# Computer Model of Coronary Heart Disease

by

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Submitted in Partial Fulfillment of the
Requirements of the University Undergraduate
Fellows Program

1980-1981

Approved by:

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I would like to acknowledge the Honors Program for allowing me to participate in the University Undergraduate Fellows Program.

I wish to thank my advisor, Dr. Gerald E. Miller, for his patience and the amount of time he spent with me.

ABSTRACT - A computer code representing fluid flow is described which calculates the pressure gradient in the entry region of a rigid tube. The velocity profile is calculated from graphical correction of known flow equations. Poiseuille's model, a simplification of the z-momentum equation, is used as a basis for the calculations. The results obtained compare favorably with the literature.

# INTRODUCTION

Atherosclerosis is responsible for greater than one-half of all deaths each year. Of these deaths, approximately two-thirds are a result of Coronary Heart Disease. Coronary Heart Disease is the degeneration of the coronary blood vessels, concluding with thrombotic events. Occlusion of the vessel leads to oxygen deprivation of the myocardium. Death generally occurs by myocardial infarction.

Atherosclerosis is principally a disease of the large arteries. Low density lipoproteins are transported across the blood vessel wall and deposited in the subintimal layer of the artery. The lipid deposits are known as atheromatous plaques. These plaques contain a high concentration of low density cholesterol.

If plaque build-up continues, the plaque will protrude through the intima into the flowing blood. An obstruction of the blood flow is termed a stenosis. A stenosis will generally lead to thrombosis. However, little is known about the origin of atherosclerosis.

Coronary Heart Disease is almost always a result of atherosclerosis. Diagnosis is usually made when a patient develops angina pectoris(with a stenosis) or myocardial infarction(with an occlusion). Angina pectoris, a cardiac pain, develops whenever the load on the heart becomes too great for the coronary blood flow to releive.

Beyond an occluded vessel little or no blood flows. The now impaired cardiac muscle function is said to be infarcted. The overall process is termed myocardial infarction( 4 ).

Hemodynamic factors probably play a decisive role in the progression of the stenosis. Severe stenoses cause turbulence in the blood flow. It has been demonstrated that turbulence in the coronary arteries causes five times as much thrombus formation as does laminar flow(8 ). Additionally, arteries are distensible; if the stenosis creates a large pressure drop(Bernoulli's effect)the artery can collapse and thus expand the effective length of the stenosis(9 ). If turbulence is present, the pressure is even greater as energy is being expended to create the turbulence.

A pressure drop across a stenosis is more likely to occur than a reducton in flow rate. May and his colleagues found that there was no reduction in flow until the stenosis reached a critical value(for the iliac artery-85% by area, 62% by diameter)(7 ).

Stenoses also cause a jet stream through the constriction. By the Bernoulli effect the velocities of the blood particulates through the stenosis are highly accelerated.

A velocity profile is a plot of stream velocity versus radius for a vessel. Previously mentioned information outlining flow characteristics of atherosclerosis demonstrates the importance of the velocity profile in the diagnosis of the disease.

Presently, clinical methods of obtaining necessary data for blood flow are both painful and costly. Additionally, the limitations of these methods are great. For example, the present state-of-the-art clinical method for obtaining velocity profiles is Pulse Doppler Ultrasound(11). This device uses high frequency waves to sample points approximately 1 mm apart in a blood vessel. For the case of the left common coronary artery - diameter of 3 mm - only three points can be sampled. This is insufficient data to properly shape a profile. Thus, a model which could provide an accurate profile would have great utility.

The objective of this project is to computer model a velocity profile, using published data for the coronary arteries.

On a gross scale, fluids are typically divided into two catagories, perfect and viscous fluids. A perfect fluid is one whose viscosity is zero. A viscous fluid is one whose viscosity is non-zero and finite. Although a perfect fluid does not exist, low viscosity fluids such as air are often modeled as perfect. Blood is a viscous fluid. As a result, the remainder of this project is concerned with the development of viscous fluids.

Viscosity can be interpreted as a force - in paticular a frictional force. When a viscous fluid enters a tube from a large reservoir its velocity profile is blunt. That is, all of its particulates have an equal velocity. As the fluid proceeds through the tube, a viscous drag shears the fluid. This drag occurs first at the wall, and progressively involves more and more layers up to the tube axis. At this point, the fluid is defined as fully developed. This term is interchangeable with Poiseuille flow. The velocity of the fluid at the wall equals zero; its shear rate is a maximum. The free stream velocity(the axis velocity) is a maximum and its shear rate equals zero.

The basic equations of viscous fluid mechanics are the Navier-Stokes equations.

For a cylindrical coordinate system, the Navier-Stokes equations are:

<u>θ - direction</u>

$$\rho\left[\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} + \frac{V_r V_\theta}{r} + V_z\frac{\partial V_\theta}{\partial z}\right] = \rho g_\theta - \frac{1}{r}\frac{\partial P}{\partial \theta}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r} r V_\theta\right) + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} + \frac{\partial^2 V_\theta}{\partial z^2}\right] \quad (1)$$

R - direction

$$\rho\left[\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta{}^2}{r} + V_z\frac{\partial V_r}{\partial z} = \rho g_r - \frac{dP}{dr}\right.$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r V_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial^2 V_r}{\partial z^2}\right] \quad (2)$$

Z - direction

$$\rho\left[\frac{\partial V_z}{\partial z} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right] = \rho g_z - \frac{dP}{dz}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\,\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right. \quad (3)$$

Assuming an axially symmetric, rigid tube and thus negligible velocities in the R and θ directions, two classic solutions exist: Wormersley's solution to a non-zero pressure gradient and Poiseuille's solution to a zero pressure gradient. Wormersly's solution is presented first.

In its general form for a Newton fluid, the z direction equation reduces to

where,

$V_z$ = axial velocity
$\mu$ = viscosity
$\rho$ = density
P = pressure
r = radius
t = time
v = kinematic viscosity

Assume $\frac{dP}{dz}$ to be harmonic, such that

$$\frac{dP}{dz} = A^* e^{iwt} \quad (4)$$

Substituting for $\frac{dP}{dz}$,

$$\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r}\frac{\partial V_z}{\partial r} - \frac{1}{V_z}\frac{\partial V_z}{\partial t} = -\frac{A^*}{\mu} e^{i\omega t} \qquad (5)$$

Assuming $V_z = ue^{iwt}$ ,

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{i\omega}{v} u = -\frac{A^*}{\mu} \qquad (6)$$

This is a form of Bessel's equation, and its solution requisite to the necessary boundary conditions, is

$$u = \frac{A^*}{iwp}\left[1 - \frac{J_o\left(r\,(w/v)^{\frac{1}{2}}\, i^{3/2}\right)}{J_o\left(R\,(w/v)^{\frac{1}{2}}\, i^{3/2}\right)}\right] \qquad (7)$$

where $J_o(xi^{3/2})$ is a Bessel function of the first kind of order zero and complex argument. Its general form is

$$J_v = X^v \sum_{m=0}^{\infty} \frac{(-1)^m X^{2m}}{2^{2m+v} m!\,\Gamma(v+m+1)} \qquad (8)$$

Using the substitution, $\alpha = R(w/v)^{\frac{1}{2}}$ (a number often tabulated) and $y = r/R$

$$V_z = \frac{A^* R^2}{iu\alpha^2}\left[1 - \frac{J_o\,(\alpha y i^{3/2})}{J_o\,(\alpha i^{3/2})}\right] e^{iwt} \qquad (9)$$

At this point the Bessel function may be represented in terms of a modulus and phase of its real part,

$$J_o\,(\alpha y i^{3/2}) = M_o(y)e^{i\theta(y)} \qquad (10)$$

$$J_o\,(\alpha i^{3/2}) = M_o e^{i\theta}. \qquad (11)$$

In the same fashion, the real part of $A^* e^{iwt}$ may be written in terms of a modulus and phase,

$$A^{*}e^{iwt} = M\cos(wt-\phi) \qquad (12)$$

Using the substitutions, and that $\delta_o = \theta - \theta(y)$

$$V_z = \frac{M}{wp}\ \sin(wt - \phi) - \frac{M_o(y)}{M_o}\sin(wt - \phi - \delta_o) \qquad (13)$$

Wormersly further reduced his equation by writing

$$h_o = \frac{M_o(y)}{M_o} \qquad (14)$$

and introduced $M_o'$ and $\varepsilon_o$ by the following definition,

$$M_o = (1 + h_o^2 - 2h_o\cos\delta_o)^{\frac{1}{2}} \qquad (15)$$

and,

$$\tan\varepsilon_o = \frac{h_o\sin\delta_o}{1-h_o\cos\delta_o} \qquad (16)$$

such that

$$V_z = \frac{M}{wp}\ M_o'\sin(wt-\phi+\varepsilon_o) \qquad (17)$$

Since $\alpha^2 = R^2\frac{wp}{u}$

$$V_z = \frac{MR^2M'}{\mu\,\alpha^2}\sin(wt-\phi+\varepsilon_o) \qquad \text{(Wormersly)}\ (18)$$

(see figure 1).

Poiseuille's model is considerably less complex. By making several assumptions, Poiseuille was able to reduce the Navier-Stokes equations to an integrable form. Poiseuille's assumptions were:

(1) steady (time invariant) flow. This implies that all time dependent variables are zero.

$$\frac{d}{dt} = 0 \qquad (19)$$

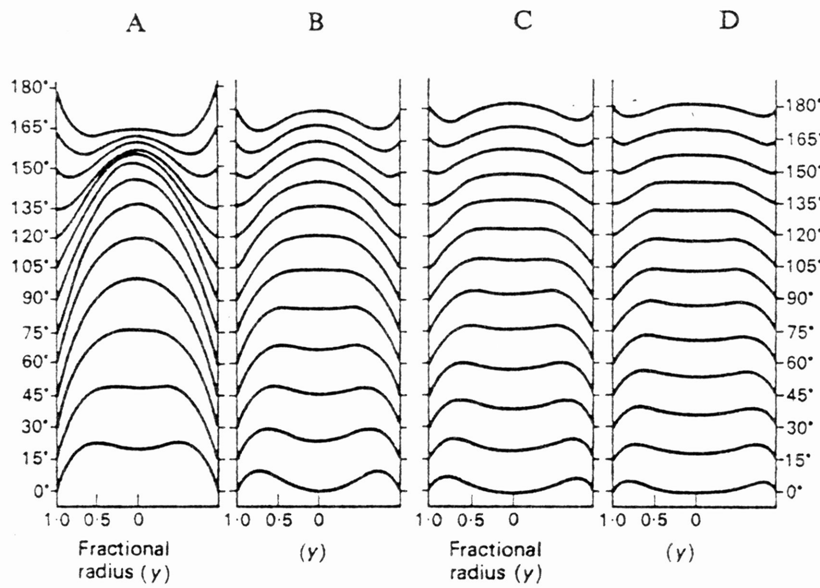



**Fig. 5.1A.** The velocity profiles, at intervals of 15°, of the flow resulting from a sinusoidal pressure-gradient (cos $\omega t$) in a pipe. In this case, $\alpha = R \cdot \sqrt{\omega/\nu} = 3{\cdot}34$, corresponding to the fundamental harmonic of the flow curves illustrated in Figs. 5.3 and 5.4. Note that reversal of flow starts in the laminae near the wall. As this is harmonic motion only half a cycle is illustrated as the remainder will be the same in form but opposite in sign, e.g. compare 180° and 0°.

**B.** A similar set of profiles for harmonic motion of double the frequency of A ($\alpha = 4{\cdot}72$). The amplitude and phase of the pressure are the same here and in C and D as in A. The effects of the larger $\alpha$ are thus seen to be a flattening of the profile of the central region, a reduction of amplitude of the flow and the rate of reversal of flow increases close to the wall.

**C.** The third harmonic with $\alpha = 5{\cdot}78$. The effects of higher frequency noted in B are here further accentuated.

**D.** The fourth harmonic ($\alpha = 6{\cdot}67$) shows the same effects again. The rapidly varying part of the flow lies between $y = 0{\cdot}8$ and $y = 1{\cdot}0$ and the central mass of the fluid reciprocates almost like a solid core.

FIG. 1

(2) uniform flow profile. This assumption in combination with assumption 4 negates a radial and angular velocity. A uniform profile ensures that no turbulence exists within the system. Additionally, uniform flow assumes that the fluid undergoes no accelerations.

$$V_r = V_z = 0 \qquad (20)$$

$$\frac{dV_z}{dz} = 0 \qquad (21)$$

(3) the fluid is incompressible, homogenous and has a constant viscosity independent of the shear rate. This is the definition of a Newtonian fluid.

$$\rho = \text{constant}$$

$$\mu = \text{constant}$$

(4) rigid, circular pipe. At the wall of the tube, any change with respect to the radial direction must be zero.

$$\frac{d}{dr} = 0 \qquad (22)$$

(5) symmetry. All functions dependent upon the theta direction are zero.

$$\frac{d}{d\theta} = 0 \qquad (23)$$

(6) fully developed flow. That is, the region of study is distal to the entry region.

