PROBABILISTIC MODELS AND RELIABILITY ANALYSIS OF SCOUR DEPTH AROUND BRIDGE PIERS

A Thesis

by

LAURA CHRISTINE BOLDUC

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2006

Major Subject: Civil Engineering

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ABSTRACT

Probabilistic Models and Reliability Analysis of Scour

Depth around Bridge Piers. (August 2006)

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Chair of Advisory Committee: Dr. Paolo Gardoni

Scour at a bridge pier is the formation of a hole around the pier due to the erosion of soil by flowing water; this hole in the soil reduces the carrying capacity of the foundation and the pier. Excessive scour can cause a bridge pier to fail without warning. Current predictions of the depth of the scour hole around a bridge pier are based on deterministic models. This paper considers two alternative deterministic models to predict scour depth. For each deterministic model, a corresponding probabilistic model is constructed using a Bayesian statistical approach and available field and experimental data. The developed probabilistic models account for the estimate bias in the deterministic models and for the model uncertainty. Parameters from both prediction models are compared to determine their accuracy. The developed probabilistic models are used to estimate the probability of exceedance of scour depth around bridge piers. The method is demonstrated on an example bridge pier. The values of the model parameters suggest that the maximum sour depth predicted by the deterministic HEC-18 Sand and HEC-18 Clay models tend to be conservative. Evidence is also found that the applicability of the HEC-18 Clay method is not limited to clay but can also be used for

other soil types. The main advantage of the HEC-18 Clay method with respect to the HEC-18 Sand method is that it predicts the depth of scour as a function of time and can be used to estimate the final scour at the end of the design life of a structure. The paper addresses model uncertainties for given hydrologic variables. Hydrologic uncertainties have been presented in a separate paper.

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NOMENCLATURE

a	Effective pier width [L]
D	Diameter of pier [L]
$f(\mathbf{\Theta})$	Posterior distribution representing the updated state of knowledge about $\mathbf{\Theta}$
$F(.)$	Fragility of bridge pier
$\hat{F}(.)$	Point estimate fragility
Fr_1	Froude number directly upstream of the pier
g	Acceleration due to gravity [L/T^2]
$g(.)$	Limit state function of achieving or exceeding of the limit state by the bridge pier
k	Normalizing factor
K_1	Correction factor for the pier shape
K_2	Correction factor for the angle of attack
K_3	Correction factor for the bed configuration
K_4	Correction factor for the sediment gradation
$L(\mathbf{\Theta})$	Likelihood function representing the objective information on $\mathbf{\Theta}$ contained in the observations
$p(\mathbf{\Theta})$	Prior distribution reflecting our state of knowledge about $\mathbf{\Theta}$ prior to obtaining the observations
P[A|**x**]	Conditional probability of event A for the given values of variables **x**
Re	Reynolds number
t	Total time for applied velocity [T]
v	Upstream velocity [L/T]
V_1	Mean upstream velocity [L/T]
x	Vector of measurable variables of bridge pier
y_1	Upstream water depth [L]
z_{final}	Final depth of scour at the bridge pier [L]
$\hat{z}_{final}$	Deterministic final depth of scour at the bridge pier [L]
z_{max}	Maximum depth of scour at the bridge pier [L]
$\hat{z}_{max}$	Deterministic maximum depth of scour at the bridge pier [L]
$\dot{z}$	Initial rate of scour [L/T]
$\nabla_{\mathbf{\Theta}}\beta(.)$	Gradient row vector of β computed at the mean value
$\beta(.)$	Reliability index corresponding to the fragility
θ_ζ	Unknown parameter in the logarithmic space
$\mathbf{\Theta}$	Vector of unknown model parameters
μ_{θ_ζ}	Mean value of θ_ζ
μ_{σ_ζ}	Mean value of σ_ζ
σ_ζ	Standard deviation in the logarithmic space

$\sigma^2_\beta(.)$	Variance around the mean point of β
$\Sigma_{\mathbf{\Theta\Theta}}$	Posterior covariance matrix of $\mathbf{\Theta}$
$\Phi(.)$	Standard normal cumulative probability
υ	Water viscosity [T/L^2]
ζ	Natural logarithmic of scour depth [L]
$\hat{\zeta}$	Deterministic value of ζ [L]
ζ_{design}	Logarithmic design depth of scour [L]
ζ_{final}	Logarithmic final depth of scour [L]
$\hat{\zeta}_{final}$	Deterministic logarithmic final depth of scour [L]

Subscripts

i	Generitc subscript indicating whether the parameter is the maximum or final depth of scour

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INTRODUCTION

Scour at a bridge pier is the formation of a hole around the pier due to the erosion of soil by flowing water; this hole in the soil reduces the carrying capacity of the foundation and the pier. Excessive scour can cause a bridge pier to fail without warning. The scour hole can be attributed to “local sour” due to the presence of the pier, “contraction scour” due to a decrease in the width of the river, and “abutment scour” which develops near the bridge abutments. This article deals with local scour only. Current predictions of the local scour depth around a bridge pier are based on deterministic models. The Geotechnics of Soil Erosion technical committee No. 33 is a committee developed of members from various countries that share their knowledge of scour. Countries involved include: Australia, Columbia, Netherlands, New Zealand, Poland, United States of America, and United Kingdom. The research in this paper focuses on methods typically used in the United States. Two common methods of prediction in the United States are described in Hydraulic Engineering Circular No. 18 (HEC-18) (Richardson and Davis 2001). One of these methods applies when the soil is sand; it was developed at Colorado State University and will be referred to as HEC-18 Sand. The other method applies when the soil is clay; it was developed at Texas A&M University by Briaud et al. (1999) and will be referred to as HEC-18 Clay. HEC-18 Sand predicts the maximum depth of scour (z_{max}) for a given set of variables; while it is based on model tests performed in

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This thesis follows the style and format of the *ASCE Journal of Geotechnical and Geoenvironmental Engineering*.

sand, it is often used in all soils as a conservative estimate. HEC-18 Clay was developed to predict the scour depth vs. time curve in soils where the rate of scour is slow (cohesive soils) and over the design life of the bridge pier. The scour depth predicted at the end of the design life of the bridge is the final depth of scour (z_{final}). Both methods make use of soil properties, water flow characteristics, and pier geometry.

Current methods to predict scour depths are deterministic and do not account for the prevailing uncertainties and errors. Also, the deterministic equations that are used do not indicate how conservative (biased) the estimated scour depth is compared to previously recorded data. By removing the inherent bias and enhancing the deterministic equations to account for uncertainties one is able to better evaluate the factor of safety that should be applied to the estimated scour depth. In this study, the bias in the deterministic equations for both HEC-18 Sand and HEC-18 Clay is corrected using a bias correction factor and the model uncertainty is included using an error term in the equation. This type of uncertainty arises when approximations are introduced in the formulation of the model. It has two essential components: error in the form of the model, e.g., a linear expression is used when the actual relation is nonlinear, and missing variables, i.e., the model contains only a subset of the variables that influence the quantity of interest. The bias correction factor and the model uncertainty are assessed by statistical analysis using three databases that include field and experimental data. A reliability analysis is performed using the developed probabilistic models to estimate the probability a specified threshold depth will be exceeded at a bridge pier. Confidence bounds on the probability estimates are developed by first-order analysis (Gardoni et al. 2002) to reflect the effect of the epistemic uncertainty present the model parameters.

As an illustration, the probability of exceedance at an example bridge pier is estimated. This paper addresses model uncertainties for given hydrologic variables. Hydrologic uncertainty is beyond the scope of this paper but has been addressed previously (Brandimarte et al. 2006).

DETERMINISTIC PREDICTIONS OF SCOUR DEPTH BY HEC-18

HEC-18 Sand was developed at Colorado State University starting in the early 80s. Over the last two decades, it has been used for all types of soils including rock even though it was developed using fine sand. Because fine sand is one of the most erodible soils, predictions using HEC-18 Sand are often conservative if not very conservative when applied to other soils. Based on the data used to develop HEC-18 Sand, its use should be limited to cohesionless soils. For a given velocity and pier geometry, HEC-18 Sand predicts the maximum depth of pier scour, z_{max}. The current HEC-18 Sand deterministic equation is

$$\frac{\hat{z}_{max}}{y_1} = 2.0 K_1 K_2 K_3 K_4 \left(\frac{a}{y_1}\right)^{0.65} Fr_1^{0.43} \tag{1}$$

where $\hat{z}_{max}$ is the deterministic prediction of maximum scour depth, y_1 is the upstream water depth, a is the effective pier width, $Fr_1 = V_1 / [(gy_1)^{1/2}]$ Froude number directly upstream of the pier, V_1 is the mean upstream velocity, g is the acceleration due to gravity, and K_1, K_2, K_3, and K_4 are correction factors for the pier shape, angle of attack, bed configuration, and sediment gradation, respectively.

HEC-18 Clay was developed at Texas A&M University starting in the early 90s and was given the name of the SRICOS-EFA method (Briaud et al. 1999, 2001a). In 2001, the new version of HEC-18 (Richardson and Davis 2001) adopted the SRICOS method to predict local bridge scour in cohesive soils, thus the SRICOS method became the HEC-18 Clay method. The HEC-18 Clay method can be used for any soil for which a representative sample can be collected and tested in the Erosion Function Apparatus or EFA (Briaud et al. 2001b). The sample is tested in the EFA to obtain the relationship between the water velocity and the erosion rate of the soil. This erosion function represents the soil input. The input for the water is in the form of the hydrograph while the input for the pier is its geometry. These three inputs are combined in an algorithm developed on the basis of experiments and numerical simulations. The ouput of this algorithm is the scour depth as a function of time over the period of the hydrograph. Therefore the main distinction between HEC-18 Sand and HEC-18 Clay is that HEC-18 Sand gives the maximum depth of scour (z_{max} is the depth reached when a constant water velocity is applied for an infinite time) while HEC-18 Clay gives the final depth of scour (z_{final} is the depth reached at the end of the design life of the structure). If the soil is sand, there is usually no difference between z_{final} and z_{max} under the maximum velocity in the hydrograph because a flood duration is usually long enough to create z_{max} in clean sand. If the soil erodes more slowly (dirty sands, silts, and clays), z_{final} can be less than z_{max} and there is an advantage in using HEC-18 Clay rather than HEC-18 Sand.

