# STREMGTHENING OF TUBULAR COMPRESSION <br> MEMBERS 

A Senior Thesis
by

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#### Abstract

Strengthening of Tubular Compression Members April 1988

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The weakening of tubular compression members in offshore oil production facilities is a constant problem the oil industry must deal with. The purpose of this project was to evaluate different methods to reinforce damaged tubular compression members. Discussed in this paper are the results of analytical and experimental investigations into several reinforcing methods.


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## CHAPTER I

## INTRODUCTION AND SUMMARY

## Tubular Compression Members

Tubular members are the most common component in offshore oil production facilities, particularly in jacket-type platform structures. Various reasons exist for the excessive use of these members: for instance, the tubular cross sections offer great local strength against sudden impact loading, have outstanding torsional rigidity, minimize the hydrodynamic forces, and minimize the surface area that is subjected to the destructive forces of corrosion. Far outweighing these characteristics, however, is their unique quality of having identical buckling strength in all directions.

These members are not indestructible, however, and they can become damaged by collisions with ships, fatigue in the steel, corrosion, or destructive weather like hurricanes. Also, more strength may be required after the structure is built due to changes in the production needs of the platform. Underwater welding to repair or further strengthen these components is very difficult, therefore, an alternate group of methods is needed to increase the
strength of these tubular compression members.

## Objectives

The purpose of this study was to evaluate different methods to reinforce tubular compression members. All the methods considered can be added to the structure before or after it has been built. These methods were:
(1) To bolt on a simple steel sleeve that will surround the damaged area;
(2) To completely fill the weakened section with concrete;
(3) To place a second tubular member into the existing tube and fill the annulus with concrete.

These methods were first evaluated analytically to determine their cross sectional characteristics; next models of the aforementioned repair techniques were taken into the laboratory and tested in compression to determine their performance. These cross sections are depicted graphically in Figure 1.1. The results of the test data were then compared to the analytical results.

## Summary of Results

Specimens from both the inelastic and elastic buckling modes were tested for hollow, concrete filled, and concrete grouted cross sections. The results showed good trends


Figure 1.1 - Cross Sections for the Proposed Repair Techniquescompared to the theoretical strengthening curves that werecalculated. All three sets of test values for the criticalbuckling loads were slightly higher than their theoreticalcounterparts, but they plotted relatively parallel to thecomputed curves. The strengthening methods offered overfifty percent increase in strength compared to theunstiffened tube.

## CHAPTER I I

## METHODS USED IN ANALYTICAL ANALYSIS

## Introduction

In the initial literature review on this strengthening problem, it became evident that a complete understanding of several analytical techniques was necessary to compute and compare these proposed strengthening techniques. These solution practices are known as Euler or elastic buckling equations and inelastic buckling techniques.
The loads considered in this analysis were of the static nature only. Dynamic forces like those due to wave impact were not dealt with. We also assumed that the ends of the members were simple pin connections.

## Analytical Techniques

## Elastic (Euler) Buckling

The swiss mathematician, Leonhard Euler (1707-1783), developed the method now widely used to solve for the critical buckling load for long slender columns. This analytical technique is used to determine the maximum load a column can support before it buckles. If one takes a simple column loaded in axial compression like the one in

Figure 2.1, imposes a displacement, and cuts out a free body diagram, the following equation can be formed to represent the internal moment, $M_{r}$, in the section:

$$
\begin{equation*}
M r=E I \frac{d^{2} y}{d x^{2}}=P(\delta-y) \tag{2.1}
\end{equation*}
$$

where $E$ is the modulus of elasticity, I is the moment of inertia, $P$ is the load, and $\delta-y$ is the deflection at the point in question. By simply rearranging this equation, the relationship can be represented by:

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}+\frac{P y}{E I}=\frac{P \delta}{E I} \tag{2.2}
\end{equation*}
$$

This equation is a second order linear differential equation with constant coefficients and a constant on the right side. Methods beyond the scope of this thesis have established a solution to equations like Equation 2.2 in the form:

$$
\begin{equation*}
y=A \sin (p x)+B \cos (p x)+C \tag{2.3}
\end{equation*}
$$

By taking the first and second derivatives of Equation 2.3, Equations 2.4 and 2.5 can be arrived at as:

$$
\begin{equation*}
\frac{d y}{d x}=p A \cos (p x)-p B \sin (p x) \tag{2.4}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}=-p^{2} A \sin (p x)-p^{2} \operatorname{Bcos}(p x) \tag{2.5}
\end{equation*}
$$

