

## Semi-Empirical Screw Compressor Chiller Model

Ian C. Nelson  
Graduate Student  
Texas A&M University  
College Station, TX USA

Charles H. Culp, Ph.D., P.E.,  
FASHRAE, LEED-AP  
Associate Professor,  
Department of Architecture  
Associate Director,  
Energy Systems Laboratory  
College Station, TX USA

Rhett D. Graves, P.E.  
Research Associate II, Department  
of Mechanical Engineering  
Mississippi State University  
Starkville, MS USA

### ABSTRACT

A screw chiller model which is based on a first principles, semi-empirical analysis that describes the system performance based on observations of the thermodynamic processes is developed. This model is a modified method to empirically derive the system irreversibilities of the Gordon-Ng first principles approach and is applied to a screw chiller. The irreversibilities were combined into one effect based on the refrigerant temperature difference between the condenser and evaporator and the water temperature difference between the outlet and inlet of the evaporator. The required measured parameters of the model are the evaporator water inlet and outlet

temperatures and mass flow, the refrigerant temperature in the condenser and evaporator, and the input power. The similarity in the results generated by the Gordon-Ng model and the semi-empirical model effectively verified both calculation methods. The semi-empirical chiller model simulated outputs were the input power, coefficient of performance and kW/ton with an accuracy of 4-7% when compared to the 4,104 data points acquired from a screw chiller system. The semi-empirical chiller model results were also compared to the Gordon-Ng model predictions as a function of the percentage chiller loading.

### NOMENCLATURE

$c_p$	specific heat of the fluid
COP	coefficient of performance
$\varepsilon$	effectiveness of the heat exchanger
$\dot{m}$	mass flow
$P_{in}$	total electrical power
$Q_{comp}^{leak}$	compressor heat leak
$Q_{cond}$	heat transfer (refrigerant to water) in the condenser
$Q_{cond}^{leak}$	condenser heat leak to the external environment
$Q_{evap}$	heat transfer (refrigerant to water) in the evaporator
$Q_{evap}^{leak}$	evaporator heat leak to the external environment
$T_{cond}^{ref}$	refrigerant temperature in the condenser
$T_{evap}^{ref}$	refrigerant temperature in the evaporator
$T_{cond}^{water,i}$	inlet temperature of the water in the condenser
$T_{evap}^{water,i}$	inlet temperature of the evaporator water
$\Delta S_{int}$	change in internal entropy generation

### INTRODUCTION

The Gordon-Ng chiller models include a fundamental (K.C. Ng 1997) model and quasi-empirical (J.M. Gordon 1995) chiller model. The models describe the

operation of chiller systems by applying first principles to each component of the chiller. In the Gordon-Ng fundamental model, the energy transfers between the black-box models of each component form the basis of

the model approach. The system performance curves are then used to empirically derive the parameters defined by the first principles approach. The empirical parameters of the Gordon-Ng fundamental chiller model represent measurable system quantities. Studies applying the Gordon-Ng chiller model to reciprocating (J.M Gordon 1997), centrifugal (J.M Gordon 1995), and absorption (J.M. Gordon 1995) chillers found the model to predict the chiller performance within 2-5% (W. Jiang 2003) of actual operational performance. This study presents a modification to the Gordon-Ng approach and shows the results of both the modified model and the Gordon-Ng model applied to a screw chiller.

A pure empirical model uses measurements to create a mathematical relationship, typically using regression techniques, that describes the performance or behavior of a system. Empirical models usually lack the capability of being applied to a variety of systems using a single set of parameters since the models were derived from data from a specific machine. A first principles study defines the physical relationships using the fundamental physics of a process and enables the development of a dominant parameter understanding. The model can then be extended to predict system performance for a wider spectrum of machines.

