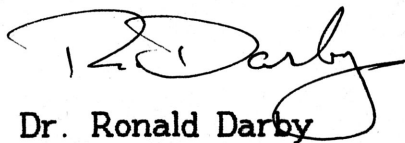# Turbulent Drag Reduction Modeling

by

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Submitted in Partial Fulfillment of the Requirements of the
University Undergraduate Fellows Program

1985-86

Approved by:

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April 1986

Abstract

A previously developed correlation that predicts the pipe friction loss for drag reducing polymer solutions in turbulent pipe flow is made more rigorous and general by relaxing some of the assumptions made in the development of the original correlation. The resulting improved correlation has 15% better precision, follows the trends in the data better, and furnishes enhanced support for the energy dissipation model used to develop the correlation. An attempt was made to extend the model to enable the prediction of a maximum drag reduction asymptote by incorporating a linear viscoelastic model having a high shear limiting viscosity. The result was successful in qualitatively predicting maximum drag reduction, but was not quantitatively consistent with observed drag reduction data.

Acknowledgment

The following references supplied much of the material presented within this paper:

1. Darby, R., "A generalized Correlation for Friction Loss in Drag Reducing Polymer Solutions", Encyclopedia of Fluid Mechanics, M.P. Cheremisinoff (ed.), Gulf Publishing Co. 1986

2. Darby, Ron and Chang, H.D., "Prediction of Turbulent Drag Reduction in Polymer Solutions from Rheological Properties" , Drag Reduction 84 Conference, University of Bristol, July 1984

3. Chang, H.D., "Correlation of Turbulent Drag Reduction in Dilute Polymer Solutions with Rheological Properties by an Energy Dissipation Model", Ph.D. Dissertation, Chemical Engineering, Texas A&M University, 1982

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Background

Adding a small quantity of a soluble high molecular weight polymer to a liquid in turbulent flow can substantially reduce the friction loss experienced by the solution.[1,2,3,4,5,6] Although the mechanism by which this drag reduction occurs has not been established, a recent study by Darby and Chang[7] explains it as a natural consequence of energy dissipation and storage by turbulent eddies in the viscoelastic fluid. A model was developed which demonstrates that the energy stored by the elastic properties of the polymer solution in turbulent flow represents energy that would otherwise be dissipated by viscous properties. Hence, the reduction in friction results from this storage of energy by the elastic properties of the solution, requiring less energy input to the fluid to sustain the flow.

Chang[8] used the above mentioned model to develop a correlation that predicts the turbulent friction loss of the dilute polymer solution from a knowledge of the nonlinear viscoelastic properties in generalized dimensionless form. Chang's correlation requires a knowledge of the friction loss in the Newtonian solvent and three rheological parameters for the solution. A linear Maxwell model is used to define a relaxation time for the viscoelastic solution, although the actual solution exhibits nonlinear

behavior. This is later accounted for by defining the relaxation time to be a function of shear rate. The correlation is valid for both fresh and shear degraded solutions of various concentrations of Separan AP-30 polyacrylamide in distilled water over a wide range of pipe sizes. The friction factor, which includes a term to account for energy storage by the solution elastic properties, reduces to the usual Fanning friction factor for Newtonian fluids.

The fluid rheological parameters $(\zeta, \Omega, \eta_o, \eta_\infty)$ which enter into the model can be evaluated from apparent viscosity, η, versus shear rate data, as shown in Figure 1 (from Chang and Darby[9]). The characteristic time constant of the fluid, ζ, is the reciprocal of the shear rate where the transition from low shear Newtonian to non-Newtonian behavior occurs. The Ω parameter equals ½ of the negative of the slope of the linear portion of the curve, η_o is the zero shear viscosity, and η_∞ is the high shear limiting viscosity.

The apparent viscosity data show that shear degradation lowers the zero shear viscosity by an order of magnitude, and increases the shear rate at which transition from low shear Newtonian behavior to non-Newtonian behavior occurs by more than one order of magnitude. Shear degradation has essentially no effect on Ω or the high shear limiting viscosity.

Typical friction loss data[8] for various Separan AP-30

polyacrylamide solutions in a smooth stainless steel pipe with a diameter of 0.460 centimeter appear in Figure 2 as the Fanning friction factor versus the Reynolds number using the solvent viscosity. The data show that the greatest degree of drag reduction occurs for the most dilute shear degraded solutions[10,11] and that the friction loss for the solvent (water) alone may be represented by the Blasius or Colebrook[12] equations.

Assuming a linear viscoelastic Maxwell model to describe the solution viscoelastic properties, the friction factor for the polymer solution relative to the friction factor of the Newtonian solvent is predicted to be

$$f_P = f_s / (1 + N^2_{DE})^{1/2} \qquad (1)$$

where the Deborah number (N_{DE}) is a dimensionless characteristic eddy frequency, equal to $\omega_P \lambda$, where ω_P is the characteristic frequency of the most dissipative turbulent eddies, and λ is the fluid relaxation time. The function $(1+N^2_{DE})^{1/2}$ can be thought of as an "energy storage function" which accounts for the energy stored by the elastic properties of polymer solution.

First order characteristics of the dissipative eddies for both Newtonian solvent and polymer solution were used to determine an

approximate expression for the eddy frequency of the Newtonian solvent:

$$\omega = 0.0166\,[8V/D]\,N_{RES}^{3/8} \tag{2}$$

Approximating ω by $8V/D$ in the denominator of the expression for the Deborah number in the form presented by Darby[11]

$$N_{DE} = \lambda\,\omega_P = \frac{\omega\lambda}{(1+\lambda^2\omega_P^2)^{1/4}}\left[\frac{N_{REP}}{N_{RES}}\right]^{1/2} \tag{3}$$

gives:

$$N_{DE} = \frac{0.0166\,N_\lambda\,N_{RES}^{3/8}\,(\mu_s/\eta_o)^{1/2}}{(1+N_\lambda^2)^{1/4}} \tag{4}$$

where $N_\lambda = 8V\lambda/D$, $N_{RES} = \rho VD/\mu_s$, and $N_{REP} = \rho VD/\eta_0$.

