DETERMINATION OF
FANNING FRICTION FACTOR
FOR POWER LAW FLUIDS

by

STEVEN W. MALLETT

Department of Chemical Engineering

Submitted in Partial Fulfillment of the Requirements of the University Undergraduate Fellows Program

1982-1983

Approved by:

R E White

Dr. R.E. White

ABSTRACT

An equation was developed that enables the calculation of the Fanning friction factor for the flow of power law fluids. The equation applies for all flow regions - laminar, transitional and turbulent. The equation is based on a weighted sum of equations for the Fanning friction factor in each of the three regions of fluid flow. Deviations between the developed equation and accepted values is on the order of five percent or less for the majority of the values for the Fanning friction factor. The equation is simple to use and is ideally suited for a computer or for a hand-held calculator. The equation can replace the current graphical methods that are used to solve power law fluid systems.

I would like to take this opportunity to thank my faculty advisors, Dr. Ron Darby and Dr. Ralph E. White, for all their guidance and help through the course of this project.

TABLE OF CONTENTS

LIST OF FIGURES

INTRODUCTION

The purpose of this paper is to present an equation for predicting the Fanning friction factor for flow of non-Newtonian fluids. The equation must be valid for laminar, transitional and turbulent flow regimes and it must be relatively simple to apply.

Non-Newtonian fluids are defined as fluids whose shear stress is not linearly proportional to its shear rate. This treatment of non-Newtonian fluids will be limited to those that can be represented by the "power law" model. The power law model relates the shear stress (τ) to the shear rate (γ) by the equation:

$$\tau = K\,(\gamma)^n \qquad (1)$$

For the special case when n is equal to 1, the power law model reduces to the equation for Newtonian fluids and K represents the fluid viscosity.

Fluids that are described by the power law model are divided into two categories. Fluids whose value for n is greater than 1 are referred to as dilatant fluids while those fluids with a value of n that is less than 1 are classified as pseudoplastics. This treatment will be limited to pseudoplastics.

As with all fluids, power law fluids possess a resistance to flow. This resistance to flow is known as the frictional loss of the fluid. The friction loss per unit mass of fluid is proportional to the square of the velocity by:

$$\mathcal{F} = 2fL\frac{v^2}{D} \tag{2}$$

The constant of proportionality -f- is referred to as the Fanning friction factor. The Fanning friction factor is not constant, but decreases with an increase in fluid velocity. For power law fluids, the Fanning friction factor is also dependent upon the parameter n from Equation (1).

Previous experimental work performed on non-Newtonian fluids by Dodge and Metzner[1] resulted in an empirical correlation for the Fanning friction factor as a function of a modified Reynolds number and the parameter n. The modified Reynolds number was defined so as to obtain a linear relation with the Fanning friction factor for fully developed laminar flow. The resulting modified Reynolds number was found to be:

$$N'_{Re} = \left(\frac{D}{2}\right)^n \frac{8\rho v^{2-n}}{m}\left(\frac{n}{3n+1}\right)^n \tag{3}$$

For the special case of a Newtonian fluid, n equal to 1, Equation (3) reduces to the general form of the Reynolds number.

From the results of their experimentation, Dodge and Metzner[1] found that the Fanning friction factor was given by

$$\frac{1}{\sqrt{f}} = \frac{4}{n^A}\log\left[N'_{Re}\cdot f^B\right] - C \tag{4}$$

where

$$A = 0.75$$
$$B = 1 - n/2$$
$$C = 0.4/n^{1.2}$$

This correlation was found to be valid only for fully developed turbulent flow and was limited by the fact that only data for n between 0.4 and 1.0 were used. Figure 1 shows a plot of the data taken and the results of the correlation for turbulent flow.

Hanks and Ricks[2] developed a similar plot by theoretical methods. This plot is based on a modification of the Prandtl[3] mixing length model. The method developed enables the determination of the friction factor for all values of the modified Reynolds number. Figure 2 plots the results of this theoretical analysis.

Figure 3 compares the Dodge-Metzner correlation for turbulent flow with the results of the Hank's method. At Reynolds numbers greater than 3000, there is good agreement between the two for values of n between 0.8 and 1.0. For values of n that are less than 0.7, substantial deviations occur between the two. These deviations can be attributed to the limited amount of data available in the development of the Dodge-Metzner correlation.

