Static load and dynamic forced performance of rigid rotor supported on short length journal bearings (includes thermal effects)

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Updated 09/04/09

$\text{ORIGIN} := 1$

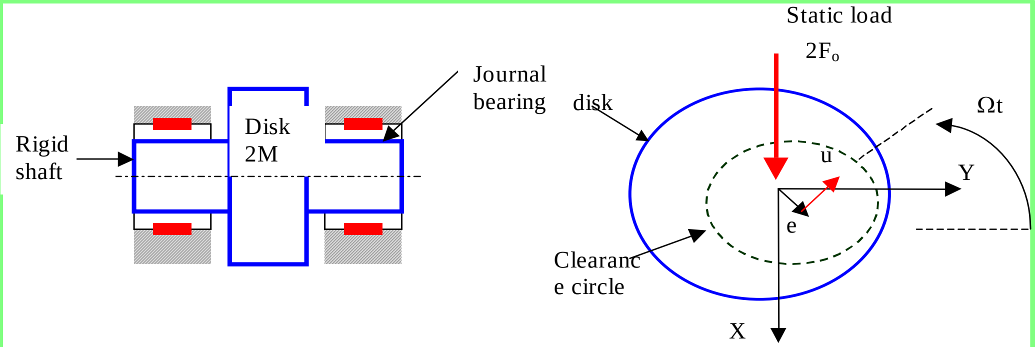
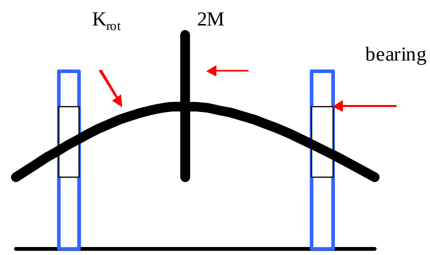


Figure 5.1. Rigid rotor supported on journal bearings. (u) imbalance, (e) journal eccentricity

rotor properties

DATA for rotor

$W_T := 500 \cdot \text{lbf}$ **Rotor weight**



$W := \frac{W_T}{2}$ **Load per bearing**

$W = 1.112 \times 10^3 \, \text{newton}$

$M := \frac{W}{g}$ $M = 250 \, \text{lb}$ **1/2 rotor mass**

$k_{shaft} := 40 \cdot 10^6 \cdot \frac{N}{m}$ **Rotor stiffness on each side of disk (at midspan)**

$k_{shaft} = 2.284 \times 10^5 \, \frac{\text{lbf}}{\text{in}}$

$\text{Rotor\_sag} := \frac{W}{k_{shaft}}$ **Rotor sag at midspan:**

$\text{Rotor\_sag} = 1.095 \times 10^{-3} \, \text{in}$

rotor properties

bearing geometry

BEARING GEOMETRY

$D := 6 \cdot \text{in}$ **journal diameter**

$R := \frac{D}{2}$ **journal radius**

$L := 2 \cdot \text{in}$ **bearing length**

$c := 0.003 \cdot \text{in}$ **radial clearance**

Static shaft deflection due to rotor weight = % of clearance

$\frac{L}{D} = 0.333$

$\frac{\text{Rotor\_sag}}{c} = 0.365$

bearing geometry

$RPM\_max := 10000$ **MAXIMUM & design speeds** # of cases for analysis $Nmax := 50$

$RPM_{design} := 7200$

$a := 0.2 \cdot c$ Amplitude of imbalance on rotor disk

Thermal model conditions

$\kappa := 0.8$ **Mechanical energy convected by lubricant.**

$\lambda := 0.70$ **Heat carry over - thermal mixing coefficient.**

$T_{supply} := 50K$ Supply Oil Temperature Take deg-K as deg-C

Lubricant properties

PROPERTIES OF LUBRICANT **MOBIL velocite No 10 (ISO VG 22)**

$\mu_{supply} := 0.0143 \cdot \frac{N \cdot s}{m^2}$ Lubricant viscosity at Tsupply in Pa-sec.

$\alpha := 0.028 \cdot \frac{1}{K}$ Alpha coefficient for viscosity equation.

$\rho := 862 \cdot \frac{kg}{m^3}$ Density in kg/m^3.

$Cp := 1880 \cdot \frac{J}{kg \cdot K}$ Specific Heat in kJ/(kg C).

Lubricant properties

Calculations

CALCULATIONS =======FIND journal eccentricity as rotor speed increases for given load

$Dr := \frac{RPM\_max}{(Nmax)}$ step in rpm $n := 1 .. Nmax$ $T_{supply} = 50\,K$

$rpm_n := Dr \cdot (n)$ rotor running speed(rpm)

$\Omega_n := rpm_n \cdot \frac{\pi}{30} \cdot \frac{rad}{sec}$ rotor speed(rad/sec) $krpm := \frac{rpm}{1000}$

$it_{max} := 10$ Max iterations for thermal loop

VISCOSITY formula

$\mu(T) := \mu_{supply} \cdot e^{-\alpha \cdot (T - T_{supply})}$

Journal eccentricity ratio and attitude angle for STATIC equilibrium

Find journal eccentricity and operating temperatures

$\varepsilon_{guess} := 0.8$ $T_{trial} := T_{supply} + 20K$

ITERATIVE LOOP - Given load, find operating eccentricity & find temperature rise

$$\text{SteadyJB}(\varepsilon, rpm, T_{guess}) :=$$

$$it \leftarrow 0$$

$$x \leftarrow \varepsilon$$

$$N \leftarrow rpm \cdot \frac{1}{60 \cdot sec}$$ Speed in rev/sec

$$\Omega \leftarrow rpm \cdot \frac{\pi}{30 \cdot sec}$$ Speed in rad/sec

$$T_{eff} \leftarrow T_{guess}$$ Guess (set) Effective temperature

$$T_{guess} \leftarrow T_{eff} \cdot 1.1$$

$$\text{while } (it < it_{max}) \wedge (|T_{eff} - T_{guess}| > 0.1K)$$

$$it \leftarrow it + 1$$

$$\mu \leftarrow \mu_{supply} \cdot e^{-\alpha \cdot (T_{eff} - T_{supply})}$$ effective viscosity

$$\sigma \leftarrow \pi \cdot \left[\frac{\mu \cdot N \cdot L \cdot D}{W} \cdot \left(\frac{R}{c}\right)^2\right] \cdot \left(\frac{L}{D}\right)^2$$ σ = short length bearing Sommerfeld #

