CONSIDERING REPRESENTATIONAL CHOICES OF FOURTH GRADERS WHEN SOLVING DIVISION PROBLEMS

A Thesis

by

MARY CHILES GILBERT

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2006

Major Subject: Curriculum & Instruction

CONSIDERING REPRESENTATIONAL CHOICES OF FOURTH GRADERS WHEN SOLVING DIVISION PROBLEMS

A Thesis

by

MARY CHILES GILBERT

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Approved by:

Chair of Committee,	Robert M. Capraro
Committee Members,	Mary M. Capraro
	Victor L. Willson
Head of Department,	Dennie Smith

May 2006

Major Subject: Curriculum & Instruction

ABSTRACT

Considering Representational Choices of Fourth Graders

When Solving Division Problems. (May 2006)

Mary Chiles Gilbert, B.A., University of Houston – Downtown

Chair of Advisory Committee: Dr. Robert M. Capraro

Students need to build on their own understanding when problem solving. Mathematics reform is moving away from skill and drill types of activities and encouraging students to develop their own approaches to problem solving. The National Council of Teachers of Mathematics emphasizes the importance of representation by including it as a process standard in Principles and Standards for School Mathematics (2000) as a means for students to develop mathematically powerful conceptualization. Students use representation to make sense of and communicate mathematical concepts. This study considers the way fourth grade students view and solve division problems and whether problem type affected the choice of strategy. This study also looked at factors that affect students' score performance. Students in extant classrooms were observed in their regular mathematics instructional settings. Data were collected and quantified from pretests and posttests using questions formatted like students see on the state assessment. The results indicate that students moved from pre-algorithmic strategies to algorithmic strategies between pretest and posttest administration. The results also indicate that problem type did not predict students' choice of strategy and did not have an affect on the students' ability to arrive at a correct solution to the problem. This study found that the students' choice of strategy did play a significant role in their quest for correct solutions. The implication is that when students are able to make sense of the problem and choose an appropriate strategy, they are able to successfully solve division problems.

DEDICATION

To my E. L. E. L. F. T. A. daughter, Beth ’97.

ACKNOWLEDGEMENTS

As I look back at the time I have spent at Texas A&M University in preparation for this project, I am humbled by the experience. I think of the many hours of work and the people who have helped facilitate this dream.

To my committee chair, Dr. Robert M. Capraro, I praise you for your support and appreciate you taking me under your wing in this endeavor. You have contributed to my personal growth as a student and professional growth as a teacher and I am very grateful. Mary Margaret Capraro, as a committee member, I am thankful for your ideas. Your smile and cheerful manner were always more welcome than you knew. Dr. Victor L. Willson, as a committee member, I am grateful for your patience and encouragement as you lead me down that sometimes dim statistical pathway. I appreciate your insight and guidance on this project, as well as your time and energy. All of you, as extraordinary educators, have made this project worthy of the effort and one that I can feel proud of.

Thank you Denise Nichols for your vivacious attitude the first time I approached Texas A&M University when considering this degree. I'm not sure what I expected in that meeting, but your overwhelming enthusiasm certainly lit a fire that couldn't be put out. Dr. Gerald Kulm, thanks for providing the encouragement and confidence I needed to continue this project when I thought I had stepped beyond my limits.

To my principal, Maureen Saenz, thank you for the support and encouragement you have offered during this project. To Tracy Sackllah '85, my math teaching partner, thanks for your support of this project. It is wonderful to work with people who are truly concerned about the future generations.

I will cherish the friendships I have made during this project. I can only hope that the hope and reassurance I felt from you was reciprocated. A project of this magnitude could not happen without friends.

To my family, Mom, Dad, Emmett, and Georgia, thanks for always being there and listening patiently to jargon that meant nothing to you. Knowing that you all care and were there behind me is a truly awesome feeling. Your encouragement is deep felt and very humbling. Mom and Dad, thanks for believing in me. I am grateful for the undying confidence you have expressed throughout this project and in my life.

To my loving and totally devoted husband, Jan. You are truly the light of my life. You have not only provided a rock solid foundation for me to stand on, you have kept open arms for me to fall into when times were tough. You have shown your unconditional love and commitment to me by picking up the slack when I was unavailable or completely worn out. I am so blessed to have you in my life and I thank God for putting you there.

To my wonderful daughter, Beth: where would I be without you? It’s been said that children are a gift from God. You are living proof. Without a doubt I could not have attained this goal without you. You have provided both the gentle nudges and good swift kicks that I’ve needed to fulfill this dream. You are, without reservation, one of the finest young women that I know. Your faith strengthens my faith more that you probably know. Thanks for being more than a daughter…thanks for being a true friend.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER I

INTRODUCTION

There are many approaches to teaching mathematical content. The National Council of Teachers of Mathematics (NCTM) included representation as a process standard in *Principles and Standards for School Mathematics* (2000). In this instance students use internal and external representation to make sense of and communicate mathematical concepts. The use of multiple representations in the teaching and learning of mathematics is considered a basic need in understanding mathematics. The purpose of this study is to explore what children understand and do when solving division problems, such as those that appear on the state minimal skills examinations like the Texas Assessment of Knowledge and Skills (TAKS). This study seeks to determine if fourth grade students choose pre-algorithmic or algorithmic division strategies based on problem type, quotitive or partitive, and their level of success with their chosen strategy.

Rationale

The significance of this study is to emphasize of the need for teachers to offer multiple representations to students when teaching mathematics. Studies have shown that representation plays an important role in problem solving and mathematical reasoning. Internal representation, such as imaging, can be the difference between good and poor problem solvers. Wheatley (1997) "…suggests that encouraging imaging can

---

The format and style of this thesis follow that of the *Journal for Research in Mathematics Education*.

result in greater success in mathematics for students at all grade levels" (p. 295). Internal and external representations are important tools that students use to make sense of mathematics.

In their work, Davis and Maher (1997) show "how representations—sometimes mental, and sometimes on paper—make it possible to think about some idea which might otherwise be labeled as 'new'" (p.114). These authors also show how classroom and everyday experiences provide a foundation for the construction of representations that may be used in mathematical problem solving. Squire and Bryant (2002) suggest that students need to be able to distinguish between the division terms of dividend, divisor, and quotient and understand the role of each in division problems. The context of the division problem can be either partitive, where portions are placed into groups, or quotitive, where the number of groups is determined by the size of the portion. NCTM suggests that students develop computational fluency as they relate pre-algorithmic strategies to algorithmic strategies (NCTM, 2000). Prior to fourth grade, students are limited to pre-algorithmic strategies to solve division problems. Pre-algorithmic strategies include drawing pictures or tables, using multiplication as the inverse operation of division, possessing knowledge of basic fact families or repeated subtraction. With the introduction of the long division algorithm, fourth graders are broadening their knowledge base of strategies that can be used to solve division problems. How students view and understand problems can be evidenced in how they solve problems. Teachers need to analyze how their students solve problems in order to understand where they make mistakes. With this type of analysis, teachers will be better able to correct the errors and misconceptions of their students and enable the students to be successful in problem solving (Ashlock, 1994).

Statement of the Problem

Do fourth grade students prefer one strategy to others in solving division problems? Will each student use the same strategy, or will their choice of strategy change from problem to problem? In Texas, long division is introduced at the fourth grade level. Before fourth grade, students are required to have some basic division fact knowledge, but are not trained in the use of the traditional division algorithm. While students seem to understand that division is putting items into groups or finding out how many groups can be made, it has been my experience that when the problem involves numbers beyond basic facts, students struggle. As a teacher, I want to understand more about how my students learn and what makes strategies contribute to correct solutions. The National Council for Teacher of Mathematics (NCTM) suggests that multiple representations should be a requirement in the mathematics classroom (NCTM, 2000). Research has shown that low-achieving students have been successful with problem solving when given specific strategy instruction (Jitendra, 2002). The intent of this study is to look at how fourth grade students view division problems. Do they choose the same strategy for all division problems, or will they use different strategies based on how the division problem is formatted?

Research Questions

The following questions are addressed in this study:

1. When fourth graders are allowed to choose their own strategy when solving division problems, what type of strategy, pre-algorithmic or algorithmic, will they choose most often?
2. Do students choose the same strategy for a partitive problem as they choose for a quotitive problem?
3. Did score performance change by instruction (time), strategy, problem type or interactions between pretest and posttest strategy, or problem type and strategy type?

