

FAULT DETECTION AND DIAGNOSIS METHOD FOR VAV TERMINAL UNITS

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Summary: This paper proposes two fault detection and diagnosis methods for VAV units without a sensor of supply air volume, and the results of applying these methods to a real building are presented. One method detects faults by applying a statistical method to four values calculated using the room air temperatures and the demand values of VAV damper opening of each unit during a steady state operation period. From the results of case studies, the method can reduce the number of units to be checked as faulty ones down to 12% of the total number and all the units that really have a fault are included in this group. The other method judges the faults by applying dynamic system analysis to the operational data when the VAV system starts up. From the result of the case studies, the method can reduce the number of units down to 30%, among which five units actually have a fault and only one faulty unit was not included in this group. Both methods can reduce time and cost for commissioning of VAV units significantly by the help of BEMS.

Keywords: Commissioning, VAV terminal unit, Fault detection and diagnosis method

1. INTRODUCTION

Variable Air Volume (VAV) Air Conditioning Systems are widely used all over the world because of the high energy saving feature. However it is reported that the possibility of faults occurrences, such as damper and actuator stuck, and defects in control logic, is unfortunately high¹⁾²⁾. Especially since the number of VAV units installed in large buildings is sometimes more than one thousand, detecting the faults is becoming an important issue. For example in a building, which the authors investigated, all the VAV units are manually and routinely checked every year spending a great deal of budget and manpower and quite many faults have been actually detected.

To cope with this issue, it is needed to develop an automatic fault detection and diagnosis (FDD) technology. In the past a few types of FDD methods, such as, an Exponentially Weighted Moving Average (EWMA) method by J. Seem et al. and an RARX model method by H. Yoshida et al., were proposed³⁾⁴⁾⁵⁾⁶⁾. These methods require the airflow rate of each VAV unit for FDD, however, it is not available in old type VAV units because of no installation of an airflow rate sensor. This paper proposes two kinds of FDD method for this type of VAV units. One method (Method A) is based on a statistical detection of outlier data among control signals of VAV opening ratio during steady state period (non-startup and lunch time period). The other method (Method B) is based on applying dynamic system analysis to startup period data. The both methods are verified using operation data obtained at a real office building.

2. FDD METHOD USING THE DATA OF STEADY STATE PERIOD (METHOD A)

Figure-1 represents a general model of a VAV system that is used to describe proposed FDD methods. The system has an air-handling unit and $M \times N$ zones, each of which has one VAV unit. Each zone is named as $Z(i, j)$ and each unit is named as $U(i, j)$ ($i=1,2,\dots,M$, $j=1,2,\dots,N$). The zones surrounding $Z(1,1)$, for example, are defined as $Z(2,1)$ and $Z(1,2)$. The zones surrounding $Z(2,1)$ are defined as $Z(1,1)$, $Z(2,2)$ and $Z(1,3)$. In the case of the zone $Z(2,2)$, the surrounding zones are defined as $Z(1,2)$, $Z(2,1)$, $Z(3,2)$ and $Z(2,3)$. All units do not have an airflow sensor. This assumption is not proper for the recent VAV products but there are still a great number of AHU systems equipped with old type VAV units like the building studied by present research.

In this paper, two kinds of fault are analyzed, which are fully-close damper and fully-open damper. The reason of selecting these two faults is that the other kinds of faults, for example damper stuck at middle-range positions, were not found in the building that the authors investigated.

2.1 Definition of FDD Parameters

The following four FDD parameters are defined for each unit using the zone air temperature $q_{r(i,j)}$, the set point of room temperature q_{rs} and the

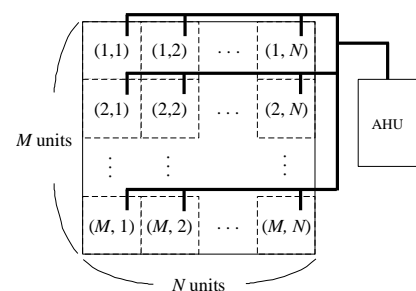


Figure-1 Assumed VAV system

demand control signal of VAV damper opening $f_{(i,j)}$ during steady state period.

a) Difference between zone temperature and temperature set point

$$X_{a(i,j)} = q_{r(i,j)} - q_{rs(i,j)} \tag{1}$$

b) Demand control signal of VAV damper opening

$$X_{b(i,j)} = \frac{f_{(i,j)} - f_{\min(i,j)}}{1 - f_{\min(i,j)}} \tag{2}$$

c) Temperature difference between the analyzed zone and the average of the surrounding zones

$$X_{c(i,j)} = q_{r(i,j)} - (\sum q_{r(k,l)}) / N_{(i,j)} \tag{3}$$

d) Difference between demand signal of VAV damper opening and the average of those of surrounding zones

$$X_{d(i,j)} = f_{(i,j)} - (\sum f_{(k,l)}) / N_{(i,j)} \tag{4}$$

One or all parameters of a faulty unit are expected to be statistically abnormal compared with those of normal units. Namely, in the case of a fully-close damper fault some of them are expected to be larger and in the case of a fully-open damper fault some of them are expected to be smaller during cooling operation.