From the above assumptions, the Navier-Stokes equations can be reduced to the following:

R - direction

$$0 = g_r - \frac{dP}{dr} = \frac{dP'}{dr} \quad , \quad P' \neq f(r) \qquad (24)$$

θ - direction

$$0 = g_o - \frac{dP}{d\theta} = \frac{dP'}{d\theta} \tag{25}$$

z - direction

$$\frac{\mu}{r}\frac{d}{dr}\left( r \frac{dV_z}{dr} \right) = \frac{dP'}{dz} \tag{26}$$

The boundary conditions for this model are:

$$(1) \qquad \left.\frac{dV_z}{dr}\right|_{r=0} = 0 \tag{27}$$

$$(2) \quad \left. V_z \right|_{r=R_o} = 0 \tag{28}$$

Solution of the above differential equation subjected to the boundary conditions will result in the Poiseuille model. Substituting $k = \frac{dP'}{dz}$ and solving,

$$\frac{rk}{\mu} = \frac{d}{dr}\left( r\frac{dV_z}{dr}\right) \tag{29}$$

Integrating,

$$C_1 + \frac{r^2k}{2\mu} = r\,\frac{dV_z}{dr} \tag{30}$$

$\left.\frac{dV_z}{dr} = 0 \right|_{r=0}$ yields $C_1 = 0$.

$$\frac{rk}{2\mu} = \frac{dV_z}{dr} \tag{31}$$

Integrating,

$$C_2 + \frac{r^2k}{4\mu} = V_z \tag{32}$$

For $r = R_o$ ,

$$0 = \frac{R_o^2}{4\mu}\frac{dP'}{dz} + C_2 \qquad (33)$$

$$C_2 = -\frac{R_o^2}{4\mu}\frac{dP'}{dz} \qquad (34)$$

Substituting,

$$V_z = \frac{R_o^2}{4\mu}\frac{dP}{dz}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (35)$$

From the resulting equation for axial velocity, it can shown that the profile will be parabolic. The profile is dependent only upon r, as the pressure gradient was assumed to be constant.

To find the volume flow rate, Q, the velocity is integrated over the area.

$$Q = \int V_z . dA \qquad (36)$$

For cylindrical coordinates,

$$Q = \int_0^{2\pi}\int_0^{R_o} V_z rdrd\theta \qquad (37)$$

$$Q = 2\pi\int_0^{R_o} rV_z dr \qquad (38)$$

$$Q = 2\pi\int_0^{R_o} r \frac{1}{4\mu}\frac{dP'}{dz}\left(r^2 - R_o^2\right) \qquad (39)$$

$$Q = \frac{2\pi}{4}\frac{dP'}{dz}\left(\frac{r^4}{4\mu} - \frac{R_o^2 r^2}{2\mu}\right)\Bigg|_0^{R_o} \qquad (40)$$

ignoring the negative sign,

$$Q = \frac{\pi R_o^4}{8\mu} \frac{dP'}{dz} \quad (41)$$

McDonald drew the following conclusions as to the validity of applying Poiseuille's equation to blood flow.

(1) The fluid is homogenous and its viscosity is constant for all shear rates. Blood is a suspension of particles - notably red blood cells. If the dimensions of the vessel are large compared with the size of red blood cells, then blood can be considered a Newtonian fluid. The size restraint imposed by McDonald is an internal radius of 0.5 mm.

(2) The liquid does not slip at the wall. This was one of Poiseuille's boundary conditions - that is, $V_z=0$ at $r=R_o$. It has been shown that this condition is universally true for all fluids(1 ,5 ). One might project that presence of a plasma skimming layer might lead to slip. This point has never been proven experimentally.

(3) The flow is laminar. At lower flow rates, laminar flow is observed. If the flow rate exceeds a critical value, flow disturbances and turbulence in the fluid are seen. In some of the large arteries, turbulence is present. But for the smaller arteries and veinous circulation, laminar flow is a fairly good assumption.

(4) The rate of fluid flow is 'steady' and is not subjected to any accelerations of decelerations. Since flow in the large arteries and the intrathoracic veins (2 ) is pulsatile, this assumption is not valid for that portion of the circulation.

(5) The tube is long compared with the region being studied. The transition region from a blunt velocity

profile to fully developed flow for a fluid is defined as its entry region. Within the entry region, the center portion of the flow is accelerated while the fluid near the wall is decelerated. Additionally, the pressure gradient in the entry region is not constant. Thus Poiseuille's law does not apply.(see figures 2 and 3)

(6) The tube is rigid; the diameter does not vary with the internal pressure. Blood vessels are distensible. Thus, the flow will not be completely determined by the pressure gradient.

A paper by Taylor and Gerrard (1977) closely parallels the intentions of this project for the non-stenosed case. Taylor and Gerrard presented a mathematical model utilizing the technique of finite differences to calculate the axial velocity as a function of time. The primary difference between their approach and the approach of this project is their inclusion of a skin friction term. This term takes into account viscous effects. It negates the importance of calculating the axial velocity as a function of the radius. Their results are presented in figure 4.

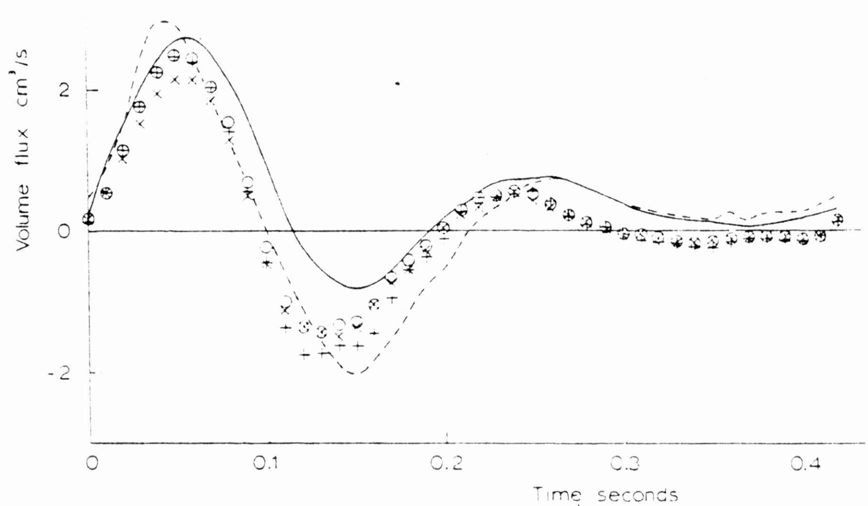



*Fig. 1 Volume flux for one period of oscillation in a canine femoral artery*
*Streeter et al. (1963)*
——— *experiment*
– – – – *calculation*

FIG. 4

*Present model*
+ *skin friction = zero frequency value, entrance-length correction included*
× *full linear treatment of the skin friction, entrance-length correction included*
. . *as last without entrance-length correction*

METHODS

An analytic solution to the general form of the Navier-Stokes equations would be extremely difficult.

A digital solution utilizing the technique of finite differences is more straight forward. Finite differences is a method for the approximation of a derivative. Central finite differences were chosen.

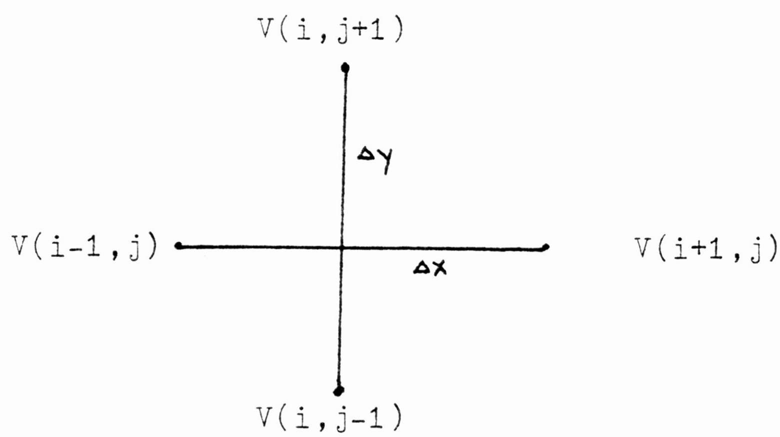
An approximation for the first derivative of a general function, Y, is:

$$\left.\frac{dY}{dx}\right|_{i,j} = \frac{Y(i+1,j) - Y(i-1,j)}{2\Delta x} \qquad (42)$$

An approximation of the second derivative is:

$$\left.\frac{d^2Y}{dx^2}\right|_{i,j} = \frac{Y(i+1,j) + Y(i-1,j) - 2Y(i,j)}{\Delta x^2} \qquad (43)$$

Recalling that symmetry was assumed, the following grid system can be used.




Similar grid systems can be chosen for approximations of $\frac{dP}{dz}$ and $\frac{dV}{dz}$.

Applying finite differences to the Navier-Stokes z-direction equation yields:

$$\rho\left[\frac{V_{ij}^{n+1}-V_{ij}^{n-1}}{2\Delta t}+V_{r_{ij}}^{n}\left(\frac{V_{i+1,j}^{n}-V_{i-1,j}^{n}}{2\Delta r}\right)+V_{ij}^{n}\left(\frac{V_{i,j+1}^{n}-V_{i,j-1}^{n}}{2\Delta z}\right)\right]$$

$$=\rho g_z-\frac{P_{i,j+1}^{n}-P_{i,j-1}^{n}}{2\Delta z}+\mu\left[\frac{1}{r}\left(\frac{V_{i,j+1}^{n}-V_{i,j-1}^{n}}{2\Delta z}\right)+\right.$$

$$\left.\frac{V_{i+1,j}^{n}+V_{i-1,j}^{n}-2V_{ij}^{n}}{\Delta r^2}+\frac{V_{i,j+1}^{n}+V_{i,j-1}^{n}-2V_{ij}^{n}}{\Delta z^2}\right] \tag{44}$$

where, i = radial increment
j = distal increment
n = time increment
t = time step size
r = radial step size
z = distal step size

solving for $V_{i,j}^{n+1}$ yields:

$$V_{i,j}^{n+1}=V_{i,j}^{n}+\frac{\Delta t}{\rho}\left[\rho g_z-\frac{\Delta P}{\ell}\right]+\frac{\Delta t}{\rho}\mu\left[\frac{1}{r}\left(\underline{V_{i,j+1}^{n}+V_{i+2,j}^{n}}\right.\right.$$

$$\left.\left.+\frac{V_{i,j}^{n}-2V_{i+1,j}^{n}}{\Delta r^2}\right)+\frac{V_{i,j+2}^{n}+V_{i,j}^{n}-2V_{i,j+1}^{n}}{\Delta z^2}\right]$$

$$-\frac{\Delta t}{\rho}V_{i,j}^{n}\left[\frac{V_{i,j+1}^{n}-V_{i,j}^{n}}{\Delta z}\right] \tag{45}$$

An iterative solution to find $V_z$ as a function of time is relatively simple. If $V_z(t=n)$ is known, then $V_z(t=n+1)$ can be found. The pressure gradient as a function of time can be approximated by obtaining two waveforms. At a particular time in the cycle, the two pressures are subtracted and the difference is divided by the length between the points of measurement. All other parameters for this difference equation are known.

However, to start the iterative process, $V_z(t=0)$ at every point in the tube must be known. However, only the boundary conditions are known.

Within the coronary arteries, blood is rarely fully developed. Thus, the Poiseuille model can not be applied directly.

Viscous fluid theory in the entry region of rigid tube violates three of Poiseuille's assumptions:

(1) $V_z$ is not a function of z. The principal difference between fully developed flow and entry region flow is that in the entry region, fluid elements experience accelerations - positive near the center of the tube, negative near the wall. Since the flow rate is a constant, no local acceleration exists. The acceleration of individual fluid elements is termed convective acceleration.

(2) The pressure gradient is a constant. The pressure gradient for a steady flow model falls linearly with distance only for Poiseuille flow. In the entry region there is a rapid non-linear pressure drop (see figure 5).