Due to the assumptions involved in its derivation, including linear viscoelastic properties and the approximation for ω, this theoretical expression for the Deborah number is only an approximation. However, using this equation as a basis for correlating drag reduction data, with the numerical coefficient and exponents determined by regression analysis, yielded

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$$N_{DE} = \frac{0.0867 \, N_\lambda \, N_{RES}^{0.34} \, (\mu_s / \eta_o)^{0.237}}{(1 + N_\lambda^2)^{0.33}} \qquad (5)$$

This equation correlated all of the data, for three concentrations of both fresh and degraded solutions in six tube sizes, with a correlation coefficient of $r^2 = 0.956$. The result is shown in Figure 3, which shows the range of data and the model correlation.

Objectives

The purpose of this study is to investigate the extent to which the above model and correlation for drag reduction can be improved by relaxing some of the approximations made in the original development. The specific approximations to be evaluated are:

1. Neglecting the effect of Reynolds number on the frequency of the dissipative eddies for a Newtonian fluid (i.e. equation (2));

2. Using the Maxwell model to define the properties of the linear viscoelastic fluid.

Also, observations have indicated that a maximum asymptote for drag reduction exists, as described by Virk[13]. An attempt will be made to modify and extend the model to enable the prediction of this maximum drag reduction asymptote.

Development

Eddy Frequency

Relaxing the approximation for the frequency of the dissipative eddies by substituting the entire expression given by equation (2) into the expression for the Deborah number (3) results in

$$N_{DE} = \frac{0.0166\, N_\lambda\, N_{RES}^{3/8}\, (\mu_s / \eta_o)^{1/2}}{[\,1 + (0.0166\, N_\lambda\, N_{RES}^{3/8})^2\,]^{1/4}} \qquad (6)$$

In order to fit the model to the data, the numerical values of .0166 in the numerator, .0166 in the denominator, 3/8, 1/4, 1/2 were replaced by parameters A, B, b, c, d, respectively, which were determined by regression of equation (6) with the data of Chang.

Both linear and nonlinear regression techniques were used to evaluate the regression parameters. In order to use the linear

technique, equation (6) was transformed into

$$\ln(N_{DE}) = \ln(A) + \ln(N_\lambda) + b\ln(N_{RES}) - c\ln(1+BN_\lambda N_{RES}) + d\ln(\mu_S/\eta_o)$$

The "REG" procedure available on the fourth release of the Statistical Analysis System (SAS) was then exercised for various arrangements of the above equation. Equation (6) was also used, in linear and nonlinear form, with the nonlinear procedure "NLIN" available on SAS, using both the Guass-Newton and Marquardt iteration methods. Selecting the result that best fit the data yielded

$$N_{DE} = \frac{0.03357 \, N_\lambda \, N_{RES}^{.375} \, (\mu_S/\eta_o)^{.296}}{\left[1 + (0.00777 \, N_\lambda \, N_{RES}^{3/8})^2\right]^{.262}} \tag{7}$$

Comparison of r^2 or F values between the original correlation and the new correlation is not possible since these statistics are not available in the "NLIN" statistical summary. However, comparison of the residuals and sum of squares is a possible measure of improvement. The corrected total sum of least squares for the new correlation was 25.4, a 84% improvement when compared to the 159.7 for the original correlation. The exponents in equation (7)

show better agreement with the exponents of the theoretical expression (equation (6)) than do these of equation (5), and, in all cases, the standard error in the parameter estimates improved. Table 1 compares the parameter estimates between the correlations.

Table 1: Comparison of Regression Parameters

Parameter	Theoretical Value	Value in Eqn. 5	Value in Eqn. 7	Percent Improvement
A	0.0166	0.087	0.034	319.3
B[†]	0.0166	–	0.0078	–
b	3/8	0.34	0.375	9.3
c	1/4	0.33	0.262	27.2
d	1/2	0.237	0.296	11.8

[†] parameter B does not exist in equation (5)

The "Percent Improvement" is the degree by which the parameter in equation (7) is closer than the parameter in equation (5) to the theoretical value of that parameter from equation (6).

Another measure of the relative goodness of fit is the comparison of the friction factor plots ($f_p \sqrt{1 + N^2_{DE}}$ versus N_{RES}) between the original correlation (equation 5) and the new correlation (equation 7). Comparing the r^2 for figure 3 (.6896)

to the r^2 for Figure 4 (.7920) indicates a 15% improvement in precision. Figure 4 is also qualitatively better than Figure 3 since the cloud of data points is better centered, fits tighter, has less curvature, and follows the trend of the Colebrook line better. Thus, including the dependence of the dissipative eddies on the solution Reynolds number has increased the accuracy of the correlation considerably, and has also provided enhanced support for the model leading to equation (6).

Viscoelastic Model

The previous model assumed that the relaxation time for the dilute polymer solution could be defined by the linear viscoelastic Maxwell model. When generalized by incorporating the nonlinear co-rotational derivative to follow continuous deformations, the Maxwell model predicts shear thinning behavior, but does not predict the high shear limiting viscosity observed in the apparent viscosity function for the solutions. This makes replacement of the Maxwell model with a Jeffreys model attractive, since the Jeffreys model accounts for the high shear limiting viscosity by including a retardation time constant.

