CAPITAL MOBILITY AND SUDDEN STOPS:
CONSEQUENCES AND POLICY OPTIONS

A Dissertation

by

CHRISTOPHER PATRICK BALL

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2003

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ABSTRACT

Capital Mobility and Sudden Stops: Consequences and Policy Options. (August 2003)
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This dissertation attempts in three essays to contribute to the growing body of research on the problems associated with sudden stops of capital inflows, known to have been at the heart of many recent emerging market crises. It does this by developing basic models that can incorporate sudden stops and hopefully make policy relevant recommendations.

The first essay develops a simple three date representative agent model of a small open endowment economy without money. It allows sudden stops to occur at date two and asks whether individuals in such a shock-prone world are still better off borrowing than in autarky. Unambiguously, this chapter shows that individuals are better off borrowing than in autarky and provides a tractable core model on which the later chapters build.

The second essay then includes a long-term borrowing option as well as country-specific risk premia based on an information asymmetry between domestic borrowers and international lenders. This allows analysis of optimal maturity choices in a meaningful way. The intent is to address questions in the literature concerning whether emerging economies could enhance welfare by imposing short-term capital controls to encourage the use of longer-maturing debt and thus avoid the sudden stop. The results imply that short-term capital controls would generally lower welfare, even when sudden stops are fully anticipated.

Finally, the third essay extends the horizon of the model and includes a much wider range of maturities. This allows one to start making sense of maturity bunching
(when a country’s debt all matures around a given date) which is known to exacerbate sudden-stop related problems. The model shows that maturity bunching can occur endogenously when both risk premia and uncertainty over the duration of the sudden stop are present.
This dissertation is dedicated to the people who truly believed in me over the years:

Emese, John and Sharon Ball, David Humphries and John Rush.
ACKNOWLEDGEMENTS

My intellectual debt is owed primarily to two people. First and foremost is Leonardo Auernheimer, my advisor and friend. I learned from him everything I know about macroeconomics and how to think about economic matters in general. Second is Curtis Taylor, my Industrial Organization professor and friend who first taught me what it means to model. To these two people I owe more than I can ever repay.

For wonderful discussions, much intellectual provocation and great companionship, special thanks to Raymond Battalio. To my close friend and colleague, who has been with me since the beginning, and without whom I would never have made it so far, many thanks to Javier Reyes. And last, but surely not least, thanks again to my wife Emese who was always there when I truly needed it most.
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CHAPTER I
INTRODUCTION

That emerging markets frequently lose access to international capital markets during times of crisis has by now been well documented empirically (Calvo and Reinhart, 2000). This phenomenon of a sudden loss of access to capital markets has been termed a “sudden stop” of capital inflows by Calvo (1998) and constitutes the core subject of study in this dissertation.

While the existence of sudden stops has been recently recognized empirically and their prevention has come to occupy the thoughts of policy makers (see Bordo 2003, IMF 1999, 2000, and 2001, Williamson 2002 and Wolf 2003 for a few examples), the theoretical underpinnings needed to better inform both policy making and empirical study are still in a state of flux. As noted by Arellano and Mendoza (2002), sudden stops are at serious at odds with the majority of existing models of current account determination which rely on perfect capital markets. This failure of existing theory has led to an active research program seeking to build models that can deliver predictions consistent with the sudden stop phenomenon. The first step has been to reconsider the conventional approach to model international capital markets as perfect mechanisms for consumption smoothing.

The aim of the work in this dissertation has been to reconsider the way in which we model small economies that smooth consumption through international capital markets in a world where sudden stops occur. Generally the cases in mind are of countries that suffer from a sudden stop due to no fault of their own. The focus is on three related questions. First is a very basic question: if sudden stops are so bad and are known to exist, why do countries continue to borrow? Answering this question while developing the basic analytical framework that will be used throughout the dissertation is the subject of Chapter II. The remaining two questions relate to the role of longer-

This dissertation follows the style and format of the American Economic Review.
term assets, a topic often touched on, but rarely elucidated in most theoretical models with sudden stops.

Second, Chapter III develops a three-date model of an economy where individuals have access to two assets, one short- and one long-term, and where sudden stops can occur. The main question here is how individuals allocate the maturity of their debt given the structure of the capital market and expectations over the sudden stop occurring. A subsidiary question is whether welfare can be increased in this environment by banning short-term borrowing, a policy frequently proposed for emerging markets.

Third, Chapter IV extends the horizon of the model and allows assets of longer maturities to exist. It then asks how individuals behave when the duration of the sudden stop is unknown. The chapter is intended as a first pass attempt to address the concerns that countries may have underestimated the duration of a recent sudden stop following Russia’s debt default in 1998. The observed anomaly that arises in this context is that countries often seem to bunch the maturity of their debt around dates that will occur during the anticipated sudden stop. The underlying assumption of most casual observers is that, if countries knew they would face a sudden stop, then they would either not borrow, or choose maturity structures that avoid the shock. Chapter IV shows that maturity bunching can arise endogenously when sudden stops are long-enough or their duration is unknown.

The remainder of this introduction is as follows. First there is a brief overview of what is known about sudden stops and the problems they pose in practice and theory. Second each of the dissertation’s main chapters and their contributions are discussed. Finally, the main results and conclusions of the dissertation as a whole are presented.

I.1 Sudden Stops: What We Know and Why They Matter

The traditional view of balance of payments crises is based on the theoretical model laid out by Paul Krugman (1979) and Flood and Garber (1984). The Krugman-
Flood-Garber (KFG) model emphasizes that countries with fixed exchange rates and domestic policies that are monetary financed, resulting in large current account deficits (CADs), will consistently lose their stock of international reserves as they buy their own currency on world markets to maintain the exchange rate. Since reserves are a fixed stock, once they depleted the exchange rate regime can no longer be maintained. Once this process of declining reserves is identified, the end of the regime, referred to as a balance of payments crisis, is a foregone conclusion and can be predicted, ceteris paribus, with complete accuracy.

This view of crises made life easy in the sense that countries simply needed to be wary of fixed exchange rates and large CADs. It was generally believed by policy makers and academic economists alike that as long as a country’s CAD was small or in surplus, they need not fear balance of payments crises (Calvo 1998 and 2003).

When Mexico ran into trouble in 1994-95, many observers immediately identified the standard elements of CADs and fixed exchange rates as the key problems. As Calvo (2003) points out,

> however, hardly any time elapsed between these conventional explanations’ reaching the printing press and the crash of Thailand, toppling in its wake mighty Korea and others in the region. This dealt a heavy blow to the conventional wisdom because, on the whole, these countries had exhibited a very solid fiscal position, sky-high saving rates, no major current account imbalance, and so on. Calvo (2003, p. 149).

While the general view was that the standard KFG-type mechanism was at play, the fact that most of the action in Mexico took place around their inability to refinance their debt led several researchers to begin questioning the conventional wisdom. In general this new skepticism focused on the role of the bond market (Flood and Marion 1998) and explored cases when a country’s borrowing constraints bind (Atkeson and Rios-Rull (1996) and Cole and Kehoe (1996), and Sachs, Tornell and Velasco (1996)), causing sudden debt refinancing problems.
The Asian Flu of 1997 then confirmed the suspicions that the conventional view was not accurate in explaining the emerging market crises occurring in the 1990s. In 1998 Calvo returned to basic macroeconomic identities and sketched out the theory of sudden stops, emphasizing the capital account as opposed to the current account as the KFG approach had done. His argument is essentially as follows. Suppose that for some given country there is a sudden lack of access to international capital markets which causes the capital inflow to slow. This necessitates a fall in the CAD. This implies an increase in the real exchange rate. To the degree that this is unexpected, hence the term sudden stop, loans to the nontradable sector extended under expectations that the previous real exchange rate would remain unchanged could become unperforming leading potentially to bankruptcies. Given that much of the action of the crises as well as subsequent discussion focused on short-term debt, Calvo (1998) made two clarifying claims about the relevance of debt maturity for his sudden stop theory that distinguish his view from that of most other authors: the first is that the theory of sudden stops (concerning the occurrence and impact of the stop) is independent of the maturity structure; and, second, that the residual debt maturity structure (i.e., the time profile of maturing debt) is relevant in assessing the largest possible short-run fall in capital inflows. Contrary to Calvo’s view, most authors (Cole and Kehoe (1996) or Cordella (1998), for example) felt sudden stops were partially caused by over-reliance on short term debt.

With the advent of the Asian Flu and the impact of Russia’s debt problems in 1998, research in the area of sudden stops began to grow in earnest. Since the sudden stop itself takes place in the international capital market, it makes sense to categorize the literature into two broad groups: the lender’s side and the borrower’s side.

The first group focuses mainly on the lender’s side of the market and addresses questions of why lender’s might simultaneously withdraw their supply of funds to a country or group of countries. In the theoretical line of this literature, Calvo and

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1 It should also be noted that the crisis in Argentina later proved Calvo’s claims to be very accurate.
Mendoza (1996 and 2000b) develop a model of rational herding and contagion based on the notion that lenders bunch countries together in order to save on information costs. The empirical side of the literature is vast and has mainly focused on contagion across countries. Two papers worth note however are Kaminsky and Reinhart (2000) and Rijckegehem, Van and Weber (2001). Both of these look specifically at sudden-stop-driven crises and take contagion arguments beyond trade linkages by attempting to determine the role played by financial sector linkages like international bank lending which is important for understanding sudden-stop related contagion. Rigobon (2001) provides a good overview of the contagion literature and its inherent problems.

The second group focuses on the borrower’s side of international capital markets. Here there is much less consensus. The empirical research is weak at best although Calvo, Izquierdo and Talvi (2002) contains some detailed empirical analysis for the case of Argentina and Calvo and Reinhart (2000) contains analysis on the overall bailout and GDP costs of many sudden stop crises. In part, this line of empirical research may still be fragmented and country-specific (see Edwards (1999) and (2000)) because the theoretical underpinnings of the phenomenon are still too under-developed to provide a good guide for researchers.

The need for better theoretical models that can make empirically relevant predictions consistent with sudden stops is indeed the call to arms discussed by Arellano and Mendoza (2002). To date, only two main authors have developed consistent and ever improving sudden stop models. The first is Calvo and the second is Mendoza.

Calvo’s view of sudden stops began to be formally presented in his (1998) paper on “Varieties of Capital-Market Crises”. Recognizing the shift in real world crises from the current to the capital account, and knowing the basic facts about sudden stop crises in general, this paper explores different types of capital-market crises and points out that all that is needed for a sudden-stop-type crisis is for the expected constant capital inflow to slowdown or cease to grow as rapidly as anticipated. In (1998) he published “Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops” which is discussed above. There is where he sketched out a new theory of sudden stops based on
macroeconomic identities. In a paper with Izquierdo and Talvi (2002) he applies his theory to the case of Argentina and puts forth the view that part of Argentina’s problem was that it underestimated the duration of the sudden stop resulting from Russia’s debt problems in 1998. The view that Argentina underestimated the sudden stop duration motivates the last chapter of this dissertation. Most recently, in a lecture at the IMF, Calvo (2002) presented the most recent version of a model of sudden stops. The model is one where growth is a negative function of the fiscal burden and where growth discontinuously switches from high to low as the fiscal burden reaches some critical level and is related to a sudden stop of capital inflows. The basic policy prescription is that policymakers should aim at improving institutions to avoid sudden stops, a theme consistent throughout his work.

Enrique Mendoza has developed models which interpret sudden stops in terms of his earlier work on small open economy real business cycles (Mendoza (1991)). In particular he views sudden stops as the result of excess volatility in a cycle-driven small open economy subject to borrowing constraints that only bind in certain regions of the state space. He develops this approach in two recent papers: Mendoza (2002) and Arellano and Mendoza (2002).

The remaining literature on sudden stops is varied and somewhat fragmented. A few of the papers are relevant, however, to the work presented in this dissertation and are therefore discussed in brief. Cole and Kehoe (1996) develop a model with borrowing constraints where the chance of a crisis depends on the level of short-term debt – particularly whether or not it is high enough to be in a pre-determined “crisis zone” – and the realization of a sunspot. Their chief policy prescription is that countries should move to longer-term horizons for their debt so that the debt level at any point in time is too low to be in the “crisis zone” and hence the economy is immune to realizations of the sunspot. The belief that switching to longer-term horizons can “save” a country from sudden stops is common, refuted repeatedly and from early on by Calvo, and is the subject of the third chapter of this dissertation.
Chang and Velasco (2000) also present a model that has come to have some repute in the sudden stop literature. Their work utilizes a Diamond-Dybvig banking structure domestically so that external sudden-stop type shocks can lead to domestic banking crises. This is an appealing feature of their model since financial fragility and banking crises are observed empirically in connection with the sudden stop phenomenon. One policy prescription that arises in this kind of environment is that capital controls might help avoid sudden stops by limiting the exposure of the domestic banking system to international shocks. This is proposed directly by Cordella (1998) using a similar Diamond-Dybvig model. Capital controls are examined directly in this dissertation in chapter three.

Of final interest has been the development of a line of research by Caballero and Krishnamurty (1998, 2000 and 2001). These authors focus on the asset-market aspect of emerging market crises and the role of domestic insurance arrangements to smooth out the impact of sudden stops. This represents one of the few domestic and non-interventionist views of a possible response to a sudden-stop-ridden world. This non-interventionist view is consistent with the general policy recommendations in this dissertation although their work bears little resemblance to mine otherwise.

I.2 Overview of Dissertation Chapters

The work in this dissertation touches on various strands of the literature briefly discussed above. My work intentionally utilizes a framework wholly consistent with Obstfeld and Rogoff’s (1999) approach to balance of payments. The extensions are that it allows for sudden stops and for multiple maturing assets in order to address issues of short- versus long-term debt and the duration of a sudden stop. In choosing a framework consistent with Obstfeld and Rogoff’s the hope is that building on my work will be easy for any economist trained in this now popular approach. Below is a brief overview of each of the chapters.
1.2.1 Chapter II

Chapter II lays out the time line and basic structure of a three-date model of an economy subject to sudden stops. The intention here is to set the stage for the rest of the dissertation and, in the context of this model, to determine whether or not countries are better off borrowing at all when sudden stops can occur. If they are not, since the focus will be on sudden-stop-ridden worlds, the optimal policy will always be to simply close off the economy and remain in autarky. If they are, then policy questions become less trivial. This question is of course not an original one. Two other recent pieces asking similar questions, from different perspectives are Stulz (2003), “Should We Fear Capital Flows?” which provides a cost-benefit analysis of capital flows and Dooley and Walsh (2003), “Capital Movements: Curse or Blessing?” which argues in favor of limited capital controls (a partial return to autarky in the context of my model).

The analytical structure here exploits a three-date representative-individual model of an endowment economy. The endowment is the same at each date so that, based on this, individuals have no desire to borrow at early dates against higher future incomes. There is one consumption good which is the numeraire for the economy. To generate a desire to borrow, individuals are assumed to be impatient relative to world capital markets. That is, their subjective rate of time preference exceeds the world real interest rate. In this world, individuals always prefer borrowing to living in autarky.

The sudden stop is viewed as a sudden increase in the base (i.e., non-risk-premium) interest rate. This amounts to a leftward shift of the supply schedule. To allow expectations to be formed over the sudden stop interest rate, the maximum interest rate charged to individuals will always be the one where the supply schedule touches the vertical axis so that at that rate the quantity demanded is zero.

Welfare under an unanticipated and fully anticipated sudden stop is calculated. It is then shown that as long as the rate of time preference exceeds the world real interest rate, the early gains from borrowing are always greater in welfare terms than the loss
suffered when the sudden stop strikes. Hence in this world it is better to borrow and lose than not to borrow at all.

I.2.2 Chapter III

Chapter III deals with the issue of short- versus long-term borrowing which continually surfaces in the discussion of sudden-stop-driven crises. As noted above, Calvo, in nearly all his work on the subject, has argued that it is not necessarily the maturity of the debt, but other factors that are important when a sudden stop strikes. The general consensus, however, seems to be that too much short-term debt is at the heart of the problem and hence encouraging longer-term maturity structures, perhaps through capital controls, would mitigate the damage resulting from sudden stops and thereby be welfare enhancing.

The problem is that most models are of economies with access to short-term borrowing exclusively\(^2\), making the study of the actual role played by longer-term borrowing very difficult. For example, Obstfeld and Rogoff’s *Foundations of International Macroeconomics* (1999), widely regarded as the standard international macroeconomics textbook today in graduate schools worldwide, does not have a single example or equation anywhere with anything other than short-term assets (i.e., assets borrowed today to be repaid tomorrow). Chapter III introduces longer-term borrowing into a standard intertemporal approach to balance of payments model under varying informational assumptions in order to better address the issues in the short-versus-long-term debate and subsequently the debate over capital controls as well.

