PERFORMANCE ANALYSIS USING SEQUENTIAL DETECTION IN A SERIAL MULTI-HOP WIRELESS SENSOR NETWORK

A Thesis

by

DAE HYUN CHOI

Submitted to the Office of Graduate Studies of Texas A&M University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2008

Major Subject: Electrical Engineering

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Approved by:

Chair of Committee,	Jean-François Chamberland
Committee Members,	Henry Pfister
	Alexander Sprintson
	Natarajan Gautam
Head of Department,	Costas Georghiades

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ABSTRACT

Performance Analysis Using Sequential Detection in a Serial Multi-Hop Wireless Sensor Network. (August 2008)

Dae Hyun Choi, B.E., Korea University, Seoul

Chair of Advisory Committee: Dr. Jean-François Chamberland

Wireless sensor networks (WSNs) have been developed for a variety of applications such as battlefield surveillance, environment monitoring, health care and so on. For such applications, the design of WSN has been limited by two main resource constraints, power and delay. Therefore, since wireless sensors with a small battery are subject to strict power constraints, the efficient usage of power is one of the important challenges. As delay-sensitive applications are emerging, they have been in demand for making a quick decision with the enhanced detection accuracy. Under above constraints, we propose a sequential detection scheme and compare it with a Fixed-sample-size (FSS) detection scheme in terms of power and delay. Our main contribution is to analyze the overall system performance of the proposed scheme in the statistical signal processing framework under of power and delay constraints.

In this thesis, we evaluate the overall system performance of sequential detection scheme in a serial multi-hop WSN topology. For sequential detection, the sensor nodes continue to relay the observations to the next node until the sequential detector makes a final decision based on the observations. On the other hand, the FSS detector waits until all the observations come to the fusion center, and then gives a final decision. For a fair comparison of the two schemes with respect to power and delay, the initial step is to find the same detection performance region between the two schemes. Detection performance is evaluated with performance measures such as false alarm, miss and prior probability. Simulation results show that each scheme has an advantage and a disadvantage concerning power and delay respectively. That is, sequential detection performs more efficiently in delay since the number of samples in sequential detection is less on average than in FSS detection to obtain the same detection performance. However, FSS detection with a small number of packet paths consumes less power than sequential detection. Through the analysis of a cost function, which is a linear combination of power and delay, we compare the cost value between the two schemes and find less region of the cost value in both schemes. This analysis will provide a good starting point and foundation for designing an efficient multi-hop WSN with small power and delay constraints. To My Family

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CHAPTER I

INTRODUCTION

Initially, wireless sensor network (WSN) technologies have emerged as a battlefield surveillance application. However, with the surge of communication research efforts, the spectrum of this application has significantly enlarged from environment monitoring into areas such as traffic control and even health care. A representative application in WSN is event tracking, which has wide usage in various settings. As such, detection accuracy has been one of the standards for evaluating system performance. WSNs faces the dual challenge of extending the network lifetimes and reducing communication delays. Since wireless sensor nodes are battery-driven and operate on a frugal energy budget, power management for WSN has become one of the primary methods for prolonging the lifetime of a network. For transferring delay-sensitive data, sensor nodes are required to rapidly detect a target and quickly relay the observation packets to a destination, which is called the fusion center. In addition, decision times at the fusion center affect overall delays. To overcome these challenges, novel detection schemes and improved analysis models are required. In this thesis, we study a sequential detection scheme and investigate the overall system performance of a sequential detection in the integrated framework of statistical signal processing with power and delay constraints.

In general, statistical signal processing refers to a methodology to infer generalizations from empirical data. Detection theory is a key aspect of statistical signal processing and has been developed over a half-century by numerous researchers [1, 2, 3]. There are two well-known representative detection schemes. First, *centralized de-*

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tection assumes that the local sensors communicate all observations to a central processing entity, called fusion center, which makes a final decision based on these observations. This scheme provides a better detection accuracy at the expense of excessive communication cost. The second scheme is decentralized or distributed de*tection* where each sensor makes a decision locally and sends the quantized data to the fusion center, which admits a final decision. This scheme has some advantages such as decreased communication bandwidth and cost. Yet, it entails additional processing power at each sensor and features a worse detection performance than the centralized detection scheme. Other schemes with random sample size draw a line between fixed-sample-size (FSS) detection and sequential detection. Unlike a FSS detector where the number of observations is predetermined, a sequential detector can make an early decision, that is to say, take less observations on average to achieve the same detection performance as a FSS detector. In most settings, the aforementioned strategies are performed by smart sensors that know the statistics of the observations. Recently, type-based detection with dumb sensors with no knowledge of the observation statistics has been proposed by Liu and Sayeed [4]. This scenario requires a relatively small bandwidth and provides detection performance similar to that of a centralized detection.

In this thesis, we emphasize power efficiency and detection accuracy rather than the efficient use of communication bandwidth. Therefore, we focus on a centralized detection framework. In this framework, a sequential detection scheme over a multihop WSN is proposed and compared with a FSS detection scheme in terms of power consumption and delay performance. There are two reasons to choose multi-hop system model and two different detection schemes. First, a WSN is usually required for multi-hop topologies because transmission power at each sensor increases nonlinearly in the distance between sensors or between a sensor and the fusion center. Moreover, the sensors with a small battery have a narrow transmission range and need relay nodes to send their observation packets to the fusion center. Also, the issue of which detection scheme is applied to a multi-hop WSN can influence power and delay performance as well as detection accuracy.

Our main work is composed of two parts. The first part is to analyze detection performance of sequential detection and compare it to FSS detection. The result of this part later provides a standard to fairly compare our schemes with each other in terms of power and delay. For the evaluation of detection performance, a Bayesian framework is selected to derive optimality criteria for both schemes. The initial step for the analysis of detection performance is to numerically derive optimal decision rules for each scheme and simulate detection performance based on the derived decision rules. The second part is to compute average power and delay in each scheme and compare them. The purpose of this part is to find which scheme consumes a smaller amount of power or has less delay while achieving the same detection performance. Simulation results show that each scheme has advantages and disadvantages with respect to power and delay. Finally, we compare the performance of both schemes in terms of a joint cost function of power and delay. This suggests that, as either delay or power are emphasized more, either sequential detection or FSS detection can be deemed more appropriate for implementation than the other.

A common and specific system model for our schemes is described as follows. A common system model is physically composed of four parts: the first sensor node (the farthest to the fusion center), the intermediate sensor nodes (between the first sensor and the last sensor), the last sensor node (the nearest to the fusion center) and the fusion center. In the case of sequential detection, at the first time slot, each sensor node transmits its own observation packet to the next sensor node toward the fusion center. From the second time slot to the last time slot, all the sensor nodes receiving observation packets from the previous node relay them to the next one until the last observation packet reaches the fusion center. On the other hand, for FSS detection, the observation taken at each sensor is combined dynamically with previous observation to decrease overhead. The observations are appended to the current observation packet as it is being relayed toward the fusion center. At the first time slot, the first sensor node transmits its own observation packet to the next one. At the second time slot, the first intermediate sensor node receiving an observation packet from the first sensor node combines it with its own observation taken and then sends the resulting packet to the next intermediate sensor node. This procedure continues until the fusion center receives the last observation packet. Here, we assume that all the sensor nodes have access to a single observation at a time, and until all the observation packets transmitted by the sensor nodes arrive at the fusion center. In addition to the above assumption, all the sensor nodes except the last sensor node are not permitted to directly transmit the observation packets to the fusion center.

Sequential detection is different from traditional detection with fixed-sample-size in that the number of observations required by the sequential test has a random value depending on the realization of the observation process. Classical sequential detection analysis dates back to Wald's work [5]. At each step, the sequential detector can either admit one of the two hypotheses (H_0, H_1) or wait and take an additional observation. The design of the sequential detector needs a sequential decision rule composed of a pair of functions: a stopping rule and a terminal decision rule. A stopping rule is a procedure that informs us when to stop taking observations, and a stopping time is a time when a stopping rule decides to finish taking observations. A terminal decision rule is a function that makes a decision when even the fusion center elects to stop taking observations. Decentralized sequential detection problems have also been well developed under the concept of classical sequential detection. It has been shown in the literature [6, 7] that the function of sequential detection can be performed at the sensor or the fusion center and in each case, different local decision rules and fusion decision rules are derived. Recently, a hybrid detection scheme has been suggested [8]; a sensor sends a one-bit local decision as in a decentralized detection scheme if the likelihood ratio function exceeds a predetermined threshold, and if not so, as in a centralized detection scheme the sensor sends all observations. In some sense, this scheme is similar to a sequential detection in that the number of observations is random according to a predetermined threshold.

There have been significant advances in the analysis of WSN. Classic detection theory, starting with a simple hypothesis test, has been incorporated into the area of decentralized detection. Numerous papers [9, 10, 11, 12, 13] have investigated decentralized detection under constraints such as bandwidth and multiple access channel (MAC). For different WSN topologies, serial network decentralized detection [14, 15, 16] and parallel network decentralized detection [17, 18] have been studied extensively. In particular, previous researchers [6, 7, 19] have conducted indepth investigations of sequential detection. Recently, decentralized detection over non-ideal channel models has emerged as an area of interest [20, 21, 22, 23].