Now if Equations 2.4 and 2.5 are substituted into Equation 2.2 and like terms are grouped, we find:

$$
\begin{equation*}
\left(-p^{2}+\frac{F}{E I}\right)(A \sin (p x)+B \cos (p x))+\frac{P C}{E I}=\frac{P \delta}{E I} \tag{2.6}
\end{equation*}
$$



FREE BODY DIAGRAM


Figure 2.1 - Euler Buckling Characteristics

Pc/EI and Pס/EI are both constants, and Asin(px)-Bcos(px) can never equal zero; therefore, $p$ must equal the square root of $P / E I$ so that the first term in Equation 2.6 will go to zero. If this is done, the remaining equation then looks like this:

$$
\begin{equation*}
\frac{P C}{E I}=\frac{P \delta}{E I} \tag{2.7}
\end{equation*}
$$

From observation of Equation 2.7, it is evident that $c$ is equal to $\delta$. Therefore, by substituting into Equation 2.3, the expression becomes:

$$
\begin{equation*}
y=A \sin \left[\left(\sqrt{\frac{P}{E I}}\right) x\right]+\operatorname{Bcos}\left[\left(\sqrt{\frac{P}{E I}}\right) x\right]+\delta \tag{2.8}
\end{equation*}
$$

The constants $A$ and $B$ can now be solved for by using the boundary conditions at the end of the column. These conditions are: at the end of the column ( $x=0$ ), the deflection is zero $(y=0)$, and the slope of the member is zero $(d y / d x=0) . \quad$ By substituting these conditions into Equation 2.8 and its derivative, the constants $A$ and $B$ are found to be zero and $-\delta$, respectively. Therefore:

$$
\begin{equation*}
y=\delta-\delta \cos \left[\left(\sqrt{\frac{F}{E} I}\right) x\right] \tag{2.9}
\end{equation*}
$$

The physical requirement $y=\delta$, at one half the length of the column, must be satisfied. For this to be true:

$$
\begin{equation*}
\cos \left[\left(\sqrt{\frac{P}{E I}}\right) \frac{L_{2}}{2}\right]=0 \tag{2.10}
\end{equation*}
$$

This is satisfied only when $\sqrt{P / E I}$ times $L / 2$ equals some multiple of $\pi / 2$.

When $\sqrt{P} / E I$ times $L / 2$ equals $\pi / 2$, the criterion for the
first mode of buckling has been met. Since this is the first mode of buckling, it is therefore the critical buckling mode. If:

$$
\begin{equation*}
\left[\left(\sqrt{\frac{P}{E} I}\right) \frac{L}{2}\right]=\frac{\pi}{2} \tag{2.11}
\end{equation*}
$$

is the critical buckling mode, then, by rearranging Equation 2.11, Euler's critical buckling load equation is obtained in terms of the cross sectional characteristics as:

$$
\begin{equation*}
P=\left(\frac{\pi}{L}\right) 2 E I=\frac{\pi^{2} E I}{L^{2}} \tag{2.12}
\end{equation*}
$$

Euler's theory is valid for long slender columns having a slenderness ratio greater than about 140 for steel columns. The slenderness ratio is:

$$
\begin{equation*}
\underset{\text { slenderness }}{\text { slio }}=\frac{k L}{r} \tag{2.13}
\end{equation*}
$$

where $L$ is the length of the member, $k$ is the effective length constant, and $r$ is the least radius of gyration of the member cross section. For members connected at both ends, the $k$ value ranges from 1 for perfectly pinned ends to 0.5 for a column with two fixed or built-in ends.

If the column is long and slender, Euler's equation can be used to calculate the critical buckling load. This load is expressed as:

$$
\begin{equation*}
P_{\text {crit }}=\frac{\pi^{2} \mathrm{EI}}{(\mathrm{~kL})^{2}} \tag{2.14}
\end{equation*}
$$

where $P_{\text {crit }}$ is the critical buckling load, $E$ is the modulus of elasticity for the material used in the column, I is the
moment of inertia for the column's cross section about its bending axis, $k$ is the effective length constant, and $L$ is the actual length of the member.

## Inelastic Buckling

Euler's theory is only valid for those columns that buckle at a stress below the elastic stress of the material. Since the actual tubular sections are geometrically and materially imperfect, the effects of inelastic buckling need to be calculated and considered in any design. The first theory presented for calculating inelastic buckling loads for short, non-Euler columns was proposed by F.R. Engesser. He called his theory the Basic Tangent Modulus Theory, and it was rooted in Euler's elastic theories. The only differences in the two concepts were the values used for the modulus of elasticity in Equation 2.14. From Engesser's Basic Tangent Modulus and Double-Modulus Theories, it was learned that the inelastic buckling modulus is not constant but instead is changing across the cross section at the time of failure. His tangent modulus is defined as the slope of the stress-strain diagram for a material at a particular stress. This modulus is therefore a function of stress for stresses beyond the elastic limit. Since the values for this modulus were hard to compute for many different materials, the American Institute of Steel Construction's

