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Optimization of Water Use Efficiency Through Trickle Irrigation and the Stress Day Index

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OPTIMAZATION OF WATER USE EFFICIENCY THROUGH TRICKLE IRRIGATION AND THE STRESS DAY INDEX

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ABSTRACT

Water deficits reduce plant growth and subsequently, crop yields. Man has relied upon irrigation to overcome these crop water deficiencies. But since the present supplies of water are limited, more efficient irrigation application methods must be developed and utilized effectively. High frequency irrigation has been shown to be an efficient means for minimizing crop water deficits while maximizing irrigation application efficiency. This research evaluated the effects of high frequency irrigation on grain sorghum growth and yield and developed guidelines for the optimal utilization of a scarce resource--water--under high frequency irrigation.

An experiment was conducted in fully instrumented field lysimeters which had undisturbed soil cores. Rainfall was eliminated as a variable by an automated movable shelter which protected the lysimeters from rain. The field measurements that were made in each lysimeter were soil water content, leaf temperature, lead resistance, leaf water potential, leaf area index, and crop height. Measurements of wind speed, air temperature, dew-point temperature, and net radiation were made above the crop.

The yeild and water use efficiency of grain sorghum under high frequency irrigation was decreased primarily by water deficits occurring during the boot-to-bloom growth period. Water use efficiency was increased when water deficits were carefully managed by applying small, frequent applications of water and avoiding large deficits during the boot-to-bloom period.

Yield models of the multiplicative- and additive-type were compared to the yield data. Only small differences between the models resulted, which were due primarily to the small set of data used to develop and test the models. However, within the range of the data, each model was an acceptable representation of the actual results of independent experiments.

An environmental model which used Monte Carlo methods to simulate temperature, rainfall, and potential evaporation was developed for Temple, Texas. This model was coupled to the Blackland soil water balance model to simulate the water use of grain sorghum under high frequency irrigation.

Stochastic dynamic programming was used to maximize the expected yield per unit of available irrigation water. The results indicated that irrigation requirements could be reduced by almost one-half without appreciable loss in yield if the irrigation water was supplied optimally throughout the season. Also, pre-irrigation (irrigation at or prior to germination) was preferred to using a like-amount of water, supplied optimally later in the season, if the soil water content was less than 150 mm.

This research demonstrated the potential of high frequency irrigation to improve water use efficiency. The application of operations

research techniques greatly improved the understanding of the interactions of a large number of components and enhanced the decision selection under uncertain outcomes.

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Symbols

Symbol	Description	Units
A	slope of SDI vs. Y curve	kg ha^{-1}
A1	leaf area expansion rate	cm^2 day^{-1}
AW	relation between Y and SW	kg ha^{-1}
B_o	dummy regression variable	
C	Cornu-criteria	
CN	SCS curve number	
CS	crop susceptibility factor	
D/D	dry day given previous day was dry	
D/W	dry day given previous day was wet	
W/D	wet day given previous day was dry	
W/W	wet day given previous day was wet	
D	available irrigation water	mm
DA	depth of active soil layer	mm
DEPTH	depth of total soil profile	mm
DRA	drainage from active layer	mm
DRP	drainage from passive layer	mm
DRAIN	drainage	mm

Symbols (continued)

Symbol	Description	Units
DZ	root zone extension rate	mm day^{-1}
E	evaporation	mm
E_o	potential evaporation	mm
ES	soil evaporation	mm
EP	plant evaporation	mm
ET	total evaporation	mm
EC	electrical conductivity	mmhos cm^{-1}
Ep	pan evaporation	mm
ES_o	potential soil evaporation	mm
EDD	evaporation for D/D	mm
EDW	evaporation for D/W	mm
EWD	evaporation for W/D	mm
EWW	evaporation for W/W	mm
F	irrigation treatment ratio	
IA	initial abstraction	mm
I	irrigation level	
IRR	irrigation amount	mm
LAI	leaf area index	
LLE_o	lower limit of SW for potential evaporation	mm
M	total number of days in a sequence (D or W)	

Symbols (continued)

Symbol	Description	Units
MD	mean deviation	
N	number of days in a given sequence	
ND	number of D/D days	
NPI	no pre-irrigation	
P	probability	
PI	pre-irrigation	
PRECIP	precipitation	mm
P1	leaf appearance rate	
Q	runoff	mm
R	rainfall	mm
R1	rainfall for the previous day	mm
R_ℓ	leaf surface resistance	$s\ mm^{-1}$
RUNOFF	runoff	mm
RV	random variable	
S	standard deviation	
SE	standard error	
SD	stress day factor	
SDI	stress day index	
SES_2	sum of stage II evaporation	mm
SP	difference of R and potential runoff	mm

Symbols (continued)

Symbol	Description	Units
SW	soil water content	mm
SWA	soil water content in active layer	mm
SWP	soil water content in passive layer	mm
Sdd	standard deviation of EDD	mm
Sdw	standard deviation of EDW	mm
Swd	standard deviation of EWD	mm
Sww	standard deviation of EWW	mm
Sd	standard deviation of TD	C
Sw	standard deviation of TW	C
TWU	total water use	mm
T	temperature	C
TD	temperature of a dry day	C
TW	temperature of a wet day	C
U	upper limit of stage I soil evaporation	mm
UL	upper limit of water holding capacity	mm
WUE	water use efficiency	$kg\ ha^{-1}\ mm^{-1}$
WN	week number	
X	dummy variable	
Y	yield	$kg\ ha^{-1}$

Symbols (continued)

Symbol	Description	Units
Y_o	potential yield	kg ha^{-1}
Z_o	roughness length	mm
b	yield sensitivity factor	
d	available irrigation water	mm
t_2	days of stage II evaporation	day
y	stage return	kg ha^{-1}
α	relation of CS to SD	
β	gamma distribution parameter	
γ	gamma distribution parameter	
δ	soil evaporation constant	mm $day^{-1/2}$
λ	yield sensitivity factor	
ψ_ℓ	leaf water potential	

Abbreviations

Abbreviation	Description
ASCII	American Standard Communication Information Interchange
BPI	Bureau of Plant Industry
CSMP	Continuous System Modeling Program
SCS	Soil Conservation Service

CHAPTER I

INTRODUCTION

Qualitative Considerations of Water Deficits

Water deficits usually reduce plant growth and subsequently, crop yields. Shaw and Laing (1966) describe the effect of water deficits on yield as follows:

> The final yield of a crop is the integrated result of a number of interrelated effects on different physiological processes. Stress (water stress) can affect the processes of photosynthesis and respiration; it can affect growth, which provides synthetic tissue; and it can affect reproduction, which provides the sink for the photosynthates.

A reduction in yield will result when the crop water balance becomes less than optimum for plant growth. A water stress condition could be caused by either an inadequate amount of precipitation in a dryland crop production system or by poor water distribution in an irrigated crop production system.

Water deficits develop when the supply furnished by the soil water is exceeded by the atmospheric evaporative demand. Water moves through the plant in response to the potential gradient that develops between the leaves and the roots. Generally, during the night the plant water potential will come to equilibrium with the soil water potential. The evaporation rate will increase simultaneously with the diurnal increase in evaporative demand; this will cause a decrease in water potential of the upper leaves and will develop a water potential gradient through the plant from the evaporating surfaces of the leaves to the absorbing surfaces of the roots [Kramer (1969) and Slatyer (1967)]. The usual result of this process is that absorption will temporarily lag behind plant evaporation. The degree of "water stress" which develops depends upon this absorption-transpiration (plant evaporation) lag and the susceptibility of the crop to a certain water deficit level at the various stages of growth.

When a plant is in a "water stressed" condition, plant evaporation may decrease. As the water deficit reaches some critical point or value, the reduced turgor pressure in the guard cells of the leaf epidermis decreases the stomatal aperture. This results in a regulation of the gas and vapor exchange between the leaf and the atmosphere (Van Bavel, et al., 1973). The increase in the resistance to evaporation serves as a control mechanism to limit dehydration of the leaf. Also, when partial or complete stomatal closure occurs, carbon dioxide uptake by the plant is reduced, and subsequently, dry matter production will be decreased. Other important metabolic reactions in plants are also believed to be affected adversely by water deficits (Slatyer, 1969).

The water deficit which develops in the plant is related to the water deficit in the soil through the root distribution pattern, soil hydraulic properties, plant characteristics, and soil water potential distribution pattern [Cowan (1965), Newman (1969), and Feddes and Rijtema (1972)]. The influence of soil water deficits on crop growth and ultimately yield are indirect, especially compared to the effects of plant water deficits on plant growth (Jordan, 1970).

Water Use Efficiency

Water use efficiency (WUE) as generally defined by Viets (1962) is the mass ratio of yield obtained to the amount of water used. Many variations of this basic definition have appeared throughout the literature. The yield can be characterized by the end product of a plant (grain, seed, lint, etc.) or by the vegetative component (leaves, stover, etc.) or combinations of both representing the total biological yield. The amount of water use can be characterized by the water stored in the plant and the plant evaporation. On the field level, plant evaporation has been difficult to measure; therefore, water use has been associated with the total evaporation. This total evaporation is generally determined by balancing the inputs (precipitation, irrigation, upward flow from water tables) and soil water storage (initial and final soil water contents) to the losses (evaporation, drainage, and runoff). Failure to consider certain terms in the water balance can give misleading conclusions when the WUE is based only on part of the balance. In this report, WUE will be defined as

$$WUE = \frac{Y}{TWU} \qquad \ldots(1)$$

where Y = the grain yield, kg ha^{-1}; TWU = the seasonal water use, mm; and WUE = the water use efficiency, kg ha^{-1} mm^{-1}. Since one ha-mm of water equals 10^4 kg of water, the mass ratio of Y to TWU is 10^{-4} times WUE defined by equation (1). TWU, as used in this report, is defined as

$$TWU = ET = SW_1 - SW_2 + PRECIP + IRR \pm DRAIN - RUNOFF \qquad \ldots(2)$$

where ET = the total evaporation, mm; SW_1 = the initial soil water content, mm; SW_2 = the final soil water content, mm; PRECIP = the precipitation total, mm; IRR = the irrigation total, mm; DRAIN = the root zone drainage ("+" upward flow to root zone or "-" downward flow from root zone), mm; RUNOFF = the field runoff, mm; and TWU = the total water use (equated to the total evaporation), mm. Figure 1 shows a schematic diagram of this simplified hydrologic system.

Maximizing WUE as pointed out by Viets (1962) may not be desirable since crops grown on dryland (no irrigation) frequently use water more efficiency than "well-watered" crops, but at much lower levels of production. Maximum yields are seldom desirable

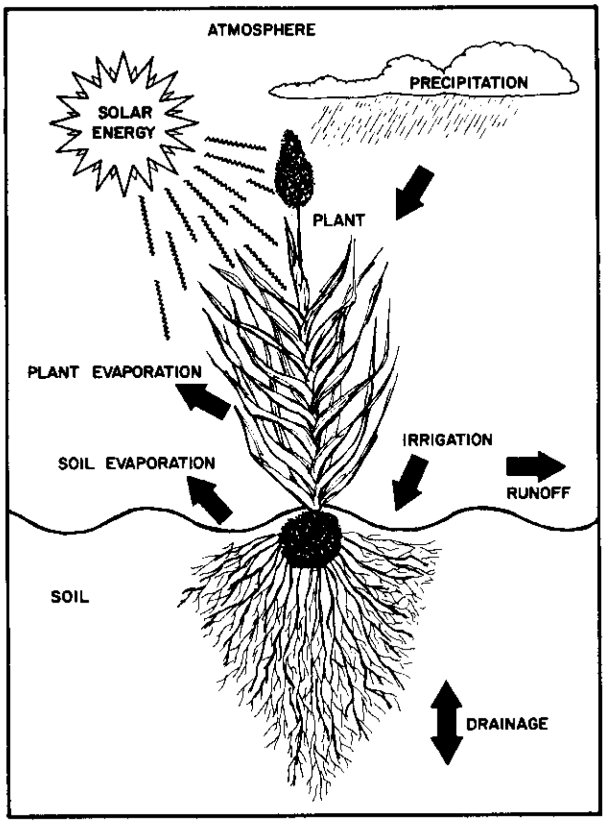

Figure 1. Simplified hydrologic cycle.

from an economic viewpoint since other resources--fertilizer, disease control, pest control, labor, etc.--are not utilized efficiently. However, an "optimum" WUE--maximum yield subject to local constraints of water availability--is a desirable tool for water planners in assessing future water requirements.

As discussed by Hillel (1972), WUE involves two interpretations: (1) the technical (or engineering) aspect of minimizing wastes of water, e.g., utilizing practices which improve water conveyance efficiency, water application efficiency, and water distribution efficiency and (2) the agronomic aspect of maximizing crop yield per unit of total water use. WUE can be increased by (1) increasing yield and maintaining equal water use or (2) maintaining equal yield and decreasing water use. Yield can be increased by better methods of pest and disease control, plant breeding, and improving supplies of sunlight and carbon dioxide to the leaves. Some of these practices will invariably also cause some change in the water use pattern which will also directly affect WUE. Hillel and Guron (1973) state that it appears more promising to attempt to increase WUE by increasing crop yields than by decreasing evapotranspiration, since plants growing in the field are subject to an externally imposed evaporative demand. This statement is valid for well-watered crop regimes but does not fully explore the possible implications of limited irrigation in regions of short and/or costly water supplies.

High Frequency Irrigation

High frequency irrigation is the daily, bi-daily, or multi-daily irrigation with small water application rates. These water application rates can approach the actual plant water use rate and thereby minimize the occurrence of plant water deficits and water losses to drainage and runoff. Irrigation techniques of trickle (drip), subsurface, and solid-set sprinkler are generally associated with high frequency irrigation. This modern irrigation technology has many advantages when compared to surface and conventional sprinkler irrigation. In addition to the above given advantages, high frequency irrigation can be easily automated (saves labor), can be used to distribute chemicals and fertilizers to plants, and makes possible the effective utilization of small irrigation water supplies not presently feasible for surface irrigation.

Application of Operations Research

Operations research (OR) is described by Hillier and Lieberman (1967) as "a scientific approach to decision making that involves the operations of organizational systems." Generally, OR is used to determine optimum allocation of scarce resources to maximize or minimize some desired objective. Four areas of OR--system simulation, inventory control, decision analysis, and dynamic programming--will be applied to optimization of high frequency irrigation applications to grain sorghum. The objective of this work will be to maximize grain yield subject to constraints of irrigation water quantity, local climatic patterns, and specific plant and soil characteristics. The application of OR to this system should evolve the following contributions (Hillier and Lieberman, 1967):

> (1) The structuring of the real life situation into a mathematical model, abstracting the essential elements so that a solution relevant to the decision-maker's objectives can be sought. This involves looking at the problem in the context of the entire system.
>
> (2) Exploring the structure of such solutions and developing systematic procedures for obtaining them.
>
> (3) Developing a solution that yields an optimal value of the system measure of desirability (or possibly comparing alternative courses of action by evaluating their measure of desirability).

Objectives

The general premise of this research is that high frequency irrigation can be utilized to apply water in limited quantities to grain sorghum thereby improving yield potentials and minimizing water losses to runoff and drainage. These results should improve the system WUE. Specific objectives are:

(1) to determine the evapotranspiration and yield of grain sorghum grown under a frequent but limited irrigation regime;

(2) to evaluate the effects of frequent, limited irrigations at specific growth stages on grain sorghum yield;

(3) to utilize an existing water balance model (Richardson and Ritchie, 1973) to separate the soil and plant evaporation components and evaluate their effect on water use efficiency;

(4) to simulate the soil water balance for grain sorghum at Temple, Texas, using stochastic input of rainfall, temperature, and potential evaporation; and

(5) to optimize the decision of irrigation quantity under high frequency irrigation using stochastic dynamic programming.

CHAPTER II

LITERATURE REVIEW

High Frequency Irrigation

High frequency irrigation is very intriguing to irrigation engineers, agronomists, and plant scientists. The idea of having a water distribution system to each plant opens up new opportunities to improve the production of many crops, while increasing the management responsibilities. The basic concepts of optimizing this new technology have yet to be formulated.

Theoretical Considerations

Rawlins (1973) presented a theoretical consideration of the laws of water movement in soils under high frequency irrigation. Since under steady-state frequent irrigation the infiltration process dominates the extraction process associated with furrow, flood or portable sprinkler irrigation, the hydraulic conductivity of the soil will adjust to precisely the application rate by adjusting the soil water content. He concluded that the water flux through the profile and not the soil water potential of the root zone should be utilized to prevent either over-irrigation or under-irrigation. Also, he found that the soil water content of the profile will remain high if the irrigation amount was equal to the evaporation flux and that the value of the water content for the top 0.6 of the effective root zone depended only slightly on the flux through the profile. However, no study was made of the effect of limited irrigation.

Experimental Studies

Hiler and Howell (1973) conducted high frequency irrigation experiments in 1971 and 1972 to evaluate the effects of limited high frequency irrigations on grain sorghum yield and water use efficiency (WUE). In 1971, different application methods were studied under an irrigation frequency of three irrigations per week and compared to a surface irrigation treatment which was irrigated when the soil matric potential reached -0.7 bar. The 1972 experiment consisted of trickle irrigation levels of 0.4, 0.7, and 1.1 times the water use rate of the "well-irrigated" control treatment. Since the 1971 subsurface results are suspect due to emitter plugging, only the trickle irrigation results will be discussed here.

The 1971 results indicated that WUE could be enhanced by the high frequency irrigations as compared to the variable frequency surface irrigated treatment. This result is probably due to the fact that at a soil matric potential of -0.7 bar in this particular soil and climatic combination, a certain degree of plant water stress was developed which limited yield. Unpublished results from a 1974 experiment* at this same site verify this conclusion. In the 1974 experiment, three trickle irrigation frequencies (1, 2, and 3 times per week) were compared in which the soil matric potential was maintained above approximately -0.4 bar. The 1974 preliminary results indicated no yield differences occurred as a result of the different frequencies.

The 1972 experiment results indicated that WUE could be increased as much as 50 percent by reducing the irrigation quantity but maintaining a high frequency of application. However, this practice did significantly reduce grain yields.

Bucks, et al. (1973) and Bucks, et al. (1974) studied the effects of varying irrigation frequency in conjunction with limited trickle irrigation applications on cotton and cabbage, respectively. With regard to cotton, trickle irrigation treatments consisted of 1.06, 0.90, and 0.72 times the present water use rate for furrow irrigation applied at three frequencies of 3, 6, and 12 days. Lint production for the full season was nearly the same when water was applied at the 1.06 and 0.90 treatments and decreased 18 percent for the 0.72 treatment. Frequency of trickle irrigation showed no significant effect on lint production between 3, 6, and 12 days for all irrigation quantities. The mean lint production for the 1.06 and 0.90 water use quantities was 1687 kg ha^{-1}, with late-season production

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*Howell, T. A. and E. A. Hiler. 1974. Unpublished data.

accounting for 25 percent to 30 percent of this yield. Results suggest that the amount of water needed by the cotton plant for high production with trickle irrigation is approximately equal to the present water use estimate for furrow irrigation, and that increased frequency of trickle irrigation may not necessarily increase yields on a fine-textured soil.

The cabbage production with trickle irrigation was the same when water was applied at 1.3 and 1.05 times the water use rate for furrow irrigation, but yields were decreased 10 and 43 percent, respectively, when 0.8 and 0.5 times the water use rate were applied. Frequency of trickle irrigation caused no difference in production between 3, 6, and 12 days at the 1.3 water use quantity, but yield reductions did result for the 3-day frequency at 1.05, 0.8, and 0.5 consumptive-use quantities, amounting to a 9 percent mean reduction for the 3-day compared to the 6-day frequency, and a 13 percent mean reduction for the 3-day compared to the 12-day frequency for the three smaller quantities.

Patterson and Hanson (1972) investigated the use of sprinkler, subsurface, and trickle irrigation on onions and sweet corn to improve WUE. Two soil matric potential values of -0.2 and -0.6 bar for each irrigation method were used to determine irrigation timing and depleted soil water was replaced. The WUE of each crop was improved, as compared to a conventional practice furrow treatment, by utilizing these more sophisticated application methods. However, trickle irrigation produced the greatest increase in WUE primarily due to its high application efficiency.

Trickle, subsurface, and sprinkler irrigation methods were compared on potato and corn crops by Brosz and Wiersma (1974). Increased yields of 5 to 15 percent were found with 20 percent less water used in the trickle and subsurface irrigation methods. They projected water savings of up to 30 to 40 percent with trickle and subsurface methods, while maintaining yields comparable to those obtained by sprinkler irrigation.

Bernstein and Francois (1973) conducted a detailed study in 1970 and 1971 comparing trickle, furrow, and sprinkler irrigation of bell peppers using two levels of water quality. The 1970 experiment provided equal water applications to each method with the trickle being irrigated daily and the furrow and sprinkler plots being irrigated when a soil matric potential of -0.5 bar was reached. For the best quality water, the trickle plots out-yielded the furrow and sprinkler plots by about 50 percent. In the 1971 experiment, each method was irrigated according to its need. The yields for the trickle plot were the largest, but only slightly larger than either the furrow or sprinkler plots for the best water quality. However, the sprinkler plots received 25 percent more water than the trickle plot while the furrow plot received 48 percent more water than the trickle plot. Bernstein and Francois (1973) concluded that "for mature crops water requirements by the three methods of irrigation (trickle, furrow, and sprinkler) are similar and water savings by drip (trickle) irrigation would depend largely on the inefficiency of the method it replaces."

Phene (1974) in a series of experiments examined the effects of high frequency irrigation by trickle (shallow buried porous tube), furrow, and sprinkler methods at -0.1, -0.2, and -0.4 bar soil matric potentials on sweet corn. The trickle treatment resulted in a water savings as large as 50 percent at the -0.1 bar control level while maintaining a small yield increase over sprinkler and furrow methods. This work demonstrated that high frequency irrigation can, rather remarkably, increase water use and nitrogen use efficiencies.

Water Balance Models

Many soil water balance models have been developed in the past. Usually each model was developed for a specific purpose. Various ones were used to compute stream flow (Hann, 1972), runoff [Richardson, et al. (1969) and Knisel, et al. (1969)], drainage (Ligon, et al., 1965), irrigation scheduling (Jensen, et al., 1970), and watershed evaporation (Richardson and Ritchie, 1973). These are only a sample of the many models which have been used. Essentially, each model attempts to calculate the variables in equation (2) from the smallest number of input data. This has resulted in many assumptions which may or may not limit the applicability of a particular model depending on its intended use. Only two of the more recent of these models [Jensen, et al., (1970) and Richardson and Ritchie (1973)] will be discussed.

Jensen Water Balance Model

The Jensen model [Jensen (1972), Heermann and Jensen (1970), Jensen (1969), Jensen and Heermann (1970), Jensen, et al., (1971), and Jensen, et al. (1970)] was developed to aid in scheduling irrigations (timing and amount). This model has been widely used in Idaho and Arizona. The evaporation from the crop is estimated by the product of a crop coefficient and the estimated potential evaporation (Penman, 1963). The crop coefficient is influenced by the crop species, surface wetness (recent rainfall or irrigation), and physiological growth pattern (time of year). Improvements are being added to the model as the technology is advanced. Recently Jensen (1972) discussed many of the refinements which are being placed in the model. These modifications include: (1) improving potential evaporation estimates; (2) projecting future rainfall and evaporation until the next irrigation; (3) calculating drainage between irrigations, and (4) calculating upward flow from water tables.

Blackland Water Balance Model

The Blackland model [Richardson and Ritchie (1973) and Ritchie (1972)] was developed to aid in the prediction of runoff from small watersheds. Most of the mathematical relationships used to compute evaporation were reported by Ritchie (1972) and Ritchie and Jordan (1972). The important contributions of this model are that soil and plant evaporation are computed separately, the evaporation is computed for both freely evaporating and limited soil water conditions, and the constants in the model have some physical significance. Soil evaporation is calculated in two stages: (1) the constant rate stage when only the supply of energy to the surface limits evaporation, and (2) the falling rate stage when water movement to evaporating surface is controlled by soil hydraulic properties. Plant evaporation is calculated by using an empirically determined relation between leaf area index and plant evaporation rate relative to potential evaporation rate. When soil water deficits cause plant evaporation to decrease, the magnitude of the soil water deficit determines the plant evaporation rate. Total evaporation rate is computed by adding soil evaporation and plant evaporation. In the present state of the model, drainage is assumed to be any excess infiltrated water above the maximum water holding capacity of the soil.

Yield Functions

The yield prediction of a crop appears to be far more art than science. As pointed out previously, the yield of a crop results from the integrated physiological processes of a plant over time and space. Many experiments have been conducted to determine the effects of many types of variables on the yield response of various crops. As a typical example, examine the hypothetical yield response to irrigation water shown in Figure 2. This particular example shows a diminishing-return type function. This function would be, of course, site-specific for given climatic, soil, and plant characteristics. Notice at zero water that there is some possible expected dryland yield.

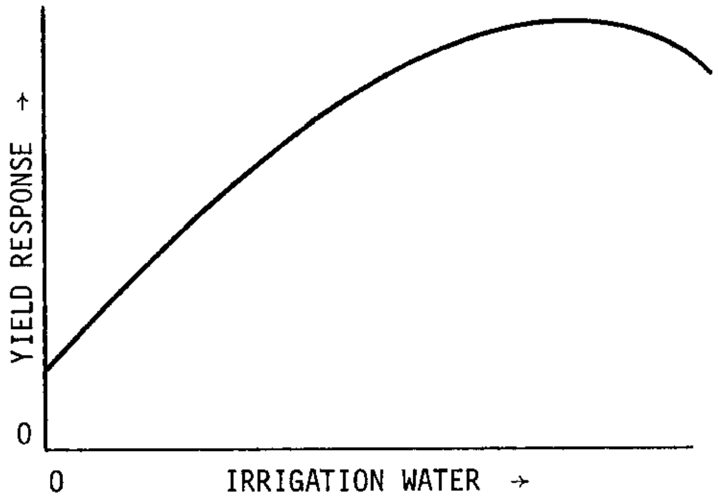


Figure 2. Hypothetical yield response to irrigation water.

Inherent in this function are the following:

1. temporal distribution of the irrigations;
2. irrigation efficiency; and
3. no year-to-year climatic variation.

In addition to the above limitations, the experimental layout for a large number of independent variables and their interactions will quickly become too large to manage.

If any empirical yield function is representative of the real system, characterization of the effects of water deficits at specific crop growth stages must be considered [Stewart, et al. (1974), Howell and Hiler (1974), Hiler and Clark (1971), Jensen (1968), Yaron (1971), Nix and Fitzpatrick (1969), Minhas, et al. (1974), Hall and Butcher (1968)]. Four of these yield models [Hiler and Clark (1971), Jensen (1968), Hall and Butcher (1968) and Minhas, et al. (1974)] will be described here.

Stress Day Index

Hiler and Clark (1971) presented an additive model of yield response. The stress day index is determined from a stress day factor and a crop susceptibility factor. The stress day factor (SD) is a measure of the degree and duration of plant water deficit. The crop susceptibility factor (CS) depends on the species and stage of development of the given crop and indicates the plant susceptibility to a given water deficit. The stress day index can be written as follows:

$$\mathrm{SDI} = \sum_{i=1}^{n} (\mathrm{SD}_i \times \mathrm{CS}_i) \qquad \ldots(3)$$

where n represents the number of growth periods considered. A discussion of possible alternative approaches for quantitative characterization of SD and CS is given in their paper.

One proposed characterization of SD was given as follows:

$$SD_i = (1.0 - \frac{E}{E_o})_i \qquad \ldots(4)$$

where E/E_o = the relative evaporation in growth stage i, and E_o = the potential evaporation, mm. If no deficit occurred, SD would equal zero. SD would take on a maximum value of 1.0.

One approach to the characterization of CS is to subject the crop to a specified critical SD value at different growth stages. The critical SD value would vary for different species and would be determined in a preliminary experiment. The primary experiment for determinations of CS would be as follows:

Treatment	Yield
I - Stress at Growth Stage I, no stress during rest of season	i
II - Stress at Growth Stage II, no stress during rest of season	ii
. . .	
M - Stress at Growth Stage M, no stress during rest of season	m
C - No stress during season	x

Then, the crop susceptibilities during each of the M growth stages would be

$$CS_I = \frac{x - i}{x},$$

$$CS_{II} = \frac{x - ii}{x} \qquad \ldots(5)$$

$$\vdots$$

$$CS_M = \frac{x - m}{x}$$

This is an experimental approach to determine CS and is best adapted to field experiments where the soil water variable can be at least partially controlled.

A linear relationship between grain sorghum yield and SDI was found as follows:

$$Y = Y_o - A\ (SDI) \qquad \ldots(6)$$

where Y_o = the potential yield, kg ha^{-1}, and A = the slope of the line, kg ha^{-1}. The relative yield is given by

$$\frac{Y}{Y_o} = 1.0 - \frac{A\ (SDI)}{Y_o} \qquad \ldots(7)$$

Figure 3 shows the relative yield for a single growth stage as a function of E/E_o and ACS/Y_o, assuming that CS is independent of SD. CS is influenced by SD as shown for southern peas by Hiler, et al. (1972), but no general functional relationship between CS and SD has been developed. If CS is assumed to be linearly related to SD ($CS = \propto SD$), which resembles the relationship found by Hiler, et al. (1972), Figure 4 shows how E/E_o and $A\propto/Y_o$ are related to Y/Y_o. To properly characterize CS, a detailed experiment (involving multiple levels of SD) similar to that reported by Hiler, et al. (1972) is required.

Jensen Yield Model

Jensen (1968) developed a multiplicative model to describe the effects of water deficits on the crop yield of determinate crops providing water is the only limiting factor. The model is expressed as

$$\frac{Y}{Y_o} = \prod_{i=1}^{n} \left(\frac{E}{E_o}\right)_i^{\lambda i} \qquad \ldots(8)$$

where Y/Y_o = the relative yield; Y_o = the potential yield with water not limiting, kg ha^{-1}; E = the actual evaporation, mm; E_o = the potential evaporation, mm; λ = the relative sensitivity of the crop to water stress during the growth stage i; and n = the number of growth stages. The right side of equation (8) is a

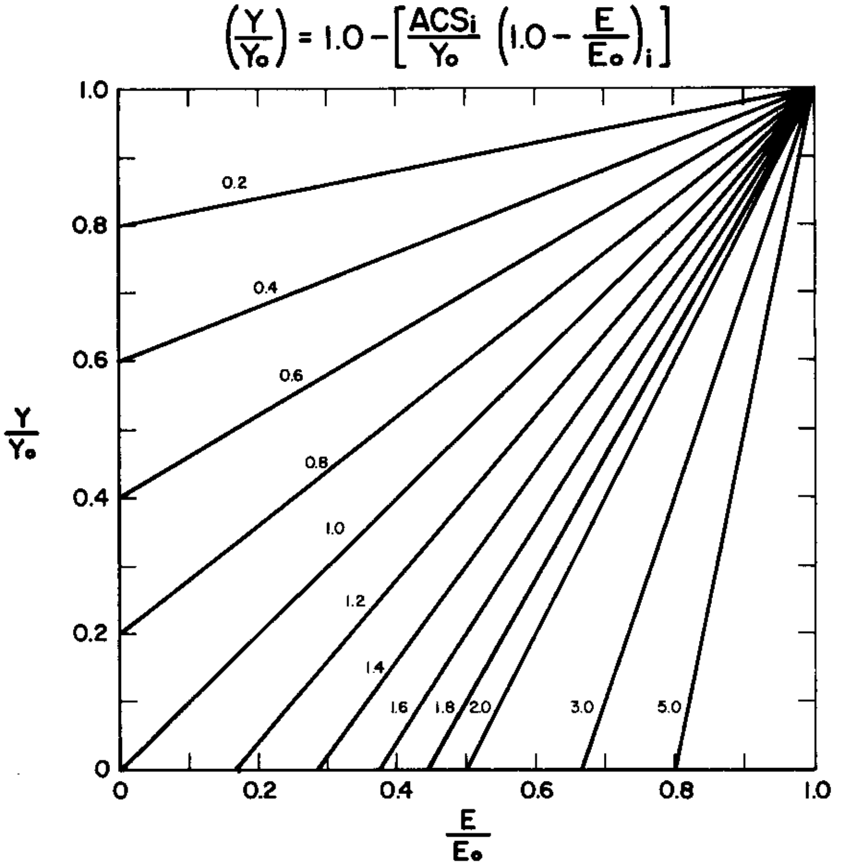


Figure 3. Stress Day Index yield reduction in a single growth period for various values of ACS/Y_0, assuming CS is independent of SD$(1.0 - E/E_0)$.

product. Figure 5 shows the influence of E/E_0 on Y/Y_0 for various values of λ for a single growth period. If λ is less than 1.0, then WUE is increased by decreasing E/E_0, but if λ is greater than 1.0, WUE can only be increased by increasing E/E_0. For grain sorghum, Jensen used $\lambda = 0.5$ for the emergence to boot and milk to harvest stages and $\lambda = 1.5$ for the boot to milk stage.

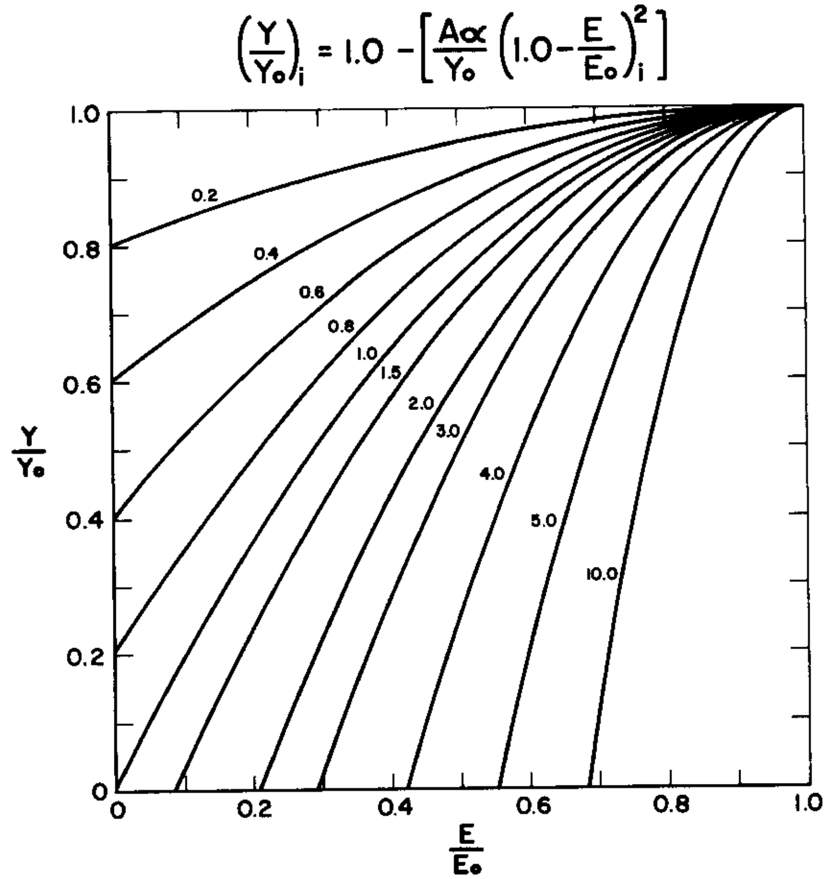


Figure 4. Stress Day Index yield reduction in a single growth period for various values of $A\alpha/Y_0$, assuming CS is linearly related to $SD(1.0 - E/E_0)$.

Hall and Butcher Yield Model

Hall and Butcher (1968) proposed a multiplicative model to be used to determine optimum irrigation timing. The model was presented as

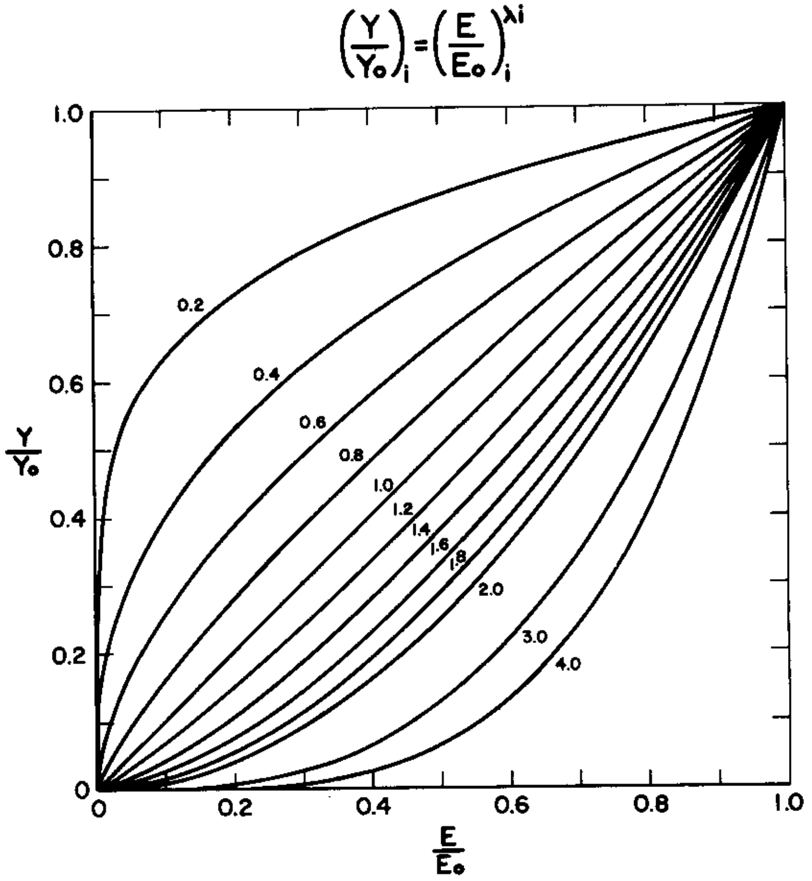


Figure 5. Jensen yield model of yield reduction in a single growth period for various values of λ.

$$\frac{Y}{Y_0} = \prod_{i=1}^{n} [AW_i(SW_i)] \qquad \ldots(9)$$

where AW = the fractional yield reduction caused by a soil water content of SW in growth stage i of n total stages and Y/Y_0 = the relative yield as defined previously. They did not present any data to substantiate the model. Stewart, et al. (1974) demonstrated a possible use of the model to describe the effect of an evaporation deficit during corn pollination on corn yield. Figure 6 shows this relationship.

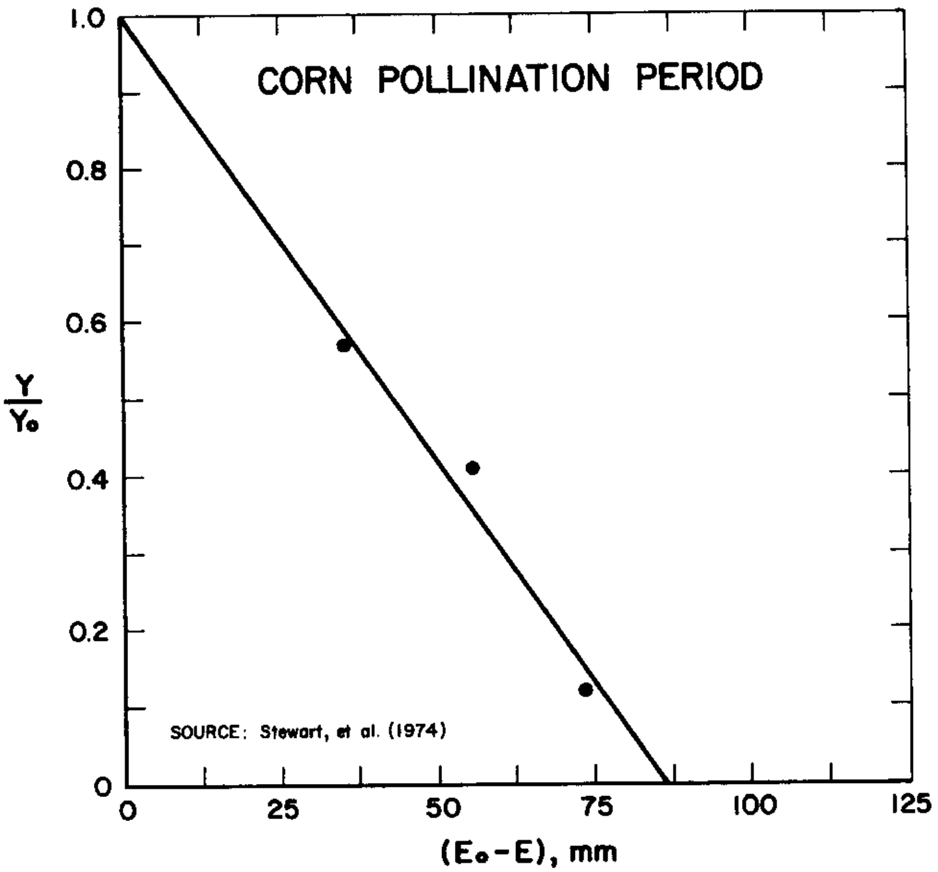


Figure 6. Effect of water deficit during pollination stage on corn yield after Stewart, et al. (1974).