2.2 Grubbs' Test

In order to detect a fault automatically, a statistical method called as Grubbs' test is applied to the above-mentioned four parameters. This test is a common statistical method to detect outliers among a set of data ⁷⁾. The hypothesis and the Grubbs' test statistic are defined as follows.

- Null hypothesis : There are no outliers in the dataset
- Alternative hypothesis : There is at least one outlier in the dataset

Grubbs' test statistic : $X_e = \frac{|x_{\max} - \bar{x}|}{s}$ (5)

Where, \bar{x} is the sample mean, x_{\max} is a sample with maximum distance from \bar{x} , and s is the standard deviation.

Grubbs' test statistic X_e is the largest absolute deviation from the sample mean in units of the sample standard deviation. When the significance level is set α , the null hypothesis of no outliers is rejected if

$$X_e > X_{SG} = \frac{(n-1)}{\sqrt{n}} \sqrt{\frac{t_{\alpha/2n}^2}{(n-2) + t_{\alpha/2n}^2}} \tag{6}$$

Where, $t_{\alpha/2n}$ is the critical value of the t -distribution with $(n-2)$ degrees of freedom and a significance level of $\alpha/2n$.

This method can check only one outlier per trial. So if x_{\max} is judged as an outlier, it is expunged from the data set and the test is continued until the null hypothesis is accepted. The procedure of Grubbs' test is shown in Figure-2.

Grubbs' test is based on the assumption of normality. That is, it must be verified whether the data set can be reasonably approximated by a normal distribution before applying this test.

2.3 FDD Procedure Using Operational Data of Steady State Period

Based on the above discussion, the FDD method using operational data of steady state period is proposed as follows.

- 1) Record $q_{r(i,j)}$, q_{rs} and $f_{(i,j)}$ during steady state period (non-startup and lunch time period).
- 2) Calculate the average of each parameter defined by Equation (1) to (4), i.e. $X_{a(i,j)}$, $X_{b(i,j)}$, $X_{c(i,j)}$ and $X_{d(i,j)}$, for each VAV unit using the data obtained at step 1).
- 3) Apply Grubbs' test to the samples of $X_{a(i,j)}$, $X_{b(i,j)}$, $X_{c(i,j)}$ and $X_{d(i,j)}$ separately, and detect an extreme value in

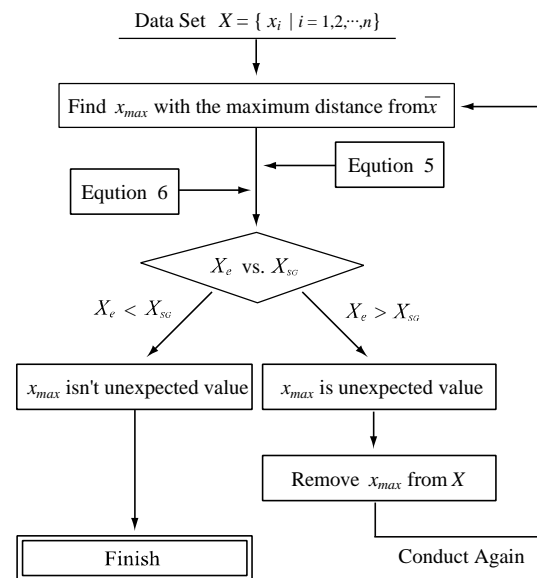


Figure-2 Flow of Grubbs' test

each data set. The VAV unit whose parameter is judged as extreme values is a faulty unit. If the extreme value is larger than the other values, the unit is suspected to have a fully-close damper fault. If the extreme value is smaller than the other, the unit is suspected to have a fully-open damper fault.

3. FDD METHOD USING DYNAMIC MODEL AND DATA OF STARTUP PERIOD (METHOD B)

In this section a FDD method based on dynamic system analysis is proposed because the FDD Method A mentioned in the previous section has a limitation in detecting faults when $f_{(i,j)}$ always sticks to the minimum value. Even though this is a kind of a control or design fault we need to detect hardware faults of VAV units. During the startup time of air conditioning, airflow rate generally begins from maximum and decreases to a certain low level. This period can be defined as a dynamic state and a new FDD method based on a dynamic system analysis can be applied.