Column Research Council used the theories of Euler and Engesser as well as actual data to developed the following equation for the allowable stress in a given cross section:

$$
\begin{equation*}
F_{a}=\frac{F y}{F S}\left[1-\frac{(\mathrm{kL} / \mathrm{r})^{2}}{2 C_{\mathrm{c}}^{2}}\right] \tag{2.15}
\end{equation*}
$$

where $F_{a}$ is the allowable stress, $F_{y}$ is the proportional limit of the material, FS is a factor of safety term, kL/r is the slenderness ratio, and $C_{C}$ is a maximum value of the slenderness ratio.
$C_{c}$ is the column formula that defines whether or not the section in question is likely to buckle in the elastic or inelastic region. This $C_{c}$ term is the slenderness ratio criterion mentioned in the Euler buckling section of this paper, and it considers the residual stress in the column. $C_{c}$ is defined as:

$$
\begin{equation*}
C_{c}=\left[\frac{2 \pi^{2} E}{F_{y}}\right] 0.5 \tag{2.16}
\end{equation*}
$$

where $E$ is the modulus of elasticity in ksi, and $F y$ is the proportional limit in ksi.

Initial Calculations for a Sample

## Tubular Section

To become familiar with the analysis techniques used in the project, buckling curves for several hypothetical cross sections were calculated. One of the cross sections that was considered had the following characteristics: an outside
diameter of the tubular cross section was assumed to be forty-eight inches and a wall thickness of one quarter of an inch; both ends were assumed to be pinned, which means that the effective length constant is one; the proportional limit, $F_{y}$, was 36 ksi ; the modulus of elasticity, $E$, was $29 \times 10^{\wedge} 6 \mathrm{psi}$; and the factor of safety, FS, was one.

After these assumptions were made, the Euler buckling curve was found by calculating the critical loads at incremental lengths of five feet starting at five feet and going to four hundred feet. For a sample calculation with an L equal to 5 feet, the area and moment of inertia must first be calculated for the cross section. The equations for the area and moment of inertia are:

$$
\begin{equation*}
\left.A=\frac{\pi(\text { Doutside } 2-\text { Dinside }}{}=2\right) \tag{2.17}
\end{equation*}
$$

and

$$
\begin{equation*}
I=\frac{\pi\left(\text { Doutside }{ }^{4}-\text { Dinside }^{4}\right)}{64} \tag{2.18}
\end{equation*}
$$

respectively. The area of the aforementioned cross section is 37.50 inches ${ }^{2}$, and its moment of inertia is $10,688.87$ inches ${ }^{4}$. The critical buckling force, using equation 2.14, is $849,820.18$ kips. Similar calculations were performed for the lengths mentioned previously. The results are displayed in Table 2.1.

The inelastic buckling curve was also found by

Table 2.1 - Euler Buckling Curve

| FOR $t=0.25$ <br> STRESS,F <br> (ksi) | L <br> (ft) <br> $=====================================$ <br> 22653.75 | 5.00 | 3.55 |
| :---: | ---: | ---: | ---: |
| 549579.10 |  |  |  |
| 5663.44 | 10.00 | 7.11 | 212394.78 |
| 1415.86 | 20.00 | 14.22 | 53098.69 |
| 629.27 | 30.00 | 21.33 | 23599.42 |
| 353.96 | 40.00 | 28.44 | 13274.67 |
| 226.54 | 50.00 | 35.55 | 8495.79 |
| 157.32 | 60.00 | 42.65 | 5899.85 |
| 115.58 | 70.00 | 49.76 | 4334.59 |
| 88.49 | 80.00 | 56.87 | 3318.67 |
| 69.92 | 90.00 | 63.98 | 2622.16 |
| 56.63 | 100.00 | 71.09 | 2123.95 |
| 46.81 | 110.00 | 78.20 | 1755.33 |
| 39.33 | 120.00 | 85.31 | 1474.96 |
| 33.51 | 130.00 | 92.42 | 1256.77 |
| 28.90 | 140.00 | 99.53 | 1083.65 |
| 25.17 | 150.00 | 106.64 | 943.98 |
| 22.12 | 160.00 | 113.74 | 829.67 |
| 19.60 | 170.00 | 120.85 | 734.93 |
| 17.48 | 180.00 | 127.96 | 655.54 |
| 15.69 | 190.00 | 135.07 | 588.35 |
| 14.16 | 200.00 | 142.18 | 530.99 |
| 12.84 | 210.00 | 149.29 | 481.62 |
| 11.70 | 220.00 | 156.40 | 438.83 |
| 10.71 | 230.00 | 163.51 | 401.50 |
| 9.83 | 240.00 | 170.62 | 368.74 |
| 9.06 | 250.00 | 177.73 | 339.83 |
| 8.38 | 260.00 | 184.83 | 314.19 |
| 7.77 | 270.00 | 191.94 | 291.35 |
| 7.22 | 280.00 | 199.05 | 270.91 |
| 6.73 | 290.00 | 206.16 | 252.55 |
| 6.29 | 300.00 | 213.27 | 235.99 |
| 5.89 | 310.00 | 220.38 | 221.01 |
| 5.53 | 320.00 | 227.49 | 207.42 |
| 5.20 | 330.00 | 234.60 | 195.04 |
| 4.90 | 340.00 | 241.71 | 183.73 |
| 4.62 | 350.00 | 248.82 | 173.38 |
| 4.37 | 360.00 | 255.92 | 163.88 |
| 4.14 | 370.00 | 263.03 | 155.15 |
| 3.92 | 380.00 | 270.14 | 147.09 |
| 3.72 | 390.00 | 277.25 | 139.64 |
| 3.54 | 400.00 | 284.36 | 132.75 |

