COORDINATION OF SUPPLY CHAIN INVENTORY SYSTEMS WITH PRIVATE INFORMATION

A Dissertation

by

CHI-LEUNG CHU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2006

Major Subject: Industrial Engineering

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ABSTRACT

Coordination of Supply Chain Inventory Systems
with Private Information. (December 2006)
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This dissertation considers the problems of coordinating different supply chain inventory systems with private information under deterministic settings. These systems studied are characterized by the following properties: (a) each facility in the system has self decision-making authority, (b) cost parameters of each facility are regarded as private information that no other facilities in the system have access to, and (c) partial information is shared among the facilities. Because of the above properties, the existing approaches for systems with global information may not be applicable. Thus, new approaches for coordinating supply chain inventory systems with private information are needed.

This dissertation first studies two two-echelon distribution inventory systems. Heuristics for finding the replenishment policy of each facility are developed under global information environment. In turn, the heuristics are modified to solve the problems with private information. An important characteristic of the heuristics developed for the private information environment is that they provide the same solutions as their global information counterpart.

Then, more complex multi-echelon serial and assembly supply chain inventory systems with private information are studied. The solution approach decomposes the problem into separate subproblems such that the private information is divided as required. Global optimality is sought with an iterative procedure in which the subproblems negotiate the material flows between facilities. At the core of the solution procedure is a node-model that represents a facility and its corresponding private information. Using the node-model as a building block, other supply chains can be formed by linking the node-models according to the product and information flows. By computational experiments, the effect of the private information on the performance of the supply chain is tested by comparing the proposed approach against existing heuristics that utilize global information. Experimental results show that the proposed approach provides comparable results as those of the existing heuristics with global information.

To my Parents

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CHAPTER I
INTRODUCTION

In the past two decades, fierce competition in the global market and rapid development in technologies have been continuously redefining the logistics practices. These new practices pose new challenges and opportunities to the researchers in both industry and academia. In response to these new practices, an increased focus has been placed on supply chain management.

Supply chain management is defined as 'the systematic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole' (Mentzer et al., 2001). One of the main areas of the supply chain management is inventory management. According to Statistical Abstract of the United States (2005), the annual investment in inventories was US$341 billion, which represented more than 10 percent of the total sales in the United States (US$3,338.4 billion) in 2004. Also, inventory represents thirty-five percent of the total logistics costs on average (Coyle et al., 2002). Because of the massive investment in inventory, proper inventory management makes good economic sense.

Coordinating the inventory systems in a supply chain, however, can be challenging. Supply chains in today's highly competitive and time-sensitive business environments are dynamic systems where alliances among companies are continuously redefined to rapidly satisfy market needs. Although the full information exchange is technically feasible, often the participants in the supply chain will be reluctant to freely share private cost information, and to let a third party dictate their inventory policies. An obvious example is a supply chain with competing retailers at the lower echelon. The

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This dissertation follows the style of International Journal of Production Economics.

prevalence of private information in a supply chain presents a monumental challenge to most existing approaches for supply chain inventory coordination where complete information sharing and existence of a centralized decision authority is assumed.

Based on the distribution of modeling information, we categorize supply chain inventory systems into two categories, namely, systems with global information and systems with private information. A supply chain inventory system with global information is characterized by a single decision-maker that has access to all the information of the system, and specifies the inventory policies for all the participating facilities. Systems with private information, on the other hand, are supply chain inventory system in which each facility possesses private modeling information that no other facilities in the system have access to, and each facility is responsible for specifying his/her own inventory policy.

Traditionally, researchers focus on supply chain inventory system with global information. While there are studies that considered coordination issues in supply chain inventory systems assuming autonomous facilities, i.e., there is no single decision-maker who can dictate the decisions for all the participating facilities; most of these studies do not deal explicitly with private information, such as the objective function and various cost parameters, of each facility in the system. Instead, private information is assumed to be available as needed, or that it can be estimated by other facilities.

In this research, we study the supply chain inventory systems with private information where the objective is to find the inventory policy for each facility in the system such that the ordering and inventory-related cost of the entire system is minimized. Specifically, we study two-echelon distribution inventory systems, and multi-echelon serial and assembly systems with private information. This study is different from the previous studies in that we assume the following information is private to each facility: (a) facility's objective function, (b) facility's setup/ordering cost, and (c) facility's inventory holding cost. Furthermore, facilities are responsible for specifying their own inventory policies; however, they will collaborate with the others to achieve a minimal system cost. The goal of this study is to develop theoretical understanding and coordination methodologies for coordinating supply chain inventory systems with private information.

1.1. Research objectives

The main objectives in this research are:

(a) To develop coordination methodologies for supply chains characterized by private information, a characteristic which is becoming more prevalent in current supply chain systems; and

(b) To investigate the effect of private information on the performance of supply chains when comparing to their counterparts with global information.

In order to achieve these objectives, we first study simple two-echelon supply chains, followed by the study of more complex multi-echelon serial and assembly supply chain configurations.

1.2. Problem statement

This research investigates the coordination of deterministic supply chain inventory systems with private information. The system with private information has the following characteristics:

(a) Each facility in the system has self decision-making authority.

(b) No single facility has complete information about the whole system. Objective function and the corresponding parameters of each facility is private information.

(c) Partial information is shared among the facilities in order to achieve close-to-optimal solutions.

Because of the above characteristics, the existing approaches for system with global information may not be applicable.

A general supply chain inventory system can be represented as a directed graph, $G(N, A)$, where each node $i \in N$ is associated with a facility, and the arc $(i, j) \in A$ represents a flow of product/information from node (facility) i to j. Fig. 1.1 shows an example of supply chain inventory system depicted as a directed graph.

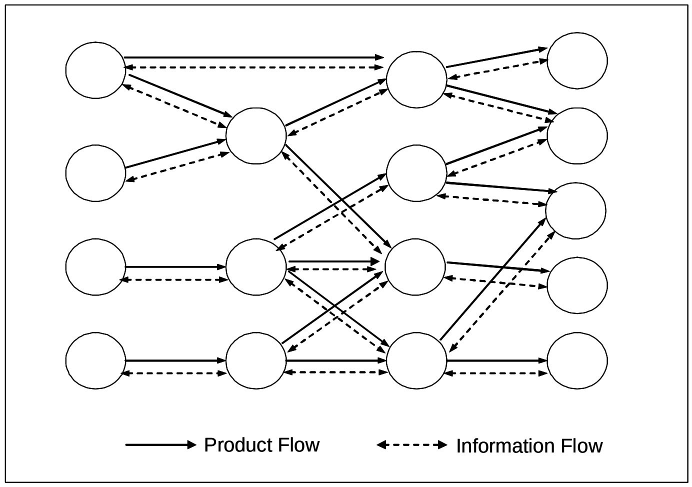


Fig. 1.1. Graph representation of a supply chain.

Each facility in the supply chain is represented by the model shown in Fig. 1.2, which is termed node-model in this study. The node-model consists of a production subsystem, a raw-material inventory subsystem, and a finished-goods inventory subsystem. A facility is responsible to specify three schedules: a delivery schedule, a production schedule, and order schedules. The delivery schedule is negotiated with the downstream facility (the customer), while the order schedules are negotiated with the upstream facilities (the suppliers). The production schedule takes into account both the delivery schedule and order schedules, such that the customer demands are met, and there is no shortage of raw materials. The work-in-process inventory subsystem is associated with the production subsystem.

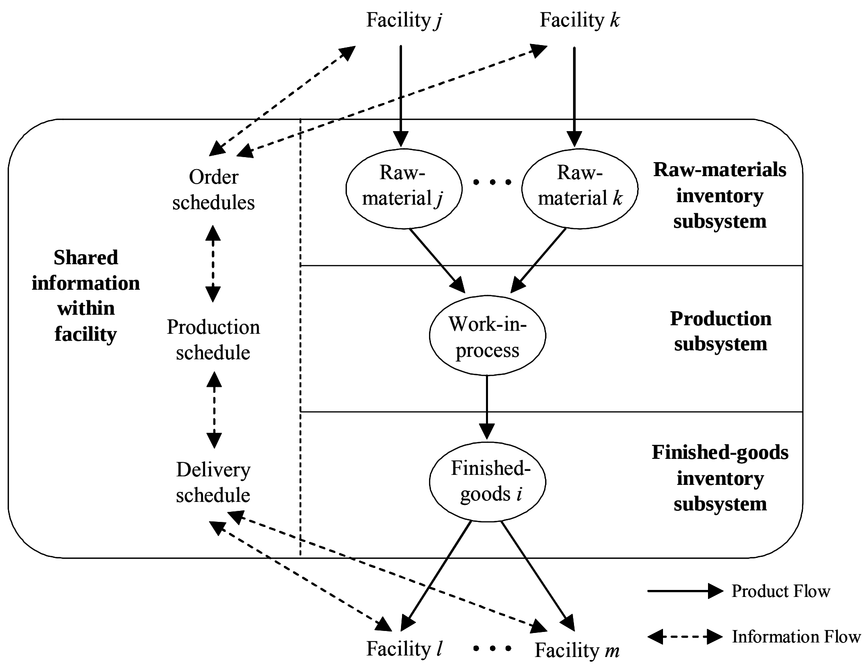


Fig. 1.2. Node model of facility *i*.

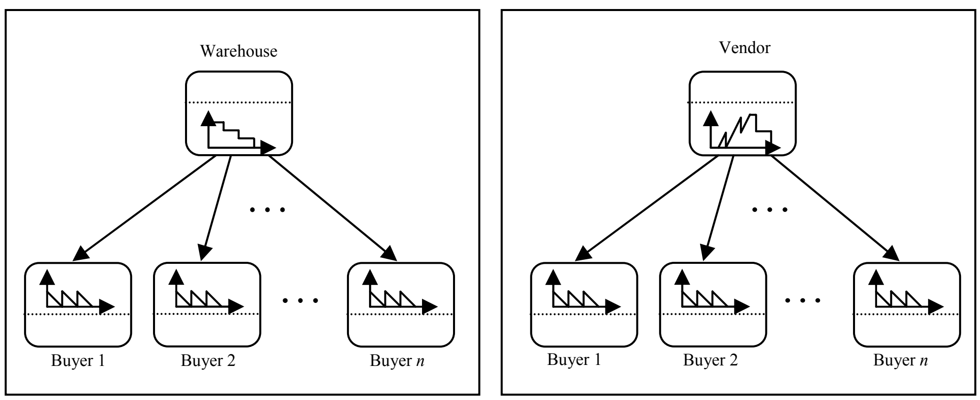
There are three salient features of the node-model in Fig. 1.2. First, it represents a general type of manufacturing organization, but can also represent a pure warehouse or retailer by simply deleting the appropriate model components. Second, it explicitly models the raw-material inventory, work-in-process inventory, and finished-goods inventory separately. Third, it can be used as an elemental building block to model complex supply chain configurations.

Using the proposed node-model, this research attempts to develop an interaction/negotiation framework in which critical system information can be recovered through negotiation among the interacting facilities.

1.2.1. Two-echelon distribution inventory systems with private information

Single-warehouse multi-buyers system (SWMB) is a simple form of supply chain inventory system where multiple facilities (buyers) draw required material from a single supplier to satisfy their given individual demands without shortage or backlogging. The supplier in turn places orders to an outside supplier to fill the orders of the buyers. It is assumed that the objective function and setup and inventory holding cost parameters of each facility are regarded as private information that no other facilities in the system have access to. Moreover, each facility is responsible to specify its own replenishment policy for a given demand. The objective is to find a replenishment policy for each facility in the system such that the total average ordering and inventory-related cost of the entire system is minimized under this restricted information environment.

A variant of SWMB is single-vendor multi-buyers system (SVMB). SVMB differs from SWMB in that the supplier is a manufacturer, which faced a finite production rate, instead of a mere warehouse, which can be viewed as having infinite production rate. Because of the constraint on the production rate, the coordination of SVMB is generally more difficult than that of SWMB.



(a) Single-Warehouse Multi-Buyers System (b) Single-Vendor Multi-Buyers System

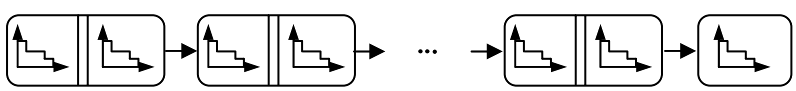
Fig. 1.3. Two-echelon distribution inventory systems.

Fig. 1.3 shows the two-echelon distribution inventory systems in this study. Two-echelon distribution inventory systems are considered because the problems have special mathematical structures that allow decomposition with regard to each facility. Further, a facility can either take the form of a warehouse or a manufacturer in a multi-echelon supply chain inventory system; and a two-echelon distribution inventory system with only one buyer is a special case of multi-echelon serial inventory system. Thus, studying two-echelon inventory systems will provide valuable insights when studying the serial and assembly inventory systems.

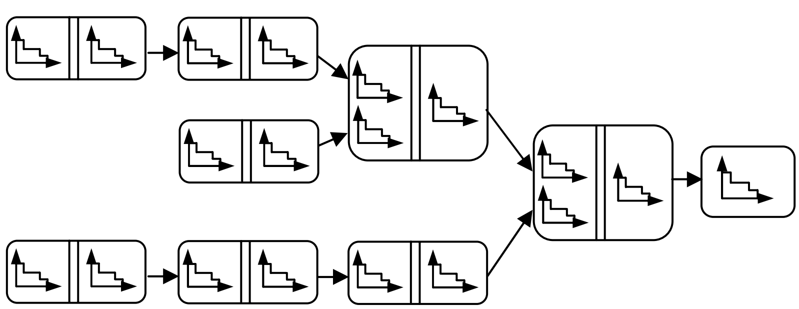
Two-echelon distribution systems are not uncommon. Most major retail chains utilize a distribution center as a warehouse to deliver products to the retail stores (Nahmias, 1997). An example of SVMB appears in the carpet and rug industry where Chesterton Carpet Mills, Inc. manufactures and supplies carpet to seven wholesalers (Kerin and Peterson, 2001).

1.2.2. Serial and assembly inventory systems with private information

Multi-echelon serial system is the simplest multi-echelon configuration where each facility except the first and the last has exactly one predecessor and one successor. Assembly system is an extension to serial system in which each intermediate facility has multiple predecessors but a single successor. In these supply chain inventory systems, each facility only produces single product. Time-dependent demands of the end product are assumed to be known over a finite horizon and must be met without backlogging. All lead times are assumed to be constant and, without loss of generality, are assumed to be zero. The coordination criterion is to minimize the total setup/ordering and inventory-related cost of the entire system over the finite horizon. These systems are shown in Fig. 1.4.



(a) Multi-Echelon Serial System



(b) An Multi-Echelon Assembly System

Fig. 1.4. Multi-echelon serial and assembly inventory systems.

These systems are considered because of their simple configurations. An example of serial system is a European printer manufacturer who assembled the printer motherboards in Europe, which are then shipped to Asia where they are integrated with the main printer housings (Simchi-Levi et al. 2004). Example of assembly system is Dell's business model where Dell's suppliers hold stock of components near Dell's assembly factory (de Kok and Graves, 2003). We believe that studying these systems can be a stepping-stone toward more complex general multi-echelon configurations.

1.3. Literature review

Modern inventory theory began with the derivation of the EOQ formula by Harris (1913). Since then, a significant amount of research has been done in the area of coordinating supply chain inventory systems in which many of these studies are extensions of Harris' basic EOQ model. Traditionally, this type of research focuses on supply chain inventory system with global information, i.e., they assumed that there is a single decision-maker who has access to all the relevant information for formulating the model and has the authority to force all the facilities to comply with his decision. Recently, there is a shift of focus from this centralized decision making paradigm to decentralized decision making paradigm. This shift of interest is partly due to the fact that typical supply chain composes of companies that are mostly independent to each other, and some basic modeling information is not readily shared among the facilities. As a result, the traditional approaches for system with global information may not be applicable. In this section, we offer a review of research conducted in the area of coordinating supply chain inventory systems.

1.3.1. Single-warehouse multi-buyers system

The SWMB problem has been studied thoroughly in the past. It is well known that the optimal policies, even with complete information, can be very complex (Graves and Schwarz 1977). Before the paper by Roundy (1985), most researchers attacked this problem by restricting themselves to special policies such as nested policy and stationary policy (Schwarz 1973, Schwarz and Schrage 1975, Graves and Schwarz 1977, and

Maxwell and Muckstadt 1985). A policy is stationary if the order intervals are constant for each facility. A policy is nested if all the facilities in the system share the same order interval. However, Roundy (1985) showed that nested policies may performed poorly as the effectiveness of an optimal nested policy can be arbitrarily small, where effectiveness is defined as the ratio of the optimal value and the heuristic value of the objective function. To overcome this drawback, Roundy (1985) introduced two types of policies, namely, the integer-ratio policy and power-of-two policy. An integer-ratio policy is a stationary policy in which the order interval of each facility in the system is an integer multiple of a base planning period. The power-of-two policy is a subset of integer-ratio policy that each facility orders at a power-of-two multiple of a base planning period. He presented a heuristic that finds an integer-ratio policy with 94% effectiveness. He also proposed a heuristic for finding a power-of-two policy with 98% effectiveness. The complexity of both heuristics are $O(n \log n)$, where n is the total number of buyers. Though the latter heuristic yields a policy with higher effectiveness, the former one provides the user with the freedom in choosing the base planning period. Nonetheless, the latter heuristic is the best approach currently available for SWMB system (Bramel and Simchi-Levi 1997).

Later, Lu and Posner (1994) revisited this problem considering integer-ratio policies. Taking advantage of the existing linear time method for finding the median, they solved the continuous relaxation of SWMB problem in $O(n)$. Based on this result, they proposed two heuristics with error bounds approach the one of Roundy's algorithm. The first heuristic explicitly evaluates a pre-defined number of points. They showed that as the number of points approaches infinitely, the error bound approaches 2.014% and the complexity of this heuristic is $O(n)$. The second heuristic finds a solution with error within ε, for any $\varepsilon > 0$, at the expense of longer running time, $O(n \log n/\sqrt{\varepsilon})$.

Wang (1995) studied the single-warehouse multiple identical buyers system from the warehouse's perspective and suggested coordinating the system using quantity discount and franchise fees. Weng and Wu (2000) studied the same problem with heterogeneous buyers. Assuming that the buyer's behavior is captured by the classical

EOQ model and that the parameters can be estimated by the warehouse, they proposed quantity discount by percentage increases instead of the traditional quantity discount by unit increase, and showed that their proposed policy is more efficient for systems with many different buyers. Chen et al. (2000) integrated the pricing decisions and replenishment strategies for the SWMB system where demand of each buyer is a general decreasing function of the retail price. Treating the warehouse as a Stackelberg game leader, they presented a heuristic in determining the wholesale price and replenishment strategies. Abdul-Jalbar et al. (2003) studied the SWMB system with integer-ratio policies assuming partial information sharing. After the buyers determine their optimal replenishment policies, the warehouse determines the planning period, clusters the buyers' replenishment periods, and calculates the demand for each interval within the planning period. By doing so, the warehouse's problem is transformed to a dynamic-lot-size problem and can be solved by the algorithm in Wagelmans et al. (1992). Jin and Wu (2002) studied the two-suppliers-single-buyer system. Assuming the participants' costs are private information and each supplier's cost is known probabilistically, they investigate different types of auctions as a means for coordination.

1.3.2. Single-vendor multi-buyers system

The literature on SVMB can be summarized into two major categories. The research in the first category studies this problem from the vendor's point of view and proposes models using quantity discount to maximize the vendor's profit. Assuming that the vendor has complete information on the buyer's cost function but has no authority on buyer's decision, Monahan (1984) analyzed the single-vendor single-buyer system as a Stackelberg game where the vendor is the game leader. The objective is to maximize the total profit of the system. Assuming lot-for-lot production, he showed that quantity discount is effective in coordinating the system. Lee and Rosenblatt (1986) generalized Monahan's model by relaxing the lot-for-lot production assumption. Parlar and Wang (1995) extended Lee and Rosenblatt (1986) model to a situation where the vendor does not have complete information on the buyer's cost parameters but knows the buyer's inventory holding costs. They proposed a quantity-discount schedule in which there is a unique per unit price for each quantity the buyer might choose. Later, Corbett and de Groote (2000) generalized the model to the case where the vendor does not have access to the cost information of the buyer but knows the holding cost probability distribution of the buyer. Wang (1995) studies the single-vendor multiple identical buyers system and suggested coordinating the system using quantity discount and franchise fees. Studying both the case when the supplier has complete information of the system and the case when the supplier does not have access to any buyer's cost information, Chen et al. (2001) generalized Wang's (1995) model to heterogeneous buyers. Considering lot-for-lot policy, Viswanathan and Piplani (2001) studied the case where a vendor only offers discount to buyers if the buyers place their order at vendor specified time.

Research in the second category studies the integrated model where the objective is to minimize the total average cost of the system. Banerjee (1986) examined the single-vendor single-buyer case and presented a 'lot-for-lot' model in which the vendor produces each buyer shipment as a separate batch. Goyal (1988) generalized Banerjee's model by relaxing the 'lot-for-lot' policy. Lu (1995) studied the problem with an additional constraint of the maximum cost that the buyer is prepared to incur. Assuming equal delivery quantity at each replenishment period, he provided an optimal solution for single-vendor single-buyer case. Later, Goyal (1995) presented an alternative policy in which successive shipments within a production batch increases by a factor equal to the production rate divided by the demand rate. Instead of using a fixed increasing factor, Hill (1997) generalized Goyal's (1995) policy such that the increasing factor is a decision variable. Goyal and Nebebe (2000) then proposed a new type of policy that ensures a quick delivery for the first shipment to the buyer. Under this policy, the first replenishment will be of small size, followed by (k-1) equal sized replenishment, where k is the number of replenishments of a production batch at the vendor. Banerjee and Burton (1994) studied the single-vendor multi-buyers system. In the situation where there is no coordination between the vendor and the buyers, each facility will operate according to his/her optimal policy. They showed that the total average cost of the system would be substantially lower through coordinated inventory control and presented an EOQ-type solution for the system for finding integer-ratio replenishment policies.

1.3.3. Serial and assembly system

Studies on dynamic lot-sizing problem of supply chain inventory system with global information can be traced back to 1960's. Zangwill (1969) is the first to solve the multi-echelon serial system by dynamic programming. Love (1972) solved the same problem by dynamic programming which takes advantage of the nested structure of the optimal solution. Veinott (1969) and Crowston and Wagner (1973) considered general multi-echelon supply chain system and solved the problems by dynamic programming. Unfortunately, the computation efforts of these dynamic programming algorithms increase exponentially with the problem size.

When considering multi-echelon assembly system, Afentakis, Gavish, and Karmarkar (1984) proposed a Lagrangian relaxation approach that decomposes the problem into a set of single-echelon problems which can be solved by Wager-Whitin algorithm (Wager and Whitin 1957). Later, Afentakis and Gavish (1986) extended the approach of Afentakis, Gavish, and Karmarker (1984) to more general assembly systems. Kuik and Salomon (1990) suggested a solution approach for a general multi-echelon supply chain inventory problem based on simulated annealing. Later, Salomon (1991) proposed a simple decomposition heuristic for the multi-echelon supply chain system in which the upper bound is obtained by modifying the balance equation.

Due to the complexity of the multi-level supply chain inventory system, different hierarchical heuristics that sequentially consider the facility from the bottom to the top of the supply chain, which are termed single-pass approaches, were also developed. New (1974) studied the performance of applying single-item lot sizing method to multi-echelon supply chain systems. Blackburn and Miller (1982) identified the potential errors in applying the single-pass heuristics and developed series of simple heuristics to modify the setup and holding costs of the problems before applying the single-item algorithm to each facility. Grave (1981) suggested an iterative approach that improved on the single pass solution for multi-echelon supply chain system.

Billington et al. (1986) studied the multi-echelon capacitated lot sizing problem with a single bottleneck and proposed a branch and bound algorithm for solving the problem. Maes and Van Wassenhove (1986) developed a so-called ABC heuristic for single-echelon capacitated lot sizing problem, and later extended it to multi-echelon serial system (Maes and Van Wassenhove, 1991). Tempelmeier and Derstroff (1996) studied the general multi-echelon supply chain structure and presented an effective Lagrangian relaxation heuristic which decomposed the problem into a set of uncapacitated single-item lot sizing problems.

Recently, the studies of serial and assembly supply chain systems are mainly focused on stochastic models. Researchers have been developed different supply chain contracts with different incentives to coordinate the supply chain inventory systems. When considering two-echelon serial system, Lee and Whang (1999) proposed coordinating the system with incentive scheme that is characterized by transfer pricing, consignment, backlog penalty, and shortage reimbursement. Proteus (2000) considered the same problem and proposed using responsibility token, which is in essence a reimbursement scheme, as the coordination mechanism. Cachon and Zipkin (1999) studied two-echelon serial system and proposed using an array of transfer payment contracts that are based on local information for channel coordination.

1.4. Organization of this dissertation

This dissertation is organized as follows. In Chapter II, the single-warehouse multi-buyers system with private information is studied and an interaction/negotiation approach is proposed. In Chapter III, the proposed approach for coordinating single-warehouse multi-buyers system with private information is extended to single-vendor multi-buyers systems. In Chapter IV, we address the coordination problems of serial and assembly systems and a Lagrangian-based decomposition framework is proposed. Finally, Chapter V presents conclusion of this study and future research directions.

CHAPTER II
SINGLE-WAREHOUSE MULTI-BUYERS SYSTEM

2.1. Introduction

In this chapter, the problem of coordinating the single-warehouse multiple-buyers (SWMB) system with private information is studied. The SWMB is a simple form of supply chain network where multiple facilities (buyers) draw required material from a single warehouse to satisfy their given individual demands. The warehouse in turn places orders to an outside supplier to fill the orders of the buyers. Specifically, we consider the SWMB system where the following information is private to each facility: (i) facility's objective function, (ii) facility's setup cost, and (iii) facility's inventory holding cost. Furthermore, we consider the case in which no facility has authority to make decision for other facilities in the system besides him/her, and each facility will collaborate with the others to achieve a minimal system cost. The objective is to find a policy that minimizes the total average ordering and inventory-related cost of the SWMB system under this restricted information environment.

Consider the simplest case of SWMB as follows: a fixed charge is incurred whenever the warehouse places an order. Similarly, a facility-dependent setup cost for each buyer is charged for each order placed. Also, there is a facility-dependent holding cost for inventory at each facility in the system. Even under a global information environment where there is a decision maker who possesses all the information about the system, there is no known polynomial method for solving this SWMB problem. The power-of-two approximation of Roundy (1985) is the best heuristic currently available. In the private information case under consideration in this study, Roundy's method or any other heuristics developed for global information environment will not be applicable.

The objective of this chapter is to develop a solution approach to address the SWMB system where the cost structure and parameters are considered private information of the corresponding facility and no other facilities have access to these pieces of information. We assume there is a fixed base planning period (T_B), and that all the order intervals are power-of-two multiples of T_B. With this additional assumption, a new method for solving the SWMB system with global information is developed. In turn, we propose an interaction model for the case with private information based on the new method such that both models will produce the same solutions. Both models are applicable to SWMB systems where the cost function of each facility is convex. The remainder of this chapter is organized as follows. In Section 2.2, solution procedure developed by Roundy (1985) is reviewed. Section 2.3 presents a new heuristic for solving the SWMB problem under global information environment. Section 2.4 contains the derivation and description of an interaction model for the SWMB system with private information. Finally, Section 2.5 presents some concluding remarks.

2.2. Roundy's algorithm

We now describe in more detail the solution procedure presented by Roundy (1985) since it motivated the methodologies developed in this study. For convenience, we term this solution procedure as Roundy's algorithm. We assume that no shortage or backlogging is allowed. Without loss of generality, replenishment is assumed to be instantaneous. Furthermore, the base planning period, T_B, is assumed to be fixed and that only power-of-two policies are employed. In other words, the order interval of each facility is a power-of-two multiple of T_B. Assuming an agree-upon base planning period is more of a practical reason than an academic one. For example, delivery may service every morning. Therefore, a natural choice of T_B is 1 day. On the other hand, if T_B is not predefined but is treated as a decision variable, the "optimal" T_B may not be in accord with the actual delivery service practice, which renders the "optimal" policy inoperable. Consider the following notations:

n : Number of buyers

K_0 : Setup cost at the warehouse

K_i : Setup cost at buyer i, $i = 1, \ldots, n$

h_0^i : Unit holding cost of item for buyer i at the warehouse, $i = 1, \ldots, n$

h_i : Unit holding cost at buyer i, $i = 1, \ldots, n$

D_i : Constant demand rate at buyer i, $i = 1, \ldots, n$

T_B : Base planning period

T_0 : Order interval at the warehouse (decision variable)

T_i : Order interval at buyer $i, i = 1, \ldots, n$ (decision variables)

Γ : System order policy, $\Gamma = \{T_0, T_1, \ldots, T_n\}$

$c(\Gamma)$: Total average cost of the system

$c_0(\Gamma)$: Total average cost of the warehouse

$c_i(T_i)$: Total average cost of buyer $i, i = 1, \ldots, n$

$g_i(T_i)$: Total average cost attributable to buyer i when $T_0 > T_i$, $i = 1, \ldots, n$

τ_i' : Optimal solution that minimizes $c_i(T_i)$, $i = 1, \ldots, n$

τ_i : Optimal solution that minimizes $g_i(T_i)$, $i = 1, \ldots, n$

t_i' : Power-of-two solution that minimizes $c_i(T_i)$, $i = 1, \ldots, n$

t_i : Power-of-two solution that minimizes $g_i(T_i)$, $i = 1, \ldots, n$

$\Delta(\Gamma^i, \Gamma^j)$: Changes in total average cost of the system if Γ^i is used instead of Γ^j, $\Delta(\Gamma^i, \Gamma^j) = c(\Gamma^i) - c(\Gamma^j)$

When determining the total average cost of the system, if $T_0 \le T_i$, then the warehouse orders for buyer i at the same time when buyer i orders. Thus, the burden of inventory holding falls solely on buyer i; and

$$c_i(T_i) = \frac{K_i}{T_i} + \frac{1}{2} h_i D_i T_i . \tag{2.1}$$

On the other hand, if $T_0 > T_i$, the warehouse orders $T_0 D_i$ for buyer *i* every T_0. Both the warehouse and buyer *i* carry inventories in this case. Thus, the average total cost attributable to buyer *i* when $T_0 > T_i$ is

$$g_i(T_i) = \frac{K_i}{T_i} + \frac{1}{2}h_i D_i T_i + \frac{1}{2}h_0^i D_i (T_0 - T_i) = c_i(T_i) + \frac{1}{2}h_0^i D_i (T_0 - T_i)\,. \tag{2.2}$$

As a result, for any T_0 and T_i, the total average cost attributable to buyer *i*, $f_i(T_0, T_i)$, is

$$\begin{aligned} f_i(T_0,T_i) &= \begin{cases} \dfrac{K_i}{T_i} + \dfrac{1}{2}h_i D_i T_i & \text{if } T_0 \le T_i \\ \dfrac{K_i}{T_i} + \dfrac{1}{2}h_i D_i T_i + \dfrac{1}{2}h_0^i D_i (T_0 - T_i) & \text{if } T_0 > T_i \end{cases}. \\ &= \max\{c_i(T_i), g_i(T_i)\} \end{aligned} \tag{2.3}$$

It follows that the SWMB problem can be express as follows.

$$\begin{aligned} \min c(\Gamma) &= \frac{K_0}{T_0} + \sum_{i=1}^{n} \frac{1}{2} h_0^i D_i (T_0 - T_i) U(T_0 > T_i) + \sum_{i=1}^{n} \frac{K_i}{T_i} + \frac{1}{2} h_i D_i T_i \\ &= \frac{K_0}{T_0} + \sum_{i=1}^{n} f_i(T_0, T_i) \end{aligned} \tag{2.4}$$

subject to:

$$T_0 = 2^{k_0} T_B \tag{2.5}$$

$$T_i = 2^{k_i} T_B \tag{2.6}$$

$$k_0, k_i : \text{integer}, i = 1 \ldots n$$

$$U(T_0 > T_i) = \begin{cases} 0 & \text{if } \quad T_0 \le T_i \\ 1 & \text{if } \quad T_0 > T_i \end{cases} \tag{2.7}$$

Consider the relaxed problem when (2.5) and (2.6) are removed, i.e., when power-of-two assumption is relaxed. Notice that, by definition, τ_i' is the optimal solution to $c_i(T_i)$, and τ_i is the optimal solution to $g_i(T_i)$, it is easy to verify that $\tau_i' \le \tau_i$. Both $c(\Gamma)$ and $f_i(T_0, T_i)$ are convex in T_0, and the optimal solution to the relaxed problem, given T_0, is

$$T_i = \begin{cases} \tau_i^{'} & \text{if } T_0 < \tau_i^{'} \\ T_0 & \text{if } \tau_i^{'} \le T_0 \le \tau_i \\ \tau_i & \text{if } \tau_i < T_0 \end{cases} . \tag{2.8}$$

As illustrated in Fig. 2.1, Roundy's algorithm starts by assuming T_0 falls within the leftmost interval, I_1. After finding the optimal T_i based on (2.8) for all buyer *i*, optimal T_0 can be calculated by solving the relaxed problem (2.4). This procedure is repeated by successively assuming T_0 falls within each interval on the right until the calculated optimal T_0 falls within the same interval, in which the optimal solution is found and the optimal power-of-two policy is obtained by rounding the solution to power-of-two multiple of T_B. Assuming a fixed T_B, this approach generates a power-of-two policy with 94% effectiveness in $O(n \log n)$ operations.

