GUT AND STANDARD-LIKE MODELS IN INTERSECTING D-BRANE WORLDS

A Dissertation

by

CHING-MING CHEN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Physics

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ABSTRACT

GUT and Standard-like Models in

Intersecting D-Brane Worlds. (August 2006)

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The main goal of string phenomenology is to find a convincing connection between realistic particle physics and string theory. An extended object called D-brane in string theory is shown as a very powerful tool to resolve phenomenology problems. D-branes, D standing for Dirichlet boundary conditions, naturally appear in the T-dual space along one of the toroidally compactified dimensions in non-perturbative Type I theory. A D-brane forms an $U(1)$ gauge group and the group structure can be enriched by Chan-Paton indices with multiple coincided D-branes and orientifold actions. Oribfolds define fixed points of the compactified space and break the theory to $N = 1$ supersymmetry, and the extended orientifold from world-sheet parity projects the brane image to help cancel the anomalies. Strings at the intersections of two D-branes (Type IIA) form massless chiral fermions as bi-fundamental representations of the gauge groups of the intersecting branes. With these properties, we construct Grand Unification Theory (GUT) and standard-like models by intersecting D-brane configuration on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Also, supergravity and geometrical fluxes are introduced to stabilize the moduli. In this dissertation, first a brief review of the D-brane theory is discussed, then the complete construction of D-brane configuration on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ is presented, and finally some realistic Trinification, Pati-Salam, $SU(5)$ and flipped $SU(5)$ models are constructed and discussed. We present the models both in D-brane wrapping numbers and the corresponding particle spectra.

To My Parents

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TABLE OF CONTENTS

CHAPTER | Page

CHAPTER Page

LIST OF TABLES

TABLE Page

TABLE Page

TABLE Page

TABLE Page

TABLE Page

LIST OF FIGURES

CHAPTER I

INTRODUCTION*

The Standard Model (SM) of particle physics is a milestone of physics and it provides a convincing way to explain the electromagnetic and nuclear interactions (strong and weak) among elementary particles. Due to its great success in confronting the experiments, all of the attempt to develop more fundamental theories must base on it. The reason that people seek for more fundamental theories is the SM suffering some problems such as the absence of gravity, the gauge hierarchy problem, the disconnected three gauge symmetries (interactions), many unconstraint parameters ranging over nine orders of magnitude, as well as cosmological problems such as the explanation of dark matter. The Grand Unification Theory (GUT) suggests a unification of these three interactions to one gauge coupling, and the simplest unification gauge group of color and flavor is $SU(5)$. Because of the large differences between the coupling strength of strong and weak interactions, this unification will not become apparent until the scale 10^{14} GeV is reached. However even at this energy scale the three interactions do not exactly unified. Supersymmetry (SUSY) and its local symmetry treatment supergravity (SUGRA) which incorporating the spacetime symmetry is introduced to extend SM and they provides a natural mechanism to solve the hierarchy problem, the unification of the gauge symmetries, and the dark matter origins from the lightest supersymmetric particle (LSP). But SUSY (or SUGRA) does not describe gravity as a quantum theory, and actually it introduces more undetermined

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†The journal model is *Nuclear Physics B*.

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parameters in model building which do not make it an ultimate theory. In the past twenty years string theory became very popular because it has a potential to resolve all the questions and probably makes some predictions of phenomenology at high energy scale.

String theory (for a review, see [1]) takes a particle as a particular vibration mode of an elementary microscopic string, and allows gravity included as a quantum theory. Supersymmetry is regarded as an ingredient of string theory (superstring theory). String theory only allows certain gauge groups which can be the origin of SUSY GUT gauge groups and may offer a possibility to calculate the Yukawa coupling constants. String theory fixed the dimensionality of spacetime to ten (superstring) rather than four so the extra dimensions may hide from plain view if they curl up into a space that is too small to be detected at low energy scale. These extra dimensions lead to additional fields known as moduli which need to be fixed in low energy physics. An extended object called D-brane [2, 3] in string theory is introduced and shown as a very powerful tool to resolve many phenomenology problems.

D-branes, D standing for Dirichlet boundary conditions, naturally appear in the T-dual space along one of the toroidally compactified dimensions in non-perturbative Type I theory. A Dp-brane is a p spatial dimensional BPS solitonic object with open string ends confined on it by Dirichlet boundary conditions. A D-brane forms an $U(1)$ gauge group and the group structure can be enriched by Chan-Paton indices with multiple coincided D-branes and orientifold actions. In the T-dual Type II theory p is only allowed even in IIA theory and odd in Type IIB theory. Oribfolds define fixed points of the compactified space and break the theory to $N = 1$ supersymmetry [4, 5], and the extended orientifold from world-sheet parity projects the brane image to help cancel the anomalies.

The fundamental goal of string phenomenology is to find a convincing connection between realistic particle physics and string theory. Previously it was thought that only models based upon weakly coupled heterotic string compactifications could achieve this. Indeed, the most realistic GUT models based on string theory may be the heterotic string-derived flipped $SU(5)$ [6] which has been studied in great detail. However, in recent years Type I and Type II compactifications involving D-branes, where chiral fermions can arise from strings stretching between D-branes intersecting at angles (Type IIA picture) [7] and in its T-dual (Type IIB) picture with magnetized D-branes [8], have provided an interesting and exciting approach to this problem.

Many consistent standard-like and grand unified theory (GUT) models were built at an early stage [9, 10, 11, 12, 13, 14] using D-brane constructions. However, these models encountered problems of supersymmetry. Furthermore, these models suffered from instability in the internal space. The quasi-realistic supersymmetric models were constructed first in Type IIA theory on a $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [15, 16, 17] and other orientifolds [18]. Following this, models with standard-like, left-right symmetric (Pati-Salam [19]), Georgi-Glashow ($SU(5)$) and flipped $SU(5)$ gauge groups have been constructed based upon this framework and systematically studied [20, 21, 22, 23, 24, 25].

Turn to the question of our preference of building flipped $SU(5)$ models. Different types of particle models have been discussed using various constructions. The minimal option is to embed just the Standard Model $SU(3) \times SU(2) \times U(1)$ gauge group, but almost every construction contains at least some extra $U(1)$ factors. Conventional GUT models such as $SU(5)$ or $SO(10)$ have been investigated, but none of them has been completely satisfactory. This triggered the motivation to consider the gauge group $SU(5) \times U(1)_X$ [6, 26, 27] as a candidate for a model derived from string. The *raison d'être* of this 'flipped' $SU(5)$ is that it requires only $\mathbf{10}$ and $\overline{\mathbf{10}}$ Higgs representations to break the GUT symmetry, in contrast to other unified models which require large and unwieldy adjoint representations. This point was given further weight when it was realized that models with adjoint Higgs representations cannot be derived from string theory with a $k = 1$ Kac-Moody algebra [28]. There are many attractive features of flipped $SU(5)$. For example, the hierarchy problem between the electroweak Higgs doublets and the color Higgs triplets is solved naturally through a 'missing partner' mechanism [6]. Furthermore, this dynamical doublet-triplet splitting does not require or involve any mixing between the Higgs triplets leading to a natural suppression of dimension 5 operators that may mediate rapid proton decay and for this reason it is probably the simplest GUT to survive the experimental limits placed upon proton lifetime [29]. Recent investigation showed that the proton could be even stable by rotating away the gauge dimension 6 contributions [30]. More recently, the cosmic microwave anisotropy $\delta T/T$ has been successfully predicted by flipped $SU(5)$, as it has been determined to be proportional to $(M/M_P)^2$ where M denotes the symmetry breaking scale and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass [31]. Finally, string-derived flipped $SU(5)$ may provide a natural explanation for the production of Ultra-High Energy Cosmic Rays (UHECRs), through the decay of super-heavy particles dubbed 'cryptons' [32] that arise in the hidden sector of the model, which are also candidates for cold-dark matter (CDM).

The heterotic string-derived flipped $SU(5)$ model was created within the context of the free-fermionic formulation, which easily yields string theories in four dimensions. This model belongs to a class of models that correspond to compactification on the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold at the maximally symmetric point in the Narain moduli space [33]. Although formulated in the context of weakly coupled heterotic string theory, it is believed that the vacuum may in fact be non-perturbative due to its proximity to special points in the moduli space and may elevate to a consistent vacuum of M-theory. For this reason, it is our hope that in searching for a realistic flipped $SU(5)$ model that we may arrive at or near the same vacuum using D-brane constructions.

However, in spite of these successes, a natural mechanism is still needed to stabilize the moduli of the compactification, although in some cases the complex structure parameters (in Type IIA picture) and dilaton fields may be stabilized due to the gaugino condensation in the hidden sector [34]. Turning on non-trivial fluxes as background of the compactification gives rise to a non-trivial low energy supergravity potential which freezes some Calabi-Yau moduli [35]. Type IIB configurations with non-trivial Ramond-Ramond (RR) and Neneu-Schwarz-Neveu-Schwarz (NSNS) fluxes together with the presence of anti-D3 branes have been studied in [36, 37, 38], and a complete analysis of Type IIA configurations with RR and NSNS and metric fluxes has been studied in [39]. These fluxes impose strong constraints on the RR tadpole cancellation since their supergravity equation of motion and the Dirac quantization conditions must be satisfied. The corresponding models are studied, for example, in [40, 41, 42, 43, 44].

This thesis is organized as follows. In Chapter II a very brief review of the required knowledge to build intersecting D-brane models is provided. By Kaluza-Klein dimension reduction the concept of duality connecting different limits in different D-brane theories is introduced as the main spirit in M-theory. This new extended object of string theory is discussed and its properties are stated concisely. In chapter III we list all the constraints such as RR-tadpole conditions, supersymmetry conditions, and K-theory constraints including non-trivial supergravity fluxes specifically in one kind of orientifold, $\mathbb{Z}_2 \times \mathbb{Z}_2$, for model building. In Chapter IV we take use of the constraints from Chapter III to construct semi-realistic Standard-Like models, especially for Pati-Salam models. And finally in Chapter V the grand unification models (GUT), are discussed and especially the flipped $SU(5)$ construction is focused on. We also show an example for the Georgi-Glashow $SU(5)$ GUT model. Chapter VI is for discussion and conclusions. We list all other similar models that are not explicitly discussed in the appendices.

CHAPTER II

D-BRANE THEORY

In this chapter we give a very brief review of D-brane theory, which basically follows [45]. D-branes are physical objects to which string endpoints are attached with Dirichlet boundary conditions satisfied. We will consider Type I and II theories for gauges from branes rather than from the anomaly cancellation in heterotic theories.

A. T-Duality

T-duality explains the equivalence among different theories in string theory. The mass spectra remain the same in the different limits of the radius of the compactfied space by interchanging the parameters of the corresponding Kaluza-Klein modes.

1. T-duality and Closed Strings

Consider first the zero modes of closed strings with the expansion

$$X^\mu(z,\bar{z}) = x^\mu + \tilde{x}^\mu - i\sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu + \tilde{\alpha}_0^\mu)\tau + \sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu - \tilde{\alpha}_0^\mu)\sigma + \text{oscillators}. \tag{2.1}$$

We know non-compact spatial directions in X^μ is single-valued. However, the oscillators are periodic by $\sigma \to \sigma + 2\pi$, so X^μ shifts by a value $2\pi\sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu - \tilde{\alpha}_0^\mu)$. Therefore by the space-time momentum $p^\mu = \frac{1}{\sqrt{2\alpha'}}(\alpha_0^\mu + \tilde{\alpha}_0^\mu)$ we have

$$\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}}p^\mu. \tag{2.2}$$

Now if we compactify X^{25} on a circle with radius R by Kaluza-Klein method, the momentum of the 25th dimension will be discrete as $p^{25} = n/R$ for n an integer. Again, if we apply $\sigma \to \sigma + 2\pi$, this time X^{25} is not single-valued anymore because of the compactification. The difference of X^{25}: $2\pi\sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu - \tilde{\alpha}_0^\mu)$ will change by a discrete value, say $2\pi\omega R$, where $\omega \in \mathbb{Z}$. Therefore we can write the zero modes of this compact dimension as

$$\begin{aligned} \alpha_0^{25} &= (\frac{n}{R} + \frac{\omega R}{\alpha'})\sqrt{\frac{\alpha'}{2}}, \\ \tilde{\alpha}_0^{25} &= (\frac{n}{R} - \frac{\omega R}{\alpha'})\sqrt{\frac{\alpha'}{2}}. \end{aligned} \tag{2.3}$$

Then the mass spectrum in terms of the zero modes with the left- and right-moving excitations L and $\bar{L}$ is

$$\begin{aligned} M^2 = -p^\mu p_\mu &= \frac{2}{\alpha'}(\alpha_0^{25})^2 + \frac{4}{\alpha'}(L-1) \\ &= \frac{2}{\alpha'}(\tilde{\alpha}_0^{25})^2 + \frac{4}{\alpha'}(\bar{L}-1). \end{aligned} \tag{2.4}$$

As we just claimed, the mass spectrum is invariant and all the interactions are identical as well [46] if we do the following exchange [47]

$$n \leftrightarrow \omega, \quad R \leftrightarrow R' \equiv \frac{\alpha'}{R}. \tag{2.5}$$

This exchange is called T-duality transformation, which is an exact symmetry of perturbative closed string theory [2, 3, 45]. If we write the radius-R theory (the effective coordinate of X^{25}) in terms of

$$X'^{25}(z,\bar{z}) = X^{25}(z) - X^{25}(\bar{z}), \tag{2.6}$$

the energy-momentum tensor and other basic properties of conformal field theory will be invariant. This can be regarded as a space-time parity transformation acting only on the right-moving modes. The only thing changes is the zero modes different by a sign:

$$\alpha_0^{25} \leftrightarrow \alpha_0^{25}, \quad \tilde{\alpha}_0^{25} \leftrightarrow -\tilde{\alpha}_0^{25} \tag{2.7}$$

This theory is very important because it provides a connection between two limits, or in other words, two geometrical structures due to the transformation, R' being proportional to the inverse of R. Consider $R \to \infty$, intuitively it implies X^{25} is not compact. So the states with $\omega \neq 0$ are infinitely massive, and the states with $\omega = 0$ provides a continuous momentum for any n. On the other hand, for $R \to 0$, states with $n \neq 0$ become extremely massive, and X^{25} are fully compactified. The other parts of the space are independent of the reduced dimension, which is the same as normal Kluza-Klein theory. However, in string theory there are other modes for ω when $n = 0$ which can be regarded as an effective momentum. Thus accompanied with this a fully uncompactified effective dimension appears.

