Downstream hydraulic geometry relations: 2. Calibration and testing

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[1] Using 456 data sets under bank-full conditions obtained from various sources, the geometric relations, derived in part 1 [*Singh et al.*, 2003], are calibrated and verified using the split sampling approach. The calibration of parameters shows that the change in stream power is not shared equally among hydraulic variables and that the unevenness depends on the boundary conditions to be satisfied by the channel under consideration. The agreement between the observed values of the hydraulic variables and those predicted by the derived relations is close for the verification data set and lends credence to the hypotheses employed in this study. *INDEX TERMS:* 1824 Hydrology: Geomorphology (1625); 1871 Hydrology: Surface water quality; *KEYWORDS:* dynamic equilibrium, hydraulic geometry, maximum entropy, minimum energy dissipation, regime equations, stream power

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1. Introduction

[2] In part 1 [*Singh et al.*, 2003], downstream hydraulic geometry relations were derived by hypothesizing that for a given flow discharge the spatial change of stream power is accomplished by the spatial change in channel form and hydraulic variables. This change in the stream power is shared by flow depth, channel width, flow velocity, and friction, depending on the boundary conditions the channel has to satisfy. One implication of this hypothesis is that the change in stream power is shared equally among hydraulic variables. However, equal sharing is seldom true and hence by introducing the concept of unequal sharing through weighting factors, this hypothesis was generalized, which then led to four families of general hydraulic geometry relations. The exponents in these relations vary continuously over a wide range, which is in agreement with the concept expounded by *Rhoads* [1991] who developed a continuously varying parameter model of the downstream hydraulic geometry.

[3] Reviewing the data reported in the literature, it is apparent that the precise information on the conditions under which data were collected, such as whether the river is in equilibrium, is often missing or is not known. Thus one usually assumes that the river under consideration is under equilibrium [*Carlston*, 1969; *Park*, 1977; *Rhodes*, 1978; *Osterkamp and Hedman*, 1982]. Indeed the basic assumption for all regime equations is that they were obtained under the equilibrium or dynamic equilibrium condition. This means that the geometric relationships at a given location and under specific hydrologic and hydraulic conditions may deviate from those given in regime equations, but the long-term average values remain fairly constant. These considerations led us to employ this same assumption in this study. Furthermore, it is not always clear as to the boundary conditions under which the data has been collected. The derived relations have weighting factors as well as morphological coefficients and exponents as parameters. When estimating these parameters, all four families of relations were employed, because there was no information on a specific set of boundary conditions in order to apply a specific family of relations. This is consistent with past investigations reported in the literature [*Park*, 1977; *Hey and Thorne*, 1986]. Thus the objective of this paper is to calibrate and test the hydraulic geometry relations, derived in the first part of the series, using a large set of data collected from various sources under a variety of hydraulic environments.

[4] The paper is organized as follows: Introducing the objective of the paper in the first section, section 2 discusses calibration of the derived hydraulic geometry relations. More specifically, it develops a procedure for estimating the weighting factors, and determines the morphological coefficients from a large set of data collected from various sources. Section 3 tests the derived equations using the split sampling approach, and section 4 analyzes the influence of different channel shapes on the derived hydraulic geometry relations which are based on V-shaped cross section. The

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results are discussed in section 5. The paper is concluded by section 6 and the cited literature.

2. Calibration of Downstream Hydraulic Geometry Equations

[5] The downstream hydraulic geometry equations derived in part 1 have two sets of parameters: (1) weighting factors, and (2) morphological coefficients and exponents. The weighting factors can be determined using the values of the hydraulic exponents, whereas the morphological coefficients and exponents can be determined from the data.

2.1. Estimation of Weighting Factors w, r, and J

[6] In the downstream hydraulic geometry equations derived in part 1 of the series, there are three weighting factors w (corresponding to possibility 1 for Manning's n and channel width B relation), r (corresponding to possibility 2 for channel width B and flow depth h relation), and J (corresponding to possibility 3 for Manning's n and flow depth h relation). It may be recalled that possibility 1 states that a channel adjusts its roughness and channel width in response to a change in its stream power. This means that weighting factors r and J will be zero in this case. According to possibility 2, the adjustment in stream power is accommodated by changes in channel width and flow depth. That means that weighting factors w and J will be zero. Likewise, possibility 3 states that the channel changes its roughness and flow depth in response to the change in its stream power. Hence weighting factors w and r will be zero. Possibility 4 involves changes in flow depth, width and roughness in response to adjustment in its stream power. This means that weighting factors, w, r and J, will have positive finite values. Theoretically, w, r and J can be zero, which means that they have no effect on the adjustment of the hydraulic geometry. In reality, they are interrelated and any weighting factor can have a very small but not a zero value. A zero value is used as a limiting case for discussion to determine the variation of other hydraulic parameters.

[7] In principle, weighting factors, w, r and J, should be determined, based on the data of specific rivers or regions under the specified possibilities (1, 2, 3, or 4). However, information on the possibility for which the data were collected and the consequent exponents b, f, and m determined is lacking. Therefore, in this part, data reported corresponding to Tables 1a and 1b were used to calibrate and test the performance of the derived equations. The average values of 40 sets of the exponents b, f, and m listed in Tables 1a and 1b are width exponent b = 0.456, depth exponent f = 0.357, and velocity exponent m = 0.202. These values were used to develop a procedure for evaluating the weighting factors.