Nevertheless, little research exists for sequential detection with power and delay constraints. Some related works [24, 25, 8] on sequential detection in the context of energy-efficient WSN design shed new light on the analysis of sequential detection with resource constraints. However, all of the previous works have focused on the distributed detection scheme. In addition, they assume that intermediate sensor nodes only relay observation packets to the next sensor node without detecting a target. Some of them [24, 25] investigate system performance for the simplified onehop WSN. The formulation we adopt in this thesis is new. Our framework seeks to address several issues and challenges: temporal and spatial correlation among the sensor observations, multi-hypothesis likelihood ratio test, various topologies with flexibility, and aggregation data toward the fusion center. Our goal is to design a WSN model with improved detection accuracy under small power and delay constraints, and to get a general formulation for performance analysis in spite of these challenges.

A. Contributions of Thesis

Our task is to analyze the system performance of a multi-hop WSN by using a sequential detection scheme with resource constraints on power and delay. Consider two proposed serial multi-hop WSN models. One is to perform sequential detection at the fusion center and the other is to conduct FSS detection at the same location. To maintain a tractable problem, we assume that the overall system is subject to ideal channels, and sensors have identical transmission power. Suppose that we have a simple binary hypothesis test where H takes on one of two possible values, and the observations cross sensors are independent and identically distributed (i.i.d.), conditioned on H. We study and compare detection performance between two models with various system parameters in the Bayesian framework and then analyze overall system performance in terms of power and delay. A linear combination of power and delay is defined as a cost function, the analysis of which will pave the way for designing efficient multi-hop WSNs that require less power and reduce delay simultaneously.

B. Organization of Thesis

The remainder of the thesis is organized as follows. In Chapter II, we study the concept of detection theory. We also discuss two general framework, the Bayesian formulation and the Neyman-Pearson formulation. Binary hypothesis test is then introduced in each formulation. Chapter III presents a description of a basic multihop WSN model and our two proposed models are described in a centralized detection scheme perspective. For the analysis of power efficiency, we borrow a transmission power formulation from information theory literature. Chapter IV discusses classical sequential detection and introduces a mathematical formulation for this problem in the Bayesian framework. In Chapter V, we derive the optimal decision rules with different observation models and we compute an expression for energy consumption. Chapter VI surveys our schemes utilizing numerical results from Chapter V, and presents a power and delay performance analysis for our systems. Finally, we provide our conclusions and future tasks in Chapter VII.

CHAPTER II

DETECTION THEORY AND OPTIMALITY CRITERIA

In general, statistical signal processing refers to the act of inferring generalizations from empirical data with uncertainty. In statistical problem, we are interested in an unknown parameter which we can denote by θ . We have access to an observation Y that provides partial information about the value of θ . The relationship between θ and Y is probabilistic in nature. The abstract framework of statistical problem can be illustrated in Figure 1. This framework is composed of three components.

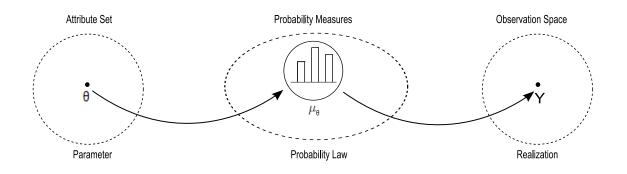


Fig. 1. Detection framework

- Attribute set: It consists of all admissible values of θ . This set is denoted by U.
- Measurable space (\mathcal{F}, Ω) : It provides a mathematical basis for the stochastic nature of the observations. Here, \mathcal{F} is the σ -algebra of all probability events and Ω represents the universal sample space.
- Observation space (Γ): It is the collection of all realizable observations.

If the attribute set is partitioned into a finite number of subsets and the objective is to identify which subset θ belongs to, this problem is called detection or hypothesis testing. We refer to the different subsets as hypotheses, and label them as H_1, H_2, \ldots If the attribute set contains only two elements, the function of the detectors is to distinguish between the corresponding two hypotheses. The ensuing problem is called binary detection (or binary hypothesis testing), which is our main focus for the analysis of our models. Figure 2 shows the abstract framework of binary detection. Based

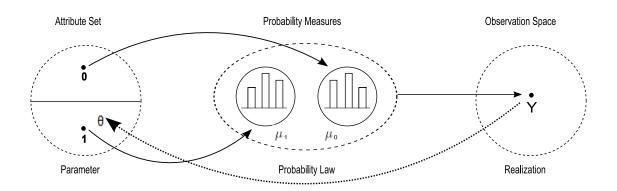


Fig. 2. Binary detection framework

on this binary detection framework, we study hypothesis testing problem under two representative settings, the Bayesian formulation and the Neyman-Pearson formulation.

A. Bayesian Hypothesis Testing

In the Bayesian framework, the attribute set is assumed to be a probability space with a known distribution. We assume that the true parameter θ is equal to 0 with probability $\gamma(0)$, and it is equal to 1 with probability $\gamma(1) = 1 - \gamma(0)$. The probability measure on the elements of U is called the *a priori distribution*, which means the knowledge we have about the parameter before getting empirical measurements. The performance criteria in the Bayes formulation is to find a detector that minimizes the Bayesian risk. The Bayesian risk is the expected value of the cost, and it can be computed as

$$R = C_{00}\gamma(0)\int_{\Gamma} \mathbf{1}_{\Gamma_{0}}d\mu_{0} + C_{10}\gamma(0)\int_{\Gamma} \mathbf{1}_{\Gamma_{1}}d\mu_{0} + C_{01}\gamma(1)\int_{\Gamma} \mathbf{1}_{\Gamma_{0}}d\mu_{1} + C_{11}\gamma(1)\int_{\Gamma} \mathbf{1}_{\Gamma_{1}}d\mu_{1}.$$
(2.1)

Here, $C_{i,j}$ is the cost incurred by choosing hypothesis H_i when hypothesis H_j is true. Using conditional probabilities defined by the above integrals, we get

$$P_F = \Pr(\hat{H} = 1 \mid H = 0) = \int_{\Gamma_1} d\mu_0$$
$$P_D = \Pr(\hat{H} = 1 \mid H = 1) = \int_{\Gamma_1} d\mu_1$$
$$P_M = \Pr(\hat{H} = 0 \mid H = 1) = \int_{\Gamma_0} d\mu_1.$$

Under the assumption that the cost of an erroneous decision is higher than the cost of a correct decision ($C_{10} > C_{00}$ and $C_{01} > C_{11}$), the optimal decision rule is equal to

$$\frac{d\mu_1}{d\mu_0}(y) \overset{H_1}{\underset{H_0}{\gtrless}} \frac{\gamma(0)(C_{10} - C_{00})}{\gamma(1)(C_{01} - C_{11})}.$$

B. Neyman-Pearson Hypothesis Testing

In some situations, it may be difficult to model parameter θ as a random variable. Also, it may be impractical to assign realistic costs to the four detection outcomes. The Neyman-Pearson formulation to binary hypothesis testing provides an alternative to the Bayesian formulation. In the Neyman-Pearson framework, θ is viewed as an unknown deterministic parameter. The problem is defined in terms of the probability of false alarm P_F and the probability of detection P_D . The goal can be stated as follows:

$$\max P_D \text{ subject to } P_F \leq \alpha.$$

Using Lagrange multiplier methods, we wish to maximize the function

$$P_D + \lambda (P_F - \alpha) = \int_{\Gamma_1} \left(\frac{d\mu_1}{d\mu_0} + \lambda \right) d\mu_0 - \lambda \alpha.$$
(2.2)

Equation (2.2) leads us to the optimal decision structure

$$\frac{d\mu_1}{d\mu_0} \underset{H_0}{\overset{H_1}{\gtrless}} \eta,$$

where η is the smallest threshold value such that

$$P_F = \int_{\Gamma_1} d\mu_0 \le \alpha;$$

and Γ_1 is the region given by

$$\Gamma_1 = \left\{ y \in \Gamma \Big| \frac{d\mu_1}{d\mu_0}(y) > \eta \right\}.$$

CHAPTER III

SYSTEM MODEL

We introduce the serial multi-hop topology we wish to use in Section A. The basic system structure is extended to our proposed schemes, as described in Section B. One of our main goals is to study the power consumption of our schemes. To this end, we develop a general formulation for system performance evaluation in Section C.

A. Basic System Structure

We consider a serial multi-hop wireless network with multiple sensors and one fusion center. All the observations taken by the sensor nodes are transmitted via intermediate relay nodes to the fusion center, which makes a final decision based on the information gathered. Our basic system model is illustrated in Figure 3.

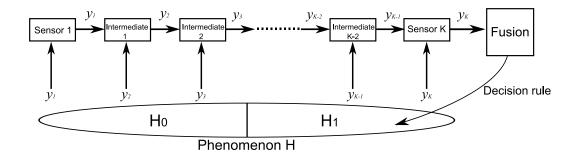


Fig. 3. Block diagram of a basic system model for a serial multi-hop WSN.

For our system model, we make the following assumptions for analytical tractability.

• The link between any two sensor nodes and the link between the sensor node and the fusion center are needed as idealized channels.

