PHENOMENOLOGY OF HETEROTIC AND TYPE II ORIENTIFOLD STRING MODELS

A Dissertation

by

VAN ERIC MAYES

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2007

Major Subject: Physics

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Approved by:

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ABSTRACT

Phenomenology of Heterotic and Type II Orientifold String Models. (August 2007)

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Chair of Advisory Committee: Dr. Dimitri Nanopoulos

Cryptons are metastable bound states of fractionally-charged particles that arise generically in the hidden sectors of models derived from heterotic string. We study their properties and decay modes in a specific flipped SU(5) model with long-lived four-particle spin-zero bound states called *tetrons*. The expected masses and lifetimes of the neutral tetrons make them good candidates for cold dark matter (CDM), and a potential source of the ultra-high energy cosmic rays (UHECRs) which have been observed, whereas the charged tetrons would have decayed in the early Universe.

We calculate the spectra of ultra-high-energy cosmic rays (UHECRs) in an explicit top-down model based on the decays of metastable neutral 'crypton' states. For all the decay operators, the total UHECR spectra are compatible with the available data. Also, the fractions of photons are compatible with all the published upper limits, but may be detectable in future experiments.

We also construct several intersecting D-brane models on a variety of orientifold backgrounds. In particular, we construct flipped SU(5), Pati-Salam, and MSSM-like models. The phenomenological properties of these models are studied. For one model in particular, we find that we may explain the quark masses and mixings, the tau lepton mass, and generate small neutrino masses via the see-saw mechanism.

To my family and friends.

ACKNOWLEDGMENTS

I would first like to acknowledge my Ph.D. advisor, Dr. Dimitri Nanopoulos, who taught me almost everything I know about particle physics and string theory. I would also like to acknowledge John Ellis, with whom I wrote my first two papers on theoretical physics and from whom I learned a great deal. I would also like to thank all the members of my doctoral committee for being willing to serve. I cannot overlook the assistance of Ching-Ming Chen, George Kraniotis, and Tianjun Li with whom I had many productive collaborations.

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CHAPTER I

INTRODUCTION

For a long time, high energy physics has been in something of a rut. Theory has far outpaced experimental progress, a situation which may finally change with the upcoming commissioning of the Large Hadron Collider (LHC) in November 2007. It has been roughly thirty years since the development of the highly successful Standard Model (SM) of elementary particles. Although this model has largely been verified, there still remain open questions. Among these is the verification of the Higgs sector, which is required to spontaneously break the electroweak symmetry and generate mass for quarks and leptons. In addition, the SM contains eighteen parameters which are not fixed by the theory.

Since the 1970's, there has been some hope that the free parameters of the SM may be fixed or at least reduced by embedding the SM gauge group in a larger structure, known as a Grand Unified Theory (GUT). Additional encouragment for this idea arose after the observation that the SM gauge couplings appear to converge and become unified at a scale $\sim 10^{15}$ GeV when extrapolated to high energies. Furthermore, the quarks and leptons which transform as representations of $SU(3)_c \times SU(2)_L \times U(1)_Y$ fit nicely inside representations of $SU(5)$. However, despite it's attractiveness, there were some additional problems introduced by grand unification. First among these is the problem of proton stability. Null experiments designed to observe proton decay have pushed the proton lifetime to much longer than what was predicted for conventional $SU(5)$ unification. The apparent convergence of the gauge coupling constants at a single point was shown to actually not occur after precision measurements by

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LEP of the gauge couplings at the Z_0 pole. Lastly, it was difficult to explain the huge difference in energy between the electroweak scale and the grand unification scale, a problem which has become known as the gauge hierarchy problem.

Amazingly, all of these problems may be ameliorated or cured altogether by incorporating $N = 1$ supersymmetry (SUSY) so that the SM becomes the Minimal Supersymmetric Standard Model (MSSM). In particular, the gauge hierarchy problem is solved and the gauge couplings become unified at an energy $\sim 2 \cdot 10^{16}$ GeV. Since the unification scale is pushed to a higher scale the predicted proton lifetime is longer and was not in conflict with experimental data that was available through the 1980's, although minimal $SU(5)$ has now been excluded. Additionally, if supersymmetry is a local symmetry, then the theory necessarily includes gravity, a development known as supergravity (SUGRA). Thus, supersymmetry which is beautiful in it's own right, economically solves many problems. Indeed it is hard to see how nature would not incorporate it.

Yet, despite all of this progress there are still problems. In unbroken SUSY, each SM particle should have a partner of equal mass. However, this is clearly in conflict with experimental observations. This problem may be circumvented by postulating that supersymmetry is broken at some scale, causing the SUSY partners to become somewhat heavier than the SM states. The mass of the SUSY partners is expected to be of the TeV scale, and should be observable at the LHC if they exist. The discovery of supergravity created much excitement and led many to believe that they were on the verge of a final 'theory of everything' (TOE). However, this excitement was soon tempered with the realization that supergravity was probably not finite, a requirement of any candidate quantum theory of gravitation.

Shortly thereafter, string theory (for reviews, see [1]) emerged as a candidate theory of not only gravitation, but of all interactions. The basic idea of string theory is that the elementary particles are not mathematical points, but rather different vibrational modes of an elementary microscopic string. Because it is based on extended objects rather than point particles, the divergences that plague supergravity and other candidate theories of quantum gravity are not present. Supersymmetry is included as an necessary ingredient of string theory (superstring theory). Furthermore, the dimensionality of spacetime is fixed to be ten. Since the number of dimensions that we observe is four rather than ten, it is supposed that the extra dimensions must have the geometry of a compact manifold, and so are unobserved. The size and shape of the compactified manifold is arbitrary, and leads to additional fields known as moduli which must be fixed in the low energy action. Alternatively, our universe may be confined to an object known as a D-brane, which is a type of topological defect than arises in string theory.

In principle, it should be possible to derive all known physics from the string, as well as potentially provide something new and unexpected. This is the goal of string phenomenology. However, in spite of this there exist many solutions that may be derived from string, all of which are consistent vacua. One of these vacua should correspond to our universe, but then the question becomes why this particular vacuum corresponds to the universe as we observe. One possible approach to this state of affairs is to statistically classify the possible vacua, in essence making a topographical map of the 'landscape'. One then attempts to assess the liklihood that vacua with properties similar to ours will arise (For example, see [2, 3, 4, 5, 6, 7]). Another approach is to take the point of view that there are unknown dynamics, perhaps involving a departure from criticality, which determine the vacuum that corresponds to our universe. Although these dynamics are presently not well understood, it is possible that by constructing models in the vicinity of this vacuum, we may gain a deeper understanding, much as the Bohr model of the hydrogen atom eventually led to quantum mechanics. Regardless of the question of uniqueness, if string theory is correct then it should be possible to find a solution which corresponds *exactly* to our universe, at least in it's low energy limit. Although there has been a great deal of progress in constructing semi-realistic models, this has not yet been achieved.

The minimal option is to embed just the Standard Model $SU(3) \times SU(2) \times U(1)$ gauge group, but almost every construction contains at least some extra $U(1)$ factors. Conventional GUT models such as $SU(5)$ or $SO(10)$ have been investigated, but none of them has been completely satisfactory. This triggered the motivation to consider the gauge group $SU(5) \times U(1)_X$ [8, 9, 10] as a candidate for a model derived from string. The *raison d'être* of this 'flipped' $SU(5)$ is that it requires only $\mathbf{10}$ and $\overline{\mathbf{10}}$ Higgs representations to break the GUT symmetry, in contrast to other unified models which require large and unwieldy adjoint representations. This point was given further weight when it was realized that models with adjoint Higgs representations cannot be derived from string theory with a $k = 1$ Kac-Moody algebra [11]. There are many attractive features of flipped $SU(5)$. For example, the hierarchy problem between the electroweak Higgs doublets and the color Higgs triplets is solved naturally through a 'missing partner' mechanism [8]. Furthermore, this dynamical doublet-triplet splitting does not require or involve any mixing between the Higgs triplets leading to a natural suppression of dimension 5 operators that may mediate rapid proton decay and for this reason it is probably the simplest GUT to survive the experimental limits placed upon proton lifetime [12]. More recently, the cosmic microwave anisotropy $\delta T/T$ has been successfully predicted by flipped $SU(5)$, as it has been determined to be proportional to $(M/M_P)^2$ where M denotes the symmetry breaking scale and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass [13]. Finally, string-derived flipped $SU(5)$ may provide a natural explanation for the production of Ultra-High Energy Cosmic Rays (UHECRs), through the decay of super-heavy particles dubbed 'cryptons' [14] that arise in the hidden sector of the model, which are also candidates for cold-dark matter (CDM).

The heterotic string-derived flipped $SU(5)$ model was created within the context of the free-fermionic formulation, which easily yields string theories in four dimensions. This model belongs to a class of models that correspond to compactification on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold at the maximally symmetric point in the Narain moduli space [15]. Although formulated in the context of weakly coupled heterotic string theory, it is believed that the vacuum may in fact be non-perturbative due to it's proximity to special points in the moduli space and may elevate to a consistent vacuum of M-theory. For this reason, it is hoped that in searching for a realistic flipped $SU(5)$ model that we may arrive at or near the same vacuum using D-brane constructions.

Previously it was thought that only models based upon weakly coupled heterotic string compactifications could achieve this. However, in recent years Type II (Type I Orientifold) compactifications involving D-branes, where chiral fermions arise from strings stretching between D-branes intersecting at angles (Type IIA picture) [16] and in its T-dual (Type IIB) picture with magnetized D-branes [17], have provided an interesting and exciting approach to this problem.

A plan of research is outlined in the following. First, the problem and Ultra-High Energy Cosmic Rays is discussed and as well as the possiblity that they are the result of crypton decay. Cryptons are a type of superheavy particle which appears generically in string models. The focus of this research will be on cryptons appearing in a particular string model, which is physically well-motivated. Secondly, the basic ideas of model building with intersecting D-branes on Type I orientifold backgrounds is discussed. Finally, the specific research goals are stated.

A. Cryptons and Ultra-High Energy Cosmic Rays

Metastable particles of mass $\mathcal{O}(10^{12-15})$ GeV whose lifetime is greater than the age of the Universe would be appealing cadidates for cold dark matter, and their decays might provide the observed ultra-high-energy cosmic rays (UHECRs) [18, 19]. A perfect candidate for such particles is provided by 'cryptons' [11, 14, 20], bound states that appear in the hidden sectors of unified superstring models. It has been pointed out that the hidden sectors of compactifications of the heterotic string generically contain fractionally-charged particles [21, 22]. Since there are very stringent limits on the abundances of fractionally-charged particles [23], it is desirable to confine them, just as occurs for quarks in QCD. This is exactly what happens to the fractionally-charged states in the flipped SU(5) free fermionic string model [8], where this solution to the problem of fractionally-charged states was first pointed out [11, 20], and which remains the only example in which this solution has been worked out in detail.

In flipped SU(5), the cryptons are bound states composed of constituents with electric charges $\pm\frac{1}{2}$ that form $\mathbf{4}$ and $\bar{\mathbf{4}}$ representations of a hidden non-Abelian gauge group, SO(6) $\sim$ SU(4) [8]. This confines the fractionally-charged states into integer-charged cryptons that may be either meson-like $\bar{\mathbf{4}}\mathbf{4}$ combinations or baryon-like states containing four $\mathbf{4}$ or $\bar{\mathbf{4}}$ states, that we term *tetrons*, at a characteric mass scale $\Lambda_4 \sim 10^{11-13}$ GeV [20]. It is known that superheavy particles X with masses in the range 10^{11} GeV $\lesssim M_X \lesssim 10^{14}$ GeV might well have been produced naturally through the interaction of the vacuum with the gravitational field during the reheating period of the Universe following inflation in numbers sufficient to provide superheavy dark matter [24]. As was pointed out in [14], cryptons have just the right properties to be produced in this way, in particular because their expected masses $\sim \Lambda_4$ fall within the preferred range.

In general, tetrons may decay through Nth-order non-renormalizable operators in the superpotential, which would yield lifetimes that are expected to be of the order of

$$\tau \approx \frac{\alpha_{string}^{2-N}}{m_X} \left(\frac{M_s}{m_X} \right)^{2(N-3)}, \tag{1.1}$$

where m_X is the tetron mass and $M_S \sim 10^{18}$ GeV is the string scale. The α-dependent factor reflects the expected dependence of high-order superpotential terms on the effective gauge coupling g. If some tetron can decay only via higher-order interactions with $N \geq 8$, the tetron might be much longer-lived than the age of the Universe, in which case it might be an important form of cold dark matter [25]. However, no significant fraction of the astrophysical cold dark matter could consist of charged tetrons, as these would have been detected directly [26, 27, 28]. If the neutral tetrons are close to the experimental limit in (m_X, τ_X) space, with lifetimes in the range 10^{15} years $\lesssim \tau_X \lesssim 10^{22}$ years [25], an additional possibility is that their decays might explain the UHECRs observed by the AGASA collaboration [18], if these turn out to exceed significantly the GZK cutoff [29, 19, 14].

The existence of cosmic rays with energies above the Greisen-Zatsepin-Kuzmin (GZK) cutoff [29] is one of the most important open problems in high-energy astrophysics [30, 31]. These ultra-high energy cosmic rays (UHECRs) may be a tantalizing hint of novel and very powerful astrophysical accelerators, or they may be harbingers of new microphysics via the decays of metastable supermassive particles. *It is remarkable that we still do not know whether the UHECRs originate from macrophysics or microphysics.* If there is no GZK cutoff, as suggested by the AGASA data [32], the sources of the UHECRs would need to be local. In this case, since local magnetic fields are unlikely to have deflected significantly their directions of propagation, the UHECRs would 'remember' the directions of their sources. Thus, one would expect some anisotropy in the arrival directions of the UHECRs, associated either with discrete energetic astrophysical sources nearby, such as BL Lac objects [31], or the distribution of (mainly galactic) superheavy dark matter. No significant anisotropy of the UHECRs has yet been seen, but the existing experiments have insufficient statistics to exclude one at the expected level [33, 34]. On the other hand, the GZK cutoff may be present in the HiRes data [19], in which case no exotic microphysics may be needed, and any astrophysical sources would be less restricted and more difficult to trace. The first batch of Auger [35] data are inconclusive on the possible existence of the GZK cutoff.

B. Intersecting D-branes

D-branes have played a critical role in elucidating the web of dualities which connects the different string theories. They are essentially topological defects upon which strings may end. Because of this, there are gauge fields which 'live ' on the brane, which has motivated the idea of brane worlds. These suggests that our universe is confined to a D-brane and provides an alternative to compactification. A stack of N D-branes will generically have a $U(N)$ gauge group in it's world-volume. Different stacks which intersect at angles will have chiral fermions transforming in the bifundamental represenation present at the intersection from Type I strings which stretch between the different stacks. The D-branes wrap homology cycles of the compactified space, and will generally intersect one another. The multiplicity of fermionic representations is then equal to the topological intersection number between the different stacks.

A generic expression for the *net* number of chiral fermions in bifundamental, symmetric, and antisymmetric representations consistent with the vanishing of RR

Table I. Net chiral matter spectrum in terms of the three-cycles defined on the orbifold space.

Representation	Multiplicity
$\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array}$	$\frac{1}{2}([\Pi^o_{a'}] \circ [\Pi^o_a] + [\Pi_{O6}] \circ [\Pi^o_a])$
$\begin{array}{\|c\|c\|}\hline & \\ \hline \end{array}$	$\frac{1}{2}([\Pi^o_{a'}] \circ [\Pi^o_a] - [\Pi_{O6}] \circ [\Pi^o_a])]$
$(\overline{\Box}_{\mathbf{a}}, \Box_{\mathbf{b}})$	$[\Pi^o_a] \circ [\Pi^o_b]$
$(\Box_{\mathbf{a}}, \Box_{\mathbf{b}})$	$[\Pi^o_{a'}] \circ [\Pi^o_b]$

tadpoles can be given in terms of the orbifold cycles [36] which is shown in Table I.

Certain conditions must be applied to construct consistent, supersymmetric vacua which are free of anomalies:

1. All RR-tadpoles and twisted charges must vanish, which ensures that all non-Abelian anomalies will cancel. This can be expressed in terms of the homology cycles as
$$\sum_a N_a \left([\Pi_a] + [\Pi_{a'}]\right) = 4\,[\Pi_{O6}]\,. \tag{1.2}$$

2. $N = 1$ supersymmetry is preserved if and only if the angles of the three-cycle wrapped by a D-brane with the $O6$-planes is an element of $SU(3)$:
$$\theta_1 + \theta_2 + \theta_3 = 0 \mod 2\pi. \tag{1.3}$$
Together with the RR-tadpole condition above, the NS-NS tadpoles will also be canceled.

3. In order to cancel the Abelian, mixed Abelian-non-Abelian, and mixed Abelian-gravitational anomalies we must use a generalized Green-Schwarz mechanism. This typically results in several $U(1)$ factors becoming massive. These remain as global symmetries to all orders in perturbation theory, and usually tightly constrain the matter couplings.

4. To cancel all of the RR-charges, we must also cancel those which arise via K-theory. D-brane charges are not classifed by homology groups, but rather by K-theory groups. Thus, the above RR-tadpole condition is not sufficient to ensure that all tadpoles are canceled.

Thus, it is a non-trivial task to construct semi-realistic models. Many consistent Standard-like and GUT models were built at an early stage [37, 38, 39, 40] using D-brane constructions. However, these models were not supersymmetric. Furthermore, these models suffered from instability in the internal space due to the unfixed moduli. The first quasi-realistic supersymmetric models were constructed in Type IIA theory on a $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [41, 42, 43]. Turning on non-trivial fluxes as background of the compactification gives rise to a non-trivial low energy supergravity potential which freezes some Calabi-Yau moduli [44]. Type IIB configurations with non-trivial Ramond-Ramond (RR) and Neneu-Schwarz-Neveu-Schwarz (NSNS) fluxes together with the presence of anti-D3 branes have been studied in [45, 46, 47], and a complete analysis of Type IIA configurations with RR and NSNS and metric fluxes has been studied in [48]. These fluxes impose strong constraints on the RR tadpole cancellation since their supergravity equation of motion and the Dirac quantization conditions must be satisfied.

CHAPTER II

FLIPPED CRYPTONS AND UHECRS

A. Introduction

Metastable particles of mass $\mathcal{O}(10^{12-15})$ GeV whose lifetime is greater than the age of the Universe would be appealing cadidates for cold dark matter, and their decays might provide the observed ultra-high-energy cosmic rays (UHECRs) [18, 19]. A perfect candidate for such particles is provided by 'cryptons' [11, 20, 49], bound states that appear in the hidden sectors of unified superstring models. It has been pointed out that the hidden sectors of compactifications of the heterotic string generically contain fractionally-charged particles [21, 22]. Since there are very stringent limits on the abundances of fractionally-charged particles [23], it is desirable to confine them, just as occurs for quarks in QCD. This is exactly what happens to the fractionally-charged states in the flipped SU(5) free fermionic string model [8], where this solution to the problem of fractionally-charged states was first pointed out [11, 20], and which remains the only example in which this solution has been worked out in detail.

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where m_X is the tetron mass and $M_S \sim 10^{18}$ GeV is the string scale. The α-dependent factor reflects the expected dependence of high-order superpotential terms on the effective gauge coupling g. If some tetron can decay only via higher-order interactions with $N \geq 8$, the tetron might be much longer-lived than the age of the Universe, in which case it might be an important form of cold dark matter [25]. However, no significant fraction of the astrophysical cold dark matter could consist of charged tetrons, as these would have been detected directly [26, 27, 28]. If the neutral tetrons are close to the experimental limit in (m_X, τ_X) space, with lifetimes in the range 10^{15} years $\lesssim \tau_X \lesssim 10^{22}$ years [25], an additional possibility is that their decays might explain the UHECRs observed by the AGASA collaboration [18], if these turn out to exceed significantly the GZK cutoff [29, 19, 49].

We make in this paper a detailed study of cryptons in the minimal flipped SU(5) string model [8]. A survey of non-renormalizable superpotential terms up to tenth order enables us to investigate whether neutral tetrons might live long enough to constitute cold dark matter, and whether charged tetrons are likely to have had lifetimes short enough to avoid being present in the Universe today. We also study whether the decays of neutral tetrons could generate the UHECR. We indeed find that charged tetrons would have decayed into neutral tetrons in the early universe through lower-order interactions, while neutral tetrons decay through higher-order interactions with a lifetime that makes them a potential source for the UHECRs, as well as being attractive candidates for cold dark matter.

B. Field and Particle Content in the Flipped $SU(5)$ Model

Already before the advent of string models, flipped $SU(5)$ attracted interest as a grand unified theory in its own right, principally because it did not require large and exotic Higgs representations and avoided the straitjacket of minimal $SU(5)$ without invoking all the extra gauge interactions required in larger groups such as $SO(10)$ [50, 51]. Interest in flipped $SU(5)$ increased in the context of string theory, since simple string constructions could not provide the adjoint and larger Higgs representations required by other grand unified theories. Moreover, it was observed that flipped $SU(5)$ provided a natural 'missing-partner' mechanism for splitting the electroweak-doublet and colour-triplet fields in its five-dimensional Higgs representations [8]. We now review the properties of the favoured version of the flipped $SU(5)$ model derived from string theory, before discussing how, as an added bonus, it can accommodate UHECRs.

In a field-theoretic 'flipped' $SU(5) \otimes U(1)$ model the Standard Model states occupy $\bar{\mathbf{5}}$, $\mathbf{10}$, and $\mathbf{1}$ representations of the $\mathbf{16}$ of SO(10), with the quark and lepton assignments being 'flipped' $u_L^c \leftrightarrow d_L^c$ and $\nu_L^c \leftrightarrow e_L^c$ relative to a conventional SU(5) GUT:

$$f_{\bar{\mathbf{5}}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L ; \quad F_{\mathbf{10}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \; d_L^c \quad \nu_L^c \right) ; \quad l_{\mathbf{1}} = e_L^c, \tag{2.2}$$

In particular, this results in the **10** containing a neutral component with the quantum numbers of ν^c_L. Spontaneous GUT symmetry breaking can be achieved by using a **10** and $\bar{\mathbf{10}}$ of superheavy Higgs where the neutral components develop a large vacuum expectation value (*vev*), $\langle \nu^c_H \rangle = \langle \bar{\nu}^c_H \rangle$,

$$H_{\mathbf{10}} = \{Q_H, d^c_H, \nu^c_H\} \quad ; \quad H_{\bar{\mathbf{10}}} = \{Q_{\bar{H}}, d^c_{\bar{H}}, \nu^c_{\bar{H}}\}\,, \tag{2.3}$$

while the electroweak spontaneous breaking occurs through the Higgs doublets $\mathbf{H}_2$ and $\mathbf{H}_{\bar{2}}$,

$$h_{\mathbf{5}} = \{\mathbf{H}_2, \mathbf{H}_3\} \quad ; \quad h_{\bar{\mathbf{5}}} = \{\mathbf{H}_{\bar{\mathbf{2}}}, \mathbf{H}_{\bar{\mathbf{3}}}\}\,. \tag{2.4}$$

The presence of a neutral component in the **10** and $\bar{\mathbf{10}}$ of Higgs fields provides a very economical doublet-triplet splitting mechanism which gives a large mass to the Higgs triplets $(\mathbf{H}_3, \mathbf{H}_{\bar{\mathbf{3}}})$ while keeping Higgs doublets $(\mathbf{H}_2, \mathbf{H}_{\bar{\mathbf{3}}})$ light through trilinear superpotential couplings of the form,

$$FFh \rightarrow d^c_H \, \langle \nu^c_H \rangle \, H_3 \tag{2.5}$$

$$\bar{F}\bar{F}\bar{h} \rightarrow \bar{d}^c_H \, \langle \bar{\nu}^c_H \rangle \, H_{\bar{3}}. \tag{2.6}$$

Thus, in constrast to GUTs based upon other groups such as SU(5), SO(10), etc., flipped SU(5) does not require any adjoint Higgs reprentations. As a direct consequence of this, it is the only unified model that can be derived from string theory with a $k = 1$ Kac-Moody algebra [11]. As an added bonus, this dynamic doublet-triplet splitting does not require or involve any mixing between the Higgs triplets leading to a natural suppression of dimension 5 operators that may mediate rapid proton decay.

String-derived flipped SU(5) was created within the context of the free-fermionic formulation, which easily yelds string theories in four dimensions. This model belongs to a class of models that correspond to compactification on the $Z_2 \times Z_2$ orbifold at the maximally symmetric point in the Narain moduli space [15]. At the string scale, the full gauge symmetry of the model is $SU(5)\otimes U(1)\otimes U(1)^4\otimes SO(10)_h\otimes SU(4)_h$, and the spectrum contains the following massless fields [8].

(i) *Observable sector*:

This comprises three **16** representations of SO(10), that contain the $SU(5)\otimes U(1)$ chiral multiplets $F_i(\mathbf{10},\frac{1}{2})$, $\overline{f}_i(\bar{\mathbf{5}},\frac{3}{2})$, $l_i^c(\mathbf{1},\frac{1}{2})(i=1,2,3)$; extra matter fields $F_4(\mathbf{10},\frac{1}{2})$, $f_4(\mathbf{5},\frac{3}{2})$, $\bar{l}_4^c(\mathbf{1},-\frac{5}{2})$ and $\bar{F}_5(\overline{\mathbf{10}},-\frac{1}{2})$, $\bar{f}_5(\bar{\mathbf{5}},-\frac{3}{2})$, $l_5^c(\mathbf{1},\frac{5}{2})$; and four Higgs-like fields in the **10** representation of SO(10), that $\supset SU(5)\otimes U(1)$ representions $h_i(\mathbf{5},-1)$, $\bar{h}_i(\bar{\mathbf{5}},1)$, $i=1,2,3,45$.

A viable string derived flipped SU(5) model must contain the Standard Model in its light, low-energy spectrum, whilst all other observable fields should have masses sufficiently high to have avoided production at particle accelerators or observation in cosmic rays. Additionally, there must be two light Higgs doublets. As we discuss below, these two objectives have been achieved in some specific variants [52, 53] of the flipped SU(5) model, although exactly the flavor assignments of these states corresponding to those of the standard model particle content is rather model-dependent. However, a convenient choice for the flavour assignments of the fields up to mixing effects is as follows:

$$\bar{f}_1:\bar{u},\tau;\quad \bar{f}_2:\bar{c},e/\mu;\quad \bar{f}_5:\bar{t},\mu/e \tag{2.7}$$

$$F_3:Q_2,\bar{s};\quad F_3:Q_1,\bar{d};\quad F_4:Q_3,\bar{b} \tag{2.8}$$

$$l_1^c:\bar{\tau};\quad l_2^c:\bar{e};\quad l_5^c:\bar{u}. \tag{2.9}$$

(ii) *Singlets*:

There are ten gauge-singlet fields ϕ_{45}, ϕ^+, ϕ^-, $\phi_i(i=1,2,3,4)$, Φ_{12}, Φ_{23}, Φ_{31}, their ten ‘barred’ counterparts, and five extra fields $\Phi_I(I=1\cdots5)$.

(iii) *Hidden Sector*:

This contains 22 matter fields in the following representations of $SO(10)_h \otimes SU(4)_h$: $T_i(\mathbf{10},\mathbf{1})$, $\Delta_i(\mathbf{1},\mathbf{6})(i=1\cdots5)$; $\tilde{F}_i(\mathbf{1},\mathbf{4})$, $\bar{\tilde{F}}_i(\mathbf{1},\bar{\mathbf{4}})(i=1\cdots6)$. Flat potential directions along which the anomalous combination of hypercharges $U(1)_A$ is cancelled induce masses that are generally near the string scale for some, but not all, of these states. Depending upon the number of T_i and Δ_i states remaining massless, the SO(10) condensate scale is 10^{14-15} GeV and the SU(4) condensate scale is 10^{11-13} [54] GeV. The $\tilde{F}_{3,5}$ and $\bar{\tilde{F}}_{3,5}$ states always remain massless down to the condensate scale. The $U(1)_i$ charges and hypercharge assignments are shown in Table II below.

In order to preserve D and F flatness, many of the singlet fields can develop vacuum expectation values, as can some of the hidden-sector fields. Many of these flat directions have been studied in detail [55]. Typically, we have

$$\left\langle \Phi_{23}, \Phi_{31}, \bar{\Phi}_{23}, \bar{\Phi}_{31}, \phi_{45}, \bar{\phi}_{45}, \phi^+, \phi^- \right\rangle \neq 0, \tag{2.10}$$

while it can be shown that there is no solution unless $\left\langle \Phi_3, \Phi_{12}, \bar{\Phi}_{12} \right\rangle = 0$. The phenomenological details of a particular model depends upon the flat direction which is chosen.