Imagine a three-date endowment economy where individuals borrow because they are impatient relative to world capital markets (i.e., their rate of time preference exceeds the world real interest rate). While individuals know their endowments, which

\(^2\) Some exceptions include authors like Barro (1979, 1995 and 1997) who look at maturity structures directly but for government debt. Since governments don’t save or otherwise behave like individuals, this line of work turns out to have little relevance to the questions addressed in this dissertation.
are constant, international lenders do not. Instead, lenders assign a probability
distribution over possible endowment realizations at each date, the mean of which equals
the actual constant endowment. Due to the uncertainty over borrowers’ ability to repay,
lenders charge residents of the country a risk premium that increases with the level of
funds borrowed and due at a given date.

In this three-date world, individuals at date one can borrow assets of two
different maturities. One asset must be repaid at date two (“short-term”) while the other
must be repaid at date three (“long-term”). The risk premium is based on the total
amount of debt due on a given date with no additional maturity-based premium on either
asset so that, from the perspective of date one, both assets initially look identical since
no debt is yet due at any date. Given this interest rate structure, two different
assumptions about information availability are assumed. First considered are cases
when both the individual and aggregate levels of borrowing are verifiable by
international lenders. This is referred to as the full verification case. With full
verification, when individuals maximize lifetime utility, they consider how the stock of
debt they chose to be due on date three will influence the interest rate on any new
borrowing contracted at date two. They then borrow accordingly given their beliefs
about possible sudden stops. Second, the case when only aggregate, but not individual
borrowing levels, can be verified. This case is referred to as the zero private verification
case. With zero private verification individuals are not charged risk premia based on
increases in their individual debt stocks at the repayment date, but rather on increases in
the aggregate stock. This causes a divergence between private and social marginal costs
where individuals borrow based on their private marginal costs which ignore the impact
their borrowing has on the aggregate stock. As a result, private marginal costs are lower
than social marginal costs and individuals borrow too much relative to the social
optimal.

When a sudden stop strikes, it affects the interest rate charged on new borrowing
at date two. Assuming full commitment to repay, individuals must repay their date one
obligations but are unable to borrow new funds. This means the endowment remaining
for consumption is less than was anticipated. In this model, because all the benefits are captured at the first date, there is still a benefit to borrowing when the sudden stop is fully anticipated. Both maturities are held because, knowing there will be zero new borrowing at date two, both assets have the same risk premium. Individuals in this case essentially take the benefits from initial borrowing, use the different maturities to minimize overall interest payments, and allocate the pain of repayment over future dates. Since individuals “over borrow” when there is zero private verification, they are hit harder by sudden stops than when there is full verification.

This model develops a method for modeling choices over different maturing assets. In particular is does this in the context of a world where sudden stops are possible. When this is done, it is argued that most other comparisons between short-versus long-term borrowing in the literature (Agenor (1999), Cole and Kehoe (1996), Cordella (1998), Kaplan and Rodrik (2001), Mussa (2000), World Bank (1999), and Zee (2000), for example) are actually comparing two constrained problems – one with only short-term assets versus one with only long-term assets. This model allows one to consider a single unconstrained problem. Doing so leads to conclusions that are sometimes different than considering two separate constrained problems, which is an argument in and of itself for the need of more rigorous treatments of different maturing debt. Short-term assets play an important role even in the face of sudden stops and therefore banning them with capital controls is generally detrimental to welfare, contrary to casual intuition.

The primary reason banning short-term flows does not enhance welfare is that there is a gain in lower total interest payments from borrowing both assets, since individuals generally desire new funds at date two when only short-term assets are available. Furthermore, when a sudden stop strikes, individuals would like to spread out the pain of repayment over several future dates, a result that is more pronounced in Chapter IV. A government considering capital controls would better increase welfare by announcing its beliefs about the occurrence of sudden stops and allowing individuals to allocate their asset maturities on their own. The only case when controls could be
helpful in this model is if the government announced its beliefs about the upcoming sudden stop, individuals do not believe the government and it has no other policy tool available. Then, and only then, can banning short-term capital flows actually be welfare enhancing in this model. A decrease in information asymmetry is always welfare enhancing in this model, a result consistent with a number of other findings (Calvo and Mendoza (2000), IMF (1999, 2000, and 2001), and Mishkin (2003), to name a few).

Appendices A – E are associated with Chapter III. Appendix A briefly reviews the basis for an upward sloping, country-specific supply curve of funds. Appendix B develops one case that arises in the full-information version of Chapter III’s model where a domestic loanable funds market emerges if the sudden stop interest rate is less than its zero-quantity-demanded level. Appendix C contains data for the simulated numerical examples in Appendix B. Appendix D contains data for the simulated numerical examples in Chapter III. And, finally, Appendix E presents graphs similar to those in Chapter III, but for an interest rate structure than includes a term premia. This is done for completeness and consistency since term premia are employed in Chapter IV.

I.2.3 Chapter IV

Many observers have noted that countries suffering from crises also have large portions of their debt coming due in bunches that happen to be exactly during times when the crisis is taking place (see Calvo, Izquierdo, Talvi (2002), Calvo and Mendoza (2000), Williamson (2002), and Wolf (2003)). This bunching of maturities during the crisis period is seen as some sort of mistake on the part of individuals or policymakers due often to an inability to precommit or to moral hazard. This mistake is believed to add gas to the fire of the crisis. Chapter IV shows that bunching maturities during a sudden stop can result as the endogenous and optimal result given a sudden stop of sufficient or unknown duration without resorting to commitment or moral hazard arguments.
The first step in Chapter IV is to extend the horizon of the problem individuals solve. It is shown how the horizon can be made infinite and any number of different maturing assets can be introduced. When this is done, a time-based premium that increases with the maturity length is required to provide a boundary to the problem. Otherwise, individuals are initially indifferent about holding an infinite number of assets. Intuitively, the rate at which this time-based premium increases can be thought of as a country difference where more mature markets have premia that increase slowly, allowing them feasible access to very long-term assets, while emerging markets have premia that increase more rapidly, allowing them access to only a few maturities. Mathematically, this is also convenient because sufficiently limiting the number of available assets makes the problem analytically solvable.

Due to the complexity of the problem, analytical solutions with assets beyond two-date maturities are unobtainable. Therefore Chapter IV contains simulation results from finite, six-date horizon problems with assets of one-, two- and three-date maturities. Results are obtained for markets with and for markets without term premia.

Individuals face an environment where they know the starting date but not the duration of the sudden stop. In this case, individuals form expectations over the continuation probability of a sudden stop given its start date. With a very low continuation probability – i.e., they believe it to be a “short-lived” event – the borrowing pattern looks like that of a one-date shock. As the expected continuation probability rises – i.e., people believe the shock will be “longer-lived” – the borrowing pattern approaches that of the fully anticipated permanent shock case.

The intended contributions of Chapter IV are two-fold. First, a framework is developed within which infinitely lived individuals can choose between multiple maturities and country differences can be introduced through the risk premium structure allowing “good” countries feasible access to very long-term maturity structures and “bad” countries access to only a few maturities. Thus, “good” countries can borrow

\footnote{Feasible here means affordable in the sense that there are cases in which individuals might feasibly purchase them. Assets of a longer maturities will never be purchased because the rate charged on them always leaves quantity demanded at zero.}
around many potential shocks, albeit at a higher cost, while “bad” countries only have that option for shocks which are known to be extremely short-lived.

The second contribution is to the maturity-bunching discussion. It shows that one does not need government intervention, moral hazard, or any other external assumption to generate maturity bunching. Maturity bunching arises endogenously in this model when individuals are given multiple maturities to choose from in a world with uncertainty over the duration of a sudden stop of capital inflows or certainty over the occurrence of a permanent sudden stop.

I.3 Conclusion

This dissertation is devoted to developing the models discussed above and then drawing some brief conclusions in Chapter V. The major intended contribution my work is in returning to a basic framework within which one can begin to think meaningfully about sudden stops and what happens when individuals have access to assets of maturity lengths longer than one date, the standard asset currently used in the literature. It is shown that doing this yields a number of results.

First, it is consistently true in the models presented in this dissertation that ex ante capital controls decrease welfare. In Chapter II, only one asset is available. Capital controls would mean raising the cost of borrowing the sole asset or forcing individuals into autarky. Both of these results unambiguously lower welfare. In Chapter III individuals have two assets and choose to hold both to minimize total interest payments when there are no sudden stops. When there is a sudden stop, individuals would still like to hold both assets to minimize interest payments, but have an additional incentive to spread out the pain of repaying given the weights they assign to utility at different dates. Thus any restriction on this choice set by either eliminating or making more
costly either asset lowers welfare\textsuperscript{4}. In Chapter IV, the model and hence the story is similar.

Second, most of the literature looks at models with only short-term assets yet frequently discusses the importance of switching to longer-term maturities. By actually modeling economies with longer-term assets, it is shown in Chapter III that the standard approach is really comparing constrained problems while the multiple-maturity models presented here better capture the unconstrained problem. Chapter IV further highlights this point since issues of maturity bunching can not even be discussed in models with assets of only one maturity length.

Finally, the hope is that this dissertation is one step in the right direction. Now that the framework for handling multiple maturing assets is elaborated and some of the issues better understood, future work will hopefully incorporate the analytical structure or at least insights from here into more complicated models with money, exchange rates and production to address broader questions of interest.

\textsuperscript{4} Unless, again, the government has better information about the probability of a sudden stop and the public refuses to believe government announcements. Then, if no other policies are available, banning short-term capital will improve welfare, \emph{ceteris paribus}. 
CHAPTER II
TO BORROW AND LOSE OR NEVER BORROW AT ALL: SUDDEN STOPS
WITH SHORT-TERM ASSETS ONLY

This chapter develops a model of an economy that is subject to sudden stops of capital inflows. The model is of a three-date, representative-individual economy with endowments, consumption and international borrowing. Sudden stops are modeled as a discrete jump in the interest rate charged to new borrowers at date two. The jump is assumed to be high enough so that individuals desire zero new borrowing. The natural initial question to ask in this environment is whether or not countries should still borrow. If not, then either we should not observe borrowing by rational agents or, if we do, governments could increase welfare by banning capital flows to protect their citizens from the harmful effects of these sudden stops. This chapter shows that, at least in this model, it is never optimal to ban capital flows, even when the sudden stop is known to occur with certainty.

II.1. The Model

Time is viewed as a series of discrete dates, or points. That is, individuals wake up at a date, perform some actions and then go back to sleep until the next date at which time they wake up, perform some more actions, go back to sleep and so on.\(^5\)

---

\(^5\) The choice of using dates as opposed to periods is arbitrary. Either way, one must specify precisely the timing of events at each point in time. The only difference is that in one case (dates) the timing of events is ordered vertically and in the other (periods) it is ordered horizontally.
Individuals awake at date one without any initial debt to repay. They learn their entire lifetime endowment path and receive their current endowment. Given this information, world real interest rate and their subjective rate of time preference, they solve their utility maximization problem to determine lifetime consumption and thus how much to borrow or lend each date. To focus on capital inflows, it will be assumed that consumers are borrowing. This is driven by their impatience relative to international capital markets (i.e., their subjective rate of time preference exceeds the world real interest rate). Thus, individuals go to the international capital markets to borrow the amount needed to satisfy their demanded date-one consumption. Finally, date one ends.

At date two, individuals receive their current date endowment, go to the international capital market to borrow enough to cover date two repayment of date one debt and date two consumption. This is the meaning of “rolling over debt” in Figure 1. Date two then ends. Finally, at date three, individuals receive their last endowment, repay their date two debt and enjoy their last supper. The world then ends.

If a sudden stop occurs, individuals are unable to “rollover their debt" at date two because the interest rate on new borrowing has risen to a level at which they choose not to borrow. With the exception of Appendix B, in this dissertation the shift will be
sufficient to intersect the demand curve at the vertical axis so that the quantity of funds demanded is zero. They must still repay date one debt (given full commitment). In this dissertation, sudden stops are viewed as exogenous leftward shifts in the supply of international funds available to individuals in this country. They are then left to consume whatever remains and live out the rest of their three-date lives in autarky.

Let $R_{t,s}$ denote the market discount factor for date $s$ consumption on date $t \leq s$.

\begin{equation}
R_{t,s} = \frac{1}{\prod_{v=t}^{s} (1 + r_v)} \quad s.t. \quad R_{t,t} = 1 & R_{t,s} = \frac{1}{(1 + r)^{s-t}} \text{ if } r_v = r \ \forall \ t
\end{equation}

where $r$ is the world real interest rate.

Individuals receive per date endowments, $Y_t$, the entire sequence of which is known with certainty from the initial date onward. It is assumed that endowments are constant over time (i.e., the endowment path is flat). Consumers then have the following intertemporal budget constraint.

\begin{equation}
\sum_{s=t}^{t+2} R_{t,s} Y_s = \sum_{s=t}^{t+2} R_{t,s} C_s
\end{equation}

where $C_s$ is per-date consumption.

Representative individuals maximize the following lifetime utility subject to constraint (2).

\begin{equation}
\sum_{s=t}^{t+2} \left( \frac{1}{1 + \rho} \right)^{s-t} u(C_s)
\end{equation}
ρ is the subjective rate of time preference, assumed to be constant. The utility function is assumed to have the standard properties needed for an internal solution to the maximization problem (i.e. it is twice continuously differentiable and concave). For simplicity, it is additionally assumed to be additively separable across time.

The optimality\textsuperscript{6} condition for such a problem is the intertemporal Euler relation

\begin{equation}
\frac{u'(C_{t+1})}{u'(C_t)} = (1 + \rho) \frac{R_{t,t+1}}{R_{t,t+1} + u'(C_t)}
\end{equation}

which says that at the point of utility maximization, the individual cannot gain from feasible shifts of consumption between dates. Note that $(1 + \rho)R_{t,t+1} = (1 + \rho)/(1 + r)$, making it clear that for $\rho > r$, the left-hand side of expression (4) is greater than the right-hand side. By concavity of the utility function, this says that $C_t > C_{t+1}$. Since individuals receive the same endowment at each date, $C_t$ can only exceed $C_{t+1}$ if they borrow at date $t$ against their future endowments. Throughout the remainder of this dissertation it will be assumed that $\rho > r$ which provides a tractable desire to borrow.

To obtain explicit solutions, let the utility function be logarithmic so that

\begin{equation}
\log(C_t) = \ln(C_t) = u(C_t)
\end{equation}

Solving equation (4) for each date $t \in \{1, 2, 3\}$ and using the lifetime budget constraint, the following consumption functions can be determined.

\begin{equation}
C_1^* = \frac{(1 + \rho)^2}{1 + (1 + \rho) + (1 + \rho)^2} \sum_{s=1}^{3} \bar{R}_{1,s} \equiv \tilde{\xi}_1 Y_1 \sum_{s=1}^{3} \bar{R}_{1,s}
\end{equation}

\begin{equation}
C_2^* = \frac{(1 + r)(1 + \rho)}{1 + (1 + \rho) + (1 + \rho)^2} \sum_{s=1}^{3} \bar{R}_{1,s} \equiv \tilde{\xi}_2 Y_1 \sum_{s=1}^{3} \bar{R}_{1,s}
\end{equation}

\textsuperscript{6} Throughout this dissertation, the word optimal refers to utility maximizing (as opposed to Pareto optimality, for example) unless otherwise stated.
where * indicates these as the benchmark levels. The second equalities indicate a notational simplification in defining all “the stuff” multiplying lifetime incomes as ζ subscript 1, 2, and 3 for each date's consumption. The interpretation of this multiplier is as a current date marginal propensity to consume out of discounted lifetime income. With these exact solutions it is possible to solve for the borrowing levels as represented by net foreign asset holdings.

Individuals borrow because the endowment is kept constant at each date and ρ > r. By Euler equation (4), individuals have a downward sloping path of lifetime consumption.

All individuals in this economy want to borrow. In aggregate they can only do this if they obtain funds from the outside world. This means that the representative individual’s flow budget constraint is the balance of payments equation for the entire economy. Let $B_{t+1}$ denote the value of the economy’s net foreign assets, chosen at date $t$ and repaid at date $t+1$. The change in the country’s net foreign asset holdings at date $t$ is then defined as

\[ \Delta B_t = B_{t+1} - B_t = Y_t + rB_t - C_t \]

which says that, at a given date $t$, a country's net foreign asset holdings are the difference between current date endowment and repayment of past borrowing and current date consumption. $B_t$ will always be a non-positive number in this dissertation because it always represents a liability to the individual.

