Theoretical Study of a Novel Control Method of VAV Air-conditioning System Based on MATLAB

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Abstract: The main purpose of this study is to put forward a novel nonlinear feedback control strategy on controlling indoor air temperature by variable air volume. A dynamic model of a typical room for a VAV air-conditioning system is established. The performance of the novel control strategy is investigated. Simulation of the controlling air temperature, on which the novel strategy is adopted, was carried out based on MATLAB in the VAV system. In order to show that the novel control strategy outperforms conventional PID control, a comparison is made between the performance of conventional PID and the novel nonlinear feedback control strategy. The results show that nonlinear feedback control strategy outperforms a conventional PID control system in terms of celerity, stability and other aspects.

Key words: VAV; differentiable homeomorphism; nonlinear feedback; simulation

1. INTRODUCTION

In order to maintain indoor air temperature at set value, there are two methods in the air-conditioning system. One method is called CAV system, which supply air temperature varies with the room load and simultaneously keeps air volume constant. The other is called VAV system, which the supply air volume varies with the room load and simultaneously keeps air temperature constant. Compared with CAV system, VAV system can save energy by 30%~70% [1], and therefore VAV is widely adopted in air-conditioning system. The performance of VAV system lies on how to vary supply air volume, that is, control strategy of air volume has a strong impact on energy consumption. Nowadays control arithmetic of VAV is designed based on the linear system. Differing from above method, a novel control strategy with differentiable geometry theory of modern math is put forward in VAV system.

2. DYNAMIC MODEL OF AIR-CONDITI-IONING ROOM

In fact air-conditioning room should be considered as a system with distribution parameter. If the model with distribution parameter is adopted, it becomes complex and is not applicable to design controller. To be simple, lump-parameter method is adopted to establish the model of air-conditioning room. Air conditioning systems provide hot or cold air to the zones to meet heating or cooling loads. According to conservation principle of energy, a heat balance on the room can be represented that change rate of energy stored in zone air is equal to the difference between energy entering the room and leaving the room [2, 3].

The equation of heat balance can be written as:

$$Q_{a,in} - Q_{a,out} + Q_{source} + Q_{transfer} = C_r \frac{dT_n}{d\tau} \quad (1)$$

Where:

τ —time, s;

T_n—Indoor air temperature, ;

C_r—Capacity for heat, J/kg;

$Q_{a,in}$—supply air enthalpy, W;

$Q_{a,out}$—enthalpy of the air leaving the zone, W;

Q_{source}—Convective internal loads, W；

$Q_{transfer}$ —Convective heat transfer from the envelope, W。

In equation (1), the difference between the supply air enthalpy and the enthalpy of the air leaving the zone can be formulated as in Equation (2):

$$Q_{a,in} - Q_{a,out} = GC_p(T_s - T_n) \quad (2)$$

Sum of Convective heat transfer from the envelope can be formulated as in equation (3):

$$Q_{transfer} = KA\left(\overline{T}_z - T_n + \frac{\alpha_n}{K}\Delta T_{n,\tau}\right) \quad (3)$$

Where:

G —supply air mass flow rate，kg/s；

C_p —Specific heat, J/kg· ；

T_s —Supply air temperature， ；

K —Coefficient of heat transfer，W/m^2· ；

A —Area of envelope，m^2;

$\overline{T}_z$ —solar-air temperature， ；

α_n —convection coefficient，W/m^2· ；

$\Delta T_{n,\tau}$ —Temperature fluctuation of interior surface， 。

Substituting equation (2) and (3) for $Q_{a,air}$, $Q_{a,out}$ and $Q_{transfer}$ in equation (1), the dynamic equation can be written as equation (4):

$$\frac{dT_n}{d\tau} = -\frac{GC_pT_n + KAT_n}{C_r} + \frac{Q_{source} + KA\left(\overline{T}_z + \frac{\alpha_n}{K}\Delta T_{n,\tau}\right)}{C_r} + \frac{GC_pT_s}{C_r} \quad (4)$$

Taking analyses for equation (4), internal heat gains, including people, electric device and other equipments, and convection heat gain of interior surface of envelope are considered as disturbances. If defining state variable as $x = T_n$, input variable as $G = u$, and disturbance variable as:

$$\xi = \frac{Q_{source} + KA\left(\overline{T}_z + \frac{\alpha_n}{K}\Delta T_{n,\tau}\right)}{\rho VC_v} \quad (5)$$