2.1.1. Possibility 1: r = J = 0

[8] According to the exponents of discharge in equations (A1) and (A2) of part 1, one can write, respectively,

$$\frac{1}{1+w} = 0.456 \quad (1)$$

$$\frac{2w}{5(1+w)} = 0.202 \quad (2)$$

Equation (1) yields the weighting factor w (designated as w_1) = 1.193 and equation (2) results in w (designated as w_2) = 1.020. One possible way to satisfy both equations (1) and (2) to the maximum extent is to take the average value of w_1 and w_2, which gives w = 1.107 and thereby exponent b (designated as b_1) = 0.475 in equation (A1) of part 1 and exponent m (designated as m_1) = 0.210 in equation (A2) of part 1. For possibility 1, the weighting factor and the corresponding exponent values are thus as follows: w = 1.107, r = J = 0, b_1 = 0.475, and m_1 = 0.210, with subscript 1 denoting the possibility 1. The value of the weighting factor 1.107 says that the channel width proportionately undergoes greater change than does the channel roughness in sharing the change of stream power.

2.1.2. Possibility 2: w = J = 0

[9] According to the exponents of discharge in equations (A10) to (A12) of part 1, one gets, respectively,

$$\frac{1}{1+r} = 0.456 \quad (3)$$

$$\frac{3r}{5(1+r)} = 0.357 \quad (4)$$

$$\frac{2r}{5(1+r)} = 0.202 \quad (5)$$

It should be noted that the left-hand sides of equations (3) to (5) are the power indexes of parameter Q (flow discharge) in equations (A10) to (A12) of part 1. Equation (3) yields the weighting factor r (designated as r_1) = 1.193, equation (4) results in r (designated as r_2) = 1.469, and equation (5) leads to r (designated as r_3) = 1.020. One possible way to satisfy equations (3) to (5) to the maximum extent is to take the average values of r_1, r_2 and r_3, which gives r = 1.227 and thereby b (designated as b_2) = 0.449 in equation (A10), f (designated as f_2) = 0.330 in equation (A11) and m (designated as m_2) = 0.220 in equation (A20) of part 1. For possibility 2, the weighting factor and the corresponding exponent values are: r = 1.227, w = J = 0, b_2 = 0.449, and m_2 = 0.220, with subscript 2 denoting the possibility 2. The value of r of 1.227 implies that the flow depth undergoes proportionately greater change than does the channel width in accomplishing the change in stream power. If possibilities 1 and 2 are considered together, then the channel flow depth and width are more important in accommodating the change in stream power than is channel roughness.

2.1.3. Possibility 3: w = r = 0

[10] According to the exponents of discharge in equations (A19) to (A20) of part 1, one gets, respectively,

$$\frac{3}{5(1+J)} = 0.357 \quad (6)$$

$$\frac{2+5J}{5(1+J)} = 0.202 \quad (7)$$

Equation (6) yields J (designated as J_1) = 0.681 and equation (7) results in J (designated as J_2) = − 0.248. As a

Table 1a. Observed Average Values of Exponents b, f, and m Gathered From the Literature: Downstream

Source	Exponent b	Exponent f	Exponent m	Drainage Area	Conditions
Leopold and Maddock [1953]	0.50	0.40	0.10		
Wolman [1955]	0.34	0.45	0.32	duration = 50	ephemeral streams (at Q equaled or exceeded at % of time)
Wolman [1955]	0.38	0.42	0.32	15	ephemeral streams (at Q equaled or exceeded at % of time)
Wolman [1955]	0.45	0.43	0.17	2	ephemeral streams (at Q equaled or exceeded at % of time)
Wolman [1955]	0.42	0.45	0.05	bank-full discharge	ephemeral streams (at Q equaled or exceeded at % of time)
Wolman [1955]	0.57	0.40	0.03	50	principal stations at Brandywine Creek and headwaters
Wolman [1955]	0.58	0.40	0.02	2	principal stations at Brandywine Creek and headwaters
Leopold and Miller [1956]	0.29	0.15	0.58	Sedalia Gully near Sedalia, Colorado	
Leopold and Miller [1956]	0.31	0.20	0.49	Sowbelly Creek near Hat Creek, Nebraska	
Miller [1958]	0.38	0.25	0.39	high mountain streams	
Brush [1961]	0.55	0.36	0.09	Appalachian streams	
Ackers [1964]	0.42	0.43	0.15		
Ackers [1964]	0.43	0.43	0.14		
Ackers [1964]	0.53	0.35	0.12		
Scott [1966]	0.69	0.12	0.19	perennial rivers	
Scott [1966]	0.03	0.48	0.45	ephemeral streams	
Carlston [1969]	0.461	0.383	0.155	10 river basins	
Carlston [1969]	0.499	0.32	0.18	Yellow River	
Thornes [1970]	0.40	0.34	0.25	Suia-Missu and Araguaia basins, Mato Gross, Brazil	minimization of error sum of squares
Thornes [1970]	0.47	0.41	0.04	Suia-Missu and Araguaia basins, Mato Gross, Brazil	Q greater than 1.94
Thornes [1970]	0.11	0.32	0.59	Suia-Missu and Araguaia basins, Mato Gross, Brazil	Q less than 1.94 cfs
Thornes [1970]	0.19	0.32	0.56	Suia-Missu and Araguaia basins, Mato Gross, Brazil	maximization of explained variance Q greater than 5.02 cfs
Thornes [1970]	0.51	0.50	0.01	Suia-Missu and Araguaia basins, Mato Gross, Brazil	Q less than 15.02 cfs
Thornes [1970]	0.14	0.36	0.54	smaller streams	
Ponton [1972]	0.60	0.40	−0.01	Green River	
Ponton [1972]	0.80	0.44	−0.23	Birkenhead River	
Knighton [1974]	0.61	0.31	0.08		
Smith [1974]	0.60	0.30	0.10		
Smith [1974]	0.54	0.23	0.23		
Smith [1974]	0.46	0.16	0.38		
Parker [1979]	0.50	0.415	0.085		
Lane and Foster [1980]	0.46	0.46	0.081		
Rhoads [1991]	0.46	0.46	-	Missouri River basin	106,155–1,358,000 sq. Km.
Rhoads [1991]	0.49	0.30		James River basin	655–55810 sq. km
Rhoads [1991]	0.51	0.37		Smokey Hill River	9207–49,880 sq. Km.
Allen et al. [1994]	0.557	0.341	0.104		674 cross sections from 15 states in the conterminous U. S.

