Downstream hydraulic geometry relations: 1. Theoretical development

Vijay P. Singh
Department of Civil and Environmental Engineering, Louisiana State University, Baton Rouge, Louisiana, USA

Chih Ted Yang[1]
Department of Civil Engineering, Colorado State University, Fort Collins, Colorado, USA

Z. Q. Deng
Department of Civil and Environmental Engineering, Louisiana State University, Baton Rouge, Louisiana, USA

Received 14 July 2003; accepted 10 September 2003; published 4 December 2003.

[1] In this study, it is hypothesized that (1) the spatial variation of the stream power of a channel for a given discharge is accomplished by the spatial variation in channel form (flow depth and channel width) and hydraulic variables, including energy slope, flow velocity, and friction, and (2) that the change in stream power is distributed among the changes in flow depth, channel width, flow velocity, slope, and friction, depending on the constraints (boundary conditions) the channel has to satisfy. The second hypothesis is a result of the principles of maximum entropy and minimum energy dissipation or its simplified minimum stream power. These two hypotheses lead to four families of downstream hydraulic geometry relations. The conditions under which these families of relations can occur in field are discussed. *INDEX TERMS:* 1871 Hydrology: Surface water quality; 1860 Hydrology: Runoff and streamflow; 1824 Hydrology: Geomorphology (1625); *KEYWORDS:* hydraulic geometry, entropy, stream power, minimum energy dissipation, principle of maximum entropy, hydraulic variables

Citation: Singh, V. P., C. T. Yang, and Z. Q. Deng, Downstream hydraulic geometry relations: 1. Theoretical development, *Water Resour. Res.*, *39*(12), 1337, doi:10.1029/2003WR002484, 2003.

1. Introduction

[2] The term "hydraulic geometry" connotes the relationships between the mean stream channel form and discharge both at-a-station and downstream along a stream network in a hydrologically homogeneous basin. The channel form includes the mean cross-section geometry (width, depth, etc.), and the hydraulic variables include the mean slope, mean friction, and mean velocity for a given influx of water and sediment to the channel and the specified channel boundary conditions. *Leopold and Maddock* [1953] expressed the hydraulic geometry relationships for a channel in the form of power functions of discharge as

$$B = aQ^b, \quad d = cQ^f, \quad V = kQ^m \tag{1a}$$

where B is the channel width, d is the flow depth, V is the flow velocity, Q is the flow discharge, and a, b, c, f, k, and m are parameters. Also added to equation (1a) are:

$$n = NQ^p, \quad S = sQ^y \tag{1b}$$

where n is Manning's roughness factor, S is slope, and N, p, s, and y are parameters. Exponents b, f, m, p and y represent, respectively, the rate of change of the hydraulic variables B, d, V, n, and S as Q changes; and coefficients a, c, k, N, and s are scale factors that define the values of B, d, V, n, and S when Q = 1. The hydraulic geometry relations (1a) and (1b) are of great practical value in prediction of channel deformation; layout of river training works; design of stable canals and intakes, river flow control works, irrigation schemes, and river improvement works. These relations through their exponents can also be employed to discriminate between different types of river sections [*Richards*, 1976] as well as in planning for resource and impact assessment [*Allen et al.*, 1994].

[3] The hydraulic variables, width, depth, and velocity, satisfy the continuity equation:

$$Q = BdV \tag{2a}$$

Therefore the coefficients and exponents in equation (1a) satisfy

$$ack = 1, \quad b + f + m = 1 \tag{2b}$$

[4] The at-a-site hydraulic geometry entails mean values over a certain period, such as a week, a month, a season, or a year. The concept of downstream hydraulic geometry involves spatial variation in channel form and process at a constant frequency of flow. *Richards* [1982] has noted that the downstream hydraulic geometry involving channel process and form embodies two types of analyses, both of

[1]Formerly at Sedimentation and River Hydraulics Group, Technical Services Center, U. S. Bureau of Reclamation, Denver, Colorado, USA.

0043-1397/03/2003WR002484$09.00

which are expressed as power functions of the form [*Rhoads*, 1991] given by equations (1a) and (1b). The first type of analysis is typified by the works of *Leopold and Maddock* [1953] and *Wolman* [1955], who formalized a set of relations, such as equations (1a) and (1b), to relate the downstream changes in flow properties (width, mean depth, mean velocity, slope, and friction) to the mean discharge. This type of analysis describes regulation of flow adjustments by channel form in response to increases in discharge downstream, and has been applied at particular cross sections as well as in the downstream direction.

[5] The second type of analysis is a modification of the original hydraulic geometry concept and entails variation of channel geometry for a particular reference discharge downstream with a given frequency. Implied in this analysis is an assumption of an appropriate discharge that is the dominant flow controlling channel dimensions [*Knighton*, 1987; *Rhoads*, 1991]. For example, for perennial rivers in humid regions, the mean discharge or a discharge that approximates bank-full flow (Q_b), such as Q_2 and $Q_{2.33}$, with a return period of 2 and 2.33 years, respectively, is often used in equations (1a) and (1b). This concept is similar to that embodied in the regime theory [*Blench*, 1952, 1969]. It should, however, be noted that the coefficients and exponents are not constrained by the continuity equation when the selected discharge substantially differs from the bank-full flow. On the other hand, *Stall and Yang* [1970] related hydraulic geometry to flow frequency and drainage area.

[6] The mean values of the hydraulic variables of equations (1a) and (1b) are known to follow, according to *Langbein* [1964] and *Yang et al.* [1981], necessary hydraulic laws and the principle of the minimum energy dissipation rate (or stream power). As a consequence, these mean values are functionally related and correspond to the equilibrium state of the channel. This state is regarded as the one corresponding to the maximum sediment transporting capacity. The implication is that an alluvial channel adjusts its width, depth, slope, velocity, and friction to achieve a stable condition in which it is capable of transporting a certain amount of water and sediment. In other words, the average river system tends to develop in such a way as to produce an approximate equilibrium between the channel and the water and sediment it must transport [*Leopold and Maddock*, 1953]. *Knighton* [1977] observed that at cross sections undergoing a systematic change, the potential for adjustment toward some form of quasi-equilibrium in the short term is related to the flow regime and channel boundary conditions; and that the approach to quasi-equilibrium or establishment of a new equilibrium position is relatively rapid.

[7] The relations of equations (1a) and (1b) have been calibrated for a range of environments, using both field observations and laboratory simulations. *Dury* [1976] confirmed the validity of power function relations for hydraulic geometry using extended sets of data at the 1.58-year mean annual discharge. *Chong* [1970] stated, without a firm basis, that hydraulic geometry relations of equations (1a) and (1b) were similar over varying environments. *Parker* [1978] analyzed the cause of this systematic behavior for gravel rivers. Thus it seems that the regional generalizations proposed in the literature are acceptable for rivers that have achieved ''graded-time'' equilibrium [*Phillips and Harlin*, 1984]. *Parker* [1979] has stated that the scale factors, a, c, and k, vary from locality to locality but the exponents, b, f, and m, exhibit a remarkable degree of consistency, and seem independent of location and only weakly dependent on channel type. From an analysis of a subalpine stream in a relatively homogeneous environment, *Phillips and Harlin* [1984] found that hydraulic exponents were not stable over space. *Knighton* [1974] emphasized variations in exponents as opposed to mean values. *Rhodes* [1978] noted that the exponent values for high-flow conditions can be vastly different than those for low-flow conditions.

[8] Using data from 318 alluvial channels in the midwestern United States and 50 Piedmont sites, *Kolberg and Howard* [1995] showed that the discharge-width exponents were distinguishable, depending on the variations in materials forming the bed and banks of alluvial channels. Both midwestern and piedmont data indicated that the width-discharge exponents ranged from 0.35 to 0.46 for groups of streams with width to depth ratios less than 45. For groups of streams with width to depth ratios greater than 45, the width-discharge exponents decreased to values below 0.15, suggesting a systematic variation in the exponents and a diminished influence of channel shape. These results are in agreement with the findings of *Osterkamp and Hedman* [1982]. *Howard* [1980] asserted that the variations among channel types are not discrete but can be viewed as continuous. This assertion was supported by *Rhoads* [1991].

[9] *Rhoads* [1991] examined the factors that produce variations in hydraulic geometry parameters. He hypothesized that the parameters are functions of channel sediment characteristics and flood magnitude, and that the parameters vary continuously rather than discretely. Analyzing the variation of channel width with downstream discharge, *Klein* [1981] found that the value $b = 0.5$ was a good average. The low b values normally occur for small basins (in lower flows) and for very big basins (in very high flows). Thus the $b = 0.5$ value, being a good average, tends to smooth out deviations from the average. The value of b ranged from 0.2 to 0.89. Klein argued that the simple power function for hydraulic geometry was valid for small basins and that did not hold over a wide range of discharges.

[10] The above discussion shows that the exponents and coefficients of hydraulic geometry relations of equations (1a) and (1b) vary from location to location on the same river and from river to river, as well as from high-flow range to low-flow range. This is because the influx of water and sediment and the constraints (boundary conditions) that the river channel is subjected to vary from location to location as well as from river to river. This means that for a fixed influx of water and sediment a channel will exhibit a family of hydraulic geometry relations in response to the constraints imposed on the channel. It is these constraints that force the channel to adjust its allowable hydraulic variables. For example, if a river is leveed on both sides, then it cannot adjust its width and is therefore left to adjust other variables, such as depth, friction, slope, and velocity. Likewise, if a canal is lined, then it cannot adjust its friction. This aspect does not seem to have been fully explored in the literature.

[11] Various approaches have been employed for deriving functional relationships among the aforementioned hydraulic variables for downstream hydraulic geometry or equations (1a) and (1b). These approaches are based on

the following theories: (1) empirical theory (e.g., regression theory [*Leopold and Maddock*, 1953], regime theory [*Blench*, 1952]), (2) tractive force theory [*Lane*, 1955] and its variants-threshold channel theory [*Li*, 1974] and stability theory [*Stebbings*, 1963], (3) hydrodynamic theory [*Smith*, 1974], (4) thermodynamic entropy theory [*Yalin and Da Silva*, 1997, 1999], (5) minimum extremal theories (e.g., minimum channel mobility theory [*Dou*, 1964], minimum energy dissipation rate theory or its simplified versions of minimum unit stream power theory [*Yang and Song*, 1986] and minimum stream power theory [*Chang*, 1980, 1988; *Yang et al.*, 1981], minimum energy dissipation theory [*Rodriguez-Iturbe et al.*, 1992], minimum energy degradation theory [*Brebner and Wilson*, 1967], minimum entropy production theory [*Leoplod and Langbein*, 1962], principle of least action [*Huang and Nanson*, 2000], and minimum variance theory [*Langbein*, 1964]), and (6) maximum extremal theories (maximum friction theory [*Davies and Sutherland*, 1983], maximum sediment discharge theory [*White et al.*, 1982], maximum sediment discharge and Froude number theory [*Ramette*, 1980], and maximum entropy theory [*Deng and Zhang*, 1994]). Each hypothesis leads to unique relations between channel form parameters and discharge, and the relations corresponding to one hypothesis are not necessarily identical to those corresponding to another hypothesis.

