

THE TRANSPORTER'S IMPACT ON CHANNEL COORDINATION AND
CONTRACTUAL AGREEMENTS

A Dissertation

by

FATIH MUTLU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Industrial Engineering

THE TRANSPORTER'S IMPACT ON CHANNEL COORDINATION AND
CONTRACTUAL AGREEMENTS

A Dissertation

by

FATİH MUTLU

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Approved by:

Chair of Committee,	Sıla Çetinkaya
Committee Members,	Georgia-Ann Klutke
	Martin A. Wortman
	Guy L. Curry
	E. Powell Robinson
Head of Department,	Brett A. Peters

August 2006

Major Subject: Industrial Engineering

ABSTRACT

The Transporter's Impact on Channel Coordination and Contractual Agreements.

(August 2006)

Fatih Mutlu, B.S., Bilkent University, Turkey

Chair of Advisory Committee: Dr. Sila Çetinkaya

This dissertation focuses on the recent supply chain initiatives, such as Vendor Managed Inventory (VMI) and Third-Party Logistics (3PL), enabling the coordination of supply chain entities; e.g., suppliers, buyers, and transporters. With these initiatives, substantial savings are realizable by carefully coordinating inventory, transportation, and pricing decisions. The impact is particularly tangible when the transporter's role and the transportation costs are explicitly incorporated into decision mechanisms that aim to coordinate the supply channel. Furthermore, expanding the perspective of channel coordination by introducing the transporter as an individual party in the channel provides tangible benefits for each member of the channel.

Recognizing the need for further analytical research in the field of multi-echelon inventory and channel coordination, we developed and solved a class of integrated inventory and transportation models with explicit shipment consolidation considerations. Moreover, we examined transporter-buyer and supplier-transporter-buyer channels and solved centralized and decentralized models for these channels with the aim of investigating the impact of transporters on channel performance. In this dissertation, we also developed efficient coordination mechanisms between the transporter and the other parties in the channel.

To my parents and my brother

ACKNOWLEDGMENTS

I am grateful to my parents for all the sacrifices they have made since I was born: I thank my father who spent his hours helping me with my classes when I was a little child and inspired me to work hard. I thank my mother for all the love she gave me and for actively supporting me.

I would like to express my gratitude to my advisor Dr. Sila Çetinkaya: She has been my mentor since the first day of my graduate-school life, not only by guiding me academically, but also by helping me whenever and wherever I needed. I also thank Dr. Georgia-Ann Klutke, Dr. Guy L. Curry, Dr. Martin A. Wortman and Dr. Powell E. Robinson for providing their valuable knowledge and for serving as members of my advisory committee. In addition, I would like to thank Dr. Lale Yurttas for being my teaching mentor and for sharing her teaching experiences with me.

Last, but not least, I owe many thanks to my friends. Without their support, I could not have done what I was able to do.

TABLE OF CONTENTS

CHAPTER		Page
I	INTRODUCTION	1
	I.1. Scope of the Dissertation	5
	I.1.1. Integrated Inventory and Transportation Problems with Explicit Shipment Consolidation Decisions	6
	I.1.2. Transporter-Buyer Channels	11
	I.1.3. Supplier-Transporter-Buyer Channels	12
	I.2. Organization of the Dissertation	14
II	LITERATURE REVIEW	15
	II.1. Integrated Inventory and Transportation Models	17
	II.1.1. Pure Consolidation Models	20
	II.1.2. Integrated Models	21
	II.2. Channel Coordination Literature	23
	II.2.1. Multi-Echelon Production/Inventory Literature	24
III	INTEGRATED INVENTORY AND TRANSPORTATION DECISIONS: A QUANTITY-BASED SHIPMENT CONSOLIDATION PROBLEM	30
	III.1. Problem Setting	31
	III.2. The Integrated Inventory/Quantity-Based Dispatch Model	33
	III.3. Analytical Comparison of Time- and Quantity-Based Policies	40
	III.4. Numerical Results	47
	III.4.1. Numerical Results for a Comparison of Time-Based and Quantity-Based Policies	48
	III.5. Managerial Takeaways and Conclusions	50
IV	ANALYSIS OF TIME- AND QUANTITY-BASED POLICIES UNDER COMMON CARRIER FREIGHT SCHEDULES	52
	IV.1. Time-Based Policy	52
	IV.1.1. Derivation of the E[DC] for the Time-Based Policy	53
	IV.1.2. Analysis of C(Q,T)	55

CHAPTER	Page
IV.2. Quantity-Based Policy	62
IV.2.1. Derivation of $E[DC]$ for Quantity-Based Policy . .	62
IV.2.2. Analysis of $C(k, q)$	63
IV.3. Summary and Conclusions	65
V A HYBRID POLICY FOR SHIPMENT CONSOLIDATION . .	67
V.1. Easily Implementable Hybrid Policies	68
V.1.1. Numerical Results Illustrating Cost and Ser- vice Performance Improvements under Hybrid Policies	71
V.2. An Analytical Model for a Pure Hybrid Consolidation Policy	75
V.2.1. An Analytical Model	76
V.2.2. Service vs Cost: A Trade-off Analysis	90
V.3. Summary	92
VI COORDINATION IN TRANSPORTER-BUYER CHANNELS .	93
VI.1. Problem Setting and Operational Characteristics	93
VI.1.1. Operational Characteristics of the Buyer	94
VI.1.2. Operational Characteristics of Transporter	95
VI.2. Single Period Problem in a Single-Buyer, Single-Transporter Setting	96
VI.2.1. Centralized Problem	96
VI.2.2. Decentralized Problems	99
VI.2.3. Channel Efficiency: Centralized vs. Decentral- ized Channels	102
VI.3. Infinite Horizon Problem with Deterministic Price Sen- sitive Demand	105
VI.3.1. Centralized Model	106
VI.3.2. Buyer Driven Decentralized Channel	116
VI.3.3. Numerical Results and Conclusions	121
VII CHANNEL COORDINATION PROBLEMS UNDER CON- STANT AND DETERMINISTIC DEMAND	124
VII.1. Operational Characteristics and Problem Setting	125
VII.1.1. Operational Characteristics of Supplier and Buyer	125
VII.1.2. Problem Setting and Notation	126
VII.2. Analysis of Costs for the Parties	132
VII.2.1. Transporter's Costs and Cost Saving Opportunities	132

CHAPTER	Page
VII.2.2. Supplier's and Buyer's Costs and Cost Saving Opportunities	133
VII.3. <i>Model I</i> - Supplier and Buyer Coordinates	140
VII.3.1. Transporter's Problem	140
VII.3.2. Coordination Mechanisms	148
VII.4. <i>Model II</i> : Supplier and Buyer Act Independently	160
VII.4.1. Transporter's Problem When Supplier Incurs the Transportation Charges	161
VII.4.2. Transporter's Problem When Buyer Incurs the Transportation Charges	162
VII.5. Conclusion	173
VII.5.1. Managerial Insights and Future Work	177
VIII SUMMARY AND CONCLUSIONS	178
REFERENCES	181
APPENDIX A	190
VITA	202

LIST OF TABLES

TABLE	Page
1	Performance Improvement for the Problems in Axsäter (2001) 49
2	% Cost Savings Using Hybrid Solutions 72
3	Average % Cost Savings from Using the Modified Hybrid Solutions 73
4	A Comparison of the Average Service and Cost Performances of <i>Hybrid Policies</i> and the <i>Quantity-Based Policy</i> 75
5	Ratio of Transportation Costs to Total Costs and Total Sales 104
6	Centralized Channel Savings over Decentralized Channels 104
7	Ratio of Transportation Costs in Numerical Examples 123
8	Centralized Channel Savings over Buyer-Driven Decentralized Channel 123
9	Summary of Coordination Mechanisms for <i>Model I</i> 176

LIST OF FIGURES

FIGURE	Page
1	$DC(q)$ as Provided by the Transporter 9
2	$DC(q)$ as Interpreted after the Bumping Clause 10
3	An Illustration of $I(t)$ and $L(t)$ 35
4	An Illustration of $G(q, t)$ for $K = 100, w = 3$ 86
5	Percentage Cost Increase with Respect to Expected Maximum Waiting Time for High K/w Ratio 91
6	Percentage Cost Increase with Respect to Expected Maximum Waiting Time for Low K/w Ratio 92
7	Centralized Profit Function - $\Pi_C(D)$ 97
8	Transporter's Cost Structure 134
9	$G_C(q, n)$ 141
10	Supplier's Discount Schedule Given by Equation (7.17) 163
11	Supplier's Discount Schedule Given by Equation (7.18) 164
12	Possible Forms of $\overline{G}_B(q)$ Depending on the Supplier's Discount . . . 165
13	An Illustration of $\overline{\overline{G}}_B(q)$ in Expression (7.31) 171
14	An Illustration of $\overline{\overline{G}}_B(q)$ in Expression (7.33) 172
15	An Illustration of $\overline{\overline{G}}_B(q)$ in Expression (7.35) 173

CHAPTER I

INTRODUCTION

Effective Supply Chain Management (SCM) is essential in today's competitive environment for two primary reasons:

Supply Chain Costs are Significant: 2001 statistics report that total supply chain costs constitute approximately 9% of the gross national product (GNP) of the U.S. (<http://www.sctrucking.org/economics/statistics.htm>). On the other hand, the average ratio of supply chain costs to sales dollars is significant in almost all industries. Even for the lowest case, it is around 10% (machinery and tools industry) and can be as high as 32%, (food and food products industry). These ratios are similar in other countries as well. In Japan, the average ratio of supply chain costs to sales rate is about 26%, whereas it is around 21% for European Union firms (See Ballou 1992, pp. 14-15).

Customer Expectations are Increasing: In today's competitive markets, customers have a wide variety of options of similar products and multiple alternatives of the same product. Hence, for companies to survive, they must provide high levels of service to their customers. The major criteria for the service level are i) availability of products, and ii) fast delivery/response times for customer orders. In order to satisfy high standards for such performance metrics, companies must evaluate every possible alternative to make their supply links more efficient.

As a consequence, companies have been developing and implementing innovative supply chain practices. Vendor Managed Inventory (VMI) and Third-Party Logis-

tics (3PL) are examples of such practices. In VMI, the vendor, i.e., the supplier, assumes the responsibility of managing the inventory of items at the retailers', i.e., the buyers', sites (Çetinkaya and Lee, 2000). This gives the supplier the opportunity to manage the overall flow of items and to control the flow of information throughout the supplier-buyer channel. One of the most successful examples of VMI practice has been implemented between Wal-Mart and Procter & Gamble. 3PL, on the other hand, means that an outside logistics company controls and manages all inventory and transportation decisions from the manufacturer's site to the warehouses and retail stores. Compaq was one of the first major companies to implement 3PL (Lee, Çetinkaya, and Jaruphongsa, 2003). Both VMI and 3PL integrate and coordinate various supply chain activities, such as inventory, transportation and pricing decisions. They also encourage the parties in the supply chain to combine their efforts and align their incentives so as to improve the overall performance of the chain.

Alongside these innovations in business practices, there has been a tremendous interest in SCM in academia within the past few decades. Supply chain research has built on an increased recognition that a well-designed plan for the chain as a whole requires coordinating different functional specialties within each firm/entity (e.g., marketing, procurement, manufacturing, distribution, etc.) as well as the individual firms/entities of the chain (Çetinkaya, 2004). Consequently, emphasis on coordination has increased in the recent years. As a result, supplier-buyer *channel coordination* has become one of the main research areas

Channel coordination is a term in the supply chain literature that refers to combining the efforts of the parties in a supplier-buyer channel for the purpose of improving the system profit in the decentralized channel and bringing it to the system profit of the centralized channel. The term *decentralized channel* refers to the supplier-buyer channel where the parties make their inventory and pricing decisions so as to maxi-

mize their own profits. In such channels, there is an inherent dominant-subordinate relationship where the dominant party's priorities lead the channel solution (Toptal, 2003). On the other hand, a *centralized channel* refers to the supplier-buyer channel where all of the inventory and pricing decisions are made to maximize the channel profits. In practice, centralized channels are very rare. This is because the parties of the channel are usually owned by different firms. Even in cases where the supplier and the buyer are different branches/departments of the same firm, each branch/department may have its individual objectives. Consequently, decentralized channels are not as efficient as centralized channels as far as the system profits are considered. *Channel coordination* research attempts to identify the inefficiencies in decentralized channels and to develop mechanisms to align the incentives for the parties with those of the centralized channel (Tsay et. al., 2000).

The main concentration of the *channel coordination* literature is on only inventory related costs and inventory decisions. However, recent supply chain initiatives, such as VMI and 3PL, enable the coordination of inventory and transportation decisions as well. Successful implementations of such practices show that substantial savings are realizable through explicit consideration of the transportation costs and the careful integration of transportation decisions with inventory decisions (Çetinkaya, 2004).

Moreover, explicit consideration of transportation costs and coordination of inventory decisions with transportation decisions are vitally important for supply chains because transportation costs constitute an important portion of the supply chain costs. Surveys indicate that almost half of the SC costs are transportation related costs, which implies that such costs are as high as 5% of the U.S. GDP for U.S. based firms. On the other hand, the ratio of transportation costs to total sales dollar varies from 1.4% (electronics industry) to 60% (food industry) and 16.64% (food and food products industry). The values are similar for other countries, also (Ballou 1992,

pp.14-15).

Furthermore, the highly deregulated transportation market in the U.S. provides many opportunities for shippers to efficiently plan their transportation activities and reduce costs. The Airline Deregulation Acts of 1977, the Staggers Rail Act of 1980, and the Motor Carrier Act of 1980 relaxed a number of regulations in the transportation industry (Ballou 1992, pp.200-202). By year 2001, the number of trucking companies exceeded 500,000, a jump from less than 20,000 before the Motor Carrier Act of 1980 (<http://www.sctrucking.org/economics/statistics.htm>). The availability of numerous freight carriers and different transportation mode alternatives allows shippers to select the most suitable alternative(s) with respect to their needs as well as to enjoy cost saving opportunities. On the other hand, the availability of many transportation alternatives brings crucial challenges for the transporters. Transporters should carefully plan their activities via coordinating with their shippers. In this regard, the incorporation of the transporters into channel coordination models provides opportunities for the transporters to benefit from the combined efforts and reduce their costs.

Based on the motivating factors described above, we set the goals of the dissertation as follows:

- to build on the theoretical framework of the existing literature in the context of integrated inventory and transportation decisions,
- to determine the impact of transportation costs on channel decisions,
- to integrate transporters as a separate entity into channel coordination models and address the cost saving opportunities that result from this integration,
- to identify effective mechanisms to coordinate supplier-transporter-buyer chan-

nels and analyze under what conditions they work.

I.1. Scope of the Dissertation

In order to achieve the goals in this dissertation, we

- provide a critical review of the related literature and identify the unexplored areas,
- develop and solve analytical models regarding integrated inventory and transportation decisions,
- develop and solve analytical models for transporter-buyer and supplier-transporter-buyer channels,
- test the performance of commonly used coordination mechanisms and develop alternative mechanisms and incentive schemes that will provide a “win” situation for all parties in the channel.

The problems that we study in this dissertation relate to various levels of decisions. The first three models regarding integrated inventory and transportation decisions specify optimal decisions at the operational level, such as when to replenish inventory and when to ship the consolidated orders to their destination. The focus of the latter models is on tactical decisions, such as price setting and contract design between the parties in the channel. These models also provide insights about strategic decisions for the firms in the context of the degree of partnership among the parties in the channel.

Next, we present some brief information about the specific problems covered in this dissertation:

I.1.1. Integrated Inventory and Transportation Problems with Explicit Shipment Consolidation Decisions

Integrated inventory and transportation decisions have received increasing attention in practical applications and academia over the past few decades. Besides the methodologically oriented (e.g., large scale mixed integer programs) work, a number of analytical studies attempt to provide general managerial insights into operational decisions under conditions of uncertainty (Federgruen and Zipkin 1984a, 1984b, 1984c, Yano and Gerchak 1989, Federgruen and Simchi-Levi 1995, Gallego and Simchi-Levi 1990, Anily and Federgruen 1993, Ernst and Pyke 1993, Viswanathan and Mathur 1997, Qu et. al. 1999, Çetinkaya and Lee 2000, Chan et. al. 2002, and Geunes and Zeng 2003). Çetinkaya (2004) provides a review of the related literature.

Integrated inventory-transportation problems involve the inbound inventory decisions of an entity, say a supplier, as well as the outbound shipments to her customer(s). Since the transportation costs reflect scale economies, it is a common practice to consolidate small orders before making an outbound shipment. Two types of consolidation policies have been widely implemented in practice and identified in the literature: (i) recurrent (ii) non-recurrent. (See Higginson 1995.)

In the non-recurrent policies, the consolidation rules are set in advance. There are two easily implementable non-recurrent consolidation policies:

- *Time-Based Consolidation Policy* A shipment is made every T time units.
- *Quantity-Based Consolidation Policy* A shipment is made when the accumulated quantity/load reaches a target value, q .

The literature also identifies a *hybrid* policy. This policy is characterized by two parameters: T represents the maximum allowable time between two shipments, and q is the target consolidation quantity. A shipment is made either when q units are

accumulated, or after T time units from the previous shipment, whichever occurs first.

There are also variants of these policies. For example, a variant of the *time-based policy* ships when the waiting time of a consolidated load reaches T . Similarly, in a variant of the *hybrid policy*, T denotes the maximum allowable waiting time of an order.

Recurrent consolidation policies do not set any policy parameters in advance. They re-evaluate the shipment release decisions several times within a consolidation cycle; for example, every time an order arrives (Higginson 1995).

In Chapter II, we present an overview of the related literature and identify the unexplored integrated inventory and transportation problems with explicit shipment consolidation considerations. These include the following:

I.1.1.1. Quantity-Based Consolidation Policy

In Chapter III, we model the inbound inventory and outbound shipment consolidation decisions of a supplier. In this model, the supplier serves a group of customers located in close proximity. Her customers are retail stores who do not wish to carry an inventory of the goods, except for display items. As demand arrives, they submit an order to the supplier. The supplier consolidates the orders according to *quantity-based policy* in order to benefit from the scale economies of transportation. The transportation cost, i.e., the shipment cost, has the form

$$C(q) = A_D + c_D q, \quad (1.1)$$

where the first term represents the fixed cost and the second represents the marginal cost. The fixed cost may include all the setup cost to make the shipment as well as the truck cost. The implicit assumption here is that the outbound shipments are made

by private fleet. In Equation (1.1), q represents the consolidation quantity which is a decision variable for the supplier. The supplier's other decision variable is her inventory replenishment quantity, Q .

Assuming the demand arrives according to a random process, we derive the expected annual cost function for the supplier and find the optimal values for q and Q so as to maximize this function.

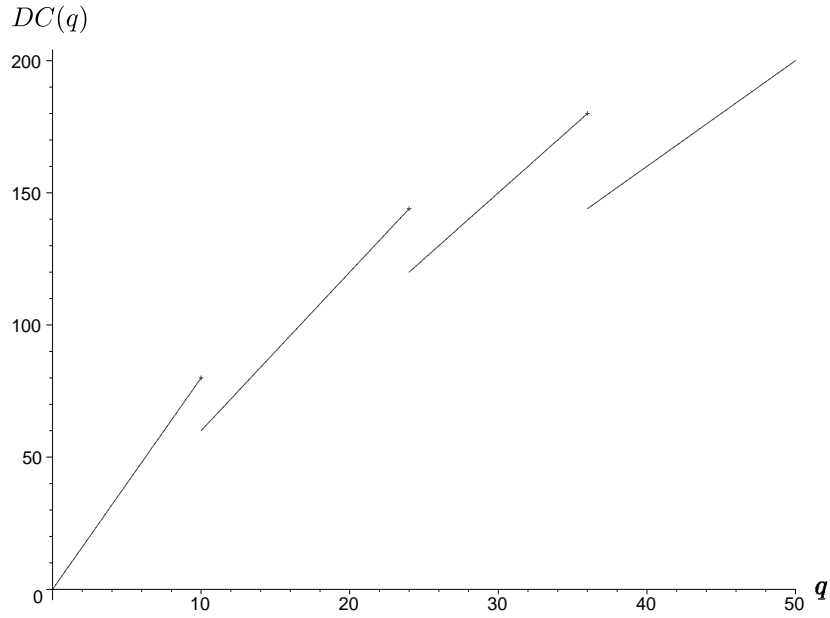
I.1.1.2. Time- and Quantity-Based Consolidation Policies with Common Carrier Freight Schedules

As mentioned above, the model in Chapter III assumes the employment of a private fleet for outbound transportation. However, in practice, most suppliers use common carriers to make such shipments. Common carrier freight rates also reflect scale economies. Under a typical a common carrier freight rate, the shipment cost is of the form

$$DC(q) = \begin{cases} c_0q & \text{if } 0 < q < q_2, \\ c_1q & \text{if } q_2 \leq q < q_4, \\ \vdots & \\ c_iq & \text{if } q_{2i} \leq q < q_{2i+2}, \\ \vdots & \\ c_Iq & \text{if } q_{2I} \leq q, \end{cases} \quad (1.2)$$

where $c_0 > \dots > c_I$ denote per unit freight rates, and $0 < q_2 < \dots < q_{2I}$ denotes the break points. Figure 1 represents the behavior of $DC(q)$.

Observe that $DC(q) > DC(q_2)$ for all $q \in [c_1q_2/c_0, q_2)$. However, it is not reasonable to pay a higher transportation cost for a smaller weight. In fact, shippers are legally allowed to over-declare the actual shipment size to overcome this situation. This

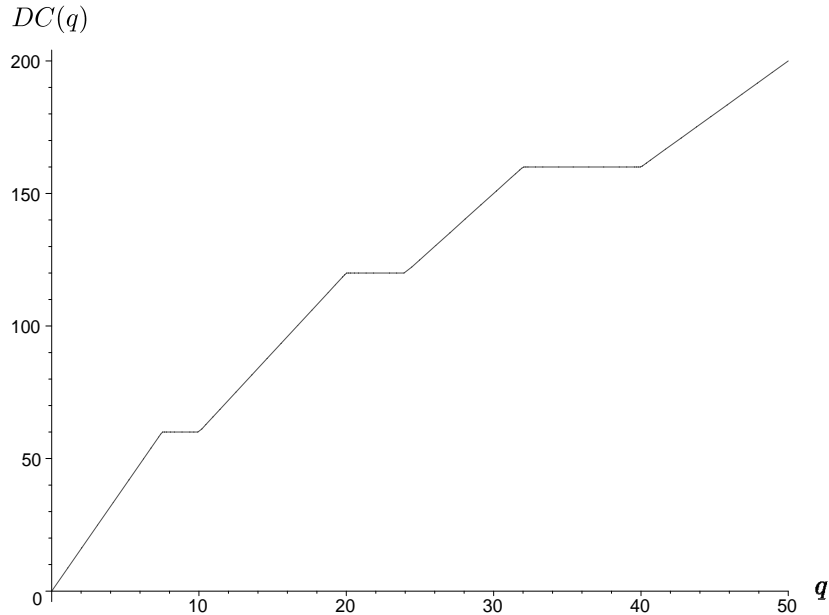
Figure 1 $DC(q)$ as Provided by the Transporter

practice is known as a *bumping clause*. As a result, the actual shipment cost takes the following form:

$$DC(q) = \begin{cases} c_0 q & \text{if } 0 < q < q_1, \\ c_0 q_1 & \text{if } q_1 \leq q < q_2, \\ \vdots & \\ c_i q & \text{if } q_{2i} \leq q < q_{2i+1}, \\ c_i q_{2i+1} & \text{if } q_{2i+1} \leq q < q_{2i+2}, \\ \vdots & \\ c_I & \text{if } q_{2I} \leq q, \end{cases} \quad (1.3)$$

where $q_{2i-1} = c_i q_{2i} / c_{i-1}$ for $i = \{1, \dots, I\}$. Figure 2 shows the dispatch cost after the bumping clause.

Figure 2 $DC(q)$ as Interpreted after the Bumping Clause



An exact analysis of the model in Chapter III with such common carrier costs has not been conducted for either a *time-based policy* or for *quantity-based policy*. In Chapter IV, we develop the expected annual cost expressions and derive the optimal policy parameters under both of these consolidation policies.

I.1.1.3. Hybrid Consolidation Policies

Hybrid consolidation policies try to utilize the cost benefits of *quantity-based policies* and the service performance of *time-based policies*. Within the setting of the previous

problems, we propose several easily implementable hybrid consolidation policies and compare their cost and service performance with the two policies mentioned above.

Computing the optimal solution of the integrated inventory and transportation problem of interest under a hybrid consolidation policy is analytically intractable. Hence, we consider the pure shipment consolidation problem under a *hybrid policy*, i.e., the case where inventory decisions are ignored, and compute the corresponding optimal policy parameters, i.e., q and T values. To the best of our knowledge, this problem has remained unexplored in the literature for several years.

I.1.2. Transporter-Buyer Channels

The focus of the integrated inventory and transportation problems of interest in this dissertation is operational level (daily) decisions. Broadening our scope, we next introduce problems which will provide results and insight about higher level decisions, such as pricing decisions and coordination with the other parties in the channel. Along with this, we not only consider explicit transportation costs in our models but also introduce the transporter as a separate entity in our models.

We start our analysis with the transporter-buyer channel¹: The buyer observes customer demand and orders from an outside supplier. The transporter is responsible for the inbound transportation. We model the demand as a function of the retail price. The main rationale for assuming a price dependent demand is to analyze the impact of transportation costs on the retail price and, thus, the demand. Channel coordination problems with price dependent demand has begun receiving increased attention since the early 90s. Abad (1994b), Parlar and Wang (1994), and Weng (1995a, 1995b)

¹In this channel, the so-called buyer is, in fact, a retailer. Hence, we could have named the channel a transporter-retailer channel. However, in the traditional channel coordination literature, the channels are referred to as supplier-buyer channels. In order to keep with this convention, we prefer to name it the transporter-buyer channel.

produced pioneering examples of such models. In an earlier paper, Porteus (1985) employs various price-demand functions in the EOQ-setting. We utilize his results in our models.

The transporter's costs are of particular importance in the problem. We assume that the transporter owns a number of trucks (the number of trucks is not a constraint), each with a certain capacity P and a fixed operating cost R_T . Hence, letting q be the buyer's order quantity, the truck cost is $\lceil q/P \rceil R_T$. We also consider a per unit transportation cost denoted by c_T . With these parameters, the total cost of the transporter for carrying q units is

$$\lceil q/P \rceil R_T + c_T q. \quad (1.4)$$

This cost structure is also known as *multiple set-up cost* structure in the classical inventory literature (Lee, 1986).

We study transporter-buyer channel via two models in Chapter VI. First one is a single period model. The buyer orders once, and the order quantity is equal to the demand. In the second model, the planning horizon is infinite, and time is considered on a continuous scale.

For both models, we study the fully coordinated scenarios where the parties operate in a centralized fashion, and uncoordinated scenarios where the parties make their decisions independent of each other. This way, we benchmark the efficiency of channel coordination.

I.1.3. Supplier-Transporter-Buyer Channels

In the literature, the typical supply chain coordination problem is modelled within the supplier-buyer context. In order to analyze the impact of transporters on channel coordination decisions, we extend the transporter-buyer channel to a supplier-

transporter-buyer channel in Chapter VII.

The supplier-transporter-buyer channel that we study is very similar to the supplier-buyer channel of Lee and Rosenblatt (1986). The operational characteristics and cost parameters of the supplier and buyer are the same as those in Lee and Rosenblatt's model. We introduce the transporter into this channel for the delivery of items from the supplier to the buyer. The transporter's cost function is given by Equation (1.4).

We consider two different models for the supplier-transporter-buyer channel. In the first model, the supplier and the buyer are controlled by a central decision maker, and these parties act as a single entity. They decide on their order quantities and frequencies so as to minimize their total annual cost based on an initial price schedule from the transporter. The second model assumes all of the parties in the channel are independent entities. In this model, the buyer chooses the order quantity to minimize his annual cost based on the initial wholesale and transportation prices. Then, both the supplier and the transporter would like to align the incentives of the buyer to change this order quantity. Based on which party, i.e., supplier or buyer, incurs the transportation charges, we study the second model in two cases.

In Chapter VII, we solve all of the resulting analytical problems regarding the optimal order quantity, i.e., inventory replenishment decisions. In addition, we also study the coordination mechanisms available to the transporter to coordinate the other parties: We first analyze how commonly used coordination mechanisms perform and identify the necessary and sufficient conditions for them to achieve channel coordination. Next, we propose several coordination mechanisms and contracts which guarantee channel coordination.

I.2. Organization of the Dissertation

Next, in Chapter II, we present an overview of the relevant literature. In Chapter III, we study the integrated inventory and transportation problem of interest under a quantity-based consolidation policy. We compare the cost performance of this policy to that of time-based policy which was earlier studied by Çetinkaya and Lee (2000). We study the same model under common carrier shipment costs in Chapter IV and present the optimal solutions for both time- and quantity-based consolidation policies. In Chapter V, we propose several hybrid policies, and through simulation, we compare the cost and service performances of the hybrid policies to time- and quantity-policies. Following this, we also study a pure consolidation model for which a hybrid consolidation policy is implemented. We derive the optimal policy parameters for this model. Chapters III- V focus mainly on the impact of transportation costs on operational decisions, such as inventory replenishment and outbound shipment decisions. In Chapters VI and VII, we widen our scope to analyze the impact of transporters in channel decisions. More specifically, in Chapter VI, we study transporter-buyer channels, and in Chapter VII, we extend this to the supplier-transporter-buyer channel.

CHAPTER II

LITERATURE REVIEW

Contemporary supply chain management research places an increased emphasis on the integration/coordination of different functional specialties along the supply chain as well as within the entities of the chain (Çetinkaya, 2004). As we explained in Chapter I, (i) the advent of new supply-chain initiatives such as VMI and 3PL, (ii) increased transportation costs, and (iii) increased competition amplifies the criticality of the integration of inventory, transportation, and pricing decisions. In this chapter, we present an overview of the supply chain coordination literature that emphasizes these issues.

We classify the related supply chain coordination literature into two areas:

Coordination of inventory-transportation decisions: Integrating inventory replenishment decisions with transportation decisions has always been one of the major concerns of logistics managers, and the topic has been extensively studied in the literature. Related literature spans a very broad range of work from network flow and large scale optimization problems to stochastic inventory problems. Within the scope of this dissertation, we focus on the gap in the literature and direct our attention to stochastic models for integrated inventory and transportation decisions under explicit shipment consolidation considerations. Çetinkaya (2004) presents a review of the class of problems in this area, reviews the literature, and identifies possible areas for future research, which provide the main motivations of the analytical models presented herein.

Channel coordination under explicit transportation costs: Channel coordination problems aim to synchronize inventory decisions in two-echelon settings,

i.e., supplier-buyer settings. In this regard, the classical multi-echelon inventory literature provides a foundation for channel coordination problems. Clark and Scarf's (1960) seminal paper is one of the first in the area of multi-echelon inventory problems. In this paper, Clark and Scarf study a periodic review inventory control problem for a serial system, i.e., a multi-echelon system, facing independent, identically distributed demand. They show that, under certain assumptions, the problem of deriving optimal inventory policies that minimize the system-wide costs can be decomposed into serial single-echelon problems. Building on Clark and Scarf's (1960) work, many of the researchers that followed them study the optimal inventory policies in multi-echelon settings (e.g., Federgruen and Zipkin 1984a, 1984b, Debo and Graves 1985, Rosling 1989). Federgruen (1993) and Muckstadt and Roundy (1993) present excellent reviews of the multi-echelon production/inventory literature spanning three decades of work since the 1960s. The classical literature in the area does not take into account the impact of transportation costs explicitly. However, the potential impact of transportation related costs and their impact on inventory decisions has been gaining increasing attention. This dissertation falls into this stream of research by addressing the impact of transportation costs on coordination. Topal's PhD dissertation (2003) also is in this category. Topal (2003) presents a critical review of the buyer-vendor coordination literature that emphasizes transportation, production/inventory and channel coordination issues which are very closely related to our work.

Next, we proceed with more detailed reviews of the two areas of literature summarized above.

II.1. Integrated Inventory and Transportation Models

The broad range of the literature on integrated inventory and transportation in supply chain coordination makes it virtually impossible to present a complete review in this dissertation. Hence, after briefly presenting an overall picture, we focus our attention on the literature most relevant to our work.

That is, we provide a detailed review of the literature on analytical models that examine the coordination of inventory and outbound shipment decisions. Consider a supplier, serving one (a group) of retailer(s), i.e., buyer(s): the supplier's inventory is depleted by the orders coming from the buyer(s) and she has to control (i) when to replenish her inventory, and (ii) how to schedule the shipment of the buyer's orders. This problem setting can also be extended to multi-echelon cases where the buyer's inventory decisions are also included in the model. In such a case, the buyer's inventory policy defines the order process for the supplier. One of the seminal works in this area is by Yano and Gerchak (1989). They consider a single-supplier, single-buyer model with stochastic demand. The supplier is a Just-In-Time (JIT) manufacturer, and the retailer implements a base-stock policy. The shipments from the supplier to the buyer can be made using a private truck fleet and a common carrier. Their assumption is that if the order quantity exceeds the total truck capacity of the private fleet, emergency shipments should be made using common carriers. They find the optimal base-stock levels for the buyer and also the number of trucks to be allocated. Ernst and Pyke (1993) extend this model to the case where the supplier also carries inventory, and her inventory cost is also included in the average cost to be minimized. They assume that the supplier also implements a base-stock policy; hence, they have an additional decision variable in their model. Geunes and Zeng (2003) consider a special case of Ernst and Pyke (1993) where the review period is

not a decision variable, and they assume that the excess demand can be backordered and expediting emergency orders avoided in most cases. They were able to show that practical cases exist where a combination of backordering and expediting emergency orders outperforms both complete backordering and complete expediting policies.

Although, the studies mentioned above consider transportation related issues, such as the type of the carrier, and the associated transportation costs, they do not investigate the impact of shipment scheduling decisions. This is because in all of the papers mentioned above, shipment scheduling is implicitly determined by the buyer's orders. The supplier has no control over delaying shipments. However, there are practical cases where the supplier has the flexibility to combine several orders from her buyers and schedule the shipments so that she can benefit from the economies of scale of transportation. This practice is known as *shipment consolidation*. Shipment consolidation models constitute an essential part of this dissertation. Hence, we proceed with a detailed discussion of the the related literature.

As defined earlier in Section I.1.1, the practice of consolidating small size orders to accumulate a larger shipment size with the aim of benefiting from the scale economies of transportation costs is called *shipment consolidation*. Transportation costs may have several forms: Equations (1.1), (1.2), and (1.4) are different examples of transportation costs. All of these examples reflect scale economies, which provide a motivation for the shippers to consolidate for large sizes at the expense of customer service.

Shipment consolidation practices first appeared in the logistics trade journals, e.g., Newbourn and Barret (1972) and Pollock (1978). The early models were based on performance testing via simulation. Some examples are Masters (1980), Jackson (1981), Cooper (1984), and Closs and Cook (1987). Most of the early papers that appeared in trade journals focused on “how to consolidate?” and identified several easily

implemented consolidation policies. The two most common shipment consolidation policies are the *time-based policy* and the *quantity-based policy*. The literature also identifies hybrid policies. In Chapter I, we explain the way these policies work. An early survey by Jackson (1981) indicates that the *time-based policy* is the most frequently implemented consolidation policy. However, there was not a large difference in percentage of usage between the three policies.

Another focal point of the studies within the shipment consolidation area is the economic justification of shipment consolidation and how this is effected by external factors. Blumenfeld et al. (1985), Burns et al. (1985), Hall (1987), Daganzo (1988), Campbell (1990), Abdelwahab and Sargious (1990), Russell and Krajewski (1991), and Pooley and Stenger (1992) are some examples.

Jackson (1981) is one of the first studies that compares consolidation policies via simulation. He compares a *time-based policy* to a *hybrid policy*. More recent studies try to compare the performances of the three consolidation policies via simulation. Higginson and Bookbinder (1994) simulate different scenarios by varying the policy parameters, T and Q , of the policies. In their model, they assume common carrier transportation. Orders arrive to the system at random times and random sizes. They identify possible situations where one policy works better than the others. They also emphasize that “knowledge of the level of service required by the customers is crucial in selecting a shipment-release policy. Customer service and order arrival rate must be examined simultaneously...”.

Neither the work that identifies and compares shipment consolidation policies nor the studies that focus on the economic justification of these policies have as their goal computing optimal policy parameters, an area of research that has gained significant attention in the shipment consolidation literature. In Sections II.1.1 and II.1.2, we will review the analytical studies that investigate the optimal policy parameters

for pure consolidation and the integrated inventory and shipment consolidation models, respectively. In pure consolidation models the decision variables are only the consolidation policy parameters; whereas, in integrated inventory and shipment consolidation models inventory replenishment quantity and frequency are the additional decision variables.

II.1.1. Pure Consolidation Models

Higginson and Bookbinder (1995) employ a Markovian Decision Process (MDP) approach to determine the optimal shipment release schedule. Such an approach is called a *recurrent approach* whereas the other policies can be classified as *non-recurrent*. The distinction between recurrent and non-recurrent approaches is first addressed by Higginson (1995). He notes that, although non-recurrent policies yield good average performance, for random orders, they may suffer from poor cost and service due to some aspects realizations of the order processes. In this regard, recurrent approaches are superior to non-recurrent ones. Higginson and Bookbinder (1995) identify the optimal recurrent strategies for both common carrier and private fleet usage.

Gupta and Baghci (1987) study an inbound freight consolidation in a Just-in-Time (JIT) environment. They adopt Stidham's (1977) results on stochastic clearing systems. Brennan's PhD Dissertation (1981) provides structural results when consolidated loads are reviewed on a periodic basis for both deterministic and stochastic demand problems. Other analytical treatments include those based on queueing theory (Powell 1985, Powell and Humblet 1986) in the setting of passenger transport; stochastic clearing systems (Stidham 1977); and dynamic vehicle dispatch (Minkoff 1993, Gans and Van Ryzin 1999).

In a recent paper, Çetinkaya and Bookbinder (2003) study the optimization of the policy parameters for time- and quantity- policies in a stochastic setting. They

analyze the problems for private carriage, and then for common carriage. In their model, both the size and the interarrival times of orders follow a probability distribution. The order arrival stream is Poisson. However, most of the results for quantity-based policy still hold for general arrival processes. They assume a fixed cost for the delivery of each order in addition to the fixed cost associated with each shipment release. Under these modelling assumptions, for private carriage, they were able to provide exact optimal solutions for the two policies. They also provide approximate solutions for common carriage, and discuss the special case with unit order sizes and provide results. Although Çetinkaya and Bookbinder (2003) study *time-based policy* and *quantity-based policy*, they do not focus on optimizing the model parameters of *hybrid policies*. To the best of our knowledge, there is no work that specifically considers this problem. In this dissertation, we fill this gap in the literature by studying the *hybrid policy* in Chapter V. On the other hand, Çetinkaya and Bookbinder's (2003) common carriage models are closely related to the models that we consider in Chapter IV. Our work differs from theirs, because we consider common carriage in an integrated inventory and shipment consolidation model, and we only study unit demand size. A special case of our work, where the supplier does not carry any inventory, is equivalent to the pure consolidation models. We were able to provide exact expected cost expressions for common carriage cases. However, for the *time-based policy*, we could not obtain analytical solutions either.

II.1.2. Integrated Models

The main objective of the studies mentioned in Section II.1.1 is obtaining analytical results for the shipment consolidation policies, only. They do not investigate the interactions between inventory and shipment consolidation decisions. Observing the need for analytical models examining these interactions, Çetinkaya and Lee (2000)

study the inventory replenishment and outbound shipment scheduling problem of a supplier in a VMI context. The supplier is authorized to manage inventories of an agreed-upon item at several retail stores located in a given geographical region. The VMI contract between the retailer and the supplier gives the supplier the liberty of controlling the downstream, i.e., retailers, resupply decisions rather than filling the orders as they arrive. The orders are assumed to arrive according to a Poisson stream, and they are of unit size. The supplier implements a *time based* consolidation policy for dispatching the retailers' orders, and the policy parameter, T , is a decision variable. The retailers agree to wait for their orders; however, they charge the supplier $\$w$ on a per unit per time basis. The supplier also has to manage her inbound inventory replenishment. She implements a periodic review, and orders Q units when the inventory level drops below 0. Q is the supplier's other decision variable. The analytical problem is deriving the optimal T and Q so as to minimize the expected annual cost. Çetinkaya and Lee (2000) provide an easily implementable approximate solution algorithm for this problem. Axsäter (2001) presents a procedure that optimally solves the problem.

Çetinkaya, Tekin, and Lee (2000) consider a similar setting. In their model, the supplier implements a *quantity-based* consolidation policy, and the target load size is q . Differently from Çetinkaya and Lee (2000), the size of each order is also a random variable. The analytical model for this problem tends to be highly complicated, which makes it impossible to obtain exact analytical solutions. Hence, the authors provide an approximate solution procedure. Based on this solution, they identify three policy forms: In *Form I*, the supplier does not carry any inventory, i.e., $Q = 0$; *Form II* represents the case where the supplier carries inventory and shipment consolidation is economically justified, i.e., $Q, q > 0$; and *Form III* is the immediate delivery case, i.e., $q = 0$. Observe that a case can be both of *Form I* and *Form III*.

The model that we consider in Chapter III can be considered a special case of Çetinkaya, Tekin, and Lee (2000) for orders with a unit size. In our model, we were able to derive exact solutions for general order arrival processes. We also compare the results of the *quantity-based policy* to those of the *time-based policy* studied by Çetinkaya and Lee (2000).

Çetinkaya and Lee's (2000) and Çetinkaya, Tekin, and Lee's (2000) models assume stochastic order arrival processes. Çetinkaya and Lee (2002), on the other hand, study deterministic orders. The order stream is modelled with a known and stationary rate D per year. The *time-based* and the *quantity-based policies* are equivalent in this setting. Çetinkaya and Lee (2002) provide exact solutions for the optimal policy parameters. They show that the optimal consolidation load is not constant. Defining the time between two inventory replenishment epochs as *replenishment cycles*, they prove that the last shipment size, i.e. the last consolidation quantity, in a *replenishment cycle* should be larger than the previous ones. Çetinkaya and Lee (2002), also, study a so-called *finite cargo capacity* model. In this model, the outbound shipments are made using a private fleet of trucks, each with certain capacity P , and a fixed cost is incurred for each truck used. This cost structure is the same transportation cost structure that we use in Chapters VI and VII. In Equation 1.4, we present an analytical expression for this type of transportation cost as a function of the shipment quantity, q .

II.2. Channel Coordination Literature

Tsay et. al. (2000) defines channel coordination, using a phrase coined in the marketing literature that applies to improving the total expected system profits in a decentralized model and to bringing them closer to those of a centralized model. A

centralized model is one where all decisions are made in order to maximize(minimize) the system profits(costs). In the decentralized models, each party in the channel makes her/his decisions so as to maximize(minimize) her/his profits(costs).

Traditionally in the literature, a *supply channel* is considered a two-echelon system with a supplier/vendor/manufacturer at the upper echelon and a buyer/retailer at the lower echelon. The problems of interest in the context of the *channel coordination* literature focus on the inventory and pricing decisions at these echelons. The centralized models provide the benchmarks for channel efficiency (Toptal, 2003). Consequently, the optimal solutions for the centralized models have a particular significance. As mentioned before, the integrated models that aim to optimize inventory decisions in a supply chain are known as *multi-echelon inventory models*. We have also noted already that the origins of the multi-echelon inventory models date back to the classical work of Clark and Scarf (1960). In the following five decades, a great body of literature has been built in this area. In the following section, we present a brief summary of this multi-echelon inventory literature.