- We have a simple binary hypothesis test, where *H* takes on one of two possible values. The observations at a sensor node and across sensor nodes are independent and identically distributed (i.i.d.) conditioned on *H*.
- For ease of computation, we only consider the transmission power at each node and assume that the sensor nodes consume an identical amount of energy per transmission.
- B. Two Serial Multi-hop WSN Models

1. Sequential Detection Topology

An abstract representation of the sequential detection scheme for five sensors is shown in Figure 4. Each arrow is labeled by the transmitted observation packet composed

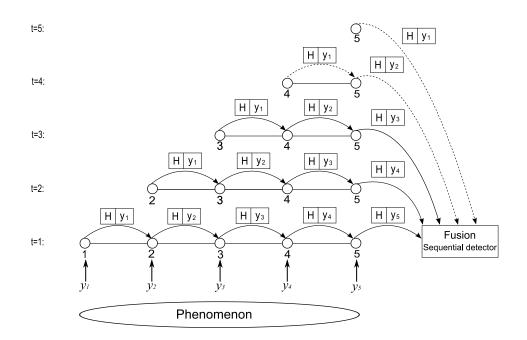


Fig. 4. Sequential detection topology.

of the packet header and the observation payload. We assume a time-slotted com-

munication scheme. In first time slot, all the sensor nodes send their observation packets to the next sensor node (or the fusion center for the last node). In all the following time-slots, sensor nodes relay the received observation packets towards the fusion center. It is important to notice that a sequential detector is not required to take all the observations sent by the sensor nodes to make a decision. In Figure 4, the solid line and the dotted line denote the actual path and the potential path of the observation packets respectively. This figure shows the case where the sequential detector makes a decision after acquires three observations. This fact follows from the definition of sequential detection; the related mathematical formulation is explained in Chapter IV.

2. FSS Detection Topology

Next, we consider the FSS detection scheme, which is depicted in Figure 5. In this

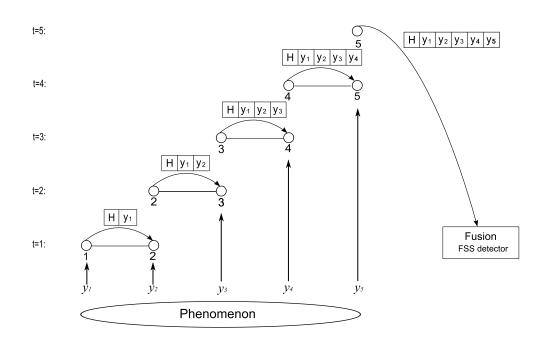


Fig. 5. FSS detection topology

scenario, there is only one packet transmission in every time-slot. Each relay node waits for an observation packet to arrive. Upon its arrival, an additional observation payload is attached to the packet header by the relay node corresponding to its gathered data. We assume that the size of the packet payload is relatively small compared to its header. Hence, we regard a packet with multiple observations having the same size as a single observation packet. In contrast to sequential detection, Figure 5 shows that FSS detection requires all the observations taken by the sensors before making a decision. Under this assumption, the FSS detection scheme is equivalent to the parallel detection scheme illustrated in Figure 6. However, the corresponding equivalence can only hold if both detectors take all the observations gathered by the sensors. For example, one of the serious problems in a serial network is a link failure between the nodes. If a link fails in a serial multi-hop network, the detector loses all previous observations and detection performance suffers. Clearly, a parallel network faces no such predicament.

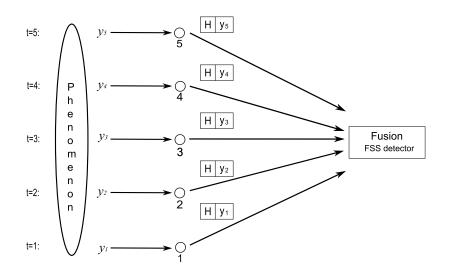


Fig. 6. Parallel detection topology equivalent to FSS detection topology.

C. Power Formulation

Energy optimization over WSN with small battery sensors plays an important role in improving the lifetime of the overall system. Since the sensor nodes are subject to strict power constraints, it is necessary to investigate the interplay between power and overall performance in these networks [26, 27]. Energy aware WSN has been discussed extensively in the literature [28]. According to a specific study with respect to the energy of WSN [28], the system architecture of a sensor is made up of four subsystems:

- MICRO CONTROLLER UNIT (MCU): This unit is in charge of controlling sensors, executing signal processing based on observed data, and it provides three modes of operating: active, idle, and sleeping.
- RADIO: It is responsible for communicating with sensors and operates in the following four modes: transmit, receive, idle and sleeping. Modulation methods, transmission rates and power, and operational duty cycles all affect power consumption of the communication unit.
- SENSOR: Sensor traducers convert a physical condition to an electrical signal and perform the following tasks: signal sampling, analog to digital conversion.
- POWER SUPPLY: It supplies the power to the other subsystems.

In the information theory literature, the relationship between achievable rates and signal power has been studied extensively [29]. For a bandlimited additive white Gaussian noise channel, the achievable rate C is given by

$$C = W \log_2 \left(1 + \frac{l(d)P}{N_0 W} \right) \text{ bits per second,}$$
(3.1)

where W is the bandwidth, N_0 is the noise spectrum density, l(d) is the path loss function with distance d between nodes, and P is the transmission power [30]. This formula determines the minimum power required to obtain a given rate. From equation (3.1), the transmission power of each sensor can be expressed as

$$P = N_0 W d^3 (2^{\frac{C(t)}{W}} - 1) \text{ watts}, \qquad (3.2)$$

where we have used the path loss function $l(d) = d^{-3}$. In our work, we assume that all the sensors have the same power. We use the transmission power of each sensor to compute the total power and provide meaningful comparisons. That is, using equation (3.2), we derive the total power employed by the proposed sequential detection and FSS detection schemes.

CHAPTER IV

SEQUENTIAL DETECTION ANALYSIS

In the current theory of hypothesis testing, the number of observations, i.e. the size of the samples on which the test is based, is treated as constant. As mentioned before, we call this fixed-sample-size (FSS) detection. An distinguishing characteristic of the sequential test from the standard test procedure is that the number of observations demanded by the sequential test depends on the outcome of the observation process and is, therefore, not predetermined, but a random value. For some observation realizations, a decision can be made taking a small number of observations, while for others, the process of taking observations is extended before making a decision. This chapter is devoted to classic sequential detector. Then, we formally define the sequential detection problem and show how it can be solved using the sequential probability ratio test (SPRT).

A. Definition of Sequential Detection

The sequential method of testing a hypothesis H can be described as follows. A rule is selected that allows one of the following three decisions at any stage of the experiment: (1) accept hypothesis H, (2) reject hypothesis H, (3) continue the experiment by taking an additional observation. After the first observation is taken, one of these three actions is performed. If a decision is made, the process is terminated. Otherwise, the detector recursively takes into account the next observation. This process is continued until either the first or the second options are selected. The number of observations, N, required by such a test procedure is a random variable because the value of N depends on the outcome of the observations.

B. Problem Formulation

We focus on sequential detection in the Bayesian framework [1]. A binary sequential detector can be described formally on a binary attribute set U. A sequential detector has access to an observation sequence $\{Y_k; k = 1, 2, ...\}$ at discrete times k. It is assumed that given the true hypothesis, the observations are conditionally independent and identically distributed according to

$$H_0: Y_k \sim \mu_0, \qquad k = 1, 2, \dots$$

 $H_1: Y_k \sim \mu_1, \qquad k = 1, 2, \dots$

The objective of a sequential detector is to decide whether to admit one of the two hypotheses upon availability of a new observation or wait for the information contained in the consecutive observation. This objective is accomplished by deriving a sequential decision rule. The sequential decision rule (Φ, Σ) is composed of a stopping rule $\Phi = \{\phi_n : n \in \mathbb{N}\}$ with $\phi_n: \{Y_1, Y_2, \ldots, Y_n\} \to \{0,1\}$, and the terminal decision rule $\Sigma = \{\sigma_n : n \in \mathbb{N}\}$ with $\sigma_n: \{Y_1, Y_2, \ldots, Y_n\} \to \{0,1\}$. The stopping rule Φ is a procedure that informs us when to stop taking observations based on the stopping time. The stopping time N is a random variable given by

$$N = \min\{n \in \mathbb{N} | \phi_n(Y_1, \dots, Y_n) = 1\}.$$
(4.1)

The terminal decision rule Σ makes a decision based on the available data. In short, the sequential decision rule (Φ, Σ) makes no decision while k < N and the detector takes new observations. The stopping rule Φ stops the process when $\phi_k(Y_1, \ldots, Y_k) =$ 1 for this time, and the terminal decision rule Σ makes a decision, H_0 or H_1 , at this time. For $i \in \{0, 1\}$, hypothesis H_i is selected if and only if $\sigma_N(Y_1, \ldots, Y_N) = i$.