Equation (4) can be rewritten as:

$$\dot{x} = -\frac{KA}{C_r}x + \left(\frac{C_pT_s}{C_r} - \frac{C_px}{C_r}\right)u + \xi \quad (6)$$

If air temperature is chosen as output variable, output function can be written as:

$$y = h(x) = x \quad (7)$$

Obviously, it is difficult to design controller and analyze control performance of the system described by equation (4). If the system can be transformed into linear system by mathematic method, control strategy will be simplified. It is noted that differential geometry theory is applicable to simplify the system described by equation (4).

3. DIFFERENTIABLE HOMEOMORPHIPHISM TRANSFORM

Firstly, some concepts of differential geometry theory need to be explained.

(1) Lie derivative [4, 5, 6, 7]

Given a smooth vector field $X : M \to TM$ and a smooth function h, Lie derivative of function h with respect to X is defined as: $L_x h(p) = X(h)(p)$.

In local coordinates, Lie derivative of function h to f can be written as:

$$L_f h(x) = \frac{\partial h}{\partial x} f(x) = \sum_{i=1}^{m} \frac{\partial h}{\partial x_i} f_i(x) \quad (8)$$

Secondly, considering a single-input single-output affine system described as in equation (9):

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (9)$$

Where $x \in R^n$ is state variable, f, g is smooth vector field, and h is a smooth nonlinear function. Assumption $x \in U$ (U is open subset of R^n) is made.

In order to study the relationship between input and output, differencing function y with respect to t can be written as:

$$\dot{y} = L_f h(x) + L_g h(x)u \quad (10)$$

Where $L_f h(x): R^n \to R$ and $L_g h(x): R^n \to R$ denote Lie derivative of function h with respect to f and g .

If $L_g h(x)$ is limitary and is not equal to 0, nonlinear feedback rule can be designed as:

$$u = \frac{1}{L_g h(x)}\left(-L_f h(x) + v\right) \quad (11)$$

The result of substituting equation (11) for u in the Equation (10) is shown in equation (12):

$$\dot{y} = v \quad (12)$$

Obviously, through nonlinear feedback transform, nonlinear system is simplified to linear system.

From above, the conclusion can be drawn that for affine system, nonlinear feedback transform can be adopted to realize linear relationship between output and input variable. It differs from local linearized method. Any higher order nonlinear term is not neglected. This method is not only exact but also applicable in the whole operating region.

4. APPLICATION IN VAV AIR-CONDITI-TIONING SYSTEM

Considering the real state space model described by equation (4), it is a single-input single-output affine system. The Lie derivative of $h(x)$ with respect to $g(x)$ and $f(x)$ can be obtained as follow:

$$L_g h(x) = \frac{\partial h}{\partial x} g(x) = \left(\frac{C_p T_s}{C_r} - \frac{C_p x}{C_r}\right) \quad (13)$$

$$L_f h(x) = \frac{\partial h}{\partial x} f(x) = -\frac{KA}{C_r} x \quad (14)$$

If $L_g h(x)$ is limitary and is not equal to 0, linear relationship between input and output vari able can be realized through nonlinear feedback. In point of fact, indoor air temperature is not eq ual to supply air temperature in equation (13), so $L_g h(x) \neq 0$. Thus the system has strict relative rank 1. According to linearization theorem, nonlin ear feedback rule can be designed as:

$$u = \frac{C_r}{C_p (T_s - x)}\left(\frac{KA}{C_r} x + v\right) \quad (15)$$

The result of substituting Equation (15) for u in the Equation (6) is shown in Equation (16):

$$\dot{y} = v \quad (16)$$

Equation (16) shows the real system is transformed into a linear system. Furthermore, for the linear system, conventional control method can be adopted to obtain good performance.

