An On-line Self-tuning Algorithm of PI Controller for the Heating and Cooling Coil in Buildings

Q. Zhou and M. Liu
Energy Systems Laboratory
Texas Engineering Experiment Station

ABSTRACT

An on-line self-tuning algorithm of PI controller for the heating and cooling coil in buildings is described in the paper. The algorithm evaluates the controller performance by the integral square error (ISE) of the coil supply air temperature, and identifies the corrections of the P and I gains by observing the spectrum distribution of the ISE. This method is easy to implement in the HVAC energy control systems due to its simplicity. This paper describes the basic feature of the on-line self-tuning algorithm for typical PI controllers in the energy management system and presents the detail of the algorithm. Demonstration example are presented as well.

INTRODUCTION

To maintain suitable indoor comfort and maximize the energy efficiency of the heating and cooling system, optimal control has to be implemented. The single input and single output (SISO) proportional and integral (PI) controllers are often used to control chilled water valves and hot water valves of AHUs. To achieve the desired performance of an HVAC system, each controller has to perform properly, or the P and I gains must be set properly in the controller. These gains are often tuned in the factory or in the field when it is installed. Unfortunately, the P and I gains of the controller should be adjusted when the ambient conditions change. For example, the SISO controller for a chilled water valve may not work properly when the chilled water differential pressure across the valve is increased from 10 psi to 30 psi. Fine tuning P and I gains is essential for maintaining the HVAC system performance.

Automatic tuning procedures have been developed to solve the problem. However, most of this tuning is either performed at the beginning of the operation or off line. Therefore, poor control performance (oscillating and drifting) is often observed in the HVAC system even when the control systems are periodically fine tuned.

A new on-line fine tuning algorithm is developed in the paper. The algorithm evaluates the current controller performance by the ISE of difference between the output signal and its setpoint, and identifies the corrections of the P and I gains by observing the spectrum distribution of the ISE. Due to its simplicity, it is easy to implement in the energy management control system of the individual controller. This paper presents a brief review of the physical model of cooling and heating coils, the model of the control loops. Then, an on-line tune up algorithm is developed for the control loops in this paper to achieve real on-line and self tuning.

REVIEW

A coil (either heating or cooling) is equipped with a control valve on the return water side. The water flow is regulated by the control valve to maintain a certain supply air temperature (set-point). The input is the control signal received by the valve. The output is the supply air temperature. When the control valve is suddenly closed or opened (step function), the supply air temperature will change gradually. A stable condition will be achieved in a few minutes. Figure 1 presents the response of the supply air temperature when a step change is applied to the control valve.

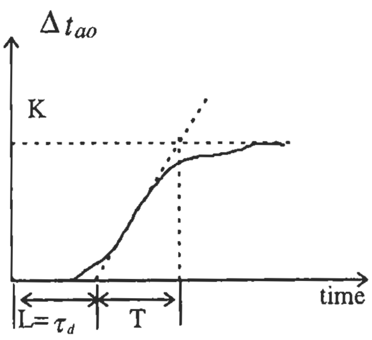

Figure 1: The process reaction curve

The coil model can be approximately as a first order system with delay and described by Equation 1 or 2 according to Ziegler:

$$\frac{\Delta t_{ao}}{\Delta C} = \frac{K_s e^{-\tau_d s}}{TS+1} \tag{1}$$

$$T\frac{d\Delta t_{ao}}{d\tau} + \Delta t_{ao} = K\Delta C|_{\tau=\tau-\tau_d} \tag{2}$$

Δt_{ao} is the supply air temperature change after the change of control valve position;
ΔC is the change of the control valve;
T is the time constant which is the time needed by the supply air temperature to achieve 63% of maximum change from the time when the change started;
τ_d is the delay time. The supply air temperature may stay unchanged after the control valve position is changed. The time between the valve position change and the supply air temperature change is approximately the time delay of the system.
K_s is the ratio of the supply air temperature change to the control valve position change. It is often called DC gain.

Equation 1 presents the coil model in a phase domain, where s is the variable. Equation 2 presents the coil model in time domain where time is the variable.

