AN EXPERIMENTAL VERIFICATION
OF
NUMERICAL WAVE MODELING

by

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ABSTRACT

The classic water-wave theories approximate waveforms and associated motions under certain boundary conditions. Only Airy linear and Stokes' second and third-order wave theories are capable of accurately describing the range of waves produced in the Maritime Systems Engineering laboratory wave tank. After a suitable wave measurement system is employed, data is generated which can be compared to theoretically predicted values. Computer analysis reveals that smaller waves with less inclined faces tend to be non-symmetrical, short and deformed. Larger, steeper-faced waves compare more favorably to the appropriate theory, although they still exhibit slight abnormalities. Water profile quality and overall wave size range in the test tank could be improved with smoother, less disturbing wave generation.

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1 INTRODUCTION

1.1 Purpose of Research

A numerical wave model is one of several mathematical representations of disturbance-induced, oscillating fluids. These theoretical creations predict various physical properties of water-waves, some of significant engineering interest. The problems associated with these wave theories stem from the fact that most natural water-wave phenomena are complex and difficult to describe. Especially when applied to the ocean, where the surface is dominated by waves whose motion is three dimensional, nonlinear and seemingly chaotic. Accurate representation of such behavior is difficult, if not impossible, in mathematical terms. Many theoretical concepts have evolved in the past two centuries for describing complex sea waves; however, complete agreement between theory and observation is not always found.

This research report compares free surface water-wave profiles predicted by the classic wave theories to measurements of wave contours generated in the Maritime Systems Engineering laboratory wave tank. The utility of the wave tank facility as a reliable engineering simulation tool is examined. The causes of error are investigated where reasonable agreement between empirical and theoretical results is not found. In such cases, it is determined whether the applied wave theory is inaccurate for the given situation, or if the source of disagreement lies within the measuring system and wave generation equipment.

1.2 History & Background

The primary numerical models which predict water-wave characteristics can be examined in order of their present usage. Theoretical development has not necessarily progressed with increasing complexity. Therefore, a potentially akward chronological listing will be avoided.

The most fundamental and elementary of the wave theories, refered to as linear wave theory or small-amplitude theory, was developed by Airy (1845). This theory's importance and popularity can be attributed to its basic simplicity and reliability over a large part of the whole wave regime. The linear wave theory is a first approximation of a total theoretical description of water-wave properties.

Stokes (1880) developed a more complete wave theory that better represents water-wave behavior in some situations. Stokes' theory is the sum of an infinite number of successive approximations, where each additional term in the series is a correction to the preceding terms. The solution of practical engineering problems rarely justifies the use of Stokes' theory beyond the fourth-order. Although some avid theorists prefer to work with eighth-order Stokes' equations.

Cnoidal wave theory was developed for use in shallow-water regions by Korteweg and De Vries (1895). Under certain conditions, it predicts waveforms and associated motions well. However, cnoidal wave theory is not popular with those who deal in practical engineering problems. This may be due to computational difficulties associated with the cnoidal theory. Masch and Wiegel (1961) have tabulated many of the cnoidal wave theory functions which has reduced the work involved considerably. However, values in graphical and tabular form aren't computer compatible.

In recent years, numerical approximations to solutions of the hydrodynamic equations describing wave motion have been proposed and developed by Dean (1965) and Monkmeyer (1970). This approach, termed a symmetric, stream function theory, is a nonlinear wave theory which is similar to higher order Stokes' theories. Stream function theory determines the coefficient of each higher order term so that a least-squares best fit is obtained.

Russell (1838) first recognized the existence of a wave which was neither oscillatory nor did it exhibit a trough. This phenomenon was originally predicted by Boussinesq (1872) in the solitary wave theory. It is difficult to form a truly solitary wave in nature. Long waves such as tsumanis sometimes behave approximatley like solitary waves. Even though solitary wave theory can describe some oscillatory waves as they move into shallow water, the overall applicability of this theory is limited.

1.3 Research Motivation

This research has been prompted by the urgent need for an evaluation of the water-wave simulation facilities in the Maritime Systems Engineering Department. The present operating condition of the laboratory wave tank equipment is needed for repair and improvement considerations which influence short-term departmental funding. The overall simulation accuracy of this facility is necessary for the long-term if a higher quality water-wave reproduction system is found to be required.

This project will hopefully stimulate increased use of this d wave tank facility. Before my research effort began in September of 1982, no serious experimental use of this equipment had been endeavored. The benefits of this facility are obvious. The comprehension of theory presented in the classroom is significanlty improved when the discussed phenomenon is visualized. A physical reconstruction of natural oceanic events is also highly motivational to undergraduate engineers who are prone to question the practicality of what they are presented in lecture.

1.4 Intended Audience

This research report is targeted to the reader who is familar with fluid mechanics, hydrodynamics and electronic instrumentation. Some technical terms are defined where needed, and discussion of data is illuminated through the use of graphics. Since this research is oriented towards the solution of practical engineering problems, the derivation of the selected wave theories is omitted. Only the resultant equations are given and discussion is focused upon their appropriate application. If detailed development is desired, reference should be made to the works of Wiegel (1964), Kinsman (1965) and Ippen (1966).

1.5 Major Topics of Research

The major accomplishments of this research are as follows:

1. Wave theory predictions
    a. Study of Airy linear wave theory
    b. Study of Stokes' second-order wave theory
    c. Study of Stokes' third-order wave theory

d. Generation of theoretical approximations by computer

2. Wave tank measurements

a. Examination of basic measuring systems

b. Identification of available laboratory equipment

c. Proposal of water-wave measuring system

d. Design and construction of wave gauge

e. Calibration of water-wave measuring system

f. Measurement of free surface wave profiles

3. Comparison and analysis of theoretical and empirical data

2. NUMERICAL WATER-WAVE MODELS

2.1 Objective

This section will:

1. Ascertain the general class of waves that will be produced in the laboratory.

3. Select the numerical wave theories mentioned in the introduction that will describe these laboratory waves.

3. Examine the equations derived from these theories and determine if they are suitable for computer use. If not, use the numerical analyses techniques necessary to make them compatible.

4. Construct a computer program for each applicable wave theory that will provide the approximations that will be used for comparison.

2.2 Selection of Applicable Theories

The purpose of this research is to compare theoretical approximations to experimentally generated data. The available laboratory equipment is not capable of producing all of the various types of waves previously mentioned. Therefore, a preliminary decision

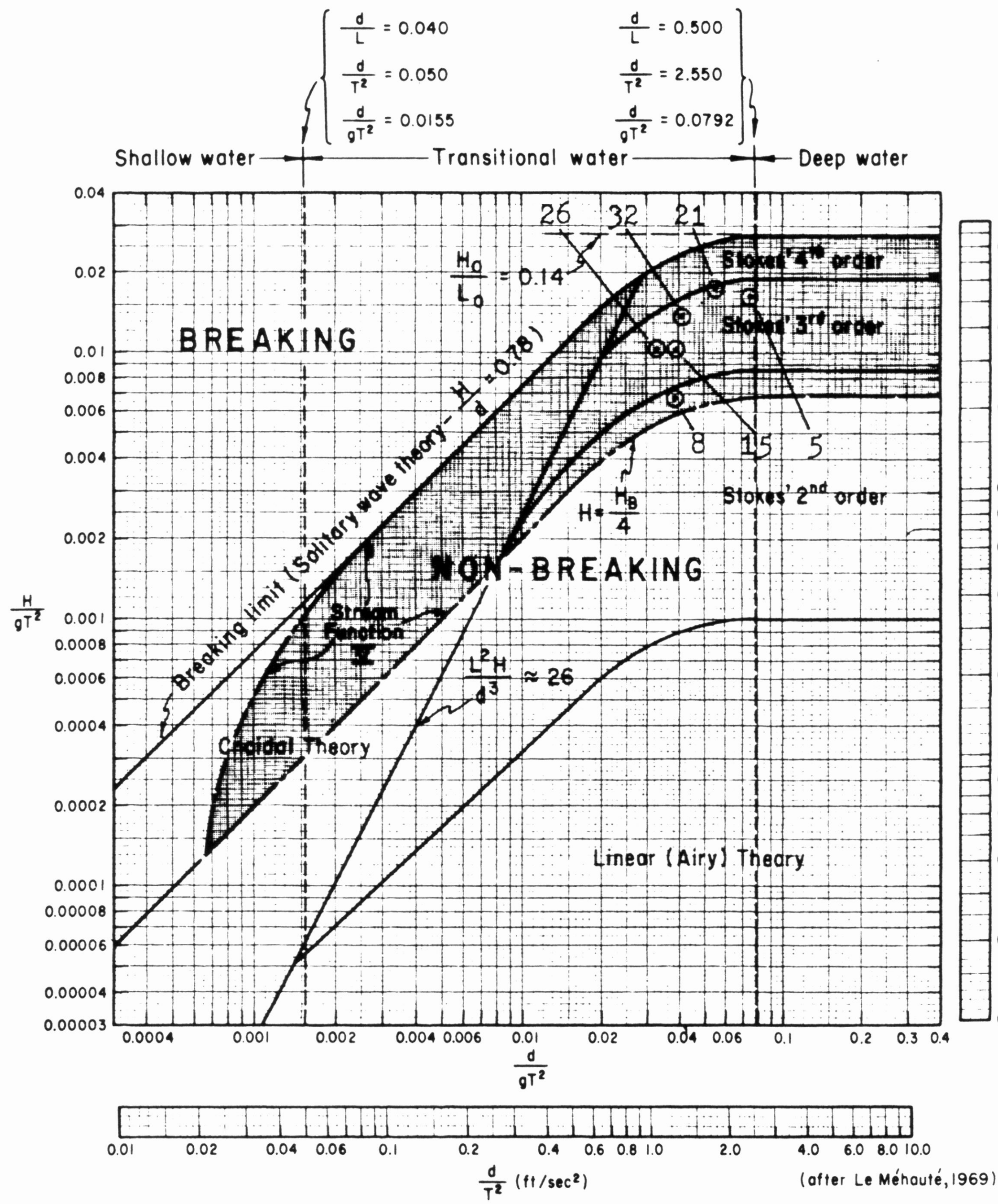

Figure 1. Regions of validity for various wave theories.

of the appropriate theories must be made. Figure 1 can be used as a guide in selecting the applicable wave theory for known values of wave height H, water depth d, and wave period T. The initial values of height, depth and period are estimated by a visual technique. Wave periods are observed with the aid of a stopwatch. The period's accuracy is improved when its value is averaged over a series of waves. A wave height estimation is made by sketching two parallel lines, one near the trough's level and one near the crest, then adjusting their position by observing passing waves. Water depth is easily measured with a rule. The initial estimates are as follows:

Wave height (H) = 3.2 in (maximum)

Water depth (d) = 10.0 in

Wave period (t) = 0.68 s (minimum)

Refering to Figure 1, the maximum vertical parameter which can be produced in the laboratory is

$$\frac{H}{gT^2} = 0.0179$$

The minimum dimensionless horizontal parameter proves to be

$$\frac{d}{gT^2} = 0.0560$$

Locating these values on Figure 1 indicates that Stokes' third-order wave theory is the highest order approximation that must be dealt with. Therefore, Stokes' third-order equations and boundary conditions will be examined. Stokes' second-order and Airy linear approximations will also be studied to provide the necessary background and may prove useful in analyzing the experimental results.

2.3 Airy first-order, progressive wave theory

The simplest of the numerical wave theories is known as the elementary progressive wave theory, the small-amplitude theory or the Airy theory. It describes the water surface profile of a wave with a single sine or cosine function.

Figure 2 illustrates a two-dimensional, small-amplitude wave moving in the positive x-direction . The variable n denotes the vertical distance (positive or negative) of the water surface relative to still water level SWL. The wavelength L is the horizontal distance between corresponding points on two sucessive waves. The height H is the vertical distance from the crest to the preceding trough. The wave's amplitude a is equal to one-half of the height. Finally, the depth d is the distance from the bed to still water level.

The speed at which a wave form propogates is termed the wave celerity C. Since the distance a wave travels in one period is equal to one wavelength, an expression of wave celerity can be shown as

$$C = \frac{L}{T}$$

Wave celerity can be written as a function of wavelength, water depth and wave period. This relation can be written as

$$C = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$

From equations 3-3 and 3-4, an expression for wavelength in terms of depth and wave period can be derived.