In generalized form the Jeffreys model can be written:

$$\tau_{ij} + \lambda \frac{D\tau_{ij}}{Dt} = \eta_o \Delta_{ij} + \lambda \eta_\infty \frac{D\Delta_{ij}}{Dt}$$

(8)

$$= \eta_o(\Delta_{ij} + \lambda_2 \frac{D\Delta_{ij}}{Dt})$$

where λ_2 is the retardation time constant equal to $\lambda\eta_\infty/\eta_o$. The Jeffreys model predicts a complex viscosity ($\eta^* = \eta' - j\eta''$) as follows:

$$\eta' = \frac{\eta_o(1 + \lambda\lambda_2\omega^2)}{1 + \lambda^2\omega^2} \quad , \quad \eta'' = \frac{\omega\eta_o(\lambda - \lambda_2)}{1 + \lambda^2\omega^2}$$

(9)

The expression for the Deborah number based on the rate of energy dissipation per unit mass for any linear viscoelastic fluid is given by

$$\dot{e} = \frac{V_o^2 \omega \eta'}{2[\eta'^2 + \eta''^2]^{1/2}}$$

(10)

which, using equation (9) for the Jeffreys model, becomes

$$\dot{e} = \frac{V_o^2 \omega}{2} \left[\frac{(1 + \lambda\lambda_2\omega^2)}{\{1 + \omega^2(\lambda^2 + \lambda_2^2) + (\lambda\lambda_2\omega^2)^2\}^{1/2}} \right]$$

(11)

The friction factor for the polymer solution (f_P) relative to that for the solvent (f_s) is, therefore:

$$\frac{f_P}{f_s} = \frac{1 + \lambda\lambda_2\omega_P^2}{\{1 + \omega_P^2(\lambda^2 + \lambda_2^2) + (\lambda\lambda_2\omega_P^2)^2\}^{1/2}}$$

$$= \frac{1 + N_{DE}^2 N_V}{[1 + N_{DE}^2(1 + N_V^2) + N_{DE}^4 N_V^2]^{1/2}} \qquad (12)$$

where $N_{DE} = \omega_P\lambda$, $N_V = \eta_\infty/\eta_o = \lambda_2/\lambda$. From equation (12), the friction factor ratio exhibits a minimum with respect to Deborah number, indicating that this equation predicts a maximum drag reduction asymptote ($\%D.R. = [1 - (f_P/f_s)] \cdot 100\%$). As demonstrated in Figure 5, which is a plot of f_P/f_s versus N_{DE} from equation (12) for a series of constant values for N_V, this agrees qualitatively with the phenomenon observed by Virk. However, equation (12) is not quantitatively consistent with the observed data in that values of N_{DE} determined from this equation and the data for f_P/f_s are not realistic.

An expression for the Deborah Number based on the Jeffreys model can be derived by following the original development.[8,11] The most dissipative eddy frequency for the Newtonian solvent[14] is given approximately by:

$$\omega = \left[\frac{2}{9}\right]^{1/2} \frac{V}{D} \sqrt{f N_{RE}} \qquad (13)$$

Substitution of the Blasius equation, $f = 0.0791/(N_{RE})^{1/4}$, into the above equation produces

$$\omega = 0.0166(8V/D)N_{RE}^{3/8} \qquad (14)$$

as the expression for the most dissipative eddy frequency. By analogy an expression similar to equation (13) is assumed for the polymer solution:

$$\omega_P = \left[\frac{2}{9}\right]^{1/2} \frac{V}{D} \sqrt{f_P N_{REP}} \qquad (15)$$

Combination of equations (12), (13) and (15) results in:

$$\frac{\omega_P}{\omega} = \left[\frac{(1 + \lambda\lambda_2\omega_P^2)}{\{1 + \omega_P^2(\lambda^2 + \lambda_2^2) + (\lambda\lambda_2\omega_P^2)^2\}^{1/2}}\right]^{1/2} \left[\frac{N_{REP}}{N_{RES}}\right]^{1/2} \qquad (16)$$

where $N_{REP}/N_{RES} = \mu_S/\eta_0$, and $N_{REP} = DV\rho/\eta_0$ is the Reynolds number for the solvent based on the limiting Newtonian viscosity.

Rearrangement of equation (14) as $1 = 0.0166(8V/D\omega)N_{RE}^{3/8}$, and

multiplying by the definition of Deborah number gives

$$N_{DE} = \omega_P\lambda = 0.0166(\omega_P/\omega)(8V\lambda/D)N_{RE}^{3/8} \tag{17}$$

as an alternate expression for the Deborah number. Substitution

of equation (16) into equation (17) produces the following expression

for the Deborah number based on the Jeffreys model

$$N_{DE} = 0.0166\,N_\lambda N_{RE}^{3/8}\left[\frac{1 + \lambda\lambda_2\omega_P^2}{\{1 + \omega_P^2(\lambda^2 + \lambda_2^2) + (\lambda\lambda_2\omega_P^2)^2\}^{1/2}}\right]^{1/2}\left[\frac{\mu_s}{\eta_o}\right]^{1/2} \tag{18}$$

where $N_\lambda = 8V\lambda/D$, $\omega_P \approx 8V/D$, $\lambda_2 = \lambda\eta_\infty/\eta_o = \lambda N_V$. Placing this

equation into dimensionless form

$$N_{DE} = 0.0166\,N_\lambda N_{RE}^{3/8} N_V^{1/2}\left[\frac{1 + N_\lambda^2 N_V}{[1 + N_\lambda^2(1 + N_V^2) + N_\lambda^4 N_V^2]^{1/2}}\right]^{1/2} \tag{19}$$

where $N_V = \eta_\infty/\eta_o \approx \mu_s/\eta_o$. Hence, the correlation for the Deborah

number should have the following form:

$$N_{DE} = \frac{A \, N_\lambda N_{RE}^b N_V^c \, (1 + N_\lambda^2 N_V)^a}{\left[1 + N_\lambda^2 (1 + N_V^2) + N_\lambda^4 N_V^2 \right]^d} \tag{20}$$

where A, a, b, c, and d are regression parameters. N_λ is evaluated

from rheological properties by $N_\lambda = [(1 + N_\zeta)^\Omega - 1]^{1/2}$, where

$N_\zeta = 8V\zeta/D$.

