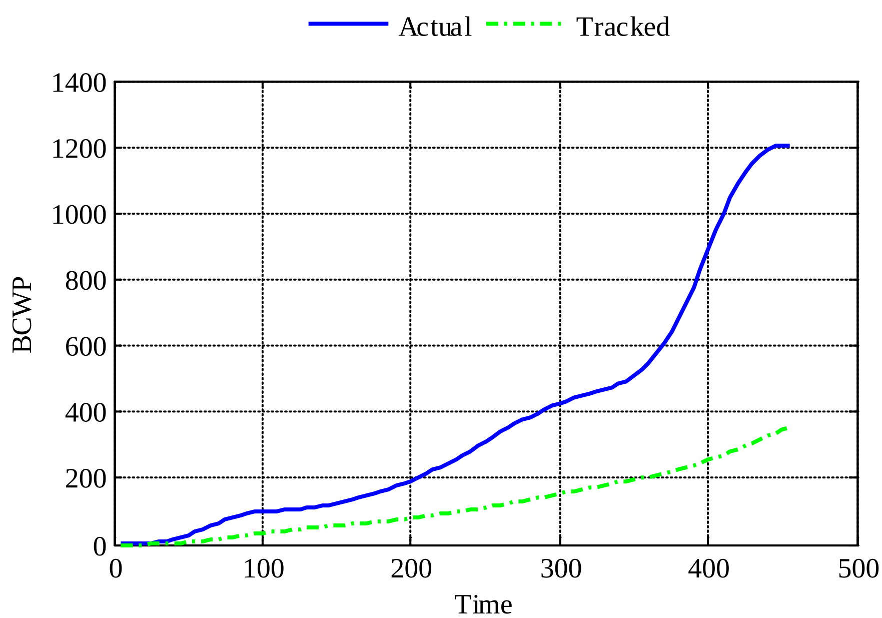


Figure 9 Tracking performance of the Polynomial Rate Model with process noise of 0.

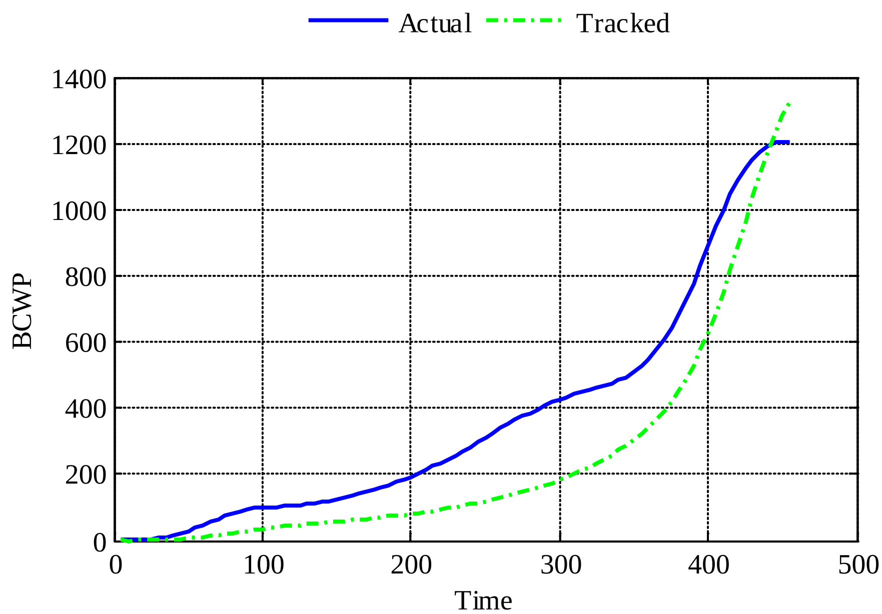


Figure 10 Tracking performance of the Polynomial Rate Model with process noise of 10^{-12}.

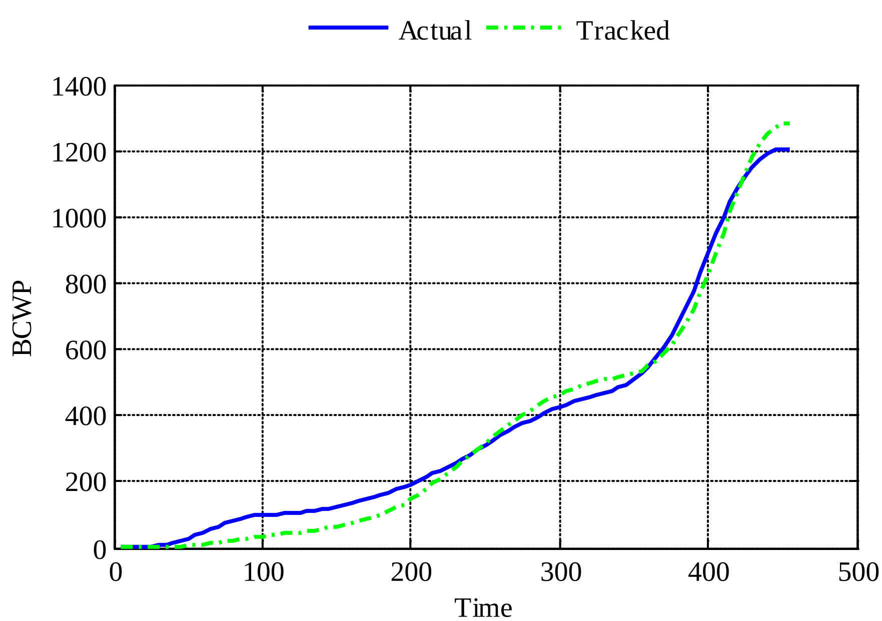


Figure 11 Tracking performance of the Polynomial Rate Model with process noise of 10^{-11}.

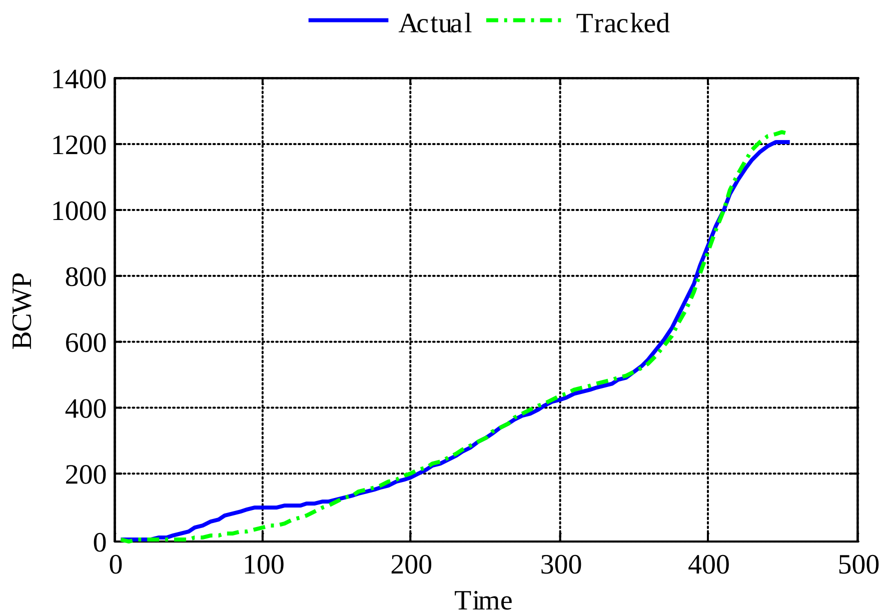


Figure 12 Tracking performance of the Polynomial Rate Model with process noise of 10^{-9}.

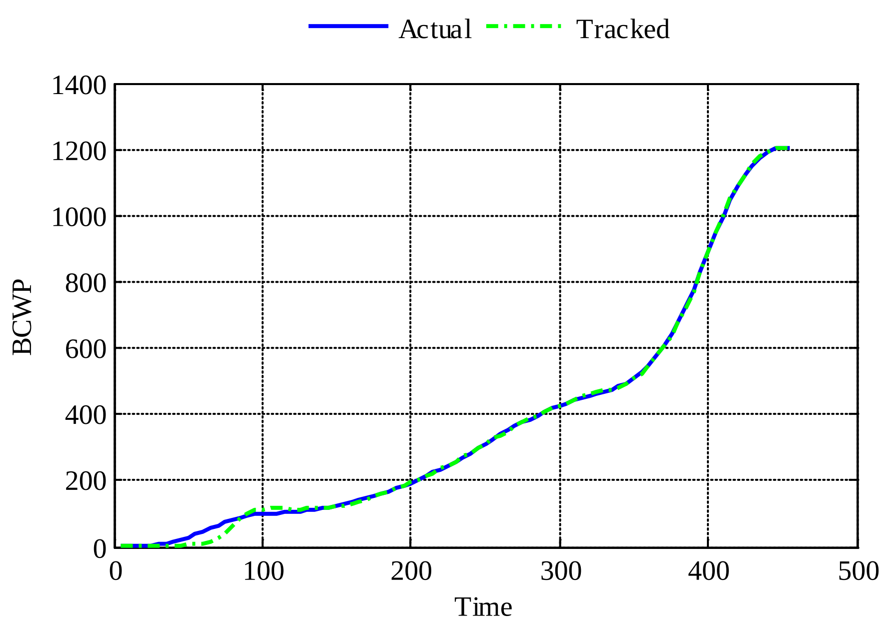


Figure 13 Tracking performance of the Polynomial Rate Model with process noise of 10^{-7}.

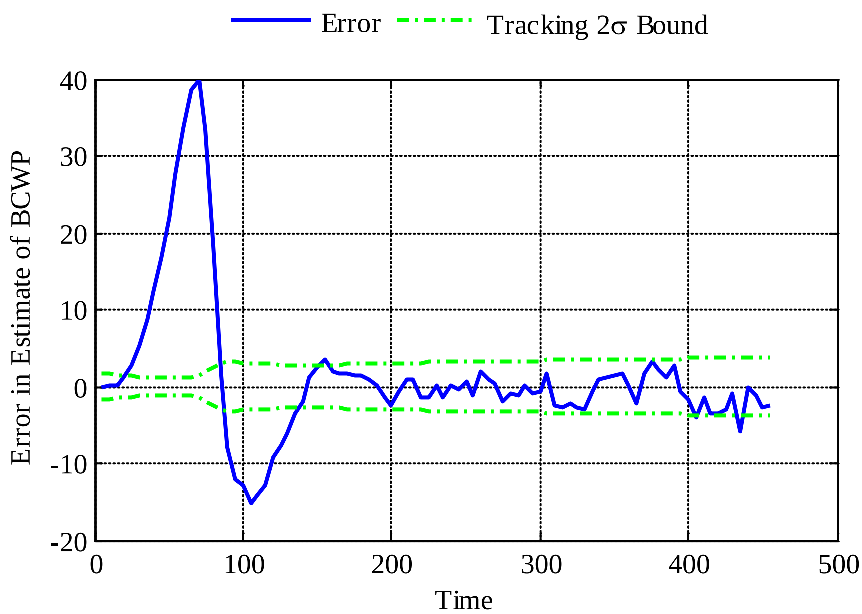


Figure 14 Tracking error and its theoretical bounds for the Polynomial Rate Model.

It can be observed that by increasing the variance of the process noise - Q - the polynomial model could be made to track the BCWP more closely, but at the expense of the involvement of the process noise term whose value is ad-hoc and not completely accounted for. Observing the tracking error in Figure 14 it can be hypothesized that the model should be suitable for representing the project only after tracking it for a certain period of time. Using the same model with EKF the estimates at completion are forecast first by tracking the actual project till time 250 and next by tracking it till time 350.

The forecast along with its 95% prediction bounds and the prediction error for the first case – tracking till time 250 – are as shown in Figure 15 and Figure 16 respectively. It can be seen that even at time 250 the forecast provided by the model is not very accurate with impractically huge forecast errors. Next, forecast along with its

95% forecast bounds and the forecast error are obtained for the second case – tracking till time 350 – are as shown in Figure 17 and Figure 18 respectively along with the actual trajectory. Again it can be seen that even at time 350 the forecast provided by the model is not very accurate with impractically huge forecast errors.

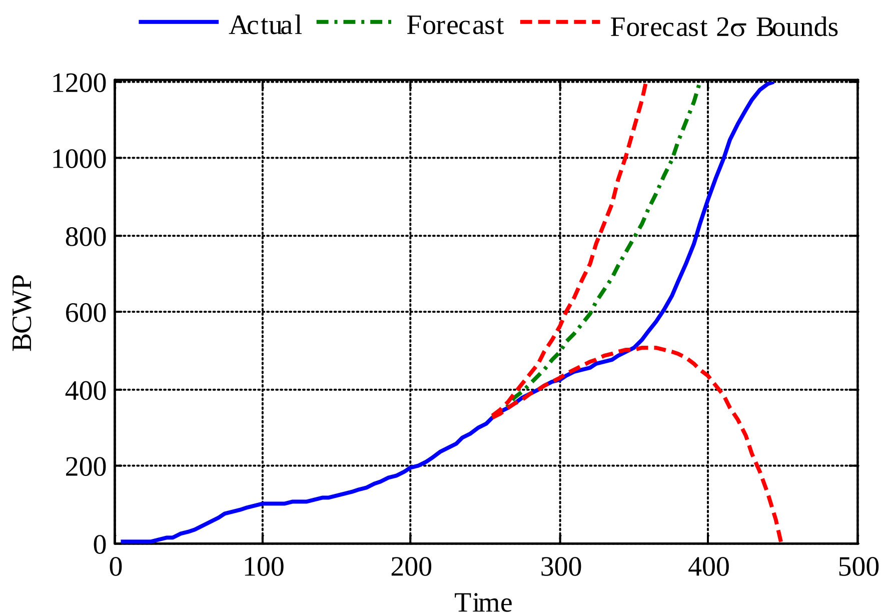


Figure 15 Forecast at time 250 and its prediction bounds – using Polynomial Rate Model.

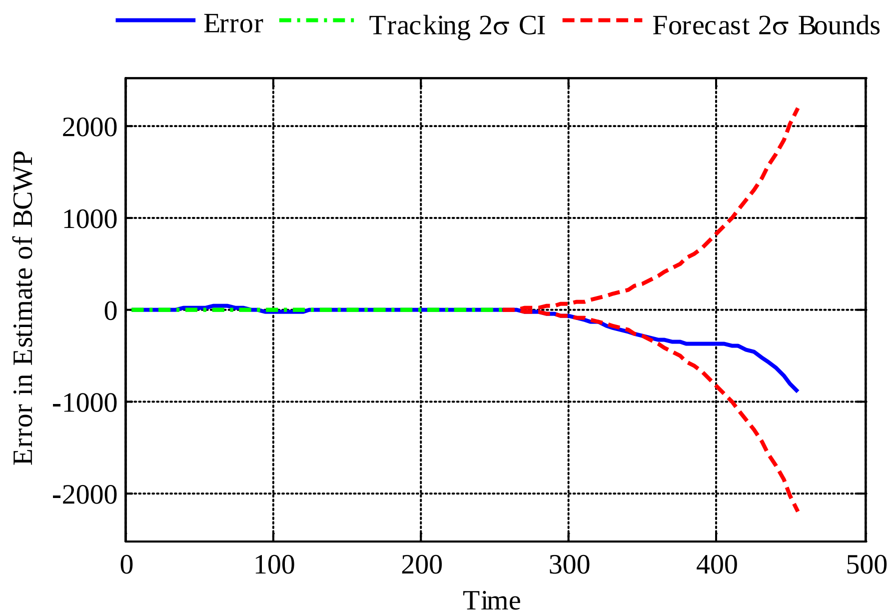


Figure 16 Forecast error and its prediction bounds for forecast at time 250 – using Polynomial Rate Model.

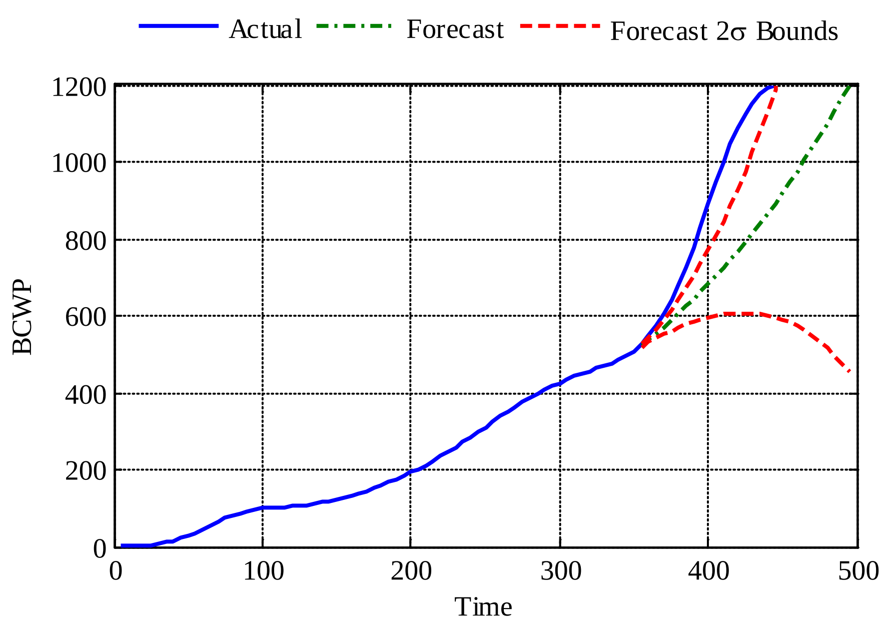


Figure 17 Forecast at time 350 and its prediction bounds – using Polynomial Rate Model.