$$\varepsilon \leftarrow \text{root}\left[\frac{(1 - x^2)^2}{x \cdot \sqrt{\pi^2 \cdot (1 - x^2) + 16 \cdot x^2}} - \sigma, x\right]$$ Root solver: find eccentricity ε fo specified Sommerfeld #

$$Power \leftarrow (\Omega)^2 \cdot \left(\mu \cdot R^3 \cdot L \cdot \frac{\pi}{c}\right) \cdot \frac{1}{(1 - \varepsilon^2)^{.5}}$$ Power loss

$$Q_0 \leftarrow c \cdot R \cdot L \cdot ((1 + \varepsilon)) \cdot \frac{\Omega}{2}$$ Qo = inlet flow rate, θ=0

$$Q_\pi \leftarrow c \cdot R \cdot L \cdot ((1 - \varepsilon)) \cdot \frac{\Omega}{2}$$ $Q\pi$ = flow rate at min film $\theta=\pi$ (start of cavitation zone

$$Q_e \leftarrow c \cdot R \cdot L \cdot \varepsilon \cdot \Omega$$ Exit flow (side flow)

$$\delta T_\pi \leftarrow \frac{\left[\frac{(\kappa \cdot Power)}{\rho \cdot Cp \cdot \left(Q_0 - \frac{Q_e}{2}\right)}\right]}{1 + \lambda \cdot \frac{Q_\pi}{Q_0}}$$ Temperature rise at $\theta=\pi$

Thermal energy equations are probably incorrect. Please check them with those in handout

$$\delta T_{in} \leftarrow \lambda \cdot \left(\frac{Q_\pi}{Q_0}\right) \cdot \delta T_\pi$$ Temperature rise at inlet plane

$$T_\pi \leftarrow T_{supply} + \delta T_\pi$$ Max temperature at $\theta=\pi$

$$T_{in} \leftarrow T_{supply} + \delta T_{in}$$

$$\delta T = T - T_S$$

$$T_{guess} \leftarrow T_{eff}$$

$$T_{eff} \leftarrow 0.5 \cdot (T_{in} + T_\pi)$$ Effective temperature

$$x \leftarrow \varepsilon$$

Supply flow rate

$$\left| \begin{array}{l} Q_{supply} \leftarrow Q_0 - \lambda \cdot Q_\pi \\ \text{return} \left(\varepsilon \quad \frac{T_\pi}{K} \quad \frac{T_{in}}{K} \quad \frac{T_{eff}}{K} \quad \frac{Power}{watt} \quad Q_{supply} \cdot \frac{s}{m^3} \quad \sigma \right) \end{array} \right.$$

$$\left(\varepsilon_n \quad T_{\pi_n} \quad T_{in_n} \quad T_{eff_n} \quad Power_n \quad Q_{supply_n} \quad \sigma_n \right) := SteadyJB\left(.1, rpm_n, T_{trial}\right)$$

convert to physical units

$Power := Power \cdot watt$ $\quad T_\pi := T_\pi \cdot 1K$ $\quad T_{eff} := T_{eff} \cdot 1K$ $\quad T_{in} := T_{in} \cdot 1K$

$Q_{supply} := Q_{supply} \cdot 60000$ $\quad Q_{sides_n} := c \cdot R \cdot L \cdot \varepsilon_n \cdot \Omega_n \cdot 60000$ $\quad$ LPM

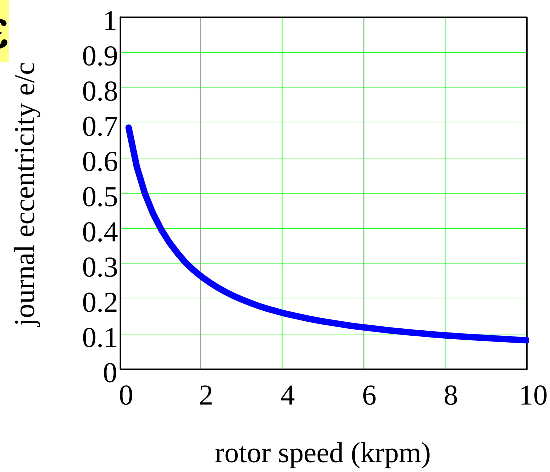
effective viscosity: $\mu\_n := \mu_{supply} \cdot e^{-\alpha \cdot \left(T_{eff_n} - T_{supply}\right)}$ deg-C (actual)

Journal Attitude angle $\phi_n := atan\left[\frac{\pi \cdot \sqrt{1 - \left(\varepsilon_n\right)^2}}{4 \cdot \varepsilon_n}\right] \cdot \frac{180}{\pi}$