weighting factor, J cannot take on a negative value and hence the negative value of J represents an unrealistic case. One interpretation of this impossibility may be that when there is a change in stream power due to influx of water and sediment, the adjustment in the hydraulic geometry through roughness and depth is not sufficient and consequently other channel geometric parameters, such as width, must change. Of course, this is limited to the data considered here and it is difficult to say whether this will universally apply. This possibility is therefore not considered in the following combined possibility.

2.1.4. Possibility 4

[11] As mentioned earlier, the exponents b = 0.456, f = 0.357, and m = 0.202 cannot satisfy the theoretical equa-

Table 1b. Average Values of Exponents b, f, and m Used in Theoretical Equations: Downstream

Source	Exponent b	Exponent f	Exponent m	Drainage Area	Conditions
Lacey [1929]	-	0.33			
Langbein [1964]	0.53	0.37	0.10		minimum variance theory
Kellerhals [1966]	0.50	0.40			regime theory
Blench [1969]	0.50	0.33			regime theory

tions simultaneously for the same possibility. In order to make the computed results more reasonable, the average values of b and m from possibilities 1 and 2 are employed in this combined possibility, i.e., b = 0.462 and m = 0.215. Exponent f can be determined based on equation (2b) of part 1 as 0.323. On the basis of these adjusted values of b, f, and m, the weighting factors for the combined possibility can be determined by noting the exponents of discharge in equations (A20) to (A30) of part 1 as follows:

$$\frac{1}{1+w+r} = 0.462 \tag{8}$$

$$\frac{3r}{5(1+r+Jr)} = 0.323 \tag{9}$$

$$\frac{3}{5}\left[\frac{2}{3}+\frac{wJ}{wJ+J+w}-\frac{2}{3(1+w+r)}\right] = 0.215 \tag{10}$$

Equations (8)–(10) result in r = 1.162 and w = J $\approx$ 0. Actually, r = 1.162 and w = J $\approx$ 0 represents possibility 2. It should be pointed out that when w and J approach zero, the numerator wJ in equation (10) converges to zero more quickly than its denominator. In reality, these weighting factors are seldom zero and their zero values represent a limiting case. Thus the term wJ/(wJ + J + w) is zero in case of w = J $\approx$ 0. It is interesting that the average values b = 0.462, f = 0.323, and m = 0.215 are in good agreement with the values of *Park* [1977] who plotted frequency distributions of the flow discharge exponents in hydraulic geometry relationships. According to these plots, the mean values of the exponents of width, depth, and velocity are 0.463, 0.327, and 0.210, respectively. Such a coincidence signifies that this possibility should be the case that occurs with a high frequency in the data employed here and possibly in nature. Therefore the verification of the theoretical equations was undertaken for this possibility only.

2.2. Data Selection

[12] In order to test the performance of the derived equations (A28) to (A31) of part 1, 456 data sets were obtained from different sources: (1) from field observations on alluvial rivers, such as Yangtze River and Yellow River, reservoir main channels, model rivers, and canals in China, Japan, Canada, Egypt, and Bangladesh from this study; (2) from *Hey and Thorne* [1986] for stable gravel bed rivers in the United Kingdom; (3) from *Higginson and Johnston* [1988] for rivers in Northern Ireland; and (4) from *Colosimo et al.* [1987, 1988] for gravel bed rivers in Calabria, southern Italy. Each data set comprises the measured values of the mean velocity V (m/s), the Manning roughness coefficient n, water surface slope S, mean flow depth h (m), and water surface width B (m), evaluated at bank-full discharge, Q. Of the 456 hydraulic geometry measurements, 277 sets of data comprise information on channel pattern and channel boundary materials, such as river bank vegetation and bed material median size diameter d_{50}. On the basis of this information, the stable channel side slope z (vertical:horizontal = 1:z) was determined as in *Deng and Zhang* [1994]. Actually, among the 277 sets of data, 109 sets of data include the measured side slope z. Thus 277 sets of data comprise the values of the side slope z, either measured or accurately determined. These 277 sets of data constitute the database A and the other 179 sets of data the database B. The values of the side slope z of database B were determined on the basis of the available data like D_{50}. For instance, 48 sets of field data from *Bray* [1979] have the same side slope of z = 3; 46 sets of field data from *Colosimo et al.* [1987, 1988] also have the same side slope of z = 3; while 68 sets of field data from *Higginson and Johnston* [1988] have the same side slope of z = 1.5; 14 sets of data measured in Japanese rivers have the side slope of z = 6 and other 3 sets of Japanese river data have z = 2. The data sets selected in this paper are by no means exhaustive but they do cover a broad range of typical flow conditions, sediment concentration range, channel boundary conditions, and river channel patterns, which most commonly occur in nature, as shown in Table 2.