C. Sequential Probability Ratio Test

Wald conjectured the structure of the sequential decision rule (Φ, Σ) [5]. He called this decision rule the sequential probability ratio test (SPRT). Let us define $p_n(Y_1, \ldots, Y_n)$ as the probability of hypothesis H_1 being the true state of attribute given n observations $\{Y_1, Y_2, \ldots, Y_n\}$

$$p_n(Y_1, \dots, Y_n) = P(H_1 \text{ is the true hypothesis } | Y_1, \dots, Y_n).$$
(4.2)

Using Bayes theorem, we can recursively compute $p_n(Y_1, \ldots, Y_n)$ as follows

$$p_n(Y_1, \dots, Y_n) = \frac{P(Y_1, \dots, Y_n \mid H_1)P(H_1)}{P(Y_1, \dots, Y_n)}$$

= $\frac{P(Y_1, \dots, Y_n \mid H_1)P(H_1)}{P(Y_1, \dots, Y_n \mid H_1)P(H_1) + P(Y_1, \dots, Y_n \mid H_0)P(H_0)}$
= $\frac{P(Y_1, \dots, Y_{n-1} \mid H_1)P(Y_n \mid H_1)P(Y_n \mid H_1)P(H_1)}{P(Y_1, \dots, Y_{n-1} \mid H_1)P(Y_n \mid H_1)P(H_0) + P(Y_1, \dots, Y_{n-1} \mid H_1)P(Y_n \mid H_0)P(H_0)}.$

The last equation is obtained because the observations are independent conditioned on the true hypothesis. If we define the likelihood ratio at the k^{th} observation as

$$L(Y_k) = \frac{P(Y_k \mid H_1)}{P(Y_k \mid H_0)} = \frac{d\mu_1}{d\mu_0}(Y_k),$$
(4.3)

then the above relation for $p_n(Y_1, \ldots, Y_n)$ can be further simplified to an iterative equation as

$$p_{n}(Y_{1},...,Y_{n}) = \frac{P(Y_{1},...,Y_{n-1} \mid H_{1})L(Y_{n})P(H_{1})}{P(Y_{1},...,Y_{n-1} \mid H_{1})L(Y_{n})P(H_{1}) + P(Y_{1},...,Y_{n-1} \mid H_{0})P(H_{0})} = \frac{L(Y_{n})P(H_{1} \mid Y_{1},...,Y_{n-1})P(Y_{1},...,Y_{n-1})}{L(Y_{n})P(H_{1} \mid Y_{1},...,Y_{n-1})P(Y_{1},...,Y_{n-1}) + P(H_{0} \mid Y_{1},...,Y_{n-1})P(Y_{1},...,Y_{n-1})} = \frac{L(Y_{n})P(H_{1} \mid Y_{1},...,Y_{n-1})}{L(Y_{n})P(H_{1} \mid Y_{1},...,Y_{n-1}) + 1 - P(H_{1} \mid Y_{1},...,Y_{n-1})}.$$

$$(4.4)$$

or simply

$$p_n(Y_1, \dots, Y_n) = \frac{L(Y_n)p_{n-1}(Y_1, \dots, Y_{n-1})}{L(Y_n)p_{n-1}(Y_1, \dots, Y_{n-1}) + 1 - p_{n-1}(Y_1, \dots, Y_{n-1})}.$$
 (4.5)

Suppose η_L and η_U are lower and upper thresholds respectively, where $\eta_L < \eta_U$. The thresholds η_L and η_U should be chosen such that the sequential decision rule yields the desired performance. The stopping rule of the SPRT for the *n*th observation is

$$\phi_n(Y_1, \dots, Y_n) = \begin{cases} 0 & \text{if } \eta_L < p_n(Y_1, \dots, Y_n) < \eta_U \\ 1 & \text{otherwise.} \end{cases}$$
(4.6)

The stopping rule can be iteratively computed using equation (4.5). We choose a stopping rule such that if $p_n(Y_1, \ldots, Y_n) \notin (\eta_L, \eta_U)$, then the process is stopped. Similarly, the terminal decision rule becomes

$$\sigma_n(Y_1, \dots, Y_n) = \begin{cases} 1 & \text{if } p_n(Y_1, \dots, Y_n) \ge \eta_U \\ 0 & \text{if } p_n(Y_1, \dots, Y_n) \le \eta_L, \end{cases}$$
(4.7)

and it can be computed using the iterative form of p_n .

In the Bayesian framework where a prior probability measure on the attribute set is given, the SPRT defined in the stopping rule (4.6) and the terminal decision rule (4.7) can be expressed in terms of the likelihood ratio of equation (4.4) and a priori probabilities $\gamma(0)$ and $\gamma(1)$. First, we need to find the iterative relation for updating $p_n(Y_1, \ldots, Y_n)$ based on the sequence of likelihood ratios and the priori probabilities,

$$p_n(Y_1, \dots, Y_n) = \frac{\gamma(1) \prod_{k=1}^n \frac{d\mu_1}{d\mu_0}(Y_k)}{\gamma(0) + \gamma(1) \prod_{k=1}^n \frac{d\mu_1}{d\mu_0}(Y_k)}$$

$$= \frac{\gamma(1) \prod_{k=1}^n L(Y_k)}{\gamma(0) + \gamma(1) \prod_{k=1}^n L(Y_k)}.$$
(4.8)

Using equation (4.8), the stopping rule (4.6) and the terminal decision rule (4.7) can

be rewritten as

$$\phi_n(Y_1, \dots, Y_n) = \begin{cases} 0 & \text{if } \tilde{\eta_L} < \prod_{k=1}^n L(Y_k) < \tilde{\eta_U} \\ 1 & \text{otherwise.} \end{cases}$$
(4.9)

Similarly, the terminal decision rule (4.7) can be expressed as

$$\sigma_n(Y_1, \dots, Y_n) = \begin{cases} 1 & \text{if } \prod_{k=1}^n L(Y_k) \ge \tilde{\eta_U} \\ 0 & \text{if } \prod_{k=1}^n L(Y_k) \le \tilde{\eta_L}, \end{cases}$$
(4.10)

where $\tilde{\eta}_L = \frac{\gamma(0)\eta_L}{\gamma(1)(1-\eta_L)}, \ \tilde{\eta}_U = \frac{\gamma(0)\eta_U}{\gamma(1)(1-\eta_U)}.$

To derive an optimal sequential detector, we assign costs to our decisions. The terminal cost is represented by nonnegative numbers c(i, j) for $i, j \in \{0, 1\}$, where c(i, j) is the cost incurred by choosing hypothesis H_i when hypothesis H_j is true. The incremental observation cost is C > 0 for each sample we take so that the cost of taking N samples is CN. The risk function of a sequential detector is the total expected cost resulting from the sequential detection procedure as follows:

$$R(\phi, \underline{\delta}) = E[c(\phi_N(Y_1, \dots, Y_N), j) + CN], \qquad (4.11)$$

where the expectation is with respect to the true hypothesis H_j and to the realization of the sequences Y_1, Y_2, \ldots Finally, we can find an optimal sequential decision rule (Φ, Σ) by minimizing the risk function.

For the problem of detecting a constant signal in additive i.i.d. noise, sample paths of stopping time given H_0 or H_1 are illustrated in Figure 7, and the results on the graphs are plotted for the parameters of Table I. We assume that observations are distributed according to a Gaussian distribution. Mathematically, Figure 7 can be translated into sequential detection rules as follows:

(1) Figure 7(a)

- Stopping rule : $\phi_0 = 0, \ldots, \phi_{12}(Y_1, \ldots, Y_{12}) = 0, \phi_{13}(Y_1, \ldots, Y_{13}) = 1$
- Decision rule : $\sigma_{13}(Y_1, \ldots, Y_{13}) = 0$;

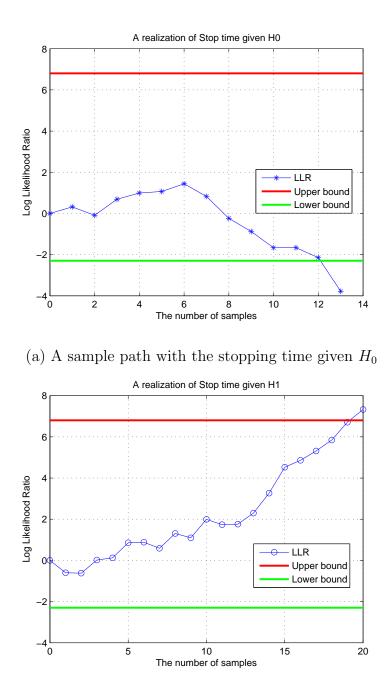
(2) Figure 7(b)

- Stopping rule : $\phi_0 = 0, \ldots, \phi_{19}(Y_1, \ldots, Y_{19}) = 0, \phi_{20}(Y_1, \ldots, Y_{20}) = 1$
- Decision rule : $\sigma_{20}(Y_1, \dots, Y_{20}) = 1$.

Figure 7(a) shows that the sequential detector takes 13 observations and makes decision H_1 . On the other hand, in Figure 7(b), the sequential detector takes 20 observations and make decision H_0 .

 $\theta = 2$ Constant signal value $\sigma^2 = 9$ Noise variance $\tilde{\eta_U} = 6.8$ Upper bound in sequential detection $\tilde{\eta_L} = -2.3$ Lower bound in sequential detection

Table I. Simulation parameters for a realization of stopping time.



(b) A sample path with the stopping time given H_1 Fig. 7. A realization of a Bayes sequential test given H_0 and H_1 .

CHAPTER V

NUMERICAL ANALYSIS

To fairly compare power and delay performance in both schemes, we need to have the same detection performance in the two cases. In this chapter, we study two specific observation models in our schemes: Gaussian observations and Bernoulli observations. In the context of centralized detection, local sensor nodes take observations and relay them to the next sensor node or to the fusion center without quantization. The fusion center performs a Likelihood Ratio Test (LRT) to make a final decision. Hence, an optimal fusion decision rule for each scheme is derived numerically to evaluate the detection performance. The derivation of this decision rule at the fusion center is conducted in the Bayesian framework assuming a uniform cost, i.e. $C_{00}=C_{11}=0$ and $C_{01}=C_{10}=1$. To analyze the sequential detector, a sequential algorithm is introduced. We assume that each sensor node shares the same bandwidth, transmits one packet per second, and is subject to the same noise spectrum density. The power consumed by each sensor node depends only on its transmission power equation (3.2). Based on these assumptions, an expression for the total power consumed by either scheme is derived.

A. Decision Rule

Since our schemes perform different detection at the fusion center, we derive one fusion decision rule per scheme. Moreover, the two different observation models require their own decision rule. Thus, four decision rules are derived in this chapter. To derive the decision rule, the first step is to obtain likelihood ratio function.