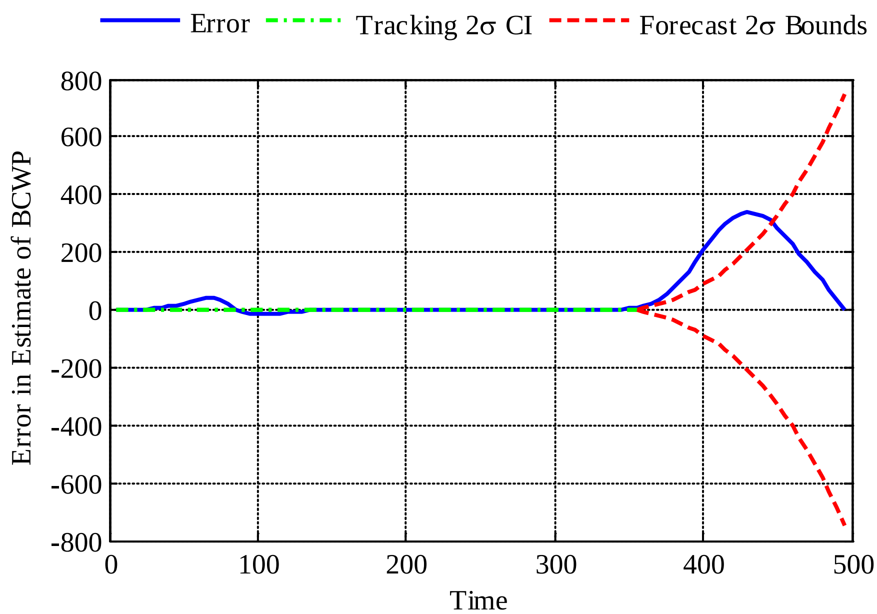


Figure 18 Forecast error and its prediction bounds for forecast at time 350 – using Polynomial Rate Model.

4.3.2 Using the Sigmoid Project Progress Model

When there is some information about the general dynamics of the system either from the plan of the project or from any other intuition, it can be used to our advantage in representing the rate of progress of the project in terms of its BCWP. For example, most of the projects seem to have a definite sigmoid rate progress rate where the rate increases till a particular period and decreases after that (Reinschmidt 2007). The rate of change of BCWP is represented by such a sigmoid rate model of the form (4.14) with two model parameters a and b.

$$\frac{d}{dt}BCWP(t) = a[OC(T_f) - BCWP(t)] + b[BCWP(t)][OC(T_f) - BCWP(t)] \quad (4.14)$$

In (4.14) $OC(T_f)$ is the cumulative indirect cost at the end of the project and is equal to 1201 for the test project. Again to account for any uncertainty in this project dynamics model it is assumed that these model parameters themselves are variable states of the project model and need to be updated to reflect the behavior of the actual project by tracking the BCWP trajectory. The project states and outputs for this system at any discrete time k are

$$x(k) = \begin{bmatrix} BCWP(k) \\ a(k) \\ b(k) \end{bmatrix} \qquad z(k) = BCWP(k) \tag{4.15}$$

The initial values of the state vectors were all set to 0 assuming there is no information about the project dynamics. The initial variance of BCWP was set to 0. It was assumed that the output measurement has a variance of 2. With this project dynamics model the BCWP of the test project was tracked over the whole duration of the project using the Extended Kalman Filtering algorithm equations (4.5) - (4.8) with a sampling time of 5. Again, as before the project is deemed complete when the BCWP is equal to the BAC derived from the project plan. The point in time where the BCWP reaches BAC is the Estimated Duration at Completion (EDAC) of the project. To observe the influence of the process noise five different trials were conducted using different values for the process noise, starting with a value of 0. The tracking performance of this model for five different values of process noise are as shown in Figures 19 to 23. It was observed that the tracking performance is best when the process noise is at least 10^{-12}. The tracking error and its theoretical 95% $(\sim 2\sigma)$ estimation bounds for a process noise value of 10^{-12} are as shown in Figure 24.

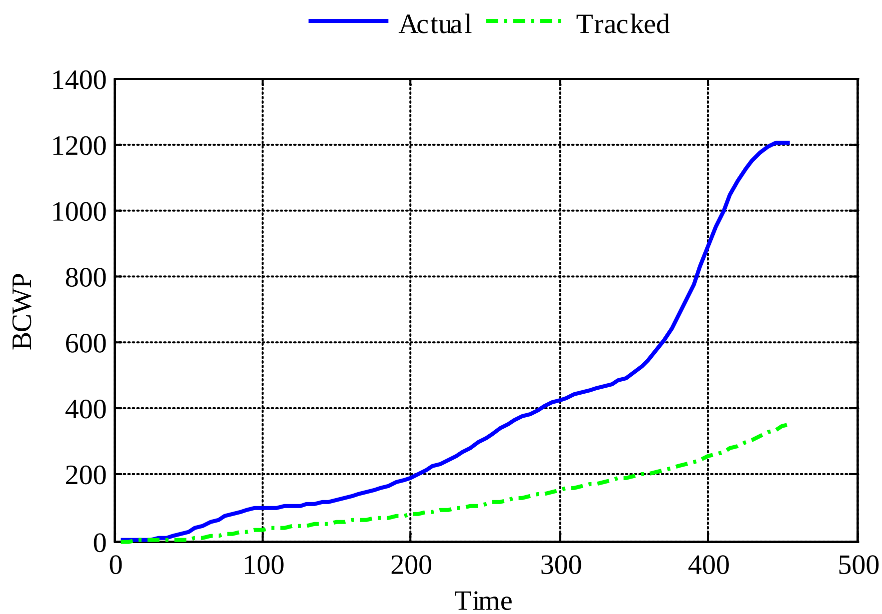


Figure 19   Tracking performance of the Sigmoid Rate Model with process noise of 0.

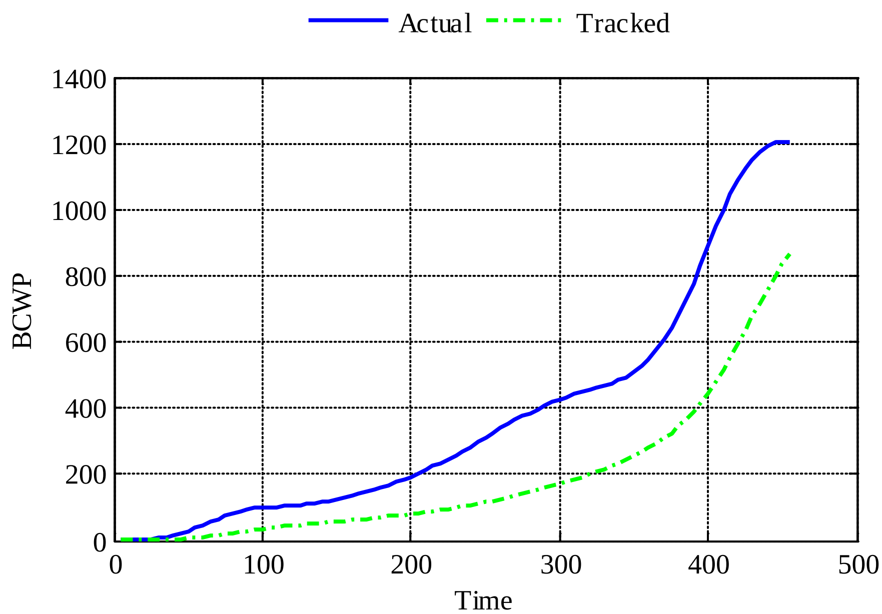


Figure 20 Tracking performance of the Sigmoid Rate Model with process noise of 10^{-20}.

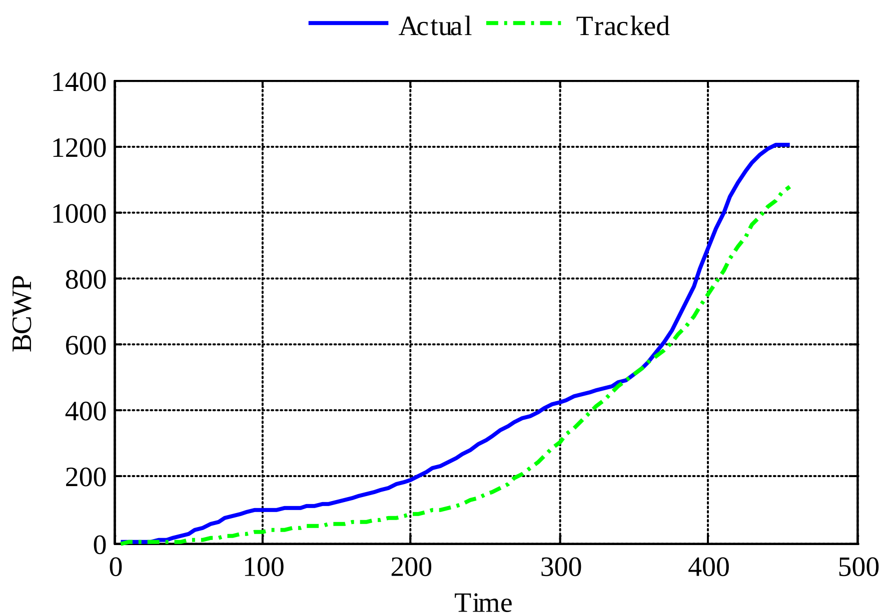


Figure 21 Tracking performance of the Sigmoid Rate Model with process noise of 10^{-18}.

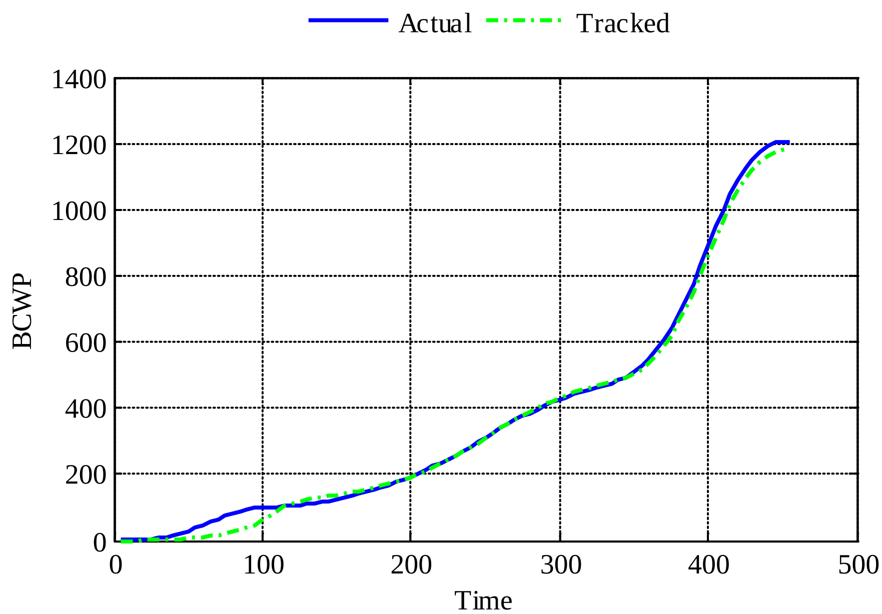


Figure 22 Tracking performance of the Sigmoid Rate Model with process noise of 10^{-15}.

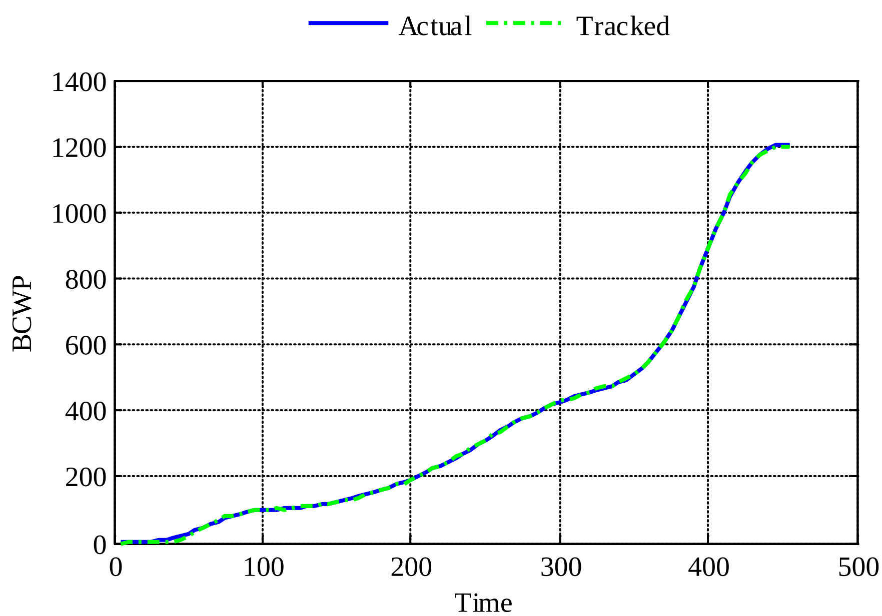


Figure 23 Tracking performance of the Sigmoid Rate Model with process noise of 10^{-12}.

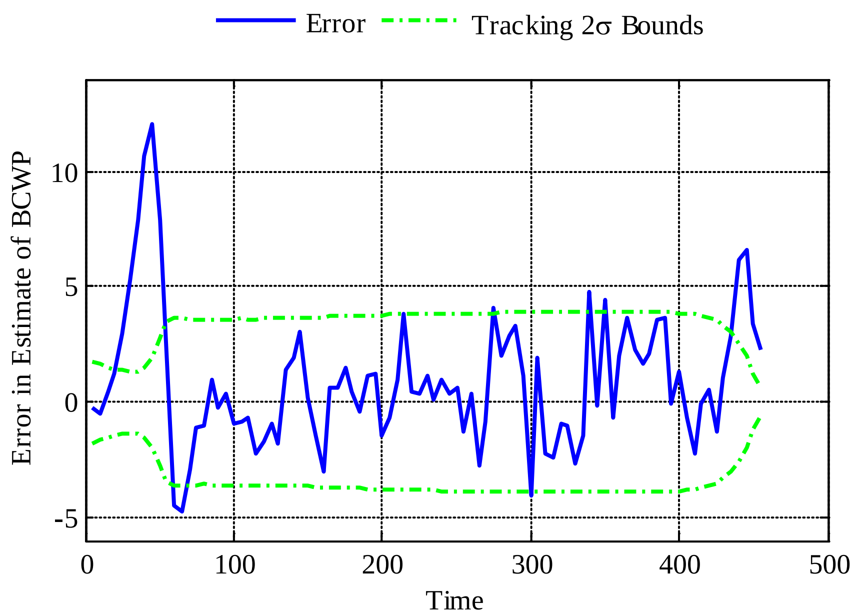


Figure 24 Tracking error and its theoretical bounds for the Sigmoid Rate Model.

It can be observed that by increasing the variance of the process noise - Q - the polynomial model could be made to track the BCWP more closely, but at the expense of the involvement of the process noise term whose value is ad-hoc and not completely accounted for. But the sigmoid model seems to perform better than the polynomial model.

Observing the tracking error in Figure 24 it can be hypothesized that the model should be suitable for representing the project only after tracking it for a certain period of time. Using the same model with EKF the estimates at completion are forecast first by tracking the actual project till time 250 and next by tracking it till time 350.

The forecast along with its 95% forecast bounds and the prediction error for the first case – tracking till time 250 – are as shown in Figure 25 and Figure 26 respectively.

It can be seen that even at time 250 the forecast provided by the model is not very accurate with impractically huge forecast errors. Next, forecast along with its 95% prediction bounds and the forecast error are obtained for the second case – tracking till time 350 – are as shown in Figure 27 and Figure 28 respectively along with the actual trajectory. Again it can be seen that even at time 350 the forecast provided by the model is not very accurate with impractically huge forecast errors.

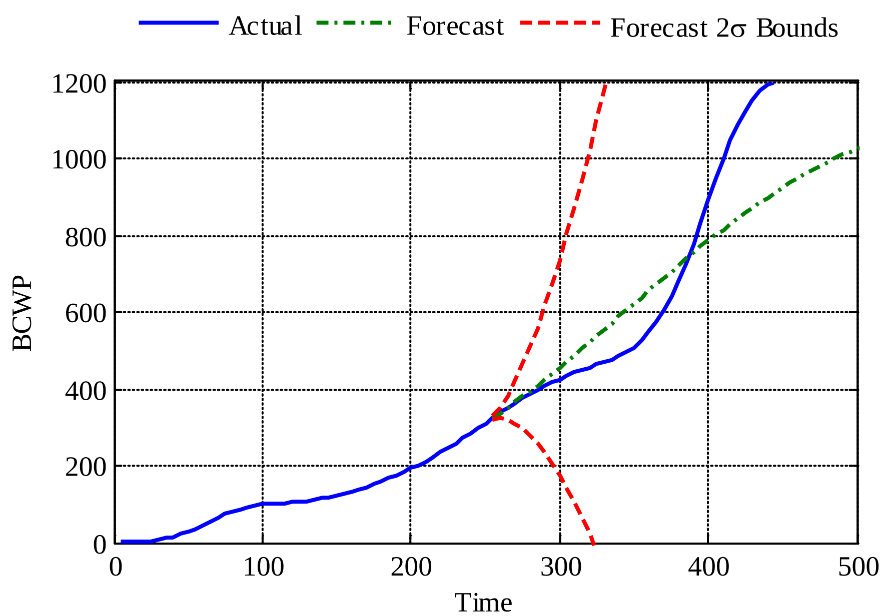


Figure 25 Forecast at time 250 and its prediction bounds – using Sigmoid Rate Model.