Using financing constraint (9) and in the absence of any sudden stops it is possible for $t = \{1,2,3\}$ to write
which is the level of net foreign asset holdings chosen at date two to be carried to date three. Substituting in for $C_1^*$ it is immediately clear that the optimal level of net foreign assets held from date one to two is

$$B_2^* = Y_1 \left( 1 - \xi_1 \sum_{s=1}^{3} R_{1,s} \right)$$

and similarly for holdings from date two to three

$$B_3^* = Y_1 \left\{ 2 + r_2 - (1 + r_2) \xi_1 \sum_{s=1}^{3} R_{1,s} - \xi_2 \sum_{s=1}^{3} R_{1,s} \right\}$$

These equations show that the optimal borrowing levels are functions of the interest rates, subjective rate of time preference, and constant level of endowment, all of which are parameters.

Welfare in this world is calculated by looking at lifetime utility. In autarky, individuals consume their endowment at each date. They still desire to borrow against their future endowments, but find no willing domestic lenders.

$$U(\text{Autarky}) = \ln(Y_i) + \frac{\ln(Y_2)}{1 + \rho} + \frac{\ln(Y_3)}{(1 + \rho)^2}$$
When the economy is opened up, domestic residents are able to borrow from international lenders and thus obtain a downward sloping consumption path that reflects their desire to borrow.

\[
U(Borrowing) = \ln(C_1^*) + \frac{\ln(C_2^*)}{1 + \rho} + \frac{\ln(C_3^*)}{(1 + \rho)^2}
\]

When capital is freely mobile and sudden stops neither occur nor are expected, \(U(Borrowing) > U(Autarky)\) for all \(r \neq \rho\) because, by definition, \(C^*\) is chosen to maximize lifetime utility and is known, by the optimality conditions, to be different than the endowment. Welfare is the same when \(r = \rho\) because individuals have no incentive to borrow, so they consume their endowments at each date. This is shown in Figure 2 which plots lifetime utility against the world real interest rate. The subjective rate of time preference, \(\rho\), is held constant at \(\rho = 0.03\). For \(\rho \neq r\) welfare is always increased by borrowing (\(\rho > r\)) or lending (\(\rho < r\)) which, for a small open economy with a representative individual can only be done through international markets.
II.2 Sudden Stops

The nature of the sudden stop is such that at date two, individuals find that lenders have decreased the supply of funds available to them in world markets. Individuals see a sudden increase in the interest rate charged to them on new borrowing. For simplicity’s sake, it will be assumed that the increase in the interest rate is high enough so that the quantity of new funds demanded by the affected individuals is zero.

This section considers two cases. In the first case, the sudden stop is unanticipated. In the second case, the sudden stop is fully anticipated. The difference is that when the sudden stop is anticipated, individuals incorporate this expectation into their date-one choice and borrow less funds initially. This limits the damage from the sudden stop since they have less to repay at date two. Most of the discussion focuses on the first case because this is the one more relevant to the discussion in the literature – the

FIGURE 2. BORROWING ALWAYS WEAKLY DOMINATES AUTARKY.
“sudden “ in sudden stop comes from it being unexpected – and because in this environment, with B3 always being zero, the anticipated problem is equivalent to the standard two-date problem that is well known and solved in texts such as Obstfeld and Rogoff (1999).

II.2.1 Unanticipated Sudden Stops

When the sudden stop is unanticipated, individuals wake up at date two, go to the world capital market and find that the world real interest rate on new borrowing has risen. They must then re-evaluate their lives by choosing their optimal consumption levels for the current and future date, taking their current level of debt (carried over from date one) and the world interest rate as given.

Their lifetime budget constraint is now

\[
\left[ Y_2 \left(1 + r_2\right) B^*_2 \right] + \frac{Y_3}{1 + r_{ss}} = C_2 + \frac{C_3}{1 + r_{ss}}
\]

The first term on the left hand side is the date two endowment less the current value of the debt that must be repaid. Since commitment is not an issue in this model, date one optimization always leaves this first term greater than or equal to zero. To distinguish it from non-sudden stop rates, the new world interest rate is denoted, \( r_{ss} \), where subscript \( ss \) indicates the "sudden stop" rate.

Individuals maximize the present value of their remaining lifetime consumption which is represented by

\[
u(C_2) + \left( \frac{1}{1 + \rho} \right) u(C_3)
\]

The optimal consumption levels, imposing logarithmic utility, are
(17) \[ C_2^{ss} = \left( Y_2 + (1 + r_2)B_2 + \frac{Y_3}{1 + r_{ss}} \right) \frac{1 + \rho}{2 + \rho} \]

(18) \[ C_3^{ss} = \left( Y_2 + (1 + r_2)B_2 + \frac{Y_3}{1 + r_{ss}} \right) \frac{1 + r_{ss}}{2 + \rho} \]

The impact of raising the world interest rate unexpectedly is unambiguously negative on current (date two) consumption and positive on date three consumption since \( Y_2 + (1 + r)B_2^* \) is positive. Intuitively, this is because individuals at date two should borrow to repay their first date obligations and consume which is now very costly due to the higher interest rate. The cost of current consumption also rose relative to date three consumption.

The level of borrowing from date two to date three is found by using the financing constraint.

(19) \[ B_3^{ss} = Y_2 + (1 + r_2)B_2 - C_2^{ss} = \frac{Y_2 + (1 + r_2)B_2}{2 + \rho} - \frac{Y_3}{1 + r_{ss}} \left( \frac{1 + \rho}{2 + \rho} \right) \]

The rise in the interest rate causes the borrowing level to fall. This is because borrowing is generated by the difference between the interest rate and the subjective rate of time preference. The higher interest rate closes that gap, thus decreasing the desire to borrow. There is, however, an additional effect due to the inherited debt that is repaid at date two. Repayment lowers the amount of the endowment remaining for date two consumption, thus causing the effective endowment path to slope upward which generates a desire to borrow. To see this, suppose that the sudden stop interest rate rose all the way to the rate of time preference. Because \( Y_2 = Y_3 \) and \( B_2 < 0 \), expression (19) would still be negative. That is, borrowing would still occur. For borrowing to cease, the sudden stop interest rate would have to be higher than \( \rho \) by enough to fully offset the upward sloping
effective endowment path. Denoting that rate as $r_{ss}^*$, it is solved for by setting (19) equal to zero.

\begin{equation}
(20)
\begin{align*}
  r_{ss}^* &= (1 + \rho) \frac{Y_3}{Y_2 + (1 + r_2)B_2^*} - 1 \\
\end{align*}
\end{equation}

To see that this interest rate is higher than the rate of time preference, move the 1 to the left-hand side of (20). The term multiplying $(1 + \rho)$ is positive since the denominator is smaller than the numerator because $Y_2 = Y_3$ by construction, $r_2 \geq 0$ and $B_2^* < 0$.

II.2.2 Fully Anticipated Sudden Stops

Suppose that at date one individuals learn that at date two there is going to be a sudden stop. Knowing that they cannot borrow again at date two, they have no means of pulling date three wealth forward in time. That is, they effectively solve the standard two-date problem although technically date three still exists.

Individuals maximize their three-date utility function subject to the following flow budget constraints:

\begin{equation}
(21)
\begin{align*}
  B_2 &= Y_1 - C_1 \\
  0 &= Y_2 + (1 + r_2)B_2 - C_2 \\
  0 &= Y_3 - C_3 \\
\end{align*}
\end{equation}

Differentiating with respect to $B_2$ yields the same Euler relations in as in (4) with $t + l = 2$ and $t = 1$.

\begin{equation}
(4')
\begin{align*}
  u'(C_2) &= (1 + \rho)R_{t+1}u'(C_1) = \frac{1 + \rho}{1 + r_2} u'(C_1) \\
\end{align*}
\end{equation}
Since $\rho > r_2$ individuals borrow at date one. The optimal consumption levels, superscripted with ESS to denote the expected sudden stop case are

$$C_{1}\text{^ESS} = \frac{1 + \rho}{2 + \rho} \left( Y_1 + \frac{Y_2}{1 + r_2} \right)$$ (22)

$$C_{2}\text{^ESS} = \frac{1 + r_2}{2 + \rho} \left( Y_1 + \frac{Y_2}{1 + r_2} \right)$$ (23)

$$C_{3}\text{^ESS} = Y_3$$ (24)

The consumption levels at dates one and two are lower than in the no sudden stop case because individuals can’t pull their date three endowment forward. Access to world markets at date one does, however, still allow them to allocate their dates one and two endowments efficiently across time. The amount they borrow is nevertheless smaller since $C_{1}\text{^ESS} < C_{1}\text{^*}$.

$$|B_{2}\text{^ESS}| = |Y_1 - C_{1}\text{^ESS}| < |Y_1 - C_{1}\text{^*}| = |B_1|$$ (25)

Because $B_{2}\text{^ESS}$ is between the two extremes of $B_2\text{^*}$ and $B_2\text{^Autarky} = 0$, to determine the maximum pain from a sudden stop, only the unanticipated case will be considered in the following welfare analysis.

II.3 Welfare Analysis

Although individuals suffering from the unanticipated sudden stop choose optimally given the new interest rate. They are caught off guard because they didn’t take $r_{ss}$ into consideration when choosing their date one consumption level. Therefore,
the utility from an unanticipated sudden stop will always be lower than it would be in the equilibrium without a sudden stop.

A policy-relevant issue arises here. If sudden stops are so bad, then perhaps it would be better not to have allowed borrowing to occur in the first place. Rather than potentially suffer the negative consequences of a sudden stop, governments could simply ban capital flows thereby forcing the economy into autarky from date one, bypassing the danger of a sudden stop altogether.

To determine whether welfare can fall below its autarkic level requires finding the $r_{ss}$ that solves the following inequality.

\[
(26) \quad U(\text{Autarky}) \equiv \ln(Y_1) + \frac{\ln(Y_2)}{1+\rho} + \frac{\ln(Y_3)}{(1+\rho)^2} > \ln(C^*_1) + \frac{\ln(C^*_2)}{1+\rho} + \frac{\ln(C^*_3)}{(1+\rho)^2} \equiv U(SS)
\]

If this can be done, then sudden stops can be bad enough to justify informed governments in their decision to ban capital flows.

Because the sudden stop only affects the borrowing rate for the affected country, the highest interest rate worth considering is $r_{ss}^*$. Any interest rate greater than $r_{ss}^*$ has the same effect since at that interest rate, borrowing at date two goes to zero. This means that $C^*_3 = Y_3$. Those terms can then be canceled out from the left and right hand sides of (26). Rearranging gives the condition

\[
(27) \quad \frac{\ln(Y_2) - \ln(C^*_2)}{1+\rho} > \ln(C^*_1) - \ln(Y_1)
\]

What condition (27) says is that for autarky to be better than the sudden stop, it must be true that the discounted value of the date two consumption loss must be large enough to offset the date one consumption gain. In this model, individuals choose their borrowing levels optimally based on the size of the difference between the rate of time preference and the world interest rate. As a result, $U(\text{Autarky})$ is always less than $U(SS)$. That is,
condition (26) never holds. The intuition behind this result is that changes in the endowment are simply level changes since the endowment path has a zero slope. For \( r \) near \( \rho \), the gain is smaller, but so is the loss and thus the same non-zero result obtains. When \( r \) is far from \( \rho \), then we are imposing a high \( \rho \) because borrowing is generated by assuming \( \rho > r \) (alternatively, \( r \) can be set to zero, but the result is the same). In that case, the loss is larger, but so is the initial gain. This is shown first graphically in Figure 3 and then algebraically below.

The key to understanding this result is to recognize that the date one gain, relative to autarky, is measured by the magnitude of the amount borrowed. The loss at date two is measured by \((1+r)\) times the amount borrowed, discounted in the lifetime utility function by \((1+\rho)\) (which is assumed to be greater than \((1+r)\)). To keep the intuition clear, it is easiest to look at absolute values.

\[(28) \quad \text{Date 1 Gain:} \quad |B^*_2| = |Y_i - C^*_1|\]
Since the sudden stop leaves individuals only able to consume their endowment at date three, we need only compare the utility value of the gain versus the utility value of the loss, discounted by the subjective rate of time preference. This comparison is

\[ U(SS) > U(Aut) \quad \text{iff} \quad u(Y_1 - B_2^*) - u(Y_i) > \frac{u(Y_2) - u(Y_2 + (1 + r_2)B_2)}{1 + \rho} \]

This condition holds for all weakly concave utility functions when \( \rho > r \). Substituting, for example, a linear function where \( u(C) = C \) into (30), the solution to the inequality is \( \rho > r \). For this dissertation, the meaning is that the condition holds for all cases where borrowing is generated by \( \rho > r \). This is the way borrowing is generated throughout the dissertation.

Since individuals are always borrowing, \( B_2^* \) is always negative. The only case for which the inequality becomes an equality is when \( \rho = r \) but then \( B = 0 \) and sudden stops have no effect.

Finally, one might consider a longer time horizon, but this only makes the difference \( U(SS) - U(Autarky) \) grow more positive. To see this, suppose we lengthen the time horizon so that the environment’s terminal date, \( T \), is \( T > 3 \). Let the sudden stop occur at time \( t = \tau \) and, to make the strongest case, suppose the shock is permanent so that \( r_{ss} \) never falls back down to its old level. Then the following conditions will hold.

\[
\begin{align*}
U(SS) > U(Autarky) & \quad \forall \, t < \tau \\
U(SS) < U(Autarky) & \quad \text{for } t = \tau \\
U(SS) = U(Autarky) & \quad \forall \, t > \tau
\end{align*}
\]

The only thing that has changed is that the time over which gains have been accruing is much longer now, so the lifetime gain is larger. The loss now occurs at a much later
date and is therefore more heavily discounted, so it matters even less for lifetime utility. In particular, the loss would be

\[ (31) \quad \text{“Loss paid at } \tau \text{”} \quad (1 + r_\tau)B_\tau = (1 + r_\tau)(Y_{\tau-1} + (1 + r_\tau)B_{\tau-1} - C_\tau) \]

But again, \( B_\tau \) is the gain at date \( \tau-1 \), so the loss is discounted at a rate \( 1 + \rho > 1 + r \) and the net gain over dates \( \tau-1 \) and \( \tau \) is still positive. If all the values after \( \tau \) are the same and all the values for \( U(SS) \) prior to \( \tau-1 \) are larger than \( U(Autarky) \), then the distance between these functions, \( U(SS) - U(Autarky) \) is growing with the length of the time horizon.

\[ (32) \quad \frac{\partial (U(SS) - U(Autarky))}{\partial T} > 0 \]

In this model then, the result that \( U(SS) \geq U(Autarky) \) holds as long as \( \rho > r \).

It is generally recognized that the policy options available to a domestic government facing sudden stops are all preventative. That is, once a country is in the midst of a sudden stop, there is little the government can do to improve matters. A preventative measure often discussed is to ban capital flows outright.

In this model, banning short-term capital flows is the same as forcing the economy into autarky beginning with date one. But we know from the above discussion that \( U(SS) > U(Autarky) \) for all relevant values of our parameters. The conclusion then is that capital controls are not welfare enhancing in this model.

II.4 Conclusion

This chapter explored a model of an economy subject to sudden stops of capital inflows. The model was of a three-date representative-agent economy with
endowments, consumption and international borrowing. Sudden stops were modeled as a discrete jump in the interest rate charged to new borrowers at date two.

The chapter showed that the pain from sudden stops can come without any appeal to relative price effects at all but that individuals are always better off borrowing. In essence, a sudden stop is simply an inability to borrow new funds. That is painful enough in and of itself to warrant concern on the part of policymakers and researchers alike. As for policy options, since it is better to have borrowed and lost than never to have borrowed at all, banning capital flows is never welfare enhancing in this model, even in the face of a fully anticipated sudden stop.
CHAPTER III
PRIVATE DEBT MATURITY CHOICES WHEN CAPITAL INFLOWS CAN SUDDENLY STOP

Most models of sudden stops, including the one developed in Chapter II of this dissertation, analyze short-term borrowing exclusively. Yet there is a common belief among many policymakers and academics alike that switching to longer-term maturities would allow countries to avoid the damage of a sudden stop (given that the duration of the shock is known). This chapter introduces long-term borrowing into an intertemporal approach to balance of payments model in order to understand its role when capital inflows can suddenly stop. The main objective of the chapter then is to analyze the role long-term borrowing plays in this environment. Since forcing individuals to borrow longer-term is frequently done by banning short-term capital controls, this chapter also addresses the potential role for such controls.

