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Optimal Use of Groundwater and Surface Water to Reduce Land Subsidence

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ABSTRACT

Optimizing Water Resource Development to Reduce
Land-Surface Subsidence

by

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An optimization model was developed to allocate surface and groundwater resources under various water management policies which might be implemented to control land-surface subsidence. This model allocated the water resources of a region so that the overall cost of water development and land-surface subsidence was minimized. To use this model, a hydrologic model was developed to predict the piezometric heads in sand and clay layers caused by groundwater pumpage. A compaction, or subsidence model, used the resulting piezometric heads from the hydrologic model to predict land-surface subsidence. A linear programming model was then developed to optimally allocate ground and surface water resources within a region so that the cost of water and land subsidence was minimized.

Using data from the land-subsidence area at Houston, Texas, a conceptual two-dimensional, vertical geologic profile was developed to describe the aquifer-aquitard system in the Houston area. This conceptual geologic profile was composed of a constant head boundary at the top, three layers of clay, two layers of sand, three layers of clay, two layers of sand, and one no-flow layer at

the bottom. Field values for hydraulic conductivity, specific storage coefficient, pumpage rate, and initial piezometric head were used in the hydrologic model.

Successive applications of the hydrologic and compaction models were made over a 10 year period for varying groundwater withdrawal rates. This information was used to develop a series of subsidence-pumpage curves for each of 15 surface grid points.

The optimization model was independent of the hydrologic and compaction models, and independent of the grid dimensions and shape. Because the resulting subsidence-pumpage curves were non-linear, separable programming was used to approximate the subsidence-pumpage curves as the sum of a set of linear functions. Thus, a linear programming formulation of the optimization problem could be used.

Three basic water use policies were evaluated and compared. Water-Use Policy I allowed groundwater pumpage to occur at each point in the model. Water-Use Policy II imposed a social constraint that no groundwater could be withdrawn from three of the grids. Water-Use Policy II-A was similar to Policy II, except that the maximum volume of groundwater that could be pumped from an area with a low subsidence cost was greatly increased.

Within the restrictions established for water demand, water availability, water cost, subsidence cost, and the social cost of subsidence, the optimization model allocated the surface and groundwater resources to various sections of the region. Other water management policies, besides those considered in this study, can be easily evaluated using this model.

Parametric analysis was used to study the effects on the optimal allocation when the coefficients in the objective function or the right hand side of the constraints were changed individually or simultaneously. This procedure allowed a systematic analysis to be made of the changes in the optimal solution caused by increased water cost, increased water demands, the volume of the surface water reserve, and the groundwater availability throughout the region.

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CHAPTER I

INTRODUCTION

To obtain an adequate supply of water, many regions of the world have continually increased their use of groundwater. In several of these areas, increased groundwater pumpage has caused land-surface subsidence. In a few select areas, such as Houston, Texas, land surface subsidence caused by groundwater withdrawals has become a critical economic, social, and political issue.

Groundwater pumpage reduces water levels and water pressures in an aquifer. The reduction in water pressure causes a corresponding increase in the effective pressure on the porous media matrix. A coarse-grained matrix such as a sand or sandstone will respond to this pressure shift immediately, and water is released from storage in the aquifer without any notable compaction. A fine-grained matrix, such as clay or shale, will react more slowly to the pressure shift and will undergo compaction. When the clay compresses, water is expelled from the clay pores and drained either upward or downward from the clay layer. As the clay undergoes the consolidation process, the thickness of the clay layer is reduced. The reduction in clay thickness is reflected at the land surface as land subsidence. Two related problems of interest also occur: (1) in a fine grained rock, the total volume of subsidence corresponds closely to the total volume of water removed from the rock and (2) the time rate of land subsidence corresponds to the time rate of flow of the rocks [Domenico, 1976].

In many areas, land subsidence has caused enormous physical and economic damage to public and private property. In particular, many sea coast communities, once safe, are now subject to flooding from high tides and hurricanes. In Houston, Texas, an entire residential community is being abandoned because land subsidence has literally made the area part of a bay.

To prevent land-subsidence, groundwater pumpage must be halted. However, if groundwater pumpage is halted, then alternate sources of water such as surface water must be made available to the community. Another alternative is to transport groundwater from areas with a small land subsidence cost to areas with a large land subsidence cost. The objective would be to optimally allocate the water resources available to a region so that the cost of land subsidence is minimized for the entire region.

Using a basic equation for the flow of fluids through a porous media and Terzaghi's theory of one-dimensional consolidation, land subsidence can be simulated and predicted using numerical techniques. Results from the land-subsidence simulation can then be utilized in an optimization model, and the water resources of a region allocated to assure a minimum water cost to the region. Use of the combined numerical and optimization models, will allow various groundwater management alternatives for controlling land subsidence to be fully evaluated.

Objectives

The overall objective of this research was to develop an optimization model which would allocate surface and groundwater resources under various management techniques so that the regional cost of land subsidence is minimized. Specifically, the research had the following objectives:

(1) Develop a numerical model for evaluating water pressure distributions in a multiaquifer system;

(2) Develop a numerical model for predicting land-surface subsidence using water pressure distributions obtained from objective 1; and

(3) Develop an optimization scheme to allocate the surface and groundwater resources of a region so that the cost of land-surface subsidence is minimized.

Methods of Investigation

The techniques of this investigation were aimed at using the computer as a model simulator. The differential equation describing one-phase fluid flow in a saturated porous media was developed and written in an implicit finite difference form and solved using Gauss elimination to give a distribution of the piezometric water heads at any time caused by various groundwater management programs.

Using the water pressure distributions to calculate effective pressure, the compaction of the various clay layers was calculated using Terzaghi's theory of consolidation. The consolidation model keeps a record of the consolidation history, and evaluates the amount of consolidation occurring at any time under various pumpage conditions. Using cost coefficients (dollars per meter of subsidence), the resulting clay compaction was transformed into a dollar value and a curve of subsidence cost versus pumpage was prepared. These curves were then approximated using linear functions so that the objective function could be minimized by linear programming techniques. Water demands and surface water reserves were used as constraints. To analyze the changes in optimality and the feasibility of solutions, parametric analysis was performed.

Field data from Houston, Texas were used to prepare a semi-hypothetical land subsidence region. The region was a two-dimensional, vertical strip which was divided into alternating layers of sand and clay. Field values of hydraulic conductivity and the coefficient of storage were smoothed considerably for use in the model. The utility of using the optimization model to allocate surface water and groundwater was illustrated using this semi-hypothetical region.

CHAPTER II

LITERATURE REVIEW

The literature concerning this research covers many topics. For purposes of presenting a summary of the available knowledge, the literature search was divided into 5 topics: (1) theoretical analysis, (2) mathematical modeling and simulation, (3) optimization techniques, (4) economic impact of land surface subsidence, and (5) legislation on land subsidence.

Theoretical Analysis

Early in this century, confined aquifers were believed to be rigid, imcompressible bodies. Consequently, changes in water pressure were not accompanied by changes in pore volume. While studying the effects of the discharge of flowing wells on the piezometric head in the Dakota sandstone, Meinzer [1928] concluded that the water discharged from wells was largely derived locally from storage within the aquifer. He found that the volume of water pumped from the aquifer could not be accounted for by the compressibility of water alone; but that it might be accounted for by considering the sandstone itself to be compressible. Meinzer [1928] suggested that the properties of compressibility and elasticity are very important considerations in aquifers with low permeabilities, slow recharge rates, and large piezometric heads.

Theis [1935] extended Meinzer's concepts of aquifer compressibility. Using an analogy between the laminar flow of water through the interstices of a porous material and the flow of heat through a solid media, Theis [1935] derived a relationship between the lowering of the piezometric surface and the rate and duration of discharge from a well. In this relationship, the concept of an aquifer storage coefficient was first introduced. The aquifer storage coefficient implies that a specific volume of water is instantaneously released from storage in the aquifer for each meter of decline in the piezometric water level. This water is derived from the elastic compression of the water-bearing beds and the expansion of the water confined in the pores of the rock.

In addition to Theis' [1935] observations, Jacob [1940] noted that the chief source of water derived from storage in an artesian aquifer is the contiguous clay or shale beds within a sandstone aquifer. Because of the low permeability of clays or shales, there is a time lag between the lowering of the piezometric head within an aquifer and the appearance of the water released from storage within the clays or shales. Assuming that the stored water is released immediately following a decline in the piezometric head and that the rate of water release is directly proportional to the rate of decline in the piezometric head, Jacob [1940] derived what he called the fundamental differential equation for the flow of a compressible viscous liquid to a well in an elastic artesian aquifer. This equation is of the form

$$r\left(\frac{\partial h}{\partial r}\right) + r^2\left(\frac{\partial^2 h}{\partial r^2}\right) = \frac{r^2 S}{Km}\left(\frac{\partial h}{\partial t}\right) \qquad (1)$$

where

r = radius from well (L),

h = piezometric head (L),

S = coefficient of storage (·),

K = hydraulic conductivity (LT^{-1}),

m = aquifer thickness (L), and

t = time (T).

A major contribution to understanding the behavior of aquifer-aquitard systems was incorporating the concept of the release of water from compressible confining layers into the equations describing unsteady flow of groundwater to a discharging well [Hantush, 1960; 1964]. In these works, Hantush defined and analyzed the concepts of storativity and coefficient of storage for aquifers.

Lohman [1961] reviewed the concepts of an elastic artesian aquifer and analyzed the equation developed by Jacob [1940] which defined the coefficient of storage, i.e.,

$$S = \gamma\phi m \left(\frac{1}{E_w} + \frac{b}{\phi E_s} \right) \qquad (2)$$

where

S = coefficient of storage (·),

m = aquifer thickness (L),

ϕ = porosity (·),

γ = specific weight of water (FL^{-3})

E_w = bulk modulus of elasticity for water (FL^{-2}),

E_s = bulk modulus of elasticity for the aquifer (FL^{-2}), and

b = the fraction of the aquifer area that responds elastically (·).

Combining Hooke's law of elasticity with equation (2), Lohman [1961] proposed an equation for determining the amount of elastic subsidence or compression.

$$\Delta m = \Delta p\left(\frac{S}{\gamma} - \phi m \beta\right) \qquad (3)$$

where

$$\beta = \frac{1}{E_s} \quad (L^2 F^{-1})$$

Δm = the reduction in aquifer thickness or the amount of elastic subsidence (L), and

Δp = a given decline in artesian pressure (FL^{-2}).

Equation (3) only gives the elastic subsidence, and this is small in many areas when compared to the greater subsidence caused by the plastic deformation of the associated clays.

Poland [1961] also reviewed Jacob's [1940] work and agreed that, due to the presence of clays, equation (2) should be written as:

$$S = \gamma\phi m \left(\frac{1}{E_w} + \frac{b}{\phi E_s} + \frac{c}{E_c} \right) \qquad (4)$$

where E_c = modulus of compression for clay beds (FL^{-2}), and

c = a dimensionless number that depends largely on the thickness, configuration, and distribution of the intercalculated clay beds (•).

Equation 4 takes care of the time-lag between the lowering of the water pressure within the aquifer-aquitard system and the appearance of that part of the water which is derived from storage in the aquitards. Poland applied the equation to the Los Banos-Kettleman city area in California, and calculated values for the three components in the brackets of equation (4). He found that the stored water released by compaction from clay beds was about 50 times greater than the volume of water released by elastic expansion of the water and elastic compression of the aquifer. Poland also noted that the volume of stored water yielded by the clays was only large during the initial decline in artesian pressure when the clays were not preconsolidated.

A major contribution to the understanding of the consolidation process was made by Terzaghi [1925], and is referred to as the one-dimensional theory of consolidation. The theory was based on the following assumptions: (1) homogeneous soil; (2) complete saturation; (3) negligible conpressibility of the soil grains and water; (4) one-dimensional compression; (5) Darcy's Law is valid; (6) one-dimensional fluid flow; (7) constant coefficient of compressibility; (8) constant hydraulic conductivity; (9) constant layer thickness; (10) at all times $p = p' + u$, where p = total pressure, p' = grain or effective pressure, and u = hydrostatic water pressure; and (11) the idealized relationship between effective pressure and voids ratio shown in Figure 1 is valid.

These assumptions are not fully met in nature, but are correct in many laboratory analysis. The first three assumptions are commonly used when analyzing most groundwater problems. The validity of the fourth and sixth assumption is questionable under most natural conditions. The fifth assumption is accepted with little question for most groundwater flow fields. Assumptions seven, eight, and nine introduce some errors, but it is believed that in most instances these errors are of minor importance. Assumption ten is of primary importance for understanding the consolidation process. Assumption eleven is a major limitation of the theory. It is highly idealized, but a more correct relationship would make the analysis excessively complex, expecially when studying natural problems.

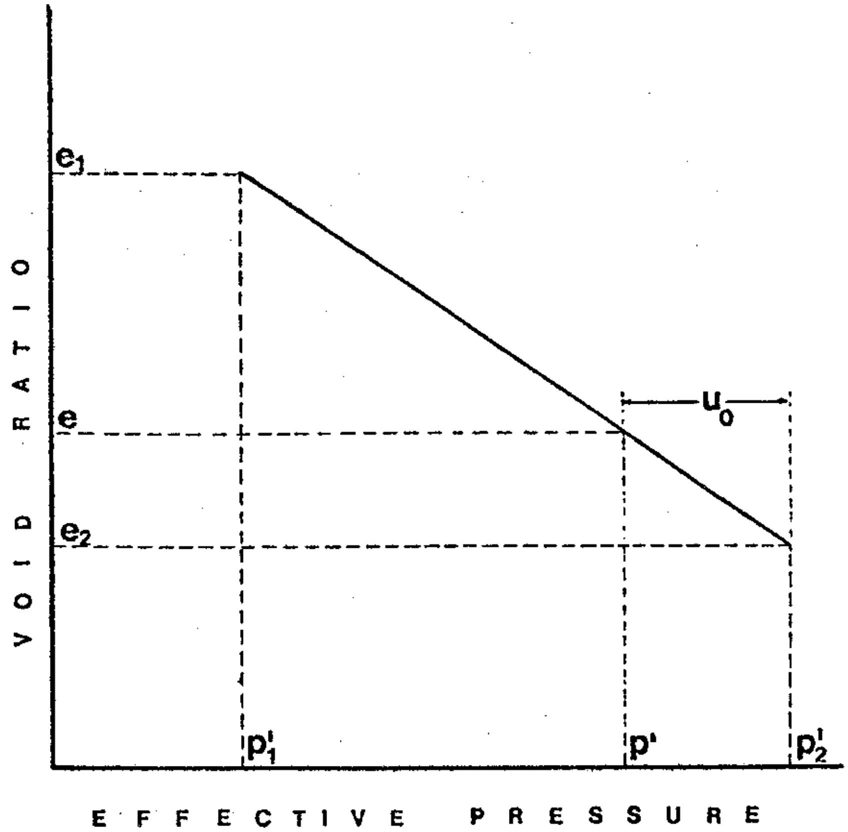


Figure 1. Idealized relationship between effective pressure and void ratio.

According to the Terzaghi Theory [Terzaghi, 1948], immediately upon applying an increment of load, $p' - p_2'$ (see Figure 1), the entire increment is registered as a hydrostatic pressure (u_0) in the pore water of the soil. The soil structure cannot take any of the load until it has experienced some compression, and the soil structure cannot experience any compression until some water has been squeezed from the soil.

An interesting mechanical analogy, shown in Figure 2, was proposed by Taylor [1948] for a better understanding of the consolidation process. In this analogy, a spring represents the compressible soil skeleton for a mass of saturated soil, and water in a cylinder represents the water in in the soil pores. A similar analogy could also be applied to the

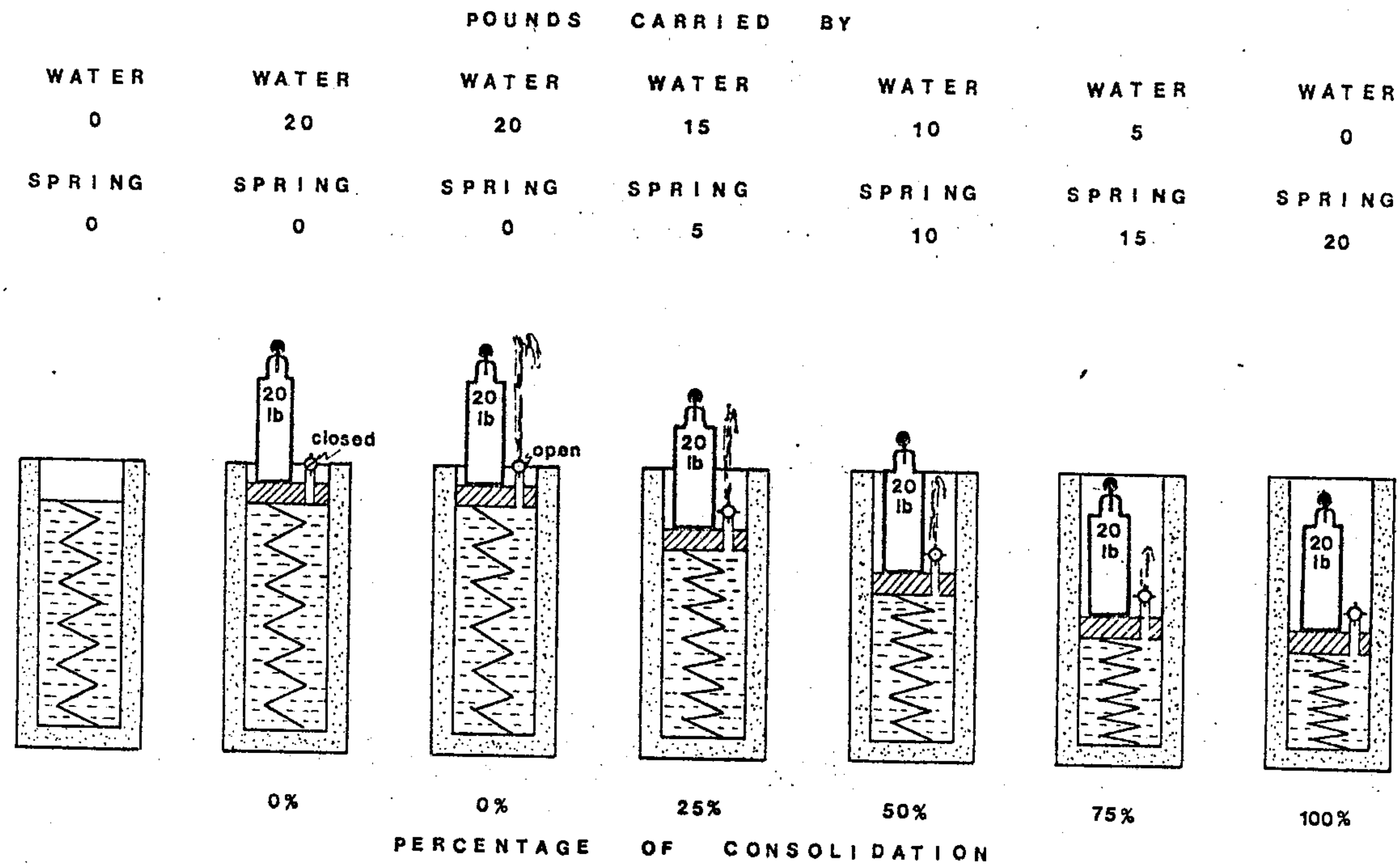


Figure 2. Taylor's mechanical analogy of the consolidation process in saturated soils [Taylor, 1948].

expansion process.

Lowe [1974] modified the unique relationship between void ratio and intergranular pressure presented in Terzaghi's theory of consolidation, and presented a third variable (the rate of strain) to describe the consolidation process. Instead of the single curve of void ratio versus intergranular pressure given by Terzaghi's theory, Lowe's modification gives a family of curves; one for each value of the rate of strain.

The differential equation describing the consolidation process resulting from linear drainage has been presented many times [Jumiskis, 1962]. Based on Darcy's Law and the continuity condition, the equation is

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \tag{5}$$

where

u = water pressure (FL^{-2}),

c_v = coefficient of consolidation (L^2T^{-1}),

z = depth coordinate (L), and

t = time (T).

Domenico and Mifflin [1965] emphasized that the concept of effective stress is essential for an understanding of natural consolidation process such as land-surface subsidence. The Terzaghi theory of consolidation notes that two stresses exist in porous water-filled sediment: (1) pressures in the solid phase (points of contact of individual grains) and (2) pressures in the water contained in the pores of the soil. The former are referred to as intergranular pressures or effective stresses and the latter are referred to as pore-water pressures or natural stresses. Natural stresses act on all sides of granular particles, but do not cause the particles to press against each other nor to appreciably deform. Thus, all measurable compaction is primarily due to changes in effective stress.

The differential equation describing effective stress as a function of time and space is [Helm, 1974]

$$\frac{\partial p'}{\partial t} = \frac{K}{S_s} - \frac{\partial^2 p'}{\partial^2 z} \qquad (6)$$

where

p' = effective stress ($ML^{-1}T^{-2}$),

t = elapsed time (T),

K = hydraulic conductivity (LT^{-1}),

S_s = coefficient of specific storage (L^{-1}), and

z = depth into the aquifer (L).

In one-dimensional consolidation theory, the z coordinate is measured downward from the surface of the clay. The total thickness of the clay layer is designated as 2H. The boundary conditions for the case of one-dimensional consolidation are as follows: (1) there is complete drainage from the top of the layer, (2) there is complete drainage from the bottom of the layer, and (3) the initial neutral pressure is equal to the intergranular pressure increment (Figure 3). Mathematically, these can be expressed as:

$$p'(2H,t) = p_1 \qquad @\ t \geq 0,$$

$$p'(0,t) = p_2 \qquad @\ t \geq 0, \text{ and}$$

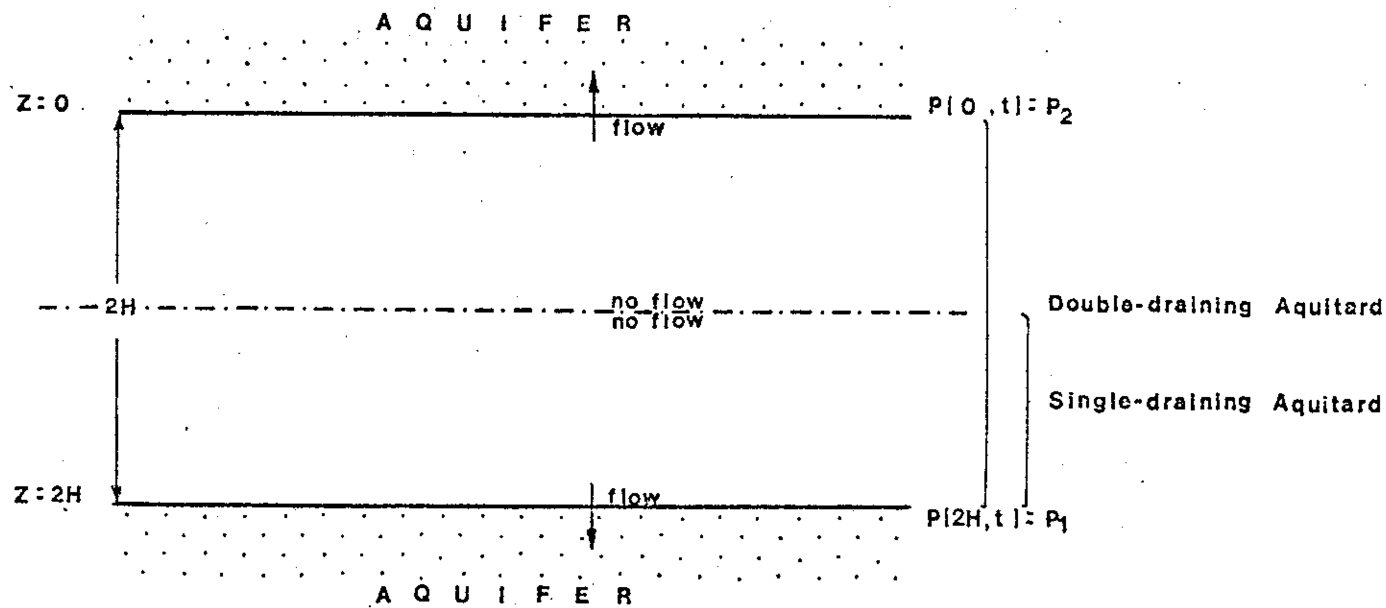


Figure 3. Boundary conditions for a draining aquitard.

$$p'(t,0) \quad = \quad \frac{p_2 - p_1}{2H} \Delta z + p_2 \qquad @ \; t = 0$$

The solution of equation (6) subject to these boundary conditions determines the degree of consolidation. As expressed by Terzaghi and Peck [1948],

$$u(\%) \quad = \quad f(T_v) \tag{7}$$

where

$$T_v \quad = \quad \text{a dimensionless time factor} \quad = \quad \frac{c_v t}{H^2} \tag{8}$$

For every open clay of thickness 2H, the relationship between u(%) and T_v is determined by a curve which changes for different conditions of loading and drainage. Domenico [1972] pointed out the importance of the one-dimensional flow equation for analysis of land subsidence problems. For additional information on the consolidation process the reader is referred to Jumiskis [1962], who presented a very complete analysis of the consolidation process.

Mathematical Modeling and Simulation

During the past decade, two symposiums on land-surface subsidence were held; one in 1970 at Tokyo, Japan, the other in 1976 at Anaheim, California, U.S.A. These two events greatly expanded the existing literature on land-surface subsidence, especially in the areas of mathematical modeling and simulation. Many approaches have been taken to model the phenomenon of land-surface subsidence, including analog models, numerical models, statistical models, and others. At this time, almost all of the mathematical models describing land-surface subsidence are direct applications of the Terzaghi consolidation theory, and are based on a solution of the one-dimensional flow equation. Only the models and examples relevant to this research are presented in this section.

Predicting land subsidence from groundwater withdrawal rates requires that a two or three dimensional hydrologic model based on standard groundwater flow equations be coupled with a consolidation model. The hydrologic model predicts the change in piezometric head in time and space caused by groundwater withdrawals. The changes in piezometric head are then used to calculate the time dependent consolidation curve at any point in the system [Gambolati and Freeze, 1973]. This approach links the fluid flow field (two or three dimensional) with a one dimensional vertical deformation model. A necessary assumption for one dimensional vertical deformation is that horizontal deformation is absent.

A complete analysis of land subsidence requires an evaluation of the three-dimensional deformation field which accompanies the three dimensional flow field. Biot [1941, 1955] presented such an approach. The rigor of Biot's model, the number of parameters required for its use in a realistic geologic configuration, and the limitations in computer storage capacity and time have reduced the usefulness of Biot's model in applied problems [Gambolati and Freeze, 1973].

An alternative to Biot's rigorous three-dimensional stress field is the use of a pseudo-three dimensional approach. This approach assumes that displacements only occur in the vertical direction, and that total stresses do not change in the vertical direction. Bredehoeft and Pinder [1970] successfully used such an approach to analyze the response of an aquifer to various hydrologic stresses. They used an implicit finite difference technique, in which several aquifers were coupled through intervening aquitards with a leakage term. Fluid flow was horizontal in the aquifers and vertical in the aquitards. Pinder and Gray [1977] proposed a solution using finite element techniques. Sandhu and Wilson [1969] and Neuman and Witherspoon [1970] also developed models using finite element techniques. Previously, Neuman and Witherspoon [1969] presented an analytical solution for a two-aquifer, one-aquitard system. However, real field problems are so complex that most analytical solutions are not feasible.

A two-dimensional vertical subsidence model was developed to simulate subsidence in Venice, Italy [Gambolati and Freeze, 1973; Gambolati et al., 1974]. In this model, a two-dimensional, radial flow equation (hydrological model) was solved by finite element techniques to obtain water table (piezometric) drawdowns caused by groundwater pumping. The subsidence model, based on a one-dimensional form of the flow equation, was solved using an implicit finite difference technique. In the subsidence model, the upper and lower boundary conditions consisted of an annual stepwise representation of the changes in hydraulic head determined from the hydrologic model. Since Gambolati's model coupled the variations in hydraulic head in space and time with a one-dimensional consolidation model, he stated that his model was more complete than models entirely based on Terzaghi's theory of consolidation. From a mathematical point of view, some limitations to Gambolati's approach were suggested by Smith [1971]. In his solution, Gambolati neglected the nonlinearity of compressibility over small pressure changes. To handle the irreversibility of the compaction process, Gambolati introduced two values of compressibility; one for compaction and the other for expansion. This model was successfully verified against field data from Venice, Italy. Due to decreases in groundwater pumpage, the piezometric heads in the area have increased and the land surface has rebounded slightly.

A simpler approach than Gambolati's was made by Helm [1974, 1975a, 1975b]. Helm's model essentially used Terzaghi's theory of consolidation. The one-dimensional equation of flow was solved using finite difference techniques to get successive new water pressure in the aquitards. Two models were studied by Helm; one is stress-dependent, (i.e., non-linear coefficients) and the other independent of stress (i.e., linear coefficients). In both models, the concept of preconsolidation pressure in fine-grained beds was applied to distinguish between present effective stress and past maximum effective stress. Helm concluded that the linear model was less time consuming, easier to work with, and nearly as accurate as the non-linear model. An important feature of Helm's approach was the concept of equivalent bed thickness. Under a given applied stress, thin beds reach 100 percent consolidation sooner than thick beds. Thus, the thickness of the various clay beds is important for an analysis of land subsidence. To calculate the cumulative compaction from a large number of clay beds would require excessive computer time. Using Helm's equivalent bed thickness, a single idealized clay bed can be used in place of the several real clay beds, and the calculated compaction would be proportional to that obtained by summing the compactions obtained from a number of actual beds. This saves computer time and makes the model more efficient. The formula for calculating an equivalent bed thickness is:

$$b'_o \text{ equiv.} = 2\left\{\left(\sum_{i=1}^{N}\left(\frac{b'_{o_i}}{2}\right)^2\right)/N\right\}^{1/2} \qquad (9)$$

where

b_{o_i} = initial thickness of the i^{th} aquitard,

N = the number of aquitards, and

b'_o equiv. = the equivalent thickness of the aquitard.

Both of Helm's models (the linear and nonlinear) were calibrated with data from the San Joaquin Valley in California. A mean error of less than two percent was obtained between predicted and observed values of subsidence, while a maximum deviation of less than seven percent was observed between individual values of subsidence.

Although several models describing land subsidence are available, attempts to forecast the subsidence resulting from different rates of water withdrawal are virtually nil. Consequently, no attempts have been made to determine the optimal amounts of water withdrawal to reduce subsidence; even though this has been suggested as a matter for further research by many people. Another problem which has been suggested for further research is to analyze the compaction resulting from seasonal variations in piezometric head. It has been suggested that subsidence would progress because of such fluctuations.

Optimization Techniques

No direct applications of optimization techniques to land-surface subsidence were found in the literature. However, numerous applications of optimization techniques to other water resource development problems have been used, and they are available for developing an optimization procedure for land subsidence problems. Linear programming, non-linear programming, dynamic programming and other mathematical techniques have been extensively applied to water development problems.

During the last decade, many water problems have been studied using a systems analysis approach. One of the first treatises on water-resource systems [Mass et al. 1966] noted the necessity of a systematic and economical approach to the analysis of water problems. Mass et al. [1966] gave a detailed description of the applicability of dynamic programming in analyzing the conjunctive use of surface water reservoirs and groundwater. Dracup [1966] applied parametric linear programming to obain an optimum use for a groundwater and surface water system. Burt [1967] used quadratic criterion functions to solve groundwater management problems.

A complete analysis of water resource problems from a systems point of view was presented by Hall and Dracup [1970]. They defined briefly, but precisely, all the concepts required to understand most of the methods and approaches used in water resource systems engineering. Excellent example problems and references were given.

Haimes [1970] presented an overall picture of system analysis, and then discussed the mathematical programming methods available. Buras [1972] provided a detailed discussion of the methodologies available for optimally allocating water resources. Buras [1972] also presented examples of the use of linear programming as applied to optimization problems on water quality management, aquifer management, and the design and operation of reservoirs. In addition, Buras [1972] illustrated the use of dynamic programming, monte carlo simulation, generation of synthetic hydrological data, and the use of stochastic techniques in water resource investigations.

Domenico [1972] analyzed many of the previously used concepts, and showed how and when the mathematical programming methods (linear programming and dynamic programming) should be used. Domenico [1972] stressed that a close relationship with economic concepts must be maintained and that clear objectives are an absolute necessity for the success of systems analysis.

Aguado and Remson [1974] used groundwater variables directly as decision variables in a linear-programming management model. Finite difference approximations of the governing differential equations were used as constraints in the linear programming formulation. Aguado et al. [1974] applied the same model to obtain optimal pumping rates for dewatering a construction site.

Stochastic programming [Chow, 1970; Curry, Helm and Clark 1972] and game theory [Clyde, 1970] have also been used to study water allocation problems. Also, parametric programming and quadratic programming have been used to formulate water management problems [Meier, Helm and Curry, 1973]. An extensive analysis of the applicability of geometric programming to water resource system optimization was made by Meier, Shih, and Wray [1971]. Separable programming was used to optimize and forecast alternatives for water supply allocation in extensive areas of Venezuela [Dérédec et al., 1975]. Howe [1976] made a valuable introduction to economic modeling by suggesting some approaches for evaluating the social aspects of water resource development.

Taha [1971] described several methods of operation research which are available. In particular, Taha [1971] described the procedures and advantages of parametric linear programming, a technique which was used extensively in this research.

Economic Impact of Land-Surface Subsidence

The economic impact of land-surface subsidence has been given some consideration, but detailed economic analysis of land subsidence is lacking. The economic impact depends on the characteristics of the subsidence area and property values. The threat to human life may be a consideration in some cases. Literature on the economics of land subsidence is notably lacking. Only one paper on economics [Jones, 1976] was presented at the last International Symposium on Land-Surface Subsidence in December of 1976.

Dramatic cases of land subsidence caused by groundwater withdrawals have been reported in Long Beach Harbor, California [Poland and Davis, 1956; Poland, 1961; IASH, 1976]; the Texas Gulf Coast [Winslow and Wood, 1959; IASH, 1976]; Savannah, Georgia [Davis et al., 1963; IASH, 1976]; Las Vegas, Nevada [Domenico et al., 1965]; and the San Joaquin Valley in California [Helm, 1974]. Around the world, the most prominent land subsidences sites are Tokyo, Japan [IASH, 1970; IASH, 1976]; Venice, Italy [IASH, 1976]; and Mexico City, Mexico [Cuevas, 1936; IASH, 1976].

In the Houston, Texas area, several valuable studies of land subsidence have been made. Gabrysch and Bonnet [1975] described in detail the land subsidence problem in the Houston region, and located the most critical areas of subsidence. Previously, Winslow and Wood [1959] analyzed the subsidence problem caused by excessive groundwater pumpage in the Houston area.

Warren et al. [1974] surveyed a 300 square mile subarea within the 3000 square mile area affected by land subsidence around Houston, Texas. Using a questionnaire method, they estimated that land subsidence caused an estimated $113 million in damages between 1943 and 1973. They also estimated that $53 million in damages were caused by a six foot tide which occurred during tropical storm Delia in 1973. A projection of their estimates indicated that similar tides would cause a loss of $27 million every five years for each foot of subsidence, and that non-tidal damages would add an additional $13 million every five years per foot of subsidence.

Jones and Larson [1975] used the method of Warren et al. [1974] in more detail, and estimated an annual cost and property value loss of over $32 million per year in a 945 square mile subsidence area near Houston, Texas. They confirmed that losses would be greater in the waterfront areas along the bays. Finally, they used a break-even analysis to estimate the public needs for importation of surface water, which would reduce the total water cost to the Houston area.

No attempts to forecast the long-term economical effects of different groundwater management alternatives have been made. Also, social factors have not been included in the economic evaluations of land-surface subsidence. However, Warren et al. [1974] and Jones and Larson [1975] classified data on volumes of water consumption per area and per type of user. This information is indispensable before attempting any consideration of social problems associated with land subsidence. Andrews [1970] made an interesting analysis of social issues in water resource development, and gave enlightening ideas for a socio-economic approach to the problem. New estimates of economic losses caused by land subsidence in the Houston area are expected after a programmed releveling survey by the National Geodetic Survey is completed [Spencer, 1977].

Legislation on Land Subsidence

Legislation to control subsidence is a much discussed topic. In both surface water and groundwater management, existing laws create conflicting situations. Biswas [1976] analyzed groundwater rights all over the United States, and noted the many problems which exist with groundwater development. Two contrasting doctrines of law were found: (1) the common law which has the basic premise that groundwater is the absolute property of the underlying land owner, in perpetuity; and (2) the doctrine of prior appropriation which has the basic premise that groundwater is the absolute property of the state and the individual appropriates a right to use for beneficial purposes a specified quantity of groundwater in perpetuity. Texas uses the common law doctrine.

From a legislative point of view, Radosevich and Sabey [1977] explored the conflict created by a reallocation of water from some users to others. They emphasized the case of the public trust doctrine, which limits private property rights and protects and strengthens social rights.

Using the San Joaquin Valley, California as a model, Singer [1976] presented the following views: (1) subsidence related to man's activities should be subject to regulation of some kind, including pecuniary liability for causing damage to another's property and the injunctive process; (2) land subsidence creates an environmental impact and should be recognized as a form of pollution so that state and federal governments could require preparation of environmental impact statements prior to undertaking any groundwater development; and (3) the federal congress and state legislatures should be made aware of the extent of the damage caused by subsidence, both physical and monetary.

In 1975, the Harris-Galveston County Subsidence District was created to study subsidence and develop rules and regulations concerning withdrawals of groundwater to control land subsidence in Harris and Galveston counties, Texas [Spencer, 1977]. This district has had a great impact on groundwater management in the Houston area. Brah and Jones [1978] working on the Upper Galveston Bay Region of Texas, made a detailed analysis of alternative arrangements of legal, economic, and political institutions with the capacity and ability to manage water resources to abate and control land subsidence.

Conclusions

From the preceding literature review on land-surface subsidence caused by groundwater withdrawals, the following facts were revealed:

1. The present knowledge on land subsidence, from a theoretical point of view, is greatly advanced and detailed, so that applied studies are feasible and should constitute the main core of any work presently being done on land subsidence.
2. Mathematical modeling has been developed from many points of view, and the prediction of land subsidence is possible with a considerable degree of accuracy.
3. Because of the lack of detailed economic information in areas affected by land-subsidence, the application of operation research techniques is difficult. More work on evaluating the economic impact of land subsidence is needed.
4. Good legislation on land-surface subsidence as a water management problem is impossible without a solid basis for decision making. This basis should be provided by studies of the subsidence problem using system analysis.
5. With sufficient economic and hydrologic information, the land subsidence problem can be mathematically formulated and simulated. Different policies of water management can then be tested, economically studied, and an optimum policy obtained.

Chapter III

MATHEMATICAL AND NUMERICAL MODELS

The formulation of an optimization scheme to reduce land-surface subsidence requires that values of piezometric head (or water pressure) and land subsidence be predicted as functions of space, time, and amount of groundwater withdrawn. Consequently, three different models must be solved successively. First, a hydrologic model is used to predict the piezometric head (water pressure) within the aquifer-aquitard system. In the present study, a two-dimensional vertical hydrological model is used. Second, a compaction model is used to evaluate the amount of subsidence caused by the changes in water pressure. Third, an optimization model is used to transform the subsidence data into dollar values, feed information into a linear programming formulation, and determine the optimal mix of surface water and groundwater which will minimize the overall water cost for a region.

Hydrologic Model

The general flow equation for predicting the piezometric head in a groundwater aquifer is derived by combining the equation of mass conservation and Darcy's Law [Domenico, 1972; Reddell and Sunada, 1970; Remson at al., 1971]. The resulting equation involves piezometric head as a dependent variable, as shown in Appendix II. The resulting vertical, two-dimensional flow equation is

$$\frac{\partial}{\partial x}\left(\Delta z\ K_x \cdot \frac{\partial H}{\partial x}\right)\Delta x + \frac{\partial}{\partial z}\left(\Delta x\ K_z \cdot \frac{\partial H}{\partial z}\right) = \rho g\ \Delta x\ \Delta z(\alpha+\phi\beta)\ \frac{\partial H}{\partial t} \pm Q, \tag{10}$$

or in vector notation

$$\nabla \cdot (K\ \nabla H) - \rho g(\phi\beta+\alpha)\ \frac{DH}{Dt} = 0, \tag{11}$$

given that

$$\alpha = \frac{\Delta e}{\Delta \delta^{I}}\ \frac{1}{1+e} \tag{12}$$

and

$$\phi = \frac{e}{1+e}\ , \tag{13}$$

where

x = horizontal coordinate (L),

z = vertical coordinate (L),

Δx = incremental horizontal distance (L),

Δz = incremental vertical distance (L),

H = piezometric head (L),

K_x = hydraulic conductivity in horizontal direction (LT^{-1}),

K_z = hydraulic conductivity in vertical direction (LT^{-1}),

ρ = fluid density (ML^{-3})

g = gravitational acceleration (LT^{-2}),

α = formation compressibility factor (L^2F^{-1})

ϕ = porosity (·),

β = compressibility of the fluid (L^2F^{-1}),

Q = fluid production term ($L^3L^{-1}T^{-1}$),

e = void ratio (·),

Δe = change in void ratio (·),

δ^1 = effective stress (FL^{-2}), and

t = time (T).

Considering that the relationship

$$S_s = \rho g(\alpha+\phi\beta) \tag{14}$$

defines the specific storage coefficient (L^{-1}), equation (10) becomes

$$\frac{\partial}{\partial x} (\Delta z \; K_x \frac{\partial H}{\partial x}) \; \Delta x + \frac{\partial}{\partial z} (\Delta x \; K_z \frac{\partial H}{\partial z}) \; \Delta z = \Delta x \; \Delta z \; S_s \frac{\partial H}{\partial t} \pm Q. \tag{15}$$

Equation (15) can be written with water pressure as the dependent variable by using the following relationship,

$$H = \frac{u}{\rho g} + z \; , \tag{16}$$

where

H = the piezometric head at a point (L),

u = the water pressure at a point (FL^{-2}), and

z = the elevation of the point above an arbitrary datum (L).

Equation (15) was used in this study to calculate changes in piezometric head caused by groundwater withdrawals. The changes in piezometric head were then related to changes in water pressure using equation (16), and the changes in water pressure were used to calculate compaction.

An implicit finite difference form of equation (15) was used to calculate the changes in piezometric head. The finite difference equation had the form:

$$\overset{+}{N_x} H^{t+\Delta t}_{i,j+1} + \overset{-}{N_x} H^{t+\Delta t}_{i,j-1} + \overset{+}{N_z} H^{t+\Delta t}_{i+1,j} + \overset{-}{N_z} H^{t+\Delta t}_{i-1,j}$$

$$-(\overset{+}{N_x} + \overset{-}{N_x} + \overset{+}{N_z} + \overset{-}{N_z} + M) H^{t+\Delta t}_{i,j} = M H^{t}_{i,j} \pm Q \; , \tag{17}$$

where $\overset{+}{N_x}$, $\overset{-}{N_x}$, $\overset{+}{N_z}$, and $\overset{-}{N_z}$ are terms of the form:

$$\overset{+}{N_x} = \frac{2K_{x_{i,j}} \; K_{x_{i,j+1}}}{K_{x_{i,j}} \cdot \Delta x_{i,j+1} + K_{x_{i,j+1}} \cdot \Delta x_{i,j}} \cdot \Delta z_{i,j+1/2} \tag{18}$$

and

$$M = \frac{(S_s \; \Delta x) i,j}{2 \; \Delta t} \; (\Delta z_{i,j+1/2} + \Delta z_{i,j-1/2}) \quad . \tag{19}$$

The development of equations (17), (18), and (19) is given in Appendix III and IV and a detailed derivation of each can be found in Reddell and Sunada [1970] and Remson et al. [1971]. The finite difference form of the flow equation as defined in equation (17) was used to calculate the distribution of piezometric heads in a flow field which was divided into a m x n grid system (Figure 4). The equation was written for the center of each grid, and resulted in a set of mn algebraic equations with mn unknown values of H. This set of equations was written in matrix form as

$$[A]\ \{H^{t+\Delta t}\} = \{rhs\} \quad , \tag{20}$$

where

[A] = a mn x mn matrix containing the coefficients of unknown piezometric heads,

$\{H^{t+\Delta t}\}$ = a mn column vector containing the unknown piezometric heads at time $(t+\Delta t)$, and

{rhs} = a mn column vector containing the piezometric heads at time (t) and production term (Q).

Equation (20) was solved implicitly using Gauss elimination. A solution of equation (20) at each time step gave values for the piezometric head at the center of each grid in Figure 4; sand layers as well as clay layers were included.

Compaction Model

To compute the compaction in a double draining aquitard caused by a known change in applied stress at the aquitard boundary, the following concepts are used:

(1) The applied stress at the boundaries of the aquitard is caused by changes in water pressure within the groundwater

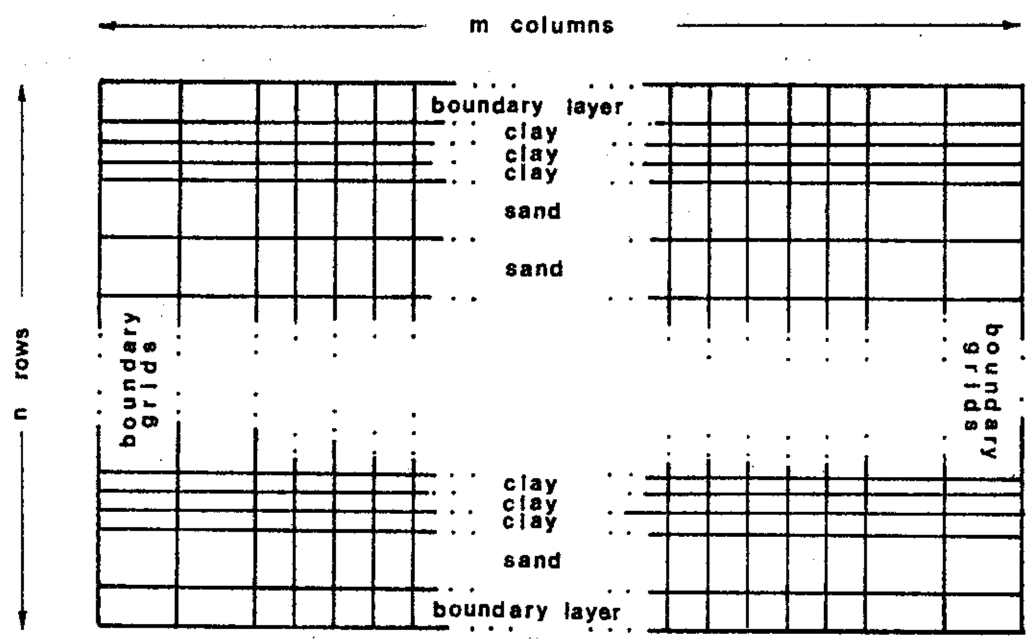


Fig. 4. m × n grid system.

flow field. The water pressures calculated using equation (16) are converted to effective stress by using the follow- relationship,

$$p' = p - u \tag{21}$$

where

p = total pressure (F/L^2),

p' = effective pressure (F/L^2), and

u = water pressure (F/L^2).

The total pressure, p, is constant with time and is determined from the initial conditions in the aquifer-aquitard system.

(2) A relationship exists between effective pressure and mean strain. Figure 5 shows the relationship between contraction (expressed as mean strain) and effective pressure for a saturated volume element of clay undergoing compaction. The solid curve in Figure 5 represents the real consolidation cycle from the virgin compression stage (i.e., compaction of a non-preconsolidated clay) to a decompression or rebound stage caused by decreasing effective pressures. A recompression stage can occur because of increasing effective pressures. In the present study the real cycle shown by the solid line in Figure 5 was represented by the linear approximation shown by the dashed line. In using these linear approximations, the hysteresis cycle between decompression and recompression is neglected.

(3) A relationship exists between compaction and the changing effective pressures. The model keeps a history of the consolidation and checks the situation at any time step. By comparing the current effective pressure with the maximum

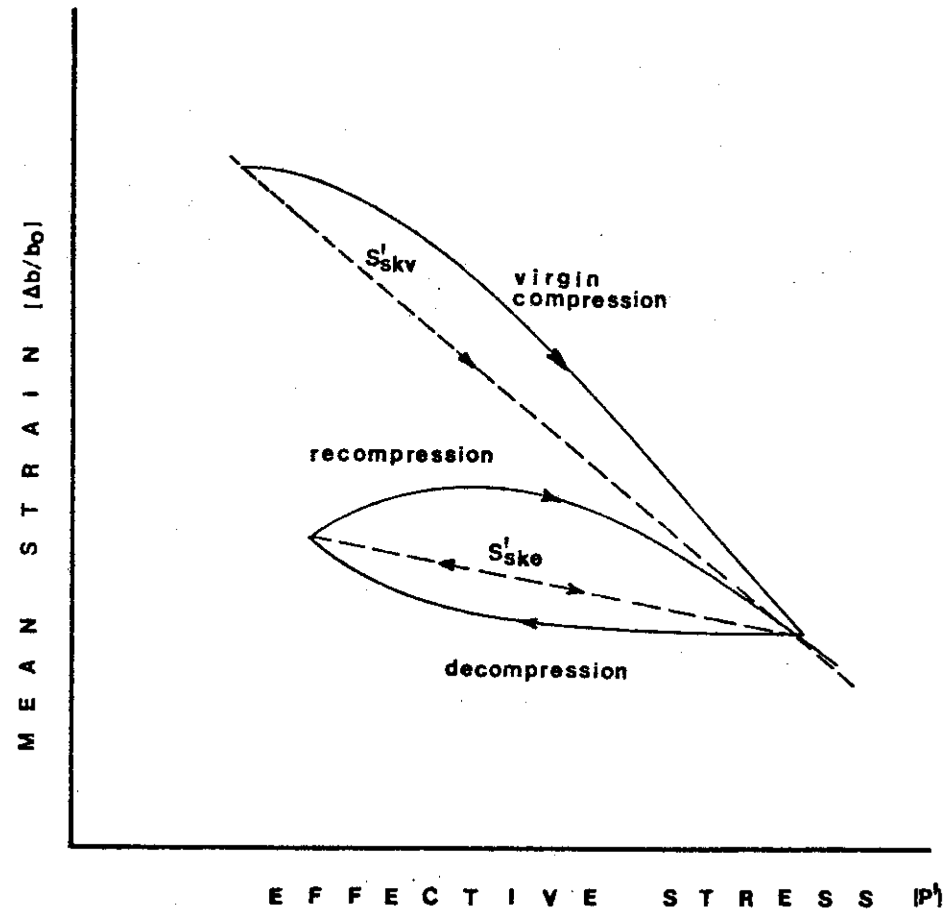


Fig. 5 Relationship between compaction (expressed as mean strain) and effective stress.