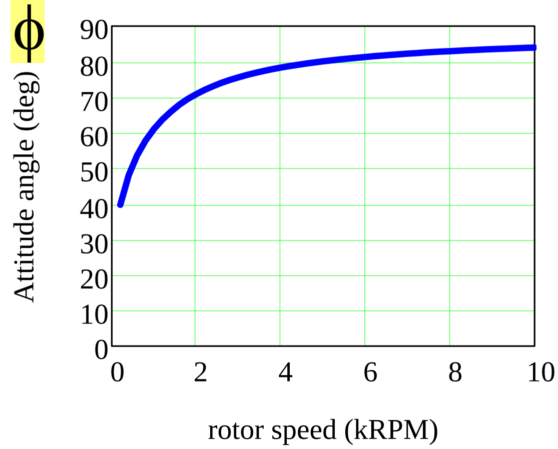
Calculations

STATIC LOAD PERFORMANCE: journal cccentricity and attitude angle

ε



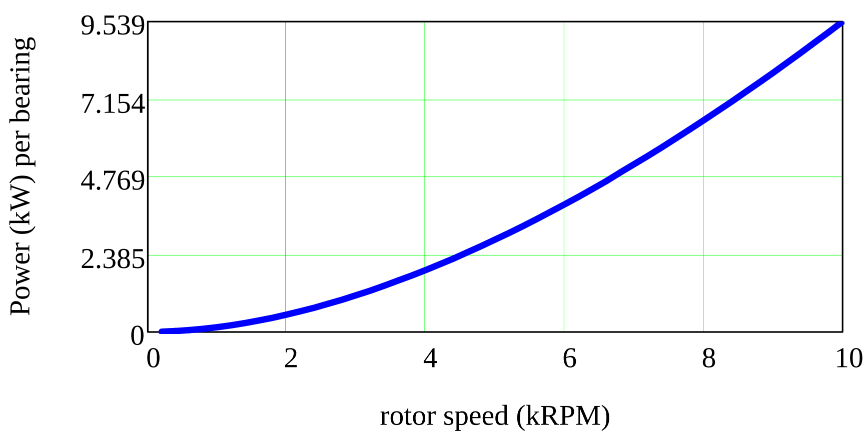
ϕ



$W = 250\,lbf$

$c = 3 \times 10^{-3}\,in$

POWER LOSS (kW) on each bearing

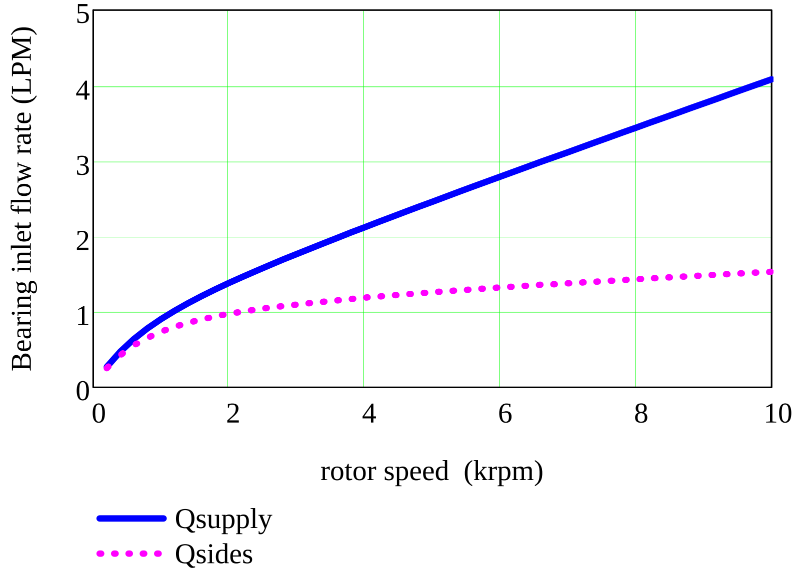


$\frac{W}{L \cdot D} = 20.833\,psi$

$L = 2\,in$

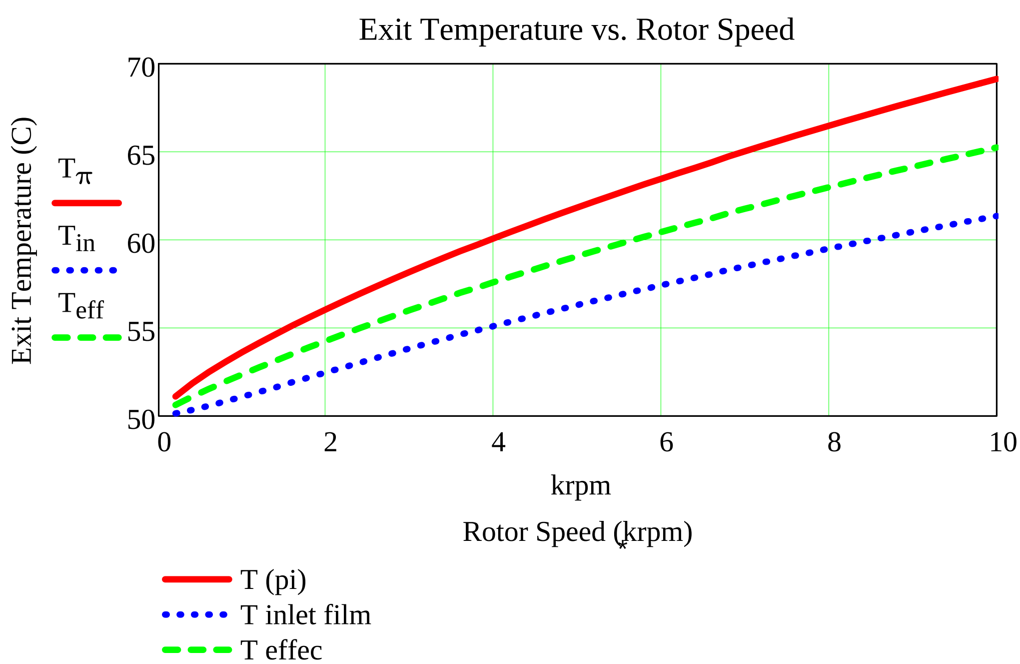
$D = 6\,in$

LUBRICANT FLOW RATE (LPM)



FILM temperatures: effective, inlet to film, exit at 180deg

$T_{supply} = 50\,K$

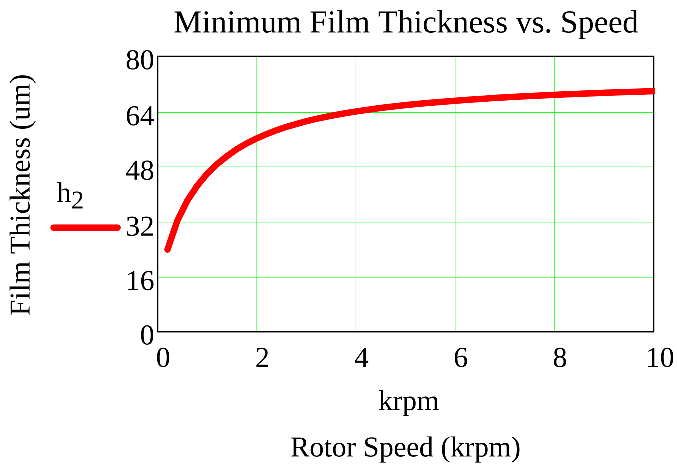


OTHER BEARING OPERATING PARAMETERS

Minimum Film Thickness

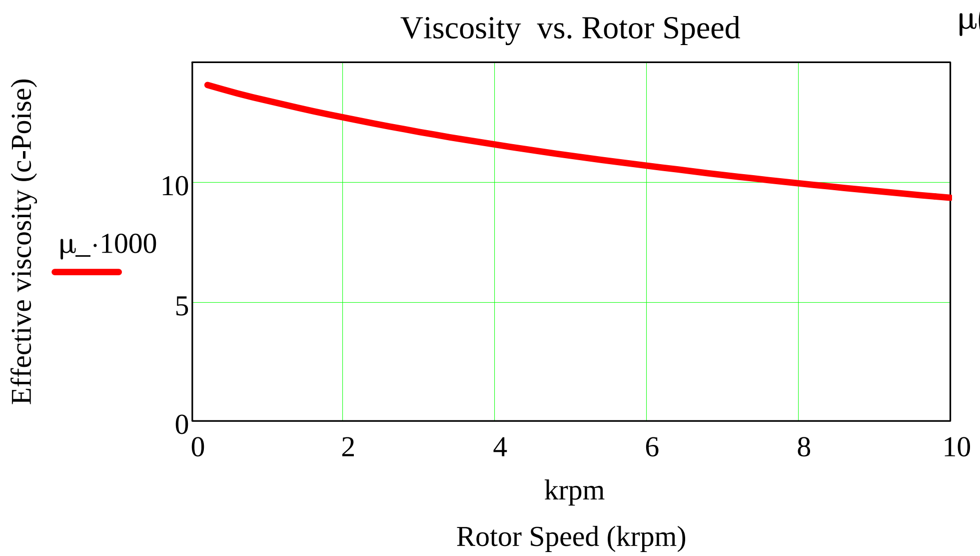
$$h_{2_n} := \left[c\cdot\left(1-\varepsilon_n\right)\right]\cdot 10^6$$