Personal Perspective

As an educator, I know that there is more than one way to solve many mathematics problems. When problem solving, I attempt to solve using an efficient strategy. For example, I know that when a problem involves equal groups, it is more efficient to either multiply or divide as opposed to using addition or subtraction. Knowing this, does it make sense to limit students to only one representation when perhaps another representation may be more efficient? Looking at the strategies fourth grade students use when solving division problems can give insight on how students view and understand division: from the quotient, the divisor, or as a multiplication problem.

Definitions of Terms

This section describes terms used in this study:

Alternate method of division is similar to the traditional long division algorithm. The students use basic knowledge of multiples of ten to use this algorithm (Wickett & Burns, 2003).

Computational errors happen when the student chooses the correct procedure or mathematical operation, but fails to arrive at the correct solution. Examples include when a student overlooks an entry, forgets to regroup, or multiplies incorrectly (Ashlock, 1994).

Computational fluency is “having and using efficient and accurate methods for computing” (NCTM, 2000, p. 32).

Conceptual error is when the student chooses an appropriate strategy but makes errors in the steps of the algorithm. These errors may be either in the way the problem is set up (placing digits in the wrong places) or by making a procedural error (Ashlock, 1994).

Conceptual understanding is when students are able to explain the strategies they use to accurately solve mathematics problems (Ashlock, 1994).

Partitive division question – when an equal number of participants share a dividend (Squire & Bryant, 2002).

Procedural errors happen when students make errors in going through the steps of the problem (Ashlock, 1994).

Quotitive division question – is where portion is determined by the number of recipients (Squire & Bryant, 2002).

Repeated subtraction algorithm is when the divisor is repeatedly subtracted from the dividend. The student then counts how many times the subtraction operation was used to determine the quotient.

Representations are what students use internally or externally to make sense of and communicate mathematical concepts.

Traditional division algorithm - focuses on the individual digits in the dividend and teaches the student to “divide, multiply, subtract, compare, and bring down.”

Limitations

1. The sample was limited to students from one elementary school in a suburban school district; this limits generalizability to fourth graders in public school in similar demographic settings. The participants in this study were students in two teachers’ extant classrooms and received mathematics instruction from them in regular mathematics classrooms. The ethnic background of students participating included only Asian, Caucasian, and Hispanic students. No African-American students chose to participate.
2. Even with students of similar ability, the background of students could vary. The role that parents play in learning outside of the regular classroom and what previous teachers introduced in their classrooms are two factors that should be considered.

CHAPTER II

REVIEW OF LITERATURE

The literature regarding the need for student knowledge of multiple representations is summarized below. The literature is presented in an attempt to relate curriculum mandates, as high as the national level, regarding not only what is taught, but how it is taught in individual classrooms. Many of the studies in the review address the use of representation in mathematics teaching and its effect on student understanding and success. This study will point out the difference in partitive and quotitive division problem types and the solution strategies that fourth grade students use to solve them. The solution strategies are categorized as pre-algorithmic, which includes basic fact knowledge, skip counting, and drawing pictures, and algorithmic, which includes the traditional division algorithm or an alternate division algorithm. A student's knowledge of various representations affords the opportunity to choose a representation when problem solving.

Student understanding and use of mathematics can be seen in the ways mathematical representation is utilized to solve problems. As students build their lexicon of mathematical representations, they extend their ability to think mathematically. With the inclusion of representation as a standard in NCTM's *Principles and Standards for School Mathematics* (NCTM, 2000), the importance of representation has been elevated. The use of representation in the teaching and learning of mathematics is considered to be a basic need in understanding mathematics. Instructional strategies have been researched in an effort to determine if a particular strategy is better than another, or if students in various demographic groups benefit from one strategy over another. Greeno and Hall (1997) suggest that representation, standard or nonstandard (drawings, non-algorithmic), should be used as tools for understanding and communicating information while conveying meaning. Teachers who offer a student more than one mode of learning are offering them more opportunities to make sense of problems.

Representation as a Part of Curriculum

National Position on Problem Solving

The United States Department of Education (US DOE) has charged the states "to develop or adopt content standards….that are rigorous and hold students to high expectations" (US DOE, n.d.). The responsibility for developing standards has fallen on the states for a long time. In light of this charge, it is reasonable that the states should develop their standards based on national standards. The National Research Council (NRC) (2001) recommends that the elementary ". . .curriculum should provide opportunities for students to develop a thorough understanding…[of] various representations" in order to "…involve connecting symbolic representations and operations with physical or pictorial representations, as well as translating between various symbolic representations" (NRC, p. 416). Students who relate pre-algorithmic strategies to algorithmic strategies are, in practice, developing a mathematical repertoire that will enable them to make strides toward computational fluency. According to NCTM (2000) "the term *representation* refers to both process and to product – in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself" (p. 67). School mathematics programs have always included various forms of representations, such as pictures, symbols, charts, graphs, and diagrams. NCTM advocates that "Instructional programs from pre-kindergarten through grade 12 should enable all students to create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena" (p. 67). *Principles and Standards for School Mathematics* (NCTM, 2000) affirms the necessity for students to understand how to perform computations in more than one way in order to achieve computational fluency. The joint position statement of NCTM and the National Association for the Education of Young Children (NAEYC) suggests that teachers of high-quality mathematics education for young children should use curriculum that strengthens the use of representation of mathematical ideas in order to acquire mathematical content knowledge. The report strongly suggests that, in order for educators to bolster interest and ability in mathematics, a variety of approaches should be used. (NAEYC/NCTM Position Statement, n. d.).

Texas Alignment to the National Position on Problem Solving

State guidelines are becoming more aligned with the national standards. With high stakes testing in many states, including Texas, teachers use their state's guidelines when planning and implementing curriculum and programs in their classrooms. The Texas Education Agency (TEA) prescribes what students should know by grade level and subject in the Texas Essential Knowledge and Skills (TEKS), which is the state mandated K-12 curriculum. For fourth graders, the State of Texas dictates “Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems” (TEA, 1998). Texas expects for educators to have fourth grade students match mathematical language and symbols that relate to informal language. The State of Texas has aligned the TEKS from one grade level to the next so that student understanding of concepts builds from year to year. In Texas, fourth grade students are required to use division to solve problems. Because the problems sometimes go beyond basic facts, it is at this point in their mathematical education that students are introduced to the traditional division algorithm.

A School District’s Implementation

With the mandate of high stakes testing and emphasis on student performance, school districts align their local curriculum and assessment to reflect that of the state. In Texas, the TEKS becomes the guide for district curriculum. Assessment at the local level is used to measure the success of students and accountability of the teachers. Teachers do have flexibility in how and what they teach, as long as they stay within the curriculum guidelines dictated by their school district.

Students come to fourth grade with some knowledge of division facts. They understand that division is the practice of putting objects into groups or finding out how many groups can be made, and that it is the opposite process of multiplication. When a division problem involves putting things into groups, it is called a partitive problem. Conversely, when a division problem involves finding out how many groups can be formed, it is called a quotitive problem. By the time students complete fourth grade, they have had exposure to additional strategies for division, including repeated subtraction, the traditional division algorithm, and alternate division algorithms. In addition, some students may use a multiplicative inverse strategy to solve division problems.

Offering Representational Choices

Representation is considered a mechanism for thinking and communicating about problem solving: meaningful representation shows what a student is thinking. Fennell and Rowan (2001) state that "Teachers can use representation to clarify mathematical ideas to students, to access students' mathematical thinking, and to help students translate a mathematical idea into a form that they can mentally or physically manipulate to gain understanding" (p. 292). Teachers need to show students how mathematical representation can be used to help them further their understanding of mathematical processes. When solving division problems, students should understand that using either pre-algorithmic or algorithmic strategies are both appropriate and acceptable.

Another aspect of teaching representation is the level of student involvement. Simple regurgitation of abstract formulas and plugging in numbers is no longer protocol in mathematics classrooms. Reform education fosters environments where students construct their own knowledge and representations when solving mathematics problems. Students should be actively engaged in the mathematical learning process because they will retain more information this way. Students should also explore, hypothesize, and be able to prove their understanding of mathematical concepts. They should be encouraged to discuss and explain the problems they solve (Schoenfeld, 1987).

In non-reform classrooms students learn mathematics from books or by doing what the teacher tells them to do. In non-reform classrooms conceptual understanding is not necessary. Teachers are sometimes referred to as ‘the sage on the stage’. When the student is allowed to construct his or her own knowledge, the teacher takes on more of a ‘guide on the side’ role. In this constructivist type of teaching, the student creates his or her own representations of the problem based on what they know, and the teacher takes on more of a coaching role in the classroom (Davis & Maher, 1997).