calculating the critical buckling loads at the incremental lengths from five feet to two hundred fifty feet at a step of five feet. A sample calculation for $L$ equal to 5 feet can be illustrated as before. The area of steel and moment of inertia for the cross section are still 37.50 inches ${ }^{2}$ and $10,688.87$ inches ${ }^{4}$. The radius of gyration is also needed for the inelastic calculation. Its formula is:

$$
\begin{equation*}
r=\left[\frac{I}{A}\right]^{0.5} \tag{2.19}
\end{equation*}
$$

and its value for this cross section is 16.88 inches. The value for $C_{c}$ in this case was found to be 126.1 using Equation 2.16, and the stress was computed as 35.99 ksi using Equation 2.15. Finally, the critical load was calculated to be $1,349.63$ kips by multiplying the stress with the area of the cross section. This was also done for the length sequence previously mentioned and the results are displayed in Table 2.2.

## Buckling Curves for a Sample

## Tubular Section

In Figure 2.2 the data in Tables 2.1 and 2.2 have been presented graphically. At a $k L / r$ value of about 130 to 140 , the two curves are tangent. This area of the curve is known as the transition zone where the column buckling ceases to be controlled by the inelastic buckling and starts to be characteristic of Euler or elastic buckling. This

Table 2.2 - Inelastic Buckling Curve

| FOR $t=0.25$ <br> STRESS, <br> (kSi) | L <br> $(\mathrm{ft})$ | $\mathrm{L} / \mathrm{R}$ | LOAD, P <br> $(\mathrm{kips})$ |
| :---: | ---: | ---: | ---: |
| $===================================$ |  |  |  |
| 35.99 | 5.00 | 3.55 | 1349.56 |
| 35.94 | 10.00 | 7.11 | 1347.95 |
| 35.77 | 20.00 | 14.22 | 1341.48 |
| 35.48 | 30.00 | 21.33 | 1330.71 |
| 35.08 | 40.00 | 28.44 | 1315.63 |
| 34.56 | 50.00 | 35.55 | 1296.24 |
| 33.93 | 60.00 | 42.65 | 1272.54 |
| 33.18 | 70.00 | 49.76 | 1244.53 |
| 32.32 | 80.00 | 56.87 | 1212.21 |
| 31.35 | 90.00 | 63.98 | 1175.58 |
| 30.25 | 100.00 | 71.09 | 1134.64 |
| 29.05 | 110.00 | 78.20 | 1089.40 |
| 27.73 | 120.00 | 85.31 | 1039.84 |
| 26.29 | 130.00 | 92.42 | 985.98 |
| 24.74 | 140.00 | 99.53 | 927.80 |
| 23.07 | 150.00 | 106.64 | 865.32 |
| 21.29 | 160.00 | 113.74 | 798.53 |
| 19.40 | 170.00 | 120.85 | 727.43 |
| 17.39 | 180.00 | 127.96 | 652.02 |
| 15.26 | 190.00 | 135.07 | 572.30 |
| 13.02 | 200.00 | 142.18 | 488.27 |
| 10.66 | 210.00 | 149.29 | 399.93 |
| 8.19 | 220.00 | 156.40 | 307.28 |
| 5.61 | 230.00 | 163.51 | 210.33 |
| 2.91 | 240.00 | 170.62 | 109.06 |
| 0.09 | 250.00 | 177.73 | 3.49 |