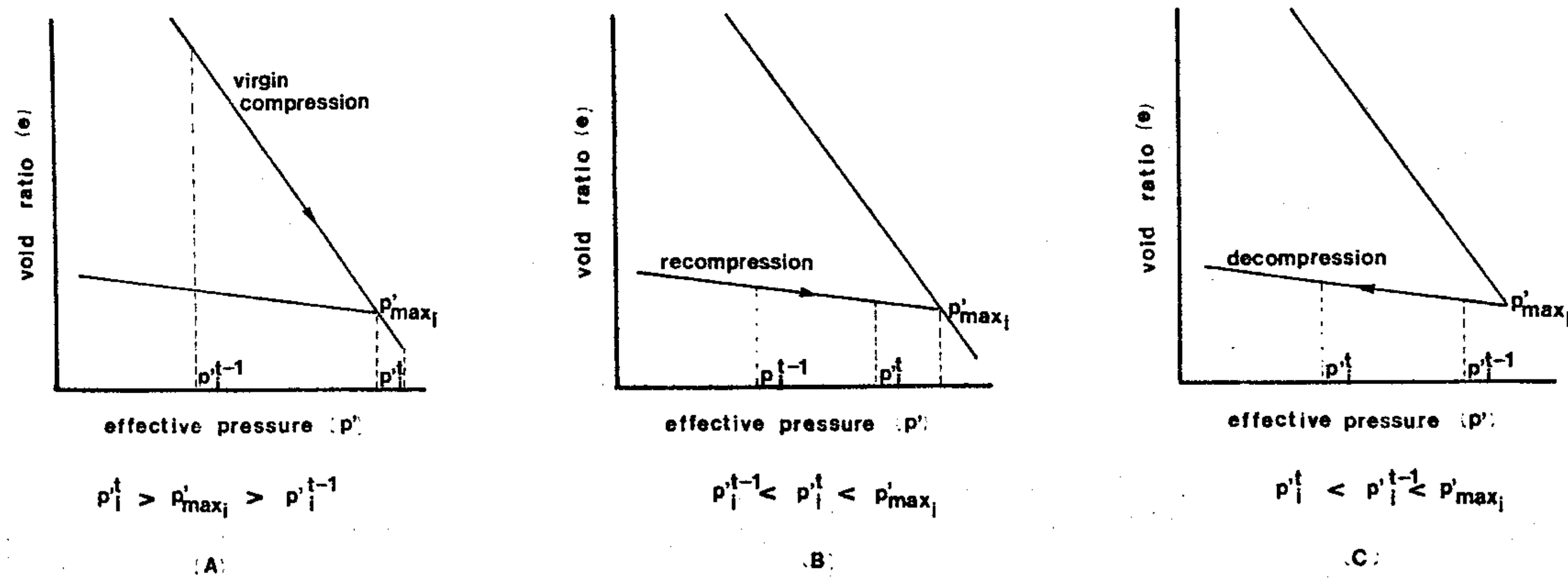


Fig. 6 Stress variations and relationships to keep a history of the consolidation process.

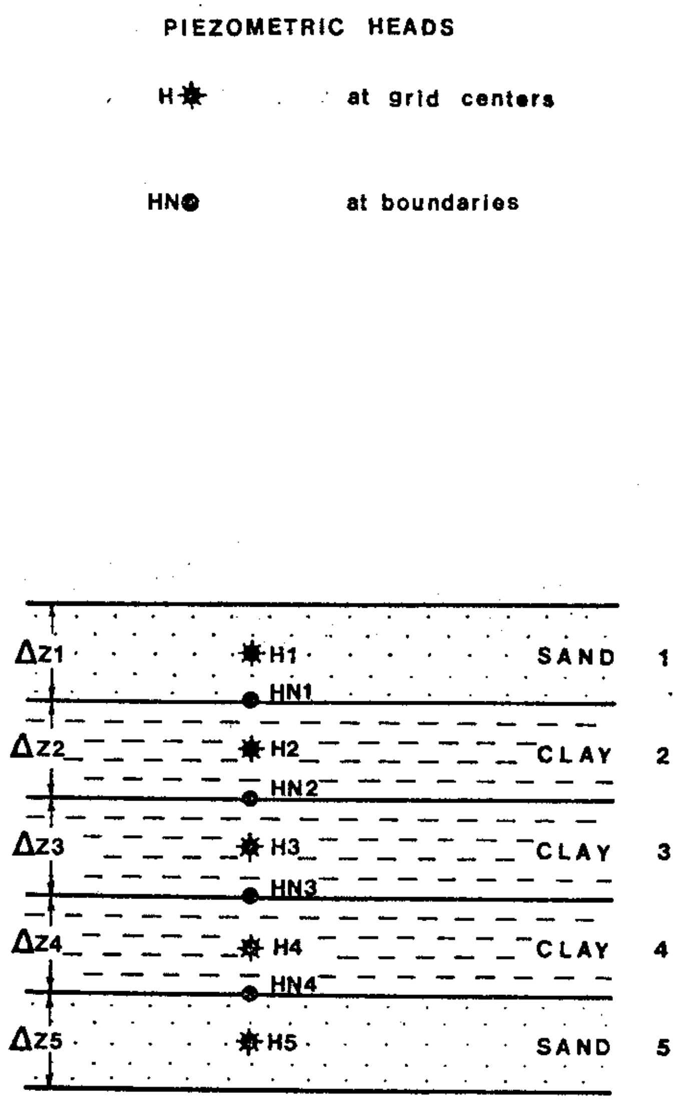


Fig. 7. Interpolation scheme used to obtain effective stresses at the boundaries of the clay layers.

The effective stress at the clay boundaries is then defined as

$$P'N_4 = P_4 - UN_4 \tag{26}$$

and

$$P'N_3 = P_3 - UN_3 \ , \tag{27}$$

where P is the total pressure obtained from the initial conditions.

(5) Each clay layer undergoes compaction due to the effective pressure gradient between its boundaries. Equation (28) was written to calculate the compaction of the clay layer at each time step, based on the concepts described above:

$$\Delta b = b[S_{skv}(P'^{t}\ max - P'^{o}\ max) - S_{ske}(P'^{t}\ max - P'^{t})] \tag{28}$$

where

Δb = compaction of a single clay layer (L),

b = thickness of clay layer ($z_4 - z_3$, etc.) (L),

S_{skv} = virgin storage coefficient (L^2/F),

S_{ske} = elastic storage coefficient (L^2/F),

P'^{t}_{max} = maximum effective pressure at time t (F/L^2),

P'^{o}_{max} = maximum effective pressure prior to time t (F/L^2), and

P'^{t} = the current effective pressure.

At each time step equation (28) is evaluated for each clay layer and summed to obtain the total compaction after that time step. The total compaction at each time step can be written as

$$\Delta z_c = \frac{1}{2} \sum_{i=1}^{I-1} (\Delta z_{i+1/2} + \Delta z_{i-1/2}) \; [S_{skv_i} \; (P'^{t} max_i - P'^{o} max_i)$$

$$- S_{ske} (P'^{t} max_i - P_i'^{t})] , \qquad (29)$$

where

Δz_c = compaction or land subsidence over several clay layers (L),

$\Delta z_{i+1/2} = z_i - z_{i+1}$,

$\Delta z_{i-1/2} = z_{i-1} - z_i$ and

S_{skv_i} = the virgin storage coefficient at node i, considered to be constant throughout the layer.

In this study the elastic storage coefficient was considered equal and constant for all the clay layers. The theoretical development of equation (29) can be found in Helm [1974]. Equation (29) was written in the following form for use in the compaction model.

$$\Delta z_c = \frac{1}{2} \sum_{i=1}^{N} (z_{i+1/2} + z_{i-1/2}) \; (S_{skv_{i,j}} \cdot \Delta PV_{i,j} - S_{ske} \cdot \Delta PE_{i,j}), \qquad (30)$$

where

$\Delta PV_{i,j} = P'^{t} max_{i,j} - P'^{o} max_{i,j}$ (FL^{-2}) , and

$\Delta PE_{i,j} = P'^{t} max_{i,j} - P_{i,j}'^{t}$ (FL^{-2}) .

Equation (30) is subject to the following constraints:

(a) If i = 1, $\Delta z_{i-1/2} = 0$, (31)

(b) If i = N, $\Delta z_{i+1/2} = 0$ (32)

(c) If $P_{i,j}'^{t} \geq p'^{t} max_{i,j}$ then $\Delta PE_{i,j} = 0$, and

$\Delta PV_{i,j} = P_{i,j}'^{t} - P'^{o} max_{i,j}$, (33)

(d) If $P'^{t}_{i,j} < P'^{t}_{i,j} \leq P'^{t}\, max_{i,j}$ then

$$\Delta PV_{i,j} = 0 \text{ , and } \Delta PE_{i,j} = P'^{t-1}_{i,j} - P'^{t}_{i,j} \text{ ,} \tag{34}$$

(e) If $P'^{t-1}_{i,j} < P'^{t}_{i,j} > P'^{t} max_{i,j}$ then

$$\Delta PV_{i,j} = P'^{t}_{i,j} - P'^{t}_{max_{i,j}} \text{ , and } \Delta PE_{i,j} = P'^{t-1}_{i,j} - P'^{t}_{max_{i,j}} \tag{35a}$$

(f) If $P'^{t}_{i,j} < P'^{t}_{i,j} < P'^{t} max_{i,j}$ then

$$\Delta PV_{i,j} = 0 \text{ , and } \Delta PE_{i,j} = P'^{t-1}_{i,j} - P'^{t}_{i,j} \text{ ,} \tag{35b}$$

(g) It was assumed that sands do not undergo compaction because of changes in effective stress.

OPTIMIZATION MODEL

In an area affected by land-surface subsidence, two principal sources of water supply are considered: groundwater and surface water. Groundwater withdrawal causes land subsidence, but it has a low direct cost (pumping and delivering) along with the possibly higher indirect cost of subsidence.

Surface water (at a reservoir) has a higher direct cost than groundwater, but it has no indirect cost for subsidence.

The breakeven value between the two water sources (the maximum amount of groundwater which can be pumped without the groundwater cost exceeding the surface water cost) can be determined by simulating subsidence at different pumping rates and transforming the subsidence values into dollar values by using local subsidence costs. The point where the cost of groundwater equals the cost of surface water determines the maximum rate of groundwater pumpage allowed at that point (breakeven point).

Formulation of the Problem

The following scheme of linear programming will be used:

$$\text{Max } Z = - \underline{C}\,\underline{X}$$

such that

$$[A]\underline{X} \leq \underline{P}_o \,,$$

and

$$\underline{X} \geq \underline{0} \,,$$

where

$\underline{C}$ = vector of cost coefficients of the objective function ($/cubic meters),

$\underline{X}$ = vector of variables (cubic meters),

$[A]$ = matrix of constant coefficients (0 or 1), and

$\underline{P}_o$ = vector of right hand side values (cubic meters).

This is equivalent to

$$\text{Min } Z = \underline{C}\,\underline{X} \tag{36}$$

such that

$$[A]\underline{X} \leq \underline{P}_o$$

and

$$\underline{X} \geq \underline{0} \,.$$

The objective function, Z, is a linear approximation of a sum of nonlinear functions representing the cost of total subsidence at the surface by varying the amount of water pumped through a fixed period of time. Consequently, the vector $\underline{C}$ contains the total cost per volume of water pumped (see Appendix V); including transportation costs and pumping costs. The vector $\underline{X}$ contains the volumes of water pumped and delivered from one point to another. Surface water is treated as an additional water source with a particular set of associated cost coefficients.

The matrix [A] is a coefficient matrix containing ones for active variables, and zeroes for non-active variables. The matrix [A] contains two types of constraints. The first type corresponds to the water needs at every point of the simulation grid. The second type corresponds to the breakeven points obtained from the linear approximation of the objective function. The vector $\underline{P}_o$ contains the values on the right hand side of the constraint equation. The solution scheme requires that the vector $\underline{X}$ have only positive or zero components.

The linear programming routine (MPSX/IBM) was used to solve the problem. This routine allows a parametric analysis of the problem. Simultaneous changes in the coefficients of the objective function ($\underline{C}$) and in the right hand side of the constraint equation ($\underline{P}_o$) will be made to illustrate the sensitivity and capabilities of the proposed scheme for evaluating various water management programs in the land subsidence area.

Optimization Procedure

The results from the hydrologic and compaction models are summarized as curves [f(x)] of total land surface subsidence costs versus the volume of water pumped during a fixed period of time (10 years). An example of this type curve is shown in Figure 8. A linear approximation of curve [f'(x)] is shown by the dotted lines. Separable programming is used to obtain values for the coefficients and break even points in such linear approximations.

The expression $F(x) = \sum_{i=1}^{N} f_i(x)$ represents the nonlinear function obtained from simulating the phenomena, and

$$f_i(x) \sim f_i'(x) = C_o + \sum_{K=1}^{N} \rho_i^K x_i^K = C_o + \rho_i^1 X_i^1 + \rho_i^2 X_i^2 + \rho_i^3 X_i^3 + \ldots\ldots \rho_i^n X_i^n \tag{37}$$

is the linear approximation of the curve, where ρ_i^k are the slopes of the approximating linear segments, a_k are the break points of the approximations, and C_o is the value of the approximated function at the lower value for x. The segment variables X_i^k must satisfy the following constraints

$$0 \leq X_i^k \leq a_i^k - a_i^{k-1} \tag{38}$$

The variable X_i must also be segmented into (N) components, representing the amount of groundwater pumped from point (i) and delivered to the other (N) points (including point (i) itself). So,

$$X_i = \sum_{j=i}^{N} X_{ij} \ . \tag{39}$$

Combining expressions (37) and (39), the result can be written as

$$f_i'(x) = C_o + \sum_{j=1}^{N} \sum_{k=1}^{N} \rho_i^k X_{ij}^k \ , \tag{40}$$

where expressions (38) results in

$$0 \leq \sum_{j=1}^{N} X_{ij}^k \leq a_i^k - a_i^{k-1} \ . \tag{41}$$

Finally, dropping constant C_o, the objective function to be minimized by the linear programming formulation can be written as

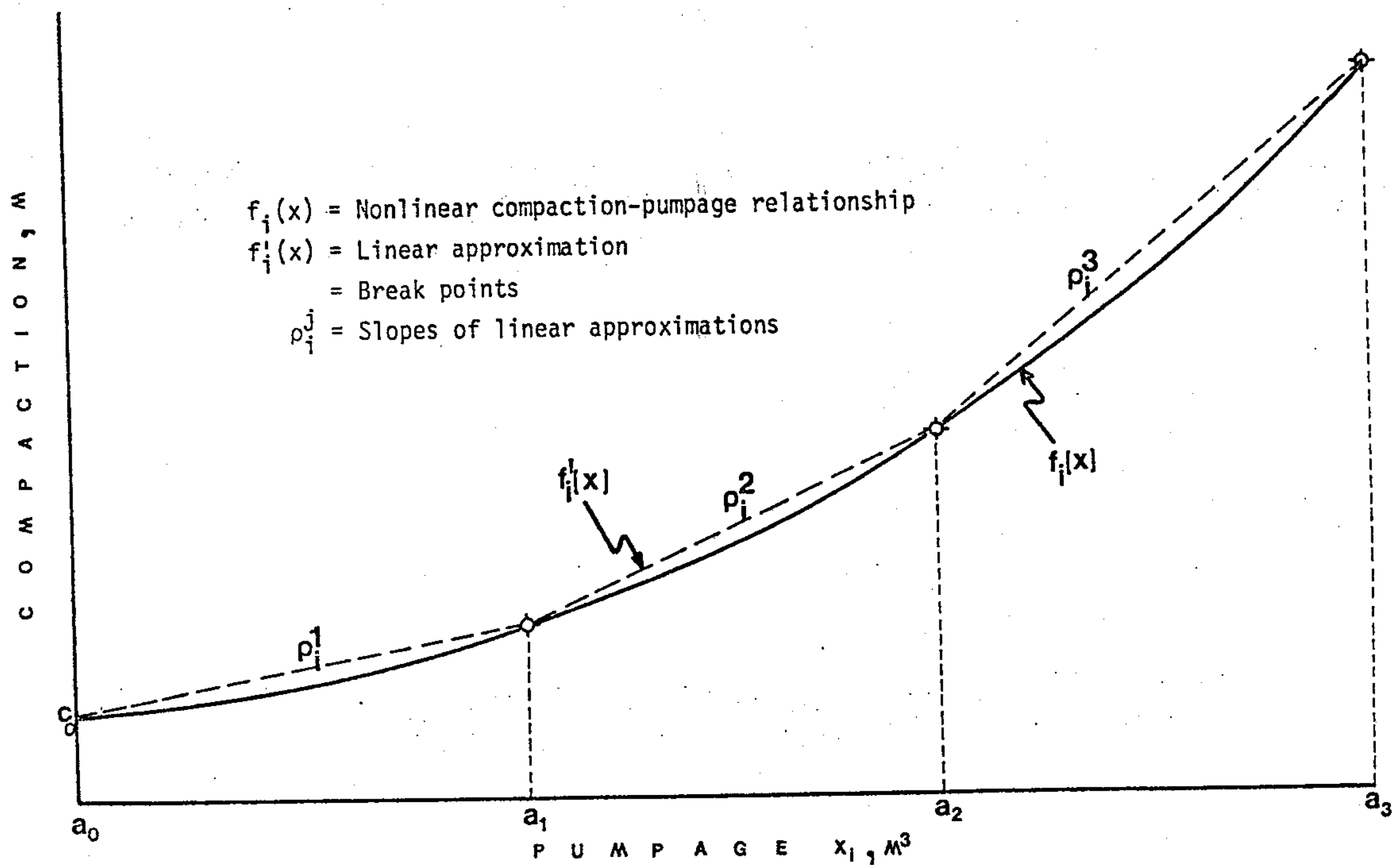


Fig. 8. Linear approximations to compaction-pumpage relationship

$$Z = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{K} (C_{ij} + G_w + \rho_i^k) X_{ij}^k + \sum_{j=1}^{N} (G_s + C_{sj}) X_{sj} \ , \qquad (42)$$

where

ρ_i^k = slopes of the approximating linear segments ($/cubic meter)

C_{ij} = cost of transporting groundwater from point (i) to point (j) ([$/km-cubic meter][distance (km)]),

G_w = cost of pumping groundwater ($/cubic meter),

G_s = cost of surface water ($/cubic meter),

C_{sj} = cost of transporting surface water to point (i) [($/km-cubic meter) x distance (km)], and

X_{sj} = amount of surface water delivered to point (j) (m^3).

The first type of constraint represents the water needs at every point (i), and may be written as

$$\sum_{i=1}^{N} \sum_{K=1}^{K} X_{ij}^k + X_{sj} \geq \text{needs (j)}. \qquad (43)$$

In addition, the surface water reserves are expressed as

$$\sum_{s} \sum_{j=1}^{N} X_{sj} \leq \text{reserve}. \qquad (44)$$

The second type of constraint is those described by expression (41) and written as

$$\sum_{j=1}^{N} \sum_{k=1}^{K} X_{ij}^k \leq a^k - a_i^{k-1} \quad , \qquad (45)$$

and represents the break points to the linear approximation of the objective function.

The fact that any X_{ij}^k , for all i,j, and k, must be greater or equal to zero, satisfies the left hand side requirement of the inequality of expression (41).

In summary, the problem can be formulated in the following way:

$$\text{Min } Z = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{K} (C_{ij} + G_w + \rho_i^k)X_{ij}^k + \sum_s \sum_{j=1}^{N} (G_s + C_{sj})X_{sj} \tag{46}$$

subject to

$$\sum_{i=1}^{N} \sum_{k=1}^{K} X_{ij}^k + X_{sj} \geq \text{needs } (j), \text{ for all } j \text{ and } s;$$

$$\sum_{j=1}^{N} X_{sj} \leq \text{reserve, for all } s$$

$$\sum_{j=1}^{N} \sum_{k=1}^{K} X_{ij}^k \leq a_i^k - a_i^{k-1} \text{ , for all } i,$$

and

$$X_{i,j}^k \geq 0 \text{ , for all } i, j, \text{ and } k.$$

The problem solved in this study was finally written as

$$\text{Min } Z = \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{K=1}^{K} (C_{ij} + G_w + \rho_i^k) X_{ij}^k + \sum_s \sum_{j=1}^{N} (G_s + C_{sj})X_{sj} \tag{47}$$

subject to

(1) N constraints expressed as

$$\sum_{i=1}^{N} \sum_{k=1}^{K} X_{ij}^k + X_{sj} \geq \text{water needs at point } j \text{ ,} \tag{48}$$

(2) one constraint expressed as

$$\sum_{j=1}^{N} X_{sj} \leq \text{surface water reserve ,} \tag{49}$$

(3) $N + 1 + \sum_{p=1}^{N} K(p)$ constraints expressed as

$$\sum_{j=1}^{N} X_{ij}^k \leq \text{upper bound at point } i \text{ and segment } k \text{ .} \tag{50}$$

The expression (50) is equivalent to expression (45), where K(p) is the number of segments at point (p)

(4) the implicitly assumed $N(1 + \sum_{p=1}^{N} K(p))$ constraints

$$X_{ij}^k \geq 0 \text{ , for all } i, j, \text{ and } k$$

$$X_{sj} \geq 0 \text{ for all } s \text{ and } j.$$

Parametric Analysis

In parametric analysis, a linear programming problem can be defined as

$$\text{Min } Z = (\underline{C + \Theta\Delta C})\underline{X} \ , \tag{51}$$

subject to

$$[A]\ \underline{X} \leq \underline{P}_o + \Theta\Delta P_o \ , \tag{52}$$

$$\underline{X} \geq \underline{0}$$

where

ΔC = a variation in the value of the objective function coefficients, and

ΔP_o = a variation in the values of the right hand side of the coefficients.

The problem is originally solved with $\Theta = 0$. Other values of Θ imply a solution to a new problem. However, the techniques of parametric analysis allows the solution to the new problem to be obtained without really solving them. The analysis may be performed on the objective function coefficients, on the right hand side coefficients of the constraint equations, or on both simultaneously. In the later case, Θ is the same for both the objection function and right hand side. When using the parametric analysis routine, an analysis may be performed using as many sets of ΔC and ΔP_o as desired; as long as they are defined in the data set. Examples of the programs used for parametric analysis in this study are given in Appendix VII.

CHAPTER IV

PROCEDURE

A system analysis approach to natural resource management includes model building, model calibration, and model validation. A tendency to focus on model building and to neglect the use of the resultant model to solve problems was discussed by Biswas [1976]. The usefulness of the model developed in this research depends on the quality of the calibration (matching historical data with simulated model variables) and validation (comparing future model forecasts with actual values) procedures.

To calibrate and validate the model, a conceptual model of the subsidence region at Houston, Texas was developed. Because this research was devoted entirely to developing a procedure for analyzing management policies associated with subsidence, the area itself was greatly simplified and groundwater data were grossly averaged over the area. Development of the model pointed out the need for new data collection efforts, such as water transportation costs, subsidence costs, etc. Lack of this information forced us to assume realistic values for many of the parameters. These approximations will be discussed later in this chapter when the model data are presented.

Model Simulation

A two-dimensional numerical simulator was programmed in WATFIV for the AMDAHL 470 computer at Texas A&M University. A flow chart of the program is shown in Appendix VI, and a reprint of the programs used to (1) solve the general flow equation for piezometric head, (2) predict subsidence, and (3) optimize the water distribution and minimize the overall water cost is given in Appendix VII.

The main program governs the entire sequence of operations. Subroutine READIN reads in the physical data needed to proceed with the simulation. It reads values for hydraulic conductivity, storage coefficient, initial piezometric head, current pumpage rate, total time of simulation, time increment, printout control, specific storage coefficient for the clays (elastic and virgin), and the acceleration of gravity. READIN also calculates the grid dimensions as well as the elevations and thicknesses of the grid elements.

Subroutine MATROP controls the printing of all the two-dimensional matrices of physical data (input data) and current values of the piezometric head and water pressure.

Subroutine MATSOL sets up the coefficient matrix, [A], and the right hand side column vector, {rhs}, for solving implicitly the piezometric head vector, {H}, at time $t+\Delta t$ using subroutine BSOLVE. Subroutine MATSOL, as presently written, calculates the grid-side dimensions and transfers all initial conditions to the array of values at the previous time step. By checking the current status of the boundary-control array, MATSOL calculates the elements of the coefficient matrix, [A], as well as the right hand side values, {rhs}, of equation (20).

For the problem used in this work, all boundary conditions are constant piezometric head boundaries. Subroutine, BSOLVE, solves equation (15) implicitly for values of the piezometric head at time $t+\Delta t$ using Gauss elimination. Values of water pressure are then obtained from the values of piezometric head using equation (16).

Subroutine COMPAC uses values of water pressure from the previous time step and the present time step to calculate subsidence. Subroutine CENTRO calculates the water pressures at the centers of each clay layer by interpolation. Subroutine DELPRE updates the values of maximum water pressures and calculates the difference in water pressure during time step Δt; determining the appropriate sign and location on the compaction curve. Subroutine SUBSID calculates the compaction within each clay layer, and the total subsidence at the land surface.

Subroutines PLOT1, PLOT2, PLOT3, and PLOT4 can be used to make plots of the results. These subroutines were very useful during calibration of the model.

Successive applications of the simulator described above for a fixed time and varying values of pumpage will give the total land surface subsidence (in meters) at every node. These subsidence-pumpage relationships are non-linear, and are externally approximated by linear functions where the subsidence data is converted to dollars.

The economic data are then fed into a linear programming routine (MPSX/IBM) where the overall cost of land surface subsidence is minimized by redistributing the water resources of the area.

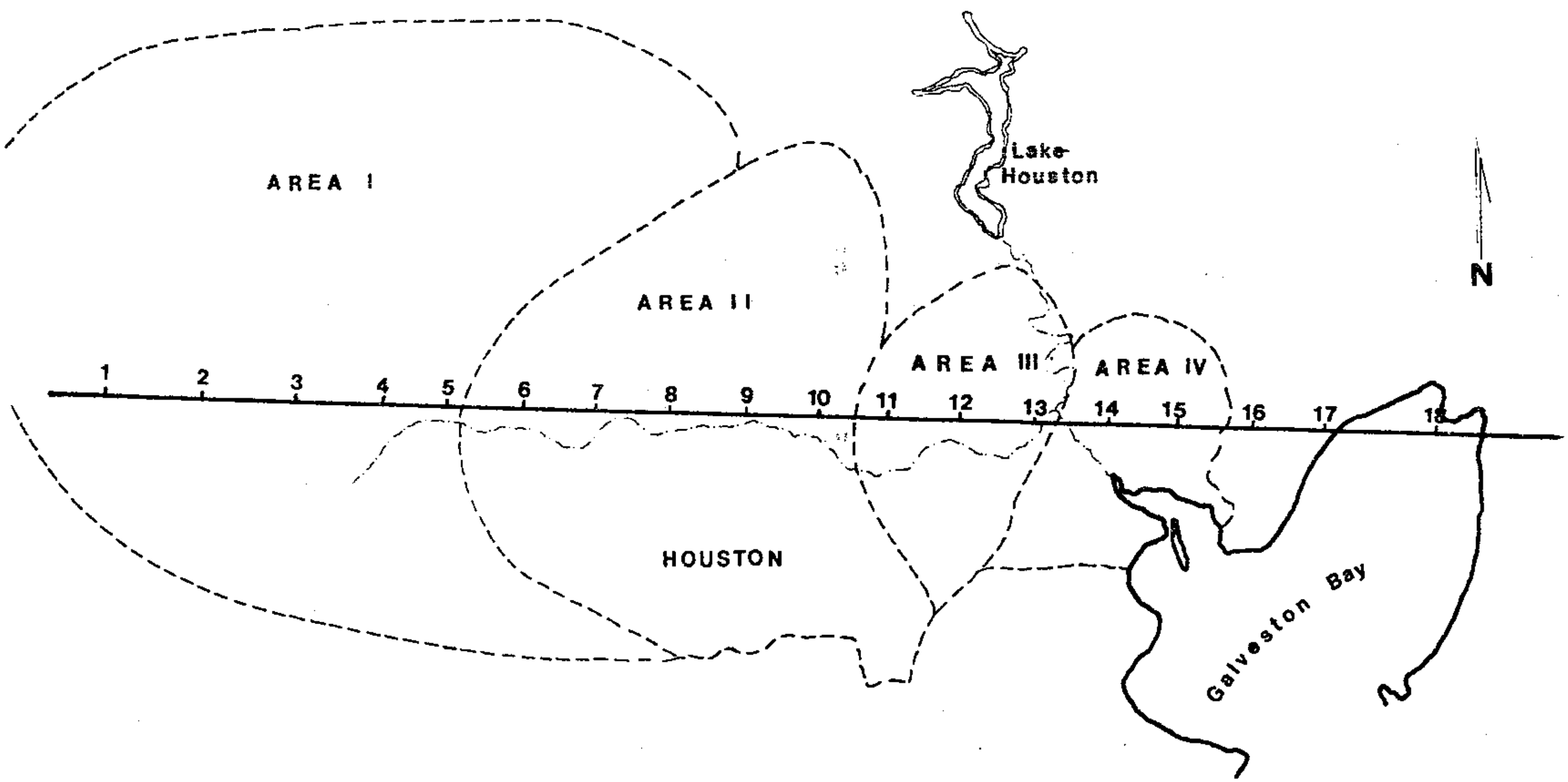
Input data are punched into 80-column cards using a routine described in Appendix VIII. The linear programming routine MPSX/IBM allows a parametric analysis to be made of the problem. Parametric analysis also allows a study to be made of the effects of discrete linear variations in: (1) the right hand side of the constraint equations; (2) the coefficients of the objective function, and (3) simultaneous variations in both the right hand side and objective function coefficients using programs described in Appendix VIII.

To illustrate the capabilities of the proposed procedure, pumpage was eliminated at certain grids by assigning extremely large values to the coefficients in the objective function. The discrete linear variations in the objective function and the right hand side of the constraint equations were arbitrarily established. Also, the water requirements and surface water availability for each node were arbitrarily established.

The linear programming formulation is independent of the method of obtaining total compaction, the number of spatial dimensions, and shape of the grid. Distances between nodes are required to calculate the transportation.

Conceptual Model of Houston Subsidence Area

To study the response of the piezometric surface to groundwater withdrawals in the Houston area, the physical characteristics of the aquitard-aquifer system and associated water withdrawal data must be known. For purposes of this study, a profile line was drawn through the area (Figure 9) and a two-dimensional vertical model was developed along the line. Water wells adjacent to this line were selected for estimating the vertical lithology of the area. In general, the area is divided into two water bearing units, which was determined at each site along the profile. Records of wells drilled in the study area were used to establish this data base



NOTE: The numbers indicate approximate location of the centers of the simulation grid with respect to the economic areas and the whole area of study.

Fig. 9. Area of Study

[Naftel et al., 1976a, 1976b, 1976c]. The elevations of the top and base of each of the water bearing units were taken from maps developed by Jorgensen [1975]. Values of transmissivity (m^2/sec.), storage coefficients, initial piezometric (meters) heads, and pumpage (cubic meter/sec.) were taken from the data used in a model study of the area by Assaf [1976]. Initial values for the specific storage coefficients (1/meter) (virgin and elastic) of the clays were taken from Helm [1974] and Jorgensen [1976].

The collection of economic information was most difficult, and in some instances was nothing more than good estimates. Data related to water consumption, areal distribution of the water consumption and the cost of subsidence were obtained from Warren et al. [1974], Jones and Larson [1975], AWWA [1970, 1976], and Gabrysch and Bonnet [1975]. All the information used was obtained by dividing the area into four sub-areas described by Gabrysch and Bonnet [1975]. In this study, these sub-areas are identified as areas I, II, III, and IV from west to east (Figure 9). To adhere to the two-dimensional nature of this model, the economic data were scaled so that it would be equivalent to a strip one meter wide across the entire area from west to east.

There is a notable lack of good water consumption and cost data for this area. Some of the economic data used in this study comes from estimates made by people in the Houston, Texas area. An example of this is the water transportation costs used as coefficients in the objective function of the linear programming formulation. In general, the economic data used in this study are believed to be realistic.

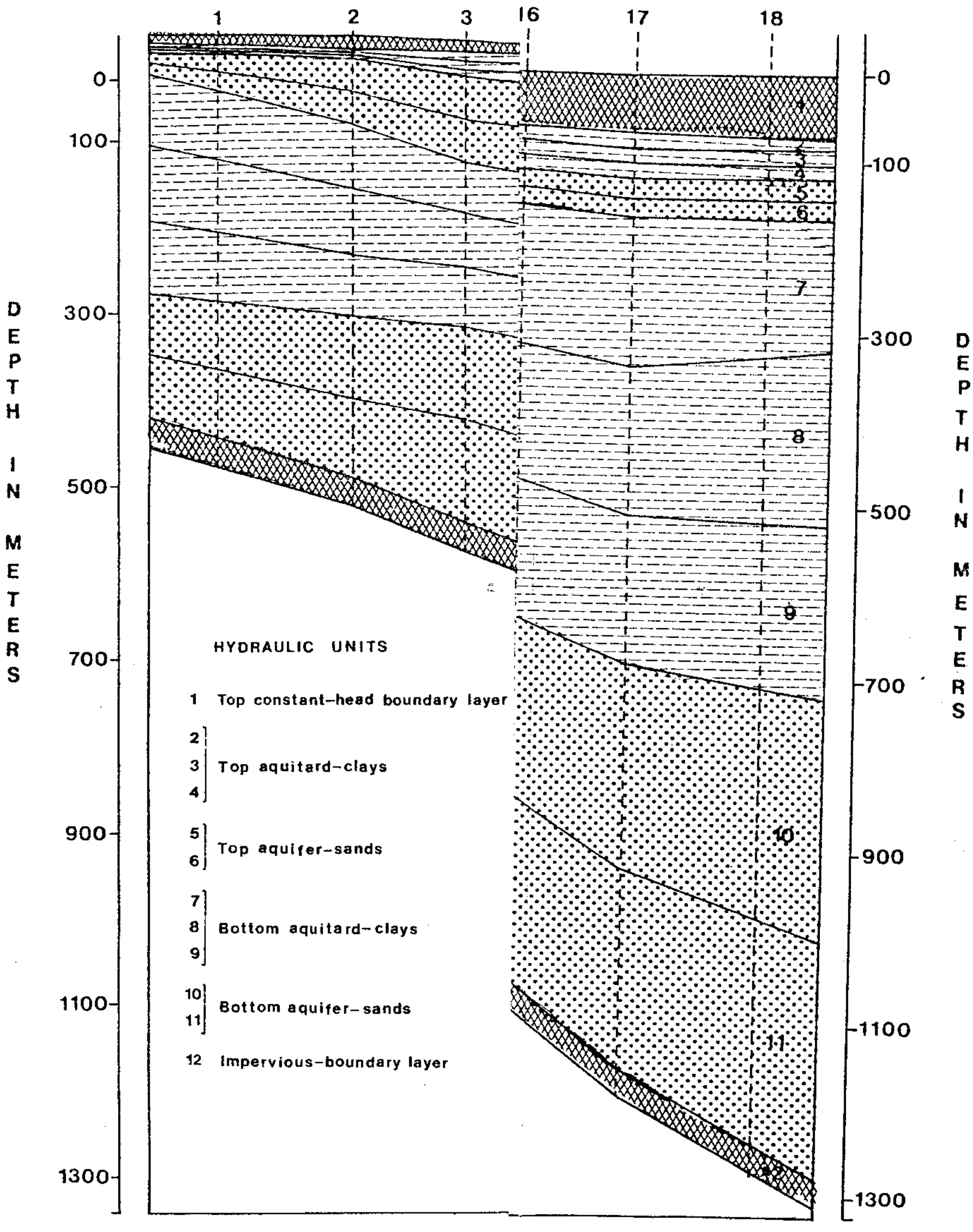
The model chosen included one dimension in the horizontal plane and one dimension in the vertical plane. It was one meter wide in the other horizontal direction and oriented west to east (Figure 9). The model was divided into 18 grids in the horizontal direction and 12 grids in the vertical direction (Figure 10). The simulated area was larger than the study area so that boundary effects could be minimized. Thus, only 15 horizontal nodes were actually considered in the optimization procedure.

The geology of the study area defines two distinct water bearing units. These will be referred to as the upper and lower water bearing units (aquifer). Both units are very similar lithologically and are composed of many alternating layers of sand and clay. It was physically and economically impossible to model each layer of sand and clay. Thus, for simulation purposes, the geological profile was altered considerably. The following procedure was followed in developing the conceptual model of the two water-bearing units as shown in Figure 11:

(1) A study of well logs along the profile provided an estimate of the percent sand and clay in each water-bearing unit.

(2) The total thickness of each water-bearing unit was obtained from maps by Jorgensen [1975].

(3) Both water-bearing units act like confined aquifers. Therefore, the conceptual model was made to simulate a confined aquifer system, with some small amounts of vertical leakage between the two units.



1
2
3
16
17
18
0
100
300
500
700
900
1100
1300
DEPTH IN METERS
-0
-100
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-500
-700
-900
-1100
-1300
DEPTH IN METERS
1
2
3
4
5
6
7
8
9
10
11
12
HYDRAULIC UNITS
1 Top constant-head boundary layer
2, 3, 4 Top aquitard-clays
5, 6 Top aquifer-sands
7, 8, 9 Bottom aquitard-clays
10, 11 Bottom aquifer-sands
12 Impervious-boundary layer

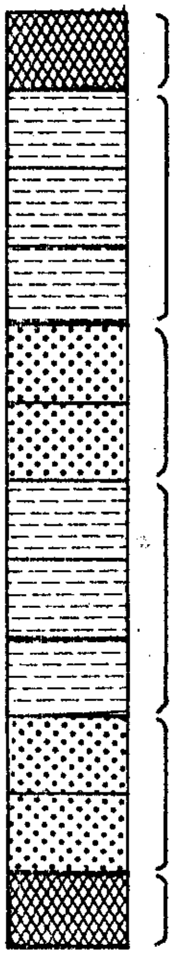


Fig. 11. Vertical profile design of the grid for simulation purposes.

(4) Using the total thickness of the upper unit and the percent sand in the upper unit, the total sand thickness in the upper unit was calculated.

(5) The total sand thickness in the upper unit was then placed into two sand layers (Layers 5 and 6) in the conceptual model.

(6) Using the percent clay and total thickness of the lower unit, the total clay thickness in the upper unit was calculated.

(7) One-half of the total clay thickness of the upper unit was then placed into three clay layers (Layers 2, 3, and 4), which overlay the sand.

(8) Using the total thickness, percent sand, and percent clay for the lower unit, the total thickness of sand and clay in the lower unit was calculated.

(9) The total sand thickness in the lower unit was then placed into two sand layers (Layers 10 and 11) in the conceptual model.

(10) The total clay thickness of the lower unit plus one-half of the total clay thickness of the upper unit were combined and placed into three clay layers (Layers 7, 8, and 9) separating the upper and lower sand units.

(11) An impermeable or no-flow boundary layer was placed along the bottom of the model (Layer 12).

(12) A shallow groundwater aquifer with a constant piezometric head exists throughout the area. To simulate this, a

constant piezometric head layer (Layer 1) was used as a boundary layer at the top of the model.

(13) The grids on the extreme left- and right-hand sides of the model were several kilometers from the pumpage centers. Therefore, they were simulated as constant piezometric heads.

(14) The estimated pumpage from the upper unit was equally divided between Layers 5 and 6.

(15) The estimated pumpage from the lower unit was assigned entirely to Layer 10. No pumpage was made from Layer 11 because of its depth in most parts of the region.

The hydraulic characteristics of each layer are shown in Figure 12.

The nodes were numbered from west to east (left to right), and the extreme nodes, 1 and 18 in Figure 10, correspond to the centers of the constant head boundary layers. All cells are 5560 meters long with the exception of cells 1, 2, 17, and 18, which are 8330 meters long. The thickness of the individual layers vary along the profile.

Linear Programming Model

As previously noted, the number of grid dimensions and their shape is immaterial to the optimization procedure. The only parameter affecting input to the optimization procedure is the total number of nodes (N), because the number of variables and constraints depends almost exclusively on the number of grids. A second factor which determines the number of variables in the linear programming

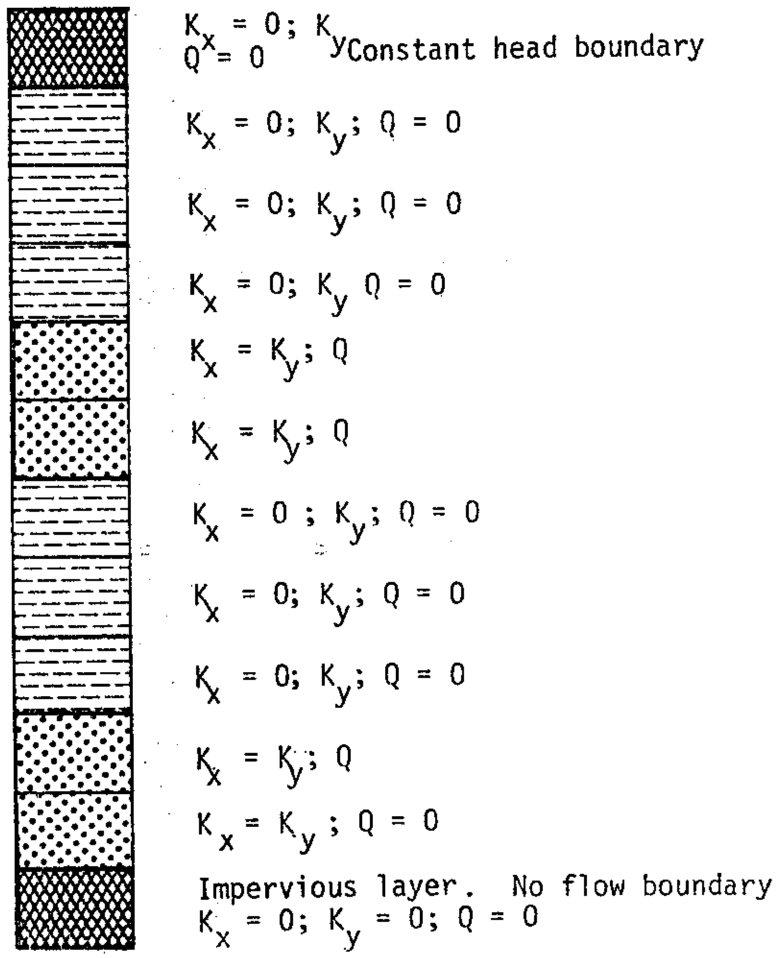


Fig. 12. Characteristics of each layer for simulation purposes.

model is the number of break points, K(N), used to linearly approximate the curve of subsidence cost versus volume of water pumped for each node. If no break-points are used in the linear approximation, the number of linear programming variables will be given by N(N+1). Otherwise, the number of variables is given by $N(1 + \sum_{p=1}^{N} K(p))$. Similarly, the number of constraints is 2N+1 (including the upper bound constraints) when no break-points are used. When breakpoints are used the number of constraints is $N + 1 + \sum_{p=1}^{N} K(p)$. If K(p) = 1 for all values of p , then both cases are the same. Table 1 shows a comparison between the number of variables and constraints for the no break-point case, the two break-point case (the average condition for this study), and the three break-point case. The runs for the most complex cases of parametric analysis were not expensive, and the inclusion of more nodes, as long as economic data allows it, is encouraged.

Table 1 . Comparison of Number of Variables and Constraints in the Optimization Model for Different Number of Break-points in the Linear Approximations.

N^{O} Nodes	N^{O} Variables			N^{O} Constraints		
	No Break-Points	Two Break-Points	Three Break-Points	No Break-Points	Two Break-Points	Three Break-Points
4 (2×2)	20	36	52	9	13	17
16 (4×4)	272	528	784	33	49	65
25 (5×5)	650	1275	1900	51	76	101
36 (6×6)	1332	2628	3924	73	109	145
49 (7×7)	2450	4851	7252	99	148	197
64 (8×8)	4160	8256	12,352	129	193	257

CHAPTER V

RESULTS AND DISCUSSION

The hydrological and compaction models were tested using historical data from the study area. The predicted total compaction values along the simulation line were considered realistic and valid as input to the optimization procedure. Three different water use policies were evaluated using the optimization procedure. The capabilities of the model and its possibilities for further use are discussed, as well as its limitations and possible improvements for more specific applications.

Simulation of Subsidence in the Area

The hydrological and compaction models were checked by observing their behavior under the following conditions:

(1) The hydrological model was run for a period of 20 years without pumpage. Under this condition, a steady-state condition was achieved, and the corresponding piezometric heads were used as initial conditions for subsequent runs.

(2) After the steady-state condition described above was reached, pumping was initiated from element 9 at the rate of 305.8 M^3/yr. This system was tested by observing the change in drawdown with distance from element 10. For the lower layer sand (Layer 10 and 11), the drawdown versus distance from the pumping center is shown in Figure 13. The effects of the constant head boundary

conditions at element 1 and 18 were easily observed.

(3) Finally, pumpage was discontinued and compaction was calculated for successive 10 year periods. Figures 14 and 15 show compaction in element 6 following periods with and without pumpage. A near zero value for compaction was achieved by the 6th year after shutting down pumpage (Figure 14). However, when pumpage was continued, compaction increased dramatically (Figure 15). Ten years was the simulation period selected, and groundwater pumpage was varied at every point from zero to 60 percent increase over the initial pumpage conditions.

(4) Table 2 compares the rates of subsidence (meter/year) for real data from the Houston area with the simulated rates of subsidence. The simulated subsidence rates for a 10 year period resemble the subsidence rates for the 11 year period of 1964-1973. The subsidence rates for the 30 year period (1943-1973) were less than the simulated rates. The differences observed were due to the fact that the average rates of pumpage used in the simulation were higher than the actual rates of pumpage for both of the time periods shown. Additional scaling and calibration of the hydrologic model could probably improve the output from the model.

The data used were initially held constant throughout each layer and later modified so that the predicted and observed values of subsidence were close throughout the modeled area. However,

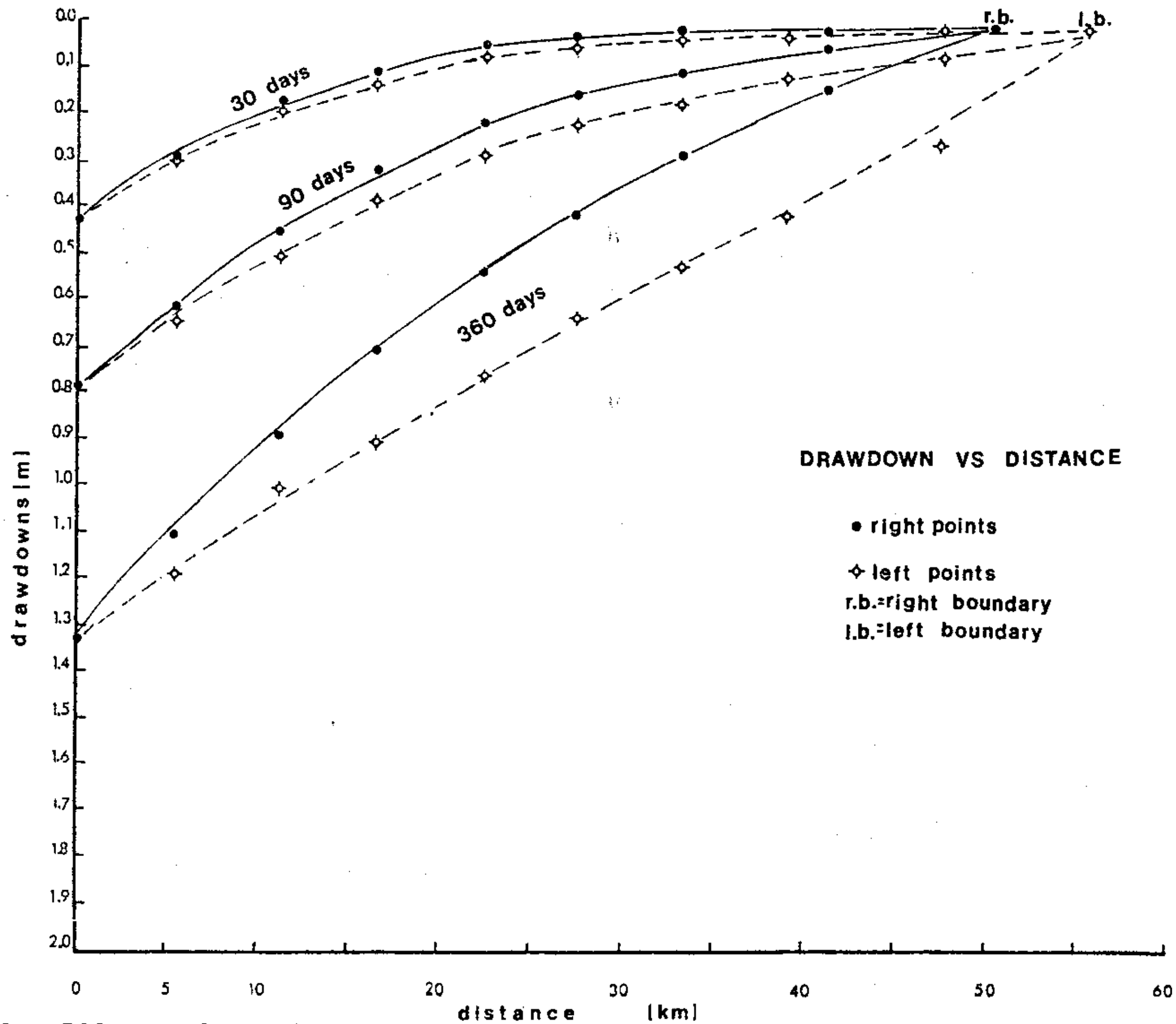


Fig. 13. Effects of boundary conditions on drawndowns vs. distance from pumpage center under confined conditions. The pumpage center was located in sands No. 10 and 11 at cell No. 10. Isotropy, homogeneity and constant layer thickness were established.