Minhas, Parikh, and Srinivasan Yield Model

Minhas, et al. (1974) developed another form of a multiplicative-type production function. Their equation was

$$\frac{Y}{Y_0} = \prod_{i=1}^{n} [1.0 - (1.0 - \frac{E}{E_0})_i^2]^{b_i} \qquad \ldots(10)$$

where E/E_0, Y/Y_0, i, and n were defined previously, and b = a sensitivity factor to water deficits in the ith period. This model, as well as each of the previous multiplicative types, has the property that if $E/E_0 = 0$, then $Y = 0$. Figure 7 shows this model as influenced by b and E/E_0 for a single growth period. For values of b less than 1.5 the solution resembles Jensen's model (Figure 5, page 25).

System Simulation

System simulation has long been an important element in design. With the advent of high speed computers, simulation of complex mathematical systems has become commonplace. Simulation has been used to investigate the outputs of irrigation systems under stochastic inputs [Dudley, et al. (1971) and Ahmed (1974)]. Dudley, et al. (1971) utilized past weather records to simulate stochastic inputs. Ahmed (1974) utilized a CSMP simulation program to compute

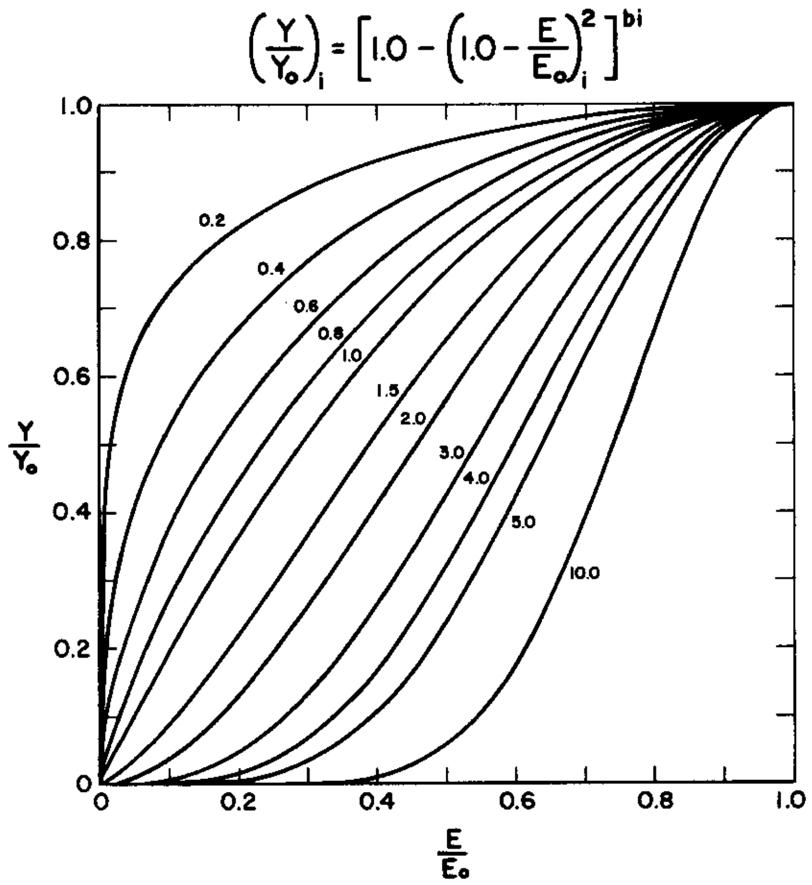


Figure 7. Minhas, et al. (1974) yield model of yield reduction in a single growth period for various values of b.

the plant water use rate under a single stochastic regime using Monte Carlo techniques (Link and Splinter, 1970).

The Monte Carlo simulation is widely used to simulate hydrologic events from known probability distributions [Wiser (1966) and Hann and Barfield (1973)]. A random event is determined by selecting a random outcome from a known distribution of the outcomes. A distribution of the random system outputs can then be determined.

An excellent example of the use of Monte Carlo methods was given by Jones, et al. (1972). A first-order Markov chain was used to calculate the probability of rainfall occurrence. A random number was drawn from a uniform probability distribution. If the random number was larger than the probability of a dry day (defined as rainfall less than 0.25 mm) depending on the rainfall state of the previous day (dry given previous day was dry or dry given previous day was wet), rainfall was assumed to occur. The rainfall quantity was then simulated from an incomplete gamma distribution. Daily temperature and pan evaporation were simulated in a similar manner using normal distributions.

The simulation of an irrigation system closely resembles a multi-period stochastic inventory problem (Hadley and Whitin, 1963). The farmer must determine "when" and "how much" to irrigate. His decisions are constrained by his available water (quantity and quality), production costs and returns, and a stochastic demand rate over which he has little control. Many times due to institutional factors, he must order his water well in advance of an anticipated application. System simulation can be quite useful in determining the optimum operating policies of such a system.

Decision Analysis and Optimization

Decision analysis (DA) is a relatively new field of operations research (OR). Essentially, DA is a technique for evaluating decision alternatives under uncertainty. Generally, DA techniques are divided into two classes: Bayesian, which requires a knowledge of the probability distributions affecting the outcome, and non-Bayesian, which does not require any knowledge of the probability distributions. Raiffa (1970) describes these two classes as follows:

> Bayesians, or subjectivists, wish to introduce intuitive judgments and feelings directly into the formal analysis of a decision problem. The non-Bayesians, or objectivists, feel that these subjective aspects are best left out of the formal analysis and should be used only, if at all, to bridge the gap between the real world and the objective results one obtains using a formal model.

The non-Bayesian analysis is given by Halter and Dean (1971) while Raiffa (1970) gives the Bayesian view. Humber (1974) utilizes both of these decision classes to make management decisions on the operation of a herd of cattle on range.

Dynamic programming (DP) has been utilized in the optimization of the allocation of irrigation water to crops [Hall and Buras (1961), Flinn and Musgrave (1967), Salcedo and Meier (1971), Hall and Butcher (1968), Windsor and Chow (1971), Dudley, et al. (1971), DeLucia (1969), Burt and Stauber (1971), Asopa, et al. (1973), and Texas Water Development Board (1972)]. DP has been utilized extensively to allocate scarce resources in multi-stage decision processes. DP techniques are based on the "Principle of Optimality" (Bellman, 1957): "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." Hillier and Lieberman (1967) provide the following characteristics of a DP problem:

> (1) The problem can be divided up into stages, with a policy decision required at each stage.
>
> (2) Each stage has a number of states associated with it.
>
> (3) The effect of the policy decision at each stage is to transform the current state into a state associated with the next stage (possibly according to a probability distribution).
>
> (4) Given the current state, an optimal policy for the remaining stages is independent of the policy adopted in previous stages.
>
> (5) The solution procedure begins by finding the optimal policy for each state of the last stage.
>
> (6) A recursive relationship is available which identifies the optimal policy for each state with n stages remaining, given the optimal policy for each state with (n - 1) stages remaining.
>
> (7) Using this recursive relationship, the solution procedure moves backward stage by stage--each time finding the optimal policy for each state of that stage--until it finds the optimal policy when starting at the initial stage.

The basic elements of a DP problem are the stages, the states, the decision variables, the transformation functions, a stage reward function and a total reward function. The stages are decision points which may be continuous or discrete and finite, denumerable, or infinitely uncountable. The states are a set of parameters, or a single parameter, which contain all the information necessary to make the current decision and all future decisions. The decision variables are a set of choices, or a single choice, at a particular stage among all the alternatives. The transformation functions (recursive relationships) describe changes in the state variables at a particular stage resulting from the decisions. These transformations can be either deterministic or stochastic. The stage reward function determines the incremental reward (or cost) associated with a particular decision when the system is in a given state of a particular stage. The total reward function gives the final reward resulting from the individual stages. This total reward function is generally referred to as the "objective function" in the DP analysis. DP can be used to solve any linear programming problem (although DP may be computationally inefficient) or any Bayesian decision analysis problem, and DP can handle stochastic transformations easily.

The objective function (or total reward function) can be any linear or non-linear function which meets the criteria of separability and monotonicity. Separability implies that the total reward function can be separated by stages as

$$Y\ [y_1, y_2, y_3, \ldots y_n] = Y_1\ [y_1, Y_2\ (y_2, y_3, \ldots y_n)] \qquad \ldots(11)$$

where Y = total reward function and y = the stage reward function. Monotonicity implies that Y_1 regarded as a function of Y_2 must be monotone non-decreasing for every value of y_1, such as if $Y_2^1\ (\ldots) \geq Y_2^2\ (\ldots)$, then

$$Y_1\ [y_1, Y_2^1\ (y_2, y_3, \ldots y_n)] \text{ must be } \geq Y_1\ [y_1, Y_2^2\ (y_2, y_3, \ldots y_n)] \qquad \ldots(12)$$

These two conditions are met for any additive objective function. However, for a multiplicative objective function to meet the monotonicity criteria, the stage return domain must be limited to the non-negative orthant on the N--dimensional Euclidean space.

Chapter Summary

The following conclusions were obtained from the review of literature relating to high frequency irrigation, soil water balance models, yield functions, and the application of OR techniques to irrigation systems:

1. High frequency irrigation implies a system capable of supplying water to a plant at any time. This usually is conceived to be a solid-set system or a movable-type system. The basic costs

of operating such a system are related to the purchase price, maintenance costs, and the cost per unit of applied water (water costs, labor costs, and operation costs). If we presume that maintenance costs are a function of system age and not use [Lacewell, et al. (1972)] and that automation will reduce labor requirements so that labor costs can be neglected (an inventory system with no set-up costs), the cost to apply one mm ten times would equal the cost to apply ten mm one time. The small, frequent irrigations would maintain a more uniform soil water content, prevent water and fertilizer losses to drainage (Phene, 1974), and increase the capture of rainfall by minimizing the chances for runoff. Therefore, it seems logical that if a high frequency irrigation method is to be of any benefit, it must be utilized to its utmost capacity, i.e., irrigate daily or several times daily.

2. Based on the experimental evidence, high frequency irrigation will increase yields only if the system to which it is compared is inefficient in maintaining a desired water balance. However, experimental evidence (Freeburg, et al., 1974) suggests that the thermal environment is improved by high frequency irrigation and in fact may be largely responsible for increasing WUE.

3. The water savings under high frequency irrigation are directly related to the inefficiency of the system it replaces.

4. To accurately model the field water status of a crop under a high frequency irrigation management system, a two-dimensional water balance model would probably be required.

5. Each of the existing soil water balance models are deficient with respect to the real system. Realizing these limitations, the Blackland water balance model has several advantages (as listed previously) over the other models.

6. Each yield function described in this chapter was empirical. Therefore, the accuracy and reliability of each depends on the range and accuracy of the data from which it was developed. In actual fact, there is only a slight difference between each. The basic difference is that the Stress Day Index is an additive model while the others are multiplicative.

7. Until the physiological processes in the plant which influence yield are more fully understood, accurate yield predictions for all circumstances cannot be made but only approximated.

8. Monte Carlo and system simulation techniques can be used to study the stochastic nature of the plant-soil-atmosphere system.

9. The irrigation system can be simulated and is analogous to a stochastic inventory system.

10. Dynamic programming can be used to simplify the decision analysis and determine the optimum irrigation policies under various constraints.

11. Decision analysis offers more methods to evaluate the system operation under different viewpoints (pessimistic or optimistic).

CHAPTER III

EXPERIMENTAL EQUIPMENT AND PROCEDURES

This chapter will describe the experimental equipment and procedures utilized to evaluate the first three objectives of this study. The experiment was conducted in field lysimeters located at the Agricultural Engineering Research Laboratory at College Station, Texas. The equipment used in this study will be described first, followed by the procedures used in obtaining the data.

Experimental Equipment

The equipment used in this study included field lysimeters, meteorological instrumentation, soil moisture instrumentation, plant-water instrumentation, and a trickle irrigation system.

Field Lysimeters

Twenty-four lysimeters with undisturbed cores of Travis fine sandy loam soil, which has a layer of fine sandy loam soil in the A horizon to a depth of 45 cm with an available water holding capacity of 0.12 cm per cm and a red sandy clay soil in the B horizon with an available water holding capacity of 0.22 cm per cm, were used in this research. This soil type is similar to Amarillo fine sandy loam found abundantly in the Texas High Plains. The lysimeters were 90 cm in diameter and 180 cm in depth. The undisturbed soil cores were obtained by applying a large static load (approximately 34,000 kg) to each empty lysimeter to force it into the ground. The lysimeters were removed from the ground by large hydraulic fork lifts. A ceramic filter system was installed in the bottom of the lysimeters with 5 cm of diatomaceous earth and 2.5 cm of fine sand placed between the subsoil and the ceramic filter. This prevented clogging of the filters and provided for a uniform removal of water. The bottom of the lysimeters was sealed. A negative potential could be applied to the soil in the lysimeters through the ceramic filter system by pumping the lysimeters with a vacuum pump. A movable shelter, automatically actuated by 0.25 mm of rainfall, protected the lysimeters from rain. An overall view of the lysimeter installation is shown in Figure 8 (page 39). Further details of the installation are given by Hiler (1969).

Meteorological Instrumentation

Wind speed, dry-bulb temperature, dew-point temperature, net radiation, solar radiation, and reflected solar radiation were measured above the crop canopy. Wind speed was measured by miniature 3-cup anemometers at heights of 3 m, 2 m, and 1 m above the ground. Dry-bulb temperatures were measured at 1 m and 2 m above the ground with aspirated copper-constantan thermocouples. Dew-point temperature at 1 m and 2 m above the ground was determined from the cavity temperature (measured by a copper-constantan thermocouple) of an electrically heated, self-regulating lithium chloride dew-point hygrometer. The dew-point sensors were periodically exchanged to prevent sensor bias. Net radiation was measured by a miniature net radiometer similar to that described by Fritschen (1965) at a height of 1.5 m above the ground. Solar and reflected solar radiation were measured with Epply pyranometers. Solar and net radiation were recorded continuously by strip chart recorders. All meteorological variables were recorded every six minutes on a data acquisition system. Meteorological instrumentation is shown in Figure 9.

Soil Water Instrumentation

Soil water content was determined by the neutron method [Van Bavel, et al. (1963)]. Each lysimeter contained an access tube extending to a depth of 1.5 m. The neutron equipment consisted of a probe and a scaler. The probe consisted of an Americium: Beryllium source with a strength of 100 millicuries, a BF_3 (boron trifluoride) counter tube, and a preamplifier which increases the voltage entering the scaler. The scaler counted the total number of pulses admitted in a predetermined increment of time. One minute count times were used. Soil water content was monitored by the neutron method in 15-cm increments in each lysimeter thrice weekly throughout the growing season.

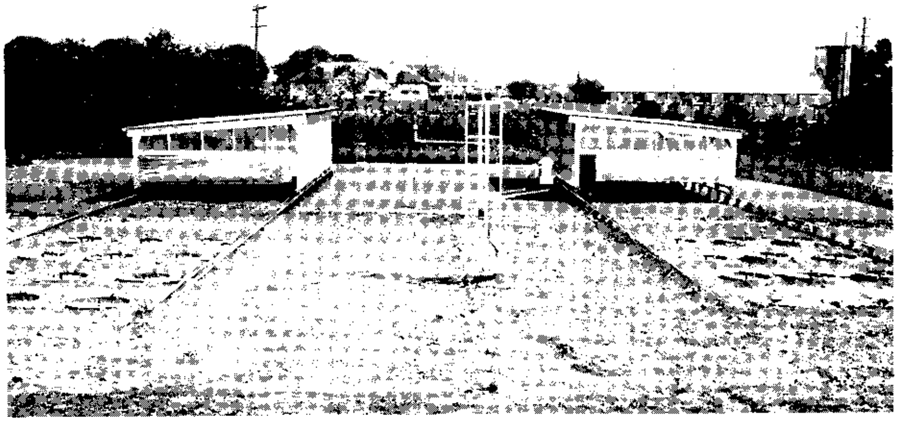


Figure 8. Overall view of lysimeter installation.

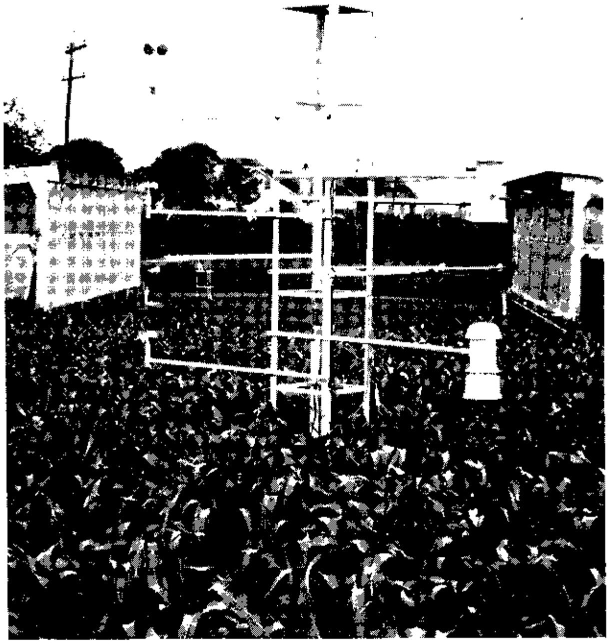


Figure 9. Meteorological instrumentation.

Surface soil water samples were taken from 0 to 10 cm deep to determine the gravimetric soil water content to supplement the neutron measurements. The samples were oven dried at 105 C for at least 30 hours.

Tensiometers were used to measure the soil matric potential. Psychrometers were used to measure the total soil water potential.

Plant Water Instrumentation

Plant water instrumentation included equipment to measure leaf temperature, leaf diffusion resistance, and leaf water potential.

Leaf temperature. Leaf temperature was measured daily during the week with an infrared radiometer. The radiometer had a field definer and measured an area approximately one cm in diameter with a 35-degree field of view. It provided a fast, accurate, non-disturbing measurement of leaf temperature. Gates (1962) discusses the design and use of this type of instrument.

Diffusion resistance. A diffusion resistance meter similar to that described by Van Bavel, et al. (1965) was used to determine the individual leaf diffusion resistances. The instrument consisted of a sensing cup, which contained a humidity sensor, and a micro-ampere meter. This instrument provided a direct and rapid determination of the diffusion resistances of leaves in the field.

Pressure bomb apparatus. Leaf water potential was measured with a pressure bomb similar to that described by Scholander, et al. (1965). The pressure bomb determined the negative hydrostatic sap pressure in the xylem. Since the osmotic potential in the xylem is usually small, it can be neglected in most instances (De Roo, 1969). Therefore, the leaf water potential can be described by the negative hydrostatic pressure in the xylem. The pressure bomb provided a rather simple field determination of the leaf water potential. A rubber gasket sealed the leaf petiole in the pressure chamber and nitrogen under pressure entered the pressure bomb. The chamber pressure was shown on a 0-300 pounds per square inch pressure gauge which was graduated in 2 pounds per square inch increments.

Trickle Irrigation System

The trickle irrigation systems used consisted of 1.58-cm (1/2 inch nominal) inside diameter black polyethylene pipe with 2 TrikLon emitters per lysimeter. This emitter is a coiled microtube approximately 2.44 m in length with a 0.89-mm inside diameter (0.035 inch) which discharged approximately 12.9 cm^3 per min (0.20 gph) at 0.69 bar (10 psi). Figure 10 (page 43) shows a photograph of this emitter. The irrigation water had an electrolyte concentration of 450 ppm, a sodium-adsorption-ratio of 40, and an electrical conductivity of 700 mmhos cm^{-1}; the water was filtered by a cartridge filter and the pressure was reduced by regulating valves. A timer was set to operate a solenoid valve on the irrigation system for each treatment to apply the calculated irrigation amount. The system application rate per lysimeter was 0.24 cm per hr. Figure 11 shows a photograph of the trickle irrigation lateral installed in a lysimeter.

Experimental Procedures

The experimental procedures used to obtain the data included the field layout, the preliminary procedures, the treatments, the plant measurements, the soil water balance measurements, and the meteorological measurements.

Field Layout

The field layout was arranged with two sheltered lysimeter installations, each containing twelve lysimeters. Figure 12 shows the arrangement of the lysimeters in a single shelter. In each shelter there were 4 lysimeter rows containing 3 lysimeters each. This arrangement provided 8 treatments with 3 replications of each. Between each lysimeter row, 2 buffer rows were used. The entire plot (approximately 40 m by 18 m) was cultivated and planted to simulate a field environment. The meteorological measurements were made on a 3-m tall tower located in the center of the plot between the two sets of lysimeters.

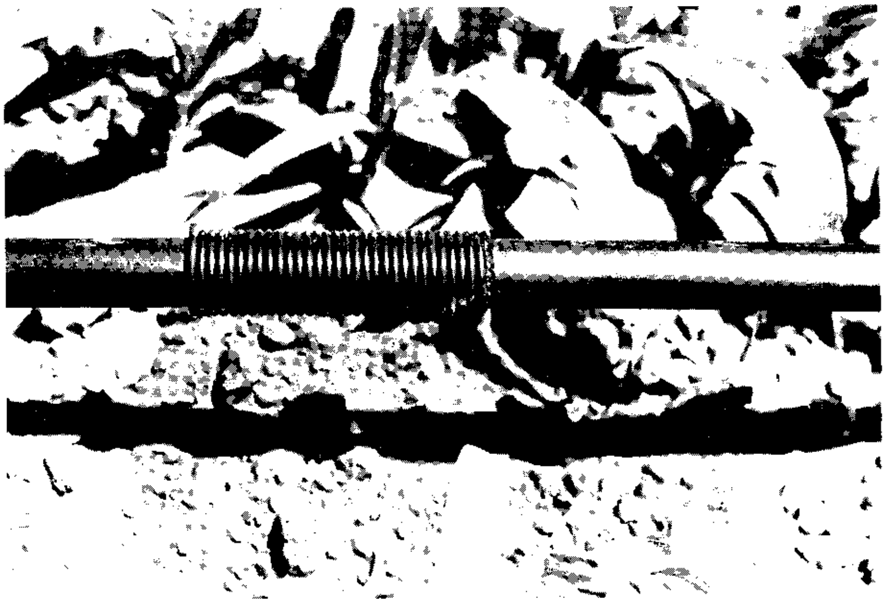
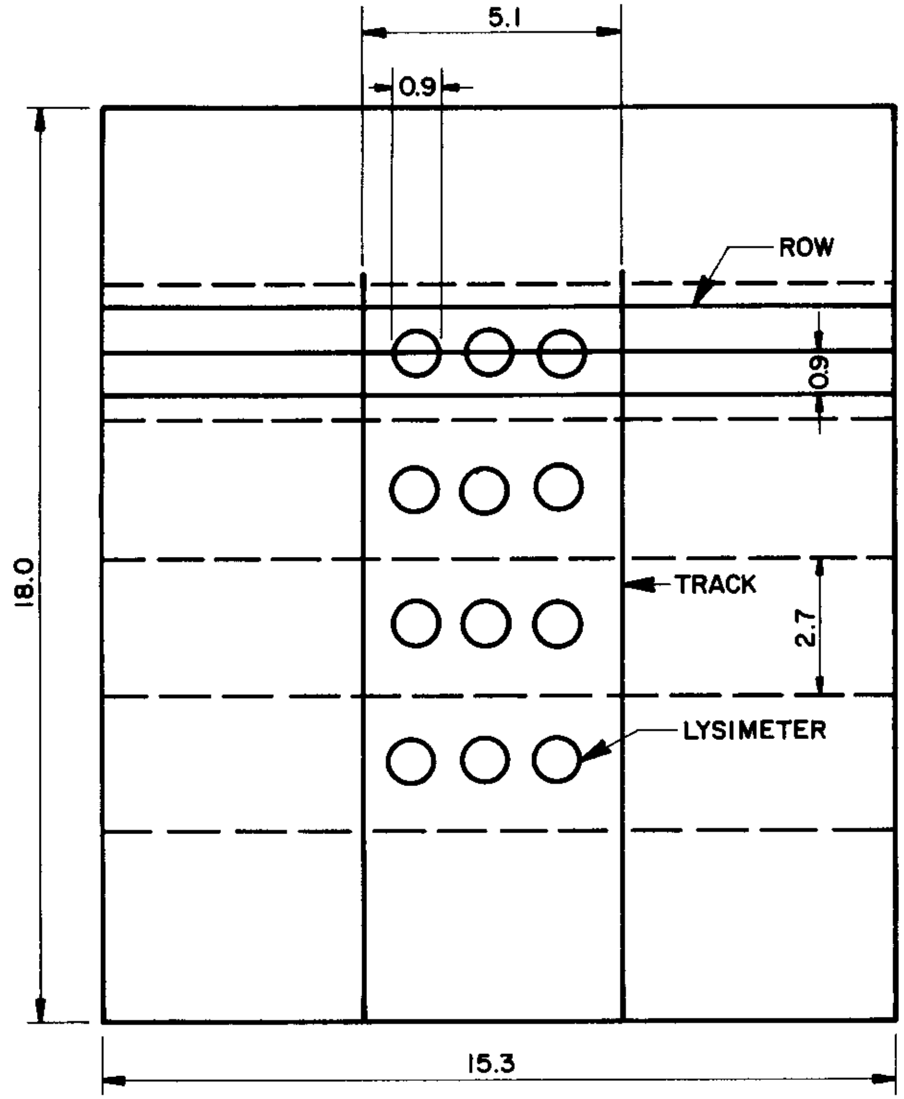


Figure 10. TrikLon trickle irrigation emitter.

Figure 11. View of a trickle irrigation lateral installed in a lysimeter.



NOTE: All dimensions are in meters.

Figure 12. Experimental arrangement of lysimeters.

Preliminary Procedures

Grain sorghum (Sorghum bicolor (L.) Moench. cv. 'Oro') was precision planted on April 13, 1973 and was harvested on August 3, 1973. Two drills 25 cm apart were planted across the center of each lysimeter. Plant populations were thinned uniformly to 18 plants per lysimeter (222,000 plants ha^{-1}). Each lysimeter received applications of N, P, and K at rates of 67 kg ha^{-1} prior to planting; 67 kg ha^{-1} of N was also added at the 4- to 6-leaf stage of plant development. All lysimeters were irrigated prior to planting in the amount necessary to replenish the top meter of soil to "field capacity".

Treatments

Eight treatments were designed to produce 2 values of water deficit in each of 3 growth stages. The treatments used were as follows:

1. Irrigated in the amount of 1.1 times the measured evaporation losses as determined by the water balance of the lysimeters and soil water content measured by the neutron method (Control);

2. Irrigated in the same amount as treatment 1 during 2 of the 3 growth stages of the crop, and irrigated in the amount of 0.4 times the measured evaporation losses in treatment 1 during the third growth stage (III-0.4);

3. Same as treatment 2 except that the irrigation amount was 0.4 during the second growth stage instead of during the third growth stage (II-0.4);

4. Same as treatment 2 except that the irrigation amount was 0.4 during the first growth stage instead of during the third growth stage (I-0.4);

5. Same as treatment 2 except that the irrigation amount was 0.1 during the third growth stage instead of 0.4 (III-0.1);

6. Same as treatment 5 except that the irrigation amount was 0.1 during the second growth stage instead of during the third growth stage (II-0.1);

7. Same as treatment 5 except that the irrigation amount was 0.1 during the first growth stage instead of during the third growth stage (I-0.1); and

8. Irrigated in the amount of 0.32, 0.64, and 0.42 times the measured evaporation losses in treatment 1 during growth stages one, two, and three, respectively, (SDI). These values of irrigation amounts were calculated to approximate a total seasonal stress of 0.5 times the potential water use rate (Hiler, et al., 1974).

The growth stages utilized were similar to those of Lewis, et al. (1974), and were as follows:

Growth Stage No.	Description
1	Late vegetative to early reproductive stage
2	Boot to bloom stage
3	Milk to soft dough stage

Growth stage one included stages 2, 3, 4, and part of stage 5 as defined by Vanderlip and Reeves (1972). This stage started on May 23, 30 days after emergence, and ended on June 8 (17 days total length). Growth stage 2 included stages 4, 5, and 6 as given by Vanderlip and Reeves (1972). This stage started on June 4, 42 days after emergence, and ended on June 29 (26 days total length). Growth stage 3 included stages 6 and 7 as defined by Vanderlip and Reeves (1972). The third stage began on June 22, 60 days after emergence and ended on July 16 (25 days total length). The growth stages overlapped each other so that differences in individual plants would be minimized.

Each treatment was irrigated 3 times a week (Monday, Wednesday, and Friday). The irrigation amount was determined as

$$IRR_{ij} = F_j \; (SW_f - SW_{i1}) \qquad \text{...(13)}$$

where IRR_{ij} = the irrigation amount for the jth treatment on day i, mm; F_j = the multiplication factor of treatment j; SW_f = the "field capacity" soil water content, taken as 215 mm; and SW_{i1} = the measured profile soil water content in treatment 1 (Control) on day i, mm.

Plant Measurements

The plant measurements included measurements of plant growth (plant height, leaf area index, and final yield) and plant-water status (leaf temperature, leaf surface resistance, and leaf water potential). The crop height and leaf area index were measured on 4 "representative" plants in each lysimeter.

Plant height. Plant height was measured twice weekly in each lysimeter. Plant height was defined as the distance from the ground to the tip of the topmost extended leaf.

Leaf area index. Leaf area index (LAI) is the ratio of leaf area to ground area subtended. LAI was measured weekly in each lysimeter. Each leaf on the 4 designated plants in each lysimeter was measured. The length and width of each leaf was recorded. Periodic leaf samples were taken from the border plants to determine a relationship between the leaf area and the product of leaf width and leaf length.

Yield. Grain yield was determined by harvesting all the heads per lysimeter. The grain was threshed by a miniature combine. The water content of the grain was determined in a moisture meter. The grain yield was corrected to 14 percent (wet basis). The stover yield was determined by harvesting all the plants per lysimeter. The plants were then cut into segments approximately two to five cm in length and dried in an oven for seven days at 70 C. The total dry matter was computed as the stover yield plus the grain yield (converted to zero water content).

Leaf temperature. Leaf temperature was measured on 2 sunlit leaves at the top of the canopy in each lysimeter each day at approximately 2:00 p.m. CDT during the week. The infrared radiometer was calibrated using a simulated "black body" in a water bath which had a known temperature.

Leaf resistance. Leaf resistance (harmonic average of diffusion resistance of adaxial and abaxial leaf surfaces) was measured at approximately 3:00 p.m. CDT on selected days. The leaf was shaded for 10 seconds prior to the measurement. The cup and meter were always protected from direct sunlight since the readings were sensitive to temperature differences between the leaf and the cup. Dr. S. D. Davis made these measurements and analyzed the data.

Leaf water potential. Leaf water potential was measured on the same leaves used for leaf resistance measurements. The leaves were cut at approximately one-half of its length. The leaf blade was slit next to the mid-rib (De Roo, 1969) and inserted into the pressure bomb. The pressure bomb was sealed by a rubber gasket which was tightened around the leaf stem until air-tight. A micrometering valve was then opened and nitrogen entered the chamber until the vascular sap had returned to the top of the cut. At this instant, the chamber pressure was read from the pressure gauge and recorded. Since it is a destructive sampling technique, only one measurement per treatment was taken at any one time. Ritchie and Hinckley (1974) reviewed the utility of the pressure bomb for ecological research.

Soil Water Balance Measurements

Soil water content in each lysimeter was measured from 15 cm to 105 cm with a 15-cm increment each Monday, Wednesday, and Friday morning. Each Monday morning, surface gravimetric samples were taken. On Tuesday, each lysimeter was pumped, the effluent weighed, and the effluent electrical conductivity (EC) was measured. Weekly water balances (Monday to Monday) were determined by utilizing the measured data of SW_1, SW_2, and DRAIN, and the total irrigation input for the week.

Meteorological Measurements

The meteorological variables were recorded at six-minute intervals by a digital data logger. The analog signals were digitized and punched on teletype tape (ASCII Code). These paper tapes (one-half roll of paper tape per day) were then recorded on magnetic tape and computer processed at the Data Processing Center at Texas A&M University. The data were reduced to one-half hour averages, and these averages were used to compute the daily totals or averages of the individual variables. The Penman (1948) and Van Bavel (1966) combination equations were utilized to compute the potential evaporation at the site. Additionally, Class A weather station data consisting of maximum and minimum temperature, temperature and relative humidity (hydrothermograph), rainfall, 2-m windspeed, and Class A pan evaporation were recorded.

CHAPTER IV

EXPERIMENTAL RESULTS AND DISCUSSION

Evaporation and Seasonal Water Use

The cumulative values of the net radiation (equivalent evaporation depth assuming 583 cal g^{-1}), Class A pan evaporation, Penman and Van Bavel potential evaporation estimates, and evapotranspiration from the Control treatment (1.1) are given in Figure 13. The three estimates of potential evaporation were systematically different, with Class A pan giving the largest values followed by the Van Bavel and Penman methods in that order. Total evapotranspiration in the Control treatment (1.1) was 478 mm in 1973 as shown in Figure 13. These results are similar to those reported for well-watered grain sorghum [Ritchie and Burnett (1971), Hanks, et al. (1969), and Jensen and Sletten (1965)].

The roughness length, Z_0, was assumed equal to 20 mm for all calculations in Van Bavel's equation. Additionally, Z_0 was assumed to be a function of wind speed above the canopy (Sziecz, et al., 1973); this resulted in an underprediction of monthly potential evaporation (compared to the Z_0 = 20 mm case) of less than 20 mm during the months of May through July while it overpredicted by as much as 42 mm during April (a period of essentially bare soil). Table A*-1 gives the potential evaporation data.

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*Refers to Appendix A.

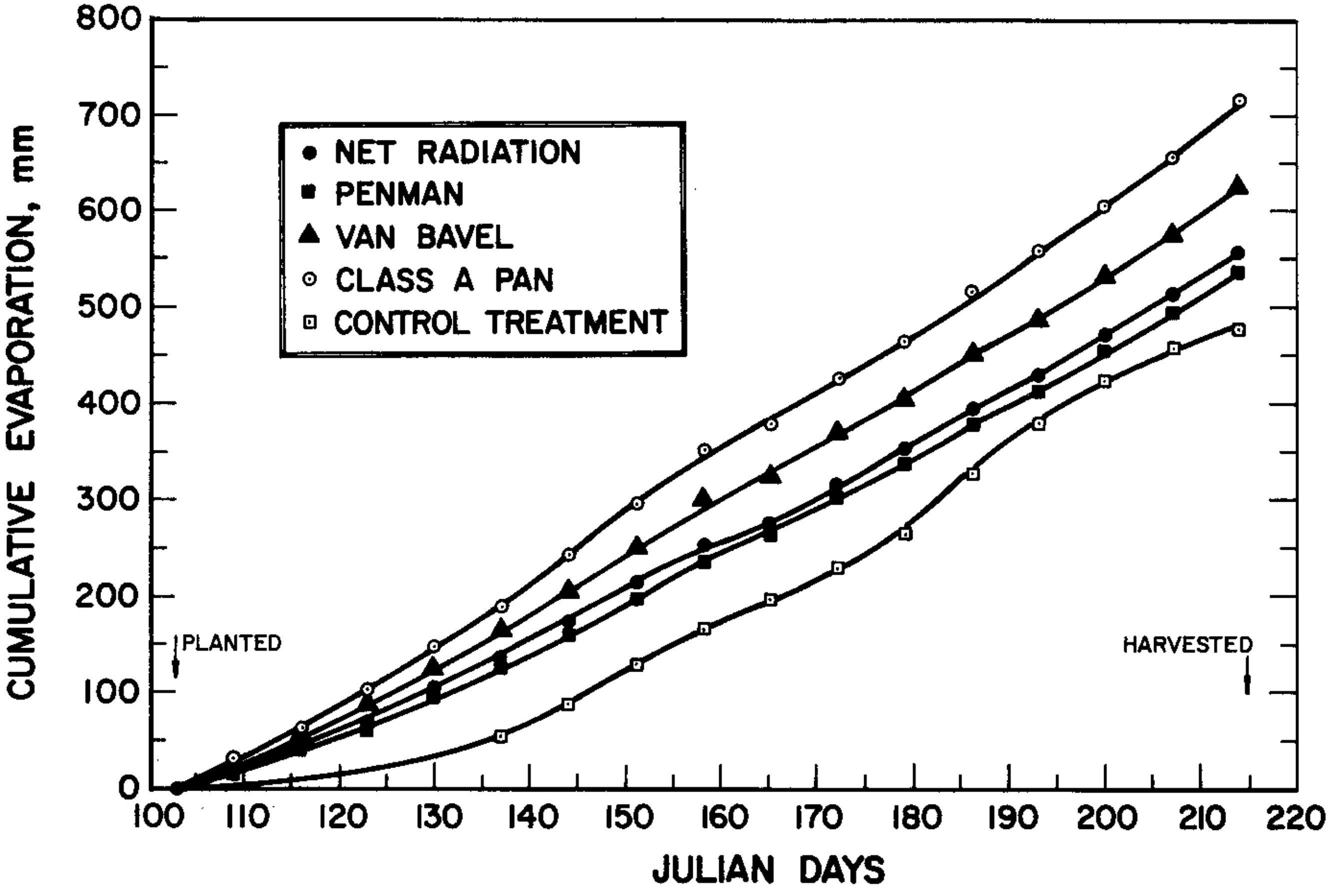

Figure 13. Cumulative net radiation (equivalent evaporation depth), Class A pan evaporation, Penman and Van Bavel potential evaporation estimates, and measured evapotranspiration from the Control treatment (1.1).

Considering the slopes of the lines in Figure 13 (page 52) (evaporation rate), Van Bavel's method appears to be a better predictor of potential evaporation rates than was Penman's method after the necessary crop canopy development (Ritchie and Burnett, 1971), which occurred on day 152. This conclusion is opposite to that reached by Richardson and Ritchie (1973) in a similar climate in Central Texas. The lysimeters in our study were subject to local advection, even though a border area was maintained, and our measurement accuracy did not approach that of Ritchie and Burnett (1968). However, since our data exhibited trends similar to past reports (Jensen, 1972) and considering reported measurement accuracies for our method (Van Bavel and Stirk, 1967) for time periods on the order of several days, I feel that this conclusion is valid for the present case; however, the improved accuracy would be small.

Cumulative values of the total water loss (includes both evapotranspiration and drainage) for each treatment are given in Figure 14. The evapotranspiration of the treatments was largely determined by the irrigation quantity. This is due primarily to the restricted root zone and limited water holding capacity in this soil type. The water use data indicated that 90 percent or more of the water used as measured on a 105-cm profile occurred above the 45 cm-depth in all the treatments. Critical values of soil water potential can develop in this type of soil in short periods of time

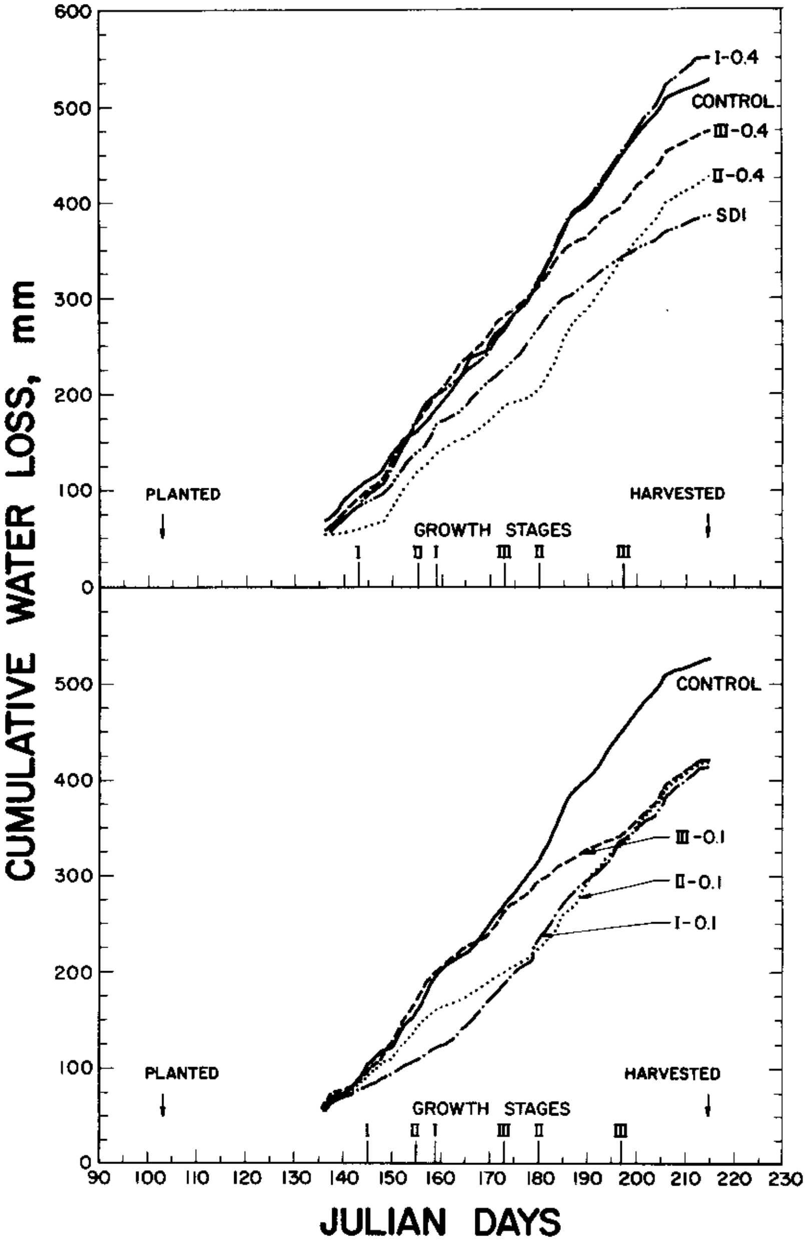

Figure 14. Cumulative water loss in each treatment.