Figure 2.2
transition is point $A$ on Figure 2.2. When the column islong enough to be called an Euler column, the member willbuckle in the elastic region before the material's yieldstrength is reached. In other words, the elastic curve tothe right of point $A$ on Figure 2.2. is the controllingfactor. To the left of this point, however, the columns aregetting shorter. This causes the columns to have thecapability of withstanding forces only up to the criticalyield stress for that particular $k L / r$ value. These columnsdo not buckle in the elastic region, instead they buckle inthe inelastic region. As one can see in Figure 2.2 , thebuckling loads calculated for a short column by Euler'sequation are much higher than the loads needed to buckle thecolumn inelastically. For this reason, the inelasticbuckling curve controls to the left of point $A$ on Figure2.2. In Figure 2.3 the non-controlling ends of theinelastic and elastic curves have been removed, and thebuckling load characteristics can be seen as a function ofthe dimensionless term $k L / r$.

Figure 2.3

## CHAPTER I I I

## THEORETICAL ANALYSIS

## Introduction

To fabricate columns that could be safely tested in the laboratory, typical copper tubing like that used in plumbing work was chosen as the model. The loads required to fail these columns were within the limits of the equipment in the laboratory and provided a comparative indication of the relative merits of the different strengthening systems. Further explanations of the cross sectional and material characteristics of this and other materials used in testing will be given in the following sections of this chapter.

## Cross Sectional and Material Characteristics

As mentioned previously, the material used for the column model was simple copper plumbing pipe obtained at a local plumbing supply company. The kind used is known as Wolverine Tube Type M rigid copper pipe with an inside diameter of three quarters of an inch. The Type $M$ refers the material's thickness which is 0.032 inches. From research in several material handbooks, a modulus of elasticity value of $17,000,000 \mathrm{psi}$ and a yield strength of

45,000 psi were obtained for use in theoretical calculations.

## Theoretical Calculations for Hollow Cross Section

The calculations made for this were very similar to
those made for the sample tube mentioned in Chapter Two.
The first items calculated were the area of copper and the
moment of inertia for the hollow cross section. They were
0.0786 inches ${ }^{2}$ and 0.006019 inches 4 , respectively, using
Equations 2.17 and 2.18 . The radius of gyration was
computed as 0.2767 inches using Equation 2.19 . With these
values and Equations 2.14 and 2.15 , the values in Table 3.1
were formulated. Finally, using the values in Table 3.1 ,
Figure 3.1 was created as a theoretical curve of buckling
loads for the hollow copper cross section.

Theoretical Calculations for Concrete Filled Cross Section
The area of copper tube for this cross section was
0.0786 inches $^{2}$ like before, but now there was an added area of concrete equal to 0.4417 inches ${ }^{2}$. To compute a moment of inertia and radius of gyration, however, it was necessary to convert the area of concrete into an equivalent area of copper. This is done by multiplying the area of concrete by the appropriate modular ratio. This ratio is no more than the modulus of elasticity of concrete divided by the modulus of elasticity of copper. The equivalent area of concrete

THEORETICAL BUCKLING CURVE

Figure 3.1
was 0.09459 inches ${ }^{2}$ of copper.
After this area was calculated, the total area, in copper terms, was found by summing the area of actual copper and the equivalent area of copper for the concrete; this came out to be 0.1732 inches ${ }^{2}$. At this point an inside diameter for this area was needed to calculate the moment of inertia for the cross section. This was done by using the known total area, the copper tube's outside diameter, Equation 2.17 and back calculating for the inside diameter. This value was found to be 0.66 inches.

Now that all the terms were known, the moment of inertia of 0.01196 inches $^{4}$ was computed using Equation 2.18 . Likewise, the radius of gyration was obtained using Equation 2.19. It was found to be 0.26276 inches.

Finally, the values for the elastic buckling loads were calculated using the information computed above and Equation 2.14. The inelastic buckling stresses were calculated for the copper tube using the radius of gyration found above and Equation 2.15. This value was then multiplied by the area of the copper and added to the real area of the concrete section multiplied by its yield stress of 4000 psi. A sample of these values are presented in Table 3.2 , and $a$ plot of the theoretical buckling curve for the concrete filled section is presented in Figure 3.2.

