INDUCED POLARIZATION METHODS

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Abstract

Investigations of the direct current resistivity theory on which induced polarization theory is partially based, have lead to development of expressions which more clearly indicate the contribution of the perturbing layers. Also, expressions which allow simplification of numerical data inversion are presented.

Some significance is given to the calculated induced polarization results and why it is unlikely that it will ever be possible to determine the specific type of ore simply from this data. However, the data still gives an indication of the presents of potentially valuable ores or their absence.

INTRODUCTION

Induced polarization is a remote sensing geophysical prospecting tool that is designed to give information on the nature of the subsurface. Induced polarization (IP) methods of prospecting are patterned after direct current (DC) resistivity investigations. To have a better understanding of the IP method, the theory and principle of DC resistivity measurements must be examined. DC resistance apparatus measures the resistivity of the earth using a direct current and the IP device measures the resistivity of the earth using alternating currents.

Using the IP method, the measurements are made at the same points at more than one frequency so more information is made available. IP data can be interpreted as having two components. The first is similar to DC resistivity data and comes from the lowest frequency IP measurement. The second is what makes IP different from DC resistivity - the frequency effect. The frequency effect involves both the lowest frequency data and all the other higher frequency data.

DC RESISTIVITY

Direct current resistivity theory has its basis in the solutions to Laplace's equation in three dimensions. Because of the complexities in solving Laplace's equation, some simplifying assumptions are made. The ground in the vicinity of the measurements is assumed to be composed of a finite number of distinct layers with horizontal planar boundaries. Each layer is further assumed to be electrically isotropic and homogeneous. Also, the bottom layer is a half space.

These assumptions allow the problem to be solved and do not seriously limit the application of the theory.

The potential or voltage associated with an electric field satisfies Laplace's equation ($\nabla^2 V=0$). When written in cylindrical coordinates

$$\frac{\delta^2}{\delta r^2} V + \frac{1}{r}\frac{\delta V}{\delta r} + \frac{\delta^2}{\delta z^2} V + \frac{1}{r^2}\frac{\delta^2}{\delta \phi^2} V = 0$$

where z is depth, r is radial distance and ϕ is the angle from an arbitrary x-axis. Since the assumptions rule out any variance of the potential with respect to any given horizontal direction, the derivative of V with respect to ϕ is zero, and the equation reduces to:

$$\frac{\delta^2}{\delta r^2} V + \frac{1}{r}\frac{\delta V}{\delta r} + \frac{\delta^2}{\delta z^2} V = 0$$

The uniqueness theorem states that a solution found by any method is the correct solution. The dependence of V on only two variables (r,z) suggest the use of the separation of variables method. Assuming a solution can be written in the form

$$V = R(r) \cdot D(z)$$

where R(r) is only a function of r, and D(z) is only a function of z. Hence, since

$$\frac{\delta^2}{\delta r^2} V = D \frac{\delta^2}{\delta r^2} R$$

$$\frac{1}{r}\left(\frac{\delta}{\delta r} V\right) = \frac{1}{r}\left(D \frac{\delta}{\delta r} R\right)$$

$$\frac{\delta^2}{\delta z^2} V = R \frac{\delta^2}{\delta z^2} D$$

the equation $\nabla^2 V=0$ becomes

$$\frac{1}{R}\left(\frac{1}{r}\frac{\delta}{\delta r} R + \frac{\delta^2}{\delta r^2} R\right) = -\frac{1}{D}\frac{\delta^2}{\delta z^2} D$$

The left side is dependent only on r and the right side has only dependence on z. The variables r and z should be able to range independently, so each side must be equal to a constant.

$$\frac{1}{R}\left(\frac{1}{r}\frac{\delta}{\delta r} R + \frac{\delta^2}{\delta r^2} R\right) = C$$

$$-\frac{1}{D}\frac{\delta^2}{\delta z^2} D = C$$

Choosing C to be negative, we set $C=-\lambda^2$, where λ is a real variable. (If we had chosen C to be positive the solution would not have worked out). Thus the separated equations given above are

$$r^2 \frac{\delta^2}{\delta r^2} R + r \frac{\delta}{\delta r} R + (\lambda r)^2 R = 0$$ (This is Bessels equation of zero'th order)

$$\frac{\delta^2}{\delta z^2} D - \lambda^2 D = 0$$

with solutions of

$$D = A_1 e^{-\lambda z} + A_2 e^{+\lambda z}$$

$$R = A_3 J_o(\lambda r) + A_4 Y_o(\lambda r)$$

A_1, A_2, A_3, A_4 are arbitrary constants

(because $Y_0(\lambda r)$ becomes infinite at $r=0$, A_4 must be zero) and further,

$$V = D \cdot R = J_o(\lambda r)\ [\theta_1 e^{-\lambda z} + \theta_2 e^{+\lambda z}]$$