The choice over maturities matters in this model because there is an asymmetry of information between domestic borrowers and international lenders. Specifically, international lenders do not know individuals' actual endowment path. This generates the need for lenders to charge domestic residents a country-specific risk premium that makes the interest rate rise the more individuals borrow of a given maturity. This generates a friction in the model and makes it generally optimal to split borrowing between short-term and long-term maturities to minimize the total cost of borrowing. Individuals weigh this cost-minimizing allocation against the probability of a sudden stop and thus the amount of short-term maturities to borrow that require rolling over.

After discussing the fixed-interest rate case where there is no information asymmetry, the remainder of the chapter deals with the asymmetric case and, in that context, discusses four cases. Broadly the cases fall into two categories, those when lenders can fully verify individual and aggregate borrowing levels (hereinafter referred to as “full verification”) and those when lenders can fully verify aggregate borrowing levels, but are unable to verify individual borrowing levels (hereinafter referred to as
“zero private verification”). In anticipation of eventually expanding the time horizon and number of maturities in the model (done in Chapter IV) additional term premia that make longer maturing assets more expensive that short-term assets are of interest. For that reason, the cases in this chapter are studied with and without term premia. In general, the purpose of this taxonomical approach is to investigate the role that different information assumptions have in this model.

Finally, all the cases with information differences are conducted with the literature in mind. Since the sudden stop literature frequently models economies with short-term assets only, but frequently infers conclusions about cases where all debt is long-term instead, three broad alternative domestic policies are compared in the second-best environment. First, banning long-term borrowing is considered (called the “B-only” case). This leaves only short-term borrowing and is intended to represent the standard models that generally do not explicitly consider the long-term borrowing option. Second, banning short-term borrowing is considered (called the “A-only” case). This leaves only long-term borrowing and represents the strongest case in this model for short-term capital controls. Finally, the option of leaving both maturities available is considered (called “A&B” or “both”). All of these cases are analyzed when there is no sudden stop, when one is unanticipated, and when one is fully anticipated.

III.1 The Model

The core model is a three-date representative agent model with one traded good, the price of which is the numeraire, and exogenous endowments. Both short and long-term borrowing in world markets allow individuals to smooth consumption over time.
Time continues to be viewed as a series of discrete dates, or points, as in Figure 4. If a sudden stop occurs, individuals are unable to rollover their debt at date two.

International lenders have a similar time line. The major difference is that they do not know with certainty the lifetime endowment path of domestic borrowers. The interest rate structure resulting from this assumption has a long history in the literature and the formulation here follows the line of Eaton and Gersovitz (1981), Aizenman (1989), Agénor (1999), Auernheimer and García-Saltos (2000), Mendoza (2002) and Stulz (2003).

III.1.1 Fully Symmetric Information: Fixed Interest Rates

The date $t$ financing constraint in this frictionless scenario where the paths of all variable are known in advance by all agents, domestic and international, is

$$(B_{t+1} - B_t) + (A_{t+2} - A_t) = Y_t + r_t B_t + [(1 + r_d)^2 - 1] A_t + (1 + r_s) S_t - C_t - S_{t+1}$$

Appendix A provides a brief overview of the logic behind the upward sloping supply curve of funds.
where $Y_t$ denotes the endowment, known with certainty by all parties so that international lenders charge only the riskless world real interest rate on loans; $B_t$ and $A_t$ are the amounts of short-term and long-term previously-contracted borrowing, respectively, that are due on date $t$ and must be repaid at interest rates $r_{At}$ and $r_{Bt}$. $S_t$ is the amount saved at interest rate $r_{St}$ from the previous date for use at date $t$. $B_{t+1}$ is the amount of new short-term borrowing contracted at date $t$ that matures at date $t+1$. $S_{t+1}$ is the amount saved at date $t$ to be carried over to date $t+1$. Finally, $A_{t+2}$ is the amount of new long-term borrowing contracted at date $t$ that matures at date $t+2$.

Individuals maximize the following lifetime utility function subject to expression (34).

\[
\sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^{t-1} u(C_t)
\]

where $\rho$ is the subjective rate of time preference, always assumed to exceed the constant world real interest rate, and $C_t$ is the amount consumed at a given date. The functional form is assumed to have all the standard properties that ensure an interior solution and is additionally assumed to be additively separable across time.

When long-term borrowing is allowed, individuals must also account for the amount of savings per date, $S_t$. As with $B_t$ and $A_t$, savings is a state variable so that at time $t$ individuals choose how much to save for the next date, $S_{t+1}$.

The other implication is that the type of the sudden stop shock now matters. Generally there are two types of interest rate shocks. The first type (Type-I Shock) is when the world real interest rate changes. This moves both the borrowing and the lending rates, allowing lenders to gain. Depending on the magnitude of the change in the interest rate, the gain for lenders can be very large while the loss to borrowers is limited by the point where borrowing is optimally set to zero.
The second type of shock (Type-II Shock) occurs when the world real interest rate remains unchanged, but the risk premium (set to zero in the first-best case) charged to borrowers of the affected country changes. This affects borrowers only, leaving long-term borrowers lending short-term at the unchanged world lending rate\(^8\). Such an asymmetry is often assumed in empirical research where a positive Type-I shock (i.e., lower world interest rates) drives a sudden surge of capital inflows and is then followed by a negative Type-II shock (i.e., higher borrowing rate premia). This is done in Calvo, Liederman and Reinhart (1996), Calvo and Reinhart (2000), Edwards (1999), and Grandes (2002)). Nevertheless, because long-term borrowing in a non-heuristic optimizing framework is not generally studied in most theoretical models, the asymmetric nature of the interest rate shock is often overlooked at the formal level.

Individuals choose the method of financing that yields the highest lifetime utility. When individuals are indifferent between choosing long-term, \(A_t\), and short-term, \(B_t\), financing, the following equality obtains

\[
(1 + r_{B_t})(1 + r_{B_t}) = (1 + r_A)^2
\]

If the equality is broken, a corner solution is obtained where one asset is not held. Because long-term financing requires per date saving, given the levels of \(r_{B_t}\) defined in (36), individuals will only hold \(A_t\) if a further condition is satisfied.

\[
(1 + r_{S_t})(1 + r_{S_t}) = (1 + r_A)^2
\]

\(^{8}\) Under the special condition when individuals can hold only one asset type (A or B, but not both), there is a range of interest rates between the world lending rate and the maximum borrowing rate for which a domestic loanable funds market emerges allowing some wealth transfers between short-term borrowers and short-term lenders. This case is considered in Appendix B.
These two equations are the no arbitrage conditions for the frictionless economy. The first, expression (35), says that, if individuals are indifferent between assets $A$ and $B$, and both yield the same per-date consumption path, then the per-date interest rates must be equal as well. This eliminates any potential arbitrage between long- and short-term maturities.

The second equation, (36), says that when individuals borrow long-term, they cannot earn positive profit by lending short term. If they could, then they would borrow an infinite amount of $A$, at the low $r_{At}$ and earn unbounded profits by lending at the higher $r_{St}$. If the per-date $r_{At}$ exceeded the per-date $r_{St}$ then individuals would lose money and hold $B$ instead.

Since the returns are equal and consumers are identical, all consumers are indifferent between holding assets $A$ and $B$. This means that in aggregate the level of each type of asset held is indeterminate. Individuals gladly hold $B=0$ and $A=0$ or any composition in between. This is the result of having costless accumulation and is no longer an issue when risk premia are included. For the moment, to restore aggregate determinacy, assume the following tie-breaking rule.

**Tie-Breaking Rule** – If an individual is indifferent between assets $A$ and $B$, then with probability $\alpha$ they choose $B$ and with probability $1-\alpha$ they choose $A$.

The parameter $\alpha$ can be interpreted as either the percentage of each individual's wealth that is comprised of $B$-holdings so that $\alpha = B/(A+B)$ or $\alpha$ can be thought of as the percentage of the population holding $B$-type assets. Either way determinacy is restored\(^9\). In the risk-free, no-shock world there is no difference between the interpretations. But when shocks are possible, the difference is not trivial. In the first case, which is the one that emerges when risk premia are included, each individual holds both assets. Therefore, when a *Type-II* shock occurs, everyone is harmed because they hold $B$-type

\(^9\) Also, for either definition, $\alpha$ is defined in terms of the choices made at date one. That keeps $\alpha$ from changing at date two if more or less short-term borrowing is chosen.
assets although no one is harmed as bad as if they only held $B$-type assets because they also have $A$-type holdings. In the second case, some individuals are harmed by the shock (because they only have $B$-holdings) and others are unscathed (because they only have $A$-holdings). The result is that domestic gains from trade can be obtained as is explained in Appendix B. Finally, independent of the interpretation of $\alpha$, when there is even an epsilon probability of a sudden stop occurring, everyone holds the long-term asset which guarantees them the optimal consumption path and the sudden stop is completely avoided. For clarity of discussion, the second interpretation of $\alpha$ will be used in what follows, but it will be assumed that the sudden stop is always sufficient to prevent the domestic market for funds from emerging. This is equivalent to assuming that the interest rate jumps to its maximum effective level which leaves the quantity demanded for new funds at zero.

The $\alpha$ percent of the population using $B$-type assets optimally borrows the following amounts.

\[ B_2^* = Y_1 - C_1^* \]

\[ B_3^* = Y_2 - C_2^* + (1 + r_b)B_2^* \]

where $C_t^*$ denotes optimal (utility-maximizing) consumption. These tell us that at date one, $B$-holders borrow enough to cover the difference between the date one endowment and optimal consumption. At date two, they borrow enough to repay their date one debt and cover the difference between their date two endowment and optimal consumption.

The remaining individuals, $(1-\alpha)$ of the population, choose $A$-type assets and borrow the following amount.

\[ B_2^* = Y_1 - C_1^* \]

\[ B_3^* = Y_2 - C_2^* + (1 + r_b)B_2^* \]

While long-term borrowers are never harmed by the sudden stop, they may actually capture a gain if the sudden stop interest rate is below its maximum in which case they can lend domestically to short-term borrowers at a rate higher than the world rate and gain from doing so. Again, see Appendix B for this case.
Since the consumption path is chosen optimally and is the same under either maturity structure, any long-term borrowing that allows for this consumption path is supportable. That is, $A_3$ is not unique when there is no premium charged based on the level of the debt borrowed. In order for both types of borrowing to be used in equilibrium, individuals must borrow at least enough to finance the optimal consumption path. Any amount beyond this amount is simply lent out, i.e., invested in world markets, to offset the accruing interest and then repaid at date three. There is no utility gain or loss to doing this. Thus, $A_3^* \in \left[ Y_1 - C_1^*, \infty \right)$ because there is no cost to higher debt levels. Since the level of $A_3$ beyond that needed to cover date one consumption is irrelevant, I will assume individuals always borrow the minimum needed. This means $S_2^* = 0$ and $S_3^* = Y_2 - C_2^*$.

This borrowing and savings pattern makes the model much more tractable because $A_3$ equals the optimal date one short-term borrowing level $B_2$\textsuperscript{11}. Short-term borrowers must repay the principal plus interest at date two and thus need to borrow more funds at that date in order to obtain the optimal level of consumption. Long-term financers borrow the same amount at date one, but don't need to repay anything at date two. They don't consume their full date two endowments because they must save in order to make date three repayments and consume the optimal amount at date three.

III.1.2 Asymmetric and Incomplete Information: Risk Premia

The source of uncertainty in this model, and the reason for risk premia, is an asymmetry of information between international lenders and domestic borrowers. Specifically, international lenders do not know the exact level of domestic endowments

\begin{equation}
A_3^* = Y_1 - C_1^* - S_2^*
\end{equation}

\textsuperscript{11} Actually, the date one net capital flow is always the same for short and long-term financing. The reason is that any $A_3$ beyond $B_2$ is loaned back out as $S_2$ since only $B_2$ is needed. The result is that net $A_3 = A_3 + S_2 = B_2$. Recall, $A$ and $B$ are negative while $S$ is positive.
although they know the distribution of them. Domestic residents have perfect information over their actual endowment path. From an international perspective, the endowment at date $t$, $Y_t$, is a random variable with p.d.f. $g_t(Y_t)$ for which there exists a minimum output $Y_{t-min} \geq 0$ and a maximum $Y_{t-max} < \infty$ such that

$$\int_{Y_{t-min}}^{Y_{t-max}} g_t(Y_t) dY_t = 1$$

Furthermore, this variable is independently and identically distributed so that no serial correlation exists (an assumption that is relaxed in Chapter IV).

Lenders are risk-neutral, can always lend elsewhere at the risk free world interest rate, $r^w$, and therefore will only make loans which guarantee them an expected rate of return at least as high as the riskless world real interest rate. This means that the more domestic individuals borrow, the higher the risk premium\(^{12}\).

The supply curve of funds is upward sloping in the level of individual borrowing, starting at the riskless world real interest rate for zero borrowing. Because the premia here are charged as a result of uncertainty over the endowment at a given date and thus the ability to repay, lenders consider the entire amount of debt due at the time of repayment. The interest rate equations that include risk premia are then functions of both types of borrowing due at each date (i.e., $A_t + B_t$).

For short-term borrowing:

$$r_{i_t} = r_{i_t} (B_t + A_t; r^w, s) \quad s.t. \quad r_1 < 0, r_2 > 0 \text{ and } r_3 > 0$$

where $s$ is the sudden stop component of the risk premium, and $r^w$ is the exogenous world real interest rate (i.e., the risk-free rate). Again, these rates rise as the level of

\(^{12}\) For a more formal treatment, see Appendix A, which in turn follows Eaton and Gersovitz (1981) very closely.
borrowing increases. Borrowing levels are negative values in this model because they represent liabilities.

For long-term borrowing:

\[ r_A = r_A (B_t + A_t; r^w) \quad \text{s.t.} \quad r_1 < 0 \text{ and } r_2 > 0 \]

Note that \( s \) is absent from this expression. Nevertheless, the sudden stop and its anticipation or lack thereof affect the interest rate on long-term assets. That is because both borrowers and lenders at date one form expectations over the amount that individuals will borrow short-term at date two.

These formulations allow one to distinguish properly between both types of sudden stops. For the first type, \( r^w \) increases, shifting the supply curve of funds up and to the left in the world market. The second type is captured by the \( s \) term. Normally \( s = 0 \), but when a sudden stop hits, \( s \) rises until the individual demand for funds and the risk-adjusted supply of funds to this economy's individuals intersect at the vertical axis. The result is that individual demand for borrowing at this and higher rates is zero.

Individuals solve the following expected utility maximization problem.

\[
\max_{\{c_t\}_{t=0}^3} E_t \left\{ \sum_{t=1}^{3} \left( \frac{1}{1+\rho} \right)^{t-1} u(C_t) \right\}
\]

subject to the following date \( t \) constraints

\[
A_{t+2} + B_{t+1} = \mu_{Y_t} + (1 + r_{B_t})B_t + (1 + r_A)A_t + (1 + r_s)S_t - S_{t+1}
\]

(i) \( \mu_{Y_t} = \int_{Y_{t_{\text{min}}}}^{Y_{t_{\text{max}}}} Y_t g(Y_t) dY_t \quad \forall t \)
(ii) \[ r_B = r_B(B_t + A_t; r^w, E_i(s)) \]

(iii) \[ r_A = r_A(B_t + A_t; r^w) \]

The first constraint is the date \( t \) financing constraint or flow constraint. \( B \) reflects borrowing that must be repaid at the next date and \( A \) reflects borrowing that must be repaid in two dates. Hence at date \( t \), individuals choose \( B_{t+1} \) and \( A_{t+2} \). This makes \( B_t \) and \( A_t \) state variables. There are three different interest rates, one for short-term borrowing (\( r_B \)), for long-term (\( r_A \)) and for saving (\( r_S \)).

The second constraint forces the endowment path to be constant. The actual endowment is always realized to be the mean of the distribution. This is known in advance to domestic residents, but not to international lenders. The individual’s problem is then a deterministic problem regarding endowments.