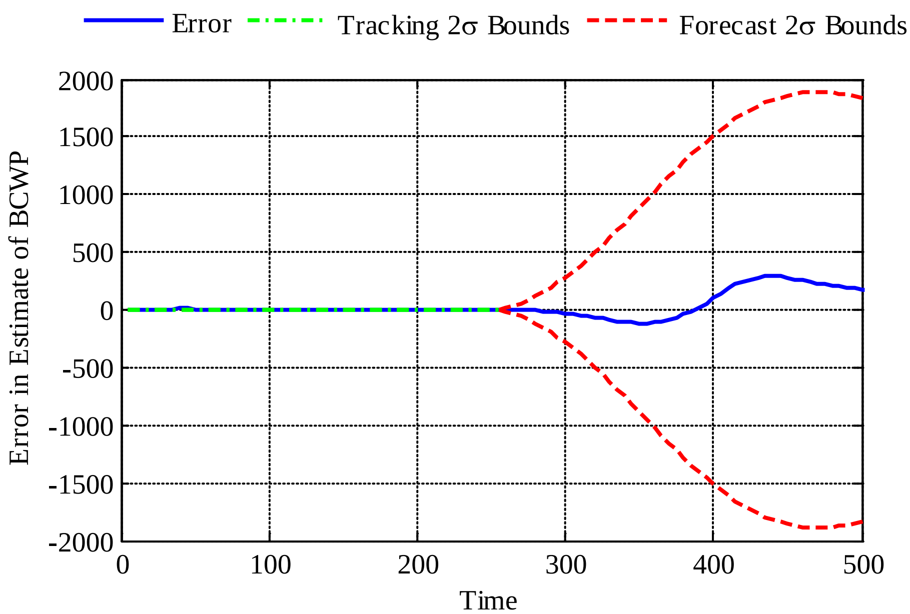


Figure 26 Forecast error and its prediction bounds for forecast at time 250 – using Sigmoid Rate Model.

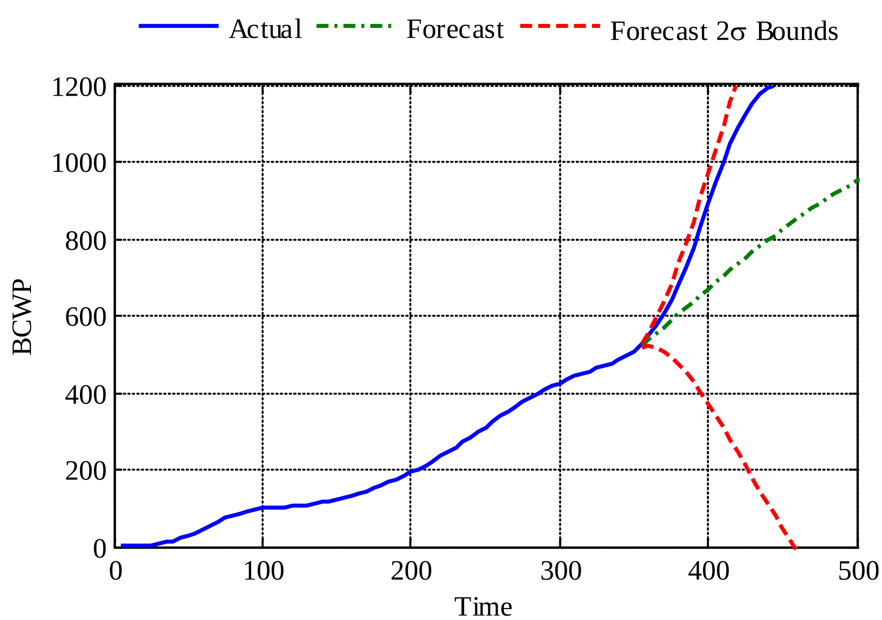


Figure 27 Forecast at time 350 and its prediction bounds – using Sigmoid Rate Model.

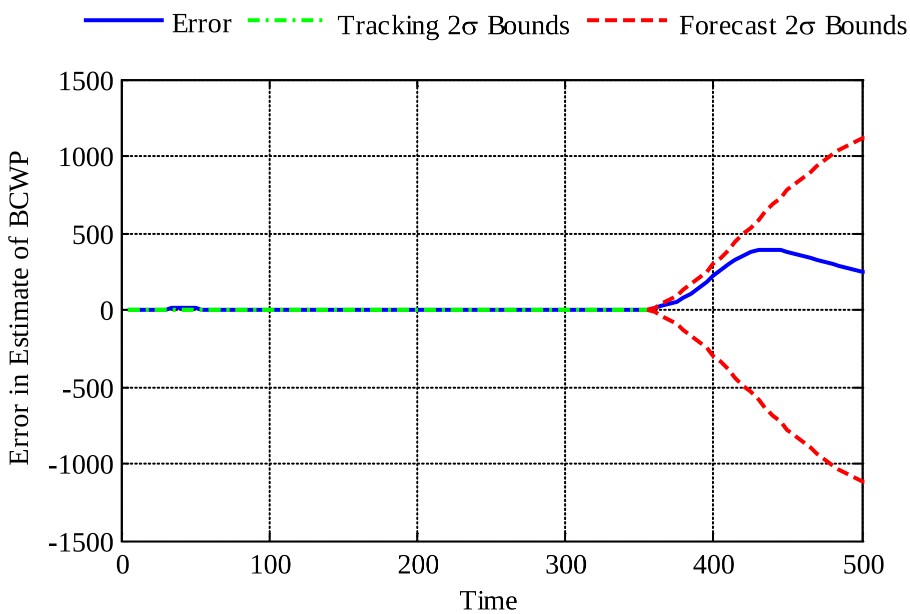


Figure 28 Forecast error and its prediction bounds for forecast at time 350 – using Sigmoid Rate Model.

With the observations from the tracking performance it can be inferred that the polynomial and sigmoid models are able to track the actual process with the use of process noise. But from the observation of forecasting performance it can be inferred that it is difficult to use the filter with these models to determine the true nature of the actual process. These generalized project models are unable to capture the actual nature of the rate of change of progress of the test project. Further, in such cases it doesn't seem to matter if the project model is making use of any information from the project dynamics as long as an appropriate value is used for process noise. The process noise is a type of fudging method accounting for the lack of knowledge of all the components of the process or system and it needs to be obtained by other means, either subjective or by analysis of past data.

To deal with this problem in the test project we need to use a model that has at least some information about the actual dynamics of the projects, especially the information about the discontinuities in the rate of progress due to the involvement of multiple activities with spatial interdependencies. This information is most often readily available from the plan of the project and can be used to make a more accurate project model. The project plan reflected by the Gantt charts provides the information about the activity interdependencies but lacks the knowledge about the progress rate of each individual task. This problem can be overcome by assuming that each sub-task follows a sigmoid or triangular progress rate. Thus it is possible to make a project model that completely incorporates both the spatial and dynamic behavior of the project. But such models are usually discontinuous and non-differentiable and cannot be used with the Extended Kalman Filter. To overcome this problem the Unscented Kalman Filter (Julier and Uhlmann 1997) is used.

4.3.3 Using the Comprehensive Project Progress Model

Now that it is possible to use discontinuous and non-differentiable models in Kalman Filters the plan of the project itself is assumed to the project model. The project plan for the test project is based on the triangular progress rate equation (4.16) for each of the sub-task x_i.

$$\dot{x}_i(t) = \begin{cases} 0.001 & x_i(t) \leq 0 \\ a_i\sqrt{x_i(t)} & 0 < x_i(t) \leq 5\ 0 \\ b_i\sqrt{100 - x_i(t)} & x_i(t) \leq 100 \end{cases} \tag{4.16}$$

Again to account for any uncertainty in this project dynamics model it is assumed that the model parameters a_i and b_i themselves are unrealized states of the project model and need to be updated to reflect the behavior of the actual project by tracking the actual project trajectory. In this case the measurement of the system is the state vector itself indicating the progress of the sub-tasks. The project states and outputs at any discrete time k are

$$X(k) = \begin{bmatrix} X_1(k) \\ X_2(k) \\ X_3(k) \end{bmatrix} \qquad Z(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

where, (4.17)

$$X_1(k) = \begin{bmatrix} x_1(k) \\ a_1(k) \\ b_1(k) \end{bmatrix} \qquad X_2(k) = \begin{bmatrix} x_2(k) \\ a_2(k) \\ b_2(k) \end{bmatrix} \qquad X_3(k) = \begin{bmatrix} x_3(k) \\ a_3(k) \\ b_4(k) \end{bmatrix}$$

For testing the tracking performance of the model the initial values of the state vector $x(k)$ were all set to 0 assuming there is no information about the system parameters. The initial variance of the individual tasks was set to 1.0. Further it was assumed that the process noise has a variance of 0 while the output measurement has a variance of 2. With this project dynamics model the actual states of the test project described earlier were first tracked over the whole duration of the project using the Unscented Kalman Filtering algorithm equations with a sampling interval of 5. The absolute tracking performance of this model is shown in Figure 29, Figure 30 and Figure

31, while the tracking error and its theoretical 95% $(\sim 2\sigma)$ tracking bounds are shown in Figure 32, Figure 33 and Figure 34.

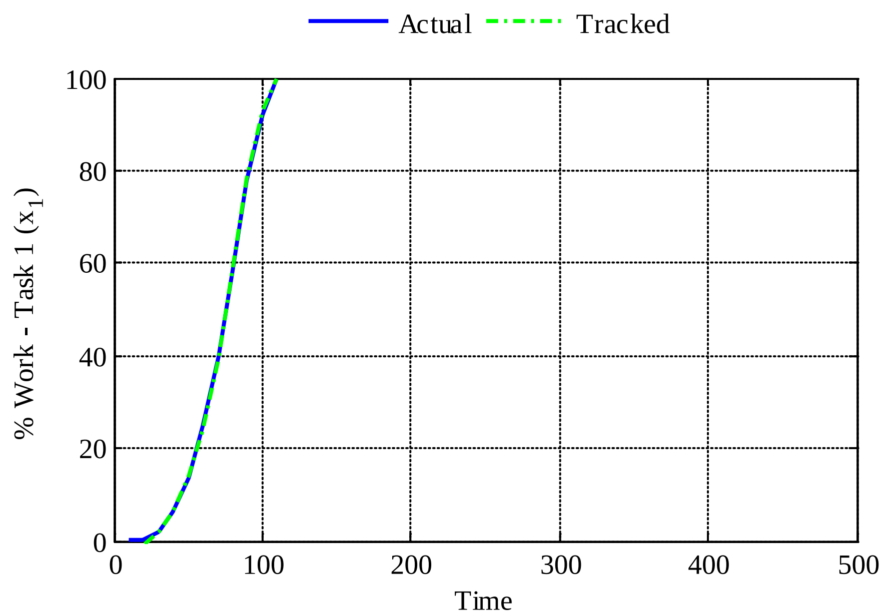


Figure 29 Tracking performance for Task 1 – using the project plan.

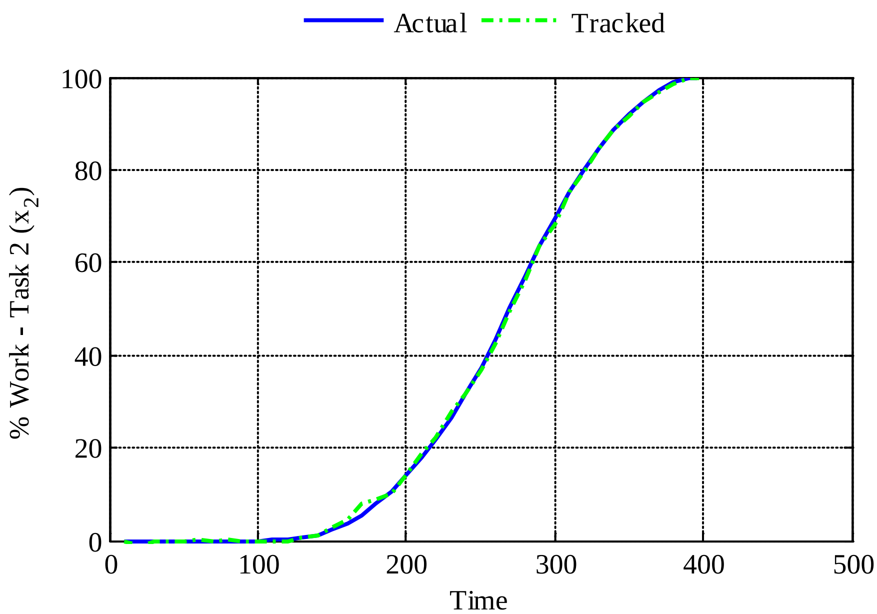


Figure 30 Tracking performance for Task 2 – using the project plan.

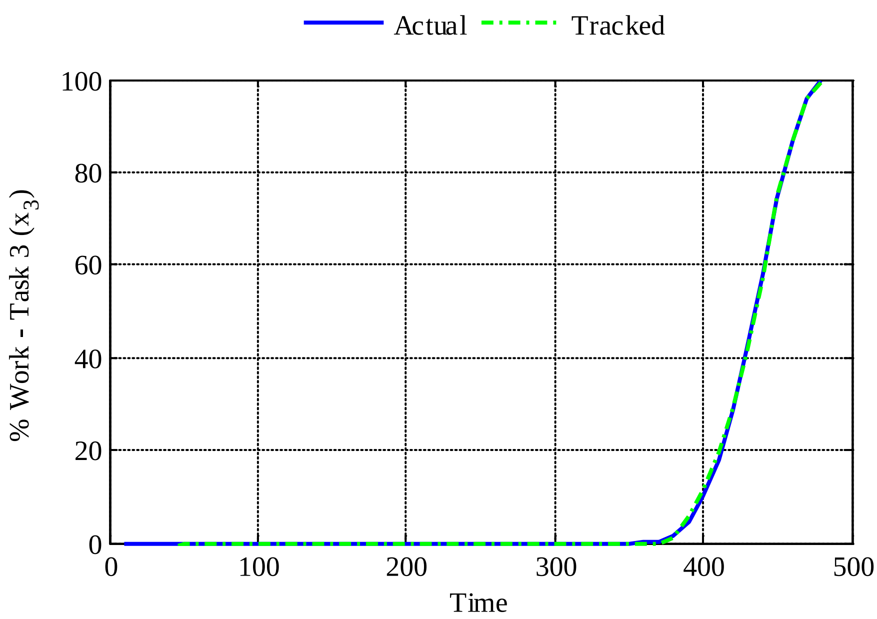


Figure 31 Tracking performance for Task 3 – using the project plan.

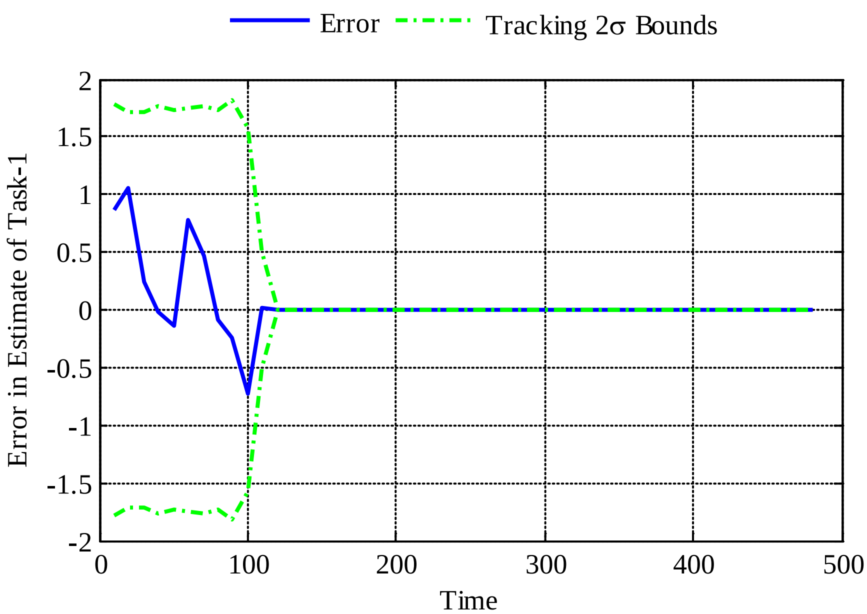


Figure 32 Tracking error for Task 1 – using the project plan.