θ_1, θ_2 are arbitrary constants containing A_1, A_2 and A_3. A linear combination of solutions of the form given allows a total solution as follows, where the values of θ_1, θ_2 will depend upon the parameter λ

$$V(r,z) = \int_o^\infty [\theta_1(\lambda) e^{-\lambda z} + \theta_2(\lambda) e^{+\lambda z}]\ J_o(\lambda r) d\lambda$$

This expression together with the boundary conditions represents the solution to the DC resistivity problem. To simplify the boundary condition only one electrode is considered. This causes no loss of generality because by using supposition we may introduce additional electrodes at a later time. The boundary conditions which must hold are:

1) The normal component of the current must be continous across an interface.
2) There must be continuity in the potential across an interface

3) The potential away from the electrode must everywhere be finite.

The first condition has a special case at the earth's surface. Since no current flows out of the ground into the air, the normal component at z=0 is equal to zero. The last of the boundary conditions has a special case in the bottom layer. Since the bottom layer is a half space, it extends to points where z is infinitely large. The potential, V, must be finite everywhere, and to insure this as $z \to \infty, \theta_2(\lambda)$ must be identically zero.

SINGLE LAYER

Consider the case of a single layer from a different point of view. Ohm's Law states:

$$-p\overline{J} = \overline{E}$$

where $\overline{E}$ is the electric field vector and $\overline{J}$ is the current density vector. So breaking this into component form

$$E_R = pJ_R$$

where E_R is the radial component of $\overline{E}$ in spherical coordinates and J_R is the radial component of $\overline{J}$.

$$V_p = \int_R^\infty E_R dR$$

$$= \int_R^\infty pJ_R dR$$

$$= p \int_R^\infty J_R dR \qquad \text{(p is constant by the assumptions)}$$

for a point electrode putting in I amps at the surface of a half space, as in Figure 1, the current flows radially outward.

$$J_R = \frac{I}{\text{surface area of half sphere}} \qquad R^2 = z^2 + r^2$$

$$J_R = \frac{I}{2\pi R^2}$$

$$V_p = \frac{pI}{2\pi} \int_R^\infty \frac{1}{R^2} dR$$

$$= \frac{pI}{2\pi} \left(\frac{1}{(z^2+r^2)^{\frac{1}{2}}}\right)$$

Since

$$(z^2+r^2)^{-\frac{1}{2}} = \int_0^\infty e^{-\lambda z} J_o(\lambda r) d\lambda \qquad \text{(Lipschitz integral identity)}$$

$$V_p(r,z) = \frac{pI}{2\pi} \int_0^\infty e^{-\lambda z} J_o(\lambda r) d\lambda$$

(The subscript p refers to the particular solution of Laplaces equation). This expression will be useful in further developing the other more general solutions.

$$V(r,z) = \int_0^\infty [\theta_1(\lambda) e^{-\lambda z} + \theta_2(\lambda) e^{+\lambda z}] \, J_o(\lambda r) d\lambda$$

can be written

$$V(r,z) = \frac{pI}{2\pi} \int_0^\infty [\phi_1(\lambda)e^{-\lambda z} + \phi_2(\lambda)e^{-\lambda z}]\ J_o(\lambda r)d\lambda$$

where

$$\phi_i \frac{pI}{2\pi} = \theta_i \qquad i=1,2$$

Adding and subtracting the solution for a half space

$$V(r,z) = \frac{pI}{2\pi} [\int_0^\infty e^{-\lambda z} J_o(\lambda r)d\lambda + \int_0^\infty [\phi_1(\lambda)-1]e^{-\lambda z}$$

$$+ \phi_2(\lambda)e^{+\lambda z} J_o(\lambda r)d\lambda]$$

$$= \frac{pI}{2\pi} [\frac{1}{(z^2+r^2)^{\frac{1}{2}}} + \int_0^\infty \{[\phi_1(\lambda)-1]e^{-\lambda z} + \phi_2(\lambda)e^{+\lambda z}\} J_o(\lambda r)d\lambda]$$

we define

$$V_f(r,z) = \frac{pI}{2\pi} [\int_0^\infty \{[\phi_1(\lambda)-1]e^{-\lambda z} + \phi_2(\lambda)e^{+\lambda z}\} J_o(\lambda r)d\lambda]$$

and

$$V_p(r,z) = \frac{pI}{2\pi} [\frac{1}{(z^2+r^2)^{\frac{1}{2}}}]$$

so that

$$V = V_p + V_f$$

Substituting

$$\kappa(\lambda) = [\phi_1(\lambda)-1]$$

$$\psi(\lambda) = \phi_2(\lambda)$$

Now we introduce a subscript on V to indicate which layer the solution is for.

