EVALUATION OF AERODYNAMIC PERFORMANCE METHODS FOR A HELICOPTER IN FORWARD FLIGHT

University Undergraduate Fellows Paper

by

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ABSTRACT

An analysis of rotor aerodynamic performance is treated by the comparison of several theoretical methods with experimental full scale rotor performance. These methods were found to accurately predict rotor aerodynamic performance; however, it was found that for the purpose of digital flight simulation there are certain advantages to using closed form equations utilizing blade element theory. Also a brief description of the limitations and assumptions for the methods is provided.

ACKNOWLEDGMENTS

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NOMENCLATURE

A_1 - lateral swashplate tilt (+ right and down)

a - lift curve slope

a_o - precone

a_1 - fore and aft (F/A) flapping (+ back and down)

B_1 - fore and aft (F/A) swashplate tilt (+ forward and down)

b - number of blades

b_1 - lateral flapping (+ right and down)

c - blade chord

C_{D_f} - section profile drag coefficient

C_H - H-force coefficient

C_L - lift coefficient

C_p - power coefficient

C_T - thrust coefficient

C_y - Y-force coefficient

C_{nm} - blade twist constants

D - drag

I_B - blade flapping inertia

L - lift

K_H - flapping spring rate

$\hat{p}$ - roll rate of mast (+ roll right)

$\hat{q}$ - pitch rate of mast (+ pitch up)

R - rotor radius

SHP - shaft horsepower

V - forward airspeed

λ - inflow ratio

γ - blade mass constant

ρ - air density

μ - advance ratio

α - blade element angle of attach

α_m - mast angle of attack (+ forward)

σ - rotor solidity

$\Theta_{0.75R}$ - blade collective pitch at 0.75 radius

Ω - rotor angular velocity

ϕ - rotor inflow angle

INTRODUCTION

The development and application of methods for the analysis of helicopter rotor performance in forward flight has been a topic of considerable interest to the helicopter industry. The complexity and cost of developing a new helicopter design requires almost all development and design work to be finalized before the actual design is built. The cost of developing a design from experimental prototypes over a period of years has become prohibitive. Therefore, simulation of a design in detail has become an important part of the design process.

Flight simulations can be accomplished primarily in two ways. The first method involves the mathematical modeling of a helicopter design on the digital computer. This method is quite satisfactory for most engineering design applications and the mathematical model can be made quite complex so than many design parameters can be investigated. However, this method limits the design process since the human elements of reaction time, design feasibility, and pilot training and evaluation can not be carried out. In the past these design criteria were evaluated using prototype aircraft and design modifications were then made. With the advent of the hybrid-digital computer, actual working cockpit mockups have become available. However, to utilize the mockups for real time flight simulations, the mathematical modeling requirements must be kept to a minimum so that pilot and simulator reaction times can be kept to time frames similar to the actual flight hardware. Therefore, the development of accurate and yet mathematically simple expressions for helicopter rotor performance have become of prime importance.

The purpose of this project has been to evaluate and compare methods for predicting rotor performance. This evaluation suggests that rotor performance prediction can be accomplished using simple closed form algebraic equations. More advanced methods which divide up the rotor blade into sections for performance calculations do not predict significantly better results. The advantage of these methods results from their ability to predict blade airloads. In most flight simulations this information is not of importance though. Therefore, only needless mathematical modeling and increased computational time results from the use of large blade element rotor performance programs.

COMPARISON OF ROTOR PERFORMANCE PREDICTION METHODS

The comparison of rotor performance prediction methods presented in this study is primarily based upon two methods. These methods are the U.S. Army - Bell Helicopter Textron developed C81 method and an advanced closed form blade element method (ACFBE). C81 is a digital flight simulation program which analyzes overall helicopter flight performance. The blade element analysis of C81 divides the blade into twenty segments, evaluates the airloads on these segments, and then for overall simulation purposes adds the twenty segments back into one blade. For real time flight simulation and most preliminary design work, the airloads data computed by C81 is not required. An analysis of the ACFBE method was made for the purpose of evaluating rotor equations which compute overall performance and do not divide the blade up into segments. The use of this type of method is based upon the prior assumption that general flight simulations normally will not require blade airloads information. Two other methods which were evaluated are the energy method and simple blade element method (Gessow and Myer Method). The energy method is an extremely simple method and the assumptions upon which it is based preclude its use for purposes other than rapid estimation of the level flight performance parameters of thrust and power required. One comparison of this method is made for general evaluation purposes. The simple blade element method was also found to be based upon assumptions which preclude its use for general flight simulation purposes. The primary limitation is a result of the elimination of cyclic control position effects from the general equations. Results using this method are not presented in this report primarily because correlation with the other methods requires knowledge of the cyclic control position effects on the rotor. A general description of these methods, their limitations, and governing assumptions is presented in Appendix A.

The basis for comparison of the methods is experimental test data on a Sikorsky H-34 rotor which was evaluated in the 40 ft. x 80 ft. NASA-AMES wind tunnel.[1,2] This fully articulated rotor system is described in Table 1. Two test conditions were evaluated for this analysis; however, due to the similarity of results only test condition one is extensively presented in this report. These two test conditions are presented in Table 2.

The results from the comparison of the rotor performance prediction methods are presented in Figure 1 through 7. Figure 1 presents an evaluation of the energy, C81, ACFBE, and experimental results for test condition one at a mast angle of attack of 0^0. The C81 and ACFBE methods yield similar results; whereas, the energy method only approximates the rotor performance. The energy method does not take into account control position and assumes the rotor lift is the required amount for level flight. It should be noted that error in measurement for the test data provided for some uncertainty in the calculation of theoretical correlations. The test data was recorded when the control positions were adjusted so that fore and aft (F/A) and lateral (LAT) flapping were within ±0.2 degrees from zero flapping. Analysis of a flapping range of this magnitude on the theoretical methods yielded thrust variations of approximately ±300 lb. The C81 method of analysis incorporates a model for stall and compressibility effects and this is

Table 1. H-34 Rotor System

The blade investigated with the fully articulated rotor system had -8° linear twist. The dimensional information related to this fully articulated rotor is listed below.[1,2] A standard H-34 transmission and rotor shaft were driven by the 1500 HP motor, and a special high-strength rotor control system was used.

Rotor radius, R, ft	28
Blade chord, c, ft	1.337
Cutout radius, ft	4.48
Rotor solidity, σ	0.062
Reference area, ft^2	153.1
Blade moment of inertia about flapping hinge, $ft\text{-}lb\text{-}sec^2$	1264
Blade weight moment about flapping hinge, lb-ft	2265
Flapping hinge offset, ft	1.0
Number of blades, b	4
Airfoil	NACA 0012
Blade taper ratio	1.0

Table 2. Test Conditions

Parameter	Condition 1	Condition 2
Advance Ratio (μ)	0.3	0.4
Tip Mach Number	0.74	0.82
Tip Speed	650 ft/sec	680 ft/sec
Airspeed	117 Kts.	161 Kts.
Density (ρ)	0.002203	0.002114