HEC-18 Clay also has an equation to predict z_{max}. This equation is based on 36 model scale flume experiments on three different clay soils. The deterministic equation to predict the maximum depth of scour for the HEC-18 Clay method is

$$\hat{z}_{max}\,(\text{mm}) = 0.18 Re^{0.635} \qquad (2)$$

where Re is the Reynolds number equal to vD/υ where v is the upstream velocity, D is the diameter of the pier, and υ is the water viscosity (10^{-6} s/m^2 at 20°C). The time dependency of the scour depth evolution is introduced in the method through a hyperbola that links the scour depth to the time a given velocity has been applied. This equation is based on a series of flume experiments (Briaud et al. 1999). The resulting deterministic final depth of scour is of the form

$$\hat{z}_{final}\,(\mathrm{mm}) = \frac{t}{\dfrac{1}{\dot{z}_i} + \dfrac{t}{\hat{z}_{max}}} \tag{3}$$

where t is the time over which a given velocity is applied, $\dot{z}_i$ is the initial rate of scour, and $\hat{z}_{max}$ is given in Eq. (2). In the more complex case of a velocity hydrograph, the scour depth accumulation process consists of juxtaposing appropriate pieces of the hyperbolas (Briaud et al. 2001b). If $\dot{z}_i$ of the clay soil is small, then it is possible for z_{final} to only be a small fraction of z_{max}.

DATABASES USED TO DEVELOP PROBABILISTIC MODELS

In this research, laboratory and full scale scour depth measurements are used to construct probabilistic scour models that correct for the inherent bias in the deterministic estimates and properly account for the model error. The Gudavalli (Gudavalli 1997) and the Landers-Mueller (Landers and Mueller 1996) databases are used to evaluate the precision of the HEC-18 Sand and HEC-18 Clay methods when predicting the maximum depth of scour z_{max}. The Kwak (Kwak 2000) database is used to evaluate the precision of the HEC-18 Clay method when predicting the final depth of scour z_{final}.

The Gudavalli database is composed of 43 laboratory flume experiments. Soil types used in this database include: porcelain clay, armstone clay, bentonite clay, and sand. The majority of the experiments were performed in porcelain clay. The bridge piers populating this database are circular with diameters ranging from 25 to 210 mm. The water velocities were measured far upstream of the bridge pier and ranged from 0.204 to 0.83 m/s and the upstream water depth ranged from 0.16 to 0.4 m. The results of the experiments consisted of the scour depth vs. time curves. These curves were fitted with a hyperbolic model to obtain the asymptotic value of the scour depth. The Gudavalli database is therefore a maximum scour depth database for piers in clay.

The original Landers-Mueller database is populated with 305 bridge pier scour depth readings at 56 bridges in the United States. Since this article is concerned only with circular piers, only 186 piers having diameters ranging from 0.61 to 4.57 m were used from the Landers-Mueller database. The majority of the soil type for the Landers-

Mueller database is cohesionless soils. However both HEC-18 Sand and HEC-18 Clay are used because HEC-18 Sand is used in practice for all soils and the authors of HEC-18 Clay state that their method is not limited to clay (Briaud et al. 1999). The water velocity recorded for this database is the velocity measured at the time the scour depth was recorded. These velocities range from 0.15 to 4.48 m/s. The upstream water depth ranges from 0.46 to 12.04 m. The velocities and the scour depths in this database were usually recorded during the later part of a flood event. However it is not known whether this velocity was the highest velocity the bridge pier had ever seen, nor is it known if the scour depth was the maximum scour depth under this velocity. Given the fact that the soils in the database were predominantly cohesionless and the scour depth measurements were taken towards the tail end of the flood, it is likely that the measured scour depth associated with the measured velocity is approaching the maximum scour depth under that velocity. The measured scour depth could be higher than the maximum scour depth under the measured velocity if a previous and higher velocity had created a deeper hole around the pier. On the other hand a previous and deeper hole would probably have been in-filled during the post flood deposition. Yet again, the soil back filling the scour hole would have a different erodibility than the parent material. All this means that the data itself generates scatter even if the prediction method was perfect. Nevertheless this database is very valuable because it is very large (four times larger than the Gudavalli database) and populated with full scale bridges.

The Kwak database compiles measurements of scour depth at ten actual bridge piers from eight bridges across the State of Texas. Bridge piers in this database are either circular or square in cross section with widths ranging from 0.36 to 0.91 m. The soil at all bridges is clay and the database is used to evaluate the final depth of scour for the HEC-18 Clay method only. Indeed this database does not give the maximum depth of scour but rather the final depth of scour for the observed bridge pier after years of water flow. Flood hydrographs from each river were used to predict the scour depth as a function of time. Then the scour depth corresponding to the time at which the measured scour depth was obtained was selected as the final depth of scour and compared to the measured final scour depth. Kwak's calculations for the velocity, shear stress, and rate of scour used in the HEC-18 Clay method are documented in his dissertation (Kwak 2000).

DATABASE RESULTS AND COMMENTS ON DETERMINISTIC METHODS

A set of measured versus predicted scour depth plots are shown on Figs. 1 to 5. In each of the figures the dots represent the data points and the dotted dashed lines are the one-to-one reference line. For a perfect model the data should lie along this line. If the data lie above the one-to-one line is an indication that the model tends to be conservative (predicts a higher scour than what actually recorded). Vice versa, if the data lie below the one-to-one line means that the models tend to underestimate the actual scour (unconservative). The deterministic predictions based on the HEC-18 Sand model should be compared to the Landers Mueller database only since it is the only database among the three where the soils are made predominantly of cohesionless materials. On the other hand, the predictions based on the HEC-18 Clay model should only be compared to the Gudavalli and Kwak databases since these are the databases where the soils are made predominantly of cohesive soils. Nevertheless, the predictions from both methods were compared to all databases to find out how they compared to each other and because there is some evidence that soil type does not influence the maximum depth of scour (Briaud et al. 1999).

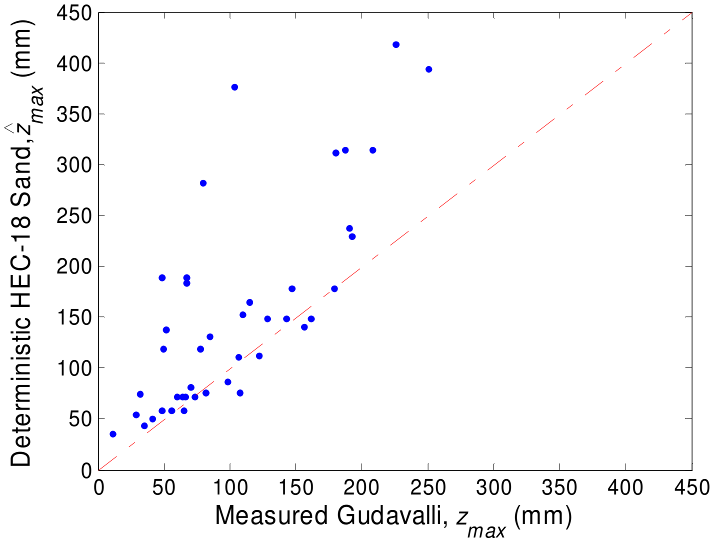

Fig. 1. Deterministic prediction of maximum scour depth for HEC-18 Sand using the Gudavalli database

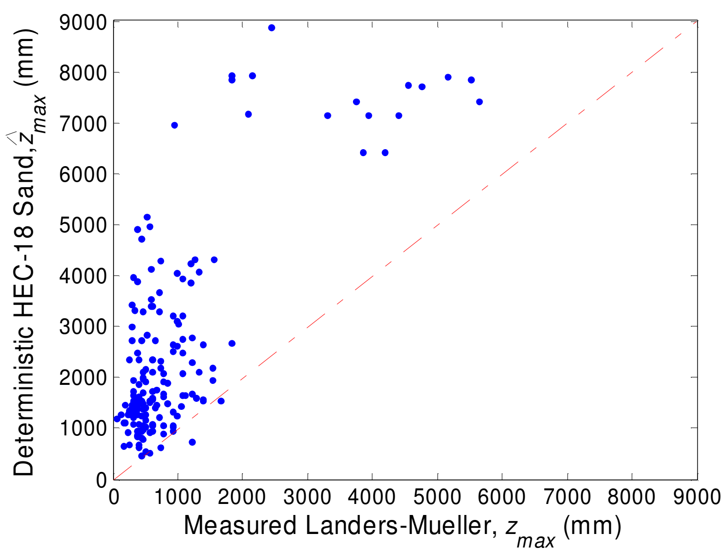

Fig. 2. Deterministic prediction of maximum scour depth for HEC-18 Sand using the Landers-Mueller database

One of the first observations is that HEC-18 Sand (Figs. 1 and 2) is more conservative than HEC-18 Clay (Figs. 3 and 4). It must be noted that HEC-18 Sand gives a predicted maximum depth of scour that is used directly in design without an additional factor of safety (design method). On the other hand HEC-18 Clay gives a predicted maximum depth of scour that has to be multiplied by a factor of safety equal to 1.5 before making use of that sour depth in design (prediction method). Considering this factor in comparing the methods, it appears that both methods are comparably conservative when calculating a maximum scour depth for use in design.

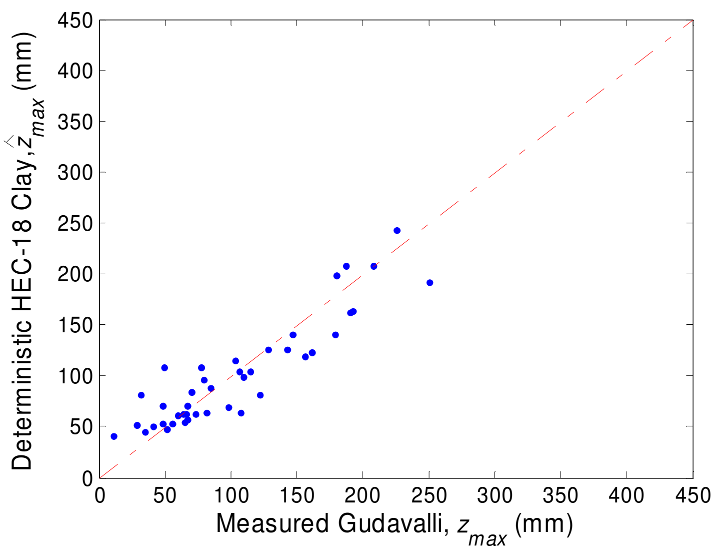

Fig. 3. Deterministic prediction of maximum scour depth for HEC-18 Clay using the Gudavalli database

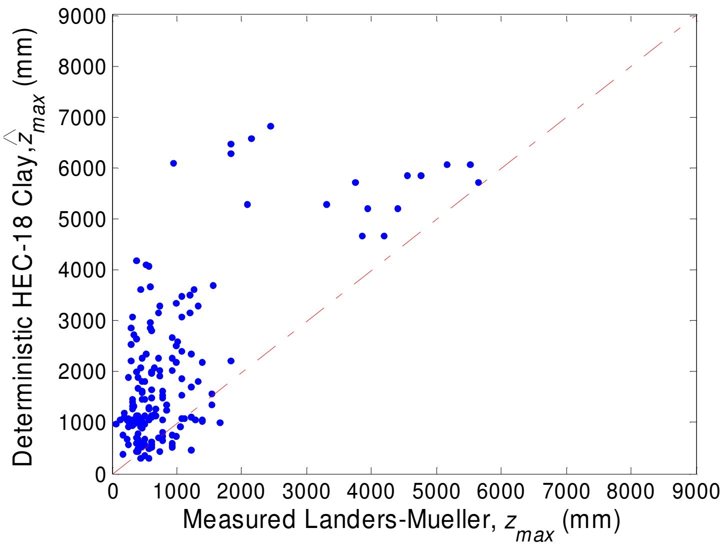

Fig. 4. Deterministic prediction of maximum scour depth for HEC-18 Clay using the Landers-Mueller database

The true advantage of HEC-18 Clay is that it can predict the time rate of scour at a bridge while HEC-18 Sand cannot. This becomes useful when the soil erodes slowly and when the duration of a flood may not generate the maximum scour depth. HEC-18 Clay has the ability of predicting the final sour depth as well as the maximum scour depth while HEC-18 Sand is limited to giving values of maximum scour depth. This is where HEC-18 Clay is more useful than HEC-18 Sand. The comparison of HEC-18 Clay predictions of final scour depth and the observed final scour depth for the Kwak database shows a good agreement (Fig. 5).