According to Vergnaud (1998), the teacher’s role “…consists mainly in helping students develop their repertory of schemes and representations…” so that “…students become able to face more and more complex situations (usually tasks and problems)” (p. 180). This is consistent with NCTM’s (2000) statement that “Representations can help students organize their thinking” (p. 68) and “Students’ use of representations can help make mathematical ideas more concrete and available for reflection” (p. 68). Teachers who offer their students multiple representations enable the students to think broadly about what makes sense in the mathematics problems they are faced with.

In his work Abrams (2001) shows that students will use mathematical skills and processes in their real life when they recognize the need for those skills. He suggests that students need to be explicitly taught how to model, or represent, everyday phenomena. Abrams believes that “Representation is the first step in using mathematics to answer realistic questions….Teachers need to identify all the skills of representation and provide their classes with a variety of contexts in which to apply them creatively” (p. 282).

When students do not fully understand a problem, they may be unsuccessful in applying problem-solving strategies or computation algorithms (Buschman, 2002). To find out what students know, problem-solving experiences should be offered to students who are then asked to choose any method and use any manipulative to solve the problem. He found that when children make their own sense of problems “their natural problem-solving abilities emerge” (p. 103).

Anghileri (2001) purports that external representations “…reveal the way children are thinking about the problems” (p. 22). These representations show students’ progress from their own naïve strategies toward using efficient solution methods. Teachers can gain access to where students’ misconceptions may be starting by analyzing the representational choices made. By doing so, the teacher is more able to guide the student to correct methods and solutions.

The type of representations that students choose can also be a contributing factor to finding correct solutions. van Garderen and Montague (2003) found that the use of schematic representation was more effective in problem solving than pictorial representation. This piece of research also showed that students of lower ability often chose pictorial representation, which may have been a contributing factor to their more unsuccessful solution rates. Hegarty and Kozhevnikov (1999) concluded “instruction should encourage students to construct spatial representation of the relations between objects in a problem and discourage them from representing irrelevant pictorial details” (p. 688).

Studies have shown that students with low mathematical ability can be instructed in the use of strategies to assist them in solving word problems. Jitendra, et al. (1998) found that for students who struggle with the ability to represent word problems, instruction in the use of the schema strategy will allow students to put their ideas on paper. More importantly, the use of the schema strategy by low ability mathematics students gives them the reassurance that they can be successful in mathematics and serves as encouragement that they can be successful in mathematics.

Representation of the problem is where many difficulties begin in problem solving. Students use representation to reflect their understanding of a problem, and if they cannot represent their understanding, there will be obstacles in reaching a solution. Representation can be an issue in problem solving if the student limits their use of various strategies. Neuman and Schwarz (2000) found that when students relied on one type of representation (creation of a table from the word problem) they might be unsuccessful in their problem solving attempts. They suggested that several strategies and differing representations might allow students more success when solving problems.

How Students View Division

Children's initial knowledge about a concept and informal strategies may be groundwork for learning formal knowledge. A young child's knowledge and understanding of sharing is related to the concept of division, and teachers usually introduce division from the standpoint of sharing. However, even when children understand sharing, they exhibit difficulties when learning to divide in school. Squire and Bryant (2002) suggest that "…the ability to distinguish these three terms (dividend, divisor and quotient) from each other, and to recognize the role of each in the division problem, is an important starting point in understanding division” (p. 454). According to these researchers, the context of division problem can be either partitive, where “…a dividend is shared equally among a certain number of recipients and the size of the portion (quotient) depends on the number of recipients (divisor)”, or quotitive, where “…the dividend is divided into fixed portions (divisor) and the number or recipients (quotient) depends on the size of the portion” (p. 454). These authors argue that children need to “…be exposed to different problem representations and problem contexts in order to improve their ability to recognize the important variables in a problem, to develop their conceptual understanding of multiplicative relations….” (p. 464).

In a Cognitively Guided Instruction (CGI) model (Carpenter, Fennema, & Franke, 1996) the teacher focuses on student thinking: what students know and understand dictates the direction of instruction in the classroom. In a CGI classroom, students learn with an understanding that connects to their own prior knowledge. “The CGI framework provides a detailed analysis of how students use concrete materials to represent problems and the meanings they attribute to them” (p. 14). This study showed how a first grade student used manipulatives to solve division problems from the measurement (quotitive) and partitive perspectives without perceiving (or knowing) that the problems were division problems. The student was able to model the problems based on the problem description.

In an elementary classroom, Moyer (2000) used the children’s book *A Remainder of One* by Elinor Princzes (1995) to introduce children to the concept of division before actually teaching division. By allowing the students to manipulate objects, they could show their mathematical thinking in this case, partitive division, without ever expressing it as a written division problem. In order to effectively problem solve, students must be able to determine which information in a problem is needed and then represent that information to create number sentences. Again, these students are gaining background knowledge about division without ever putting a pencil to paper.

Further, Li and Silver (2000) studied third grade students' ability to solve division with remainder problems without knowledge of the formal division algorithm and found that they were successful. These students were able to use non-division solution strategies (i.e., multiplication or repeated addition or subtraction) to solve the problem based on the context of the problem. The non-division solution strategies included repeated addition and subtraction which are simply "more mathematically primitive than the long division algorithm" (p. 235). Because the students were able to make sense of the problems, they were successful in solving the problems. Results from this study also imply that student knowledge of alternative solution strategies can be beneficial in problem solving situations.

Mauro, LeFevre, and Morris (2003) explored the idea that the concept of division is often represented by multiplication. With representation being a foundation for mathematical understanding, it is important to know that operations may sometimes be processed through their inverse. This finding is supportive of the need for students to have multiple solution approaches (representations) when problem solving.

Is the traditional division algorithm obsolete? Addington poses this question in NCTM's *Mathematics Education Dialogues* (1998). In her response, she points out that "…long division gives a construction (in the mathematical, not the educational, sense) way of obtaining a quotient of integers" (Addington & Willoughby, 1998, p. 16). She concludes that the mechanical teaching of long division is obsolete, but the conceptual understanding that students have when they do a long division problem is not. She is a proponent of calculators, only because of their efficiency. Willoughby agrees with that position and further states "They [students] should also understand division. Whether they become proficient at long division is probably of very little consequence as long as they don't spend too much time learning it" (p. 16). He believes "…that mathematics is something to understand and think about rather than something to be memorized and regurgitated" (p. 18).

Summary

National, state, and local governing agencies influence what is being taught in classrooms. There is considerable interest in students' ability to solve and view problems in a variety of ways. Anghileri (2001) discusses the need for teachers to allow students to build on their own understanding when problem solving. Mathematics reform moves teachers away from skill and drill types of activities and toward a focus on allowing students to develop their own approaches to problem solving. Current reform in mathematics instruction recognizes the need for students to have a conceptual understanding of mathematics so that they can make sense of the problem and be successful in problem resolution. Students will struggle when their own understanding of a problem conflicts with the algorithm. The way a problem is worded may interfere with the student’s understanding of the problem. Without understanding, students are unable to reconstruct the steps to the algorithm. To this end, research is needed to support the idea that a student’s knowledge of multiple representations of a problem facilitates their ability to problem solve.

CHAPTER III

METHODOLOGY

This chapter outlines the research design for this study. The design includes quantitative data collected from the pretest and posttest and includes an analysis of the strategies used by fourth graders in solving division problems. This study seeks to answer the following research questions: When fourth graders are allowed to choose their own strategy for solving division problems, what type of strategy, pre-algorithmic or algorithmic, will they choose most often? Do students choose the same strategy for a partitive problem as they choose for a quotitive problem? Did score performance change by instruction (time), strategy, problem type or interactions between pretest and posttest strategy, or problem type and strategy type?

Additionally, this chapter seeks to communicate the method of data collection, information regarding the participants, procedures used to collect the data, and the instrument used to analyze data.

Participants

All fourth grade students enrolled in a neighborhood school located in a middle-class suburban subdivision of Houston were invited to participate in the study. The participants were comprised of students from two mathematics teachers' extant classrooms and were students of the primary researcher and a collaborative partner. All students who participated in the study signed a student assent form and their parents or guardians signed a consent form. Students who receive mathematics instruction in a resource setting were not included in this study. According to the teachers, formal procedures for computing long division had not yet been taught to these students prior to participating in the study.