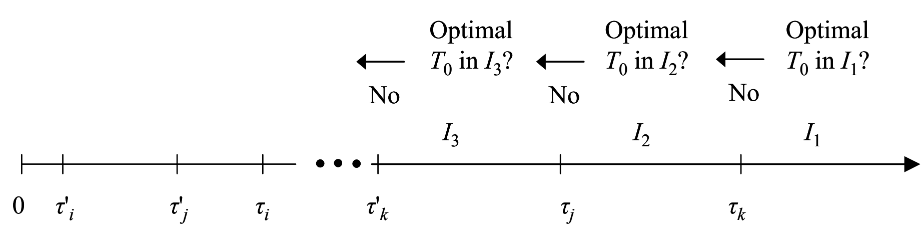


Fig. 2.1. Roundy's algorithm.

2.3. SWMB with global information (SWMB-GI)

This section presents an alternate method, termed SWMB-GI, for solving the SWMB system with global information. A feature of SWMB-GI is that it will lead to the development of an interaction model applicable to environments characterized by partial information sharing in Section 2.4.

2.3.1. Heuristic development

Distinct to Roundy's algorithm, the proposed method considers only feasible power-of two policies (of T_B). Instead of successively checking whether the optimal T_0

falls within a certain interval, the proposed method takes advantage of the property that $c(\Gamma)$ is convex in T_0 and monitors the slope of the total average cost by varying T_0. Starts by letting T_0 be a power-of-two policy, the proposed method finds the corresponding optimal power-of-two policy, T_i, for each buyer i, $i = 1,\ldots,n$, and calculates the corresponding total average cost of the system; and then successively increases T_0 to the next power-of-two period until the total average cost of the system increases at which point the optimal power-of-two policy is found; as illustrated in Fig. 2.2.

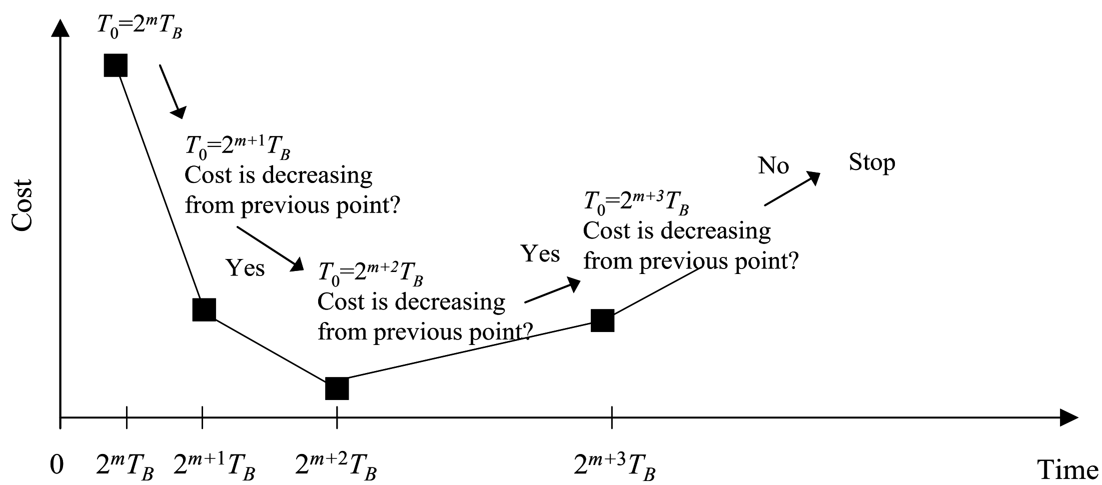


Fig. 2.2. Proposed solution approach.

Since the proposed method only considers power-of-two policies, the optimal power-of-two solution, t_i' and t_i, are used instead of τ_i' and τ_i. By definition, $t_i' = 2^{k_i} T_B$ and $t_i = 2^{m_i} T_B$ are the optimal power-of-two solutions to $c_i(T_i)$ and $g_i(T_i)$, respectively. To obtain t_i' and t_i, we need to round τ_i' and τ_i to the power-of-two multiple of T_B. The logic of the rounding procedure is as follows. Since $c_i(T_i)$ is convex, if $2^{k_i-1}T_B \le \tau_i' \le t_i' = 2^{k_i}T_B$, it must be that $c(2^{k_i-1}T_B) \ge c(\tau_i')$ and $c(2^{k_i}T_B) \ge c(\tau_i')$. It is easy to verify that k_i must satisfy the following condition: $2^{k_i-1}\sqrt{2} \le \tau_i'/T_B \le 2^{k_i}\sqrt{2}$. Similarly, m_i must satisfy the condition that $2^{m_i-1}\sqrt{2} \le \tau_i/T_B \le 2^{m_i}\sqrt{2}$. For details of this rounding procedure, please refer to Roundy (1985).

Proposition 2.1: For a given T_0, the optimal power-of-two policy is given by

$$T_i = \begin{cases} t_i' & \text{if } T_0 < t_i' \\ T_0 & \text{if } t_i' \le T_0 \le t_i \\ t_i & \text{if } t_i < T_0 \end{cases}. \tag{2.9}$$

Proof: As shown in (2.3), the total average cost attributable to buyer *i* is

$$f_i(T_0, T_i) = \begin{cases} c_i(T_i) & \text{if } T_0 \le T_i \\ g_i(T_i) & \text{if } T_0 > T_i \end{cases}. \tag{2.10}$$

Since both $c_i(T_i)$ and $g_i(T_i)$ are convex in T_i, they intersect at a single point $T_0 = T_i$. Also, they are piecewise convex in $T_i = 2^m T_B$. For any given T_0, buyer *i* must belong to either one of the following three sets: $G(T_0) = \{i : T_0 < t_i'\}$, $E(T_0) = \{i : t_i' \le T_0 \le t_i\}$, and $L(T_0) = \{i : t_i < T_0\}$.

<u>Case 1</u>: $i \in G(T_0)$

In this case, $c_i(t_i')$ is on the feasible solution frontier and $c_i(t_i') < g_i(T_i)$ for $T_i < T_0$. Thus, the optimal T_i is t_i'. Fig. 2.3 shows the average total cost functions in this case.

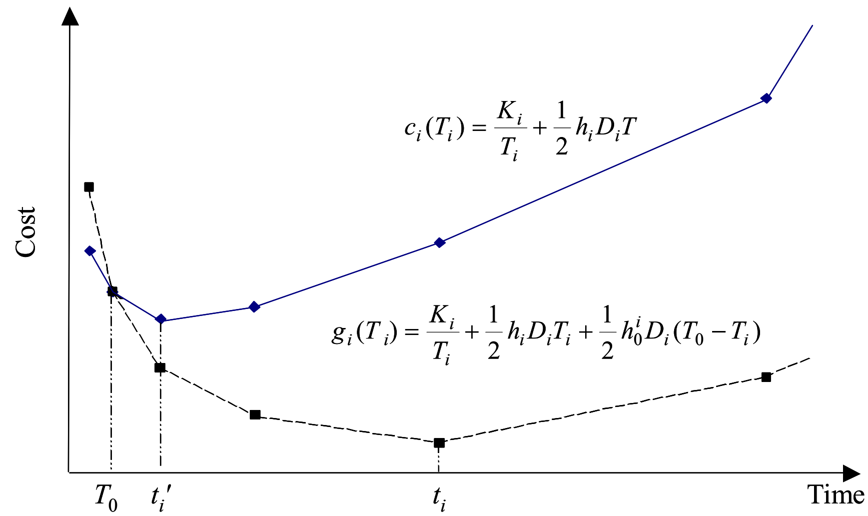


Fig. 2.3. Average total cost of buyer *i* when $T_0 \le t_i'$.

Case 2: $i \in E(T_0)$

In this case, $g_i(T_i)$ is decreasing in the region of $0 < T_0 \le t_i$ and $c_i(T_i)$ is increasing in the region of $t_i' \le T_0 < \infty$, the optimal T_i is the intersection point of $g_i(T_i)$ and $c_i(T_i)$, i.e., T_0. Fig. 2.4 exemplifies this case.

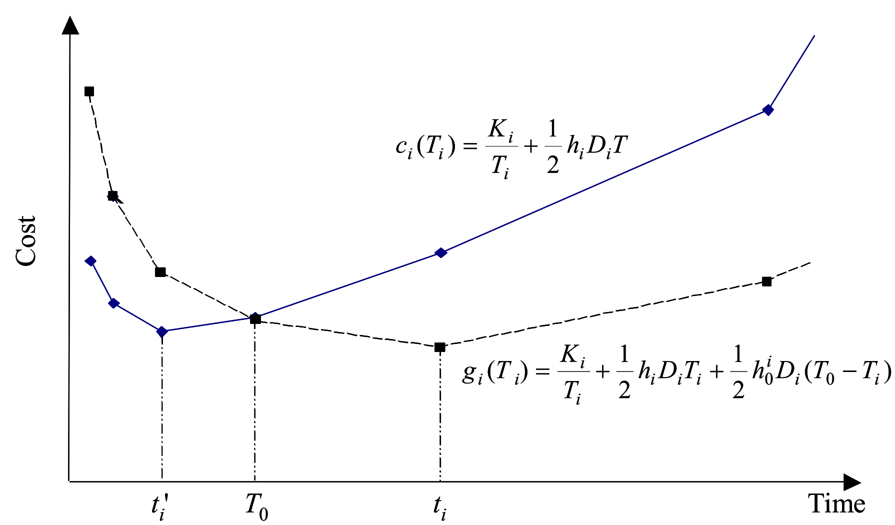


Fig. 2.4. Average total cost of buyer *i* when $t_i' \le T_0 < t_i$.

Case 3: $i \in L(T_0)$

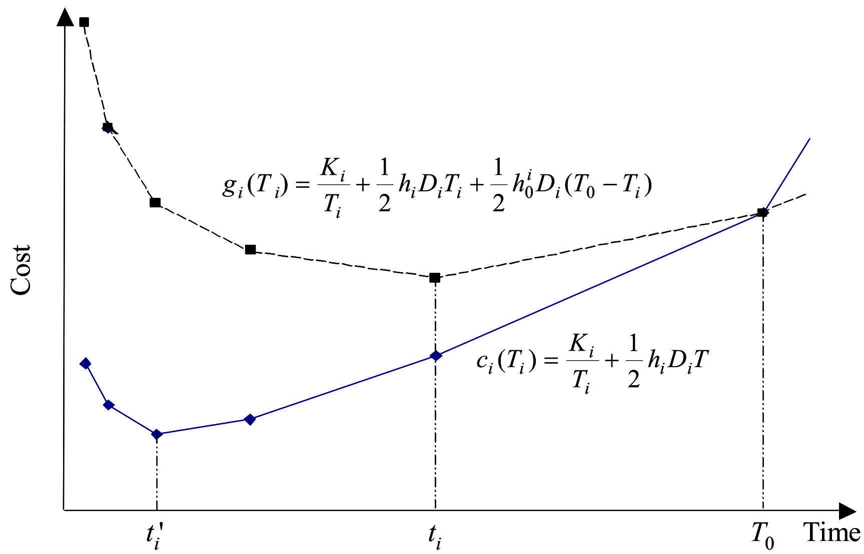


Fig. 2.5. Average total cost of buyer *i* when $t_i < T_0$.

As shown in Fig. 2.5, $g_i(t_i)$ is on the feasible solution frontier and $g_i(t_i) < g_i(T_0) \le c_i(T_i)$, for $T_i \ge T_0$. Thus, the optimal solution in this case is t_i. *Q.E.D.*

Based on Proposition 2.1, and the fact that $c(\Gamma)$ is convex in T_0, we propose an iterative heuristic that monitors the changes in total average cost if policy Γ^i is used instead of policy Γ^j, i.e., $\Delta(\Gamma^i, \Gamma^j) = c(\Gamma^i) - c(\Gamma^j)$. Define $t_{min} = \min\{t_i' : i = 1, \cdots, n\}$ and $t_{max} = \max\{t_i : i = 1, \cdots, n\}$, the heuristic for SWMB system with global information (SWMB-GI) that exploits Proposition 2.1 is summarized as follows.

SWMB-GI Heuristic

Step 0: Calculate t_i' and t_i for $i = 1, \ldots, n$. Find $t_{min} = \min\{t_i' : i = 1, \cdots, n\}$ and $t_{max} = \max\{t_i : i = 1, \cdots, n\}$. Let $j = 0$, $T_0 = t_{min}$, $\Gamma^0 = \{\phi\}$, and $c(\Gamma^0) = \infty$.

Step 1: Choose T_i according to condition (2.9).

Let $\Gamma^j = \{T_0, T_1, \ldots, T_n\}$ and calculate $c(\Gamma^j)$ using (2.4).

If $\Delta(\Gamma^{j}, \Gamma^{j-1}) < 0$, go to step 2.

Otherwise, stop, and the best power-of-two policy is $\Gamma^{*} = \Gamma^{j-1}$.

Step 2: If $T_0 < t_{max}$, set $j = j + 1$, $T_0 = 2T_0$ and go to step 1.

If $T_0 = t_{max}$, it means that the optimal T_0 falls in the region [t_{max}, ∞]. Since for any $T_0 > t_{max}$, the optimal T_i, for all i, remains the same. Therefore, given the optimal T_i, for all i, T_0 can be found by first minimizing (2.4) with respect to T_0 and then rounding the solution such that $2^{k-1}\sqrt{2}T_B \leq T_0 = kT_B \leq 2^{k}\sqrt{2}T_B$.

In SWMB-GI, we start searching T_0 from t_{min} in step 0. It is because, for any $T_0 < t_{min}$, a policy developed according to condition (2.9) will have a higher average total cost than a policy with $T_0 = t_{min}$ since the only difference in average total cost of these two policies is the setup cost of the warehouse. Also, because $c(\Gamma)$ is convex in T_0, the optimal power-of-two policy can be found by successively increasing T_0 until $c(\Gamma)$ starts to increase, i.e, $\Delta(\Gamma^{j}, \Gamma^{j-1}) > 0$. The effectiveness of SWMB-GI is given in Theorem 2.1.

Theorem 2.1: The power-of-two policy obtained from the proposed heuristic, Γ^{*}, has 94% effectiveness.

Proof: Since (2.4) is convex, the heuristic stops when the average total cost increases. Since the policy obtained at each iteration is an optimal power-of-two policy with respect to T_0, by Proposition 2.1, Γ^{*} is the optimal power-of-two policy for given T_B. Note that Roundy (1985) proves that a power-of-two policy obtained from his approach has 94% effectiveness for a fixed T_B. Therefore, as the optimal power-of-two policy, Γ^{*} will not perform worse than that of Roundy's algorithm and thus has 94% effectiveness. *Q.E.D.*

Note that the maximum number of points needed to be examined in our proposed method is $m = ln(t_{max} - t_{min})$. Since each step can be finished in linear time, therefore, the complexity of the proposed heuristic is $O(m)$.

2.3.2. Numerical example

To illustrate the method, we consider a simple example of SWMB system with the parameters as shown in Table 2.1.

Table 2.1
Parameters of the numerical example

	Warehouse		Buyer 1	Buyer 2	Buyer 3
K_0/order	\$500	K_i/order	\$100	\$300	\$600
h_0^i/*unit/year* (i=1,2,3)	\$2	h_i/unit/year	\$4	\$4	\$3
		D_i/year	4000	6000	4000

Let $T_B = 1$ month.

<u>*Initialization*</u>

Step 0: $\tau' = \{\tau_1', \tau_2', \tau_3'\} = \{1.34,\ 1.89,\ 3.79\}$ months, and $t' = \{t_1', t_2', t_3'\} = \{1,\ 2,\ 4\}$ months.

$\tau = \{\tau_1, \tau_2, \tau_3\} = \{1.90,\ 2.68,\ 6.57\}$ months, and $t = \{t_1, t_2, t_3\} = \{2,\ 2,\ 8\}$ months.

t_{min} = 1 months, t_{max} = 8 months, $T_0 = 1$ months, and $c(\Gamma^0) = \infty$

<u>*Iteration 1*</u>

Step 1: $\Gamma^1 = \{T_0, T_1, T_2, T_3\} = \{1,\ 1,\ 2,\ 4\}$, $c(\Gamma^1) = 15{,}467$, and $\Delta(\Gamma^1, \Gamma^0) < 0$.

Step 2: Set $T_0 = 2$ months.

<u>*Iteration 2*</u>

Step 1: $\Gamma^2 = \{2,\ 2,\ 2,\ 4\}$, $c(\Gamma^2) = 12{,}533$, and $\Delta(\Gamma^2, \Gamma^1) = -2934 < 0$.

Step 2: Set $T_0 = 4$ months.

Iteration 3

Step 1: $\Gamma^3 = \{4,\ 2,\ 2,\ 4\}$, $c(\Gamma^3) = 12{,}700$, and $\Delta(\Gamma^3, \Gamma^2) = 167 \geq 0$. Since $\Delta(\Gamma^3, \Gamma^2) \geq 0$, the optimal policy is $\Gamma^2 = \{2,\ 2,\ 2,\ 4\}$, i.e., $T_0 = 2$ month, $T_1 = 2$ month, $T_2 = 2$ month, and $T_3 = 4$ month, with average total cost, \$12,533.

2.4. Heuristic for SWMB with private information (SWMB-PI)

In this section, we study the SWMB system with private information. First we restate the SWMB under private information problem; second, a solution procedure which finds the same solution as that of SWMB-GI is developed, and finally a numerical example is presented.

2.4.1. Problem description and heuristic development

The SWMB system with private information has the following characteristics:

(a) each facility in the system has self decision-making authority,

(b) no single facility has complete information about the whole system, and

(c) partial information is shared among the facilities in order to achieve close-to-optimal solutions.

Specifically, the following information is considered private for the problem under consideration:

(a) The exact form of the objective function of each facility is private information; and the only assumption imposed on $c(\Gamma)$ and $f_i(T_0, T_i)$ is that of convexity.

(b) Each facility only views its local objective coefficients.

To employ Roundy's algorithm, one needs to know the exact objective function and cost parameters of the warehouse and those of every buyer in order to calculate τ_i', τ_i, and T_0. Therefore, Roundy's algorithm is not applicable to the private information environment considered here. Next, the problem is analyzed to uncover relevant structural properties that are helpful in developing an interaction model based on SWMB-GI.

Note that our analysis in Section 2.3 is developed on the fact that $c(\Gamma)$ and $f_i(T_0,T_i)$ are convex. In other words, the results obtained in Section 2.3 are applicable to the SWMB systems where $c(\Gamma)$ and $f_i(T_0,T_i)$ are convex, $i=1,\ldots,n$. To develop an interaction model for SWMB system with private information, we assume that the average total cost of the warehouse is convex in T_0 and that of buyer *i* is convex in T_i, $i=1,\ldots,n$. Also, warehouse's inventory holding cost attributable to buyer *i*, $H_i(T_i)$ is convex in T_i for given T_0. These assumptions guarantee $c(\Gamma)$ and $f_i(T_0,T_i)$ are all convex. For example, buyer *i* may have a unit holding cost schedule as follows: $2/unit for 0-100 units, and $3/unit for 101-200 units. In this case, $c(\Gamma)$ and $f_i(T_0,T_i)$ are convex and thus the analysis in Section 2.3 is still valid.

Consider a simple interactive process in which the warehouse and the buyers negotiate on the replenishment policies, T_i, $i=1,\ldots,n$. The negotiation process starts with buyer *i* proposing t_i' and D_i to the warehouse, $i=1,\ldots,n$; and the warehouse determines a power-of-two replenishment period, T_0 with respect to buyers' proposed replenishment policies. Then, the warehouse successively modifies T_0 to the next power-of-two period, finds the corresponding optimal power-of-two policy T_i and proposes it to buyer *i*; buyer *i* in turn proposes a compensation amount required from the warehouse should (s)he use the compromised T_i instead of t_i'; and the negotiation repeats until a compromised policy is obtained.

It is interesting that the above negotiation process is similar to SWMB-GI. Specifically, after initialization, SWMB-GI also successively increases T_0 and determines the corresponding T_i for buyer *i*, $i=1,\ldots,n$, at each iteration until the best policy is obtained. We will show that SWMB-GI can be adapted to simulate the above negotiation process.

To apply SWMB-GI in this limited information environment, two main ingredients are missing: (i) the changes in total average cost of the system as T_0

increases; and (ii) optimal T_i for buyer *i* given T_0, $i = 1,\ldots,n$. Though t_i and $c_i(T_i)$ are unknown to other facilities except buyer *i*, and no facility has knowledge on $c(\Gamma)$, (i) and (ii) can be determined with partial information sharing by the warehouse. We first show how to determine (i).

In the negotiation process mentioned earlier, buyer *i* will ask for compensation if (s)he uses T_i, instead of t_i', as the replenishment policy. We define the compensation requested by buyer *i*, $comp_i(T_i)$, as follows.

$$comp_i(T_i) = c_i(T_i) - c_i(t_i') \tag{2.11}$$

Thus, $comp_i(T_i)$ is simply the difference in the total average cost of buyer *i* when T_i is used instead of the optimal power-of-two solution, t_i'. In other words, it is the minimum compensation for buyer *i* so that (s)he considers both policies (replenishes at T_i and t_i') as indifferent. We assume that all the facilities are willing to disclose this piece of information honestly. Using $comp_i(T_i)$, the difference in total average cost of any two policies, Γ^1 and Γ^2, can be restated as follows.

$$\begin{aligned}\Delta(\Gamma^1, \Gamma^2) &= c(\Gamma^1) - c(\Gamma^2) \\ &= \left(c_0(\Gamma^1) + \sum_{i=1}^{n} c_i(T_i^1)\right) - \left(c_0(\Gamma^2) + \sum_{i=1}^{n} c_i(T_i^2)\right) \\ &= \left(c_0(\Gamma^1) - c_0(\Gamma^2)\right) + \sum_{i=1}^{n} \left\{\left(c_i(T_i^1) - c_i(t_i')\right) - \left(c_i(T_i^2) - c_i(t_i')\right)\right\} \\ &= \left(c_0(\Gamma^1) - c_0(\Gamma^2)\right) + \sum_{i=1}^{n} \left(comp(T_i^1) - comp(T_i^2)\right) \end{aligned} \tag{2.12}$$

Therefore, with the additional piece of information on $comp_i(\cdot)$, the warehouse is now in the position to determine the differences in total average cost of any two policies.

Next, we show how the warehouse determines T_i, $i = 1,\ldots,n$, for given T_0. Given T_0, buyer *i* must belong to one of the following three sets: $G(T_0) = \{i : T_0 < t_i'\}$, $E(T_0) = \{i : t_i' \le T_0 \le t_i\}$, and $L(T_0) = \{i : t_i < T_0\}$. Since buyer *i* will request to order according to his/her optimal power-of-two policy initially, t_i' is known to the warehouse.

However, there is no facility in the system possesses the information on t_i, and one may think that no facility in the system is able to determine whether buyer i belongs to $E(T_0)$ or $L(T_0)$ if $t_i' < T_0$. As mentioned in Section 2.3, T_0 will not be smaller than $t_{min} = \min\{t_i' : i = 1, \cdots, n\}$. At the beginning of the negotiation, if the warehouse starts by setting $T_0 = t_{min}$, all the buyers can be grouped into $G(T_0)$ and $E(T_0)$; and $L(T_0) = \{\phi\}$. Thus, given $T_0 = t_{min}$, the optimal solution is simply t_i', $i = 1, \ldots, n$ accordingly to Proposition 2.1. Define $T_i(T_0)$ as the optimal T_i given T_0. As the warehouse successively increases T_0, optimal T_i and classification of buyer i can be determined according to the following proposition.

Proposition 2.2: For a given T_0,

(a) The optimal power-of-two policy is given by

$$T_i = \begin{cases} t_i' & \text{if } i \in G(T_0/2) \\ T_0 & \text{if } i \in E(T_0/2) \text{ and } \Delta_i(T_0, T_0/2) \le 0 \\ T_0/2 & \text{if } i \in E(T_0/2) \text{ and } \Delta_i(T_0, T_0/2) > 0 \\ T_i(T_0/2) & \text{if } i \in L(T_0/2) \end{cases} \tag{2.13}$$

(b) After finding T_i by (2.13), classification of buyer i can be determined by the following rule:

$$\begin{aligned} &i \in G(T_0): \text{ if } T_i > T_0 \\ &i \in E(T_0): \text{ if } T_i = T_0 \\ &i \in L(T_0): \text{ if } T_i < T_0 \end{aligned} \tag{2.14}$$

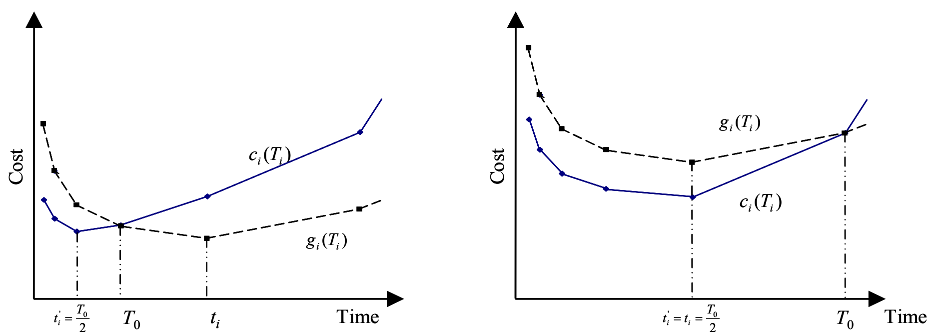
Proof: To prove Proposition 2.2, we consider three cases, namely, $i \in G(T_0/2)$, $i \in E(T_0/2)$, and $i \in L(T_0/2)$ given T_0.

<u>Case 1</u>: $i \in G(T_0/2)$

Since only power-of-two policy is considered, for any given T_0, if $i \in G(T_0/2)$, t_i' is either larger than T_0 or equal to T_0. Either case, the optimal T_i is still t_i'. In the former case, $T_i > T_0$ and $i \in G(T_0)$; in the latter case, $T_i = T_0$ and $i \in E(T_0)$.

<u>Case 2</u>: $i \in E(T_0/2)$

If $i \in E(T_0/2)$, we have two subcases: (i) $i \in E(T_0)$, i.e, $t_i' < T_0 \le t_i$, and (ii) $i \in L(T_0)$, i.e., $t_i < T_0$. Fig. 2.6 shows the average total cost attributable to buyer i for a given T_0 for the two sub-cases.



(i) $t_i' < T_0 \le t_i$ (ii) $t_i < T_0$

Fig. 2.6. Cases when $i \in E(T_0/2)$.

Since $f_i(T_0, T_i) = \max\{c_i(T_i), g_i(T_i)\}$, the solution frontier to the left of T_0 is formed by $g_i(T_i)$ in both subcases as illustrated in Fig. 2.6. Recall that both the warehouse's inventory holding cost attributable to buyer *i*, $H_i(T_i)$, and the total average cost of buyer *i* are convex, so is the total average cost attributable to buyer *i*, $g_i(T_i)$.

In subcase (i), $T_0 \le t_i$. Thus, $g_i(\cdot)$ decreases as T_0 increases from $T_0/2$. Define $\Delta_i(T_0, T_0/2) = g_i(T_0) - g_i(T_0/2)$ as changes in total average cost attributable to buyer *i* if

T_0 is used instead of $T_0/2$ as the replenishment policy for buyer *i*. Then, $\Delta_i(T_0, T_0/2) \le 0$ in this case.

In subcase (ii), $i \in E(T_0/2)$ and $i \in L(T_0)$. Since only power-of-two policies are considered, it must be that $t_i = T_0/2$ and $g_i(\cdot)$ is increasing to the left of $t_i = T_0/2$, i.e., $\Delta_i(T_0, T_0/2) > 0$. Therefore, it is suffice to check changes in $g_i(T_i)$ to determine buyer *i* as $i \in E(T_0)$ or $i \in L(T_0)$.

Since $comp_i(T_i)$ is assumed to be shared between buyer *i* and the warehouse, and $H_i(T_i)$ is known to the warehouse, $\Delta_i(T_0, T_0/2)$ can be calculated as follows.

$$\begin{aligned}
\Delta_i(T_0, T_0/2) &= g_i(T_0) - g_i(T_0/2) \\
&= (c_i(T_0) + H_i(T_0)) - (c_i(T_0/2) + H_i(T_0/2)) \\
&= (c_i(T_0) - c_i(T_0/2)) + (H_i(T_0) - H_i(T_0/2)) \\
&= (c_i(T_0) - c_i(t_i')) - (c_i(T_0/2) - c_i(t_i')) + (H_i(T_0) - H_i(T_0/2)) \\
&= (comp_i(T_0) - comp_i(T_0/2)) + (H_i(T_0) - H_i(T_0/2))
\end{aligned} \tag{2.15}$$

Therefore, if the warehouse has the information on the compensation amount of each buyer, (s)he will be able to determine $\Delta_i(T_0, T_0/2)$. In summary, if $\Delta_i(T_0, T_0/2) \le 0$, it must be that $i \in E(T_0)$ and the optimal solution is $T_i = T_0$; otherwise, $i \in L(T_0)$ and the optimal solution is $T_i = t_i = T_0/2$.

<u>Case 3</u>: $i \in L(T_0/2)$

In this case, $t_i < T_0/2 < T_0$. According to Proposition 2.1, the optimal solution is still t_i, which is $T_i(T_0/2)$. Since $t_i = T_i(T_0/2) < T_0$, $i \in L(T_0)$. *Q.E.D.*

Before presenting the procedure for solving SWMB-PI, we illustrate the information and material flow of the proposed model in Fig. 2.7. Specifically, D_i and t_i' are passed from buyer *i* to the warehouse at the beginning of the negotiation. Then, at each iteration, the warehouse increases T_0 and proposes T_i to buyer *i*; and buyer *i* in turn passes information on $comp_i(T_i)$ to the warehouse and the negotiation process repeats.