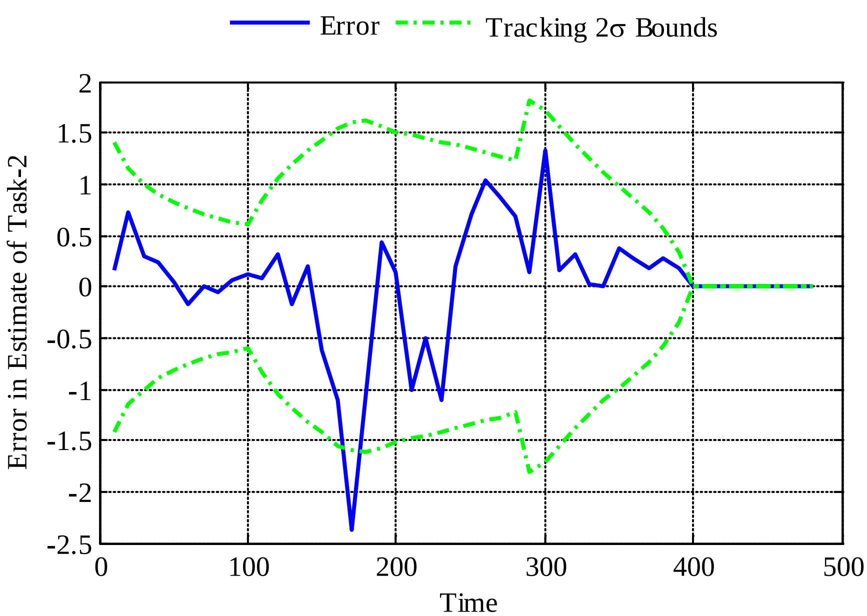


Figure 33 Tracking error for Task 2 – using the project plan.

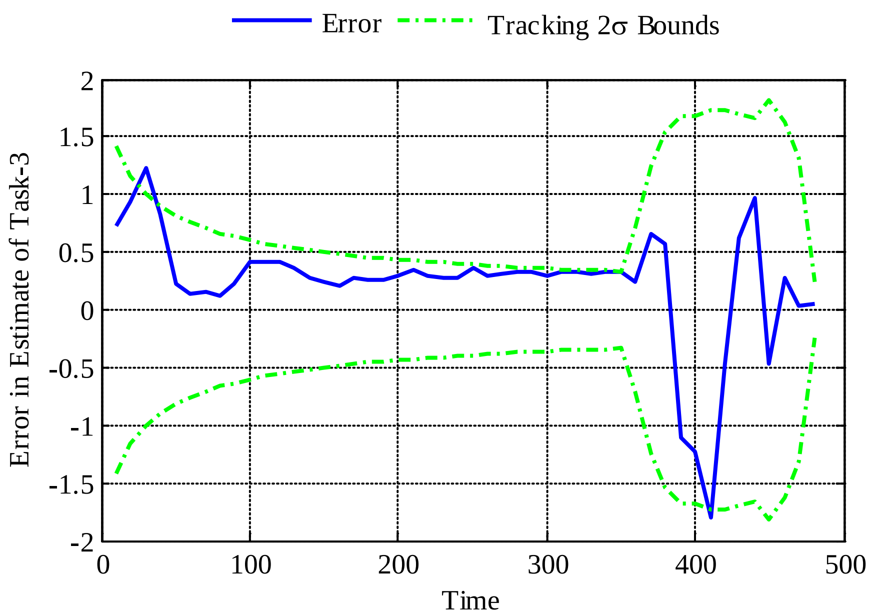


Figure 34 Tracking error for Task 3 – using the project plan.

It can be observed that the Unscented Kalman Filter can be used to easily update the system parameters of the project plan when the actual data is available. But this updating is only possible for the system parameters that are involved in the tracking process and needed to be updated to reflect the actual behavior of the project. In this initial tracking test all the system parameters were involved in the updating process. Hence it was possible to update all the model parameters for the initial values of 0. The parameters not involved in the tracking process will not be updated, indicating that there was no need for their revision. This perfectly reflects the actual project dynamics where a task may be a delay due to delays in its dependencies, but there might be no effect on the rate of progress of the task. Hence for forecasting the system it is assumed that the initial state vector is same as that of the project plan.

Using the same model with UKF the state and earned value estimates at completion are forecast first by tracking the actual project states till time 200 and then extrapolating till the end. The forecast and its 2σ forecast bounds for the progress of Task 2 and Task 3 are shown in Figure 35 and Figure 36 respectively along with the actual trajectory. The forecast and its 2σ forecast bounds for the project cost are shown in Figure 37. The actual task and project trajectories are well within the 2σ forecast bounds of the forecast. Further discussion of the performance and verification of the Kalman-Filter estimation algorithm is discussed in the Sections 4.3.4 and 4.3.5.

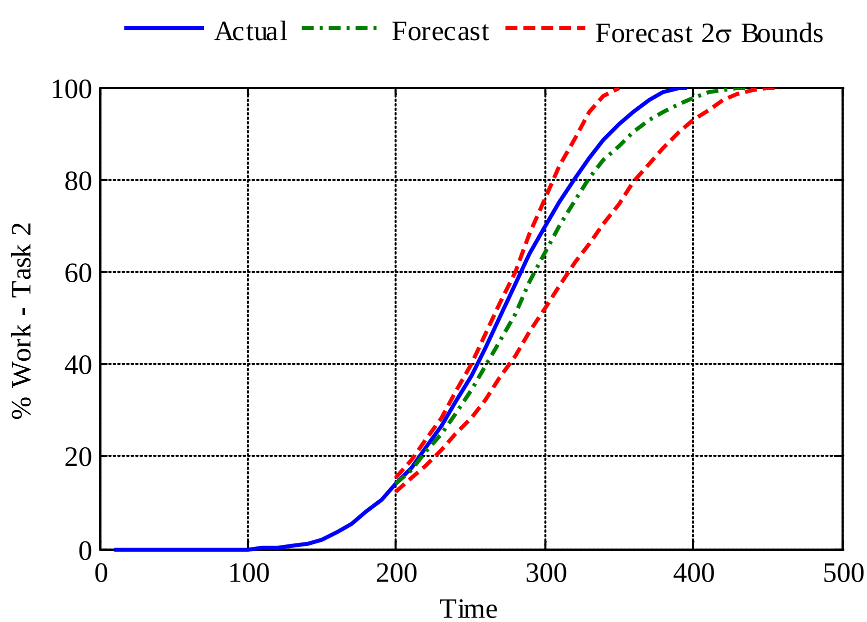


Figure 35 Forecast for Task 2 at time 200 along with forecast bounds – using the project plan.

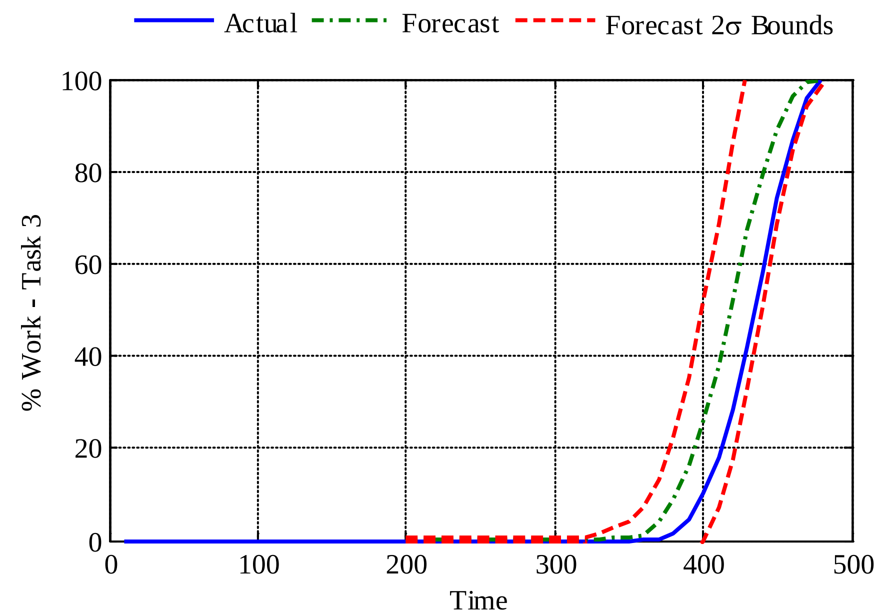


Figure 36 Forecast for Task 3 at time 200 along with forecast bounds – using the project plan.

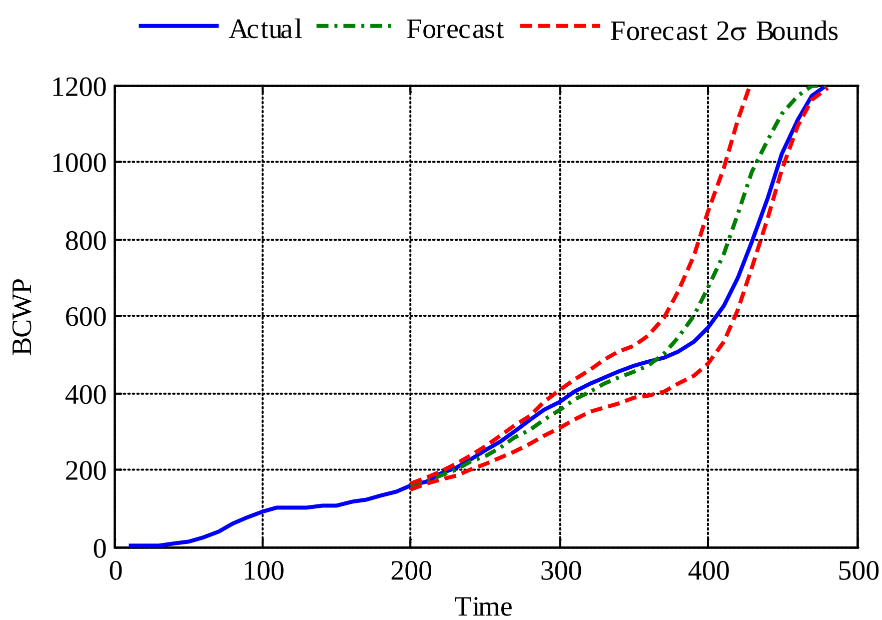


Figure 37 Forecast for BCWP at time 200 along with forecast bounds – using the project plan.

4.3.4 Performance of the Estimation Algorithm for Deterministic Project

The performance of the estimation algorithm – the Kalama-Filtering algorithm in this case – can be verified and qualified using various criteria. For example, if the chosen model has the exact structure of the true system, then the algorithm can be verified by the convergence of the estimated model parameters to their true values. Further the performance of the algorithm can be qualified by the rate of this convergence and the robustness of the convergence to various errors, noise and initial assumptions. In the present research, since the actual system – the test project – was simulated using the equation (3.8), the project model using the same equation can be used verify the performance of the estimation algorithm.

Initially it is assumed that the actual project has deterministic values for the parameters a_i and b_i , and are same as those used for the original project plan as shown in Table 1. For the project model these initial values of these parameters are all set to 0 and the initial values of the variance of error of estimation are all set to 0.1. The initial values of the variance of error of estimation of the state of the sub-tasks are set to 1. For the process of verification the project model was made to track the actual project till the end of the project using the Kalman-Filter for estimation of the model parameters a_i and b_i. The observed convergences of the estimated value of the parameters for the three sub-tasks are as shown in Figure 38, Figure 39 and Figure 40.

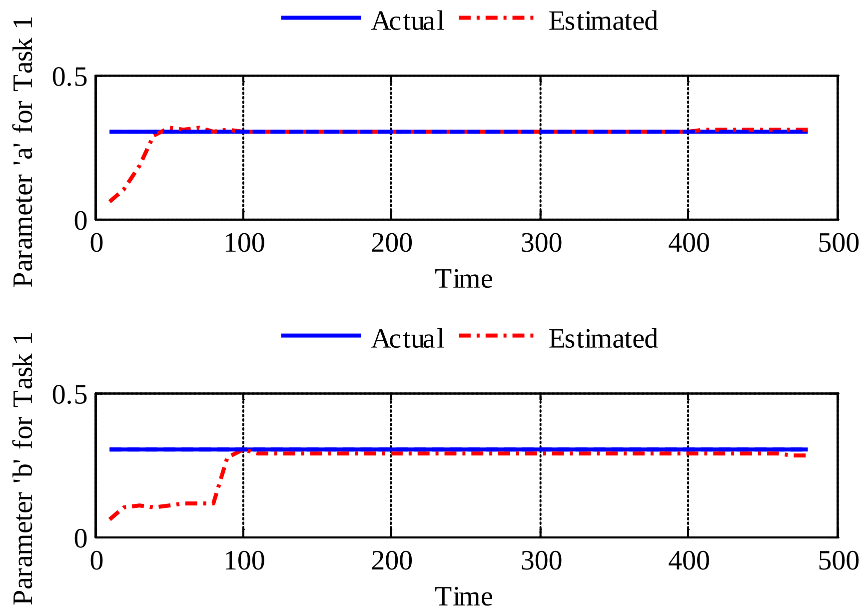


Figure 38 Convergence of the estimates for Task 1 in a deterministic system – initial value of parameters and variance error of estimation set to (0.0, 0.0) and (0.1, 0.1).

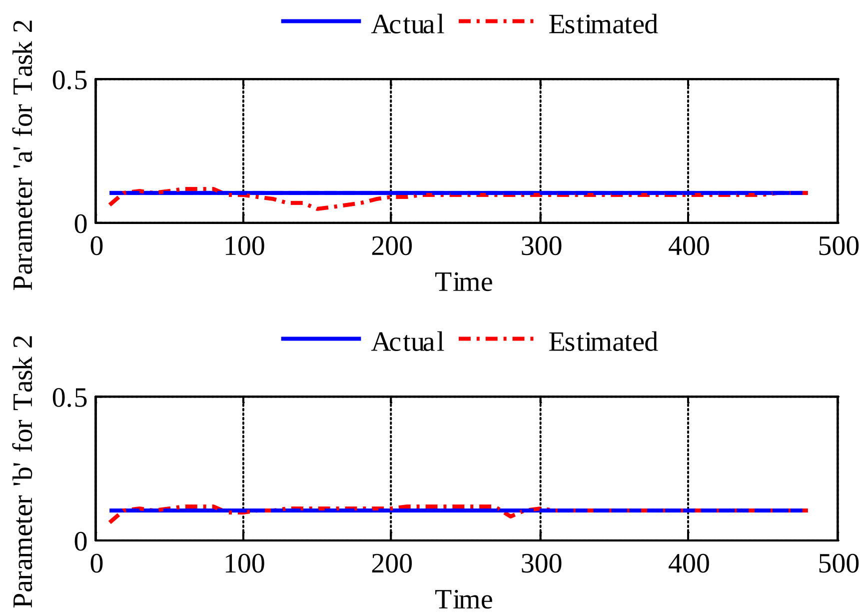


Figure 39 Convergence of the estimates for Task 2 in a deterministic system – initial value of parameters and variance error of estimation set to (0.0, 0.0) and (0.1, 0.1).

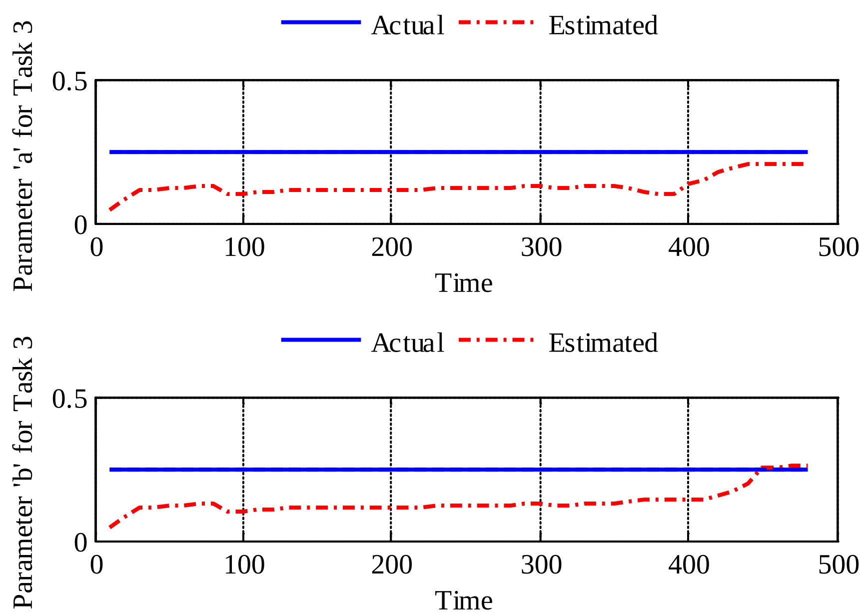


Figure 40 Convergence of the estimates for Task 3 in a deterministic system – initial value of parameters and variance error of estimation set to (0.0, 0.0) and (0.1, 0.1).

It can be observed that for each task - after the availability of data about the actual progress of the task - the estimated values of the model parameters converge to values close to the true values. Further it can also be observed that the parameter a_i for the Task *i* converges earlier than the parameter b_i. This is expected due to the nature of the true system where the parameter a_i influences the rate of the first half of the task and the parameter b_i the later half.

To check the robustness of the algorithm to the variation in the initial assumptions, the initial values of the variance of error in estimation are increased to 1.0. The observed convergences of the estimated value of the parameters for the three sub-tasks are as shown in Figure 41, Figure 42 and Figure 43.

