AN EIGHTH GRADE CURRICULUM INCORPORATING

LOGICAL THINKING AND ACTIVE LEARNING

A Thesis

by

MARTA ANNA KOBIELA

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2006

Major Subject: Mathematics

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Approved by:

Chair of Committee,   Philip Yasskin
Committee Members,   Susan Geller
                    Christine Stanley
Head of Department,   Albert Boggess

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ABSTRACT

An Eighth Grade Curriculum Incorporating Logical Thinking and Active Learning.

(August 2006)

Marta Anna Kobiela, B.S., Texas A&M University

Chair of Advisory Committee: Dr. Philip Yasskin

With the increasing stress on teachers and students to meet and raise mathematics standards in schools, especially in the secondary level, the need for strong curricula and supporting materials for teachers has grown. A good curriculum, however, must do more than align with state standards and teach to the state exams; it must encourage students to enjoy mathematics. In an effort to help ease the plague of math anxiety, this thesis presents an eighth grade curriculum, called MathTAKStic, not only directly aligning with the Texas state standards, the Texas Essential Knowledge Skills (TEKS), but also encouraging students to pursue higher level thinking through active learning and logical thinking. To test the curriculum and find out its usefulness, several lessons were taught at a middle school.   Although the scores of those learning with the curriculum were not always better than others, MathTAKStic led to a greater increase in students' performance compared to those who were not exposed to the lessons, an increased interest in math and a plethora of ideas for the future. These results were concluded based on a comparison of students' scores from the previous year to the current year on the Texas standardized test. Overall, the increase in passing scores of MathTAKStic students preceded other classes in the same school.

To my mother and father

ACKNOWLEDGMENTS

I would like to thank my committee chair, Dr. Yasskin, for his unending patience and support, and my committee members, Dr. Geller and Dr. Stanley, for their guidance throughout this learning experience.

Thanks also to Dr. Johnson and Dr. Klemm from Texas A&M's NSF GK-12 Program for their support and willingness to assist in my data collection. Moreover, I owe many thanks to Deborah Parker, my mentor teacher at Jane Long Middle School, for her assistance and encouragement and for all she taught me during my tenure in the GK-12 Program. And, of course, I could not have succeeded without the support of my friends and family.

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INTRODUCTION

This thesis presents a curriculum for eighth grade mathematics. Several of the lessons of the curriculum and their supplemental materials are contained in the appendix. The full set of lessons will appear on the following websites: http://peer.tamu.edu and http://www.math.tamu.edu/outreach/mathtakstic. The lessons focus strongly on active learning. Several also encourage students to engage in logical thinking. The usage of the lessons and the results are discussed later in the thesis.

Two years ago, I began working for Texas A&M University's NSF-funded GK-12 program. Such programs have been created at many institutions across the nation, hiring graduate students to work in elementary and secondary math, science, engineering and technology classrooms. The graduate students serve as resources to the teachers, providing updated content to the classroom and helping to harness an enriched environment through the creation of lessons, labs, after-school programs and much more. The graduate fellows also act as role models for the students, dissolving many falsely held myths and beliefs about the character of scientists and mathematicians. The GK-12 program exists as a method to improve students' attitudes towards scientific fields and to prove to them that they, too, can be mathematicians, engineers, and scientists.

Through my own experiences as a Resident Mathematician at a high-stakes

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This thesis follows the style of *Journal for Research in Mathematics Education*.

middle school, I soon came to understand first-hand the stress and difficulties teachers face. As part of our program at Texas A&M University, we were encouraged to create lessons and activities that supported inquiry-based learning. Understanding how to foster inquiry in the science classrooms soon made sense, but creating similar lessons for mathematics proved much more challenging. For science, the popular, visually stimulating, Discrepant Events quickly became a favorite of the Resident Scientists. These demonstrations introduce the students to bizarre phenomena, leaving them with the mystery of wondering how and why the action could have occurred. Students leave the classroom and brainstorm their own hypotheses to the question. In about one week, the teacher revisits the problem, asking the students for the solution, often with several eager answers. However, similar equally astonishing demonstrations, equipped with lights and explosions, are hard to come by in mathematics classrooms.

To make the task even more vexing, Texas teachers, like many others in the United States, experience a great deal of pressure from authorities demanding that state standardized test scores rise. Originating from the No Child Left Behind Act (U.S. Department of Education), schools feel increased pressure from Washington to ensure that the state standards are met. When these standards are not adequate for as few as four years (regardless of improvement), the school is subject to replacing staff or implementing a brand new plan of education (Stronger Accountability Questions). Many teachers fear their jobs are on the line based on their students' performance.

Texas administers the Texas Assessment of Knowledge and Skills (TAKS). This exam, given once a year in April, tests students in the major academic areas. The criteria

to cover, called the Texas Essential Knowledge and Skills (TEKS) is the defining outline of what teachers must minimally teach. In the spring of 2005, only sixty-one percent of Texas eighth graders passed the Mathematics TAKS as satisfactory (Texas Educational Agency). This implies not only that thirty-nine percent of the eighth graders have not mastered close to half of the content of the school year, but they will continue to struggle to greater degrees in future math courses.

Then, is learning in a more active, inquiry-based manner more conducive to students' performance? According to research, inquiry learning not only accommodates a diversified environment of learning styles, but also engages students intensively because they are central to the lesson. Inquiry lessons often create great group projects because they allow students to brainstorm and discuss all possible solutions (Youth Learn Initiative). Students are forced to ask, "Why?" with inquiry learning. They, thus, indirectly fine-tune their problem-solving skills in the process, all while become better debaters and teammates.

Preparation time for creating inquiry lessons is considerably long. This task is all the more daunting when one considers a teacher's busy schedule. Thus, with impending deadlines, emotional students, and plenty to grade, teachers often have to sacrifice a basic component of their classroom environment: their lessons. One might think that with the added pressure to improve, teachers would try new ideas. However, according to Schorr, Firestone, and Monfils (2003), the teachers they interviewed in New Jersey simply kept their old activities and lessons despite pressure from the standardized tests.

Another major important mathematical focus that can be easily skipped by teachers when creating lessons is that of reasoning and proof. Although such logical thinking is included in the TEKS, it is not heavily tested in the TAKS. When teachers are stressed for classroom time, this point is often tossed away. Since reasoning and proof are essentially mathematical thinking, strengthening this skill automatically makes the student a better mathematician.

Thus, through my own difficulties in creating such lessons, I decided to take on the challenge to create an eighth grade curriculum, MathTAKStic, that incorporated inquiry-based activities, active learning, logical reasoning and proof. Although there are a plethora of great mathematics lessons that cover this range of thinking, many do not apply to the content the students must learn for their TAKS test. I knew then that my curriculum must follow the TEKS and that reasoning and proof must be inserted into as many lessons as possible so that it would not be skimmed over later.

One other prominent question one might ask is why create a new curriculum when other well-established curricula exist. The answer is to make the materials easier to use. The more the lessons are correlated to the Texas state standards (which are presently more significant for Texas teachers than national standards) the easier it will be for a teacher to implement them in her/his classroom. Some districts have strict layouts of what to teach and for how long, so a curriculum that is molded to Texas will prove easier to use.

In creating the MathTAKStic lessons, I am following a theme to ensure that inquiry-based learning or logical thinking exists at some level. The students begin each

lesson by **Understand**ing a problem or question that the teacher poses. After their interest is peaked, they **Investigate** the possibilities. This portion of the lesson is often lab-like and entails the students generating definitions and hypotheses as to what the answer could possibly be. Once an idea is formed, the students then **Discover** the solution by deriving a formula or proving a theorem. Finally, the students must **Apply** the knowledge. Often this entails practicing problems with the "tool" they developed, or working in groups on an in-class project.

I have tried to incorporate ample detail into the curriculum. Many of the materials are included and vivid descriptions, including questions to pose to students, are given. In a sense, MathTAKStic tries to embody not only lessons for students, but also for the teacher who will use them. All lessons include practice TAKS-like questions that the teacher may assign for homework. Additionally, all lab worksheets and supplementary materials are included.

BACKGROUND

As the United States wages global battles in technology and industry, politicians are putting increased pressure on improving the country's mathematics education. However, before immediately looking for solutions, one must step back and look at the underlying issues affecting children's learning of math. Many students today experience what experts have coined "math anxiety." Ashcraft (2002) defines math anxiety "as a feeling of tension, apprehension, or fear that interferes with math performance" (p. 181). Those plagued with this disease not only perform poorly in math, but consequently, they also avoid it both consciously and unconsciously (Ashcraft, 2002). Math anxiety has many causes. A lack of understanding and poor self-esteem are common sources (Fiore, 1999). Additionally, technical aspects surrounding math assessment, such as the stress on correctness and strict timing of tests, can lead to students' unease (Harper and Daane, 1998). Unfortunately, the disease is contagious, and often spreads from teachers and parents to students. Much math anxiety results from the way math is taught rather than the content itself (Fiore, 1999). Thus, by improving teaching, a great deal of the math anxiety epidemic could be cured.

However, with many teachers also affected by math anxiety, focus must also lie on helping them overcome their own fears. Teachers who are math anxious (or might have been taught math by such individuals) tend to teach more traditionally, through a lecture-style. As Harper and Daane (1998) note in their research, to reverse such habits, teachers require adequate training and knowledge. Specifically, they observed significant improvement in preservice teachers' attitudes towards mathematics after a

methods course that emphasized group work and active learning. However, with or without such professional development, teachers require ample materials, with games, activities and plenty of manipulatives for students to use (Harper and Daane, 1998). Black (2004) advises schools to give new teachers curriculum resources that include assessments and materials.

To add another dimension of stress, teachers and students have the burden of standardized testing and its consequences weighing upon them. Reys, Reys, Lapan, Holliday and Wasman (2003) found that by using curriculum materials focused around mathematics national standards, schools improved their performance on statewide tests. Thus, with an increased focus on aligning curricula to standards, students and teachers will increase their performance, resulting in improved attitudes.

Obviously, standards-based curricula, rich with materials, are essential to helping improve mathematics attitudes and performance. To further increase likelihood of improvement, Fiore (1999) stresses the importance of helping teachers and students understand why mathematical concepts are true. By encouraging explanation and avoiding shallow memorization, students will feel more comfortable with the material. Active learning, which includes discovery learning, hands-on learning, and problem-based learning, allows students to become personally engaged in the content. Students tend to better understand reasons behind the techniques they use. Inquiry or discovery learning helps students connect with their material. In this method, students find out for themselves how the mathematics works, rather than reading about it in a textbook (Tress, 1999). Problem-based learning is a form of discovery learning in which the students

learn by focusing on a specific problem. Cerezo (2004) notes that problem-based learning helps students become more independent, thus increasing confidence. Moreover, active learning is often group oriented, and the nurturing of peer relationships can increase mathematical performance (Cerezo, 2004).

A curriculum with plenty of teacher support, explanation and resources and one that encourages active learning and logical thinking can ease students' anxieties in mathematics, thus improving their performance. MathTAKStic, with its included emphasis on the Texas state standards, incorporates all these elements with the hope of improving eighth graders' TAKS scores and their attitudes towards mathematics.

Despite all the positive points about MathTAKStic, certain barriers are unavoidable. Just making curriculum materials, without supplemental support, may not be sufficient. As Stonewater (2005) points out, effective teacher preparation is essential for mathematics teaching. More specifically, by influencing a teacher's philosophy and understanding of mathematics, one can directly affect his or her teaching methods. Reys, Reys, Lapan, Holliday and Wasman (2003) also emphasize the need for professional development to effectively implement curriculum.

Moreover, implementing higher-level thinking in eighth grade after so many years of algorithmic math and math anxiety is challenging. The students are expected to perform cognitively differently. Researchers have found that longer implementation of good curriculum can lead to greater improvement in the students and fix many of these issues (Reys, Reys, Lapan, Holliday and Wasman, 2003). Since this project has a short time-frame, its results may be skewed.

PROCEDURE

The MathTAKStic curriculum consists of a collections of lessons grouped into units of related topics. Several lessons appear in Appendices B1 through B16. Supplemental materials that are used in multiple lessons are featured in B0. The full set of lessons will be posted on the websites, http://peer.tamu.edu and http://www.math.tamu.edu/outreach/mathtakstic. A few lessons reteach prior information, covering sixth or seventh grade TEKS. Most lessons teach one or more of the eighth grade TEKS. The eighth grade TEKS are listed in Appendix A1 and the covered sixth and seventh grade TEKS are covered in Appendix A2. The eighth grade lessons and their corresponding TEKS are shown in Figure 1. The reteach lessons and their TEKS are in Figure 2. Most lessons were original or developed in collaboration with my advisor and my mentor teacher at the middle school. The former suggested ideas while the latter gave input as to how the lessons would function in a real classroom. A few lessons have borrowed ideas which have been revised and adapted to fit into MathTAKStic. Their contributors are footnoted underneath Figure 1.

Each lesson covering eighth grade material consists of the following four parts to emphasize discovery learning: understanding the problem, investigating the possibilities, discovering the connections, and applying the knowledge. For example, in the Pythagorean Theorem lesson, the teacher begins by asking the students the answer to a real-world math problem. The problem involves understanding the Pythagorean Theorem although the students have yet to realize that. After generating curiosity, the

TEKS Used in Lessons

	1a,b c,d	2a,b c,d	3a,b	4a	5a,b	6a,b	7a,b c,d	8a,b c	9a,b	10a, b	11a, b,c	12a, b,c	13a, b	14a, b,c,d	15a. b	16a, b
Unit 0																
1. (B1) An Introduction to Problem-Solving														**b,c**		
Unit 1																
6. Decimals: Real-Life Applications	**b**	a,b												b,c		
8. Fractions: Multiplying & Dividing in Real Life Applications	**b**	a,b												b,c		
9. Fractions: Adding & Subtracting in Real Life Applications	**b**	a,b												b,c		
10. Ordering Between Fractions, Decimals & Percents	**a**				a									b,c		

Figure 1. TEKS used in lessons.

Bold letter indicate TEKS which are the primary focus of the lesson.
Those lessons which appear in Appendix B are indicated in parentheses.

This lesson was influenced by an activity developed by the NSF GK-12 Program, found at http://peer.tamu.edu/DLC/NSF_Resources.asp

	1a,b c,d	2a,b c,d	3a,b	4a	5a,b	6a,b	7a,b c,d	8a,b c	9a,b	10a, b	11a, b,c	12a, b,c	13a, b	14a, b,c,d	15a. b	16a, b
Unit 2																
11. Understanding Patterns																a
12. Working with Algebraic Expressions of Sequences◊*				a	b									a,b,c		a
13. Using Formulas & Equations					a									b,c		
14. Solving Algebraic Equations⊥														b,c		
Unit 3																
16. Statistics of Data	a	a,b										a,c		b,c	b	
17. Plotting Data	a	a		a								b,c		b,c		
18. Scientific Notation	d													b,c		
19. Evaluating Data	a												a,b	b,c		

Figure 1. Continued

◊ This lesson was influenced by a similar lesson or activity by Dr. Philip Yasskin
* Blue lessons emphasize logical thinking.
⊥ This lesson was influenced by a similar lesson by Deborah Parker.

	1a,b c,d	2a,b c,d	3a,b	4a	5a,b	6a,b	7a,b c,d	8a,b c	9a,b	10a, b	11a, b,c	12a, b,c	13a, b	14a, b,c,d	15a. b	16a, b
Unit 4																
20. Sizing Up the Human Body: An Investigation into Proportions		d	a				b							a,b,c		
21. Understanding Proportions	b		a											b,c		
22. Percents as Proportions	b													b,c		
23. Similar Shapes		d							b					b,c		
24. (B5) Applications of Similar Shapes	b	d	b						b					a,b,c		
Unit 5																
25. Squares and Square Roots⊥	c													b,c		
26. Estimating Square Roots	c													b,c		
28. (B6) The Pythagorean Theorem*	c			a			c							b,c		a,b

Figure 1. Continued

⊥ This lesson is influenced by a similar lesson by Deborah Parker.

* Blue lessons emphasize logical thinking.

	1a,b c,d	2a,b c,d	3a,b	4a	5a,b	6a,b	7a,b c,d	8a,b c	9a,b	10a, b	11a, b,c	12a, b,c	13a, b	14a, b,c,d	15a. b	16a, b
29. (B7) Applications of the Pythagorean Theorem	c								a					a,b,c		
Unit 6																
30. An Expedition in the Coordinate Plane				a			**d**							b,c		
31. Dynamic Dilations		d				**a,b**	b							b,c	a	a
32. (B8) Terrific Translations*				a		**b**								b,c	a	a
33. (B9) Radical Reflections*		b		a		**b**								b,c	a	a
Unit 7																
34. (B10) Architecture 101: An Adventure in 3-D Visualization							**a,**b							a,b,c		
35. Surface Area of Prisms*								**a,**c						b,c		
36. Surface Area of Pyramids*								**c**						b,c		

Figure 1. Continued

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* Blue lessons emphasize logical thinking.

	1a,b c,d	2a,b c,d	3a,b	4a	5a,b	6a,b	7a,b c,d	8a,b c	9a,b	10a, b	11a, b,c	12a, b,c	13a, b	14a, b,c,d	15a. b	16a, b
37. Surface Area of Cylinders*								**a**,c						b,c		
38. Surface Area of Cones*								c						b,c		
Unit 8																
39. Volume of Prisms*								**b**						b,c		
40. (B11) Volume of Pyramids*		b		a				**b**						b,c	a	a,b
41. (B12) Volume of Cylinders*		b		a				**b**						b,c	a	a,b
42. (B13) Volume of Cones*		b		a				**b**						b,c	a	a,b
Unit 9																
43. Change in Perimeter*				a						**a**				b,c		a,b
44. Change in Area*				a						**a**				b,c		a,b

Figure 1. Continued

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* Blue lessons emphasize logical thinking.

	1a,b c,d	2a,b c,d	3a,b	4a	5a,b	6a,b	7a,b c,d	8a,b c	9a,b	10a, b	11a, b,c	12a, b,c	13a, b	14a, b,c,d	15a. b	16a, b
45. Change in Volume*				a				c		b				b,c		a,b
Unit 10																
47. Independent Probability											a,b			b,c		
48. Dependent Probability											a,b			b,c		
Unit 11																
49. Estimation		c	b		a			c						b,c,d		

Figure 1. Continued

TEKS Used in Reteach Lessons

	6.1a	6.9b	7.1a,c	7.2e	7.9a
Unit 1					
2. (B2) Integers: Ordering	a		a		
3. (B3) Integers: Adding & Subtracting			c		
4. (B4) Integers: Multiplying & Dividing			c		
5. Decimals: Ordering	a				
7. Fractions: Ordering	a				
Unit 2					
15. Order of Operations				e	
Unit 5					
23. Area of 2-Dimensional Shapes[*]					a
Unit 10					
46. Simple Probability		b			

Figure 2. TEKS used in reteach lessons.

Those lessons which appear in Appendix B are indicated in parentheses.

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[*] Blue lessons emphasize logical thinking.

class investigates possibilities for the solution through an experiment. They measure the side lengths of right triangles, plotting the data in a table, and computing their squares (as recommended by the teacher). To discover connections, the class interprets the data, looking for patterns between the squares of the side lengths of the right triangles. Eventually, the class states the Pythagorean Theorem. This discovery should be followed by a proof of why this theorem is always true, so as to help students generalize their findings. Two hands-on proofs are provided, both involving cutting and pasting of geometrical shapes. An additional algebraic approach is given for the Algebra I students. Finally, the students practice applying their new theorem through additional problems.

On the other hand, those lessons that reteach TEKS from sixth or seventh grade have a different format. The lessons are divided into two parts: refresh and practice. Rather than focus on inquiry, the refresh portions are geared to hands-on learning, to help reinforce concepts. For the practice part, supplemental materials, in the form of games or activities, are included within each lesson to help students master the content.

It should be noted that one of the eighth grade TEKS standards has remained uncovered in the curriculum. It is TEKS 11(c). This TEKS was unclear and had no obvious TAKS questions to follow. Additionally, all the lessons (except Evaluating Algebraic Equations) focus on teaching specific TEKS. However, many lessons review or use material from other TEKS. Those TEKS that are primary for each lesson are in bold in Figure 1. Evaluating Algebraic Equations does not teach an eighth grade TEKS

since it is typically covered in Algebra I. This lesson is included since many TAKS questions require the ability to solve simple algebraic equations.

Although the units suggest an ordering of the lessons, they may be done in different orderings. The flow chart in Figure 3 illustrates which units are necessary predecessors for other units. In Figure 3, the reteach lessons are marked inside of a rectangular, rather than oval box. Additionally, in all the figures, those lessons that focused on logical thinking are in blue and have an asterisk(*). All the lessons incorporated some form of active learning.

Flowchart of Lessons

Unit 1

An Introduction to Problem-Solving

Fractions: Ordering

Fractions: Adding & Subtracting in Real Life Applications

Fractions: Multiplying & Dividing in Real Life Applications

Decimals: Ordering

Decimals: Real-Life Applications

Ordering Between Fractions, Decimals & Percents

Integers: Ordering

Integers: Multiplying & Dividing

Integers: Adding & Subtracting

Unit 2

Using Formulas and Equations

Solving Algebraic Equations

Order of Operations

Understanding Patterns

Working with Algebraic Expressions

Unit 4

Unit 10

Unit 3

Statistics of Data

Plotting Data

Scientific Notation

Evaluating Data

Figure 3. Flowchart of lessons.

Unit 2

Unit 3

Unit 4

Sizing Up the Human Body: An Investigation into Proportions

Understanding Proportions

Similar Shapes

Percents as Proportions

Applications of Similar Shapes

Unit 5

Squares and Square Roots

Estimating Square Roots

Area of 2-Dimensional Shapes

Pythagoras's Theorem

Applications of the Pythagorean Theorem

Unit 7

Unit 6

An Expedition in the Coordinate Plane

Terrific Translations

Radical Reflections

Dynamic Dilations

Figure 3 Continued

Figure 3 Continued

IMPLEMENTATION, OBSERVATIONS, AND ASSESSMENT

In planning for this project, I intended to teach at least one lesson from MathTAKStic every week to my mentor teachers' regular eighth grade math class. In a large middle school such as Jane Long, the teacher does not have as much control of her schedule. The school and the district both administer benchmark tests to assess students' progress, and sometimes little notice was given. Additionally, teachers would have to shift their timing of lessons based on students' understanding. Being a fulltime graduate student, I could only come to Jane Long twice a week, and often lessons we had planned for me to teach had to be canceled. In the end, I taught 30) An Expedition in the Coordinate Plane, 28) Pythagorean Theorem, 24) Applications of Similar Shapes, 43) Change in Perimeter, 44) Change in Area, 45) Change in Volume, and 47) Independent Probability to either the eighth grade regular class or the eighth grade Algebra I class. Moreover, I adapted ideas of lessons I had helped Deborah teach to the eighth graders. These include 3) Integers: Adding and Subtracting, 4) Integers: Multiplying and Dividing, 16) Statistics of Data, and 25) Squares and Square Roots. In addition, I taught parts of 1) An Introduction to Problem-Solving, 15) Order of Operations, 23) Similar Shapes, and 32) Terrific Translations. The lesson, 12) Working with Algebraic Expressions of Sequences, was one I had taught several times in Texas A&M's summer math camp for middle school students.

Teaching the lessons was a greater challenge than expected. Last year I had only worked with seventh grade pre-AP classes and eighth grade Algebra I classes. When I planned to make this curriculum, I had those students in mind. Working with the eighth

grade regular class was a completely different matter. These students cared little about school. Enticing them to do work was difficult. They did not care about grades and showed little respect for teachers. They were not excited about playing games or doing active math. Even rewards had little impact. The entire first semester progressed in this manner. However, somehow, many of them matured during the winter break. Most of their attitudes improved by the beginning of the second semester, and they participated more in class, although it was still a challenge.

The Pythagorean Theorem lesson was the first lesson I taught to these students. This lesson took three full days (with ninety minute classes) to complete while I imagined it to take half the time. My mentor teacher taught the beginning part of the lesson, introducing the concept and measuring the triangles. The work load was too much for the students and they became frustrated and annoyed. I had severely underestimated the difficulty of the lesson. During my portion to teach, we made posters that illustrated how squaring the sides of right triangles could be seen as the area of squares. Initially, in my original lesson plan, the students were to have five triangles illustrated on their posters. Many students did not finish their posters in the ninety minute class. The best students in the class were annoyed with the work. Worst of all, the purpose of the activity, for the students to see why the theorem works, was lost in the frustration.

After this initial disaster, the following lessons were toned down. When teaching for the eighth grade regular class, I focused on simple active lessons. In the spring, when I taught the Applications of the Similar Shapes, the lesson went much more

smoothly. The lesson was shorter and required less of the students. They also enjoyed the adventure associated with using flashlights in a dark classroom. Some worked really well, but as always, several students refused to do any work.

The eighth grade regular class probably enjoyed the Independent Probability lesson the best. They loved doing math with the M&M® candies, and were incredibly focused during this portion. When we asked them to play the dice game with a partner, we offered additional rewards for the winner in each pair, a method that proved to work well. They were more focused for me than I had ever observed during a lesson. To understand these students required much more than understanding mathematics. Studying psychology and at-risk students in greater depth would have helped me create MathTAKStic.

Working with the eighth grade Algebra I students, although in the same grade and same school, was an entirely different situation. When we did the Change in Perimeter, Change in Area, and Change in Volume lessons, they responded enthusiastically and often had incredibly intuitive contributions. Games were always accepted eagerly and they loved the competition.

To assess how MathTAKStic affected the students, the eighth grade students' TAKS scores from last year when they were in seventh grade are compared to this year in eighth grade (see Table 1). Table 1 shows scores for those 17 students who were in the regular math class and those 37 students who were in the Algebra I class the entire year. These are also compared to the scores of all students at Jane Long. Although the MathTAKStic regular students have fewer passing scores than the school's overall

eighth grade, their growth from the previous year is dramatically better. Additionally, the school average accounts for the higher scores of those in the Pre-AP and Algebra I classes which contain students who are often gifted and talented. These students obviously pull the school average up considerably. Besides the regular students, many more Algebra I students improved their TAKS scores.

Table 1
Table of TAKS scores for Class versus Jane Long before and after MathTAKStic

	TAKS 2005 7th Grade		TAKS 2006 8th Grade	
Jane Long Middle School All Students	N= 304 students	42.0% passing	N= 350* students	48.0% passing
Regular Class with MathTAKStic	N= 17 students	12.0% passing	N= 17 students	35.0% passing
Algebra I Class with MathTAKStic	N= 37 students	81.1% passing	N= 37 students	94.6% passing

Of course, this improvement for all the classes influenced by MathTAKStic is not only a consequence of MathTAKStic, especially since it was not regularly implemented in their classroom. Other factors, such as the mentor teacher, supplemental instruction (tutoring), and home life all are essential in students' performance. To know exactly

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* Number of students is approximate

what affected each would take in-depth analysis of each student. Table 2 also provides a

breakdown of the eighth grade regular class based on gender and race.

Table 2
Table of breakdown of gender and ethnicity

	Male	Female	Hispanic	African-American	White
8th Grade Regular Class Percentage Passing	44	25	30	50	33

All of the students also took a pre-survey assessing attitudes and beliefs towards

mathematics. The survey, administered by the NSF GK-12 program, was given in

September. In April, the students were expected to take an equivalent post-survey.

Although the Algebra I students all completed this assessment, the regular eighth grade

class chose not to. The assessment of this data will appear in the future dissertation of

Shannon Degenhart and will provide extra insight into the effectiveness of

MathTAKStic.

Overall, MathTAKStic, after its simplification, proved enjoyable and effective in

the classroom. However, to have adequate proof attaining to its power, more research is

needed.

SUMMARY AND FUTURE RESEARCH

The MathTAKStic curriculum was created to help eighth grade teachers incorporate more active learning and logical thinking into their classrooms without sacrificing their focus on the TAKS test. MathTAKStic lessons cover most of the eighth grade TEKS plus a few of the sixth and seventh grade TEKS. Most of the materials required for the curriculum are included. Those lessons and materials not included in the appendix will be posted at http://www.math.tamu.edu/outreach/mathtakstic and http://peer.tamu.edu.

When a few of the lessons were taught at Jane Long Middle School, they became increasingly more popular with the students. Overall their TAKS tests scores improved from the previous year although many factors might have contributed to this phenomenon.

In creating the curriculum, several complications and areas for improvement became apparent. First of all, little research has been done illustrating the positive effects of logical thinking. Although many people believe that logical thinking is important, proving it has yet to be done. Such proof would support MathTAKStic's implementation. Work in this area might also contribute to the debate about what sort of mathematics should be taught.

Additionally, the research discussed earlier about teacher preparation proved to be a continuous barrier in writing the lessons. Often times, good ideas were not included because of their foreign nature to most middle school teachers, and those that were incorporated might be too challenging. For example, when I showed several

teachers at Jane Long my geometric proofs for the Pythagorean Theorem, they dismissed them immediately as too difficult and confusing. Without proper teacher preparation, many of the difficult concepts found in MathTAKStic might yield unease. A workshop for the curriculum would be ideal to have prior to its use.

Moreover, the curriculum was not used often enough or for long enough. Most of the eighth graders had already decided that math was too difficult and a waste of their time. Their background in elementary mathematics was so weak that working middle school math problems frustrated them. If active learning, such as MathTAKStic, began in an earlier year, as Reys, Reys, Lappan, Holliday and Wasman (2003) suggest, by the time students reach eighth grade, they would be more ready to handle its rigor.

Another difficulty in creating the curriculum was the way the TAKS test is formatted. Although the TEKS encourage higher level thinking, the TAKS test, in a multiple-choice format is not an appropriate assessment. Until a better assessment of logical thinking is encouraged, a curriculum encouraging such will not be embraced.

In all, the creation of MathTAKStic was an excellent catalyst for future research ideas, both directly and indirectly associated with its effectiveness for eighth grade classrooms.

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APPENDIX A

TEKS

Appendix A1: Eighth Grade TEKS

(1) Number, operation, and quantitative reasoning. The student understands that different forms of numbers are appropriate for different situations. The student is expected to:

>   (A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;
>
>   (B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;
>
>   (C) approximate (mentally and with calculators) the value of irrational numbers as they arise from problem situations (p, Ö2); and
>
>   (D) express numbers in scientific notation, including negative exponents, in appropriate problem situations using a calculator.

(2) Number, operation, and quantitative reasoning. The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:

>   (A) select and use appropriate operations to solve problems and justify the selections;
>
>   (B) add, subtract, multiply, and divide rational numbers in problem situations;
>
>   (C) evaluate a solution for reasonableness; and
>
>   (D) use multiplication by a constant factor (unit rate) to represent proportional relationships; for example, the arm span of a gibbon is about 1.4 times its height, $a = 1.4h$.

(3) Patterns, relationships, and algebraic thinking. The student identifies proportional relationships in problem situations and solves problems. The student is expected to:

>   (A) compare and contrast proportional and non-proportional relationships; and
>
>   (B) estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates.

(4) Patterns, relationships, and algebraic thinking. The student makes connections among various representations of a numerical relationship. The student is expected to generate a different representation given one representation of data such as a table, graph, equation, or verbal description.

(5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to:

> (A) estimate, find, and justify solutions to application problems using appropriate tables, graphs, and algebraic equations; and

> (B) use an algebraic expression to find any term in a sequence.

(6) Geometry and spatial reasoning. The student uses transformational geometry to develop spatial sense. The student is expected to:

> (A) generate similar shapes using dilations including enlargements and reductions; and

> (B) graph dilations, reflections, and translations on a coordinate plane.

(7) Geometry and spatial reasoning. The student uses geometry to model and describe the physical world. The student is expected to:

> (A) draw solids from different perspectives;

> (B) use geometric concepts and properties to solve problems in fields such as art and architecture;

> (C) use pictures or models to demonstrate the Pythagorean Theorem; and

> (D) locate and name points on a coordinate plane using ordered pairs of rational numbers.

(8) Measurement. The student uses procedures to determine measures of solids. The student is expected to:

> (A) find surface area of prisms and cylinders using concrete models and nets (two-dimensional models);

> (B) connect models to formulas for volume of prisms, cylinders, pyramids, and cones; and

(C) estimate answers and use formulas to solve application problems involving surface area and volume.

(9) Measurement. The student uses indirect measurement to solve problems. The student is expected to:

(A) use the Pythagorean Theorem to solve real-life problems; and

(B) use proportional relationships in similar shapes to find missing measurements.

(10) Measurement. The student describes how changes in dimensions affect linear, area, and volume measures. The student is expected to:

(A) describe the resulting effects on perimeter and area when dimensions of a shape are changed proportionally; and

(B) describe the resulting effect on volume when dimensions of a solid are changed proportionally.

(11) Probability and statistics. The student applies concepts of theoretical and experimental probability to make predictions. The student is expected to:

(A) find the probabilities of compound events (dependent and independent);

(B) use theoretical probabilities and experimental results to make predictions and decisions; and

(C) select and use different models to simulate an event.

(12) Probability and statistics. The student uses statistical procedures to describe data. The student is expected to:

(A) select the appropriate measure of central tendency to describe a set of data for a particular purpose;

(B) draw conclusions and make predictions by analyzing trends in scatterplots; and

(C) construct circle graphs, bar graphs, and histograms, with and without technology.

(13) Probability and statistics. The student evaluates predictions and conclusions based on statistical data. The student is expected to:

    (A) evaluate methods of sampling to determine validity of an inference made from a set of data; and

    (B) recognize misuses of graphical or numerical information and evaluate predictions and conclusions based on data analysis.

(14) Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:

    (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;

    (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;

    (C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and

    (D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.

(15) Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to:

    (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and

    (B) evaluate the effectiveness of different representations to communicate ideas.

(16) Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:

   (A) make conjectures from patterns or sets of examples and nonexamples;

   (B) validate his/her conclusions using mathematical properties and relationships.