The Gordon-Ng fundamental model takes a first principles approach to derive the basic chiller model. Gordon-Ng then includes an empirical parameterization to customize the model. This type of model development utilizes a hybrid methodology. The system performance curves are used to empirically derive the parameters defined by the first principles approach. The empirical parameters of the Gordon-Ng fundamental chiller model represent measurable system quantities.

A chiller system consists of a compressor, condenser, expansion device, and evaporator. Compressor types

commonly found in chiller systems include reciprocating, scroll, centrifugal, and screw compressors. Screw compressors range in size between 50-300 ton capacities. This capacity range represents a very large demographic in building size, and over the years screw chillers have become increasingly common. Existing empirical models (B. Solati 2003), (C.V. Le 2005), (L. Fu 2002) characterize the operation of screw chillers based solely on regression of measured parameters. Although these existing models do effectively model the operation of specific chillers, empirical models are difficult to generalize. Modeling a different system requires additional data sets of measured parameters.

The semi-empirical approach uses first principles and then empirically models specific parameters that cannot be generalized. The application of empirically derived parameters can directly model system irreversibilities. The modification of the Gordon-Ng fundamental model applied to a screw chiller in this study used a first principles approach to define the system with an empirical relationship for the system losses. This semi-empirical approach produced a model of the screw chiller system with 4-7% agreement in calculated performance to measured data.

The calculation procedure of the Gordon-Ng chiller model is shown in Figure 1. Gordon and Ng developed their model from a three-parameter multi-linear regression based on the inlet and outlet water temperatures in the evaporator, thermal capacitance of the evaporator, inlet water temperature in the condenser, and the chiller COP. The model used in this study was a two-parameter multi-linear regression based on the water temperature difference from the inlet to outlet in the evaporator and the difference in refrigerant temperatures between the evaporator and condenser, illustrated in Figure 2.

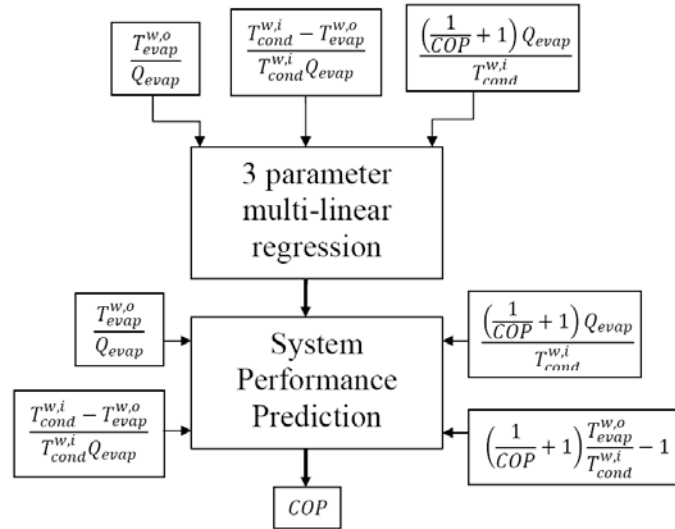


Figure 1. Gordon-Ng three-parameter chiller model calculation procedure.

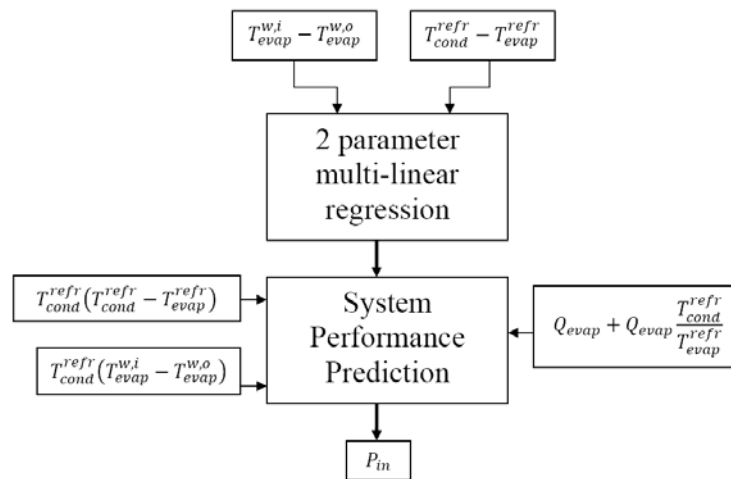


Figure 2. Two-parameter screw chiller model calculation procedure.