[12] The objective of this first part of the two-part paper is to apply the principles of minimum energy dissipation rate and maximum entropy to derive downstream hydraulic geometry relations. Inherent in the derivation is an explanation for self-adjustment of channel morphology. It is shown that by combining the hypotheses based on the principles of maximum entropy and minimum energy dissipation rate a family of hydraulic geometry relations is obtained. This family may encompass many of the hydraulic geometry relations corresponding to other hypotheses. The paper is organized as follows. Introducing hydraulic geometry relations in section 1, derivation of hydraulic geometry relations using the principles of maximum entropy and minimum energy dissipation rate is presented in section 2. The discussion of the derived equations is given in section 3. The paper is concluded in section 4, followed by an appendix and the cited literature.

2. Derivation of Hydraulic Geometry Relations

[13] *Langbein* [1964] and *Yang et al.* [1981] emphasized that equations (1a) and (1b) corresponds to the case when the channel is in equilibrium state. Langbein hypothesized that when a channel adjusts its hydraulic variables corresponding to this state, the adjustment is shared equally among the hydraulic variables. Employing the principle of maximum entropy and minimum stream power, *Deng and Zhang* [1994] derived morphological equations, assuming that for a given discharge the flow depth and width were independent variables among five hydraulic variables. However, in practice the channel is seldom in an equilibrium state and this means that the adjustment among hydraulic variables will be unequal. It is not clear as to the exact proportion in which the adjustment will be shared among variables. Nevertheless, two points can be made. First, there will be a family of hydraulic geometry relations, depending on the adjustment of hydraulic variables. Second, the adjustment can explain the variability in the parameters (scale and exponents) of these relations. These two points will be pursued in what follows.

[14] *Yang* [1972] defined the unit stream power (USP) as the time rate of potential energy expenditure per unit weight of water in an alluvial channel. Simply put, the unit stream power is the velocity-slope product having the dimensions of power per unit weight of water. Thus USP, denoted as P_w, is expressed as

$$P_w = VS \tag{3a}$$

where V is the average flow velocity, and S is the energy slope. Stream power (SP) is the rate of energy dissipation due to water:

$$\mathrm{SP} = Q\gamma S \tag{3b}$$

where γ is the weight density of water and Q is discharge of water. It should be noted that SP can be obtained by integrating USP over a given cross section. A channel responds to the influx of water and sediment coming from its watershed by the adjustment of SP. Indeed, *Yang* [1972] found USP to be the dominating factor in determination of the total sediment concentration. *Yang* [1986, 1996] also related sediment load and channel geometry adjustment to SP. Thus the spatial rate of adjustment of SP along a river, R_s, can be expressed as

$$R_S = \frac{d(SP)}{dx} = \frac{d(Q\gamma S)}{dx} \tag{3c}$$

where x is the space coordinate along the direction of flow. *Cheema et al.* [1997] determined stable width of an alluvial channel using the hypothesis that an alluvial channel attains a stable width when the rate of change of unit stream power with respect to its width is a minimum. This means that an alluvial channel with stable cross section has the ability to vary its width at a minimum consumption of energy per unit width per unit time.

[15] If a channel is assumed rectangular with h as the depth of flow and B as the width of flow, then the flow cross-sectional area A = Bh, the wetted perimeter P = B + 2h, and the hydraulic radius R = A/P = (Bh)/(B + 2h). If the channel is wide rectangular, then R ≅ h = depth of flow. The flow discharge in equation (3b) can be obtained from either Manning's or Chezy's or the Darcy-Weisbach equation. For wide rectangular channels, these equations can be written, respectively, as

$$Q = \frac{1}{n} AR^{2/3} S^{1/2} = \frac{1}{n} Bh^{5/3} S^{1/2} \tag{4a}$$

$$Q = CA\sqrt{RS} = CBh^{3/2}\sqrt{S} \tag{4b}$$

$$Q = 2\sqrt{\frac{2g}{f}} A\sqrt{RS} = 2\sqrt{\frac{2g}{f}} Bh^{3/2}\sqrt{S} \tag{4c}$$

where n is Manning's roughness coefficient, C is Chezy's roughness coefficient, and f is the Darcy-Weisbach friction factor, and g is acceleration due to gravity. Clearly,

$$C = \frac{1}{n} h^{1/6}; \quad C = 2\sqrt{2g/f} \tag{4d}$$

Equations (4a) to (4c) can be expressed in a general from as:

$$Q = \alpha B h^{\beta} \sqrt{S} \tag{4e}$$

in which α is a roughness measure, and β is an exponent. For Manning's equation, $\alpha = 1/n$, and $\beta = 5/3$; for Chezy's equation, $\alpha = C$, and $\beta = 3/2$; and for Darcy-Weisbach equation, $\alpha = 2(2g/f)^{0.5}$, and $\beta = 3/2$.

[16] The energy slope S can be expressed from equation (4e) as

$$S = \frac{Q^2}{\alpha^2 B^2 h^{2\beta}} \tag{5}$$

Thus, using equations (3b) and (5), the stream power of a channel is expressed as

$$SP = \frac{\gamma Q^3}{\alpha^2 B^2 h^{2\beta}} \tag{6}$$

In equation (6), there are five variables: Q, S, B, α and h; of these variables, Q, α, h, and B are on the right side of the equation, and Q and S through SP on the left side. Three of these variables, including α, B, and h, are controlling variables or constraints for a given discharge. It may be noted that the slope term S is not an independent variable here, because it is imbedded in the stream power and hence it is not considered as a controlling variable. Furthermore, from a practical point of view a natural river can easily adjust its width, depth, velocity, and roughness due to changing discharge. The longitudinal slope takes a very long time, say years if not centuries, to adjust [*Yang*, 1996]. Therefore we generally treat the longitudinal profile or slope as constant over a short period of time. Because of this timescale difference S is not considered as a variable when compared with velocity, depth, width and roughness. Thus it is hypothesized that for a given influx of discharge from the watershed the channel will adjust or minimize its stream power by adjusting these three controlling variables. This hypothesis is similar to the one proposed by *Langbein* [1964] in his theory of minimum variance. Therefore substitution of equation (6) in equation (3c) yields

$$R_s = \frac{d(Q\gamma S)}{dx} = \gamma \frac{d}{dx}\left(\frac{Q^3}{\alpha^2 B^2 h^{2\beta}}\right) = \gamma Q^3 \frac{d}{dx}\left(\frac{1}{\alpha^2 B^2 h^{2\beta}}\right) \tag{7}$$

Equation (7) gives

$$R_s = -\frac{2\gamma Q^3}{\alpha^3 B^2 h^{2\beta}} \frac{d\alpha}{dx} - \frac{2\gamma Q^3}{\alpha^2 B^3 h^{2\beta}} \frac{dB}{dx} - \frac{2b\gamma Q^3}{\alpha^2 B^2 h^{2\beta+1}} \frac{dh}{dx} \tag{8}$$

The right side of equation (8) has three parts, designated as R_1, R_2, R_3:

$$R_1 = -\frac{2\gamma Q^3}{\alpha^3 B^2 h^{2\beta}} \frac{da}{dx} \tag{9}$$

$$R_2 = -\frac{2\gamma Q^3}{\alpha^2 B^3 h^{2\beta}} \frac{dB}{dx} \tag{10}$$

$$R_3 = -\frac{2\beta\gamma n^2 Q^3}{\alpha^2 B^2 h^{2\beta+1}} \frac{dh}{dx} \tag{11}$$

[17] Equation (9) can be interpreted as the spatial rate of adjustment of friction, equation (10) the spatial rate of adjustment of width, and equation (11) the spatial rate of adjustment of flow depth. Dividing equations (9) to (11) by the total spatial rate of adjustment of SP, one gets

$$P_\alpha = \frac{R_1}{R_s} = \frac{2\gamma Q^3}{\alpha^3 B^2 h^{2\beta}} \frac{[d\alpha/dx]}{[d(SP)/dx]} \tag{12}$$

$$P_B = \frac{R_2}{Rs} = \frac{2\gamma Q^3}{B^3 h^{2\beta}} \frac{[dB/dx]}{[d(SP)/dx]} \tag{13}$$

$$P_h = \frac{R_3}{R_s} = \frac{2\beta\gamma Q^3}{\alpha^2 B^2 h^{2\beta+1}} \frac{[dh/dx]}{[d(SP)/dx]} \tag{14}$$

Equation (12) can be interpreted as the proportion of the adjustment of stream power by friction, equation (13) the proportion of the adjustment of stream power by channel width, and equation (14) the proportion of the adjustment of stream power by flow depth.

[18] According to the principle of maximum entropy [*Jaynes*, 1957], any system in equilibrium state under steady constraints tends to maximize its entropy. When a river reaches a dynamic (or quasi-dynamic) equilibrium, the entropy should attain its maximum value. The principle of maximum entropy (POME) states that the entropy of a system is maximum when all probabilities are equal, i.e., the probability distribution is uniform. Applying this principle to a river in its dynamic equilibrium, the following must therefore be true:

$$P_\alpha = P_B = P_h \tag{15}$$

Equation (15) holds, of course, under the stipulation that there are no constraints imposed on the channel and can be interpreted to mean that the self-adjustment of SP (γQS) is equally shared among α, B, and h. This interpretation is supported by *Williams* [1967, 1978] who found from an analysis of data from 165 gaging stations that a channel adjusted all its hydraulic parameters (B, h, S, V) in response to changes in the influx of water and sediment and that self-adjustments were realized in an evenly distributed manner among factors. Equation (15) is similar to the concept embodied in the minimum variance theory [*Langbein*, 1964].