Appendix A2: Sixth and Seventh Grade TEKS

(6.1) Number, operation, and quantitative reasoning. The student represents and uses rational numbers in a variety of equivalent forms. The student is expected to:

(A) compare and order non-negative rational numbers;

(6.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, and divides to solve problems and justify solutions. The student is expected to:

(A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers;

(B) use addition and subtraction to solve problems involving fractions and decimals;

(6.9) Probability and statistics. The student uses experimental and theoretical probability to make predictions. The student is expected to:

(B) find the probabilities of a simple event and its complement and describe the relationship between the two.

(6.10) Probability and statistics. The student uses statistical representations to analyze data. The student is expected to:

(B) use median, mode, and range to describe data;

(7.1) Number, operation, and quantitative reasoning. The student represents and uses numbers in a variety of equivalent forms. The student is expected to:

(A) compare and order integers and positive rational numbers;

(C) use models to add, subtract, multiply, and divide integers and connect the actions to algorithms;

(7.2) Number, operation, and quantitative reasoning. The student adds, subtracts, multiplies, or divides to solve problems and justify solutions. The student is expected to:

(B) use addition, subtraction, multiplication, and division to solve problems involving fractions and decimals;

(E) simplify numerical expressions involving orderof operations and exponents

(7.9)   Measurement. The student solves application problems involving estimation and measurement.

   The student is expected to estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.

APPENDIX B

LESSON PLANS

Appendix B0: Supplemental Materials

Appendix B0 contains supplemental materials that are used in more than one lesson. They are organized by the units in which they appear. These materials include the following:

- Unit 1: Integer Dots
  - Used in 3) (Appendix B3) Integers: Adding & Subtracting
  - Used in 4) (Appendix B4) Integers: Multiplying & Dividing
- Unit 5: Pythagorean Triangles
  - Used in 23) Area of 2-Dimensional Shapes
  - Used in 28) (Appendix B7) The Pythagorean Theorem
- Unit 6: Grid Paper
  - Used in 32) (Appendix B9) Terrific Translations
  - Used in 33) (Appendix B10) Radical Reflections
- Unit 6: Grid Paper 2
  - Used in 32) (Appendix B9) Terrific Translations
  - Used in 33) (Appendix B10) Radical Reflections
- Units 7 & 8: Prisms
  - Used in 35) Surface Area of Prisms
  - Used in 39) Volume of Prisms
- Units 7 & 8: Pyramids
  - Used in 36) Surface Area of Pyramids
  - Used in 40) (Appendix B14) Exploring the Volume of Pyramids
- Units 7 & 8: Cylinders
  - Used in 37) Surface Area of Cylinders
  - Used in 41) (Appendix B15) Volume of Cylinders
- Units 7 & 8: Cones
  - Used in 38) Surface Area of Cones
  - Used in 42) (Appendix B16) Exploring the Volume of Cones

Besides using these materials for these specific lessons, they are useful for many other classroom activities, as the teacher wishes.

The items in Appendix B0 have been reduced in size to fit into the margins of the thesis. Because of this, the proportions have been skewed and may not work accurately in the lessons, as intended.

MathTAKStic!
UNIT 1

integer
dots

1

2

3

4

5

6

MathTAKStic!
UNIT 6

grid
paper

MathTAKStic!
UNIT 6

grid
paper 2

1

I

2

2

1

I

1

1

2

2

3

3

1

2

3
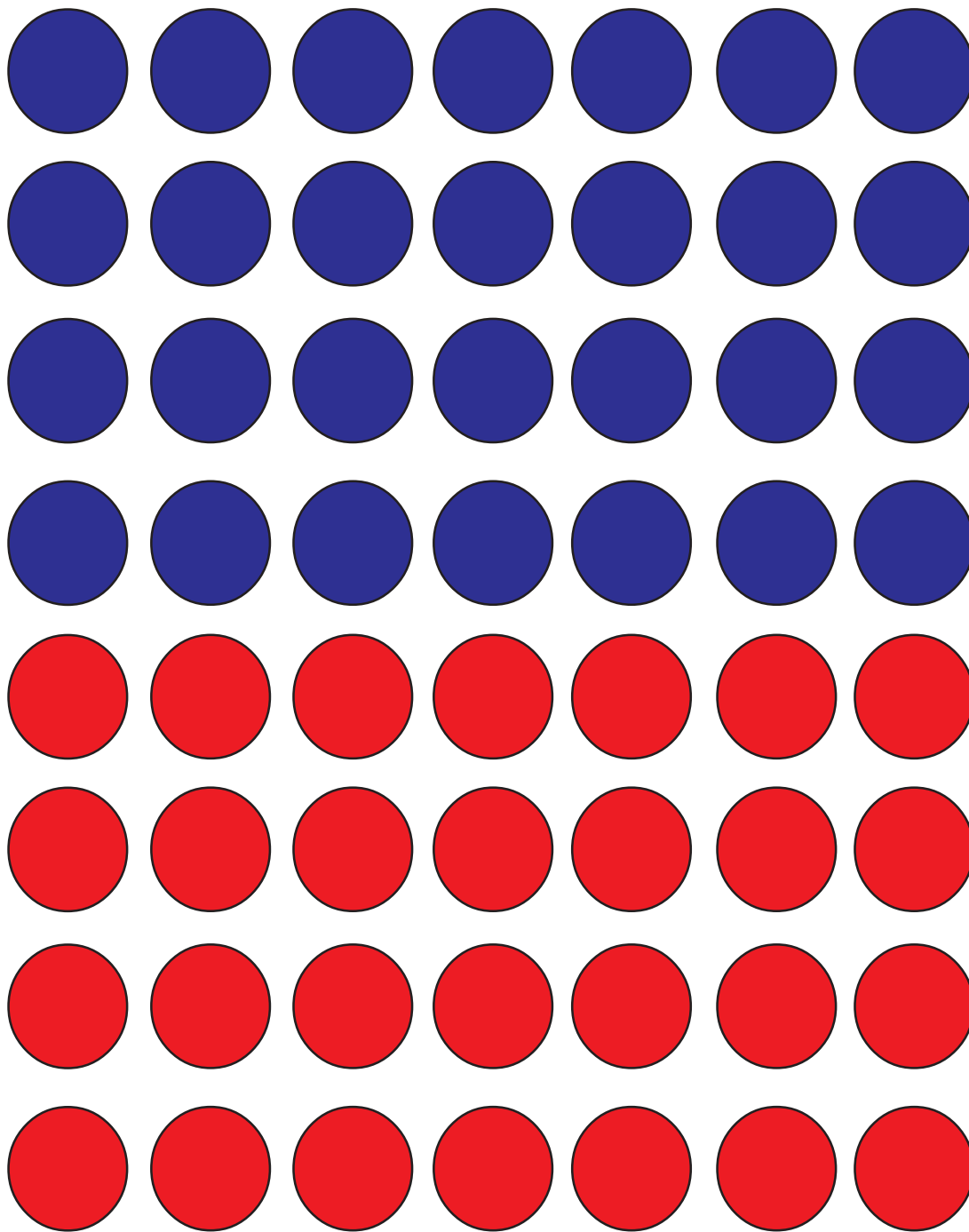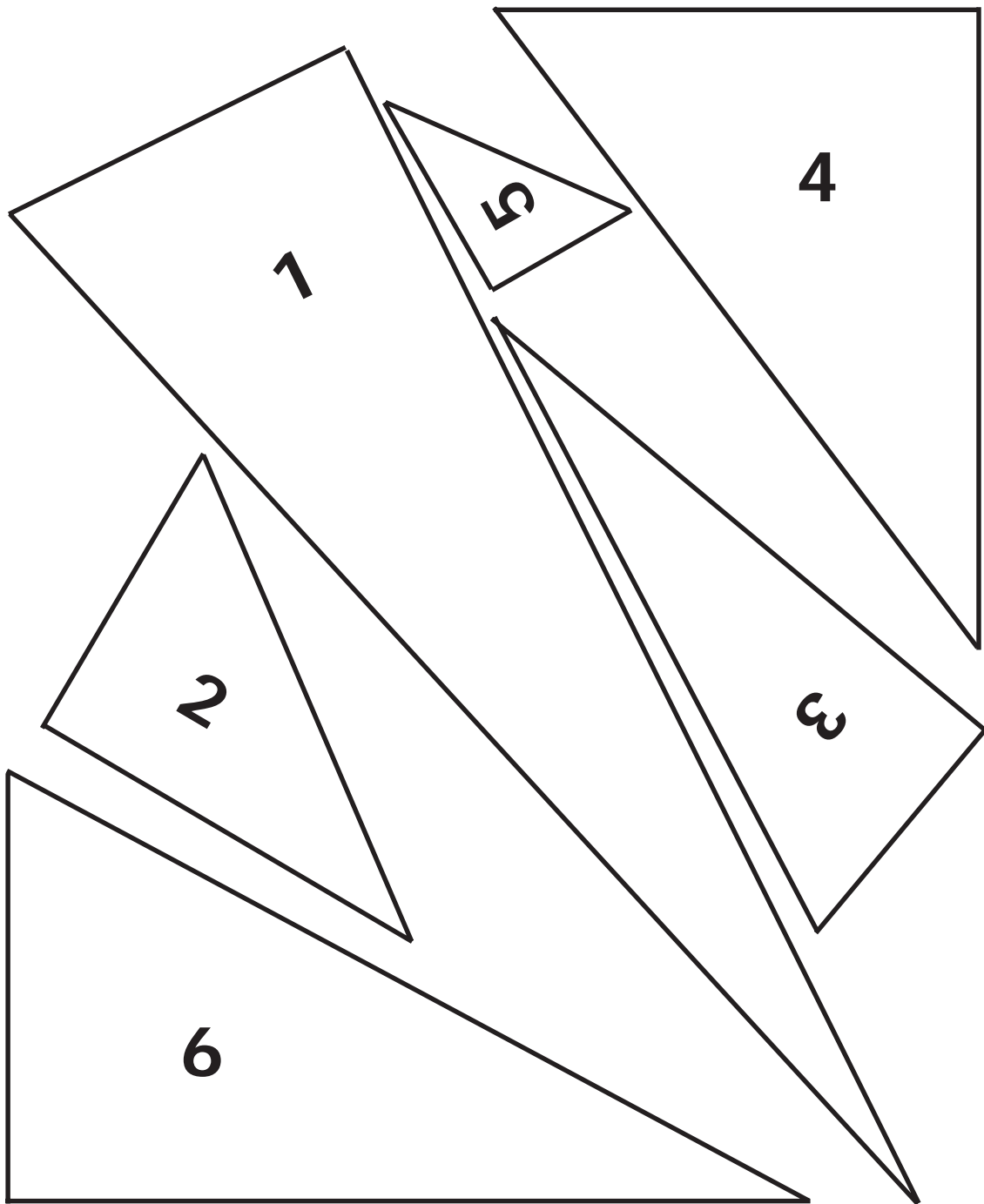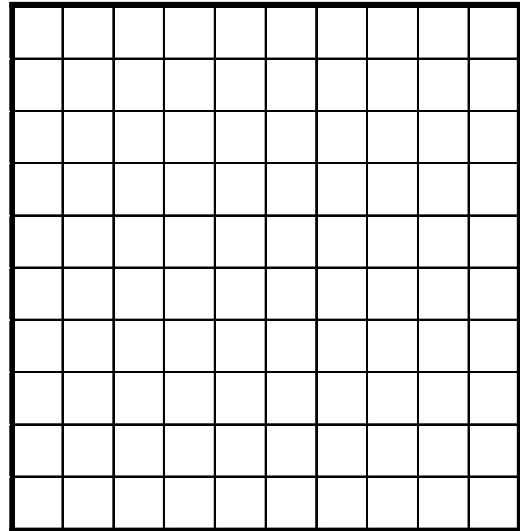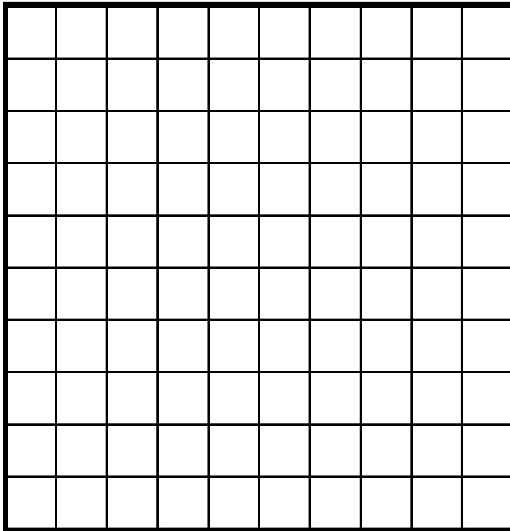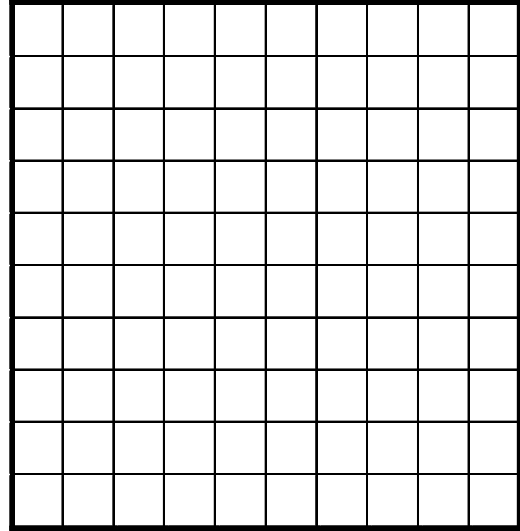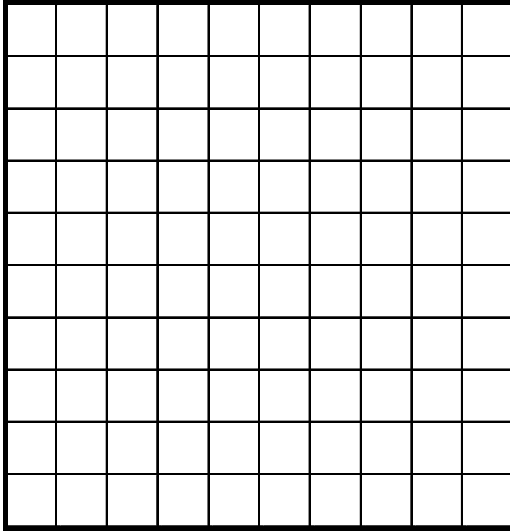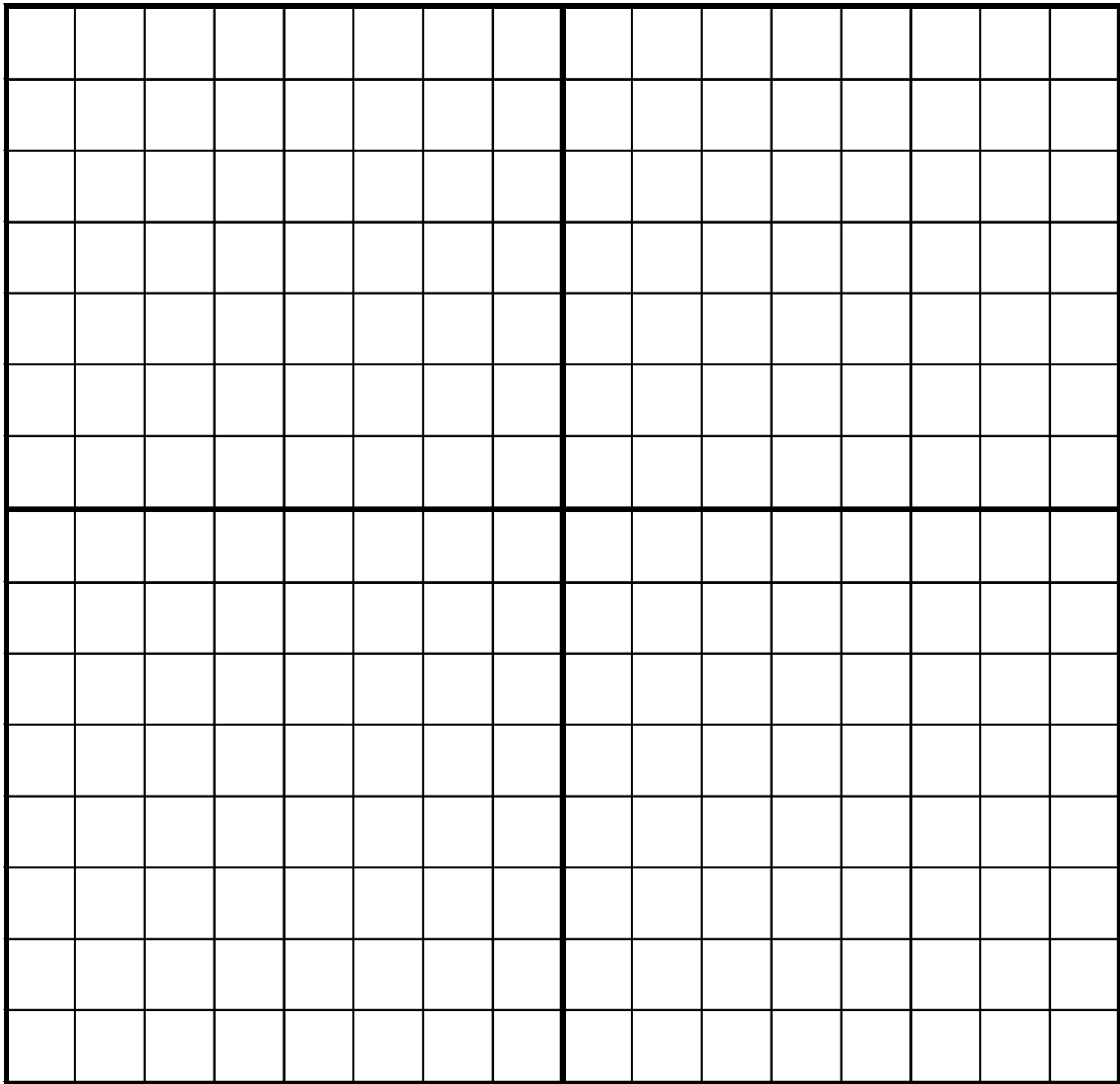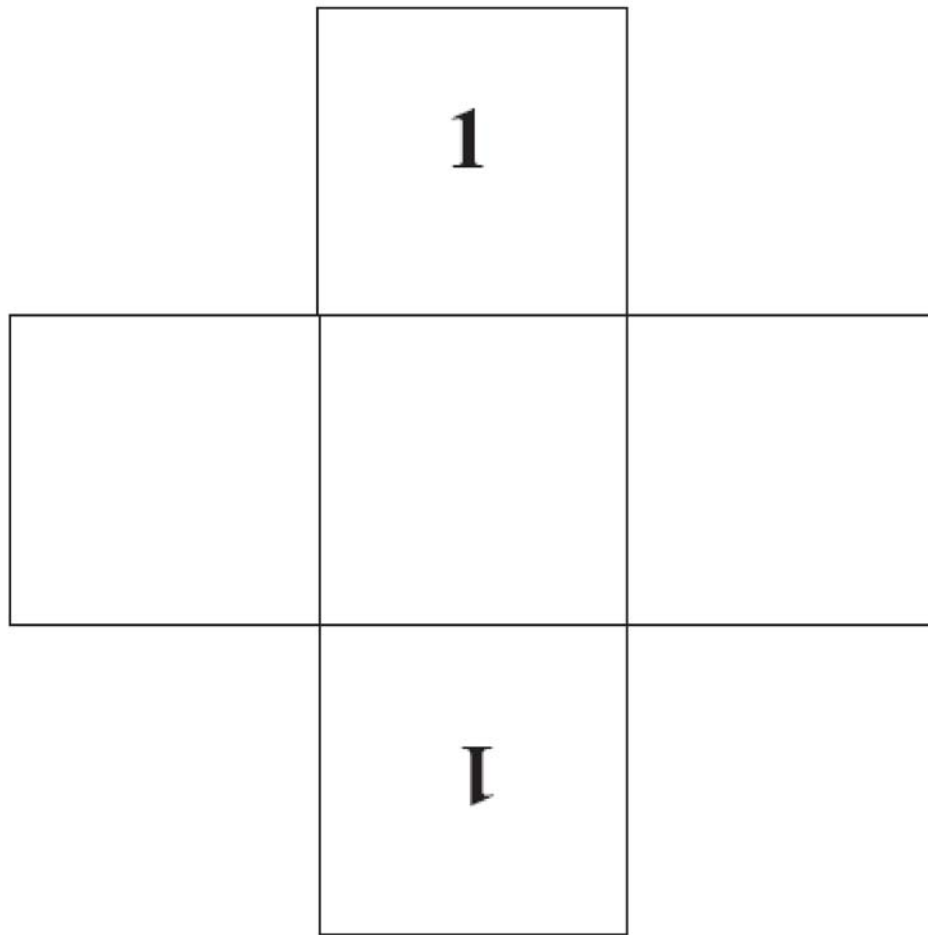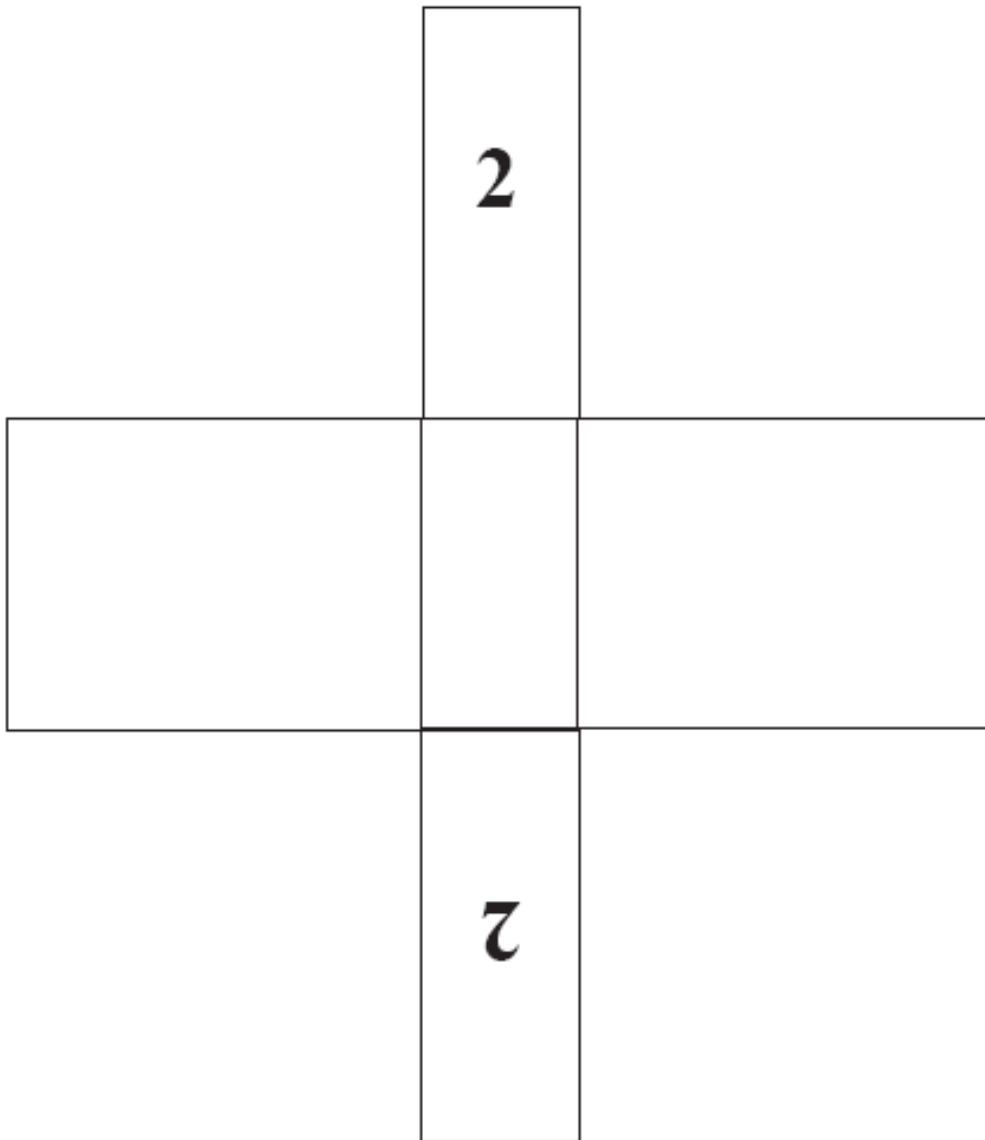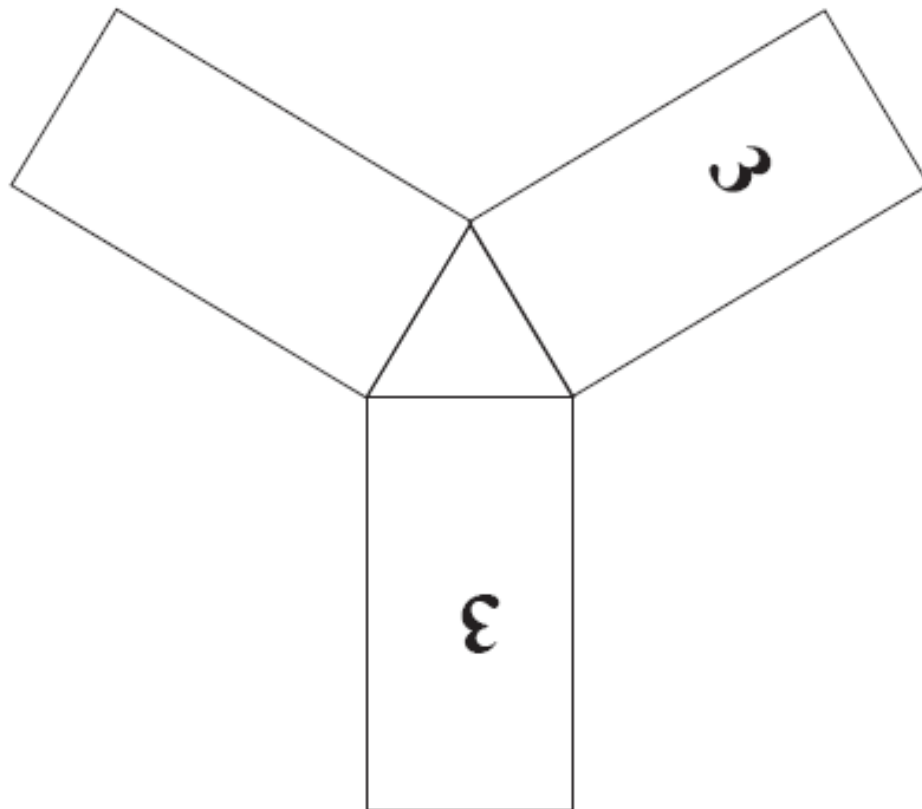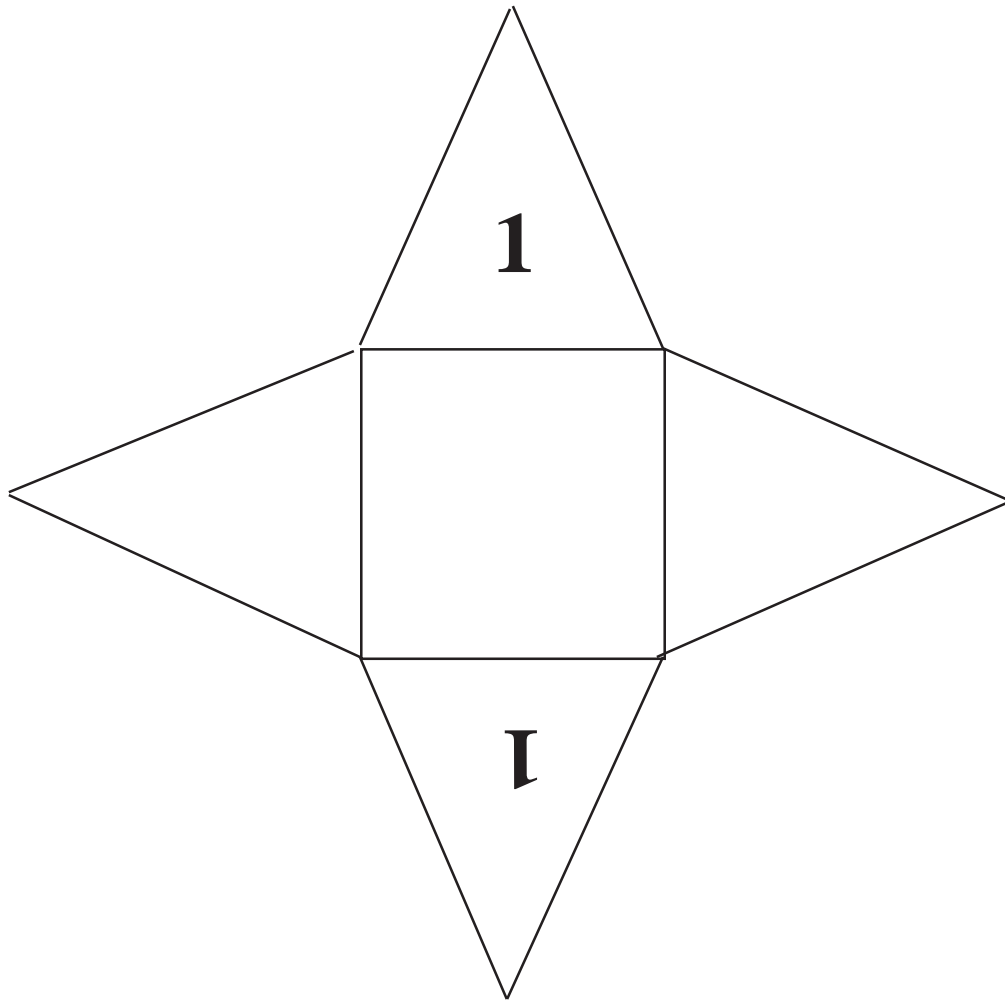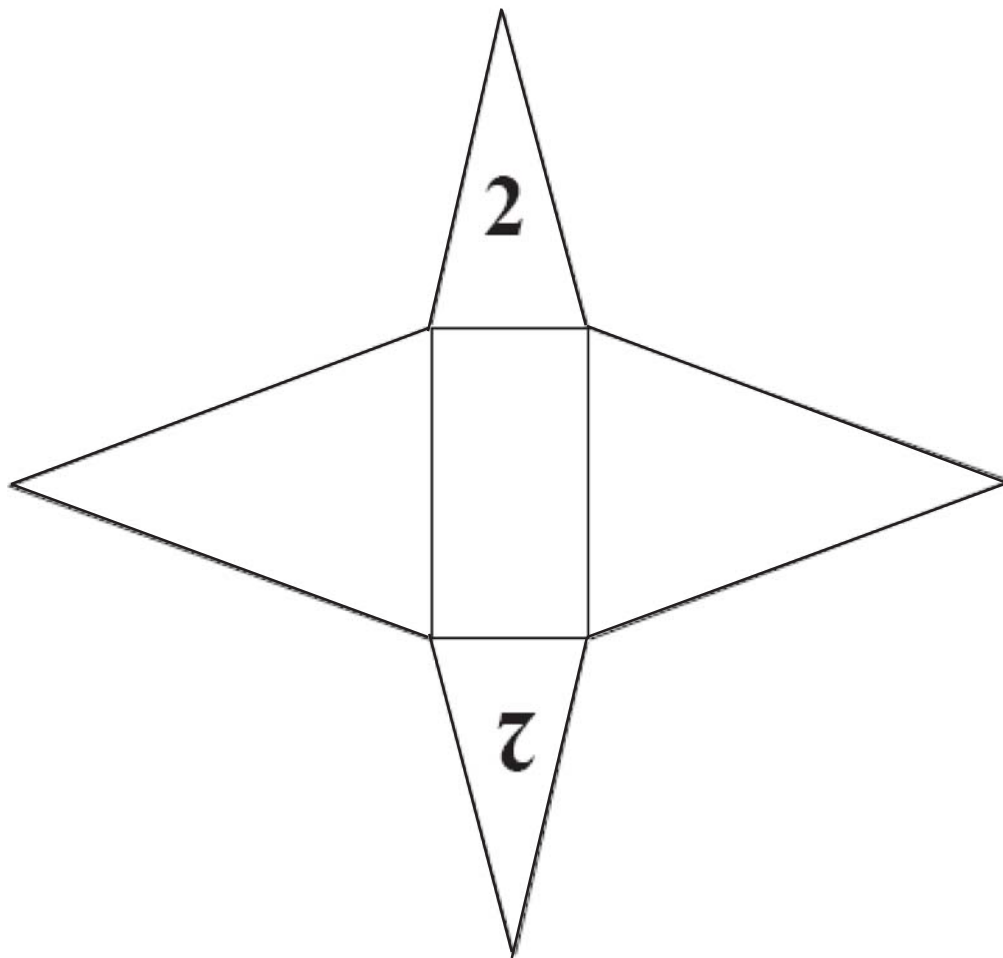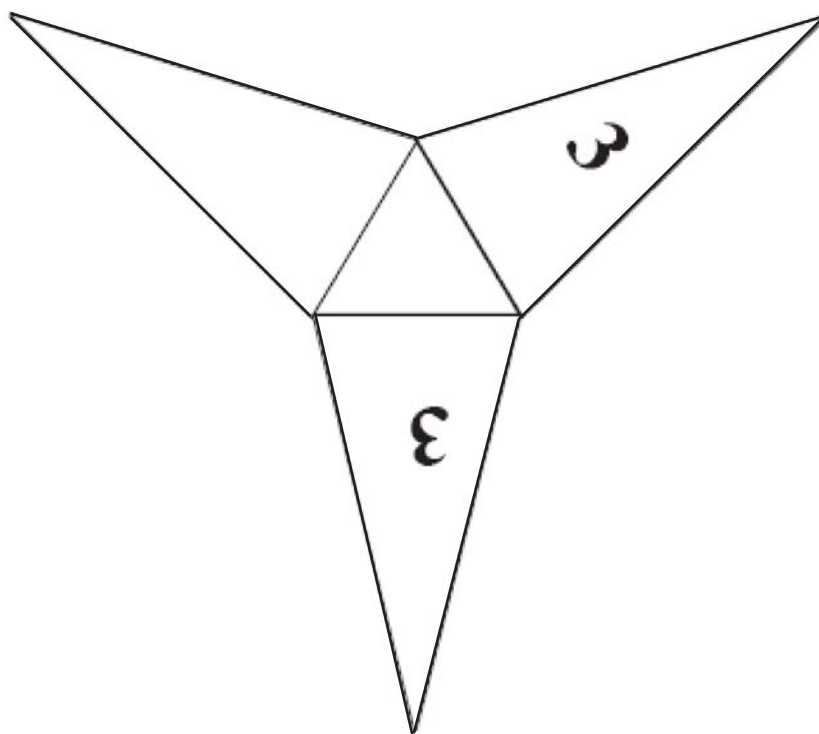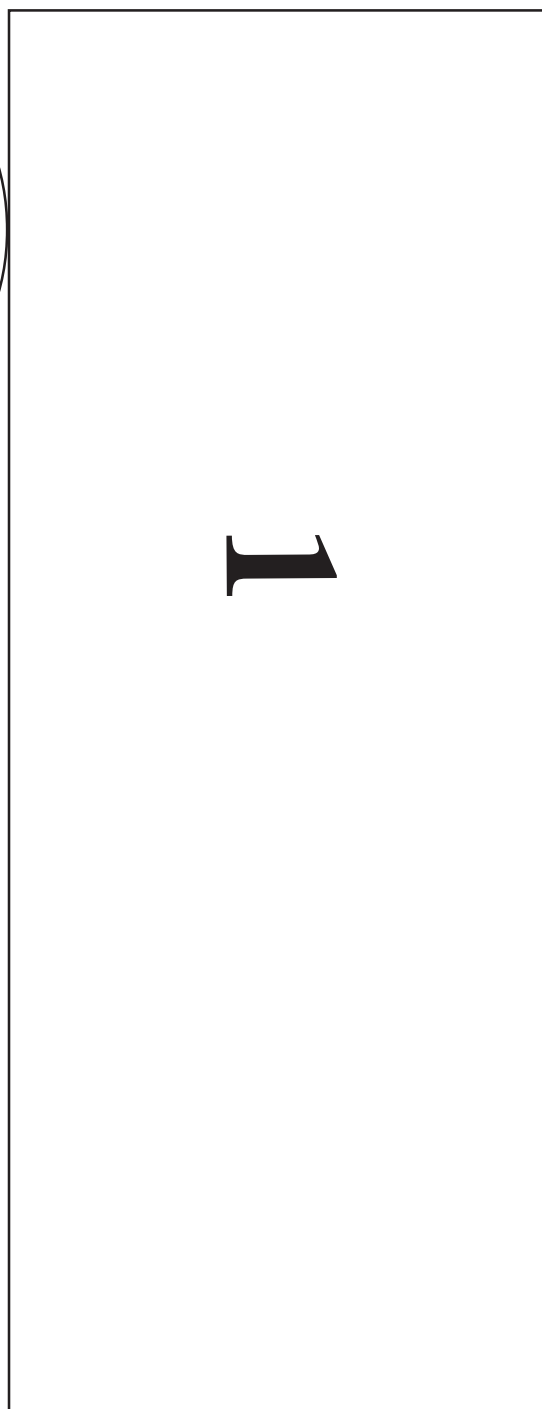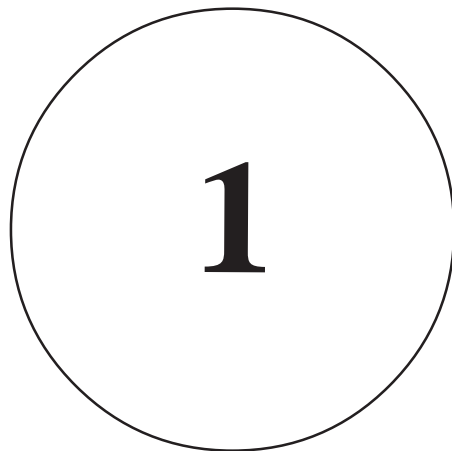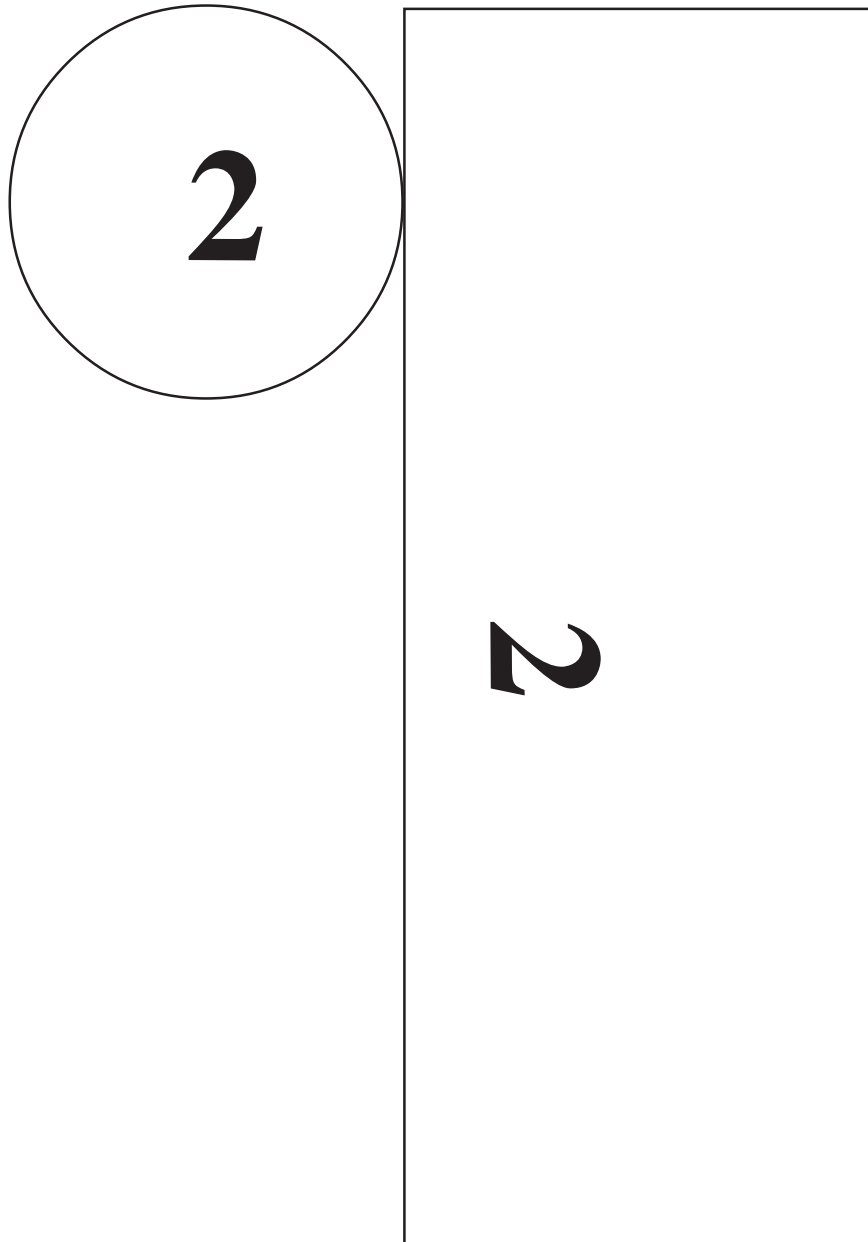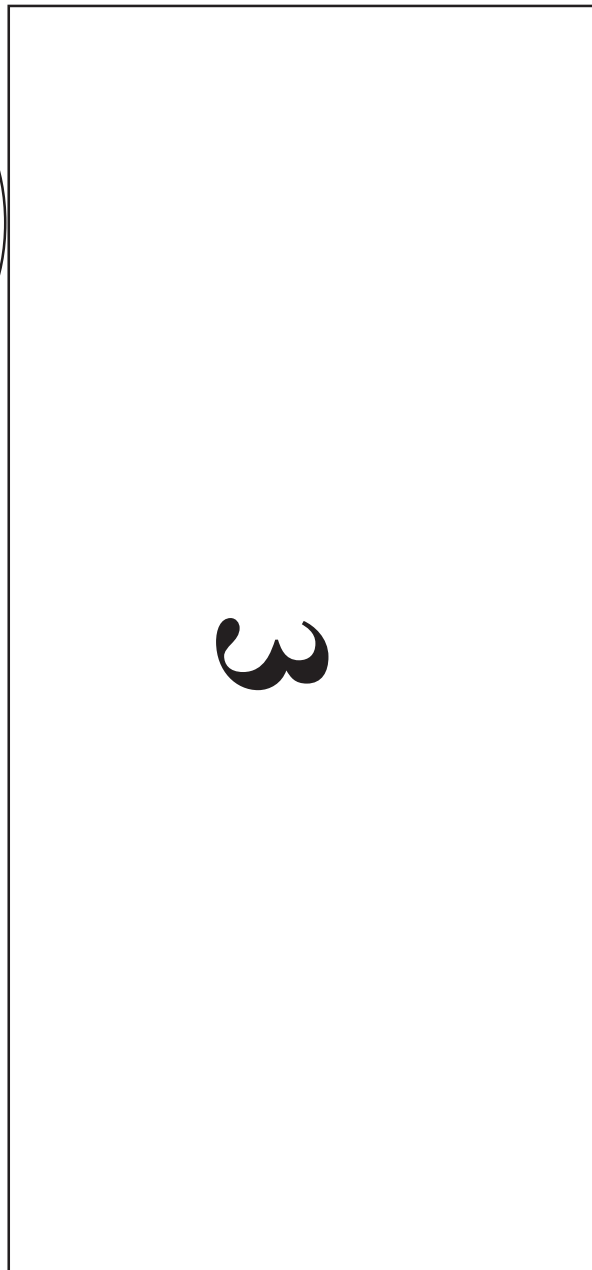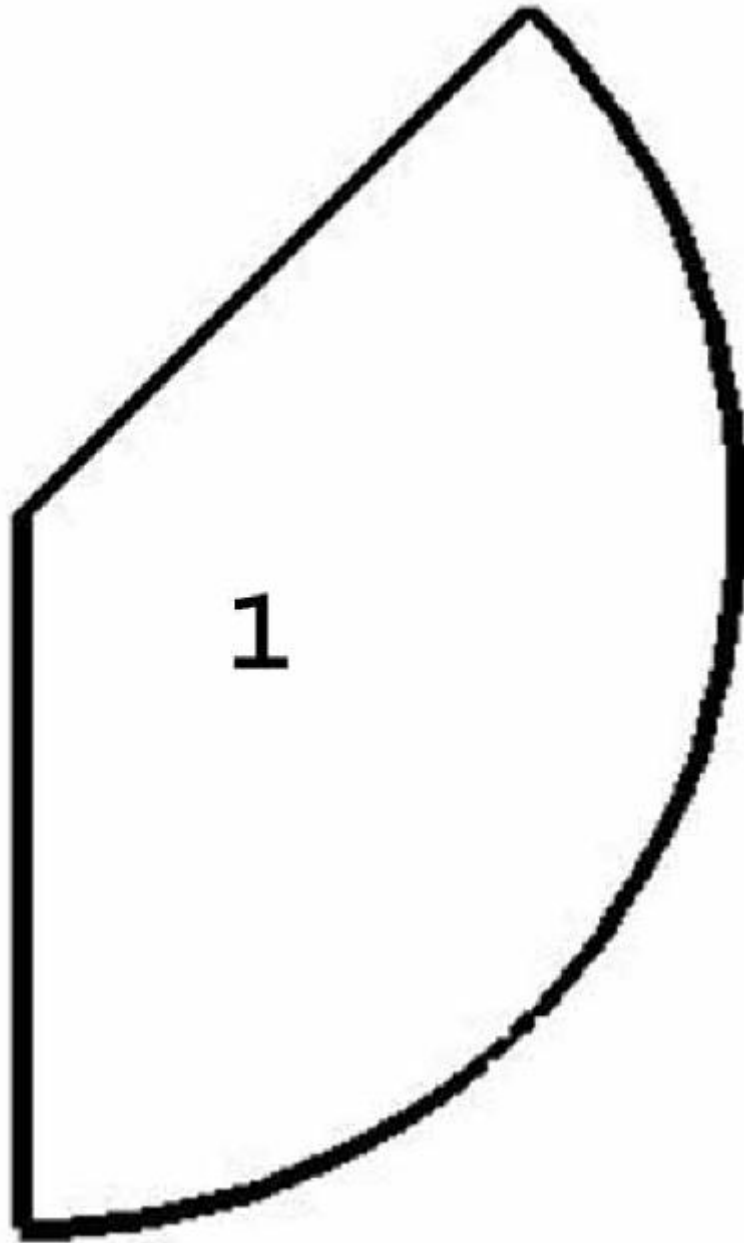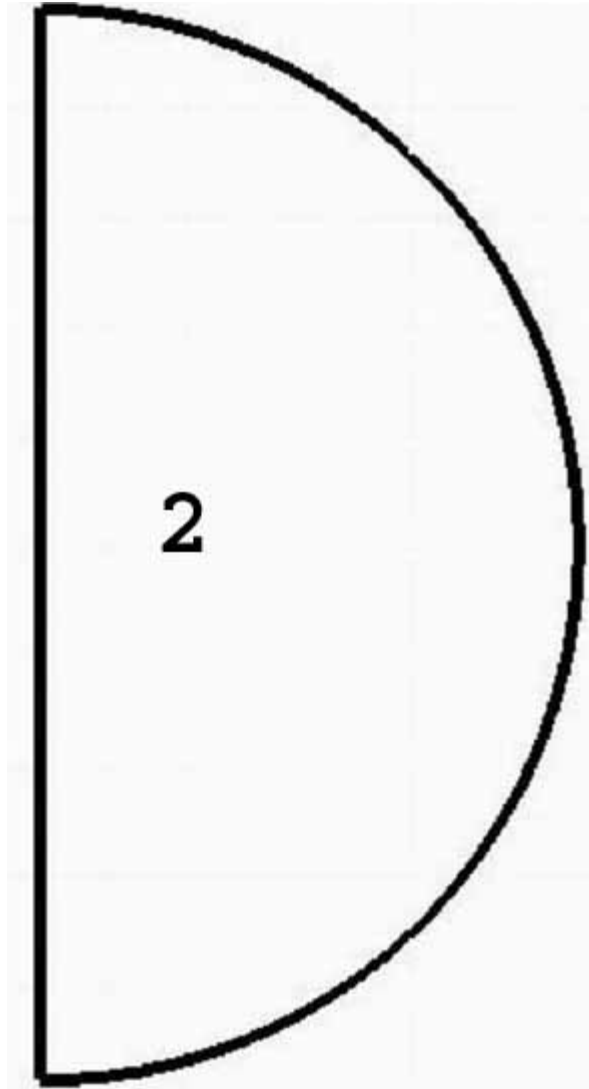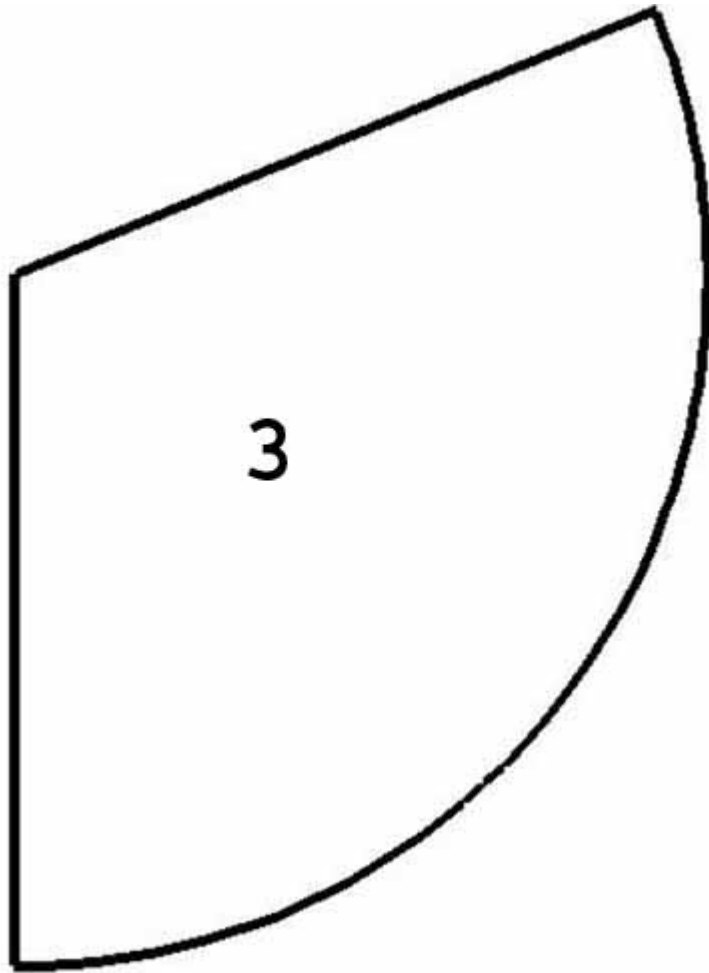
Appendix B1: An Introduction to Problem-Solving