2. T-duality, Open Strings, and D-branes

Now turn to apply T-duality on open strings. The mode expansion of X^{25} is

$$\begin{aligned} X^{25}(z) &= \frac{x^{25}}{2} + C - i\alpha' p^{25} \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \frac{\alpha_m^\mu}{m z^m}, \\ X^{25}(\bar{z}) &= \frac{x^{25}}{2} - C - i\alpha' p^{25} \ln \bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{m\neq 0} \frac{\alpha_m^\mu}{m \bar{z}^m}. \end{aligned} \tag{2.8}$$

Again compactify X^{25} on a circle with radius R so $p^{25} = n/R$, then write the radius-R theory in terms of

$$\begin{aligned} X'^{25}(z,\bar{z}) = X^{25}(z) - X^{25}(\bar{z}) &= 2C - i\alpha' p^{25} \ln \frac{z}{\bar{z}} + \text{oscillators} \\ &= 2C + 2n\frac{\alpha'}{R}\sigma + \text{oscillators}. \end{aligned} \tag{2.9}$$

At the endpoints $\sigma = 0, \pi$ the oscillator terms vanish, and the remaining zero mode is independent of τ. Therefore the endpoints of the string do not move on the effect coordinate X'^{25} [2, 3]. In other words, in the dual theory the endpoints are fixed on certain points on X'^{25}, $X'^{25}(\sigma = 0) = 2C$ and $X'^{25}(\sigma = \pi) = 2C + 2\pi n R'$, but they are still able to move freely on other 24 spatial coordinates and wind n times on space-time circle. This 24-dimensional hyperplane where the string endpoints lie on is a dynamical object, and is called D(irichlet)-brane because in the T-dual space the boundary condition is Dirichlet [2, 48, 49]:

$$\begin{aligned} 0 &= \left[\partial_n X^{25}(z,\bar{z}) = \partial_z X^{25}(z) + \partial_{\bar{z}} X^{25}(\bar{z})\right]_{\sigma=0,\pi} \\ &= \left[\partial_t X'^{25}(z,\bar{z}) = \partial_z X^{25}(z) - (-\partial_{\bar{z}} X^{25}(\bar{z}))\right]_{\sigma=0,\pi}. \end{aligned} \tag{2.10}$$

Note from (2.9) by the same analysis when $R \to 0$ there is no ω for new continuous states. Therefore this is a dimension reduction case and it seems inconsistent that open strings live in a $D-1$ dimensional space rather than a D dimensional space where closed strings live. The answer is they both live in the space with same dimensions because only the endpoints of open strings live on the $D-1$ space, which is the D-brane.

To summarize, T-duality interchanges Neumann and Dirichlet boundary conditions and define D-branes in the dual space. And continue taking T-duality in a direction tangent to a Dp-brane reduce this brane to a D_{p-1}-brane, while taking T-duality in a direction orthogonal to this brane makes it into a D_{p+1}-brane.

B. Gauge Groups from D-branes

1. Chan-Paton Factor and Oriented Open Strings

Consider an oriented open string, it is still consistent with space-time Poincaré invariance and world-sheet conformal invariance to add non-dynamical degrees of freedom at the ends. The ends of the string will not change their status. If one labels the two ends i and j and each runs from 1 to N, the two variables can form an $N \times N$ matrix λ^a_{ij} which is a basis for a string wave function $|k, a\rangle = \sum_{i,j} |k, ij\rangle \lambda^a_{ij}$. These fields are Chan-Paton factors [50]. For the graviton must be real, the Chan-Paton factors should be Hermitian. Each vertex carries such a factor, so if we consider an interaction with four oriented open strings at tree level, under the conformal transform we can see the right end of string 1 must have the same value as the left end of string 2 (If string 1 is at the left of string 2), for Chan-Paton factors are non-dynamical. Therefore, the net effect of this scattering is [51]

$$\sum \lambda^1_{ij}\lambda^2_{jk}\lambda^3_{kl}\lambda^4_{li} = Tr(\lambda^1\lambda^2\lambda^3\lambda^4). \tag{2.11}$$

This trace factor appear in the amplitude, and is invariant under $U(N)$ symmetry on the world-sheet. It also takes into account the massless vertex operator by $V^{a\mu} = \lambda^a_{ij}\partial_t X^\mu \exp(ikX)$, so the vertex operator transforms as the adjoint under the $U(N)$ symmetry. In other words, the global symmetry of the world-sheet is promoted to a gauge symmetry in space-time [51].

2. Unoriented Strings

If we apply the world-sheet parity Ω on the open string as $z \leftrightarrow -\bar{z}$, we reflect rihgt-moving modes into left-moving modes. The open string tachyon survives under this discreet symmetry, but the photon does not. Chan-Paton factors on the string ends provide an additional structure to the photon [52]. World-sheet reverses Chan-Paton factors on the two ends of the string, and have some additional action [51]:

$$\Omega\lambda_{ij}|k, ij\rangle \rightarrow \lambda'_{ij}|k, ij\rangle, \quad \lambda' = M\lambda^T N. \tag{2.12}$$

Here $N = M^{-1}$ because (2.11) should be satisfied. Acting Ω twice to the identity, the states are invariant under

$$\lambda \rightarrow MM^{-T}\lambda M^T M^{-1}. \tag{2.13}$$

If strings with λ_{ik} and λ_{jl} are in the spectrum with any k and l, then so is the state with λ_{ij}. λ_{jl} implies λ_{lj} by CPT, and a splitting-joining interaction in the middle gives $\lambda_{ik} \oplus \lambda_{lj} \to \lambda_{ij} \oplus \lambda_{lk}$. By Schur's lemma MM^{-1} is proportional to identity, so M is either symmetric or antisymmetric, and there are two choice of basis [53]:

- $M = M^T = \mathbf{1}_N$. For the photon to be even under Ω and survive it is required $\lambda = -\lambda^T$, then the gauge group is $SO(N)$.
- $M = -M^T = i \begin{pmatrix} 0 & \mathbf{1}_{N/2} \\ -\mathbf{1}_{N/2} & 0 \end{pmatrix}$. $\lambda = -M\lambda^T M$ in this case, and the gauge symmetry is $USp(N)$.

Consider unoriented closed strings. The theory is invariant under a world-sheet parity symmetry, which reverses the right- and left-moving oscillators. If we gauge this global symmetry, states which are symmetric survive such as graviton and dilaton, and the antisymmetric tensor is projected out.

3. Chan-Paton Factors and Wilson Lines

Consider space-time has the non-trivial topology of a circle on coordinate X^{25} with radius R. Take the simplest case with gauge group $U(1)$, then we choose a constant background gauge potential

$$A_{25}(X^\mu) = -\frac{\theta}{2\pi R} = -i\Lambda^{-1}\frac{\partial \Lambda}{\partial X^{25}}, \tag{2.14}$$

where $\Lambda(X^{25}) = \exp(\frac{i\theta X^{25}}{2\pi R})$. This is a pure local gauge. The Wilson Line preserve a charge q of this gauge:

$$W_q = \exp\left(iq\oint dX^{25} A_{25}\right) = e^{-iq\theta}. \tag{2.15}$$

So if an object does a loop along X^{25}, W_q just get a phase factor. If we gauge away A by Λ^{-1}, it means this object with a charge q will pick up a phase $e^{iq\theta}$ when moving

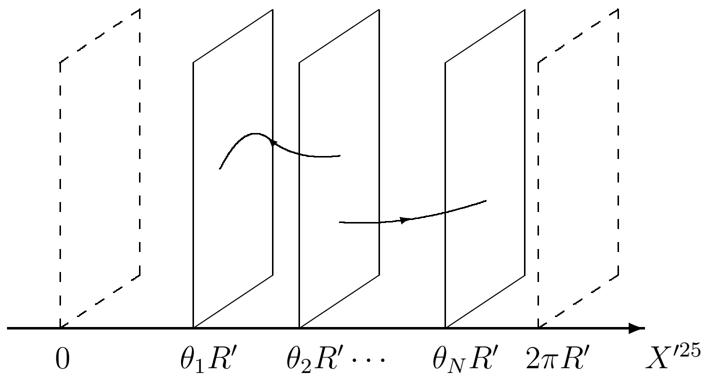

Fig. 1. N hyperplanes at different positions along X^{25}

along the circle.

We can generate the case for an $U(N)$ group. Now the gauge potential A_{25} is

$$A_{25} = \frac{1}{2\pi R}\text{diag}\{\theta_1, \theta_2, \cdots \theta_N\} = -i\Lambda^{-1}\partial_{25}\Lambda, \tag{2.16}$$

where $\Lambda = \text{diag}\{e^{iX^{25}\theta_1/2\pi R}, e^{iX^{25}\theta_2/2\pi R}, \cdots e^{iX^{25}\theta_N/2\pi R}\}$. The gauge generically breaks $U(N)$ to $U(1)^N$ if θ_i are different. So similarly if we gauge A^{25} away the fields have a phase

$$\text{diag}\{e^{-i\theta_1}, e^{-i\theta_2}, \cdots e^{-i\theta_N}\} \tag{2.17}$$

under $X^{25} \to X^{25} + 2\pi R$. Therefore, for a string state $|ij\rangle$ charged under $U(N)$ will have a shift on the canonical momentum by $p^{25} = \frac{n}{R} + \frac{\theta_j - \theta_i}{2\pi R}$. The endpoints are no longer on the same hyperplane, but with a shift as

$$X'^{25}(\pi) - X'^{25}(0) = (2\pi n + \theta_j - \theta_i)R'. \tag{2.18}$$

There will be N hyperplanes at different positions along X^{25}, as shown in Figure 1.

C. D-brane Dynamics

It is interesting to see if several coordinates $X^m = \{X^{25}, X^{24}, \cdots X^{p+1}\}$ are periodic, and then write X^m in terms of the dual coordinate. So there are N $p+1$-dimensional hyperplanes on which open string endpoints attach. Then we apply T-duality, which interchanges the Neumann conditions and Dirichlet conditions on the world-sheet. The $(p+1)$-dimensional hypersurface is the world-volume of a p-dimensional extended object called D-brane.

Take the mass spectrum with only one coordinate periodic, i.e., a D24-brane as an example,

$$M^2 = \left\{ \frac{[2\pi n + (\theta_i - \theta_j)]R'}{2\pi\alpha'} \right\}^2 + \frac{1}{\alpha'}(L-1) \tag{2.19}$$

$[2\pi n + (\theta_i - \theta_j)]R'$ is the minimum length of the string. he massless states are from non-winding ($n = 0$) open strings with two endpoints on the same D-brane. One of the massless states is with the gauge field in the directions transverse to the D-brane, and the other is with the gauge field in the compact direction of the original theory, which is the position of the D-brane in the dual theory.

If there is no D-brane coincide, we know $U(N)$ is breaking into $U(1)^N$. But when m D-branes coincide, there are new massless states since the length of strings stretching on these D-branes vanishes. So there will be m^2 vectors, forming the adjoint of a $U(m)$ gauge group. Furthermore, m^2 massless scales will appear, and the m positions are promoted to a matrix.

D. Orbifolds

To explain the real world in the $D = 4$ space-time we need to compactify the extra dimensions, for instance, C_6 for superstring theory. However C_6 cannot be any random manifold for keeping $N = 1$ supersymmetry, breaking gauge symmetry down to the

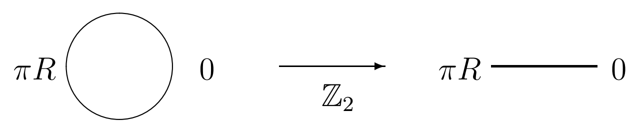

Fig. 2. Orbifold as a discrete symmetry.

Standard Model, and obtaining chiral fermions. From the early day C_6 is taken as a Calabi-Yau manifold for all the properties satisfied. But it is very difficult to construct Calabi-Yau manifold and so far only very few examples are presented. Therefore to investigate physics in our space-time we need to find a simple way to compactify C_6 with flat space solutions.

Toroidal compactification [54] is a good candidate for the analysis. The $N = 1$ $D = 10$ theory is broken down to $N = 4$ $D = 4$ theory, which is still far away from our requirement $N = 1$ supersymmetry. A discrete symmetry acting on the compactified space denoted as C_6/G_D with fixed points is introduced so then the space is named "orbifold" [55] instead of a general "manifold". This orbifold indeed satisfy the requirements mentioned and most important is it is easy to construct.

Take one compact dimension X^{25} on a circle S_1 with radius R as an example. Consider the discrete symmetry $\mathbb{Z}_2$ with the action $X^{25} \to -X^{25}$, by fixing the two points 0 and πR all other points are projected on a line, as shown in Figure 2. The string modes are different by a sign under this discrete symmetry. So for a closed string, we have known $X^{25} \to X^{25} + 2\pi\omega R$ when $\sigma \to \sigma + 2\pi$, so now under the orbifold symmetry $X^{25} = -X^{25}$ we find there is neither zero modes nor winding at the fixed points. This means there are two identical copies of these "twisted sectors" corresponding to strings trapped at $X^{25} = 0, \pi R$ in space-time with zero momentum. Therefore these trapped strings satisfy equations of motion and the boundary conditions should be included in the spectrum.

E. Unoriented Strings and Orientifolds

The $R \to 0$ limit of unoriented string compactification also leads to new objects. The effect of T-duality can be regarded as a one-sided parity transformation [51]. The action of world-sheet parity reversal Ω is to exchange $X^m(z)$ and $X^m(\bar{z})$, so the dual coordinate is different by a sign under Ω acting on the original compact space for closed strings:

$$X'^m(z, \bar{z}) \leftrightarrow -X'^m(z, \bar{z}). \tag{2.20}$$

In the effective coordinate it is a result of the product of a world-sheet and a space-time parity. This implies that in the dual theory the transformation has to be this product rather than Ω only to make strings invariant. The space-time parity is the same transformation as the orbifold construction. With the additional world-sheet parity this object an orbifold with orientation reversal. This generalization of the usual unoriented theory is an *orientifold* [2, 56].