### SCREW CHILLER LOSSES

H.T Chua et al (H.T. Chua 1996) showed that the irreversible losses, component and cycle losses, must be included in chiller modeling. Irreversible losses in vapor compression systems are due to the compressor, the heat exchangers and the connecting pipes. Compressor operation results in entropy generation categorized as internal loss, independent from external losses. Irreversibilities are primarily caused by friction in centrifugal and reciprocating compressors. Screw compressors are unique since they also generate entropy by way of refrigerant leakage through the clearance between the rotors [10]. The loss increases with an increasing pressure ratio due to an increase in the refrigerant temperature difference between the condenser and evaporator and

distinguishes screw compressor loss analysis from reciprocating and centrifugal compressors.

The losses within the heat exchangers result from a temperature gradient between the thermal reservoirs and a pressure drop through the heat exchangers. The second law of thermodynamics can be applied to the heat exchangers to calculate the amount of loss associated with the difference between the two reservoirs. The losses due to pressure drop are small and can be neglected. Additionally, the piping and tubing connecting the chiller components provide opportunity for heat leaks into and out of the system. The mechanisms responsible for entropy generation in the chiller components are shown in Figure 3.

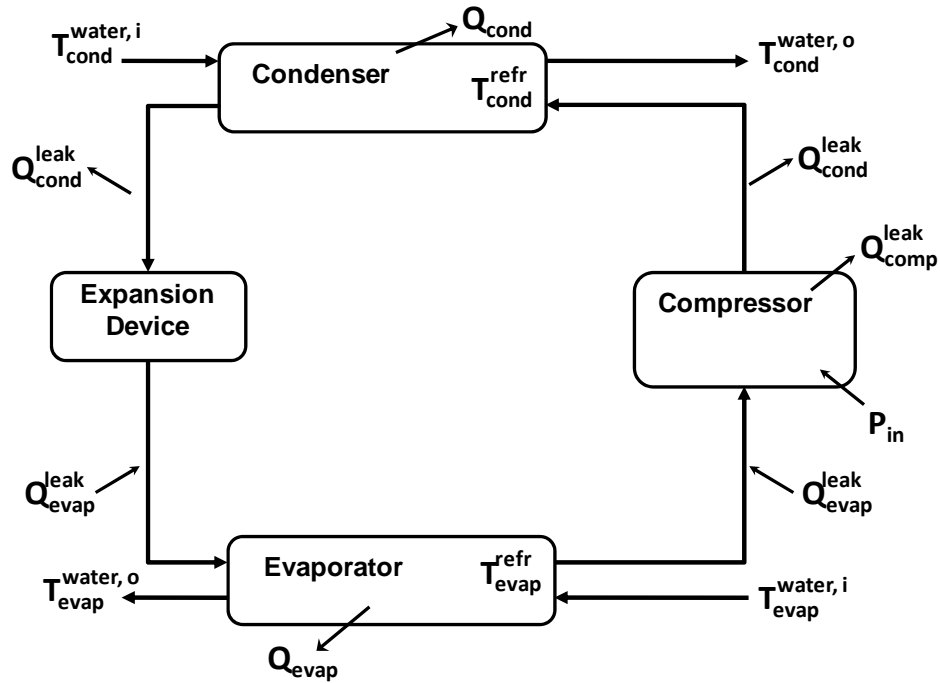


Figure 3. Chiller system components with heat transfers, temperatures and power input shown.