1. Gaussian Observations

Consider the problem of detecting a constant signal in additive noise where the observations are conditionally independent and identically distributed according to

$$H_0: Y_k = N_k,$$
 $k = 1, 2, ...$
 $H_1: Y_k = N_k + \theta,$ $k = 1, 2, ...$

with $\theta > 0$ and $\{N_k\}_{k=1}^{\infty}$ is a sequence of i.i.d. Gaussian random variables, each with distribution, $\mathcal{N}(0, \sigma^2)$. For FSS detection, the decision rule is derived as follows,

$$\lambda_n(y_1, \dots, y_n) = \prod_{k=1}^n \left[\frac{d\mu_1(y_k)}{d\mu_0(y_k)} \right]$$

=
$$\prod_{k=1}^n \left[\frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(y_k - \theta)^2}{2\sigma^2}\right) \right] / \left[\frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{y_k^2}{2\sigma^2}\right) \right] \qquad (5.1)$$

=
$$\exp\left[\frac{\theta}{\sigma^2} \sum_{k=1}^n \left(y_k - \frac{\theta}{2} \right) \right] \stackrel{H_1}{\gtrless} \frac{\gamma(0)}{\gamma(1)} = \eta.$$

where $d\mu_0$, $d\mu_1$ and $\gamma(0), \gamma(1)$ are conditional probability distributions and priori distributions corresponding to H_0 and H_1 , respectively; and η is a threshold. Taking the logarithm of both sides, we get

$$\Lambda_n(y_1,\ldots,y_n) = \frac{\theta}{\sigma^2} \sum_{k=1}^n \left(y_k - \frac{\theta}{2} \right).$$
 (5.2)

Since $\theta > 0$, we have

$$Y_{n} = \sum_{k=1}^{n} y_{k} \underset{H_{0}}{\overset{H_{1}}{\gtrless}} \frac{\sigma^{2}}{\theta} \ln \eta + \frac{\theta}{2} n = \eta'$$

$$(5.3)$$

with

$$Y_n \sim \begin{cases} \mathcal{N}(0, n\sigma^2) \text{ under } H_0 \\ \mathcal{N}(n\theta, n\sigma^2) \text{ under } H_1. \end{cases}$$
(5.4)

Using (5.3) and (5.4), we obtain expressions for the false alarm probability P_F ,

the detection probability P_D in terms of the new threshold η' ,

$$P_F = P(Y_n > \eta' \mid H_0) = Q\left(\frac{\eta'}{\sqrt{n\sigma^2}}\right)$$
(5.5)

$$P_D = P(Y_n > \eta' \mid H_1) = Q\left(\frac{\eta' - n\theta}{\sqrt{n\sigma^2}}\right)$$
(5.6)

$$\eta' = \frac{\sigma^2}{\theta} \ln \eta + \frac{\theta}{2} n.$$
 (5.7)

We can write the probability of error ${\cal P}_e$ as

$$P_e = \gamma(0)Q\left(\frac{\frac{\sigma^2}{\theta}\ln\eta + \frac{\theta}{2}n}{\sqrt{n\sigma^2}}\right) + \gamma(1)\left[1 - Q\left(\frac{\frac{\sigma^2}{\theta}\ln\eta - \frac{\theta}{2}n}{\sqrt{n\sigma^2}}\right)\right].$$
 (5.8)

For the derivation of the decision rule in sequential detection, the log-likelihood ratio function of (5.2) can also be used. However, this function has different thresholds from FSS detector. Both a lower bound $\tilde{\eta}_L$ and an upper bound $\tilde{\eta}_U$ are needed. The corresponding sequential detector is given by

$$\tilde{\eta}_L \leq \Lambda_n(y_1, \dots, y_n) \leq \tilde{\eta}_U,$$
 Take more samples
 $\Lambda_n(y_1, \dots, y_n) \leq \tilde{\eta}_L,$ Decision H_0 (5.9)
 $\Lambda_n(y_1, \dots, y_n) \geq \tilde{\eta}_U,$ Decision $H_1.$

It is difficult to derive P_F and P_M analytically since there is no closed-form expression for the distribution of the stopping time N. Since N is a random variable, P_e is derived differently from the FSS case. We can write it as

$$P_{e} = \sum_{n=1}^{\infty} E[\mathbf{1}_{\{N=n\}} \mathbf{1}_{\{\sigma_{n} \neq H\}}]$$

$$= \sum_{n=1}^{\infty} [\{1 - \gamma(1)\} E[\mathbf{1}_{\{N=n\}} \mathbf{1}_{\{\sigma_{n} = 1\}} | H = H_{0}] + \gamma(1) E[\mathbf{1}_{\{N=n\}} \mathbf{1}_{\{\sigma_{n} = 0\}} | H = H_{1}]]$$

$$= \sum_{n=1}^{\infty} [\{1 - \gamma(1)\} E_{0}[\mathbf{1}_{\{N=n\}} \sigma_{n}] + \gamma(1) E_{1}[\mathbf{1}_{\{N=n\}} (1 - \sigma_{n})]]$$

$$= \{1 - \gamma(1)\} E_{0}[\sigma_{N}] + \gamma(1) E_{1}[(1 - \sigma_{N})]$$

$$= \{1 - \gamma(1)\} E_{0}[\sigma_{N}] + \gamma(1) (1 - E_{1}[\sigma_{N}])$$

$$= \{1 - \gamma(1)\} P_{F} + \gamma(1) P_{M}.$$
(5.10)

We resort to Monte Carlo simulation to evaluate and hence estimate approximate values for P_F and P_M , P_e . Next, we compute E[N] and $E[N^2]$ since these values are used to compute the total power of our schemes. We can express them for our schemes as

$$E[N] = \left[\sum_{i=1}^{n_{\text{seq}}-1} iP_N(i)\right] + n_{\text{seq}} \sum_{i=n_{\text{seq}}}^{\infty} P_N(i)$$
(5.11)

$$E[N^2] = \left[\sum_{i=1}^{n_{\text{seq}}-1} i^2 P_N(i)\right] + (n_{\text{seq}})^2 \sum_{i=n_{\text{seq}}}^{\infty} P_N(i),$$
(5.12)

where P_N is the PMF of N and n_{seq} is the number of sensors involved in sequential detection. The results of equations (5.11) and (5.12) are due to the fact that, for a multi-hop network, the number of sensors is always greater or equal to the stopping time. Hence, a new stopping time \tilde{N} is defined as

$$\tilde{N} = \min(n_{\text{seq}}, N). \tag{5.13}$$

In our simulation, we set $m = 10^5$ as the number of rounds. Using the recursive formula of equation (5.14), we compute the values of the sequential log-likelihood

function exceeding the thresholds and then obtain the values of P_F , P_M , $E[\tilde{N}]$ and $E[\tilde{N}^2]$.

$$\Lambda_{k+1}(y_1,\ldots,y_{k+1}) = \Lambda_k(y_1,\ldots,y_k) + \frac{\theta}{\sigma^2} \left(y_{k+1} - \frac{\theta}{2} \right).$$
 (5.14)

More specifically, the sequential detection algorithm is operated as follows:

- 1. Fix upper lower bounds.
- 2. Generate Gaussian distribution given H_0 and H_1 .
- 3. Compute and update the log-likelihood ratio function as follows
 - while $(\Lambda(N) > \tilde{\eta}_L \text{ and } \Lambda(N) < \tilde{\eta}_U)$ N = N + 1;Update $\Lambda(N)$ according to equation (5.14) end;
- 4. Stopping time is computed and decision rule is applied
 - if $(N > n_{seq})$ $\tilde{N} = n_{seq};$

Apply FSS decision rule;

 $\bullet~{\rm else}$

 $\tilde{N} = N;$

Apply Sequential decision rule;

- 5. Compute average value of P_F , P_M and $E[\tilde{N}]$, $E[\tilde{N}^2]$ with 10⁵ iterations.
- 6. Go back to step 1 and increase lower bound.

2. Bernoulli Observations

We consider a sequence of Bernoulli observations distributed according to

$$P(Y_k = 1|H_0) = p_0$$

 $P(Y_k = 1|H_1) = p_1.$

For sequential detection in the Bayesian framework, the decision rule is derived as follows. For a single sample, the likelihood ratio function is

$$\lambda(y_i) = \left(\frac{p_1}{p_0}\right)^{y_i} \left(\frac{1-p_1}{1-p_0}\right)^{1-y_i} = \left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right]^{y_i} \left(\frac{1-p_1}{1-p_0}\right).$$
(5.15)

Suppose that the observations are conditionally i.i.d. random variables, then the likelihood ratio function for n observations becomes

$$\lambda_n(y_1, \dots, y_n) = \prod_{k=1}^n \left[\frac{p_1(1-p_0)}{p_0(1-p_1)} \right]^{y_i} \left(\frac{1-p_1}{1-p_0} \right)$$

= $\left[\frac{p_1(1-p_0)}{p_0(1-p_1)} \right]^{\sum_{i=1}^n y_i} \left(\frac{1-p_1}{1-p_0} \right)^n.$ (5.16)

Taking the logarithm of both sides, the log-likelihood ratio becomes

$$\Lambda_n(y_1, \dots, y_n) = \ln \left[\frac{p_1(1-p_0)}{p_0(1-p_1)} \right] \sum_{i=1}^n y_i + n \ln \left(\frac{1-p_1}{1-p_0} \right).$$
(5.17)