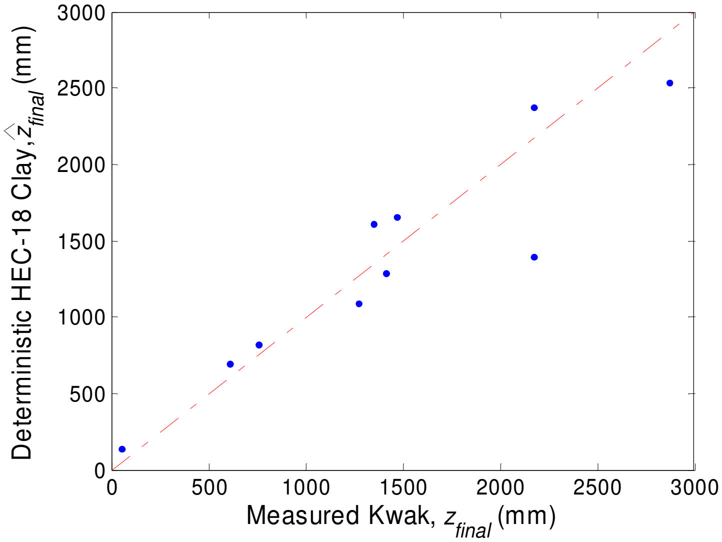

Fig. 5. Deterministic prediction of final scour depth for HEC-18 Clay using the Kwak database

PROBABILISTIC MODELS FOR SCOUR PREDICTION

Probabilistic models are formulated based on the deterministic HEC-18 Sand and HEC-18 Clay models and consider a multiplicative correction factor to account for the bias inherent in the deterministic models. Figs. 1 through 5 show that the data scatter increases, opening up, as the values of scour increase. The opening of the data indicates a non-constant variance referred to as heteroskedasticity (Stone 1996). To account for the uncertainty in the model a multiplicative error term is considered. So, the probabilistic models are formulated as

$$z_i = \theta_z \hat{z}_i(\mathbf{x}) e \tag{4}$$

where θ_z is an unknown model parameter (correction factor), $\hat{z}_i$ is the deterministic prediction (i.e., HEC-18 Sand or Clay) where the subscript i indicates whether the model is for the maximum or final scour depth, $\mathbf{x}$ is a vector of inputs into the deterministic prediction (i.e., pier geometry, fluid properties, etc.), and e is the unit-median error term that describes the uncertainty in the probabilistic model.

Following Gardoni et al. (2002), a logarithmic transformation of Eq. (4) is used

$$\zeta_i(\mathbf{x},\mathbf{\Theta}) = \theta_\zeta + \hat{\zeta}_i(\mathbf{x}) + \sigma_\zeta \varepsilon \tag{5}$$

where $\zeta_i = \ln(z_i)$, $\mathbf{\Theta} = (\theta_\zeta, \sigma_\zeta)$ denotes the set of unknown model parameters, with $\theta_\zeta = \ln(\theta_z)$, $\hat{\zeta}_i = \ln(\hat{z}_i)$, $\sigma_\zeta\varepsilon$ is the random error in the model, ε is a random variable with zero mean and unit variance, and σ_ζ represents the standard deviation of the model error. So for given $\mathbf{x}$ and $\mathbf{\Theta} = (\theta_\zeta, \sigma_\zeta)$, $\text{Var}[\zeta_i(\mathbf{x},\mathbf{\Theta})] = \sigma_\zeta^2$. The logarithmic transformation is used to approximately satisfy the following assumptions (a) the model variance σ_ζ^2 is independent of $\mathbf{x}$ (homoskedasticity assumption), and (b) ε has the normal distribution (normality assumption). Diagnostic plots of the data and the residuals against model predictions and individual regressors (Rao and Toutenburg 1997) have been used to verify the suitability of the logarithmic transformation.

Parameters in Eq. (5) are updated by use of the well-known Bayesian updating rule (Box and Tiao 1992)

$$f(\mathbf{\Theta}) = kL(\mathbf{\Theta})\,p(\mathbf{\Theta}) \qquad (6)$$

where $f(\mathbf{\Theta})$ is the posterior distribution representing the updated state of knowledge about $\mathbf{\Theta}$; $L(\mathbf{\Theta})$ is the likelihood function representing the objective information on $\mathbf{\Theta}$ contained in the data; $p(\mathbf{\Theta})$ is the prior distribution reflecting our state of knowledge about $\mathbf{\Theta}$ prior to obtaining the data; and $k = \left[\int L(\mathbf{\Theta})\,p(\mathbf{\Theta})d\mathbf{\Theta}\right]^{-1}$ is a normalizing factor. The posterior distribution represents a compromise between the prior information and the data. Having no prior information on these parameters, a noninformative prior distribution is used. The assumption on the prior distribution does not significantly affect the posterior distributions and the final results for a large or even moderate sized database (Box and Tiao 1992).

PROBABILISTIC MODEL FOR THE HEC-18 SAND METHOD

In layman terms, the probabilistic models aim at explaining the data better than the deterministic models. They fit the data better by splitting the data evenly and by documenting the extent of the error associated with the prediction. This process is associated with the data and as such the probabilistic model is tied to each database. The following are the equations for the resulting probabilistic models associated with the HEC-18 Sand method in the original space, where θ_z is replaced with the exponent of the posterior mean of θ_ζ:

Gudavalli Database $$z_{max} = 0.690 \hat{z}_{max \text{ HEC-18 Sand}} e \quad (7)$$

Landers-Mueller Database $$z_{max} = 0.331 \hat{z}_{max \text{ HEC-18 Sand}} e \quad (8)$$

The Gudavalli database is a database populated with high confidence values of the maximum scour depths in cohesive soils and for flume scale experiments. The Landers-Mueller database is a database populated with measured scour depths that are estimates of the maximum scour depth in predominantly cohesionless soils and for full scale bridges.

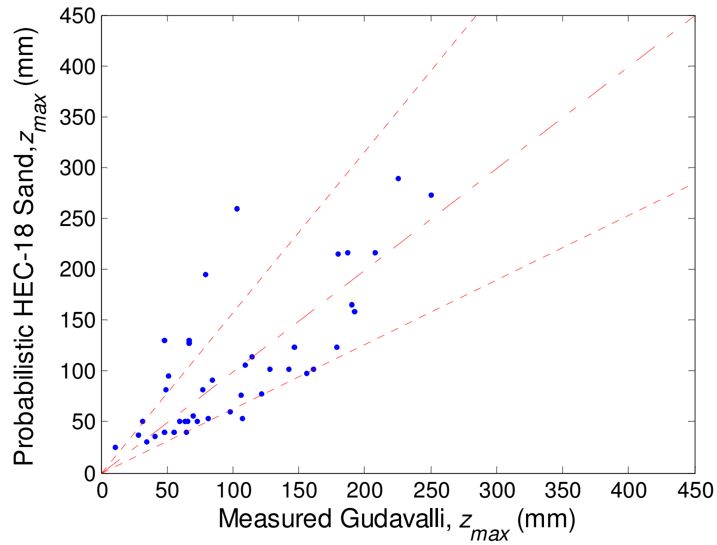

Fig. 6. Probabilistic prediction of maximum scour depth for HEC-18 Sand using the Gudavalli database

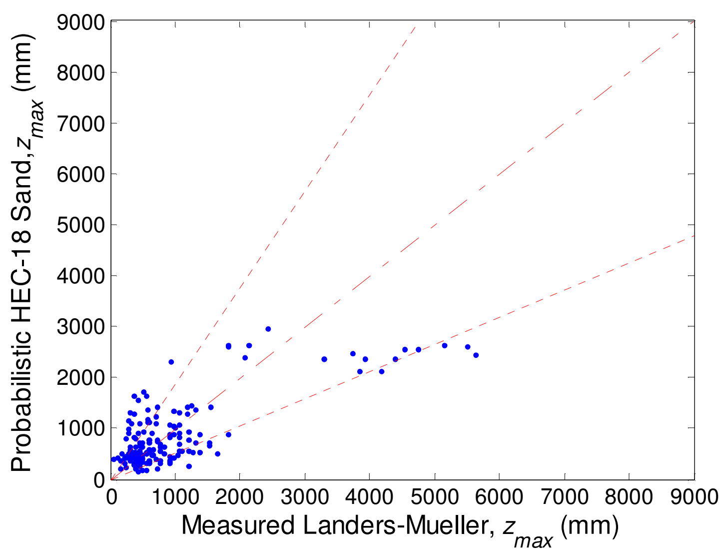

Fig. 7. Probabilistic prediction of maximum scour depth for HEC-18 Sand using the Landers-Mueller database

Figs. 6 and 7 show the median predictions ($e = 1$) from the probabilistic models for HEC-18 Sand using the Gudavalli and Landers-Mueller databases, respectively. Both probabilistic models improve the fit significantly compared to the deterministic models (Figs. 1 and 2). They also give an indication of the amount of conservatism in the deterministic models as well as the extent of the associated scatter. The dotted lines in Figs. 6 and 7 indicate the lines at one standard deviation from the one-to-one line. Tables 1, 2, and 3 summarize the probabilistic model parameters.