Table 3.2 - Buckling Curves for Concrete Filled Members EULER CURVE

| $\begin{gathered} \text { LOAD, } \\ (\mathrm{lbs}) \end{gathered}$ | $\begin{gathered} \mathrm{L} \\ (\mathrm{in}) \end{gathered}$ | L/r | $\begin{gathered} \text { LOAD, P } \\ (l \mathrm{~b} \text { ) }) \end{gathered}$ | STRESS, F (ksi) |
| :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 5304.83 | 45.00 |
| 80260.81 | 5 | 19.03 | 5218.94 | 43.91 |
| 20065.20 | 10 | 38.06 | 4961.27 | 40.63 |
| 8917.87 | 15 | 57.09 | 4531.83 | 35.17 |
| 5016.30 | 20 | 76.11 | 3930.61 | 27.52 |
| 3210.43 | 25 | 95.14 | 3157.60 | 17.69 |
| 2229.47 | 30 | 114.17 | 2212.82 | 5.67 |
| 1637.98 | 35 | 133.20 | 1096.27 | -8.53 |
| 1254.08 | 40 | 152.23 | -192.07 | -24.92 |
| 990.87 | 45 | 171.26 | -1652.18 | -43.49 |
| 802.61 | 50 | 190.29 | -3284.07 | -64.25 |
| 663.31 | 55 | 209.31 | -5087.74 | -87.20 |
| 557.37 | 60 | 228.34 | -7063.19 | -112.32 |
| 474.92 | 65 | 247.37 | -9210.42 | -139.64 |
| 409.49 | 70 | 266.40 | -11529.42 | -169.13 |
| 356.71 | 75 | 285.43 | -14020.21 | -200.82 |


Figure 3.2

Theoretical Calculations for Concrete Grouted Cross Section


#### Abstract

As in the two previous cross sections, the standard copper tube with a cross sectional area equal to 0.0786 inches ${ }^{2}$ was once again used. In addition to this copper piece, a second copper tube with an inside diameter of one quarter of an inch and a thickness of 0.030 inches was used for the center pile. It was found to have an area of 0.0264 inches ${ }^{2}$ using Equation 2.17. The annulus, with a computed area of 0.3663 inches ${ }^{2}$ using Equation 2.17 , between the inner pile and the tubular member was filled with concrete. Its equivalent area in terms of copper was found to be 0.0784 inches $^{2}$ using the method described in the previous section. The total area for the cross section, in terms of copper, was 0.1834 inches².


Following these initial calculations, the circular dimensions for the equivalent concrete area were needed to calculate the moment of inertia of the cross section. This was done by assuming that the outside diameter of the equivalent concrete section was equal to the inside diameter of the outer tubular member. Using this value, three quarters of an inch, as the outside diameter, the inside diameter of 0.68 inches for the equivalent concrete section was computed using Equation 2.17.

The moment of inertia for the cross section were now calculated by using Equation 2.18 for each of the three
cross sections: the outside copper tube, the equivalent concrete annulus, and the inside copper tube. These values were then summed to obtain a value for the moment of inertia for the cross section equal to 0.0113 inches ${ }^{4}$. With the moment of inertia and the area for the cross section, the radius of gyration was then computed, using Equation 2.19, to be 0.248384 inches.

After these values were established, the elastic buckling curve was computed using Equation 2.14. The inelastic buckling stresses were calculated as in the previous section and once again multiplied by the area of copper in the cross section. This area is equal to the area of the copper tubular member and the area of the inner pile. The critical buckling loads were found by summing the load calculated above with the load value of the concrete. This concrete load was computed by multiplying the real area of the concrete and its yield stress together. An sample of the values for various lengths of columns are given in Table 3.3. The theoretical curve is graphically depicted in Figure 3.3.

## Observations Based on Theoretical Analysis

The first comparison of the two strengthening techniques was done in the creation of Figure 3.4. As one can see, this is a plot of all three proposed test samples

Table 3.3 - Buckling Curves for Concrete Grouted Members EULER CURVE

INELASTIC CURVE

| $\begin{gathered} \text { LOAD, P } \\ (\mathrm{lbs}) \end{gathered}$ | $\begin{gathered} \mathrm{L} \\ \text { (in) } \end{gathered}$ | L/r | $\begin{gathered} \text { LOAD, P } \\ \text { (lbs) } \end{gathered}$ | STRESS, F (ksi) |
| :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 6070.42 | 45.00 |
| 75952.13 | 5 | 19.14 | 5942.03 | 43.78 |
| 18988.03 | 10 | 38.28 | 5556.88 | 40.11 |
| 8439.13 | 15 | 57.43 | 4914.95 | 34.00 |
| 4747.01 | 20 | 76.57 | 4016.26 | 25.44 |
| 3038.09 | 25 | 95.71 | 2860.79 | 14.43 |
| 2109.78 | 30 | 114.85 | 1448.55 | 0.98 |
| 1550.04 | 35 | 134.00 | -220.46 | -14.91 |
| 1186.75 | 40 | 153.14 | -2146.23 | -33.25 |
| 937.68 | 45 | 172.28 | -4328.78 | -54.04 |
| 759.52 | 50 | 191.42 | -6768.10 | -77.27 |
| 627.70 | 55 | 210.56 | -9464.19 | -102.94 |
| 527.45 | 60 | 229.71 | -12417.05 | -131.06 |
| 449.42 | 65 | 248.85 | -15626.68 | -161.63 |
| 387.51 | 70 | 267.99 | -19093.08 | -194.64 |
| 337.57 | 75 | 287.13 | -22816.25 | -230.10 |