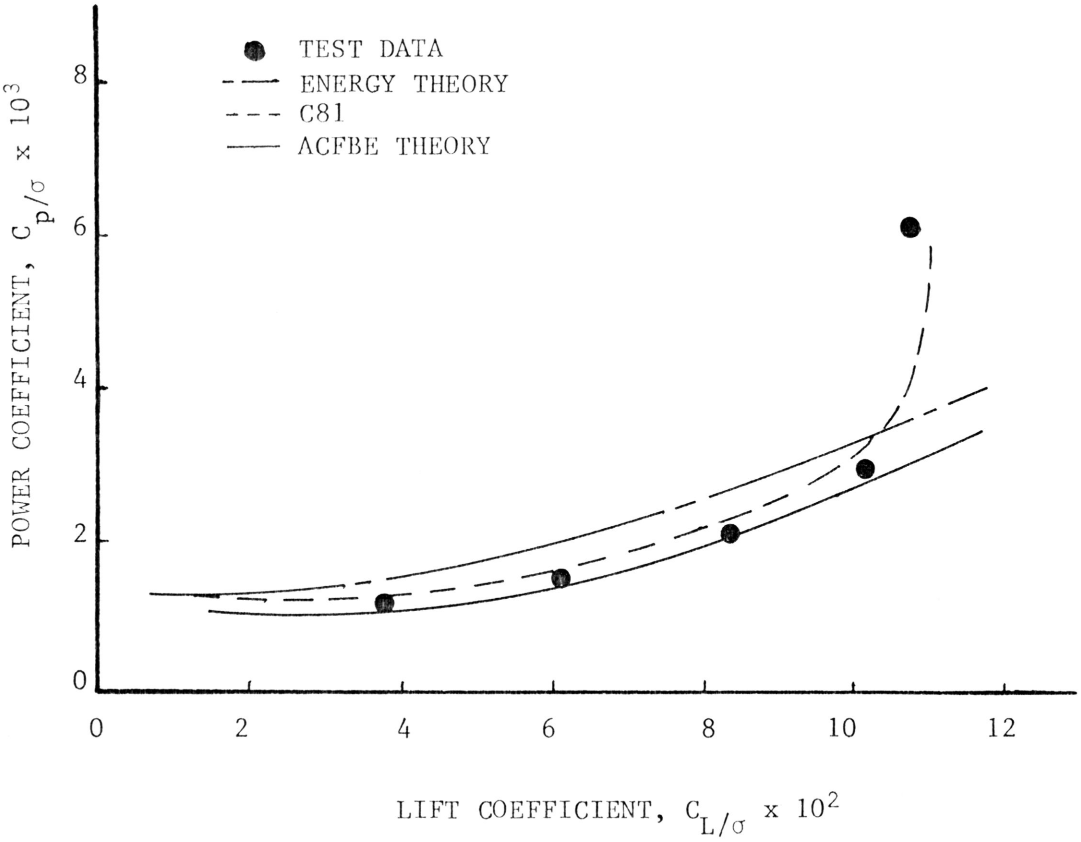


Figure 1. A Comparison of Rotor Lift Coefficient Versus Power Coefficient for $\mu=0.3$, $\alpha_m=0^0$

evidenced in Figure 1 at a collective pitch of ten degrees. The ACFBE model was not developed to this extent and the power correlations in this study were not good above approximately ten degrees for any mast position. However, incorporation of a model for stall and compressibility effects has been successfully attempted without increasing computer computation time significantly.[10] Figures 2 through 4 present correlations of lift and power for test condition two and varying mast angles of attack for condition one. Figure 2 is the only presented comparison of results for test condition two due to the similarity of correlation trends with test condition one. Figures 5 through 7 compare rotor lift, drag, and power correlations for mast angles of 0^0, 10^0, and -5^0 respectively. The lift predictions are similar for both theoretical methods and correlate closely with test data when the experimental error is considered. Drag predictions seem to be better predicted by the ACFBE method for all mast angles of attack except 10^0. At this condition both methods seem to be in contradiction with theoretical results. Prediction of power using the C81 method compares quite well with experimental data. The lack of stall and compressibility effects modeling in the ACFBE method leaves something to be desired however. Correlation is good at low angles of collective pitch, but as stall and compressibility become important the prediction of power is low.

Except for the stall-compressibility problem the evaluation of the C81 and ACFBE methods shows little significant difference in prediction capability. Computational time for the C81 method is on the order of six to ten times longer than the time required for the ACFBE method. For real time flight simulations where accurate wind tunnel data is

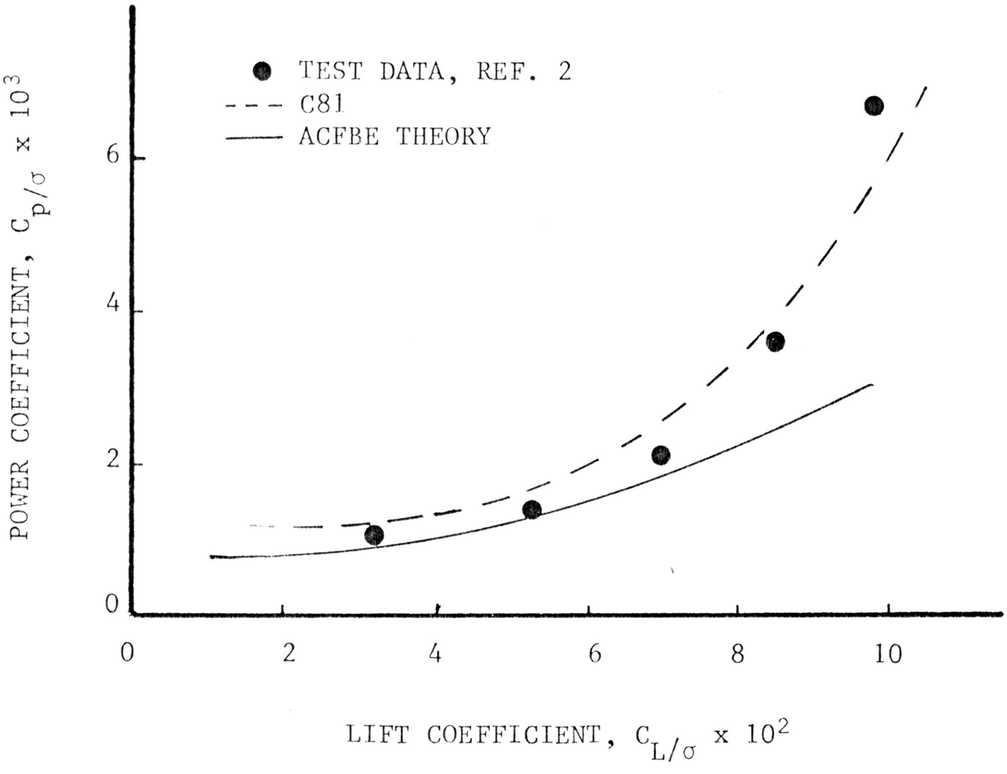
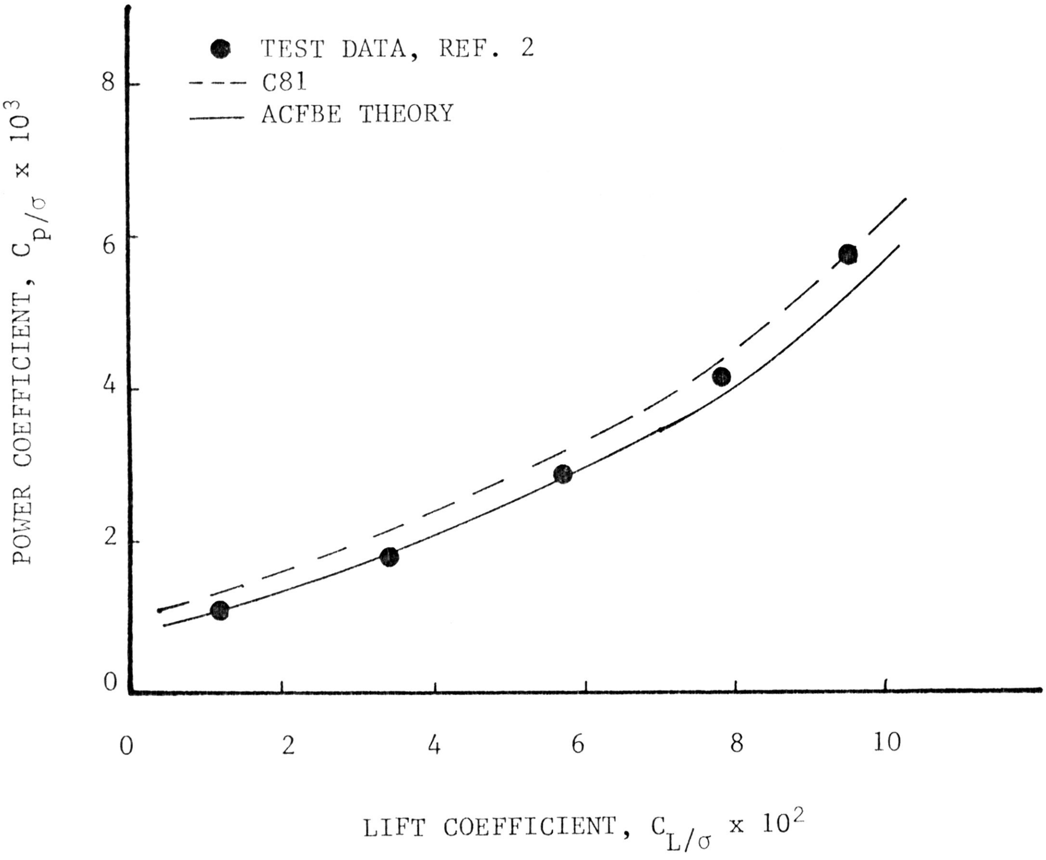


Figure 2. A Comparison of Rotor Lift Coefficient Versus Power Coefficient for $\mu=0.4$, $\alpha_m=0^0$