The superheavy Higgs $H_{\mathbf{10}}$ can in general be a linear combination of F_1, F_2, F_3, and F_4 , while $H_{\bar{\mathbf{10}}} = \bar{F}_5$. The Higgs doublet matrix takes the following form, including terms up to $5th$ order in the superpotential:

$$m_h = \begin{pmatrix} 0 & \Phi_{12} & \bar{\Phi}_{31} & T_5^2\bar{\phi}_{45} \\ \bar{\Phi}_{12} & 0 & \Phi_{23} & \Delta_4^2\bar{\phi}_{45} \\ \Phi_{31} & \bar{\Phi}_{23} & 0 & \bar{\phi}_{45} \\ \Delta_5^2 & T_4^2\phi_{45} & \phi_{45} & 0 \end{pmatrix} \tag{2.11}$$

If only all-order contributions generated by singlet vevs are considered, H_1, $H_{245} =$

Table II. Charges and hypercharges for crypton fields in flipped SU(5).

State	$SU(4) \otimes SO(10)$	$U_1(1)$	$U_2(1)$	$U_3(1)$	$U_4(1)$
Δ_1	$(\mathbf{6},\mathbf{1})^0$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0
Δ_2	$(\mathbf{6},\mathbf{1})^0$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0
Δ_3	$(\mathbf{6},\mathbf{1})^0$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$
Δ_4	$(\mathbf{6},\mathbf{1})^0$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0
Δ_5	$(\mathbf{6},\mathbf{1})^0$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
T_1	$(\mathbf{1},\mathbf{10})^0$	0	$-\frac{1}{2}$	$\frac{1}{2}$	0
T_2	$(\mathbf{1},\mathbf{10})^0$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0
T_3	$(\mathbf{1},\mathbf{10})^0$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$
T_4	$(\mathbf{1},\mathbf{10})^0$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
T_5	$(\mathbf{1},\mathbf{10})^0$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0
$\tilde{F}_1$	$(\mathbf{4},\mathbf{1})^{+5/4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$
$\tilde{F}_2$	$(\mathbf{4},\mathbf{1})^{+5/4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$
$\tilde{F}_3$	$(\mathbf{4},\mathbf{1})^{-5/4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$
$\tilde{F}_4$	$(\mathbf{4},\mathbf{1})^{+5/4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$
$\tilde{F}_5$	$(\mathbf{4},\mathbf{1})^{+5/4}$	$-\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0
$\tilde{F}_6$	$(\mathbf{4},\mathbf{1})^{+5/4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$
$\tilde{\bar{F}}_1$	$(\bar{\mathbf{4}},\mathbf{1})^{-5/4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
$\tilde{\bar{F}}_2$	$(\bar{\mathbf{4}},\mathbf{1})^{-5/4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$
$\tilde{\bar{F}}_3$	$(\bar{\mathbf{4}},\mathbf{1})^{+5/4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$
$\tilde{\bar{F}}_4$	$(\bar{\mathbf{4}},\mathbf{1})^{-5/4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$
$\tilde{\bar{F}}_5$	$(\bar{\mathbf{4}},\mathbf{1})^{-5/4}$	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0
$\tilde{\bar{F}}_6$	$(\bar{\mathbf{4}},\mathbf{1})^{-5/4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$

$cos\ \theta H_2 - sin\ \theta H_{45}$, $\bar{H}_{12} = cos\ \bar{\theta}\bar{H}_1 - sin\ \bar{\theta}\bar{H}_2$, and $\bar{H}_{45}$ light, where $tan\ \theta = \langle\Phi_{23}\rangle / \langle\phi_{45}\rangle$ and $tan\ \bar{\theta} = \langle\Phi_{31}\rangle / \langle\bar{\Phi}_{23}\rangle$. The $\langle TT\rangle$ in the Higgs doublet matrix give additional structure. With the choice $\langle\Phi_{12}, \bar{\Phi}_{12}\rangle = 0$ and with the additional constraints $\langle\Delta_i^2\rangle = 0$ and $\langle T_i^2\rangle = 0$, the massless Higgs doublet eigenstates are identified as $H_{\mathbf{2}} = H_1$ and $H_{\bar{\mathbf{2}}} = \bar{H}_{45}$. Similarly, the Higgs triplet mass matrix can be formed, and it is found that all of the Higgs triplets become massive [52].

If the state $F_\beta \propto -\langle F_3\rangle F_1 + \langle F_1\rangle F_3$ is the linear combination that does not receive a vev, the flavour identification of the quarks and leptons with the specific string representations can be made:

$$t\ b\ \tau\ \nu_\tau : \quad Q_4\ d_4^c\ u_5^c\ L_1\ l_1^c, \tag{2.12}$$

$$c\ s\ \mu\ \nu_\mu : \quad Q_2\ d_2^c\ u_2^c\ L_2\ l_2^c, \tag{2.13}$$

$$u\ d\ e\ \nu_e : \quad Q_\beta\ d_\beta^c\ u_1^c\ L_5\ l_5^c. \tag{2.14}$$

In addition to the above states which have been identified with those of the Standard Model, there are extra states $\bar{f}_3$ and l_3^c, as well as 'exotic' states f_4 and $\bar{l}_4^c$ which should not appear in the light spectrum. In particular, there are $5th$ order superpotential terms that contain $\bar{f}_3$ and l_3^c which can generate dimesion-five operators leading to rapid proton decay. Fortunately, there are superpotential terms [52] of the form

$$f_4 \sum_i \alpha_i \bar{f}_i, \qquad \bar{l}_4^c \sum_i \alpha_i l_i^c \tag{2.15}$$

which allow these states to become heavy.

The singlet fields also may potentially obtain masses. The relevant trilinear couplings involving the singlet fields are

$$\frac{1}{2}(\phi_{45}\bar{\phi}_{45}\Phi_3 + \phi^+\bar{\phi}^+\Phi_3 + \phi^-\bar{\phi}^-\Phi_3 + \phi_i\bar{\phi}_i\Phi_3) + (\phi_1\bar{\phi}_2 + \bar{\phi}_1\phi_2)\Phi_4 + \\ (\Phi_{12}\Phi_{23}\Phi_{31} + \Phi_{12}\phi^+\phi^- + \Phi_{12}\phi_i\phi_i + h.c.), \tag{2.16}$$

from which it is clear that having $\langle\Phi_3\rangle \neq 0$ would give trilinear mass terms for ϕ_{45}, ϕ^+, ϕ^-, ϕ_i and their barred counterparts. However, $\langle\Phi_3\rangle = 0$ is required. Moreover, we have the result [52]

$$\phi^N = 0, \;\; N \geq 4, \tag{2.17}$$

Hence, we expect that most of the singlet fields will remain light.

C. Crypton Bound States

Since the strong-interaction scale for the SU(4) factor in the hidden sector is expected to lie below that for the SO(10) factor, we concentrate on the states bound by the hidden-sector SU(4) interactions. These include 'holomorphic' 'mesons' with the contents T_iT_j, $\Delta_i\Delta_j$ and $\tilde{F}_i\tilde{\bar{F}}_j$, 'non-holomorphic' mesons with the contents $T_iT_j^*$, $\Delta_i\Delta_j^*$ and $\tilde{F}_i\tilde{F}_j^*$, 'baryons' with the contents $\tilde{F}_i\tilde{F}_j\Delta_k$ and $\tilde{\bar{F}}_i\tilde{\bar{F}}_j\Delta_k$, and quadrilinear *tetrons*, with the contents of four $\tilde{F}_i$ and/or $\tilde{\bar{F}}_i$ fields and/or their complex conjugates. We assume that the baryons are heavier than the lightest tetrons, which are expected to be BPS-like 'holomorphic' states with the quantum numbers of $\tilde{F}_i\tilde{F}_j\tilde{F}_k\tilde{F}_l$ and $\tilde{\bar{F}}_i\tilde{\bar{F}}_j\tilde{\bar{F}}_k\tilde{\bar{F}}_l$, where $i,j,k,l = 3,5$. 'Non-holomorphic' tetrons with the quantum numbers of $\tilde{F}_i\tilde{F}_j\tilde{F}_k(\tilde{\bar{F}}_l)^*$, $\tilde{F}_i\tilde{F}_j(\tilde{\bar{F}}_k)^*(\tilde{\bar{F}}_l)^*$, etc., are generally expected to be heavier, although this remains to be proved. We assume that, by analogy with QCD, these excited states have short lifetimes.

Crypton bound states occur in 'cryptospin' multiplets with different permutations of confined constituents, analogous to the flavour SU(3) and SU(4) multiplets of bound states in QCD. We recall that the observable-sector non-Abelian gauge interactions do not act on the hidden-sector supermultiplets, and assume masses $\gg \Lambda_4$ for all the U(1) gauge supermultiplets except that in the Standard Model, in which case they also do not contribute significantly to the cryptospin mass splittings. Two classes of diagrams are likely to contribute to the mass differences between 'cryptospin' partners: electromagnetic 'self-energy' diagrams and the photon-exchange 'Coulomb potential' diagrams. We do not enter here into a discussion which of these classes of diagrams is likely to dominate for which cryptospin multiplets, as this is not essential for our purposes.

We expect these diagrams to have the following orders of magnitude:

$$\mathcal{O}(\frac{\alpha}{\pi})\Lambda_4 \times \{(\mathrm{a})\, \Sigma_i Q_i^2,\ (\mathrm{b})\, Q_T^2\}, \tag{2.18}$$

where the Q_i are the charges of the tetron constituents, and Q_T is the total tetron charge. It is easy to check the well-known fact that both of these terms make positive contributions to both the $\pi^+ - \pi^0$ and $p - n$ mass differences. The former agrees with experiment in sign and order of magnitude, and the difference of the latter from experiment is explained by the difference between the u and d quark masses, so one may have some confidence in the qualitative estimates in (2.18).

Each of the dependences in (2.18) would give $m_{T^{++}} > m_{T^+} > m_{T^0}$. We therefore expect the doubly-charged tetrons

$$\Psi^{--} = \tilde{F}_3\tilde{F}_3\tilde{F}_3\tilde{F}_3,\ \ \Psi^{++} = \tilde{F}_5\tilde{F}_5\tilde{F}_5\tilde{F}_5, \tag{2.19}$$

$$\bar{\Psi}^{++} = \bar{\tilde{F}}_3\bar{\tilde{F}}_3\bar{\tilde{F}}_3\bar{\tilde{F}}_3,\ \ \bar{\Psi}^{--} = \bar{\tilde{F}}_5\bar{\tilde{F}}_5\bar{\tilde{F}}_5\bar{\tilde{F}}_5, \tag{2.20}$$

to be heavier than the singly-charged states

$$\Psi^{+} = \tilde{F}_3\tilde{F}_5\tilde{F}_5\tilde{F}_5,\ \ \Psi^{-} = \tilde{F}_5\tilde{F}_3\tilde{F}_3\tilde{F}_3, \tag{2.21}$$

$$\bar{\Psi}^- = \tilde{\bar{\bar{F}}}_3\tilde{\bar{\bar{F}}}_5\tilde{\bar{\bar{F}}}_5\tilde{\bar{\bar{F}}}_5, \;\; \bar{\Psi}^+ = \tilde{\bar{\bar{F}}}_5\tilde{\bar{\bar{F}}}_3\tilde{\bar{\bar{F}}}_3\tilde{\bar{\bar{F}}}_3, \tag{2.22}$$

which are in turn expected to be heavier than the neutral states

$$\Psi^0 = \tilde{F}_3\tilde{F}_3\tilde{F}_5\tilde{F}_5, \;\; \bar{\Psi}^0 = \tilde{\bar{\bar{F}}}_3\tilde{\bar{\bar{F}}}_3\tilde{\bar{\bar{F}}}_5\tilde{\bar{\bar{F}}}_5. \tag{2.23}$$

Just like the proton in QCD, the lowest-lying neutral tetrons can decay only via higher-order operators in the superpotential, as we discuss below. This may make them good candidates for cold dark matter as well as providing via their decays a possible source of the UHECRs [49].

D. The Decays of the Lightest SU(4) Mesons

We first discuss the decays of the lightest hidden-sector SU(4) bound-state mesons. In analogy with QCD chiral symmetry breaking, it is expected that there will be an isotriplet of cryptopions that could play the role of pseudo-Nambu-Goldstone bosons, with masses that are small compared to Λ_4. Specifically, the charged SU(4) pion states

$$\pi^{\pm} = (\tilde{F}_3\tilde{\bar{\bar{F}}}_5, \tilde{\bar{\bar{F}}}_3\tilde{F}_5). \tag{2.24}$$

are expected to have masses

$$m^2_{\pi^{\pm}} = \Lambda_4 \times (m_3 + m_5), \tag{2.25}$$

where $m_{3,5}$ are the bare masses of the fractionally-charged constituents, which are expected to be $< \Lambda_4$, as we discussed above. The neutral SU(4) pion state

$$\pi^0 = \frac{1}{\sqrt{2}}(\tilde{F}_3\tilde{\bar{\bar{F}}}_3 - \tilde{F}_5\tilde{\bar{\bar{F}}}_5). \tag{2.26}$$

is expected to be lighter by an amount

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = (\frac{\alpha}{\pi})\Lambda_4^2 ln(\Lambda_4^2/m_{\pi^0}^2).) \tag{2.27}$$

The cryptospin-zero state

$$\eta^0 = \frac{1}{\sqrt{2}}(\tilde{F}_3\bar{\tilde{F}}_3 + \tilde{F}_5\bar{\tilde{F}}_5) \tag{2.28}$$

is expected to be significantly heavier because of a $U_A(1)$ anomaly.

We find that there are $N = 3$ superpotential terms of the form

$$\tilde{F}_3\bar{\tilde{F}}_3\Phi_3 - \tilde{F}_5\bar{\tilde{F}}_5\bar{\Phi}_{12} \tag{2.29}$$

that would allow the crypto-π^0 and -η^0 mesons to decay very rapidly. Additionally, we expect the crypto-π^0 and -η^0 states to have couplings to pairs of photon supermultiplets, analogous to those of the QCD π^0 and $\eta^0 \to \gamma\gamma$. These couplings would be described in an effective supergravity lagrangian by terms in the chiral gauge kinetic function f of the form $\alpha\pi/\Lambda_4$ and $\alpha\eta/\Lambda_4$, where Π, η denote composite superfields and Λ_4 is the scale at which the hidden-sector SU(4) interactions become strong. As in the case of the QCD π^0 decaying to $\gamma\gamma$, these couplings would give very short lifetimes for the crypto-π^0 and -η^0 states. It is also possible that in some variant models the crypto-π^0 and -η^0 might have additional decays, analogous to those of the QCD η^0, which would further shorten their lifetimes.

In the case of the charged cryptopions, we find terms of the form

$$\pi^-(F_2F_2F_3\bar{h}_{45} + F_3F_4\phi_2f_4 + F_3F_4h_1l_5^c + F_3\bar{h}_{45}\bar{f}_2l_2^c + F_3\bar{h}_{45}\bar{f}_5l_5^c), \tag{2.30}$$

and

$$\pi^+(F_4\Phi_{31}\bar{f}_3\bar{f}_5 + \bar{\phi}_{45}f_4\bar{f}_3\bar{l}_4^c + \bar{\phi}_{45}\bar{l}_4^c\bar{l}_4^c l_3^c), \tag{2.31}$$

that would allow the $\pi^\pm$ states to decay fairly rapidly.

There would also be a complex spectrum of heavier ‘non-holomorphic’ SU(4) bound-state mesons, analogous to the ρ and heavier mesons of QCD, but we expect them all to be very unstable, and do not discuss them further. Likewise, we do not discuss mesons made of the higher SU(4) representations Δ_i, or $FF\Delta$ cryptobaryons, or SO(10) bound states, as these have been studied previously in [20].

E. The Fate of the Neutral Tetrons

As discussed above, we expect the lightest tetrons to be the electrically neutral states. These can decay only through higher-order non-renormalizable superpotential terms, for which the first candidates appear at eighth order:

$$\Psi^0 \qquad F_4\phi^-\bar{h}_2\bar{f}_5, \tag{2.32}$$

$$\bar{\Psi}^0 \qquad \phi^+\bar{h}_{45}f_4\bar{l}^c_4. \tag{2.33}$$

At ninth order, we find terms involving neutral tetrons of the following forms:

$$\bar{\Psi}^0 \qquad (\Phi_{31}f_4f_4\bar{f}_3\bar{f}_3 + \Phi_{31}f_4\bar{f}_3\bar{l}^c_4l^c_3 + \Phi_{31}\bar{l}^c_4\bar{l}^c_4l^c_3l^c_3), \tag{2.34}$$

$$\Psi^0 \quad (F_1\phi_1\phi^-\bar{h}_2\bar{f}_1 + F_2\phi_4\phi^-\bar{h}_1\bar{f}_2 + F_2\bar{\phi}_4\phi^-\bar{h}_2\bar{f}_2 + F_2\bar{\phi}_4\phi^-\bar{h}_2\bar{f}_2). \tag{2.35}$$

All of these $8th$ and $9th$ order terms contain fields which are expected to have large masses, so we do not expect that these decay modes would be kinematically accessible. The next terms yielding possible neutral tetron decays are of tenth order. There are a large number of such terms, of which the following are those containing only fields that are light in the model:

$$\Psi^0[F_2F_2\bar{\Phi}_{31}\bar{\phi}_{45}\phi^-h_1 + F_2F_2\Phi_{23}\bar{\phi}_{45}\bar{\phi}^+h_1 + F_2F_3F_3\phi_4\bar{\phi}_{45}\bar{f}_2 + F_4\Phi_{23}\bar{\phi}_{45}\phi^-\bar{h}_{45}\bar{f}_5 + (\bar{\Phi}_{31}\bar{\phi}_{45}\phi^- + \Phi_{23}\bar{\phi}_{45}\bar{\phi}^+)h_1(\bar{f}_2l_2^c + \bar{f}_5l_5^c) + \Phi_{23}\bar{\phi}_{45}\phi^-h_1\bar{f}_1l_1^c]. \tag{2.36}$$

$$\bar{\Psi}^0[F_2F_2\Phi_{31}\phi_{45}\bar{\phi}^-h_1 + F_2F_2\bar{\Phi}_{23}\phi_{45}\phi^+h_1 + F_2F_2\bar{\phi}^-h_1h_1\bar{h}_{45} + F_4F_4\Phi_{31}\phi_{45}\phi^+h_1 + F_4F_4\phi^+h_1h_1\bar{h}_{45} + F_4\Phi_{31}\phi_{45}\phi^+\bar{h}_{45}\bar{f}_5 + F_4\phi^+h_1\bar{h}_{45}\bar{h}_{45}\bar{f}_5 + F_4\bar{\phi}^-h_1h_1h_1l_5^c + (\Phi_{31}\phi_{45}\phi^+h_1 + \phi^+h_1h_1\bar{h}_{45})\bar{f}_1l_1^c + (\Phi_{31}\phi_{45}\bar{\phi}^-h_1 + \bar{\Phi}_{23}\phi_{45}\phi^+h_1 + \phi^+\bar{h}_{45}\bar{h}_{45}\bar{h}_{45} + \bar{\phi}^-h_1h_1\bar{h}_{45})(\bar{f}_2l_2^c + \bar{f}_5l_5^c)]. \tag{2.37}$$

Using the flavour identifications we outlined above, these operators would give rise to the following neutral tetron decay modes:

$$\Psi^0 \to \tau\ \tau^c\ h_d\ \phi^3, \Psi^0 \to e/\mu\ e^c/\mu^c\ h_d\ \phi^3, \Psi^0 \to b\ b^c\ h_d\ \phi^3, \Psi^0 \to b\ b^c\ h_d\ h_d\ h_u\phi, \Psi^0 \to t\ t^c\ h_u\ \phi^3, \Psi^0 \to t\ t^c\ h_u\ h_u\ h_d\ \phi, \Psi^0 \to c\ c^c\ d\ d^c\ \phi^2, \Psi^0 \to s\ s^c\ h_d\ \phi^3. \tag{2.38}$$

These $10th$ order interactions would have a lifetime $\sim 10^{17} - 10^{52}$ years for the mass range $\sim \Lambda_4 = 10^{12} - 10^{13}$ GeV and $M_s = 10^{17} - 10^{18}$ GeV. These interactions involve multi-particle decays involving both particles and SUSY partners, within the constraints of R-parity and charge conservation. Although there are many of these decay interactions some general comments can be made. Almost all of them contain Higgs fields which would tend to decay (depending upon what the mass of the Higgs turns out to be) into $W^{\pm}$, quark-antiquark pairs (neutral Higgs) or τ leptons (charged Higgs), or remain as LSP if they are Higgsinos, assuming Higgsinos compose a fraction of LSP. Since the Higgs couple to heavier particles, we would expect $\bar{H}_{45}$ to decay most strongly to a pair the heaviest up-type quark allowed by kinematics, which is expected to be the c-quark. Similarly, we would expect the H_1 to decay most strongly to $\tau^{\pm}$, and to pairs of b-quarks. Furthermore, most of the decay interactions contain many Higgs fields as well as $\mathbf{10}$ and $\bar{\mathbf{5}}$ fields which may also produce quarks and antiquarks. Thus, several such pairs are expected to be created. These decay interactions also all involve several singlet fields which could decay into observable particles if their mass is great enough, or remain as hot-dark matter if is not.

F. The Fate of the Charged Tetrons

The lifetimes and abundances of charged tetrons have recently been discussed by Coriano *et.al.* [56], who have raised questions about their lifetimes and abundances relative to those of the neutral tetrons. In particular, they pointd out that if the only ways for the charged tetrons to decay are through the the same higher-order non-renormalizable operators that govern the decays of the neutral tetrons, then, if the neutral tetrons are long-lived, so also would be the charged tetrons, and they would probably have comparable cosmological abundances. Since there are very strong constraints on stable charged matter [26, 27, 28], it was argued in [56] that tetrons could not be good candidates for dark matter.

Indeed, we do find ninth-order superpotential terms involving charged tetrons that correspond to the annihilations of their constituents:

$$\bar{\Psi}^{++} \qquad (\Phi_{31}\bar{\phi}^{-}\bar{\phi}^{-}\bar{l}_4^c\bar{l}_4^c + \bar{\Phi}_{23}\phi^{+}\bar{\phi}^{-}\bar{l}_4^c\bar{l}_4^c), \tag{2.39}$$

$$\bar{\Psi}^{-} \qquad F_3\phi^{+}h_1 f_4 f_4, \tag{2.40}$$

$$\begin{aligned} \Psi^{--} \qquad & (\bar{\Phi}_{31}\bar{\phi}^{+}\phi^{-}l_2^c l_2^c + \bar{\Phi}_{31}\bar{\phi}^{+}\phi^{-}l_5^c l_5^c + \bar{\Phi}_{31}\phi^{-}\phi^{-}l_1^c l_1^c + \\ & \Phi_{23}\bar{\phi}^{+}\bar{\phi}^{+}l_2^c l_2^c \Phi_{23}\bar{\phi}^{+}\bar{\phi}^{+}l_5^c l_5^c + \Phi_{23}\bar{\phi}^{+}\phi^{-}l_1^c l_1^c), \end{aligned} \tag{2.41}$$

$$\begin{aligned} \Psi^{-} \qquad & (F_1F_1F_3\phi^{-}\bar{h}_2 + F_2F_2F_3\bar{\phi}^{+}\bar{h}_2 + F_2F_2F_3\phi^{-}\bar{h}_1 + \\ & F_3F_4F_4\phi^{-}\bar{h}_2 + F_3F_4\phi^{-}h_{45}l_5^c + F_3\bar{\phi}^{+}\bar{h}_2\bar{f}_2 l_2^c \\ & F_3\bar{\phi}^{+}\bar{h}_2\bar{f}_5 l_5^c + F_3\phi^{-}\bar{h}_1\bar{f}_2 l_2^c + F_3\phi^{-}\bar{h}_1\bar{f}_5 l_5^c + F_3\phi^{-}\bar{h}_2\bar{f}_1 l_1^c). \end{aligned} \tag{2.42}$$

$$\bar{\Psi}^{+} \qquad (\bar{F}_5\bar{F}_5\Phi_{31}\phi^{+}\bar{f}_3 + \Phi_{31}\bar{\phi}^{-}\bar{l}_4^c\bar{l}_4^c l_3^c + \Phi_{31}\bar{\phi}^{-}f_4\bar{f}_3\bar{l}_4^c + \bar{\Phi}_{23}\phi^{+}\bar{l}_4^c\bar{l}_4^c l_3^c). \tag{2.43}$$

Thus, if these non-renormalizable interactions were the only ways for charged tetrons to decay, they would have lifetimes similar to those of the neutral tetrons. Moreover, there are no superpotential terms corresponding to decays of Ψ^{++}, $\bar{\Psi}^{--}$, Ψ^{+}, or $\bar{\Psi}^{-}$ states that appear before tenth order, which would correspond to even longer lifetimes.

However, there is another mechanism which enables the heavier (charged) members of cryptospin multiplets to decay relatively rapidly into the lightest (neutral) isospin partner, analogous to the β decay of the neutron into its lighter isospin partner in QCD, the proton. We recall that neutron decay is generated by a four-fermion interaction of the type $(\bar{d}u\bar{\nu}e)/m_W^2$, which leads to an effective neutron-decay interaction of the form $(\bar{n}p\bar{\nu}e)/m_W^2$. This then leads to a neutron decay rate $\Gamma_n \sim (\delta m)^5/m_W^4$, where δm is the neutron-proton mass difference. In the case of charged-crypton decay, we expect there to exist a crypto-strong interaction of the form

$$\frac{\bar{C}^{+}C^{0}(\pi^{+}\partial\pi^{0})}{\Lambda_4^2}, \tag{2.44}$$

where the $C^{+,0}$ are charged and neutral crypton fields [1]. If the $C^{+,0}$ mass difference ΔM were larger than $m_{\pi^+} + m_{\pi^0}$, the C^+ decay rate would be very rapid: $\Gamma_{C^+} \sim (\Delta M)^5/\Lambda_4^4$. However, we expect that $\Delta M < m_{\pi^+}, m_{\pi^0}$, in which case the two crypto-pions must be virtual. In this case, the lowest-order decay interaction becomes

$$\alpha \Delta M \frac{\bar{C}^+ C^0 F \tilde{F} B_1 B_2}{m_{\pi^+}^2 m_{\pi^0}^2 \Lambda_4 M_s}, \tag{2.45}$$

where F denotes the Maxwell field strength and $\tilde{F}$ its dual, $B_{1,2}$ denote generic MSSM bosons, M_s is the string scale, and $\alpha = \alpha(\Lambda_4)$. If the π^+ can only decay through higher-order interactions, (2.45) would be replaced by effective interactions with more inverse powers of M_s. Setting $\Delta M \sim \alpha \Lambda_4$ as suggested by (2.18), and assuming the minimum values $m_{\pi^+}^2, m_{\pi^0}^2 \sim \alpha \Lambda_4^2$ allowed by (2.27), interactions of the form (2.45) would yield decay rates of order

$$\Gamma_{C^+} \sim \frac{\Delta M^{11}}{\Lambda_4^8 M_s^2} \sim \frac{\alpha^{11} \Lambda_4^3}{M_s^2}, \tag{2.46}$$

with additional factors of $(\Delta M/M_s)^2 \sim (\alpha \Lambda_4/M_s)^2$ for higher-order π^+ decay interactions. In the case of the interactions (18) in our particular flipped SU(5) model, we would pick up an extra factor of $(\alpha \Lambda_4/M_s)^4$.

In this case, we estimate a charged crypton lifetime $\tau^{\pm} \sim 10^2 - 10^9$ years for $\Lambda_4 \sim 10^{13} - 10^{12}$ GeV and $M_s \sim 10^{17}$ GeV. For the same range of Λ_4 and $M_s = 10^{18}$ GeV, we estimate a charged crypton lifetime of $\tau^{\pm} \sim 10^8 - 10^{14}$ years. These charged-tetron lifetimes are much shorter than what we expect for the neutral tetrons. For comparison, with the same values of Λ_4 and M_s, taking $M_X = \Lambda_4$ and assuming a ninth-order neutral-crypton decay interaction, we estimate a neutral crypton lifetime

[1] In QCD, the W^- couples to $\bar{n}p$ via a strongly-interacting vector meson ρ^-. By an analogous vector-meson dominance argument, one could consider the interaction (2.44) as being mediated by the exchange of a 'non-holomorphic' crypto-ρ meson.