3.1 Single-Input Single-Output Dynamic System

A single-input and single-output (SISO) dynamic system is considered as shown in Figure-3. The relationship between the input u and the output y of the system can be expressed by the following equation if taking the system as the first order differential equation by approximation.

$$T_c \frac{dy}{dt} + y = Ku \quad (7)$$

When the initial value of y equals zero, the general solution of Equation (7) is shown in Equation (8). The solution of an impulse response y_i and a step response y_s are expressed by Equation (9) and (10) respectively.

$$y(t) = \int_0^t \frac{K}{T_c} e^{-\frac{(t-t')}{T_c}} u(t') dt' \quad (8)$$

$$y_i = \frac{K}{T_c} e^{-\frac{t}{T_c}} \quad (9)$$

$$y_s = K(1 - e^{-\frac{t}{T_c}}) \quad (10)$$

Figure-4 and Figure-5 show the behavior of the impulse response and the step response. In the Figure-5, K shows the steady state value of y and T_c shows the time constant of the system.

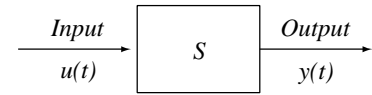


Figure-3 SISO dynamic system

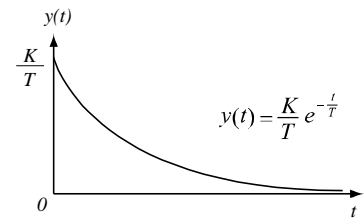


Figure-4 Impulse response

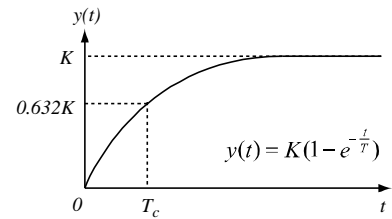


Figure-5 Step response

3.2 FDD Method Using the Response Function

The response of zone air temperature during VAV systems startup period is similar to the step response of SISO system. Therefore the step response Equation (10) is used in this FDD method. The VAV systems startup period is defined as the period during which zone air temperature goes down or up until it becomes steady. The required time is defined as t_s and the steady zone air temperature is defined as $q_r(\infty)$. We define supply air sensible heat as the input and room air temperature as the output. The input u , the output y and the steady state value K are expressed as follows.

$$u(t) = c_p r V_{\max} f(t)(q_r(0) - q_s(t)) \quad (0 \leq t \leq t_s) \quad (11)$$

$$y(t) = q_r(0) - q_r(t) \quad (0 \leq t \leq t_s) \quad (12)$$

$$K = q_r(0) - q_r(\infty) \quad (13)$$

For different VAV systems, the value of $q_r(\infty)$ and t_s are generally different. In order to determine the value of $q_r(\infty)$ and t_s suitable for the systems, the temperature of all measured zones is averaged and the average temperature $\bar{q}(t)$ is approximated using the following equation.

$$\bar{q}(t) \cong ae^{-bt} + c \quad (14)$$

a , b and c of the equation are fitted using the measured temperature by the least square method. As shown in Figure-6, $q_r(\infty)$ is defined as the value of c , and t_s is defined as the time when the difference between $\bar{q}(t)$ and convergent value c becomes 1% of K .

Although the behavior of zone air temperature is assumed to be the step response as shown in Equation (10), it is not the

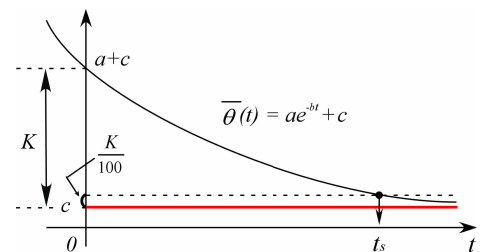


Figure-6 Convergence condition

step response exactly. In order to simulate the behavior of y more accurately, the following equation that combines the impulse response and the supply air sensible heat is used instead of Equation (10).

$$y(t) = \int_0^{\infty} y_i(\mathbf{t})u(t-\mathbf{t})d\mathbf{t} \cong \sum_{j=0}^{\infty} \frac{K}{T_c} e^{-\frac{j\Delta t}{T_c}} u(t-j\Delta t) \tag{15}$$

Using the measured data $\hat{y}(t)$ and $\hat{u}(t)$, T_c is calculated by the least square method. In general, it takes a few minutes that zone air temperature gets down after the VAV system starts up. L is defined as the time delay and T_c is estimated using the operational data after VAV system starts up except the data during the time delay period L (5 ~ 15 min).