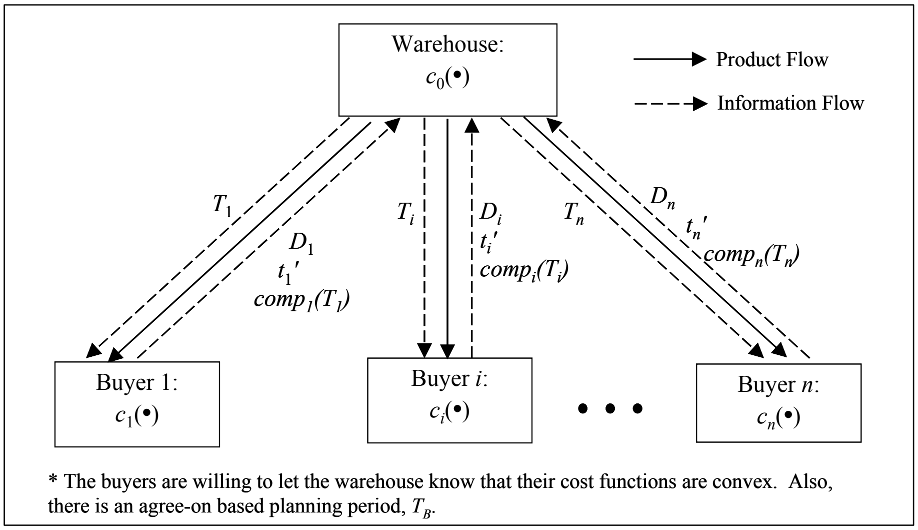


Fig. 2.7. Information and material flow.

Based on the previous discussion, an interaction model, termed SWMB-PI, for the SWMB system with private information is presented as follows.

SWMB-PI Heuristic

Step 0: Initialization

Buyer i, $i = 1, \ldots, n$:

Calculate t_i', and submit it, as well as D_i, to the warehouse.

Warehouse:

(a) Find $t_{min} = \min\{t_i' : i = 1, \cdots, n\}$, set $T_0 = t_{min}$, and $j = 1$.

(b) Initialize:

(i) $G(T_0/2) = \{i : i = 1, \ldots, n\}$, $E(T_0/2) = \{\phi\}$, and $L(T_0/2) = \{\phi\}$.

(ii) $\Gamma^0 = \{\phi\}$, $c_0(\Gamma^0) = \infty$, and $comp_i(T_0/2) = 0$, $i = 1, \ldots, n$.

Step 1: Negotiation

Warehouse:

(a) Choose T_i for buyer i, $i = 1, \ldots, n$, as follows:

$$T_i = \begin{cases} t_i' & \text{if } i \in G(T_0/2) \\ T_0 & \text{if } i \in E(T_0/2) \\ T_i(T_0/2) & \text{if } i \in L(T_0/2) \end{cases} \tag{2.16}$$

(b) Propose the chosen T_i to buyer *i*, $i \in G(T_0/2) \cup E(T_0/2)$.

Buyer *i*, $i \in G(T_0/2) \cup E(T_0/2)$:

Calculate $comp_i(T_i)$ as in (2.11) and submit it to the warehouse.

Step 2: Updating

Warehouse:

(a) For each buyer *i*, $i \in E(T_0/2)$, calculate $\Delta_i(T_0, T_0/2)$ as in (2.15) and update T_i according to condition (2.13). Set $\Gamma^j = \{T_0, T_1, \ldots, T_n\}$.

(b) Calculate $\Delta(\Gamma^j, \Gamma^{j-1})$ using (2.12).

(c) If $\Delta(\Gamma^j, \Gamma^{j-1}) < 0$, update $G(T_0)$, $E(T_0)$, and $L(T_0)$ according to (2.14). Set $j = j + 1$, $T_0 = 2T_0$ and go to Step 1.

Otherwise, stop the negotiation process, and the best power-of-two policy is Γ^{j-1}, and the warehouse announces the corresponding optimal power-of-two policy, T_i, to buyer *i*, $i = 1, \ldots, n$.

2.4.2. Discussion

First, the reader should notice that the warehouse proposes T_i to buyer *i* according to (2.16) but not (2.13) of Proposition 2.2 in Step 1 of SWMB-PI. The reason is that if $i \in E(T_0/2)$, the optimal T_i will be either T_0 or $T_0/2$. As shown previously, the optimal T_i can be determined if the warehouse knows $comp_i(T_0)$ and $comp_i(T_0/2)$. Since the warehouse already knows $comp_i(T_0/2)$ from the previous negotiation, (s)he needs to get $comp_i(T_0)$ by proposing T_0 to buyer *i*. After obtaining this piece of information, the optimal policy, given T_0, is then derived according to (2.13) in Step 2.

Furthermore, the negotiation process excludes buyer *i*, $i \in L(T_0/2)$. As discussed previously, the optimal solution T_i for $i \in L(T_0/2)$ is t_i, and will not change as T_0 increases. Thus, there is no need for the warehouse to further negotiate with buyer *i* in this group.

At first glance, it seems that there involves many iterations before an optimal power-of-two policy is settled. Theoretically, there are infinite potential candidates for T_0. Since only power-of-two policies are considered, the number of T_0 needed to be examined is very small. For example, if $T_B = 1$ minute, only 20 points are needed to cover a year's interval.

One may think that, through negotiation, the warehouse will be able to calculate the cost parameters of buyer *i*, $i = 1, \ldots, n$. Recall that, by assumption, the forms of the objective functions, in addition to the cost parameters, of the buyers are unknown to the warehouse. Without complete knowledge on the form of the cost function of buyer *i*, the warehouse will not even know all the cost parameters, not to mention calculating them.

Also, the above model works exactly the same as the method proposed in Section 2.3. Thus, there is no loss of performance quality when using SWMB-PI instead of SWMB-GI. When the cost functions and model parameters are the same as described in Roundy (1985), the policy found using SWMB-PI will also have 94% effectiveness.

Finally, for further research, SWMB-PI may be expanded to include buyer's willingness to cooperate. Recall that one of the crucial pieces of information of SWMB-PI is $comp_i(T_i)$. In a boarder sense, it can be viewed as the compensation buyer *i* requested to stay cooperation with the warehouse. Thus, buyer *i* can reflect his/her willingness to cooperate by adding an extra penalty term to the compensation term. For example, if it is buyer *i* is only willing to reorder at interval smaller than a certain interval *T*, then for any $T_i > T$ proposed by the warehouse, (s)he can submit a very large number as the compensation term.

2.4.3. Numerical example

Here, we illustrate the model using the same example in 2.3.2. Let $T_B = 1$ month.

Step 0: Initialization

Buyer 1: Submit $t_1' = 1, D_1 = 4{,}000$ to the warehouse.

Buyer 2: Submit $t_2' = 2, D_2 = 6{,}000$ to the warehouse.

Buyer 3: Submit $t_3' = 4, D_3 = 4{,}000$ to the warehouse.

Warehouse: (a) Set $T_0 = t_{min} = 1$ months, and $j = 1$.

(b) Initialize:

(i) $G(T_0/2) = \{1,2,3\}$, $E(T_0/2) = \{\phi\}$, and $L(T_0/2) = \{\phi\}$.

(ii) $\Gamma^0 = \{\phi\}$, $c_0(\Gamma^0) = \infty$, and $comp_i(T_0/2) = 0$, $i = 1,2,3$.

<u>*Iteration 1*</u>

Step 1: Negotiation

Warehouse: Propose $T_1 = 1, T_2 = 2, T_3 = 4$ to buyers 1, 2, and 3, respectively.

Buyer *i*: Compute $comp_i(T_i) = 0$ and submit to the warehouse, $i = 1,2,3$.

Step 2: Updating

Warehouse: Since $E(T_0/2) = \{\phi\}$, set $\Gamma^1 = \{T_0, T_1, T_2, T_3\} = \{1,\ 1,\ 2,\ 4\}$.

Calculate $\Delta(\Gamma^1, \Gamma^0) = -\infty < 0$. Update $G(T_0) = \{2,3\}$, $E(T_0) = \{1\}$, and $L(T_0) = \{\phi\}$. Set $j = 2$, $T_0 = 2T_0 = 2$.

<u>*Iteration 2*</u>

Step 1: Negotiation

Warehouse: Propose $T_1 = 2, T_2 = 2, T_3 = 4$ to buyer 1,2, and 3, respectively.

Buyer 1: Compute $comp_1(2) = 66$ and submit to the warehouse.

Buyer 2: Compute $comp_2(2) = 0$ and submit to the warehouse.

Buyer 3: Computer $comp_3(4) = 0$ and submit to the warehouse.

Step 2: Updating

Warehouse: (a) Since buyer 1 belongs to $E(T_0/2)$, calculate $\Delta_1(T_0, T_0/2)$ as in (2.15):

$$\begin{aligned}\Delta_1(T_0, T_0/2) \\ &= (comp_1(T_0) - comp_1(T_0/2)) + (H_1(T_0) - H_1(T_0/2)) \\ &= (66 - 0) + \left(0 - \tfrac{1}{2}(2)(4000)\left(\tfrac{2}{12} - \tfrac{1}{12}\right)\right) \\ &= -267 < 0\end{aligned}$$

According to (2.13), the optimal policy is $T_1 = T_0 = 2$. Thus, set $\Gamma^2 = \{2, 2, 2, 4\}$.

(b) Calculate $\Delta(\Gamma^2, \Gamma^1)$ using (2.12), i.e.,

$$\begin{aligned}\Delta(\Gamma^2, \Gamma^1) &= (c_0(\Gamma^2) - c_0(\Gamma^1)) + \sum_{i=1}^{n}(comp_i(T_i^2) - comp_i(T_i^1)) \\ &= \left(\tfrac{500}{4/12} - \tfrac{500}{2/12}\right) + ((66 - 0) + (0 - 0) + (0 - 0)) \\ &= -2934 < 0.\end{aligned}$$

(c) Update $G(T_0) = \{3\}$, $E(T_0) = \{1,2\}$, and $L(T_0) = \{\phi\}$. Set $j = 3$, $T_0 = 2T_0 = 8$.

Iteration 3

Step 1: Negotiation

Warehouse: Propose $T_1 = 4, T_2 = 4, T_3 = 4$ to buyer 1, 2, and 3, respectively.

Buyer 1: Compute $comp_1(4) = 1100$ and submit to the warehouse.

Buyer 2: Compute $comp_2(4) = 1100$ and submit to the warehouse.

Buyer 3: Computer $comp_3(4) = 0$ and submit to the warehouse.

Step 2: Updating

Warehouse: (a) Both buyers 1 and 2 belong to $E(T_0/2)$.

(i) Buyer 1: $\Delta_1(T_0, T_0/2) = 367 > 0$. According to (2.13), the optimal policy is $T_1 = T_0/2 = 2$.

(ii) Buyer 2: $\Delta_2\left(T_0, T_0/2\right) = 100 > 0$. Thus, optimal policy is

$$T_2 = T_0/2 = 2.$$

(iii)Set $\Gamma^3 = \{4, 2, 2, 4\}$.

(b) $c_0(\Gamma^3) = \frac{500}{4/12} + \frac{1}{2}(2)\left(4000 + 6000\right)\left(\frac{4}{12} - \frac{2}{12}\right) = 3{,}167$

$c_0(\Gamma^2) = \frac{500}{2/12} = 3{,}000$

Since $T_i^3 = T_i^2, i = 1,2,3$, thus $\sum_{i=1}^{n}\left(comp_i(T_i^3) - comp_i(T_i^2)\right) = 0$.

$$\begin{aligned}\Delta\left(\Gamma^3, \Gamma^2\right) &= \left(c_0(\Gamma^3) - c_0(\Gamma^2)\right) + \sum_{i=1}^{n}\left(comp_i(T_i^3) - comp_i(T_i^2)\right) \\ &= \left(3{,}167 - 3000\right) + \left((66 - 66) + (0 - 0) + (0 - 0)\right) \\ &= 167 > 0.\end{aligned}$$

The average total cost is increasing. Thus, the optimal power-of-two policy is $\Gamma^2 = \{2, 2, 2, 4\}$, which is the same as the solution obtained in Section 2.3.2.

2.5. Conclusion

In this chapter, we consider the problem of coordinating SWMB system where the objective is to find a replenishment policy for each facility such that the total average ordering and inventory-related cost of the system is minimized. This problem has been studied thoroughly in the past under the global information environment. This study is different from the previous studies in that we consider the SWMB system in which there is no single facility that has complete information about the whole system and each facility has self decision-making authority. Specifically, the objective function and cost parameters of each facility are regarded as private information that no other facilities in the system have access to.

In this study, we approach the problem by first modeling the problem under centralized decision-making paradigm and develop heuristic for solving the problem. Then, the proposed heuristic is modified for SWMB system with private information. As a result, two heuristics for finding power-of-two policies are developed. The first heuristic, termed SWMB-GI, is developed assuming complete information access. The effectiveness of SWMB-GI is 94% and the complexity is $O(m)$ where $m=ln(t_{max}-t_{min})$.

The second procedure, termed SWMB-PI, exploits relevant properties of the problem and solves the problem through negotiation and partial information sharing such that there is no performance loss for the system when SWMB-PI is used instead of SWMB-GI. In a system where all the model parameters are the same as the one studied in Section 2.3, SWMB-PI will find a power-of-two policy with 94% effectiveness.

An important result of this study is that we demonstrate that knowledge of the system can be partially recovered through negotiation and partial information sharing. In next section, we will extend our knowledge on SWMB system to the single-vendor multi-buyers system with private information.

CHAPTER III
SINGLE-VENDOR MULTI-BUYERS SYSTEM

3.1. Introduction

In this chapter, we study the problem of coordinating the single-vendor multiple-buyer (SVMB) inventory system. In a SVMB system, a single vendor receives the order from multiple buyers and produces batches of product at a constant rate to satisfy the buyers' demands without backlogging. The objective is to find a production schedule for the vendor and replenishment policies for the buyers that minimize the long-run total average setup/ordering and inventory-related cost of the system. The SVMB inventory system is studied under two scenarios, namely, SVMB with global information and SVMB with private information. In the former case, there is a single decision-maker who has all the information about the system. In the latter case, no facility has complete access to the information of the model parameters. Specifically, the following information is private to each facility: (a) facility's objective function, (b) facility's setup/ordering cost, and (c) facility's inventory holding cost. Furthermore, facilities are responsible for specifying their own inventory policies; however, they will collaborate with the others to achieve a minimal system cost.

To develop solution approaches for coordinating SVMB inventory system, we consider two nested and stationary policies. The first policy is proposed by Banerjee and Burton (1994) and assumes that all the buyers must replenish simultaneously. While Banerjee and Burton (1994) considered integer-ratio policies, this chapter considers only power-of-two polices. This policy is termed "common replenishment period policy" (CRPP) in this chapter. The second policy is an extension of CRPP that allows buyers to replenish asynchronously. This policy is termed "asynchronous replenishment period policy" (ARPP) in this chapter.

Assuming that there is a fixed base-planning period (T_B), and that all the order intervals are power-of-two multiples of T_B, two heuristics are developed for finding CRPP and ARPP for the global information case. In turn, the proposed heuristics are modified for the SVMB inventory with private information. The remainder of this chapter is organized as follows. The problem is described in Section 3.2. In Section 3.3, we analyze the SVMB system under global information environment and present heuristics for finding CRPP and ARPP. Next, two interaction models for SVMB inventory system coordination with private information are presented in Section 3.4. Then, computational results are given in Section 3.5, followed by conclusion in Section 3.6.

3.2. Problem description

The following notation is used throughout the chapter:

n:	Number of buyers
P:	Constant production rate at the vendor
D_i:	Constant demand rate at buyer i, $i = 1,\ldots,n$
T_B:	Base planning period
T_S:	Decision variable: production start time
T_0:	Decision variable: production cycle for the vendor
T_R:	Decision variable: common replenishment interval for buyers
T_i:	Decision variable: replenishment interval for buyer i, $i = 1,\ldots,n$
Γ:	Policy, $\Gamma = \{T_S, T_0, T_1, \ldots, T_n\}$
$K_0(\Gamma)$:	Setup cost function of the vendor
$H_0(\Gamma)$:	Inventory holding cost function of the vendor
$K_i(T_i)$:	Setup cost function of buyer i, $i = 1,\ldots,n$
$H_i(T_i)$:	Inventory holding cost function of buyer i, $i = 1,\ldots,n$
$c_0(\Gamma)$:	Cost function of the vendor, $c_0(\Gamma) = K_0(\Gamma) + H_0(\Gamma)$

$c_i(T_i)$: Cost function of buyer *i*, $c_i(T_i)=K_i(T_i)+H_i(T_i)$, $i=1,\ldots,n$

$c(\Gamma)$: Cost function of the system, i.e., $c(\Gamma)=c_0(\Gamma)+\sum_{i=1}^{n}c_i(T_i)$.

$\Delta(\Gamma^i,\Gamma^j)$: Changes in total average cost of the system if Γ^i is used instead of Γ^j, $\Delta(\Gamma^i,\Gamma^j)=c(\Gamma^i)-c(\Gamma^j)$

τ_i: Optimal solution that minimizes $c_i(T_i)$, $i=1,\ldots,n$

t_i: Power-of-two solution that minimizes $c_i(T_i)$, $i=1,\ldots,n$

The SVMB is an inventory system where a single vendor is the sole supplier of *n* buyers. The buyers are retailer-type facilities that do not involve production. Each buyer *i* faces an external demand, D_i, that must be met without shortage or backlogging. The vendor receives orders from each buyer and produces the product in batches. The vendor's production rate, *P*, and the buyer *i*'s demand rate D_i, $i=1,\ldots,n$, are assumed to be given and constant, and $P\geq\sum_{i=1}^{n}D_i$. The replenishment at each buyer is assumed to be instantaneous. The objective is to find a policy, Γ, that minimizes the total average setup/ordering and inventory-related cost of the system, $c(\Gamma)$.

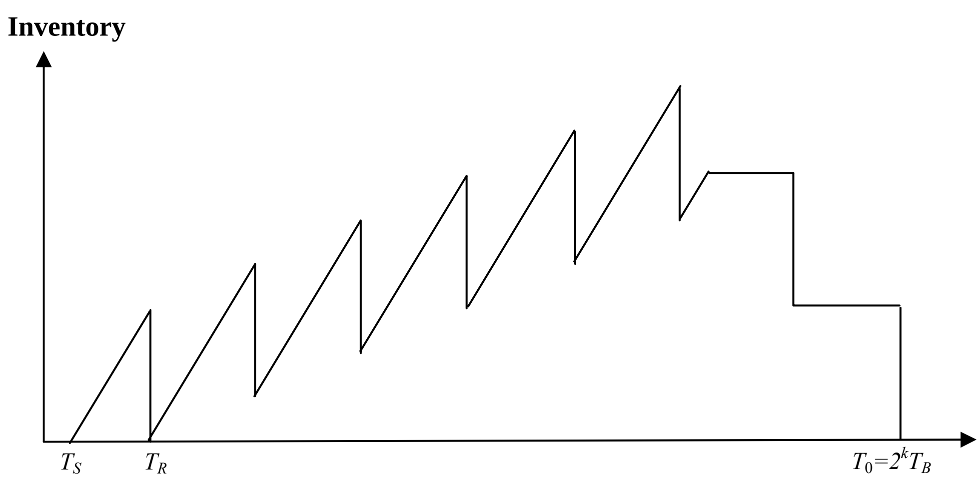


Fig. 3.1. Inventory level at the vendor under CRPP.

Similar to other studies (Banerjee 1986, Goyal 1988, Banerjee and Burton 1994, Goyal 1995, Hill 1997, and Goyal and Nebebe 2000), the vendor is assumed to begin production at time $T_S = T_R\left(1 - \sum_{i=1}^{n} D_i / P\right)$ such that the vendor will have no inventory at time T_R (Fig. 3.1). The rationale of this assumption is that, for any given T_R and T_0, the inventory level of the vendor will be higher if (s)he starts the production at $T < T_S$. Therefore, (s)he will be better off to start the production at T_S.

To coordinate the SVMB inventory system, we have the following additional assumptions:

(a) The total average cost of the vendor is convex in T_0 and T_i and that of buyer *i* is convex in T_i, $i = 1,\ldots,n$.

(b) Only power-of-two policies with a fixed base planning period, T_B, are considered.

(c) A maximum replenishment period, $T_{\max}$, is given.

Assumption (a) guarantees $c(\Gamma)$ is convex. Assumption (b) states that the replenishment policy of each facility is a power-of-two multiple of T_B. Assumption (c) states that no buyer should have replenishment period longer than $T_{\max}$, which is reasonable in practice because it is rare for a buyer to have an extremely long replenishment period such as 2 years.

To determine the average inventory holding cost for the vendor, $H_0(\Gamma)$, and that of buyer *i*, $H_i(T_i)$, $i = 1,\ldots,n$, we need to obtain the average inventory level of each facility in the system. Hill (1997) described a simple method to determine the average inventory level of the vendor and the system for single-vendor single-buyer system. His approach is extended here to the single-vendor multi-buyers case as follows.

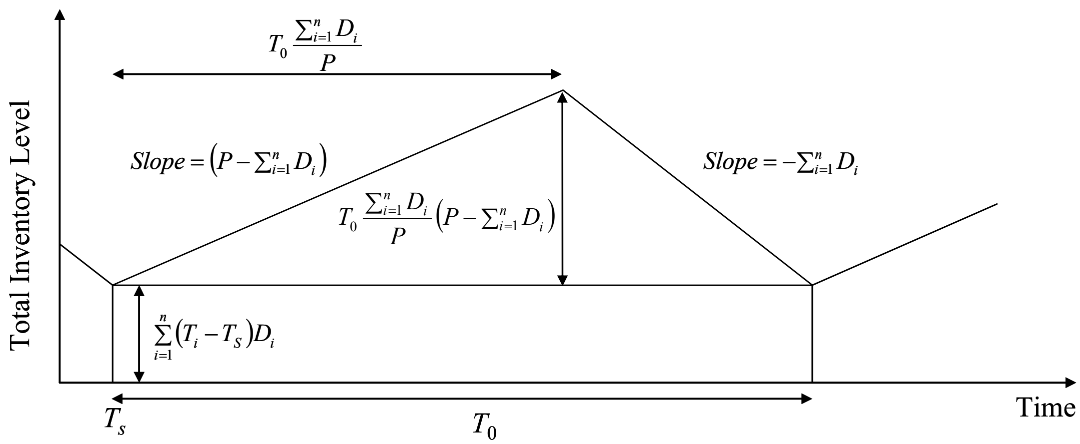


Fig. 3.2. Total system inventory.

Fig. 3.2 presents the total system inventory versus time for an arbitrary policy, Γ. The total inventory in the system is minimum at T_S, where the vendor has no on-hand inventory, and buyer *i*, $i = 1, \ldots, n$, has inventory just enough to satisfy the demand until the next replenishment. Therefore, the inventory level at buyer *i* at T_S is $(T_i - T_S)D_i$, and the inventory level of the system is $\sum_{i=1}^n (T_i - T_S)D_i$. Once the production batch starts at the vendor, the production will continue until the total units produced equal to the total demands of all the buyers for the interval of T_0. The production period is thus $T_0\left(\sum_{i=1}^n D_i / P\right)$. Since the total inventory in the system increases at a rate of $P - \sum_{i=1}^n D_i$ during the production period, it follows that the height of the upper triangle in Fig. 3.2 is $T_0\left(\sum_{i=1}^n D_i / P\right)\left(P - \sum_{i=1}^n D_i\right)$. Using this model the average inventory of the system, I_{Sys}, is

$$I_{Sys} = \frac{1}{2} T_0 \sum_{i=1}^n D_i \left(1 - \sum_{i=1}^n D_i / P\right) + \sum_{i=1}^n (T_i - T_S) D_i \,. \tag{3.1}$$

Since buyer *i* is assumed to be retailer-type facility with no production, the average inventory of buyer *i*, $I_i, i = 1, \ldots, n$ is

$$I_i = \frac{1}{2} T_i D_i \,. \tag{3.2}$$

It follows that the average inventory of the vendor, I_v, is

$$\begin{aligned} I_v &= I_{Sys} - \sum_{i=1}^{n} I_i \\ &= \frac{1}{2} T_0 \sum_{i=1}^{n} D_i \left(1 - \sum_{i=1}^{n} D_i / P\right) + \sum_{i=1}^{n} (T_i - T_S) D_i - \sum_{i=1}^{n} \frac{1}{2} T_i D_i \\ &= \frac{1}{2} T_0 \sum_{i=1}^{n} D_i \left(1 - \sum_{i=1}^{n} D_i / P\right) + \sum_{i=1}^{n} \left(\frac{T_i}{2} - T_S\right) D_i \ . \end{aligned} \tag{3.3}$$

Once the average inventory levels of all facilities are calculated as shown above, the corresponding average inventory holding cost can be determined.

3.3. SVMB with global information

The SVMB system with global information is characterized by a single decision maker that has access to all the model information, and has the authority to specify the inventory policies for all the facilities in the system. In this section, a search method for finding a CRPP for SVMB system with global information is proposed. Then, by relaxing the common replenishment assumption, we extend our solution approach to finding an ARPP. The solution methodologies are carefully designed such that they can be used as a stepping-stone for the development of an interaction model applicable to SVMB inventory system with private information.

3.3.1. CRPP with global information

To find a CRPP, there are only two variables needed to be determined, namely, T_0 and T_R. Due to convexity of $c(\Gamma)$, the optimal T_0 for given T_R can be found by successively increasing T_0 until the total average cost of the system increases. Initially setting T_0 equal to T_R, the proposed heuristic calculates the corresponding total average cost of the system, $c(\Gamma)$; and then successively increases T_0 to the next power-of-two period until $c(\Gamma)$ increases. At this point, the optimal T_0 for a given T_R is found. In turn, T_R is successively increased to the next power-of-two period and the procedure of finding the optimal T_0 is repeated. Since there are only finite possible values for T_R, the optimal policy can be found by repeating the same procedures for all possible T_R. The reader should note that the policy found in each step is a feasible power-of-two policy.

Defining $\Gamma(T_R)$ as the least total average cost policy given T_R and Γ^{OPT} as the least total average cost policy found, the heuristic for finding the optimal CRPP for SVMB system with global information (CRPP-GI) is presented as follows.

CRPP-GI Heuristic

Step 0: Initialization

Set $\Gamma^{OPT} = \{T_S = \infty, T_i = \infty : i = 0, \cdots, n\}$, and $T_R = T_B$.

Step 1: Find optimal T_0 given T_R

1.0 Set $T_0 = T_R$, $T_S = T_R\left(1 - \sum_{i=1}^{n} D_i / P\right)$, $\Gamma^0 = \{T_S, T_0, T_i = \infty : \forall i\}$, $c(\Gamma^0) = \infty$, $\Gamma(T_R) = \Gamma^0$, and $k = 1$.

1.1 Set $\Gamma^k = \{T_S, T_0, T_i = T_R : i = 1, \cdots, n\}$. Calculate $\Delta(\Gamma^k, \Gamma^{k-1})$.

1.2 If $\Delta(\Gamma^k, \Gamma^{k-1}) < 0$, the total average cost is decreasing. Set $T_0 = 2T_0$, $k = k + 1$ and go to Step 1.1.

Otherwise, the total average cost ceases to decrease and optimal T_0 is found for current T_R. Set $\Gamma(T_R) = \Gamma^{k-1}$ and go to Step 2.

Step 2: Updating

2.1 Calculate $\Delta(\Gamma(T_R), \Gamma^{OPT})$.

2.2 If $\Delta(\Gamma(T_R), \Gamma^{OPT}) < 0$, update $\Gamma^{OPT} = \Gamma(T_R)$, set $T_R = 2T_R$ and continue to Step 2.3.

Otherwise, the total average cost ceases to decrease as T_R increase. The optimal policy is Γ^{OPT} and STOP.

2.3 If $T_R > T_{\max}$, the optimal policy is Γ^{OPT} and STOP.

Otherwise, go to Step 1.

The reader should note that the heuristic can be sped up by taking advantage of the convexity property of $c(\Gamma)$. Let $T_0(T_R)$ be the optimal T_0 given T_R. For every given T_R in Step 1, the search for the optimal T_0 can be started from $T_0 = T_0(T_R/2)$ instead of $T_0 = T_R$ because $c(\Gamma)$ is convex in both T_0 and T_R. However, since there are not many possible T_R points, time-savings in implementing this modification will not be significant. Thus, we presented CRPP-GI in a straightforward manner as above.

3.3.2. ARPP with global information

In this section we show that an ARPP can be obtained by relaxing the common replenishment assumption for any given CRPP. As it will become evident later in this chapter, the benefits of relaxing the common replenishment assumption are that: (a) the total average cost of the system is less than or equal to that of CPRR; and (b) each buyer's demand is satisfied using a replenishment policy that is as close to his/her individual optimal replenishment policy as possible.

Since power-of-two policies are considered, the individual optimal replenishment period for buyer *i*, τ_i, must be rounded to power-of-two multiple of T_B, i.e., $t_i = 2^{m_i} T_B$ where m_i satisfies the following condition: $2^{m_i - 1}\sqrt{2} \le \tau_i / T_B \le 2^{m_i}\sqrt{2}$. For details of the power-of-two rounding procedure, please refer to Roundy (1985). For any given CRPP (i.e., given T_0 and T_R), buyer *i* must belong to either one of the following three sets: $E(T_R) = \{i : t_i = T_R\}$, $L(T_R) = \{i : t_i < T_R\}$, and $G(T_R) = \{i : t_i > T_R\}$.

<u>Case 1</u>: $i \in E(T_R)$

In this case, buyer *i* is served according to his/her optimal power-of-two policy by replenishing at T_R.

<u>Case 2</u>: $i \in L(T_R)$

Assume that buyer *i*, $i \in L(T_R)$, replenishes according to his/her optimal replenishment solution, t_i, instead of T_R. In this case, buyer *i* clearly will incur lower total average cost. Furthermore, the vendor will have less average on-hand inventory as vendor delivers more frequently. So, the vendor will incur lower inventory holding cost while the setup cost remains the same for given T_0 and will lead to lower $c_0(\Gamma)$. It is easy to verify from (3.3) that the changes in average inventory of the vendor is $(t_i - T_R)D_i/2$ should buyer *i* replenish at t_i instead of T_R. Thus, the vendor, the buyer *i*, $i \in L(T_R)$, and the system will be beneficial if buyer *i* replenishes at t_i instead of T_R.

Fig. 3.3 showed the inventory level versus time at the vendor if buyer *i* replenishes at t_i instead of T_R (the solid line). Also presented is the inventory level against time at the vendor under the original CRPP (the dash line).

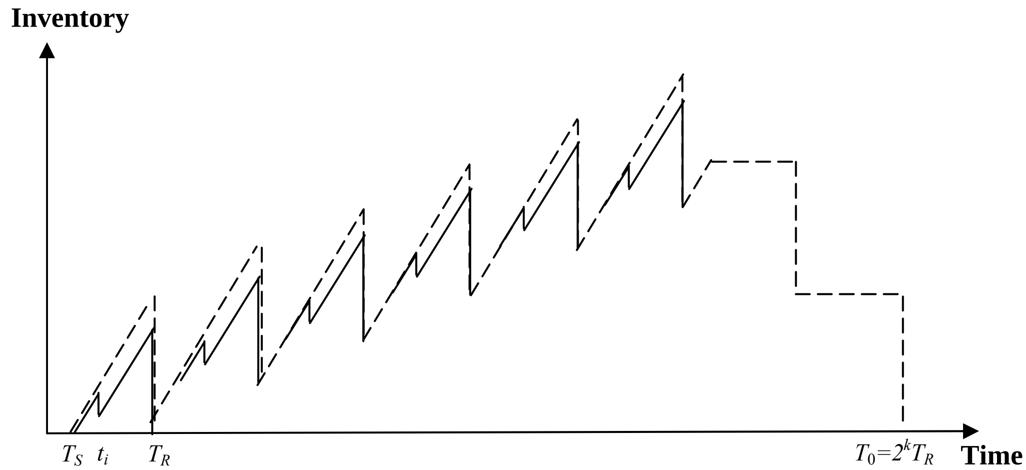


Fig. 3.3. Inventory at the vendor.