[19] Equation (15) involves probabilities of three variables, meaning that any two of the three cases of adjustment in hydraulic variables may coexist as well as that of all three cases may co-exist. These configurations of adjustment do indeed occur in nature [*Wolman*, 1955]. Thus the equality among three probabilities raises four possibilities and hence leads to four sets of equations: (1) $P_\alpha = P_B$, (2) $P_B = P_h$, (3) $P_\alpha = P_h$, and (4) $P_\alpha = P_B = P_h$. It should be noted that all four possibilities can occur in the same river in different

reaches or in the same reach at different times, or in different rivers at the same time or at different times. In order to enumerate the consequences of these possibilities, one can either employ the general discharge equation (4e) or employ either Manning's equation (4a) or Chezy's equation (4b) or Darcy-Weisbach equation (4c). It is however more informative to use a specific discharge-resistance relation than the general discharge-resistance relation. Survey of literature shows that Manning's equation is more commonly employed in hydraulic and river engineering in general and more specifically in investigations on hydraulic geometry [see, e.g., *Leopold and Wolman*, 1957; *Wolman and Brush*, 1961; *Stall and Fok*, 1968; *Bray*, 1982]. Furthermore, the data that could be found on alluvial rivers and canals contained Manning's n a lot more than Chezy's C or Darcy-Weisbach's f. For these reasons, Manning's equation was used in this study. Consequently, possibility P_α is replaced by P_n. Expressing then equations (12) to (14) for Manning's equation,

$$P_n = \frac{R_1}{R_s} = \frac{2n\gamma Q^3}{B^2 h^{10/3}} \frac{[dn/dx]}{[d(SP)/dx]} \tag{12'}$$

$$P_B = \frac{R_2}{Rs} = -\frac{2\gamma n^2 Q^3}{B^3 h^{10/3}} \frac{[dB/dx]}{[d(SP)/dx]} \tag{13'}$$

$$P_h = \frac{R_3}{R_s} = -\frac{10\gamma n^2 Q^3}{3B^2 h^{13/3}} \frac{[dh/dx]}{[d(SP)/dx]} \tag{14'}$$

2.1. Primary Morphological Equations

[20] The four possibilities for spatial stream power adjustment lead to primary morphological equations which are needed for deriving the downstream hydraulic geometry relations. The morphological equations are therefore derived first.

2.1.1. Possibility 1: $P_B = P_n$

[21] Here P_n is given by equation (12′) and P_B by equation (13′). Equating these two equations, one gets

$$\frac{dn}{dx} = -\frac{n}{B}\frac{dB}{dx} \tag{16a}$$

Equation (16a) hypothesizes that the spatial change in stream power is accomplished by an equal spatial adjustment between flow width B and resistance expressed by Manning's n. This possibility occurs in wide rectangular channels where the flow depth is not a controlling variable but the roughness and the flow width are. The downstream end of the Brahmaputra River before joining the Bay of Bengal in India and the Mississippi River before joining the Gulf of Mexico in the United States are examples. The hypothesis can be considered as a limiting case and will presumably hold under the equilibrium condition. However, such a condition is not always achieved and therefore the spatial change in stream power will be accomplished by an unequal adjustment between B and n. To that end, equation (16a) is modified as

$$\frac{dn}{dx} = -\frac{wn}{B}\frac{dB}{dx} \tag{16b}$$

where w is a weighting factor, $0 \leq w$, which accounts for the proportion in which the adjustment in stream power is shared between B and n. For the special case, where the adjustment is shared equally between B and n, w = 1.

[22] Integration of equation (16b) yields

$$nB^w = C_1 \quad or \quad B = C_1{*}\, n^{-1/w} \tag{17a}$$

where C_1 and $C_1{*}$ are constants of integration. For the limiting case (w = 1), equation (17a) becomes

$$nB = C_1 \quad or \quad B = C_1{*}\, n^{-1} \tag{17b}$$

Parameter C_1 or $C_1{*}$ can be labeled as a primary morphological coefficient and equation (17a) or (17b) as a primary morphological equation.

2.1.2. Possibility 2: $P_B = P_h$

[23] Here P_B is given by equation (13′) and P_h by equation (14′). Equating these two equations, one gets

$$\frac{dh}{dx} = \frac{3}{5}\frac{h}{B}\frac{dB}{dx} \tag{18a}$$

Equation (18a) hypothesizes that the spatial variation in stream power is accomplished by an equal spatial adjustment between flow depth and flow width. This possibility occurs in channels where the roughness is fixed, say by lining, and the controlling variables are flow depth and width. Examples of such cases are the channels employed for recreation, and trapezoidal channels which have attained a kind of equilibrium condition. The hypothesis can be considered as a limiting case and will hold under the equilibrium condition. Such a condition is however seldom achieved and therefore the spatial change in stream power is accomplished by an unequal adjustment between h and B. To that end, equation (18a) is modified as

$$\frac{dh}{dx} = \frac{3r}{5}\frac{h}{B}\frac{dB}{dx} \tag{18b}$$

where r is a weighting factor, $0 \leq r$, which accounts for the proportion in which the adjustment of stream power is shared between h and B. For the special case, where the adjustment is equally shared, r = 1.

[24] Integration of equation (18b) yields

$$\frac{B}{h^{\frac{5}{3r}}} = C_2 \ or\ h = C_2{*}B^{\frac{3r}{5}} \tag{19a}$$

where C_2 and $C_2{*}$ are constants of integration. For the limiting case (r = 1), equation (19a) reduces to

$$\frac{B}{h^{5/3}} = C_2 \quad or \quad h = C_2{*}B^{3/5} \tag{19b}$$

Parameter C_2 or $C_2{*}$ can be designated as a primary morphological coefficient and equation (19a) or (19b) as a primary morphological equation. It should interesting to note that equation (19a) resembles the basic form of regime equation expressed as

$$\frac{B^\phi}{h} = \varphi \tag{19c}$$

where $\phi = 3/5$ and $\varphi = 1/C_2{*}$.

2.1.3. Possibility 3: $P_n = P_h$

[25] Here P_n is given by equation (12') and P_h by equation (14′). Equating these two equations, one gets

$$\frac{dn}{dx} = -\frac{5n}{3h}\frac{dh}{dx} \tag{20a}$$

Equation (20a) hypothesizes that the spatial variation in stream power is accomplished by an equal spatial adjustment between flow depth and resistance. Examples of such a possibility are a laboratory flume with fixed walls, lined canals, leveed rivers, and so on. This hypothesis can be considered as a limiting case and will hold under the equilibrium condition. Such a condition is not always attained and therefore the spatial change in stream power is accomplished by an unequal adjustment between h and n. To that end, equation (20a) is modified as

$$\frac{dn}{dx} = -\frac{5nJ}{3h}\frac{dh}{dx} \tag{20b}$$

where J is a weighting factor, $0 \leq J$, which accounts for the proportion in which the adjustment of stream power is shared between n and h. For the special case, where the adjustment is equally shared, J = 1.

[26] Integration of equation (20b) yields

$$n = C_3 h^{-\frac{5J}{3}} \quad or \quad h = C_3{*}n^{-\frac{3}{5J}} \tag{21a}$$

where C_3 and C_3* are constants of integration. For the limiting case (J = 1), equation reduces to

$$nh^{5/3} = C_3 \quad or \quad h = C_3{*}n^{-3/5} \tag{21b}$$

Parameter C_3 or C_3* can be considered as a primary morphological coefficient and equation (21a) or (21b) as a primary morphological equation.

2.1.4. Possibility 4: $P_n = P_b = P_h$

[27] Equation (17a) relates n and B, equation (19a) relates B and h, and equation (21a) relates n and h. The first two equations can be employed to eliminate n and h in equation (4a) and express B as a function of Q. Similarly, equations (19a) and (21a) can be used to eliminate B and n in equation (4a) and express h as a function of Q. Likewise, all three equations can be used to express V as a function of Q.

[28] Thus three primary morphological equations (17a) or (17b), (19a) or (19b), and (21a) or (21b); and their three corresponding primary morphological coefficients, C_1, C_2, and C_3 (or C_1*, C_2*, and C_3*), are obtained. It should be noted that equation (19b) can also be obtained by combining equations (17b) and (21b) or equation (17b) can be obtained by combining equations (19b) and (21b).

2.2. Downstream Hydraulic Geometry Equations for a Given Discharge

[29] If the discharge Q and slope S of a river are known, then substitution of primary morphological equations (17a), (19a), and (21a) in equation (4a) leads to equations for hydraulic geometry of the river. These equations under all four possibilities are derived in what follows.

2.2.1. Possibility 1: $P_B = P_n$, $nB^w = C_1$

[30] This possibility leads to the hydraulic geometry relations for B, V, and n. To derive these relations, three steps are involved: (1) Substitution of equation (17a) in equation (4a) leading to expressions for B, V, and n in terms of Q and S. (2) Elimination of S in these expressions using a sediment transport relation. With use of the Engelund and Hansen sediment transport equation [*Engelund and Hansen*, 1967], the channel slope S can be expressed in terms of discharge Q [*Knighton*, 1998] as

$$S = C_s Q^z \tag{22}$$

where C_s is a coefficient, $z = -2/5$ for gravel rivers, and $z = -1/6$ for sandy rivers. Such a relation is useful if S is unknown. This step leads to one set of hydraulic geometry relations for gravel rivers and another set for sandy or alluvial rivers. (3) Substitution of equation (22) which leads to expressions for B, V, and n in terms of Q alone. These three steps lead to the following hydraulic geometry relations:

$$B = C_{BS} Q^{\frac{6}{5(1+w)}} \tag{23a}$$

for gravel rivers

$$B = C_{BS} Q^{\frac{13}{12(1+w)}} \tag{23b}$$

for sandy rivers

$$V = C_{VS} Q^{\frac{5w-1}{3(1+w)}} \tag{24a}$$

for gravel rivers

$$V = C_{VS} Q^{\frac{60w-5}{36(1+w)}} \tag{24b}$$

for sandy rivers

$$n = C_{nS} Q^{\frac{-6w}{5(1+w)}} \tag{25a}$$

for gravel rivers

$$n = C_{nS} Q^{\frac{-13w}{12(1+w)}} \tag{25b}$$

for sandy rivers, where $C_{BS} = \frac{C_B}{C_S^{1/2(1+w)}}$, $C_B = \left(C_1 h^{-5/3}\right)^{1/(1+w)}$, $C_{VS} = C_V C_S^{5/[6(1+w)]}$, $C_V \left[C_n (C_B)^{2/3}\right]^{-3/5}$, $C_{ns} = C_n C_S^{w/[2(1+w)]}$, and $C_n = (C_1)^{1/(1+w)} h^{5w/[3(1+w)]}$. For the special case when the weighting factor is unity, special forms of equations (23a)–(26b) are obtained by inserting w = 1. Derivation of these equations and their special forms is given in Appendix A.