$$\min_{T_c} \sum_{t=L}^{t_s} \left\{ \hat{y}(t) - \sum_{\mathbf{t}=0}^{t-L} \frac{K}{T_c} e^{-\frac{t-\mathbf{t}-L}{T_c}} \hat{u}(\mathbf{t}+L) \right\} \tag{16}$$

Because T_c might be different on different days and for different systems, T_c is standardized using the average \mathbf{m}_c and the standard deviation \mathbf{s}_c of a system on a same measured day as shown in Equation (17).

$$X_t = \frac{T_c - \mathbf{m}_c}{\mathbf{s}_c} \tag{17}$$

Faults of VAV units are detected and diagnosed using the difference of X_t between the analyzed unit and the surrounding units. The definition of the surrounding units is the same as the previous section.

$$\Delta X_{t(i,j)} = X_{t(i,j)} - (\sum X_{t(k,l)}) / N_{(i,j)} \tag{18}$$

If a unit has a fully-close damper fault, ΔX_t of the unit is larger than the other units because the speed of response is small. If the unit has a fully-open damper fault, ΔX_t should be smaller than the other units.

3.3 FDD Procedure Using the Dynamic State Data

Based on the above discussion, the FDD method using the dynamic state data is proposed as follows.

- 1) Record $\mathbf{q}_r(t)$, $\mathbf{q}_s(t)$, $\mathbf{f}(t)$ in dynamic state. Recommended data sampling time is 60 seconds.
- 2) Calculate $\mathbf{q}(t)$ and determine t_s and $\mathbf{q}_r(\infty)$ using Equation (14).
- 3) Calculate T_c of each unit using the operational data in t_s minutes after VAV system starts up.
- 4) Calculate $\overline{\Delta X_t}$, the average of ΔX_t in all measured day. If $\overline{\Delta X_t}$ is larger than the other units, the unit might have fully-close damper fault. If $\overline{\Delta X_t}$ is smaller, the unit might have fully-open damper fault.

4. VERIFICATION FOR THE TWO FDD METHODS

The two FDD methods are validated using the data collected from VAV systems in a real building in Tokyo Japan. The information on the VAV systems is described in the following section.

4.1. Information of the VAV Systems for Verification

In order to verify the proposed methods, operational data of the VAV systems in a large office building were collected. The configuration of the building together with duct works, VAV units and the air-handling unit are shown in Figure-7. Every floor is divided into four zones and each zone has a VAV air handling unit (AHU) system (WN, WS, EN and ES). Each AHU system has 15 VAV units (W1, W2, ..., E5). The total number of VAV units in the building is about 1,000 and no unit is equipped with an airflow rate sensor.

Measurement was conducted from August to October 2003. The details of measurement are shown in Table-1. In this

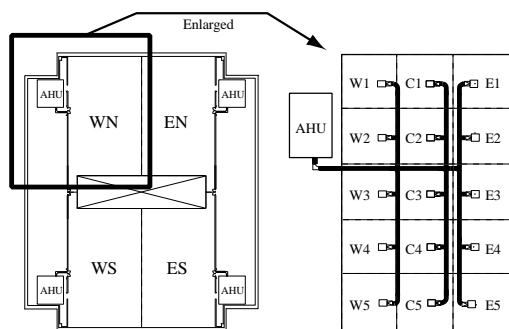


Figure-7 Building plan and VAV system plan

Table-1 Outlines of measured data

Floor	System	Amount of measured data	Real faulty unit	
11th	ES	7 days	W3(close)	
12th	ES	4 days	C3(open)	
13th	EN	3 days	W1(close)	
15th	WS	9 days	W2(open)	
16th	WS	10 days	C3(close)	E5(close)
5th	WN	9 days	none	
6th	WN	5 days	none	
10th	ES	7 days	none	
12th	EN	5 days	none	
13th	ES	5 days	none	

