Upward Continuation of Single Profiles of Vertical Magnetic Data

## A Paper <br> by

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## Abstract

This paper considers the upward continuation of single profiles of vertical magnetic data taken across models where all the information is contained in a single profile. The single profile upward continuation integral is:

$$
B(\vec{x})=\frac{\left(z_{2}-z_{1}\right)}{\pi} \int_{-\infty}^{\infty} \frac{B_{z}\left(\overrightarrow{x^{\prime}}\right) d x^{\prime}}{\left|\vec{x}-\vec{x}^{\prime}\right|^{2}} .
$$

This upward continuation method is applied to such models as the infinite line current and the infinite horizontal cylindrical shell. When applied to these models, the method works quite well. Some error results from information lost because the data profiles are not infinitely long. Also the method does not work well if the separation between the data profile and the upward continued profile is too small.

Upward continuation is the calculation of a potential field at an elevation higher than that at which the field is known. That is, potential field data taken at one level is used to predict what the potential field will be at a higher level.

Usually, the problem considered is the upward continuation of a plane of data to another plane further away from the source producing the potential field. The equation used for upward continuation in the case of the vertical magnetic field is:

$$
\begin{equation*}
B_{z}(\vec{x})=\frac{\left(z_{2}-z_{1}\right)}{2 \pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{B_{z}\left(\vec{x}^{\prime}\right) d x^{2}}{\left|\vec{x}-\vec{x}^{\prime}\right|^{3}} \tag{seeFig.1}
\end{equation*}
$$

(Grant and West)
$\left(z_{2}-z_{p}\right)$ is the vertical distance between the two planes, $\vec{X}^{\prime}\left(x^{\prime}, y^{\prime}, z^{\prime}\right)$ is the source position vector, and $\vec{X}(x, y, z)$ is the observation position vector. $B_{z}\left(\vec{x}^{\prime}\right)$ is the vertical component of the magnetic field measured on the lower plane, and $B_{z}(\vec{x})$ is the upward continued field. This integral is called the upward continuation integral and can be solved numerically on the computer.

Now consider the situation where all the information concerning the vertical magnetic field can be obtained by using a single profile. This situation occurs where the source is infinitely long with no variation of physical properties along its length, such as an infinite line current and an infinite horizontal cylindrical shell of magnetic material. Every profile of data taken across the source with the
same orientation and distance with respect to the source would be identical. Such a profile of vertical magnetic data can be taken across the source at one level, then upward continued to another, higher level by the equation:

$$
\begin{equation*}
B_{z}(\vec{x})=\frac{\left(z_{2}-z_{1}\right)}{\pi} \int_{-\infty}^{\infty} \frac{B_{z}\left(\vec{x}^{\prime}\right) d x^{\prime}}{\left|\vec{x}-\vec{x}^{\prime}\right|^{2}} \tag{seeFig.2}
\end{equation*}
$$

$\left(z_{2}-z_{1}\right)$ is the vertical separation of the two levels at which the profiles are made. $\vec{X}(x, z)$ is the observation position vector and $\vec{X}^{\prime}\left(x^{\prime}, z^{\prime}\right)$ is the source position vector. $B_{z}\left(\vec{x}^{\prime}\right)$ is the vertical component of the magnetic field measured on the lower profile, while $B_{z}(\vec{x})$ is the upward continued field. This integral can also be done numerically.

The remainder of this paper concerns itself with the upward continuation of a single profile. The numerical technique used will be discussed, with special focus on problems of accuracy. Then the single profile upward continuation method will be applied to two different models, the infinite line current and the infinite horizontal cylindrical shell. These models will be used to generate data for the lower profile. The data will be upward continued to another, higher level. Then the upward continued results will be compared with the model applied at the higher level. Also, some field data taken across a long, horizontal cylindrical shell, a culvert, will be examined. The problems occurring with this data bring out some of the limitations of the particular upward continuation method being used.