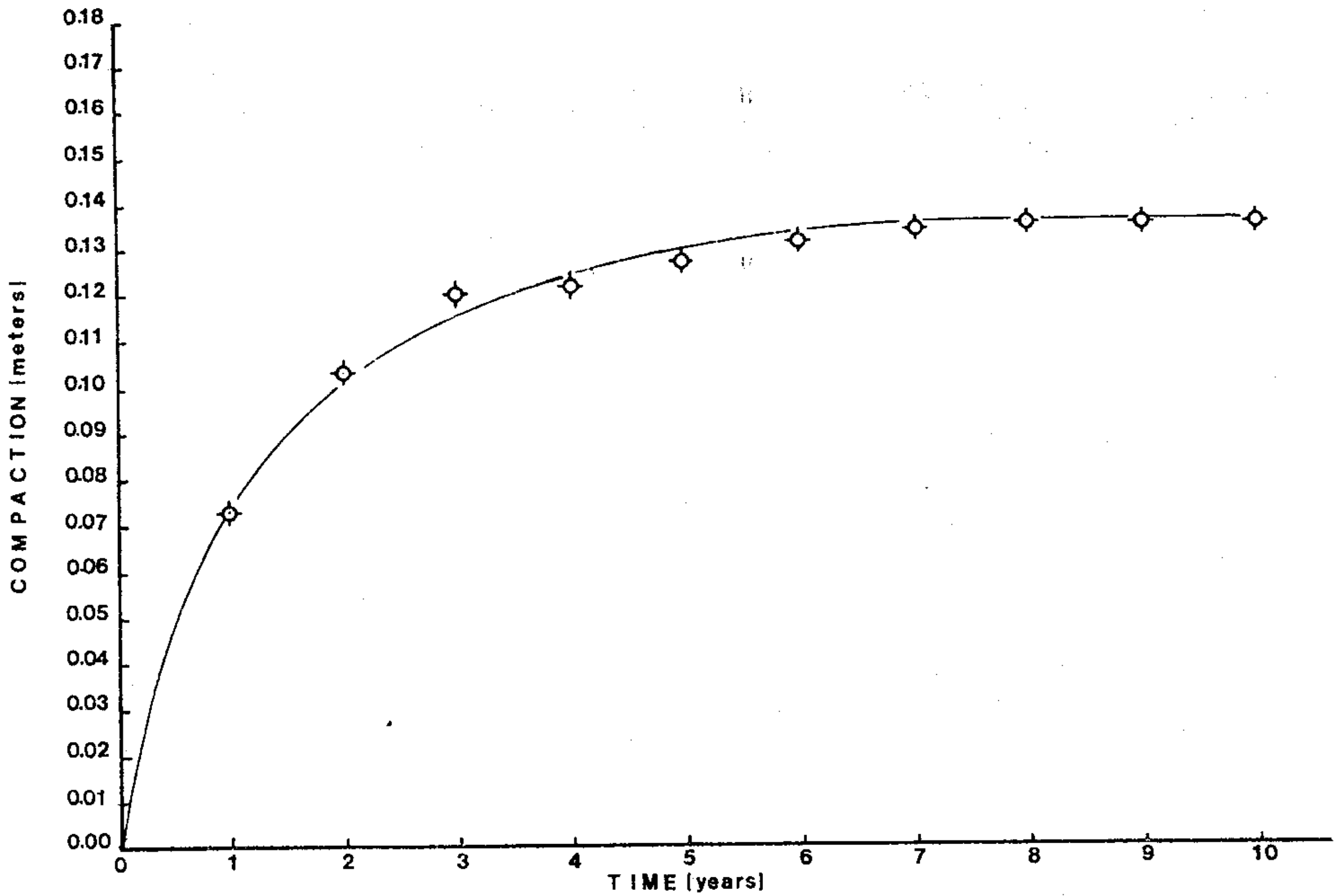


Fig. 14. Compaction at point 6, during ten years after shutting down pumpage.

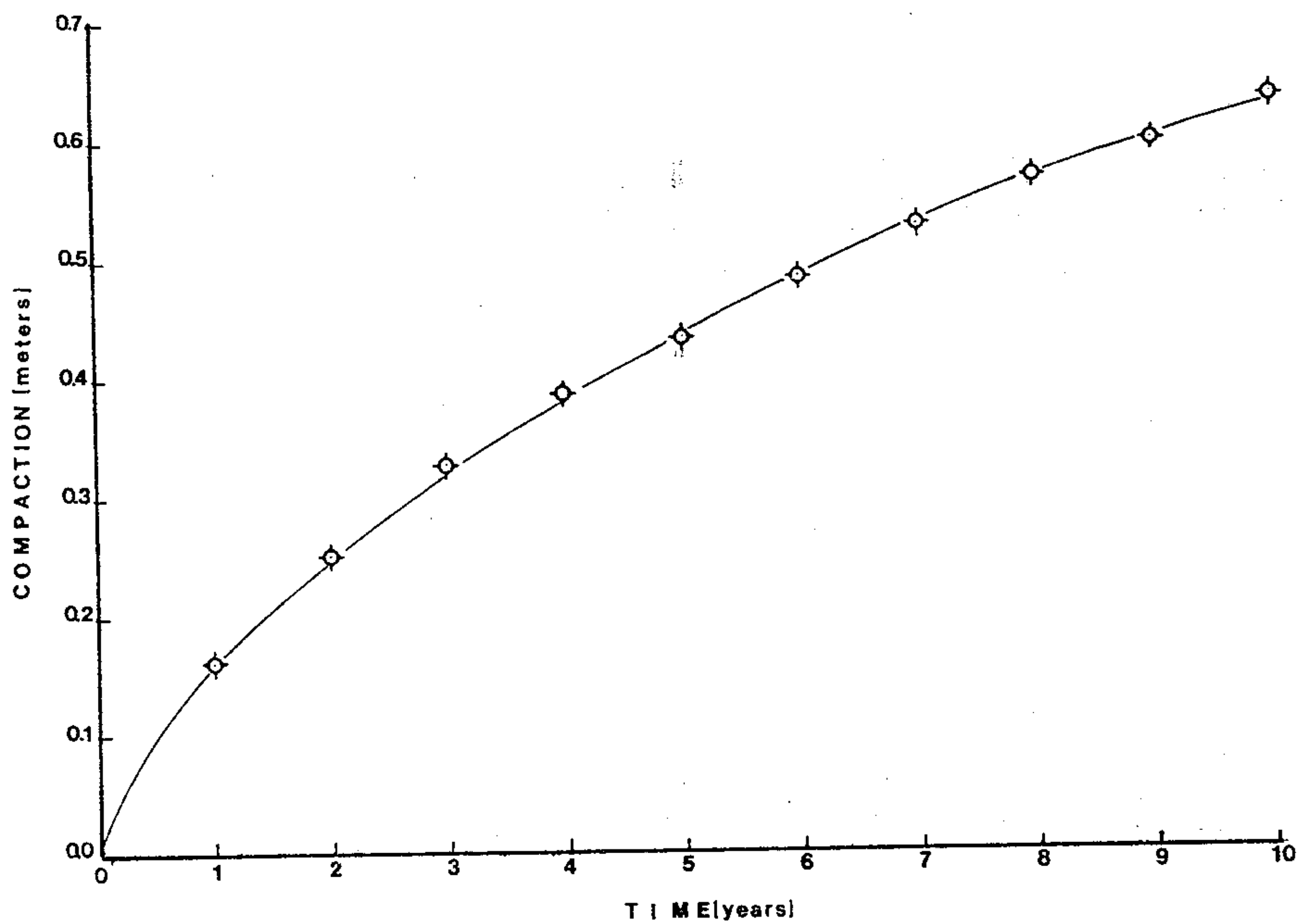


Fig. 15. Compaction at point 6 during ten years keeping the pumpage constant at 1,507 cubic meters per year.

TABLE 2. Average Rates of Subsidence (meters/year)

Point	30 Years (43-73)	11 Years (64-73)	Simulated (10 years)
2	.007	.013	.041
3	.009	.012	.027
4	.010	.013	.039
5	.012	.020	.037
6	.020	.027	.040
7	.030	.030	.037
8	.033	.048	.041
9	.033	.054	.060
10	.040	.054	.070
11	.051	.054	.084
12	.071	.082	.130
13	.066	.096	.081
14	.060	.069	.022
15	.051	.054	.043
16	.030	.027	.035

these values were determined from only a small amount of good quality field data. Additional field data should be obtained if more accuracy in the calibration and validation of the model is to be achieved. Even though the predicted values of total compaction at every point were in the expected range, the compaction values obtained from the upper clays were small, and when pumpage was less than the initial pumpage rates, land elevations actually increased slightly.

Optimization Procedure

Based on results from the compaction simulator and using available economic data, linear approximations to the compaction curves shown in Appendix X were transformed into total cost vs. pumpage curves. These curves were used to determine the slopes (ρ_i^k) used to calculate the coefficients for the objective function, represented by equation (25), and for estimating the upper bound values defined by equations (24) and (29). Water needs at each point (equation 27) and surface water reserves (equation 28) were arbitrarily set.

A test run was made to verify that the optimization scheme was able to allocate groundwater withdrawals and surface water use to minimize land subsidence and the overall cost of water supply. A second run was designed to test a specific water use policy; i.e., to stop pumpage at designed points and observe the alternatives chosen to minimize subsidence costs. A third run was designed to illustrate the effects of varying the right hand side values of the constraints represented by equation (27). A fourth run analyzed the effects of varying the values of the coefficients in the objective function. A fifth run simultaneously varied the right hand side values of the constraints and the coefficients in the objective function. A sixth run, similar to the fifth but varying surface water costs using Policy II, was also made. In a seventh run, the upper bounds on groundwater pumpage at some points were relaxed, and in an eighth run the cost of surface water was allowed to increase with water demands. To understand the results obtained, values for the parameters used in the runs must be examined.

Coefficients of the Objective Function

(1) Slopes of the linear approximations

Previously, the meaning of the slopes (ρ_i^k) of the approximating linear segments to the compaction-pumpage curve was explained in Figure 8. However, for the curves shown in Appendix X to be useful in calculating the coefficients of the objective ($\$/M^3$) function, the slopes obtained must be multiplied by a subsidence cost factor, i.e., the cost per meter of subsidence ($/M) at that point. Values for the factors used in this study are given in Table 3.

TABLE 3. Cost per Meter of Subsidence Used at Every Point in the Area of Study

Point	Area	Cost ($/M)*
2	I	786
3,4,5	I	525
6,7,8,9,10	II	1394
11,12,13	III	4024
14,15,16	IV	3146

*These values are scaled to a one meter wide strip of land along the line of simulation, and to a specific grid length for each grid.

By multiplying the slopes (ρ_i^k) by the corresponding subsidence cost, the component of the corresponding objective function coefficients were obtained.

(2) Groundwater pumpage cost

A basic cost for groundwater pumpage was assumed. It was identified as G_w in equations (42) and (46). The value used in this study was 0.01852 $\$/M^3$.

(3) Surface water cost

The cost of treating and obtaining surface water was assumed constant, and is identified as G_s in equations (42) and (46). In this study, G_s has a value of 0.05812 $\$/M^3$.

(4) Transportation cost

A fourth term in the objective function (equation 46) is the transportation cost. The transportation cost is a product of the water delivery distance and the transportation cost (0.000528 $\$/km \cdot M^3$). In the case of groundwater, the delivery distance is the distance between the center of the pumped cell and the center of the destination cell. In the case of surface water, the delivery distance is the distance between the reservoir and the center of the destination cell. Only one source of surface water was considered in this study. Other surface water reservoirs could easily be added.

Water Demands and Reserves

The water demands at every point were initially set and remained constant throughout most of this study. To evaluate the effects of increasing water demands (parametric analysis of the right hand side of the constraints), a vector of water demand increases was defined. The values are shown in Table 4.

The surface water reserve was varied between 200,000 cubic meters and 500,000 cubic meters. These bounds were obtained from a parametric analysis trial run on the corresponding constraint.

TABLE 4. Water Demands and Vector of Increases Per Point.

Point	Area	Demand (M^3)	Vector (M^3)
2	I	15,000	600
3	I	15,000	400
4	I	7,500	400
5	I	15,000	400
6	II	15,000	900
7	II	7,000	900
8	II	20,000	900
9	II	20,000	900
10	II	5,000	900
11	III	10,000	300
12	III	11,000	300
13	III	20,000	300
14	IV	20,000	200
15	IV	20,000	200
16	IV	20,000	200

Water demands and increases for the ten years of simulation period.

Upper Bounds

The upper bounds used on the right hand side of the constraints defined in equation (45) resulted from the selection of break points which divided the curves of compaction versus pumpage (shown in Appendix X) into a series of linear approximations to the curves. These values were arbitrarily picked for this study to minimize model size and computer time. A more accurate optimization scheme can be developed by increasing the number of linear approximations to these curves. The value of the right hand side boundary to the compaction curves, i.e., that value indicating the maximum amount of water to be pumped (a_3 in Figure 8), corresponds to a 60 percent increase over the initial pumpage value. This value has a special meaning and effect on the results obtained in this study, and it will be discussed in detail later in this chapter. Table 5 contains values of the maximum pumpage and compaction at each grid.

Increases in Coefficients in the Objective Function

The variation of the coefficients in the objective function were as defined in Table 6. The cost of water extraction was kept constant while the transportation cost and subsidence cost were increased by a percentage of the original values.

TABLE 5. Maximum Pumpage, Compaction, and Ratio of Compaction/Pumpage.

Point (Area)	Pumpage (*) (M^3)	Compaction (M)	Ratio (M/M^3) x 10^{-4}
2 (I)	7,616 (7.6)	.63	.82
3 (I)	9,024 (9.0)	.518	.57
4 (I)	3,633 (3.6)	.662	1.82
5 (I)	8,720 (8.7)	.670	.76
6 (II)	17,187 (17.2)	.750	.43
7 (II)	10,617 (10.6)	.730	.68
8 (II)	18,064 (18.0)	.860	.47
9 (II)	19,300 (19.3)	1.157	.59
10 (II)	4,896 (4.8)	1.35	2.75
11 (III)	108,352 (108.3)	1.76	.16
12 (III)	154,880 (154.8)	2.38	.15
13 (III)	21,088 (21.0)	1.38	.65
14 (IV)	15,760 (15.7)	.28	.17
15 (IV)	7,968 (7.9)	.78	.97
16 (IV)	3,984 (3.9)	.50	1.25

(*) In parenthesis the rounded off values ($x10^3M^3$), as used in Tables and Figures.

Pumpage and compaction values correspond to a 10 year simulation period.

TABLE 6. Increases in the Objective Function Coefficients Used in the Parametric Analysis.

Point	Area	Increase ($/M)*
2	I	24
3,4,5	I	16
6,7,8,9,10	II	70
11,12,13	III	80
14,15,16	IV	63

*The transportation cost was increased by 30% to a value of .00686 $\$/km \cdot M^3$.

Discussion of Results

Allocation of Water Resources

The optimization scheme allocates water resources by minimizing the total water cost to the area while satisfying certain constraints. Three water management policies were compared and analyzed in this study. The first water management policy (POLICY I) allows groundwater pumpage at all points in the system. The second water management policy (POLICY II) eliminated groundwater pumpage at points 14, 15, and 16 by assigning extremely large values to the corresponding coefficients in the objective function (1000 $\$/M^3$). This large cost can be looked at as a social value for not using groundwater in critical areas. The third water management policy (POLICY II-A), in addition to the criteria used in Policy II, allowed the maximum amounts of groundwater to be pumped from points 2, 3, 4, and 5 to increase to 15,000 M^3. In all three policies, the surface water reserve was assumed to be 500,000 cubic meters.

There is enough surface water to satisfy any water demands. Results of the allocation from these three policies are shown in Figure 16, and Tables 7 and 8 (which are a representation of the results shown in Appendix I, Tables I-1, I-2, and I-9, for $\Theta = 0$).

The fact that large values of pumpage occurred at points where large values of compaction also occurred was due to the small value of the ratio of compaction/pumpage at these points. This allowed the optimization scheme to load them with higher values of pumpage. Another important factor is the small values of maximum pumpage used for points 2, 3, 4, and 5 (see Table 5). These values are the result of current groundwater use in the area and perhaps a larger maximum pumpage value should be allowed in the area since the subsidence cost in Area I was relatively small. However, the maximum pumpage values should be developed from the hydraulic properties of the aquifer in the area. The effect of relaxing these maximum pumpage values will be analyzed later. The fact that the value of the objective function varied from $25,691 in Policy I to $26,083 in Policy II indicated that Policy II carried a social benefit as well as economic benefit for the area.

Increasing Water Demands

Using water use Policy I, water demands were allowed to increase. The results of a parametric analysis of the right hand side constraint values are summarized in Appendix I, Table I-3. For this analysis, surface water reserves were held constant at 200,000 cubic meters. The decrease in surface water reserves from 500,000 to 200,000 cubic meters caused a slight increase in the value of the objective

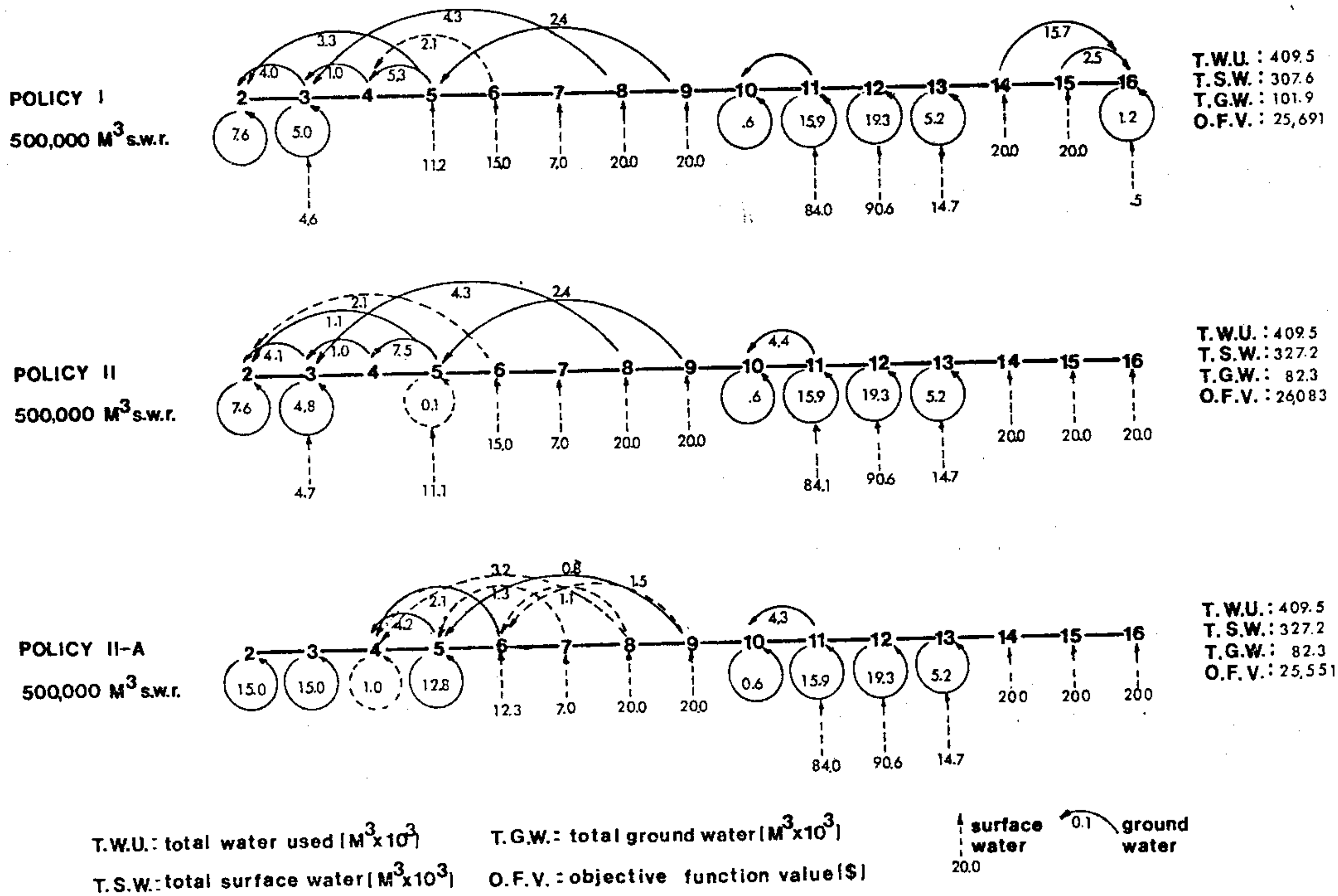


Fig. 16 Comparison of water resources allocations for different water policies.

TABLE 7. Amount of Water Pumped (M^3x10^3) Per Point Using Two Different Policies of Water Use 500,000 Cubic Meters of Surface Water Reserve.

Policy	Point														
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
I	7.6	9.10	1.0	8.6	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	15.7	2.5	1.3	101.1
II	7.6	8.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	-	-	-	82.3
II-A	15.0	15.0	1.0	13.9	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	-	-	-	100.9

TABLE 8. Amount of Surface Water (M^3x10^3) Delivered to Every Point Using Two Different Policies of Water Use 500,000 Cubic Meters of Surface Water Reserve.

Policy	Point															
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total*
I	-	4.6	-	11.2	15.0	7.0	20.0	20.0	-	84.1	90.6	14.7	20.0	20.0	.5	308.4
II	-	4.7	-	11.1	15.0	7.0	20.0	20.0	-	84.1	90.6	14.7	20.0	20.0	20.0	327.2
II-A	-	-	-	-	12.3	7.0	20.0	20.0	-	84.0	90.6	14.7	20.0	20.0	20.0	308.6

*The total values represent the minimum amount of surface water needed to minimize overall cost.

Note: The objective function values are: Policy II, $26,083; Policy I, $25,691; Policy II-A, $25,551.

function (from \$25,569 for 500,000 cubic meters to \$25,977 for 200,000 cubic meters). To improve the solution, surface water reserves should be a minimum of 308,000 cubic meters as shown in Table 8.

As Θ increases, water demands are increased and the number of source-destined combinations is also increased. Thus, the volume of surface water delivered per point changes. Partial results showing the volume of groundwater pumped and the volume of surface water used are shown in Tables 9 and 10, respectively.

Table 11 shows that some points, such as points 5, 7, 8, and 9, have some pumpage, but the water is exported to other grids (in this case to grid 5. All the water demand for points 6, 7, 8, and 9 are supplied by the surface water source. This situation is potentially a very grave political and social problem because residents of these areas must carry some of the burden of subsidence without getting to use the groundwater in their areas.

Simultaneous Increases in Water Demands and Costs

Simultaneous increases in water demands and water costs is a very realistic situation caused by simultaneous economic growth, urban growth, and inflation. The use of parametric analysis on elements of the objective function in conjunction with water needs in the right hand side constraint elements allows makers of water policy to estimate and compare alternatives. Using water use Policy II, a simultaneous parametric analysis of the objective function and the right hand side constraints was made with limited surface water reserves (300,000 M^3) and with adequate surface water reserves

TABLE 9. Variations of Total Groundwater Pumpage (M^3x10^3) for Different Values of Θ for Policy I. Right-hand Side Parametric Analysis with an Initial Surface Water Reserve of 200,000 Cubic Meters.

Value of Θ	Point														
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	7.6	9.0	1.0	8.7	17.2	1.3	4.3	7.4	.6	20.0	112.2	5.3	15.7	2.5	1.3	209.5
5	7.6	9.0	1.0	8.7	17.2	1.3	4.3	7.4	.6	20.0	151.1	5.3	15.7	2.5	1.3	248.5
10	7.6	9.0	1.0	8.7	17.2	1.3	17.9	7.2	.6	37.0	154.8	5.3	15.7	2.5	1.3	287.5
15	7.6	9.0	1.0	8.7	17.2	1.3	18.0	7.2	.6	76.0	154.8	5.3	15.7	2.5	1.3	326.5
20	7.6	9.0	3.5	8.7	17.2	5.4	18.0	7.2	.6	108.3	154.8	5.3	15.7	2.5	1.3	365.5

TABLE 10. Variations of Surface Water Use (M^3x10^3) for Different Values of Θ for Policy I. Right-hand Side Parametric Analysis with an Initial Surface Water Reserve of 200,000 Cubic Meters.

Value of Θ	Point															
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	-	-	-	.9	15.0	7.0	20.0	20.0	-	81.9	-	14.7	20.0	20.0	20.0	200.0
5	-	-	-	9.9	19.5	11.5	24.5	24.5	-	50.5	-	16.2	21.0	21.0	21.0	200.0
10	-	-	-	.2	24.0	16.0	29.0	29.0	-	37.5	-	17.7	27.0	22.0	27.0	200.0
15	-	-	-	9.2	28.5	20.5	33.5	33.5	-	6.0	-	19.2	23.0	23.0	23.0	200.0
20	-	-	-	-	33.0	25.0	38.0	38.0	-	-	-	13.5	24.0	24.0	24.0	200.0

(500,000 M^3). Results from these runs are shown in Tables I-2 and I-4, respectively, in Appendix I. Partial results showing the groundwater pumped and the surface water per point in this case (Policy II with 300,000 M^3 of surface water reserve) are shown in Tables 12 and 13, respectively. Similarly, results for the case of 500,000 M^3 of surface water reserve are shown in Tables 14 and 15.

TABLE 11. Illustration of Water Resource Allocation in Which Groundwater is Pumped from One Area and Transported to Another Area for Use.

Point	Point Source						Water
Destination	5	6	7	8	9	Surface	Needs
5	1.2	4.9	1.3	4.3	2.4	.9	15.0

When surface water is limited, the stress of increasing future water needs is placed on groundwater, independent of the growth in water costs. This is illustrated by the increase in groundwater pumpage at points 11 and 12 in Table 12. Surface water distribution is adjusted to meet the total water demands at a minimum cost.

When an excess of surface water is available, the cost of pumping groundwater can exceed the cost of surface water. Thus, all the water used will be surface water (Tables 14 and 15). If surface water reserves do not grow, then groundwater must ultimately be used again to meet the increased water demands. This situation is dramatically illustrated in Tables 14 and 15.

Increase in Surface Water Costs

In the two preceeding examples, the cost of surface water was held constant. Increased surface water cost will certainly increase

TABLE 12. Variations of Total Groundwater Pumpage (M^3x10^3) for Different Values of Θ for Policy II. Objective Function and Right-hand Side Parametric Analysis with an Initial Surface Water Reserve of 300,000 Cubic Meters.

Value	Point														
of Θ	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	7.6	9.0	1.1	8.7	17.2	1.3	4.3	2.4	.6	20.4	31.7	5.3	--	--	--	109.5
5	7.6	9.0	1.1	8.7	17.2	1.3	4.3	2.4	.6	20.4	70.7	5.3	--	--	--	148.5
10	7.6	9.0	1.1	8.7	17.2	1.3	4.3	2.5	.6	20.4	109.6	5.3	--	--	--	187.5
15	7.6	9.0	1.1	8.7	17.1	1.3	4.3	2.5	.6	20.4	148.6	5.3	--	--	--	226.5
20	7.6	9.0	1.1	8.7	17.1	1.3	18.1	7.2	.6	34.4	154.8	5.3	--	--	--	205.5

TABLE 13. Variation of Surface Water Use (M^3x10^3) for Different Values of Θ for Policy II. Objective Function and Right-hand Side Parametric Analysis with Initial Surface Water Reserve of 300,000 Cubic Meters.

Value	Point															
of Θ	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	-	-	-	1.0	15.0	7.0	20.0	20.0	-	84.0	78.3	14.7	20.0	20.0	20.0	300.0
5	1.3	15.2	-	-	15.2	9.1	24.5	24.5	-	90.0	40.3	16.2	21.0	21.0	21.0	300.0
10	4.4	18.5	-	2.6	19.7	13.5	29.0	29.0	-	96.1	3.4	17.8	22.0	22.0	22.0	300.0
15	7.3	21.0	1.5	4.6	24.2	18.0	33.5	33.5	-	67.9	-	19.2	23.0	23.0	23.0	300.0
20	10.3	23.0	3.5	6.6	28.7	8.8	38.0	33.1	-	55.0	-	20.7	24.0	24.0	24.0	300.0

TABLE 14. Variations of Total Groundwater Pumpage ($M^3 x 10^3$) for Different Values of Θ for Policy II. Objective Function and Right-hand Side Parametric Analysis with an Initial Surface Water Reserve of 500,000 Cubic Meters.

Value of Θ	Point 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	7.6	8.9	1.0	8.7	2.1	1.3	4.3	2.4	1.0	20.3	19.3	5.3	--	--	--	82.3
5	-	-	-	-	-	-	-	-	--	--	--	--	--	--	--	--
10	-	-	-	-	-	-	-	-	--	--	--	--	--	--	--	--
15	-	1.1	-	1.1	2.1	1.3	4.3	-	.6	15.9	--	--	--	--	--	26.5
20	1.9	9.0	1.1	1.1	2.1	1.3	4.3	2.4	.6	20.3	19.3	--	--	--	--	65.5

TABLE 15. Variation of Surface Water Use ($M^3 x 10^3$) for Different Values of Θ for Policy II. Objective Function and Right-hand Side Parametric Analysis with an Initial Surface Water Reserve of 500,000 Cubic Meters.

Value of Θ	Point 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	-	4.7	-	11.1	15.0	7.0	20.0	20.0	--	84.1	90.6	14.7	20.0	20.0	20.0	327.2
5	18.0	17.0	9.5	17.0	19.5	11.5	24.5	24.5	9.5	101.5	111.5	26.5	21.0	21.1	21.1	448.5
10	21.0	19.0	11.5	19.0	21.5	13.5	26.5	26.5	11.5	103.0	113.0	23.0	22.0	22.0	22.0	487.5
15	22.8	21.1	10.2	19.6	24.2	20.5	33.5	33.5	1.9	104.5	114.5	24.5	23.0	23.0	23.0	500.0
20	16.0	23.0	11.2	21.6	28.7	22.5	38.0	38.0	2.0	106.0	96.6	24.1	24.0	24.0	24.0	500.0

the total water cost of the area and will alter the distribution of water throughout the area. The total water cost under Policy II for the two preceeding examples is shown in Table 16. Note that the total water cost is substantially less with the large value of surface water reserve when surface water costs are held constant.

TABLE 16. Values of the Objective Function for Water Use Policy II with Different Values of Θ.

Value of Θ	Policy II (500,000 M^3 of SWR)	Policy II (300,000 M^3 of SWR)
0	$26,083	$26,140
5	$32,699	$44,138
10	$35,739	$69,933
15	$45,238	$103,590
20	$64,833	$145,729

Tables 17 and 18 show the effects of increasing the cost of surface water under water use Policy II. The increased cost of surface water was set at 0.02 $\$/M^3$, which is less than the groundwater cost variables. Under these conditions, the use of surface water increased much slower for the different values of Θ. However, the total volume of groundwater used was constant for any value of Θ. This situation occurred because surface water costs did not allow new combinations of source-delivery to take place or for increases in the present volumes of groundwater pumpage to occur because it was uneconomical. The values of the objective function for this case are given in Table 19 and they are considerably higher than those shown in Table 16 because of the higher water costs.

TABLE 17. Variations of Total Groundwater Pumpage ($M^3 x 10^3$) for Different Values of Θ for Policy II. Objective Function and Right Hand Side Parametric Analysis with an Initial Surface Water Reserve of 500,000 Cubic Meters (surface water cost increasing).

Value of Θ	Point 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	7.6	8.9	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	--	--	--	82.2
5	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	--	--	--	82.2
10	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	--	--	--	82.2
15	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	--	--	--	82.2
20	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	--	--	--	82.2

TABLE 18. Variations of Surface Water Use ($M^3 x 10^3$) for Different Values of Θ for Policy II. Objective Function and Right Hand Side Parametric Analysis with an Initial Surface Water Reserve of 500,000 Cubic Meters (surface water cost increasing).

Value of Θ	Point 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	-	4.7	-	11.1	15.0	7.0	20.0	20.0	--	84.0	90.6	14.7	20.0	20.0	20.0	327.3
5	1.3	14.8	-	15.3	15.2	9.0	24.5	24.5	--	90.0	92.1	16.2	21.0	21.0	21.0	366.3
10	4.3	18.5	-	17.6	19.7	13.5	29.0	29.0	--	96.0	93.6	17.7	22.0	22.0	22.0	405.3
15	7.3	21.0	1.5	19.6	24.2	18.0	33.5	33.5	--	102.0	95.1	19.2	23.0	23.0	23.0	444.3
20	10.3	23.0	3.5	21.6	28.7	22.5	38.0	38.0	--	106.0	96.6	20.7	24.0	24.0	24.0	483.3

TABLE 19. Values of the Objective Function for Water Use Policy II When Increasing Surface Water Cost for Different Values of Θ.

Value of Θ	Policy II
0	$ 26,083
5	73,972
10	129,658
15	193,144
20	264,432

Results of these runs are shown in Table I-5, Appendix I.

Relaxation of Upper Bounds on Groundwater Pumpage

Results were finally obtained by relaxing the upper bounds on groundwater pumpage at points 2, 3, 4, and 5 (Policy II-A). Results from these runs are shown in Table I-6 of Appendix I, and partial results are given in Tables 20 and 21. The total surface water reserve was held at 300,000 cubic meters and should be compared with the results from Policy II shown in Tables 12 and 13. For the first two values of Θ, points 2 through 5 get their water demand from pumpage within the grid. This makes more surface water available for distribution, and consequently, less groundwater pumpage is needed so that subsidence is less at most of the points. For larger values of Θ, water needs are much larger and groundwater pumpage is activated at most of the points, and subsidence is increased to the same levels as shown in Table 12.

The value of the objective function is lower for Policy II-A than the one obtained for Policy II, as shown in Table 22. Consequently, the largest upper bound possible on pumpage will provide an

TABLE 20. Variations of Total Groundwater Pumpage ($M^3 x 10^3$) for Different Values of Θ for Policy II-A. Objective Function and Right-hand Side Parametric Analysis with an Initial Surface Water Reserve of 300,000 Cubic Meters.

Value	Point														
of Θ	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	15.0	15.0	1.0	13.7	10.8	1.3	4.3	2.4	.6	20.2	19.3	5.2	--	--	--	109.5
5	15.0	15.0	1.0	13.8	17.0	1.3	4.3	2.4	.6	20.2	52.0	5.2	--	--	--	148.5
10	15.0	15.0	1.0	13.8	17.1	1.3	4.3	2.4	.6	20.2	91.1	5.2	--	--	--	187.5
15	15.0	15.0	1.0	13.8	17.1	1.3	4.3	2.4	.6	20.2	130.1	5.2	--	--	--	226.5
20	15.0	15.0	1.0	13.8	17.1	1.3	13.7	2.2	.6	20.3	154.9	5.2	--	--	--	265.5

TABLE 21. Variations of Surface Water Use ($M^3 x 10^3$) for Different Values of Θ for Policy II-A. Objective Function and Right-hand Side Parametric Analysis with an Initial Surface Water Reserve of 300,000 Cubic Meters.

Value	Point															
of Θ	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total
0	-	-	-	-	3.5	7.0	20.0	20.0	--	84.0	90.6	14.7	20.0	20.0	20.0	300.0
5	-	-	-	-	13.2	9.1	24.5	24.5	--	90.0	59.3	16.2	21.0	21.0	21.0	300.0
10	-	7.0	-	-	19.7	13.5	29.0	29.0	--	96.0	21.8	17.7	22.0	22.0	22.0	300.0
15	-	13.9	-	2.0	24.2	18.1	33.5	33.5	--	86.4	--	19.2	23.0	23.0	23.0	300.0
20	-	18.9	-	6.1	28.7	13.2	38.0	33.1	--	69.2	--	20.7	24.0	24.0	24.0	300.0

improvement in the solution.

TABLE 22. Values of the Objective Function for Water Use Policy II and II-A with Different Values of Θ.

Value of Θ	Policy II	Policy II-A
0	$ 26,140	$ 25,569
5	44,138	28,700
10	69,933	69,340
15	103,590	102,958
20	145,729	144,658

Both policies are with 300,000 cubic meters of surface water reserve.

Effects of the Water Use Policies on Total Subsidence

So far, no mention has been made of additional subsidence directly caused by implementing one of the Water Use Policies. The effects of the water-use policies on subsidence will be discussed in this section.

The first relevant fact is that the same volume of groundwater pumpage kept reoccurring at many of the points, no matter what the water use policy was. These values of pumpage and the cause of their persistant reoccurrence are summarized in Table 23.

The points which have upper bound values on groundwater pumpage are those with the lowest value of the ratio of subsidence/compaction, as shown in Table 5. Thus, they will enter into the solution at their upper bounds when the water demands are increased. The rest of the points consistently had similar values because of the physical reasons shown in the compaction-pumpage curves of Appendix X. The

TABLE 23. Persistant Values of Pumpage (M^3x10^3) Which Frequently Reoccurred, No Matter What the Water Policy Was.

Point	Pumpage (M^3x10^3)	Cause of Persistancy
2	7.6 or 15.0	Upper bounds on groundwater pumpage
3	7.6 or 15.0	Upper bounds on groundwater pumpage
4	1.0	Low subsidence cost
5	8.7 or 13.8	Upper bound on groundwater pumpage
6	17.2	Upper bound on groundwater pumpage
7	1.3	No subsidence cost (rebound)
8	4.3	No subsidence cost (rebound)
9	2.4	Low subsidence cost
10	.6	No subsidence cost
11	20.3	No subsidence cost
12	154.8	Upper bound on groundwater pumpage
13	5.3	Low subsidence cost
14	15.7	Upper bound on groundwater pumpage
15	2.5	Low subsidence cost
16	1.3	Low subsidence cost

fact that points 7, 8, 10, and 11 had no compaction at the proposed levels of groundwater pumpage makes them enter the solution as a first alternative. The points with a low subsidence cost also entered the solution before the pumpage was increased at points with higher ratios of compaction/pumpage. Tables 24, 25, and 26 show the additional subsidence which resulted at each point when the different water use policies ($\Theta = 0$) used in this study were implemented. These subsidence values were obtained by interpolating

TABLE 24. Values of Additional Subsidence (meters) Resulting from Water Use Policy I and II with 500,000 Cubic Meters of Surface Water Reserve.

Policy	Point														
Θ = 0	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Cost($)
I	0.63	0.52	0.14	0.67	0.02	-	-	-	--	--	0.56	0.18	0.28	0.20	0.20	25,569
II	0.63	0.52	0.14	0.67	0.02	-	-	-	--	--	0.56	0.18	0.13	0.16	0.18	26,083

TABLE 25. Values of Additional Subsidence (meters) Resulting from Water Use Policy I with 200,000 Cubic Meters of Surface Water Reserve.

Policy	Point															
Θ = 0	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Cost($)
I	0.63	0.52	0.14	0.67	0.75	-	-	-	--	--	1.58	0.18	0.28	0.20	0.20	25,977

TABLE 26. Values of Additional Subsidence (meters) Resulting from Water Use Policy II-A with 300,000 Cubic Meters of Surface Water Reserve.

Policy	Point															
Θ = 0	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Cost($)
II-A	1.12	0.85	1.82	1.04	0.37	-	-	-	--	--	0.43	0.18	0.13	0.16	0.18	25,569

If the surface water reserves are relaxed to 500,000 M^3, the subsidence at each point is the same except at point 5 where subsidence reaches 0.82 m. The objective function in that case is $25,551.

the curves shown in Appendix X and they are represented in Figure 17. The following relevant facts are obtained from the analysis of Figure 17.

(1) The final subsidence for points 14, 15, and 16 do not change significantly under different water policies, but there is an improvement by suspending pumpage at those points.

(2) Subsidence at point 12 is highly reduced when the surface water reserves on water policy I are increased. In fact, given the high ratio of compaction to pumpage at point 12, when surface water is available to satisfy the water demands at that point, pumpage is reduced and so is subsidence.

(3) A similar case to that in (2) appears if the surface water reserve is relaxed from 300,000 cubic meters to 500,000 cubic meters when using water policy II-A at $\Theta = 0$. The total subsidence at each time remains the same in each case, except at point 5 where the surface water relaxation implies a reduction of 0.22 meters in final subsidence.

(4) The effects of relaxing the upper bounds of pumpage at points 2, 3, 4, and 5 increased the land-subsidence at these points, with lower values at the critical points 12, 13, 14, 15, and 16.

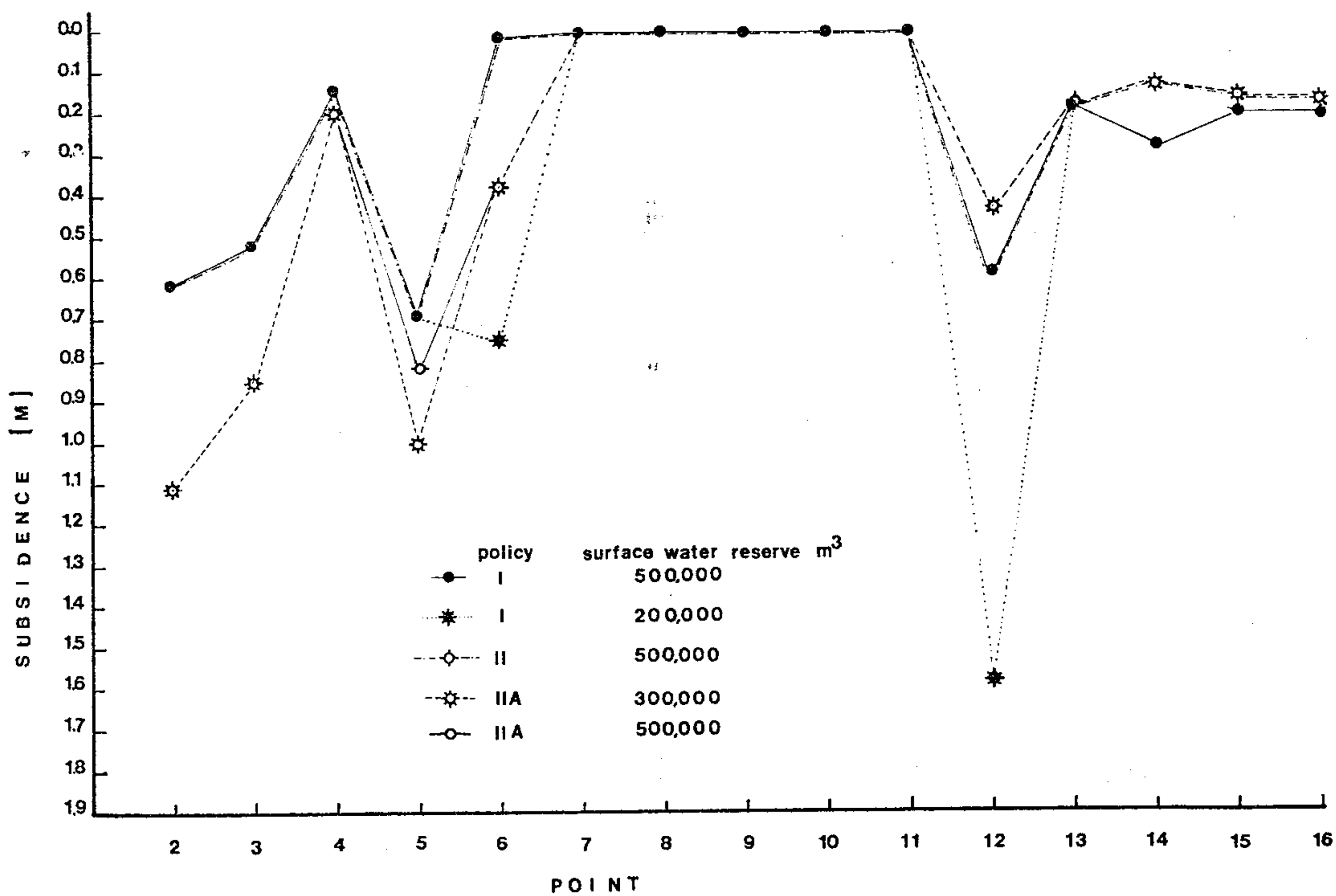


Fig. 17 Land-surface subsidence after ten years for different water policies.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

A two dimensional hydrologic model describing variations of the flow field in a heterogeneous isotropic porous media was developed. Also, a compaction model based on the linear consolidation theory was formulated. Finite difference forms of these two equations were developed. The two models were linked by the effective pressures, which were calculated from the piezometric heads at each time step.

A computer program was written in FORTRAN IV to solve the hydrologic and compaction equations in a nonhomogeneous isotropic porous media. The computer program may be used to handle a diversity of boundary conditions; both constant head and no-flow boundaries were used. In the hydrologic model, the two dimensional fluid flow equation was solved implicitly using Gauss elimination. Water pressures from the hydraulic model were used to solve the linear compaction equation at each time step. Both equations were tested independently with success.

An optimization scheme based on linear programming techniques was independently developed to minimize the total regional water cost and land-surface subsidence. Because the subsidence-pumpage relationship is non-linear, separable programming was used to approximate the subsidence-pumpage relationship as the sum of a set of linear functions.

Using existing data as the initial conditions, the simulator was used to obtain a "realistic" relationship between subsidence and pumpage by varying the volume of water pumped per point from total shut down to 60 percent of the initial pumpage. The total subsidence was calculated for a period of ten years for each increment of pumpage. Curves of subsidence versus pumpage were drawn for each grid point and successive linear approximations to those curves were made. Separable programming was used to approximate the subsidence pumpage curves as the sum of the set of linear approximations which also were used to define a set of constraints stating the maximum pumpage allowed per point. Water demands and surface water reserves were used as additional constraints.

CONCLUSIONS

The following specific conclusions can be drawn as a result of this study:

1. Results from the compaction simulator used in this study may be improved with a more detailed calibration and validation of the hydrologic model. However, the time required for this work is beyond the scope of this study. Results using the data available at this time were from currently available field data from Houston, Texas.
2. The optimization scheme proposed in this study is independent of the subsidence simulator. Consequently, any simulator capable of predicting land-surface subsidence as a function of pumpage rates can be used to generate the information required in the optimization model.

3. As long as the distance between nodes can be estimated, the proposed optimization scheme is independent of the simulation grid type or size and from the geometric configuration of the grid. The number of grid nodes in the simulation is a decisive factor in determining the number of variables and constraints required in the optimization scheme.
4. To investigate the effects of different water-use policies, three different policies were devised. The first policy allowed each point in the region to pump groundwater, receive surface water, receive groundwater from other points, and export groundwater to other points. The second policy restricted groundwater pumpage at some points where subsidence was socially unacceptable. The third policy used the second policy criteria while relaxing maximum groundwater pumpage at some other points. Results from the three policies were compared. The optimization procedure establishes and modifies water source-water destination combinations to achieve a minimum overall water cost while satisfying local water demands and minimizing the regional effects of subsidence.
5. The water-use policies can reflect the social value of water use in critical areas by assigning large subsidence costs in those areas where subsidence is socially and politically unacceptable. This prevents these areas from entering the solution, and keeps land subsidence

at a minimum. Also, additional subsidence costs can be added in those areas where it is socially desirable to reduce subsidence.

6. After a water-use policy has been defined, it is often desirable to know how the solution would change if water costs, subsidence costs, or water demands should increase. Parametric analysis of the objective function and the right-hand side of the constraints provides these answers. Examples and illustrations of this procedure were supplied by this study.
7. Results from this study showed that some areas consistently reported the same volume of groundwater pumpage, no matter what the water policy. By relaxing the upper bounds on the maximum volume of water to be pumped from each area, the objective function was appreciably reduced. As long as these upper bounds do not exceed the physical limitations of the aquifer to supply water on a sustained basis, exportation of groundwater from areas with a low subsidence cost to areas with a high subsidence cost is feasible.
8. The optimization scheme adheres to the characteristics of the subsidence pumpage curves by keeping subsidence as low as possible in areas. The possibility exists for establishing a water-use policy where the effects of aquifer recharge could be defined by including a rebound-recharge curve. This curve would be similar

to the subsidence-pumpage curves, and negative values would be assigned to the coefficients in the objective function. This water-use policy was not evaluated in this study, but could easily be done.

9. This model allows a water manager to allocate the surface water and groundwater resources of a region to meet economic constraints, increases in taxes, subsidence, changes in water costs, penalties for groundwater use, increased subsidence costs, social constraints, and political constraints. To be effectively used in Texas, a change in groundwater and surface water law would be required and institutional arrangements for administering it would be required.

Recommendations

Further work is recommended in the following areas:

1. The accuracy of the results from this model depend upon the quality and quantity of economic data available. Adequate economic data were not found for use in this study. The following types of economic data must be developed:
   a. The cost of land-surface subsidence per unit of land area must be evaluated as a function of time. This will be a consequence of economic growth in the area itself.
   b. Land-surface subsidence should be monitored through

more frequent land surveys to determine the actual subsidence-pumpage curves.

c. An analysis of the water transportation facilities in the area should be made. As a result of this study, the cost of transporting water should be more clearly defined in terms of dollars per distance per volume of water transported. Also, the cost of treating water for consumption should be updated.

d. The future water demands should be evaluated for each of the areas. This would be fundamental in establishing the necessary constraints related to water use, delivery, and surface water reserve.

e. The hydrologic characteristics of each area should be evaluated from pumping tests to determine the maximum rates at which groundwater can be withdrawn without causing physical damage to the aquifer.

2. The type of water user should be considered in the model because the social value of water will vary with the type of user. The social costs can be adjusted at will, so that equity is achieved by almost any criteria. Even though this will increase the number of variables in the model, it would be a solid step toward quantifying the social constraints.

3. The present model is static, but it can be solved for different points in time. Therefore, the average economic data presently used would need to be modified for the

time period selected.

4. The political configuration of the area will be a fundamental factor in the feasibility of transporting groundwater from one point to another. The existance of several cities within the same metropolitan area is an example of this situation. The optimization scheme should be modified to handle this complicating factor.
5. The present model considers the combined subsidence and pumpage from both the upper and lower aquifer units. The operation of the model so that groundwater pumpage from each aquifer and its resulting subsidence would add to the size of the model, but it would make the model much more realistic.

REFERENCES

Aguado, E., and I. Remson, Groundwater hydraulics in aquifer management. Journal of Hydraulics Division, ASCE 100(HY1):103-117, 1974.

Aguado, E., I. Remson, J. F. Pikul, and W. A. Thomas, Optimal pumping for aquifer dewatering. Journal of Hydraulics Division, ASCE,IDO(Hy7):869-877, 1974.

Andrews, W. H., Social issues in water resources development. In: Systems analysis of hydrologic problems, 446-452. Proceedings of the Second International Seminar for Hydrology Professors, Utah, 1970.

Assaf, Karem, Digital simulation of aquifer response to artificial groundwater recharge. Ph.D. dissertation. School of Public Health, University of Texas, Houston, 1976.