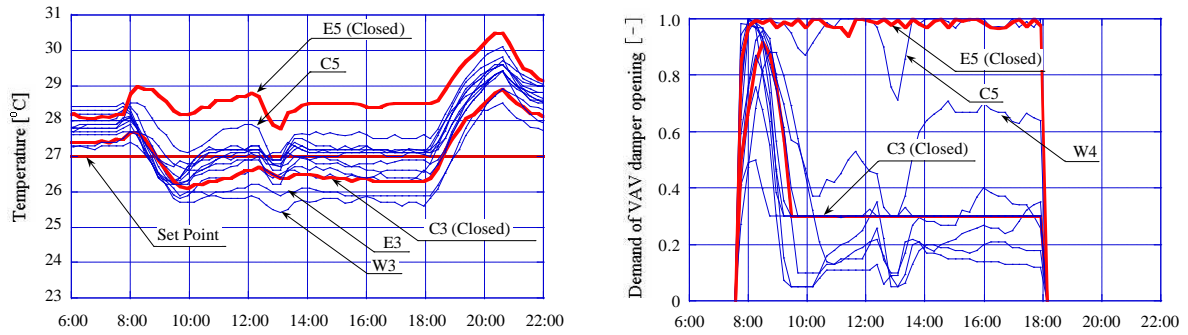


Figure-8 Operational data in WS system on 16th floor (2003/08/29)

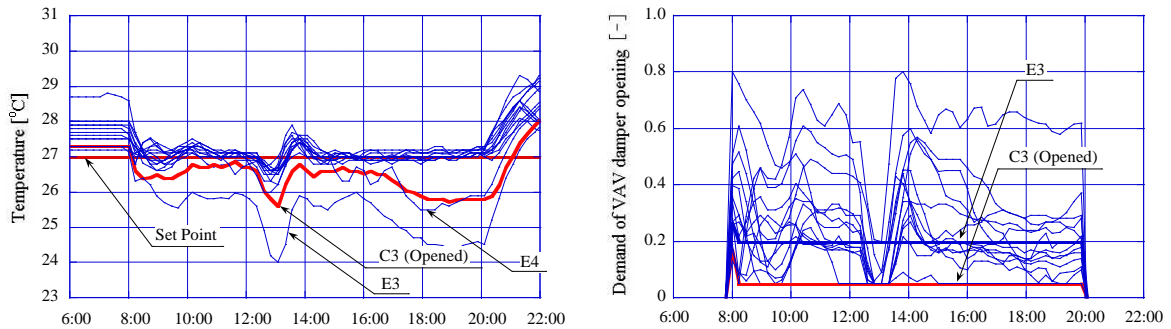


Figure-9 Operational data in ES system on 12th floor (2003/09/09)

building, about 5 to 10 % of the total units have been found to be faulty every year by routine manual check about the airflow rate of the VAV units. In the present research, based on the results of the check conducted in April 2003, we chose 10 systems and carried out the measurement. 5 systems of which included faulty units and the other 5 systems didn't include any faulty unit. The kinds of recorded data are room air temperature, set point of room temperature, demand signal of VAV damper opening, and supply air temperature of each unit and the sampling time is 1 minute.

Figure-8 and Figure-9 show the example of the measurement results. Figure-8 shows the operational data of WS system on 16th floor and this system has two faulty units with fully-close malfunction. One is Unit-E5, which substantially affects the room temperature. The other is Unit-C3, which doesn't much affect the room temperature because the zone is surrounded by eight units with no fault. Figure-9 shows the data of ES system on 12th floor and the system has one faulty unit with fully-open malfunction. The zone temperature of faulty Unit-C3 is lower than the temperature set point because of the fault. But the temperature of Zone-E3 that is bounded by Zone-C3 is lower than that of Zone-C3. The faulty unit sometimes affects not only the its zone temperature but also those of the surround zones.

The measured data shows that the demand signals of VAV damper opening of most units are saturated at a minimum value set for each unit. This means that the airflow rate of each unit is not well controlled as intended.

4.2 Verification of the FDD Method A

In order to verify the effectiveness of Method A, the method is applied to the real operational data mentioned above. FDD test is conducted on 150 VAV units in 10 AHU systems. The existence of four VAV units with a fully-close damper fault and two units with a fully-open damper fault out of 150 VAV units had been reported by the preliminary manual check to all units. Although the locations of the faulty units are known, however, the FDD test is conducted to the system assuming they are unknown. The periods of steady state are defined as the period from two hours after AHU system startup until shutdown except lunchtime (12:00 ~ 14:00).

In this research, the significance level α for the Grubbs' test is 0.1. The general value of α is 0.01 or 0.05, but the faults couldn't be detected when we used these values. If α was set larger than 0.1, the number of false alarm got larger and the reliance of alarm was down. From the test, we recommend that the suitable value of α is 0.1.