2.2.2. Possibility 2: $P_B = P_h$, $B = C_2\, h^{5/3r}$

[31] This possibility leads to the hydraulic geometry relations for B, h, and V in terms of Q. Following the same three steps as under possibility 1, these relations are derived from equations (19a), (4a) and (22) for gravel and alluvial rivers:

$$B = C_{BS} Q^{\frac{6}{5(1+r)}} \tag{26a}$$

for gravel rivers

$$B = C_{BS} Q^{\frac{13}{12(1+r)}} \quad (26b)$$

for sandy rivers

$$h = C_{hS} Q^{\frac{18r}{25(1+r)}} \quad (27a)$$

for gravel rivers

$$h = C_{hS} Q^{\frac{13r}{20(1+r)}} \quad (27b)$$

for sandy rivers

$$V = C_{VS} Q^{\frac{7r-5}{25(1+r)}} \quad (28a)$$

for gravel rivers

$$V = C_{VS} Q^{\frac{21r-5}{60(1+r)}} \quad (28b)$$

for sandy rivers, where $C_{BS} = \frac{C_B}{C_S^{1/2(1+r)}}$, $C_B = [n(C_2)^r]^{1/[1+r]}$, $C_{hs} = C_h C_S^{-3r/[2(5+5r)]}$, $C_h = (nC_2)^{3r/[5+5r]}$, $C_{VS} = C_V C_S^{[3(1+r)+2]/[10(1+r)]}$, and $C_V = n^{-3/5}[C_B]^{-2/5}$. Derivation of equations (26a) to (28b) and their special form for r = 1 is given in Appendix A.

2.2.3. Possibility 3: $P_n = P_h$, $C_3 = n\ h^{5J/3}$

[32] Under this possibility, the hydraulic geometry relations result for h, B, and n in terms of Q. Following the same three steps as under possibility 1, these relations are derived from equations (21a), (4a) and (22) for gravel and alluvial rivers:

$$h = C_{hS} Q^{\frac{18}{25(1+J)}} \quad (29a)$$

for gravel rivers

$$h = C_{hS} Q^{\frac{13}{20(1+J)}} \quad (29b)$$

for sandy rivers

$$V = C_{VS} Q^{\frac{7+25J}{25(1+J)}} \quad (30a)$$

for gravel rivers

$$V = C_{VS} Q^{\frac{20J+7}{20(1+J)}} \quad (30b)$$

for sandy rivers

$$n = C_{nS} Q^{\frac{-6}{5(1+J)}} \quad (31a)$$

for gravel rivers

$$n = C_{nS} Q^{\frac{-13}{12(1+J)}} \quad (31b)$$

for sandy rivers, where $C_{hS} = C_h C_S^{-3/[2(5+5J)]}$, $C_h = (C_B)^{3/[5+5J]} B^{-3/[5+5J]}$, $C_{VS} = C_V C_S^{3/[10(1+J)]}$, $C_V = (C_n)^{-3/5} B^{-2/5}$, $C_{nS} = C_n C_S^{1/[2(1+J)]}$, and $C_n = (C_3)^{1/[1+J]} B^{1/[1+J]}$. Derivation of equations (29a) to (31b) and their special forms for J = 1 is given in Appendix A.

2.2.4. Possibility 4

[33] The objective is to derive hydraulic geometry relations for B, h, V, and n in terms of Q under this possibility. Following the same three steps as under possibility 1, these relations are derived from equations (17a), (19a), (21a), (4a) and (22) for gravel and alluvial rivers:

$$B = C_{BS} Q^{\frac{6}{5(1+w+r)}} \quad (32a)$$

for gravel rivers

$$B = C_{BS} Q^{\frac{13}{12(1+w+r)}} \quad (32b)$$

for sandy rivers

$$h = C_{hS} Q^{\frac{18r}{25(1+Jr+r)}} \quad (33a)$$

for gravel rivers

$$h = C_{hS} Q^{\frac{13r}{20(1+Jr+r)}} \quad (33b)$$

for sandy rivers

$$V = C_{VS} Q^{\frac{3}{25}\left(\frac{7}{3}+\frac{6wJ}{wJ+J+w}-\frac{4}{1+w+r}\right)} \quad (34a)$$

for gravel rivers

$$V = C_{VS} Q^{\frac{1}{5}\left(\frac{7}{4}+\frac{13wJ}{4(wJ+J+w)}-\frac{13}{6(1+w+r)}\right)} \quad (34b)$$

for sandy rivers

$$n = C_{nS} Q^{\frac{-6wJ}{5(wJ+J+w)}} \quad (35a)$$

for gravel rivers

$$n = C_{nS} Q^{\frac{-13wJ}{12(wJ+J+w)}} \quad (35b)$$

for sandy rivers, where $C_{BS} = \frac{C_B}{C_S^{1/2(1+w+r)}}$, $C_B = [C_1(C_2)^r]^{1/[1+w+r]}$, $C_{hS} = C_h C_S^{-3r/[2(5+5Jr+5r)]}$, $C_h = \left[C_3(C_2)^{-1}\right]^{3r/[5+5Jr+5r]}$, $C_{VS} = C_V(C_S)^{\frac{3}{5}\left[\frac{1}{2}-\frac{wJ}{2(wJ+J+w)}+\frac{1}{3(1+w+r)}\right]}$, $C_V = \left[C_n(C_B)^{2/3}\right]^{-3/5}$, $C_{nS} = C_n C_S^{wJ/[2(wJ+J+w)]}$, and $C_n = \left[(C_1)^{1/w}(C_3)^{1/J}\right]^{wJ/[wJ+J+w]}$. Derivation of equations (32a) to (35b) and their special forms with w = r = J = 1 is give in Appendix A.

[34] The coefficients, C_B, C_h, C_V and C_n, in equations (32a) to (35b) are defined in terms of morphological coefficients C_1, C_2, and C_3 as

$$C_B = [C_1\, C_2]^{1/3} \quad (36)$$

$$C_h = C_2^{-0.2}\, C_3^{0.2} \quad (37)$$

Table 1. Values of Exponents b, f, m, and p for Three Limiting Cases When the Weighting Factors are Zero, Unity, and Infinity for Different Possibilities, With Slope Explicitly Appearing in Hydraulic Geometry Relations[a]

		Possibility			
	Weighting Factor	1	2	3	1 + 2 + 3
b for B	0	1 (w = 0)	1 (r = 0)		1 (w = r = 0)
b for B	1	0.5 (w = 1)	0.5 (r = 1)		1/3 (w = r = 1)
b for B	∞	0 (w = ∞)	0 (r = ∞)		0 (w = r = ∞)
f for h	0		0 (r = 0)	3/5 (J = 0)	0 (J = r = 0)
f for h	1		3/10 (r = 1)	3/10 (J = 1)	1/5 (J = r = 1)
f for h	∞		3/5 (r = ∞)	0 (J = ∞)	0 (J = r = ∞)
m for V	0	0 (w = 0)	0 (r = 0)	2/5 (J = 0)	0 (w = J = r = 0)
m for V	1	1/5 (w = 1)	1/5 (r = 1)	7/10 (J = 1)	7/15 (w = J = r = 1)
m for V	∞	2/5 (w = ∞)	2/5 (r = ∞)	1 (J = ∞)	1 (w = J = r = ∞)
p for n	0	0 (w = 0)		−1 (J = 0)	0 (J = w = 0)
p for n	1	−0.5 (w = 1)		−0.5 (J = 1)	−1/3 (J = w = 1)
p for n	∞	−1 (w = ∞)		0 (J = ∞)	−1(J = w = ∞)
Equation numbers		(A1)–(A3)	(A10)–(A12)	(A19)–(A21)	(A28)–(A31)

[a]Usually, these factors will have values between the limiting values.

$$C_n = C_1^{1/3} C_3^{1/3} \tag{38}$$

$$C_V = \left[C_1^{1/3} C_2^{2/5} C_3^{1/5}\right]^{-1} \tag{39}$$

Here C_B, C_h, C_V, and C_n are the coefficients associated with flow width, depth, velocity, and Manning's n, respectively, and depend on C_1, C_2, and/or C_3, given, respectively, by equations (17b), (19b) and (21b). It is to be noted that since the discharge Q = BhV, equations (32a)–(35b) show that the sum of exponents of Q equals 1. Similarly, the sum of exponents of S from these equations equals 0.

3. Discussion of Derived Equations

[35] Four sets of hydraulic geometry expressions have been derived. In the first set are expressions corresponding to possibility 1 wherein the channel adjusts its width, roughness and velocity to accommodate changes in discharge and sediment load. These expressions are given by equations (23a)–(25b) (or equations (A7a) to (A9c) for the special case, w = 1). The second possibility corresponds to the case where the channel adjusts its depth, width and velocity to accommodate changes in discharge. Under this possibility the hydraulic geometry expressions are given by equations (26a) to (28b) (or equations (A16a) to (A18c) for the special case r = 1). In the third set are expressions, given by equations (29a) to (A31b) (or equations (A25a) to (A27c) for the special case J = 1), wherein the channel adjusts its depth, roughness and velocity for accommodating changes in discharge. The fourth possibility is most general and leads to hydraulic geometry expressions given by equations (32a) to (35b) (or equations (A36a) to (A39c) for the special case, w = r = J= 1), where the channel adjusts its width, depth, velocity and roughness to accommodate changes in discharge. A short discussion of each set of hydraulic geometry relations is in order. To facilitate discussion, the values of exponents, b, f, m, and p for three cases (one special and two limiting cases) when the weighting factors are zero, unity, and infinity, are tabulated for all three possibilities and their combination in Tables 1, 2a, and 2b. It should be pointed out that the limiting case of infinity is only a theoretically generalized case for the factors r, w, and J, respectively, for the lack of knowledge of the values of their upper limits, which should be far less than infinity.