### CHILLER MODEL DEVELOPMENT

The development of the chiller model in this section applies the approach of the Gordon-Ng fundamental model with modification for a screw chiller. The screw chiller model development began with the application of the first and second law of thermodynamics to the chiller system as shown in Equations 1 and 2, respectively,

$$\Delta E = 0 = Q_{cond} + Q_{cond}^{leak} - Q_{evap} - Q_{evap}^{leak} - P_{in} + Q_{comp}^{leak} \quad (1)$$

$$\Delta S = 0 = \left( \frac{Q_{cond} + Q_{cond}^{leak}}{T_{cond}^{refr}} \right) - \left( \frac{Q_{evap} + Q_{evap}^{leak}}{T_{evap}^{refr}} \right) - \Delta S_{internal} \quad (2)$$

Combining Equations 1 and 2 and solving for the input power yielded Equation 3,

$$P_{in} = Q_{evap} \left( \frac{T_{cond}^{refr}}{T_{evap}^{refr}} - 1 \right) + Q_{evap}^{leak} \left( \frac{T_{cond}^{refr}}{T_{evap}^{refr}} - 1 \right) + T_{cond}^{refr} \Delta S_{internal} + Q_{comp}^{leak} \quad (3)$$

The coefficient of performance (COP) is defined in Equation 4 as,

$$COP = \frac{Q_{evap}}{P_{in}} \quad (4)$$

Inserting Equation 4 into Equation 3 yielded Equation 5,

$$\frac{1}{COP} = -1 + \frac{T_{cond}^{refr}}{T_{evap}^{refr}} + \frac{T_{cond}^{refr} \Delta S_{internal}}{Q_{evap}} + \frac{T_{cond}^{refr} \Delta S_{leak}}{Q_{evap}} \quad (5)$$

where  $\Delta S_{leak}$  is defined in Equation 6 as,

$$\Delta S_{leak} = \frac{Q_{comp}^{leak}}{T_{cond}^{refr}} + Q_{evap} \left( \frac{1}{T_{evap}^{refr}} - \frac{1}{T_{cond}^{refr}} \right) \quad (6)$$

Equation 5 characterizes the performance of a chiller in terms of the condenser and evaporator temperatures, the heat transfer through the evaporator, the internal entropy production, and the entropy associated with heat leaks. The equation was modified in order to calculate the COP as a function of measurable parameters. Using the heat transfer in the evaporator and condenser and the conservation of energy approach to replace the refrigerant temperatures in the condenser and evaporator,  $Q_{cond}$  (Equation 7) and  $Q_{evap}$  (Equation 8) were defined.

$$Q_{cond} = (\dot{m} c_p \varepsilon)_{cond} (T_{cond}^{refr} - T_{cond}^{water,i}) \quad (7)$$

$$Q_{evap} = (\dot{m} c_p \varepsilon)_{evap} (T_{evap}^{water,i} - T_{evap}^{refr}) \quad (8)$$

Substituting into Equation 5, chiller performance was expressed in terms of both the coolant and refrigerant temperatures which yielded Equation 9,

$$\frac{1}{COP} = -1 + \frac{T_{cond}^{refr}}{T_{evap}^{refr}} + \frac{T_{cond}^{refr} \Delta S_{internal}}{(\dot{m}c_p)_{evap} (T_{evap}^{water,i} - T_{evap}^{water,o})} + \frac{T_{cond}^{refr} \Delta S_{leak}}{Q_{evap}} \quad (9)$$

In terms of the input power, Equation 9 can be transformed to Equation 10,

$$P_{in} = -Q_{evap} + \frac{Q_{evap} T_{cond}^{refr}}{T_{evap}^{refr}} + T_{cond}^{refr} \Delta S_{internal} + T_{cond}^{refr} \Delta S_{leak} \quad (10)$$

Combining the entropy terms yielded Equation 11,

$$P_{in} = -Q_{evap} + \frac{Q_{evap} T_{cond}^{refr}}{T_{evap}^{refr}} + T_{cond}^{refr} \Delta S_{total} \quad (11)$$