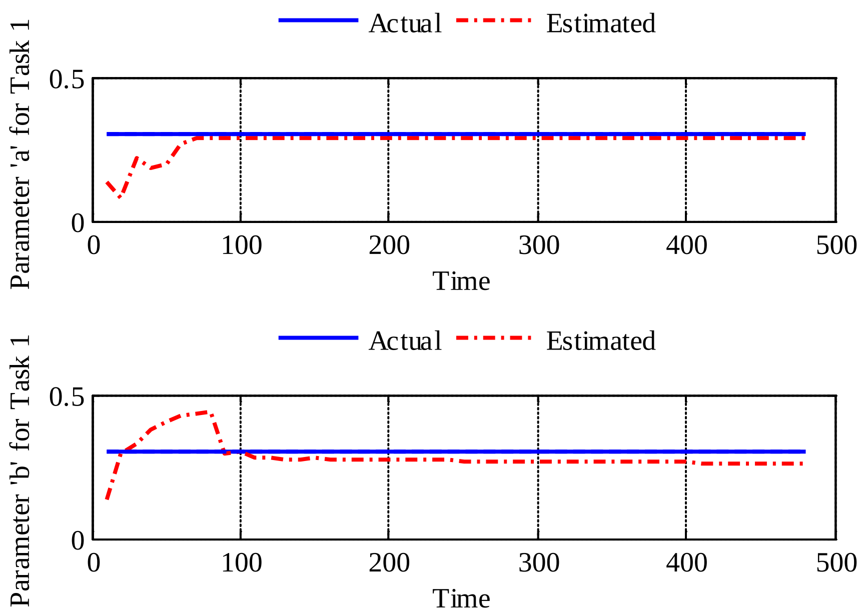


Figure 41 Convergence of the estimates for Task 1 in a deterministic system – initial value of parameters and variance error of estimation set to (0.0, 0.0) and (1.0, 1.0).

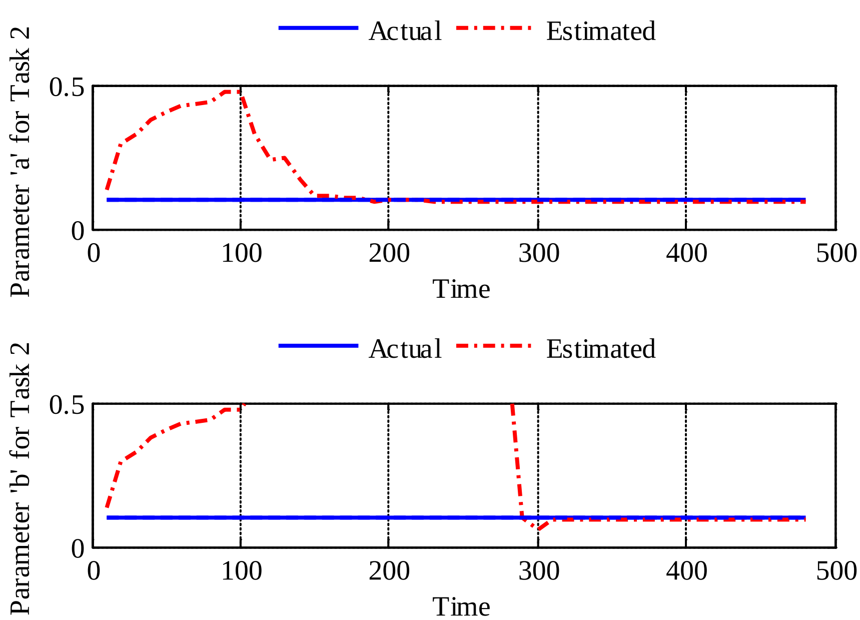


Figure 42 Convergence of the estimates for Task 2 in a deterministic system – initial value of parameters and variance error of estimation set to (0.0, 0.0) and (1.0, 1.0).

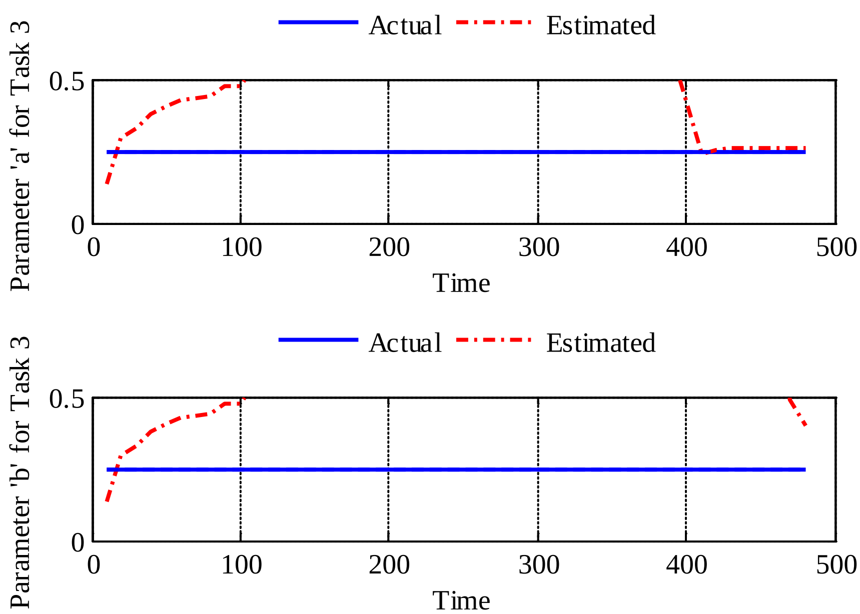


Figure 43 Convergence of the estimates for Task 3 in a deterministic system – initial value of parameters and variance error of estimation set to (0.0, 0.0) and (1.0, 1.0).

Again it can be observed that except for b_3, the estimated values of the model parameters for each task that converge to values close to the true values. Parameter b_3 started to converge, but the rate of convergence was slow.

4.3.5 Performance of the Estimation Algorithm for Stochastic Project

Actual construction activities are usually stochastic in nature. Many external disturbances and noise influence the rates of progress of the subtasks. To investigate the performance of the implemented estimated algorithms with such stochastic projects, the actual system – the test project – was simulated with the parameters a_i and b_i taking on random values from a Gaussian distribution with mean or true values as shown in Table 1. To prevent the occurrence of negative values for the parameters – negative values indicate negative rate of progress, which is not possible - the distribution is truncated at 0 to the left resulting in a truncated normal distribution that has a positive probability of having a value of 0. Figure 44 shows such a distribution for the rate of progress parameters of Task 1 with a Coefficient of Variation (COV) of 0.1. For the project model these initial values of these parameters are all set to 0 and the initial values of the variance of error of estimation are all set to 0.1. The initial values of the variance of error of estimation of the state of the sub-tasks are set to 1.

For the process of verification the project model is made to track the actual project till the end of the project using the Kalman-Filter for estimation of the model parameters a_i and b_i, checking for the convergence of the estimates of the parameters to their true values. The observed convergences of the estimated value of the parameters for the three sub-tasks for a test project with COV of 0.2 are as shown in Figure 45, Figure 46 and Figure 47.

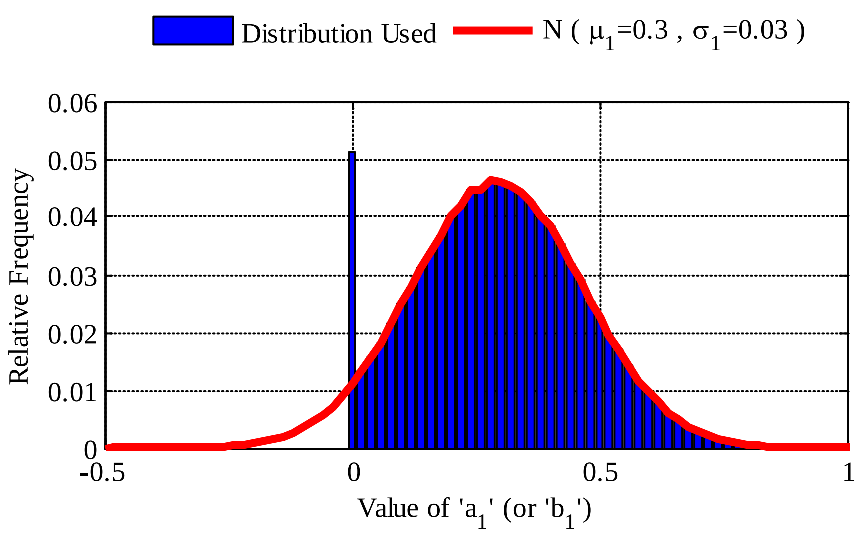


Figure 44 Distribution of the rate of progress parameters for Task 1 with COV 0.1.

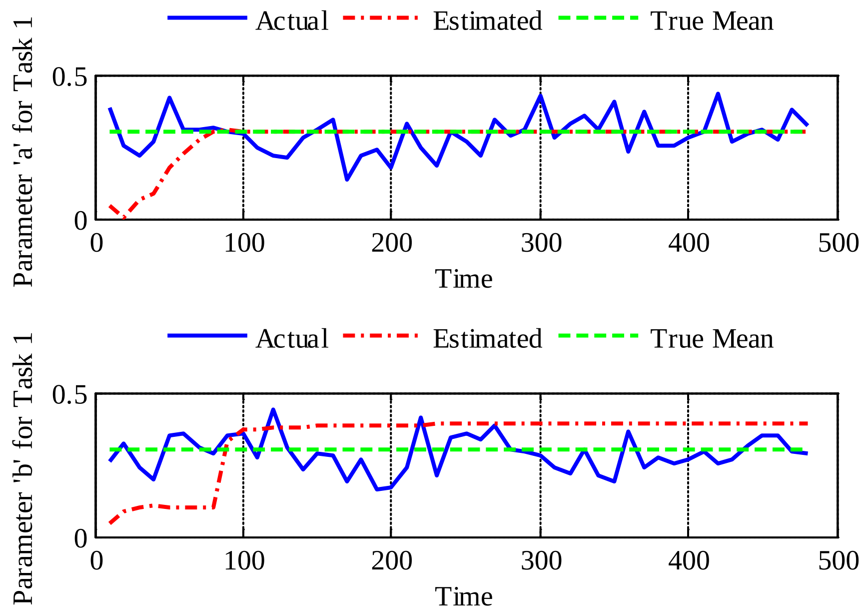


Figure 45 Convergence of the estimates for Task 1 in a stochastic system with COV 0.2.

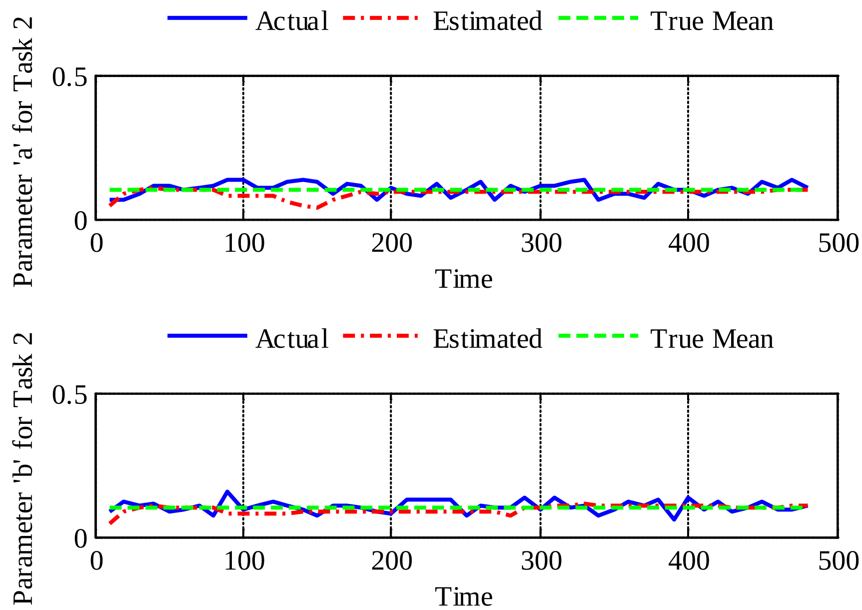


Figure 46 Convergence of the estimates for Task 2 in a stochastic system with COV 0.2.

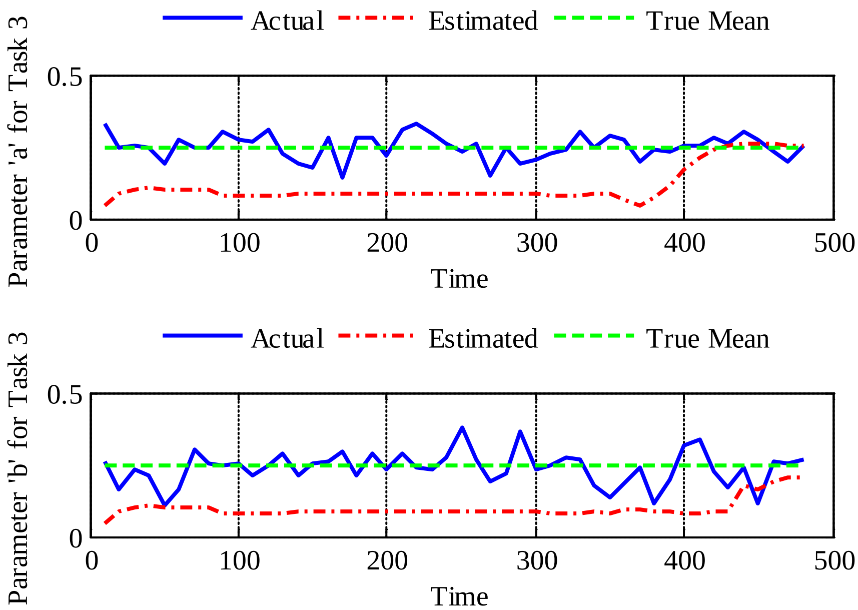


Figure 47 Convergence of the estimates for Task 3 in a stochastic system with COV 0.2.

The observed convergences of the estimated value of the parameters for the three sub-tasks for a test project with COV of 0.5 are as shown in Figure 48, Figure 49 and Figure 50.

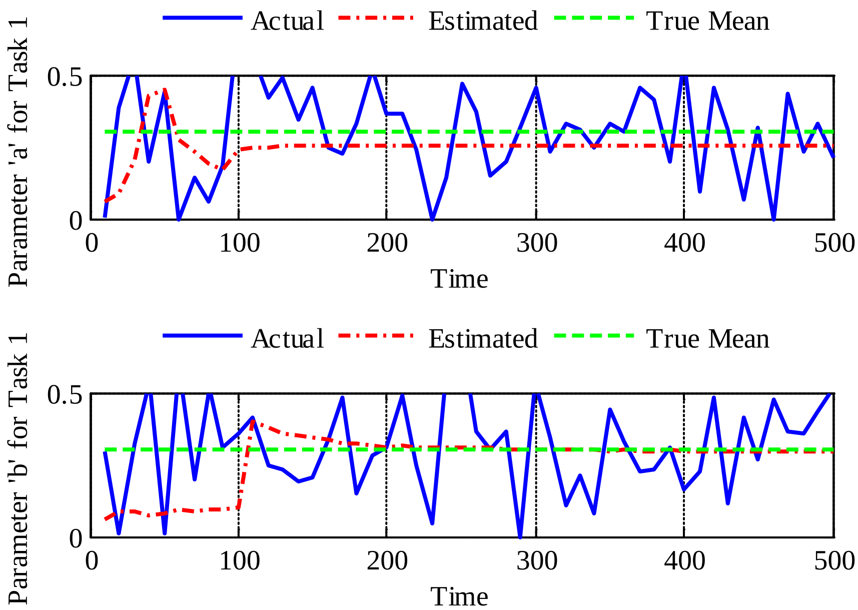


Figure 48 Convergence of the estimates for Task 1 in a stochastic system with COV 0.5.

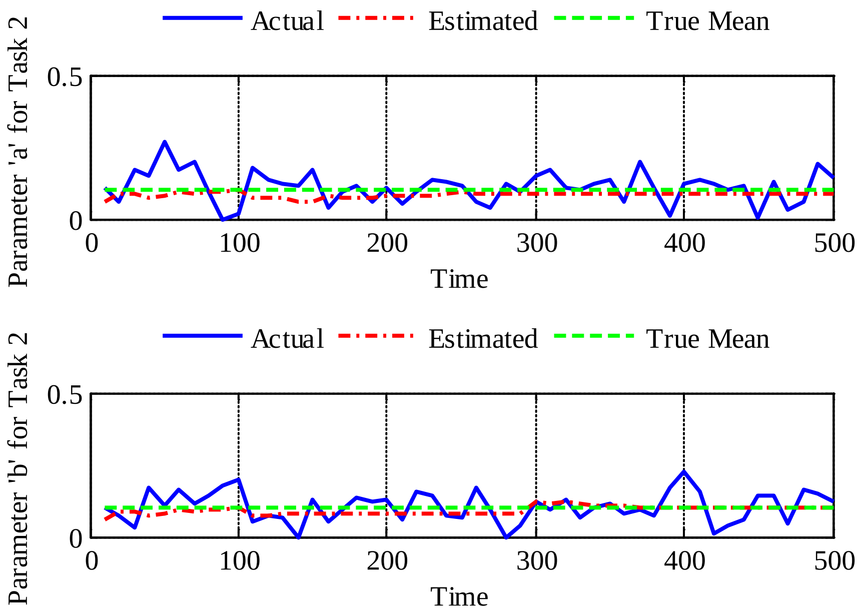


Figure 49 Convergence of the estimates for Task 2 in a stochastic system with COV 0.5.