AWWA, Statistical Report #20112. Operating data for water utilities 1970 and 1975.

AWWA, Statistical Report. Operating data survey 1976, 1977.

Biot, M. A., General theory of three dimensional consolidation. Journal of Applied Physics 12(2)155-164, 1941.

Biot, M. A., Theory of elasticity and consolidation for a porous anisotropic solid. Journal of Applied Physics 26(2):182-185, 1955.

Biswas, A. K. Systems approach to water management. McGraw Hill Co., New York, 1976.

Brah, W. J., and L. L. Jones, Institutional arrangements for effective groundwater management to halt land subsidence. Texas Water Resources Institute, Texas A&M University, 1978.

Bredehoeft, J., and G. Pinder, Digital analysis of aereal flow in multiaquifer groundwater systems: a quasi-three dimensional model. Water Resources Research 4(3):884-888, 1970.

Buras, N., Scientific allocations of water resources. American Elsevier Publishing Co., New York, 1972.

Burt, O., Groundwater management under quadratic criterion function. Water Resources Research 3(3):673-682, 1967.

Chow, V. T., An introduction to systems analysis of hydrologic problems. In: Systems analysis of hydrologic problems, 15-44. Proceedings of the Second International Seminar for Hydrology Professors, Water Research Laboratory, Utah, 1970.

Clyde, C. G., Game theory. In: Systems analysis of hydrologic problems, 223,230. Proceedings of the Second International Seminar for Hydrology Professors. Water Research Laboratory, Utah, 1970.

Cuevas, J. A., Foundation conditions in Mexico City. Proceedings of the International Conference of Soil Mechanics 3. Cambridge, Massachusetts, 1936.

Curry, G. L., J. C. Helm, and R. A. Clark, A model for a linked system of multipurpose reservoirs with stochastic inflow and demands. Technical Report No. 41. Texas Water Resources Institute, 1972.

Davis, G. H., J. B. Small, and H. B. Counts, Land subsidence related to decline of artesian pressure in the Ocala limestone of Savannah, Georgia, Engineering Geological Case Histories 4. Geological Society of America:1-8, 1963.

Dérédec, A., J. M. Mejía, G. Pineiro, and H. Silvestre, Un modelo para la determinación del esquema óptimo de abstecimiento de agua potable en una región. Reporte especial. Ministerio de Obras Publicas. Venezuela, 1975.

Domenico, P. A., Concepts of models in groundwater hydrology. McGraw Hill Co., New York, 1972.

Domenico, P. A., and M. D. Mifflin, Water from low permeability sediments and land subsidence. Water Resources Research 4:563-576, 1965.

Dracup, J., The optimum use of a groundwater and surface water system--a parametric linear programming approach. University of California (Berkeley). Water Res. Contrib. No. 107, 1966.

Gabrysch, R. K., and C. W. Bonnet, Land-surface subsidence in the Houston-Galveston region, Texas. Report No. 188. Texas Water Development Board, 1975.

Gambolati, G., and R. A. Freeze, Mathematical simulation of the subsidence of Venice I. Theory. Water Resources Research 9(3): 721-733, 1973

Gambolati, G., P. Gatto, and R. A. Freeze, Mathematical simulation of the subsidence of Venice 2. Results. Water Resources Research 10(3):563-577, 1974.

Haimes, Y. Y., Mathematical programming in water resources. In: Systems analysis of hydrological problems, 122-171. Proceedings of the Second International Seminar for Hydrology Professors, Water Research Laboratory, Utah, 1970.

Hall, W., and J. A. Dracup, Water resources systems engineering. McGraw Hill Book Co., 1970.

Hantush, M. S., Modification of the theory of leaky aquifers. Journal of Geophysical Research 65:3713-3725, 1960.

Hantush, M. S., Hydraulics of wells. Advances of Hydroscience 1:281-482. Academic Press, New York, 1964.

Helm, D. C., Evaluation of stress-dependent aquitard parameters by simulating observed compaction from known stress history. Ph.D. dissertation. University of California, Berkeley, 1974.

Helm, D. C., One dimensional simulation of aquifer system compaction near Pixley, California 1. Constant parameters. Water Resources Research 11(3):465-478, 1975a.

Helm, D. C., One dimensional simulation of aquifer system compaction near Pixley, California 2. Stress dependent parameters. Water Resources Research 12(3):375-391, 1975b.

Howe, C. W., Economic models. In: A. K. Biswas (ed.) Systems approach to water management. McGraw Hill Co., New York, 1976.

IASH, Land subsidence. Proceedings of the Tokyo Symposium. International Association of Scientific Hydrology. Pub. No. 88. Two volumes, 1970.

IASH, Land subsidence symposium. IASH-AISH Pub. No. 121, Anaheim, California, 1976.

IBM, Mathematical programming system--extended (MPSX) and generalized upper bounding (GUB), program descriptions. Publication SH20-0968-1, 1972.

Jacob, C. E., On the flow of water in an elastic artesian aquifer. Transactions of the AGU 2:574-586, 1940.

Jones, L. L., External costs of surface subsidence: upper Galveston Bay, Texas. IASH Pub. No. 121, p. 617, 1976.

Jones, L. L., and J. Larson, Economic effects of land subsidence due to excessive groundwater withdrawal in the Texas Gulf Coast area. Technical Report No. 67. Texas Water Resources Institute, 1975.

Jorgensen, D. G., Analog-model studies of groundwater hydrology in the Houston District, Texas, Texas Water Development Board, Report #190, 1975.

Jumiskis, A., Soil mechanics. Princeton, N.J., 1962.

Lohman, S. W., Compression of elastic artesian aquifers. U. S. Geological Survey. Professional Paper 424-B:47-78, 1961.

Lowe, J., III, New concepts in consolidation and settlement analysis. Terzaghi Lectures. A.S.C.E. 337-378, 1974.

Maas, A., M. Hufschmidt, R. Dorfman, H. Thomas, S. Marglin, and G. Maskew, Design of water-resource systems. Harvard University Press, 1966.

Meier, W. L., C. S. Shih, and D. J. Wray, Water resource system optimization by geometric programming. Technical Report No. 34. Texas Water Resources Institute, 1971.

Meier, W. L., J. C. Helm, and G. L. Curry, Development of a dynamic management policy for Texas. Technical Report No. 52. Texas Water Resources Institute, 1973.

Meinzer, O. E., Compressibility of elastic artesian aquifers. Society of Economic Geology 23:263-291, 1928.

Naftel, W. L., B. Fleming, and K. Vaught, Records of wells, driller's logs, water-level measurements, and chemical analyses of ground water in Brazoria, Fort Bend and Waller Counties, Texas, 1966-74. Texas Water Development Board, Report #201, 1976a.

Naftel, W. L., B. Fleming, and K. Vaught, Records of wells, driller's logs, water-level measurements, and chemical analyses of ground water in Chambers, Liberty, and Montgomery Counties, Texas, 1966-74. Texas Water Development Board, Report #202, 1976b.

Naftel, W. L., B. Fleming, and K. Vaught, Records of wells, driller's logs, water-level measurements, and chemical analyses of ground water in Harris and Galveston Counties, Texas, 1970-74. Texas Water Development Board, Report #203, 1976c.

Neuman, S. P., and P. A. Whitherspoon, Theory of flow in a confined two aquifer system. Water Resources Research, 5(4), 1969.

Neuman, S. P., and P. A. Whitherspoon, Variation principles for confined and unconfined flow of groundwater. Water Resources Research 6(5):1376-1382, 1970.

Pinder, G., and W. Gray, Finite element simulations in surface and subsurface hydrology. Academic Press, 1977.

Poland, J. F., and G. H. Davis, Subsidence of the land surface in Tulare-Wasco (Delano) and Los Banos-Keetleman City areas, San Joaquin Valley, California. Transactions of AGU 37:287-296, 1956.

Poland, J. F., The coefficient of storage in a region of major subsidence caused by compaction of an aquifer system. U. S. Geological Survey. Professional Paper 424-B:52-54, 1961.

Radosevich, G. W., and M. E. Sabey, Water rights, eminent domain, and the public trust. Water Resources Bulletin 13(4):747-758, 1977.

Reddell, D. L., and D. K. Sunada, Numerical simulation of dispersion in groundwater aquifers. Colorado State University, Hydrology Papers, No. 41, 1970.

Remson, I., G. M. Hornberger, and F. J. Molz, Numerical methods in subsurface hydrology. Wiley-Interscience, Inc., New York, 1971.

Sandhu, R. S., and E. L. Wilson, Finite element analysis of land subsidence. In: Land subsidence. IASH. Pub. No. 88. Proceedings of the Tokyo Symposium 2:393-400, 1969.

Singer, R., Legal implications of land subsidence in the San Joaquin Valley. IASH Pub. No. 121, p. 609, 1976.

Smith, J. E., Shale compactions, paper SPE 3633 9 pp., AIME Soc. Petrol. Eng., New Orleans, 1971.

Spencer, G. W., The fight to keep Houston from sinking. Environmental Design/Engineered Construction, Civil Engineering. ASCE 47(7): 69-71, 1977.

Taha, H., Operations research--An introduction. The Macmillan Company, New York, 1971.

Taylor, D. W., Fundamentals of soil mechanics, John Wiley and Sons, New York, 1948.

Terzaghi, K., Settlement and consolidation of clays, Engineering News Rec., Nov. 26:874-878, 1925.

Terzaghi, K., and R. B. Peck, Soil mechanics in engineering practice. John Wiley and Sons, New York, 1948.

Theis, C. V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, Trans. Amer. Geophys. Union, vol. 2:514-529, 1935.

Warren, J. P., L. L. Jones, and W. L. Griffin, and R. D. Lacewell, Cost of land subsidence due to groundwater withdrawal. Technical Report No. 57. Texas Water Resources Institute, 1974.

Winslow, A. G., and L. A. Wood, Relation of land subsidence to groundwater withdrawals in the upper gulf coastal region of Texas. Mining Division, Mining Engineering, AIME:1030-1034, 1959.

APPENDIX I

RESULTS OF THE OPTIMIZATION MODEL

The results of diverse applications of the optimization model are provided in table form. For proper interpretation of those tables, the meaning of the headings in each table have to be explained.

(1) Water volume units: in thousand of cubic meters (M^3x10^3).

(2) Example No.: refers to the number of computer run as explained in the introductory section of Chapter IV.

(3) Parametric analysis: refers to the type of parametric analysis performed on such example. PARARIM = parametric analysis of simultaneous variations of coefficient of the objective function and right hand side of the constraints; PARARHS = parametric analysis of the right hand side elements of the constraints (water needs and surface water reserve), and PARAOBJ = parametric analysis of coefficients of the objective function.

(4) Θ = value of the parameter in the parametric analysis.

(5) Activity: value of the objective function at that Θ value.

(6) Surface water reserve: maximum volume of surface water available.

(7) Policy: type of policy used;

I = all points allowed to pump groundwater.

II = all points, but point 14, 15 and 16, allowed to pump groundwater.

II-A = policy II, relaxing maximum values of pumpage for points 2, 3, 4 and 5.

To get information from tables, if reading Table I-1, $\theta = 0$, the following point should be followed.

<u>Source</u>: are the points from which water is pumped.

<u>Destination</u>: are the points which receive water.

For example: Source #5 pumps water to DESTINATION #2 ($3.3 \times 10^3 M^3$), and to DESTINATION #4 ($5.3 \times 10^3 M^3$).

<u>Surface</u>: are the volumes of surface water delivered to destination points.

For example: DESTINATION #3 receives $4.6 \times 10^3 M^3$, while DESTINATION #4 does not use surface water at all.

<u>Total delivered</u>: is the row for total volumes of water pumped at the sources. Those values are used to obtain the total compaction.

<u>Total used</u>: is the column for water demands at each destination point.

The interception between surface column total delivered row shows the total volumn of surface water used.

The interceptor between total used column and total delivered row, shows the total volume demanded in the area.

TABLE I-1 Water Distribution for Example No. 5, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARIM Θ: 0 Activity: $25,691

Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	Total Used
2	7.6	6.0		3.3												-0-	15.0
3		5.0	1.0				4.3									4.6	15.0
4				5.3	2.1											-0-	7.5
5						1.3		2.4								11.2	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.4						-0-	5.0
11										15.9						84.0	100.0
12											19.3					90.6	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16													15.7	2.5	1.2	.5	20.0
Total Delivered	7.6	9.0	1.0	8.6	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	15.7	2.5	1.2	307.6	409.5

TABLE I-1 Water Distribution for Example No. 5, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 5 Activity: $32,699
Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE																Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2																18.0	18.0
3																17.0	17.0
4																9.5	9.5
5																17.0	17.0
6																19.5	19.5
7																11.5	11.5
8																24.5	24.5
9																24.5	24.5
10																9.5	9.5
11																101.5	101.5
12																111.5	111.5
13																21.5	21.5
14																21.0	21.0
15																21.0	21.0
16																21.0	21.0
Total Delivered	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	448.5	448.5

TABLE 1-1 Water Distribution for Example No. 5, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 10 Activity: $35,739
Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE																Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2																21.0	21.0
3																19.0	19.0
4																11.5	11.5
5																19.0	19.0
6																24.0	24.0
7																16.0	16.0
8																29.0	29.0
9																29.0	29.0
10																14.0	14.0
11																103.0	103.0
12																113.0	113.0
13																23.0	23.0
14																22.0	22.0
15																22.0	22.0
16																22.0	22.0
Total Delivered	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	487.5	487.5

TABLE I-1 Water Distribution for Example No. 5, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 15 Activity: \$45,238
Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE																Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2		1.1														22.9	24.0
3																21.0	21.0
4				1.1	2.1											10.2	13.5
5						1.3										19.6	21.0
6							4.3									24.2	28.5
7																20.5	20.5
8																33.5	33.5
9																33.5	33.5
10									.6	15.9						1.9	18.5
11																104.5	104.5
12																114.5	114.5
13																24.5	24.5
14																23.0	23.0
15																23.0	23.0
16																23.0	23.0
Total Delivered	0.0	1.1	0.0	1.1	2.1	1.3	4.3	0.0	.6	15.9	0.0	0.0	0.0	0.0	0.0	500.00	526.5

TABLE I-1 Water Distribution for Example No. 5, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARIM Θ: 20 Activity: $64,833

Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE																Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	1.9	9.0														16.0	27.0
3																23.0	23.0
4			1.0	1.1	2.1											11.2	15.5
5						1.3										21.6	23.0
6							4.3									28.7	33.0
7																25.0	25.0
8																38.0	38.0
9								2.4								35.5	38.0
10									.6	20.3						2.1	23.0
11																106.0	106.0
12											19.3					96.6	116.0
13												1.9				24.1	26.0
14																24.0	24.0
15																24.0	24.0
16																24.0	24.0
Total Delivered	1.9	9.0	1.0	1.1	2.1	1.3	4.3	2.4	.6	20.3	19.3	1.9	0.0	0.0	0.0	500.0	565.5

TABLE I-2 Water Distribution for Example No. 6, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARIM: Θ: 0 Activity: $26,083

Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	4.1		1.1	2.1											-0-	15.0
3		4.8	1.0				4.3									4.7	15.0
4				7.5												-0-	7.5
5				.1		1.3		2.4								11.1	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.4						-0-	5.0
11										15.9						84.1	100.0
12											19.3					90.6	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered	7.6	8.9	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	0.0	0.0	0.0	327.2	409.5

TABLE I-2 Water Distribution for Example No. 6, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 5 Activity: $32,699
Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2																18.0	18.0
3																17.0	17.0
4																9.5	9.5
5																17.0	17.0
6																19.5	19.5
7																11.5	11.5
8																24.5	24.5
9																24.5	24.5
10																9.5	9.5
11																101.5	101.5
12																115.0	115.0
13																21.5	21.5
14																21.0	21.0
15																21.0	21.0
16																21.0	21.0
Total Delivered	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	448.5	448.5

TABLE I-2 Water Distribution for Example No. 6, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 10 Activity: $35,739
Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2																21.0	21.0
3																19.0	19.0
4																11.5	11.5
5																19.0	19.0
6																24.0	24.0
7																16.0	16.0
8																29.0	29.0
9																29.0	29.0
10																14.0	14.0
11																103.0	103.0
12																113.0	113.0
13																23.0	23.0
14																22.0	22.0
15																22.0	22.0
16																22.0	22.0
Total Delivered	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	487.5	487.5

TABLE I-2 Water Distribution for Example No. 6, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 15 Activity: $45,238
Surface Water Reserve: 500,000M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2		1.1														22.8	24.0
3																21.0	21.0
4				1.1	2.1											10.2	13.5
5						1.3	4.3									19.6	21.0
6																24.2	28.5
7																20.5	20.5
8																33.5	33.5
9																33.5	33.5
10									.6	15.9						1.9	18.5
11																104.5	104.5
12																114.5	114.5
13																24.5	24.5
14																23.0	23.0
15																23.0	23.0
16																23.0	23.0
Total Delivered	0.0	1.1	0.0	1.1	2.1	1.3	4.3	0.0	0.6	15.9	0.0	0.0	0.0	0.0	0.0	500.0	126.5

TABLE I-2 Water Distribution for Example No. 6, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 20 Activity: $64,833
Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	Total Used
2	1.9	9.0														16.0	27.0
3																23.0	23.0
4			1.1	1.1	2.1											11.2	15.5
5						1.3										21.6	23.0
6							4.3									28.7	33.0
7								2.4								22.5	25.0
8																38.0	38.0
9																38.0	38.0
10									.6	20.3						2.0	23.0
11																106.0	106.0
12											19.3					96.6	116.0
13												1.9				24.0	26.0
14																24.0	24.0
15																24.0	24.0
16																24.0	24.0
Total Delivered	1.9	9.0	1.1	1.1	2.1	1.3	4.3	2.4	.6	20.3	19.3	1.9	0.0	0.0	0.0	500.0	565.5

TABLE I-3 Water Distribution for Example No. 3, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARHS Θ: 0 Activity: $25,977
Surface Water Reserve: 200,000 M^3 Policy: I

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	7.4														-0-	15.0
3		1.6	1.1		12.3											-0-	15.0
4				7.5												-0-	7.5
5				1.2	4.9	1.3	4.3	2.4								.9	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.4						-0-	5.0
11										15.9	2.2					81.9	100.0
12											110.0					-0-	110.0
13												5.3				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16													15.7	2.5	1.3	.5	20.0
Total Delivered	7.6	9.0	1.1	8.8	17.2	1.3	4.3	2.4	.6	20.3	112.3	5.3	15.7	2.5	1.3	200.0	409.5

TABLE I-3 Water Distribution for Example No. 3, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARHS θ: 5 Activity: $29,184
Surface Water Reserve: 200,000 M^3 Policy: I

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	7.9			1.6		.9									-0-	18.0
3		1.1	.8		15.1											-0-	17.0
4			.2	8.7	.6											-0-	9.5
5						1.3	3.4	2.4								9.9	17.0
6																19.5	19.5
7																11.5	11.5
8																24.5	24.5
9																24.5	24.5
10									.6	8.9						-0-	9.5
11										11.4	39.6					50.5	101.5
12											111.5					-0-	111.5
13												5.3				16.2	21.5
14																21.0	21.0
15																21.0	21.0
16													15.7	2.5	1.3	1.5	21.0
Total Delivered	7.6	9.0	1.0	8.7	17.3	1.3	4.3	2.4	.6	20.3	151.1	5.3	15.7	2.5	1.3	200.0	448.5

TABLE I-3 Water Distribution for Example No. 3, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARHS Θ: 10 Activity: $33,110

Surface Water Reserve: 200,000 M^3 Policy: I

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	7.9					5.5									-0-	21.0
3		1.1	1.0		15.0		1.7									-0-	19.0
4				3.2			8.2									-0-	11.5
5				5.5	2.1	1.3	2.5	7.2								.2	19.0
6																16.0	16.0
7																29.0	29.0
8																29.0	29.0
9									.6	13.4						-0-	14.0
10										23.6	41.9					37.5	103.0
11											113.0					-0-	113.0
12												5.2				17.7	23.0
13																22.0	22.0
14																22.0	22.0
15													15.7	2.5	1.2	2.5	23.0
16																	
Total Delivered	7.6	9.0	1.0	8.7	17.1	1.3	17.8	7.2	.6	37.0	154.8	5.2	15.7	2.5	1.2	200.0	487.5

TABLE I-3 Water Distribution for Example No. 3, Expressed as $M^3 \times 10^{-3}$

Parametric Analysis: PARARHS θ: 15 Activity: $37,234

Surface Water Reserve: 200,000 M^3 Policy: I

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	7.9	.5	.06			7.9									-0-	24.0
3	1.1	1.1	.5		15.0		4.3									-0-	21.0
4				7.6			5.8									-0-	13.5
5				1.0	2.1	1.3		1.2								9.2	21.0
6																28.5	28.5
7																20.5	20.5
8																33.5	33.5
9																33.5	33.5
10									.6	17.9						-0-	18.5
11										58.1	40.4					6.0	104.5
12											114.5					-0-	114.5
13												5.2				19.2	24.5
14																23.0	23.0
15																23.0	23.0
16													15.7	2.5	1.2	23.5	23.0
Total Delivered	7.6	9.0	1.0	8.7	17.1	1.3	18.0	7.2	.6	76.0	154.9	5.2	15.7	2.5	1.2	200.0	526.5

TABLE I-3 Water Distribution for Example No. 3, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARHS: Θ: 20 Activity: $41,451

Surface Water Reserve: 200,000 M^3 Policy: I

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	7.8	2.5			4.1	4.8									-0-	27.0
3		1.1	1.0		15.0		4.3			1.4						-0-	23.0
4				6.5			8.9									-0-	15.5
5				2.1	2.1	1.3		7.2	.6	9.5						-0-	23.0
6																33.0	33.0
7																25.0	25.0
8																38.0	38.0
9																38.0	38.0
10										23.0						-0-	23.0
11										74.3	31.6					-0-	106.0
12											116.0					-0-	116.0
13											7.2	5.2				13.5	26.0
14																24.0	24.0
15																24.0	24.0
16													15.7	2.5	1.2	4.5	24.0
Total Delivered	7.6	8.9	3.5	8.7	17.1	5.4	18.0	7.2	.6	108.2	154.8	5.2	15.7	2.5	1.2	200.0	565.5

TABLE I-4 Water Distribution for Example No. 2, Expressed as M^3 x-10^3
Parametric Analysis: PARARIM Θ: 0 Activity: $26,140
Surface Water Reserve: 300,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	7.4														-0-	15.0
3		1.6	1.1		12.3		.1									-0-	15.0
4				1.1	2.2		4.2									-0-	7.5
5				7.6	2.7	1.3		2.4								1.0	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.4						-0-	5.0
11										16.0						84.0	100.0
12											31.7					78.3	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered	7.6	9.0	1.1	8.7	17.2	1.3	4.3	2.4	.6	20.4	31.7	5.2	0.0	0.0	0.0	300.0	409.5

TABLE I-4 Water Distribution for Example No. 2, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 5 Activity: $44,138.6
Surface Water Reserve: 300,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														1.3	18.0
3			1.1	.7												15.2	17.0
4				7.9	1.5											-0-	9.5
5					15.7	1.3										-0-	17.0
6							4.3									15.2	19.5
7								2.4								9.1	11.5
8																24.5	24.5
9																24.5	24.5
10									.6	8.9						-0-	9.5
11										11.5						90.0	101.5
12											70.7					40.8	111.5
13												5.2				16.2	21.5
14																21.0	21.0
15																21.0	21.0
16																21.0	21.0
Total Delivered	7.6	9.0	1.1	8.6	17.2	1.3	4.3	2.4	.6	20.4	70.7	5.2	0.0	0.0	0.0	300.00	448.5

TABLE I-4 Water Distribution for Example No. 2, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 10 Activity: $69,933
Surface Water Reserve: 300,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														4.4	21.0
3			.4													18.5	19.0
4			.6	8.7	2.1											-0-	11.5
5					15.1	1.3										2.6	19.0
6							4:3									19.7	24.0
7								2.5								13.5	16.0
8																29.0	29.0
9																29.0	29.0
10									.6	13.4						-0-	14.0
11										6.9						96.1	103.0
12											109.6					3.4	113.0
13												5.2				17.8	23.C
14																22.0	22.0
15																22.0	22.0
16																22.0	22.0
Total Delivered	7.6	9.0	1..	8.7	17.2	·1.3	4.3	2.5	.6	20.4	109.6	5.2	0.0	0.0	0.0	300.0	487.5

TABLE I-4 Water Distribution for Example No. 2, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 15 Activity: $103,590
Surface Water Reserve: 300,000 M^3 Policy: II

Destination	SOURCE																Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														7.3	24.0
3																21.0	21.0
4			1.0	8.7	2.1											1.5	13.5
5					15.0	1.3										4.6	21.0
6							4.3									24.2	28.5
7								2.4								18.0	20.5
8																33.5	33.5
9																33.5	33.5
10									.6	17.8						-0-	18.5
11										2.4	34.1					67.9	104.5
12											114.5					-0-	114.5
13												5.3				19.2	24.5
14																23.0	23.0
15																23.0	23.0
16																23.0	23.0
Total Delivered	7.6	9.0	1.1	8.7	17.1	1.3	4.3	2.4	.6	20.4	148.6	5.3	0.0	0.0	0.0	300.0	526.5

TABLE I-4 Water Distribution for Example No. 2, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 20 Activity: $145,729
Surface Water Reserve: 300,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														10.3	27.0
3																23.0	23.0
4			1.0	8.6	2.1											3.5	15.5
5					15.0	1.3										6.6	23.0
6							4.3									28.7	33.0
7							13.8	2.4								8.8	25.0
8																38.0	38.0
9								4.8								33.1	38.0
10									.6	22.3						-0-	23.0
11										12.1	38.8					55.0	106.0
12											116.0					-0-	116.0
13												5.2				20.7	26.0
14																24.0	24.0
15																24.0	24.0
16																24.0	24.0
Total Delivered	7.6	9.0	1.0	8.6	17.1	1.3	18.1	7.2	.6	34.4	154.8	5.2	0.0	0.0	0.0	300.0	565.5

TABLE I-5 Water Distribution for Example No. 8, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 0 Activity: $26,083
Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	Total Used
2	7.6	4.1		1.1	2.1											-0-	15.0
3		4.8	1.0				4.3									4.7	15.0
4				7.5												-0-	7.5
5				.1		1.3		2.4								11.1	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.3						-0-	5.0
11										15.9						84.0	100.0
12											19.3					90.6	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered	7.6	8.9	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	0.0	0.0	0.0	327.3	409.5

TABLE I-5 Water Distribution for Example No. 8, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 5 Activity: $73,972
Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE																Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														1.3	18.0
3			1.0	1.1												14.8	17.0
4				7.3	2.1											-0-	9.5
5				.2		1.3										15.3	17.0
6							4.3									15.2	19.5
7								2.4								9.0	11.5
8																24.5	24.5
9																24.5	24.5
10									.6	8.8						-0-	9.5
11										11.4						90.0	101.5
12											19.3					92.1	111.5
13												5.2				16.2	21.5
14																21.0	21.0
15																21.0	21.0
16																21.0	21.0
Total Delivered	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	0.0	0.0	0.0	366.3	448.5

TABLE I-5 Water Distribution for Example No. 8, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 10 Activity: $129,658
Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														4.3	21.0
3			.4													18.5	19.0
4			.6	8.7	2.1											-0-	11.5
5						1.3										17.6	19.0
6							4.3									19.7	24.0
7								2.4								13.5	16.0
8																29.0	29.0
9																29.0	29.0
10									.6	13.3						-0-	14.0
11										6.9						96.0	103.0
12											19.3					93.6	113.0
13												5.2				17.7	23.0
14																22.0	22.0
15																22.0	22.0
16																22.0	22.0
Total Delivered	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.2	19.3	5.2	0.0	0.0	0.0	405.3	487.5

TABLE I-5 Water Distribution for Example No. 8, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 15 Activity: $193,144
Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE																Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														7.3	24.0
3																21.0	21.0
4			1.0	8.7	2.1											1.5	13.5
5						1.3										19.6	21.0
6							4.3									24.2	28.5
7								2.4								18.0	20.5
8																33.5	33.5
9																33.5	33.5
10									.6	17.9						-0-	18.5
11										2.4						102.0	104.5
12											19.3					95.1	114.5
13												5.2				19.2	24.5
14																23.0	23.0
15																23.0	23.0
16																23.0	23.0
Total Delivered	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	0.0	0.0	0.0	444.3	526.5

TABLE I-5 Water Distribution for Example No. 8, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARIM: Θ: 20 Activity: $264,432

Surface Water Reserve: 500,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	9.0														10.3	27.0
3																23.0	23.0
4			1.0	8.7												3.5	15.5
5						1.3										21.6	23.0
6							4.3									28.7	33.0
7								2.4								22.5	25.0
8																38.0	38.0
9																38.0	38.0
10									.6	20.3						2.0	23.0
11																106.0	106.0
12											19.3					96.6	116.0
13												5.2				20.7	26.0
14																24.0	24.0
15																24.0	24.0
16																24.0	24.0
Total Delivered	7.6	9.0	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	0.0	0.0	0.0	483.3	565.5

TABLE I-6 Water Distribution for Example No. 7, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 0 Activity: $25,569
Surface Water Reserve: 300,000 M^3 Policy: II-A

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	15.0															-0-	15.0
3		15.0														-0-	15.0
4			1.0	4.2	2.1											-0-	7.5
5				9.5	1.6	1.3		2.4								-0-	15.0
6					7.1		4.3									3.5	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.3						-0-	5.0
11										15.9						84.0	100.0
12											19.3					90.6	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered	15.0	15.0	1.0	13.7	10.8	1.3	4.3	2.4	.6	20.2	19.3	5.2	0.0	0.0	0.0	300.0	409.5

TABLE I-6 Water Distribution for Example No. 7, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM: Θ: 5 Activity: $28,700
Surface Water Reserve: 300,000 M^3 Policy: II-A

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	15.0	3.0														-0-	18.0
3		12.0	1.0	3.9												-0-	17.0
4				7.3	2.1											-0-	9.5
5				2.6	13.0	1.3										-0-	17.0
6					1.9		4.3									13.2	19.5
7								2.4								9.1	11.5
8																24.5	24.5
9																24.5	24.5
10									.6	8.8						-0-	9.5
11										11.4						90.0	101.5
12											52.0					59.3	111.5
13												5.2				16.2	21.5
14																21.0	21.0
15																21.0	21.0
16																21.0	21.0
Total Delivered	15.0	15.0	1.0	13.8	17.0	1.3	4.3	2.4	.6	20.2	52.0	5.2	0.0	0.0	0.0	300.0	448.5

TABLE I-6 Water Distribution for Example No. 7, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARIM: Θ: 10 Activity: $69,340

Surface Water Reserve: 300,000 M^3 Policy: II-A

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	15.0	6.0														-0-	21.0
3		9.0	1.0	1.9												7.0	19.0
4				9.3	2.1											-0-	11.5
5				2.6	15.0	1.3										-0-	19.0
6							4.3									19.7	24.0
7								2.4								13.5	16.0
8																29.0	29.0
9																29.0	29.0
10									.6	13.3						-0-	14.0
11										6.9						96.0	103.0
12											91.1					21.8	113.0
13												5.2				17.7	23.0
14																22.0	22.0
15																22.0	22.0
16																22.0	22.0
Total Delivered	15.0	18.0	1.0	13.8	17.1	1.3	4.3	2.4	.6	20.2	91.1	5.2	0.0	0.0	0.0	300.0	487.5

TABLE I-6 Water Distribution for Example No. 7, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 15 Activity: $102,958
Surface Water Reserve: 300,000 M^3 Policy: II-A

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	15.0	9.0														-0-	24.0
3		6.0	1.0													13.9	21.0
4				11.3	2.1											-0-	13.0
5				2.5	15.0	1.3										2.0	21.0
6							4.3									24.2	28.5
7								2.4								18.1	20.5
8																33.5	33.5
9																33.5	33.5
10									.6	17.8						-0-	18.5
11										2.4	15.6					86.4	104.5
12											114.5					-0-	114.5
13												5.2				19.2	24.5
14																23.0	23.0
15																23.0	23.0
16																23.0	23.0
Total Delivered	15.0	15.0	1.1	13.8	17.1	1.3	4.3	2.4	.6	20.2	130.1	5.2	0.0	0.0	0.0	300.0	576.5

TABLE I-6 Water Distribution for Example No. 7, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARIM Θ: 20 Activity: $144,658

Surface Water Reserve: 300,000 M^3 Policy: II-A

Destination	SOURCE 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	Total Used
2	15.0	12.0														-0-	27.0
3		3.0	1.0													18.9	23.0
4				13.3	2.1											-0-	15.5
5				.5	15.0	1.3										6.1	23.0
6							4.3									28.7	33.0
7							9.4	2.4								13.2	25.0
8																38.0	38.0
9								4.8								33.1	38.0
10									.6	20.3	2.1					-0-	23.0
11											36.8					69.2	106.0
12											116.0					-0-	116.0
13												5.2				20.7	26.0
14																24.0	24.0
15																24.0	24.0
16																24.0	24.0
Total Delivered	15.0	15.0	1.0	13.8	17.1	1.3	13.7	7.2	.6	20.3	154.9	5.2	0.0	0.0	0.0	300.0	565.5

TABLE I-7 Water Distribution for Example No. 4, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARAOBJ Θ: 0 Activity: $25,691
Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	4.0		3.3												-0-	15.0
3		5.0	1.0				4.3									4.6	15.0
4				5.2	2.1											-0-	7.5
5						1.3		2.4								11.2	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.4						-0-	5.0
11										15.9						84.0	100.0
12											19.3					90.6	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16													15.7	2.5	1.2	20.0	20.0
Total Delivered	7.6	9.0	1.0	8.5	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	15.7	2.5	1.2	407.8	409.5

TABLE I-7 Water Distribution for Example No. 4, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARAOBJ Θ: 20 Activity: $29,659
Surface Water Reserve: 500,000 M^3 Policy: I

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2																15.0	15.0
3																15.0	15.0
4																7.5	7.5
5																15.0	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10																5.0	5.0
11																100.0	100.0
12																110.0	110.0
13																20.0	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered																409.5	409.5

TABLE I-8 Water Distribution for Example No. 8, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARHS Θ: 0 Activity: $26,140
Surface Water Reserve: 300,000 M^3 Policy: II

Destination	SOURCE 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	Total Used
2	7.6	6.2														-0-	15.0
3		1.6			12.2		1.1									-0-	15.0
4			1.1	1.1	2.1		3.2									-0-	7.5
5				7.6	2.7	1.3		2.4								.8	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.3						-0-	5.0
11										15.9						84.1	100.0
12											31.6					78.3	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered	7.6	7.8	1.1	6.7	17.0	1.3	4.3	2.4	.6	20.2	31.6	5.2	0.0	0.0	0.0	300.0	409.5

TABLE I-8 Water Distribution for Example No. 8, Expressed as $M^3 \times 10^3$

Parametric Analysis: PARARHS: Θ: 50 Activity: $26,083

Surface Water Reserve: 2,800,000 M^3 Policy: II

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	7.6	7.2		.1												-0-	15.0
3		1.7	1.0	1.1	2.1	1.3	4.3	2.4								.9	15.0
4				7.5												-0-	7.5
5																15.0	15.0
6																15.0	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.4						-0-	5.0
11										15.9						84.1	100.0
12											19.3					90.6	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered	7.6	8.9	1.0	8.7	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	0.0	0.0	0.0	327.3	409.5

TABLE I-9 Water Distribution for Example No. 9, Expressed as $M^3 \times 10^3$
Parametric Analysis: PARARIM Θ: 0 Activity: $25,551
Surface Water Reserve: 500,000 M^3 Policy: II-A

Destination	SOURCE															Total Used
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Surface	
2	15.0															-0-	15.0
3		15.0														-0-	15.0
4			1.0	1.1	2.1		3.2									-0-	7.5
5				12.8		1.3		.8								-0-	15.0
6							1.1	1.5								12.3	15.0
7																7.0	7.0
8																20.0	20.0
9																20.0	20.0
10									.6	4.3						-0-	5.0
11										15.9						84.0	100.0
12											19.3					90.6	110.0
13												5.2				14.7	20.0
14																20.0	20.0
15																20.0	20.0
16																20.0	20.0
Total Delivered	15.0	15.0	1.0	13.9	2.1	1.3	4.3	2.4	.6	20.3	19.3	5.2	0.0	0.0	0.0	327.2	409.5

APPENDIX II

DERIVATION OF THE GENERAL FLOW EQUATION

The fundamental flow equation is derived for water flowing through a saturated porous media. A mass balance in two dimensions is combined with Darcy's Law and some equations of state to obtain the final equation.

The principle of mass conservation, when applied to a differential volume element of porous media fixed in space may be stated as:

(Rate of mass inflow) - (Rate of mass outflow) =

Rate of change of mass inside the volume element.

Applying this principle to the volume element shown in Figure II-1, results in the following equation:

$$\left(Mx - \Delta x/2\right) + \left(Mx + \Delta x/2\right) + \left(Mz - \Delta z/2\right) - \left(Mz + \Delta z/2\right) = \frac{\partial\ Mve}{\partial t} \pm Mp\ , \qquad \text{(II-1)}$$

or

$$\frac{\partial Mx}{\partial x}\,\Delta x = \frac{\partial Mz}{\partial z}\,\Delta z = -\,\frac{\partial\ Mve}{\partial t} \pm Mp\ , \qquad \text{(II-2)}$$

where $Mx-\Delta x/2$, $Mz-\Delta z/2$, $Mx+\Delta x/2$, and $Mz+\Delta z/2$ are the rates of mass inflow and outflow across the faces of the volume element. Mve is the mass contained inside the volume element and Mp is a mass source or sink term which is negative for a source and positive for a sink. The terms $\frac{\partial Mx}{\partial x}\Delta x$ $\frac{\partial Mz}{\partial z}\Delta z$, and $\frac{\partial Mve}{\partial t}$ represent the changes in mass with respect to space and time.

Expressing individual mass flow components in terms of the fluid density, the dimensions of the volume element, and the volume flux, we

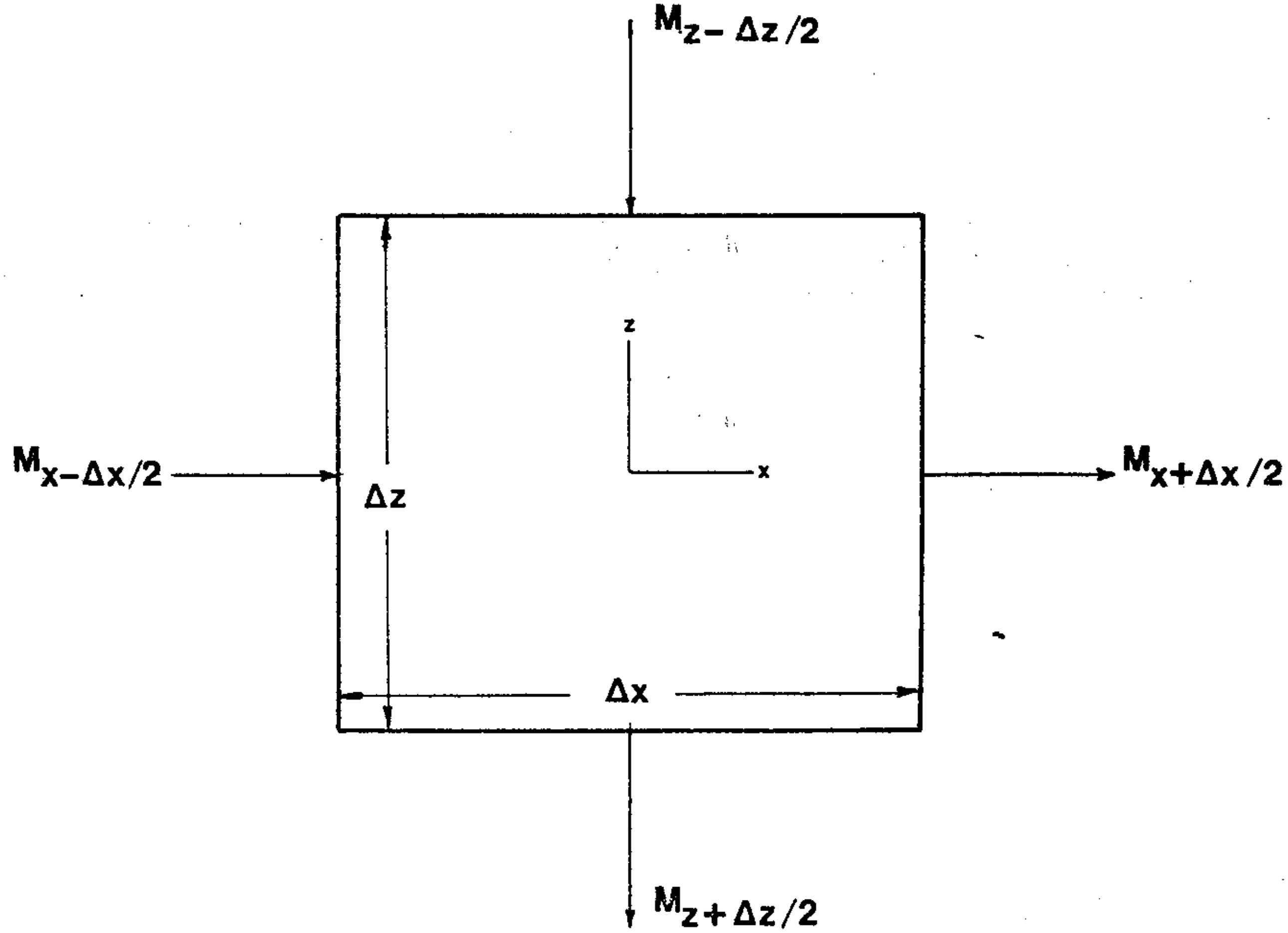


Fig. II-1 Volume element

get

$$Mx = \rho\, q_x\, \Delta z(1)\ ,$$

$$Mz = \rho\, q_z\, \Delta z(1)$$

$$MVE = \rho\phi\, \Delta x\, \Delta z(1)\ , \text{ and}$$

$$Mp = \rho Q$$

where

ρ = fluid density (M/L^3)

Δx = volume element's side in the x direction

Δz = volume element's side in the z direction

ϕ = porosity, and

Q = source on sink term (L^3/T) .

Substituting these relationships into equation II-2, we get

$$\frac{\partial}{\partial}(\rho q_x\, \Delta z(1))\, \Delta_x + \frac{\partial}{\partial z}(\rho\, qz\, \Delta x(1))\Delta z = -\frac{\partial}{\partial t}(\rho\phi\, \Delta x\, \Delta z(1)) \pm \rho Q \qquad \text{(II-3)}$$

According to Darcy's law,

$$q_x = -K_x \frac{\partial H}{\partial x} \quad \text{and} \quad q_z = -K_z \frac{\partial H}{\partial z} \qquad \text{(II-4)}$$

where

q_x = volume flux across z(1) face (L/T),

q_z = volume flux across z(1) face (L/T),

K_x = hydraulic conductivity in (x) direction (L/T),

K_y = hydraulic conductivity in (z) direction (L/T),

$\frac{\partial H}{\partial x}$ = head gradient in (x) direction (.), and

$\frac{\partial H}{\partial z}$ = head gradient in (z) direction (.).

Substituting equations (A-4) into equation (II-3), we obtain

$$\frac{\partial}{\partial x}(\rho\ K_x \Delta z(1)\frac{\partial H}{\partial x}\Delta x + \frac{\partial}{\partial z}(\rho\ K_z \Delta x(1)\frac{\partial H}{\partial z})\Delta z = \frac{\partial}{\partial t}(\rho\phi\ \Delta x\ \Delta z(1)) \pm \rho Q\ . \tag{II-5}$$

The first term on the right-hand side of equation II-5 can be expanded as follows:

$$\frac{\partial}{\partial t}(\rho\phi\ \Delta x\ \Delta z(1)) = \rho\phi\ \Delta z\frac{\partial(\Delta x)}{\Delta t} + \rho\phi\ \Delta x\frac{\partial(\Delta z)}{\partial t} + \rho\ \Delta x\ \Delta z\frac{\partial\rho}{\partial t} + \phi\ \Delta x\ \Delta z\frac{\partial\rho}{\partial t}\ . \tag{II-6}$$

The resulting four terms deal with the time rate of change of the volume element's horizontal and vertical dimensions, porosity, and fluid density. It is assumed that no horizontal changes occur, so the first term is neglected. The second and third terms are related to vertical compression of the porous media. They may be expressed in terms of intergranular pressure, and ultimately, fluid head. The fourth term may also be expressed in terms of fluid head by using a density-pressure relationship.

The second term in equation II-6 is related to the vertical compressibility of the aquifer as follows:

$$\alpha = \frac{1}{E_s}\ , \tag{II-7}$$

where α is the formation compressibility factor, and E_s is the bulk modulus of compression of the rock skeleton defined by

$$E_s = \frac{\Delta\ \text{stress}}{\Delta\ \text{strain}} = -\frac{\Delta\bar{\sigma}}{d(\Delta z/\Delta z)}\ , \tag{II-8}$$

where $\Delta\bar{\sigma}$ is the change in intergranular pressure, and $d(\Delta z/\Delta z)$ is the unit change in the vertical dimension of the volume element.

Combining equations II-7 and II-8 yields,

$$d(\Delta z) = -\alpha \; \Delta z \; d\bar{\sigma} \; ,$$

or in terms of the time derivative,

$$\frac{\partial(\Delta z)}{\partial t} = -\alpha \; \Delta z \; \frac{\partial \sigma}{\partial t} \; . \qquad \text{(II-9)}$$

The third term of equation II-6 expresses the change in porosity, which is also related to vertical compression. The volume of solids in the volume element can be related to the vertical dimension only as

$$Vs = (1-\sigma)\Delta z \; , \qquad \text{(II-10)}$$

where

Vs = the volume of solids.

Assuming that Vs remains constant with time,

$$dVs = d((1-\phi) \; \Delta z) = 0 \; . \qquad \text{(II-11)}$$

Carrying out the differentiation,

$$dVs = -d\phi \; \Delta z + (1-\phi)d(\Delta z) = 0 \; . \qquad \text{(II-12)}$$

Consequently,

$$\frac{\partial \phi}{\partial t} = \frac{(1-\phi)}{\Delta z} \; \frac{\partial \Delta z}{\partial t} \; , \qquad \text{(II-13)}$$

and using equation II-9

$$\frac{\partial \phi}{\partial t} = - \; (1-\phi)\alpha \; \frac{\partial \bar{\sigma}}{\partial t} \; ,$$

The fourth term in equation II-6 refers to the changes in fluid density with respect to time and is related to the fluid compressibility (β) , where $\beta = 1/Ew$. Ew is the bulk modulus of compression for water and is defined as

$$Ew = - \frac{dp}{d(\Delta V)/\Delta V} = \frac{1}{\beta} \quad , \qquad \text{(II-15)}$$

where dp = the change in fluid pressure, and

$d(\Delta V)/\Delta V$ = the change in fluid volume per unit volume.

Rearranging terms in equation II-15 yields $d(\Delta V) = - \beta \Delta V dp$. (II-16)

By the conservation of mass principle,

$$\rho_1 \Delta V_1 = \rho_2 \Delta V_2 \quad \text{or} \quad \rho d(\Delta V) + \Delta V \, d\rho = 0 \; . \qquad \text{(II-17)}$$

Substituting equation II-16 into equation II-17 and taking the time derivative, we get

$$\rho\beta \frac{dp}{dt} = \frac{d\rho}{dt} \; .$$

Now, substituting equations II-18 , II-14 and II-9 into equation (II-6) yields

$$\frac{\partial(\rho\sigma \; \Delta x \Delta z(1)}{\partial t} = -\rho\alpha \; \Delta z \; \Delta x \; \frac{\partial\bar{\sigma}}{\partial t} \; -\alpha\rho \; \Delta x \; \Delta z(1-\phi) \; \frac{\partial\bar{\sigma}}{\partial t} +$$

$$+ \phi \; \Delta x \; \Delta z \; \rho\beta\frac{dp}{dt} = \text{RHS} \; . \qquad \text{(II-19)}$$

since $d\bar{\sigma} = - dp$, (II-20)

equation II-19 becomes

$$\text{RHS} = \frac{\partial p}{\partial t}(\rho\alpha \; \Delta z \; \Delta x\phi + \alpha\rho \; \Delta x \; \Delta z(1-\phi) + \phi\rho\beta \; \Delta x \; \Delta z) \; ,$$

or

$$RHS = \frac{\partial p}{\partial t} \Delta x \, \Delta z \, \rho(\alpha + \phi\beta) \quad . \tag{II-21}$$

Since the piezometric head is defined as

$$H = \frac{p}{\rho g} + z \tag{II-22}$$

then the change in H with respect to time is

$$\frac{\partial H}{\partial t} = \frac{\partial z}{\partial t} + \frac{1}{\rho g} \frac{\partial p}{\partial t} \tag{II-23}$$

and given that $\partial Z/\partial t = 0$ (II-24)

we obtain

$$RHS = \rho^2 g \Delta x \Delta z (\alpha + \phi\beta) \frac{\partial H}{\partial t} \quad , \tag{II-25}$$

where $\rho g(\alpha + \phi\beta)$ is the coefficient of specific storage S_s (L/T^2). Thus,

$$RHS = \rho S_s \, \Delta x \, \Delta z \frac{\partial H}{\partial t} \quad . \tag{II-26}$$

Assuming that density does not vary in space,

$$\frac{\partial}{\partial x}(K_x \, \Delta z \, \frac{\partial H}{\partial x}) \Delta x + \frac{\partial}{\partial z}(K_z \, \Delta x \, \frac{\partial H}{\partial z}) \Delta z = \Delta x \, \Delta z \, S_s \, \frac{\partial H}{\partial t} + Q \quad . \tag{II-27}$$

APPENDIX III

FINITE DIFFERENCE FORM OF THE GENERAL FLOW EQUATION

The general flow equation II-27 used to calculate piezometric heads is written as

$$\frac{\partial}{\partial x}\left(K_x \, \Delta z \, \frac{\partial H}{\partial x}\right)\Delta x + \frac{\partial}{\partial z}\left(K_z \, \Delta x \, \frac{\partial H}{\partial z}\right)\Delta z = \Delta x \, \Delta x \, S_s \, \frac{\partial H}{\partial t} \pm Q \; . \qquad \text{(III-1)}$$

A centered-in-space finite difference grid system (Figure III-1) is used to develop a finite difference equation for Equation III-1

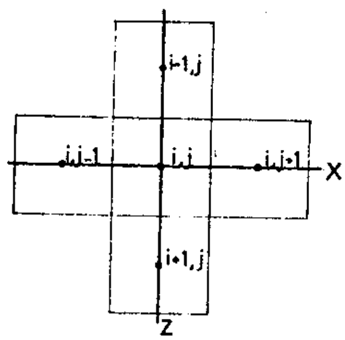


Fig. III-1 Finite difference grid system.