$$V_n(r,z) = \frac{pI}{2\pi}\left\{\frac{1}{(z^2+r^2)^{\frac{1}{2}}} + \int_0^\infty [\kappa_n(\lambda)e^{-\lambda z} + \psi_n(\lambda)e^{+\lambda z}]J_o(\lambda r)d\lambda\right\}$$

or alternately

$$V_n(r,z) = \frac{pI}{2\pi}\left\{\int_0^\infty e^{-\lambda z}J_o(\lambda r)d\lambda + \int_0^\infty [\kappa_n(\lambda)e^{-\lambda z}\right.$$

$$\left. + \psi_n(\lambda)e^{+\lambda z}]J_o(\lambda r)d\lambda\right\}$$

It can be seen that the integral involving $\kappa_m(\lambda)$ and $\psi_m(\lambda)$ represents the difference between the voltage that would be measured if the layer was not present and the actual voltage. This difference is the perturbing or anomalous effect. Applying the boundary conditions at the surface: (continuity of the normal component of current at the interface)

$$\frac{1}{p_1}\frac{\delta V_1}{\delta z}\bigg|_{z=0} = \frac{1}{p_o}\frac{\delta V_o}{\delta z}\bigg|_{z=0} = 0$$

since, the resistivity of the air above the top layer, is infinite.

Therefore,

$$\int_0^\infty \lambda[\kappa_1(\lambda)-\psi_1(\lambda)]J_o(\lambda r)d\lambda = 0.$$

If the integral is equal to zero, then the integrand must be equal to zero and since λ and $J_o(\lambda r)$ are not everywhere zero,

$$\kappa_1(\lambda) = \psi_1(\lambda).$$

Putting this back into the equation for V_1

$$V_1(r,z)= \frac{p_1 I}{2\pi} \{\frac{1}{(z^2+r^2)^{\frac{1}{2}}} + \int_0^\infty \kappa_1(\lambda)[e^{-\lambda z} + e^{+\lambda z}]J_o(\lambda r)d\lambda\}$$

However, all measurements are made at the surface so V_1 (r,z=0) is what is most important, so

$$V_1(r,0) = \frac{p_1 I}{2\pi} \{\frac{1}{r} + 2 \int_0^\infty \kappa_1(\lambda)J_o(\lambda r)d\lambda\}$$

MULTI-LAYER CASE

Now the problem has been reduced to finding $K_1(\lambda)$. This is commonly called the Kernel function. Applying the boundary conditions to an multiple layer case,

$$V_n = \frac{p_1 I}{2\pi} \int_0^\infty \{[\kappa_n(\lambda)+1]e^{-\lambda z} + \psi_n(\lambda)e^{-\lambda z}\}J_o(\lambda r)d\lambda$$

$$V_{n+1} = \frac{p_1 I}{2\pi} \int_0^\infty \{[\kappa_{n+1}(\lambda)+1]e^{-\lambda z} + \psi_{n+1}(\lambda)e^{+\lambda z}\}J_o(\lambda r)d\lambda$$

where V_n is the potential in the nth layer.

For the case of N layers, as in Figure 2

$$V_n(z=h_n) = V_{n+1}(z=h_n)$$

$$\frac{p_1 I}{2\pi} \int_0^\infty \{[\kappa_n(\lambda)+1]e^{-\lambda h_n} + \psi_n(\lambda)e^{+\lambda h_n}\}J_o(\lambda r)d\lambda \quad =$$

$$\frac{p_1 I}{2\pi} \int_0^\infty \{[\kappa_{n+1}(\lambda)+1]e^{-\lambda h_n} + \psi_{n+1}(\lambda)e^{+\lambda h_n}\}J_o(\lambda r)d\lambda$$

$$(1 \leq n \leq N - 1)$$

Moving all the expressions to one side yields:

$$\frac{p_1 I}{2\pi} \int_0^\infty (\{[\kappa_n(\lambda)+1] - [\kappa_{n+1}(\lambda)+1]\}e^{-\lambda h_n} + [\psi_n(\lambda)$$

$$- \psi_{n+1}(\lambda)]e^{+\lambda h_n})J_o(\lambda r)d\lambda = 0$$

Again an integral is equal to zero so the integrand is equal to zero.

$$[\kappa_n(\lambda) - \kappa_{n+1}(\lambda)] + [\psi_n(\lambda)-\psi_{n+1}(\lambda)]e^{+2\lambda h_n} = 0$$

This gives N-1 equations relating the variables. Recalling in the general development we had one additional equation:

$$\kappa_1(\lambda) = \psi_1(\lambda)$$

thus, giving a total of N equations so far.