$\tau^0 \sim 10^{13} - 10^{26}$ years for $\Lambda_4 = 10^{13} - 10^{12}$ GeV and $M_s = 10^{17}$ GeV, and $\tau^0 \sim 10^{25} - 10^{38}$ years for the same range of tetron mass and $M_s = 10^{18}$ GeV. In particular, $\tau^{\pm} \lesssim 10^5$ years and $\tau^0 > 10^{10}$ years if $3 \cdot 10^{12}$ GeV $\leq \Lambda_4 \leq 2 \cdot 10^{13}$ GeV with $M_s = 10^{17}$ GeV and $3 \cdot 10^{13}$ GeV $\leq \Lambda_4 \leq 2 \cdot 10^{14}$ GeV with $M_s = 10^{18}$ GeV. In fact, it is possible to choose a value for Λ_4 in the expected range such that $\tau^{\pm} \lesssim 10^5$ years and $\tau^0 > 10^{10}$ years for all values of M_s between $10^{17} - 10^{18}$ GeV. Thus, it is always possible to choose reasonable values of these parameters such that neutral tetrons will have a lifetime longer than the present age of the universe while the charged tetrons will have decayed prior to photon-matter decoupling. Therefore, neutral tetrons can be in existence today as cold dark matter unencumbered by any constraints due to charged dark matter.

G. Generic Super-heavy Relic Decay

The basic idea in generic 'top-down' explanations (see [57, 58]) is that the UHECR are produced via the decay of some relic particles or topological defects left over from the inflationary epoch and which are locally clustered in the galactic halo as cold, dark matter with an over-density $n_X/n_X^{cos} \sim 10^{4-5}$. The lifetime of such relics must exceed the present age of the universe in order for them to exist today in sufficient abundance, however the lifetime must not be too large so that the decay rates produced are too small to produce the UHECR. Furthermore, the relic mass must be at least $M_X > 10^{12}$ GeV in order to produce the UHECR energies observed. Typically, the lifetime of a particle is expected to be inversely proportional to it's mass, $\tau \sim 1/M$. Clearly it is not easy to have a particle with both a large enough mass and a decay lifetime in the right range to produce the UHECR. However, as pointed out in the Introduction, flipped $SU(5)$ cryptons satisfy both of these criteria, which makes them very attractive as a top-down explanation of the UHECR. Indeed the 'flipped' crypton is probably the most natural and most physically motivated of any top-down candidate.

There are three generic statements that can be presently made about a decaying super-heavy X particle explanation for the UHECR:

1. Since the super-heavy relics may accumulate locally in the galactic halo with an over-density $\sim 10^{4-5}$ over the cosmological average, they may avoid the GZK cutoff.

2. Due to the displacement of our solar system with respect to the galactic plane, there should be some anisotropy in the arrival directions of the UHECR with respect to the galactic center.

3. Photons tend to be the dominant component of the UHECR flux produced by the super-heavy X decay. However, the photons may scatter off the galactic radio background, which is poorly measured, and thus may be somewhat attenuated.

The injection spectrum produced by such a decaying super-heavy relic X-particles with number density n_X and lifetime τ_X is proportional to the inclusive decay width:

$$\Phi^{halo}(E) = \frac{n_X}{\tau_X}\frac{1}{\Gamma_X}\frac{d\Gamma(X \to g_1 + \cdots)}{dE}. \tag{2.47}$$

If a spherical halo of radius R_{halo} and uniform number density n_X is assumed, then the galactic halo contribution to the UHECR will be given by

$$J^{halo} = \frac{1}{4\pi}R_{halo}\Phi^{halo}(E). \tag{2.48}$$

In general, the X-particles will decay into one or more partons which hadronize into other particles g of the MSSM, which carry a fraction x of the maximum available momentum $M_X/2$ and a fraction z of the parton momentum. For such a decay, the inclusive decay width can be factored as

$$\frac{1}{\Gamma_X}\frac{d\Gamma(X \to g_1 + \cdots)}{dE} = \sum_a \int_0^x \frac{1}{\Gamma_a}\frac{d\Gamma_a(y, \mu^2, M_X^2)}{dy}\bigg|_{y=x/z} D_a^g(z, \mu^2), \tag{2.49}$$

where $D_a^g(z, \mu^2)$ is the fragmentation function (FF) for particles of type g into particles of type a, and μ is the energy scale, most appropriately taken to be equal to the X particle mass, $\mu = M_X$. The evolution of the fragmentation function is governed by the DGLAP equations, which may be extended to include the MSSM. Thus, the determination of the expected UHECR flux from the super-heavy X decay essentially becomes the problem of starting with a set of initial decay partons and evolving the decay cascades via the fragmentation functions to find the end decay products and energy distribution. To evolve the fragmentation functions up to the energy of the super-heavy X decay, the DGLAP equations must be solved numerically. Several groups have done such calculations for generic initial decay partons (usually into a quark-antiquark pair) [59, 60, 61, 62]. Perhaps the best such code is `SHdecay` [63]. This code calculates the fragmentation into the seven stable MSSM particles (p, γ, e, neutralino LSP χ, ν_e, ν_μ, ν_τ) for any given initial decay parton. In the case of flipped $SU(5)$ cryptons, we have a specific model where the initial decay partons in the cascade are known.

In addition to the UHECR flux produced by the super-heavy decay from X particles clustered in our galactic halo, there may be a background flux from sources outside of our galaxy, perhaps super-heavy X decay in *other* galactic halos or in intergalactic regions. Generally, this flux is assumed to be due to a homogenous distribution of sources and exhibits a characteristic GZK pileup due to the fact that they are produced *non-locally* [64]. Since the GZK attenuation is much more severe for photons than for nucleons, this background should be comprised primarily of nucleons. Thus, in this scenario the UHECR flux observed on Earth will be the sum of this extragalactic background and the local galactic flux from decaying relics clustered in our galactic halo. Due to the extragalactic component, super-heavy X particle decay may only unambiguously explain the UHECR flux for energies $E > 4 \cdot 10^{19}$ eV. However, we note that the extragalactic component may also be due partially to *non-local* super-heavy X decay, as well from astrophysical sources ('bottom-up' production). Thus, it is more accurate to say that a distinct signal of super-heavy X decay within our galactic halo would be the existence of an excess of events above this energy. The lack of any events above this energy would not rule out the presence of a top-down component, but it would not provide an unambiguous reason for the introduction of such a mechanism.

We have previously found [65] the following 10^{th}-order superpotential operators through which the neutral tetrons may decay. These operators would give rise to the neutral tetron decay modes

$$\Psi^0 \to \tau\ \tau^c\ h_d\ \phi^3, \Psi^0 \to e/\mu\ e^c/\mu^c\ h_d\ \phi^3, \Psi^0 \to b\ b^c\ h_d\ \phi^3, \tag{2.50}$$
$$\Psi^0 \to b\ b^c\ h_d\ h_d\ h_u\phi, \Psi^0 \to t\ t^c\ h_u\ \phi^3, \Psi^0 \to t\ t^c\ h_u\ h_u\ h_d\ \phi,$$
$$\Psi^0 \to c\ c^c\ d\ d^c\ \phi^2, \Psi^0 \to s\ s^c\ h_d\ \phi^3.$$

We note that there are several different possible decay modes, any of which may be dominant, depending on unknown features of the model dynamics that determine the relative values of their coefficients. In particular, the most important tetron decays could be into either leptons or quarks, and there are many different possibilities for the dominant flavours.

We plot in Figs. 1 to 8 below the expected UHECR energy spectra of photons and nucleons due to each of these possible tetron decay modes, as well as the maximum photon fractions expected. The energy spectra were calculated for a mass $M_X = 2 \cdot 10^{13}$ GeV, using the fragmentation functions $D^i(x, M_X^2)$ generated by the code `SHdecay` [63]. This code calculates the fragmentation into the seven stable MSSM particles (p, γ, e, neutralino LSP χ, ν_e, ν_μ, ν_τ) for any given initial decay parton. The many-body decays distribute the total decay energy M_X among the different particles. We include Higgs decays, but we ignore the decays of the singlet fields, except to take into account their kinematical effects on the primary quark and lepton spectra. We follow [66] in estimating the probability density $\rho_n(z)$ that one decay parton carries off a fraction z of the total available decay energy M_X:

$$\rho_n(z) = (n-1)(n-2)z(1-z)^{n-3} \tag{2.51}$$

for $n \geq 3$ decay partons. The resulting flux from the emission of a given decay parton is then

$$E^3 J^i(E) = Bx^3 \int_x^1 \frac{dz}{z} \rho_n\left(\frac{x}{z}\right) D^i(z, M_x^2), \tag{2.52}$$

where $i = (p, \gamma, e, \chi, \nu_e, \nu_\mu, \nu_\tau)$.

To obtain the total UHECR spectrum, we add to this the background flux of nucleons that would be expected to result from a homogenous distribution of extra-galactic sources that exhibits the distinctive pile-up due to the GZK effect [64] [2]. The constant B in (2.52) is a normalization coefficent determined by the tetron number density and lifetime, *viz*

$$B \sim R^{halo} \frac{n_X}{\tau_X} \frac{1}{M_X} \tag{2.53}$$

This dimensional coefficient B is not determined *a priori*, and must be fitted to the

[2] However, we note that this model is likely to come under pressure from upper limits on high-energy cosmic-ray neutrinos [67] - private communication from S. Sarkar, see also http://www-thphys.physics.ox.ac.uk/users/SubirSarkar/talks/munich05.pdf.

experimental data. In each of Figs. 1 to 8, we show the total spectrum obtained by summing the background and the fluxes of nucleons and photons resulting from tetron decay, and in a second panel we display the gamma fractions: $\gamma/(\gamma+p)$. We have assumed no photon attentuation in the calculated spectra, although a strong attenuation cannot be excluded [68], because the galactic radio background has never been accurately measured and its intensity is largely unknown [69].

Figs. 1 to 6 are for quark primaries, and are ordered according to the masses of the quarks involved. In the case of the b quark, we show in Figs. 3 and 4 plots for two superpotential operators with different numbers of accompanying Higgs fields: the two plots are rather similar, and the same is true of the two plots shown in Figs. 5 and 6 for primary t quarks. We are thus led to hope that including the (model-dependent) decays of the singlets ϕ would not have large effects. The plots for lepton primaries shown in Figs. 7 and 8 are more distinctive, in that the photon fractions rise to much larger values at energies above 10^{20} eV [3].

In Fig. 9 we compare the spectrum for one of the operators with primary b quarks, calculated for a crypton mass of 10^{13} GeV, with experimental data from the Fly's Eye, HiRes, AGASA, and Auger experiments [30, 19, 32, 35]. The AGASA flux has been scaled by a factor of 0.55 for consistency with the other data, and the normalizations for the crypton decay contributions to these spectra has been adjusted for the different crypton masses. The limited statistics for UHECRs with energies $\geq 10^{19}$ eV available in the present data sets do not offer any clear discrimination between crypton masses in the range 2×10^{13} GeV $\geq M_X \geq 10^{12}$ GeV. In the case of a crypton mass $\sim 10^{12}$ GeV there is no clear signal of a crypton contribution to the

[3] The photon fractions for second-generation quark primaries look somewhat flatter than those for b and t quarks, but this difference is probably within the modelling uncertainties.

UHECR since the flux from such a decay is essentially buried within the background from homogenous extragalatic sources. A clear signal of crypton decay, at least in this model, would require a lower limit on the crypton mass $M_X \geq 5 \cdot 10^{12}$ GeV in order to provide an excess of events above $4 \cdot 10^{19}$ eV that could not be attributable to extragalactic astrophysical acceleration mechanisms.

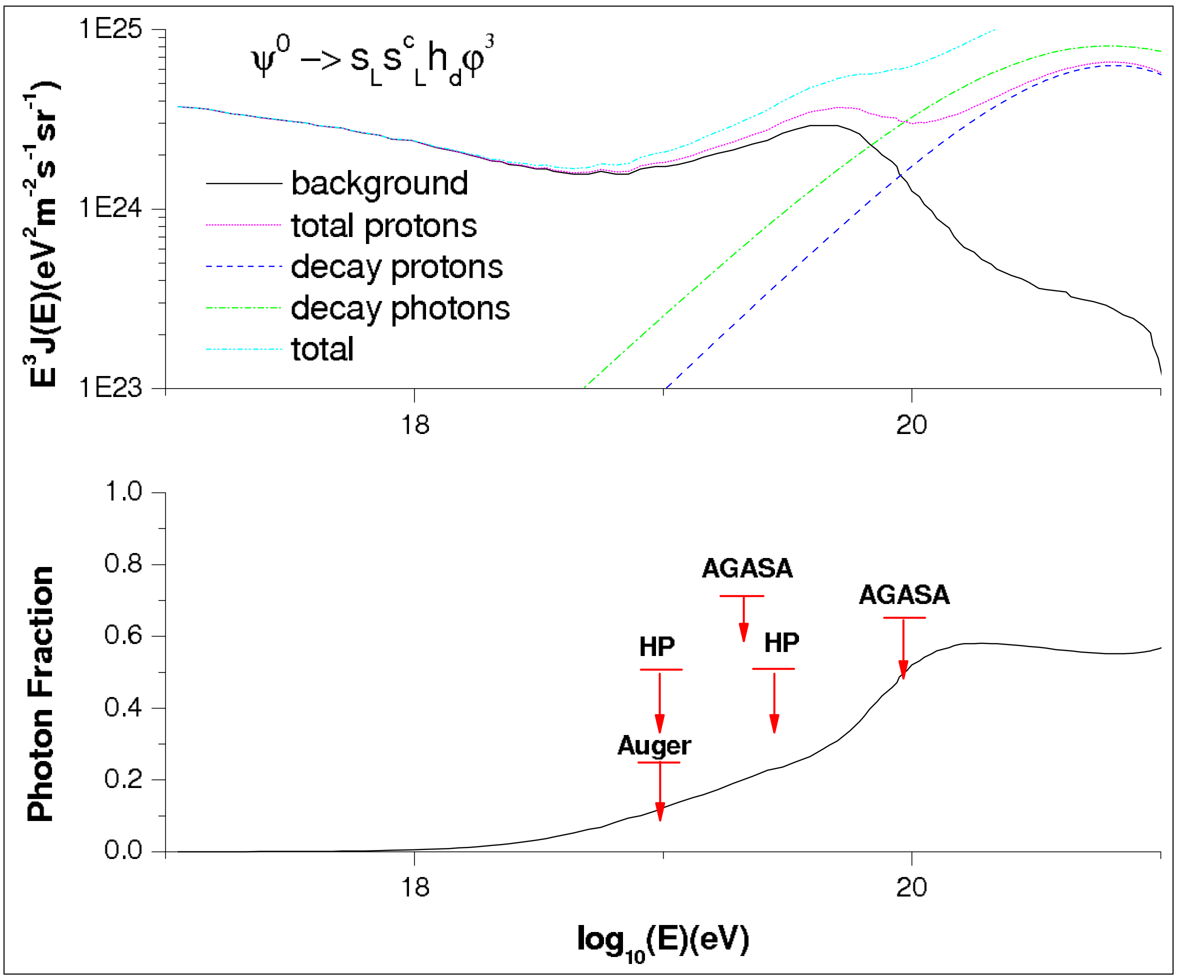

Fig. 1. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \rightarrow s\ s^c\ h_d\ \phi^3$.

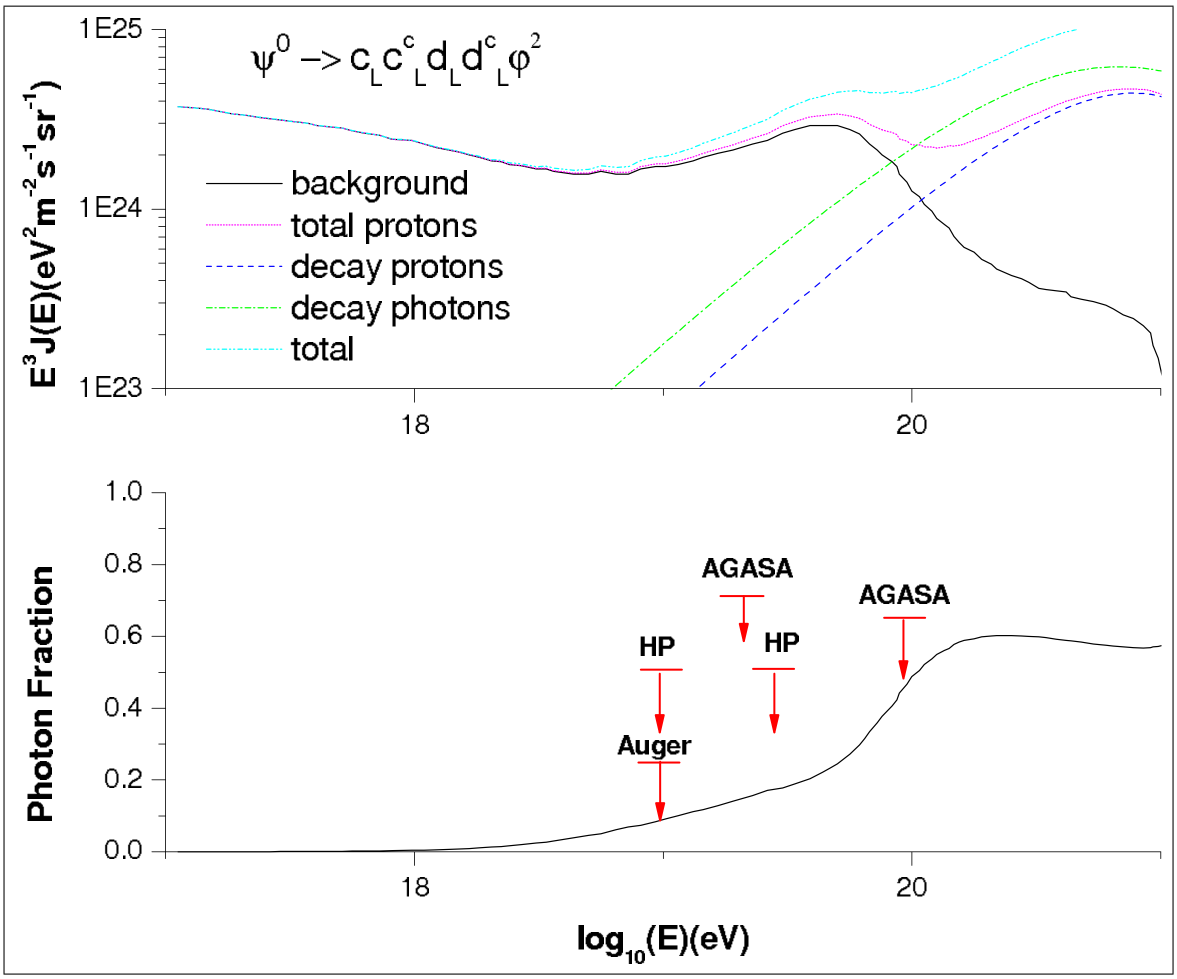

Fig. 2. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \to c\ c^c\ d\ d^c\ \phi^2$.

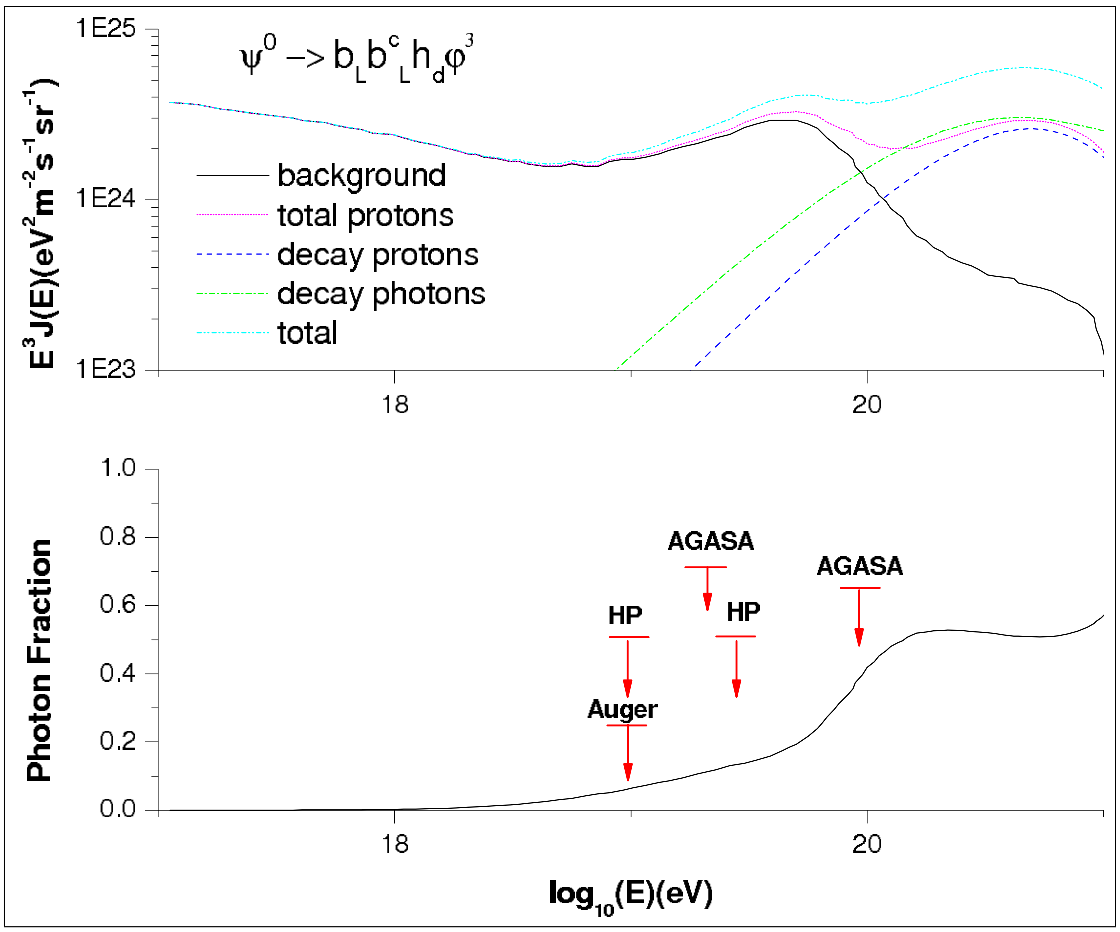

Fig. 3. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \to b\ b^c\ h_d\ \phi^3$.

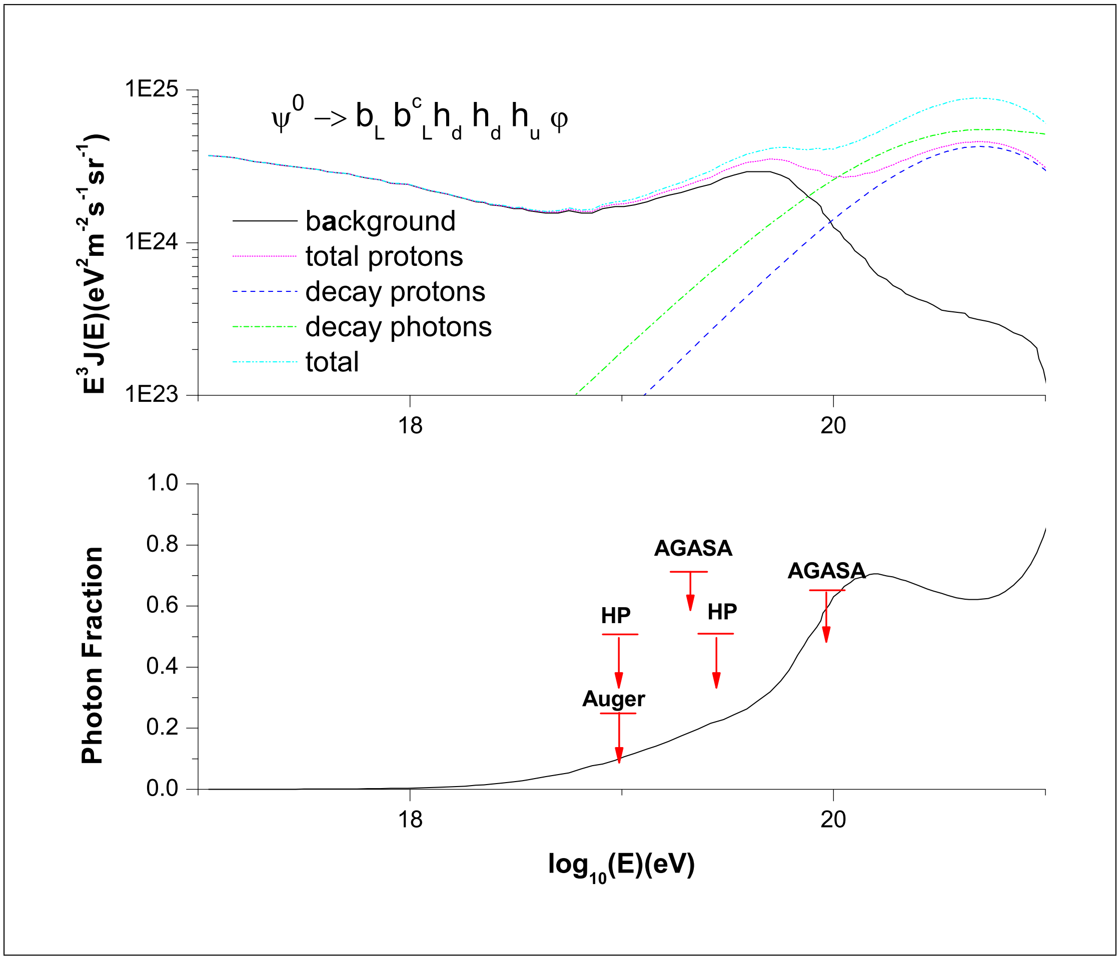

Fig. 4. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \to b\ b^c\ h_d\ h_d\ h_u \phi$.

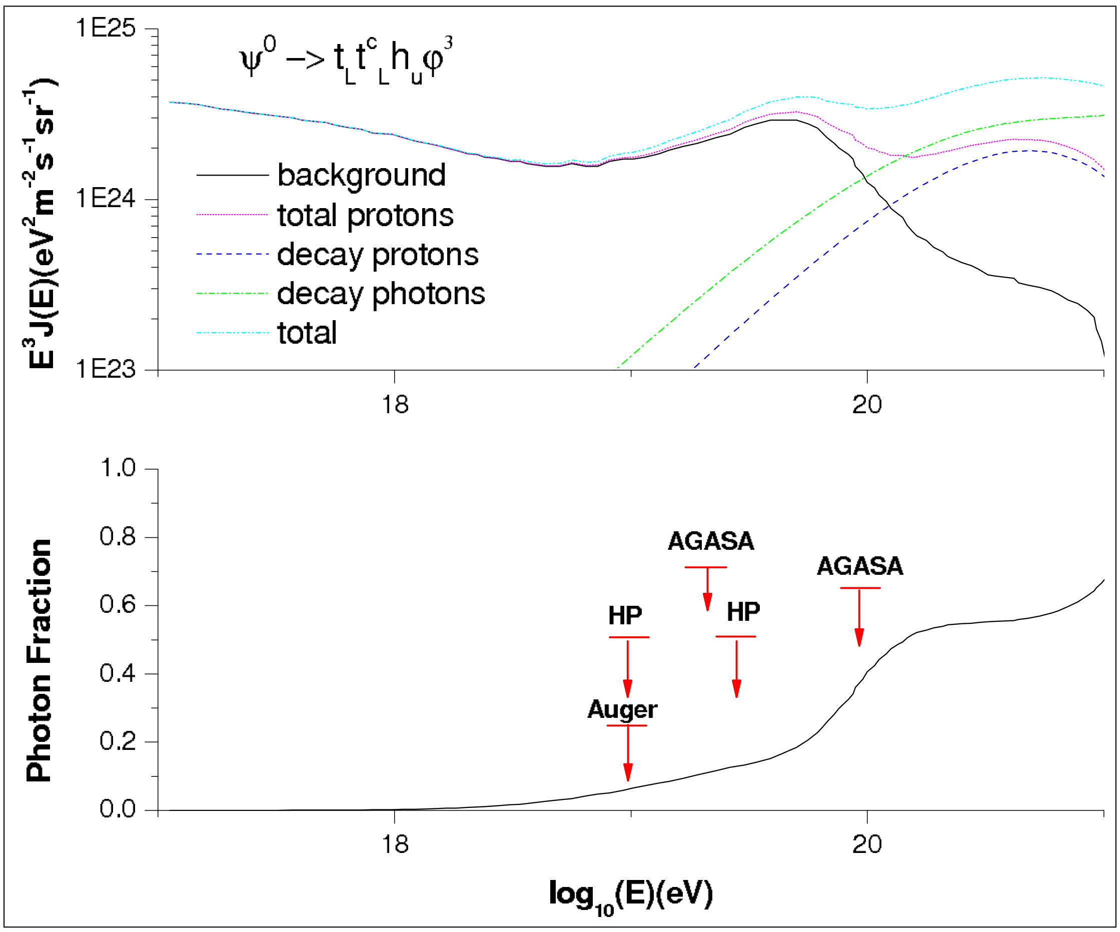

Fig. 5. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \rightarrow t\ t^c\ h_u\ \phi^3$.