3.1. Possibility 1: Hydraulic Geometry Relations for Width, Roughness, and Velocity

[36] In this possibility the change in stream power is accomplished by the adjustment between channel width and roughness. The resulting general equations are equations (23a) to (25b) and their specialized forms (w = 1) are given by equations (A7a) to (A9c). For three cases, the values of exponents, b, p and m, are given in Tables 1, 2a, and 2b. Equations (23a) and (23b) show that the channel width varies with discharge raised to the power (b = 6/{5(1 + w)} for gravel rivers and b = 13/{12(1 + w)} for sandy rivers) from some positive value greater than zero to a value of 1, and the scale factor C_B varies with flow depth. Thus one can infer that the b exponent has a range of 0 to 1. The precise value of b depends on the value of w, meaning the proportion in which the spatial change of stream power is accomplished by the adjustment between B, n, and V. When w = 1, the channel width varies with the discharge raised to the power of 0.5 as shown by equation (A7a) for gravel rivers and 0.6 for sandy rivers as shown by. equation (A7b). This exponent value of 0.5 is about the average value reported in the literature [*Klein*, 1981]. However, one should note that in equation (A1), slope also appears with an exponent of $-1/[2(1 + w)]$. If the channel slope is constant then the slope component of the equation will merge with coefficient C_B. This shows that the scale factor C_B varies from one location to another and also with time through flow depth. Otherwise, slope can be expressed as a function of discharge raised to the power of z = −2/5 and −1/6 for gravel rivers and for sandy rivers, respectively. Then, in this case under the special condition with weighting factor w = 1, the width will vary with discharge raised to the power of 0.6 and 0.5, for sandy

Table 2a. Values of Exponents b, f, m, and p for Three Limiting Cases When the Weighting Factors are Zero, Unity, and Infinity for Different Possibilities, With Slope Expressed as a Function of Discharge With the Power of −1/6[a]

	Weighting Factor	Possibility 1	2	3	1 + 2 + 3
b for B	0	13/12 (w = 0)	13/12 (r = 0)		13/12 (w = r = 0)
b for B	1	13/24 (w = 1)	13/24 (r = 1)		13/36 (w = r = 1)
b for B	∞	0 (w = ∞)	0 (r = ∞)		0 (w = r = ∞)
f for h	0		0 (r = 0)	13/20 (J = 0)	0 (J = r = 0)
f for h	1		13/40 (r = 1)	13/40 (J = 1)	13/60 (J = r = 1)
f for h	∞		13/20 (r = ∞)	0 (J = ∞)	0 (J = r = ∞)
m for V	0	−5/36 (w = 0)	−1/12 (r = 0)	7/20 (J = 0)	−1/12 (J = r = 0)
m for V	1	55/72 (w = 1)	2/15 (r = 1)	27/40 (J = 1)	19/45 (J = r = 1)
m for V	∞	5/3 (w = ∞)	7/20 (r = ∞)	1 (J = ∞)	1 (w = J = r = ∞)
p for n	0	0 (w = 0)		−13/12 (J = 0)	0 (J = w = 0)
p for n	1	−13/24 (w = 1)		−13/24 (J = 1)	−13/36 (J = w = 1)
p for n	∞	−13/12 (w = ∞)		0 (J = ∞)	−13/12(J = w = ∞)
Equation numbers		(A4a), (A5a), and (A6a)	(A13a), (A14a), and (A15a)	(A22a), (A23a), and (A24a)	(A32a), (A33a), (A34a), and (A35a)

[a]Usually these factors will have values between these limiting values.

rivers and for gravel rivers, respectively, which exponent values also fall within the range reported in the literature.

[37] The average flow velocity varies with the discharge raised to the power from zero to 2/5, as shown in Table 1, and the scale factor C_V varies with flow depth. The precise value of exponent m depends on the value of the weighting factor w. For the limiting case w = ∞, m = 2/5, and for the special case w = 1, m is 1/5 as shown by equation (A8a), which is in the range of the values reported in the literature. In this case, slope also appears in equations (A2) and (A8a), in which case the power of slope varies from 0 to 5/6. If the slope is expressed as a function of discharge with the power of −1/6, then the exponent m varies from −5/36 (−0.14) to 5/3. Most of the values reported in the literature lie within the derived range.

[38] Manning's n varies with discharge raised to the power varying from −1 to 0, as shown in Table 1, and the scale factor C_n varies with flow depth. The exponent p in this case depends on the value of w. The exponent value of −0.5 is for the special case w = 1. This range of exponent values encompasses the values reported in the literature. Again, if S appearing in equation (25a) is expressed as a function of Q with the exponent of −1/6, then the exponent of Q for n varies from −13/12 to 0.0. In this case, the reported range of exponent values is −0.54 to 0.03 [*Knighton*, 1975].

[39] The above discussion shows that the exponent values of b, m and p do not possess fixed values; rather they vary over certain ranges dictated by the way the adjustment of stream power is distributed among variables. Depending on the value of w, the derived exponent values encompass the whole ranges of values reported in the literature. Furthermore, the scale parameters are variant, depending on the channel hydraulics. Indeed this observation should help with regionalization of scale parameters.

3.2. Possibility 2: Hydraulic Geometry Relations for Width, Depth, and Velocity

[40] This is the most investigated possibility. In this case, the exponent, b, of discharge is found to vary from 0 to 1, as shown in Table 1, and the scale factor C_B varies with flow

Table 2b. Values of Exponents b, f, m, and p for Three Limiting Cases When the Weighting Factors are Zero, Unity, and Infinity for Different Possibilities, With Slope Expressed as a Function of Discharge With the Power of −2/5[a]

	Weighting Factor	Possibility 1	2	3	1 + 2 + 3
b for B	0	6/5 (w = 0)	6/5 (r = 0)		6/5 (w = r = 0)
b for B	1	3/5 (w = 1)	3/5 (r = 1)		2/5 (w = r = 1)
b for B	∞	0 (w = ∞)	0 (r = ∞)		0 (w = r = ∞)
f for h	0		0 (r = 0)	18/25 (J = 0)	0 (J = r = 0)
f for h	1		9/25 (r = 1)	9/25 (J = 1)	6/25 (J = r = 1)
f for h	∞		18/25 (r = ∞)	0 (J = ∞)	0 (J = r = ∞)
m for V	0	−1/3 (w = 0)	−1/5 (r = 0)	7/25 (J = 0)	−1/5 (J = r = 0)
m for V	1	2/3 (w = 1)	1/25 (r = 1)	16/25 (J = 1)	9/25 (J = r = 1)
m for V	∞	5/3 (w = ∞)	7/25 (r = ∞)	1 (J = ∞)	1 (w = J = r = ∞)
p for n	0	0 (w = 0)		−6/5 (J = 0)	0 (J = w = 0)
p for n	1	−3/5 (w = 1)		−3/5 (J = 1)	−2/5 (J = w = 1)
p for n	∞	−6/5 (w = ∞)		0 (J = ∞)	−6/5(J = w = ∞)
Equation numbers		(A4a), (A5a), and (A6a)	(A13a), (A14a), and (A15a)	(A22a), (A23a), and (A24a)	(A32a), (A33a), (A34a), and (A5a)

[a]Usually the values of these factors will have values between the limiting values.

resistance. The precise value of b depends on the weighting factor r which specifies the proportion for adjustment of stream power between B, h and V. For the special case r = 1, where the adjustment is equally proportioned, b = 0.5 as seen from equation (A16a). The width-discharge relation is found to depend on the slope of the channel, S. If S is expressed as a function of discharge with an exponent of −1/6, then the range of b becomes 0 to 13/12. These values of b encompass the entire range of values reported in the literature.

[41] The value of exponent f varies from 0 to 3/5 (when r ranges from 0 to ∞) shown in Table 1, with the scale factor C_h being dependent on the flow resistance. The precise value depends on the value of r. For the special case r = 1, the value of f is 3/10 as shown by equation (A17a). It is to be noted that equations (A11) and (A17a) contain a slope term. If the slope is expressed in terms of discharge with the power of −1/6, then the value of f ranges from 0 to 13/20 (when r ranges from 0 to ∞). These derived exponent values encompass the reported range.

[42] The value of exponent, m, varies from 0 to 2/5, shown in Table 1, with the scale factor C_V being dependent on the flow depth. The exact value of m depends on the value of r. For the special case, r = 1, the value of m is 1/5, as exhibited by equation (A18a). If the slope, appearing in equations (A15a) and (A18a) is expressed in terms of discharge with the power of −1/6, then the m exponent varies from −1/12 to 7/20 (when r ranges from 0 to ∞). Thus the derived exponent values are seen to envelope the reported range.

[43] For the downstream geometry of 72 streams from a variety of exponents, *Park* [1977] reported the range of b as 0.03 to 0.89 with modal class as 0.4 to 0.5; the range of f as 0.09 to 0.70 with modal class as 0.3 to 0.4; and the range of m as −0.51 to 0.75 with modal class as 0.1 to 0.2. Thus the derived exponents are in the reported ranges. The above discussion illustrates that the values of exponents, b, f, and m, do not possess fixed values; rather they vary over certain ranges dictated by the way the adjustment of stream power is distributed among variables. Furthermore, the scale parameters are variant, depending on the channel hydraulics. This observation should be helpful with regionalization of scale factors.

3.3. Possibility 3: Hydraulic Geometry Relations for Depth, Roughness, and Velocity

[44] The value of exponent, f, varies from 0 to 3/5 (when J ranges from ∞ to 0), as exhibited in Table 1, and the scale factor C_h depends on the channel width. The exact value depends on the value of the weighting factor J. For the special case, J = 1, the value of f becomes 3/10, as shown by equation (A25a). Equations (A19) and (A25) contain a slope term. When the slope is expressed in terms of discharge with the power of −1/6, then the f exponent varies from 0 to 13/20 (when J ranges from 0 to ∞), as seen in Table 2a. The exponent values thus derived cover the whole range reported in the literature.

[45] The value of exponent m varies from 2/5 to 1, as exhibited in Table 1, and the scale factor C_V depends on the channel width. This exact value depends on the value of the weighting factor J. For the special case, J = 1, the exponent m assumes the value of 7/10, as shown by equation (A26a). Equations (A23a) and (A26a) contain a slope term. When this slope term is expressed in terms of discharge with the power of −1/6, the m exponent varies from 7/20 to 1, as exhibited in Table 2a. These exponent values encompass the range reported in the literature.