II.2.1. Multi-Echelon Production/Inventory Literature

After Clark and Scarf (1960) showed that, under certain conditions, multi-echelon inventory problems can be solved as a series of single-echelon inventory problems, interest in these problems surged. Clark and Scarf's model assumed stochastic demand, which relates the multi-echelon problem to the theory of stochastic inventory. Another line of work in this area treats demand as deterministic and relates the problems to the theory of deterministic inventory and production. In our review, we classify the multi-echelon literature according to the demand assumptions and mention only the most seminal work in each of these groups.

Although the theoretical implications of Clark and Scarf's (1960) solution are

very promising, the technique is computationally intense. Federgruen and Zipkin (1984a, 1984b) provide approximations and computationally effective tools for computing the optimal base-stock levels for the problem studied by Clark and Scarf. All of these models assume periodic review of the inventory levels and use of (s, S) policy for inventory replenishments. Axsäter (1997, 1998a, 1998b), Axsäter and Rosling (1993), and Chen (2000) study multi-echelon serial systems in a continuous review setting and derive the optimal values for the (Q, R) policies. In addition, Axsäter (1993), Cachon (1995), and Chen (2000) study the impact of batch-ordering policies for serial systems. Axsäter (1995) derives the approximate cost functions for a single-supplier multi-retailer model with batch ordering policies. The literature on stochastic demand, multi-echelon inventory models is vast, and an extensive review is beyond the scope of our research. We refer to Axsäter (2000) for a review of broad range of papers that study multi-echelon problems under various assumptions.

On the other hand, the deterministic-demand multi-echelon inventory started flourishing in the early 1970s. The pioneers of these studies are Schwarz (1973) and Goyal (1976). In fact, Goyal (1976) is one of the first to introduce the “centralized” and “decentralized” models. Goyal’s (1976) problem setting is very similar to the supplier-buyer setting which we consider in Chapter VII. Both the supplier and the buyer incur fixed costs for replenishing inventory, A_S and A_B , respectively; and both incur a per unit per time inventory carrying cost, h_S and h_B , respectively. Demand is known and has a stationary rate D . In our problem, we formulate the annual cost as a function of the order quantities. Goyal (1976) builds the annual cost function based on the times between orders. We note that, under these demand assumptions, both formulations are equivalent. In addition, although Goyal models the supplier as a manufacturer, he assumes such a high production rate that the manufacturer’s inventory is instantaneously replenished.

Both Schwarz (1973) and Goyal (1976) consider a two-echelon setting with a single supplier and single buyer. Single supplier - multiple buyer problems have also been a major avenue of research. The most notable results in this area are by Roundy (1985, 1986) who develops 98% effective nested delivery policies, which are known as *power-of-two policies*, for a single-supplier multi-retailer model. Banerjee and Burton (1994), and Lu (1995) other examples.

An important generalization of the deterministic two-echelon inventory models is the case where the supplier is a manufacturer having a finite production rate (Banerjee 1986b, Goyal 1988, Banerjee and Burton 1994, Goyal 1995, Viswanathan 1998, Hill 1999). Toptal (2003) provides a detailed review of the production/inventory models in two-echelon settings. We refer to Toptal (2003) for further information and proceed with the review of channel coordination models.

II.2.1.1. Channel Coordination Models

The concept of channel coordination is adopted from the marketing literature. The idea of using quantity discounts as a mechanism to influence the buyer's ordering decisions constitutes the roots of channel coordination. Quantity discounts have been frequently used by marketing managers for a very long time. However, the role of discounts was not clearly understood by many until Dolan (1987) presented a detailed analysis of the motivations of quantity discounts. Jeuland and Shugan's (1983) study is another example from the marketing literature that discusses how price discounts can be implemented for the purpose of sharing profits and achieving channel coordination. They propose channel coordination mechanisms for single supplier-single buyer channels as well as single supplier- multiple buyer channels. The coordination mechanisms for single supplier - multiple buyer channels provided by Jeuland and Sguan (1983) require that the supplier offers different price schedules to different buyers.

However, such an implementation does not comply with the Robinson Patman Act, which aims to assure that suppliers do not price differentiate their buyers. Lal and Staelin (1984) and Hoffman (2000) study the channel coordination mechanisms that comply with the Robinson Patman Act.

Dolan (1987) and Jeuland and Shugan (1983) offer meritorious examples of channel coordination ideas in the marketing literature. The pioneers in adopting channel coordination in the supply chain management literature are, on the other hand, Monahan (1984) and Lal and Staelin (1984).

Monahan (1984) considers a single supplier single buyer channel where the buyer observes a deterministic and constant demand. The buyer's goal is to minimize his annual inventory and ordering costs. In this sense, the buyer's problem is identical to the classical EOQ problem. On the other hand, the supplier's replenishment size is equal to the buyer's order quantity. Hence, the supplier's annual cost depends solely on the buyer's order size. This type of a setting represents a decentralized model with sequential decisions. In this particular case, the buyer is the leader, and the supplier is the follower. Monahan identifies how the supplier's profit can be increased if the buyer's order size increases by a factor K . He shows that the optimal value of K is $\sqrt{1 + A_S/A_B}$, where A_S and A_B are the fixed replenishment costs of the supplier and the buyer, respectively. In fact, the resulting order quantity is the optimal order quantity for the centralized model. Later Toptal (2003) generalizes Monahan's result by proving that when the supplier's profit is an increasing function of the buyer's order quantity, then the buyer's optimal order quantity is always less than the optimal order quantity in the centralized model. Monahan (1984) also shows that a quantity discount with a single break point suffices to align the buyer's incentives so as to change his order quantity from q_B to $q_S = K^* q_B$. Banerjee (1986a) extends Monahan's (1984) model to incorporate the inventory carrying costs of the supplier. In Banerjee's

(1986a) model, the supplier is a manufacturer with a finite production rate, R . He shows that there are cases for which decreasing the buyer's order quantity increases the channel profits. Banerjee finds that the K^* for this setting is $\sqrt{(1 + \alpha)/(1 + \beta)}$ where $\alpha = A_S/A_B$, and $\beta = Dh_S/Rh_B$. Lee and Rosenblatt (1986) extend Monahan's (1984) model to the case where the supplier also carries inventory. Unlike Banerjee (1986), the supplier, in their model, is not a manufacturer. Their model is similar to the supplier-buyer model that we study in Chapter VII. They show that under certain conditions, Monahan's deduction that the system-wide optimal order quantity is larger than the buyer's EOQ is correct.

Dada and Srikanth (1987), Goyal (1987), Joglekar (1988), Kohli and Park (1989), Abad (1994a), Joglekar and Kelly (1998), Klastorin, Moinzadeh and Son (2002) offer other examples of channel coordination models. All of these channel coordination models we have discussed so far assume constant demand. Weng (1995a) studies the channel coordination problem with price dependent demand and shows that neither an all-units nor an incremental quantity discount is sufficient to coordinate the channel at the system optimal. However, he shows that both the supplier's and the buyer's profits can be improved through a carefully designed quantity discount schedule. Abad (1994b), Parlar and Wang (1994), Weng (1995b), Weng and Zeng (2001), Ertek and Griffin (2002), and Lau and Lau (2003) also study channel coordination under price dependent demand.

None of the studies that we mentioned here explicitly consider transportation costs and their impact on channel coordination with the exceptions of Toptal, Çetinkaya, and Lee (2003), and Toptal and Çetinkaya (2004, 2006). Toptal, Çetinkaya, and Lee (2003) consider a centralized supplier-buyer channel in a deterministic demand setting. They assume that both the supplier's and buyer's order cost have the generic form presented by Equation (1.4). Toptal and Çetinkaya (2004) study the corre-

sponding decentralized channels and coordination mechanisms. Toptal and Çetinkaya (2006), on the other hand, study counterpart centralized and decentralized channels and coordination mechanisms in a stochastic demand environment. Neither of these studies models the transporter as a separate entity in the channel. However, our work is unique in the literature in that we model the transporter as a separate entity in the channel and propose a wider perspective for supply channels, extending them from supplier-buyer channels to supplier-transporter-buyer channels. Our work differs from these studies in the sense that we introduce the transporter as a separate entity to the channel. Hence, beyond analyzing the impact of transportation costs on channel decisions, we also analyze the role of transporters in channel coordination.

CHAPTER III

INTEGRATED INVENTORY AND TRANSPORTATION DECISIONS: A
QUANTITY-BASED SHIPMENT CONSOLIDATION PROBLEM

In this chapter, we study a joint inventory replenishment and outbound dispatch scheduling problem identified in a recent paper by Çetinkaya and Lee (2000). The problem of interest arises in the context of vendor-managed supply arrangements where the vendor is authorized to manage the supply of agreed upon items at a group of downstream supply-chain members, i.e., a group of retailers located in a given geographical region. By retrieving demand information from the retailers, the vendor makes decisions regarding the quantity and timing of re-supply. Hence, the vendor has the autonomy to consolidate small orders from the retailers until a larger dispatch quantity accumulates. This practice is known as temporal shipment/load consolidation. If a temporal shipment consolidation policy is in place, then the actual inventory requirements at the vendor are, in part, specified by the timing and quantity of dispatch decisions. In this context, the vendor's problem is the computation of an integrated inventory replenishment and outbound dispatch policy.

As discussed in Axsäter (2001), two different types of temporal shipment consolidation routines are popular in transportation and logistics applications (also see Çetinkaya and Bookbinder 2003, Higginson and Bookbinder 1994, Higginson and Bookbinder 1995.) These are i) *time-based*, and ii) *quantity-based* dispatch policies. A *time-based policy* ships accumulated loads (clears all outstanding orders) every T periods whereas a *quantity-based policy* ships an accumulated load when an economical dispatch quantity, say q , is available. Çetinkaya and Lee (2000) develop a model for joint optimization of inventory replenishment and shipment release decisions where the vendor implements a time-based dispatch policy, and they present

an approximate optimization technique. In a recent note, Axsäter (2001) provides an exact optimization procedure for Çetinkaya and Lee’s (2000) model.

Building on Çetinkaya and Lee (2000), we develop here a model for the case where the vendor implements a quantity-based dispatch policy. We develop an exact optimization procedure for the quantity-based model. We also present analytical proofs as well as numerical results showing that the cost savings in using this model, rather than the time-based model, can be substantial.

This chapter is organized as follows: The quantity-based dispatch model and its exact optimal solution are discussed in Section III.2. In Section III.3, we provide some analytical results proving that the cost savings obtained by using the quantity-based model are substantial. In Section III.4, we present results of our numerical study that compares time- and quantity-based policies.

III.1. Problem Setting

As we have already mentioned, the problem setting considered here is the same as the one in Çetinkaya and Lee (2000). As an example of this setting, consider a product that is clearly unreasonable for the retailers to keep in stock—say office photocopy machines, expensive laptop computers, etc. The retailers may own some display models of the product, but, typically, they act as sales agents who help customers decide what type of product best suits their needs and offer after-sales service so that customer orders are placed from, and delivered to, retail locations. These situations are particularly common for businesses selling high-tech, bulky, or expensive items through retail stores where the inventory holding cost for the retailer is high and the customer waiting cost is modest for “reasonable” time intervals so the retailer does not have to carry inventory and the vendor satisfies orders from end-customers placed

through a retailer. Before releasing an outbound shipment to the retailers, the vendor has the liberty of consolidating several small orders/deliveries so that transportation scale economies can be achieved.

For the particular practical application that motivated this study, the vendor is a distributor (i.e., a third-party logistics provider operating under the terms of a service contract for warehousing and distribution), and the cost of replenishing inventory at the distributor's warehouse (i.e., the cost of an inbound shipment) is a fixed cost and the corresponding delivery lead time is negligible. Note that these assumptions are valid provided that the two parties are located within a certain proximity, say 200-300 miles, so that a fast delivery with a negligible lead-time is possible. Under the terms of a service contract between the two companies (i.e., the manufacturer and the distributor), the manufacturer pays the distributor for storage and distribution based on the number of demands that are satisfied and the costs incurred. As a result, both the manufacturer and the distributor would like to take advantage of cost saving opportunities at the distributor's location. In this context, an important cost saving opportunity is realized through shipment consolidation in outbound transportation.

The distributor serves a group of downstream retailers with stochastic demands that are located in a given geographical region. The orders from the geographical region (i.e., retailers) arrive at the distributor's site possibly through an electronic data interchange (EDI) link. After consolidating these orders, the distributor releases a combined dispatch quantity to the region to satisfy the outstanding orders of all the retailers in the area. Since the major cost of a dispatch is a fixed term associated with launching a truck to the geographical area and this cost does not drastically change based on which or how many retailers are visited, we model the cumulative demand of the region as a convolution of the individual retailers' demands. Hence, in the remainder of the paper, we refer to a single retailer with this revised demand distri-

bution. Since demand is modeled as a pure Poisson process, our results are applicable regardless whether the geographical region includes a single retailer or multiple retailers whose demands arrive according to independent Poisson processes. We note that this approach is viable if different geographical regions are served individually or by different distributors, while the case where multiple geographical regions are served simultaneously is a topic for future research.

In sum, replenishment decisions at the distributor's warehouse contribute to a build up of inventory, which is depleted later by outbound dispatch decisions, for delivery to the retail location. As a result, the actual inventory requirements and the costs of the distributor are dictated by the dispatch schedules. Hence, we consider alternative integrated inventory and outbound dispatch models for the purpose of investigating the impact of practical dispatch policies on the cost efficiency. We also note that the results of this paper can be used by the distributor for operational planning or by the manufacturer for the purpose of estimating the distribution contract value/cost for a particular geographical area of interest.

III.2. The Integrated Inventory/Quantity-Based Dispatch Model

We begin our analysis by developing an analytical model for the case where the vendor releases a shipment as soon as the size of an outbound load waiting to be released reaches a *critical dispatch quantity* denoted by q . In this context, the time between two successive outbound dispatch decisions is called a *dispatch cycle*, and all orders arriving during a dispatch cycle are combined to form a large outbound load. Our assumptions regarding the characteristics of the underlying demand and inventory processes are as follows:

- As we have already mentioned, since one of the objectives of this paper is to provide a comparative analysis of the model in Çetinkaya and Lee (2000), we

concentrate on the case where demand arrivals form a Poisson process with interarrival times $X_i, i = 1, 2, \dots$ where $E[X_i] = 1/\lambda$ so that the arrival rate is λ . However, we note that the quantity-based dispatch model developed in this section is also applicable for general unit arrival processes.¹ Letting $S_0 = 0$ and $S_i = \sum_{j=1}^i X_j$, we define $N(t) = \sup \{i : S_i \leq t\}$ as the underlying demand arrival process.

- We let $L(t)$ denote the size of the accumulated/consolidated outbound load waiting to be released at time t and let $I(t)$ denote the inventory level at the vendor's warehouse at time t . Under these assumptions, the vendor's inventory and dispatch decisions can be made in the following manner:

- $L(t)$ is updated each time a customer order is received. This way, the time that the accumulated outbound load reaches q for the first time in a dispatch cycle can be registered immediately.
- The vendor employs a special kind of (s, S) policy with $s = 0$ and $S = Q$. Since the replenishment lead-time at the vendor is negligible, there is no need to replenish stock if $I(t) \geq 0$ after a shipment is dispatched. Hence, an (s, S) policy with $s = 0$ is appropriate. It follows that at the end of a dispatch cycle, say at t , a replenishment quantity of size $Z(t)$ is ordered where

$$Z(t) = \begin{cases} Q + q - I(t), & \text{if } I(t) < q, \\ 0, & \text{if } I(t) \geq q. \end{cases}$$

The time between two successive inventory replenishment decisions is called an *inventory replenishment cycle*.

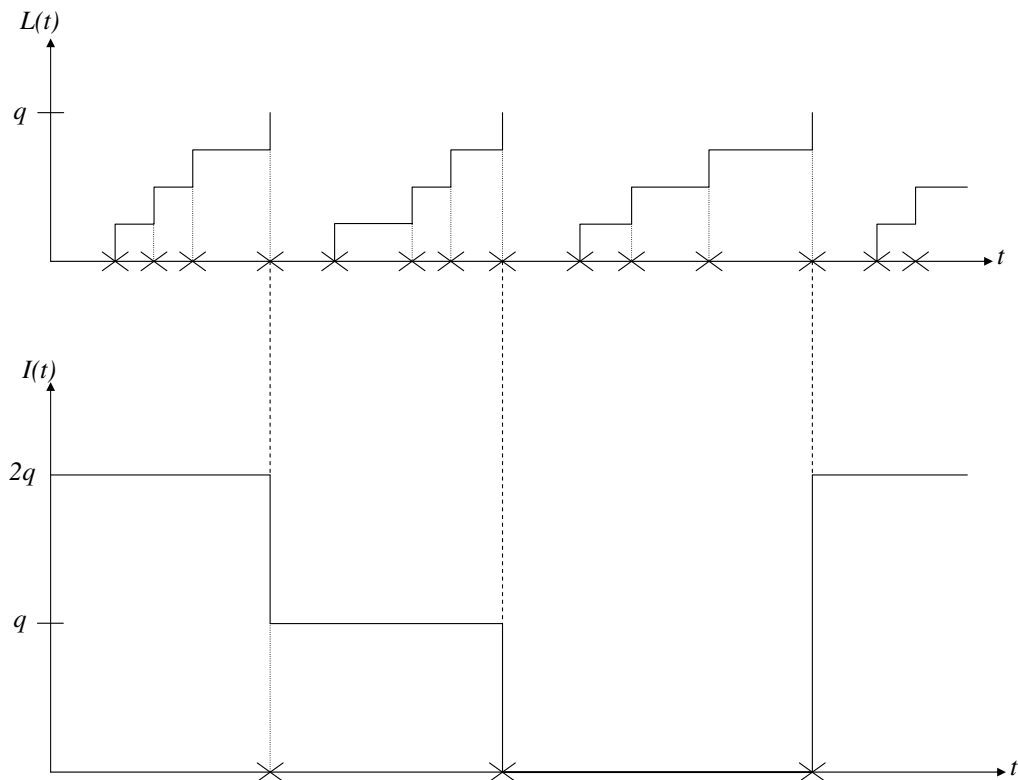
- Upon the receipt of $Z(t)$, a load containing $L(t) = q$ units is dispatched

¹The case of bulk arrival processes is considerably more complicated and is modeled in Çetinkaya, Tekin and Lee (2000).

instantaneously. A new dispatch cycle begins with $Y(t)$ units of inventory where

$$Y(t) = \begin{cases} Q, & \text{if } I(t) < q, \\ I(t) - q, & \text{if } I(t) \geq q. \end{cases}$$

Figure 3 An Illustration of $I(t)$ and $L(t)$.



As a result, the vendor's problem is to compute the optimal q and Q values minimizing the total replenishment, transportation, inventory carrying, and waiting costs. Observe that one can safely substitute $Q = (k - 1)q$ where k is an integer denoting the number of dispatch cycles within an inventory replenishment cycle so that there is no inventory at the vendor during the last dispatch cycle of an inventory replenishment cycle. That is, the maximum inventory at the vendor's warehouse is

$(k-1)q$ whereas the order quantity is kq . Also, it is important to note that, unlike in the case of the time-based dispatch policy in Çetinkaya and Lee (2000), the number of dispatch cycles within an inventory replenishment cycle is no longer a random variable after we fix Q and q . Under these assumptions, a realization of $L(t)$ and $I(t)$ is depicted in Figure 3.

Let RCost, DCost, HCost, and WCost denote the replenishment, dispatch, holding, and waiting costs per replenishment cycle, respectively. Then, by letting c_R denote the unit procurement cost; A_R the fixed cost of replenishing the inventory, h the inventory carrying cost per unit per unit time, c_D the unit shipment cost; A_D the fixed cost of dispatching; and w the customer waiting penalty per unit per unit time, it is easy to show

$$\begin{aligned}
E[\text{RCost}] &= c_R kq + A_R, \\
E[\text{DCost}] &= c_D kq + kA_D, \\
E[\text{HCost}] &= hE [qS_q + qS_{2q} + \cdots + qS_{(k-1)q}] \\
&= hE \left[q \left((k-1) \sum_{i=1}^q X_i + (k-2) \sum_{i=q+1}^{2q} X_i + \cdots + \sum_{i=(k-2)q+1}^{(k-1)q} X_i \right) \right] \\
&= hE \left[q \sum_{j=1}^{k-1} (k-j) \sum_{i=(j-1)q+1}^{jq} X_i \right] = hq \sum_{j=1}^{k-1} (k-j) \sum_{i=(j-1)q+1}^{jq} E[X_i] \\
&= \frac{hk(k-1)q^2}{2\lambda}, \\
E[\text{WCost}] &= kE \left[w \sum_{i=0}^{q-1} iX_{i+1} \right] = kw \sum_{i=0}^{q-1} iE[X_{i+1}] = k \frac{w(q-1)q}{2\lambda},
\end{aligned}$$

so that

$$\begin{aligned}
E[\text{Replenishment cycle cost}] &= E[\text{RCost}] + E[\text{DCost}] + E[\text{HCost}] + E[\text{WCost}] \\
&= c_R k q + A_R + c_D k q + k A_D + \frac{h k (k-1) q^2}{2\lambda} \\
&\quad + \frac{w k (q-1) q}{2\lambda}.
\end{aligned}$$

It is straightforward to show that

$$\begin{aligned}
E[\text{Replenishment cycle length}] &= k E[\text{Dispatch cycle length}] \\
&= k E \left[\sum_{i=1}^q X_i \right] = k q E[X_i] = k q / \lambda,
\end{aligned}$$

and, hence, using the Renewal Reward Theorem, the expected total long-run average cost per unit-time, denoted by $C(k, q)$, is given by

$$\begin{aligned}
C(k, q) &= \frac{E[\text{Replenishment cycle cost}]}{E[\text{Replenishment cycle length}]} \\
&= c_R \lambda + \frac{A_R \lambda}{k q} + c_D \lambda + \frac{A_D \lambda}{q} + \frac{h(k-1)q}{2} + \frac{w(q-1)}{2}. \quad (3.1)
\end{aligned}$$

Consequently, the problem reduces to

$$\begin{aligned}
\min \quad & C(k, q) \\
\text{s.to} \quad & q, k : \text{positive integers.}
\end{aligned}$$

Let k^* and q^* denote the optimal k and q values, respectively. We define

$$k_0 = \sqrt{\frac{A_R(w-h)}{A_D h}}.$$

Also, let $\lfloor x \rfloor$ denote the largest integer less than, or equal to, x and $\lceil x \rceil$ denote the smallest integer greater than, or equal to, x where x is a real number. Although $C(k, q)$ is not necessarily jointly convex in k and q , the exact solution of the problem can be computed very easily using Proposition 1. Also, note that although the

motivations of the underlying problems are different, the mathematical formulation of the quantity-based dispatch model considering stochastic demands is similar to Goyal's (1976) and Schwarz's (1973) formulations for the deterministic buyer-vendor coordination problem which received significant academic attention. That is, the cost function given by (3.1) exhibits the same characteristics as the cost function of the deterministic buyer-vendor coordination problem with the exception that q is a continuous variable and $w > h$ in Goyal (1976) and Schwarz (1973). Although the proof is straightforward, these two papers do not prove that this function has a unique finite minimizer which is the underlying idea we use for developing a simple solution. For our purposes, both q and k are discrete variables, and computing the optimal solution is simple as described in Proposition 1. The proof of the proposition is presented here for the sake of completeness.

PROPOSITION 1 • *If $w \leq h$, then $k^* = 1$ and q^* is either $\lfloor \sqrt{2(A_R + A_D)\lambda/w} \rfloor$ or $\lceil \sqrt{2(A_R + A_D)\lambda/w} \rceil$ depending on which one yields a lower value of $C(1, q)$.*

• *If $w > h$, then the optimal solution is given by*

$$\min \{C(k_1, q_1), C(k_1, q_2), C(k_2, q_3), C(k_2, q_4)\} \text{ where}$$

$$k_1 = \lfloor k_0 \rfloor, \quad k_2 = \lceil k_0 \rceil,$$

$$q_1 = \lfloor q(k_1) \rfloor, \quad q_2 = \lceil q(k_1) \rceil, \quad q_3 = \lfloor q(k_2) \rfloor, \quad q_4 = \lceil q(k_2) \rceil, \quad \text{and}$$

$$q(k) = \sqrt{\frac{2(A_R + kA_D)\lambda}{k[w + h(k-1)]}}. \quad (3.2)$$

Proof:

Let us treat q as a continuous variable momentarily. For a fixed $k = 1, 2, \dots$, the function $C(k, q)$, given by (3.1), is convex in q . Solving $\partial C(k, q)/\partial q = 0$ for q , we obtain (3.2). Thus, for a fixed $k = 1, 2, \dots$, the optimal value of q is computed using

$q(k)$ in (3.2). Let us define $F(k) = C(k, q(k))$ so that

$$F(k) = \sqrt{\frac{2(A_R + kA_D)\lambda[h(k-1) + w]}{k}}.$$

Also, let k^* denote the optimal solution of $\min_{k \in \mathbb{Z}^+} F(k)$. Consequently, the optimal solution for the quantity-based dispatch model is given by k^* and $q(k^*)$ where $q(k^*)$ is computed using (3.2). Therefore, in order to complete the proof, it is sufficient to compute k^* .

Case 1: $w \leq h$. In this case,

$$\begin{aligned} F(k) &= \sqrt{\frac{2(A_R + kA_D)\lambda[h(k-1) + w]}{k}} \\ &\geq \sqrt{(A_R + kA_D)\frac{2\lambda kw}{k}} = \sqrt{(A_R + kA_D)2\lambda w}, \quad \forall k \geq 1. \end{aligned}$$

Also, we can easily show that

$$\sqrt{2(A_R + A_D)\lambda w} \geq C(1, q(1) + 1) \geq C(1, \lceil q(1) \rceil) \geq \min\{C(1, \lceil q(1) \rceil), C(1, \lfloor q(1) \rfloor)\}.$$

It follows that $k^* = 1$.

Case 2: $w > h$. Rewriting $F(k)$, it is easy to show that $F(k) = \sqrt{a + bk + c/k}$ where $a = 2A_R\lambda h + 2A_D\lambda(w - h)$, $b = 2A_D\lambda h$, and $c = 2A_R\lambda(w - h)$. Let us treat k as a continuous variable momentarily. Observe that $a + bk + c/k$ is a convex function of k with a minimizer at $k_0 = \sqrt{c/b} = \sqrt{A_R(w - h)/A_D h}$. On the other hand, $F(k) = \sqrt{a + bk + c/k}$ is not necessarily convex in k . However, interestingly, k_0 also minimizes $F(k)$ as we prove below. Observe that the first derivative of $F(k)$ is given by

$$F'(k) = \frac{-c/k^2 + c}{2\sqrt{a + bk + c/k}}.$$

Since $w > h$, we have $c > 0$. Thus, for $k < k_0$, we have $-c/k^2 + b < -c/k_0^2 + b = 0$.

It follows that $F(k)$ is a strictly decreasing function of k for $k < k_0$. In a similar

fashion, it can be easily shown that $F(k)$ is a strictly increasing function of k for $k \geq k_0$. Therefore, $k^* = \arg \min \{F(\lfloor k_0 \rfloor), F(\lceil k_0 \rceil)\}$, and this completes the proof.

■

Once k^* and q^* are computed, the optimal Q , denoted by Q^* , is given by $(k^* - 1)q^*$. Proposition 1 implies that if the cost of waiting is less than the cost of holding, then there is no incentive to carry inventory at the vendor, and the warehouse can either be operated as a transshipment terminal or closed. On the other hand, if the cost of holding is less than the cost of waiting, then the warehouse is operated as a break-bulk terminal where we replenish stock in bulk, carry inventory, and dispatch several outbound shipments in a replenishment cycle.

III.3. Analytical Comparison of Time- and Quantity-Based Policies

Proposition 1 states that the optimal Q and q values imply one of the following two forms for the integrated inventory/quantity-based dispatch policy under consideration:

Form I. *There is a single dispatch cycle within a replenishment cycle. This is the case if $k^* = 1$, and, hence, $Q^* = 0$, and $q^* = \sqrt{2(A_R + A_D)\lambda/w}$.*

Form II. *There are multiple dispatch cycles within a replenishment cycle. This is the case when $k^* > 1$, q^* is computed using (3.2), and $Q^* = (k^* - 1)q^* > 0$.*

Proposition 1 suggests that if $w \leq h$, then the optimal policy is of Form I. However, even if $w > h$, such a policy may still be optimal, e.g., the problem instance $h = 1, \lambda = 16, A_R = 40, A_D = 20$, and $w = 2$. We also introduce one more policy type, and we call it Form III:

Form III. *If $q^* = 1$, then we say that the optimal policy is of Form III. Hence, Form III represents the class of immediate delivery policies.*

Note that, although a Form I policy cannot be of Form II at the same time (and vice versa), both Form I and Form II policies can also be of Form III.

Interpretations of these three different policy forms may lead to insightful results about distribution system design in the following manner:

- If the optimal integrated inventory/quantity-based dispatch policy is of Form I, then no inventory is held at the vendor's warehouse, i.e., the vendor's warehouse should be either operated as a transshipment point for consolidating orders over time or closed. Since each replenishment cycle consists of a single dispatch cycle, this policy implies that it is more economical to ship the consolidated orders directly from the manufacturer (vendor's supplier) to the customer.
- If the optimal integrated inventory/quantity-based dispatch policy is of Form II, then the vendor's warehouse is operated as a break-bulk terminal so that several outbound shipments are dispatched from the warehouse during a given replenishment cycle. In this case, the suggested vendor-managed supply agreement makes perfect economical sense.
- If the optimal integrated inventory/quantity-based dispatch policy is of Form III then each order is shipped individually without consolidation. An optimal policy is of Form III if the suggested quantity-based dispatch arrangement does not make economical sense.

Let T and Q_T denote the dispatch interval and the vendor's order-up-to level for the time-based dispatch model in Çetinkaya and Lee (2000), and let function $C_T(Q_T, T)$ denote the underlying expected long-run total average cost per unit-time. We note that in the time-based dispatch model, as in the quantity-based dispatch model presented in Section III.2, inventory is only replenished if the current load

before a dispatch is larger than the inventory on hand, and, hence, the underlying inventory policy is again an (s, S) policy with $s = 0$ and $S = Q$. For specific details of the time-based dispatch model, the reader may refer to Çetinkaya and Lee (2000). Also, let T^* and Q_T^* denote the *optimal* dispatch interval and the vendor's *optimal* order-up-to level. Hence, $C_T(Q_T^*, T^*)$ denotes the optimal cost for the time-based dispatch model. It is worth noting that the optimal integrated inventory/time-based dispatch policies can also be classified in a similar fashion. However, the very basic assumptions of the time-based dispatch model excludes the class of immediate dispatch policies. Hence, the resulting integrated inventory/time-based dispatch policies are either of Form I, representing the case $Q_T^* = 0$, or Form II, representing the case $Q_T^* > 0$.

In fact, for a given problem data set, if the optimal integrated inventory/time-based dispatch policy is of Form I, then we can provide an analytical proof showing that the optimal integrated inventory/quantity-based dispatch policy performs better in terms of costs (see Propositions 2–5 below.) This is simply because if $Q_T^* = 0$, then we can easily obtain the closed form expressions of T^* and $C_T(0, T^*)$. In other cases, i.e., those problem instances where $Q_T^* > 0$, we rely on the overwhelming numerical evidence, discussed in Section III.4.1, illustrating that the cost savings resulting from using the optimal integrated inventory/quantity-based dispatch policy are substantial.

The following propositions identify those cases where the optimal integrated inventory/quantity-based dispatch policy is proved to be superior to the time-based counterpart. These propositions also provide an analytical basis for Observations 1 through 3 in Section III.4.1 where we provide numerical results for a comparison of the two dispatch policies.

PROPOSITION 2 For a given parameter set, if $Q_T^* = 0$, then

$$C_T(Q_T^*, T^*) - C(k^*, q^*) \geq 0,$$

i.e., the optimal integrated inventory/quantity-based dispatch policy is always superior to the optimal integrated inventory/time-based dispatch policy.

Proof: Using the development in Çetinkaya and Lee (2000), it can be shown that

$$C_T(0, T) = c_R\lambda + \frac{A_R}{T} + c_D\lambda + \frac{A_D}{T} + \frac{w\lambda T}{2}. \quad (3.3)$$

Hence, if $Q_T^* = 0$, then $T^* = \sqrt{2(A_R + A_D)/(\lambda w)}$ and

$$C_T(0, T^*) = \sqrt{2(A_R + A_D)\lambda w} + c_R\lambda + c_D\lambda. \quad (3.4)$$

Let $C^* = C(k^*, q^*)$ denote the optimal value of the cost function $C(k, q)$ in (3.1). For the sake of simplicity, let us treat q as a continuous variable momentarily. Under this assumption, Proposition 1 implies that, if $w < h$, then $C^* = \sqrt{2(A_R + A_D)\lambda w} + c_R\lambda + c_D\lambda - w/2$, and if $w \geq h$, then

$$C^* \leq C\left(1, \sqrt{2(A_R + A_D)/(\lambda w)}\right) = \sqrt{2(A_R + A_D)\lambda w} + c_R\lambda + c_D\lambda - w/2.$$

Therefore, under the assumption that q is continuous, if $Q_T^* = 0$, then $C_T(0, T^*) \geq C^*$.

Since q is not a continuous variable, in order to prove $C_T(0, T^*) \geq C^*$, we show that

$$\sqrt{2(A_R + A_D)\lambda w} + c_R\lambda + c_D\lambda \geq C(1, q^*(1)) \geq C^*, \quad (3.5)$$

where $q^*(1)$ is either $\left\lfloor \sqrt{2(A_R + A_D)\lambda/w} \right\rfloor$ or $\left\lceil \sqrt{2(A_R + A_D)\lambda/w} \right\rceil$ depending on which one yields a lower value of $C(1, q)$.

The inequality on the right hand side of (3.5) is trivial. Recalling (3.1), the inequality on the left hand side of (3.5) can be written as

$$\frac{A_R\lambda}{q^*(1)} + \frac{A_D\lambda}{q^*(1)} + \frac{w(q^*(1) - 1)}{2} \leq \sqrt{2(A_R + A_D)\lambda w}. \quad (3.6)$$

Let us define

$$G(q) = \frac{A_R\lambda}{q} + \frac{A_D\lambda}{q} + \frac{wq}{2}, \text{ and } \bar{q} = \left\lceil \sqrt{\frac{2(A_R + A_D)\lambda}{w}} \right\rceil.$$

Observe that, in order to show (3.6), it is sufficient to show

$$G(\bar{q}) - \frac{w}{2} \leq \sqrt{2(A_R + A_D)\lambda w}.$$

Since $G(\cdot)$ is an EOQ type convex function with a minimizer at $\sqrt{2(A_R + A_D)\lambda/w}$, we have

$$G(\lceil q \rceil) \leq G(q + 1), \quad \forall q \geq \sqrt{2(A_R + A_D)\lambda/w},$$

and

$$\frac{G(q)}{\sqrt{2(A_R + A_D)\lambda w}} = \frac{1}{2} \left(\frac{q}{\sqrt{\frac{2(A_R + A_D)\lambda}{w}}} + \frac{\sqrt{\frac{2(A_R + A_D)\lambda}{w}}}{q} \right).$$

Therefore,

$$\begin{aligned} G(\bar{q}) - \frac{w}{2} &\leq \frac{1}{2} \sqrt{2(A_R + A_D)\lambda w} \left(\frac{\sqrt{\frac{2(A_R + A_D)\lambda}{w}} + 1}{\sqrt{\frac{2(A_R + A_D)\lambda}{w}}} + \frac{\sqrt{\frac{2(A_R + A_D)\lambda}{w}}}{\sqrt{\frac{2(A_R + A_D)\lambda}{w}} + 1} \right) - \frac{w}{2} \\ &\leq \frac{1}{2} \sqrt{2(A_R + A_D)\lambda w} \left(2 + \frac{1}{\sqrt{2(A_R + A_D)\lambda/w}} \right) - \frac{w}{2} \\ &= \sqrt{2(A_R + A_D)\lambda w} + \frac{w}{2} - \frac{w}{2} = \sqrt{2(A_R + A_D)\lambda w}, \end{aligned}$$

It follows that if $Q_T^* = 0$, then the *quantity-based policy* outperforms the *time-based policy*, and this completes the proof of Proposition 2. ■

The following intuitive explanation of Proposition 2 also underlies its proof. If $Q_T^* = 0$ and $Q^* = 0$, then the corresponding time-based and quantity-based models reduce to pure shipment consolidation models where inventory at the vendor has no effect on the cost because it is nonexistent. In this case, the optimal *quantity-based policy* always outperforms the optimal *time-based policy*, i.e., the inequality in (3.5) is correct. The intuition is based on the fact that if we restrict ourselves to a fixed time interval, then any time we release a shipment at the end of this interval, we could have dispatched it earlier (i.e., when the previous demand arrived,) or later (i.e., when the next demand arrived.) One of these options may be cheaper than dispatching at fixed intervals, since, in the first case, we did not realize the scale economies associated with larger dispatch quantities, and, in the second case, we unnecessarily held inventory too long.

Based on Proposition 2, we know that if $Q_T^* = 0$, then the cost savings resulting from a *quantity-based policy* is always positive. Now we discuss those cases where the actual savings are less than $w/2$, in excess of 50%, depending on the values of T^* and Q^* .

PROPOSITION 3 *For a given parameter set, if both $Q_T^* = 0$ and $Q^* = 0$, then*

$$0 \leq C_T(Q_T^*, T^*) - C(k^*, q^*) \leq w/2.$$

Proof of Proposition 3: Now, suppose that both $Q_T^* = 0$ and $Q^* = 0$. Hence, Proposition 1 implies $k^* = 1$ and q^* is either $\left\lceil \sqrt{2(A_R + A_D)\lambda/w} \right\rceil$ or $\left\lceil \sqrt{2(A_R + A_D)\lambda/w} \right\rceil$

depending on which one yields a lower value of $C(1, q)$. Therefore,

$$C^* \geq \sqrt{2(A_R + A_D)\lambda w} + c_R\lambda + c_D\lambda - w/2. \quad (3.7)$$

Using (3.4) and (3.7), we can write $C_T(0, T^*) - C^* \leq w/2$, and this completes the proof. It is worth noting that in the numerical results that will be discussed in Section III.4.1, the smallest cost savings (e.g., % Savings $< 5\%$ as shown in the table on p.49) correspond to such problem instances. ■

PROPOSITION 4 *For a given parameter set, if $Q_T^* = 0$ and $\lambda T^* \leq 1$, then the percentage of cost savings resulting from using the optimal integrated inventory/quantity-based dispatch policy is at least 50%.*

Proof of Proposition 4: Assuming that $Q_T^* = 0$ and $\lambda T^* < 1$, we compare the *operational* cost savings resulting from using the quantity-based model. That is, we ignore the fixed terms $c_R\lambda$ and $c_D\lambda$ in the cost expressions C_T and C , and compute

$$\frac{C_T(Q_T^*, T^*) - C^*}{C_T(Q_T^*, T^*)}.$$

Recalling (3.3), we have

$$T^* = \sqrt{2(A_R + A_D)/(\lambda w)} \quad (3.8)$$

so that $\lambda T^* < 1$ implies $\sqrt{w} > \sqrt{2(A_R + A_D)\lambda}$. Also, recalling (3.1) and noting that $C^* \leq C(1, 1) = (A_R + A_D)\lambda$, we can write

$$\frac{C_T(0, T^*) - C(1, 1)}{C_T(0, T^*)} = 1 - \frac{(A_R + A_D)\lambda}{\sqrt{2(A_R + A_D)\lambda w}} > 1 - \frac{(A_R + A_D)\lambda}{2(A_R + A_D)\lambda} = 0.5.$$

The intuition behind this proposition is based on the fact that if the dispatch frequency is smaller than the arrival rate, then a regular dispatch schedule does not make economical sense. ■

PROPOSITION 5 *For a given parameter set, if $Q_T^* = 0$ and $\lambda T^* \leq 1.5$, then the percentage of cost savings resulting from using the optimal integrated inventory/quantity-based dispatch policy is at least 25%.*

Proof of Proposition 5: Again, using (3.8), $\lambda T^* < 1.5$ leads to

$$\sqrt{w} > \sqrt{2(A_R + A_D)\lambda}/1.5.$$

Hence,

$$\frac{C_T(0, T^*) - C(1, 1)}{C_T(0, T^*)} = 1 - \frac{(A_R + A_D)\lambda}{\sqrt{2(A_R + A_D)\lambda w}} > 1 - \frac{(A_R + A_D)\lambda}{2(A_R + A_D)\lambda/1.5} = 0.25.$$

It is worth noting that in the numerical results discussed in Section III.4.1, the largest cost savings correspond to such problem instances. ■

III.4. Numerical Results

In this section, we first provide numerical results for a comparison of the *time-based* and *quantity-based policies*, and, then, we proceed with a demonstration of the cost and service performance improvements achievable using hybrid policies. For these purposes, we use two data sets:

- The data used in Axsäter (2001). (Since this is a smaller data set, we use it for tabulating specific results.)
- A set of 1024 problem instances was generated using factorial design and considering $c_R = 0$; $c_D = 0$; $A_R = 40, 80, 160, 320$; $A_D = 5, 10, 20, 40$; $h = 1, 2, 4, 8$; $w = 2, 4, 8, 16$; and $\lambda = 2, 4, 8, 16$. (This data set is used for providing stronger numerical evidence.)

III.4.1. Numerical Results for a Comparison of Time-Based and Quantity-Based Policies

Proposition 1 implies that the exact optimization of the quantity-based dispatch model is easy, i.e., it can be performed manually with minimal effort whereas the exact optimization of the time-based dispatch model cannot be presented in closed form for the reasons addressed in Çetinkaya and Lee (2000). As we have already mentioned, a numerical technique for exact optimization of the time-based dispatch model is developed in Axsäter (2001). Hence, for the purpose of comparing the performance of the time-based and quantity-based models numerically, we have utilized the exact optimization technique in Axsäter (2001) and Proposition 1.

Considering the numerical examples presented in Axsäter (2001), we first obtained the results in Table 1 where the cost savings realized by using the suggested quantity-based dispatch model are as high as 14.19%. For the purpose of providing a comprehensive comparative analysis of time-based and quantity-based dispatch models, in addition to the results in Table 1, we have conducted an extensive numerical study considering the 1024 problem instances discussed above. We have observed that the average cost savings under the quantity-based dispatch model is 6.58%; the maximum saving is 25.79%; and the minimum saving is 0.66%.

Next, we present some observations regarding the results of the numerical study. (These observations hold for both of the examples presented in Table 1 and the 1024 problems solved.)

OBSERVATION 1 In general, i.e., regardless of the form of the optimal policy, the quantity-based dispatch model performs better than the time-based dispatch model in terms of the resulting expected total long-run average cost per unit-time.