DEVELOPMENT OF METHOD

The method to be used to determine the overall equation for the Fanning friction factor was developed by Churchill and Usagi[4]. The method is based on the weighting of two asymptotic solutions to obtain one equation for the entire range. The weighting of the two asymptotic solutions is achieved by using an equation of the form:

$$Y = \left(y(z \to 0)^n + y(z \to \infty)^n \right)^{1/n} \tag{5}$$

where $y(z \to 0)$ and $y(z \to \infty)$ represents the asymptotic solutions as z approaches zero and infinity respectively.

This method can be applied to the calculation of the overall Fanning friction factor by weighting expressions for the factor in the laminar and turbulent flow regions. The generalized form of the resulting equation would be:

$$f = \left(f_L^a + f_T^a \right)^{1/a} \tag{6}$$

where f_L and f_T are the equations for the Fanning friction factor in the laminar and turbulent regions respectively.

Equation (6) was successively used by Churchill[5] to calculate the Fanning friction factor for Newtonian fluids. Churchill's analysis resulted in an equation of the form:

$$f = \left[\left(\frac{16}{N_{Re}} \right)^{12} + \left(\frac{256}{A + B} \right)^{3/2} \right]^{1/12}$$

where

$$A = 2.457 \ln \left[\frac{1}{\left(\frac{7}{N_{Re}} \right)^{0.9} + 0.27 \frac{\epsilon}{D}} \right] \tag{7}$$

$$B = \left(\frac{37,530}{N_{Re}} \right)^{16}$$

This same method was applied to Bingham plastics by Darby and Melson[6]. They reported that the Fanning friction factor can be calculated from Equation (6) where:

$$f_L = \frac{16}{N_{Re}}\left[1 + \frac{N_{He}}{6N_{Re}} - \frac{N_{He}^4}{3f_L^3 N_{Re}}\right]$$

$$f_T = 10^m N_{Re}^{-0.193} \qquad (8)$$

$$m = -1.378\left[1 + 0.146\, e^{-2.9\times10^{-5} N_{He}}\right]$$

$$a = 1.7 + 40{,}000/N_{Re}$$

RESULTS

The development of the overall equation for the Fanning friction factor is dependent upon the individual equations in each flow regime. For laminar flow, an exact description of the Fanning friction factor was developed by Metzner and Reed[7]. This result is expressed as

$$f_L = 16/N'_{Re} \tag{9}$$

where N'_{Re} is the modified Reynolds number as defined by Equation (3).

For turbulent flow, the Dodge-Metzner correlation was used. This correlation deviated significantly from the Hanks method and was therefore modified to provide better agreement between the two. The modified form of the Dodge-Metzner equation is of the same general form as Equation (4) except that the constants A, B and C have been redefined as:

$$\begin{aligned} A &= .7n + .866 \\ B &= 1.02 - .02\,e^{3.258\,n} \\ C &= .329\left(e^{4.45(1-n)} + .216\,n\right) \end{aligned} \tag{10}$$

Figure 4 shows a comparison of the modified Dodge-Metzner equation and the Hanks plot. The modified form provides a substantial decrease in the deviations from the Hanks plot.

Equation (4) is implicit in f, while it is desired to have an explicit function for the Fanning friction factor. An approximation of the modified Dodge-Metzner equation was found by utilizing a partial trial and error process. The process is initiated by assuming an initial value for f and then calculating a new f from Equations (4) and (10). This new f is then used as the next assumed value in order to obtain a more accurate value of f. The initial value of f was assumed to be 0.005 and two iterations were required in order to obtain acceptable agreement with the true values calculated from Equations (4) and (10). The resulting explicit equation for can be written as:

$$\frac{1}{\sqrt{f}} = \frac{4}{n^A}\left\{\log N'_{Re} - 2B\log\left[\frac{4}{n^A}\left(\log N'_{Re} - 2.3B\right) - C\right]\right\} - C \qquad (11)$$

where the constants A, B and C are given by Equation (10).

In order to obtain an accurate description for the Fanning friction factor over the entire range of Reynolds numbers, it was necessary to include an expression for the transitional flow region. Churchill[5] determined an empirical equation for Newtonian fluids in the transitional flow region and this was modified so as to apply to pseudoplastics. The resulting equation was found to be:

$$f_{TR} = 1.42\times10^{-9}\, N'^{\,2}_{Re}\,(.6n+.4) \qquad (12)$$

This equation provided reasonable results for Reynolds numbers less than 4000.

The transitional and turbulent equations for the Fanning friction factor were then combined to get a single equation for these two regions. Equations (11) and (12) were used and the weighting factor as defined by Equation (5) was found to be -4. This single equation was then combined with Equation (9) to obtain the overall form of the equation. In this case, a weighting factor of 5 was found to give the best overall fit. The final form of the equation can be written as:

$$f = \left[f_L^5 + \left(f_{TR}^{-4} + f_{TB}^{-4} \right)^{-5/4} \right]^{1/5} \qquad (13)$$

where f_L, f_{TR} and f_{TB} are given by Equations (9), (12) and (11) respectively.