(3) Radial and/or angular velocities are zero. From the continuity equation (written for an incompressible fluid),

$$\frac{1}{r}\frac{\partial V_r}{\partial r} + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z} = 0 \qquad (46)$$

It can be demonstrated that both $V_r$ and $V_o$ can not be zero. If so,

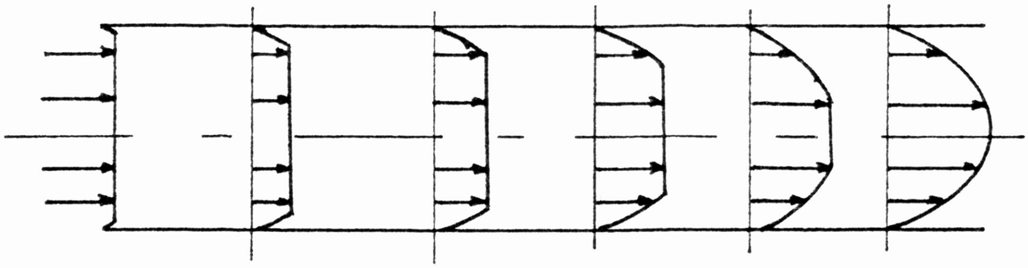


Fig. 2. The change in the velocity profile along the tube in steady flow indicating the growth of the boundary layer. The initially flat profile becomes modified to form the parabolic profile at a critical distance from the inlet.

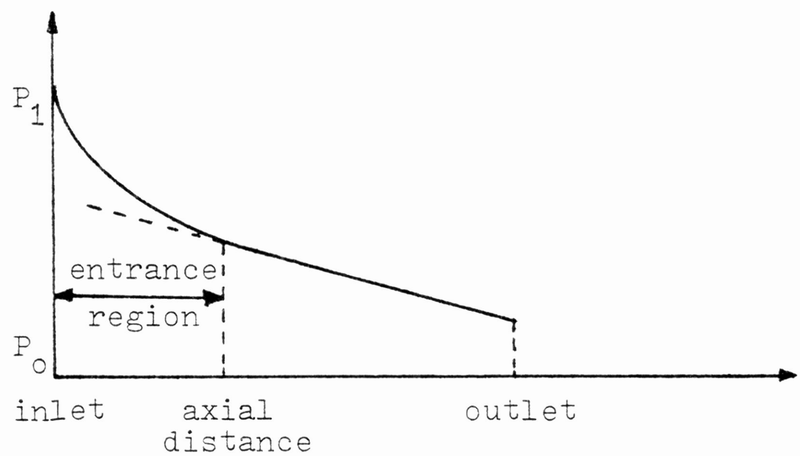



Fig. 3. The variation in local pressure as a function of distance from the inlet. In the entrance region the pressure falls rapidly with distance. Beyond the entry region, the pressure falls linearly with distance.

$$\frac{dV_z}{dz} = 0 \quad ; \quad V_z = \text{constant} \qquad (47)$$

This is an obvious contradiction of condition 1.

If the remainder of Poiseuille's assumptions are kept and a radial velocity is assumed to exist rather than an angular velocity, the Navier-Stokes equations reduce to,

$$\rho\left[V_r \frac{\partial V_r}{\partial r}\right] = \rho g_r - \frac{dP}{dr} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r V_r}{\partial r}\right)\right] \qquad (48)$$

$$\rho\left[V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z}\right] = \rho g_z - \frac{dP}{dz} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial z}\right) + \frac{\partial^2 V_z}{\partial z^2}\right] \qquad (49)$$

Solution of these coupled differential equations would prove difficult.

An important concept in viscous fluid theory is the development of the boundary layer. In the entry region fluid elements are sheared; the sheared elements become part of the boundary layer. The non-sheared elements comprise the free stream. As the fluid proceeds through the tube, a greater portion of its elements are sheared and thus its boundary layer grows. This is shown in figure 2.

The boundary layer thickness is defined as 99% of the radial distance from the vessel wall to the blunt portion of the profile. Obviously, the boundary layer thickness is a minimum for a totally blunt profile, and is a maximum for fully developed flow. The boundary layer thickness, $\delta$ , may be calculated from,

$$\delta = K\sqrt{\frac{\mu x}{\rho U}} \qquad 50$$

where, k = experimentally determined proportionality constant

x = distance from entrance

An important parameter of a fluid system is the Reynolds number. The Reynolds number is a dimensionless number representing the ratio of inertial to viscous forces. It is used as an indication to the state of a fluid. In general, laminar flow is observed for Reynolds numbers less than 2000. Turbulent flow is observed for Reynolds numbers greater than 3000. A transition region is seen for Reynolds numbers between 2000 and 3000. The Reynolds number, Re, can be calculated from:

$$Re = \frac{\rho U d}{\mu} \tag{51}$$

where d = diameter.

The inlet length defines the axial distance from the tube entrance at which the fluid first becomes fully developed. The inlet length, XFULL, can be calculated from:

$$XFULL = .03dRe \tag{52}$$

Poiseuille's solution to the Navier-Stokes equations is rather simple. Recall from equation 35 and 40 that the formulas to find axial velocity and flow rate are,

$$V_z = \frac{R_o^2}{4\mu}\frac{dP}{dz}\left[1 - \left(\frac{r}{R_o}\right)^2\right] \tag{35}$$

$$Q = \frac{2\pi}{4\mu}\frac{dP}{dz}\left(\frac{R_o^2 r^2}{2} - \frac{r^4}{4}\right)\Bigg|_0^{R_o} \tag{40}$$

The mechanism for finding $V_z$ and $\frac{dP}{dz}$ as functions of z is as follows:

(1) A value for $\delta$ , the boundary layer thickness is calculated from equation 50 . The free stream velocity, U, from the profile of the previous distal increment was used, such that

$$\delta = K\sqrt{\frac{\mu X}{\rho U(x-1)}} \qquad (53)$$

(2) Proceeding from the wall toward the tube center Poiseuille flow was assumed, and velocities were calculated from equation at particular radial increments. When the boundary layer thickness was reached, a final application of the equation was used.

(3) To find the free stream velocity the Poiseuille profile was integrated from the boundary layer thickness to the radius, RAD. This flow was termed QPAR. Assuming the volume flow rate, Q, to be constant, the free stream velocity can now be found from:

$$U = \frac{Q-QPAR}{(RAD-\delta)^2} \qquad (54)$$

However, this method assumes a constant pressure gradient equal to the pressure of fully developed flow. This is clearly incorrect due to the discontinuity in the waveform (see figure 5).

A correct general shape for a velocity profile in entry region of a tube is shown in figure 6 (3 ).

A method of graphical correction of figure 5 to obtain figure 6 was used.

(4) The values of the slopes from point 1 to point 2 and from point 2 to point 3 are calculated. These are designated SLOPE1 (points 1 to 2), and SLOPE2 (points 2 to 3).

(5) Note in figure 6 that SLOPE1 is greater than SLOPE2. However, note also that SLOPE2 is not negative.

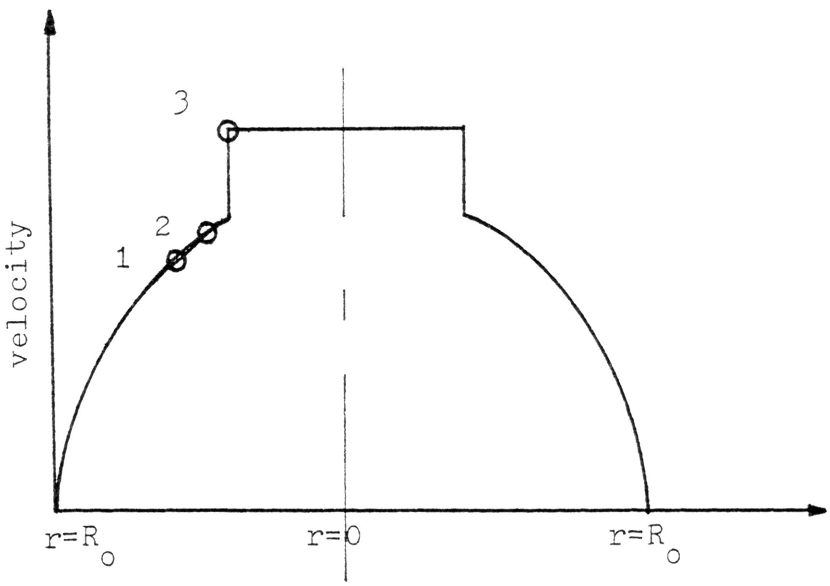



Fig. 5.

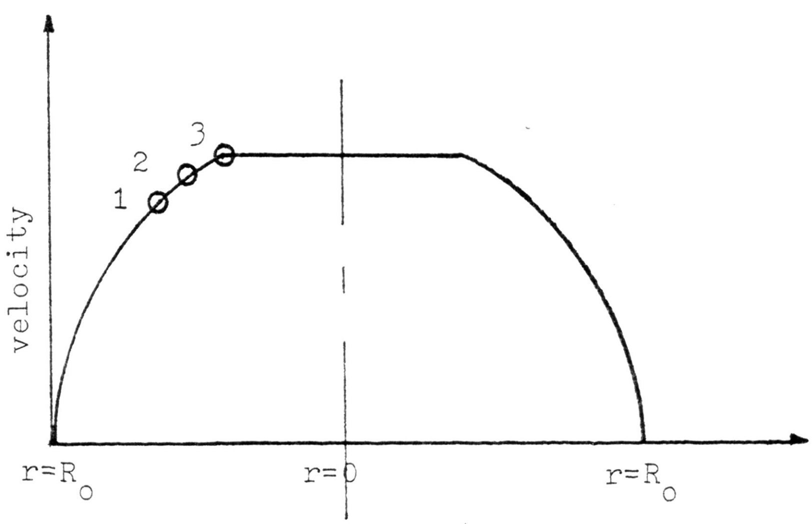



Fig 6.

In figure 5, SLOPE2 is obviously much greater than SLOPE1. By increasing the pressure gradient, a new velocity profile may be calculated. If once again, step 3 is applied, a new free stream velocity is found.

(6) SLOPE1 and SLOPE2 are recalculated. If SLOPE1 is still less than SLOPE2, the pressure gradient is incremented, and step 3 is repeated (followed by step 6). If SLOPE2 is negative, i.e., the free stream velocity is less than the velocity at the boundary layer thickness, the pressure gradient is decreased.

(7) The iterative process continues until SLOPE1 is greater than SLOPE2, and SLOPE2 is non-negative. The result is a value for $\frac{dP}{dz}$ which gives a continuous waveform.

A computer program was written using OS - WATFIV, the listing of which is located in appendix A. The program was run on the Amdahl 470V/6. The program used 15.11 seconds of CPU time. Computer plots of the output are listed in appendix B.

## RESULTS AND CONCLUSIONS

Characteristics of the coronary arteries were found through out the literature. The following data were used. The volume flow rate was assumed to be 50 ml/min. The diameter of the left common coronary was assumed to be 3 mm.

Blood was assumed to have a density of 1.05g/cc and a viscosity of 3.0cp.

The pressure for fully developed flow was calculated by knowing the flow rate. The pressure gradient allowed calculation of the velocity profile for Poiseuille flow. The free stream velocity was found to be 23.576 cm/sec.

The Reynolds number was calculated to be 123.80.

The inlet length equaled 1.114 cm.

Realizing the free stream velocity for fully developed flow, the inlet length and that the boundary layer thickness for Poiseuille flow equals the radius, the proportionality constant, CST, was calculated. CST was found to be 4.042.

The results obtained are consistent with theory and with the literature.

In fact, the pressure gradient dropped rapidly in the entry region (see figure 7, table 1). The gradient eventually leveled at the fully developed value of 125.75 dynes/cc.

If additional computer money had been available, the more interesting region of the tube (the initial .07 cm) could have been investigated more rigorously. A significant step down in distal increment size may have shown a pressure gradient greater than the value of 183.33 dynes/cc listed. An attempt at a 100 increment code rather than the 40 increment code used was made. But execution time exceeded 40 seconds. Although this is not an immense job, funds were limited.

Appendix B shows computer plots of velocity profiles taken once every 4 distal increments. The profiles demonstrate that the process of graphical correction did, in fact, work. The profile developed quite rapidly. This was the most surprising portion of the program. Apparently, blood flow is considerably more dependent upon viscous effects than on its inertial effects.

At certain points, identical profiles were obtained for different values of the boundary layer thickness. This error is easily explained. Due to the magnitude of the radial step size, it was possible for two (or more) values of the boundary thickness to share a radial point as a cut-off for the Poisuille profile.