Table 2. Variability of Selected River Parameters and Conditions

Variables	Range
Bank-full discharge Q, m^3/s	0.006–60,000
Slope S	0.000004–0.0241
Water surface width B, m	0.54–4,000
Mean flow depth, m	0.03–45
Mean velocity V, m/s	0.182–4.2
Manning roughness coefficient n	0.005–0.143
Median suspended load size d_{50}, mm	0.004–0.055
Median bed material size D_{50}, mm	1.3–175.8
Sediment concentration, kg/m^3	0.06–196.5
Critical side slope z	0.9–12
Channel pattern	straight, meandering, braided, and wandering

2.3. Determination of Morphological Coefficients and Calibration of Derived Equations

[13] The generalized theoretical equations (A28) to (A31) of part 1 contain four coefficients C_B, C_h, C_v, and C_n, which are defined in terms of morphological coefficients C_1, C_2, C_3 by equations (A40) to (A43). Thus the coefficients C_1, C_2, and C_3 need to be determined.

2.3.1. Determination of Coefficient C_1

[14] Equation (17a) of part 1 leads to

$$B = \left(\frac{C_1}{n}\right)^{1/w} \tag{11}$$

Substitution of equation (11) into equation (4) yields

$$h = \eta_B \frac{Q^{3/5}}{S^{3/10}} \tag{12}$$

where

$$\eta_B = \left(\frac{n^{(1+w)/w}}{C_1^{1/w}}\right)^{3/5} \tag{13}$$

For a V-shaped cross section, the water surface width B is linearly related to the mean flow depth h and the channel side slope z as

$$B = 4zh \tag{14}$$

Substitution of equation (12) into equation (14) yields

$$\eta_B = \frac{1}{z}\left(\frac{BS^{3/10}}{4Q^{3/5}}\right) \tag{15}$$

Equations (13) and (15) give

$$C_1 = \frac{n^{1+w}}{\eta_B^{5w/3}} \tag{16}$$

Equation (16) is an explicit relation for the morphological coefficient C_1 in terms of Manning's n, channel width, slope, discharge, channel side slope and the weighting factor. One can also derive equations similar to equations (15) and (16) for other geometric shapes, such as trapezoidal, but explicit expressions become complicated. Therefore a sensitivity analysis was performed to evaluate the validity of an assumed channel shape, which is described in the next section.

2.3.2. Determination of Coefficient C_2

[15] Equation (19a) of part 1 leads to

$$h = \left(\frac{B}{C_2}\right)^{3r/5} \tag{17}$$

Substitution of equation (17) into equation (4) yields

$$B = \xi_{B1}\frac{Q^{1/(1+r)}}{S^{1/2(1+r)}} \tag{18}$$

where

$$\xi_{B1} = \left(nC_2^r\right)^{1/(1+r)} \tag{19}$$

For a V-shaped cross section, substitution of equation (18) into equation (14) yields

$$\xi_{B1} = z\left(4h\frac{S^{1/2(1+r)}}{Q^{1/(1+r)}}\right) \tag{20}$$

Let

$$\eta_h = 4h\frac{S^{1/[2(1+r)]}}{Q^{1/(1+r)}} = \frac{B}{z}\frac{S^{1/[2(1+r)]}}{Q^{1/(1+r)}} \tag{21}$$

Then, equations (19) to (21) result in

$$C_2 = \frac{(z\eta_h)^{(1+r)/r}}{n^{1/r}} \tag{22}$$

Equation (22) is an explicit relation for the morphological coefficient C_2 in terms of Manning's n, channel width, bed slope, side slope, discharge and weighting factor.

2.3.3. Determination of Coefficient C_3

[16] Equation (21a) of part 1 leads to

$$h = \left(\frac{C_3}{n}\right)^{3/5J} \tag{23}$$

Substitution of equation (23) into equation (4) yields

$$B = \xi_{B2}\frac{Q}{S^{1/2}} \tag{24}$$

where

$$\xi_{B2} = \frac{n^{(1+J)/J}}{C_3^{1/J}} \tag{25}$$

For a V-shaped cross section, substitution of equation (24) into equation (14) yields

$$\xi_{B2} = z\left(4h\frac{S^{1/2}}{Q}\right) \tag{26}$$

Let

$$\eta_n = 4h\frac{S^{1/2}}{Q} = \frac{B}{z}\frac{S^{1/2}}{Q} \tag{27}$$

Then, equations (25) to (27) result in

$$C_3 = \frac{n^{1+J}}{(z\eta_n)^J} \tag{28}$$

Equation (28) is an explicit relation for the morphological coefficient C in terms of Manning's n, channel width, side slope, bed slope, discharge and weighting factor.

2.3.4. Determination of Coefficients C_B, C_h, C_v, and C_n

[17] With C_1, C_2, and C_3 known, C_B, C_h, C_v, and C_n follow from equations (A28) to (A31) of part 1.

$$C_B = \left(C_1C_2^r\right)^{1/(1+w+r)} = \left(z^{1+r}n^w\frac{\eta_h^{1+r}}{\eta_B^{5w/3}}\right)^{1/(1+w+r)} \tag{29}$$

$$C_h = \left(\frac{C_3}{C_2}\right)^{3r/[5(1+r+rJ)]} = \left(\frac{n}{z}\right)^{3/5}\left(\eta_n^{rJ}\eta_h^{1+r}\right)^{-3/[5(1+r+rJ)]} \tag{30}$$

$$C_n = \left(C_1^{1/w}C_3^{1/J}\right)^{wJ/(w+J+wJ)} = \frac{n^{1+\frac{wJ}{w+J+wJ}}}{\left(z\eta_n\eta_B^{5/3}\right)^{\frac{wJ}{w+J+wJ}}} \tag{31}$$

$$C_v = \left(C_nC_B^{2/3}\right)^{-3/5} = \frac{\eta_B^{\frac{wJ}{w+J+wJ}+\frac{2w}{3(1+w+r)}}\eta_n^{\frac{3wJ}{5(w+J+wJ)}}}{n^{\frac{3(w+J+2wJ)}{5(w+J+wJ)}+\frac{2w}{5(1+w+r)}}z^{\frac{2(r+1)}{5(1+w+r)}-\frac{3wJ}{5(w+J+wJ)}}\eta_h^{\frac{2(1+r)}{5(1+w+r)}}} \tag{32}$$