Replacing  $Q_{evap}$  in Equation 11 and solving for the total loss yielded Equation 12,

$$\Delta S_{total} = \frac{P_{in}}{T_{cond}^{refr}} + \frac{(\dot{m}c_p)_{evap} (T_{evap}^{water,i} - T_{evap}^{water,o})}{T_{evap}^{refr}} - \frac{(\dot{m}c_p)_{evap} (T_{evap}^{water,i} - T_{evap}^{water,o}) T_{cond}^{refr}}{T_{evap}^{refr}} \quad (12)$$

The two-parameter model in this study used the difference in water temperatures in the evaporator and the difference in refrigerant temperatures between the evaporator and condenser. The refrigerant temperature difference was used to evaluate the entropy change due to the pressure change across the compressor. Since the evaporation and condensation in the heat exchangers occur at nearly constant pressure and temperature, the refrigerant condensation and evaporation temperatures correlate with the refrigerant condensation and evaporation pressures, respectively. The difference in refrigerant pressures between the evaporator and condenser drive the leakage through the rotors, effecting the total entropy change in the compressor.

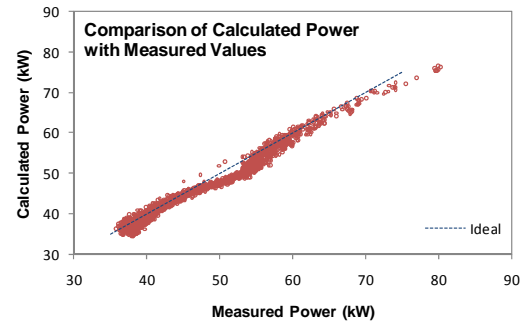
The difference in the inlet and outlet water temperatures of the evaporator correlates with the effect on the total entropy change due to the thermal reservoirs. Since a change in the log mean temperature difference (LMTD) changes the effectiveness of the heat exchanger, the difference in chilled water entering and leaving temperatures also affects the total entropy change, which was included in the two-parameter model.

## RESULTS

Equation 12 is a first principles derivation of the total entropy change in the system. Using a measured data set for a 200-ton screw machine, the total entropy change based on measured system characteristics was calculated which used the refrigerant temperatures in the evaporator and condenser, and the inlet and outlet water temperature in the evaporator. A multivariate regression analysis yielded Equation 13,

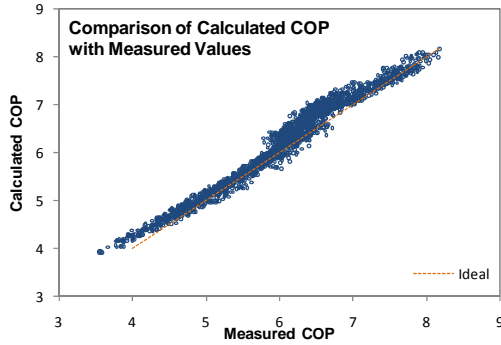
$$\Delta S_{total} = 0.0001608(T_{evap}^{refr} - T_{cond}^{refr}) + 0.00176(T_{evap}^{w-out} - T_{evap}^{w-in}) + 0.05605 \quad (13)$$

By equating the two-parameter total system loss with the total system loss derived from the fundamental model, the modeled input power of the system was derived and  $\Delta S_{total}$  was replaced with the above expression in Equation 11. The calculated power of the system using the parameterized change in total entropy as a function of the measured power is shown in Figure 4. The dashed line represents the ideal relationship between the simulated and observed power. At values less than 50 kW, the simulated results lie within 5.5-10.9%, and above 55 kW between 4.4-7.0% with a CV(RMSE) of 3.9%.