Since $c(\Gamma)$ decreases if buyer *i* replenishes at $t_i < T_R$, it is beneficial to replenish all buyers in $L(T_R)$ according to their corresponding optimal replenishment policies. In general, however, the vendor may not have enough on-hand inventory to satisfy the demands of all the buyers in $L(T_R)$ at time T, $T < T_R$. Firstly, since T_S is fixed, at any time T, $T < T_S$, the vendor obviously does not have any on-hand inventory because the production has not started yet. Secondly, at any time T, $T_S < T < T_R$, the total units produced by vendor is $(T - T_S)P$. If the total demands for all the buyers who request to replenish at time T is larger than $(T-T_S)P$, the vendor will not be able to satisfy all the demands. Therefore, some buyers in $L(T_R)$ have to replenish at a time greater than t_i.

For any T such that $t_i \le T \le T_R$, the vendor and the buyer *i*, $i \in L(T_R)$, will incur less average cost if buyer *i* replenishes at T instead of T_R. Therefore, we should try to replenish the buyers as close to their optimal replenishment policies as possible. Since T_B is predetermined and the replenishment period must be power-of-two multiple of T_B, there are finite possible "replenishment time slots" between T_S and T_R. Thus, we can restate the problem as to assign the buyers to different "replenishment time slots" such that the total decrease in the total average cost of the system is maximized. Regarding each time-slot as a knapsack and the available on-hand inventory at the vendor as the capacity of the knapsack, we propose a myopic approximation strategy that solves a "0-1" knapsack problem for each "replenishment time slot", $2^m T_B$ where $T_S < 2^m T_B < T_R$, for finding the replenishment policies for the buyers in $L(T_R)$.

Before presenting the "0-1" knapsack problem, we define the following additional notation.

$\Delta c_{0,i}\left(T_i^1, T_i^2\right)$: Changes in total average cost of the vendor if T_i^1 is used instead of T_i^2 as buyer *i*'s replenishment policy

$\Delta c_i\left(T_i^1, T_i^2\right)$: Changes in total average cost of buyer *i* if T_i^1 is used instead of T_i^2 as buyer *i*'s replenishment policy

$\Delta c_{Total,i}\left(T_i^1, T_i^2\right)$: Changes in total average cost of the system if T_i^1 is used instead of T_i^2 as buyer *i*'s replenishment policy, $\Delta c_{Total,i}\left(T_i^1, T_i^2\right) = \Delta c_{0,i}\left(T_i^1, T_i^2\right) + \Delta c_i\left(T_i^1, T_i^2\right)$

$A\left(2^m\right)$: Set of buyers who are assigned to the replenishment time slot $2^m T_B$

$\overline{A}$: Set of buyers who have not been assigned to any replenishment time slot

B: Set of buyers who are eligible to be assigned to the current replenishment time slot, $2^m T_B$, $B = \{i : i \in \overline{A} \text{ and } t_i \leq 2^m T_B\}$

$I_B(2^m)$: Available inventory at the vendor for buyers in set B for replenishment time slot 2^m

x_i: 0-1 variable in which $x_i = 1$ if buyer i is assigned to the current replenishment time slot; $x_i = 0$, otherwise.

The "0-1" knapsack problem that assign buyers to a given replenishment time slot, $2^m T_B$, is formulated as follows:

$$\text{(KP)} \quad \min \quad \sum_{i \in B} \Delta c_{Total,i}\left(2^m T_B, T_R\right) x_i = \sum_{i \in B} \left\{\Delta c_{0,i}\left(2^m T_B, T_R\right) + \Delta c_i\left(2^m T_B, T_R\right)\right\} x_i \tag{3.4}$$

$$\text{s.t.} \quad \sum_{i \in B} \left(2^m T_B D_i\right) x_i \leq I_B\left(2^m\right) \tag{3.5}$$

$$x_i \in \{0,1\} \tag{3.6}$$

Since $\sum_{i \in B} \Delta c_{Total,i}\left(2^m T_B, T_R\right) x_i \leq 0$, objective function (3.4) states the changes in total average cost of the system is minimized. Constraint (3.5) guarantees that the vendor must have sufficient on-hand inventory to satisfy the demands of the buyers who use $2^m T_B$ as the replenishment policy.

At time $2^m T_B$, the total production quantities are $P\left(2^m T_B - T_S\right)$, and the number of units that are required to satisfy the demands for the buyers who have been assigned to replenishment time slot, $T < 2^m T_B$, are $2^m T_B \sum_{k=s}^{m-1} \sum_{i \in A(2^k)} D_i$, where s is the smallest integer such that $2^s T_B > T_S$. It follows that

$$I_B\left(2^m\right) = P\left(2^m T_B - T_S\right) - 2^m T_B \sum_{k=s}^{m-1} \sum_{i \in A(2^k)} D_i \,. \tag{3.7}$$

The myopic approximation strategy for finding the replenishment policy for buyer i, $i \in L(T_R)$, is to solve (KP) successively for each replenishment time slot.

Though (KP) is known to be *NP*-hard, it has been studied extensively in the past and different solution approaches can be found in the literature. To find the exact solution, dynamic programming approaches (Bellman 1957, Toth 1980) and branch-and-bound algorithms (Nauss 1976, Fayard and Plateau 1982, Martello and Toth 1990, etc.) are developed. Besides, there are several fast approximation algorithms available (Dantzig 1957, Ibarra and Kim 1975, Sahni 1975, Pisigner 1997). In the experiments provided in later section, a simple $O(n)$ approximation algorithm (algorithm *L*) proposed by Sahni (1975) yielded good results for this application.

Given T_R, the replenishment policy for buyer *i*, $\forall i \in L(T_R)$, is determined by the following heuristic.

Knapsack Heuristic (KH)

Step 0: Let $2^s T_B$ be the first power-of-two replenishment period after the production batch starts, i.e., $2^{s-1} T_B < T_S < 2^s T_B$.

0.1: Initialize $\overline{A} = L(T_R)$.

0.2: For $m = s$ to T_R / T_B, initialize $A(2^m) = \phi$.

0.3: Let $m = s$.

Step 1: Update $B = \{i : i \in \overline{A} \text{ and } t_i \le 2^m T_B\}$ and calculate $I_B(2^m)$ as in (3.7).

Step 2: For each $i \in B$, calculate

$$\Delta c_{Total,i}(2^m T_B, T_R) = \Delta c_{0,i}(2^m T_B, T_R) + \Delta c_i(2^m T_B, T_R).$$

Step 3: Solve (KP) to find the set of buyers to be replenished at every $2^m T_B$.

Step 4: Update $A(2^m) = \{i : i \in B \text{ and } x_i = 1\}$ and $\overline{A} = \overline{A} - A(2^m)$.

Step 5: Set $T_i = 2^m T_B, \forall i \in A(2^m)$, and $m = m + 1$.

Step 6: If $2^m T_B < T_R$, go to Step 1. Otherwise, set $T_i = T_R, \forall i \in \overline{A}$, and STOP.

<u>Case 3</u>: $i \in G(T_R)$

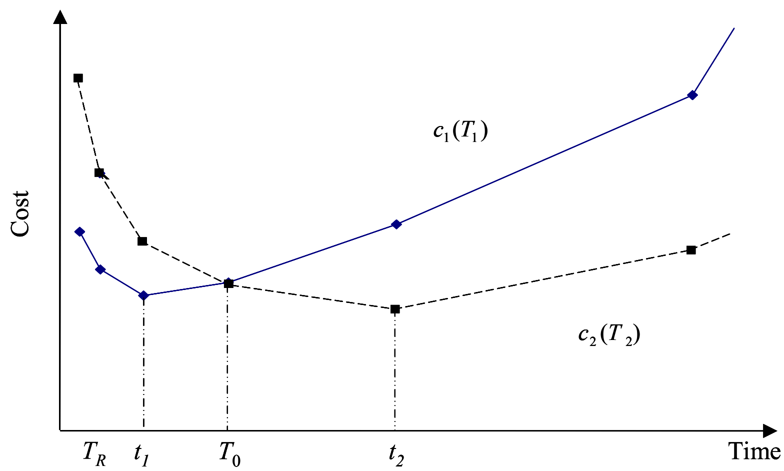


Fig. 3.4. Average cost function of two different buyers.

Consider two buyers in $G(T_R)$ with different average cost functions as shown in Fig. 3.4. Given T_0 and T_R, $c_1(T_1)$ is at its minimum if buyer 1 replenishes at $T_1 = t_1$. On the other hand, buyer 2's replenishment policy is constrained by T_0 because only nested policies are considered. Nonetheless, buyer 2 will incur lower average cost if (s)he replenishes at $T_2 = T_0$ because $c_2(T_2)$ is convex in T_2. Thus, buyers in $G(T_R)$ will benefit from replenishing less often.

However, a buyer's improvement in average cost is at the expense of the vendor because of the increase in on-hand inventory at the vendor. If buyer *i* replenishes at $T_i > T_R$ instead of T_R, it is easy to verify from (3.1) to (3.3) that the changes in average inventory of the system and that of the vendor are $(T_i - T_R)D_i > 0$ and $(T_i - T_R)D_i/2 > 0$, respectively. Therefore, the system will improve only if the additional holding cost incurred at the vendor is smaller than the savings of buyer *i*. Since $c(\Gamma)$ is convex in T_i and there are finite number of possible T_i between T_R and T_0, we can find the optimal power-of-two policy for buyer *i*, given T_R and T_0, by first setting $T_i = 2T_R$ and successively increasing T_i to the next power-of-two period until the changes in total average cost of the system starts to increase, i.e., $\Delta c_{Total,i}(T_i, T_i/2) > 0$. The heuristic to determine T_i for buyer *i* is as follows.

Replenishment Heuristic (RH)

Step 0: Initialize $T_i = 2T_R$.

Step 1: If $T_i \geq T_0$, set $T_i = T_0$. STOP. Otherwise, continue to Step 2.

Step 2: Calculate $\Delta c_{Total,i}(T_i, T_i/2)$.

Step 3: If $\Delta c_{Total,i}(T_i, T_i/2) > 0$, the total average cost is increasing. Set $T_i = T/2$. STOP.

Otherwise, set $T_i = 2T_i$ and go to Step 1.

From the analysis of the three cases above, an ARPP can be found for any given T_R and T_0. Similar to CRPP-GI, for given T_R, the optimal T_0 can be found as follows. Start with $T_0 = T_R$ and assign each buyer to one of the three sets, namely, $E(T_R)$, $L(T_R)$, and $G(T_R)$, find the corresponding replenishment policy for each buyer. Then successively increase T_0 to the next power-of-two period and determine the replenishment policy for each buyer until the total average cost increases. At this point, the optimal T_0 for given T_R is found. The procedure is repeated for all possible T_R.

Though the replenishment policies for buyers in $E(T_R)$ and $L(T_R)$ are not affected by T_0, it is not the case for the buyers in $G(T_R)$. Defining $T_i(T_0, T_R)$ as the optimal replenishment policy for buyer *i* given T_0 and T_R, if $T_i(T_0, T_R) < T_0$, the optimal solution for buyer *i* given T_R will not change as T_0 increases due to the convexity of $c(\Gamma)$. Thus, only buyer *i*, $i \notin \{i : T_i(T_0, T_R) < T_0\}$, may change his/her replenishment policy as T_0 increases. For easy reference for this group of buyers in the proposed heuristic, we define the set $G(T_0, T_R) = G(T_R) - \{i : T_i(T_0, T_R) < T_0\}$, which is the set of buyers whose optimal policies have not been determined yet given T_0 and T_R. The proposed heuristic for finding an ARPP that coordinate the SVMB system with global information, termed ARPP-GI, is presented as follows.

ARPP-GI Heuristic

Step 0: Initialization

For each buyer *i*, calculate t_i. Set $\Gamma^{OPT} = \{T_S = \infty, T_i = \infty : i = 0, \cdots, n\}$, and $T_R = T_B$.

Step 1: Find optimal T_0 given T_R

1.0 Initialize sets $E(T_R)$, $L(T_R)$, and $G(T_R)$, and set $T_S = T_R\left(1 - \sum_{i=1}^{n} D_i / P\right)$, and $T_0 = T_R$.

1.1 $\forall i \in E(T_R)$, set $T_i = T_R$.

1.2 $\forall i \in L(T_R)$, set T_i by Procedure KH.

1.3 $\forall i \in G(T_R)$, set $T_i = T_R$, $\Gamma^0 = \{T_S, T_0, T_i : i = 1, \ldots, n\}$, $\Gamma(T_R) = \Gamma^0$, $c(\Gamma^0) = \infty$, $G(T_0/2, T_R) = G(T_R)$, and $k = 1$.

1.4 $\forall i \in G(T_0/2, T_R)$

1.4.1 Set T_i by Procedure RH.

1.4.2 $\forall i \in G(T_R) - G(T_0/2, T_R)$, set $T_i = T_i(T_0/2, T_R)$.

1.4.3 Update $G(T_0, T_R) = G(T_R) - \{i : T_i(T_0, T_R) < T_0\}$.

1.4.4 Update Γ^k and calculate $\Delta(\Gamma^k, \Gamma^{k-1})$.

1.4.5 If $\Delta(\Gamma^k, \Gamma^{k-1}) < 0$, the total average cost is decreasing. Set $T_0 = 2T_0$, $k = k + 1$ and go to Step 1.4.

Otherwise, the total average cost ceases to decrease and optimal T_0 is found for current T_R. Set $\Gamma(T_R) = \Gamma^{k-1}$ and go to Step 2.

Step 2: Updating

2.1 Calculate $\Delta(\Gamma(T_R), \Gamma^{OPT})$.

2.2 If $\Delta\left(\Gamma(T_R),\Gamma^{OPT}\right)<0$, update $\Gamma^{OPT}=\Gamma(T_R)$ and set $T_R=2T_R$.

2.3 If $T_R>T_{\max}$, the optimal policy is Γ^{OPT} and STOP.

Otherwise, go to Step 1.

The reader should note that RH is presented in general for any given T_R and T_0 such that it starts searching the optimal T_i by setting $T_i=2T_R$ and increasing T_i successively. In other words, RH is designed to find the optimal T_i for buyer *i*, $i\in G(T_R)$ for any given CRPP. However, when RH is applied in Step 1.4 of ARPP-GI, only the replenishment policy for buyer *i*, $i\in G(T_0/2,T_R)$ is needed to determine. Since $c(\Gamma)$ is convex, the optimal replenishment policy for buyer *i*, $i\in G(T_0/2,T_R)$, must be $T_i\geq T_0/2$. It follows that there are only two possible replenishment time slots for T_i, namely, $T_0/2$ and T_0. Thus, the procedure can be sped up by setting $T_i=T_0$ at step 0 of RH when calling RH from ARPP-GI and perform one iteration to determine the optimal replenishment policy for buyer *i*.

3.4. SVMB with private information

In this section, we study the SVMB system with private information. First we restate the SVMB under private information problem and present essential characteristics of the problem.

3.4.1. Problem description and analysis

The SVMB system with private information has the following characteristics:

(a) Each facility in the system has self decision-making authority.

(b) No single facility has complete information about the whole system.

(c) Objective function and the corresponding parameters of each facility is private information.

(d) Partial information is shared among the facilities in order to achieve close-to-optimal solutions.

(e) All facilities agree on the use of power-of-two policy with regard to a fixed base-planning period, T_B.

(f) All facilities agree on a maximum replenishment period, $T_{\max}$.

Consider a simple interactive process in which the vendor and the buyers negotiate on the replenishment policies, T_i, $i = 1,\ldots,n$. The negotiation process starts with buyer *i*, $i = 1,\ldots,n$, proposing t_i and D_i to the vendor; and the vendor determines a power-of-two production cycle, T_0. Then, the vendor successively modifies T_0 to the next power-of-two period, finds the corresponding power-of-two replenishment period T_i and proposes it to buyer *i*; buyer *i* in turn proposes a compensation amount required from the vendor should (s)he use the compromised T_i instead of t_i; and the negotiation repeats until a compromised policy is obtained.

This negotiation process is similar to CRPP-GI and ARPP-GI in that they all successively increase T_0 and determine the corresponding T_i for buyer *i*, $i = 1,\ldots,n$ at each iteration until the best policy is obtained. However, direct applications of CRPP-GI and ARPP-GI in this limited information environment is not possible because no facility in the system has complete knowledge on $c(\Gamma)$ and the changes in total average cost of the system when different policies are used.

In the negotiation process mentioned earlier, buyer *i* will ask for compensation during each round of the negotiation to make it attractive for him/her to move away from t_i. As discussed in Chapter II, $comp_i(T_i)$, which is the difference in the total average cost of buyer *i* when T_i is used instead of his/her local optimal power-of-two solution t_i, can be used as the compensation for buyer *i*. Mathematically,

$$comp_i(T_i) = c_i(T_i) - c_i(t_i). \tag{3.8}$$

This compensation can also be viewed as the minimum compensation for buyer *i* so that (s)he considers both policies (replenishes at T_i and t_i) as indifferent. Assuming that all the facilities are willing to disclose this piece of information honestly, the difference in total average cost of any two policies, Γ^1 and Γ^2, can be restated as follows.

$$\Delta\left(\Gamma^1,\Gamma^2\right)=c(\Gamma^1)-c(\Gamma^2)=\left(c_0(\Gamma^1)-c_0(\Gamma^2)\right)+\sum_{i=1}^{n}\left(comp(T_i^1)-comp(T_i^2)\right) \qquad (3.9)$$

.

Note that $c_0(\Gamma)$ is known to the vendor as it is regarded as his/her private information. With the additional piece of information on $comp_i(T_i)$ for all i, the vendor is in the position to calculate $\Delta\left(\Gamma^1,\Gamma^2\right)$ as in (3.9).

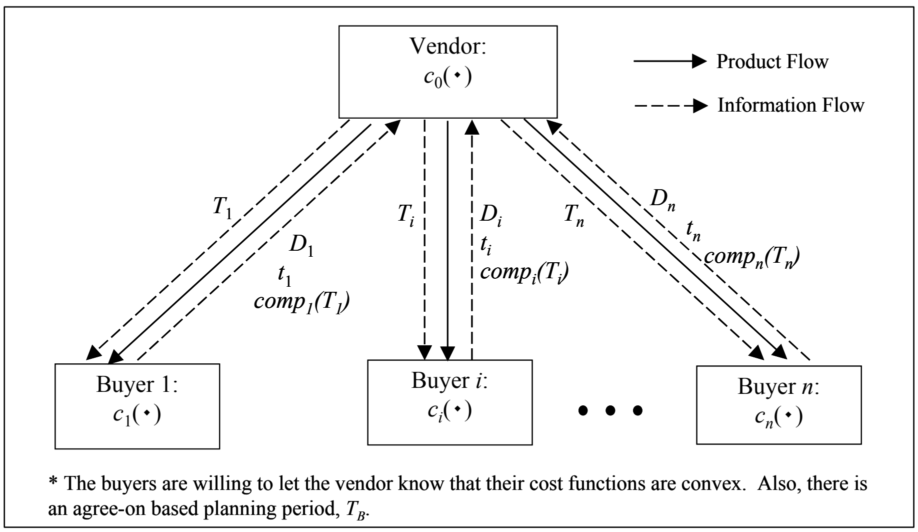


Fig. 3.5. Information and material flow of the system.

Next, CRPP-GI and ARPP-GI are adapted to implement the negotiation process with the additional piece of information on $comp_i(T_i)$. The information and material flow of the negotiation process is illustrated in Fig. 3.5. Specifically, buyer i determines his/her optimal replenishment policy, t_i, and submits a request of replenishing every t_i at rate of D_i to the vendor at the beginning of the negotiation. At each iteration, the vendor first determines T_R and $T_S=T_R\left(1-\sum_{i=1}^{n}D_i/P\right)$, and finds the optimal T_0 by successively increasing it. Then, (s)he proposes T_i to buyer i; and buyer i in turn calculates $comp_i\left(T_i\right)=c_i\left(T_i\right)-c_i\left(t_i\right)$ and passes $comp_i\left(T_i\right)$ to the vendor; and the negotiation process repeats.

3.4.2. CRPP with private information

To find CRPP with private information, we must be able to determine $\Delta\left(\Gamma^1,\Gamma^2\right)$ for any two policies, Γ^1 and Γ^2, if CRPP-GI is adopted. As discussed earlier, if buyer *i* provides the information on *comp*$_i$(T_i), then the vendor can evaluate $\Delta\left(\Gamma^1,\Gamma^2\right)$. Using the negotiation process mentioning in Section 3.4.1 where information is exchanged as shown in Fig. 3.5, CRPP-GI can be modified to accommodate this limited information environment. The procedure, termed CRPP-PI, is presented as follows.

CRPP-PI Heuristic

Step 0: Initialization

Buyer *i*, $\forall i$: Calculate t_i and submit it, as well as D_i, to the vendor.

Vendor: Set $\Gamma^{OPT} = \{T_S = \infty, T_i = \infty : i = 0,\cdots,n\}$, and $T_R = T_B$.

Step 1: Negotiation

Vendor: 1.1 Set $T_S = T_R\left(1 - \sum_{i=1}^{n} D_i / P\right)$.

Initialize $T_0 = T_R$, $\Gamma^0 = \{T_S, T_0, T_i = \infty : i = 1,\cdots,n\}$, $\Gamma(T_R) = \Gamma^0$, $c_0\left(\Gamma^0\right) = \infty$, $comp_i(\infty) = \infty$, $i = 1,\cdots,n$, and $k = 1$.

1.2 Propose $T_i = T_R$ to buyer *i*.

Buyer *i*, $\forall i$: Calculate $comp_i(T_i)$ as in (3.8) and submit it to the vendor.

Step 2: Find optimal T_0 given T_R (Vendor only)

2.1 Set $\Gamma^k = \{T_S.T_0, T_i = T_R : i = 1,\cdots,n\}$. Calculate $\Delta\left(\Gamma^k,\Gamma^{k-1}\right)$ as in (3.9).

2.2 If $\Delta\left(\Gamma^k,\Gamma^{k-1}\right) < 0$, the total average cost is decreasing. Set $T_0 = 2T_0$, $k = k+1$ and go to Step 2.1.

Otherwise, the total average cost ceases to decrease and optimal T_0 is found for current T_R. Sets $\Gamma(T_R) = \Gamma^{k-1}$ and go to Step 3.

Step 3: Updating (Vendor only)

3.1 Calculate $\Delta\left(\Gamma\left(T_R\right),\Gamma^{OPT}\right)$ as in (3.9).

3.2 If $\Delta\left(\Gamma\left(T_R\right),\Gamma^{OPT}\right) < 0$, update $\Gamma^{OPT} = \Gamma\left(T_R\right)$, set $T_R = 2T_R$ and continue to Step 3.3.

Otherwise, the total average cost ceases to decrease as T_R increases. The optimal policy is Γ^{OPT} and STOP.

3.3 If $T_R > T_{\max}$, the optimal policy is Γ^{OPT} and STOP.

Otherwise, continue the negotiation by going to Step 1.

Comparing CRPP-PI with CRPP-GI, one can observe that they are very similar except for how the vendor calculates $\Delta\left(\Gamma^1,\Gamma^2\right)$. In CRPP-PI, $\Delta\left(\Gamma^1,\Gamma^2\right)$ is calculated in terms of $comp_i\left(T_i\right)$ thus protecting information that is private to buyer *i*. So, if each buyer *i* discloses $comp_i\left(T_i\right)$ honestly, CRPP-PI will generate the same result as that of CRPP-GI. Nonetheless, CRPP-PI may be affected by the quality of the information exchanged between the vendor and the buyers.

3.4.3. ARPP with private information

This section discusses how the vendor determines T_i for each buyer *i* with private information during the negotiation process and presents ARPP-PI.

For given T_R, T_0, the following three cases are considered: $E(T_R) = \{i : t_i = T_R\}$, $L(T_R) = \{i : t_i < T_R\}$, and $G(T_R) = \{i : t_i > T_R\}$.

<u>Case 1</u>: $i \in E(T_R)$

In this case, buyer *i* is served according to his/her optimal power-of-two policy by replenishing at T_R.

Case 2: $i \in L(T_R)$

In this case, the replenishment policy for buyer i can be determined by KH. However, the implementation of KH requires the knowledge on $\Delta c_{Total,i}\left(T_i^1, T_i^2\right)$ for any policies, T_i^1 and T_i^2. Similar to (3.9), it is easy to verify that

$$\begin{aligned}\Delta c_{Total,i}\left(T_i^1, T_i^2\right) &= \Delta c_{0,i}\left(T_i^1, T_i^2\right) + \Delta c_i\left(T_i^1, T_i^2\right) \\ &= \Delta c_{0,i}\left(T_i^1, T_i^2\right) + \left(comp_i\left(T_i^1\right) - comp_i\left(T_i^2\right)\right) \qquad (3.10)\end{aligned}$$

Thus, the vendor is able to determine $\Delta c_{Total,i}\left(T_i^1, T_i^2\right)$ with the additional piece of information on $comp_i\left(T_i\right)$ instead of the private information $c_i(T_i)$. Recall that ARPP-GI starts with $T_R = T_B$ and successively increases T_R. Therefore, for given T_R, the vendor already knows $comp_i\left(T\right)$, where T is any power-of-two period such that $T_S < T < T_R$. As a result, the vendor can implement KH to determine T_i by means of (3.10) without modification.

Case 3: $i \in G(T_R)$

As discussed in Section 3.3.2, given T_R, the optimal replenishment policies for the buyers who do not belong to the set $G\left(T_0, T_R\right) = G\left(T_R\right) - \left\{i : T_i\left(T_0, T_R\right) < T_0\right\}$ are already specified. Therefore, only buyers who belong to $G\left(T_0, T_R\right)$ are required to participate in the negotiation process. Furthermore, the only piece of information required for finding the replenishment policy for buyer i in $G\left(T_0, T_R\right)$ given T_R and T_0 is $\Delta c_{Total,i}\left(T_i^1, T_i^2\right)$, which can be determined as in (3.10) with the knowledge of $comp_i\left(T_i\right)$. Therefore, the replenishment policy for buyer i can be found by the following procedure.

Replenishment Heuristic (RH-PI)

Step 1: Negotiation

Vendor: Set $T_i = T_0$ and propose to buyer i.

Buyer i, $\forall i$: Calculate $comp_i\left(T_i\right)$ as in (3.8) and submit it to the vendor.

Step 2: Vendor: Updating

2.1 Calculate $\Delta c_{Total,i}\left(T_i, T_i/2\right)$ as in (3.10).

2.2 If $\Delta c_{Total,i}\left(T_i, T_i/2\right) > 0$, policy with T_i incurs larger total average cost. Thus, set $T_i = T_i/2$.

Based on the discussion above, an interaction model, termed ARPP-PI, for finding ARPP for the SVMB system with private information is as follows.

ARPP-PI

Step 0: Initialization

Buyer *i*, $\forall i$: Calculate t_i and submit it, as well as D_i, to the vendor.

Vendor: Set $\Gamma^{OPT} = \{T_S = \infty, T_i = \infty : i = 0, \cdots, n\}$, and $T_R = T_B$.

Step 1: Negotiation

Vendor: Propose $T_i = T_R$ to buyer *i*.

Buyer *i*, $\forall i$: Calculate $comp_i\left(T_i\right)$ as in (3.8) and submit it to the vendor.

Step 2: Find optimal T_0 given T_R

Vendor: 2.0 Initialize sets $E(T_R)$, $L(T_R)$, and $G(T_R)$, and set $T_S = T_R\left(1 - \sum_{i=1}^{n} D_i/P\right)$ and $T_0 = T_R$.

2.1 $\forall i \in E(T_R)$, set $T_i = T_R$.

2.2 $\forall i \in L(T_R)$, set T_i by Procedure KH.

2.3 $\forall i \in G(T_R)$, set $T_i = T_R$, $\Gamma^0 = \{T_S, T_0, T_i : i = 1, \ldots, n\}$, $\Gamma\left(T_R\right) = \Gamma^0$, $c_0\left(\Gamma^0\right) = \infty$, $G\left(T_0/2, T_R\right) = G\left(T_R\right)$, and $k = 1$.

2.4 $\forall i \in G\left(T_0/2, T_R\right)$

2.4.1 Propose $T_i = T_0$.

Buyer $i, i \in G\left(T_0/2, T_R\right)$:

2.4.2 Calculate $comp_i(T_i)$ as in (3.8) and submit it to the vendor.

Vendor: 2.4.3 Set T_i by Procedure RH-PI.

2.4.4 $\forall i \in G(T_R) - G(T_0/2, T_R)$, set $T_i = T_i(T_0/2, T_R)$.

2.4.5 Update $G(T_0, T_R) = G(T_R) - \{i : T_i(T_0, T_R) < T_0\}$.

2.4.6 Update Γ^k and calculate $\Delta(\Gamma^k, \Gamma^{k-1})$ as in (3.9).

2.4.7 $\Delta(\Gamma^k, \Gamma^{k-1}) < 0$, the total average cost is decreasing. Set $T_0 = 2T_0$, $k = k+1$ and go to Step 2.4.

Otherwise, the total average cost ceases to decrease and optimal T_0 is found for current T_R. Set $\Gamma(T_R) = \Gamma^{k-1}$ and go to Step 3.

Step 3: Vendor: Updating

3.1 Calculate $\Delta(\Gamma(T_R), \Gamma^{OPT})$ as in (3.9).

3.2 If $\Delta(\Gamma(T_R), \Gamma^{OPT}) < 0$, update $\Gamma^{OPT} = \Gamma(T_R)$ and set $T_R = 2T_R$.

3.3 If $T_R > T_{\max}$, the optimal policy is Γ^{OPT} and STOP.

Otherwise, continue the negotiation by going to Step 1.

The implementation of ARPP-PI is basically the same as that of ARPP-GI. However, the calculation of $\Delta c_{Total,i}(T_i^1, T_i^2)$, which is required for implementing RH-PI to find the replenishment policy for buyer *i*, $i \in G(T_R)$, and the calculation of $\Delta(\Gamma^1, \Gamma^2)$ which is used in KH-PI and the negotiation and updating steps in ARPP-PI are different from those of ARPP-GI. When global information is available, $\Delta c_{Total,i}(T_i^1, T_i^2)$ and $\Delta(\Gamma^1, \Gamma^2)$ can be calculated directly by using the cost functions of the corresponding facilities. Under private information environment, $\Delta c_{Total,i}(T_i^1, T_i^2)$ and $\Delta(\Gamma^1, \Gamma^2)$ can only be calculated with the additional information, $comp_i(T_i)$, which is provided by buyer i to the vendor through negotiation. Thus, same as CRPP-PI, ARPP-PI may be affected by the quality of the information exchanged between the vendor and the buyers. If each buyer i discloses $comp_i(T_i)$ honestly, ARPP-PI will generate the same result as that of ARPP-GI.

3.5. Computational experiment

This section describes an experimental study on the performance of the heuristics proposed in this chapter. Since CRPP-PI and ARPP-PI work exactly the same as their counterparts under global information environment, it is sufficed to study CRPP-PI and ARPP-PI.

In this experiment, we study the performance of CRPP-PI and ARPP-PI against existing approach that utilizes global information. Specifically, the performance of the proposed heuristics is compared to that of the CRPP by Banerjee and Burton (1994). The performance improvement of ARPP-PI over CRPP-PI is also studied.

As Banerjee and Burton (1994) considered integer-ratio CRPP under global information environment, the performance of their policies must be at least as good as that of the power-of-two CRPP obtained by CRPP-PI. On the other hand, ARPP-PI provides a policy that must be at least as good as that of CRPP-PI. Therefore, ARPP-PI may outperform Banerjee and Burton's CRPP. However, the improvement of an ARPP over CRPP is influenced by two factors: (1) degree of heterogeneity of the buyers, which is defined by the spread of the buyers' optimal replenishment policies; and (2) vendor utilization, which is defined as $\sum_{i=1}^{n} D_i / P$.