A total of 72 students (26 male, 46 female) were invited to participate. Thirty-two students (11 male, 21 female) returned signed parent/guardian consent and student assent forms to participate in the pretest and posttest. Twenty-nine students (10 male, 19 female) returned signed parent/guardian consent and student assent forms allowing background information. Of the consent/assent forms returned, one student received instruction in a resource setting and that data was not included in the study. In addition, there were two students who enrolled in fourth grade at this school after the pretest was given. Their posttest scores were not included in this study. The age of the students participating in this study ranges from 9 years 1 month to 10 years 3 months (mean 9 years 9 months). The ethnic background of fourth grade students in this school is 5.6% African American, 11.1% Hispanic, 31.9% Asian, and 51.4% Caucasian. The ethnic background of the fourth grade students participating in this study and who gave permission for background information is 0.0% African American, 6.9% Hispanic, 27.6% Asian, and 65.5% Caucasian. The ethnic background of fourth grade students participating in this study and who gave permission to use pretest and posttest information is 0.0% African American, 6.9% Hispanic, 27.6% Asian, and 65.5% Caucasian.

Procedure

All students in the study received the same instruction. Each student was given a pretest and posttest that consisted of twenty division questions which were formatted like those on the previous state assessments. The pretest was given to students before division instruction began and the posttest was given at the end of division instruction; there was a seven week period between pretest and posttest administrations. All students received ninety minutes of mathematics instruction every day. The lessons described below were taught to all students.

Description of the Course

The purpose of each class is to facilitate learning and conceptual understanding of mathematics concepts. Each class received ninety minutes of mathematics instruction every school day. The daily routine of each class included daily warm-up activities that included problem solving, and then a review of the previous night's homework assignment. The class was then given whole group instruction of the day's lesson including guided practice. While some students worked on independent practice or math centers, some students were pulled by the teacher into small groups for confirmation of their understanding of the task at hand. Students occasionally worked in small groups to collaboratively solve problems. Some concepts or activities included the use of manipulatives to facilitate student understanding. It was a goal of the class that all students actively participate in their own learning.

Traditional Division Algorithm Including a Mnemonic

The traditional division algorithm students are taught requires them to divide the divisor into the dividend without reference to the actual place value of the digits. A mnemonic is used to assist the students in remembering the steps: Divide, Multiply, Subtract, Compare, Bring Down. (See Appendix A.)

Alternate Method of Division Algorithm

The alternate method of division is a combination of the repeated subtraction algorithm and the traditional division algorithm. The students use basic knowledge of multiples of ten. The students multiply the divisor by multiples of ten and subtract from the dividend. The student then adds up how many times they multiplied the divisor to find the quotient. The students will be given a separate lesson to reinforce the concept of multiples of ten before this lesson is taught. (See Appendix A.)

Analysis of the Data

This study was a quantitative analysis of student use of strategies based on differing methods of instruction. To provide information to answer each research question, quantitative data was taken from the pretest and posttest. The test was designed to enable the researcher to look at what strategies are used before and after strategy instruction for different question types (quotitive and partitive).

To answer question one, a frequency distribution was prepared to see how the strategies were distributed on the pretest and posttest. Data was gathered using a Data Gathering Tool, which included the student code, problem number, problem type, strategy choice for each pretest and posttest item, and a score code for each pretest and posttest item. (See Appendix B.)

To answer question two, the same Data Gathering Tool was used. Items included in the test instrument were paired so that for a given partitive item, there was a similar quotitive item. For each item, the student could make a choice of strategy. The strategy choice for quotitive problems was associated with the strategy choice for the partitive problems using a chi-square analysis across the paired items.

To answer question three, student change in score between the pretest and posttest was computed using the formula Posttest – Pretest = Change. Change in score was used as the dependent variable. An ANOVA was run to determine how much problem type, problem number (difficulty of question), pretest strategy, and posttest strategy contributed to student change in score. In addition, an analysis was performed on the interaction of problem type and pretest strategy, problem type and posttest strategy, and pretest and posttest strategy.

Instruments/Test Development

In order to answer the research question, quantitative data and error analysis was examined using the pretest and posttest (Appendix C). The test was designed to follow the format of division questions that fourth grade students see on the state assessment such as the Texas Assessment of Knowledge and Skills (TAKS). Division items from the 2003 and 2004 TAKS test are shown in Appendix D. Division items from the 2005 TAKS test will not be released from the State of Texas, so therefore these are not included in this study. Differing values were used for divisor and dividend. Half of the questions were formatted from the quotitive perspective, and the other half were formatted from the partitive perspective. Samples of quotitive and partitive questions, including test format, can be found in Appendix E. To control for any knowledge of strategies for division, a pretest was administered to students before division instruction in algorithmic strategies began. The posttest was given at the end of division instruction.

There was a 7 week period between the pretest and posttest. Correctness of selection of strategy was correlated with problem score (0-4) using a dependent t-test for the difference in score across items.

The students were instructed to complete all questions, show their work, and give a brief explanation of why they chose a particular strategy. The purpose for the explanation is to understand if students used different strategies for different types of test questions (quotitive or partitive), and to determine if students were successful across problem types with the chosen strategy(ies).

Strategy Coding

A coding scheme was used to examine the strategy used. The solution strategies were defined as pre-algorithmic strategies and algorithmic strategies. Pre-algorithmic strategies include drawing pictures or tables to represent the problem, use of multiplication as the opposite process of division, and repeated subtraction. Pre-algorithmic strategies also included skip counting, using fact families or basic facts, and drawing a T-chart. A problem that did not have any strategy applied to it was coded as pre-algorithmic. Algorithmic strategies were defined as the traditional division algorithm and the alternative division algorithm. If the student wrote the division sign and then just wrote an answer without any work, the problem was scored as pre-algorithmic. This is an indicator that the student used knowledge of basic facts to solve the problem (Goldin & Kaput, 1996; Manon, Capraro, & Kulm, 2004).

Test Scoring

A scoring rubric was used to determine student success and conceptual understanding. The purpose of using a scoring rubric was to allow the researcher to more fully comprehend student understanding of the problem. The following scoring rubric was used for scoring student success:

Zero – no attempt made to solve the problem;

One – attempted to solve the problem, but student used a strategy that did not lead to a correct answer;

Two – attempted to solve the problem, appropriate strategy, but conceptual error made;

Three – attempted to solve the problem, appropriate strategy, but computational error lead to an incorrect solution; and

Four – attempted to solve the problem, appropriate strategy, correct solution.

A student received zero points if there was not an attempt made to solve the problem. A student who wrote notes such as “I don’t understand” or “Don’t know” fit this category.

A student received one point if he attempted to solve the problem but used a strategy that did not lead to a correct solution. Correct strategies include drawing a picture involving grouping, multiplying, repeated subtraction, or using a division algorithm. An example of a solution that received one point on problem number one is as follows:

1. Logan baked 96 brownies for a bake sale. If he puts 4 brownies in a bag, how many bags of brownies will he have?

Solution:
$$\begin{array}{r} 96 \\ \underline{-\ \ 4} \\ 92 \end{array}$$

This student chose the operation of subtraction to solve the problem. However, a correct strategy would be repeated subtraction where the student could then count how many times the subtraction operation was utilized to arrive at the solution. For this student, on this problem, a score of one point was given.

A student who attempted to solve the problem, used an appropriate strategy, but made a conceptual error received two points. The following example from problem fifteen received two points:

15. Mrs. Raska divided 42 flowers equally in 3 vases. How many flowers were in each vase?

Solution:
$$\begin{array}{r} 31 \\ \underline{\div\ 42} \\ 2 \end{array}$$

This student knows to use division, but has no conceptual understanding of how to write the problem or where to place the numbers. For this student, on this problem a score of two points was given.

A score of three points was awarded to a student who attempted to solve the problem with an appropriate strategy, but a computational error lead to an incorrect solution. An example of a solution receiving three points on problem two is as follows:

> 2. Jennifer has 132 stickers. She decides to put even amounts on each page of her memory book. If she has 12 pages, how many stickers will be on each page of her memory book?
>
> Solution: $\begin{array}{r} 13 \\ 12\overline{)132} \end{array}$ 12, 24, 36, 48, 50, 62, 74, 86, 98, 100, 112, 124, 132

The strategy used, counting by 12s, was correct and should lead to a correct solution. However, the student did not regroup when adding twelve to forty-eight or when adding twelve to ninety-eight. In addition, the student forced the last sum of 132 to fit the problem. For this student, on this problem, a score of three points was given.

A student who arrived at a correct solution, regardless of strategy, was awarded four points. There were no errors in these solutions.