[46] The value of exponent, p, varies from 0 to −1 (as J ranges from 0 to ∞), as shown in Table 1, and the scale factor depends on the channel width. The precise value depends on the value of J. For the special case, J = 1, the p exponent becomes −0.5, as shown by equation (A27a). Equations (A21) and (A27a) contain a slope term which when expressed in terms of discharge with the power of −1/6 result in the value of p ranging from −13/12 to 0 (as J ranges from 0 to ∞), as shown in Table 2a. The exponent values thus derived encompass the reported range.

[47] The above discussion shows that exponents, f, m, and p, do not possess fixed values; rather they vary over certain ranges, depending on the way the adjustment of stream power is distributed among variables. Furthermore, the scale parameters are variant, depending on channel hydraulics, and this observation should help with regionalization of scale parameters.

3.4. Possibility 4: Hydraulic Geometry Relations for Depth, Width, Roughness, and Velocity

[48] The value of exponent, b, varies from 0 to 1, as exhibited in Table 1. The exact value depends on the values of the weighting factors. For the special case w = r = 1, the value of b becomes 1/3, as shown by equation (A36a). Equations (A28a) and (A36a) contain a slope term; when this slope is expressed in terms of discharge with a power of −1/6, then the exponent varies from 0 to 13/12, as shown in Table 2a. These values encompass the range reported in the literature.

[49] The value of exponent, f, varies from 0 to 1/5 as shown in Table 1. For the special case, J = r = 1, the value of f becomes 1/5, as shown by equation (A37a). Equations (A29) and (A37a) contain a slope term. When slope is expressed in terms of Q with the power of −1/6, the exponent f varies from 0 to 13/60, as shown in Table 2a. These exponent values cover the reported range.

[50] The value of exponent m varies from 0 to 1, as seen in Table 1. For the limiting case, w = J = r =1, the m exponent value becomes 7/15, as shown by equation (A38a). Equations (A30) and (A38a) contain a slope term. When the slope is expressed in terms of discharge with the power of −1/6, the value of the m exponent varies from −1/12 to 1, as shown in Table 2a. These exponent values cover the range reported in the literature.

[51] The value of exponent, p, varies from −1 to 0, as shown in Table 1. For the special case, J = w = 1, the value of p becomes −1/3, as shown by equation (A39a). Equations (A31a) and (A39a) contain a slope term which, when expressed in terms of Q with the power of −1/6, leads to the value of p to range from −13/12 to 0, as seen in Table 2a.

[52] The above discussion shows that the values of the b, f, m, and p exponents are not fixed; rather they vary over certain ranges, as exhibited by equations (32a) to (35b). The variation is indeed continuous and is dictated by the way the adjustment of stream power is distributed among variables. Under this possibility the adjustment occurs simultaneously in river width, depth, velocity and roughness.

4. Conclusions

[53] The following conclusions are drawn from this study: (1) The application of the principles of the minimum

energy dissipation rate or its simplified minimum stream power and maximum entropy lead to a family of hydraulic geometry relations. These relations correspond to four different possibilities, depending on the way the spatial change in stream power is distributed among variables. (2) The exponent values are not fixed, rather they have ranges dictated by the value of the associated weighting factors. The exponent values vary continuously. (3) The exponent values derived here encompass the reported ranges in the literature. (4) The scale factors are not fixed but vary with hydraulic variables and their variation may be helpful with regionalization.

Appendix A: Derivation of Equations for Hydraulic Geometry Relations

A1. Possibility 1: $P_B = P_n$

[54] Recall that $nB^w = C_1$. The objective is to determine the downstream hydraulic geometry relations for B, V, and n in terms of Q. To that end, substitution of equation (17a) in equation (4a) and a little algebraic manipulation yield:

$$B = C_B Q^{1/(1+w)} S^{-1/[2(1+w)]}, \quad C_B = \left(C_1 h^{-5/3}\right)^{1/(1+w)} \tag{A1}$$

$$V = C_V Q^{2w/[5(1+w)]} S^{5/[6(1+w)]}, \quad C_V = \left[C_n (C_B)^{2/3}\right]^{-3/5} \tag{A2}$$

$$n = C_n Q^{-w/(1+w)} S^{w/[2(1+w)]}, \quad C_n = (C_1)^{1/(1+w)} h^{5w/[3(1+w)]} \tag{A3}$$

Equations (A1) to (A3) contain S which can be eliminated with the use of a sediment transport relation. For the Engelund and Hansen sediment transport equation, the channel slope S can be expressed in terms of discharge Q by equation (22) [*Knighton*, 1998]. Introducing equation (22) in equation (A1), one obtains

$$B = C_{BS} Q^{\frac{(1-z/2)}{1+w}}, \; C_{BS} = \frac{C_B}{C_S^{1/2(1+w)}} \tag{A4a}$$

Substitution of z = −2/5 and z = −1/6 into equation (A4a) yields

$$B = C_{BS} Q^{\frac{6}{5(1+w)}} \tag{A4b}$$

for gravel rivers

$$B = C_{BS} Q^{\frac{13}{12(1+w)}} \tag{A4c}$$

for sandy rivers

Likewise, with use of equation (22), equations (A2) and (A3) can be simplified as

$$V = C_{VS} Q^{\frac{5w-1}{3(1+w)}} \quad \left(C_{VS} = C_V C_S^{5/[6(1+w)]}\right) \tag{A5a}$$

for gravel rivers

$$V = C_{VS} Q^{\frac{60w-5}{36(1+w)}} \tag{A5b}$$

for sandy rivers

$$n = C_{nS} Q^{\frac{-6w}{5(1+w)}} \quad \left(C_{nS} = C_n C_S^{w/[2(1+w)]}\right) \tag{A6a}$$

for gravel rivers

$$n = C_{nS} Q^{\frac{-13w}{12(1+w)}} \tag{A6b}$$

for sandy rivers

For the special case, w =1, equations (A4a) to (A6b) reduce, respectively, to

$$B = C_B \frac{Q^{0.5}}{S^{1/4}}, C_B = (C_1)^{0.5} h^{-5/6} \tag{A7a}$$

$$B = C_{BS} Q^{\frac{3}{5}} \quad \left(C_{BS} = \frac{C_B}{C_S^{1/2(1+w)}}\right) \tag{A7b}$$

for gravel rivers

$$B = C_{BS} Q^{\frac{13}{24}} \tag{A7c}$$

for sandy rivers

$$V = C_V Q^{1/5} S^{5/12}, \quad C_V = \left(C_n^{3/5} C_B^{2/5}\right)^{-1} \tag{A8a}$$

$$V = C_{VS} Q^{\frac{2}{3}} \quad \left(C_{VS} = C_V C_S^{5/12}\right) \tag{A8b}$$

for gravel rivers

$$V = C_{VS} Q^{\frac{55}{72}} \tag{A8c}$$

for sandy rivers

$$n = C_n \frac{S^{1/4}}{Q^{1/2}}, \quad C_n = C_1^{1/2} h^{5/6} \tag{A9a}$$

$$n = C_{nS} Q^{\frac{-3}{5}} \quad \left(C_{nS} = C_n C_S^{1/4}\right) \tag{A9b}$$

for gravel rivers

$$n = C_{nS} Q^{\frac{-13}{24}} \tag{A9c}$$

for sandy rivers

General equations (A4a) to (A6b) or their special forms (A7a) to (A9c) express B, V, and n as functions of Q only.

A2. Possibility 2: $P_B = Ph$, $B = C_2 h^{5/3r}$

[55] Substitution of equation (19a) in equation (4a) and a little algebraic manipulation yield:

$$B = C_B Q^{1/(1+r)} S^{-1/[2(1+r)]} hbox, C_B = [n(C_2)^r]^{1/[1+r]} \tag{A10}$$

$$h = C_h Q^{3r/[5+5r]} S^{-3r/[2(5+5r)]}, \quad C_h = (nC_2)^{3r/[5+5r]} \tag{A11}$$

$$V = C_V Q^{2r/[5(1+r)]} S^{[3(1+r)+2]/[10(1+r)]}, \quad C_V = n^{-3/5}[C_B]^{-2/5} \tag{A12}$$

Substitution of equation (22) with z = −2/5 and z = −1/6 into equations (A10) to (A12) yields

$$B = C_{BS} Q^{\frac{6}{5(1+r)}} \quad \left(C_{BS} = \frac{C_B}{C_S^{1/2(1+r)}} \right) \tag{A13a}$$

for gravel rivers

$$B = C_{BS} Q^{\frac{13}{12(1+r)}} \tag{A13b}$$

for sandy rivers

$$h = C_{hS} Q^{\frac{18r}{25(1+r)}} \quad \left(C_{hS} = C_h C_S^{-3r/[2(5+5r)]} \right) \tag{A14a}$$

for gravel rivers

$$h = C_{hS} Q^{\frac{13r}{20(1+r)}} \tag{A14b}$$

for sandy rivers

$$V = C_{VS} Q^{\frac{7r-5}{25(1+r)}} \quad \left(C_{VS} = C_V C_S^{[3(1+r)+2]/[10(1+r)]} \right) \tag{A15a}$$

for gravel rivers

$$V = C_{VS} Q^{\frac{21r-5}{60(1+r)}} \tag{A15b}$$

for sandy rivers
For the special case, r = 1, equations (A10)–(A15b) reduce, respectively, to

$$B = C_B \frac{Q^{1/2}}{S^{1/4}}, \quad C_B = (nC_2)^{1/2} \tag{A16a}$$

$$B = C_{BS} Q^{\frac{3}{5}} \quad \left(C_{BS} = \frac{C_B}{C_S^{1/4}} \right) \tag{A16b}$$

for gravel rivers

$$B = C_{BS} Q^{\frac{13}{24}} \tag{A16c}$$

for sandy rivers

$$h = C_h \frac{Q^{3/10}}{S^{3/20}}, C_h = C_2^{-3/10} n^{3/10} \tag{A17a}$$

$$h = C_{hS} Q^{\frac{9}{25}} \quad \left(C_{hS} = C_h C_S^{-3/20} \right) \tag{A17b}$$

for gravel rivers

$$h = C_{hS} Q^{\frac{13}{40}} \tag{A17c}$$

for sandy rivers

$$V = C_V Q^{1/5} S^{2/5}, \quad C_V = \left[(C_n)^{3/5} (C_B)^{2/5} \right]^{-1} \tag{A18a}$$

$$V = C_{VS} Q^{\frac{1}{25}} \quad \left(C_{VS} = C_V C_S^{2/5} \right) \tag{A18b}$$

for gravel rivers

$$V = C_{VS} Q^{\frac{2}{15}} \tag{A18c}$$

for sandy rivers

A3. Possibility 3: $P_n = P_h$, $C_3 = n\ h^{5J/3}$

[56] Substitution of equation (21a) in equation (4a) and a little algebraic manipulation yield:

$$h = C_h Q^{3/[5+5J]} S^{-3/[2(5+5J)]}, \; C_h = (C_B)^{3/[5+5J]} B^{-3/[5+5J]} \tag{A19}$$

$$V = C_V Q^{(2+5J)/[5(1+J)]} S^{3/[10(1+J)]}, \quad C_V = (C_n)^{-3/5} B^{-2/5} \tag{A20}$$

$$n = C_n Q^{-1/(1+J)} S^{1/[2(1+J)]}, \quad C_n = (C_3)^{1/[1+J]} B^{1/[1+J]} \tag{A21}$$