Table 1 Performance Improvement for the Problems in Axsäter (2001)

$c_R = 0, c_D = 0.$											
A_R	λ	h	A_D	w	Q_T^*	T^*	$C_T(Q_T^*, T^*)$	Q^*	q^*	C^*	%Saving
125	1	1	10	10	14	1.42	29.43	18	2	25.25	14.19
125	1	1	25	10	14	2.24	37.63	12	2	32.43	13.82
125	1	3	10	10	7	1.44	39.91	8	2	34.50	13.55
125	1	3	25	10	7	2.27	47.99	6	3	41.22	14.11
125	10	1	10	10	47	0.45	94.18	45	5	87.50	7.10
125	10	1	25	10	45	0.71	120.14	42	7	112.22	6.59
125	10	3	10	10	26	0.45	129.63	25	5	119.17	8.07
125	10	3	25	10	24	0.71	155.42	18	9	141.07	9.23
125	20	1	10	10	66	0.32	133.42	63	7	125.79	5.72
125	20	1	25	10	65	0.5	170.12	60	10	160.71	5.53
125	20	3	10	10	37	0.32	183.99	32	8	170.50	7.33
125	20	3	25	10	35	0.5	220.46	26	13	201.56	8.57
125	10	7	50	10	0	1.87	187.08	0	19	182.11	2.66
100	10	7	50	10	0	1.73	173.21	0	17	168.24	2.87
150	10	7	50	10	0	2.00	200.00	0	20	195.00	2.50
125	5	7	50	10	0	2.65	132.29	0	13	127.31	3.76
125	15	7	50	10	0	1.53	229.13	0	23	224.13	2.18
125	10	5	50	10	0	1.87	187.08	12	12	178.75	4.45
125	10	9	50	10	0	1.87	187.08	0	19	182.11	2.66
125	10	7	25	10	0	1.73	173.21	10	10	167.50	3.29
125	10	7	75	10	0	2.00	200.00	0	20	195.00	2.50
125	10	7	50	8	0	2.09	167.33	0	21	163.33	2.39
125	10	7	50	12	0	1.71	204.94	0	17	198.94	2.93
Average Saving										6.35%	

OBSERVATION 2 The cost savings are particularly large, e.g., $\geq 5\%$, if the time-based solution suggests $Q_T^* > 0$ rather than $Q_T^* = 0$. The case $Q_T^* > 0$ represents those problem instances where it is optimal to operate the vendor's warehouse as a break-bulk terminal and dispatch several outbound shipments from the warehouse during a given replenishment cycle. On the contrary, the case $Q_T^* = 0$ represents those problem instances where no inventory is carried at the vendor's warehouse, so that the warehouse, in fact, acts as a transshipment point for consolidating orders

over time. Considering the set of 1024 scenarios, for those problems where $Q_T^* > 0$, the average cost savings in using the quantity-based dispatch policy is 9.31% whereas the overall average (including the case $Q_T^* = 0$) is 6.58%.

OBSERVATION 3 For a given set of problem data, the cost savings decrease as λ and λT^* increase. However, the magnitude of the cost savings also depends on the relative values of the h/w and A_R/A_D ratios as well as the individual values of A_R , A_D , h , and w ; and it is not straightforward to make conclusive comments about the magnitude of the cost savings depending on a particular cost parameter value. On the other hand, we can simply conclude that the cost savings are tangible in general.

OBSERVATION 4 We note that the expected average dispatch cost per-unit time under the quantity-based dispatch model is in most cases smaller than the one under its time-based counterpart. This result is rather intuitive since one expects the *quantity-based policy* to realize transportation economies of scale to a greater extent. On the other hand, it is difficult to make general conclusive comments about the relative values of the other expected average per unit-time cost components, i.e., replenishment, holding, and waiting costs.

III.5. Managerial Takeaways and Conclusions

In this chapter, we demonstrate that significant cost savings can be achieved by using the suggested *quantity-based policy* rather than the exact solutions of time-based dispatch models. In particular, our numerical results indicate that quantity-based dispatch policies are always superior to time-based policies.

In practical applications, it is important to take into account the simplicity and periodic delivery advantages of time-based dispatch policies in evaluating the cost improvements obtained through *quantity-based policy*. That is, in practice, it may be

easier to schedule deliveries so that a shipment is released on a periodic-basis, rather than on as-needed basis. It is worth noting that, both time-based and quantity-based dispatch policies are popular in practice, and they are incorporated in VMI contracts for the purposes of achieving timely delivery and load optimization, respectively. Typically, *time-based policies* are used for A-class (lower volume, higher value) items, such as expensive hardware in the computer industry, to guarantee timely delivery. *Quantity-based policies* are used for B-class and C-class (higher volume, lower value) items, such as peripheral computer equipment. On the other hand, our numerical results show that *quantity-based policies* also offer large savings for lower volume, lower value items, i.e., those items where λ and h^2 are smaller.

²Note that h is typically a percentage of the per unit procurement cost/value.

CHAPTER IV

ANALYSIS OF TIME- AND QUANTITY-BASED POLICIES UNDER COMMON CARRIER FREIGHT SCHEDULES

In Chapter III, we studied a joint inventory and shipment consolidation model. In this model, the outbound shipment cost implied the use of a private fleet. Use of common carriers for outbound transportation is also very common in practice. In this chapter, we study the model where a common carrier is employed for outbound transportation. We analyze this model for both the *time-based policy* and the *quantity-based policy*.

The characteristics of the model are the same as those in Chapter III except for the outbound shipment, i.e., dispatch, cost structure. The dispatch cost for this model is presented in Chapter I by Equation(1.3).

Çetinkaya and Bookbinder (2003) study a similar problem. However, their focus is on the shipment consolidation only, i.e., the inventory replenishment decisions are not incorporated. In that sense, their model is a special case of our model where the inventory replenishment quantity is equal to the dispatch quantity. In our model, we identify this case as a “single dispatch case.” On the other hand, Çetinkaya and Bookbinder consider a more general order process in which the order sizes are also random.

IV.1. Time-Based Policy

The decision variables for this model under the *time-based policy* are

Q is the order up-to level for replenishing the inventory. (Here we note that Q should not be confused with the inventory replenishment size that we used for the *quantity-based policy* model in Chapter III.)

T is the consolidation cycle length, i.e., the time between two dispatches.

All of the model parameters are the same as those in the model in Chapter III.

As we mentioned before, this model is studied by Çetinkaya and Lee (2000) excluding the common carrier charges. They show that the system stochastically regenerates itself at every inventory replenishment epoch. Hence, we consider the time between two replenishment epochs as a replenishment cycle. To find the long run average of the total expected cost, it suffices to calculate the total expected cost for one replenishment cycle and divide this by the expected cycle length. This argument is a direct consequence of the Renewal Reward Theorem.

We refer to Çetinkaya and Lee (2000) for the calculation of the $E[HC]$, $E[RC]$ and $E[WC]$ and also the expected cycle length and use their results. However, the derivation of the $E[DC]$ is different in our problem, and we present a detailed derivation of the $E[DC]$ below.

IV.1.1. Derivation of the $E[DC]$ for the Time-Based Policy

Before going into the details of the calculation, we note that:

$$c_0q_1 \approx c_1q_2.$$

If q were a continuous variable, then the above relation would hold an exact equality for all cases. For the rest of the chapter, we assume that $c_0q_1 = c_1q_2$. Otherwise, it would be very difficult to make analytical comments on the problem.

We define the time between two dispatches as a dispatch cycle. For the *time-based policy*, the number of dispatch cycles within a replenishment cycle is a random variable denoted by K . In order to compute $E[DC]$, we first compute the Expected Dispatch Cost for one dispatch cycle and then multiply it by $E[K]$. (Derivation of

$E[K]$ is given by Çetinkaya and Lee 2000.) The Expected Dispatch Cost for one dispatch cycle, $E[DCC]$, is given by:

$$\begin{aligned} E[DCC] = & c_0 \sum_{x=1}^{q_1} x p_x + c_0 q_1 \sum_{x=q_1+1}^{q_2} p_x + \dots + c_i \sum_{x=q_{2i}+1}^{q_{2i+1}} x p_x \\ & + c_i q_{2i+1} \sum_{x=q_{2i+1}+1}^{q_{2i+2}} p_x + \dots + c_I \sum_{x=q_{2I}+1}^{\infty} x P_x, \end{aligned}$$

where $p_x = (\lambda T)^x e^{-\lambda T} / x!$.

More explicitly;

$$\begin{aligned} E[DCC] &= \sum_{i=0}^{I-1} \left\{ c_i \sum_{x=q_{2i}}^{q_{2i+1}} x \frac{(\lambda T)^x e^{-\lambda T}}{x!} + c_i q_{2i+1} (F_{q_{2i+2}} - F_{q_{2i+1}}) \right\} \\ &+ c_I \sum_{x=q_{2I}+1}^{\infty} x \frac{(\lambda T)^x e^{-\lambda T}}{x!} \\ &= \sum_{i=0}^{I-1} \left\{ c_i \lambda T \sum_{x=q_{2i}-1}^{q_{2i+1}-1} \frac{(\lambda T)^x e^{-\lambda T}}{x!} + c_i q_{2i+1} (F_{q_{2i+2}} - F_{q_{2i+1}}) \right\} \\ &+ c_I \lambda T \sum_{x=q_{2I}}^{\infty} \frac{(\lambda T)^x e^{-\lambda T}}{x!} \\ &= \sum_{i=0}^{I-1} \left\{ c_i \lambda T (F_{(q_{2i+1}-1)} - F_{(q_{2i}-1)}) + c_i q_{2i+1} (F_{q_{2i+2}} - F_{q_{2i+1}}) \right\} \\ &+ c_I \lambda T (1 - F_{(q_{2I}-1)}) \\ &= \sum_{i=0}^{I-1} \left\{ \lambda T (c_i F_{(q_{2i+1}-1)} - c_{i+1} F_{(q_{2i+2}-1)}) + c_i q_{2i+1} (F_{q_{2i+2}} - F_{q_{2i+1}}) \right\} + c_I \lambda T, \end{aligned}$$

where F_x represents the cumulative poisson distribution function value for x .

The Expected Dispatch Cost for one replenishment cycle, $E[DC]$, is simply obtained by multiplying the above the expression by $E[K]$.

IV.1.2. Analysis of $C(Q, T)$

So far, we have derived the expression for the $E[DC]$. We use the results of Çetinkaya and Lee (2000) for $E[RC]$, $E[HC]$, $E[WC]$ and $E[K]$:

$$E[RC] = A_R + c_R \lambda E[K]T,$$

$$E[HC] = hQT + \frac{hQ(Q+1)}{2\lambda},$$

$$E[WC] = \frac{w\lambda T^2}{2}, \text{ and}$$

$$E[K] \approx \begin{cases} 1 & \text{if } Q+1 \leq \lambda T, \\ \frac{Q+1}{\lambda T} & \text{if } Q+1 > \lambda T. \end{cases}$$

Now we apply the Renewal Reward Theorem to obtain the long run average of the Expected Cost, $C(Q, T)$:

$$C(Q, T) = \frac{E[RC] + E[HC] + E[DC] + E[WC]}{E[K]T}. \quad (4.1)$$

The numerator in Equation(4.1) is the total replenishment cycle cost and the denominator is the replenishment cycle length. Substituting the expressions for $E[RC]$, $E[HC]$, $E[WC]$, $E[WC]$ and $E[K]$, we obtain the following for $C(Q, T)$:

$$C(Q, T) = \begin{cases} \frac{A_R}{T} + c_R \lambda + hQ + \frac{hQ(Q+1)}{2\lambda T} \\ + \sum_{i=0}^{I-1} \lambda (c_i F_{(q_{2i+1}-1)} - c_{i+1} F_{(q_{2i+2}-1)}) \\ + \sum_{i=0}^{I-1} \lambda \frac{c_i q_{2i+1}}{T} (F_{q_{2i+2}} - F_{q_{2i+1}}) \\ + c_I \lambda + \frac{w\lambda T}{2} & \text{if } Q + 1 \leq \lambda T, \\ \frac{A_R \lambda}{Q+1} + c_R \lambda + h\lambda T - \frac{h\lambda T}{Q+1} + \frac{hQ}{2} \\ + \sum_{i=0}^{I-1} \lambda (c_i F_{(q_{2i+1}-1)} - c_{i+1} F_{(q_{2i+2}-1)}) \\ + \sum_{i=0}^{I-1} \lambda \frac{c_i q_{2i+1}}{T} (F_{q_{2i+2}} - F_{q_{2i+1}}) \\ + c_I \lambda + \frac{w\lambda T}{2} & \text{if } Q + 1 > \lambda T. \end{cases}$$

Then, we formulate our problem as

$$\begin{aligned} \min \quad & C(Q, T) \\ \text{s.t} \quad & Q \in \mathbb{N}^+, \quad T > 0. \end{aligned}$$

When $Q + 1 \leq \lambda T$, the optimal Q is 0. This means that in one replenishment cycle, there is only one dispatch cycle. We call this case the *Single Dispatch Case* and the other case the *Multi Dispatch Case*. We optimize both cases separately and then

pick the solution with the lower cost as the optimal solution. Since $c_R\lambda$ and $c_I\lambda$ are constants, we will not consider those cost components in the rest of the paper.

IV.1.2.1. Single Dispatch Case

In this case, the objective function depends on only one decision variable, T . Hence, we use $C(T)$ instead of $C(Q, T)$. We can easily derive $C(T)$ from $C(Q, T)$ by substituting $Q = 0$:

$$C(T) = \frac{A_R}{T} + \sum_{i=0}^{I-1} \left\{ \lambda(c_i F_{(q_{2i+1}-1)} - c_{i+1} F_{(q_{2i+2}-1)}) + \frac{c_i q_{2i+1}}{T} (F_{q_{2i+2}} - F_{q_{2i+1}}) \right\} + \frac{w\lambda T}{2}.$$

If the function is convex in the region where $\lambda T \geq 1$, then it suffices to find the T value that solves for the first derivative.

LEMMA 1 *The first derivative of $C(T)$ is given by:*

$$\frac{dC(T)}{dT} = \frac{w\lambda}{2} - \frac{A_R}{T^2} - \sum_{i=0}^{I-1} \frac{c_i q_{2i+1}}{T^2} (F_{q_{2i+2}} - F_{q_{2i+1}}). \quad (4.2)$$

PROOF: Here we present a sketch of the proof. Initially we start with $I = 1$. Then our cost expression for the single dispatch case will be as follows:

$$C_1(T) = \frac{A_R}{T} + \frac{w\lambda T}{2} + \lambda(c_0 F_{(q_1-1)} - c_1 F_{(q_2-1)}) + \frac{c_0 q_1}{T} (F_{q_2} - F_{q_1}) + c_1 \lambda.$$

Below is the procedure to find the derivative of the above function with respect to T :

First, we take the derivative of $\lambda(c_0 F_{(q_1-1)} - c_1 F_{(q_2-1)})$. Let's call it $A(T)$. Then

$$\frac{dA(T)}{dT} = \lambda \left\{ c_0 \sum_{x=0}^{q_1-1} \frac{d}{dT} \left(\frac{(\lambda T)^x e^{-\lambda T}}{x!} \right) - c_1 \sum_{x=0}^{q_2-1} \frac{d}{dT} \left(\frac{(\lambda T)^x e^{-\lambda T}}{x!} \right) \right\}. \quad (4.3)$$

To make things easier, we compute the following generic expression:

$$\begin{aligned} \sum_{x=0}^k \frac{d}{dT} \left(\frac{(\lambda T)^x e^{-\lambda T}}{x!} \right) &= \lambda \sum_{x=0}^{k-1} \frac{(\lambda T)^x e^{-\lambda T}}{x!} - \lambda \sum_{x=0}^k \frac{(\lambda T)^x e^{-\lambda T}}{x!} \\ &= -\lambda p_k. \end{aligned}$$

When we substitute the above expression in (4.3), we obtain the following:

$$\frac{dA(T)}{dT} = \lambda^2 (c_1 p_{(q_2-1)} - c_0 p_{(q_1-1)}). \quad (4.4)$$

Next, we compute the derivative of $(c_0 q_1 / T)(F_{q_2} - F_{q_1})$:

$$\begin{aligned} \frac{d}{dT} \left(\frac{c_0 q_1}{T} (F_{q_2} - F_{q_1}) \right) &= -\frac{c_0 q_1}{T^2} (F_{q_2} - F_{q_1}) + \frac{c_0 q_1}{T} \left[\sum_{x=q_1+1}^{q_2} \frac{d}{dT} \frac{(\lambda T)^x e^{-\lambda T}}{x!} \right] \\ &= -\frac{c_0 q_1}{T^2} (F_{q_2} - F_{q_1}) \\ &\quad + \frac{c_0 q_1}{T} \left[\sum_{x=q_1+1}^{q_2} -\lambda \frac{(\lambda T)^x e^{-\lambda T}}{x!} + \sum_{x=q_1}^{q_2-1} \lambda \frac{(\lambda T)^x e^{-\lambda T}}{x!} \right] \\ &= -\frac{c_0 q_1}{T^2} (F_{q_2} - F_{q_1}) - \lambda \frac{c_0 q_1}{T} [F_{q_2} - F_{q_1} - F_{(q_2-1)} + F_{(q_1-1)}] \\ &= -\frac{c_0 q_1}{T^2} (F_{q_2} - F_{q_1}) + \lambda \frac{c_0 q_1}{T} (F_{q_1} - F_{(q_1-1)}) \\ &\quad - \lambda \frac{c_0 q_1}{T} (F_{q_2} - F_{(q_2-1)}) \\ &= -\frac{c_0 q_1}{T^2} (F_{q_2} - F_{q_1}) + \lambda \frac{c_0 q_1}{T} p_{q_1} - \lambda \frac{c_0 q_1}{T} p_{q_2} \\ &= -\frac{c_0 q_1}{T^2} (F_{q_2} - F_{q_1}) + \lambda \frac{c_0 q_1}{T} p_{q_1} - \lambda \frac{c_1 q_2}{T} p_{q_2} \\ &= -\frac{c_0 q_1}{T^2} (F_{q_2} - F_{q_1}) + c_0 \lambda^2 p_{(q_1-1)} - c_1 \lambda^2 p_{(q_2-1)}. \end{aligned}$$

When we add the above expression and the expression in (4.4), we obtain

$$-(c_0 q_1 / T^2)(F_{q_2} - F_{q_1}).$$

The derivative of the rest of the $C(T)$ is $w\lambda/2 - A_R/T^2$, and we obtain

$$\frac{dC_1(T)}{dT} = \frac{w\lambda}{2} - \frac{A_R}{T^2} - \frac{c_0 q_1}{T^2}(F_{q_2} - F_{q_1}).$$

Finally, we extend the procedure to the general case of I and obtain Equation(4.2).

This completes the sketch of the proof. ■

To check convexity, we look at the second derivative. We state the second derivative without proof:

$$\frac{d^2 C(T)}{dT^2} = \frac{2A_R}{T^3} + \sum_{i=0}^{I-1} \left\{ \frac{2c_i q_{2i+1}}{T^3} (F_{q_{2i+2}} - F_{q_{2i+1}}) + \frac{c_i q_{2i+1}}{T^2} \lambda (p_{q_{2i+2}} - p_{q_{2i+1}}) \right\}. \quad (4.5)$$

The second derivative is always positive when $p_{q_{2i+2}} \geq p_{q_{2i+1}}$ for all i . The worst case occurs when $p_{q_{2i+2}} < p_{q_{2i+1}}$ for all i ; however, we can still state a sufficient condition in this case which is: $2(F_{q_{2i+2}} - F_{q_{2i+1}})/p_{q_{2i+2}} - p_{q_{2i+1}} \geq \lambda T$.

So far, we have given the sufficient conditions for convexity. Hence the next step is to solve for the T that makes the first derivative 0 and check to see whether or not it is convex at that point.

IV.1.2.2. Multi Dispatch Case

In this case, the cost function is given as follows:

$$\begin{aligned} C(Q, T) &= \frac{A_R \lambda}{Q+1} + \frac{Q h \lambda T}{Q+1} + \frac{h Q}{2} \\ &\quad + \sum_{i=0}^{I-1} \left\{ \lambda (c_i F_{(q_{2i+1}-1)} - c_{i+1} F_{(q_{2i+2}-1)}) + \frac{c_i q_{2i+1}}{T} (F_{q_{2i+2}} - F_{q_{2i+1}}) \right\} + \frac{w \lambda T}{2}. \end{aligned}$$

The cost function has 2 decision variables, Q and T . One way to minimize

$C(Q, T)$ is to look at the stationary points where both $\partial C(Q, T)/\partial Q$ and $\partial C(Q, T)/\partial T$ are 0. However, for such a point, if it exists, to be the minimizer, the function must be jointly convex in Q and T . This requires a Hessian check. Below we present the first and second order partial derivatives:

$$\frac{\partial C(Q, T)}{\partial T} = h\lambda - \frac{h\lambda}{Q+1} + \frac{w\lambda}{2} - \sum_{i=0}^{I-1} \frac{c_i q_{2i+1}}{T^2} (F_{q_{2i+2}} - F_{q_{2i+1}}),$$

$$\frac{\partial C(Q, T)}{\partial Q} = \frac{h}{2} + \frac{h\lambda T - A_R \lambda}{(Q+1)^2},$$

$$\frac{\partial^2 C(Q, T)}{\partial T^2} = \sum_{i=0}^{I-1} \left\{ \frac{2c_i q_{2i+1}}{T^3} (F_{q_{2i+2}} - F_{q_{2i+1}}) + \frac{c_i q_{2i+1}}{T^2} \lambda (p_{q_{2i+2}} - p_{q_{2i+1}}) \right\},$$

$$\frac{\partial^2 C(Q, T)}{\partial Q^2} = \frac{2(A_R \lambda - h\lambda T)}{(Q+1)^3},$$

$$\frac{\partial^2 C(Q, T)}{\partial Q \partial T} = \frac{h\lambda}{(Q+1)^2}.$$

We compute the determinant of the Hessian matrix as follows:

$$\begin{aligned} |H| &= \sum_{i=0}^{I-1} \left\{ \frac{4c_i q_{2i+1} \lambda (A_R - hT) (F_{q_{2i+2}} - F_{q_{2i+1}})}{(Q+1)^3 T^3} + \frac{2c_0 q_1 \lambda^2 (A_R - hT) (p_{q_{2i+2}} - p_{q_{2i+1}})}{(Q+1)^3 T^2} \right\} \\ &\quad - \frac{(h\lambda)^2}{(Q+1)^4}. \end{aligned} \tag{4.6}$$

From (4.6) it is not easy to say anything about the convexity of $C(Q, T)$. Furthermore, $C(Q, T)$ is not necessarily convex in T either. That is why we must make a search over all possible values of Q and T . The search algorithm proceeds as follows:

ALGORITHM 1 Step 0: Set $Q = 0$, $C^* = M$, where M is a very large number.

Step 1: $Q = Q + 1$ and $T_Q^m = (Q + 1)/\lambda$.

Step 2: If $Q \geq Q^s$ then **STOP**, else go to Step 3.

Step 3: $grid = \min(0.01, T_Q^m/50)$.

Step 4: for $T = 0.01$ to $T \leq T_Q^m$, T by grid; evaluate $C(Q, T)$.

Step 5: If $C(Q, T) < C^*$, set $T^* = T$ and $Q^* = Q$. Go to Step 1.

The Q^s value in this procedure is the stopping Q value for the search. The following Lemma helps determine Q^s :

LEMMA 2 *The stopping value of Q , namely Q^s , is*

$$Q^s = \sqrt{\frac{2A_R\lambda}{h}} - 1. \quad (4.7)$$

PROOF : Let Q^s be a stopping value. Then for any $Q = Q^s + m$, $m \geq 0$, $C(Q, T)$ must be greater than $C(Q^s, T)$ for all T ; in other words, $C(Q, T) - C(Q^s, T) \geq 0$:

$$C(Q, T) - C(Q^s, T) = \frac{hm}{2} - A_R\lambda \frac{m}{(Q^s + 1 + m)(Q^s + 1)} + h\lambda T \frac{m}{(Q^s + 1 + m)(Q^s + 1)}.$$

We guarantee that the above expression is nonnegative when $h/2 \geq A_R\lambda/(Q + 1)^2$.

This completes the proof. ■

REMARK 1 *Although this search procedure does not provide an optimal solution, it still is a good approximation, because the grid size for T is always less than, or equal to, 0.01.*

IV.1.2.3. The Solution

After solving the Single Dispatch and Multi Dispatch problems, we compare both solutions and pick the one with the minimum cost.

IV.2. Quantity-Based Policy

Similar to the procedure that we followed for the *time-based policy*, we first express the cost function in terms of the decision variables. Recall from Chapter III that there are two decision variables in this case: k and q . Also, we refer to Chapter III for the derivation of the expected annual cost excluding the expected annual dispatch cost. Next, we derive the expected annual cost:

IV.2.1. Derivation of $E[DC]$ for Quantity-Based Policy

For the *quantity-based policy*, the dispatch quantity is no longer a random variable. Hence, we can express the expected cost for a shipment as follows:

$$E[DCC] = \begin{cases} c_0q & \text{if } 1 \leq q < q_1, \\ c_0q_1 & \text{if } q_1 \leq q < q_2, \\ \vdots & \\ c_iq & \text{if } q_{2i} \leq q < q_{2i+1}, \\ c_iq_{2i+1} & \text{if } q_{2i+1} \leq q < q_{2i+2}, \\ \vdots & \\ c_I & \text{if } q_{2I} \leq q. \end{cases}$$

Then the expected dispatch cost for one replenishment cycle, $E[DC]$, is simply given by multiplying the above expression by $kE[Dispatch\ Cycle\ Length]$. Note that

since $E[DC]$ is a piecewise function, $C(k, q)$ is also a piecewise function:

$$C(k, q) = \begin{cases} \frac{[h(k-1)+w]q}{2} + \frac{A_R\lambda}{kq} - \frac{w}{2} + c_0\lambda & \text{if } 1 \leq q < q_1, \\ \frac{[h(k-1)+w]q}{2} + \frac{A_R\lambda}{kq} - \frac{w}{2} + \frac{A_D^0\lambda}{q} & \text{if } q_1 \leq q < q_2, \\ \vdots & \\ \frac{[h(k-1)+w]q}{2} + \frac{A_R\lambda}{kq} - \frac{w}{2} + c_i\lambda & \text{if } q_{2i} \leq q < q_{2i+1}, \\ \frac{[h(k-1)+w]q}{2} + \frac{A_R\lambda}{kq} - \frac{w}{2} + \frac{A_D^i\lambda}{q} & \text{if } q_{2i+1} \leq q < q_{2i+2}, \\ \vdots & \\ \frac{[h(k-1)+w]q}{2} + \frac{A_R\lambda}{kq} - \frac{w}{2} + c_I\lambda & \text{if } q_{2I} \leq q. \end{cases}$$

where we define A_D^i to be $c_i q_{2i+1}$.

IV.2.2. Analysis of $C(k, q)$

We state our problem as:

$$\begin{aligned} \min \quad & C(k, q) \\ \text{s.t} \quad & k, q \in \mathbb{N}^+. \end{aligned}$$

Although the cost function has many pieces, we can see that there are two types of functions, one with a fixed dispatch cost (having $A_D^i\lambda/q$ term) and the other is without a fixed dispatch cost but with a per unit dispatch cost (having $c_i\lambda$ term). We name these functions $FC_i(k, q)$ and $LC_i(k, q)$ respectively:

$$FC_i(k, q) = \frac{[h(k-1)+w]q}{2} + \frac{A_R\lambda}{kq} - \frac{w}{2} + \frac{A_D^i\lambda}{q}, \quad (4.8)$$

$$LC_i(k, q) = \frac{[h(k-1)+w]q}{2} + \frac{A_R\lambda}{kq} - \frac{w}{2} + c_i\lambda. \quad (4.9)$$

The resulting subproblems are

$$\min \quad FC_i(k, q) \quad (4.10)$$

$$\text{s.t} \quad q_{2i+1} \leq q < q_{2i+2}$$

$$k, q \in \mathbb{N}^+,$$

$$\min \quad LC_i(k, q) \quad (4.11)$$

$$\text{s.t} \quad q_{2i} \leq q < q_{2i+1}$$

$$k, q \in \mathbb{N}^+.$$

Observe that both $FC_i(k, q)$ and $LC_i(k, q)$ have a structure similar to $C(k, q)$, which is given by Equation (3.1) in Chapter III. However, we cannot simply follow the solution technique that we developed in Chapter III, because these subproblems have constraints on q . Next, we present the solution procedures for $FC_i(k, q)$ and $LC_i(k, q)$, respectively:

IV.2.2.1. Analysis of $FC(k, q)$

Observe that, $FC_i(k, q)$ is convex in k . Momentarily assuming k as a continuous variable, we can easily show that for a given q , the optimal k , namely $k^*(q)$ is

$$k^*(q) = \frac{1}{q} \sqrt{\frac{2A_R \lambda}{h}}.$$

Note that, $k^*(q)$ is decreasing in q . Hence, $k^*(q_{2i+1}) \geq k^*(q_{2i+2})$. On the other hand, for a given k , $FC_i(k, q)$ is convex in q and is minimized at $q^*(k)$ where

$$q^*(k) = \sqrt{\frac{2\lambda(A_D^i + A_R/k)}{h(k-1) + w}}.$$

Using these results, we propose the following procedure to solve the problem:

ALGORITHM 2 Let, $\underline{k} = \lfloor k^*(q_{2i+2}) \rfloor$, and $\bar{k} = \lceil k^*(q_{2i+1}) \rceil$.

Step 0: Set $FC_i^* = \infty$ and $k = \underline{k}$.

Step 1: Find $q^*(k)$. If $q^*(k) \notin [q_{2i+1}, q_{2i+2}]$, set $q^*(k)$ to the closest boundary point.

Step 2: Compute $FC_i(k, q^*(k))$. If $FC_i(k, q^*(k)) < FC_i^*$, then set $FC_i^* = FC_i(k, q^*(k))$.

Step 2: Increment k by 1. If $k \leq \bar{k}$, go to **Step 1**.

IV.2.2.2. Analysis of $LC_i(k, q)$

The analysis is similar to that of $FC_i(k, q)$. For $LC_i(k, q)$,

$$k^*(q) = \frac{1}{q} \sqrt{\frac{2A_R \lambda}{h}} \quad q^*(k) = \sqrt{\frac{2\lambda(A_D^i + A_R/k)}{h(k-1) + w}}$$

$$\underline{k} = \lfloor k^*(q_{2i+1}) \rfloor, \quad \bar{k} = \lceil k^*(q_{2i}) \rceil.$$

We simply implement Algorithm 2 with the updated values to minimize $LC_i(k, q)$.

After solving each subproblem, we simply pick the solution which yields the minimum expected cost as the optimal solution of $C(k, q)$.

IV.3. Summary and Conclusions

In this chapter, we characterized the solutions for the integrated inventory and shipment consolidation problem with common carrier charges. We developed solution procedures for both the *time-based policy* and *quantity-based policy*.

As was also noted by Çetinkaya and Bookbinder (2003), it is not possible to obtain exact analytical solutions for the *time-based policy* with common carrier charges. We have provided an extensive search procedure and upper and lower bounds for the ranges of the search. However, in a pure consolidation setting, Çetinkaya and Bookbinder's approximate solutions can also be used.

An immediate extension to this problem is to study the consolidation policies under different transportation cost structures, e.g., incremental freight discounts.

CHAPTER V

A HYBRID POLICY FOR SHIPMENT CONSOLIDATION

In Chapter III, we solved the integrated inventory and dispatch consolidation problem under a quantity-based consolidation regime. We also showed the cost superiority of *quantity-based policy* over *time-based policy*.

Under a *time-based policy*, each order is dispatched by a pre-specified shipment release date, even though the dispatch quantity does not necessarily realize transportation scale economies. On the other hand, under a *quantity-based policy*, the dispatch quantity assures transportation scale economies, but a specific dispatch time cannot be guaranteed. An alternative to these two policies is a hybrid routine aimed at balancing the tradeoff between the timely delivery advantages of time-based policies and the transportation cost savings associated with quantity-based policies. Under a *hybrid policy*, the objective is to consolidate an economical dispatch quantity, denoted by q_H . However, if this quantity does not accumulate within a reasonable time window, denoted by T_H , then a shipment of smaller size may be released. A dispatch decision is made either when the size of a consolidated load exceeds q_H or when the time since the last dispatch exceeds T_H .

Through a numerical investigation, we show that although the resulting costs of *hybrid policies* are higher than those of *quantity-based policies*, they lead to better service for the retailer when service is measured by the long-run average cumulative waiting time. This is simply because a *hybrid policy* imposes an upper bound on the maximum waiting time for an order by specifying the maximum time between two successive dispatch decisions. We also illustrate that the main advantage of *hybrid policies* over *time-based policies* is that they may lead to lower expected total average costs per unit-time. Hence, *hybrid policies* are attractive in the sense that they are

cost-wise superior to *time-based policies* and service-wise superior to *quantity-based policies*.

In the numerical study, we select the parameters of the *hybrid policy* using the optimal policy parameters of the corresponding *time-based policy* and *quantity-based policy*. This is mainly because it is analytically intractable to express the expected annual cost function of the *hybrid policy* and optimize its parameters under the model assumptions of Chapter III. However, computing the optimal policy parameters of a *hybrid policy* is both an analytically challenging problem and an unexplored area of research. In this chapter, we also model a pure *hybrid policy*, derive the expected annual cost function, and find the optimal policy parameters.

This chapter is organized as follows: The operational characteristics of the proposed hybrid policies are discussed in Section V.1. We present our numerical results in Section V.1.1 where we provide a comparison of the cost and service performances of *time-based* -, *quantity-based* -, and *hybrid policies*. Section V.2 studies an analytical model for a pure *hybrid policy*. Finally, Section V.3 concludes the study with a discussion of important observations and generalizations.

V.1. Easily Implementable Hybrid Policies

We suggest that a cost effective hybrid solution for the inventory system under consideration can be obtained using our results for the models with *time-based* - and *quantity-based* - policies. Under the suggested *hybrid policy*, the goal is to consolidate an outbound load of size q_H . However, if the time since the last dispatch exceeds T_H before an economical dispatch quantity of q_H accumulates, then a dispatch decision should be made immediately. We argue that, if q_H and T_H are specified carefully, then the corresponding *hybrid policy* exhibits the cost advantages associated with

quantity-based policies as well as the timely delivery advantages associated with *time-based policies*. For this purpose, letting Q_H denote the order-up-to level for inventory replenishments under a hybrid solution, we propose two easily implementable hybrid solutions.

1. **Hybrid-quantity-based solution:** This solution is specified by the following three parameters: $Q_H = Q^*$, $T_H = T^*$, and $q_H = q^*$.
2. **Hybrid-time-based solution:** This solution is specified by the following three parameters: $Q_H = Q_T^*$, $T_H = T^*$, and $q_H = q^*$.

Both of these hybrid solutions require the exact optimization of Çetinkaya and Lee's (2000) time-based model using the exact numerical optimization technique in Axsäter (2001) so that we have the numerical values for Q_T^* and T^* . For implementation purposes, it is important to note that the exact optimization of this model is not as simple as the exact optimization of the quantity-based model in Section III.2 where we have a closed form solution. Hence, the approximate technique presented in Çetinkaya and Lee (2000) can be utilized to form easily implementable *hybrid policies*. This approximate technique is appealing since it can be implemented on a spreadsheet with minimal effort. The implementation details for using the approximate technique and computing the costs of the hybrid solutions are discussed next.

Some of the numerical examples in Axsäter (2001) seem to imply that the approximate approach in Çetinkaya and Lee (2000) may lead to errors, particularly, if $Q_T^* = 0$, i.e., if it is optimal to operate the vendor's warehouse as a transshipment point under a time-based dispatch model. It is worth noting that this is, in fact, the only case that the approximate technique developed in Çetinkaya and Lee (2000) may overlook, and that the technique performs remarkably well in all other problem instances. Further, the approximate technique can be modified in a trivial manner

so that its performance is excellent in general (See Çetinkaya, Mutlu, and Lee 2006). The modification suggests the following: i) Compute the corresponding exact costs of the solutions given by $Q_T = 0$ and $T = \sqrt{2(A_R + A_D)/w}$ and the original approximate solution suggested in Çetinkaya and Lee (2000), and ii) pick the one producing a smaller exact cost value as the solution to the time-based dispatch model. Since the modified approximation works remarkably well and is easy to implement, we also suggest the use of the approximate approach described above in specifying the parameters of easily-implementable hybrid solutions. Let \tilde{Q}_T and \tilde{T} denote the resulting approximate order-up-to level for the vendor and the dispatch frequency, respectively, under the time-based dispatch model. The approximate hybrid solutions are obtained as follows.

1. **Approximate hybrid-quantity-based solution:** This solution is specified by $Q_H = Q^*$, $T_H = \tilde{T}$, and $q_H = q^*$.
2. **Approximate hybrid-time-based solution:** This solution is specified by $Q_H = \tilde{Q}_T$, $T_H = \tilde{T}$, and $q_H = q^*$.

In order to compare the cost implications of the suggested hybrid solutions, we need to develop a technique for computing the expected total average cost per unit-time under these solutions, namely $C_H(Q, q, T)$. Under a hybrid policy, identifying the regenerative cycles and computing the expected number of dispatch cycles within a replenishment cycle is a challenging task. Therefore, obtaining a closed form analytical formula for $C_H(Q, q, T)$ is not practical for our purposes. In order to overcome this hurdle, we have computed the empirical cost of the hybrid solutions using simulation.

V.1.1. Numerical Results Illustrating Cost and Service Performance Improvements under Hybrid Policies

The numerical results presented in this section illustrate that, in terms of the resulting average cost, the *hybrid policies* may be superior to a *time-based policy*, but a *quantity-based policy* is the best. Furthermore, the numerical results also indicate that the *hybrid policies* are superior to the quantity policies in terms of service, i.e., average waiting times of customers. As a result, the *hybrid policies* provide reasonable alternatives by improving on the cost performance of the *time-based policy* and the service measures of the *quantity-based policy*. As we demonstrate in the following discussion, these improvements are significant, and, hence, the hybrid policies provide favorable alternatives for practical problems.

Using the numerical examples considered in Axsäter (2001), we first obtain the results presented in Table 2. Hence, the problem instances considered in Table 2 are the same as those considered in Table 1. The % cost savings in Table 2 represent the savings from using the corresponding hybrid-solution rather than the time based solution, i.e.,

$$\% \Delta C_H(\cdot, \cdot, \cdot) = \frac{C_T(Q_T^*, T^*) - C_H(\cdot, \cdot, \cdot)}{C_T(Q_T^*, T^*)} \times 100\%.$$

For this particular data set, Table 2 suggests that the cost savings resulting from a hybrid solution can be in excess of 12%.

Although the approximate technique suggested for the time-based dispatch model performs remarkably well in estimating the exact solution (see Table 2), and the results are promising in general, one should be cautious in using this approximate technique in forming a hybrid policy. This is because, based on our numerical results and the examples in Table 2, we conclude that while the hybrid-quantity-based solution with $T_H = T^*$ does reduce the cost, the hybrid-quantity-based solution with

Table 2 % Cost Savings Using Hybrid Solutions

$\% \Delta C_H(Q^*, q^*, T^*)$	$\% \Delta C_H(Q^*, q^*, \tilde{T})$	$\% \Delta C_H(Q_T^*, q^*, T^*)$	$\% \Delta C_H(\tilde{Q}_T, q^*, \tilde{T})$
12.84	12.78	13.97	13.85
9.05	8.74	9.66	9.29
10.13	9.94	9.80	9.94
6.29	5.56	6.61	5.93
2.71	1.96	2.70	2.01
1.68	0.76	1.94	0.93
2.00	0.27	1.83	0.42
-0.38	-3.08	1.12	-1.21
1.76	0.91	1.79	1.04
1.25	0.43	1.32	0.55
0.46	-1.30	0.98	-0.67
-0.78	-3.27	0.66	-1.30
1.33	1.17	1.17	1.17
1.21	1.41	1.41	1.41
1.28	1.08	1.08	1.08
1.56	1.60	1.60	1.60
1.00	1.06	1.06	1.06
4.22	4.22	-7.66	-7.66
1.17	1.17	1.17	1.17
3.18	3.18	-12.74	-12.74
1.08	1.08	1.08	1.08
1.19	1.19	1.19	1.19
1.25	1.25	1.25	1.25
2.85	2.27	1.87	1.36

$T_H = \tilde{T}$ may not. On the other hand, both of the hybrid-time-based solutions, with $T_H = T^*$ or $T_H = \tilde{T}$, seem worse than the hybrid-quantity-based solution in terms of their worst case performance.

Considering the average % of cost savings in the last row of Tables 1 and 2, the *quantity-based policy* outperforms the hybrid-quantity-based solution which outperforms the *time-based policy*. Given that a hybrid solution offers timely delivery advantages similar to those of a *time-based policy*, the cost improvements gained by using a hybrid-quantity-based solution are still significant although they are not as

great as those realized by using a *quantity-based policy*. Our numerical results for the 1024 problem instances¹ also demonstrate this fact.

A natural extension of the hybrid policies studied above arises when we start counting time after the first order is realized, rather than right after the last dispatch, since this approach uses more demand information. In fact, time-based policies can also be modified in a similar fashion, but since the above hybrid policies already improve on cost performance while achieving the same timely delivery advantages, we only discuss the impact of this modification on the cost performance of the *hybrid policies*. Our results based on this modification are summarized in Table 3². where $\% \Delta C_{HM}(\cdot, \cdot, \cdot)$ represents the cost savings from using the corresponding modified hybrid solution rather than the time-based solution. After this modification, the cost performance of the *hybrid policies* improves. However, this improvement is not substantial.

Average $\% \Delta C_{HM}(Q^*, q^*, T^*)$	Average $\% \Delta C_{HM}(Q^*, q^*, \tilde{T})$	Average $\% \Delta C_{HM}(Q_T^*, q^*, T^*)$	Average $\% \Delta C_{HM}(\tilde{Q}_T, q^*, \tilde{T})$
3.78%	3.30%	2.52%	2.31%

We also have tested the sensitivity of the cost performance of the *hybrid policies* to the T_H and q_H values. We have found the intuitive result that the cost decreases as T_H increases, and the cost increases as T_H decreases. This is simply because for larger (resp. smaller) values of T_H , the dispatch load is more (resp. less) likely to be equal to q_H for most dispatch cycles, which implies that the *hybrid policy* works almost like

¹These problems are the ones that we generated in Chapter III.

²Problem settings in Table 3 are the same as in Table 1

a *quantity-based policy* (resp. *time-based policy*). In fact, for smaller values of T_H , we are, in fact, imposing a more strict time constraint on the problem, and, hence, the cost performance deteriorates. The relationship between the cost performance of the *hybrid policy* and the underlying q_H value is more complicated. For some problem instances, the cost increases as we deviate from q_H because q_H becomes too large and unlikely to accumulate during a dispatch cycle so that the *hybrid policy* effectively reduces to a *time-based policy*. In other problem instances, the cost may increase first and then decrease. The reason behind this result is that we use a heuristic approach to specify the q_H and T_H values, and for a given T_H , there may be a more suitable q_H value that we are underestimating.

Finally, we argue that although the cost performance of the *quantity-based policy* is superior to the cost performance of the *hybrid policies*, the *hybrid policies* lead to higher service measures. In this context, the service measure we consider is the long-run average of cumulative waiting, i.e.,

$$\lim_{t \rightarrow \infty} \frac{\text{sum of waiting times of orders in } [0, t]}{t},$$

which has been estimated using simulation for the problem settings in Table 1 as well as the 1024 additional problem instances solved. Table 4 provides some results for a comparison of the average service and cost performances of hybrid and quantity-based solutions for the 1024 problem instances.

It is worth noting that the hybrid policies lead to higher service measures than the *quantity-based policy* because they impose an upper bound on the waiting times of orders. Also, by definition, the resulting service measure of a *hybrid policy* is the same as the service measure of the underlying *time-based policy* used to set the value of T_H . Naturally, our numerical results are consistent with these intuitive expectations about the service and cost performance of the *hybrid policies*.

Table 4 A Comparison of the Average Service and Cost Performances of *Hybrid Policies* and the *Quantity-Based Policy*

	Average % service measure improvement	Average % cost increase
Hybrid quantity p. vs. quantity p.	16.20	3.82
Hybrid time p. vs. quantity p.	16.20	4.46
Appr. Hybrid quantity p. vs. quantity p.	22.85	4.46
Appr. Hybrid time p. vs. quantity p.	22.82	5.37

V.2. An Analytical Model for a Pure Hybrid Consolidation Policy

In Chapter III, we derived the optimal policy parameters for an integrated inventory and shipment consolidation model under a *quantity-based* consolidation regime. We also compared the cost performance of this consolidation practice to that of a *time-based* consolidation policy. *Quantity-based* and *time-based policies* are the most common consolidation practices. In addition to these, there are mixed consolidation strategies that utilize both the time and quantity aspects. In the first section of this chapter, we characterized some of these hybrid consolidation strategies whose policy parameters are derived from the optimal policy parameters of the *quantity-based* and *time-based policies*, and we have compared the cost and service performances of these *hybrid policies* in an integrated inventory and shipment consolidation model. We also noted that, deriving an analytical expression for the expected cost of the *hybrid policy* in such a model is analytically intractable. However, in order to have a better understanding of the performance of *hybrid policies*, we can isolate the shipment consolidation decisions from inventory decisions and build a pure consolidation model. In this section, we build a pure consolidation model with a *hybrid consolidation policy* and analytically derive the optimal policy parameters.