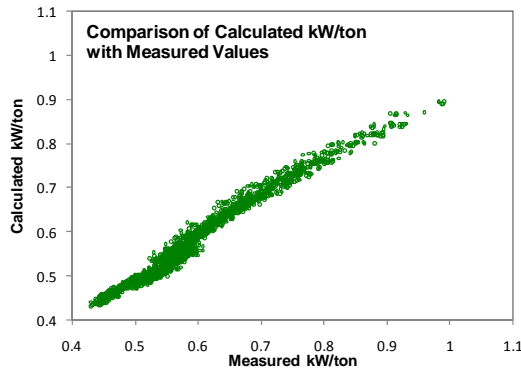


**Figure 4.** Input power measured and calculated data of screw chiller with parameterized model fit.

The calculated COP compared to measured COP values provided similar results as shown in Figure 5. The calculated and measured values are within 6.7-10.0% for a COP below 6, and between 7.2-9.4% for a COP above 6. The calculated results yielded a CV of 3.8% and compare to the results found by Jiang and Reddy (W. Jiang 2003) with an average CV among the screw chillers of 2.1%. The comparison of the kW/ton calculated with measured data is shown in Figure 6 and has a CV of 4.0%.

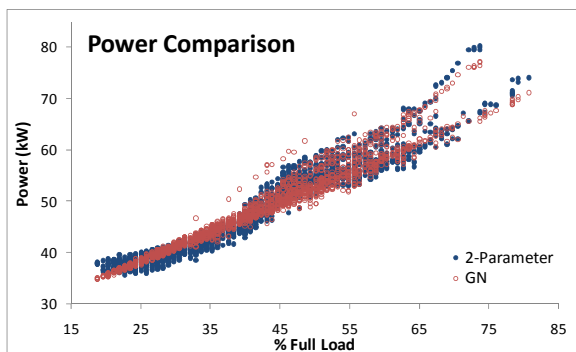


**Figure 5.** Performance data of screw chiller with parameterized model fit.



**Figure 6.** kW/ton of screw chiller with parameterized model fit

The Gordon-Ng model was used to simulate the chiller performance for comparison with the results generated by the two-parameter model calculations. Three comparisons as a function of the percentage load of the chiller are presented below: the calculated power (Figure 7), the ratio of the input power to the total cooling capacity (Figure 8), and the coefficient of performance (Figure 9).

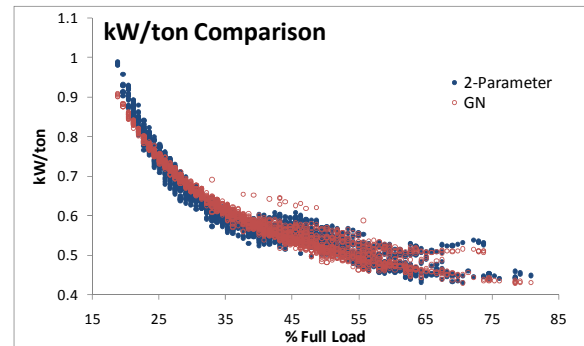


**Figure 7.** Power consumption comparison of 2-Parameter model with the Gordon-Ng model.

The simulations of both models are shown in Figure 7 and compare the power consumption. A pattern was seen in Figure 7 that is also present in the

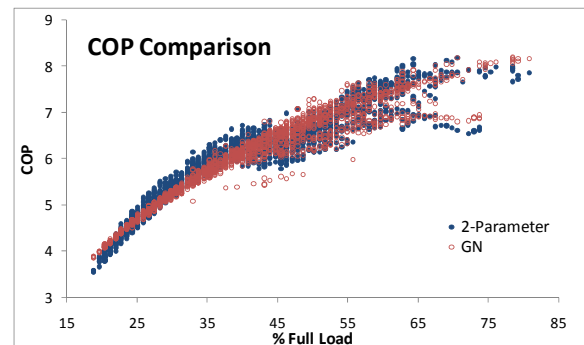
subsequent plots. As the chiller loading approaches 65% of full load, the power predictions from both models split into two different mappings.