The procedure would be to determine values of N_{DE} from the

friction loss data f_p/f_s by solving equation (12) for N_{DE} for each

data point. The resulting values could be fit by equation (20) by

regression to determine the regression parameters. Solving

equation (12) for the Deborah number is equivalent to finding the

inverse function of (12). Unfortunately, this "inverse" function

does not map from real space into real space for the given domain

of the input friction factor ratios, but instead maps from real

space to complex space. Examination of equation (12) reveals the

reason for this problem.

Analysis of the Jeffreys Model Based Deborah Number Expression

Equation (12) from the above analysis based upon the Jeffreys

model is repeated below:

$$\frac{f_P}{f_s} = \frac{1 + N_{DE}^2 N_V}{\left[1 + N_{DE}^2 (1 + N_V^2) + N_{DE}^4 N_V^2\right]^{1/2}} \qquad (12)$$

Equation (12) can be rearranged as follows:

$$N_{DE}^4 + N_{DE}^2 \frac{\left\{2N_V - \left[\frac{f_P}{f_s}\right]^2 \left[1 + N_V^2\right]\right\}}{N_V^2 \left[1 - \left[\frac{f_P}{f_s}\right]^2\right]} + \frac{1 - \left[\frac{f_P}{f_s}\right]^2}{N_V^2 \left[1 - \left[\frac{f_P}{f_s}\right]^2\right]} = 0 \qquad (21)$$

After letting $Y = N_{DE}^2$, equation (21) becomes $Y^2 + bY + c = 0$, which is a quadratic in Y. The solution of the quadratic for Y (and hence N_{DE}) is real if and only if $b^2 - 4c \geq 0$. Thus,

$$\left[\frac{\left\{2N_V - \left[\frac{f_P}{f_s}\right]^2 \left[1 + N_V^2\right]\right\}}{N_V^2 \left[1 - \left[\frac{f_P}{f_s}\right]^2\right]}\right]^2 - \frac{4}{N_V^2} \geq 0 \qquad (22)$$

must be satisfied if N_{DE} is to be real. Solving this equation for f_P/f_s gives

$$\left[\frac{f_P}{f_s}\right] \geq \sqrt{\frac{2N_V - \sqrt{4N_V^2}}{N_V^2 - \sqrt{4N_V^2} + 1}} \qquad (23)$$

where the positive root of equation (23) is taken since f_p/f_s must be positive. If the positive root of $\sqrt{4N_V^2}$ is taken, then equation (23) says that $f_p/f_s \geq 0$, which is trivial. Taking the negative root of $\sqrt{4N_V^2}$, equation (23) becomes:

$$\left[\frac{f_p}{f_s}\right] \geq 2\sqrt{\frac{N_V}{(N_V + 1)^2}} \tag{24}$$

Equation (24) mandates that the minimum friction factor ratio that will still give real Deborah numbers occurs when equation (24) is an equality. From the properties of the six polymer solutions, the right hand side of equation (24) was evaluated and is shown in Table 2, along with the minimum values of f_p/f_s observed for these solutions. Examination of Table 2 points out the incompatibility of the data and the model: the data for the shear degraded solutions indicate values of f_p/f_s less than the minimum ratio allowed by equation (24) for N_{DE} to be real.

Table 2: Comparison of Minimum f_p/f_s Ratios

Solution (ppm.)	Min ratio allowed by Eqn. 24	Min ratio observed in data
100 fresh	0.214	0.220
250 fresh	0.179	0.228
500 fresh	0.149	0.312
100 sheared	0.643	0.203
250 sheared	0.532	0.228
500 sheared	0.397	0.257

Attempted Modifications

Two alterations of equation (12) were tested in order to determine whether or not the Jeffreys model could be made to predict real valued Deborah numbers and whether or not a correlation based upon the Jeffreys model could be made quantitatively consistent with the data.

Equation (24) and the shape of the curves in Figure 5 suggested that an adjustable scaling parameter (β), which would lower the friction factor ratio predicted by the model to that observed in the data, be inserted into equation (12). The exact manifestation of the scaling parameter β in equation (12) was selected in order to keep the mathematics of the subsequent development manageable.

For one such case, equation (12) becomes

$$\frac{f_P}{f_S} = \frac{1 + \beta^2 N_{DE}^2 N_V}{\left[1 + \beta^4 N_{DE}^2 (1 + N_V^2) + \beta^4 N_{DE}^4 N_V^2\right]^{1/2}} \qquad (25)$$

Following the previous argument from equation (21) to equation (24) yields an analogous relationship for the minimum allowable friction factor ratio.

$$\left[\frac{f_P}{f_S}\right] \geq \sqrt{\frac{4\beta^2 N_V}{\beta^2 N_V + 2 N_V + \beta^2}} \qquad (26)$$

Since N_V is known, β may be found by solving equation (26) once a method for finding f_P/f_S that is consistent with, but independent of, Chang's data has been created.