From equation (5.17), we obtain a recursive formula for the log-likelihood ratio,

$$\Lambda_{k+1}(y_1, \dots, y_{k+1}) = \Lambda_k(y_1, \dots, y_k) + \ln\left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right] y_{k+1} + \ln\left(\frac{1-p_1}{1-p_0}\right).$$
 (5.18)

For deriving the decision rule of the FSS detector in the Bayesian framework, we go back to equation (5.16), where $\sum_{i=1}^{n} y_i$ denotes the number of "1" out of the total number of samples "n". Therefore, it can also be expressed as follows,

$$\lambda_n(y_1, \dots, y_n) = \left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right]^t \left[\frac{1-p_1}{1-p_0}\right]^n \underset{H_0}{\overset{H_1}{\geq}} \frac{\gamma(0)}{\gamma(1)} = \eta,$$
(5.19)

where $\sum_{i=1}^{n} y_i = t$. Taking the logarithm of both sides, we get

$$t \underset{H_0}{\gtrless} \left[\ln\left(\frac{\gamma(0)}{\gamma(1)}\right) + n \ln\left(\frac{1-p_0}{1-p_1}\right) \right] / \ln\left[\frac{p_1(1-p_0)}{p_0(1-p_1)}\right] = \eta'.$$
(5.20)

Since a detector makes a decision based on the number of "1" out of a total samples "n", the decision rule is a function of the binomial distribution. The probabilities P_F and P_M are

$$P_F = P(t > \eta' \mid H_0) = \sum_{i=\eta'}^n \binom{n}{i} p_0^i (1 - p_0)^{n-i}$$
(5.21)

$$P_M = P(t < \eta' \mid H_1) = \sum_{i=0}^{\eta'-1} \binom{n}{i} p_1^i (1-p_1)^{n-i}$$
(5.22)

and therefore

$$P_e = \gamma(0) \sum_{i=\eta'}^{n} \binom{n}{i} p_0^i (1-p_0)^{n-i} + \gamma(1) \sum_{i=0}^{\eta'-1} \binom{n}{i} p_1^i (1-p_1)^{n-i}.$$
 (5.23)

The procedure to estimate P_F , P_M , E[N], and $E[N^2]$ is based on the sequential detection algorithm, except that the observations are generated using a Bernoulli distribution and equation (5.18).

B. Total Power Analysis

We only consider the transmission power in analyzing the total power consumption of our schemes. The transmission power is proportional to distance d between adjacent sensors,

$$P(d) = Ad^{\alpha} \tag{5.24}$$

where A is constant and α is the path loss exponent. For FSS detection, the total number of observations is simply equal to the number of sensors. Under the assumption that all sensors use identical transmission power, the total power with the number of sensor nodes n_{fix} can be computed as:

$$P_{\text{Total},\text{FSS}} = \sum_{k=1}^{n_{\text{fix}}} A d^{\alpha}$$

= $n_{\text{fix}} A d^{\alpha}$. (5.25)

The other scheme is to perform sequential detection at the fusion center and to consecutively relay packets. Hence, the total power depends on $E[\tilde{N}]$ and n_{seq} :

$$E[P_{\text{Total,Seq}}] = E\left\{\sum_{k=1}^{\tilde{N}} [n_{\text{seq}} - (k-1)]Ad^{\alpha}\right\}$$

= $\frac{Ad^{\alpha}}{2}E[\tilde{N}(2n_{\text{seq}} - \tilde{N} + 1)]$
= $\frac{Ad^{\alpha}}{2}\{(2n_{\text{seq}} + 1)E[\tilde{N}] - E[\tilde{N}^{2}]\}.$ (5.26)

In our simulation, we choose $\alpha = 3$ and A = 1. When the spacing between sensor nodes is identical, the value of the total power can be simplified to the total number of hops necessary for packets to reach the fusion center. That is, computing the total power is equal to counting the total number of hops traversed by packets to arrive at the fusion center, until the detector makes a final decision. We apply this assumption to our schemes for numerical results.

CHAPTER VI

SIMULATION RESULTS

In this chapter, the overall system performance of sequential detection is simulated and compared with FSS detection. Section A shows general features of a centralized sequential detection. In Section B, we evaluate the detection accuracy, the power and the delay performance of our proposed sequential detection scheme. In addition, the results of the cost analysis reveal which scheme consumes less cost than the other as either the delay or power are emphasized.

A. General Features of Sequential Detection

The results of our simulation in Section A are obtained from the Gaussian observation model explained in Chapter V and the parameters of Table II . Figures 8 and 9 show

Table II. Simulation parameters: Gaussian observations for the sequential detection scheme.

$\gamma(0) = \frac{1}{2}, \gamma(1) = \frac{1}{2}$	Prior distributions
$\theta = 1$	Constant signal value
$\ln \tilde{\eta_U} = 0.1$	Upper bound in sequential detection
$\ln(\tilde{\eta_L}) = \ln[0.1:0.05:0.95]$	Lower bound range in sequential detection

the relationship between the lower bound and the average stopping time E[N], and they also demonstrate the relationship between the lower bound and the probability of error P_e . In Figure 8, it is observed that an increase in the lower bound reduces E[N]. It is consistent with our expectation that, as the gap between the bounds narrows, the sequential detector needs less observations to make a decision.

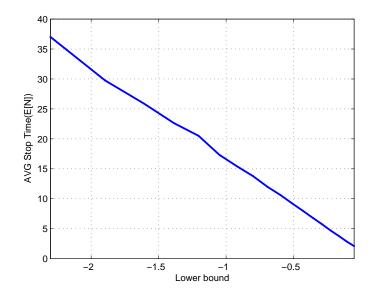


Fig. 8. Lower bound vs average stopping time with SNR=0.01.

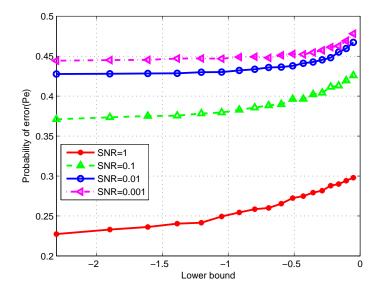


Fig. 9. Lower bound vs probability of error for sequential detection with varying SNR from 0.001 to 1.

Figure 9 shows that as the lower bound increases, P_e also increases. This fact can be explained by the results of Figure 8. An increase in the lower bound reduces E[N] and consequently results in worse detection performance. As expected, P_e has a smaller value in the high SNR than in the low SNR. Finally, Figure 10 shows that, on

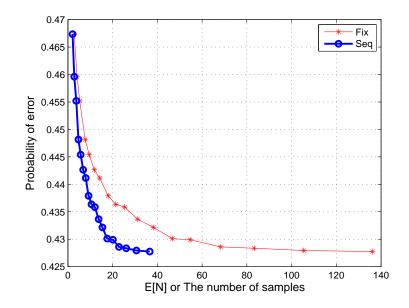


Fig. 10. Average number of samples vs probability of error: Gaussian observation with SNR=0.01.

average, sequential detection requires less observations than FSS detection to obtain the same detection performance. It indicates that using sequential detection reduces waiting time, and consequently decreases overall delay. The fact that this result is also maintained in a multi-hop WSN will be discussed in the next section.

B. Performance Analysis of Proposed Sequential Detection

In this section, we focus our analysis on the comparison of our schemes with respect to power and delay. In Section 1, we simulate detection and power performance as we change the number of sensor nodes in the sequential detection setting. Two observation models based on the Gaussian distribution and the Bernoulli distribution are considered, along with the parameters of Table III and Table IV. It is verified that the proposed sequential detection inherits the general properties of a centralized sequential detection scheme through Section 1. Delay performance is also investigated and simulated in Section 2.

1. Detection and Power Performance

First, we study detection performance in the Gaussian observation case. As n_{seq} varies from 2 to 6 with fixed SNR, sequential detection performance is compared with FSS detection performance. Similar to the result of Figure 10, Figure 11(a) shows that to obtain the same detection performance, sequential detection requires less observations on average than FSS detection, irrespective of n_{seq} . This figure also shows that, as n_{seq} increases, the detection performance improves. Referring to our sequential detector to take on more observations for an unlikely outcomes thereby improving performance over the FSS detector. Even when SNR changes, the relationship between the two detection schemes remains the same, as seen in Figure 11(a). These results are depicted in Figures 12(a), 13(a), 14(a). In addition, we expect an increase in SNR to improve detection performance in sequential detection, and we verify this intuition in Figure 15, including FSS detection. Figure 16 shows a tradeoff between power and detection performance for the case $n_{seq}=6$. As expected, sequential detection consumes less power as SNR increases.

Another observation with respect to detection performance is that as SNR decreases, the probability of error of the sequential detection procedure converges to that of FSS detection with the same number of sensor nodes as n_{seq} . This fact is

$\gamma(0) = \frac{2}{3}, \gamma(1) = \frac{1}{3}$	Prior distributions
$\theta = 1$	Constant signal value
$\ln(\tilde{\eta_U}) = \ln(e)$	Upper bound in sequential detection
$\ln(\tilde{\eta_L}) = \ln[0.1:0.05:0.95]$	Lower bound range in sequential detection

Table III. Simulation parameters: Gaussian observation for proposed schemes.

Table IV. Simulation parameters: Bernoulli observation for proposed schemes.

$\gamma(0) = \frac{2}{3}, \gamma(1) = \frac{1}{3}$	Prior distributions
$p_0 = \frac{1}{5}, p_1 = \frac{3}{5}$	Conditional distributions
$\ln(\tilde{\eta_U}) = \ln(99)$	Upper bound in sequential detection
$\ln(\tilde{\eta_L}) = [-8:0.5:-0.5]$	Lower bound range in sequential detection

also explained by our sequential detection algorithm. The decrease in SNR causes the sequential detector to need more observations before making a decision, and this fact then prompts the detector to operate in a regime closer to FSS detection than the sequential detection. This result is depicted in Figure 14(a).