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$

Equation 3-5 is a transcendental relation with unkwon wavelength L appearing on both sides of the equality sign. Wiegel (1948) tabulated values which simplifies the solution of this equation. Wiegel's tables are of little use in modern computer programming (Lighthill, 1978). The method of dealing with equation 3-5 evolved into one of a number of

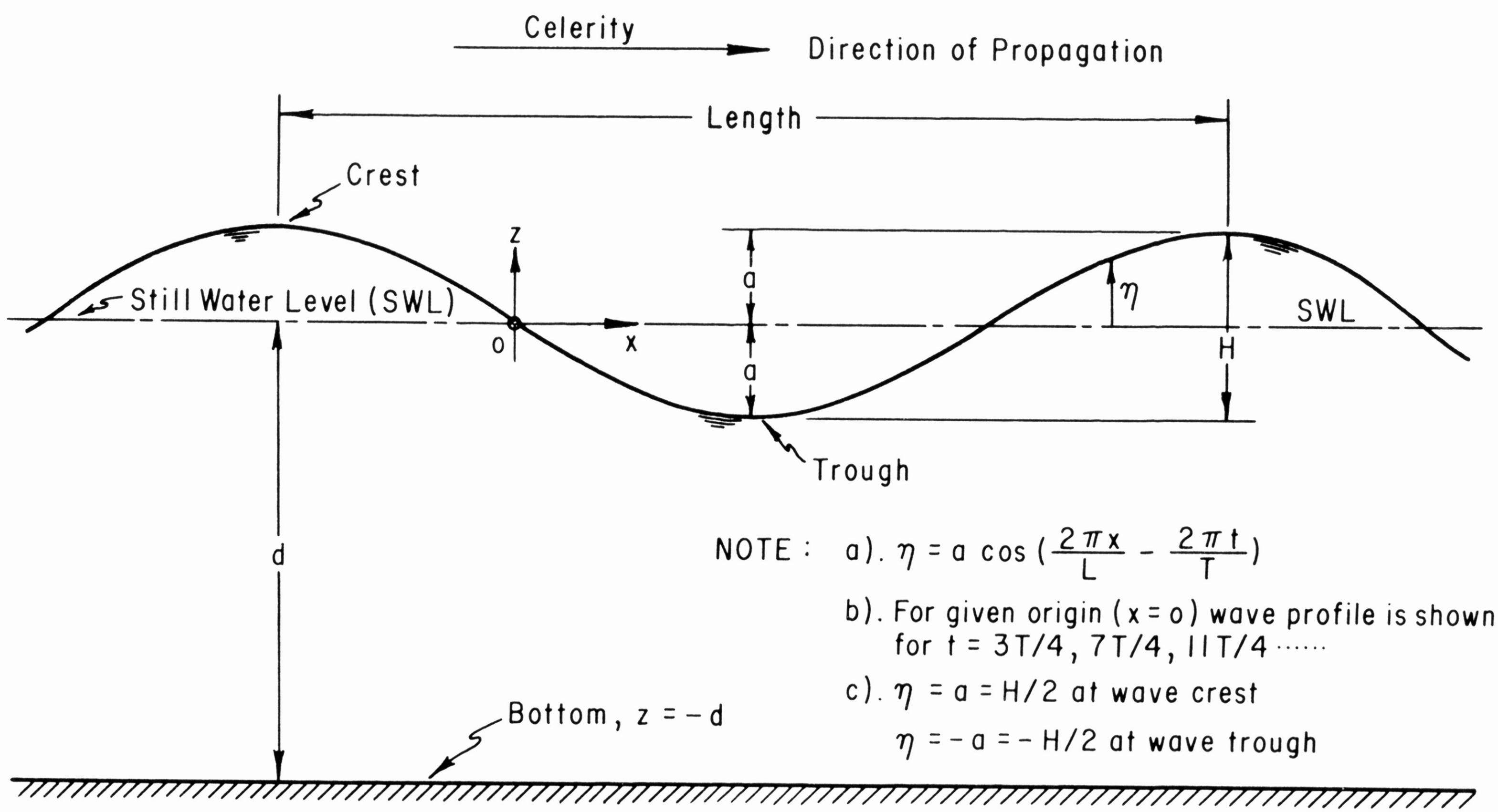

Figure 2. Definition of terms for Airy linear waves (U.S. Army Coastal Engineering Research Center, 1977).

numerical analysis techniques requiring iterative solutions. The derivations as well as the examples of several of these techniques can be found in a work by Hamming (1973).

Hunt (1979) has devised a direct method for calculating wavelength which significanlty reduces expensive computer time. Hunt uses a Pade approximation in which

$$L \cong T\sqrt{\frac{gd}{F}}$$

when

$$F = G + \frac{1}{1.0 + .6522G + .4622G^2 + .0675G^4 + .0864G^3}$$

and

$$G = \left(\frac{2\pi}{T}\right)^2\left(\frac{d}{g}\right)$$

The wavelength obtained using the approximation has an error on the order of 0.1 to 0.2 percent. More than accurate enough for most practical purposes.

The profile of the free surface of a simple progressive wave traveling in the positive x-direction can be written as a function of time t and horizontal distance x.

$$\eta = a \cos\theta$$

Where the phase angle θ can be written as

$$\theta = \left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

Airy linear equations for local fluid velocities and accelerations, fluid particle displacements and subsurface pressure are shown in Figure 3. Because of limitations of time and experimental equipment, an attempt to verify these relations is not made. However, theoretical values for these water-wave characteristics can be easily calculated with the information provided.

The small-amplitude theory is divided into regions of relative depth, allowing for some equation reduction. This is done to quicken

RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave Profile	Same As →	$\eta = \frac{H}{2} \cos \left[\frac{2\pi x}{L} - \frac{2\pi t}{T} \right] = \frac{H}{2} \cos \theta$	Same As ←
2. Wave Celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \text{ tann} \left(\frac{2\pi d}{L} \right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wave Length	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group Velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2} \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right] \cdot C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water Particle Velocity a) Horizontal	$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \cos \theta$
b) Vertical	$w = \frac{H\pi}{T} \left(1 + \frac{z}{d}\right) \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \sin \theta$
6. Water Particle Accelerations a) Horizontal	$a_x = \frac{H\pi}{T} \sqrt{\frac{g}{d}} \sin \theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$a_x = 2H \left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \sin \theta$
b) Vertical	$a_z = -2H \left(\frac{\pi}{T}\right)^2 \left(1 + \frac{z}{d}\right) \cos \theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$a_z = -2H \left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \cos \theta$
7. Water Particle Displacements a) Horizontal	$\xi = -\frac{HT}{4\pi} \sqrt{\frac{g}{d}} \sin \theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin \theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin \theta$
b) Vertical	$\zeta = \frac{H}{2} \left(1 + \frac{z}{d}\right) \cos \theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos \theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos \theta$
8. Subsurface Pressure	$p = \rho g (\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

Figure 3. Summary of Airy linear wave theory characteristics (U.S. Army Coastal Engineering Research Center, 1977).

hand-calculations, but is not necessary when working with high-speed computers.

The end product of this examination of the Airy linear theory is a computer program which provides surface profile values that can be compared to experimentally obtained data. This program, which utilizes the Hunt (1979) method for calculating wavelength, is listed in appendix A.

2.4 Stokes' finite-amplitude theory

2.41 General

Stokes (1880) presented a finite-amplitude theory which described wave behavior more completely than any approximation up to that time. It consists of an infinite number of successive approximations, where each additional term in the series is a correction to the preceding terms. The general expression for the free surface profile according to the Stokes' finite-amplitude theory is

$$\begin{aligned} n = {} & a\cos\theta + a^2 B_2(L,d)\cos 2\theta \\ & + a^3 B_3(L,d)\cos 3\theta + \ldots + a^n B_n(L,d)\cos(n\theta) \end{aligned}$$

The order of approximation is determined by the highest order term of the series considered.

2.42 Stokes' second-order wave theory

The Stokes' finite-amplitude theory which considers the first two terms on the right side of equation 3-11 is examined here. Recall, Figure 1 suggests that Stokes' second-order theory approximates steeper waves than the Airy linear theory in deep water. Oddly enough, it can be shown that the second-order equations for wave celerity and wavelength are identical to the relations given by the first-order Airy approximation.

$$C = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$

Therefore, the Hunt (1979) wavelength calculation method applies to second-order Stokes' as well.

Unlike the wavelength and celerity, the first and second-order equations for the free surface profiles differ. The Stokes' second-order equation contains an additional term.

$$n = a\cos\Theta + \left(\frac{\pi H^2}{8L}\right)\frac{\cosh(2\pi d/L)}{\sinh(2\pi d/L)}\left[2 + \cosh\left(\frac{4\pi d}{L}\right)\right]\cos 2\Theta$$

Additional numerical methods, beyond the Pade approximation, are not required for computer compatibility. The Stokes' second-order theory computer program is listed in appendix B.

2.43 Stokes' third-order wave theory

Third-order Stokes' considers the first three terms on the right side of equation 3-11. This theory predicts waves with steepness beyond the second-order range while still in deep water.

The equations for the third-order approximations of wave celerity and wavelength contain two more terms than the Airy first-order theory. The expression for Stokes' third-order wavelength is given by

$$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)\left[1 + \left(\frac{\pi H}{L}\right)^2\left(\frac{5 + 2\cosh(4\pi d/L) + 2\cosh^2(4\pi d/L)}{8\sinh^4(2\pi d/L)}\right)\right]$$

Notice that this equation is also transcendental. An iterative solution of this formula is required since a direct method cannot be found. Hopefully work towards such a method is underway. A direct solution would prove to be quite time-saving.

The equation for the free water suface, coorected to the third-order, is given by

$$\eta = a\cos\theta + \frac{\pi a^2}{L} f_2\left(\frac{d}{L}\right)\cos 2\theta + \frac{\pi^2 a^3}{L^2} f_3\left(\frac{d}{L}\right)\cos 6\theta$$

when

$$f_2\left(\frac{d}{L}\right) = \frac{\left(2 + \cosh\frac{4\pi d}{L}\right)\cosh\frac{2\pi d}{L}}{2\sinh^3\left(\frac{2\pi d}{L}\right)}$$

and

$$f_3\left(\frac{d}{L}\right) = \frac{3}{16}\,\frac{\left(1 + 8\cosh^6(2\pi d/L)\right)}{\sinh^6(2\pi d/L)}$$

The third-order expression relating the wave height to the amplitude adds further complication. Unlike first and second-order equations, where the amplitude is simply one-half the wave height, the Stokes' third-order equation is

$$H = 2a + 2\frac{\pi^2}{L^3}a^3 f_3\left(\frac{d}{L}\right)$$

The solution of equation 3-17 will also require repetitiious compuatations as well as a wavelength value corrected to the third-order. The Stokes' third-order wave theory computer program is listed in appendix C.

3. EXPERIMENTAL VERIFICATION

3.1 Objective

This section will

1. Examine basic measuring systems.
2. Identify and list the available laboratory equipment.
3. Propose a measuring system, while accounting for equipment and facility limitations, that will be capable of sensing changes in water-surface level with time.
4. Design and construct a transducer and a wave gauge that will be able to react to the range of waves produced in the laboratory.
5. Calibrate the assembled measuring system.

3.2 Basic measuring system

Any electronic measuring system has a minimum of four components as shown in Figure 4. These components include:

1. The transducer - converts a physical quantity into an electrical signal.
2. The signal conditioner - converts the transducer output into a form acceptable to the display device.
3. The power supply - provides the necessary electrical power for the transducer, signal conditioner and the display device.
4. The display - visualizes the desired information about the physical quantity.