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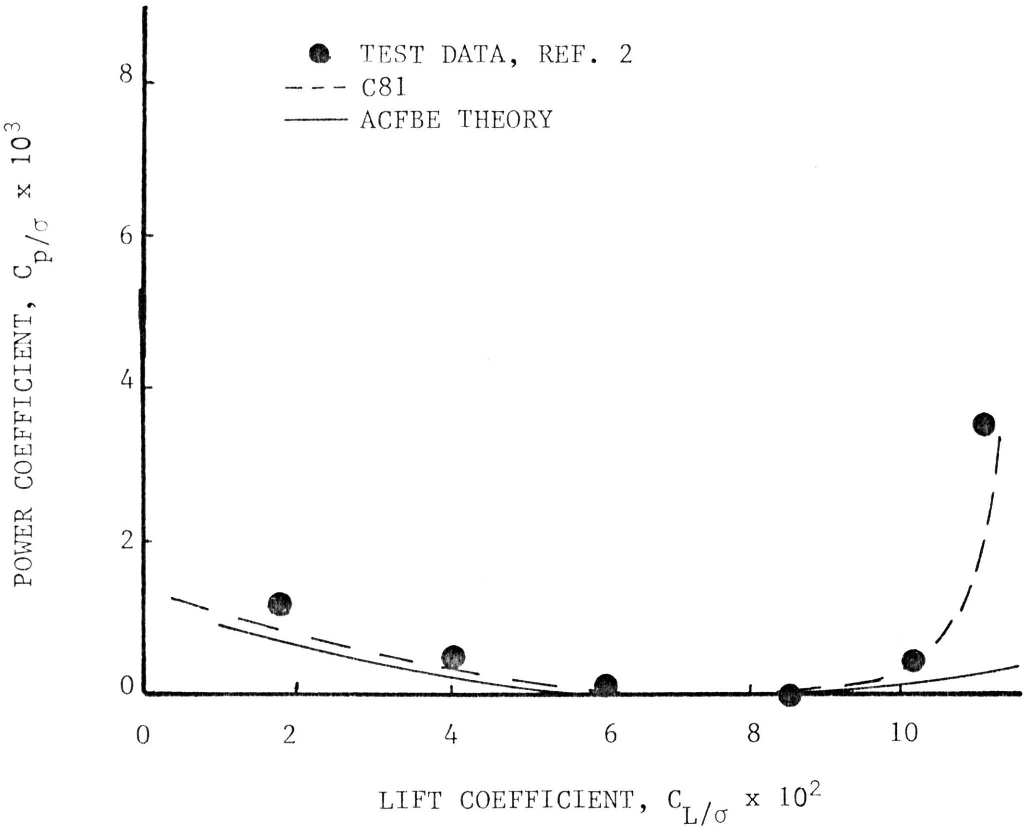


Figure 4. A Comparison of Rotor Lift Coefficient Versus Power Coefficient for $\mu=0.3$, $\alpha_m=-5^0$

available, the input data for the ACFBE method should be capable of being idealized to provide for highly realistic and accurate responses. This should provide for a mathematical model yielding results as accurate as larger, more detailed models such as C81 without the loss of computer capability and increased computation time.

In conclusion to the results presented, it should be noted that another method for comparison was discovered too late for evaluation; but nevertheless should be mentioned. This method is referenced as the Y-92 computer program and was developed by Boeing-Vertol. A comparison of results with C81 is presented in reference 13 for the B-0105 soft-in-plane hingeless rigid rotor. Evaluation of the computer math model and required computational times was not available.

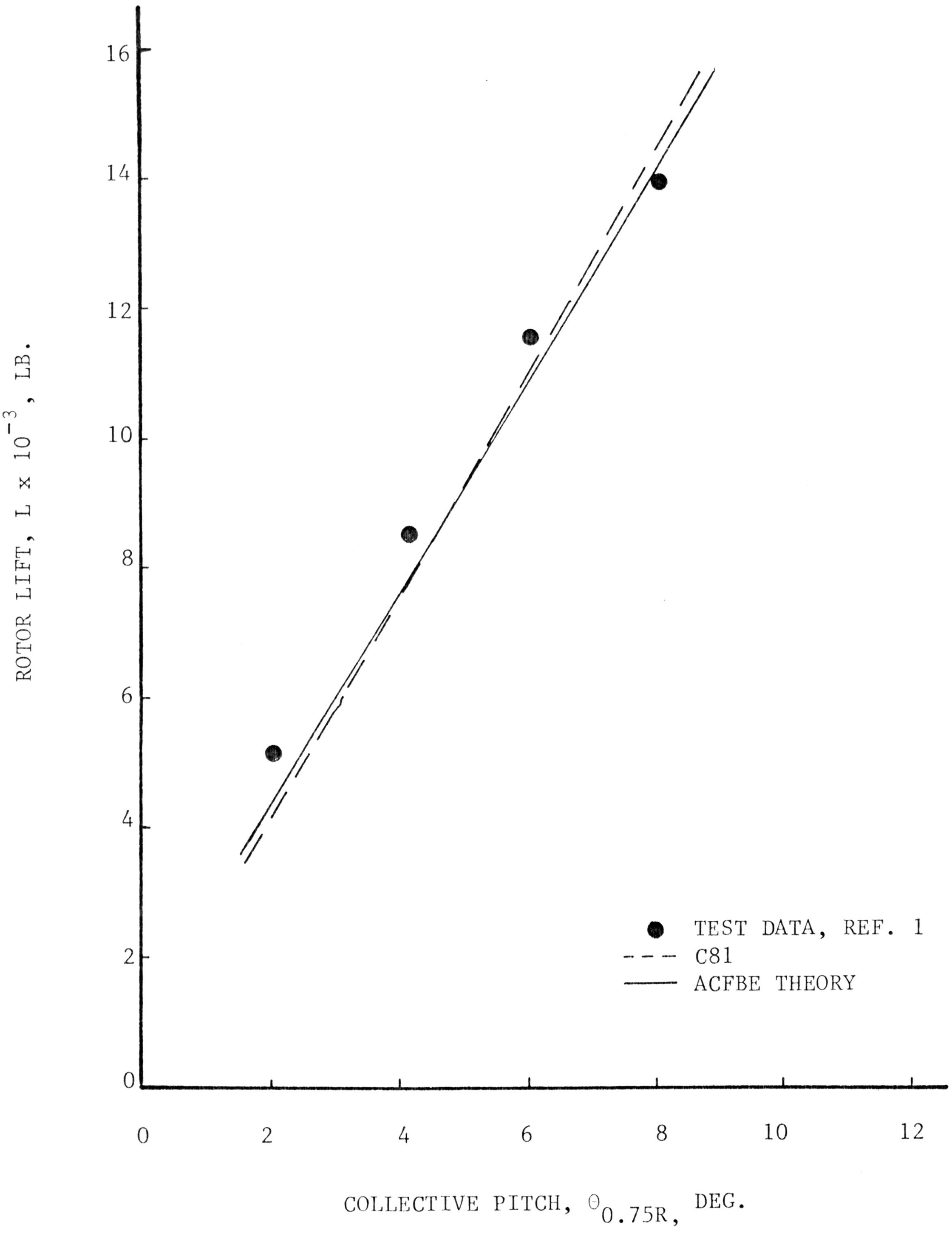
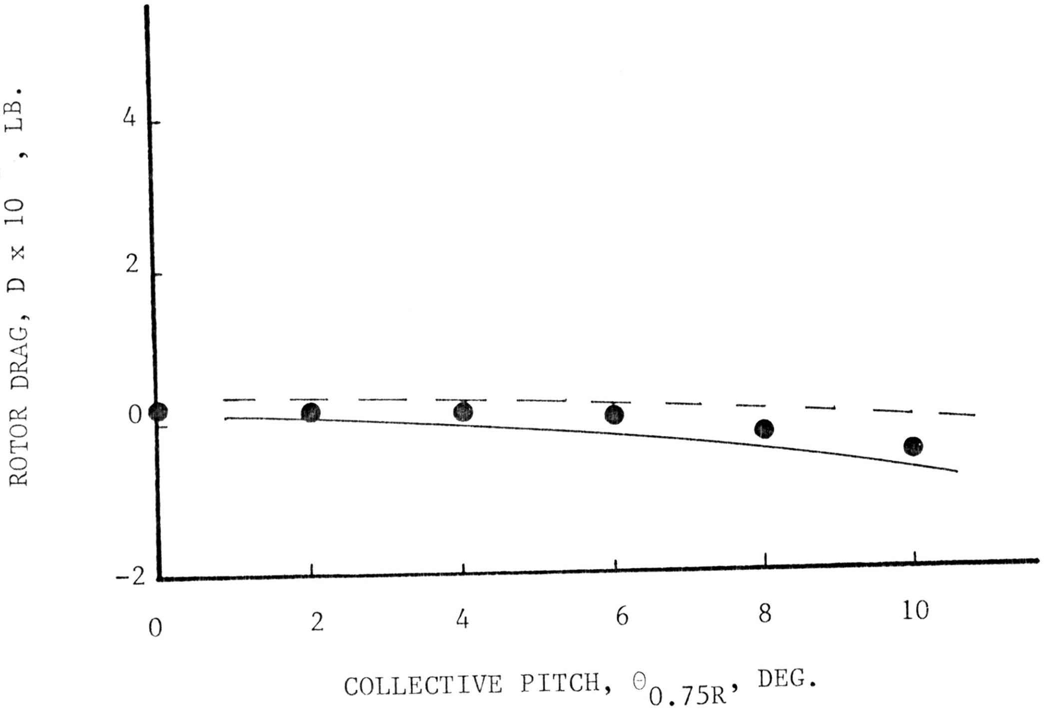