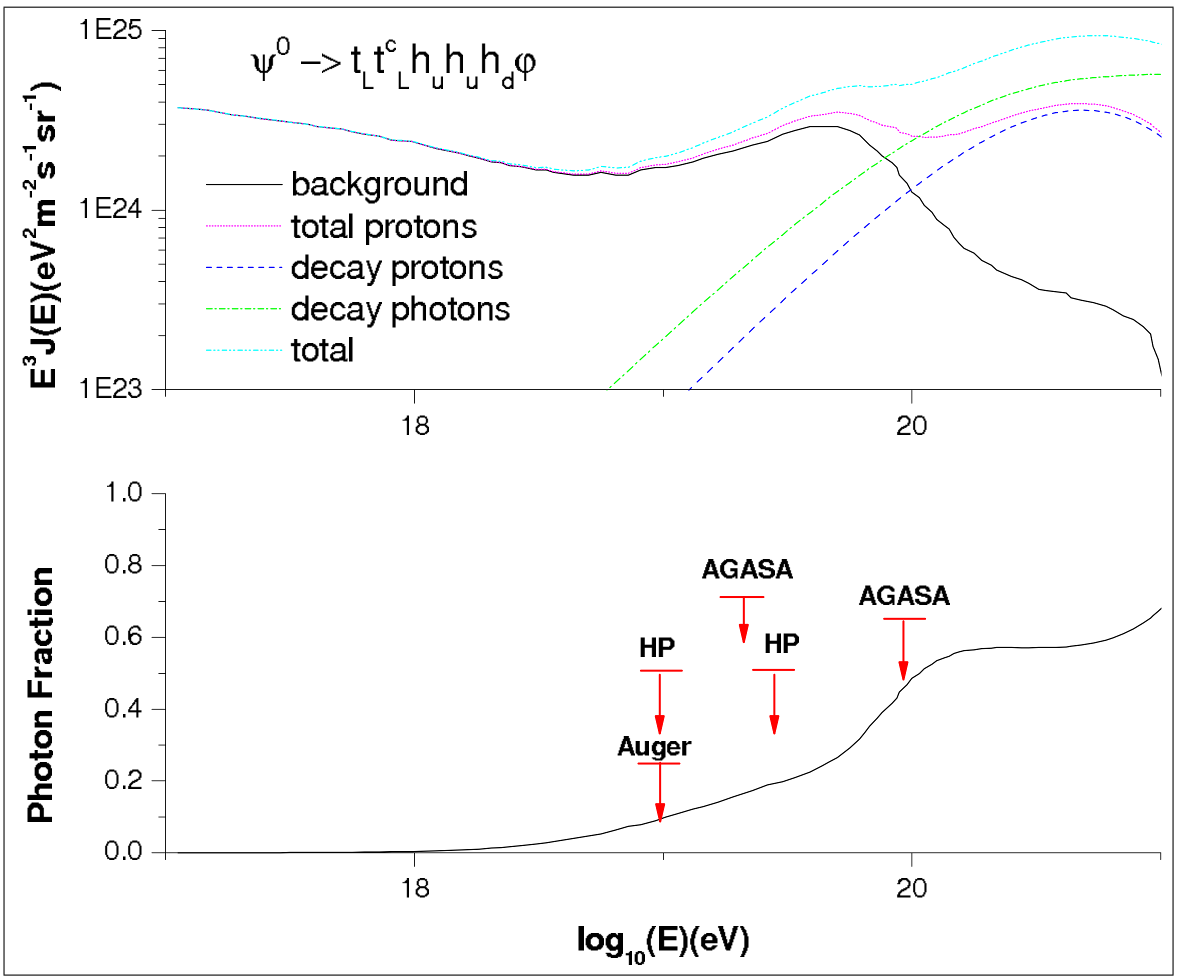

Fig. 6. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \to t\ t^c\ h_u\ h_u\ h_d\ \phi$.

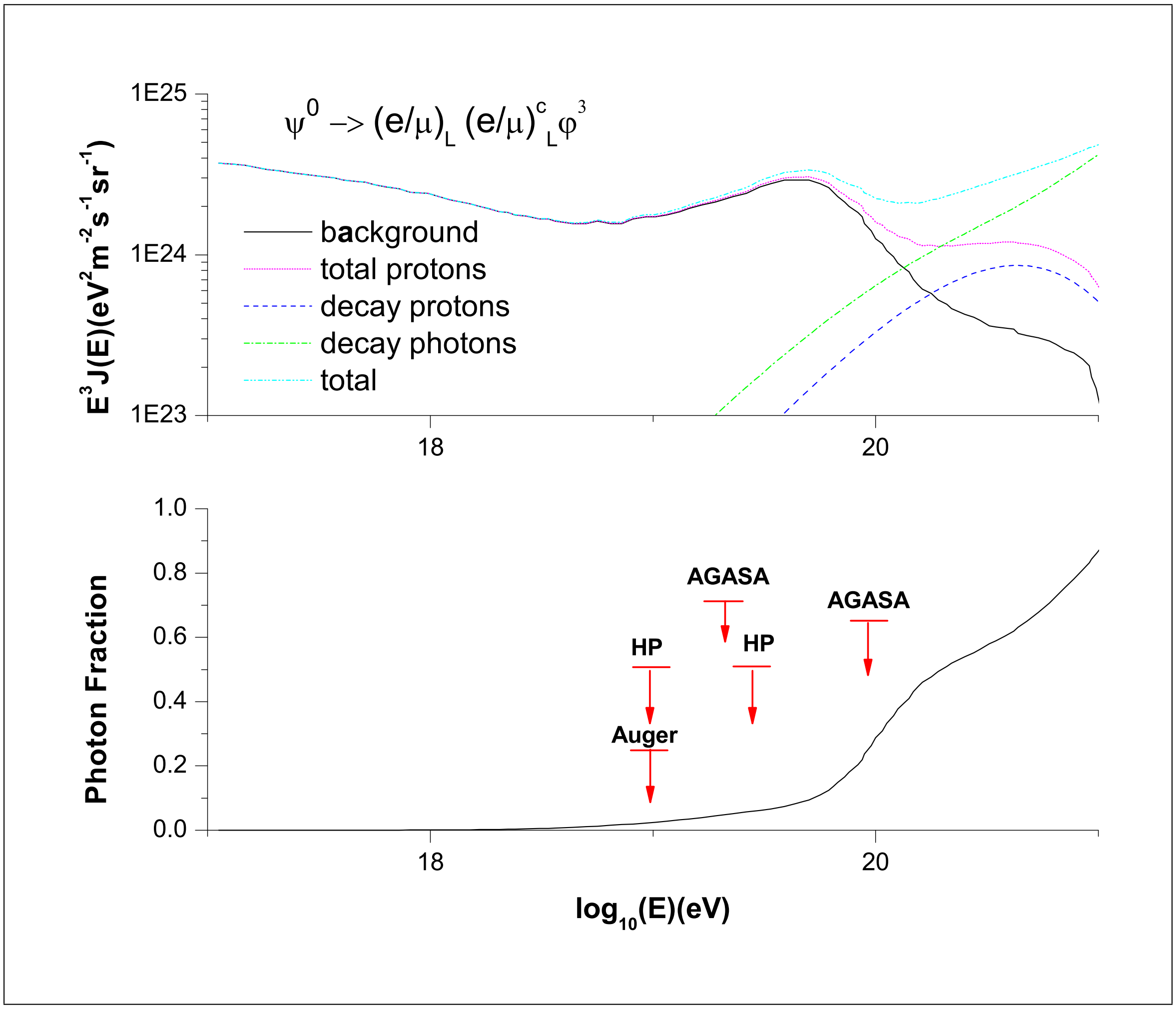

Fig. 7. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \rightarrow e/\mu\ e^c/\mu^c\ h_d\ \phi^3$.

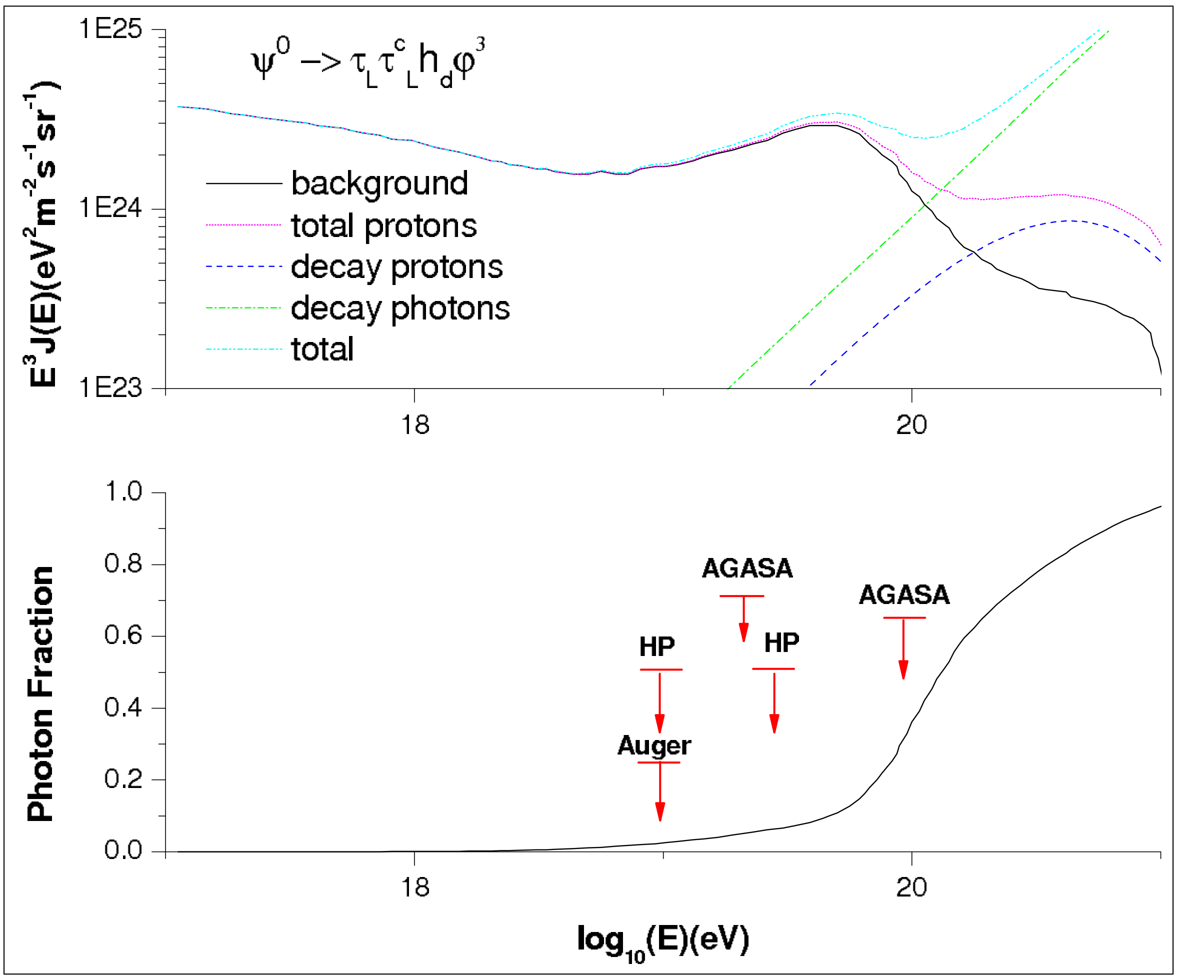

Fig. 8. The top panel shows the total UHECR spectrum and the bottom panel the photon fraction for the decay mode $\Psi^0 \rightarrow \tau\ \tau^c\ h_d\ \phi^3$.

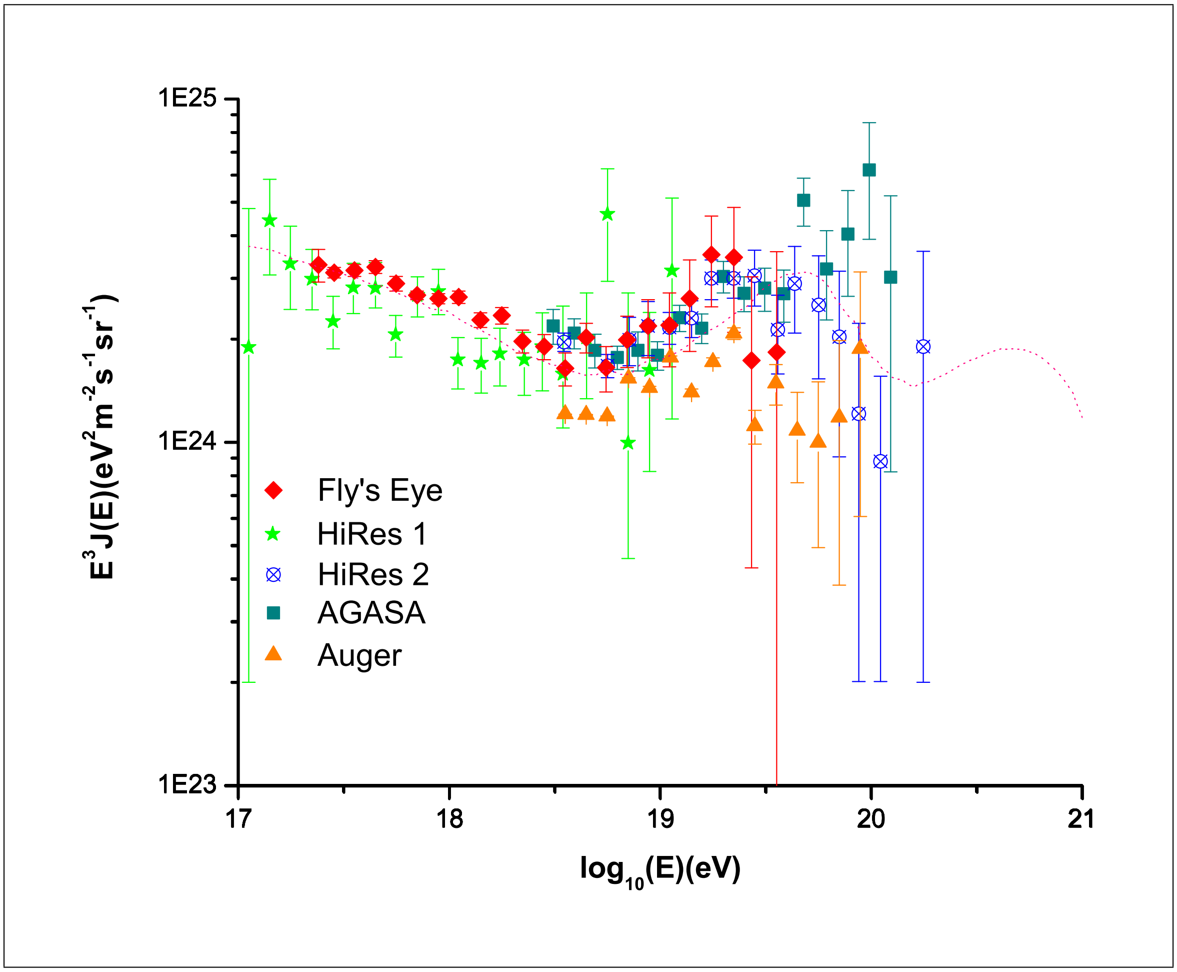

Fig. 9. A comparison with the available data on the UHECRs from the Fly's Eye, HiRes, AGAS and Auger experiments with the crypton decay model $\Psi^0 \to b\ b^c\ h_d\ \phi^3$ for $M_X = 10^{13}$ GeV.

CHAPTER III

A SUPERSYMMETRIC FLIPPED $SU(5)$ INTERSECTING BRANE WORLD

A. Introduction

The intersecting D-brane world approach [70, 71, 72, 73] plays a prominent role in the attempts of string phenomenologists to reproduce the standard model physics in a convincing way from type II string theory.

A number of consistent non-supersymmetric three-generation standard-like models have been constructed in [74, 75] (for a complete set of references the reader should consult the excellent reviews [76, 77, 78, 79, 80]). Open strings that begin and end on a stack of M D-branes generate the gauge bosons of the group $U(M)$ living in the world volume of the D-branes. So the standard approach is to start with one stack of 3 D-branes, another of 2, and n other stacks each having just 1 D-brane, thereby generating the gauge group $U(3) \times U(2) \times U(1)^n$. The D4-, D5- or D6-branes wrap the three large spatial dimensions and respectively 1-, 2- or 3-cycles of the six-dimensional internal space (typically a torus T^6 or a Calabi-Yau 3-fold). Fermions in bi-fundamental representations of the corresponding gauge groups can arise at the multiple intersections of such stacks [70]. For D4- and D5-branes, to get $D = 4$ *chiral* fermions the intersecting branes should sit at a singular point in the space transverse to the branes, an orbifold fixed point, for example. In general, intersecting-brane configurations yield a non-supersymmetric spectrum, so to avoid the hierarchy problem the string scale associated with such models must be no more than a few TeV. Gravitational interactions occur in the bulk ten-dimensional space, and to ensure that the Planck mass has its observed large value, it is necessary that there are large dimensions transverse to the branes [81]. Thus getting the correct Planck scale effectively means that only D4- and D5-brane models are viable, since for D6-branes there is no dimension transverse to all of the intersecting branes. However, a generic feature of these models is that flavour changing neutral currents are generated by four-fermion operators induced by string instantons. Although such operators allow the emergence of a realistic pattern of fermion masses and mixing angles, the severe experimental limits on flavour changing neutral currents require that the string scale is rather high, of order 10^4 TeV [82]. In a non-supersymmetric theory the cancellation of the closed-string (twisted) Ramond-Ramond (RR) tadpoles does *not* ensure the cancellation of the Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles. There is a resulting instability in the complex structure moduli [83]. One way to stabilise some of the (complex structure) moduli is to use an orbifold, rather than a torus, for the space wrapped by the D-branes.

If the embedding is supersymmetric, then the instabilities including the gauge hierachy problem are removed. In this case, one in general has to introduce in addition to D6-branes orientifold O6-planes, which can be regarded as branes of negative RR-charge and tension. For a general Calabi-Yau compact space these orientifold planes wrap special Lagrangian 3-cycles calibrated with respect to the real part of the holomorphic 3-form Ω_3 of the Calabi-Yau compact space [1].

This has been studied [84], using D6-branes and a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, but it has so far proved difficult to get realistic phenomenology consistent with experimental data from such models. Further progress has been achieved using D6-branes and a $\mathbb{Z}_4$ [85], $\mathbb{Z}_4 \times \mathbb{Z}_2$ [86] or $\mathbb{Z}_6$ [87] orientifold. Although a semi-realistic three-generation model has been obtained this way [87], it has non-minimal Higgs content, so it too will have flavour changing neutral currents [88] (for recent progress in orientifolds of

[1]In this case, the gauge hierarchy problem can be addressed with soft supersymmetry-breaking terms.

Gepner models see [89, 90]).

An alternative approach in this framework is to start engineering a grand unified gauge symmetry which subsequently breaks down to the standard model gauge group [91]. This possibility is not available in standard type IIB orientifolds, due to the difficulty in getting adjoint representations to break the GUT group to the Standard Model [92]. A well motivated example is the flipped $SU(5) \times U(1)_X$ model [93, 94], which had been extensively studied in the closed string era of the heterotic compactifications [95, 96]. From the theoretical point of view this motivation was coming from the fact that its symmetry breaking requires only $\mathbf{10}$ and $\overline{\mathbf{10}}$ representations at the grand unification scale, as well as $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations at the electroweak scale, and these were consistent with the representations of $SU(5)$ allowed by the unitarity condition with gauge group at level 1 [97, 98] [2]. From the phenomenological point of view flipped $SU(5) \times U(1)_X$ [93, 94] has a number of attractive features in its own right [99]. For example, it has a very elegant missing-partner mechanism for suppressing proton decay via dimension-5 operators [94], and is probably the simplest GUT to survive experimental limits on proton decay [100]. These considerations motivated the derivation of a number of flipped $SU(5)$ models from constructions using fermions on the world sheet [95, 96]. Consistency of the low energy values of the gauge coupling constants with string unification at about 10^{18} GeV (in the absence of large string loop threshold corrections) required the existence of extra matter, besides that of the supersymmetric standard model [101, 102].

Non-supersymmetric flipped $SU(5)$ models have been produced in [103] using

[2]Thus attempts to embed conventional grand unified theory (GUT) groups such as $SU(5)$ or $SO(10)$ in heterotic string required more complicated compactifications, but none of these has been completely satisfactory. Constructions with the minimal option to embed just the standard model gauge group, were plagued with at least extra unwanted U(1) factors.

D6-branes wrapping toroidal 3-cycles and also when the wrapping space is the $T^6/\mathbb{Z}_3$ orbifold [3]. It is therefore of interest to search for supersymmetric flipped $SU(5)$ models from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at general angles.

The wrapping numbers of the various stacks are constrained by the requirement of RR-tadpole cancellation as well as the supersymmetry conditions. Tadpole cancellation ensures the absence of non-abelian anomalies in the emergent low-energy quantum field theory. A generalised Green-Schwarz mechanism ensures that the gauge bosons associated with all anomalous $U(1)$s acquire string-scale masses [104], but the gauge bosons of some non-anomalous $U(1)$s can also acquire string-scale masses [105]; in all such cases the $U(1)$ group survives as a global symmetry. Thus we must also ensure the flipped $U(1)_X$ group remains a gauge symmetry by requiring that its gauge boson does *not* get such a mass.

The material of this Letter is organized as follows. In section 2 we provide all the necessary formalism for constructing a consistent string supersymmetric model on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. This formalism includes the RR tadpole consistency conditions and the restrictions placed on each stack of D6-branes for preservation of supersymmetry as well as the generalised Green-Schwarz anomaly cancellation mechanism and the requirements we impose such that the flipped $U(1)_X$ remains a gauge symmetry.

In section 3, for the convenience of the reader, we first provide the minimal field-theory content of the flipped $SU(5)$ model and then we proceed to derive a string model consistent with the rules described in section 2. This is a three-generation model, whose gauge symmetry includes $SU(5) \times U(1)_X$, however with a non-minimal

[3]The $T^6/\mathbb{Z}_3$ orbifold is not suitable for supersymmetric model building.

matter content.

Finally, we use section 4 for our discussions and conclusions.

B. Search for Supersymmetric Flipped $SU(5)\times U(1)_X$ Brane Models on a $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ Orientifold

We have several choices at our disposal in attempting to build a four-dimensional three-generation GUT flipped $SU(5)$ model. A flipped $SU(5)$ model was successfully built in [103] on a $\mathbb{Z}_3$ orientifold but it was not supersymmetric. So, in this paper we will focus on the supersymmetric type IIA orientifold on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at generic angles. This choice has the feature that $\mathbb{Z}_2$ actions do not constrain the ratio of the radii on any 2-torus. Additionally, the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold has only bulk cycles, contrasting the cases of $\mathbb{Z}_4$ and $\mathbb{Z}_6$ orientifolds where exceptional cycles also necessarily exist and generally increase the difficulty of satisfying the Ramond-Ramond tadpole condition. However, as we shall see only a limited range of ratio of the complex structure moduli is consistent with the supersymmetry conditions.

This $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ structure was first introduced in [84] and further studied in [91] [4], and we will use the same notations here. Consider type IIA theory on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ generators θ, ω act on the complex coordinates (z_1, z_2, z_3) of $T^6 = T^2 \times T^2 \times T^2$ as

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \end{aligned} \tag{3.1}$$

We implement an orientifold projection ΩR, where Ω is the world-sheet parity, and

[4] See also [106].

R acts as

$$R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \tag{3.2}$$

Although the complex structure of the tori is arbitrary under the action of $\mathbb{Z}_2 \times \mathbb{Z}_2$, it must be assigned consistently with the orientifold projection. Crystallographic action of the complex conjugation R restricts consideration to just two shapes. We may take either a rectangular toroidal cell or a very specific tilted variation. Define here a canonical basis of homology cycles ($[a_i]$, $[b_i]$) lying respectively along the $(\hat{x}_i, \mathrm{i}\hat{y}_i)$ coordinate directions, where $i = 1, 2, 3$ labels each of the three 2-tori. Next, consider K different stacks of N_a D6-branes wrapping on ($[a_i]$, $[b_i]$) with integral coefficients (n_a^i, m_a^i), where $a = 1, 2, ...K$. For the tilted complex structure variants the toroidal cell is skewed such that an alternate homology basis is required to close cycles spanning the displaced lattice points. Specifically, we must consider the cycle $[a_i'] \equiv [a_i] + \frac{1}{2}[b_i]$, so that the tilted wrapping is described by $n_a^i[a_i'] + m_a^i[b_i] = n_a^i[a_i] + (n_a^i/2 + m_a^i)[b_i]$. For convenience, define the effective wrapping number l_a^i as $l_a^i \equiv m_a^i$ for rectangular and $l_a^i \equiv 2m_a^i + n_a^i$ for tilted tori.

With these definitions the homology three-cycles for a stack a of D6-branes and its orientifold image a' are given by

$$[\Pi_a] = \prod_{i=1}^{3}(n_a^i[a_i] + 2^{-\beta_i} l_a^i [b_i]), \quad [\Pi_{a'}] = \prod_{i=1}^{3}(n_a^i[a_i] - 2^{-\beta_i} l_a^i [b_i]) \tag{3.3}$$

where $\beta_i = 0$ if the ith torus is not tilted and $\beta_i = 1$ if it is tilted.

There are four kinds of orientifold 6-planes associated with the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$. The homology three-cycles which they wrap are [91]

$$\Omega R : [\Pi_1] = 2^3[a_1][a_2][a_3], \qquad \Omega R\omega : [\Pi_2] = -2^{3-\beta_2-\beta_3}[a_1][b_2][b_3]$$
$$\Omega R\theta\omega : [\Pi_3] = -2^{3-\beta_1-\beta_3}[b_1][a_2][b_3], \quad \Omega R\theta : [\Pi_4] = -2^{3-\beta_1-\beta_2}[b_1][b_2][a_3] \tag{3.4}$$

This represents the fact that 180° rotation *plus* conjugate reflection produce 'vertical', i.e. $[b_i]$-oriented, invariant cycles, while the operator R alone preserves certain cycles along the 'horizontal', or $[a_i]$ axis. Each two-torus yields always a pair of such cycles, with the exception of the $[b_i]$-type tilted scenario where only a single invariant wrapping exists. This explains then the normal counting of $8 = 2^3$ distinct combinations, halved for each application of tilting in the vertically aligned case.

The total effect of these four planes should be combined, so we define $[\Pi_{O6}] = \sum_i [\Pi_i]$ [91]. In addition, a set of new parameters which are convenient in the following discussion are introduced [91]:

$$\begin{aligned} &A_a = -n_a^1 n_a^2 n_a^3,\ B_a = n_a^1 l_a^2 l_a^3,\ C_a = l_a^1 n_a^2 l_a^3,\ D_a = l_a^1 l_a^2 n_a^3 \\ &\tilde{A}_a = -l_a^1 l_a^2 l_a^3,\ \tilde{B}_a = l_a^1 n_a^2 n_a^3,\ \tilde{C}_a = n_a^1 l_a^2 n_a^3,\ \tilde{D}_a = n_a^1 n_a^2 l_a^3 \end{aligned} \tag{3.5}$$

With the basic definitions in hand, we can continue working on the global constraints of this model.

1. RR-tadpole Consistency Conditions

The Ramond-Ramond tadpole cancellation requires the total homology cycle charge of D6-branes and O6-planes to vanish [72]. The resulting equation

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0 \tag{3.6}$$

can be expressed in terms of the parameters defined in (3.5) as

$$\sum_a N_a A_a = \sum_a N_a B_a = \sum_a N_a C_a = \sum_a N_a D_a = -16 \tag{3.7}$$

It should be stressed that the tadpole condition is independent of the selected tilting. However, these coupled constraints are generally quite difficult to satisfy. The introduction of so called 'filler branes' [91] which wrap along the O6-planes can help somewhat. Such branes automatically preserve supersymmetry, so that they can be selected with only an eye for independent saturation of each RR-tadpole condition. If $N^{(i)}$ branes wrap along the i^{th} O6-plane, (3.7) is updated to

$$\begin{aligned} -2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a = \\ -2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16 \end{aligned} \tag{3.8}$$

Here $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted tori.