(5 to 7 days) when the plant water use in large (7 to 10 mm day^{-1}). Table 1 gives the computed relative evapotranspiration values for each growth period. The largest evapotranspiration value of all the treatments at a particular growth stage was used as the value of E_0 for that growth stage. The growth period iniation was defined in terms of the Control treatment growth stage. This resulted in spreading a portion of the actual water deficit to the period prior to the intended deficit when the growth periods were defined in this manner. The data in Table 1 show the effect of

Table 1. Relative evapotranspiration data.

Treatment	Relative Yield Y/Y_0	Growth Period I	Growth Period II	Growth Period III
		Relative Evapotranspiration E/E_0		
1.1 (Control)	0.98	0.99	0.96	1.00
I-0.4	0.90	0.63	1.00	1.00
I-0.1	0.79	0.39	0.80	0.80
II-0.4	0.67	0.68	0.55	0.83
II-0.1	0.59	0.61	0.54	0.76
III-0.4	0.94	1.00	0.91	0.64
III-0.1	0.78	0.96	0.91	0.42
SDI I-0.32 II-0.64 III-0.42	0.83	0.75	0.85	0.64

these growth period definitions. This effect is not important as will be seen later.

Plant Measurements

Measurements of leaf resistance (R_ℓ) and leaf water potential (ψ_ℓ) were taken three times weekly (Tuesday, Thursday, and Saturday) on three irrigation treatments at approximately 1:00 to 4:00 p.m. CDT. The treatments measured were the Control (1.1), 0.4, and 0.1 treatments at the respective growth stages. Plants from only one lysimeter per treatment were sampled. Four well-illuminated, fully-expanded leaves were selected for R_ℓ measurements and one leaf per treatment for the ψ_ℓ measurement. While making the R_ℓ measurements, the leaf was shaded for 10 seconds prior to attaching the cup and both the cup and leaf were shaded during the measurement. Figure 15 shows that the only significant difference in R_ℓ between treatments occurred in stage I where treatment I-0.1 experienced partial stomatal closure. This was verified by visual observation of wilting in treatment I-0.1, as opposed to I-0.4 and Control which showed no external signs of stress. It is interesting that this wilting and partial stomatal closure of I-0.1 occurred at a relatively high ψ_ℓ (-13 to -16 bars) and that the ψ_ℓ was not significantly different than other treatments which experienced little or no stress. With the exception of R_ℓ in the I-0.1 treatment there was essentially no difference in R_ℓ (Figure 15) or in ψ_ℓ among treatments throughout

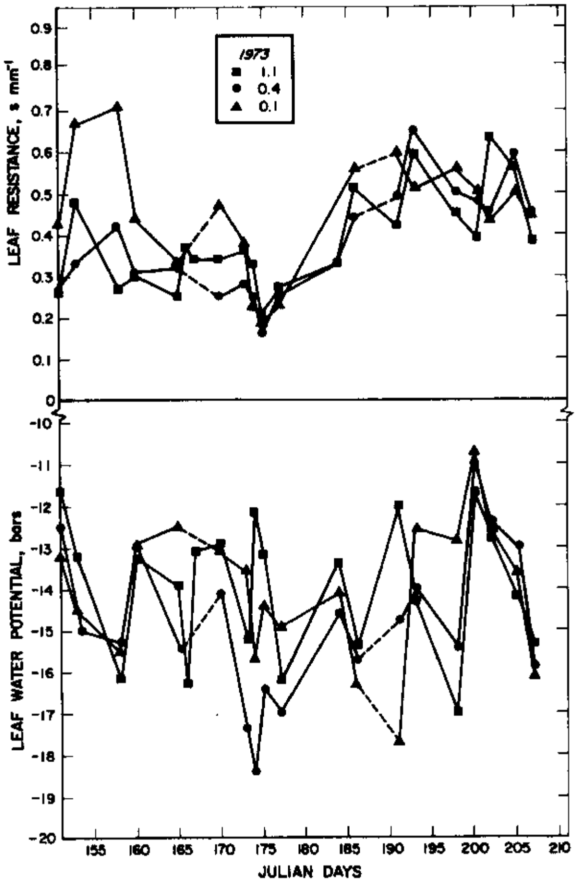

Figure 15. Leaf resistance and leaf water potential throughout the growing season.

the remaining growth stages (stages II and III). It was evident that water never became limiting enough to affect either ψ_ℓ or R_ℓ and any differences in yield between treatments was therefore due to some factor other than stomatal closure (the only exception to this would be treatment I-0.1 where differences in yield might be due partially to closure of stomates during stage I). An increasing trend was observed for R_ℓ in all treatments during the latter stages of growth (Figure 15, page 57). This suggests a possible aging effect on the ability of grain sorghum leaves to maintain completely open stomates under favorable conditions.

The results of leaf temperature measurements indicated rather small differences between treatments. These differences never exceeded 1 C as would be expected since the stomata apparently seldom closed under these particular soil water conditions.

Table 2 gives the measured values of leaf area index (LAI). The maximum variance in the LAI data was ±0.49 which was due to the small sample size. Therefore, some of the indicated differences in Table 2 are not significant. However, the treatment influence was apparent in most instances. The LAI data showed that all the treatments except I-0.1 had reached the critical LAI value of 2.7 (Ritchie and Burnett, 1971) necessary for potential evaporation by June 5. Only small differences in crop height were observed. Table 3 gives the measured values of crop height.

Soil Water Measurements

Measurements of the quantity and state (potential) of the soil water were made. The average soil water matric potentials measured by tensiometers (referenced at the soil surface) for the 30 cm-depth are shown in Figure 16. The Control (1.1) treatment was maintained between -0.1 and -0.2 bar. The influence of each treatment was evident by the marked decrease in potential. This particular soil

Table 2. Leaf area index values for the growing season.

Date	Treatments 1.1 (Control)	I-0.4	II-0.4	III-0.4	I-0.1	II-0.1	III-0.1	SDI I-0.32 II-0.64 III-0.42
May 16	0.32	0.50	0.25	0.35	0.27	0.22	0.25	0.32
May 22	1.12	1.45	0.95	1.18	0.82	0.68	0.92	0.84
May 29	2.39	2.35	1.74	2.32	1.57	1.66	2.14	1.91
June 5	3.81	3.48	2.66	3.29	2.19	2.75	4.11	3.30
June 12	4.98	4.46	4.05	4.92	3.04	4.09	4.31	4.99
June 19	5.12	4.21	4.09	5.05	3.44	4.69	4.28	5.01
June 26	5.15	4.46	4.04	5.08	3.64	4.45	4.53	4.90
July 3	4.89	4.27	3.62	4.38	3.51	3.89	4.23	4.76
July 10	4.46	4.04	3.52	3.77	3.49	3.45	3.64	4.35
July 17	3.35	3.63	2.84	2.81	3.54	3.10	3.01	3.20

Table 3. Plant height during the growing season.

Date	Plant Height, cm							
	Treatments							
	1.1 (Control)	I-0.4	I-0.1	II-0.4	II-0.1	III-0.4	III-0.1	SDI I-0.32 II-0.64 III-0.42
May 16	27	34	26	27	24	28	25	28
May 18	32	40	29	33	27	34	28	31
May 22	48	54	37	49	38	50	42	42
May 25	64	62	49	63	50	66	59	55
May 29	75	76	58	75	68	76	73	69
June 1	85	85	65	84	77	84	82	79
June 5	90	90	70	90	83	89	87	84
June 8	94	95	72	94	90	94	90	91
June 12	97	97	77	98	95	101	95	97
June 15	100	101	80	103	97	104	97	98
June 19	102	102	84	104	97	106	98	100
June 22	103	105	84	106	99	106	99	100
June 26	103	105	86	106	99	106	99	100
June 29	103	105	87	105	99	106	99	103
July 6	103	104	86	105	97	106	98	102
July 10	103	104	88	104	101	106	102	101
July 13	102	103	87	103	98	104	100	101
July 17	102	102	87	103	96	104	99	101
July 24	101	101	87	103	97	104	97	100

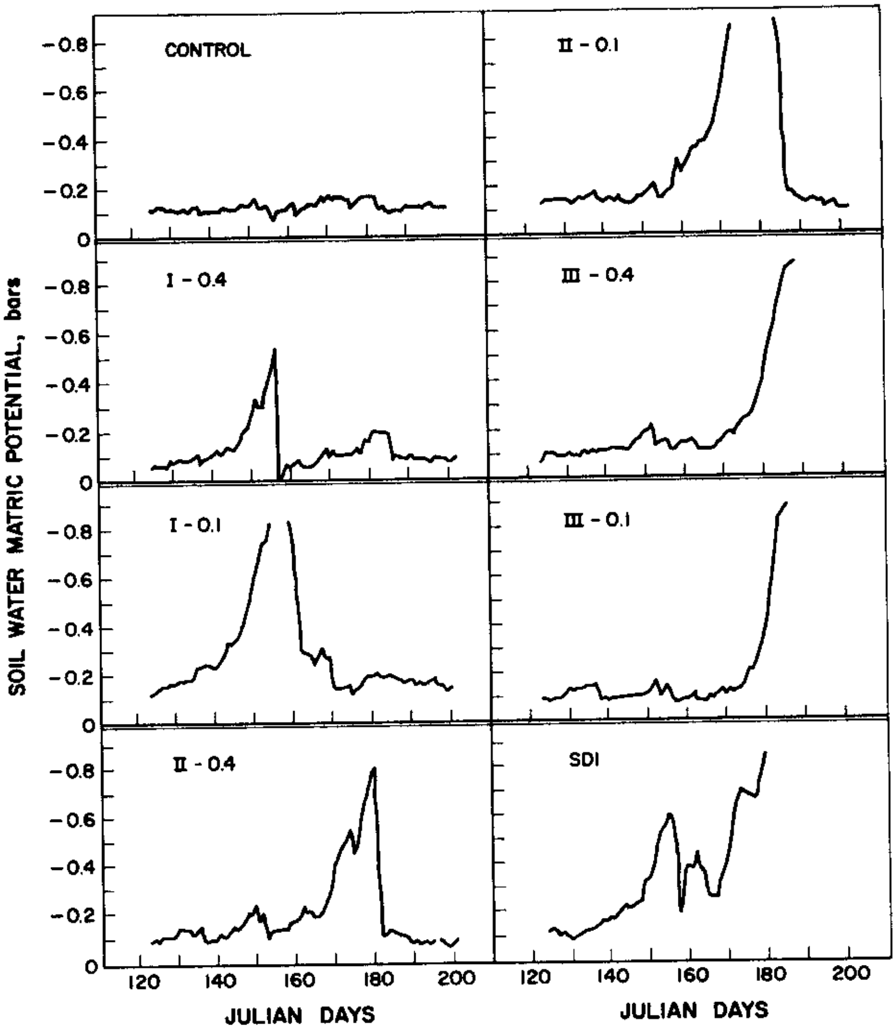

Figure 16. Soil water potential for each treatment at 30 cm depth.

layer has very little water which can be extracted by plants at a matric potential of -1 bar. Even though small, frequent applications were supplied to the soil surface, this water was soon consumed by the plants.

The average available soil water content of the lysimeters are shown in Figure 17. This soil can hold at "field capacity" approximately 240 mm of water in the top meter. Approximately 120 mm of this water is unavailable due to the soil hydraulic properties and root distribution pattern. These data indicate the degree to which soil water was depleted at each growth period by the respective treatments.

Yield and Water Use Efficiency

The yield and water use efficiency results are given in Table 4. Analyses of variance were performed on the yields; in all cases, variance between treatments was significant at the 99 percent level. Test of difference between treatment means was done by Duncan's multiple range test. Table A-2 gives the analysis of variance of the yield and water use efficiency results. Only water deficits prior to the milk to soft dough stage (stage III) affected yield and WUE to a large extent. However, the SDI treatment in 1973 produced the greatest WUE (see Hiler, et al., 1974, for a discussion of the SDI concept). This treatment was depleted at a variable rate, but maintained adequate soil water until the third growth

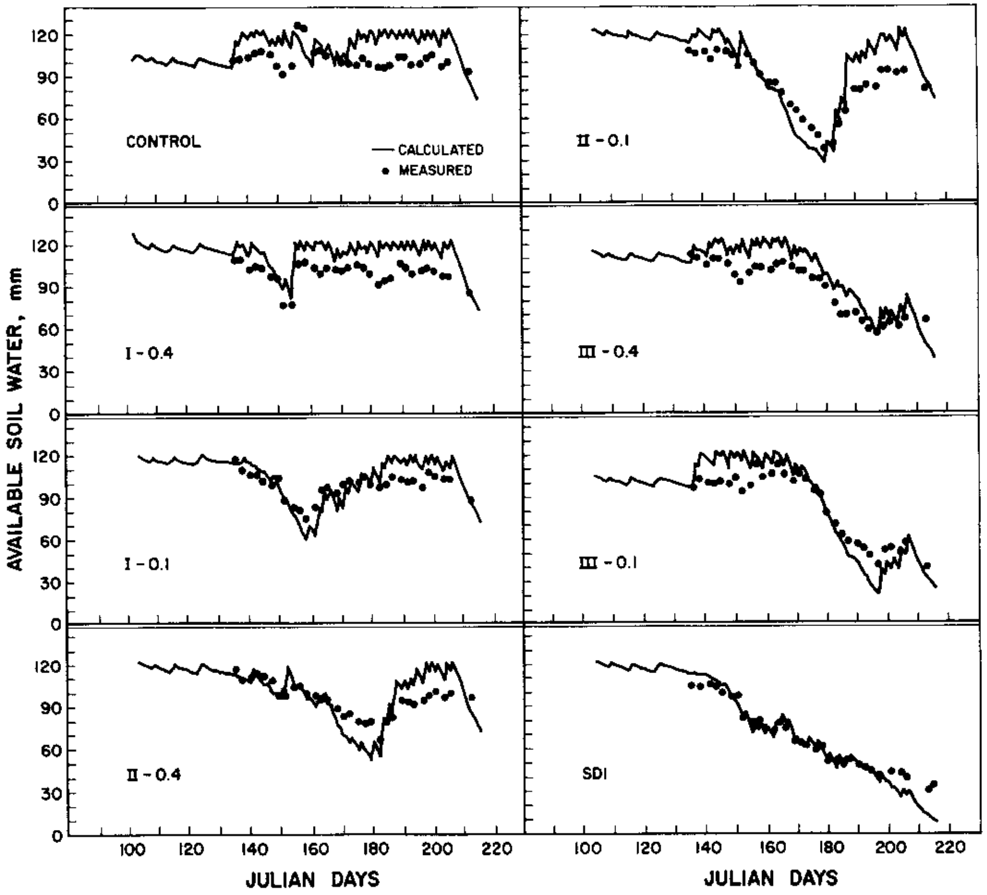

Figure 17. Comparison of measured soil water content with the results of the soil water balance calculations.

Table 4. Yield, water use, and water use efficiency results.

Treatment	Dry Matter Yield kg ha^{-1}	Grain Yield[1] Y kg ha^{-1}	Irrigation mm	Drainage mm	Evapotranspiration ET mm	Total Losses mm	Water Use Efficiency kg ha^{-1} mm^{-1} Y/ET
1.1 (Control)	16,245	8,650 a[2]	540	58	478 a	536	18.1
I-0.4	15,770	7,880 ab	516	81	470 a	551	16.8
I-0.1	14,036	6,910 c	378	35	378 c	413	18.3
II-0.4	12,940	5,898 d	388	36	384 c	420	15.4
II-0.1	11,972	5,196 d	370	32	390 c	422	13.3
III-0.4	15,652	8,240 a	407	37	436 ab	473	18.9
III-0.1	14,621	6,890 b	362	17	402 bc	419	17.1
SDI I-0.32 II-0.64 III-0.42	15,002	7,272 bc	239	37	345 d	382	21.1

[1] Grain yield corrected to 14 percent moisture content wet basis.

[2] Differences between means followed by the same letter are not statistically significant at the 0.05 level by the Duncan multiple range test.

stage. These results explain why the limited irrigations used by Hiler and Howell (1973) resulted in increased WUE. In their study in 1972, the limited irrigations allowed an adequate soil water level until the third growth period was reached. It is apparent that if a water deficit condition is to occur, it should be spread over the early vegetative growth period and the period past heading. Water deficits during the boot to bloom period should be avoided. The above increase in WUE in the SDI treatment resulted, however, in lower production (less marketable yield). These trends are similar to results of Bucks, et al. (1973), Bucks, et al. (1974), and Patterson and Hanson (1972) with other crop species.

Comparison to Blackland Water Balance Model

The soil water balance model of Richardson and Ritchie (1973) and Ritchie (1972) was utilized to calculate the evapotranspiration during the growing season and to calculate separately the soil and plant evaporation components. Climatic data, initial soil water content, leaf area index, irrigation quantities, and physical soil constants were required in the model.

Figure 17 (page 63) shows comparisons of the results of the model calculations and the actual measured soil water content. The calculated values were within ±20 mm of the measured data on 95 percent of the days. The correlation coefficient of calculated and measured soil water was 0.95. Most errors were near the limit of accuracy of the soil water measurement. Table 5 gives the calculated water use for each treatment. Figure 18 shows the relationship between the calculated evapotranspiration and total losses and their measured values. The largest error was 10.0 percent in the evapotranspiration and 8.4 percent in the total water loss. Based on previous results, the model accuracy might be increased by using the Van Bavel method to estimate potential evaporation.

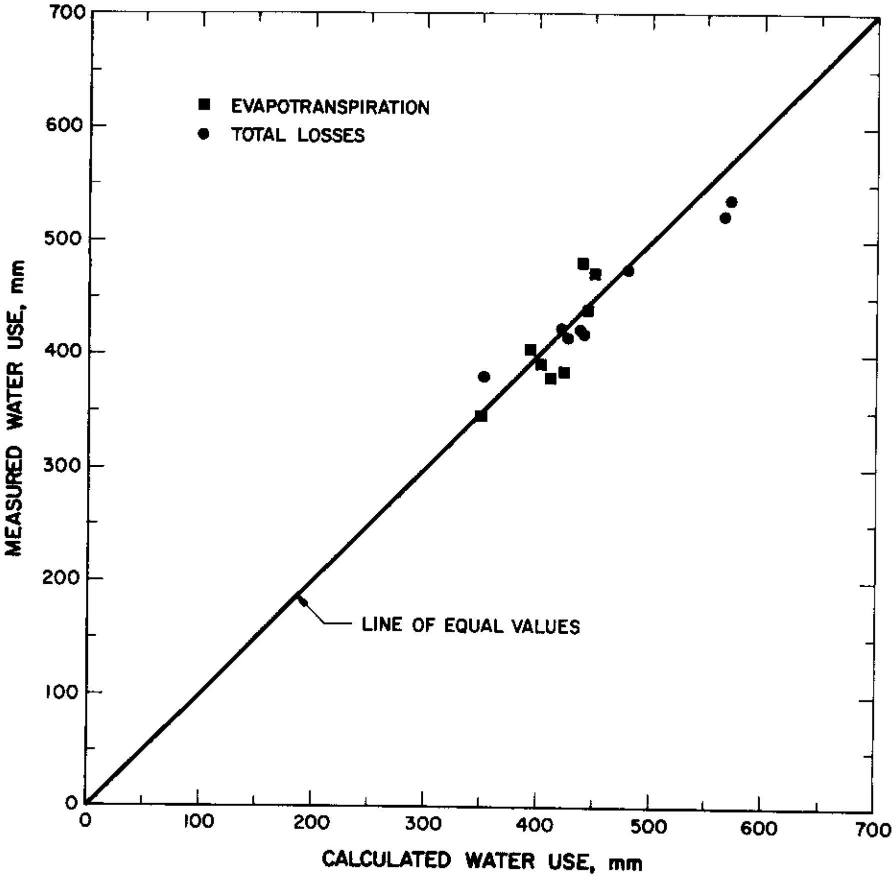

Figure 18. Relationship between calculated evapotranspiration and total losses and their measured values.

Table 5. Comparison of measured and calculated water use data.

Treatment	Measured		Calculated				Water Use Efficiency Y/EP kg ha^{-1} mm^{-1}
	Evapotranspiration ET mm	Total Losses mm	Evaporation		Evapotranspiration ET mm	Total Losses mm	
			Plant EP mm	Soil ES mm			
1.1 (Control)	478	536	321	124	445	570	26.9
I-0.4	470	551	323	127	450	566	24.4
I-0.1	378	413	293	118	411	426	23.6
II-0.4	384	420	295	128	423	437	20.0
II-0.1	390	422	269	133	402	420	19.3
III-0.4	436	473	311	130	441	480	26.5
III-0.1	402	419	261	130	391	439	26.7
SDI I-0.32 II-0.64 III-0.42	345	382	238	111	349	350	30.5

For the plant and soil evaporation components given in Table 5 (page 67), the ratio of soil evaporation to evapotranspiration (ES/ET) varied from maximum of 0.33 to minimum of 0.28. Using the calculated plant evaporation (EP), water use efficiencies based solely on plant evaporation are presented also in Table 5 (page 66). Those values show that only water deficits in growth stage II caused reductions in water use efficiency of grain sorghum production which confirmed the earlier results.

Yield Model

The yield models of Hiler and Clark (1971), Jensen (1968), and Minhas, et al. (1974) were tested on the experimental data. The coefficients in each model were calculated with multi-variable regression techniques. Due to the limited data base, effects of the water deficit (E/E_0) on these coefficients could not be determined. For this reason all the treatments, except the SDI treatment, were lumped for computational purposes. The SDI treatment data, data of Hiler and Howell (1973), Lewis, et al. (1974), and data of Stewart, et al. (1974) were utilized for comparison to the results.

Stress Day Index

The stress day index model, given previously by equation (6), can be written as

$$Y_j = Y_o - ACS_1\ (SD_1)_j - ACS_2\ (SD_2)_j \ldots - ACS_n\ (SD_n)_j \qquad \ldots(14)$$

Equation (14) is now written in the familiar multi-variable regression form where the SD_i's are the independent variables. To solve equation (14) with n = 3, at least four sets of data are required if Y_o is unknown. Table 6 shows the regression coefficients. The correlation coefficient was 0.962, and the regression was significant at the 95 percent level.

Table 6. Stress Day Index regression coefficients.

Y_o	ACS_1	ACS_2	ACS_3
8,836	-1,569*	-4,946*	-1,858+

*Indicates 95 percent level of significance

+Indicates 90 percent level of significance

Figure 19 shows a comparison of the model to the experimental data. The range of the data was limited by the individual experimental designs. All but one data point are within ±20 percent of the predicted values. Part of this variation could possible be accounted for by slight differences in growth stage definition in the literature data and also the assumption that CS was not dependent on SD.

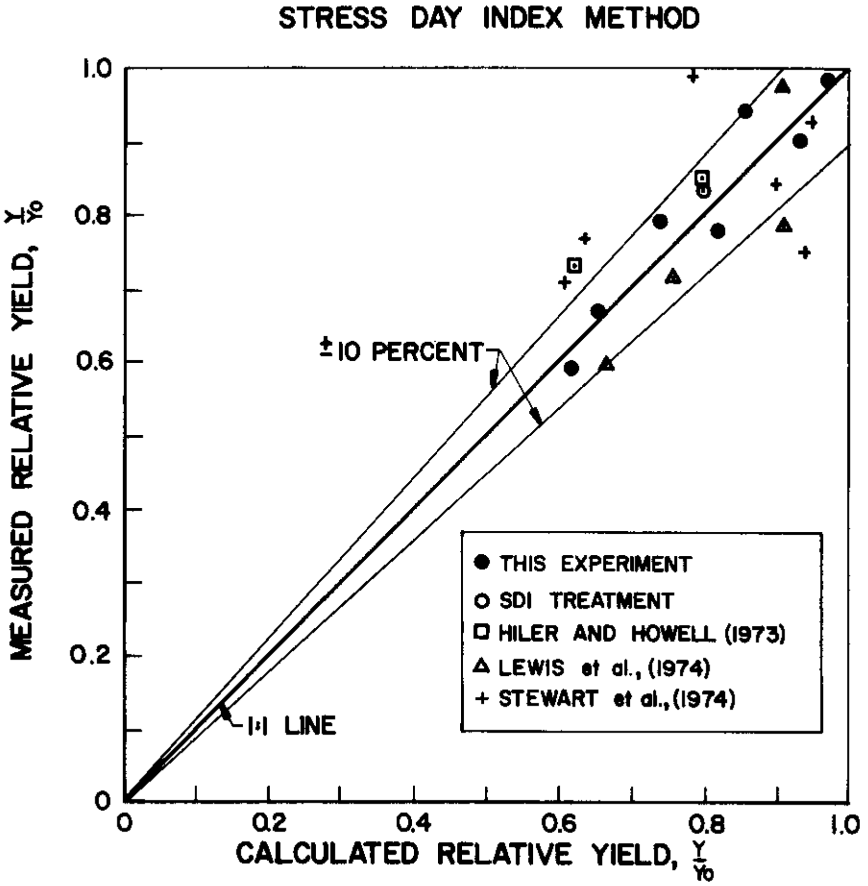

Figure 19. Comparison of Stress Day Index to experimental model.

Jensen Yield Model

The Jensen model given previously by equation (8) can be written as

$$\ln(\frac{Y}{Y_o}) = B_o + \lambda_1 \ln(\frac{E}{E_o})_1 + \lambda_2 \ln(\frac{E}{E_o})_2 + \ldots \lambda_n \ln(\frac{E}{E_o})_n \qquad \ldots(15)$$

This equation can be evaluated by a multi-variable regression to determine λ_i. $(E/E_o)_i$'s are the independent variables. Table 7 shows the regression coefficients. The correlation coefficient was 0.966 and the regression was significant at the 95 percent level.

Table 7. Jensen yield model regression coefficients.

B_o	λ_1	λ_2	λ_3
5.169×10^{-3}	0.117^{+}	0.604*	0.178

*Indicates 95 percent level of significance

$^{+}$Indicates 90 percent level of significance

Figure 20 shows a comparison of the model to the experimental data. B_o was not included in the original equation and its effect is very limited. The exponential of B_o is approximately 1.0 and therefore does not affect the solution.

Minhas, Parikh, and Srinivasan Yield Model

The Minhas, et al. (1974) yield model given previously by equation (10) can be written as

$$\ln(\frac{Y}{Y_o}) = B_o + b_1 \ln[1.0 - (1.0 - \frac{E}{E_o})_1{}^2] + b_2 \ln[1.0 - (1.0 - \frac{E}{E_o})_2{}^2] + \ldots b_n \ln[1.0 - (1.0 - \frac{E}{E_o})_n{}^2] \quad \ldots(16)$$

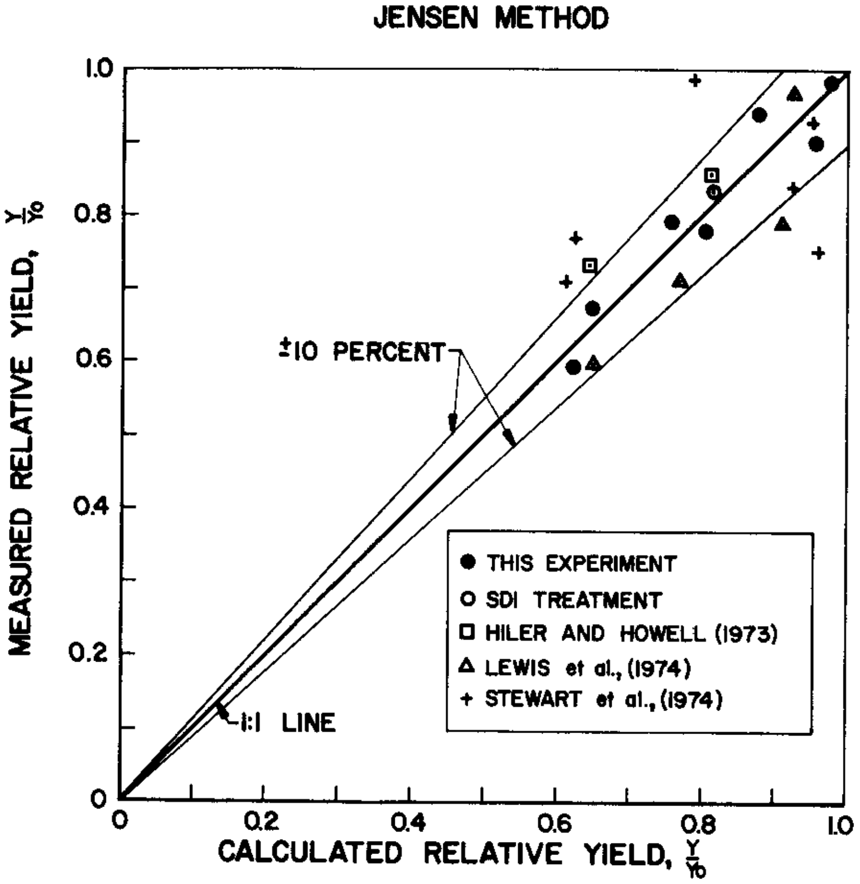

Figure 20. Comparison of Jensen yield model to experimental data.

Thus equation (16) can now be evaluated using multi-variable regression techniques. Table 8 shows the regression coefficients. The exponential of B_0 was 0.98, so its effect on the solution was small. The correlation coefficient was 0.986, and the regression was significant at the 99 percent level.

Table 8. Minhas, Parikh, and Srinivasan yield model regression coefficients.

B_0	b_1	b_2	b_3
-0.016	0.323^{+}	1.638**	0.503*

*Indicates significance at 95 percent level

**Indicates significance at 99 percent level

$^{+}$Indicates significance at 90 percent level

Figure 21 shows the comparison of this model to the experimental data. All of the data points are within the ±20 percent range. However, the improvement over the previous models was small.

Summary

As pointed out previously each yield model simulated the experimental results equally well. Figures 19 through 21 verified this conclusion. Table 9 shows the explained variance (correlation coefficient squared) for each set of data and model. The low correlation of the data of Stewart, et al. (1974) could result from differences of varietal response or in characterization of the growth periods. If the growth periods of Stewart, et al. (1974) were shifted to correspond to the growth stages used in this report, a marked improvement was obtained as illustrated by Table 9. Thus,

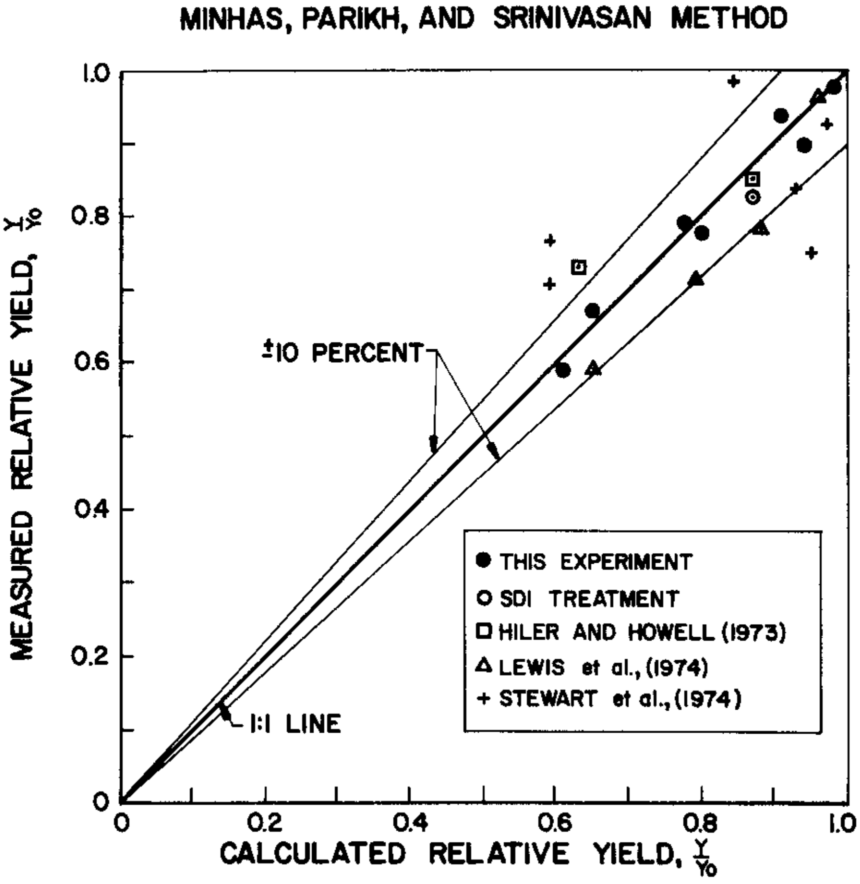

Figure 21. Comparison of Minhas, Parikh, and Srinivasan yield model to experimental data.

the comparison of one experiment to another, depends heavily on the growth period definitions. The data from Lewis, et al. (1974) were from a surface irrigation experiment at the same site as Hiler and Howell (1973). The agreement among the data of Lewis, et al., (1974), Hiler and Howell (1973), and this experiment was acceptable. The requirement of data for E/E_0 for growth periods

Table 9. Percent of variance explained by each yield model.

Data	Yield Model		
	Minhas, et al.	Jensen	Stress Day Index
Lewis, et al. (1974)	92	83	76
Hiler and Howell (1973) and SDI treatment	85	98	98
Stewart, et al. (1974)	6	8	19
Stewart, et al. (1974)*	50	68	66

*Data were shifted by one growth stage to correspond better to the growth periods used in this study.

and yield data limited the sources of data to test the models. Based on these limited results, the yield model of Jensen (1968) was utilized for the remainder of this work because this model explained usually the largest percentage of the variance in each set of data.

Chapter Summary

Evaluating the effect of limited irrigation on grain sorghum production has been of interest to many researchers, especially in

Texas [Bonnen, et al. (1952), Swanson and Thaxton (1957), Newman (1960), Jensen and Sletten (1965), Musick and Dusek (1969 and 1970), Shipley and Regier (1970), and Lewis, et al. (1974)]. Most field experiments were subject to local rainfall patterns and deficiencies in measurement of crop water use which confounded many direct conclusions of that research. Yet the trend is apparent; grain sorghum is tolerant to limited "water deficits" at specific growth stages. The degree of the tolerance as reflected by the crop yield depends on the timeliness of certain rainfall or irrigations. The WUE can be increased as compared to a well-watered control treatment, by allowing selected crop growth stages (early vegetative and late grain filling) to be water-deficit periods while adequately irrigating the crop during certain "critical" periods (boot to heading) of crop development.

This research demonstrates a potential for increasing WUE of grain sorghum by utilizing high frequency irrigation to apply frequent, but small, irrigation quantities and limiting these applications according to the stage of plant development. The findings of this research indicate that water deficits which occur during the boot-to-bloom stage of grain sorghum development can reduce yield and water use efficiency. However, careful regulation of the irrigation quantity to minimize water deficits during this period can increase WUE. The important conclusion of this work is that WUE is directly affected by the timeliness of a specific water deficit to a much larger extent than the magnitude of the water deficit. A small water deficit occurring during the boot-to-bloom period can reduce the yield as much or more than a large water deficit occurring during either the early vegetative period or late grain filling period.

CHAPTER V

SYSTEM SIMULATION PROCEDURE

This chapter will consider the simulation procedure of high frequency irrigation (daily irrigation) on the soil water balance of a Houston Black Clay soil with a grain sorghum crop and climatic conditions for Temple, Texas, This particular soil and climatic site has been described by a detailed series of papers [Templin, et al. (1956), Kunze and Templin (1956), Godfrey, et al. (1960), Ritchie, et al. (1972a), Ritchie (1971), Ritchie and Burnett (1971), Ritchie, et al. (1972b), Ritchie and Jordan (1972), and Ritchie (1973)]. The system simulation involved the coupling of an environmental model (rainfall, temperature, and potential evaporation) to the Blackland Water Balance Model. The water balance model was improved to account for root zone extension, runoff, and leaf area index simulation (Ritchie and Arkin, 1973).

Environmental Model

The environmental model was patterned after work by Jones, et al. (1972) to simulate rainfall, temperature, and potential evaporation. Following Jones, et al. (1972), it was hypothesized that rainfall is the most basic weather variable, independent of evaporation and temperature, and dependent only upon the time of the year and the tendency for the persistency of rainfall described by a Markov chain.

Temperature was postulated to be dependent upon the time of year and rainfall occurrence. Potential evaporation is largely controlled by solar radiation, but solar radiation data were not available. It was postulated that potential evaporation could be estimated from the time of year, pan evaporation, rainfall occurrence on the present day, and rainfall occurrence on the previous day.

Potential evaporation was determined as pan evaporation times a coefficient (Richardson and Ritchie, 1973). It was recognized that this method of calculating potential evaporation would be poor in the case of actual data (Van Bavel, et al., 1967); however, since the purpose was to generate a distribution of potential evaporation values, this method was quite satisfactory. Further, it was assumed throughout this analysis that all the days in each week were a stationary homogeneous sample. Therefore, all the daily data for each week could be grouped in one sample.

Rainfall

The rainfall model was postulated to be of the following functional form:

$$R = f(WN, R1, RV) \qquad \ldots(17)$$

where R = the daily rainfall, mm; WN = time of the year (week number beginning March 1); R1 = the previous day's rainfall, mm; and RV = other random variables. RV was simulated using the

Monte Carlo method [Link and Splinter (1970), de Roccaferrera (1964), and Jones, et al. (1972)].

Historical rainfall records for Temple, Texas, were used to compute the rainfall Markov probabilities and the probability distribution of daily rainfall quantities. Data from 1948 to 1972 (25 years) were utilized.

Markov probabilities. A Markov process can be described as any process in which the conditional probability of any future event, given any past event and the present state, is independent of the past event and dependent only on the present state of the process. A Markov process is said to have no "memory". Markov chains are quite common in hydrology to describe the persistence of an event [Gabriel and Neumann (1962), Wiser (1965), Wiser (1966), Hann and Barfield (1973), and Heermann, et al. (1971)].

For this work, a dry day was defined as any day with less than 0.25 mm of rainfall. For each week of the year, all the days were arranged in a sequence of D/D (dry day given previous day was dry) and D/W (dry day given previous day was wet). The probability of D/D sequence for a given week was the number of D/D sequences divided by the number of previous days which were dry. The probability of a D/W sequence was the number of D/W sequences divided by the number of previous days which were wet. This can be expressed as

$$P_i(D/D) = \frac{N_i(D/D)}{ND_i} \;, \qquad \ldots(18)$$

$$P_i(D/W) = \frac{N_i(D/W)}{(M-ND_i)} \;, \qquad \ldots(19)$$

$$P_i(W/D) = 1.0 - P_i(D/D), \text{ and} \qquad \ldots(20)$$

$$P_i(W/W) = 1.0 - P_i(D/W) \qquad \ldots(21)$$

where P_i = the probability of selected sequence for the ith week; N_i = the number of particular sequences in the ith week; ND_i = the number of previously dry days in week i; and M = the number of individual days in week i. The complementary probabilities [P(D/D), P(W/D) and P(D/W), P(W/W)] must add to one.