$c = 0.076\,mm$



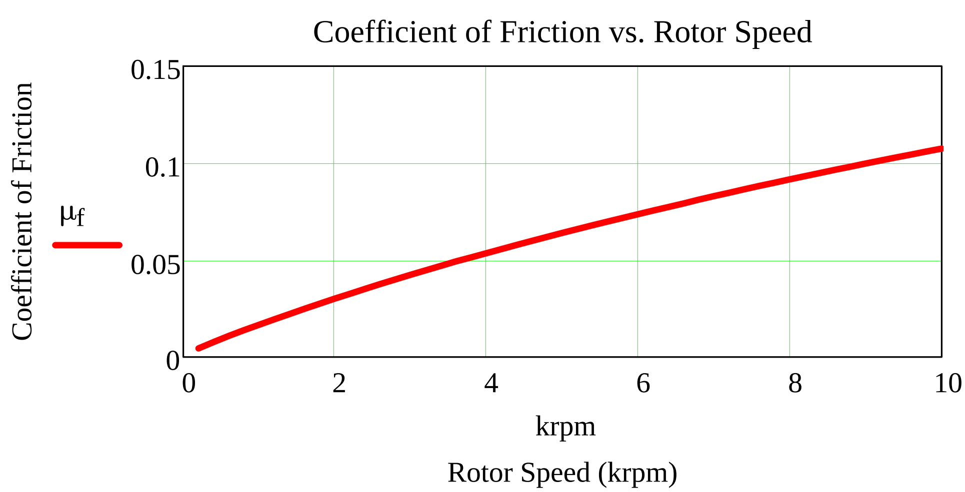
Effective viscosity (C-Poise)

$\mu\left(T_{supply}\right) = 0.014\,N\cdot\dfrac{s}{m^2}$



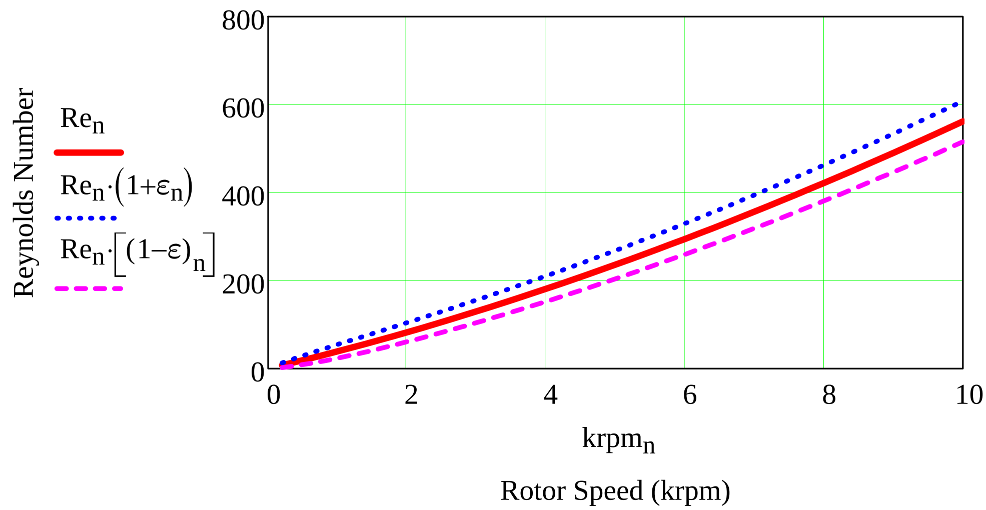
Friction Coefficient

$$\mu_{f_n} := \frac{Power_n}{\Omega_n\cdot R\cdot W}$$



Reynolds Number

$$Re_n := \rho \cdot \Omega_n \cdot R \cdot \frac{c}{\mu\_n}$$



Force coefficients evaluation

Short length journal bearing: rotordynamic force coefficients at equilibrium position

$$f_{ro_n} := \frac{4 \cdot \sigma_n \cdot (\varepsilon_n)^2}{\left[1-(\varepsilon_n)^2\right]^2} \qquad f_{to_n} := \frac{\pi \cdot \sigma_n \cdot \varepsilon_n}{\left[1-(\varepsilon_n)^2\right]^{1.5}}$$

Dimensionless Stiffness

$$k_{xx_n} := \frac{f_{ro_n}}{\varepsilon_n \cdot \left[1-(\varepsilon_n)^2\right]} \cdot \left[\left(f_{ro_n}\right)^2 + 1 + 2 \cdot (\varepsilon_n)^2\right]$$

$$k_{yy_n} := \frac{f_{ro_n}}{\varepsilon_n \cdot \left[1-(\varepsilon_n)^2\right]} \cdot \left[\left(f_{to_n}\right)^2 + 1 - (\varepsilon_n)^2\right]$$

$$k_{yx_n} := \frac{f_{to_n}}{\varepsilon_n \cdot \left[1-(\varepsilon_n)^2\right]} \cdot \left[\left(f_{ro_n}\right)^2 - 1 + (\varepsilon_n)^2\right]$$

$$k_{xy_n} := \frac{f_{to_n}}{\varepsilon_n \cdot \left[1-(\varepsilon_n)^2\right]} \cdot \left[\left(f_{ro_n}\right)^2 + 1 + 2 \cdot (\varepsilon_n)^2\right]$$