THEORETICAL BUCKLING CURVE
FOR CONCRETE GROUTED TUBE

THEORETICAL BUCKLING CURVE
FOR ALL THREE TEST MEMBERS

Figure 3.4
superimposed on each other. From this graph, severalconclusions were reached. First, the grouted tube appearedto be a better strengthening technique than the concretefilled tube. Both methods of reinforcement, however,created stronger members than the original plain hollowsection. The two repair methods also gave relativelysimilar results in the elastic region of the curve, and thisincrease in strength over the plain hollow tube was minimal.From the theoretical analysis of the hollow crosssection, the concrete filled cross section, and the concretegrouted cross section, the repair techniques proposed showedmarked improvement to column strength in the inelasticbuckling region of the buckling curves. In the elasticbuckling region, however, the strength gained by the repairmethods was minimal.

## CHAPTER IV

## TEST PROCEDURES AND RESULTS

## Sample Characteristics

As mentioned in the previous chapter, the compressionmembers used to model the strengthening techniques weresimple copper plumbing pipe with an inside diameter of threequarters of an inch and a wall thickness of 0.032 inches.In addition to the main hollow tubular member, in thegrouted column, a copper pile member was chosen with aninside diameter of one quarter of an inch and a wallthickness of 0.030 inches. The last material that was usedwas portland cement concrete purchased at the local hardwarestore. The manufacturer's specifications were found to be$3.64 \times 10^{\wedge} 6 \mathrm{psi}$ for the modulus and 4000 psi for the yieldstrength.The sample lengths were decided upon as $36,28,20,12$, 6 inches. A sample of each of the three test specimens, the hollow cross section, the concrete filled cross section, and the concrete grouted cross section, were prepared for each of the five test lengths. These lengths were selected so that a representative test curve, with points in the inelastic, transition, and elastic zones of the buckling
could be created for the test data and later compared to the appropriate theoretical curves.

## Test Procedures

The test specimens, fifteen in all, were tested in the structures testing laboratory on the first floor of the Wisenbaker Engineering Research Center on the Texas A\&M University campus. The testing apparatus used was an Instron compression/tension testing machine with a 20 kip load cell. Each of the specimens were tested in compression by displacing them at a rate of 0.02 inches per minute. This loading machine had a chart that plotted the results on graph paper. For testing purposes, the instron plotted load versus time on the chart plotter.

For each sample, the test procedure was as follows. The specimen was placed into the machine, and the compression heads were placed against the ends of the test specimen. The instron's control board was then cleared and checked to make sure the setting were correct. The chart and compression heads were then started simultaneously, and the chart and specimen were observed closely. After the load peaked on the chart and a general decreasing trend was established, the loading was halted, the specimen was removed. This process was repeated for all of the test specimens.

## Test Results

A table of the results is presented in Table 4.1. This table was created by using the graphs plotted by the instron to obtain the peak loads for each test. Samples of these plots are presented in Figures $4.1,4.2$, and 4.3 . The results obtained in the tests of the samples supported the original theoretical hypotheses. The concrete grouted tubular members buckled at higher loads than the concrete filled columns, and they both proved to be stronger than their hollow section counterparts.

All three sets of test data results showed good trends, that is , they paralleled their theoretical compliments. The one exception to this was the six inch concrete filled sample. Its buckling load seemed to be too low and did not follow the trend represented by the rest of the data for this type of column. This anomaly was probably due to the specimen being released from the loading too early. On these concrete filled specimens, the test loading curve peaked once initially, but after falling off, the loads increased once again to a higher crest than the first one encountered. From observations of all other test data collected, it was determined that this test sample was only allowed to reached one peak before the load was released. The test was probably stopped too soon, which can account for the low critical buckling load obtained for this

Table 4.1 - Test Results

| LENGTH | HOLLOW | FILLED | GROUTED |
| :---: | :---: | :---: | :---: |
| 6 in. | 4700 lbs | 5300 lbs | 6800 lbs |
| 12 in . | 4500 lbs | 6200 lbs | 5600 lbs |
| $20 \mathrm{in}$. | 3600 lbs | 4800 lbs | 4400 lbs |
| 28 in. | 3000 lbs | 3200 lbs | 3700 lbs |
| $36 \mathrm{in}$. | 2300 lbs | 2600 lbs | 2800 lbs |