The model assumptions are same as those of Chapter III. The *hybrid policy* works

as follows: A shipment is released either when the accumulated load exceeds q , i.e., reaches $q + 1$, or the waiting time of the first order exceeds T . We define a *shipment consolidation cycle* to be the time between two consecutive shipment epochs.

We note here that T is not the maximum shipment cycle length, but the maximum waiting time of the first order. This is slightly different from Çetinkaya and Lee's (2000) model. In their model, the shipment cycle length is set to T time units. Such an implementation ensures periodic shipments but does not necessarily realize the scale economies. On the other hand, in the *hybrid* policy, periodic shipments cannot be guaranteed by setting a maximum cycle length. Hence, in order to ensure the scale economies, we define T to be the maximum waiting time of an order. The related costs are as follows:

- \tilde{K} : Fixed cost for each shipment.
- c : Shipment cost per unit of item.
- w : Cost of delaying the shipment of one unit of item for one unit of time.

Since the annual demand rate is constant, c , the per unit shipment cost, does not affect the choice of q and T . Hence, without loss of generality, we assume $c = 0$ in the rest of the paper.

In the next section, we derive the long run average of the expected cost of the system for unit demand sizes.

V.2.1. An Analytical Model

First, we state some assumptions about the underlying demand and inventory processes and introduce some notation:

- We let $X_i, i = 1, 2, \dots$ denote the interarrival time of the i^{th} order in a shipment cycle. Since demand is Poisson, the X_i 's follow an Exponential distribution with mean $E[X_i] = 1/\lambda$.

- Also let $S_0 = 0$, $S_i = \sum_{j=1}^i X_j$, and $N(t) = \sup \{i : S_i \leq t\}$ as the underlying demand arrival process in a shipment cycle. Observe that $N(t)$ is the counting process that registers the number of Poisson arrivals within the first t time units of the cycle. For any given t , the distribution of $N(t)$ is Poisson with parameter λt . Recall that, T is the decision variable that represents the maximum tolerable waiting time for a customer. Since we use the probability distribution of $N(T)$ very frequently in the paper, we define the following shorthand notation:

$$p_n = P\{N(T) = n\} = \frac{e^{-\lambda T} (\lambda T)^n}{n!}.$$

We also define

$$F(x) = \sum_{n=0}^x p_n \text{ and,}$$

$$\bar{F}(x) = \sum_{n=x+1}^{\infty} p_n.$$

- Each time a customer order is received, we update the size of the accumulated/consolidated outbound load waiting to be released at time t . This way, the time when the accumulated outbound load reaches $q + 1$ for the first time in a shipment cycle can be registered immediately.

In order to calculate the long run average of the expected cost, we employ the renewal reward theorem. For this, we should first identify the regeneration epochs, and define the regenerative cycles. Since the orders arrive according to a Poisson Process, no matter what triggers the shipment decision, each shipment stochastically clears the system, i.e., each shipment is a regeneration epoch.³ Using the Renewal Reward Theorem, we can express the long run average of the expected cost, $\tilde{G}(q, T)$

³We note that, this argument would not have held if the order stream were not Poisson.

as follows:

$$\tilde{G}(q, T) = \frac{E[C_C]}{E[L]}. \quad (5.1)$$

Here, $E[C_C]$ represents the expected cost for a shipment cycle, and $E[L]$ represents the expected shipment cycle length. The expression for the latter, required to derive the former, is presented first.

LEMMA 3 *The expected cycle length is given by*

$$E[L] = \frac{1}{\lambda} + \frac{q}{\lambda} \bar{F}(q) + TF(q-1). \quad (5.2)$$

Proof: The cycle length is

$$(L|S_1 = s_1) = \begin{cases} s_1 + (S_{q+1} - S_1)|S_1 = s_1 & \text{if } S_{q+1}|S_1 - s_1 \leq T, \\ s_1 + T & \text{otherwise.} \end{cases} \quad (5.3)$$

Using expectations, we can write

$$E[L|S_1 = s_1] = s_1 + E[\min((S_{q+1} - S_1)|S_1 = s_1), T)]. \quad (5.4)$$

We know that S_1 follows an exponential distribution with mean $1/\lambda$. We observe that the distribution of $(S_{q+1} - S_1)|S_1 = s_1$ has Erlang(q, λ). For notational clarity, we define $Y_q := (S_{q+1} - S_1)|S_1 = s_1$, and rewrite Equation (5.4) as

$$E[L|S_1 = s_1] = s_1 + E[\min(Y_q, T)]. \quad (5.5)$$

We can derive $E[\min(Y_q, T)]$, by conditioning on Y_q :

$$E[\min(Y_q, T)] = \int_0^T t f_{Y_q}(t) dt + TP\{Y_q > T\} \quad (5.6)$$

$$= \int_0^T t \lambda (\lambda t)^{q-1} \frac{e^{-\lambda t}}{(q-1)!} dt + TF(q-1). \quad (5.7)$$

Here, we note that

$$\begin{aligned}
\int_0^T (\lambda t)^q e^{-\lambda t} dt &= (\lambda T)^q \frac{e^{-\lambda T}}{-\lambda} - \int_0^T q \lambda^q t^{q-1} \frac{e^{-\lambda t}}{-\lambda} dt \\
&= -(\lambda T)^q \frac{e^{-\lambda T}}{\lambda} + q \int_0^T (\lambda t)^{q-1} \frac{e^{-\lambda t}}{\lambda} dt \\
&= -(\lambda T)^q \frac{e^{-\lambda T}}{\lambda} - q(\lambda T)^{q-1} \frac{e^{-\lambda T}}{\lambda} + q(q-1) \int_0^T (\lambda t)^{q-2} \frac{e^{-\lambda t}}{\lambda} dt \\
&\vdots \\
&= -\frac{e^{-\lambda T}}{\lambda} \left(\sum_{n=0}^{q-1} \frac{q! (\lambda T)^{q-n}}{(q-n)!} \right) + q! \int_0^T e^{-\lambda t} dt \\
&= -\frac{e^{-\lambda T}}{\lambda} \left(\sum_{n=0}^{q-1} \frac{q! (\lambda T)^{q-n}}{(q-n)!} \right) - \frac{q! e^{-\lambda T}}{\lambda} + \frac{q!}{\lambda} \\
&= -\frac{e^{-\lambda T}}{\lambda} \left(\sum_{n=0}^q \frac{q! (\lambda T)^{q-n}}{(q-n)!} \right) + \frac{q!}{\lambda} \\
&= \frac{q!}{\lambda} \left(1 - \sum_{n=0}^q p_n \right) \\
&= \frac{q!}{\lambda} \overline{F}(q).
\end{aligned}$$

Thus,

$$\int_0^T (\lambda t)^q \frac{e^{-\lambda t}}{(q-1)!} dt = \frac{q}{\lambda} \overline{F}(q). \quad (5.8)$$

Substituting Equation (5.8) into Equation (5.6), we obtain

$$E[\min(Y_q, T)] = \frac{q}{\lambda} \overline{F}(q) + TF(q-1). \quad (5.9)$$

We, next, substitute Equation (5.9) into (5.5) and obtain

$$E[L|S_1 = s_1] = s_1 + \frac{q}{\lambda} \overline{F}(q) + TF(q-1). \quad (5.10)$$

Note that $E[L] = E_{S_1}[E[L|S_1]]$. By taking the expectation of the expression (5.10) over S_1 , we obtain Equation (5.2). ■

Next, note that the cost per cycle has two components, namely the shipment costs, C_D , and the customer waiting cost, C_W . C_D is simply a setup cost \tilde{K} associated with each shipment, which is constant over all realizations of the shipment cycle. $E[C_W]$ is found in the following lemma:

LEMMA 4 *The expected customer waiting cost per cycle is given by*

$$E[C_W] = w \left(\frac{q}{\lambda} \overline{F}(q) + TF(q-1) + \frac{(q-1)q}{2} \frac{\overline{F}(q)}{\lambda} + \frac{T}{2} \sum_{n=0}^{q-1} np_n \right). \quad (5.11)$$

Proof: Observe that, the expected waiting time for the first order in a shipment cycle is $E[L] - E[S_1]$. By using the expression of $E[L]$ in Equation (5.2), we obtain the expected waiting time of the first customers as

$$\frac{q}{\lambda} \overline{F}(q) + TF(q-1). \quad (5.12)$$

Next, we explain how to derive the total waiting times of other orders in the cycle. Let W be the random variable to denote the total waiting times of those orders, and \tilde{W} be $W|S_1$. Let us also define

$$Y_i = (S_{i+1} - S_1)|S_1 = s_1, \text{ for } i = 1 \dots q.$$

Each Y_i follows Erlang(i, λ).

Further, let τ be the random variable for denoting the time that elapses between the arrival of the first order and the shipment conditional on S_1 . Recalling the proof of Lemma 3, τ is in fact identical to $\min(Y_q, T)$, i.e.,

$$\tau = \begin{cases} Y_q & \text{if } Y_q \leq T, \\ T & \text{otherwise.} \end{cases} \quad (5.13)$$

Equation (5.13) basically says that, if the time it takes to accumulate q units is less than T , then τ is equal to that time; otherwise, $\tau = T$. Utilizing that equation, we find $E[\tilde{W}]$ by conditioning on the value of τ :

$$E[\tilde{W}] = E_\tau[E[\tilde{W}|\tau]] = \int_0^T E[\tilde{W}|\tau = t]f_\tau(t)dt + E[\tilde{W}|\tau = T]P\{\tau = T\}. \quad (5.14)$$

The two pieces in (5.14) pertain, respectively, to the cases when q units are consolidated in a time less than T and when fewer than q orders arrive within an interval T . The first piece is

$$\begin{aligned} \int_0^T E[\tilde{W}|\tau = t]f_\tau(t)dt &= \int_0^T E[\tilde{W}|\tau = t]f_\tau(t)dt \\ &= \int_0^T E\left[\sum_{n=1}^{q-1}(t - Y_n|Y_q = t)\right] f_\tau(t)dt \\ &= \int_0^T \left((q-1)t - E\left[\sum_{n=1}^{q-1}(Y_n|Y_q = t)\right]\right) f_\tau(t)dt. \end{aligned} \quad (5.15)$$

By Lemma 4.5.1 in Resnick (2002), if $Y_q = t$, then the joint distribution of Y_n for $n < q$, is the same as the joint distribution of $\{U_{(1)}, \dots, U_{(n)}\}$, the order statistics of a sample of size n from the Uniform distribution on $(0, t)$. Also, since $\sum_{n=1}^q Y_n$ is symmetric in the elements of the summand, the following assertion holds:

$$E\left[\sum_{n=1}^{q-1}(Y_n|Y_q = t)\right] = E\left[\sum_{n=1}^{q-1}U_{(n)}\right] = \frac{(q-1)t}{2}. \quad (5.16)$$

We next substitute Equation (5.16) into (5.15), and obtain

$$\begin{aligned} \int_0^T E[W|\tau = t]f_\tau(t)dt &= \int_0^T \frac{(q-1)t}{2}f_\tau(t)dt \\ &= \frac{(q-1)}{2} \int_0^T t\lambda(\lambda t)^{q-1} \frac{e^{-\lambda t}}{(q-1)!} dt \end{aligned}$$

Using Equation (5.8), we obtain

$$\int_0^T E[W|\tau = t]f_\tau(t)dt = \frac{(q-1)q}{2} \frac{q}{\lambda} \bar{F}(q). \quad (5.17)$$

Continuing, we calculate the second piece in Equation (5.14):

$$\begin{aligned} E[\tilde{W}|\tau = T]P\{\tau = T\} &= \sum_{n=0}^{q-1} E[\tilde{W}|N(t, T+t) = n]P\{N(s_1, T+s_1) = n\} \\ &= \sum_{n=0}^{q-1} E[\tilde{W}|N(T) = n]p_n \\ &= \sum_{n=0}^{q-1} E\left[\sum_{i=1}^n (T - Y_i|N(T) = n)\right] p_n \\ &= \sum_{n=1}^{q-1} \left(nT - E\left[\sum_{i=1}^n Y_i|N(T) = n\right]\right) p_n \\ &= \sum_{n=1}^{q-1} \left(nT - E\left[\sum_{i=1}^n Y_i|N(T) = n\right]\right) p_n. \end{aligned} \quad (5.18)$$

Using Theorem 4.5.2 of Resnick (2002) and the symmetric property of $\sum_{i=1}^n Y_i$ in Y_i , we find

$$E\left[\sum_{i=1}^n Y_i|N(T) = n\right] = E\left[\sum_{i=1}^n U_{(i)}\right] = E\left[\sum_{i=1}^n U_i\right] = \frac{nT}{2}. \quad (5.19)$$

Substituting Equation (5.19) into (5.18) results in

$$E[\tilde{W}|\tau = T]P\{\tau = T\} = \sum_{n=1}^{q-1} \frac{nT}{2} p_n = \sum_{n=0}^{q-1} \frac{nT}{2} p_n. \quad (5.20)$$

Combining Equations (5.17) and (5.20) in Equation (5.14), we obtain the following:

$$E[\tilde{W}] = \frac{(q-1)q}{2} \frac{q}{\lambda} P\{N(T) > q\} + \frac{T}{2} \sum_{n=0}^{q-1} np_n. \quad (5.21)$$

Since $E[W] = E_{S_1}[\tilde{W}]$, we compute $E[W]$ by taking the expectation of Equation (5.21) over S_1 . However, since this equation is independent of S_1 , we conclude that

$E[W]$ is given by the same expression.

Recall that Equation (5.12) gives the expected waiting time of the first order, and now we have Equation (5.21), the sum of the expected waiting times of the other orders in the cycle. Hence, summation of (5.12) and (5.21) yields Equation (5.11). ■

Using Equations (5.2) and (5.11) in Equation (5.1), we find

$$\tilde{G}(q, T) = \frac{\tilde{K} + w \left(\frac{q}{\lambda} \bar{F}(q) + TF(q-1) + \frac{(q-1)}{2} \frac{q}{\lambda} \bar{F}(q) + \frac{T}{2} \sum_{n=0}^{q-1} np_n \right)}{\frac{1}{\lambda} + \frac{q}{\lambda} \bar{F}(q) + TF(q-1)}. \quad (5.22)$$

The following remark simplifies the objective function in Expression (5.22).

REMARK 2 *Minimizing $\tilde{G}(q, T)$ is equivalent to minimizing*

$$G(q, t) = \frac{K + wq(q+1)\bar{F}(q, t) + wt[\sum_{n=0}^{q-1} np_n + 2F(q-1, t)]}{1 + q\bar{F}(q, t) + tF(q-1, t)}, \quad (5.23)$$

where $K := 2\tilde{K}\lambda$, $t := \lambda T$, and $G(q, t) = 2\tilde{G}(q, T)$.

Proof: Substituting $t = \lambda T$ in Expression (5.22), we have

$$\begin{aligned} \tilde{G}(q, t) &= \frac{\tilde{K}\lambda}{1 + q\bar{F}(q) + tF(q-1)} + \frac{w}{2} \frac{(q(q+1)\bar{F}(q) + t \sum_{n=0}^{q-1} (n+2)p_n)}{1 + q\bar{F}(q) + tF(q-1)} \\ &= \frac{1}{2} \left(\frac{K + wq(q+1)\bar{F}(q, t) + wt[\sum_{n=0}^{q-1} np_n + 2F(q-1, t)]}{1 + q\bar{F}(q, t) + tF(q-1, t)} \right). \end{aligned}$$

Having shown $2\tilde{G}(q, t) = G(q, t)$, we can say that the (q, t) that minimizes $G(q, t)$ also minimizes $\tilde{G}(q, t)$. ■

Before attempting to minimize $G(q, t)$, we consider two limiting cases:

1. $t \rightarrow \infty$: When t is very large, the shipments are always triggered when the target load is reached. Hence, the *hybrid policy* reduces to the *quantity-based policy*. In this case, $G(q, t)$ is

$$G(q, \infty) = \frac{K}{q+1} + wq. \quad (5.24)$$

It is trivial to show that $G(q, \infty)$ is minimized at

$$q^* = \sqrt{\frac{K}{w}} - 1 \text{ and,} \quad (5.25)$$

$$G(q^*, \infty) = 2\sqrt{Kw} - w. \quad (5.26)$$

In practice, q^* is rounded to one of the closest integer, since it is in fact an integer variable.

2. $q \rightarrow \infty$: When q is very large, the target load can never be consolidated, and shipments are released T time units after the arrival of the first order. Thus, the *hybrid policy* becomes *time-based policy*, and $G(q, t)$ is

$$G(\infty, t) = \frac{K}{1+t} + wt \left(1 + \frac{1}{1+t}\right).$$

One can show that $G(\infty, t)$ is minimized at

$$t^* = \sqrt{\frac{K-w}{w}} - 1 \text{ and,} \quad (5.27)$$

$$G(\infty, t^*) = 2\sqrt{(K-w)w}. \quad (5.28)$$

Next, we provide the following Remark which compares $G(q^*, \infty)$ and $G(\infty, t^*)$.

We will use this result for minimizing $G(q, t)$.

REMARK 3 For $K > 2w$, the expected average cost of the optimal quantity-based policy is lower than that of the optimal time-based policy.

Proof: We proceed to the proof by contradiction. Let us assume that $G(\infty, t^*) < G(q^*, \infty)$. Using Equations (5.26) and (5.28), we derive

$$\begin{aligned} G(\infty, t^*) &= 2\sqrt{(K-w)w} < 2\sqrt{Kw} - w = G(q^*, \infty) \\ w &< 2(\sqrt{Kw} - \sqrt{(K-w)w}) \\ \sqrt{w} &< 2(\sqrt{K} - \sqrt{(K-w)}). \end{aligned}$$

Multiplying both sides by $(\sqrt{K} + \sqrt{(K-w)})$ gives

$$\sqrt{Kw} + \sqrt{(K-w)w} < 2w.$$

Since $\sqrt{(K-w)w} < \sqrt{Kw}$, we can say

$$\sqrt{(K-w)w} < w.$$

However, by the assumption of $K > 2w$, we already know that

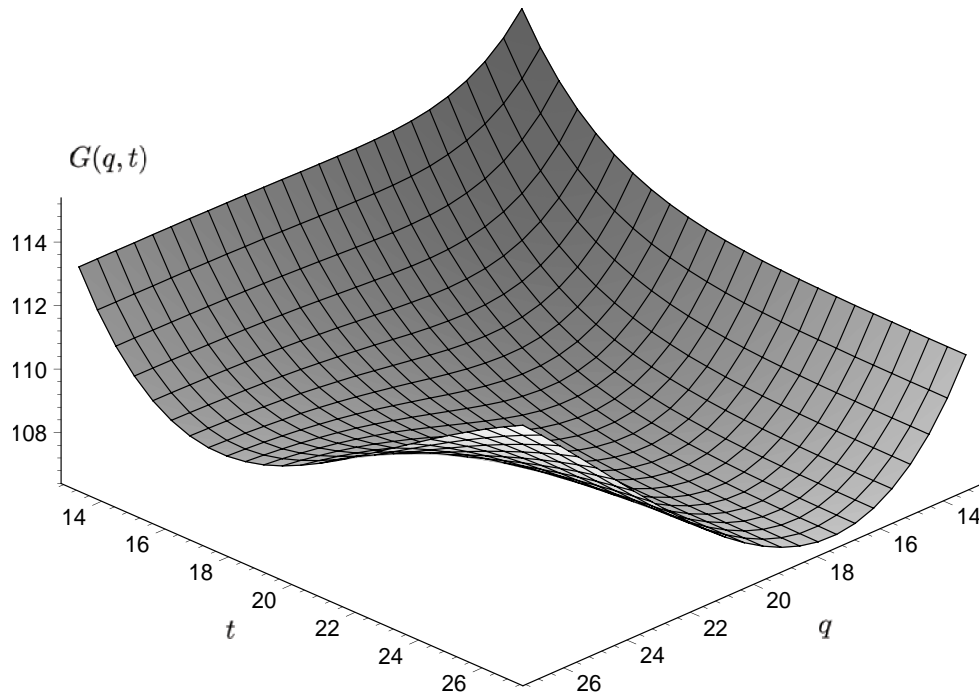
$$\sqrt{(K-w)w} > \sqrt{w^2} = w.$$

This contradiction completes the proof. ■

Having looked at the extreme cases, we turn back to minimizing $G(q, t)$. It is possible to show that the function is neither jointly convex nor convex in any of the variables. Such characteristics make the optimization problem a rather challenging task. (Figure 4 illustrates the behavior of $G(q, t)$ for a given problem instance.) On the other hand, the optimization problem is even more complicated under the integrality constraint on q . However, since consolidation policies make economic sense only for large quantities, we may concentrate on those applications where we expect the optimal q to be large. To be able to derive some optimality characteristics for such

cases, we will assume that q is a continuous variable where necessary in the rest of the paper.

Figure 4 An Illustration of $G(q, t)$ for $K = 100$, $w = 3$



Assuming a continuous q , we obtain the necessary first order conditions for the optimal (q, t) :

$$\frac{\partial G(q, t)}{\partial q} = \frac{\bar{F}(q, t)}{E[L(q, t)]} (w(2q + 1) - G(q, t)) = 0, \quad (5.29)$$

$$\frac{\partial G(q, t)}{\partial t} = \frac{2w[tF(q - 2) + F(q - 1)] - F(q - 1)G(q, t)}{E[L(q, t)]} = 0. \quad (5.30)$$

Equation (5.29) provides us with very useful information about the stationary points and brings us to the following lemma:

LEMMA 5 *For a given t , $G(q, t)$ has either a unique minimizer, $q(t)$ which satisfies*

$$w(2q(t) + 1) = G(q(t), t), \quad (5.31)$$

or $q(t)$ is at infinity.

Proof: Equation (5.29) shows that any q that satisfies $w(2q(t) + 1) = G(q(t), t)$ is a stationary point. Furthermore, one can observe that such a point is a local minimum because the second derivative of $G(q, t)$, with respect to q at that point is nonnegative:

$$\frac{\partial^2 G(q, t)}{\partial q^2} = \frac{2w\bar{F}(q, t)}{E[L(q, t)]} - (w(2q + 1) - G(q, t)) \left\{ \frac{p_q}{E[L]} + 2\frac{\bar{F}^2(q, t)}{E^2[L]} \right\}.$$

The next step is to show that the minimizer is unique. For this, observe that each local minimum satisfies $w(2q(t) + 1) = G(q(t), t)$. Hence, the smallest of the q values that satisfy this equality is the minimizer.

If no finite value of q can satisfy that equality, then the stationary point is in the limit where q approaches infinity, because in Equation (5.29),

$$\lim_{q \uparrow \infty} \frac{\bar{F}(q, t)}{E[L(q, t)]} = 0.$$

This completes the proof. ■

PROPOSITION 6 *(Assuming a continuous solution for q) For $K > 2h$, $G(q, t)$ is minimized at (q^*, ∞) where q^* is given by Equation (5.25).*

Proof: We know from Lemma 5 that, for any given t , if there is a finite q that minimizes G , it should satisfy Equation (5.29). By substituting the expression of

$G(q, t)$ given by Equation (5.23) into (5.29), we obtain the following:

$$w(2q + 1) = \frac{K + wq(q + 1)\overline{F}(q, t) + wt[\sum_{n=0}^{q-1} np_n + 2F(q - 1, t)]}{1 + q\overline{F}(q, t) + tF(q - 1, t)}. \quad (5.32)$$

After multiplying both sides of the equation by the denominator of the right hand side and doing some simplifications, we obtain

$$w(2q + 1) = K - wq^2\overline{F}(q) + wt\left(\sum_{n=0}^{q-1} np_n - (2q - 1)F(q - 1)\right). \quad (5.33)$$

Let $g(q, t)$ denote the right side of Equation (5.33). Note that for the optimal solution with a finite q for a given t , $g(q, t) = G(q, t)$. We can also show that $g(q, t)$ is decreasing in t , since

$$\begin{aligned} \frac{\partial g(q, t)}{\partial t} &= w \left[\sum_{n=0}^{q-1} np_n - (2q - 1)F(q - 1) \right] + wt [F(q - 2) + qp_{q-1}] - wq^2p_q \\ &= w \left\{ \sum_{n=0}^{q-1} np_n - (2q - 1)F(q - 1) + tF(q - 2) + tqp_{q-1} - q^2p_q \right\} \\ &= w \left\{ \sum_{n=0}^{q-1} np_n - \sum_{n=0}^{q-1} 2qp_n + \sum_{n=0}^{q-1} p_n + \sum_{n=0}^{q-2} tp_n \right\} \\ &= w \left\{ \sum_{n=0}^{q-1} np_n - \sum_{n=0}^{q-1} 2qp_n + \sum_{n=0}^{q-1} p_n + \sum_{n=1}^{q-1} np_n \right\} \\ &= w \left\{ \sum_{n=0}^{q-1} 2(n - q)p_n + \sum_{n=0}^{q-1} p_n \right\} \leq 0. \end{aligned}$$

Since g is decreasing in t for any given q , g is minimized when t is at infinity. The function $g(q, \infty)$ takes the form

$$g(q, \infty) = K - wq^2. \quad (5.34)$$

Note that $q^* = \sqrt{K/w} - 1$ is the unique solution to

$$w(2q + 1) = g(q, \infty).$$

At this point, we claim that (q^*, ∞) minimizes $G(q, t)$. The proof proceeds with a contradiction argument:

Suppose there exists a stationary point (q°, t°) where q° is finite and $G(q^\circ, t^\circ) < G(q^*, \infty)$. By Lemma 5, (q°, t°) should satisfy Equality (5.31). Thus, q° should be less than q^* . Then by the implication of Equation (5.34)

$$g(q^\circ, \infty) > g(q^*, \infty), \quad (5.35)$$

because Equation (5.34) shows that $g(q, \infty)$ is decreasing in q for $q > 0$. On the other hand,

$$g(q^\circ, t^\circ) \geq g(q^\circ, \infty), \quad (5.36)$$

because g is decreasing in t . Combining the Inequalities (5.35) and (5.36), we obtain

$$G(q^\circ, t^\circ) = g(q^\circ, t^\circ) \geq g(q^\circ, \infty) > g(q^*, \infty) = G(q^*, \infty).$$

This contradiction shows that there is not a finite (q°, t°) that leads to a lower expected cost than $G(q^*, \infty)$.

Now, let us investigate if there is any (q°, t°) where $q^\circ = \infty$ and $G(q^\circ, t^\circ) < G(q^*, \infty)$. When $q^\circ = \infty$, the consolidation policy reduces to a *time-based policy*. Then, the best value of t° is t^* - the optimal policy parameter of the *time-based policy* - given by Equation (5.27). However, in Remark 3, we showed that $G(q^*, \infty) < G(\infty, t^*)$ when $K > 2h$.

Having shown that no (q°, t°) pair could lead to a better expected cost than $G(q^*, \infty)$, we conclude that (q^*, ∞) is the unique minimizer of $G(q, t)$. ■

V.2.2. Service vs Cost: A Trade-off Analysis

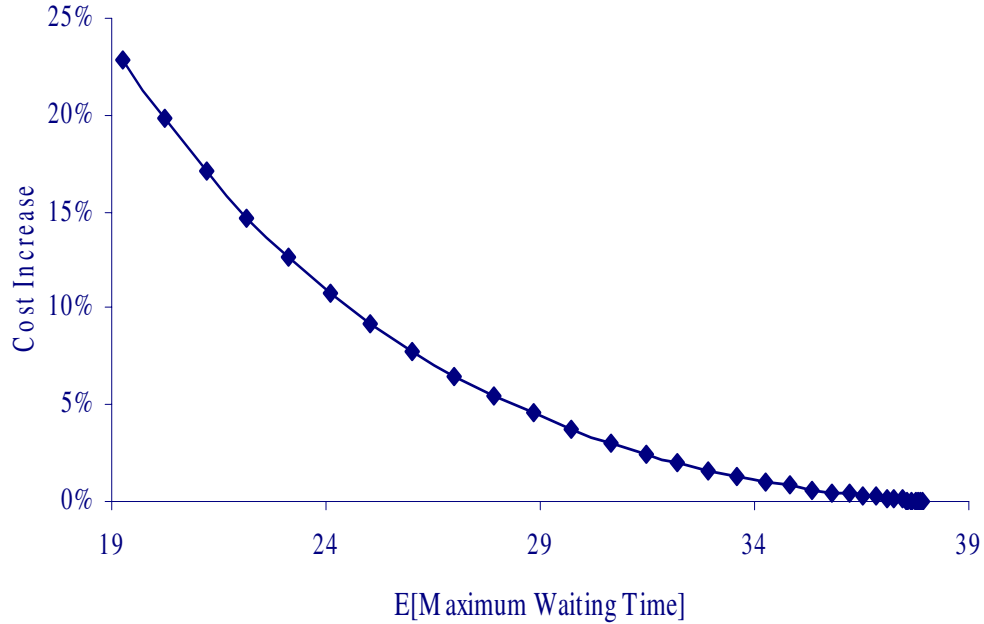
Proposition 6 can be interpreted as showing the cost-wise superiority of the *quantity-based policy* compared to both the *time-based policy* and the *hybrid policy*. However, the proposition refers to an unconstrained problem; there is no upper bound on the maximum waiting time. Because customers may object to a lead time beyond \bar{T} for receipt of their orders, the hybrid policy is attractive for practical applications. Recall that Lemma 5 provides the best q value for any given T . We can derive $q(\bar{T})$, corresponding to a *hybrid policy*, with time parameter \bar{T} .

The service measure, “maximum waiting time,” demonstrates the responsiveness of the supplier to its customers. Intuitively, any improvement in service performance results in additional cost. We now numerically analyze the trade-off between maximum waiting time and expected annual cost.

For a given problem instance, i.e., given K and w values (note that we can always assume a normalized demand for which $\lambda = 1$), we first derive q^* and the corresponding expected annual cost, $C(q^*, \infty)$. Next, we calculate $q(T)$ for different values of T , i.e., the maximum tolerable waiting time, and derive the percentage increase in the cost. In fact, T and $q(T)$ also imply an expected maximum waiting time which can be expressed as $E[\min(S_{q+1} - S_1, T)]$. Figure 5 shows the percentage increase in the expected cost with respect to the decrease in expected maximum waiting time. In this example, the model parameters are $K = \$616$, and $w = \$0.41$.

An interesting observation in Figure 5 is that even when the expected maximum waiting time changes by almost 50%, the average cost only increases by around 23%. One can argue that this is due to the fact that $K \gg w$. However, even when K/w is not this high, the increase in the cost is still limited. Figure 6 shows such an example

Figure 5 Percentage Cost Increase with Respect to Expected Maximum Waiting Time for High K/w Ratio

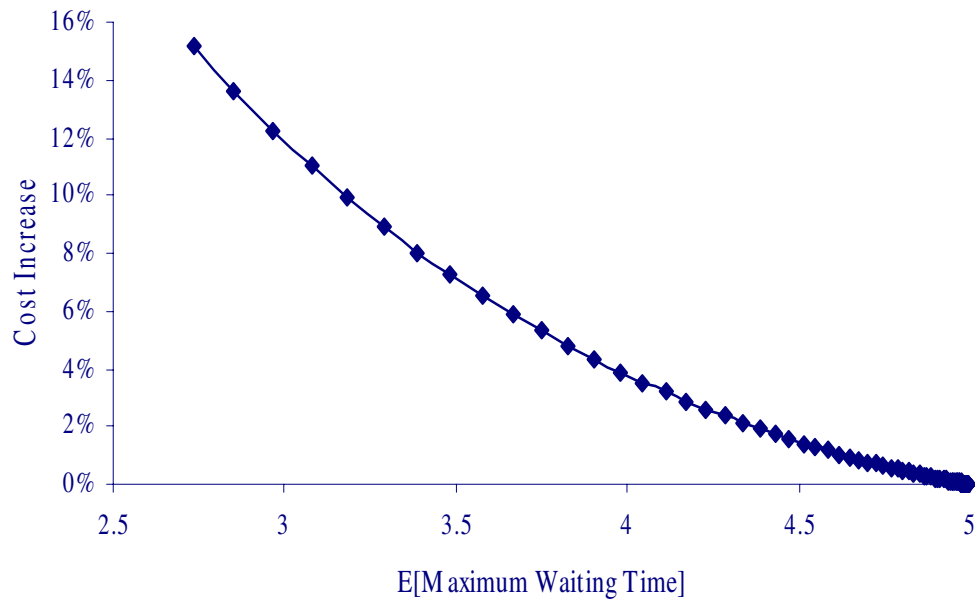


where $K = \$48$, and $w = \$1.49$.

Our numerical study over 650 problems⁴ indicates that the increase in cost is always less than 25% when the expected maximum waiting time decreases by half. This relative insensitivity of expected cost to maximum waiting time is very similar to the cost sensitivity observed in the EOQ-Model, and in the present case it shows the benefits and near optimality of a hybrid policy.

⁴We have generated the problems through a factorial design of the model parameters K and w . K varies from 40 to 4000 with a factor of 1.2, and w varies from 0.2 to 40 with a factor of 1.2.

Figure 6 Percentage Cost Increase with Respect to Expected Maximum Waiting Time for Low K/w Ratio



V.3. Summary

In this chapter, we introduced several hybrid consolidation policies and demonstrated that *hybrid policies* can not only achieve cost savings over time policies but are also superior to *quantity-based policies* in terms of customer service. We also studied an analytically pure consolidation model with a *hybrid policy*, and showed that the optimal *hybrid policy* is, in fact, a *quantity-based policy*. This coincides with the earlier numerical results which clearly imply that a *quantity-based policy* leads to the lowest expected cost.

CHAPTER VI

COORDINATION IN TRANSPORTER-BUYER CHANNELS

In the previous three chapters, we have studied the impact of transportation costs on operational decisions. Starting with this chapter, we expand our scope and begin analyzing the impact of transporters on supply channels.

The major focus of the channel coordination literature has been on supplier-buyer interactions. Although recent studies (See Toptal 2003) take into account transportation related costs in channel coordination decisions, there is not any work in the literature that treats the transporter as a member of the channel. However, increased transportation costs and new innovative practices such as 3PL and VMI necessitate a broader look at supply channels by integrating transportation costs and transporters into these channels. In this chapter, we take the first steps in this direction by introducing transporter-buyer channels and showing the substantial savings that can be achieved through coordination of these parties.

The organization of this chapter is as follows: In Section VI.1, we explain the problem setting and introduce the notation. We study transporter-buyer channels for two different models: (i) A single period model is studied in Section VI.2. (ii) An infinite planning horizon model is presented in Section VI.3.

VI.1. Problem Setting and Operational Characteristics

We consider a buyer who orders a certain type of item from an external source and a transporter who ships these orders to the buyer's site. The buyer needs to decide about his order size, order frequency, and retail price. On the other hand, the transporter's decision variable is the freight rate that he charges to the buyer.

The buyer observes a deterministic price dependent customer demand. Eco-

conomic theory suggests an inverse relation between the retail price and the demand of an item. Demand can be represented as a decreasing function of price. Two of the most commonly used price-demand functions are *linear demand* and *iso-elastic demand*. For *linear demand*, the demand linearly decreases with the retail price, and the function is given by $D(p) = a - bp$. For *iso-elastic demand*, the elasticity of the demand is constant, and the function is given by $D(p) = ap^{-b}$. In our analysis, we consider these two type of functions.

For certain industries and product types transportation costs are relatively high. For example, in the food industry, transportation costs account for around 16% the sales dollar of the products on the average. This value is more than 13% for chemicals and rubber products, and 11% for wood products and furniture. (Ballou 1992, pp.15) For such items, transportation costs play a significant role in the retail price. On the other hand, in the marketing and operations management literature, the retail prices of items are usually modelled as a function of the wholesale prices. Retailers often apply a price multiplier on the wholesale price and a markup price to cover the operating costs. In this dissertation, we assume that the transportation price is a per unit price, and we model the retail price as a function of both the wholesale price and the transportation price. Hence, the demand is indirectly affected by the per unit transportation rate. By considering price dependent demand, we intend to analyze the impact of transportation costs on pricing and ordering decisions.

The operational characteristics of the parties are explained below.

VI.1.1. Operational Characteristics of the Buyer

For a particular type of item, the buyer observes a deterministic price dependent demand. He orders from an outside source. Loss of sales or backorders are not allowed, which means the buyer has to order enough to satisfy all the demand. Every

time the buyer places an order, he incurs a fixed cost, denoted by A_B , possibly representing the the sum of administrative, receiving, and inspection costs. Each unit of item costs the buyer ν dollars. This cost includes the wholesale price, and handling charges. In addition to this, there is also a per unit shipment price, p_T , that the buyer pays to the transporter. The buyer also incurs a per unit, per unit-time inventory holding cost denoted by h_B .

The buyer has two decision variables: order size, q , and retail price p .

For the single period problem, the fixed ordering cost, A_B , is irrelevant to the buyer's decision variables, because A_B is independent of the order size. Thus, for this problem, we exclude A_B from the analytical model. Since the planning horizon is one period, the per unit per time inventory carrying cost, h_B , is also irrelevant for this problem, and is also excluded from the model.

VI.1.2. Operational Characteristics of Transporter

We consider a transporter operating a fleet of vehicles, say trucks. The size of the fleet. i.e., the number of trucks in the fleet, is sufficiently large, and each truck has a finite capacity of P . Hence, we say that the fleet size for each dispatch has installable capacity in increments of P . Naturally, there is a fixed cost, denoted by R_T , associated with using each truck of size P . This cost includes the driver's hourly wages, fueling costs associated with the shipment from the supplier's location to the buyer's location, etc. Additionally, the transporter also incurs a fixed cost, denoted by A_T , associated with each dispatch regardless of the fleet size used or the dispatch quantity. This cost may include fixed costs of administrative paperwork and bundling. We note that in the rest of the paper, the transporter's fixed cost A_T is assumed to be zero for the sake of simplicity. However, our analysis can easily be extended to consider the case where $A_T > 0$. Finally, the transporter may incur a cost per unit

item transferred, c_T .

VI.2. Single Period Problem in a Single-Buyer, Single-Transporter Setting

In this section, we consider the single period transporter-buyer channel coordination problem.

In order to simplify the notation, we define

$$w := p_T + \nu,$$

$$c := \nu + c_T.$$

The buyer's profit function is

$$\Pi_B(p) = (a - bp)(p - w).$$

On the other hand, the transporter's profit function is

$$\Pi_T(\tilde{p}_T) = (a - bp)(p_T - c_T) - \left\lceil \frac{(a - bp)}{P} \right\rceil R_T.$$

We can also express the profits as functions of demand. Observe that in the single period problem, the demand is also the order quantity:

$$\begin{aligned} \Pi_B(D) &= D \left(\frac{a - D}{b} - \omega \right), \\ \Pi_T(D) &= D(p_T - c_T) - \left\lceil \frac{D}{P} \right\rceil R_T. \end{aligned}$$

VI.2.1. Centralized Problem

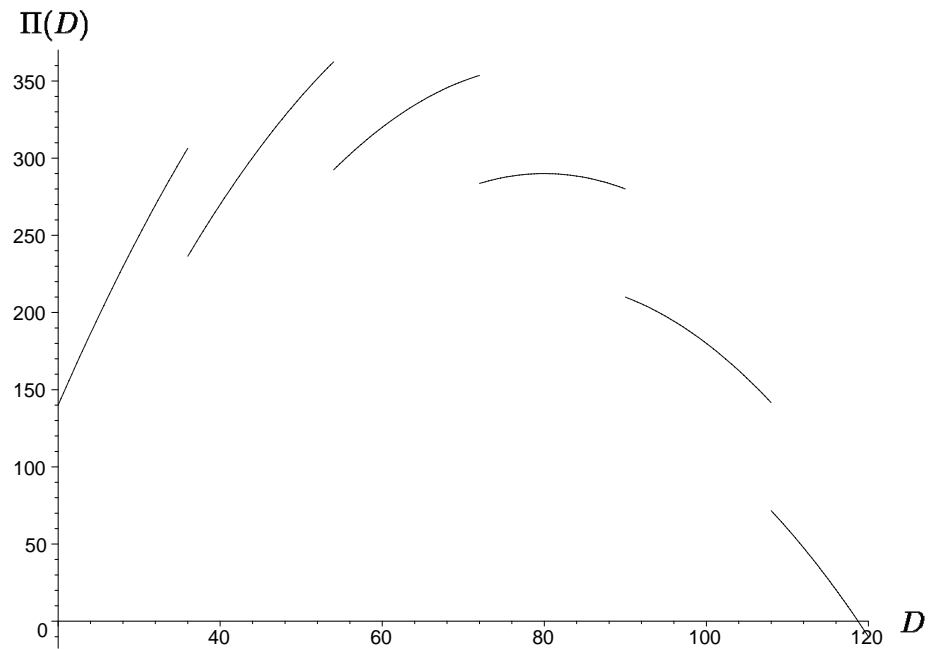
We first consider a centralized model where both parties are branches of the same company, and the objective is to maximize the total system profit. The system profit

is

$$\Pi_C(D) = D \left(\frac{a-D}{b} - c \right) - \left\lceil \frac{D}{P} \right\rceil R_T.$$

$\Pi_C(D)$ consists of two parts: the first part which we call $\phi(D)$ is a concave function of D , and the second part, namely $\varphi(D)$, is a step function which causes discontinuities. We can write $\Pi_C(D)$ as $\Pi_C(D) = \phi(D) - \varphi(D)$. Figure 7 shows the behavior of $\Pi_C(D)$.

Figure 7 Centralized Profit Function - $\Pi_C(D)$



Recall that, $\phi(D)$ is concave in D . and its maximizer is

$$D^\circ := \frac{a - bc}{2}.$$

The corresponding retail price is

$$p^\circ := \frac{a + bc}{2b}.$$

Before going further in the analysis, we also define:

$$p^k := \frac{a - kP}{b}.$$

In the above expression, p^k corresponds to the retail price that sets the total demand, D^k to kP , an integer multiple of the truck load.

When D decreases from D^k to D^{k-1} , $\varphi(D)$ decreases by R_T . Thus, we can observe jumps with a magnitude of R_T in $\Pi_C D$ at D^k values. This observation has several implications. The first immediate implication is presented in the following lemma:

LEMMA 6 *The maximizer of $\Pi_C(D)$ is less than, or equal to, D° .*

PROOF: Let k° be the number of trucks needed to ship D° units. Consider $D^{k^\circ-1}$. If we slightly increase the demand, Π_C will decrease by R_T units. As we continue to increase D , the function will increase and reach its local maximum at D° . However, the total increase can be less than R_T . Thus, it is possible to have $\Pi_C(D^{k^\circ-1}) > \Pi_C(D^\circ)$. In other words, one can find a $D < D^\circ$ which leads to a higher function value.