One method may be derived as follows. Virk's empirical equation for the maximum drag reduction asymptote, which is consistent with the maximum drag reduction from Chang's data, may be used as an expression for the minimum value of f_P:

$$f_P = \frac{1}{\left[19 \log\left(N_{RE} \sqrt{f}\right) - 32.4\right]^2} \qquad (27)$$

The $\sqrt{f}$ in the right hand side of equation (27) may be replaced by an equation of the same form as the Blasius equation

$$f_P = a/N_{RE}^b \qquad (28)$$

where a and b are determined by best fit to Virk's maximum drag reduction asymptote in the turbulent region. Making this substitution for $\sqrt{f}$ in equation (27) results in:

$$f_P = \frac{1}{\left[19\log\left(.892 N_{RE}^{.459}\right) - 32.4\right]^2} \qquad (29)$$

The Deborah number may now be evaluated by the following method: 1) Calculate f_P from equation (29) and the friction factor for the Newtonian solvent f_S from either the Colebrook equation ($f_S = 0.41*[\log_e(N_{RES}/7)]^{-2}$) or the Blasius equation; 2) from N_V, which is known for each solution, and f_P/f_S , calculate β from equation (26); 3) using this β, calculate N_{DE} from equation (25)

Real values for the Deborah number for all of the solutions were successfully obtained by this scheme. However, these numbers were of an unacceptable magnitude and trend. That is,

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the Deborah number would be expected to be on the order of $0 < N_{DE} < 10$, since a purely viscous fluid has a Deborah number of zero, and should increase with increasing Reynolds number, since larger Deborah numbers imply greater energy storage by the elastic properties (i.e. drag reduction). The Deborah numbers ranged from 10^0 to 10^3 when either the Colebrook or the Blasius equation was used for f_s. For all of the solutions, the Deborah number found by this method decreased with increasing Reynolds number, which is opposite to the trend predicted by the models and correlations described previously.

The second alteration was an attempt to weight the importance of the η'' term of the complex viscosity, which is equivalent to putting a weighting factor on the elastic properties of the solution. For this alteration, the energy per unit mass of the linear viscoelastic fluid becomes

$$\dot{e} = \frac{V_0^2 \omega \eta'}{2[\eta'^2 + \beta^2 \eta''^2]^{1/2}} \tag{30}$$

where β is the weighting factor. With this modification, equation (12) becomes

$$\frac{f_P}{f_s} = \frac{1 + N_{DE}^2 N_V}{\left[1 + 2 N_{DE}^2 N_V (1 - \beta^2) + \beta^2 N_{DE}^2 (1 + N_V^2) + N_{DE}^4 N_V^2\right]^{1/2}} \tag{31}$$

and the criterion for the minimum allowable friction factor analogous to equation (24) is

$$\left[\frac{f_P}{f_s}\right] \geq \sqrt{\frac{4 N_V}{\beta^2 + 4 N_V - \beta^2 N_V}} \tag{32}$$

The Deborah numbers obtained by using equations (31) and (32) in the procedure outlined earlier suffered the same problems as before, but to an even greater extent.

With these and the previous results in mind, it is concluded that a correlation based on the linear viscoelastic *Jeffreys* model cannot be made quantitatively consistent with the data by the procedures reported herein. It is recommended that alternative models that display the proper qualitative behavior (i.e. utilizing the high shear limiting viscosity and being able to predict the maximum drag reduction asymptote) be investigated.

Summary

An improved version of an existing generalized correlation that predicts the pipe friction loss of drag reducing polymer solutions has been developed. This correlation accounts for the reduced energy dissipation observed in the viscoelastic solutions by including a dimensionless Deborah number, which represents the energy storage by elastic deformation in the turbulent eddies, in the expression for the friction factor. By using an expression for the frequency of the dissipative eddies for a Newtonian fluid which is more rigorous than used previously, the precision of the correlation was improved 15%. Also, the values of the exponents calculated for the improved correlation agree more closely with the theoretically predicted exponents than the previous correlation. These results provide enhanced support for the model leading to the correlation's development.

The final correlation requires only a knowledge of friction loss for the Newtonian solvent for a given flow rate and tube diameter and three rheological parameters of the polymer solution, which can be obtained directly from the apparent viscosity function. Use of the correlation is demonstrated below:

For example, a fresh 100 ppm Separan AP-30 solution is to be pumped in a 0.25 meter smooth horizontal pipe at a velocity of 1

meter/second. Calculate the friction factor for the polymer solution and the percent drag reduction.

1. The rheological parameters are evaluated from apparent viscosity versus shear rate data. As read from Figure 1: η_0, the low shear viscosity = .1113 Pa-s; $\Omega = (1-n)/2$, where $(n-1)$ is the slope of the linear portion of the curve in the intermediate shear rate range, = .266 ; ζ , the reciprocal of the shear rate where the low shear viscosity and the linear portion of the curve in the intermediate shear rate range intersect on the figure, = 11.89 sec.

2. From the specified diameter and velocity, the dimensionless groups $N_\zeta, N_\lambda, N_{RES}, N_{DE}$, f_P, f_s , are calculated.

$$N_\zeta = \frac{8V\zeta}{D} = \frac{(8)(4m/s)(11.89 \text{ sec})}{(0.5 \text{ m})} = 761$$

$$N_\lambda = [(1 + N_\zeta^2)^\Omega - 1]^{1/2}$$
$$= [(1 + (761)^2)^{.266} - 1]^{1/2} = 5.754$$

$$N_{RES} = \frac{DV\rho}{\mu_s} = \frac{.25m}{.001 \text{ Pa-s}} \left| \frac{1m}{\text{sec}} \right| \frac{1000 Kg.}{1 m^3} \left| \frac{Pa\ m^2}{N} \right| \frac{N\ sec^2}{kg\ m}$$

$$= 2.5 \times 10^5$$

$$N_{DE} = \frac{0.03357 \, N_\lambda \, N_{RES}^{.375} \, (\mu_s/\eta_o)^{.296}}{\left[1 + (0.00777 \, N_\lambda \, N_{RES}^{3/8})^2 \right]^{.262}}$$

$$= \frac{(.03357)(5.754)(2.5 \times 10^5)^{.375}(.001/.1113)^{.296}}{\left[1 + ((.00777)(5.754)(2.5 \times 10^5)^{.375})^2 \right]^{.262}}$$

$$= 2.218$$

$$f_s = \frac{.41}{[\log_e(N_{RES}/7)]^2} = \frac{.41}{[\log_e(2.5 \times 10^5/7)]^2}$$

$$= 3.73 \times 10^{-3}$$

$$f_P = \frac{f_s}{\sqrt{1 + N_{DE}^2}} = \frac{3.73 \times 10^{-3}}{\sqrt{1 + (2.218)^2}} = 1.53 \times 10^{-4}$$

$$\%D.R. = [1-(f_P/f_s)] \cdot 100\% = [1-.41] \cdot 100\% = 59\%$$

Thus, adding 100 ppm of Separan AP-30 achieves 59% drag reduction as compared to the solvent alone.