In the case of a single compact dimension, $X'^{25} \in [0, \pi R']$, and at the two ends are dimension 24 spatial planes. These orientifold planes are not dynamical and no string modes tied on them. The local physics is unoriented on the O-planes, but oriented away from these fixed planes. Unlike the theory with only orbifold where the space-time parity projects away half of the states, the world-sheet orientifold relate the strings to their images.

For open strings the situation is similar that the orientifold fixed at 0 and $\pi R'$ in the dual space of X^{25}. From the orientifold action the Wilson-line has a negative

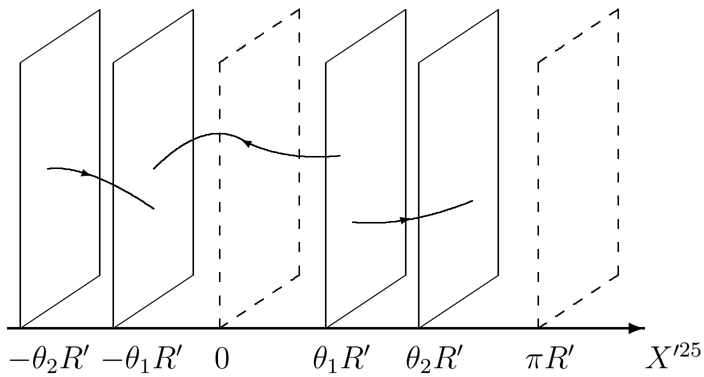

Fig. 3. D-branes and their images projected by the orientifold plane.

projection of each θ_i on X'^{25}:

$$\mathrm{diag}\{\theta_1, -\theta_1, \theta_2, -\theta_2, \cdots, \theta_{N/2}, -\theta_{N/2}\}. \tag{2.21}$$

So there are $\frac{N}{2}$ D-branes in the segment $[0, \pi R']$ and $\frac{N}{2}$ of their image branes on the negative side of the dual compact space, as shown in Figure 3. Strings can not only stretch between the ordinary D-branes but also stretch between D-branes and their images. The generic gauge group is then therefore $U(1)^{N/2}$. And for the oriented strings, before orientifold is introduced if m D-branes coincide they form a $U(m)$ group. Now if these m branes coincide at one of the fixed points, the strings between these branes and their images are massless as well, so there are effectively $2m$ branes coincide and thus form an $SO(2m)$ group. Same analysis can apply on the $USp(N)$ groups.

F. The D-brane Action

1. The D-brane Tension

D-brane is dynamical so it feels the gravity force, and the tension on it controls its behavior responding to the outside influence. Denote the coordinates ξ^a for $a = 0, \cdots, p$ on the D-brane. The fields imbedding on it could be a string $X^m(\xi^a) = 2\pi\alpha'\Phi^m$ and a gauge field $A_a(\xi^a)$ as mentioned. We first write down the D-brane action, and will discuss the gauge field later:

$$S_p = -T_p \int d^{p+1}\xi e^{\Phi} \sqrt{(\det G_{ab})}. \tag{2.22}$$

G_{ab} is the induced metric on the D-brane, T_p is the tension, and the dilaton dependence $e^{\Phi} = g_s^{-1}$ is from an open string tree level action. The mass of a Dp-brane wrapping around a p-torus is $T_p e^{-\Phi} \prod_{i=1}^{p} 2\pi R_i$ [57].

Taking T-duality on a direction of the Dp-brane and the transformation of the dilaton, we then can find a recursing relation for the tension:

$$T_p = \frac{T_{p-1}}{2\pi\sqrt{\alpha'}}. \tag{2.23}$$

Therefore we can see that a string stretching between two parallel separated D-branes with non-zero tension is massive.

2. Tilted D-branes

Now we include the gauge field. Consider a D2-brane neglecting other components on a surface extended by the X^1 and X^2 directions, and then introduce a constant gauge field strength $F_{12} = B_3$ regarded as a 'magnetic' field orthogonal to the surface [58, 59]. The gauge field can be chosen as $A_2 = X^1 F_{12}$. After taking T-duality along

X^2, its dual coordinate turns out

$$X'^2 = 2\pi\alpha' X^1 F_{12}. \tag{2.24}$$

It is a D1-brane, just as we mentioned that if we take T-duality along one direction of a D-brane it results in a dimension reduction, forming an angle $\theta = \tan^{-1}(2\pi\alpha' F_{12})$ in the dual space, as shown in Figure 4. This D1-brane world-volume action can be written as

$$S \sim \int_{D1} ds = \int_{D1} \frac{1}{\cos\theta} = \int dX^1 \sqrt{1+\tan^2\theta} = \int dX^1 \sqrt{1+(2\pi\alpha' F_{12})^2} \tag{2.25}$$

By boosting the D-brane to be along the coordinate axes and rotating $F_{\mu\nu}$ to a block-diagonal form, we can generalize the above analysis and write the action as

$$S \sim \int d^D X \sqrt{\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}. \tag{2.26}$$

This is so called Born-Infeld action [60].

Note the background space-time anti-symmetric tensor $B_{\mu\nu}$ should be considered in the action as well because the space-time gauge invariance should be preserved. Consider the action on world-sheet for B and A:

$$\frac{1}{2\pi\alpha'} \int_{\mathcal{M}} B + \int_{\partial\mathcal{M}} A. \tag{2.27}$$

If this action is invariant under space-time gauge transformation $\delta B = d\zeta$ it must be cancelled by $\delta A = -\zeta/2\pi\alpha'$. So if the total gauge $\mathcal{F}$ is conserved, it should be a combination of $B_{\mu\nu}$ and A: $\mathcal{F} = B + 2\pi\alpha' F$ where $F = dA$. Then the total action is a Dirac-Born-Infeld action:

$$S \sim \int d^{p+1}\xi e^{-\Phi} \sqrt{\det(G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}. \tag{2.28}$$

This tilting mechanism induced by gauge fields (magnetic) moves the D-branes

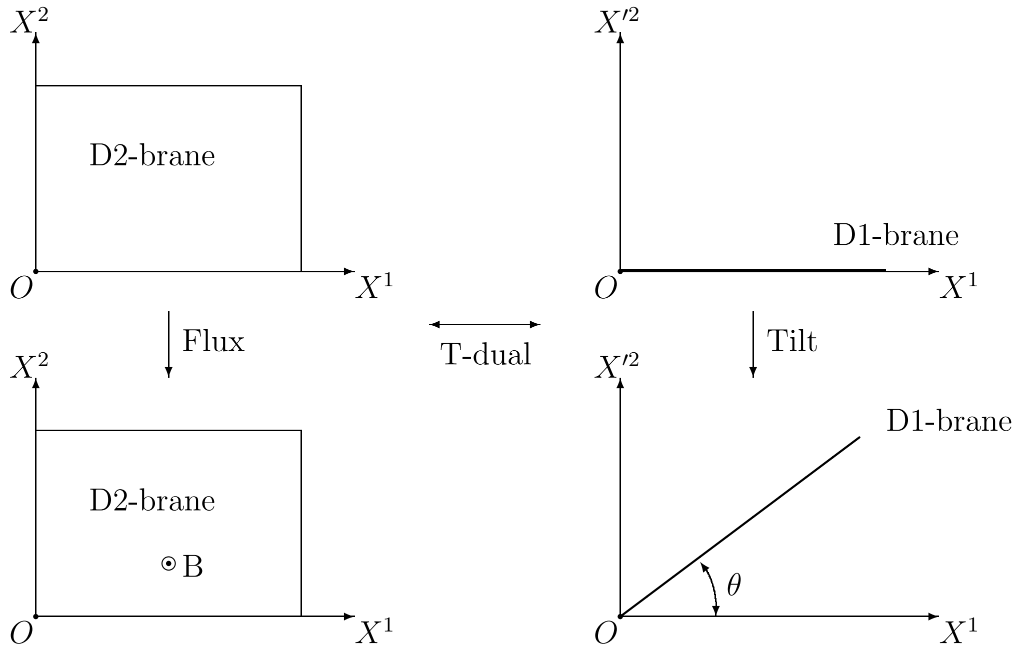

Fig. 4. A D2-brane with a flux is tilted by an angle in its T-dual picture.

away from the original position and separates them due to different gauge field strengthes. The magnetic fluxes can split the energies (masses) of different spinors so then distinguish the chirality of the strings (in superstring), which is similar to the phenomena of Zeeman effect. We may take this as 'brane Zeeman effect' [61]. In superstring theory different tilted D-branes may intersect and the strings stretched between two intersecting D-branes are massless at the intersection point, see Figure 5. These chiral massless states are bi-fundamental representations and are the particles we look for in model building from D-brane constructions.

G. Superstring Intersecting D-branes

To apply D-brane theory in phenomenology it is natural to consider the superstring construction since we live in a world with fermions. We mainly consider Type II the-

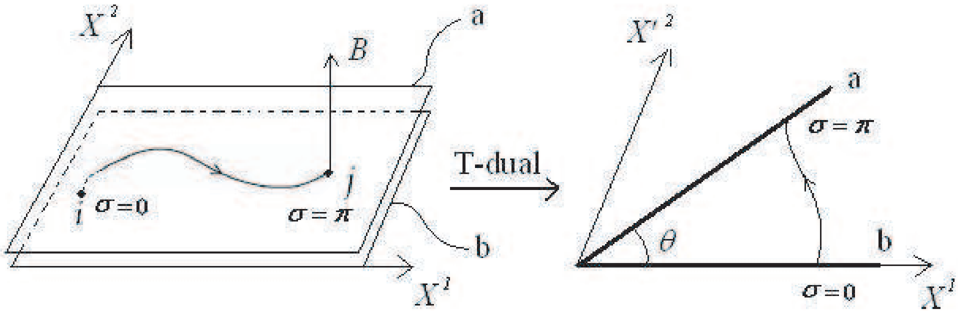

Fig. 5. A massless chiral string state is from the intersection after a flux is turned on in the dual space.

ory, where IIA picture with D(3+n)-branes wrapping on n-cycles on a $2n$-dimensional torus $\mathbf{T^{2n}}$ or IIB picture with D($3+2n$)-branes wrapping on the same space. We choose factorized $\mathbf{T^{2n}}$ as a product of n rectangular $\mathbf{T^2}$ tori. We will see why we only consider the $n = 3$ case, *i.e.*, D6-branes in IIA picture.

Type IIA theory provides a more a clear geometric picture. Chiral fermions can only arise from the sector of open strings stretched between two D-branes. If D_a-brane set makes an angle θ^I_a from one of the canonical basis of the torus $\mathbf{T^2_I}$, then we define $\theta^I_{ab} = \theta^I_a - \theta^I_b$ the angle difference between stack a and b. The mass operator for a string in such sector is [62]

$$\alpha' M^2_{ab} = \frac{Y^2}{4\pi^2\alpha'} + N_\nu + \nu(\sum_I \theta^I_{ab} - 1), \tag{2.29}$$

where Y is the length of the stretched string and N_0 and $N_{\frac{1}{2}}$ are the Ramond and Neveu-Schwarz number operators of oscillations. These oscillators can be modified to investigate the open string spectrum in a simple way. The mass operator then turns out [11]

$$\alpha' M_{ab}^2 = \frac{Y^2}{4\pi^2\alpha'} + N_{bos}(\theta) + \frac{(r+v_\theta)^2 - 1}{2} + E_{ab}, \tag{2.30}$$

where N_{bos} stands for the bosonic oscillator contribution and E_{ab} is the vacuum energy. The twist vectors of the D6-brane case ($n = 3$) are bosonic states from the NS sector

$$\begin{aligned} r_{NS} + v_\theta &= (-1+\theta^1, \theta^2, \theta^3, 0), \\ & (\theta^1, -1+\theta^2, \theta^3, 0), \\ & (\theta^1, \theta^2, -1+\theta^3, 0), \\ & (-1+\theta^1, -1+\theta^2, -1+\theta^3, 0), \end{aligned} \tag{2.31}$$

and a massless fermionic state from the Ramond sector

$$r_R + v_\theta = (-\frac{1}{2}+\theta^1, -\frac{1}{2}+\theta^2, -\frac{1}{2}+\theta^3, +\frac{1}{2}), \tag{2.32}$$

in the four dimensional space. We focus on the D6-brane case because it does not have vector-like fermions and tachyons in the spectrum.

Now we have all the background for the intersecting D-brane theory. The next step is choosing a proper orientifold to analyze its properties. As we will see in the next chapter a $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold proposed in [17] is fully discussed. Based on this scenario we will build Standard-like and GUT models as we claimed.

CHAPTER III

D-BRANE CONSTRUCTIONS

There are many ways to compactify the internal space. We are especially interested in Type II theory compactified on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. It provides not only a solution to compensate the RR charges of D-branes but a minimum constraint on model building. A supersymmetric Type IIA intersecting D6-brane construction is T-dual to the open-string sector of a Type IIB theory with magnetized intersecting D3-, D5-, D7-, D9-branes, thus they share similar properties in model building. The configuration of D-brane construction is highlighted in the following subsections.

A. Type IIA Construction*

We have several choices of compactification at our disposal in attempting to build a four-dimensional three-generation model, but we will focus on the supersymmetric type IIA orientifold on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at generic angles. This choice has the feature that $\mathbb{Z}_2$ actions do not constrain the ratio of the radii on any 2-torus, i.e., fix the complex moduli, and the four orientifold planes provide opposite charges to the four Ramond-Ramond charges. Additionally, the $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold has only bulk cycles, contrasting the cases of $\mathbb{Z}_4$ and $\mathbb{Z}_6$ orientifolds where exceptional cycles also necessarily exist and generally increase the difficulty of satisfying the Ramond-Ramond tadpole conditions. However, as we shall see only a limited range of ratio of the complex structure moduli is consistent with the supersymmetry conditions.