For the case of r = 1.162 and w = J = 0, the fractions comprised of only w and J converge to zero and both equation (A31) of part 1 and equation (31) lead to C_n = n. This means that roughness coefficient n is a constant independent of discharge Q and bed slope S. Under the condition of n and z known, η_n, η_h, and η_B need to be determined further to solve C_B, C_h, and C_v. To that end, database A is used to determine parameters η_B, η_h, and η_n. It is found that the closer to 0.462 the exponent b of flow discharge is, the closer to a constant of 0.5 the parameters

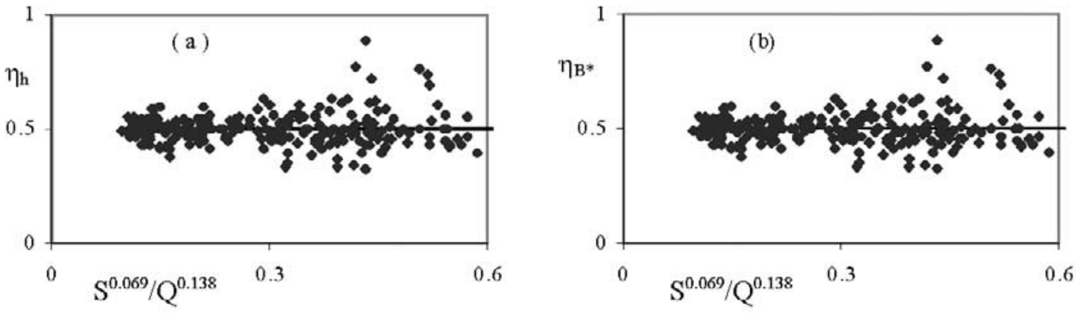

Figure 1. Calibration of parameters (a) η_h and (b) η_{B*}.

are. η_n and η_B are independent of parameters r, J, and w, whereas η_h is dependent on r. The three parameters, η_B, η_h, and η_n, are defined, respectively, as:

$$\eta_B = 0.5\frac{S^{0.069}}{Q^{0.138}} \quad (33)$$

$$\eta_n = 0.5\frac{S^{0.269}}{Q^{0.538}} \quad (34)$$

$$\eta_h = 0.5\frac{S^{\frac{1}{2(1+r)}-0.231}}{Q^{\frac{1}{1+r}-0.462}} \quad (35)$$

If

$$\eta_{B*} = \eta_B\frac{Q^{0.138}}{S^{0.069}} \quad (36)$$

$$\eta_{n*} = \eta_n\frac{Q^{0.538}}{S^{0.269}} \quad (37)$$

$$\eta_{h*} = \eta_h\frac{Q^{\frac{1}{1+r}-0.462}}{S^{\frac{1}{2(1+r)}-0.231}} \quad (38)$$

then η_{B*}, η_{h*}, and η_{n*} assume a constant value of 0.5. A comparison between equations (36) to (38) and the database A consisting of 277 sets of data is shown in Figure 1. It is noted that η_h is equal to a constant of 0.5 for r = 1.162. η_{n*} is the same with η_{B*} shown in Figure 1b if it is plotted against $S^{0.269}/Q^{0.538}$, that is $\eta_{n*} = \eta_{B*} = \eta_h = 0.5$.

3. Verification of Derived Equations

[18] For r = 1.162, w = $\eta_{n*} = \eta_{B*} = \eta_h = 0.5$, equations (A28) to (A30) of part 1 are simplified as follows:

$$B = 0.50z\frac{Q^{0.462}}{S^{0.231}} \quad (39)$$

$$h = 1.52\left(\frac{n}{z}\right)^{0.6}\frac{Q^{0.323}}{S^{0.161}} \quad (40)$$

$$V = \frac{1.32}{n^{0.6}z^{0.4}}Q^{0.215}S^{0.393} \quad (41)$$

Equations (39) to (41) were verified using database A. The results are plotted in Figure 2 for the values of width B, depth h, and velocity V, respectively. Figures 2a–2c illustrate that the agreement between measured and predicted values of width, flow depth, and velocity is satisfactory, with only one predicted width B out of the range of $0.5 < B_{Measured}/B_{Predicted} < 2.0$ and $R^2 = 0.990$, only one predicted depth h out of the range of $0.5 < h_{Measured}/h_{Predicted} < 2.0$ and $R^2 = 0.9939$, and three predicted values of velocity V out of the range of $0.5 < V_{Measured}/V_{Predicted} < 2.0$ and $R^2 = 0.897$, in a total of 277 data sets.

[19] Database B was employed for an independent validation of the derived equations (39) to (41). The validation results are shown in Figures 3a–3c. Figure 3 demonstrates that the theoretically predicted values of channel width, flow depth and velocity are in good agreement with the field observed values, with two predicted values of width B out of the range of $0.5 < B_{Observed}/B_{Predicted} < 2.0$ and $R^2 = 0.939$, no predicted depth h out of the range of $0.5 < h_{Observed}/h_{Predicted} < 2.0$ and $R^2 = 0.933$, and two predicted velocity V out of the range of $0.5 < V_{Observed}/V_{Predicted} < 2.0$ and $R^2 = 0.835$, in a total of 179 data sets.