Dimensional Stiffness (Fo=W)

$$F_{o_n} := \frac{\mu\_n \cdot \Omega_n \cdot \left(\frac{L}{c}\right)^2 \cdot L \cdot R}{4 \cdot \sigma_n}$$

$$K_{xx_n} := k_{xx_n} \cdot \frac{F_{o_n}}{c}$$

$$K_{yy_n} := k_{yy_n} \cdot \frac{F_{o_n}}{c}$$

$$K_{yx_n} := k_{yx_n} \cdot \frac{F_{o_n}}{c}$$

$$K_{xy_n} := k_{xy_n} \cdot \frac{F_{o_n}}{c}$$

MATRIX of STIFFNESSES

$$K_{b_n} := \begin{pmatrix} K_{xx_n} & K_{xy_n} \\ K_{yx_n} & K_{yy_n} \end{pmatrix}$$

Dimensionless Damping

$$c_{xx_n} := \frac{2 \cdot f_{to_n}}{\varepsilon_n \cdot \left[1 - (\varepsilon_n)^2\right]} \cdot \left[\left(f_{ro_n}\right)^2 \cdot \left[2 + (\varepsilon_n)^2\right] + 1 - (\varepsilon_n)^2\right]$$

$$c_{yy_n} := \frac{2 \cdot f_{to_n}}{\varepsilon_n \cdot \left[1 - (\varepsilon_n)^2\right]} \cdot \left[\left(f_{to_n}\right)^2 \cdot \left[2 + (\varepsilon_n)^2\right] - 1 + (\varepsilon_n)^2\right]$$

$$c_{xy_n} := \frac{2 \cdot f_{ro_n}}{\varepsilon_n \cdot \left[1 - (\varepsilon_n)^2\right]} \cdot \left[\left(f_{to_n}\right)^2 \cdot \left[2 + (\varepsilon_n)^2\right] - 1 + (\varepsilon_n)^2\right]$$

$$c_{yx_n} := c_{xy_n}$$

Dimensional Damping Coefficients

$$C_{xx_n} := c_{xx_n} \cdot \frac{F_{o_n}}{c \cdot \Omega_n}$$

$$C_{yy_n} := c_{yy_n} \cdot \frac{F_{o_n}}{c \cdot \Omega_n}$$

$$C_{xy_n} := c_{xy_n} \cdot \frac{F_{o_n}}{c \cdot \Omega_n}$$

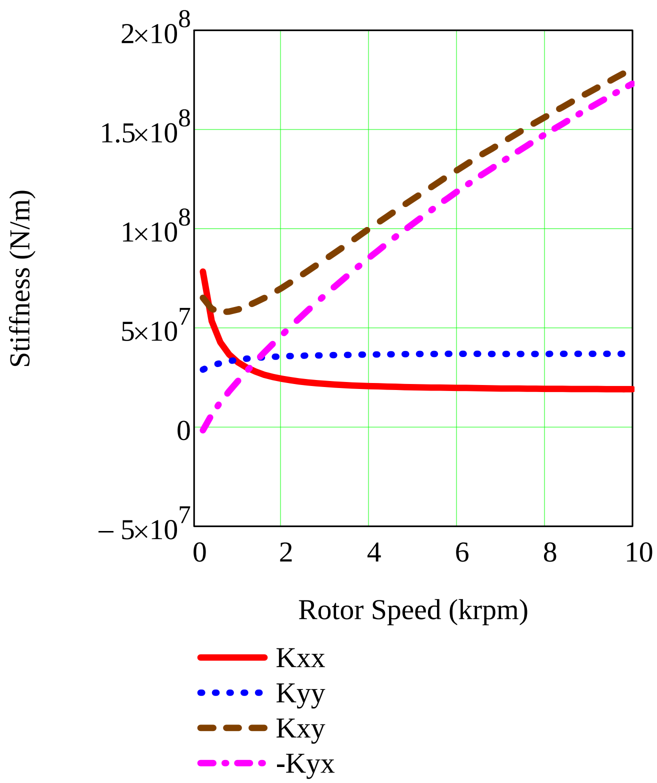
$$C_{yx_n} := C_{xy_n}$$

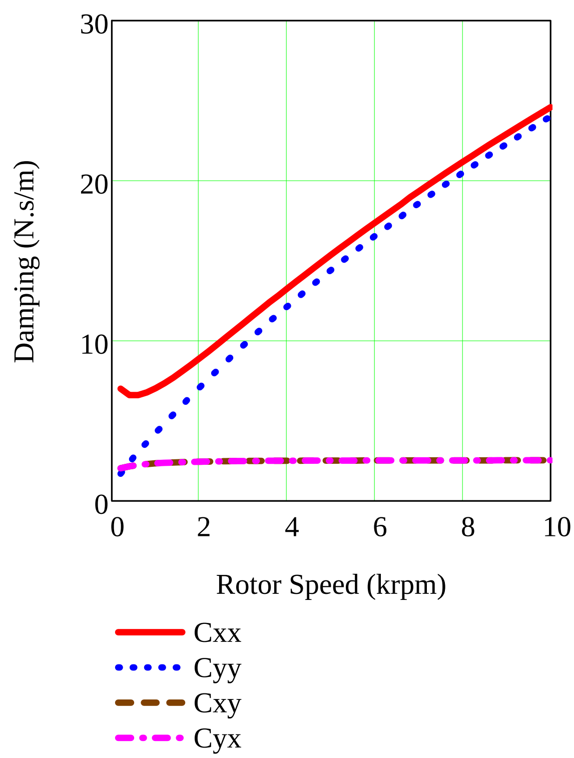
MATRIX OF DAMPING COEFFS

$$C_{b_n} := \begin{pmatrix} C_{xx_n} & C_{xy_n} \\ C_{yx_n} & C_{yy_n} \end{pmatrix}$$