5. SIMULATION

A typical air-conditioning room is chosen as a simulation object. The volume of the typical room is $V = 1440 m^3$. The heat capacity per unit temperature difference is given by $KA = 0.5 kW / K$. The density of air is $\rho = 1.2 kg / m^3$. The specific heat of air is $C_p = 1.01 kW / (kg \cdot K)$. The disturbances contain periodically varying in ambient temperature and internal heat gain, including people, electric device and other equipments. Outdoor disturbance can be simply represented as a first order normal wave, taking the city of Qingdao as example,

$T_w = 27.2 + 1.8\cos(15\tau - 225)$. Internal disturbance can be expressed as: $Q_{source} = 16.4kW + 4.1kW \times random(\cdot)$. Supply air mass flow rate is limited between $2\ kg/s$ to $4\ kg/s$. In order to compare with conventional PID controller, a simulation model based on MATLAB/SIMLINK is established. Nonlinear feedback controller and PID controller are adopted to act on VAV system respectively. The control block diagram is shown in Fig1 and Fig2.

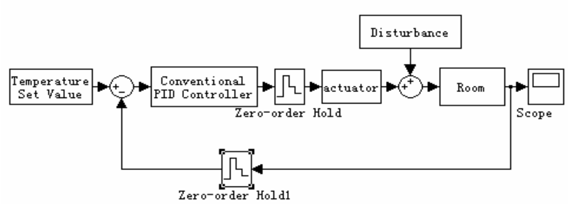

Fig.1 Control diagram of conventional PID

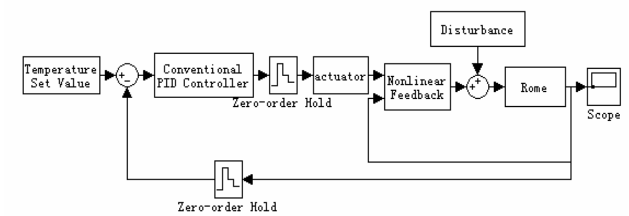

Fig.2 Control diagram of nonlinear

It is assumed that the initial indoor temperature is 30 , and simulation time is 18000s. Initial air set temperature is $T_{set} = 26$; at the time of 4000s air temperature sets to 23 and sets back to 26 at the time of 8000s. Response time, overshoot and steady error are investigated respectively. Using above two control strategies, the system response curves to variety in set temperature are shown in Fig 3.

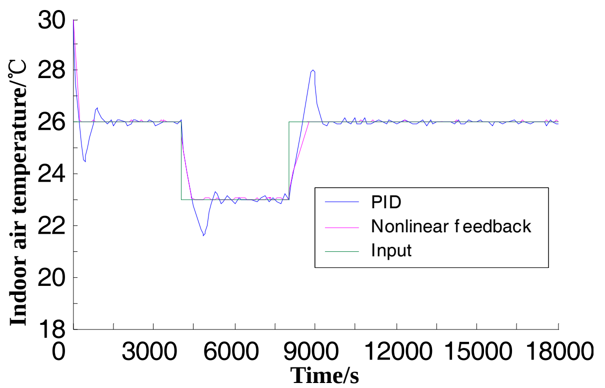

Fig.3 Simulated responses to variety in temperature

6. RESULTS

Fig.3 shows the simulated response of indoor air temperature to variety in set value with conventional PID control and nonlinear feedback control respectively. It is note that oscillation of indoor air temperature takes place about set value at transitional stage using conventional PID controller. But nonlinear feedback controller can drive air temperature to the set temperature with no overshoot. Furthermore conventional PID control system has large overshoot, and indoor air temperature fluctuates largely at the steady state on the condition of the same input signal. It is also noted that rangeability of air temperature is very small and have lesser adjust time using nonlinear feedback control. For the following twice variations in set value, the conventional controller and nonlinear feedback controller can trail after input signal. The nonlinear feedback controller, however, reaches the set value with no overshoot. The conventional PID controller goes to set value with oscillation. It can be concluded that nonlinear feedback controller can trail after the variations in set value and damp the ambient and indoor disturbance more effectively than conventional PID controller.

7. CONCLUSION

Modern mathematic, differential geometry theory, is adopted as analyses tool to simplified air-conditioning system. Through nonlinear feedback single-input single-output linear system can be obtained. Fourthermore conventional control strategy can be adopted to design controller. In order to make a comparison between conventional PID control and nonlinear feedback control strategy, the simulation that applies the two control strategy is carried out. The simulation results show the nonlinear feedback control strategy can provide an excellent means of maintaining environmental conditions in buildings.

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