Table 1. Posterior statistics for the HEC-18 Sand method using the Gudavalli database

Parameter	Mean	Standard Deviation	Correlation Coefficient	
			θ_ζ	σ_ζ
θ_ζ	−0.380	0.059	1	−0.21
σ_ζ	0.452	0.044	−0.21	1

Table 2. Posterior statistics for the HEC-18 Sand method using the Landers-Mueller database

Parameter	Mean	Standard Deviation	Correlation Coefficient	
			θ_ζ	σ_ζ
θ_ζ	−1.11	0.047	1	0.01
σ_ζ	0.632	0.034	0.01	1

Table 3. Posterior statistics and percent error results for the probabilistic models in the original space

Database	Equation	HEC-18 Sand		HEC-18 Clay	
		θ_z	% Error	θ_z	% Error
Gudavalli	z_{max}	0.690	41.3	0.955	28.7
Landers-Mueller	z_{max}	0.331	60.4	0.447	69.33
Kwak	z_{final}	--	--	0.919	28.2

These parameters indicate that:

1. The HEC-18 Sand deterministic model is conservative. On the average it predicts scour depths that are 3.02 times larger than the measured scour depths for the full scale bridges and 1.45 times larger than the flume test database. The better fit with the flume tests reminds us that the HEC-18 Sand model was developed on the basis of flume tests and that the extrapolation to full scale may be flawed. It is also possible, although not as likely in the authors opinion, that the velocity measured in that database are too low compared to the velocity which truly created the observed depth of scour.
2. The HEC-18 Sand model exhibits more scatter with the Landers-Mueller database than with the Gudavalli database. This is attributed to the fact that there is more uncertainty with the data in the Landers-Mueller database than in the Gudavalli database.
3. The percent error in the HEC-18 Sand model is 60.4% for the Landers-Mueller database and 41.3% for the Gudavalli database. In fact, the scatter around the mean prediction (represented by the estimated standard deviation

of the probabilistic models) is larger when using the Landers-Mueller database ($\sigma_\zeta = 0.632$) than based on the Gudavalli database ($\sigma_\zeta = 0.452$). Some of the predicted values with the HEC-18 Sand deterministic model are 20 times higher than the measured values.

PROBABILISTIC MODEL FOR THE HEC-18 CLAY METHOD

The following are the equations for the resulting probabilistic models associated with the HEC-18 Clay method in the original space, where θ_z is replaced with the exponent of the posterior mean of θ_ζ:

Gudavalli Database $$z_{max} = 0.955\hat{z}_{max\ \text{HEC-18 Clay}}e \quad (9)$$

Landers-Mueller Database $$z_{max} = 0.447\hat{z}_{max\ \text{HEC-18 Clay}}e \quad (10)$$

Kwak Database $$z_{final} = 0.919\hat{z}_{final\ \text{HEC-18 Clay}}e \quad (11)$$

Other probabilistic parameters are given in Tables 3 through 6. As already noted, the Gudavalli database is a database populated with high confidence values of the maximum scour depths in cohesive soils and for flume scale experiments. Furthermore it was the database used in the development of the HEC-18 Clay method, therefore it is not surprising that the correction factor, θ_z, value is close to 1.

Table 4. Posterior statistics for the HEC-18 Clay method using the Gudavalli database

Parameter	Mean	Standard Deviation	Correlation Coefficient	
			θ_ζ	σ_ζ
θ_ζ	–0.046	0.051	1	0.01
σ_ζ	0.353	0.036	0.01	1

Table 5. Posterior statistics for the HEC-18 Clay method using the Landers-Mueller database

Parameter	Mean	Standard Deviation	Correlation Coefficient	
			θ_ζ	σ_ζ
θ_ζ	–0.805	0.048	1	–0.12
σ_ζ	0.698	0.025	–0.12	1

Table 6. Posterior statistics for the HEC-18 Clay method using the Kwak database

Parameter	Mean	Standard Deviation	Correlation Coefficient	
			θ_ζ	σ_ζ
θ_ζ	–0.085	0.125	1	0.05
σ_ζ	0.407	0.091	0.05	1

While the Landers-Mueller database is a database populated with measured scour depths that are estimates of the maximum scour depth in predominantly cohesionless soils and for full scale bridges. Figs. 8 through 10 show the median predictions ($e = 1$) from the probabilistic models for HEC-18 Clay using the Gudavalli, Landers-Mueller, and Kwak databases, respectively.

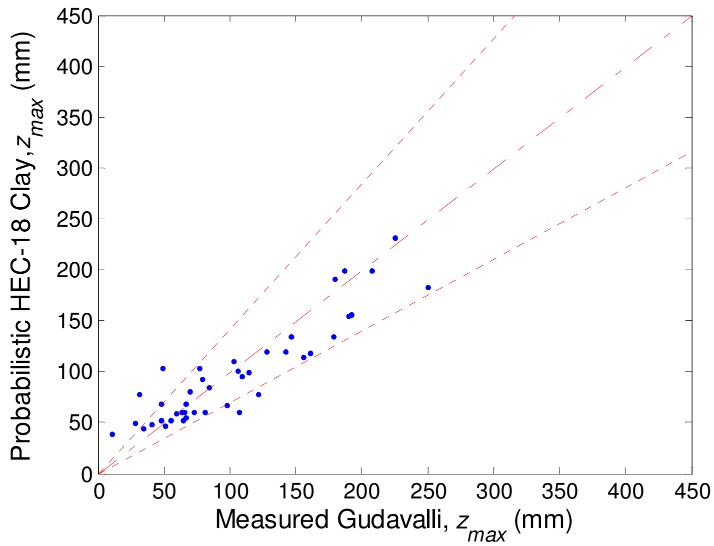

Fig. 8. Probabilistic prediction of maximum scour depth for HEC-18 Clay using the Gudavalli database

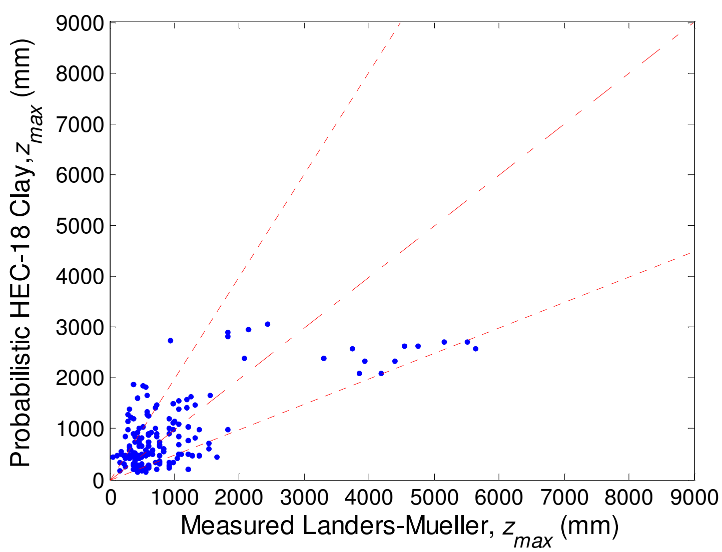

Fig. 9. Probabilistic prediction of maximum scour depth for HEC-18 Clay using the Landers-Mueller database

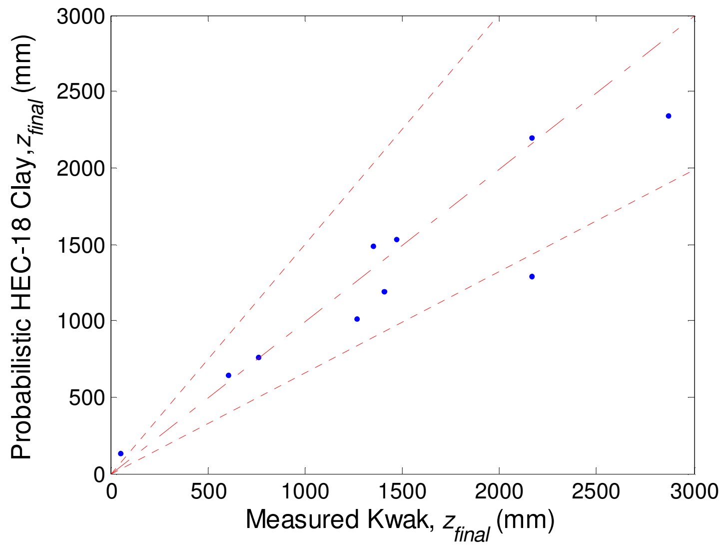

Fig. 10. Probabilistic prediction of final scour depth for HEC-18 Clay using the Kwak database

Both probabilistic models significantly improve the fit compared to the deterministic models (Figs. 3 and 4). They also give an indication of the extent of the scatter associated with each probabilistic model. The parameters summarized in Tables 3 through 6 indicate that:

1. The HEC-18 Clay deterministic model is conservative when compared to the full scale bridge database. On average it predicts scour depths that are 2.24 times larger than the measured scour depths for the full scale bridges and 1.05 times larger than the flume test database. The better fit with the flume tests is simply due to the fact that the HEC-18 Clay model was developed on the basis of these flume tests.
2. The HEC-18 Clay model exhibits a similar amount of scatter as the HEC-18 Sand model when compared to the Landers-Mueller database. This may be a

confirmation that a good part of the scatter in the predictions is in fact due to the data itself. It is also an indication that the HEC-18 Clay and HEC-18 Sand method are equally applicable to that database confirming that the HEC-18 Clay method is not limited to clays.

3. Considering that there is little difference between the HEC-18 Sand and HEC-18 Clay method on the independent Landers-Mueller database, one wonders when to use one or the other method. The answer is that the main advantage of the HEC-18 Clay method is that it offers the engineer a way to predict the depth of scour in cases where the soil erodes more slowly than fine sands do and therefore allows the engineers to get estimates of scour depth (z_{final}) that are more realistic. If an engineer wishes to obtain the maximum depth of scour (z_{max}), it appears that he or she can choose either method.
4. The HEC-18 Sand method requires no factor of safety on the predicted z_{max} value (i.e., predicted value = design value) while the authors of the HEC-18 Clay method recommend a 1.5 factor of safety to go from the predicted value to the design value of z_{max} (i.e., design value = 1.5 times the predicted value). This distinction seems to be confirmed to some extent by the fact that the ratio between the θ_z values for the two methods and the Landers-Mueller database is 0.447/0.331 = 1.3.

5. The HEC-18 Clay compares very favorably with the full scale bridge Kwak database. This database is populated of final depths of scour instead of maximum depths of scour and therefore the data is compared to the HEC-18 Clay predictions and not the HEC-18 Sand predictions. Indeed only HEC-18 Clay permits such predictions. Note also that the Kwak database is an independent database that was not used to develop the HEC-18 Clay method.