Figure 5. A Comparison of Rotor Lift, Drag, and Power Versus Collective Pitch at $\mu=0.3$, $\alpha_m=0^0$



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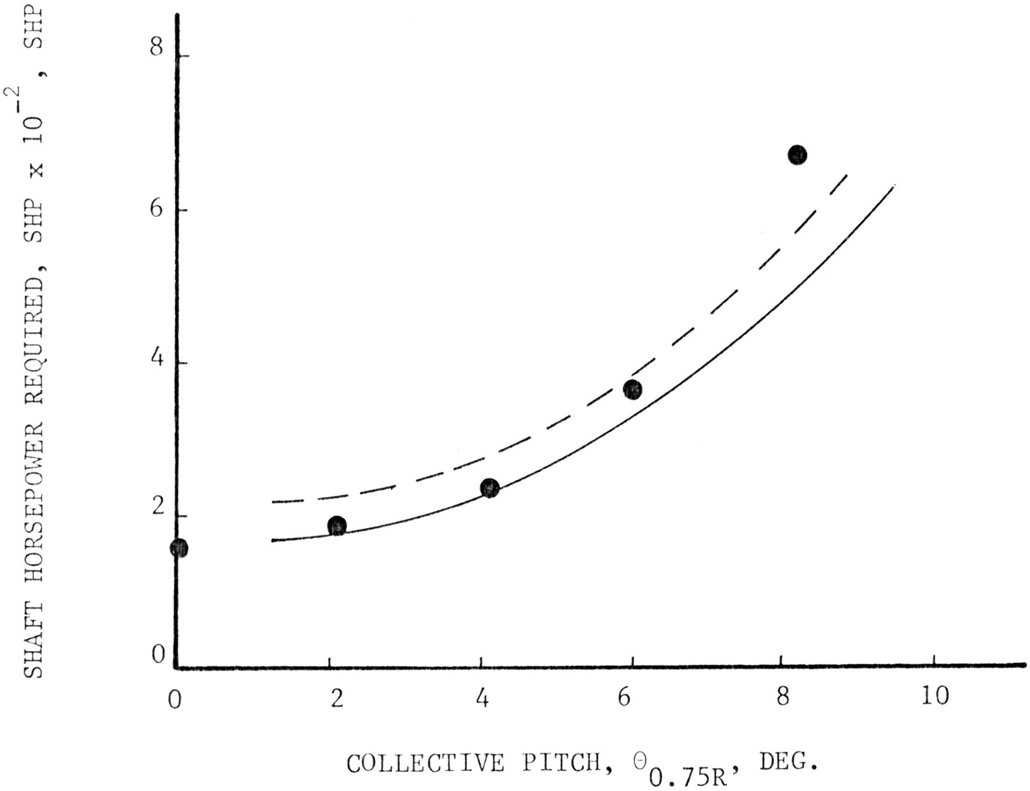

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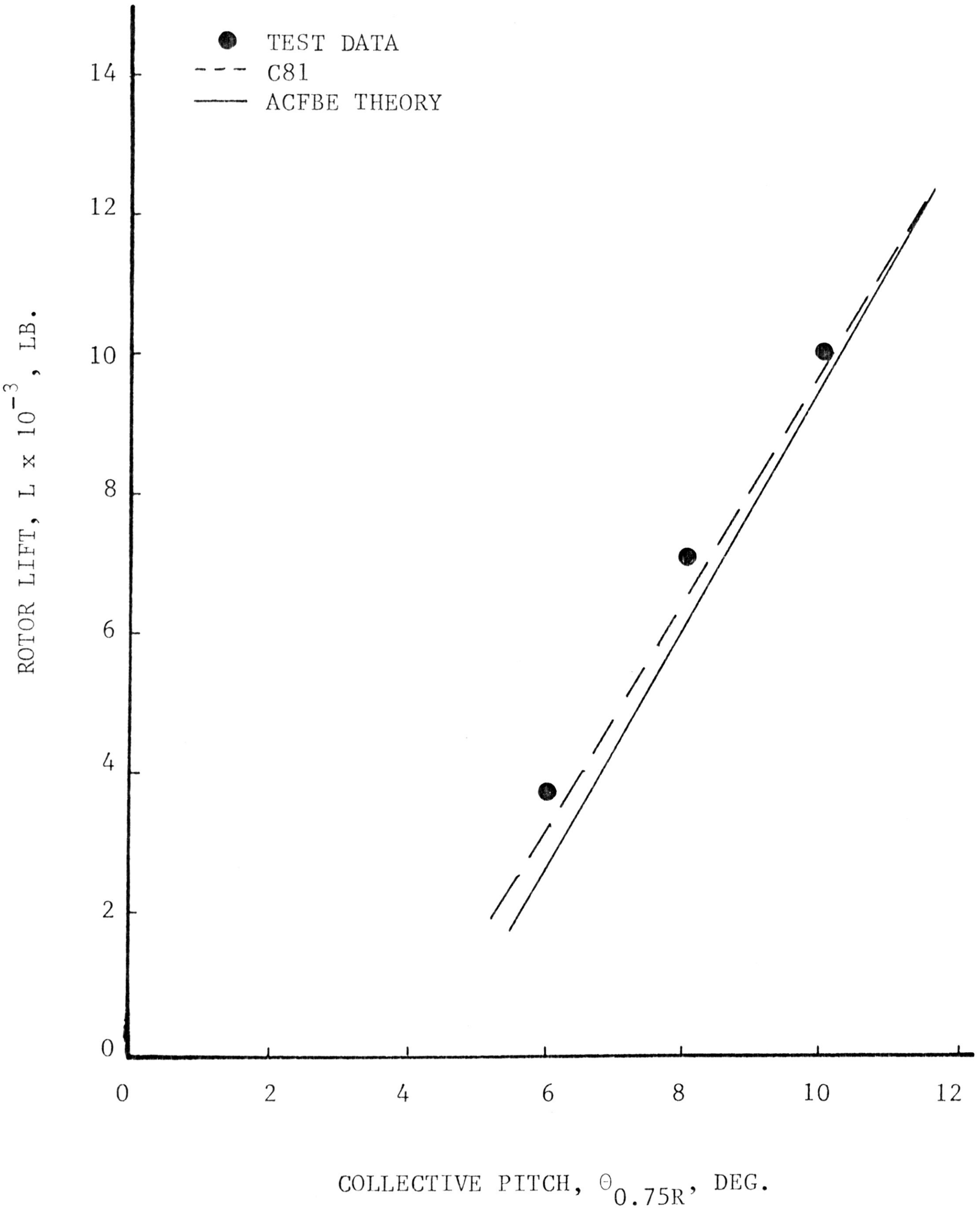