A controller must be designed and used to position the control valve in order to maintain the setpoint of the supply air. The typical controller is proportional and integral type (PI) since the coils have high inertia. The PI controller uses both the instant bias and integral bias from the set point to determine the new valve position. The propagation of instant bias is called proportional gain (P gain); the propagation of integral bias is called integral gain (I gain). P gain can be determined by Equation 3 based on coil characteristics: the time constant, delay time and DC gain while I gain can be determined by Equation 4 with a decay ratio of 0.25(Ziegler).

$$K_p = \frac{0.9T}{K\tau_d} \tag{3}$$

$$K_i = \frac{0.27T}{K{\tau_d}^2} \tag{4}$$

Figure 2 presents the system models of a coil with its controller. The controller measures the supply air temperature, calculates the bias using P gain, I gain and setpoint. A correction signal is sent out to the control valve to modulate the water flow rate.

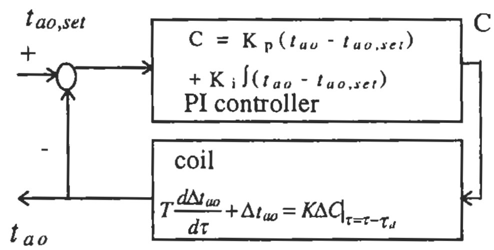

Figure 2: Cooling coil feedback control system

Different coils have different P and I gains. The gains also change with the valve position, the differential pressure across the valve and other parameters. For example, the P and I gain can change from 6.23 and 0.14 to 0.79 and 0.02 respectively for a typical cooling coil when the valve position is changed from full open to 50% open. When inappropriate P and I gains are used, the supply air temperature cannot be controlled properly. A typical problem is the supply air temperature fluctuates more than 10°F and creates comfort complaints. Figures 3 and 4 demonstrate the impacts of P and I gains on the supply air temperature for a typical coil.

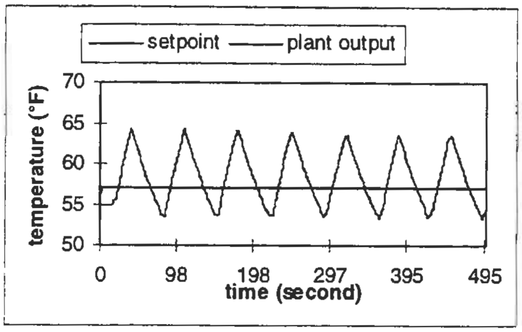

Figure 3: The step response of the system when inappropriate P and I gain are applied

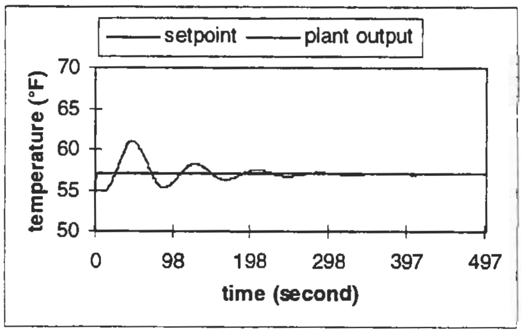

Figure 4: The step response of the system when appropriate P and I gain are applied.

Figure 3 shows that the discharge air temperature fluctuates more than 10°F around setpoint when the inappropriate P and I gains are applied. The setpoint is matched quickly in Figure 4 when appropriate P and I gain are applied. Therefore, the P and I gains have to be tuned in order to maintain the supply air temperature at setpoint.

The self-tuning method based on Equations 3 and 4 is used widely in industry (Wallenborg, Zhang, Franklin). However, the values of input and output of the system may not change much during the system's normal operation or the signals are corrupted by disturbance and noise. The time constant T, delay time τ_d and the constant K are not easy to determine for a certain coil. Wallenborg has proposed to apply an unit relay signal to the setpoint, record both the input and output signals of the plant and then use these data to identify K, T and τ_d by doing time series analysis. K_p and K_i can be calculated and applied on line. This method applies an unit relay signal to the setpoint and thus, interrupts the normal operation. A more practical and simple algorithm needs to be developed.

RANGE OF P AND I GAINS DUE TO THE SIMILARITY OF COILS

The range of P and I gains depends on the coil and its ambient conditions, such as water temperature, valve position, differential pressure, air flow, air temperature etc. The ambient conditions can be represented by the time constant, time delay and DC gain. For example, hot water will increase the supply air temperature significantly when air flow is extremely low. However, when the air flow is very high, the temperature rise will be much smaller. The DC gain in the formal case is much higher than the later case. According to our experience, we suggest the following ranges for typical heating and cooling coils.

$$100\ \text{sec} < T < 300\text{sec} \tag{5}$$

$$20\ \text{sec} < \tau_d < 40\text{sec} \tag{6}$$

$$1°F < K_s < 5°F \tag{7}$$

for 1% valve position change

In order to determine the range of P and I gain the closed loop coil transfer function is developed:

$$\frac{Y}{r} = \frac{\left(K_p + \frac{K_i}{s}\right)\left(\frac{K_s}{Ts+1}\right)e^{-\tau_d s}}{1 + \left(K_p + \frac{K_i}{s}\right)\left(\frac{K_s}{Ts+1}\right)e^{-\tau_d s}} \tag{8}$$

Y is the output;
r is the setpoint;
K_p is the proportional gain for the PI controller;
K_i is the integral gain for the PI controller;
K_s is the DC gain;
T is the time constant;
τ_d is the delay time.