The spatial derivative of H at a point on the boundary between grids i,j and i,j+1 may be approximated by

$$\left.\frac{\partial H}{\partial x}\right)_{i,j+1/2} = \frac{H_{i,j+1} - H_{i,j}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j+1}}{2}} \quad , \qquad \text{(III-2)}$$

likewise, for a point on the boundary between grids i,j and i,j-1

$$\left.\frac{\partial H}{\partial x}\right)_{i,j-1/2} = \frac{H_{i,j} - H_{i,j-1}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j-1}}{2}} \quad . \qquad \text{(III-3)}$$

for a point between grids i,j and i+1,j

$$\left.\frac{\partial H}{\partial z}\right)_{i+1/2,j} = \frac{H_{i+1,j} - H_{i,j}}{\frac{\Delta z_{i,j}}{2} \quad \frac{\Delta z_{i+1,j}}{2}} \quad , \tag{III-4}$$

and for a point between grids i,j and i-1,j

$$\left.\frac{\partial H}{\partial z}\right)_{i-1/2,j} = \frac{H_{i,j} - H_{i-1,j}}{\frac{\Delta z_{i,j}}{2} - \frac{\Delta z_{i-1,j}}{2}} \quad . \tag{III-5}$$

The left-hand side (LHS) of equation (III-1) can be written as

$$\begin{aligned} LHS = & \frac{1}{\Delta x_{i,j}} \left[\left(K_x \; \Delta z \; \frac{\partial H}{\partial x} \right)_{i,j+1/2} - \left(K_x \; \Delta z \; \frac{\partial H}{\partial x} \right)_{i,j-1/2} \right] \Delta x_{i,j} \\ & + \frac{1}{\Delta z_{i,j}} \left[\left(K_z \; \Delta x \; \frac{\partial H}{\partial z} \right)_{i-1/2,j} - \left(K_z \; \Delta x \; \frac{\partial H}{\partial z} \right)_{i-1/2,j} \right] \Delta z_{i,j} \quad . \end{aligned} \tag{III-6}$$

Substituting equations III-2, III-3, III-4 and III-5 into equation III-6 and manipulating terms, we get

$$\begin{aligned} LHS = & (K_x \; \Delta z)_{i,j+1/2} \left[\frac{H_{i,j+1} - H_{i,j}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j+1}}{2}} \right] \\ & - (K_x \; \Delta z)_{i,j-1/2} \left[\frac{H_{i,j} - H_{i,j-1}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i-1,j}}{2}} \right] \\ & + (K_z \; \Delta x)_{i+1/2,j} \left[\frac{H_{i+1,j} - H_{i,j}}{\frac{\Delta z_{i,j}}{2} + \frac{\Delta z_{i-1,j}}{2}} \right] \end{aligned} \tag{III-7}$$

$$LHS = N\overset{+}{x}\ H_{i,j+1} + N\overset{-}{x}\ H_{i,j-1} + N\overset{+}{z}\ H_{i+1,j} + N\overset{-}{z}\ H_{i-1,j}$$

$$- (N\overset{+}{x} + N\overset{-}{x} + N\overset{+}{z} + N\overset{-}{z})H_{i,j} \ . \qquad \text{(III-13)}$$

The time derivative on the right-hand side (RHS) of equation (III-1) then be expressed as

$$\frac{\partial H}{\partial t}_{i,j} = \frac{H_{i,j}^{t+\Delta t} - H_{i,j}^{t}}{\Delta t} \ , \qquad \text{(III-14)}$$

and the factor $(\Delta z)_{i,j}$ as

$$(\Delta z)_{i,j} = \frac{\Delta z_{i,j+1/2} + \Delta z_{i,j-1/2}}{2} \ . \qquad \text{(III-15)}$$

Thus, the right-hand side becomes

$$RHS = \frac{1}{2\Delta t} (\Delta z_{i,j+1/2} + \Delta z_{i,j-1/2})(S \ \ \Delta x)_{i,j} \left[H_{i,j}^{t+\Delta t} - H_{i,j}^{t} \right] + Q \ . \qquad \text{(III-16)}$$

We then define

$$M = \frac{(Ss \ \Delta x)_{i,j} \ (\Delta z_{i,j+1/2} + \Delta z_{i,j-1/2})}{2\Delta t} \ , \qquad \text{(III-17)}$$

and the right hand side may be written as

$$RHS = M \ H_{i,j}^{t+1} - M \ H_{i,j}^{t} \ . \qquad \text{(III-18)}$$

The final form of equation (III-1) is then written as

$$N_x^{+} H_{i,j+1}^{t+1} + N_x^{-} H_{i,j-1}^{t+1} + N_z^{+} H_{i+1,j}^{t+1} + N_z^{-} H_{i-1,j}$$

$$- (N_x^{+} + N_x^{-} + N_z^{+} + N_z^{-} + M) H_{i,j}^{t+1} = -M H_{i,j}^{t} \pm Q \ . \qquad \text{(III-19)}$$

APPENDIX IV

AVERAGE OF CONDUCTIVITIES

The derivation of equation III-8 is based on an application of the continuity law and Darcy's law to a two-grid system like the one in Figure IV-1 where values of Δz and Δx are not equal.

Let (1) and (0) denote the centers of the two grids.

The flow rate from grid (1) to grid (0) can be expressed according to Darcy's law as

$$Q = Kx_1 \cdot \Delta z_{1/2} \cdot \frac{H_1 - H_{1/2}}{\Delta x_{1/2}} \quad . \qquad \text{(IV-1)}$$

Similarly the flow rate coming to grid (0) from grid (1) must also be equal to

$$Q = Kx_2 \cdot \Delta z_{1/2} \cdot \frac{H_{1/2} - H_0}{\Delta x_{0/2}} \quad . \qquad \text{(IV-2)}$$

Manipulating both equations IV-1 and IV-2, we get

$$H_{1/2} = - \frac{Q.\Delta x_1}{2Kx_1 \ \Delta z_{1/2}} + H_1 \qquad \text{(IV-3)}$$

and

$$H_{1/2} = + \frac{Q\Delta x_0}{2 \ Kx_0 \Delta z_{1/2}} + H_0 \qquad \text{(IV-4)}$$

Substracting IV-3 from IV-4, we can write

$$H_0 - H_1 = \frac{Q}{2 \ \Delta z_{1/2}} \left[\frac{\Delta x_1}{Kx_1} + \frac{\Delta x_o}{Kx_0} \right] \quad , \qquad \text{(IV-5)}$$

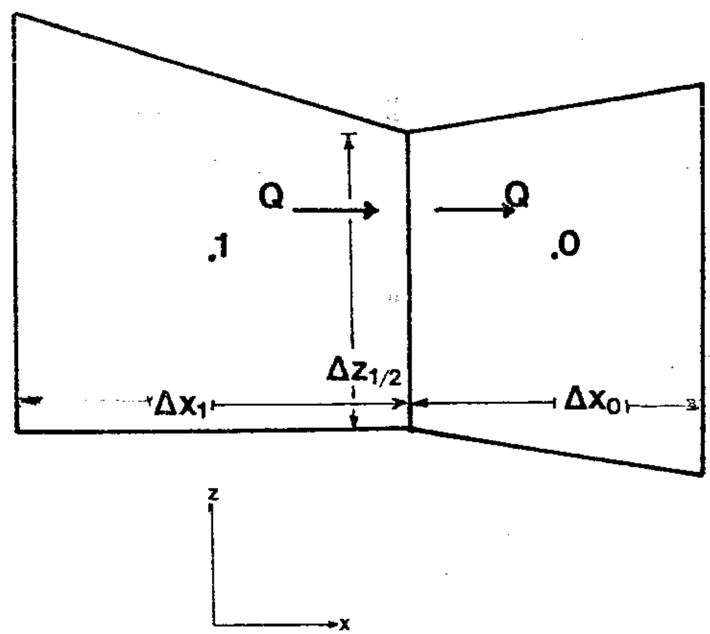


Fig. IV-1 Two grid system for evaluation of the conductivities average between both cells.

since by continuity Q is the same on both sides of the grid interface. Rearranging and solving for Q we get

$$Q = \left[\frac{2\ K_{x_1} . K_{x_0}}{\Delta x_1 K_{x_0} + \Delta x_0 K_{x_1}}\right] \Delta z_{1/2} (H_0 - H_1) \quad . \qquad \text{(IV-6)}$$

Thus, the first term of equation III-7 can be written as

$$\left[\frac{(k_x)_{i,j+1/2}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j+1}}{2}}\right] [H_{i,j+1} - H_{i,j}] \quad \Delta z_{i,j+1/2} \qquad \text{(IV-7)}$$

$$= \frac{2K_{x_{i,j}} . K_{x_{i,j+1}} . \Delta z_{i,j+1/2}}{(\Delta x_{i,j}\ K_{x_{i,j-1}} + \Delta x_{i,j-1}\ K_{x_{i,j}}} \quad [H_{i,j+1} - H_{i,j}] \quad .$$

The other terms of equation III-7 can be written in a similar fashion, as was expressed in equation (III-8).

APPENDIX V

ECONOMICAL DATA

The economical data used in this study is shown in Table V-1. The sources of that data are the following:

(1) Warren et al. [1974]

(2) Jones, L. L. and T. Larson [1975]

(3) AWWA. Statistical Reports [1975, 1977]

To illustrate how to calculate the coefficients of the objective function, consider a variable X^1_{23} for example. This variable coefficient will indicate how much in costs to pump water from point 2 to point 3; by delivery an amount of water less or equal to the upper bound of the first breakpoint in the linear approximations made to the compaction pumpage relationship at point 2. The slope of that corresponding to break point is $\rho^1_2(M/M^3)$. From Table V-1 the average cost of subsidence for ten years can be obtained as follows:

(1) average losses (\$/year-sq. km) x time (years) x area of the grid where water is pumped (sq. km) in numbers.

Lets say it is 109.5 \$ the total cost

(2) Divide that cost by the average subsidence of the area and multiply the result (\$/M) by the slope $\rho^1_2(M/M^3)$

$$109.4(\$) \div 0.210(M)\text{x}0.0473\text{x}10^{-3}(M/M^3) = 0.0010882\ (\$/M^3)$$

(3) Calculate transportation cost by multiplying the distance (km) between grid centers 3 and 3 by the transport cost (\$/M -km)

$$6{,}945(km)\text{x}0.000528(\$/M^3\text{-}km) = 0.003667(\$/M^3)$$

TABLE V-1 Economic Data Used to Evaluate Coefficients and the Objective Function

Type of Information	Area I	Area II	Area III	Area IV
Estimated annual average cost ($)	For the whole area of study: $31,705,040			
Average losses ($/year/sq. km)x$10^6$	.00039	.0032	.035	.039
Area (sq. km)	3523	1697	488	403
Average Subsidence (M)	.213	1.127	1.676	2.377
Groundwater extracting cost ($/M^3)	.015852	.015852	.015852	.015852
Surface water cost ($/M^3)	.05812	.05812	.05812	.05812
Transportation Cost $/km/M^3)	.000528	.000528	.000528	.000528

(4) Add the cost of extracting groundwater to the results obtained in (2) and (3).

$$0.015851 + 0.003667 + 0.0010882 = 0.02067 \ (\$/M^3)$$

The water transport cost was estimated for data obtained from the statistical report of the American Water Works Association, 1975 for the city of Houston.

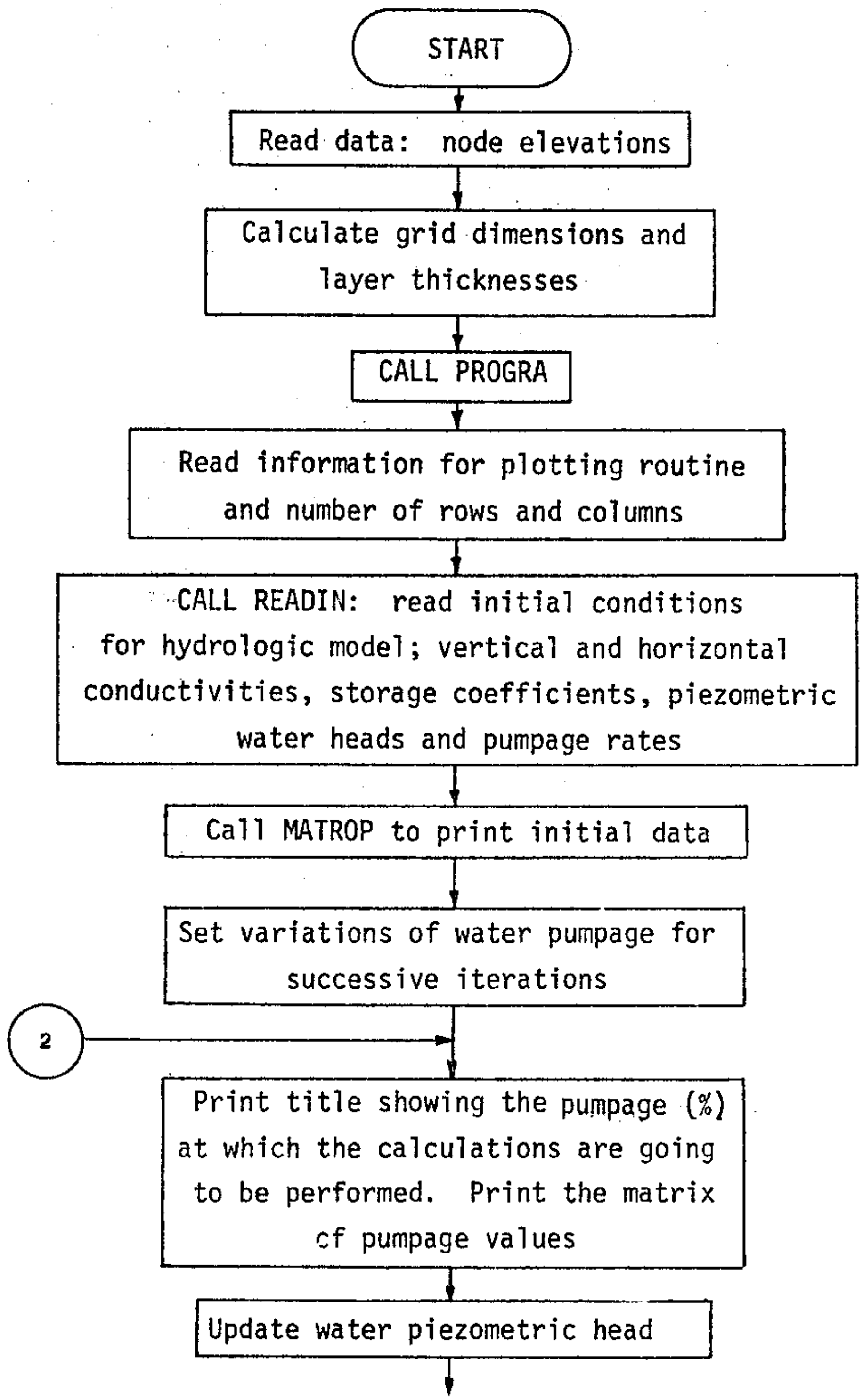
Miles of lines servicing the city:	81 miles
Total water delivered:	$98{,}144 \times 10^3$ gallons
Total capital expenditure:	$11{,}981 \times 10^3$ dollars.

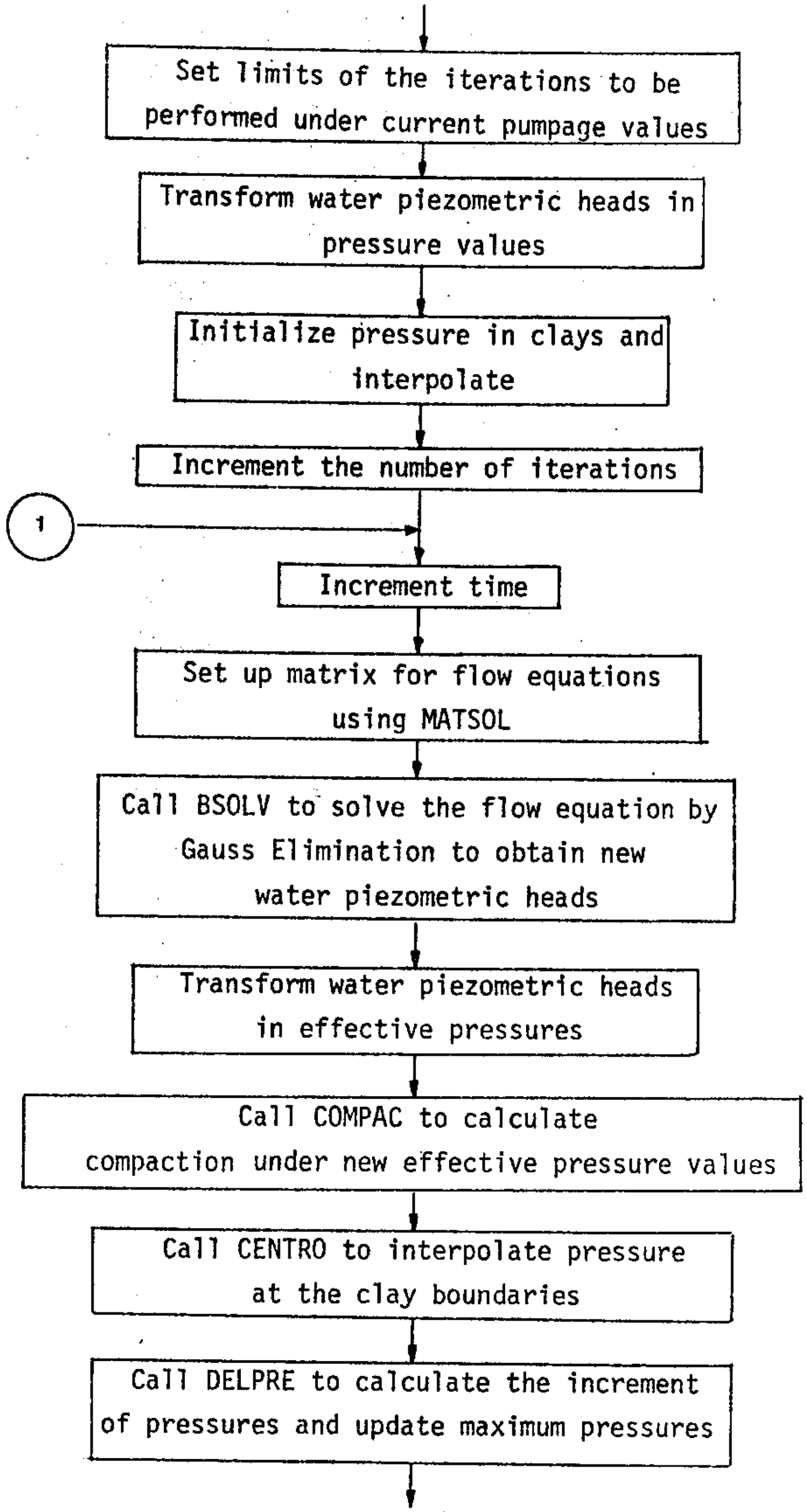
The estimated water transport cost is expressed in Table V-1 using metric units.

Those values shown in Table V-1 were used as a guide to estimate the values used in the study, and to conclude what kind of economical information is necessary to be obtained to have a much better estimate of the total cost when using the optimization model to analyze water management policy.

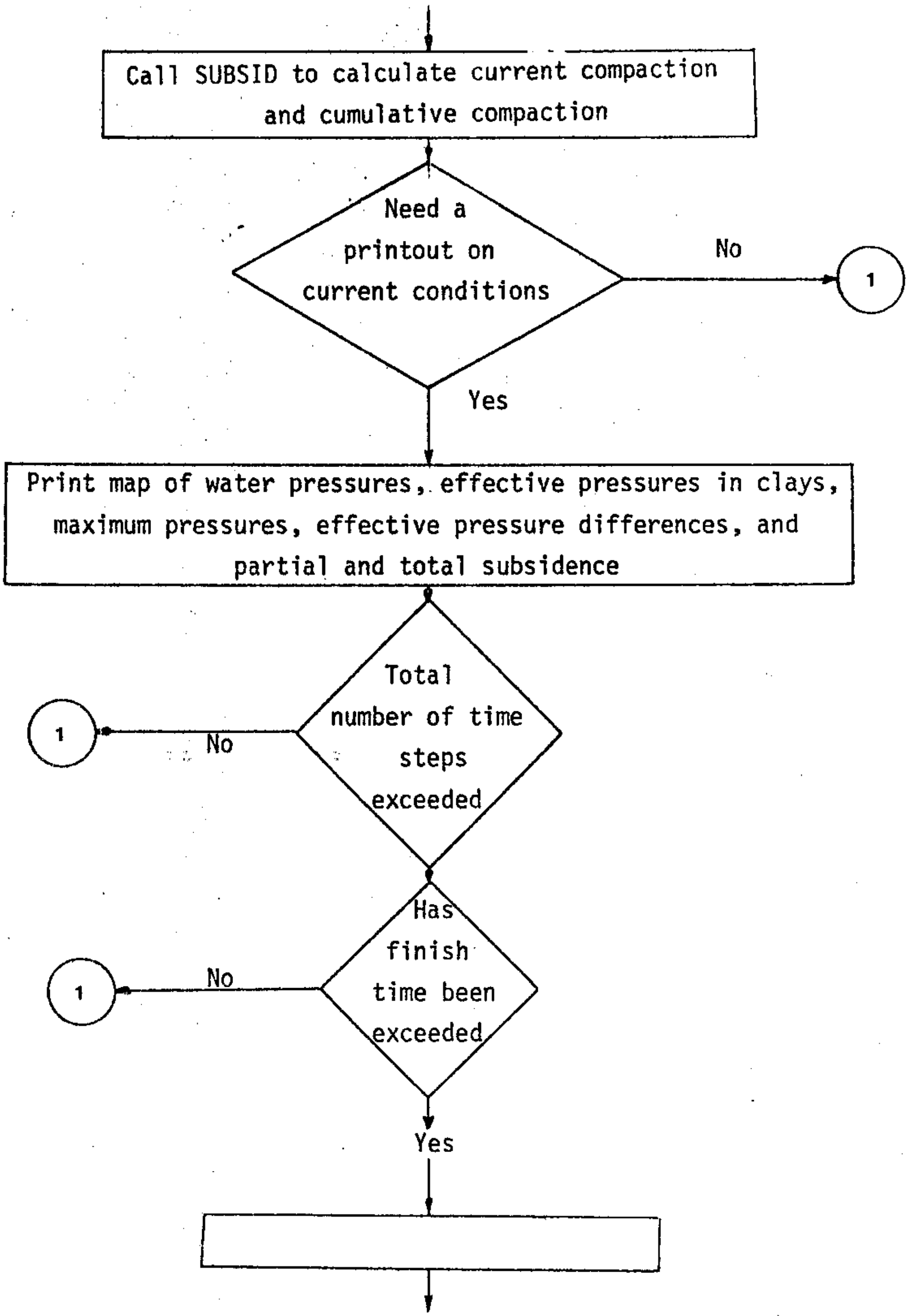
APPENDIX VI

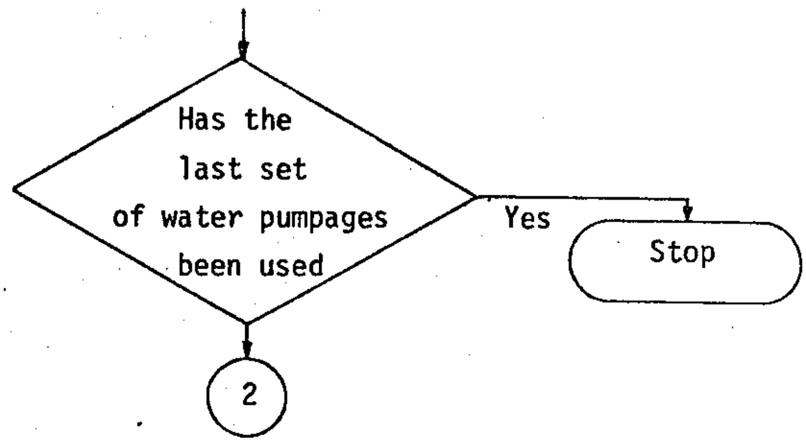
FLOW CHART OF PROGRAM





Set limits of the iterations to be performed under current pumpage values
Transform water piezometric heads in pressure values
Initialize pressure in clays and interpolate
Increment the number of iterations
1
Increment time
Set up matrix for flow equations using MATSOL
Call BSOLV to solve the flow equation by Gauss Elimination to obtain new water piezometric heads
Transform water piezometric heads in effective pressures
Call COMPAC to calculate compaction under new effective pressure values
Call CENTRO to interpolate pressure at the clay boundaries
Call DELPRE to calculate the increment of pressures and update maximum pressures


Call SUBSID to calculate current compaction and cumulative compaction
Need a printout on current conditions
No
1
Yes
Print map of water pressures, effective pressures in clays, maximum pressures, effective pressure differences, and partial and total subsidence
Total number of time steps exceeded
No
1
Has finish time been exceeded
No
1
Yes


Has the
last set
of water pumpages
been used
Yes
Stop
2

APPENDIX VII

COMPUTER PROGRAMS

(1) Hydrological and Compaction Models

(2) MPSX/IBM Programs

(3) MPSX-Data Punching Program

```
C*******************************************************************************
C*******************************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C         NUMERICAL SIMULATION OF ONE-PHASE FLUID FLOW AND COMPACTION OF
C         A SATURATED POROUS MEDIA
C
C
C
C.....THIS PROGRAM SIMULATES ONE DIMENSIONAL FLUID FLOW THROUGH A
C      SATURATED NONHOMOGENEOUS,ISOTROPIC POROUS MEDIA.  THE BOUNDARY CONDI-
C      TIONS ARE: INFILTRATING CONSTANT-HEAD UPPER BOUNDARY; CONSTANT HEAD
C      GRIDS ON THE LEFT AND RIGHT HAND SIDES OF THE GRID SYSTEM, AND
C      IMPERVIOUS CONSTANT HEAD LOWER BOUNDARY.
C      THE INITIAL CONDITIONS ARE: NON-UNIFORM PIEZOMETRIC WATER HEADS, NON-
C      UNIFORM STORAGE COEFFICIENTS, NON-UNIFORM VERTICAL AND HORIZONTAL
C      HYDRAULIC CONDUCTIVITIES, NON-UNIFORM PUMPAGE, VARIABLE HORIZONTAL AN
C      VERTICAL GRID DIMENSIONS.
C
C
C.....THE PROGRAM LISTING THAT FOLLOWS AND THE DIMENSION STATEMENTS ARE
C      SPECIFIC TO THE FLOW PROBLEM.
C
C.....NEXT TO THE WATER PRESSURE CALCULATIONS, THE SUBROUTINE COMPAC
C      CONTROLS THE PROCEDURE FOR CALCULATING THE CURRENT COMPACTION.
C      THE NODE NUMBERS ARE REDEFINED INTERNALLY TO FACILITATE COMPUTATIONS.
C      AGAIN THE DIMENSION STATEMENTS ARE SPECIFIC TO THE PROBLEM.
C
C
C.....THE PROGRAM WAS WRITTEN IN FORTRAN IV LANGUAGE AND RUN ON
C      AMDHAL 470V/6 AT TEXAS A&M UNIVERSITY DATA PROCESSING CENTER.
C
C
C.....THE FOLLOWING VARIABLES ARE USED THROUGHOUT THE MAIN PROGRAM
C      AND SUBROUTINES
C
C      NC    = NUMBER OF COLUMNS
C      NR    = NUMBER OF ROWS
C      ELEV  = ELEVATION OF NODES(S.L.) (METER)
C      HC    = ELEVATION OF GRID CENTERS (METER)
C      CX    = HORIZONTAL HYDRAULIC CONDUCTIVITY (METER/SEC)
C      CY    = VERTICAL HYDRAULIC CONDUCTIVITY (METER/SEC)
C      H     = INITIAL PIEZOMETRIC HEAD (METER)
C      HP    = PIEZOMETRIC HEAD AT PREVIOUS TIME STEP (METER)
C      HT    = PIEZOMETRIC HEAD AT PRESENT TIME STEP (METER)
C      DHT   = PIEZOMETRIC WATER HEAD DRAWDOWN (METER)
C      S     = SPECIFIC STORAGE COEFFICIENT (1/METER)
C      DH    = GRID THICKNESS (METER)
C      DELX  = HORIZONTAL GRID SPACEMENT (KILOMETERS)
C      DELZ  = VERTICAL GRID SPACEMENT (METER)
C      Q     = PUMPAGE RATE (CUBIC METER/SEC)
C      QR    = PUMPAGE ( CUBIC METER)
C      P     = INITIAL WATER PRESSURE (KG-F/ SQ METER)
C      PD    = WATER PRESSURE AT PREVIOUS TIME STEP (KG-F/ SQ METER)
C      PT    = WATER PRESSURE AT PRESENT TIME STEP (KG-F/ SQ METER)
C      PN    = EFFECTIVE PRESSURE AT PRESENT TIME STEP (KG-F/ SQ METER)
C      PP    = EFFECTIVE PRESSURE AT PREVIOUS TIME STEP (KG-F/SQ METER)
C      PMAX  = MAXIMUM EFFECTIVE PRESSURE AT PRESENT TIME STEP (KG-F/ SQ METE
C      PMAXI= MAXIMUM EFFECTIVE PRESSURE EVER (KG-F/ SQ METER)
```

```
C       DPV   = INCREMENT OF VIRGIN COMPACTING PRESSURE (KG-F/SQ METER)
C       DPE   = INCREMENT OF ELASTIC COMPACTING PRESSURE (KG-F/SQ METER)
C       SS    = SPECIFIC STORAGE COEFFICIENT (M/ SQ SEC)
C       SUSK = PARTIAL COMPACTION PER LAYER (METER)
C       SUSE = TOTAL COMPACTION  AT PRESENT TIME STEP (METER)
C       STOT = TOTAL CUMULATIVE COMPACTION (METER)
C       DZZ   = CLAY THICKNESS (METER)
C       ST    = TOTAL TIME (DAYS)
C       DELT = TIME INCREMENT (DAYS)
C       G     = ACCELERATION OF GRAVITY M/ SQ SEC)
C       RHO   = FLUID DENSITY (KG/ CUBIC METER)
C       VIS
C       SSKV = VIRGIN SPECIFIC STORAGE COEFFICIENT (1/METER)
C       SSKE = ELASTIC SPECIFIC STORAGE COEFFICIENT (1/METER)
C       FWTOP= PRINT CONTROL. TO BE EXACT MULTIPLE OF TIME STEP
C       KC    = COMPACTION HISTORY CONTROL ARRAY
C
C
C.....XMAX,XMIN,YMAX,YMIN,NSCALE,LABEL,DEC,LABEO,IMAGE, ARE
C       VARIABLES USED IN THE BUILT IN PLOTTING ROUTINE
C
C
C*************************************************************************
C*************************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C*********MAIN PROGRAM
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C*************************************************************************
C
C
      DIMENSION KC(8,18)
      DOUBLE PRECISION ELEV(13,19),HC(13,18),CX(12,18),CY(12,18),
     1                 HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                 DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                 SU(19),GH(19),DHT(12,18),SUSK(8,18),SUSE(2,18),
     4                 DZZ(6,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                 HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                 DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                SSKV,SSKE,FWTOP

      DOUBLE PRECISION STOT(1,18)
      COMMON ELEV,HC,CX,CY,HT,H,S,DH,DELX,DELZ,Q,HP,SU,GH,SSKV,DELT,ST
      COMMON G,FWTOP,RHO
      COMMON/SUBS1/SUSK,SUSE,SSKE
      COMMON/COMP1/PT,PD,PN,PP
      COMMON/COMP2/PMAX,PMAX1,DPV,DPE,KC
      COMMON/COMP3/SS,STOT
C
C       READ NODE ELEVATIONS (FEET)
C
      READ(5,1) ((ELEV(I,J),I=1,13),J=1,19)
C
C       CONVERT TO METRIC SYSTEM (METERS)
C
      DO 2 I=1,13
      DO 2 J=1,19
    2 ELEV(I,J)= ELEV(I,J)*.3048 + 304.8
      DO 12 I=1,13
      DO 12 J=1,18
```

```
C
C       CALCULATE GRID DIMENSIONS
C
C**     CALCULATE THICKNESS OF INDIVIDUAL LAYERS
        HC(I,J)=(ELEV(I,J+1)+ELEV(I,J))/2.
   12   CONTINUE
        DO 28 I=1,12
        DO 28 J=1,19
        DH(I,J)=HC(I,J)-HC(I+1,J)
   28   CONTINUE
C
        DO 15 J=1,18
        DO 15 I=1,12
        DELZ(I,J)=DH(I,J)
   15   CONTINUE
        DO 17 I=1,12  .
        DO 16 J=3,16
        DELX(I,J)=5.56*1000
   16   CONTINUE
        DELX(I,1)=8.33*1000.
        DELX(I,2)=8.33*1000.
        DELX(I,17)=8.33*1000.
        DELX(I,18)=8.33*1000.
   17   CONTINUE
        CALL PROGRA
        STOP
    1   FORMAT(13F6.0)
        END
C
C
C***********************************************************************
C***********************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

        SUBROUTINE PROGRA
        DOUBLE PRECISION ELEV(13,19),HC(13,18),CX(12,18),CY(12,18),
     1                  HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                  DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                  SU(19),GH(19),DHT(12,18),SUSK(8,18),SUSE(2,18),
     4                  DZZ(5,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                  HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                  DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                 SSKV,SSKE,FWTOP

        DOUBLE PRECISION X(18),Y(18),U(18),CL(18),CU(18),CT(18),
     9                  XMAX,XMIN,YMAX,YMIN
        DOUBLE PRECISION DELT1,ST1
        DOUBLE PRECISION QR(12,18)
        DOUBLE PRECISION STOT(1,18)
        DOUBLE PRECISION QI(12,18)
        DOUBLE PRECISION PCNT
        DIMENSION KC(8,18),NSCALE(5),LABEL(5),DCD(13),LABEO(5),IMAGE(1400)
        COMMON ELEV,HC,CX,CY,HT,H,S,DH,DELX,DELZ,Q,HP,SU,GH,SSKV,DELT,ST
        COMMON G,FWTOP,RHO
        COMMON/SUBS1/SUSK,SUSE,SSKE
        COMMON/COMP1/PT,PD,PN,PP
        COMMON/COMP2/PMAX,PMAX1,DPV,DPE,KC
        COMMON/COMP3/SS,STOT
C
C       READ INFORMATION FOR PLOTS
```

```
C
      READ(5,200)(LABEL(I),I=1,5),(DCD(K),K=1,13),(LABEO(J),J=1,5)
  200 FORMAT(5A4,13A1,5A4)
C
C     READ NUMBER OF ROWS AND COLUMNS
C
      READ(5,12) NR,NC
      NA=(NR-2)*(NC-2)
      NB=(2*NR)-3
      WRITE(6,11)
C
C
C     READ INITIAL DATA
C
      CALL READIN(NR,NC,NA,NB)
      READ(5,13) VIF,SSKV,SSKE
C
C     PRINT INITIAL DATA
C
      WRITE(6,31)VIF,SSKV,SSKE
      WRITE(6,11)
      WRITE(6,20)
      CALL MATROP(NR,NC,CX)
      WRITE(6,11)
      WRITE(6,21)
      CALL MATROP(NR,NC,CY)
      WRITE(6,11)
      WRITE(6,22)
      CALL MATROP(NR,NC,S)
      WRITE(6,11)
      WRITE(6,23)
      CALL MATROP(NR,NC,H)
      WRITE(6,11)
      WRITE(6,24)
      CALL MATROP(NR,NC,DELX)
      WRITE(6,11)
      WRITE(6,25)
      CALL MATROP(NR,NC,DELZ)
      WRITE(6,11)
      WRITE(6,26)
      CALL MATROP(NR,NC,Q)
      WRITE(6,11)
C
C     CALCULATE ELEVATIONS OF GRID CENTERS
C
      DO 70 I=1,12
      DO 70 J=1,18
   70 HE(I,J)=(HC(I,J)+HC(I+1,J))/2.
C
C     CONVERT UNITS OF SPECIFIC STORAGE COEFFICIENTS OF CLAYS
C
      DO 79 J=1,18
      SS(1,J)=S(2,J)/(RHO*G)
      SS(2,J)=S(2,J)/(RHO*G)
      SS(3,J)=S(3,J)/(RHO*G)
      SS(4,J)=S(4,J)/(RHO*G)
      SS(5,J)=S(7,J)/(RHO*G)
      SS(6,J)=S(7,J)/(RHO*G)
      SS(7,J)=S(8,J)/(RHO*G)
      SS(8,J)=S(9,J)/(RHO*G)
   79 CONTINUE
```

```
      SSKE=SSKE/(RHO*G)
C
C     CONVERT TIME CONTROL VARIABLES TO SECONDS
C
      DELT=DELT*86400
      ST=ST*86400
C
C     DEFINE NUMBER OF INCREASES IN PUMPAGE TO BE PERFORMED
C
      DO 997 I=1,12
      DO 997 J=1,18
  997 QI(I,J)=Q(I,J)
      DO 998 KK=5,6
      INCRS=KK*20-80
      DO 995 I=1,8
      DO 995 J=1,18
  995 KC(I,J)=0
      WRITE(6,218)
      WRITE(6,217) INCRS
      WRITE(6,11)
C
C     INITIALIZE SUBSIDENCE ARRAYS
C
      DO 75 J=1,18
      DO 74 I=1,8
   74 SUSK(I,J)=0.
      DO 75 I=1,2
   75 SUSE(I,J)=0.
C***  VARY PUMPAGE RATE  ********
      DO 999 I=1,12
      DO 999 J=1,18
      Q(I,J)=QI(I,J)*(100+INCRS)*.01
  999 CONTINUE
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C     CALCULATE TOTAL PUMPAGE FOR THE TOTAL SIMULATION TIME
C
      DO 889 I=1,12
      DO 889 J=1,18
      QR(I,J)=Q(I,J)*86400.*365.*10.
  889 CONTINUE
      WRITE(6,11)
      WRITE(6,29)
C
      CALL MATROP(NR,NC,QR)
C
      DELT1=DELT
      ST1=ST
      DO 27 I=1,12
      WRITE(6,11)
      DO 27 J=1,18
   27 HT(I,J)=H(I,J)
      DO 300 J=1,18
      DO 300 I=1,8
  300 KC(I,J)=0
C
C     DEFINE NUMBER OF ITERATIONS TO BE PERFORMED AT THE CURRENT
C     PUMPAGE
C
      LOOPUL=ST1/DELT1 + .5
      PCNT=1.0
```

```
      DO 77 I=1,12
C
C     CALCULATE WATER PRESSURES
C
      DO 77 J=1,18
      P(I,J)=(H(I,J)-HE(I,J))*RHO*G
      PT(I,J)=P(I,J)
      PD(I,J)=P(I,J)
   77 CONTINUE
C
C     INITIALIZE MAXIMUM  EFECTIVE PRESSURES
C
      CALL CENTRO
C
      DO 78 I=1,8
      DO 78 J=1,18  .
      PMAX(I,J)=0.0
      PMAX1(I,J)=0.0
   78 CONTINUE
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C     INITIATE ITERATIONS
C
C
      TIME=0.0
      DO 8 ILAST=1,LOOPUL
      TIME=TIME + DELT1
C
C     SET MATRIX AND VECTORS TO SOLVE FLOW EQUATION
C
      CALL MATSOL(NP,NC,NA,NB)
C
C     CALCULATE EFFECTIVE PRESSURES
C
      DO 71 I=1,12
      DO 71 J=1,18
      PD(I,J)=P(I,J)-(HP(I,J)-HE(I,J))*RHO*G
      PT(I,J)=P(I,J)-(HT(I,J)-HE(I,J))*RHO*G
   71 CONTINUE
C
C     CALCULATE COMPACTION
C
      CALL COMPAC
C
      IPCNT=PCNT + .5
      IFWTOP=FWTOP + .5
C
C     ASK IF A PRINTOUT IS DESIRED
C
      IF(IPCNT.EQ.IFWTOP) GO TO  3
      GO TO 9
    3 TIMEX=TIME/86400
C
C     CALCULATE WATER DRAWDOWNS
C
      DO 100 I=1,12
      DO 100 J=1,18
  100 DHT(I,J)=HT(I,J)-H(I,J)
C
C     PRINT CURRENT INFORMATION
```

```
C
      WRITE(6,91)
      WRITE(6,10) TIMEX
      CALL MATROP(NR,NC,HT)
      WRITE(6,11)
      CALL MATROP(NR,NC,DHT)
      WRITE(6,11)
      WRITE(6,76)
      CALL MATROP(NR,NC,PT)
      WRITE(6,11)
      WRITE(6,210)
      CALL MATROP(8,18,PN)
      WRITE(6,11)
      WRITE(6,211)
      CALL MATROP(8,18,PMAX)
      WRITE(6,11)
      WRITE(6,213)
      CALL MATROP(8,18,DPV)
      WRITE(6,11)
      WRITE(6,214)
      CALL MATROP(8,18,DPE)
      WRITE(6,11)
      WRITE(6,215)
      CALL MATROP(8,18,SUSK)
      WRITE(6,11)
      WRITE(6,18) TIMEX
      CALL MATROP(02,NC,SUSE)
      WRITE(6,216)
      CALL MATROP(1,18,STOT)
      WRITE(6,11)
      WRITE(6,11)
C
C     ASK IF IT IS TIME TO PLOT
C
      IF(TIME.GE.ST) GO TO 53
      GO TO 52
   53 CONTINUE
      NSCALE(1)=1
      NSCALE(2)=0
      NSCALE(3)=3
      NSCALE(4)=0
      NSCALE(5)=0
      NHL=10
      NVL=18
      NSBH=4
      NSBV=5
      YMAX=360.
      YMIN=0.0
      XMAX=18
      XMIN=1
      BCD=DCD(5)
      WRITE(6,91)
   52 CONTINUE
C
C     ASK IF COUNTER CAN BE SET AT (1) AGAIN
C
    9 IF(IPCNT.EQ.IFWTOP) GO TO  6
      GO TO 8
C
C     INCREMENT COUNTER
C
```

```
  6   PCNT=0.0
  8   PCNT=PCNT + 1.0
 998 CONTINUE
      RETURN
C
CC
C
C
  10  FORMAT(1H0,47X,'NEW WATER HEADS ( METERS)AFTER',2X,F8.0,' DAYS')
  11  FORMAT('1')
  12  FORMAT(2I4)
  13  FORMAT(3D12.5)
  18  FORMAT(1H0,47X,'CURRENT COMPACTION(METERS) AFTER',F8.0,' DAYS')
  20  FORMAT(1H0,40X,'HORIZONTAL CONDUCTIVITIES  (METER/SEC) ')
  21  FORMAT(1H0,40X,' VERTICAL CONDUCTIVITIES   (METER/SEC) ')
  22  FORMAT(1H0,40X,'SPECIFIC STORAGE COEFFICIENTS  (1/METER)')
  23  FORMAT(1H0,40X,'HYDRAULIC WATER HEADS (METER)  - 305 METER BSL ')
  24  FORMAT(1H0,40X,'HORIZONTAL GRID SPACEMENT  (METERS)')
  25  FORMAT(1H0,40X,' VERTICAL GRID SPACEMENT  (METERS)')
  26  FORMAT(1H0,40X,' PUMPAGE  (CUBIC METERS/SEC) ')
  29  FORMAT(1H0,47X,'TOTAL PUMPAGE  (CUBIC METERS)  IN TEN YEARS ')
  31  FORMAT(1H0,' VERTICAL INFLOW = ',2X,D10.3,2X,'METERS/SEC',///,
     1'  SPECIFIC STORAGE COEFFICIENT OF CLAYS =',2X,D9.3,2X,
     2' 1/METER',///,'  ELASTIC STOTAGE COEFFICIENT OF CLAYS =',
     32X,D9.3,2X,'1/METER')
  76  FORMAT(1H0,47X,' CORRESPONDING PRESSURES')
  91  FORMAT('1')
 210  FORMAT(1H0,47X,'PRESSURES IN CLAYS')
 211  FORMAT(1H0,47X,'MAXIMUM PRESSURES')
 213  FORMAT(1H/,47X,'VIRGIN PRESSURES DIFFERENCES')
 214  FORMAT(1H0,47X,'ELASTIC PRESSURES DIFFERENCES')
 215  FORMAT(1H0,47X,'PARTIAL COMPACTION VALUES')
 216  FORMAT(1H0,47X,' TOTAL SUBSIDENCE AT THE SURFACE ')
 217  FORMAT(1H0,47X,'ANALYSIS FOR A',///,55X,I4,///,20X,'PERCENT INCREA
     1SE OF THE PUMPAGE RATE AT EVERY POINT')
 218  FORMAT(//////////////////////////////////////////////////////////)
      END
C
C
C
C
C*************************************************************************
C*************************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

      SUBROUTINE READIN(NR,NC,NA,NB)
C
C     THIS ROUTINE READS INITIAL DATA
C
      DIMENSION KC(8,18)
      DOUBLE PRECISION ELEV(13,19),HC(13,18),CX(12,18),CY(12,18),
     1                 HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                 DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                 SU(19),GH(19),DHT(12,18),SUSK(8,18),SUSE(2,18),
     4                 DZZ(6,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                 HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                 DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                SSKV,SSKE,FWTOP

      COMMON ELEV,HC,CX,CY,HT,H,S,DH,DELX,DELZ,Q,HP,SU,GH,SSKV,DELT,ST
```

```
      COMMON G,FWTOP,RHO
      COMMON/SUBS1/SUSK,SUSE,SSKE
      COMMON/COMP1/PT,PD,PN,PP
      COMMON/COMP2/PMAX,PMAX1,DPV,DPE,KC
C
C
      READ(5,1) DELT,ST,FWTOP
      READ(5,2) G,BETA,RHO
      WRITE(6,41)G,BETA,RHO
      WRITE(6,40) DELT,ST,FWTOP
      READ(5,4)((CX(I,J),J=1,NC),I=1,NR)
      READ(5,4)((CY(I,J),J=1,NC),I=1,NR)
      READ(5,4)(( S(I,J),J=1,NC),I=1,NR)
      READ(5,4)(( H(I,J),J=1,NC),I=1,NR)
      READ(5,4)(( Q(I,J),J=1,NC),I=1,NR)
      DO 990 I=1,NR
      DO 990 J=1,NC
  990 H(I,J)=H(I,J)*1000.
      RETURN
    1 FORMAT(3D10.3)
    2 FORMAT(3D10.3)
    4 FORMAT(6D13.7)
   40 FORMAT(1H0,'TIME INCREMENT =',F10.3,2X,'DAYS',//,
     11X,'TOTAL TIME = ',F10.3, 2X,'DAYS',//,
     21X,'PRINTOUT CONTROL =',F10.3,//)
   41 FORMAT(1H ,'ACCELERATION OF GRAVITY =',F10.3,'METER/SQ SEC',/,
     11X,'FLUID COMPRESSIBILITY=',F10.3,' SQ METER/KG ',/,
     21X,'DENSITY OF THE FLUID =',F10.3,2X,' KG/CUBIC CM',/)
      END
C
C
C
C*************************************************************************
C*************************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

      SUBROUTINE MATROP(NR,NC,B)
C
C***  THIS ROUTINE PRINTS INITIAL DATA AND CURRENT STATE INFORMATION
C
      DOUBLE PRECISION B(NR,NC),A(12)
      DIMENSION LC(18)
      WRITE(6,13)
      KONT=1
      KL=9
      KF=1
      DO 30 J=1,12
      LC(J)=J
   30 CONTINUE
      DO 11 I=1,NC,9
      IN=I/9
      DO 9 J=1,NR
      IF(J.EQ.1) GO TO 14
      KKT=0
      GO TO 15
   14 KKT=1
   15 IF((IN+1)*9.GE.NC) GO TO 3
    1 DO 2 JJ=1,9
      JJJ=IN*9+JJ
    2 A(JJ)=B(J,JJJ)
```

```
      GO TO 6
    3 LL=NC-IN*9
      KF=KL+1
      KL=KL+9
      IF(4.EQ.1) KKT=-2
      DO 4 JJ=1,9
      JJJ=IN*9+JJ
    4 A(JJ)=B(J,JJJ)
      LL=LL+1
      IF(LL.GE.9) GO TO 6
      DO 5 JJ=LL,9
    5 A(JJ)=9999999999999.
    6 CONTINUE
      DO 22 II=1,9
   22 CONTINUE
    7 IF(KKT) 71,70,71
   71 WRITE(6,121)(K,K=KF,KL)
      WRITE(6,24)
   70 IF(KONT) 20,20,20
   20 WRITE(6,35) LC(J),(A(II),II=1,9),J
      GO TO 9
    9 KONT=1
      WRITE(6,23)
   11 CONTINUE
      RETURN
   13 FORMAT(1H0,///)
   23 FORMAT('0')
   24 FORMAT(' ')
   35 FORMAT(1H ,9X,I2,7X,9(D10.3,2X),4X,I2)
  121 FORMAT(1H ,8X,'LAYER/POINT',4X,I2,8(10X,I2))
      END
C
C
C*****************************************************************************
C*****************************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

      SUBROUTINE MATSOL(NR,NC,NA,NB)
C
C***  THIS ROUTINE SOLVES THE FLOW EQUATION BY GAUSS ELIMINATION
C
      DOUBLE PRECISION CMATRX(160,21),CR(160),BB(12,18),DZ(12,19),
     1                 DX(13,19),PARAM,TIMAM,AK1,AK2,DS1,DS2,DS3,
     2                 DT1,AS1,DXC(12,18),DZC(12,18)
      DOUBLE PRECISION ELEV(13,19),HG(13,18),CX(12,18),CZ(12,18),
     1                 HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                 DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                 SU(19),GH(19),DHT(12,18),SUSK(8,18),SUSE(2,18),
     4                 DZZ(6,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                 HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                 DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                SSKV,SSKE,FWTOP

      COMMON ELEV,HG,CX,CZ,HT,H,S,DH,DXC ,DZC ,Q,HP,SU,GH,SSKV,DELT,ST
      COMMON G,FWTOP,RHO
C     HT= HEAD @ CENTER AT PRESENT TIME STEP
C     HP= HEAD @ CENTER AT PREVIOUS TIME STEP
C     H= HEAD AT INITIAL TIME STEP
C     DZ= VERTICAL DIMENSION @ GRID INTERFACES
C     DZC= VERTICAL DIMENSION @ GRID CENTER
```

```
C       CZ= VERTICAL CONDUCTIVITY
C       CX= HORIZONTAL CONDUCTIVITY
C       S= STORAGE COEFFICIENT
C       BB= ARRAY OF VALUES TO INDICATE BOUNDARY CONDITIONS
C       CR= VECTOR OF KNOWN VALUES
C       CMATRX= MATRX OF COEFFICIENTS
C
C
        PARAM(AK1,AK2,DS1,DS2,DS3)=(2.0*AK1*AK2*DS1)/((AK1*DS2)+(AK2*DS3))
        TIMAM(AS1,DS1,DS2,DS3,DT1)=((.5*AS1*DS1)*(1./DT1)*(DS2+DS3))
C
C       SET BOUNDARY CONTROL ARRAY
        DO 23 I=1,12
        DO 23 J=1,18
   23   BB(I,J)=15.
        DO 24 I=2,11
        DO 24 J=2,17
   24   BB(I,J)=0.0
C
        DO 1 J=1,NB
        DO 1 I=1,NA
    1   CMATRX(I,J)=0.0
C
C       CALCULATE GRID SIDE DIMENSIONS
        DO 20 J=1,19
        DO 20 I=1,12
   20   DZ(I,J)=ELEV(I,J)-ELEV(I+1,J)
        DO 21 J=1,18
        DO 21 I=1,12
   21   DX(I,J)=DXC(I,J)
        DO 22 J=1,18
   22   DX(13,J)=DX(12,J)
        DO 28 I=1,12
        DO 28 J=1,18
   28   HP(I,J)=HT(I,J)
        NT=0
        NC1=NC-1
        NR1=NR-1
        IB=NR-2
        IM=IB+1
        IC=IM+1
        ID=2*IB+1
        DO 12 J=2,NC1
        DO 12 I=2,NR1
        NT=NT+1
        CR(NT)=0.0
        IF(BB(I,J).GE.10.0) GO TO 11
        IF(CX(I,J).EQ.0.0.AND.CX(I,J-1).EQ.0.0) GO TO 30
    2   CMATRX(NT,1)=PARAM(CX(I,J),CX(I,J-1),DZ(I,J),DXC(I,J),DXC(I,J-1))
   30   IF(CZ(I,J).EQ.0.0.AND.CZ(I-1,J).EQ.0.0) GO TO 31
        CMATRX(NT,IB)=PARAM(CZ(I,J),CZ(I-1,J),DX(I,J),DZC(I,J),DZC(I-1,J))
   31   IF(CZ(I,J).EQ.0.0.AND.CZ(I+1,J).EQ.0.0) GO TO 32
        CMATRX(NT,IC)=PARAM(CZ(I,J),CZ(I+1,J),DX(I+1,J),DZC(I,J),DZC(I+1,
       1J))
   32   IF(CX(I,J).EQ.0.0.AND.CX(I,J+1).EQ.0.0) GO TO 33
        CMATRX(NT,ID)=PARAM(CX(I,J),CX(I,J+1),DZ(I,J+1),DXC(I,J),DXC(I,
       1J+1))
   33   IF(BB(I,J-1).LT.10.0) GO TO 4
    3   CR(NT)=CR(NT)-(CMATRX(NT,1)*HT(I,J-1))
        CMATRX(NT,IM)=CMATRX(NT,IM)-CMATRX(NT,1)
        CMATRX(NT,1)=0.0
```

```
    4   IF(BB(I-1,J).LT.10.0) GO TO 6
    5   CR(NT)=CR(NT)-(CMATRX(NT,IB)*HT(I-1,J))
        CMATRX(NT,IM)=CMATRX(NT,IM)-CMATRX(NT,IB)
        CMATRX(NT,IB)=0.0
    6   IF(BB(I+1,J).LT.10.) GO TO 8
    7   CR(NT)=CR(NT)-(CMATRX(NT,IC)*HT(I+1,J))
        CMATRX(NT,IM)=CMATRX(NT,IM)-CMATRX(NT,IC)
        CMATRX(NT,IC)=0.0
    8   IF(BB(I,J+1).LT.10.) GO TO 10
    9   CR(NT)=CR(NT)-(CMATRX(NT,ID)*HT(I,J+1))
        CMATRX(NT,IM)=CMATRX(NT,IM)-CMATRX(NT,ID)
        CMATRX(NT,ID)=0.0
   10   CMATRX(NT,IM)=CMATRX(NT,IM)-(CMATRX(NT,1)+CMATRX(NT,IB)+CMATRX
       1(NT,IC)+CMATRX(NT,ID)+TIMAM(S(I,J),DXC(I,J),DZ(I,J),DZ(I,J+1),
       2DELT))
        CR(NT)=CR(NT)-(TIMAM(S(I,J),DXC(I,J),DZ(I,J),DZ(I,J+1),DELT)*
       1HT(I,J))+Q(I,J)
        GO TO 12
   11   CMATRX(NT,IM)=1.0
        CR(NT)=HT(I,J)
   12   CONTINUE
C
        CALL BSOLVE(CMATRX,NA,NB,CR)
C
        NT=0
        DO 13 J=2,NC1
        DO 13 I=2,NR1
        NT=NT+1
   13   HT(I,J)=CR(NT)
        RETURN
        END
C
C
C***********************************************************************
C***********************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

        SUBROUTINE BSOLVE(C,N,M,V)
C
C***    CALL TRAPS... CALL A PROTECTIVE ROUTINE (WATFIV) TO SKIP
C                     UNDERFLOW AND OVERFLOW SITUATIONS
C
        DOUBLE PRECISION C(N,M),V(N),TEMP
        LR=(M-1)/2
        CALL TRAPS(1,1,1,1,1)
        DO 2 L=1,LR
        IM=LR-L+1
        DO 2 I=1,IM
        DO 1 J=2,M
    1   C(L,J-1)=C(L,J)
        KN=N-L
        KM=M-I
        C(L,M)=0.0
    2   C(KN+1,KM+1)=0.0
        LR=LR+1
        IM=N-1
        DO 10 I=1,IM
        NPIV=I
        LS=I+1
        DO 3 L=LS,LR
```

```
      IF(DABS(C(L,1)).GT.DABS(C(NPIV,1))) NPIV=L
    3 CONTINUE
      IF(NPIV.LE.I) GO TO 6
    4 DO 5 J=1,M
      TEMP=C(I,J)
      C(I,J)=C(NPIV,J)
    5 C(NPIV,J)=TEMP
      TEMP=V(I)
      V(I)=V(NPIV)
      V(NPIV)=TEMP
    6 V(I)=V(I)/C(I,1)
      DO 7 J=2,M
    7 C(I,J)=C(I,J)/C(I,1)
      DO 9 L=LS,LR
      TEMP=C(L,1)
      V(L)=V(L)-TEMP*V(I)
      DO 8 J=2,M
      CALL TRAPS(1,1,1,1,1)
      C(L,J-1)=C(L,J)-TEMP*C(I,J)
    8 CONTINUE
    9 C(L,M)=0.0
      IF(LR.LT.N) LR=LR+1
   10 CONTINUE
      V(N)=V(N)/C(N,1)
      JM=2
      DO 12 I=1,IM
      L=N-I
      DO 11 J=2,JM
      CALL TRAPS(1,1,1,1,1)
      KM=L+J
   11 V(L)=V(L)-C(L,J)*V(KM-1)
      IF(JM.LT.M) JM=JM+1
   12 CONTINUE
      RETURN
      END
C
C
C*************************************************************************
C*************************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

      SUBROUTINE COMPAC
C
C***  THIS  ROUTINE CONTROLS THE CALCULATIONS OF COMPACTION DUE TO
C     CHANGING EFFECTIVE STRESSES
C
      DIMENSION KC(8,18)
      DOUBLE PRECISION ELEV(13,19),HC(13,18),CX(12,18),CY(12,18),
     1                 HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                 DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                 SU(19),GH(19),DHT(12,18),SUSK(6,18),SUSE(2,18),
     4                 DZZ(6,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                 HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                 DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                SSKV,SSKE,FWTOP

      DOUBLE PRECISION STOT(1,18)
      COMMON ELEV,HC,CX,CY,HT,H,S,DH,DELX,DELZ,Q,HP,SU,GH,SSKV,DELT,ST
      COMMON G,FWTOP,RHO
      COMMON/SUBS1/SUSK,SUSE,SSKE
```

```
      COMMON/COMP1/PT,PD,PN,PP
      COMMON/COMP2/PMAX,PMAX1,DPV,DPE,KC
      COMMON/COMP3/SS,STOT
C
      CALL CENTRO
C
      CALL DELPRE
C
      CALL SUBSID
C
C
      RETURN
      END
C
C
C
C*****************************************************************************
C*****************************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      SUBROUTINE CENTRO
C
C***  THIS ROUTINE SETS EFFECTIVE PRESSURES AT CLAY BOUNDARIES
C
      DIMENSION KC(8,18)
      DOUBLE PRECISION CENTRE,PT1,PT2,DR1,DR2
      DOUBLE PRECISION ELEV(13,19),HC(13,18),CX(12,18),CY(12,18),
     1                 HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                 DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                 SU(19),GH(19),DHT(12,18),SUSK(8,18),SUSE(2,18),
     4                 DZZ(6,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                 HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                 DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                SSKV,SSKE,FWTOP
      COMMON ELEV,HC,CX,CY,HT,H,S,DH,DELX,DELZ,Q,HP,SU,GH,SSKV,DELT,ST
      COMMON G,FWTOP,RHO
      COMMON/SUBS1/SUSK,SUSE,SSKE
      COMMON/COMP1/PT,PD,PN,PP
      COMMON/COMP2/PMAX,PMAX1,DPV,DPE,KC
      CENTRE(PT1,PT2,DR1,DR2)=PT1+(DR1*(PT2-PT1))/(DR1+DR2)
      DO 1 J=1,18
      PN(1,J)=PT(1,J)
      PP(1,J)=PD(1,J)
      PN(2,J)=CENTRE(PT(2,J),PT(3,J),DH(2,J),DH(3,J))
      PP(2,J)=CENTRE(PD(2,J),PD(3,J),DH(2,J),DH(3,J))
      PN(3,J)=CENTRE(PT(3,J),PT(4,J),DH(3,J),DH(4,J))
      PP(3,J)=CENTRE(PD(3,J),PD(4,J),DH(3,J),DH(4,J))
      PN(6,J)=CENTRE(PT(7,J),PT(8,J),DH(7,J),DH(8,J))
      PP(6,J)=CENTRE(PD(7,J),PD(8,J),DH(7,J),DH(8,J))
      PN(7,J)=CENTRE(PT(8,J),PT(9,J),DH(8,J),DH(9,J))
      PP(7,J)=CENTRE(PD(8,J),PD(9,J),DH(8,J),DH(9,J))
      PN(4,J)=PT(5,J)
      PP(4,J)=PD(5,J)
      PN(5,J)=PT(6,J)
      PP(5,J)=PD(6,J)
      PN(8,J)=PT(10,J)
      PP(8,J)=PD(10,J)
    1 CONTINUE
      RETURN
```

```
      END
C
C
C***********************************************************************
C***********************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * :
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * :

      SUBROUTINE DELPRE
C
C***  THIS ROUTINE CALCULATES VARIATIONS OF EFFECTIVE PRESSURES AT CLAY
C     BOUNDARIES BY REVIEWING THE CONSOLIDATION HISTORY OF THE CLAYS
C
      DIMENSION  KC(8,18)
      DOUBLE PRECISION ELEV(13,19),HC(13,18),CX(12,18),CY(12,18),
     1                 HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                 DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                 SU(19),GH(19),DHT(12,18),SUSK(8,18),SUSE(2,18),
     4                 DZZ(6,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                 HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                 DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                SSKV,SSKE,FWTOP

      COMMON ELEV,HC,CX,CY,HT,H,S,DH,DELX,DELZ,Q,HP,SU,GH,SSKV,DELT,ST
      COMMON G,FWTOP,RHO
      COMMON/SUBS1/SUSK,SUSE,SSKE
      COMMON/COMP1/PT,PD,PN,PP
      COMMON/COMP2/PMAX,PMAX1,DPV,DPE,KC
      DO 200 I=1,8
      DO 200 J=1,18
      IF(PN(I,J).GE.PMAX(I,J)) GO TO 201
      KC(I,J)=-1
      IF(PN(I,J).LT.PP(I,J)) GO TO 202
      DPE(I,J)=PP(I,J)-PN(I,J)
      DPV(I,J)=0.
      GO TO 203
  202 DPV(I,J)=0.
      DPE(I,J)=PP(I,J)-PN(I,J)
  203 PMAX1(I,J)=PMAX(I,J)
      GO TO 204
  201 CONTINUE
      IF(KC(I,J)) 206,205,205
  206 DPE(I,J)=PP(I,J)-PMAX(I,J)
      PMAX(I,J)=PN(I,J)
      PMAX1(I,J)=PN(I,J)
      GO TO 204
  205 PMAX(I,J)=PN(I,J)
      DPV(I,J)=PN(I,J)-PP(I,J)
      PMAX1(I,J)=PN(I,J)
      DPE(I,J)=0.
  204 CONTINUE
  200 CONTINUE
      RETURN
      END
C
C
C***********************************************************************
C***********************************************************************
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * :
C* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * :
```

```
      SUBROUTINE SUBSID
C
C***  THIS ROUTINE CALCULATES PARTIAL,TOTAL AND CUMULATIVE COMPACTION AT
C     PRESENT TIME STEP
C
      DOUBLE PRECISION DR
      DIMENSION KC(8,18)
      DOUBLE PRECISION SUBS,Z1,Z2,S1,S2,R1,R2
      DOUBLE PRECISION ELEV(13,19),HC(13,18),CX(12,18),CY(12,18),
     1                 HT(12,18),H(12,18),S(12,18),DH(12,18),
     2                 DELX(12,18),DELZ(12,18),Q(12,18),HP(12,18),
     3                 SU(19),GH(19),DHT(12,18),SUSK(8,18),SUSE(2,18),
     4                 DZZ(6,18),PD(12,18),PT(12,18),PN(8,18),PP(8,18),
     5                 HE(12,18),PMAX(8,18),PMAX1(8,18),DPE(8,18),
     6                 DPV(8,18),SS(8,18),P(12,18),DELT,ST,G,RHO,VIS,VIF
     7,                SSKV,SSKE,FWTOP
      DOUBLE PRECISION STOT(1,18)
      COMMON ELEV,HC,CX,CY,HT,H,S,DH,DELX,DELZ,Q,HP,SU,GH,SSKV,DELT,ST
      COMMON G,FWTOP,RHO
      COMMON/SUBS1/SUSK,SUSE,SSKE
      COMMON/COMP1/PT,PD,PN,PP
      COMMON/COMP2/PMAX,PMAX1,DPV,DPE,KC
      COMMON/COMP3/SS,STOT
      SUBS(Z1,Z2,S1,S2,R1,R2)=.5*(Z1+Z2)*(S1*R1-S2*R2)
      DR=0.
      DO 2 J=1,18
      DZZ(1,J)=DH(2,J)
      DZZ(2,J)=DH(3,J)
      DZZ(3,J)=DH(4,J)
      DZZ(4,J)=DH(7,J)
      DZZ(5,J)=DH(8,J)
      DZZ(6,J)=DH(9,J)
    2 CONTINUE
      DO 4 I=1,4
      DO 4 J=1,18
      IF(I.EQ.1) GO TO 7
      IF(I.EQ.4) GO TO 7
      SUSK(I,J)=SUBS(DZZ(I-1,J),DZZ(I,J),SS(I,J),SSKE,DPV(I,J),DPE(I,J))
      GO TO 4
    7 IF(I.EQ.1) SUSK(I,J)=SUBS(DR,DZZ(1,J),SS(I,J),SSKE,DPV(I,J),
     *DPE(I,J))
      IF(I.EQ.4) SUSK(I,J)=SUBS(DZZ(3,J),DR,SS(I,J),SSKE,DPV(I,J),
     *DPE(I,J))
    4 CONTINUE
      DO 10 I=5,8
      DO 10 J=1,18
      IF(I.EQ.5) GO TO 11
      IF(I.EQ.8) GO TO 11
      SUSK(I,J)=SUBS(DZZ(I-2,J),DZZ(I-1,J),SS(I,J),SSKE,DPV(I,J),DPE(I,J
     1))
      GO TO 10
   11 IF(I.EQ.5) SUSK(I,J)=SUBS(DR,DZZ(4,J),SS(I,J),SSKE,DPV(I,J),DPE(I,
     1J))
      IF(I.EQ.8) SUSK(I,J)=SUBS(DZZ(6,J),DR,SS(I,J),SSKE,DPV(I,J),DPE(I,
     1J))
   10 CONTINUE
      DO 6 J=1,18
      DO 5 I=1,4
    5 SUSE(1,J)=SUSE(1,J)+SUSK(I,J)
      DO 6 I=5,8
```

```
    6  SUSE(2,J)=SUSE(2,J)+SUSK(I,J)
       DO 20 J=1,18
       STOT(1,J)=SUSE(1,J)+SUSE(2,J)
   20  CONTINUE
       RETURN
       END

//$DATA
```