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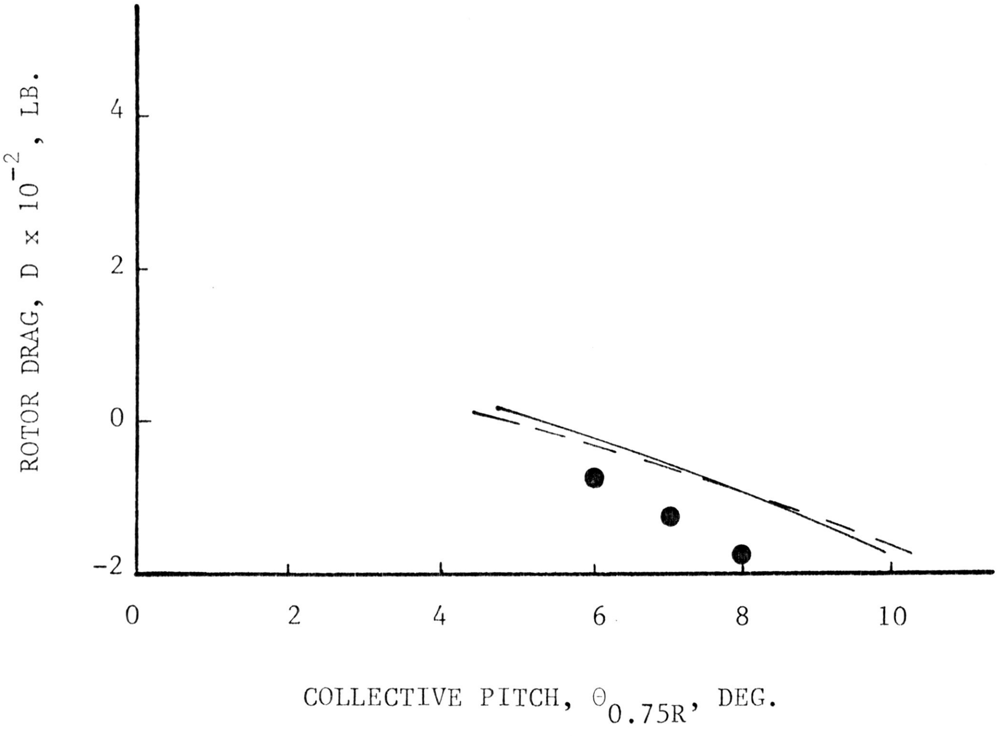

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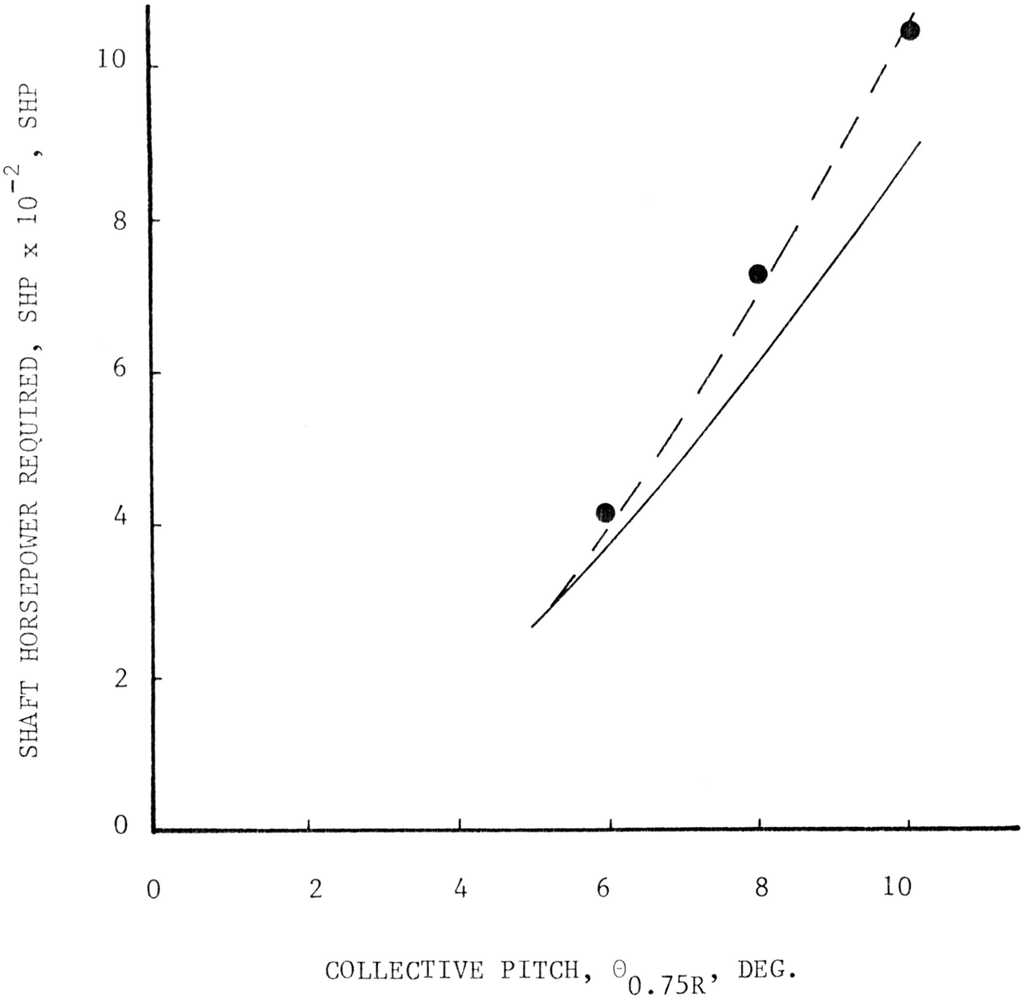

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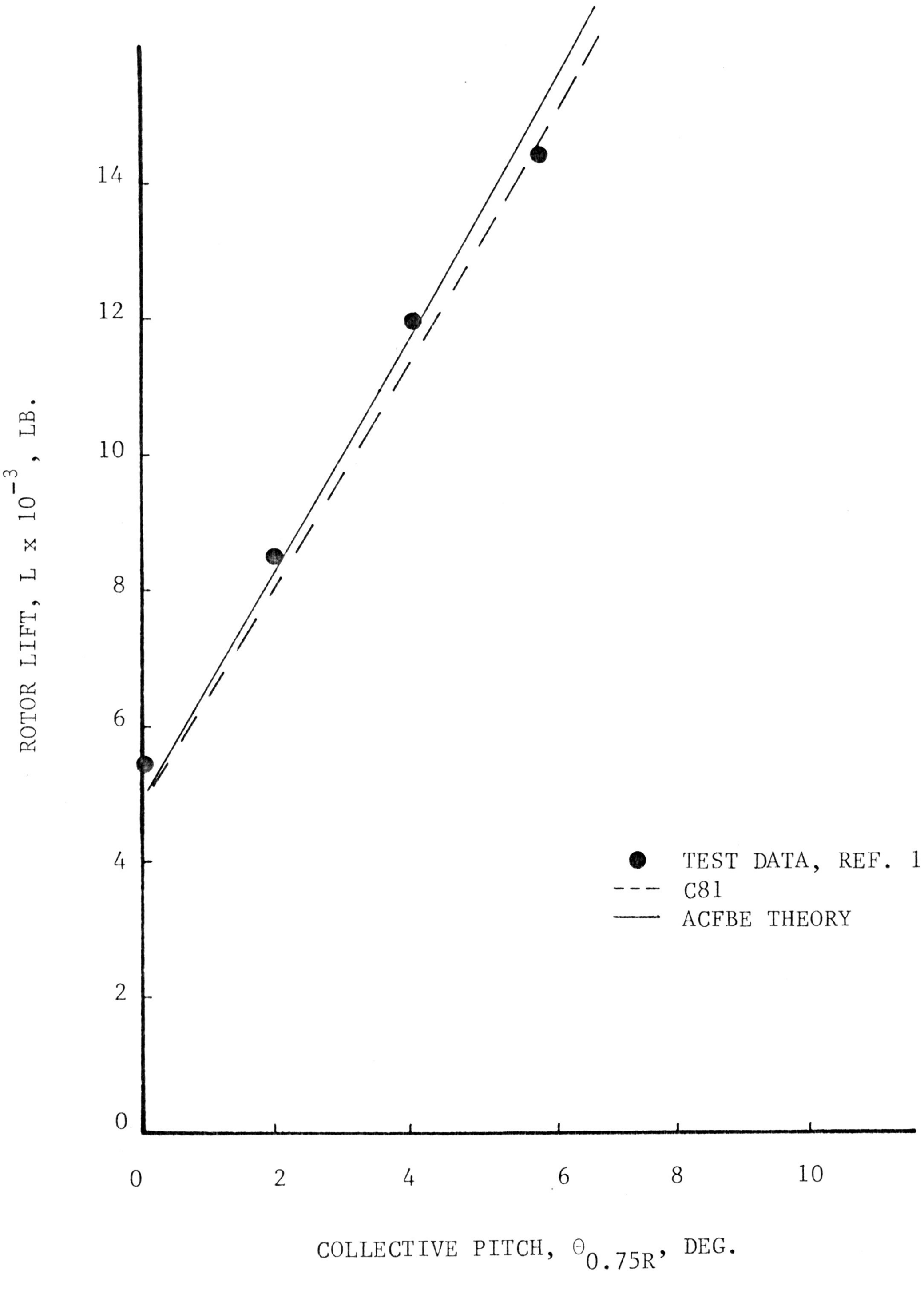
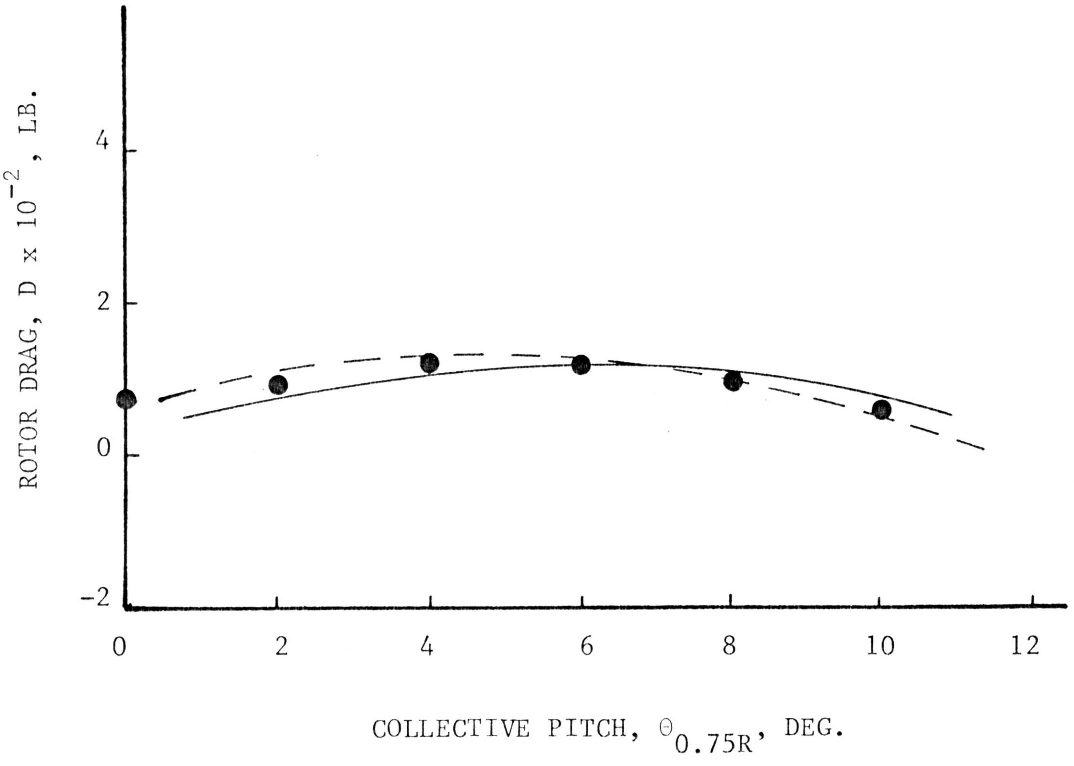