an introduction to problem-solving

lesson summary

This lesson not only introduces students to a problem-solving model, but allows the teacher to begin the school year with a fun, engaging activity. In the first part of the activity, the students compete to build the tallest tower out of gum drops and toothpicks. Reflecting back on the process, the students, together with the teacher, develop a problem-solving model which they proceed to use in a follow-up activity.

u nderstand

To introduce the activity, the teacher must explain to the students that they will work with a group to build the tallest tower in the class. The teacher must also point out that the following rules apply:

- ■ Only toothpicks and gum drops may be used to build the tower.
- ■ The tower will be measured from the bottom of the tower to the top of it.
- ■ The tower must be able to stand on its own for 20 seconds.
- ■ The tower may not lean on anything.'
- ■ You will not get extra supplies. You must use what you have.
- ■ You will have 30 minutes to build your tower.

i nvestigate

The teacher now breaks the students into groups of three. He/she hands out the supplies to the groups and indicates that the students may begin. The students have thirty minutes to construct their towers.

During this time, the teacher should observe the students' behavior. He/she may reference certain actions during the **discover** portion of the lesson.

MathTAKStic!
UNIT 0

TEKS: 14b,c

Time Required:
135 minutes

Learning Objectives:

- ■ To engage in the problem-solving process

- ■ To develop a problem-solving model to use in the school year

- ■ To spark students' interests in mathematics and mathematics class

- ■ To create an engaging class environment and help the students become comfortable in the class and with their classmates

Materials:

- Gum drops (about 50 per group)

- Toothpicks (about 250-300 per group)

- Poster Board (one piece per group-optional)

- Markers or other poster making supplies

Support and Attachments:

- General Review Questions

- Problem Solving Model

After the thirty minute time limit finishes, the students are told to stop work on their towers. The teacher then circulates from group to group, ensuring that the tower can stand unaided for twenty seconds and measuring the height of the tower. The teacher then announces the winner of the competition.

d iscover

The teacher now gathers the students' attention and asks them to reflect on what went well during the building experience, what did not go well, and what could be improved. He/she may begin by asking the students to individually (and silently) reflect for two minutes by writing down any notable points.

After the students have collected their thoughts, the teacher may make a list on the board of strategies that worked versus those that did not work. Using this list, the teacher, along with the students may create a method for tackling problems. Although the idea is for the students, to create a plan from their own experience, the teacher should ensure that the class incorporates the following four steps:

- **Look** at the problem at hand: highlight any relevant points. In a math problem, this would include any significant words, any numerical values, and what the problem is asking. In a problem such as the tower-building excercise, students might consider what supplies they have and what their instructions are.

- **Plan** out a strategy for solving the problem. This may be as simple as guessing and checking or as complex as an intricate blueprint.

- **Use** the strategy you chose to solve the problem. In this step, showing the steps taken may prove helpful.

- **Reflect** back on the answer or result obtained. Does this product correspond with the intitial question? Does it make sense?

To help the students understand these steps, the teacher should illustrate how they might be applied to the tower-building excercise. Perhaps a group incorporated these steps and had success in their building, and the teacher may reference this group.

problem solving

If the class has time, the teacher may have each group make a poster using poster board, markers and any other supplies available. The poster should display all four components. The students may illustrate each step and/or give examples for each step. Afterwards, the posters may be hung up in the classroom to serve as a reminder to the stuents.

Likewise, the teacher may ask the students to create mini-poster illustrating all four steps. These cards may be kept with the students' notes for easy reference.

apply

Once the students understand the problem-solving model, they must practice using it in new problem scenarios. To begin, the students engage in a short activity.

The teacher divides the students into groups of three. Each group receives a copy of the **Problem Solving Model** worksheet. Additionally, the teacher may give the students colored paper. He or she then poses the following problem:

> You are given an 11 gallon bucket and a 6 gallon bucket. You need to measure exactly 8 gallons using the two buckets. How can you do this?

The teacher should require each group to fill out their **Problem Solving Model** while solving the problem. Plans for solving strategies might include the following: cutting out 11 squares of paper and 6 squares of paper and rearranging them, creating a table, or, if possible, actually experimenting.

Whenever the students have completed the activity, they may further their practice by completing the **General Review Questions**. Each question provides a model to fill out with the four steps.

lesson extensions

The teacher may have the students write a short journal entry or essay on the experience. This allows students to reflect back on what methods worked and what methods did not work. They may also create a problem scenario and explain how the problem may be solved using the model created in class.

Vocabulary / Definitions:

- **Problem**: A question or situation which needs a solution.

Assessment:

General Review Questions

Problem Solving Model worksheet

MathTAKStic!
UNIT 0

problem
solving model

MathTAKStic!
UNIT 0

review
questions

Review Questions

INSTRUCTIONS: Show all work.
Be sure to show all of the problem-solving steps.
You may put one step in each of the four boxes.

1. Juan and Samuel are splitting a pie and a half.
    How much pie does each of them get?

problem solving model

2. Terry and Samantha are buying groceries. They buy two loaves of bread for $1.50 each and one pound of apples for $3.00. If Terry and Samantha are splitting the cost, how much will each of them spend?

LOOK	PLAN
USE	**REFLECT**

MathTAKStic!
UNIT 0

review
questions

3. How much is 20 meters in centimeters?

4. What are all the divisors of 64?

problem solving model

5. What is the next number in the sequence below:

36, 12, 4, 1 1/3, …

Appendix B2: Integers: Ordering

reteach
integers:
ordering

lesson summary

This **RETEACH** lesson focuses on the ordering of integers. In **Part One** of the lesson, the students relearn how to use a number line in an interactive activity. The students are assigned integer numbers and line up in their corresponding order. The focus is for the *students* to reteach each other the ordering of integers.

In **Part Two** of the lesson, the students apply their knowledge of integers through a fun, quick game.

part one
refresh

anticipatory set

Before class, the teacher creates a giant number line on the board. The number line should have no numbers or only a few numbers already defined on it. As the students walk in, he or she hand each student a post-it note with an integer written on it. As the teacher hands the post-it note to the student, he or she gives the following instructions:

> "Find where your integer should be placed on the number line on the board and stick it in that place. Make sure to be aware of what integers others have already put up there."

The purpose is two-fold. The students should have to jog their memories to remember how to order integers. Additionally, the students should generate a discussion on where to place the integers. If such interaction does not occur, and integers are misplaced, the teacher may suggest the following to a student who *did* place their integer correctly:

TEKS: 6.1(a), 7.1(a)

Time Required:
Part One: 45 minutes
Part Two: 45 minutes

Objectives:

- The students will learn how to order integers.

- The students will apply the ordering of integers to real-life scenarios.

- The students will practice problem-solving involving integers.

Prerequisites for this Lesson:

- A basic understanding of what integers are.

Materials for Part One:

- Post-It Notes

- Integer Cards (available in the lesson materials)

- Number Strips (available in the lesson materials)

Support for Part One:

- Integer Cards Handout

- Number Strips Handout

- Integers Ordering: Basic Practice

"Perhaps you could help out the others in placing their integers. Try to explain why you are ordering them in the way you are."

At the end of the activity, once all the integers are placed correctly, the teacher may comment on what was done correctly and what common mistakes were made. He or she may remind the students of the following:

- Negative numbers start at zero, but count left rather than right.
- If one were to remove the negatives and positives, the number line is symmetric about zero. This can be demonstrated by folding a number line in half.

The teacher then tells the students that in today's activity, the class will be remembering and reteaching each other how to order the integers.

procedure

For the main activity, the teacher hands each student a notecard with an integer written on it. The teacher then instructs the students to order themselves into a single line so that their integers are in order from least to greatest. When they are finished, the teacher hands out another set of integers and the students repeat the activity. A set of these integers in included in the **Integer Cards Handouts** found in this section.

For extra incentive, the teacher may time the students and ask them to complete the activity as quickly as possible. If done several times, the teacher may take the fastest of the times (or the average). The class with the best time may be rewarded with a prize.

Alternatively, the teacher may divide the class in half and have them race.

integers:

ordering

closure

The teacher closes the activity by asking the students to take notes on a number line strip. These are available for the teacher in the **Number Strips Handout**. It is recommended that the teacher copy these strips on cardstock.

On the front of the number strip, the teacher has the students copy the integers from -20 to +20 to have as a reference. It is recommended that the teacher do the same on the overhead projector or on the board to ensure that the students know the correct order.

Afterwards, the students flip over the number strip and copy these notes down from the teacher:

- An integer is a whole number that is positive or negative or zero. Integers may not be fractions or decimals.
- We say a number is less than another number using the symbol <. For example, -3 < +2.
- We say a number is greater than another number using the symbol >. For example, +2 > -3.

If the class is blocked, then the teacher may proceed to **Part Two** of the lesson.

If the class is not blocked, then the teacher may hand out the Integers: Ordering Basic Practice for in-class work to be completed for homework.

Vocabulary/ Definitions:

- **Integer**: All positive and negative whole numbers, including zero. This excludes fractions and decimals.

Assessment:

- Integers: Ordering Basic Practice

integer cards handout

-1	**-2**
-3	**-4**
-5	**-6**
-7	**-8**
-9	**-10**
-11	**-12**

-13	**-14**
-15	**-16**
-17	**-18**
-19	**-20**
-25	**-28**
-30	**-32**

integer cards handout

0	**1**
2	**3**
4	**5**
6	**7**
8	**9**
10	**11**

12	13
14	15
16	17
18	19
20	21
22	23

integer cards handout

integer cards handout

24	25
26	27
28	29
30	31
32	33
34	35
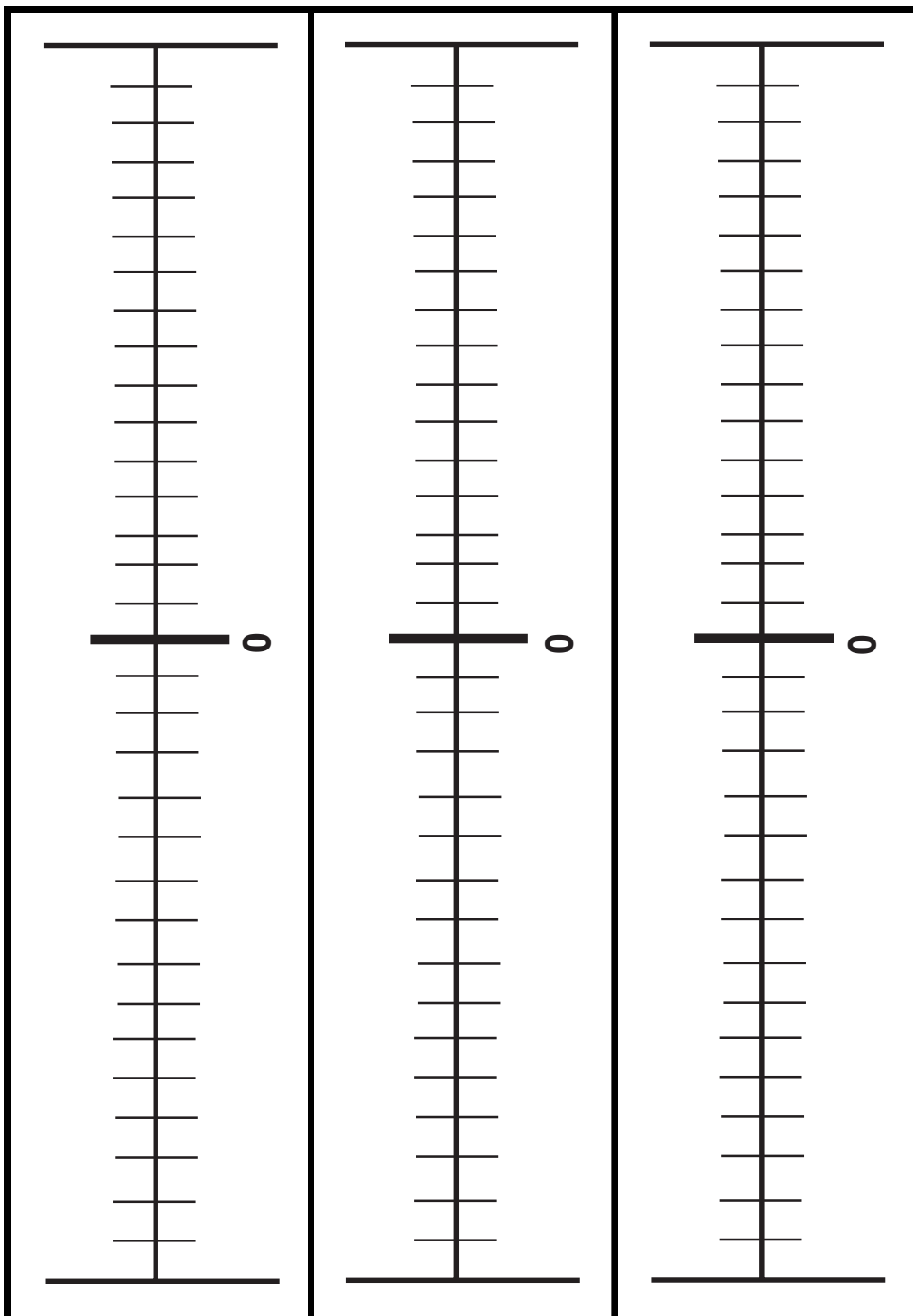
Integers Ordering: Basic Practice

1. Write the integers 1, 7, -5, -2, 3, -7, 6, 0, -4, 2, 4, -6, -3, -1 on the number line below.

2. Order the following integers from least to greatest: -2, -7, -5, -12, -3, -6.

3. Order the following integers from least to greatest: -2, -8, 5, 20, 0, -17, 13, -5, -12.

4. Order the following integers from greatest to least: -100, 30, 1, -3, -17.

5. What integer lies between -4 and -2?

5. List all the integers in order between -2 and +5.

part two

p r a c t i c e

Materials for Part Two:

- Ordering Integers Game Board

Support for Part Two:

- Ordering Integers Game Board

- Ordering Integers Game Pieces

- Ordering Integers Game Rules

- Integers: Ordering TAKS Practice

Assessment:

- Integers: Ordering TAKS Practice

anticipatory set

The teacher begins by reminding the students about the order of integers. He or she might also point out that integers may be ordered greatest to least or least to   greatest. The teacher may do a few problems with the class to make sure their understanding is correct.

procedure

For the game, the students should be placed in pairs. Each pair receives one **Ordering Integers Game Board**, one **Ordering Integers Game Pieces** set, and one **Ordering Integers Game Rules** handout. The teacher should explain the rules and possibly demonstrate. The rules are explained in the **Ordering Integers Game Rules** handout.

The students should practice the game several times. To change up the pace, the teacher should have half the students rotate after every game so as to play with as many students as possible.

closure

After the game, the teacher should ask students to reflect on the game: what did they find worked well and what didn't? By thinking through the game strategies, they may understand better about how to order integers.

Ordering Integers Game Rules

1. Lay the game pieces face down on the table. Each player should take exactly half of the pieces.

2. The players should decide who will go first and who will go second.

3. The playera alternate placing one of his or her game pieces face up on the game board in a circle.

4. The goal of the game is to get 4 pieces in a row so that they are in order from least to greatest or greatest to least. The four pieces may be diagonal, horizontal or vertical.