To maintain a stable system, the P and I gains must satisfy Equation 9-11 according to Nyquist criterion (Franklin).

$$\frac{k_s k_p}{T} + \frac{1}{T} > 0 \tag{9}$$

$$\frac{k_s k_i}{T} > 0 \tag{10}$$

$$\tau_d \sqrt{\frac{\sqrt{\left(\left(\frac{k_s k_p}{T}\right)^2 - \left(\frac{1}{T}\right)^2\right)^2 + 4\left(\frac{k_s k_i}{T}\right)^2} - \left(\frac{1}{T}\right)^2 + \left(\frac{k_s k_p}{T}\right)^2}{2}} < \arccos \frac{\frac{k_s k_i}{T} - \frac{1}{T}\frac{k_s k_p}{T}}{\left(\frac{1}{T}\right)^2 + \frac{\sqrt{\left(\left(\frac{k_s k_p}{T}\right)^2 - \left(\frac{1}{T}\right)^2\right)^2 + 4\left(\frac{k_s k_i}{T}\right)^2} - \left(\frac{1}{T}\right)^2 + \left(\frac{k_s k_p}{T}\right)^2}{2}} \tag{11}$$

To satisfy Equation 9 and 10, both P and I gains must be greater than zero. If we assume the time delay and the product of I gain and DC gain as constants, we can express Equation 11 in Figure 5

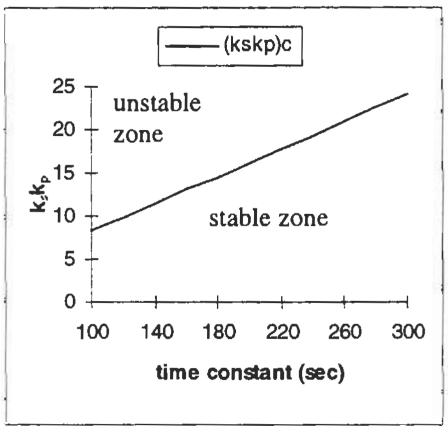

Figure 5: Stable and unstable zone for the system closed-loop dynamic behavior

According to Figure 5, the controller is not stable if the $k_s k_p$ is larger than $(k_s k_p)_c$ and the supply air temperature is fluctuating at this case. The controller can become stable by reducing the P gain. When different values of time delay and $k_i k_s$ are applied, a similar figure is produced. It appears that the system oscillating can be eliminated by decreasing P gain when I gain is set properly.

The stable and unstable zone can be discerned on-line by observing the movement of the control valve since the control valve changes from full open to full close periodically when the system is unstable and the $k_p k_s$ is higher than the critical valve.

The theoretical analysis indicated that the system can be stable as soon as the P gain or $k_p k_s$ is less than the critical value. However, when a change is applied to the system, for example, the supply air temperature setpoint is changed from 55°F to 60°F, the system may oscillate for a much longer period if the P gain is not the optimal value. Therefore, we introduce another concept of the maximum integral square error (ISE). The ISE is defined by Equation 12:

$$J = \int_0^\infty e(t)^2 dt = \int_0^\infty (y(t) - r(t))^2 dt \tag{12}$$

If ISE is less than 80 under a step change (1°F) of setpoint, the system can reach a new setpoint in 5 minutes for most cooling and heating coils. Consequently we define the maximum ISE as 80. The ISE can be expressed as a function of τ_d, T, $k_s k_p$ and $k_s k_i$ as Equation 13(Marshall):