2. Conditions for Supersymmetric Brane Configurations

The condition to preserve $N = 1$ supersymmetry in four dimensions is that the rotation angle of any D-brane with respect to the orientifold plane is an element of $SU(3)$ [70, 84]. Consider the angles between each brane and the R-invariant axis of i^{th} torus θ_a^i, we require $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \bmod 2\pi$. This means $\sin(\theta_a^1 + \theta_a^2 + \theta_a^3) = 0$ and $\cos(\theta_a^1 + \theta_a^2 + \theta_a^3) = 1 > 0$. We define

$$\tan \theta_a^i = \frac{2^{-\beta_i} l_a^i R_2^i}{n_a^i R_1^i} \tag{3.9}$$

where R_2^i and R_1^i are the radii of the i^{th} torus. Then the above supersymmetry conditions can be recast in terms of the parameters defined in (3.5) as follows [91]:

$$\begin{aligned} x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0 \\ A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0 \end{aligned} \tag{3.10}$$

where x_A, x_B, x_C, x_D are complex structure parameters, all of which share the same sign. These parameters are given in terms of the complex structure moduli $\chi_i =$

(R^i_2/R^i_1) by

$$x_A = \lambda, \;\; x_B = \lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3, \;\; x_C = \lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3, \;\; x_D = \lambda 2^{\beta_1+\beta_2}/\chi_1\chi_2 \qquad (3.11)$$

The positive parameter λ was introduced in [91] to put all the variables A, B, C, D on an equal footing. However, among the x_i only three are independent.

3. Intersection Numbers

The initial $U(N_a)$ gauge group supported by a stack of N_a identical D6-branes is broken down by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry to a subgroup $U(N_a/2)$ [84]. Chiral matter particles are formed from open strings with two ends attaching on different stacks. By using Grassmann algebra $[a_i][b_j] = -[b_j][a_i] = \delta_{ij}$ and $[a_i][a_j] = -[b_j][b_i] = 0$ we can calculate the intersection numbers between stacks a and b and provide the multiplicity ($\mathcal{M}$) of the corresponding bi-fundamental representation:

$$\mathcal{M}(\frac{N_a}{2}, \frac{\overline{N_b}}{2}) = I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n^i_a l^i_b - n^i_b l^i_a) \qquad (3.12)$$

Likewise, stack a paired with the orientifold image b' of b yields

$$\mathcal{M}(\frac{N_a}{2}, \frac{N_b}{2}) = I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3} (n^i_a l^i_b + n^i_b l^i_a) \qquad (3.13)$$

Strings stretching between a brane in stack a and its mirror image a' yield chiral matter in the antisymmetric and symmetric representations of the group $U(N_a/2)$ with multiplicities

$$\mathcal{M}((\mathrm{A}_a)_L) = \frac{1}{2} I_{aO6}, \;\; \mathcal{M}((\mathrm{A}_a + \mathrm{S}_a)_L) = \frac{1}{2}(I_{aa'} - \frac{1}{2} I_{aO6}) \qquad (3.14)$$

so that the net total of antisymmetric and symmetric representations are given by

$$\begin{aligned}\mathcal{M}(\text{Anti}_a) &= \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6}) = -2^{1-k}[(2A_a - 1)\tilde{A}_a - \tilde{B}_a - \tilde{C}_a - \tilde{D}_a] \\ \mathcal{M}(\text{Sym}_a) &= \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6}) = -2^{1-k}[(2A_a + 1)\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a]\end{aligned} \tag{3.15}$$

where

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k}\prod_{i=1}^{3} n_a^i l_a^i \tag{3.16}$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a) \tag{3.17}$$

This distinction is critical, as we require independent use of the paired multiplets such as $(\mathbf{10}, \overline{\mathbf{10}})$ which are masked in expression (3.15). In what follows we consider the case $k = 0$.

4. Generalized Green-Schwarz Mechanism

Although the total non-Abelian anomaly in intersecting brane world models cancels automatically when the RR-tadpole conditions are satisfied, there may be additional mixed anomalies present. For instance, the mixed gravitational anomalies which generate massive fields are not trivially zero [84]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism which involves untwisted Ramond-Ramond forms. The couplings of the four untwisted Ramond-Ramond forms B_2^i to the $U(1)$ field strength F_a of each stack a are

$$\begin{aligned} &N_a l_a^1 n_a^2 n_a^3 \int_{M4} B_2^1 \wedge \text{tr}F_a, \quad N_a n_a^1 l_a^2 n_a^3 \int_{M4} B_2^2 \wedge \text{tr}F_a \\ &N_a n_a^1 n_a^2 l_a^3 \int_{M4} B_2^3 \wedge \text{tr}F_a, \quad -N_a l_a^1 l_a^2 l_a^3 \int_{M4} B_2^4 \wedge \text{tr}F_a \end{aligned} \tag{3.18}$$

These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G-S mechanism. In flipped $SU(5) \times U(1)_X$, the symmetry $U(1)_X$ must remain a gauge symmetry so that it may remix to help generate the standard model hypercharge after the breaking of $SU(5)$. Therefore, we must ensure that the gauge boson of the flipped $U(1)_X$ group does not receive such a mass. The $U(1)_X$ is a linear combination (to be identified in section 3.2) of the $U(1)$s from each stack :

$$U(1)_X = \sum_a C_a U(1)_a \tag{3.19}$$

The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand :

$$\begin{aligned} \sum_a C_a N_a \tilde{B}_a = 0, \quad \sum_a C_a N_a \tilde{C}_a = 0 \\ \sum_a C_a N_a \tilde{D}_a = 0, \quad \sum_a C_a N_a \tilde{A}_a = 0 \end{aligned} \tag{3.20}$$

C. Flipped $SU(5) \times U(1)_X$ Model Building

In the previous section we have outlined all the necessary machinery for constructing an intersecting-brane model on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Our goal now is to realize a supersymmetric $SU(5) \times U(1)_X$ gauge theory with three generations and a complete GUT and electroweak Higgs sector in the four-dimensional spacetime. We also try to avoid as much extra matter as possible.

1. Basic Flipped $SU(5)$ Phenomenology

In a flipped $SU(5) \times U(1)_X$ [93, 94] unified model, the electric charge generator Q is only partially embedded in $SU(5)$, *i.e.*, $Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$, where Y' is the $U(1)$ internal $SU(5)$ and $\tilde{Y}$ is the external $U(1)_X$ factor. Essentially, this means that the photon is 'shared' between $SU(5)$ and $U(1)_X$. The Standard Model (SM) plus right handed neutrino states reside within the representations $\bar{\mathbf{5}}$, $\mathbf{10}$, and $\mathbf{1}$ of $SU(5)$, which are collectively equivalent to a spinor **16** of $SO(10)$. The quark and lepton assignments are flipped by $u_L^c \leftrightarrow d_L^c$ and $\nu_L^c \leftrightarrow e_L^c$ relative to a conventional $SU(5)$ GUT embedding:

$$\bar{f}_{\bar{\mathbf{5}},-\frac{\mathbf{3}}{\mathbf{2}}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L \;; \quad F_{\mathbf{10},\frac{\mathbf{1}}{\mathbf{2}}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \; d_L^c \;\; \nu_L^c \right) ; \quad l_{\mathbf{1},\frac{\mathbf{5}}{\mathbf{2}}} = e_L^c \tag{3.21}$$

In particular this results in the **10** containing a neutral component with the quantum numbers of ν_L^c. We can break spontaneously the GUT symmetry by using a **10** and $\overline{\mathbf{10}}$ of superheavy Higgs where the neutral components provide a large vacuum expectation value, $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle$,

$$H_{\mathbf{10},\frac{\mathbf{1}}{\mathbf{2}}} = \{Q_H,\, d_H^c,\, \nu_H^c\} \,; \quad \bar{H}_{\overline{\mathbf{10}},-\frac{\mathbf{1}}{\mathbf{2}}} = \{Q_{\bar{H}},\, d_{\bar{H}}^c,\, \nu_{\bar{H}}^c\} \,. \tag{3.22}$$

The electroweak spontaneous breaking is generated by the Higgs doublets H_2 and $\bar{H}_{\bar{2}}$

$$h_{\mathbf{5},-\mathbf{1}} = \{H_2, H_3\} \,; \quad \bar{h}_{\bar{\mathbf{5}},\mathbf{1}} = \left\{\bar{H}_{\bar{2}}, \bar{H}_{\bar{3}}\right\} \tag{3.23}$$

Flipped $SU(5)$ model building has two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet (H_2)-triplet(H_3) splitting problem of the electroweak Higgs pentaplets $h, \bar{h}$ through the trilinear coupling of the Higgs fields: $H_{\mathbf{10}} \cdot H_{\mathbf{10}} \cdot h_{\mathbf{5}} \rightarrow \langle \nu_H^c \rangle \, d_H^c H_3$, and (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields ϕ, $F_{\mathbf{10}} \cdot \bar{H}_{\overline{\mathbf{10}}} \cdot \phi \rightarrow \left\langle \nu_{\bar{H}}^c \right\rangle \nu^c \phi$.

The generic superpotential W for a flipped $SU(5)$ model will be of the form :

$$\lambda_1 FFh + \lambda_2 F\bar{f}\bar{h} + \lambda_3 \bar{f} l^c h + \lambda_4 F\bar{H}\phi + \lambda_5 HHh + \lambda_6 \bar{H}\bar{H}\bar{h} + \cdots \in W \tag{3.24}$$

the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino mass and the last two terms are responsible for the doublet-triplet splitting mechanism [94].

2. Model Building

We first consider a stack with ten D6-branes to form the desired $U(5)$ group, and then determine additional stacks of two branes which provide $U(1)$ group factors and are compatible with the supersymmetry conditions of the 10-brane stack. To have enough but not too many copies of the antisymmetric and symmetric representation in the first stack a to satisfy the tadpole conditions, it is reasonable to consider the case of no tilted tori ($k = 0$) and we choose a set of proper wrapping numbers to make $\mathcal{M}((\mathrm{A}_a)_L) = 4$ and $\mathcal{M}((\mathrm{A}_a + \mathrm{S}_a)_L) = -2$. Under this setting, one wrapping number is zero and it makes two of the RR-tadpole parameters A, B, C, D zero with the remaining two negative, which forces the structure parameters x_A, x_B, x_C, x_D to be all positive by the SUSY conditions. Then the rest of the 2-brane stacks are chosen in accordance with our requirements.

Because of the combined constraints from RR-tadpole and SUSY conditions, it is harder to get negative values than to get positive values or zero for I_{ab} and $I_{ab'}$ to generate the required bi-fundamental representations. Generally when a negative number is needed, the absolute value cannot be large enough to alone provide three generations of chiral matter. This suggests the consideration of multiple two-brane stacks to share the burden of this task.

Next we turn to the question of the number of stacks we need. Generally speaking a case with three stacks is enough to provide all the required matter to construct a normal $SU(5)$ GUT model. However, as we mentioned we have to ensure that the $U(1)_X$ remains a gauge symmetry after the application of the G-S mechanism. It is clear that at least two more stacks are needed if all the couplings to the four RR forms are present.

The pentaplet $\bar{f}$ which contains Standard Model fermions is different from the Higgs pentaplet $\bar{h}$ resulting from the 'flipped' nature of the model as we saw in section 3.1. For example, if we take $U(1)_X$ for $(\mathbf{10}, \mathbf{1})$ in both SM and Higgs spectrum as $1/2$, then it is $-3/2$ for $(\bar{\mathbf{5}}, \mathbf{1})$ in SM, $5/2$ for $(\mathbf{1}, \mathbf{1})$ in SM, $-1/2$ for $(\overline{\mathbf{10}}, \mathbf{1})$ in Higgs, 1 or -1 for $(\bar{\mathbf{5}}, \mathbf{1})$ and $(\mathbf{5}, \mathbf{1})$ in Higgs, and 0 for $(\mathbf{1}, \mathbf{1})$ in Higgs. These constrain some coefficients of $U(1)$s from the stacks involving the SM and Higgs spectra, and may require more stacks in addition to the five mentioned above for obtaining the correct $U(1)_X$ charge for all the matter and Higgs representations. In this paper we present an example with seven stacks.

However, with seven stacks it was still difficult to find chiral bi-fundamental representations to be identified with the electroweak Higgs pentaplets, $h, \bar{h}$ and at the same time for the $U(1)_X$ group to remain a gauge symmetry. This directed us towards the most natural choice of identifying our Higgs pentaplets as well as some matter representations from intersections which provide non-chiral matter. After all, the Higgs $\mathbf{5}$ and the $\bar{\mathbf{5}}$ construct the vector-like $\mathbf{10}$ representation of $SO(10)$. A zero intersection number between two branes implies that the branes are parallel on at least one torus. At such kind of intersection additional non-chiral (vector-like) multiplet pairs from $ab+ba$, $ab'+b'a$, and $aa'+a'a$ can arise [107][5]. The multiplicity of these non-chiral multiplet pairs is given by the remainder of the intersection product, neglecting the null sector. For example, if $(n_a^1 l_b^1 - n_b^1 l_a^1) = 0$ in $I_{ab} = [\Pi_a][\Pi_b] =$

[5]Representations $(\mathrm{Anti}_a + \overline{\mathrm{Anti}}_a)$ occur at intersection of a with a' if they are parallel on at least one torus.

$2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i)$,

$$\mathcal{M}\left[\left(\frac{N_a}{2},\frac{\overline{N_b}}{2}\right)+\left(\frac{\overline{N_a}}{2},\frac{N_b}{2}\right)\right]=\prod_{i=2}^{3}(n_a^i l_b^i - n_b^i l_a^i) \qquad (3.25)$$

This is useful since we can fill the spectrum with this matter without affecting the required global conditions because the total effect of the pairs is zero. For instance in our model, besides the (ae') intersection which provides a vector-like pair of Higgs pentaplets, the intersection (ef') delivers the fermion (singlet under the $SU(5)$ group) $l_{\mathbf{1},\frac{\mathbf{5}}{\mathbf{2}}}$ particles.

In Table III we present a consistent model compatible with the constraints we described. Note that this is a (7+1)-stack model, with one stack of two filler branes wrapped along the first orientifold plane and two sets of parallel branes; the latter provide several non-chiral pairs. The gauge symmetry associated with the two filler branes is $Usp(2) \cong SU(2)$.

The Result The gauge symmetry of the (7+1)-stack model in Table III is $U(5) \times U(1)^6 \times Usp(2)$, and the structure parameters of the wrapping space are

$$x_A = 1, \;\; x_B = 2, \;\; x_C = 8, \;\; x_D = 1 \qquad (3.26)$$

which means

$$\frac{R_2^1}{R_1^1} = \frac{1}{2}, \;\; \frac{R_2^2}{R_1^2} = 2, \;\; \frac{R_2^3}{R_1^3} = \frac{1}{4} \qquad (3.27)$$

The intersection numbers are listed in Table IV, and the resulting spectrum in Table V. We have a complete Standard Model sector plus right handed neutrinos in three copies, a complete Higgs spectrum, and in addition extra *exotic* matter which includes two $(\overline{\mathbf{15}},\mathbf{1})$.

The $U(1)_X$ is

$$U(1)_X = \frac{1}{12}(3U(1)_a - 20U(1)_b + 45U(1)_d - 15U(1)_e - 15U(1)_f - 20U(1)_g) \quad (3.28)$$

while the other two anomaly-free and massless combinations $U(1)_Y$ and $U(1)_Z$ are

$$\begin{aligned} U(1)_Y &= U(1)_b + U(1)_c - 6U(1)_d + 3U(1)_e + 3U(1)_f + 2U(1)_g \\ U(1)_Z &= U(1)_b - U(1)_c + U(1)_e - U(1)_f \end{aligned} \quad (3.29)$$

These two gauge symmetries can be spontaneously broken by assigning vacuum expectation values to singlets from the intersection (bg). Thus, the final gauge symmetry is $SU(5) \times U(1)_X \times Usp(2)$.

The remaining four global $U(1)$s from the Green-Schwarz mechanism are given respectively by

$$\begin{aligned} U(1)_1 &= -10U(1)_a + 2U(1)_b + 2U(1)_c - 2U(1)_d - 8U(1)_g \\ U(1)_2 &= -2U(1)_b - 2U(1)_c + 2U(1)_g \\ U(1)_3 &= 6U(1)_b + 6U(1)_c + 4U(1)_d + 2U(1)_e + 2U(1)_f \\ U(1)_4 &= 20U(1)_a + 6U(1)_b + 6U(1)_c - 2U(1)_e - 2U(1)_f. \end{aligned} \quad (3.30)$$

Table III. Wrapping numbers and their consistent parameters.

stack	N_a	(n_1, l_1)	(n_2, l_2)	(n_3, l_3)	A	B	C	D	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$
a	$N = 10$	(0,-1)	(-1,-1)	(-1,-2)	0	0	-2	-1	2	-1	0	0
b	$N = 2$	(-1,-1)	(-1, 1)	(1, 3)	-1	-3	3	-1	3	1	-1	3
c	$N = 2$	(-1,-1)	(-1, 1)	(1, 3)	-1	-3	3	-1	3	1	-1	3
d	$N = 2$	(-1, 1)	(1, 0)	(-1,-2)	-1	0	-2	0	0	-1	0	2
e	$N = 2$	(-1, 1)	(1,-1)	(0,-1)	0	-1	-1	0	-1	0	0	1
f	$N = 2$	(-1, 1)	(1,-1)	(0,-1)	0	-1	-1	0	-1	0	0	1
g	$N = 2$	(1,-1)	(-4,-1)	(-1, 0)	-4	0	0	-1	0	-4	1	0
filler	$N^{(1)} = 2$	(1, 0)	(1, 0)	(1, 0)	-1	0	0	0	0	0	0	0

Table IV. List of intersection numbers. The number in parenthesis indicates the multiplicity of non-chiral pairs.

stk	N	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	f1
a	10	2	-2	-2	0(5)	-2	0(5)	0(1)	4	-2	0(1)	-2	0(1)	6	10	2
b	2	24	0	-	-	0(0)	24	2	0(5)	0(2)	0(2)	0(2)	0(2)	30	0(9)	3
c	2	24	0	-	-	-	-	2	0(5)	0(2)	0(2)	0(2)	0(2)	30	0(9)	3
d	2	2	-2	-	-	-	-	-	-	0(1)	-2	0(1)	-2	0(2)	4	0
e	2	0	0	-	-	-	-	-	-	-	-	0(0)	0(4)	0(5)	-6	-1
f	2	0	0	-	-	-	-	-	-	-	-	-	-	0(5)	-6	-1
g	2	-6	6	-	-	-	-	-	-	-	-	-	-	-	-	0

Table V. The spectrum of $U(5)\times U(1)^6\times Usp(2)$, or $SU(5)\times U(1)_X\times U(1)_Y\times U(1)_Z\times Usp(2)$, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like non-chiral pairs.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_g$	$12U(1)_X$
$(10,1)$	3	2	0	0	0	0	0	0	6
$(\bar{5}_a,1_e)$	2	-1	0	0	0	1	0	0	-18
$(\bar{5}_a,1_f)$	1	-1	0	0	0	0	1	0	-18
$(\bar{1}_e,\bar{1}_f)^\star$	3	0	0	0	0	-1	-1	0	30
$(10,1)$	1	2	0	0	0	0	0	0	6
$(\overline{10},1)$	1	-2	0	0	0	0	0	0	-6
$(5_a,1_e)^\star$	1	1	0	0	0	1	0	0	-12
$(\bar{5}_a,\bar{1}_e)^\star$	1	-1	0	0	0	-1	0	0	12
$(1_b,\bar{1}_g)$	4	0	1	0	0	0	0	-1	0
$(\overline{15},1)$	2	-2	0	0	0	0	0	0	-6
$(\overline{10},1)$	1	-2	0	0	0	0	0	0	-6
$(\bar{5}_a,1_c)$	2	-1	0	1	0	0	0	0	-3
$(5_a,1_d)$	4	1	0	0	1	0	0	0	48
$(\bar{5}_a,1_b)$	2	-1	1	0	0	0	0	0	-23
$(\bar{5}_a,1_f)$	1	-1	0	0	0	0	1	0	-18
$(5_a,\bar{1}_g)$	6	1	0	0	0	0	0	-1	23
$(5_a,1_g)$	10	1	0	0	0	0	0	1	-17
$(1_b,1_c)$	24	0	1	1	0	0	0	0	-20
$(1_b,\bar{1}_d)$	2	0	1	0	-1	0	0	0	-65
$(1_b,\bar{1}_g)$	26	0	1	0	0	0	0	-1	0
$(1_c,\bar{1}_d)$	2	0	0	1	-1	0	0	0	-45
$(1_c,\bar{1}_g)$	30	0	0	1	0	0	0	-1	20
$(\bar{1}_d,\bar{1}_e)$	2	0	0	0	-1	-1	0	0	-30
$(\bar{1}_d,\bar{1}_f)$	2	0	0	0	-1	0	-1	0	-30
$(1_d,1_g)$	4	0	0	0	1	0	0	1	25
$(\bar{1}_e,\bar{1}_g)$	6	0	0	0	0	-1	0	-1	35
$(\bar{1}_f,\bar{1}_g)$	6	0	0	0	0	0	-1	-1	35
$(\bar{1},\bar{1})$	2	0	0	0	-2	0	0	0	-90
$(1,1)$	6	0	0	0	0	0	0	2	-40
$(1_e,1_f)^\star$	4	0	0	0	0	1	1	0	-30
$(\bar{1}_e,\bar{1}_f)^\star$	1	0	0	0	0	-1	-1	0	30
Additional non-chiral Matter									
Usp(2) Matter									

CHAPTER IV

FLIPPED $SU(5)$ FROM D-BRANES WITH TYPE IIB FLUXES

A. Introduction

The fundamental goal of string phenomenology is to find a convincing connection between realistic particle physics and string theory. Previously it was thought that only models based upon weakly coupled heterotic string compactifications could achieve this. Indeed, the most realistic model based on string theory may be the heterotic string-derived flipped $SU(5)$ [8] which has been studied in great detail. However, in recent years Type I and Type II compactifications involving D-branes, where chiral fermions can arise from strings stretching between D-branes intersecting at angles (Type IIA picture) [16] and in its T-dual (Type IIB) picture with magnetized D-branes [17], have provided an interesting and exciting approach to this problem.

Many consistent standard-like and grand unified theory (GUT) models were built at an early stage [37, 38, 39, 40] using D-brane constructions. However, these models were not supersymmetric. Furthermore, these models suffered from instability in the internal space. The first quasi-realistic supersymmetric models were constructed in Type IIA theory on a $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [41, 42]. Following this, models with standard-like, left-right symmetric (Pati-Salam), Georgi-Glashow ($SU(5)$) and flipped $SU(5)$ gauge groups have been constructed based upon the same framework and systematically studied [43, 108, 109].

However, in spite of these successes, a natural mechanism is still needed to stabilize the moduli of the compactification, although in some cases the complex structure parameters (in Type IIA picture) and dilaton fields may be stabilized due to the gaugino condensation in the hidden sector [110]. Turning on RR and NSNS fluxes as background of the compactification gives rise to a non-trivial low energy supergravity potential which freezes some Calabi-Yau moduli [44]. Type IIB configurations with non-trivial RR and NSNS fluxes together with the presence of anti-D3 branes have been studied in [45, 46]. These fluxes impose strong constraints on the RR tadpole cancellation by giving large positive D3 RR charges since their supergravity equation of motion and the Dirac quantization conditions must be satisfied.

In the closed string sector, generic choices of the fluxes do not preserve supersymmetry. This leads to soft supersymmetry breaking terms at a mass scale $M_{soft} \sim \frac{M_{string}^2}{M_{Pl}}$ which implies an intermediate string scale or an inhomogeneous warp factor in the internal space to stabilize the electroweak scale [111, 112, 113]. On the other hand, for the string scale to be close to the Planck scale, supersymmetry in the open string sector must be preserved by fixing the Kähler toroidal moduli [113], which is T-dual to the supersymmetry consistency conditions in Type IIA theory. Recently D-brane constructions corresponding to models with magnetized D-branes where the role of the intersection angles is played by the magnetic fluxes on the D-branes on the Type IIB $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold have been studied [113, 47, 114, 115, 116].

As previously mentioned, there are only a few specific choices of fluxes which are supersymmetric in the closed string sector, which is interesting from a phenomenological point of view. In general, non-supersymmetric fluxes lead to soft supersymmetry breaking terms in the effective action of open string fields. Detailed studies of the soft-breaking mechanism and some trial investigations in the effective low energy scenario were explored in [112, 117]. Combined with an analysis of the Yukawa couplings [118], these studies may provide a clear picture of the low energy physics in the intersecting D-brane configuration which is worthwhile for future work. On the other hand, if supersymmetry is required to be conserved both in the closed and open string sectors, it has been recently shown that the RR, NSNS and metric fluxes could contribute negative D6-brane charges in Type IIA orientifold with flux compactifications, which makes it easier to satisfy the RR tadpole cancellation conditions [48]. We will presently not consider this, but plan to investigate this possibility in the future.

In this paper we search for consistent flipped $SU(5)$ models on a Type IIB $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with supergravity fluxes turned on. As mentioned before, due to the difficulty to impose supersymmetric fluxes in the closed string sector with consistent RR tadpole conditions, we do not insist that the fluxes be supersymmetric and consider all possible fluxes in constructing flipped $SU(5)$ models. However, supersymmetry in the open string sector is still preserved for a reasonable string scale. By requiring that the gauge bosons coupled to $U(1)_X$ do not acquire a string scale mass via a generalized Green-Schwarz mechanism, which has four constraints in $(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold construction, we find that the models must have at least five stacks of D-branes. In addition, there are K-theory constraints which must be imposed to avoid the anomaly classified by the discrete symmetry $\mathbb{Z}_2$. Some K-theory properties are modified by the NSNS fluxes, but this is currently regarded to have no effect on phenomenology [114].

Next, we turn to the question of our motivation in building flipped $SU(5)$ models. Different types of particle models have been discussed using various constructions. The minimal option is to embed just the Standard Model $SU(3)\times SU(2)\times U(1)$ gauge group, but almost every construction contains at least some extra $U(1)$ factors. Conventional GUT models such as $SU(5)$ or $SO(10)$ have been investigated, but none of them has been completely satisfactory. This triggered the motivation to consider the gauge group $SU(5) \times U(1)_X$ [8, 9, 10] as a candidate for a model derived from string. The *raison d'être* of this 'flipped' $SU(5)$ is that it requires only $\mathbf{10}$ and $\overline{\mathbf{10}}$ Higgs representations to break the GUT symmetry, in contrast to other unified models which require large and unwieldy adjoint representations. This point was given further weight when it was realized that models with adjoint Higgs representations cannot be derived from string theory with a $k = 1$ Kac-Moody algebra [11]. There are many attractive features of flipped $SU(5)$. For example, the hierarchy problem between the electroweak Higgs doublets and the color Higgs triplets is solved naturally through a 'missing partner' mechanism [8]. Furthermore, this dynamical doublet-triplet splitting does not require or involve any mixing between the Higgs triplets leading to a natural suppression of dimension 5 operators that may mediate rapid proton decay and for this reason it is probably the simplest GUT to survive the experimental limits placed upon proton lifetime [12]. More recently, the cosmic microwave anisotropy $\delta T/T$ has been successfully predicted by flipped $SU(5)$, as it has been determined to be proportional to $(M/M_P)^2$ where M denotes the symmetry breaking scale and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass [13]. Finally, string-derived flipped $SU(5)$ may provide a natural explanation for the production of Ultra-High Energy Cosmic Rays (UHECRs), through the decay of super-heavy particles dubbed 'cryptons' [14] that arise in the hidden sector of the model, which are also candidates for cold-dark matter (CDM).

The heterotic string-derived flipped $SU(5)$ model was created within the context of the free-fermionic formulation, which easily yields string theories in four dimensions. This model belongs to a class of models that correspond to compactification on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold at the maximally symmetric point in the Narain moduli space [15]. Although formulated in the context of weakly coupled heterotic string theory, it is believed that the vacuum may in fact be non-perturbative due to it's proximity to special points in the moduli space and may elevate to a consistent vacuum of M-theory. For this reason, it is our hope that in searching for a realistic flipped $SU(5)$ model that we may arrive at or near the same vacuum using D-brane constructions.

We organize this letter in the following way. In section 2 a brief but complete construction of D-branes compactified on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with Type IIB RR and NSNS supergravity fluxes is provided. In section 3 a short review of basic flipped $SU(5)$ phenomenology is presented. Section 4 contains the discussion of D-brane model building with fluxes, and we provide a few examples including a complete spectrum of a flipped $SU(5)$ model. We present our conclusions in section 5.