The third constraint reflects two things. First it highlights that the source of uncertainty for the individual is uncertainty over the sudden stop. In particular, individuals know the magnitude of the sudden stop, but not whether it will occur or not. The magnitude \( s^* \) is defined to leave new borrowing optimally at zero. Actually, \( s^* \) is the minimum level required for a full sudden stop because all higher borrowing rates also yield zero optimal borrowing. The probability that a sudden stop occurs is \( \lambda \) and the probability that it does not is \( 1 - \lambda \). Secondly, this and the last constraint show that the risk premium charged to an individual depends on the individual’s total amount of borrowing due at the time of repayment.

Individuals maximize expected lifetime utility expressed in equation (43) subject to the constraints in (44). Most importantly, the interest rate levels charged to borrowers are now increasing functions of the total amount borrowed that matures on a given date \( (D_t = B_t + A_t) \). This adds a cost to choosing between levels within a maturity and across maturities. The relevant optimality conditions for this problem are
\[ u'(C_t) = \frac{u'(C_{t+1})}{1 + \rho} (1 + r_{S_{t+1}}) \]

\[ u'(C_t) = E_t \left\{ \frac{u'(C_{t+1})}{1 + \rho} \left[ (1 + r_{B_{t+1}} (D_{t+1})) + \frac{\partial r_{B_{t+1}}}{\partial B_{t+1}} B_{t+1} \right] \right\} \]

\[ u'(C_{t+1}) = E_t \left\{ \frac{u'(C_{t+2})}{1 + \rho} \left[ (1 + r_{B_{t+2}} (D_{t+2})) + \frac{\partial r_{B_{t+2}}}{\partial B_{t+2}} B_{t+2} + 2 A_{t+2} (1 + r_{A_{t+2}} (D_{t+2})) \frac{\partial r_{A_{t+2}}}{\partial A_{t+2}} A_{t+2} + \frac{\partial r_{B_{t+2}}}{\partial A_{t+2}} B_{t+2} \right] \right\} \]

\[ u'(C_{t+2}) = E_t \left\{ \frac{u'(C_{t+3})}{(1 + \rho)^2} \left[ (1 + r_{A_{t+2}} (D_{t+2}))^2 + 2(1 + r_{A_{t+2}} (D_{t+2})) \frac{\partial r_{A_{t+2}}}{\partial A_{t+2}} A_{t+2} + \frac{\partial r_{B_{t+2}}}{\partial A_{t+2}} B_{t+2} \right] \right\} \]

The first condition reflects the fact that only Type-II shocks are considered, so \( S_{t+1} \) is the amount saved / lent in world markets at the constant risk-free rate. Because the lending rate is less than the borrowing rate, \( S_t = 0 \) is optimal for all \( t \). \( S_2 = 0 \) because the total desired borrowing is the difference between date one endowment and date one consumption. \( S_3 = 0 \) because the lending that would normally occur at date two is now consumed so that less short-term borrowing is required which in turn lowers the total amount that must be repaid at date three, and consequently the amount must be saved for repayment. The second condition reflects the fact that there is uncertainty over whether a shock occurs or not and that, independent of the shock, the interest rate is now a function of the quantity borrowed. Additionally, since the interest rate charged individuals on long-term borrowing is based on the stock due at date three, and because lenders can rationally anticipate the level individuals will borrow short-term at date two, individuals must consider the impact of additional borrowing at date two on the long-term interest rate. Condition three shows that new date \( t+1 \) short-term borrowing directly influences the interest rate charged to long-term borrowers as discussed above. The last condition reflects that, when considering long-term borrowing, the source of uncertainty is how
much is anticipated to be borrowed short-term at date $t+1$. Since the date $t+1$ short-term interest rate will depend on the amount of long-term borrowing contracted at date $t$, individuals must keep expected date $t+1$ borrowing in mind when choosing how much to borrow long-term. This is how expectations over the sudden stop influence the amount of long-term borrowing even though the long-term borrowing rate is unaffected by the shock directly.

III.1.3 Full Verification Versus Zero Private Verification

Throughout the above discussion it was implicitly assumed that lenders could fully verify both private and aggregate borrowing levels. In that case, when individuals go to the market to borrow, they fully take into account how marginal additions to the amount they would like to borrow will influence the interest rate that is charged to them. Mathematically this is reflected in the derivative terms that show up in the right-hand side brackets of the optimality conditions. The result is that private and social marginal costs are equated so that the private optimal solution also solves the social optimization problem.

If, on the other hand, lenders can not verify private borrowing levels but can verify aggregate borrowing levels for the economy as a whole, then when lenders arrive at the market, they are quoted a fixed interest rate and they chose to borrow or not at that rate. They do not in this case take into account how their own borrowing adds to the aggregate and consequently affects the interest rate charged to all borrowers in the economy. The result is that individuals base their borrowing decisions on their private marginal costs which are actually the social average costs and are consequently different than (specifically, they are lower) social marginal costs. Mathematically, this is reflected in the partial derivatives in the optimality conditions all being set to zero. Intuitively, this is an over congestion problem where, due to a divergence between marginal private and social costs, individuals over engage in an activity. In the textbook case of highway congestion this leads to an inefficiently high number of cars on the
freeway. In the case of borrowing here considered, it leads to an inefficiently high level of borrowing (i.e., individuals over borrow relative to the social optimal).

The optimality conditions become

\[ u'(C_t) = \frac{u'(C_{t+1})}{1 + \rho} (1 + r_{s_{t+1}}) \]  

(49)

\[ u'(C_t) = E_t \left\{ \frac{u'(C_{t+1})}{1 + \rho} \left[ (1 + r_{b_{t+1}} (D_{t+1})) \right] \right\} \]

(50)

\[ u'(C_{t+1}) = E_t \left\{ \frac{u'(C_{t+2})}{1 + \rho} \left[ (1 + r_{b_{t+2}} (D_{t+2})) \right] \right\} \]

(51)

\[ u'(C_t) = E_t \left\{ \frac{u'(C_{t+2})}{(1 + \rho)^2} \left[ (1 + r_{a_{t+2}} (D_{t+2})) \right]^2 \right\} \]

(52)

where \( D_t = B_t + A_t \) is the total stock of debt due at the time of repayment, as before. These are simply equations (45) – (48) without the additional derivatives for the effect of marginal units of borrowing on interest rates.

After solving for the individual’s optimal levels of borrowing, given the interest rate, the equilibrium interest rates are solved for by substituting the borrowing levels into the economy’s overall resource constraint. The resource constraints in this case are the interest rate equations (44) (ii) and (iii).

As a foreshadow of results to come, interest rates must now be equalized. To see this, note that for all optimal solutions individuals must be indifferent between the assets actually held in equilibrium. Since this condition never holds for savings except in the fixed interest rate case with only long-term borrowing considered at the beginning of this chapter, indifference here requires equality of the first order conditions for the problem. While this always holds for optimal solutions (since the first order conditions must all be
equated with zero), in the zero private verification case it implies that the product of the two short-term rates must equal the square of the long-term rate, a condition that was seen also in the fixed rate case before. This further requires that $B_2 = B_3 + A_3$. The result is that the market forces the amount of the long-term asset borrowed, $A_3$, to influence directly the interest rate charged on initial short-term borrowing, $B_2$. This was not the case with full verification which meant that the interest rate on initial short-term borrowing, $B_2$, was always lower than the long-term rate. Since individuals would like to borrow both short-term and long-term in both the full verification and the zero private verification cases, this means that the level of the long-term asset borrowed is much lower in the zero private verification case than it is in the full verification case. This can be seen by looking at the values of the long-term asset, $A_3$, chosen in each case and reported in Appendix D.

Since the only qualitative difference between the full verification and the zero private verification cases is that the level of the long-term asset is lower and borrowing is higher, individuals are holding relatively more and absolutely “too much” of the short-term asset. When the sudden stop is unanticipated then, the damage is worse in the zero private verification case than in the full verification case. Again, the reader is referred to the values reported in Appendix D.

The major results of this chapter will be presented in terms of the full verification case and the zero private verification case. Appendix E contains the results for the case with term premia. That case has been included since a term-premia case will also be considered in Chapter IV. The degree to which term premia have an impact on the results presented will be noted as necessary throughout the remaining discussion.

III.2 Response to Shocks: Simulations and Welfare Analysis

For simulations, specific functional forms and parameter values are assumed. For a complete list of the parameter values, see Appendix D. Utility is logarithmic and the interest rate equations are
\[(53) \quad r_B = r^w - \gamma (B_{t,t} + A_{t,t}) + \lambda (s = s^*) + (1 - \lambda)(s = 0) \quad \& \quad r_A = r^w - \varphi (E_{t,1} \{ B_{t,t} \} + A_{t,t}) \]

where \( \gamma > 0 \) and \( \varphi > 0 \) are exogenously determined parameters, \( s \) is the risk-premium's shift parameter that departs from zero during sudden stops, and \( \lambda \) is the probability of a sudden stop (i.e., \( s = s^* \)). Because \( A_t \) and \( B_t \) are negative, the risk premia rises as individuals borrow more\(^{13}\).

There are three distinct cases to consider. First, the case of no shock is considered. This provides the benchmark. Second, the case of an unanticipated shock is considered. Third, anticipated shocks are discussed. Here the case where the shock is unanticipated (i.e., assigned a probability weight of zero) and the case with probability one (“fully anticipated”) are both considered.

### III.2.1 No Shock Benchmark

In Figure 5.A and Figure 5.B, the steepest path (“Both”) represents the complete case where both assets are available. There the ability to finance consumption through both assets lowers individuals’ initial interest payment. The lower cost is translated into higher consumption and, thus, this path yields the highest utility of the three. The kink at date two in Figure 5.A is from the higher premium resulting from the long-term borrowing already due at date three.

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\(^{13}\) To avoid the impression of any bias in my results, I generally let \( \gamma = \varphi \). Intuitively, one might assume that \( \varphi > \gamma \) since uncertainty is generally higher the farther out into the future on looks although nothing in the structure of the model suggests this should be so. That intuition is captured in Chapter IV and Appendix E by considering the case with explicitly included term premia.
FIGURE 5.A. FULL VERIFICATION CONSUMPTION WITHOUT A SHOCK

FIGURE 5.B. ZERO PRIVATE VERIFICATION CONSUMPTION WITHOUT A SHOCK
The other two consumption paths are where only short-term, B, assets are available (e.g., no long-term, A, option is available) which is the case commonly studied in the literature and the case where only A assets are available (no short-term options exist) which is intended to capture the “capital controls” case where short-term borrowing is banned.

The difference between the full verification case and the zero private verification case is that when there is zero private verification very little of the long-term asset is held. As discussed in the previous section, this is because the market forces the long-term asset to exert an influence on the level of the interest rate charged on individuals’ initial holding of the short-term asset chosen at date one. The result is that, graphically, the difference between the “B-only” and “Both” consumption paths is nearly imperceptible.

III.2.2 Unanticipated Shock

When an unanticipated shock occurs at date two, individuals must re-optimize, updating their plans given the new interest rate. To do this, they solve a two date problem by maximizing

\[
\sum_{t=2}^{3} \left( \frac{1}{1+\rho} \right)^{t-2} u(C_t)
\]

subject to their remaining lifetime budget constraint

\[
\mu_{Y_2} + (1+r_{B_2}(B_2))B_2 + \frac{\mu_{Y_3} + \left(1 + r_{A_3}(E_1\{B_3\} + A_3)\right)^2 A_3}{1+r_{3ss}} = C_2 + \frac{C_3}{1+r_{3ss}}
\]

where \(B_2\) and \(A_3\) are pre-determined variables from the individual's perspective at date two. The interest rate between dates two and three is now the sudden stop rate, \(r_{3ss}\), which, by definition, solves optimally for \(B_3 = 0\). Technically, one could write the
interest rate charged to borrowers as a function of $B_3$ because the sudden stop is merely a level shift of the supply curve upwards. Nevertheless, because it is assumed that the shift is far enough that the new intersection with individual demand occurs on the vertical axis, no non-zero amounts are demanded, so the slope term can be dropped for notational simplicity. The sudden stop interest rate, $r_3^{ss}$, solves

$$B_3^{ss} = 0 = \mu_{y_2} - \left( \frac{1+p}{2+p} \right) \left[ \mu_{y_2} + (1 + r_{B_2}(B_2))B_2 + \frac{\mu_{y_2} + (1 + r_{B_2}(B_2))(E_2 \{B_2\} + A_3)^2}{1 + r_3^{ss}} \right]$$

That is,

$$r_3^{ss} = \frac{\mu_{y_2} + (1 + r_{B_2}(B_2))(E_2 \{B_2\} + A_3)^2}{\left( \frac{2+p}{1+p} \right)\mu_{y_2} - \left[ \mu_{y_2} + (1 + r_{B_2}(B_2))B_2 \right]} - 1$$

The assumption that the sudden stop rate is always the one that leaves zero borrowing optimal is a simplification, but without any loss of generality. All interest rates between the maximum sudden stop rate calculated here and the no-shock rate would allow some borrowing, but not as much as before. Thus, the maximum sudden stop rate defines the lower bound on utility while the no-shock rate defines the upper bound. All other rates allow for a utility level somewhere in between. For rates exceeding the maximum sudden stop rate borrowing is still zero. So the “sudden stop rate” here can also be thought of as the minimum rate required for zero borrowing.
FIGURE 6.A. FULL VERIFICATION CONSUMPTION FOR AN UNANTICIPATED SHOCK

FIGURE 6.B. ZERO PRIVATE VERIFICATION CONSUMPTION FOR AN UNANTICIPATED SHOCK
It is a picture like that in Figure 6.A or Figure 6.B that many authors probably have in mind when recommending longer-term borrowing as a precaution against sudden stops as well as when justifying capital controls. The consumption path when only long-term borrowing (“A only “ in Figure 6.A and Figure 6.B) is available guarantees individuals a consumption path and level of utility that does not depend on the sudden stop.

With full verification when both assets are available (“Both” in Figure 6.A) the hit of the sudden stop is not as bad as when only short-term borrowing is available (“B only” in Figure 6.A). This is due to the presence of long-term holdings in the individual’s portfolio. Notice also that date three consumption is highest for “B only” because in such a world individuals have nothing to repay at date three. This is not necessarily good when considered relative to higher consumption at other dates. Higher date three consumption contributes the least to lifetime utility.

Again, when there is zero private verification, the difference between the “Both” and “B-only” paths in Figure 6.B is nearly zero. The damage, however, is worse than in the full verification case since individuals hold too much short-term debt which is precisely the asset hit hardest by the sudden stop.

III.2.3 Anticipated Shock

Figure 7.A and Figure 7.B show how individuals take account of their full anticipation of the shock. While the “A only” case is the same as before, utility is higher here in the “Both” and “B only” cases than it was in those cases when the shock was unanticipated. By assigning positive and accurate probabilities to the sudden stop, individuals are able to obtain the ex post optimal consumption path. The more accurate the probability assignment (here it has 100% accuracy while the unanticipated case was “completely inaccurate”), the better. It is in this sense that one policy implication of this model is that better information for domestic residents about international lenders (i.e., a more accurate estimates of \( \lambda \)) is welfare enhancing.
Because individuals at date one anticipate the inability to borrow short-term at date two, when free to choose (i.e., the “Both” case) they adjust their relative holdings at date one and hold more of the long-term asset. This diminishes the qualitative differences between the full verification and zero private verification environments as can be seen by comparing Figures 7.A and 7.B.
The major message of this chapter is best seen in Figure 8.A and Figure 8.B. For a moment ignore $U(\text{Both})$ and consider instead the $U(\text{B-only})$ versus $U(\text{A-only})$ comparison. This is the one commonly made in the literature. Standard procedure is first to conduct the entire analysis for the $U(\text{B-only})$ case only. Clearly when looking at sudden stops, the outcome for the B-only case is quite bleak. The next step is generally to point out that $U(\text{A-only})$ would be obtained if all debts were converted to a longer-maturity. The analysis usually stops there because it is clear that long-term financing guarantees a utility level that is always better than that obtained with short-term financing when sudden stops are present. This, however, is not the correct comparison and wrongly implies that in such a world, capital controls that ban short-term borrowing are generally to be recommended.
When considering banning short-term capital flows, the correct comparison is between $U(\text{Both})$ and $U(\text{A-only})^{14}$. The case for ex ante capital controls disappears under this comparison. The only case when an argument for banning short-term capital remains is when the shock is unanticipated. Then ex ante if one assumes that the government has better information than individuals, the optimal policy would be for the government to announce its expectations and leave individuals free to allocate their resources as they choose. If individuals do not believe the government and the government has no other policy tool available, then they best enhance welfare by banning short-term borrowing.