3.3 Available laboratory equipment and facilities

Before any measuring system is considered, the existing experimental tools and facilities need to be noted. The dimensions and limitations of the available equipment will determine the general type and necessary complexity of the wave measuring system. The available

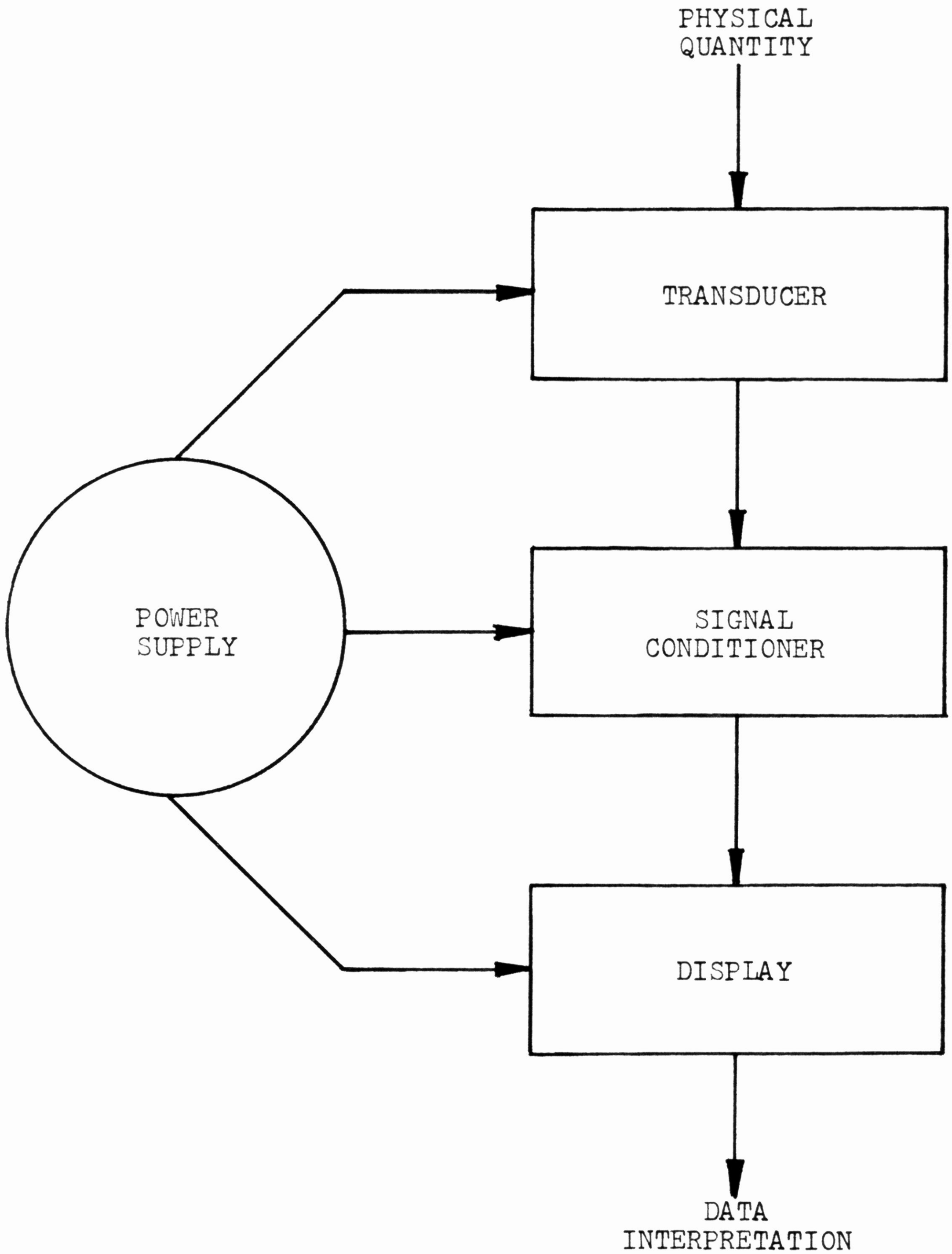

Figure 4. Basic electronic measuring system.

facilities and equipment in the Maritime Systems Engineering laboratory include:

1. A water-wave flume - consisting of a plunger-type wave generator and an inclined wave reflection supressor on the downstream end. The dimensions are (24 ft(length) x 14.5 in(depth) x 16 in(width)).

2. Three multimeters - capable of measuring DC and AC(rms) voltage, current and resistance.

3. An AC signal generator - produces either sine, square or delta waves at variable frequency (0-100kHz) and variable amplitude (15v p-p maximum).

4. Three oscilloscopes.

5. A four-channel oscillographic recorder - contains its own excitation source (DC preamplifiers) with variable chart speed.

6. A microprocessor system - consisting of the main microprocessor unit with accompanying CRT, printer and an analog/digital converter.

3.4 Initial system proposal

With the available equipment in mind, an initial proposal for the water-wave measuring system is made as shown in Figure 5. A transducer will be placed into the wave tank to produce an electrical n output at some relation to the water elevation. The AC siganl generator will be in-circuit to excite the transducer. A resistor R(c) will be placed in the circuit to insure varying output voltage, not a constant AC signal from the generator. The microprocessor accepts only positive DC voltage. Therefore, the transducer circuit's alternating current will enter an AC/DC converter to produce the suitable voltage. An analog/digital converter will then transform the continous (analog) DC signal into an on-off type (digital) signal as required by the microprocessor. The microprocessor refers this variable DC voltage to

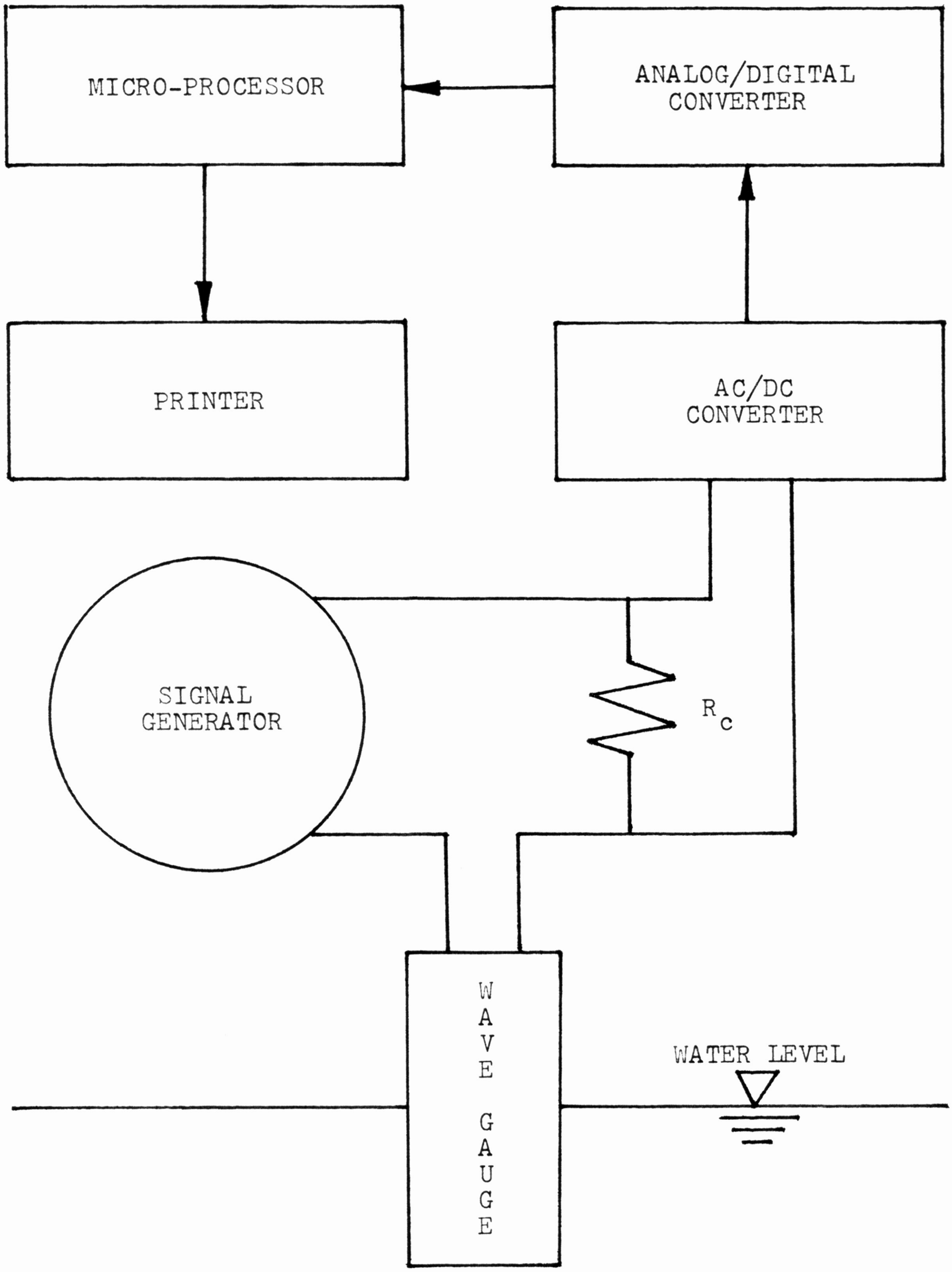

Figure 5. Initial water-wave measuring system proposal.

an output versus water level curve stored in a computer program. The appropriate water elevation values are sent to the printer which types this information as a series of numbers.

3.5 Transducer development

The transducer is the only major component in the measuring system which has to be designed and constructed. Its development is described in this section.

Since the number of types of transducers from which to choose is large, the selectin process is required to be subjective in nature. Transducers that rely on moving parts are not considered. Any water-wave measuring system has its ultimate application in the ocean. This harsh and incredibly corrosive environment fouls moving parts and necessitates frequent mechanical maintenance. The selection process now focuses on transducers based upon purely electronic principles. Of these, the two most widely used in liquid level detection are the resistive and the capactive types. Research within the Maritime Systems Engineering Department of capacitive transduction is already underway, therefore the resistive type will be chosen. In particular, a step resistance type wave gauge which is currently in wide use in Japan (Horikawa,1978).

A step resistance wave gauge is comprised of a series of electrical probes spaced evenly apart as shown in Figure 6. An electrical ground, which is placed equidistant from each probe, completes the circuit when the gauge is immersed in water. A resistor is placed between each probe to accentuate the output change with the varying water level.

In this circuit, the probe resistance R(1), the circuit resistance

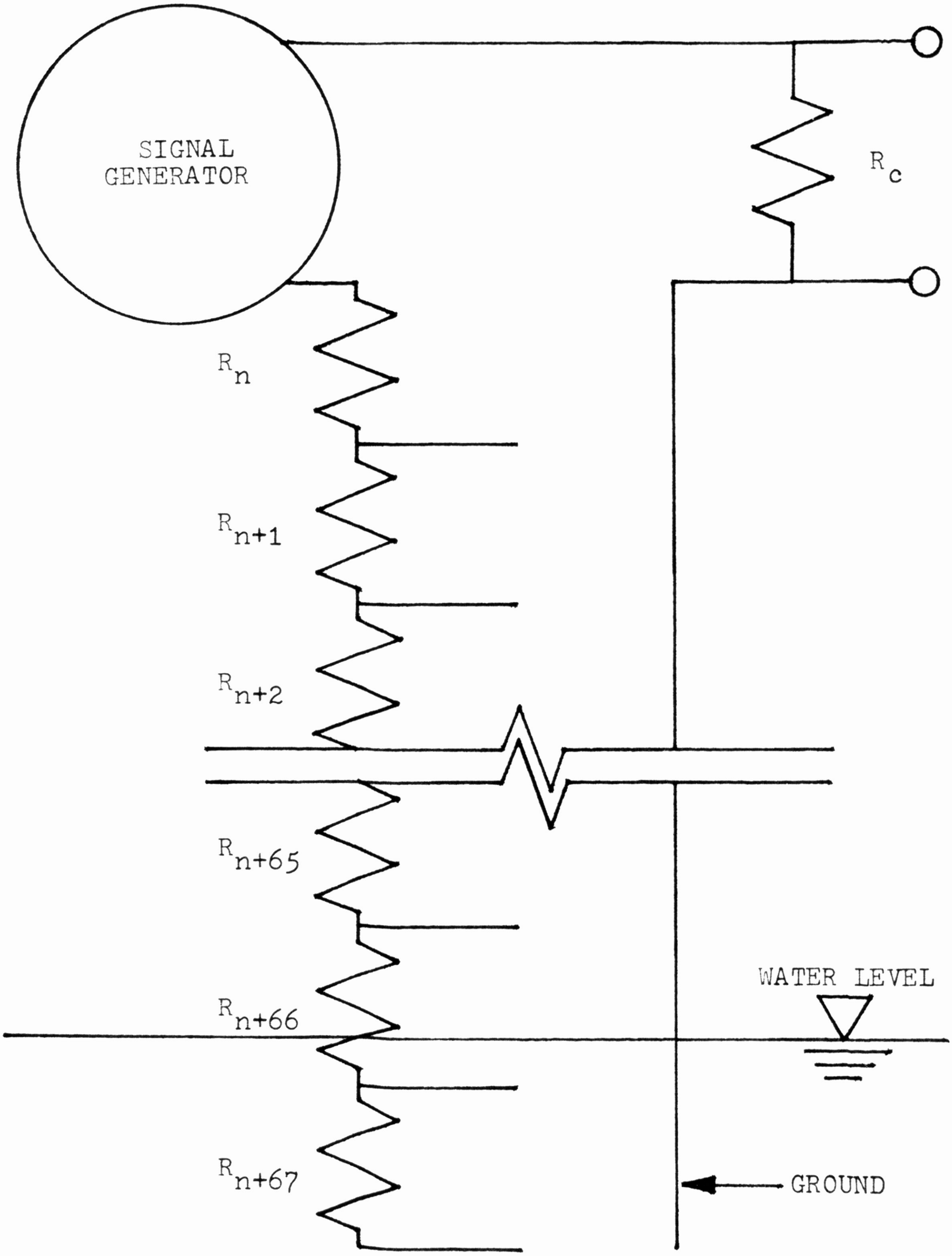

Figure 6. Step resistance wave gauge.