5. The player that wins is he or she who places the fourth piece down that creates such an ordering.
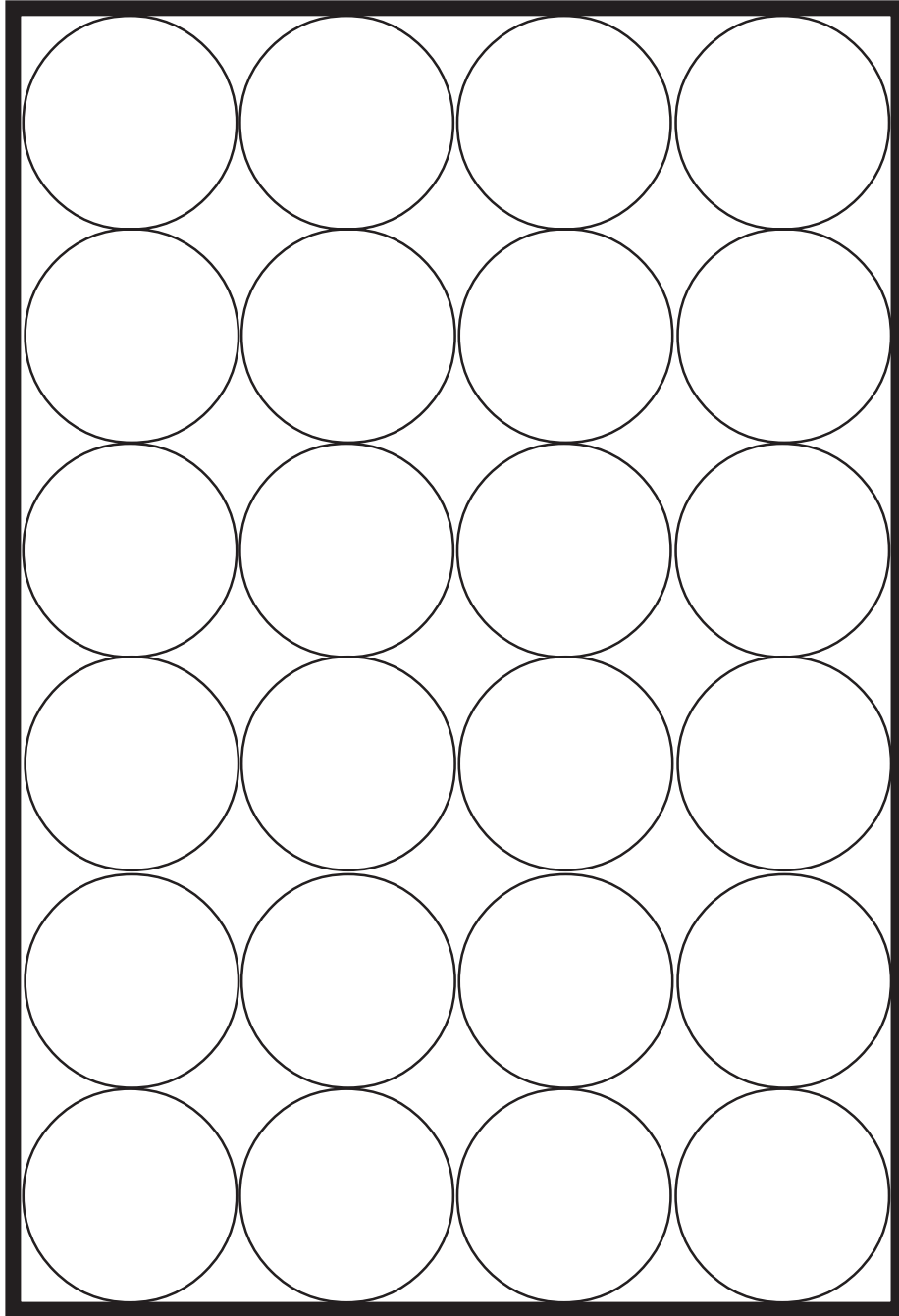
ordering integers game rules

ordering integers game pieces

-12	-6	0	6
-11	-5	1	7
-10	-4	2	8
-9	-3	3	9
-8	-2	4	10
-7	-1	5	11
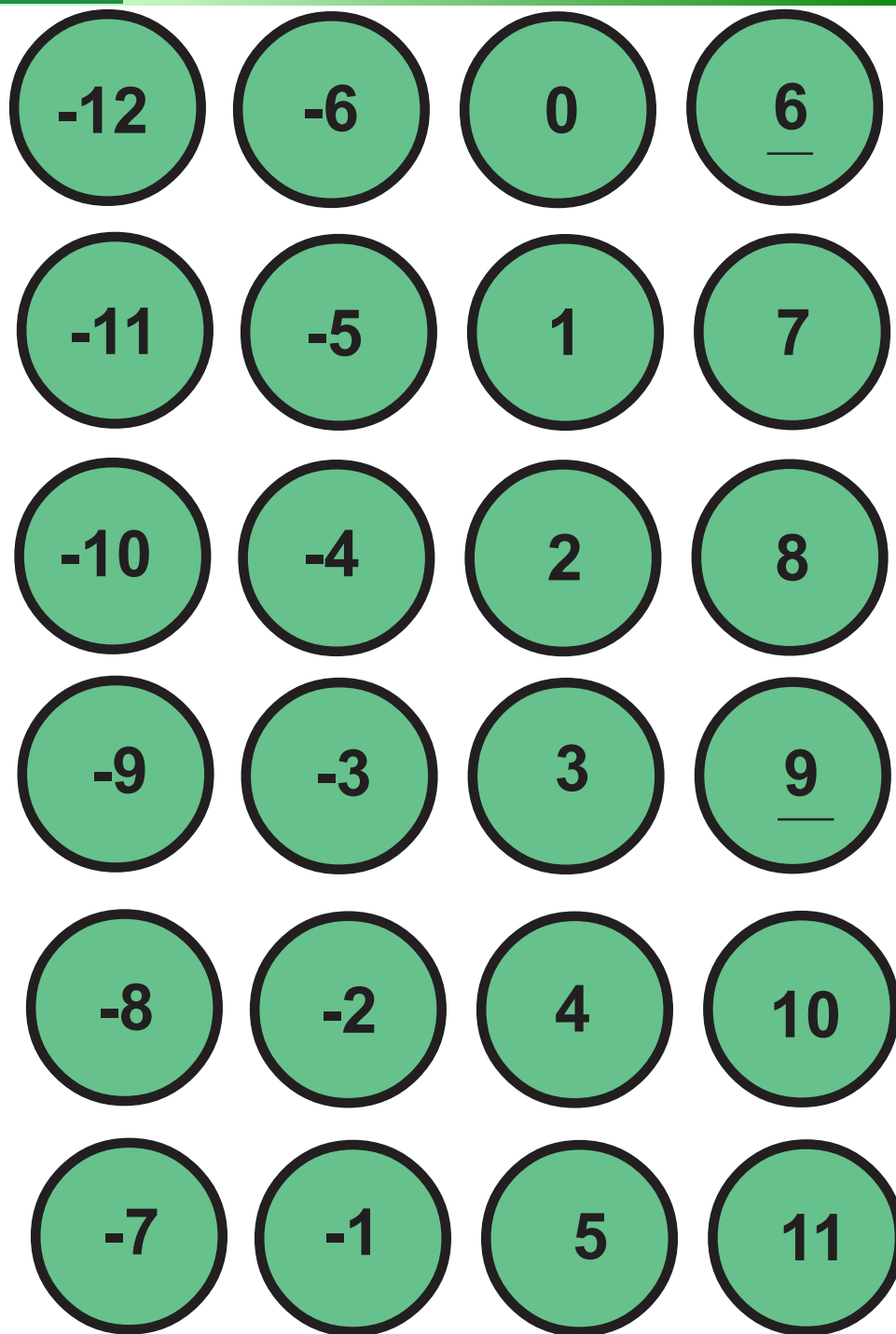
Appendix B3: Integers: Adding and Subtracting

reteach
integers:
adding/subtracting

TEKS: 7.1(a)

Time Required:
Part One: 90 minutes
Part Two: 45 minutes

lesson summary

Although students learn how to add and subtract integers in seventh grade, many often still struggle with this concept when they enter eighth grade. In the first part of this lesson, the teacher guides the class through two activities: one using the number line and the other using positive and negative integer dots. The second part of the lesson allows the students to practice their addition and subtraction skills through a short game. The game helps students see addition and subtraction with reference to a number line.

part one

r e f r e s h

anticipatory set

If the students worked on the previous lesson about ordering integers, then they will already be familiar with the concept of an integer. The teacher should ask the students to recognize places where adding or subtracting integers might be of use. Possible answers might be: finding the change in temperature, adding deposits and withdrawals to and from a bank account, and considering depth under or above sea level.

Objectives:

- The students will be able to add and subtract positive and negative integers using a number line.

- The students will be able to model the addition and subtraction of integers, both positive and negative.

- The students will understand how to add and subtract positive integers.

Prerequisites for this Lesson:

- A basic understanding of what integers are

- The ability to order integers.

MathTAKStic!
UNIT 1

Materials for Part One:

- Magnets, Post-It Notes or something similar

- Place indicators for the students

- Integer "dots"

Support for Part One:

- Adding & Subtracting Integers Notes Handout

- Adding/ Subtracting Integers: Basic Practice

Vocabulary/ Definitions:

- **Integers:** All positive and negative whole numbers and zero. This excludes fractions and decimals.

Assessment:

- Adding & Subtracting Integers: Basic Practice

procedure

USING A NUMBER LINE:

For this part of the lesson, the teacher must create a large number line on the board. He/she should also prepare some sort of place indicator, either a magnet, a Post-It Note or anything else that will attach to the board and can be easily moved.

The teacher should give the students the **Adding & Subtracting Integers Notes Handout**, available in this section. The teacher is also encouraged to provide the students with their own place indicator to use on the number line on the handout. A place indicator may be a small candy, a piece of paper, or anything readily available.

The teacher leads the class through the notes as directed on the **Adding & Subtracting Integers Notes Handout Solutions**. The solutions provide a guideline of what the teacher should say to the class and what the students should write on their worksheet. It might be helpful for the teacher to make an overhead transparency of the **Adding & Subtracting Integers Notes Handout** to fill out with the students.

The teacher may do multiple examples to ensure that the students understand the concepts. Additionally, he or she may assess the students' knowledge by posing simple addition and subtraction problems and allow them to solve the problems using the number line.

Note that the teacher should model the problems on the board or overhead projector. This will help those students who are struggling.

At the end of the notes, the teacher should make sur e that students understand that adding a negative number is the same as subtracting a positive integer and that subtracting a negative integer is the same as adding a positive integer.

$$-10 \quad -9 \quad -8 \quad -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$$
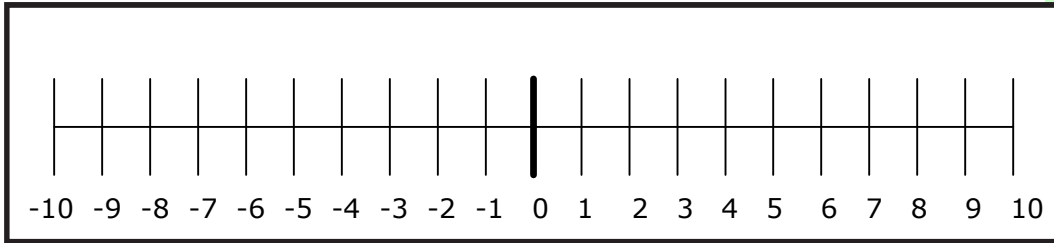
STARTING WITH A POSITIVE INTEGER

1. Adding a Positive Integer: 1 + 5

2. Subtracting a Positive Integer:

   o Case 1: 3 -1

   o Case 2: 3 - 6

3. Adding a Negative Integer:

   o Case 1: 3 + (-2)

   o Case 2: 3 + (-5)

4. Subtracting a Negative Integer: 6 – (-4)

adding & subtracting integer notes

integers:

adding/subtracting

adding & subtracting integer notes

```
  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0  1  2  3  4  5  6  7  8  9  10
```
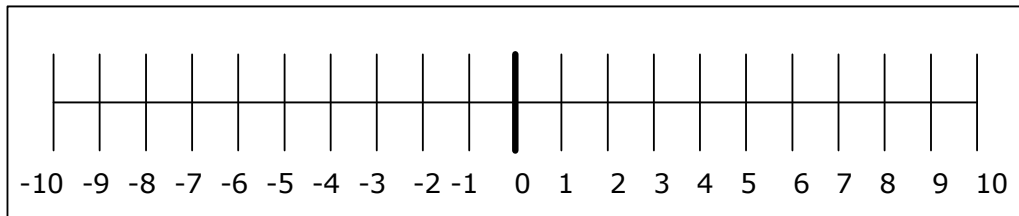
STARTING WITH A NEGATIVE INTEGER

1. Adding a Positive Integer:

   o Case 1: -6 + 2

   o Case 2: -6 + 9

2. Subtracting a Positive Integer: -4 – 5

3. Adding a Negative Integer: -2 + (-4)

4. Subtracting a Negative Integer:

   o Case 1: -5 – (-4)

   o Case 2: -5 – (-7)

STARTING WITH A POSITIVE INTEGER

1. Adding a Positive Integer:

    VERBAL QUESTION: Put your <place indicator> on the 1. Add 5 to 1. In doing so, what are you doing to your <place indicator>? What is our solution?

    > **SOLUTION (TO BE SPOKEN):** We move the <place indicator> to the right by 5.

    > **SOLUTION (TO BE WRITTEN):** EX: 1 + 5 = 6

    VERBAL QUESTION: What happens to a positive integer when we add another positive integer?

    > **SOLUTION (TO BE WRITTEN):** The integer increases by the number we are adding (it moves to the right on the number line by that number).

2. Subtracting a Positive Integer:

    ·   Case 1:

    VERBAL QUESTION: Remember that subtracting is a change in direction. Put your <place indicator> on the 3. Subtract 1 from 3. In doing so, what are you doing to your <place indicator>?   Is the answer positive or negative?

    > **SOLUTION (TO BE SPOKEN):** We move the <place indicator> to the left by 1. The answer is positive.

    > **SOLUTION (TO BE WRITTEN):** EX: 3 – 1 = 2

    ·   Case 2:

    VERBAL QUESTION: Put your <place indicator> on the 3 again. Now, Subtract 6 from 3. In doing so, what are you doing to your <place indicator>? Is the answer positive or negative?

    > **SOLUTION (TO BE SPOKEN):** We move the <place indicator> to the left by 6. The answer is negative.

    > **SOLUTION (TO BE WRITTEN):** EX: 3 – 6 = -3

    VERBAL QUESTION: What happens to a positive integer when we subtract another positive integer?

    > **SOLUTION (TO BE WRITTEN):** The integer decreases by the integer we are subtracting (it moves to the left on the number line by that integer value). The answer may be negative or positive depending on which number is bigger.

    *EXTRA VERBAL QUESTION: If you start at 6, what positive integers can you subtract and still be positive?

adding & subtracting integer notes solutions

adding & subtracting integer notes solutions

3. Adding a Negative Integer:

· Case 1:

VERBAL QUESTION: Adding a negative is like subtractign so it also needs a change in direction. Put your <place indicator> on the 3. Add -2 to the 3. Where did your <place indicator> move? Is the answer positive or negative?

SOLUTION (TO BE SPOKEN): We move the <place indicator> to the left by 2. The answer is positive.

SOLUTION (TO BE WRITTEN): EX: 3 + -2 = 1

· Case 2:

VERBAL QUESTION: Put your <place indicator> on the 3. Now, add -5 to the 3. Where did your <place indicator> move? Is the answer positive or negative?

SOLUTION (TO BE SPOKEN): We move the <place indicator> to the left by 5. The answer is negative.

SOLUTION (TO BE WRITTEN): EX: 3 + -5 = -2

VERBAL QUESTION: What happens to a positive integer when we add a negative integer?

SOLUTION (TO BE WRITTEN): The integer decreases by the value we are adding (it moves to the left on the number line by that value). The answer may be negative or positive.

*EXTRA VERBAL QUESTION: If you start at 6, what negative integers can you add and still be positive?

4. Subtracting a Negative Integer:

VERBAL QUESTION: Subtacting a negative needs two changes of direction, and they cancel out. Put your <place indicator> on the 6. Subtract -4 from the 6. Where did your <place indicator> move?

SOLUTION (TO BE SPOKEN): Our <place indicator> moved to the right by 4.

SOLUTION (TO BE WRITTEN): EX: 6 – (-4) = 10

VERBAL QUESTION: What happens to a positive integer when we subtract a negative integer?

SOLUTION (TO BE WRITTEN): The integer increases by the value we are subtracting (it moves to the right on the number line by that integer value).

integers:

adding/subtracting

STARTING WITH A NEGATIVE INTEGER

1. Adding a Positive Integer:

    ·   Case 1:

        VERBAL QUESTION: Put your <place indicator> on the -6. Add 2 to -6. In doing so, what are you doing to your <place indicator>? What is our solution? Is the answer positive or negative?

                SOLUTION (TO BE SPOKEN): We move the <place indicator> to the right by 2. The answer is negative.

                SOLUTION (TO BE WRITTEN): EX: -6 + 2 = -4

    ·   Case 2:

        VERBAL QUESTION: Put your <place indicator> on the -6. Now, add 9 to -6. In doing so, what are you doing to your <place indicator>? What is our solution? Is the answer positive or negative?

                SOLUTION (TO BE SPOKEN): We move the <place indicator> to the right by 9. The answer is negative.

                SOLUTION (TO BE WRITTEN): EX: -6 + 9 = 3

    VERBAL QUESTION: What happens to a negative integer when we add a positive integer?

            SOLUTION (TO BE WRITTEN): The integer increases by the number we are adding (it moves to the right on the number line by that number). The answer may be positive or negative.

    *EXTRA VERBAL QUESTION: If you start at -4, what positive integers can you add and still be negative?

2. Subtracting a Positive Integer:

    VERBAL QUESTION: Put your <place indicator> on the -4. Subtract 5 from -4. In doing so, what are you doing to your <place indicator>?

            SOLUTION (TO BE SPOKEN): We move the <place indicator> to the left by 5.

            SOLUTION (TO BE WRITTEN): EX: -4 – 5 = -9

    VERBAL QUESTION: What happens to a negative integer when we subtract a positive integer?

            SOLUTION (TO BE WRITTEN): The integer decreases by the integer we are subtracting (it moves to the left on the number line by that integer value).

adding & subtracting integer notes solutions

adding & subtracting integer notes solutions

3. Adding a Negative Integer:

VERBAL QUESTION: Put your <place indicator> on the -2. Add -4 to the -2. Where did your <place indicator> move? Is the answer negative or positive?

> **SOLUTION (TO BE SPOKEN):** We move the <place indicator> to the left by 4. The answer is negative.

> **SOLUTION (TO BE WRITTEN):** EX: -2 + -4 = -6

VERBAL QUESTION: What happens to a negative integer when we add another negative integer?

> **SOLUTION (TO BE WRITTEN):** The integer decreases by the value we are adding (it moves to the left on the number line by that value).

4. Subtracting a Negative Integer:

· Case 1:

VERBAL QUESTION: Put your <place indicator> on the -5. Subtract -4 from the -5. Where did your <place indicator> move?

> **SOLUTION (TO BE SPOKEN):** Our <place indicator> moved to the right by 4.

> **SOLUTION (TO BE WRITTEN):** EX: -5 – (-4) = -1

· Case 2:

VERBAL QUESTION: Put your <place indicator> on the -5. Now, subtract -7 from the -5. Where did your <place indicator> move?

> **SOLUTION (TO BE SPOKEN):** We move the <place indicator> to the right by 7.

> **SOLUTION (TO BE WRITTEN):** EX: -5 – (-7) = 2

VERBAL QUESTION: What happens to a negative integer when we subtract a negative integer?

> **SOLUTION (TO BE WRITTEN):** The integer increases by the value we are subtracting (it moves to the right on the number line by that integer value). The answer may be positive or negative.

*EXTRA VERBAL QUESTION: If you start at -4, what integers (positive, negative, and zero) can you add and still be negative? What integers can you subtract and still be negative?

integers:
adding/subtracting

USING POSITIVE AND NEGATIVE INTEGER DOTS:

The teacher transitions to this part of the lesson by letting the students know that they will now be learning about some of the interesting properties of integers and how they can help in adding and subtracting. He or she should have a class set of integer "dots." Integer "dots" must be two different colors. They may be any materials that the teacher has handy: bingo chips, flat marbles, or, if nothing else, the teacher may laminate and cut out the dots available in the appendix.

Each student should receive at least ten dots of each color. It is suggested that the teacher has dots he or she can stick of the board (with tape) to provide a visual assistance to the students.

The teacher informs the class that one color will represent the positives and the other color will represent the negatives. Each dot has value 1. In the following explanation, black will be positive and white will be negative.

The teacher begins by posing a problem on the board: 3 + 5.
He or she asks, "Can you show me this problem using your dots?"
The teacher waits for a student to show that one may select 3 black dots and then 5 black dots, creating a total of 8 black dots. In other words, one has an answer of +8.

"Now, what will happen if we add negative numbers to the positive numbers?" The teacher poses this problem on the board: -3 + 5.
The students should show 3 white dots and 5 black dots on their desks.

The teacher points out an important property.
"What happens if a -1 is added to a +1?" He or she allows the students to suggest answers. Eventually, a student should point out that they cancel to make zero.
"So, you just told me that a white dot and a black dot make zero. So, if we pair each white dot with a black dot, those pairs are each equal to zero."

The teacher demonstrates on the board by pairing together every white dot with a black dot and removing them from the problem scenario.
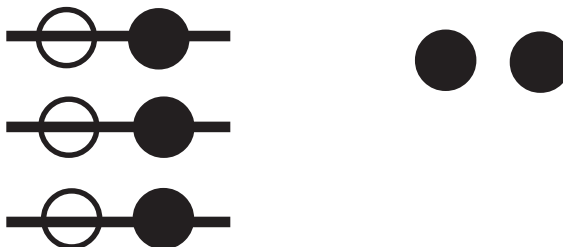
"What is left?" The students should reply that 2 black dots are left.
"What does that mean in integers?" The students now reply that +2 is the answer.

The teacher should repeat with several more examples, including ones like 1 + (-4) which produces a negative. This allows the students to work on their own and raise their hand when they know the correct answer. The teacher may then walk around the classroom, checking for individual understanding.

After the teacher feels that the students have a solid understanding of the addition, he or she may now pose a subtraction problem: 5 - 3.
The students might show a variety of models. Probably, the students will show 5 black dots and take away 3 of them. But, the teacher is hoping to help students understand that taking away a positive is the same as adding a negative. He or she poses the following thought:

"You all are correct in saying that I can take away 3 black dots from my 5 black dots. But, what if I start with 5 black dots, and I ask you to adding something to these 5 black dots to get the same answer?"

The teacher may help the students until they realize that by adding 3 white dots they will obtain the same answer. The teacher writes on the board the following:

$$5 - 3 = 5 - {}^+3 = 5 + {}^-3$$

It is encouraged to do at least one more example of subtracting a positive from another positive. When the students are more comfortable with this concept, the teacher may pose the following problem: 5 - (-3).

He or she lets the students experiment with the dots, looking for a correct answer. The teacher must ensure that the students understand, in the end, that subtracting a negative is the same as adding a positive.

integers:
adding/subtracting

closure
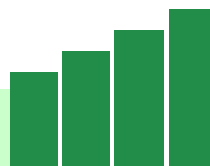
The teacher should make a connection between the two methods dis-
cussed in class. Students should understand that when subtracting an
integer, one "switches" directions on the number line. For instance, when
subtracting a negative integer, rather than moving left, we move right.
This is the same idea as changing the sign of the value, much like when
using the integer dots.

The teacher may check for understanding by asking the students to
complete the Adding & Subtracting Integers: Basic Practice worksheet.

adding & subtracting integers: basic practice

Adding & Subtracting Integers: Basic Practice

1. 4 + 6 = _____

2. 12 + 22 = _____

3. 12 – 10 = _____

4. 12 + -10 = _____

5. 20 + - 25 = _____

6. -8 + -10 = _____

7. -26 + -14 = _____

8. 4 – 12 = _____

9. 5 – (-18) = _____

10. 22 – (-6) = _____

11. -6 – 10 = _____

12. -14 – 2 = _____

13. -35 – (-15) = _____

14. -3 – (-7) = _____

-20 -15 -10 -5 0 5 10 15 20 25
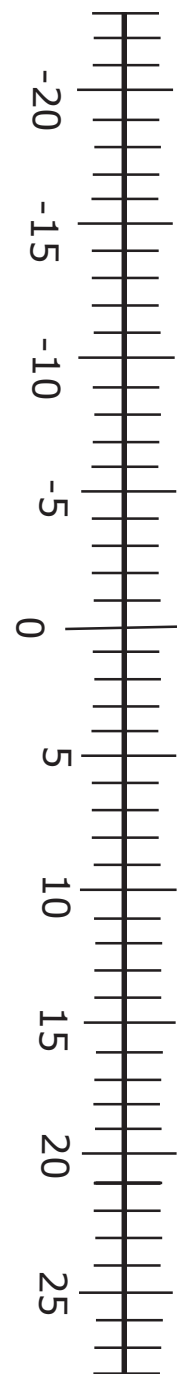
integers:

adding/subtracting

part two

p r a c t i c e

anticipatory set

The teacher should begin the lesson with a quick reminder of what the students learned in the previous portion of the lesson. A warm-up or a quick quiz might be good reminders for the students.

Afterwards, the teacher asks the class the following four questions to make sure they are prepared for the game:

- When I add a positive integer, what direction do I move on the number line?
- When I add a negative integer, what direction do I move on the number line?
- When I subtract a positive integer, what direction do I move on the number line?
- When I subtract a negative integer, what direction do I move on the number line?

procedure

Prior to class, the teacher must prepare the game pieces. He or she must cut out the **Adding & Subtracting Integers Strategy Game Pieces**. In class, the teacher begins by pairing off the students. Each pair receives one **Adding & Subtracting Integers Strategy Game Board**. Each person receives 20 game pieces numbering -10 through 10. The instructions for the game appear on **Adding & Subtracting Integers Strategy Game Rules**, on the following page.

Materials for Part Two:

- Adding & Subtracting Integers Strategy Game Pieces

- Adding & Subtracting Integers Strategy Game Board

Support for Part Two:

- Adding & Subtracting Integers Strategy Game Pieces

- Adding & Subtracting Integers Strategy Game Board

- Adding & Subtracting Integers Strategy Game Rules

Assessment:

Adding & Subtracting Integers

MathTAKStic!
UNIT 1

closure

After the students have played several games, the teacher may ask them to reflect on what they learned and if they found any strategies to be particularly effective. The teacher may note that each game piece could be used in two ways. For instance, if one desired to subtract 4, but did not have a +4 game piece, he or she could instead add -4.

Adding & Subtracting Integers
Strategy Game Rules

1. The game is to be played in pairs.

2. Each pair receives a game board.

3. Each person receives 20 game pieces.

4. The first person starts at zero. He or she selects a game piece and either adds or subtracts that amount from zero. He or she then places the game piece on the new value. For instance, if one were to use the -4 game piece, one could add -4 to 0 to land on -4 or one could subtract -4 from 0 to land on 4.

5. The second person selects a game piece to add or subtract to the integer that the other person landed on.

6. The game continues in this manner, always starting from where the other person landed.

7. The game ends when a person cannot make another move. The winner is the other person.
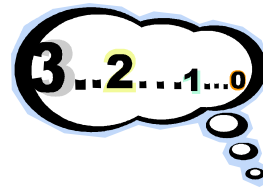
MathTAKStic!
UNIT 1

adding & subtracting integers strategy game board

-22	-21	-20	-19	-18
-17	-16	-15	-14	-13
-12	-11	-10	-9	-8
-7	-6	-5	-4	-3
-2	-1	0	1	2
3	4	5	6	7
8	9	10	11	12
13	14	15	16	17
18	19	20	21	22

integers:

adding/subtracting

-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
1	2	3	4	5	6	7	8	9	10
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
1	2	3	4	5	6	7	8	9	10
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
1	2	3	4	5	6	7	8	9	10
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
1	2	3	4	5	6	7	8	9	10
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
1	2	3	4	5	6	7	8	9	10

adding & subtracting integers strategy game pieces
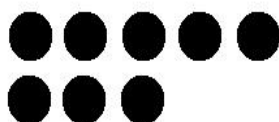
adding & subtracting integers practice

1. Each white circle equals +1 and each black circle equals -1. Which of the following represents 5 + -3?

A) ⚪⚪⚪⚪
⚫⚫

B) ⚪⚪⚪⚪⚪
⚪⚪⚪

C) ⚪⚪⚪⚪⚪
⚫⚫⚫

D) ⚫⚫⚫⚫⚫
⚫⚫⚫

2. In the model below, each white square equals +1 and each gray square equals -1.

▢▢▢
▨▨▨▨▨
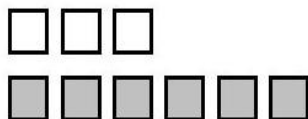
Which equation does the model *not* represent?

A)   3 + -6
B)   3 – 6
C)   3 – -6
D)   -6 + 3

3. Mary wants to model the equation -2 – 4. She has red and blue chips she may use. She decides to represent the negative numbers with blue and the positives with red. Which of the following describes how Mary should model the equation?

A) She should lay out 2 red chips and 4 blue chips
B) She should lay out 2 red chips and then 4 more red chips
C) She should lay out 2 blue chips and 4 red chips
D) She should lay out 2 blue chips and then 4 more blue chips

4. In the model below, each striped square equals +2 and each gray square equals -1. Which integer does this model represent?

▤▤▤▤
▨▨▨▨▨▨▨▨
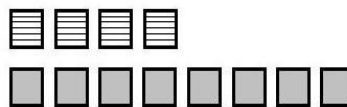
A)   -4
B)   0
C)   4
D)   12

5. Which equation below is equivalent to the equation -2 – -3?

A)   -2 – 3
B)   -2 + 3
C)   -3 + -2
D)   -3 – 2
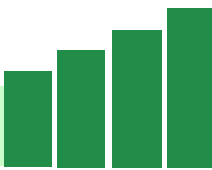
Appendix B4: Integers: Multiplying and Dividing

reteach
integers:
multiplying/dividing

TEKS: 7.1(c)

Time Required:
Part One: 45 minutes
Part Two: 45 minutes
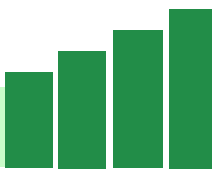
lesson summary

After students have mastered adding and subtracting integers, learning the multiplication and division of integers is often less challenging. The teacher presents one or more simple methods to remember how to multiply and divide integers. Afterwards, the students play a game of bingo to practice all integer arithmetic.

Objectives:

- To learn how to multiply both positive and negative integers.

- To practice integer arithmetic.

part one
r e f r e s h

anticipatory set

To prepare the students for the lesson, the teacher may set up an imaginary scenario. The scenario is the following:

- A girl likes a boy. He likes her too. Is this a good situation or a bad situation? (Good)
- A girl likes a boy. He does not like her though. Is this a good situation or a bad situation? (Bad)
- A girl dislikes a boy. He, however, likes her. Is this a good situation or a bad situation? (Bad)
- A girls dislikes a boy. He also dislikes her. Is this a good situation or a bad situation? (Good-the feelings are mutual)

The teacher explains that the scenario will relate to the lesson. He or she proceeds to hand out the **Multiplying & Dividing Integers Notes**.

Prerequisites for this Lesson:

- An ability to order both positive and negative integers.

- An understanding of how to add and subtract integers.

MathTAKStic!
UNIT 1

Materials for Part One:

- Multiplying & Dividing Integers Notes

- Multiplying & Dividing Integers Notes Solutions

Support for Part One:

- Multiplying & Dividing Integers Notes

- Multiplying & Dividing Integers Notes Solutions

- Multiplying & Dividing Integers Basic Practice

Vocabulary/ Definitions:

- **Integers:** All positive and negative whole numbers, and zero. This excludes fractions and decimals.

Assessment:

- Multiplying & Dividing Integers Basic Practice

procedure

Once the students have the **Multiplying & Dividing Integers Notes**, the teacher may help the students fill them out. The **Multiplying & Dividing Integer Notes Solutions** outlines how to fill in the worksheet and what to say to the students to help them remember the rules.

The teacher should be sure to work plenty of examples and have the students work examples. Once the teacher is sure that they have mastered this concept, they may practice all integer arithmetic with the **Integer Bingo Game** in the **Practice Section** of this lesson plan.

Multiplying & Dividing Integers Notes

STEP ONE: Don't look at the signs and just multiply the numbers. Write down what you get.

STEP TWO: Now, you have to figure out if your answer is positive or negative. Here are the rules:

- If you have a _____ and a _____,

   your answer is _____.

- If you have a _____ and a _____,

   your answer is _____.

- If you have a _____ and a _____,

   your answer is _____.

- If you have a _____ and a _____,

   your answer is _____.

EXAMPLES:

- 5 x 6 =

- 3 x -4 =

- -6 ÷ 2 =

- -10 x -5 =

integers:

multiplying/dividing

Multiplying & Dividing Integers Notes SOLUTIONS

Indented parts denote ideas the teacher should say to help students understand the concepts.

Bold denotes those parts that the students should write down on their paper.

STEP ONE: Don't look at the signs and just multiply the numbers. Write down what you get.

STEP TWO: Now, you have to figure out if your answer is positive or negative. Here are the rules:

**A girl likes a boy (this is POSITIVE). The boy also likes the girl (another POSITIVE). They both like each other. This is GOOD, which is POSITIVE.
- If you have a _**POSITIVE**__ and a _**POSITIVE**_,

  your answer is __**POSITIVE**_____.

**A girl likes a boy (this is POSITIVE). The boy does not like the girl (a NEGATIVE). One of them does not like the other. This is BAD, which is NEGATIVE.
- If you have a __**POSITIVE**__ and a ____**NEGATIVE**__,

  your answer is _____**NEGATIVE**_____.

**A girl does not like a boy (this is NEGATIVE). The boy, however, does like the girl (a POSITIVE). One of them does not like the other. This is BAD, which is NEGATIVE.
- If you have a __**NEGATIVE**__ and a __**POSITIVE**____,

  your answer is _____**NEGATIVE**_____.

**A girl does not like a boy (this is NEGATIVE). The boy also does not like the girl (another NEGATIVE). They both do not like each other. Since the feeling is mutual, this is GOOD, which is POSITIVE.

- If you have a __**NEGATIVE**__ and a __**NEGATIVE**___,

  your answer is ____**POSITIVE**_____.

EXAMPLES:

- 5 x 6 =

  5 x 6 = 30
  Positive and Positive = Positive

  Answer = +30

- 3 x -4 =

  3 x 4 = 12
  Positive and Negative = Negative

  Answer = -12

- -6 ÷ 2 =

  6 ÷ 2 = 3
  Negative and Positive = Negative

  Answer = -3

- -10 x -5 =

  10 x 5 = 50
  Negative and Negative = Positive

  Answer = +50

multiplying & dividing integer notes solutions

integers:

multiplying/dividing

Multiplying & Dividing Integers: Basic Practice

1. 2 x 5 = _____

2. -2 x 5 = _____

3. -1 x 3 = _____

4. 6 x -4 = _____

5. 3 x -9 = _____

6. -6 x -8 = _____

7. -2 ÷ 1 = _____

8. 24 ÷ -3 = _____

9. 81 ÷ 9 = _____

10. -25 ÷ -5 = _____

11. 8 ÷ -4 = _____

12. -64 ÷ 8 = _____

13. -42 ÷ -6 = _____

14. 100 ÷ -10 = _____

integers:

multiplying/dividing

part two

practice

anticipatory set

The teacher should do examples of several types of problems involving adding, subtracting, multiplying and dividing integers to help the students sort out all the rules and methods.

procedure

To set up the game, the teacher should give each student an Integer Bingo Card. Additionally, each student must receive about 20 game pieces. Game pieces can be bingo chips, flat marbles, torn or the **Integer Dots** found in Appendix B0. If the teacher chooses to use the cards only once, the students can use highlighters also. For a permanent set of cards, the teacher may laminate the cards, preferably printed on cardstock.