PROBABILITY OF EXCEEDANCE VS. CORRECTION FACTOR

A value of the correction factor, θ_z, equal to 0.331 in Eq. (8) means that approximately 50% of the data in the Landers-Mueller database are above the median prediction based on HEC-18 Sand and 50% are below. Fig. 11 shows the probability that a data point in this database is above (exceeds) the median prediction as a function of θ_z. For a small value of θ_z, most of the data are above the median prediction, the number of data above the median prediction decrease as θ_z increases. As noted earlier, for $\theta_z = 0.331$, approximately half of the data are above and half are below the median prediction.

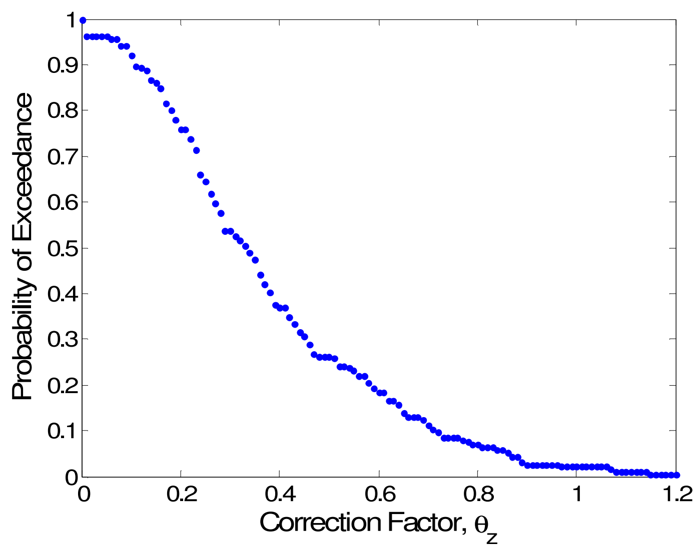

Fig. 11. Probability of exceeding the median predicted scour depth vs. the correction factor, θ_z, for the HEC-18 Sand method with the Landers-Mueller database

Similarly, using Eqs. (10) and (11) approximately 50% of the data are above (and 50% are below) the median predictions based on HEC-18 Clay for the Landers-Mueller and Kwak database, respectively. Figs. 12 and 13 show the probability that a data point in Landers-Mueller database and Kwak database, respectively, will be above the median prediction for the HEC-18 Clay method as a function of θ_z. The same trend as in Fig. 11 can be observed.

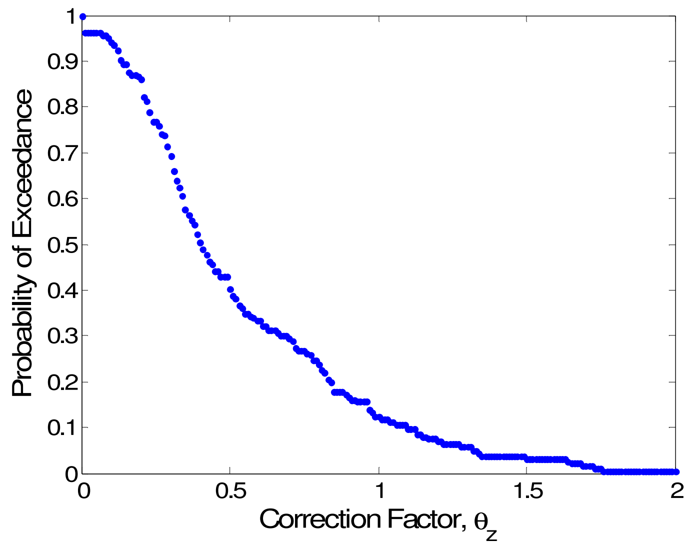

Fig. 12. Probability of exceeding the median predicted scour depth vs. the correction factor, θ_z, for the HEC-18 Clay method with the Landers-Mueller database

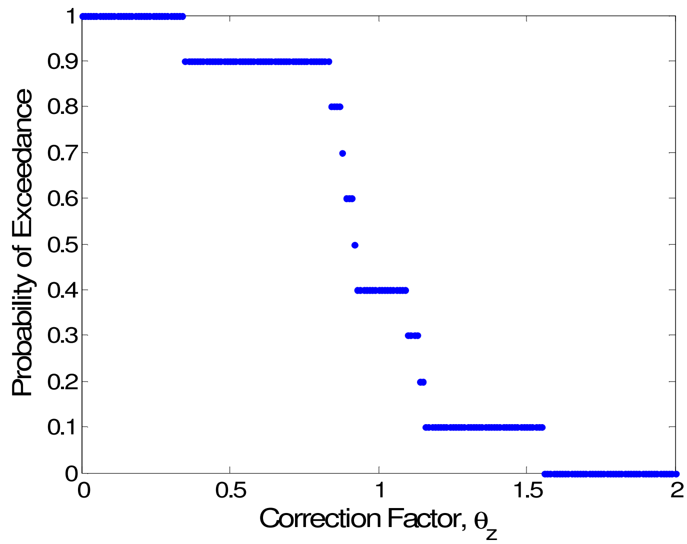

Fig. 13. Probability of exceeding the median predicted scour depth vs. the correction factor, θ_z, for the HEC-18 Clay method with the Kwak database

Figs. 11, 12 and 13 can be used to select the appropriate multiplicative correction factor, θ_z, for the deterministic HEC-18 models, based on the desired level of safety.

PROBABILITY OF EXCEEDANCE VS. TIME AND DESIGN SCOUR DEPTH

Following the conventional notation in reliability theory (Ditlevsen and Madsen 1996), a limit state function $g(\cdot)$ can be defined such that the event $\{g(\cdot) \leq 0\}$ denotes the attainment or exceedance of a design scour depth, ζ_{design}. Using the probabilistic model described in Eq. (5), a limit state function can be written as

$$g\left(\zeta_{design}, \mathbf{x}, \mathbf{\Theta}\right) = \zeta_{design} - \zeta_{final}\left(\mathbf{x}, \mathbf{\Theta}\right) \tag{12}$$

A conditional probability of exceedance for given measurable variables, $\mathbf{x}$, and model parameters, $\mathbf{\Theta}$, can then be computed as

$$P\left[g\left(\zeta_{design}, \mathbf{x}, \mathbf{\Theta}\right) \leq 0 \middle| \mathbf{x}, \mathbf{\Theta}\right] \tag{13}$$

where $\mathrm{P}[A|B]$ denotes the conditional probability of event A for the given value of variable B. The reliability (or safety) index (Ditlevsen and Madsen 1996) corresponding to the probability in Eq. (13) is

$$\beta\left(\zeta_{design}, \mathbf{x}, \mathbf{\Theta}\right) = \Phi^{-1}\left[1 - P\left[g\left(\zeta_{design}, \mathbf{x}, \mathbf{\Theta}\right) \leq 0 \middle| \mathbf{x}, \mathbf{\Theta}\right]\right] \tag{14}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of the standard normal cumulative probability.

The uncertainty in the model parameters is reflected in the probability distribution of $P[g(\zeta_{design}, \mathbf{x}, \mathbf{\Theta}) \leq 0 \mid \mathbf{x}, \mathbf{\Theta}]$ relative to $\mathbf{\Theta}$. Exact evaluation of this distribution requires nested reliability calculations (Der Kiureghian 1989). A point estimate of the probability of exceedance can be computed by ignoring the uncertainty in the model parameters and using a point estimate $\hat{\mathbf{\Theta}}$ in place of $\mathbf{\Theta}$. Using the posterior mean value as the point estimate leads to the closed form solution of Eq. (13) that is written as

$$P\left[g\left(\zeta_{design},\mathbf{x},\hat{\mathbf{\Theta}}\right)\leq 0\middle|\mathbf{x}\right]=1-\Phi\left(\frac{\zeta_{design}-\hat{\mu}_{\theta_\zeta}-\hat{\zeta}_{final}\left(\mathbf{x}\right)}{\hat{\mu}_{\sigma_\zeta}}\right) \tag{15}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative probability, and $\hat{\mu}_{\theta_\zeta}$ and $\hat{\mu}_{\sigma_\zeta}$ are the estimated mean values of the model parameters θ_ζ and σ_ζ, respectively. The reliability index that corresponds to the closed form solution in Eq. (15) is then

$$\hat{\beta}\left(\zeta_{design},\mathbf{x}\right)=\frac{\zeta_{design}-\hat{\mu}_{\theta_\zeta}-\hat{\zeta}_{final}\left(\mathbf{x}\right)}{\hat{\mu}_{\sigma_\zeta}} \tag{16}$$

Following Gardoni et al. (2002), a first-order analysis is used to obtain approximate confidence bounds that reflects the uncertainty in the model parameters. In general, $\beta(\zeta_{design},\mathbf{x},\mathbf{\Theta})$ is less nonlinear in $\mathbf{\Theta}$ than $P[g(\zeta_{design},\mathbf{x},\mathbf{\Theta})\leq 0 \mid \mathbf{x},\mathbf{\Theta}]$, so bounds are constructed around $\hat{\beta}(\zeta_{design},\mathbf{x})$ and then transformed into the probability space. Using a first-order Taylor series expansion around the mean point, the variance of $\beta(\zeta_{design},\mathbf{x},\mathbf{\Theta})$ is approximated as

$$\sigma_\beta^2\left(\zeta_{design},\mathbf{x}\right)\approx\nabla_{\mathbf{\Theta}}\beta\left(\zeta_{design},\mathbf{x}\right)\Sigma_{\mathbf{\Theta\Theta}}\nabla_{\mathbf{\Theta}}\beta\left(\zeta_{design},\mathbf{x}\right)^T \tag{17}$$

where $\nabla_{\mathbf{\Theta}}\beta(\zeta_{design},\mathbf{x})$ is the gradient row vector of $\beta(\zeta_{design},\mathbf{x},\mathbf{\Theta})$ computed at the mean point, and $\Sigma_{\mathbf{\Theta\Theta}}$ denotes the posterior covariance matrix of $\mathbf{\Theta}$. Approximate bounds on the conditional probability of exceedance can be expressed in terms of one standard deviation away from the mean as

$$\left\{\Phi\left[-\hat{\beta}\left(\zeta_{design},\mathbf{x}\right)+\sigma_{\beta}\left(\zeta_{design},\mathbf{x}\right)\right],\ \Phi\left[-\hat{\beta}\left(\zeta_{design},\mathbf{x}\right)-\sigma_{\beta}\left(\zeta_{design},\mathbf{x}\right)\right]\right\} \tag{18}$$

These bounds approximately correspond to 15% and 85% probability levels.