R(c) and the input voltage V(i) are all unknown. A circuit output with the widest possible range and the most linear relation between output and depth is desired. It must be determined at what values of probe resistance, circuit resistance and input voltage this occurs.

If it is assumed that the gauge has 67 probes and the resistance of the water R(w) is 3000 ohms, the equation for the total resistance at any probe level R(n) is

$$R(n) = R_1(69 - n) + R_{xn}$$

when

$$RX(n) = \frac{R_w(R_1 + R_{x(n-1)})}{R_w + R_1 + R_{x(n-1)}}$$

and given that

$$RX(1) = 0$$

$$RX(2) = R_w$$

Relations between total output resistance R(n) and the number of probes submerged n are as shown in Figure 7. Recall that the criteria specifies linear output. Notice that curves 1, 2 and 3 all satisfy this requirement. As it turns out, any value of probe resistance R(1) produces a curve which follows these guidelines. It appears that any probe resistance R(1) value will be suitable. But the circuit resistance and input voltage have not been considered.

The transducer circuit output is eventually read by the microprocessor which only accepts 0-5 volts. The equation for this circuit ouput voltage V(c), with R(n) calculated as above, is

$$V(c) = \frac{V_i}{1 + R_n/R_c}$$

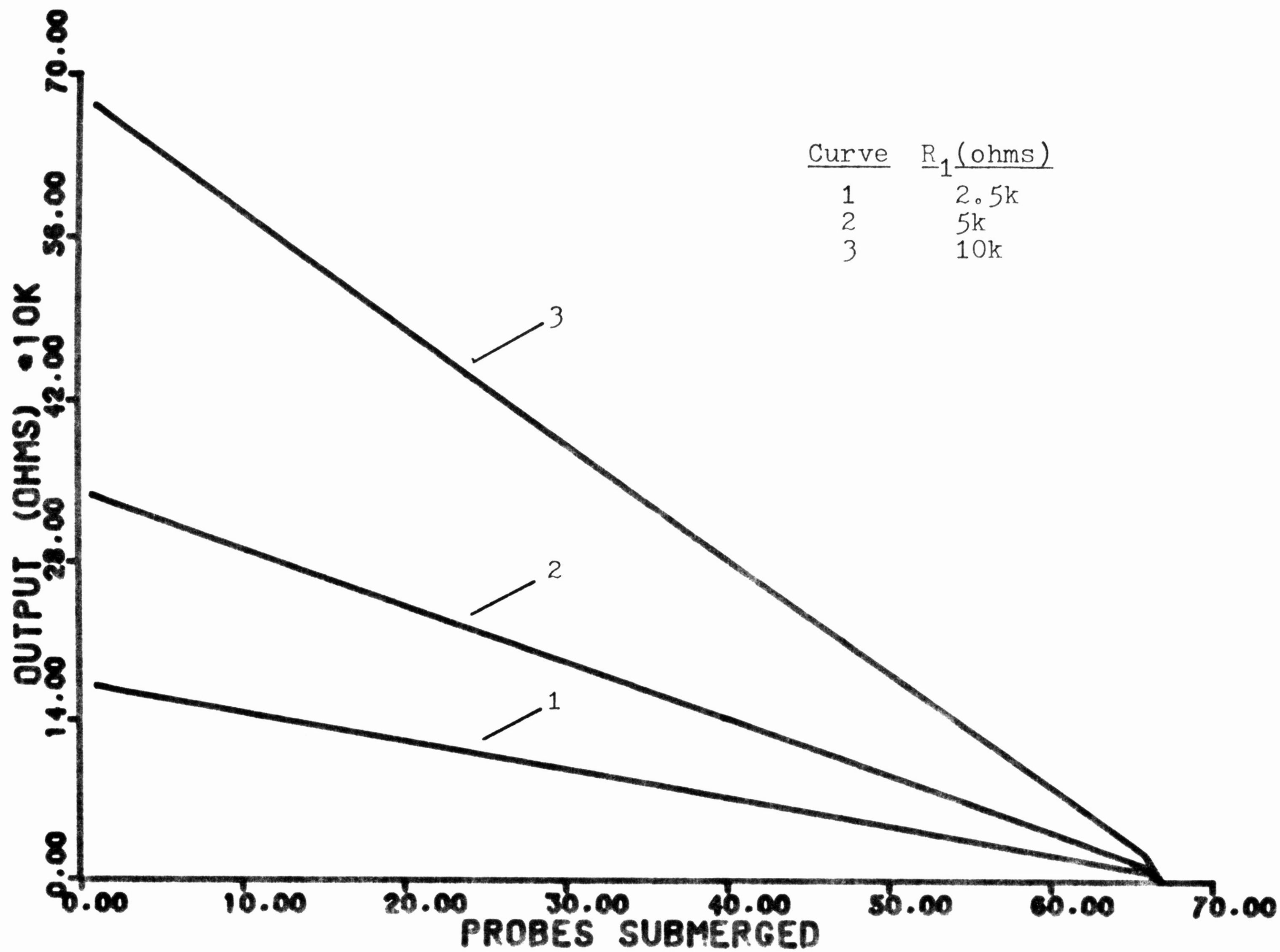

Figure 7. Selected resistance output vs. number of probes submerged curves.

The probe resistance R(1), the input voltage V(i) and the circuit resistance R(c) are arbitrarily chosen as 10k ohms, 10v and 10k ohms, respectively. The voltage output V(c) versus depth relation for these values is represented as curve 1 in Figure 8. This curve is by no means linear. In fact, its usable range is quite limited. This begins a trail-and-error procedure in which various values of probe resistance R(1), circuit resistance R(c) and input voltage V(i) are used. Curve 4 is not acceptable because it violates the 0-5v voltage output restriction. Curves 2 and 3 fit the criteria well, but curve 3 has the widest range. Curve 3 is chosen for this reason. The theoretically predicted circuit values are

R(1) = 10k ohms

V(i) = 5.5 volts

R(c) = 100k ohms

3.6 Calibration

Calibration is a test during which known values of the physical quantities to be measured are applied to the transducer and the corresponding output readings are recorded. A static calibration is performed under room conditions and in the absence of acceleration, shock or vibration. A single performance of this test over the entire range of the transducer, once in ascending mode and once in descending mode, is a calibration cycle.

A static calibration test for the step resistance wave measuring system consists of simply raising and lowering the probe into a large graduated cylinder filled with water. The values of output voltage for the particular probe level are recorded and a static calibration curve is constructed.

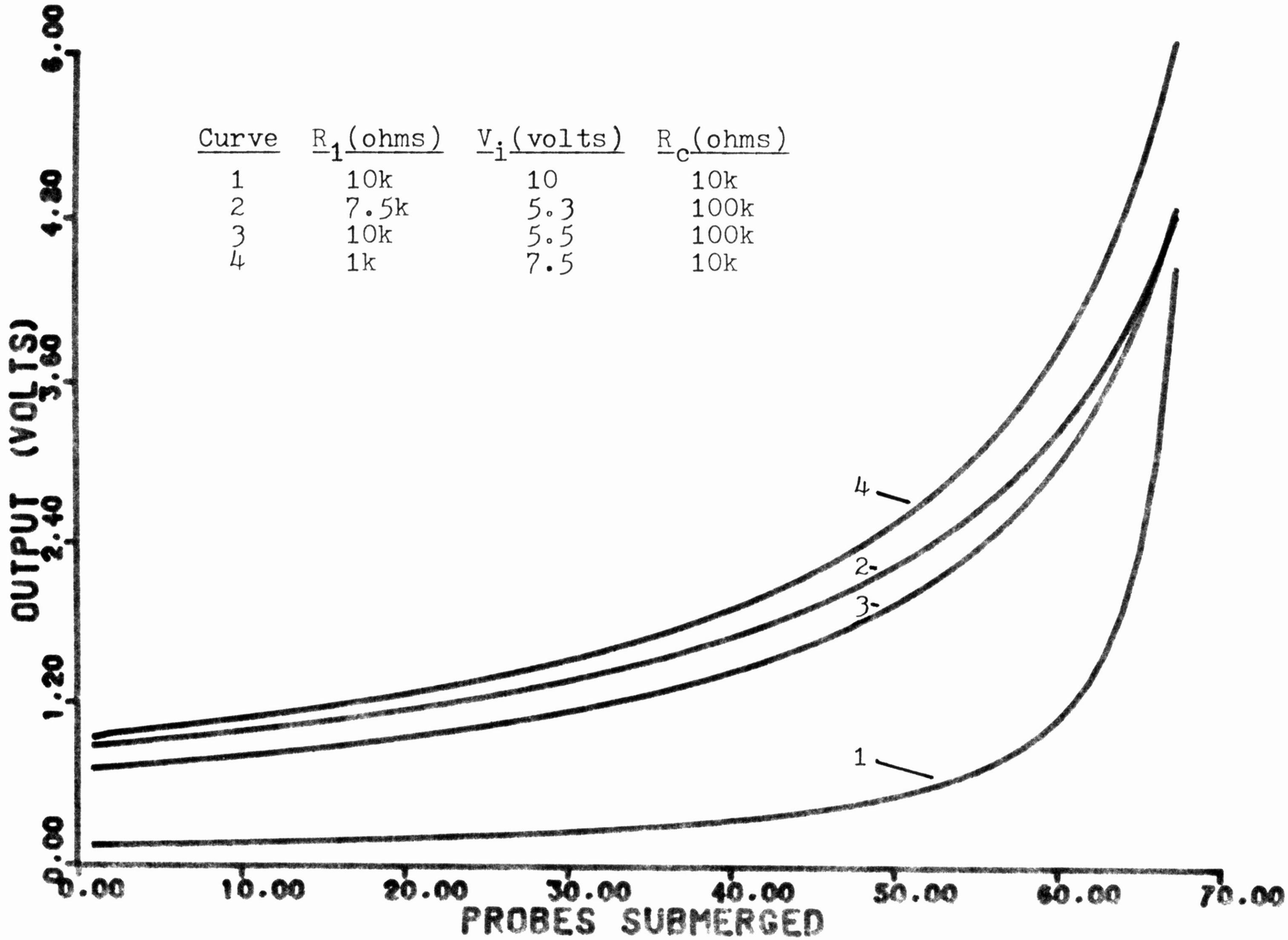

Figure 8. Selected voltage output vs. number of probes submerged curves.

The static calibration curve for the wave measuring system when using the prescribed values for probe resistance, input voltage and circuit resistance is shown in Figure 9 and is labeled as curve 1. Notice that it is far less linear than was expected. In actuality, it is seen that the theoretically predicted resistance and voltage values do not satisfy the prestated criteria.

Again, by a trail-and-error method, different values of circuit resistance R(c) and input voltage V(i) are used to optimize the system's output range and linearity. Calibration curves 2, 3 and 4 represent great improvements in range and linearity over the intial attempt. Of these three, calibration curve 3 best fits the criteria. The final circuit values are

R(1) = 10k ohms

V(i) = 5.5 volts

R(c) = 300k ohms

When the complete calibration cycle of the final circuit values is plotted, a difference in the increasing and the decreasing mode values is observed as shown in Figure 10. This phenomenon, known as hysteresis, is caused by a certain amount of lag in the sensing element. Hysteresis is defined as the maximum difference in any pair of output readings so obtained during any one calibration cycle, usually expressed in percent of full-scale output FSO.