The teacher may use the Integer Bingo Problems or make up his or her own. To play the game, the class should follow the instructions given on **Integer Bingo Instructions**.

Assessment:

Multiplying & Dividing Integers TAKS Questions

closure

After the game, the teacher should be sure that the students are truly confident in integer arithmetic, as it is incredibly fundamental for a lot of eighth grade math and future mathematics.

integers:

multiplying/dividing

INTEGER BINGO INSTRUCTIONS

1. The teacher reads out an integer problem and writes the problem on the board or overhead projector.

2. The students should solve the problem and search for the answer on their card. If the card has the answer, the student may place a game piece on that rectangle. Everybody is welcome to place a game piece on the free space.

3. There are several versions of the game. In one, a person must get 5 game pieces in a row (horizontally, vertically, or diagonally) on his or her card to win. In another, a person must have all four corners covered. In yet another, one must fill the entire board with game pieces. This last version is often called "black-out."

4. The first person to obtain one of the above layouts may receive a small prize, such as a piece of candy, bonus points, or a pencil. At this point, the game may end or may continue, either with the boards cleared or as they are.

MathTAKStic!
UNIT 1

-1	-8	-21	-5	15
14	-7	0	19	11
-22	-3	FREE SPACE	-15	1
-6	10	-30	12	18
-29	-9	-19	5	16

multiplying & dividing integer notes solutions

integers:

multiplying/dividing

integer bingo blank card
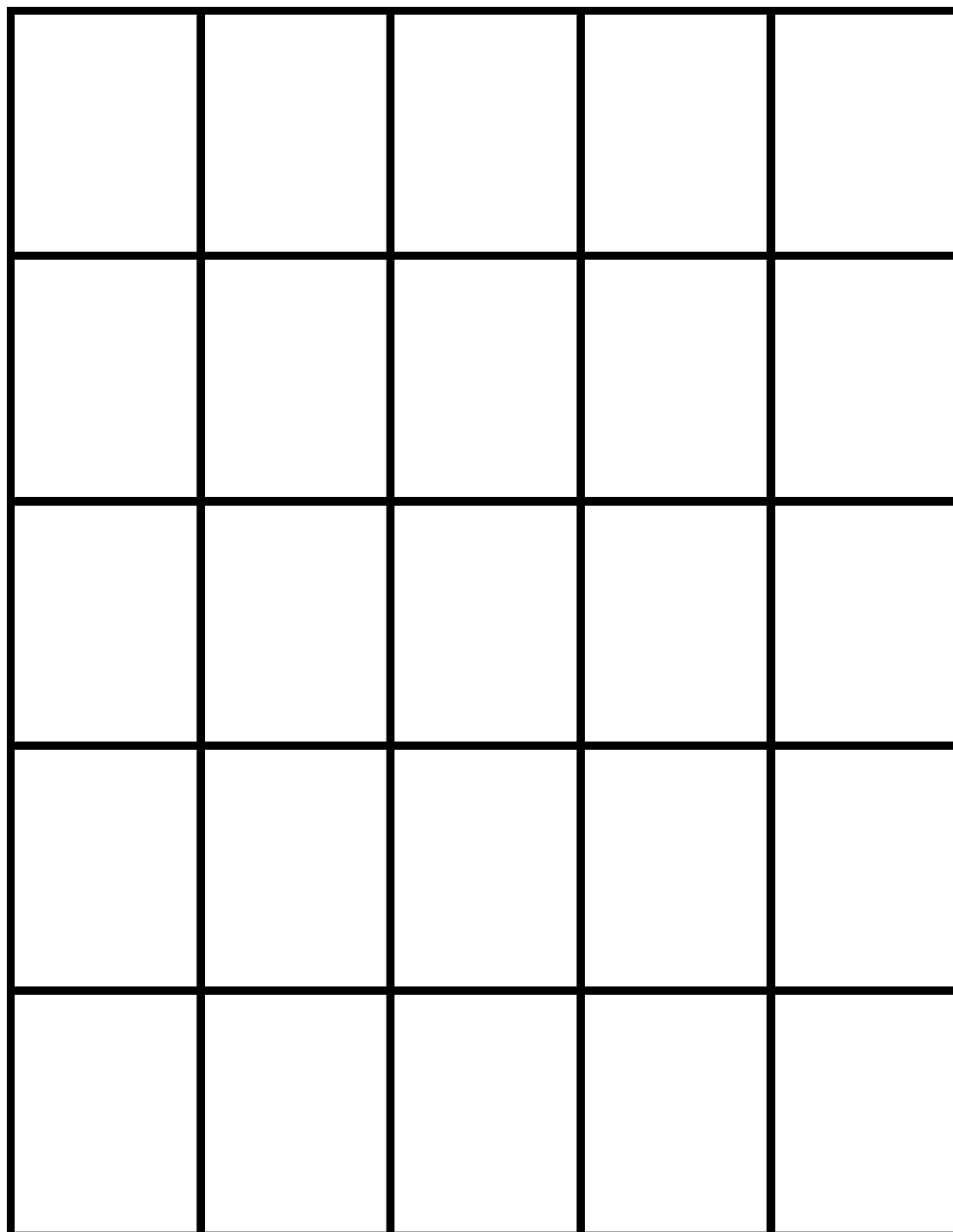
INTEGER BINGO PROBLEMS

0 = -15 + 15
1 = -4 – (-5)
2 = -4 ÷ -2
3 = 1 x 3
4 = -2 x -2
5 = -1 + 6
6 = 48 ÷ 8
7 = 17 + (-10)
8 = -64 ÷ -8
9 = -43 – (-52)
10 = -5 x -2
11 = 5 + 6
12 = 60 ÷ 5
13 = -26 ÷ -2
14 = 20 + (-6)
15 = -45 ÷ -3
16 = -4 x -4
17 = -24 + 41
18 = -2 – (-20)
19 = -38 ÷ -2
20 = -4 x -5
21 = -7 x -3
22 = -11 x -2
23 = -10 –(-33)
24 = 6 x 4
25 = -125 ÷ -5
26 = 44 + (-18)
27 = 11 +16
28 = 32 + (-4)
29 = 40 + (-11)
30 = 5 x 6
31 = 17 + 14

32 = -8 x -4
33 = -3 x -11
34 = -6 –(-40)
35 = 5 x 7
36 = -6 x -6
-1 = -33 ÷ 33
-2 = 48 ÷ -24
-3 = 10 - 13
-4 = 2 x -2
-5 = 45 ÷ -9
-6 = -2 x 3
-7 = -2 + -5
-8 = -48 ÷ 6
-9 = 3 x -3
-10 = 1000 ÷ -100
-11 = -55 ÷ 5
-12 = -3 x 4
-13 = -30 – (-17)
-14 = 2 x -7
-15 = -5 x 3
-16 = -4 x 4
-17 = 25 - 42
-18 = 30 - 48
-19 = -25 + 6
-20 = -4 x 5
-21 = -12 + -9
-22 = 66 ÷ - 3
-23 = -26 – (-3)
-24 = 3 x -8
-25 = -200 ÷ 8
-26 = -36 + 10
-27 = -17 + -10

-28 = -7 x 4
-29 = - 40 –(-11)
-30 = -6 x 5
-31 = -40 + 9
-32 = -96 ÷ 3
-33 = -38 + 5
-34 = 20 - 54
-35 = 5 x -7
-36 = -52 + 16

multiplying & dividing integer notes solutions

multiplying & dividing integers practice

1. The black squares in the model below each represent -1.
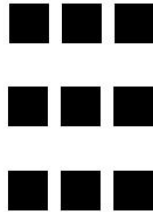
   Which expression is NOT modeled by this picture?

   A) -3 + -3 + -3
   B) -3 x 3
   C) (-1 + -1 + -1) x 3
   D) 3 x 3

2. Ivan spent $10 every month for 5 months. This money was subtracted from his bank account. Which expression represents the total change in Ivan's bank account?

   A) 5 – 10
   B) -10
   C) -10 x 5
   D) -10 ÷ 5

3. In the model below, black squares represent -1 and white squares represent +1.
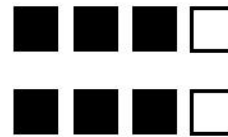
   Which expression is NOT modeled by this picture?

   A) (-3 + 1) x 2
   B) -3 + 1 – 3 + 1
   C) 8 ÷ 4
   D) -6 + 2

4. The temperature dropped a total of 18 degrees in three months. Which expression represents the average change of temperature each month?

   A) -18 ÷ 3
   B) -18 x 3
   C) -18 + 3
   D) -18 – 3

5. Franz lost 2 pounds in January, 4 pounds in February, 1 pound in March and 2 pounds in April. In May, Franz gained 3 pounds back. Which expression represents his average weight change per month?

   A) (-2 + -4 + -1 + -2 + -3) ÷ 5
   B) (2 + 4 + 1 + 2 + 3) ÷ 5
   C) (-2 + -4 + -1 + -2 + 3) ÷ 5
   D) (-2 + -4 + -1 + -2 + 3) x 5

Appendix B5: Applications of Similar Shapes

applications of
similar shapes

TEKS: 2d, 9b, 14b, 14c

Time Required:
90 minutes

lesson summary

In this lesson, students will solidify their understanding of similar shapes by working through a real-world problem. The students step into the problem at hand by actively seeking out data and interpreting it. Two versions are presented: one that is outside and one that is indoors.

Objectives:

- Students will be able to apply their understanding of similar shapes to real-world applications.

u nderstand

The teacher begins by introducing the problem at hand. For the **outdoor** version of the lesson, he or she begins by asking students how they would go about measuring a very tall object: one that is too tall to reach. The teacher may allow the students to discuss the problem and offer various suggestions. The teacher then informs the students that today they in fact will be doing just that: measuring a tree.

For the **indoor** version, the teacher should also ask the students how they would go about measuring a very tall object. After a discussion, the teacher will inform the students that today they will be measuring the heights of objects in the classroom.

Prerequisites for this Lesson:

- Knowledge of how the proportions between similar shapes correspond.

i nvestigate

Outdoor Version: The teacher pairs up the students. Each pair receives a ruler (or measure tape) and a copy of the **Applications of Similar Shapes Lab Worksheet-Outdoor**. The students then go outside. They are instructed to find a tree whose height is too tall for them to measure in the conventional manner. The teacher asks for at least two pairs of students to share the same tree. The pair of students must then stand in the location of their tree. They must measure their own heights and the lengths of their shadows. Then, they must also measure the length of their tree's shadow.

UNIT 4

Materials:

- Meter sticks (or measuring tape)

- Flashlights (for indoor version)

- Ramps (for indoor version)

Support and Attachments:

- Applications of Similar Shapes Lab Worksheet-Indoor

- Applications of Similar Shapes Lab Worksheet-Outdoor

- Applications of Similar Shapes TAKS Questions

Indoor Version: Thisis version is better if because of weather or other circumstances, the class cannot go outside. The teacher begins by pairing off the students and giving each student a copy of the **Applications of Similar Shapes Lab Worksheet-Indoor**. Each group should receive a flashlight and a ruler or meter stick. Additionally, if possible, the teacher should hand out mini wooden ramps (which could be created by the school's wood shop). If ramps are not available for the students to use, the teacher may ask the students to place a book (or two) underneath the flashlight. This is to help the students keep the angle consistent.

The students are to begin by picking three items in the class that are directly above one another. The lowest object should be low enough to measure. The students measure this height using the ruler and write that value in the table. Then, they rest their flashlight on the ramp and move the ramp backwards or forwards until the light is shining on the object. The students measure the distance from the end of the ramp to the object and write this in the table. The students proceed by moving the ramp backwards until the light shines on the middle object. They measure this distance and write it in the table. Finally, they move it back one more time until the light shines on the tallest object. The students measure this distance and write it in the table. See the drawing below.
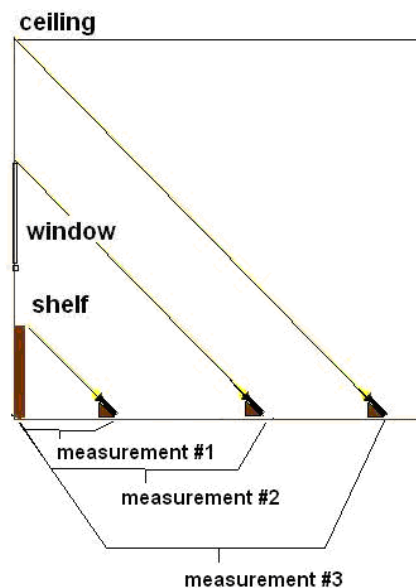
applications of
similar shapes

d iscover

Outdoor Version: Once all the data is recorded, the class goes inside. With their data, the students are instructed to calculate how tall the tree is. The teacher may point out that the students were creating similar triangles while gathering data.

Indoor Version: After finding these measurements, the students must then use their understanding of similar shapes to calculate the height of the second and third objects. The teacher may point out that the students were creating similar triangles while gathering data.

a pply

After having completed one of the above activities, the students should be able to understand how useful similar triangles are. However, students need additional practice applying similar shapes (not just triangles) in other situations. The Applications of Similar Shapes TAKS Questions provides such reinforcement.

Vocabulary / Definitions:

- **Similar Shape**—Two shapes are similar if they are they have the same shape. They do not have to be the same size.

Assessment:

Applications of Similar Shapes TAKS Questions

applications
of similar shapes
lab worksheet-outdoor

How tall is that tree?!

Instructions:

1. When you are outside, you and your partner must pick a tree to measure.

2. Stand by your tree.
    a. Measure your height in centimeters.
    b. Measure the length of your shadow in centimeters.
    c. Measure the length of the shadow of the tree in centimeters.

    Write these **3** values in the table below:

Your Height	_____ centimeters
Length of Your Shadow	_____ centimeters
Length of the Shadow of the Tree	_____ centimeters

3. Draw a picture of you and your shadow:

4. Draw a picture of the tree and its shadow:

5. Using similar triangles, calculate the **height of the tree**.

MathTAKStic!
UNIT 4

applications
of similar shapes
lab worksheet-indoor

Flashlight Measurements

Instructions:
1. Pick three objects in the classroom to measure. These objects must be directly above one another.
2. Write the names of the objects in the table below.
3. Measure the height of the object that is closest to the ground. Write this value in the table below.
4. Lean your flashlight on your ramp. Turn it on.
5. Move the flashlight back until the light is shining on the top of the **shortest** object. Measure how far away the back of your ramp is from the wall. Write this in the table below.
6. Move the flashlight back again until the light is shining on the top of the **middle** object.   Measure how far away the back of your ramp is from the wall. Write this in the table below.
7. Move the flashlight back one more time until the light is shining on the top of the **tallest** object.   Measure how far away the back of your ramp is from the wall. Write this in the table below.
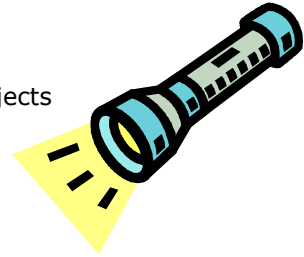
Name of Object	Distance from the Object to the End of the Flashlight	Height of Object
	Measure this:	Measure this:
	Measure this:	Calculate this:
	Measure this:	Calculate this:

8. Draw a picture of the flashlight shining on the wall. What shape does the light beam, the ground, and the wall make?

9. Calculate the height of the middle object and the tallest object using the data in your table and your knowledge of similar shapes. Show your work below.

1. Hector wants to measure the height of his door using a flashlight, as illustrated in the picture below. The doorknob is 3 feet from the ground. He positions the flashlight 6 feet away from the door and shines the light on the doorknob. Then, he keeps the flashlight at the same angle and moves back until he is 15 feet away from the door so that the light is shining on the top of the door. How tall is the door?
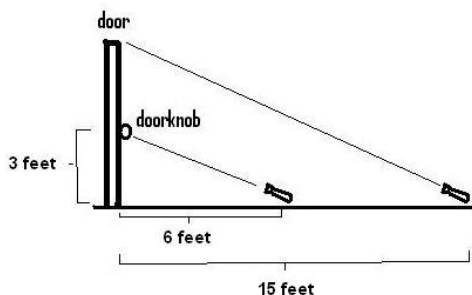
A) 2 feet
B) 3 feet
C) 7.5 feet
D) 9 feet

2. Ms. Jones made a scale drawing of a room. Her drawing has a width of 5 inches and a length of 8 inches. Ms. Jones' actual room has a width of 4 meters. What is the length of Ms. Jones' room?

A) 2.5 meters
B) 6.4 meters
C) 8.3 meters
D) 10 meters

3. Sandra has a height of 5 feet. Her shadow measures 2 feet long. A tree she is standing next to has a shadow measuring 10 feet long. How tall is the tree?

A) 1 foot
B) 4 feet
C) 20 feet
D) 25 feet

4. A factory is making two sizes of trapezoid-shaped tables. The tables have the same shape. The smaller table has bases of lengths 80 centimeters and 132 centimeters. The larger table has a shorter base length equal to 180 centimeters. What is the longer base length of the larger table?

A) 63
B) 109
C) 297
D) 392

5. A company produces rectangular cookies and rectangular cakes with similar dimensions. The cakes have lengths of 12 inches and widths of 8 inches. The cookies have widths of 1.2 inches. How long are the cookies?

A) 1.8 inches
B) 2.8 inches
C) 4 inches
D) 13.3 inches

Appendix B6: The Pythagorean Theorem

The Pythagorean Theorem

TEKS: 1c, 4a, 7c, 14b, 14c, 16a, 16b

Time Required:
90 minutes

Objectives:
- To foster an understanding of what the Pythagorean Theorem is and why it is true.

- To develop students' skill in logical thinking and proof-based arguments.

- To enrich students historical knowledge.

- To develop students' capabilities to solve basic problems using the Pythagorean Theorem.

Prerequisites for this Lesson:

- An understanding of squares of numbers.

- Ability to calculate the area of basic shapes.

- An understanding of what a right triangle is.

- Capability of calculating square roots.

lesson summary

Pythagoras's Theorem is one of the most useful and well-known theorems in geometry. In this lesson, students not only learn *what* the theorem is, but also *why* it is true. The activity helps students formulate a solid understanding of the theorem while building logical thinking and problem solving skills. Moreover, the lesson includes a history of Pythagoras, serving as further motivation for the students and lending to their understanding of mathematical history.

U nderstand

The teacher presents a hypothetical situation, presented below, whose solution requires the Pythagorean Theorem. The teacher simply presents the problem, but leaves the solution unanswered, generating curiosity in the students.

> **PROBLEM:** Suppose you are running late for school. You have only 5 minutes until the bell rings. You could walk around the courtyard, on the sidewalk, or you could cut across the courtyard. If the courtyard looks like the picture below, which is the shorter route?

Materials:

- Rulers: one for each pair or group (about $0.10 each)

- Triangles: one set for each pair or group (the teacher may make a class set-found in Appendix B0)

- Pythagorean Lab Worksheet

- Pythagorean Pieces (one set for each student)

- Pythagorean Worksheet

- Scissors (one for each student in the class)

- Projector and Computer (for the optional **Pythagoras History** PowerPoint)

Support and Attachments:

- History of Pythagoras PowerPoint

- Teacher Guide

- Pythagorean Pieces

- Pythagorean TAKS Questions

- Triangles Handout

- Pythagorean Lab Worksheet

The teacher should continue with the problem by stating this observation to the students:

> Obviously, we could measure this distance, but you don't have time for that. How do we know what the distance could be?

He or she should allow the students to think of ideas.

i nvestigate

Now that the students' curiosity is peaked, the teacher asks the students to experiment to find the answer. She/he hands out the **Pythagorean Lab Worksheet** along with a set of triangles from Appendix B0. The teacher may even encourage students to bring in their own triangles, rectangles (can measure the diagonal), rectangular shaped objects or triangular shaped objects from home (provided the triangles are right triangles). Students work in pairs or in groups of three, measuring the triangles and filling out the charts on the worksheet.

d iscover

Once students have completed the Pythagorean Lab worksheet, the teacher brings the class together again. He/she asks the class what trends are apparent in the table. How do a^2 and b^2 relate to c^2? The students should eventually come to the conclusion that $a^2 + b^2 = c^2$. Now, the teacher may note that this is indeed a great *hypothesis*, but how does one know that it is *always* true?

The teacher may explain that in mathematics, a hypothesis is called a *conjecture*. Here, the class has conjectured that $a^2 + b^2 = c^2$. In order to show that it is true, the teacher must demonstrate a proof. The teacher proceeds to pass out the **Pythagorean Pieces** handouts (either Proof #1 or Proof #2, at the discretion of the teacher). The students cut out the pieces. Then, the teacher guides them through the proof (explained in **Teacher Guide**). At the end of the proof, the students should understand that it validated the Pythagorean Theorem for *all* triangles.

The Pythagorean Theorem

Finally, the teacher may conclude by defining the Pythagorean Theorem formally, by name:

In a right triangle with legs of length a and b, and hypotenuse of length c, the following is true:

$$a^2 + b^2 = c^2$$

The teacher should reinforce the theorem by having students copy it in their notes.

The teacher may go over some simple examples (from **Teacher Guide**) of how to use the theorem in geometry problems. A **Pythagorean Worksheet** is included to be used for in class or out of class work.

Another Version (for a less advanced class): After the students have completed the first part of the lesson, creating a conjecture, the teacher may provide a simpler "proof" for the student. Each student picks one of the triangles used in the first part (making sure there are a variety of triangles chosen). The student is instructed to measure the sides using grid paper and create squares out of the grid paper. Thus, each side of the right triangle should have a corresponding square. The students then glue their triangle on a piece of construction paper and glue the squares along side each corresponding side, as illustrated below. The students should see that the area of the two smaller squares (which can be obtained simply by counting grid squares) is equal to the area of the largest square, showing that the formula works for this particular triangle. Although this "proof" only verifies the theorem for specific cases, it helps aid students to thinking logically.
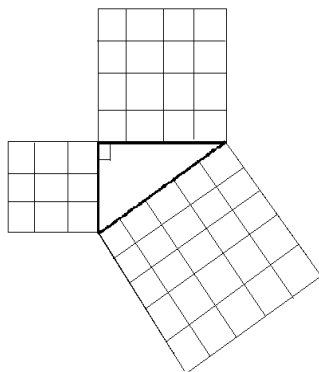
Vocabulary / Definitions:

- **Right Triangle**— A triangle with an angle of measure 90 degrees.

- **Hypotenuse**— The longest side of a right triangle.

- **Legs**—The shorter two sides of a right triangle.

- **Pythagorean Triple**—A triple of positive integers satisfying the Pythagorean Theorem.

- **Conjecture**—A mathematical hypothesis

- **Proof**—A logical, step-by-step argument for showing that something is true.

Assessment:

Pythagorean TAKS Questions

The Pythagorean Theorem

a pply

Now, the teacher may return to the example problem given at the very beginning of the lesson. He or she may conclude the problem as so:

With the Pythagorean Theorem, this problem becomes easy. One can see that:

$$5^2 + 12^2 = 169 = 13^2$$

Therefore, the distance to the door is 13 meters, which is much shorter than 12 + 5 = 17 meters (especially when running late!).

Additionally, the teacher may go over some simple examples (from **Teacher Guide**) of how to use the theorem in geometry problems. A **Pythagorean Practice Worksheet** is included to be used for in class or out of class work.

lesson extensions

Students learn about the mathematician Pythagoras by going over the PowerPoint slideshow provided (included in supplemental CD). This integration of history into mathematics makes the subject more real for some students and simply more fascinating for others.

The Teacher Guide includes an extra proof that the teacher may use. These might may serve as an extra credit project for a student, as further reinforcement of concepts for the class, or as means for a class competition.

The Pythagorean Theorem

teacher guide

This Teacher Guide includes Example Problems and explanations of three proofs of the Pythagorean Theorem.

The Example Problems should be done after the presentation of the proof, depending on how much reinforcement and practice the students need.

EXAMPLE PROBLEMS:

PROBLEM #1:

The teacher presents the following scenario:

Suppose you are in a park and there is a lamp post casting a shadow to the ground, like in the picture below.

Suppose you know that the height of the lamp post is 4 meters and you measure that the length of the shadow is 3 meters. But, what if you needed to know the distance from the top of the lamp post to the end of the shadow? There is no easy way of measuring this!

This is where the Pythagorean Theorem becomes incredibly handy.   What shape does the lamp post, its shadow, and the distance from the top of the lamp post to the end of the shadow make?

It makes a triangle! And, what do we notice about this triangle? Does it have any special properties?

Yes! This triangle is a right triangle! Now, if Pythagoras were here right now, he would remind us of the following:

$$a^2 + b^2 = c^2$$

What does this have to do with our problem?
What is the length of the legs of our triangle?—3 meters and 4 meters
What is the length of the hypotenuse?—We don't know, but this represents c in the formula.

So, according to Pythagoras's Theorem, $3^2 + 4^2$ = (the length between the top of the lamp post and the end of the shadow)^2. This means that the length we are looking for has a square of 25. What could that be?—5 meters.

_____

PROBLEM #2:

Using the triangle shown, verify that the triangle is, in fact, a right triangle by using the Pythagorean Theorem.
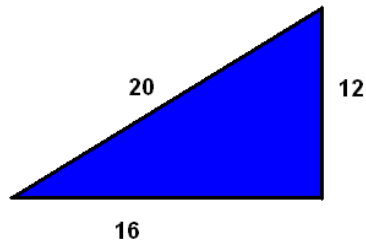
SOLUTION:

$$12^2 = 144$$
$$16^2 = 256$$

Since 144 + 25 6= 400, and 20^2 = 400, then the triangle is a right triangle.

PROBLEM #3:

The triangle below shows the values for the lengths of the legs of a right triangle. What is the length of the hypotenuse?
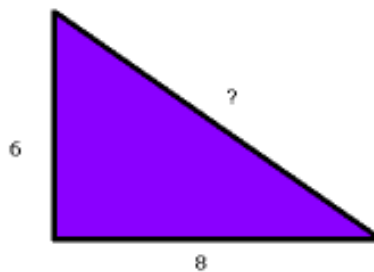
SOLUTION: Using the Pythagorean Theorem,
$$6^2 + 8^2 = 36 + 64 = 100$$

What squared will make 100? 10^2 = 100.
Thus, the hypotenuse has length 10.

Pythagorean lab worksheet

What's Up With These Right Triangles!?

Measure the sides of your triangles with a ruler.

Let **a** and **b** be the shorter sides of your triangle.
Let **c** be the longest side of your triangle (the hypotenuse).

Enter the values you get in the table below.
Calculate the values for the squared terms.
Do you see a pattern in the squares?

Triangle Number	a	b	c	a^2	b^2	c^2

PROOFS OF THE PYTHAGOREAN THEOREMS:

PROOF #1:

For this proof, students use the Pythagorean Pieces for Proof #1 on the following page. Instruct them to cut out the two squares. The students may also create their own squares to use in the proof. They then can follow the following sequence to prove the Pythagorean Theorem:

b

b

Mark off length a
on the square with
side length b.

b

a

a

Move the small square and tape together.
Since the total length is a + b and we marked off a, the rest
is b.

b c

c

a

a   b

Draw the two diagonals. You have
created two right triangles with legs
a and b. Label each hypotenuse c.

b c

c

a

a   b

Cut on the diagonals and
move the two triangles.

c

c

You have created a
square with side c from
two squares with sides a
and b.

So, we have that:
$c^2 = a^2 + b^2$
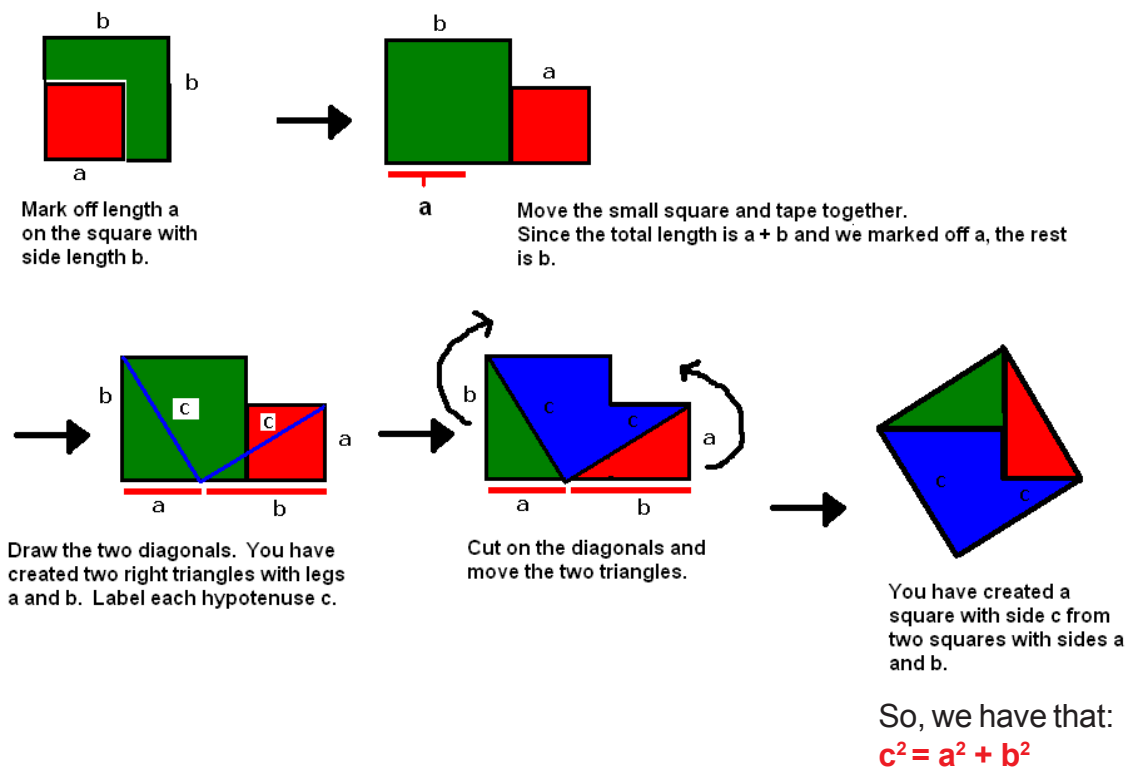
The Pythagorean Theorem

PROOF #1:

a

b
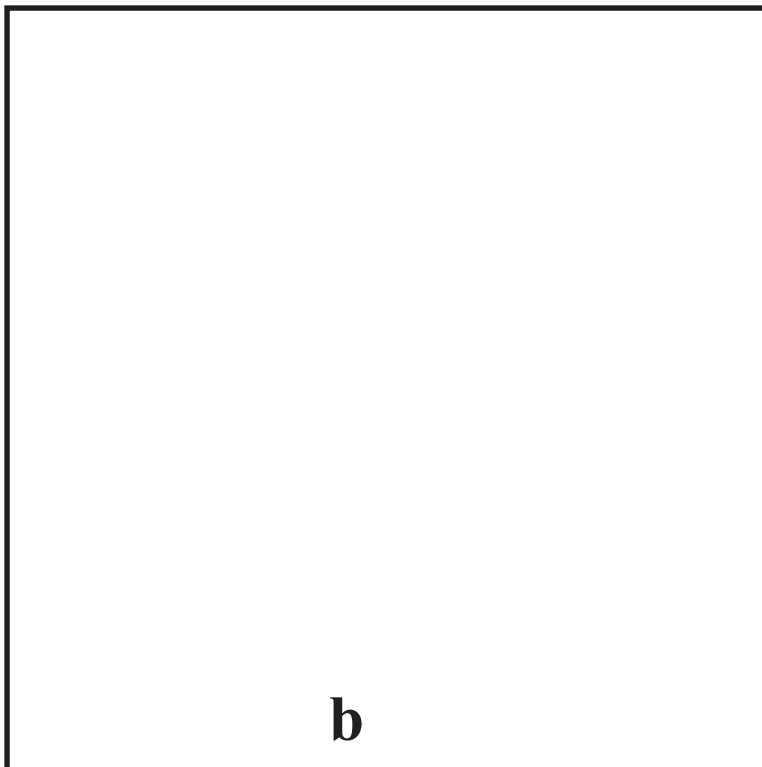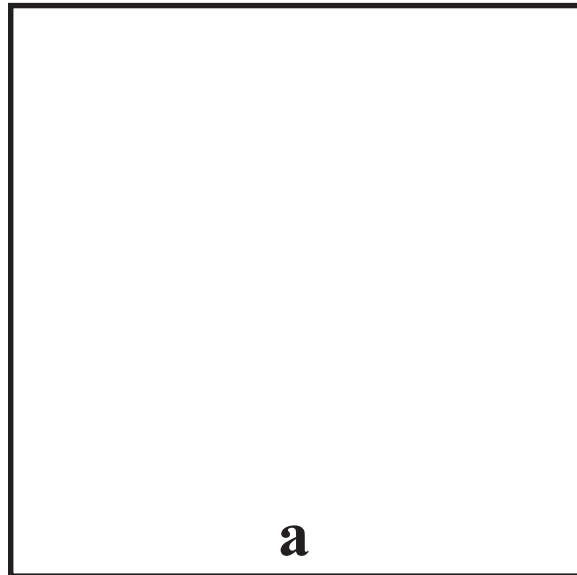
PROOF #2:

This proof may be done purely geometrically, or may include algebraic concepts for advanced students. Thus, it serves as a great enrichment activity for an Algebra I student.

VERSION #1:

Students first start out with the square given in the Pythagorean Pieces worksheet shown below.
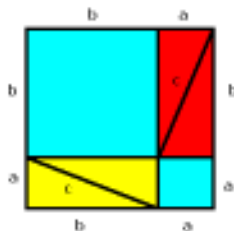
The teacher explains the relations of the lengths to the students as they are noted on the worksheet. They must then cut out the pieces of this square.

First, the teacher inquires as to what the areas of the two squares are. Students should notice that the smaller square has area a^2 and the bigger square has area b^2. The teacher instructs the students to put aside the two squares. The teacher then asks the students what the base and height of the 4 triangles are (they are a and b, respectively). Students must also understand that the total area of the large square is compromised of the areas of the two squares and the triangles.