## Numerical Techniques

Gaussian-Legendre integration is the method used in this paper for evaluating the upward continuation integral. The form of this method is:

$$
\int_{a}^{b} f(x) d x=\frac{b-a}{2} \sum_{i=1}^{n} W_{i} f\left(x_{i}\right)
$$

where $a$ and $b$ are the end points of the profile. $X_{i}$ is where the field is measured, relative to the endpoints, and $w_{i}$ is a weighting coefficient. The $x_{i}$ 's and $w_{i}$ 's vary with the number of points ( $n$ ) used, and can be found in published tables. Since the integral used in upward continuation goes from $-\infty$ to $\infty$, it is important that the contribution of the endpoints in the numerical integration, be small. In Figure 3, the integral of curve $G_{7}$ from $-\infty$ to $\infty$ is essentially the same as the integral from a to b. But for curve $F$, the integral from a to $b$ is not the same as the integral from $-\infty$ to $\infty$. The shaded area would produce a difference in the finite integral and the infinite integral.

## Upward Continuation of Potential Fields Due to Line Currents

The first example of upward continuation along a profile will be the vertical magnetic field about a line current. This field can be determined by use of the Biot-Savart law and is given by:

$$
B_{z}(\vec{x})=\frac{\mu_{0} I}{2 \pi} \frac{x}{\left(x^{2}+z^{2}\right)}
$$

$x=$ horizontal position with respect to the line current $z=$ vertical position with respect to the line current $\mu_{0}=$ permeability of free space $=4 \pi \cdot 10^{-7}$ newton/ampere ${ }^{2}$ $\mathrm{I}=$ current (amperes)

The field is determined in units of Gauss. With this equation, data was generated on a horizontal profile a distance $z_{p}$ above the line current (see Table 1), then used in the upward continuation integral to predict what the vertical magnetic field would be at the new level, $z_{2}$. The Biot-Savart law was again applied, this time at the higher level, and the results were compared with the upward continued field values (see Table 2 and Fig. 4). Far from the endpoints (i.e., near the center of the profile), the upward continued field agrees very well with the actual field found using the Biot-Savart law. Closer to the endpoints, the results are less accurate. It can be seen from the plots of the integrand why this is the case (see Fig. 5). The integrand for the observation point at the center of the profile approaches zero within the integration interval. So nearly all the area under the curve is integrated. But for the observation point near the end of the profile, the integrand does not go to zero within the integration interval. This means that a significant portion of the area under the curve is not included in the numerical integration, thus causing error.

Upward Continuation of Potential Fields Due to an Infinite Horizontal Cylindrical Shell

Another example of upward continuation of a line profile is for the vertical magnetic field of an infinite horizontal cylindrical shell. The equation for this field is:

$$
B(\vec{x})=\frac{4 \pi a K_{S} H}{\left(x^{2}+z^{2}\right)}\left[\sin I\left(x^{2}-z^{2}\right)-2 x z \cos I \sin \alpha\right]
$$

$a=$ radius of the cylinder
$\mathrm{K}_{\mathrm{s}}=$ magnetic susceptibility of the cylinder
$H=$ strength of the earth's magnetic field at the cylinder
I = inclination of the earth's field
$\alpha=$ declination of the cylinder's axis with the earth's field $x=$ horizontal position with respect to the cylinder's axis $z=$ depth to the cylinder
(see Fig. 6)

Using this equation, data was generated in the same manner as for the infinite line current, then upward continued to a new level. The infinite cylindrical shell equation was then used to determine the accuracy of the upward continued field (see Fig. 7 and Table 3). This example demonstrates the importance of both the number of points used in the Gaussian integration, and the length of the profile. In Fig. 7 we see that for a profile 40 feet in length, the 16 point integration gives a fairly good fit but is inaccurate at both the peak and the ends of the profile. When 48 points are used to integrate
the profile, the fit is much better, especially at the peak. At the endpoints, the fit is still not very good. When 48 points are applied to a longer profile with length $=60$ feet, the fit is better further from the peak than for the profile of length 40 feet. Therefore one should take into consideration the degree of integration as well as the amount of available data in order to produce accurate upward continued fields.