Figure 7. A Comparison of Rotor Lift, Drag, and Power Versus Collective Pitch at $\mu=0.3$, $\alpha_m=-5^0$



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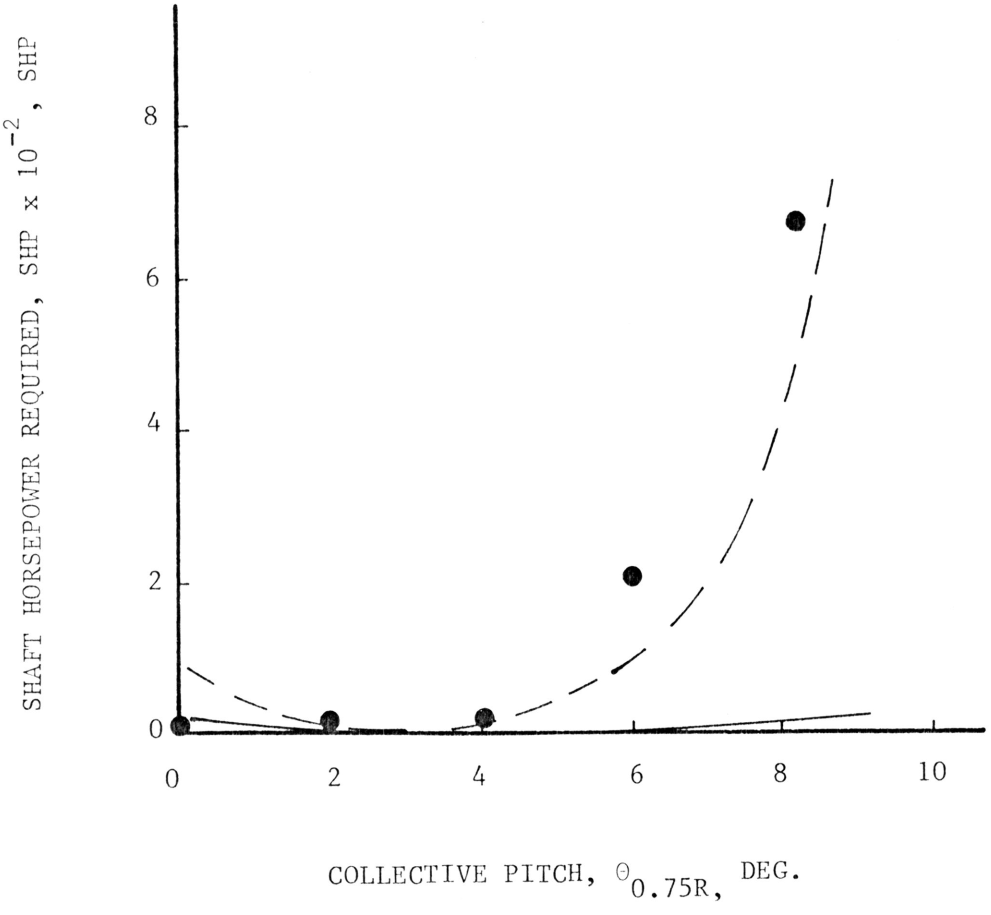

Figure 7. Continued

CONCLUSIONS

A comparison of several rotor performance prediction methods and correlation with experimental results has been accomplished. Methods involving energy theory or blade element theory, without incorporation of collective and cyclic controls, are inadequate for flight simulation. The use of these methods should be confined to problems where an evaluation of general rotor performance is desired. Methods involving blade element theory where the rotor blade is divided into many segments for performance evaluation are to complex for flight simulation programs unless blade airloads data is a requirement. These methods in general require computation times which exceed the requirements which must be met for real time flight simulation.

Methods incorporating closed form blade element theory predict overall rotor performance accurately for simulation programs in approximately one-sixth to one-tenth the time required for programs which evaluate a blade in approximately twenty segments (C81 type). However, mathematical simulation of compressibility and stall effects must be carefully incorporated into these expressions if they are to be utilized to their full potential. Accomplishment of this task can be made without significantly increasing computer computational times. Therefore, until technology produces significantly more advanced computer designs, closed form blade element theory should be one of the most efficient ways to predict overall rotor performance for flight simulation purposes.

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APPENDIX A

Description of Performance Prediction Methods

Energy Method

A relatively simple approach to forward flight rotor performance calculation is the energy method.[4,5,6,7] The power required by the rotor may be divided into essentially two elements for level flight. These elements are the power needed to sustain lift, induced power, and the power required to overcome drag, profile power. References 4 and 5 provide an extensive development of the energy equations. Reference 6 presents a very elementary analysis of energy methods and is recommended for students who have never worked with helicopter aerodynamics before. Reference 7 provides for analysis of effects due to mach number and maneuvering using energy methods. General assumptions made in using advanced energy methods are

1. Triangular flow distributions through the rotor disc
2. Simple corrections for the lift lost due to reversed flow effects on retreating blades
3. Average values for blade drag coefficients and induced velocity across the rotor disc
4. Empirical values for tip loss and compressibility effects for the rotor disc.

In using energy theory, there are also certain limitations which must be noted, these are

1. There are no provisions for analysis of rotor parameters such as twist or control setting
2. There are no provisions for analysis of airflow parameters to calculate loads
3. Values for overall rotor drag forces, side forces, and blade flapping cannot be obtained.

In conclusion, energy methods are ideal for making first hand approximations of rotor thrust and powered required. However, beyond this, the method is extremely limited.