To make a fair power comparison between two schemes, we find the region where the two schemes have the same detection performance. It is observed in Figure 11(b) that the total power of sequential detection is always greater than that of FSS detection for the same detection performance. For example, in Figure 11(a).(1) indicates the same detection performance region between sequential detection with $n_{\text{seq}}=3$ and FSS detection with $n_{\text{fix}}=2$. In particular, as previously mentioned, the total power in FSS detection is proportional to the number of sensor nodes. At this point, the value of E[N] in sequential detection is equal to 1.5076 and the total power computed at this value is 3.8803, as Figure 11(a).(2) indicates in Figure 11(b). Clearly, sequential detection consumes more power than FSS detection. Even when both SNR and n_{seq} change, this result remains true. These findings are depicted in Figures 12(b),13(b), and 14(b).

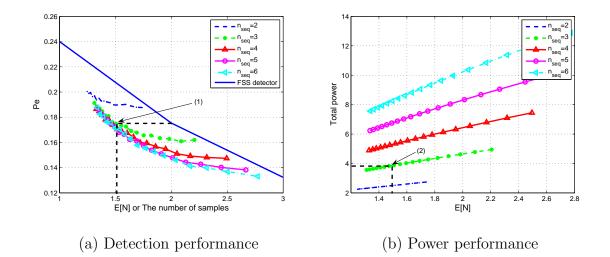


Fig. 11. Comparing detection and power performance between sequential detection and FSS detection: Gaussian observations with SNR=1.5.

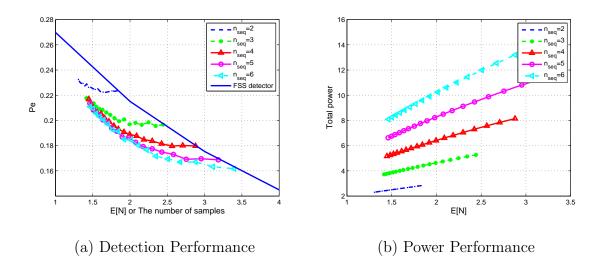
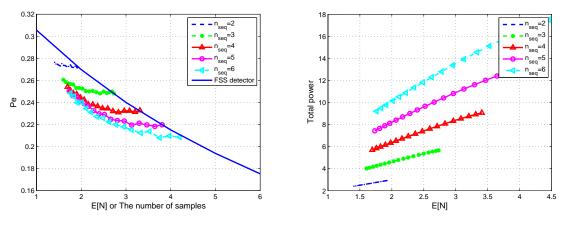


Fig. 12. Comparing detection and power performance between sequential detection and FSS detection: Gaussian observations with SNR=1.



(a) Detection Performance

(b) Power Performance

Fig. 13. Comparing detection and power performance between sequential detection and FSS detection: Gaussian observations with SNR=0.5.

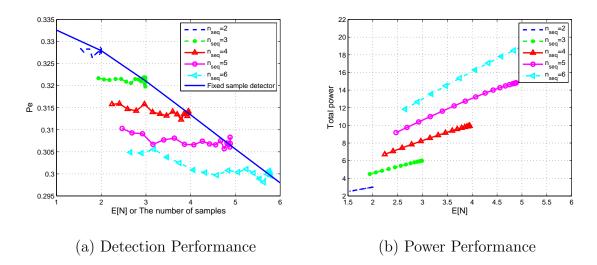


Fig. 14. Comparing detection and power performance between sequential detection and FSS detection: Gaussian observations with SNR=0.1.

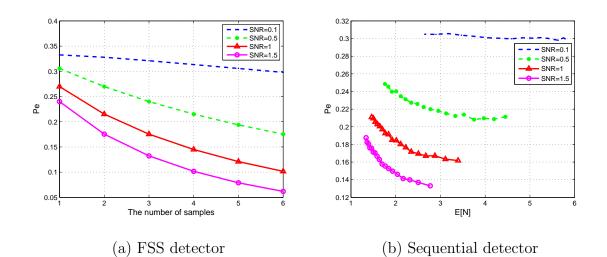


Fig. 15. Comparing detection performance between sequential detection and FSS detection: Gaussian observations with $n_{seq}=6$ and different SNRs.

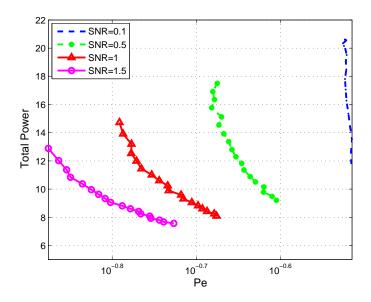


Fig. 16. Total power in sequential detection: Gaussian observations with $n_{seq}=6$ and different SNRs.

Using the same scenario as the Gaussian observation case, the Bernoulli observation case is simulated. The only difference is that conditional probabilities p_0 and p_1 vary instead of the SNR value of the Gaussian observation case, and n_{seq} varies from 5 to 10. Figures 17, 18, and 19 show that the conclusions of our simulations are the same as for the Gaussian case. That is, sequential detection requires fewer observations on average to make a decision at the cost of more power consumption. Figure 20 shows detection performance for both detectors with different conditional probabilities. Similar to the Gaussian case, a tradeoff between power and detection performance is demonstrated, and as the conditional probability increases, sequential detection saves more power. This result is depicted in Figure 21.

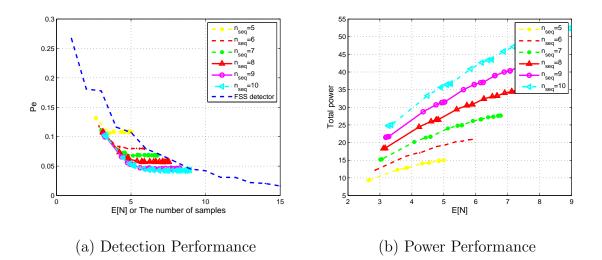
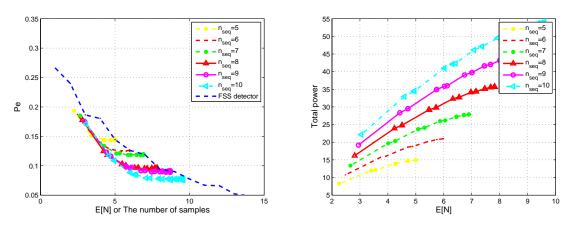


Fig. 17. Comparing detection and power performance between sequential detection and FSS detection: Bernoulli observations with $p_0=0.3$, $p_1=0.8$.



(a) Detection Performance

(b) Power Performance

Fig. 18. Comparing detection and power performance between sequential detection and FSS detection: Bernoulli observations with $p_0=0.2$, $p_1=0.6$.

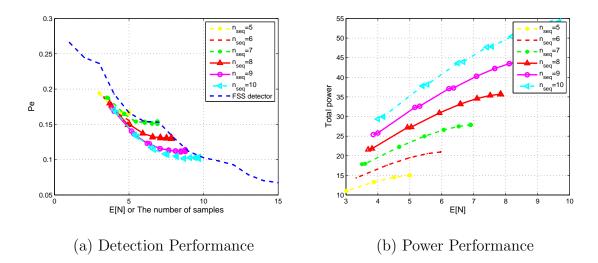


Fig. 19. Comparing detection and power performance between sequential detection and FSS detection: Bernoulli observations with $p_0=0.1$, $p_1=0.4$.

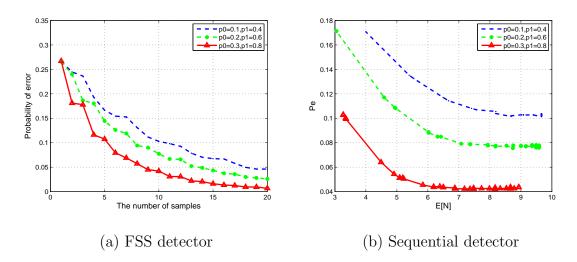


Fig. 20. Comparing detection performance between sequential detection and FSS detection: Bernoulli observations with $n_{seq}=10$ and different conditional distributions.

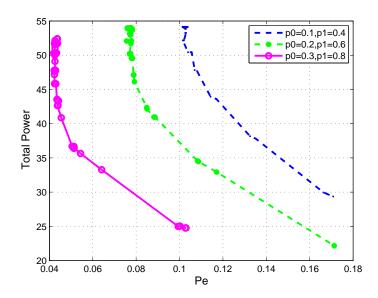


Fig. 21. Total power in sequential detection: Bernoulli observations with $n_{seq}=10$ and different conditional distributions.

2. Delay Performance

Next, we perform a simulation study for delay performance. Let us consider the Gaussian observation model with SNR=1.5. The most noticeable observation with respect to delay performance is depicted in Figure 22(a). This figure shows that sequential detection features shorter delays when it is compared to the FSS detection under the same detection performance. The same trend as Figure 22(a) is shown in Figures 22(b), (c), and (d). We also observe that, as the probability of error increases, delay decreases. This means that there exists a tradeoff between detection performance and delay. This is a natural result for our system model since a small delay implies that the detector takes fewer observations, which in turn, causes poor detection performance. In addition, a decrease in SNR prompts the detector to take more observations, and consequently, reduces its advantage in delay efficiency. After

all, when the SNR is within some small range, the expected delay of the sequential detection scheme converges to that of FSS detection and P_e decreases. This is shown in Figure 22(d). The Bernoulli observation case shows the same behavior as the Gaussian observation case. Simulation results are illustrated in Figure 23, with changing conditional probability.