Rainfall distribution. The quantity of rainfall for a given rainfall event is a function of the geographic location in relation to the major climatic interactions at that site and the specific meteorological state at that time. Generally, fitting this given rainfall pattern to a specific distribution has been difficult. Wyrick (1974) utilized the Weibull distribution; Jones, et al. (1972) used the incomplete gamma distribution, and Marshall and Shaw (1974) used the square-root normal distribution. Many other statistical distributions have been used.

For this study the square-root normal, log-normal, and the incomplete gamma distribution were tested. The statistical properties of the data for W/D and W/W sequences of each week were computed. For the square-root normal distribution, governing equations are

$$\overline{\sqrt{X}}_i = \frac{1}{n} \sum_{j=1}^{n} \sqrt{X} \text{ , and} \qquad \ldots(22)$$

$$S_{ri} = \sqrt{\frac{\sum_{j=1}^{n} \left(\overline{\sqrt{X}}_i - \sqrt{X}_j\right)^2}{n-1}} \qquad \ldots(23)$$

where $\overline{\sqrt{X}}$ = the mean of the square-roots of the daily rainfall amounts for week i, $mm^{1/2}$, and S_{ri} = the standard deviation of the square-roots of the daily rainfall for week i, $mm^{1/2}$. For the log-normal distribution,

$$\overline{\ln X_i} = \frac{1}{n} \sum_{j=1}^{n} \ln X_j \text{, and} \qquad \ldots(24)$$

$$S_{\ell i} = \sqrt{\frac{\sum_{j=1}^{n} (\overline{\ln X_i} - \ln X_j)^2}{n-1}} \qquad \ldots(25)$$

where $\overline{\ln X}$ = the mean logarithm of the daily rainfall amount for week i, mm, and $S_{\ell i}$ = the standard deviation of the logarithms of daily rainfall amounts for week i, mm. For the incomplete gamma distribution,

$$\overline{X}_i = \frac{1}{n} \sum_{j=1}^{n} X_j \qquad \ldots(26)$$

$$\gamma_i = \frac{1 + \sqrt{1 + \frac{4}{3}\left[\ln(\overline{X}_i) - \frac{1}{n} \sum_{j=1}^{n} \ln(X_j)\right]}}{4\left[\ln(\overline{X}_i) - \frac{1}{n} \sum_{j=1}^{n} \ln(X_j)\right]} \qquad \ldots(27)$$

$$\beta_i = \frac{\overline{X}_i}{\gamma_i} \qquad \ldots(28)$$

where $\overline{X}_i$ = the mean daily rainfall amount for week i, mm, and γ_i and β_i and the gamma distribution parameters.

The data of the square-root normal and log-normal distributions were tested for normality by the Cornu-criteria and the skewness test. The Cornu-criteria is calculated as

$$C = \frac{|MD|}{S} \qquad \ldots(29)$$

where $|MD|$ = the mean absolute value of the deviation about the mean; S = the standard deviation, and C = the Cornu-criteria which should be about 0.80 for a normal distribution. The probability limits for this criterion are a function of the sample size, with the range of the limits decreasing as the sample size increases

(Snedecor and Cochran, 1967). The skewness test compares the distribution skewness to the skewness times the standard error. The standard error, SE, can be approximated by

$$SE = \sqrt{\frac{6}{n}} \qquad \ldots(30)$$

where n = the sample size. The sample skewness must be less than 1.96 times its standard error to be normal at the 95 percent level.

The Chi-square test was also used to test the distribution. But due to the small sample size (usually less than 35), the test gave inconclusive results. This was also found by Tucker and Griffiths (1965).

Temperature

Following Jones, et al. (1972), the temperature model was postulated to be of the form:

$$T = f(WN, R, RV) \qquad \ldots(31)$$

where T = the average daily temperature (average of minimum and maximum temperatures), C, and WN, R, and RV were defined previously. The temperature for the day was assumed to be influenced by the rainfall on that day. Historical records of daily maximum and minimum temperatures for the same time period as the rainfall data for Temple, Texas, were utilized. The temperature was assumed to be normally distributed (Jones, et al. (1972). The data were arranged by weeks (beginning on March 1) and grouped according to whether the day was dry or wet. The mean and standard deviation were computed for each set of data for each week. Polynomial equations were then fitted to the weekly data.

Potential Evaporation

Potential evaporation was postulated to be given in a functional relationship as

$$E_o = f(WN, E_p, R, R1, RV) \qquad \ldots(32)$$

where E_o = the potential evaporation, mm; E_p = the pan evaporation, mm; and WN, R, R1, and RV were defined previously. Coefficients were utilized to convert the pan evaporation to potential evaporation (Richardson and Ritchie, 1973). Historical data of BPI pan evaporation for Temple, Texas, were used. The data were converted to Class A pan evaporation by multiplying the BPI value by 1.28 (Bloodgood, et al., 1954). The pan evaporation was assumed to be normally distributed (Jones, et al., 1972). The pan evaporation data were arranged by weeks (beginning March 1) and grouped according to rainfall data (D/D, D/W, W/W, W/D). The mean and standard deviation were computed for each set of data for each week. Polynomial equations were then fitted to the weekly statistics. The pan evaporation data could then be calculated for each day and multiplied times the pan coefficient to estimate the potential evaporation.

Soil Water Balance Model

The soil water balance model used was basically the Blacklands Water Balance Model (Richardson and Ritchie, 1973) which was modified to allow for root zone extension and runoff. A grain sorghum leaf area index model (Ritchie and Arkin, 1973) was added as a subroutine. The model was programmed in FORTRAN IV (LEVEL G). The basic components of the model were soil evaporation, plant evaporation, runoff, drainage, soil water balance, and leaf area index simulation.

Soil Evaporation

In the first part of the growing season, the soil may be bare. When the soil is bare, the maximum evaporation rate is defined by the potential evaporation rate, E_o. As the crop grows the soil becomes shaded and the potential soil evaporation is reduced because of increased humidity in the air close to the soil, decreased net radiation, and decreased wind speed. The potential soil evaporation can be given as

$$ES_o = E_o \, e^{-0.40 \, LAI} \qquad \ldots(33)$$

where ES_0 = the potential soil evaporation, mm, and LAI = the leaf area index.

The soil evaporation process is considered to take place in two stages: the first stage is termed the constant-rate stage in which the evaporation rate is controlled by the potential rate, and the second stage is termed the falling-rate stage in which the evaporation rate is controlled largely by the hydraulic properties of the soil. The amount of water which can evaporate under stage one conditions is defined as U. Therefore, stage one soil evaporation is given as

$$ES_1 = ES_0, \; 0 \leq SES_1 \leq U \qquad \ldots(34)$$

where SES_1 = the cumulative first stage soil evaporation, and ES_1 = the first stage soil evaporation.

Soil evaporation during the second stage is computed as

$$SES_2 = \delta t_2^{0.5} \qquad \ldots(35)$$

where SES_2 = the cumulative stage two soil evaporation, mm; δ = the soil type evaporation coefficient, mm $day^{-1/2}$; and t_2 = time in stage two evaporation, days. The daily soil evaporation in stage 2 is calculated by computing SES_2 for the given day and subtracting the SES_2 for the previous day. Ritchie (1972) presents more details and conditions for calculations when rainfall occurs. Irrigation was considered as rainfall in computing the soil evaporation.

Plant Evaporation

The plant evaporation was determined as a function of the leaf area index (LAI). If the LAI was less than 3.0 and the soil water was not limiting, the plant evaporation was

$$EP = E_o \; (-0.21 + 0.70 \sqrt{LAI}) \qquad \ldots(36)$$

for $0.1 \leq LAI \leq 3.0$. The sum of ES and EP was limited to E_o each day. For $LAI > 3.0$, and soil water not limiting,

$$EP = E_o - ES. \qquad \ldots(37)$$

If soil water is limiting,

$$EP = E_o \frac{SW}{LLE_o} \qquad \ldots(38)$$

where SW = the soil water content, mm, and LLE_o = the lower limit of soil water for potential evaporation, mm. Details are given by Richardson and Ritchie (1973).

Runoff

Runoff was computed with a modified SCS equation (Mockus, 1972). The initial abstraction was estimated as

$$IA = 116 - 0.41\ SW \qquad \text{...(39)}$$

where IA = the initial abstraction, mm. The SCS curve number was estimated as

$$CN = 50 + 0.15\ SW \qquad \text{...(40)}$$

where CN = the SCS curve number. The maximum potential difference between rainfall and runoff starting at the beginning of the storm, SP, in mm, is given by

$$SP = \frac{25400}{CN} - 254 \qquad \text{...(41)}$$

The runoff volume is then given by

$$Q = \frac{(R - IA)^2}{R + SP - IA} \qquad \text{...(42)}$$

where Q = the runoff volume, mm, and R = the rainfall, mm. Figure 22 shows the runoff function computed for various soil water levels and rainfall. This relationship resembles the SCS curves for different curve numbers (Mockus, 1972).

Soil Water Balance

The bulk soil profile was separated into two layers, an active layer from which plants could extract water and a passive layer from which only drainage could occur (either into or out of). If the upper layer became saturated, water could flow to the passive

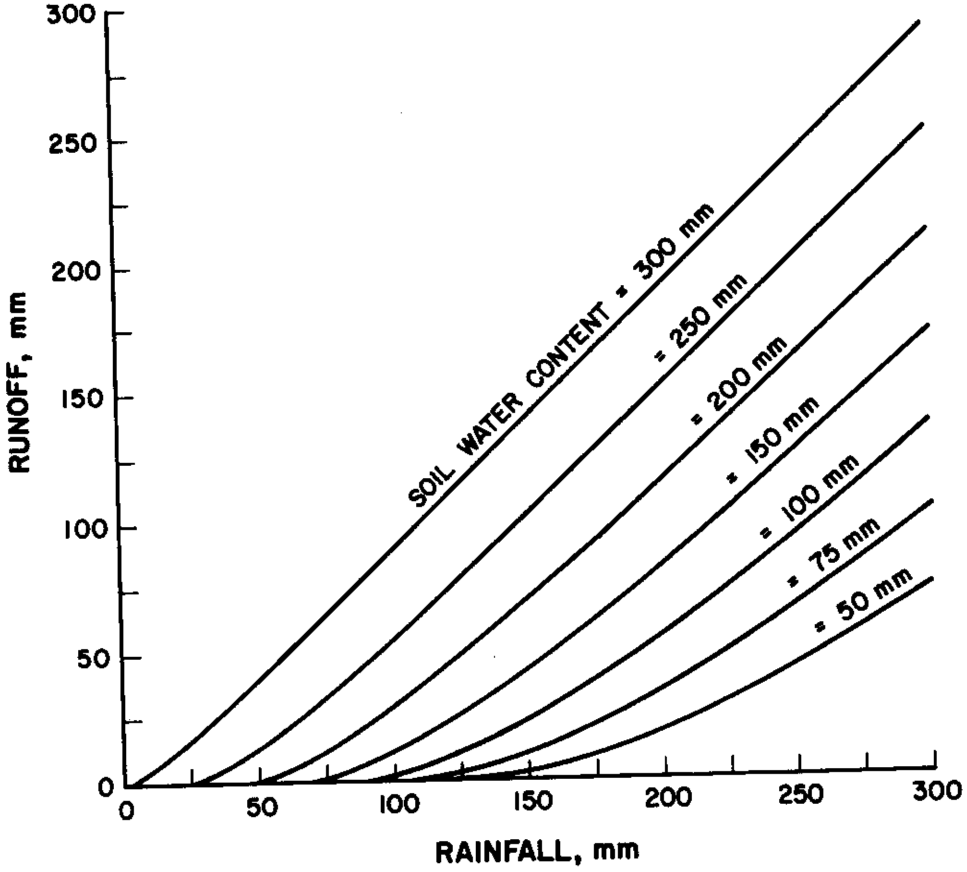

Figure 22. Runoff calculated as a function of soil water content and rainfall amount.

layer. If this drainage rate exceeded 25 mm day^{-1}, the excess was added to the runoff. The active layer was assumed to linearly increase in depth each day following germination until the depth of the active layer was equal to the entire profile depth. This could be considered a root zone extension without any water stress feedback. Each day the root zone increased, an additional small quantity of water became available to the plants.

The soil water balance equation for the active layer is

$$SWA_i = SWA_{i-1} + \frac{(DZ\ SWP_{i-1})}{DEPTH - DA_{i-1}} + R_i + IRR_i - DRA_i - Q_i - (ES + EP)_i \qquad ...(43)$$

where $DA_{i-1} < DEPTH$; SWA_i = the soil water content in the active layer on day i, mm; SWA_{i-1} = the soil water content in the active layer on day i-1; DZ = the root zone extension, mm day^{-1}; SWP_{i-1} = the soil water content in the passive layer on day i-1, mm; DEPTH = the total root zone depth, mm; DA_{i-1} = the depth of the active layer on day i-1 ($DA_i = DA_{i-1} + DZ$), mm; R = the rainfall on day i, mm; IRR = the irrigation on day i, mm; DRA_i = the active layer drainage on day i, mm; Q_i = the runoff on day i, mm; ES_i = the soil evaporation on day i, mm; and EP_i = the plant evaporation on day i, mm. The water balance for the passive layer is

$$SWP_i = SWP_{i-1} + DRA_i - (\frac{DZ\ SWP_{i-1}}{DEPTH - DA_{i-1}}) - DRP_i \qquad \ldots(44)$$

where DRP_i = the drainage from the passive layer on day i, mm; $DA_{i-1} < DEPTH$, and SWP_i = the soil water content in the passive layer on day i, mm.

Drainage

The drainage was assumed to be any excess water which had infiltrated over the upper soil water content (UL). If the drainage exceeded the saturated conductivity of 25 mm day^{-1} (Ritchie, et al., 1972b), the amount of drainage over 25 mm day^{-1} was added to the runoff. This provided a check on the runoff function.

Leaf Area Index

The leaf area index of grain sorghum was simulated using the model of Ritchie and Arkin (1973). The model is based solely on temperature with a slight feedback for aging and soil water deficit. The concept is based on the determinate nature of grain sorghum to produce only a predetermined number of leaves. The maximum size of each leaf is known. The rate at which these leaves appear and expand is determined by the temperature. The germination date and plant population are also inputs.

The maximum leaf sizes are shown in Table 10 for a medium-maturing sorghum variety. The leaf appearance rate is given by

$$Pl_i = Pl_{i-1} + 0.03\ T - 0.3 \qquad \text{...(45)}$$

where Pl = the leaf appearance rate, and T = the daily temperature, C. The actual leaf number which is appearing is the integer value of Pl. Therefore, a leaf cannot start expanding until the next integer value of Pl is reached, i.e., if Pl = 10.5, the eleventh leaf does not appear until Pl $\geq$ 11.0. The leaf expansion rate is given by

$$Al = 5.1\ (T - 12) \qquad \text{...(46)}$$

where Al = the leaf expansion rate, $cm^2\ day^{-1}$. Each leaf on the plant which is not fully expanded is assumed to expand at the rate given by equation (46). If the maximum area for a specific leaf is reached, the size is limited to that leaf's maximum size. If the last leaf (the flag leaf) has expanded to at least half of its maximum size and soil water is limiting, the bottom leaves begin to drop off the plant. The leaf aging effect is determined by multiplying the leaf size by 0.994 each day. This accounts for the gradual decrease in leaf area as a result of "die-back" around the leaf edges. The leaf area index is then determined by the sum of the leaf area per plant times the plant population (plants per cm^2 of land area).

Table 10. Maximum leaf size for a medium-maturing grain sorghum variety.

Maximum Leaf Size for Grain Sorghum					
Leaf Number	Leaf Size cm^2	Leaf Number	Leaf Size cm^2	Leaf Number	Leaf Size cm^2
1 (bottom)	10	7	30	13	375
2	12	8	40	14	410
3	14	9	60	15	425
4	16	10	100	16	400
5	18	11	200	17	350
6	20	12	300	18	240
				19 (Flag)	180

This model is quite simple, but works very well. In the future, a better feedback loop will be utilized between limiting soil water and leaf size*. This will involve decreasing the potential size of a leaf by the effect of a water deficit in a particular growth stage.

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*Personal communication with Dr. J. T. Ritchie and Dr. G. F. Arkin, August, 1974.

Simulation Parameters

To proceed to the simulation process, growth period delineation, irrigation levels, and soil water states had to be defined. These values were defined for maximum utility of the model and computational tractability.

Growth Periods

The grain sorghum growing season was divided into five growth periods of 25 days each. These correspond closely to growth periods determined in Chapter IV for College Station. Allowing the periods to be equal in length was for simplicity, but was by no means a requirement. Grain sorghum was assumed to germinate on April 10 each year (Julian day 100). A plant population of 22 plants per m^2 (approximately that used in the study at College Station) was utilized.

The first growth period from April 10 to May 4 was when the plants were just beginning to grow and ground cover was sparse. The leaf area index at the end of this period was about 0.3. The second period was from May 5 to May 29. This was the period of rapid vegetative growth and formation of reproductive tissues on the shoot apex. The leaf area index at the end of this period was approximately 3.5. As far as evaporation was concerned, the canopy was developed. This period corresponded to period I in the previous work (Chapters III and IV).

Growth period three began on May 30 and ended on June 23. The LAI had increased only to about 4.0. This period was defined as boot to bloom, which corresponded to period II in the previous work. The fourth growth period was started on June 24 and ended on July 18. The LAI had generally decreased to approximately 3.3. This period was defined as the milk to soft dough stage which corresponded to period III in the previous work. Growth period five began on July 19 and ended on August 13. The LAI had dropped to below 3.0. This period was grain maturity. Since no experimental data were available on the effects of water deficits in either period one or five, yield was assumed unaffected by water deficits in these periods.

Irrigation Levels

Irrigation was assumed to occur every day (high frequency irrigation). The irrigation quantity was a function of the actual water use. Fractions of this use rate were replenished. The irrigation quantities of 0, 0.25, 0.50, 0.75, and 1.00 times actual water use on the previous day were replenished. Naturally, if the irrigation ratio was allowed to be less than 1.0, the water required by the plant had to come either from rainfall, root extension, or soil water storage. If the irrigation ratio was 1.0, the plant was guaranteed that the soil water storage could not decrease.

Leaching was not considered in this study (irrigation ratios greater than 1.0).

Soil Water States

The soil water parameters used in this study were those given by Richardson and Ritchie (1973). In this soil approximately 75 percent of the available soil water can be extracted by a fully expanded canopy. If 75 percent of the available soil water in the active layer was extracted, the plant evaporation as determined by equation (36) or equation (37) was reduced by use of equation (38) where the ratio SW/LLE_0 was replaced by $(SWA)(DEPTH)/(DA)(LLE_0)$. The plant root zone was assumed to be 150 mm in depth and expanded at the rate of 32 mm day^{-1} to a maximum value of 1750 mm (Ritchie, et al., 1972a). The given initial soil water content was assumed uniformly distributed over depth.

The soil water content was divided into six discrete soil water states. The states were devised as given in Table 11. The states were divided in that manner since yield will only be affected when soil water becomes limiting. The states were simulated using 200, 150, 100, 75, 50, and 25 mm as the beginning soil water contents in each stage.

Table 11. Soil water state definitions.

Soil Water States	Soil Water Content, mm
1	$270 \geq SW > 175$
2	$175 \geq SW > 125$
3	$125 \geq SW > 87.5$
4	$87.5 \geq SW > 62.5$
5	$62.5 \geq SW > 37.5$
6	$37.5 \geq SW \geq 0$

CHAPTER VI

SIMULATION RESULTS

In this chapter, the results of the environmental simulation will be presented and discussed and a specific soil water balance simulation will be given for illustrative purposes. Then the results from the total simulation will be presented and discussed.

Environmental Simulation

The environmental model consisted of rainfall, temperature, and potential evaporation. The temperature and potential evaporation calculations were dependent on the rainfall model. The model parameters were computed as given in the previous chapter and two simulated records of 10 years and 30 years were computed. These simulated records were then compared to an independent record (data from the same site but for different years from which the simulated data were synthesized). It was believed that this last test would be more critical than simply comparing the simulated data to the original data set.

Rainfall

The Markov probabilities for a W/W and W/D sequence were computed for each week (beginning on March 1). A considerable cyclic variation was present in data particularly in the W/W data.

The period of this cycle was approximately 6 weeks. This trend was also present in similar data for Temple (Heermann, et al., 1971). Rather than use polynomial equations fitted to this data, the data were smoothed with a variable-weighted three point filter (weights of 0.25, 0.50, and 0.25) and tabulated for use as given in Table 12.

At the onset of this work, it was anticipated that the rainfall distribution parameters would depend on the rainfall of the previous day and the time of year. Three distributions--the incomplete gamma, the log-normal, and the square-root normal--were utilized to compare to the data. Based on the test of normality performed (previous chapter) the square-root transformation was not effective in normalizing the rainfall pattern. The log-normal transformation was effective in normalizing the data for every week. No tests were conducted to determine the degree of fit for the incomplete gamma distribution. However, when the different distribution parameters were plotted, the random scatter of the data indicated no apparent trend with regard to time of year or the influence of the rainfall for the previous day. Thus, the conclusion was that the rainfall amount was independent of the time of year and previous rainfall history. This was a significant simplification resulting in the use of a single probability distribution for the entire year.

Figures 23 and 24 show the assumed log-normal distribution with mean 4.2 mm plotted with the actual data for 1967 and 1958

Table 12. Rainfall probabilities (Markov) for rainfall greater than 0.25 mm at Temple, Texas.

Week	P(W/W)[1]	P(W/D)[2]
1	39.00	17.00
2	37.25	16.75
3	33.50	15.75
4	25.25	14.00
5	22.00	14.85
6	33.25	13.50
7	45.75	18.75
8	48.75	22.00
9	40.75	23.75
10	35.25	20.50
11	37.00	18.25
12	30.75	20.50
13	27.50	21.25
14	32.25	16.25
15	41.75	10.75
16	50.50	9.75
17	50.75	11.50
18	43.25	10.75
19	43.75	7.75
20	44.50	8.00

---

[1] Wet day given that the previous day was wet.
[2] Wet day given that the previous day was dry.

Table 12. (continued)

Week	P(W/W)	P(W/D)
21	41.75	9.50
22	45.75	8.00
23	50.75	7.25
24	45.75	9.75
25	40.75	12.25
26	39.25	13.50
27	35.50	15.25
28	38.75	16.00
29	52.25	15.00
30	54.75	13.75
31	40.00	12.75
32	32.50	12.50
33	37.75	11.00
34	45.25	10.00
35	49.50	11.25
36	48.75	12.50
37	42.75	14.00
38	38.00	14.75
39	42.25	13.75
40	49.50	13.25

Table 12. (continued)

Week	P(W/W)	P(W/D)
41	52.50	15.00
42	44.75	16.25
43	35.75	15.50
44	40.00	15.25
45	45.75	15.75
46	44.75	17.00
47	45.25	17.25
48	48.25	16.75
49	48.00	17.00
50	45.75	18.75
51	45.00	21.50
52	45.00	22.50

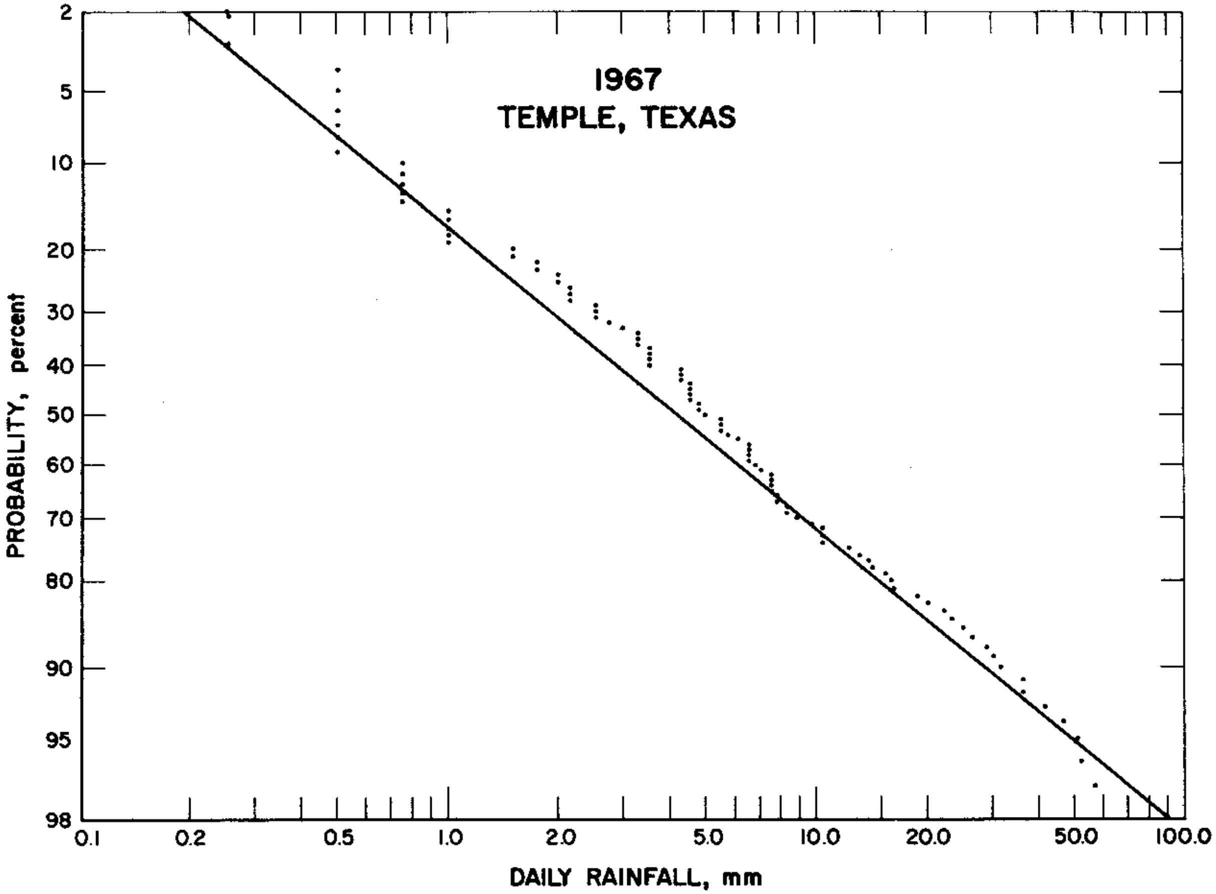


Figure 23. Cumulative daily rainfall probability distribution for 1967 at Temple, Texas.

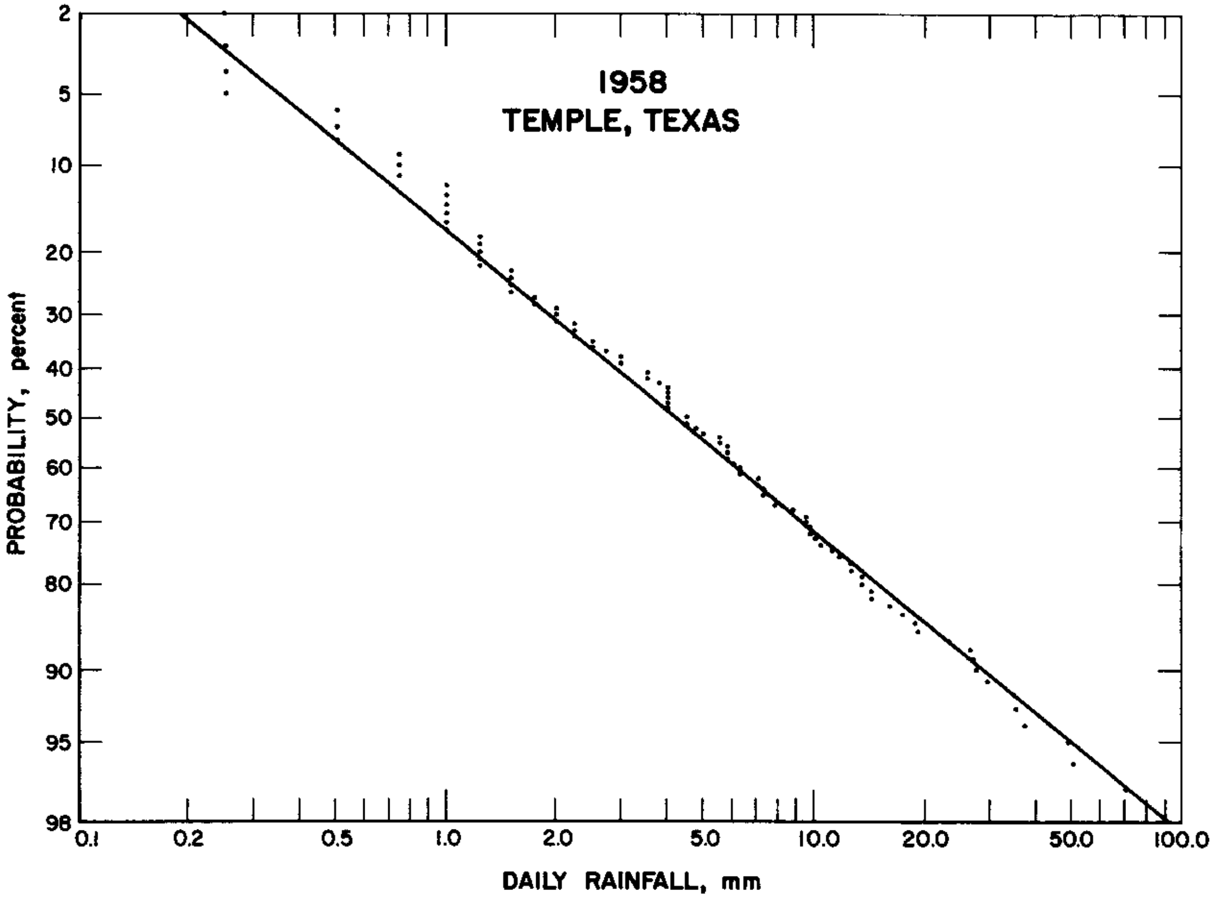


Figure 24. Cumulative daily rainfall probability distribution for 1958 at Temple, Texas.

(these years were chosen at random). This distribution fits the data adequately. Therefore, since the log-normal distribution fit the historical data adequately and is easier to simulate, the log-normal distribution was adopted for this work rather than the incomplete gamma distribution.

The rainfall events were simulated by drawing a random number from a uniform probability distribution. If this number was greater than the probability of dry day given the rainfall condition for the previous day, then a rainfall event occurred. The rainfall quantity was simulated by drawing a random rainfall logarithm from a log-normal distribution with mean 1.4274 and standard deviation 1.5044. The magnitude of the event was equal to the exponential of the random rainfall event.

Table 13 shows the results for the two simulated runs for 10 and 30 years. There was more deviation between the actual data and the simulated data on a monthly basis than was desired. However, the annual totals agreed fairly well with approximately a 5.3 percent decrease for the 10-year simulation and a 1.4 percent decrease for the 30-year simulation. Figure 25 shows the 10-year simulated length of wet runs (consecutive wet days) and actual data for a 10-year period. The number of wet runs were accurately simulated. Figure 26 shows the length of dry runs (consecutive dry days) for the 10-year simulation and the actual data for a 10-year period. In the simulated data, 676 rainfall events occurred

Table 13. Results of the rainfall simulation.

Month	Actual[1] Rainfall mm	10-Year Simulated Rainfall mm	30-Year Simulated Rainfall mm
March	57.4	61.7	71.6
April	104.8	85.1	96.5
May	109.3	96.3	64.3
June	73.3	82.6	48.8
July	48.3	68.3	36.6
August	44.8	37.3	48.5
September	83.2	37.1	67.8
October	67.6	61.5	61.5
November	68.0	46.2	53.1
December	68.4	36.8	55.4
January	65.1	56.6	119.9
February	58.8	134.4	112.8
Annual Average	849.0	803.9	836.8

[1] Thirty-nine year record (1915-1953) rainfall average (Bloodgood, et al., 1954).

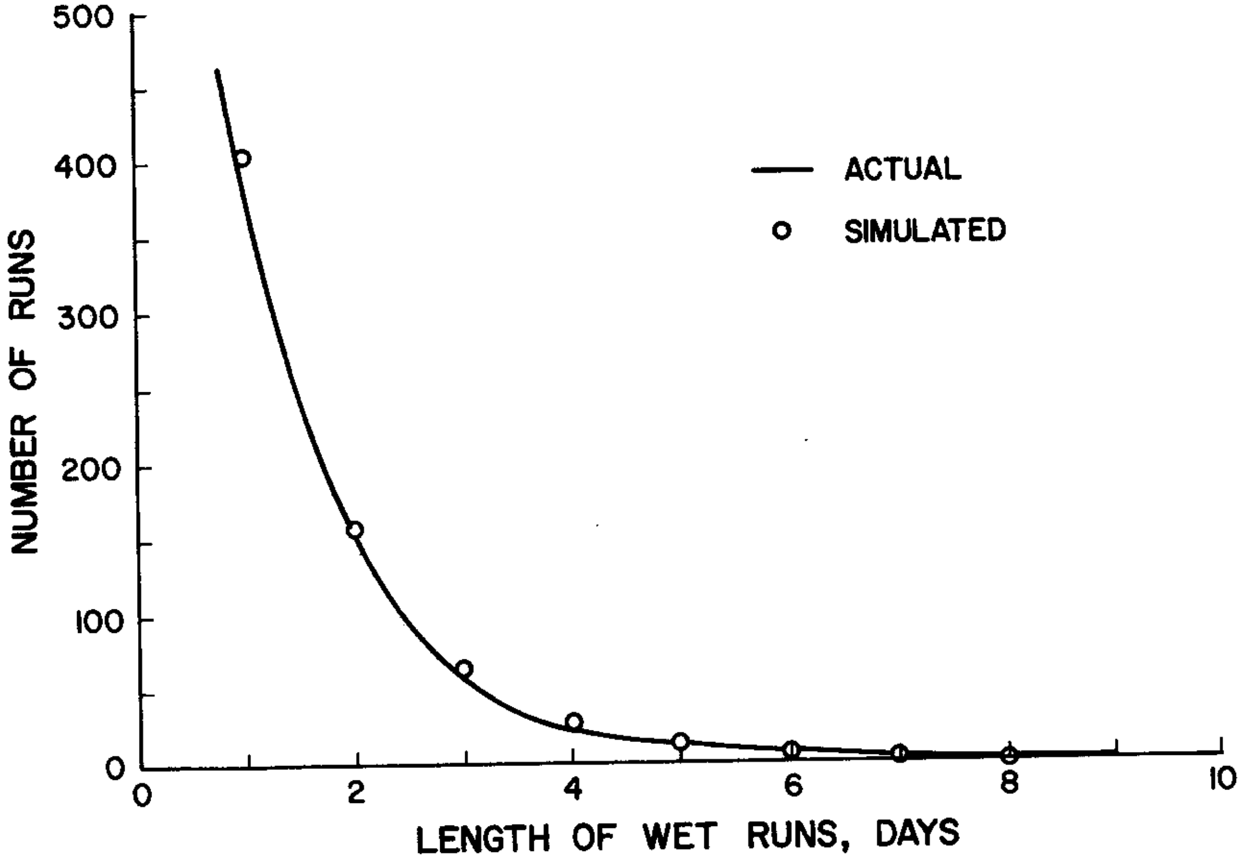


Figure 25. Comparison of simulated length of wet runs to actual data for ten years.

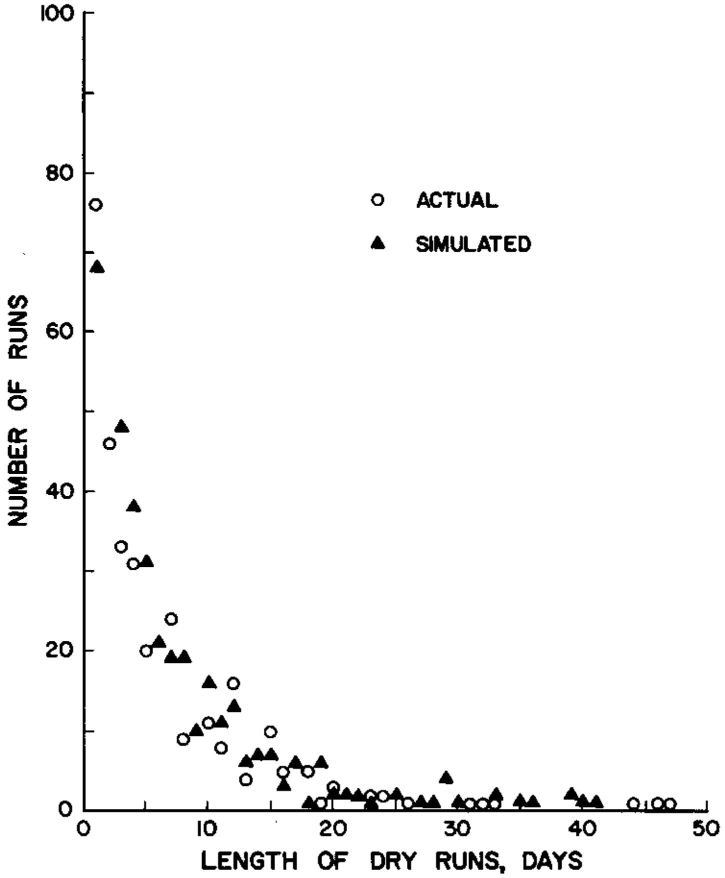


Figure 26. Comparison of simulated length of dry runs to actual data for ten years.

compared to 641 for the actual data in the 10-year period. The longest dry run in the simulated data is 41 days while the longest dry run in the actual data was 47 days in the 10-year period. Most of these errors were small and should decrease if more years were simulated.

Based on these results, the simulation of rainfall occurrence was satisfactory. Therefore, the differences in the monthly rainfall quantities is probably due to the use of only a single rainfall probability distribution. However, the error appears to be of the same order of magnitude as that found by Wyrick (1974).

Temperature

A similar procedure was used to develop the temperature model. Temperature was assumed to be normally distributed (Jones, et al., 1972). The dry days were separated from the wet days in each week and the mean and standard deviation of each class were determined. Then polynomial equations were fitted to these statistics. The following equations resulted:

$$TD(WN) = 0.556[(49.3 + 1.6WN + 1.2 \times 10^{-1}WN^2 - 7.2 \times 10^{-3}WN^3 + 7.6 \times 10^{-5}WN^4 + 1.2 \times 10^{-7}WN^5) - 32] \quad \ldots(47)$$

$$TW(WN) = 0.556[(54.4 + 1.7WN + 3.2 \times 10^{-1}WN^2 - 1.6 \times 10^{-3}WN^3 - 4.6 \times 10^{-5}WN^4 + 1.1 \times 10^{-6}WN^5) - 32] \quad \ldots(48)$$

$$S_d(WN) = 0.556[7.6 + 1.9 \times 10^{-1}WN - 7.7 \times 10^{-2}WN^2 + 4.1 \times 10^{-3}WN^3 - 7.7 \times 10^{-5}WN^4 + 4.8 \times 10^{-7}WN^5] \quad \ldots(49)$$

$$S_w(WN) = 0.556[8.6 - 1.4WN + 1.3 \times 10^{-1}WN^2 - 6.0 \times 10^{-3}WN^3 + 1.4 \times 10^{-4}WN^4 - 1.2 \times 10^{-6}WN^5] \quad \ldots(50)$$

where TD = the mean temperature of a dry day in week number WN (beginning on March 1), C; TW = the mean temperature of a wet day, C; S_d = the standard deviation of dry-day temperature, C; and S_w = the standard deviation of temperature on a wet day. Equations (47), (48), (49), and (50) were all highly significant with correlation coefficients of 0.97, 0.94, 0.99, and 0.93, respectively.

Table 14 shows the results of the 10- and 30-year simulations. The simulated annual average temperature was only 5 percent below the 39-year average. The monthly distribution was reasonably similar to the actual data, particularly from May through September, the major portion of the growing season.

Potential Evaporation

The potential evaporation model was similar in development to the temperature model. However, since no potential evaporation data were available for comparison, only the Class A pan evaporation simulation will be discussed here. Like temperature, pan evaporation was assumed normally distributed (Jones, et al., 1972). The D/D, D/W, W/D, and W/W sequences were separated and weekly means and standard deviations were computed for each class. Then polynomial

Table 14. Results of the temperature simulation.