B. D-branes with Type IIB Flux on the $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

1. Magnetized D-branes in Type IIB Theory

We begin with the Type IIB theory on the $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where $\mathbf{T^6}$ is product of three two-tori and the two orbifold group generators θ, ω act on the complex coordinates (z_1, z_2, z_3) as

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3) \end{aligned} \tag{4.1}$$

This construction contains a $D = 4$, $N = 2$ supergravity multiplet, the dilaton hypermultiplet, h_{11} hypermultiplets, and h_{21} vector multiplets which are all massless. For the orbifold with discrete torsion the Hodge numbers from both twisted and untwisted sectors are $(h_{11}, h_{21}) = (3, 51)$. In order to include the open string sector, orientifold planes are introduced by an orientifold projection ΩR, where Ω is the world-sheet parity and R acts as

$$R : (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, -z_3) \tag{4.2}$$

There will then be 64 $O3$-planes and 4 $O7_i$-planes, which are transverse to the $\mathbf{T}^2_i$. Thus ΩR projects the $N = 2$ spectrum to an $N = 1$ supergravity multiplet, the dilaton chiral multiplet, and 6 untwisted and 48 twisted geometrical chiral multiplets. [113, 115]

We need $D(3+2n)$-branes to fill up the four-dimensional Minkowski space-time and wrapping the $2n$-cycles on a compact manifold in type IIB theory. The introduction of magnetic fluxes provides more flexibility in constructing models. For one stack of N_a D-branes wrapping m_a^i times on $\mathbf{T}_i^2$, n_a^i denotes the units of magnetic fluxes F_a^i turned on each $\mathbf{T}_i^2$, thus

$$m_a^i \frac{1}{2\pi} \int_{\mathbf{T}_i^2} F_a^i = n_a^i \tag{4.3}$$

To write down an explicit description of D-brane topology we introduce the even homology classes $[\mathbf{0}_i]$ and $[\mathbf{T}_i]$ for the point and the two-torus. Then the vectors of RR charges (corresponding to Type IIA homology cycles) of a^{th} stack D-brane and its image are [114]

$$[\Pi_a] = \prod_i^3 (n_a^i[\mathbf{0}_i] + m_a^i[\mathbf{T}_i]),\ [\Pi_a'] = \prod_i^3 (n_a^i[\mathbf{0}_i] - m_a^i[\mathbf{T}_i]) \tag{4.4}$$

The O3- and O7$_i$-planes of $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ resulting from the orientifold action ΩR, $\Omega R\omega$, $\Omega R\theta\omega$ and $\Omega R\theta$ can be written as

$$\begin{aligned}
\Omega R: &\quad [\Pi_{O3}] = [\mathbf{0}_1][\mathbf{0}_2][\mathbf{0}_3] \\
\Omega R\omega: &\quad [\Pi_{O7_1}] = -[\mathbf{0}_1][\mathbf{T}_2^2][\mathbf{T}_3^2] \\
\Omega R\theta\omega: &\quad [\Pi_{O7_2}] = -[\mathbf{T}_1^2][\mathbf{0}_2][\mathbf{T}_3^2] \\
\Omega R\theta: &\quad [\Pi_{O7_3}] = -[\mathbf{T}_1^2][\mathbf{T}_2^2][\mathbf{0}_3]
\end{aligned} \tag{4.5}$$

where the total effect is the sum of the above O-planes: $[\Pi_{O_p}] = [\Pi_{O3}] + [\Pi_{O7_1}] + [\Pi_{O7_2}] + [\Pi_{O7_3}]$.

2. The Fermionic Spectrum

Table VI. Spectrum of bi-fundamental representations.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet and 3 adjoint chiral multiplets
$ab+ba$	$\mathcal{M}(\frac{N_a}{2},\frac{\overline{N_b}}{2}) = I_{ab} = \prod_{i=1}^{3}(n_a^i m_b^i - n_b^i m_a^i)$
$ab'+b'a$	$\mathcal{M}(\frac{N_a}{2},\frac{N_b}{2}) = I_{ab'} = -\prod_{i=1}^{3}(n_a^i m_b^i + n_b^i m_a^i)$
$aa'+a'a$	$\mathcal{M}(\mathrm{Anti}_a) = \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO})$
	$\mathcal{M}(\mathrm{Sym}_a) = \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO})$

Chiral matter arises from open strings with two ends attaching on different stacks. The multiplicity ($\mathcal{M}$) of the corresponding bi-fundamental representation is given by the 'intersection' number (as in Type IIA theory) between different stacks of branes. The initial $U(N_a)$ gauge group supported by a stack of N_a identical D6-branes is broken down by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry to a subgroup $U(N_a/2)$. However a model may contain additional non-chiral (vector-like) multiplet pairs from $ab+ba$, $ab'+b'a$, and $aa'+a'a$ if the branes are parallel on at least one torus. The multiplicity of these non-chiral multiplet pairs is given by the remainder of the intersection product, neglecting the null sector. For example, if $(n_a^1 m_b^1 - n_b^1 m_a^1) = 0$ in $I_{ab} = [\Pi_a][\Pi_b] = \prod_{i=1}^{3}(n_a^i m_b^i - n_b^i m_a^i)$,

$$\mathcal{M}\left[\left(\frac{N_a}{2},\frac{\overline{N_b}}{2}\right) + \left(\frac{\overline{N_a}}{2},\frac{N_b}{2}\right)\right] = \prod_{i=2}^{3}(n_a^i m_b^i - n_b^i m_a^i) \tag{4.6}$$

The multiplicity of bi-fundamental as well as symmetric and antisymmetric representations are shown in Table VI.

3. Turning on Type IIB Fluxes

Turning on supergravity fluxes for closed string fields provides a possible way to stabilize the compactification moduli; however it also naturally breaks space-time supersymmetry in the bulk as well as contribute to the RR charges. Thus, specific solutions are needed to preserve supersymmetry.

The Type IIB non-trivial RR 3-form F_3 and NSNS 3-form H_3 fluxes compactified on Calabi-Yau threefold X_6 need to obey the Bianchi identities and be quantized [45]:

$$dF_3 = 0, \quad dH_3 = 0 \tag{4.7}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_{X6} F_3 \in \mathbf{Z}, \quad \frac{1}{(2\pi)^2\alpha'}\int_{X6} H_3 \in \mathbf{Z} \tag{4.8}$$

When the two fluxes are turned on, they induce a covariant field $G_3 = F_3 - \tau H_3$ and contribute to the D3-brane RR charges

$$N_{flux} = \frac{1}{(4\pi^2\alpha')^2}\int_{X_6} H_3 \wedge F_3 = \frac{1}{(4\pi^2\alpha')^2}\frac{i}{2\mathrm{Im}(\tau)}\int_{X_6} G_3 \wedge \bar{G}_3 \tag{4.9}$$

where $\tau = a + i/g_s$ being the Type IIB axion-dilaton coupling.

A complex cohomology basis can be utilized to describe the 3-form flux G_3 on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$:

$$\begin{aligned}
\omega_{B_0} &= dz^1 \wedge dz^2 \wedge dz^3, \quad \omega_{A_1} = d\bar{z}^1 \wedge dz^2 \wedge dz^3, \\
\omega_{B_1} &= dz^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3, \quad \omega_{A_2} = dz^1 \wedge d\bar{z}^2 \wedge dz^3, \\
\omega_{B_2} &= d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^3, \quad \omega_{A_3} = dz^1 \wedge dz^2 \wedge d\bar{z}^3, \\
\omega_{B_3} &= d\bar{z}^1 \wedge d\bar{z}^2 \wedge dz^3, \quad \omega_{A_0} = d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3
\end{aligned} \tag{4.10}$$

where $dz^i = dx^i + U_i dy^i$, U_i are complex structure moduli. Here ω_{B_0} corresponds to the (3,0) of the flux, ω_{B_i} with i =1, 2, 3 correspond to (1,2) of the flux, ω_{A_i} with i =1, 2, 3 correspond to (2,1), and ω_{A_0} is (0,3) component of the flux. Then the untwisted 3-form G_3 takes the form:

$$\frac{1}{(2\pi)^2\alpha'}G_3 = \sum_{i=0}^{3}(A^i\omega_{A_i} + B^i\omega_{B_i}) \tag{4.11}$$

Therefore the contribution of the fluxes to the RR tadpole condition N_{flux} can be calculated in terms of the basis defined above:

$$N_{flux} = \frac{1}{(4\pi^2\alpha')^2}\frac{i}{2\mathrm{Im}(\tau)}\int_{X_6} G_3 \wedge \bar{G}_3 = \frac{4\prod_{i=1}^{3}\mathrm{Im}(U^i)}{\mathrm{Im}(\tau)}\sum_{j=0}^{3}(|A^i|^2 - |B^i|^2) \tag{4.12}$$

The choice of fluxes may be positive (ISD-fluxes[1]) or negative (IASD-fluxes). However, in order to satisfy the supergravity equation of motion, the BPS-like self-dual condition $*_6 G_3 = iG_3$ demands N_{flux} to be positive [46, 112, 113]. The quantization conditions of F_3 and H_3 fluxes require that N_{flux} be a multiple of 64.

4. Supersymmetry Conditions

$D = 4$ $N = 1$ supersymmetric vacua from flux compactification require 1/4 supercharges of the ten-dimensional Type I theory be preserved both in the open and closed string sectors [113]. The supersymmetry constraints in the open string sector are from the world-volume magnetic field and those in the closed string sector induced by the fluxes.

a. Supersymmetry Conditions in the Closed String Sector

In the closed string sector, to ensure that the RR and NSNS fluxes are supersymmetric, the primitivity condition $G_3 \wedge J = 0$ should be satisfied [46]. Here J is the

[1] Imaginary self dual fluxes, lead to zero or negative cosmological constant(to lowest order).

general Kähler form of $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [114]:

$$J = J_1 dz^1 \wedge d\bar{z}^1 + J_2 dz^2 \wedge d\bar{z}^2 + J_3 dz^3 \wedge d\bar{z}^3 \tag{4.13}$$

We list a few solutions below. We also require that the turned on fluxes are as small as possible to avoid too large RR charge and satisfy the above requirements.

(2, 1)-Flux

(1) A specific supersymmetric solution for G_3 is (2, 1)-form given in [112] as

$$\frac{1}{(2\pi)^2\alpha'} G_3 = -4\omega_{A_2} - 4\omega_{A_3} \tag{4.14}$$

where the complex structure U^i and the dilaton coupling τ stabilize at $U^1 = U^2 = U^3 = \tau = i$. This solution gives the flux RR tadpole contribution:

$$N_{flux} = 128 \tag{4.15}$$

(2) Another specific supersymmetric solution for (2, 1)-form is given in [114] as

$$\frac{1}{(2\pi)^2\alpha'} G_3 = \frac{8}{\sqrt{3}} e^{-\pi i/6} (\omega_{A_1} + \omega_{A_2} + \omega_{A_3}) \tag{4.16}$$

The fluxes stabilize the complex structure toroidal moduli at values $U^1 = U^2 = U^3 = \tau = e^{2\pi i/3}$. Thus, the flux contributes to the RR tadpole contribution an amount:

$$N_{flux} = 192 \tag{4.17}$$

Non-SUSY This solution has the smallest contribution to the D3 RR charge. Although it is not supersymmetric due to the existence of (0, 3) component, it is still worthy of study since we do not observe supersymmetry at low energies. The 3-form flux is

$$\frac{1}{(2\pi)^2\alpha'}G_3 = 2(\omega_{A_0} + \omega_{A_1} + \omega_{A_2} + \omega_{A_3}) \tag{4.18}$$

with $U^1 = U^2 = U^3 = \tau = i$. The flux induced RR charge is then

$$N_{flux} = 64 \tag{4.19}$$

b. Supersymmetry Conditions in the Open String Sector

In order to preserve $N = 1$ supersymmetry in the open string sector, a constraint must be placed upon the D-brane world-volume magnetic fields $F^i = n^i/m^i\chi^i$ associated with each two-torus $\mathbf{T_i^2}$ which can be expressed in terms of an 'angle' θ_i (as in the Type IIA picture) on each torus, as $\sum_i \theta_i = 0 \bmod 2\pi$ [114], where $\tan\theta_i = (F^i)^{-1} = \frac{m^i\chi^i}{n^i}$ and $\chi^i = R_1^i R_2^i$ the area of the $\mathbf{T}_i^2$ in α' units. Then we can write it in a form that is similar to the constraints in Type IIA picture as [42]

$$\begin{aligned} -x_A m_a^1 m_a^2 m_a^3 + x_B m_a^1 n_a^2 n_a^3 + x_C n_a^1 m_a^2 n_a^3 + x_D n_a^1 n_a^2 m_a^3 = 0 \\ -n_a^1 n_a^2 n_a^3/x_A + n_a^1 m_a^2 m_a^3/x_B + m_a^1 n_a^2 m_a^3/x_C + m_a^1 m_a^2 n_a^3/x_D < 0 \end{aligned} \tag{4.20}$$

where $x_A = \lambda$, $x_B = \lambda/\chi^2\chi^3$, $x_C = \lambda/\chi^1\chi^3$, $x_D = \lambda/\chi^1\chi^2$, and λ is a normalization constant used to keep the variables on an equal footing.

5. RR Tadpole Cancellation and K-theory Constraints

The RR charges of the magnetized D-brane associated homology classes and the contribution from the orientifold planes as well as the effect of the fluxes must be cancelled, namely we demand

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi'_a] + \sum_p N_{O_p} Q_{O_p}[\Pi_{O_p}] + N_{flux} = 0 \tag{4.21}$$

where $[\Pi_{O_p}]$ are the sum of the orientifold planes listed in (4.5), and $N_{O_p}Q_{O_p} = -32$ in D_p-branes for S_p-type O-planes. N_{flux} is the amount of flux turned on, and is quantized in units of the elementary flux as discussed above [112, 113, 114]. Filler branes wrapping cycles along the O-planes can also be introduced here to reduce the difficulty of satisfying this condition. Thus the RR tadpole cancellation condition can be simplified as

$$\begin{aligned} -N^{(O3)} - \sum_a N_a n_a^i n_a^2 n_a^3 - \frac{1}{2} N_{flux} &= -16 \\ -N^{(O7_1)} + \sum_a N_a n_a^1 m_a^2 m_a^3 &= -16 \\ -N^{(O7_2)} + \sum_a N_a m_a^1 n_a^2 m_a^3 &= -16 \\ -N^{(O7_3)} + \sum_a N_a m_a^1 m_a^2 n_a^3 &= -16 \end{aligned} \tag{4.22}$$

In addition to the RR-tadpole condition the discrete D-brane RR charges classified by $\mathbb{Z}_\mathbf{2}$ K-theory groups in the presence of orientifolds, which are invisible by the ordinary homology [115, 119, 120, 121], should be also taken into account [115, 120].

In Type I superstring theory there exist non-BPS D-branes carrying non-trivial K-theory $\mathbb{Z}_\mathbf{2}$ charges. To avoid this anomaly it is required that in compact spaces these non-BPS branes must exist in an even number [120]. In Type IIB picture, these Type I non-BPS p-branes can be regarded as a pair of Dp-brane and it's world-sheet parity image. For example, $\widehat{\mathrm{D7}}|_{\mathrm{I}} = (\overline{\mathrm{D7}} + \overline{\mathrm{D7}}/\Omega)|_{\mathrm{IIB}}$. We need to consider the effects both from D3- and D7-branes since they do not contribute to the standard RR charges. The K-theory conditions for a $\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2}$ orientifold were derived in [115] and are given by

$$\sum_a N_a m_a^1 m_a^2 m_a^3 = 0 \bmod 4, \quad \sum_a N_a m_a^1 n_a^2 n_a^3 = 0 \bmod 4,$$
$$\sum_a N_a n_a^1 m_a^2 n_a^3 = 0 \bmod 4, \quad \sum_a N_a n_a^1 n_a^2 m_a^3 = 0 \bmod 4. \tag{4.23}$$

Furthermore, D-brane states are classified by the K-theory group due to the presence of NSNS 3-form fluxes as well. This requires adding additional D-branes to preserve the homological charges and the possibility of instanton mediating D-branes and fluxes [114]. These properties do not affect the main constraints, and they are not presently well known and need further study.

6. The Green-Schwarz Mechanism for Flipped $SU(5)$ GUT Construction

Although the total non-Abelian anomaly cancels automatically when the RR-tadpole conditions are satisfied, additional mixed anomalies like the mixed gravitational anomalies which generate massive fields are not trivially zero [42, 122]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism which involves untwisted Ramond-Ramond forms. The couplings of the four untwisted Ramond-Ramond forms B_2^i to the $U(1)$ field strength F_a are [38]

$$N_a m_a^1 n_a^2 n_a^3 \int_{M4} B_2^1 \wedge \mathrm{tr} F_a, \quad N_a n_a^1 m_a^2 n_a^3 \int_{M4} B_2^2 \wedge \mathrm{tr} F_a$$
$$N_a n_a^1 n_a^2 m_a^3 \int_{M4} B_2^3 \wedge \mathrm{tr} F_a, \quad -N_a m_a^1 m_a^2 m_a^3 \int_{M4} B_2^4 \wedge \mathrm{tr} F_a \tag{4.24}$$

These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G-S mechanism. In flipped $SU(5) \times U(1)_X$, the symmetry $U(1)_X$ must remain a gauge symmetry so that it may remix to help generate the standard model hypercharge after the breaking of $SU(5)$. Therefore, we must ensure that the gauge boson of the flipped $U(1)_X$ group does not receive such a mass. The

$U(1)_X$ is a linear combination of the $U(1)$s from each stack :

$$U(1)_X = \sum_a c_a U(1)_a \tag{4.25}$$

The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand :

$$\begin{aligned} \sum_a c_a N_a m_a^1 n_a^2 n_a^3 = 0, \quad \sum_a c_a N_a n_a^1 m_a^2 n_a^3 = 0 \\ \sum_a c_a N_a n_a^1 n_a^2 m_a^3 = 0, \quad \sum_a c_a N_a m_a^1 m_a^2 m_a^3 = 0 \end{aligned} \tag{4.26}$$

The G-S mechanism will be considered only after the coefficients of $U(1)_X$ are determined.

C. Flipped $SU(5) \times U(1)_X$ Model Building

In the previous section we have outlined all the necessary machinery for constructing models as Type IIB flux vacua on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Our goal now is to realize a supersymmetric $SU(5) \times U(1)_X$ gauge theory with three generations and a complete GUT and electroweak Higgs sector in the four-dimensional spacetime. We also try to avoid as much extra matter as possible.

1. Basic Flipped $SU(5)$ Phenomenology

In a flipped $SU(5) \times U(1)_X$ [8, 9, 10] unified model, the electric charge generator Q is only partially embedded in $SU(5)$, *i.e.*, $Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$, where Y' is the $U(1)$ internal $SU(5)$ and $\tilde{Y}$ is the external $U(1)_X$ factor. Essentially, this means that the photon is 'shared' between $SU(5)$ and $U(1)_X$. The Standard Model (SM) plus right handed neutrino states reside within the representations $\bar{\mathbf{5}}$, $\mathbf{10}$, and $\mathbf{1}$ of $SU(5)$, which are collectively equivalent to a spinor $\mathbf{16}$ of $SO(10)$. The quark and lepton assignments are flipped by $u_L^c \leftrightarrow d_L^c$ and $\nu_L^c \leftrightarrow e_L^c$ relative to a conventional $SU(5)$ GUT embedding:

$$\bar{f}_{\bar{\mathbf{5}},-\frac{\mathbf{3}}{\mathbf{2}}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L \; ; \quad F_{\mathbf{10},\frac{\mathbf{1}}{\mathbf{2}}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L \; d_L^c \;\; \nu_L^c \right) ; \quad l_{\mathbf{1},\frac{\mathbf{5}}{\mathbf{2}}} = e_L^c \tag{4.27}$$

In particular this results in the **10** containing a neutral component with the quantum numbers of ν_L^c. We can spontaneously break the GUT symmetry by using a **10** and $\overline{\mathbf{10}}$ of superheavy Higgs where the neutral components provide a large vacuum expectation value, $\langle\nu_H^c\rangle = \langle\bar{\nu}_H^c\rangle$,

$$H_{\mathbf{10},\frac{\mathbf{1}}{\mathbf{2}}} = \{Q_H, \; d_H^c, \; \nu_H^c\} \, ; \quad \bar{H}_{\overline{\mathbf{10}},-\frac{\mathbf{1}}{\mathbf{2}}} = \{Q_{\bar{H}}, \; d_{\bar{H}}^c, \; \nu_{\bar{H}}^c\} \, . \tag{4.28}$$

The electroweak spontaneous breaking is generated by the Higgs doublets H_2 and $\bar{H}_{\bar{2}}$

$$h_{\mathbf{5},-\mathbf{1}} = \{H_2, H_3\} \, ; \quad \bar{h}_{\bar{\mathbf{5}},\mathbf{1}} = \left\{\bar{H}_{\bar{2}}, \bar{H}_{\bar{3}}\right\} \tag{4.29}$$

Flipped $SU(5)$ model building has two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet (H_2)-triplet(H_3) splitting problem of the electroweak Higgs pentaplets $h, \bar{h}$ through the trilinear coupling of the Higgs fields: $H_{\mathbf{10}} \cdot H_{\mathbf{10}} \cdot h_{\mathbf{5}} \rightarrow \langle\nu_H^c\rangle \, d_H^c H_3$, and (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields ϕ, $F_{\mathbf{10}} \cdot \bar{H}_{\overline{\mathbf{10}}} \cdot \phi \rightarrow \left\langle\nu_{\bar{H}}^c\right\rangle \nu^c\phi$.

The generic superpotential W for a flipped $SU(5)$ model will be of the form :

$$\lambda_1 FFh + \lambda_2 F\bar{f}\bar{h} + \lambda_3 \bar{f} l^c h + \lambda_4 F\bar{H}\phi + \lambda_5 HHh + \lambda_6 \bar{H}\bar{H}\bar{h} + \cdots \in W \tag{4.30}$$

the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino mass and the last two terms are responsible for the doublet-triplet splitting mechanism [8].

D. Some Models with Fluxes

1. $N_{flux} = 192$

The most ideal situation is to preserve supersymmetry both in the closed string and open string sectors in the spirit of this flux construction. However we found that it is difficult to achieve. An example of this is shown in Table VII. Although this example is supersymmetric both in the open and closed string sectors, satisfies the conditions for cancellation of RR charges, and yields a three generation flipped $SU(5)$ model with a complete but extended Higgs sector, it does not satisfy the K-theory constraints.

Table VII. List of wrapping numbers and intersection numbers for three-fluxes $N_{flux} = 192$. The number in parenthesis indicates the multiplicity of non-chiral pairs. Here $x_A = 62$, $x_B = 1$, $x_C = 1$, and $x_D = 2$. Clearly, the first K-theory constraint is not satisfied.

stk	N	$(n_1, m_1)(n_2, m_2)(n_3, m_3)$	b	b'	c	c'	d	d'	e	e'	f	f'	$D7_2$
a	10	(1, 0) (-1,-1) (-2, 1)	-12	24	1	-3	1	-3	0(1)	-2	0(1)	-2	2
b	2	(3,-1) (-5, 1) (4,-1)	-	-	7	15	7	15	12	16	12	16	12
c	2	(-2, 1) (2, 1) (-1, 0)	-	-	-	-	0(0)	0(16)	0(0)	0(9)	0(0)	0(9)	2
d	2	(-2, 1) (2, 1) (-1, 0)	-	-	-	-	-	-	0(0)	0(9)	0(0)	0(9)	2
e	2	(-1, 1) (1, 1) (-1, 0)	-	-	-	-	-	-	-	-	0(0)	0(4)	1
f	2	(-1, 1) (1, 1) (-1, 0)	-	-	-	-	-	-	-	-	-	-	1
$O7_2$	6	(0, 1) (1, 0) (0,-1)	-	-	-	-	-	-	-	-	-	-	-

2. $N_{flux} = 128$

We present an example for $N_{flux} = 128$ with four stacks of magnetized D-branes as well as two filler branes presented in Table VIII. Although this particular model does not contain flipped $SU(5)$ symmetry, it is a consistent solution of the RR tadpole conditions and the K-theory constraints, and is supersymmetric both in the open and closed string sectors. The gauge symmetry is

$$U(5) \times U(1) \times USp(4) \times USp(4) \tag{4.31}$$

Table VIII. $N_{flux} = 128$. The number stacks is only two plus two filler branes, though it has very few exotic particles, we have too few stacks to complete the cancellation of $U(1)_X$ mass. Here $x_A = 27$, $x_B = 1$, $x_C = 1$, and $x_D = 2$.

stk	N	$(n_1, m_1)(n_2, m_2)(n_3, m_3)$	A	S	b	b'	$D3$	$D7_2$
a	10	(1, 0) (-1,-1) (-2, 1)	2	-2	-16	24	0(1)	2
b	2	(3,-2) (-3, 1) (4,-1)	374	202	-	-	-2	12
$O3$	4	(1, 0) (1, 0) (1, 0)	-	-	-	-	-	-
$O7_2$	4	(0, 1) (1, 0) (0,-1)	-	-	-	-	-	-

3. $N_{flux} = 1 \times 64$

Table IX. List of intersection numbers for $N_{flux} = 64$ with gauge group $U(5) \times U(1)^5$. The number in parenthesis indicates the multiplicity of non-chiral pairs.

stk	N	$(n_1, m_1)(n_2, m_2)(n_3, m_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	10	(1, 0) (-1,-1) (-2, 1)	2	-2	-8	12	-8	12	0(0)	0(8)	0(1)	4	0(1)	4
b	2	(1,-1) (-3, 1) (4,-1)	84	12	-	-	0(0)	96	8	12	4	0(6)	4	0(6)
c	2	(1,-1) (-3, 1) (4,-1)	84	12	-	-	-	-	8	12	4	0(6)	4	0(6)
d	2	(-1, 0) (1, 1) (-2, 1)	2	-2	-	-	-	-	-	-	0(1)	4	0(1)	4
e	2	(1, 1) (1, 0) (2,-1)	2	-2	-	-	-	-	-	-	-	-	0(0)	8
f	2	(1, 1) (1, 0) (2,-1)	2	-2	-	-	-	-	-	-	-	-	-	-

In this example, we use two sets of parallel D-branes and all conditions are satisfied. No filler brane is needed, and $x_A = 22$, $x_B = 1$, $x_C = 1$, and $x_D = 2$. The complete (n_a^i, m_a^i) and $SU(5) \times U(1)_X$ spectrum are listed in Tables IX and X, and $U(1)_X$ is

$$U(1)_X = \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c - 5U(1)_d + 5U(1)_e - 5U(1)_f) \qquad (4.32)$$

The four global $U(1)$s from the Green-Schwarz mechanism are given respectively:

$$\begin{aligned} U(1)_1 &= 24U(1)_b + 24U(1)_c + 4U(1)_e + 4U(1)_f \\ U(1)_2 &= 20U(1)_a + 8U(1)_b + 8U(1)_c + 4U(1)_d \\ U(1)_3 &= -10U(1)_a + 6U(1)_b + 6U(1)_c - 2U(1)_d - 2U(1)_e - 2U(1)_f \\ U(1)_4 &= -2U(1)_b - 2U(1)_c \end{aligned} \qquad (4.33)$$

From Table X we found that none of the global $U(1)$s from the G-S anomaly cancellation mechanism provides Yukawa couplings required for generation of mass terms in superpotential (4.30). However, $U(1)_X$ admits these Yukawa couplings, and if we require the other anomaly-free and massless combination $U(1)_Y$ does as well, two conditions can be considered. The first one is to demand all the Yukawa couplings from the assigned intersections, and an example of the $U(1)_Y$ and the corresponding combinations of representations are listed as follows:

$$U(1)_Y^1 = 5U(1)_a - 25U(1)_b + 25U(1)_c - 25U(1)_d - 38U(1)_e + 38U(1)_f \qquad (4.34)$$

$$
\begin{aligned}
FFh &\rightarrow (\mathbf{10},\mathbf{1})(\mathbf{10},\mathbf{1})(\mathbf{5}_a,\mathbf{1}_d)^\star \\
F\bar{f}\bar{h} &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{5}}_a,\mathbf{1}_b)(\overline{\mathbf{5}}_a,\overline{\mathbf{1}}_d)^\star \\
\bar{f}l^c h &\rightarrow (\overline{\mathbf{5}}_a,\mathbf{1}_b)(\mathbf{1}_c,\overline{\mathbf{1}}_d)(\mathbf{5}_a,\mathbf{1}_d)^\star \\
F\bar{H}\phi &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\mathbf{1}_b,\mathbf{1}_c) \\
HHh &\rightarrow (\mathbf{10},\mathbf{1})(\mathbf{10},\mathbf{1})(\mathbf{5}_a,\mathbf{1}_d)^\star \\
\bar{H}\bar{H}\bar{h} &\rightarrow (\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{5}}_a,\overline{\mathbf{1}}_d)^\star
\end{aligned}
\tag{4.35}
$$

If we do not require the Higgs pentaplet $\bar{h}'$ coupled with the chiral fermions in the term $F\bar{f}\bar{h}'$ to be the same as the Higgs pentaplet $\bar{h}$ coupled to $\bar{H}$, then we expect a mixture state $\bar{h}_x = c\bar{h}' + s\bar{h}$ of these two different Higgs pentaplets in the Higgs sector, therefore

$$U(1)_Y^2 = U(1)_b - U(1)_c + U(1)_e - U(1)_f \tag{4.36}$$

$$
\begin{aligned}
F\bar{f}\bar{h}' &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{5}}_a,\mathbf{1}_b)(\overline{\mathbf{5}}_a,\mathbf{1}_c) \\
\bar{H}\bar{H}\bar{h} &\rightarrow (\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{5}}_a,\overline{\mathbf{1}}_d)^\star
\end{aligned}
\tag{4.37}
$$

We should also notice that the superfluous $\bar{\mathbf{5}}, \mathbf{5}$, and $\overline{\mathbf{10}}$ representations may be ostracized from the low energy spectrum through trilinear couplings of the generic form $\bar{\mathbf{5}}\cdot\mathbf{5}\cdot\mathbf{1}$ and $\overline{\mathbf{10}}\cdot\mathbf{10}\cdot\mathbf{1}$ satisfying the gauged $U(1)$ symmetries, where the singlets are assumed to acquire string scale *vev*s.