MPSX/IBM PROGRAMS

The program shown in Appendix VIII is used to prepare the input data to feed the MPSX/IBM routine used to (1) solve linear programming problems, and (2) perform parametric analysis. Before getting into descriptions of MPSX programs, it will be useful to explain how to set up a MPSX problem. Figure IX-1 shows the general deck set up for using MPSX/IBM routine. In general the use of accuracy control parameters is encouraged when those parameters are required in the program set up. The use of them will avoid being caught in infinite iterative do loops when optimal values of objective function are bounded by feasible non-optimal values.

The programs used in this study were the following:

(1) To solve PRIMAL problems:

```
PROGRAM
INITIALZ
MOVE(XDATA,'MINSUB')
MOVE(XPBNAME, 'PBFILE')
CONVERT('SUMMARY')
BCDOUT
SETUP('MAX')
MOVE(XDBJ,'OBJ')
MOVE(XRHS,'WAT')
CRASH
PRIMAL
SOULTION
```

EXIT

PEND

where OBJ contains the value of the coefficient of the objective function and WAT contains the right-hand side value of the constraints.

(2) To perform PARAOBJ (parametric analysis of variations of the coefficients of the objective function). Include in the program (1) right after SOLUTION the following instructions:

MOVE(XCHROW,'ROW50')

where

ROW50 contains the vector of increments of the objective function coefficients for the parametric analysis.

XPARAM = 0.0

XPARMΔX = 20.0

XPARDELT = 5.0

SOLUTION

where XPARAM, XPARMAX and XPARDELT are the control parameters to determine initial value of Θ, final value of Θ, and frequency for solution printouts.

(3) To perform PARARHS (parametric analysis of variations of right hand side values of the constraints) included in program (1) right after SOLUTION the following instructions

MOVE(XCHCOL,'WAT1')

XPARAM = 0.0

XPARMAX = 20.0

XPARDELT = 5.0

SOLUTION

where

WAT1 contains the increments of the right hand side values of the constraints for the parametric analysis.

(4) To perform PARARIM (parametric analysis of simultaneous variations of coefficients of the objective function and right hand side values of the constraints) included in program (1) right after SOLUTION the following instructions.

```
MOVE(XCHROW,'ROW50')
MOVE(XCHCOL,'WAT1'')
XPARAM = 0.0
XPARMAX = 20.0
XPARDELT = 5.0
SOLUTION
```

The names MINSUB, OBJ, WAT, ROW50 and WAT1 are specific of the problem solved in this study and must be defined by the user.

```
C
      DIMENSION M(15),XNEED(15),RHO(15,3),C(5),P(5),BOUND(15,3),KL(15)
      DIMENSION TR(15,15),COEF(15,15,3),SUB(5),DIST(15)
      DIMENSION XPARED(15),SUBX(5),RHA(15,3),COEFX(15,15,3)
C
C    READ SURFACE WATER COST  AND  GROUNDWATER COST
      READ,SWC,GWC
C
C
C    READ COST OF SUBSISENCE (DOLLARS PER METER) PER PPINT
      READ,(SUB(I),I=1,5)
C
C   READ WATER USE INCREASE PER POINT
      READ,(XPARED(I),I=1,15)
C
      NV=15
C   READ DISTANCES FROM SURFACE WATER RESERVOIR(PER POINT)
C
      READ,(DIST(I),I=1,NV)
C
      NVS=NV+1
C
C   READ WATER USE PER POINT
C
      READ,(XNEED(J),J=1,NV)
      SUB(1)=SUB(1)*1.5
      SUB(2)=SUB(2)*1.5
      DO 452 I=1,4
      XNEED(I)=XNEED(I)*1.5
  452 CONTINUE
C
C   READ NUMBER OF BREAKS PER POINT
C    THIS ARE FOR THE LINEAR APPROXIMATIONS OF THE OBJECTIVE FUNCTION
C   READ SURFACE WATER RESERVE
      READ23,(M(I),I=1,NV),RESER
      DO 21 I=1,NV
      MN=M(I)
C
C
C   READ COMPACTION(METERS) AND WATER USE BREAKS PER POINT(CUBIC METERS)
      READ,(C(J),J=1,MN),(P(K),K=1,MN)
C
C   CALCULATE SLOPES FOR LINEAR APPROXIMATIONS OF THE OBJECTIVE FUNCTION
      KN=MN-1
      KL(I)=KN
      DO 21 K=1,KN
      RHO(I,K)=(C(K+1)-C(K))/(P(K+1)-P(K))
      IF(I.EQ.1) RHO(I,K)=RHO(I,K)*SUB(1)
      IF(I.GT.1.AND.I.LE.4) RHO(I,K)=RHO(I,K)*SUB(2)
      IF(I.GT.4.AND.I.LE.9) RHO(I,K)=RHO(I,K)*SUB(3)
      IF(I.GT.9.AND.I.LE.12) RHO(I,K)=RHO(I,K)*SUB(4)
      IF(I.GT.12) RHO(I,K)=RHO(I,K)*SUB(5)
C
C   CALCULATE BOUNDS OF BREAKS(CUBIC METERS)
      BOUND(I,K)=P(K+1)-P(K)
   21 CONTINUE
C
C
C
```

```
C  CALCULATE COEFFICIENT INCREASES FOR PARAMETRIC ANALYSIS(ROW50). SUBSIDEI
C   COMPONENT. IT ARRAY MAY BE READ DIRECTLY
      DO 210 I=1,NV
      KN=KL(I)
      DO 210 K=1,KN
C
      IF(I.GE.1.AND.I.LE.4) RHA(I,K)=RHO(I,K)*.3*.1
      IF(I.GT.4.AND.I.LE.9) RHA(I,K)=RHO(I,K)*.5 *.1
      IF(I.GT.9.AND.I.LE.12) RHA(I,K)=RHO(I,K)*.2 *.1
      IF(I.GT.12) RHA(I,K)=RHO(I,K)*.2 *.1
  210 CONTINUE
      DO 451 I=1,NV
      KP=KL(I)
      DO 451 K=1,KP
C
C  PRINT SLOPES AND BOUNDS
      PRINT24,RHO(I,K),BOUND(I,K)
  451 CONTINUE
C
C  READ TRANSPORTATION COST AND DISTANCE CONSTANTS
      READ,TRANSP,DELT1,DELT2
C
C
      NV1=NV-1
      DO 25 I=1,NV
  25  TR(I,I)=0.0
C
C
C  CALCULATE DISTANCES BETWEEN POINTS
C
      DO 26 I=1,NV1
      IF(I.NE.1) GO TO 35
      TR(1,2)=DELT1
      DO 28 J=3,NV
  28  TR(I,J)=TR(I,J-1)+DELT2
      GO TO 26
  35  K=I+1
      DO 29 J=K,NV
  29  TR(I,J)=TR(I,J-1)+DELT2
  26  CONTINUE
C
C
C  PRINT DISTANCES BETWEEN POINTS
      DO 31 I=1,NV
      DO 31 J=1,NV
  31  TR(J,I)=TR(I,J)
C
      PRINT30,((TR(I,J),I=1,NV),J=1,NV)
      DO 27 I=1,NV
      DO 27 J=1,NV
C
C
C  CALCULATE TRANSPORTATION COSTS OF GROUNWATER
  27  TR(I,J)=TR(I,J)*TRANSP
      DO 300 I=1,NV
      DO 300 J=1,NV
      KN=KL(I)
      DO 300 K=1,KN
C
C
C  CALCULATE COEFFICIENTS OF THE OBJECTIVE FUNCTION(ROW50)
```

```
      COEFX(I,J,K)=RHA(I,K)+TR(I,J)*1.3 + GWC
C
  300 COEF(I,J,K)=RHO(I,K)+TR(I,J) + GWC
C
C  PRINT COEFFICIENTS
C
      DO 301 I=1,NV
      DO 301 J=1,NV
      KP=KL(I)
  301 PRINT,(COEF(I,J,K),K=1,KP)
C
C
C
C***  START PUNCHING DATA ***. IF THE INFORMATION IS DESIRED TO BE RECORDEC
C       IN TAPE OR DISK, PROVISION SHOULD BE TAKEN FOR INCLUDING THE CARDS N
C       FOR THE USE OF THE  MPSX/IBM ROUTINE.I.E ROW CARDS,COLUMN CARD, AND
C       CARD
C
C    IF "C1" VALUES ARE PUNCHED POSITIVE, PROVISIONS SHOULD BE TAKEN FOR
C    SET UP 'MAX' THE MPSX/IBM ROUTINE
C
      N9=0
      DO 100 I=1,NV
      N5=16
      DO 100 J=1,NV
      KN=KL(J)
      N1=J
      N2=I
      DO 100 K=1,KN
      N3=K
      C1=-COEF(J,I,K)
C
C*** IF A SHUT DOWN PUMPAGE POLICY IS GOING TO BE TESTED, THE "C1" COEFFICI
C    OF THE POINTS INVOLVED MUST BE GIVEN 1000 VALUE
      IF(J.GE.13) C1=-1000.
      CALL PRESS1(N1,N2,N3,C1,N9)
      N4=I
      C1=1.
      CALL PRESS(N1,N2,N3,N4,C1,N9)
      N5=N5+1
      C1=1.
      CALL PRESS(N1,N2,N3,N5,C1,N9)
      N6=50
      C1=-COEFX(J,I,K)
      CALL PRESS(N1,N2,N3,N6,C1,N9)
  100 CONTINUE
      DO 101 I=1,NV
      C1=-(SWC+DIST(I)*TRANSP)
      N1=16
      N3=0
      N2=I
      CALL PRESS1(N1,N2,N3,C1,N9)
      C1=1
      N2=I
      N4=I
      CALL PRESS(N1,N2,N3,N4,C1,N9)
      N4=16
      CALL PRESS(N1,N2,N3,N4,C1,N9)
  101 CONTINUE
C
      DO 200  I=1,NV
```

```
      N1=I
      C1=XNEED(I)
  200 CALL PRESS2(N1,C1)
      N1=16
      C1=RESER
      CALL PRESS2(N1,C1)
      DO 201 I=1,NV
      KN=KL(I)
      DO 201 K=1,KN
      N1=N1+1
      C1=BOUND(I,K)
  201 CALL PRESS2(N1,C1)
      DO 203 I=1,NV
      N1=I
      C1=XPARED(I)
  203 CALL PRESS3(N1,C1)
      M1=13
      M2=15
      NT=16
      DO 701 J=1,M1
  701 NT=NT+KL(J)
C
   23 FORMAT(15I2,F10.3)
   24 FORMAT(F10.6,5X,F11.2)
   30 FORMAT(5(F12.2,2X))
      STOP
      END

      SUBROUTINE PRESS(N1,N2,N3,N4,C1,N9)
      IF(N1.LT.10.AND.N2.LT.10.AND.N4.LT.10) PRINT10,N9,N1,N9,N2,N3,N4,C
     11
      IF(N1.LT.10.AND.N2.LT.10.AND.N4.GE.10) PRINT11,N9,N1,N9,N2,N3,N4,C
     11
      IF(N1.LT.10.AND.N2.GE.10.AND.N4.LT.10) PRINT12,N9,N1,N2,N3,N4,C1
      IF(N1.LT.10.AND.N2.GE.10.AND.N4.GE.10) PRINT13,N9,N1,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.LT.10.AND.N4.LT.10) PRINT14,N1,N9,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.LT.10.AND.N4.GE.10) PRINT15,N1,N9,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.GE.10.AND.N4.LT.10) PRINT16,N1,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.GE.10.AND.N4.GE.10) PRINT17,N1,N2,N3,N4,C1
      IF(N1.LT.10.AND.N2.LT.10.AND.N4.LT.10) PUNCH10,N9,N1,N9,N2,N3,N4,C
     11
      IF(N1.LT.10.AND.N2.LT.10.AND.N4.GE.10) PUNCH11,N9,N1,N9,N2,N3,N4,C
     11
      IF(N1.LT.10.AND.N2.GE.10.AND.N4.LT.10) PUNCH12,N9,N1,N2,N3,N4,C1
      IF(N1.LT.10.AND.N2.GE.10.AND.N4.GE.10) PUNCH13,N9,N1,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.LT.10.AND.N4.LT.10) PUNCH14,N1,N9,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.LT.10.AND.N4.GE.10) PUNCH15,N1,N9,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.GE.10.AND.N4.LT.10) PUNCH16,N1,N2,N3,N4,C1
      IF(N1.GE.10.AND.N2.GE.10.AND.N4.GE.10) PUNCH17,N1,N2,N3,N4,C1
   10 FORMAT(4X,'P',5I1,T15,'ROW',I1,T25,F5.2)
   11 FORMAT(4X,'P',5I1,T15,'ROW',I2,T25,F5.2)
   12 FORMAT(4X,'P',2I1,I2,I1,T15,'ROW',I1,T25,F5.2)
   13 FORMAT(4X,'P',2I1,I2,I1,T15,'ROW',I2,T25,F5.2)
   14 FORMAT(4X,'P',I2,3I1,T15,'ROW',I1,T25,F5.2)
   15 FORMAT(4X,'P',I2,3I1,T15,'ROW',I2,T25,F5.2)
   16 FORMAT(4X,'P',2I2,I1,T15,'ROW',I1,T25,F5.2)
   17 FORMAT(4X,'P',2I2,I1,T15,'ROW',I2,T25,F5.2)
      RETURN
      END

      SUBROUTINE PRESS1(N1,N2,N3,C1,N9)
```

```
      IF(N1.LT.10.AND.N2.LT.10) PRINT10,N9,N1,N9,N2,N3,C1
      IF(N1.LT.10.AND.N2.GE.10) PRINT11,N9,N1,N2,N3,C1
      IF(N1.GE.10.AND.N2.LT.10) PRINT12,N1,N9,N2,N3,C1
      IF(N1.GE.10.AND.N2.GE.10) PRINT13,N1,N2,N3,C1
      IF(N1.LT.10.AND.N2.LT.10) PUNCH10,N9,N1,N9,N2,N3,C1
      IF(N1.LT.10.AND.N2.GE.10) PUNCH11,N9,N1,N2,N3,C1
      IF(N1.GE.10.AND.N2.LT.10) PUNCH12,N1,N9,N2,N3,C1
      IF(N1.GE.10.AND.N2.GE.10) PUNCH13,N1,N2,N3,C1
 10   FORMAT(4X,'P',5I1,T15,'OBJ',T25,F10.4)
 11   FORMAT(4X,'P',2I1,I2,I1,T15,'OBJ',T25,F10.4)
 12   FORMAT(4X,'P',I2,3I1,T15,'OBJ',T25,F10.4)
 13   FORMAT(4X,'P',2I2,I1,T15,'OBJ',T25,F10.4)
      RETURN
      END

      SUBROUTINE PRESS2(N1,C1)
      IF(N1.LT.10) PRINT10,N1,C1
      IF(N1.GE.10) PRINT11,N1,C1
      IF(N1.LT.10) PUNCH10,N1,C1
      IF(N1.GE.10) PUNCH11,N1,C1
 11   FORMAT(4X,'WAT',7X    ,'ROW',I2,T25,F10.2)
 10   FORMAT(4X,'WAT',7X    ,'ROW',I1,T25,F10.2)
      RETURN
      END

      SUBROUTINE PRESS3(N1,C1)
      IF(N1.LT.10) PRINT10,N1,C1
      IF(N1.GE.10) PRINT11,N1,C1
      IF(N1.LT.10) PUNCH10,N1,C1
      IF(N1.GE.10) PUNCH11,N1,C1
 11   FORMAT(4X,'WAT1',6X   ,'ROW',I2,T25,F10.2)
 10   FORMAT(4X,'WAT1',6X   ,'ROW',I1,T25,F10.2)
      RETURN
      END
```

APPENDIX VIII

INITIAL DATE TO THE HYDROLOGIC MODEL

```
ACCELERATION OF GRAVITY =      9.800METER/SQ SEC
FLUID COMPRESSIBILITY=     0.000 SQ METER/KG
DENSITY OF THE FLUID =    999.730   KG/CUBIC  M

TIME INCREMENT =     60.000  DAYS

TOTAL TIME =     360.000  DAYS

PRINTOUT CONTROL =      6.000
```

VERTICAL GRID SPACEMENT (METERS)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.160D 02	0.160D 02	0.137D 02	0.991D 01	0.686D 01	0.991D 01	0.190D 02	0.274D 02	0.328D 02
2	0.305D 01	0.457D 01	0.914D 01	0.114D 02	0.991D 01	0.152D 02	0.183D 02	0.152D 02	0.160D 02
3	0.229D 01	0.457D 01	0.991D 01	0.130D 02	0.122D 02	0.145D 02	0.175D 02	0.152D 02	0.145D 02
4	0.381D 01	0.305D 01	0.914D 01	0.137D 02	0.122D 02	0.152D 02	0.168D 02	0.137D 02	0.145D 02
5	0.198D 02	0.366D 02	0.480D 02	0.518D 02	0.610D 02	0.495D 02	0.396D 02	0.472D 02	0.434D 02
6	0.213D 02	0.366D 02	0.488D 02	0.533D 02	0.632D 02	0.533D 02	0.419D 02	0.488D 02	0.434D 02
7	0.785D 02	0.724D 02	0.617D 02	0.617D 02	0.617D 02	0.762D 02	0.930D 02	0.952D 02	0.101D 03
8	0.808D 02	0.732D 02	0.625D 02	0.617D 02	0.617D 02	0.770D 02	0.109D 03	0.110D 03	0.998D 02
9	0.785D 02	0.716D 02	0.685D 02	0.701D 02	0.625D 02	0.777D 02	0.792D 02	0.800D 02	0.101D 03
10	0.815D 02	0.907D 02	0.107D 03	0.114D 03	0.126D 03	0.114D 03	0.101D 03	0.105D 03	0.111D 03
11	0.831D 02	0.914D 02	0.116D 03	0.125D 03	0.127D 03	0.112D 03	0.105D 03	0.111D 03	0.111D 03
12	0.305D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.343D 02	0.343D 02	0.389D 02	0.450D 02	0.556D 02	0.610D 02	0.594D 02	0.655D 02	0.701D 02
2	0.175D 02	0.152D 02	0.137D 02	0.152D 02	0.160D 02	0.160D 02	0.168D 02	0.175D 02	0.168D 02
3	0.152D 02	0.137D 02	0.145D 02	0.160D 02	0.130D 02	0.145D 02	0.175D 02	0.168D 02	0.160D 02
4	0.160D 02	0.137D 02	0.137D 02	0.152D 02	0.137D 02	0.145D 02	0.158D 02	0.168D 02	0.152D 02
5	0.373D 02	0.450D 02	0.411D 02	0.267D 02	0.206D 02	0.206D 02	0.229D 02	0.229D 02	0.251D 02
6	0.351D 02	0.419D 02	0.396D 02	0.267D 02	0.213D 02	0.183D 02	0.190D 02	0.206D 02	0.205D 02
7	0.109D 03	0.115D 03	0.122D 03	0.131D 03	0.139D 03	0.148D 03	0.156D 03	0.172D 03	0.189D 03
8	0.108D 03	0.114D 03	0.120D 03	0.132D 03	0.144D 03	0.149D 03	0.155D 03	0.173D 03	0.191D 03
9	0.108D 03	0.112D 03	0.120D 03	0.128D 03	0.138D 03	0.149D 03	0.158D 03	0.173D 03	0.190D 03
10	0.117D 03	0.123D 03	0.133D 03	0.146D 03	0.171D 03	0.197D 03	0.211D 03	0.235D 03	0.251D 03
11	0.119D 03	0.122D 03	0.132D 03	0.149D 03	0.175D 03	0.197D 03	0.207D 03	0.234D 03	0.264D 03
12	0.305D 02	0.320D 02	0.320D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02	0.305D 02

HORIZONTAL GRID SPACEMENT (METERS)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
2	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
3	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
4	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
5	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
6	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
7	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
8	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
9	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
10	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
11	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04
12	0.833D 04	0.833D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
2	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
3	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
4	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
5	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
6	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
7	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
8	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
9	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
10	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
11	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04
12	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.556D 04	0.833D 04	0.833D 04

PIEZOMETRIC WATER HEADS - referred to a 305 m. BSL datum
(METERS)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.354D 03	0.352D 03	0.345D 03	0.336D 03	0.327D 03	0.325D 03	0.325D 03	0.322D 03	0.319D 03
2	0.344D 03	0.333D 03	0.331D 03	0.325D 03	0.318D 03	0.313D 03	0.306D 03	0.298D 03	0.292D 03
3	0.325D 03	0.324D 03	0.319D 03	0.312D 03	0.307D 03	0.299D 03	0.287D 03	0.281D 03	0.275D 03
4	0.303D 03	0.317D 03	0.307D 03	0.298D 03	0.295D 03	0.284D 03	0.270D 03	0.266D 03	0.259D 03
5	0.289D 03	0.282D 03	0.276D 03	0.271D 03	0.265D 03	0.260D 03	0.251D 03	0.244D 03	0.238D 03
6	0.289D 03	0.282D 03	0.276D 03	0.271D 03	0.265D 03	0.260D 03	0.251D 03	0.244D 03	0.238D 03
7	0.289D 03	0.281D 03	0.275D 03	0.269D 03	0.263D 03	0.257D 03	0.248D 03	0.241D 03	0.235D 03
8	0.287D 03	0.279D 03	0.271D 03	0.265D 03	0.258D 03	0.249D 03	0.237D 03	0.230D 03	0.225D 03
9	0.286D 03	0.275D 03	0.267D 03	0.260D 03	0.253D 03	0.244D 03	0.235D 03	0.230D 03	0.225D 03
10	0.285D 03	0.273D 03	0.264D 03	0.257D 03	0.250D 03	0.245D 03	0.240D 03	0.236D 03	0.234D 03
11	0.285D 03	0.273D 03	0.264D 03	0.257D 03	0.250D 03	0.245D 03	0.240D 03	0.236D 03	0.234D 03
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.316D 03	0.312D 03	0.310D 03	0.309D 03	0.309D 03	0.310D 03	0.310D 03	0.309D 03	0.308D 03
2	0.288D 03	0.283D 03	0.279D 03	0.277D 03	0.279D 03	0.286D 03	0.292D 03	0.295D 03	0.305D 03
3	0.269D 03	0.266D 03	0.362D 03	0.260D 03	0.266D 03	0.275D 03	0.282D 03	0.288D 03	0.299D 03
4	0.251D 03	0.250D 03	0.246D 03	0.245D 03	0.253D 03	0.263D 03	0.272D 03	0.282D 03	0.295D 03
5	0.232D 03	0.228D 03	0.226D 03	0.235D 03	0.245D 03	0.255D 03	0.265D 03	0.277D 03	0.291D 03
6	0.232D 03	0.228D 03	0.226D 03	0.235D 03	0.245D 03	0.255D 03	0.265D 03	0.277D 03	0.291D 03
7	0.229D 03	0.222D 03	0.222D 03	0.235D 03	0.244D 03	0.255D 03	0.265D 03	0.277D 03	0.289D 03
8	0.218D 03	0.207D 03	0.213D 03	0.235D 03	0.237D 03	0.254D 03	0.264D 03	0.276D 03	0.286D 03
9	0.220D 03	0.211D 03	0.219D 03	0.237D 03	0.239D 03	0.255D 03	0.263D 03	0.274D 03	0.284D 03
10	0.232D 03	0.231D 03	0.233D 03	0.239D 03	0.247D 03	0.256D 03	0.264D 03	0.273D 03	0.282D 03
11	0.232D 03	0.231D 03	0.233D 03	0.239D 03	0.247D 03	0.256D 03	0.264D 03	0.273D 03	0.282D 03
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

SPECIFIC STORAGE COEFFICIENTS (1/METER)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
2	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
3	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
4	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
5	0.782D-02	0.400D-02	0.271D-02	0.251D-02	0.213D-02	0.262D-02	0.191D-02	0.500D-03	0.900D-04
6	0.726D-02	0.400D-02	0.257D-02	0.244D-02	0.206D-02	0.244D-02	0.180D-02	0.500D-03	0.900D-04
7	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
8	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
9	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
10	0.100D-03	0.125D-04	0.850D-04	0.800D-04	0.700D-04	0.800D-04	0.950D-04	0.899D-04	0.900D-04
11	0.100D-03	0.125D-04	0.850D-04	0.800D-04	0.700D-04	0.800D-04	0.950D-04	0.899D-04	0.900D-04
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
2	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
3	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
4	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
5	0.400D-03	0.300D-03	0.900D-02	0.200D-02	0.500D-03	0.500D-03	0.500D-03	0.500D-03	0.500D-03
6	0.400D-03	0.300D-03	0.900D-02	0.200D-02	0.500D-03	0.500D-03	0.500D-03	0.500D-03	0.500D-03
7	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
8	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
9	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03	0.755D-03
10	0.100D-03	0.900D-03	0.300D-03	0.200D-03	0.100D-04	0.500D-04	0.500D-04	0.500D-04	0.500D-03
11	0.100D-03	0.900D-03	0.300D-03	0.200D-03	0.100D-04	0.500D-04	0.500D-04	0.500D-04	0.500D-03
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

VERTICAL CONDUCTIVITIES (METER/SEC)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.215D-09	0.862D-09	0.101D-08	0.120D-08	0.122D-08	0.137D-08	0.113D-08	0.971D-09	0.918D-09
2	0.215D-09	0.862D-09	0.101D-08	0.120D-08	0.122D-08	0.137D-08	0.113D-08	0.971D-09	0.918D-09
3	0.215D-09	0.862D-09	0.101D-08	0.120D-08	0.122D-08	0.137D-08	0.113D-08	0.971D-09	0.918D-09
4	0.215D-09	0.362D-09	0.101D-08	0.120D-08	0.122D-08	0.137D-08	0.113D-08	0.971D-09	0.918D-09
5	0.141D-01	0.870D-02	0.748D-02	0.807D-02	0.693D-02	0.824D-02	0.930D-02	0.681D-02	0.712D-02
6	0.141D-01	0.370D-02	0.748D-02	0.807D-02	0.693D-02	0.824D-02	0.930D-02	0.681D-02	0.712D-02
7	0.830D-07	0.351D-07	0.190D-07	0.196D-07	0.167D-07	0.115D-07	0.157D-07	0.173D-07	0.197D-07
8	0.830D-07	0.351D-07	0.190D-07	0.196D-07	0.167D-07	0.115D-07	0.157D-07	0.173D-07	0.197D-07
9	0.830D-07	0.351D-07	0.190D-07	0.196D-07	0.167D-07	0.115D-07	0.157D-07	0.173D-07	0.197D-07
10	0.351D-02	0.345D-02	0.287D-02	0.282D-02	0.384D-02	0.491D-02	0.547D-02	0.558D-02	0.615D-02
11	0.351D-02	0.345D-02	0.287D-02	0.282D-02	0.384D-02	0.491D-02	0.547D-02	0.558D-02	0.615D-02
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.923D-08	0.862D-09	0.796D-09	0.834D-09	0.962D-09	0.143D-08	0.197D-08	0.301D-08	0.437D-08
2	0.923D-08	0.862D-09	0.796D-09	0.834D-09	0.962D-09	0.143D-08	0.197D-08	0.301D-08	0.437D-08
3	0.923D-08	0.362D-09	0.796D-09	0.834D-09	0.962D-09	0.143D-08	0.197D-08	0.301D-08	0.437D-08
4	0.923D-08	0.862D-09	0.796D-09	0.830D-07	0.351D-07	0.190D-07	0.196D-07	0.167D-07	0.115D-07
5	0.906D-02	0.733D-02	0.804D-02	0.937D-01	0.182D-01	0.208D-01	0.193D-01	0.186D-01	0.213D-01
6	0.906D-02	0.733D-02	0.804D-02	0.937D-01	0.182D-01	0.208D-01	0.193D-01	0.186D-01	0.213D-01
7	0.189D-07	0.133D-07	0.245D-07	0.259D-06	0.226D-07	0.166D-06	0.107D-06	0.109D-06	0.113D-06
8	0.189D-07	0.133D-07	0.245D-07	0.259D-06	0.226D-07	0.166D-06	0.107D-06	0.109D-06	0.113D-06
9	0.189D-07	0.133D-07	0.245D-07	0.259D-06	0.226D-07	0.166D-06	0.107D-06	0.109D-06	0.113D-06
10	0.539D-02	0.494D-02	0.442D-02	0.352D-02	0.265D-02	0.304D-02	0.298D-02	0.289D-02	0.298D-02
11	0.539D-02	0.494D-02	0.442D-02	0.352D-02	0.265D-02	0.304D-02	0.298D-02	0.289D-02	0.298D-02
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

HORIZONTAL CONDUCTIVITIES (METER/SEC)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
2	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
3	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
4	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
5	0.141D-02	0.870D-03	0.749D-03	0.807D-03	0.693D-03	0.824D-03	0.930D-03	0.681D-03	0.712D-03
6	0.141D-02	0.870D-03	0.749D-03	0.807D-03	0.693D-03	0.824D-03	0.930D-03	0.681D-03	0.712D-03
7	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
8	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
9	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
10	0.351D-03	0.345D-03	0.257D-03	0.282D-03	0.348D-03	0.491D-03	0.547D-03	0.558D-03	0.615D-03
11	0.351D-03	0.345D-03	0.257D-03	0.282D-03	0.348D-03	0.491D-03	0.547D-03	0.558D-03	0.615D-03
12	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
2	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
3	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
4	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
5	0.906D-03	0.733D-03	0.804D-03	0.137D-02	0.182D-02	0.208D-02	0.193D-02	0.186D-02	0.213D-02
6	0.906D-03	0.733D-03	0.804D-03	0.137D-02	0.182D-02	0.208D-02	0.193D-02	0.186D-02	0.213D-02
7	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
8	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
9	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
10	0.539D-03	0.494D-03	0.442D-03	0.352D-03	0.265D-03	0.304D-03	0.298D-03	0.289D-03	0.298D-03
11	0.539D-03	0.494D-03	0.442D-03	0.352D-03	0.265D-03	0.304D-03	0.293D-03	0.289D-03	0.298D-03
12	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00

PUMPAGE RATE (CUBIC METERS/SEC)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
2	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
3	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
4	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
5	0.000D 00	0.465D-05	0.535D-05	0.319D-05	0.516D-05	0.178D-05	0.223D-05	0.424D-05	0.216D-05
6	0.000D 00	0.465D-05	0.535D-05	0.319D-05	0.516D-05	0.178D-05	0.223D-05	0.424D-05	0.216D-05
7	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
8	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
9	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
10	0.000D 00	0.576D-05	0.520D-05	0.795D-06	0.695D-05	0.305D-04	0.166D-04	0.273D-04	0.339D-04
11	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
2	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
3	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
4	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
5	0.103D-05	0.159D-04	0.579D-04	0.209D-04	0.137D-04	0.788D-05	0.000D 00	0.000D 00	0.000D 00
6	0.103D-05	0.159D-04	0.579D-04	0.209D-04	0.137D-04	0.788D-05	0.000D 00	0.000D 00	0.000D 00
7	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
8	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
9	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
10	0.763D-05	0.193D-03	0.191D-03	0.817D-05	0.383D-05	0.000D 00	0.000D 00	0.000D 00	0.000D 00
11	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

APPENDIX IX

HYDROLOGIC AND COMPACTION MODEL OUTPUT

NEW WATER HEADS (METERS)AFTER 1800. DAYS

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.3540 03	0.3520 03	0.3450 03	0.3360 03	0.3270 03	0.3250 03	0.3250 03	0.3220 03	0.3190 03
2	0.3440 03	0.3320 03	0.3310 03	0.3250 03	0.3180 03	0.3130 03	0.3060 03	0.2980 03	0.2920 03
3	0.3250 03	0.3240 03	0.3190 03	0.3120 03	0.3070 03	0.2990 03	0.2880 03	0.2820 03	0.2760 03
4	0.3030 03	0.3160 03	0.3060 03	0.2980 03	0.2950 03	0.2840 03	0.2700 03	0.2670 03	0.2610 03
5	0.2890 03	0.2800 03	0.2740 03	0.2690 03	0.2640 03	0.2590 03	0.2510 03	0.2450 03	0.2400 03
6	0.2890 03	0.2800 03	0.2740 03	0.2690 03	0.2640 03	0.2590 03	0.2510 03	0.2450 03	0.2400 03
7	0.2880 03	0.2800 03	0.2740 03	0.2680 03	0.2620 03	0.2570 03	0.2490 03	0.2420 03	0.2370 03
8	0.2870 03	0.2770 03	0.2700 03	0.2640 03	0.2570 03	0.2500 03	0.2390 03	0.2330 03	0.2280 03
9	0.2860 03	0.2740 03	0.2660 03	0.2580 03	0.2510 03	0.2430 03	0.2340 03	0.2230 03	0.2250 03
10	0.2850 03	0.2710 03	0.2600 03	0.2510 03	0.2410 03	0.2340 03	0.2280 03	0.2230 03	0.2190 03
11	0.2850 03	0.2710 03	0.2600 03	0.2510 03	0.2410 03	0.2340 03	0.2280 03	0.2230 03	0.2190 03
12	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.3160 03	0.3120 03	0.3100 03	0.3090 03	0.3090 03	0.3100 03	0.3100 03	0.3090 03	0.3080 03
2	0.2880 03	0.2930 03	0.3030 03	0.2740 03	0.2750 03	0.2810 03	0.2870 03	0.2920 03	0.3050 03
3	0.2710 03	0.2660 03	0.3000 03	0.2500 03	0.2550 03	0.2650 03	0.2740 03	0.2840 03	0.2990 03
4	0.2550 03	0.2510 03	0.2700 03	0.2340 03	0.2440 03	0.2560 03	0.2660 03	0.2790 03	0.2950 03
5	0.2350 03	0.2280 03	0.2250 03	0.2340 03	0.2430 03	0.2550 03	0.2650 03	0.2770 03	0.2910 03
6	0.2350 03	0.2280 03	0.2250 03	0.2340 03	0.2430 03	0.2550 03	0.2650 03	0.2770 03	0.2910 03
7	0.2330 03	0.2240 03	0.2240 03	0.2340 03	0.2430 03	0.2550 03	0.2650 03	0.2770 03	0.2890 03
8	0.2220 03	0.2110 03	0.2170 03	0.2330 03	0.2390 03	0.2550 03	0.2640 03	0.2760 03	0.2860 03
9	0.2210 03	0.2130 03	0.2190 03	0.2310 03	0.2400 03	0.2540 03	0.2630 03	0.2740 03	0.2840 03
10	0.2160 03	0.2130 03	0.2160 03	0.2290 03	0.2410 03	0.2520 03	0.2620 03	0.2720 03	0.2820 03
11	0.2160 03	0.2130 03	0.2160 03	0.2290 03	0.2410 03	0.2520 03	0.2620 03	0.2720 03	0.2820 03
12	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00