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This $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ structure was first introduced in [17, 63] and further studied in [21] *, and we will use the same notations here. Consider type IIA theory on the $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ generators θ, ω act on the complex coordinates (z_1, z_2, z_3) of $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$ as

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \rightarrow (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \rightarrow (z_1, -z_2, -z_3) \end{aligned} \tag{3.1}$$

We implement an orientifold projection ΩR, where Ω is the world-sheet parity, and R acts as

$$R : (z_1, z_2, z_3) \rightarrow (\overline{z}_1, \overline{z}_2, \overline{z}_3) \tag{3.2}$$

Although the complex structure of the tori is arbitrary under the action of $\mathbb{Z}_2 \times \mathbb{Z}_2$, it must be assigned consistently with the orientifold projection. Crystallographic action of the complex conjugation R restricts consideration to just two shapes. We may take either a rectangular toroidal cell or a very specific tilted variation which can be taken in another point of view, with an angle between the two vectors of the basis. Define here the canonical basis of homology cycles ($[a_i]$, $[b_i]$) lying respectively along the $(\hat{x}_i, \mathrm{i}\hat{y}_i)$ coordinate directions, where $i = 1, 2, 3$ labels each of the three 2-tori. Next, consider K different stacks of N_a D6-branes wrapping on ($[a_i]$, $[b_i]$) with integral coefficients (n_a^i, m_a^i), where $a = 1, 2, ...K$. For the tilted complex structure variants the toroidal cell is skewed such that an alternate homology basis is required to close cycles spanning the displaced lattice points. Specifically, we must consider the cycle $[a_i'] \equiv [a_i] + \frac{1}{2}[b_i]$, so that the tilted wrapping is described by $n_a^i[a_i'] + m_a^i[b_i] = n_a^i[a_i] + (n_a^i/2 + m_a^i)[b_i]$. For convenience, define the effective wrapping number l_a^i as $l_a^i \equiv m_a^i$ for rectangular and $l_a^i \equiv 2m_a^i + n_a^i$ for tilted tori.

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*See also [4, 15].

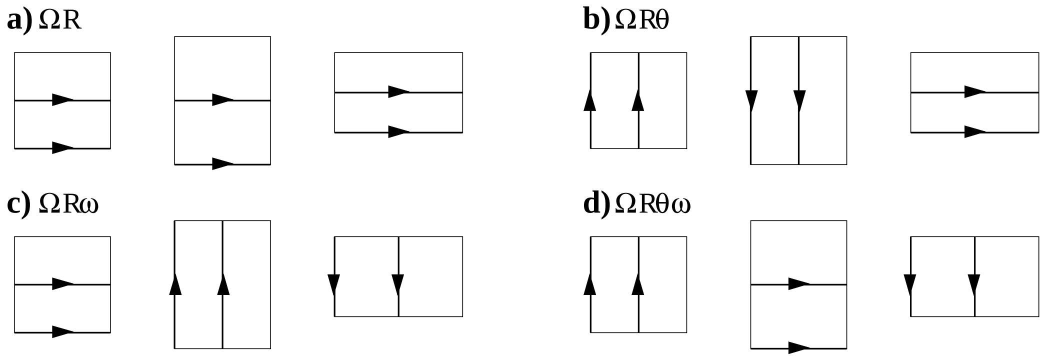

Fig. 6. The four O6-planes in $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [17].

With these definitions the homology three-cycles for a stack a of D6-branes and its orientifold image a' are given by

$$[\Pi_a] = \prod_{i=1}^{3}(n_a^i[a_i] + 2^{-\beta_i} l_a^i[b_i]), \quad [\Pi_{a'}] = \prod_{i=1}^{3}(n_a^i[a_i] - 2^{-\beta_i} l_a^i[b_i]) \tag{3.3}$$

where $\beta_i = 0$ if the ith torus is not tilted and $\beta_i = 1$ if it is tilted.

There are four kinds of orientifold 6-planes associated with the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$, which are shown in Figure 6. The homology three-cycles which they wrap are [21]

$$\Omega R : [\Pi_1] = 2^3[a_1][a_2][a_3], \qquad \Omega R\omega : [\Pi_2] = -2^{3-\beta_2-\beta_3}[a_1][b_2][b_3]$$
$$\Omega R\theta\omega : [\Pi_3] = -2^{3-\beta_1-\beta_3}[b_1][a_2][b_3], \quad \Omega R\theta : [\Pi_4] = -2^{3-\beta_1-\beta_2}[b_1][b_2][a_3] \tag{3.4}$$

This represents the fact that 180° rotation *plus* conjugate reflection produce 'vertical', i.e. $[b_i]$-oriented, invariant cycles, while the operator R alone preserves certain cycles along the 'horizontal', or $[a_i]$ axis. Each two-torus yields always a pair of such cycles, with the exception of the $[b_i]$-type tilted scenario where only a single invariant wrapping exists. This explains then the normal counting of $8 = 2^3$ distinct combinations, halved for each application of tilting in the vertically aligned case.

Table I. Wrapping numbers of D-branes on the four O6-planes.

Orientifold Action	O6-Plane	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$
ΩR	1	$(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$
$\Omega R\omega$	2	$(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$
$\Omega R\theta\omega$	3	$(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$
$\Omega R\theta$	4	$(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$

The total effect of these four planes should be combined, so we define $[\Pi_{O6}] = \sum_i [\Pi_i]$ [21]. In addition, a set of new parameters which are convenient in the following discussion are introduced [21]:

$$\begin{aligned} &A_a = -n_a^1 n_a^2 n_a^3,\ B_a = n_a^1 l_a^2 l_a^3,\ C_a = l_a^1 n_a^2 l_a^3,\ D_a = l_a^1 l_a^2 n_a^3 \\ &\tilde{A}_a = -l_a^1 l_a^2 l_a^3,\ \tilde{B}_a = l_a^1 n_a^2 n_a^3,\ \tilde{C}_a = n_a^1 l_a^2 n_a^3,\ \tilde{D}_a = n_a^1 n_a^2 l_a^3 \end{aligned} \tag{3.5}$$

With the basic definitions in hand, we can continue working on the global constraints of this model.

1. RR-tadpole Consistency Conditions

The Ramond-Ramond tadpole cancellation requires the total homology cycle charge of D6-branes and O6-planes to vanish [64]. The resulting equation

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0 \tag{3.6}$$

can be expressed in terms of the parameters defined in (3.5) as

$$\sum_a N_a A_a = \sum_a N_a B_a = \sum_a N_a C_a = \sum_a N_a D_a = -16 \tag{3.7}$$

It should be stressed that the tadpole condition is independent of the selected tilting. However, these coupled constraints are generally quite difficult to satisfy. The introduction of so called 'filler branes' [21] which wrap along the O6-planes can help somewhat. Such branes automatically preserve supersymmetry, so that they can be selected with only an eye for independent saturation of each RR-tadpole condition. If $N^{(i)}$ branes wrap along the ith O6-plane, they generate $USp(N^{(i)})$ groups and (3.7) is updated to

$$\begin{aligned} &-2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a = \\ &-2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16 \end{aligned} \tag{3.8}$$

Here $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted tori. The wrapping numbers of these filler branes on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ are listed in Table I.

2. Conditions for Supersymmetric Brane Configurations

The condition to preserve $N = 1$ supersymmetry in four dimensions is that the rotation angle of any D-brane with respect to the orientifold plane is an element of $SU(3)$ [7, 17, 63]. Consider the angles between each brane and the R-invariant axis of i^{th} torus θ_a^i, we require $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \bmod 2\pi$. This means $\sin(\theta_a^1 + \theta_a^2 + \theta_a^3) = 0$ and $\cos(\theta_a^1 + \theta_a^2 + \theta_a^3) = 1 > 0$. We define

$$\tan \theta_a^i = \frac{2^{-\beta_i} l_a^i R_2^i}{n_a^i R_1^i} \tag{3.9}$$

where R_2^i and R_1^i are the radii of the i^{th} torus. Then the above supersymmetry conditions can be recast in terms of the parameters defined in (3.5) as follows [21]:

$$x_A\tilde{A}_a + x_B\tilde{B}_a + x_C\tilde{C}_a + x_D\tilde{D}_a = 0$$
$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0 \quad (3.10)$$

where x_A, x_B, x_C, x_D are complex structure parameters, all of which share the same sign. These parameters are given in terms of the complex structure moduli $\chi_i = (R_2^i/R_1^i)$ by

$$x_A = \lambda, \;\; x_B = \lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3, \;\; x_C = \lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3, \;\; x_D = \lambda 2^{\beta_1+\beta_2}/\chi_1\chi_2 \quad (3.11)$$

The positive parameter λ was introduced in [21] to put all the variables A, B, C, D on an equal footing. However, among the x_i only three are independent.

B. Type IIB Construction*

In Type IIB theory the orbifold group of $\mathbb{Z}_2 \times \mathbb{Z}_2$ are the same as the one defined in (3.1). This construction contains a $D = 4$, $N = 2$ supergravity multiplet, the dilaton hypermultiplet, h_{11} hypermultiplets, and h_{21} vector multiplets which are all massless. For the orbifold with discrete torsion the Hodge numbers from both twisted and untwisted sectors are $(h_{11}, h_{21}) = (3, 51)$. In order to include the open string sector, orientifold planes are introduced by an orientifold projection ΩR, where Ω is the world-sheet parity and R acts as

$$R : (z_1, z_2, z_3) \to (-z_1, -z_2, -z_3) \quad (3.12)$$

There will then be 64 $O3$-planes and 4 $O7_i$-planes, which are transverse to the $\mathbf{T}_i^2$. Thus ΩR projects the $N=2$ spectrum to an $N=1$ supergravity multiplet, the dilaton chiral multiplet, and 6 untwisted and 48 twisted geometrical chiral multiplets. [40, 41, 42]

We need $D(3+2n)$-branes to fill up the $D=4$ Minkowski space-time and wrapping the $2n$-cycles on a compact manifold in type IIB theory. The introduction of magnetic fluxes provides the T-dual consistency to Type IIA theory. For a stack of N_a D-branes wrapping m_a^i times on $\mathbf{T}_i^2$, n_a^i denotes the units of magnetic fluxes F_a^i on $\mathbf{T}_i^2$, thus

$$m_a^i \frac{1}{2\pi}\int_{\mathbf{T}_i^2} F_a^i = n_a^i \tag{3.13}$$

To write down an explicit description of D-brane topology we introduce the even homology classes $[\mathbf{0}_i]$ and $[\mathbf{T}_i]$ for the point and the two-torus. Then the vectors of RR charges (corresponding to Type IIA homology cycles) of a^{th} stack D-brane and its image are (for simplicity, regardless the tilted cases) [65]

$$[\Pi_a] = \prod_i^3 (n_a^i[\mathbf{0}_i] + m_a^i[\mathbf{T}_i]),\ [\Pi_a'] = \prod_i^3 (n_a^i[\mathbf{0}_i] - m_a^i[\mathbf{T}_i]) \tag{3.14}$$

The O3- and O7_i-planes of $\mathbf{T^6}/(\mathbb{Z}_2\times\mathbb{Z}_2)$ resulting from the orientifold action ΩR, $\Omega R\omega$, $\Omega R\theta\omega$ and $\Omega R\theta$ can be written as

$$\begin{aligned} \Omega R:\ \ [\Pi_{O3}] &= \quad [\mathbf{0}_1][\mathbf{0}_2][\mathbf{0}_3],\ \ \Omega R\omega : [\Pi_{O7_1}] = -[\mathbf{0}_1][\mathbf{T}_2^2][\mathbf{T}_3^2] \\ \Omega R\theta\omega:\ \ [\Pi_{O7_2}] &= -[\mathbf{T}_1^2][\mathbf{0}_2][\mathbf{T}_3^2],\ \ \Omega R\theta : [\Pi_{O7_3}] = -[\mathbf{T}_1^2][\mathbf{T}_2^2][\mathbf{0}_3] \end{aligned} \tag{3.15}$$

1. RR-tadpole Consistency Conditions

It is the same in Type IIB theory that the total (associated) homology cycle RR charge of D-branes and orientifold planes must vanish since the RR field flux lines are conserved. This implies cancellation of all $D=4$ non-Abelian gauge anomalies from triangular diagrams, such as $SU(N)^3$ and $U(1)$ anomalies. That is, [17, 20]

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4\sum_p [\Pi_{O_p}] = 0 \tag{3.16}$$

Additional objects called filler branes on top of the O-planes can be introduced again to reduce the difficulty of satisfying this condition.

2. Conditions for Supersymmetric Brane Configurations

We have known that the condition in Type IIA theory to preserve $N = 1$ supersymmetry in four dimensions is that each rotation angle θ^i between each D6-brane and the R-invariant axis of ith torus is an element of $SU(3)$ [7, 17, 63]. On the other hand, in Type IIB theory to satisfy $N = 1$ supersymmetry in the open-string sector these "angles" of each torus determined by the world-volume magnetic fields are defined as $\tan\theta_i = (F^i)^{-1} = \frac{m^i \chi'^i}{n^i}$, where $\chi'^i = R_1^i R_2^i$ is the area of the $\mathbf{T}_i^2$ in α' units, then we can write it in a form that is similar to the constraints in Type IIA picture as [17]

$$\begin{aligned} -x_A m_a^1 m_a^2 m_a^3 + x_B m_a^1 n_a^2 n_a^3 + x_C n_a^1 m_a^2 n_a^3 + x_D n_a^1 n_a^2 m_a^3 = 0 \\ -n_a^1 n_a^2 n_a^3/x_A + n_a^1 m_a^2 m_a^3/x_B + m_a^1 n_a^2 m_a^3/x_C + m_a^1 m_a^2 n_a^3/x_D < 0 \end{aligned} \tag{3.17}$$

where $x_A = \lambda$, $x_B = \lambda/\chi^2\chi^3$, $x_C = \lambda/\chi^1\chi^3$, $x_D = \lambda/\chi^1\chi^2$, and λ is a normalization constant used to keep the variables on an equal footing. It is not a surprise since they are T-dual to each other.

C. The K-theory Conditions*

In the previous sections, the consistency conditions for having a model free of RR tadpoles were stated. These conditions essentially translate into constraints on the allowed homology cycles. However, it has been argued that it is K-theory which fully classifies the RR-charges of D-branes and not the ordinary homology theory [24, 41, 66, 67, 68]. In addition to the RR-tadpole condition the discrete D-brane RR charges classified by $\mathbb{Z}_\mathbf{2}$ K-theory groups in the presence of orientifolds, which are invisible by the ordinary homology [40, 41, 66, 67, 68], should be also taken into account [40, 41, 65].