First, if all the buyers have the same optimal replenishment policies, ARPP will reduce to power-of-two CRPP. Second, if $\sum_{i=1}^{n} D_i / P \leq 1/2$, the production must start at T_S , $T_R/2 \leq T_S < T_R$, because only power-of-two policies are considered; buyers in the set $L(T_R)$ are forced to replenish at T_R if $\sum_{i=1}^{n} D_i / P \leq 1/2$. As vendor utilization, $\sum_{i=1}^{n} D_i / P$, increases, more buyers in the set $L(T_R)$ can be replenished according to their optimal policies and the improvement of ARPP over CRPP will increase. Therefore, we use these two factors for the experiment.

In this experiment, a single-vendor twenty-buyer system is considered. For convenience, all the facilities in the system are assumed to adopt the classical EOQ assumptions, and T_B is assumed to be one day. The vendor's setup cost is generated randomly from a uniform distribution with minimum 50 and maximum 100, and his/her holding cost, h_0, is generated from a uniform distribution with minimum 1 and maximum 20. Demand rate of each buyer is generated independently from uniform distribution over [200,600]. To adapt the value-added concept as mentioned by Banerjee and Burton (1994), we assume $h_i > h_0, \forall i$. The holding cost of buyer *i*, h_i, $i = 1, \ldots, n$, is obtained from echelon holding cost (e_i), which is defined as $e_i = h_i - h_0$. Once h_0 and e_i is generated, h_i can be calculated. Echelon holding costs are generated randomly from a uniform distribution with minimum 1 and maximum 20.

Three different degrees of heterogeneity of the buyers, namely, low, medium, and high, are considered. To generate different degree of heterogeneity of the buyers, we generate the optimal time between orders, τ_i. Given D_i, h_i and τ_i, setup/ordering cost of buyer *i*, K_i, $i = 1, \ldots, n$, is computed based on EOQ model as follows (Salomon 1991):

$$K_i = 0.5 \times h_i \times D_i \times \tau_i^2 \tag{3.11}$$

From the vendor's point of view, the degree of heterogeneity is determined by the spread of power-of-two policies of the buyers. The reader can verify that the optimal policy of buyer *i* is rounded to its corresponding power-of-two policy as follows.

$$t_i = \begin{cases} 1 & \text{if } \tau_i \in [0,1.41] \\ 2 & \text{if } \tau_i \in [1.41,2.83] \\ 4 & \text{if } \tau_i \in [2.83,5.66] \\ 8 & \text{if } \tau_i \in [5.66,11.31] \\ 16 & \text{if } \tau_i \in [11.31,22.63] \\ 32 & \text{if } \tau_i \in [22.63,45.25] \\ 64 & \text{if } \tau_i \in [45.25,90.51] \end{cases} \tag{3.12}$$

Therefore, the goal of generating τ_i is to uniformly assign the buyers to each power-of-two period according to (3.12). For low-degree of heterogeneity, buyers' uncoordinated optimal power-of-two policies will be one of the first three power-of-two periods (1,2, and 4). Similarly, buyers' uncoordinated optimal power-of-two polices will be one of the first five (from 1 to 16) and one of the seven (from 1 to 64) power-of-two periods for medium- and high-degree of heterogeneity, respectively. The following procedure is used to generate τ_i for low-degree of heterogeneity buyer.

Step 1: Generate random number, R, from [0,3).

Step 2: $$\tau_\text{i} \text{ (in days)} = \begin{cases} \text{Uniform}(0.1,1.41) & \text{if } 0 \le R < 1 \\ \text{Uniform}(1.42,2.83) & \text{if } 1 \le \text{R} < 2 \\ \text{Uniform}(2.84,5.66) & \text{if } 2 \le R < 3 \end{cases} \tag{3.13}$$

where Unfiorm(a,b) denotes that a number is generated from a uniform distribution with minimum a and maximum b.

Similarly, τ_i for medium- and high-degree of heterogeneity buyers are generated using the same procedure with different number of power-of-two periods.

Also, three vendor utilizations are considered: 0.4, 0.6 and 0.8. Since we consider three degree of heterogeneity of buyers and three vendor utilizations, there are 9 different scenarios in total. For each scenario, 100 problems are generated using the stated parameters. These problems are solved using Banerjee and Burton's method (BB), CRPP-PI and ARPP-PI. The 0-1 Knapsack problem of KH is solved using the approximation algorithm (algorithm L) proposed by Sahni (1975). We define c_{BB}, c_{CRPP}

and c_{ARPP} as the total average of cost of the policy generated by BB, CRPP-PI and ARPP-PI, respectively.

Two performance measures are used for comparing our proposed heuristics against BB, namely, efficiency of CRPP-PI over BB which is defined as $Eff_{CRPP} = 100 \times (c_{BB} - c_{CRPP}) / c_{BB}$, and efficiency of ARPP-PI over BB, $Eff_{ARPP} = 100 \times (c_{BB} - c_{ARPP}) / c_{BB}$ Table 3.1 presents the average performance measures, $\overline{Eff}_{CRPP}$ and $\overline{Eff}_{ARPP}$.

Table 3.1
Average performance measure

		Degree of Heterogeneity					
		Low		Medium		High	
		$\overline{Eff}_{CRPP}$	$\overline{Eff}_{ARPP}$	$\overline{Eff}_{CRPP}$	$\overline{Eff}_{ARPP}$	$\overline{Eff}_{CRPP}$	$\overline{Eff}_{ARPP}$
Vendor Utilization	0.4	-1.9	0.8	-1.9	6.2	-1.7	11.5
	0.6	-2.1	3.15	-1.8	12.1	-1.6	20.8
	0.8	-1.6	5.9	-1.5	16.9	-1.4	27.1

From the experiment, it is observed that the $\overline{Eff}_{CRPP}$ is very stable at around -2%. In other words, CRPP-PI is not sensitive to both degree of heterogeneity and vendor utilization when comparing to BB.

It is interesting to notice that the average efficiencies of ARPP-PI of all the scenarios are positive, i.e., ARPP-PI on average outperforms BB. As the degree of heterogeneity of the buyers increases, the improvement of using ARPP over CRPP increases significantly. Even for the scenarios where vendor utilization is 0.4, the $\overline{Eff}_{ARPP}$ ranges from 0.8% to 11.5%. Recall that when $\sum_{i=1}^{n} D_i / P \le 0.5$, the benefit of using ARPP is solely due to the buyers in the set $G(T_R)$ as $T_R/2 < T_S < T_R$. In other words, RH-PI alone has significant impact on the total average cost if ARPP is used. For the scenarios where $\sum_{i=1}^{n} D_i / P = 0.6$, $\overline{Eff}_{ARPP}$ ranges from 3.15% to 20.8%. Finally, when $\sum_{i=1}^{n} D_i / P = 0.8$, ARPP-PI outperforms BB by 5.9% to 27.1%. Similarly, for a given degrees of heterogeneity of the buyers, as $\sum_{i=1}^{n} D_i / P$ increases, the performance of using of ARPP-PI instead of BB prevails with increasing margins.

Though the performance improvement of using ARPP-PI over CRPP-PI can be inferred from Table 3.1, we present the performance improvement in Table 3.2 to demonstrate the benefits of using ARPP-PI over CRPP-PI. The performance measure is efficiency of using ARPP-PI over CRPP-PI, which is defined as $Eff_{AC} = 100 \times (c_{CRPP} - c_{ARPP}) / c_{CRPP}$. The average performance measure, $\overline{Eff}_{AC}$, are presented in Table 3.2.

Table 3.2
Average efficiency of using ARPP-PI over CRPP-PI

		Degree of Heterogeneity		
		Low	Medium	High
Vendor Utilization	0.4	2.7	7.9	12.9
	0.6	5.2	13.6	22.0
	0.8	7.3	18.2	28.1

Similar to the results of $\overline{Eff}_{ARPP}$, $\overline{Eff}_{AC}$ increases significantly from 2.7% to 12.9% in the case of $\sum_{i=1}^{n} D_i / P = 0.4$, from 5.2% to 22.0% in the case of $\sum_{i=1}^{n} D_i / P = 0.6$, and from 7.3% to 28.1% in the case of $\sum_{i=1}^{n} D_i / P = 0.8$ as degree of heterogeneity increases from low to high. Also, for given degree of heterogeneity of buyers, $\overline{Eff}_{AC}$ increases as vendor utilization increases. In summary, $\overline{Eff}_{AC}$ is positively related to both the degree of heterogeneity of the buyers and vendor utilization.

To further investigate the performance of ARPP-PI, we examined the minimum Eff_{ARPP} and maximum Eff_{ARPP} as presented in Table 3.3. Though ARPP-PI generates only power-of-two policies while BB considers integer-ratio policies, it is interesting to observe that ARPP-PI not only outperforms BB on average, but on every single case for medium- and high-degree of heterogeneity when vendor utilizations are 0.6 and 0.8. Even for the scenarios of low-degree of heterogeneity, Eff_{ARPP} is -3% in the worst case among test problems. These results demonstrated the advantage of using ARPP over CRPP.

Table 3.3
Minimum and maximum efficiency of ARPP-PI

		Degree of Heterogeneity					
		Low		Medium		High	
		Min	Max	Min	Max	Min	Max
Vendor Utilization	0.4	-3.0	6.8	0.3	20.2	-1.0	29.4
	0.6	-1.2	6.9	4.2	21.9	7.5	40.6
	0.8	1.0	9.2	7.3	26.9	14.9	41.0

3.6. Conclusion

In this chapter, we consider the problem of coordinating the SVMB inventory system with private information where the objective is to find the replenishment policies for the buyers and the production schedule for the vendor such that the total average setup and inventory related cost of the system is minimized. This study is different from the previous studies in that we consider the SVMB system in which there is no single facility has complete information about the whole system and each facility has self decision-making authority. Specifically, the objective function and cost parameters of each facility are regarded as private information that no other facilities in the system have access to.

Two different types of nested and stationary policies, namely, CPRR and ARPP, are considered for coordinating the inventory system. We first develop two heuristics, CRPP-GI and ARPP-GI, for finding CRPP and ARPP to coordinate the SVMB inventory system assuming complete information is available. Then, these heuristics are modified for finding the CRPP and ARPP for the SVMB system with private information. Computational experiments are conducted to compare the performance of CRPP-PI and ARPP-PI where the control variables are degree of heterogeneity of the buyers and the total demand to production ratio. The results show that there is a positive relationship between the percentage of improvement of using ARPP instead of CRPP and the two control factors.

An important result of this study is that we demonstrate that knowledge of the system can be recovered through negotiation and partial information sharing and it is possible to develop approach for system with private information such that there is no performance loss when comparing to its counterpart with global information. We believe that it is promising to extend the framework of SVMB-PI to more complex supply chain networks.

CHAPTER IV
MULTI-ECHELON SERIAL AND ASSEMBLY SYSTEMS

4.1. Introduction

In this chapter, we consider the multi-echelon uncapacitated dynamic lot-sizing problem where the following information is private to each facility: (a) facility's objective function, (b) facility's setup/ordering cost, and (c) facility's inventory holding cost. Furthermore, each facility is responsible for specifying its own inventory policy. Also, it is assumed that facilities willingly collaborate with the others to achieve a minimal overall system cost. The objective of this chapter is to develop a scalable solution methodology to address the dynamic lot-sizing problem of serial and assembly supply chains with private information.

Motivated by the research results from the studies of coordination with private information problems (Barbarosoğlu and Özgür 1999, Kutanoglu and Wu 1999, Ertogral and Wu 2000, Jeong and Leon 2002, Jeong and Leon 2003, Jeong and Leon 2005) and the satisfactory results reported by using Lagrangian relaxation heuristics developed for coordinating supply chain inventory systems with global information (Afentakis and Gavish 1986, Afentakis, Gavish and Karmarkar 1984, Salomon 1991), we proposed a hierarchical Lagrangian-based decomposition methodology to coordinate supply chain inventory system with private information. A node-model that represents a facility in the supply chain is developed; and an interaction protocol for a supplier-buyer pair using the corresponding subproblems is proposed. By applying the interaction protocol of each supplier-buyer pair in the supply chain from downstream to upstream, a system-wide solution that is close to optimal is obtained. The remainder of the chapter is organized as follows. In Section 4.2, the mathematical model for serial/assembly inventory system is formulated. Interaction model for two-echelon serial inventory system is proposed in Section 4.3; followed by the development of interaction model for two-echelon assembly inventory system in Section 4.4. Then these models are extended for the multi-echelon serial/assembly inventory systems in Section 4.5. In Section 4.6, results on computational experiments are reported, followed by a conclusion in Section 4.7.

4.2. Problem description and model formulation

The supply chain inventory systems with private information under study are characterized by the following properties:

(a) Each facility in the system has self decision-making authority.

(b) No single facility has complete information about the whole system. Specifically, objective function and the corresponding parameters of each facility is private information.

(c) Partial information is shared among the facilities in order to achieve close-to-optimal solutions.

Two special supply chain inventory systems with private information are studied in this chapter, namely, a serial system, in which each facility has at most one predecessor and one successor; and an assembly system, in which each facility has multiple predecessors but at most one successor.

Each facility in the supply chain is represented by the model shown in Fig. 4.1, which is termed node-model in this chapter. The node-model consists of a production subsystem, a raw-material inventory subsystem, and a finished-goods inventory subsystem. A facility is responsible to specify three schedules: a delivery schedule, a production schedule, and order schedules. The delivery schedule is obtained by negotiating with the downstream facility (the customer), while the order schedules are found through negotiations with the upstream facilities (the suppliers). The production schedule takes into account the delivery schedule and order schedules such that the customer demands are met, and there is no shortage of raw materials. The work-in-process inventory subsystem is associated with the production subsystem.

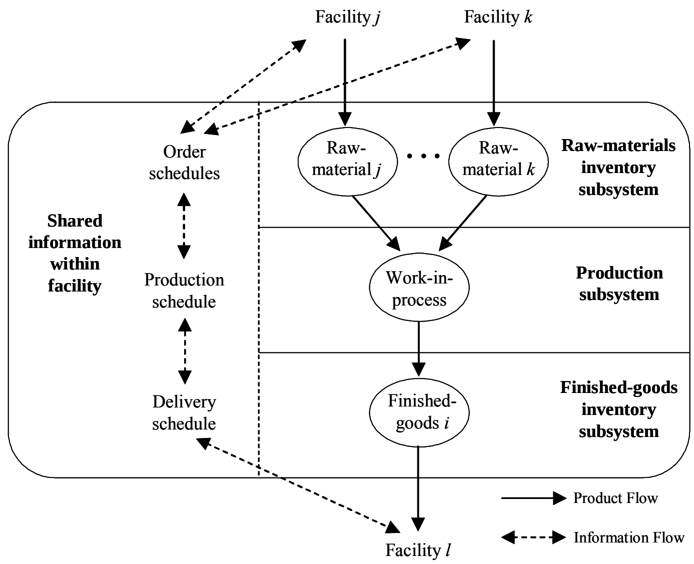


Fig. 4.1. Node-model of facility i.

The proposed node-model is the basis for the scalability of the coordination approach developed in this chapter. By linking the node-models according to the product and information flows associated with the supply chain under consideration, a supply chain inventory system can be represented as a directed graph, $G(N, A)$, where each node $i \in N$ is associated with a facility (node-model), and the arc $(i, j) \in A$ represents a flow of production/information from node (facility) i to j. Fig. 4.2 shows an example of assembly system depicted as a directed graph.

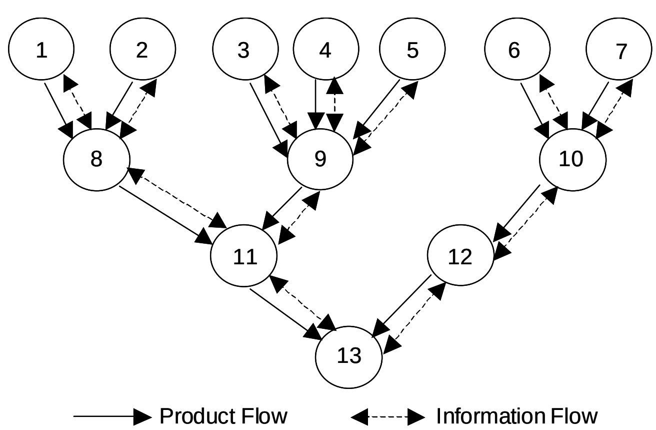


Fig. 4.2. An example of assembly system.

The following assumptions are made in this chapter: (i) each facility produces a single product; (ii) time-dependent demands of the end product are known over a finite horizon and must be met without backlogging; and (iii) products are produced in batches and production lead times are constant - thus, the total work-in-process for given demands over a finite horizon is constant and is disregarded in the model. Without loss of generality, production and delivery lead times are assumed to be zero.

The following notation is used in this chapter:

n: Total number of facilities in the system.

T: The number of planning periods.

$S^R_{i,j,t}(x)$: Ordering/setup cost of facility i for ordering x units of raw-material j from facility j at period t.

$S^F_{i,t}(y)$: Setup cost of facility i for producing y units of finished-goods i at period t.

$H^R_{i,j,t}(u)$: Holding cost of facility i for having u units of raw-material j ending inventory at period t.

$H^F_{i,t}(u)$: Holding cost of facility i for having u units of finished-goods i ending inventory at period t.

$I^R_{i,j,t}$: Ending inventory of raw-material j at facility i at period t.

$I^F_{i,t}$: Ending inventory of finished-goods at facility i at period t.

$m_{i,j}$: Number of raw-material j required to produce one unit of finished-goods i at facility i.

d_t: External demand of the end-product at period t.

$x_{i,j,t}$: Decision variable: Lot size of raw-material j transfers from facility j to facility i at period t.

$y_{i,t}$: Decision variable: Production quantities at facility i at period t.

$B(i)$: Set of indices of immediate predecessors of facility *i*.

$a(i)$: Index of the immediate successor of facility *i*.

By numbering the facilities from the higher-tier to lower-tier in the supply chain in which facility *i* is labeled with an integer $v(i)$ such that for arc (i, j), $v(i) < v(j)$ (Fig. 4.2), the problem of coordinating the multi-echelon supply chain inventory system can be modeled as the following mixed-integer program.

$$\text{(SC)} \quad \min \quad \sum_{i=1}^{n}\sum_{t=1}^{T}\left\{\sum_{i\in B(i)}\left(S_{i,j,t}^{R}\left(x_{i,j,t}\right)+H_{i,j,t}^{R}\left(I_{i,j,t}^{R}\right)\right)+\left(S_{i,t}^{F}\left(y_{i,t}\right)+H_{i,t}^{F}\left(I_{i,t}^{F}\right)\right)\right\} \tag{4.1}$$

s.t.

$$I_{i,j,t-1}^{R} + x_{i,j,t} - I_{i,j,t}^{R} = m_{i,j} y_{i,t} \qquad i = 1,\ldots,n;\ j \in B(i);\ t = 1,\ldots,T \tag{4.2}$$

$$I_{i,t-1}^{F} + y_{i,t} - I_{i,t}^{F} = x_{a(i),i,t} \qquad i = 1,\ldots,n;\ t = 1,\ldots,T \tag{4.3}$$

$$x_{n+1,n,t} = d_t \qquad t = 1,\ldots,T \tag{4.4}$$

$$I_{i,j,t}^{R} \geq 0 \qquad i = 1,\ldots,n;\ j \in B(i);\ t = 1,\ldots,T \tag{4.5}$$

$$I_{i,t}^{F} \geq 0 \qquad i = 1,\ldots,n;\ t = 1,\ldots,T \tag{4.6}$$

$$y_{i,t} \geq 0 \qquad i = 1,\ldots,n;\ t = 1,\ldots,T \tag{4.7}$$

$$x_{i,j,t} \geq 0 \qquad i = 1,\ldots,n;\ j \in B(i);\ t = 1,\ldots,T \tag{4.8}$$

Private information constraints:

$S_{i,j,t}^{R}(\cdot), S_{i,t}^{F}(\cdot), H_{i,j,t}^{R}(\cdot), H_{i,t}^{F}(\cdot)$, $m_{i,j}$ and $y_{i,t}$ are private to facility *i*. (4.9)

The objective function (4.1) states that the total ordering/setup and inventory-related cost is minimized. Constraint set (4.2) describes the mass-balance relation between ordering quantities, production, and end-of-period inventory of the raw material inventory subsystem; while constraint set (4.3) states the balance between production, delivery quantities, and end-of-period inventory of the finish-goods inventory subsystem. Constraint set (4.4) simply states the given external demand must be satisfied. Constraint sets (4.5) and (4.6) express that no backlogging is allowed. Constraint sets (4.7) and (4.8) are the non-negativity constraints. Constraint set (4.9) states that the setup/ordering cost functions, holding cost functions, raw-material requirements for producing finished goods, and the production schedule are private to the corresponding facility.

Because of the private information constraint set (4.9), most existing approaches for supply chain inventory coordination that assume complete information sharing and the existence of a centralized decision authority will not be applicable. In this case, to coordinate the supply chain inventory with private information, it is necessary to decompose (SC) at the facility level such that the private information is divided as required.

4.3. Two-echelon serial inventory system

Consider a two-echelon system with a single supplier and a single buyer. For convenience, the buyer is indexed as *b*, and the supplier is indexed as *s*. The general idea of the inventory coordination methodology is to first decompose (SC) into buyer and supplier subproblems in which the private information is separated according to the conditions in (4.9), and then use an interactive procedure where the buyer and the supplier negotiate on the best order/delivery policy. The methodologies developed in this chapter are aimed at finding globally optimal or close-to-optimal compromised order/delivery policies.

Note that (SC) cannot be decomposed into buyer and supplier subproblems because (4.2) and (4.3) are the coupling constraints. Specifically, the material transfers between facilities, $x_{i,j,t}$, make these subproblems interdependent. To decompose (SC), we create two auxiliary variables, termed negotiation variables:

$r_{b,s,t}$: Negotiation variable: Buyer's proposed order quantities of raw material for period *t*.

$p_{s,b,t}$: Negotiation variable: Supplier's proposed delivery quantities of raw material to the buyer for period *t*.

Substituting $x_{b,s,t}$ with $r_{b,s,t}$ in (4.2) for the buyer, and $x_{b,s,t}$ with $p_{s,b,t}$ in (4.3) for the supplier, (SC) can be separated at the facility level and resulted in the following non-coordinated subproblems of the buyer (NCB) and the supplier (NCS).

(NCB) min $\quad c_{NCB}=\sum_{t=1}^{T}\left\{\left(S_{b,s,t}^{R}\left(r_{b,s,t}\right)+H_{b,s,t}^{R}\left(I_{b,s,t}^{R}\right)\right)+\left(S_{b,t}^{F}\left(y_{b,t}\right)+H_{b,t}^{F}\left(I_{b,t}^{F}\right)\right)\right\}$ (4.10)

$$I_{b,s,t-1}^{R}+r_{b,s,t}-I_{b,s,t}^{R}=m_{b,s}y_{b,t} \qquad t=1,\dots,T \tag{4.11}$$

s.t. (4.3)-(4.7) with $i=b$.

(NCS) min $\quad c_{NCS}=\sum_{t=1}^{T}\left\{\left(S_{s,s-1,t}^{R}\left(x_{s,s-1,t}\right)+H_{s,s-1,t}^{R}\left(I_{s,s-1,t}^{R}\right)\right)+\left(S_{s,t}^{F}\left(y_{s,t}\right)+H_{s,t}^{F}\left(I_{s,t}^{F}\right)\right)\right\}$ (4.12)

s.t. $\quad I_{s,t-1}^{F}+y_{s,t}-I_{s,t}^{F}=p_{s,b,t} \qquad t=1,\dots,T$ (4.13)

(4.2), (4.5)-(4.7) with $i=s$.

Now that the subproblems can be solved separately, it is possible to devise an interactive procedure where the buyer and the supplier negotiate on the ordering and delivery quantities that will minimize (4.1). In the proposed procedure described later, in the k^{th} iteration, each facility sends the other the desired quantities (from a local perspective) and a compensation term which will entice the other one to accept the proposed quantities. This procedure is illustrated in Fig. 4.3.

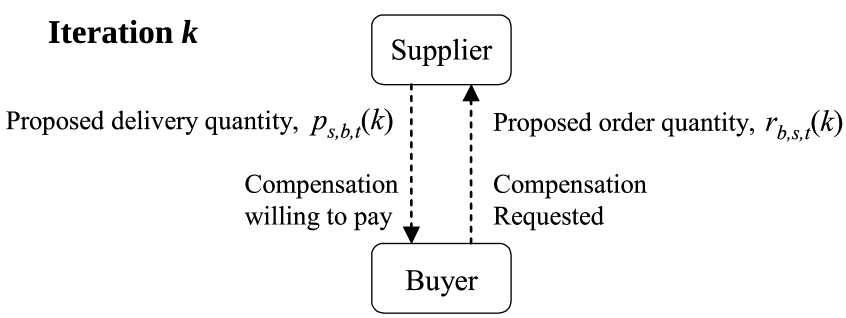


Fig. 4.3. Interaction procedure.

In general, $r_{b,s,t}$ and $p_{s,b,t}$ will be different during the negotiation, however, they must converge to the same value at the end of the procedure since they represent the same flow of material between the buyer and the supplier in period t; i.e. $r_{b,s,t}=p_{s,b,t}=x_{b,s,t}$. In order to force $r_{b,s,t}$ and $p_{s,b,t}$ to converge, convenient redundant constraints are introduced, and relaxed with Lagrangian multipliers into the objective function. These redundant constraints must capture the differences between $r_{b,s,t}$ and $p_{s,b,t}$. Specifically, for the buyer the constraint set

$$\left|\sum_{\tau=1}^{t} r_{b,s,\tau} - \sum_{\tau=1}^{t} p_{s,b,\tau}\right| = 0 \qquad t = 1,\ldots,T \tag{4.14}$$

states that the cumulative order quantities proposed by the buyer and the cumulative delivery quantities proposed by the supplier must be the same. Relaxing these constraints into the objective function with Lagrangian multipliers result in the penalty term

$$\mu_{b,s,t}^{R}(k)\left|\sum_{\tau=1}^{t} r_{b,s,\tau}(k) - \sum_{\tau=1}^{t} p_{s,b,\tau}(k)\right| \qquad t = 1,\ldots,T \tag{4.15}$$

This term serves as the buyer's penalty of deviation in the cumulative order and delivery quantities proposed at iteration *k* of the interactive procedure, while $\mu_{b,s,t}^{R}(k)$ denotes the buyer's unit cost of deviation in buyer's proposed cumulative order quantities and the supplier's proposed cumulative delivery quantities at iteration *k*. Also, $r_{b,s,t}(k)$, $p_{s,b,t}(k)$ denote $r_{b,s,t}$ and $p_{s,b,t}$ in iteration *k* of the interactive procedure. Since a positive penalty should incur when there is any deviation in the buyer's and supplier's proposal, (4.14) is presented in absolute value. One should note that $p_{s,b,t}(k)$ is known to the buyer at iteration *k* of the interaction process because the supplier proposed the delivery policy at each iteration first.

Similarly, for the supplier the redundant constraint set,

$$\left|\sum_{\tau=1}^{t} p_{s,b,\tau} - \sum_{\tau=1}^{t} r_{b,s,\tau}\right| = 0 \qquad t = 1,\ldots,T \tag{4.16}$$

is added to (SC), and

$$\mu_{s,b,t}^{F}(k)\left|\sum_{\tau=1}^{t} p_{s,b,\tau}(k) - \sum_{\tau=1}^{t} r_{b,s,\tau}(k-1)\right| \qquad t = 1,\ldots,T \tag{4.17}$$

can be viewed as the supplier's penalty function if a compromised order/delivery schedule is not found at iteration *k*, where $\mu_{s,b,t}^{F}(k)$ is the penalty cost for unit deviation of the buyer's proposed cumulative order and the supplier's proposed cumulative delivery quantities. In this case, $r_{b,s,t}(k-1)$ is the order quantities received from the buyer in the previous iteration.

The penalty function is enhanced by accounting for a special case in which (4.15) and (4.17) do not detect differences between $r_{b,s,t}$ and $p_{s,b,t}$. Consider a five-period problem where the buyer's proposed order policy is 5 units per period, and the supplier's proposed delivery policy is to deliver 15 units at period 1 and 10 units at period 4 at iteration *k*. In this example, (4.15) and (4.17) have positive values in periods 1, 2, and 4. Though the deviation in period delivery quantities and period order quantities of period 3 and period 5 are considered in the penalty functions of periods 1 and 2 and period 4, (4.15) and (4.17) do not provide penalty for periods 3 and 5 because there is no deviation in the cumulative delivery and order quantities proposed at periods 3 and 5. As a result, periods 3 and 5 are more attractive to invite deviation in proposed period delivery quantities and proposed period order quantities than they should be in the next iteration. To consider this special case, we add the redundant constraint set, $\left|r_{b,s,t} - p_{s,b,t}\right| = 0$, $t = 1,\ldots,T$, which states that the supplier's proposed delivery quantities coincide with the buyer's proposed order quantities in each period. This will result in the additional penalty term

$$U\left(\sum_{\tau=1}^{t} r_{b,s,\tau}(k) = \sum_{\tau=1}^{t} p_{s,b,\tau}(k)\right)\lambda_{b,s,t}^{R}\left|r_{b,s,t}(k) - p_{s,b,t}(k)\right| \tag{4.18}$$

where, $\lambda_{b,s,t}^{R}(k)$ can be viewed as the penalty for unit deviation of the buyer's proposed order quantities and supplier's proposed delivery quantities for period *t*; $U(\cdot) = 1$ if condition in the parenthesis is true, and $U(\cdot) = 0$ otherwise. $U(\cdot)$ is needed to avoid double counting the penalties in time periods where (4.15) and (4.17) are already active.

Similarly for the supplier, the additional penalty function is

$$U\left(\sum_{\tau=1}^{t} p_{s,b,\tau}(k) = \sum_{\tau=1}^{t} r_{b,s,\tau}(k-1)\right)\lambda_{s,b,t}^{F}\left|p_{s,b,t}(k) - r_{b,s,t}(k-1)\right| \tag{4.19}$$

where, $\lambda_{s,b,t}^{F}(k)$ is viewed as the penalty for unit deviation of the proposed order and delivery quantities for period *t*.

Finally, to guarantee that the delivery policy proposed by the supplier is feasible to the buyer during the negotiation procedure, the following constraints are added to (SC).

$$\sum_{\tau=1}^{t} p_{s,b,\tau} \geq \sum_{\tau=1}^{t} r_{b,s,\tau} \qquad t = 1,\ldots,T \tag{4.20}$$

$$r_{b,s,t} \geq 0 \qquad t = 1,\ldots,T \tag{4.21}$$

$$p_{s,b,t} \geq 0 \qquad t = 1,\ldots,T \tag{4.22}$$

Based on the above discussion, (SC) is decomposed into the following buyer node-model (BN) and supplier node-model (SN).

$$\text{(BN)}\quad \min \quad g_{BN} = \sum_{t=1}^{T} \left\{ \begin{array}{l} \left(S_{b,s,t}^{R}\left(r_{b,s,t}\right) + H_{b,s,t}^{R}\left(I_{b,s,t}^{R}\right) + \mu_{b,s,t}^{R} \left| \sum_{\tau=1}^{t} r_{b,s,\tau} - \sum_{\tau=1}^{t} p_{s,b,\tau} \right| \right. \\ \left. \quad + U\left(\sum_{\tau=1}^{t} r_{b,s,\tau} = \sum_{\tau=1}^{t} p_{s,b,\tau} \right) \lambda_{b,s,t}^{R} \left| r_{b,s,t} - p_{s,b,t} \right| \right) \\ \quad + \left(S_{b,t}^{F}\left(y_{b,t}\right) + H_{b,t}^{F}\left(I_{b,t}^{F}\right) \right) \end{array} \right\} \tag{4.23}$$

s.t. (4.3)-(4.7) with $i = b$, (4.11) and (4.21).

$$\text{(SN)}\quad \min \quad g_{SN} = \sum_{t=1}^{T} \left\{ \begin{array}{l} \left(S_{s,s-1,t}^{R}\left(x_{s,s-1,t}\right) + H_{s,s-1,t}^{R}\left(I_{s,s-1,t}^{R}\right) \right) \\ \quad + \left(S_{s,t}^{F}\left(y_{s,t}\right) + H_{s,t}^{F}\left(I_{s,t}^{F}\right) + \mu_{s,b,t}^{F} \left| \sum_{\tau=1}^{t} p_{s,b,\tau} - \sum_{\tau=1}^{t} r_{b,s,\tau} \right| \right. \\ \left. \quad + U\left(\sum_{\tau=1}^{t} p_{s,b,\tau} = \sum_{\tau=1}^{t} r_{b,s,\tau} \right) \lambda_{s,b,t}^{F} \left| p_{s,b,t} - r_{b,s,t} \right| \right) \end{array} \right\} \tag{4.24}$$

s.t. (4.2), (4.5)-(4.7) with $i = s$, (4.13), (4.20) and (4.22).