DRAWDOWNS (METERS)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
2	0.000D 00	-0.677D 00	-0.349D 00	-0.218D 00	0.467D 00	-0.933D-01	-0.153D 00	0.328D 00	0.680D-01
3	0.000D 00	-0.471D 00	-0.426D 00	-0.108D 00	0.246D 00	-0.500D 00	0.529D 00	0.912D 00	0.512D 00
4	0.000D 00	-0.858D 00	-0.742D 00	-0.413D 00	-0.124D 00	-0.134D 00	0.423D 00	0.122D 01	0.152D 01
5	0.000D 00	-0.154D 01	-0.163D 01	-0.212D 01	-0.141D 01	-0.108D 01	0.145D 00	0.847D 00	0.195D 01
6	0.000D 00	-0.154D 01	-0.163D 01	-0.212D 01	-0.141D 01	-0.108D 01	0.145D 00	0.847D 00	0.195D 01
7	0.000D 00	-0.948D 00	-0.136D 01	-0.115D 01	-0.108D 01	-0.196D 00	0.803D 00	0.970D 00	0.212D 01
8	0.000D 00	-0.155D 01	-0.625D 00	-0.101D 01	-0.683D 00	0.750D 00	0.204D 01	0.261D 01	0.324D 01
9	0.000D 00	-0.122D 01	-0.140D 01	-0.179D 01	-0.193D 01	-0.911D 00	-0.907D 00	-0.112D 01	-0.235D 00
10	0.000D 00	-0.228D 01	-0.418D 01	-0.648D 01	-0.863D 01	-0.108D 02	-0.120D 02	-0.130D 02	-0.150D 02
11	0.000D 00	-0.228D 01	-0.418D 01	-0.648D 01	-0.863D 01	-0.108D 02	-0.120D 02	-0.130D 02	-0.150D 02
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
2	0.439D 00	0.896D-01	0.244D 02	-0.340D 01	-0.445D 01	-0.547D 01	-0.473D 01	-0.336D 01	0.000D 00
3	0.212D 01	0.438D 00	-0.621D 02	-0.104D 02	-0.108D 02	-0.101D 02	-0.752D 01	-0.446D 01	0.000D 00
4	0.372D 01	0.123D 01	0.240D 02	-0.110D 02	-0.902D 01	-0.736D 01	-0.553D 01	-0.349D 01	0.000D 00
5	0.331D 01	-0.264D 00	-0.717D 00	-0.128D 01	-0.150D 01	-0.335D 00	0.437D 00	0.431D 00	0.000D 00
6	0.331D 01	-0.264D 00	-0.717D 00	-0.128D 01	-0.150D 01	-0.335D 00	0.437D 00	0.431D 00	0.000D 00
7	0.351D 01	0.200D 01	0.192D 01	-0.127D 01	-0.513D 00	-0.304D 00	0.409D 00	0.358D 00	0.000D 00
8	0.390D 01	0.375D 01	0.422D 01	-0.189D 01	0.178D 01	0.569D 00	0.308D 00	-0.261D 00	0.000D 00
9	0.591D 00	0.217D 01	-0.182D-01	-0.589D 01	0.122D 01	-0.120D 01	0.579D-01	-0.217D 00	0.000D 00
10	-0.158D 02	-0.181D 02	-0.171D 02	-0.101D 02	-0.557D 01	-0.359D 01	-0.237D 01	-0.972D 00	0.000D 00
11	-0.158D 02	-0.181D 02	-0.171D 02	-0.101D 02	-0.557D 01	-0.359D 01	-0.237D 01	-0.973D 00	0.000D 00
12	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00

PARTIAL COMPACTION VALUES (meters)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00
2	0.0000 00	0.583D-04	0.952D-04	0.624D-04	-0.222D-07	0.831D-04	-0.159D-06	0.0000 00	0.0000 00
3	0.0000 00	0.645D-04	0.162D-03	0.0000 00	0.642D-07	0.132D-03	-0.271D-06	-0.441D-06	-0.319D-06
4	0.0000 00	0.497D-04	0.178D-03	0.268D-03	0.193D-03	0.153D-03	-0.255D-07	-0.614D-06	-0.615D-06
5	0.0000 00	0.118D-02	0.120D-02	0.121D-02	0.979D-03	0.763D-03	-0.142D-06	-0.199D-06	-0.333D-06
6	0.0000 00	0.261D-02	0.212D-02	0.247D-02	0.208D-02	0.378D-06	-0.417D-05	-0.138D-05	-0.231D-05
7	0.0000 00	0.289D-02	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.474D-05	-0.632D-05	-0.909D-05
8	0.0000 00	0.156D-02	0.251D-02	0.345D-02	0.379D-02	0.540D-02	0.633D-02	0.521D-05	0.125D-05
								0.721D-02	0.103D-01

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00	0.0000 00
2	-0.654D-06	-0.253D-06	0.365D-02	0.181D-02	0.156D-02	0.156D-02	0.138D-02	0.701D-03	0.0000 00
3	-0.770D-06	-0.436D-06	0.378D-02	0.139D-02	0.929D-03	0.859D-03	0.771D-03	0.392D-03	0.0000 00
4	-0.400D-06	-0.129D-06	0.726D-04	0.0000 00	0.0000 00	0.0000 00	0.248D-06	0.129D-06	0.0000 00
5	-0.272D-05	-0.109D-05	0.645D-03	0.0000 00	0.0000 00	0.0000 00	0.231D-05	0.133D-05	0.0000 00
6	-0.134D-04	-0.128D-04	-0.990D-05	0.0000 00	-0.952D-06	0.682D-05	0.257D-05	0.223D-05	0.0000 00
7	-0.203D-05	-0.690D-05	-0.875D-06	0.0000 00	-0.495D-05	0.0000 00	0.328D-05	0.157D-02	0.0000 00
8	0.126D-01	0.151D-01	0.136D-01	0.112D-01	0.0000 00	0.558D-02	0.369D-02	0.189D-02	0.0000 00

CORRESPONDING PRESSURES (N/m^2)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00
2	-0.0000 00	0.663D 04	0.342D 04	0.214D 04	-0.458D 04	0.914D 03	0.150D 04	-0.321D 04	-0.666D 03
3	-0.0000 00	0.462D 04	0.417D 04	0.106D 04	-0.241D 04	0.490D 04	-0.518D 04	-0.893D 04	-0.502D 04
4	-0.0000 00	0.841D 04	0.727D 04	0.405D 04	0.121D 04	0.131D 04	-0.415D 04	-0.119D 05	-0.148D 05
5	-0.0000 00	0.151D 05	0.160D 05	0.207D 05	0.138D 05	0.105D 05	-0.142D 04	-0.830D 04	-0.191D 05
6	-0.0000 00	0.151D 05	0.160D 05	0.207D 05	0.139D 05	0.105D 05	-0.142D 04	-0.830D 04	-0.191D 05
7	-0.0000 00	0.929D 04	0.133D 05	0.113D 05	0.105D 05	0.192D 04	-0.787D 04	-0.950D 04	-0.208D 05
8	-0.0000 00	0.151D 05	0.612D 04	0.994D 04	0.669D 04	-0.735D 04	-0.200D 05	-0.256D 05	-0.317D 05
9	-0.0000 00	0.120D 05	0.137D 05	0.175D 05	0.189D 05	0.893D 04	0.889D 04	0.110D 05	0.231D 04
10	-0.0000 00	0.224D 05	0.410D 05	0.635D 05	0.846D 05	0.106D 06	0.118D 06	0.127D 06	0.147D 06
11	-0.0000 00	0.224D 05	0.410D 05	0.635D 05	0.846D 05	0.106D 06	0.118D 06	0.127D 06	0.147D 06
12	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00
2	-0.430D 04	-0.878D 03	-0.239D 06	0.333D 05	0.436D 05	0.536D 05	0.463D 05	0.330D 05	-0.0000 00
3	-0.207D 05	-0.429D 04	0.609D 06	0.102D 06	0.106D 06	0.986D 05	0.737D 05	0.437D 05	-0.0000 00
4	-0.365D 05	-0.120D 05	-0.235D 06	0.108D 06	0.884D 05	0.721D 05	0.542D 05	0.342D 05	-0.0000 00
5	-0.324D 05	0.259D 04	0.702D 04	0.125D 05	0.147D 05	0.328D 04	-0.428D 04	-0.422D 04	-0.0000 00
6	-0.324D 05	0.259D 04	0.702D 04	0.125D 05	0.147D 05	0.328D 04	-0.428D 04	-0.422D 04	-0.0000 00
7	-0.344D 05	-0.196D 05	-0.138D 05	0.125D 05	0.503D 04	0.298D 04	-0.400D 04	-0.351D 04	-0.0000 00
8	-0.382D 05	-0.367D 05	-0.414D 05	0.186D 05	-0.175D 05	-0.557D 04	-0.301D 04	0.256D 04	-0.0000 00
9	-0.579D 04	-0.212D 05	0.178D 03	0.577D 05	-0.120D 05	0.118D 05	-0.567D 03	0.213D 04	-0.0000 00
10	0.155D 06	0.177D 06	0.168D 06	0.991D 05	0.546D 05	0.351D 05	0.232D 05	0.953D 04	-0.0000 00
11	0.155D 06	0.177D 06	0.168D 06	0.991D 05	0.546D 05	0.351D 05	0.232D 05	0.953D 04	-0.0000 00
12	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00

MAXIMUM PRESSURES (N/m²)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00
2	0.000D 00	0.562D 04	0.378D 04	0.163D 04	0.000D 00	0.296D 04	0.000D 00	0.000D 00	0.000D 00
3	0.000D 00	0.689D 04	0.578D 04	0.251D 04	0.000D 00	0.315D 04	0.000D 00	0.000D 00	0.000D 00
4	-0.000D 00	0.151D 05	0.160D 05	0.207D 05	0.138D 05	0.105D 05	0.000D 00	0.200D 03	0.000D 00
5	-0.000D 00	0.151D 05	0.160D 05	0.207D 05	0.139D 05	0.105D 05	0.000D 00	0.200D 03	0.000D 00
6	0.000D 00	0.122D 05	0.975D 04	0.106D 05	0.861D 04	0.000D 00	0.000D 00	0.000D 00	0.000D 00
7	0.000D 00	0.135D 05	0.973D 04	0.135D 05	0.127D 05	0.748D 03	0.000D 00	0.000D 00	0.000D 00
8	-0.000D 00	0.224D 05	0.410D 05	0.635D 05	0.846D 05	0.106D 06	0.118D 06	0.127D 06	0.147D 06

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00	-0.000D 00
2	0.580D 03	0.000D 00	0.173D 06	0.669D 05	0.780D 05	0.773D 05	0.597D 05	0.385D 05	0.000D 00
3	0.000D 00	0.000D 00	0.175D 06	0.105D 06	0.973D 05	0.854D 05	0.637D 05	0.390D 05	0.000D 00
4	0.000D 00	0.790D 04	0.702D 04	0.125D 05	0.147D 05	0.328D 04	0.000D 00	0.000D 00	-0.000D 00
5	0.000D 00	0.790D 04	0.702D 04	0.125D 05	0.147D 05	0.328D 04	0.000D 00	0.000D 00	-0.000D 00
6	0.000D 00	0.000D 00	0.000D 00	0.155D 05	0.000D 00	0.000D 00	0.000D 00	0.000D 00	0.000D 00
7	0.000D 00	0.000D 00	0.000D 00	0.384D 05	0.000D 00	0.311D 04	0.000D 00	0.234D 04	0.000D 00
8	0.155D 06	0.177D 06	0.168D 06	0.991D 05	0.546D 05	0.351D 05	0.232D 05	0.953D 04	-0.000D 00

PRESSURES IN CLAYS (N/m^2)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00
2	0.0000 00	0.562D 04	0.378D 04	0.163D 04	-0.360D 04	0.296D 04	-0.191D 04	-0.607D 04	-0.295D 04
3	0.0000 00	0.689D 04	0.578D 04	0.251D 04	-0.598D 03	0.315D 04	-0.465D 04	-0.105D 05	-0.993D 04
4	-0.0000 00	0.151D 05	0.160D 05	0.207D 05	0.138D 05	0.105D 05	-0.142D 04	-0.830D 04	-0.191D 05
5	-0.0000 00	0.151D 05	0.160D 05	0.207D 05	0.139D 05	0.105D 05	-0.142D 04	-0.830D 04	-0.191D 05
6	0.0000 00	0.122D 05	0.975D 04	0.106D 05	0.861D 04	-0.269D 04	-0.134D 05	-0.169D 05	-0.263D 05
7	0.0000 00	0.135D 05	0.973D 04	0.135D 05	0.127D 05	0.748D 03	-0.327D 04	-0.434D 04	-0.148D 05
8	-0.0000 00	0.224D 05	0.410D 05	0.635D 05	0.846D 05	0.106D 06	0.118D 06	0.127D 06	0.147D 06

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00	-0.0000 00
2	-0.131D 05	-0.267D 04	0.173D 06	0.669D 05	0.780D 05	0.773D 05	0.597D 05	0.385D 05	0.0000 00
3	-0.284D 05	-0.815D 04	0.175D 06	0.105D 06	0.973D 05	0.854D 05	0.637D 05	0.390D 05	0.0000 00
4	-0.324D 05	0.259D 04	0.702D 04	0.125D 05	0.147D 05	0.328D 04	-0.428D 04	-0.422D 04	-0.0000 00
5	-0.324D 05	0.259D 04	0.702D 04	0.125D 05	0.147D 05	0.328D 04	-0.428D 04	-0.422D 04	-0.0000 00
6	-0.363D 05	-0.282D 05	-0.302D 05	0.155D 05	-0.604D 04	-0.128D 04	-0.351D 04	-0.482D 03	0.0000 00
7	-0.220D 05	-0.289D 05	-0.205D 05	0.384D 05	-0.147D 05	0.311D 04	-0.180D 04	0.234D 04	0.0000 00
8	0.155D 06	0.177D 06	0.168D 06	0.991D 05	0.546D 05	0.351D 05	0.232D 05	0.953D 04	-0.0000 00

VIRGIN PRESSURES DIFFERENCES (N/m^2)

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
2	0.0000D 00	0.1660D 03	0.1310D 03	0.6650D 02	0.0000D 00	0.7260D 02	0.0000D 00	0.0000D 00	0.0000D 00
3	0.0000D 00	0.2200D 03	0.2210D 03	0.0000D 00	0.0000D 00	0.1160D 03	0.0000D 00	0.0000D 00	0.0000D 00
4	0.0000D 00	0.4230D 03	0.5060D 03	0.5080D 03	0.4120D 03	0.2600D 03	0.0000D 00	0.0000D 00	0.0000D 00
5	0.0000D 00	0.4230D 03	0.5060D 03	0.5080D 03	0.4120D 03	0.2600D 03	0.0000D 00	0.0000D 00	0.0000D 00
6	0.0000D 00	0.4660D 03	0.4430D 03	0.5200D 03	0.4370D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
7	0.0000D 00	0.5190D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
8	0.0000D 00	0.5640D 03	0.9490D 03	0.1280D 04	0.1570D 04	0.1800D 04	0.2070D 04	0.2340D 04	0.2640D 04

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
2	0.0000D 00	0.0000D 00	0.3360D 04	0.1510D 04	0.1400D 04	0.1330D 04	0.1040D 04	0.5300D 03	0.0000D 00
3	0.0000D 00	0.0000D 00	0.3480D 04	0.1150D 04	0.9040D 03	0.7700D 03	0.5840D 03	0.3040D 03	0.0000D 00
4	0.0000D 00	0.0000D 00	0.1370D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
5	0.0000D 00	0.0000D 00	0.1370D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
6	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
7	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.1180D 03	0.0000D 00
8	0.3010D 04	0.3490D 04	0.2950D 04	0.2260D 04	0.0000D 00	0.9690D 03	0.6080D 03	0.2830D 03	0.0000D 00

ELASTIC PRESSURES DIFFERENCES (N/m^2) .

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
2	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.1400D 02	0.0000D 00	0.620D 02	0.203D 03	0.146D 03
3	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	−0.369D 02	0.0000D 00	0.111D 03	0.297D 03	0.297D 03
4	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.213D 02	0.203D 03	0.322D 03
5	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.213D 02	0.203D 03	0.322D 03
6	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	−0.346D 02	0.289D 03	0.430D 03	0.634D 03
7	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	−0.353D 03	−0.382D 03	−0.871D 02
8	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
2	0.279D 03	0.123D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
3	0.345D 03	0.222D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00
4	0.350D 03	0.132D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	−0.207D 03	−0.108D 03	0.0000D 00
5	0.350D 03	0.132D 03	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	−0.207D 03	−0.108D 03	0.0000D 00
6	0.864D 03	0.780D 03	0.572D 03	0.0000D 00	0.470D 02	−0.321D 03	−0.116D 03	−0.905D 02	0.0000D 00
7	0.132D 03	0.427D 03	0.510D 02	0.0000D 00	0.246D 03	0.0000D 00	−0.146D 03	0.0000D 00	0.0000D 00
8	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00	0.0000D 00

CURRENT COMPACTION(METERS) AFTER 1800. DAYS

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.000D 00	0.578D-02	0.127D-01	0.125D-01	0.650D-02	0.132D-01	-0.180D-04	0.625D-04	-0.467D-04
2	0.000D 00	0.248D 00	0.193D 00	0.271D 00	0.277D 00	0.347D 00	0.359D 00	0.393D 00	0.573D 00

LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.599D-03	0.415D-02	0.382D 00	0.207D 00	0.187D 00	0.186D 00	0.163D 00	0.101D 00	0.000D 00
2	0.645D 00	0.800D 00	0.806D 00	0.489D 00	-0.418D-03	0.202D 00	0.141D 00	0.947D-01	0.000D 00

TOTAL SUBSIDENCE AT THE SURFACE

LAYER/POINT	1	2	3	4	5	6	7	8	9
1	0.000D 00	0.254D 00	0.206D 00	0.284D 00	0.284D 00	0.360D 00	0.359D 00	0.393D 00	0.573D 00

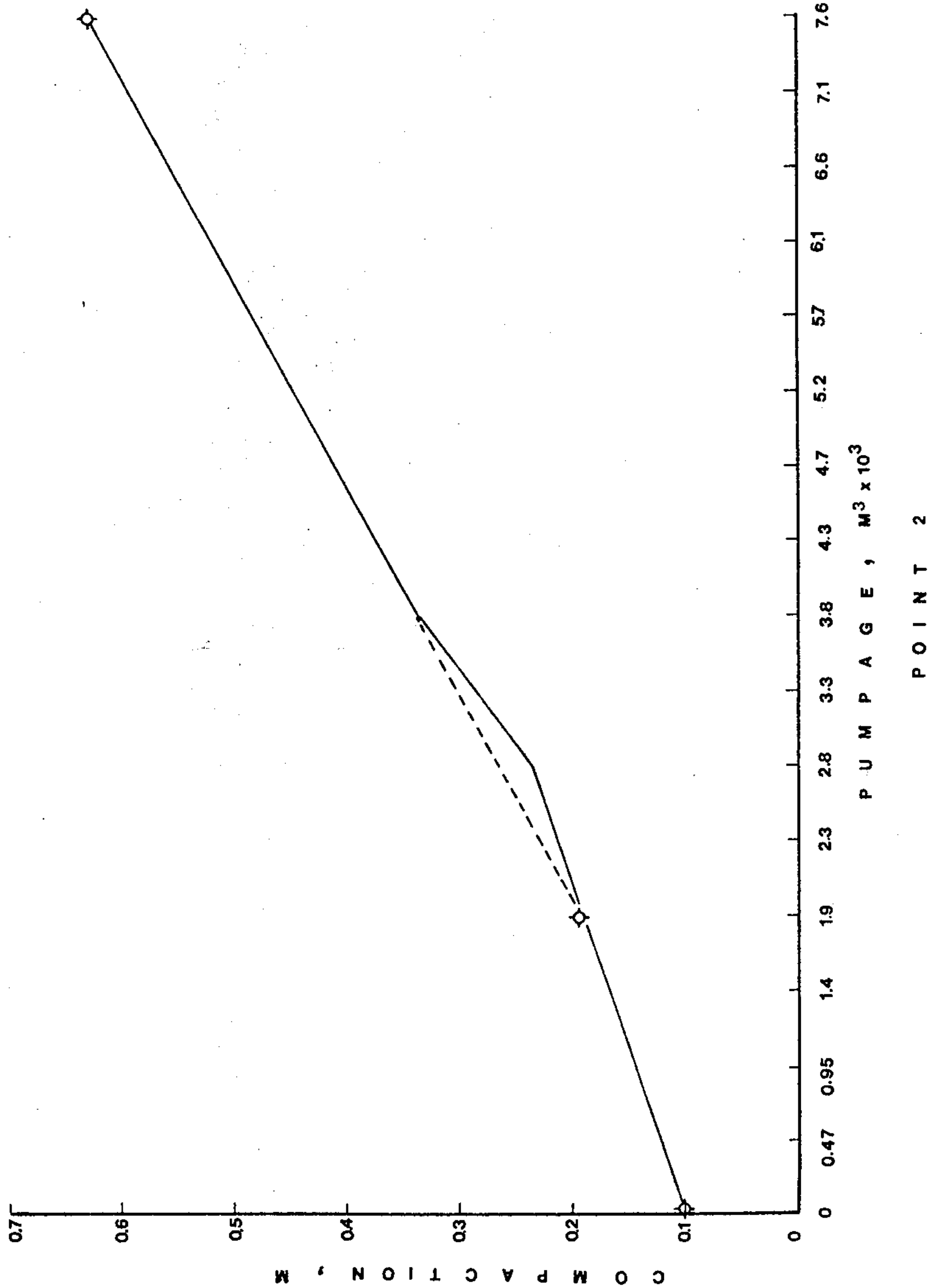
LAYER/POINT	10	11	12	13	14	15	16	17	18
1	0.646D 00	0.804D 00	0.119D 01	0.696D 00	0.187D 00	0.388D 00	0.304D 00	0.196D 00	0.000D 00

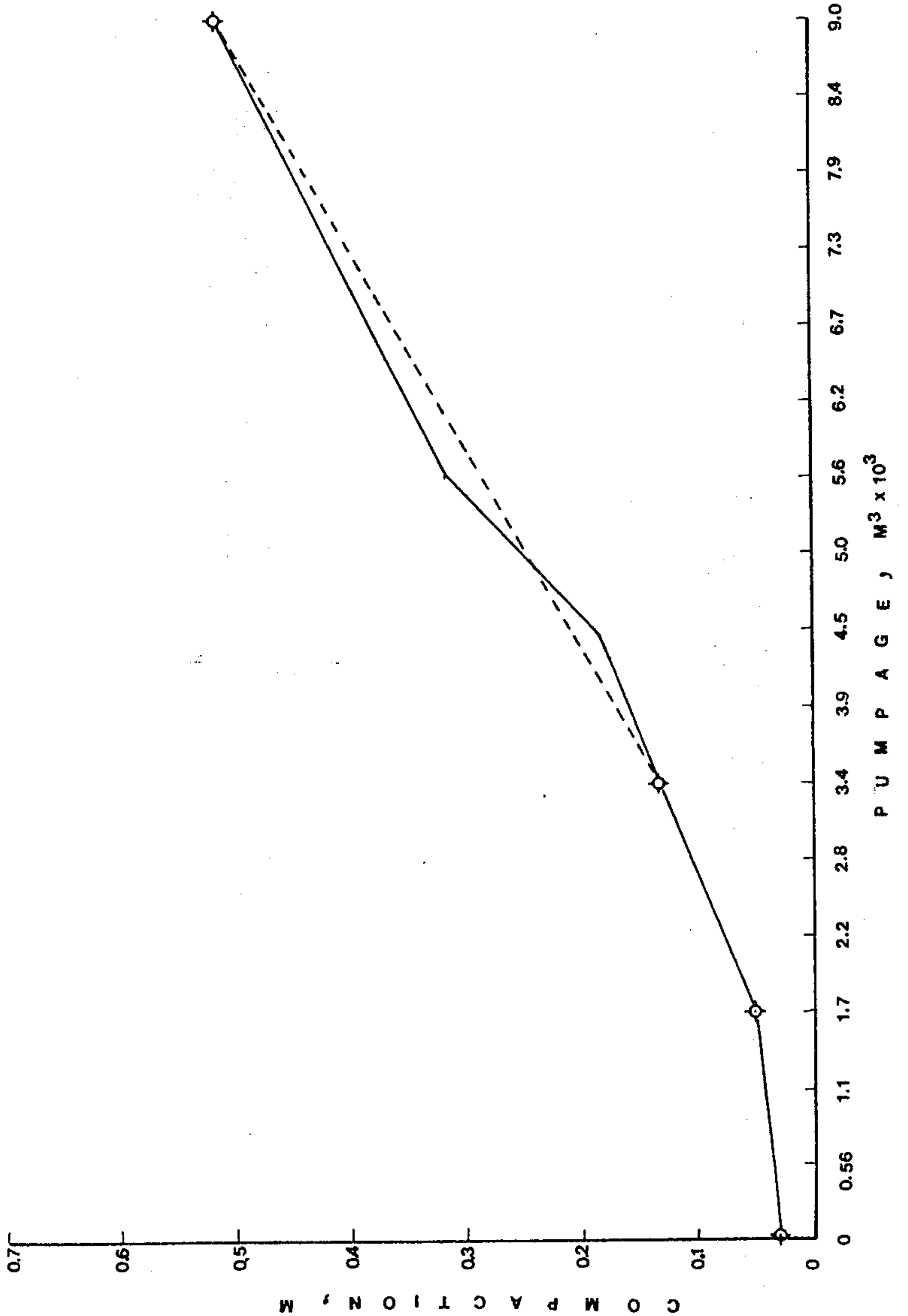
APPENDIX X

PUMPAGE-COMPACTION CURVES

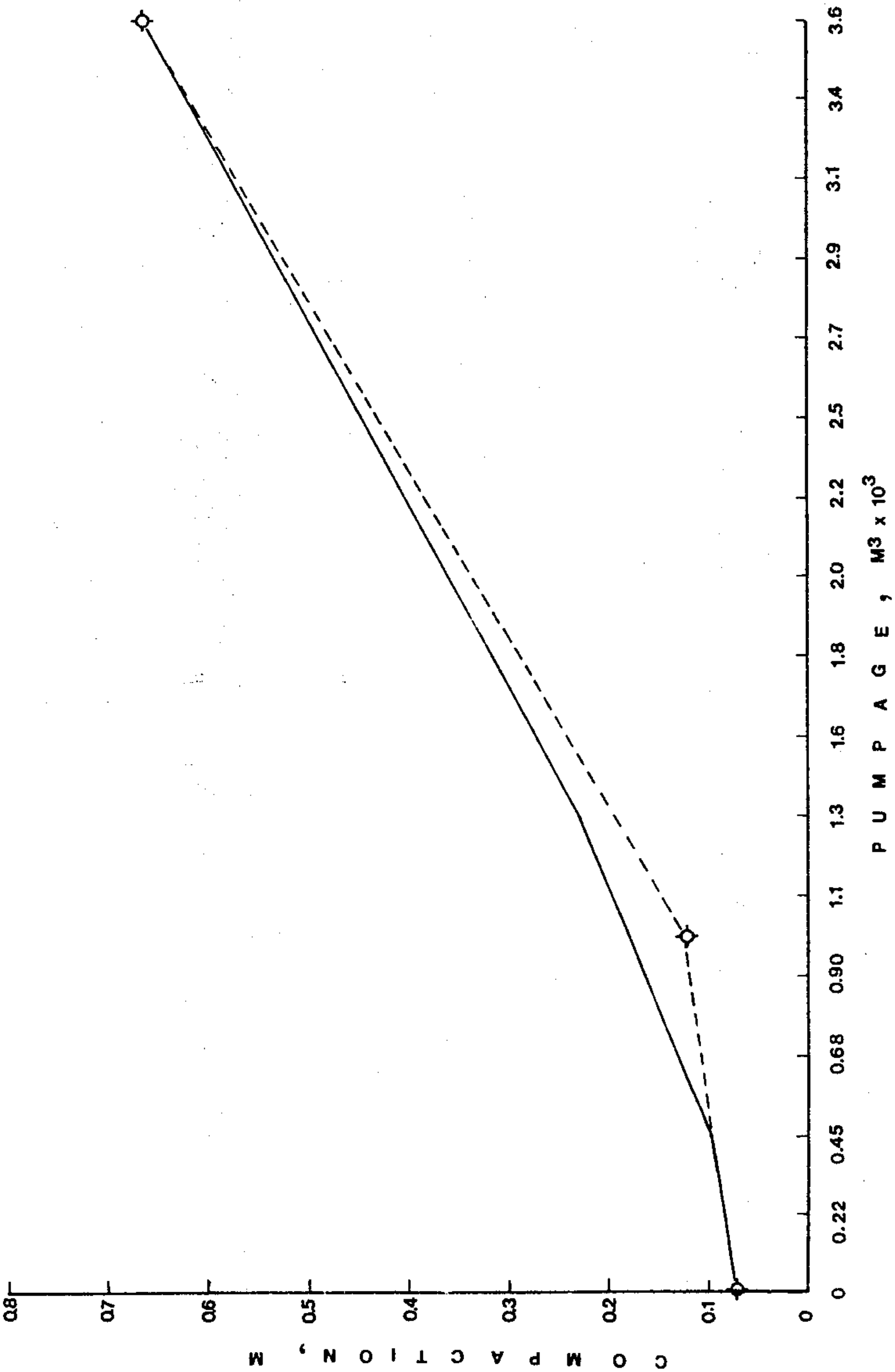
Breakpoints for the approximating linear segments.

---- Approximating linear segments when they do not coincide with the original curve.

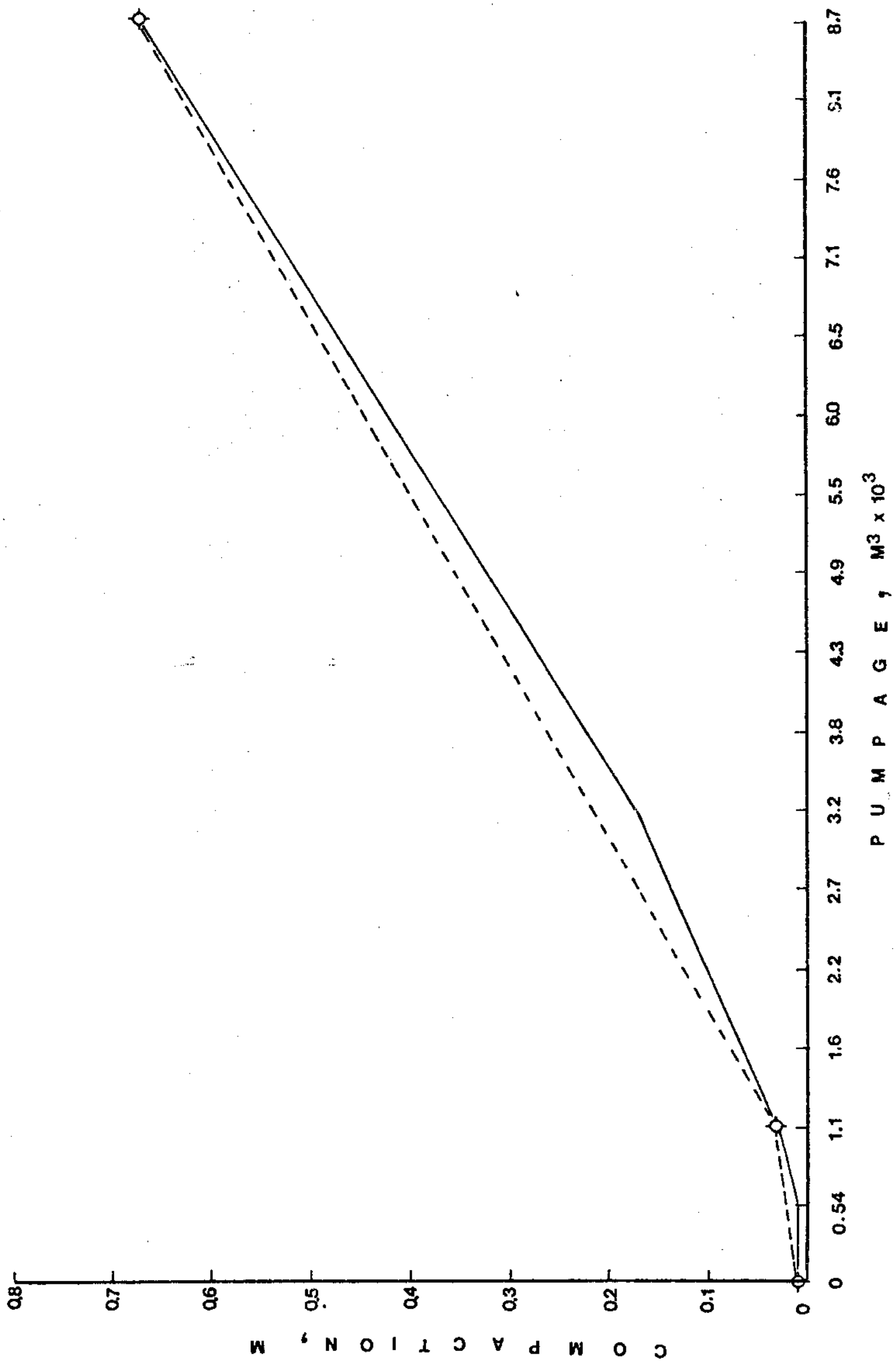

C O M P A C T I O N , M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
0.47
0.95
1.4
1.9
2.3
2.8
3.3
3.8
4.3
4.7
5.2
57
6.1
6.6
7.1
7.6
P U M P A G E , M³ x 10³
P O I N T 2


COMPACTION, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
0.56
1.1
1.7
2.2
2.8
3.4
3.9
4.5
5.0
5.6
6.2
6.7
7.3
7.9
8.4
9.0
PUMPAGE, $M^3 \times 10^3$

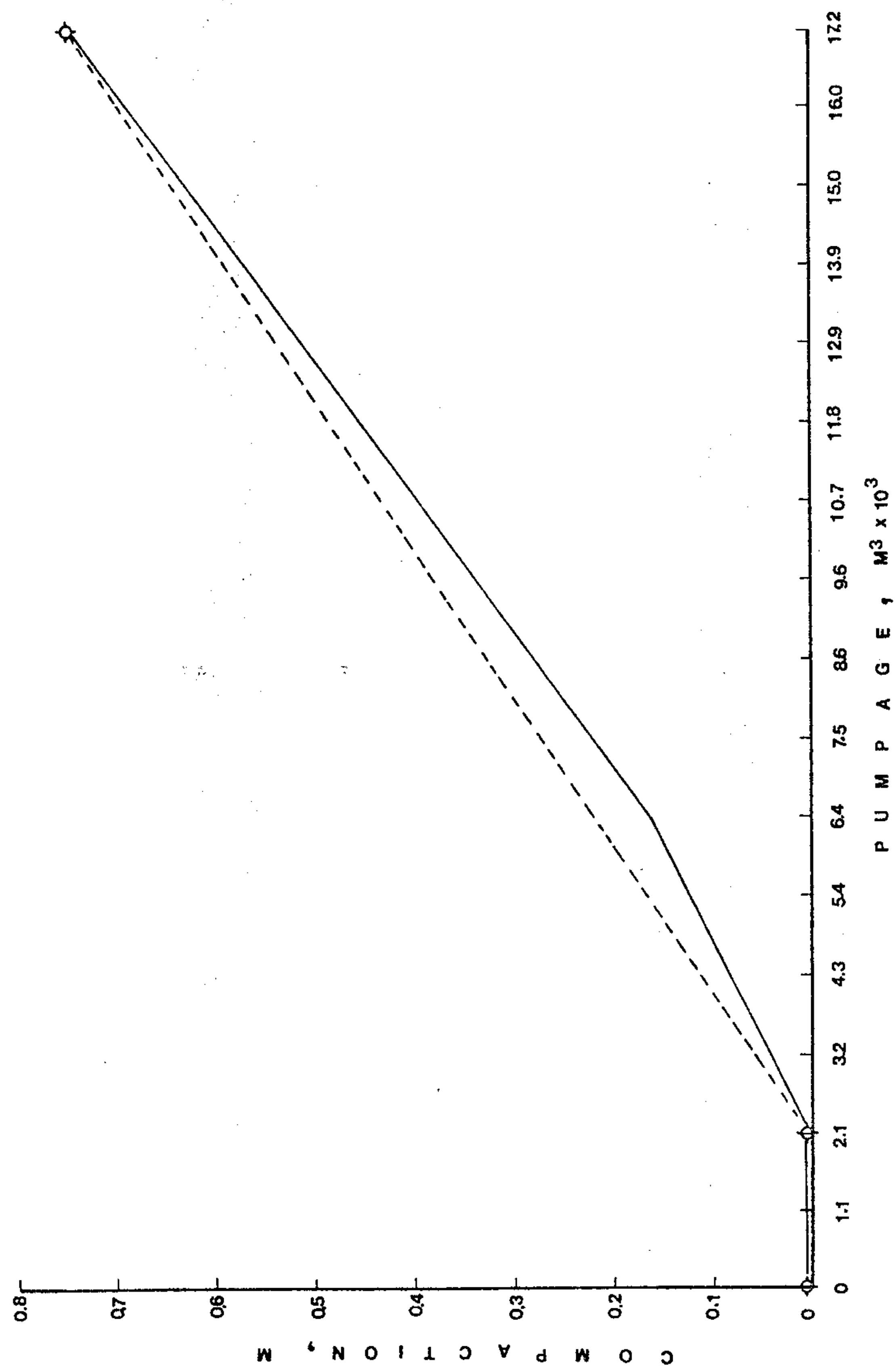
POINT 3


COMPACTION, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
0.22
0.45
0.68
0.90
1.1
1.3
1.6
1.8
2.0
2.2
2.5
2.7
2.9
3.1
3.4
3.6
PUMPAGE, M3 x 10^3

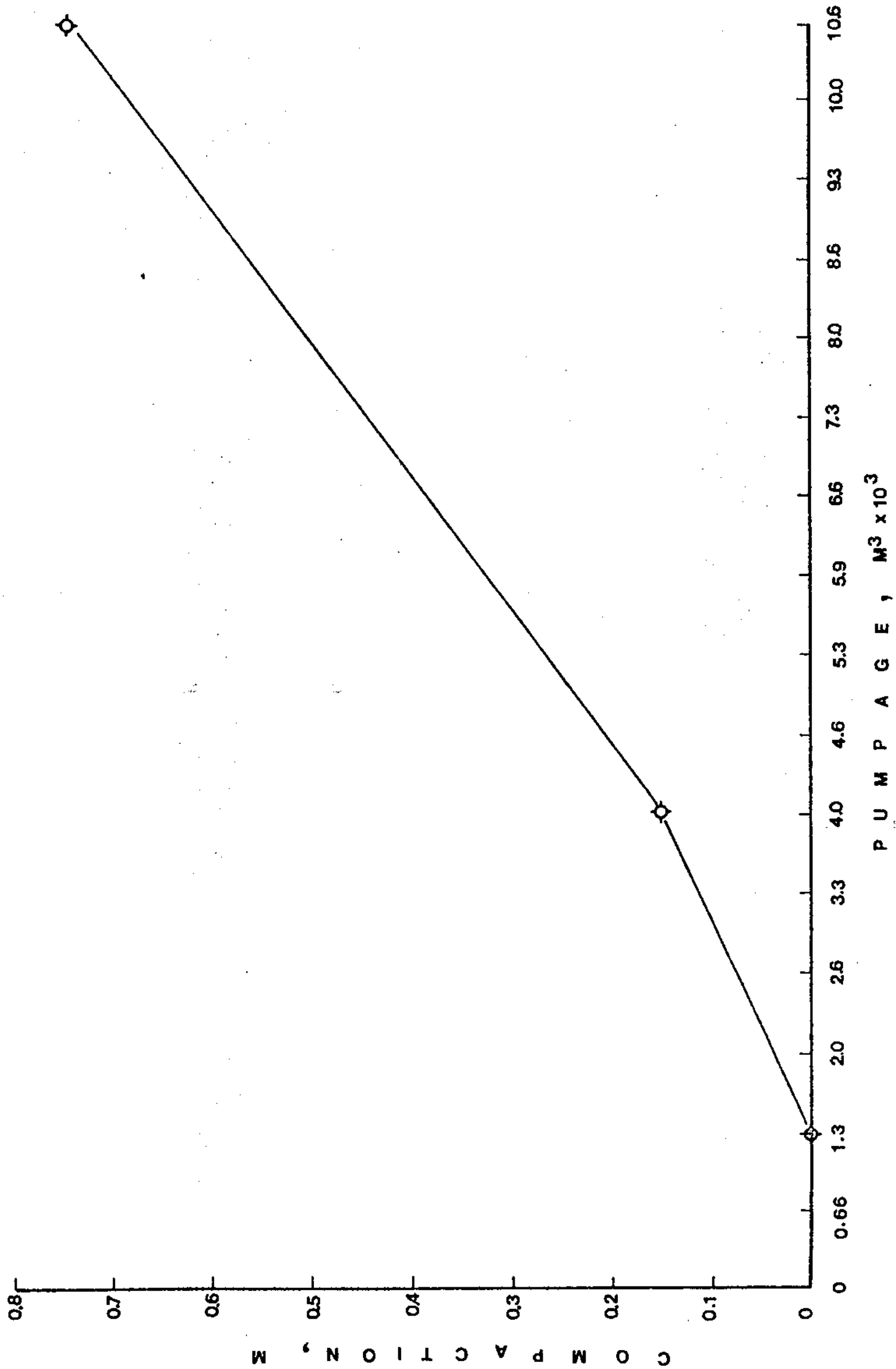
POINT 4


COMPACTION, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
PUMPAGE, M3 x 10^3
0
0.54
1.1
1.6
2.2
2.7
3.2
3.8
4.3
4.9
5.5
6.0
6.5
7.1
7.6
8.7

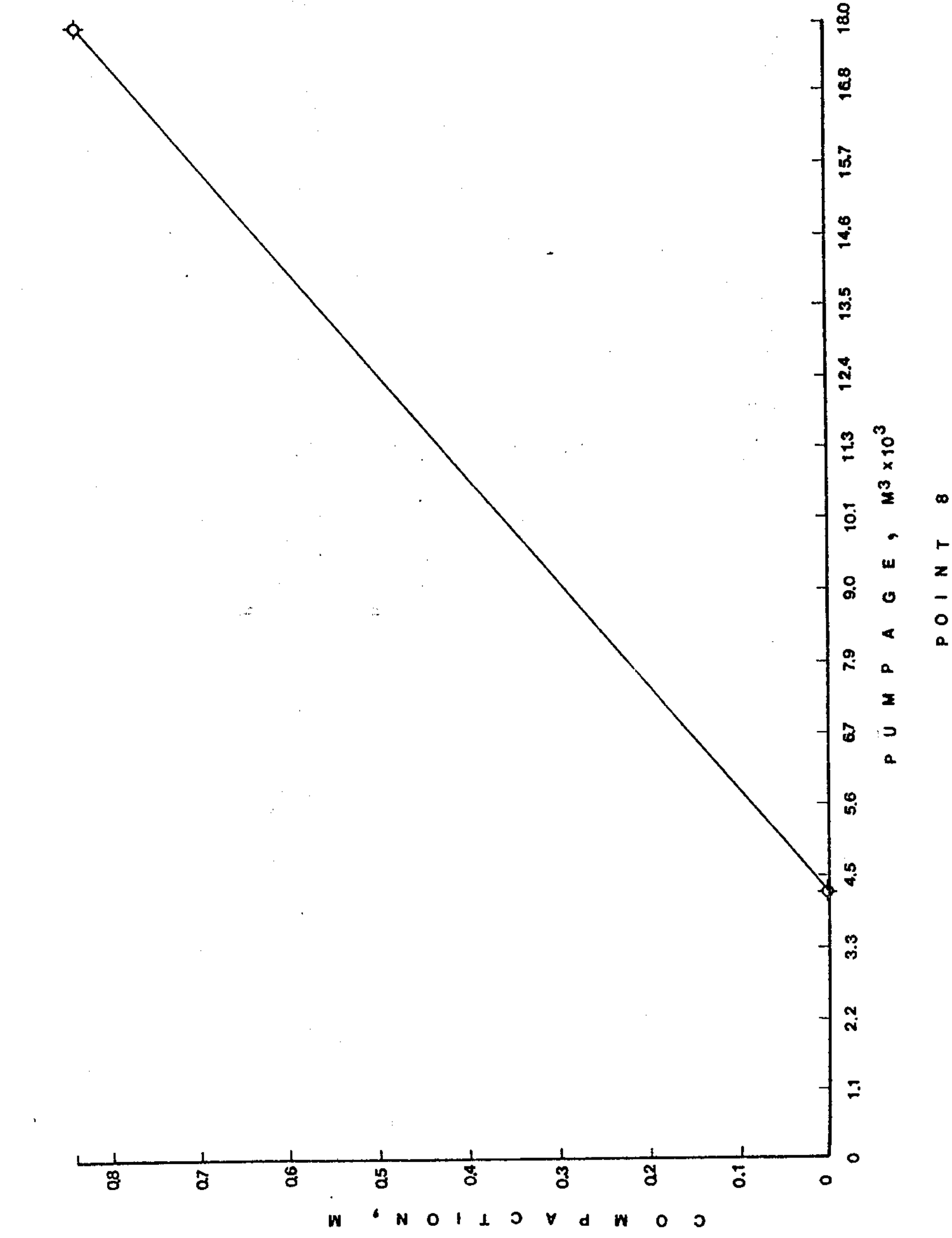
POINT 5

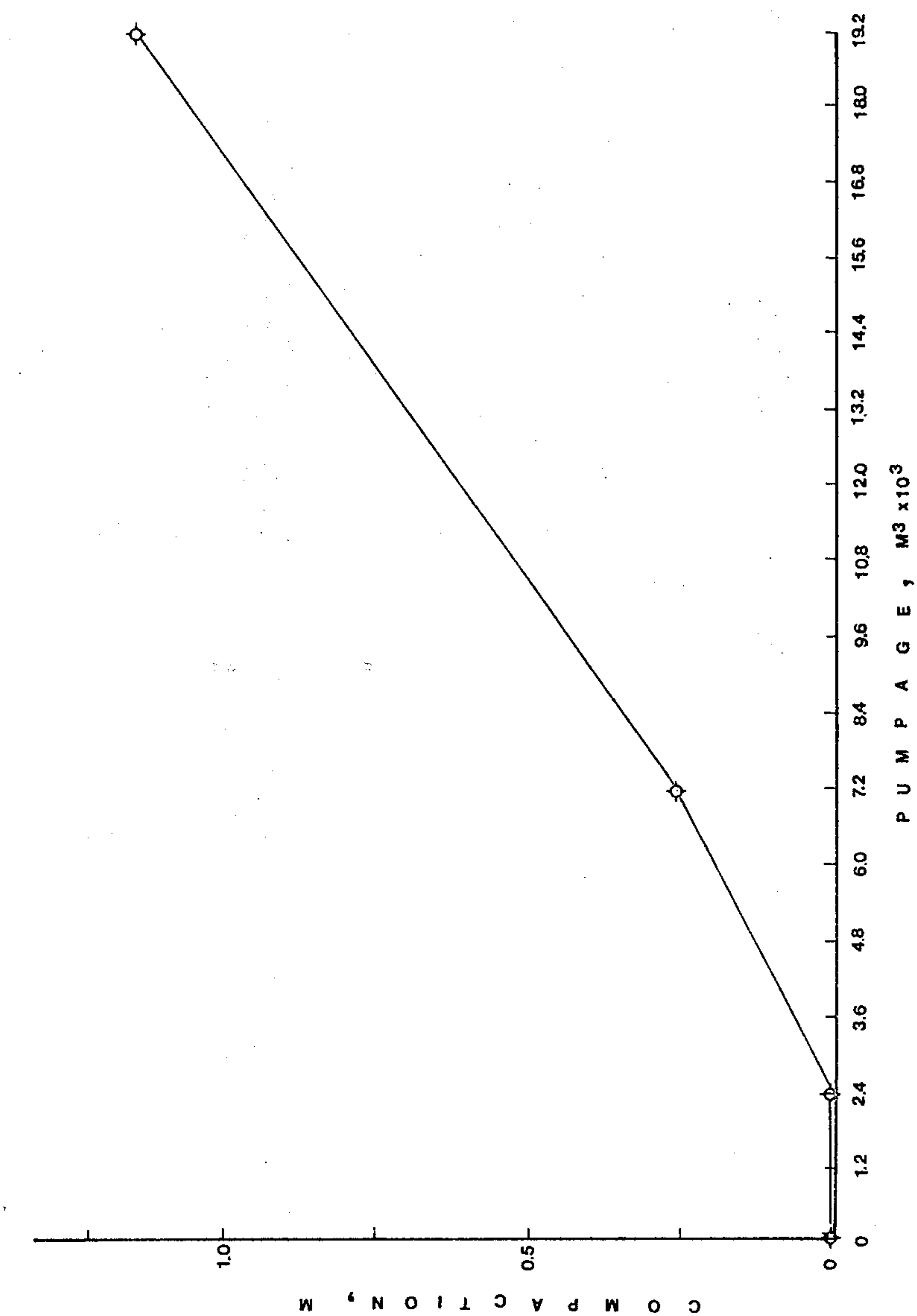

COMPACTION, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
PUMPAGE, M3 x 10^3
0
1.1
2.1
3.2
4.3
5.4
6.4
7.5
8.6
9.6
10.7
11.8
12.9
13.9
15.0
16.0
17.2