On the other hand, $\Pi_C(D)$ is decreasing for $D > D^\circ$ including the discontinuity points. This completes the proof. ■

In the following proposition, we quantify the net change in $\Pi_T(p)$ when D increases from D^{k-1} to D^k , and characterize the system-wide optimal solution:

PROPOSITION 7 *The centralized solution is given by the following:*

$$D^C = \begin{cases} D^\circ & \text{if } k_C^* > k^\circ, \\ \arg \max\{\Pi_C(D^\circ), \Pi_C(k_C^*P)\} & \text{otherwise,} \end{cases}$$

where,

$$k_C^* = \max_{k \in \mathbb{Z}^+} \left\{ k : k < \frac{a - bc}{2P} + \frac{1}{2} - \frac{bR_T}{2P^2} \right\}.$$

PROOF: As mentioned in Lemma 6, when p changes from D^{k-1} to D^k , the amount of decrease in $\varphi(D)$ is R_T . Mathematically,

$$\Delta_\varphi(k) := \varphi(D^k) - \varphi(D^{k-1}) = R_T.$$

On the other, we can quantify the change in $\phi(D)$ as follows:

$$\Delta_\phi(k) := \phi(D^k) - \phi(D^{k-1}) = \frac{aP + P^2}{b} - \frac{2kP^2}{b} - Pc.$$

In sum, the change in $\Pi_C(D)$ is

$$\begin{aligned} \Delta_\Pi(k) &= \Delta_\phi(k) - \Delta_\varphi(k) \\ &= \frac{aP + P^2}{b} - \frac{2kP^2}{b} - Pc - R_T. \end{aligned}$$

Note that $\Delta_\Pi(k)$ is linearly decreasing in k . One can check that $\Delta_\Pi(k)$ is negative for $k > k_C^*$; there is a maximum k value for which $\Delta_\Pi(k) < 0$. If $0 < k_C^* < k^\circ$, $D^{k_C^*}$ is a candidate to maximize Π_C . This completes the proof. ■

VI.2.2. Decentralized Problems

The previous section solves the centralized problem which represents full coordination between the transporter and the buyer. The solution to the centralized problem is the benchmark for system-wide profit. In this section, we investigate the cases where there is no coordination among the parties. In such cases, each party makes his/her own

decisions by considering the other party's possible decisions. We model decentralized cases as Stackelberg games in which one of the parties is making the first move, i.e., leading the channel. In the subsequent analysis, we study transporter driven and buyer driven channels, respectively.

VI.2.2.1. Transporter Driven Non-Cooperative Decentralized Problem

In the transporter driven channel, the transporter moves first and declares the per unit transfer price p_T . Then, the buyer decides the retail price, p . We recall the buyer's profit function:

$$\Pi_B(D) = D \left(\frac{a - D}{b} p - w \right).$$

It is easy to check that $\Pi_B(p)$ is concave in D .

$$\Pi'_B(D) = \frac{a - bw}{b} - \frac{2D}{p}, \text{ and}$$

$$\Pi''_B(D) = \frac{-2}{p} < 0.$$

The buyer's profit is maximized at

$$D^B = \frac{a - bw}{2}. \tag{6.1}$$

Next, we substitute buyer's optimal solution into the transporter's profit function:

$$\Pi_T(p_T) = \frac{a - bw}{2} (p_T - c_T) - \left[\frac{a - bw}{2P} \right] R_T.$$

Since w and p_T differ by only a constant, i.e., $w = p_T + c$, and the relation between D and w is given by Equation (6.1), we can rewrite Π_T as a function of D :

$$\Pi_T(D) = D \left(\frac{a - 2D}{b} - c \right) - \left\lceil \frac{D}{P} \right\rceil R_T.$$

Observe that, $\Pi_T(D)$ has a similar structure to $\Pi_C(D)$. We provide the optimal solution to the transporter's problem in the following proposition.

PROPOSITION 8 *The optimal D in the transporter driven channel is*

$$D^T = \begin{cases} D_T^\circ & \text{if } k_T^* > k^\circ, \\ \arg \max\{\Pi_T(D_T^\circ), \Pi_T(k_T^*P)\} & \text{otherwise,} \end{cases} \quad (6.2)$$

where,

$$\begin{aligned} D_T^\circ &= \frac{a - bc}{4}, \\ k^\circ &= \left\lceil \frac{D_T^\circ}{P} \right\rceil, \\ k_T^* &= \max_{k \in \mathbb{Z}^+} \left\{ k : k < \frac{a - bc}{4P} + \frac{1}{2} - \frac{bR_T}{4P^2} \right\}. \end{aligned}$$

Transporter's optimal price is

$$p_T^* = \frac{a - 2D^T}{b} - \nu. \quad (6.3)$$

PROOF: The proof follows the same steps as of the proof of Proposition 7. ■

VI.2.2.2. Buyer Driven Non-Cooperative Decentralized Problem

In Section 4.3 we considered the decentralized model where the transporter moves first. In this section, we consider a similar model where the buyer moves first.

The buyer announces a non-negative price multiplier, $\hat{\alpha}$. In other words, he declares that the retail price, p , is going to be $\hat{\alpha}w$. For a given w , the total demand is $a - b\hat{\alpha}w$. For notational simplicity, we define $\alpha := b\hat{\alpha}$. On the other hand, since w and p_T differ by only a constant, we can write the transporter's problem as a function

of w . (Recall that $w = p_T + \nu$.) Thus, the transporter's profit function is

$$\Pi_T(w) = (a - \alpha w)(w - c) - \left\lceil \frac{a - \alpha w}{P} \right\rceil R_T.$$

If we want to write Π_T as a function of D , then it takes the following form:

$$\Pi_T(D) = D \left(\frac{a - D}{\alpha} - c \right) - \left\lceil \frac{D}{P} \right\rceil R_T.$$

Observe that $\Pi_T(D)$ has the same structure as $\Pi_C(D)$ where the only difference is that we have b in Π_C and w in Π_T . Hence, we mimic the optimal solution to the centralized model and present the solution to the transporter's problem:

PROPOSITION 9 *The optimal response of the transporter to a given α is given by the following:*

$$D_T^B = \begin{cases} D_B^\circ & \text{if } k_B^* > k^\circ, \\ \arg \max \{ \Pi_T(D_B^\circ), \Pi_T(k_B^* P) \} & \text{otherwise,} \end{cases}$$

where,

$$D_B^\circ = \frac{a - \alpha c}{2} \quad \text{and,}$$

$$k_B^* = \max_{k \in \mathbb{Z}^+} \left\{ k : k < \frac{a - \alpha c}{2P} + \frac{1}{2} - \frac{\alpha R_T}{2P^2} \right\}.$$

It is not possible to derive the buyer's optimal response, α , analytically. Hence we propose a numerical search over α .

VI.2.3. Channel Efficiency: Centralized vs. Decentralized Channels

We have finished modelling and deriving the optimal decision variables in centralized and decentralized buyer-transporter channels. Although it is not possible to derive theoretical results to quantify the efficiency of channel coordination, we can still compare the system profit of centralized channels to that of decentralized chan-

nels through a numerical study. Furthermore, a numerical study also can provide a comparison between transporter driven and buyer driven channels. For this purpose, we generated 8821 problem instances via a factorial design of the model parameters. The model parameters are chosen as follows:

1. *Demand Function*: Recall that the relation between demand and price is given by $D(p)a - bp$. In our numerical study
 - a varies from 300 to 1300 with increments of 200.
 - b varies from 6 to 26 with increments of 4.
2. *Wholesale Price*: ν varies from 3 to 12 with increments of 3.
3. *Truck Cost*: R varies from 30 to 270 with increments of size 30.
4. *Truck Capacity*: P varies from 40 to 110 with increments of size 10.
5. *Per Unit Transportation Cost*: c_T is set equal to \$2 in all of the problem instances.

From these problems, the ones that cannot generate positive profit are eliminated. Table 5 provides insight about the cost parameters of the 8600 problems that we solved. The average proportion of transportation costs to total costs is about 38.26%, which is close to the overall average of transportation costs to logistics costs as this ratio is stated to be approximately 36% (Ballou 1992, pp.15). Furthermore, the ratio of the transportation costs to sales dollars is 11.21 % on the average for our problem set. This ratio varies from 1% to 60%. By looking at these figures, we can claim that our numerical examples are both large enough to cover a broad range of possible situations and reasonable enough to derive insights.

Table 5 Ratio of Transportation Costs to Total Costs and Total Sales

	Avg. Ratio	Min. Ratio	Max. Ratio
Transportation Costs / Total Costs	38.26%	16.08 %	75.23%
Transportation Costs / Total Sales	11.23%	1.38%	60.58%

In Table 6, we compare the average savings that can be achieved through coordination in the two decentralized cases. One of the most important insights that we can obtain from comparing these savings results is that decentralized channels are very inefficient. Even the buyer driven channel, which is by far better than the transporter driven channel in terms of system profit, the average inefficiency is more than 12%. In other words, if the parties can align their incentives, the system profit can be increased by more than 12%.

Table 6 Centralized Channel Savings over Decentralized Channels

	Avg. Savings	Min. Saving	Max. Saving
Transporter Driven Channel	34.07%	0%	90.22%
Buyer Driven Channel	12.60%	0%	44.93%

Even though centralized solution yields the highest profit most of the time, there are a few cases where the buyer driven channel is as efficient as the centralized channel. Moreover, for certain cases, buyer driven channel can be better than the transporter driven channel for the transporter. In fact, in all of these problem instances, profit margins are very low, and the transporter driven channel is often not efficient enough to generate positive profit. However, if the buyer leads the channel, then both parties can derive profit.

VI.3. Infinite Horizon Problem with Deterministic Price Sensitive Demand

In this section, we extend the single period problem to the infinite horizon. The time is modelled on a continuous scale. In this model, we introduce some more cost parameters: A_B is the fixed cost that the buyer incurs when he submits an order, and h_B is the per unit per time inventory carrying cost.

One of the major differences in this model from the single period model is the demand. We assume that demand in this model occurs continuously at a constant rate, D . Recall that in the single period model, the retail price implies the demand, i.e., the order quantity. In this model, the retail price determines the annual demand rate. Once the price is set, the demand rate is known. In this model, the buyer submits an order every time he runs out of inventory. The buyer has a constant order size, q , and he orders every q/D years.

Even though we analyze supplier-transporter-buyer channels in the next chapter, the transporter-buyer model that we study here can be extended to the supplier-transporter-buyer channel under the following conditions:

- i. The supplier and buyer are controlled by a central decision maker,
- ii. the supplier's inventory replenishment quantity is an integer multiple of the buyer's order quantity, say n , and n is set exogenously.

If these conditions hold, one can redefine

$$A_B = A_B + A_S \text{ and, } h_B = h_B + (n - 1)h_S,$$

and apply the solution techniques that we develop in this section.

VI.3.1. Centralized Model

In this section, we derive the optimal solution to the centralized transporter-buyer channel. The problem is to simultaneously select the delivery quantity, q , and the sales price, p , so as to maximize the annual channel profit. The objective function is given by

$$\begin{aligned} \Pi^C(q, p) = & D(p)(p - \nu - c_T) \\ & - \left\{ \frac{A_B D(p)}{q} + \frac{h_B q}{2} + \frac{[q/P] D(p) R_T}{q} \right\}. \end{aligned} \quad (6.4)$$

Since demand is a function of the price, we can also express Π^C as a function of q , quantity, and D , demand. In fact, working with D is usually easier than working with p . That is why we prefer expressing Π^C as follows:

$$\Pi^C(q, D) = D(p(D) - c) - \left\{ \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{[q/P] D R_T}{q} \right\}. \quad (6.5)$$

We divide our analysis into two cases depending on the value of the order quantity, q^C :

1. The order quantity is an integer multiple of the truck load, i.e., $q^C = kP$.
2. The order quantity is not necessarily an integer multiple of the truck load, but the number of trucks used for each dispatch is known, i.e., $(k-1)P < q^C \leq kP$.

VI.3.1.1. Case 1 - $q = kP$:

The objective function has the following form:

$$\mathcal{P}_{Ik} : \max_D \Pi_{Ik}^C(D) = (p(D) - c)D - \left\{ \frac{A_B D}{kP} + \frac{h_B kP}{2} + \frac{D R_T}{P} \right\}.$$

We, next, write the first and second derivatives of $\Pi_{Ik}^C(D)$ with respect to D :

$$\Pi_{Ik}^{C'}(D) = p(D) + p'(D)D - \gamma_k,$$

$$\Pi_{Ik}^{C''}(D) = 2p'(D) + p''(D)D,$$

where

$$\gamma_k := c + \frac{A_B}{kP} + \frac{R_T}{P}.$$

Observe that, it is necessary (but not sufficient) to have $p(D) > \gamma_k$ for the system to generate a positive annual profit.

One can easily see that for linear demand $\Pi_{Ik}^C(D)$ is concave. However, the same is not true for iso-elastic demand. Hence, we study the two different demand functions separately:

(a) Linear Demand: We have already mentioned that for linear demand, i.e., $p(D) = (a - D)/b$, $\Pi_{Ik}^C(D)$ is concave in D , and the stationary point, which is also the maximizer, is given by

$$D_{Ik}^C := \frac{a - b\gamma_k}{2}.$$

$\Pi_{Ik}^C(D_{Ik}^C)$ is

$$\Pi_{Ik}^C(D_{Ik}^C) = \frac{1}{4b}(a^2 - b^2\gamma_k^2) - \frac{h_B k P}{2}.$$

(b) Iso-Elastic Demand: When demand is of form $D(p) = ap^{-b}$, the first and second derivative of the objective function are

$$\Pi_{Ik}^{C'}(D) = \hat{a}D^{-1/b}(1 - 1/b) - \gamma_k \text{ and,}$$

$$\Pi_{Ik}^{C''}(D) = \frac{\hat{a}}{b^2}(1 - b)D^{-\frac{b+1}{b}},$$

where $\hat{a} := a^{1/b}$. The characteristics of $\Pi_{Ik}^C(D)$ vary with respect to b :

(i) $b \leq 1$: In this case, the objective function is convex and decreasing in D . Hence,

it is optimal to set $D = \underline{D}$, where \underline{D} is the lowest possible value of D . Observe that the lowest possible value of D corresponds to the highest possible value of p . Hence, when $b \leq 1$, it is optimal for Π_{Ik}^C to set the price as high as possible.

(ii) $b > 1$: The objective function is concave in D , and it is maximized at

$$D_{Ik}^\circ = \left(\frac{\hat{a}(1 - 1/b)}{\gamma_k} \right)^b.$$

The objective function value at this point is

$$\Pi_{Ik}^C(D_{Ik}^\circ) = \frac{a}{b} \left(\frac{1 - 1/b}{\gamma_k} \right)^{b-1} - \frac{h_B k P}{2}.$$

VI.3.1.2. Case 2 - $(k - 1)P < q \leq kP$:

The objective function has the following form:

$$\begin{aligned} \mathcal{P}_{IIk} &: \max_{D,q} \Pi_{IIk}^C(D, q) = (p(D) - c)D - \left\{ \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{k D R_T}{q} \right\} \\ \text{s.t.} & \quad (k - 1)P < q \leq kP. \end{aligned}$$

LEMMA 7 *In order to solve \mathcal{P}_{IIk} , it suffices to solve*

$$\begin{aligned} \tilde{\mathcal{P}}_{IIk} &: \max_D \tilde{\Pi}_{IIk}^C = (p(D) - c)D - \sqrt{2(A_B + kR_T)Dh_B} \\ \text{s.t.} & \quad \underline{D}_k \leq D \leq \overline{D}_k, \end{aligned} \quad (6.6)$$

where

$$\begin{aligned} \underline{D}_k &= \frac{(k - 1)^2 P^2 h_B}{2(A_B + kR_T)} \text{ and,} \\ \overline{D}_k &= \frac{k^2 P^2 h_B}{2(A_B + kR_T)}. \end{aligned}$$

PROOF: Observe that q appears only in the second piece of $\Pi_{IIk}^C(D, q)$, and this piece is an EOQ-type function. Hence, for a given D , $\sqrt{2(A_B + kR_T)D/h_B}$ maximizes $\Pi_{IIk}^C(D, q)$. Substituting it for q gives $\tilde{\Pi}_{IIk}^C$ of Equation (6.6).

In \mathcal{P}_k^{II} , the lower bound for q is $(k-1)P$, and the upper bound is kP . By substituting $\sqrt{2(A_B + kR_T)D/h_B}$ with the lower and upper bounds, we obtain \underline{D}_k and \overline{D}_k , respectively. Observe that $\tilde{\mathcal{P}}_{IIk}$ imposes bounds on D but not on q , while $\tilde{\mathcal{P}}_{IIk}$ does vice versa. At this point, one can argue that a (D^a, q^a) pair exists where $D^a \notin (\underline{D}_k, \overline{D}_k]$, $q^a \neq \sqrt{2(A_B + kR_T)D^a/h_B}$, and (D^a, q^a) yields a higher objective function value than the optimal solution of (6.6): Suppose that such a point exists, and let $\lceil q^a/P \rceil$ be equal to k^a . It is trivial that the pair (D^a, k^aP) yields a better annual profit than (D^a, q^a) . However, the solution to the problem $\mathcal{P}_{k^a}^I$ is always superior to (D^a, k^aP) .

Note that, the feasible region in the original formulation is $((k-1)P, kP]$, which is a half open set. The corresponding feasible region with variable D should be $(\underline{D}_k, \overline{D}_k]$. On the other hand, it is always more convenient to work with closed sets in optimization problems. In this case, we can extend the feasible set to $[\underline{D}_k, \overline{D}_k]$ without loss of generality by including \underline{D}_k in the feasible set. Although the quantity that \underline{D}_k implies - $k-1$ trucks to be used (whereas all the other D values in the constraint set imply the quantities that require k trucks)- we can still assume that k trucks are used when $D = \underline{D}_k$. If \underline{D}_k is the optimal solution to \mathcal{P}_{IIk} , it is obvious that $\Pi_{IIk}^C(\underline{D}_k)$ underestimates the annual profit. However, this would not cause incorrect conclusions at the end, because $\underline{D}_k = \overline{D}_{k-1}$, and \mathcal{P}_{k-1}^{II} gives the correct annual profit value for \overline{D}_{k-1} . ■

Problem $\tilde{\mathcal{P}}_k^{II}$ is similar to the EOQ-problem with price dependent demand. This problem has been widely studied in the literature for several demand functions and under various assumptions. However, to the best of our knowledge, the existing literature is not complete enough to provide all the necessary solutions for the problems we consider. Hence, we next discuss the solution procedures of $\tilde{\mathcal{P}}_k^{II}$ given by Equation 6.6 for different demand functions:

In order to find the maximizer of $\tilde{\Pi}_{IIk}^C$, we first investigate its behavior, i.e., we compute the first two derivatives:

$$\tilde{\Pi}_{IIk}^{C'}(D) = p'(D)D + p(D) - c - \frac{\sqrt{2(A_B + kR_T)h_B}}{2\sqrt{D}},$$

$$\tilde{\Pi}_{IIk}^{C''}(D) = p''(D)D + 2p'(D) + \frac{\sqrt{2(A_B + kR_T)h_B}}{4\sqrt{D}^3}.$$

Without further information about $p(D)$ it is not possible to derive conclusions about the concavity, or convexity of $\tilde{\Pi}_{IIk}^C(D)$. For this reason, at this point, we continue our analysis by considering the demand functions that we introduced before. We proceed first with *linear demand* and then *iso-elastic demand*.

Linear Demand: For a given annual demand rate D , we can express price as

$$p = \frac{a - D}{b}.$$

When we substitute this into (6.6), the objective function becomes

$$\begin{aligned} \max \quad & \tilde{\Pi}_{IIk}^C(D) = D \left(\frac{a - D}{b} - c \right) - \sqrt{2(A_B + kR_T)Dh_B} \\ \text{s.to} \quad & D \in (\underline{D}_k, \overline{D}_k]. \end{aligned}$$

The feasible set of this problem is a half-closed set, and working with such constraint sets in an optimization problem can cause technical difficulties. (Consider the case where the optimal solution is at the left end point.) In order to overcome that problem, we extend the feasible set to $[\underline{D}_k, \overline{D}_k]$. We can always do this extension without loss of generality. Suppose that the optimal solution for $\tilde{\Pi}_{IIk}^C$ happens to be \underline{D}_k , and say the objective function value at this point is M . In fact, the actual annual profit function at this point is $M + R_T/\underline{D}_k$. However, this value was already observed in problem $\tilde{\Pi}_{II(k-1)}^C$.

Porteus (1984) solves a relaxation of this problem where the only constraint is the nonnegativity of the demand. In the following proposition, we utilize his results to obtain the solution of the above problem:

PROPOSITION 10 $\tilde{\Pi}_{IIk}^C(D)$ is maximized at

$$D_{IIk}^C = \begin{cases} \hat{D}_k, & \text{if } \hat{D}_k \in [\underline{D}_k, \overline{D}_k], \\ \arg \min\{\underline{D}_k, \overline{D}_k\}, & \text{otherwise,} \end{cases}$$

where,

$$\begin{aligned} \hat{D}_k &= \frac{4\tilde{D}}{3} \cos^2(\phi/3), \\ \phi &:= \arccos\left(\frac{-bK}{4(\tilde{D}/3)^{3/2}}\right), \text{ and} \\ \tilde{D} &= \frac{a-bc}{2}. \end{aligned}$$

An approximation of \hat{D}_k is given by

$$\hat{D}_k \cong \tilde{D} - \frac{2bK\tilde{D}}{4\tilde{D}^{3/2} - bK},$$

where $K = \sqrt{(A_B + kR_T)h_B/2}$.

PROOF: The first two derivatives of $\tilde{\Pi}_{IIk}^C(D)$ are as follows:

$$\begin{aligned} \tilde{\Pi}_{IIk}^{C'}(D) &= -\frac{2D}{b} + \left(\frac{a}{b} - c\right) - \frac{\sqrt{2(A_B + kR_T)h_B}}{2\sqrt{D}}, \\ \tilde{\Pi}_{IIk}^{C''}(D) &= -\frac{2}{b} + \frac{\sqrt{2(A_B + kR_T)h_B}}{4\sqrt{D}^3}. \end{aligned}$$

By looking at the second derivative, we can see that $\tilde{\Pi}_{IIk}^C(D)$ is convex on $[0, D_k^\circ]$ and concave on $[D_k^\circ, \infty)$, where $D_k^\circ = [b\sqrt{2(A_B + kR_T)h_B}/8]^{2/3}$. The next step is to find the stationary point of $\tilde{\Pi}_{IIk}^C(D)$, which is a candidate for the maximizer. We can

rewrite the first derivative as

$$\tilde{\Pi}_{IIk}^{C'}(D) = -\frac{2}{b\sqrt{D}} \left\{ \sqrt{D}^3 - \frac{a-bc}{2}\sqrt{D} + b\sqrt{(A_B + kR_T)h_B/8} \right\}.$$

The above is a cubic polynomial of \sqrt{D} . Porteus shows that this polynomial has 3 real roots, and one of them belongs to $[D_k^\circ, \infty)$. Hence the maximizer is either the larger root or one of the end points. \hat{D}_k is the larger of the positive roots of the cubic polynomial. This completes the first part of the proof.

The approximation of \hat{D}_k comes from the first order Taylor expansion of $\tilde{\Pi}_{IIk}^{C'}(D) = 0$ as stated in Porteus (1984). ■

Iso-Elastic Demand: Substituting

$$p(D) = \hat{a}D^{-1/b}$$

into (6.6), the objective function becomes

$$\begin{aligned} \max \quad & \tilde{\Pi}_{IIk}^C(D) = D(\hat{a}D^{-1/b} - c) - \sqrt{2(A_B + kR_T)Dh_B} \\ \text{s.to} \quad & D \in (\underline{D}_k, \overline{D}_k]. \end{aligned}$$

The first and second derivatives of the objective function take the following forms:

$$\tilde{\Pi}_{IIk'}^C(D) = -\frac{\hat{a}}{b}D^{-1/b} + \hat{a}D^{-1/b} - c - D^{-1/2}\sqrt{2(A_B + kR_T)h_B}/2,$$

$$\tilde{\Pi}_{IIk''}^C(D) = D^{-3/2}\sqrt{2(A_B + kR_T)h_B}/4 - D^{-(1+\frac{1}{b})}\hat{a}(b-1)b^{-2}.$$

Now we look at 4 different cases:

$b \in (0, 1]$:

PROPOSITION 11 *When $b \in (0, 1]$, $D_{IIk}^C = \underline{D}_k$.*

PROOF: $\tilde{\Pi}_{IIk'}^C(D) < 0$ which implies that $\Pi_{IIk}^C(D)$ is decreasing in D . Hence $\tilde{\Pi}_{IIk}^C(D)$ achieves its maximum at $D = \underline{D}_k$. ■

$b \in (1, 2)$:

Notice that the second derivative is increasing in D . It is equal to 0 at $D = D_k^\circ$ where

$$D_k^\circ = \left(\frac{\hat{a}(b-1)(1/b^2)}{\sqrt{2(A_B + kR_T)h_B/4}} \right)^{\frac{2b}{2-b}}.$$

The next proposition characterizes the optimal solution to the maximization problem:

PROPOSITION 12 *When $b \in (1, 2)$, the optimal demand rate that maximizes $\tilde{\Pi}_{IIk}^C$ is*

$$D_{IIk}^C = \begin{cases} \hat{D}_k, & \text{if } \hat{D}_k \in [\underline{D}_k, \overline{D}_k], \\ \arg \min\{\underline{D}_k, \overline{D}_k\}, & \text{otherwise,} \end{cases}$$

where \hat{D}_k is the unique root to $\Pi_{IIk'}^C(D) = 0$.

PROOF: As we mentioned above, $\tilde{\Pi}_{IIk}^C(D)$ is concave over $[0, D_k^\circ]$ and convex over $[D_k^\circ, \infty)$. Also, we can easily show that $\tilde{\Pi}_{IIk'}^C(0^+) > 0$ and that $\lim_{D \uparrow \infty} \Pi_{IIk'}^C(D) < 0$. All these imply that $\tilde{\Pi}_{IIk}^C(D)$ reaches its local maximum in the concave region and the local maximum is, in fact, the global maximum over $[0, \infty)$. ■

In fact, it is not easy to find D_k^* analytically. One should appeal to numerical methods to find its value.

$b = 2$:

This special case can be solved analytically and leads to closed form results. The following proposition presents the joint optimal solution for $b = 2$.

PROPOSITION 13 When $b = 2$, the maximizer of $\tilde{\Pi}_{IIk}^C$ is given by

$$D_{IIk}^C = \left(\frac{\sqrt{a} - \sqrt{2(A_B + kR_T)h_B}}{2c} \right)^2,$$

if $a > 2(A_B + kR_T)h_B$. Otherwise $D_k^C = \underline{D}_k$.

PROOF: By substituting 2 for b in the first and second derivatives of Π_{IIk}^C we obtain

$$\tilde{\Pi}_{IIk'}^C(D) = -c + D^{-1/2} \left[\frac{\sqrt{a}}{2} - \frac{\sqrt{2(A_B + kR_T)h_B}}{2} \right],$$

$$\tilde{\Pi}_{IIk''}^C(D) = -D^{-3/2} [\sqrt{a} - \sqrt{2(A_B + kR_T)h_B}] / 4.$$

If $a > 2(A_B + kR_T)h$, then the objective function is concave and the maximizer is the root of the first derivative, which is given by

$$\left(\frac{\sqrt{a} - \sqrt{2(A_B + kR_T)h_B}}{2c} \right)^2.$$

However, if the above inequality does not hold, then the objective function is convex and decreasing. Hence the maximizer is \underline{D} which implies that the price should be set to its highest possible value. ■

$(b > 2)$:

When $b > 2$, the objective function is convex over $[0, D_k^\circ)$ and concave afterwards, where

$$D_k^\circ = \left(\frac{\hat{a}(b-1)(1/b^2)}{\sqrt{2(A_B + kR_T)h_B}/4} \right)^{\frac{2b}{b-2}}.$$

Hence, similarly to the linear demand case, the maximizer is the larger root of $\Pi_{IIk'}^C = 0$.

VI.3.1.3. Solution of the Centralized Problem

So far, we have completed the solution of \mathcal{P}_k^I and $\tilde{\mathcal{P}}_k^{II}$. In order to solve the centralized problem, we must solve the two sets of sub-problems for a range of values of k . We would normally start the enumeration from $k = 1$. The question that arises at this point is what should the upper and lower bounds on k be. Next, we discuss this question.

We derive the bounds on k for \mathcal{P}_k^I separately for *linear* and *iso-elastic* demand functions.

In the *Linear Demand* case, for a given k , Π_{Ik}^C is maximized at $D_{Ik}^C = (a - b\gamma_k)/2$, and

$$\Pi_{Ik}^C(D_{Ik}^C) = \frac{1}{4}b(a^2 - b^2\gamma_k^2) - \frac{h_B k P}{2}.$$

Momentarily assume k is continuous, and consider the above expression as a function of k , say $\Pi_I(k)$. Its first derivative with respect to k is

$$\Pi'_I(k) = \frac{A_B b}{2P} \frac{1}{k^2} \left(\frac{A_B/P}{k} + c + R_T/P \right) - \frac{h_B P}{2}.$$

Observe that $\Pi'_I(k)$ decreases as k increases. Hence, if $\Pi'_I(1) > 0$, then there exists a k^* such that

$$k^* = \arg \max\{k | \Pi'_I(k) > 0\}.$$

It suffices to make a search over the values of $k \leq k^*$.

In the *Iso-Elastic Demand* case, following an analysis similar to the one we did for *Linear Demand*, we have

$$\begin{aligned} \Pi_{Ik}^C(D_{Ik}^C) &= \frac{a}{b} \left(\frac{1 - 1/b}{\gamma_k} \right)^{b-1} - \frac{h_B k P}{2} \text{ and,} \\ \Pi'_{Ik}(D_{Ik}^C) &= \frac{A_B a (b-1)^b}{P b^b} \frac{k^{b-2}}{[A_B/P + k(c + R_T/P)]^b} - \frac{h_B P}{2}. \end{aligned}$$

For $b \leq 1$, the first derivative is always negative. Hence, it suffices to solve \mathcal{P}_k^I for $k = 1$ and then compare it to $k = 0$.

For $b > 1$, one can observe that the first derivative is decreasing in k . Hence, the analysis is similar to the linear demand case. This concludes the derivation of bounds on k for \mathcal{P}_k^I .

Even though we can derive finer upper bounds on k for \mathcal{P}_k^{II} , for both linear and iso-elastic demand functions, it suffices to stop enumeration at the specific k value which satisfies $\underline{D}_k < \tilde{D} \leq \overline{D}_k$ where $\tilde{D} = D(c)$.

VI.3.2. Buyer Driven Decentralized Channel

As the centralized solution represents the fully coordinated channel and benchmarks the channel efficiency, we next look at the uncoordinated channel.

In the decentralized channel, the buyer's problem is to decide on the order quantity and the retail price, and the transporter's problem is to determine the per unit transportation price. We model this problem as a two stage Stackelberg Game. The sequence of the events is as follows:

1. The buyer announces his per unit profit margin, $1 - \hat{\alpha}$. In other words, he declares that $p = \hat{\alpha}(\nu + p_T) = \hat{\alpha}w$.
2. The transporter announces his per unit freight rate, p_T .

Such a Stackelberg setting is called a *buyer driven channel* in the supply chain literature. In our case, it is reasonable to assume a buyer driven channel because, typically, the shippers exert power on their carriers.

We divide our analysis into two, representing the two different demand functions: *Linear Demand*: If we express the demand rate in terms of the $\hat{\alpha}$, we obtain $D = a - b\hat{\alpha}w$. For notational simplicity, we introduce $\alpha := b\hat{\alpha}$. Then, the transportation

price can be written in terms of demand:

$$p_T = (a - D)/\alpha - \nu.$$

We first characterize the transporter's response to a quoted α .

$$\begin{aligned} \Pi_T(D) &= D\left(\frac{a - D}{\alpha} - \nu - c_T\right) - \frac{\left\lceil \frac{\sqrt{2A_B D}}{P} \right\rceil R_T D}{\sqrt{\frac{2A_B D}{h}}} \\ &= D\left(\frac{a - D}{\alpha} - c\right) - \frac{\left\lceil \frac{\sqrt{2A_B D}}{P} \right\rceil R_T D}{\sqrt{\frac{2A_B D}{h}}}. \end{aligned}$$

Suppose that the number of trucks used for each dispatch is k , i.e.,

$$(k - 1)P < \sqrt{\frac{2A_B D}{h}} \leq kP.$$

For this to be true, the following inequality should hold:

$$\frac{h\ell^k}{2A_B} < D \leq \frac{hu^k}{2A_B},$$

where $\ell^k := [(k - 1)P]^2$ and $u^k := [kP]^2$. For notational simplicity, we define \underline{D}^k as the left-hand-side of the above inequality and \overline{D}^k as the right-hand-side. When $D \in (\underline{D}^k, \overline{D}^k]$, the transporter's annual profit function is

$$\Pi_T^k(D) = D\left(\frac{a - D}{\alpha} - c\right) - \frac{kR_T\sqrt{D}}{\sqrt{2A_B/h}}.$$

The next two lemmas provide helpful insights about the behavior of $\Pi_T^k(D)$.

LEMMA 8 $\Pi_T^k(D)$ is convex for $k \leq k_1$, where

$$k_1 = \arg \max \left\{ k : k^2 \leq \frac{\alpha R_T A_B}{4P^3 h} = \Phi \right\}.$$

PROOF: When $D \in (\underline{D}^k, \overline{D}^k]$, the second derivative of $\Pi_T(D)$ is

$$\Pi_T^{k''}(D) = \frac{-2}{\alpha} + \frac{kR_T}{4\sqrt{2A_B/h}} \frac{1}{D^{3/2}}.$$

On this interval, $\Pi_T^{k''}(D)$ is decreasing in D . On the other hand, the function is not continuous at $D = \underline{D}^k$. However, we can show that

$$\lim_{D \uparrow \overline{D}^k} \Pi_T^{k''}(D) = \frac{-2}{\alpha} + \frac{kR_T}{4\sqrt{2A_B/h}} \frac{1}{\overline{D}^{k3/2}} < \frac{-2}{\alpha} + \frac{(k+1)R_T}{4\sqrt{2A_B/h}} \frac{1}{\overline{D}^{k3/2}} = \lim_{D \downarrow \overline{D}^k} \Pi_T^{k''}(D).$$

Also, note that

$$\lim_{D \uparrow \overline{D}^k} \Pi_T^{k''}(D) = \frac{-2}{\alpha} + \frac{kR_T}{4\sqrt{2A_B/h}} \frac{1}{\overline{D}^{k3/2}} = \frac{-2}{\alpha} + \frac{R_T}{4P\overline{D}^k}.$$

This indicates that $\lim_{D \uparrow \overline{D}^k} \Pi_T^{k''}(D)$ is decreasing in k . Hence, if $\lim_{D \uparrow \overline{D}^{k^*}} \Pi_T^{k''}(D) > 0$, then, $\lim_{D \uparrow \overline{D}^k} \Pi_T^{k''}(D) > 0$ for all $k < k^*$.

In order to derive the expression for k_1 , one needs to check

$$\frac{-2}{\alpha} + \frac{R_T}{4P\overline{D}^k} < 0$$

by substituting

$$\overline{D}^k = \frac{h(kP)^2}{2A_B}.$$

The proof is complete. ■

LEMMA 9 $\Pi_T^k(D)$ is piecewise concave for $k \geq k_2$ where

$$k_2 = \arg \min \left\{ k : \frac{(k-1)^3}{k} \geq \frac{\alpha A_B R_T}{4P^3 h} = \Phi \right\}.$$

PROOF: On $(\underline{D}^k, \overline{D}^k]$, $\sup \Pi_T^{k''}(D) = \lim_{D \downarrow \underline{D}^k} \Pi_T^{k''}(D)$, and

$$\lim_{D \downarrow \underline{D}^k} \Pi_T^{k''}(D) = \frac{-2}{\alpha} + \frac{R_T k}{4\sqrt{2A_B/h} \underline{D}^{k3/2}}.$$

We substitute

$$\underline{D}^k = \frac{(k-1)^2 P^2 h}{2A_B}$$

into the above equation and obtain

$$\lim_{D \downarrow \underline{D}^k} \Pi_T^{k''}(D) = \frac{-2}{\alpha} + \frac{A_B R_T k}{2P^3 h (k-1)^3}.$$

Notice that this expression is decreasing in k . Hence, if it is negative for a value of k , say k^* , then it should be negative for all $k > k^*$. As it is the supremum of the second derivative over the interval $(\underline{D}^k, \overline{D}^k]$, the second derivative is always negative. This completes the proof. ■

LEMMA 10 $\Pi_T^k(D)$ is decreasing for $k \geq k_3$ where

$$k_3 = \arg \min \left\{ k : k^2 > \frac{A_B [2P(a - c\alpha) - R_T \alpha]}{2hP^3} = \Phi_3 \right\}.$$

PROOF: We first note that the function is decreasing at the discontinuity points. In order to complete the proof, we need to show that the function is decreasing over $(\underline{D}^k, \overline{D}^k]$ for all $k > k_3$. First, we show that if $\lim_{D \uparrow \overline{D}^k} \Pi_T^{k'}(D) < 0$, then the same assertion holds for $k+1$. $\lim_{D \uparrow \overline{D}^k} \Pi_T^{k'}(D)$ is given by

$$\lim_{D \uparrow \overline{D}^k} \Pi_T^{k'}(D) = \left(\frac{a}{\alpha} - c \right) - \frac{2\overline{D}^k}{\alpha} - \frac{R_T}{2P}.$$

By looking at this expression, one can trivially observe that it is decreasing in k .

Furthermore,

$$\lim_{D \downarrow \overline{D}^k} \Pi_T^{k'}(D) = \left(\frac{a}{\alpha} - c \right) - \frac{2\overline{D}^k}{\alpha} - \frac{R_T \sqrt{D^{k+1}}}{2P \sqrt{D^k}} < 0,$$

and

$$\Pi_T^{k'}(D) = \left(\frac{a}{\alpha} - c \right) - \frac{2}{\alpha} - \frac{R_T \sqrt{D^{k+1}}}{2P \sqrt{D}} < 0, \quad \forall D \in (\underline{D}^{k+1}, \overline{D}^{k+1})$$

for every k such that $\lim_{D \uparrow \overline{D}^k} \Pi_T^{k'}(D) < 0$. In other words, $\Pi_T^k(D)$ is decreasing on

the interval $(\underline{D}^k, \overline{D}^k]$ if $\lim_{D \uparrow \overline{D}^k} \Pi_T^{k'}(D) < 0$.

Hence, we have shown that $\Pi_T^k(D)$ is decreasing for $[\overline{D}^k, \infty)$ for k such that $\lim_{D \uparrow \overline{D}^k} \Pi_T^{k'}(D) < 0$. One can easily check that k_3 is the minimum of such k 's. ■

The next proposition identifies the solution to the transporter's optimal response:

PROPOSITION 14 *One can derive the transporter's optimal response to a given α as follows:*

- For $k = \{1, \dots, k_3 - 1\}$:
 - if $k \leq k_1$, evaluate $\Pi_T^k(D)$ at \overline{D}^k .
 - if $k_1 < k \leq k_2$, find the stationary points of $\Pi_T^k(D)$. If the larger of the stationary points lies on $(\underline{D}^k, \overline{D}^k]$ select that. Otherwise select \overline{D}^k .
 - if $k > k_2$, find the unique stationary point of $\Pi_T^k(D)$. If the stationary point is in $(\underline{D}^k, \overline{D}^k]$, select the stationary point. Otherwise select \overline{D}^k .

Out of the possible candidates, pick the one that leads to the highest profit.

Proposition 14 suggests finding a stationary point of $\Pi_T(D)$ on the interval $(\underline{D}^k, \overline{D}^k]$ for which $\Pi_T^k(D)$ is concave. Note that the first derivative of $\Pi_T^k(D)$ on such an interval can be rewritten as a function of \sqrt{D} :

$$\Pi_T^k(D) = \frac{1}{\sqrt{D}} \left\{ (a/\alpha - c)\sqrt{D} - 2\sqrt{D}^3 - \frac{kR_T}{2\sqrt{2A_B/h}} \right\}.$$

Note that, $\Pi_T^{k'}(D)$ is a cubic polynomial of \sqrt{D} and that finding the roots of this polynomial is similar to the task that we discussed in the previous section. Using the results of that section, an approximation of the local maximizer of Π_T^k is given by:

$$D_k^* = \check{D} - \left(\frac{2\alpha\rho_k\check{D}}{4\check{D}^{3/2} - \alpha\rho_k} \right),$$

where

$$\check{D} = \frac{a - \alpha c}{2} \text{ and,}$$

$$\rho_k = \sqrt{\frac{k^2 R_T^2 h}{8A_B}}.$$

Iso-Elastic Demand: When $D = ap^{-b}$, we can model D as a function of $\hat{\alpha}$ as follows:

$$D = ap^{-b} = a(\hat{\alpha}(p_T + \nu))^{-b} := \tilde{\alpha}(p_T + \nu)^{-b}.$$

Using the above expression, we can express p_T as

$$p_T = \tilde{\alpha}^{1/b} D^{-1/b} - \nu := \alpha D^{-1/b} - \nu.$$

For a given α , the transporter's response function takes the following form:

$$\Pi_T^k(D) = D(\alpha D^{-1/b} - c) - \frac{\left[\frac{\sqrt{\frac{2A_B D}{h}}}{P} \right] R_T D}{\sqrt{\frac{2A_B D}{h}}}.$$

Following an analysis similar to the one we did in the linear demand case, when the order quantity requires k trucks, the transporter's annual profit function is

$$\Pi_T^k(D) = D(\alpha D^{-1/b} - c) - \frac{k R_T \sqrt{D}}{\sqrt{2A_B/h}}.$$

In fact, this function demonstrates the same characteristics as Π_{IIk}^C .

VI.3.3. Numerical Results and Conclusions

In order to quantify the channel coordination benefits, we generate a set of problem instances by altering the model parameters. Following an approach similar to the one we used in generating single period problems, the model parameters are chosen as follows:

Demand Function: We use linear demand in the numerical examples. Recall that the relation between demand and price is given by $D(p)a - bp$. In our numerical study

- a varies from 300 to 1300 with increments of 250.
- b varies from 6 to 24 with increments of 6.

Wholesale Price: c varies from \$3 to \$12 with increments of \$3.

Buyer's Set-up Cost: A_B varies from \$150 to \$900 with increments of \$150.

Buyer's Inventory Carrying Rate: h_B varies from \$0.6 to \$3.0 with increments of \$0.8.

Truck Cost: R varies from \$30 to \$180 with increments of size \$30.

Truck Capacity: P varies from 40 to 100 with increments of size 15.

Per Unit Transportation Cost: c_T is set equal to \$2 in all of the problem instances.

After eliminating the ones that cannot generate positive profit, we solve a total of 28,850 problem instances. Table 7 gives an overview of the cost parameters. By looking at the table, we can see that the selection of model parameters is reasonable and that it reflects the industry averages. In fact, the ratio of transportation costs to annual sales dollars is lower than the quoted average.

Even though the ratio of transportation costs to annual sales volume is lower than the average, the amount of savings that can be obtained through coordination is promising. As indicated in Table 8, full coordination brings about an average of 12.75% savings. Compared to the savings that can be realized in the single period

Table 7 Ratio of Transportation Costs in Numerical Examples

The Ratio of Transportation Costs to	Avg.	Min.	Max
Total Costs	18.19%	9.16%	36.43%
Annual Sales Dollars	5.80%	1.71%	21.67%

models, coordination is even more efficient in long term business relationships. The maximum savings in these examples is 753.56%. This figure seems to be very high, but it corresponds to the case where the decentralized channel generates very low profit and the centralized channel justifies the business. We identified 238 problem instances in which the decentralized channel cannot generate positive profit but the centralized channel can.

Table 8 Centralized Channel Savings over Buyer-Driven Decentralized Channel

	Avg. Savings	Min. Saving	Max. Saving
Buyer Driven Channel	12.75%	0%	753.56%

CHAPTER VII

CHANNEL COORDINATION PROBLEMS UNDER CONSTANT AND
DETERMINISTIC DEMAND

In Chapter VI, we took the first steps to introduce transporters into channel coordination by considering a transporter-buyer channel. The theoretical and numerical results clearly show that channel performance improves with coordination. Motivated by this, we continue to study the impact of transporters and transportation costs on channel coordination via analyzing supplier-transporter-buyer channels.