PROBABILITY OF EXCEEDANCE FOR AN EXAMPLE BRIDGE PIER

The probabilistic HEC-18 Clay model developed in Eq. (11) can be used to assess the probability that the final scour will exceed a design depth, ζ_{design}, at any circular bridge pier with specified geometry, and for specified water velocity and upstream water depth. The previous section describes the computational framework for this purpose. In particular Eq. (15) can be used to construct a point estimate of the probability of exceedance and Eq. (18) can be used to construct confidence bounds that reflect the uncertainty in the model parameters. As an example, a bridge pier with circular cross section of 2 m in diameter is considered. It is also assumed that the pier is subject to a constant flood velocity of 3 m/s, and the upstream water depth is 5 m.

The probability of exceedance for the example pier is a function of the design scour depth, ζ_{design} (or z_{design}), and of the time, t, over which the pier is subject to the constant flood velocity. Fig. 14 provides a conceptual three-dimensional plot of the probability of exceedance vs. z_{design} and t. The figure shows that at a specified time the probability of exceedance decreases as z_{design} increases, and at a specified z_{design} the probability of exceedance increases with t.

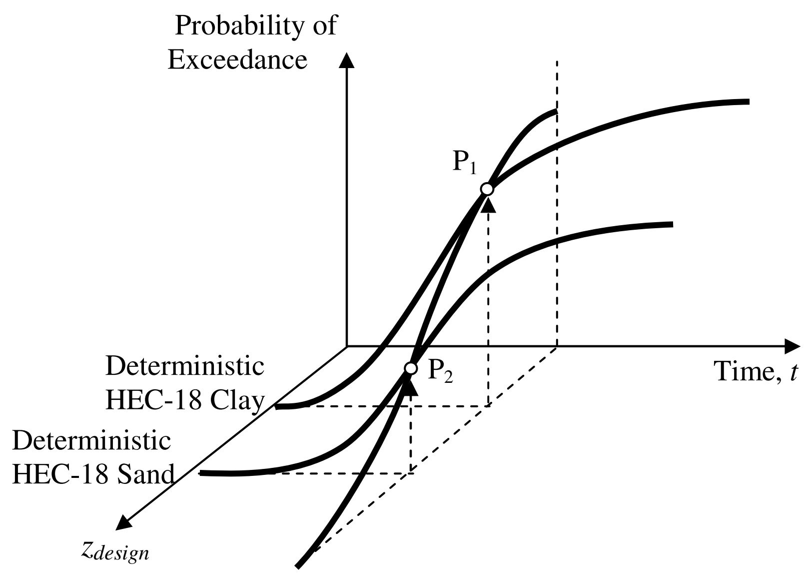

Fig. 14. Concept of the probability of exceedance, time, and design scour depth for the HEC-18 methods

Fig. 15 shows the probability of exceedance vs. z_{design} at $t = 600$ hours. The solid line represents the point estimate of the probability of exceedance and the dashed lines indicate the 15 and 85% confidence bounds relative to the uncertainty in the model parameters. The dispersion indicated by the slope of the solid curve represents the effect of the uncertainty capture by the model error $\sigma_{\zeta}\varepsilon$. Using the deterministic HEC-18 Clay method, $z_{design} = \hat{z}_{final\ \text{HEC-18 Clay}} = 1770$ mm, this corresponds to a probability of exceedance of 0.45. Using the deterministic HEC-18 Sand method, $z_{design} = \hat{z}_{max\ \text{HEC-18 Sand}} = 4211$ mm, this corresponds to a probability of exceedance of 0.01. The deterministic HEC-18 Sand method gives a lower probability of exceedance than the deterministic HEC-18 Clay method, because, as previously shown, HEC-18 Sand is a design method and tends to be more conservative, while HEC-18 Clay is a prediction method.

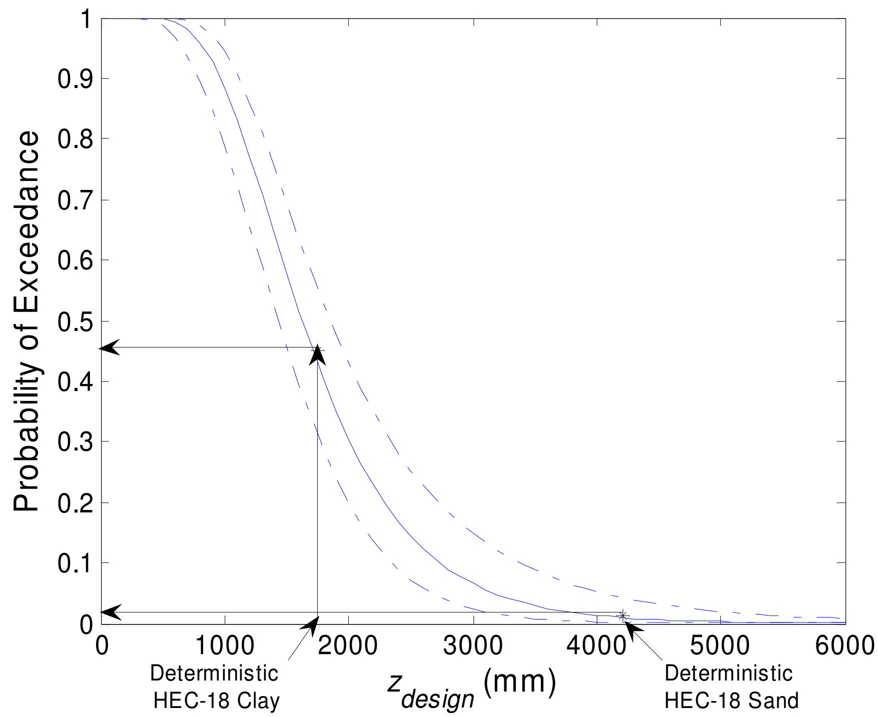

Fig. 15. Probability of exceedance vs. z_{design} for $t = 600$ hours

Figs. 16 and 17 show the probability of exceedance vs. t at $z_{design} = \hat{z}_{final\ \text{HEC-18 Clay}} = 1770$ and $z_{design} = \hat{z}_{max\ \text{HEC-18 Sand}} = 4211$ mm, respectively. For the given velocity, the majority of the scour occurs in the first 4000 hours. If the velocity were lower, the time needed to reach the same levels of probability of exceedance would be longer. The probabilities of exceedance over time for $z_{design} = \hat{z}_{max\ \text{HEC-18 Sand}}$ are lower than for $z_{design} = \hat{z}_{final\ \text{HEC-18 Clay}}$. This is consistent with the more conservative nature of the deterministic HEC-18 Sand with respect to the deterministic HEC-18 Clay, already observed in Fig. 15.

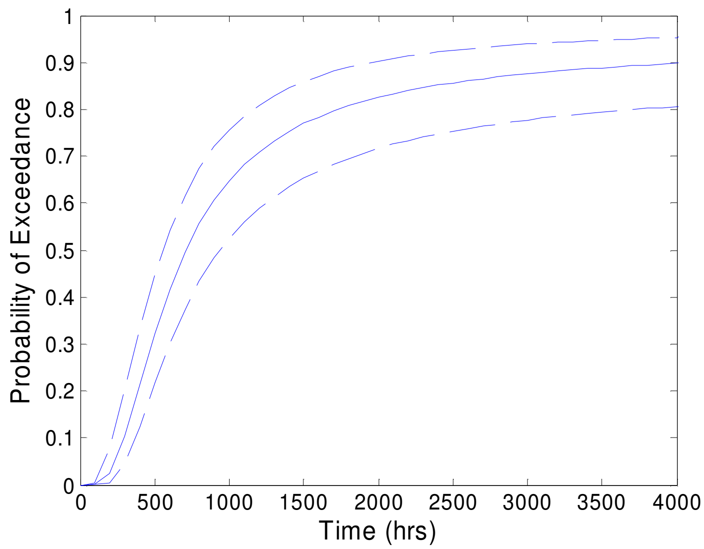

Fig. 16. Probability of exceeding $z_{design} = \hat{z}_{final\ \text{HEC-18 Clay}}$ vs. time

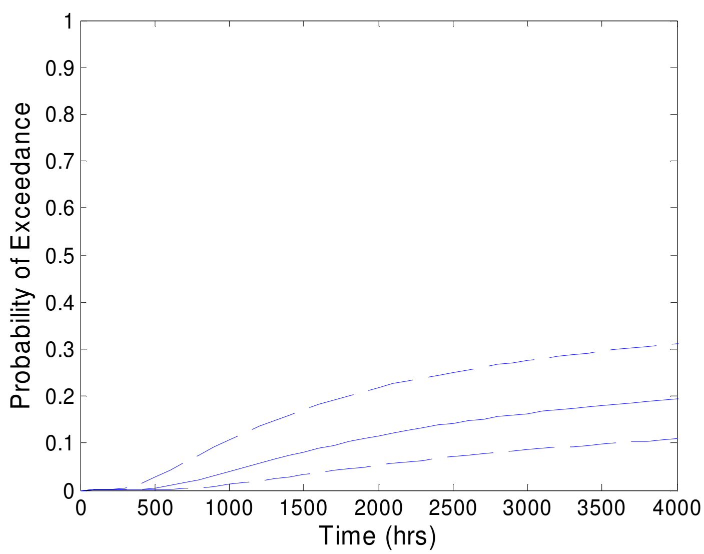

Fig. 17. Probability of exceeding $z_{design} = \hat{z}_{max\ \text{HEC-18 Sand}}$ vs. time

CONCLUSIONS

Probabilistic models are formulated based on the deterministic HEC-18 Sand and HEC-18 Clay models. The developed probabilistic models are unbiased and account for the inherent model uncertainty. In particular, bias correction factors are assessed by Bayesian statistical analysis using field and experimental data. The values of the model parameters suggest that the maximum sour depth predicted by the deterministic HEC-18 Sand and HEC-18 Clay models tend to be conservative. Evidence is also found that the applicability of the HEC-18 Clay method is not limited to clay but can also be used for other soil types. The main advantage of the HEC-18 Clay method with respect to the HEC-18 Sand method is that it predicts the depth of scour as a function of time and can be used to estimate the final scour at the end of the design life of a structure. The final scour depth predictions based on the HEC-18 Clay method compare well with the data, showing no significant bias.

The developed probabilistic model for the final scour depth is used in a formulation to assess the probability that a specified threshold depth is exceeded at a bridge pier for given hydrologic variables. Confidence bounds on the probability estimates are developed by first-order analysis to reflect the effect of the epistemic uncertainty present in the model parameters. As an illustration, the probability of exceedance of a threshold depth at an example bridge pier is estimated.

REFERENCES

Box, G. E. P., and Tiao, G. C. (1992). *Bayesian inference in statistical analysis.* Addison-Wesley, Reading, Mass.