4. Shape Modification and Sensitivity Analysis

[20] All the theoretical relationships for the hydraulic geometry were derived under the assumption of a V-shaped cross section. However, natural channels rarely possess triangular cross sections. Although the numerical coefficients of equations (39) to (41) obtained by calibration reflect the influence of the shape of natural channels to some extent, it is still necessary and desirable to include the influence of natural channel shape in the hydraulic geometry relations or at least evaluate the sensitivity of different channel shapes to the above-derived results. It is apparent that a sound equation for describing the cross-sectional shape of natural channels is essential to the modification of the derived relations and the sensitivity analysis. *Deng et*

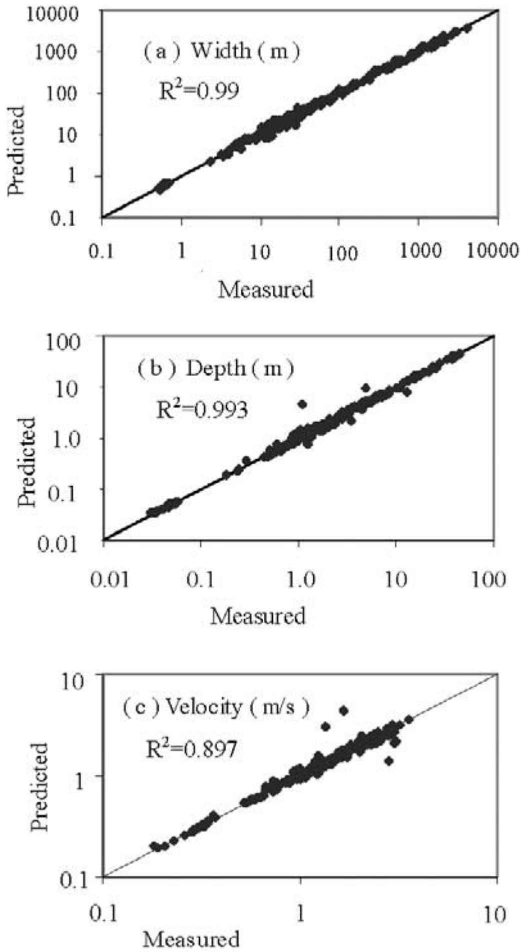

Figure 2. Validation of derived equations with database A.

al. [2001] presented a versatile transverse profile equation of channel shape and thereby derived the following channel shape equation and the relationship between the cross-section-averaged flow depth h and the maximum (center) flow depth h_c in a straight channel:

$$\frac{h(y)}{h_c} = 1 - \left(\frac{y}{B/2}\right)^{\beta} \quad (42a)$$

$$h = \frac{\beta}{\beta+1} h_c \quad (42b)$$

in which h(y) is the local flow depth and the channel shape parameter β is defined as

$$\beta = \ln\left(\frac{B}{h}\right) \quad (42c)$$

Owing to its adaptability to variable channel shapes, equation (42) is a useful mathematical relation for describing the cross-sectional channel shape of natural rivers. For instance, equation (42a) represents a triangular shape for β = 1, a parabolic shape for β = 2, an approximate natural channel shape with a flat-bed region and two curving bank regions for β > 2, and a rectangular shape for β = ∞. Equation (42) is able to reflect different cross-sectional shapes of channels with size ranging from small canals to large rivers. It was therefore employed to analyze the sensitivity of channel shape to hydraulic geometry relationships derived in this paper.

[21] For a V-shaped channel, equation (14) can also be expressed by the center flow depth h_c as

$$B = 2zh_c \quad (43)$$

Solving h_c from equation (42b) and then substituting it into equation (43), one obtains

$$B = 4\left(\frac{\beta+1}{2\beta}z\right)h \quad (44)$$

Let

$$\alpha = \frac{\beta+1}{2\beta} \text{ and } z_0 = \alpha z \quad (45)$$

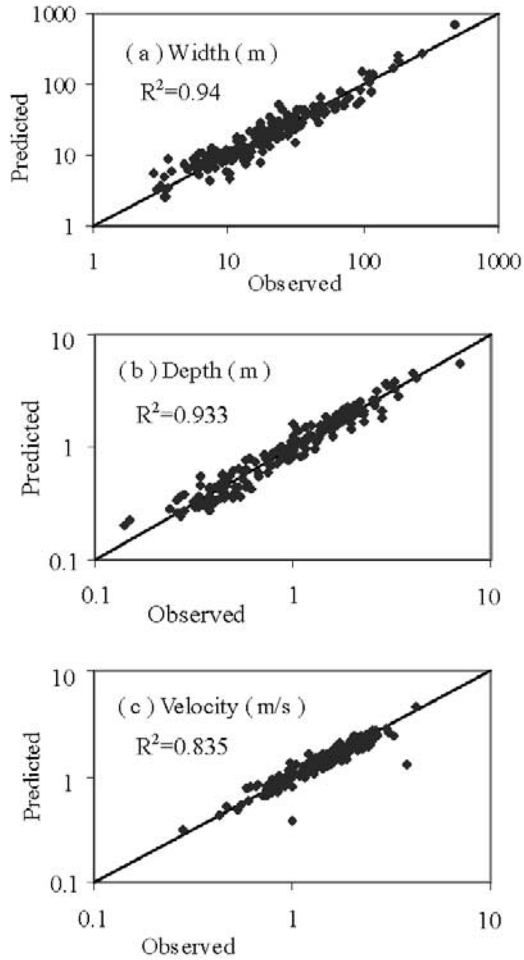

Figure 3. Validation of derived equations with database B.