Figure-10 to Figure-13 show the histogram of \bar{X}_a , \bar{X}_b , \bar{X}_c and \bar{X}_d . The threshold between normal units and faulty units defined by the Grubbs' test are shown on the figures. From Figure-11, it can be said that the distribution of \bar{X}_b is not normal distribution and Grubbs' test should not be applied to the parameter. But there are 11 units whose \bar{X}_b value is zero. It means the demand signal of VAV damper opening is always zero over the measuring period. As we can easily say that it is abnormal, such units can be judged to have fully-open damper fault. Similarly, when the

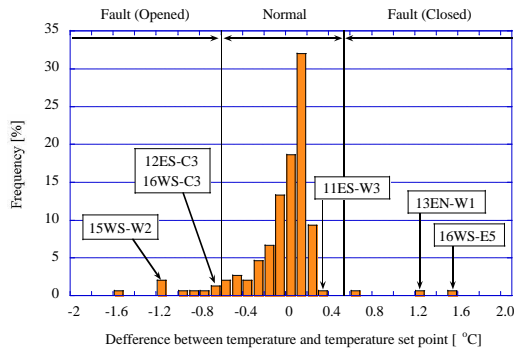


Figure-10 Histogram of \bar{X}_a

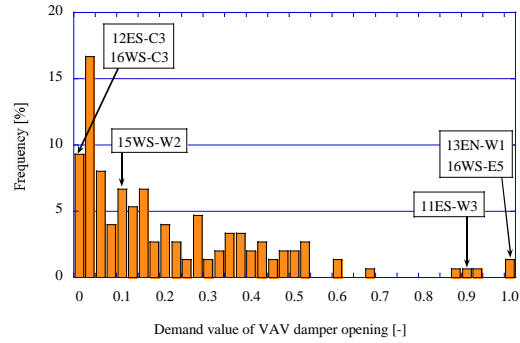


Figure-11 Histogram of \bar{X}_b

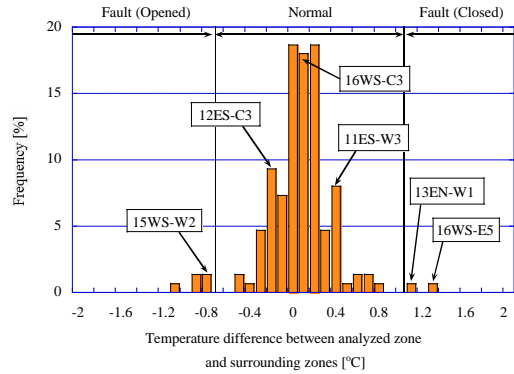


Figure-12 Histogram of \bar{X}_c

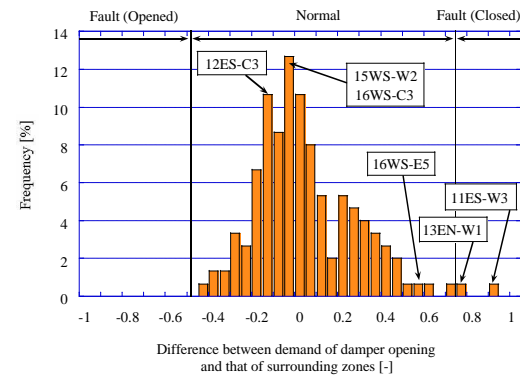


Figure-13 Histogram of \bar{X}_d

Table-2 FDD result of Method A

Method	Fully-closed damper fault			Fully-opened damper fault		Number of alarms	Number of false alarms	Number of real faulty unit that can't be detected			
	11ES-W3	13EN-W1	16WS-C3	16WS-E5	12ES-C3				15WS-W2		
Parameter a)		■	□	■	■	15	10%	10	67%	1	17%
Parameter b)			□		■	11	7%	9	82%	4	67%
Parameter c)		■		■		7	5%	4	57%	3	50%
Parameter d)	■	■				2	1%	0	0%	4	67%
a) U b)		■	□	■	■	17	11%	12	71%	1	17%
c) U d)	■	■		■		8	5%	4	50%	2	33%
a) U b) U c) U d)	■	■	□	■	■	18	12%	12	67%	0	0%

value of \bar{X}_b is one, those units can be judged to have fully-close damper fault.

Table-2 shows whether the true faulty units can be identified and diagnosed correctly or not by the present test method. The amounts of alarms, false alarms, and the true faulty units that were detected are listed in Table-2. In the table, the mark of "■" shows the VAV units whose fault can be identified and diagnosed correctly by the test and the mark of "□" shows the faulty units that can be identified but cannot be diagnosed correctly.