Inter-rater Reliability

For results to be reliable there must be a consistency in the coding: therefore inter-rater reliability studies were conducted (Huck, 2004; Mertens, 2005). Another person knowledgeable about the research topic was trained using the work of two students. To measure the inter-rater reliability of the solution coding and the strategy coding, a random sample of 10 students was selected and re-analyzed by the trained person. A 90 percent or higher level of agreement was used as the acceptable boundary for inter-rater agreement. A 96.75% agreement between raters was calculated using the formula (agreements ÷ (agreement + disagreements) x 100). To measure the agreement of scoring between the raters, a comparison of scoring the times was calculated using Cohen's kappa. At .841, the value of kappa is statistically significant suggesting that the raters' scores are largely similar, with few exceptions. This test was needed to demonstrate that score scaling of 0-4 was consistent with an independent expert.

CHAPTER IV

RESULTS

This study was designed to investigate what strategies fourth grade students use when solving division problems, and specifically if their strategy choice is associated with the type of problem, partitive or quotitive. This study also sought to determine other factors that impact student success when solving division problems. Is there a significant difference in the way students use strategies to solve problems depending on the context of the problem? What factors contribute to student change in score when solving division problems? This chapter presents analyses of the data as it pertains to the research questions in Chapter 1.

The pretest/posttest design of the study provided a way to quantify differences between subjects. The test consisted of twenty problems. Half of the problems were formatted as quotitive problems and half were formatted as partitive problems. Each problem was coded for students' use of pre-algorithmic or algorithmic strategies. In addition, each problem was scored with a possible range of zero to four points. This design allowed for the comparison of strategy choice over time, as well as student change in score over time. In addition, other effects that impact student change in score were observed. The statistical software used to analyze data was the SPSS, version 13.0. An alpha level of a .05 level of significance was used for all statistical tests.

Student Selected Strategies Used to Solve Division Problems

Research question 1 raised the issue of students' choice of strategy when solving division problems. In this study when the fourth graders were allowed to choose their own strategy for solving division problems on the pretest, they most often chose pre-algorithmic strategies. On the posttest, they most often chose an algorithmic strategy to solve the problems.

To see how strategies were distributed in the pretest, a frequency distribution was prepared. Table 1 shows the distribution of pre-algorithmic and algorithmic strategies chosen on the pretest. On the pretest students chose a pre-algorithmic strategy of 587 out of 640 times, or 91.7% of the time. An algorithmic strategy was chosen 53 times or (8.3% of the time) on the pretest.

Table 1
Strategies selected on pretest.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Pre-algorithmic	587	91.7	91.7	91.7
	Algorithmic	53	8.3	6.3	100.0
	Total	640	100.0	100.0	

Strategies were categorized as either pre-algorithmic or algorithmic. To better interpret the table, a pie chart was constructed to facilitate comparison to posttest strategy choices. Figure 1 illustrates the frequency distribution of pre-algorithmic and algorithmic strategies chosen on the pretest.

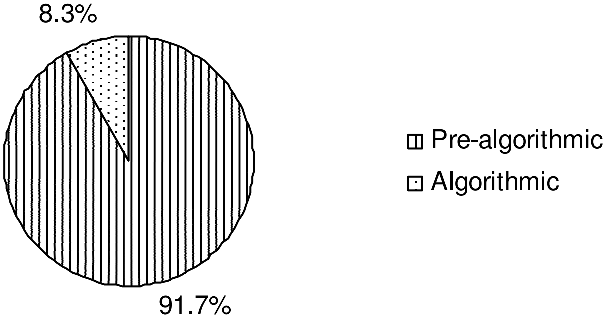

Figure 1. **Strategies selected on the pretest.**

On the posttest, students chose a pre-algorithmic strategy of 157 out of 640 times or 24.5% of the time. An algorithmic strategy was chosen 483 times or (75.5% of the time) on the posttest. Table 2 shows the distribution of the pre-algorithmic and algorithmic strategies chosen on the posttest.

Table 2
Strategies selected on posttest.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Pre-algorithmic	157	24.5	24.5	24.5
	Algorithmic	483	75.5	75.5	100.0
	Total	640	100.0	100.0	

On the posttest students had a choice of pre-algorithmic strategies or algorithmic strategies. Figure 2 illustrates the frequency distribution of pre-algorithmic and algorithmic strategies chosen on the posttest.

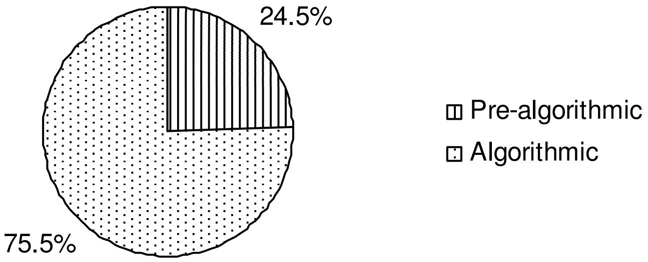

Figure 2. **Strategies selected on the posttest.**

Association of Strategy Choice to Problem Type

Research question 2 requires the exploration of students' choice of solution strategy based on problem type, partitive or quotitive. In this study students' strategy choice was not dependent on problem type. For partitive questions on the pretest, pre-algorithmic strategies were chosen 90.9% of the time and algorithmic strategies were chosen 9.1% of the time. For quotitive questions on the pretest, pre-algorithmic strategies were chosen 92.5% of the time and algorithmic strategies were chosen 7.5% of the time. Table 3 presents the cross-tabulation of pretest strategies with problem type.

Table 3

Cross-tabulation of pretest strategies with problem type.

			Strat_Pre		
			pre-algorithmic	algorithmic	Total
Prob_Type	partitive	Count	291	29	320
		Expected Count	293.5	26.5	320.0
		% within Prob_Type	90.9%	9.1%	100.0%
		% within Strat_Pre	49.6%	54.7%	50.0%
		% of Total	45.5%	4.5%	50.0%
	quotitive	Count	296	24	320
		Expected Count	293.5	26.5	320.0
		% within Prob_Type	92.5%	7.5%	100.0%
		% within Strat_Pre	50.4%	45.3%	50.0%
		% of Total	46.3%	3.8%	50.0%
Total		Count	587	53	640
		Expected Count	587.0	53.0	640.0
		% within Prob_Type	91.7%	8.3%	100.0%
		% within Strat_Pre	100.0%	100.0%	100.0%
		% of Total	91.7%	8.3%	100.0%

The pretest strategy choice for quotitive problems was associated with the pretest strategy choice for partitive problems using a chi-square analysis across the paired items. The chi-square test was assumed to be significant at the .05 level or less. The chi-square test revealed that problem type did not predict pretest strategy, χ^2 (1, N = 640) = .514, *p* = .567. The corresponding chi-square table is presented in Table 4.

Table 4

Chi-square test associating strategy code with problem type on pretest.

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.514(b)	1	.473		
Continuity Correction(a)	.329	1	.566		
Likelihood Ratio	.515	1	.473		
Fisher's Exact Test				.567	.283
Linear-by-Linear Association	.513	1	.474		
N of Valid Cases	640				

a Computed only for a 2x2 table

b 0 cells (.0%) have expected count less than 5. The minimum expected count is 26.50.

For partitive questions on the posttest, pre-algorithmic strategies were chosen 25.9% of the time and algorithmic strategies were chosen 74.1% of the time. For quotitive questions on the posttest, pre-algorithmic strategies were chosen 23.1% of the time and algorithmic strategies were chosen 76.9% of the time. Table 5 presents the cross-tabulation of posttest strategies with problem type.

Table 5

Cross-tabulation of posttest strategies with problem type.

			Strat_Post		Total
			pre-algorithmic	algorithmic	
Prob_Type	partitive	Count	83	237	320
		Expected Count	78.5	241.5	320.0
		% within Prob_Type	25.9%	74.1%	100.0%
		% within Strat_Post	52.9%	49.1%	50.0%
		% of Total	13.0%	37.0%	50.0%
	quotitive	Count	74	246	320
		Expected Count	78.5	241.5	320.0
		% within Prob_Type	23.1%	76.9%	100.0%
		% within Strat_Post	47.1%	50.9%	50.0%
		% of Total	11.6%	38.4%	50.0%
Total		Count	157	483	640
		Expected Count	157.0	483.0	640.0
		% within Prob_Type	24.5%	75.5%	100.0%
		% within Strat_Post	100.0%	100.0%	100.0%
		% of Total	24.5%	75.5%	100.0%

The posttest strategy choice for quotitive problems was associated with the posttest strategy choice for partitive problems using a chi-square analysis across the paired items. The chi-square test was assumed to be significant at the .05 level or less. The chi-square test revealed that problem type did not predict posttest strategy, χ^2 (1, N = 640) = .684, *p* = .462. The corresponding chi-square table is presented in Table 6.

Table 6

Chi-square test associating strategy code with problem type on posttest.