In type I superstring theory there exist non-BPS D-branes carrying non-trivial K-theory $\mathbb{Z}_\mathbf{2}$ charges. To avoid this anomaly it is required that in compact spaces these non-BPS branes must exist in an even number [67]. If we consider a type I non-BPS D7-brane ($\widehat{\mathrm{D7}}$-brane), we may regard it as a pair of D7-brane and its world-sheet parity image $\overline{\mathrm{D7}}$-brane in type IIB theory, i.e. $\widehat{\mathrm{D7}} = \mathrm{D7} + \overline{\mathrm{D7}}/\Omega$. There are three different kinds of non-BPS ($\widehat{\mathrm{D7}}$)-branes, denoted as $\widehat{\mathrm{D7}_i}$, where $i = 1, 2, 3$ labels the two-torus where the $\widehat{\mathrm{D7}}$ does not wrap. By construction there are three pairs $\mathrm{D7}_i$, $\overline{\mathrm{D7}}_i$ in type IIB theory [41]. These D7-brane pairs, as well as other D-brane pairs in type IIB theory, can be explicitly expressed by the homology 3-cycles in type IIA theory as listed in Table II.†.

It is reasonable to take the branes in Table II as a basis of a magnetized model (obviously they are in terms of the homology one-cycles). We can see that a general D6-brane three-cycle in type IIA theory is composed of these brane pairs, i.e., a

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†In type IIB picture $\mathrm{D5}_i$ stands for a D5-brane wrapping the i^{th} two torus.

Table II. Brane pairs of Type IIB theory without B-field and their corresponding homology classes of 3-cycles in Type IIA picture.

D3-brane	$\Pi_{D3} = ([b_1])([b_2])([b_3])$	$\Pi_{\overline{D3}} = (-[b_1])(-[b_2])(-[b_3])$
D5-brane	$\Pi_{D5_1} = ([a_1])([b_2])([b_3])$	$\Pi_{\overline{D5}_1} = ([a_1])(-[b_2])(-[b_3])$
	$\Pi_{D5_2} = ([b_1])([a_2])([b_3])$	$\Pi_{\overline{D5}_2} = (-[b_1])([a_2])(-[b_3])$
	$\Pi_{D5_3} = ([b_1])([b_2])([a_3])$	$\Pi_{\overline{D5}_3} = (-[b_1])(-[b_2])([a_3])$
D7-brane	$\Pi_{D7_1} = ([b_1])([a_2])([a_3])$	$\Pi_{\overline{D7}_1} = (-[b_1])([a_2])([a_3])$
	$\Pi_{D7_2} = ([a_1])([b_2])([a_3])$	$\Pi_{\overline{D7}_2} = ([a_1])(-[b_2])([a_3])$
	$\Pi_{D7_3} = ([a_1])([a_2])([b_3])$	$\Pi_{\overline{D7}_3} = ([a_1])([a_2])(-[b_3])$
D9-brane	$\Pi_{D9} = ([a_1])([a_2])([a_3])$	$\Pi_{\overline{D9}} = ([a_1])([a_2])([a_3])$

general D6-brane is a linear combination of these brane pairs, which is why we should take the K-theory constraints into account since the numbers of the pairs given by wrapping numbers are not trivially even.

We do not have to worry about the K-theory charge contributed by D5 and D9-branes since the RR-tadpole conditions guarantee the even numbers if we choose the number of the filler branes to be even, which is not difficult to achieve. The real problem comes from D3 and D7-branes, though they do not contribute to the standard RR charges. The K-theory conditions for a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold were derived in [41] and are given by [40, 41, 65]

$$\sum_a 2^{-k} N_a l_a^1 l_a^2 l_a^3 = \sum_a 2^{-k} N_a \tilde{A}_a = 0 \bmod 4$$
$$\sum_a 2^{-\beta_1} N_a l_a^1 n_a^2 n_a^3 = \sum_a 2^{-\beta_1} N_a \tilde{B}_a = 0 \bmod 4$$

$$\sum_a 2^{-\beta_2} N_a n_a^1 l_a^2 n_a^3 = \sum_a 2^{-\beta_2} N_a \tilde{C}_a = 0 \bmod 4$$
$$\sum_a 2^{-\beta_3} N_a n_a^1 n_a^2 l_a^3 = \sum_a 2^{-\beta_3} N_a \tilde{D}_a = 0 \bmod 4 \qquad (3.18)$$

These constraints turn out to be more clear if the additional three D5-branes or D9-brane are introduced as "probes" [67]. These branes wrap cycles along the O6-planes so they satisfy supersymmetry automatically and form USp groups. The sum of intersection numbers between these probe branes and the general D6-branes should be even (mod 4 in our case) in order to cancel the global gauge anomaly [69]. For example,

$$\sum_a N_a [\Pi_{D5_1}][\Pi_a] = \sum_a N_a l_a^1 n_a^2 n_a^3 = 0 \bmod 4 \qquad (3.19)$$

which is exactly the same as the second equation in (3.18). Though we add these extra branes to detect the K-theory charges, they are still exterior to our original model and do not contribute to the determined RR-tadpole cancellation configuration.

D. Intersection Numbers and the Spectra*

The initial $U(N_a)$ gauge group supported by a stack of N_a identical D6-branes is broken down by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry to a subgroup $U(N_a/2)$ [17, 63]. Chiral matter particles are formed from open strings with two ends attaching on different stacks. In Type IIA point of view the bi-fundamental fields are from the strings near the intersection of two D-branes, so the number of the generation is the intersection number. In Type IIA picture, by using the algebra $[a_i][b_j] = -[b_j][a_i] = \delta_{ij}$ and $[a_i][a_j] = -[b_j][b_i] = 0$ we can calculate the intersection numbers between stacks a and b and provide the multiplicity ($\mathcal{M}$) of the corresponding bi-fundamental repre-

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sentation:

$$\mathcal{M}(\frac{N_a}{2},\overline{\frac{N_b}{2}}) = I_{ab} = [\Pi_a][\Pi_b] = 2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i) \tag{3.20}$$

Likewise, stack a paired with the orientifold image b' of b yields

$$\mathcal{M}(\frac{N_a}{2},\frac{N_b}{2}) = I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i + n_b^i l_a^i) \tag{3.21}$$

Strings stretching between a brane in stack a and its mirror image a' yield chiral matter in the antisymmetric and symmetric representations of the group $U(N_a/2)$ with multiplicities

$$\mathcal{M}((\mathrm{A}_a)_L) = \frac{1}{2}I_{aO6}, \;\; \mathcal{M}((\mathrm{A}_a+\mathrm{S}_a)_L) = \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6}) \tag{3.22}$$

so that the net total of antisymmetric and symmetric representations are given by

$$\begin{aligned}\mathcal{M}(\mathrm{Anti}_a) &= \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6}) = -2^{1-k}[(2A_a-1)\tilde{A}_a - \tilde{B}_a - \tilde{C}_a - \tilde{D}_a] \\ \mathcal{M}(\mathrm{Sym}_a) &= \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6}) = -2^{1-k}[(2A_a+1)\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a]\end{aligned} \tag{3.23}$$

where

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k}\prod_{i=1}^{3} n_a^i l_a^i \tag{3.24}$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a) \tag{3.25}$$

This distinction is critical, as we require independent use of the paired multiplets such as $(\mathbf{10},\overline{\mathbf{10}})$ in Flipped $SU(5)$ models which are masked in expression (3.23).

A zero intersection number between two branes implies that the branes are parallel on at least one torus. At such kind of intersection additional non-chiral (vector-like) multiplet pairs from $ab+ba$, $ab'+b'a$, and $aa'+a'a$ can arise [63]*. The multiplicity of these non-chiral multiplet pairs is given by the remainder of the

*Representations $(\mathrm{Anti}_a + \overline{\mathrm{Anti}}_a)$ occur at intersection of a with a' if they are parallel on at least one torus.

Table III. Spectra of bi-fundamental representations.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet and 3 adjoint chiral multiplets
$ab+ba$	$\mathcal{M}(\frac{N_a}{2},\frac{\overline{N_b}}{2}) = I_{ab} = \prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i)$
$ab'+b'a$	$\mathcal{M}(\frac{N_a}{2},\frac{N_b}{2}) = I_{ab'} = -\prod_{i=1}^{3}(n_a^i l_b^i + n_b^i l_a^i)$
$aa'+a'a$	$\mathcal{M}(\mathrm{Anti}_a) = \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO})$
	$\mathcal{M}(\mathrm{Sym}_a) = \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO})$

intersection product, neglecting the null sector. For example, if $(n_a^1 l_b^1 - n_b^1 l_a^1) = 0$ in $I_{ab} = [\Pi_a][\Pi_b] = 2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i)$,

$$\mathcal{M}\left[\left(\frac{N_a}{2},\frac{\overline{N_b}}{2}\right) + \left(\frac{\overline{N_a}}{2},\frac{N_b}{2}\right)\right] = \prod_{i=2}^{3}(n_a^i l_b^i - n_b^i l_a^i) \tag{3.26}$$

The spectra from Type IIB orientifold are identical to that from Type IIB. A summary of the representations of both picture is listed in Table III.

E. Generalized Green-Schwarz Mechanism*

Although the total non-Abelian anomaly in intersecting brane world models cancels automatically when the RR-tadpole conditions are satisfied, there may be additional mixed anomalies present. For instance, the mixed gravitational anomalies which generate massive fields are not trivially zero [17, 63]. These anomalies are cancelled

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by a generalized Green-Schwarz (G-S) mechanism which involves untwisted Ramond-Ramond forms. The couplings of the four untwisted Ramond-Ramond forms B_2^i to the $U(1)$ field strength F_a of each stack a are [11]

$$\begin{array}{ll} N_a l_a^1 n_a^2 n_a^3 \int_{M4} B_2^1 \wedge \mathrm{tr} F_a, & N_a n_a^1 l_a^2 n_a^3 \int_{M4} B_2^2 \wedge \mathrm{tr} F_a \\ N_a n_a^1 n_a^2 l_a^3 \int_{M4} B_2^3 \wedge \mathrm{tr} F_a, & -N_a l_a^1 l_a^2 l_a^3 \int_{M4} B_2^4 \wedge \mathrm{tr} F_a \end{array} \tag{3.27}$$

These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G-S mechanism. If in some models a combined gauge group $U(1)_X$ is required to remain a gauge symmetry so that for example it may remix to help generate the standard model hypercharge, we must ensure that the gauge boson of the $U(1)_X$ group does not receive such a mass. The $U(1)_X$ is a linear combination of the $U(1)$s from each stack :

$$U(1)_X = \sum_a c_a U(1)_a \tag{3.28}$$

The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand :

$$\begin{array}{ll} \sum_a c_a N_a \tilde{B}_a = 0, & \sum_a c_a N_a \tilde{C}_a = 0, \\ \sum_a c_a N_a \tilde{D}_a = 0, & \sum_a c_a N_a \tilde{A}_a = 0. \end{array} \tag{3.29}$$

F. Turning on Fluxes

Although intersecting D-brane construction is successful in building low energy physics, the moduli are left to be stabilized, which is a main problem of string theory. Turning on supergravity RR and NS three-form fluxes provides a possible way of moduli stabilization, and it has been fully studied in Type IIB orientifolds where the Kähler moduli are fixed at string scale via non-perturbative effects and the dilaton is of order one [37, 38, 39, 40, 41, 42, 65, 70]. The Type IIB fluxes contribute only to one of the tadpoles and the contribution is typically large due to the quantization condition. In Type IIA theory the fluxes can be both even and odd ranks, so both complex and Kähler moduli can be fixed by the fluxes perturbatively at the same time. The metric fluxes which are from the T-duality of NS fluxes can be included to couple with the moduli. The Type IIA fluxes contribute to all of the tadpoles and the solutions are vacua dependent. Both naturally break space-time supersymmetry in the bulk, thus, specific solutions are needed in order to preserve supersymmetry. We provide brief description of Type IIB and Type IIA fluxes below.

1. Type IIB Fluxes*

The Type IIB non-trivial RR 3-form F_3 and NSNS 3-form H_3 fluxes compactified on Calabi-Yau threefold X_6 need to obey the Bianchi identities and be quantized [36]:

$$dF_3 = 0, \quad dH_3 = 0 \tag{3.30}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_{X6} F_3 \in \mathbf{Z}, \quad \frac{1}{(2\pi)^2\alpha'}\int_{X6} H_3 \in \mathbf{Z} \tag{3.31}$$

When the two fluxes are turned on, they induce a covariant field $G_3 = F_3 - \tau H_3$ and contribute to the D3-brane RR charges

$$N_{flux} = \frac{1}{(4\pi^2\alpha')^2}\int_{X_6} H_3 \wedge F_3 = \frac{1}{(4\pi^2\alpha')^2}\frac{i}{2\mathrm{Im}(\tau)}\int_{X_6} G_3 \wedge \bar{G}_3 \tag{3.32}$$

where $\tau = a + i/g_s$ being the Type IIB axion-dilaton coupling.

A complex cohomology basis can be utilized to describe the 3-form flux G_3 on

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$\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$:

$$\begin{aligned}
\omega_{B_0} &= dz^1 \wedge dz^2 \wedge dz^3, \ \ \omega_{A_1} = d\bar{z}^1 \wedge dz^2 \wedge dz^3, \\
\omega_{B_1} &= dz^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3, \ \ \omega_{A_2} = dz^1 \wedge d\bar{z}^2 \wedge dz^3, \\
\omega_{B_2} &= d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^3, \ \ \omega_{A_3} = dz^1 \wedge dz^2 \wedge d\bar{z}^3, \\
\omega_{B_3} &= d\bar{z}^1 \wedge d\bar{z}^2 \wedge dz^3, \ \ \omega_{A_0} = d\bar{z}^1 \wedge d\bar{z}^2 \wedge d\bar{z}^3
\end{aligned} \tag{3.33}$$

where $dz^i = dx^i + U_i dy^i$, U_i are complex structure moduli. Here ω_{B_0} corresponds to the (3,0) of the flux, ω_{B_i} with i =1, 2, 3 correspond to (1,2) of the flux, ω_{A_i} with i =1, 2, 3 correspond to (2,1), and ω_{A_0} is (0,3) component of the flux. Then the untwisted 3-form G_3 takes the form:

$$\frac{1}{(2\pi)^2\alpha'} G_3 = \sum_{i=0}^{3} (A^i \omega_{A_i} + B^i \omega_{B_i}) \tag{3.34}$$

Therefore the contribution of the fluxes to the RR tadpole condition N_{flux} can be calculated in terms of the basis defined above:

$$N_{flux} = \frac{1}{(4\pi^2\alpha')^2} \frac{i}{2\mathrm{Im}(\tau)} \int_{X_6} G_3 \wedge \bar{G_3} = \frac{4\prod_{i=1}^{3} \mathrm{Im}(U^i)}{\mathrm{Im}(\tau)} \sum_{j=0}^{3} (|A^i|^2 - |B^i|^2) \tag{3.35}$$

The choice of fluxes may be positive (ISD-fluxes*) or negative (IASD-fluxes). However, in order to satisfy the supergravity equation of motion, the BPS-like self-dual condition $*_6 G_3 = iG_3$ demands N_{flux} to be positive [37, 42, 76]. The quantization conditions of F_3 and H_3 fluxes require that N_{flux} be a multiple of 64.