Since $r_{b,s,t}$ and $p_{s,b,t}$ are shared information between the two facilities and other parameters of (BN) and (SN) are private to the buyer and the supplier, respectively, thus, the buyer and the supplier have all the information to solve their corresponding (BN) and (SN). Also, any solution to (BN) and (SN) with $r_{b,s,t} = p_{s,b,t}$, $t = 1,\ldots,T$ is feasible to the original problem (SC).

Remark. When developing the penalty functions for guiding the buyer and the supplier to reach a feasible solution, an obvious choice is to add and dualize the auxiliary period-by-period deviation constraint $\left|r_{b,s,t} - p_{s,b,t}\right| = 0, t = 1,\ldots,n$ (period-by-period penalty). Through experiments, we found that the period-by-period penalty approach, while providing comparable performance as the proposed cumulative penalty approach for the two-echelon cases, its performance degraded quicker than our approach as the number of echelon increases. Our conjecture is that the cumulative deviation $\left|\sum_{\tau=1}^{t} r_{b,s,\tau} - \sum_{\tau=1}^{t} p_{b,s,\tau}\right|,\ t = 1,\ldots,n$ utilizes information from period 1 to period *t*, while the period-by-period deviation only employs information of the current period. Thus, the proposed cumulative penalty approach has more look-ahead capability than the period-by-period approach. Since the solution domains of the facilities at the higher-tiers are affected by the facility at the lower-tier, the effect of the solution of last echelon buyer ripples up through the supply chain. Because of the proposed cumulative penalty approach provides more stable performance than the period-by-period penalty approach, we use the proposed cumulative penalty functions in this study.

4.3.1. Compensation determination

To find a compromised solution, we assume that the buyer will request for a compensation if an order policy other than the desired order policy is used, and the supplier will propose the compensation (s)he is willing to pay if a delivery policy other than the one according to the buyer's order policy during the interactive procedure. When studying two-echelon inventory systems in Chapters II and III, we suggest that the compensation provided by the supplier should be large enough for the buyer to consider his/her reference policy and the supplier's proposed policy as indifferent. In this section, we extend this idea to determine the compensation requested by the buyer *b*, $comp_b(k)$, and the compensation proposed by the supplier *s*, $comp_s(k)$, at iteration *k* during the interaction.

To determine $comp_b(k)$ and $comp_s(k)$, reference policies of the buyer and the supplier are needed. The optimal order quantities, $r_{b,s,t}^{*}$, $t = 1,\ldots,T$, obtained by solving the non-coordinated problem (NCB) can be served as the reference policy for the buyer.

Upon receiving $r^*_{b,s,t}$, $t=1,\ldots,T$, from the buyer, the supplier determines his/her optimal order policy by solving the non-coordinated problem (NCS) with $p_{s,b,t} = r^*_{b,s,t}$, $t=1,\ldots,T$. The resulting order and delivery policies can be served as the reference policies for the supplier.

Let c^*_{NCB} be the objective value of (NCB) with respect to the buyer's reference policy and c^*_{NCS} be the objective value of (NCS) with respect to the supplier's reference policy. Also define $g_{BN}(k)$ and $g_{SN}(k)$ as the objective values of (BN) and (SN) at iteration *k*, respectively; the compensation requested by the buyer, $comp_b(k)$ is

$$comp_b(k) = g_{BN}(k) - c^*_{NCB}. \tag{4.25}$$

And the compensation the supplier is willing to pay, $comp_s(k)$ is

$$comp_s(k) = c^*_{NCS} - g_{SN}(k). \tag{4.26}$$

It is clear that if the buyer's proposed order policy coincides with the supplier's proposed delivery policy, $comp_b(k)$ is simply the difference in total cost of the buyer if the proposed order policy is used instead of buyer's noncoordinated optimal order policy. In other words, it is the minimum compensation for buyer *i* so that (s)he considers both policies as indifferent. Similarly, $comp_s(k)$ is the difference in total cost of the supplier if the proposed delivery policy is used instead of delivering according to the buyer's non-coordinated optimal order policy. Thus, it is the maximum compensation (s)he is willing to pay to consider both policies indifference.

4.3.2. Lagrangian multipliers updates

Restating the above interaction procedure in the Lagrangian-based heuristic context, the interactive procedure starts with the initialization process that finds the reference solutions for the buyer and the supplier by solving (NCB) and (NCS) with $p_{s,b,t} = r^*_{b,s,t}, t=1,\ldots,T$, respectively. After the initialization, in each iteration *k*, the supplier solves (SN) with respect to the buyer's proposed order policy. (S)he then updates the Lagrangian multipliers and proposes a delivery policy and $comp_s(k)$ to the buyer. In turn, the buyer solves (BN), updates the corresponding Lagrangian multipliers, and proposes an order policy and $comp_b(k)$ to the supplier, and the interactive procedure repeats until a compromised order policy is obtained.

In each iteration of the above interactive procedure, the Lagrangian multipliers are updated using subgradient optimization. Let *w*(*k*) be a positive scale step size used in iteration *k*, θ be a user-defined scalar, *UB* and *LB* be the upper- and lower-bound solutions of the system. Then,

$$\mu^R_{b,s,t}(k+1) = \mu^R_{b,s,t}(k) + w\left|\sum_{\tau=1}^{t} r_{b,s,\tau}(k) - \sum_{\tau=1}^{t} p_{s,b,\tau}(k)\right| \tag{4.27}$$

$$\lambda^R_{b,s,t}(k+1) = \lambda^R_{b,s,t}(k) + w\left\{U\left(\sum_{\tau=1}^{t} r_{b,s,\tau}(k) = \sum_{\tau=1}^{t} p_{s,b,\tau}(k)\right)\left|r_{b,s,t}(k) - p_{s,b,t}(k)\right|\right\} \tag{4.28}$$

$$\mu^F_{s,b,t}(k+1) = \mu^F_{s,b,t}(k) + w\left|\sum_{\tau=1}^{t} p_{s,b,\tau}(k) - \sum_{\tau=1}^{t} r_{b,s,\tau}(k-1)\right| \tag{4.29}$$

$$\begin{aligned}\lambda^F_{s,b,t}(k+1) = {} & \lambda^F_{s,b,t}(k) + \\ & w\left\{U\left(\sum_{\tau=1}^{t} p_{s,b,\tau}(k) = \sum_{\tau=1}^{t} r_{b,s,\tau}(k-1)\right)\left|p_{s,b,t}(k) - r_{b,s,t}(k-1)\right|\right\}\end{aligned} \tag{4.30}$$

$$w(k) = \frac{\theta(UB - LB)}{\sum_{t=1}^{T} G_t} \tag{4.31}$$

$$\begin{aligned}G_t = {} & \left\{\sum_{\tau=1}^{t} p_{s,b,\tau}(k) - \sum_{\tau=1}^{t} r_{b,s,\tau}(k-1)\right\}^2 + \left\{\sum_{\tau=1}^{t} r_{b,s,\tau}(k) - \sum_{\tau=1}^{t} p_{s,b,\tau}(k)\right\}^2 \\ & + \left\{U\left(\sum_{\tau=1}^{t} p_{s,b,\tau}(k) = \sum_{\tau=1}^{t} r_{b,s,\tau}(k-1)\right)\left(p_{s,b,t}(k) - r_{b,s,t}(k-1)\right)\right\}^2 \\ & + \left\{U\left(\sum_{\tau=1}^{t} r_{b,s,\tau}(k) = \sum_{\tau=1}^{t} p_{s,b,\tau}(k)\right)\left(r_{b,s,t}(k) - p_{s,b,t}(k)\right)\right\}^2\end{aligned} \tag{4.32}$$

To determine *w*(*k*) in (4.31), the upper- and lower-bound solutions of the system are required. Note that in the private information environment, the supplier and the buyer only know their corresponding upper- and lower-bound solutions, and not the overall system upper- and lower-bound solutions. Therefore, an alternate method to calculate *w*(*k*) is derived here.

Since the solution of (NCB) and that of (NCS) by setting $p_{s,b,t} = r^*_{b,s,t}, t = 1,\ldots,T$ is also feasible to (SC), thus $UB = c^*_{NCB} + c^*_{NCS}$ is an upper bound. Also, solving (BN) and (SN) in iteration *k* provides the lower bound for the problem; i.e. $LB = g_{NB}(k) + g_{NS}(k)$. It follows that

$$\begin{aligned} UB - LB &= \left(c^*_{NCB} + c^*_{NCS}\right) - \left(g_{NB}(k) + g_{NS}(k)\right) \\ &= \left(c^*_{NCS} - g_{SN}(k)\right) - \left(g_{BN}(k) - c^*_{NCB}\right) \\ &= comp_s(k) - comp_b(k) \end{aligned} \tag{4.33}$$

Thus, (4.31) can be restated as

$$w(k) = \frac{\theta(UB - LB)}{\sum_{t=1}^{T} G_t} = \frac{\theta(comp_s(k) - comp_b(k))}{\sum_{t=1}^{T} G_t} \tag{4.34}$$

Since $comp_b(k)$ and $comp_s(k)$ are known to both the buyer and the supplier, $w(k)$ can now be calculated without private information using (4.34).

4.3.3. Solution procedure

In the k^{th} iteration, let $R_{b,s}(k) = \{r_{b,s,t}(k), t = 1,\ldots,T\}$ be the order policy proposed by buyer *b* to supplier *s* and $P_{s,b}(k) = \{p_{s,b,t}(k), t = 1,\ldots,T\}$ be the delivery policy proposed by supplier *s* to buyer *b*. Assuming that all the facilities exchange information honestly, the interactive model for two-echelon serial system for buyer *b* and supplier *s*, SIM(*b*,*s*), is stated as follows.

SIM(*b*,*s*)

Step 0: Initialization

Buyer:

(a) Initialize $\lambda^R_{b,s,t}(0) = 0$, $\mu^R_{b,s,t}(0) = 0$, $t = 1,\ldots,T$.

(b) Determine the proposed order policy, $R_{b,s}(0)$, and c^*_{NCB} by solving (NCB), and submit $R_{b,s}(0)$ to the supplier. Set $k_b = 1$.

Supplier:

(c) Initialize $\lambda^F_{s,b,t}(0)=0$, $\mu^F_{s,b,t}(0)=0$, $t=1,\ldots,T$.

(d) Solve (NCS) with respect to $R_{b,s}(0)$ to obtain c^*_{NCS}. Set $k_s=1$.

Step 1: Negotiation

Supplier:

(e) Given $R_{b,s}(k_s-1)$, solve (SN) to obtain $P_{s,b}(k_s)$.

(f) Determine $comp_s(k_s)$ as in (4.26).

(g) If $P_{s,b}(k_s)=R_{b,s}(k_s-1)$ a compromised solution is found. Inform the buyer and stop.

(h) Otherwise, propose $P_{s,b}(k_s)$ and $comp_s(k_s)$ to the buyer. Set $k_s=k_s+1$.

Buyer:

(i) Given $P_{s,b}(k_b)$ and $comp_s(k_b)$, solve (BN) to obtain $R_{b,s}(k_b)$.

(j) Determine $comp_b(k_b)$ as in (4.25).

(k) If $R_{b,s}(k_b)=P_{s,b}(k_b)$, a compromised solution is found. Inform the supplier and stop.

(l) Otherwise, propose $R_{b,s}(k_b)$ and $comp_b(k_b)$ to the supplier. Set $k_b=k_b+1$.

(m) If iteration limit is exceeded, go to Step 3. Otherwise, go to Step 2.

Step 2: Update Lagrangian multipliers

Supplier:

(n) Update $\mu^F_{s,b,t}(k_s)$ and $\lambda^F_{s,b,t}(k_s)$ as in (4.29) and (4.30).

Buyer:

(o) Update $\mu^R_{b,s,t}(k_b)$ and $\lambda^R_{b,s,t}(k_b)$ as in (4.27) and (4.28).

(p) Go to Step 1.

Step 3: Restore feasibility

Buyer:

(q) Use $R_{b,s}(k_b - 1)$ as the lot sizing policy and propose to the supplier.

Supplier:

(r) Set $p_{b,s,t} = r_{b,s,t}(k_s - 1), t = 1, \ldots, T$. Solve (NCS) and stop.

Since orders are originated from the buyer, if a compromised policy is not found after reaching the pre-specified iteration limit, preference is given to the buyer and the buyer's last proposed order policy is used as the order/delivery policy as in Step 3.

In order to provide more specifics about how to solve the subproblems (SN), (BN), (NCB) and (NCS) in the procedure SIM(*b*,*s*), it is necessary to further specify the components of the objective function; i.e., $S^R_{i,j,t}(x)$, $S^F_{i,t}(y)$, $H^R_{i,j,t}(u)$, and $H^F_{i,t}(u)$. Solution procedures for common forms of these components are described in the following sections.

4.3.4. Procedure for cases with concave cost functions

In this section, we consider the special case in which the ordering/setup cost functions are concave on $[0,\infty)$ for all *i* and *t*; and the inventory holding cost functions are linear on $[0,\infty)$ and are nondecreasing from upstream to downstream facilities. With these concavity assumptions, Veinott (1969) showed that there is at least one optimal solution for the multi-echelon assembly system that must satisfy the zero inventory property. Love (1972) proved the nested property of an optimal policy for serial system. The nested property states that if facility *i* orders/produces in period *t*, all facilities downstream of *i* must order/produce. The nested property for assembly system is later proved by Crowston and Wagner (1973). This study considers nested policies that satisfy the zero inventory property for solving (BN) and (SN). Next, dynamic programming heuristics and fast Silver-Meal-based heuristics are proposed for solving the buyer subproblem (BN) and the supplier subproblem (SN).

4.3.4.1. Dynamic programming heuristics

Let's start by defining the necessary additional notation:.

$F(t)$: Minimum total cost from period t to T.

$C^R(t,u)$: Total cost of the raw-material inventory subsystem of ordering at period t to meet requirements through period u-1.

$F_{t,u}^F(k)$: Minimum total cost of the finished-goods inventory subsystem from period k to u-1 if the facility orders raw material at period t and produces at period k to meet requirements through period u-1.

$R(t,u)$: Buyer's total proposed order quantities from period t to u-1, $R(t,u) = \sum_{m=t}^{u-1} r_{b,s,m}$.

$P(t,u)$: Supplier's total proposed delivery quantities from period t to u-1, $P(t,u) = \sum_{m=t}^{u-1} p_{s,b,m}$.

Buyer Subproblem (BN)

Given that only nested policies that satisfy zero inventory property are considered, (BN) can be solved by the following recursive equations.

$$F(T+1) = 0$$

$$F(t) = \min\left\{F(u) + C^R(t,u) + F_{t,u}^F(t) : t < u\right\} \tag{4.35}$$

When ordering raw material at period t to meet requirements through period u-1, the total quantities of the raw material ordered is $m_{b,s} \sum_{v=t}^{u-1} y_{b,v}$ and the total inventory holding cost is $\sum_{v=t}^{u-2} H_{b,s,v}^R \left(m_{b,s} \sum_{k=v}^{u-2} y_{b,k+1}\right)$. Also, the cumulative proposed order quantities from period 1 to period t-1 is $m_{b,s} \sum_{k=1}^{t-1} y_{b,k}$ Thus,

$$C^R(t,u) = S^R_{b,s,t}\left(m_{b,s}\sum_{v=t}^{u-1} y_{b,v}\right) + \sum_{v=t}^{u-2} H^R_{b,s,v}\left(m_{b,s}\sum_{k=v}^{u-2} y_{b,k+1}\right)$$
$$+ \sum_{v=t}^{u-2} \mu_{b,s,v}\left(\left|m_{b,s}\sum_{k=1}^{t-1} y_{b,k} + R(t,v) - P(1,v)\right|\right) \qquad (4.36)$$
$$+ U\left(m_{b,s}\sum_{k=1}^{t-1} y_{b,k} + R(t,u-1) = P(1,u-1)\right)\lambda_{b,s,u-1}\left(\left|r_{b,s,u-1} - p_{s,b,u-1}\right|\right)$$

Also, total production from period v to u-1 is $\sum_{k=v}^{u-1} d_k$ and total holding cost for the finished goods from period t to period u-1 is $\sum_{v=t}^{u-2} H^F_{b,v}\left(\sum_{k=v}^{u-2} d_{k+1}\right)$. Therefore, $F^F_{t,u}(t)$ can be found by the Wagner-Whitin algorithm (Wagner and Whitin 1958) with the following recursive equations.

$$F^F_{t,u}(u) = 0$$

$$F^F_{t,u}(v) = \min\left\{F^F_{t,u}(j) + S^F_{b,v}\left(\sum_{k=v}^{u-1} d_k\right) + \sum_{v=t}^{u-2} H^F_{b,v}\left(\sum_{k=v}^{u-2} d_{k+1}\right) : v < j\right\} \qquad (4.37)$$

For given t and u, it takes $O\left((u-t)^2\right)$ time to compute $F^F_{t,u}(t)$. Since there are $O\left(T^2\right)$ operations for the recursive equation (4.35), the computational complexity is $O\left(T^4\right)$.

Numerical Example

To illustrate the dynamic programming heuristic for solving the buyer subproblem, we consider a simple three-period example with the parameters shown in Table 4.1.

Table 4.1
Parameters for the buyer subproblem of two-echelon serial system

	Raw-material inventory subsystem	Finished-goods inventory subsystem
Setup cost/order	\$400 (K^R)	\$200 (K^F)
Unit holding cost	\$2 (h^R)	\$5 (h^F)

$$D = \{d_1, d_2, d_3\} = \{200, 460, 350\}$$

$$\mathrm{P} = \{p_{s,b,1}, p_{s,b,2}, p_{s,b,2}\} = \{1010, 0, 0\}$$

$$\mathrm{M} = \{\mu_{b,s,1}, \mu_{b,s,2}, \mu_{b,s,3}\} = \{0.55, 0.22, 0\}$$

$$\Lambda = \{\lambda_{b,s,1}, \lambda_{b,s,2}, \lambda_{b,s,3}\} = \{0, 0, 0.22\}$$

Then,

(a) $F(4) = 0$

(b) $F(3) = F(4) + C^R(3,4) + F^F_{3,4}(3)$

(i) $C^R(3,4) = K^R + \lambda_{b,s,3}(350) = 477$

(ii) $F^F_{3,4}(4) = 0$

(iii) $F^F_{3,4}(3) = \min\{F^F_{3,4}(4) + K^F\} = 200$

$F(3) = 0 + 477 + 200 = 677$

(c) $F(2) = \min\begin{Bmatrix} F(4) + C^R(2,4) + F^F_{2,4}(2) \\ F(3) + C^R(2,3) + F^F_{2,3}(2) \end{Bmatrix}$

(i) $C^R(2,4) = K^R + h^R(350) + \lambda_{b,s,2}(810) = 1100$

(ii) $F^F_{2,4}(4) = 0$

(iii) $F^F_{2,4}(3) = 0 + 200 = 200$

(iv) $F^F_{2,4}(2) = \min\begin{Bmatrix} F^F_{2,4}(4) + 200 + 350(5) \\ F^F_{2,4}(3) + 200 \end{Bmatrix} = \min\begin{Bmatrix} 1950 \\ 490 \end{Bmatrix} = 490$

(v) $C^R(2,3) = K^R + \mu_{b,s,2}(350) = 477$

(vi) $F^F_{2,3}(3) = 0$

(vii) $F^F_{2,3}(2) = 0 + 200 = 200$

$F(2) = \min\begin{Bmatrix} 0 + 1100 + 490 \\ 677 + 477 + 200 \end{Bmatrix} = \min\begin{Bmatrix} 1590 \\ 1354 \end{Bmatrix} = 1354$

(d) $$F(1)=\min\begin{Bmatrix} F(4)+C^{R}(1,4)+F_{1,4}^{F}(1) \\ F(3)+C^{R}(1,3)+F_{1,3}^{F}(1) \\ F(2)+C^{R}(1,2)+F_{1,2}^{F}(1) \end{Bmatrix}$$

(i) $C^{R}(1,4)=K^{R}+h^{R}(460+350)+h^{R}(350)=2720$

(ii) $F_{1,4}^{F}(4)=0$

(iii) $F_{1,4}^{F}(3)=0+200=200$

(iv) $$F_{1,4}^{F}(2)=\min\begin{Bmatrix} F_{1,4}^{F}(4)+200+350(5) \\ F_{1,4}^{F}(3)+200 \end{Bmatrix}=\min\begin{Bmatrix} 1950 \\ 400 \end{Bmatrix}=400$$

(v) $$F_{1,4}^{F}(1)=\min\begin{Bmatrix} F_{1,4}^{F}(4)+200+(460+350)(5)+350(5) \\ F_{1,4}^{F}(3)+200+460(5) \\ F_{1,4}^{F}(2)+200 \end{Bmatrix}$$

$$=\min\begin{Bmatrix} 6000 \\ 2700 \\ 400 \end{Bmatrix}=400$$

(vi) $C^{R}(1,3)=K^{R}+h^{R}(350)+\mu_{b,s,1}(350)+\mu_{b,s,2}(350)=1370$

(vii) $F_{1,3}^{F}(3)=0$

(viii) $F_{1,3}^{F}(2)=0+200=200$

(ix) $$F_{1,3}^{F}(1)=\min\begin{Bmatrix} F_{1,3}^{F}(3)+200+460(5) \\ F_{1,3}^{F}(2)+200 \end{Bmatrix}=\min\begin{Bmatrix} 2500 \\ 400 \end{Bmatrix}=400$$

(x) $C^{R}(1,2)=K^{R}+\mu_{b,s,1}(460+350)=846$

(xi) $F_{1,2}^{F}(2)=0$

(xii) $F_{1,2}^{F}(1)=200$

$$F(1)=\min\begin{Bmatrix} F(4)+C^{R}(1,4)+F_{1,4}^{F}(1) \\ F(3)+C^{R}(1,3)+F_{1,3}^{F}(1) \\ F(2)+C^{R}(1,2)+F_{1,2}^{F}(1) \end{Bmatrix}=\min\begin{Bmatrix} 0+2720+400 \\ 677+1370+400 \\ 1354+846+200 \end{Bmatrix}=\min\begin{Bmatrix} 3120 \\ 2447 \\ 2400^{*} \end{Bmatrix}$$

Thus, the production schedule is $\{200, 460, 350\}$ and the proposed order policy is also $D = \{d_1, d_2, d_3\} = \{200, 460, 350\}$.

Supplier Subproblem (SN)

Similar to (BN), (SN) can be solved by the following recursive equations.

$$F(T+1) = 0$$

$$F(t) = \min\{F(u) + C^R(t,u) + F^R_{t,u}(t) : t < u\} \tag{4.38}$$

In (4.38), $C^R(t,u)$ is simply the sum of ordering cost at period t and the holding cost from period t to u-1, $C^R(t,u) = S^R_{s,s-1,t}\left(m_{s,s-1}\sum_{v=t}^{u-1} y_{s,v}\right) + \sum_{v=t}^{u-2} H^R_{s,s-1,v}\left(m_{s,s-1}\sum_{k=v}^{u-2} y_{s,k+1}\right)$.

To find $F^P_{t,u}(t)$, we need to determine both the production and delivery policies. We define the following additional notations for developing the recursive equations for obtaining $F^P_{t,u}(t)$.

$G_{v,j}(k)$: Minimum total inventory holding and penalty cost of the finished-goods inventory subsystem from period k to j-1 if the supplier produces at period v to meet buyer's proposed requirements through period j-1.

$g_{v,j}(k,l)$: Total inventory holding and penalty cost of the finished-goods inventory subsystem from period k to l-1 if the supplier produces at period v to meet the buyer's proposed requirements through period j-1 and delivers at period k to meet requirements from period k through l-1.

Then, $F^F_{t,u}(t)$ can be found by the following recursive equations.

$$F^F_{t,u}(u) = 0$$

$$F^F_{t,u}(v) = \min\left\{F^F_{t,u}(j) + S^F_{s,v}\left(\sum_{k=v}^{u-1} r_{b,s,k}\right) + G_{v,j}(v) : v < j\right\} \tag{4.39}$$

In (4.39), $G_{v,j}(v)$ is used to determine the delivery policy and can be found using the following recursive equations.

$$G_{v,j}(j)=0$$
$$G_{v,j}(k)=\min\{G_{v,j}(l)+g_{v,j}(k,l):k<l\} \tag{4.40}$$

If the supplier produces at period v to meet buyer's proposed requirements through period j-1, the total units produced are $\sum_{q=v}^{j-1} r_{b,s,q}$. If the supplier delivers at period k to satisfy requirements from period k to l-1, there will be $\sum_{q=l}^{j-1} r_{b,s,q}$ units remained in the finished-goods inventory. Since there will be no delivery from period k+1 to l-1 due to zero inventory property, the total inventory holding cost of finished goods from period k to l-1 is $\sum_{m=k}^{l-1} H_{s,m}^{F}\left(\sum_{q=l}^{j-1} r_{b,s,q}\right)$. Also, the cumulative proposed delivery quantities from period 1 to period k is $R(1,k)$. Thus, $g_{v,j}(k,l)$ is calculated as follows.

$$\begin{aligned} g_{v,j}(k,l)= & \sum_{m=k}^{l-1} H_{s,m}^{F}\left(\sum_{q=l}^{j-1} r_{b,s,q}\right)+\sum_{m=k}^{l-1}\mu_{s,m}\left(\left|P(k,m)-R(k,m)\right|\right) \\ & +U\left(P(k,l-1)=R(k,l-1)\right)\lambda_{s,l-1}\left(\left|p_{s,b,l-1}-r_{b,s,l-1}\right|\right) \end{aligned} \tag{4.41}$$

Given v and j, it takes $O\left((j-v)^2\right)$ time to compute $G_{v,j}(v)$; and given t and u, there are $O\left((u-t)^2\right)$ operations to compute $F_{t,u}^{F}(t)$. Since there are $O\left(T^2\right)$ operations for the recursive equation (4.39), hence, the computational complexity is $O\left(T^6\right)$.

<u>Numerical Example</u>

We demonstrate the dynamic programming heuristic for solving the supplier problem by presenting a portion of the heuristic using a simple three-period example with the parameters shown in Table 4.2.

Table 4.2
Parameters for the supplier subproblem of two-echelon serial system

	Raw-material inventory subsystem	Finished-goods inventory subsystem
Setup cost/order	\$400 (K^R)	\$200 (K^F)
Unit holding cost	\$1 (h^R)	\$1.5 (h^F)

$$R = \{r_{b,s,1}, r_{b,s,2}, r_{b,s,3}\} = \{200, 460, 350\}$$

$$\mathrm{M} = \{\mu_{s,b,1}, \mu_{s,b,2}, \mu_{s,b,3}\} = \{0.55, 0.22, 0\}$$

$$\Lambda = \{\lambda_{s,b,1}, \lambda_{s,b,2}, \lambda_{s,b,3}\} = \{0, 0, 0.22\}$$

(a) $F(4) = 0$

(b) $F(3) = F(4) + C^R(3,4) + F^F_{3,4}(3)$

(i) $C^R(3,4) = K^R = 400$

(ii) $F^F_{3,4}(3) = K^R = 200$

$F(3) = 0 + 400 + 200 = 600$

(c) $F(2) = \min\begin{Bmatrix} F(4) + C^R(2,4) + F^F_{2,4}(2) \\ F(3) + C^R(2,3) + F^F_{2,3}(2) \end{Bmatrix}$

(i) $C^R(2,4) = K^R + h^R(350) = 750$

(ii) $F^F_{2,4}(4) = 0$

(iii) $F^F_{2,4}(3) = 0 + 200 = 200$

(iv) $F^F_{2,4}(2) = \min\begin{Bmatrix} F^F_{2,4}(4) + 200 + G_{2,4}(2) \\ F^F_{2,4}(3) + 200 + G_{2,3}(2) \end{Bmatrix}$

- $G_{2,4}(4) = 0$
- $G_{2,4}(3) = 0 + g_{2,4}(3,4) = 0$
- $G_{2,4}(2) = \min\begin{Bmatrix} G_{2,4}(4) + g_{2,4}(2,4) \\ G_{2,4}(3) + g_{2,4}(2,3) \end{Bmatrix}$

- $g_{2,4}(2,4) = \mu_{b,s,2}(350) + \lambda_{b,s,3}(350) = 154$
- $g_{2,4}(2,3) = 0$

- $G_{2,4}(2) = \min\{154,0\} = 0$
- $G_{2,3}(2) = 0 + g_{2,4}(3,4) = 0$
- $F^{F}_{2,4}(2) = \min\begin{Bmatrix} F^{F}_{2,4}(4) + 200 + G_{2,4}(2) \\ F^{F}_{2,4}(3) + 200 + G_{2,3}(2) \end{Bmatrix} = \min\begin{Bmatrix} 0 + 200 + 0 \\ 200 + 200 \end{Bmatrix} = 200$

$$F(2) = \min\begin{Bmatrix} F(4) + C^{R}(2,4) + F^{F}_{2,4}(2) \\ F(3) + C^{R}(2,3) + F^{F}_{2,3}(2) \end{Bmatrix} = \min\begin{Bmatrix} 0 + 750 + 200 = 950 \\ F(3) + C^{R}(2,3) + F^{F}_{2,3}(2) \end{Bmatrix}$$

If the heuristic continues, it will terminate at $F(1) = 1200$. The optimal order schedule is $\{1010, 0, 0\}$, i.e., an order of 1010 units is placed at period 1. Coincidentally, the optimal production schedule and the proposed delivery schedule are also $\{1010, 0, 0\}$.

4.3.4.2. Silver-Meal-based heuristics

Because of the computational complexity, the dynamic programming heuristics may not be efficient for problem with long planning horizon. A fast single-pass heuristic based on Silver-Meal heuristic that sequentially determines the policy of the finished-goods inventory subsystem and the policy of the raw-material inventory subsystem is proposed. In developing the single-pass heuristic, we use the cost modification method proposed by New (1974) in which the echelon holding cost is used for the finished-goods inventory subsystem. The holding cost of the raw material inventory subsystem is not modified as the facility does not have access to the cost parameters of his/her supplier.

<u>Buyer Subproblem (BN)</u>

Since the raw material inventory subsystem and the finished-goods inventory subsystem are considered separately, the production policy of the finished goods inventory subsystem can be determined using the traditional Silver-Meal heuristic. Once the production policy is determined, we proposed finding the ordering policy of the raw material inventory subsystem as follows. Define $C(t,u)$ as the average cost of the raw material subsystem if an order is placed at period *t* to meet requirements through period *u* with respect to (BN). Then, per period average cost can be calculated as

$$C(t,u)=\frac{1}{u-t+1}\left\{\begin{array}{l} S^{R}_{b,s,t}\left(m_{b,s}\sum_{v=t}^{u}y_{b,v}\right)+\left(\sum_{v=t}^{u-1}H^{R}_{b,s,v}\left(m_{b,s}\sum_{k=v}^{u-1}y_{b,k+1}\right)\right) \\ +\left(\sum_{v=t}^{u}\mu_{b,s,v}\left(\left|m_{b,s}\sum_{k=1}^{t-1}y_{b,k}+R(t,v)-P(1,v)\right|\right)\right) \\ +U\left(m_{b,s}\sum_{k=1}^{t-1}y_{b,k}+R(t,u)=P(1,u)\right)\lambda_{b,s,u}\left(\left|r_{b,s,u}-p_{s,b,u}\right|\right) \end{array}\right\}, \quad (4.42)$$

and the usual Silver-Meal heuristic can be applied directly.