The students then cut out the square labeled with side length c. The teacher gives the students the task of filling in the grey square provided on the worksheet with the four triangles they cut out and the square with side length c. If they do it correctly, they will obtain the following:

What does this tell us? That,

*A square with area **a** + a square with area **b** + the 4 triangles = A square with area **c** + the 4 triangles*

Thus, since the 4 triangles are in both sides of the equality, this leaves us with $a^2 + b^2 = c^2$.

For those students who need algebra enrichment, they may write down the measurements of the triangles and that of the big square (each side has length *a+b*). Then, the students must calculate the area of the large square. They should find that the area is:

$$(a + b)^2 = a^2 + 2ab + b^2$$

Next, the students must calculate the area of the individual pieces inside the square. They should find that the area of each triangle is ab/2. Because there are four of them, their total area is 4(ab/2) = 2ab. The area of the square is c^2.
Thus, they may conclude that:

$$a^2 + 2ab + b^2 = 2ab + c^2$$

or

$$a^2 + b^2 = c^2$$

because the 2ab's cancel.

VERSION #2:

In this version, the students start with this square:
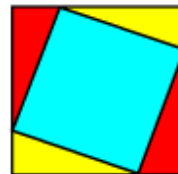
They cut up this square into pieces. They then cut out the square with side length *a* and the square with side length *b*. They are then instructed to fit the square with side length *a* and the square with side length *b* and the four triangles into the grey square. They should obtain the following:
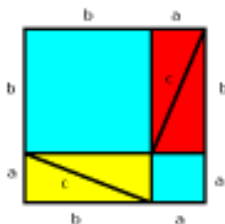
The Pythagorean Theorem follows, just as in Version #1 of this proof.
_____

NOTE: It is recommended to the teacher to print pages 1 and 2 on different colored paper (for Version 1) and pages 4 and 5 different colors (Version 2) to help students better visualize the theorem.

PROOF #2: page 1   **VERSION 1**
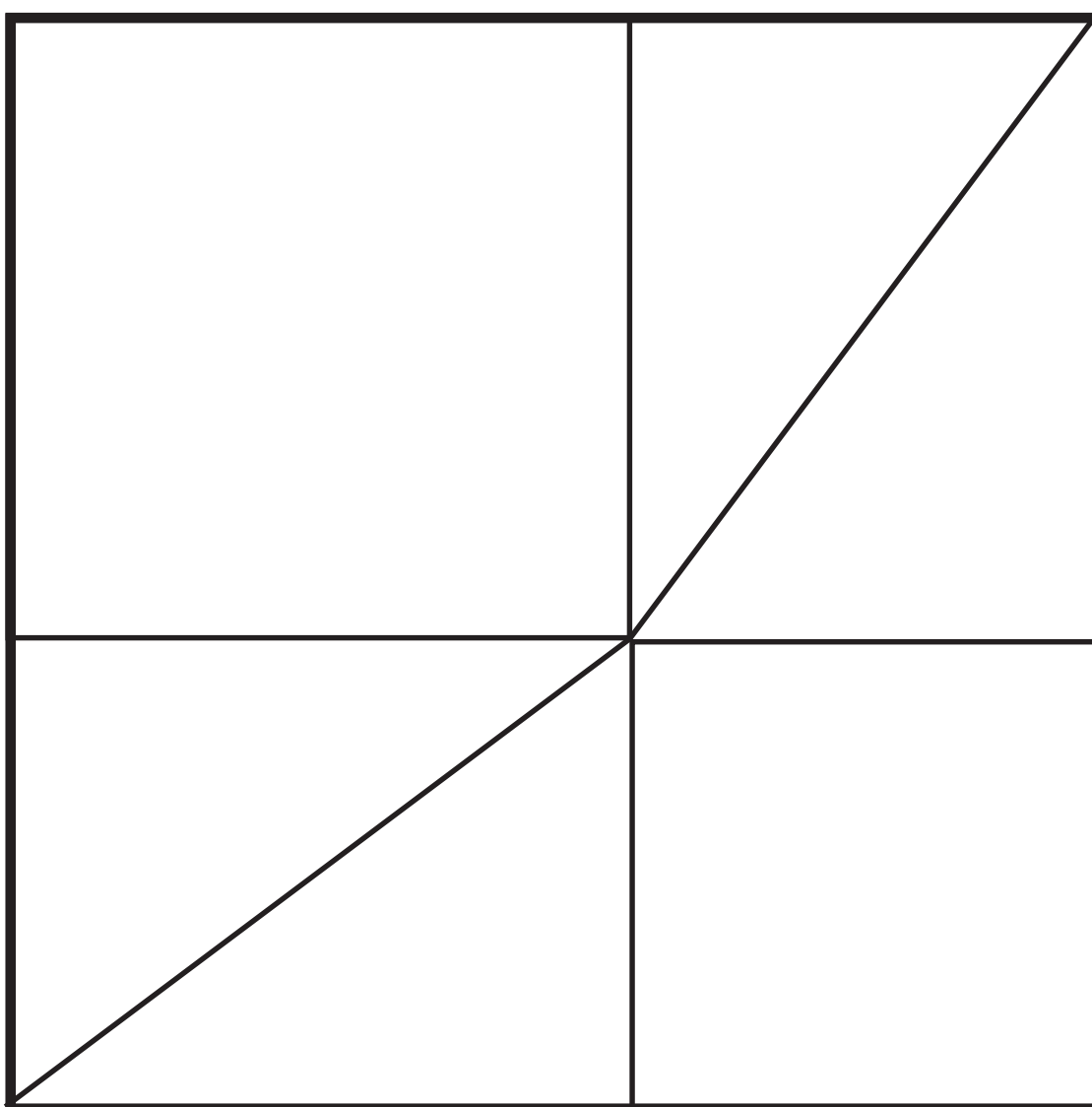
Cut out these pieces.

teacher guide

The Pythagorean Theorem

PROOF #2: page 2   **VERSION 1**
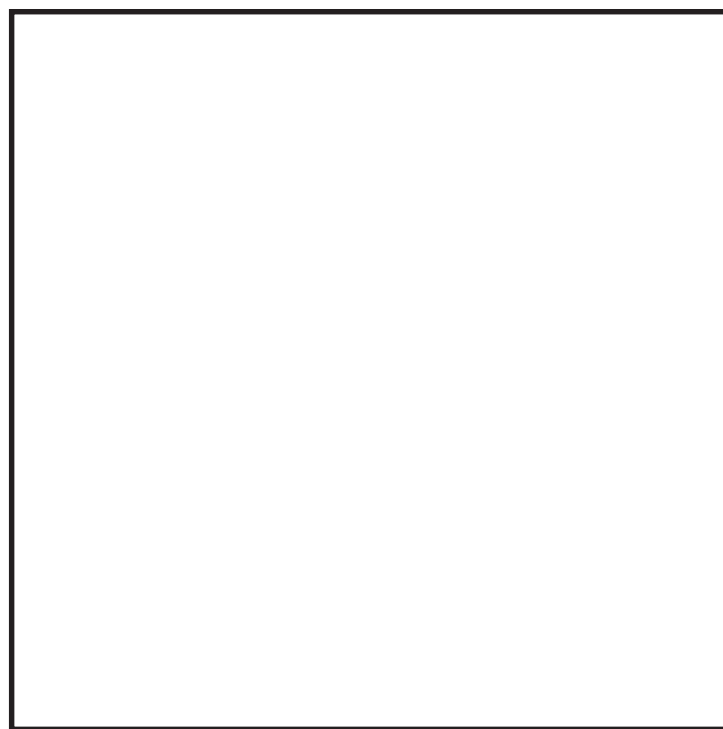
Cut out this piece.

MathTAKStic!
UNIT 5

PROOF #2: page 3   **VERSION 1**
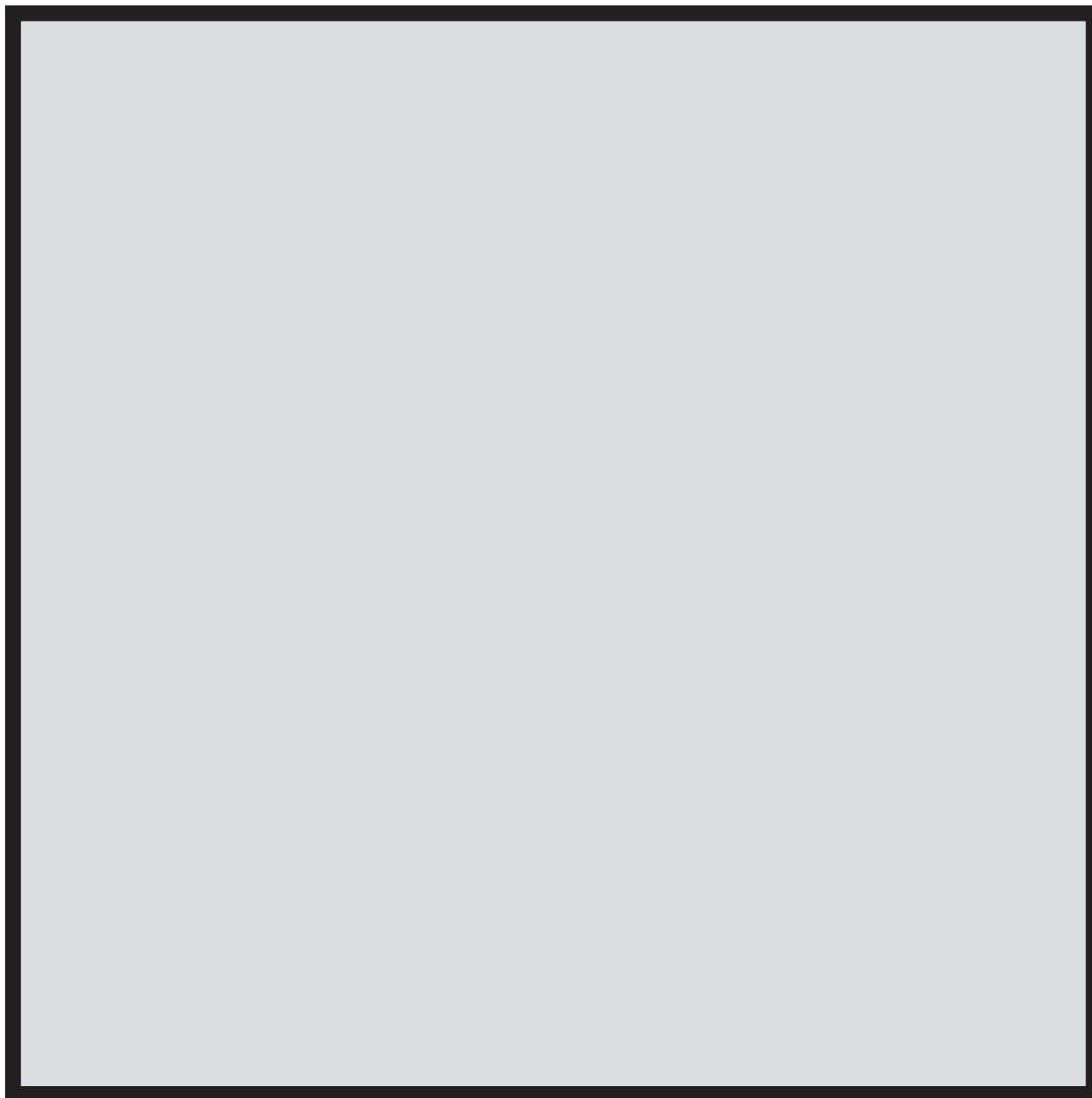
Do not cut out this piece.

teacher
guide

The Pythagorean
Theorem

PROOF #2: page 4   **VERSION 2**
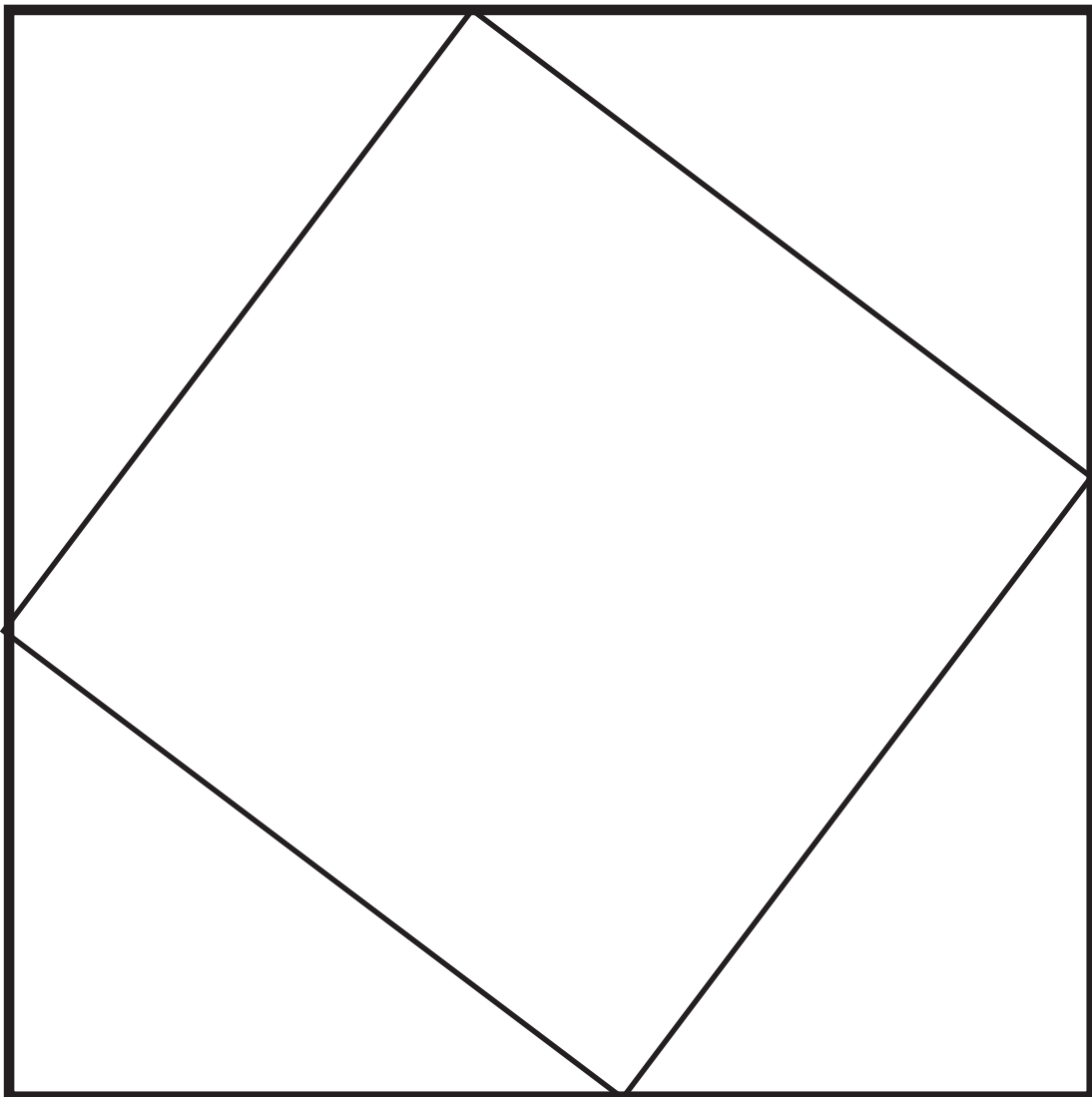
Cut out these pieces.

PROOF #2: page 5   VERSION 2
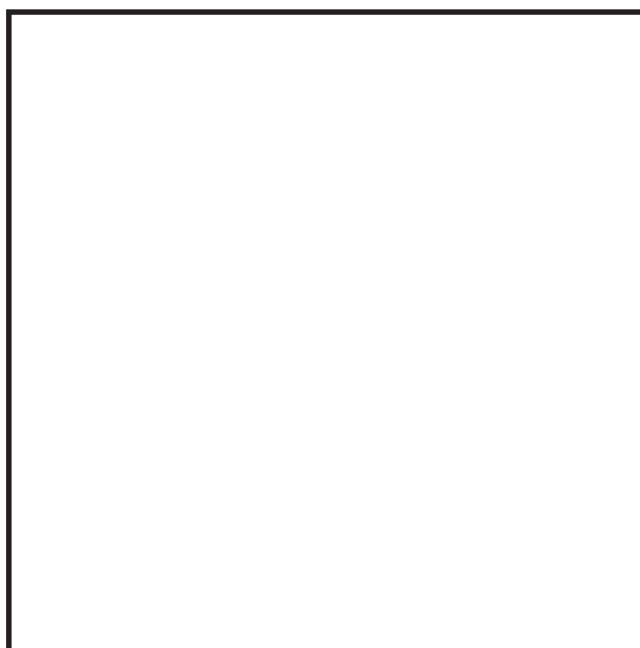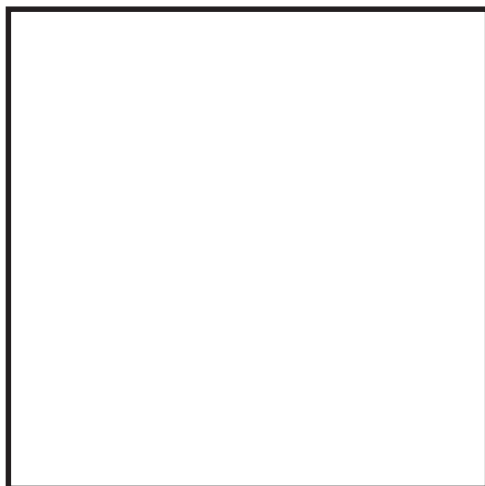
Cut out these pieces.

teacher guide

The Pythagorean Theorem

PROOF #2: page 6

VERSION 2

Do not cut out this piece.

1. The triangle below is a right triangle. One side has length 3 and the other has length 4. What is the length of the longest side?

4

3

A 5

B 6

C 7

D 10

2. The triangle below is a right triangle. One side has length 5 and the other has length 12. What is the length of the third side?

12

5

A 10

B 13

C 15

D 17
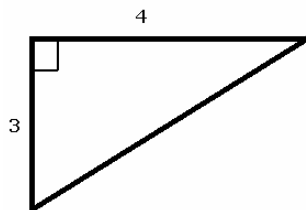
3. The triangle below is a right triangle. One side has length 8 and the other has length 10. What is the length of the third side?

8

10

A 2

B 4

C 6

D 9
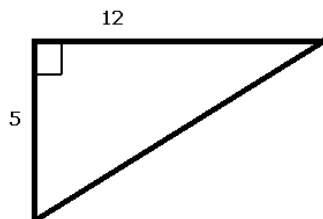
4. The triangle below is a right triangle. One side has length 21 and the other has length 29. What is the length of the third side?

21

29

A 10

B 15

C 20

D 25
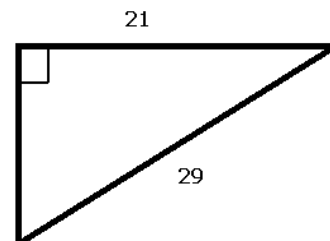
5. The sides of three squares can be used to form triangles. The areas of the squares that form right triangles have a special relationship.

   The triangles in the drawing below is a right triangle.

   What must be the area of square 2 for the right triangle to exist?

   ?
   Square 2

   49
   Square 1

   625
   Square 3
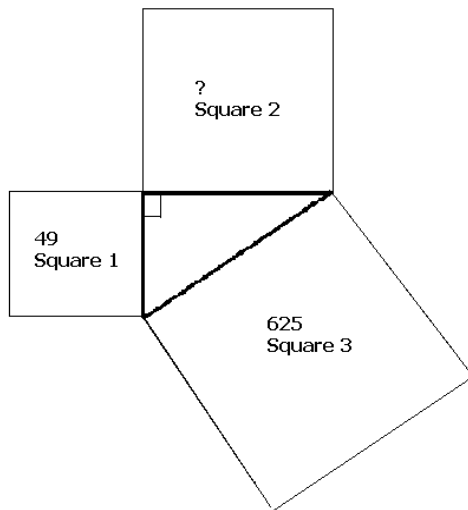
   A 75

   B 126

   C 344

   D 576

6. The sides of three squares can be used to form triangles. The areas of the squares that form right triangles have a special relationship.

   The triangles in the drawing below is a right triangle.

   What must be the area of square 3 for the right triangle to exist?

   576
   Square 2

   100
   Square 1

   ?
   Square 3
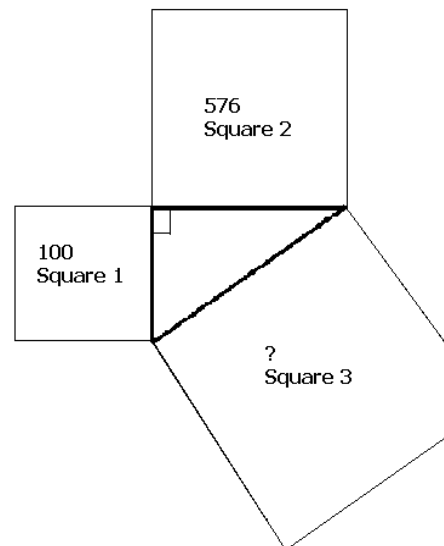
   A 470

   B 600

   C 676

   D 830

Appendix B7: Applications of the Pythagorean Theorem

applications of
the Pythagorean Theorem

TEKS: 1c, 9a, 14a, 14b, 14c

Time Required:
90 minutes

lesson summary

Prior to this lesson, students have learned what the Pythagorean Theorem is and how to use it, but they have little exposure to its importance in real life. In a collaborative group project, students design their own real-world problem by collecting data from their surroundings at school. When all the problems are designed and written, the teacher distributes them to the entire class to be worked on. In this manner, students are able to have an integral part in the creation of the mathematics and the lesson.

U nderstand

To introduce the activity, the teacher may ask students, "Where do you think the Pythagorean Theorem might be useful in real life?"

If this question elicits no response, the teacher may instead ask a broader question, "Where do you see right triangles around you?"

The teacher may create a list of student responses on the board or on an overhead projector. The list may include the following:

- ■ Calculating the distance from the top of a tree to the tip of its shadow
- ■ Calculating the diagonal distance across a large field
- ■ Calculating the Diagonal of a T.V. (when T.V.s are sold, they are labeled by this distance)
- ■ Calculating the length of a fishing line needed to catch a fish a certain distance away
- ■ Calculating the radius of a circle
- ■ Calculating the distance from Home Plate to Second Base in a baseball diamond (a base ball diamond is actually a square)
- ■ Calculating the length of a ladder needed to reach a specific height or measuring the height based on a specific ladder length
- ■ Calculating how fast a car is traveling using police radar

Learning Objectives:

- ■ To see first-hand how the Pythagorean Theorem can be useful in everyday life.
- ■ To practice writing and developing a mathematics problem.
- ■ To practice linear measurement using tools.
- ■ To practice real-world problems involving the use of the Pythagorean Theorem.

Prerequisites for this Lesson:

- ■ An understanding of squares and square roots of numbers.
- ■ An understanding of how to use the Pythagorean Theorem

Materials:

- Rulers: one for each pair or group (about $0.10 each)

- Calculators

- Computers and Printer (optional)

Support and Attachments:

- Pythagorean Applications Lab Worksheet

- Pythagorean Applications TAKS Questions

d iscover

Since the students have brainstormed possible real-world scenarios involving the Pythagorean Theorem, they are now ready to embark on their own mission: to find a problem around their own school!

The teacher begins by dividing up the class into groups of three. The teacher explains to the students that they must find somewhere where the Pythagorean Theorem could be used to find a certain distance. Depending on the circumstances, the teacher will define an area to search for such a problem. If the students are well-behaved, he or she might send groups to particular areas, such as the cafeteria or the library.

Students must "determine" their problem scenario, and make a note of it on their **Pythagorean Applications Lab Worksheet**. The teacher should encourage students to find problems which require one to calculate the legs of the right triangle rather than the hypotenuse.

i nvestigate

Once students have found the problem they wish to expand on, they must collect the data required to create their problem. Using the Pythagorean Applications Lab Worksheet as a guide, students use the rulers or other available measurement tools to find any appropriate measurements.

For instance, suppose a group chose to create a problem based on finding the diagonal length of the classroom. The students would first measure the length and width of the room and write these on the worksheet.

a pply

Once the students have collected their data, they now must actually construct the problem. Students are encouraged to use creativity in the creation of their word problem. If the students have access to a computer, they may type up their word problem. Once the problem is written, each group must also solve their problem and write out the solution, with all work included, on the **Pythagorean Applications Lab Worksheet** to turn in to the teacher.

applications of the Pythagorean Theorem

lesson extensions

To finalize the lesson, the teacher may make a worksheet containing all the problems created by the students. The students may work on the assignment in class. Afterwards, as a check, one member from each group may present the solution and how to obtain the solution.

In solving the problems, the teacher may want to choose to not allow calculator usage. On the TAKS test, students must be able to estimate an approximate whole number solution to similar problems.

For instance, consider the following problem:

Roger is buying a television. Televisions are sold according to the length of their diagonal. He wants a television that measures 16 inches by 20 inches. What size of television does Roger need to buy?

SOLUTION: $16^2 + 20^2$ = the square of the diagonal

656 = the square of the diagonal

I know that $20^2 = 400$. This is smaller than 656, so our diagonal must be larger than 20.

I guess that 23 might work, so I multiply 23 by 23 to get 529. This is still too small.

I now try 25 x 25 = 625. This looks right.

26 x 26 = 676. So I know my diagonal is between the length of 25 inches and 26 inches.

Obviously, this is not the most efficient method of obtaining an answer. The teacher may want to point out that on the TAKS test, the student will have solutions to work from. That is, they can square the given solutions to see which comes out the closest.

Vocabulary / Definitions:

- **Right Triangle**—A triangle with an angle of measure 90 degrees.

- **Hypotenuse**—The longest side of a right triangle.

- **Legs**—The shorter two sides of a right triangle.

Assessment:

Pythagorean Applications Lab Worksheet

Pythagorean Applications TAKS Questions

MathTAKStic!
UNIT 5

applications of the
Pythagorean Theorem
lab worksheet

Task: You and your group must create a math problem using the following criteria:

- The problem must be a real-world problem.
- The problem must be solved using the Pythagorean Theorem.

Follow these steps to help you complete your task.

STEP ONE: Pick a problem topic

What is your problem going to be about? _____

What will you need to measure to create

your problem? _____

STEP TWO: Make Measurements

Measure the object that you need to measure.
Draw a picture of your object or scenario,
labeling it with your measurements.

STEP THREE: Write your problem.

Write your problem. Make sure to include a lot of detail!

STEP FOUR: Solve your problem.

Solve your problem. Show all work.

1. Doug is walking his dog with a leash measuring 10 feet long. He is holding the leash 4 feet above the ground. Approximately how far away is the dog from Doug?

10 feet
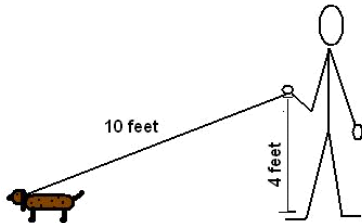
4 feet

A   4 feet

B   6 feet

C   9 feet

D   12 feet

2. A television screen has a diagonal length of 20 inches. If the width of the screen is 16 inches, how tall is the screen?

16 inches

A   10 inches

B   12 inches

C   16 inches

D   18 inches

3. WenYen, Richard and Asha are playing a game of freeze tag. Wenyen has already tagged Asha. Richard would like to tag Asha to unfreeze her. If the three make a right triangle, as shown below, and Richard is 14 meters away from WenYen and WenYen is 8 meters away from Asha, about how far does Richard have to run to unfreeze Asha?

WenYei

14 meters

8 meters

Richard

?

Asha
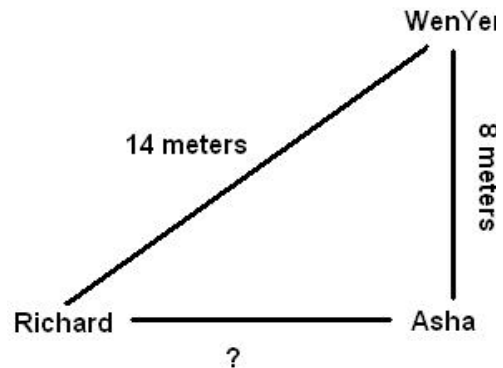
A   9 meters

B   11 meters

C   13 meters

D   15meters

applications of the
pythagorean theorem practice

4. A lighthouse shines its light all the way around, creating a circle. It spots two different towers in the outer rim of the light, ninety degrees apart. If these towers are 8 miles apart, what is the approximate length of the light beam?

8 miles

lighthouse
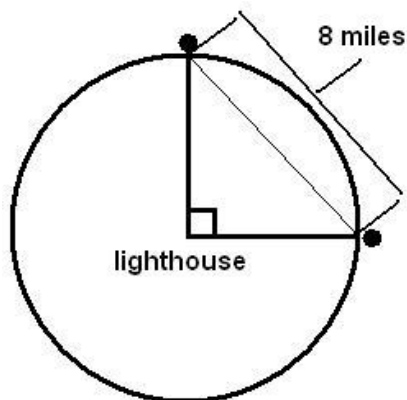
A 5.7 miles

B 6.4 miles

C 7.8 miles

D 8.3 miles

5. A boy is fishing. If the boy wants to catch a fish with a line 26 feet long and the end of the pole is 10 feet above the water, what is the farthest away that the fish could be.
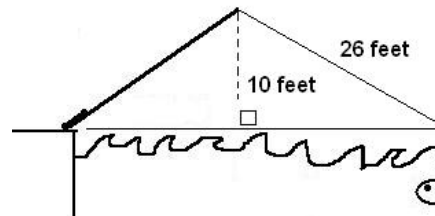
26 feet

10 feet

A 12 feet

B 16 feet

C 20 feet

D 24 feet

6. Frederick is working on the roof of his house. The base of the roof is 8 meters from the ground. If he plans to put the base of the ladder 4 meters from the house, at least how long of a ladder does Frederick need?
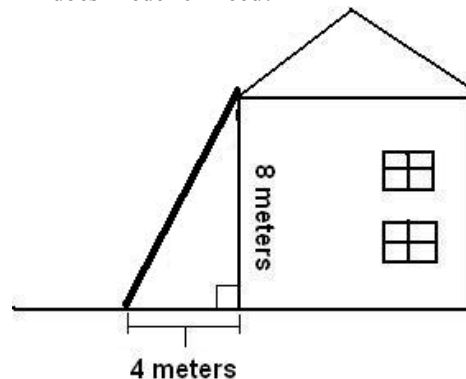
8 meters

4 meters

A 9 meters

B 11 meters

C 13 meters

D 15 meters

Appendix B8: Terrific Translations

terrific
translations

TEKS: 4a, 6b, 14b, 14c, 15a, 16a

Time Required:
180 minutes

lesson summary

In this lesson, students learn what a translation is and generate an understanding between the visual translation and the computational translation. Students also have the opportunity to create their own work of art and experience the connections between art and math.

U nderstand

The teacher shows the class a famous tessellation on the overhead. She/He then takes a cut out replica of the shape being translated and moves it on the overhead for a visual and loose description of translation. To find such a tessellation, the teacher may look up the artwork of M.C. Escher. A good website is http://www.mcescher.com/.

After the introduction, the students spend about 15-20 minutes developing a visual understanding of translations. The teacher may begin by giving the students a definition of translation. Then, he/she gives each student a copy of the **Grid Paper** handout and a piece of patty paper (available in teacher supply catalogues). Afterwards, the teacher leads the students through the **Translations Problems** (he/she may choose to give the students a copy of this). Students are expected to solve them by tracing the shapes on the patty paper, moving the patty paper the designated distance and, finally, retracing the shape onto the coordinate grid. Students may answer the questions verbally, on handheld white boards (if they are available) or on a piece of paper, at the teacher's discretion. Through this activity students should learn the following:

■   What a translation visually looks like

■   A translated shape maintains the same appearance and thus the same area (it is not stretched or shrinked)

■   How to describe a translation in words

Objectives:

■   To understand what a translation is and how to perform a translation on a shape

■   To understand the connections between the transforming coordinates one-by-one to applying a generalization to an entire set of coordinate points.

Prerequisites for this Lesson:

■   An understanding of translations of how to calculate area of rectangles and triangles

■   An ability to plot coordinate points in all four quadrants

■   An ability to add and subtract integers

Materials:

- Tesselation

- Overhead Projector

- Grid Paper (available in appendix B0)

- Grid Paper 2 (available in appendix B0)

- Patty Paper

- Notecards (one per student)

- Scissors (one per student)

- Tape (two pieces per student)

- Coloring Utensils

Support and Attachments:

- Grid Paper

- Grid Paper 2

- Translations Problems

- Translations Lab Worksheet

- Translations TAKS Questions

i nvestigate

To create an understanding of how to compute the new points in a translation, students participate in a lab activity. Each student receives a copy of the **Translations Lab Worksheet**, a piece of **Grid Paper 2** and more patty paper, if needed. The student is to draw any shape on the coordinate plane and then translate it however he/she pleases. If time is an issue, the teacher may choose to restrict what the student draws by limiting the shapes to only particular ones or to shapes with a certain amount of sides or less. The student then proceeds to fill out the chart on the worksheet, thus completing data collection.

d iscover

In order for the students to develop a method of computing translated coordinates, they answer the questions in the **Translations Lab Worksheet**. The questions are designed to help students discover that a translation is performed by adding a certain amount of units to each coordinate to obtain the new coordinates. Students should also see that all the geometric properties of the original shapes are preserved during translation.

a pply

To further practice using translations, students create their own art work by making a tessellation. For this activity, students will each need a square (it may be cut out from note cards or cardstock), a piece of blank paper, tape, and coloring utensils. The teacher may make the activity as simple or as complicated as he or she chooses. In a more complicated activity, after the students complete their tesselations, they may draw a coordinate grid on them. This would allow them to acurately calculate how many units the shape was translated.