*Imaginary self dual fluxes, lead to zero or negative cosmological constant(to lowest order).

2. Supersymmetry Conditions for Type IIB Fluxes*

$D = 4$ $N = 1$ supersymmetric vacua from flux compactification require 1/4 supercharges of the ten-dimensional Type I theory be preserved both in the open and closed string sectors [42]. The supersymmetry constraints in the open string sector are from the world-volume magnetic field which has been discussed in the general case without flux above, and those in the closed string sector induced by the fluxes.

In the closed string sector, to ensure that the RR and NSNS fluxes are supersymmetric, the primitivity condition $G_3 \wedge J = 0$ should be satisfied [37]. Here J is the general Kähler form of $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [65]:

$$J = J_1 dz^1 \wedge d\bar{z}^1 + J_2 dz^2 \wedge d\bar{z}^2 + J_3 dz^3 \wedge d\bar{z}^3 \tag{3.36}$$

We list a few solutions below. We also require that the turned on fluxes are as small as possible to avoid too large RR charge and satisfy the above requirements.

a. (2, 1)-Flux

(1) A specific supersymmetric solution for G_3 is (2, 1)-form given in [76] as

$$\frac{1}{(2\pi)^2\alpha'} G_3 = -4\omega_{A_2} - 4\omega_{A_3} \tag{3.37}$$

where the complex structure U^i and the dilaton coupling τ stabilize at $U^1 = U^2 = U^3 = \tau = i$. This solution gives the flux RR tadpole contribution:

$$N_{flux} = 128 \tag{3.38}$$

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(2) Another specific supersymmetric solution for (2, 1)-form is given in [65] as

$$\frac{1}{(2\pi)^2\alpha'}G_3 = \frac{8}{\sqrt{3}}e^{-\pi i/6}(\omega_{A_1} + \omega_{A_2} + \omega_{A_3}) \tag{3.39}$$

The fluxes stabilize the complex structure toroidal moduli at values $U^1 = U^2 = U^3 = \tau = e^{2\pi i/3}$. Thus, the flux contributes to the RR tadpole contribution an amount:

$$N_{flux} = 192 \tag{3.40}$$

b. Non-SUSY

This solution has the smallest contribution to the D3 RR charge. Although it is not supersymmetric due to the existence of (0, 3) component, it is still worthy of study since we do not observe supersymmetry at low energies. The 3-form flux is

$$\frac{1}{(2\pi)^2\alpha'}G_3 = 2(\omega_{A_0} + \omega_{A_1} + \omega_{A_2} + \omega_{A_3}) \tag{3.41}$$

with $U^1 = U^2 = U^3 = \tau = i$. The flux induced RR charge is then

$$N_{flux} = 64 \tag{3.42}$$

3. Type IIA Fluxes*

Recently the techniques for consistent flux compactifications on Type IIA orientifolds were developed [39, 71]. RR and NSNS fluxes and metric fluxes can be turned on on to stabilize the moduli. By using the effective flux-induced superpotential, there are four classes of (non-singular) vacua which correspond to $N = 1$ supersymmetric Minkowski vacua, Minkowski no-scale vacua, AdS vacua as well as non-supersymmetric AdS vacua. We are especially interested in the AdS vacua with metric fluxes because the

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fluxes contribute negative charges to *all* RR tadpole cancellation conditions, which means that not only the RR tadpole constraints are relaxed but the fluxes play the role of the O6-planes. Therefore in some cases simpler orientifold like $\mathbf{T^6}$ instead of $\mathbf{T^6}/\mathbb{Z}_2 \times \mathbb{Z}_2$ could be used to construct models to avoid too large fluxes confined by the quantization conditions. Then supersymmetric models with fluxes on Type IIA orientifolds are mainly constrained only by $D = 4$ $N = 1$ supersymmetry conditions.

Unlike the complex cohomology basis for the 3-form fluxes in Type IIB theory defined in (3.33), we will use even and odd 3-forms as basis in Type IIA theory defined as follows:

$$\begin{aligned}
\alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3, \quad \beta_0 = dy^1 \wedge dy^2 \wedge dy^3, \\
\alpha_1 &= dx^1 \wedge dy^2 \wedge dy^3, \quad \beta_1 = dy^1 \wedge dx^2 \wedge dx^3, \\
\alpha_2 &= dy^1 \wedge dx^2 \wedge dy^3, \quad \beta_2 = dx^1 \wedge dy^2 \wedge dx^3, \\
\alpha_3 &= dy^1 \wedge dy^2 \wedge dx^3, \quad \beta_3 = dx^1 \wedge dx^2 \wedge dy^3,
\end{aligned} \tag{3.43}$$

where $\int_{\mathbf{T^6}} \alpha_I \wedge \beta_J = \delta_{IJ}$. The NSNS flux H_3 which is odd under orientifold action and the RR fluxes F can be written as [39]

$$H_3 = \sum_{L=0}^{h_{12}} h_L \beta_L. \tag{3.44}$$

$$\begin{aligned}
F_0 &= -m, \qquad F_6 = e_0 d\mathrm{V}_6, \\
F_2 &= \sum_{A=0}^{h_{\bar{1}1}} q_A \xi_A, \qquad F_4 = \sum_{A=0}^{h_{\bar{1}1}} e_A \tilde{\xi}_A,
\end{aligned} \tag{3.45}$$

where ξ and $\tilde{\xi}$ are

$$\xi_i = -dx^i \wedge dy^i, \quad \tilde{\xi}_i = dx^j \wedge dy^j \wedge dx^k \wedge dy^k, \quad i \neq j \neq k \neq i. \tag{3.46}$$

The same, the fluxes should be quantized.

Superpotential terms mixing moduli can be generated by metric fluxes, which appear naturally in the context of Scherk-Schwarz reductions [72]. We define

$$d\eta^P = -\frac{1}{2}\omega^P_{MN}\eta_M \wedge \eta_N, \tag{3.47}$$

where η^P is tangent 1-form and the metric fluxes ω^P_{MN} should satisfy $\omega^P_{[MN}\omega^S_{R]P} = 0$. Further constraint applied on metric fluxes is the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry and after introducing new variables the metric fluxes can be written as

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} \omega^1_{56} \\ \omega^2_{64} \\ \omega^3_{45} \end{pmatrix}, \quad \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} -\omega^1_{23} & \omega^4_{53} & \omega^4_{26} \\ \omega^5_{34} & -\omega^2_{31} & \omega^5_{61} \\ \omega^6_{42} & \omega^6_{15} & -\omega^3_{12} \end{pmatrix}. \tag{3.48}$$

The Jacobi identities imply constraints

$$b_{ij}a_j + b_{jj}a_i = 0, \; i \neq j; \;\; b_{ik}b_{kj} + b_{kk}a_{ij} = 0, \; i \neq j \neq k \neq i. \tag{3.49}$$

Some obvious solutions can be given like [39] (1) $b_{ij} = 0$, (2) $a_i = 0, \; b_{ij} = b_i\delta_{ij}$, (3) $a_i = a, \; b_{ij} = b, \; b_{ii} = -b, \; i \neq j$.

The total RR charges from the D6-branes and O6-planes and from the metric, NSNS, and RR fluxes must vanish since the RR field flux lines are conserved. With the filler branes on the top of the four O6-planes, we obtain the RR tadpole cancellation conditions [39, 71]:

$$2^k N^{(1)} - \sum_a N_a A_a + \frac{1}{2}(h_0 m + a_1 q_1 + a_2 q_2 + a_3 q_3) = 16, \tag{3.50}$$

$$-2^{\beta_1} N^{(2)} + \sum_a 2^{-\beta_2-\beta_3} N_a B_a + \frac{1}{2}(m h_1 - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) = -2^{4-\beta_2-\beta_3}, \tag{3.51}$$

$$-2^{\beta_2} N^{(3)} + \sum_a 2^{-\beta_1-\beta_3} N_a C_a + \frac{1}{2}(m h_2 - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) = -2^{4-\beta_1-\beta_3}, \tag{3.52}$$

$$-2^{\beta_3}N^{(4)} + \sum_a 2^{-\beta_1-\beta_2}N_aD_a + \frac{1}{2}(mh_3 - q_1b_{13} - q_2b_{23} - q_3b_{33}) = -2^{4-\beta_1-\beta_2}, \quad (3.53)$$

where $2N^{(i)}$ are the number of filler branes wrapping along the i-th O6-plane which is defined in Table I. In addition, a_i and b_{ij} arise from the metric fluxes, h_i arise from the NSNS fluxes, and m and q_i arise from the RR fluxes. We consider these fluxes (a_i, b_{ij}, h_i, m and q_i) quantized in units of 8 so that we can avoid the problems with flux Dirac quantization conditions.

We will concentrate on the supersymmetric AdS vacua with metric, NSNS and RR fluxes [39]. For simplicity, we assume that the Kähler moduli T_i satisfy $T_1 = T_2 = T_3$, then we obtain $q_1 = q_2 = q_3 \equiv q$ from superpotential in [39]. To satisfy the Jacobi identities for metric fluxes, we consider the solution $a_i = a$, $b_{ii} = -b_i$, and $b_{ji} = b_i$ in which $j \neq i$ [39].

To have supersymmetric minima [39], we obtain that

$$3a\mathrm{Re}S \;=\; b_i\mathrm{Re}U_i \ , \quad \text{for} \quad i = 1,\ 2,\ 3, \quad (3.54)$$

where

$$\mathrm{Re}S \;\equiv\; \frac{e^{-\phi}}{\sqrt{\chi_1\chi_2\chi_3}} \ , \ \mathrm{Re}U_i \;\equiv\; e^{-\phi}\sqrt{\frac{\chi_j\chi_k}{\chi_i}} \ , \quad (3.55)$$

where S and U_i are respectively dilaton and complex structure moduli, ϕ is the four-dimensional T-duality invariant dilaton, and $i \neq j \neq k$. And then we have

$$b_1 = \frac{3a}{\chi_2\chi_3} \ , \ b_2 = \frac{3a}{\chi_1\chi_3} \ , \ b_3 = \frac{3a}{\chi_1\chi_2} \ . \quad (3.56)$$

Moreover, there are consistency conditions

$$3h_ia + h_0b_i = 0 \ , \quad \text{for} \quad i = 1,\ 2,\ 3, \quad (3.57)$$

So we have

$$h_1 = -\frac{h_0}{\chi_2\chi_3} \ , \ h_2 = -\frac{h_0}{\chi_1\chi_3} \ , \ h_3 = -\frac{h_0}{\chi_1\chi_2} \ . \quad (3.58)$$

Thus, the RR tadpole cancellation conditions can be rewritten as following

$$2^k N^{(1)} - \sum_a N_a A_a + \frac{1}{2}(h_0 m + 3aq) = 16, \tag{3.59}$$

$$-2^{\beta_1} N^{(2)} + \sum_a 2^{-\beta_2-\beta 3} N_a B_a - \frac{1}{2\chi_2\chi_3}(h_0 m + 3aq) = -2^{4-\beta_2-\beta_3}, \tag{3.60}$$

$$-2^{\beta_2} N^{(3)} + \sum_a 2^{-\beta_1-\beta_3} N_a C_a - \frac{1}{2\chi_1\chi_3}(h_0 m + 3aq) = -2^{4-\beta_1-\beta_3}, \tag{3.61}$$

$$-2^{\beta_3} N^{(4)} + \sum_a 2^{-\beta_1-\beta_2} N_a D_a - \frac{1}{2\chi_1\chi_2}(h_0 m + 3aq) = -2^{4-\beta_1-\beta_2}. \tag{3.62}$$

Therefore, if $(h_0 m + 3aq) < 0$, the supergravity fluxes contribute negative D6-brane charges to all the RR tadpole cancellation conditions, and then, the RR tadpole cancellation conditions give no constraints on the consistent model building because we can always introduce suitable supergravity fluxes and some stacks of D6-branes in the hidden sector to cancel the RR tadpoles. Also, if $(h_0 m + 3aq) = 0$, the supergravity fluxes do not contribute to any D6-brane charges, and then do not affect the RR tadpole cancellation conditions.

In addition, the Freed-Witten anomaly cancellation condition is

$$-2^{-k} h_0 \tilde{A}_a + 2^{-\beta_1} h_1 \tilde{B}_a + 2^{-\beta_2} h_2 \tilde{C}_a + 2^{-\beta_3} h_3 \tilde{D}_a = 0. \tag{3.63}$$

We can show that if Eqs. (3.17), (3.54) and (3.57) are satisfied, the Freed-Witten anomaly is automatically cancelled. So, in our model building, we will not consider the Freed-Witten anomaly.

Table IV. The general spectrum for the intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications.

Sector	Representation
aa	$U(N_a)$ vector multiplet and 3 adjoint chiral multiplets
$ab+ba$	I_{ab} $(N_a, \overline{N_b})$ chiral multiplets
$ab'+b'a$	$I_{ab'}$ (N_a, N_b) chiral multiplets
$aa'+a'a$	$\frac{1}{2}(I_{aa'}+I_{a,O6})$ anti-symmetric chiral multiplets
	$\frac{1}{2}(I_{aa'}-I_{a,O6})$ symmetric chiral multiplets

4. Type IIA Theory on $\mathbf{T^6}$ Orientifold*

The intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications is similar to that on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2}\times\mathbb{Z}_\mathbf{2})$ orientifold. For model building rules in above subsection, we only need to make the following changes: (1) For a stack of N_a D6-branes and its ΩR image, we have $U(N_a)$ gauge symmetry, while for a stack of N_a D6-branes and its ΩR image on the top of O6-plane, we obtain $USp(2N_a)$ gauge symmetry. Also, we present the general spectrum of D6-branes' intersecting at generic angles in Type IIA theory on $\mathbf{T^6}$ orientifold in Table IV; (2) We only have the ΩR O6-planes, so, $[\Pi_{O6}]=[\Pi_{\Omega R}]$ in (3.16), and the right-hand sides of (3.51), (3.52) and (3.53) are zero; (3) The metric, NSNS and RR fluxes (a_i, b_{ij}, h_i, m and q_i) are quantized in units of 2.