To apply the maximum ISE value to Equation 13, two $k_p k_s$ solution are obtained when we assume fixed values of T and $k_i k_s$. The results are shown in Figure 6 as $(k_s k_p)_{min}$ and $(k_s k_p)_{max}$. Figure 6 divides the $k_s k_p$ into four zones. In zone 1, the $k_p k_s$ is higher than the critical stable value, the system will oscillate after being disturbed. In zone 2, the value of $k_p k_s$ is in the stable zone. However, the system will oscillate more than 5 minutes before reaching the new set-point. If the $k_p k_s$ is in zone 3, the setpoint will be achieved within 5 minutes. In zone 4, the $k_p k_s$ is too low to reach the setpoint within 5 minutes. Although oscillation does not occur, a constant bias exist for a longer period. Therefore, the goal of the fine tuning is to move the $k_p k_s$ to zone 3. It is important to note that the system can be stable as long as the P gain is within ±90% of the true value.

$$
J=\begin{cases}
\dfrac{1}{2\sqrt{\left({\frac{k_sk_p}{T}}^2-{\frac{1}{T}}^2\right)^2+4{\frac{k_sk_i}{T}}^2}} \\[1ex]
\left[\left(\left(\frac{1}{T}\right)^2-\rho^2\right)\dfrac{\frac{k_sk_i}{T}-\frac{1}{T}\frac{k_sk_p}{T}+\left(\left(\frac{1}{T}\right)^2-\rho^2\right)\cosh\rho\tau_d}{\frac{k_sk_p}{T}\rho^2-\frac{1}{T}\frac{k_sk_i}{T}-\rho\left(\left(\frac{1}{T}\right)^2-\rho^2\right)\sinh\rho\tau_d}\right. & \text{when}\ k_p\neq T\,k_i \\[1ex]
\left.+\left({\frac{1}{T}}^2+\sigma^2\right)\dfrac{\frac{k_sk_i}{T}-\frac{1}{T}\frac{k_sk_p}{T}+\left(\left(\frac{1}{T}\right)^2+\sigma^2\right)\cos\sigma\tau_d}{\frac{k_sk_p}{T}\sigma^2+\frac{1}{T}\frac{k_sk_i}{T}-\sigma\left({\frac{1}{T}}^2+\sigma^2\right)\sin\sigma\tau_d}\right] \\[2ex]
\dfrac{1}{2\sigma}\dfrac{1+\sin\sigma\tau_d}{\cos\sigma\tau_d} & \text{when}\ k_p=T\,k_i
\end{cases}
\qquad (13)
$$

where $\rho=\sqrt{\dfrac{\sqrt{\left({\frac{k_sk_p}{T}}^2-{\frac{1}{T}}^2\right)^2+4{\frac{k_sk_i}{T}}^2}+{\frac{1}{T}}^2-\sigma^2}{2}}$ and $\sigma=\sqrt{\dfrac{\sqrt{\left({\frac{k_sk_p}{T}}^2-{\frac{1}{T}}^2\right)^2+4{\frac{k_sk_i}{T}}^2}-{\frac{1}{T}}^2+{\frac{k_sk_p}{T}}^2}{2}}$.

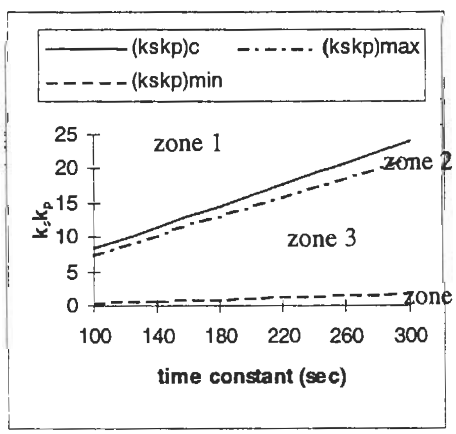

Figure 6: Zone division based on the system dynamic performance

IDENTIFYING STATUS OF THE CONTROLLER

When the system is in zone 3, the ISE is less than the critical value. When the ISE is higher than the critical value, the system can be in either zone 2 or zone 4. This section provides a methods to identify the status (zone 2 or zone 4) of the system.

The difference of output and reference signal under unit step input can be written in s-domain as:

$$
e(s)=\frac{Ts+1}{Ts^2+s+(K_ps+K_i)K_se^{-\tau_ds}} \qquad (14)
$$

Then the magnitude of spectrum of the error signal under different frequency is:

$$
|X(w)|=\left|e(s)|_{s=jw}\right|
=\frac{\sqrt{1+(Tw)^2}}{\sqrt{\begin{array}{l}(K_ik_s\cos(\tau_dw)+K_pk_s\sin(\tau_dw)-Tw^2)^2\\+(w+K_pk_s\cos(\tau_dw)-K_ik_s\sin(\tau_dw))^2\end{array}}} \qquad (15)
$$

When the system is in zone 2, $(k_sk_p)_{max}<k_sk_p<(k_sk_p)_c$, the major energy of the signal is within the high frequency range; When the system is in zone 4, $k_sk_p<(k_sk_p)_{min}$, the major energy of the signal is within the low frequency range.