Supplier-transporter-buyer channels encompass the supplier-buyer channels which are the primary focus of the channel coordination literature and can help us see the additional benefits that transporters can bring to the channel. Also, by expanding the scope of transporter-buyer channels, the study of supplier-transporter-buyer channels provides a more realistic perspective on the impact of transporters.

In this chapter, we study the transporter's impact on channel performance under deterministic and constant demand. The remainder of the chapter is organized as follows. We start by introducing the general operational characteristics of the parties in the channel and the problem setting. Next, we discuss the cost saving opportunities of the parties in the channel. We introduce two models that demonstrate the interactions between the supplier and buyer, and we study these models in detail before concluding with managerial insights.

Our assumptions regarding the operational characteristics of the supplier-buyer system under consideration are similar to those of the classical problem described in Goyal (1976) except for the fact that, eventually, we aim to analyze the transporter's impact on the costs associated with the link from supplier to buyer. In the following, we briefly describe the operational characteristics of the buyer and the supplier, and

then we elaborate on the operational characteristics of the transporter.

VII.1. Operational Characteristics and Problem Setting

We consider a supplier-buyer channel in which a supplier sells a certain product to a buyer. The buyer observes a deterministic and constant customer demand with an annual rate of D . The transporter is responsible for delivery of the items from the supplier's site to the buyer's site.

The operational characteristics of the transporter are the same as those in Chapter VI. Next, we explain the operational characteristics of the supplier and the buyer.

VII.1.1. Operational Characteristics of Supplier and Buyer

For a particular type of item, the buyer observes a deterministic constant demand with an annual rate of D . He periodically replenishes his inventory from the supplier. The supplier also replenishes her inventory from an outside supplier or produces the items with an infinite production rate. Given the costs for both parties associated with replenishing (including setup/procurement and transportation costs) and holding the inventory, the problem is to compute the replenishment quantities for the supplier and the buyer, denoted by Q and q , respectively. It follows that the supplier's replenishment cycle length is Q/D and the buyer's replenishment cycle length is q/D . Since, no lost-sales or backorders are allowed, $Q = nq$ where n is a positive integer representing the number of buyer replenishments, i.e., dispatches, within one supplier replenishment cycle.

Considered in the supplier's costs is a fixed term denoted by A_S , possibly representing the setup cost associated with instantaneously producing or purchasing Q units, as well as a per unit, per unit-time inventory holding cost denoted by h_S . Sim-

ilarly, considered in the buyer's inventory replenishment cost is a fixed term, denoted by A_B , possibly representing the fixed costs associated with ordering/receiving q units, such as the sum of administrative, receiving, and inspection costs. The buyer also incurs a per unit, per unit-time inventory holding cost denoted by h_B . Additionally, the supplier may incur a per unit production or purchase cost, c_S , whereas the buyer may incur a per unit purchase cost, c_B . Since the demand rate is constant, these costs do not have an impact on determining the values of Q and q . However, supplier-buyer coordination can be achieved by adjusting the value of c_B . In the classical channel coordination literature, it is argued that all-units or incremental quantity discounts can coordinate the system in several situations. In Appendix A we discuss the role of quantity discounts under several different transportation price structures.

VII.1.2. Problem Setting and Notation

For this problem, we consider two different models. In *Model I*, the buyer and the supplier are interested in designing an efficient channel coordination mechanism for improving their supply-chain efficiency. In *Model II*, there is virtually no coordination between the buyer and the supplier; however, the supplier tries to align the buyer's decision by arranging her own wholesale price structure.

VII.1.2.1. Interactions Between Supplier and Buyer

Model I assumes an integration between supplier and buyer. In other words, the supplier and buyer act as a single unit and they operate at their joint optimal solution. There are several cases where such an integration can occur. The model where both the supplier and the buyer are branches of one company and controlled by a central decision maker is one example as is the model where both parties agree on a contract that binds them both to operate at the system optimal. For our purposes, it does

not make a significant difference how coordination is achieved.

On the other hand, in *Model II*, the supplier and buyer make their decisions independently with each trying to maximize(minimize) his/her own profit(cost). In such a situation, it is the buyer who makes the final decision about the order quantity. The buyer's decision depends on the transportation price (if he incurs transportation charges) and the wholesale price. The supplier can coordinate the buyer by changing the wholesale price schedule. As shown in Appendix A, it is always optimal for the supplier to coordinate the buyer at their joint optimal level.

We can see that in either model the order quantity is going to be q_J . Then the obvious question arises: what is the difference between these two models? The answer lies in the stability of the supplier-buyer solution. More specifically, if the transporter wants to change this order quantity, in the first model the transporter has to negotiate a contract with the buyer and/or the supplier. However, in the second model the transporter can easily change the order quantity by adjusting the transportation price.

Next, we explain how the supplier and buyer decide about the order quantity in *Model I*. As mentioned earlier, there is not much difference between the way these two models work.

Although the supplier and buyer do not have information about the transporter's actual operating expenses, they can easily obtain information about possible pricing schedules, and their preliminary analysis regarding the dispatch service they request from the transporter is based on this information. For this purpose, given some initial transportation pricing information, they develop a coordinated solution specifying the value of the coordinated dispatch quantity denoted by q_J . During this process, they may encounter different transportation pricing structures including the following:

- p_T : a per unit price for each item transferred.

- A_p : a fixed price for each dispatch.
- M_p : a fixed price for each truck used.
- $\{p_T^1, q_1, p_T^2\}$: a per unit price with freight discounts.

In fact, any combination of the above pricing structures is a possibility depending on the transportation mode or the transporter's operational policy, etc. For example, if the transporter is an LTL, or overnight, carrier, he may prefer a per unit based pricing scheme. If the transporter dedicates some of his fleet to serving a specific customer, preferably by using FTL shipments, he/she may charge a fixed price for each truck and/or each dispatch.

We can prove that under the transportation pricing schedules listed above, the supplier-buyer system can be coordinated by developing win-win solutions for both of the parties¹. In fact, the coordinated dispatch quantities leading to win-win solutions, are equal to the corresponding q values of the centralized optimal solutions minimizing the system-wide costs of the supplier-buyer system, namely q_J . On the other hand, q_J may not be the best choice as far as the transporter's actual operating costs are concerned. For example, q_J may not be sufficiently large for the transporter to enjoy the benefits of scale economies or increased resource utilization. More specifically, the transporter may improve his/her cost efficiency and resource utilization, possibly, by trying to influence q_J via offering a contractual dispatch quantity, denoted by q_T , applicable under a new pricing schedule. However, he should convince both the buyer and the supplier to shift from q_J because any deviation from q_J leads to an increase in the (ideal) system-wide annual cost of the supplier-buyer system. Hence, the transporter should compensate for this increase in cost. The problem associated with developing a compensation mechanism for this purpose is called the transporter's

¹Formal proofs are presented in Appendix A.

coordination problem.

The interactions between the supplier and buyer in *Model II* is slightly different. These interactions also depend on which party is incurring the transportation charges. Below we summarize how these interactions are shaped in either case.

If the buyer incurs the transportation charges, he has preliminary information about the transportation price as in the first model. Based on this preliminary information and the initial wholesale price, he can choose his order quantity to minimize his annual average cost. However, the transporter also has the same preliminary information; therefore, he can arrange the wholesale price structure in such a way that the solution to the buyer's problem is q_J , the joint optimal solution of the supplier-buyer problem.

If the supplier incurs transportation charges, then buyer's problem is the same regardless of the structure of transportation price. This will allow the supplier to align the buyer at the system optimal by providing a quantity discount schedule. Details of the theory of quantity discounts are provided in the appendix.

VII.1.2.2. Interactions Between Transporter and the Party Who Incurs Transportation Charges

The order quantity agreed upon between supplier and buyer is not always the most desirable quantity for the transporter. Hence, he may want to induce the other parties to alter this quantity. This is explained in detail in Section VII.3.1.

After solving his problem, the transporter has to find effective mechanisms to convince the other parties to operate under his decision. The structure of such mechanisms is discussed in Sections VII.3.2 and VII.4.2.

Before proceeding with further details of the problem, let us summarize the notation used so far and introduce additional notation that will be used in our analysis.

Model Parameters

- D : Buyer's annual demand.
- A_S : Supplier's fixed replenishment cost.
- A_B : Buyer's fixed replenishment cost.
- h_S : Supplier's inventory holding cost per unit per year.
- h_B : Buyer's inventory holding cost per unit per year.
- Q : Supplier's replenishment quantity.
- q : Buyer's replenishment quantity, i.e., dispatch quantity.
- n : # of dispatches in a supplier's replenishment cycle.
- c : Supplier's per unit purchasing cost.
- p_S : per unit wholesale price paid by the buyer to the supplier.
- A_T : Transporter's fixed cost per shipment.
- R_T : Transporter's cost for launching one truck.
- P : Truck capacity.
- c_T : Transporter's per unit transfer cost.

Cost, Revenue, and Profit Functions

- $P_S(q)$: The initial total revenue of the supplier
 $\bar{P}_S(q)$: Total revenue of the supplier after coordinating/negotiating with the buyer.
 $P_T(q)$: Price that the transporter charges for each dispatch before coordination.
 $\bar{P}_T(q)$: Price that the transporter charges for each dispatch after coordination.
 $G_B(q)$: Buyer's average annual operating cost.
 $G_S(q, n)$: Supplier's average annual operating cost.
 $G_C(q, n)$: Average annual cost of the centralized system.
 $G_J(q, n)$: Joint average annual cost of the supplier-buyer system before transporter coordination.
 $\hat{G}_B(q)$: Buyer's average annual operating cost excluding the transportation costs.
 $\hat{G}_S(q, n)$: Supplier's average annual operating cost excluding the transportation costs.
 $\hat{G}_J(q, n)$: Joint average annual cost of the supplier-buyer system excluding the transportation costs.
 $\bar{G}_J(q, n)$: Joint average annual cost of the supplier-buyer system after transporter coordination.
 $G_{SB}(q, n)$: Sum of supplier's and buyer's average annual operating costs.
 $G_T(q)$: Transporter's average annual operating cost.

Decision Variables

- p_T : Per unit transfer price.
 A_p : Fixed price per each dispatch.
 M_p : Price charged for launching one truck.
 q_B : Buyer's optimal order quantity.
 (q_J, n_J) : Optimal solution to the joint supplier-buyer problem.
 (q_{SB}, n_{SB}) : Coordinated/negotiated solution for the supplier-buyer system, i.e., the outcome of channel coordination between the buyer and the supplier.
 (q_T, n_T) : Coordinated solution proposed by the transporter, i.e., the solution of the transporter's coordination problem or outcome of channel coordination between the three parties.

At this point, we make a distinction between G_J and G_{SB} . The first term implies that the supplier and the buyer are either controlled by a central decision-maker or are in strategic partnership, whereas the latter implies that the supplier and the buyer act independently, and they may or may not be coordinated under a contract.

VII.2. Analysis of Costs for the Parties

VII.2.1. Transporter's Costs and Cost Saving Opportunities

Under the assumptions of our model, given the dispatch quantity q and the annual volume D , the transporter's average annual cost can be expressed as

$$G_T(q) = \frac{[q/P]R_T D}{q} + \frac{A_T D}{q} + c_T D.$$

As mentioned earlier, the per unit transfer cost is irrelevant for our purposes, that is why from now on we will express $G_T(q)$ as

$$G_T(q) = \frac{\lceil q/P \rceil R_T D}{q} + \frac{A_T D}{q}.$$

Figure 8 illustrates the behavior of $G_T(q)$ for the case where $A_T = 0$. The first term represents the average annual truck cost, with a minimum value equal to $R_T D/q$, at $q = kP$ for all positive integer k values. Also, for each positive integer k , this term is a decreasing convex function of q over $(k-1)P < q \leq kP$. It follows that the transporter's average annual cost is minimized only when full trucks are dispatched, i.e., truck capacities are fully utilized.

VII.2.2. Supplier's and Buyer's Costs and Cost Saving Opportunities

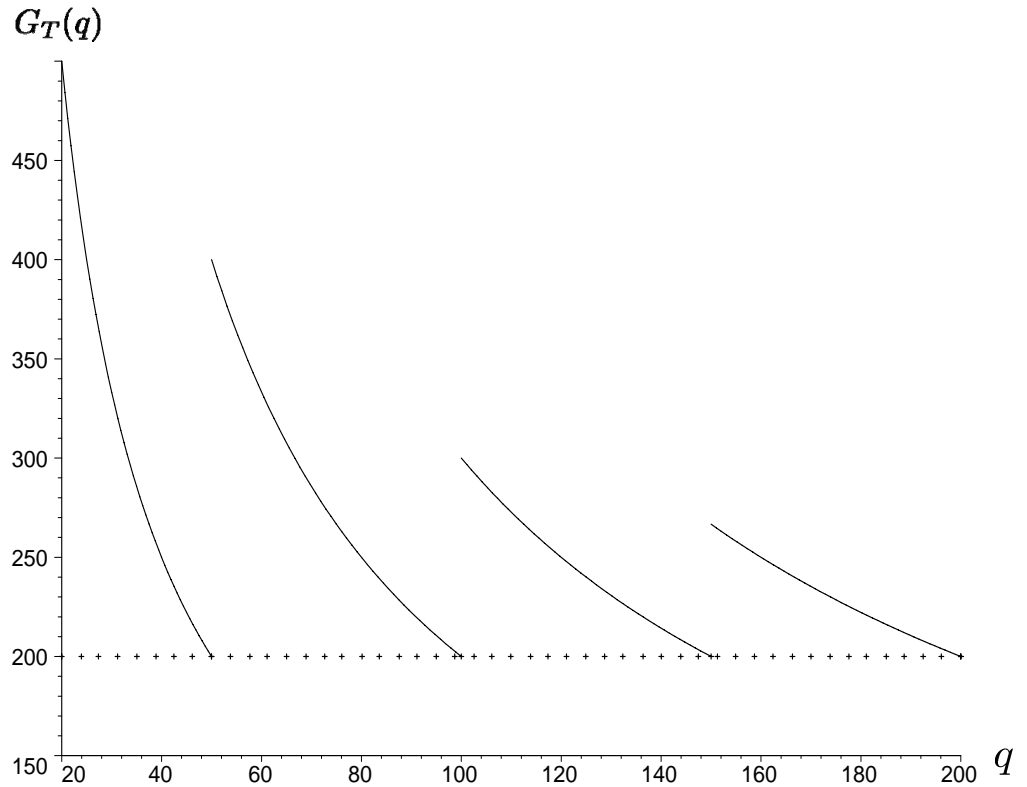
As we have already discussed in Section VII.1, according to the first model, given some initial transportation pricing information, the supplier and buyer develop a coordinated solution specifying the value of the coordinated dispatch quantity, q_J . For this purpose, they need to compute their centralized solution by solving the following problem:

$$\mathcal{P}_J : \min_{q \geq 0, n \in \mathbb{Z}^+} G_J(q, n) = \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{A_B D}{q} + \frac{h_B q}{2} + P_T(q). \quad (7.1)$$

The characteristics of \mathcal{P}_J depend on the form of $P_T(q)$:

- **Case 1** - $P_T(q) = p_T q$: If the initial pricing is based on only a per unit item charge then the transporter's average annual revenue is independent of q , i.e., $P_T(q) = p_T D$.

Figure 8 Transporter's Cost Structure



- **Case 2** - $P_T(q) = p_T q + A_p$: If the initial pricing is based on a fixed price for each dispatch as well as a per unit item charge $P_T(q) = p_T D + A_p D/q$.
- **Case 3** - $P_T(q) = p_T q + A_p + M_p \lceil q/P \rceil$: If the initial pricing also involves a fixed price for each truck used, then

$$P_T(q) = p_T D + \frac{A_p D}{q} + \left\lceil \frac{q}{P} \right\rceil \frac{M_p D}{q}.$$

- **Case 4 - All units freight discount:** Per unit transfer price is p_T^1 for $q \in (0, q_2)$ and p_T^2 for $q \geq q_2$ where $p_T^1 > p_T^2$. Hence,

$$P_T(q) = \begin{cases} p_T^1 q, & \text{if } 0 < q < q_2, \\ p_T^2 q, & \text{if } q \geq q_2. \end{cases}$$

As explained in Chapter I, this price schedule is often interpreted as

$$P_T(q) = \begin{cases} p_T^1 q, & \text{if } 0 < q < q_1, \\ p_T^1 q_1 & \text{if } q_1 \leq q < q_2, \\ p_T^2 q, & \text{if } q \geq q_2, \end{cases}$$

where $q_1 \in (0, q_2)$ such that $p_T^1 q_1 = p_T^2 q_2$. Note that we can also consider more general discount schedules with multiple breaks.

For the first two cases listed above, the minimization problem given in Expression (7.1) reduces to the problem solved in Lee and Rosenblatt (1986). We present the solution for these two cases in Appendix A.

For **Case 3**, the problem given in (7.1) leads to

$$\begin{aligned} \min_{q \geq 0, n \in \mathbb{Z}^+} G_J(q, n) &= \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{A_p D}{q} \\ &+ \left\lceil \frac{q}{P} \right\rceil \frac{M_p D}{q} + p_T D, \end{aligned} \quad (7.2)$$

which corresponds to a variation of the class of minimization problems studied in Çetinkaya and Lee (2002) and Toptal et. al. (2003). Motivated by practical applications of inventory and transportation planning, these two papers develop finite time minimization procedures for functions corresponding to special cases of the following

functional form:

$$\begin{aligned} \mathcal{G}(\underline{q}_k, n) = & \frac{\left(A_1 + \left\lceil \frac{\sum_{k=1}^{k=n} q_k}{P_1} \right\rceil R_1 \right) D}{\sum_{k=1}^{k=n} q_k} + \frac{h_1 \sum_{k=2}^{k=n} q_k \left(\sum_{i=1}^{i=k-1} q_i \right)}{\sum_{k=1}^{k=n} q_i} \\ & + \frac{\left(nA_2 + \sum_{k=1}^{k=n} \left\lceil \frac{q_k}{P_2} \right\rceil R_2 \right) D}{\sum_{k=1}^{k=n} q_k} + \frac{h_2 \sum_{k=1}^{k=n} q_k^2}{2 \sum_{k=1}^{k=n} q_k}. \end{aligned} \quad (7.3)$$

In particular, the technique developed in Çetinkaya and Lee (2002) focuses on the case

$$M_1 = 0, \quad A_2 = 0, \quad A_1 \geq M_2, \quad q_1 = q_2 = \dots = q_{n-1} \leq q_n,$$

whereas the two techniques developed in Toptal et. al. (2003) concentrate on the cases

$$M_1 = M_2, \quad P_1 = P_2, \quad q_1 = q_2 = \dots = q_n,$$

respectively. Observe that the function in Expression (7.2) is also a special case of the functional form in Expression (7.3) where $q_1 = q_2 = \dots = q_n$, A_1 and A_2 can take any positive value, as well as

$$M_1 = 0, \quad M_2 > 0,$$

and, hence, the techniques developed in Çetinkaya and Lee (2002) and Toptal et. al. (2003) are not directly applicable for our purposes. On the other hand, utilizing some of the fundamental ideas in Çetinkaya and Lee (2002) and Toptal et. al. (2003), we are able to develop a finite time exact procedure for minimizing $G_J(q, n)$ given by Equation (7.2). This problem is, in fact, equivalent to minimizing the system-wide annual cost for the supplier-transporter-buyer system, which we solve in Section VII.3.1.

Unlike in *Model I*, in *Model II* the supplier and buyer do not engage in cooperation but make their decisions independently. The buyer always tries to choose the order quantity that minimizes his annual cost. However, this order quantity may not always be the most desirable quantity for supplier. She may be better off if the order

quantity changes. Therefore she may try to affect the buyer's decision and coordinate him at a different order quantity. The only way the supplier can achieve coordination is to change the wholesale pricing structure. We can formally state the supplier's problem as follows:

“By changing the initial pricing structure, $P_S(q)$, to a new pricing schedule, $\bar{P}_S(q)$, find the order quantity and the corresponding n value

- (i) that maximizes her annual profit (*Condition 1*) and,
- (ii) that minimizes buyer's annual cost under the new pricing schedule. (*Condition 2*)”

The supplier should also take care of the incentive compatibility constraint for the buyer which says that the buyer's optimal annual cost under this new pricing should not exceed his optimal annual cost under the initial pricing.

In fact, the second condition is related to the structure of $\bar{P}_S(\cdot)$. While formulating the supplier's problem, we omit this condition momentarily. We discuss the structure of $\bar{P}_S(\cdot)$ later.

We formulate supplier's problem for two different cases: (i) the supplier incurs transportation charges, (ii) the buyer incurs transportation charges.

Case 1: Supplier Incurs Transportation Charges

Given the initial wholesale price, $P_S(q) = p_S q$, the buyer's annual cost is

$$G_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + p_S D.$$

The minimizer of this function, i.e. the buyer's optimal order quantity is given by q_B . We also define $\tilde{G}_B(q)$ as

$$\tilde{G}_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2}.$$

The supplier's problem can be formulated as follows:

$$\begin{aligned} \max_{q,n} \quad & \frac{\bar{P}_S(q)D}{q} - G_S(q, n) \\ \text{s.t.} \quad & \tilde{G}_B(q) + \frac{\bar{P}_S(q)D}{q} = \tilde{G}_B(q_B) + p_S D. \end{aligned}$$

Equivalently,

$$\begin{aligned} \max_{q,n} \quad \tilde{G}_B(q_B) + p_S D - \tilde{G}_B(q) - G_S(q, n) &= G_B(q_B) - \tilde{G}_B(q) - G_S(q, n) \\ &= G_B(q_B) - G_{SB}(q) \\ &\equiv \min_{q,n} G_{SB}(q). \end{aligned}$$

Case 2: Buyer Incurs Transportation Charges

Given the initial wholesale price, $P_S(q) = p_S q$, and the initial transportation price, $P_T(q)$, the buyer's annual cost is

$$G_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{P_T D}{q} + p_S D.$$

The minimizer of this function, i.e., the buyer's optimal order quantity, is given by q_B . We also note that, the buyer's annual cost excluding the purchase cost, $\tilde{G}_B(q)$, is

$$\tilde{G}_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{P_T D}{q}.$$

Using the above information, the supplier's problem can be formulated as follows:

$$\begin{aligned} \max_{q,n} \quad & \frac{\overline{P}_S(q)D}{q} - G_S(q, n) \\ \text{s.t.} \quad & \tilde{G}_B(q) + \frac{\overline{P}_S(q)D}{q} = \tilde{G}_B(q_B) + p_S D. \end{aligned}$$

From the above constraint, we substitute the value of $\overline{P}_S(q)$ into the objective function so that

$$\begin{aligned} \max_{q,n} \quad & \tilde{G}_B(q_B) + p_S D - \tilde{G}_B(q) - G_S(q, n) \\ & = G_B(q_B) - \tilde{G}_B(q) - G_S(q, n) \\ & = G_B(q_B) - G_{SB}(q) \\ & \equiv \min_{q,n} G_{SB}(q). \end{aligned}$$

As we can see from these results, the supplier's problem, no matter who incurs the transportation charges, is essentially to minimize the joint annual cost of the buyer and supplier. Recall that it is the same problem that the buyer and supplier face in *Model I*. Depending on the structure of the transportation price function, this problem can be solved using the appropriate techniques developed in Appendix A.

The supplier's next problem is to find a new pricing schedule, $\overline{P}_S(q)$, which satisfies *Condition 2*. The structure of this new price function depends on the transportation price and on which party incurs the transportation price. In Appendix A, we discuss this problem for each possible case.

In conclusion, whichever model we consider, given the preliminary information about the parties' price schedules, the buyer's order quantity is always the joint optimal order quantity of the supplier-buyer system.

VII.3. Model I - Supplier and Buyer Coordinates

As mentioned in Section VII.1, this model assumes either that the buyer and supplier are controlled by a central decision maker or that they align their incentives in a long-term strategic partnership and operate on a contractual basis. In either instance, the order quantity between the buyer and the supplier will be their joint optimal quantity, q_J , and the supplier's replenishment quantity is $(n_J - 1)q_J$. We should note that the joint optimal solution of the supplier-buyer problem, (q_J, n_J) depends on the initial transportation price or the preliminary information that the buyer and/or supplier have about the transportation price.

In this section, based on the joint optimal solution of the buyer and supplier, we introduce the transporter's problem, formulate and solve it, and finally discuss how the transporter can coordinate the supplier-buyer system at his solution.

VII.3.1. Transporter's Problem

Suppose that the buyer and the supplier have already decided on a dispatch quantity, q_J , based on initial transportation pricing information. As we have already mentioned, this quantity may not be the best choice as far as the transporter's actual operating costs are concerned. Hence, the transporter is interested in influencing q_J via offering a contractual dispatch quantity, denoted by q_T . Preferably, the transporter's annual cost is reduced when the order quantity is changed from q_J to q_T , i.e., $G_T(q_T) \leq G_T(q_J)$. On the other hand, deviating from the optimal quantity increases the annual cost of the supplier and buyer, i.e., $\hat{G}_J(q_J, n_J) \geq \hat{G}_J(q_T, n_T)$ ². The transporter should also compensate this increase in order to be able to coordinate the channel at q_T .

²When the buyer's order quantity changes to q_T , the supplier also changes her replenishment quantity so as to reduce the costs. $n_T q_T$ is the replenishment quantity that minimizes the joint annual cost of the supplier and buyer when buyer's order quantity is q_T .

Formally, q_T is the quantity that solves the following problem:

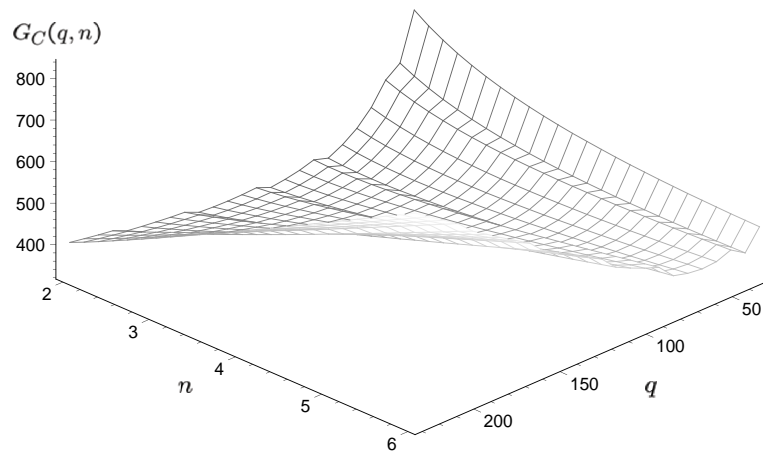
$$\mathcal{P}_T : \max_{q \geq 0, n \in \mathbb{Z}^+} (G_T(q_J) - G_T(q)) - \left(\hat{G}_J(q, n) - \hat{G}_J(q_J, n_J) \right), \quad (7.4)$$

or

$$\begin{aligned} \mathcal{P}_T : \min_{q \geq 0, n \in \mathbb{Z}^+} G_C(q, n) &= \hat{G}_J(q, n) + G_T(q) \\ &= \frac{(A_S + nA_B)D}{nq} + \frac{(h_B + (n-1)h_S)q}{2} + \frac{[q/P]R_T D}{q} \\ &\quad + \frac{A_T D}{q}. \end{aligned} \quad (7.5)$$

Hence, the transporter's problem is equivalent to minimizing the system-wide costs, $G_C(q, n)$. The structure of $G_C(q, n)$ makes it very difficult to find the minimizer directly. Figure 9 displays the behavior of the continuous approximation of $G_C(q, n)$.

Figure 9 $G_C(q, n)$



Since it is rather difficult to directly minimize $G_C(q, n)$, we stick to the first formulation and develop a technique for maximizing (7.4) under $A_T = 0$ (Equivalently minimizing (7.5)). In the next section, we will generalize this technique to the case where $A_T > 0$.

The summary of the technique is as follows. First, we find the best q value for the transporter for any given value of n . Then, we do an enumeration over a set of possible values of n .

Step-I: Optimizing over q for a fixed n

- We first analyze the increase in the joint cost of the supplier-buyer when n is fixed and q is changed:

For a fixed value of n , the structure of $\hat{G}_J(q, n)$ is identical to an EOQ-type function in q . That is, defining

$$A = \frac{A_S + nA_B}{nD} \quad \text{and} \quad B = \frac{(n-1)h_S + h_B}{2},$$

we have

$$\hat{G}_J(q) = \frac{A}{q} + Bq.$$

Now suppose we deviate from $q^*(n)$ by a factor K , keeping n constant. The following property helps us find the amount of increase in $\hat{G}_J(q, n)$.

PROPERTY 1 *In an EOQ-Type function, $f(\cdot)$, with minimizer x^* , $f(Kx^*) - f(x^*) = f(x^*)(K - 1)^2/2K$.*

Proof: A Typical EOQ-Type function has the form $f(x) = a/x + bx$. The minimizer of this function, x^* , is given by $\sqrt{a/b}$; hence, the minimum value of

f is $2\sqrt{ab}$. Noting that

$$\begin{aligned} f(Kx^*) &= \frac{a}{Kx^*} + bKx^* = \frac{a}{K\sqrt{a/b}} + Kb\sqrt{a/b} = \sqrt{ab}\left(\frac{1}{K} + K\right) \\ &= f(x^*)\frac{1+K^2}{2K}, \end{aligned}$$

we have $f(Kx^*) - f(x^*) = f(x^*)(K-1)^2/2K$. ■

Given that the change in q is u units - either an increase or a decrease - K is equal to

$$\frac{q+u}{q}.$$

On the other hand, for any given n , the q that minimizes $\hat{G}_J(q, n)$ is:

$$q^*(n) = \sqrt{\frac{2(A_S + nA_B)D}{n(h_B - h_S + nh_S)}}. \quad (7.6)$$

Once this value is substituted into $\hat{G}_J(q, n)$, $\hat{G}_J(q, n)$ can be expressed as only a function of n , namely $\phi(n)$:

$$\phi(n) = \sqrt{2(A_S + nA_B)D(h_B - h_S + nh_S)/n}.$$

With these expressions, we are now ready to state the expression for the change in $\hat{G}_J(q, n)$ when the dispatch quantity is changed by u :

$$\Delta_J(u) = \frac{u^2}{2q(q+u)}\phi(n).$$

- Next, we investigate the decrease in the transporter's cost when q is changed by u units. For this purpose, we present the following observations:
 - The transporter's annual cost is minimized when $q = kP$, $k \in \mathbb{N}$.
 - The value of k above does not affect the annual cost. As a consequence of this, we have the following property: If q is not an integer multiple of P ,

we specify the following two possible actions that a transporter can take:

Action 1: try to decrease q to $P\lceil q/P \rceil$. The decrease in cost is

$$\frac{R_T D(P\lceil q/P \rceil - q)}{qP}.$$

Action 2: try to increase q by u units where $0 \leq u \leq P\lceil q/P \rceil - q$.

The decrease in cost is

$$\frac{\lceil q/P \rceil R_T D u}{q(q+u)}.$$

The observations indicate that, given any dispatch quantity, there will be one and only one truck which is not completely full unless the $q = kP$ (we call this truck *the last truck*), and there is always room for decreasing the transporter's cost either by decreasing the dispatch quantity so that *the last truck* is cancelled and the other trucks have full load or by increasing the dispatch quantity so that the utilization of *the last truck* is increased.

There are also two other actions that the transporter can take:

Action 3: Decrease the quantity more than $(R_T D(P\lceil q/P \rceil - q))/qP$.

Action 4: Increase the quantity more than $P\lceil q/P \rceil - q$.

However, these two actions are dominated by the first two actions. The annual transportation cost under *Action 3* is always greater than, or equal to, the transportation cost under *Action 2*, because the annual transportation cost under *Action 2* is the minimum annual cost that can be achieved. Similarly, the annual transportation cost under *Action 4* is always greater than, or equal to, the annual transportation cost under a particular choice of *Action 4* where the last truck is filled completely.

On the other hand, the increase in the joint annual cost of the supplier and

buyer is more under *Actions 3* and *4*, because they require a higher deviation from the joint optimal quantity $q(n)$.

Using the above results, the transporter's problem can be formulated as follows:

$$\begin{aligned} \max \quad & \Pi_{q,n}(u) = \Delta_T(u) - \Delta_J(u), \\ \text{s.t} \quad & u \in \{P\lfloor q/P \rfloor - q\} \cup [0, P\lceil q/P \rceil - q]. \end{aligned}$$

We call this problem \mathcal{PZERCO} . Once we substitute the values of $\Delta_T(u)$ and $\Delta_J(u)$, we obtain

$$\Pi_{q,n}(u) = \begin{cases} \frac{R_T D(P\lceil q/P \rceil - q)}{qP} - \frac{(q - P\lfloor q/P \rfloor)^2}{2q(P\lfloor q/P \rfloor)} \phi(n) & \text{if } u = P\lfloor q/P \rfloor - q, \\ \frac{\lceil q/P \rceil R_T D u}{q(q+u)} - \frac{u^2}{2q(q+u)} \phi(n) & \text{if } 0 \leq u \leq P\lceil q/P \rceil - q. \end{cases}$$

The first piece of the function is a single value (we call this value $\alpha(q, n)$ from now on), and the latter piece is concave in u . The solution to the problem \mathcal{PZERCO} is given by the following Proposition:

PROPOSITION 15 *The optimal value of u is given by:*

$$u_{q,n}^* = \begin{cases} P\lfloor q/P \rfloor - q & \text{if } \alpha(q, n) > \max_{(u \in [0, P\lceil q/P \rceil - q])} \Pi_{q,n}(u), \\ \bar{u}_{q,n} & \text{if } \alpha(q, n) \leq \Pi_{q,n}(\bar{u}) \text{ and } \bar{u} \leq P\lceil q/P \rceil - q, \\ P\lceil q/P \rceil - q & \text{otherwise,} \end{cases}$$

where \bar{u} is the stationary point of the second piece, and its value is given by

$$\bar{u}_{q,n} = q \sqrt{1 + \frac{2\lceil q/P \rceil R_T D}{\phi(n)q}} - q.$$

Proof: Note that $(d\Pi_{q,n}(u)/du)|_{u=0} > 0$. Hence $\bar{u}_{q,n}$ is the global maximizer over $[0, \infty)$. Then over $[0, P\lceil q/P \rceil - q]$, the function attains its maximum if $\bar{u}_{q,n}$ is feasible,

otherwise the maximizer is the right end point. Finally, we compare the maximum value of $\Pi_{q,n}(u)$ to $\alpha(q, n)$. ■

Step-II: Search Over n

So far, we have solved the transporter's problem while assuming that n is fixed. However, it is quite possible that a change in q would lead the supplier to change n . So the solution we have found may not reflect the best decision that the transporter could make. To compensate for this, we propose a search procedure over the possible values of n :

ALGORITHM 3

Step 0: Start with n_J and q_J . Find Π_{q_J, n_J} and find u_{q_J, n_J}^* . Set $n_T = n_J$, $q_T = q_J + u_{q_J, n_J}^*$, and $C^* = \hat{G}_J(q_T, n_T) + G_T(q_T)$.

Step 1: Set $i = 1$. While $\hat{G}_J(q(n_J + i), n_J + i) + DR_T/P \leq C^*$:

- Calculate $\Delta_i = (G_T(q(n_J + i)) - G_T(q(n_J))) - (\hat{G}_J(q(n_J + i), n_J + i) - \hat{G}_J(q_T, n_T))$.
- Find $u_{q,n}^*$ and $\Pi_{q,n}(u)$ for $q = q(n_J + i)$, and $n = n_J + i$.
- Add Δ_i to $\Pi_{q,n}(u)$. This is the net savings that are obtained by changing n from n_J to $n_J + i$. If this net savings leads to less total cost than the current optimal solution, then update q_T , n_T and C^* accordingly.
- increase i by 1.

Step 2: Repeat Step 1 this time by starting with $i = -1$ and decrease n at each sub-step.

Algorithm 3 is a finite-step algorithm. This is easy to prove, but first we must justify the stopping criterion: $\hat{G}_J(q(n_J + i), n_J + i) + DR_T/P$ is the lower bound for

the system-wide cost when $n = n_J + i$. Obviously, if this cost is greater than the current optimal cost, then for this value of n , we cannot achieve a better solution. That justifies the stopping criterion.

Next we show that the algorithm is a finite-step algorithm. Recall that $\hat{G}_J(q(n), n)$ is given by $\sqrt{2(A_S + nA_B)D(h_B - h_S + nh_S)/n}$. We know that this expression has a unique minimizer, n_J , and that it is increasing for $n > n_J$ and $n < n_J$. Hence, if for a particular value of n , say $\bar{n} > n_J$, the lower bound is greater than the current optimal solution, then for all other $n > \bar{n}$, the lower bound should be greater than the current optimal. Similarly, if for a particular $\underline{n} < n_J$, the lower bound is greater than the current optimal, then for all $n < \underline{n}$, the lower bound will be greater than the current optimal. We also know that

$$\lim_{n \rightarrow \infty} \sqrt{2(A_S + nA_B)D(h_B - h_S + nh_S)/n} = \infty,$$

so there exists an n_{min} such that

$$\sqrt{2(A_S + nA_B)D(h_B - h_S + nh_S)/n} + \frac{DR_T}{P} \geq \hat{G}_J(q_J, n_J) + \frac{[q_J/P]R_T}{q_J} \quad \forall n \geq n_{min}.$$

The right hand side of the inequality is $G_C(q_J, n_J)$; its value is always greater than, or equal to, the current optimal solution to the transporter's problem in any iteration. This proves the existence of a finite $n > n_J$ after which the procedure can be stopped. For $n < n_J$, a bound can be derived in a similar manner. In addition, $n = 1$ is already a lower bound for such $n < n_J$.

VII.3.1.1. Solution to the Transporter's Problem When $A_T > 0$

When $A_T > 0$, the observations of the possible actions of the transporter do not hold directly. However, we can still solve the transporter's problem by utilizing the ideas developed for the $A_T = 0$ case. We have a two step procedure. First, we solve the

transporter's problem by assuming that he has no cargo cost, i.e. he has only a fixed cost for each dispatch. Under this assumption, the transporter's problem is identical to the supplier-buyer problem where the buyer's fixed dispatch cost is $A_B + A_T$. We denote the solution of this problem as (q'_T, n'_T) . In the second step, we assume that the joint solution of the supplier-buyer is (q'_T, n'_T) and that the transporter has only cargo costs. Rewriting (7.5), we have

$$\begin{aligned}
 \mathcal{P}_T : \min_{q \geq 0, n \in \mathbb{Z}^+} G_C(q, n) &= \hat{G}_J(q, n) + G_T(q) \\
 &= \frac{(A_S + n(A_B + A_T))D}{nq} + \frac{(h_B + (n-1)h_S)q}{2} + \frac{[q/P]R_T D}{q} \\
 &= \frac{(A_S + nA_B)D}{nq} + \frac{(h_B + (n-1)h_S)q}{2} + \frac{[q/P]R_T D}{q} + \frac{A_T D}{q},
 \end{aligned}$$

and, hence, this second problem is identical to the problem that we solved in the previous section.

VII.3.2. Coordination Mechanisms

So far, we have developed the solution to the transporter's problem. The next step is to find an answer as to how the transporter can coordinate the buyer and the supplier to his solution. While structuring the transporter's problem, we mentioned that the transporter has to compensate the cost increase for the supplier and buyer system. The transporter can provide contractual mechanisms, which compensate the supplier-buyer system and coordinate the channel at (q_T, n_T) . In this dissertation, we discuss two different types of contractual mechanisms:

Transportation Price Contracts: The transporter changes the initial price schedule, and offers a new price schedule, namely $\bar{P}_T(q)$, as a contract to the supplier and buyer. The supplier and the buyer solve their joint optimization problem with the new price schedule. When providing the new price schedule, the trans-

porter has to make sure that the new joint solution of the supplier and the buyer is (q_T, n_T) and the increase in the joint cost of the supplier and buyer is compensated.

Order Quantity Contracts: The transporter offers a contract to the supplier and buyer that sets the order quantity to q_T . Keeping the initial pricing schedule, the transporter compensates the cost increase of the supplier-buyer system through fixed annual payments.

VII.3.2.1. Coordination by Transportation Price Contracts

The structure of the *transportation price contracts* may depend on the initial pricing structure. Hence, we consider each case separately. Before going into the details of each case, we mention once more that in this model, it does not matter which party is incurring the transportation charges since they act as a single entity.

Case 1: $P_T(q) = p_T q$:

We derive the joint solution of the supplier-buyer problem with the transportation price schedule in Appendix A. The transporter's new price schedule, $\bar{P}_T(q)$ has to satisfy similar conditions as *Condition 1* and *Condition 2*, which are given in the VII.2.2. This new pricing schedule implies a new annual revenue for the transporter and a new annual transportation cost for the supplier-buyer. Below we adapt *Condition 1* and *Condition 2* for the transporter's problem:

- i- $\hat{G}_J(q_T, n_T) + \bar{P}_T(q_T)D/q_T \leq \hat{G}_J(q_J, n_J) + P_T(q_J)D/q_J$. (*Condition 1*), and
- ii- $(q_T, n_T) = \arg \min \hat{G}_J(q, n) + \bar{P}_T(q)D/q$. (*Condition 2*)

One of the most commonly used pricing mechanisms used for channel coordination is quantity discounts. In the transportation industry, quantity discounts are

referred to as freight discounts. We next discuss, how the freight discounts can be employed as new price schedules to coordinate the supplier and the buyer.

We first consider incremental freight discounts. Incremental freight discount schedules can be of various types. Here we present three types of incremental discount schedules:

$$\bar{P}_T(q) = \begin{cases} p_T q & \text{if } q < q_1, \\ p_T q_1 + p'_T(q - q_1) & q_1 \leq q, \end{cases} \quad (7.7)$$

$$\bar{P}_T(q) = \begin{cases} 0 & \text{if } q < q_0, \\ p'_T(q - q_0) & \text{if } q_0 \leq q < q_1, \\ p'_T q_1 + p_T(q - q_1) & q_1 \leq q. \end{cases} \quad (p'_T > p_T) \quad (7.8)$$

Equation (7.7) represents two types of incremental discounts. $p_T > p'_T$ represents the standard form of the incremental discount. If $p_T < p'_T$, then the price schedule offers a negative discount. In other words, higher volumes are discouraged. On the other hand, equation (7.8) represents a negative discount with a fixed reward for each dispatch.

Following this brief preliminary information about incremental discounts, we next analyze how they can be used for coordination purposes. Consider the second condition, which requires (q_T, n_T) to be the joint optimal solution of the buyer and supplier problem. Momentarily assume that the transporter is charging a fixed price, A_p , for each dispatch in his modified price schedule. There are three necessary and sufficient conditions for (q_T, n_T) :

$$q_T = \sqrt{\frac{2(A_S + n_T(A_B + A_p))D}{n_T(h_B - h_S + n_T h_S)}}, \quad (7.9)$$

$$Ah_S + (A_B + A_p)h_S - \frac{A_S h_e}{n_T(n_T + 1)} \geq 0, \quad (7.10)$$

$$\frac{A_S h_e}{n_T(n_T + 1)} - (A_B + A_p)h_S - Ah_S \geq 0, \quad (7.11)$$

where $h_e := h_B - h_S$.

Equation (7.9) is a direct consequence of equation (7.6). Inequalities (7.10) and (7.11) result from the difference equations, and they ensure the optimality of n_T . From (7.9), we can derive the value of A_p as follows:

$$A_p = \frac{q_T^2 n_T (h_e + n_T h_S) / 2D - A_S}{n_T} - A_B. \quad (7.12)$$

After finding the value of A_p , we need to check whether or not the other two conditions are satisfied. If they are also satisfied, then imposing this fixed price, A_p , will coordinate the supplier-buyer system at (q_T, n_T) .