Figure 5 shows the values calculated from Equation (13) and the values calculated from Hanks method. There is excellent agreement between the two for values of n between 1.0 and 0.4. Slight deviations begin to appear for Reynolds numbers greater than 50,000 and values of n greater than 0.4. The deviations are on the order of 5% or less. Significant deviations do occur for values of n that are less than 0.4. At Reynolds numbers greater than 5000, Equation (13) calculates substantially higher values for the Fanning friction factor at values of 0.2 and 0.3 for n. The deviations in this area are in excess of 20%.

CONCLUSION

Equation (13) is an accurate and convenient replacement for the Hanks plot in determining the Fanning friction factor for flow of power law fluids. The equation is much simpler to use than Hanks' mixing length model analysis. Also, the equation accurately applies over the entire range of Reynolds numbers and is not limited to turbulent flow as is the Dodge-Metzner equation. The only limitation to the application of Equation (13) is if the fluid of interest has a very low value of n from Equation(1). The accuracy of the equation for these small values of n is marginal. But for the most part, Equation (13) is accurate, simple to use and ideally suited for computer design of power law fluid systems.

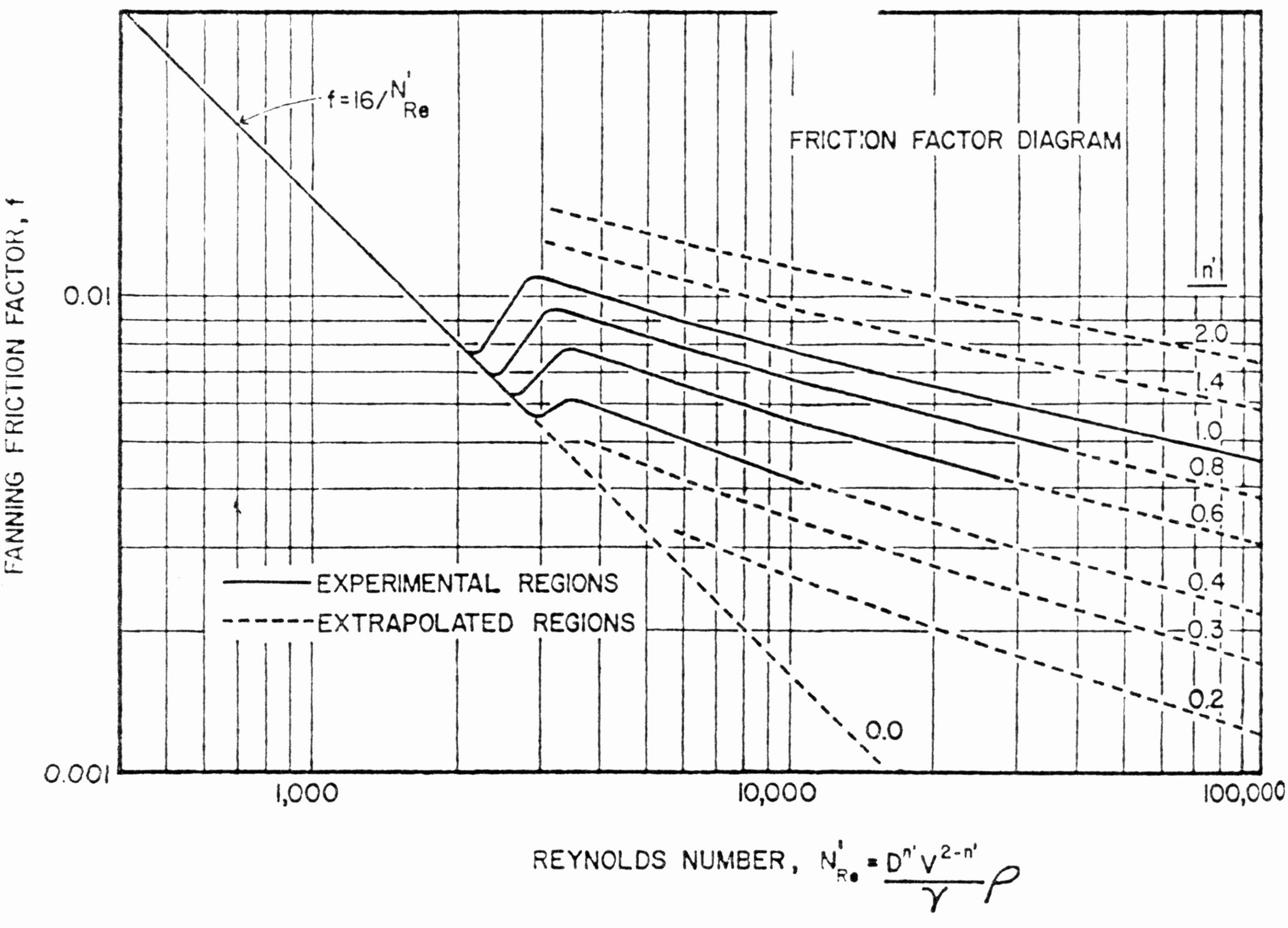

FIGURE 1 DODGE-METZNER CORRELATION FOR TURBULENT FLOW

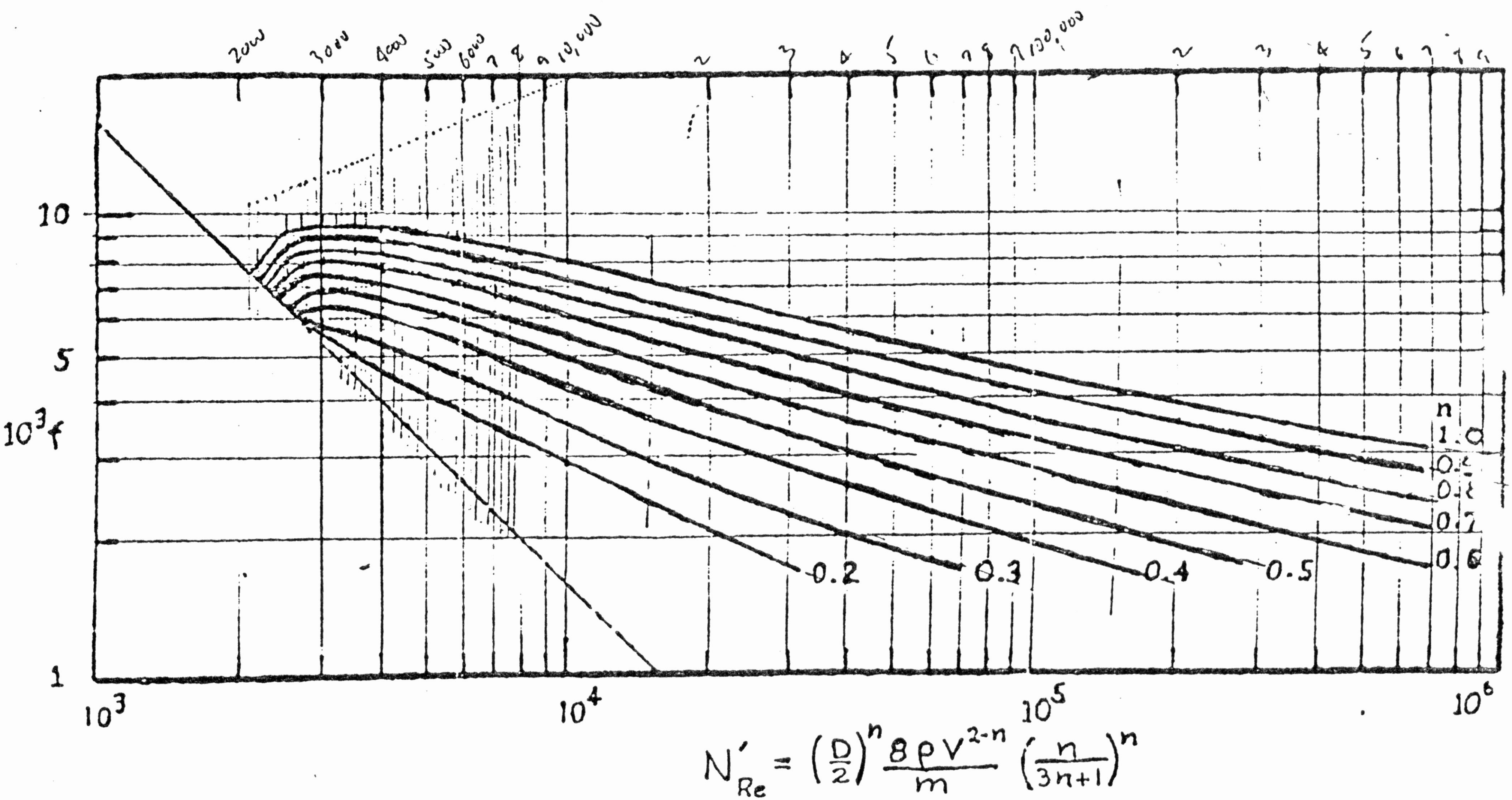

FIGURE 2 RESULTS OF HANKS' MIXING LENGTH MODEL ANALYSIS

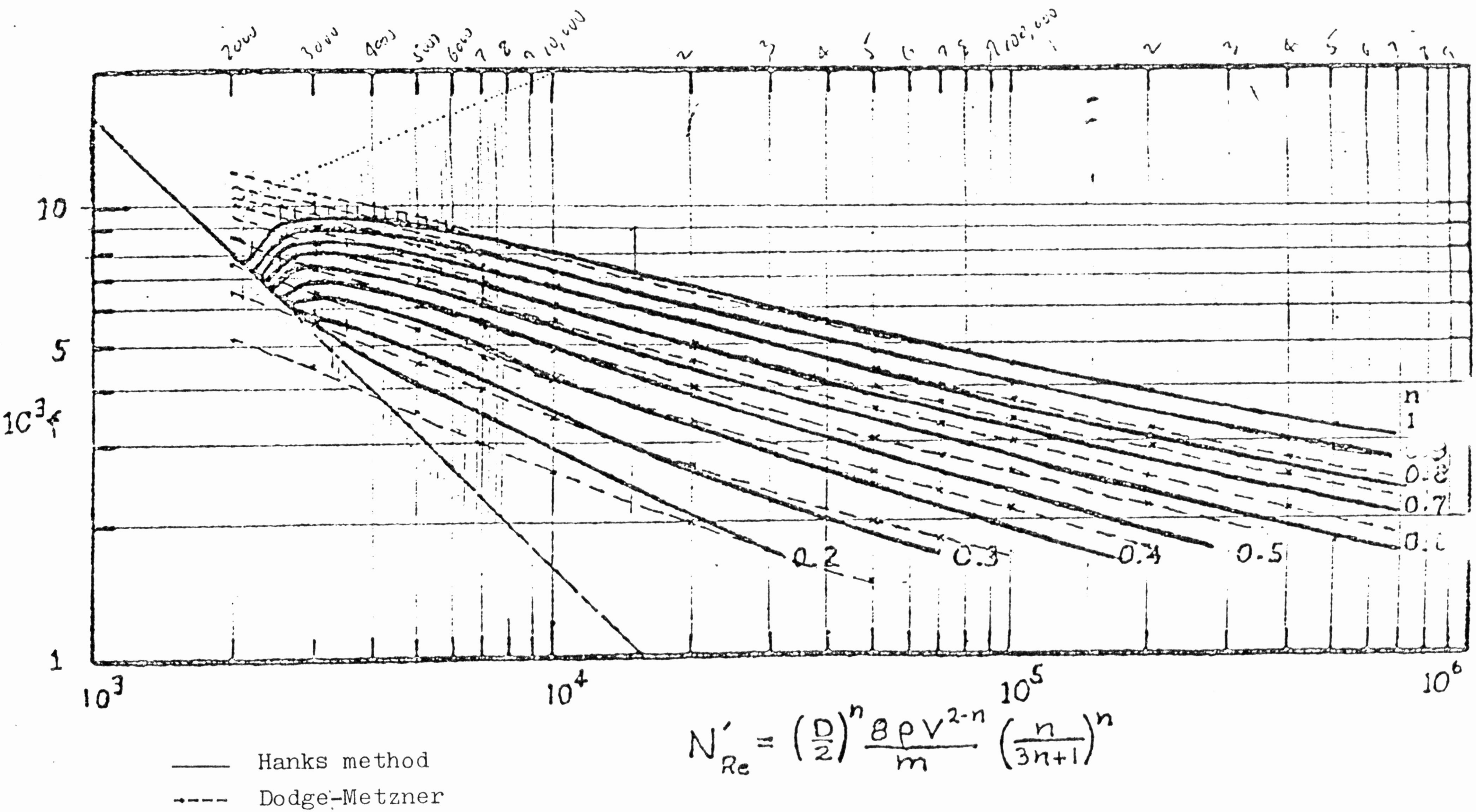

FIGURE 3 COMPARISON OF DODGE-METZNER CORRELATION AND HANKS METHOD

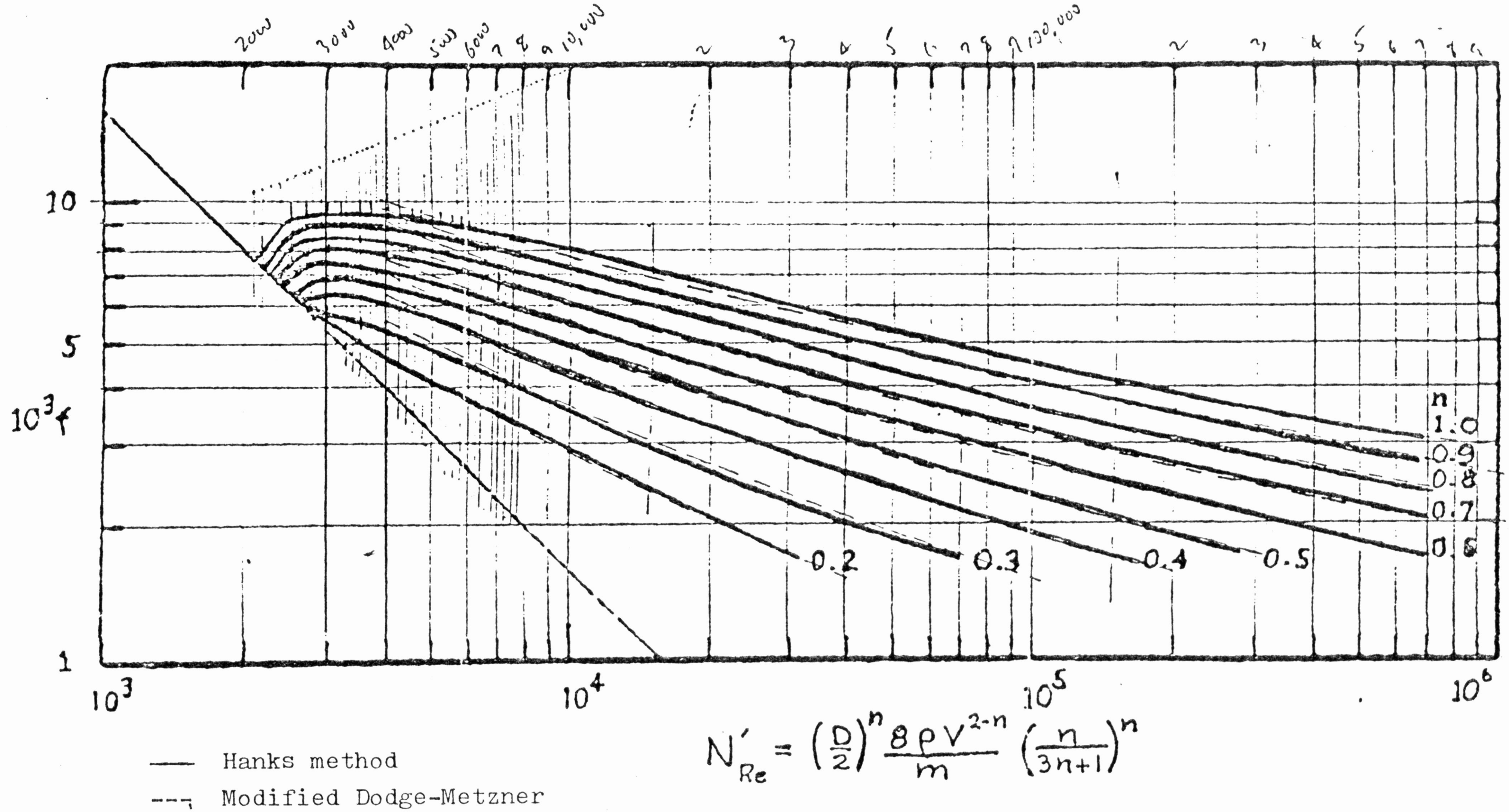