## Field Data Across a Long Culvert

Two profiles of data were taken across the center of a buried steel culvert, of length 40 feet, radius 2.2 feet, and buried a depth 2 feet. The culvert is located about $\frac{1}{2}$ mile west of the Texas A\&M campus. A good approximation of an infinite cylindrical shell is made by the culvert when the data profile is taken across the center of the culvert. The first profile was taken 1.7 feet above the culvert for 30 feet on each side of it. Then a second profile was made, 3.4 inches above the first. The lower profile was upward continued to the level of the higher profile. Unfortunately, the results of the upward continuation were completely wrong. Using the same parameters and distances as for the field data, the infinite horizontal cylindrical shell model was applied to compare with the field data. A profile was generated 1.7 feet above the cylinder, then upward continued the same distance as the field data, 3.4 inches. Again the results are completely wrong. So the numerical method used here for upward continuation is unstable for small separations. So when applying this upward continuation method in the field, care must be taken to be
sure that the separation between the lower profile and the upward continued profile is large enough, especially since the field data obviously contains a fair amount of noise.

## Conclusion

The upward continuation of single profiles works very well for the models used in this paper. It is important to remember that these models are infinitely long and uniform. So to use this method in the field, the source must be of great enough extent to approximate an infinite model. Some geological examples might be, a fault or an elongate ore body. If the data is taken on a profile across these sources, the single profile upward continuation should work very well. The profiles need to be long enough to record most of the contribution of the source to the magnetic field. Also, the vertical separation between the data profile and the upward continued profile needs to be large enough for the numerical methods to work.

## References

Grant, F.S., and West, G.F., 1965, Interpretation Theory in Applied Geophysics: New York, McGraw-Hill.
TABLE 1

$$
\begin{aligned}
& \text { along a profile at level } z_{1} \text { by use of the Biot- } \\
& \text { or the upward continuation in Table 2. } z_{1}=10 \mathrm{~cm} \text {. }
\end{aligned}
$$

Vertical Magnetic

$$
\begin{aligned}
& \text { Field Found Using } \\
& \text { the Biot-Savart law } \\
& \hline
\end{aligned}
$$








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Figure 1: UPWARD CONTINUATION OF A PLANE OF VERTICAL MAGNETIC DATA.
$z_{1}$ is the vertical location of the lower plane. $z_{2}$ is the vertical location of the higher plane. The field data for the lower plane is used to predict what the field is at the higher plane.


Figure 2: UPWARD CONTINUATION OF A SINGLE PROFILE OF VERTICAL MAGNETIC DATA.
The profile of data taken at $z_{1}$ is used to predict what the field will be at $z_{2}$.


Figure 3: AREAS UNDER CURVES.
These curves demonstrate that a finite length integral may not include all the area under the curve.


Figure 4: UPWARD CONTINUED FIELD AND MODEL FIELD FOR A LINE CURRENT. $\quad\left(z_{2}-z_{1}\right)=4 ; \frac{\mu_{0} I}{4 \pi}=1.0$


Figure 5: INTEGRANDS FOR THE UPWARD CONTINUATION OF A LINE CURRENT.
This graph shows the integrands used for several values of $x$ in upward continuation of a profile.


Fiqure 6: INFINITE HORIZONTAL CYLINDRICAL SHELL MODEL. $I=$ inclination, $H=$ magnetic field strength, $\alpha=$ declination of the cylinder's axis with the earth's magnetic field.


Figure 7: PLOT OF INFINITE CYLINDER UPWARD CONTINUATIONS AND MODEL COMPARISON. These are plots of the upward continuation of a profile of vertical magnetic data using the infinite horizontal cylindrical shell model.