Force coefficients evaluation

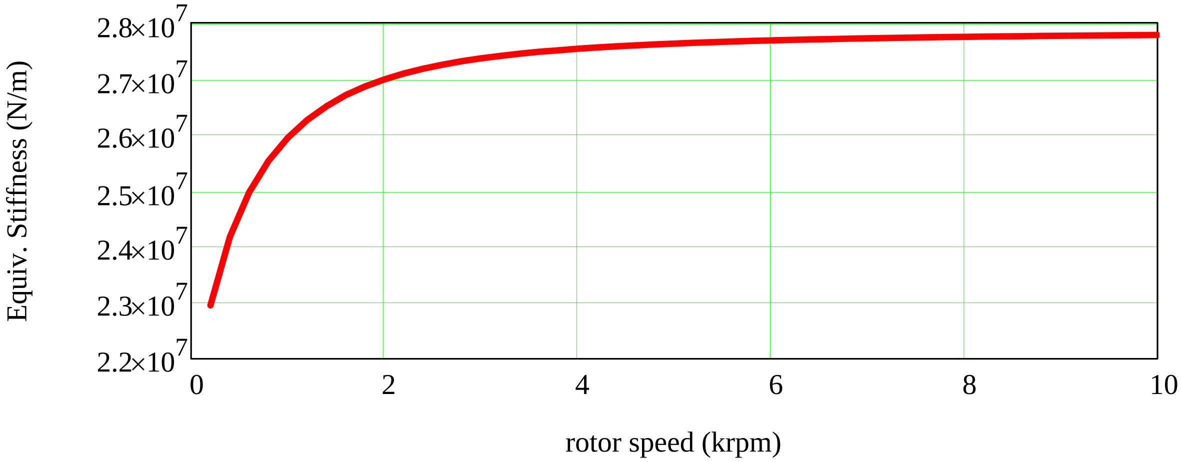
STIFFNESS AND DAMPING COEFFICIENTS





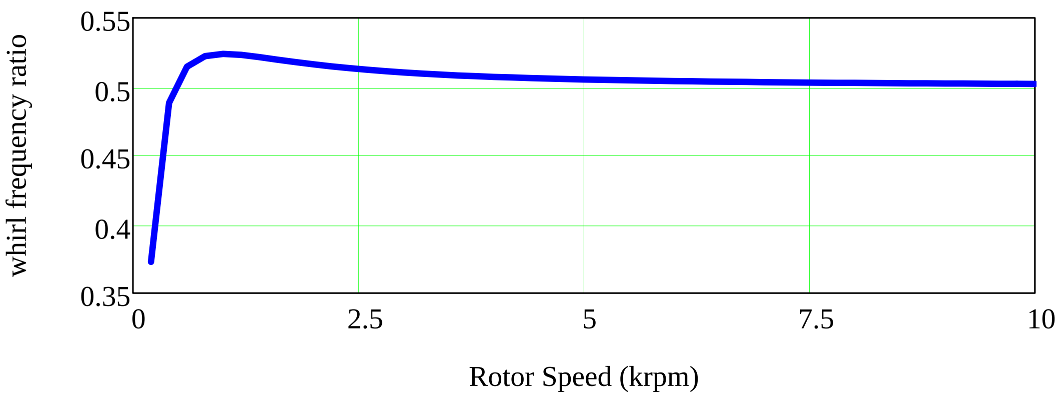
Equivalent bearing stiffness for rigid rotor

$$k_{eq_n} := \frac{k_{xx_n} \cdot c_{yy_n} + k_{yy_n} \cdot c_{xx_n} - c_{yx_n} \cdot k_{xy_n} - c_{xy_n} \cdot k_{yx_n}}{c_{xx_n} + c_{yy_n}}$$



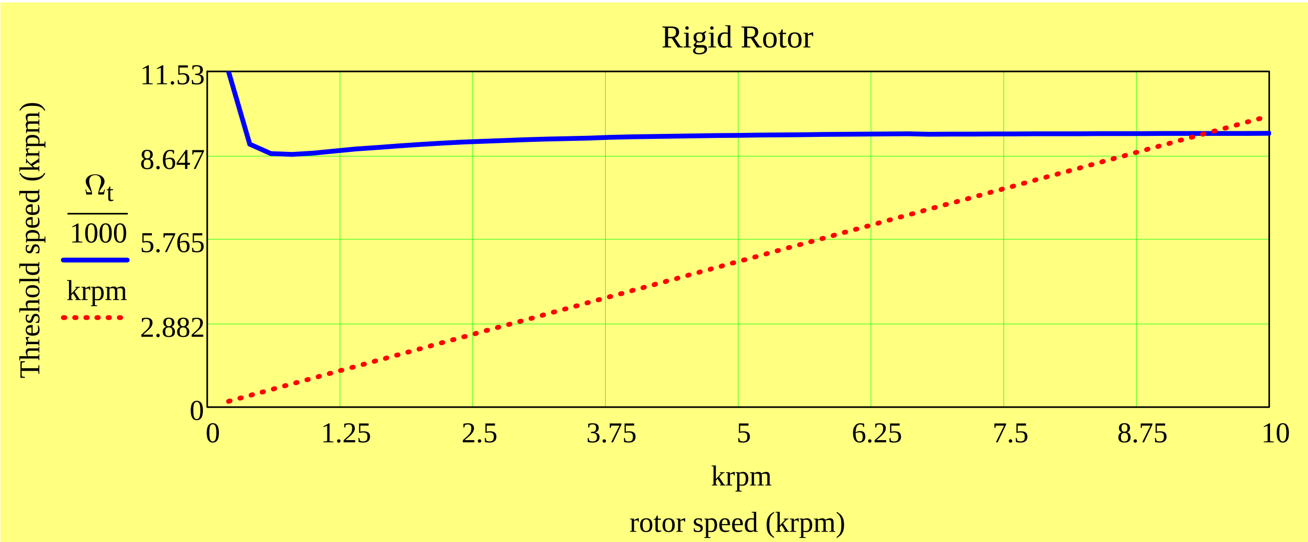
Whirl frequency ratio

$$WFR_n := \sqrt{\frac{\left(k_{eq_n} - k_{xx_n}\right) \cdot \left(k_{eq_n} - k_{yy_n}\right) - k_{xy_n} \cdot k_{yx_n}}{c_{xx_n} \cdot c_{yy_n} - c_{xy_n} \cdot c_{yx_n}}}$$



Threshold speed of instability(krpm)

$$\Omega_{t_n} := \sqrt{\frac{k_{eq_n} \cdot F_{o_n}}{M \cdot c}} \cdot \frac{\left(\frac{30}{\pi}\right)}{WFR_n}$$