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CHAPTER IV

STANDARD-LIKE MODELS

A. A Trinification Model*

The $SU(3)_C \times SU(3)_L \times SU(3)_R$ trinification model, as a candidate for a grand unified theory, was proposed by de Rújula, Georgi, and Glashow [73] (see also [74]). Although no one has considered such models, the trinification model is quite interesting for the intersecting D-brane model building because all the left-handed quarks Q^i_L, the right-handed quarks Q^i_R, the leptons L^i, and the Higgs fields H^k, which are listed in Table V, belong to the bi-fundamental representations.

Let us briefly review the trinification model. The electric charge generator Q_{EM} is given by

$$Q_{EM} \equiv I_{3L} + \frac{Y}{2} = I_{3L} - \frac{Y_L}{2} + I_{3R} - \frac{Y_R}{2} \ , \tag{4.1}$$

where the generators for $U(1)_{I_{3L}}$ and $U(1)_{I_{3R}}$, and $U(1)_{Y_L}$ and $U(1)_{Y_R}$ in $SU(3)_L$ and $SU(3)_R$ gauge symmetries are

$$\mathbf{T}_{U(1)_{I_{3L,R}}} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \ , \tag{4.2}$$

$$\mathbf{T}_{U(1)_{Y_{L,R}}} = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} \end{pmatrix} \ . \tag{4.3}$$

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Table V. The particle contents in the $SU(3)_C \times SU(3)_L \times SU(3)_R$ model.

Particles	Representation
Q_L^i	$(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$
Q_R^i	$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3})$
L^i or H^k	$(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$

And the explicit particle components in the $(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})$, $(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3})$, and $(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$ representations are

$$(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}): \quad Q_L^i = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \tag{4.4}$$

$$(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}): \quad Q_R^i = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}, \tag{4.5}$$

$$(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}): \quad L^i \text{ or } H^k = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix}. \tag{4.6}$$

The $SU(3)_C \times SU(3)_L \times SU(3)_R$ gauge symmetry can be broken down to the SM gauge symmetry by giving the vacuum expectation values (VEVs) to ν^c and S, *i. e.*,

$$\langle \nu^c \rangle \neq 0 \,, \quad \langle S \rangle \neq 0 \,. \tag{4.7}$$

The electric charges for h and h^c are respectively $-\frac{1}{3}$ and $\frac{1}{3}$, for E and E^c are respectively -1 and 1; and for N, N^c, and S are zero.

Table VI. Wrapping and intersection numbers in the $SU(3)_C \times SU(3)_L \times SU(3)_R$ model.

stack	N_a	$(n_1, l_1)(n_2, l_2)(n_3, l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	$f^{(1)}$	$f^{(3)}$
a	6	(0, 1) (-1,-1) (2, 1)	-2	2	3	1	-3	-1	-3	-3	-3	-3	-2	0
b	6	(-1,-1) (-2, 1) (1, 0)	2	-2	-	-	4	0(2)	6	0(5)	6	0(5)	2	-1
c	6	(-1, 1) (0, 1) (-1, 1)	0	0	-	-	-	-	0(-2)	0(2)	0(-2)	0(2)	0	1
d	2	(-1, 1) (-1, 2) (1, 1)	-16	0	-	-	-	-	-	-	0(0)	-16	-1	-2
e	2	(-1, 1) (-1, 2) (1, 1)	-16	0	-	-	-	-	-	-	-	-	-1	-2
$fil^{(2)}$	2	(1, 0) (0, 1) (0,-1)	-	-	-	-	-	-	-	-	-	-	-	-
$fil^{(3)}$	6	(0, 1) (1, 0) (0,-1)	-	-	-	-	-	-	-	-	-	-	-	-

With above background, we can construct an intersecting D6-brane trinification model. The bi-fundamental representation with one fundamental and one anti-fundamental indices is different from the bi-fundamental representation with two fundamental (or anti-fundamental) indices, for example, $(\mathbf{3}, \overline{\mathbf{3}}, \mathbf{1})$ and $(\mathbf{3}, \mathbf{3}, \mathbf{1})$ (or $(\overline{\mathbf{3}}, \overline{\mathbf{3}}, \mathbf{1})$). So, we can contruct the trinification models with three families of the SM fermions and without tilted two-torus.

There are three $SU(3)$ groups in the trinification model, so three stacks of six D6-branes are required. Additional stacks with $U(1)$ group and filler branes are also used to satisfy the RR tadpole cancellation conditions. In our model building, we require the intersection numbers to satisfy

$$I_{ab} = 3; \quad I_{ac} = -3; \quad I_{bc} \geq 4 \ , \tag{4.8}$$

where $I_{ab} = 3$ and $I_{ac} = -3$ give us three families of the left-handed quarks and three families of the right-handed quarks, respectively, and $I_{bc} \geq 4$ gives us three families of the leptons and $(I_{bc} - 3)$ Higgs field(s).

We have large RR charges from three $SU(3)$ groups, so it is not easy to construct a trinification model without RR tadpoles. After careful searches, we find a super-

Table VII. The spectrum in the $SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)^5 \times USp(2) \times USp(6)$ model with four global $U(1)$s from the G-S mechanism.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$
$(3_a, \bar{3}_b)$	3	1	-1	0	0	0	-24	6	0	6
$(\bar{3}_a, 3_c)$	3	-1	0	1	0	0	12	6	0	-12
$(3_b, \bar{3}_c)$	3	0	1	-1	0	0	12	-12	0	6
$(3_b, \bar{3}_c)$	1	0	1	-1	0	0	12	-12	0	6
$(3_a, 3_b)$	1	1	1	0	0	0	0	-6	0	6
$(\bar{3}_a, \bar{3}_c)$	1	-1	0	-1	0	0	12	-6	0	0
$(\bar{3}_a, 1_d)$	3	-1	0	0	1	0	10	-4	2	-10
$(\bar{3}_a, \bar{1}_d)$	3	-1	0	0	-1	0	14	4	-2	-2
$(\bar{3}_a, 1_e)$	3	-1	0	0	0	1	10	-4	2	-10
$(\bar{3}_a, \bar{1}_e)$	3	-1	0	0	0	-1	14	4	-2	-2
$(3_b, \bar{1}_d)$	6	0	1	0	-1	0	14	-2	-2	4
$(3_b, \bar{1}_e)$	6	0	1	0	0	-1	-2	-2	4	4
$(\bar{1}_d, \bar{1}_e)$	16	0	0	0	-1	-1	4	8	-4	8
A_a	2	-2	0	0	0	0	24	0	0	-12
A_b	2	0	2	0	0	0	24	-12	0	0
S_a	2	2	0	0	0	0	-24	0	0	12
S_b	2	0	-2	0	0	0	-24	12	0	0
Additional non-chiral and $USp(2)$ & $USp(6)$ Matter										

symmetric intersecting D6-brane trinification model which satisfies the RR tadpole cancellation conditions and K-theory conditions. This model is originally discussed in the case without turning on any flux, however one can still turn on the Type IIA fluxes in AdS vacua and choose appropriate fluxes to contribute zero to the RR tadpole conditions and then fix the moduli. We present its complete wrapping numbers and intersection numbers in Table VI and its spectrum in Table VII. In this model, we have three families of the SM fermions including the right-handed neutrinos, one pair of Higgs doublets, H_u and H_d, one field ν^c and one field S.

1. Gauge Symmetry Breaking

The $U(3)_C \times U(3)_L \times U(3)_R$ gauge symmetry is broken down to the $SU(3)_C \times SU(3)_L \times SU(3)_R$ gauge symmetry due to the G-S mechanism. And the $SU(3)_C \times SU(3)_L \times SU(3)_R$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{I_{3R}} \times U(1)_{Y_R}$ gauge symmetry by the splittings of the $U(3)_L$ and $U(3)_R$ stacks of the D6-branes. Giving VEVs to the singlet Higgs fields ν^c and S, we can break the $U(1)_{Y_L} \times U(1)_{I_{3R}} \times U(1)_{Y_R}$ gauge symmetry down to the $U(1)_Y$ hypercharge interaction. The complete gauge symmetry breaking chains are

$$\begin{aligned} & SU(3)_C \times SU(3)_L \times SU(3)_R \\ \stackrel{Splitting}{\longrightarrow} \quad & SU(3)_C \times SU(2)_L \times U(1)_{Y_L} \times U(1)_{I_{3R}} \times U(1)_{Y_R} \\ \stackrel{VEVs}{\longrightarrow} \quad & SU(3)_C \times SU(2)_L \times U(1)_Y \ . \end{aligned} \tag{4.9}$$

We assume the low scale supersymmetry in this paper. Then the VEVs for ν^c and S should be around the TeV scale because their Higgs mechanism can not preserve the D-flatness and F-flatness and then breaks four-dimensional $N = 1$ supersymmetry.

2. Fermion Masses and Mixings

The quark Yukawa couplings $y_{ijk} Q_L^i Q_R^j H^k$ are allowed by the anomalous $U(1)$ gauge symmetries in the intersecting D6-brane trinification model, while the lepton and neutrino Yukawa couplings $y'_{ijk} L_i L_j H^k$ are forbidden by the anomalous $U(1)_L \times U(1)_R \subset U(3)_L \times U(3)_R$ gauge symmetry.

In our model, only one family of the SM quarks can obtain masses because Q_L^i arise from the intersections on the second two-torus, while Q_R^i arise from the intersections on the third two-torus, or because we only have one pair of Higgs doublet fields.

B. Pati-Salam-Like Models

1. A P-S Model from Type IIA D-branes*

In the previous model building with or without fluxes, it is very difficult to generate suitable three-family SM fermion masses and mixings. In the $SU(5)$ models and flipped $SU(5)$ models, the up-type quark Yukawa couplings and the down-type quark Yukawa couplings are forbidden by anomalous $U(1)$ gauge symmetries. And for the Pati-Salam like models, although all the Yukawa couplings could be allowed in principle, it is very difficult to construct three-family models which can give suitable masses and mixings to three families of the SM fermions because the left-handed fermions, the right-handed fermions and the Higgs fields in general arise from the intersections on different two-tori. Moreover, if supersymmetry is broken by supergravity fluxes, it seems that the masses for the massless SM fermions may not be generated from radiative corrections because the supersymmetry breaking trilinear soft terms are universal and the supersymmetry breaking soft masses for the left/right-chiral squarks and sleptons are also universal [75, 76, 77]. Thus, how to construct the Standard-like models, which can give suitable fermion masses and mixings for three families, is an interesting problem.

To solve this problem, we construct another class of supersymmetric Pati-Salam models without RR tadpoles and K-theory anomaly. In particular, under the $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, we consider the particles with quantum numbers $(\mathbf{4}, \overline{\mathbf{2}}, \mathbf{1})$ and $(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ as the SM fermions while we consider the particles with quantum numbers $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}})$ as exotic particles because these particles are dis-

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Table VIII. Wrapping and intersection numbers in the $U(4)_C \times U(2)_L \times U(2)_R \times U(1)^4$ Model.

stack	N_a	$(n_1, l_1)(n_2, l_2)(n_3, l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f	g	g'
a	8	(-1, 0) (-1, 1) (1, 1)	0	0	3	1	-3	-1	-3	-1	-3	-1	1	3	1	3
b	4	(1,-1) (1, 2) (1, 0)	2	-2	-	-	6	0(5)	8	0(4)	8	0(4)	-2	0(1)	-2	0(1)
c	4	(1, 1) (2, 1) (0,-1)	-2	2	-	-	-	-	0(1)	2	0(1)	2	0(-4)	-8	0(-4)	-8
d	2	(-1,-1) (-1, 0) (1,-2)	-2	2	-	-	-	-	-	-	0(0)	0(0)	0(5)	-6	0(5)	-6
e	2	(-1,-1) (-1, 0) (1,-2)	-2	2	-	-	-	-	-	-	-	-	0(5)	-6	0(5)	-6
f	2	(1, 1) (0, 1) (-2,-1)	-2	2	-	-	-	-	-	-	-	-	-	-	0(0)	0(0)
g	2	(1, 1) (0, 1) (-2,-1)	-2	2	-	-	-	-	-	-	-	-	-	-	-	-

tinguished by anomalous $U(1)$ gauge symmetries (In the trinification models, these kinds of particles are obviously different.). With this convention, we can construct the three-family Pati-Salam models without tilted two-torus where the left-handed and right-handed SM fermions and the Higgs fields arise from the intersections on the same two-torus, and then, we may explain three-family SM fermion masses and mixings.

In this kind of Pati-Salam model building, we require the intersection numbers to satisfy

$$I_{ab} = 3; \quad I_{ac} = -3; \quad I_{bc} \geq 1 \ , \tag{4.10}$$

where $I_{ab} = 3$ and $I_{ac} = -3$ give us three families of the left-handed fermions and three families of the right-handed fermions, respectively, and $I_{bc} \geq 1$ gives us bidoublet Higgs field(s) with allowed Yukawa couplings.

We present one concrete model whose wrapping numbers and intersection numbers are given in Table VIII. In this model, the absolute values of the intersection numbers on the seond two-torus between the $U(4)_C$ and $U(2)_L$ stacks of D6-branes, between the $U(4)_C$ and $U(2)_R$ stacks of D6-branes, and between the $U(2)_L$ and $U(2)_R$ stacks of D6-branes are all three, and all the Yukawa couplings are allowed by anomalous $U(1)$ gauge symmetries. Therefore, we may explain the masses and mixings for three families of the SM fermions. Note that we have four additional D6-brane stacks d, e, f and g in this model, for which one can arbitrarily substitute them into filler brane stacks (USp groups).

In general, the $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry due to the G-S mechanism and the splittings of the $U(4)_C$ and $U(2)_R$ stacks of D6-branes. In our model, the $U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry can only be broken down to the $U(1)_Y$ gauge symmetry by giving VEVs to the scalar components of the right-handed neutrino superfields or the neutral component in the multiplet $(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}})$ from $I_{ac'}$ intersection. However, this Higgs mechanism can not preserve the D-flatness and F-flatness, and then breaks four-dimensional $N = 1$ supersymmetry. Therefore, the $U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry breaking scale should be around the TeV scale.