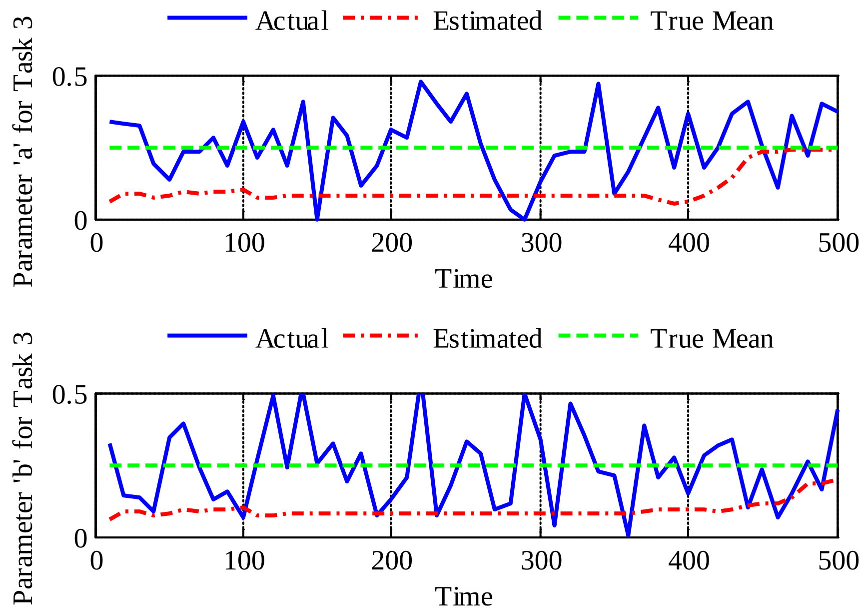


Figure 50 Convergence of the estimates for Task 3 in a stochastic system with COV 0.5.

The observed convergences of the estimated value of the parameters for the three sub-tasks for a test project with COV of 1.0 are as shown in Figure 51, Figure 52 and Figure 53.

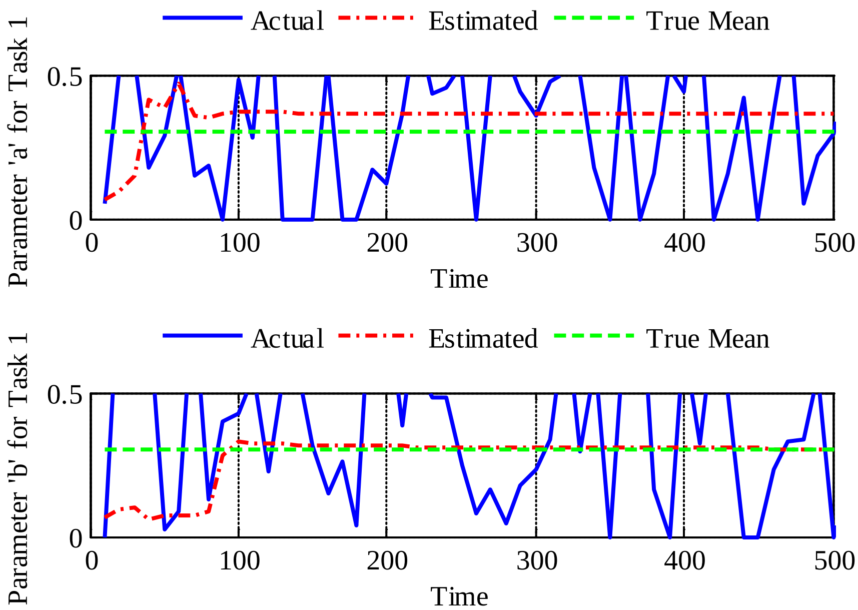


Figure 51 Convergence of the estimates for Task 1 in a stochastic system with COV 1.0.

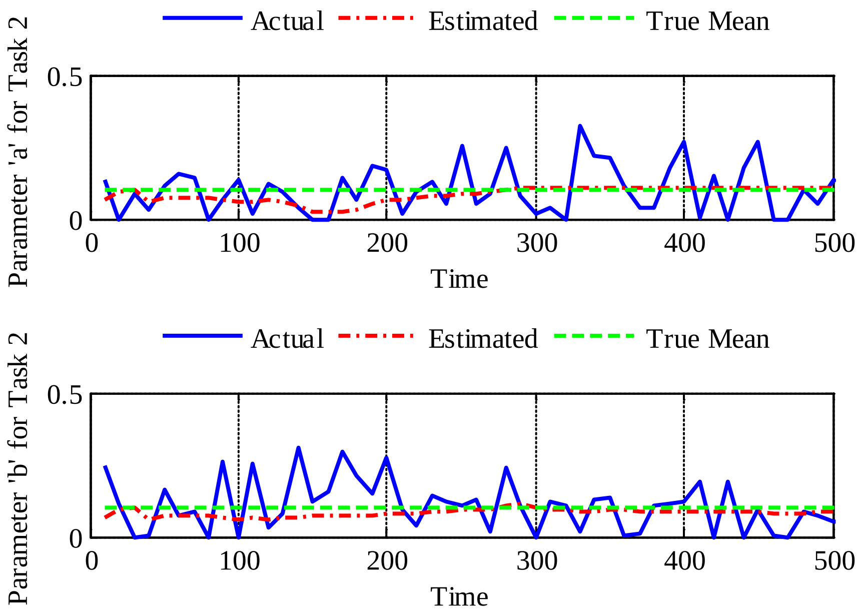


Figure 52 Convergence of the estimates for Task 2 in a stochastic system with COV 1.0.

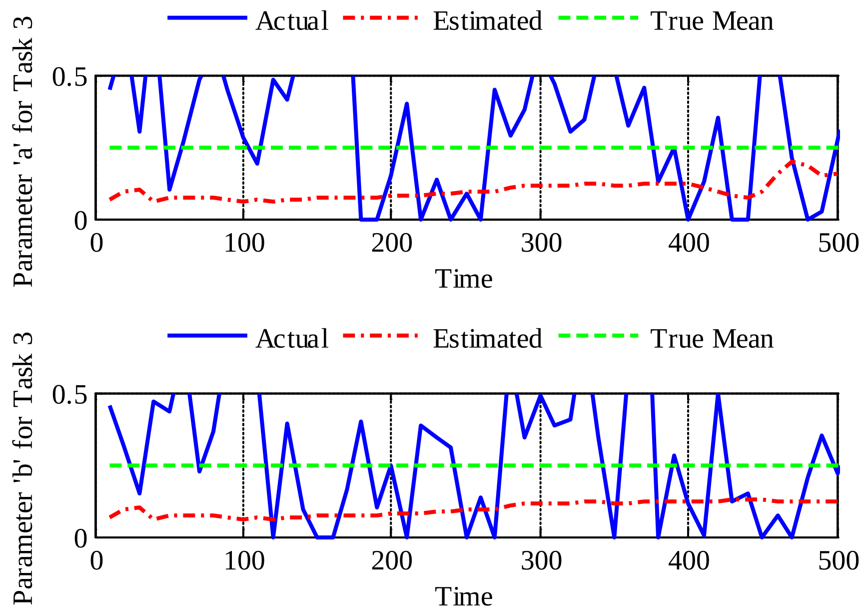


Figure 53 Convergence of the estimates for Task 3 in a stochastic system with COV 1.0.

It can be observed that in most of the cases the estimated values of the model parameters for each task converge to values close to the true values. Some estimation bias remains in some cases. Such a bias can be as observed in the estimates of parameters for Task 1 and Task 3 for a process with COV 1.0. This estimation bias could be due to the inclusion of measurement noise.

From all the above observations it can be inferred that the implemented Kalman Filter algorithm is capable of estimating the true values of the model parameters for both the deterministic and stochastic construction processes provided the process is within certain some process limits.

5. OPTIMAL CONTROL OF PROJECTS

The purpose of a project controller is to determine the resource inputs for the project that produce the desired progress performance over the remaining duration of the project. For construction projects the progress performance measures are the EAC and the EDAC. The desired project performance is: a) conformance to the project schedule requirements, i.e. the EDAC should be the same or – or at least close to – the planned duration or deadline, and b) minimum possible cost at completion (EAC). The implementation of the main component of a stochastic optimal controller - the objective based forecasting algorithm that can forecast or predict the above mentioned estimates-at-completion of the project given a proposed resource allocation schedule – has been discussed in the previous section. This section discusses the methodology for optimizing these estimates at completion.

It is not uncommon for construction projects to deviate from the planned or desired performance. The common cause of these deviations are: a.) the presence of errors in the original project plan or schedule due to uncertainties and errors in the assumed project models , and b.) unforeseen delays during the implementation phase. It can be assumed that the model uncertainty is limited to the corresponding model parameters – the progress model parameters a_i and b_i in (3.8), the control model parameter c_i in (3.7), and the cost model parameters rc_i and oc_i in (3.9) & (3.10).

Usually these deviations and hence the model uncertainties or errors are apparent only after some part of the project has been completed. Because of this assumption that the model error is limited to the model parameters it is possible that there exists a correlation between these deviations and errors in the model parameters. Hence by recursively using an appropriate filtering technique the assumed models can be refined by updating the corresponding model parameters so that they reflect the behavior of the actual project progress. Presently in this research it is assumed that the uncertainty is limited to the project progress model alone i.e. the progress model parameters a_i and b_i are assumed to be uncertain or unknown. These parameters a_i and b_i are considered to be the variable states of the project model and need to be estimated or updated to reflect the behavior of the actual project by tracking the actual progress trajectories of the sub-tasks using the Unscented Kalman Filter approach. Hence the project state $X(k)$ and measurable output $Z(k)$ at any discrete time k are as shown in (5.1).

$$X(k) = \begin{bmatrix} X_1(k) \\ X_2(k) \\ X_3(k) \end{bmatrix} \qquad Z(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

where, (5.1)

$$X_1(k) = \begin{bmatrix} x_1(k) \\ a_1(k) \\ b_1(k) \end{bmatrix} \qquad X_2(k) = \begin{bmatrix} x_2(k) \\ a_2(k) \\ b_2(k) \end{bmatrix} \qquad X_3(k) = \begin{bmatrix} x_3(k) \\ a_3(k) \\ b_4(k) \end{bmatrix}$$

As discussed in the previous sections the Kalman Filter forecasting method can be used to update the assumed progress model and forecast the EAC and EDAC at any point of time along the course of the project given a proposed resource allocation schedule for the remaining duration of the project. The objective of the optimal project controller is to find the resource allocation schedule that leads to the optimal values of EAC and EDAC as predicted by the Kalman Filter forecasting method. The optimal value of EAC is the minimum possible value. And the optimal value of EDAC is the closest value to DAC. Hence the general objective will be to minimize the EAC as well as the deviation of EDAC from DAC subject to any resource constraints. Using a penalty for the deviation of EDAC the formulation this optimization problem is (5.2).

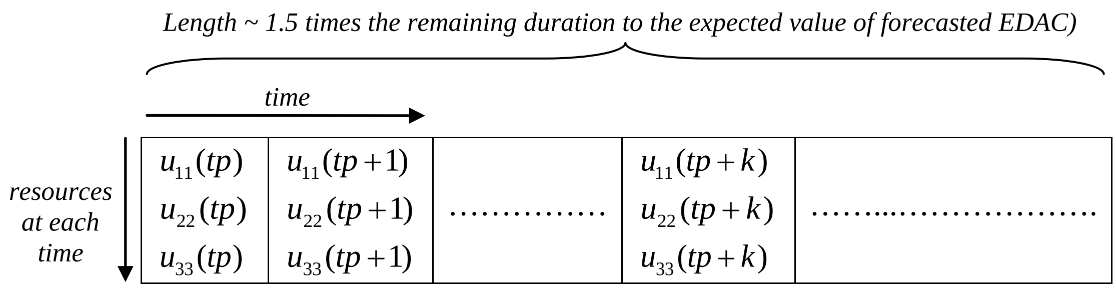
$$
\begin{aligned}
&\underset{U(tp)}{Minimize} \qquad EAC + M\left(EDAC - DAC\right)^2 \\
&s.t \qquad 0 \le U(tp) \le UU(tp) \\
&where, \\
&\quad U(tp) \triangleq \{u(tp), u(tp+1), \ldots\} \qquad \text{(the future resouce allocation)} \\
&\quad UU(tp) \triangleq \{uu(tp), uu(tp+1), \ldots\} \qquad \text{(the resouce constraints)} \\
&\quad M = 100(BAC) \quad for \ \text{EDAC} > \text{DAC} \quad \text{(the penalty)} \\
&\quad\quad = 0 \qquad for \ \text{EDAC} \le \text{DAC}
\end{aligned}
\tag{5.2}
$$

By iteratively varying the resource allocation plan for the remaining duration of the project it is quite possible to obtain a resource plan or schedule which leads to a forecast of minimum EAC, and EDAC close to DAC. Any of the evolutionary optimization algorithms would be an ideal and obvious choice for this purpose due their efficiency in dealing with large and complex nonlinear objective functions with multiple local optima. The following sections describe and demonstrate the use of such an optimal project controller for iteratively calculating the optimal resource allocation schedule using the evolutionary optimization algorithm known as Covariance Matrix Adaption Evolution Strategy (CMA-ES).

5.1 Covariance Matrix Adaption Evolution Strategy (CMA-ES)

CMA-ES is meant for iteratively and adaptively searching the solution space. The search step-size along each dimension is also evolved iteratively governed by a self-adaption technique (Hansen et al. 1995). Further, the correlations between the step-sizes along the various search dimensions are also evolved using a Covariance Matrix Adaption (Hansen and Ostermeier 1996). As such there is no particular reason for preferring CMA-ES over other evolutionary optimization algorithms. But, in this study it was observed that CMA-ES consistently outperforms Genetic Algorithm (GA) – the other commonly used evolutionary optimization algorithm – in terms of the rate of convergence to the solution as well as the efficiency in working with larger resource allocation schedules. Further, unlike GA, the particular CMA-ES used in this research is perfectly capable of choosing the best internal search strategy parameters with the exception of the population size.

To use the CMA-ES (or any other evolutionary algorithm) the first step is to define the structure of the population to be evolved. The Kalman Filter forecasting method has the resource allocation schedule as its input. Hence a population individual structure reflecting the time variation of the resources for each of the sub-task is the ideal and obvious choice. Further, for the working of the CMA-ES algorithm with various scenarios, the length of the string structure should be large enough to accommodate solutions – resource allocation schedules - which have EDAC much beyond the scheduled project duration or deadline. Hence the length – representing the time dimension – of the population structure is allowed to be 50% larger than the length required for the expected value of forecasted EDAC. Considering all these aspects, the structure of the individual solution used in the implementation is as shown in Figure 54.



Where, 'tp' is the present time – the end of model revision and time of forecast

Figure 54 The structure of the solution individual for the CMA-ES.

5.2 Application and Discussions

The main tracking, filtering and forecasting system based on Kalman Filter approach has been programmed in MATLAB. The project progress model (3.8) along with the project control model (3.6)&(3.7) has been programmed in FORTRAN and compiled as separate executables that can be called from the tracking and forecasting system in MATLAB. For the optimization method, an already available CMA-ES algorithm programmed in MATLAB (Hansen 2008) has been used as it is.

For first testing and then implementing the proposed optimal control method two different scenarios are considered for the actual progress of the project:

a) 200 time units have passed since the start of the test project, with the data about the actual project progress available with a sampling period of 10 time units; the assumed future resource allocation schedule is not optimal with resources allotted to the tasks even after their finish times, as shown in Figure 55, and

b) 200 time units have passed since the start of the test project, with the data about the actual project progress available with a sampling period of 10 time units; the Task 1 has been delayed beyond the planned schedule, assuming the actual values for both the parameters a_1 and b_1 as 0.20.

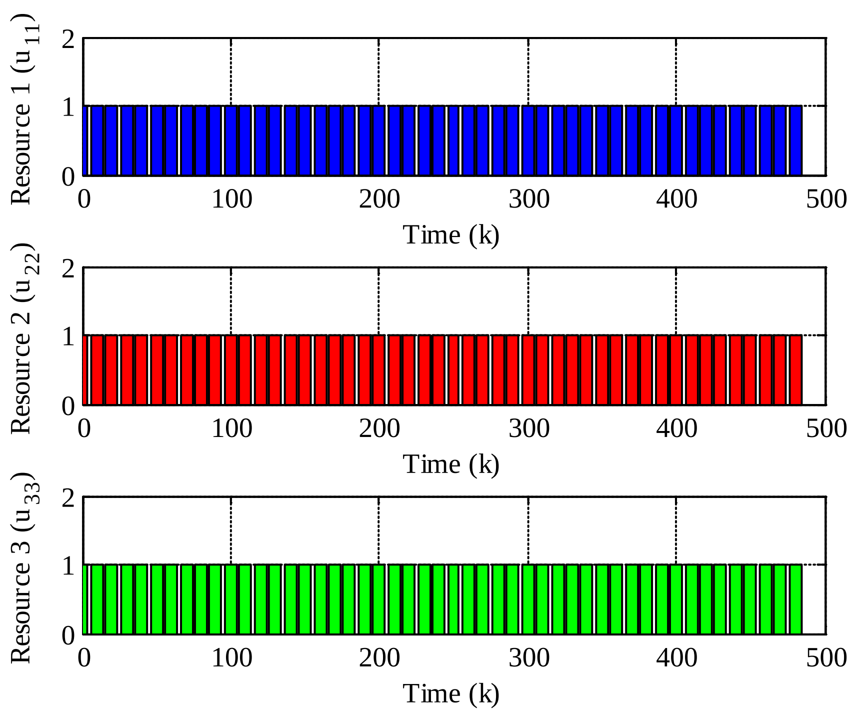


Figure 55 The future non-optimal resource allocation schedule for the test project.