TABLE 1

| INLET LENGTH | BOUNDARY LAYER THICKNESS | PRESSURE GRADIENT |
|---|---|---|
| 0.02717 | 0.032798 | 183.33 |
| 0.05434 | 0.041033 | 165.06 |
| 0.08151 | 0.049064 | 148.16 |
| 0.10869 | 0.054724 | 144.21 |
| 0.13586 | 0.060493 | 138.28 |
| 0.16304 | 0.064858 | 135.95 |
| 0.19021 | 0.069320 | 134.06 |
| 0.21738 | 0.073372 | 132.23 |
| 0.24456 | 0.077015 | 131.01 |
| 0.27173 | 0.080424 | 129.97 |
| 0.29890 | 0.083623 | 129.06 |
| 0.32607 | 0.086594 | 128.29 |
| 0.35325 | 0.089369 | 128.29 |
| 0.38042 | 0.092743 | 127.65 |
| 0.40759 | 0.095210 | 127.14 |
| 0.43477 | 0.097569 | 126.76 |
| 0.46194 | 0.099868 | 126.76 |
| 0.48911 | 0.10276 | 126.51 |
| 0.51628 | 0.10498 | 126.51 |
| 0.54346 | 0.10770 | 126.26 |
| 0.57063 | 0.10964 | 126.13 |
| 0.59780 | 0.11174 | 126.13 |
| 0.62497 | 0.11426 | 126.00 |
| 0.65215 | 0.11615 | 126.00 |
| 0.67932 | 0.11854 | 125.88 |
| 0.70649 | 0.12016 | 125.82 |
| 0.73367 | 0.12192 | 125.82 |
| 0.76084 | 0.12416 | 125.82 |
| 0.78801 | 0.12628 | 125.82 |
| 0.81518 | 0.12844 | 125.76 |
| 0.84236 | 0.12969 | 125.76 |
| 0.86953 | 0.13177 | 125.75 |

| INLET LENGTH | BOUNDARY LAYER THICKNESS | PRESSURE GRADIENT |
|---|---|---|
| 0.89670 | 0.13356 | 125.75 |
| 0.92388 | 0.13557 | 125.75 |
| 0.95105 | 0.13740 | 125.75 |
| 0.97822 | 0.13935 | 125.75 |
| 1.0054 | 0.14116 | 125.75 |
| 1.0326 | 0.14305 | 125.75 |
| 1.0597 | 0.14485 | 125.75 |
| 1.0869 | 0.14670 | 125.75 |

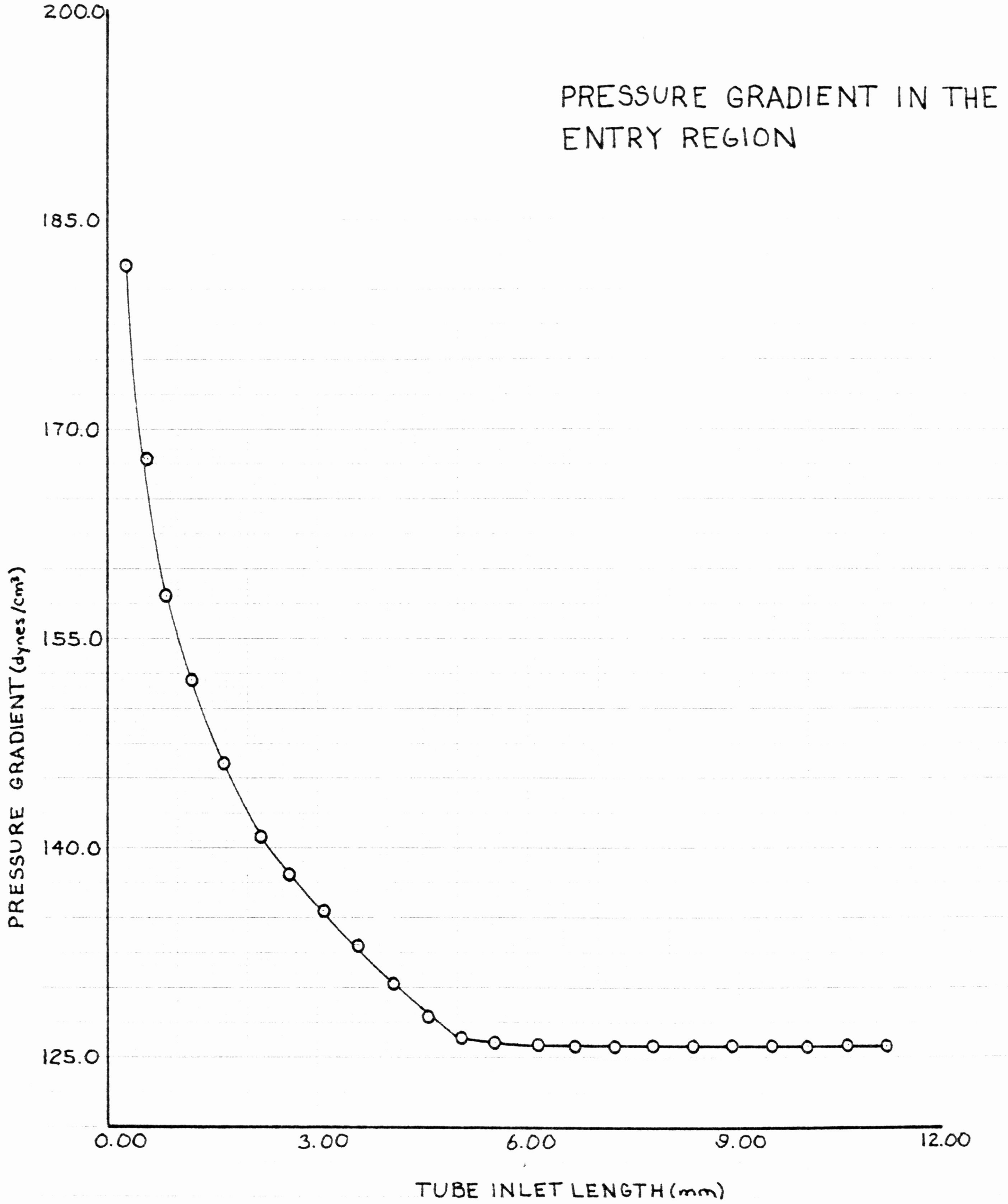

PRESSURE GRADIENT IN THE ENTRY REGION
200.0
185.0
170.0
155.0
140.0
125.0
PRESSURE GRADIENT (dynes/cm³)
0.00
3.00
6.00
9.00
12.00
TUBE INLET LENGTH (mm)

FIG. 7

This necessarily forced an identical profile and thus an identical pressure gradient. A simple solution to this error would be to reduce the radial step size.

## REFERENCES

1. Birchoff, G.(1960). Hydrodynamics. Pronceton University Press.

2. Brecher, G.A.(1956). Venous Return. New York; Grune and Stratton.

3. Caro, C.G., Pedley, T.J., Schroeter,R.C., and Seed, W.A. The Mechanics of the Circulation. New York: Oxford University Press, 1978, pp.44-55.

4. Guyton, A.C., M.D. Medical Physiology. Philadelphia W.B. Saunders Company, 1976, pp.324-330.

5. Kaufmann, W.(1963). Fluid Mechanics. translated by E.C.Chilton. New York: McGraw Hill.

6. McDonald, D.A.,Blood Flow in Arteries. Baltimore: Williams and Wilkins Company, 1974, pp.27-30.

7. May, A.G., DeWeese, J.A., and Rob, C.G.: Hemodynamics effects of arterial stenosis. Surgery, 53:513,1963.

8. Smith, R.L., Blick, E.F., Coalson, J. and Stein, P.D.: Thrombus production by turbulence. J Appl Physiol, 32:261, 1972

9. Stehbens, W.E., editor. Hemodynamics and the Blood Vessel Wall. Springfield, Ill.: Charles C. Thomas, 1979, pp.447-455.

10. Taylor, L.A., and Gerrard, J.H.: Mathematical model representing blood flow in arteries. Med & Biol Eng. & Comput, 15:611-617, 1977.

11. Webster, John G., editor. Medical Instrumentation: Application and Design. Boston: Houghton Mifflin Company, 1978, pp.409-412.