Table X. The spectrum of $U(5) \times U(1)^5$, or $SU(5) \times U(1)_X \times U(1)_Y$, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter. We list the two cases for the $U(1)_Y$.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_X$
$(10, 1)$	3	2	0	0	0	0	0	1
$(\bar{5}_a, 1_b)$	3	-1	1	0	0	0	0	-3
$(1_c, \bar{1}_d)$	3	0	0	1	-1	0	0	5
$(10, 1)$	1	2	0	0	0	0	0	1
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	-1
$(5_a, 1_d)^\star$	1	1	0	0	1	0	0	-2
$^1(\bar{5}_a, \bar{1}_d)^\star/^2\bar{h}_x$	1	^{1}-1	^{1}0	^{1}0	^{1}-1	^{1}0	^{1}0	2
$(1_b, 1_c)$	4	0	1	1	0	0	0	0
$(\overline{15}, 1)$	2	-2	0	0	0	0	0	-1
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	-1
$(\bar{5}_a, 1_b)$	5	-1	1	0	0	0	0	-3
$(5_a, 1_b)$	12	1	1	0	0	0	0	-2
$(\bar{5}_a, 1_c)$	8	-1	0	1	0	0	0	2
$(5_a, 1_c)$	12	1	0	1	0	0	0	3
$(5_a, 1_e)$	4	1	0	0	0	1	0	3
$(5_a, 1_f)$	4	1	0	0	0	0	1	-2
$(1_b, 1_c)$	92	0	1	1	0	0	0	0
$(1_b, \bar{1}_d)$	8	0	1	0	-1	0	0	0
$(1_b, 1_d)$	12	0	1	0	1	0	0	-5
$(1_b, \bar{1}_e)$	4	0	1	0	0	-1	0	-5
$(1_b, \bar{1}_f)$	4	0	1	0	0	0	-1	0
$(1_c, \bar{1}_d)$	5	0	0	1	-1	0	0	5
$(1_c, 1_d)$	12	0	0	1	1	0	0	0
$(1_c, \bar{1}_e)$	4	0	0	1	0	-1	0	0
$(1_c, \bar{1}_f)$	4	0	0	1	0	0	-1	5
$(1d, 1_e)$	4	0	0	0	1	1	0	0
$(1d, 1_f)$	4	0	0	0	1	0	1	-5
$(1, 1)$	12	0	2	0	0	0	0	-5
$(1, 1)$	12	0	0	2	0	0	0	5
$(\bar{1}, \bar{1})$	2	0	0	0	-2	0	0	5
$(\bar{1}, \bar{1})$	2	0	0	0	0	-2	0	-5
$(\bar{1}, \bar{1})$	2	0	0	0	0	0	-2	5
$(5_a, 1_d)^\star$	7	1	0	0	1	0	0	-2
$(\bar{5}_a, \bar{1}_d)^\star$	7	-1	0	0	-1	0	0	2

CHAPTER V

MSSM VIA PATI-SALAM FROM INTERSECTING BRANES ON $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2')$

A. Introduction

At the present, string theory is the only framework for realization of a unification of gravitation with gauge theory and quantum mechanics. In principle, it should be possible to derive all known physics from the string, as well as potentially provide something new and unexpected. This is the goal of string phenomenology. However, in spite of this there exist many solutions that may be derived from string, all of which are consistent vacua. One of these vacua should correspond to our universe, but then the question becomes why this particular vacuum is selected. One possible approach to this state of affairs is to statistically classify the possible vacua, in essence making a topographical map of the 'landscape'. One then attempts to assess the liklihood that vacua with properties similar to ours will arise.[1] Another approach is to take the point of view that there are unknown dynamics, perhaps involving a departure from criticality, which determine the vacuum that corresponds to our universe. Regardless of the question of uniqueness, if string theory is correct then it should be possible to find a solution which corresponds *exactly* to our universe, at least in it's low energy limit. Although there has been a great deal of progress in constructing semi-realistic models, this has not yet been achieved.

An elegant approach to model construction involving Type I orientifold (Type II) compactifications is where chiral fermions arise from strings stretching between D-branes intersecting at angles (Type IIA picture) [16] or in its T-dual (Type IIB) picture with magnetized D-branes [17]. Many consistent standard-like and grand

[1] For example, see [2, 3, 4, 5, 6, 7].

unified theory (GUT) models have been constructed [37, 38, 39, 123] using D-brane constructions. The first quasi-realistic supersymmetric models were constructed in Type IIA theory on a $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [42, 41]. Following this, models with standard-like, left-right symmetric (Pati-Salam), unflipped $SU(5)$ gauge groups were constructed based upon the same framework and systematically studied [43, 124, 108, **?**]. In addition, several different flipped $SU(5)$ [9, 10, 8] models have also been built using intersecting D-brane constructions [123, 109, 125, 126, 127, 128].[2]

Although much progress has been made, none of these models have been completely satisfactory. Problems include extra chiral and non-chiral matter, and the lack of a complete set of Yukawa couplings, which are typically forbidden by global symmetries. In addition to the chiral matter which arises at brane intersections, D-brane constructions typically will have non-chiral open string states present in the low-energy spectrum associated with the D-brane position in the internal space and Wilson lines. This results in adjoint or additional matter in the symmetric and antisymmetric representations unless the open string moduli are completely frozen. These light scalars are not observed and are not present in the MSSM. While it is possible that these moduli will obtain mass after supersymmetry is broken, it would typically be of the TeV scale. While this would make them unobservable in present experiments, the succesful gauge unification in the MSSM would be spoiled by their presence. While it may be possible to find some scenarios where the problems created by these fields are ameliorated, it is much simpler to eliminate these fields altogether. One way to do this is to this is to construct intersecting D-brane models where the D-branes wrap rigid cycles.[3] Another motiviation for the absence of these adjoint states

[2]For excellent reviews, see [129] and [130].

[3]This possibility was first explored in [131] and [132].

is that this is consistent with a $k = 1$ Kac-Moody algebra in models constructed from heterotic string, some of which may be dual.

In this letter, we construct an intersecting D-brane model on the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold background, also known as the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with discrete torsion, where the D-branes wrap rigid cycles thus eliminating the extra adjoint matter. This letter is organized as follows: First, we briefly review intersecting D-brane constructions on the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold. We then proceed to construct a supersymmetric four-generation MSSM-like model obtained from a Pati-Salam model via spontaneous gauge symmetry breaking. All of the required Yukawa couplings are allowed by global symmetries present in the movel. We find that the tree-level gauge couplings are unified at the string scale.

B. Intersecting Branes on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ Orientifold with and without Discrete Torsion

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold has been the subject of extensive research, primarily because it is the simplest background space which allows supersymmetric vacua. We will essentially follow along with the development given in [132]. The first supersymmetric models based upon the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold were explored in [42, 41, 43, 124]. In Type IIA theory on the $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, the $\mathbf{T^6}$ is product of three two-tori and the two orbifold group generators θ, ω act on the complex coordinates (z_1, z_2, z_3) as

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \end{aligned} \tag{5.1}$$

while the antiholomorphic involution R acts as

$$R(z_1, z_2, z_3) \to (\bar{z}_1, \bar{z}_2, \bar{z}_3). \tag{5.2}$$

As it stands, the signs of the θ action in the ω sector and vice versa have not been specified, and the freedom to do so is referred to as the choice of discrete torsion. One choice of discrete torsion corresponds to the Hodge numbers $(h_{11}, h_{21}) = (3, 51)$ and the corresponding to $(h_{11}, h_{21}) = (51, 3)$. These two different choices are referred to as with discrete torsion ($\mathbb{Z}_2 \times \mathbb{Z}_2'$) and without discrete torsion ($\mathbb{Z}_2 \times \mathbb{Z}_2$) respectively. To date, most phenomenological models that have been constructed have been without discrete torsion. Consequently, all of these models have massless adjoint matter present since the D-branes do not wrap rigid 3-cycles. However, in the case of $\mathbb{Z}_2 \times \mathbb{Z}_2'$, the twisted homology contains collapsed 3-cycles, which allows for the construction of rigid 3-cycles.

D6-branes wrapping cycles are specified by their wrapping numbers (n^i, m^i) along the fundamental cycles $[a^i]$ and $[b^i]$ on each torus. However, cycles on the torus are, in general, different from the cycles defined on the orbifold space. In the case of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold, all of the 3-cycles on the orbifold are inherited from the torus, which makes it particulary easy to work with. The $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold contains 16 fixed points, from which arise 16 additional 2-cycles with the topology of $\mathbf{P}^1 \cong S^2$. As a result, there are 32 collapsed 3-cycles for each twisted sector. A $D6$-brane wrapping collapsed 3-cycles in each of the three twisted sectors will be unable to move away from a particular position on the covering space $\mathbf{T^6}$, which means that the 3-cycle will be rigid.

A basis of twisted 3-cycles may be denoted as

$$[\alpha^{\theta}_{ij,n}] \;=\; 2[\epsilon^{\theta}_{ij}] \otimes [a^3] \qquad [\alpha^{\theta}_{ij,m}] = 2[\epsilon^{\theta}_{ij}] \otimes [b^3], \tag{5.3}$$

$$[\alpha^{\omega}_{ij,n}] \;=\; 2[\epsilon^{\omega}_{ij}] \otimes [a^1] \qquad [\alpha^{\omega}_{ij,m}] = 2[\epsilon^{\omega}_{ij}] \otimes [b^1], \tag{5.4}$$

$$[\alpha^{\theta\omega}_{ij,n}] \;=\; 2[\epsilon^{\theta\omega}_{ij}] \otimes [a^2] \qquad\qquad [\alpha^{\theta\omega}_{ij,m}] = 2[\epsilon^{\theta\omega}_{ij}] \otimes [b^2]. \tag{5.5}$$

where $[\epsilon^{\theta}_{ij}]$, $[\epsilon^{\omega}_{ij}]$, and $[\epsilon^{\theta\omega}_{ij}]$ denote the 16 fixed points on $\mathbf{T}^2 \times \mathbf{T}^2$, where $i, j \in 1, 2, 3, 4$.

A fractional D-brane wrapping both a bulk cycle as well as the collapsed cycles may be written in the form

$$\Pi^F_a = \frac{1}{4}\Pi^B + \frac{1}{4}\left(\sum_{i,j\in S^a_\theta} \epsilon^{\theta}_{a,ij}\Pi^{\theta}_{ij,a}\right) + \frac{1}{4}\left(\sum_{j,k\in S^a_\omega} \epsilon^{\omega}_{a,jk}\Pi^{\omega}_{jk,a}\right) + \frac{1}{4}\left(\sum_{i,k\in S^a_{\theta\omega}} \epsilon^{\theta\omega}_{a,ik}\Pi^{\theta\omega}_{ik,a}\right). \tag{5.6}$$

where the $D6$-brane is required to run through the four fixed points for each of the twisted sectors. The set of four fixed points may be denoted as S^g for the twisted sector g. The constants $\epsilon^{\theta}_{a,ij}$, $\epsilon^{\omega}_{a,jk}$ and $\epsilon^{\theta\omega}_{a,ki}$ denote the sign of the charge of the fractional brane with respect to the fields which are present at the orbifold fixed points. These signs, as well as the set of fixed points, must satisfy consistency conditions. However, they may be chosen differently for each stack.

A bulk cycle on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold space consist of the toroidal cycle wrapped by the brane D_a and it's three orbifold images:

$$\left[\Pi^B_a\right] \;=\; (1 + \theta + \omega + \theta\omega)\,\Pi^{T^6}_a. \tag{5.7}$$

Each of these orbifold images in homologically identical to the original cycle, thus

$$\left[\Pi^B_a\right] \;=\; 4\left[\Pi^{T^6}_a\right]. \tag{5.8}$$

If we calculate the intersection number between two branes, we will find

$$\left[\Pi^B_a\right] \circ \left[\Pi^B_b\right] \;=\; 4\,\left[\Pi^{T^6}_a\right] \circ \left[\Pi^{T^6}_b\right] \tag{5.9}$$

which indicates that the bulk cycles $\left[\Pi^B_a\right]$ do not expand a unimodular basis for the homology lattice $H_3(M, Z)$. Thus, we must normalize these purely bulk cycles as $[\Pi_a^o] = \frac{1}{2}\left[\Pi_a^B\right]$ [129, 132]. So, in terms of the cycles defined on the torus, the normalized purely bulk cycles of the orbifold are given by

$$[\Pi_a^o] \;=\; \frac{1}{2}\left(1+\theta+\omega+\theta\omega\right)\left[\Pi_a^{T^6}\right] = 2\left[\Pi_a^{T^6}\right]. \tag{5.10}$$

Due to this normalization, a stack of N $D6$-branes wrapping a purely bulk cycle will have a $U(N/2)$ gauge group in its world-volume. However, this does not apply to a brane wrapping collapsed cycles, so that a stack of N branes wrapping fractional cycles as in eq. 5.6 will have in its world-volume a gauge group $U(N)$.

Since we will have $D6$-branes which are wrapping fractional cycles with a bulk component as well as twisted cycles, we will need to be able to calculate the intersection numbers between pairs of twisted 3-cycles. For the intersection number between two twisted 3 cycles of the form $[\Pi_{ij,a}^g] = n_a^{I_g}[\alpha_{ij,n}] + m_a^{I_g}[\alpha_{ij,m}]$ and $[\Pi_{kl,b}^h] = n_b^{I_h}[\alpha_{kl,n}] + m_b^{I_h}[\alpha_{kl,m}]$ we have

$$[\Pi_{ij,a}^g]\circ[\Pi_{kl,b}^h] \;=\; 4\delta_{ik}\delta_{jl}\delta^{gh}(n_a^{I_g}m_b^{I_g} - m_a^{I_g}n_b^{I_g}) \tag{5.11}$$

where I_g corresponds to the torus left invariant by the action of the orbifold generator g; specifically $I_\theta = 3$, $I_\omega = 1$, and $I_{\theta\omega} = 2$.

Putting everything together, we will find for the intersection number between a brane a and brane b wrapping fractional cycles we will have

$$\begin{aligned}\Pi_a^F\circ\Pi_b^F = \frac{1}{16}[\Pi_a^B\circ\Pi_b^B &+ 4(n_a^3m_b^3 - m_a^3n_b^3)\sum_{i_aj_a\in S_\theta^a}\sum_{i_bj_b\in S_\theta^b}\epsilon_{a,i_aj_a}^{\theta}\epsilon_{b,i_bj_b}^{\theta}\delta_{i_ai_b}\delta_{j_aj_b} + \\ &4(n_a^1m_b^1 - m_a^1n_b^1)\sum_{j_ak_a\in S_\omega^a}\sum_{j_bk_b\in S_\omega^b}\epsilon_{a,j_ak_a}^{\omega}\epsilon_{b,j_bk_b}^{\omega}\delta_{j_aj_b}\delta_{k_ak_b} + \\ &4(n_a^2m_b^2 - m_a^2n_b^2)\sum_{i_ak_a\in S_{\theta\omega}^a}\sum_{i_bk_b\in S_{\theta\omega}^b}\epsilon_{a,i_ak_a}^{\theta\omega}\epsilon_{b,i_bk_b}^{\theta\omega}\delta_{i_ai_b}\delta_{k_ak_b}].\end{aligned} \tag{5.12}$$

The 3-cycle wrapped by the $O6$-planes is given by

$$2q_{\Omega R}[a^1][a^2][a^3] - 2q_{\Omega R\theta}[b^1][b^2][a^3] - 2q_{\Omega R\omega}[a^1][b^2][b^3] - 2q_{\Omega R\theta\omega}[b^1][a^2][b^3]. \tag{5.13}$$

where the cross-cap charges $q_{\Omega Rg}$ give the RR charge and tension of a given orientifold plane g, of which there are two types, $O6^{(-,-)}$ and $O6^{(+,+)}$. In this case, $q_{\Omega Rg} = +1$ indicates an $O6^{(-,-)}$ plane, while $q_{\Omega Rg} = -1$ indicates an $O6^{(+,+)}$ while the choice of discrete torsion is indicated by the product

$$q = \prod_g q_{\Omega Rg}. \tag{5.14}$$

The choice of no discrete torsion is given by $q = 1$, while for $q = -1$ is the case of discrete torsion, for which an odd number of $O^{(+,+)}$ must be present.

The action of ΩR on the bulk cycles is the same in either case, and is essentially just changes the signs of the wrapping numbers as $n_a^i \rightarrow n_a^i$ and $m_a^i \rightarrow -m_a^i$. However, in addition, there is an action on the twisted 3 cycle as

$$\alpha_{ij,n}^g \rightarrow -q_{\Omega R} q_{\Omega Rg} \alpha_{ij,n}^g, \quad \alpha_{ij,m}^g \rightarrow q_{\Omega R} q_{\Omega Rg} \alpha_{ij,m}^g. \tag{5.15}$$

Using these relations, one can work out the intersection number of a fractional cycle with it's ΩR image, we have

$$\Pi_a'^F \circ \Pi_a^F = q_{\Omega R}\left(2q_{\Omega R}\prod_I n_a^I m_a^I - 2q_{\Omega R\theta} n_a^3 m_a^3 - 2q_{\Omega R\omega} n_a^1 m_a^1 - 2q_{\Omega R\theta\omega} n_a^2 m_a^2\right) \tag{5.16}$$

while the intersection number with the orientifold planes is given by

$$\Pi_{O6} \circ \Pi_a^F = 2q_{\Omega R}\prod_I m_a^I - 2q_{\Omega R\theta} n_a^1 n_a^2 m_a^3 - 2q_{\Omega R\omega} m_a^1 n_a^2 n_a^3 - 2q_{\Omega R\theta\omega} n_a^1 m_a^2 n_a^3. \tag{5.17}$$

A generic expression for the *net* number of chiral fermions in bifundamental, symmetric, and antisymmetric representations consistent with the vanishing of RR tadpoles can be given in terms of the three-cycles cycles [36] which is shown in Table

Table XI. Net chiral matter spectrum in terms of three-cycles.

Representation	Multiplicity
$\Box\!\!\!\Box$ (antisymmetric)	$\frac{1}{2}([\Pi^o_{a'}] \circ [\Pi^o_a] + [\Pi_{O6}] \circ [\Pi^o_a])$
$\Box\Box$	$\frac{1}{2}([\Pi^o_{a'}] \circ [\Pi^o_a] - [\Pi_{O6}] \circ [\Pi^o_a])]$
$(\overline{\Box}_{\mathbf{a}}, \Box_{\mathbf{b}})$	$[\Pi^o_a] \circ [\Pi^o_b]$
$(\Box_{\mathbf{a}}, \Box_{\mathbf{b}})$	$[\Pi^o_{a'}] \circ [\Pi^o_b]$

XI.

C. Consistency and SUSY conditions

Certain conditions must be applied to construct consistent, supersymmetric vacua which are free of anomalies, which we discuss in the following sections.

1. RR and Torsion Charge Cancellation

With the choice of discrete torsion $q_{\Omega R} = -1$, $q_{\Omega R\theta} = q_{\Omega R\omega} = q_{\Omega R\theta\omega} = 1$, the condition for the cancellation of RR tadpoles becomes

$$\sum N_a n^1_a n^2_a n^3_a = -16, \quad \sum N_a m^1_a m^2_a n^3_a = -16, \tag{5.18}$$
$$\sum N_a m^1_a n^2_a m^3_a = -16, \quad \sum N_a n^1_a m^2_a m^3_a = -16.$$

whilst for the twisted charges to cancel, we require

$$\sum_{a,ij\in S^{\omega}} N_a n_a^1 \epsilon^{\omega}_{ij,a} = 0, \qquad \sum_{a,jk\in S^{\theta\omega}} N_a n_a^2 \epsilon^{\theta\omega}_{jk,a} = 0, \qquad \sum_{a,ki\in S^{\theta}} N_a n_a^3 \epsilon^{\theta}_{ki,a} = 0. \tag{5.19}$$

where the sum is over *each* each fixed point $[e^g_{ij}]$. As stated in Section 2, the signs $\epsilon^{\theta}_{ij,a}$, $\epsilon^{\omega}_{jk,a}$, and $\epsilon^{\theta\omega}_{ki,a}$ are not arbitrary as they must satisfy certain consisitency conditions. In particular, they must satisfy the condition

$$\sum_{ij\in S^g} \epsilon^g_{a,ij} = 0 \mod 4 \tag{5.20}$$

for each twisted sector. Additionally, the signs for different twisted sectors must satisfy

$$\begin{aligned} \epsilon^{\theta}_{a,ij}\epsilon^{\omega}_{a,jk}\epsilon^{\theta\omega}_{a,ik} &= 1, \\ \epsilon^{\theta}_{a,ij}\epsilon^{\omega}_{a,jk} &= \text{constant } \forall\, j. \end{aligned} \tag{5.21}$$

Note that we may choose the set of signs differently for each stack provided that they satisfy the consistency conditions. A trivial choice of signs which satisfies the constraints placed on them is just to have them all set to +1,

$$\epsilon^{\theta}_{a,ij} = 1\ \forall\ ij, \qquad \epsilon^{\omega}_{a,jk} = 1\ \forall\ jk, \qquad \epsilon^{\theta\omega}_{a,ki} = 1\ \forall\ ki. \tag{5.22}$$

Another possible non-trivial choice of signs consistent with the constraints is given by

$$\epsilon^{\theta}_{a,ij} = -1\ \forall\ ij, \qquad \epsilon^{\omega}_{a,jk} = -1\ \forall\ jk, \qquad \epsilon^{\theta\omega}_{a,ki} = 1\ \forall\ ki. \tag{5.23}$$

Note that a fractional three cycle is invariant under the transformations

$$\begin{aligned} (n_a^1, m_a^1) \otimes (n_a^2, m_a^2) \otimes (n_a^3, m_a^3) &\Rightarrow (-n_a^1, -m_a^1) \otimes (n_a^2, m_a^2) \otimes (-n_a^3, -m_a^3), \\ \epsilon^{\theta}_{a,ij} &\Rightarrow -\epsilon^{\theta}_{a,ij}, \\ \epsilon^{\omega}_{a,jk} &\Rightarrow -\epsilon^{\omega}_{a,jk}. \end{aligned} \tag{5.24}$$

More general sets of these signs may be found in [132].

2. Conditions for Preserving $N = 1$ Supersymmetry

The condition to preserve $N = 1$ supersymmetry in four dimensions is that the rotation angle of any D-brane with respect to the orientifold plane is an element of $SU(3)$ [16, 42]. Essentially, this becomes a constraint on the angles made by each stack of branes with respect to the orientifold planes, *viz* $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \bmod 2\pi$, or equivalently $\sin(\theta_a^1 + \theta_a^2 + \theta_a^3) = 0$ and $\cos(\theta_a^1 + \theta_a^2 + \theta_a^3) = 1$. Applying simple trigonometry, these angles may be expressed in terms of the wrapping numbers as

$$\tan \theta_a^i = \frac{m_a^i R_2^i}{n_a^i R_1^i} \tag{5.25}$$

where R_2^i and R_1^i are the radii of the i^{th} torus. We may translate these conditions into restrictions on the wrapping numbers as

$$\begin{aligned} x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0 \\ A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0 \end{aligned} \tag{5.26}$$

where we have made the definitions

$$\tilde{A}_a = -m_a^1 m_a^2 m_a^3, \quad \tilde{B}_a = n_a^1 n_a^2 m_a^3, \quad \tilde{C}_a = m_a^1 n_a^2 n_a^3, \quad \tilde{D}_a = n_a^1 m_a^2 n_a^3, \tag{5.27}$$

$$A_a = -n_a^1 n_a^2 n_a^3, \quad B_a = m_a^1 m_a^1 n_a^3, \quad C_a = n_a^1 m_a^1 m_a^3, \quad D_a = m_a^1 n_a^1 m_a^3. \tag{5.28}$$

and the structure parameters related to the complex structure moduli are

$$x_a = \lambda, \quad x_b = \frac{\lambda}{\chi_2 \cdot \chi_3}, \quad x_c = \frac{\lambda}{\chi_1 \cdot \chi_3}, \quad \frac{\lambda}{\chi_1 \cdot \chi_2}. \tag{5.29}$$

where λ is a positive constant. One may invert the above expressions to find values for the complex structure moduli as

$$\chi_1 = \lambda, \quad \chi_2 = \frac{x_c}{x_b} \cdot \chi_1, \quad \chi_3 = \frac{x_d}{x_b} \cdot \chi_1. \tag{5.30}$$

3. The Green-Schwarz Mechanism

Although the total non-Abelian anomaly cancels automatically when the RR-tadpole conditions are satisfied, additional mixed anomalies like the mixed gravitational anomalies which generate massive fields are not trivially zero [42]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism which involves untwisted Ramond-Ramond forms. Integrating the G-S couplings of the untwisted RR forms to the $U(1)$ field strength F_a over the untwisted cycles of $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}'_\mathbf{2})$ orientifold, we find

$$\int_{D6_a^{untw}} C_5 \wedge \mathrm{tr} F_a \sim N_a \sum_i r_{ai} \int_{M_4} B_2^i \wedge \mathrm{tr} F_a, \tag{5.31}$$

where

$$B_2^i = \int_{[\Sigma_i]} C_5, \quad [\Pi_a] = \sum_{i=1}^{b_3} r_{ai} [\Sigma_i], \tag{5.32}$$

and $[\Sigma_i]$ is the basis of homology 3-cycles, $b_3 = 8$. Under orientifold action only half survive. In other words, $\{r_{ai}\} = \{\tilde{B}_a, \tilde{C}_a, \tilde{D}_a, \tilde{A}_a\}$ in this definition. Thus the couplings of the four untwisted RR forms B_2^i to the $U(1)$ field strength F_a are [38]

$$\begin{aligned} &N_a \tilde{B}_a \int_{M_4} B_2^1 \wedge \mathrm{tr} F_a, \qquad N_a \tilde{C}_a \int_{M_4} B_2^2 \wedge \mathrm{tr} F_a, \\ &N_a \tilde{D}_a \int_{M_4} B_2^3 \wedge \mathrm{tr} F_a, \qquad N_a \tilde{A}_a \int_{M_4} B_2^4 \wedge \mathrm{tr} F_a. \end{aligned} \tag{5.33}$$

Besides the contribution to G-S mechanism from untwisted 3-cycles, the contribution from the twisted cycles should be taken into account. As in the untwisted case we integrate the Chern-Simons coupling over the exceptional 3-cycles from the twisted sector. We choose the sizes of the 2-cycles on the topology of S^2 on the orbifold singularities to make the integrals on equal foot to those from the untwisted sector. Consider the twisted sector θ as an example,

$$\int_{D6_a^{tw,\theta}} C_5 \wedge \mathrm{tr} F_a \sim N_a \sum_{i,j \in S_\theta^a} \epsilon_{a,ij}^{\theta} m_a^3 \int_{M_4} B_2^{\theta ij} \wedge \mathrm{tr} F_a. \tag{5.34}$$

where $B_2^{\theta ij} = \int_{[\alpha_{ij,m}^{\theta}]} C_5$, with orientifold action taken again. Although i, j can run through each run through $\{1-4\}$ for each of the four fixed points in each sector, these are constrained by the wrapping numbers from the untwisted sector so that only four possibilities remain. A similar argument may be applied for ω and $\theta\omega$ twisted sectors:

$$\int_{D6_a^{tw,\omega}} C_5 \wedge \mathrm{tr} F_a \sim N_a \sum_{j,k \in S_\omega^a} \epsilon_{a,jk}^{\omega} m_a^1 \int_{M_4} B_2^{\omega jk} \wedge \mathrm{tr} F_a. \tag{5.35}$$

$$\int_{D6_a^{tw,\theta\omega}} C_5 \wedge \mathrm{tr} F_a \sim N_a \sum_{i,j \in S_{\theta\omega}^a} \epsilon_{a,ik}^{\theta\omega} m_a^2 \int_{M_4} B_2^{\theta\omega ik} \wedge \mathrm{tr} F_a. \tag{5.36}$$

In summary, there are twelve additional couplings of the Ramond-Ramond 2-forms B_2^i to the $U(1)$ field strength F_a from the twisted cycles, giving rise to massive $U(1)$'s. However from the consistency condition of the ϵ's (see section 3.1) related to the discrete Wilson lines they may be dependent or degenerate. So even including the couplings from the untwisted sector we still have an opportunity to find a linear combination for a massless $U(1)$ group. Let us write down these couplings of the twisted sector explcitly:

$$N_a\epsilon^{\theta}_{a,ij}m^3_a\int_{M_4}B^{\theta ij}_2\wedge \mathrm{tr}F_a, \quad N_a\epsilon^{\omega}_{a,jk}m^1_a\int_{M_4}B^{\omega jk}_2\wedge \mathrm{tr}F_a,$$
$$N_a\epsilon^{\theta\omega}_{a,ik}m^2_a\int_{M_4}B^{\theta\omega ik}_2\wedge \mathrm{tr}F_a. \tag{5.37}$$

Checking the mixed cubic anomaly by introducing the dual field of B^i_2 in the diagram, we can find the contribution from both untwisted and twisted sectors having a intersection number form and which is cancelled by the RR-tadpole conditions mentioned. These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G-S mechanism. Thus, in constructing MSSM-like models, we must ensure that the gauge boson of the hypercharge $U(1)_Y$ group does not receive such a mass. In general, the hypercharge is a linear combination of the various $U(1)$s generated from each stack :

$$U(1)_Y=\sum_a c_a U(1)_a \tag{5.38}$$

The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand

$$\sum_a c_aN_a\epsilon^{\omega}_{a,jk}m^1_a = 0, \quad \sum_a c_aN_a\epsilon^{\theta\omega}_{a,ki}m^2_a=0, \quad \sum_a c_aN_a\epsilon^{\theta}_{a,ij}m^3_a=0, \tag{5.39}$$

for the twisted couplings as well as

$$\sum_a c_aN_a\tilde{A}_a = 0, \quad \sum_a c_aN_a\tilde{B}_a=0, \quad \sum_a c_aN_a\tilde{C}_a=0, \quad \sum_a c_aN_a\tilde{D}_a=0, \tag{5.40}$$

for the untwisted.