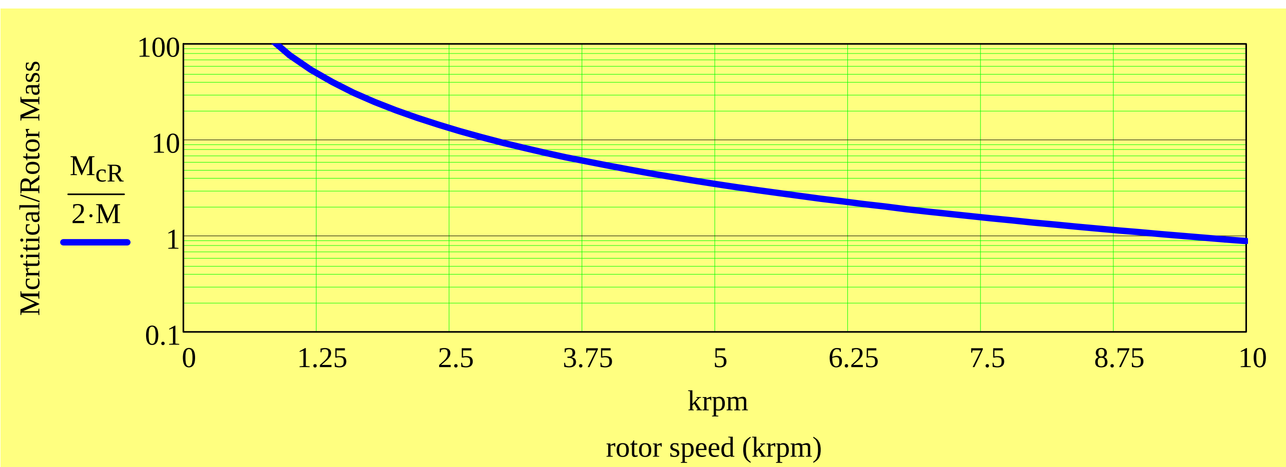


Critical Rotor Mass for rigid rotor

$$M_{c_n} := \frac{k_{eq_n} \cdot F_{o_n}}{c \cdot (\Omega_n)^2 \cdot (WFR_n)^2}$$

Recall the rotor mass: $2 \cdot M = 226.796\,kg$

$$M_{cR_n} := 2 \cdot M_{c_n}$$



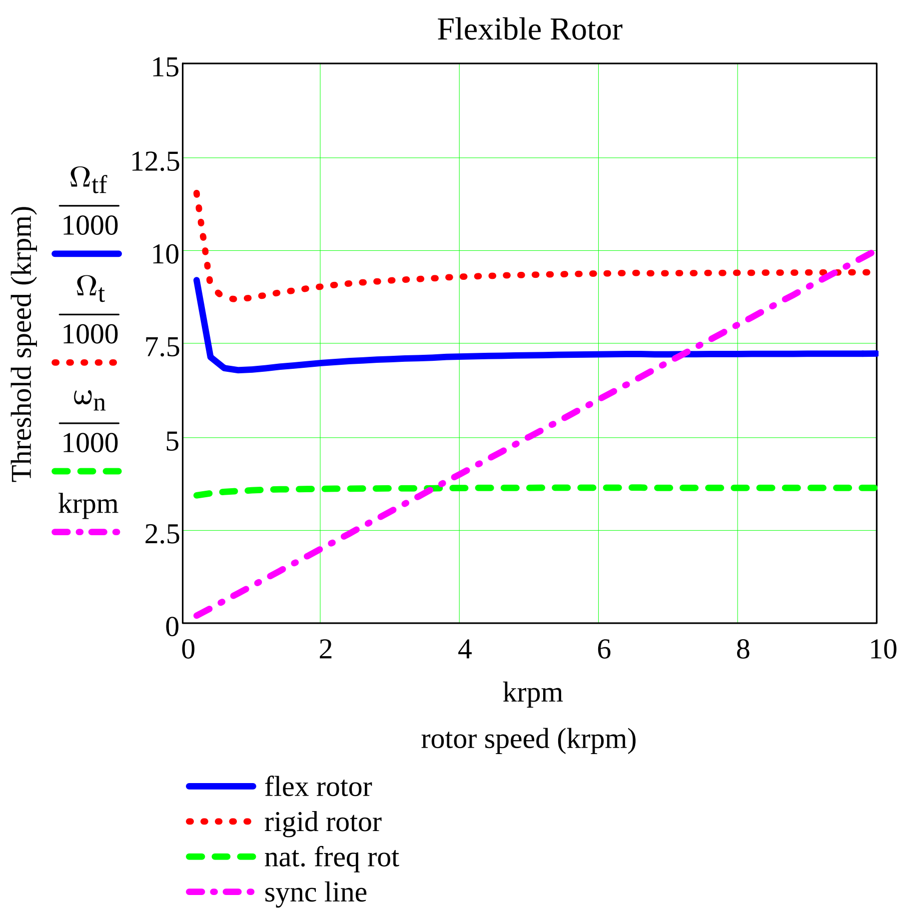
Effect of shaft flexibility on threshold speed of instability:

Threshold speed of instability calculated including rotor sag at midspan.

$$\Omega_{tf_n} := \frac{\Omega_{t_n}}{\left(1 + k_{eq_n} \cdot \frac{Rotor\_sag}{c}\right)^{\frac{1}{2}}}$$

Natural Frequency (rpm) of ***Flexible*** Shaft

$$\omega_{n_n} := WFR_n \cdot \Omega_{tf_n}$$



$$k_{shaft} = 4 \times 10^7 \frac{N}{m}$$

$$\frac{Rotor\_sag}{c} = 0.365$$

The threshold speed of instability is lower for the flexible rotor than for the rigid rotor model.

Synchronous response of flexible rotor due to imbalance

The equations of motion for both rotor and journal bearings are given below. The coordinates of rotor and disk motion have origin at the static equilibrium position.

Let: $k := k_{shaft}$

ROTOR

$$m\cdot\left(\frac{d^2\cdot X}{dt^2}\right) + k\cdot(X - x) = m\cdot a\cdot\omega^2\cdot\cos(\omega\cdot t)$$

$$m\cdot\left(\frac{d^2\cdot Y}{dt^2}\right) + k\cdot(Y - y) = m\cdot a\cdot\omega^2\cdot\sin(\omega\cdot t)$$

MASSLESS BEARINGS

$$\left(C_{I,J}\right)\cdot\begin{pmatrix}\frac{dx}{dt}\\ \frac{dy}{dt}\end{pmatrix} + \left(K_{I,J}\right)\cdot\begin{pmatrix}x\\ y\end{pmatrix} = -k\cdot\begin{pmatrix}x - X\\ y - Y\end{pmatrix}$$

$a = 1.524\times10^{-5}\,m$ **imbalance displacement** $\frac{a}{c} = 0.2$

The rotor disk (X,Y) and journal center displacements (x,y) are synchronous with the imbalance excitation, i.e.

$$X = X_c\cdot\cos(\omega\cdot t) + X_s\cdot\sin(\omega\cdot t) \qquad Y = Y_c\cdot\cos(\omega\cdot t) + Y_s\cdot\sin(\omega\cdot t)$$

$$x = x_c\cdot\cos(\omega\cdot t) + x_s\cdot\sin(\omega\cdot t) \qquad y = y_c\cdot\cos(\omega\cdot t) + y_s\cdot\sin(\omega\cdot t)$$

Find the solution.