Brandimarte, L., Montanari, A., Briaud, J.-L., and D'Odorcio, P. (2006). "Stochastic flow analysis for predicting scour of cohesive soils." *Journal of Hydraulic Engineering*. 132(5), 493 – 500.

Briaud, J.-L., Chen, H. C., Kwak, K. W., Han, S. W., and Ting, F. C. K. (2001a). "Multiflood and multilayer method for scour rate prediction at bridge piers." *Journal of Geotechnical and Environmental Engineering*, 127(2), 114 – 123.

Briaud, J.-L., Ting, F. C. K., Chen, H. C., Cao, Y., Han, S. W., and Kwak, K. W. (2001b). "Erosion function apparatus for scour rate predictions." *Journal of Geotechnical and Environmental Engineering*, 127(2), 105 – 113.

Briaud, J.-L., Ting, F. C. K., Chen, H. C., Gudavalli, S. R., Perugu, S., and Wei, G. (1999). "SRICOS: Prediction of scour rate in cohesive soils at bridge piers." *Journal of Geotechnical and Environmental Engineering*, 125(4), 237 – 246.

Der Kiureghian, A. (1989). "Measures of structural safety under imperfect states of knowledge." *Journal of Structural Engineering*, 115(5), 1119 – 1140.

Ditlevsen, O., and Madsen, H. O. (1996). *Structural reliability methods*, Wiley, New York, New York.

Gardoni, P., Der Kiureghian, A., Mosalam, K. M. (2002). "Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations." *Journal of Engineering Mechanics*, 128(10), 1024 – 1038.

Gudavalli, S. R. (1997). "Prediction model for scour rate around bridge piers in cohesive soil on the basis of flume tests." PhD Dissertation, Department of Civil Engineering, Texas A&M University, College Station, Tex.

Kwak, K. (2000). "Prediction of scour depth versus time for bridge piers in cohesive soils in the case of multi-flood and soil systems." PhD Dissertation, Department of Civil Engineering, Texas A&M University, College Station, Tex.

Landers, M. N., and Mueller, D. S. (1996). "Channel scour at bridges in the United States." *Report No. FHWA-RD-95-184*, Federal Highway Administration, Washington, D. C.

Moody, L. F. (1944). "Friction factors for pipe flow." *Trans. Am. Soc. of Mech. Engrs.*, 66, 671-678.

Rao, C. R. and Toutenburg, H. (1997). *Linear models, least squares and alternatives*. Springer, New York.

Richardson, E. V., and Davis, S. R. (2001). “Evaluating scour at bridges.” *Hydraulic Engineering Circular No. 18, FHWA NHI 01-001*, 4th Ed., U.S. Department of Transportation, Washington, D.C.

Stone, J.C. (1996). *A course in probability and statistics*. Duxbury, Belmont, Calif.

APPENDIX A

DETAILED REVIEW OF THE DETERMINISTIC HEC-18 CLAY METHOD

The HEC-18 Clay method is a multi-step set of calculations to determine the final depth of scour a bridge pier will experience over its lifespan. A summary of the basic HEC-18 Clay method (Briaud 1999) consists of:

1. Collecting Shelby tube samples near the bridge pier
2. Test the Shelby tube samples in the EFA (Erosion Function Apparatus) to obtain the erosion rate $\dot{z}$ (mm/hr) versus hydraulic shear stress τ (N/m^2) curve
3. Calculating the maximum hydraulic shear stress τ_{max} around the pier before scour starts
4. Reading the initial erosion rate $\dot{z}_i$ (mm/hr) corresponding to τ_{max} on the $\dot{z}$ versus τ curve
5. Calculating the maximum depth of scour $\dot{z}_{max}$
6. Construct the scour depth z versus time t curve

The EFA works by allowing water to flow at a constant velocity, over a 1 mm Shelby tube sample, to determine the rate of erodibility of the soil. Once the Shelby tube sample has been placed in the EFA a piston pushes the soil sample up 1 mm into the pipe where the water flows (Fig. A1). As water flows through the pipe at a constant velocity v, the time it takes to erode the 1 mm of soil is recorded. The rate of erosion, $\dot{z}$, is then established as $1/t$ in mm/hr. The hydraulic shear stress experienced by the soil from the flowing water is calculated with the aid of the Moody Chart (Moody 1944). Various velocities are used to test the soil sample and for each test a I and a τ value are collected to build the $\dot{z}$ versus τ curve (Fig. A1).

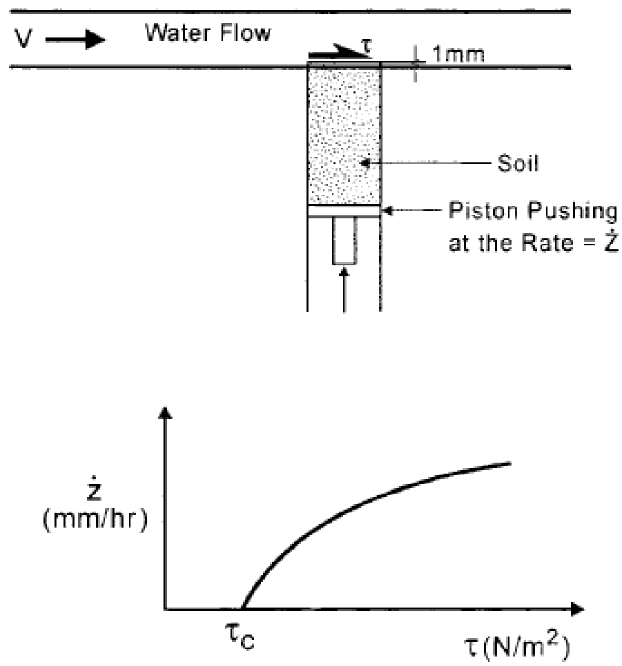

Fig. A1. Diagram and result of EFA (Erosion Function Apparatus) (Briaud et al. 1999)

Maximum shear stress around bridge piers is a function of the diameter of the pier, shape of the pier, and approach velocity. Three-dimensional numerical simulations of water flowing past a cylindrical pier of diameter *B* were used in development of the maximum shear stress equation (Briaud et al. 1999). A flat bed with a large water depth (greater than 1.5*B*) was the soil bed condition for the simulations. From simulations the τ_{max} equation is as follows:

$$\tau_{max}\left(N/m^2\right)=0.094\rho_w v^2\left(\frac{1}{\log \mathrm{Re}}-\frac{1}{10}\right) \tag{19}$$

where ρ_w = density of water (999.972 kg/m^3 at 20°C), v = average velocity in the river (without the bridge pier) at the bridge pier location, and Re = Reynolds number of the bridge pier (vB/υ) where B = pier diameter, and υ = kinematic viscosity of water (10^{-6} m^2/s at 20°C). Using τ_{max} and the $\dot{z}$ versus τ curve developed from the EFA, the initial rate of scour $\dot{z}_i$ is read off the curve given the τ_{max} value calculated in Eq. (19).

Briaud et al. (1999) used 36 model scale flume experiments on three different clay soils to develop the maximum depth of scour z_{max} equation. Results from the experiments gave the following relationship for maximum depth of scour:

$$z_{max}\left(mm\right)=0.18\,\mathrm{Re}^{0.635} \tag{20}$$

where Re is defined the same as in Eq. (19). Seven sand model scale flume experiments conducted by Gudavalli (1997) and previous research by Landers and Muller (1996) confirm Eq. (20) is valid for sand and clay soils.

Clay soil scale flume experiments were carried out over several days with high velocities (simulating flooding) to attempt to reach the maximum scour depth (Briaud et al. 1999). Time dependency of scour depth evolution is introduced in the method through a hyperbola which links the scour depth to the time a given velocity has been applied. The resulting final depth of scour is of the form

$$z_{final}\left(mm\right)=\frac{t}{\frac{1}{\dot{z}_i}+\frac{t}{z_{\max}}} \tag{21}$$

where t is time in hours and $\dot{z}_i$ and z_{max} are previously defined. If $\dot{z}_i$ of the clay soil is small then it is possible for z_{final} to only be a small portion of z_{max}. Kwak's database measured final scour depth using live bridge pier data for cohesive soils.

APPENDIX B

INTERMEDIATE HEC-18 CLAY PROBABILISTIC MODELS

A probabilistic equation for each intermediate step leading up to the final depth of scour prediction involved in the HEC-18 Clay method is evaluated to predict the accuracy of each step and to determine the error. Gudavalli (1997) performed the original research that developed the HEC-18 Clay equations. Thus, databases from Gudavalli (1997) are used to evaluate the model parameters in each of the probabilistic models.

All data points used to develop τ_{max} deterministically are used to compute the probabilistic τ_{max} model which takes the form:

$$\tau_{max}\,(N/m^2) = \frac{C_f \rho_w v^2}{2} \tag{22}$$

where ρ_w and v are defined in (19) and

$$C_f = \frac{\theta_1}{\log \mathrm{Re}} + \theta_2 + \sigma_1 \varepsilon \tag{23}$$

where θ_1, θ_2, σ_1 are all random variables, Re is defined in (19), and ε denotes a random variable with zero mean and unit standard deviation. Table B1 lists the posterior statistics of the parameters $\mathbf{\Theta_1} = (\theta_1, \theta_2, \sigma_1)$ for the τ_{max} model. Note that the correlation coefficient matrix is symmetrical in all presented models.

Table B1. Posterior statistics of parameters in HEC-18 Clay model for τ_{max}

Parameter	Mean	Standard Deviation	Correlation Coefficient		
			θ_1	θ_2	σ_1
θ_1	0.189	0.016	1		
θ_2	-0.019	0.003	-0.97	1	
σ_1	0.002	0.005	0.10	-0.12	1

The coefficient C_f is used in the determination of τ_{max} as done in the original work (Gudavalli 1996). Fig. B1 depicts the comparison between the deterministic and probabilistic model for C_f. The correlation between the two models is good, providing a small standard deviation. For all figures dots indicate data points, dashed lines indicate plus or minus one standard deviation from the one-to-one line where the one-to-one line is displayed by a dotted dashed line. It is observed that the deterministic model is accurate based on the relationship between the location of the data points and the one-to-one line (left side of Fig. B1). Therefore, it is reasonable for the bounds of the probabilistic curve (right side of Fig. B1) to be close to the one-to-one line.