Therefore, introduce the energy ratio of the low frequency energy to the total signal energy:

$$K = \frac{\int_0^{w_c} [e(w)]^2 dw}{\int_0^{\infty} [e(w)]^2 dw} \qquad (16)$$

K is larger than a threshold k_c when the system is in the zone 2. K is less than this critical value when system is in zone 4. The critical frequency point w_c in Equation 16 has been found to be 0.02 rad/sec based on the numerical analysis of Equation 15 for heating and cooling coils in buildings which has time constant, delay time and DC gain under certain range as Equations 5-7; meanwhile the critical threshold k_c is found to be 0.5. It is hold that

$K = \frac{\int_0^{0.02} [e(w)]^2 dw}{\int_0^{\infty} [e(w)]^2 dw} > 0.5$ when the system is in zone 2 and $K = \frac{\int_0^{0.02} [e(w)]^2 dw}{\int_0^{\infty} [e(w)]^2 dw} < 0.5$ when the system is in zone 4.

The signal needs to be separated into two frequency ranges of $w>w_c$ and $w<w_c$; so that the zone 2 and zone 4 can be identified. It is convenient that we use a high pass filter and a low pass filter which both have cutoff frequency at w_c to separate the signal into two signals at different frequency intervals. Then we can analyze the ratio of integral square error of these two signals to get the value of K in Equation 16. There are several types of commonly used filters such as: butterworth filters, chebyshev filters and elliptic filters, etc. A normalized low pass analog filter $H_{NLP}(s)$ (shown in Equation 17), cutoff frequency is 1 rad/sec, with amplitude response of -20dB at the normalized frequency of 1.3 is designed in the format of butterworth filters, chebyshev filters and elliptic filters respectively. The frequency response of the three filters are shown in Figures 7, 8 and 9. We can see that all three filters do the job. However, the order of filters is different: the order of the butterworth filter is 19, the order of the chebyshev filter is 7 and the order of the elliptic filter is 4. It is clear that the elliptic filter is the most efficient filter and we use it in the algorithm.

$$H_{NLP}(s) = \frac{Y(s)}{X(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \ldots + a_n}{s^n + a_1 s^{n-1} + \ldots + b_n} \qquad (17)$$

a_i, i=1,2,...,n are constants;
b_i, i=1,2,...,n are constants;
n is the order of the filter.

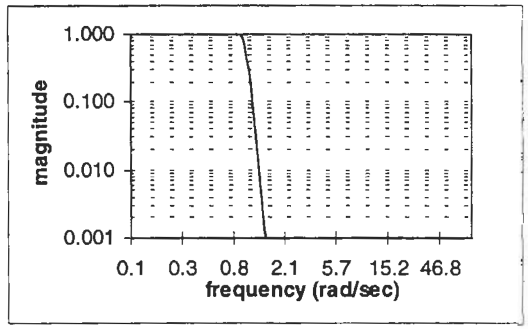

Figure 7: Normalized butterworth filter with the order of 19
(larger than -3db in when freq. < 1.0 rad/sec and less than -20 dB when freq. >1.2 rad/sec)

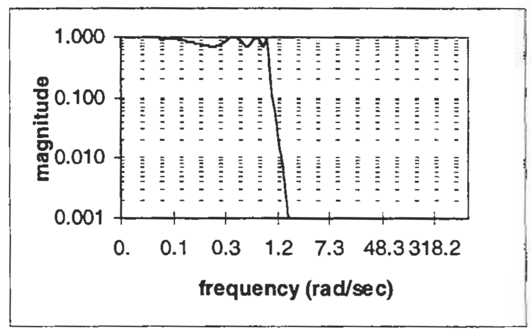

Figure 8: Normalized chevolev filter with the order of 7
(larger than -3db in when freq. < 1.0 rad/sec and less than -20 dB when freq. >1.2 rad/sec)

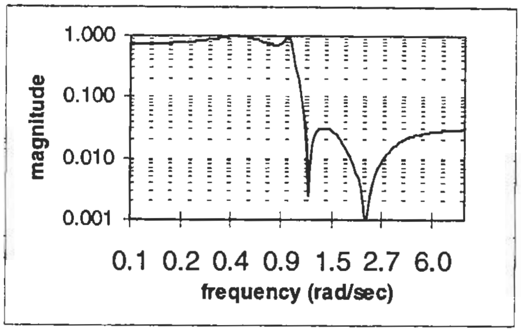

Figure 9: Normalized elliptic filter with the order of 4
(larger than -3db in when freq. < 1.0 rad/sec and less than -20 dB when freq. >1.2 rad/sec)

Low pass digital filter and high pass digital filter at different cutoff frequencies and different sampling intervals can be obtained by doing certain transformation from the analog normalized low pass filter (Proakis).