12. Wormersley,J.R.(1955b). Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known J Appl Physiol.,127:553-563.

Appendix A

Computer Code Listing

```
//$OPTIONS      LIBLIST
C*************************************************************
C
C
C
C     A COMPUTER CODE HAS BEEN WRITTEN WHICH CALCULATES THE
C
C     VELOCITY PROFILE AND PRESSURE GRADIENT IN A RIGID TUBE
C
C     AS A FUNCTION OF THE TUBE LENGTH.  THE METHOD BY WHICH
C
C     THIS WAS ACCOMPLISHED IS DESCRIBED IN THE TEXT.
C
C
C
C*************************************************************
C
C
C
C                  TABLE OF VARIABLES
C
C
C  CST - PROPORTIONALITY CONSTANT RELATING THE BOUNDARY LAYER
C        THICKNESS TO THE FREE STREAM VELOCITY AND THE INLET
C        LENGTH.
C
C  DELMAX - EQUAL TO THE RADIUS AND USED TO CALCULATE THE INLET
C           LENGTH.  IT REPRESENTS THE MAXIMUM BOUNDARY LAYER
C           THICKNESS.
C
C  DELTA - THE BOUNDARY THICKNESS
C
C  DIAM - TUBE DIAMETER
C
C  DINCR - THE DISTAL INCREMENT
C
C  DPDZ - THE PRESSURE GRADIENT WITH RESPECT TO THE AXIAL
C         DIRECTION.
C
C  FRVEL - THE ENTERING FREE STREAM VELOCITY.  IT IS FOUND
C          BY DIVIDING THE VOLUME FLOW RATE BY THE AREA.
C
C  INIT - SUBROUTINE WHICH CALCULATES THE VELOCITY PROFILE
C         AND SUBSEQUENTLY THE PRESSURE GRADIENT.
C
C  MU - BLOOD VISCOSITY
C
C  NO - SEE COMMENTED SUBROUTINE PLOTLE
C
C  NPLOTS - SEE COMMENTED SUBROUTINE PLOTLP
C
C  NPTS - SEE COMMENTED SUBROUTINE PLOTLP
C
C  PLOTLP - SUBROUTINE WHICH PLOTS THE VELOCITY AS A FUNCTION
C           OF THE RADIUS.
C
C  Q - VOLUME FLOW RATE.
C
C  QA - USED TO CALCULATE THE POISEUILLE FLOW RATE DUE TO VISCOUS
```

```
C          EFFECTS.
C
C     QPAR - USED TO REPRESENT THE TOTAL VOLUME FLOW DUE TO VISCOUS
C              EFFECTS OUTSIDE OF THE BLUNT FLOW.
C
C     QR - USED TO CALCULATE THE POISEUILLE FLOW RATE DUE TO VISCOUS
C          EFFECTS.
C
C     R - REPRESENTS A GIVEN RADIUS FOR INTEGRATION PURPOSES.
C
C     RAD - TUBE RADIUS
C
C     RE - THE REYNOLDS NUMBER (SEE TEXT)
C
C     RINCR - RADIAL INCREMENT
C
C     RHO - BLOOD DENSITY
C
C     SL1 - SLOPE 1.  USED TO FIT A VELOCITY PROFILE.
C
C     SL2 - SLOPE 2.  USED TO FIT A VELOCITY PROFILE.
C
C     STORE - PARAMETER USED TO STORE THE POISEUILLE PRESSURE GRADIENT.
C
C     VEL - MATRIX USED TO STORE VALUES FOR THE VELOCITY AS A FUNCTION
C           OF THE RADIUS.
C
C     VELOC - REPRESENTS THE BLUNT STREAM VELOCITY AS THE FLOW DEVELOPS.
C
C     VPEAK - FREE STREAM VELOCITY FOR POISEUILLE FLOW.
C
C     XFULL - TUBE INLET LENGTH.
C
C     XINC - SEE COMMENTED SUBROUTINE PLOTLP.
C
C     XSTART - SEE COMMENTED SUBROUTINE PLOTLP.
C
C     XX - REPRESENTS THE DISTAL LOCATION WITHIN THE TUBE.
C
C
C
C
C**********************************************************************
      DOUBLE PRECISION VEL(42,101,1),MU,DPDZ,FRVEL,RE,XFULL,
     *RINCR,DINCR,VPEAK,DELMAX,CST,XX,DELTA,STORE,R,QR,QA,QPAR,VELOC,
     *SL1,SL2,RHO,RAD,DIAM,C,PI
      REAL PC(1,41)
      DATA MU,RHO,RAD,DIAM,C,PI/3.0D-2,1.05D0,1.4D-1,3.0D-1,5.0D1,
     *3.14159D0/
C
C*********************************************************************
C
C     PARAMETERS OF THE FREE STREAM FLOW ARE CALCULATED.  POISEUILLE'S
C     MODEL IS ALSO USED TO FIND VARIOUS PARAMETERS.
C
C*********************************************************************
      Q=Q/6.001
      DPDZ=Q*8.0D0*MU/(PI*RAD**4)
      FRVEL=Q/(PI*RAD**2)
      RE=RHO*FRVEL*DIAM/MU
```

```
      XFULL=3.0D-2*DIAM*RE
      WRITE(6,100)DPDZ,FRVEL,RE,XFULL
  100 FORMAT('1',//////////,T20,'THE PRESSURE GRADIENT FOR FULLY
     *DEVELOPED FLOW,DPDZ =',D13.4,2X,'DYNES/CUBIC CM',///,T20,'THE
     *FREE STREAM VELOCITY FOR A BLUNT PROFILE,FRVEL =',D12.4,2X,
     *'CM/SEC',///,T20,'THE REYNOLDS NUMBER,RE =',D14.1///T20,
     *'THE DISTANCE FROM THE BLUNT PROFILE TO FULLY DEVELOPED FLOW,
     *XFULL =',D11.4,2X,'CM')
C
C*************************************************************************
C
C  THE IS PARTITIONED INTO 40 DISTAL AND RADIAL SEGMENTS.
C
C*************************************************************************
C
      RINCR=RAD/40
      DINCR=XFULL/41
      CALL INIT(VEL,RINCR,DINCR,DPDZ,RAD,DIAM,MU,RHO,XFULL,FRVEL
     *,ARRAY,X,COEF,TOT,Q,PI)
C
C  THE PARAMETERS NECESSARY TO PLOT ARE SET.
C
      XSTART=0.0
      XINC=.00375
      NPLOTS=1
      NPTS=41
      ND=1
      DO 10 K=2,41,4
      DO 20 I=1,41
        PC(1,I)=VEL(I,K,1)
   20 CONTINUE
      WRITE(6,225)
  225 FORMAT('1')
      CALL PLOTLP(XSTART,XINC,PC,NPLOTS,NPTS,ND)
   10 CONTINUE
      STOP
      END
      SUBROUTINE INIT(VEL,RINCR,DINCR,DPDZ,RAD,DIAM,MU,RHO,XFULL,
     1FRVEL,ARRAY,X,COEF,TOT,Q,PI)
      DOUBLE PRECISION VEL(42,101,1),MU,VPEAK,DELMAX,CST,XX,DELTA,
     *STORE,R,DR,QA,QPAR,VELCC,SL1,SL2,RAD,RHO,DIAM,DPDZ,RINCR,DINCR,
     *XFULL,FRVEL,Q,PI
C
C
C  CST IS CALCULATED BY KNOWING POISEUILLES MODEL AND THE
C  RADIUS OF THE VESSEL.
C
      VPEAK=RAD**2*DPDZ/(4.D0*MU)
      DELMAX=RAD*9.9D-1
      CST=DELMAX/(MU*XFULL/RHO/VPEAK)**.5
      WRITE(6,200)CST
  200 FORMAT('-',/////,T30,'CONSTANT =',D12.4)
      DO 10 I=1,41
      VEL(I,1,1)=FRVEL
   10 CONTINUE
C
C   A VALUE FOR THE BOUNDARY LAYER THICKNESS IS FOUND FOR A
C   PARTICULAR DISTAL LENGTH.
C
```

```
      DO 30 JJ=2,41
         XX=(JJ-1)*DINCR
         DELTA=CST*(MU*XX/(VEL(41,JJ-1,1)*RHO))**.5
         WRITE(6,600)XX,DELTA
  600    FORMAT('1'//////////,T20,'FOR A TUBE INLET LENGTH OF',
     *   D12.5,2X,'THE BOUNDARY LAYER THICKNESS,DELTA =',D12.5,2X,
     *   'CM'/////)
  700    FORMAT(' ',///T33,'INLET LENGTH (CM)',T65,'RADIAL DISTANCE ('
     *   ,'CM)',T98,'VELOCITY (CM/SEC)',///)
      STORE=DPDZ
C
C   POISEUILLE'S MODEL IS USED TO CALULATE THE VELOCITY AS A
C   FUNCTION OF THE RADIUS.
C
      DO 60 LL=1,500
      DO 40 II=1,41
         L=41-II
         R=L*RINCR
         VEL(II,JJ,1)=RAD**2*DPDZ*(1-(R/RAD)**2)/(4*MU)
  500 FORMAT(' ',T34,D13.5,T72,D13.5,T98,D15.6)
      IF(II.GE.41)GOTO88
C
C   AN IF STATEMENT IS USED TO DETERMINE IF THE BLUNT PROFILE HAS
C   BEEN REACHED YET.
C
         IF(R.LE.RAD-DELTA)GOTO 22
   40 CONTINUE
C
C   THE VELOCITY IS INTEGRATED OVER THE AREA TO FIND THE VOLUME
C   FLOW RATE.
C
   22 QR=PI*DPDZ/(2.D0*MU)*(RAD**4/2.D0-RAD**4/4.D0)
      QA=PI*DPDZ/(2.D0*MU)*(RAD**2*R**2/2.D0-R**4/4.D0)
      QPAR=QR-QA
      VELOC=VEL(II+1,JJ,1)=(Q-QPAR)/(PI*R**2)
*EXTENSION*  OTHER COMPILERS MAY NOT ALLOW MULTIPLE ASSIGNMENT STATEMENTS
      IF(II.GE.40)GOTO88
      IK=II+2
      DO 50 K=IK,41
         R=R-RINCR
         VEL(K,JJ,1)=VELOC
   50 CONTINUE
C
C   THE SLOPES AROUND THE TRANSITION TO THE BLUNT PROFILE ARE
C   FOUND.  THESE ARE USED TO FIT THE PROFIL TO A CONTINUOUS
C   WAVEFORM.
C
      SL1=(VEL(II,JJ,1)-VEL(II-1,JJ,1))/RINCR
      SL2=(VEL(II+1,JJ,1)-VEL(II,JJ,1))/RINCR
      IF(SL2.LT.0)GOTO44
      IF(SL1.GT.SL2)GOTO88
      DPDZ=1.001*DPDZ
      GOTO60
   44 DPDZ=DPDZ/1.0015005
      GOTO60
   88 R=RAD
      WRITE(6,540)DPDZ
        WRITE(6,700)
  540 FORMAT('0',///,T50,'DPDZ =',D14.5,2X,'DYNES/CUBIC CM')
      DO 70 MM=1,41
```

```
            WRITE(6,500)XX,R,VEL(MM,JJ,I)
            R=R-RINCR
   70    CONTINUE
         DPDZ=STORE
         GOTO 30
   60    CONTINUE
   30    CONTINUE
         RETURN
         END
*EXTENSION*   PSEUDO VARIABLE DIMENSIONING ASSUMED FOR ARRAY VEL
//$DATA
C
C-----------------------------------------------------------------------
C
C     SUBROUTINE PLOTLP
C
C     PURPOSE
C        PLOT 1 TO 8 CURVES SIMULTANEOUSLY DOWN THE PAGE (S) IN A 121
C        SPACE FIELD.
C
C     USAGE
C        CALL PLOTLP (XSTART, XINC, YY, NPLOTS, NPTS, ND)
C
C     PARAMETER DESCRIPTION
C
C        XSTART - THE STARTING 'X' VALUE PRINTED IN THE LEFT MARGIN.
C
C        XINC   - THE INCREMENT TO BE ADDED TO CREATE THE 'X' VALUE
C                 TO BE PRINTED IN THE LEFT MARGIN OF THE NEXT LINE.
C
C        YY     - A DOUBLY SUBSCRIPTED ARRAY CONTAINING THE VALUES TO
C                 BE PLOTTED.  THE FIRST DIMENSION IS THE CURVE NUMBER,
C                 (MAX = 8), WHILE THE 2ND DIMENSION IS THE POINT NUMBER
C                 FOR THE CURVE.  AS AN EXAMPLE, YY(2,43) REFERS TO
C                 THE 43RD VALUE OF CURVE NUMBER 2.
C
C        NPLOTS - THE NUMBER OF CURVES TO BE PLOTTED.  MUST BE .LE. ND.
C
C        NPTS   - NUMBER OF POINTS TO BE PLOTTED FOR EACH CURVE
C                 (ASSUMED TO BE THE SAME FOR EACH CURVE).
C
C        ND     - MUST BE EQUAL TO THE ACTUAL FIRST DIMENSION OF THE
C                 ARRAY WHICH CORRESPONDS TO ARRAY YY IN THE USERS
C                 CALLING ROUTINE.  E.G. IF THE USERS DIMENSION STATE-
C                 MENT IS DIMENSION Y(3,50).... THEN ND MUST EQUAL 3.
C
C     REMARKS
C        THE PLOT ROUTINE DETERMINES THE MAXIMUM AND MINIMUM YY VALUES
C        AND SCALES THE 121 PRINT POSITIONS ACCORDINGLY.  THESE MAX AND
C        MIN VALUES, ALONG WITH THE VALUE CORRESPONDING TO EACH PRINT
C        POSITION ARE PRINTED PRIOR TO PLOTTING THE GRAPHS.  AN 'X'
C        VALUE IS PRINTED IN THE LEFT HAND MARGIN OF EACH LINE.  THE
C        CURVES ARE SUPERIMPOSED ON A 20 HORIZONTAL SPACE BY 10 VERTICAL
C        LINE GRID COMPOSED OF PERIODS.
C
C        VALUES TO BE PLOTTED MAY BE POSITIVE OR NEGATIVE BUT MUST BE
C        PRINTABLE WITH AN F10.3 FORMAT.  X VALUES MUST BE PRINTABLE
C        WITH AN F7.3 FORMAT.
C
```