The logical assumption is that the probes are retaining water. Since the wave gauge itself has no moving parts, the hysteresis cannot be attributed to frictional error. The wave gauge is responding well as the water level is rising. When the level begins to fall, apparently some water is clinging to the probes for a fraction of a second. This causes the microprocessor to read a higher water surface

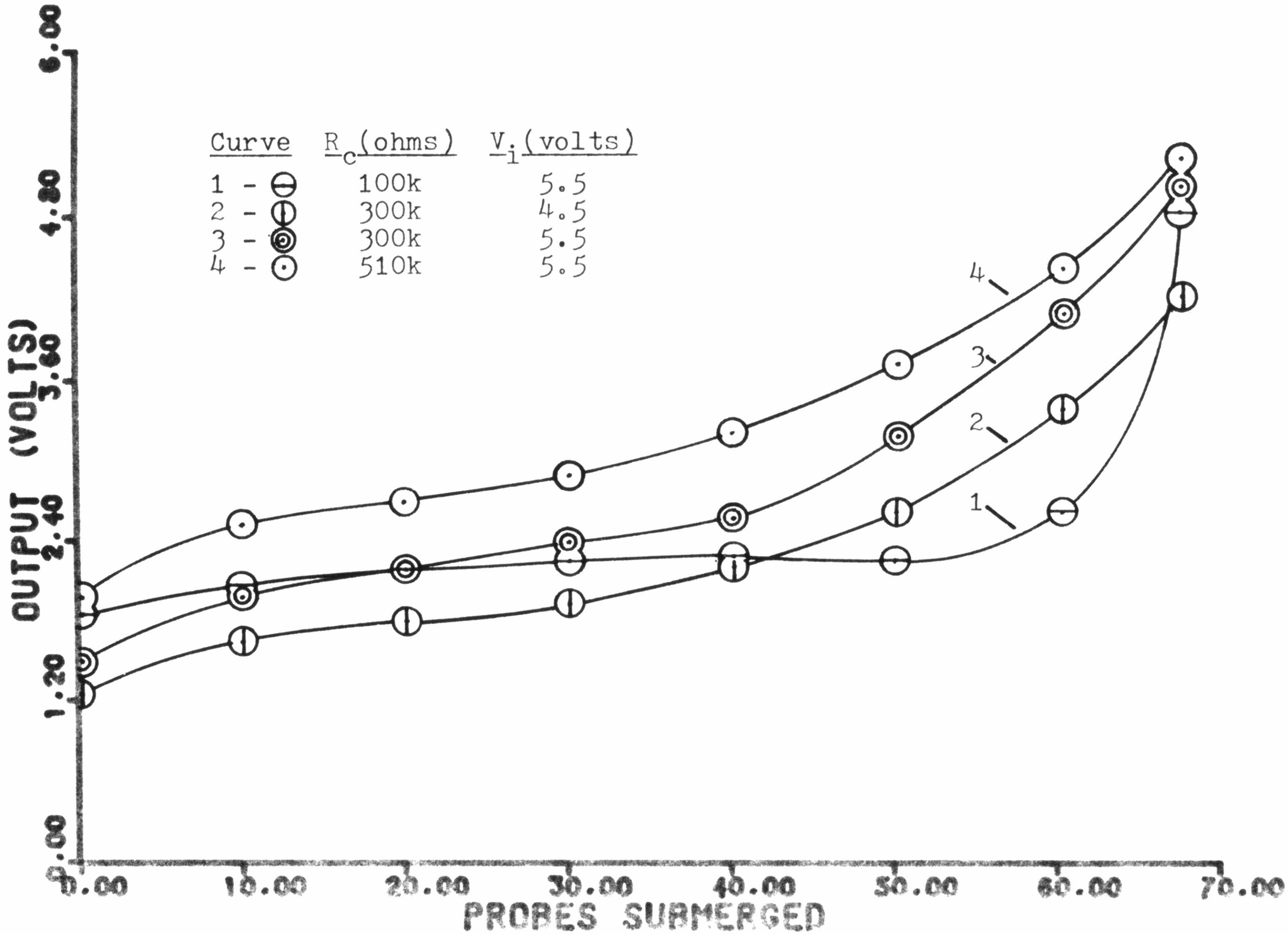

Figure 9. Calibration curves for the step resistance wave measuring system.

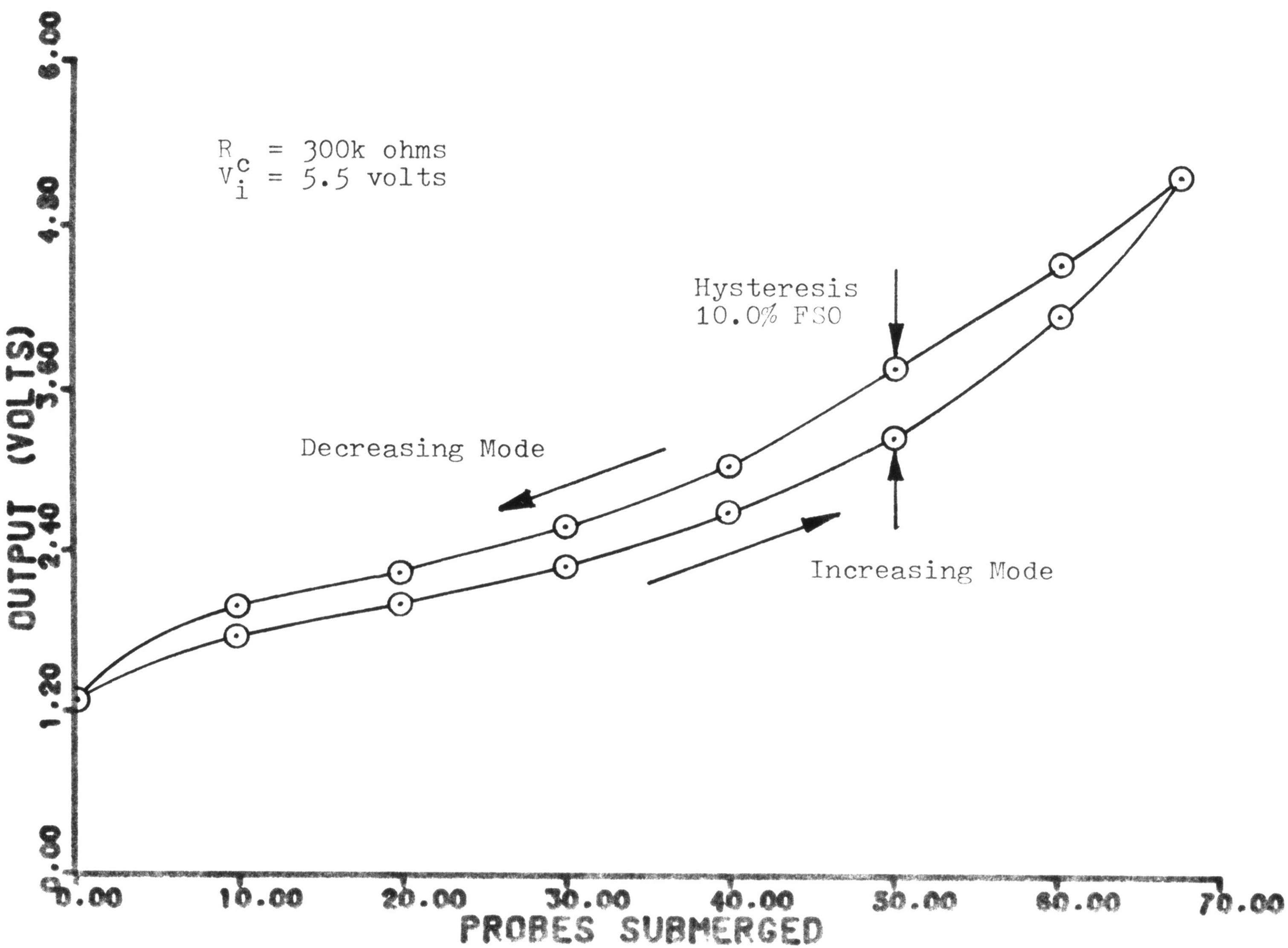

Figure 10. Calibration cycle curve for the step resistance wave measuring system.

level than actually exists.

This problem can be minimized by constructing a computer program which contains both curves. When the microprocessor recieves either increasing or decreasing output voltage, it will refer to the appropriate mode calibration curve to calculate an accurate water surface level value.

4. Data interpretation

This section will compare the wave profiles predicted by the selected wave theories to the free surface measurements taken in the Maritime Systems Engineering laboratory wave tank. Figures 11 through 16 describe six representative wave tests for various values of wave period and wave height. Each graph contains plotted experimental data points. The Airy linear theory curve is plotted for comparison purposes only, since none of the water-waves generated lie within its region of validity (see Figure 1). Stokes' third-order wave theory curve is included because all but one of the waves produced lie within its regime. Stokes' second-order is omitted because it is not applicable to most of the waves tested. Also an attempt is made to avoid crowding in the graphs.

4.1 Wave theory prediction

All of the figures presented illustrate the basic differences between the the two wave theories well. Figure 11 is the most extreme case of overall wave profile variance between Stokes' nonlinear and Airy linear theories. Notice the Airy curve is perfectly symmetrical about both the vertical and horizontal axes. The relative amplitude is always +0.5 for the crest and -0.5 for the trough regardless of period, height or depth (examine Figures 11 through 16).

The Stokes' curve is Figure 11 is quite different from the Airy curve. Although Stokes' waves are also symmetrical about the vertical axis, they are obviously not symmetrical with respect to the horizontal axis. This Stokes' curve has a relative amplitude of -0.4, while its crest is over +0.6. All of Stokes' waves shown demonstrate this variance to some degree.

Figure 11 also demonstrates the characteristically steep and pointed crest of Stokes' waves. Notice the wide, flat trough. Stokes' wave crests have greater cross-sectional areas above the still water level than its troughs do below water level. Airy waves have equal amounts of area on either side because of its symmetry.

Wave number 21 has the greatest variation between the Stokes' and Airy surface profile curves. Wave number 8 demonstrates the smallest difference as shown in Figure 13. Refer again to Figure 1 and locate wave numbers 8 and 21. Wave number 21 is almost on the Stokes' third/fourth-order boundary, while wave number 8 is in Stokes' second-order regime. A fundamental property of Stokes' wave theory is seen here. Stokes' wave number 21 is not merely larger in magnitude than Stokes' wave number 8, its trough is flatter and its crest is steeper and more pointed.

4.2 Measured data

Several definite features are witnessed when the data measured in the wave tank is examined. The lack of vertical symmetry is demonstrated by the experimental data from every test. Wave number 15 is a good case in point as shown in Figure 14. Since this wave is propogating from right to left, the right slope of the curve is the leading edge. Notice that a visualized data point curve has a steeper leading edge than its corresponding trailing edge. This feature is seen in every graph from Figure 11 through 16. This property is totally unpredicted since both the Airy linear and Stokes' nonlinear theories always produce vertically symmetrical waves.

It is initially hypothesized that the crest of these waves are traveling faster than the troughs. This could account for the steepened face as the crest slowly overruns the trough. The celerity of shallow water-waves is a function of depth only as shown in Figure 3. Since the wave's crest has a longer column of water beneath it than its trough, it is in deeper water. Consequently, theory predicts the crest to travel faster than its trough. But wait, these hypothesis is incorrect because it is based upon the false assumption that these waves travel in shallow water. Figure 1 shows that all of the waves produced are in transitional - almost deep water. Depth here is described in a relative sense because the wave tank has less than ten inches of water in it.

The actual cause of this steepening effect can be found when the wave generator is examined. Figure 17 is a diagram of the motion of the plunger type wave generator installed above the laboratory tank. The effect of the plunger (g) on two water particles (p1 and p2) at different depths (0.5L and L) will be studied.

at t = 0 :

d(p1) = 0

d(p2) = 0

d(g) = 0

at t = t(1) :

d(p1) = d

d(p2) = 0

d(g) = d

at t = t(2) :

d(p1) = 2d

d(p2) = d

d(g) = 2d

Now the displacmement of p1 and p2 over time is examined.

velocity = displacement/time

velocity of p1 = $\dfrac{2d}{t_1 + t_2}$

velocity of p2 = $\dfrac{d}{t_2}$

Assume a constant downward velocity of the plunger.

then t(1) = t(2)

$$\frac{2d}{t_1 + t_2} = \frac{2d}{2t_2} = \frac{d}{t_2}$$

velocity of p1 = velocity of p2

Then all particles are equally accelerated by the plunger. But due to the cyclical nature of the drive wheel, the downward velocity of the plunger is <u>not</u> constant. The generator slows at the bottom of the cycle.

then t(2)>t(1)

and

$$\frac{2d}{t_1 + t_2} > \frac{d}{t_2}$$

so

velocity of p2 > velocity of p1

Therefore, the particles at the surface are accelerated faster than particles below them. The wave's crest <u>is</u> traveling faster than the trough, but it is due to the motion of the plunger type wave generator and not the water depth.