Although it has the proper qualitative characteristics, the use of the linear Jeffreys model to define the energy storage in the viscoelastic fluid fails to produce an equation which can represent the data quantitatively. Analysis reveals that the predictions based on the Jeffreys model are inconsistent with the amount of drag reduction observed in the data, and the model cannot be manipulated to correct this behavior.

Notation

A,B,a,b,c,d – regression parameters

D   –   pipe diameter, m

D/Dt –   Jaumann or co-rotational time derivative, s^{-1}

%D.R –   percent drag reduction

$\dot{e}$ –   energy dissipation per unit mass, w/kg

f   –   Fanning Friction Factor

n   –   power law flow index

N_{DE} –   Deborah number = $\omega_p \lambda$

N_{RE} –   Reynolds number

N_λ –   dimensionless relaxation time

N_ζ –   dimensionless fluid time constant, $8V\zeta/D$

N_V –   dimensionless viscosity ratio, η_∞/η_0

V   –   average tube velocity, m/s

V_0 –   amplitude of oscillating velocity, m/s

β –   scaling or weighting parameter

λ –   relaxation time constant, s

λ_2 –   retardation time constant, $\lambda\eta_\infty/\eta_0$

Δ_{ij} –   rate of strain tensor, s^{-1}

ζ – time constant of polymer solution, s

ρ – density kg/m^3

η – apparent viscosity, Pa-s

η^* – complex viscosity, Pa-s

η' – real part of complex viscosity, Pa-s

η'' – imaginary part of complex viscosity, Pa-s

μ – Newtonian viscosity, Pa-s

τ_{ij} – shear stress tensor, Pa

ω – frequency of oscillation, s^{-1}

Ω – rheological parameter related to slope of curve in the intermediate shear rate range of apparent viscosity versus shear rate data

subscripts

p   – polymer solution

s   – solvent

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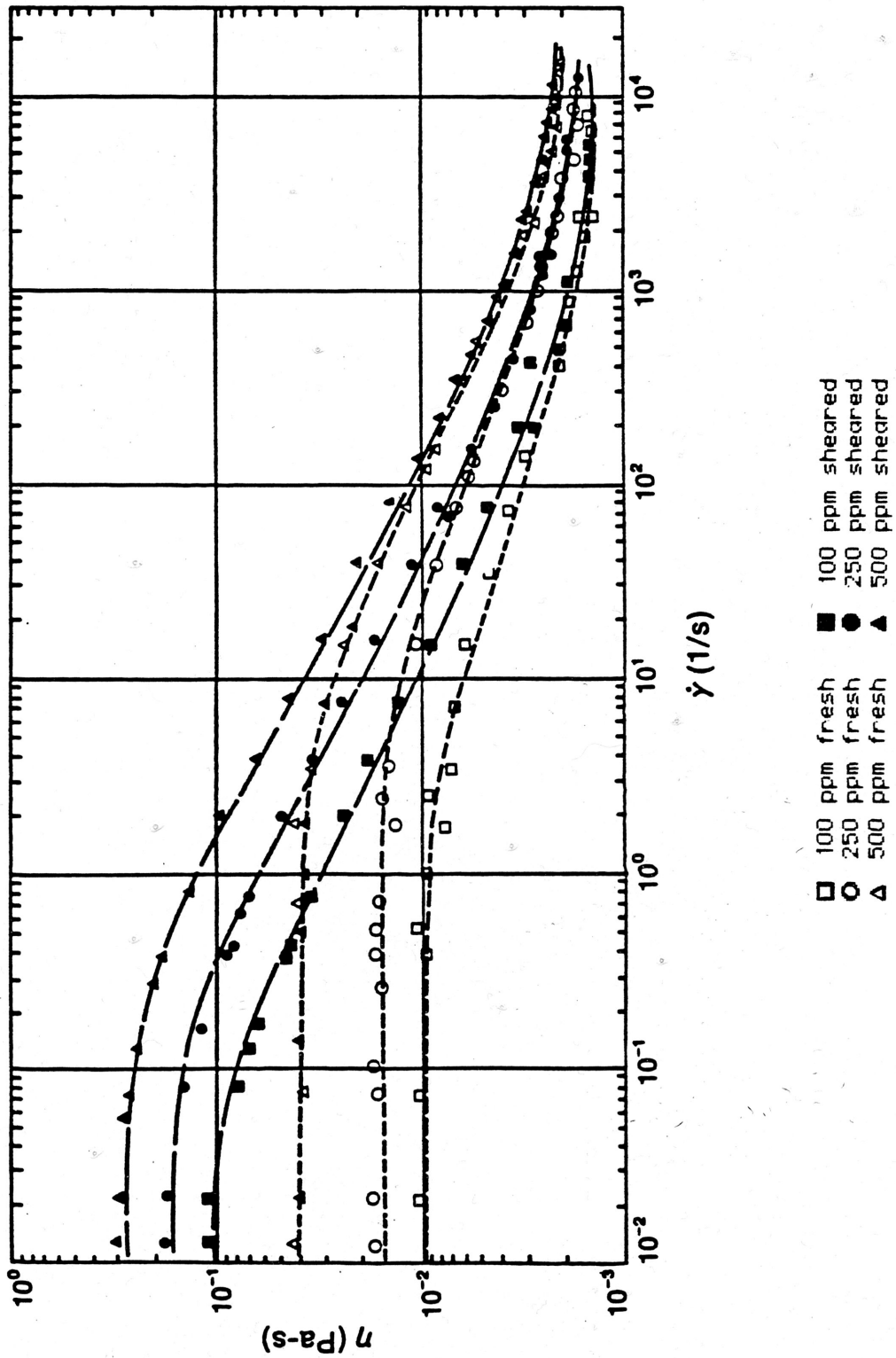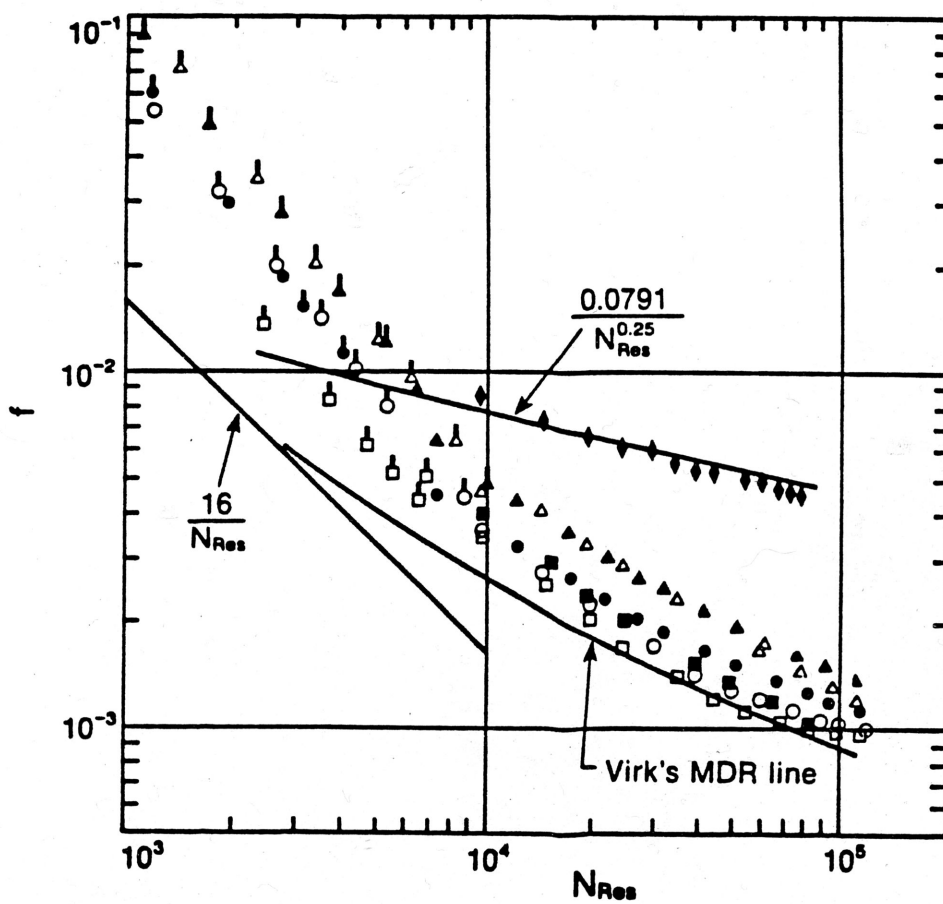# FIGURE 1: APPARENT VISCOSITY DATA FOR SEPARAN AP-30 SOLUTIONS