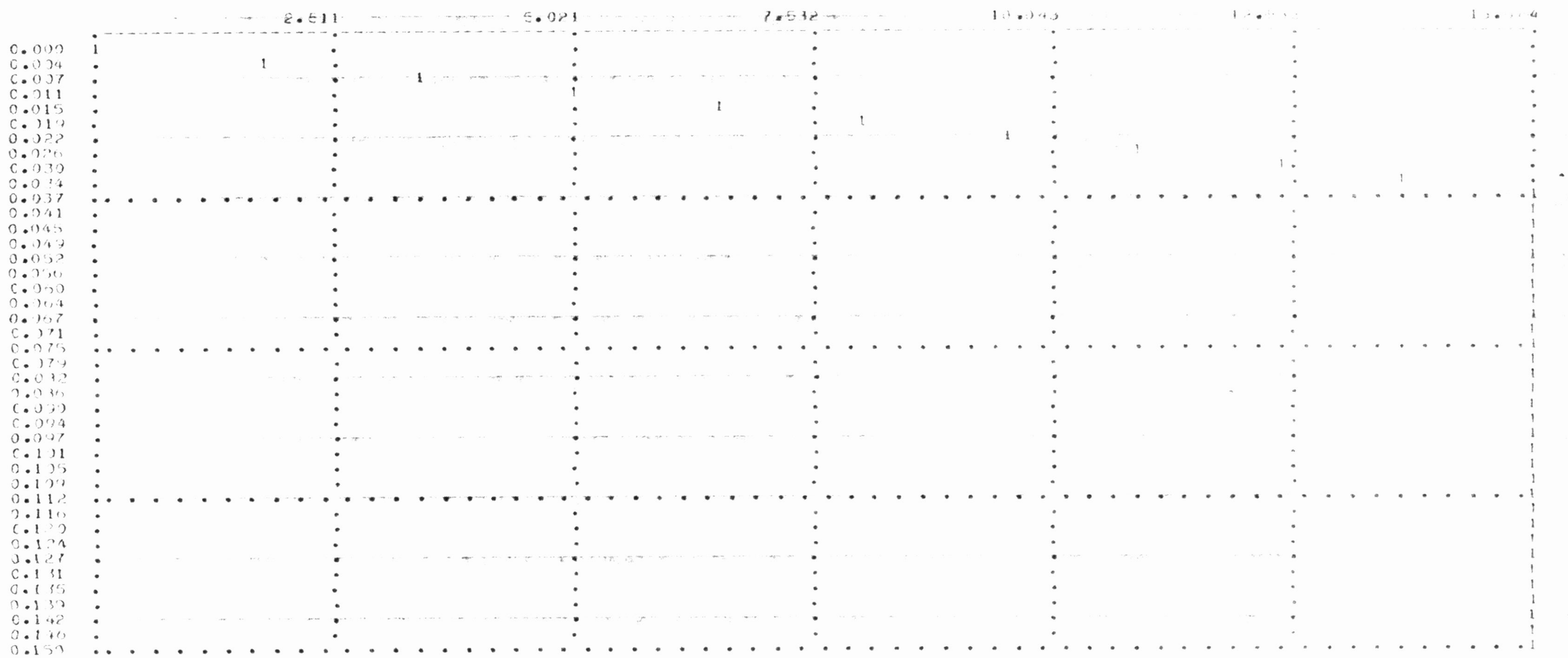
```
C          THE CURVES ARE IDENTIFIED BY THE PRINT CHARACTERS USED, WHICH    00004900
C          ARE THE NUMBERS 1 THRU 8.  SHOULD TWO OR MORE CURVES HAVE        00005000
C          VALUES WHICH OCCUPY THE SAME PRINT POSITION, THE LARGEST         00005100
C          NUMERIC CHARACTER IS PRINTED.                                    00005200
C                                                                           00005300
C          NO PAGE EJECT IS MADE PRIOR TO PRINTING, SO USER MUST PROVIDE    00005400
C          ONE IF HE DESIRES TO HAVE PLOT ON 'CLEAN' SHEET OF PAPER.        00005500
C                                                                           00005600
C       REFERENCE                                                           00005700
C          ADAPTED FROM UT - ARLINGTON PLOT PACKAGE BY R. R. WIER            00005800
C          TAMU COMPUTER SCIENCE DEPARTMENT - 10 APRIL, 1978                00005900
C                                                                           00006000
C-----------------------------------------------------------------------00006100
C                                                                           00006200
       SUBROUTINEPLOTLP(XSTART,XINC,YY,NPLOTS,NPTS,ND);DIMENSIONYY(ND,NPT00006300
      *S),LINE(121),MARK(121),IP(8),KEEP(8),HORIZ(8);INTEGERBLANK,DASH,DO00006400
      *T,SIGN(8),PR,UNDRLN;DATABLANK/' '/,DASH/'-'/,DOT/'.'/,SIGN/'1','2'00006500
      *,'3','4','5','6','7','8'/,UNDRLN/'_'/;DATAPR/6/                      00006600
C                                                                           00006700
C-----------------------------------------------------------------------00006800
C       CHECK ARGUMENTS FOR VALIDITY.                                       00006900
C-----------------------------------------------------------------------00007000
       IF(NPTS.LT.1.OR.NPLOTS.LT.1.OR.NPLOTS.GT.8.OR.ND.LT.NPLOTS)GOTO10  00007100
       GOTO90;10:WRITE(PR,6);RETURN                                         00007200
C-----------------------------------------------------------------------00007300
C       FIND MIN AND MAX YY VALUES.                                         00007400
C-----------------------------------------------------------------------00007500
 90    BIGY=YY(1,1);SMALLY=BIGY;DO110I=1,NPLOTS;DO110J=1,NPTS;IF(YY(I,J).00007600
      *LE.BIGY)GOTO100;BIGY=YY(I,J);GOTO110;100:IF(YY(I,J).LT.SMALLY)SMAL00007700
      *LY=YY(I,J);110:CONTINUE                                              00007800
C-----------------------------------------------------------------------00007900
C       COMPUTE HORIZONTAL INCREMENT AND WRITE SCALE.                       00008000
C-----------------------------------------------------------------------00008100
       SPACE=(BIGY-SMALLY)/120.;X=0.;DO115I=1,6;X=X+20.;115:HORIZ(I)=SMAL00008200
      *LY+X*SPACE;WRITE(PR,1)SMALLY,BIGY,SPACE,(HORIZ(I),I=1,6)             00008300
C-----------------------------------------------------------------------00008400
C       CREATE DIVIDING MARKS.                                              00008500
C-----------------------------------------------------------------------00008600
       DO130I=1,121;LINE(I)=BLANK;130:MARK(I)=DASH;MARK(1)=DOT;LINE(1)=DO00008700
      *T;DO140I=21,121,20;LINE(I)=DOT;140:MARK(I)=DOT                       00008800
C-----------------------------------------------------------------------00008900
C       WRITE FIRST DIVIDING MARK.                                          00009000
C-----------------------------------------------------------------------00009100
       WRITE(PR,2)(MARK(I),I=1,121);DO150I=2,120,2;MARK(I+1)=BLANK;       00009200
 150   MARK(I)=DOT;MARK(121)=DOT                                            00009300
C-----------------------------------------------------------------------00009400
C       PRINT X AND PLOT YY VALUES.  DIVIDING MARKS ARE REPEATED EACH 10 L00009500
C-----------------------------------------------------------------------00009600
       X=XSTART;DO210K=1,NPTS;DO160J=1,NPLOTS;160:IP(J)=IFIX(((YY(J,K)-SM00009700
      *ALLY)/SPACE)+1.5);IF(K.GT.1.AND.MOD(K-1,10).EQ.0)GOTO180             00009800
C-----------------------------------------------------------------------00009900
C       WRITE REGULAR LINE.                                                 00010000
C-----------------------------------------------------------------------00010100
       DO165I=1,NPLOTS;L=IP(I);165:KEEP(I)=LINE(L);DO170I=1,NPLOTS;L=IP(I00010200
      *);170:LINE(L)=SIGN(I);WRITE(PR,3)X,(LINE(I),I=1,121);173:DO175I=1,00010300
      *NPLOTS;L=IP(I);175:LINE(L)=KEEP(I);GOTO200                           00010400
C-----------------------------------------------------------------------00010500
C       WRITE LINE WITH DIVIDING MARKS.                                     00010600
C-----------------------------------------------------------------------00010700
 180   DO185I=1,NPLOTS;L=IP(I);185:KEEP(I)=MARK(L);DO190I=1,NPLOTS;L=IP(I00010800
```

```
              *);190:MARK(L)=SIGN(I);WRITE(PR,3)X,(MARK(I),I=1,121);195:DO200I=1,00010900
              *NPLOTS;L=IP(I);200:MARK(L)=KEEP(I);KK=K;209:KK=K;210:X=X+XINC      00011000
        C-----------------------------------------------------------------------00011100
        C          PRINT LOWER X BORDER IF NECESSARY                             00011200
        C-----------------------------------------------------------------------00011300
153            IF(KK.GT.1.AND.MOD(KK-1,10).NE.0)GOTO214;GOTO220;214:DO215I=1,121 00011400
156            MARK(I)=UNDRLN;215:CONTINUE;MARK(1)=DOT;DO216I=21,121,20;MARK(I)=00011500
              *DT;216:CONTINUE;WRITE(PR,2)(MARK(I),I=1,121);220:RETURN           00011600
        C-----------------------------------------------------------------------00011700
        C      FORMATS                                                           00011800
        C-----------------------------------------------------------------------00011900
164      1     FORMAT(' ',10X,'PARAMETER RANGE  ',F8.3,' TO ',F8.3,50X,'EACH HORI00012000
              *ZONTAL SPACE IS ',F9.4,///,11X,6(10X,F10.4))                      00012100
165      2     FORMAT(10X,121A1)                                                 00012200
166      3     FORMAT(1X,F7.3,2X,121A1)                                          00012300
167      6     FORMAT(' A CALL TO SUBROUTINE PLOTLP HAS INVALID ARGUMENT (S).',' 00012400
              * CHECK VALUES OF 4TH, ETF, AND 6TH ARGUMENTS.'/)                  00012500
168            END                                                               00012600
```