The complete picture then tells us that while exclusively borrowing long-term can completely avoid a sudden stop, it is not always optimal to do so. Both assets should generally be allowed so individuals can reduce their borrowing costs in general, independent of whether sudden stops are possible or not.

The difference between the full verification (Figure 8.A) and zero private verification (Figure 8.B) cases is only that when there is no sudden stop (neither anticipated nor realized) then, because individuals choose to hold so little of the long-term asset it appears in the simulation that individuals are indifferent between “Both”, “A-only” and “B-only”. None of the policy implications or other results are in any way affected.

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14 $U(\text{B-only})$ is actually the case where long-term flows are banned.
FIGURE 8.A. FULL VERIFICATION LIFETIME UTILITY FOR THE THREE CASES STUDIED

FIGURE 8.B. ZERO PRIVATE VERIFICATION LIFETIME UTILITY FOR THE THREE CASES STUDIED
III.3 Conclusion

Long-term borrowing helps, but is not a saving grace that would allow economies to emerge with maximum utility, unscathed from sudden-stop-type crises. Furthermore, in a world with information asymmetries, individuals do not optimally drop all short-term borrowing to avoid crises, even when the crisis is fully anticipated.

The implication, of course, is that banning short-term capital flows would not generally be optimal. In particular, it was shown that $U(\text{Both}) > U(\text{A-only}) \geq U(\text{B-only})$ where there is no sudden stop or when it is fully anticipated. While banning short-term flows would insulate the economy from shocks, it would not generally be utility maximizing. Again the unanticipated case when government is not believed by the public and has no other policy tool is an exception.

An additional implication is that international policies that increase information availability are beneficial in this framework. In this framework, the first best policy would actually eliminate the information asymmetry driving the need for risk premia in general. Second best policies then might aim at moving from zero private verification to full verification. This is in general agreement with the widely-held view that rating agencies for countries and other efforts to improve information on the international lender's side are to be encouraged. The effects of such efforts in this model would be to eliminate the risk premia charged, which is always welfare enhancing. As a complement to better information for international investors, this paper also argues that there is a need for better information for domestic borrowers about the international lenders. In this model, that would mean the provision of accurate information about the probability of a sudden stop so that individuals could better choose their optimal plans. This is consistent with the recommendations of the IMF (1999, 2000, and 2001) that there be greater transparency on the part of international lending organizations in order to better ascertain their portfolio positions and potential for herd-type behavior. Additionally, this model implies that if a government is concerned about the possibility of sudden stops,
welfare would best be increased if the government publicly announced its expectations and allowed individuals to adjust on their own.

A further lesson learned by including long-term borrowing as an option is that models focusing solely on short-term borrowing miss completely the role of savings. This short-term lending by long-term borrowers means two things. First, that there is an asymmetric effect between shocks to the world borrowing rate charged to a specific country and shocks to the world real interest rate that has largely been overlooked by theoretical models. And, second, the effect that saving has on balance of payments accounts and measures of current account sustainability can be important. When savings occurs, or can occur, it lowers the net amount borrowed short-term at some dates. This lowers the current account. Further research on this effect could prove useful in explaining why many Asian economies had small current account deficits and even some surpluses, yet were still caught by the crises of 1997, as noted by Calvo (1998) and others.

Finally, this provides a simple and tractable means of determining jointly the term structure of interest rates and optimal maturity structure of private debt endogenously and within a framework based on Obstfeld-and-Rogoff (1999) - type models currently popular in the open economy literature. Appendix E presents results for the cases in this chapter but when term premia are also charged.. Chapter IV presents a different longer-horizon modified version of the model with more assets.
CHAPTER IV
ENDOGENOUS MATURITY BUNCHING: SUDDEN STOPS OF UNKNOWN DURATION

One issue raised somewhat parenthetically in the literature on sudden stops, although addressed directly by Calvo and Mendoza (2000) and Calvo, Izqueirdo and Talvi (2002), is that the damage from sudden stops can be made worse if a country chooses repayment dates for its debt that are all bunched together, a phenomenon called “maturity bunching”. If the maturities are bunched around a date that happens to occur while a sudden stop is in effect, then the impact of the shock is much larger than normal because the amount being repaid is so much larger.

Calvo and Mendoza (2000) write that

“[b]unching of debt maturity at short maturities is a serious problem under imperfect information and lack of government precommitment... In particular, maturity bunching opens the possibility that contagion triggers a major financial crisis if contagion hits when a large debt stock is due for refinancing. Combined with the sudden-stop effect, this suggests that preventing maturity-bunching can be welfare-improving. The trade-off is that lengthening debt maturity generally increases debt-servicing costs. This is an important topic for future research.”

The reasons given for why maturity bunching might occur are generally not treated analytically. This chapter extends the horizon of the model from the previous chapters to allow individuals to choose from a greater number of maturities (i.e., more than two). With a longer time horizon, a more accurate description of the timing and duration of the sudden stop is needed. It is shown that maturity bunching occurs as the utility maximizing outcome when individuals are uncertain about the duration of a sudden stop.
IV.1 The Model

In the previous chapters, individuals maximized expected lifetime utility over three dates. They had access to two assets, a short-term asset which was to be repaid one date after contracting and a long-term asset which was to be repaid after two dates. Sudden stops had the specific meaning that, when they occurred, the interest rate on all new borrowing at date two increased so that the quantity demanded of new funds was zero. This specified the timing of the shock as occurring at date two. The matter of duration was irrelevant since the second date was the last date during which individuals had the chance to borrow new funds. Allowing the time horizon to be infinite leaves open the question of the timing and duration of the shock as well as how many assets individuals can access.

Letting $T$ denote the longest duration asset available, the date $t$ financing constraint for individuals becomes

\[
\sum_{s=t+1}^{T} B_s = Y_t - C_t + \sum_{s=0}^{t-1} (1 + r_t(D_t))^{t-s} B_s + (1 + r^{\infty}_{t-1}) S_{t-1} - S_{t+1} \tag{58}
\]

where all state variables are prescripted by the date at which they were contracted and post scripted by the date of their maturity. Thus, the left hand side of the constraint shows all the new assets chosen at $t$ to be repaid at all dates in the future from $t+1$ to $T$. Generally one thinks of $T$ as being some finite number, but technically it could be infinity (a perpetuity) as well. $Y_t$ is the individual’s constant endowment, $C_t$ represents consumption of the traded good, $D_t$ is the total stock of debt outstanding at the repayment date as in Chapter III and $S$ is one-date savings assumed to yield the world real interest rate.

Individuals maximize the following concave and time separable expected lifetime utility function subject to constraint (58).
As in Chapter III, a fundamental asymmetry of information between domestic borrowers and international lenders is assumed to exist. However, in this chapter, only the full verification case will be considered.

Interest rates are determined according to the following equation

\[ r_t \equiv r_{t,B} \left( B_{t+1} + \frac{1}{1+\rho} B_{t+2} + \phi(x_t) \right) \]

where \( \phi(x) \) is the time-dependent risk premium discussed below and \( D_t = B_{t+1} + \frac{1}{1+\rho} B_{t+2} + \phi(x_t) \) is the total stock of outstanding funds at repayment date \( t \).

The major difference between this and previous chapters is that with longer horizons, it makes less sense to assume that the risk premium is constant over time. That is, from the perspective of date \( t \), there is actually greater uncertainty concerning the exact level of date \( t+6 \)'s endowment than there is concerning date \( t+1 \)'s endowment. This is true because even though the mean is constant, the variance need not be the same (assuming, the endowments follow, say, a pure random walk). As a result, in this chapter it will be assumed that lenders charge a higher premium, \( \phi \), on longer-maturities and that the premium grows linearly over time, \( \phi(t) \). For an example of this same term premia in the exact model of Chapter III, see the Appendix E where Chapter III cases are reconsidered with term premia.

This structure of the risk premium provides a convenient way to limit the number of assets actually held by individuals. Borrowing in this model, as in the previous chapters, is driven by the difference between the rate of time preference, \( \rho \), and the interest rate charged for borrowing international funds, \( r \). That at \( r = \rho \), the quantity of funds demanded by borrowers is zero is exploited here. The time-dependent risk premium used in this chapter will be
where $x$ is the time away from the current date and $T+1$ is the date after the longest maturity date which will still be feasibly held. For example, suppose an individual at date one is choosing an asset and the time-dependent risk premium increases so that for this individual $r = \rho$ for a six-date asset. Then, for $t = 1$, if the individual were to look at a six-date asset, $iB_7$, with $D_1 = 0$ for simplicity (i.e., assume no other debt will ever be held), the interest rate, setting the world interest rate equal to zero, would already be $\rho$:

(62) \[ r(iB_7) = r^w - \gamma D_1 + \varphi(6) = 0 - \gamma 0 + \frac{6}{6} \rho = \rho \]

and the amount of $iB_7$ desired at this rate is zero. But, a five-date asset, $iB_6$, is affordable in small quantities at $r = -\gamma iB_6 + \frac{5}{6} \rho$. Given zero initial debt the cheapest asset available to individuals, given this positive time-based risk premium is the shortest maturity possible, $iB_{t+1}$, since the time-based risk premium is only $\frac{5}{6} \rho$. It is important to keep in mind here that this entire chapter is in terms of anticipated borrowing levels and is all considered in expectation terms from the perspective of date 1. In the future, when the actual state of nature has been realized, given the levels of the state variables, an interest rate just equal to the subjective rate of time preference may not be sufficient to stifle all new demand. To see exact examples of that one only need look at Appendix D to Chapter III. There the interest rate required for a full sudden stop where new borrowing is zero is frequently greater than the rate of time preference. In this chapter, the term premia will be defined such that four-date maturing assets would not be held initially. It will also be assumed that the longest maturity available to this economy is a three-date asset. Cause and effect should not be inferred between these two assumptions.

Subject to the above cause and effect caveat, in general, one of the more appealing interpretations of this approach to modeling multiple maturities is that it says
all maturities are available to everyone all the time. The only difference between
countries in this environment is how steep the slope of their time-based risk premium is.
If it is relatively flat \((T+1)\) is large) then they will generally hold longer term assets than a
country with a relatively steep slope (small \(T+1\)). The question becomes how lenders
determine the slopes. Some possible explanations might be history of interaction with
the borrowing country, credibility of the borrowing country, and other information
regarding borrowers’ ability to repay over time. Determining how the slope is actually
determined is left for future research.

IV.2 Solving the Model

Individuals maximize \((59)\) subject to \((58)\) and \((60)\). The conditions required for
optimality in for this problem are:

\[
\begin{align*}
(63) \quad & u'(C_t) = E_t \left\{ u'(C_{t+1}) \left( \frac{1}{1 + \rho} \right) \left[ 1 + r^w \right] \right\} \\
(64) \quad & u'(C_t) = E_t \left\{ u'(C_{t+1}) \left( \frac{1}{1 + \rho} \right) \left[ 1 + r_{i,t} (D_{t+1}) + \frac{\partial r_{i,t} (D_{t+1})}{\partial B_{t+1}} B_{t+1} \right. \right. \\
& \quad \left. \left. + 2 r_{i,t} B_{t+1} \left( 1 + r_{i,t} (D_{t+1}) \right) \frac{\partial r_{i,t} (D_{t+1})}{\partial B_{t+1}} + 3 r_{i,t} B_{t+1} \left( 1 + r_{i,t} (D_{t+1}) \right) \frac{\partial r_{i,t} (D_{t+1})}{\partial B_{t+1}} \right] \right\} \\
(65) \quad & u'(C_t) = E_t \left\{ u'(C_{t+2}) \left( \frac{1}{1 + \rho} \right) \left[ 1 + r_{i,t,2} (D_{t+2}) \right]^2 \right. \\
& \quad \left. + 2 \left( 1 + r_{i,t,2} (D_{t+2}) \right) \frac{\partial r_{i,t,2} (D_{t+2})}{\partial B_{t+2}} B_{t+2} \right. \\
& \quad \left. + \frac{\partial r_{i,t,2} (D_{t+2})}{\partial B_{t+2}} r_{i,t,2} B_{t+2} + 3 r_{i,t,2} B_{t+2} \left( 1 + r_{i,t,2} (D_{t+2}) \right) \frac{\partial r_{i,t,2} (D_{t+2})}{\partial B_{t+2}} \right] \right\} 
\end{align*}
\]
Because some assets have maturity lengths greater than one date, standard numerical methods based on the Bellman equation are not useful. Bellman-based dynamic programming relies on the one-date Euler equation which does not correctly reflect the optimality conditions required in the multiple-maturity environment, as is shown in equations (65) and (66). As a first pass attempt to simulate this model, the simulations in this paper were done for finite horizons with discretized control and state space. Lifetime utility for individuals is calculated, given all their budget constraints, over all possible combinations of state variables. The combination that yields the highest lifetime utility is selected. All simulations were done using Excel 2000’s Visual Basic for Applications (VBA).

That the borrowing levels rise and then fall reflects both the desire to bring future endowments forward in time (i.e., impatience relative to the market) and that the transversality condition, \( B_{T+1} = 0 \), is binding. This prevents individuals from non-credibly borrowing infinite amounts that are never repaid, Ponzi financing.

IV.3 Sudden Stops

The no-sudden stop world is not really the case of interest. The real question is what maturity structure individuals chose when there is a sudden stop. This section examines that question by looking at numerical simulations over a finite horizon version of the model. Analyzed first is the allocation during a permanent and fully anticipated sudden stop known to start at date \( t = 1 \) and, then, during a sudden stop known to start at date \( t = 1 \) but with an unknown duration.
IV.3.1 Permanent Sudden Stop

When the sudden stop is fully anticipated and permanent, individuals allocate their maturities so as to minimize interest payments and spread out the pain of repayment over several dates. This is shown in Figure 9.A and Figure 9.B. There are two cases to consider here. First, for both the term premia and no term premia cases, due to their impatience relative to world markets, individuals desire a downward-sloping and smooth consumption path (see Appendix F for the consumption paths). Second, for the term premia case, individuals trade off consumption smoothing against the cost of borrowing longer-term assets. In both cases, to the outside observer it would appear that individuals “foolishly” bunch up their debt around a date (date three in Figure 9.B and date two in Figure 9.A) when they know the sudden stop will be in force. This occurs endogenously without resorting to exogenous influences like government distortions, lack of commitment, etc., highlighting that these influences need not be the first places to look when searching for the causes of maturity bunching.
FIGURE 9.A. NO TERM PREMIA BORROWING AND PERMANENT ANTICIPATED SUDDEN STOP

FIGURE 9.B. TERM PREMIA BORROWING AND PERMANENT ANTICIPATED SUDDEN STOP
IV.3.2 Sudden Stops of Unknown Duration

Suppose now that the sudden stop is known to begin at date one but its duration is not known with certainty. Individuals must now form expectations over the probability of the sudden stop’s continuation. Let $s$ denote the sudden stop’s level effect on the base interest rate.

For simplicity, assume that individuals assign a constant probability weight to the sudden stop remaining in effect once it has begun. Denote as $\lambda_t$ the conditional probability that the sudden stop will still be in effect at date $t$, given that it was in effect at date $t-1$. That is,

$$(67) \quad \lambda_t = \Pr(SS_t | SS_{t-1})$$

Since this probability is kept constant, $\lambda = \lambda_t$ for all $t$. Individuals now maximize lifetime expected utility subject to the constraints in (68) below. These show that individuals have access to all three assets at all dates except four and five since the horizon is finite. The sudden stop occurs at date two for certain and shifts the supply schedule of funds to the left by the amount $s$ which is sufficient to set the quantity of new funds demanded at date two to zero. To minimize notation, in equations (68) the expectations operator has been dropped, but all borrowing levels contracted after date one are actually those levels expected from the perspective of date one. That this is so is reflected in the interest rate equations where the standard interest rate is modified by the conditional probability that a sudden stop remains in effect. Note also that when the sudden stop ends $s = 0$, so the probability terms associated with this state of the world drop out of the equations and allow the interest rates to be written in terms of $\lambda$ and $s > 0$ only.
Figure 10.A and Figure 10.B show the borrowing levels individuals select for different conditional probability weights, \( \lambda \), assigned to the sudden stop’s continuation when no term premia are charged. In these figures, the expectations operator has been included notationally to remind the reader which are expected and which are actually realized values. That the assets contracted at date one and two are not expected but actual reflects that individuals know that the sudden stop starts at date two and are only uncertain about the duration.

The different levels of \( \lambda \) show how individual allocations are influenced by expectations over the duration of the sudden stop. For a low \( \lambda \), say 0.5, the expected interest rate is low and rapidly falling over time. As a result, thinking the chances that the sudden stop is over by date three are high enough, individuals avoid any repayment during date two when the stop is known to be in effect for sure.