Substitution of equation (22) with z = −2/5 and z = −1/6 into equations (A19) to (A21) yields

$$h = C_{hS} Q^{\frac{18}{25(1+J)}} \quad \left(C_{hS} = C_h C_S^{-3/[2(5+5J)]} \right) \tag{A22a}$$

for gravel rivers

$$h = C_{hS} Q^{\frac{13}{20(1+J)}} \tag{A22b}$$

for sandy rivers

$$V = C_{VS} Q^{\frac{7+25J}{25(1+J)}} \quad \left(C_{VS} = C_V C_S^{3/[10(1+J)]} \right) \tag{A23a}$$

for gravel rivers

$$V = C_{VS} Q^{\frac{20J+7}{20(1+J)}} \tag{A23b}$$

for sandy rivers

$$n = C_{nS} Q^{\frac{-6}{5(1+J)}} \quad \left(C_{nS} = C_n C_S^{1/[2(1+J)]} \right) \tag{A24a}$$

for gravel rivers

$$n = C_{nS} Q^{\frac{-13}{12(1+J)}} \tag{A24b}$$

for sandy rivers
For the special case, J = 1, equations (A19)–(A24b) reduce to

$$h = C_h \frac{Q^{3/10}}{S^{3/20}}, \quad C_h = B^{-3/10} C_3^{3/10} \tag{A25a}$$

$$h = C_{hS}Q^{\frac{9}{25}} \quad \left(C_{hS} = C_h C_S^{-3/20}\right) \tag{A25b}$$

for gravel rivers

$$h = C_{hS}Q^{\frac{13}{40}} \tag{A25c}$$

for sandy rivers

$$V = C_V Q^{7/10} S^{3/20}, \quad C_V = C_n^{-3/5} B^{2/5} \tag{A26a}$$

$$V = C_{VS}Q^{\frac{16}{25}} \quad \left(C_{VS} = C_V C_S^{3/20}\right) \tag{A26b}$$

for gravel rivers

$$V = C_{VS}Q^{\frac{27}{40}} \tag{A26c}$$

for sandy rivers

$$n = C_n \frac{S^{0.25}}{Q^{0.5}}, \quad C_n = B^{0.5} C_3^{0.5} \tag{A27a}$$

$$n = C_{nS}Q^{\frac{-3}{5}} \quad \left(C_{nS} = C_n C_S^{1/4}\right) \tag{A27b}$$

for gravel rivers

$$n = C_{nS}Q^{\frac{-13}{24}} \tag{A27c}$$

for sandy rivers

A4. Possibility 4

[57] Substitution of equations (17a), (19a) and (21a) in equation (4a) and a little rearrangement result in

$$B = C_B Q^{1/[1+w+r]} S^{-1/[2(1+w+r)]}, \quad C_B = [C_1 (C_2)^r]^{1/[1+w+r]} \tag{A28}$$

$$h = C_h Q^{3r/[5+5Jr+5r]} S^{-3r/[2(5+5Jr+5r)]}, \; C_h = \left[C_3 (C_2)^{-1}\right]^{3r/[5+5Jr+5r]} \tag{A29}$$

$$V = C_V Q^{\frac{3}{5}\left[\frac{2}{3}+\frac{wJ}{wJ+J+w}-\frac{2}{3(1+w+r)}\right]} S^{\frac{3}{5}\left[\frac{1}{2}-\frac{wJ}{2(wJ+J+w)}+\frac{1}{3(1+w+r)}\right]}, \quad C_V = \left[C_n (C_B)^{2/3}\right]^{-3/5} \tag{A30}$$

$$n = C_n Q^{-wJ/[wJ+J+w]} S^{wJ/[2(wJ+J+w)]}, \quad C_n = \left[(C_1)^{1/w} (C_3)^{1/J}\right]^{wJ/[wJ+J+w]} \tag{A31}$$

Substitution of equation (22) with z = −2/5 and z = −1/6 into equations (A28) to (A29) yields

$$B = C_{BS}Q^{\frac{6}{5(1+w+r)}} \quad \left(C_{BS} = \frac{C_B}{C_S^{1/2(1+w+r)}}\right) \tag{A32a}$$

for gravel rivers

$$B = C_{BS}Q^{\frac{13}{12(1+w+r)}} \tag{A32b}$$

for sandy rivers

$$h = C_{hS}Q^{\frac{18r}{25(1+Jr+r)}} \quad \left(C_{hS} = C_h C_S^{-3r/[2(5+5Jr+5r)]}\right) \tag{A33a}$$

for gravel rivers

$$h = C_{hS}Q^{\frac{13r}{20(1+Jr+r)}} \tag{A33b}$$

for sandy rivers

$$V = C_{VS}Q^{\frac{3}{25}\left(\frac{7}{3}+\frac{6wJ}{wJ+J+w}-\frac{4}{1+w+r}\right)} \quad \left(C_{VS} = C_V (C_S)^{\frac{3}{5}\left[\frac{1}{2}-\frac{wJ}{2(wJ+J+w)}+\frac{1}{3(1+w+r)}\right]}\right) \tag{A34a}$$

for gravel rivers

$$V = C_{VS}Q^{\frac{1}{5}\left(\frac{7}{4}+\frac{13wJ}{4(wJ+J+w)}-\frac{13}{6(1+w+r)}\right)} \tag{A34b}$$

for sandy rivers

$$n = C_{nS}Q^{\frac{-6wJ}{5(wJ+J+w)}} \quad \left(C_{nS} = C_n C_S^{wJ/[2(wJ+J+w)]}\right) \tag{A35a}$$

for gravel rivers

$$n = C_{nS}Q^{\frac{-13wJ}{12(wJ+J+w)}} \tag{A35b}$$

for sandy rivers

For the special case, w = J = r =1, equations (A28) to (A35b) reduce to

$$B = C_B \frac{Q^{1/3}}{S^{1/6}}, \quad C_B = (C_1 C_2)^{1/3} \tag{A36a}$$

$$B = C_{BS}Q^{\frac{2}{5}} \quad \left(C_{BS} = \frac{C_B}{C_S^{1/6}}\right) \tag{A36b}$$

for gravel rivers

$$B = C_{BS}Q^{\frac{13}{36}} \tag{A36c}$$

for sandy rivers

$$h = C_h \frac{Q^{1/5}}{S^{1/10}}, C_h = \left(C_2^{-0.2} C_3^{0.2}\right) \tag{A37a}$$

$$h = C_{hS}Q^{\frac{6}{25}} \quad \left(C_{hS} = C_h C_S^{-1/10}\right) \tag{A37b}$$

for gravel rivers

$$h = C_{hS} Q^{\frac{13}{60}} \quad \text{(A37c)}$$

for sandy rivers

$$V = C_V Q^{7/15} S^{4/15}, \quad C_V = \left[(C_n)^{3/5}(C_B)^{2/5}\right]^{-1} \quad \text{(A38a)}$$

$$V = C_{VS} Q^{\frac{9}{25}} \quad \left(C_{VS} = C_V C_S^{4/151}\right) \quad \text{(A38b)}$$

for gravel rivers

$$V = C_{VS} Q^{\frac{19}{45}} \quad \text{(A38c)}$$

for sandy rivers

$$n = C_n \frac{S^{1/6}}{Q^{1/3}}, C_n = [C_1 C_3]^{1/3} \quad \text{(A39a)}$$

$$n = C_{nS} Q^{\frac{-2}{5}} \quad \left(C_{nS} = C_n C_S^{1/6}\right) \quad \text{s} \quad \text{(A39b)}$$

for gravel river

$$n = C_{nS} Q^{\frac{-13}{36}} \quad \text{(A39c)}$$

for sandy rivers. The coefficients, C_B, C_h, C_V and C_n, in equations (A36) to (A39c) are defined in terms of morphological coefficients C_1, C_2, and C_3 as

$$C_B = [C_1 C_2]^{1/3} \quad \text{(A40)}$$

$$C_h = C_2^{-0.2} C_3^{0.2} \quad \text{(A41)}$$

$$C_n = C_1^{1/3} C_3^{1/3} \quad \text{(A42)}$$

$$C_V = \left[C_1^{1/3} C_2^{2/5} C_3^{1/5}\right]^{-1} \quad \text{(A43)}$$

Here C_B, C_h, C_V, and C_n are the coefficients associated with flow width, depth, velocity, and Manning's n, respectively, and depend on C_1, C_2, and/or C_3, given, respectively, by equations (17b), (19b) and (21b). It is to be noted that since the discharge Q = BhV, equations (A28) to (A35b) or (A36a) to (A39c) show that the sum of exponents of Q equals 1. Similarly, the sum of exponents of S from these equations equals 0.