FIGURE 2: FANNING FRICTION FACTOR VERSUS SOLVENT REYNOLDS NUMBER

D = .460 cm

□	100 ppm fresh	■	100 ppm sheared
○	250 ppm fresh	●	250 ppm sheared
△	500 ppm fresh	▲	500 ppm sheared

FIGURE 3: FRICTION FACTOR PLOT
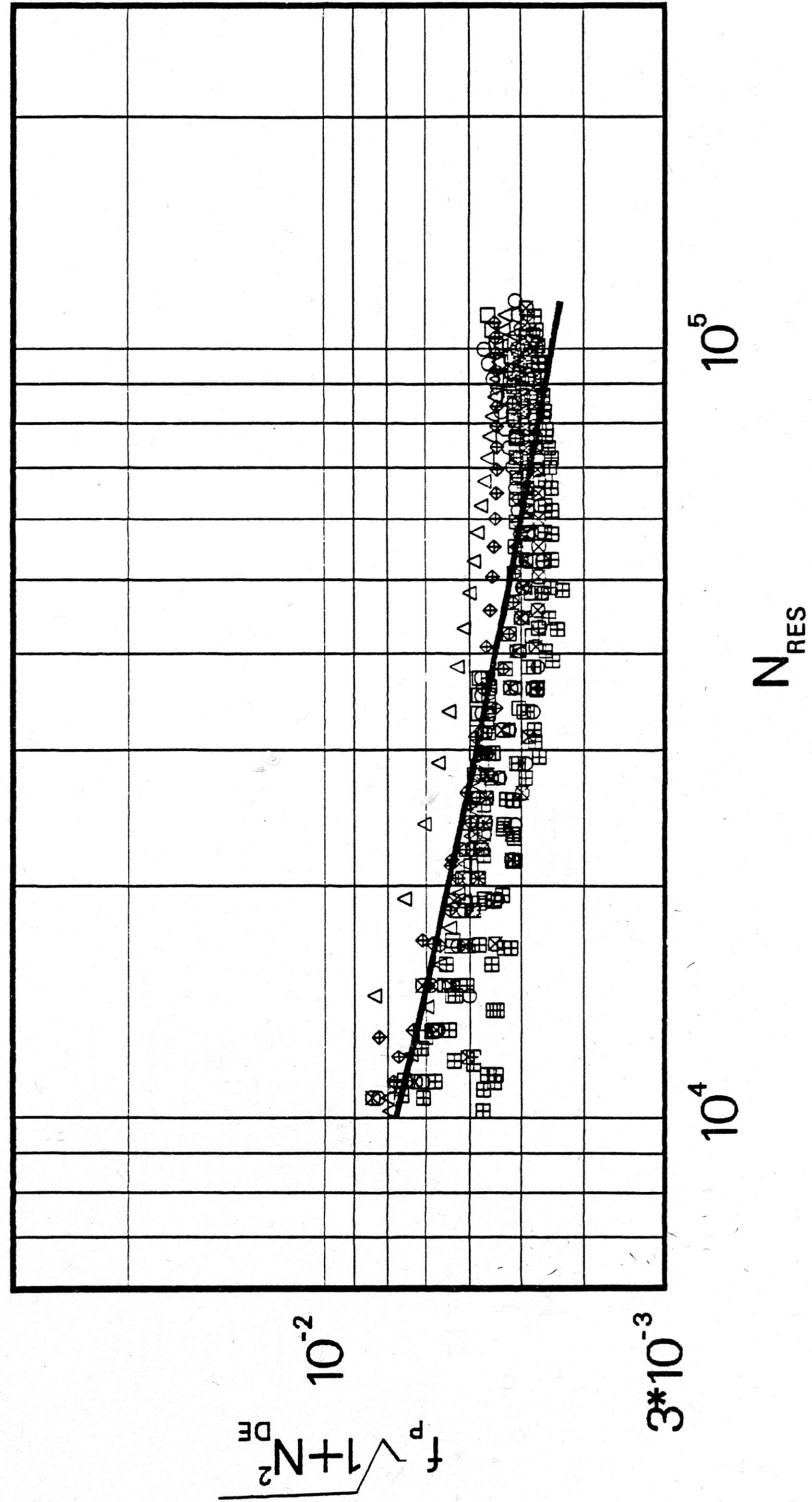FOR ORIGINAL CORRELATION