Month	Actual[1] Temperature C	10-Year Simulated Temperature C	30-Year Simulated Temperature C
March	17.2	13.3	12.4
April	19.2	18.3	18.2
May	23.1	23.2	23.3
June	26.7	27.1	27.0
July	28.9	29.0	28.9
August	29.2	28.6	28.4
September	25.6	25.8	25.7
October	24.7	21.2	21.0
November	15.0	15.7	15.4
December	10.6	11.0	10.8
January	10.8	8.2	8.1
February	11.9	10.4	10.3
Annual Average	20.2	19.3	19.1

[1] Thirty-nine year record (1915-1953) of average of maximum and minimum temperature (Bloodgood, et al., 1954).

equations were fitted to these statistics. The following equations resulted:

$$\begin{aligned} EDD(WN) = 25.4[&1.8 \times 10^{-1} - 2.9 \times 10^{-3}WN + 1.9 \times 10^{-3}WN^2 - \\ &9.4 \times 10^{-5}WN^3 + 1.4 \times 10^{-6}WN^4 - 4.6 \times 10^{-9}WN^5] \end{aligned} \qquad \ldots(51)$$

$$\begin{aligned} EWD(WN) = 25.4[&1.5 \times 10^{-1} - 1.8 \times 10^{-2}WN + 3.3 \times 10^{-3}WN^2 - \\ &1.5 \times 10^{-4}WN^3 + 2.7 \times 10^{-6}WN^4 - 1.5 \times 10^{-8}WN^5] \end{aligned} \qquad \ldots(52)$$

$$\begin{aligned} EWW(WN) = 25.4[&7.5 \times 10^{-2} + 4.6 \times 10^{-3}WN + 6.6 \times 10^{-4}WN^2 - \\ &4.8 \times 10^{-5}WN^3 + 9.2 \times 10^{-7}WN^4 - 5.3 \times 10^{-9}WN^5] \end{aligned} \qquad \ldots(53)$$

$$\begin{aligned} EDW(WN) = 25.4[&7.7 \times 10^{-2} + 1.6 \times 10^{-2}WN + 2.0 \times 10^{-4}WN^2 - \\ &4.6 \times 10^{-5}WN^3 + 9.4 \times 10^{-7}WN^4 - 5.1 \times 10^{-9}WN^5] \end{aligned} \qquad \ldots(54)$$

where EDD = the mean daily pan evaporation on a D/D day in week WN, mm; EWD = the mean daily pan evaporation on a W/D day, mm; EWW = the mean daily pan evaporation on a W/W day, mm; and EDW = the mean daily pan evaporation on a D/W day, mm. Equations (51), (52), (53), and (54) were highly significant with correlation coefficients of 0.99, 0.95, 0.90, and 0.97, respectively. However, the standard deviation statistics were random and poorly correlated to the time of year. Therefore, the standard deviations S_{dd}, S_{wd}, S_{ww}, and S_{dw} were set equal to their mean values which were 1.9, 1.7, 1.9, and 1.8 mm, respectively. Since the standard deviations were rather large, the minimum simulated pan evaporation was set equal to zero. However, only a small number of zero evaporation rates were simulated (generally less than 2 per year).

Table 15 shows the simulated results for the 10- and 30-year periods. The simulated annual average pan evaporation was 4.8 and 5.7 percent above the actual data for the 30-year and 10-year simulations, respectively. However, the data agree closely between March and September (the major portion of the growing season).

Soil Water Balance Simulation Example

The soil water balance of grain sorghum at Temple, Texas, with a beginning soil water content of 200 mm distributed uniformly with irrigation levels of 0.50, 0.50, 0.75, 0.25, and 0.0 times the evaporation for the 5 respective growth stages, was simulated. This particular simulated growing season started with a large soil water content (or water inventory). The irrigation rates used in all the periods resulted in reduced soil water content when the rainfall input was small.

Tables 16 through 20 show the computer printed output for this example for the 5 growth periods, respectively. Appendix B gives

Table 15. Results of pan evaporation simulation.

Month	Actual Pan[1] Evaporation mm	10-Year Simulated Evaporation mm	30-Year Simulated Evaporation mm
March	135.4	132.3	128.5
April	159.3	155.4	151.4
May	183.9	200.4	197.9
June	225.3	222.3	228.1
July	256.5	255.3	258.8
August	257.8	248.4	247.1
September	192.8	204.5	199.9
October	150.9	164.6	167.1
November	99.6	118.6	119.9
December	72.6	93.2	85.9
January	68.3	87.6	86.9
February	83.6	111.0	104.1
Annual Average	1,886.0	1,993.6	1,975.6

[1] Thirty-nine year record (1915-1953) BPI data converted to Class A pan evaporation (Bloodgood, et al., 1954).

Table 16. Simulation of growth period one.

```
                                  YEAR =  1

                             GROWTH PERIOD =  1


                 PAN     AVG.  LEAF                                       POT.   POT.PANT    PLANT    SOIL      TOTAL  AVAIL.SOIL  SOIL
           JUL.  EVAP.   TEMP. AREA   RAIN  IRRIG. RUNOFF DRAINAGE   EVAP.    EVAP.     EVAP.    EVAP.     EVAP.     WATER    WATER
MO DAY YR DATE (MM/DAY)  (C)   INDEX (MM)   (MM)    (MM)    (MM)   (MM/DAY) (MM/DAY)  (MM/DAY) (MM/DAY) (MM/DAY)    (MM)     (MM)
--------------------------------------------------------------------------------------------------------------------------------------


 5  10  1 100    4.2     12.  0.00   0.0    0.5    0.0     0.0      3.8      0.0       0.0      1.6      1.6        20.     199.
 5  11  1 101    9.8     11.  0.00   0.0    0.8    0.0     0.0      8.8      0.0       0.0      1.7      1.7        22.     198.
 5  12  1 102    7.0     19.  0.02  30.2    0.9    0.0     0.0      6.3      0.0       0.0      6.2      6.2        38.     223.
 5  13  1 103    4.9     17.  0.02   0.0    3.1    0.0     0.0      4.4      0.0       0.0      3.8      3.8        41.     222.
 5  14  1 104    3.8     15.  0.02   0.0    1.9    0.0     0.0      3.4      0.0       0.0      2.4      2.4        45.     222.
 5  15  1 105    1.7     17.  0.02   0.0    1.2    0.0     0.0      1.6      0.0       0.0      1.6      1.6        48.     221.
 5  16  1 106    3.9     20.  0.05   0.0    0.8    0.0     0.0      3.5      0.0       0.0      2.9      2.9        50.     219.
 5  17  1 107    3.4     17.  0.05   2.6    1.5    0.0     0.0      3.0      0.0       0.0      1.9      1.9        56.     221.
 5  18  1 108    6.3     23.  0.05   0.0    0.9    0.0     0.0      5.7      0.0       0.0      3.1      3.1        58.     219.
 5  19  1 109    5.9     14.  0.05   0.0    1.6    0.0     0.0      5.3      0.0       0.0      2.9      2.9        60.     218.
 5  20  1 110    7.2     21.  0.08   0.0    1.5    0.0     0.0      6.5      0.0       0.0      1.2      1.2        65.     218.
 5  21  1 111    7.5     17.  0.08   0.0    0.6    0.0     0.0      6.7      0.0       0.0      1.7      1.7        67.     217.
 5  22  1 112    4.2     25.  0.08   0.0    0.8    0.0     0.0      3.8      0.0       0.0      1.8      1.8        70.     216.
 5  23  1 113    4.8     22.  0.11   0.0    0.9    0.0     0.0      4.3      0.1       0.1      1.7      1.8        73.     215.
 5  24  1 114    7.2     20.  0.11   0.0    0.9    0.0     0.0      6.5      0.2       0.2      1.7      1.8        77.     214.
 5  25  1 115    2.2     21.  0.11  39.1    0.9    1.8     0.0      2.0      0.0       0.0      1.9      1.9       102.     251.
 5  26  1 116    3.7     23.  0.15   1.0    1.0    0.0     0.0      3.4      0.2       0.2      3.2      3.4       105.     249.
 5  27  1 117    8.0     22.  0.15   0.0    1.7    0.0     0.0      7.2      0.4       0.4      5.9      6.3       105.     245.
 5  28  1 118    2.9     19.  0.15   0.0    3.1    0.0     0.0      2.6      0.2       0.2      2.2      2.3       110.     245.
 5  29  1 119    5.2     17.  0.19   0.0    1.2    0.0     0.0      4.6      0.4       0.4      2.9      3.3       112.     243.
 5  30  1 120    6.7     21.  0.19   0.0    1.7    0.0     0.0      6.0      0.6       0.6      3.6      4.2       114.     241.
 5  31  1 121    4.5     23.  0.19   1.4    2.1    0.0     0.0      4.0      0.4       0.4      2.8      3.2       119.     241.
 6   1  1 122    6.8     23.  0.23   8.2    1.6    0.0     0.0      6.1      0.5       0.5      5.6      6.1       127.     245.
 6   2  1 123    5.4     16.  0.23   0.0    3.1    0.0     0.0      5.8      0.7       0.7      4.6      5.3       129.     242.
 6   3  1 124    5.1     23.  0.23  21.3    2.6    0.5     0.0      4.6      0.4       0.4      4.2      4.6       147.     261.


PERIOD                                                                                                                          E/EO
TOTALS         133.3               104.0   36.8    2.3     0.0    120.0      4.2       4.2     73.0     77.1                    1.00


GROWTH PERIOD =  1
IRRIGATION AMOUNT = 0.50*ET (MM/DAY)
LEAF AREAS FOR LEAF 1(BOTTOM) TO 19(FLAG)
   0.0  10.7  12.8  14.9  17.1  19.3  29.5   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
```

Table 17. Simulation of growth period two.

```
                                        YEAR =  1

                                  GROWTH PERIOD =  2


                   PAN     AVG.  LEAF                                        POT.    POT.PANT     PLANT     SOIL        TOTAL  AVAIL.SOIL  SOIL
           JUL.   EVAP.   TEMP.  AREA   RAIN   IRRIG.  RUNOFF DRAINAGE      EVAP.     EVAP.       EVAP.     EVAP.       EVAP.    WATER     WATER
MO DAY YR  DATE (MM/DAY)   (C)  INDEX  (MM)    (MM)    (MM)     (MM)     (MM/DAY)  (MM/DAY)    (MM/DAY)  (MM/DAY)   (MM/DAY)    (MM)      (MM)
--------------------------------------------------------------------------------------------------------------------------------------------------

 6   4  1 125    4.2      22.  0.29  15.5     2.3     1.2      4.1        3.8        0.4         0.4       3.4        3.8       152.      270.
 6   5  1 126    5.4      22.  0.29   0.0     1.9     0.0      0.0        4.8        0.5         0.5       4.3        4.8       154.      267.
 6   6  1 127    8.0      24.  0.29   0.0     2.4     0.0      0.0        7.2        1.2         1.2       4.9        6.1       155.      263.
 6   7  1 128    4.0      20.  0.35   0.0     3.1     0.0      0.0        3.6        0.7         0.7       2.2        3.0       160.      263.
 6   8  1 129    9.6      19.  0.39   0.0     1.5     0.0      0.0        8.6        2.0         2.0       4.4        6.4       160.      259.
 6   9  1 130    7.9      26.  0.52   0.0     3.2     0.0      0.0        7.1        2.1         2.1       2.6        4.6       153.      257.
 6  10  1 131    6.5      24.  0.58   0.0     2.3     0.0      0.0        5.8        1.9         1.9       1.9        3.7       167.      256.
 6  11  1 132    8.0      25.  0.57   0.0     1.9     0.0      0.0        7.2        2.3         2.3       3.4        5.7       168.      252.
 6  12  1 133    5.2      23.  0.65   0.0     2.8     0.0      0.0        4.7        1.7         1.7       2.3        3.9       172.      251.
 6  13  1 134    6.7      23.  0.78  59.9     2.0    26.4     10.2        6.0        1.6         1.5       4.4        6.0       196.      270.
 6  14  1 135    2.1      23.  0.98   6.8     3.0     0.1      7.8        1.9        0.6         0.5       1.3        1.9       201.      270.
 6  15  1 136    3.6      24.  1.18   0.9     0.9     0.0      0.0        3.2        1.2         1.2       2.0        3.2       204.      269.
 6  16  1 137    4.9      19.  1.26   0.0     1.6     0.0      0.0        4.4        1.7         1.7       2.7        4.4       207.      266.
 6  17  1 138    4.5      22.  1.42   0.0     2.2     0.0      0.0        4.1        1.9         1.8       2.3        4.1       210.      264.
 6  18  1 139    7.1      22.  1.63   0.0     2.0     0.0      0.0        6.4        3.1         3.1       3.3        6.4       210.      260.
 6  19  1 140    7.6      24.  1.86   0.0     3.2     0.0      0.0        6.9        3.6         3.5       3.3        6.9       212.      256.
 6  20  1 141    8.5      23.  2.01   0.0     3.4     0.0      0.0        7.7        4.2         4.2       3.5        7.7       212.      252.
 6  21  1 142    2.7      24.  2.27   4.5     3.8     0.0      0.0        2.5        1.5         1.5       1.0        2.5       273.      258.
 6  22  1 143    8.1      28.  2.68   0.0     1.2     0.0      0.0        7.3        4.9         4.8       2.5        7.3       222.      251.
 6  23  1 144    9.0      30.  3.12   0.1     3.7     0.0      0.0        8.1        5.8         5.8       2.3        8.1       222.      247.
 6  24  1 145    9.1      28.  3.44   0.0     4.1     0.0      0.0        8.2        6.1         5.1       2.1        8.2       223.      243.
 6  25  1 146   10.2      23.  3.77   0.0     4.1     0.0      0.0        6.6        5.2         5.2       1.5        6.6       226.      241.
 6  26  1 147    4.4      25.  3.76   0.0     3.3     0.0      0.0        2.9        2.7         2.2       0.6        2.9       231.      241.
 6  27  1 148    9.5      26.  4.17   0.0     1.4     0.0      0.0        6.2        5.0         5.0       1.2        6.2       231.      236.
 6  28  1 149    6.3      24.  3.93   0.0     3.1     0.0      0.0        4.1        3.2         3.2       0.9        4.1       235.      235.


PERIOD                                                                                                                                       E/EO
TOTALS         163.2                 87.7    64.5    27.6     22.2      139.2       64.4        64.4      64.1      128.5                     1.00

GROWTH PERIOD =  2
IRRIGATION AMOUNT = 0.50*ET (MM/DAY)
LEAF AREAS FOR LEAF 1(BOTTOM) TO 19(FLAG)
   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0 361.7 400.2 419.9 330.3 195.2  63.1   0.0
```

Table 18. Simulation of growth period three.

```
                                          YEAR =  1

                                   GROWTH PERIOD =   3


                     PAN      AVG.  LEAF                                         POT.    POT.PANT     PLANT     SOIL        TOTAL   AVAIL.SOIL  SOIL
             JUL.   EVAP.     TEMP. AREA  RAIN   IRRIG. RUNOFF DRAINAGE   EVAP.     EVAP.      EVAP.     EVAP.       EVAP.     WATER     WATER
MO DAY YR DATE (MM/DAY)   (C)   INDEX (MM)    (MM)    (MM)     (MM)    (MM/DAY) (MM/DAY)  (MM/DAY) (MM/DAY) (MM/DAY)     (MM)       (MM)
--------------------------------------------------------------------------------------------------------------------------------------------

 6  29  1 150     5.1      25.  4.34   0.9     3.1    0.0      0.0      3.3       2.7        2.7      0.6        3.3       236.       236.
 6  30  1 151     9.2      27.  4.04   0.0     2.5    0.0      0.0      6.0       4.8        4.8      1.2        6.0       232.       232.
 7   1  1 152     9.0      25.  4.28   0.0     4.5    0.0      0.0      5.9       4.8        4.8      1.1        5.9       231.       231.
 7   2  1 153    10.7      28.  4.35   0.0     4.4    0.0      0.0      7.0       5.7        5.7      1.2        7.0       228.       228.
 7   3  1 154     5.9      27.  4.32   1.5     5.2    0.0      0.0      3.9       3.2        3.2      0.7        3.9       231.       231.
 7   4  1 155     2.7      27.  4.29   0.8     2.9    0.0      0.0      1.8       1.4        1.4      0.3        1.8       233.       233.
 7   5  1 156     7.0      27.  4.27   0.0     1.3    0.0      0.0      4.5       3.7        3.7      0.8        4.5       230.       230.
 7   6  1 157     7.5      27.  4.24   0.4     3.4    0.0      0.0      4.9       4.0        4.0      0.9        4.9       229.       229.
 7   7  1 158     4.6      25.  4.22   7.3     3.6    0.0      0.0      3.0       2.5        2.5      0.6        3.0       237.       237.
 7   8  1 159     6.5      28.  4.19   0.0     2.3    0.0      0.0      4.2       3.4        3.4      0.8        4.2       235.       235.
 7   9  1 160     6.0      28.  4.17   0.0     3.2    0.0      0.0      3.9       3.2        3.2      0.7        3.9       234.       234.
 7  10  1 161     9.2      26.  4.14   0.0     2.9    0.0      0.0      6.0       4.9        4.8      1.1        6.0       231.       231.
 7  11  1 162     5.3      30.  4.12   0.0     4.5    0.0      0.0      3.4       2.8        2.8      0.7        3.4       232.       232.
 7  12  1 163     8.6      30.  4.09   0.0     2.6    0.0      0.0      5.6       4.5        4.5      1.1        5.6       229.       229.
 7  13  1 164     5.8      27.  4.07   0.0     4.2    0.0      0.0      3.8       3.0        3.0      0.8        3.8       229.       229.
 7  14  1 165     7.9      28.  4.04   0.0     2.8    0.0      0.0      5.1       4.1        4.1      1.0        5.1       227.       227.
 7  15  1 166     8.8      27.  4.02   0.0     3.8    0.0      0.0      5.7       4.6        4.5      1.2        5.7       225.       225.
 7  16  1 167     4.5      29.  3.99   0.0     4.3    0.0      0.0      2.9       2.3        2.3      0.6        2.9       227.       227.
 7  17  1 168     9.2      26.  3.97   0.1     2.2    0.0      0.0      5.3       4.2        4.2      1.1        5.3       224.       224.
 7  18  1 169     7.4      28.  3.95   0.0     4.0    0.0      0.0      4.8       3.8        3.8      1.0        4.8       223.       223.
 7  19  1 170     8.5      30.  3.92   0.0     3.6    0.0      0.0      5.6       4.4        4.4      1.2        5.6       221.       221.
 7  20  1 171     4.8      27.  3.90   0.0     4.2    0.0      0.0      3.1       2.4        2.4      0.7        3.1       222.       222.
 7  21  1 172    10.8      29.  3.88   0.0     2.3    0.0      0.0      7.0       5.5        5.5      1.5        7.0       217.       217.
 7  22  1 173    10.2      26.  3.85   0.0     5.2    0.0      0.0      6.7       5.2        5.2      1.4        6.7       216.       216.
 7  23  1 174     5.7      28.  3.83   0.0     5.0    0.0      0.0      3.7       2.9        2.9      0.8        3.7       217.       217.


PERIOD                                                                                                                                E/EO
TOTALS          190.0                 11.0    88.1    0.0      0.0    117.0      94.0       94.0     23.0      117.0                  1.00

GROWTH PERIOD =   3
IRRIGATION AMOUNT = 0.75*ET (MM/DAY)
LEAF AREAS FOR LEAF 1(BOTTOM) TO 19(FLAG)
    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0 344.3 351.3 346.2 304.8 209.0 157.7
```

Table 19. Simulation of growth period four.

```
                                         YEAR =  1

                                   GROWTH PERIOD =  4


                 PAN    AVG.  LEAF                                        POT.   POT.PANT    PLANT    SOIL       TOTAL  AVAIL.SOIL   SOIL
         JUL.   EVAP.   TEMP. AREA  RAIN   IRRIG. RUNOFF DRAINAGE        EVAP.    EVAP.      EVAP.    EVAP.      EVAP.    WATER      WATER
MO DAY YR DATE (MM/DAY)  (C)  INDEX (MM)    (MM)   (MM)     (MM)      (MM/DAY) (MM/DAY)  (MM/DAY) (MM/DAY) (MM/DAY)    (MM)       (MM)
-----------------------------------------------------------------------------------------------------------------------------------------

 7  24  1 175    9.1     27.  3.81   0.0    0.9    0.0      0.0       5.9      4.6        4.6      1.3       5.9       212.       212.
 7  25  1 176    8.3     29.  3.78   0.0    1.5    0.0      0.0       5.4      4.2        4.2      1.2       5.4       208.       208.
 7  26  1 177    9.2     30.  3.76   0.0    1.3    0.0      0.0       6.0      4.6        4.5      1.3       6.0       204.       204.
 7  27  1 178   11.2     29.  3.74   0.0    1.5    0.0      0.0       7.3      5.7        5.7      1.6       7.3       199.       198.
 7  28  1 179    7.3     30.  3.72   0.0    1.8    0.0      0.0       4.8      3.7        3.7      1.1       4.8       195.       195.
 7  29  1 180   11.2     29.  3.69   0.0    1.2    0.0      0.0       7.3      5.6        5.6      1.7       7.3       189.       189.
 7  30  1 181   10.6     27.  3.67   0.0    1.8    0.0      0.0       6.9      5.3        5.3      1.6       6.9       184.       184.
 7  31  1 182    5.5     30.  3.65   0.0    1.7    0.0      0.0       3.6      2.8        2.8      0.8       3.6       182.       182.
 8   1  1 183    9.0     26.  3.63   0.3    0.9    0.0      0.0       5.9      4.5        4.5      1.4       5.9       177.       177.
 8   2  1 184    4.5     30.  3.61   0.0    1.5    0.0      0.0       2.9      2.2        2.2      0.7       2.9       176.       176.
 8   3  1 185   10.0     28.  3.58   0.0    0.7    0.0      0.0       6.5      4.9        4.9      1.5       6.5       170.       170.
 8   4  1 186    7.1     27.  3.56   0.0    1.6    0.0      0.0       4.6      3.5        3.5      1.1       4.6       167.       157.
 8   5  1 187   11.0     32.  3.54   0.0    1.2    0.0      0.0       7.2      5.4        5.4      1.7       7.2       161.       161.
 8   6  1 188   10.9     29.  3.52   0.0    1.8    0.0      0.0       7.1      5.3        5.3      1.7       7.1       156.       156.
 8   7  1 189   11.0     30.  3.50   0.0    1.8    0.0      0.0       7.1      5.4        5.4      1.8       7.1       150.       150.
 8   8  1 190    8.5     30.  3.48   0.0    1.8    0.0      0.0       5.6      4.2        4.2      1.4       5.6       147.       147.
 8   9  1 191    8.1     29.  3.46   0.0    1.4    0.0      0.0       5.3      3.9        3.9      1.3       5.3       143.       143.
 8  10  1 192    4.3     27.  3.44  19.0    1.3    0.0      0.0       2.8      2.1        2.1      0.7       2.8       160.       160.
 8  11  1 193    6.0     29.  3.42   0.0    0.7    0.0      0.0       3.9      2.9        2.9      1.0       3.9       157.       157.
 8  12  1 194    7.2     30.  3.40   0.0    1.0    0.0      0.0       4.7      3.5        3.5      1.2       4.7       153.       153.
 8  13  1 195   10.9     30.  3.37   0.0    1.2    0.0      0.0       7.1      5.2        5.2      1.9       7.1       147.       147.
 8  14  1 196   10.1     31.  3.35   0.0    1.8    0.0      0.0       6.6      4.8        4.8      1.7       6.6       143.       143.
 8  15  1 197    6.4     32.  3.33   0.0    1.6    0.0      0.0       4.1      3.0        3.0      1.1       4.1       140.       140.
 8  16  1 198   12.1     29.  3.31   0.0    1.0    0.0      0.0       7.9      5.8        5.8      2.1       7.9       133.       133.
 8  17  1 199   10.1     31.  3.29   0.0    2.0    0.0      0.0       6.6      4.8        4.8      1.8       6.6       129.       129.

PERIOD                                                                                                                              E/EO
TOTALS         219.7                19.3   35.0    0.0      0.0     142.8    107.9      107.9     34.9     142.8                    1.00


GROWTH PERIOD =  4
IRRIGATION AMOUNT = 0.25*ET (MM/DAY)
LEAF AREAS FOR LEAF 1(BOTTOM) TO 19(FLAG)
    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0 296.2 310.8 297.8 262.2 179.8 135.7
```

Table 20. Simulation of growth period five.

```
                                      YEAR =  1

                                  GROWTH PERIOD =  5


                   PAN      AVG.  LEAF                                         POT.   POT.PANT     PLANT    SOIL      TOTAL  AVAIL.SOIL  SOIL
            JUL.  EVAP.     TEMP. AREA   RAIN  IRRIG. RUNOFF DRAINAGE   EVAP.    EVAP.       EVAP.    EVAP.      EVAP.     WATER      WATER
MO DAY YR DATE (MM/DAY)  (C)  INDEX  (MM)   (MM)    (MM)     (MM)    (MM/DAY) (MM/DAY)  (MM/DAY) (MM/DAY) (MM/DAY)    (MM)       (MM)
---------------------------------------------------------------------------------------------------------------------------------------

 8  18  1 200    7.7      29.  3.27   0.0    0.0    0.0      0.0      5.0      3.6        3.6      1.4       5.0      124.       124.
 8  19  1 201    8.9      30.  3.26   0.0    0.0    0.0      0.0      5.8      4.2        4.2      1.6       5.8      118.       118.
 8  20  1 202    9.2      29.  3.24   0.0    0.0    0.0      0.0      6.0      4.7        4.7      1.3       6.0      112.       112.
 8  21  1 203   11.8      28.  3.22   0.0    0.0    0.0      0.0      7.6      5.5        5.5      2.1       7.6      104.       104.
 8  22  1 204   10.5      32.  3.20   0.0    0.0    0.0      0.0      6.8      5.1        5.1      1.8       6.8       97.        97.
 8  23  1 205    6.4      27.  3.18  17.2    0.0    0.0      0.0      4.2      3.0        3.0      1.2       4.2      110.       110.
 8  24  1 205    7.5      27.  3.16   0.0    0.0    0.0      0.0      4.9      3.5        3.5      1.4       4.9      105.       106.
 8  25  1 207    8.9      28.  3.14   0.0    0.0    0.0      0.0      5.8      4.1        4.1      1.7       5.8      100.       100.
 8  26  1 208   10.6      30.  3.12   0.0    0.0    0.0      0.0      6.9      5.0        5.0      1.9       6.9       93.        93.
 8  27  1 209   12.1      33.  3.10   0.0    0.0    0.0      0.0      7.9      5.6        5.6      2.3       7.9       85.        85.
 8  28  1 210    6.9      31.  3.08   0.0    0.0    0.0      0.0      4.5      3.2        3.2      1.3       4.5       81.        81.
 8  29  1 211    9.5      28.  3.06   0.0    0.0    0.0      0.0      6.2      4.8        4.8      1.4       5.2       74.        74.
 8  30  1 212    6.4      27.  3.05   0.0    0.0    0.0      0.0      4.2      3.1        3.1      1.1       4.2       70.        70.
 8  31  1 213    8.2      29.  3.03   4.6    0.0    0.0      0.0      5.3      3.7        3.7      1.6       5.3       70.        70.
 9   1  1 214    4.9      28.  3.01   0.9    0.0    0.0      0.0      3.1      2.2        2.2      0.9       3.1       67.        67.
 9   2  1 215    5.5      29.  2.99   0.0    0.0    0.0      0.0      3.6      2.5        2.5      1.1       3.6       64.        64.
 9   3  1 216    7.3      31.  2.97   0.0    0.0    0.0      0.0      4.7      3.5        3.4      1.2       4.6       59.        59.
 9   4  1 217    5.9      28.  2.96   0.0    0.0    0.0      0.0      4.5      3.5        3.0      1.0       4.1       55.        55.
 9   5  1 218    9.1      30.  2.94   0.0    0.0    0.0      0.0      5.9      5.1        4.2      0.9       5.0       50.        50.
 9   6  1 219    9.8      28.  2.92   0.0    0.0    0.0      0.0      6.4      5.6        4.1      0.8       4.9       45.        45.
 9   7  1 220   10.5      28.  2.90   8.9    0.0    0.0      0.0      6.8      5.3        3.2      1.5       4.7       49.        49.
 9   8  1 221    6.6      31.  2.89   0.0    0.0    0.0      0.0      4.3      2.9        1.7      1.4       3.2       46.        46.
 9   9  1 222    7.6      29.  2.87   0.0    0.0    0.0      0.0      4.9      3.4        1.9      1.6       3.5       42.        42.
 9  10  1 223    6.5      28.  2.85   0.0    0.0    0.0      0.0      4.2      2.9        1.4      1.4       2.8       40.        40.
 9  11  1 224    5.4      31.  2.83   0.0    0.0    0.0      0.0      4.2      3.1        1.5      1.1       2.5       37.        37.


PERIOD                                                                                                                        E/EO
TOTALS         205.7                 31.6    0.0    0.0      0.0    133.7     98.9       88.3     34.8     123.1                0.92


GROWTH PERIOD =  5
IRRIGATION AMOUNT = 0.00*ET (MM/DAY)
LEAF AREAS FOR LEAF 1(BOTTOM) TO 19(FLAG)
   0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0    0.0 254.9 257.4 256.2 225.6 154.7 115.7
```

a complete listing of the program. The simulated totals of rainfall, potential evaporation, actual evaporation, runoff, drainage, and irrigation are given in Table 21. The first period received 104 mm of rain of which 2 mm were runoff. The plant size had increased to a LAI of 0.23. The soil evaporation was the dominating evaporative loss (the soil evaporation was approximately 95 percent of the total evaporation). Note the trend of soil evaporation after the rainfall events. During this period the soil water content (water inventory) increased by 61 mm even though the total irrigation quantity was less than 50 percent of the actual water use.

The second period received 88 mm of rainfall with 28 mm of runoff and 22 mm of drainage. The LAI increased from 0.23 to 3.93 indicating that with soil water not limiting, the potential evaporation rate was reached. The soil water content decreased by 35 mm during this period. The LAI data indicated that the next leaf to appear was leaf number 19 (the flag leaf). The soil water content decreased by 19 mm in the third period due to the small amount of rain (11 mm) even though the irrigation level was large (0.75 ET). The LAI reached a maximum value of 4.29. Also, the LAI data showed that the last leaf was fully expanded, indicating that the sorghum head was present and probably blooming.

The fourth period received 35 mm of rainfall with no runoff. The soil water content was being depleted at a rapid rate since the irrigation input was small (0.25 ET). The soil water content

Table 21. Simulated seasonal totals of water balance components.

Water Balance Component	Simulated Seasonal Total mm
Rainfall	253.6
Runoff	29.9
Potential Evaporation	652.7
Actual Evaporation	588.5
Irrigation	224.4
Drainage	32.2
Soil Water Content Change	162.6

decreased by 83 mm. The LAI had declined to 3.29. During period five, no irrigation occurred, so the soil water content declined to 37 mm due to the fact that only 32 mm of rain occurred. The LAI further declined to 2.83. A water deficit of 0.92 (E/E_0) developed in period five but would not affect yield. In this particular year with the given irrigation levels, practically no water deficits developed.

Simulation Results

The simulation results were summarized by the "expected" soil water state transition probabilities (Markov process) and the "expected" yield, evaporation, and irrigation for each growth period, each soil water state, and each irrigation level. The simulated cases (5 growth periods, 6 soil water states, and 5 irrigation decisions) were simulated for 30 years. The "expected" values which result do not refer to any specific year, but depict the average value which would result over the 30-year period.

Table 22 gives the Markov probabilities which resulted from the simulation. These data show the chances for changing from a specific soil water state at the beginning of a certain growth period to any other soil water state at the end of that period as a result of a specified irrigation decision. Several points of interest in these results are that in growth stage one, soil water state one ($270 \geq SW > 175$ mm) is an "absorbing state". Given that the soil water content is in that state at the beginning of stage one (no matter how the soil water content got to that state), there is <u>no</u> chance of leaving that state regardless of the irrigation decision. This results because the evaporation is small in this stage. For each growth stage soil water content state one is an absorbing state for irrigation level 5 (1.00 ET) because the soil water content cannot decrease with this irrigation level.

As an example of the use of Table 22, what would be the chance of going from soil water state 3 ($125 \geq SW > 87.5$ mm) to soil water state 6 ($37.5 \geq SW \geq 0$) in growth stage 3 using irrigation level 2 (0.25 ET)? Table 22 shows this probability to be 50 percent (answer marked with *). What are the chances for the same condition

Table 22. Soil water state transition (Markov) probabilities.

Growth Period 1

Irrigation Level 1
(0.00 ET) [Dryland]

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	30.0	70.0	0	0	0	0
3	6.7	13.3	53.3	26.7	0	0
4	6.7	13.3	30.0	40.0	10.0	0
5	0	0	13.3	33.3	46.7	6.7
6	0	0	3.3	0	30.0	66.7

Irrigation Level 2
(0.25 ET)

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	33.3	66.7	0	0	0	0
3	10.0	23.3	43.3	23.3	0	0
4	0	13.3	23.3	36.7	26.7	0
5	3.3	6.7	10.0	26.7	53.3	0
6	0	3.3	13.3	13.3	23.3	46.7

Irrigation Level 3
(0.50 ET)

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	20.0	80.0	0	0	0	0
3	20.0	13.3	53.3	13.3	0	0
4	0	13.3	30.0	53.3	3.3	0
5	0	13.3	20.0	20.0	43.3	3.3
6	3.3	3.3	10.0	13.3	30.0	40.0

Irrigation Level 4
(0.75 ET)

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	53.3	46.7	0	0	0	0
3	13.3	26.7	60.0	0	0	0
4	10.0	20.0	46.7	23.3	0	0
5	6.7	13.3	13.3	10.0	56.7	0
6	0	10.0	20.0	10.0	23.3	36.7

Irrigation Level 5
(1.00 ET)

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	50.0	50.0	0	0	0	0
3	33.3	46.7	20.0	0	0	0
4	20.0	13.3	53.3	13.3	0	0
5	3.3	16.7	40.0	23.3	16.7	0
6	3.3	6.7	26.7	20.0	33.3	10.0

Table 22. (continued)

Growth Period 2

Irrigation Level 1
(0.00 ET) [Dryland]

SW_2 SW_1	1	2	3	4	5	6
1	43.3	30.0	26.7	0	0	0
2	20.0	26.7	20.0	26.7	6.7	0
3	6.7	13.3	10.0	6.7	50.0	13.3
4	0	6.7	20.0	6.7	36.7	30.0
5	10.0	0	3.3	10.0	16.7	60.0
6	0	3.3	6.7	13.3	23.3	53.3

Irrigation Level 2
(0.25 ET)

SW_2 SW_1	1	2	3	4	5	6
1	56.7	43.3	0	0	0	0
2	40.0	26.7	16.7	16.7	0	0
3	3.3	13.3	16.7	13.3	40.0	13.3
4	6.7	6.7	13.3	16.7	33.3	23.3
5	3.3	0	13.3	6.7	36.7	40.0
6	3.3	10.0	10.0	10.0	30.0	36.7

Irrigation Level 3
(0.50 ET)

SW_2 SW_1	1	2	3	4	5	6
1	86.7	13.3	0	0	0	0
2	33.3	30.0	36.7	0	0	0
3	16.7	6.7	26.7	26.7	23.3	0
4	13.3	16.7	3.3	20.0	36.7	10.0
5	6.7	13.3	16.7	13.3	36.7	13.3
6	6.7	10.0	13.3	13.3	26.7	30.0

Irrigation Level 4
(0.75 ET)

SW_2 SW_1	1	2	3	4	5	6
1	96.7	3.3	0	0	0	0
2	56.7	33.3	10.0	0	0	0
3	13.3	36.7	36.7	13.3	0	0
4	20.0	13.3	23.3	33.3	10.0	0
5	10.0	6.7	13.3	26.7	36.7	13.3
6	6.7	10.0	20.0	20.0	26.7	16.7

Irrigation Level 5
(1.00 ET)

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	73.3	26.7	0	0	0	0
3	46.7	36.7	16.7	0	0	0
4	23.3	16.7	40.0	20.0	0	0
5	10.0	46.7	23.3	20.0	0	0
6	6.7	10.0	16.7	36.7	23.3	6.7

Table 22. (continued)

Growth Period 3

Irrigation Level 1
(0.00 ET) [Dryland]

SW_2 SW_1	1	2	3	4	5	6
1	13.3	10.0	30.0	40.0	6.7	0
2	13.3	6.7	20.0	26.7	26.7	6.7
3	6.7	6.7	3.3	6.7	13.3	63.3**
4	3.3	6.7	3.3	6.7	23.3	56.7
5	10.0	0	0	3.3	10.0	76.7
6	3.3	0	3.3	3.3	10.0	80.0

Irrigation Level 2
(0.25 ET)

SW_2 SW_1	1	2	3	4	5	6
1	20.0	36.7	43.3	0	0	0
2	10.0	10.0	13.3	36.7	30.0	0
3	6.7	0	0	26.7	16.7	50.0*
4	6.7	0	3.3	10.0	30.0	50.0
5	0	0	10.0	3.3	23.3	63.3
6	0	0	10.0	6.7	23.3	60.0

Irrigation Level 3
(0.50 ET)

SW_2 SW_1	1	2	3	4	5	6
1	53.3	46.7	0	0	0	0
2	26.7	10.0	53.3	10.0	0	0
3	6.7	6.7	16.7	30.0	40.0	0
4	10.0	10.0	3.3	20.0	40.0	16.7
5	3.3	10.0	3.3	13.3	33.3	36.7
6	3.3	0	6.7	6.7	13.3	70.0

Irrigation Level 4
(0.75 ET)

SW_2 SW_1	1	2	3	4	5	6
1	76.7	23.3	0	0	0	0
2	20.0	53.3	26.7	0	0	0
3	10.0	26.7	33.3	30.0	0	0
4	10.0	13.3	20.0	20.0	36.7	0
5	16.7	3.3	20.0	16.7	26.7	16.7
6	3.3	6.7	3.3	13.3	16.7	56.7

Irrigation Level 5
(1.00 ET)

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	43.3	56.7	0	0	0	0
3	10.0	33.3	56.7	0	0	0
4	20.0	16.7	36.7	26.7	0	0
5	13.3	10.0	20.0	23.3	33.3	0
6	3.3	13.3	13.3	16.7	30.0	23.3

Table 22. (continued)

Growth Period 4

Irrigation Level 1
(0.00 ET) [Dryland]

SW_1 / SW_2	1	2	3	4	5	6
1	3.3	20.0	30.0	33.3	13.3	0
2	3.3	6.7	6.7	20.0	30.0	33.3
3	0	0	6.7	3.3	13.3	76.7
4	0	0	6.7	3.3	10.0	80.0
5	6.7	6.7	3.3	6.7	10.0	66.7
6	0	0	3.3	0	6.7	90.0

Irrigation Level 2
(0.25 ET)

SW_1 / SW_2	1	2	3	4	5	6
1	13.3	40.0	46.7	0	0	0
2	3.3	13.3	10.0	26.7	46.7	0
3	0	0	3.3	16.7	6.7	73.3
4	0	3.3	3.3	6.7	10.0	76.7
5	0	3.3	3.3	6.7	10.0	76.7
6	0	0	3.3	0	16.7	80.0

Irrigation Level 3
(0.50 ET)

SW_1 / SW_2	1	2	3	4	5	6
1	23.3	76.7	0	0	0	0
2	13.3	23.3	33.3	30.0	0	0
3	3.3	13.3	20.0	16.7	36.7	10.0
4	13.3	6.7	20.0	6.7	26.7	26.7
5	0	3.3	20.0	3.3	30.0	43.3
6	0	3.3	0	6.7	23.3	66.7

Irrigation Level 4
(0.75 ET)

SW_1 / SW_2	1	2	3	4	5	6
1	83.3	16.7	0	0	0	0
2	16.7	43.3	40.0	0	0	0
3	13.3	16.7	36.7	33.3	0	0
4	0	3.3	16.7	36.7	43.3	0
5	6.7	3.3	10.0	13.3	20.0	26.7
6	0	0	10.0	10.0	30.0	50.0

Irrigation Level 5
(1.00 ET)

SW_1 / SW_2	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	43.3	56.7	0	0	0	0
3	20.0	20.0	60.0	0	0	0
4	10.0	16.7	50.0	23.3	0	0
5	13.3	3.3	26.7	13.3	43.3	0
6	3.3	0	16.7	16.7	53.3	10.0

Table 22. (continued)

Growth Period 5

Irrigation Level 1
(0.0 ET) [Dryland]

SW_2 SW_1	1	2	3	4	5	6
1	10.0	13.3	30.0	46.7	0	0
2	3.3	6.7	6.7	20.0	33.3	30.0
3	0	0	13.3	16.7	10.0	60.0
4	3.3	0	0	6.7	3.3	86.7
5	6.7	3.3	0	3.3	10.0	76.7
6	0	3.3	0	3.3	10.0	83.3

Irrigation Level 2
(0.25 ET)

SW_2 SW_1	1	2	3	4	5	6
1	10.0	40.0	43.3	6.7	0	0
2	3.3	13.3	20.0	26.7	36.7	0
3	13.3	0	10.0	3.3	20.0	53.3
4	6.7	3.3	3.3	6.7	20.0	60.0
5	0	3.3	3.3	10.0	10.0	73.3
6	3.3	0	0	3.3	6.7	86.7

Irrigation Level 3
(0.50 ET)

SW_2 SW_1	1	2	3	4	5	6
1	30.0	70.0	0	0	0	0
2	6.7	23.3	36.7	33.3	0	0
3	6.7	3.3	16.7	10.0	53.3	10.0
4	6.7	10.0	6.7	10.0	26.7	30.0
5	0	6.7	3.3	10.0	33.3	46.7
6	0	0	6.7	10.0	10.0	73.3

Irrigation Level 4
(0.75 ET)

SW_2 SW_1	1	2	3	4	5	6
1	73.3	26.7	0	0	0	0
2	23.3	43.3	33.3	0	0	0
3	13.3	10.0	33.3	43.3	0	0
4	13.3	13.3	13.3	26.7	33.3	0
5	3.3	6.7	13.3	20.0	20.0	36.7
6	0	10.0	3.3	3.3	36.7	46.7

Irrigation Level 5
(1.00 ET)

SW_2 SW_1	1	2	3	4	5	6
1	100.0	0	0	0	0	0
2	60.0	40.0	0	0	0	0
3	13.3	26.7	60.0	0	0	0
4	20.0	13.3	36.7	30.0	0	0
5	3.3	3.3	20.0	26.7	46.7	0
6	0	0	10.0	16.7	50.0	23.3

happening under dryland? Table 22, page 126, shows the probability to be 63.3 percent (answer marked with **). Thus, the simulation model has allowed many of the uncertainties of irrigation water allocation to be placed in a single table which described the expected changes in soil water content.