Notation

A	cross-sectional area.
a, b, c, k, m, N, p, s, y	numerical constants.
b, f, m, p, y	exponents of flow discharge Q in empirical hydraulic geometry relationships.
B	water surface width.
C	Chezy's roughness coefficient.
C_1, C_{1*}, C_2, C_{2*}, C_3, C_{3*}	primary morphological coefficients.
C_s, C_{BS}, C_{VS}, C_{nS}, C_{hS}, C_B, C_h, C_V, C_n	coefficients of morphological equations.
d	mean flow depth.
f	Darcy-Weisbach friction factor.
g	acceleration due to gravity.
h	mean flow depth.
J	weighting factor.
m	exponent in Manning formula.
n	Manning's roughness coefficient.
P	wetted perimeter.
P_w	unit stream power.
P_α, P_n	proportion of the adjustment of stream power by friction.
P_B	proportion of the adjustment of stream power by channel width.
P_h	proportion of the adjustment of stream power by flow depth.
Q	flow discharge.
R	hydraulic radius.
R_1	spatial rate of adjustment of friction.
R_2	spatial rate of adjustment of channel width.
R_3	spatial rate of adjustment of flow depth.
r	weighting factor.
S	channel slope.
SP	stream power.
V	average flow velocity.
w	weighting factor.
x	distance along the flow direction.
z	exponent of discharge Q in empirical relation between S and Q.
α	roughness measure.
β	exponent of flow depth h.
ϕ, φ	parameters in basic form of regime equation.
γ	weight density of water.

References

Allen, P. M., J. G. Arnold, and B. W. Byars, Downstream channel geometry for use of in planning-level models, *Water Resour. Bull.*, *30*(4), 663–671, 1994.

Blench, T., Regime theory for self-formed sediment bearing channels, *Trans. Am. Soc. Civil Eng.*, *117*, 383–408, 1952.

Blench, T., Coordination in mobile-bed hydraulics, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *95*(HY6), 1871–1988, 1969.

Bray, R. D., Regime equations for gravel-bed rivers, in *Gravel-Bed Rivers*, edited by R. D. Hey, J. C. Bathurst, and C. Thorne, pp. 517–544, John Wiley, Hoboken, N. J., 1982.

Brebner, A., and K. C. Wilson, Derivation of the regime equations from relationships for pressurized flow by use of the principle of energy-degradation rate, *Proc. Inst. Civ. Eng.*, *36*, 47–62, 1967.

Chang, H. H., Geometry of gravel stream, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *106*(HY9), 1443–1456, 1980.

Chang, H. H., *Fluvial Processes in River Engineering*, John Wiley, Hoboken, N. J., 1988.

Cheema, M. N., M. A. Marino, and J. J. DeVries, Stable width of an alluvial channel, *J. Irrig. Drain. Eng.*, *123*(1), 55–61, 1997.
Chong, S. E., The width, depth and velocity of Sungei Kimla, Perak, *Geographica*, *6*, 63–72, 1970.
Davies, T. R. H., and A. J. Sutherland, Extremal hypotheses for river behavior, *Water Resour. Res.*, *19*, 141–148, 1983.
Deng, Z., and K. Zhang, Morphologic equations based on the principle of maximum entropy, *Int. J. Sediment. Res.*, *9*(1), 31–46, 1994.
Dou, G. R., Hydraulic geometry of plain alluvial rivers and tidal river mouth (in Chinese), *J. Hydraul. Eng.*, *2*, 1–13, 1964.
Dury, G. H., Discharge prediction, present and former, from channel dimensions, *J. Hydrol.*, *30*, 219–245, 1976.
Engelund, F., and E. Hansen, *A Monograph on Sediment Transport in Alluvial Streams*, Teknisk, Copenhagen, 1967.
Howard, A. D., Thresholds in river regimes, in *Thresholds in Geomorphology*, edited by D. R. Coates and J. D. Vitek, pp. 227–258, Allen and Unwin, Concord, Mass., 1980.
Huang, H. W., and G. C. Nanson, Hydraulic geometry and maximum flow efficiency as products of the principle of least action, *Earth Surf. Processes Landforms*, *25*, 1–16, 2000.
Jaynes, E. T., Information theory and statistical mechanics, I, *Phys. Rev.*, *106*, 620–630, 1957.
Klein, M., Drainage area and the variation of channel geometry downstream, *Earth Surf. Processes Landforms*, *6*, 589–593, 1981.
Knighton, A. D., Variation in width-discharge relation and some implications for hydraulic geometry, *Geol. Soc. Am. Bull.*, *85*, 1069–1076, 1974.
Knighton, A. D., Variations in at-a-station hydraulic geometry, *Am. J. Sci.*, *275*, 186–218, 1975.
Knighton, A. D., Alternative derivation of the minimum variance hypothesis, *Ecolog. Soc. Am. Bull.*, *83*, 3813–3822, 1977.
Knighton, A. D., River channel adjustment—The downstream dimension, in *River Channels: Environment and Process*, edited by K. S. Richards, pp. 95–128, Blackwell, Malden, Mass., 1987.
Knighton, A. D., *Fluvial Forms and Processes: A New Perspective*, Edward Arnold, London, 1998.
Kolberg, F. J., and A. D. Howard, Active channel geometry and discharge relations of U. S. piedmont and midwestern streams: The variable exponent model revisited, *Water Resour. Res.*, *31*, 2353–2365, 1995.
Lane, E. W., Design of stable channels, *Trans. Am. Soc. Civ. Eng.*, *120*, 1234–1260, 1955.
Langbein, W. B., Geometry of river channels, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *90*(HY2), 301–311, 1964.
Leopold, L. B., and T. J. Maddock, Hydraulic geometry of stream channels and some physiographic implications, *U.S. Geol. Surv. Prof. Pap.*, *252*, 55 pp., 1953.
Leopold, L. B., and W. B. Langbein, The concept of entropy in landscape evolution, *U.S. Geol. Survey Prof. Pap.*, *500-A*, 1962.
Leopold, L. B., and L. B. Wolman, River channel patterns: Braided, meandering and straight, *U.S. Geol. Surv. Prof. Pap.*, *282-B*, 1990.
Li, R. M., Mathematical modeling of response from small watershed, Ph.D. dissertation, 212 pp., Colo. State Univ., Fort Collins, 1974.
Osterkamp, W. R., and E. R. Hedman, Perennial-streamflow characteristics related to channel geometry and sediment in Missouri River basins, *U.S. Geol. Surv. Prof. Pap.*, *1242*, 37 pp., 1982.
Park, C. C., World-wide variations in hydraulic geometry exponents of stream channels: An analysis and some observations, *J. Hydrol.*, *33*, 133–146, 1977.
Parker, G., Self-formed rivers with equilibrium banks and mobile bed: Part II. The gravel river, *J. Fluid Mech.*, *89*(1), 127–148, 1978.
Parker, G., Hydraulic geometry of active gravel rivers, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *105*(HY9), 1185–1201, 1979.
Phillips, P. J., and J. M. Harlin, Spatial dependency of hydraulic geometry exponents in a subalpine stream, *J. Hydrol.*, *71*, 277–283, 1984.
Ramette, M., A theoretical approach on fluvial processes, paper presented at International Symposium on River Sedimentation, Chin. Acad. of Hydraul. Eng., Beijing, 1980.
Rhoads, B. L., A continuously varying parameter model of downstream hydraulic geometry, *Water Resour. Res.*, *27*, 1865–1872, 1991.
Rhodes, D. D., World wide variations in hydraulic geometry exponents of stream channels: An analysis and some observations—Comments, *J. Hydrol.*, *33*, 133–146, 1978.
Richards, K. S., Complex width-discharge relations in natural river sections, *Geol. Soc. Am. Bull.*, *87*, 199–206, 1976.
Richards, K. S., *Rivers: Form and Process in Alluvial Channels*, Metheun, London, 1982.
Rodriguez-Iturbe, I., A. Rinaldo, R. Rigon, R. L. Bras, A. Marani, and E. J. Ijjasz-Vasquez, Energy dissipation, runoff production and the three dimensional structure of river basins, *Water Resour. Res.*, *28*, 1095–1103, 1992.
Smith, T. R., A derivation of the hydraulic geometry of steady-state channels from conservation principles and sediment transport laws, *J. Geol.*, *82*, 98–104, 1974.
Stall, J. B., and Y. S. Fok, Hydraulic geometry of Illinois rivers, *Water Resour. Res. Cent. Rep. 15*, Univ. of Ill., Urbana, July 1968.
Stall, J. B., and C. T. Yang, Hydraulic geometry of 12 selected stream systems of the United States, *WRC Res. Rep.*, *32*, 1970.
Stebbings, J., The shape of self-formed model alluvial channels, *Proc. Inst. Civ. Eng.*, *25*, 485–510, 1963.
White, W. R., R. Bettess, and E. Paris, Analytical approach to river regime, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *108*(HY10), 1179–1193, 1982.
Williams, G. P., Flume experiments on the transport of a coarse sand, *U.S. Geol. Surv. Prof. Pap.*, *562-B*, 1967.
Williams, G. P., Hydraulic geometry of river cross-sections—Theory of minimum variance, *U.S. Geol. Surv. Prof.*, *1029*, 1978.
Wolman, M. G., The natural channel of Brandywine Creek, Pennsylvania, *U.S. Geol. Surv. Prof.*, 271, 1955.
Wolman, M. G., and L. M. Brush, Factors controlling the size and shape of stream channels in coarse noncohesive sands, *U.S. Geol. Surv. Prof. Pap.*, *282-G*, 183–210, 1961.
Yalin, M. S., and A. M. F. Da Silva, On the computation of equilibrium channels in cohesionless alluvium, *J. Hydrosci. Hydraul. Eng.*, *15*(2), 1–13, 1997.
Yalin, M. S., and A. M. F. Da Siva, Regime channels in cohesionless alluvium, *J. Hydraul. Res.*, *37*(6), 725–742, 1999.
Yang, C. T., Unit stream power and sediment transport, *J. Hydraul. Div. Am. Soc. Civ. Eng.*, *98*(HY10), 1805–1826, 1972.
Yang, C. T., Dynamic adjustment of rivers, paper presented at 3rd International Symposium on River Sedimentation, Univ. of Miss., Jackson, Miss., 1986.
Yang, C. T., *Sediment Transport Theory and Practice*, McGraw-Hill, New York, 1996.
Yang, C. T., and C. C. S. Song, Theory of minimum energy and energy dissipation rate, in *Encyclopedia of Fluid Mechanics*, chap. 11, Gulf, Houston, Tex., 1986.
Yang, C. T., C. C. Song, and M. T. Woldenberg, Hydraulic geometry and minimum rate of energy dissipation, *Water Resour. Res.*, *17*, 877–896, 1981.

Z. Q. Deng and V. P. Singh, Department of Civil and Environmental Engineering, Louisiana State University, Baton Rouge, LA 70803, USA. (cesing@lsu.edu)

C. T. Yang, Department of Civil Engineering, Colorado State University, Fort Collins, CO 80523-1372, USA.