By taking the union of the four tests, i.e. $(a \cup b \cup c \cup d)$, the number of suspected faulty units can be reduced from 150 (100%) to 18 (12%) and all the units that truly have faults are included in the 18 units. This result means that Method A can reduce the required time, and manpower or cost for commissioning by about 90% compared with those needed by the present manual check, which is generally done by testing all the units. Moreover, if the union of the two tests, namely considering the comparison with the surrounding units $(c \cup d)$, is applied, we can identify and diagnose the faults that significantly affect the room air temperature by checking only 8 units (5%). If identifying just the units that significantly affect room temperature is enough, this method is appropriate. In all test methods, the fault of Unit-C3 in WS system on 16th floor, which is surrounded by many normally operated units,

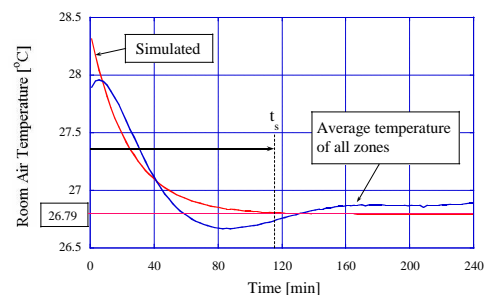


Figure-14 Fitting result of \bar{q}

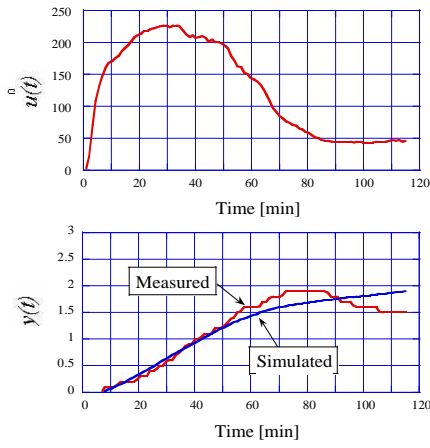


Figure-15 Unit-C3 in WN system on 6th floor (2003/09/01)

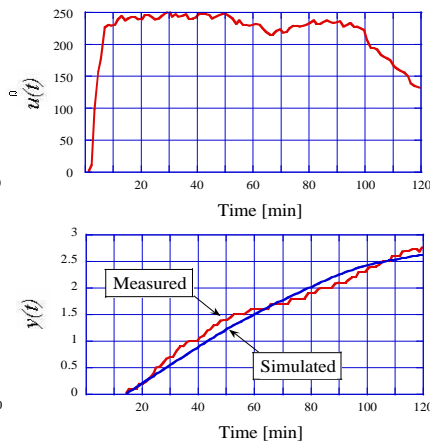


Figure-16 Unit-C3 in WN system on 16th floor (2003/08/27)

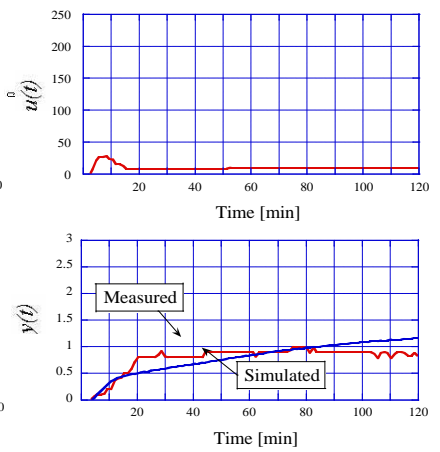


Figure-17 Unit-C3 in WN system on 12th floor (2003/09/10)

cannot be identified or diagnosed correctly. When a faulty unit is surrounded by many normally operated units, the present method sometime fails.

4.3 Verification of the FDD Method B

Method B is verified by applying it to the real operational data of the building formerly mentioned. At FDD step 2), t_s and $q_r(\infty)$ of the system is decided. Figure-14 shows the result of the calculation. t_s of the system is 115 minutes and $q_r(\infty)$ is 26.79 °C.

T_c is calculated per measured day per unit. As the example of the calculation, the result of Unit-C3 in WN system on 6th floor, Unit-C3 in WS system on 16th floor and Unit-C3 in ES system on 12th floor are shown in Figure-15 to Figure-17. The figures show that the accuracy of the simulation is not good because of the assumption that this system is the first order system.

Figure-18 shows the histogram of $\overline{\Delta X}_t$ and Table-3 shows the number of alarms, false alarms and undetectable real faulty units when the threshold changes from $m \pm s$, $m \pm 2s$, $m \pm 3s$. It is clear that all real faulty units except for Unit-C3 in WS system on 16th floor can be detected if the threshold is $m \pm s$, but Unit-W2 in WS system on 15th floor cannot be diagnosed correctly. The number of alarm 46 (31% of all units) is larger than that of Method A and the reduction of cost and time for commissioning is also smaller than Method A. If the threshold is $m \pm 3s$, the method gives the alarms for four units and three of the alarms are right, and the other 3 faulty units cannot be detected.