Making use of the continuity of the normal current at the nth interface

$$\frac{1}{p_n}\frac{\partial V_n}{\partial z}\Big|_{z=h_n} = \frac{1}{p_{n+1}}\frac{\partial V_{n+1}}{\partial z}\Big|_{z=h_n}$$

$$\frac{p_1 I}{2\pi}\int_0^\infty \lambda J_o(\lambda r)\{[\frac{\kappa_n(\lambda)+1}{p_n} - \frac{\kappa_{n+1}(\lambda)+1}{p_{n+1}}]e^{-\lambda h_n}$$

$$- [\frac{\psi_n(\lambda)}{p_n} - \frac{\psi_{n+1}(\lambda)}{p_{n+1}}]e^{+\lambda h_n}\}d\lambda = 0$$

Again the integral equals zero, so that

$$\{\kappa_n(\lambda) - (\frac{p_n}{p_{n+1}})\kappa_{n+1}(\lambda)\} - \{\psi_n(\lambda) - (\frac{p_n}{p_{n+1}})\psi_{n+1}(\lambda)\}e^{+2\lambda h_n} = \frac{p_n}{p_{n+1}} - 1$$

$$(1 \leq n \leq N-1)$$

This again gives N-1 equations as n varies.

Now applying the remaining boundary condition that V must be finite, as $z\to\infty$, thus $\psi_N(\lambda) = 0$. This is the last equation necessary to solve the system of equations, because we now have 2N equations and 2N unknowns (a $\kappa(\lambda)$ and $\psi(\lambda)$ for each of the N layers).

Solving the equations to find K_1 can be done but there is a general formula for K_1 for any number of layers. It is easiest to arrive at the general formula by examining the 2 layer and 3 layer cases and utilizing mathematical induction for N layers. For the 2 layer case, N=2, the following set of equations results: h_1 is the depth to the top of the second layer

$$\kappa_1(\lambda) - \kappa_2(\lambda) + e^{+2\lambda h_1}\,\psi_1(\lambda) - e^{+2\lambda h_1}\,\psi_2(\lambda) = 0$$

$$\kappa_1(\lambda) \qquad - \qquad \psi_1(\lambda) \qquad = 0$$

$$\kappa_1(\lambda) - \frac{p_1}{p_2}\kappa_2(\lambda) - e^{+2\lambda h_1}\,\psi_1(\lambda) + \frac{p_1}{p_2}e^{+2\lambda h_1}\,\psi_2(\lambda) = \frac{p_1}{p_2} - 1$$

$$\psi_2(\lambda) = 0$$

Solving for $\kappa_1(\lambda)$, we get

$$\kappa_1(\lambda) = \frac{-u_{12}e^{-2\lambda h_1}}{1+u_{12}e^{-2\lambda h_1}}$$

where

$$u_{12} = \left(\frac{p_1-p_2}{p_1+p_2}\right)$$

The surface potential distribution for a two layer case is

$$V_1(r,0) = \frac{p_1 I}{2\pi}\left\{\frac{1}{r} - 2\int_0^\infty \left(\frac{u_{12}e^{-2\lambda h_1}}{1+u_{12}e^{-2\lambda h_1}}\right)J_o(\lambda r)\,d\lambda\right\}$$

For the three layer case N=3, the following set of expressions result:

h_1 = depth to top of second layer

h_2 = depth to top of third layer

$$\begin{aligned}
&\kappa_1(\lambda) - \kappa_2(\lambda) + e^{+2\lambda h_1}\psi_1(\lambda) - e^{+2\lambda h_1}\psi_2(\lambda) = 0\\
&\kappa_2(\lambda) - \kappa_3(\lambda) + e^{+2\lambda h_2}\psi_2(\lambda) - e^{+2\lambda h_2}\psi_3(\lambda) = 0\\
&\kappa_1(\lambda) - \psi_1(\lambda) = 0\\
&\kappa_1(\lambda) - \frac{p_1}{p_2}\kappa_2(\lambda) - e^{+2\lambda h_1}\psi_1(\lambda) + \frac{p_1}{p_2}e^{+2\lambda h_1}\psi_2(\lambda) = \frac{p_1}{p_2} - 1\\
&\kappa_2(\lambda) - \frac{p_2}{p_3}\kappa_3(\lambda) - e^{+2\lambda h_2}\psi_2(\lambda) + \frac{p_2}{p_3}e^{+2\lambda h_2}\psi_3(\lambda) = \frac{p_2}{p_3} - 1\\
&\psi_3(\lambda) = 0
\end{aligned}$$

again solving for $\kappa_1(\lambda)$ we get

$$\kappa_1(\lambda) = \frac{-u(\lambda)e^{-2\lambda h_1}}{1+u(\lambda)e^{-2\lambda h_1}}$$