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	.684(b)	1	.408		
Continuity Correction(a)	.540	1	.462		
Likelihood Ratio	.684	1	.408		
Fisher's Exact Test				.462	.231
Linear-by-Linear Association	.683	1	.409		
N of Valid Cases	640				

a Computed only for a 2x2 table

b 0 cells (.0%) have expected count less than 5. The minimum expected count is 78.50.

Factors Contributing to Student Score Performance

Research question 3 sought to determine score performance change based on instruction (time), strategy, problem type, or interactions between pretest and posttest strategies or problem type and strategy type. In this study score performance was not changed by problem type, but it was impacted by the difficulty of problem and the choice of strategy. Using change in score as the dependent variable, an analysis of variance (ANOVA) was run to determine how much fixed factors of problem type, problem number (difficulty of question), pretest strategy and posttest strategy contributed to student gain. The ANOVA also tested the interaction of problem type with strategies chosen on the pretest and the interaction of problem type with strategies chosen on the posttest.

The effect of problem type on student change in score was not statistically significant, $F(1,585) = .390$, $p = .532$. This indicated that problem type did not have an effect on the students' ability to solve the problem. The effect of difficulty of the problem on student change in score was statistically significant, $F(18,585) = 6.312$, $p < .001$. This indicated that problem number (difficulty of the problem) did have an effect on students' ability to solve the problem.

The effect of the strategy chosen on the pretest on student change in score was statistically significant, $F(1,585) = 8.689$, $p = .003$. Additionally, the effect of the strategy chosen on the posttest on student change in score was found to be statistically significant, $F(1,585) = 11.823$, $p = .001$. This means that students' choice of strategy contributed to their ability to solve the problem. A second analysis investigating the interaction of strategies chosen on the pretest and strategies chosen on the posttest was not statistically significant, $F(1,585) = 1.059$, $p = .304$.

The interaction of problem type and strategies chosen on the pretest was not statistically significant, $F(1,585) = .322$, $p = .646$. The interaction of problem type by strategies chosen on the posttest was not statistically significant, $F(1,585) = 1.059$, $p = .164$. The ANOVA table for the data analysis of contributions to student change from pretest to posttest score is presented in Table 7.

Table 7
ANOVA table showing effects in change in student pre- and posttest scores.

Source	Type I Sum of Squares	df	Mean Square	F	Sig.	Effect Size	Observed Power
PT[1]	.564	1	.564	.390	.532	.003	.096
Problem Difficulty	164.203	18	9.122	6.312	<.001	.825	.999
Pretest Strategy	12.558	1	12.558	8.689	.003	.063	.837
Posttest Strategy	17.088	1	17.088	11.823	.001	.086	.930
PT by Pretest Strategy	.306	1	.306	.212	.646	.002	.075
PT by Posttest Strategy	2.809	1	2.809	1.944	.164	.014	.285
Pretest by Posttest Strategy	1.531	1	1.531	1.059	.304	.008	.177

Note: [1]PT = Problem Type

Table 8 presents the means and standard deviations of item test scores on the pretest and posttest by problem type. The pretest scores for partitive and quotitive question types showed very little difference. While scores did improve for both problem types on the posttest, the gain was negligible. There is a measurable gain for both partitive and quotitive question types from pretest to posttest. The difference in standard deviation for mean item scores on the pretest and posttest indicates a more narrow distribution of item scores on the posttest.

Table 8
Means and standard deviations of pre- and posttest scores by problem type.

	Quotitive		Partitive	
	Pretest	Posttest	Pretest	Posttest
Item score	3.15	3.92	3.11	3.82
	(1.460)	(.422)	(1.399)	(.562)

The parameter estimates indicate that change in student score is most affected when students used a pre-algorithmic strategy on the pretest. The effect size is greatest for students who chose a pre-algorithmic strategy on the pretest. Table 9 shows the parameter estimates of the effects of student use of strategies on change in score.

Table 9

Parameter estimates for student change in score across problem types.

Parameter	B	Std. Err	t	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power(a)
Pretest Strategy = pre-algorithmic	.921	.344	2.674	.008	.012	2.674	.761
Pretest Strategy = algorithmic	0(b)	.	.	.	.	.	.
Posttest Strategy = pre-algorithmic	-.325	.206	-1.575	.116	.004	1.575	.349
Posttest strategy = algorithmic	0(b)	.	.	.	.	.	.

b This parameter is set to zero because it is redundant

CHAPTER V

CONCLUSIONS

Summary

This study sought to determine if fourth grade students choose pre-algorithmic or algorithmic strategies when they solve division problems and whether problem type, partitive or quotitive, affected the choice of strategy. This study also looked at factors that affect the score performance of fourth grade students' division problems. As teachers consider the types of strategies fourth graders use when solving division problems, they can develop an understanding of the students' conceptual understanding and adjust the way they teach. Students were observed in their regular mathematics instructional setting. The following questions were asked in this study:

1. When fourth graders are allowed to choose their own strategy when solving division problems, what type of strategy, pre-algorithmic or algorithmic, will they choose most often?
2. Do students choose the same strategy for a partitive problem as they choose for a quotitive problem?
3. Did score performance change by instruction (time), strategy, problem type or interactions between pretest and posttest strategy, and problem type and strategy type?

Quantitative methods were chosen to answer all research questions and data for students was collected using a pretest and posttest. The design of the test followed the format of division questions that fourth grade students see on the state assessment or

TAKS. The items were paired so that for a given partitive problem type there was a similar quotitive problem type.

Student Selected Strategies Used to Solve Division Problems

When fourth graders are allowed to choose their own strategy when solving division problems, what type of strategy, pre-algorithmic or algorithmic, will they choose most often?

The answer to question 1 was needed to see where the students started in their knowledge of division strategies and to find out where they progressed to. Students were asked to use any strategy they were comfortable with when solving the division problems. On the pretest, students chose a pre-algorithmic strategy 90.9% of the time. Before fourth grade, students use a variety of pre-algorithmic strategies to solve division problems, including skip counting, repeated subtraction, and drawing pictures or charts. Students coming to fourth grade also know that division and multiplication are opposite operations and they will employ their knowledge of basic facts and fact families to solve division problems. With this in mind, it was not surprising that the students chose a pre-algorithmic strategy most of the time when solving division problems on the pretest.

Research has shown that children attempt to make sense of mathematics and that, over time, build up their ideas about mathematics (Mayer & Martino, 1996). Buschman (2002) observed that students moved through stages as they developed into problem solvers. In this study, the students' movement from pre-algorithmic to algorithmic strategies supports these earlier findings. On the posttest, students chose an algorithmic strategy 74.1% of the time. This was no surprise, as formal long division strategies are introduced to students during fourth grade. The algorithmic strategies presented to this group of fourth graders were the traditional division algorithm and an alternative division algorithm. The traditional division algorithm focuses on the individual digits in the dividend, and the students are instructed to use a mnemonic to recall the process of "divide, multiply, subtract, compare, bring down." The alternate division algorithm is similar to the traditional division algorithm in process, but students use their knowledge of multiples of tens when solving.

Understanding all the components of the division process is necessary if students are going to be successful in doing long division. The National Research Council (2001) recommends that students should be able to translate between various symbolic representations through opportunities presented in the curriculum. NCTM (2000) agrees that students' move toward computational fluency is necessitated by their understanding of how to perform calculations in more than one way. As students develop their repertoire of schemes and representations and make sense of the problem at hand, the teacher is able to take on the role of a facilitator.

The TEKS in Texas are aligned from grade level to grade level so that student understanding of concepts builds from year to year. Looking at the strategies that students use before and after division instruction enables teachers to understand how students have progressed in their understanding and use of more efficient solution methods. In this study, many students chose to move from pre-algorithmic strategies to algorithmic strategies. This supports Anghileri's (2001) notion that students' efficiency gains are built on understanding that is introduced progressively.

Association of Strategy Choice to Problem Type

Do students choose the same strategy for a partitive problem as they choose for a quotitive problem? This research question was investigated to determine if students use different strategies to solve problems based on the format, partitive or quotitive, of the problem. Previous studies addressing this question have found that students' choice of strategy may be influenced by the partitive or quotitive problem type (Anghileri, 2001; Carpenter, Fennema, & Franke, 1996; Moyer, 2000; Squire & Bryant, 2002). In their Cognitively Guided Instruction model, Capenter, Fennema, and Franke (1996) suggest that teachers need to be able to assess where students' misconceptions are in order to make instruction decisions. They give examples that show students using different strategies to solve division problems based on the problem type, quotitive (referred to in their article as a measurement problem) or partitive. Li and Silver (2000) suggest that third grade students can be successful solving division problems when non-division strategies are applied. Neuman and Schwartz (2000), on the other hand, suggest that even when 9th grade students have a strategy for solving word problems, moving information to an algebraic expression could prove problematic if the student does not understand how to represent the problem. In their 2002 study of 5 to 9-year-old students, Squire and Bryant highlight the idea that how the problem is presented, partitive or quotitive, impacts problem difficulty, even though the mathematical structure is the same. Squire and Bryant's 2003 study presented on 5 to-8-year olds suggests that students "…have a misconception about the inverse relation between divisor and quotient" (p. 522).