THE NEW ALGORITHM

The new algorithm has been developed based on the above theoretical analysis of the coil feedback control system. The new algorithm decides whether P and I gains need to be adjusted and how to adjust P and I gain by on-line observing the system performance. The procedure of the new self-tuning algorithm is illustrated in Figure 10. It consists of two steps: the first step is to determine whether P and I gains need to be adjusted; the second step is to determine how to adjust P and I gains.

The first step is conducted by observing the ISE between the system output and its setpoint. The P gain and I gain must be adjusted when the ISE is higher than its thresholds. The threshold can be determined based on experience.

In the second step, we first observe the valve position. If the valve position oscillates from full open to full close, the system is in zone 1 in Figure 6 (unstable zone). P gain needs to be decreased. When the system is in the stable zone, the output signal is sent to the high and low pass filters. Then, the ratio of ISE in the low frequency is calculated. The system is in zone 2 (stable but P gain too big.) if the ratio is bigger than 0.5. P gain needs to be decreased. The system is in zone 4 (stable but P gain too small) if the ratio is smaller than 0.5. P gain needs to be increased.

The new method tunes up the system by directly observing the system performance. It can be implemented on-line without the interruption of the coil operation.

APPLICATION OF THE ALGORITHM

Two tests are conducted to demonstrate the algorithm by using a cooling coil and control valve in a real AHU(shown in Figure 11).

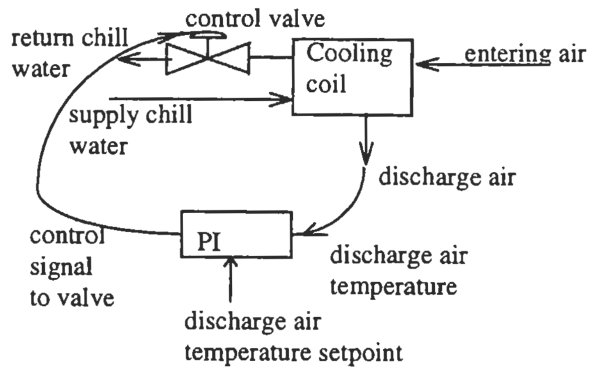

Figure 11: Diagram for the cooling coil with PI controller

The first test is shown in Figures 12a and 12b. At the beginning, a high P gain (7.5) replaced the optimal value of 3. Oscillation occurred as soon as the high P gain was used. After 20 minutes, the algorithm decreased the P gain to 3.75. Then the oscillation disappeared in 10 minutes. It appears that this algorithm is capable of identifying the problem of high P gain and correcting it automatically.

The second test is shown in Figures 13a and 13b. A low P gain (0.6) was assigned first. The drifting occurred when the low P gain was used. After 20 minutes, the algorithm increased the P gain to 1.2. Drifting was depressed. Then the algorithm increased the P gain to 2.4. The drifting disappeared finally. It appears that the algorithm can identify low P gains and correct them as well.