As the conditional probability rises, the allocation approaches the fully anticipated permanent sudden stop allocation. For longer horizon problems, a larger difference in expected later-date borrowing levels would emerge – as long as \( \lambda \) is less than 1 – than it does in the six-date version of the problem presented here. Appendix F includes the intermediate steps toward convergence with \( \lambda = 0.6 \) and 0.8.
FIGURE 10.A. NO TERM PREMIA BORROWING LEVELS WITH $\lambda = 0.5$

FIGURE 10.B. NO TERM PREMIA BORROWING LEVELS WITH $\lambda = 0.7$
Figure 11.A and Figure 11.B show the maturity structure when term premia are charged. Now individuals weigh postponing repayment by one additional date due to lower expected interest rates against the higher term premium associated with doing so.

The affect is that individuals now “bunch” their maturities around an earlier date (date three). Adding an additional cost to the term premia then generally has two effects. First it lowers welfare by increasing financing costs and, second, it shifts the bunching date backwards to earlier dates. Qualitatively though the results and policy implications remain unchanged because “bunching” still occurs and occurs during dates when individuals assign positive probability weight to the sudden stop occurring, contrary to casual intuition.

FIGURE 11.A. TERM PREMIA BORROWING LEVELS WITH $\lambda = 0.5$
IV.4 Conclusion

This chapter has outlined an approach to modeling economies with multiple maturing assets available to them. In particular, the first challenge is simply to find a means of making the problem solvable. The approach used here was to allow individuals access to all maturities, but to add a time-dependent component to the risk premium charged so that longer-term assets were more expensive than short-term assets. This is intuitively appealing since it seems reasonable that longer maturity structures are not observed to be used as often in reality not because they don’t or can’t exist, but rather because they are more expensive and, in maximizing their utility subject to some budget constraints, individuals choose the cheaper assets instead. This approach has one technically appealing feature as well. It places natural and finite boundaries on the number of assets under consideration.

The main research question addressed here is how maturity bunching can arise. Casual intuition suggests that there is something “irrational” about maturity bunching or
that it occurs for some external reason like distortions from government intervention or the inability to commit. The results from this model are that maturity bunching arises endogenously when the duration of the sudden stop is not known or when the duration is known and is longer than the longest maturity length affordably available to individuals. There is no need to resort to stories about moral hazard or government commitment to obtain maturity bunching as the optimal result when capital inflows can suddenly stop. Furthermore, instead of preventing maturity bunching to enhance welfare, as Calvo and Mendoza (2000) suggest, this chapter suggests that the bunching patterns are already optimal responses and might better be thought of as signaling borrowers’ beliefs about the duration of a sudden stop.
CHAPTER V
CONCLUSION

This dissertation is devoted to developing models of economies with access to multiple maturing assets but that is subject to sudden stops of capital inflows. The major intended contribution is in returning to a basic framework within which one can begin to think meaningfully about sudden stops and what happens when individuals have access to assets of maturity lengths longer than one date, the standard asset currently used in the literature. It is shown that doing this yields a number of results.

First, it is consistently true in the models presented in this dissertation that capital controls generally decrease welfare. In Chapter II, only one asset is available. Capital controls would mean raising the cost of borrowing the sole asset or forcing individuals into autarky. Both of these results unambiguously lower welfare. In Chapter III individuals have two assets and choose to hold both to minimize total interest payments when there are no sudden stops. When there is a sudden stop, individuals would still like to hold both assets to minimize interest payments, but have an additional incentive to spread out the pain of repaying given the weights they assign to utility at different dates. Thus restrictions on this choice set by either eliminating or making more costly either asset generally lowers welfare. In Chapter IV, the story is similar. Restricting the choice set always lowers welfare. Thus, throughout this dissertation, the case in favor of capital controls due to volatile international capital markets where sudden stops are possible is not well supported. Again, a single case in Chapter III emerges when the shock is unanticipated, the government has better information than individuals about the sudden stop occurring, individuals refuse to believe the government and the government has no other policy option. In that single case, capital controls banning short-term capital can raise welfare relative to doing nothing at all.

Second, most of the literature looks at models with only short-term assets yet frequently discusses the importance of switching to longer-term maturities. By actually modeling economies with longer-term assets, it is shown in Chapter III that the standard
approach is really comparing constrained problems while the multiple-maturity models presented here better capture the unconstrained problem. Chapter IV further highlights this point since issues of maturity bunching can not even be discussed in models with assets of only one maturity length.

Finally, the hope is that this dissertation is one step in the right direction. Now that the framework for handling multiple maturing assets is elaborated and some of the issues better understood, future work will hopefully incorporate the analytical structure or at least insights from here into more complicated models with money, exchange rates and production to address broader questions of interest.
REFERENCES


Following Eaton and Gersovitz (1981), lenders are risk-neutral, can always lend elsewhere at the risk free world interest rate, $r^w$, and therefore will only make loans which guarantee them an expected rate of return at least as high as the riskless world real interest rate. That is

\[(A.1) \quad \left[1 - \sigma(R(b_t))\right]R(b_t) = (1 + r^w)b_t\]

where $b_t$ is the amount borrowed at date $t$, $R(b_t)$ is the return to lenders on borrowing which is generally equal to $(1+r)b_t$ (i.e., the interest rate charged plus the principle). The probability of default, $\sigma$, is increasing in $R(b_t)$. As Eaton and Gersovitz (1981) show, if $\sigma$ is also differentiable, then $R(b_t)$ is defined by the differential equation

\[(A.2) \quad R'(b_t) = \frac{1 + r^w}{1 - \sigma(R) - \sigma'(R)R(b_t)}\]

since the probability of default when borrowing is zero is also zero, then substituting $\sigma(0) = 0$ into the differential equation yields

\[(A.3) \quad R'(0) = 1 + r^w\]

which says that loan amounts near zero are charged the riskless world interest rate and therefore is rising for all nonzero borrowing amounts.
APPENDIX B
SUDDEN STOPS AND THE DOMESTIC LOANABLE FUNDS MARKET

Borrowing long-term exclusively in Chapter III’s model implies lending short-term. For a shock to world interest rates, therefore, there is a range of sudden stop interest rates for which long-term borrowers can lend domestically to those borrowing short-term. This appendix deals with the special case when this market emerges. The presence of long-term financing options in this case increases welfare because some portion of domestic residents avoid the sudden stop and, for a range of sudden stop interest rates, an additional gain is made by some that further increases aggregate welfare beyond what is normally expected.

B.1 The Model

When there are no exogenous shocks and no risk premia individuals are indifferent between short and long-term financing. Interpret $\alpha$ as the percent of the domestic residents, normalized to unity in aggregate, financing with short-term borrowing. Let pre-sudden stop equilibrium variables be denoted with a superscript “o” and sudden stop variables be superscripted with “ss”.

Sudden stops are a rise in the date two borrowing rate. The exact interest rate is, however, no longer assumed to be the maximum one that keeps borrowing from dates two to three optimally at zero. Now it will be within a range where the upper bound is the zero-borrowing rate and the lower bound is the no-sudden-stop rate. This allows a wider range of results to be examined. Denoting the sudden stop interest rate, $s$, we have

$$s \in (s - \min, s^*) \text{ where } s - \min = r_{ss} \text{ and } s^* \text{ solves } B^s_3 = 0$$
where \( r_{B3} \) is known from date one onward and \( B_3^{ss} \) is the optimal level of short-term borrowing given the new interest rate, \( s \). The left bracket is open because \( s = s-min \) is not considered a sudden stop.

**B.1.1 Short-Term Financing Response**

The sudden stop primarily affects short-term borrowers who need to roll over their debt at date two and consume. The effect of the sudden stop is that individuals can not borrow as much as anticipated. When the sudden stop strikes those financing consumption through short-term borrowing must re-optimize given the new interest rate. They now take date one debt plus the accrued interest as given and maximize the following two-date utility function

\[
(B.2) \quad \sum_{t=2}^{3} \left( \frac{1}{1 + \rho} \right)^{t-2} u(C_t)
\]

subject to

\[
(B.3) \quad Y_2 + (1 + r_{B2})B_2^0 + \frac{Y_3}{1 + s} = C_2 + \frac{C_3}{1 + s}
\]

the subscripts on endowments are there for tractability's sake. While the endowments themselves are actually equal, the effect of the required repayment is equivalent here to the "effective" endowment at date two being lower than that in the date three. This adds to the desire to borrow and means that even if the sudden stop rate rose all the way to equal \( \rho \), individuals would still borrow as a result of the sloped "effective" endowment path.

The consumption functions resulting from the solution to this problem are
The optimal level of short-term borrowing becomes

\begin{equation}
C_{2,B}^{ss} = \left( Y_2 + (1 + r_{B_2})B_{2}^0 + \frac{Y_3}{1 + s}\right) \left( \frac{1 + \rho}{2 + \rho}\right)
\end{equation}

\begin{equation}
C_{3,B}^{ss} = \left( Y_2 + (1 + r_{B_2})B_{2}^0 + \frac{Y_3}{1 + s}\right) \left( \frac{1 + s}{2 + \rho}\right)
\end{equation}

and represents the individual's demand for short-term funds as a function of $s$ through $C_{2B}^{ss}$.
Figure B.1 shows that higher interest rates result in less borrowing until, at \( s \) near 0.04, borrowing ceases altogether. For consumption, this means that date two consumption is falling as interest rates rise. The negative relationship between the sudden stop interest rate and date two consumption can also be seen by taking a derivative of equation (B.4) with respect to \( s \) or \((1+s)\). In welfare terms, any \( s > r_{B2} \) results in lower utility by lowering \( C_{ss}^{2B} \). \( C_{ss}^{3B} \) rises until it reaches a maximum at the level of date three's endowment.

### B.1.2. Long-Term Financing Response

For the maximum-interest-rate sudden stop, long-term borrowers are unaffected. They obtain their maximum utility from the optimal consumption path. This is the case considered in general.

When the interest rate at date two does not rise to its maximum, however, long-term borrowers need to re-optimize given the new environment since a domestic market for their funds now exists and pays up to the sudden stop interest rate. The interest rate paid to lenders in that market is the sudden stop rate which is higher than the interest rate paid in the world market since the sudden stop by definition only affects this country's borrowers. Thus, for \( s \in (s-min, s^*) \) the relevant problem for long-term borrowers is the following.

\[
\begin{align*}
\max_{C_2, C_1} & \sum_{t=2}^{3} \left( \frac{1}{1+r_t} \right)^{t-2} u(C_t) \\
\text{subject to} & \\
Y_2 + \frac{Y_3 + (1+r_3)^2 A_3^0}{1+s} \frac{C_2}{1+s} &= C_2 + \frac{C_3}{1+s}
\end{align*}
\]
where the superscript “o” on \( A_3 \) denotes that this is a predetermined variable from date two's perspective. This budget constraint differs from the short-term borrowers' constraint because the burden of repayment is shifted forward one date. The implication is that the “effective” endowment path for these individuals is downward sloping, which generates a desire to lend despite the fact that the rate of time preference exceeds the interest rate. Normally, these two desires offset each other and the result is short-term lending in the no-sudden-stop, exclusive-long-term-borrowing equilibrium. Denoting that level of date two lending, \( S_3^o \) (because it is a claim on date three payment), the result of the sudden stop on long-term financers is that the desired \( S^{es}_3 > S^o_3 \). This is because the downward sloping “effective” endowment path and the higher short-term interest rate reinforce each other to generate a greater desire to lend.

The consumption functions resulting from optimization are

\begin{equation}
C_{2,4}^{es} = \left( Y_2 + \frac{Y_3 + (1 + r_A)A_3}{1 + s} \right) \left( \frac{1 + \rho}{2 + \rho} \right)
\end{equation}

(B.9)

\begin{equation}
C_{3,4}^{es} = \left( Y_2 + \frac{Y_3 + (1 + r_A)A_3}{1 + s} \right) \left( \frac{1 + s}{2 + \rho} \right)
\end{equation}

(B.10)

The level of borrowing, \( A3 \) is given, but the amount lent becomes an increasing function of the sudden stop rate, \( s \). This is true up to the maximum \( s, s^* \), at which point the domestic market ceases to exist and the lenders are left in the same position they were in prior to the sudden stop.

The amount saved at date two is

\begin{equation}
S_3^{es} = Y_2 - C_{2,4}^{es} > 0
\end{equation}

(B.11)
Savings is now a positive function of the sudden stop interest rate because by equation (B.9), $C_d^{ss}$ is a decreasing function of the sudden stop interest rate. Figure B.2 displays this relationship.

![Figure B.2. S3 RESPONSE TO SS INTEREST RATES](image)

**B.2 The Domestic Loanable Funds Market**

In the domestic market, equation (B.11) defines the supply of funds as a function of the interest rate. Welfare for lenders will depend on the amount actually traded domestically. This was not the case for short-term borrowers because they always borrow at the rate charged by the world markets. Nevertheless, equation (B.6) continues to represent the demand curve for funds whether they are obtained domestically or from international markets. It is precisely the fact that this is a small open economy that the results in this market are obtained. Borrowers always bid the domestic interest rate up to the world sudden stop rate, but never higher because they
can always turn to world markets for funds. Lenders would be willing to charge a rate up to the world sudden stop rate and competition in this market keeps them from ever charging a rate lower. In order to plot both graphs in the positive orthant of Cartesian space, $-B_3^{ss}$ is plotted in Figure B.3.

![Figure B.3. Domestic Loanable Funds Market](image)

This market functions just like the small open economy loanable funds market found in any basic textbook on the subject. For all sudden stop interest rates below the equilibrium interest rate in this market, there is excess demand. Lenders are thus able to lend all their savings to domestic borrowers at the sudden stop interest rate. Borrowers obtain the excess from world markets.

When the sudden stop rate exactly equals the market clearing equilibrium, the market clears. Lenders sell all of their funds to borrowers and no one ever turns to the world market. This interest rate represents the rate at which long-term borrowers / short-term lenders obtain their maximum utility.
As sudden stop rates rise past equilibrium and up to the maximum rate, lenders can sell quantities up to the demand curve to borrowers. This results in an excess supply of funds which lenders cannot get from world markets because the sudden stop rate is only charged to domestic borrowers. They can still, however, always receive the old lending rate. Thus, they sell all they can to domestic borrowers at the sudden stop rate and the excess supply remains unsatisfied. The reason is that

\[ (B.12) \quad \text{for } s \in [r_{B_1}, s'] \quad -B_3^{ss}(s) - S_3(r_{B_1}) \geq 0 \]

where \( s' \) is the interest rate at which this equality hold. The quantity at which this happens in the above graph is \(-B_3^{ss}(s') = .01604961\) and the interest rate; \( s' = .0398817405 \).

\[ (B.13) \quad \text{for } s \in (s', s^*) \begin{cases} \text{Domestic Lending} = -B_3^{sw}(s) \\ \text{Lending to World} = S(r_{B_2}) + B_3^{sw}(s) \end{cases} \]

At \( s = s^* \), the demand for borrowing is zero. Lenders then have no domestic market to profit from and lend in the world markets at \( r_{B_2} \).

### B.3 Conclusion

The first point worth noting is that although the sudden stop hurts some, it helps others. Under the assumption that one-half of the population borrows long-term and one-half borrows short-term, the maximum gain for lenders is not sufficient to offset the loss to borrowers. The difference is, however, small and in the simulation, the difference in the change in utility (i.e., rise for lenders and fall for borrowers) is only .0009525. It is not clear that this would change if a greater percentage of people borrowed long-term because this would be a simultaneous rightward shift in the supply curve and leftward
shift in the demand curve. Thus, the peak interest rate would fall implying a smaller 
maximum gain while the number of individuals capturing the gain would be higher. In 
the limit, for example, everyone borrows long-term and thus there is no domestic 
market.

The importance of accounting for this gain is that (1) it is unexpected in general 
and unaccounted for in the literature; (2) it represents a re-distribution of wealth within 
the domestic economy; (3) it lessens the aggregate cost of a sudden stop by more than 
anticipated; and, (4), it implies that policies like those in Argentina in 2001 where bank 
accounts were frozen do more damage than expected at first glance because they also 
close down this domestic market through which the aggregate cost of a sudden stop is 
mitigated. That policy is discussed in further detail below.