where

$$u(\lambda) = \frac{p_1 - p_2\left(\dfrac{1-u_2e^{-2\lambda h_2}}{1+u_2e^{-2\lambda h_2}}\right)}{p_1 + p_2\left(\dfrac{1-u_2e^{-2\lambda h_2}}{1+u_2e^{-2\lambda h_2}}\right)}$$

and

$$u_2 = \frac{p_2 - p_3}{p_2 + p_3}$$

Therefore, by using mathematical induction for the N layer case we get

$$\kappa_1(\lambda) = \frac{-u_{1\ldots N} e^{-2\lambda h_1}}{1+u_{1\ldots N} e^{-2\lambda h_1}}$$

$$u_{1\ldots N} = \frac{p_1 - p_2 \; k_{2\ldots N}}{p_1 + p_2 \; k_{2\ldots N}}$$

$$k_{(m-1)\ldots N} = \frac{1-u_{(m-1)\ldots N} e^{-2\lambda h_{m-1}}}{1+u_{(m-1)\ldots N} e^{-2\lambda h_{m-1}}} \qquad m=3\ldots(N-1)$$

$$u_{(m-1)\ldots N} = \frac{p_{m-1} - p_m \; k_{m\ldots N}}{p_{m-1} + p_m \; k_{m\ldots N}} \qquad m=3\ldots(N-1)$$

$$k_{(N-1)N} = \frac{1-u_{(N-1)N} e^{-2\lambda h_{N-1}}}{1+u_{(N-1)N} e^{-2\lambda h_{N-1}}}$$

$$u_{(N-1)\,N} = \frac{p_{N-1} - p_N}{p_{N-1} + p_N}$$

and the potential at the top of the surface layer is

$$V_1(r,0) = \frac{p_1 I}{2\pi} \left\{ \frac{1}{r} - 2 \int_0^\infty \left(\frac{u_{1\ldots N} e^{-2\lambda h_1}}{1+u_{1\ldots N} e^{-2\lambda h_1}} \right) J_o(\lambda r) d\lambda \right\}$$

where $u_{1\ldots N}$ is defined as before. By double subscripting $\kappa_1(\lambda)$ and $V_1(z=0)$ this is finally written in a general form:

$$V_{1,N}(r,0) = \frac{p_1 I}{2\pi} \left\{ \frac{1}{r} - 2 \int_0^\infty \kappa_{1,N}(\lambda) J_o(\lambda r) d\lambda \right\}$$

Note that this is still for only one electrode.

Using a more normal four electrode set up with two current electrodes and two measuring electrodes the total potential is:

$$V_{total} = V_{1,N}(r_1,0) - V_{1,N}(r_2,0) - V_{1,N}(r_3,0) + V_{1,N}(r_4,0)$$

Using the LIpschitz identity an alternate form can be derived.

Since

$$\frac{1}{(z^2+r^2)^{\frac{1}{2}}} = \int_0^\infty e^{-\lambda z} J_o(\lambda r)d\lambda$$

setting z=0 gives

$$\frac{1}{r} = \int_0^\infty J_o(\lambda r)d\lambda$$

Hence,

$$V_{1,N}(r,0) = \frac{p_1 I}{2\pi}\left\{\int_0^\infty J_o(\lambda r)d\lambda + \int_0^\infty 2\kappa_{1,N}(\lambda)J_o(\lambda r)d\lambda\right\}$$

$$V_{1,N}(r,0) = \frac{p_1 I}{2\pi}\int_0^\infty [2\kappa_{1,N}(\lambda)+1]J_o(\lambda r)d\lambda$$

$$2\kappa_{1,N}(\lambda) + 1 = 1-2\,\frac{u_{1...N}e^{-2\lambda h_1}}{1+u_{1...N}e^{-2\lambda h_1}}$$

$$= \frac{1-u_{1...N}e^{-2\lambda h_1}}{1+u_{1...N}e^{-2\lambda h_1}}$$

$$= k_{1...N}$$

$$V_{1,N}(r,0)=\frac{p_1 I}{2\pi}\int_0^\infty k_{1\ldots N} J_1(\lambda r)d\lambda$$

We now can write the total potential for the four electrode array as

$$V_{total}^{(r,0)} = \frac{p_1 I}{2\pi}\int_0^\infty k_{1\ldots N}[J_1(\lambda r_1) - J_o(\lambda r_2) - J_o(\lambda r_3)$$

$$+ J_o(\lambda r_4)]d\lambda$$

This expression contains an element related only to the electrode geometry, $[J_o(\lambda r_1)-J_o(\lambda r_2)-J_o(\lambda r_3)+J_o(\lambda r_4)]$, and an element related only to the earth's layering $(k_{1\ldots N})$.