The first sub-section discusses the results obtained from the optimization method for project progress scenario ‘a’ and their comparisons with the original optimal schedule. The second sub-section discusses the implementation of the optimal controller for progress scenario ‘b’ and the results obtained. The final sub-section discusses the generation of the Pareto Optimal Solution for the trade-off between EDAC and EAC. In all the tests and demonstrations the number of generations or iterations used in CMA-ES is 2000.

5.2.1 Testing the Optimization Method

After the initial tracking and revision period of 200 time units, the revised project progress model is used for forecasting the EAC and EDAC. The first scenario ‘a’ is chosen to test the proposed optimization method for the rate of convergence and effectiveness in optimizing the resource allocation schedule, before using it for optimal control. In this scenario both the original planned EAC and the expected value of the forecasted EAC were found to be 1691 cost units – as expected, higher than the optimal value of 1523. The theoretical optimal resource schedule and cost trajectory for the remaining duration is depicted in Figure 56 and Figure 57. The forecasted EDAC is 480. The planned, actual and forecasted project-cost trajectories for this scenario are shown in Figure 58.

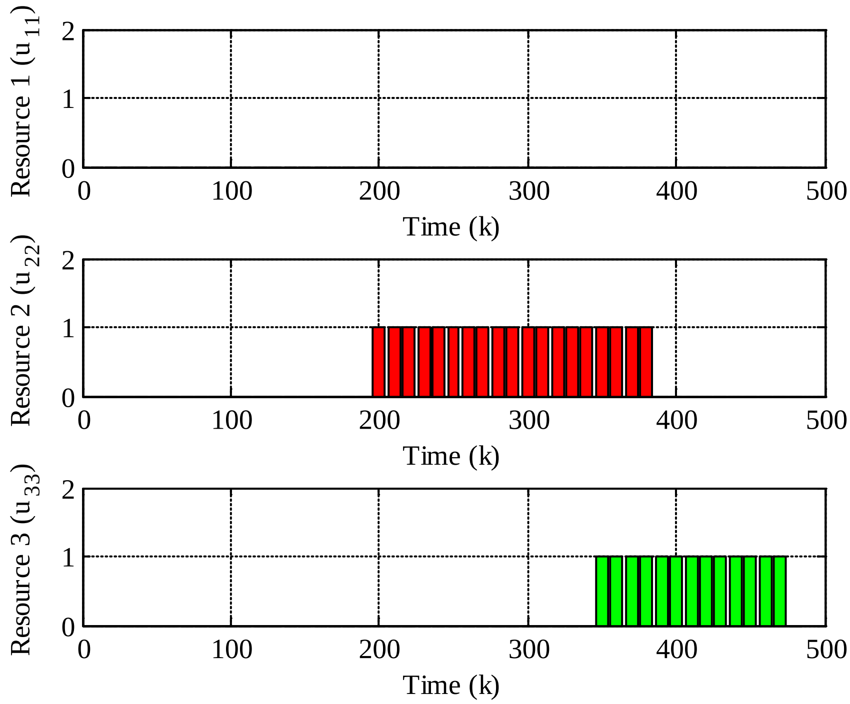


Figure 56 The theoretical optimal resource allocation schedule for rest of the project.

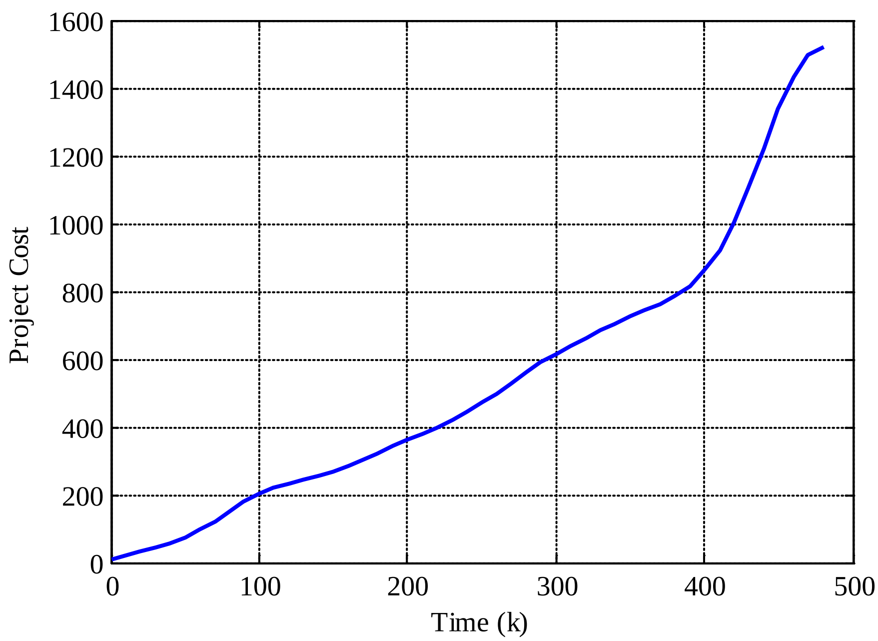


Figure 57 The theoretical optimal project-cost trajectory for the test project with scenario ‘a’ for actual progress where the future resource allocation is non-optimal.

The optimization algorithm is setup to minimize the expected value of the forecasted EAC with the constraint that the EDAC is closest to the originally planned duration of 480 time units while the resources have an upper constraint of 2. The non-optimal resource allocation schedule shown in Figure 55 is used as the initial solution for the CMA-ES optimization algorithm. The resultant optimal resource allocation schedule for the rest of the duration of the project is as shown in Figure 59. The planned, actual and optimized project-cost trajectories are shown in Figure 60.

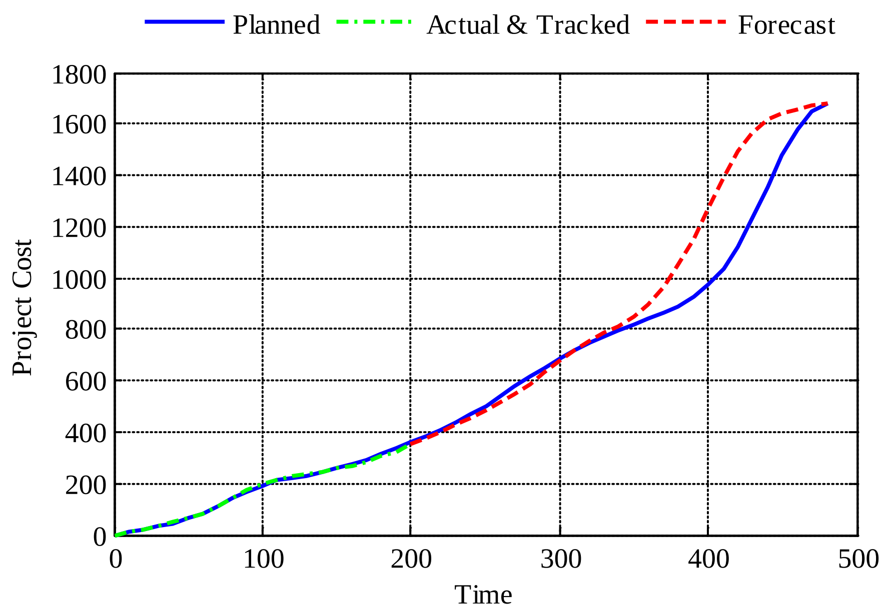


Figure 58 The planned, actual and forecasted project-cost trajectories for the test project with scenario ‘a’ for actual progress where the planned resource allocation is non-optimal.

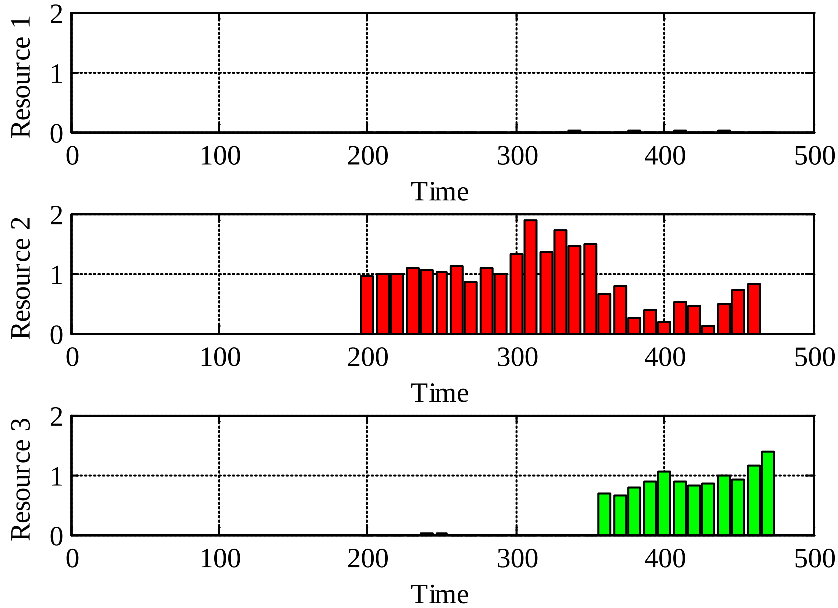


Figure 59 Optimal resource allocation schedule without leveling.

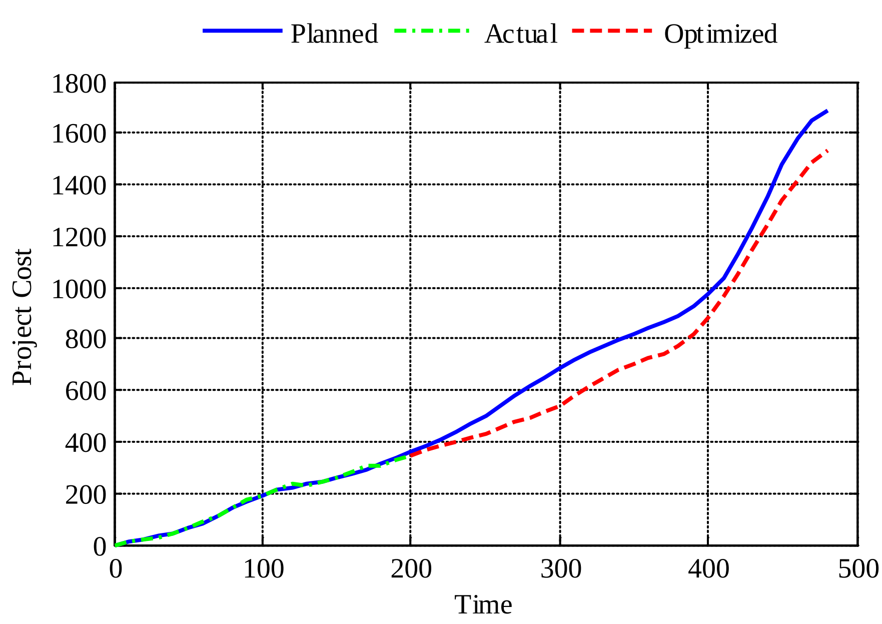


Figure 60 Planned, actual and optimized project-cost trajectories for optimal resource allocation without leveling – optimized expected EAC of 1531 against planned EAC of 1691 and theoretical optimal EAC of 1523.

The optimized cost of 1531 is much lower than the original planned cost of 1691 and closer to the theoretical optimal value of 1523 implying that the optimization algorithm is able to derive an optimal solution. It can be noted from Figure 59 that the resource allocation schedule for Task 1 has a 0 level throughout, which is expected.

The start and finish times for planned task progress and resources match for Task 3. But the resources for Task 2 as seen in Figure 59 seem to be much beyond the planned progress. The progress of Task 2 beyond the planned duration can be seen Figure 61. The reason for this behavior seems to be the dependency between Task 2 and Task 3. Since Task 3 can start after 90% of Task 2 has been complete, the optimization algorithm seems to have allotted a resource schedule that forces a high rate of progress of for Task 2 till 90% of it is complete and a very low rate after that till the end. Further, the finish time of Task 2 and Task 3 are the same as that of the EDAC.

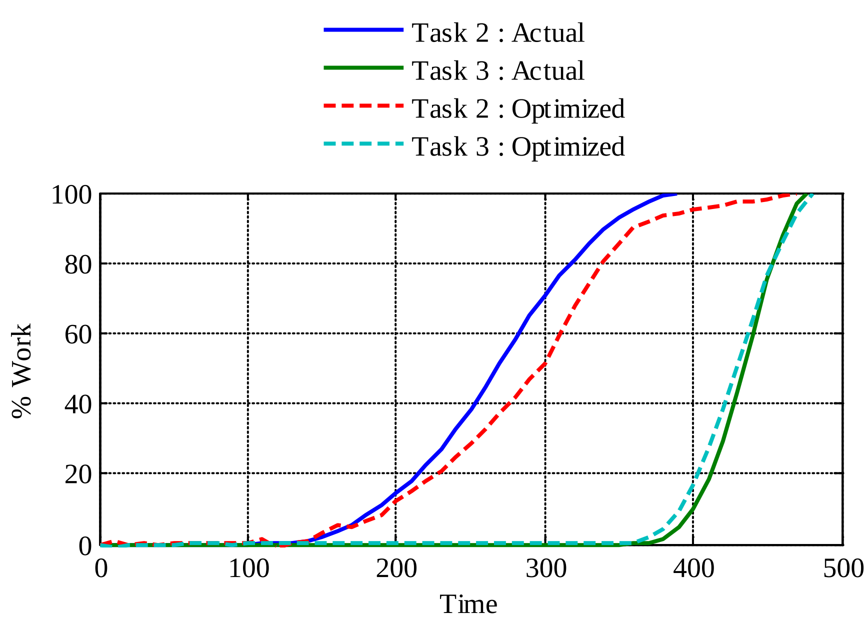


Figure 61 Optimized progress trajectories for Task 2 and Task 3.

In this first optimization setup, it can be noticed from Figure 59 that the resource allocation schedule is not a smooth graph. In reality it will be impractical to implement such a schedule since there is allocation and de-allocation of resources in every time period. Usually a smooth graph is desired with monotonously increasing or decreasing resource schedule. For implementing this aspect, a penalty was setup in the objective function that increases with the square of deviation between the allocated schedule and a smoothened - a moving average of the allocated schedule - form of the same. The resultant resource schedules and cost trajectories for a smoothing period of 5 time periods and 10 time periods are depicted in Figure 62 & Figure 63, and Figure 64 & Figure 65 respectively. It can be seen from these graphs that using a larger smoothing window size does give a smoother resource schedule but at a slightly higher estimated cost at completion.

Now that the results obtained are close and according to the theoretical results and explanations, it has been concluded that the optimization algorithm used is working as expected and can effectively be used for minimizing the cost to completion. Further other necessary aspects like the implementation of resource leveling and project schedule constraints have also been implemented and verified.

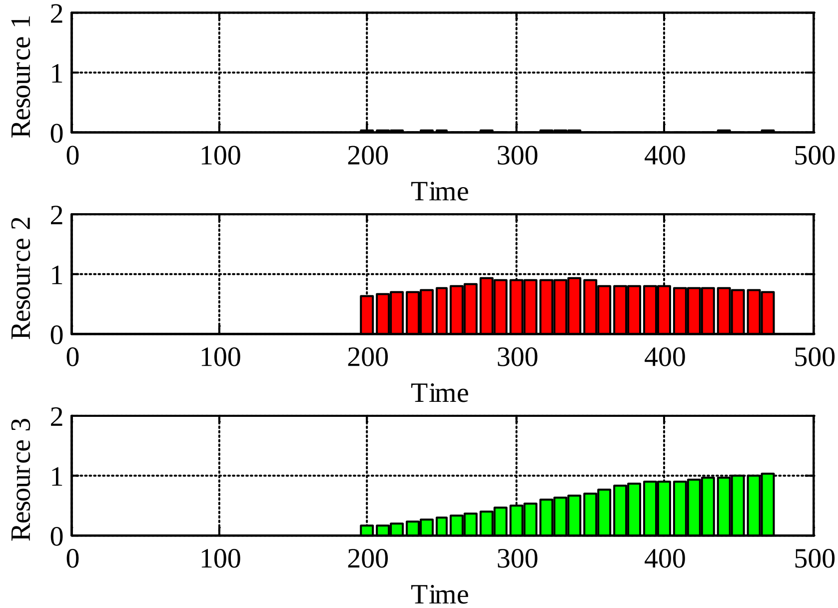


Figure 62 Optimal resource allocation schedule with leveling (smoothing over 5 periods).