Simple Blade Element Theory

Gessow and Myers[8] have compiled and combined the theory and improvements of many early rotor performance investigators. The equations derived are for a helicopter rotor as a function of collective pitch for an untwisted, untapered rotor blade. The equations presented and the method of overall integration into a performance prediction tool as presented in reference 8 should be considered ideal for rotor analysis when an approximation for rotor drag, side force, and blade flapping is required along with the prediction of rotor thrust and power required. Assumptions made in the theory as presented by Gessow and Myer are

1. The blades are untapered and untwisted
2. Radial components of air velocity along the blade are neglected
3. The induced velocity is constant across the rotor disc
4. The blade flapping and air inflow angles are small and small angle assumptions can be made
5. The second and higher harmonies of the blade flapping Fourier series are negligible
6. The blades continue into the center of rotation and the flapping hinge is located on the axis of rotation
7. The profile drag coefficient and lift curve slope are assumed constant along the blade span
8. The effect of the reversed flow region is negligible
9. The blades are infinitely rigid in all directions

10. Tip losses are ignored unless corrected by empirical methods.

General theory limitations may be summarized as follows

1. The effects of cyclic controls on the rotor are not analyzed
2. Provisions for analysis of airloads along the blade are not included.

In conclusion, simple blade element theory is quite accurate for first hand approximation of basic helicopter rotor performance. However, for use in flight simulation studies, the general lack of cyclic control provisions requires further development of blade element theory.

Advanced Closed Form Blade Element Theory

Development of simple blade element theory by Ferguson[9] provides for analysis of cyclic as well as collective control position effects on the rotor in forward flight. These equations can be incorporated to provide for inclusion of non-linear twisted blades, and a more detailed analysis of the general effects of compressibility.[10] This method provides for closed form expressions for rotor thrust, power required, drag, side force, and blade flapping. These equations have been integrated into a simple rotor performance program for comparison in this report with other data. Since this method is the primary method of importance in this report, a listing of these equations is presented in Appendix B. General assumptions made in the method are

1. The blades are untapered but may be twisted
2. Average values for lift curve slope and profile drag coefficient are used over the span of the blade
3. The blade angle of attack α is approximated by $\sin\alpha$. Substitution of $\sin\alpha$ for α in the blade

element equations makes it possible to develop equations for rotor forces without restricting blade pitch, Θ, and inflow angle, ϕ, to small angles.

4. Blade stall and compressibility effects 10 are approximated by limiting the maximum rotor thrust coefficient as a function of advance ratio
5. Blade flapping with respect to the mast is assumed small such that small angle approximations are made and higher order harmonics are ignored
6. The induced velocity is constant across the rotor disc
7. The flapping hinge is located on the center of rotation
8. The blades are infinitely rigid in all directions
9. Tip losses are empirically derived.[8]

General theory limitations may be summarized as follows

1. Provisions for simple analysis of airloads along the blade cannot be provided for the equations in the closed form expressions
2. Studies of effects on individual blade sections such as unsteady aerodynamics cannot be evaluated.

In conclusion, advanced closed form blade element theory provides for prediction of overall steady equilibrium rotor performance in forward flight and should be considered ideal for use in flight simulation programs where accurate approximations to overall rotor performance are required. For detail rotor aerodynamic studies however, other rotor performance calculation methods should be used.

Advanced Rotor Performance Prediction

There are several methods developed for detail rotor performance evaluation. One of the most widely circulated rotor performance programs is the Rotorcraft Flight Simulation Program (C81) which was developed by Bell Helicopter Textron.[11] Program C81 is a general purpose rotorcraft flight simulation program; however, for this study only the rotor simulation capability was used. The rotor simulation is divided into a trim procedure and a loads calculation portion. The limitations and assumptions made in the program are user optional and cannot be described in this report. Reference 11 (a three volume series) provides for a complete description of the program.

This program can be considered the present state of the art in rotor performance prediction for flight simulation purposes. However, if loads predictions are not desired, the equations developed for this program provide for analysis which need not be required for general flight simulation.

APPENDIX B

Advanced Closed Form Blade Element Equations

Using the simplified theory of Gessow and Myer, the following closed form expressions have been developed.[9] These equations require the use of references 8 and 10 for full integration into numerical flight simulation programs. Forces and sign conventions are defined in Figure 8.

Thrust

$$\begin{aligned}\frac{2C_T}{\sigma a} &= C_{S2} + \frac{1}{2}\mu^2 C_{S0} - (\lambda - \frac{1}{2}\mu\hat{p})(C_{C1} + C_{D_f}) \\ &- \frac{A_1}{2}[a_0\mu C_{S1} - b_1(C_{S2} + \frac{1}{4}\mu^2 C_{S0}) - \hat{q}C_{S2}] \\ &- \frac{B_1}{2}[2\mu C_{C1} + a_1(C_{S2} - \frac{1}{4}\mu^2 C_{S0}) + \mu\lambda C_{S0} - \hat{p}C_{S2}]\end{aligned}$$

Power

$$\begin{aligned}\frac{2C_P}{\sigma a} &= C_{C1}[-\lambda(\lambda-\mu a_1) - \frac{\mu^2}{2}(a_0^2 - \frac{1}{4}b_1^2 - \frac{3}{4}a_1^2)] \\ &+ C_{C2}\mu a_0(b_1+\hat{q}) - \frac{1}{2}C_{C3}[(b_1+\hat{q})^2 + (a_1-\hat{p})^2] \\ &+ C_{S2}(\lambda - \frac{1}{2}\mu\hat{p}) + \frac{1}{2}C_{D_f}(1+\mu^2) \\ &+ A_1\{\frac{1}{8}C_{C1}\mu^2 b_1 - \frac{1}{2}C_{C2}a_0\mu + \frac{1}{2}C_{C3}(b_1+\hat{q}) \\ &- \mu a_0 C_{S1}(\lambda - \frac{3}{4}\mu a_1) + C_{S2}[\lambda(b_1+\hat{q}) \\ &- \frac{1}{2}\mu a_1 b_1 - \frac{1}{4}\mu b_1\hat{p} - \frac{3}{4}\mu a_1\hat{q}]\}\end{aligned}$$

(continued next page)

$$+ \quad B_1\{-\frac{1}{2}\mu(\lambda-\frac{1}{4}\mu a_1)C_{C1} - \frac{1}{2}C_{C3}(a_1-\hat{p})$$

$$- \quad C_{S2}[(\lambda-\frac{1}{4}\mu a_1)(a_1-\hat{p}) + \frac{1}{4}\mu b_1(b_1+\hat{q}+\frac{1}{4}C_{S1}\mu^2 a_0 b_1]\}$$

H-Force

$$\frac{2C_H}{\sigma a} = C_{C1}[-\frac{3}{2}\lambda a_1 + \lambda\hat{p} - \frac{1}{8}\mu(b_1\hat{q}+a_1\hat{p}) + \frac{1}{2}\mu(\frac{a_1{}^2}{4}+a_0{}^2)]$$

$$- \quad \frac{1}{2}C_{C2}a_0(b_1+\hat{q}) + \frac{1}{2}C_{S0}\mu\lambda + C_{S2}(a_1-\frac{1}{2}\hat{p})$$

$$+ \quad C_{D_f}[\mu-\frac{1}{2}\lambda a_1 + \frac{1}{2}\mu(a_0{}^2+\frac{1}{2}a_1{}^2+\frac{1}{2}b_1{}^2) - \frac{1}{3}a_0 b_1(1+\frac{3}{2}\mu^2)$$

$$- \quad \frac{1}{3}a_0\hat{q} + \frac{1}{8}\mu(b_1\hat{q} + a_1\hat{p})]$$

$$+ \quad \frac{1}{2}A_1\{\frac{1}{4}C_{C1}\mu\hat{q} + a_0 C_{C2} + \frac{1}{4}C_{S0}\mu\lambda b_1 + a_0 C_{S1}(\lambda-\frac{7}{4}\mu a_1$$

$$+ \quad \frac{1}{4}\mu\hat{p}) + C_{S2}(a_1 b_1 + \frac{5}{4}a_1\hat{q} - \frac{1}{4}b_1\hat{p} - \frac{1}{2}\hat{p}\hat{q})\}$$

$$- \quad \frac{1}{2}B_1\{C_{C1}(\lambda+\mu a_1 - \frac{3}{8}\mu\hat{p}) + \lambda C_{S0}(\lambda-\frac{1}{4}\mu a_1) - \frac{1}{4}C_{S1}\mu a_0$$

$$(q-b_1) + C_{S2}(a_1{}^2 - \frac{7}{4}a_1\hat{p} + \frac{1}{4}b_1\hat{q} + \frac{1}{4}\hat{q}^2 + \frac{3}{4}\hat{p}^2)\}$$