Table 23 gives the expected relative yield, relative evaporation, water use (evaporation), and irrigation water requirement for each growth period, soil water level, and irrigation decision level, I. Since no data were available to properly define the yield reduction caused by water deficits in growth periods one and five, the reduction in yield was assumed to be zero. The potential evaporation value used to compute the E/E_0 ratio were 117, 141, 143, 147, and 150 mm for the respective growth periods. These values were the largest evaporation quantities simulated for growth period one through five, respectively. The conditions necessary for these maximum rates to occur are adequate soil water levels, irrigation, and large potential evaporation (in this simulation corresponds to small rainfall and large pan evaporation). These define the consequences of the expected soil water content changes which were given in Table 22 (page 124). For example, if the soil water content was 75 mm at the beginning of growth stage 3 (Table 23, page 132), the expected relative yields for this period would be 0.75, 0.79, 0.86, 0.91, and 0.91 for the respective irrigation treatments of 0.0, 0.25, 0.50, 0.75, and 1.00 times the actual water use rate.

Table 23. Expected relative yield, relative evaporation, water use, and irrigation water requirement.

Growth Period 1

SW = 25 mm					SW = 50 mm				
I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm	I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.23	27	0	1	1.00	0.28	33	0
2	1.00	0.35	41	10	2	1.00	0.40	47	12
3	1.00	0.42	49	25	3	1.00	0.49	57	28
4	1.00	0.57	67	50	4	1.00	0.58	68	51
5	1.00	0.87	102	102	5	1.00	0.92	108	108
SW = 75 mm					SW = 100 mm				
I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm	I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.32	37	0	1	1.00	0.30	35	0
2	1.00	0.42	49	12	2	1.00	0.37	43	11
3	1.00	0.48	56	28	3	1.00	0.50	59	30
4	1.00	0.69	80	60	4	1.00	0.63	74	56
5	1.00	0.92	108	108	5	1.00	0.95	111	111
SW = 150 mm					SW = 200 mm				
I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm	I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.32	37	0	1	1.00	0.30	35	0
2	1.00	0.42	49	12	2	1.00	0.41	48	12
3	1.00	0.48	56	28	3	1.00	0.51	60	30
4	1.00	0.62	72	54	4	1.00	0.62	72	54
5	1.00	0.91	106	106	5	1.00	0.92	108	108

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual evaporation.

Table 23. (continued)

Growth Period 2

SW = 25 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.90	0.42	59	0
2	0.92	0.51	72	18
3	0.93	0.55	78	39
4	0.95	0.64	90	68
5	0.96	0.70	99	99

SW = 50 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.93	0.53	75	0
2	0.95	0.64	90	23
3	0.96	0.71	100	50
4	0.97	0.74	105	79
5	0.99	0.89	125	125

SW = 75 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.94	0.61	86	0
2	0.96	0.73	103	26
3	0.97	0.80	113	56
4	0.98	0.87	122	92
5	0.99	0.94	133	133

SW = 100 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.96	0.70	99	0
2	0.97	0.75	106	27
3	0.98	0.84	118	59
4	0.99	0.89	126	94
5	0.99	0.91	128	128

SW = 150 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.96	0.72	102	0
2	0.98	0.80	113	28
3	0.98	0.84	118	59
4	0.99	0.91	129	96
5	1.00	0.96	135	135

SW = 200 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.96	0.73	103	0
2	0.97	0.78	110	28
3	0.98	0.84	118	59
4	0.99	0.89	125	94
5	1.00	0.95	134	134

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual evaporation.

Table 23. (continued)

Growth Period 3

SW = 25 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.49	0.32	46	0
2	0.56	0.38	55	14
3	0.55	0.38	54	27
4	0.57	0.41	58	43
5	0.69	0.54	77	77

SW = 50 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.61	0.44	63	0
2	0.65	0.50	71	18
3	0.75	0.62	88	44
4	0.81	0.71	101	76
5	0.86	0.77	110	110

SW = 75 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.75	0.62	89	0
2	0.79	0.68	97	24
3	0.86	0.78	111	56
4	0.91	0.85	122	91
5	0.91	0.86	123	123

SW = 100 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.80	0.69	99	0
2	0.85	0.76	109	27
3	0.91	0.85	122	61
4	0.92	0.87	124	93
5	0.93	0.88	126	126

SW = 150 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.91	0.85	122	0
2	0.93	0.88	126	32
3	0.92	0.87	125	63
4	0.93	0.88	126	95
5	0.93	0.88	126	126

SW = 200 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.93	0.90	128	0
2	0.92	0.87	124	31
3	0.92	0.87	124	62
4	0.93	0.88	126	95
5	0.92	0.87	125	125

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual evaporation.

Table 23. (continued)

Growth Period 4

SW = 25 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.78	0.27	39	0
2	0.82	0.34	50	12
3	0.84	0.39	57	29
4	0.87	0.46	68	51
5	0.90	0.57	84	84

SW = 50 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.87	0.46	68	0
2	0.88	0.50	73	18
3	0.93	0.65	96	48
4	0.93	0.68	100	75
5	0.96	0.78	115	115

SW = 75 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.90	0.57	84	0
2	0.92	0.64	94	24
3	0.96	0.79	116	58
4	0.97	0.86	127	95
5	0.98	0.90	132	132

SW = 100 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.93	0.69	102	0
2	0.95	0.75	110	28
3	0.97	0.86	126	63
4	0.98	0.88	130	97
5	0.98	0.89	131	131

SW = 150 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.97	0.85	125	0
2	0.98	0.91	134	34
3	0.98	0.89	131	66
4	0.98	0.92	135	101
5	0.98	0.91	133	133

SW = 200 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	0.98	0.91	134	0
2	0.98	0.90	132	33
3	0.98	0.91	133	66
4	0.98	0.89	131	98
5	0.98	0.88	130	130

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual evaporation.

Table 23. (continued)

Growth Period 5

SW = 25 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.29	43	0
2	1.00	0.26	39	10
3	1.00	0.31	47	24
4	1.00	0.43	65	49
5	1.00	0.51	76	76

SW = 50 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.41	62	0
2	1.00	0.47	71	18
3	1.00	0.55	82	41
4	1.00	0.66	99	74
5	1.00	0.75	112	112

SW = 75 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.52	78	0
2	1.00	0.63	94	24
3	1.00	0.75	112	56
4	1.00	0.83	125	94
5	1.00	0.89	134	134

SW = 100 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.66	99	0
2	1.00	0.77	115	29
3	1.00	0.84	126	63
4	1.00	0.87	131	98
5	1.00	0.87	131	131

SW = 150 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.84	126	0
2	1.00	0.88	132	33
3	1.00	0.88	132	66
4	1.00	0.88	132	99
5	1.00	0.86	129	129

SW = 200 mm

I	$\frac{Y}{Y_o}$	$\frac{E}{E_o}$	E mm	IRR mm
1	1.00	0.88	132	0
2	1.00	0.89	133	33
3	1.00	0.87	131	65
4	1.00	0.87	130	98
5	1.00	0.87	131	131

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual evaporation.

In this case, irrigation level 3 (0.50 ET) produced the largest marginal return in relative yield per unit of water.

CHAPTER VII

OPTIMIZATION AND DECISION ANALYSIS

The dynamic programming (DP) formulation is developed in this chapter to find the irrigation decisions which maximize the crop yield subject to constraints in irrigation water. These results are presented and discussed in the context of decision analysis.

Dynamic Programming

This section will present the dynamic programming formulation which follows closely that developed by the Texas Water Development Board (1972). The resulting solutions for the specific case of grain sorghum at Temple, Texas, are presented.

Dynamic Programming Formulation

The optimization problem treated in this report can be stated mathematically as

$$\text{MAX } \frac{Y}{Y_o} = \prod_{i=1}^{n} \left(\frac{E}{E_o}\right)_i^{\lambda_i} \qquad \ldots(55)$$

This optimization is subject to constraints involving limitations on the total water quantity, D, available for irrigation during the growing season;

$$D \geq \sum_{i=1}^{n} d_i \qquad \ldots(56)$$

where d = the amount of water allocated to growth period i, mm, plus possible limitations on variables such as irrigation quantity per irrigation, soil water content, and yield which might arise from contractual arrangements.

The state return function is given as

$$\bar{y}_N^*(SW_N, I_N, d_N) = MAX\ [\bar{y}_N\ (SW_N, I_N, d_n)] \qquad \ldots(57)$$

subject to $0 \leq d_N \leq D_N$; where $\bar{y}_N^*$ = the optimum (note that any variables with a * superscript refer to optimum values and bar variables refer to expected values) expected yield ratio for the state variables of soil water content (SW) over irrigation level (I), and N = the dynamic programming stage (this goes backward from growth period n to 1). The decision variable is I, the choice of irrigation level. The expected quantity of water to be used in irrigation is a function of the decision as well as the soil water state. For N = 1, the stage return is given by

$$\bar{y}_1(SW_1, I_1) = \sum_{m=1}^{6} P\ (SW_m, I_1)\ \frac{Y}{Y_o}\ (SW_N, I_1) \qquad \ldots(58)$$

where P = the transition probability from soil water state N to m using decision I which resulted in the particular value of Y/Y_o.

These data were given in Tables 22 (page 124) and 23 (page 130), respectively. For N greater than one, the stage return is given by

$$\bar{y}_N\ (SW_N,\ I_N) = \sum_{m=1}^{6} P\ (SW_m,\ I_N)\ \frac{Y}{Y_o}\ (SW_N,\ I_N)\ \bar{y}^*_{N-1}(SW_m) \quad \text{...(59)}$$

The state transitions are given by

$$D_N = D_{N-1} - d_N \quad \text{...(60)}$$

$$SW = SW_{N-1} + \underline{R} + I\ \underline{ET} + RZ - \underline{ET} - Q - DRAIN \quad \text{...(61)}$$

where RZ = the water added to the root zone by expansion, mm. The underlined variables in equation (61) are stochastic.

The irrigation quantity, D, is a continuous variable; however, to further simplify the problem a limitation was employed which allowed the irrigation water to be dispensed in increments of 25 mm. This constraint, as investigated later, was not very restrictive.

Dynamic Programming Results

Since growth period five would not affect the optimal decisions, it was excluded from this analysis. The expected relative yield and optimum irrigation decisions are shown in Table 24. This table shows generally the importance of irrigation timing as opposed to irrigation quantity. The results indicate that if the available irrigation water is limited (less than 150 mm) the optimum decision is to apply water in growth periods 2 and 3 with growth period 3

Table 24. Solution of dynamic programming problem.

Growth Period 1

D mm	SW = 25 mm		SW = 50 mm		SW = 75 mm		SW = 100 mm		SW = 150 mm		SW = 200 mm	
	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*
0	.448	1	.487	1	.563	1	.579	1	.728	1	.780	1
25	.502	1	.543	1	.611	1	.619	1	.739	1	.784	1
50	.551	1	.600	1	.659	1	.667	1	.785	1	.824	1
75	.599	1	.638	1	.694	1	.701	1	.809	1	.842	1
100	.623	3	.681	1	.743	1	.757	1	.840	1	.860	1
125	.699	1	.760	1	.791	1	.789	1	.851	1	.866	1
150	.732	3	.782	1	.821	1	.826	1	.868	1	.880	1
175	.764	1	.812	1	.837	1	.840	1	.880	1	.889	1
200	.796	1	.836	1	.857	1	.863	1	.892	1	.896	1
225	.821	1	.857	1	.868	1	.871	1	.893	1	.898	1
250	.839	3	.867	1	.878	1	.881	1	.901	1	.904	1
275	.847	3	.875	1	.885	1	.888	1	.903	1	.906	1
300	.855	3	.882	1,2	.892	1	.895	1	.908	1	.909	1
325	.863	3	.888	2	.895	1	.896	1,2	.908	2	.909	2
350	.867	3	.891	1,2	.900	3	.899	1,2	.907	3	.909	3
375	.871	4	.896	2	.901	1	.901	1,4	.908	4	.909	4
400	.877	5	.898	2	.901	1	.902	1,2	.907	1,2	.907	1,2
425	.884	5	.898	2,4	.900	1,2,3,4	.905	5	.908	5	.909	5
450	.887	5	.899	4	.902	4	.904	5	.907	3,4	.907	3,4
475	.891	5	.899	4	.903	4	.905	5	.907	4	.907	4
500	.897	5	.901	5	.902	5	.906	5	.907	5	.907	5
525	.892	5	.901	5	.902	5	.906	5	.907	5	.907	5
550	.892	5	.900	5	.900	5	.907	5	.901	5	.900	5
575			.900	5	.900	5	.907	5	.901	5		

Table 24. (continued)

Growth Period 2

D	SW = 25 mm		SW = 50 mm		SW = 75 mm		SW = 100 mm		SW = 150 mm		SW = 200 mm	
mm	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*
0	.438	1	.459	1	.509	1	.557	1	.706	1	.780	1
25	.498	2	.503	1	.589	2	.588	1	.721	1	.784	1
50	.534	2	.579	3	.627	2	.642	1	.768	2	.824	2
75	.573	2	.615	4	.670	2	.675	1	.795	1	.842	3
100	.604	2	.650	3	.713	2	.748	4	.832	1	.860	4
125	.673	2	.749	5	.790	2	.765	1	.845	1	.866	2
150	.700	3	.760	5	.817	2	.815	5	.863	2	.880	3
175	.743	4	.805	4	.827	2	.833	4	.876	4	.869	4
200	.775	5	.835	4	.838	2	.865	4	.890	4	.896	4
225	.800	5	.863	5	.855	4	.871	4	.891	4,5	.898	4
250	.808	5	.871	5	.867	5	.881	4	.900	5	.904	5
275	.816	5	.880	5	.875	5	.889	5	.902	5	.906	5
300	.826	5	.884	5	.884	5	.896	5	.907	5	.907	5
325	.831	5	.889	5	.889	5	.897	5	.905	5	.906	5
350	.833	5	.895	5	.894	5	.899	5	.906	5	.906	5
375	.834	5	.895	5	.899	5	.901	5	.907	5	.907	5
400	.831	5	.895	5	.900	5	.902	5	.907	5	.907	5
425					.900	5	.899	5	.902	5	.900	5
450					.900	5	.899	5	.902	5	.900	5

Table 24. (continued)

Growth Period 3

D mm	SW = 25 mm $\frac{Y^*}{Y_o}$	SW = 25 mm I*	SW = 50 mm $\frac{Y^*}{Y_o}$	SW = 50 mm I*	SW = 75 mm $\frac{Y^*}{Y_o}$	SW = 75 mm I*	SW = 100 mm $\frac{Y^*}{Y_o}$	SW = 100 mm I*	SW = 150 mm $\frac{Y^*}{Y_o}$	SW = 150 mm I*	SW = 200 mm $\frac{Y^*}{Y_o}$	SW = 200 mm I*
0	.408	1	.506	1	.635	1	.669	1	.825	1	.866	1
25	.461	2	.538	2	.664	2	.689	1	.835	1	.874	1
50	.480	4	.640	3	.683	2	.714	2	.850	1	.882	1
75	.495	4	.655	3	.761	3	.822	3	.876	3	.902	1
100	.605	5	.726	4	.831	4	.864	4	.894	4	.909	1,4
125	.615	5	.782	5	.862	5	.869	4	.894	4	.911	1
150	.630	5	.789	5	.866	5	.863	5	.904	5	.914	1
175	.642	5	.806	5	.872	5	.892	4	.907	4	.911	4
200	.650	5	.818	5	.888	5	.898	4	.908	4,5	.911	4
225	.656	5	.823	5	.894	5	.905	5	.910	4	.912	4
250	.656	5	.830	5	.895	5	.908	5	.911	4	.912	4
275			.832	5	.897	5	.909	5	.911	5	.905	5
300							.910	5	.911	5		
Growth Period 4												
0	.785	1	.867	1	.905	1	.935	1	.972	1	.983	1
25	.820	2	.880	2	.924	2	.935	1	.972	1	.983	1
50	.839	3	.926	3	.924	2	.949	2	.979	2	.983	1
75	.870	4	.933	4	.958	3	.973	3	.980	3	.983	1
100	.904	5	.933	4	.975	4	.978	4	.980	3	.983	1
125			.958	5	.975	4	.978	4	.984	4	.983	1
150					.981	5	.980	5	.985	5	.983	1

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual water use, respectively.

(boot to bloom) predominating. These decisions were affected by the soil water states, as would be expected; however, the influence of growth periods 2 and 3 was quite evident. The expected relative yield is shown for each beginning soil water state in Figure 27. This figure gives the expected relative yield if the water available is allocated optimally throughout the season as given by Table 24 (page 139). These values do not show the yield of any specific year but the expected yield of a large number of years. It is interesting to note a specific example in Figure 27. Consider the worst and best soil water states at the beginning of the season (25 mm and 200 mm, respectively).and assume 250 mm of irrigation water is available. The expected yield of the worst soil water state is 93.3 percent (0.84/0.90) of the best expected yield. However, if 500 mm of irrigation water are available, the expected yield of the worst soil water state is 98.9 percent (0.90/0.91). The irrigation water requirement doubled, but the yield increased only 6 percent. These results were quite similar to those of Minhas, et al. (1974) for wheat; however, they had no way to account for the effect of the dynamic aspects of rainfall and soil water content on the results. For the drier soil water states (SW < 100 mm) the greatest marginal increase in yield per unit of water occurred at approximately 100 to 125 mm of available irrigation water. As the soil water content increased to 200 and 150 mm, the largest

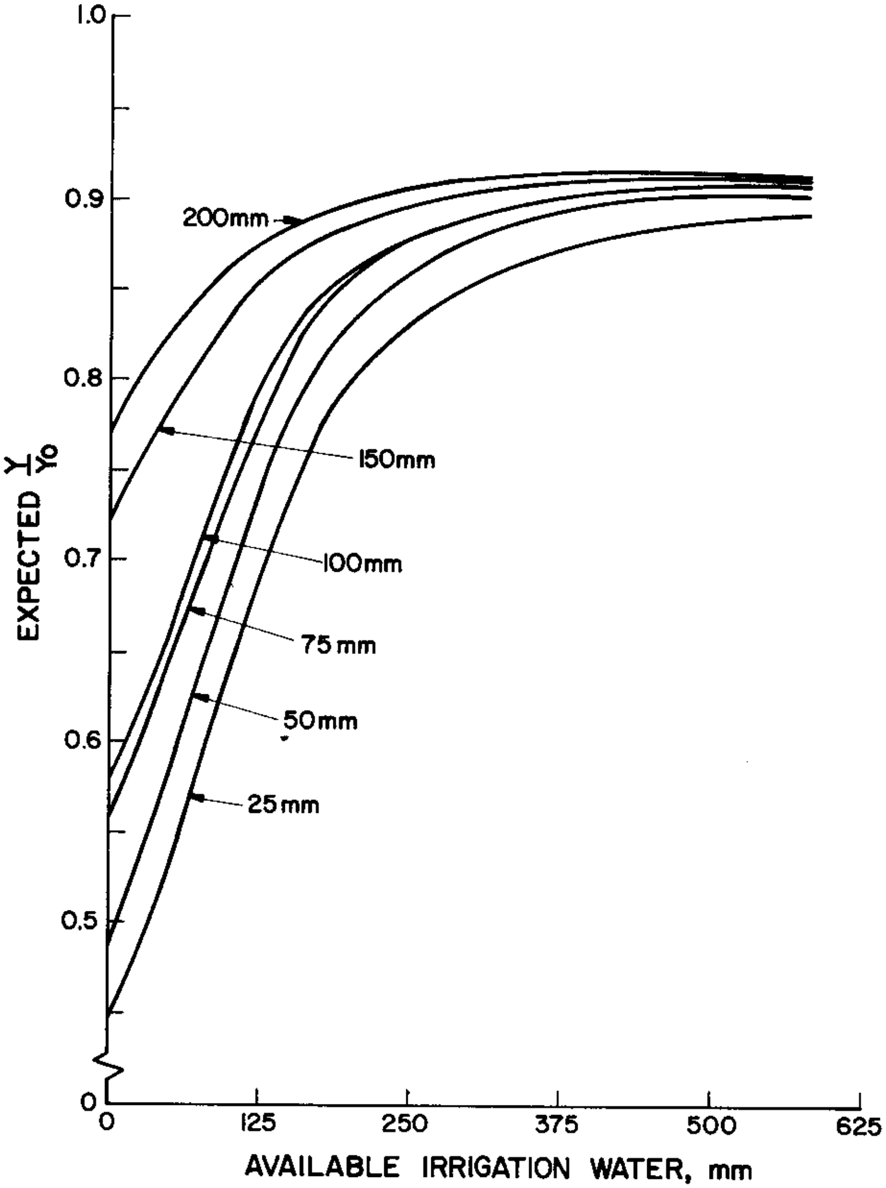


Figure 27. Maximum expected yield at the beginning of growth period one as affected by available irrigation water and beginning soil water content.

marginal yield increase per unit of water was at approximately 25 to 50 mm of available irrigation water.

The sometimes erratic change in irrigation decisions in Table 24 (page 139), i.e., going from I = 1 at D = 75 mm to I = 3 at 100 mm and then back to I = 1 at D = 125 mm, results from the narrow range in expected yield at this site in combination with a slight instability of transition probabilities. If more years were simulated these changes would have become smaller. Table A-3 gives the complete tabular solution of this problem. These tables give the optimum decision choices if the beginning soil water content of a particular period is known.

To be more realistic of the actual system, the dynamic programming analysis should have contained a state variable relating to the plant growth, such as LAI. The LAI model used in simulation was not sensitive to the soil water deficit so LAI was not included as a state variable. Yet its inclusion in the future would not be difficult but would increase the dimensionality of the problem.

Dynamic Programming Example

This example problem will demonstrate the stochastic dynamic programming solution method. This particular example will determine the dryland (irrigation level one) expected yield at growth period 2 (N = 3) with a beginning soil water content of 75 mm. As shown in Table A-3 (page 187) and Table 24 (page 140), the expected yield at growth period 2 with 75 mm of soil water is 0.509. Equation (59) will be used to solve this problem.

The probabilities for the soil water state transition are 0.000, 0.067, 0.200, 0.067, 0.367, and 0.300 for going from soil water state 4 ($87.5 \geq SW > 37.5$) to soil water states 1, 2, 3, 4, 5, and 6, respectively [Table 22 (page 125)]. The expected yield at growth period 2 for the beginning soil water state 4 is 0.94 for the dryland irrigation level [Table 23 (page 131)]. The maximum expected yields at growth period 3 ($N = 2$) are shown in Table 24 (page 141) and Table A-3 (pages 191-196). For the dryland case ($I = 1$), the expected yields are 0.87, 0.83, 0.67, 0.64, 0.51, and 0.41 for the soil water states 1, 2, 3, 4, 5, and 6, respectively. The calculation process follows below:

		(1)	(2)	(3)
m	SW_m	$P(SW_m, 1)$	$\frac{Y}{Y_o}(SW_N, 1)$	$\frac{Y}{Y_o}^*(SW_m)$
1	200	0.000	0.94	0.87
2	150	0.067	0.94	0.83
3	100	0.200	0.94	0.67
4	75	0.067	0.94	0.64
5	50	0.367	0.94	0.51
6	25	0.300	0.94	0.41

Sum of (1) x (2) x (3) = 0.509

Thus, the expected yields and optimum irrigation decisions can be determined for each soil water state at a given growth stage and irrigation water constraint.

Decision Analysis

The decision analysis problem can be formulated in a decision tree. Figure 28 gives the complete decision tree for this problem. The squares represent decision points and the circles represent chance nodes over which the decision maker has no recourse. The first decision is the choice of pre-irrigation amount. This amount was constrained to be less than 200 mm minus the soil water state. The data presented in Table A-3 can be utilized to make this decision. The assumption that pre-irrigated water was 100 percent efficient in raising the soil water state was made. This implied that germination occurred immediately following pre-irrigation and that no evaporation occurred prior to germination. This assumption simply defines the starting conditions.

The decision to be made is if a given quantity of water is available at the beginning of the season, what portion if any of that water should be allocated to pre-irrigation. Note that no assumption has been made of the effect of soil water content on germination; however, this effect could be added. Figure 27 (page 143) shows that if the soil water content is less than 150 mm the optimum decision is to apply as much water as available at the start

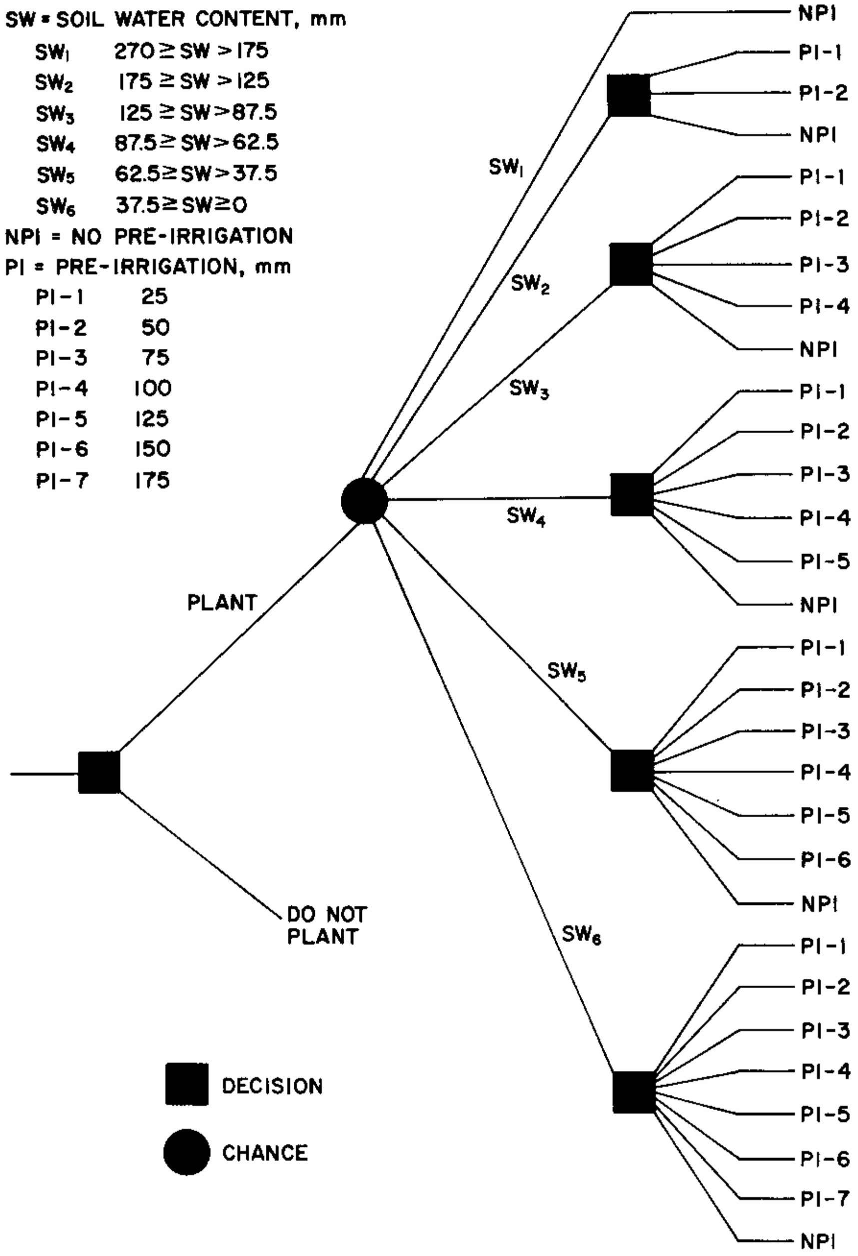


Figure 28. Irrigation decision tree.

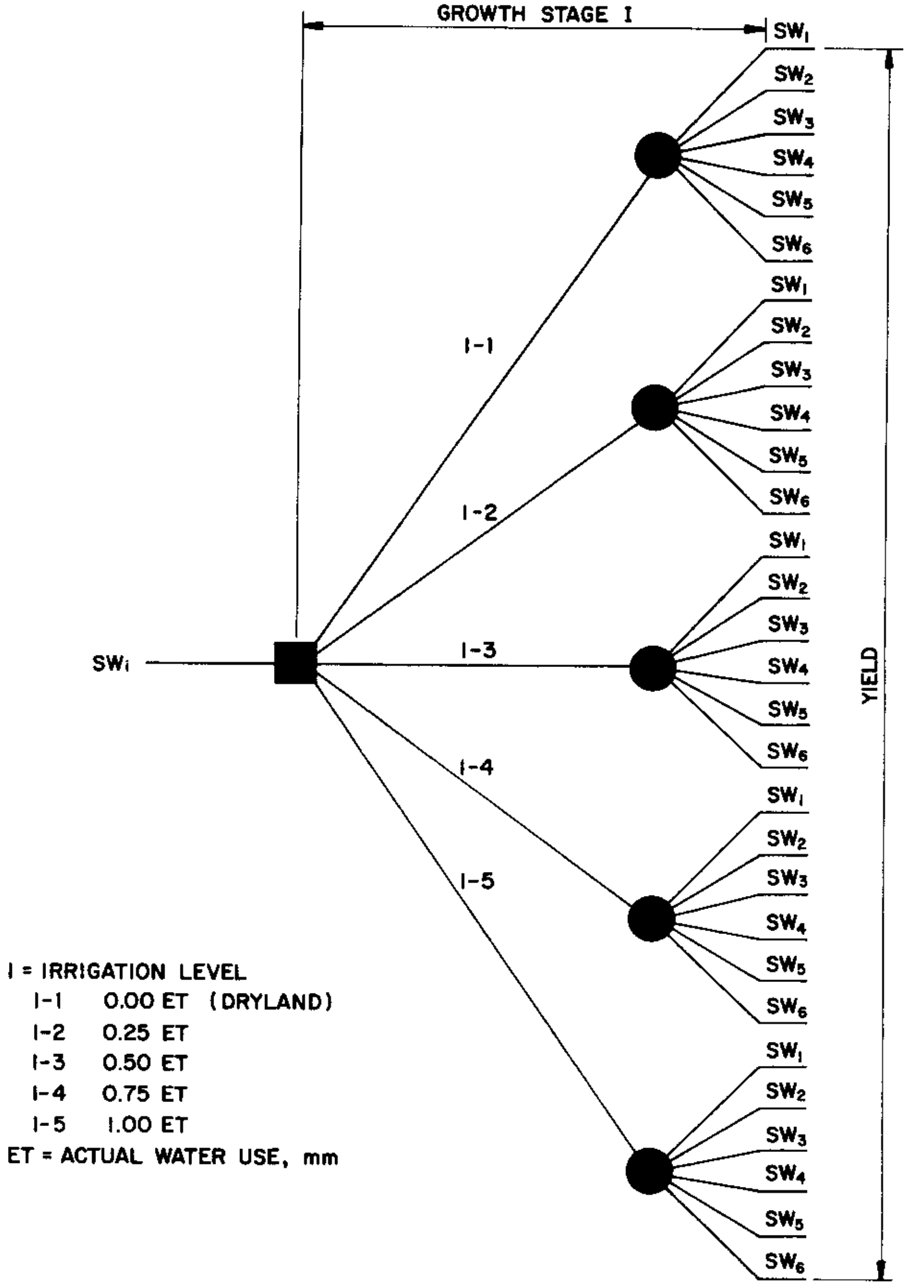
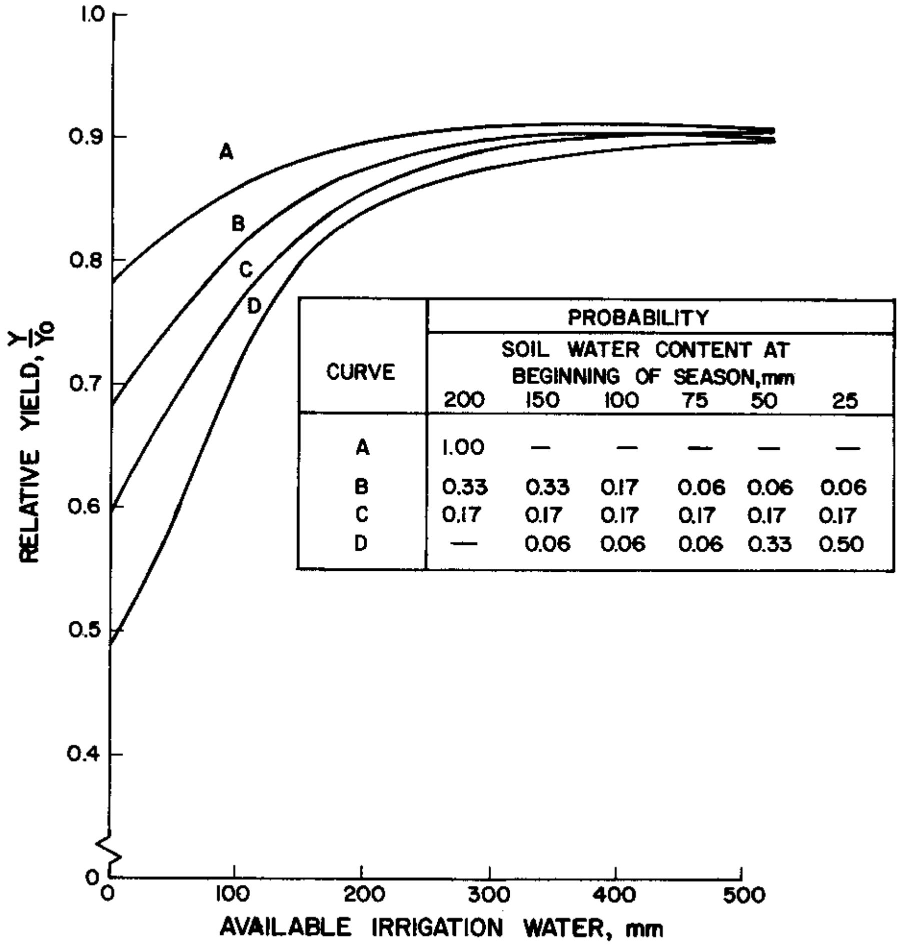


Figure 28. (continued)

of the season (until the constraint of 200 mm of total water inventory is reached) and apply the remaining portion optimally throughout the season as given in Table 24 (page 139).

The expected relative yield was computed for a wide range of probability values for the chances of being in a specific soil water state at the beginning of the season. Several of these cases are plotted in Figure 29. Curve C represents the case when each soil water class is equally likely to occur. Curve D shows the unlikely case when five out of six years the beginning soil water content would be less than 37.5 mm. Curve A shows a case when the beginning soil water content would always be larger than 175 mm. Curve B shows a case when the beginning soil water content was larger than 125 mm two out of three years. It can be seen that to appreciably affect the system, a large change in these beginning probabilities is required. The true expected curve would most likely fall in the region between curves A and C since the early spring rainfall at Temple, Texas, is usually enough to replenish a major portion of the crop root zone. Curve B should be close to the long-term expected average.

Two decision analysis methods--MAXIMAX and MAXIMIN--were utilized to determine the range of the decisions under very optimistic and pessimistic viewpoints. These methods did not consider any of the subjective or objective probabilities given previously. The MAXIMAX criterion is a very optimistic view of the future. This



CURVE	PROBABILITY SOIL WATER CONTENT AT BEGINNING OF SEASON,mm 200	150	100	75	50	25
A	1.00	—	—	—	—	—
B	0.33	0.33	0.17	0.06	0.06	0.06
C	0.17	0.17	0.17	0.17	0.17	0.17
D	—	0.06	0.06	0.06	0.33	0.50

Figure 29. Effect of subjective soil water state probabilities on the expected yield at the beginning of growth period one.

criterion assumes that nature will provide the best possible states, and the object is to find the alternative which maximizes the return. This criterion provides an upper limit to the expected outcomes. The MAXIMIN criterion provides a very pessimistic outlook. This criterion assumes that the very worst state occurs each time, and the object is to find the alternate decision which maximizes the return given that the worst state always occurs. This value determines the lower limit of the expected outcomes provided each unit of water is allocated optimally throughout the season.

Figure 30 shows the relative yield for the dynamic programming solution (Bayesian) and the MAXIMAX and MAXIMIN solutions (non-Bayesian). The expected yield approaches the MAXIMAX value as the available water increases. This results because as irrigation water is added optimally, the maximum yield potential is reached. Also, the chances for the MAXIMIN to occur must be small since expected values are so much larger. This demonstrates that range of possible outcomes for different values of irrigation input. The fact that the maximum relative yield seldom goes above 0.90 at the beginning of the season is indicative of the fact that a perfect season (in regard to solar radiation, etc.) is rare. Tables 25 and 26 show the MAXIMAX and MAXIMIN decisions, respectively.

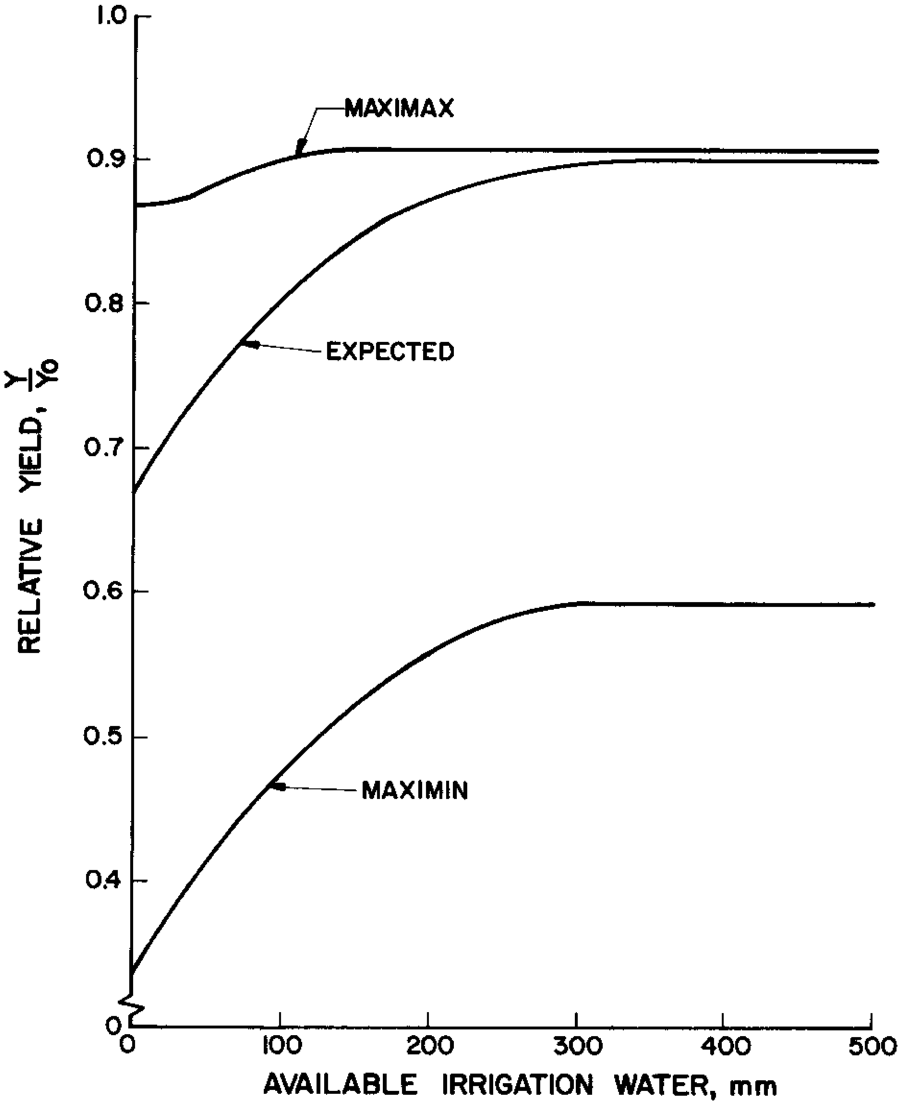


Figure 30. Expected, MAXIMAX, and MAXIMIN relative yields as affected by available irrigation water at the beginning of growth period one.