Define UNIT matrix $I_2 := \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

Rotor (complex) displacements

$$\begin{pmatrix}X_n\\ Y_n\end{pmatrix} := \left[-\left(\Omega_n\right)^2\cdot M\cdot I_2 + \left[\left[\left(C_{b_n}\cdot i\cdot\Omega_n\right) + K_{b_n}\right]^{-1} + \frac{1}{k_{shaft}}\cdot I_2\right]^{-1}\right]^{-1}\cdot\begin{bmatrix}M\cdot a\cdot\left(\Omega_n\right)^2\\ -i\cdot\left[M\cdot a\cdot\left(\Omega_n\right)^2\right]\end{bmatrix}$$

Journal (complex) displacements

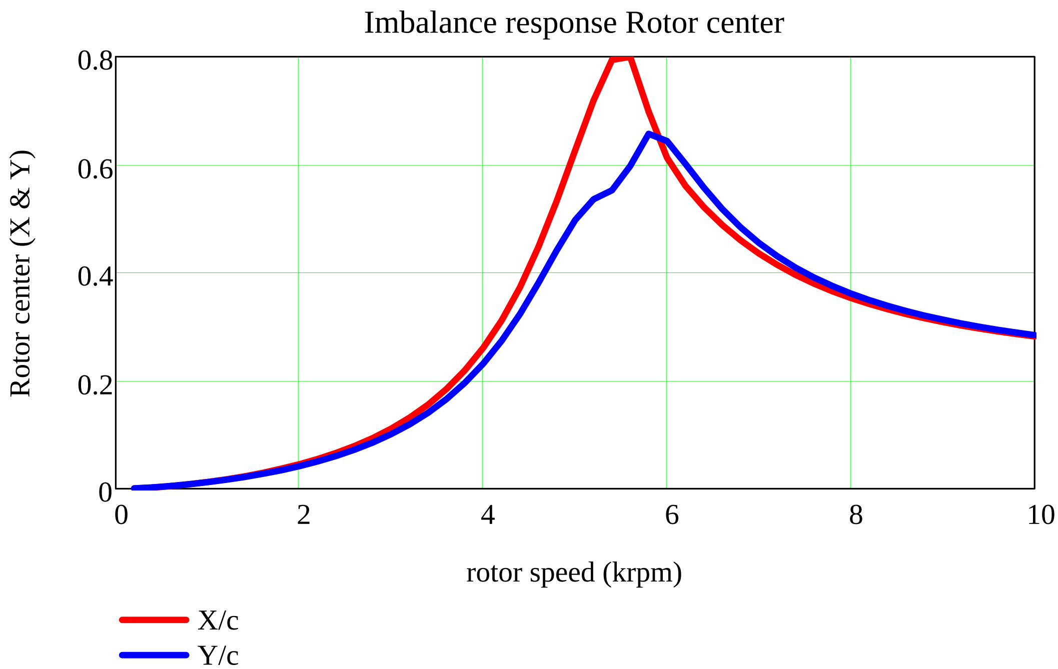
$$\begin{pmatrix}x_n\\ y_n\end{pmatrix} := \left[k_{shaft}\cdot I_2 + \left[\left(C_{b_n}\cdot i\cdot\Omega_n\right) + K_{b_n}\right]\right]^{-1}\cdot k_{shaft}\cdot\begin{pmatrix}X_n\\ Y_n\end{pmatrix}$$

Amplitudes of rotor and journal motion

$$X_{m_n} := \left|X_n\right| \qquad Y_{m_n} := \left|Y_n\right|$$

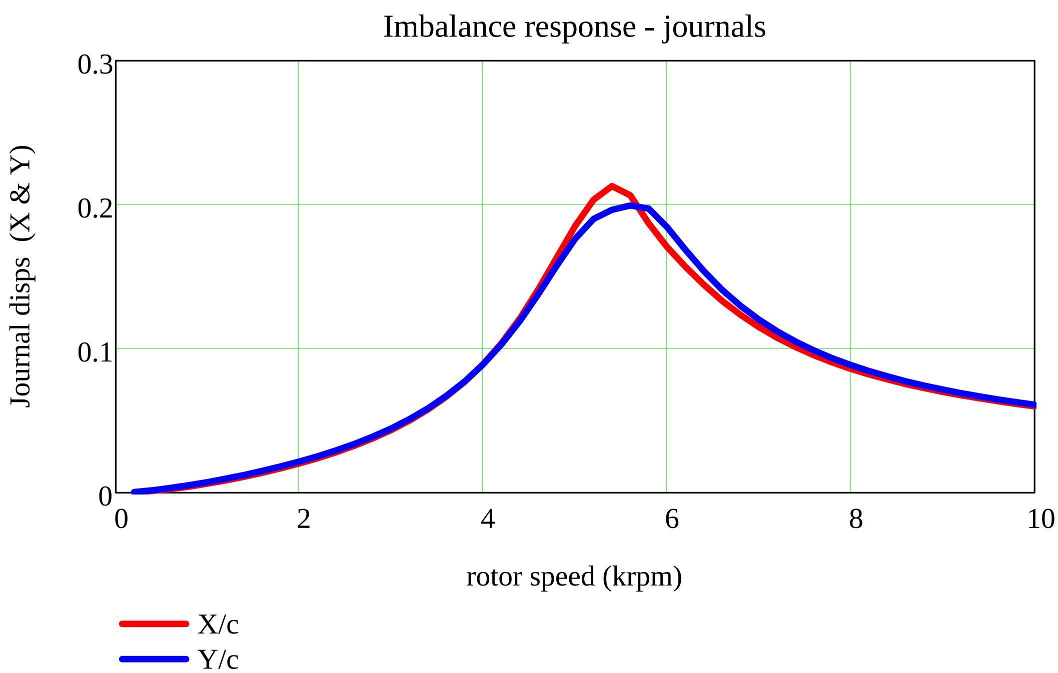
$x_{m_n} := |x_n| \qquad y_{m_n} := |y_n|$

show responses in dimensionless form (amplitude/clearance)



ROTOR CENTER

$\frac{a}{c} = 0.2$



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Exercise: Calculate the major and minor axes of the ellipses describing the (X,Y) motions. See Appendix A of Childs' Rotordynamics Book:.