Also, using the alternate form:

$$V_{1,N(total)}^{(r,0)} = \frac{p_1 I}{2\pi}\{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}$$

$$-2\int_0^\infty \kappa_{1,N}(\lambda)[J_o(\lambda r_1)-J_o(\lambda r_2)-J_o(\lambda r_3)-J_o(\lambda r_4]d\lambda\}$$

<u>DATA MANIPULATION</u>

When data is collected, it is usually expressed as apparent resistivity (p_a) which is defined as:

$$p_a = 2\pi\frac{V}{I}\left[\frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}}\right]$$

where V and I are measured quantities and the electrode geometry r_o defined as

$$r_o \equiv \frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}}$$

The quantity r_o is tabulated for the four geometries most generally used with IP equipment presently available. See Figure 3 and the attached table.

Rewritting p_a as

$$p_a = 2\pi\frac{V}{I} r_o$$

$$= p_1 + 2p_1 r_o \int_0^\infty K_{1,N}(\lambda)[J_o(\lambda r_1) - J_o(\lambda r_2) - J_o(\lambda r_3)$$

$$+ J_o(\lambda r_4)]d\lambda$$

This form perhaps shows better that p_a is equal to p_1 plus a perturbing factor due to the other layers beneath layer one.

Using the other expression for V:

$$p_a = p_1 r_o \int_0^\infty k_{1 \ldots N}[J_o(\lambda r_1) - J_o(\lambda r_2) - J_o(\lambda r_3)$$

$$+ J_o(\lambda r_4)]d\lambda$$

If the data is collected such that a V(r) curve is generated for a single current electrode, then the Fourier-Bessel transform pairs can be used to invert the data.

$$k_{1\ldots N} = \frac{2\pi}{Ip_1} \quad \lambda \int_0^\infty r \; \underline{V(r)} J_o(\lambda r)dr$$

$$\underline{V(r)} = \frac{Ip_1}{2\pi} \int_0^\infty k_{1\ldots N} J_o(\lambda r)d\lambda$$

Substituting the alternate expressions for k and V

$$(2\kappa_{1,N}(\lambda)+1) = \lambda \int_0^\infty [1+2r \int_0^\infty \kappa_{1,N}(\lambda)J_o(\lambda r)d\lambda]J_o(\lambda r)dr$$

$$= \lambda \int_0^\infty J_o(\lambda r)dr + \lambda \int_0^\infty r[\int_0^\infty 2\kappa_{1,N}(\lambda)J_o(\lambda r)d\lambda]J_o(\lambda r)dr$$

$$= \lambda \quad (\frac{1}{\lambda}) + \lambda \int_0^\infty r[\frac{2\pi}{p_1 I} \; V_f(r,o)]J_o(\lambda r)dr$$

$$2\kappa_{1,N}(\lambda) = \lambda \int_0^\infty r[\frac{2\pi}{p_1 I} \; V_f(r,0)]J_o(\lambda r)dr$$

where

$$V_f(r,0) = \frac{p_1 I}{2\pi} \int_0^\infty 2\kappa_{1,N}(\lambda)J_o(\lambda r)d\lambda$$

Defining $V' = \frac{V_{f(z=0)}}{2}$

and

$$\kappa' = \kappa_{1,N}(\lambda)$$

The following new transform pair results

$$\kappa' = \frac{2\pi}{p_1 I} \lambda \int_0^\infty r\, V'\, J_o(\lambda r) dr$$

$$V' = \frac{p_1 I}{2\pi} \int_0^\infty \kappa' J_o(\lambda r) d\lambda$$

Where V(r,0) can be calculated using the equation

$$V(r,0) = 2V' + \frac{p_1 I}{2\pi r}$$

The purpose behind this change is to make theoretical V(r) curves easier to calculate. J_o is basically an oscillating function and multiplying by κ makes it a decreasing oscillating function which will converge faster. This completes the DC resistivity theory.

INDUCED POLARIZATION ELEMENTS

To characterize an electrical circuit whose output depends on frequency, one generally wishes to know how it will react to any frequency input. This would require inputting all frequencies individually and noting the output associated with each. An alternative method is inputting an impulse, which is a superposition of all frequencies, and from its output determining the individual frequency responses. Measuring responses for an infinite number of frequencies is not possible, nor is making a perfect impulse to input. So measurements are made at a very small number of frequencies and the total approximate response is extrapolated from that information or an imperfect impulse is used and the approximate responses derived from it.