Next, we explain the analogy between imposing a fixed price and offering incremental freight discounts. Consider the region where $q > q_1$ in Equation (7.7). In this region, we can rewrite $\bar{P}_T(q)$ as $q_1(p_T - p'_T) + qp'_T$. The first term is constant; hence, if the dispatch quantity is greater than q_1 , this price schedule is equivalent to offering a fixed price for each dispatch and a per unit price. Note that, depending on the relations between p_T and p'_T , the fixed term can be negative. On the other hand, consider the region where $q_0 \leq q < q_1$ in Equation (7.8). Rewriting $\bar{P}_T(q)$ in this region we obtain $p'_T q - p'_T q_0$. Note that the second term represents a fixed reward for each dispatch.

If $q_J < q_T$, the transporter may offer the incremental discount schedule as in Equation (7.7); if $q_J > q_T$, then a discount schedule as in Equation (7.8) would be appropriate. The transporter has to determine the value of q_1, p'_T , and/or q_0 . p'_T can

be derived from *Condition 1* as follows:

$$p'_T D = \hat{G}_J(q_J, n_J) + p_T D - \hat{G}_J(q_T, n_T) - A_p D / q_T.$$

After finding p'_T , the value of q_1 can be derived by setting $q_1(p_T - p'_T)$ equal to A_p for the case $q_J < q_T$. For $q_J > q_T$, q_0 can be derived from $q_0 = -A_p/p'_T$, and q_1 is obtained from $p'_T(q_1 - q_0) = p_T q_1$.

Below we state the necessary and sufficient conditions for the incremental discount to coordinate the supplier-buyer system:

1- $q_J < q_T$:

The freight schedule in expression (7.7) coordinates the supplier-buyer system if the following conditions hold:

(i1C1) Expressions (7.10) and (7.11).

(i1C2) $q_1 \leq q_T$.

(i1C3) $p'_T D = \hat{G}_J(q_J, n_J) + p_T D - \hat{G}_J(q_T, n_T) - A_p D / q_T$ and $q_1(p_T - p'_T) = A_p$ where A_p is given by equation (7.12).

2- $q_J > q_T$:

The freight schedule in expression (7.8) coordinates the supplier-buyer system if the following conditions hold:

(i2C1) Expressions (7.10) and (7.11).

(i2C2) $q_1 \geq q_T$.

(i2C3) $p'_T D = \hat{G}_J(q_J, n_J) + p_T D - \hat{G}_J(q_T, n_T) - A_p D / q_T$, $q_0 = -A_p/p'_T$, and $q_1 = \frac{p'_T q_0}{p_T - p'_T}$ where A_p is given by equation (7.12).

Next, we consider *all-units freight discounts*. Consider the following all-units freight discount schedule:

$$\bar{P}_T(q) = \begin{cases} p_T q & \text{if } q < q_2, \\ p'_T q & \text{if } q_2 \leq q. \end{cases} \quad (7.13)$$

In this expression, we assume $p_T > p'_T$. As explained in Appendix A, this discount schedule is interpreted as follows:

$$\bar{P}_T(q) = \begin{cases} p_T q & \text{if } q < q_1, \\ p'_T q_2 & \text{if } q_1 \leq q < q_2, \\ p'_T q & \text{if } q_2 \leq q, \end{cases}$$

where $q_1 = (c_T^2 q_2)/c_T^1$. We now define $[0, q_1]$ as *Region I*, (q_1, q_2) as *Region II*, and $[q_2, \infty)$ as *Region III*. As explained in Appendix A, given this price schedule, the joint annual cost of the supplier and buyer, namely $\bar{G}_J(q, n)$, becomes

$$\bar{G}_J(q, n) = \begin{cases} \frac{(A_S + nA_B)D}{nq} + \frac{(h_S(n-1) + h_B q)}{2} + p_T D & \text{if } q < q_1, \\ \frac{[A_S + n(A_B + c_T^2 q_2)]D}{nq} + \frac{(h_S(n-1) + h_B q)}{2} + \frac{p'_T q_2 D}{q} & \text{if } q_1 \leq q < q_2, \\ \frac{(A_S + nA_B)D}{nq} + \frac{(h_S(n-1) + h_B q)}{2} + p'_T D & \text{if } q_2 \leq q. \end{cases}$$

Recall that the joint annual cost of the supplier-buyer under the initial pricing is

$$G_J(q, n) = \frac{(A_S + nA_B)D}{nq} + \frac{(h_S(n-1) + h_B q)}{2} + p_T D.$$

The structure of $\bar{G}_J(q, n)$ in *Regions I* and *III* is the same as the structure of $G_J(q, n)$.³

After presenting some preliminary information, we next summarize the necessary and sufficient conditions for such an all-units discount schedule to be optimal. We divide this analysis into two cases:

³*Regions I, II, and III* refer to $q < q_1$, $q_1 \leq q < q_2$, and $q \geq q_2$, respectively.

1- $q_J < q_T$:

If the transporter wants to coordinate the buyer and the supplier, q_T cannot be in *Region I*. Hence, we need to consider two regions only.

First, we let q_T be in *Region II* and state the necessary and sufficient conditions for (q_T, n_T) to be optimal:

(IIC1) p'_T and q_2 has to satisfy

$$p'_T q_2 = \frac{q_T^2 n_T (h_e + n_T h_S) / 2D - A_S}{n_T} - A_B.$$

(IIC2) $\bar{G}_J(q_T, n_T) \leq \phi(n) + c_2 q_2 D / q(n) \quad \forall n$ such that $q(n)$ is in *Region II*.

(IIC3) $\bar{G}_J(q_T, n_T) \leq \bar{G}_J(q_1, n(q_1))$ and $\bar{G}_J(q_T, n_T) \leq \bar{G}_J(q_2, n(q_2))$.

(IIC4) $q_J < q_T$.

(IIC5) $\bar{G}_J(q_T, n_T) \leq \bar{G}_J(q(n_{max}), n_{max})$ where $n_{max} = \arg \max \{q(n) \geq q_2\}$.

(IIC6) If $q_J \leq q_1$, the discounted freight price has to satisfy

$$p'_T q_2 D / q_T \leq \hat{G}_J(q_J, n_J) + p_T D - \hat{G}_J(q_T, n_T).$$

On the other hand, if $q_J > q_1$, it should satisfy

$$p'_T q_2 D (1/q_T - 1/q_J) \leq \hat{G}_J(q_J, n_J) - \hat{G}_J(q_T, n_T).$$

The inequalities in the conditions imply that the transporter can share some of the system savings with the buyer and the supplier.

Using (IIC1), the transporter can determine $p'_T q_2$ and then he can also derive the value of q_1 from $p_T q_1 = p'_T q_2$. That also specifies the location of q_J .

In order to satisfy (IIC5), the transporter can restrict the discounted price to an interval, i.e. he can set his discount schedule as

$$\bar{P}_T(q) = \begin{cases} p_T q & \text{if } q < q_2, \\ p'_T q & \text{if } q_2 \leq q, q_3, \\ p_T q & \text{if } q_3 \leq q, \end{cases}$$

for some q_3 .

Next, letting q_T be in *Region III*, we state the necessary and sufficient conditions for (q_T, n_T) to be the minimizer of G_J , i.e. to satisfy *Condition 2*:

(IIIC1) $n(q_T) = n_T$.

(IIIC2) Either $q_T = q_2$ and $n(q_T) = n_T$ or $q_T = q(n_{max})$ where $n_{max} = \arg \max\{q(n) \geq q_2\}$.

(IIIC3) $q_J < q_2$.

(IIIC4) If $q_T = q_2$, then $\bar{G}_J(q_T, n_T) \leq \phi(n_{max}) + p'_T D$.

(IIIC5) $\bar{G}_J(q_T, n_T)$ should not lead to a higher joint cost than the minimizer of $\bar{G}_J(q, n)$ in *Region II*.

(IIIC6) If q_J is in *Region I*, the discounted price, p'_T has to satisfy

$$p'_T D \leq \hat{G}_J(q_J, n_J) + p_T D - \hat{G}_J(q_T, n_T).$$

If q_J is in *Region II*, p'_T and q_2 have to satisfy

$$p'_T D \leq \hat{G}_J(q_J, n_J) + p'_T q_2 D / q_J - \hat{G}_J(q_T, n_T).$$

The inequalities indicate that the transporter may be willing to share some of the system savings in order to achieve coordination. The transporter can

restrict his discount structure only to an interval as in the case where q_T is in *Region II*.

In conclusion, the transporter has to either satisfy (IIC1)-(IIC6) or (IIIC1)-(IIIC6) to achieve coordination. Because he can only control the values of p'_T and q_2 , he may not always be able to satisfy those conditions. If those conditions cannot be satisfied, we conclude that the all-units discount schedule cannot coordinate the supplier-buyer system when $q_J < q_T$.

2- $q_J > q_T$:

In this case, the quantity discount schedule as given in expression (7.13) will not coordinate the supplier-buyer system. Instead, a discount schedule of the following form would do so:

$$\bar{P}_T(q) = \begin{cases} p'_T q & \text{if } q \leq q_2, \\ p_T q & \text{if } q_2 < q. \end{cases} \quad (7.14)$$

In this schedule, there are only two regions, *Region I* and *Region II*. The necessary and sufficient conditions for price schedule (7.14) to be optimal are:

(IC1) Either $q_T = q$ and $n(q_T) = n_T$ or $q_T = n_{max}(q)$ where $n_{max} = \arg \max\{q(n) \leq q_2\}$.

(IC2) p'_T has to satisfy

$$p'_T D \leq \hat{G}_J(q_J, n_J) + p_T D - \hat{G}_J(q_T, n_T).$$

It is very unlikely that (IC1) will be satisfied. Also, note that the transporter cannot control the related variables.

Case 2. $P_T(q) = p_T q + A_p$

In Appendix A, we analyze the solution to the supplier-buyer problem under the stated pricing schedule, i.e. finding (q_J, n_J) . The transporter will try to change the supplier-buyer decision, so that they are operating at q_T . As in Section VII.3.2, he has two choices: either change the price, or make a contract with the supplier and buyer.

If the transporter decides to change the price, he can offer an incremental, or an all units, freight discount. The mechanics of the freight discount are same as in Case 1.

The transporter can also induce the supplier and buyer by changing the fixed dispatch price A_p to A'_p such that (q_T, n_T) is the minimizer of the resulting joint annual cost function of the supplier-buyer system. In fact, this is similar to offering incremental discounts. The necessary and sufficient conditions for this to work are either (i1C1), (i1C3) or (i2C1), (i2C3). With the new fixed price, A'_p , the joint annual cost function is given by

$$\bar{G}_J(q, n) = \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{(A_B + A'_p)D}{q} + \frac{h_B q}{2}.$$

If the transporter wants (q_T, n_T) to be the minimizer of this function, the three conditions given by (7.9)-(7.11) have to be satisfied, and the value of A'_p can be found by equation (7.12).

If one of these conditions is not satisfied, then changing the fixed price may not be sufficient to align the incentives of the supplier and the buyer. The transporter also needs to change the value of p_T to p'_T in order to keep the joint annual cost of the supplier-buyer system the same. Recall that in the initial pricing schedule, this cost is equal to $\hat{G}_J(q_J, n_J) + A_p D/q_j + p_T D$. With the modified price schedule, it becomes

$\hat{G}_J(q_T, n_T) + A'_p D/q_T + p'_T D$. The value of p'_T can be solved by equating these two quantities, that is

$$p'_T = \frac{\hat{G}_J(q_J, n_J) - \hat{G}_J(q_T, n_T)}{D} + \frac{A_p}{q_j} - \frac{A_p}{q_T} - p_T.$$

Case 3 - $P_T(q) = p_T q + A_p + [q/P] M_p$

In Appendix A, we analyze the solution of the supplier-buyer problem when the transportation price includes a per truck cost.

Next, we discuss how the transporter could switch his solution from (q_J, n_J) to (q_T, n_T) . The transporter may try to offer an all-units freight discount, or he may change the fixed price for this purpose. The mechanics of these methods are similar to those explained in Section VII.3.2. We now propose a third method that the transporter might try to implement. This is to change the per truck price.

Consider the following situation. Suppose that the transporter set $A_p = 0$.⁴ Then, the supplier-buyer problem is formulated as follows:

$$\min_{q,n} \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{[q/P] M_p D}{q} + p_T D. \quad (7.15)$$

On the other hand, using (7.5), we can write the transporters problem as follows:

$$\min_{q,n} \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{[q/P] R_T D}{q}. \quad (7.16)$$

The structure of these two problems are identical. They differ only by a constant term $p_T D$, which has no effect on the problem and by the per truck component. That is why, if the transporter wants the solution of (7.15) to be equal to the solution of (7.16), he should set $M_p = R_T$.

Under this new pricing schedule, $A_p = 0$ and $M_p = R_T$, the joint annual cost of

⁴If $A_T > 0$, then set $A_p = A_T$.

the supplier-buyer system will change. In order to keep the cost at the same level, the transporter should set the new per unit price p'_T to

$$p'_T = \frac{\hat{G}_J(q_J, n_J) - \hat{G}_J(q_T, n_T)}{D} + \frac{\lceil q_J/P \rceil M_p}{q_J} - \frac{\lceil q_T/P \rceil R_T}{q_T} + p_T.$$

Unlike the other pricing incentives, setting the per truck price equal to the per truck cost, $A_p = A_T$, always ensures aligning the supplier-buyer system to operate at q_T .

VII.3.2.2. Coordination by Order Quantity Contracts

We have seen in the previous section that if the transporter wants to align the supplier-buyer system by changing the transportation price schedule, he may not always be able to force the system to operate at q_T . Changing the price schedule will always work only if it exactly reflects the transporter's costs.

In this section, we provide another contracting mechanism to help the transporter coordinate the system. This mechanism is to offer a contract to the supplier-buyer system. Keeping the transportation price schedule the same, the terms of the contract should include two parameters:

1. order quantity,
2. the compensation amount to be made by the transporter to the supplier-buyer system, namely ν .

The order quantity should be set to q_T for that is the quantity that minimizes the total cost of all the parties. The supplier will also change her replenishment quantity to $(n_T - 1)q_T$ so as to minimize her costs. With (q_T, n_T) , the joint annual cost of the supplier-buyer becomes $G_J(q_T, n_T)$. However, their initial joint annual cost was $G_J(q_J, n_J)$, which is lower than $G_J(q_T, n_T)$ because (q_J, n_J) is the minimizer

of $G_J(q, n)$. The buyer and supplier will only be willing to sign this contract if *at least* the increase in their joint cost is compensated. Hence, ν should at least be $G_J(q_T, n_T) - G_J(q_J, n_J)$. The compensation might be higher if the buyer and the supplier also want to allocate some of the system savings.

The transporter can pay ν as a franchise fee every year. Another way to pay this compensation, which may be more practical, is to offer a per unit discount for the items transferred. Since the annual demand is constant, this discount can substitute for a fixed payment. The value of the discount is ν/D . This type of a freight discount is different from the other discount schedule that we discussed in Section VII.3.2. In the previous one, the supplier-buyer system can independently decide the order quantity; however in the latter case, they are bound by a contract which has already set the order quantity at a specific value.

VII.4. Model II: Supplier and Buyer Act Independently

In this section, we again consider the transporter's problem and investigate the coordination mechanisms under the assumption that the supplier and buyer do not act as a single unit but are both trying to minimize their own annual costs.

Even though the supplier and buyer behave independently, the supplier still tries to induce the buyer's decisions by changing the wholesale price schedule and coordinating him at their joint optimal solution. Recall that in Section VII.2.2 where we stated and formulated the supplier's problem, we showed that the supplier's problem is essentially to minimize the total annual cost of the buyer and the supplier, $G_{SB}(q, n)$. The solution to this problem, which is denoted by (q_{SB}, n_{SB}) , depends on the structure of the transportation price function. For each possible transportation price function, the solution procedures are explained in Appendix A. The supplier

also has to find a new wholesale pricing structure, $\overline{P}_S(q)$, so that when the buyer minimizes his total annual cost under this price, the minimizer will be q_{SB} . The structure and the parameters of $\overline{P}_S(q)$ depend on the structure of the transportation price function and on which party incurs the transportation charges. We discuss how to develop practical $\overline{P}_S(q)$ structures in Appendix A .

As in *Model I*, after determining the order quantity of the buyer, the next step is to construct the transporter's problem. The transporter's problem and coordination mechanisms depend on which party is incurring the transportation costs. That is why, we divide the analysis of *Model II* into two cases:

1. The transporter's problem when the supplier incurs the transportation charges
2. The transporter's problem when the buyer incurs the transportation charges

VII.4.1. Transporter's Problem When Supplier Incurs the Transportation Charges

The analysis of this case is more straightforward. First of all, we should note that regardless of the structure of the transportation price, the buyer's annual cost function is an EOQ type function given by

$$\hat{G}_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + p_S D.$$

As explained in Appendix A, the supplier can change the buyer's decision to any quantity she wishes by offering an all-units quantity discount. Notice that the buyer always wants to choose his optimal order quantity, q_B . The supplier, depending on her preliminary information about the transportation price schedule, finds an optimal order quantity, q_{SB} , that minimizes the sum of the annual costs of the buyer and supplier and then offers a quantity discount schedule to the buyer.

In this case, the transporter negotiates or coordinates with the supplier because the supplier incurs the transportation charges. Since the supplier always tries to minimize the joint supplier-buyer cost, the transporter's problem is exactly the same as the transporter's problem in the first model that was introduced, formulated and solved in Section VII.3.1. Furthermore, the coordination mechanisms that the transporter should follow are exactly the same as the ones in *Model I*. The reader may refer to Section VII.3.2 for details.

VII.4.2. Transporter's Problem When Buyer Incurs the Transportation Charges

The analysis of this case is perhaps the most complicated one, because, in this case, both the supplier and the transporter try to align the buyer's incentives. At this point, we make an assumption and model this problem as a Stackelberg Game in which the supplier leads and the transporter follows. The game is described as follows:

- First, the supplier offers an all-units quantity discount schedule to the buyer. Depending on the relative location of the discount quantity, q_{SB} , to the buyer's optimal order quantity q_B , the discount schedule takes different forms. If $q_B < q_{SB}$, then it becomes as follows:

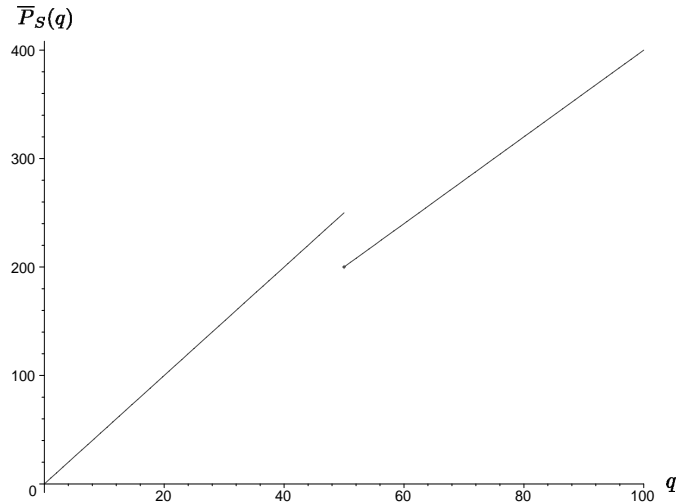
$$\bar{P}_S(q) = \begin{cases} p_S & \text{if } q < q_{SB}, \\ p'_S & \text{if } q_{SB} \leq q. \end{cases} \quad (7.17)$$

If $q_{SB} < q_B$, then

$$\bar{P}_S(q) = \begin{cases} p'_S & \text{if } q \leq q_{SB}, \\ p_S & \text{if } q_{SB} < q. \end{cases} \quad (7.18)$$

Figures 10 and 11 graphically show the discounts.

Figure 10 Supplier's Discount Schedule Given by Equation (7.17)



- Then the transporter offers a freight schedule to further change the buyer's order quantity to q_T .

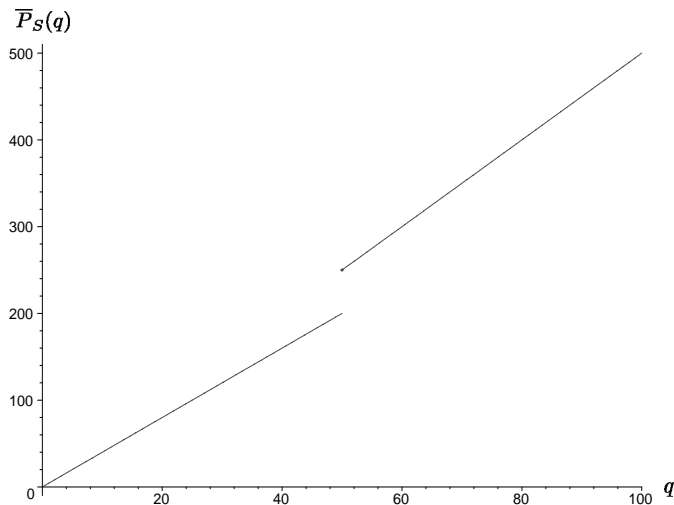
In the remainder of this section, we analyze the equilibrium strategies of the supplier and the transporter:

VII.4.2.1. Transporter's Problem

Suppose that the supplier provides one of the discount schedules given by (7.17) and (7.18) to the buyer. The buyer's respective annual cost functions, excluding the transportation costs, $\bar{G}_B(q)$ takes one of the following forms.

$$\bar{G}_B(q) = \begin{cases} \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + p_T)D, & \text{if } 0 < q < q_{SB}, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p'_S + p_T)D & \text{if } q_{SB} \leq q, \end{cases}$$

Figure 11 Supplier's Discount Schedule Given by Equation (7.18)



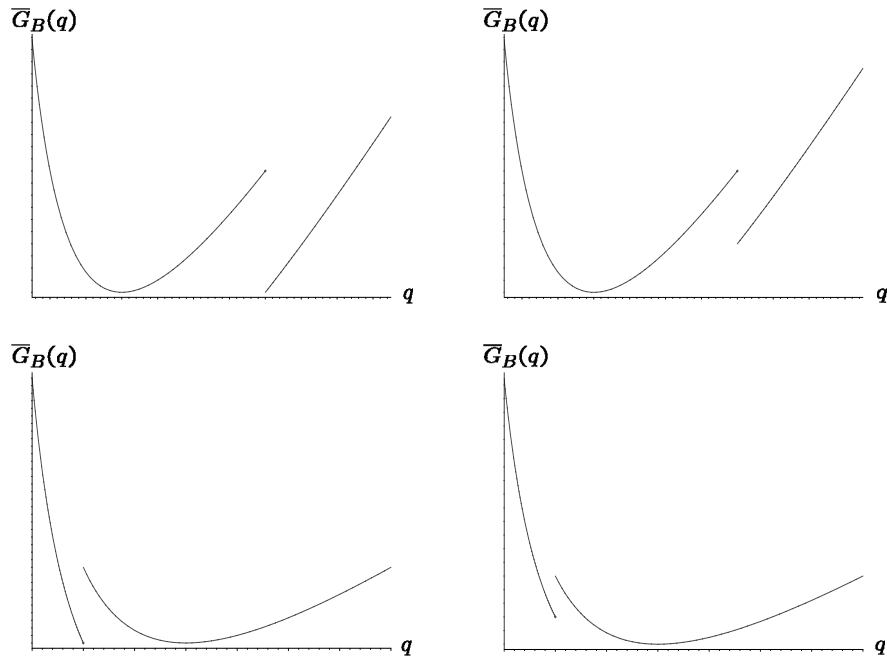
for $q_{SB} \geq q_B$. On the other hand, when $q_{SB} < q_B$, the buyer's annual cost is

$$\bar{G}_B(q) = \begin{cases} \frac{A_B D}{q} + \frac{h_B q}{2} + (p'_S + p_T)D, & \text{if } 0 < q \leq q_{SB}, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + p_T)D & \text{if } q_{SB} < q. \end{cases}$$

In either case, $\bar{G}_B(q)$ is a piecewise continuous function, and its minimizer, namely q_B^* , is either q_B , or the discontinuity point q_{SB} . (See Figure 12.)

Next, we formulate the transporter's problem in a similar manner as we did for *Model I*. First, find the order quantity, namely q_T , that will maximize the decrease in the transporter's annual cost minus the increase in the buyer's annual cost. Stated mathematically,

$$\max_q [G_T(q_B^*) - G_T(q)] - [\bar{G}_B(q) - \bar{G}_B(q_B^*)]. \quad (7.19)$$

Figure 12 Possible Forms of $\bar{G}_B(q)$ Depending on the Supplier's Discount

Since there are two candidates for q_B^* , we can decompose the transporter's problem into two problems:

$$\max_q [G_T(q_B) - G_T(q)] - [\bar{G}_B(q) - \bar{G}_B(q_B)], \quad (7.20)$$

$$\begin{aligned} \max_q [G_T(q_{SB}) - G_T(q)] - [\bar{G}_B(q) - \bar{G}_B(q_{SB})] \\ - [G_B(q_{SB}) - G_B(q_B) - (p_S - p'_S)D]. \end{aligned} \quad (7.21)$$

We explain the last term in the second problem as follows. The supplier offers a quantity discount p'_S to the buyer. If the buyer's order quantity is q_{SB} , then his annual cost reduces by $(p_S - p'_S)D$ due to the discount. On the other hand, his annual cost increases by $G_B(q_{SB}) - G_B(q_B)$ because of deviating from his optimal solution. Thus, $G_B(q_{SB}) - G_B(q_B) - (p_S - p'_S)D$ represents the net change in the

buyer's annual cost if he changes his order quantity from q_B to q_{SB} . In the classical channel coordination problems where there is no transporter, the net change should at most be 0 for the buyer to change his order quantity. However, now the supplier knows that the transporter also provides a freight discount, she does not have to set her discount high enough to make it 0. That is why, the transporter should also compensate for this difference. Note that, given the supplier's discount schedule, $G_B(q_{SB}) - G_B(q_B) - (p_S - p'_S)D$ is constant. We call this constant δ_S from now on.

Let us consider the first problem and restate it as

$$\begin{aligned} \max_{\omega} \quad \Gamma(\omega) &:= [G_T(q_B) - G_T(q_B + \omega)] - [\bar{G}_B(q_B + \omega) - \bar{G}_B(q_B)] \\ &:= \Gamma_T(\omega) - \Gamma_B(\omega). \end{aligned} \quad (7.22)$$

Examining (7.22) carefully, the above problem has the same structure as the transporter's problem, given by equation (7.7), in Section VII.3.1. In this case, $\Gamma_T(\omega)$ represents the decrease in the transporter's annual cost, and $\Gamma_B(\omega)$ represents the increase in the buyer's annual cost. In order to solve this problem, we use the same techniques. As in that problem, the transporter has two best possible actions:

1. change q_B to $q_B^l = \lfloor q_B/P \rfloor P$, i.e. eliminate the last truck.
2. increase q_B by u where $u \in [0, q_B^f - q_B]$, i.e. increase the utilization of the last truck, where $q_B^f = \lceil q_B/P \rceil P$.

By using this information, we rewrite the transporter's problem as

$$\max_{\omega} \quad \Gamma(\omega) = \begin{cases} \Gamma_1(q_B^l - q_B) = \Gamma_T^1(q_B^l - q_B) - \Gamma_B^1(q_B^l - q_B) & \text{if } \omega = q_B^l - q_B, \\ \Gamma_2(\omega) = \Gamma^2(\omega) = \Gamma_T^2(\omega) - \Gamma_B^2(\omega) & \text{if } 0 \leq \omega \leq q_B^f - q_B. \end{cases} \quad (7.23)$$

Observe that

$$\Gamma_T^1(q_B^l - q_B) = \frac{[q_B/P]R_T D}{q_B} - \frac{R_T D}{P}, \quad (7.24)$$

$$\Gamma_B^1(q_B^l - q_B) = \sqrt{2A_B D h_B} \frac{(q_B^l - q_B)^2}{2q_B^l q_B} = \frac{h_B (q_B^l - q_B)^2}{2q_B^l}, \quad (7.25)$$

whereas

$$\begin{aligned} \Gamma_T^2(\omega) &= \frac{[q_B/P]R_T D}{q_B} - \frac{[(q_B + \omega)/P]R_T D}{q_B + \omega} \\ &= [q_B/P]R_T D \left(\frac{1}{q_B} - \frac{1}{q_B + \omega} \right) \quad ([q_B/P] = [(q_B + \omega)/P]) \\ &= \frac{[q_B/P]R_T D \omega}{q_B(q_B + \omega)}, \end{aligned} \quad (7.26)$$

$$\Gamma_B^2(\omega) = \sqrt{2A_B D h_B} \frac{\omega^2}{2q_B(q_B + \omega)} = \frac{h_B \omega^2}{2(q_B + \omega)}. \quad (7.27)$$

When we substitute expressions (7.24)-(7.27) into (7.23), we obtain the following:

$$\max_{\omega} \Gamma(\omega) = \begin{cases} \Gamma_1(\omega) = \frac{[q_B/P]R_T D}{q_B} - \frac{R_T D}{P} - \frac{h_B (q_B^l - q_B)^2}{2q_B^l} & \text{if } \omega = q_B^l - q_B, \\ \Gamma_2(\omega) = \frac{[q_B/P]R_T D \omega}{q_B(q_B + \omega)} - \frac{h_B \omega^2}{2(q_B + \omega)} & \text{if } 0 \leq \omega \leq q_B^f - q_B. \end{cases}$$

We can easily calculate the value of $\Gamma_1(\omega)$, say γ_1 . On the other hand, $\Gamma_2(\omega)$ is a concave function over its feasible region. The first and second derivatives with respect to ω are

$$\frac{d\Gamma_2(\omega)}{d\omega} = \frac{[q_B/P]R_T D + \sqrt{2A_B D h_B}/2}{(q_B + \omega)^2} - \frac{q_B^2 h_B}{2},$$

$$\frac{d^2\Gamma_2(\omega)}{d\omega^2} = -\frac{([q_B/P]R_T D + \sqrt{2A_B D h_B}/2)}{(q_B + \omega)^3}.$$

We can also show $\frac{d\Gamma_2(\omega)}{d\omega}|_{\omega=0^+}$, which implies that $\Gamma_2(\omega)$ is increasing at the left end point of the feasible region. As a consequence of these properties of $\Gamma_2(\omega)$, its maximizer, say ω^0 , is $\min\{\bar{\omega}, q_B^f - q_B\}$ where $\bar{\omega}$ is the stationary point of $\Gamma_2(\omega)$, and its

value is given by

$$\bar{\omega} = \sqrt{\frac{[q_B/P]R_T D + q_B^2 h_B/2}{h_B/2}} - q_B = q_B \sqrt{1 + \frac{[q_B/P]R_T}{A_B}} - q_B.$$

Finally, the maximizer of $\Gamma(\omega)$, say ω^* , is either $q_B^l - q_B$ or w^0 , whichever leads to more savings. Using these results, we next present the lemma to characterize the solution to the first problem:

LEMMA 11 *The solution to the first problem given by (7.20), namely q_T^1 , is given by*

$$q_T^1 = \arg \min \{q_B^l, \min \{q_B \sqrt{\frac{A_B + [q_B/P]R_T}{A_B}} \wedge q_B^f\}\}. \quad (7.28)$$

Next, we present the solution of the second problem, (7.21), in a similar manner. The solution procedure is very similar to the solution procedure in (7.20). Hence, we skip the details and restate the problem as:

$$\max_u \Lambda(u) = \begin{cases} \Lambda_1(u) = \frac{R_T D [q_{SB}/P]}{q_{SB}} - \frac{R_T D}{P} - \frac{h_B (q_{SB}^l - q_B)^2}{2q_{SB}^l} \\ \quad - 1_{q_{SB} > q_B} \delta_S^- \\ \quad - 1_{q_{SB} > q_B} \left(\delta_S - \frac{h_B (q_{SB} - q_B)^2}{2q_{SB}} \right) & \text{if } u = q_{SB}^l - q_{SB}, \\ \Lambda_2(u) = \frac{[q_{SB}/P]R_T D u}{q_{SB}(q_{SB} + u)} - \frac{h_B (q_{SB} + u - q_B)^2}{2(q_{SB} + u)} \\ \quad - 1_{q_{SB} > q_B} \left(\delta_S - \frac{(q_{SB} - q_B)^2}{2q_{SB}} \right) \\ \quad - 1_{q_{SB} < q_B} \delta_S^- & \text{if } 0 \leq u \leq q_{SB}^f - q_{SB}. \end{cases}$$

$\Lambda_1(u)$ is a constant value, and we can easily compute it. The second piece, $\Lambda_2(u)$, is a concave function of u . (Observe that the relative locations of q_{SB} and q_B do not change the structure of $\Lambda_2(u)$.) The first derivative and second derivative with respect to u are given by

$$\frac{d\Lambda_2(u)}{du} = \frac{1}{(q_{SB} + u)^2} \left[[q_{SB}/P]R_T D - \frac{h_B}{2}(q_{SB} + q_B + u)(q_{SB} - q_B + u) \right],$$

$$\frac{d^2\Lambda_2(u)}{du^2} = \frac{-2D(\lceil q_{SB}/P \rceil R_T + A_B)}{(q_{SB} + u)^3}.$$

$\Lambda_2^{(2)}(u) < 0$ for $u \geq 0$; and this ensures the concavity. The maximizer of $\Lambda_2(u)$, namely u_2^* , is either the stationary point, or one of the end points, of the feasible region. If $\frac{d\Lambda_2(u)}{du}|_{u=0^+} < 0$, then $u_2^* = 0$ because $\frac{d\Lambda_2(u)}{du}|_{u=0^+} < 0$ shows that $\Lambda_2(u)$ is decreasing over the feasible region. $\frac{d\Lambda_2(u)}{du}|_{u=0^+}$ is

$$\frac{d\Lambda_2(u)}{du}|_{u=0^+} = \frac{\lceil q_{SB}/P \rceil R_T D - (h_B/2)(q_{SB}^2 - q_B^2)}{q_{SB}^2}.$$

If $\frac{d\Lambda_2(u)}{du}|_{u=0^+} \geq 0$, then the maximizer is $u^0 = \min\{\bar{u}, q_{SB}^f - q_{SB}\}$ where \bar{u} is the stationary point and its expression is

$$\bar{u} = q_B \sqrt{1 + \frac{\lceil q_{SB}/P \rceil R_T}{A_B}} - q_{SB}.$$

The solution of this problem, say u^* , is either $q_{SB}^l - q_{SB}$ or u^0 , whichever leads to higher savings. We are now ready to state our next lemma that characterizes the solution to the second problem:

LEMMA 12 *The solution to the problem in (7.21), namely q_T^2 , is given by*

$$q_T^2 = \arg \min\{q_{SB}^l, \max\{q_{SB}, \min\{q_B \sqrt{\frac{A_B + \lceil q_{SB}/P \rceil R_T}{A_B}}, q_{SB}^f\}\}\}. \quad (7.29)$$

The solution of the transporter's problem, i.e., the transporter's optimal response to the supplier's quantity discount schedule, is $\arg \min\{q_T^1, q_T^2\}$.

VII.4.2.2. Freight Discount Schedule

The next step for the transporter is to find the mechanism that will induce a change in the buyer's annual cost function so that the minimizer of the resulting function is q_T . Freight discounts work for this purpose. We define p_T' as the discounted per unit

transfer charge. At this point, we should note that the interpretation of this freight discount by the buyer will be different from the interpretation of the freight discounts in the case where the supplier and buyer act as a single unit or where the supplier incurs transportation charges. The reader may refer to Appendix A for a detailed discussion of this.

Depending on the value of q_T , we propose three different discount mechanisms for the transporter:

1. $q_T < q_B$: The following quantity discount will suffice to align the buyer:

$$\bar{P}_T(q) = \begin{cases} p'_T q & \text{if } q \leq q_T, \\ p_T q & \text{if } q_T < q. \end{cases} \quad (7.30)$$

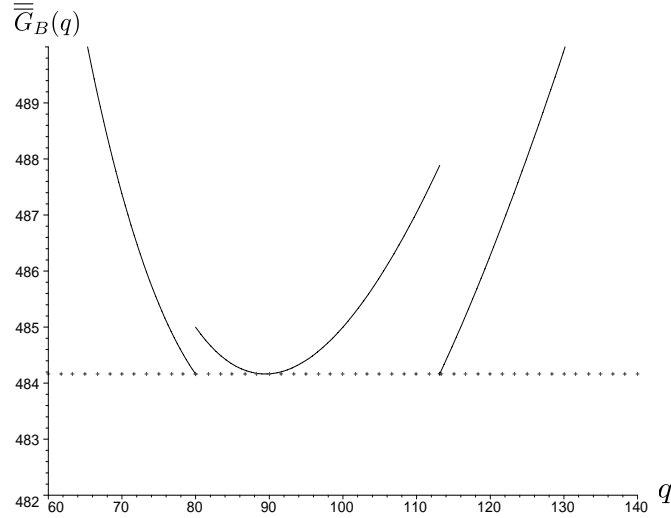
Then, the buyer's annual cost, $\bar{G}_B(q)$, is given by

$$\bar{G}_B(q) = \begin{cases} \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + p'_T)D, & \text{if } 0 < q \leq q_T, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + p_T)D, & \text{if } q_T < q < q_{SB}, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p'_S + p_T)D, & \text{if } q_{SB} \leq q. \end{cases} \quad (7.31)$$

Figure 13 provides an illustration of $\bar{G}_B(q)$ in this case.

2. $q_B \leq q_T < q_{SB}$: In this case the transporter would choose to offer a partial freight discount, whose structure can be given by

$$\bar{P}_T(q) = \begin{cases} p_T q & \text{if } q < q_T, \\ p'_T q & \text{if } q_T \leq q < q_{SB}, \\ p_T q & \text{if } q_{SB} \leq q. \end{cases} \quad (7.32)$$

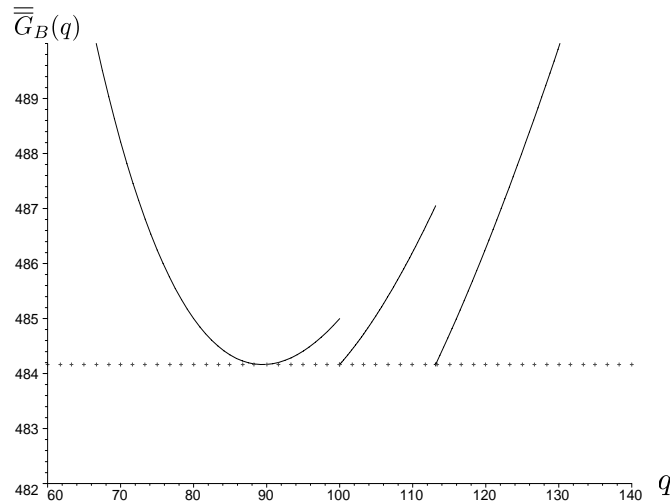
Figure 13 An Illustration of $\overline{\overline{G}}_B(q)$ in Expression (7.31)

The buyer's annual cost, $\overline{\overline{G}}(q)$, is given by

$$\overline{\overline{G}}_B(q) = \begin{cases} \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + p_T)D, & \text{if } 0 < q < q_{SB}, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + p'_T)D, & \text{if } q_T \leq q < q_{SB}, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p'_S + p_T)D, & \text{if } q_{SB} \leq q. \end{cases} \quad (7.33)$$

Figure 14 provides an illustration of $\overline{\overline{G}}_B(q)$ in this case.

3. $q_T \geq q_{SB}$: We know that $\overline{\overline{G}}_B(q)$ is increasing over $[q_{SB}, \infty)$. When $q_T \geq q_{SB}$, it will be sufficient for the transporter to offer a freight discount starting from q_T in order to coordinate the buyer at q_T . Under this discount, the transportation

Figure 14 An Illustration of $\overline{\overline{G}}_B(q)$ in Expression (7.33)

price schedule as a function of the shipment quantity, namely \overline{P}_T , is

$$\overline{P}_T(q) = \begin{cases} p_T q & \text{if } q < q_T, \\ p'_T q & \text{if } q_T \leq q. \end{cases} \quad (7.34)$$

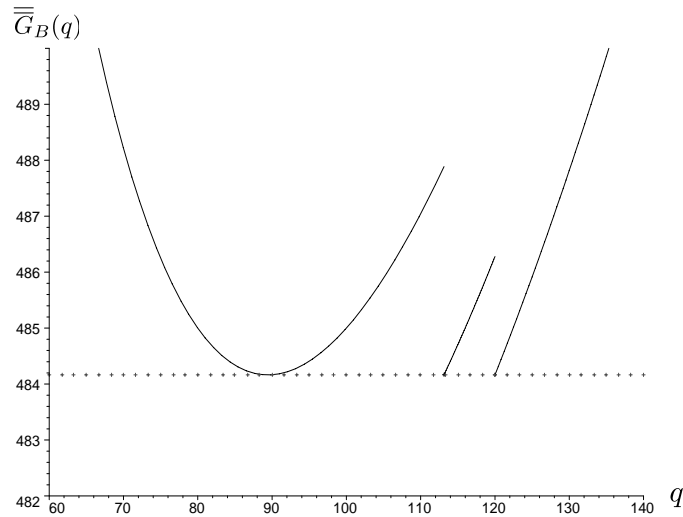
Then, the buyer's annual cost, $\overline{\overline{G}}_B(q)$, is given by

$$\overline{\overline{G}}_B(q) = \begin{cases} \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + p_T)D, & \text{if } 0 < q < q_{SB}, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p'_S + p_T)D, & \text{if } q_{SB} \leq q < q_T, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p'_S + p'_T)D, & \text{if } q_T \leq q. \end{cases} \quad (7.35)$$

Figure 15 provides an illustration of $\overline{\overline{G}}_B(q)$ in this case.

VII.4.2.3. Supplier's Optimal Strategy

It is not possible to characterize a closed form expression of the supplier's optimal strategy, namely q_{SB} and p'_S . We derive the supplier's optimal response numerically.

Figure 15 An Illustration of $\overline{\overline{G}}_B(q)$ in Expression (7.35)

VII.5. Conclusion

In this paper, we set a framework for incorporating transporters into supply chain coordination. We considered a two-echelon setting that includes one supplier and one buyer. Customer demand is deterministic, stationary and observed only at the buyer's site. The buyer replenishes his inventory from the supplier, and the transporter is responsible for the delivery of the items from the supplier to the buyer. In the classical channel coordination literature, the emphasis is on supplier-buyer coordination, i.e. finding the system optimal solution and coordination mechanisms that the supplier can employ in order to coordinate the buyer. In this study, we initiate a different perspective on channel coordination, and we study the transporter's coordination problem and the mechanisms that the transporter can use to coordinate the system.

In our analysis, we decomposed the basic setting into two models which differ based on the interactions between the buyer and supplier. For both models, we

formulated and solved the transporter's problem. Then, we identified some of the possible coordination mechanisms that the transporter can use in order to coordinate the buyer and/or supplier.

Model I assumes that supplier and buyer act as a single unit. That is, either they are controlled by a central decision maker or they are coordinated by a contract. Hence, given an initial transportation price schedule, they choose the order quantity that minimizes their joint annual cost. Intuitively, this order quantity is not always the most desirable load for the transporter. Therefore, he may want to try to induce the supplier-buyer system to order at a different quantity level. Notice that because the supplier-buyer unit already operates at their joint optimal, any other order quantity will increase their average annual cost. The transporter must take this fact into account while trying to induce the buyer and the supplier.