It should be mentioned that many waves normally reproducible in a tank of this size were not realized. There is a fairly narrow band of the motor's velocity that will create smooth profiles. If it is slowed, secondary waves created by the vacuum of the plunger are witnessed. When the generator is set higher than this velocity, the steepened face of the wave becomes so pronounced that they break.

4.3 Comparison of theoretical and empirical results

There is a definite relation between the agreement of measured and predicted values and the wave's steepness. In Figure 11, wave number 21 demonstrates the best fit of the wave tests. Wave number 21 is also the steepest wave produced with a relative crest amplitude of +0.624. The wave in Figure 13, on the other hand, represents the worst fit. This wave is the flattest of the tests with a relative crest amplitude of +0.55. All of the waves follow this pattern. The steeper the wave, the closer the measured profile is to the Stokes' third-order prediction. This is puzzling since the steepness of the waves with good empirical-theoretical agreement approach the third/fourth-order Stokes' boundary as shown in Figure 1. Their appears to be a restraining force at play which is restricting the wave steepness - surface tension perhaps.

Another universal feature of the waves produced not predicted by theory is reduced wave height. Waves in Figures 11 through 16 have wave heights which are less than third-order Stokes' theory approximates. Even Figure 11, which demonstrates the closest fit obtained, does not have quite the wave height that theory would predict. Again, some restrictive action is occuring which is preventing the formation of natural crests and troughs. This effect is pronounced in the flatter waves. So much so that the Airy linear theory seems to be able to predict their amplitudes accurately as shown in Figure 13. The use of Airy theory for these waves in amplitude related problems would prove time-saving.

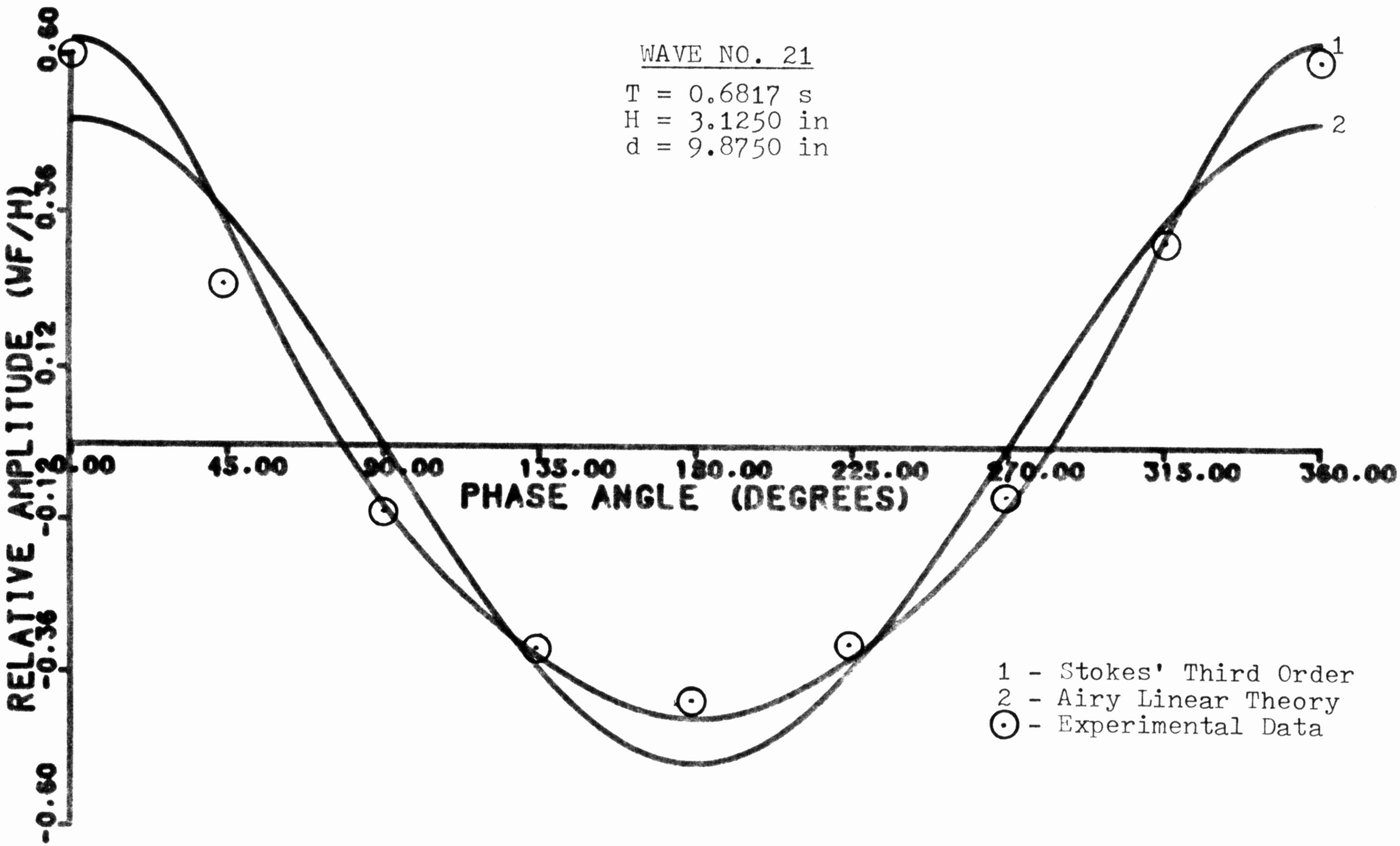

Figure 11. Wave No. 21: Comparison of laboratory water surface with Stokes' third-order and Airy linear wave theories.

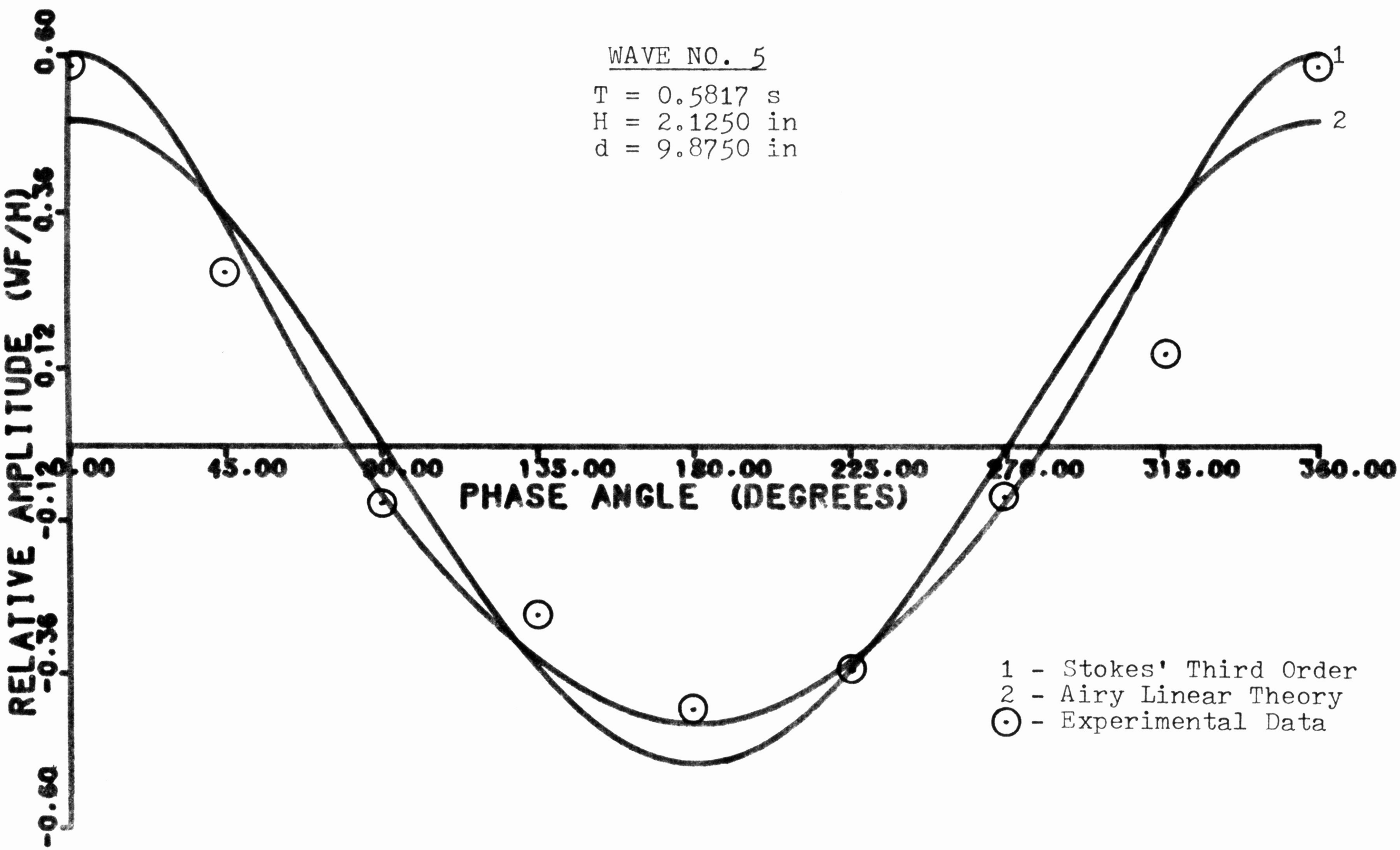

Figure 12. Wave No. 5: Comparison of laboratory water surface with Stokes' third-order and Airy linear wave theories.

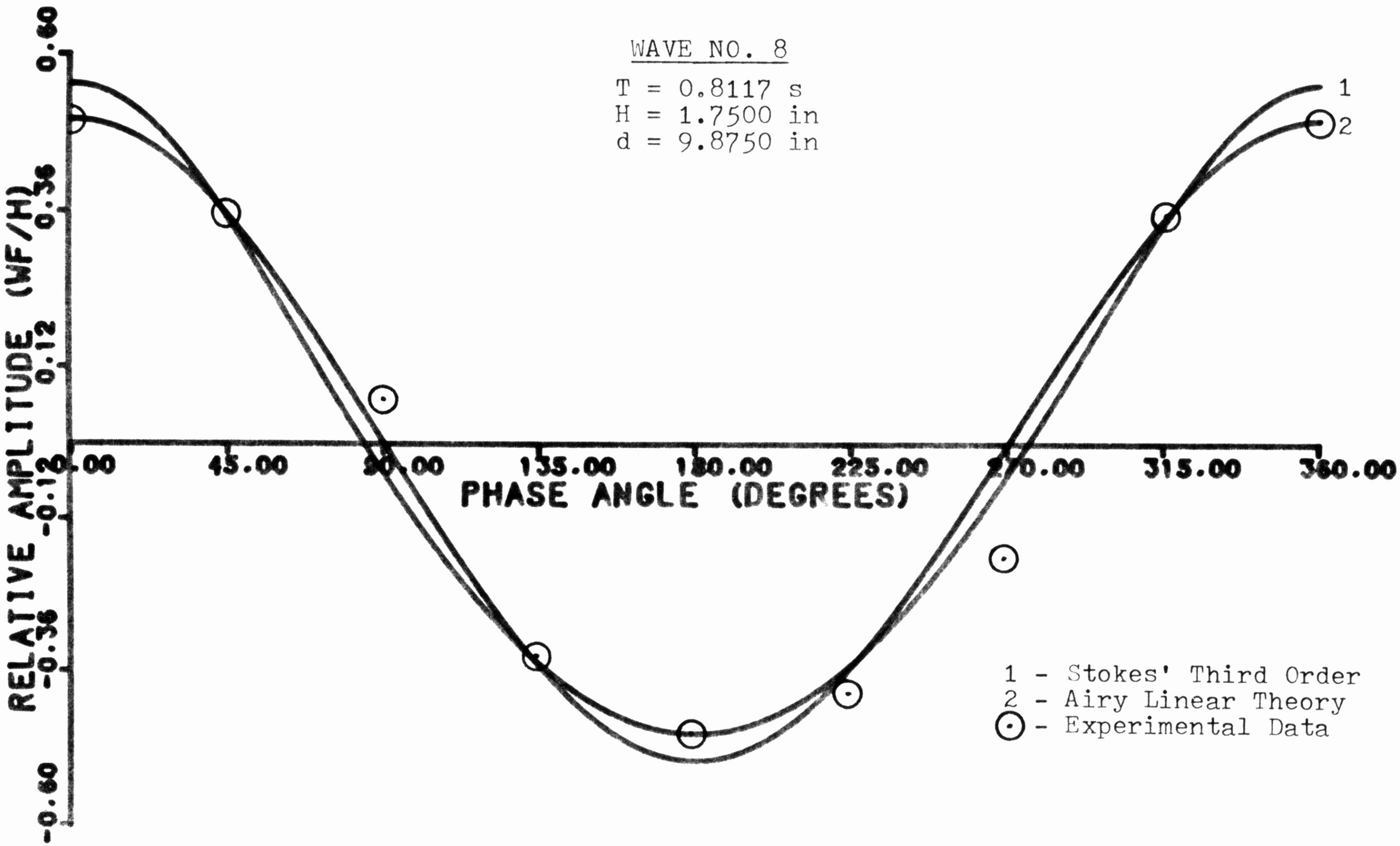

Figure 13. Wave No. 8: Comparison of laboratory water surface with Stokes' third-order and Airy linear wave theories.