Y-Force

$$\frac{2C_y}{\sigma a} = \frac{1}{2}C_{S0}\mu^2 b_1 - \frac{3}{2}C_{S1}\mu a_0 + C_{S2}(b_1+\frac{1}{2}\hat{q}) + \frac{3}{2}C_{C0}\mu a_0(\lambda-\frac{3}{2}\mu a_1)$$

$$- \quad C_{C1}[\frac{3}{2}b_1(\lambda-\frac{1}{3}\mu a_1) + \lambda\hat{q} - \frac{1}{8}\mu(5b_1\hat{p} + 3a_1\hat{q})]$$

$$+ \quad \frac{1}{2}C_{C2}a_0(a_1-\hat{p}) + C_{D_f}[\mu a_0(\lambda-\frac{1}{2}\mu a_1) - \frac{1}{2}\lambda b_1 + \frac{1}{8}\mu$$

$$(a_1\hat{q} + 3b_1\hat{p}) + \frac{1}{3}a_0(a_1 - \hat{p})]$$

(continued)

$$+ \quad \frac{1}{2}A_1\{C_{S0}[(\lambda - \frac{7}{4}\mu a_1) + \frac{1}{4}\mu^2(4a_0{}^2 + b_1{}^2 + 3a_1{}^2)]$$

$$- \quad \frac{1}{8}C_{S1}\mu a_0(9b_1 + 5\hat{q}) + C_{S2}[b_1{}^2 + \frac{7}{4}b_1\hat{q}$$

$$+ \quad \frac{1}{4}a_1\hat{p} + \frac{1}{4}(3\hat{q}^2 + \hat{p}^2)] + C_{C1}(\lambda - \mu a_1 - \frac{1}{4}\mu\hat{p})\}$$

$$+ \quad \frac{1}{2}B_1\{- \frac{5}{4}C_{S0}\mu b_1(\lambda - \frac{2}{5}\mu a_1) + a_0C_{S1}(\lambda + \frac{3}{4}\mu a_1 - \frac{5}{4}\mu\hat{p})$$

$$- \quad C_{S2}(a_1b_1 + \frac{1}{4}a_1\hat{q} - \frac{5}{4}\hat{p}b_1 - \frac{1}{2}\hat{p}\hat{q})$$

$$+ \quad C_{C0}a_0\mu^2 - \mu C_{C1}(2b_1 + \frac{1}{4}\hat{q}) + a_0C_{C2}\}$$

Blade Flapping

$$\left[\begin{array}{c:c:c} \frac{4}{\gamma} + \frac{1}{2}\mu A_1 C_{S2} & \frac{1}{2}B_1(C_{S3} - \frac{1}{4}\mu^2 C_{S1}) & -\frac{1}{2}A_1(C_{S3} + \frac{1}{4}\mu^2 C_{S1}) \\ \frac{1}{4}\mu^2 A_1 C_{S1} & (C_{C3} - \frac{1}{4}\mu^2 C_{C1}) + \frac{1}{2}\mu B_1 C_{S2} + \frac{1}{2}C_{D_f}(1-\mu^2) & -\frac{1}{2}\mu A_1 C_{S2} - 2K_H \\ -\mu C_{C2} - \frac{1}{4}\mu^2 B_1 C_{S1} - \frac{2}{3}\mu C_{D_f} & \frac{1}{2}\mu A_1 C_{S2} + 2K_H & (C_{C3} + \frac{1}{4}\mu^2 C_{C1}) + \frac{1}{2}\mu B_1 C_{S2} + \frac{1}{2}C_{D_f}(1+\mu^2) \end{array}\right] \left\{\begin{array}{c} a_o \\ a_1 \\ b_1 \end{array}\right\}$$

$$\left[\begin{array}{l} (C_{S3} + \frac{1}{2}\mu^2 C_{S1}) - (\lambda - \frac{1}{2}\mu\hat{p})C_{C2} + \frac{1}{2}\hat{q}A_1 C_{S3} - \frac{1}{2}B_1(2\mu C_{C2} + \mu\lambda C_{S1} - \hat{p}C_{S3}) - \frac{1}{3}C_{D_f}(2\lambda - \mu\hat{p}) - \frac{4M_w}{\gamma} \\ \hdashline 2\mu C_{S2} - \lambda\mu C_{C1} + \hat{p}C_{C3} + \frac{1}{4}\mu\hat{q}A_1 C_{S2} - \frac{1}{2}B_1[C_{C3} + \frac{3}{2}\mu^2 C_{C1} + C_{S2}(\lambda - \frac{3}{2}\mu\hat{p})] - C_{D_f}(\lambda\mu - \frac{1}{2}\hat{p}) - \frac{8\hat{q}}{\gamma} \\ \hdashline -\hat{q}C_{C3} + A_1[C_{C3} + \frac{1}{4}\mu^2 C_{C1} + C_{S2}(\lambda - \frac{1}{4}\mu\hat{p})] - \frac{1}{4}\mu\hat{q}B_1 C_{S2} - \frac{1}{2}\hat{q}C_{D_f} - \frac{8\hat{p}}{\gamma} \end{array}\right]$$

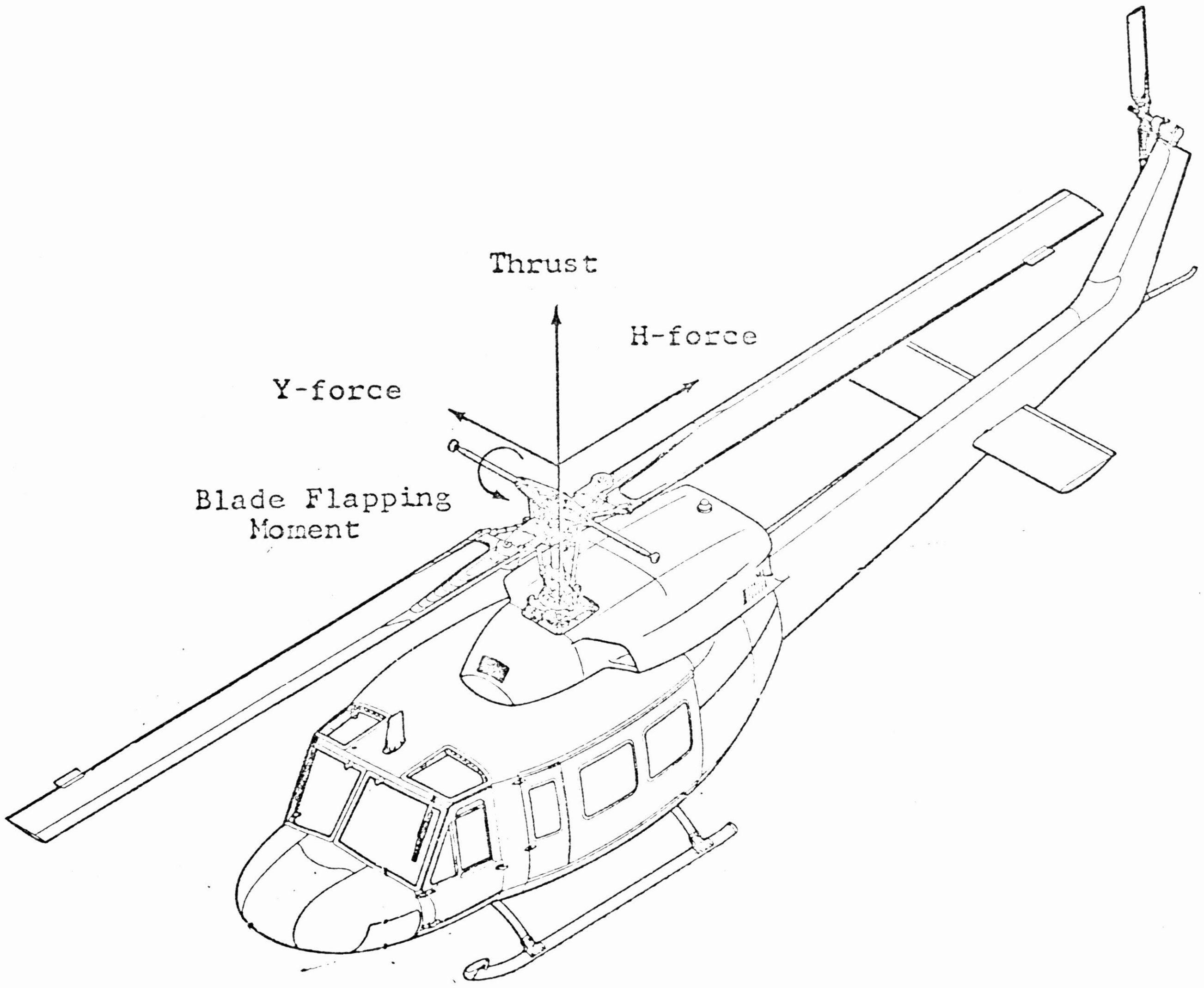

Figure 8. Forces on the Helicopter