If the shock is anticipated at all, then the expected rate on short-term borrowing 
rises and everyone switches to long-term financing since there is no cost to doing so. 
Whether a sudden stop occurs or not, the consumption path is guaranteed and no 
problem arises. An alternative extension in this context might be to allow the 
government with some known probability to tax savings (or, in the extreme, freeze bank 
accounts as was done in Argentina in 2001). Then everyone would switch to short-term 
borrowing which doesn't require saving. When both risks are present (sudden stops and 
savings taxes), individuals weigh the relative probabilities of each and the associated 
losses in welfare. They then choose the “better” option. Of course, for a certain set of 
parameters, the risks could exactly offset one another and individuals will again be 
indifferent between the two assets.
APPENDIX C
SIMULATION PARAMETERS FOR THE DOMESTIC LOANABLE FUNDS MARKET IN APPENDIX B

The simulations that generated the graphs in Appendix B were done in Maple 7.

Exogenous Parameters

\[ \rho = 0.03 \]
world interest rates \((r_a, r_s^2, r_s^3, r_b^2, r_b^3) = 0.02\)
\[ Y_1 = Y_2 = Y_3 = 100 \]

Pre-Sudden Stop Values

Consumption Levels Under Either Financing Structure

\[ C_1 = 100.9641852 \]
\[ C_2 = 99.98395039 \]
\[ C_3 = 99.01323243 \]

Short-Term Borrowing

\[ B_2^o = -0.9641852 \]
\[ B_3^o = -0.967419294 \]

Long-Term Borrowing and Savings

\[ A_3 = -0.9641852 \]
\[ S_2^o = 0 \]
\[ S_3^o = .01604961 \]

**Ranges for Sudden Stop Interest Rates (s)**

- \( s\text{-min} = .02 \)
- \( s\text{-max} = .04023034210 \)
- \( s\text{-eqm} = .02989820231 \)

where \( s\text{-min} \) is set equal to the constant world interest rates, \( s\text{-max} \) is the short-term borrowing rate required for \( B_3 = 0 \), and \( s\text{-eqm} \) is the equilibrium sudden stop interest rate that exactly clears the domestic loanable funds market. Any rate below \( s\text{-eqm} \) results in excess demand that is met by world markets and any rate above this results in excess supply that is not met.
APPENDIX D

SIMULATION VALUES AND RESULTS FOR THE COMPLETE MODEL

General Values for Simulations in Maple 7:

\[ t = \{1,2,3\} \]
\[ \mu_{Yt} = 1000 \]
\[ r^w = 0 \]
\[ \rho = 0.03 \]
\[ \gamma = \varphi = 0.0003 \]

D.1. Full Verification Without Term Premia

TABLE D.1. FULL VERIFICATION WITHOUT TERM PREMIA NO SUDDEN STOP

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D.2. Full Verification With Term Premia

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### TABLE D.6. FULL VERIFICATION WITH TERM PREMIA FULLY ANTICIPATED

<table>
<thead>
<tr>
<th></th>
<th>A and B both</th>
<th>B only</th>
<th>A only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_3$</td>
<td>-15.65785</td>
<td>n.a.</td>
<td>-17.19975</td>
</tr>
<tr>
<td>$B_2$</td>
<td>-5.03646</td>
<td>-10.97650</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E{B_3}$</td>
<td>0.00000</td>
<td>0.00000</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.00251</td>
<td>0.00429</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E{r_3}$</td>
<td>0.01880</td>
<td>0.04148</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_{A3}$</td>
<td>0.00670</td>
<td>n.a.</td>
<td>0.00716</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1020.69431</td>
<td>1010.97650</td>
<td>1017.19975</td>
</tr>
<tr>
<td>$E{C_2}$</td>
<td>994.95089</td>
<td>988.97638</td>
<td>1000.00000</td>
</tr>
<tr>
<td>$E{C_3}$</td>
<td>984.13172</td>
<td>1000.00000</td>
<td>982.55307</td>
</tr>
<tr>
<td>Utility</td>
<td>20.12603</td>
<td>20.12569</td>
<td>20.12600</td>
</tr>
</tbody>
</table>

### TABLE D.7. ZERO PRIVATE VERIFICATION WITHOUT TERM PREMIA

<table>
<thead>
<tr>
<th></th>
<th>A and B both</th>
<th>B only</th>
<th>A only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_3$</td>
<td>-0.12206</td>
<td>n.a.</td>
<td>-22.63086</td>
</tr>
<tr>
<td>$B_2$</td>
<td>-22.64546</td>
<td>-22.74759</td>
<td>n.a.</td>
</tr>
<tr>
<td>$B_3$</td>
<td>-22.52340</td>
<td>-22.63789</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.00679</td>
<td>0.00682</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_3$</td>
<td>0.00679</td>
<td>0.00679</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_{A3}$</td>
<td>0.00679</td>
<td>n.a.</td>
<td>0.00679</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1022.76751</td>
<td>1022.74759</td>
<td>1022.63086</td>
</tr>
<tr>
<td>$C_2$</td>
<td>999.72410</td>
<td>999.73506</td>
<td>1000.00000</td>
</tr>
<tr>
<td>$C_3$</td>
<td>977.19987</td>
<td>977.20837</td>
<td>977.06080</td>
</tr>
<tr>
<td>Utility</td>
<td>20.12604</td>
<td>20.12604</td>
<td>20.12604</td>
</tr>
</tbody>
</table>

D.3. Zero Private Verification Without Term Premia
### TABLE D.8. ZERO PRIVATE VERIFICATION WITHOUT TERM PREMIA
#### UNANTICIPATED SUDDEN STOP

<table>
<thead>
<tr>
<th></th>
<th>A and B both</th>
<th>B only</th>
<th>A only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_3$</td>
<td>-0.12206</td>
<td>n.a.</td>
<td>-22.63086</td>
</tr>
<tr>
<td>$B_2$</td>
<td>-22.64546</td>
<td>-22.74759</td>
<td>n.a.</td>
</tr>
<tr>
<td>$B_3$</td>
<td>-22.52340</td>
<td>-22.63789</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.00679</td>
<td>0.00682</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_3^{ss}$</td>
<td>0.05390</td>
<td>0.05414</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_{A3}$</td>
<td>0.00679</td>
<td>n.a.</td>
<td>0.00679</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1022.76751</td>
<td>1022.74759</td>
<td>1022.63086</td>
</tr>
<tr>
<td>$C_2$</td>
<td>977.20070</td>
<td>977.09717</td>
<td>1000.00000</td>
</tr>
<tr>
<td>$C_3$</td>
<td>999.87628</td>
<td>1000.00000</td>
<td>977.06080</td>
</tr>
<tr>
<td>Utility</td>
<td>20.12554</td>
<td>20.12553</td>
<td>20.12604</td>
</tr>
</tbody>
</table>

### TABLE D.9 ZERO PRIVATE VERIFICATION WITHOUT TERM PREMIA FULLY
#### ANTICIPATED SUDDEN STOP

<table>
<thead>
<tr>
<th></th>
<th>A and B both</th>
<th>B only</th>
<th>A only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_3$</td>
<td>-3.74787</td>
<td>n.a.</td>
<td>-22.63086</td>
</tr>
<tr>
<td>$B_2$</td>
<td>-21.23502</td>
<td>-12.8325</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E{B_3}$</td>
<td>0.00000</td>
<td>0.0000</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.00112</td>
<td>0.0038</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E{r_3}$</td>
<td>0.01164</td>
<td>0.0434</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_{A3}$</td>
<td>0.00637</td>
<td>n.a.</td>
<td>0.00679</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1024.98290</td>
<td>1012.83250</td>
<td>1022.63086</td>
</tr>
<tr>
<td>$E{C_2}$</td>
<td>996.24791</td>
<td>987.11810</td>
<td>1000.00000</td>
</tr>
<tr>
<td>$E{C_3}$</td>
<td>978.49356</td>
<td>1000</td>
<td>977.06080</td>
</tr>
<tr>
<td>Utility</td>
<td>20.12607</td>
<td>20.12570</td>
<td>20.12604</td>
</tr>
</tbody>
</table>
### D.4. Zero Private Verification With Term Premia

#### TABLE D.10 ZERO PRIVATE VERIFICATION WITH TERM PREMIA NO SUDDEN STOP

<table>
<thead>
<tr>
<th></th>
<th>A and B both</th>
<th>B only</th>
<th>A only</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₃</td>
<td>0.00000</td>
<td>n.a.</td>
<td>-21.08465</td>
</tr>
<tr>
<td>r₂</td>
<td>0.00759</td>
<td>0.00759</td>
<td>n.a.</td>
</tr>
<tr>
<td>r₃</td>
<td>0.00756</td>
<td>0.00756</td>
<td>n.a.</td>
</tr>
<tr>
<td>rₐ₃</td>
<td>0.00856</td>
<td>n.a.</td>
<td>0.00833</td>
</tr>
<tr>
<td>C₁</td>
<td>1021.97127</td>
<td>1021.97127</td>
<td>1021.08465</td>
</tr>
<tr>
<td>C₂</td>
<td>999.73732</td>
<td>999.73732</td>
<td>1000.00000</td>
</tr>
<tr>
<td>C₃</td>
<td>977.95918</td>
<td>977.95918</td>
<td>978.56281</td>
</tr>
<tr>
<td>Utility</td>
<td>20.12601</td>
<td>20.12601</td>
<td>20.12597</td>
</tr>
</tbody>
</table>

#### TABLE D.11. ZERO PRIVATE VERIFICATION WITH TERM PREMIA UNANTICIPATED SUDDEN STOP

<table>
<thead>
<tr>
<th></th>
<th>A and B both</th>
<th>B only</th>
<th>A only</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₃</td>
<td>0.00000</td>
<td>n.a.</td>
<td>-21.08465</td>
</tr>
<tr>
<td>r₂</td>
<td>0.00759</td>
<td>0.00759</td>
<td>n.a.</td>
</tr>
<tr>
<td>r₃</td>
<td>0.00756</td>
<td>0.00756</td>
<td>n.a.</td>
</tr>
<tr>
<td>rₐ₃</td>
<td>0.00856</td>
<td>n.a.</td>
<td>0.00833</td>
</tr>
<tr>
<td>C₁</td>
<td>1021.97127</td>
<td>1021.97127</td>
<td>1021.08465</td>
</tr>
<tr>
<td>C₂</td>
<td>977.86194</td>
<td>977.86194</td>
<td>1000.00000</td>
</tr>
<tr>
<td>C₃</td>
<td>1000.00000</td>
<td>1000.00000</td>
<td>978.56281</td>
</tr>
<tr>
<td>Utility</td>
<td>20.12553</td>
<td>20.12553</td>
<td>20.12597</td>
</tr>
<tr>
<td></td>
<td>A and B both</td>
<td>B only</td>
<td>A only</td>
</tr>
<tr>
<td>------------</td>
<td>--------------------</td>
<td>-------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>$A_3$</td>
<td>-19.58848</td>
<td>n.a.</td>
<td>-21.08465</td>
</tr>
<tr>
<td>$B_2$</td>
<td>-4.01304</td>
<td>-12.39539</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E{B_3}$</td>
<td>0.00000</td>
<td>0.00000</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.00220</td>
<td>0.00472</td>
<td>n.a.</td>
</tr>
<tr>
<td>$E{r_3}$</td>
<td>0.01358</td>
<td>0.04299</td>
<td>n.a.</td>
</tr>
<tr>
<td>$r_{A3}$</td>
<td>0.00788</td>
<td>n.a.</td>
<td>0.00833</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1023.60153</td>
<td>1012.39539</td>
<td>1021.08465</td>
</tr>
<tr>
<td>$E{C_2}$</td>
<td>995.97811</td>
<td>987.54612</td>
<td>1000.00000</td>
</tr>
<tr>
<td>$E{C_3}$</td>
<td>980.10172</td>
<td>1000.00000</td>
<td>978.56281</td>
</tr>
<tr>
<td>Utility</td>
<td>20.12601</td>
<td>20.12569</td>
<td>20.12597</td>
</tr>
</tbody>
</table>
APPENDIX E

CHAPTER III CASES WITH TERM PREMIA

The intuition behind using term premia is simply that the future is less certain than the present. That longer-term loans tend to have higher premia than shorter-term loans is a generally observed phenomenon and one that is expected to have important implication when the horizon of the model from Chapter III is extended and more maturities are included. A first step at addressing longer-horizon questions is the subject of Chapter IV. For consistency, it is important that term premia also be introduced formally into Chapter III’s model as well. This appendix presents in brief the results from simulating the model in Chapter III where term premia have been included.

E.1. The Model

Formally, the term premia are simply added to the interest rate equation so that longer-term interest rates are higher than shorter-term interest rates even at zero levels of borrowing. Because the intuition is that all terms have some premia, but that they increase with the term length, short-term borrowing is also charged an additional premia, \( \phi(1) \) to denote a “one-date” premium, otherwise not present in the core model of Chapter III. A premium is charged to long-term borrowing and this premium, \( \phi(2) \), exceeds the short-term premium, \( \phi(2) > \phi(1) \).

The only modification to the individual’s maximization problem is the presence of the premia in the interest rate equations.

(E.1) \[ r_B^* = r_B^* (B_t + A_t; r^*, s, \phi(1)) \quad s.t. \quad r_1 < 0 , r_2 > 0 , r_3 > 0 and r_4 > 0 \]

and

(E.2) \[ r_A^* = r_A^* (B_t + A_t; r^*, \phi(2)) \quad s.t. \quad r_1 < 0 , r_2 > 0 and r_3 > 0 \]
The effect of including these premia is that the long-term asset is now inherently more expensive than the short-term asset. This means that in general welfare will be lower in all cases because the cost has risen for obtaining the same funds as before. The damage from the sudden stop is also greater now because now only are funds more expensive, but there is also an incentive to borrow more short-term funds than before relative to long-term funds. Of course, when combined with zero private verification, individuals are “overborrowing” due to the lack of private verification, which also encourages borrowing more short- than long-term, an effect that is enhanced when term premia are added. This is the worst of all worlds considered in Chapter III.

E.2. Simulation Results

The full verification versus zero private verification dichotomy is maintained here, but the discussion of the results is shortened since all of the qualitative results remain. The magnitudes of the levels do change and, as mentioned above, utility is generally lower. The specific values for the simulations presented here can be found in Appendix D.
FIGURE E.1.A. FULL VERIFICATION CONSUMPTION WITHOUT A SHOCK

FIGURE E.1.B ZERO PRIVATE VERIFICATION CONSUMPTION WITHOUT A SHOCK
FIGURE E.2.A  FULL VERIFICATION CONSUMPTION WITH AN UNANTICIPATED SHOCK

FIGURE E.2.B ZERO PRIVATE VERIFICATION CONSUMPTION WITH AN UNANTICIPATED SHOCK
FIGURE E.3.A. FULL VERIFICATION CONSUMPTION WITH AN ANTICIPATED SHOCK

FIGURE E.3.B. ZERO PRIVATE VERIFICATION CONSUMPTION WITH AN ANTICIPATED SHOCK
FIGURE E.4.A. FULL VERIFICATION LIFETIME UTILITY

FIGURE E.4.B. ZERO PRIVATE VERIFICATION LIFETIME UTILITY
APPENDIX F
REMAINING CASES AND CONSUMPTION PATHS FOR CHAPTER IV

This appendix includes the remaining cases of term premia and no term premia for \( \lambda = 0.6 \) and \( \lambda = 0.8 \) as well as the consumption paths for the cases considered in that chapter.

![Borrowing Levels with \( \lambda = 0.6 \)](image)

**FIGURE F.1. NO TERM PREMIA BORROWING LEVELS WITH \( \lambda = 0.6 \)**
FIGURE F.2. TERM PREMIA BORROWING LEVELS WITH $\lambda = 0.6$

FIGURE F.3. NO TERM PREMIA BORROWING LEVELS WITH $\lambda = 0.8$
FIGURE F.4. TERM PREMIA BORROWING LEVELS WITH $\lambda = 0.8$

FIGURE F.5. NO TERM PREMIA CONSUMPTION FOR PERMANENT ANTICIPATED SHOCK
FIGURE F.6. TERM PREMIA CONSUMPTION FOR PERMANENT ANTICIPATED SHOCK

FIGURE F.7. NO TERM PREMIA CONSUMPTION FOR $\lambda = 0.5, 0.6, 0.7, 0.8$
FIGURE F.8. TERM PREMIA CONSUMPTION FOR $\lambda = 0.5, 0.6, 0.7, 0.8$
VITA

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Professional Experience

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Teaching / Research Assistant, Department of Economics
John O. Crane Memorial Fellow
Project Director