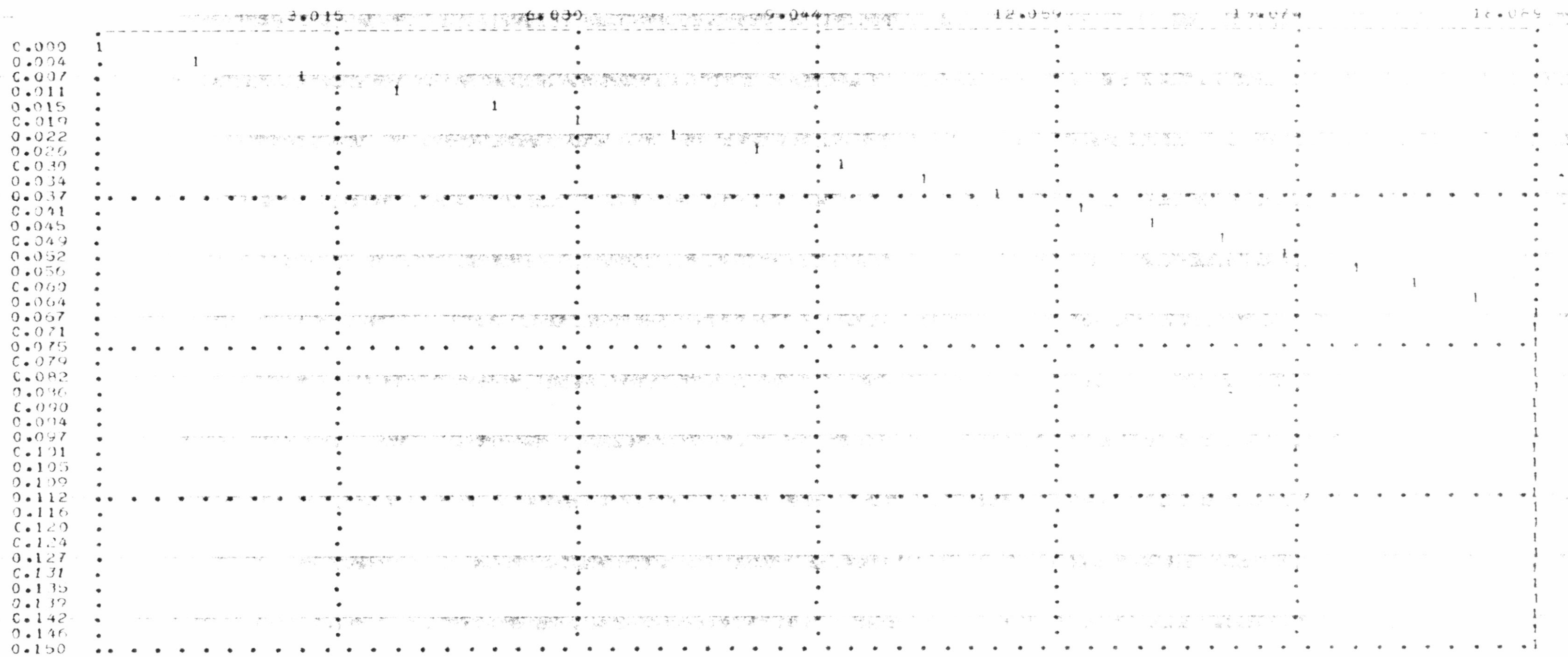
Appendix B

Computer Plots

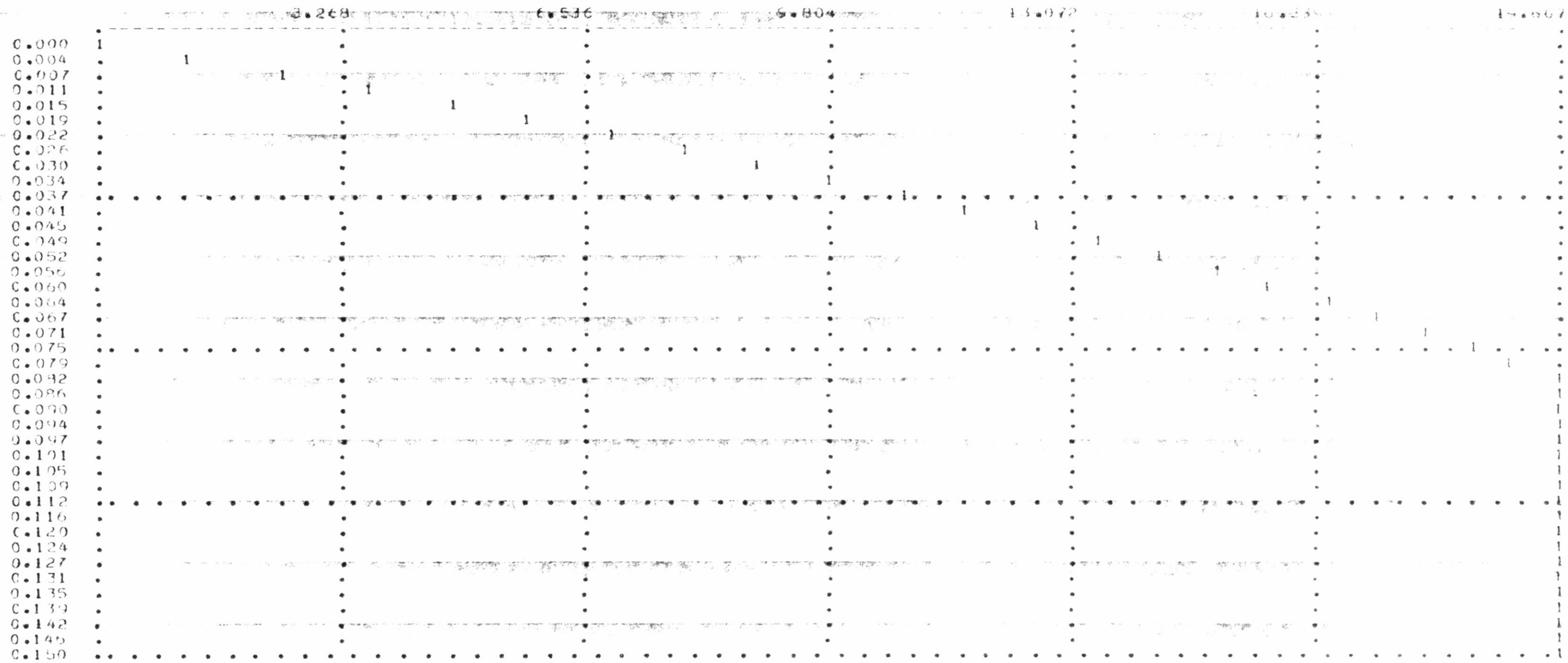
PARAMETER RANGE 0.000 TO 15.064



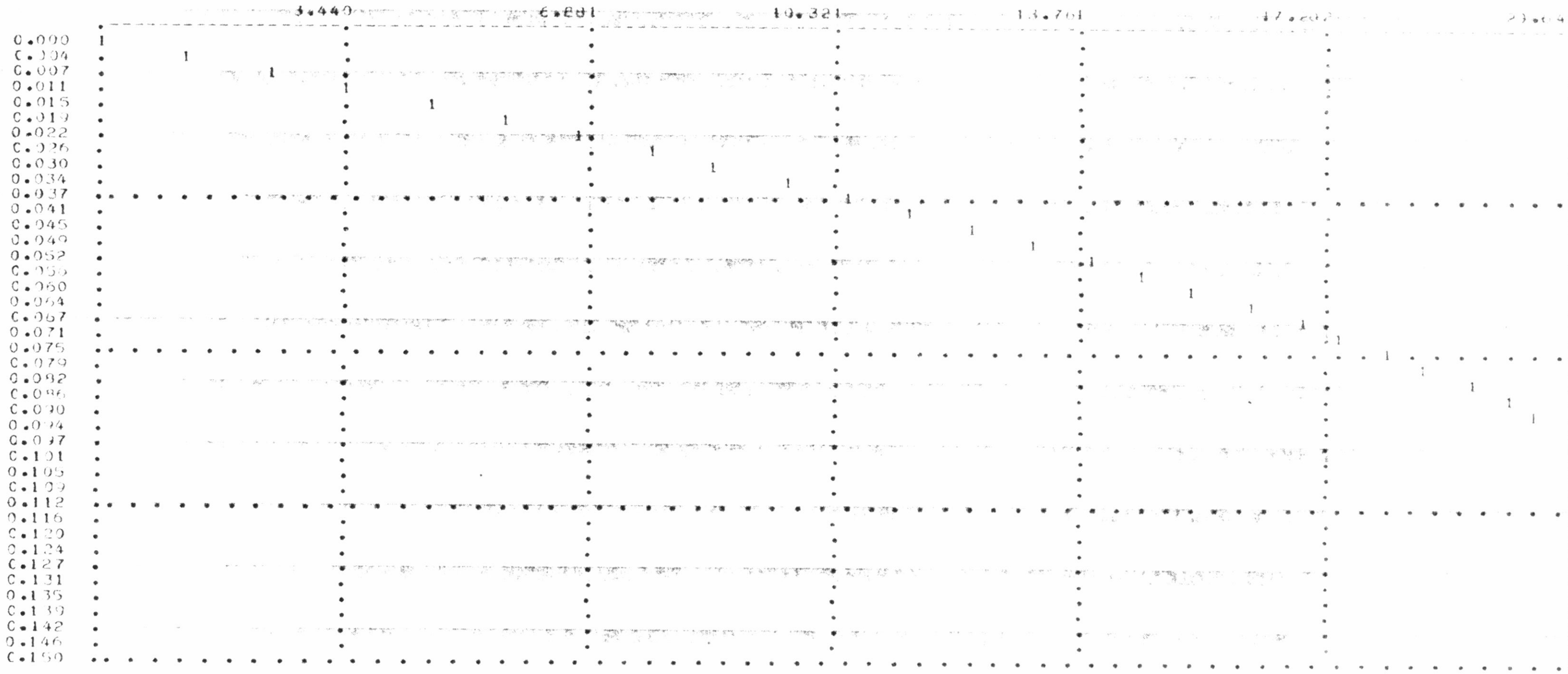
PARAMETER RANGE 0.000 TO 18.089 EACH HORIZONTAL SPACE IS 0.1507