4. K-Theory Constraints

RR charges are not fully classified by homological data, but rather by K-theory. Thus, to cancel all charges including those visible by K-theory alone, we require the wrapping numbers to satisfy certain constraints. We will not state these constraints here, but we will refer the reader to [132] where they are given explicitely.

D. MSSM via Pati-Salam

We begin with the seven-stack configuration of D-branes with the bulk wrapping numbers shown in Table XII, which produce the intersection numbers shown in Tables XIII and XIV. We make the choice of cross-cap charges $q_{\Omega R} = -1$, $q_{\Omega R\theta} = q_{\Omega R\omega} = q_{\Omega R\theta\omega} = 1$, and assume for simplicity that each stack passes throught the same set of fixed points. The resulting gauge group is that of a four generation Pati-Salam model. The 'observable' matter spectrum is presented in Table XV.

For Pati-Salam models constructed from bulk D-branes wrapping non-rigid cycles, the gauge symmetry may be broken to the MSSM by the process of brane splitting, which corresponds to assigning a VEV to an adjoint scalar in the field theoretic description. However, this option is not available in the present construction since the adjoint fields have been eliminated due to the rigidization of the cycles.

Although the adjoint fields have been eliminated by splitting the bulk D-branes into their fractional consituents, light non-chiral matter in the bifundamental representation may still appear between pairs of fractional branes [132]. These non-chiral states smoothly connect the configuration of fractional D-branes to one consisting of non-rigid D-branes. In the present case, all of the fractional D-branes are wrapping bulk cycles which are homologically identical, but differ in their twisted cycles. As

Table XII. Stacks, wrapping numbers, and torsion charges for a Pati-Salam model. With the choice of structure parameters $x_a = \sqrt{3}, x_b = x_c = x_d = \sqrt{3}/3$, $N = 1$ SUSY will be preserved. The cycles pass through the same set of fixed points for each stack.

Stack	N	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)	$\epsilon^{\theta}_{ij}\ \forall\ ij$	$\epsilon^{\omega}_{jk}\ \forall\ jk$	$\epsilon^{\theta\omega}_{ki}\ \forall\ kl$
α	4	(-1,-1)	(-1,-1)	(-1,-1)	-1	-1	1
β	2	(-1,-1)	(-1,-1)	(-1,-1)	1	1	1
γ	2	(1, 1)	(1, 1)	(-1,-1)	1	1	1
1	2	(1, 1)	(1, 1)	(-1,-1)	-1	-1	1
2	2	(-1,-1)	(1, 1)	(1, 1)	-1	-1	1
3	2	(1,-1)	(1,-1)	(-1, 1)	-1	-1	1
4	2	(1,-1)	(-1, 1)	(1,-1)	-1	-1	1

Table XIII. Intersection numbers between different stacks giving rise to fermions in the bifundamental representation. The result is a four-generation Pati-Salam model.

	α	β	γ	1	2	3	4	α'	β'	γ'	$1'$	$2'$	$3'$	$4'$
α	0	0	0	0	0	0	0	-8	4	-4	0	0	0	0
β	-	0	0	0	0	-4	-4	0	-8	0	0	0	0	0
γ	-	-	0	0	0	4	-4	0	0	-8	0	0	0	0
1	-	-	-	0	0	-8	0	0	-4	4	-8	0	0	0
2	-	-	-	-	0	0	0	0	-4	-4	0	-8	0	0
3	-	-	-	-	-	0	0	0	0	0	0	0	8	0
4	-	-	-	-	-	-	0	0	0	0	0	0	0	8

Table XIV. Intersection numbers between different stacks and their images giving rise to antisymmetric and symmetric representations for a Pati-Salam model.

Stack	Antisymmetric	Symmetric
α	8	0
β	8	0
γ	8	0
1	8	0
2	8	0
3	-8	0
4	-8	0

Table XV. The 'observable' spectrum of $SU(4) \times SU(2)_L \times SU(2)_R \times [U(2)^4 \times U(1)^3]$. The $\star'd$ representations indicate light, non-chiral matter which is present between pairs of fractional branes which wrap homologically identical bulk cycles, but differ in their twisted cycles.

Rep.	Multi.	$U(1)_\alpha$	$U(1)_\beta$	$U(1)_\gamma$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	Field
$(\mathbf{4}_{\alpha'}, \mathbf{2}_\gamma)$	4	1	0	1	0	0	0	0	Matter
$(\bar{\mathbf{4}}_{\alpha'}, \bar{\mathbf{2}}_\beta)$	4	-1	-1	0	0	0	0	0	Matter
$(\mathbf{2}_\beta, \bar{\mathbf{2}}_\gamma)^\star$	-	0	1	-1	0	0	0	0	EW Higgs
$(\bar{\mathbf{4}}_\alpha, \mathbf{2}_\gamma)^\star$	-	-1	0	1	0	0	0	0	GUT Higgs
$(\mathbf{4}_\alpha, \bar{\mathbf{2}}_\beta)^\star$	-	1	-1	0	0	0	0	0	GUT Higgs
$(\mathbf{6}_{\alpha'\alpha})$	8	2	0	0	0	0	0	0	-
$(\mathbf{1}_{\beta'\beta})$	8	0	2	0	0	0	0	0	$\phi_{\beta\beta}$
$(\mathbf{1}_{\gamma'\gamma})$	8	0	0	2	0	0	0	0	$\phi_{\gamma\gamma}$

discussed in [132], one may compute the overlap between two such boundary states:

$$\tilde{A}_{a_i a_j} = \int_0^\infty dl \, \langle a_i | \, e^{-2\pi l H_{cl}} \, | a_j \rangle + \int_0^\infty dl \, \langle a_j | \, e^{-2\pi l H_{cl}} \, | a_i \rangle \, . \tag{5.41}$$

Due to the different signs for the twisted sector, it is found that in the loop channel amplitude

$$A_{a_i a_j} = \int_0^\infty \frac{dl}{l} Tr_{ij+ji} \left(\frac{1+\theta+\omega+\theta\omega}{4} e^{-2\pi l H_{cl}} \right) \tag{5.42}$$

one massless hypermultiplet appears. Thus, the required states to play the role of the Higgs fields are present in this non-chiral sector.

In principle, one should determine that there are flat directions that can give the necessary VEV's to these states. This process would correspond geometrically to a particular brane recombination, where the CFT techniques fail and only a field theory analysis of D- and F-flat directions is applicable. For instance, a configuration of fractional branes in which one of these states receives a VEV should smoothly connect this configuration to one in which there is a stack of bulk D-branes wrapping a non-rigid cycle that has been split by assigning a VEV to an ajoint scalar. Such computations are technically very involved and beyond the scope of the present work, and we defer this for later work.

In Tables XVI, XVII, XVIII, and XIX we present an MSSM model which is obtained from the above Pati-Salam model by separating the stacks as

$$\alpha \to \alpha_B + \alpha_L, \qquad \beta \to \beta_{r1} + \beta_{r2}. \tag{5.43}$$

This does not mean that the stacks are located at different positions in the internal space. After-all, there are no adjoint scalars which may receive a VEV. Rather, this separation reflects that there has been a spontaneous breaking of the Pati-Salam gauge symmetry down to the MSSM by the Higgs mechanism, where we have identified the

Table XVI. Stacks, wrapping numbers, and torsion charges for an MSSM-like model. The cycles pass through the same set of fixed points for each stack.

Stack	N	(n_1, m_1)	(n_2, m_2)	(n_3, m_3)	$\epsilon^{\theta}_{ij}\ \forall\ ij$	$\epsilon^{\omega}_{jk}\ \forall\ jk$	$\epsilon^{\theta\omega}_{ki}\ \forall\ kl$
α_B	3	(-1,-1)	(-1,-1)	(-1,-1)	-1	-1	1
α_L	1	(-1,-1)	(-1,-1)	(-1,-1)	-1	-1	1
β_{r1}	1	(-1, -1)	(-1, -1)	(-1, -1)	1	1	1
β_{r2}	1	(-1, -1)	(-1, -1)	(-1, -1)	1	1	1
γ	2	(1, 1)	(1, 1)	(-1,-1)	1	1	1
1	2	(1, 1)	(1, 1)	(-1,-1)	-1	-1	1
2	2	(-1,-1)	(1, 1)	(1, 1)	-1	-1	1
3	2	(1,-1)	(1,-1)	(-1, 1)	-1	-1	1
4	2	(1,-1)	(-1, 1)	(1,-1)	-1	-1	1

Higgs states with $(\mathbf{4}, \mathbf{2}, 1)$ and $(\bar{\mathbf{4}}, 1, \mathbf{2})$ representations of $SU(4) \times SU(2)_L \times SU(2)_R$ present in the non-chiral sector. The resulting gauge group of the model is then given by $SU(3) \times SU(2)_L \times U(1)_Y \times SU(2)^4 \times U(1)^8$, and the MSSM hypercharge is found to be

$$Q_Y = \frac{1}{6}\left(U(1)_{\alpha_B} - 3U(1)_{\alpha_L} - 3U(1)_{\beta_{r1}} + 3U(1)_{\beta_{r2}}\right). \tag{5.44}$$

Of course, this is just

$$Q_Y = \frac{Q_B - Q_L}{2} + Q_{I_{3R}}, \tag{5.45}$$

where Q_B and Q_L are baryon number and lepton number respectively, while $Q_{I_{3R}}$ is like the third component of right-handed weak isospin.

As discussed, up to twelve $U(1)$ factors may obtain a mass *via* the GS mechanism.

Table XVII. Intersection numbers between different stacks giving rise to fermions in the bifundamental representation. The result is a four-generation MSSM-like model.

	α_B	α_L	β_{r1}	β_{r2}	γ	1	2	3	4	α'_B	α'_L	β'_{r1}	β'_{r2}	γ'	$1'$	$2'$	$3'$	$4'$
α_B	0	0	0	0	0	0	0	0	0	-8	-8	4	4	-4	0	0	0	0
α_L	-	0	0	0	0	0	0	0	0	0	-8	4	4	-4	0	0	0	0
β_{r1}	-	-	0	0	0	0	0	-4	-4	0	0	-8	-8	0	0	0	0	0
β_{r2}	-	-	-	0	0	0	0	-4	-4	0	0	0	-8	0	0	0	0	0
γ	-	-	-	-	0	0	0	4	-4	0	0	0	0	-8	0	0	0	0
1	-	-	-	-	-	0	0	-8	0	0	0	-4	-4	4	-8	0	0	0
2	-	-	-	-	-	-	0	0	0	0	0	-4	-4	-4	0	-8	0	0
3	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	8	0
4	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0	8

Table XVIII. Intersersection numbers between different stacks and their images giving rise to antisymmetric and symmetric representations for an MSSM-like model.

Stack	Antisymmetric	Symmetric
α_B	8	0
α_L	8	0
β_{r1}	8	0
β_{r2}	8	0
γ	8	0
1	8	0
2	8	0
3	-8	0
4	-8	0

Table XIX. The 'observable' spectrum of $[SU(3) \times SU(2)_L \times U(1)_Y] \times U(2)^4 \times U(1)^4$. The $\star' d$ representations indicate light, non-chiral matter which exist between pairs of fractional branes which wrap identical bulk cycles, but differ in their twisted cycles.

Rep.	Multi.	$U(1)_{\alpha_B}$	$U(1)_{\alpha_L}$	$U(1)_{\beta_{r1}}$	$U(1)_{\beta_{r2}}$	$U(1)_{\gamma}$	Q_Y	Field
$(\mathbf{3}_{\alpha'_B}, \mathbf{2}_{\gamma})$	4	1	0	0	0	1	1/6	Q
$(\bar{\mathbf{3}}_{\alpha'_B}, \mathbf{1}_{\beta_{r2}})$	4	-1	0	0	-1	0	-2/3	U^c
$(\bar{\mathbf{3}}_{\alpha'_B}, \mathbf{1}_{\beta_{r1}})$	4	-1	0	-1	0	0	1/3	D^c
$(\mathbf{1}_{\alpha'_L}, \mathbf{2}_{\gamma})$	4	0	1	0	0	1	-1/2	L
$(\mathbf{1}_{\alpha'_L}, \mathbf{1}_{\beta_{r1}})$	4	0	-1	-1	0	0	1	E^c
$(\mathbf{1}_{\alpha_L}, \mathbf{1}_{\beta_{r2}})$	4	0	-1	0	-1	0	0	N
$(\mathbf{1}_{\beta_{r1}}, \bar{\mathbf{2}}_{\gamma})^{\star}$	-	0	0	1	0	-1	-1/2	H_d
$(\bar{\mathbf{2}}_{\gamma}, \mathbf{1}_{\beta_{r2}})^{\star}$	-	0	0	0	1	-1	1/2	H_u
$(\mathbf{1}_{\gamma'\gamma})$	8	0	0	0	0	-2	0	$\phi_{\gamma\gamma}$
$(\mathbf{1}_{\beta'_{r1}}, \mathbf{1}_{\beta_{r2}})$	8	0	0	1	1	0	0	$\phi_{\beta_{r1r2}}$
$(\mathbf{3}_{\alpha'_B}, \mathbf{1}_{\alpha_L})$	8	1	1	0	0	0	-1/3	D_1
$(\bar{\mathbf{3}}_{\alpha'_B\alpha_B})$	8	2	0	0	0	0	1/3	D_2^c

In order for the hypercharge to remain massless, it must be orthogonal to each of these factors. In this case, there are only four due to the degeneracy of the stacks. These $U(1)$'s remain to all orders as global symmetries and are given by

$$\begin{aligned} U(1)_A = 3U(1)_{\alpha_B} - U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} + 2U(1)_\gamma - 2U(1)_1 \\ +2U(1)_2 + 2U(1)_3 + 2U(1)_4, \end{aligned} \tag{5.46}$$

$$\begin{aligned} U(1)_B = -3U(1)_{\alpha_B} + U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} + 2U(1)_\gamma + 2U(1)_1 \\ +2U(1)_2 - 2U(1)_3 + 2U(1)_4, \end{aligned}$$

$$\begin{aligned} U(1)_C = 3U(1)_{\alpha_B} - U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} - 2U(1)_\gamma + 2U(1)_1 \\ -2U(1)_2 - 2U(1)_3 + 2U(1)_4, \end{aligned}$$

$$\begin{aligned} U(1)_D = -3U(1)_{\alpha_B} + U(1)_{\alpha_L} - U(1)_{\beta_{r1}} - U(1)_{\beta_{r2}} - 2U(1)_\gamma - 2U(1)_1 \\ -2U(1)_2 + 2U(1)_3 + 2U(1)_4. \end{aligned}$$

Note that the hypercharge orthogonal to each of these $U(1)$ factors and so will remain massless. The 'observable' sector basically consists of a four-generation MSSM plus right-handed neutrinos. The rest of the spectrum primarily consists of vector-like matter, many of which are singlets under the MSSM gauge group. Using the states listed in Table XIX, we may construct all of the required MSSM Yukawa couplings,

$$W_Y = y_u H_u Q U^c + y_d H_d Q D^c + y_l H_d L E^c \tag{5.47}$$

keeping in mind that all of the MSSM fields are charged under the global symmetries defined in eqns. 45. Typically, this results in the forbidding of some if not all of the desired Yukawa couplings. In this case, all of the Yukawa couplings are allowed by the global symmetries including a trilinear Dirac mass term for neutrinos,

$$W_D = \lambda_\nu L N H_u. \tag{5.48}$$

By itself, such a term would imply neutrino masses of the order of the quarks and charged leptons. However, if in addition there exist a Majorana mass term for the right-handed neutrinos,

$$W_m = M_m N N, \tag{5.49}$$

a see-saw mechanism may be employed. Such a mass term may in principle be generated by $E2$ instanton effects [133, 134]. This mechanism may also be employed to generate a μ-term of the order of the EW scale.

In addition to the matter spectrum charged under the MSSM gauge groups and total gauge singlets, there is additional vector-like matter transforming under the 'hidden' gauge group $U(2)_1 \otimes U(2)_2 \otimes U(2)_3 \otimes U(2)_4$. By choosing appropriate flat directions, we may deform the fractional cycles wrapped by these stacks into bulk cycles such that

$$U(2)_1 \otimes U(2)_2 \to U(1); \qquad U(2)_3 \otimes U(2)_4 \to U(1). \tag{5.50}$$

Thus, matter transforming under these gauge groups becomes a total gauge singlet or becomes massive and disappears from the spectrum altogether. The remaining eight pairs of exotic color triplets present in the model resulting from the breaking $\mathbf{6} \to \mathbf{3} \oplus \bar{\mathbf{3}}$, while not truly vector-like due to their different charges under the global symmetries, may in principle become massive via instanton effects in much the same way a μ-term may be generated.

E. Gauge Coupling Unification

The MSSM predicts the unification of the three gauge couplings at an energy $\sim 2 \times 10^{16}$ GeV. In intersecting D-brane models, the gauge groups arise from different stacks of branes, and so they will not generally have the same volume in the compactified space. Thus, the gauge couplings are not automatically unified.

The low-energy $N = 1$ supergravity action is basically determined by the Kähler potential K, the superpotential W and the gauge kinetic function f. All of these functions depend on the background space moduli fields. For branes wrapping cycles not invariant under ΩR, the holomorphic gauge kinetic function for a D6 brane stack P is given by [130]

$$f_P = \frac{1}{2\pi l_s}\left[e^{\phi}\int_{\Pi_P} \mathrm{Re}(e^{-i\theta_P}\Omega_3) - i\int_{\Pi_P} C_3\right] \tag{5.51}$$

from which it follows (with $\theta_P = 0$ for $\mathbb{Z}_2 \times \mathbb{Z}_2$)

$$f_P \;=\; (n_P^1\, n_P^2\, n_P^3\, s - n_P^1\, m_P^2\, m_P^3\, u^1 - n_P^2\, m_P^1\, m_P^3\, u^2 - n_P^3\, m_P^1\, m_P^2\, u^3) \tag{5.52}$$

where u^i and s are the complex structure moduli and dilaton in the field theory basis. The gauge coupling constant associated with a stack P is given by

$$g_{D6_P}^{-2} \;=\; |\mathrm{Re}\,(f_P)|. \tag{5.53}$$

Thus, we identify the $SU(3)$ holomorphic gauge function with stack α_B, and the $SU(2)$ holomorphic gauge function with stack γ. The Q_Y holomorphic gauge function is then given by taking a linear combination of the holomorphic gauge functions from all the stacks.[4] In this way, it is found [135] that

$$f_Y = \frac{1}{6}f_{\alpha_B} + \frac{1}{2}f_{\alpha_L} + \frac{1}{2}f_{\beta_{r1}} + \frac{1}{2}f_{\beta_{r2}}. \tag{5.54}$$

Thus, it follows that the tree-level MSSM gauge couplings will be unified at the string scale

$$g_s^2 = g_w^2 = \frac{5}{3}g_Y^2 \tag{5.55}$$

since each stack will have the same gauge kinetic function.

[4]Note that we have absorbed a factor of 1/2 in the definition of Q_Y so that the electric charge is given by $Q_{em} = T_3 + Q_Y$.

CHAPTER VI

CONCLUSIONS

We have made a detailed study of crypton decays in a specific flipped SU(5) string model. We have shown that there are neutral tetrons that are naturally metastable in this string model, with lifetimes long enough to make perfect candidates for cold dark matter and possibly act as sources of UHECRs. Moreover, their charged 'cryptospin' partners naturally decay much more rapidly, with lifetimes that may be much shorter than the age of the Universe. Thus, the flipped SU(5) string model does not predict the existence of any charged cold dark matter. Time will tell whether the UHECRs are in fact due to the decays of ultraheavy particles, but the flipped SU(5) string model seems to have the appropriate characteristics for this to be possible, as well as providing possible cold dark matter candidates in the forms of its neutral tetron bound states. We believe that these properties along with the other successes of string-derived flipped SU(5) such as dynamic double-triplet splitting and natural suppression of dimension-5 operators that mediate rapid proton decay make this model particularly attractive and should strongly motivate future study.

We have carried as far as is possible at present the modelling of flipped crypton decay contributions to UHECRs, including all the possible 10^{th}-order superpotential operators. The experimental data presently available are consistent with all the decay modes possible in this crypton framework. The total UHECR spectra are consistent with a contribution from cryptons weighing between 2×10^{13} GeV and 10^{12} GeV, although only a crypton mass $M_X \geq 5 \cdot 10^{12}$ GeV would provide an unambiguous signal over conventional explanations. The available upper limits on the possible photon fraction do not exclude any of the crypton models we have studied.

In the future, the larger data set expected from Auger may be able to discriminate between crypton decays and other models of UHECRs, and also among different crypton models themselves. Greater statistics will enable the UHECR anisotropy to be measured with sufficient accuracy to discriminate crypton decay from a uniform distribution of astrophysical sources, and more accurate measurements of the photon fraction at higher energies might offer some discrimination between models with lepton and quark primaries, as seen by comparing Figs. 1 to 6 with Figs. 7 and 8 above.

Thus there is hope that, in the near future, we may finally learn whether UHECRs have a macrophysical origin or a microphysical origin and, in the latter case, may start to discriminate between different microphysical models.

In addition, we have constructed a particular $N = 1$ supersymmetric three-family model whose gauge symmetry includes $SU(5) \times U(1)_X$, from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at general angles. The spectrum contains a complete grand unified theory and electroweak Higgs sector. In addition, it contains extra exotic matter both in bi-fundamental and vector-like representations as well as two copies of matter in the symmetric representation of $SU(5)$. Chiral matter charged under both the $SU(5) \times U(1)_X$ and $USp(2)$ gauge symmetries is also present, as is evident from Table 2. Furthermore, three adjoint ($N = 1$) chiral multiplets are provided from the aa sector [84]. We also note that the low energy spectrum of the model we constructed is free from any $SU(2)$ global anomalies since the number of the corresponding fermion doublets is even [136]. Nevertheless, although the massless spectrum is free from such global anomalies it does not satisfy all the additional constraints arising from the K-theory interpretation of D-branes [137, 138]. This issue will be investigated in a future publication.

The global symmetries, that arise after the G-S anomaly cancellation mechanism, forbid some of the Yukawa couplings required for mass generation, for instance terms like FFh. However, by the same token the term HHh is also forbidden. We note that such a term is essential for the doublet-triplet splitting solution mechanism in flipped $SU(5)$. Nevertheless, it should not escape our notice that while these global $U(1)$ symmetries are exact to all orders in perturbation theory, they can be broken explicitly by non-perturbative instanton effects [76, 139]. Thus, providing us with the possibility of recovering the appropriate superpotential couplings. Another interesting approach toward generating these absent Yukawa couplings may entail the introduction of type IIB flux compactifications [140].

Then, we built flipped $SU(5)$ GUT models using D-brane constructions on a Type IIB $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with supergravity fluxes turned on. We considered both supersymmetric and non-supersymmetric fluxes in the closed string sector, and we claim that only the non-supersymmetric (soft-breaking) cases of flipped $SU(5)$ we have found are consistent with all the constraints of string theory including K-theory and supersymmetry in the open string sector.

One of he models that we have presented in contains three-generations of chiral fermions and a complete GUT and electroweak Higgs sector. It also includes extra matter such as two copies of the symmetric representation of $SU(5)$ as well as many extra bi-fundamental and vector-like representations, which result from the large D9-brane co-prime numbers $(n_a^i, m_a^i)_{D9_a}$ needed for the required compensation of the induced three-form flux contributions to the D3 RR charge.

As mentioned, the non-supersymmetric flux ($N_{flux} = 64$) in this particular flipped $SU(5)$ model breaks supersymmetry in the closed string sector. This leads to a mechanism of soft supersymmetry breaking at a mass scale $M_{soft} \sim \frac{M_{string}^2}{M_{Pl}}$ which implies an intermediate string scale or an inhomogeneous warp factor in the internal space to stabilize the electroweak scale [111, 112, 113]. With this non-supersymmetric flux present, soft supersymmetry breaking terms may be manifested in the effective action of open string fields. Detailed studies in soft-breaking mechanism and some trial investigations into the effective low energy scenario were studied in [112, 117]. Combined with a Yukawa coupling analysis [118], this may provide a clear picture of the low energy physics which we defer for future work.

The four global $U(1)$ symmetries from the G-S anomaly cancellation forbid all the Yukawa couplings necessary for the generation of quark and lepton masses, although if we ignore these global $U(1)$ factors and focus only on the $U(1)_X$ and $U(1)_Y$ symmetries, then we find that all of the required Yukawa couplings in (4.30) are present, as well as those needed for making the extra matter in the model obtain mass $\mathcal{O}(M_{string})$. We need to keep in mind that global $U(1)$ symmetries are valid to all orders in perturbation theory, and can be broken by non-perturbative instanton effects [141]. To solve this problem without these instanton effects, one possibility one may entertain is to use singlets, suitably charged, to trigger spontaneous breaking of global $U(1)$s as well as of the local $U(1)_Y$ at the string scale, while leaving $U(1)_X$ intact. In the case of global $U(1)$s one may hope that we will end up with invisible axion-like bosons. The interested reader may check from Table X that such singlets with appropriate charges do exist. Another possibility is that we may need a new D-brane configuration. It has been recently shown that the RR, NSNS and metric fluxes could contribute negative D6-brane charges in the Type IIA orientifold with flux compactifications, and thus relax the RR tadpole cancellation conditions [48], which is a good basis for future work as well as providing a solution to the problem of finding a compatible set of global $U(1)$s on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with the Yukawa couplings.

We also constructed an intersecting D-brane model on the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold background, also known as the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold with discrete torsion, where the D-branes wrap rigid cycles, thus eliminating the extra adjoint matter. The model constructed is a supersymmetric four generation MSSM-like model obtained from a spontaneously broken Pati-Salam, with a minimum of extra matter. All of the required Yukawa couplings are allowed by global symmetries which arise via a generalized Green-Schwarz mechanism. In addition, we find that the tree-level gauge couplings are unified at the string scale with a canonical normalization.

The main drawback of this model is that there are four generations of MSSM matter. However, the existence of a possible fourth generation is rather tightly constrained, although it is not completely ruled out. Of course, the actual fermion masses await a detailed analysis of the Yukawa couplings. The emergence of three light generations may in fact be correlated with the existence of three twisted sectors. If there turns out to be a fourth generation, then it would almost certainly be discovered at LHC within the next few years. Another interesting possibility is that the presence of discrete torsion will complexify the Yukawa couplings and thereby introduce CP violation into the CKM matrix [142]. Clearly, there is much work to be done to work out the detailed phenomenology of this model and we plan to return to this topic in the near future. With the LHC era just around the corner, it would be nice to have testable string models in hand.

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