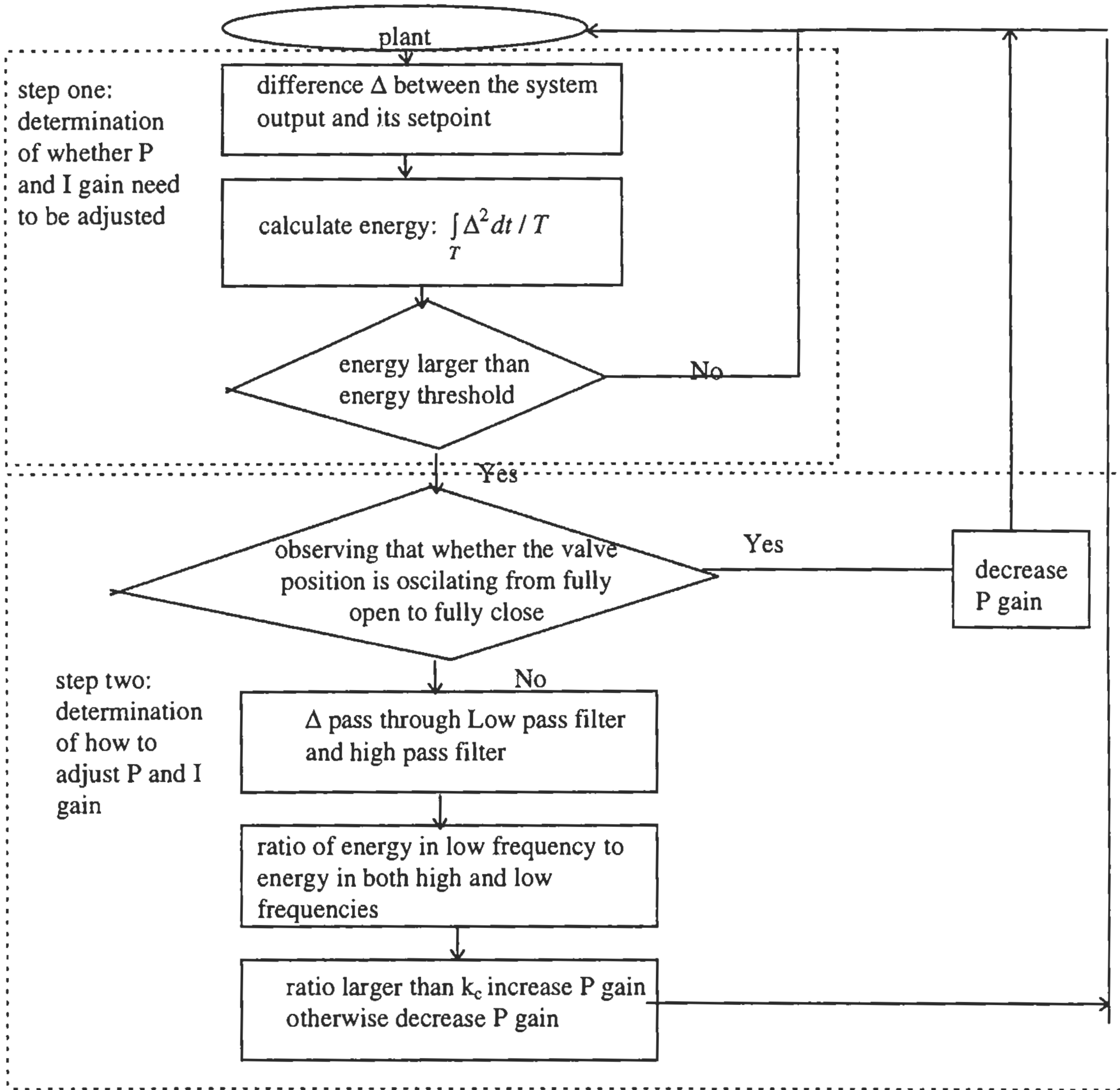

Figure 10: The procedure of the new self-tuning algorithm

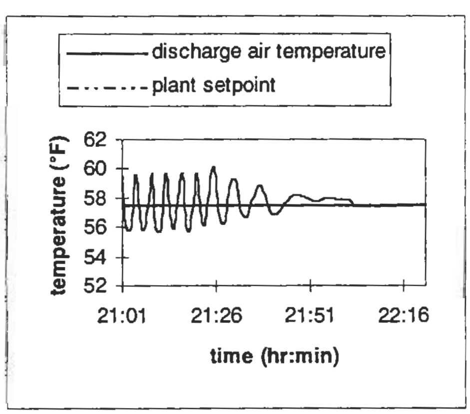

Figure 12a: The dynamic response of the plant under different P gain in test 1

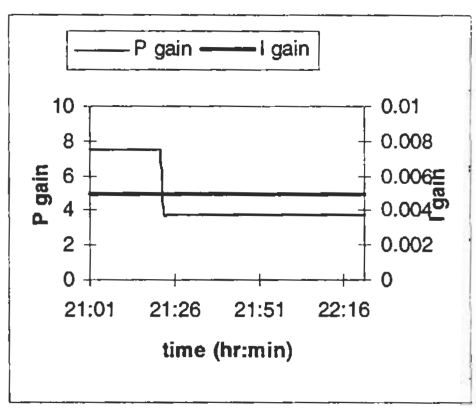

Figure 12b: The change of P gain and I gain in test 1

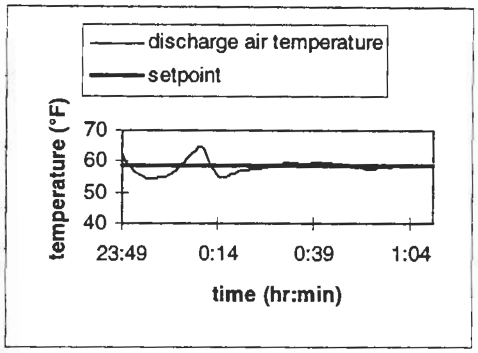

Figure 13a: The dynamic response of the plant under different P gain in test 2

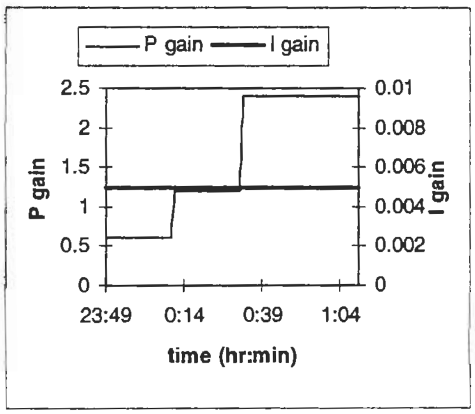

Figure 13b: The change of P gain and I gain in test 2

CONCLUSION

An on-line self-tuning algorithm for the heating and cooling system in buildings has been developed. The algorithm evaluates the controller performance by the integral square error (ISE) between the coil output signal and its setpoint, and identifies the corrections of the P and I gain by observing the spectrum distribution of the ISE. The method turns out to be simple and practical when the digital filters are applied to do spectral analysis instead of Fourier transformation. The algorithm has been tested in the field and its feasibility has been proved.

REFERENCE

Astrom, K. J., and T. Hagglund. 1984. "Automatic tuning of simple regulators with specifications on phase and amplitude margins." *Automatica*, Vol. 20, No. 5, pp. 645-651.

Astrom, K. J., and T. Hagglund. 1988. "A new auto-tuning design." *IFAC International Symposium on Adaptive Control of Chemical Processes,* Copenhagen.

Chantre, P., Maschke, B. M., Barthelemy, B., 1994 "Physical modeling and parameter identification of a heat exchanger." *IEEE 0-7803-1328-3*

Clark, D. R., 1985, "HVACSIM+ building systems and equipment simulation program - users guide." US Department of Commerce.