N_{RES}

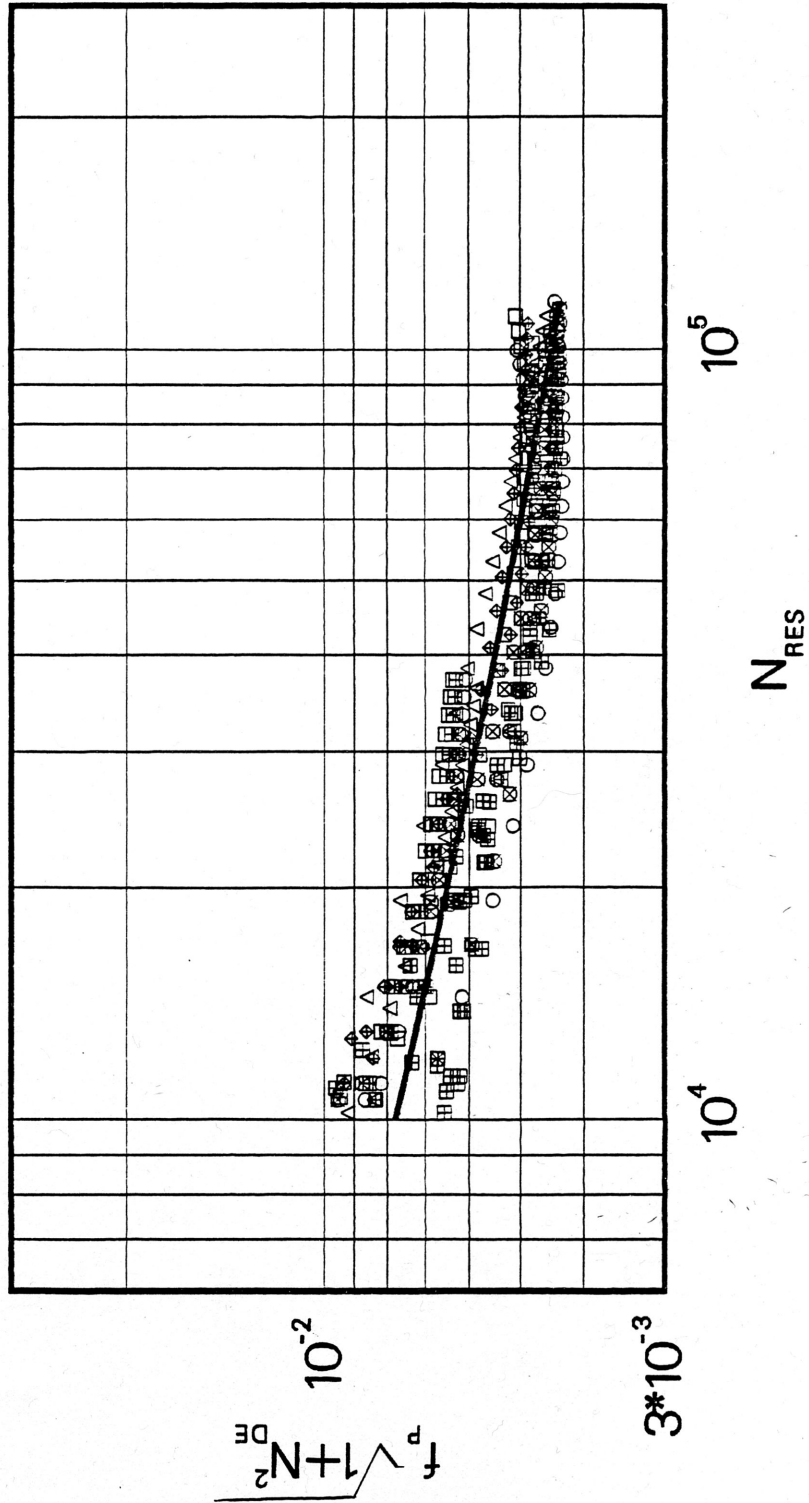$f_P \sqrt{1+N_{DE}^2}$

FIGURE 4: FRICTION FACTOR PLOT
FOR IMPROVED CORRELATION

FIGURE 5: PLOT OF EQUATION (10)