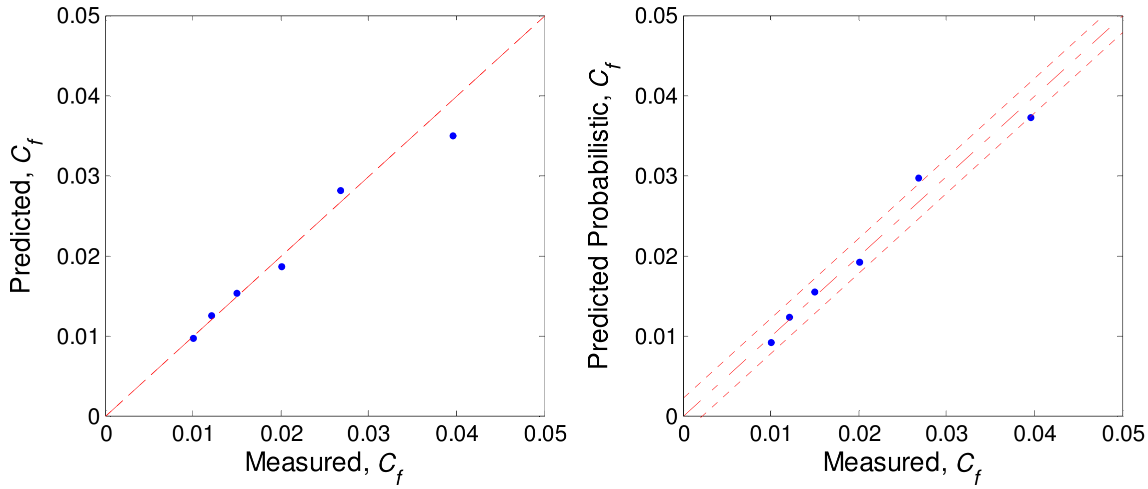

Fig. B1. Comparison between measured and predicted C_f values use to predict the maximum shear stress of the bridge pier based on deterministic (left) and probabilistic (right) HEC-18 Clay models

The deterministic HEC-18 method uses a graphical approach to determine the rate of scour, $\dot{z}$. Before evaluating the HEC-18 Clay method an analytical probabilistic model must be developed for the rate of scour. An equation form for the probabilistic model of scour rate, $\dot{z}$, is assumed to represent a typical porcelain clay soil as used in Gudavalli (1997). Based on this soil the following form for the probabilistic model of $\dot{z}$ is:

$$\dot{z}\,(mm/hr) = \begin{cases} 0 & \tau_{\max} \le \theta_4 \\ \theta_3\left(\tau_{\max} - \theta_4\right)^{\theta_5} + \sigma_2 \varepsilon & \text{Otherwise} \end{cases} \tag{24}$$

where θ_3, θ_4, θ_5, σ_2 are all random variables, τ_{max} is defined in Eq. (19), and ε denotes a random variable with zero mean and unit standard deviation. Critical shear stress is defined as the shearing stress where erosion of the soil begins. Critical shear stress is taken into account by θ_4. Thus the rate of scour is zero if θ_4 is less than the critical shear stress. It is important to recognize that for each modeled bridge pier, $\mathbf{\Theta_2} = (\theta_3, \theta_4, \theta_5, \sigma_2)$ must be redeveloped to reflect the corresponding soil. Posterior statistics for our example clay soil are shown in Table B2. These values can not be compared to any deterministic values since this is the first equation for the rate of scour.

Table B2. Posterior statistics of parameters in HEC-18 Clay model for $\dot{z}$

Parameter	Mean	Standard Deviation	Correlation Coefficient			
			θ_3	θ_4	θ_5	σ_2
θ_3	1.14	0.188	1			
θ_4	3.00	0.008	-0.64	1		
θ_5	0.45	0.047	-0.97	0.63	1	
σ_2	1.10	0.125	-0.27	0.60	0.26	1

A measured versus predicted plot for the probabilistic rate of scour model is given in Fig. B2 and can only be compared to the accuracy of reading the rate of scour from a graph in the deterministic model. Fig. B3 displays the deterministic and mean value probabilistic HEC-18 Clay method prediction of rate of scour versus shear stress. From Fig. B3 it is shown that the assumed model in Eq. (24) is reasonable for the given soil sample. This can also be determined by the standard deviation of the model as given in Table B2. For each soil sample a new probabilistic rate of scour equation must be developed and evaluated to determine the error in the model for that particular soil sample.

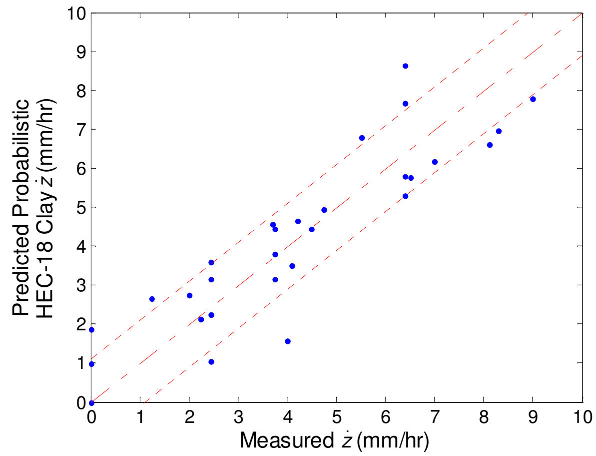

Fig. B2. Measured versus predicted rate of scour for bridge pier based on probabilistic HEC-18 Clay model

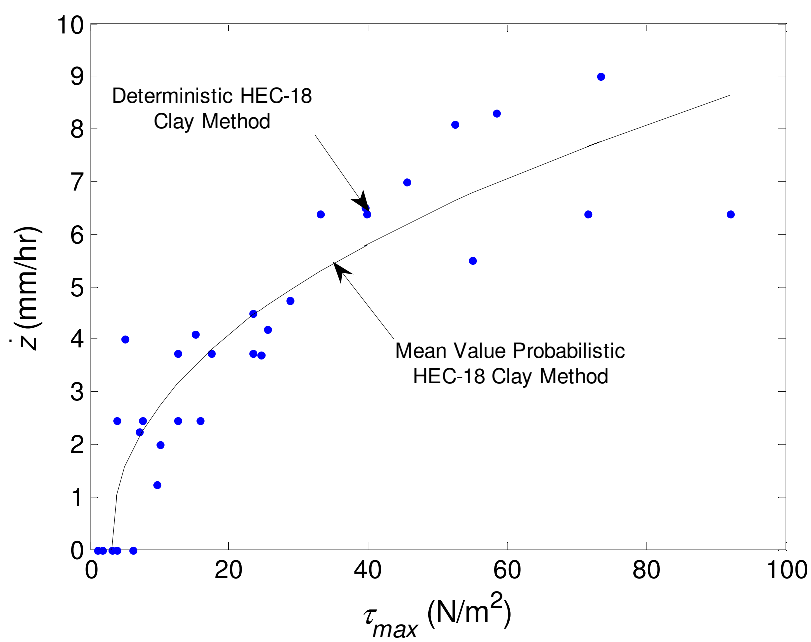

Fig. B3. Comparison between deterministic and mean value probabilistic HEC-18 Clay method of rate of scour, $\dot{z}$

As discussed previously, the deterministic z_{max} equation in HEC-18 Clay can be used for both clay and sand soils. Therefore, all 43 scale model flume experiments (Gudavalli 1997) are used in the prediction of the parameters for the probabilistic z_{max} equation. The z_{max} equation takes the form of

$$z_{max}\,(mm) = \theta_6\,\mathrm{Re}^{\theta_7} + \sigma_3 \varepsilon \tag{25}$$

where θ_6, θ_7, σ_3 are random variables, Re is previously defined, and ε denotes a random variable with zero mean and unit standard deviation. By looking at the mean values of θ_6 or θ_7 in Table B3 it is evident that additional sand flume experiments did not significantly alter the corresponding deterministic values. This confirms the original statement that the z_{max} equation is valid for both sand and clay soils.

Table B3. Posterior statistics of parameters in HEC-18 Clay Model for z_{max}

Parameter	Mean	Standard Deviation	Correlation Coefficient		
			θ_6	θ_7	σ_3
θ_6	0.191	0.051	1		
θ_7	0.635	0.026	-0.99	1	
σ_3	25.5	2.85	0.08	-0.08	1

Based on the measured versus predicted plots provided in Fig. B4 it is clear that the deterministic maximum prediction of scour depth is accurate. The linear relationship in the deterministic plot indicates the deterministic model form is good.

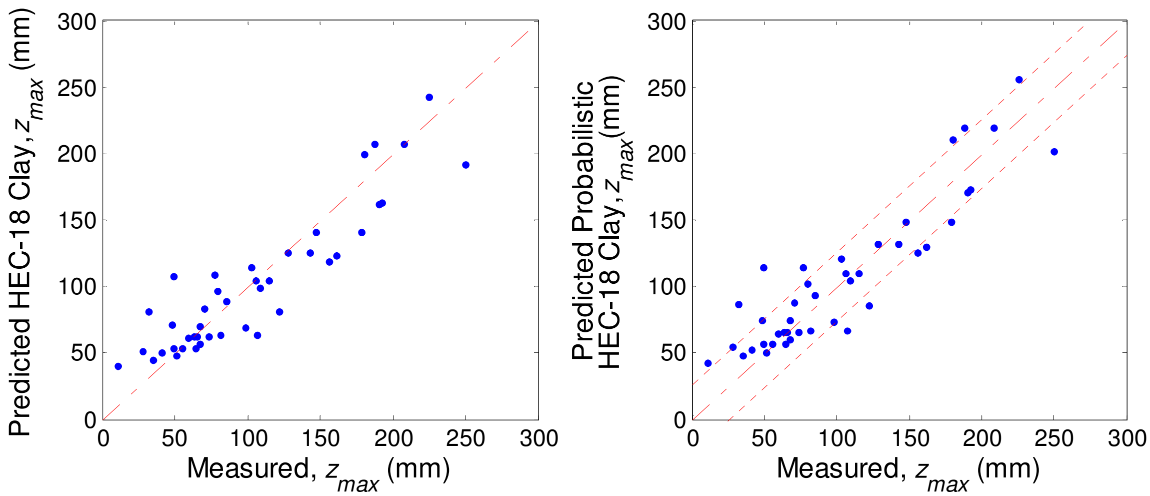

Fig. B4. Measured versus predicted maximum depth of scour for bridge piers based on probabilistic HEC-18 Clay model

The probabilistic models developed for the intermediate steps of the HEC-18 Clay method confirm that the deterministic models are good approximations of the indicated quantities. The benefit of having the error term for each method is to tell which step in the method contributes the most error in the final prediction of scour. Based on the results of this analysis the maximum scour depth equation contributes the most error to the final scour depth equation. This is shown by the standard deviation given in the provided tables. The soil and probabilistic model form for the rate of scour will change with each analysis. Therefore, the rate of scour may contribute more error to the final scour depth equation depending on the probabilistic model form of $\dot{z}$.

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