Table 25. Optimum irrigation decisions for the MAXIMAX criterion.

D mm	Growth Period 1		Growth Period 2		Growth Period 3		Growth Period 4	
	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*
0	.87	1	.87	1	.91	1	.98	1
25	.87	1,2	.87	1	.91	1	.98	1
50	.88	1	.88	2	.91	1	.98	1,2
75	.89	1	.89	3	.91	1	.98	1,2,3
100	.90	1	.90	4	.91	1,4	.98	1,2,3,4
125	.90	1,2	.90	4	.91	1,4	.98	1,2,3,4
150	.91	1	.91	5	.91	1,4	.98	1,2,3,4,5
175	.91	1,2	.91	5	.91	4		
200	.91	1,2,3	.91	5	.91	4		
225	.91	1,2,3,4	.91	5	.91	4		
250	.91	1,2,3,4	.91	5	.91	4		
275	.91	1,2,3,4,5	.91	5	.90	5		
300	.91	1,2,3,4,5	.91	5				
325	.91	1,2,3,4,5	.91	5				
350	.91	1,2,3,4,5	.91	5				
375	.91	1,2,3,4,5	.91	5				
400	.91	1,2,3,4,5	.91	5				
425	.91	2,3,4,5	.90	5				
450	.91	3,4,5						
475	.91	4,5						
500	.91	5						
525	.91	5						
550	.91	5						
575								
600								

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual water use, respectively.

Table 26. Optimum irrigation decisions for the MAXIMIN criterion.

	Growth Period 1		Growth Period 2		Growth Period 3		Growth Period 4	
D mm	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*	$\frac{Y^*}{Y_o}$	I*
0	.34	1	.34	1	.38	1	.78	1
25	.40	1	.40	1	.44	2	.82	2
50	.41	1	.41	1	.46	2	.84	3
75	.42	1	.42	1,2	.47	2	.87	4
100	.49	1	.49	1	.54	5	.90	5
125	.51	1	.51	1	.57	5		
150	.52	1	.52	1,2	.58	5		
175	.54	1	.54	1	.60	5		
200	.56	1	.56	1	.62	5		
225	.57	1	.57	2				
250	.58	1	.58	3				
275	.59	1	.59	4				
300	.60	1	.60	5				
325	.59	2,3						
350	.60	2,3						
375	.60	4						
400	.59	5						

Irrigation levels 1, 2, 3, 4, and 5 are 0.00, 0.25, 0.50, 0.75, and 1.00 times the actual water use, respectively.

Application Example

This section will apply the solution obtained by dynamic programming to a grain sorghum field near Temple, Texas. This example assumes that this farmer's objective is to maximize yield subject to water constraints and all economic factors are neglected. The following assumptions are made:

Maximum yield, Y_0 = 7,000 kg ha^{-1}

Maximum available irrigation water, D = 150 mm

Beginning soil water content, SW = 100 mm.

Refer to Figure 28 (page 147), chance has taken this field to the SW_3 branch. The first decision is whether to pre-irrigate or not and how much to pre-irrigate if that choice is made. Table A-3 (pages 182-184) shows that if this farm has 150 mm of irrigation water starting with a soil water content of 100 mm, the maximum expected yield is 5782 kg ha^{-1} (0.826 x 7,000) [page 182]. If 50 mm of water were used for pre-irrigation, the expected yield would be 5,880 kg ha^{-1} (0.840 x 7,000) [page 183]. If 100 mm of pre-irrigation water were used, the expected yield would be 5,768 kg ha^{-1} (0.824 x 7,000) [page 184]. If 25 mm and 75 mm of pre-irrigation water were used the expected yields could be estimated (interpolated) as 5,831 kg ha^{-1} (0.833 x 7,000) and 5,824 kg ha^{-1} (0.832 x 7,000), respectively. The choice of pre-irrigation should be made and a quantity of 50 mm should be applied. This means that the starting soil water content in growth stage one is 150 mm with 100 mm of irrigation water available. Table 24 (page 139) shows that with these conditions the best choice is not to irrigate during this first growth period.

The soil water content will be assumed to stay at the same state during this first stage. The probability for this to occur is 70 percent (Table 22, page 124). Thus, the beginning soil water content at growth stage 2 is 150 mm with 100 mm of irrigation water available. The optimum decision at this point is again to apply no irrigation water; however, the expected yield for the remaining stages has been reduced by 17 percent (Table 24, page 140). Table 23 (page 131) shows that the expected yield has declined to 6,720 kg ha^{-1} (0.960 x 7,000) after the first two growth stages. The soil water content will be assumed to decrease to 75 mm which has a 26.7 percent chance of happening (Table 22, page 125) at the beginning of growth stage 3 with 100 mm of irrigation water remaining. Table 24 (page 141) shows that the optimum decision is to apply all the water [expected irrigation of 91 mm, Table 23 (page 132)] at this time. No more irrigation water is left to be allocated. Table 22 (page 126) shows that the probability of decreasing to a lower soil water state is 36.7 percent. This lowers the expected final yield to 5,584 kg ha^{-1} (0.96 x 0.831 x 7,000) depending of course on the actual soil water state in growth period 4.

The above example has demonstrated how the dynamic programming solution can be used to determine optimum allocation of irrigation water throughout the season. The beginning soil water content and available irrigation water are all that must be known to make the best decision.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

Summary

This study was developed to combine experimental knowledge on the subjects of crop yield, crop water use, and hydrology at the field level into a managable system whereby the influence of high frequency irrigation on the system could be examined and eventually optimized with regard to water use efficiency. Experimental data were obtained during 1973 which related the plant water deficit E/E_0 for specific crop growth periods to the eventual grain yield of grain sorghum. These data were presented by Howell and Hiler (1974) and summarized in this dissertation. The results from that work indicated that water deficits during the boot to bloom period were the most pronounced in reducing the yield of grain sorghum. However, that work indicated that small, frequent irrigations could maintain a desirable root zone water balance with a minimum of wasted water (runoff and drainage). The experimental results were verified at least in part by the use of a water balance model (Richardson and Ritchie, 1973) which showed that the reduced evaporation resulted from a decrease in plant transpiration. When these results were compared to the yield data, only evaporation reductions in the boot to bloom period significantly affected yield.

An environmental simulation model (temperature, rainfall, and potential evaporation) was developed for Temple, Texas. The model was similar to that developed by Jones, et al. (1972) for State College, Mississippi. Using Monte Carlo techniques, these daily environmental data could be simulated for input to the soil water balance model. The simulation results adequately represented the stochastic serial-correlation of these variables.

The soil water balance model of Richardson and Ritchie (1973) was modified to allow for root zone extension, runoff calculations, and grain sorghum leaf area index simulation (Ritchie and Arkin, 1973). The model can separately calculate the plant and soil evaporation components. The root zone was assumed to linearly expand from 150 mm at germination to 1750 mm at the rate of 32 mm day^{-1}. Runoff was computed by the SCS equation. The model was programmed in FORTRAN (LEVEL G).

The grain sorghum soil water balance at Temple, Texas, was simulated for 30 years in each of 5 growth periods for 6 soil water contents and 5 irrigation levels. The soil water content transition probabilities were determined, and the expected yield and evaporation quantities were determined for each growth period, soil water content state, and irrigation level.

Stochastic dynamic programming was used to maximize yield subject to water availability constraints. These results show the proper irrigation decision for each period if the soil water content and amount of irrigation water at the start of the period are known.

Finally, these results demonstrated that the irrigation amount could be substantially reduced without a large decrease in expected yield if the irrigation water was distributed optimally over the season. Also that even in this area of high rainfall (850 mm annual average with 285 mm occurring between January and May), if the soil water content is less than 150 mm at planting time, pre-plant irrigation is preferred over using a like-amount of water later in the season, provided that soil evaporation is small.

Application of Results

These results in the strictest sense are applicable to grain sorghum grown at Temple, Texas, on a Houston black clay soil assuming that water is the only limiting factor in production. The methods used in this study can also be applied to other regions and irrigation criteria. The simulation technique can quickly simulate a complete irrigation experiment in only seconds at a fraction of the cost to conduct the experiment. The expected yield data can be quickly transformed to expected profit by including cost factors for the irrigations and return values for the yield.

Conclusions

This study demonstrated the tremendous possibilities of applying the techniques of system simulation, dynamic programming, inventory control, and decision analysis to the irrigation problem. Specific conclusions of this study were:

1. Irrigation timing is more critical to WUE than is irrigation amount.

2. Pre-irrigation is preferred to seasonal irrigation with a like-amount of water if the soil water content is low.

3. WUE can be increased significantly by utilizing small, frequent irrigations to minimize losses to drainage and runoff and by allowing water deficits to occur in noncritical periods rather than in critical periods.

4. An irrigation system can be simulated as a stochastic inventory system.

5. Decision analysis techniques offer many tools with which to evaluate decisions under uncertainty which directly apply to agriculture.

Suggestions for Future Research

Throughout this project many subjects were reviewed and studied. During this study the following subjects were found to need additional study and possible improvements:

1. Solar radiation was not a direct input to this simulation. Possibly the environmental model could be improved by simulating solar radiation as a function of cloudcover and time of year. Net radiation could then be computed in a manner similar to Ahmed (1974).

2. The calculation of runoff and drainage in this simulation was at best only approximate. Better methods to estimate these two important parameters would be desirable.

3. The leaf area index simulation could be improved to allow a more direct feedback to the soil or plant water status.

4. Better understanding of the mechanisms in the plant which cause reduced yields is imperative if yield is to be accurately predicted.

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APPENDIX A

Supplementary Data

Table A-1. Daily potential evaporation for the 1973 growing season.

Month	Day	Air Temperature C	Vapor Pressure mb	Wind Speed km day^{-1}	Net Radiation mm day^{-1}	Potential Evaporation		Class A Pan Evaporation mm day^{-1}
						Penman mm day^{-1}	Van Bavel mm day^{-1}	
4	13	21	19.4	159.7	4.7	4.2	4.9	4.4
4	14	20	19.2	361.6	1.8	2.3	3.9	3.1
4	15	19	19.8	207.0	0.9	1.1	1.5	5.8
4	16	18	16.9	211.9	2.8	2.5	3.3	5.7
4	17	16	16.9	87.2	1.2	1.1	1.1	2.3
4	18	20	10.8	289.2	5.3	4.5	5.6	5.2
4	19	25	29.2	284.3	3.4	3.0	3.7	3.3
4	20	23	26.7	316.2	1.3	1.2	1.6	0.1
4	21	22	24.5	283.7	1.4	1.5	2.1	1.4
4	22	23	24.6	160.3	1.4	1.5	1.9	0.9
4	23	24	23.8	226.7	3.1	3.4	4.6	3.6
4	24	23	25.5	120.6	2.1	1.8	1.9	5.8
4	25	21	16.7	139.2	6.1	5.5	6.3	15.8
4	26	16	12.0	203.4	4.3	4.0	5.3	4.7
4	27	15	10.8	136.5	5.9	4.9	5.7	7.4
4	28	19	15.3	193.3	6.4	5.6	6.8	6.0
4	29	21	17.1	271.2	2.2	3.2	5.3	2.4
4	30	22	17.9	290.0	1.9	3.3	5.8	10.7
5	1	23	23.6	96.1	3.6	3.0	3.2	0.6
5	2	17	12.9	213.5	3.2	3.5	5.0	3.5
5	3	17	13.4	204.0	3.9	3.8	5.1	7.5
5	4	19	13.3	130.3	6.4	5.6	6.4	5.7
5	5	21	21.3	246.3	2.2	2.4	3.4	1.8
5	6	21	23.3	144.1	1.6	1.3	1.4	6.2
5	7	21	14.6	91.3	7.4	6.3	6.7	10.0
5	8	23	18.7	256.4	6.6	6.4	8.4	6.9

Table A-1. (continued)

Month	Day	Air Temperature C	Vapor Pressure mb	Wind Speed km day^{-1}	Net Radiation mm day^{-1}	Potential Evaporation Penman mm day^{-1}	Potential Evaporation Van Bavel mm day^{-1}	Class A Pan Evaporation mm day^{-1}
5	9	23	22.4	189.9	6.2	5.5	6.5	8.8
5	10	25	22.7	197.5	5.6	5.5	6.9	5.8
5	11	24	26.7	204.7	3.8	3.2	3.7	5.7
5	12	19	16.1	224.8	2.6	2.9	4.1	3.8
5	13	20	12.0	150.2	5.8	5.7	7.1	7.6
5	14	17	11.2	179.1	2.8	3.4	4.8	4.9
5	15	18	12.6	100.8	5.0	4.4	4.9	6.5
5	16	20	13.5	141.5	5.9	5.6	6.7	6.9
5	17	23	19.9	111.7	5.9	5.2	5.7	6.6
5	18	25	21.2	306.1	6.9	7.2	10.1	6.7
5	19	27	21.7	56.2	6.2	5.8	5.7	7.9
5	20	25	23.8	163.7	6.0	5.6	6.6	13.9
5	21	26	24.0	237.2	4.9	5.2	6.8	8.8
5	22	27	25.3	18.0	4.6	4.2	3.7	5.5
5	23	27	22.0	209.5	6.0	6.6	8.7	4.6
5	24	26	28.2	74.6	3.2	3.0	3.0	6.2
5	25	25	29.4	189.1	2.6	2.2	2.5	3.1
5	26	28	28.3	95.3	5.8	5.5	5.8	6.3
5	27	23	20.7	169.4	6.8	6.0	7.0	10.9
5	28	22	15.3	161.6	6.6	6.5	7.9	10.2
5	29	23	18.3	75.0	5.4	4.9	5.0	7.2
5	30	24	17.9	71.2	6.8	6.2	6.4	7.5
5	31	26	22.7	83.9	6.6	6.1	6.4	7.2
6	1	27	26.1	237.8	3.4	4.2	6.0	6.6
6	2	28	21.3	293.4	7.1	8.2	11.8	9.9
6	3	26	22.0	177.6	4.4	5.1	6.6	7.7

Table A-1. (continued)

Month	Day	Air Temperature C	Vapor Pressure mb	Wind Speed km day^{-1}	Net Radiation mm day^{-1}	Potential Evaporation		Class A Pan Evaporation mm day^{-1}
						Penman mm day^{-1}	Van Bavel mm day^{-1}	
6	4	28	15.9	184.4	6.3	6.9	8.8	6.4
6	5	24	22.7	147.2	3.6	3.5	4.2	9.9
6	6	23	23.5	95.9	7.0	5.6	5.8	6.3
6	7	25	18.0	98.2	6.4	6.2	6.7	7.8
6	8	25	21.2	169.2	3.8	4.3	5.5	3.7
6	9	24	26.3	72.9	3.2	2.8	2.8	2.2
6	10	27	26.1	9.5	3.5	3.3	2.8	8.1
6	11	23	27.2	116.6	1.2	1.0	1.1	2.9
6	12	23	24.6	109.0	2.5	2.2	2.4	6.8
6	13	23	26.3	88.5	2.9	2.3	2.3	1.8
6	14	27	29.7	147.4	7.4	6.3	6.9	5.3
6	15	28	28.6	174.9	6.2	6.0	7.0	7.1
6	16	28	27.8	235.9	6.9	6.9	8.8	8.6
6	17	29	27.8	224.1	7.0	7.1	8.9	8.2
6	18	29	28.9	226.2	6.8	6.8	8.6	4.5
6	19	27	27.1	170.2	7.0	6.5	7.5	7.9
6	20	25	27.5	101.8	3.2	2.9	3.1	3.5
6	21	24	24.8	64.9	3.2	3.0	3.0	2.8
6	22	26	22.0	87.5	5.4	5.2	5.5	13.3
6	23	26	23.3	56.6	4.2	4.1	4.0	1.0
6	24	26	24.2	51.1	4.4	4.2	4.1	0.8
6	25	23	25.7	62.5	2.3	2.0	2.0	1.7
6	26	26	23.8	65.5	6.7	6.1	6.1	7.6
6	27	27	26.5	164.8	7.0	6.6	7.6	7.8
6	28	28	28.6	158.2	5.9	5.6	6.5	7.3
6	29	28	25.1	100.1	5.7	5.7	6.2	5.8
6	30	28	28.6	149.3	5.7	5.5	6.2	7.6

Table A-1. (continued)

Month	Day	Air Temperature C	Vapor Pressure mb	Wind Speed km day^{-1}	Net Radiation mm day^{-1}	Potential Evaporation Penman mm day^{-1}	Potential Evaporation Van Bavel mm day^{-1}	Class A Pan Evaporation mm day^{-1}
7	1	29	25.7	122.5	5.7	5.8	6.6	9.3
7	2	28	27.8	76.2	5.8	5.5	5.6	7.7
7	3	28	27.8	70.5	5.9	5.5	5.6	7.0
7	4	29	25.0	105.4	7.4	7.2	7.8	8.8
7	5	28	28.5	168.3	6.8	6.5	7.6	8.4
7	6	27	28.4	55.3	1.8	2.0	1.9	3.0
7	7	26	25.3	2.3	2.9	2.8	2.2	5.4
7	8	28	27.1	108.6	2.4	3.1	3.6	5.4
7	9	28	25.7	141.9	6.8	6.6	7.5	7.7
7	10	28	27.1	118.9	6.5	6.3	6.9	6.6
7	11	29	27.1	91.5	6.2	6.0	6.4	6.6
7	12	28	26.4	140.9	6.6	6.3	7.1	9.5
7	13	27	24.4	118.7	7.0	6.6	7.3	9.0
7	14	28	27.1	230.9	6.3	6.6	8.5	6.3
7	15	28	27.1	181.0	3.4	3.9	5.1	8.7
7	16	27	28.9	113.4	4.5	4.3	4.6	4.6
7	17	28	25.0	121.7	6.9	6.7	7.4	4.5
7	18	28	25.7	81.3	6.8	6.4	6.6	7.5
7	19	29	25.7	108.2	7.1	6.8	7.4	8.5
7	20	29	25.0	126.5	6.6	6.6	7.5	6.7
7	21	28	27.8	100.8	6.2	5.9	6.3	8.9
7	22	29	26.4	84.3	6.5	5.5	5.8	8.7
7	23	28	27.0	78.8	5.3	5.1	5.3	6.4
7	24	29	25.0	48.0	6.2	6.0	5.8	7.3
7	25	29	26.6	70.6	4.9	5.0	5.1	7.8

Table A-1. (continued)

Month	Day	Air Temperature C	Vapor Pressure mb	Wind Speed km day^{-1}	Net Radiation mm day^{-1}	Potential Evaporation Penman mm day^{-1}	Potential Evaporation Van Bavel mm day^{-1}	Class A Pan Evaporation mm day^{-1}
7	26	29	25.9	133.5	6.7	6.9	7.9	4.9
7	27	29	28.2	71.8	5.4	5.1	5.2	9.7
7	28	29	29.2	96.5	7.1	6.5	6.9	8.7
7	29	29	26.4	162.2	5.9	6.2	7.5	7.0
7	30	30	26.4	163.1	7.2	7.4	8.9	11.6
7	31	29	28.6	107.9	4.8	4.8	5.2	6.2
8	1	27	27.1	104.6	4.2	4.1	4.4	5.5
8	2	26	25.3	165.2	6.3	5.8	6.7	8.1
8	3	26	23.4	139.6	5.6	5.4	6.3	6.9

Table A-2. Analysis of variance of yield and water use efficiency data.

Analysis of Variance Yield

kg ha^{-1}

Source	Degrees of Freedom	Sum of Squares	Mean Square	F Ratio
Treatment	7	2.848×10^7	4.069×10^6	17.589**
Error	16	3.701×10^6	2.313×10^5	
Total	23	3.218×10^7		

Analysis of Variance Water Use Efficiency

kg ha^{-1} mm^{-1}

Source	Degrees of Freedom	Sum of Squares	Mean Square	F Ratio
Treatment	7	243.646	34.807	12.481**
Error	16	44.620	2.788	
Total	23	288.266		

**Indicates significance at the 99 percent level

Table A-3. Tabular solution of the dynamic programming problem.

Growth Period 1

SW = 25 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.448					.448	1
25	.502	.471	.485			.502	1
50	.551	.530	.537	.501		.551	1
75	.589	.578	.587	.549		.589	1,3
100	.623	.616	.624	.599		.624	1,3
125	.699	.655	.662	.635	.520	.699	1
150	.722	.724	.732	.677	.566	.732	3
175	.764	.750	.756	.738	.622	.764	1,3
200	.796	.784	.790	.765	.657	.796	1
225	.821	.812	.817	.797	.703	.821	1,3
250	.829	.834	.839	.825	.764	.839	3
275	.838	.843	.847	.843	.791	.847	3
300	.846	.850	.855	.852	.818	.855	3
325	.851	.858	.863	.860	.843	.863	2,3,4
350	.854	.863	.867	.867	.860	.867	3,4
375	.855	.866	.870	.871	.870	.881	2,3,4,5
400	.855	.867	.872	.874	.877	.877	5
425	.855	.868	.872	.875	.884	.884	4,5
450	.855	.867	.871	.875	.887	.887	5
475		.867	.871	.875	.891	.891	5
500				.875	.893	.893	5
525					.892	.892	5
550					.892	.892	5

Table A-3. (continued)

Growth Period 1

SW = 50 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.487					.487	1
25	.543	.509				.543	1
50	.600	.558	.520			.600	1
75	.638	.619	.566	.531		.638	1
100	.681	.655	.624	.571		.681	1
125	.760	.696	.659	.634	.562	.760	1
150	.782	.772	.704	.668	.603	.782	1
175	.812	.792	.770	.708	.656	.812	1
200	.836	.821	.793	.776	.689	.836	1
225	.859	.844	.821	.795	.741	.857	1
250	.867	.865	.846	.826	.784	.867	1
275	.875	.874	.864	.851	.816	.875	1
300	.882	.882	.873	.869	.835	.882	1,2
325	.886	.888	.881	.878	.859	.888	2
350	.891	.891	.887	.885	.870	.891	1,2
375	.893	.896	.890	.890	.880	.896	2
400	.894	.898	.894	.893	.887	.898	2
425	.893	.898	.896	.898	.893	.898	2,4
450	.893	.897	.896	.899	.895	.899	4
475		.897	.895	.899	.898	.899	4
500			.895	.897	.901	.901	5
525				.897	.901	.901	5
550					.900	.900	5

Table A-3. (continued)

Growth Period 1

SW = 75 mm

	$\frac{Y}{Y_o}$					$\frac{Y}{Y_o}$ *	I*
D mm \ I	1	2	3	4	5		
0	.563					.563	1
25	.611	.533				.611	1
50	.659	.583	.547			.659	1
75	.694	.636	.603	.598		.694	1
100	.743	.673	.648	.634		.743	1
125	.791	.720	.686	.682	.614	.791	1
150	.821	.781	.737	.714	.644	.821	1
175	.837	.807	.788	.768	.693	.839	1
200	.857	.829	.820	.797	.723	.857	1
225	.868	.850	.833	.832	.776	.868	1
250	.878	.865	.852	.845	.798	.878	1
275	.885	.876	.864	.867	.834	.885	1
300	.892	.883	.875	.874	.847	.892	1
325	.895	.890	.882	.884	.870	.895	1
350	.898	.893	.900	.890	.876	.900	3
375	.901	.897	.893	.897	.885	.901	1
400	.901	.899	.896	.898	.891	.901	1
425	.900	.900	.900	.900	.898	.900	1,2,3,4
450	.900	.899	.900	.902	.898	.902	4
475		.899	.900	.903	.900	.903	4
500			.897	.900	.902	.902	5
525				.900	.902	.902	5
550					.900	.900	5

Table A-3. (continued)

Growth Period 1

SW = 100 mm

	$\frac{Y}{Y_o}$						
D mm \ I	1	2	3	4	5	$\frac{Y}{Y_o}^*$	I*
0	.579					.579	1
25	.619	.602				.619	1
50	.667	.638	.614			.667	1
75	.701	.686	.644	.626		.701	1
100	.757	.718	.693	.649		.757	1
125	.789	.769	.723	.700	.701	.789	1
150	.826	.800	.776	.729	.715	.826	1
175	.840	.832	.798	.785	.761	.840	1
200	.863	.845	.834	.800	.787	.863	1
225	.871	.867	.847	.836	.825	.871	1
250	.881	.874	.870	.851	.836	.881	1
275	.888	.884	.876	.876	.859	.888	1
300	.895	.890	.885	.880	.871	.895	1
325	.896	.896	.891	.890	.887	.896	1,2
350	.899	.897	.898	.895	.889	.899	1,2
375	.901	.899	.898	.901	.898	.901	1,4
400	.902	.902	.900	.900	.901	.902	1.2
425	.900	.902	.902	.902	.905	.905	5
450	.900	.900	.902	.903	.904	.904	5
475		.900	.900	.904	.905	.905	5
500			.900	.900	.906	.906	5
525				.900	.906	.906	5
550					.907	.907	5

Table A-3. (continued)

Growth Period 1

SW = 150 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.728					.728	1
25	.739	.730				.739	1
50	.785	.742	.721			.785	1
75	.809	.787	.734	.745		.809	1
100	.840	.811	.779	.754		.840	1
125	.851	.841	.804	.798	.743	.851	1
150	.868	.852	.838	.820	.752	.868	1
175	.880	.869	.849	.847	.796	.880	1
200	.892	.880	.866	.856	.819	.892	1
225	.893	.892	.879	.872	.846	.893	1
250	.901	.893	.891	.883	.856	.901	1
275	.903	.901	.892	.893	.872	.903	1
300	.908	.903	.901	.895	.883	.908	1
325	.905	.908	.903	.902	.893	.908	2
350	.906	.905	.907	.904	.895	.907	3
375	.907	.906	.905	.908	.902	.908	4
400	.907	.907	.906	.906	.904	.907	1,2
425	.901	.907	.907	.906	.908	.908	5
450	.901	.901	.907	.907	.906	.907	3,4
475		.901	.902	.907	.906	.907	4
500			.902	.901	.907	.907	5
525				.901	.907	.907	5
550					.901	.901	5

Table A-3. (continued)

Growth Period 2

SW = 25 mm

	$\frac{Y}{Y_o}$					$\frac{Y}{Y_o}$ *	I*
D mm \ I	1	2	3	4	5		
0	.438					.438	1
25	.480	.498				.498	2
50	.515	.534	.531			.534	2
75	.541	.573	.563	.572		.573	2
100	.520	.604	.597	.600	.603	.604	2
125	.641	.673	.636	.635	.629	.673	2
150	.651	.695	.700	.684	.662	.700	3
175	.662	.705	.721	.743	.719	.743	4
200	.671	.715	.731	.765	.775	.775	5
225	.676	.723	.740	.774	.800	.800	5
250	.678	.728	.748	.785	.808	.808	5
275	.678	.730	.753	.792	.816	.816	5
300	.678	.731	.755	.797	.826	.826	5
325		.731	.756	.799	.831	.831	5
350			.756	.800	.833	.833	5
375				.800	.834	.834	5
400					.834	.834	5
425							
450							

Table A-3. (continued)

Growth Period 2

SW = 50 mm

D mm \ I	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
	1	2	3	4	5		
0	.459					.459	1
25	.503	.480				.503	1
50	.527	.518	.579			.579	3
75	.555	.564	.606	.615		.615	4
100	.636	.596	.650	.648		.650	3
125	.653	.672	.693	.690	.749	.749	5
150	.664	.698	.750	.741	.760	.760	5
175	.674	.708	.776	.805	.771	.805	4
200	.682	.720	.785	.835	.831	.835	4
225	.687	.729	.795	.843	.863	.863	5
250	.688	.735	.803	.854	.871	.871	5
275	.688	.737	.808	.864	.880	.880	5
300	.688	.738	.811	.869	.884	.884	5
325		.738	.812	.872	.889	.889	5
350			.812	.873	.895	.895	5
375				.873	.895	.895	5
400					.895	.895	5
425							
450							

Table A-3. (continued)

Growth Period 1

SW = 200 mm

	$\frac{Y}{Y_o}$						
D mm \ I	1	2	3	4	5	$\frac{Y^*}{Y_o}$	I*
0	.780					.780	1
25	.784	.780				.784	1
50	.824	.784	.780			.824	1
75	.842	.824	.784	.780		.842	1
100	.860	.842	.824	.784		.860	1
125	.866	.860	.842	.824	.780	.866	1
150	.880	.866	.860	.842	.784	.880	1
175	.889	.880	.866	.860	.824	.889	1
200	.896	.889	.880	.866	.842	.896	1
225	.898	.896	.889	.880	.860	.898	1
250	.904	.898	.896	.889	.866	.904	1
275	.906	.904	.898	.896	.880	.906	1
300	.909	.906	.904	.898	.889	.909	1
325	.906	.909	.906	.904	.896	.909	2
350	.906	.906	.909	.906	.898	.909	3
375	.907	.906	.906	.909	.904	.909	4
400	.907	.907	.906	.906	.906	.907	1,2
425	.900	.907	.907	.906	.909	.909	5
450	.900	.900	.907	.907	.906	.907	3,4
475		.900	.900	.907	.906	.907	4
500			.900	.900	.907	.907	5
525				.900	.907	.907	5
550					.900	.900	5

Table A-3. (continued)

Growth Period 2

SW = 75 mm

			$\frac{Y}{Y_o}$			$\frac{Y}{Y_o}$ *	I*
D mm \ I	1	2	3	4	5		
0	.509					.509	1
25	.541	.589				.589	2
50	.589	.627				.627	2
75	.625	.670	.607			.670	2
100	.694	.713	.638	.692		.713	2
125	.719	.790	.676	.710		.790	2
150	.730	.817	.716	.723	.739	.817	2
175	.741	.827	.771	.794	.746	.827	2
200	.749	.839	.799	.838	.750	.838	2
225	.755	.848	.806	.855	.832	.855	4
250	.758	.854	.815	.862	.867	.867	5
275	.759	.857	.823	.868	.875	.875	5
300	.759	.858	.827	.875	.884	.884	5
325		.858	.830	.880	.889	.889	5
350			.831	.882	.894	.894	5
375			.831	.882	.899	.899	5
400				.882	.900	.900	5
425					.900	.900	5
450					.900	.900	5

Table A-3. (continued)

Growth Period 2

SW = 100 mm

	$\frac{Y}{Y_o}$					$\frac{Y}{Y_o}^*$	I*
D mm \ I	1	2	3	4	5		
0	.557					.559	1
25	.588					.588	1
50	.642	.591				.642	1
75	.675	.599	.655			.675	1
100	.734	.648	.674	.748		.748	4
125	.765	.689	.701	.754		.765	1
150	.773	.749	.768	.764	.815	.815	5
175	.784	.777	.816	.833	.821	.833	4
200	.793	.786	.838	.865	.806	.865	4
225	.797	.796	.846	.871	.870	.871	4
250	.801	.805	.853	.881	.887	.887	4
275	.802	.810	.862	.886	.889	.889	5
300	.802	.813	.867	.890	.896	.896	5
325		.814	.869	.895	.897	.897	5
350		.815	.870	.896	.899	.899	5
375			.870	.896	.901	.901	5
400				.896	.902	.902	5
425					.899	.899	5
450					.899	.899	5

Table A-3. (continued)

Growth Period 2

SW = 150 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.706					.706	1
25	.721					.721	1
50	.731	.768				.768	2
75	.795	.779	.776			.795	1
100	.832	.769	.778	.826		.832	1
125	.845	.838	.774	.833		.845	1
150	.852	.863	.847	.811	.851	.863	2
175	.857	.870	.869	.876	.859	.876	4
200	.863	.877	.872	.890	.825	.890	4
225	.867	.879	.881	.891	.891	.891	4,5
250	.868	.882	.884	.898	.900	.900	5
275	.868	.885	.886	.898	.902	.902	5
300	.868	.886	.890	.899	.907	.907	5
325		.884	.891	.901	.905	.905	5
350		.884	.890	.901	.906	.906	5
375			.890	.897	.907	.907	5
400				.897	.907	.907	5
425					.902	.902	5
450					.902	.902	5

Table A-3. (continued)

Growth Period 2

SW = 200 mm

D mm \ I	$\frac{Y}{Y_o}$ 1	2	3	4	5	$\frac{Y}{Y_o}$*	I*
0	.780					.780	1
25	.784					.784	1
50	.773	.824				.824	2
75	.841	.833	.842			.842	3
100	.860	.811	.851	.852		.860	4
125	.864	.866	.808	.860		.866	2
150	.870	.877	.880	.811	.861	.880	3
175	.872	.878	.887	.889	.869	.889	4
200	.874	.884	.889	.896	.817	.896	4
225	.897	.884	.894	.898	.897	.898	4
250	.879	.884	.891	.901	.904	.904	5
275	.875	.886	.891	.898	.906	.906	5
300	.875	.886	.893	.898	.909	.909	5
325		.882	.893	.899	.906	.906	5
350		.882	.887	.899	.906	.906	5
375			.887	.893	.907	.907	5
400				.893	.907	.907	5
425					.900	.900	5
450					.900	.900	5

Table A-3. (continued)

Growth Period 3

SW = 25 mm

			$\frac{Y}{Y_o}$			$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.408					.408	1
25	.416	.461				.461	2
50	.426	.475	.453	.480		.480	4
75	.440	.488	.468	.495		.495	4
100	.454	.502	.479	.506	.605	.605	5
125	.455	.514	.494	.516	.615	.615	5
150	.455	.517	.508	.532	.630	.630	5
175		.518	.510	.534	.642	.642	5
200			.510	.535	.650	.650	5
225					.656	.656	5
250					.656	.656	5
275							
300							

Table A-3. (continued)

Growth Period 3

SW = 50 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.506					.506	1
25	.513	.538				.538	2
50	.525	.555	.640			.640	3
75	.540	.570	.655			.655	3
100	.557	.587	.672	.726		.726	4
125	.558	.601	.686	.736	.782	.782	5
150	.558	.605	.697	.751	.789	.789	5
175		.606	.704	.765	.806	.806	5
200			.705	.773	.818	.818	5
225				.778	.823	.823	5
250				.779	.830	.830	5
275					.832	.832	5
300							

Table A-3. (continued)

Growth Period 3

SW = 75 mm

			$\frac{Y}{Y_o}$			$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.635					.635	1
25	.643	.664				.664	2
50	.659	.683				.683	2
75	.676	.702	.761			.761	3
100	.692	.719	.774	.831		.831	4
125	.696	.734	.794	.838	.862	.862	5
150	.696	.740	.807	.857	.866	.866	5
175		.740	.815	.870	.872	.872	5
200			.825	.874	.888	.888	5
225			.826	.883	.894	.894	5
250				.885	.895	.895	5
275					.897	.897	5
300							

Table A-3. (continued)

Growth Period 3

SW = 100 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.669					.669	1
25	.689					.689	1
50	.704	.714				.714	2
75	.723	.734	.822			.822	3
100	.741	.754	.832	.864		.864	4
125	.744	.772	.851	.869		.869	4
150	.745	.788	.867	.875	.883	.883	5
175		.795	.872	.892	.883	.892	4
200		.795	.882	.898	.892	.898	4
225			.884	.899	.905	.905	5
250				.901	.908	.908	5
275					.909	.909	5
300					.910	.910	5

Table A-3. (continued)

Growth Period 3

SW = 150 mm

			$\frac{Y}{Y_o}$				
D mm \ I	1	2	3	4	5	$\frac{Y}{Y_o}$*	I*
0	.825					.825	1
25	.835					.835	1
50	.850	.846				.850	1
75	.866	.832	.876			.876	3
100	.871	.871	.877	.894		.894	4
125	.880	.887	.885	.894		.894	4
150	.882	.894	.900	.900	.904	.904	5
175		.901	.904	.907	.904	.907	4
200		.903	.904	.908	.908	.908	4,5
225			.906	.910	.909	.910	4
250				.911	.909	.911	4
275					.911	.911	5
300					.911	.911	5

Table A-3. (continued)

Growth Period 3

SW = 200 mm

D mm \ I	$\frac{Y}{Y_o}$ 1	2	3	4	5	$\frac{Y}{Y_o}$*	I*
0	.866					.866	1
25	.874					.874	1
50	.882	.878				.882	1
75	.902	.878	.897			.902	1
100	.909	.885	.897	.909		.909	1,4
125	.911	.895	.900	.909	.905	.911	1
150	.914	.897	.901	.911	.905	.914	1
175		.899	.901	.911	.905	.911	4
200		.900	.902	.911	.905	.911	4
225			.903	.912	.905	.912	4
250				.912	.905	.912	4
275					.905	.905	5
300							

Table A-3. (continued)

Growth Period 4

SW = 25 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.785					.785	1
25	.785	.820				.820	2
50	.785	.820	.839			.839	3
75	.785	.820	.839	.870		.870	4
100	.785	.820	.839	.870	.904	.904	5
125							
150							

SW = 50 mm

	$\frac{Y}{Y_o}$					$\frac{Y^*}{Y_o}$	I*
D mm \ I	1	2	3	4	5		
0	.867					.867	1
25	.867	.880				.880	2
50	.867	.880	.926			.926	3
75	.867	.880	.926	.933		.933	4
100	.867	.880	.926	.933		.933	4
125	.867	.880	.926	.933	.958	.958	5
150							

Table A-3. (continued)

Growth Period 4

SW = 75 mm

D mm \ I	$\frac{Y}{Y_o}$ 1	2	3	4	5	$\frac{Y}{Y_o}*$	I*
0	.905					.905	1
25	.905	.924				.924	2
50	.905	.924				.924	2
75	.905	.924	.958			.958	3
100	.905	.924	.958	.975		.975	4
125	.905	.924	.958	.975		.975	4
150	.905	.924	.958	.975	.981	.981	5

SW = 100 mm

D mm \ I	$\frac{Y}{Y_o}$ 1	2	3	4	5	$\frac{Y}{Y_o}*$	I*
0	.935					.935	1
25	.935					.935	1
50	.935	.949				.949	2
75	.935	.949	.973			.973	3
100	.935	.949	.973	.978		.978	4
125	.935	.949	.973	.978		.978	4
150	.935	.949	.973	.978	.980	.980	5

Table A-3. (continued)

Growth Period 4

SW = 150 mm

	$\frac{Y}{Y_o}$						
D mm \ I	1	2	3	4	5	$\frac{Y}{Y_o}*$	I*
0	.972					.972	1
25	.972					.972	1
50	.972	.979				.979	2
75	.972	.979	.980			.980	3
100	.972	.979	.980			.980	3
125	.972	.979	.980	.984		.984	4
150	.972	.979	.980	.984	.985	.985	5

SW = 200 mm

	$\frac{Y}{Y_o}$						
D mm \ I	1	2	3	4	5	$\frac{Y}{Y_o}*$	I*
0	.983					.983	1
25	.983					.983	1
50	.983	.980				.983	1
75	.983	.980	.982			.983	1
100	.983	.980	.982	.980		.983	1
125	.983	.980	.982	.980		.983	1
150	.983	.980	.982	.980	.978	.983	1

APPENDIX B

Simulation Program Listing and Documentation

General Program Information

Title: Optimization of Grain Sorghum Water Use Efficiency Under High Frequency Irrigation by System Simulation and Stochastic Dynamic Programming

Author: T. A. Howell

Installation: IBM 360/65 at Texas A&M University

Programming Language: S/360 FORTRAN (LEVEL G)

Date Written: Fall 1974

Remarks: Maximum Core Used--110K

The simulation model used Monte Carlo methods to simulate the environmental data of rainfall, temperature, and potential evaporation. These data were then input to the water balance model to compute plant and soil evaporation. The model then computed the updated inventory of soil water. The results were printed for each day with the option of plotting the results. The total of the water balance components were punched out on cards for each growth period.