Then, equation (44) can be written in the same form as equation (14):

$$B = 4z_0 h \tag{46}$$

As mentioned earlier, for a V-shaped channel $\beta = 1$ and then $z_0 = z$ and equation (46) recovers equation (14). For other channel shapes, replacing equation (14) by equation (46) and following the same procedure, one gets:

$$B = z_* \frac{Q^{0.462}}{S^{0.231}}, \quad (z_* = \alpha z) \tag{47}$$

$$h = \left(\frac{n}{z_*}\right)^{0.6} \frac{Q^{0.323}}{S^{0.161}} \tag{48}$$

$$V = \frac{Q^{0.215} S^{0.393}}{z_*^{0.4} n^{0.6}} \tag{49}$$

Equations (47) to (49) reduce to equations (39) to (41) when $\alpha = 0.5$ or $z_* = 0.5z$. Actually it can be seen from equation (45) that $\alpha = 0.5$ corresponds to $\beta = \infty$ and thus it represents rectangular channels. Since the empirical constants in equations (39) to (41) are based on database A and database B, it means that most natural channels can effectively be treated as rectangular in shape and their hydraulic geometry relationships can efficiently be described by equations (39) to (41). This is consistent with the usual practice of assuming most natural rivers as wide rectangular channels. Nevertheless, it is important to know the condition under which a channel can be simplified to have a rectangular shape.

[22] Figure 4 illustrates the sensitivity of the shape factor α to the variation of the channel width to the depth ratio B/h. Figure 4 shows that the shape factor α is sensitive to the variation of the channel width to the depth ratio when $B/h < 10$ and α gradually becomes insensitive to the variation of channel shape when $B/h > 10$. This means that all natural channels with $B/h > 10$ can be considered to be effectively rectangular in shape. In other words, equations (39) to (41) are applicable to various channel shapes with $B/h > 10$, although these equations are derived under the V-shaped channel assumption. Fortunately, natural rivers and streams generally possess a width to depth ratio of $B/h > 10$. This is the reason that equations (39) to (41) fit the measured data quite well with constant numerical coefficients. It should be noted that theoretically a rectangular channel shape can be formed under the condition of $\alpha \rightarrow 0.5$, i.e., $\beta \rightarrow \infty$ or $B/h \rightarrow \infty$, as shown in Figure 4, and it appears that $\alpha \approx 0.6$ for the frequently occurring range of B/h. However, as just mentioned, actual channels with B/h can be approximated as rectangular in shape and α can be taken roughly as 0.5. For natural channels with a $B/h < 10$, the shape factor α should be treated as a variable and equations (47) to (49) become implicit expressions. Consequently, all morphologic coefficients are actually uniquely determined by the shape factor α. For most natural channels with $B/h > 10$, $\alpha = 0.5$, leading to the constant coefficients of 0.5, 1.52, and 1.32 in the equations of B, h, and V, respectively. For channels with $B/h < 10$, α and thus all the coefficients should be determined by equation (45); then, the trial-and-error method is needed to determine parameters B and h. Therefore equations (47) to (49) can be employed as a set of generalized hydraulic geometry relationships applicable to various channel shapes.

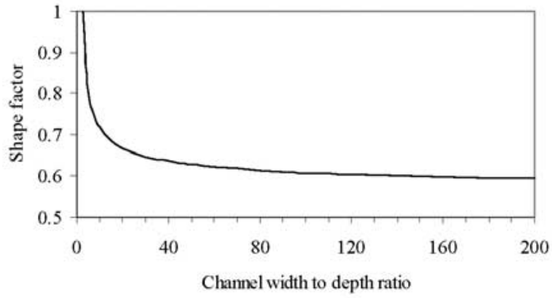

Figure 4. Variation of the shape factor α with channel width to depth ratio B/h.

5. Discussion of Results

[23] One of the main differences between this paper (along with its companion) and the papers reported in the literature lies in the introduction of the four possibilities and three weighting factors, r, w, and J. However, these possibilities themselves do not occur with the same frequency and thus these factors should have different values of weights or importance. As mentioned in the part 1, the change in the stream power (SP) or the unit stream power (USP) is the driving force causing the variation of the downstream hydraulic geometry. From equation (6) in part 1, it can be seen that the flow depth h is the most sensitive parameter to SP since h has the greatest power index $2\beta = 10/3$, while the other three parameters Q, B, and α have the same power index of 2 for the use of Manning's equation. Then, it can be seen from equation (A29) of part 1 that r and J are the two controlling weighting factors for h.

[24] In order to further compare the relative importance or sensitivity of h to r and J, the variation of the common exponent part $CP = r/(1 + r + Jr)$ of both Q and S is considered. To that end, we analyze two extreme cases: (1) J is a nonzero constant and r tends to zero. Then, CP also tends to zero. (2) J is a constant and r tends to infinity. Then, CP tends to unity. This means that in case of a nonzero constant J parameter, r is the dominant weighting factor. On the other hand, if r is a nonzero constant and J tends to zero, then CP is fully determined by r. It is almost impossible for J to tend to infinity according to the results of verification achieved in the paper. Consequently, r is the most important weighting factor. It implies that possibility 2 represents the governing mechanism responsible for the change of the stream power and thus occurs most frequently. This explains the reason that the averaged weighting factors $r = 1.162$, $w = J \approx 0$.