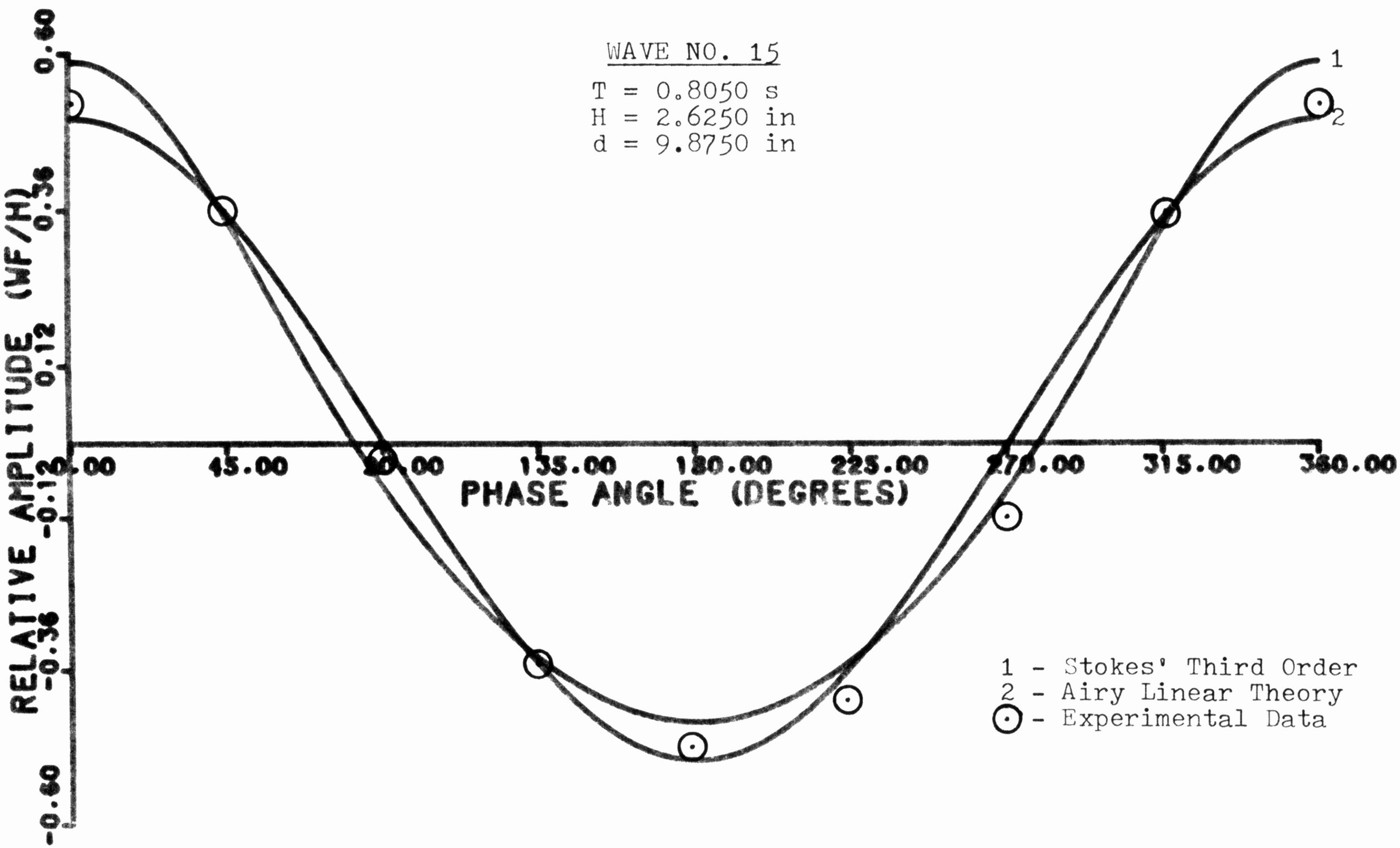

Figure 14. Wave No. 15: Comparison of laboratory water surface with Stokes' third-order and Airy linear wave theories.

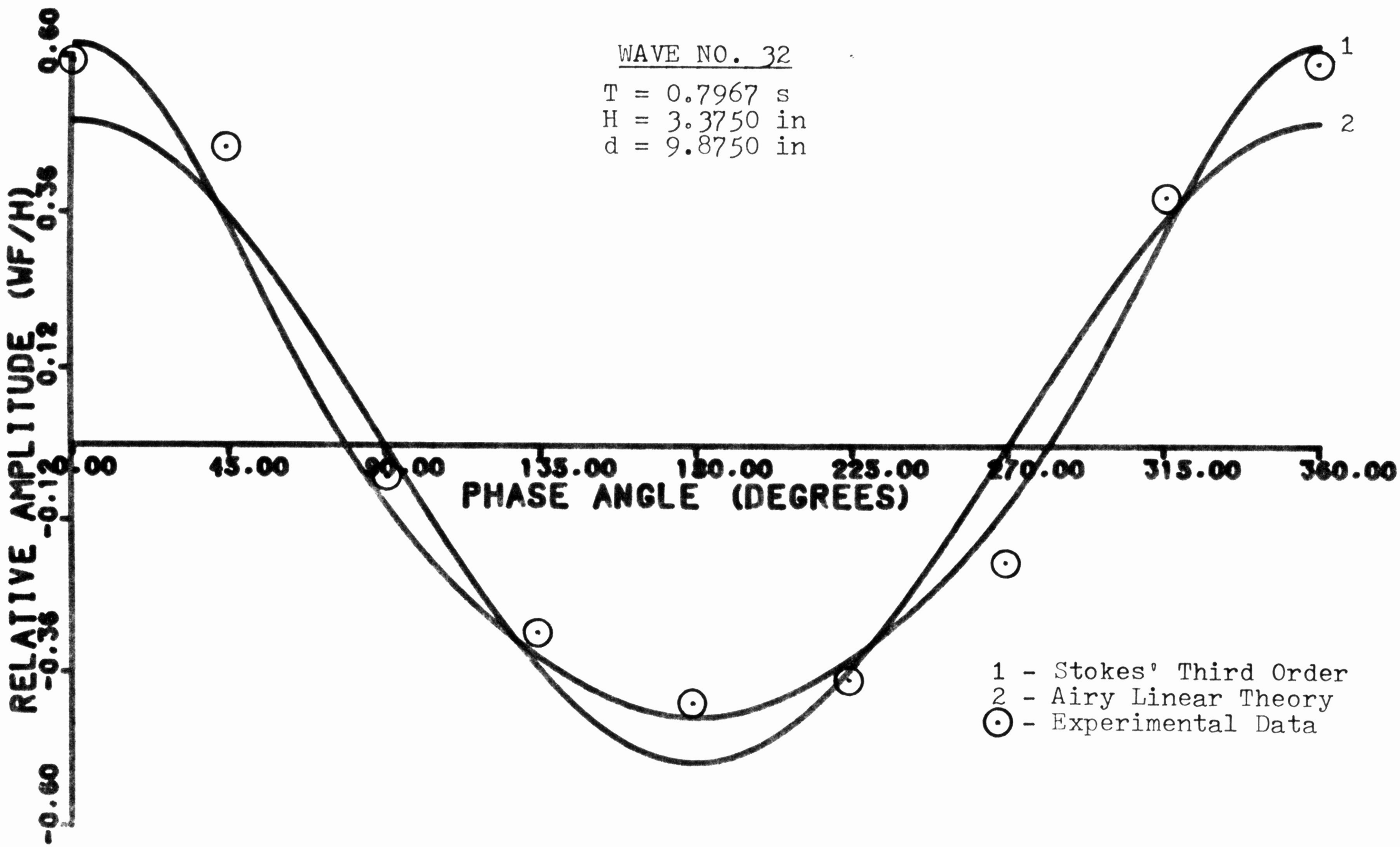

Figure 15. Wave No. 32: Comparison of laboratory water surface with Stokes' third-order and Airy linear wave theories.

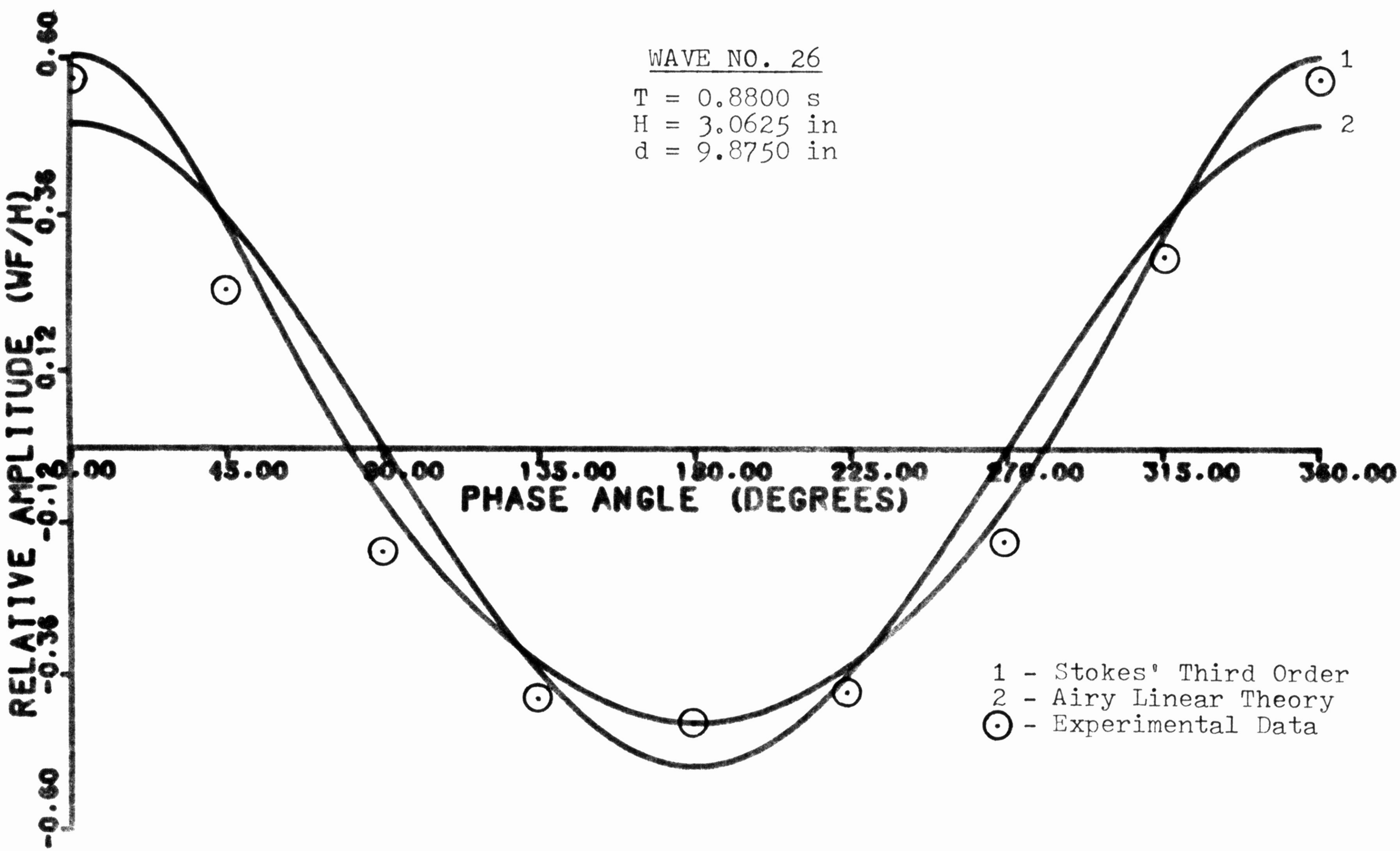

Figure 16. Wave No. 26: Comparison of laboratory water surface with Stokes' third-order and Airy linear wave theories.