This study found that partitive and quotitive problem types did not predict the strategies students chose to solve the problems on either the pretest or posttest. It may be that developmentally this group of students was able to recognize the nature of the problem to be division, and therefore were, able to apply their knowledge of constructing a division problem to solve the problem. Squire and Bryant (2003) suggest that the students' age positively impacts their performance. Vergnaud (1998) suggests that teachers should be mediators and help students develop their schemes so that the students can handle more complex problems or tasks. It might be that the group of students in this study have had a lot of exposure to both partitive and quotitive problem types and are therefore comfortable with either problem type.

Factors Contributing to Student Score Performance

Did score performance change by instruction (time), strategy, problem type or interactions between pretest and posttest strategy or problem type and strategy type? This question investigated what factors impacted student success. Carpenter, Fennema, and Franke (1996) suggest in their Cognitively Guided Instruction model that students do use different strategies to solve division problems based on problem type. Other research indicates that young learners that may not understand division are actually able to solve division problems based on the context of the problem (Li & Silver, 2000). Students' ability to problem solve is ultimately determined by their ability to make sense of the problem. Wheatley (1997) rationalizes that teachers need to move beyond the procedural component in mathematical problem solving and allow their students to make sense of problems with imaging. NCTM considers the use of multiple representations in the teaching and learning of mathematics to be a basic need in understanding mathematics. Greeno and Hall (1997) contend that non-standard use of representation should be allowed in classrooms if it promotes student understanding. The results of this study regarding the use of strategy based on problem type support these earlier studies. Surprisingly to this author, problem type did not have an effect on the students' ability to arrive at a correct solution to the problem. It was not surprising that the difficulty of the problem had an impact on the students' ability to arrive at a correct solution. It makes sense that harder problems, usually those involving larger numbers, would have an impact on student success.

What I have observed in classroom discussion of division problems is that students seem more eager to solve the partitive problem type, where objects are being placed into groups. Conversely, when they are asked to solve a quotitive type of problem there is hesitation and uncertainty. The scores by problem type for this group of students support what I have observed in the classroom. While overall there was not a significant difference in item score by problem type, the students did not score as well on the quotitive problems as they did on the partitive problems on either the pretest or posttest.

This study implies that if the student is able to make sense of the problem they will be able to solve it. The students' ability to solve division problems successfully on the pretest and the posttest was dependent on their competence to successfully choose an appropriate strategy.

Implications for Further Research

This study was intended to examine how fourth grade students view division problems and if their choice of solution strategy is dependent on the context of the problem. While for this group of students problem type did not impact their choice of strategy, it would be interesting to repeat the testing over a longer time period to see if and when the students who used the pre-algorithmic strategies move to the algorithmic strategies.

This study also sought to understand what types of interactions impacted student scores. In this age of high stakes testing, it is natural for teachers to want to understand what factors play a role in their students' ability to successfully solve problems. What this study implies is that it is the students' basic understanding of how to solve the problem that impacts their ability to solve the problems correctly. Do they know how to set the problem up and are they able to use a process that leads them to a correct solution?

It is important for teachers to understand that not all strategies are good for all students, and that when students are exposed to a variety of strategies they will choose a strategy that makes the most sense. Are we providing our students with examples from many contexts so that they can develop their understanding of all types of problems? As teachers, we would like to think that our students would take the most efficient route in problem solving. However, what we must understand about our students is that "most efficient" doesn't always mean "easiest to understand." Our students need be given latitude in the ways they choose to solve problems so that they can become successful in their efforts and confident in their ability.

REFERENCES

Abrams, J. P. (2001). Teaching mathematical modeling and the skills of representation. In A. A. Cuoco & F. R. Curcio (Eds.), *The roles of representation in mathematics: 2001 yearbook* (pp. 269-282). Reston, VA: National Council of Teachers of Mathematics.

Addington, S. & Willoughby, S. S. (1998). Response to is long division obsolete? *Mathematics Education Dialogues, 1,* 15-18.

Anghileri, J. (2001). A study of progression in written division calculation strategies for division. *Support for Learning, 16(1),* 17-22.

Ashlock, R. B. (1994). *Error patterns in computation* (6th ed.). New York: Macmillan.

Buschman, L. (2002). Becoming a problem solver. *Teaching Children Mathematics, 9,* 98-103.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996) Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary Science Journal, 97,* 3-20.

Davis, R. B., & Maher, C. A. (1997). How students think: The roles of representations. In L. E. English (Ed.). *Mathematical reasoning: Analogies, metaphors, and images* (pp. 93-115). Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Fennell, F., & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics, 7,* 288-292.

Goldin, G. A., & Kaput, J. J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 397-430). Mahwah, NJ: Erlbaum.

Greeno, J. G., & Hall, R. P. (1997). Practicing representation. *Phi Delta Kappan, 78,* 361-367.

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spatial representations and mathematical problem solving. *Journal for Educational Psychology, 91,* 684-689.

Huck, S. W. (2004). *Reading statistics and research* (4th ed.). Boston: Pearson.

Jitendra, A. (2002). Teaching student math problems solving through graphic representation. *Teaching Exceptional Children, 34(4)*, 34-38.

Jitendra, A. K., Griffin.C. C., McGoey, K., Gardill, M. C., Bhat, P., & Riley, T. (1998). Effects of mathematical word problem solving by students at risk or with mild disabilities. *Journal for Educational Research, 91,* 345-355.

Li, Y., & Silver, E.A. (2000). Can younger students succeed where older students fail? An examination of third graders' solutions of division-with-remainder (DWR) problem. *Journal of Mathematical Behavior, 19,* 233-246.

Maher, C. A., & Martino, A. M. (1996). Young children invent methods of proof: The gang of four. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 431-447). Mahwah, NJ: Erlbaum.

Manon, J., Capraro, R. M., & Kulm, G. (2004, April). *Underlying structures of mathematical representation: A theoretical perspective*. Paper presented at the American Educational Research Association Conference. San Diego, CA.

Mauro, D. G., LeFevre, J., & Morris, J. (2003). Effect of problem format on division and multiplication performance: Division facts are mediated via multiplication-based representations. *Journal of Experimental Psychology: Learning, Memory, & Cognition, 29,* 163-170.

Mertens, D. M. (2005). *Research and evaluation in education and psychology* (2nd ed.). Thousand Oaks, CA: Sage.

Moyer, P. S. (2000). A remainder of one: Exploring partitive division. *Teaching Children Mathematics, 6,* 517-521.

National Association for the Education of Young Children & National Council of Teachers of Mathematics' *Early childhood mathematics: Promoting good beginnings, a joint position statement*. (n.d.). Retrieved October 23, 2005, from http://www.naeyc.org/about/positions/pdf/Mathematics_Exec.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, D. C.: National Academy Press.

Neuman, Y., & Schwarz, B. (2000). Substituting one mystery for another: The role of self-explanations in solving algebra word problems. *Learning and Instruction, 10,* 203-220.

Princzes, E. (1995). *A remainder of one.* New York: Houghton Mifflin Company.

Schoenfeld, A. H. (1987). Cognitive science and mathematics education: An overview. In A. H. Schoenfeld (Ed.). *Cognitive science and mathematics education* (pp. 1-31). Hillsdale, NJ: Lawrence Erlbaum.

Squire, S., & Bryant, P. (2002) From sharing to dividing: Young children's understanding of division. *Developmental Science. 4,* 452-466.

Squire, S., & Bryant, P. (2003) Children's understanding and misunderstanding of the inverse relation in division. *British Journal of Developmental Psychology, 21,* 507-526.

Texas Education Agency. (1998). *Texas essential knowledge and skills*. Retrieved October 8, 2005, from http://www.tea.state.tx.us/rules/tac/chapter 111/ch111a.html

U. S. Department of Education. (n.d.). *Guidance on standards, assessments, and accountability: Archived information.* Retrieved October 22, 2005, from http://www.ed.gov/policy/elsec/guid/standardsassessment/guidance_pg3.html#content

van Garderen, D., & Montague, M. (2003). Visual-spatial representation, mathematical problem solving, and students of varying abilities. *Learning Disabilities Research & Practice, 18,* 246-254.