[25] If a river is fully alluvial and under equilibrium, then the adjustment of the stream power should be at least equally shared by the changes of B and h, i.e., $r = 1$. However, $r = 1.162$ means that the share of adjustment of flow depth h is greater than that of channel width B. In other words, the channel bank is not fully alluvial due to bank vegetation, channelization, and so on, in general, for the available hydraulic geometry data. Therefore the purpose of introduction of the weighting factors r, w, and J is also to

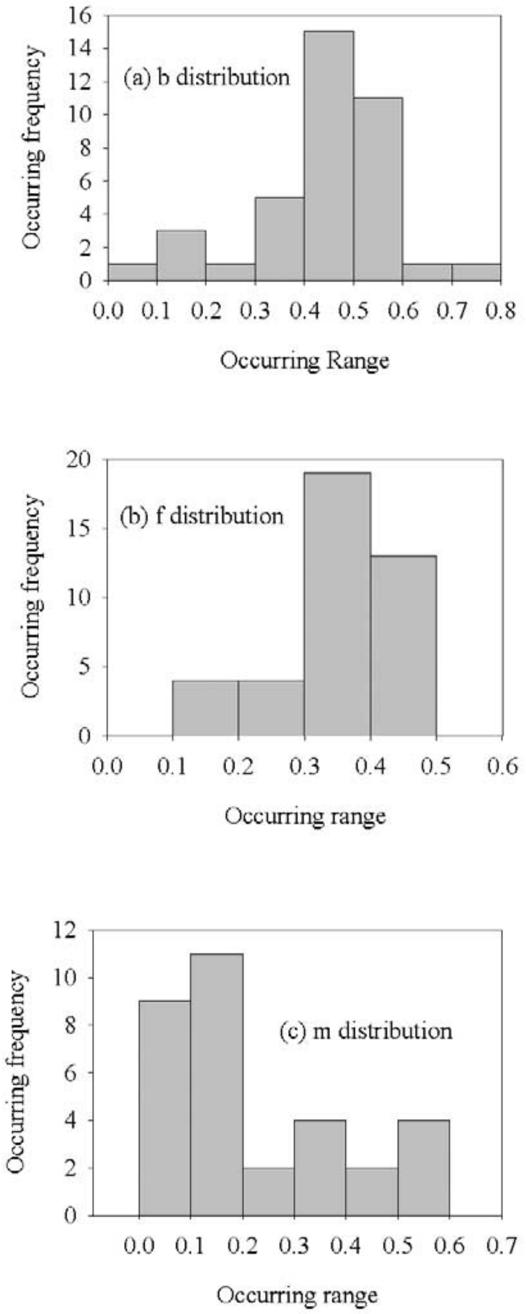

Figure 5. Distribution of occurring frequency of exponents.

take account of the influence of partially alluvial actual rivers on the adjustment of the stream power. The specific values of r, w, and J should be determined according to the local conditions of each river. The equations can be used as a framework and the values of r, w, and J as a reference for their localization. It means that different rivers should follow the same form of the hydraulic geometry equations derived in part 1 but they have their specific values of r, w, and J due to the variability in their boundary conditions.

[26] In principle the derived hydraulic geometry equations should be verified under each possibility using the corresponding data. However, the available data contain no information about the possibilities. Therefore the best available way to test the efficacy of the derived equations is deemed to be the one that corresponds to the most prevalent possibility: Possibility 2. This possibility should have a unique set of values of the exponent parameters b, f, and m. However, it can be seen from Table 1 that (1) the observed values of exponents b, f, and m vary significantly; (2) these values occur with different frequencies in different value ranges; and (3) most of the values concentrate in a narrow range, as shown in Figures 5a–5c plotted in the form of histograms. If the number of b, f, and m values were sufficiently large, their histogram shapes would exhibit that their mean values would possess the maximum frequency of occurrence. Consequently, the mean values of the exponents, b, f, and m, were determined in this paper by averaging their observed values.

[27] Ideally, it would be desirable to have data corresponding to each possibility but such data could not be found and has not been reported in the literature, to the best of our knowledge. Analysis of all possibilities based on the data employed in this study suggests that only possibility 1, possibility 2, and their combination occur in nature. Further analysis shows that the change in stream power is accomplished most frequently by adjustment of channel width and flow depth. The contribution by the change in roughness is much less. This corresponds to possibility 2. In alluvial rivers this is to be expected.

6. Conclusions

[28] The following conclusions are drawn from this study: (1) The spatial change of stream power is not shared equally among hydraulic variables, as demonstrated by unequal values of the weighting factors. (2) The exponent values greatly depend on the boundary conditions to be satisfied by the channel. (3) The morphological coefficients are functions of hydraulic and channel variables as well as the weighting factors. (4) Predicted channel width, depth and velocity are in excellent agreement with 456 sets of field observations. (5) Field data show that possibility 2 with the condition corresponding to r = 1.162 and w = J = 0 is most prevalent in natural rivers. (6) The hydraulic geometry relationships derived in this paper are applicable to various channel shapes.

Notation

B	water surface width.
C_1, C_2, C_3	primary morphological coefficients.
C_B, C_h, C_V, C_n	coefficients of morphological equations.
h	mean flow depth.
h(y)	local flow depth.
h_c	center flow depth in a straight channel.
J	weighting factor.
n	Manning's roughness coefficient.
Q	flow discharge.
R^2	root-mean square.
r	weighting factor.
S	channel slope.
V	average flow velocity.

w weighting factor.
y lateral coordinate.
z channel side slope.
α shape factor.
β channel shape parameter.
η_B, η_n, η_h, η_{B*}, η_{n*}, η_{h*} numerical constant.
ξ_{B1}, ξ_{B2} numerical constant.

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