FIGURE 4 COMPARISON OF MODIFIED DODGE-METZNER AND HANKS METHODS

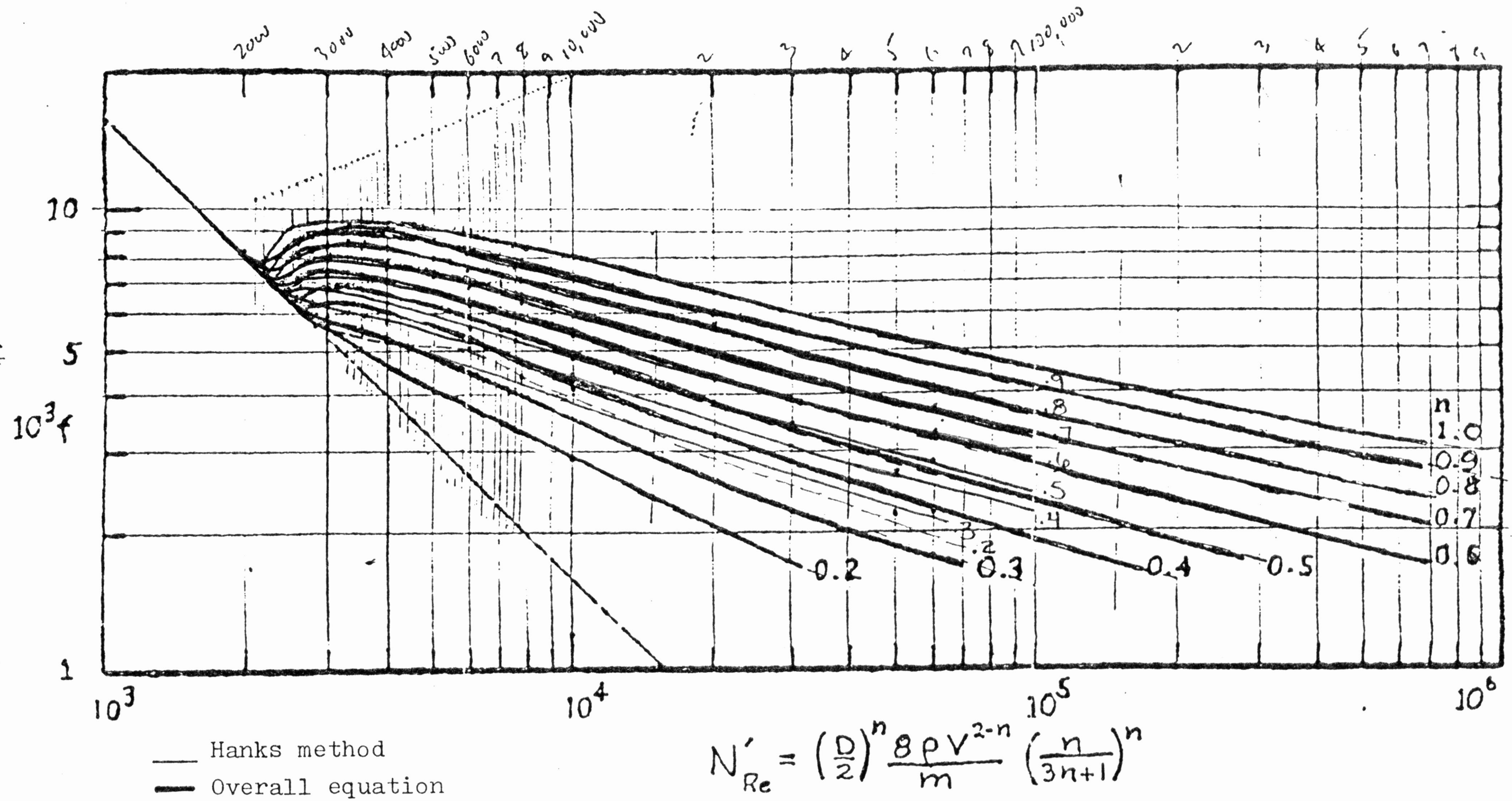

FIGURE 5 COMPARISON OF OVERALL EQUATION AND HANKS' VALUES

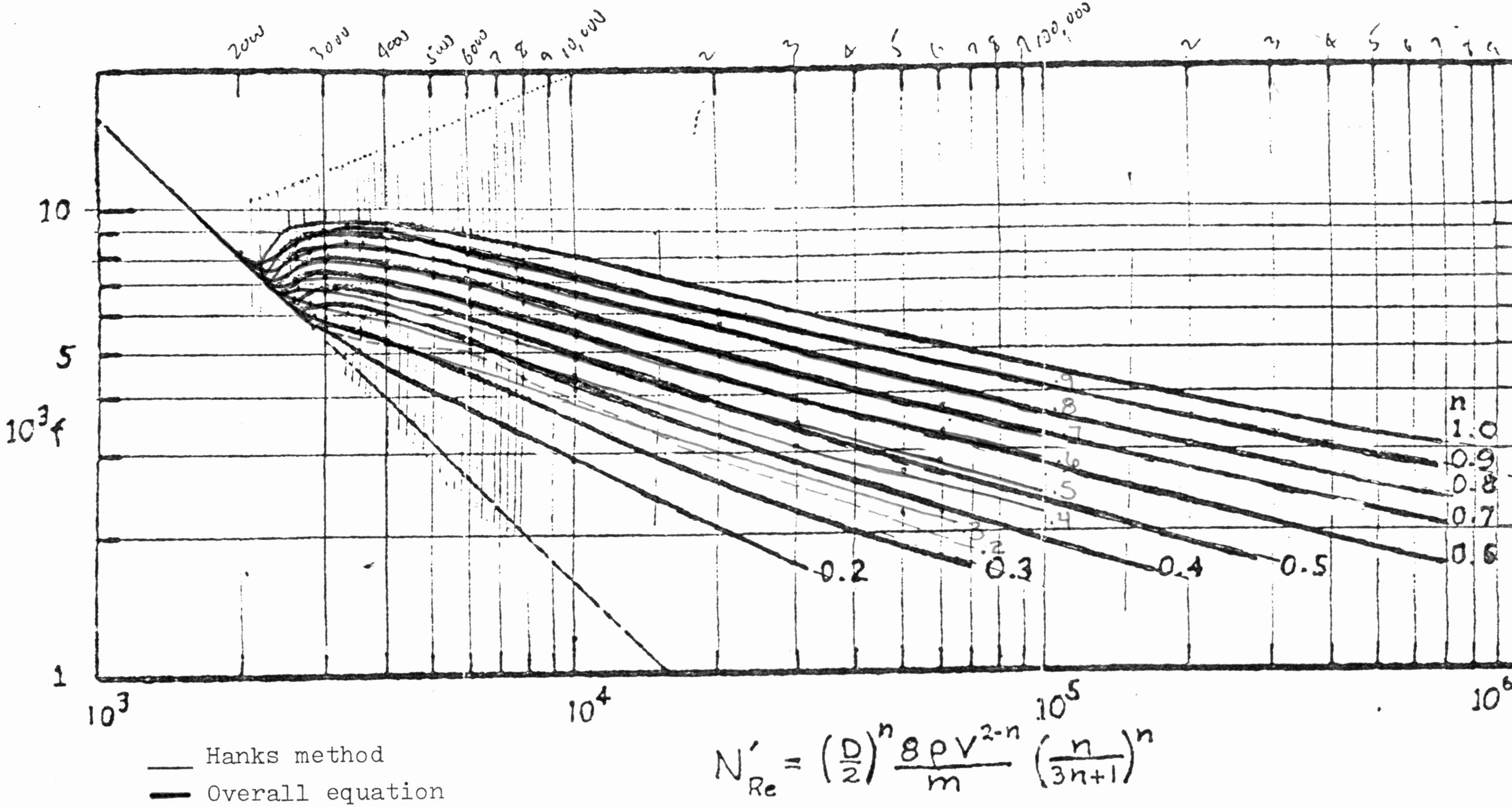

FIGURE 5 COMPARISON OF OVERALL EQUATION AND HANKS' VALUES

REFERENCES

1. Dodge, D.W. and Metzner, A.B., "Turbulent Flow of Non-Newtonian Systems," A.I.Ch.E. Journal, Vol. 5, No. 2, 1959, pp. 189-204.

2. Hanks, R.W. and Ricks, B.L., "Transitional and Turbulent Pipeflow of Pseudoplastic Fluids," Journal of Hydronautics, Vol. 9, No. 1, January 1975, pp.39-44.