The earth is like an electrical circuit in that its electrical resistivity is dependent on frequency. IP measurements characterize the earth's approximate resistivity versus the frequency response. IP measurements are of two different but equivalent types: 1) measurement of resistivities at a few selected frequencies, 2) measurement of approximate DC resistivities and the response to a step function. The latter is referred to as time domain measurements; the former is referred to as frequency domain measurement. The theoretical relationship between the two methods is given by the Fourier transform:

$$F(s) = \int_{-\infty}^{+\infty} e^{-i2\pi st} f(t)dt;$$

$$f(t) = \int_{-\infty}^{+\infty} e^{+i2\pi st} F(s)ds$$

where f(t) is the time domain data and F(s) is the frequency domain data. F(s) has two components and can be expressed as a product of an amplitude function and a phase function.

$$F(s) = A(s)e^{-i\phi(s)}$$

where A(s) is the amplitude spectra and $\phi(s)$ is the phase shift spectra. Since phase shift spectra is very small, usually less than one degree, the phase shift function approximates 1 so $F(s) \cong A(s)$ (Madden and Cantwell, 1967). For this reason, A(s) is frequently referred to as the frequency domain data. This change simplifies the integrals somewhat to:

$$A(s) \cong \int_{-\infty}^{+\infty} e^{-i2\pi st} f(t)\, dt; \quad f(t) \cong \int_{-\infty}^{+\infty} e^{+i2\pi st} A(s)\, ds$$

These are however, theoretical in nature because the data collected as field measurements is insufficient to allow numerical integration of its integrals. It is enough to know that the data collection methods are equivalent.

IP DATA PRESENTATION

In the time domain the actual quantities measured are 1) the size of the voltage change for the step function (ΔV), 2) the current change when the step function steps (ΔI), and 3) voltage decay time after the step occurs $V(t)$. From these the following calculated data is generated:

A) The apparent resistivity (p_a) is defined as

$$\frac{\Delta V}{\Delta I} (2\pi) (r_o) = p_a$$

where r_o is a geometry factor associated with the electrode array $\frac{1}{r_o} = \frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}$ just as in DC resistivity.

B) The percent induced polarization (%IP) is defined as follows:

$$\frac{V(q)}{\Delta V} (100) = \%IP$$

where (q) is some small time constant; ideally q=0, but this is not obtainable.

In the frequency domains the quantities measured are 1) the transmitting current (I), 2) the received voltage V(s) and 3) the frequency (s). It is from these quantities that the calculated data is derived.

A) The apparent resistivity $p(s_1)$ is defined as:

$$p(s_1) = \frac{V(s_1)}{I} (2\pi)(r_o)$$

s_1 is the lowest frequency at which a measurement was made.

B) The percent frequency effect (PFE) is defined as:

$$\frac{p(s_1)-p(s_2)}{p(s_1)} (100) = PFE \qquad s_2>s_1.$$

Also the metal conduction factor (MCF) could be used:

$$MCF = \frac{PFE}{p(s_2)} (2\pi \times 10^5)$$

Time domain instruments are simpler and less costly to build but frequency domain instruments have the ability to filter the data better and make it more useful. For this reason, the latter method will be explored further.

INTERPRETATION

The actual importance of the parameters PFE and MCF has been explored. It was shown that the MCF is a measure of the conductivity of the metallic content of the rock but it is influenced greatly by the pore-fluid conductivity. The MCF anomalies with significant corresponding PFE anomalies have been seen in rocks without any mineralization. So the PFE is useful also and should be considered in conjunction with MCF in interpreting results (Madden and Cantwell, 1967). Also, a reason for the lack of ability to identify a specific mineral ore from IP data is considered. It appears that PFE and MCR can be predicted from only one measurement (Madden and Cantwell, 1967).

The advantage of IP measurements over DC resistivity measurements are that in doing an IP survey you have collected the same information that would be collected in an DC resistivity survey plus the MCF and the PFE give some indication of the amount of conducting metals that are present. As an exploration tool IP surveys have a definite advantage in prospecting for ore containing conductors scattered throughout a more resistive media.