Vergnaud, G. (1998). A comprehensive theory of representation for mathematics education, *Journal of Mathematical Behavior, 17,* 167-181.

Wheatley, G. (1997). Reasoning with images in mathematical ability. In L. E. English (Ed.), *Mathematical reasoning: Analogies, metaphors, and images. Studies in mathematical thinking and learning* (pp. 281-297). Mahwah, NJ: Erlbaum.

Wickett, M., & Burns, M. (2003). *Lessons for extending division, grades 4/5.* Sausalito, CA: Math Solutions Publications.

APPENDIX A

ALGORITHMIC DIVISION STRATEGIES TAUGHT

Traditional Division Algorithm including a Mnemonic

$$\begin{array}{r} 025 \\ 5\overline{)125} \\ \underline{-\ \ 10} \\ 25 \\ \underline{-\ \ 25} \\ 00 \end{array}$$

Alternate Method of Division Algorithm

$$\begin{array}{rl} \left.\begin{array}{r} 1 \\ 2 \\ 10 \\ \underline{10} \end{array}\right\} & 23r7 \\ 16\overline{)375} & \\ \underline{-\ \ 160} & \\ 215 & \\ \underline{-\ \ 160} & \\ 55 & \\ \underline{-\ \ 32} & \\ 23 & \\ \underline{-\ \ 16} & \\ 7 & \end{array}$$

OR

$$\begin{array}{r} 16\overline{)375} \\ \underline{-\ \ 160} \\ 215 \\ \underline{-\ \ 160} \\ 55 \\ \underline{-\ \ 32} \\ 23 \\ \underline{-\ \ 16} \\ 7 \end{array} \quad \begin{array}{l} \\ \left.\begin{array}{r} 10 \\ \\ 10 \\ \\ 2 \\ \\ 1 \end{array}\right\} 23r7 \\ \\ \end{array}$$

APPENDIX B

DATA COLLECTION TOOL

Student Code:

		Strategy Code				Score Code		
Problem #	Problem Type	Pre	Post	Pre-Post		Pre	Post	Pre-Post
1	Q							
2	P							
3	P							
4	Q							
5	P							
6	Q							
7	Q							
8	P							
9	P							
10	Q							
11	P							
12	Q							
13	P							
14	Q							
15	P							
16	Q							
17	P							
18	Q							
19	P							
20	Q							

Strategy Code:
1 = Pre-algorithmic
2 = Algorithmic

Score Code:
0 = no attempt to solve
1 = attempt made, but used a strategy that does not lead to correct answer
2 = attempt made, appropriate strategy, but conceptual error made
3 = attempt made, correct strategy, computational error leads to incorrect solution
4 = attempt made, correct strategy, correct solution

APPENDIX C

PRETEST/POSTTEST QUESTIONS

1. Logan baked 96 brownies for a bake sale. If he puts 4 brownies in a bag, how many bags of brownies will he have? (quotitive)
2. Jennifer has 132 stickers. She decides to put even amounts on each page of her memory book. If she has 12 pages, how many stickers will be on each page of her memory book? (partitive)
3. Slade had 242 marbles. He decided to share them with six other friends. How many marbles will each person, including Slade, receive? (partitive)
4. Karalyn buys 84 cans of food for her cats each week. How many cans of food do her cats eat each day? (quotitive)
5. Sue made 36 cookies for her scout group. If there are 12 girls in the scout troop, how many cookies will each girl get? (partitive)
6. George had a large math homework assignment and wasn't sure if he could finish. It takes him 3 minutes to work each problem. How many problems can he do in 60 minutes? (quotitive)
7. Mike has a very nice garden in his back yard. He has 54 plants that he wants to plant on 6 rows. How many plants will Mike plant on each row? (quotitive)
8. Adam paid $125 for 5 rodeo tickets. How much did each ticket cost? (partitive)
9. Maggie and her sister went to visit their favorite aunt twice during summer vacation. They traveled a total of 408 miles when making the trips. How far did Maggie and her sister travel on one round trip to their favorite aunt's house? (partitive)
10. The fourth grade class was going on a field trip to a museum. A total of 72 students had permission to go and ride in 6 vans. How many students would need to ride in each van? (quotitive)
11. Mr. Floyd gave out 54 pencils to 9 students. How many pencils did each student receive? (partitive)
12. Georgia was a contestant in the school spelling bee. She needed to learn 120 words. If she can learn 12 words in a day, how many days must she study to learn all of the words? (quotitive)
13. At the county fair, 11 children won a total of 121 prizes. How many prizes did each student win? (partitive)
14. Laura was making friendship bracelets for her friends. Each bracelet needed 7 beads. If Laura had 56 beads, how many bracelets could she make? (quotitive)
15. Mrs. Raska divided 42 flowers equally in 3 vases. How many flowers were in each vase? (partitive)
16. The high school marching band has 156 members. If they march in rows of 6, how many rows will there be? (quotitive)
17. The librarian proudly announced that she had received 162 new books for the school library. She decided to display them on 9 tables for the students to see before they were placed on the library shelves. How many books were on each table? (partitive)
18. Mr. Hemme drove 330 miles to visit his sister in Corpus Christi. If he drove for 5 hours at the same speed, how fast was he traveling? (quotitive)
19. Blake has a collection of 234 marbles. He decided to put an equal number of marbles into each of 3 leather pouches. How many marbles will Blake put in each pouch? (partitive)
20. A restaurant has 96 lemons in their warehouse. It takes juice from 4 lemons to make a lemon pie. How many lemon pies can the restaurant make? (quotitive)

APPENDIX D

2003 AND 2004 SAMPLE TAKS DIVISION QUESTIONS

2003 SAMPLE QUESTIONS

8 There were 30 cookies on a platter for 9 children. If each child ate the same number of whole cookies, how many whole cookies did each child eat?

33 Alex bought lemons that were priced at 2 lemons for 18¢. What was the total cost of 5 lemons?

35 Danny's dog ate 56 cans of food in 4 weeks. If the dog eats the same amount each week, which number sentence can be used to find the number of cans of food the dog eats in one week?

2004 SAMPLE QUESTIONS

13 Every day Khari and her family have 2 newspapers delivered to their house. When they came back from a trip, there were 14 newspapers waiting for them. Which number sentence can be used to find the number of days they were gone?

18 When Maggie went to her sister's graduation, she saw that 300 students were graduating. Maggie noticed that equal numbers of graduating students were seated in 5 different sections of the auditorium. How many graduating students were seated in 1 section?

35 Luis has 4 paint sets. There are 12 jars of paint in each set. Which number sentence can be used to find the total number of jars of paint Luis has?

APPENDIX E

SAMPLE OF QUOTITIVE AND PARTITIVE QUESTIONS INCLUDING TEST FORMAT

1. Logan baked 96 brownies for a bake sale. If he puts 4 brownies in a bag, how many bags of brownies will he have?	
Solve the problem. Show all your work.	Explain why you used this strategy.

2. Jennifer has 132 stickers. She decides to put even amounts on each page of her memory book. If she has 12 pages, how many stickers will be on each page of her memory book?	
Solve the problem. Show all your work.	Explain why you used this strategy.

VITA

Mary Chiles Gilbert
1323 Vickery Drive
Sugar Land, TX 77478

Education

2006 Texas A&M University College Station, TX
M.S. Curriculum and Instruction, Mathematics Education

2001 University of Houston-Downtown Houston, TX
B.A. Interdisciplinary Studies, emphasis in Mathematics

Experience

August 2005 through present - Fort Bend Independent School District Sugar Land, TX
Classroom Teacher, Grade 4 - Math, Science, Social Studies

August 2001 through May 2005 - Stafford Municipal School District Stafford, TX
Classroom Teacher, Grade 4 and 5 - Math, Science, Language Arts

Summer 2000 and 2001 - Stafford Municipal School District Stafford, TX
Summer School Teacher, 2nd and 5th grade remedial reading

1998 – 2000 - Fort Bend I.S.D. / Stafford M.S.D.
Substitute Teacher, grades PK through 12

Presentation

Teacher representational choices effect student mathematical understandings. *The Annual Meeting of the Educational Research Exchange,* College Station, TX, January, 2005. (award for Excellence in Research from Texas A&M University)

Professional Organizations

National Council of Teachers of Mathematics
Kappa Delta Pi honor society
Phi Kappa Phi honor society
Chancellor's List 2006
Who's Who in American Teachers, 2005 and 2006