Numerical Example

Using the same example for the buyer subproblem and assuming the production policy is $\{200, 460, 250\}$, the Silver-Meal-based heuristic is demonstrated as follows.

(a) $C(1,1)=400$

(b) $C(1,2)=0.5\times\{K^{R}+h^{R}(460)+\mu_{b,s,1}(350)+\mu_{b,s,2}(350)\}=795>C(1,1)$

(c) $C(2,2)=400$

(d) $C(2,3)=0.5\times\{K^{R}+h^{R}(350)+\lambda_{b,s,2}(350)\}=639>C(2,2)$.

Therefore, the proposed order policy is $\{200, 460, 250\}$.

Supplier Subproblem (SN)

When interacting with the buyer, the supplier incurs no penalty of raw material inventory. For the finished-goods inventory subsystem, the supplier needs to determine both the production policy and the delivery policy. At any period *u*, the requirement of period *u* can be satisfied by delivering the required amount at any period *k*, $k \le u$. To simplify the decision rule, we assume that the requirement of period *u* is satisfied either by delivering at period *u* or at period *k*, where period *k* is the last period that a delivery is scheduled. Define

$C(t,u)$: The average cost of the finished-goods inventory subsystem if the supplier produces at period t to meet requirements through period u with respect to (SN).

$F(t,u,k)$: The total cost of the finished-goods inventory subsystem if the supplier produces at period t to meet requirements through period u and the requirements of period u is satisfied by the delivery at period k.

$e^{F}_{s,t}$: Unit echelon holding of supplier s's finished goods inventory at period t.

Given $C(t,u-1)$, $F(t,u,k)$ can be determined by adjusting the setup, inventory holding, and penalty cost as follows.

$$\begin{aligned}F(t,u,k) = {} & (u-t)C(t,u-1) - S^{F}_{s,t}\left(\sum_{v=t}^{u-1} r_{b,s,v}\right) + S^{F}_{s,t}\left(\sum_{v=t}^{u} r_{b,s,v}\right) \\ & + r_{s,b,u}\left(\sum_{v=t}^{k-1} e^{F}_{s,v}\right) + \left(\sum_{v=k}^{u-1} \mu_{s,b,v}\left(r_{b,s,u}\right)\right) \\ & - U(u-k>1)\lambda_{s,b,u-1}\left(\left|p_{s,b,u-1} - r_{b,s,u-1}\right|\right) \\ & + U\left(P(t,u)=R(t,u)\right)\lambda_{s,b,u}\left(\left|p_{s,b,u} - r_{b,s,u}\right|\right)\end{aligned} \tag{4.43}$$

Equation (4.43) is derived based on the following observations.

(a) Setup cost must be recalculated.

(b) If the supplier decides to produce the requested quantities of period u at period t and deliver at period k,

(i) the finished-goods inventory will increased by $r_{s,b,u}$ from period t to k-1. Thus, the increase in echelon holding cost is $\left(r_{s,b,u}\sum_{v=t}^{k-1} e^{F}_{s,v}\right)$;

(ii) the cumulative proposed delivery quantities will be increased by $r_{s,b,u}$ from period k to u-1, where the cumulative proposed delivery quantities and cumulative requested quantities will be the same at period u.

(c) If requirement of period u-1 is not satisfied by a new batch, a penalty $\lambda_{s,b,u-1}\left(\left|p_{s,b,u-1} - r_{b,s,u-1}\right|\right)$ is incurred. In this case, if the requirement of period u is satisfying by a new delivery at *u*, this penalty cost should not be adjusted. However, if the requirement of period *u* is combined to previous delivery *k*, i.e., $u-k>1$, the cumulative proposed order and delivery quantities at period *u*-1 will not be the same and the penalty $\lambda_{s,b,u-1}\left(\left|p_{s,b,u-1}-r_{b,s,u-1}\right|\right)$ must be subtracted.

Since the requirement of period *u* is satisfied either by delivering at period *u* or at period *k*, where period *k* is the last period that a delivery is scheduled, then

$$C(t,u)=\frac{\min\{F(t,u,k),F(t,u,u)\}}{u-t+1} \tag{4.44}$$

And the usual Silver-Meal heuristic can be applied directly with the extra bookkeeping on *k*.

<u>Numerical Example</u>

The Silver-Meal-based heuristic for solving the supplier subproblem is illustrated using the same example shown in Section 4.3.4.1.

(a) $C(1,1)=F(1,1,1)=K^R=200$

(b) $C(1,2)=\min\{F(1,2,1),F(1,2,2)\}/2$

 (i) $F(1,2,1)=C(1,1)+\mu_{s,b,1}(460)+\mu_{s,b,2}(460)=554$

 (ii) $F(1,2,2)=C(1,1)+e_s^F(460)=430$

(c) $C(1,2)=\min\{F(1,2,1),F(1,2,2)\}/2=215>C(1,1)$

(d) $C(2,2)=200$

(e) $C(2,3)=\min\{F(2,3,2),F(2,3,3)\}/2$.

 (i) $F(2,3,2)=C(2,2)+\lambda_{s,b,3}(350)=277$

 (ii) $F(2,3,3)=C(2,2)+e_s^F(350)=375$

(f) $C(2,3)=\min\{F(2,3,2),F(2,3,3)\}/2=F(2,3,2)/2=138<C(2,2)$.

Thus, both the production policy and the proposed delivery policy are $\{200,810,0\}$.

4.3.5. Numerical example of SIM(b,s)

In this section, we present a step-by-step illustration of SIM(*b*,*s*). A simple example of a two-echelon serial system is considered. Buyer and supplier subproblems are solved using the dynamic programming heuristics presented in Section 4.3.4.1. The parameters of the problem are summarized in Table 4.3.

Table 4.3
Parameters of the numerical example for two-echelon serial system

	Supplier		Buyer	
	Raw-material inventory subsystem	Finished-goods inventory subsystem	Raw-material inventory subsystem	Finished-goods inventory subsystem
Setup cost/order	\$211.2	\$132	\$171.6	\$310.2
Unit holding cost	\$0.32	\$0.52	\$0.78	\$1.25

Demand of the end-product is $D = \{d_1, d_2, d_3, d_4\} = \{345, 304, 272, 409\}$. For convenience, we define $\mathrm{M}_b(k) = \{\mu_{b,s,t}(k), t = 1,\ldots,4\}$, $\Lambda_b(k) = \{\lambda_{b,s,t}(k), t = 1,\ldots,4\}$, $\mathrm{M}_s(k) = \{\mu_{s,b,t}(k), t = 1,\ldots,4\}$ and $\Lambda_s(k) = \{\lambda_{s,b,t}(k), t = 1,\ldots,4\}$.

Step 0: Initialization

Buyer: Set $\mathrm{M}_b(0) = \{0, 0, 0, 0\}$, $\Lambda_b(0) = \{0, 0, 0, 0\}$.

Solve (NCB) and get $R_{b,s}(0) = \{345, 574, 0, 409\}$, $c^*_{NCB} = 1785$

Supplier: Set $\mathrm{M}_s(0) = \{0, 0, 0, 0\}$ and $\Lambda_s(0) = \{0, 0, 0, 0\}$.

Solve (NCS) to get $c^*_{NCS} = 986$.

<u>Iteration 1</u>

Step 1: Negotiation

Supplier: Solve (SN).

$P_{s,b}(1) = \{1330, 0, 0, 0\}$, $g_{NS}(1) = 343$, $comp_s(1) = 986 - 343 = 643$.

Buyer: Solve (BN).

$R_{b,s}(1) = \{345, 574, 0, 409\}$, $g_{NB}(1) = 1785$, $comp_b(1) = 0$.

Step 2: Update Lagrangian multipliers

Supplier: $w(1) = 0.0006$

$\mathrm{M}_s(1) = \{0.24, 0.24, 0, 0.24\}$ and $\Lambda_s(0) = \{0.24, 0.24, 0, 0.24\}$.

Buyer: $w(1) = 0.0006$

$\mathrm{M}_b(1) = \{0.24, 0.24, 0, 0.24\}$ and $\Lambda_b(0) = \{0.24, 0.24, 0, 0.24\}$.

<u>Iteration 2</u>

Supplier: Solve (SN).

$P_{s,b}(2) = \{345, 574, 0, 409\} = R_{b,s}(1)$. Inform the buyer and stop.

4.4. Two-echelon assembly inventory system

Two-echelon assembly inventory system is a more general case of the two-echelon serial system in which there is a single buyer and multiple suppliers.

Similar to the analysis on two-echelon serial system, the buyer is indexed as b in this section for convenience. Applying the same modifications and decomposition procedure to (SC) as described previously will result in a buyer subproblem and $|B(b)|$ supplier subproblems. To facilitate asynchronous interaction in which the buyer can deal with each supplier separately, the production schedule of the buyer is assumed to be fixed. As a result, the buyer subproblem can be further decomposed into $|B(b)|$ sub-problems, one for each supplier. The subproblem of the buyer with respect to supplier s, (BN(s)) is as follows.

$$\text{(BN(s)) min} \quad g_{BN(s)} = \sum_{t=1}^{T} \left\{ \begin{aligned} &\Bigg(S^R_{b,s,t}\left(r_{b,s,t}\right) + H^R_{b,s,t}\left(I^R_{b,s,t}\right) + \mu^R_{b,s,t}\left|\sum_{\tau=1}^{t} r_{b,s,\tau} - \sum_{\tau=1}^{t} p_{s,b,\tau}\right| \\ &\quad + U\left(\sum_{\tau=1}^{t} r_{b,s,\tau} = \sum_{\tau=1}^{t} p_{s,b,\tau}\right)\lambda^R_{b,s,t}\left|r_{b,s,t} - p_{s,b,t}\right| \Bigg) \end{aligned} \right\} \tag{4.45}$$

s.t. (4.3)-(4.7) with $i = b$, (4.11) and (4.21).

Similarly, the supply subproblem of supplier s, $s \in B(b)$ is

$$\text{(SN(s)) min} \quad g_{SN(s)} = \sum_{t=1}^{T} \left\{ \begin{array}{l} \sum_{k \in B(s)} \left(S_{s,k,t}^{R}(x_{s,k,t}) + H_{s,k,t}^{R}(I_{s,k,t}^{R}) \right) \\ + \left(S_{s,t}^{F}(y_{s,t}) + H_{s,t}^{F}(I_{s,t}^{F}) + \mu_{s,b,t}^{F} \left| \sum_{\tau=1}^{t} p_{s,b,\tau} - \sum_{\tau=1}^{t} r_{b,s,\tau} \right| \right. \\ \left. + U\left(\sum_{\tau=1}^{t} p_{s,b,\tau} = \sum_{\tau=1}^{t} r_{b,s,\tau} \right) \lambda_{s,b,t}^{P} \left| p_{s,b,t} - r_{b,s,t} \right| \right) \end{array} \right\} \tag{4.46}$$

$$\text{s.t.} \quad I_{s,k,t-1}^{R} + x_{s,k,t} - I_{s,k,t}^{R} = m_{s,k} y_{s,t} \qquad k \in B(s); t = 1,\ldots,T \tag{4.47}$$

(4.5)-(4.7) with $i = s$, (4.13), (4.20) and (4.22).

As stated in the previous section, $r_{b,s,t}$ and $p_{s,b,t}$ are known by both parties. Also, all other parameters of (BN(s)) and (SN(s)) are private to the buyer and the supplier *s*, $s \in B(b)$, thus they have all the information to formulate their corresponding (BN(s)) and (SN(s)).

After the decomposition, the inventory coordination can be achieved by an interaction model similar to SIM. The interactive procedure starts with the initialization process that finds the reference solutions for the buyer and each supplier *s*, $s \in B(b)$, assuming there is no coordination. In other words, the buyer first determines his/her optimal order quantities, $r_{b,s,t}^{*}, s \in B(b)$ and $t = 1,\ldots,T$, based on the known external demand by solving his/her non-coordinated problem:

$$\text{(NCBA) min} \quad c_{NCBA} = \sum_{t=1}^{T} \left\{ \sum_{s \in B(b)} \left(S_{b,s,t}^{R}(r_{b,s,t}) + H_{b,s,t}^{R}(I_{b,s,t}^{R}) \right) + \left(S_{b,t}^{F}(y_{b,t}) + H_{b,t}^{P}(I_{b,t}^{F}) \right) \right\} \tag{4.48}$$

s.t. (4.3)-(4.7) with $i = b$, and (4.11).

Upon receiving the optimal order quantities, $r_{b,s,t}^{*}$, $t = 1,\ldots,T$, from the buyer, supplier *s*, $s \in B(b)$, sets $p_{s,b,t} = r_{b,s,t}^{*}, t = 1,\ldots,T$ and determines his/her optimal order policy by solving his/her non-coordinated problem:

$$\text{(NCS(s)) min} \quad c_{NCS(s)} = \sum_{t=1}^{T} \left\{ \begin{array}{l} \sum_{k \in B(s)} \left(S_{s,k,t}^{R}(x_{s,k,t}) + H_{s,k,t}^{R}(I_{s,k,t}^{R}) \right) \\ + \left(S_{s,t}^{F}(y_{s,t}) + H_{s,t}^{F}(I_{s,t}^{F}) \right) \end{array} \right\} \tag{4.49}$$

s.t. (4.2), (4.5)-(4.7) with $i = s$ and (4.13).

Then the interactive procedure is repeated for each supplier s, $s \in B(b)$ as follows. At each iteration, supplier s solves (SN(s)) with respect to buyer's proposed order policy. (S)he then updates the Lagrangian multipliers and proposes a delivery policy and the corresponding compensation (s)he is will to pay to the buyer. The buyer then solves (BN(s)), updates the corresponding Lagrangian multipliers, and proposes an order policy and the corresponding compensation requested to supplier s. This negotiation process repeats until a compromised order/delivery policy is obtained.

4.4.1. Compensation determination

For $s \in B(b)$, let $c^*_{NCBA(s)}$ be the optimal objective value of (NCBA) with respect to supplier s and $c^*_{NCS(s)}$ be the objective value of (NCS(s)) with respect to supplier s's reference policy. Also, we define $g_{BN(s)}(k)$ and $g_{SN(s)}(k)$ as the objective value of (BN(s)) and that of (SN(s)) in iteration k, respectively. Furthermore, we define $comp_{b,s}(k)$ and $comp_{s,b}(k)$ as the compensation requested that is submitted to supplier s by buyer b and the compensation revealed by supplier s to buyer b in iteration k during the negotiation, respectively. Using the approach mentioned in Section 4.3.1, we have

$$comp_{b,s}(k) = g_{BN(s)}(k) - c^*_{NCBA(s)} \tag{4.50}$$

$$comp_{s,b}(k) = c^*_{NCS(s)} - g_{SN(s)}(k) \tag{4.51}$$

4.4.2. Lagrangian multipliers updates

During the negotiation between the buyer and supplier s, the Lagrangian multipliers $\mu^R_{b,s,t}(k)$, $\lambda^R_{b,s,t}(k)$, $\mu^F_{s,b,t}(k)$, and $\lambda^F_{s,b,t}(k)$ can be updated as in (4.27)-(4.30) in iteration k after the step size corresponding to supplier s, $w_s(k)$, is determined.

Similar to the approach used for the two-echelon serial inventory system, $w_s(k)$ can be obtained by

$$w_s(k) = \frac{g(UB - LB)}{\sum_{t=1}^{T} G_t} = \frac{\theta\left(comp_{s,b}(k) - comp_{b,s}(k)\right)}{\sum_{t=1}^{T} G_t} \tag{4.52}$$

4.4.3. Solution procedure

As stated in Section 4.3.3, preference are given to the buyer if a compromised policy is not reached after hitting the iteration limit, and the buyer's last proposed order policy is used as the final order/delivery policy. Now, we are ready to state the interaction procedure for coordinating the two-echelon inventory system with buyer *b* and his/her supplier set *B*(*b*), AIM(*b*, *B*(*b*)), as follows.

AIM(*b, B*(*b*))

Step 0: Initialization

Buyer:

(a) Initialize $\lambda^R_{b,s,t}(0)=0$, $\mu^R_{b,s,t}(0)=0$, $t=1,\ldots,T$.

(b) Determine the proposed order policies, $R_{b,s}(0)$, $s \in B(b)$, and c^*_{NCBA} by solving (NCBA). Repeat Step 1 for each supplier *s*, $s \in B(b)$.

Step 1: Negotiation between buyer *b* and supplier *s*

Step 1.1: Local initialization

Buyer:

(c) Submit $R_{b,s}(0)$ to the supplier *s*. Set $k_b=1$.

Supplier *s*:

(d) Initialize $\lambda^F_{s,b,t}(0)=0$, $\mu^F_{s,b,t}(0)=0$, $t=1,\ldots,T$.

(e) Solve (NCS(s)) to obtain $c^*_{NCS(s)}$. Set $k_s=1$.

Step 1.2: Negotiation

Supplier *s*:

(f) Solve (SN(s)) to obtain $P_{s,b}(k_s)$.

(g) Determine $comp_{s,b}(k_s)$ as in (4.51).

(h) If $P_{s,b}(k_s)=R_{b,s}(k_s-1)$, a compromised solution is found. Inform the buyer and go to Step 1.5.

(i) Propose $P_{s,b}(k_s)$ and $comp_{s,b}(k_s)$ to the buyer. Set $k_s=k_s+1$.

Buyer:

(j) Solve (BN(s)) to obtain $R_{b,s}(k_b)$.

(k) Determine $comp_{b,s}(k_b)$ as in (4.50).

(l) If $R_{b,s}(k_b) = P_{s,b}(k_b)$, a compromised solution is found. Inform the supplier and go to Step 1.5.

(m) Propose $R_{b,s}(k_b)$ and $comp_{b,s}(k_b)$ to supplier *s*. Set $k_b = k_b + 1$.

(n) If iteration limit is exceeded, go to Step 1.4. Otherwise, go to Step 1.3.

Step 1.3: Update Lagrangian multipliers

Supplier *s*:

(o) Update $\mu^F_{s,b,t}(k_s)$ and $\lambda^F_{s,b,t}(k_s)$ as in (4.29) and (4.30).

Buyer:

(p) Update $\mu^R_{b,s,t}(k_b)$ and $\lambda^R_{b,s,t}(k_b)$ as in (4.27) and (4.28).

(q) Go to Step 1.2.

Step 1.4: Restore feasibility

Buyer:

(r) Use $R_{b,s}(k_b - 1)$ as the lot sizing policy and propose to the supplier.

Supplier *s*:

(s) $p_{b,s,t} = r_{b,s,t}(k_s - 1), t = 1,\ldots,T$. Solve (NCS(s)).

Step 1.5: Buyer continues to next supplier, go to Step 1.

In order to provide more specifics about how to solve the subproblems (NCBA), (NCS(s)), (BN(s)), and (SN(s)) in the procedure AIM(*b*,*B*(*b*)), it is necessary to further specify the components of the objective function; i.e., $S^R_{i,j,t}(x)\, S^F_{i,t}(y)\, H^R_{i,j,t}(u)\, H^F_{i,t}(u)$. Solution procedures for common forms of these components are described in the following sections.

4.4.4. Procedure for cases with concave cost functions

Similar to Section 4.3.4, we consider the same special case in which the ordering/setup cost functions are concave on $[0,\infty)$ for all *i* and *t*; and the inventory holding cost functions are linear on $[0,\infty)$ and are nondecreasing from upstream to downstream facilities. We consider nested policies that satisfy the zero inventory property for solving (BN(s)) and (SN(s)). Recall that the Silver-Meal-based heuristics developed in the previous section are one-pass heuristics which consider the finished-goods inventory subsystem and raw-material inventory subsystem sequentially, they can be applied to (BN(s)) and (SN(s)) without modification. In this section, dynamic programming heuristics are developed for solving the buyer subproblem (BN(s)) and the supplier subproblem (SN(s)).

4.4.4.1. Buyer subproblem

Define $F(t)$ as the minimum total cost from period *t* to *T* with respect to (BN(s)), and $C^R(t,u)$ as the total cost of the raw-material inventory subsystem of ordering at period *t* to meet requirements through period *u*-1 with respect to (BN(s)). Since (BN(s)) only considers the raw-material inventory subsystem, the problem can be solved by the recursive equations as follows.

$$F(T+1)=0$$
$$F(t)=\min\{F(u)+C^R(t,u):t<u\} \tag{4.53}$$

where $C^R(t,u)$ is the same as in (4.36).

In this case, the computational complexity is $O(T^2)$.

4.4.4.2. Supplier subproblem

(SN(s)) is more complicated than (BN(s)) for there are multiple raw-materials inventory subsystems supplying the single finished-goods inventory subsystem. To solve (SN(s)), the dynamic programming approach for assembly system proposed by Crowston and Wagner (1973) is adopted. For convenience, inventory subsystems of the suppliers are numbered as 1 to k, where 1 to k-2 are the raw-material inventory subsystems, k-1 is the finished-goods inventory subsystem, and k is a dummy subsystem that is responsible for determining the delivery policy. Thus, the predecessor of the dummy unit k is the finished-goods inventory subsystem. Using the same notation as used in Crowston and Wagner (1973), let

- π: ordering/production/delivery policy, $\pi = \{\pi_t : t = 1, \ldots, T\}$, where $\pi_t = 1$ if there is an order/production/delivery at time t,
- $N(\pi)$: set of policy profiles that nest π, i.e., $\pi' \in N(\pi)$ if and only if $\pi_t - \pi_t' \geq 0$, $t = 1, \ldots, T$.
- $f^i(\pi)$: minimum total cost of inventory unit i and all the predecessors of i
- $b(i)$: set of immediate predecessors, in term of inventory units, of inventory unit i

Additionally, we define $c^i(\pi)$ as total cost of inventory unit i if policy π is used. In other words, if $i = k$, $c^i(\pi)$ is total penalty cost determined by the Lagrangian multipliers; otherwise, $c^i(\pi)$ is the total ordering/setup and inventory holding cost of the inventory subsystem i. Crowston and Wagner (1973) showed that $f^i(\pi)$ can be found by the following recursive equation.

$$f^i(\pi) = c^i(\pi) + \sum_{m \in b(i)} \min\{f^m(\pi') : \pi' \in N(\pi)\} \tag{4.54}$$

The worse case computational complexity is $O(2^{kT})$. For details, please refer to Crowston and Wagner (1973).

Numerical Example

A simple four-period problem for a supplier with two raw-material inventory systems is used to exemplify the dynamic programming heuristic. Parameters of the problem are shown in Table 4.4.

Table 4.4
Parameters for the supplier subproblem of two-echelon assembly system

	Supplier		
	Raw-material inventory subsystem 1	Raw-material inventory subsystem 2	Finished-goods inventory subsystem
Setup cost/order	\$400 $\left(K^{R1}\right)$	\$400 $\left(K^{R2}\right)$	\$200 $\left(K^{F}\right)$
Unit holding cost	\$2 $\left(h^{R1}\right)$	\$2 $\left(h^{R2}\right)$	\$5 $\left(h^{F}\right)$
Unit echelon holding cost	\$2 $\left(e^{R1}\right)$	\$2 $\left(e^{R2}\right)$	\$1 $\left(e^{F}\right)$

$$R = \{r_{b,s,1}, r_{b,s,2}, r_{b,s,3}\} = \{200, 460, 350\}$$

$$\mathrm{M} = \{\mu_{s,b,1}, \mu_{s,b,2}, \mu_{s,b,3}\} = \{0.55, 0.22, 0\}$$

$$\Lambda = \{\lambda_{s,b,1}, \lambda_{s,b,2}, \lambda_{s,b,3}\} = \{0, 0, 0.22\}$$

For convenience, define $\pi^0 = \{1, 0, 0\}$, $\pi^1 = \{1, 1, 0\}$, $\pi^2 = \{1, 0, 1\}$ and $\pi^3 = \{1, 1, 1\}$. We illustrate the heuristic by showing the steps for finding the solution for π^2.

(a) $$f^4\left(\pi^2\right) = c^4\left(\pi^2\right) + \sum_{m \in b(4)} \min\left\{f^m\left(\pi'\right) : \pi' \in N\left(\pi^2\right)\right\}$$

$$= \mu_{s,b,1}(460) + \lambda_{s,b,2}(460) + \min\left\{f^3\left(\pi^0\right), f^3\left(\pi^2\right)\right\}$$

(b) $$f^3\left(\pi^2\right) = c^3\left(\pi^2\right) + \sum_{m \in b(3)} \min\left\{f^m\left(\pi'\right) : \pi' \in N\left(\pi^2\right)\right\}$$

$$= 2K^F + \min\left\{f^1\left(\pi^0\right), f^1\left(\pi^2\right)\right\} + \min\left\{f^2\left(\pi^0\right), f^2\left(\pi^2\right)\right\}$$

(i) $f^2\left(\pi^2\right) = 2K^{R2} = 800$.

(ii) $f^2\left(\pi^0\right) = K^{R2} + e^{R2}(350) + e^{R2}(350) = 1800$

(iii) $f^1\left(\pi^2\right) = 2K^{R1} = 800$.

(iv) $f^1\left(\pi^0\right) = K^{R1} + e^{R1}(350) + e^{R1}(350) = 1800$

$$f^3\left(\pi^2\right) = 2K^F + \min\left\{f^1\left(\pi^0\right), f^1\left(\pi^2\right)\right\} + \min\left\{f^2\left(\pi^0\right), f^2\left(\pi^2\right)\right\}$$

$$= 400 + \min\{1800, 800\} + \min\{1800, 800\} = 2000.$$

(c) $$f^3\left(\pi^0\right) = c^3\left(\pi^0\right) + \sum_{m \in b(3)} \min\left\{f^m\left(\pi'\right) : \pi' \in N\left(\pi^0\right)\right\}$$

$$= K^F + e^F(350) + e^F(350) + f^1\left(\pi^0\right) + f^2\left(\pi^0\right)$$

$= 200 + 350 + 350 + 1800 + 1800 = 4500$

(d) $f^4(\pi^2) = \mu_{s,b,1}(460) + \lambda_{s,b,2}(460) + \min\{f^3(\pi^0), f^3(\pi^2)\}$

$= 0.55(460) + 0(460) + \min\{4500, 2000\} = 2253$.

Thus, if the supplier decides to use delivery policy π^2, the optimal order policy and production policy are also π^2 in this example.

4.4.5. Numerical example of AIM(b,B(b))

To illustration AIM(b,B(b)), we consider a two-echelon assembly. Buyer and suppliers subproblems are solved using the dynamic programming heuristics presented in the previous section. The parameters of the problem are summarized in Table 4.5 and Table 4.6 .

Table 4.5
Buyer's parameters of the numerical example for two-echelon assembly system

	Buyer		
	Raw-Material Inventory System 1	Raw-Material Inventory System 2	Finished-goods inventory system
Setup Cost/order	\$171.6	\$363	\$310.2
Unit holding cost	\$0.78	\$0.42	\$1.25

Table 4.6
Suppliers' parameters of the numerical example for two-echelon assembly system

	Supplier 1		Supplier 2	
	Raw-Material Inventory System	Finished-goods inventory system	Raw-Material Inventory System	Finished-goods inventory system
Setup Cost/order	\$211.2	\$132	\$303.58	\$363
Unit holding cost	\$0.32	\$0.52	\$0.2	\$0.3

Demand of the end product is $D = \{345, 304, 272, 409\}$.

Step 0: Initialization

Buyer: Set $\mathrm{M}_b(0) = \{0, 0, 0, 0\}$, $\Lambda_b(0) = \{0, 0, 0, 0\}$.

Solve (NCBA).

$$R_{b,s1}(0) = \{649, 0, 681, 0\},\ R_{b,s2}(0) = \{649, 0, 681, 0\}.$$

Step 1: Negotiation between buyer and supplier 1.

Step 1.1: Local initialization

Buyer: Submit $R_{b,s1}(0)$, to the supplier 1.

Supplier 1:

Set $\mathrm{M}_{s1}(0) = \{0, 0, 0, 0\}$ and $\Lambda_{s1}(0) = \{0, 0, 0, 0\}$.

Solve (NCS(s)) to get $c^{*}_{NCS} = 686$.

<u>Iteration 1</u>

Step 1.2: Negotiation

Supplier 1: Solve (SN(s)). $P_{s1,b}(1) = \{1330, 0, 0, 0\}$.

$g_{SN(s1)}(1) = 343$, $comp_{s1,b}(1) = 686 - 343 = 343$.

Buyer: Solve (BN(s)). $R_{b,s1}(1) = \{649, 0, 681, 0\}$.

$g_{BN(s1)}(1) = 343$, $comp_{b,s1}(1) = 0$.

Step 1.3: Update Lagrangian multipliers

Supplier 1: $w_{s1}(1) = 0.0007$

$\mathrm{M}_{s1}(1) = \{0.5, 0.5, 0, 0\}$ and $\Lambda_{s1}(0) = \{0, 0, 0.5, 0\}$.

Buyer: $w_{s1}(1) = 0.0007$

$\mathrm{M}_{b}(1) = \{0.5, 0.5, 0, 0\}$ and $\Lambda_{b}(0) = \{0, 0, 0.5, 0\}$.

<u>Iteration 2</u>

Step 1.2: Negotiation

Supplier 1: Solve (SN(s)). $P_{s1,b}(2) = \{649, 0, 681, 0\} = R_{b,s1}(1)$.

Inform the buyer.

Step 1: Negotiation between buyer and supplier 2.

Step 1.1: Local initialization

Buyer: Submit $R_{b,s2}(0)$, to the supplier 2.

Supplier 2:

Set $\mathrm{M}_{s2}(0) = \{0, 0, 0, 0\}$ and $\Lambda_{s2}(0) = \{0, 0, 0, 0\}$.

Solve (NCS(s)) to get $c^*_{NCS} = 1453$.

<u>Iteration 1</u>

Step 1.2: Negotiation

Supplier 2: Solve (SN(s)). $P_{s2,b}(1) = \{1330, 0, 0, 0\}$.

$g_{SN(s2)}(1) = 727$, $comp_{s2,b}(1) = 1453 - 727 = 726$.

Buyer: Solve (BN(s)). $R_{b,s2}(1) = \{649, 0, 681, 0\}$.

$g_{BN(s2)}(1) = 726$, $comp_{b,s2}(1) = 0$.

Step 1.3: Update Lagrangian multipliers

Supplier 2: $w_{s2}(1) = 0.00026$

$\mathrm{M}_{s2}(1) = \{0.18, 0.18, 0, 0\}$ and $\Lambda_{s2}(0) = \{0, 0, 0.18, 0\}$.

Buyer: $w_{s2}(1) = 0.0007$

$\mathrm{M}_{b}(1) = \{0.18, 0.18, 0, 0\}$ and $\Lambda_{b}(0) = \{0, 0, 0.18, 0\}$.

<u>Iteration 2</u>

Step 1.2: Negotiation

Supplier 2: Solve (SN(s)). $P_{s2,b}(2) = \{1330, 0, 0, 0\}$.

$g_{SN(s2)}(2) = 969$, $comp_{s2,b}(2) = 484$.

Buyer: Solve (BN(s)). $R_{b,s2}(2) = \{649, 0, 681, 0\}$.

$g_{BN(s2)}(2) = 968$, $comp_{b,s2}(1) = 968 - 726 = 242$.