Instructions for creating a tesselation are described on the following page.

terrific translations

1. On the square, draw a line, in any fashion from one corner to another. This will create two of the edges of your tessellation.
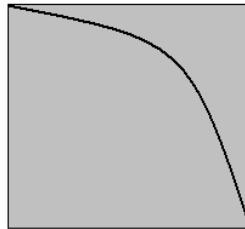
2. Cut along this line with the scissors.
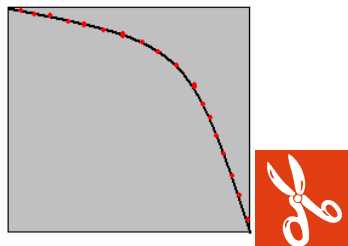
3. Tape one of the straight edges of the square alongside the opposite edge of the square, as pictured below.
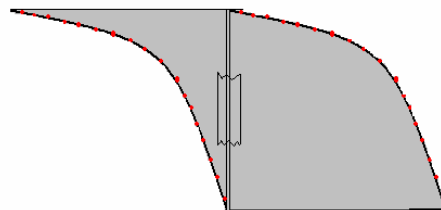
4. This becomes a tessellation model. On the blank sheet of paper, trace this design repeatedly, so that the pieces fit into one another.
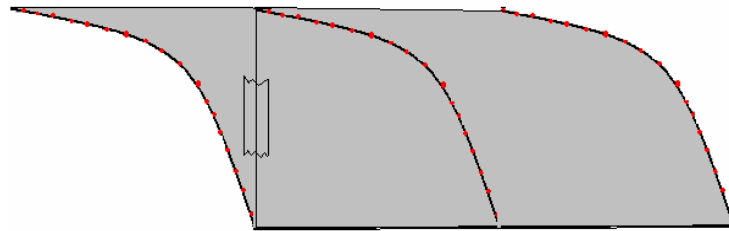
5. If the student likes, he or she may cut out another piece from the top of the square and glue it to the bottom of the square to create a more intricate model.

MathTAKStic! UNIT 6

translations problems
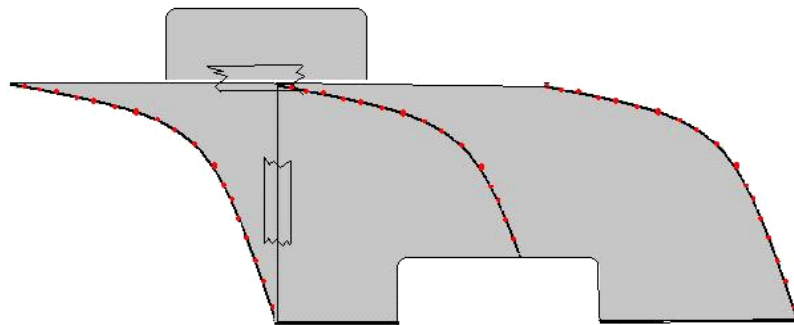
PROBLEM #1:

- o On your graph paper, plot the following points: (1, 1), (4, 1), (4, 4), (1, 4).

- o Trace the rectangle you just drew with the tracing paper.

- o Slide the shape 4 units down and 5 units to the right.
  What are the new coordinates of the translated rectangle?

PROBLEM #2:

- o Plot the following points on a new section of your graph paper:
  P: (-2, -2), Q: (0, 2), R: (2, -2).

- o Trace this triangle with the tracing paper.

- o Use your tracing paper to help you solve the following problem:

  We want to translate the triangle so that the coordinates for vertex P are (4, -3).

  How would you tell somehow how to slide the triangle?

  What are the new coordinates of the vertices Q and R?

PROBLEM #3:

- o Draw a Rectangle with Area = 12.

- o What are the coordinates of the corners of your rectangle?

- o Translate the rectangle however you like.

   How did you translate it?

   What are the new coordinates of the corners?

   What is the area of the new rectangle?

PROBLEM #4:

- o Draw the triangle with vertices A:(1,0), B:(3,0) and C:(1,2).

- o Translate each vertex 2 to the right and 1 up.

- o **What are the coordinates of the vertices of the new triangle?**

   What is the area of the original triangle?

   What is the area of the new triangle?

Making Connections: Translations

On your coordinate plane, draw any shape you like. Translate your shape however you like. Fill out the table below.
The very last row asks you to generalize your translation to a point with coordinates (n, m).

Original Coordinate Points of the Shape	The Coordinate Points AFTER Translation	Relation Between the x-Coordinates of the Two Points	Relation Between the y-Coordinates of the Two Points	How Much Did You Move the Shape Horizontally? In what direction?	How Much Did You Move the Shape Vertically? In what direction?
(n, m)					

translations lab worksheet

ITERPRETING THE DATA:

After you have filled out the above chart,
answer the following questions:

1. How many sides did your original shape
   have before you translated it? _____

   How many sides did your shape have after it was translated?
   _____

   What can you conclude about the relation of the number of
   sides a shape has before and after a translation?

2. How many angles did your original shape have before you
   translated it? _____

   How many angles did your shape have after it was translated?
   _____

   What can you conclude about the relation of the number of
   angles a shape has before and after a translation?

3. If you move a point to the right, what happens to the
   coordinates of the point? (HINT: which coordinate value
   would change? Would it increase or decrease?)

translations lab worksheet

terrific translations

4. If you move a point to the left, what happens to the coordinates of the point? (HINT: which coordinate value would change? Would it increase or decrease?)

5. If you move a point up, what happens to the coordinates of the point? (HINT: which coordinate value would change? Would it increase or decrease?)

6. If you move a point down, what happens to the coordinates of the point? (HINT: which coordinate value would change? Would it increase or decrease?)

translations
practice

1. Polygon LMNOPQ is shown on the coordinate grid below.
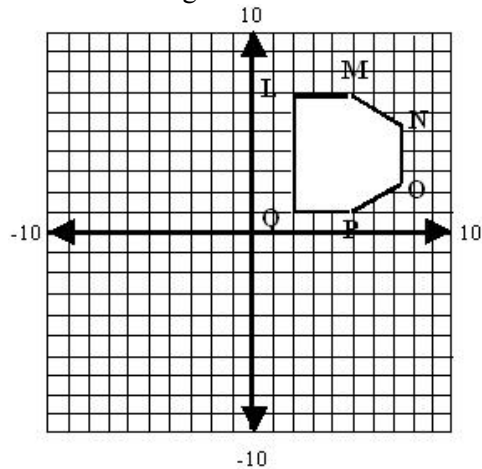
Which coordinate grid shows the translation of polygon LMNOPQ 4 units left and 5 units down?

A)

B)

C)

D)
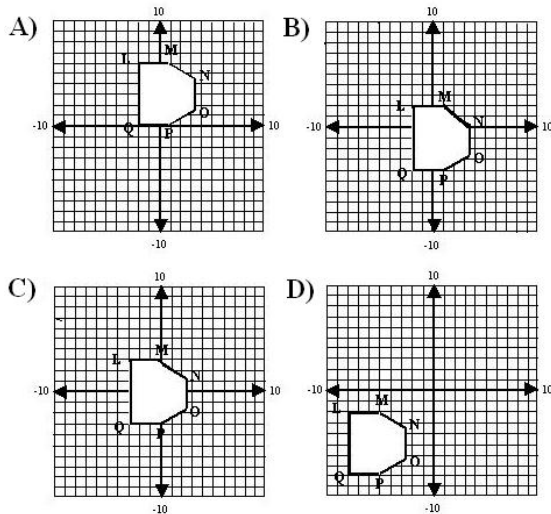
2. If triangle ABC is translated 3 units up and 6 units right, what are the coordinates of point B?
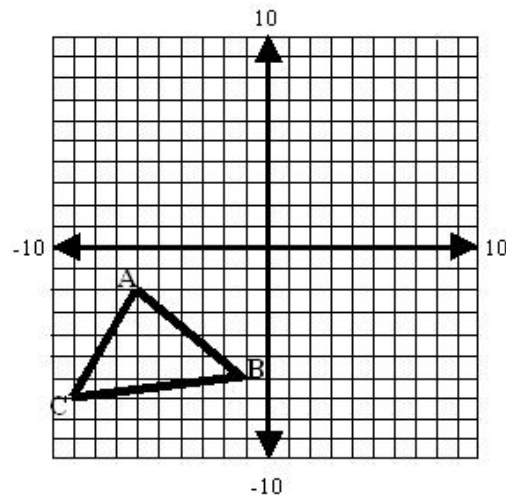
A) (2, 0)
B) (0, 2)
C) (1, -6)
D) (5, -3)

3. Point A has coordinates (-5, -6). If A is translated 4 units up and 6 units right, what will be the new coordinates of A?

A) (5, -6)
B) (0, 0)
C) (1, -2)
D) (-5, 6)

4. A circle with radius 4 units has its center at (2, -4) on a coordinate grid.
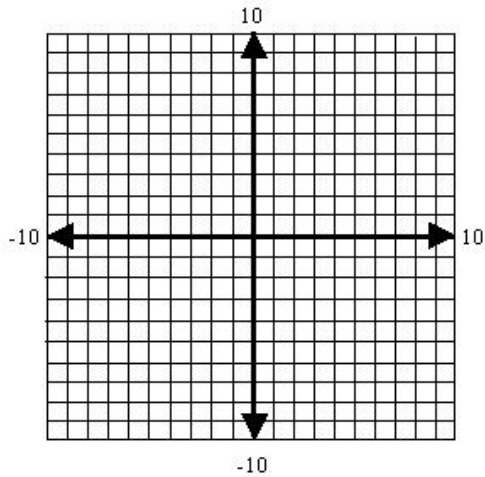
If the circle is translated 2 units down and 8 units left, what will be the coordinates of the new center?

A) (-6, -4)
B) (2, -6)
C) (-2, -4)
D) (-6, -6)

5. If rectangle LMNO is translated 4 units left and 2 units down, what will be the new coordinates of point N?
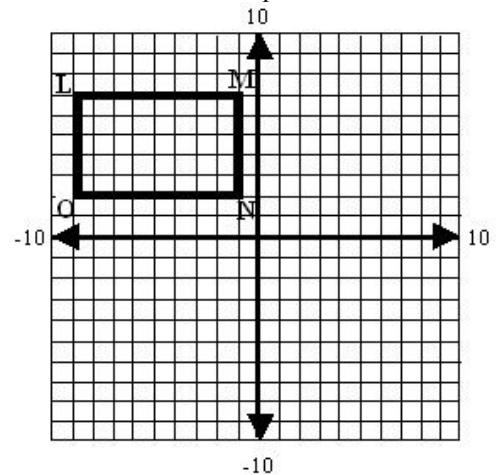
A) (-5, 2)
B) (-5, 0)
C) (-1, 0)
D) (-1, -2)

Appendix B9: Radical Reflections

radical
reflections

TEKS: 2b, 4a, 6b, 14b, 14c, 15a, 16a

Time Required: 135 minutes

lesson summary

In this lesson, students learn both visual and computational methods for finding the coordinate points of a reflected shape. Afterwards, they apply their new knowledge, along with their understanding of translations, to actively engage in a coordinate plane treasure hunt: one in which the classroom is the coordinate plane.

U nderstand

The teacher may do a demonstration to help students realize the connections between reflections in everyday life (such as mirrors) and mathematical reflections. He or she must have a mirror and a wet erase marker. The teacher asks the class what a reflection is. When students respond with answers about mirrors, the teacher then asks the class, "what does a reflection look like in relation to the original?" To demonstrate the correct answer, the teacher traces the reflection of an non-symmetric object on the mirror with the wet erase marker. When finished, he or she holds up the reflection alongside the original object. Students should then be able to see that the reflection is the original flipped along the vertical line separating the two versions of the object. The teacher finally poses the question: "how are reflections used in mathematics?"

To help students visually understand the answer to the question, the teacher guides the students through a short conceptual excercise. To begin, the teacher gives each student a copy of the **Grid Paper** and a piece of patty paper (found in teacher supply stores). The teacher then guides the students through the **Reflections Problems**. The students are expected to graph the points on the **Grid Paper**, trace them with the patty paper, flip the patty paper across the line of reflection, and then retrace the new points. Students may communicate their answers with the teacher verbally, on miniature white boards (if available) or on paper.

NOTE: Problems 3 and 4 may not be appropriate for all students or classes. The treasure hunt that appears in the Apply section will have to be modified accordingly if you don't use problems 3 and 4.

Objectives:

- To understand what a mathematical reflection is.

- To be able to solve problems involving reflections using both visual methods and computational methods.

Prerequisites for this Lesson:

- An understanding of translations

- An ability to plot coordinate points in all four quadrants

- An ability to add and subtract integers

Materials:

- Mirror

- Wet Erase Marker

- Grid Paper (one per student-found in appendix B0)

- Patty Paper (one piece per student)

- Rulers (one per group)

Support and Attachments:

- Grid Paper

- Grid Paper 2

- Reflections Problems

- Picture Reflections handout

- Sample Treasure Hunt Instructions

- Reflections TAKS Questions

i nvestigate

After the students grasp the visual concept of a reflection, they may explore the computational versions. The teacher now assigns the students into groups of 3 and either collects all the patty paper or asks the students to put it away. Each group then receives one set of the **Picture Reflections** handouts and three rulers. Each student in the group selects one of the pictures and is asked to follow the instructions on the page. The group members are encouraged to check each others' work and help each other accomplish the task. The goal is to have the students use computational methods, although they may not even realize what they are doing. After all the group members are done, the teacher gives the group patty paper. The students may then trace their picture and flip it along the line of reflection to check if their work was correct.

NOTE: The pencil reflection is good enrichment for more advanced students. It may not be appropriate for all students or classes.

d iscover

The teacher must now help the students understand the mathematical method for finding reflections. He or she should ask the series of questions below:

- I want you to find the coordinate points of two points on each picture. Then, find the coordinate values of the reflections of those points. For instance, what is the coordinate point of the tip of the carrot? What is the coordinate point of the tip of the reflected carrot?

- Look at your points for the carrot. List off some points and their reflections for me <teacher writes them on board>. What do you notice about these points? What axis were we reflecting about?

- Look at your points for the pig. List off some points and their reflections for me <teacher writes them on board>. What do you notice about these points? What axis were we reflecting about?

radical reflections

■ Look at your points for the pencil. List off some points and their reflections for me <teacher writes them on board>. What do you notice about these points? What axis were we reflecting about?

a pply

The teacher assigns the students into groups of three. Prior to the class the teacher must sketch a representation of the classroom layout on **Grid Paper 2**. He/she must also hide pieces of paper color coded for each group in the classroom. These pieces of paper may be taped underneath tables, desks, on the chalkboard, on the ceiling, etc. At the beginning of the activity, each group receives a copy of the map of the classroom and a numbered set of instructions. A **Sample Treasure Hunt Instructions** is included, and may be used, depending on the classroom layout. The groups then begin at separate starting points and follow the instructions to translate and reflect about the lines on the grid to find their final location. They are to determine the color of the piece of paper at their final location. If the paper is found in the correct manner, the group may receive a prize. The teacher should have a sheet of paper with the number of the instruction sheet and its color so the students can check at the end. Note that the same set of instructions may be used granted that the groups start at different locations. Also, if the teacher is worried about discipline, he/she may require the groups to complete the problems on paper first, check for validity with the teacher, and then search the classroom. In this case, the students may work the problems on plain grid paper and then receive the map of the classroom only when the correct answer is shown to the teacher.

Assessment:

Participation in Treasure Hunt

Reflections TAKS Questions

reflections
problems

PROBLEM ONE:

- On your graph paper, plot the following points:
  (0,2), (0,-2), (-3,2), (-3,-2)

- Connect these points in order. What letter of the alphabet does this shape make?

- Trace the shape with your patty paper so that the right edge of the shape is at the right edge of your patty paper.

- Flip the patty paper over along the right edge. You should be able to see your shape on the other side of the y-axis. Draw this new shape.

- What are the new points of the reflected shape?

- Is the reflected shape the same letter of the alphabet?

PROBLEM TWO:

- On your next grid, plot the triangle with vertices:
  (2,1), (4,2), and (3,3). Connect these points.

- Next, plot the triangle with vertices: (2,-1), (4,-2), and (3,-3). Connect these points.

- Use your patty paper to determine what axis the triangle was reflected about.

reflections problems

radical reflections

PROBLEM THREE:

- On your next grid, again plot the triangle with vertices: (2, 1), (4, 2) and (3, 3).

- Next plot the triangle with vertices: (2, -5), (4, -6) and (3, -7).

- Use your patty paper to determine what axis the triangle was reflected about.

PROBLEM FOUR:

- Pick any 4 points on the left side of the y-axis. List them here. Connect them to make a shape.

- Trace the shape with your patty paper and reflect it about the line x=2. Draw the reflected shape on the grid.

- What are the new coordinate points of your shape?

- How are the new coordinates related to the old coordinates?

picture
reflections
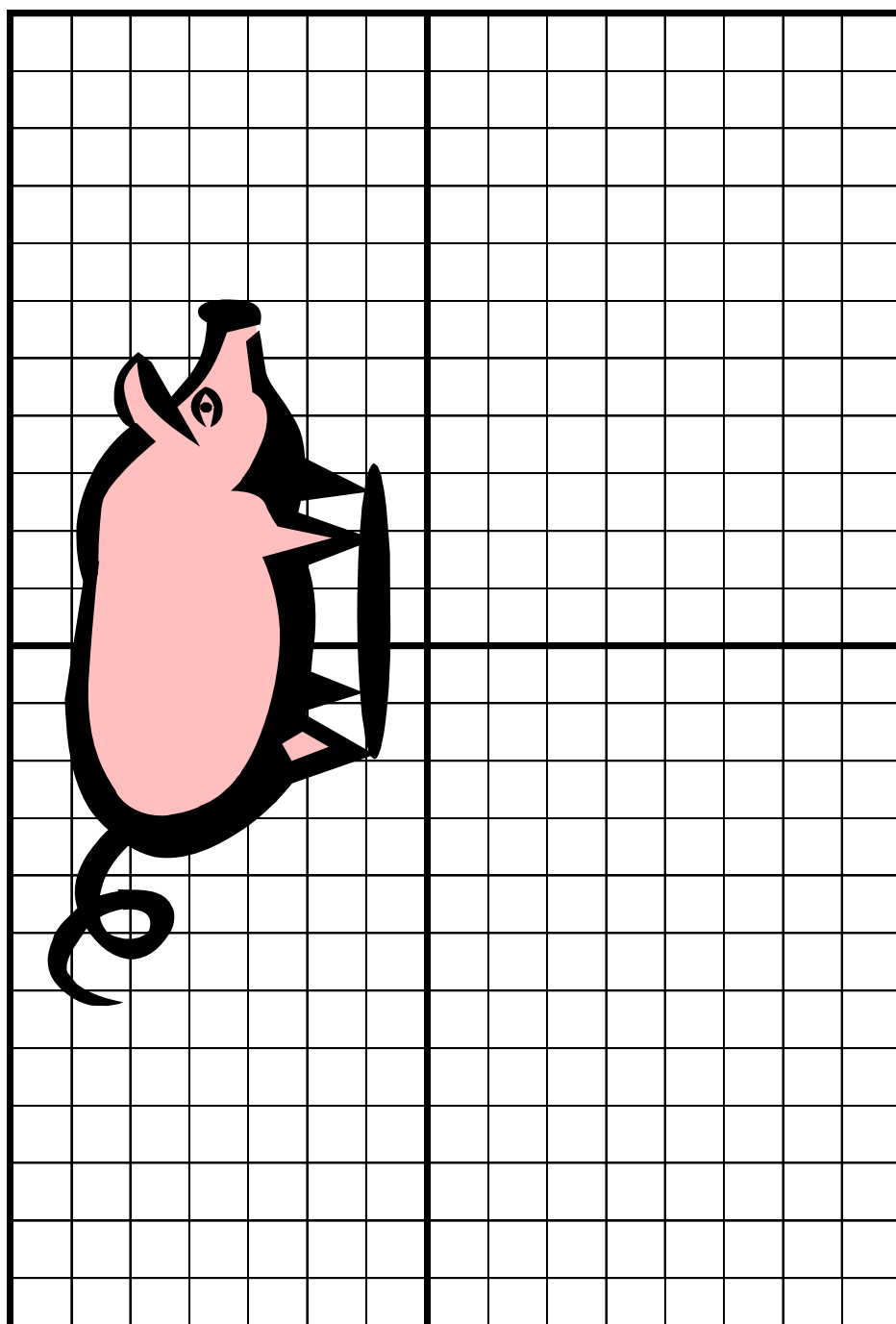
Directions: Reflect the picture about the x-axis.
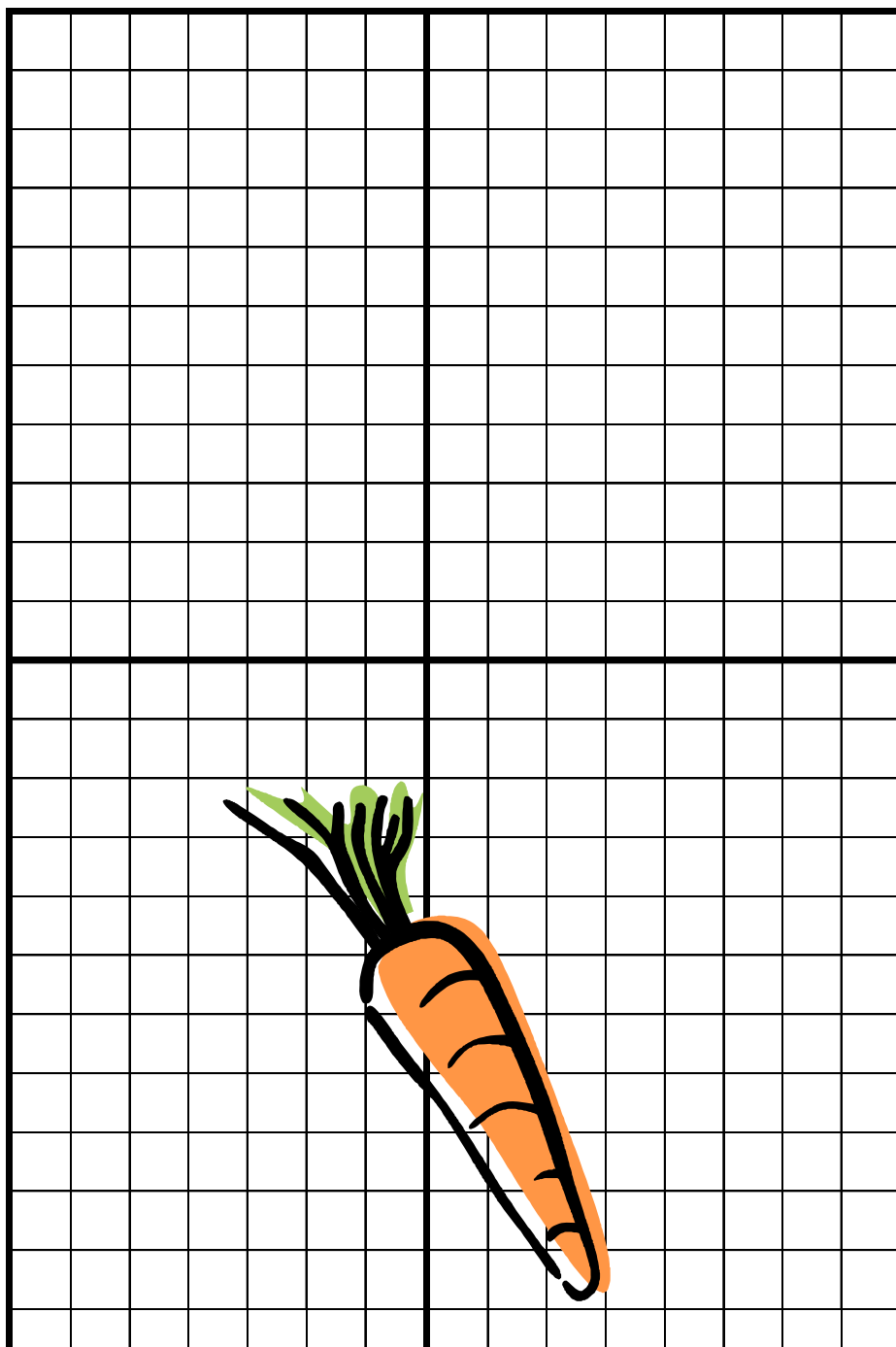
radical
reflections

Directions: Reflect the picture about the y-axis.
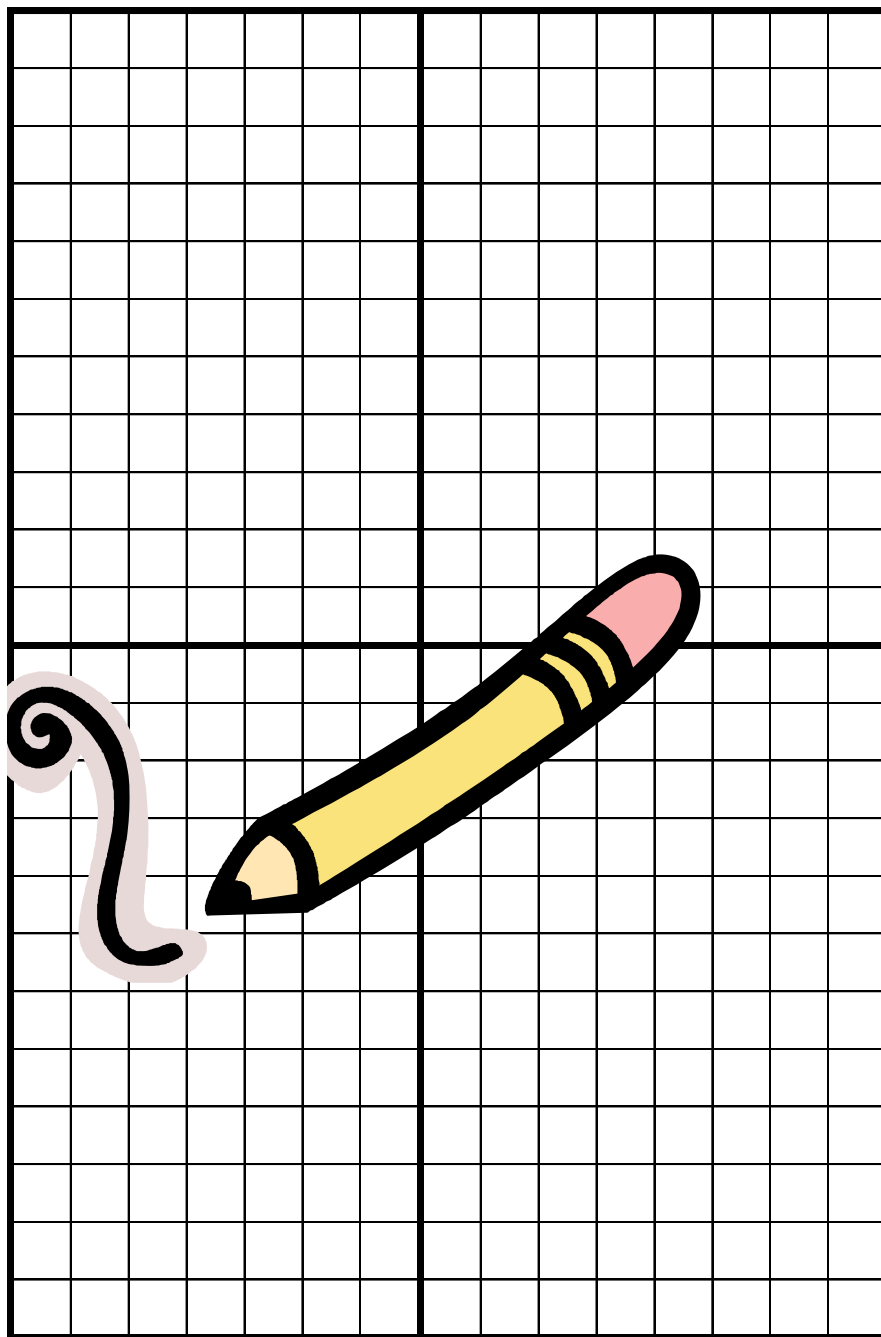
x

Find the Treasure!

Group Number: _____

Follow these instructions to find your treasure. Mark all the answers to the questions below on your grid.

1. Start at the point (3,-2). Reflect about the x-axis. Where are you now?

2. Now, translate 4 units left and 5 units up. Where are you now?

3. You need to translate again. This time you must translate left by the GCF of 8 and 26. Where are you now?

4. Reflect about the line y=1. Where are you now?

5. Finally, reflect about the line x=-1. This is where your treasure lies.

6. Record the color of your treasure: _____

1. Polygon RSTUV is shown on the coordinate grid below.

Which coordinate grid shows the reflection of polygon RSTUV across the x-axis?

A)

B)

C)

D)


2. A circle has its center at (-2, 4) on a coordinate grid.
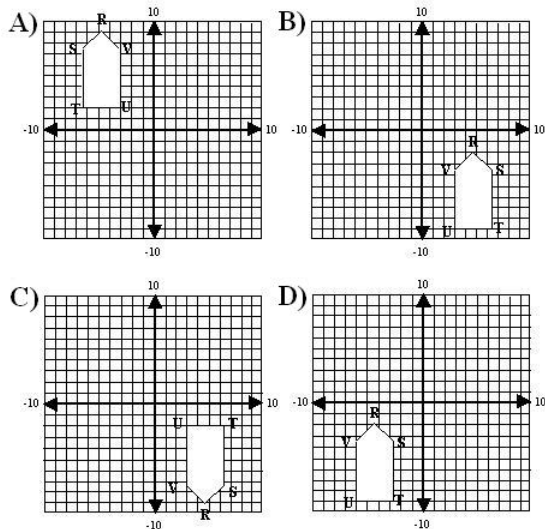
If the circle is reflected about the y-axis, what will be the new coordinates of its center?

A) (2, -4)
B) (2, 4)
C) (-2, -4)
D) (-2, -2)

reflections practice

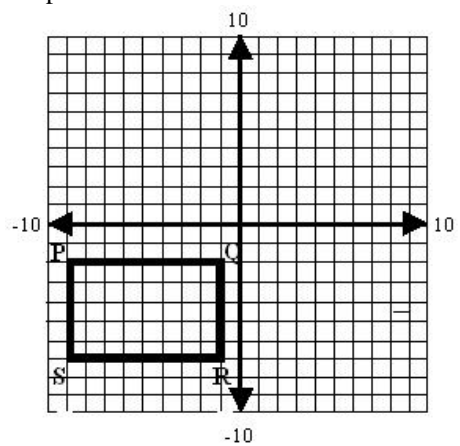# radical reflections

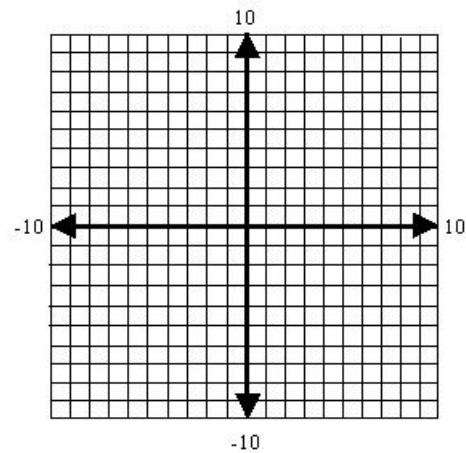3. If triangle XYZ is reflected across the x-axis, what are the coordinates of point Y?

A) (6, 3)
B) (8, 2)
C) (-6, 3)
D) (6, -3)

5. Triangle MNO has coordinate points M (0, 0), N (3, 3), and O (8, 1). If triangle MNO is reflected across the x-axis, what will be the new coordinates of point N?

A) (0, 0)
B) (3, -3)
C) (-3, 3)
D) (8, -1)
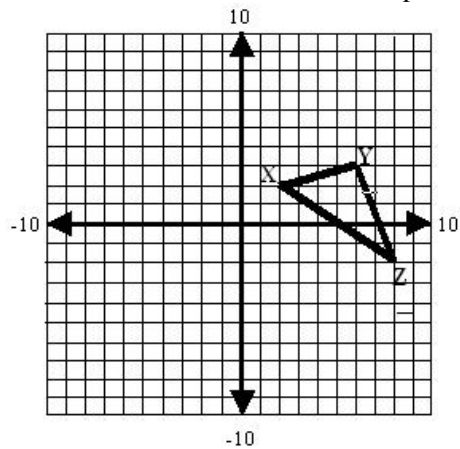
4. If rectangle PQRS is reflected across the y-axis, what will be the coordinates of point P?

A) (9, -2)
B) (-9, 2)
C) (-9, -2)
D) (9, 2)

Appendix B10: Architecture 101: An Adventure in 3-D Visualization

architecture 101:
an adventure in
3-D visualization

TEKS: 7a, 7b, 14a, 14b, 14c

Time Required:
90 minutes

lesson summary

Students learn how to draw 3-dimensional objects in a 2-dimensional representation through a real-world activity that introduces the students to the career of architecture.

Objectives:

- To be able to accurately represent a 3-dimensional model on paper in a 2-dimensional replication.

- To be able to draw each face of a 3-dimensional model.

- To understand how perspective geometry is used in real-world situations (specifically in architecture).

Understand

The teacher is encouraged to discuss architecture with the students, asking them what it is and what it entails. The teacher may then ask students how architects make their ideas into final products. The goal is for students to realize that architects must make blue prints, or 2-dimensional representations, of their ideas before a 3-dimensional model can be built. The teacher explains that the students will be architects for the day and, in doing so, will have to work with 2-dimensional drawings of 3-dimensional objects.

investigate

The teacher begins by assigning students into groups of three. Each group has the assignment of designing a building just as an architect would. They must first complete the **Brainstorming Worksheet** to help them formulate their ideas. This worksheet helps the students realize all the components that may affect the design of their building. After they have finished this, the group should show it to the teacher in order to receive the **Blueprint Paper**. On the **Blueprint Paper**, the group must draw the front, top, back and side views of the building. They are also asked to create a 3-dimensional sketch of the entire building.