POINT 6


COMPACTION, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
PUMPAGE, M3 x 10^3
0
0.66
1.3
2.0
2.6
3.3
4.0
4.6
5.3
5.9
6.6
7.3
8.0
8.6
9.3
10.0
10.6

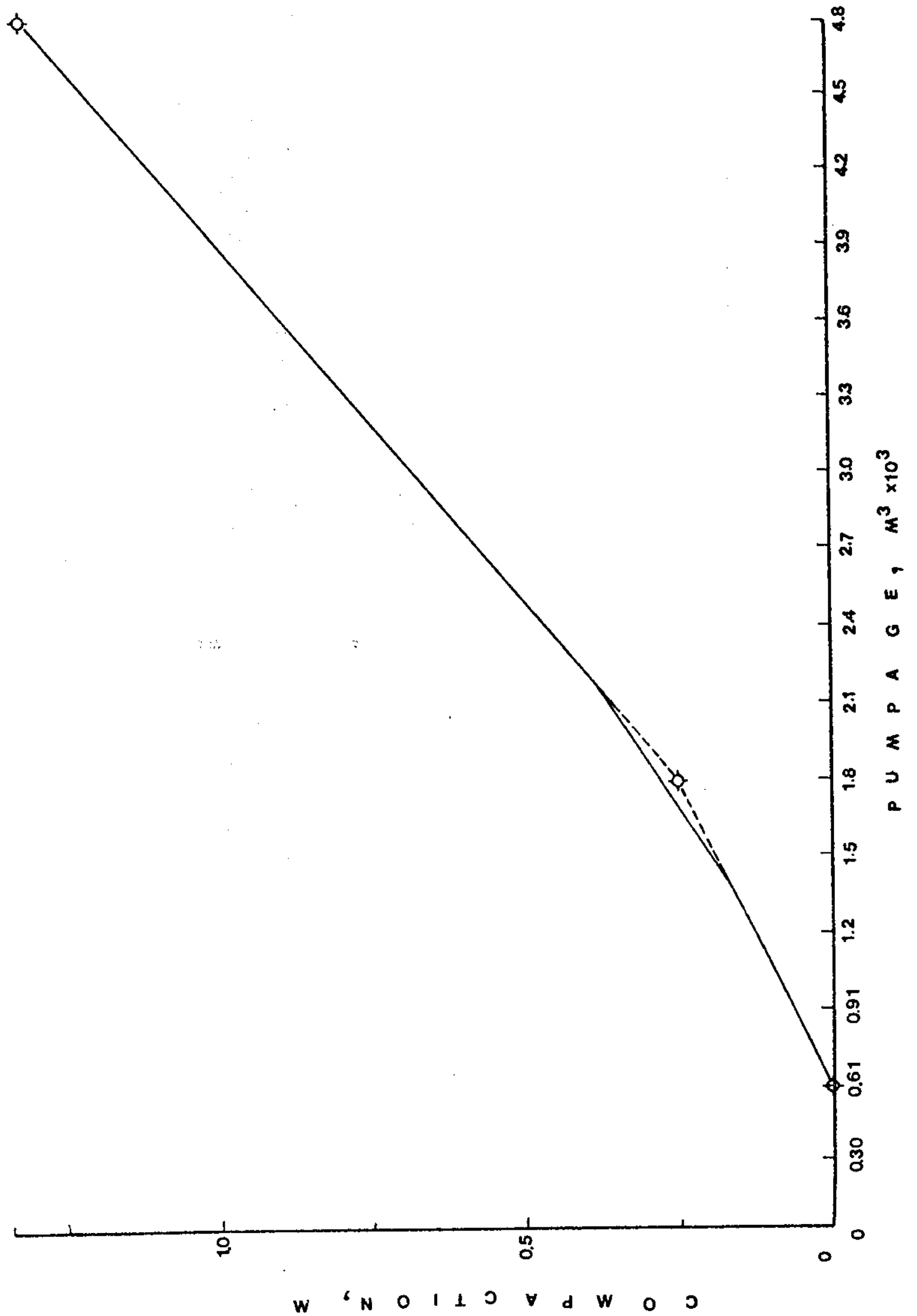
POINT 7


0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
COMPACTION, M
0
1.1
2.2
3.3
4.5
5.6
6.7
7.9
9.0
10.1
11.3
12.4
13.5
14.6
15.7
16.8
18.0
PUMPAGE, M3 x10^3
POINT 8



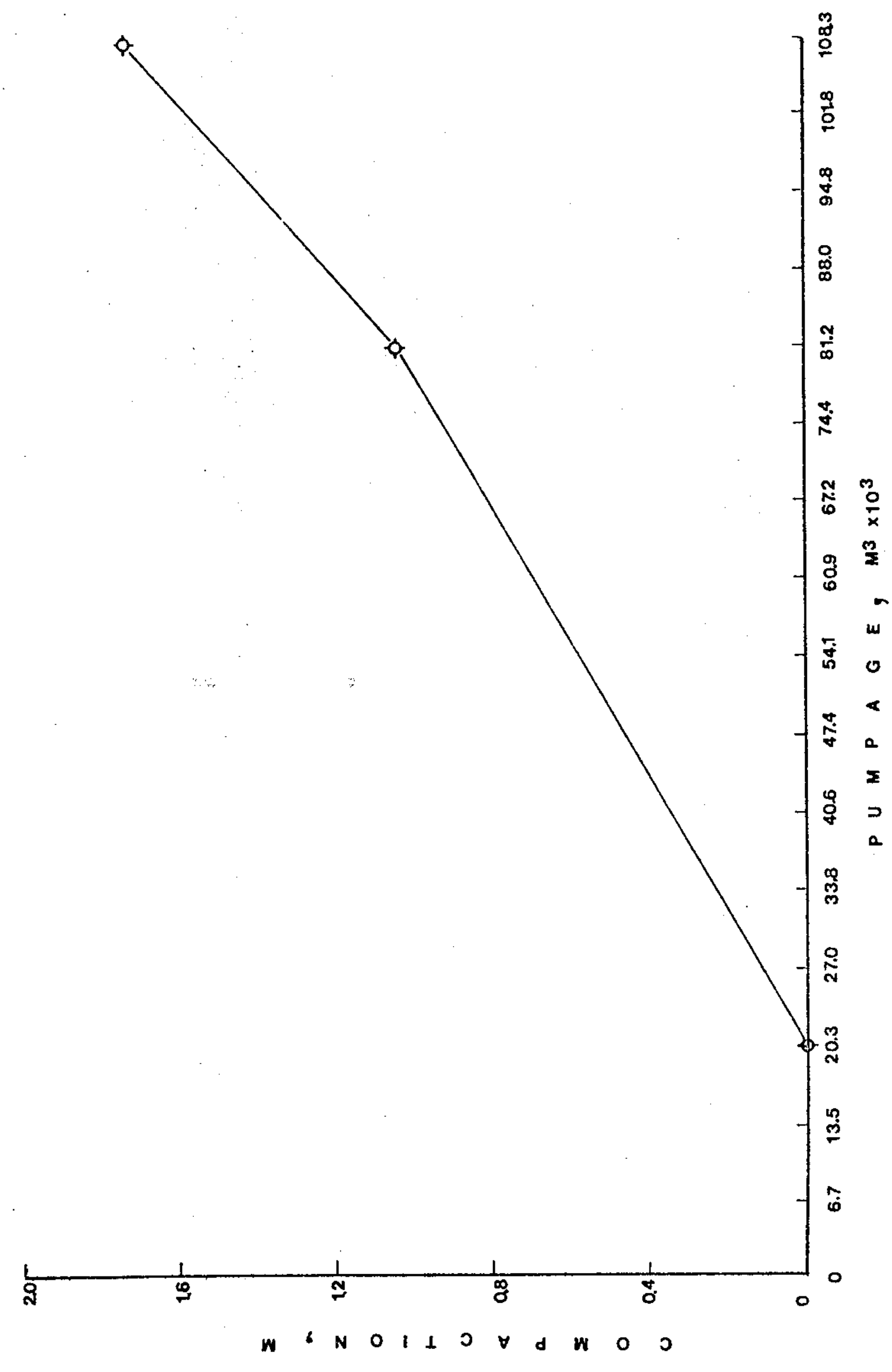
C O M P A C T I O N, M
0
0.5
1.0
P U M P A G E, M3 x10^3
0
1.2
2.4
3.6
4.8
6.0
7.2
8.4
9.6
10.8
12.0
13.2
14.4
15.6
16.8
18.0
19.2

POINT 9

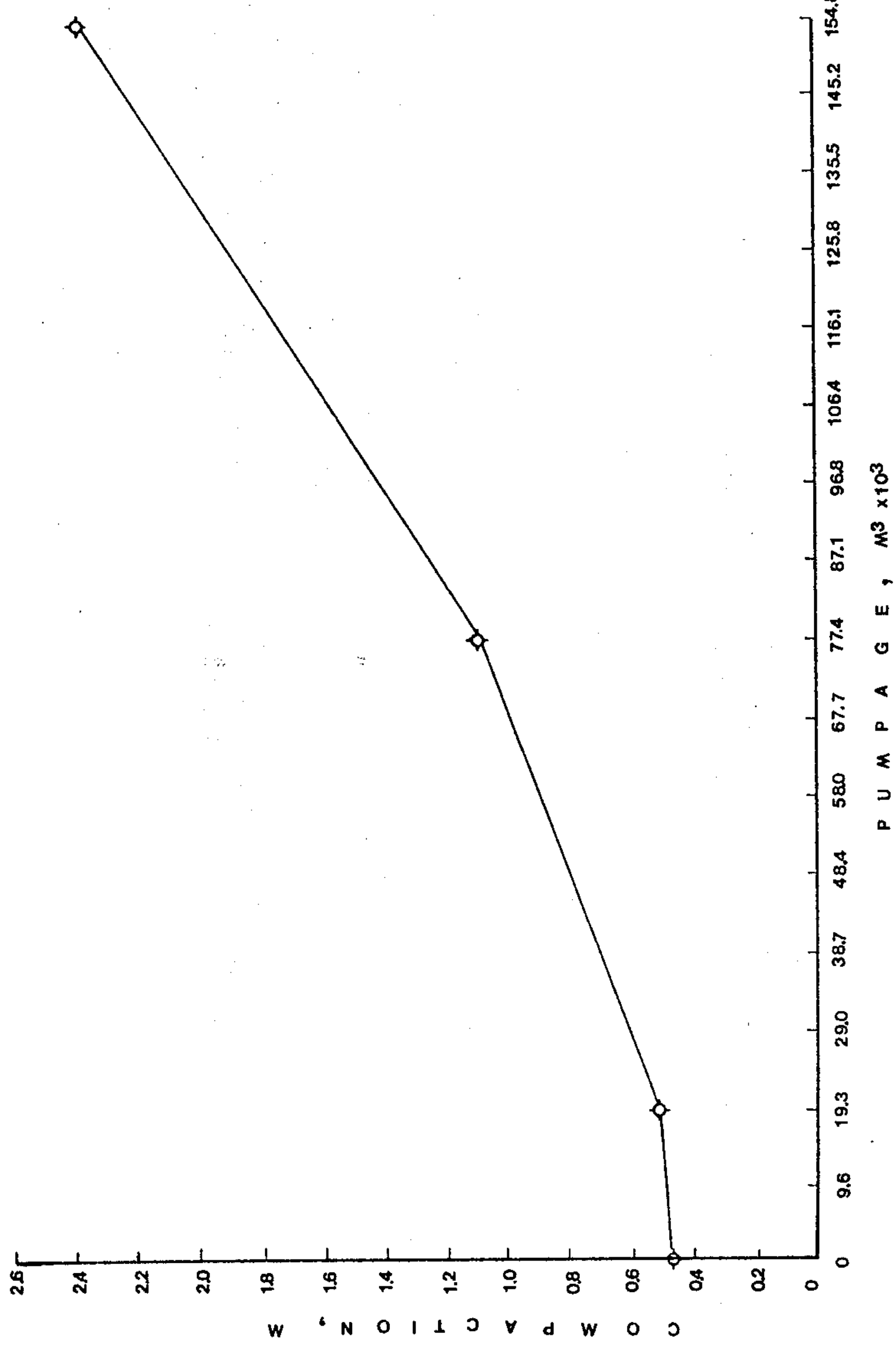


COMPACTION, M
0
0.5
1.0
PUMPAGE, M³ x10³
0
0.30
0.61
0.91
1.2
1.5
1.8
2.1
2.4
2.7
3.0
3.3
3.6
3.9
4.2
4.5
4.8

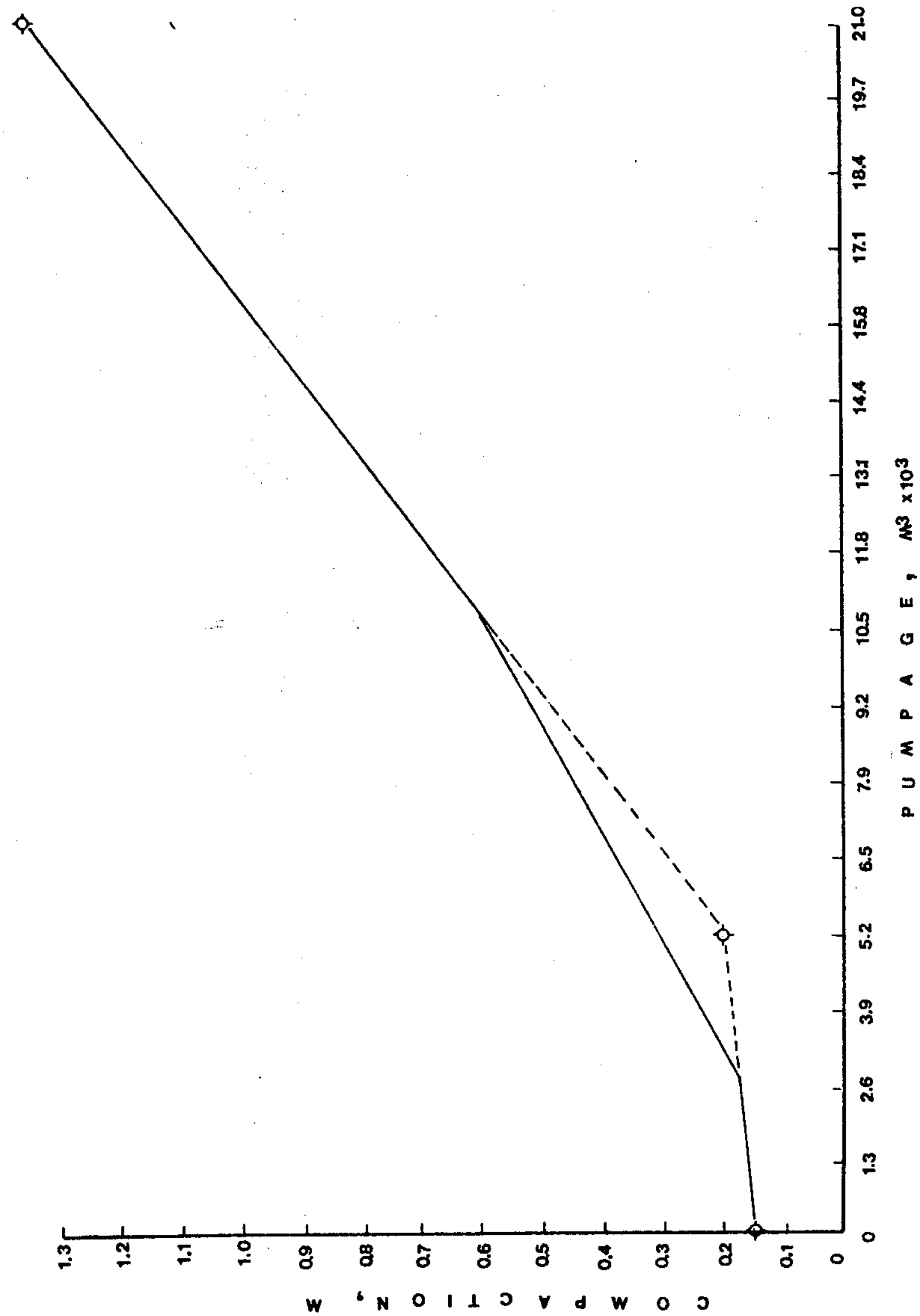
POINT 10


COMPACTION, M
0
0.4
0.8
1.2
1.6
2.0
PUMPAGE, M3 x10³
0
6.7
13.5
20.3
27.0
33.8
40.6
47.4
54.1
60.9
67.2
74.4
81.2
88.0
94.8
101.8
108.3

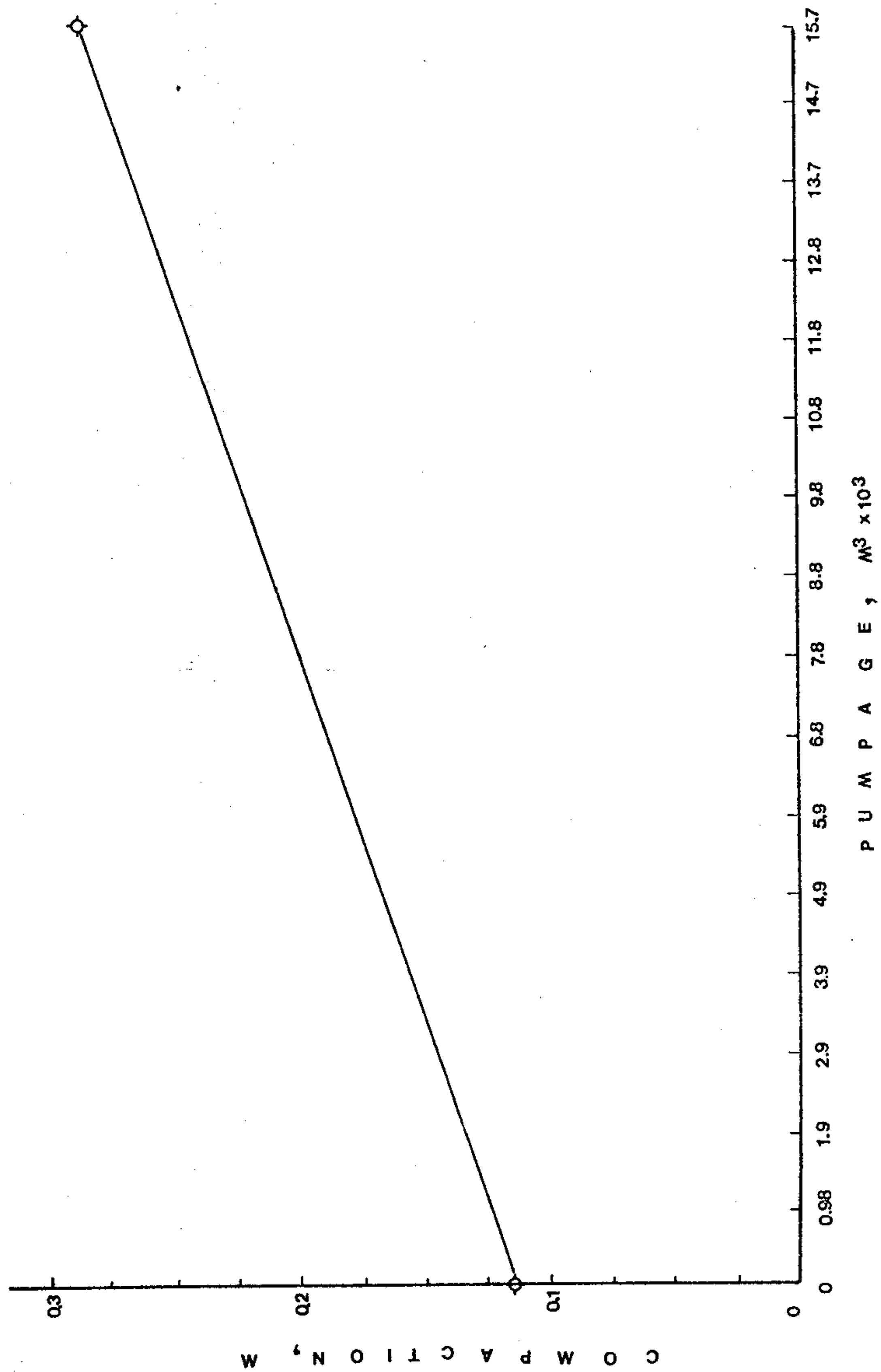
POINT 11


COMPACTION, M
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
PUMPAGE, M^3 x10^3
0
9.6
19.3
29.0
38.7
48.4
58.0
67.7
77.4
87.1
96.8
106.4
116.1
125.8
135.5
145.2
154.8

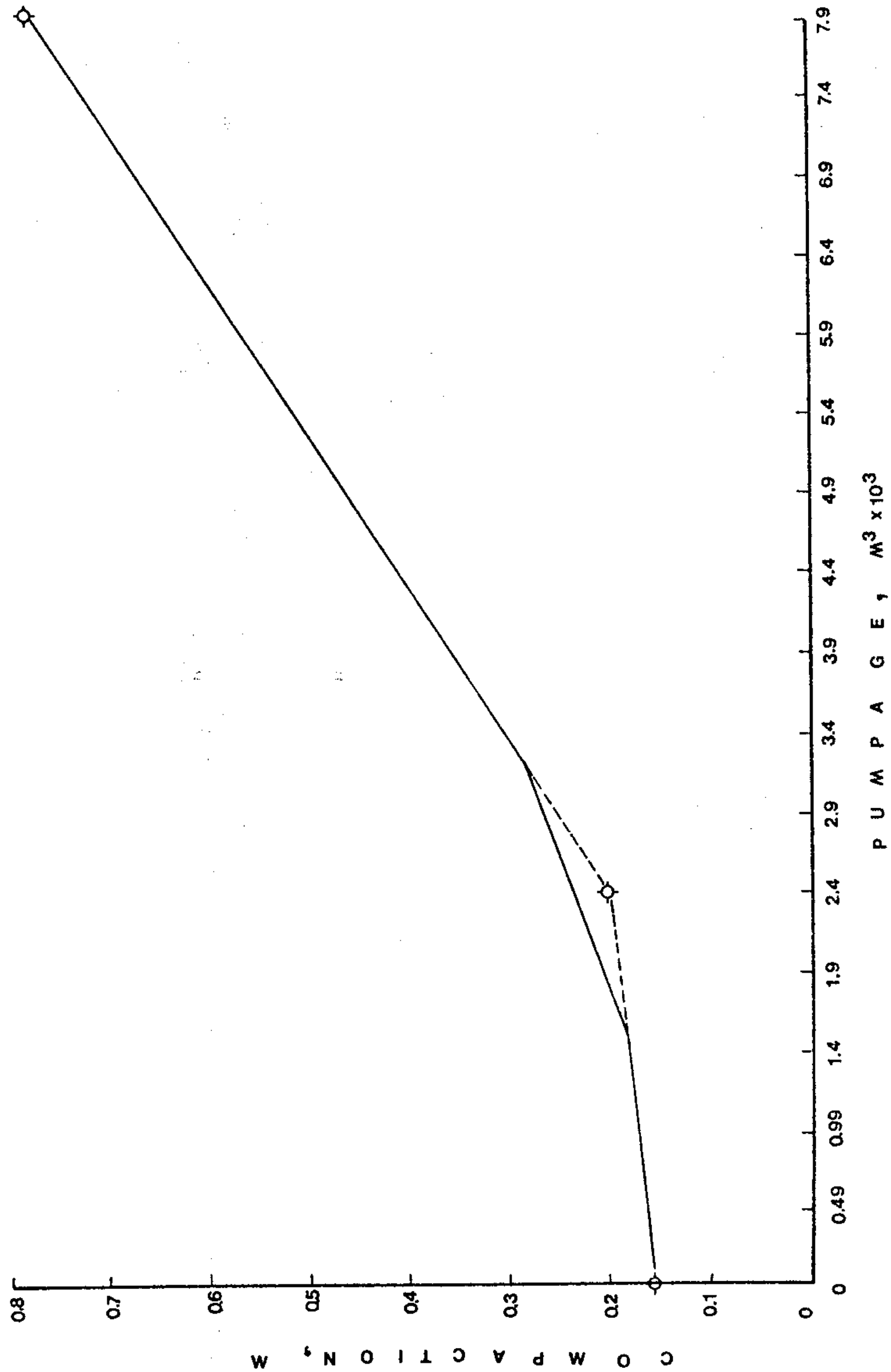
POINT 12


C O M P A C T I O N, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
0
1.3
2.6
3.9
5.2
6.5
7.9
9.2
10.5
11.8
13.1
14.4
15.8
17.1
18.4
19.7
21.0
P U M P A G E, M3 x103

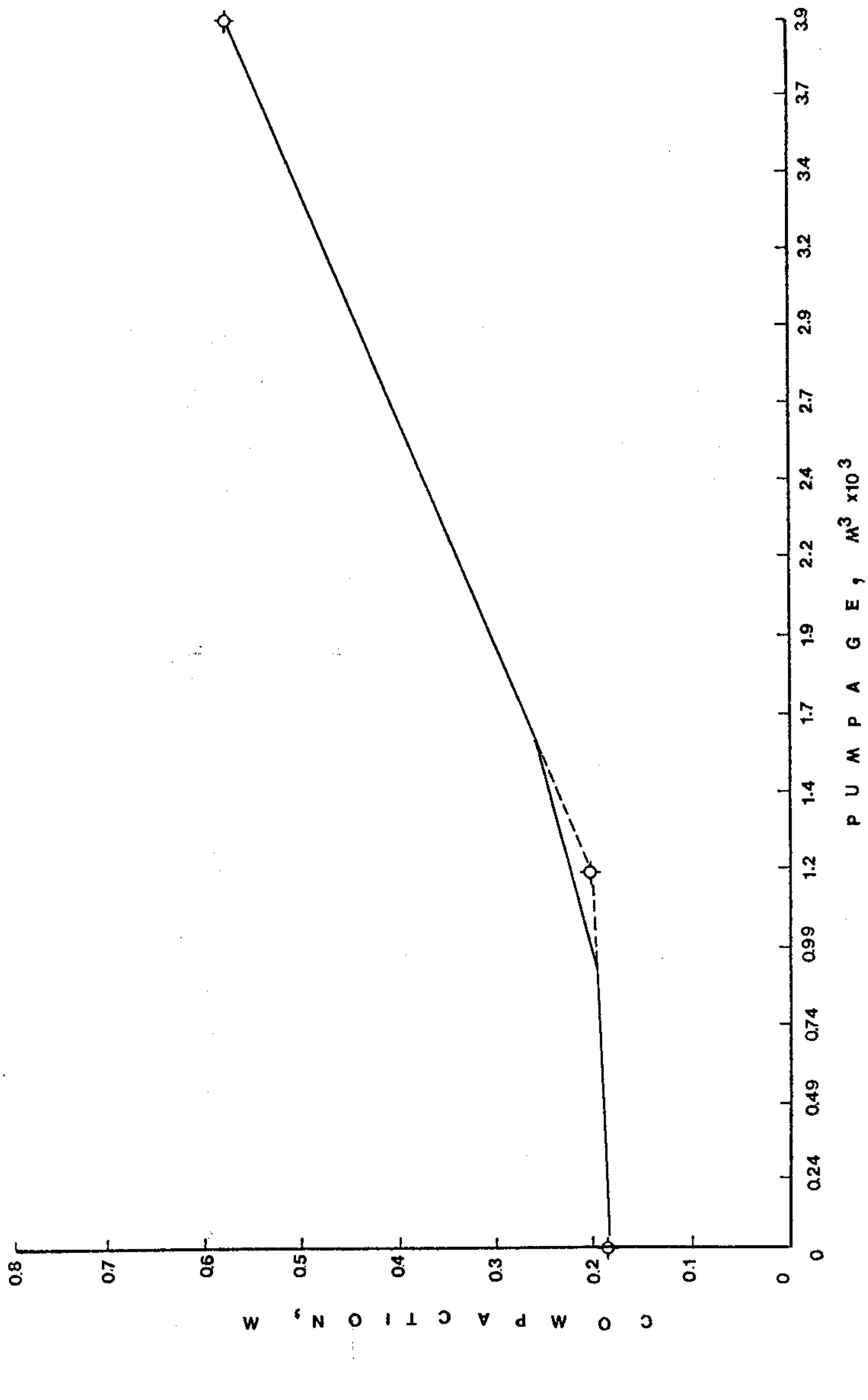
POINT 13


C O M P A C T I O N, M
0
0.1
0.2
0.3
P U M P A G E, M3 x 10^3
0
0.98
1.9
2.9
3.9
4.9
5.9
6.8
7.8
8.8
9.8
10.8
11.8
12.8
13.7
14.7
15.7

POINT 14


COMPACTION, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
PUMPAGE, M³ x10³
0
0.49
0.99
1.4
1.9
2.4
2.9
3.4
3.9
4.4
4.9
5.4
5.9
6.4
6.9
7.4
7.9

POINT 15


C O M P A C T I O N, M
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
P U M P A G E, M^3 x10^3
0
0.24
0.49
0.74
0.99
1.2
1.4
1.7
1.9
2.2
2.4
2.7
2.9
3.2
3.4
3.7
3.9

POINT 16

APPENDIX II

DERIVATION OF THE GENERAL FLOW EQUATION

The fundamental flow equation is derived for water flowing through a saturated porous media. A mass balance in two dimensions is combined with Darcy's Law and some equations of state to obtain the final equation.

The principle of mass conservation, when applied to a differential volume element of porous media fixed in space may be stated as:

(Rate of mass inflow) - (Rate of mass outflow) =

Rate of change of mass inside the volume element.

Applying this principle to the volume element shown in Figure II-1, results in the following equation:

$$\left(Mx - \Delta x/2\right) \pm \left(Mx + \Delta x/2\right) + \left(Mz - \Delta z/2\right) - \left(Mz + \Delta z/2\right) = \frac{\partial\ Mve}{\partial t} \pm Mp \ , \tag{II-1}$$

or

$$\frac{\partial Mx}{\partial x}\ \Delta x = \frac{\partial Mz}{\partial z}\ \Delta z = -\ \frac{\partial\ Mve}{\partial t}\ \pm Mp \ , \tag{II-2}$$

where $Mx - \Delta x/2$, $Mz{-}\Delta z/2$, $Mx{+}\Delta x/2$, and $Mz{+}\Delta z/2$ are the rates of mass inflow and outflow across the faces of the volume element. Mve is the mass contained inside the volume element and Mp is a mass source or sink term which is negative for a source and positive for a sink. The terms $\frac{\partial Mx}{\partial x}\Delta x$ $\frac{\partial Mz}{\partial z}\Delta z$, and $\frac{\partial Mve}{\partial t}$ represent the changes in mass with respect to space and time.

Expressing individual mass flow components in terms of the fluid density, the dimensions of the volume element, and the volume flux, we

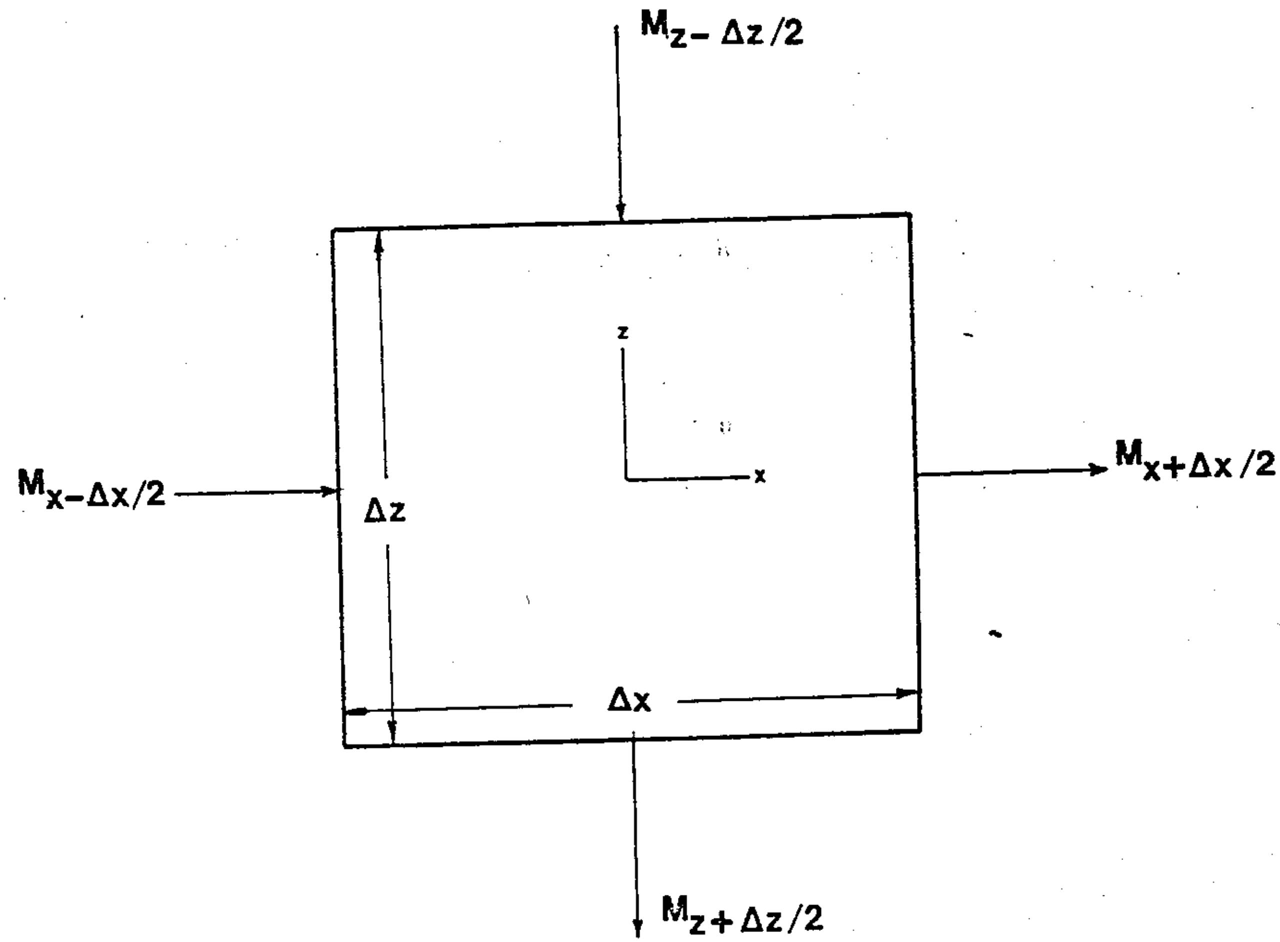


Fig. II-1 Volume element

get

$$Mx = \rho\ q_x\ \Delta z(1)\ ,$$

$$Mz = \rho\ q_z\ \Delta z(1)$$

$$MVE = \rho\phi\ \Delta x\ \Delta z(1)\ ,\ \text{and}$$

$$Mp = \rho Q$$

where

ρ = fluid density (M/L^3)

Δx = volume element's side in the x direction

Δz = volume element's side in the z direction

ϕ = porosity, and

Q = source on sink term (L^3/T) .

Substituting these relationships into equation II-2, we get

$$\frac{\partial}{\partial}(\rho q_x\ \Delta z(1))\ \Delta_x + \frac{\partial}{\partial z}(\rho\ qz\ \Delta x(1))\Delta z = -\frac{\partial}{\partial t}(\rho\phi\ \Delta x\ \Delta z(1)) \pm \rho Q \qquad \text{(II-3)}$$

According to Darcy's law,

$$q_x = -K_x \frac{\partial H}{\partial x} \quad \text{and} \quad q_z = -K_z \frac{\partial H}{\partial z} \qquad \text{(II-4)}$$

where

q_x = volume flux across z(1) face (L/T),

q_z = volume flux across z(1) face (L/T),

K_x = hydraulic conductivity in (x) direction (L/T),

K_y = hydraulic conductivity in (z) direction (L/T),

$\frac{\partial H}{\partial x}$ = head gradient in (x) direction (.), and

$\frac{\partial H}{\partial z}$ = head gradient in (z) direction (.).

Substituting equations (A-4) into equation (II-3), we obtain

$$\frac{\partial}{\partial x}(\rho\ K_x \Delta z(1)\frac{\partial H}{\partial x}\Delta x + \frac{\partial}{\partial z}(\rho\ K_z \Delta x(1)\frac{\partial H}{\partial z})\Delta z = \frac{\partial}{\partial t}(\rho\phi\ \Delta x\ \Delta z(1)) \pm \rho Q\ . \tag{II-5}$$

The first term on the right-hand side of equation II-5 can be expanded as follows:

$$\frac{\partial}{\partial t}(\rho\phi\ \Delta x\ \Delta z(1)) = \rho\phi\ \Delta z\frac{\partial(\Delta x)}{\Delta t} + \rho\phi\ \Delta x\frac{\partial(\Delta z)}{\partial t} + \rho\ \Delta x\ \Delta z\frac{\partial\rho}{\partial t} + \phi\ \Delta x\ \Delta z\frac{\partial\rho}{\partial t}\ . \tag{II-6}$$

The resulting four terms deal with the time rate of change of the volume element's horizontal and vertical dimensions, porosity, and fluid density. It is assumed that no horizontal changes occur, so the first term is neglected. The second and third terms are related to vertical compression of the porous media. They may be expressed in terms of intergranular pressure, and ultimately, fluid head. The fourth term may also be expressed in terms of fluid head by using a density-pressure relationship.

The second term in equation II-6 is related to the vertical compressibility of the aquifer as follows:

$$\alpha = \frac{1}{E_s}\ , \tag{II-7}$$

where α is the formation compressibility factor, and E_s is the bulk modulus of compression of the rock skeleton defined by

$$E_s = \frac{\Delta\ \text{stress}}{\Delta\ \text{strain}} = -\frac{\Delta\bar{\sigma}}{d(\Delta z/\Delta z)}\ , \tag{II-8}$$

where $\Delta\bar{\sigma}$ is the change in intergranular pressure, and $d(\Delta z/\Delta z)$ is the unit change in the vertical dimension of the volume element.

Combining equations II-7 and II-8 yields,

$$d(\Delta z) = -\alpha \; \Delta z \; d\bar{\sigma} \;,$$

or in terms of the time derivative,

$$\frac{\partial(\Delta z)}{\partial t} = -\alpha \; \Delta z \; \frac{\partial \sigma}{\partial t} \;. \tag{II-9}$$

The third term of equation II-6 expresses the change in porosity, which is also related to vertical compression. The volume of solids in the volume element can be related to the vertical dimension only as

$$Vs = (1-\sigma)\Delta z \;, \tag{II-10}$$

where

Vs = the volume of solids.

Assuming that Vs remains constant with time,

$$dVs = d((1-\phi) \; \Delta z) = 0 \;. \tag{II-11}$$

Carrying out the differentiation,

$$dVs = -d\phi \; \Delta z + (1-\phi)d(\Delta z) = 0 \;. \tag{II-12}$$

Consequently,

$$\frac{\partial \phi}{\partial t} = \frac{(1-\phi)}{\Delta z} \; \frac{\partial \Delta z}{\partial t} \;, \tag{II-13}$$

and using equation II-9

$$\frac{\partial \phi}{\partial t} = - \; (1-\phi)\alpha \; \frac{\partial \bar{\sigma}}{\partial t} \;,$$

The fourth term in equation II-6 refers to the changes in fluid density with respect to time and is related to the fluid compressibility (β), where $\beta = 1/Ew$. Ew is the bulk modulus of compression for water and is defined as

$$Ew = - \frac{dp}{d(\Delta V)/\Delta V} = \frac{1}{\beta} \quad , \tag{II-15}$$

where dp = the change in fluid pressure, and

$d(\Delta V)/\Delta V$ = the change in fluid volume per unit volume.

Rearranging terms in equation II-15 yields $d(\Delta V) = - \beta \Delta V dp$. (II-16)

By the conservation of mass principle,

$$\rho_1 \Delta V_1 = \rho_2 \Delta V_2 \quad \text{or} \quad \rho d(\Delta V) + \Delta V \, d\rho = 0 \; . \tag{II-17}$$

Substituting equation II-16 into equation II-17 and taking the time derivative, we get

$$\rho\beta \frac{dp}{dt} = \frac{d\rho}{dt} \quad .$$

Now, substituting equations II-18 , II-14 and II-9 into equation (II-6) yields

$$\frac{\partial(\rho\sigma \; \Delta x \Delta z(1)}{\partial t} = -\rho\alpha \; \Delta z \; \Delta x \; \frac{\partial\bar{\sigma}}{\partial t} \; -\alpha\rho \; \Delta x \; \Delta z(1-\phi) \; \frac{\partial\bar{\sigma}}{\partial t} +$$

$$+ \; \phi \; \Delta x \; \Delta z \; \rho\beta \frac{dp}{dt} = \text{RHS} \; . \tag{II-19}$$

since $d\bar{\sigma} = - \, dp$, (II-20)

equation II-19 becomes

$$\text{RHS} = \frac{\partial p}{\partial t}(\rho\alpha \; \Delta z \; \Delta x\phi \; + \; \alpha\rho \; \Delta x \; \Delta z(1-\phi) \; + \; \phi\rho\beta \; \Delta x \; \Delta z) \; ,$$

or

$$RHS = \frac{\partial p}{\partial t} \Delta x \, \Delta z \, \rho(\alpha + \phi\beta) \quad . \tag{II-21}$$

Since the piezometric head is defined as

$$H = \frac{p}{\rho g} + z \tag{II-22}$$

then the change in H with respect to time is

$$\frac{\partial H}{\partial t} = \frac{\partial z}{\partial t} + \frac{1}{\rho g} \frac{\partial p}{\partial t} \tag{II-23}$$

and given that $\partial Z / \partial t = 0$ (II-24)

we obtain

$$RHS = \rho^2 g \Delta x \Delta z (\alpha + \phi\beta) \frac{\partial H}{\partial t} \quad , \tag{II-25}$$

where $\rho g(\alpha+\phi\beta)$ is the coefficient of specific storage S_s (L/T^2). Thus,

$$RHS = \rho S_s \, \Delta x \, \Delta z \frac{\partial H}{\partial t} \quad . \tag{II-26}$$

Assuming that density does not vary in space,

$$\frac{\partial}{\partial x}(K_x \, \Delta z \, \frac{\partial H}{\partial x}) \Delta x + \frac{\partial}{\partial z}(K_z \, \Delta x \, \frac{\partial H}{\partial z}) \Delta z = \Delta x \, \Delta z \, S_s \frac{\partial H}{\partial t} \pm Q \quad . \tag{II-27}$$

APPENDIX III

FINITE DIFFERENCE FORM OF THE GENERAL FLOW EQUATION

The general flow equation II-27 used to calculate piezometric heads is written as

$$\frac{\partial}{\partial x}\left(K_x \Delta z \frac{\partial H}{\partial x}\right)\Delta x + \frac{\partial}{\partial z}\left(K_z \Delta x \frac{\partial H}{\partial z}\right)\Delta z = \Delta x \; \Delta x \; S_s \frac{\partial H}{\partial t} \pm Q \; . \qquad \text{(III-1)}$$

A centered-in-space finite difference grid system (Figure III-1) is used to develop a finite difference equation for Equation III-1 .

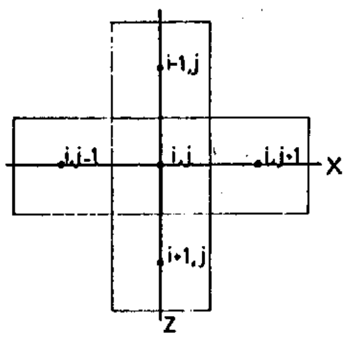


Fig. III-1 Finite difference grid system.

The spatial derivative of H at a point on the boundary between grids i,j and i,j+1 may be approximated by

$$\left.\frac{\partial H}{\partial x}\right)_{i,j+1/2} = \frac{H_{i,j+1} - H_{i,j}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j+1}}{2}} \; , \qquad \text{(III-2)}$$

likewise, for a point on the boundary between grids i,j and i,j-1

$$\left.\frac{\partial H}{\partial x}\right)_{i,j-1/2} = \frac{H_{i,j} - H_{i,j-1}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j-1}}{2}} \; . \qquad \text{(III-3)}$$

for a point between grids i,j and i+1,j

$$\left.\frac{\partial H}{\partial z}\right)_{i+1/2,j} = \frac{H_{i+1,j} - H_{i,j}}{\frac{\Delta z_{i,j}}{2} \quad \frac{\Delta z_{i+1,j}}{2}} \quad , \qquad \text{(III-4)}$$

and for a point between grids i,j and $i-1,j$

$$\left.\frac{\partial H}{\partial z}\right)_{i-1/2,j} = \frac{H_{i,j} - H_{i-1,j}}{\frac{\Delta z_{i,j}}{2} - \frac{\Delta z_{i-1,j}}{2}} \quad . \qquad \text{(III-5)}$$

The left-hand side (LHS) of equation (III-1) can be written as

$$\begin{aligned} LHS = & \frac{1}{\Delta x_{i,j}} \left[\left(K_x \ \Delta z \ \frac{\partial H}{\partial x} \right)_{i,j+1/2} - \left(K_x \ \Delta z \ \frac{\partial H}{\partial x} \right)_{i,j-1/2} \right] \Delta x_{i,j} \\ & + \frac{1}{\Delta z_{i,j}} \left[\left(K_z \ \Delta x \ \frac{\partial H}{\partial z} \right)_{i-1/2,j} - \left(K_z \ \Delta x \ \frac{\partial H}{\partial z} \right)_{i-1/2,j} \right] \Delta z_{i,j} \quad . \end{aligned} \qquad \text{(III-6)}$$

Substituting equations III-2, III-3, III-4 and III-5 into equation III-6 and manipulating terms, we get

$$\begin{aligned} LHS = & (K_x \ \Delta z)_{i,j+1/2} \left[\frac{H_{i,j+1} - H_{i,j}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j+1}}{2}} \right] \\ & - (K_x \ \Delta z)_{i,j-1/2} \left[\frac{H_{i,j} - H_{i,j-1}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i-1,j}}{2}} \right] \\ & + (K_z \ \Delta x)_{i+1/2,j} \left[\frac{H_{i+1,j} - H_{i,j}}{\frac{\Delta z_{i,j}}{2} + \frac{\Delta z_{i-1,j}}{2}} \right] \end{aligned} \qquad \text{(III-7)}$$

$$- (K_z \;\Delta x)_{i-1/2,j}\left[\frac{H_{i,j} - H_{i-1,j}}{\frac{\Delta z_{i,j}}{2} + \frac{\Delta z_{i-1,j}}{2}}\right]$$

Equation III-7 may be simplified by substituting equivalent terms of the following form

$$(K_x \,\Delta z)_{i,j+1/2}\left[\frac{H_{i,j+1} - H_{i,j}}{\frac{\Delta x_{i,j}}{2} + \frac{\Delta x_{i,j+1}}{2}}\right] = \frac{2 \cdot K_{x_{i,j}} \cdot K_{x_{i,j+1}} \cdot \Delta z_{i,j+1/2}}{K_{x_{i,j}} \cdot \Delta x_{i,j+1} + K_{x_{i,j+1}} \cdot \Delta x_{i,j}}$$

$$[H_{i,j+1} - H_{i,j}] \,. \qquad \text{(III-8)}$$

The derivation of equations of the same type as equation III-8 is given in Appendix IV.

We can then make the following definitions:

$$N_x^+ = \frac{2\, K_{x_{i,j}}\; K_{x_{i,j+1}}\;\; \Delta z_{i,j+1/2}}{K_{x_{i,j}} \cdot \Delta x_{i,j+1} + K_{x_{i,j+1}}\;\Delta x_{i,j}} \qquad \text{(III-9)}$$

$$N_x^- = \frac{2K_{x_{i,j}} \cdot K_{x_{i,j-1}}\;\Delta z_{i,j-1/2}}{K_{x_{i,j}} \cdot \Delta x_{i,j-1} + K_{x_{i,j-1}}\;\Delta x_{i,j}} \qquad \text{(III-10)}$$

$$N_z^+ = \frac{2K_{z_{i,j}}\;\; K_{z_{i+1,j}}\;\; \Delta x_{i+1/2,j}}{K_{x_{i,j}}\Delta z_{i-1,j} + K_{z_{i+1,j}}\Delta z_{i,j}} \qquad \text{(III-11)}$$

$$N_z^- = \frac{2K_{z_{i,j}}\; K_{z_{i-1,j}}\;\; \Delta x_{i-1/2,j}}{K_{z_{i,j}}\;\Delta z_{i-1,j} + K_{z_{i-1,j}}\Delta x_{i,j}} \qquad \text{(III-12)}$$

Substituting equations III-8, III-9, III-10, III-11, and III-12 into III-7 gives

$$LHS = N\overset{+}{x}\, H_{i,j+1} + N\bar{x}\, H_{i,j-1} + N\overset{+}{z}\, H_{i+1,j} + N\bar{z}\, H_{i-1,j}$$

$$- (\overset{+}{Nx} + \overset{-}{Nx} + \overset{+}{Nz} + \overset{-}{Nz})H_{i,j} \; . \qquad \text{(III-13)}$$

The time derivative on the right-hand side (RHS) of equation (III-1) then be expressed as

$$\left.\frac{\partial H}{\partial t}\right|_{i,j} = \frac{H_{i,j}^{t+\Delta t} - H_{i,j}^{t}}{\Delta t} \; , \qquad \text{(III-14)}$$

and the factor $(\Delta z)_{i,j}$ as

$$(\Delta z)_{i,j} = \frac{\Delta z_{i,j+1/2} + \Delta z_{i,j-1/2}}{2} \; . \qquad \text{(III-15)}$$

Thus, the right-hand side becomes

$$RHS = \frac{1}{2\Delta t} (\Delta z_{i,j+1/2} + \Delta z_{i,j-1/2})(S \; \Delta x)_{i,j} \left[H_{i,j}^{t+\Delta t} - H_{i,j}^{t} \right] \pm Q \; . \qquad \text{(III-16)}$$

We then define

$$M = \frac{(Ss \; \Delta x)_{i,j} \; (\Delta z_{i,j+1/2} + \Delta z_{i,j-1/2})}{2\Delta t} \; , \qquad \text{(III-17)}$$

and the right hand side may be written as

$$RHS = M \, H_{i,j}^{t+1} - M \, H_{i,j}^{t} \; . \qquad \text{(III-18)}$$

The final form of equation (III-1) is then written as