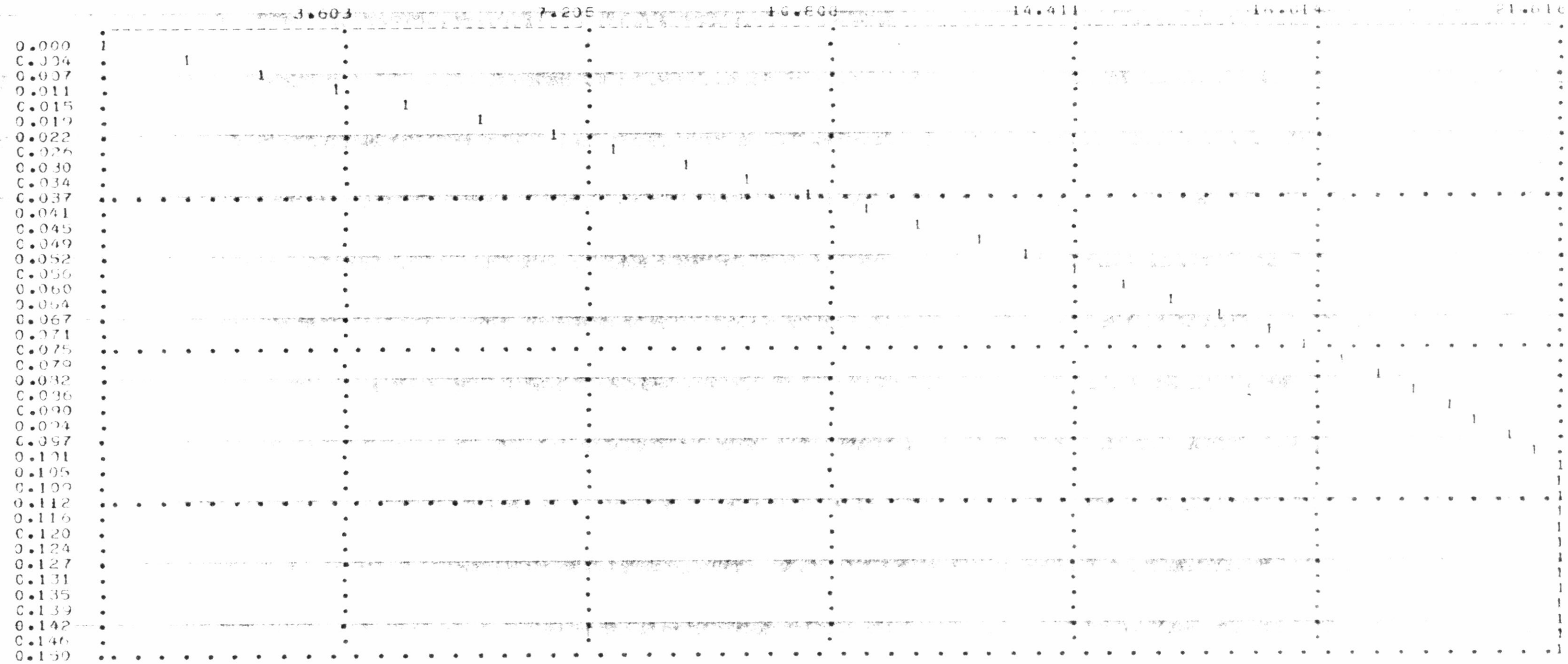
PARAMETER RANGE 0.000 TO 19.607



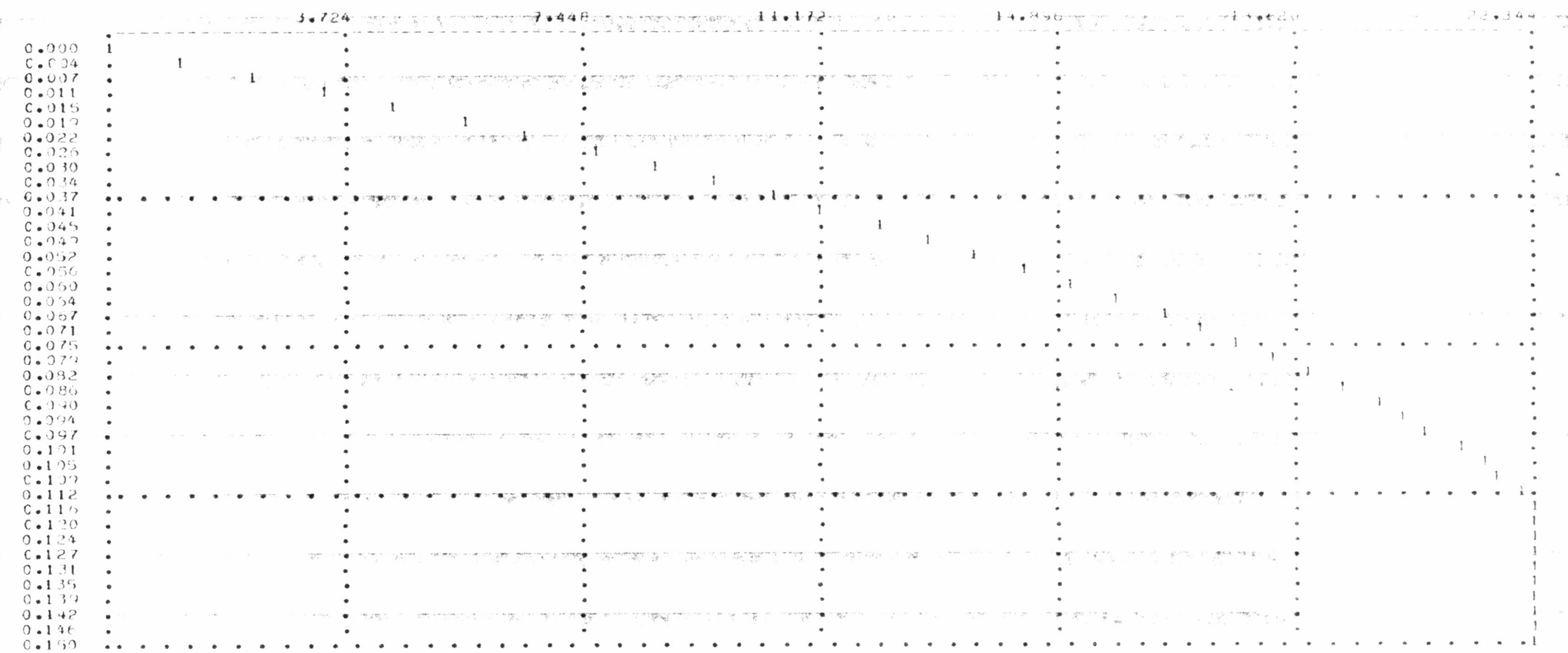
PARAMETER RANGE 0.000 TO 20.642



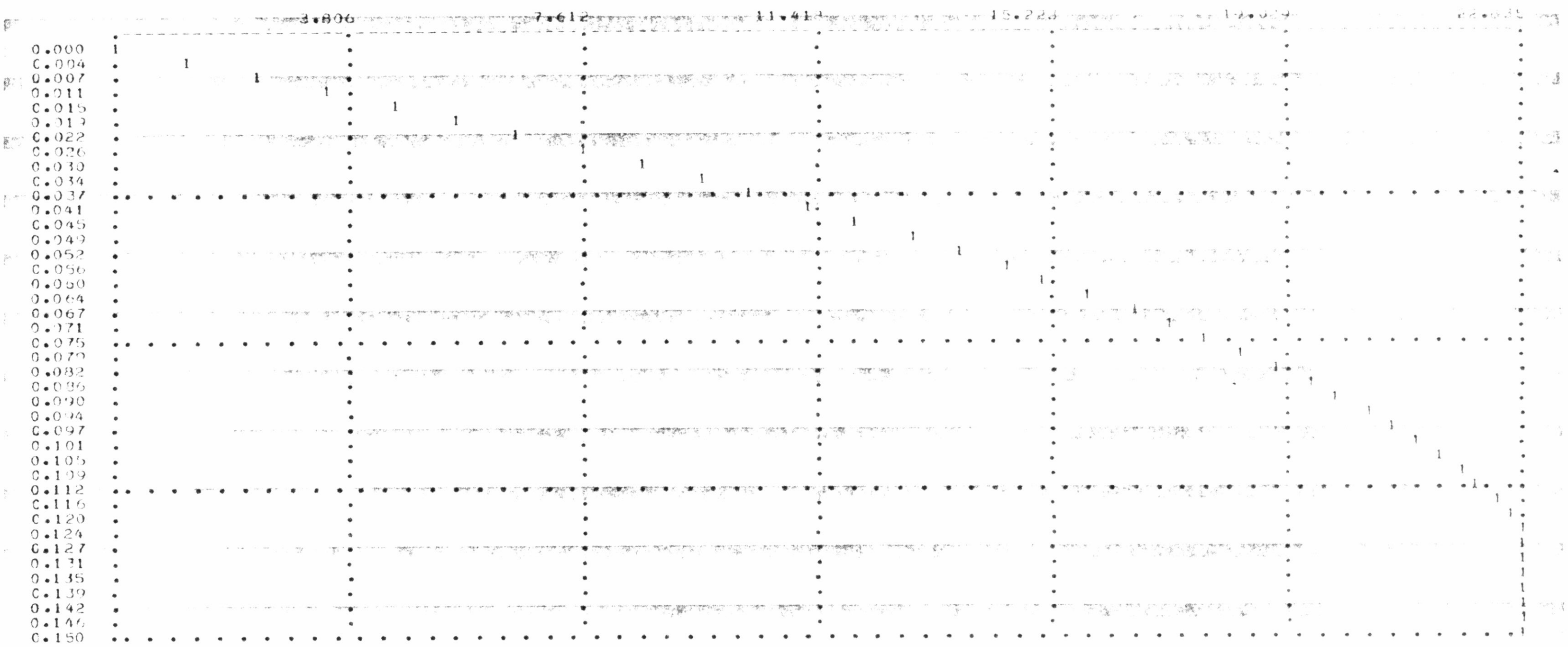
PARAMETER RANGE 0.000 TO 21.616



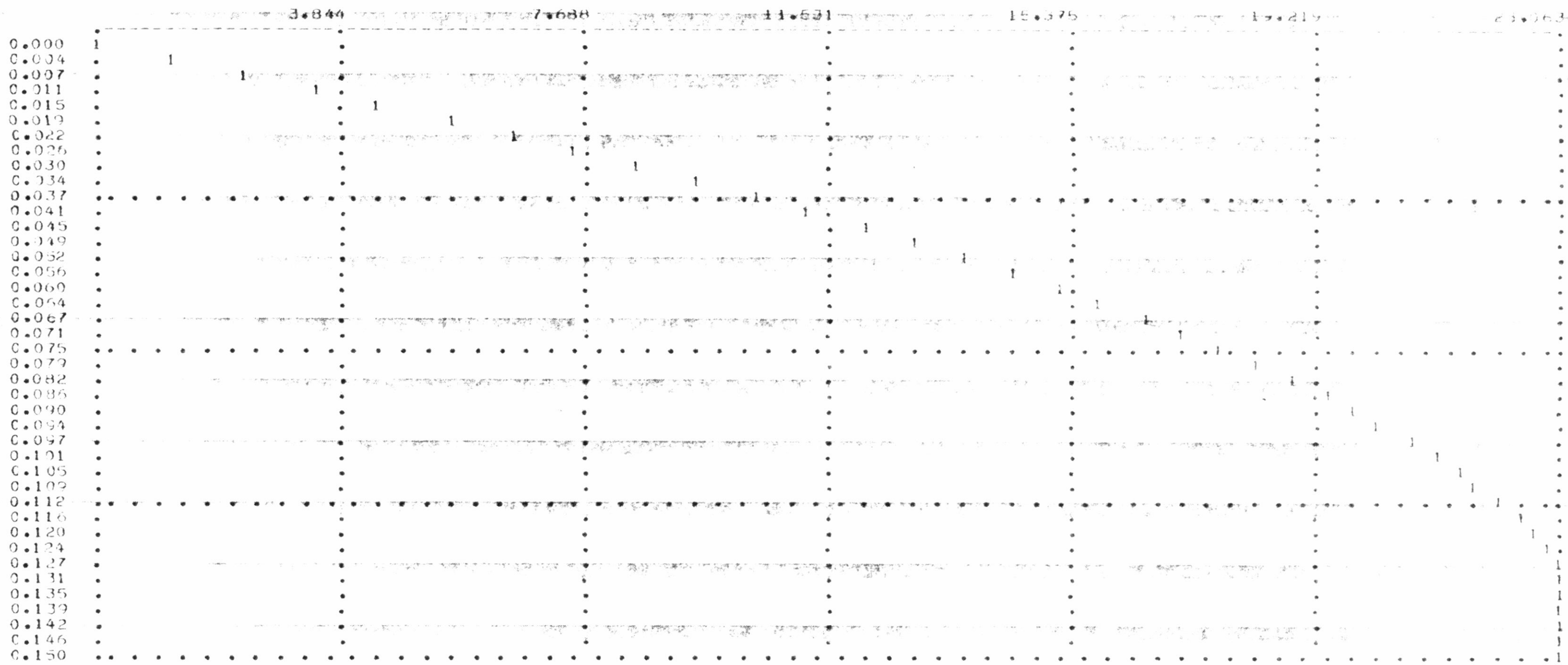
PARAMETER RANGE 0.000 TO 22.344



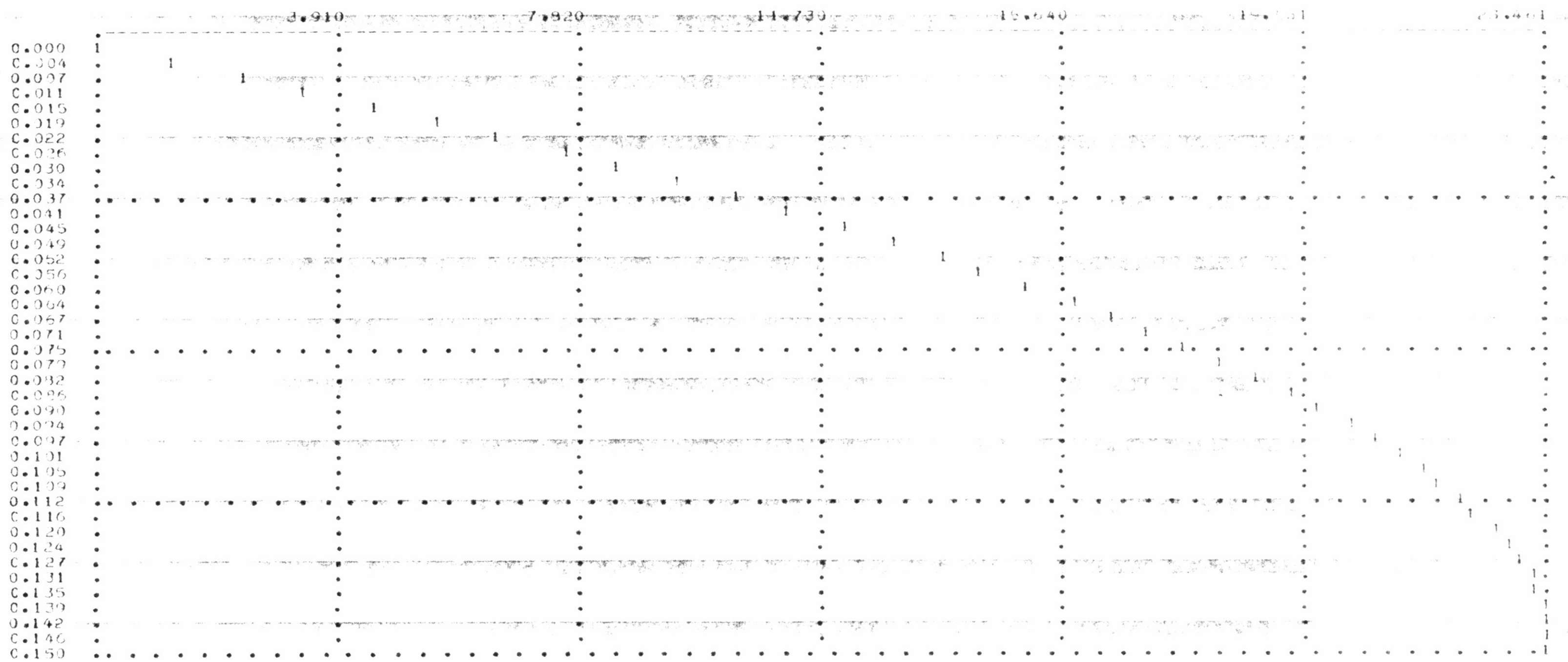
PARAMETER RANGE 0.000 TO 22.835



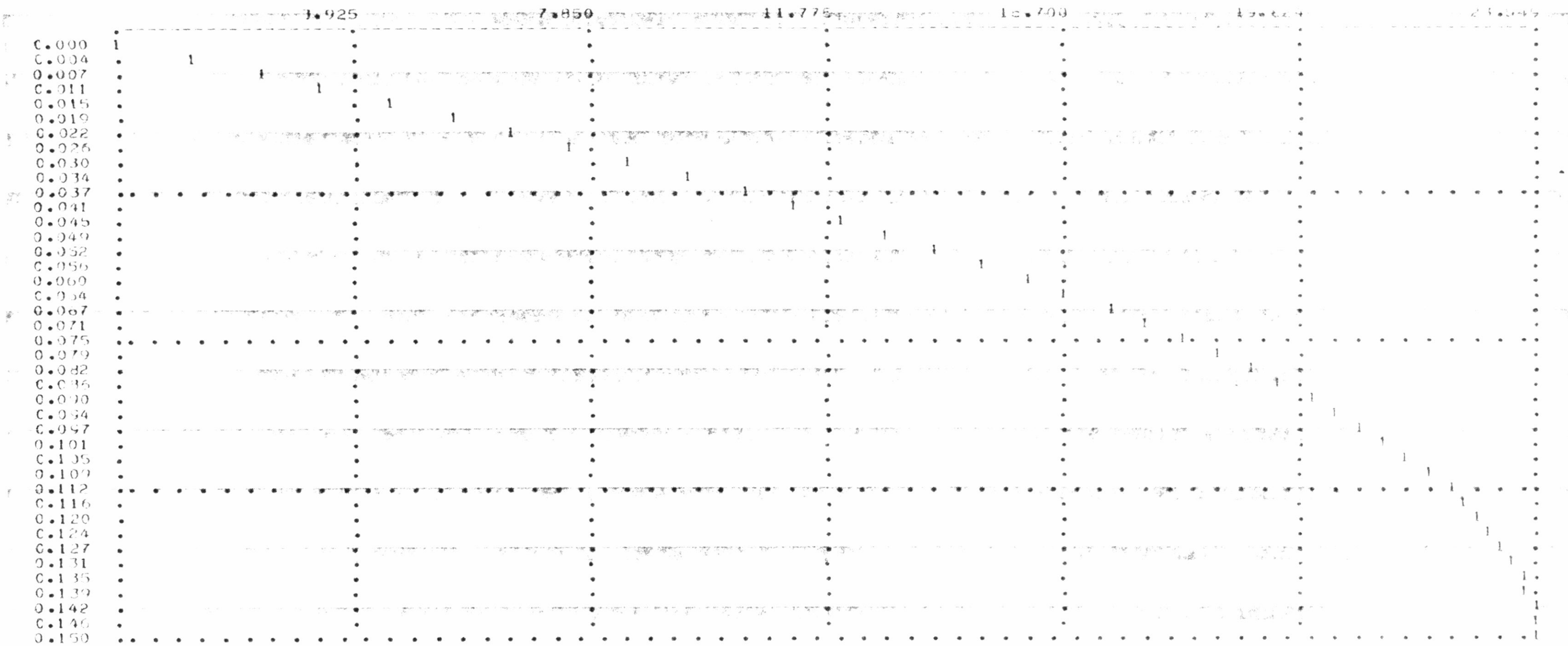
PARAMETER RANGE 0.000 TO 23.063



PARAMETER RANGE 0.000 TO 23.461



PARAMETER RANGE 0.000 TO 23.545




CORE USAGE OBJECT CODE= 14488 BYTES,ARRAY AREA= 35156 BYTES,TOTAL AREA AVAILABLE= 91136 BYTES

DIAGNOSTICS NUMBER OF ERRORS= 0, NUMBER OF WARNINGS= 0, NUMBER OF EXTENSIONS=

COMPILE TIME= 0.16 SEC,EXECUTION TIME= 6.57 SEC, 13.26.05 THURSDAY 20 APR 81 WATFIV - JUN 1977 V1L6

//$STOP