This trend is also seen in Figure 8, which is a comparison of the cooling efficiency simulations for both models. Similarly, the predictions are almost identical and as the chiller loading approaches 65%, the mapping begins to split into two different patterns.



**Figure 8.** Cooling efficiency comparison of the 2-Parameter model with the Gordon-Ng model.

This split is more pronounced in Figure 9, with a difference of up to 15% between the two paths. A root cause study of this behavior determined that at compressor loads above 65%, the refrigerant temperature in the condenser followed two separate patterns with a deviation of almost 2%, or 5 °F difference at the same chiller loading. The evaporator refrigerant temperature did not show this same behavior. The likely explanation is that the cooling tower water outlet temperature was the parameter which caused the refrigerant condenser temperature to display this behavior.



**Figure 9.** COP comparison of the 2-Parameter model with the Gordon-Ng model.

The two-parameter modeling approach differs from the Gordon-Ng model as shown in Equation 11, combining the irreversibilities into one effect based

on the refrigerant temperature difference between the condenser and evaporator and the water temperature difference between the outlet and inlet of the evaporator. The Gordon-Ng approach continues to separate and distinguish the irreversibilities at this point in the derivation and develops three irreversibility parameters independent of the refrigerant temperatures.

The similarity in simulation results between both models verifies the calculation methods of the Gordon-Ng model and offers a more direct approach for modeling irreversibilities in a screw chiller with the semi-empirical model presented in this study.

### CONCLUSIONS

This study presents an approach to modeling a chiller system with application to a screw chiller. The model was based on a first principles, semi-empirical analysis that described the system performance based on observations of the thermodynamic processes. The two-parameter chiller simulation determined performance to an accuracy of 4-7% by using parameterized screw chiller data in terms of the difference between the inlet and outlet temperatures of the chilled water in the evaporator and the difference in refrigerant temperatures between the condenser and evaporator. The required measured parameters of the model include the evaporator water inlet and outlet temperatures and mass flow, the refrigerant temperature in the condenser and evaporator, and the input power. The similarity in the results generated by the Gordon-Ng model and the two-parameter model effectively verifies the calculation method of the Gordon-Ng approach.

### REFERENCES

B. Solati, R. Z., F. Haghghat (2003). "Correlation based models for the simulation of energy performance of screw chillers." Energy Conversion and Management **44**: 1903-1920.

- C.V. Le, P. K. B., J.D. Tedford (2005). "Simulation model of a screw liquid chiller for process industries using local heat transfer integration approach." Process Mechanical Engineering **219**(E): 95-107.
- H.T. Chua, K. C. N., J.M. Gordon (1996). "Experimental study of the fundamental properties of reciprocating chillers and their relation to thermodynamic modeling and chiller design." International Journal of Heat and Mass Transfer **39**(11): 2195-2204.
- J.M Gordon, K. C. N., H.T. Chua (1995). "Centrifugal chillers: thermodynamic modelling and a diagnostic case study." International Journal of Refrigeration **18**(4): 253-257.
- J.M Gordon, K. C. N., H.T. Chua (1997). "Optimizing chiller operation based on finite-time thermodynamics: universal modeling and experimental confirmation." International Journal of Refrigeration **20**(3): 191-200.
- J.M. Gordon, K. C. N. (1995). "Predictive and diagnostic aspects of a universal thermodynamic model for chillers." International Journal of Heat and Mass Transfer **38**(5): 807-818.
- K.C. Ng, H. T. C., W. Ong, S.S. Lee, J.M. Gordon (1997). "Diagnostics and optimization of reciprocating chillers: theory and experiment." Applied Thermal Engineering **17**(3): 263-276.
- L. Fu, G. D., Z. Su, G. Zhao (2002). "Steady-state simulation of screw liquid chillers." Applied Thermal Engineering **22**: 1731-1748.
- W. Jiang, T. A. R. (2003). "Reevaluation of the Gordon-Ng performance models for water-cooled chillers." ASHRAE Transactions: 272-287.