Materials:

- Blueprint Paper

- Sugar Cubes (approximately 30 per group)

- Glue

Support and Attachments:

- Brainstorming Worksheet

- Blueprint Paper

- 3-Dimensional Perspective TAKS Questions

d iscover

Once they have completed their blueprint and shown it to the teacher, they may receive the building supplies. For this, the teacher may give sugar cubes and glue or any other cubed shaped object. It is suggested to limit the students to 30 cubes. Using their drawings, the students must build their models. By doing so, they are practicing matching their 2-dimensional drawings to a 3-dimensional construction. They may find that what they drew on paper does not equate to a buildable object. Through this learning process, students should gain a better vision of 2-dimensional representations.

a pply

To practice summarizing the qualities of a 3-dimensional object, each group may create a diagram of the number of blocks in their building, as in question 6 of **3-Dimensional Perspective TAKS Question**s. Additionally, the students may work the problems in **3-Dimensional Perspective TAKS Questions**.

lesson scaling

For a less advanced class, the teacher may give the students pictures of 3-dimensional constructions of blocks. The students may then use a set of blocks to practice building the 3-D objects.

For another version, students may work in pairs. One student receives a 3-D picture, the other the 2-dimensional representations. They both build the object to their best abilities and then compare to see how similar their constructions are.

architecture 101:
an adventure in 3-D visualization

lesson extensions

For an advanced, well-behaved class the teacher may have students engage in a competition. Each student has a partner. The teacher then divides the class into two sections, placing students on opposite sides of the classroom so that no student is on the same side as his or her partner. Then, the teacher provides each side with a different model built out of blocks. The teacher must ensure that the two sides do not see each other's model. The teacher then gives the students 5-10 minutes to write a short paper describing how to build their model. This paper may not include diagrams. When time is up, the teacher hides the models and has each student switch papers with his/her partner. Then, the teacher gives the students 10-15 minutes to build (using the same sort of blocks) the model described on their papers to the best of their abilities. The winning pair is that who have both built the most accurate representations of the original models. The goal is for students to practice describing the 3-dimensional figures in words, thus forcing them to break the figure down into its basic elements. They will also see that "a picture is worth a thousand words."

Vocabulary / Definitions:

- **Face**—A flat surface of a polyhedron.

- **Dimension**— the number of measurements that can be taken on a figure. For example, a 2-dimensional object may have measurements length and width, while a 3-dimensional object may have measurements length, width, and depth.

Assessment:

3-Dimensional Perspective TAKS Questions

MathTAKStic!
UNIT 7

brainstorming worksheet

BECOMING AN ARCHITECT
BRAINSTORMING WORKSHEET

1. What sort of building do you want to build? What is its purpose? (for example, is it going to be a house or a store or something else?)

   _____

   _____

2. Make a sketch of how you want the building to look like on the back of this worksheet.

3. Why did you choose the design you drew?

   _____

   _____

4. Are you going to make your building hollow or filled in? _____

5. About how many blocks will you need...

   For the front side of your building? _____

   For the back of the building? _____

   For the sides of the building? _____

   For the top of the building? _____

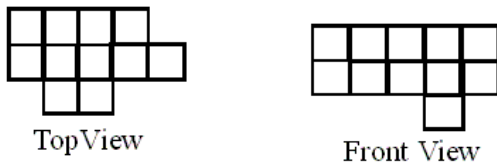   For the bottom of the building? _____

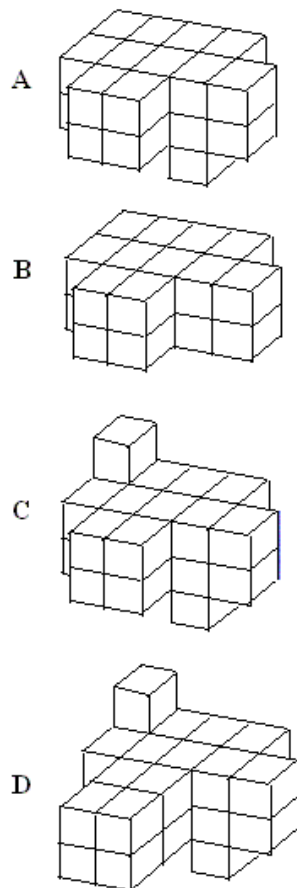   Total number of blocks _____ (Must be less than 30)

MathTAKStic!
UNIT 7

blueprint
paper

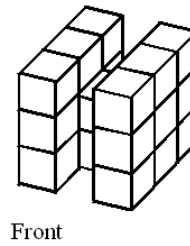# 3-dimensional visualization practice
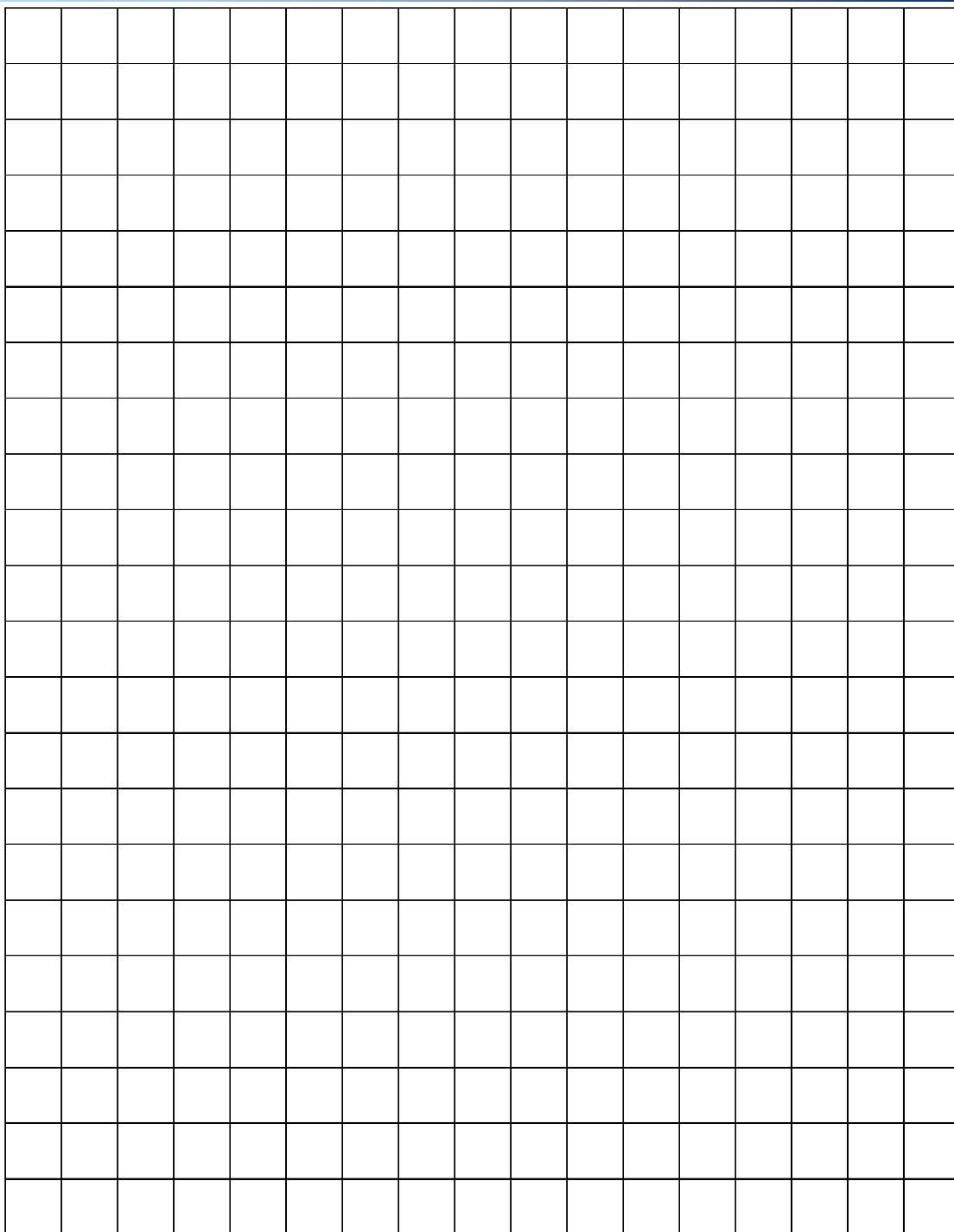
1. The drawings show the top view and the front view of a solid figure built with cubes.
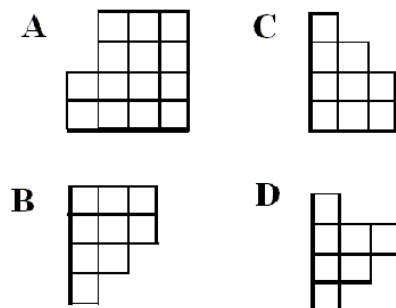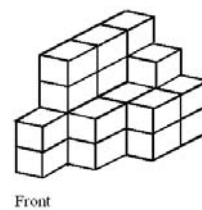

TopView

Front View

Which drawing shows a 3-dimensional view of the solid figure represented above?

A

B

C

D

2. The solid figure is built with cubes. Which could represent the shape of the solid figure when viewed from directly above?
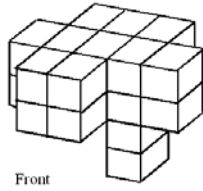

Front

A   C

B   D
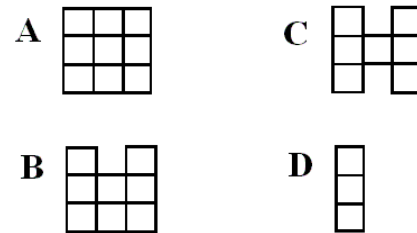
3. Look at the drawing of the solid below. Which of the following is not a top, front, or side view of this solid?
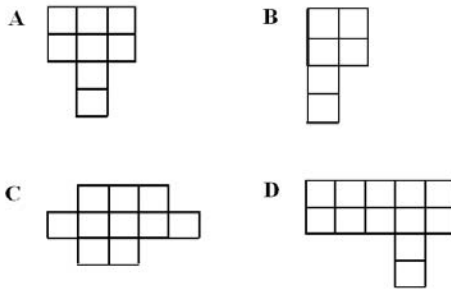

Front

A   C

B   D

3-dimensional visualization practice

architecture 101:
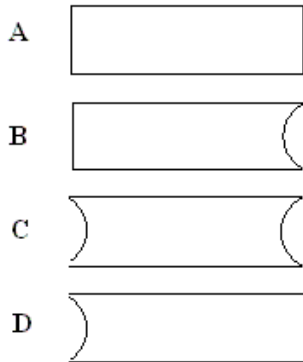## an adventure in 3-D visualization

4. The drawing shows a solid figure built with rectangular prisms.

Front

Which drawing below represents a view of the solid figure from the front?

A

B

C

D

6. The drawing shows the top view of a solid figure made of stacked cubes. The numbers in the squares identify the number of cubes in each stack.

2	2	3
2	2	2
2	1	1

Which drawing shows a 3-dimensional view of this solid figure?

A

B

C

D

5. The picture below shows a water trough. Which drawing best represents a top view of the water trough?

A

B

C

D
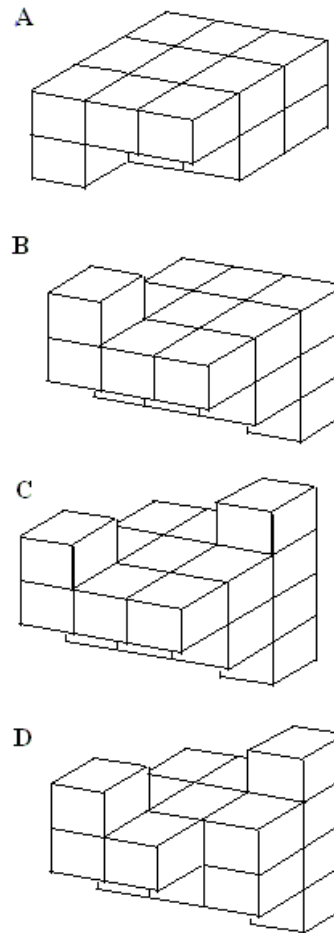
Appendix B11: Exploring the Volume of Pyramids

exploring the volume of pyramids

TEKS: 2b, 4a, 8b, 14b, 14c, 15a, 16a, 16b

Time Required: 90 minutes

lesson summary

Once students have learned about how to calculate the volume of a prism, they may then do a fun lab activity in which they compare volumes of prisms and pyramids of the same height and base areas. By doing this, students discover for themselves that the volume of a pyramid is one third the volume of a prism with the same base and height. This lesson encourages experimentation and higher level thinking.

u nderstand

The teacher begins by refreshing the students' minds on how to calculate the volume of a prism. She/he may do an example with the students as a warm-up exercise.

The teacher then assigns the students into groups of two or three. Each group receives a set of scissors, glue sticks (or tape), and the **Prisms** and **Pyramids** worksheets (found in Appendix B0). First the students construct the pyramids and prisms with guidance from the teacher. Although this is not necessary, it helps students understand the basic breakdown of the solids. If the students have done the lesson on surface area of prisms and pyramids, then they may use the solids they constructed then.

Once the pyramid construction is complete, the teacher should have students make observations about the solids. Most importantly, they should notice that solids with corresponding numbers have the same base and height. For example, Pyramid 1 and Prism 1 both have the same base and the same height.

The teacher explains that the students will be using their knowledge of the volume of prisms to find the formula for the volume of pyramids.

Objectives:
- To be able to calculate the volume of pyramids

- To understand the derivation of the formula for the volume of a pyramid

- To develop experimental thinking

Prerequisites for this Lesson:

- An ability to calculate the Volume of a prism

- An abliity to calculate the Area of a rectangle

Assessment:

Volume of Pyramids Lab Questions

Volume of Pyramids TAKS Questions

MathTAKStic! UNIT 8

Materials:

- Prisms and Pyramids Cut-Outs (found in Appendix B0)

- Rice, Beans, Sand, or similar substance

- Container (one per group)

- Tape

- Scissors

Support and Attachments:

- Volume of Pyramids TAKS Questions

- Prisms and Pyramids cut-outs

- Volume of Pyramids TAKS Questions

Vocabulary / Definitions:

- **Pyramid**—A solid shape with a polygon as a base and triangular lateral faces that taper to a point.

- **Prism**—A solid figure with two bases, parallel to one another, with the same size and shape.

- **Volume**—A measure of 3-dimensional space.

i nvestigate

Each student then receives the **Volume of Pyramids Lab Worksheet**. Still working in groups, the students proceed to fill out the worksheet. The first part of the lab asks students to measure all the prisms and pyramids to see that the prisms and pyramids with corresponding numbers have the same base areas and the same heights. For the second part of the lab, students will pour a substance (beans, rice, beads, sand or something similar) from the pyramids into the prisms to estimate volume. The goal is for them to notice that a pyramid will pour into the corresponding prism 3 times. This explains why the formula for volume of a pyramid is one third the volume of the prism with same base and height.

d iscover

Using the knowledge found in the experiment, the **Volume of Pyramids Lab Worksheet** guides the students into creating the formula for the volume of a pyramid based on their knowledge of the formula of a prism. Once the students have concluded this fact, the teacher may want to reiterate the statement to make sure that the students completely understand it.

a pply

If the teacher has additional pyramids, the students may measure their dimensions and then calculate volume using their new formula. Additionally, the Volume of Pyramids TAKS Questions are available for practice.

volume of pyramids lab

COMPARING PRISMS AND PYRAMIDS

You have been given several pyramids and prisms. Follow the instructions on the following page in order to fill out the table. Make all measurements in centimeters. Round any measurements to the nearest centimeter.

SOLID	WIDTH OF BASE	LENGTH OF BASE	HEIGHT OF SOLID	AREA OF BASE	VOLUME OF SOLID
PRISM 1					
PRISM 2					
PRISM 3					
PYRAMID 1					
PYRAMID 2					
PYRAMID 3					

volume of pyramids lab

COMPARING PRISMS AND PYRAMIDS

STEP 1:
Measure each prism and pyramid to find the **length** and **width** of
its base and its **height**. Write your measurements in the table.
Do all measurements in centimeters.

STEP 2:
Use the **length** and **width** you found to compute the **area** of each base.
Write your answers in the table.

STEP 3:
Using the formula for the volume of a prism, $V=Bh$, compute the volumes
of all 3 prisms. Write your answers in the table.

STEP 4:
Now, you are going to compare the volumes of the solids.

Fill Pyramid 1 and pour it into Prism 1. Repeat this until you have filled
Prism 1 completely.

- How many times could you pour a filled Pyramid 1 into Prism 1?_____

Fill Pyramid 2 and pour it into Prism 2. Repeat this until you have filled
Prism 2 completely.

- How many times could you pour a filled Pyramid 2 into Prism 2?_____

Fill Pyramid 3 and pour it into Prism 3. Repeat this until you have filled
Prism 3 completely.

- How many times could you pour a filled Pyramid 3 into Prism 3? _____

volume of pyramids lab

exploring the volume of pyramids

STEP 5:

In general, how many pyramid volumes fit into a prism volume? _____

STEP 6:

What fraction of the prism volume is the pyramid volume? Write this in the box:

Pyramid Volume = ⬜ **x Prism Volume**

STEP 7:

Using the formula for volume of a prism, V=Bh, what is the formula for volume of a pyramid? White this below:

Pyramid Volume = _____

STEP 8:

Fill in the remaining boxes in the table for the volume of the three pyramids.

1. A square pyramid is shown below with it's dimensions. What is the volume of the pyramid, rounded to the nearest tenth of a cubic centimeter?

4 centimeters
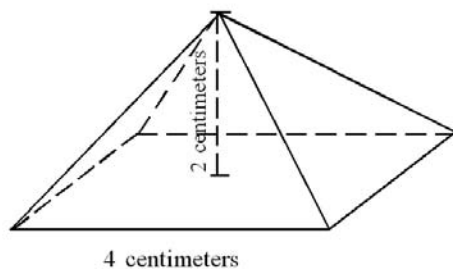
A 2.7 cm.^{3}

B 6.8 cm.^{3}

C 10.7 cm.^{3}

D 32.0 cm.^{3}

2. A pyramid and a rectangular prism have the same dimensions of their bases. They also share the same height. What is the ratio of the volume of the pyramid to the volume of the rectangular prism.

A 1 : 2

B 1 : 3

C 2 : 1

D 3 : 1

3. A drawing of a house is shown below, with the dimensions in feet. The house is constructed with a rectangular prism and a pyramid.
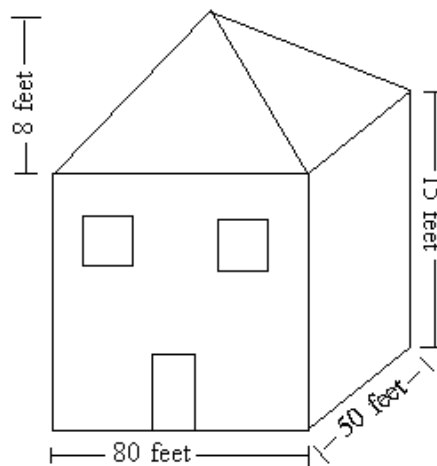
What is the volume of the house, in cubic feet?

A 60,000 ft.^{3}

B 65,330 ft.^{3}

C 70,667 ft.^{3}

D 92,000 ft.^{3}

volume of pyramids practice

exploring the volume of pyramids

4. The base of a triangular pyramid is shown below.
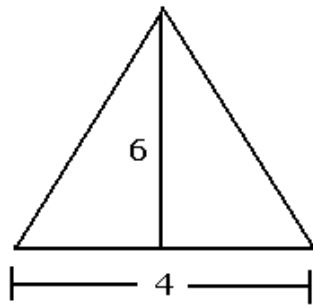
If the height of the pyramid is 10, what is the volume of the pyramid?

A 40

B 50

C 60

D 120

5. A pyramid has a base with area 20 cubic meters. What is height of this pyramid if the volume is 120 cubic meters?

A 6 cu. m.

B 18 cu. m.

C 20 cu. m.

D 24 cu. m.

Appendix B12: Volume of Cylinders

volume of cylinders

TEKS: 2b, 4a, 8b, 14b, 14c, 15a, 16a, 16b

Time Required:
90 minutes

lesson summary

In this lesson, students perform a lab activity that helps them derive the formula for the volume of a cylinder.

Objectives:

- To derive the formula for the volume of a cylinder.

- To understand how to compute volume of a cylinder.

understand

The day before the lesson, the teacher may ask students to think of cylinders in everyday life. Where have they seen them before? The students may be assigned to bring in cylinders from their homes that may be filled with water. For example, students may bring empty cans or coffee mugs.

On the day of the lesson, the teacher begins by asking the students what sorts of cylinders they brought in to see the variety and creativity available. The teacher should have several cylinders available if the students do not bring in enough.

The teacher explains that the class will use their cylinders to find a formula for the volume of any cylinder.

Prerequisites for this Lesson:

- An ability to calculate the area of Circles

- An ability to make measurements in centimeters

investigate

Assessment:

Volume of Cylinders TAKS Questions

Volume of Cylinders Lab Worksheet

Students are divided into groups of 2 or 3. Each group should have 3-5 cylinders (the teacher may provide extra), a ruler, a graduated cylinder (measuring in milliliters), a pitcher of water (or a faucet if available), and **Volume of Cylinders Lab Worksheets** (one for each student). The students are asked to first measure the cylinders they have been given. They then must compute areas of the bases. Afterwards, they measure how much water (in centimeters cubed) fits into each of the cylinders and write the information down.

MathTAKStic! UNIT 8

Materials:

- Rulers: one for each pair or group (about $0.10 each)

- Cylinders (coffee mugs, cans, etc)

- Pitcher of water (one per group)

- Graduate Cylinder in millimeters (one per group)

Support and Attachments:

- Volume of Cylinders Lab Worksheet

- Volume of Cylinders TAKS Questions

Vocabulary / Definitions:

- **Cylinder**—A solid shape with one curved surface and two equal circular faces.

- **Volume**—A measure of 3-dimensional space.

d iscover

Once all the data is connected, the students continue through their worksheet to find what the formula for volume of a cylinder is. Essentially, students are looking at patterns between the dimensions of a cylinder and its measured volume.

a pply

To practice using their formula, the students may calculate the volumes of the cylinders they used in the lab. Additionally, the students may work on the Volume of Cylinders TAKS Questions.

volume of cylinders lab

MathTAKStic!
UNIT 8

TAKING MEASUREMENTS

Follow the steps on the following page to fill out the table below.

CYLINDER	DIAMETER OF BASE	RADIUS OF BASE	HEIGHT OF CYLINDER	AREA OF BASE	VOLUME OF CYLINDER	PROCESS—how can you get from the Height and the Area to the Volume
1						
2						
3						
4						
5						
General			*h*	*B*		

volume of cylinders lab

STEP ONE:

Measure the diameter and height of each cylinder. Put these values into your table.
Do all measurements in centimeters.

STEP TWO:

Using the *diameter* you measured, calculate the radius and area of the base of
each cylinder. Write the answers in your table.

STEP THREE:

Using the graduated cylinder, measure how many milliliters fit into each of the
cylinders. This is equal to the amount of cubic centimeters (the **volume**) of each
can. Write these values in the table.

STEP FOUR:

Look at the values in your table. For each cylinder, look at the patterns between
the heights and the areas of the bases compared to the volumes. Based on your
observation, fill out the box below with a mathematical operator (+, -, x, or ÷) so
that the equation will make sense. The values may be close, but not exact.

**HEIGHT OF CYLINDER** ☐ **AREA OF BASE = VOLUME OF CYLINDER**

STEP FIVE:

Fill out the last column in your table based on your observation in STEP FOUR.

STEP SIX:

Fill out the final row, for the general case. By doing this, you are creating a formula
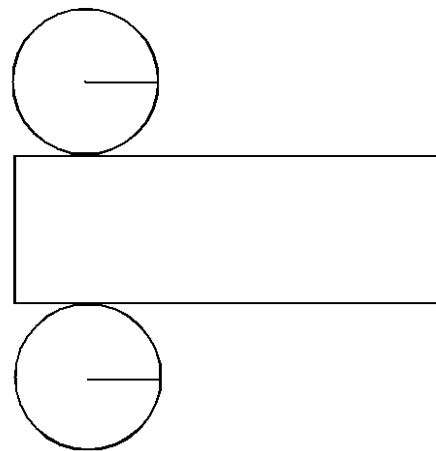to find the volume of a cylinder.

1. A water trough is a rectangular prism with a half cylinder on bottom. The base of the half cylinder has a radius of 3 inches.

10 in

5 in

3 in
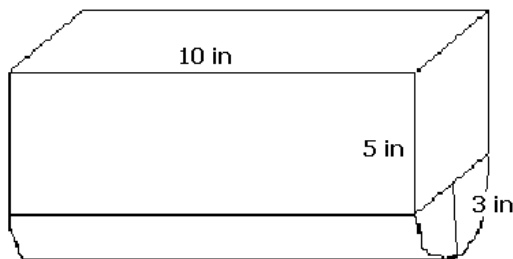
What is the approximate volume of the water trough?

A 250 in^3

B 300 in^3

C 435 in^3

D 570 in^3

2. Mary is filling a jar with cookies. The jar is shaped like a cylinder and has a base with diameter of 6 inches and a height of 12 inches. How much volume, in cubic inches, can the jar hold? Round your answer to the nearest cubic inch.

A 72 cu. in.

B 339 cu. in.

C 432 cu. in.

D 1356 cu. in.

3. Mrs. Lopez is making a cylindrical pin cushion using the net shown below. Use the ruler on the Mathematics Chart to measure the dimensions of the net in centimeters.

Which is the closest to the volume of the cylindrical pin cushion?

A 5 cm^3

B 10 cm^3

C 20 cm^3

D 40 cm^3

volume of cylinders practice

4. A cylinder is shown below with it's dimensions. What is the volume of the cylinder, in cubic feet?

4 feet

1.5 feet
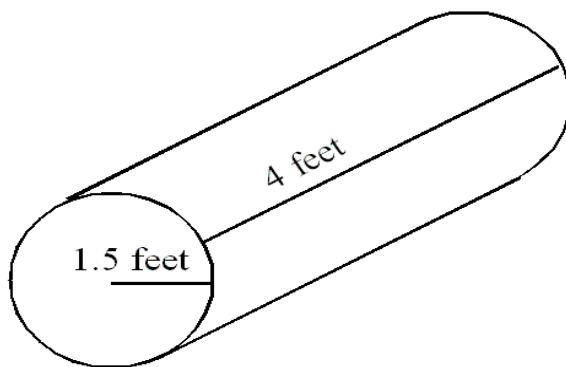
A   6 cubic feet

B   9 cubic feet

C   28.26 cubic feet

D   113.04 cubic feet

5. Anastasia is making cylindrical beads out of clay. Each bead has a diameter of 1 centimeter and a height of 0.25 centimeters. If she stacked 12 beads, one on top of another, what would be the total volume of the stack of beads?

A   2.35 cm^3

B   3.2 cm^3

C   9 cm^3

D   36 cm^3

Appendix B13: Exploring the Volume of Cones

exploring the
volume of cones

UNIT 8

TEKS: 2b, 4a, 8b, 14b, 14c, 15a, 16a, 16b

Time Required:
90 minutes

lesson summary

Once students have learned about how to calculate the volume of a cylinder, they may then do a fun lab activity in which they compare volumes of cylinders and cones of the same height and base areas. By doing this, students discover for themselves that the volume of a cone is one third the volume of a cylinder with the same base and height. This lesson encourages experimentation and higher level thinking.

U nderstand

The teacher begins by refreshing the students' minds on how to calculate the volume of a cylinder. She/he may do an example with the students as a warm-up exercise.

The teacher then assigns the students into groups of two or three. Each group receives a set of scissors, glue sticks (or tape), and the **Cylinders** and **Cones** worksheets (found in Appendix B0). First the students construct the cones and cylinders with guidance from the teacher. Although this is not necessary, it helps students understand the basic breakdown of the solids. If the students have done the lesson on surface area of cones and cylinders, they may use the solids constructed then. Be sure the students cut on the outside of the thick black line.

Once the cone construction is complete, the teacher should have students make observations about the solids. Most importantly, they should notice that solids with corresponding numbers have the same base circle and height. For example, Cone 1 and Cylinder 1 both have the same base and the same height.

The teacher explains that the students will be using their knowledge of the volume of cylinders to find the formula for the volume of cones. If the students have already completed the lesson for the volume of pyramids, they will be familiar with this process, and the teacher may encourage them to make a conjecture as to what the formula will be.

Objectives:

- To be able to calculate the volume of cones

- To understand the derivation of the formula for the volume of a cone

- To develop experimental thinking

Prerequisites for this Lesson:

- An ability to calculate the Volume of a cylinder

- An ablility to calculate the Area of a circle

Assessment:

Volume of Cones Lab Questions

Volume of Cones TAKS Questions

Materials:

- Cylinders and Cones Cut-Outs (found in Appendix B0)

- Rice, Beans, Sand, or similar substance

- Container (one per group)

- Tape or glue

- Scissors

Support and Attachments:

- Volume of Cones TAKS Questions

- Cylinders cut-outs

- Cones cut-outs

- Volume of Cones TAKS Questions

Vocabulary / Definitions:

- **Cone**—A solid shape with an elliptical or circular base and a curved surface that tapers to a point.

- **Cylinder**—A solid shape with one curved surface and two equal circular faces.

- **Volume**—A measure of 3-dimensional space.

i nvestigate

To test their hypotheses, each student receives the **Volume of Cones Lab Worksheet**. Still working in groups, the students proceed to fill out the worksheet. The first part of the lab asks students to measure all the cones and cylinders to see that the cylinders and cones with corresponding numbers have the same base areas and the same heights. For the second part of the lab, students will pour a substance (beans, rice, beads, sand or something similar) from the cones into the cylinders to estimate volume. The goal is for them to notice that a cone will pour into the corresponding cylinder 3 times. This explains why the formula for volume of a cone is one third the volume of the cylinder with same base and height.

Recommendations when making the measurements: The teacher should tell students to round their measurements to the nearest centimeter. The teacher may use this as an opportunity to talk about approximation in mathematics and why it is useful.

d iscover

Using the data collected, the **Volume of Cones Lab Worksheet** guides students into creating a formula for the volume of a cone based on their knowledge of the volume of cylinders. The teacher should reinforce this concept, ensuring that the students understand the derivation. The class may also reflect back on their conjectures to study how accurate their estimates were.

a pply

If the teacher has extra cones available for measurement, the class may practice finding volumes of more cones. Additionally, they may work the problems on the **Volume of Cones TAKS Questions.**

volume of cones lab

COMPARING CYLINDERS AND CONES

You have been given several cones and cylinders. Follow the instructions on the following page in order to fill out the table. Make all measurements in centimeters. Round any measurements to the nearest centimeter.

SOLID	DIAMETER OF BASE	RADIUS OF BASE	HEIGHT OF SOLID	AREA OF BASE	VOLUME OF SOLID
CYLINDER 1					
CYLINDER 2					
CYLINDER 3					
CONE 1					
CONE 2					
CONE 3					

volume of cones lab

COMPARING CYLINDERS AND CONES

STEP 1:
Measure each cylinder and cone to find the diameter of its base and its height. Write your measurements in the table. Do all measurements in centimeters.

STEP 2:
Use the diameter you measured to compute the radius and the area of the base of each solid. Write your answers in the table.

STEP 3:
Using the formula for the volume of a cylinder, $V=Bh$, compute the volumes of all 3 cylinders. Write your answers in the table.

STEP 4:
Now, you are going to compare the volumes of the solids.

Fill Cone 1 and pour it into Cylinder 1. Repeat this until you have filled Cylinder 1 completely.

- How many times could you pour a filled Cone 1 into Cylinder 1?_____

Fill Cone 2 and pour it into Cylinder 2. Repeat this until you havefilled Cylinder 2 completely.

- How many times could you pour a filled Cone 2 into Cylinder 2?_____

Fill Cone 3 and pour it into Cylinder 3. Repeat this until you have filled Cylinder 3 completely.

- How many times could you pour a filled Cone 3 into Cylinder 3? _____

volume of cones lab

exploring the volume of cones

STEP 5:
In general, how many cone volumes fit into a cylinder volume? _____

STEP 6:
What fraction of the cylinder volume is the cone volume. Write this in the box:

Cone Volume = [ ] **x Cylinder Volume**

STEP 7:
Using the formula for volume of a cylinder, $V=Bh$,, what is the formula for volume of a cone.
White this below:

Cone Volume = _____

STEP 8:
Fill in the remaining boxes in the table for the volume of the three cones.

volume of
cones practice

1. A cone is shown below with it's dimensions. What is the approximate volume of the cone, in cubic centimeters?

4 centimeters
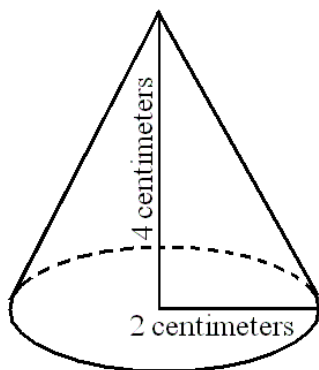
2 centimeters

A 4 cm. ^{3}

B 10 cm. ^{3}

C 16 cm. ^{3}

D 48 cm. ^{3}

2. An ice cream cone has a base with a radius of 4 centimeters and a height of 10 centimeters. What is the volume of the cone?

A 40 cu. cm.

B 160 cu. cm.

C 240 cu. cm.

D 480 cu. cm.

3. Jonathan is pouring water out of a paper cup shaped like a cone and into a glass cup shaped like a cylinder. Both the cylinder cup and the cone cup have the same areas of their bases and have the same height. How many times can Jonathan pour the cone cup into the cylinder cup before the cylinder cup is full?

A 0.5

B 1

C 2

D 3

4. The base of a cone with height 3 centimeters is shown below. Use the ruler on the Mathematics Chart to measure the dimensions of the base in centimeters.
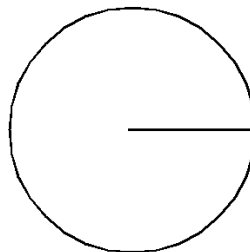
Calculate the volume of the cone to the nearest centimeter.

A 6 centimeters cubed

B 12 centimeters cubed

C 38 centimeters cubed

D 69 centimeters cubed

5. A cone has a base with diameter 10 feet. The height of the cone is 25 feet. How would you calculate the volume of the cone?

A V = 3.14 x 5 x 5 x 25

B V = 3.14 x 10 x 10 x 25

C $V = \dfrac{3.14 \times 10 \times 10 \times 25}{3}$

D $V = \dfrac{3.14 \times 5 \times 5 \times 25}{3}$

VITA

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