Dexter, A. L. and Jota, F. G., 1985, 'Multi-loop self-tuning control of an air-conditioning plant." *Proc. Int. Symposium on Recent Advances in Control and Operation of Building HVAC Systems*, Trondheim, Norway, 113-123.

Franklin, Gene F. and Powell, J. David and Emami-Naeini, Abbas, 1995, *Feedback Control of Dynamic Systems*, Addison-Wesley Publishing Company, Inc.

Jota, F. G. and Dexter, A. L., 1988, "Self-tuning control of an air-handling plant." *IEEE International Conference*

Kraus, T. W., and T. Myron. 1984. " Self-tuning PID controller uses pattern recognition approach." *Control Engineering*, June, pp.106-111

Marshall, J.E., 1992, *Time-delay systems,* Ellis Horwood LTD., UK

Proakis, John G. and Manolakis, Dimitris G., 1996, *Digital signal processing*, Prentice-Hall, Inc.

Rahman, M.H.R. Fazlur and Devanathan, 1994, "Modeling and dynamic feedback linearisation of a heat exchanger model." *IEEE 0-7803-1872-2*

Wallenborg, A. O. 1992. "A new self-tuning controller for HVAC systems." *ASHRAE Transactions: Research*, Vol. 98 pp. 19-25.

Xia, L., Abreu-Garcia, J. A. D., and Hartley, Tom T., 1991, "Modeling and simulation of a heat exchanger." *IEEE International Conference on Systems Engineering*. (Ohio), pp. 453-456,

Zhang, Zhihui, 1992, *Thermodynamic Measurement and Automatic Control*, Chinese Architecture Industrial Publishing Co.

Ziegler, J. G., and N. B. Nichols, 1942. "Optimum settings for automatic controllers," *Trans. ASME*, Vol. 64, pp. 759-768.

Ziegler, J. G., and N. B. Nichols, 1943, "Process lags in automatic control circuits." *Trans. ASME*, vol. 65, no. 5, pp. 433-444.