Step 1.3: Update Lagrangian multipliers

Supplier 2: $\mathrm{M}_{s2}(1) = \{0.24, 0.24, 0, 0\}$ and $\Lambda_{s2}(0) = \{0, 0, 0.24, 0\}$.

Buyer: $\mathrm{M}_{b}(1) = \{0.24, 0.24, 0, 0\}$ and $\Lambda_{b}(0) = \{0, 0, 0.24, 0\}$.

(After several iterations, the negotiation between buyer and supplier 2 does not reach a compromised feasible solution. Therefore, Step 1.4 is invoked.)

Step 1.4: Restore feasibility

Buyer: Use $R_{b,s2}(k)=\{649,0,681,0\}$ as the order policy.

Supplier 2: Set $P_{s2,b}(k)=R_{b,s2}(k)=\{649,0,681,0\}$. Solve (NCS(s)).

4.5. Extension to multi-echelon serial/assembly inventory system

To coordinate a general multi-echelon serial/assembly inventory system, we propose using a hierarchical approach that uses SIM(b,s) and AIM(b,B(b)) sequentially starting from the last echelon buyer to the top of the system. Recall that the facilities are numbered from the top to the bottom, the hierarchical Lagrangian-based heuristic, HLH, is as follows.

HLH

For i=n down to 2,

If facility i has single supplier, invoke SIM(i,i-1).

Otherwise, if facility i has multiple suppliers, invoke AIM(i,B(i)).

For example, consider the case where the example in Section 4.4.5 is extended such that one of the suppliers also has a single supplier as in Fig. 4.4. In this case, after the negotiation with the buyer, supplier 1 has a local optimal order policy. Then (s)he invokes a negotiation with supplier 3 by sending the order policy to supplier 3. As such, supplier 1 plays the role of a buyer and supplier 3 plays the role of a supplier and a feasible policy can be found by using SIM.

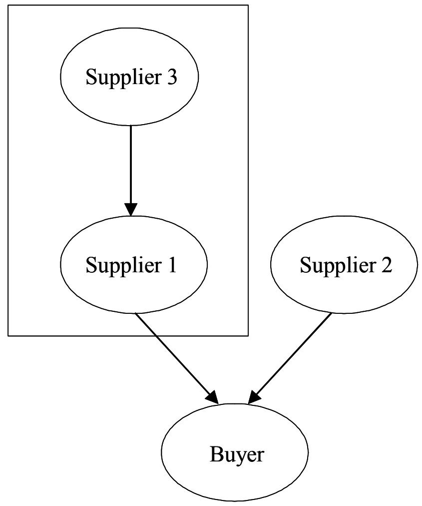


Fig. 4.4. Multi-echelon supply chain.

4.6. Computational experiments

The performance of the proposed heuristic HLH is evaluated on a set of twelve serial/assembly systems. Specifically, we evaluate the followings.

(a) The effectiveness of HLH with respect to the optimal solution.

(b) The solution quality of HLH when comparing to existing heuristics.

(c) The robustness of the solution of HLH if the facilities are not honest in exchanging information.

(d) Convergence performance of the coordination approach for each supplier-buyer node-model pair.

(e) The robustness of the solution of HLH when actual demands deviate from the original forecasted demands.

4.6.1. Test problems

For evaluation purpose, we consider the systems in which the ordering/setup cost of each inventory subsystem are constant over time regardless of the ordering/production quantities; and the holding cost function of each inventory subsystem is linear with fixed unit holding cost. Define

$K_{i,j}^{R}$: Setup/ordering cost of facility i for ordering from facility j.

$h^R_{i,j}$: Unit holding cost of facility i for raw material j.

K^F_i : Production setup cost of facility i.

h^F_i : Unit holding cost of facility i for his/her finished goods inventory.

Then,

$$S^R_{f,i,t}\left(x_{f,i,t}\right)=\begin{cases}K^R_{f,i} & \text{if } x_{f,i,t}>0\\ 0 & \text{otherwise}\end{cases} \tag{4.55}$$

$$S^F_{f,t}\left(y_{f,t}\right)=\begin{cases}K^F_{f} & \text{if } y_{f,i,t}>0\\ 0 & \text{otherwise}\end{cases} \tag{4.56}$$

$$H^R_{f,i,t}\left(I^R_{f,i,t}\right)=h^R_{f,i}I^R_{f,i,t} \tag{4.57}$$

$$H^F_{f,t}\left(I^F_{f,t}\right)=h^F_{f}I^F_{f,t} \tag{4.58}$$

Moreover, echelon holding cost is assumed to be positive, which is simply consistent with the value-added concept. With these settings, the optimal solutions of the assembly systems tested can be found using the dynamic programming approach by Crowston and Wagner (1978).

Twelve different supply chain structures are considered, namely, serial/assembly systems with two-, three-, four-, and five-echelon with one-, two-, and three-in-tree structures, where the one-in-tree structure is a serial system. For all the test problems, the length of the planning horizon is twelve periods. The demand for the end-item in each period is generated randomly from a uniform distribution over [200,400]. Echelon holding cost are generated uniformly from [0.3,0.5]. Setup/Ordering cost of each inventory unit, K, is computed based on the EOQ formula as follows (Salomon 1991).

$$K = 0.5\times \text{Echelon Holding Cost}\times \text{Average Demand}\times (TBO)^2 \tag{4.59}$$

where TBO is the time between ordering/production, and average demand is the average of the twelve-period demands generated. Thus, TBO is an approximation of the true time between ordering/production in a multi-echelon supply chain inventory system.

For each supply chain structure, four scenarios are considered in which the setup/ordering cost of each inventory unit is generated using different TBO, which translates to 48 different problem sets. *TBO*s of scenarios 1 to 3 are the same for all the facilities and equals to two, three, and four, respectively. In scenario 4, *TBO*s are different for each inventory unit and are generated from uniform distribution from [2,4]. This experiment design allows us to test the performance of HLH with various setup/ordering cost and holding cost combinations.

For each problem set, 100 test problems are randomly generated. The subproblems of HLH are solved using two heuristics, namely, dynamic programming heuristics proposed in Section 4.3.4.1 and Section 4.4.4 (HLH-DP), and Silver-Meal-based heuristics developed in Section 4.3.4.2 (HLH-SM). For comparison, the test problems are solved using three single-pass heuristics. The first single-pass heuristic (SP-WW) finds the lot size of each inventory unit using Wagner-Whitin algorithm (Wagner-Whitin 1957). The second heuristic (SP-KCC) is proposed by Blackburn and Millen (1982) which uses modified setup cost and modified echelon holding cost to determine the lot size of each inventory unit using Wagner-Whitin algorithm. The third heuristic, SP-MS, is proposed by McLaren (1976) in which the setup/ordering cost is adjusted and the lot size of each inventory unit is determined using Wagner-Whitin algorithm. It should be noted that SP-KCC requires access of setup/ordering cost and echelon holding cost of facilities in other echelon, while SP-MS needs the information on setup/ordering cost of facilities in other echelon for cost modifications. These cost adjustments methods are briefly described in Appendix.

Also, to test the sensitivity of HLH with regards to untruthful behavior of the facilities, HLH-DP and HLH-SM are repeated for all the test problems assuming the suppliers will understate and the buyers will overstate the corresponding compensation by 10% (HLH-DP(10) and HLH-SM(10)); and by 20% (HLH-DP(20) and HLH-SM(20)).

To assess the convergence performance, the number of iterations for the coordination approach for each supplier-buyer is recorded. Both HLH-DP and HLH-SM are tested using the two-echelon serial and assembly inventory problems of Scenario 4.

To evaluate (e), ordering schedules of all the facilities are assumed to be fixed by contract after solving the problems by means of the tested heuristics. Thus, the impacts of the error in demand forecast are absorbed by the lowest echelon facility alone; and it is suffice to consider two echelon systems. To this end, the hundred two-echelon serial inventory problems of Scenario 4 are used. For each problem, thirty sets of actual demand of the end-item are generated after the ordering/production schedules are determined. Actual demand of end-item of each period is generated as $d_t^{Act} = d_t^{Fore}\left(1 \pm \varepsilon_t\right)$, where d_t^{Act} is the actual demand of period *t*, d_t^{Fore} is the forecasted demand of period *t*, and ε_t is the variation parameter. Two cases are considered. In the first case, the parameter, ε_t, is generated from uniform distribution of [0,0.4]; while ε_t is generated randomly from uniform distribution of [0,0.8] in the second case.

4.6.2. Results and analyses

To evaluate (a) to (c), the performance measure, $Dev = (Z_H - Z_{OPT})/Z_{OPT} \times 100$, is used; where Z_{OPT} is the optimal total cost and Z_H is the total cost obtained by the one of the heuristics, *H*, tested. In other words, the performance measure is simply the percentage deviation of the heuristic solution from the optimal solution. The average performance measures, $\overline{Dev}$, and the corresponding sample standard deviation (in parenthesis) of the four scenarios are summarized in Table 4.7 to Table 4.10.

From Table 4.7 to Table 4.9, the average performance measures, $\overline{Dev}$, of SP-KCC are close to zero in all cases. It is because *TBO* of Scenarios 1 to 3 are constant in all the inventory units. It follows that that the setup/ordering cost to echelon holding cost ratios of all the facilities are the same. As a result, KCC method will generate a delivery/production policy for each inventory unit that is very close to the optimal solution. Consequently, SP-KCC is not considered for comparison in Scenario 1 to 3.

Table 4.7
Average performance measure of scenario 1 (*TBO*=2)

No. of Predecessor	2-Echelon			3-Echelon			4-Echelon			5-Echelon		
	1	2	3	1	2	3	1	2	3	1	2	3
HLH-DP	6(3)	8(3)	8(2)	14(3)	11(2)	11(1)	17(3)	13(1)	10(1)	20(3)	13(2)	10(1)
HLH-DP(10)	6(3)	8(3)	8(2)	14(3)	12(2)	11(1)	18(3)	13(1)	10(1)	21(3)	13(2)	10(1)
HLH-DP(20)	6(3)	8(3)	8(2)	14(3)	12(2)	11(1)	18(3)	14(1)	10(1)	21(3)	14(2)	10(1)
HLH-SM	10(7)	13(8)	13(7)	12(7)	12(6)	12(6)	11(6)	11(5)	10(6)	10(5)	11(5)	9(6)
HLH-SM(10)	10(7)	13(8)	13(7)	12(7)	12(6)	12(6)	11(6)	11(5)	10(6)	11(5)	11(5)	9(6)
HLH-SM(20)	10(7)	13(8)	13(7)	12(7)	12(6)	12(6)	11(6)	11(5)	10(6)	11(5)	12(5)	10(6)
SP-WW	18(3)	16(3)	16(2)	23(3)	19(2)	18(2)	25(3)	20(2)	17(2)	27(3)	20(2)	17(1)
SP-KCC	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
SP-MS	11(8)	11(5)	9(3)	17(5)	13(2)	9(2)	19(4)	13(1)	10(1)	22(4)	14(1)	10(1)

*Each cell shows the average performance measures, $\overline{Dev}$, and the corresponding sample standard deviation (shown in parentheses).

Table 4.8
Average performance measure of scenario 2 (*TBO*=3)

No. of Predecessor	2-Echelon			3-Echelon			4-Echelon			5-Echelon		
	1	2	3	1	2	3	1	2	3	1	2	3
HLH-DP	4(2)	5(2)	6(2)	9(2)	12(3)	12(2)	14(4)	17(2)	15(2)	20(5)	19(2)	16(1)
HLH-DP(10)	4(2)	6(2)	7(2)	9(2)	14(3)	12(2)	14(4)	17(2)	15(2)	20(5)	19(2)	16(1)
HLH-DP(20)	4(2)	6(2)	7(2)	9(2)	14(3)	12(2)	14(4)	17(2)	16(2)	20(5)	19(2)	17(1)
HLH-SM	12(4)	13(3)	15(3)	11(5)	14(4)	15(4)	9(4)	13(3)	14(2)	8(3)	13(4)	13(2)
HLH-SM(10)	12(4)	13(3)	15(3)	11(5)	15(4)	15(4)	9(4)	13(3)	14(2)	8(3)	13(4)	13(2)
HLH-SM(20)	12(4)	13(3)	15(3)	11(5)	15(4)	15(4)	9(4)	13(3)	15(2)	9(3)	13(4)	14(2)
SP-WW	16(4)	18(3)	27(3)	25(6)	26(3)	24(3)	32(6)	30(3)	25(2)	43(6)	32(3)	26(2)
SP-KCC	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
SP-MS	8(5)	8(4)	8(3)	11(3)	13(2)	12(2)	15(5)	18(2)	15(2)	20(5)	20(2)	16(1)

Table 4.9
Average performance measure of scenario 3 (*TBO*=4)

No. of Predecessor	2-Echelon			3-Echelon			4-Echelon			5-Echelon		
	1	2	3	1	2	3	1	2	3	1	2	3
HLH-DP	5(2)	6(2)	6(2)	10(3)	11(2)	12(3)	15(4)	19(2)	15(2)	19(3)	22(2)	16(2)
HLH-DP(10)	5(2)	6(2)	7(2)	10(3)	11(2)	12(3)	15(4)	19(2)	15(2)	19(3)	22(2)	16(2)
HLH-DP(20)	5(2)	6(2)	7(2)	10(3)	11(2)	13(3)	15(4)	19(2)	16(2)	20(3)	22(2)	17(2)
HLH-SM	11(9)	13(8)	15(9)	9(9)	12(9)	12(6)	11(6)	12(7)	12(8)	7(5)	13(8)	16(10)
HLH-SM(10)	11(9)	13(8)	15(9)	9(9)	12(9)	12(6)	11(6)	12(7)	13(8)	7(5)	13(8)	17(10)
HLH-SM(20)	11(9)	13(8)	16(9)	9(9)	12(9)	13(6)	11(6)	12(7)	13(8)	7(5)	14(8)	17(10)
SP-WW	16(4)	18(3)	19(3)	24(5)	28(3)	27(3)	31(5)	35(3)	31(2)	37(7)	39(3)	31(3)
SP-KCC	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
SP-MS	9(6)	9(5)	8(3)	13(6)	13(3)	13(2)	17(4)	19(2)	16(1)	21(4)	23(2)	17(2)

Table 4.10
Average performance measure of scenario 4 (*TBO*=2,3,or 4)

No. of Predecessor	2-Echelon			3-Echelon			4-Echelon			5-Echelon		
	1	2	3	1	2	3	1	2	3	1	2	3
HLH-DP	3(3)	4(4)	5(6)	5(4)	8(3)	8(3)	9(5)	13(4)	12(2)	12(5)	16(3)	13(2)
HLH-DP(10)	3(3)	4(4)	5(6)	5(4)	8(3)	9(3)	9(5)	13(4)	13(2)	12(5)	16(3)	14(2)
HLH-DP(20)	3(3)	4(4)	6(6)	5(4)	8(3)	9(3)	9(5)	14(4)	13(2)	12(5)	16(3)	15(2)
HLH-SM	12(9)	15(8)	15(9)	12(9)	14(6)	16(7)	23(9)	17(8)	17(7)	13(9)	16(5)	16(6)
HLH-SM(10)	12(9)	15(8)	15(9)	12(9)	14(6)	16(7)	23(9)	18(8)	17(7)	13(9)	16(5)	16(6)
HLH-SM(20)	12(9)	15(8)	15(9)	12(9)	15(6)	16(7)	23(9)	18(8)	18(7)	13(9)	17(5)	17(6)
SP-WW	7(5)	9(5)	8(4)	12(6)	15(5)	14(4)	17(7)	20(5)	18(2)	21(8)	23(4)	18(2)
SP-KCC	2(2)	1(2)	1(2)	5(5)	3(3)	2(2)	6(5)	4(3)	3(2)	9(7)	5(2)	4(2)
SP-MS	9(13)	7(10)	5(8)	6(6)	7(5)	6(2)	9(5)	10(3)	8(2)	12(5)	12(2)	9(2)

4.6.2.1. HLH-DP

From the experiment, we observed that HLH-DP provides improvement over SP-WW. The average improvement of HLH-DP over SP-WW in term of *Dev* is in the neighborhood of eight percent for the systems with different number of echelons. Though it may be inferred from Table 4.7 to Table 4.10, the improvement of HLH-DP over SP-WW is shown in Table 4.11.

Table 4.11
Average percentage improvement of HLH-DP over SP-WW

Predecessor	2-Echelon	3-Echelon	4-Echelon	5-Echelon	Average
1	8	9	9	10	9
2	8	9	8	8	8
3	7	8	7	7	7
Average	8	8	8	8	8

When compared to SP-MS, HLH-DP outperforms SP-MS for two-echelon system and provides comparative performance as that of SP-MS in other systems for all scenarios. When compared to SP-KCC in scenario 4, SP-KCC is found to perform extremely well in which $\overline{Dev}$ is well below ten percent for all the problem sets. For the problem sets in this scenario, on average, HLH-DP is three percent, four percent, six percent, and seven percent higher in $\overline{Dev}$ when comparing to SP-KCC for the two-, three-, four-, and five-echelon inventory systems.

Also noted from the experiment results is that the solution quality of HLH-DP is only slightly affected by the untruthful behavior of the facilities. Out of the 48 problem sets, HLH-DP(10) results in one percent higher in $\overline{Dev}$ in nine problem sets and two percent higher in only one problem set. Similarly, HLH-DP(20) has higher $\overline{Dev}$ in twenty out of 48 problem sets in which eighteen of them are one percent higher and two of them are two percent higher in $\overline{Dev}$ when compare to HLH-DP where all the facilities are honest.

To evaluate the convergent performance, number of iterations until the compromise solution is obtained are recorded for the two-echelon serial and assembly systems of scenario 4. As shown in Table 4.12, the average number of iterations of the interaction process between each pair of buyer and supplier is 20, 26, and 34 for the serial system, assembly system with 2 suppliers and assembly system with 3 suppliers, respectively. Running on a computer with a 1.7 GHz CPU, it takes on average 2.30 millisecond CPU time per iteration for serial system and 143.77 millisecond CPU time per iteration for the assembly systems. The fast convergence of HLH-DP and its satisfactory performance with respect to the optimal solution suggest that HLH-DP is a promising approach in coordinating supply chain inventory system.

Table 4.12
Convergence performance of HLH-DP

Number of Predecessors	Average number of iterations
1	20
2	26
3	34

4.6.2.2. HLH-SM

From the experiments, we found that the average performance of HLH-SM is fairly stable for different problem sets. The $\overline{Dev}$ ranges from six to sixteen percent for all the problem sets. While all other heuristics tested in this experiment exhibits positive relationships on the performance measure and the number of echelons, HLH-SM shows no such relationship. However, the sample standard deviations of HLH-SM for all of the problem sets are higher than those of other heuristics, suggesting that while stable on average, the quality of the individual solution obtained by HLH-SM varies quite a bit.

From the experiments, we found that HLH-SM performs worse than SP-WW in only five out of the 48 problem sets, while three of them are two-echelon inventory systems and the remaining two are three-echelon inventory systems. The reason is that HLH-SM employs Silver-Meal-based approach for solving the subproblems instead of by dynamic programming heuristic. As a result, its performance may be worse than that of SP-WW, which uses dynamic programming approach for finding the ordering/production policy. Nonetheless, HLH-SM, on average, shows seven percent improvement over SP-WW. Table 4.13 presents the average percentage improvement of HLH-SM over SP-WW.

Table 4.13
Average percentage improvement of HLH-SM over SP-WW

Predecessor	2-Echelon	3-Echelon	4-Echelon	5-Echelon	Average
1	2	8	12	17	10
2	1	7	10	11	7
3	0	5	7	7	5
Average	1	7	10	12	7

When comparing to HLH-DP, HLH-SM is inferior to HLH-DP in the two-echelon systems. Its performance is comparable to HLH-DP for the three-echelon systems, and it starts to provide better solutions than those of HLH-DP for four- and five-echelon systems. Intuitively, the reason that HLH-SM starts to gain advantage over HLH-DP as the number of echelon increases may be that HLH-SM tends to find policy with longer time between ordering/production than that of individual facility-based optimal policy. As a result, time between each order will be longer with larger quantity per order for facilities at the upper echelons, and causes the option of holding inventory at the higher echelon facility less attractive. In this case, some facilities at the upper echelon may benefit from this. As the number of echelons increases, the number of facilities that may benefit from the long ordering period will increase. Therefore, HLH-SM has advantage over HLH-DP as the number of echelons increases. Similar observations are found when comparing HLH-SM to SP-MS.

Furthermore, the experiment results suggest that the solution quality of HLH-SM is only slightly affected by the untruthful behavior of the facilities. HLH-SM(10) results in one percent higher in $\overline{Dev}$ in five of the 48 problem sets. Similarly, HLH-SM(20) has one percent higher in $\overline{Dev}$ in seventeen problem sets when compare to HLH-SM where all the facilities are honest.

Table 4.14 summarized the results of the convergence performance of HLH-SM. It is found that the coordination approach converges rather fast with average number of iterations in the neighborhood of 55. Running on a computer with 1.7GHz CPU, the average CPU time per iteration is 0.59 milliseconds.

When comparing this result to the HLH-DP counterpart, we found that HLH-SM, on average, requires more iterations to reach a compromised solution between the buyer and supplier. However, the Silver-Meal-based heuristics have much better time complexity than that of the dynamic programming heuristics at each iteration, which justifies the use of HLH-SM, especially for the problem with long planning horizon.

Table 4.14
Convergence performance of HLH-SM

Number of Predecessors	Average number of iterations
1	56
2	51
3	54

4.6.2.3. Demand forecast error

To test the robustness of our heuristic due to forecasting error in demand, the ordering/production policies obtained by HLH-DP, HLH-SM, SP-WW, SP-KCC, and SP-MS for the two-echelon serial inventory problems of scenario 4 are used. As stated previously, actual demand of period *t* are generated by $d_t^{Act} = d_t^{Fore}(1 \pm \varepsilon_t)$. Two cases are considered. In the first case (Case 1), the parameter, ε_t is generated from uniform distribution of [0,0.4]. In the second case (Case 2), ε_t is generated randomly from uniform distribution of [0,0.8].

The performance with respect to demand uncertainty is quantified by two performance measures. The first performance measure is service level, which is defined as the percentage of demands that are met from stock. Since the ordering policy is assumed to be fixed by contract, the service level of the upstream supplier will be 100%. Thus, only the service level of the downstream facility is considered. The second performance measure compares the actual ordering/setup and inventory-related cost of the downstream facility. To this regard, $Dev^{FE} = (Z_H^{FE} - Z_{HLH-DP}^{FE}) / Z_{HLH-DP}^{FE} \times 100$ is employed as the second performance measure where Z_{HLH-DP}^{FE} is the actual cost incurred by the downstream facility when the policy generated by HLH-DP is used, and Z_H^{FE} is the actual cost of the downstream facility when the policy obtained by the one of the heuristics, *H*, is used. Table 4.15 summaries the results on mean service level and Dev^{FE} for the heuristics tested.

Table 4.15
Mean service level and mean cost deviation from HLH-DP

Actual Demand Variation	0%-40% (Case 1)		0%-80% (Case 2)	
	Mean Service Level	$\overline{Dev^{FE}}$	Mean Service Level	$\overline{Dev^{FE}}$
HLH-DP	96.02%		92.39%	
HLH-SM	96.19%	11.83%	92.73%	8.94%
SP-WW	95.87%	3.12%	92.10%	4.09%
SP-KCC	96.04%	3.30%	92.43%	2.63%
SP-MS	96.01%	2.33%	92.37%	2.00%

From the experiment, we found that all the heuristics provide similar service levels. Also, the mean service levels of the heuristics tested are larger than 90%. In case 1, the service levels range from 95.87% to 96.19; while in case 2, the service levels are in the neighborhood of 92%. It is interesting to observe from Table 4.15 that, on average, HLH-DP provides the lowest actual cost while HLH-SM has the highest actual cost for both cases 1 and 2.

4.7. Conclusion

In this chapter, we study the problems of coordinating serial and assembly inventory systems with private information. The significance of this study is that we assume that the objective function and cost parameters of each facility are regarded as private information that no other facilities in the system have access to; and high quality solution is obtained through series of negotiation. Here, negotiation protocols in which the suppler and buyer negotiate on the order schedule and the corresponding monetary compensation due to deviations from their corresponding noncoordinated optimal schedules are developed for two-echelon serial and assembly system. Based on the proposed negotiation protocol, we develop a hierarchical Lagrangian-based decomposition methodology to coordinate a supply chain inventory system where the negotiations are sequentially invoked from the facility at the bottom to those at the top. The Lagrange multipliers are updated using the information obtained through negotiation. Computational results on 48 problem sets with 100 problems each showed that the performance of HLH is comparable to the existing single-pass heuristics that employ cost modification techniques. Also, the computational results showed that the untruthful behavior does not have significant effects on the solution quality of HLH. Finally, when testing the robustness of the heuristics due to forecasting error, we found that HLH-DP incurs the lowest actual cost while the service level of each heuristic is about the same.

CHAPTER V
CONCLUSIONS AND FUTURE RESEARCH

5.1. Conclusions

This dissertation investigates the coordination of supply chain inventory systems with private information; specifically, the single-warehouse multi-buyers system, single-vendor multi-buyers system, serial system and assembly system with private information. The objectives of this research are: (a) to develop coordination methodologies for supply chains characterized by private information, and (b) to investigate the effect of private information on the performance of supply chains.

To achieve these objectives, different supply chain structures are studied and heuristics are developed with private information.

In Chapter II, a simple two-echelon distribution system, single-warehouse-multi-buyers system, is studied. Specifically, we employed power-of-two inventory theory and developed an interaction/negotiation framework, SWMB-PI for coordinating the inventory system with private information. In addition, a heuristic, SWMB-GI, is developed for coordinating the system with global information. We demonstrate that critical system information can be recovered through negotiation with monetary compensation; and thus there is no loss in performance of SWMB-PI when compared to SWMB-GI.

In Chapter III, single-vendor multi-buyers system with private information is studied. Two power-of-two nested and stationary policies are developed. The first policy, termed common replenishment period policy (CRPP), assumes that all the buyers must replenish simultaneously. The second policy, termed asynchronous replenishment period policy (ARPP) is a more general case where the common replenishment assumption is relaxed. Heuristics for finding these policies with private information, namely CRPP-PI and ARPP-PI, are developed. In addition, this study provides another example that that critical system information can be recovered through negotiation with monetary compensation; and thus it is viable, under private information environment, to develop heuristics that perform as good as heuristics for the global information environment.

Chapter IV addresses the problem of coordinating multi-echelon serial and assembly inventory systems. Using scalable node-model, a hierarchical Lagrangian-based decomposition methodology where the negotiations are sequentially invoked from the facility at the bottom to those at the top is developed with private information. Furthermore, the effect of the private information on the performance of the supply chain is tested by computational experiments. When testing the proposed heuristic against existing heuristics that utilize global information, the experimental results show that the proposed methodology provides comparable results. When testing the robustness of the heuristics due to forecasting error, we found that the proposed heuristic incurs the lowest actual cost while the service level of each heuristic is about the same.

To summarize, this research contributes to the supply chain inventory society in that

(a) The development of an interaction/negotiation procedure for distributed system with constant demand rate under private information environment.

(b) The development of a scalable interaction/negotiation procedure for two adjacent echelons with time-dependent demand under private information environment.

(c) Demonstrate on

 (i) information can be separate such that each facility will make use of the available information for decision-making; and

 (ii) information required for implementing the proposed solution approach can be recovered through negotiation.

5.2. Future research directions

In this section, we outline the following possible future research direction.

5.2.1. General supply chain inventory system with private information

In this research, special supply chain configurations are studied. A possible research direction is to extend the current model to general supply chain inventory system with time-dependent demands.

In a serial/assembly system, the coordination can be decomposed such that the proposed interaction/negotiation framework can be applied to each supply-buyer pair. However, in general supply chain inventory system, negotiation may involve more than two facilities. In this case, a facility's change in cost is a result of the combined effects of two or more interacting facilities. As such, the facility may not be able to determine the appropriate monetary compensation attributable to each interacting facility. To evaluate whether it is possible to extend our model to general supply chain configurations, we may need to develop different allocation schemes that allocate the monetary compensation to each interacting facility and test their corresponding performance.

5.2.2. Capacity constraints

This dissertation only addresses the uncapacitated supply chain inventory system. However, facility may have scarce capacity and thus the problem must be modeled with the capacity constraints. Due to the capacity limitation, the infeasibility of an upstream facility will ruin the solutions of all the facilities in the lowest echelon. For the system with private information, the feasibility restoration is a real challenge because no single facility has global information of the system.

5.2.3. Stochastic demands

The studies in this dissertation deal with the system with known demands; however, stochastic models provide better representations of the real world. In Chapter IV, a simple experiment for testing the quality of the solution of our heuristic under stochastic demands is performed, and the solutions of deterministic model may be an approximation for the stochastic demands case. However, better understanding of coordinating inventory system under stochastic demands will be obtained by developing formal stochastic models.

Extending the proposed models for stochastic demands case is challenging. It is because there will be backlog or shortage under stochastic environment, which is not considered in our deterministic model. Also, since the success of the proposed models depend on exchanging information on compensation, it may not be easy to determine the compensation under stochastic environment.

5.2.4. Work-in-process inventory and variable lead times

In this research, though work-in-process inventory system is modeled in the proposed node-model, it is disregarded because the production is assumed to be produced in batch and the production lead time is constant. However, work-in-process may have a significant impact on the inventory policy if these assumptions are violated. Therefore, it is necessary to extend the methodologies developed in this research to address the work-in-process inventory explicitly.

One of the difficulties of addressing the work-in-process inventory system is that that the inclusion of work-in-process inventory system will increase the complexity of the node-model. Also, if the production lead time is not constant, the behavior of the inventory level of the work-in-process, and thus the related inventory systems, will be complicated.

5.2.5. Variable lead times

When studying the supply chain systems in this research, production lead time and delivery lead time is assumed to be constant. However, there are always options for the facility to minimize lead time. For example, production lead time can be shortened by using a faster machine at a higher cost; and delivery lead time can be shortened by choosing different carrier services at different cost.

Extending the models to consider variable lead times provides an interesting challenge to the researcher. For example, by allowing the options of using different machines at different cost, we may have an embedded scheduling problem. Including the carrier services options will also increase the complexity of the problem significantly. Besides, questions on whether the facilities can negotiate on what delivery mode must be address under private information environment. Therefore, the effect of variable lead times is worth investigating.

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APPENDIX

A.1. The Constrained-K Method (KCC) (Blackburn and Millen 1982)

Define K_i and e_i as the setup/ordering cost and echelon holding cost of inventory unit *i*; and $\hat{K}_i$ and $\hat{e}_i$ as the revised setup/ordering cost and echelon holding cost of inventory unit *i*. Also, define $B(i)$ as the set of indices of immediate predecessors of inventory unit *i*, and $a(i)$ as index of the immediate successor of inventory unit *i*. KCC adjusts the setup cost and holding cost for each inventory unit as follows.

$$\hat{K}_i = K_i + \sum_{j \in B(i)} \frac{\hat{K}_j}{k_j} \tag{A1}$$

$$\hat{e}_i = e_i + \sum_{j \in B(i)} \hat{e}_j k_j \tag{A2}$$

where

$$k_j = \max\left\{ \frac{\hat{K}_j}{K_{a(j)}} + \frac{e_{a(j)}}{\hat{e}_j}, 1 \right\} \tag{A3}$$

A.2. McLaren's Setup Cost Adjustment Method (MS) (McLaren 1976)

Define h_i as the holding cost of inventory unit *i*, MS adjusts only the setup/ordering cost but not the holding cost and the adjustment procedure is as follows.

$$\hat{K}_i = K_i + \sum_{j \in B(i)} \frac{K_j}{k_j} \tag{A4}$$

where

$$k_j = \frac{K_j}{K_{a(j)}} + \frac{h_{a(j)}}{h_j}.$$

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