From the case studies using the real office building, Method B is not so efficient from the view point of the number of alarms and undetectable units compared with Method A. We conducted case studies in variable condition, for example changing the amount of data or load condition, but no ascendancy of Method B over Method A was found. The methods may not detect and diagnose faults correctly if the cooling load characteristics of a zone are largely different from the surrounding zones. In this case, the operational data of the units in such zone must be removed from the data set for analyzing. And the methods are based on the assumption that room temperature sensors are free of faults. It must be checked whether the temperature sensors indicate proper values before applying the FDD methods.

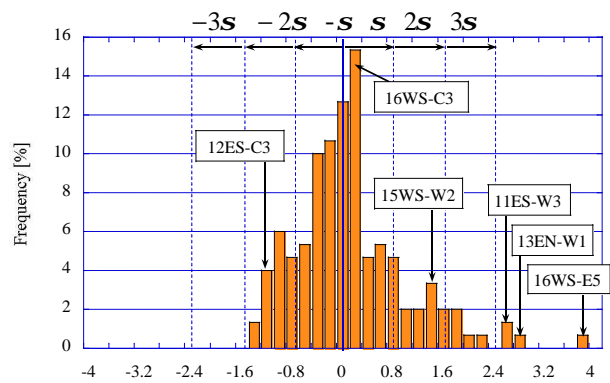


Figure-18 Histogram of $\overline{\Delta X}_t$

Table-3 FDD results using Method B

Threshold	Number of alarms	Number of false alarms	Number of real faulty unit that can't be detected
$\mu \pm s$	46	31%	41
$\mu \pm 2s$	11	7%	8
$\mu \pm 3s$	4	3%	1

5. CONCLUSIONS

Two FDD methods for VAV units are proposed in the present work. Both methods only require simple data, room temperature, demand control signal of VAV damper opening

and supply air temperature, which are easily obtained from BEMS. The followings are the conclusions based on the case studies using real building operational data.

- 1) A method is proposed to detect faults by applying the Grubbs' test to four parameters calculated from the room air temperature and the demand control signal of VAV damper opening of each unit under steady state operation. When combining all the FDD results of the four parameters, the number of suspected faulty units can be reduced down to 12% of the total number and every unit that truly has a fault is included in the units. Thus, the 90% cost or manpower for the commissioning can be reduce compared with the present test procedure through which all units are manually checked.
- 2) When combining the FDD results of the two parameters based on comparing the room temperature and the demand signals with those of the surrounding VAV units, the number of the suspected faulty units can be reduced down to 5% of the total number. In this case however, a faulty unit that does not significantly affect the room air temperature surrounded by normal units is very difficult to be detected.
- 3) A FDD method using dynamic system analysis is proposed. In this method the operation during AHU startup periods is regarded dynamic. The number of suspected units by this method is about 30% of the whole VAV units and five units with a true fault can be detected out of six units. However detecting a faulty unit surrounded by a few normal units is also difficult.

NOMENCLATURE

a, b, c	: Fitting parameters of Equation (14) [-]	y_0	: Initial value of y
c_p	: Specific heat of air [kJ/kg * K]	X_t	: Standardization value of T_c
K	: Gain of a first order system	\bar{m}	: Average of T_c
L	: Time delay of SISO system [min]	s_c	: Standard deviation of T_c
n	: Number of sample in data set X [-]	$q_{r(i,j)}$: Zone air temperature of $Z(i, j)$ [°C]
$N_{(i,j)}$: Number of zones surrounding $Z(i, j)$ [-]	q_{rs}	: Set point of room air temperature [°C]
$t_{a/2n}$: Critical Value of t distribution with $(n-2)$ degrees of freedom and a significance level $a/2n$	$q_{r(k,l)}$: Temperature of the zones surrounding $Z(i, j)$ [°C]
T_c	: Time constant of a first order system	$\bar{q}(t)$: Average air temperature in all measured days [°C]
u	: Input of SISO system	q_s	: Supply air temperature [°C]
s	: Standard deviation of data set X	$f_{(i,j)}$: Demand control signal of VAV damper opening of $U(i, j)$ [-]
V_{\max}	: Maximum value of supply air volume [m ³ /h]	$f_{(k,l)}$: Demand control signal of VAV damper opening of the zones surrounding $U(i, j)$ [-]
\bar{x}	: Sample mean of data set X	$f_{\min(i,j)}$: Minimum value of VAV damper opening [-]
x_{\max}	: Sample with the maximum distance from \bar{x} in data set X	r	: Air density [kg/m ³]
y	: Output of SISO system		

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