> Figure 4.1 - Example of Instron Test Chart For Hollow 6" Sample


## Figure 4.2 - Example of Instron Test Chart For Concrete Filled 20" Sample

$$
\begin{aligned}
& \text { LOAD } \\
& \text { (2000 lbs/in) } \\
& \text { TIME } \\
& \text { (0.05 in/min) }
\end{aligned}
$$

Figure 4.3 - Example of Instron Test Chart For Concrete Grouted 12" Sample


TIME
(0.05 in/min)
specimen.
Overall, the test result were good, and the Figures $4.4,4.5$, and 4.6 were obtained by superimposing the test data points on the theoretical curves for each of the three cross sections tested. In each of the three cases, the test data was also slightly higher than the theoretical curve. Conclusions for these results and recommendations for the use of these strengthening techniques are presented in Chapter Five.

Flgure 4.4
TEST DATA PLOT
FOR CONCRETE FILLED


Figure 4.6

## CHAPTER V

## CONCLUSIONS AND RECOMMENDATIONS

## Conclusions

The test results obtained from the testing of the samples described in Chapter Four supported the original hypothesis that the grouted tubular cross section was a better repair technique, in terms of strength gained, than the concrete filled cross section. This data also indicated that strength was increased from ten to forty-five percent by the filling and grouting methods as compared to the simple hollow compression member.

The data gained from testing was also higher than the calculated critical loads in all cases. Several possible explanations could account for this increase. First, the material properties of modulus of elasticity and yield strength were found in reference tables and volumes. The variation in material characteristics could have had something to do with these increases in buckling load as compared to theoretical values. Secondly, the concrete, since it is confined by the copper tubular members, could withstand pressure beyond its yield strength.

This increase in yield strength of the concrete can be
explained as follows. If a deflection analysis is computed for the filled concrete section, it is found that the concrete yield controls the yield of the column. From deflection analysis, the deflection of the concrete has to equal the deflection in the copper. This is explained by the condition:

$$
\begin{equation*}
\delta_{\text {concrete }}=\delta_{\text {copper }} \tag{5.1}
\end{equation*}
$$

where $\delta$ concrete is the deflection in the concrete and $\delta_{\text {copper }}$ is the deflection in the copper. By rewriting Equation 5.1 in terms of cross sectional characteristics, the equation becomes:

$$
\begin{equation*}
\frac{\sigma c L}{E c}=\frac{\sigma c u L}{E c u} \tag{5.2}
\end{equation*}
$$

where $\sigma c$ is the yield stress for concrete, $\sigma c u$ is the yield stress for copper, $L$ is the length of the member before compression, Ec is the modulus of elasticity of concrete, and Ecu is the modulus of elasticity of copper. If the appropriate modulus values are substituted into Equation 5.2, and it is rearranged, the relationship becomes:

$$
\begin{equation*}
\sigma c=0.2141 \sigma c u \tag{5.3}
\end{equation*}
$$

When the yield stress of 45 ksi for copper is substituted into Equation 5.3, the yield stress in the concrete is calculated to be 9.64 ksi This is greater than the theoretical yield stress in the concrete, therefore, the yield stress of 4 ksi in the concrete controls failure.

From this knowledge, the stress that the copper reaches at the point the concrete fails can be found. Using Equation 5.3 and the 4 ksi theoretical yield stress of the concrete to back calculate for the stress in the copper at failure. This was computed to be 18.68 ksi , well under the theoretical yield stress of the copper. With this being the case, the critical buckling load can be found by multiplying these stresses of 4 ksi for the concrete and 18.68 ksi for the copper with their appropriate cross sectional areas. If this is done, the maximum critical buckling load is 3236 pounds.

From real experience, however, the buckling load of 3236 pounds is well under the actual value. One hypothesis as to why this is the case is that the copper confines the concrete and allows it to take higher loads that are well above the yield point of the concrete. This could also explain why the test results were higher than the theoretical values, since these theoretical values were calculated using 4 ksi as the yield stress of the concrete.

## Recommendations

To conclude, I will offer several recommendations for
further research and applications of these strengthening
techniques. Firstly, the strengthening techniques proposed,
filling the cross section with concrete and grouting the
cross section, do in fact strengthen the tubular compression member. This strengthening is considerable in the inelastic regions of column failure, but minimal for the elastic failure. Secondly, the grouting method is a better strengthening technique than filling the tube with concrete when the tubular members are short inelastic buckling columns. When these techniques are done on the longer elastic buckling columns, however, the differences in strength are almost nonexistent. Lastly, further testing could be done to determine whether or not the concrete does withstand yield stresses above its limit. A strengthening technique in which the cross section was simply filled with sand would also provide for an interesting comparison with the techniques researched above.

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