For the program listed, the average execution time per simulated growth season (125 days) was 0.10 min using HEX code for the subroutines. The following subroutines were utilized:

PRECIP - simulated rainfall
TEMP - simulated temperature
EPP - simulated pan evaporation
RUNOFF - computed runoff

LAI - computed leaf area index

EVAP - computed plant and soil evaporation assuming soil water was not limiting

SOLWAT - computed water balance, evaporation with water limiting, and profile drainage

CSMPLT - produced a CSMP-likeplot

RANDU - generated a uniformly distributed random number

GAUSS - generated a normally distributed random number

The input and output variable names are defined in the program listing.

The program listed was used to produce the simulation example results as given in Chapter IV. In actual practice the program used was modified from the one listed to calculate the water balance of a given period for each starting water content and irrigation level.

```
C*******************************************************************************
C                                                                              *
C                    OPTIMIZATION OF GRAIN SORGHUM WATER                       *
C                    USE EFFICIENCY UNDER HIGH FREQUENCY                       *
C               IRRIGATION BY SYSTEM SIMULATION AND STOCHASTIC                 *
C                            DYNAMIC PROGRAMMING                               *
C                                                                              *
C                              TERRY A. HOWELL                                 *
C                                                                              *
C                                                                              *
C                           SOIL WATER BALANCE MODEL                           *
C                               DEVELOPED AT THE                               *
C                    BLACKLAND CONSERVATION RESEARCH CENTER                    *
C                                 TEMPLE, TEXAS                                *
C                                                                              *
C                                                                              *
C        THE VARIABLES INPUT ARE------                                         *
C                                                                              *
C               R1 = RAINFALL DAY BEFORE FIRST DAY, INCHES                     *
C               ET = EVAPORATION DAY BEFORE FIRST DAY, MM                      *
C              IS1 = FIRST RANDOM NUMBER SEED                                  *
C             ULLT = SOIL WATER UPPER LIMIT, MM                                *
C             PLLT = SOIL WATER LIMIT FOR POTENTIAL EVAPORATION, MM            *
C                U = UPPER LIMIT FOR STAGE 1 SOIL EVAPORATION, MM              *
C             CONA = CONSTANT FOR SOIL EVAPORATION EQUATION                    *
C              SW1 = AVAILABLE SOIL WATER IN THE UPPER LAYER, MM               *
C              SW2 = AVAILABLE SOIL WATER IN THE LOWER LAYER, MM               *
C            DEPTH = DEPTH OF SOIL PROFILE, MM                                 *
C              DF1 = DEPTH OF UPPER LAYER, MM                                  *
C              SWT = TOTAL PROFILE AVAILABLE WATER, MM                         *
C               DZ = ROOT ZONE EXPANSION RATE, MM/DAY                          *
C            RI(I) = IRRIGATION RATIO OR EFFICIENCY                            *
C               A4 = PLANT SPACING, CM**2/PLANT                                *
C               NM = LEAF AREA INDEX DATA                                      *
C               N1 = LEAF AREA INDEX DATA                                      *
C               M3 = LEAF AREA INDEX DATA                                      *
C               M3 = LEAF AREA INDEX DATA                                      *
C               M1 = LEAF AREA INDEX DATA                                      *
C               P1 = LEAF AREA INDEX DATA                                      *
C               A3 = LEAF AREA INDEX                                           *
C               C8 = LEAF AREA INDEX DATA                                      *
C              AIN = LEAF SIZE INPUT                                           *
C               J1 = GERMINATION DATE, JULIAN DAYS                             *
C               J2 = STARTING DAY, JULIAN DAYS                                 *
C               J3 = DAYS PER PERIOD                                           *
```

```
C                    J4 = WEEK NUMBER                                             *
C                    J5 = DAY OF WEEK                                             *
C                    MO = MONTH (STARTING WITH MARCH)                             *
C                    MI = MONTH BY CALENDAR                                       *
C                 PC(I) = EVAPORATION PAN COEFFICIENTS FOR I TH PERIOD            *
C                 PC(I) = CONVERTS CLASS A PAN EVAPORATION TO POT. EVAPORATION    *
C             TITLEK(I) =  TITLE OF K TH PRINT PLOT                               *
C                  M(I) = MAXIMUM LEAF AREAS, CM**2                               *
C                PWW(I) = RAINFALL PROBABILITIES FOR A WET DAY FOLLOWING A WET    *
C                          DAY FOR WEEK I                                         *
C                PWD(I) = RAINFALL PROBABILITIES FOR A WET DAY FOLLOWING A DRY    *
C                          DAY FOR WEEK I                                         *
C                DPM(I) = DAYS PER MONTH FOR THE I TH MONTH STARTING WITH MARCH   *
C                   KYS = NUMBER OF YEARS OF SIMULATION                           *
C                 KPLOT = PLOT SIGNAL, IF GREATER THAN 0 WILL PLOT                *
C                         EVAPORATION COMPONENTS AND IRRIGATION AMOUNTS           *
C                                                                                 *
C**********************************************************************************
      DIMENSION  DAY(150),ETT(150),AIRR(150),ALAI(150),SW(150),TITLE1(20
     1),TITLE2(20),TITLE3(20),TITLE4(20),DPM(12),RI(6),AIN(19),AOUT(19),
     2M(19),PC(3),PWW(52),PWD(52),ESS(150),EPL(150),DEPTHS(150),TITLE5(2
     10),TITLE6(20),TITLE7(20),AA(19)
      DIMENSION SOILW(6),SOW(30),AIRG(30),RUNO(30),DRAIN(30),ETOT(30),EP
     1O(30),RAT(30),RAINF(30)
C
C****
C  INPUT DATA AND INITIAL PARAMETERS
C****
C
      READ(5,3) KYS,KPLOT,DF1,DEPTH
    3 FORMAT(2I5,2F10.2)
      READ(5,4)J1,J2,J3,J4,J5,MO,MI
    4 FORMAT(7I5)
      READ(5,1) NM,N1,M3,M1,P1,A3,C8,(AIN(I),I=1,19)
    1 FORMAT(4I4,3F5.1,/,(10F6.1))
      DATA RAIN,ET,IS1/0.,1.,1234567/
      DATA ULLT,PLLT,U,CONA,DZ/270.,65.,6.,3.5,32./
      DATA RI(1),RI(2),RI(3),RI(4),RI(5)/0.0,0.25,0.50,0.75,1.00/
      DATA A4/450./
      DATA PC(1),PC(2),PC(3)/0.90,0.65,0.80/
      APE=0.
      APR=0.
      API=0.
      APQ=0.
      APD=0.
```

```
      APEO=0.
      APEPEO=0.
      APEP=0.
      APES=0.
      APET=0.
      SUMES1 = 6.0
      SUMES2 = 5.0
      T2 = 2.
C
C****
C   READ PRINT-PLOT TITLES
C****
C
      READ(5,700) (TITLE1(I),I=1,20)
      READ(5,700) (TITLE2(I),I=1,20)
      READ(5,700) (TITLE3(I),I=1,20)
      READ(5,700) (TITLE4(I),I=1,20)
      READ(5,700) (TITLE5(I),I=1,20)
      READ(5,700) (TITLE6(I),I=1,20)
      READ(5,700) (TITLE7(I),I=1,20)
C
C****
C   READ MAXIMUM LEAF AREAS
C****
C
      READ(5,701) (M(I),I=1,19)
C
C****
C   READ WEEKLY RAINFALL PROBABILITIES
C****
C
      READ(5,702) (PWW(I),PWD(I),I=1,52)
      DO 699 I=1,52
      PWW(I) = PWW(I)/100.0
      PWD(I) = PWD(I)/100.0
  699 CONTINUE
C
C****
C   READ DAYS PER MONTH BEGINING WITH MARCH
C****
C
      READ(5,703) (DPM(I),I=1,12)
      KKK=6
      SOILW(1)=25.
      SOILW(2)=50.
```

```
      SOILW(3)=75.
      SOILW(4)=100.
      SOILW(5)=150.
      SOILW(6)=200.
      SWT=SOILW(KKK)
      SW1=SWT*DF1/DEPTH
      SW2=SWT-SW1
C****
C   WRITE INPUT DATA
C****
C
      WRITE(6,705) ULLT,PLLT,U,CONA,SW1,SW2,DF1,DEPTH,DZ
      PP=10000./A4
      WRITE(6,706) PP,J1,(M(I),I=1,19)
      WRITE(6,1)NM,N1,M3,M1,P1,A3,C8,(AIN(I),I=1,19)
      WRITE(6,707) (PWW(I),I=1,52),(PWD(I),I=1,52)
      II=J2
      LW=J5
      N=J4
      KY=0
      KY=KY+1
      I=0
      DO 100 IP=1,5
 5000 IF(IP.EQ.1) DOM=9.
      IF(IP.EQ.1) IR=3
      IF(IP.EQ.2) IR=3
      IF(IP.EQ.3) IR=4
      IF(IP.EQ.4) IR=2
      IF(IP.EQ.5) IR=1
      IF(KY.GT.KYS) GO TO 1010
      DO 100 MM=1,J3
      I=I+1
      LW=LW+1
      IF(LW-8) 21,22,22
   22 LW=1
      N=N+1
   21 CONTINUE
      II=II+1
      DOM=DOM+1
      DAY(II-J2)=II
      NN=II-J2
      IF(II.EQ.100) GO TO 10
      IF(II.EQ.125) GO TO 10
      IF(II.EQ.150) GO TO 10
      IF(II.EQ.175) GO TO 10
```

```
      IF(II.EQ.200) GO TO 10
      GO TO 11
C
C****
C   WRITE OUTPUT HEADINGS
C****
C
  10  WRITE(6,1005) KY,IP
      WRITE(6,1000)
  11  CONTINUE
      IF(DPM(MO)-DOM) 12,13,13
  12  MO=MO+1
      MI=MI+1
      DOM=1.
  13  CONTINUE
      W=N
      PW=PWW(N)
      PD=PWD(N)
C
C****
C   SIMULATE ENVIRONMENTAL DATA--RAIN,TEMPERATURE, AND PAN EVAPORATION
C****
C
      RY=RAIN
      CALL PRECIP(PW,PD,RY,IS1,RAIN)
      R=RAIN
      CALL TEMP(R    ,W,IS1,T)
      R=RAIN
      CALL EPP(RY,R    ,W,IS1,E)
      R=RAIN
      IF(RAIN.LE.0.0) GO TO 16
      CALL RUNOFF(RAIN,SWT,RO)
      GO TO 17
  16  RO=0.0
  17  CONTINUE
      ULL=ULLT*DF1/DEPTH
      IF(DF1.GT.DEPTH) ULL=ULLT
      IF(II.LE.145) EO=E*PC(1)
      IF(II.GT.145.AND.II.LE.248) EO=E*PC(2)
      IF(II.GT.248) EO=E*PC(3)
      W1=SW1/ULL
C
C****
C   SIMULATE LEAF AREA INDEX
C****
```

```
C
      CALL LAI(AIN,J1,A4,M,T,W1,NM,P1,C9,N1,A3,C6,C8,M3,M1,II   ,AOUT,RLA
     1I)
      AIRR(II-J2)=RI(IR)*ET
      PUT=AIRR(II-J2)+RAIN-RO
      ALAI(II-J2)=RLAI
      DO 15 NI=1,19
      AIN(NI)=AOUT(NI)
   15 CONTINUE
C
C****
C   COMPUTE PLANT AND SOIL EVAPORATION
C****
C
   35 CALL EVAP(EO,RLAI,II,U,SUMES1,SUMES2,   PUT,CONA,  T2,ES,EP,ET,EPEO)
C
C****
C   COMPUTE WATER BALANCE AND EVAPORATION UNDER LIMITED SOIL WATER
C****
C
      CALL SOLWAT(EO,EP,ES,ET,SW1,SW2,   PUT,II,ULLT,PLLT,RLAI,DF1,DEPTH,
     1DZ,D1,D2,SWT,EQ,ASW)
      EPL(II-J2)=EP
      DEPTHS(II-J2)=DF1
      ESS(II-J2)=ES
      SW(II-J2)=SWT
      ETT(II-J2)=ET
      Q=RO+EQ
   30 IDOM=DOM
C
C****
C   WRITE OUTPUT DAILY DATA
C****
C
      WRITE(6,1001) MI,IDOM,KY,II,E,T,RLAI,RAIN,AIRR(II-J2),Q,D2,EO,EPEO
     1,EP,ES,ET,ASW,SWT
      APE=APE+E
      APR=APR+R
      API=API+AIRR(II-J2)
      APQ=APQ+Q
      APD=APD+D2
      APEO=APEO+EO
      APEPEO=APEPEO+EPEO
      APEP=APEP+EP
      APES=APES+ES
```

```
      APET=APET+ET
      IF(II.EQ.124) GO TO 50
      IF(II.EQ.149) GO TO 50
      IF(II.EQ.174) GO TO 50
      IF(II.EQ.199) GO TO 50
      IF(II.EQ.224) GO TO 50
      GO TO 51
   50 RATIO=APET/(APEPEO+APES)
C
C****
C   WRITE GROWTH PERIOD TOTALS
C****
C
      WRITE(6,1002) APE,APR,API,APQ,APD,APEO,APEPEO,APEP,APES,APET,RATIO
      WRITE(6,1004) IP,RI(IR),(AIN (IN),IN=1,19)
      WRITE(6,1008) NM,P1,N1,M3,M1,A3,C8
      WRITE(7,1007) NM,N1,M3,M1,P1,A3,C8,(AIN(I),I=1,19)
      RAINF(KY)=APR
      SOW(KY)=SWT
      AIRG(KY)=API
      RUNO(KY)=APQ
      DRAIN(KY)=APD
       ETOT(KY)=APET
      EPO(KY)=APEO
      RAT(KY)=RATIO
      APE=0.
      APR=0.0
      API=0.
      APQ=0.
      APD=0.
      APEO=0.
      APEPEO=0.
      APEP=0.
      APES=0.
      APET=0.
   51 CONTINUE
  100 CONTINUE
C
C****
C   PLOT DATA
C****
C
      CALL CSMPLT( DAY,SW,NN,300.,TITLE1)
      IF(KPLOT.EQ.0) GO TO 302
      CALL CSMPLT( DAY,ALAI,NN, 8.0,TITLE4)
```

```
      CALL CSMPLT(DAY,ESS,NN,10.0,TITLE5)
      CALL CSMPLT(DAY,EPL,NN,10.0,TITLE6)
      CALL CSMPLT( DAY,ETT,NN,10.0,TITLE2)
      CALL CSMPLT( DAY,AIRR,NN,10.0,TITLE3)
      CALL CSMPLT(DAY,DEPTHS,NN,2000.,TITLE7)
  302 CONTINUE
 700  FORMAT(20A4)
 701  FORMAT(19I4)
 702  FORMAT(5X,2F5.0)
 703  FORMAT(12F4.0)
  705 FORMAT(1H1,//,10X,'** SOIL PARAMETERS **',//13X,'AVAILABLE WATER =
     1',F4.0,' MM',/13X,'LOWER LIMIT POT.EVAP. =',F3.0,' MM',/13X,'MAX.
     2STAGE I SOIL EVAP. =',F2.0,' MM',/13X,'STAGE II SOIL EVAP. =',F3.1
     3,'* T **0.5',/13X,'BEGINING UPPER SOIL WATER =',F4.0,' MM',/13X,
     4'BEGINING LOWER SOIL WATER =',F4.0,' MM',/13X,'BEGINING PROFILE DE
     5PTH =',F5.0,' MM',/13X,'TOTAL PROFILE DEPTH =',F5.0,' MM'
     6                                           ,/ ,13X,'ROOT ZONE EXPANSION',F
     74.0,' MM/DAY'/)
  706 FORMAT(10X,'** PLANT PARAMETERS **',//13X,'PLANT POPULATION =',
     1F5.2,'PLANTS/SQ.METER',/13X,'GERMINATION DATE =',I3,'JULIAN DAYS'/
     213X,
     2'MAX.LEAF AREAS--LEAF 1(BOTTOM) TO 19(FLAG) IN CM**2'/20X19(I5)/)
  707 FORMAT(10X,'** RAINFALL PROBABILITIES **'//13X'PROB. FOR W/W(WET
     1GIVEN PREVIOUS DAY WET)'/19X,'FOR W/D(WET  GIVEN PREVIOUS DAY DRY)
     2'/19X,'FOR WEEK 1 (MAR.1) TO 52 (FEB.21)'/10X,'W/W',20F5.2,/13X,20
     3F5.2,/13X,12F5.2//10X,'W/D',20F5.2,/13X,20F5.2,/13X,12F5.2,//)
 1000 FORMAT(     5(/),T22,'PAN',T29,'AVG.',T35,'LEAF',T72,'POT.',T79,
     1                                                                 'POT
     1.PANT',T91,'PLANT',T99,'SOIL',T109,'TOTAL',T116,'AVAIL.SOIL',T128,
     2'SOIL',/T15,'JUL.',T21,'EVAP.',T29,'TEMP.',T35,'AREA',T41,'RAIN',T
     347,'IRRIG.',T54,'RUNOFF',
     4                        T61,'DRAINAGE',T72,'EVAP.',T81,'EVAP.',T91
     4,'EVAP.',T99,'EVAP.',T109,'EVAP.',T118,'WATER',T127,'WATER',/T5,'M
     5O DAY YR DATE (MM/DAY)  (C)  INDEX (MM)    (MM)    (MM)     (MM)    (M
     5M/DAY) (MM/DAY)  (MM/DAY) (MM/DAY) (MM/DAY)      (MM)       (MM)'/4X12
     67(1H-)/)
 1001 FORMAT(4X,I2,1X,2I3,1X,I3,3X,F4.1,4X,F4.0,1X,F5.2,1X,F5.1,2(2X,F5.
     11),3X,F5.1,4X,F4.1,5X,F4.1,1X,3(5X,F4.1),2X,2(5X,F4.0))
 1002 FORMAT(//4X,'PERIOD',T128,'E/EO',/4X,'TOTALS',9X,F5.1,T38,3(2X,F5.
     11),3X,F5.1,3X,F5.1,4X,F5.1,1X,3(4X,F5.1),16X,F4.2)
 1004 FORMAT(//4X,'GROWTH PERIOD = ',I2,/4X,'IRRIGATION AMOUNT = ',F4.2,
     1'*ET (MM/DAY)',/4X,'LEAF AREAS FOR LEAF 1(BOTTOM) TO 19(FLAG)',/4X
     2,19F6.1)
 1005 FORMAT(1H1   ,T61,'YEAR = ',I2,//,T56,'GROWTH PERIOD = ',I2)
 1007 FORMAT(4I2,3F4.1,(10F6.1))
```

```
 1008 FORMAT(//6X,'NM    P1  N1  M3  M1  A3  C8'/4X,I4,F6.1,3I4,F5.1,F4.
     11)
 1009 FORMAT(I10,2F10.2,/,(8F10.4))
 1010 WRITE(7,1009) IP,RI(IR),ST,(RAINF(I),AIRG(I),RUNO(I),DRAIN(I),
     1 ETOT(I),EPO(I),SOW(I),RAT(I),I=1,KYS)
      STOP
      END
      SUBROUTINE PRECIP(PWW,PWD,R1,IS1,R)
C***********************************************************************
C                                                                      *
C        THIS SUBROUTINE USES MARKOV PROBABILITIES TO SIMULATE         *
C        THE OCCURRANCE OF RAIN AND THEN SIMULATES THE RAINFALL        *
C        QUANTITY BY A LOG-NORMAL DISTRIBUTION.                        *
C                                                                      *
C             VARIABLES---                                             *
C             PWW(I)    =  PROBABILITY OF WET DAY FOLLOWING A          *
C                          WET DAY IN WEEK I                           *
C             PWD(I)    =  PROBABILITY OF WET DAY FOLLOWING A          *
C                          DRY DAY IN WEEK I                           *
C             R1        =  RAINFALL YESTERDAY, MM                      *
C             R         =  RAINFALL TODAY, MM                          *
C             IS1       =  RANDOM NUMBER SEED                          *
C                                                                      *
C***********************************************************************
      RLBAR=-1.80734
      RLSD=1.50436
      R1=R1/25.4
      IF(R1.LT.0.01) GO TO 1
      P=1.0-PWW
      GO TO 2
 1    P=1.0-PWD
 2    CONTINUE
      CALL RANDU(IS1,IS,UNP)
      IS1=IS
      IF(UNP.GT.P) GO TO 3
      R=0.0
      RETURN
 3    CONTINUE
      CALL GAUSS(IS1,RLSD,RLBAR,RL)
      R=EXP(RL)
      IF(R.LE.0.0) GO TO 3
      R=R*25.4
      RETURN
      END
      SUBROUTINE EPP( A,B,WKN,IS1,E)
```

```
C***********************************************************************
C                                                                      *
C        THIS SUBROUTINE SIMULATES THE PAN EVAPORATION BASED ON        *
C        THE NORMAL DISTRIBUTION, PAST RAINFALL HISTORY, AND           *
C        TIME OF YEAR.                                                 *
C                                                                      *
C            VARIABLES---                                              *
C            A         = RAINFALL YESTERDAY, MM                        *
C            B         =  RAINFALL TODAY, MM                           *
C            WKN       =  WEEK NUMBER                                  *
C            IS1       =  RANDOM NUMBER SEED                           *
C            E         =  PAN EVAPORATION, MM                          *
C                                                                      *
C***********************************************************************
      EDD=(1.7958E-1)+(-2.8690E-3)*WKN+(1.9318E-3)*WKN**2+(-9.3897E-5)*W
     1KN**3+(1.3522E-6)*WKN**4+(-4.6192E-9)*WKN**5
      EDW=(7.7356E-2)+(1.5783E-2)*WKN+(2.9775E-4)*WKN**2+(-4.6439E-5)*WK
     1N**3+(9.4478E-7)*WKN**4+(-5.1172E-9)*WKN**5
      EWW=(7.4646 E-2)+(4.5619E-3)*WKN+(6.6367E-4)*WKN**2+(-4.6439E-5)*W
     1KN**3+(9.1927E-7)*WKN**4+(-5.2867E-9)*WKN**5
      EWD=(1.5252E-1)+(-1.8303E-2)*WKN+(3.2969E-3)*WKN**2+(-1.5481E-4)*W
     1KN**3+(2.6919E-6)*WKN**4+(-1.5626E-8)*WKN**5
      EDDSD=0.075
      EWWSD=0.066
      EWDSD=0.076
      EDWSD=0.070
      B=B/25.4
      IF(A.LT.0.01.AND.B.LT.0.01) GO TO 1
      IF(A.GE.0.01.AND.B.LT.0.01) GO TO 2
      IF(A.GE.0.01.AND.B.GE.0.01) GO TO 3
      EBAR=EWD
      ESD=EWDSD
      GO TO 4
  1    EBAR=EDD
      ESD=EDDSD
      GO TO 4
 2    EBAR=EDW
      ESD=EDWSD
      GO TO 4
  3   EBAR=EWW
      ESD=EWWSD
 4    CONTINUE
      CALL GAUSS(IS1,ESD,EBAR,E)
      IF(E.LE.0.0) GO TO 4
      E=E*25.4
```

```
      RETURN
      END
      SUBROUTINE TEMP(C,WKN,IS1,T)
C************************************************************************
C                                                                       *
C        THIS SUBROUTINE SIMULATES THE AVERAGE DAILY TEMPERATURE        *
C        BASED ON THE NORMAL DISTRIBUTION AND TIME OF YEAR.             *
C                                                                       *
C             VARIABLES---                                              *
C             C         =  RAINFALL TODAY, MM                           *
C             WKN       =  WEEK NUMBER                                  *
C             IS1       =  RANDOM NUMBER SEED                           *
C             T         =  TEMPERATURE, C                               *
C                                                                       *
C************************************************************************
      TD=(4.9266E1)+(1.5587)*WKN+(1.2205E-1)*WKN**2+(-7.1678E-3)*WKN**3+
     1(7.5931E-5)*WKN**4+(1.2059E-7)*WKN**5
      TW=(5.4419E1)+(1.6757)*WKN+(3.1644E-2)*WKN**2+(-1.6497E-3)*WKN**3+
     1(-4.6321E-5)*WKN**4+(1.0511E-6)*WKN**5
      TDSD=(7.605)+(1.9067E-1)*WKN+(-7.7305E-2)*WKN**2+(4.1217E-3)*WKN**
     13+(-7.6936E-5)*WKN**4+(4.8226E-7)*WKN**5
      TWSD=(8.5600)+(-1.4307)*WKN+(1.2773E-1)*WKN**2+(-6.0347E-3)*WKN**3
     1+(1.4051E-4)*WKN**4+(-1.1873E-6)*WKN**5
      C=C/25.4
      IF(C.LT.0.01) GO TO 1
      TBAR=TW
      TSD=TWSD
      GO TO 2
 1    TBAR=TD
      TSD=TDSD
 2    CONTINUE
      CALL GAUSS(IS1,TSD,TBAR,T)
      T=(T-32.)*5./9.
      RETURN
      END
      SUBROUTINE RUNOFF(R,SW,RO)
C************************************************************************
C                                                                       *
C        THIS SUBROUTINE COMPUTES RUNOFF BASED ON THE SCS METHOD        *
C        WITH THE INITIAL ABSTRACTION AND CURVE NUMBER EMPIRICALLY      *
C        ESTIMATED AS A FUNCTION OF SOIL WATER.                         *
C                                                                       *
C             VARIABLES---                                              *
C             R         =  RAINFALL, MM                                 *
C             SW        =  SOIL WATER, MM                               *
```

```
C             RO         =  RUNOFF, MM                                *
C             AIA        =  INITIAL ABSTRACTIONS, MM                  *
C                                                                     *
C***********************************************************************
      IF(R) 1,1,2
    2 AIA=116.-0.41*SW
      IF(AIA.LT.0.0) AIA=0.0
      IF(R-AIA) 1,1,3
    3 CN=50.0+0.15*SW
      S=(25400./CN)-254.
      RO=(R-AIA)**2/(R-AIA+S)
      GO TO 4
    1 RO=0.
    4 CONTINUE
      RETURN
      END
      SUBROUTINE LAI(AIN,J1,A4,M,T,W1,N,P1,C9,N1,A3,C6,C8,M3,M1,IDAY,ADOU
     1T,RLAI)
      DIMENSION AIN(19),AOUT(19),M(19)
C***********************************************************************
C                                                                     *
C        THIS SUBROUTINE SIMULATES THE LEAF AREA INDEX OF GRAIN       *
C        SORGHUM.                                                     *
C                                                                     *
C             VARIABLES---                                            *
C             AIN(I)     =  INPUT LEAF AREAS OF LEAF I, CM**2         *
C             J1         =  GERMINATION DATE, JULIAN DAYS             *
C             A4         =  PLANT SPACING, CM**2/PLANT                *
C             SW         =  SOIL WATER, MM                            *
C             ULL        =  SOIL WATER UPPER LIMIT, MM                *
C             T          =  TEMPERATURE, C                            *
C             M(I)       =  MAXIMUM LEAF AREA OF LEAF I, CM**2        *
C             IDAY       =  DAY                                       *
C             AOUT(I)    =  NEW SIZE OF LEAF I, CM**2                 *
C             RLAI       =  LEAF AREA INDEX                           *
C                                                                     *
C***********************************************************************
      M4=M3+7
      DO 700 I=1,M1
      AOUT(I) = AIN(I)
  700 CONTINUE
      IF(C8.EQ.1.) GO TO 940
      IF(J1.GT.IDAY) GO TO 1060
C
C****
```

```
C  LEAF APPEARANCE RATE
C****
C
      P1=P1+0.03*T-0.3
      MM=P1
      IF(MM.GT.M1) GO TO 750
      GO TO 760
  750 MM=M1
C
C****
C  LEAF EXPANSION RATE
C****
C
  760 A1=5.1*(T-12.)
      IF(A1.LT.0.0) A1 = 0.
      IF(N.GT.MM) GO TO 850
      DO 830 I=N,MM
      AOUT(I) = AOUT(I) + A1
      IF(AOUT(I).GT.M(I)) GO TO 810
      GO TO 830
  810 AOUT(I)=M(I)
      N1=N1+1
  830 CONTINUE
      N=N1
  850 IF(MM.EQ.M3.AND.MM.LT.M4) GO TO 870
      GO TO 890
  870 M3=M3+1
      AOUT(M3-7)=0.
  890 IF(MM.GT.M4) GO TO 910
      GO TO 1000
  910 R5=0.1*A3**2
      M5=R5
      AOUT(M5+M4-7)=0.
      IF(AOUT(M1).GT.0.5*M(M1)) GO TO 936
      GO TO 940
 936  C8=1.
  940 W1=W1-0.
      IF(W1.LT.0.2) GO TO 970
      GO TO 1000
  970 R6=8.-40.*W1
      M6=R6
      AOUT(M5+M4-7+M6)=0.
 1000 A2=0.
      DO 1040 I=1,M1
      AOUT(I)=AOUT(I)*0.994
```

```
      A2=A2+AOUT(I)
 1040 CONTINUE
      RLAI=A2/A4
      A3=RLAI
      GO TO 1070
 1060 RLAI=0.
 1070 RETURN
      END
      SUBROUTINE EVAP(EO,RLAI,IDAY,U,SUMES1,SUMES2,  PUT,CONA,T2, ES,EP,
     1ET,EPEO)
C*******************************************************************************
C                                                                              *
C        THIS SUBROUTINE COMPUTES THE DAILY PLANT AND SOIL EVAPORATION         *
C        ASSUMING SOIL WATER IS NOT LIMITED.                                   *
C                                                                              *
C           THE VARIABLES ARE--                                                *
C              EOS = POTENTIAL EVAPORATION BELOW THE PLANT CANOPY, MM/DAY.     *
C               ES = SOIL EVAPORATION, MM/DAY.                                 *
C           SUMES1 = CUMULATIVE SOIL EVAPORATION IN STAGE I, MM.               *
C           SUMES2 = CUMULATIVE SOIL EVAPORATION IN STAGE II, MM.              *
C               EP = PLANT EVAPORATION, MM/DAY.                                *
C               ET = TOTAL EVAPORATION (ES+EP), MM/DAY.                        *
C                                                                              *
C*******************************************************************************
C**** COMPUTE SOIL EVAPORATION ****
C****
      P=PUT
      EOS=EO*EXP(-0.398*RLAI)
   99 IF(SUMES1 - U) 1,2,2
    1 IF(P - SUMES1) 3,4,4
    3 SUMES1 = SUMES1 - P
      GO TO 5
    4 SUMES1 = 0.
    5 SUMES1 = SUMES1 + EOS
      IF(SUMES1 - U) 6,6,7
    6 ES = EOS
      GO TO 24
    7 ES = EOS - 0.4*(SUMES1 - U)
      SUMES2 = 0.6*(SUMES1 - U)
      T2= (SUMES2/CONA)**2
      GO TO 24
    2 IF(P - SUMES2) 9,8,8
    8 P = P - SUMES2
      SUMES1 = U - P
      T2= 0.
```

```
      IF(P- U) 5,5,4
    9 T2= T2+ 1.
      ES =CONA*T2**0.5 - SUMES2
      IF(P .GT. 0.) GO TO 10
      IF(ES .GT. EOS) ES = EOS
      GO TO 11
   10 ESX = 0.8*P
      IF(ESX .LE.ES) ESX = ES + P
      IF(ESX .GT. EOS) ESX = EOS
      ES = ESX
   11 SUMES2 = SUMES2 + ES - P
      T2= (SUMES2/CONA) **2
C****
C**** COMPUTE PLANT EVAPORATION
C****
   24 IF(ES .LT. 0.) ES = 0.
      IF(RLAI.GT.3.0) GO TO 26
      IF(RLAI.LT.0.0) GO TO 51
      EP=(-0.21+0.70*(RLAI **0.5))*EO
      IF(RLAI) 60,60,61
  60  EPEO=0.0
      GO TO 62
  61  EPEO=EP
  62  CONTINUE
      IF(EP.LT.0.0) EPEO = 0.0
      GO TO 50
51    EP = 0.0
      EPEO=0.0
50    CONTINUE
      IF (EP .LT. 0.) EP = 0.
      GO TO 25
   26 EP = EO - ES
      EPEO=EP
C****
C**** COMPUTE TOTAL EVAPORATION ****
C****
   25 ET = ES + EP
      IF(EO-ET) 39,41,41
   39 ET = EO
      EP = ET - ES
      EPEO=EP
   41 RETURN
      END
      SUBROUTINE SOLWAT(EO,EP,ES,ET,SW1,SW2,  PUT,II,ULLT,PLLT,RLAI,DF1,
     1DEPTH ,DZ,D1,D2,SWT,EQ,ASW)
```

```
C
C************************************************************************
C                                                                       *
C        THIS SUBROUTINE COMPUTES THE SOIL WATER BALANCE AND PROFILE    *
C        DRAINAGE AND EVAPORATION WITH SOIL WATER LIMITED.              *
C                                                                       *
C             VARIABLES---                                              *
C             SW1       =  UPPER SOIL WATER CONTENT, MM                 *
C             SW2       =  LOWER SOIL WATER CONTENT, MM                 *
C             ULLT      =  MAXIMUM AVAILABLE SOIL WATER, MM             *
C             PLLT      =  MAXIMUM AVAIL. WATER FOR POT. EVAP., MM      *
C             DF1       =  ROOT ZONE DEPTH, MM                          *
C             DEPTH     =  PROFILE DEPTH, MM                            *
C             D1        =  DRAINAGE BELOW ROOT ZONE, MM/DAY             *
C             D2        =  DRAINAGE BELOW PROFILE, MM/DAY               *
C             DZ        =  ROOT ZONE EXTENSION, MM/DAY                  *
C             ASW       =  AVAILABLE SOIL WATER, MM                     *
C                                                                       *
C************************************************************************
      EQ=0.
      DF2=DF1
      IF(RLAI.LE.0.0) GO TO 1
      DF1=DF1+DZ
   1  IF(DEPTH-DF1) 100,100,200
  200 ALLEO=PLLT*DF1/DEPTH
      SW1 = SW1 +  SW2*(DF1-DF2)/(1750.-DF2)
      SW2=SWT-SW1
      ULL=ULLT*DF1/DEPTH
      IF(SW1-ALLEO) 8,9,9
  8   ET=EO*SW1/ALLEO
      IF(RLAI-0.0) 2,2,3
  2   EP=0.
      ES=ET
      IF(ET.GT.EO) ES=EO
      ET=ES
      GO TO 11
  3   IF(ET-EO) 6,6,7
  7   IF(EO-ES) 21,21,12
  21  ES=EO
      EP=0.0
      ET = EO
      GO TO 11
  12  IF(EP-EO+ES) 25,25,26
  26  EP=EO-ES
      GO TO 11
```

```
25   ET= EP+ES
     GO TO 11
6    IF(ET-ES) 13,13,14
13   ES=EO
     EP=0.0
     GO TO 11
14   IF(EP-ET+ES) 4,4,27
27   EP=ET-ES
     GO TO 11
 4   ET=EP+ES
     GO TO 11
9    CONTINUE
5    ET=ES+EP
     IF(EO-ET) 10,11,11
10   ET=EO
     EP=ET-ES
11   SW1=SW1-ET+   PUT
     IF(SW1-ULL) 15,15,16
16   D1=SW1-ULL
     SW1=ULL
     GO TO 22
15   D1=0.0
22   CONTINUE
     IF(D1 -25.) 20,20,30
30   XTRAQ=D1-25.
     EQ=EQ+XTRAQ
     D1=25.
20   CONTINUE
     SW2=SW2+D1
     IF(SW2-ULLT+ULL) 17,17,18
18   D2=SW2-ULLT+ULL
     SW2=ULLT-ULL
     GO TO 19
17   D2=0.0
19   CONTINUE
     SWT=SW1+SW2
     IF(SWT-ULLT) 50,50,40
40   D2=D2+SWT-ULLT
50   ASW=SW1
     IF(D2.GE.25.) GO TO 60
     GO TO 70
60   EQ=EQ+D2-25.
     D2=D2-25.
  70 CONTINUE
     RETURN
```

```
 100 EO=0.
     SW2=0.
     DF1 = DEPTH
     SW1 = SWT
2000 IF(SW1-PLLT) 80,90,90
 80  ET=EO*SW1/PLLT
     IF(RLAI-0.0) 202,202,203
 202 EP=0.0
     ES=ET
     IF(ET.GT.EO) ES=EO
     ET=ES
     GO TO 1100
 203 IF(ET-EO) 206,206,207
 207 IF(EO-ES) 210,210,212
 210 ES=EO
     EP=0.0
     GO TO 1100
 212 IF(EP-EO+ES) 250,250,260
 260 EP=EO-ES
     GO TO 1100
 250 ET=EP+ES
     GO TO 1100
 206 IF(ET-ES) 213,213,214
 213 ES=EO
     EP=0.0
     GO TO 1100
 214 IF(EP-ET+ES) 401,401,270
 270 EP=ET-ES
     GO TO 1100
 401 ET=EP+ES
     GO TO 1100
 90  CONTINUE
     ET=ES+EP
     IF(EO-ET) 1000,1100,1100
1000 ET=EO
     EP=ET-ES
1100 SW1=SW1-ET+   PUT
     IF(SW1-ULLT) 150,160,160
 160 D2=SW1-ULLT
     SW1=ULLT
     GO TO 170
 150 D2=0.
 170 CONTINUE
     SWT=SW1
     IF(SWT-ULLT) 500,500,400
```

```
  400 D2=D2+SWT-ULLT
      SW1=ULLT
      SWT=ULLT
  500 ASW=SWT
      IF(D2.GE.25.) GO TO 600
      GO TO 700
  600 EQ=EQ+D2-25.
      D2=D2-25.
  700 CONTINUE
      D1=0.
      RETURN
      END
      SUBROUTINE CSMPLT(X,Y,N,VMAX,TITLE)
C
C****
C   PRINT PLOTS VARIABLE Y VS VARIABLE X FOR N
C****
C
      DIMENSION X(N),DELY(100),CHAR(3), Y(N),TITLE(20)
      DATA CHAR /'-','+',' '/
      YMAX = VMAX
      YMIN = 0.
      DO 1 I = 2,N
      IF(Y(I).GT.YMAX) YMAX = Y(I)
      IF(Y(I).LT.YMIN) YMIN = Y(I)
    1 CONTINUE
      RANGE = YMAX - YMIN
      IF(RANGE.LE.0.0) RANGE = 1.0E-20
      WRITE(6,400) (TITLE(I), I = 1,20)
      WRITE(6,500) YMIN, YMAX
      DO 8 J = 1,N
      DO 2 I = 1,100
      DELY(I) = CHAR(1)
    2 CONTINUE
      INDEX = 100.0*(Y(J) - YMIN)/RANGE + 0.5
      IF(INDEX.GE.100) GO TO 4
      IF(INDEX.LE.1) GO TO 5
      DELY(INDEX) = CHAR(2)
      INDEX1 = INDEX + 1
      DO 3 K = INDEX1,100
      DELY(K) = CHAR(3)
    3 CONTINUE
      GO TO 7
    4 DELY(100) = CHAR(2)
      GO TO 7
```

```
  5    CONTINUE
       DO 6 K = 1,100
       DELY(K) = CHAR(3)
  6    CONTINUE
       DELY(1) = CHAR(2)
  7    CONTINUE
       WRITE(6,600) X(J),Y(J),(DELY(K), K = 1,100)
  8    CONTINUE
  400  FORMAT(1H1,40X,20A4)
  500  FORMAT(    T48,'                                                    ',
      1'            ', /  ,T4,'  PERIOD ',T19,'  VALUE  ',T32,
      3'MINIMUM'T126,'MAXIMUM',/,29X,1PE10.3,T123,1PE10.3,/,T33,'I',T132,
      4'I')
  600  FORMAT(2X,1PE10.3,5X,1PE10.3,5X,100A1)
       RETURN
       END
       SUBROUTINE RANDU(IX,IY,YFL)
C****
C   GENERATES A UNIFORMILY DISTRIBUTED RANDOM NUMBER BETWEEN 0 AND 1
C****
       IY=IX*65539
       IF(IY) 5,6,6
 5     IY=IY+2147483647+1
 6     YFL=IY
       YFL=YFL*.4656613E-9
       RETURN
       END
       SUBROUTINE GAUSS(IX,S,AM,V)
C****
C   GENERATES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH MEAN(AM) AND ST.DEV(S)
C****
       A=0.0
       DO 50 I=1,12
       CALL RANDU(IX,IY,Y)
       IX=IY
 50    A=A+Y
       V=(A-6.0)*S+AM
       RETURN
       END
```