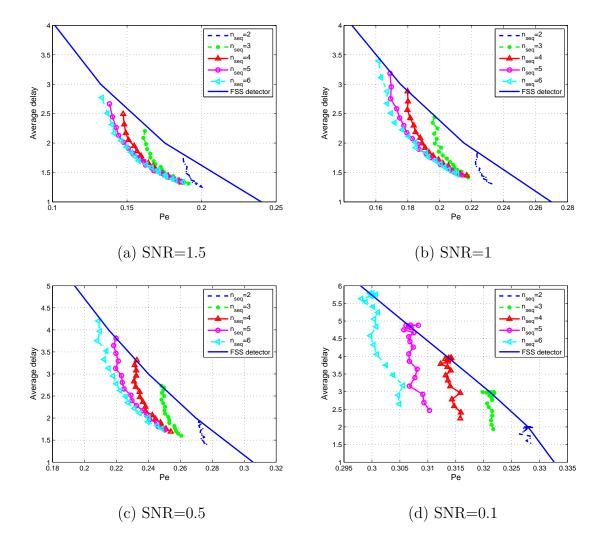


Fig. 22. Total delay vs probability of error between sequential detection and FSS detection: Gaussian observations with different SNRs.

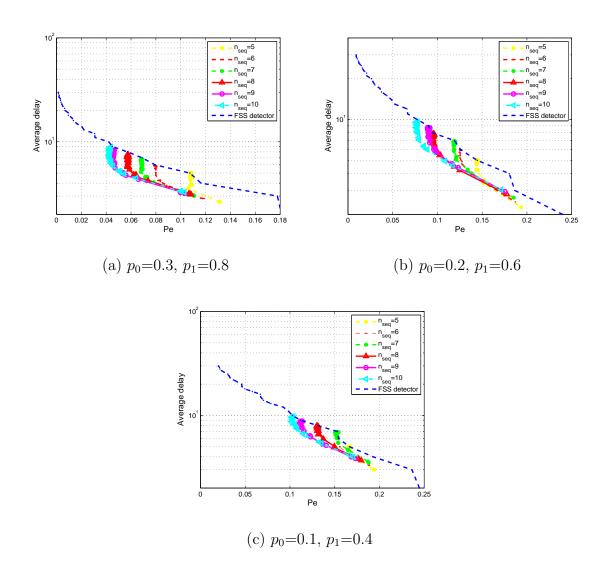


Fig. 23. Total delay vs probability of error between sequential detection and FSS detection: Bernoulli observations with different conditional probabilities.

Based on the simulation results of Section 1 and Section 2, we conclude that the proposed sequential detection scheme has an advantage and a disadvantage over FSS detection concerning power and delay. Sequential detection requires fewer observations on average to make a decision, and consequently it has lower expected delay than FSS detection. However, FSS detection proves to be more power efficient than sequential detection. In a multi-hop WSN that demands a quick decision with a small delay, sequential detection is preferred over FSS detection. On the other hand, FSS detection is adequate for designing a power-efficient WSN with a frugal power budget. In the next section, the relationship between the two schemes is investigated by combining the results of Section 1 and Section 2.

3. Cost Analysis

For the analysis of power and delay simultaneously, we define a cost function, which is a linear combination of power and delay:

$$f(E[D], E[P]) = \kappa E[D] + (1 - \kappa) \log E[P].$$

Here, E[D] and E[P] denote average delay and average power respectively, and the value of κ is between 0 and 1. For FSS detection, delay and power are both proportional to n_{fix} . The variable κ determines the relative importance of power and delay. In a WSN with small power and delay constraints, the design objective is to minimize the value of the cost function. For example, a WSN can be designed focusing on power and delay equally with κ =0.5. If the value of κ is less than 0.5, the WSN designer is placing more emphasis on power consumption. Whereas, delay is considered a more important factor in the design of the WSN when $\kappa > 0.5$. Usually, it is difficult to decide the value of κ in a two-objective optimization problem since the two functions of power and delay have different scales of value. However, our task is to compare

the cost value in both schemes as the value of κ varies and find which scheme consumes less cost value than the other. From equation (5.26) of Chapter V, the average power in sequential detection increases non-linearly as the average delay increases linearly because of the average delay square term. Hence, we take a logarithm of the average power term to compensate for the scale mismatch. Also, the power term associated with FSS detection is taken a logarithm to be fairly compared with sequential detection. Figure 24 shows the cost value for each scheme with the Gaussian observation model as the value of κ varies. The cost value of sequential detection with $n_{\text{seq}} = 3, 4, 5$ is compared with that of FSS detection with $n_{\text{fix}} = 2$. For a fair comparison of the cost value in both schemes, we conduct an evaluation of the cost based on the same detection performance, $P_e = 0.174$ and the same SNR=1.5. As κ increases, the cost function of sequential detection and FSS detection respectively increases and decreases. Sequential detection has a higher cost than FSS detection until κ arrives at a crossing point for both cost function. Furthermore, the gap between the cost values of two schemes becomes smaller. We define this crossing point as an equivalent point of a cost value in both schemes. After κ passes the equivalent point, the cost value of sequential detection becomes less than that of FSS detection and the gap between the two curves becomes larger. This result is consistent with our previous simulation results, which indicate that if the delay factor is considered more crucially than the power factor, our proposed sequential detection is superior to FSS detection. Another observation is that as n_{seq} increases, the equivalent point of the cost value in both schemes moves to the right. Two schemes have equivalent points of the cost value at $\kappa = 0.57, 0.65, 0.7$ in cases where $n_{\text{seq}} = 3, 4, 5$ respectively. It is due to the fact that the increase of average power is greater than the decrease of average delay. Hence, the increase of n_{seq} reduces the range of κ where the cost value of sequential detection is less than that of FSS detection. In short, we see again

that as either the delay or power factors are emphasized in different WSN, either a sequential detection or an FSS detection scheme can be deemed more appropriate for implementation than the other. However, the values of κ at the equivalent points of the cost are always greater than 0.5. This result implies that for a WSN which weighs power and delay equally, FSS detection is better than sequential detection.

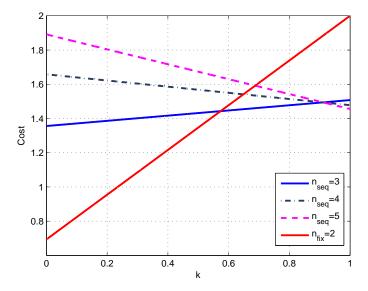


Fig. 24. Comparison of a cost value between sequential detection and FSS detection with varying κ from 0 to 1.

CHAPTER VII

CONCLUSIONS AND FUTURE WORKS

We have studied a centralized sequential detection scheme in multi-hop wireless sensor networks, and compared it with FSS detection with respect to power and delay. For the analysis of detection performance, we derived the optimal decision rule in each scheme for two observation distributions. Then, through simulations, we compared the detection performance with each other in terms of average size of observations and probability of error. Simulation results have shown that sequential detection outperforms FSS detection when both schemes use the same expected number of observations. Moreover, we verified that, as the number of sensor nodes n_{seq} increases, detection performance improves. In the Gaussian observation case, it was observed that a decrease in SNR causes the probability of error of sequential detection to converge to that of FSS detection with the same number of sensor nodes. Based on these results, the comparison of power and delay performance was conducted. The two schemes have an advantage and a disadvantage with respect to power and delay. That is, sequential detection has a shorter decision time on average than FSS detection and consequently reduces the overall delay at the cost of consuming more power. On the other hand, FSS detection performs more efficiently in terms of power than sequential detection. However, this scheme waits for all the observation packets taken by the sensor nodes to make a decision. This fact increases network delay.

Finally, we investigated the relationship between sequential detection and FSS detection considering power and delay simultaneously. The value of a joint power and delay in both schemes is captured as a cost function, which is a linear combination

of average power E[P] and average delay E[D]:

$$f(E[D], E[P]) = \kappa E[D] + (1 - \kappa) \log E[P].$$

From the analysis of the cost function, we concluded that sequential detection and FSS detection are best suited for networks demanding less delay and power, respectively. Simulation results show that there exist equivalent points of the cost value between the two schemes only if $\kappa > 0.5$. This result means that the minimum cost region of sequential detection is smaller than that of FSS detection. Hence, we need to find a better analysis model using sequential detection or a novel detection scheme for reducing power and delay. Moreover, our system model for the two detection schemes have been simplified under some assumptions: ignoring noise and fading over the channel, disregarding the spatial and temporal correlations between the observations and considering only transmission power to compute the total power. Nevertheless, the comparison of our two schemes provides a new direction for designing efficient WSNs. Our goal is to find a novel detection scheme with an efficient detection performance for minimizing power and delay, and to construct a rigorous framework where we can analyze overall system performance.

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VITA

Dae Hyun Choi graduated from Korea University, Korea in February 2002 with a bachelor's degree in Radio Sciences and Engineering. His undergraduate work was in wireless communications. He worked as a researcher in the Convergence Laboratory at Korea Telecom, Seoul, Korea from 2002 to 2006. He began pursuing his Master of Science in Electrical Engineering at Texas A&M University, College Station in Aug 2006. He can be reached at *(cdh*8954@*ece.tamu.edu)* or at,

183 Pyeongchang-dong, Jongno-gu, Seoul, Korea.