For this model, we showed that the transporter's problem is to minimize the system-wide costs, regardless of his initial price schedule. We developed a solution procedure to solve the joint minimization problem of system-wide costs. Then, we proposed several coordination mechanisms for different initial transportation price schedules. When the initial price schedule was on a per unit basis, we studied the effects of incremental and all-units freight discounts on system coordination. We have derived the necessary and sufficient conditions that allow these discounts to coordinate the supplier-buyer system. Our results show that both the structure and the performance of these mechanisms are highly data dependent. Depending on the relative locations of q_T and q_{SB} , the discount schedules can take different forms such as negative discounts which discourages high volumes, discounts with a fixed reward, or partial discounts which are only valid for one interval. For instances where the necessary conditions are not satisfied, freight discount schedules do not coordinate the system. To make a comparison between the relative performances of incremental

and all-units discounts, we can say that incremental discounts perform much better in coordinating the system because they provide an opportunity to the transporter to change the structure of the joint annual cost of the supplier-buyer system. When the initial price schedule includes a fixed price per each dispatch in addition to a per unit price, freight discounts could still coordinate the buyer and the supplier in the same manner. In addition, changing the value of the fixed price may also work for coordination. When the initial pricing structure also includes a per truck price, the safest way to coordinate the system is to bind the per truck price to the per truck cost and the fixed price to the fixed cost. Unlike other coordination mechanisms, this mechanism would always work.

Regardless of the initial pricing structure, transporter can offer a contract to the buyer and the supplier, which specifies the order quantity, order frequency, transportation price, and compensation amount to be made by the transporter, in order to coordinate the system to his solution. We name such contracts as *order quantity contracts*. These contracts always coordinate the system.

All of the coordination mechanisms that we propose in this chapter provide a win- situation for all of the parties as long as the necessary and sufficient conditions are satisfied. Table 9 summarizes the results for *Model I*.

Model II assumes the supplier and the buyer act independently. The analysis of this model depends on which party is incurring the transportation charges. If the supplier is incurring the transportation charges, the transporter's problem and the coordination mechanisms are identical to the ones in the first model. On the other hand, if the buyer is responsible for the transportation charges, then both the supplier and the transporter will try to affect his decisions. We modelled this case as a Stackelberg Game, where the supplier leads the game and offers a quantity discount schedule. Based on the discount schedule, the transporter offers a freight discount

Table 9 Summary of Coordination Mechanisms for *Model I*

Initial Price Schedule	Coordination Mechanism		
	Mechanism	Structure	N&S Conditions
$P_T(q) = p_T q$	<u>Incremental Discount</u>	$q_J < q_T$: Equation (7.7) $q_J > q_T$: Equation (7.8)	i1C1-i1C3 i2C1-i2C3
	<u>All-Units Discount</u>	$q_J < q_T$: Equation (7.13) $q_J > q_T$: Equation (7.14)	either IIC1-IIC6 or IIIC1-IIIC6 IC1-IC2
	<u>Contract</u>	(q, FP) : $q = q_T$ $FP = G_{SB}(q_T, n_T) - G_{SB}(q_J, n_J)$	always works
$P_T(q) = p_T q + A_p$	<u>Incremental Discount</u>	$q_J < q_T$: Equation (7.7) $q_J > q_T$: Equation (7.8)	i1C1-i1C3 i2C1-i2C3
	<u>All-Units Discount</u>	$q_J < q_T$: Equation (7.13) $q_J > q_T$: Equation (7.14)	either IIC1-IIC6 or IIIC1-IIIC6 IC1-IC2
	<u>Change A_p</u>	A'_p given by Equation (7.12)	either i1C1, i1C3 or i2C1, i2C3
	<u>Contract</u>	(q, FP) : $q = q_T$ $FP = G_{SB}(q_T, n_T) - G_{SB}(q_J, n_J)$	always works
$P_T(q) = p_T q + A_p + [q/P] M_p$	<u>Reflect Costs</u>	$p'_T q + A_T + [q/P] R_T$	always works
	<u>Quantity Contracts</u>	(q, FP) : $q = q_T$ $FP = G_{SB}(q_T, n_T) - G_{SB}(q_J, n_J)$	always works

schedule. As a result, the buyer observes a cost structure (which we represented by $\overline{\overline{G}}_B(q)$) that has both quantity discounts and freight discounts. The buyer chooses the order quantity that minimizes this cost.

VII.5.1. Managerial Insights and Future Work

One of the most important results of this study is that the transporter is better off by being integrated into the channel coordination, which also reduces the system-wide costs. As a consequence, the supplier and buyer are better off sharing the system savings. Hence, it is always beneficial for the whole chain to incorporate the transporter into the channel coordination mechanisms.

An intuitive coordination mechanism that the transporter can implement is reflecting the actual transportation costs and imposing a profit margin in the price schedule, which is always successful for coordinating supplier and buyer. By doing this, the transporter can make himself look as if he is being used as a private carriage. Offering freight discounts is another way to coordinate the channel. Typically, transportation companies offer freight discounts in order to increase the volume of transferred items. The results of this study indicate that freight discounts can also be used for coordination purposes.

Another interesting result of this study is that sometimes it is effective for the transporter to offer negative freight discounts and/or partial freight discounts. Such negative discounts may result from the transporter's cost structure. For the transporter, it is not always less more desirable to have more volume to ship. Often it is more important for the transporter to better utilize his trucks.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

This dissertation investigates the impact of transportation costs and transporters in supply chain coordination. The goals of the dissertation are to build on the theoretical framework of the existing literature in the context of joint inventory and transportation decisions and to address the impact of transporters in channel coordination by integrating them in the coordination processes.

As discussed in Chapter I, transportation costs realize scale economies. Therefore it is a common logistics practice to consolidate small size shipments in order to benefit from the reduced fares for larger shipment sizes. *Time-based*, *quantity-based*, and *time-and-quantity* based consolidation policies are the most commonly implemented consolidation regimes. Both the analytical treatment for optimizing the policy parameters of these consolidation regimes and the synchronization of inventory replenishment and shipment consolidation decisions have received significant interest in the literature. Such problems have also practical importance; since, the recent supply chain innovations such as 3PL and VMI encourage the integration of different supply chain activities. In Chapter II, we identify the unexplored research problems in this area.

The earlier work by Çetinkaya and Lee (2000) studies an integrated inventory and shipment consolidation model under a *time-based* consolidation policy. However, it is noted in the literature that the *quantity-based* consolidation policies perform better in realizing the scale economies. In order to compare the performances of both policies, in Chapter III, we study the same model under a *quantity-based* policy. We optimize the model parameters for the *quantity policy* that we consider. We show, theoretically that when there is no inventory considerations the *quantity policy* outperforms the

time-policy in terms of the expected cost. We also show numerically, that the *quantity policy* outperforms the *time-policy* in terms of the expected cost even when there is inventory considerations. On the other hand, *time policies* are superior to *quantity policies* in terms of customer service as they guarantee timely deliveries.

The model that we consider in Chapter III, assumes the use of a private fleet for outbound shipments of the supplier. However, the use of common-carriers are also available for such shipments. Furthermore, common carrier rates also reflect the scale economies. There are some studies in the literature that analyze the shipment consolidation policies under common carrier rates. However, to the best our knowledge, joint inventory and shipment models have not been studied under common carrier rates in the literature. In Chapter IV, we provide optimal solution procedures for the model in Chapter III under common carrier rates for both *time-policy* and *quantity-policy*.

Although there are several studies on the analytical treatment of *time-* and *quantity-based* consolidation policies, the analytical treatment of *hybrid policies* is missing in the literature. However *hybrid policies*, which are also known as *time- and quantity policies*, balance the tradeoff between the timely delivery advantages of *time policies* and the transportation cost savings associated with *quantity policies*. In Chapter V, we propose several easily implementable *hybrid policies* and compare the cost and service performances of these hybrid policies with the *time-* and *quantity-* policies. We also present an analytical model for computing the optimal policy parameters for a hybrid policy.

Chapters III through V aim to study the impact of transportation cost regarding the operational supply chain decisions such as shipment scheduling and inventory replenishment. In Chapters VI and VII, we broaden our scope by introducing the transporter as a separate entity to a supply channel. Channel coordination literature focuses only on the interactions between the supplier and the buyer. However, we show

that substantial savings can be achieved if the cost parameters of the transporters are included in the decision process. With this motivation, in Chapter VI, we study a transporter-buyer channel, and benchmark the channel efficiency. In Chapter VII, we extend the transporter-buyer channel to the supplier-transporter-buyer channel. We derive the optimal policy parameters so as to minimize the system-wide costs. We also develop efficient coordination mechanisms, which the transporter can employ to align the incentives of the supplier and the buyer.

We believe that, apart from its practical contributions, this dissertation makes several theoretical contributions in modelling and optimization. Chapters III and IV build on the existing models but are important for their contribution to deterministic optimization. Chapter V is important for both its contribution to stochastic modelling and deterministic optimization. Chapters VI and VII introduce a new perspective to the channel coordination literature by introducing supply channel models with transporters. They are also important for their contribution to optimization.

REFERENCES

- Abad, P.L. 1994a. Supplier pricing when the buyer's annual requirement are fixed. *Comput. Oper. Res.* **21-2** 155-167.
- Abad, P.L. 1994b. Supplier pricing and lot sizing when demand is price sensitive. *Eur. J. Oper. Res.* **78**334-354
- Abdelwahab, W. M. and M. Sargious. 1990. Freight rate structure and optimal shipment size in freight transportation. *Logist. Transp. Rev.* **6-3** 271-292.
- Anily, S., A. Federgruen. 1990. One warehouse multiple retailer systems with vehicle routing costs. *Management Sci.* **36-1** 92-114.
- Anily, S., A. Federgruen. 1993. Two-echelon distribution systems with vehicle routing costs and central inventories. *Oper. Res.* **41** 37-47.
- Axsäter, S. 1990. Simple solution procedures for a class of two-echelon inventory problems *Oper. Res.* **38** 64-69.
- Axsäter, S. 1995. Approximate evaluation of batch-ordering policies for a one-warehouse, N non-identical retailer system under compound poisson demand. *Naval Res. Logist.* **42** 807-819.
- Axsäter, S. 1997. Simple evaluation of echelon stock (R, Q) policies for two-level inventory systems. *IIE Trans.* **29** 661-669.
- Axsäter, S. 1998. Evaluation of installation stock based (R, Q) -policies for two-level inventory systems for poisson demand. *Oper. Res.* **46** 135-145.
- Axsäter, S. 2000. Exact analysis of continuous review (R, Q) policies in two-echelon inventory systems with compound poisson demand. *Oper. Res.* **48-5** 686-696.
- Axsäter, S. 2000. *Inventory Control*. Kluwer Academic Publishers, Norwell, MA.

- Axsäter, S. 2001. A Note on “Stock replenishment and shipment scheduling for vendor-managed inventory systems. *Management Sci.* **47-9** 1306–1310.
- Axsäter, S., K. Rosling. 1993. Installation vs. echelon stock policies for multilevel inventory control. *Management Sci.* **39** 1274–1280.
- Ballou, R. H. 1992. *Business Logistics Management*. Prentice Hall, Englewood Cliffs, NJ.
- Banerjee, A. 1986a. On a quantity discount pricing model to increase vendor profits. *Management Sci.* **32** 1513–1517.
- Banerjee, A. 1986b. A joint economic-lot-size model for purchaser and vendor. *Decision Sci.* **17** 292–311.
- Banerjee, A., S. Burton. 1994. Coordinated vs. independent inventory replenishment policies for a vendor and multiple buyers. *Internat. J. Production Economics* **35** 215–222.
- Blumenfeld, D. E. , L. D. Burns, C. F. Daganzo, M. C. Frick, and R. W. Hall 1987. Reducing logistics costs at general motors. *Interfaces.* **17-1** 26–47.
- Brennan, J.J. 1981. Models and analysis of temporal consolidation. Doctoral Dissertation. University of California at Los Angeles.
- Burns, L. D., R. W. Hall, D. E. Blumenfeld and C. F. Daganzo. 1985. Distribution Strategies That Minimize Transportation and Inventory Costs. *Oper. Res.* **33-3** 469–490.
- Cachon, G.P. 1995. Exact Evaluation of Batch Ordering Inventory Policies in Two-Echelon Supply Chains with Periodic Review. *Oper. Res.* **49-1** 79–98.
- Campbell, J. F. 1990. Designing logistics systems by analyzing transportation, inventory and terminal cost tradeoffs. *J. Bus. Logist.* **11-1** 159–179.
- Çetinkaya, S. 2004. Coordination of inventory and shipment consolidation decisions: A review of premises, models, and justification, in *Applications of Sup-*

- ply Chain Management and E-Commerce Research in Industry*. J. Geunes et. al., eds., Kluwer Series: Applied Optimization. **92** 1–50.
- Çetinkaya, S., J.H. Bookbinder. 2003. Stochastic models for the dispatch of consolidated shipments. *Transp. Res. B* **37-8** 747–768.
- Çetinkaya, S., C.-Y. Lee. 2000. Stock replenishment and shipment scheduling for vendor managed inventory systems. *Management Sci.* **46** 217–232.
- Çetinkaya, S., C.-Y. Lee. 2002. Outbound transportation: Modeling inventory and cargo capacity. *Naval Res. Logist.* **49** 531–556.
- Çetinkaya, S., E. Tekin, C.-Y. Lee. 2000. A stochastic VMII model for joint inventory replenishment and outbound shipment release decisions. *Working Paper*. A synopsis of this paper has appeared in *MSOM Conference Proceedings*, Ann Arbor, Michigan.
- Chan, L.M.A., A. Muriel, Z.-J. Shen, D. Simchi-Levi, C.-P. Teo. 2002. Effective zero inventory ordering policies for the single-warehouse multi-retailer problem with piecewise linear cost structures. *Management Sci.* **48** 1446–1460.
- Clark, A. J., H. Scarf. 1960. Optimal policies for a multiechelon inventory problem. *Management Sci.* **6** 475–490.
- Closs, D. J. and R. L. Cook. 1987. Multi-stage transportation consolidation analysis using dynamic simulation. *Internat. J. Physical Dist. and Materials Management* **17-3** 28–45.
- Cooper, M.C. 1984. Cost and delivery time implications of freight consolidation and warehouse strategies. *Internat. J. Physical Dist. and Materials Management* **14-6** 47–67.
- Dada, M. K.N. Srikanth. 1987. Pricing policies for quantity discounts. *Management Sci.* **33-10** 1247–1252.

- Daganzo, C. F. 1988. Shipment composition enhancement at a consolidation center. *Transp. Res. B* **22B-2** 103–124.
- Debodt, M., S.C. Graves. 1985. Continuous review policies for a multi-echelon inventory problem with stochastic demand. *Management Sci.* **31** 1286–1299.
- Dolan, R. J. 1987. Quantity discounts: Managerial issues and research opportunities. *Marketing Sci.* **6** 1–23.
- Ernst, R., D.F. Pyke. 1993. Optimal base stock policies and truck capacity in a two-echelon system. *Naval Res. Logist.* **40** 879–903.
- Ertek, G., P.M. Griffin. 2002. Supplier- and buyer-driven channels in a two-stage supply chain. *IIE Trans.* **34** 691–700.
- Federgruen, A. 1993. Centralized planning models for multi-echelon inventory systems under uncertainty, in *Logistics of Production and Inventory*. S. C. Graves, A. H. G. Rinnooy Kan, P. H. Zipkin, eds., Elsevier, New York, NY, 133–173.
- Federgruen, A., D. Simchi-Levi. 1995. Analytical Analysis of Vehicle Routing and Inventory Routing Problems, in *Handbooks on Operations Research and Management Science*, the volume on *Network Routing*. M. Ball, T. Magnanti, C. Monma, and G. Nemhauser, eds., North-Holland, Amsterdam, 297–373.
- Federgruen, A., P. Zipkin. 1984a. Approximations of dynamic multilocation production and inventory problems. *Management Sci.* **30** 69–84.
- Federgruen, A., P. Zipkin. 1984b. Computational issues in an infinite horizon multi-echelon inventory model. *Oper. Res.* **32** 218–236.
- Federgruen, A., P. Zipkin. 1984c. A combined vehicle routing and inventory allocation problem. *Oper. Res.* **32-5** 1019–1037.
- Gallego, G., D. Simchi-Levi. 1990. On the effectiveness of direct shipping strategy for the one-warehouse multi-retailer R-systems. *Management Sci.* **36-2** 240–243.

- Gans, N. and G. van Ryzin. 1999. Dynamic vehicle dispatching: Optimal heavy traffic performance and practical insights. *Oper. Res.* **47** 675–692.
- Geunes, J., A.Z. Zeng. 2003. Optimizing supply shortage decisions in base stock distribution operations. *J. Global Optim.* **26** 25–42.
- Goyal, S. K. 1976. An integrated inventory model for a single supplier single customer problem. *Internat. J. Production Res.* **15** 107–111.
- Goyal, S.K. 1987. Comment on: A generalized quantity discount pricing model to increase supplier's profits. *Management Sci.* **33** 1635–1636.
- Goyal, S.K. 1988. A joint economic-lot-size model for purchaser and vendor: A comment. *Decision Sci.* **19** 236–241.
- Gupta, Y. P. and P. K. Bagchi. 1987. Inbound freight consolidation under just-in-time procurement: Application of clearing models. *J. Bus. Logist.* **8-2** 74–94.
- Hall, R. W. 1987. Consolidation strategy: Inventory, vehicles, and terminals. *J. Bus. Logist.* **8-2** 57–73.
- Higginson, J.K. 1995. Recurrent decision approaches to shipment release timing in freight consolidation. *Internat. J. Physical Dist. and Logist. Management* **25-5** 3–23.
- Higginson, J. K. and J. H. Bookbinder. 1994. Policy recommendations for a shipment consolidation program. *J. Bus. Logist.* **15-1** 87–112.
- Higginson, J. K. and J. H. Bookbinder. 1995. Markovian decision processes in shipment consolidation. *Transp. Sci.* **29-3** 242–255.
- Hill, R. M. 1999. The optimal production and shipment policy for the single-vendor single-buyer integrated production inventory problem. *Internat. J. Production Res.* **97** 2463–2475.
- Hofmann, C. 2000. Supplier's pricing policy in a just-in-time environment. *Comp. Oper. Res.* **27** 1357–1373.

- Jackson, G.C. 1981. Evaluating order consolidation strategies using simulation. *J. Bus. Logist.* **2-2** 110–138.
- Jeuland, A. P., S. M. Shugan. 1983. Managing channel profits. *Marketing Sci.* **2** 239–272.
- Joglekar, P. N. 1988. Comments on a quantity discount pricing model to increase vendor profits. *Management Sci.* **34** 1391–1400.
- Joglekar, P. N., J. M. Kelly. 1998. Volume discount decisions: Wealth maximization or EOQ approach?. *Naval Res. Logist.* **45** 377–389.
- Klastorin, T.D., K. Moinzadeh, J. Son. 2002. Coordinating orders in supply chains through price discounts. *IIE Trans.* **34** 679–689.
- Kohli, R., H. Park. 1989. A cooperative game theory model of quantity discounts. *Management Sci.* **35-6** 693–707.
- LaL, R., R. Staelin. 1984. An approach for developing an optimal discount pricing policy. *Management Sci.* **30-12** 1524–1539.
- Lau, A.H.H., H.-S. Lau. 2003. Effects of a demand-curve's shape on the optimal solutions of a multi-echelon inventory/pricing model. *Eur. J. of Oper. Res.* **147** 530–548.
- Lee, C.-Y. 1986. The economic order quantity for freight discount costs. *IIE Trans.* **18** 318–320.
- Lee, C.-Y., S. Çetinkaya, W. Jaruphongsa. 2003. A dynamic model for inventory lot-sizing and outbound shipment scheduling at a third party warehouse. *Oper. Res.* **51-5** 735–747.
- Lee, H. L., M. J. Rosenblatt. 1986. A generalized quantity discount pricing model to increase supplier's profits. *Management Sci.* **32** 1177–1185.
- Lu, L. 1995. A one-vendor multi-buyer integrated inventory model. *Eur. J. Oper. Res.* **81** 312–323.

- Masters, J. M. 1980. The effects of freight consolidation on customer service. *J. Bus. Logist.* **2-1** 55–74.
- Minkoff, A. S. 1993. A Markov decision model and decomposition heuristic for dynamic vehicle dispatching. *Oper. Res.* **41-1** 77–90.
- Monahan, J. P. 1984. A quantity discount pricing model to increase vendor profits. *Management Sci.* **30** 720–726.
- Muckstadt, J. A., R.O. Roundy. 1993. Analysis of multistage production systems, in *Handbook of Operations Research and Management Science*. S. C. Graves, A. H. G. Rinnooy Kan, P. H. Zipkin, eds., North-Holland, Amsterdam, 59–132.
- Newbourne, M. J. and C. Barrett. 1972. Freight consolidation and the shipper. Parts 1 to 5. *Transportation and Distribution Management*, 12, Numbers 2 to 6.
- Parlar, M., Q. Wang. 1994. Discounting decisions in a supplier-buyer relationship with a linear buyer's demand. *IIE Trans.* **26-234**–41
- Pollock, T. 1978. A management guide to LTL consolidation. *Traffic World* **April 3** 29–35.
- Pooley, J. and A. J. Stenger. 1992. Modeling and evaluating shipment consolidation in a logistics system. *J. Bus. Logistics* **13-2** 153–174.
- Porteus, E.L. 1985. Investing in reduced setup in the EOQ model *Management Sci.* **31-8** 998–1010.
- Powell, W. B. 1985. Analysis of vehicle holding and cancellation strategies in bulk arrival, bulk service queues. *Transp. Sci.* **19-4** 352–377.
- Powell, W.B. and P. Humblet. 1986. The bulk service queue with general control strategy: Theoretical analysis and a new computational procedure. *Oper. Res.* **34-2** 267–275.

- Qu, W.W., J.H. Bookbinder, P. Iyogun. 1999. An integrated inventory-transportation system with modified periodic policy for multiple products. *Eur. J. Oper. Res.* **115** 254–269.
- Resnick, S.I. 2002. *Adventures In Stochastic Processes*. Birkhäuser,0-8176-3591-2, Rennsealer, NY.
- Rosling, K. 1989. Optimal inventory policies for assembly systems under random demand. *Oper. Res.* **37** 565–579.
- Roundy, R. 1985. 98%-effective integer-ratio lot-sizing for one-warehouse multi-retailer systems. *Management Sci.* **31** 1416–1430.
- Roundy, R. 1986. A 98%-effective lot-sizing rule for a multi-product, multi-stage production/inventory system. *Math. Oper. Res.* **11** 699–727.
- Russell, R. M. and L. Krajewski. 1991. Optimal purchase and transportation cost lot-sizing for a single Item. *Decision Sci.* **22** 940–952.
- Schwarz, L. B. 1973. A simple continuous review deterministic one-warehouse N -retailer inventory problem. *Management Sci.* **19** 555–566.
- Stidham, S., Jr. 1977. Cost models for stochastic clearing systems. *Oper. Res.* **25-1** 100–127.
- Toptal, A. 2003. Generalized models and benchmarks for channel coordination. Doctoral Dissertation. Texas A&M University at College Station.
- Toptal, A., S. Çetinkaya. 2004. Decentralized models for buyer-vendor coordination under generalized replenishment costs. *Proceedings of IIE Conference*, Houston, TX.
- Toptal, A., S. Çetinkaya. 2006. Contractual agreements for coordination and vendor-managed delivery under explicit transportation considerations. *Naval Res. Logist.* **53-5** 397–417.

- Toptal, A., S. Çetinkaya, C.-Y. Lee. 2003. The buyer vendor coordination problem: Modeling inbound and outbound cargo capacity and costs. *IIE Trans. Scheduling Logist.* **35** 987–1002
- Tsay, A., S. Nahmias, N. Agrawal. 2000. Modeling supply chain contracts: A review. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Norwell, MA, 299–330.
- Viswanathan, S. 1998. Optimal strategy for integrated vendor-buyer inventory model. *Eur. J. Oper. Res.* **105** 38–42.
- Viswanathan, S. and K. Mathur. 1997. Integrating routing and inventory decisions in one-warehouse multi-retailer multi-product distribution systems. *Management Sci.* **43-3** 294–312.
- Weng, Z. K. 1995a. Channel coordination and quantity discounts. *Management Sci.* **41** 1509–1522.
- Weng, Z.K. 1995b. Modeling quantity discounts under general price-sensitive demand functions: Optimal policies and relationships. *Eur. J. Oper. Res.* **86** 300–314.
- Weng, Z.K., A.Z. Zeng. 2001. The role of quantity discounts in the presence of heterogenous buyers. *Annals Oper. Res.* **107** 369–383.
- Yano, A.C., Y. Gerchak. 1989. Transportation contracts and safety stocks for just-in-time deliveries. *J. Manuf. Oper. Management* **2** 314–330.

APPENDIX A

COORDINATION MECHANISMS FOR SUPPLIER-BUYER SYSTEM

In Section VII.2.2, we discuss the supplier's problem of influencing the buyer's decisions. There, we show that the best order quantity for the supplier is in fact the joint optimal quantity of the supplier-buyer problem which is denoted by q_J . The corresponding n value is denoted by n_J . This conclusion is independent of the transportation price or of the party that is incurring the transportation cost.

The supplier also has to find effective mechanisms to align the buyer's decisions. She can do this in two ways:

- Sign a contract with the buyer,
- Change the wholesale price structure, so that the optimal solution to the buyer's problem under this new price is q_J .

In either approach, the supplier has to compensate for the increase in the buyer's total annual cost.

In this paper, we consider two models. In *Model II*, we assume that there is virtually no coordination between the supplier and the buyer. Hence, in this model, signing a contract with the buyer is not an option. As a result, the only thing that the supplier can do is to change her wholesale price schedule.

In this Appendix, we provide solution methods for minimizing the joint annual cost of the supplier and the buyer, $\tilde{G}_{SB}(q, n)$, and we also provide pricing mechanisms that the supplier can employ to align the buyer. Minimization procedures and the structure of the pricing mechanisms depend on two factors:

- i The structure of the transportation price,

ii Which party, buyer or supplier, incurs the transportation charges.

Therefore, we present our results for four different transportation price schedules: *per unit price*; *per unit price and fixed dispatch price*; *per unit price with fixed dispatch price and fixed truck price*; and *per unit price with freight discounts*.

Case 1: $P_T(q) = p_T q$

Under this transfer pricing structure, the annual transportation cost of the supplier-buyer is $p_T D$. With this term, \mathcal{P}_{SB} is

$$\mathcal{P}_{SB} : \min_{q \geq 0, n \in \mathbb{Z}^+} = \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{A_B D}{q} + \frac{h_B q}{2} + p_T D.$$

This problem is identical to the problem in Banerjee (1986). The closed form solution is as follows:

- n_{SB} is one of the closest integers to $n^0 = \sqrt{Av(h_B - h_S)/(A_B h_S)}$, whichever leads to a smaller G_{SB}
- For a given n_J :

$$q_{SB} = \sqrt{\frac{2(A_S + n_{SB} A_B) D}{n_{SB}(h_B - h_S + n_{SB} h_S)}}.$$

Coordination Mechanism if Supplier Incurs $P_T(q) = p_T q$

We know that given all the parameters, it is the buyer who makes the final decision about his order quantity, and the buyer always chooses the order quantity that minimizes his annual cost $G_B(q)$. Before coordinating with the supplier, his annual cost function is:

$$G_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + p_S D,$$

and his optimal order quantity is

$$q_B = \sqrt{\frac{2A_B D}{h_B}}.$$

Now, if the supplier wants to coordinate the buyer at q_J , she needs to change the pricing structure, $P_S(q)$, to $\overline{P}_S(q)$. We define the buyer's annual cost under this new pricing schedule as $\overline{G}_B(q)$, and it is given by

$$\overline{G}_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{\overline{P}_S(q) D}{q}.$$

The supplier should set her new pricing schedule such that

- i- $\overline{G}_B(q_J) \leq G_B(q_B)$,
- ii- $q_{SB} = \arg \min \overline{G}_B(q)$.

The first condition sets the limits of the modified pricing structure, $\overline{P}_S(q)$, such that the buyer's optimal annual cost under this modified price should not exceed his initial optimal annual cost. The second condition implies that the buyer's solution under the modified pricing structure should be equal to the joint solution.

In fact, to collect all the system savings, the supplier would want the above inequality to be equality. We can also characterize this as the supplier providing a reward to the buyer if the buyer operates at q_{SB} . The amount of the reward can be found by:

$$G_B(q_{SB}) - \text{reward} = G_B(q_B).$$

There are several ways for the supplier to decide on the reward's structure. A linear incentive would work. We know the value of $\Delta = G_B(q_{SB}) - G_B(q_B)$. We can express Δ in terms of D , that is $\Delta = Dr$, where $r \geq 0$. The r value can be interpreted as a per unit reward. Hence, if the new per unit price is set to $c_{S'} = p_S - r$, then the first condition is satisfied. The second condition remains to be satisfied. Notice that,

with the new per unit price, the annual cost function for the supplier does not change its structure but only shifts downwards by Δ . That is why the buyer's optimal solution is still q_B . However, if the per unit reward is given only when the order quantity is greater than some q value where $q > q_B$, then that q value will minimize the annual cost of the buyer since the G_B is increasing over $[q_B, \infty)$. Accounting for this fact, the q value where the per unit reward starts should be q_{SB} .

As a result of the above discussion, the following pricing scheme satisfies the two conditions, i.e. coordinates the buyer at q_{SB} :

$$\bar{P}_S(q) = \begin{cases} p_S q, & \text{if } 0 < q < q_{SB}, \\ p'_S q & \text{if } q_{SB} \leq q, \end{cases} \quad (\text{A.1})$$

where

$$p'_S = p_S - \{G_B(q_{SB}) - G_B(q_B)\}/D. \quad (\text{A.2})$$

In the classical channel coordination literature, this pricing schedule is known as an all-units quantity discount.

Coordination Mechanism if the Buyer Incurs $P_T(q) = p_T q$

When the buyer incurs the transportation charges, his annual cost is

$$G_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + p_S D + p_T D.$$

Essentially this cost is identical to the one when the supplier incurs the transportation charges. The only difference is an extra constant term, $p_T D$, which does not affect any decision.

Since everything remains the same, the coordination mechanism is the same as in the previous case where the transportation charge was on the supplier. The discount schedule can be found using equations (A.1) and (A.2).

Case 2: $P_T(q) = p_Tq + A_p$

With this pricing schedule, the annual revenue of the transporter, i.e. the annual transportation cost of the supplier (or the buyer), is

$$\frac{A_p D}{q} + p_T D.$$

Then, the joint optimization problem of the supplier-buyer, \mathcal{P}_{SB} becomes:

$$\mathcal{P}_{SB} : \min_{q \geq 0, n \in \mathbb{Z}^+} = \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{A_p D}{q} + p_T D.$$

It can be easily noticed that the structure of the above problem is identical to the \mathcal{P}_{SB} of **Case 1**. The only difference is that now we have $A_B + A_p$ instead of A_B . Hence, the expressions of (q_{SB}, n_{SB}) can be given as in that case, with $A_B + A_p$ replacing A_B .

Coordination Mechanism if the Supplier Incurs $P_T(q) = p_Tq + A_p$

The buyer's annual cost is independent of the transportation price; therefore, q_B remains the same as in **Case 1**. The quantity discount works to coordinate the system, and the discount schedule can be found using equations (A.1) and (A.2).

Coordination Mechanism if the Buyer Incurs $P_T(q) = p_Tq + A_p$

When the buyer incurs the transportation charges, his optimal order quantity is

$$q_B = \sqrt{\frac{2(A_B + A_p)D}{h_B}}.$$

The quantity discount still works in the same way and the discount schedule can be found using equations (A.1) and (A.2).

Case 3 - $P_T(q) = p_Tq + A_p + \lceil q/P \rceil M_p$

The implied annual transfer price is

$$p_TD + A_pD/q + \lceil q/P \rceil M_pD/q.$$

With this $\Pi(q)$, the supplier-buyer problem \mathcal{P}_{SB} becomes

$$\begin{aligned} \mathcal{P}_{SB} : \min_{q \geq 0, n \in \mathbb{Z}^+} &= \frac{A_S D}{nq} + \frac{h_S(n-1)q}{2} + \frac{A_B D}{q} + \frac{h_B q}{2} + \frac{A_p D}{q} \\ &= + \frac{\lceil q/P \rceil M_p D}{q} + p_T D. \end{aligned}$$

This problem is identical to the transporter's problem that we discuss in detail in Section VII.3.1. Hence, the algorithm given in that section can be used to solve it. We cannot obtain closed form expressions for q_{SB} and n_{SB} .

Coordination Mechanism if the Supplier Incurs $P_T(q) = p_Tq + A_p + \lceil q/P \rceil M_p$

As mentioned earlier, the buyer's annual cost remains an EOQ type function when the supplier incurs the transportation charges. Therefore, a quantity discount coordinates the system as in the other cases and the discount schedule can be found using Equations (A.1) and (A.2).

Coordination Mechanism if the Buyer Incurs $P_T(q) = p_Tq + A_p + \lceil q/P \rceil M_p$

With this transportation price, the buyer's annual cost $G_B(q)$ is

$$G_B(q) = \frac{A_B D}{q} + \frac{h_B q}{2} + p_S D + p_T D + A_p D/q + \lceil q/P \rceil M_p D/q.$$

The minimization of $G_B(q)$ is given in Lee (1986). Unfortunately there is no closed form expression for the solution.

This is one of the cases where an all-units quantity discount may not work. We

next present a case where a quantity discount definitely does not work.

Using Lee (1986), let q_{EOQ} be the EOQ for the buyer, when the truck cost is ignored. Let i be such that $iP < q_{EOQ} \leq (i+1)P$. Define q_B as the EOQ for the buyer when he has an additional fixed cost of $(i+1)M_p$. The optimal q for the buyer is $\arg \min\{G_B(iP), G_B(q_B), G_B((i+1)P)\}$. Let's assume that iP is the buyer's optimal order quantity and call this quantity q'_B . Also assume that $q'_B < q_{SB} < q_B < (i+1)P$. Under this scenario, a quantity discount will not be able to coordinate the system under q_{SB} . The reason for this is as follows: $G_B(q)$ is decreasing on $(q'_B, q_B]$ but $G_B(q_B) > G_B(q'_B)$. For a simple all-units quantity discount to work, the discount should start at q_{SB} and the annual cost of the buyer under the discount, should be at most equal to $G_B(q'_B)$, i.e., $\bar{G}_B(q_{SB}) \leq G_B(q_B)$. However, since the discount also applies to all $q \geq q_{SB}$, and $\bar{G}_B(q)$ is decreasing on $(q'_B, q_B]$, the buyer's annual cost achieves its minimum at q_B . Hence, he will want to operate at q_B instead of q_{SB} .

Under such a case, coordination at q_{SB} can best be satisfied with a special contract. Fixed annual payment contracts from the supplier to the buyer are always sufficient for this purpose. The fixed annual payment amount is given by $G_B(q_{SB}) - G_B(q_B)$.

However, one should be careful about the interpretation and implementation of contracts. The contract that we propose should offer a fixed annual payment only if the buyer's order quantity is set to q_{SB} . That is what differentiates the contract from a quantity discount. Remember that quantity discounts are given for all quantities that are greater than some q value. The other important point about the contract is the implementation process, which involves specifically deciding on how these annual payments are to be made. A very practical method is to decrease the per unit wholesale price which looks similar to the quantity discounts, but the new wholesale price can be found by equation (A.2).

Case 4: Price Schedule with Freight Discounts

In this case, the per unit transportation price is

$$p_T(q) = \begin{cases} c_T^1 & \text{if } q < q_2, \\ c_T^2 & \text{if } q_2 \leq q, \end{cases} \quad (\text{A.3})$$

where $c_T^1 > c_T^2$. Hence, $P_T(q)$ has the following form:

$$P_T(q) = \begin{cases} c_T^1 q & \text{if } q < q_2, \\ c_T^2 q & \text{if } q_2 \leq q. \end{cases}$$

Notice that the cost of shipping any q units where $q \in ((c_T^2 q_2)/c_T^1, q_2)$ is more costly than shipping q_2 units. That is why, if the dispatch quantity is in that region, the buyer or the supplier can always make their dispatch quantity look as if it were q_2 by adding a dummy load. Hence, the actual transportation cost for them becomes

$$P_T(q) = \begin{cases} c_T^1 q & \text{if } q < q_1, \\ c_T^2 q_2 & \text{if } q_1 \leq q < q_2, \\ c_T^2 q & \text{if } q_2 \leq q, \end{cases} \quad (\text{A.4})$$

where $q_1 = (c_T^2 q_2)/c_T^1$. Note that $c_T^1 q_1 = c_T^2 q_2$.

If we look at the joint annual cost of the buyer and supplier under this transportation pricing schedule, we can see that it has a piecewise form with three pieces:

$$\mathcal{P}_{SB} : \min_{q \geq 0, n \in \mathbb{Z}^+} = \begin{cases} \frac{(A_S + nA_B)D}{nq} + \frac{(h_S(n-1) + h_B q)}{2} + (c_T^1 + p_T)D & \text{if } q < q_1, \\ \frac{[A_S + n(A_B + c_T^2 q_2)]D}{nq} + \frac{(h_S(n-1) + h_B q)}{2} + p_T D & \text{if } q_1 \leq q < q_2, \\ \frac{(A_S + nA_B)D}{nq} + \frac{(h_S(n-1) + h_B q)}{2} + (c_T^2 + p_T)D & \text{if } q_2 \leq q. \end{cases}$$

Although the function has three pieces, the first and second pieces differ only by a constant. Hence the unconstrained minimizers of those are the same. We call that minimizer (q_0, n_0) . We also call the unconstrained minimizer of the second piece

(q_f, n_f) . Before going into the analysis, we name the three regions of q as I , II , and III . We also define two values: q_F is the constrained optimal solution of region II , and q_L is the constrained optimal solution of regions I and III .

1. **Search for q_F :**

- If $q_f \in II$, then $q_F = q_f$.
- If $q_f \in I$, check for the maximum value of n , n_{max} , such that $n < n_f$ and $q(n_{max}) \in II$. Set $q_F = q(n_{max})$. If there is no such n value, then q_F is not set.
- If $q_f \in III$, check for the minimum value of n , n_{min} , such that $n > n_f$ and $q(n_{min}) \in II$. Set $q_F = q(n_{min})$. If there is no such n value, then q_F is not set.

2. **Search for q_L :**

- If $q_0 \in I$, find the maximum value of n , \hat{n}_{max} , such that $n < n_0$ and $q(n) \in III$. If such \hat{n}_{max} exists, compare the corresponding joint annual cost for it and compare it to the joint annual cost for (q_0, n_0) . Set the q_L value to the q value that leads to the minimum joint annual cost.
- If $q_0 \in II$, find maximum value of n , \hat{n}_{max} , such that $n < n_0$ and $q(n) \in III$. Also find the minimum n , \hat{n}_{min} , such that $n > n_0$ and $q(n) \in I$. If such \hat{n}_{max} and \hat{n}_{min} values exist, calculate the corresponding joint annual costs and set q_L to the one that leads to the minimum cost.
- If $q_0 \in III$, then set $q_L = q_0$.

3. Compute the joint costs for q_1 and q_2 .¹

Out of q_F , q_L , q_1 , and q_2 , choose the one that leads to the minimum joint cost.

Coordination Mechanism if the Supplier Incurs $P_T(q)$

As in the other cases, a quantity discount is enough to coordinate the buyer at the joint optimal. The discount schedule can be found using equations (A.1) and (A.2).

Coordination Mechanism if the Buyer Incurs $P_T(q)$

With the piecewise structure of the transportation charges, the buyer's annual cost takes the following form:

$$G_B(q) = \begin{cases} \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + c_T^1)D & \text{if } q < q_1, \\ \frac{(A_B + c_T^2 q_2)D}{q} + \frac{h_B q}{2} + p_S D & \text{if } q_1 \leq q < q_2, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + c_T^2)D & \text{if } q_2 \leq q. \end{cases}$$

Notice that the first and third pieces have the same structure, i.e. the unconstrained optima for both are the same, say q_0^B . On the other hand, the unconstrained minimizer of the second piece, q_f^B , is always larger than q_0^B . Using these properties, we can find the optimal solution as follows:

- Define q_L^B to be the constrained minimizer of regions *I* and *III* and q_F^B to be the constrained minimizer of region *II*.
- If $q_0^B \in II$, then q_L^B is not set to anything. Otherwise, set $q_L^B = q_0^B$.
- If $q_f^B \in II$, then set $q_F^B = q_f^B$. Otherwise it is not set to anything.

¹This step is not necessary all the time. If $q_f \in II$ and $q_0 \in III$, then the other q values won't lead to a lower joint cost.

- Calculate the $G_B(q)$ for q_1 and q_2 .

Out of q_L^B, q_F^B, q_1 and q_2 q_B is the one that leads to the minimum G_B value.

So far, we have derived the optimal order quantity for the buyer when he incurs the transportation charges. The next step is to look at how the supplier could coordinate the buyer to operate at q_{SB} . All units quantity discounts may not always work for this purpose. Here is an example.

Suppose that $q_0^B \in I$, $q_L^B = q_0^B$, and $q_F^B = q_f^B \in II$. Further assume that $q_B = q_L^B$ and $q_{SB} \in (q_B, q_F^B)$. If the quantity discount is given, under the discounted schedule, q_F^B will lead to a lower annual cost than q_{SB} for the buyer. In such a case, the supplier can coordinate the buyer by using multiple-break quantity discounts which possibly include negative discounts. For example, consider the above situation and further suppose that $q_{SB} \in II$. Then $G_B(\cdot)$ is decreasing at q_{SB} . Hence any quantity discount that starts at q_{SB} will help the buyer to choose a higher quantity and that will not be what the supplier wants. We propose that the supplier start the discount at q_1 , and the amount of the discount is $(G(q_{SB}) - G(q_B))/D$. Also, let the discount to be active only for the q values, such that $q \in [q_1, q_{SB}]$. Then, as it can be seen from Figure 11, this quantity discount schedule coordinates the buyer at q_{SB} .

At this point, we question the interpretation of the freight discount when the transportation charges are incurred by the buyer, and there is no coordination between the supplier and the buyer. We said earlier that when the transporter offers a quantity discount as in (A.3), it is interpreted as (A.4). Recall that if the order quantity is between q_1 and q_2 , the supplier can add a dummy load to increase the load to q_2 units because it is always cheaper to send q_2 units. However, when the buyer incurs the transportation charges and there is no coordination between the buyer and the supplier, she will have no motivation to add a dummy load to the buyer's orders.

Hence, the buyer cannot interpret the freight discount schedule as (A.4). For this reason, his total annual cost will have the following form:

$$G_B(q) = \begin{cases} \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + c_T^1)D & \text{if } q < q_2, \\ \frac{A_B D}{q} + \frac{h_B q}{2} + (p_S + c_T^2)D & \text{if } q_2 \leq q. \end{cases}$$

Finding the minimizer of this function is fairly straightforward, and the procedure is available in many textbooks about inventory theory.

As in the previous interpretation, the supplier may not be able to induce the buyer with an all units quantity discount having a single break. Consider the situation where $q_B < q_{SB} < q_2$. If the supplier offers the discount schedule as in (A.1) and (A.2), then $\tilde{G}_B(q_2) < \tilde{G}_B(q_{SB})$ and the buyer's order quantity will be q_2 instead of q_{SB} . In such a situation the supplier's discount schedule should be of the following form:

$$\bar{P}_S(q) = \begin{cases} p_S q & \text{if } q < q_{SB}, \\ p'_S q & \text{if } q_{SB} \leq q \leq q_2, \\ p_S q & \text{if } q_2 \leq q, \end{cases}$$

where c'_S is given by (A.2).

VITA

Fatih Mutlu received his B.S. in industrial engineering from Bilkent University, Turkey in 2000. In September 2000, he enrolled in the doctoral program at Texas A&M University, Department of Industrial and Systems Engineering. His research interests are in supply chain management, inventory theory, stochastic modeling, and optimization. His permanent address is Cumhuriyet Mahallesi, Samsun Caddesi, No:314, 55600 Terme-Samsun, TURKEY.