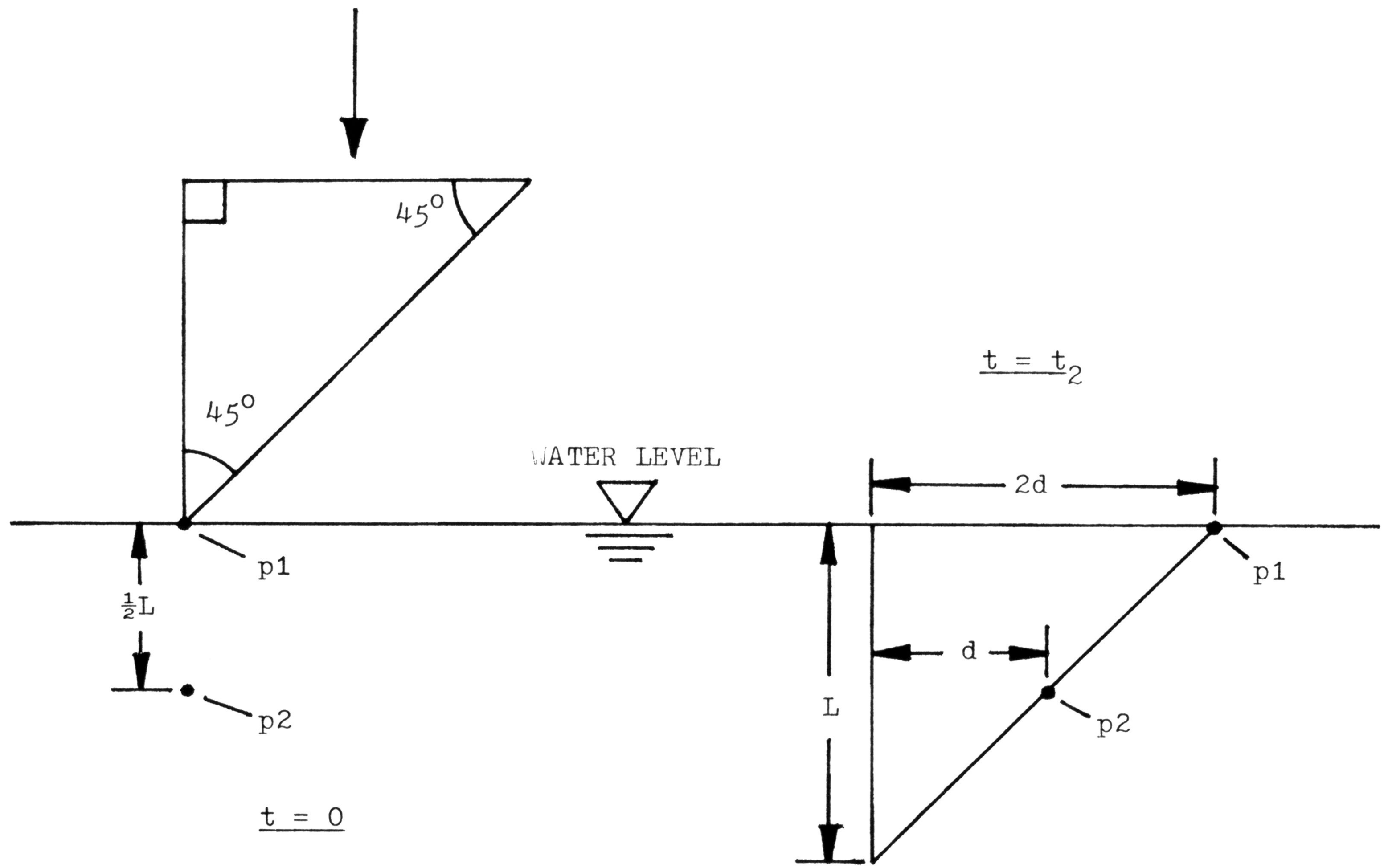

Figure 17. Schematic of wave generator motion.

5. CONCLUSION

5.1 Summary of findings

In this research report the classic wave features predicted by the dominant theories were reproduced accurately ,but waves generated in the laboratory exhibited several unforseen properties. The symmetry about both the horizontal and vertical axes proposed by Airy was confirmed. The Stokes' waves behaved as they were supposed by demonstrating symmetry with respect to the vertical axis only. The steep crests and flat troughs were found in the nonlinear theory as well. A curious lack of symmetry was discovered in all of the measured waves. These unpredicted free surface profiles were characterized by a steepened leading and a flattened trailing slope. It was determined that these deformities were caused partially by the plunger type wave generator and not the shallowness of the wave tank. The inability of the wave tank to produce a desired range of waves was noted. The experimentally produced waves with the highest steepness more closely resembled Stokes' approximations than did the smaller, flatter profiles. The existence of a restraining force that supresses the formation of steep crests was hypothesized. A universal feature of reduced wave height was also noticed. This abnormality was so prevalent in the smaller waves that Airy linear theory predicted their heights reasonably well.

5.2 Comprehensive interpretation of findings

The Maritime Systems Engineering laboratory wave tank performs well for only a narrow range of wave sizes. It was shown that even in this small range of reproducible profiles, only the larger and steeper waves accurately followed theory. The plunger type wave generator was proven to be the major destabilizing factor in the system. If this wave facility is to be a useful engineering simulation device its wave range must be expanded.

The Primos computer system and the Calcomp plotter proved to be excellent in the handling of even the difficult nonlinear wave theories. The calculation and graphics program will prove quite useful in future wave simulation. The differing profile features for the respective wave theories is brilliantly displayed by the plotter.

5.3 Recommendations

The Maritime Systems Engineering laboratory wave tank can be significantly improved if the plunger type wave generator is replaced by a less disturbing unit. Perhaps even a squirrel-cage wind wave generator could be installed. The overall tank length itself is prohibitive and needs to be extended.

Finally, I suggest that a permanent Maritime Systems Engineering laboratory computer file be established. Programs for the various needed wave theories could be stored and used for reference. Information and calibration curves for the numerous types of equipment could be filed. This laboratory library could enable upper-class students to carry on and extend the research of the preceding class.

APPENDIX A:
AIRY THEORY COMPUTER PROGRAM

```
      DIMENSION PA(362),WF(362),YSH(362)
      PRINT 1
 1    FORMAT(////,38X,'AIRY (FIRST ORDER) WAVE PROFILE PROGRAM'///,
     *'INPUT:'/,
     *8X,'WATER DEPTH (INCHES)'/,
     *8X,'WAVE PERIOD (SECONDS)'/,
     *8X,'WAVE HEIGHT (INCHES)')
      READ(1,*)DA,T,HA
      D=DA/12.0
      H=HA/12.0
      GR=32.174
      PI=3.1416
      Q=(((2.0*PI)**2)*D)/((T**2)*GR)
      FR=Q+1.0/(1.0+0.6522*Q+0.4622*(Q**2)+0.0864*(Q**3)+0.0675*(Q**4))
      WL=T*SQRT(GR*D/FR)
      A=H/2.0
      DO 3 J=1,361
      PA(J)=((J-1)*PI)/180
      WF(J)=A*COS(PA(J))
      YSH(J)=WF(J)/H
 3    CONTINUE
      DO 4 K=1,361
      WRITE(6,5)YSH(K)
 5    FORMAT(F10.7)
 4    CONTINUE
      CALL EXIT
      END
OK,
```

APPENDIX B:
STOKES' SECOND-ORDER THEORY
COMPUTER PROGRAM

```
      DIMENSION PA(362),WF(362),YSH(362)
      PRINT 1
 1    FORMAT(////,38X,'STOKES (SECOND ORDER) WAVE PROFILE PROGRAM'///,
     *'INPUT:'/,
     *8X,'WATER DEPTH (INCHES)'/,
     *8X,'WAVE PERIOD (SECONDS)'/,
     *8X,'WAVE HEIGHT (INCHES)')
      READ(1,*)DA,T,HA
      D=DA/12.0
      H=HA/12.0
      GR=32.174
      PI=3.1416
      Q=(((2.0*PI)**2)*D)/((T**2)*GR)
      FR=Q+1.0/(1.0+0.6522*Q+0.4622*(Q**2)+0.0864*(Q**3)+0.0675*(Q**4))
      WL=T*SQRT(GR*D/FR)
      A=H/2.0
      R=(2.0*PI*D)/WL
      SHR=(EXP(R)-EXP(-R))/2.0
      CHR=(EXP(R)+EXP(-R))/2.0
      CHRS=(EXP(2.0*R)+EXP(-2.0*R))/2.0
      DO 3 J=1,361
      PA(J)=((J-1)*PI)/180
      WF(J)=A*COS(PA(J))+(PI*(H**2)*CHR*(2.0+CHRS)*COS(2.0*PA(J)))/
     *(8.0*WL*(SHR**3))
      YSH(J)=WF(J)/H
 3    CONTINUE
      DO 4 K=1,361
      WRITE(6,5)YSH(K)
 5    FORMAT(F10.7)
 4    CONTINUE
      CALL EXIT
      END
OK,
```

APPENDIX C:
STOKES' THIRD-ORDER THEORY
COMPUTER PROGRAM

```
C     STOKES (THIRD ORDER) WAVE PROFILE PROGRAM
C
C
      DIMENSION WL(1000),R(1000),SHR(1000),CHR(1000),CHRS(1000),
     *QB(1000),F(1000),RF(1000),A(1000),AF(1000)
      PRINT 8
 8    FORMAT('INPUT:',/
     *'WATER DEPTH (INCHES)',/
     *'WAVE PERIOD (SECONDS)',/
     *'WAVE HEIGHT (INCHES)',///)
      READ(1,*)DA,T,HA
      D=DA/12.0
      H=HA/12.0
      GR=32.174
      PI=3.1416
      Q=(((2.0*PI)**2)*D)/((T**2)*GR)
      FR=Q+1.0/(1.0+0.6522*Q+0.4622*(Q**2)+0.0864*(Q**3)+0.0675*
     *(Q**4))
      WL(1)=T*SQRT(GR*D/FR)
      A(2)=-0.2
      DO 5 N=2,1000
 6    WL(N)=WL(N-1)+A(N)
      R(N-1)=(2.0*PI*D)/WL(N-1)
      R(N)=(2.0*PI*D)/WL(N)
      SHR(N-1)=(EXP(R(N-1))-EXP(-R(N-1)))/2.0
      SHR(N)=(EXP(R(N))-EXP(-R(N-1)))/2.0
      CHR(N-1)=(EXP(R(N-1))+EXP(-R(N-1)))/2.0
      CHR(N)=(EXP(R(N))+EXP(-R(N)))/2.0
      CHRS(N-1)=(EXP(2.0*R(N-1))+EXP(-2.0*R(N-1)))/2.0
      CHRS(N)=(EXP(2.0*R(N))+EXP(-2.0*R(N)))/2.0
      QA=(GR*(T**2))/(2.0*PI)
      QB(N-1)=(PI*H/WL(N-1))**2
      QB(N)=(PI*H/WL(N))**2
      F(N-1)=((QA*SHR(N-1))/CHR(N-1))*(1.0+QB(N-1)*((5.0+2.0*CHRS(N-1)
     *+2.0*(CHRS(N-1)**2))/(8.0*(SHR(N-1)**4))))
      F(N)=((QA*SHR(N))/CHR(N))*(1.0+QB(N)*((5.0+2.0*CHRS(N)+2.0*
     *(CHRS(N)**2))/(8.0*(SHR(N)**4))))
      RF(N)=F(N)*F(N-1)
      AF(N)=ABS(RF(N))
      IF(AF(N).LT.0.001)GO TO 7
      IF(RF(N).LT.0.000)GO TO 6
      A(N+1)=A(N)/2.0
 5    CONTINUE
 7    WL(1)=WL(1)*12.0
      WL(N)=WL(N)*12.0
      WRITE(1,9)WL(1),WL(N)
 9    FORMAT(2(4X,F7.3))
      CALL EXIT
      END
OK,
```

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