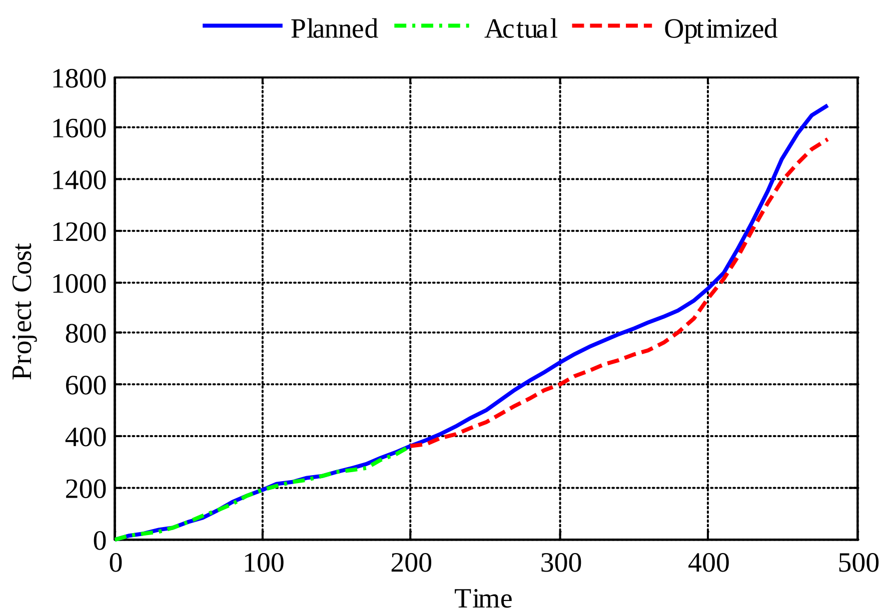


Figure 63 Planned, actual and optimized project-cost trajectory - optimal resource allocation schedule with leveling (smoothing over 5 periods).

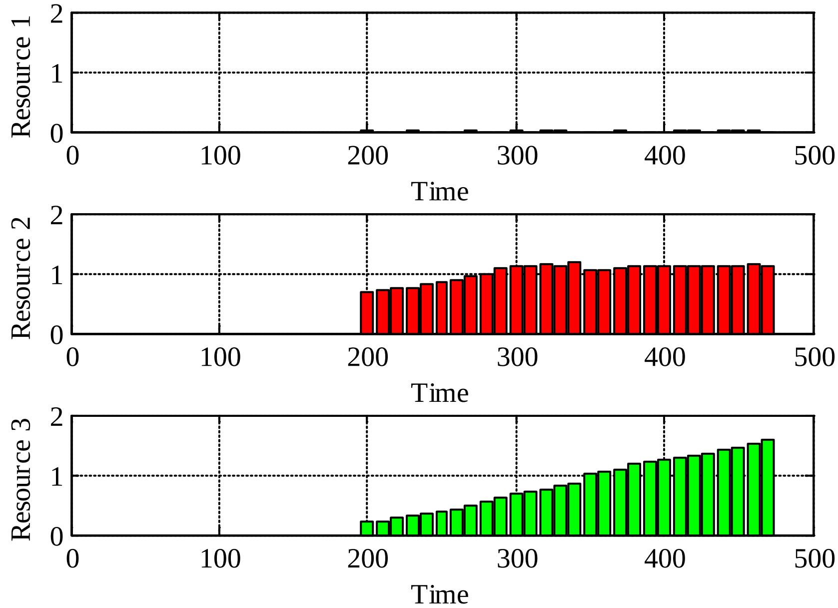


Figure 64 Optimal resource allocation schedule with leveling (smoothing over 10 periods).

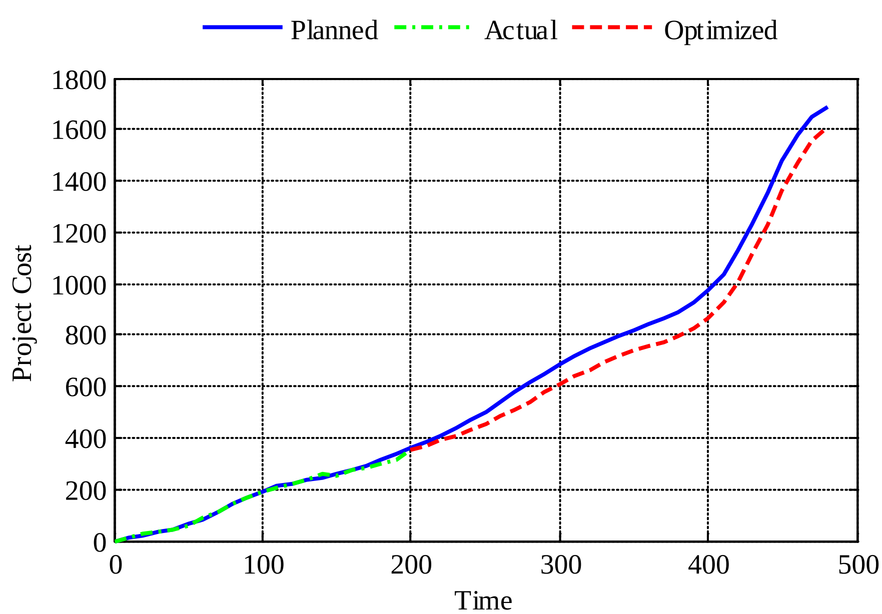


Figure 65 Planned, actual and optimized project-cost trajectory - optimal resource allocation schedule with leveling (smoothing over 10 periods).

5.2.2 *Optimal Project Control – Minimum Cost Control*

The second scenario 'b' is chosen for testing the ability of the controller to yield an optimal control policy in the form of an optimal resource allocation schedule that minimizes the overall project cost while ensuring conformance to the originally scheduled project duration. Due to delay in the Task 1 the resource allocation schedule shown in Figure 6 is not applicable and hence the schedule in scenario 'a' (Figure 55) is assumed as the revised future resource schedule. In this case the expected value of the forecasted EDAC is 570 and the expected value of the forecasted EAC is 1771. The planned, actual and forecasted BCWP trajectories for this scenario are shown in Figure 66. The use of the optimization algorithm for both scenarios is discussed in the subsequent section.

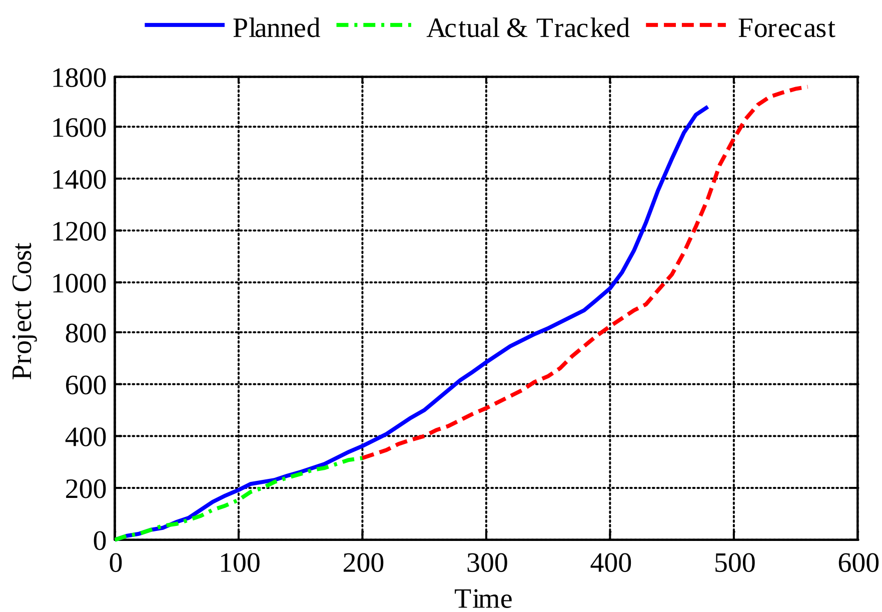


Figure 66 The planned, actual and forecasted project-cost trajectories for the test project with scenario 'b' for actual progress where the Task 1 had been delayed.

The optimization algorithm is setup to minimize the expected value of the forecasted estimated cost to completion with the constraint that the EDAC is closest to the originally planned duration of 480 time units while the resources have an upper contain of 2 units. The non-optimal resource allocation schedule shown in Figure 55 is used as the initial solution for the CMA-ES optimization algorithm. The resultant optimal resource allocation schedule for the rest of the duration of the project with a smoothing of 5 time periods is as shown in Figure 67. The planned, actual and controlled project-cost trajectories are shown in Figure 68.

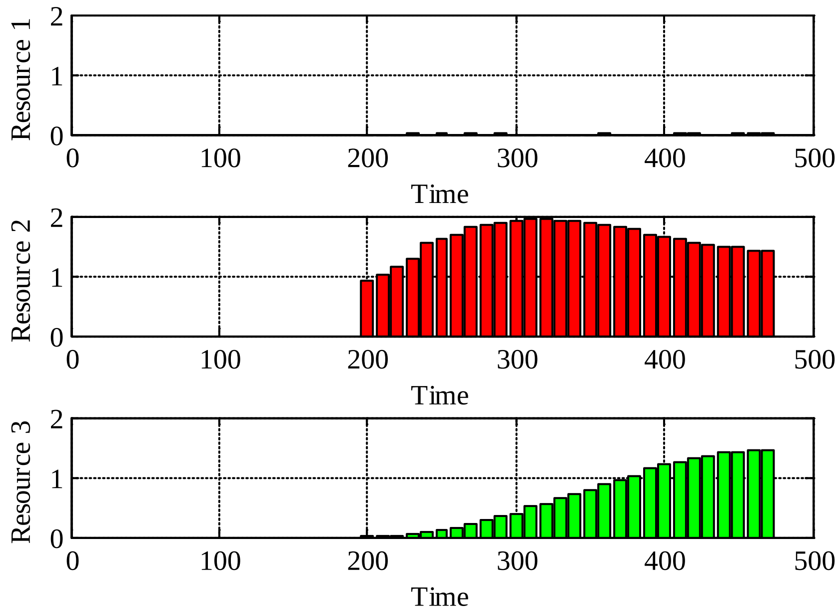


Figure 67 Optimal resource allocation schedule for optimal project control of test project.

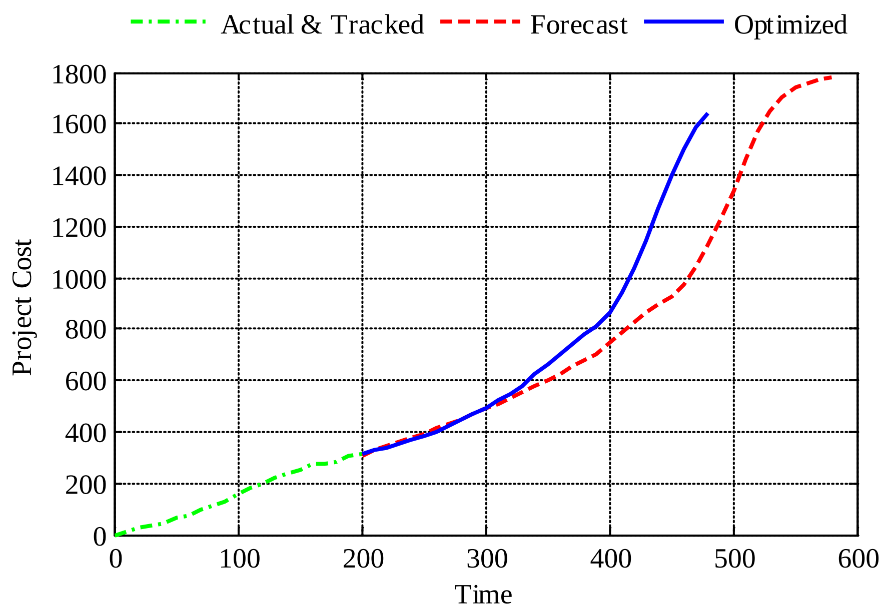


Figure 68 Planned, actual and controlled project-cost trajectories for optimal resource allocation for optimal project control of test project.

From these figures it can be noted that the optimal project control was able to allocate an optimal resource allocation schedule that controlled the project in such a way that the expected EDAC is the same as the planned value of 480.

5.2.3 Optimal Project Control – Time and Cost Trade-off Control

The Pareto Optimal Set representing the least feasible cost for a given project duration depicts the Time-Cost trade off for the project control. This is obtained by using a multi-objective CMA-ES to minimize the expected EAC and EDAC together. The time-cost trade-off for the test project as the Pareto Front between expected EAC and EDAC for actual project progress scenario 'b' is Figure 69.

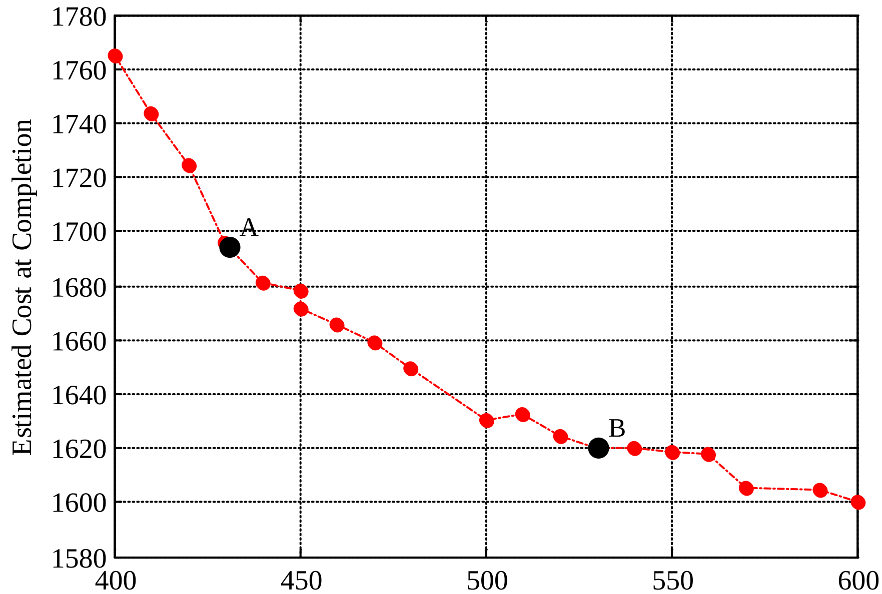


Figure 69 The Time-Cost trade-off for optimal control of the test project.

This time-cost trade-off curve can be useful for managers in making control decisions. The negative value of the slope tangent of the Pareto Front at any point represents the stability of that optimal control solution. A point with steeper slope (point A) for tangent implies a more unstable or risky solution in terms of the project cost since any deviation in the solution at this point might lead to a high variation in the project cost. A point with small slope (point B) for tangent implies a more stable solution in terms of the project cost since any deviation in the solution at this point might lead to a comparatively smaller variation in the project cost.

6. SUMMARY, CONCLUSIONS AND FUTURE WORK

6.1 Summary of the Research

1) The research has formulated and demonstrated a methodology for use of Kalman Filter for updating the project progress models. Three different types of project progress model have been tested for their tracking performance as well as suitability for forecasting. The implementation of the Kalman Filter was verified for both deterministic and stochastic project.
2) A general framework for optimal control of projects has been devised using an evolutionary optimization method – the CMA-ES – in conjunction with the Kalman Filter forecasting method. An implementation of the framework was demonstrated and tested using a hypothetical project as a numerical example.

6.2 Conclusions

1) The research has shown that any approximate project progress model can be updated by using the Kalman Filter for tracking and filtering the actual progress of the project. The tracking performance of the model depends on its ability to represent the dynamics of the project. A Sigmoid curve was observed to perform better than a polynomial model. But a model using the project Gantt chart as the basis has a much better tracking and forecasting

performance. Further it was shown that the process noise can be used as a fudging factor accounting for model specification error.

2) The general framework developed for optimal control of projects has shown to be very effective and useful as tool for devising and evaluating project control decisions. The use of evolutionary algorithm in conjunction Kalman Filter forecasting approach provides a robust framework that can be implemented for any kind of complex project model for yielding the optimal control policies.

6.3 Future Work

1) In the present research the proposed optimal control framework has be formulated for any kind of project with multiple spatially interdependent sub-tasks and the implementation has been verified to work with deterministic and stochastic systems. But the framework has only been demonstrated on a simple hypothetical project. Before formulating and implementing it for actual projects further verification is need using the data from any actual construction project.

2) The research only covered the formulation and implementation of optimal control in which only the project progress model is updated or revised using the observable data from the actual process. It was assumed that there is no error in the specification of the project control and project cost models. But to

make the framework more effective and extensive, it should be extended to include the revision of the project control and project cost models.

3) The optimal controller has been implemented to yield the minimum-cost as well as the cost-duration trade-off control. Since the optimization method is based on evolutionary techniques, it would be feasible to implement the minimum variance control by including the project control and project cost models into the Kalman Filter forecasting method. The minimum variance control can be devised to yield the trade-off relation between the cost, duration, cost-variance and the duration-variance.

4) In the present research it was demonstrated that using an appropriate value of process noise simpler polynomial and sigmoid models can be used to represent the progress of the construction process. The process noise and measurement noise used in the Kalman Filter were assumed to be known. Further, in the case of a stochastic process it is not possible to estimate the error in estimation along with the estimates of the values of the parameters. An appropriate future work would be to address the problem of estimating the process and measurement noise either form subjective estimates of the process or from the actual data available during the progress of the project.

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