3. Prandtl, L., "Neuere Ergebnisse der Turbulenzforschung," Zeitschrift Verein Deutscher Ingenieure, Vol. 77, 1933, p. 105.

4. Churchill, S.W. and Usagi, R., "A General Expression for the Correlation of Rates of Transfer and Other Phenomena," A.I.Ch.E. Journal, Vol. 18, No. 6, November, 1972, pp 1121-1127.

5. Churchill, S.W., "Friction-factor Equation Spans All Fluid-flow Regimes," Chemical Engineering, November 7, 1977, pp. 91-92.

6. Darby, R. and Melson, J., "How to Predict the Friction Factor for Flow of Bingham Plastics," Chemical Engineering, December 28, 1981, pp. 59-61.

7. Metzner, A.B. and Reed, J.C., "Flow of Non-Newtonian Fluids - Correlation of the Laminar, Transitional, and Turbulent Flow Regions," A.I.Ch.E. Journal, Vol. 1, No. 4, 1955, pp.434-440.

NOMENCLATURE

D	Diameter of pipe m
f	Fanning friction factor
$\mathcal{F}$	Frictional loss cal/kg
L	Length of pipe m
m	Power law constant
N_{He}	Hedstrom number
N_{Re}	Reynolds number
N_{Re}	Modified Reynolds number
τ	Shear stress N/m^2
ϵ	Effective roughness m
ρ	Fluid density kg/m^3
γ	Shear rate s^{-1}
v	Fluid velocity m/s