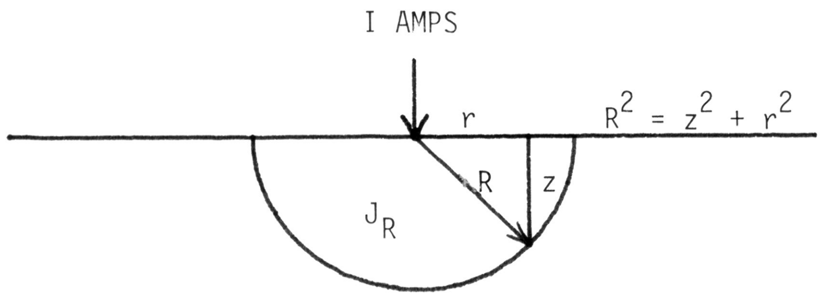


FIGURE 1

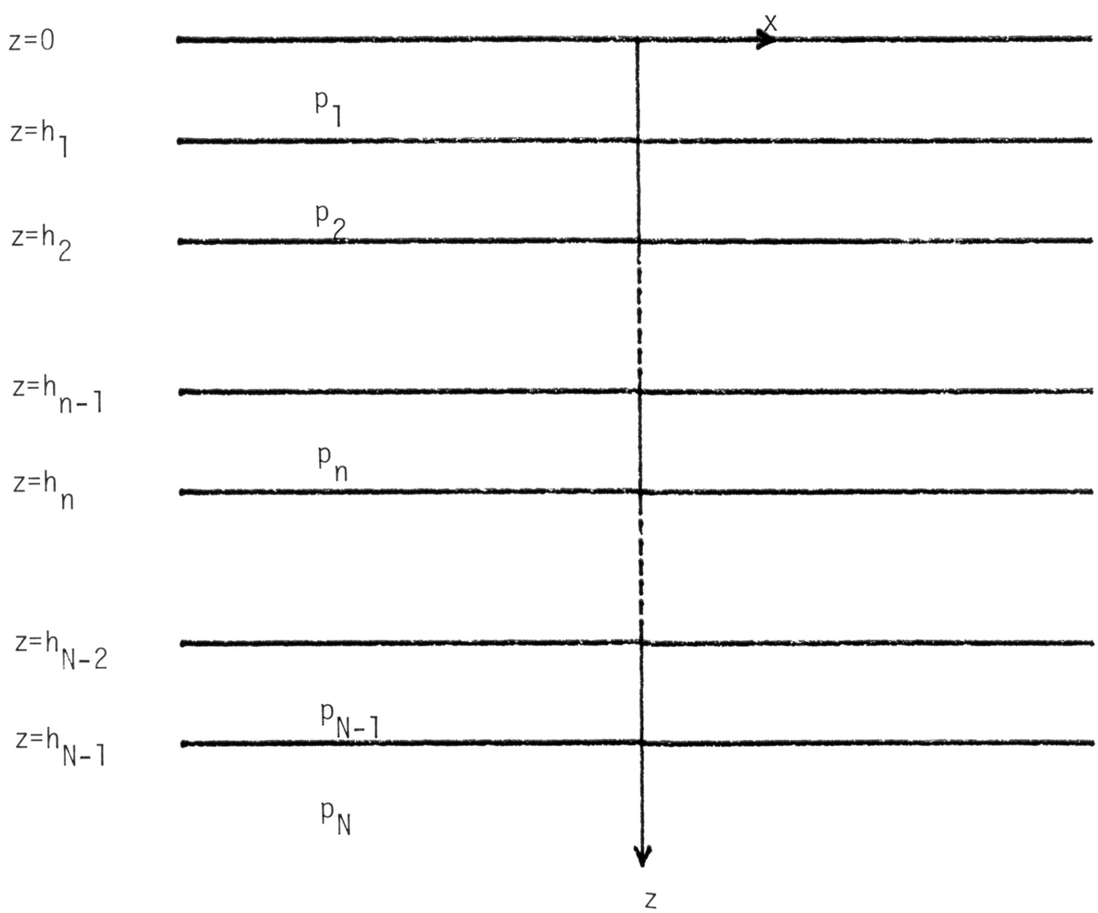
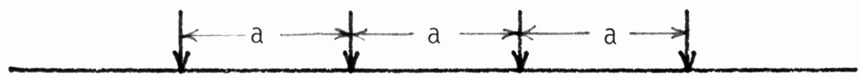
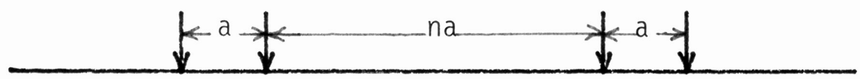


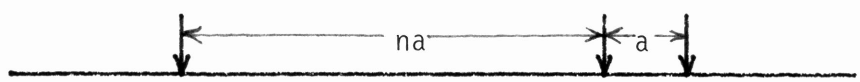
FIGURE 2



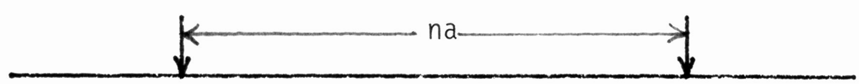
WENNER



DIPOLE - DIPOLE



MONOPOLE - DIPOLE



MONOPOLE - MONOPOLE

Type	r_o
WENNER	a
DIPOLE-DIPOLE	n(n+1)(n+2)a/2
MONOPOLE-DIPOLE	n(n+1)a
MONOPOLE-MONOPOLE	na

FIGURE 3

References

Madden, T.R., and Cantwell, T., 1967, Induced Polarization, A Review in Mining Geophysics, Vol. II, Published by The Society of Exploration Geophysicists, p. 373-400.