2. An $U(4)_C \times U(2)_L \times U(1)' \times U(1)''$ Model from Type IIA D-branes*

In all the previous Pati-Salam like model building, the $U(1)_{I_{3R}}$ arises from the non-Abelian gauge symmetry, for example, $U(2)_R$ or $USp(2f)_R$. However, $U(1)_{I_{3R}}$ may come from a linear combination of $U(1)$ gauge symmetries.

In our model building, we require

$$I_{ab} = 3 \ ; \quad I_{ac} = I_{ad} = -3 \ ; \quad I_{bc} \geq 1 \ ; \quad I_{bd} \geq 1 \ , \tag{4.11}$$

where $I_{ab} = 3$ and $I_{ac} = I_{ad} = -3$ give us three families of the left-handed fermions and three families of the right-handed fermions, respectively, and $I_{bc} \geq 1$ gives us

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Table IX. Wrapping and intersection numbers in the $U(4)_C \times U(2)_L \times U(1)' \times U(1)''$ $\times U(1)_e \times U(1)_f \times USp(4)^2$ model.

stack	N_a	$(n_1, l_1)(n_2, l_2)(n_3, l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$fil^{(1)}$	$fil^{(3)}$
a	8	(-1, 0) (-1, 1) (1, 1)	0	0	3	1	-3	-1	-3	-1	-3	-1	1	3	-1	1
b	4	(1,-1) (1, 2) (1, 0)	2	-2	-	-	8	0(4)	9	-5	8	0(4)	-3	-1	2	0
c	2	(1, 1) (1, 0) (1,-2)	-2	2	-	-	-	-	1	3	0(0)	0(0)	5	-9	0	-2
d	2	(2, 1) (2, 1) (0,-1)	-6	6	-	-	-	-	-	-	-1	3	0(-4)	-16	0	-4
e	2	(-1,-1) (-1, 0) (1,-2)	-2	2	-	-	-	-	-	-	-	-	5	-9	0	-2
f	2	(2, 1) (0,-1) (2, 1)	-6	6	-	-	-	-	-	-	-	-	-	-	-4	0
$fil^{(3)}$	4	(0, 1) (1, 0) (0,-1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$fil^{(4)}$	4	(0, 1) (0,-1) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-

bidoublet Higgs fields with allowed Yukawa couplings.

Let us give a concrete supersymmetric model without the RR tadpoles and K-theory anomaly. We present the wrapping numbers and intersecting numbers in Table IX, and the spectrum in Tables XXVIII and XXIX in Appendix A. In particular, the c and d stacks of D6-branes are not T-dual to each other (If c and d stacks of D6-branes are T-dual to each other, the gauge symmetry in fact is $U(4)_C \times U(2)_L \times U(2)_R$.). There are totally six $U(1)$ gauge symmetries where four combinations of them are global and their gauge fields obtain masses by the G-S mechanism. The rest two combinations, which are the massless anomaly-free $U(1)_{I_{3R}}$ and $U(1)_X$ gauge symmetries, are given by

$$U(1)_{I_{3R}} = \frac{1}{2}(U(1)_a + U(1)_b + 2U(1)_c) \ , \tag{4.12}$$

$$U(1)_X = \frac{1}{2}(U(1)_a - U(1)_b + 2U(1)_d - 2U(1)_e - 2U(1)_f) \ . \tag{4.13}$$

In addition, the $U(4)_C \times U(2)_L \times U(1)' \times U(1)'' \times U(1)_e \times U(1)_f$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \times U(1)_X$ gauge symmetry by the G-S mechanism and the splitting of the $U(4)_C$ stack of D6-branes.

Furthermore, the $U(1)_X$ gauge symmetry can be broken by giving VEVs to the SM singlets 1_d and 1_e (or 1_f) which are charged under $U(1)_X$ (see the spectrum in Table XXIX in Appendix A). And the $U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry can be broken down to the $U(1)_Y$ gauge symmetry by giving VEVs to $(\bar{4}_a, 1_e)$ (or $(\bar{4}_a, \bar{1}_e)$) and $(4_a, 1_f)$ (or $(4_a, \bar{1}_f)$). Because these Higgs mechanism can keep the D-flatness and F-flatness and then preserve four-dimensional $N = 1$ supersymmetry, these gauge symmetry breaking scales can be close to the string scale. However, only one family of the SM fermions can obtain masses.

3. A P-S Model with Type IIB Fluxes*

In this Section, we shall consider the Pati-Salam like models on Type IIB orientifold with flux compactifications, which are very interesting because the supergravity fluxes can stabilize the dilaton and the complex structure parameters.

For the trinification models, we already have quite large RR charges due to the three $SU(3)$ groups. With Type IIB supergravity fluxes, the RR tadpole cancellation conditions are much more difficult to be satisfied. And in our detail calculations, we find that it may be impossible to find such a model.

For the Pati-Salam like models, the three-family and four-family Standard-like models with one unit of quantized flux and with the electroweak sector from USp groups were obtained [40, 41, 70] by introducing magnetized D9-branes with large negative D3-brane charges in the hidden sector, and many supersymmtric and non-supersymmetric $U(4)_C \times U(2)_L \times U(2)_R$ models were constructed by considering the magnetized D9-branes with large negative D3-brane charges in the SM observable

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sector [42]. Here, we consider a new flux model with $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry where the magnetized D9-branes with large negative D3-brane charges are introduced in the hidden sector. This kind of models has not been studied previously because it is very difficult to have supersymmetric D-brane configurations with more than three stacks of $U(n)$ branes.

In the model building, we require the intersection numbers to satisfy the conditions in Eq. (4.10). We find a model with one unit of flux, and its wrapping numbers and intersection numbers are given in Table X. Interestingly, no filler branes are needed so we do not have any USp groups. The two extra $U(1)$ gauge symmetries are utilized to compensate the large positive D3-brane charges due to the supergravity fluxes. The $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry by the G-S mechanism and the splittings of the $U(4)_C$ and $U(2)_R$ stacks of D6-branes.

However, the $U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry can only be broken down to the $U(1)_Y$ gauge symmetry at the TeV scale by giving VEVs to the scalar components of the right-handed neutrino superfields or the neutral component in the multiplet $(\mathbf{\overline{4}}, \mathbf{1}, \mathbf{\overline{2}})$ from $I_{ac'}$ intersection because this Higgs mechanism can not preserve the D-flatness and F-flatness, and then breaks four-dimensional $N = 1$ supersymmetry. Also, we can only give masses to one family of the SM fermions.

4. P-S Models with Type IIA Fluxes*

In this section, with the help of Type IIA fluxes, we would like to construct the Pati-Salam models in AdS vacua with the following properties:

- Three families of the SM fermions.

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Table X. Wrapping and intersection numbers in the $U(4)_C \times U(2)_L \times U(2)_R \times U(1)^2$ model with one unit of flux.

stack	N_a	$(n_1, l_1)(n_2, l_2)(n_3, l_3)$	A	S	b	b'	c	c'	d	d'	e	e'
a	8	(-1, 0) (-1, 1) (1, 1)	0	0	3	1	-3	-1	3	-3	3	-3
b	4	(1,-1) (1, 2) (1, 0)	2	-2	-	-	8	0(4)	9	-5	9	-5
c	4	(1, 1) (1, 0) (1,-2)	-2	2	-	-	-	-	5	-9	5	-9
d	2	(2, 1) (-2,-1) (2, 1)	-54	-10	-	-	-	-	-	-	0(0)	-64
e	2	(2, 1) (-2,-1) (2, 1)	-54	-10	-	-	-	-	-	-	-	-

- The Pati-Salam gauge symmetry can be broken down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the SM gauge symmetry around the string scale via supersymmetry preserving Higgs mechanism.

- The SM fermion Yukawa couplings must be allowed. We consider two kinds of Pati-Salam models: in the first kind of models, we can give the suitable masses to three families of the SM fermions at stringy tree level; and in the second kind of models, we can only give masses to the third family of the SM fermions at tree level while we assume that the masses for the first two families of the SM fermions may be generated due to quantum corrections.

To break the Pati-Salam gauge symmetry down to the SM gauge symmetry via D6-brane splittings and supersymmetry preserving Higgs mechanism, the $SU(4)_C$ and $SU(2)_R$ gauge symmetries must come from $U(4)_C$ and $U(2)_R$ gauge symmetries (see the following discussions). Thus, we introduce three stacks of D6-branes, a, b, and c with number of D6-branes 8, 4 (or 2), and 4 in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold, or with number of D6-branes 4, 2 (or 1), and 2 in Type IIA theory on $\mathbf{T^6}$ orientifold. So, a, b, and c stacks of D6-branes give us the gauge symmetries $U(4)_C$, $U(2)_L$ (or $USp(2)_L$) and $U(2)_R$, respectively. The anomalies from global $U(1)$s are cancelled by the generalized Green-Schwarz mechanism, and the gauge fields of these $U(1)$s obtain masses via the linear $B \wedge F$ couplings. So, the effective gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R$. In addition, we require that the intersection numbers satisfy

$$I_{ab} \;=\; 3 \;,\; I_{ab'} \;=\; 0 \text{ if } \mathrm{SU(2)_L} \text{ from } \mathrm{U(2)_L} \;, \tag{4.14}$$

$$I_{ac} \;=\; -3 \;,\; I_{ac'} \;=\; 0 \;. \tag{4.15}$$

The conditions $I_{ab} = 3$ and $I_{ac} = -3$ give us three families of the SM fermions with quantum numbers $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ under $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry. $I_{ac'} = 0$ implies that a stack of D6-branes is parallel to the ΩR image c' of the c stack of D6-branes along at least one two-torus, for example, the third two-torus. So, if a and c' stacks of D6-branes are on the top of each other on the third two-torus, we obtain the $I_{ac'}^{(1,2)}$ pairs of the vector-like chiral multiplets with quantum numbers $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$ where $I_{ac'}^{(1,2)}$ is the product of intersection numbers for a and c' stacks of D6-branes on the first two two-tori. These particles are the Higgs fields which can break the Pati-Salam gauge symmetry down to the SM gauge symmetry, and preserve the D- and F-flatness, *i. e.*, preserve supersymmetry. Also, the conditions in Eq. (4.15) imply that the $SU(4)_C$ and $SU(2)_R$ gauge symmetries must come from $U(4)_C$ and $U(2)_R$ gauge symmetries, respectively.

In addition, for the first kind of Pati-Salam models, we require that

$$I_{bc} \;\geq\; 2 \;, \tag{4.16}$$

and all the SM fermions and at least two bidoublet Higgs fields arise from the intersections on the same two-torus so that the suitable three-family SM fermion masses and mixings can be generated at stringy tree level. And for the second kind of Pati-Salam models, we require that

$$I_{bc} \; \geq \; 1 \; , \tag{4.17}$$

and the SM fermions and Higgs fields do not arise from the intersections on the same two-torus. Then we can only give masses to the third family of the SM fermions at tree level, and we assume that the first two families of the SM fermions can have masses from quantum corrections.

In order to break the Pati-Salam gauge symmetry, we split the a stack of D6-branes into a_1 and a_2 stacks with respectively 6 (3) and 2 (1) D6-branes, split the c stack of D6-branes into c_1 and c_2 stacks with 2 (1) D6-branes for each one on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold (Type IIA $\mathbf{T^6}$ orientifold). And then, the Pati-Salam gauge symmetry is broken down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$. To break this gauge symmetry down to the SM gauge symmetry, we assume that the a_2 and c'_1 (ΩR image of c_1) stacks of D6-branes are parallel and on the top of each other on the third two-torus as an example, and then we obtain $I^{(1,2)}_{a_2 c'_1}$ pairs of vector-like chiral multiplets with quantum numbers $(\mathbf{1}, \mathbf{1}, \mathbf{-1}, \mathbf{1/2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{-1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry where $I^{(1,2)}_{a_2 c'_1}$ is the product of intersection numbers for a_2 and c'_1 stacks on the first two two-tori. These particles can break the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry down to the SM gauge symmetry and preserve supersymmetry in the mean time because their quantum numbers are the same as those of the right-handed neutrino and its Hermitian conjugate. In summary, the complete gauge symmetry breaking chains are

$$\begin{aligned} SU(4)_C \times SU(2)_L \times SU(2)_R \quad & \xrightarrow{a \rightarrow a_1 + a_2} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ & \xrightarrow{c \rightarrow c_1 + c_2} SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \\ & \xrightarrow{\text{Higgs Mechanism}} SU(3)_C \times SU(2)_L \times U(1)_Y \; . \end{aligned} \tag{4.18}$$

Table XI. The particle spectrum in the observable sector in Model TI-U-4 with gauge symmetry $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(10)]_{hidden}$.

Representation	Multiplicity	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$
$(4_a, \bar{2}_b)$	3	1	-1	0	0
$(\bar{4}_a, 2_c)$	3	-1	0	1	0
$(2_b, \bar{2}_c)$	6	0	1	-1	0
$(4_a, 2_c)$	3	1	0	1	0
$(\bar{4}_a, \bar{2}_c)$	3	-1	0	-1	0
6_a	1	2	0	0	0
$\overline{10}_a$	1	-2	0	0	0
1_c	2	0	0	-2	0
3_c	2	0	0	2	0

In this Section, we present the first and second kinds of Pati-Salam models. Because the supergravity fluxes on Type IIA $\mathbf{T^6}/(\mathbb{Z_2} \times \mathbb{Z_2})$ orientifold contribute large negative D6-brane charges due to the Dirac quantization conditions if we want to use them to relax the RR tadpole cancellation conditions, many D6-branes in the hidden sector need to be introduced so that the RR tadpoles can be completely cancelled. Then there may exist a lot of exotic particles. Therefore, we mainly consider the Pati-Salam models on Type IIA $\mathbf{T^6}$ orientifold with flux compactifications. Also, we emphasize that the Pati-Salam models on Type IIA $\mathbf{T^6}$ orientifold can be constructed similarly on Type IIA $\mathbf{T^6}/(\mathbb{Z_2} \times \mathbb{Z_2})$ orientifold by introducing more stacks of D6-branes in the hidden sector. In addition, we determine the complex structure parameters via supersymmetric D6-brane configurations in our model building. Similar to [39], all the moduli may be stabilized in our models.

a. The First Kind of Pati-Salam Models

We present the D6-brane configurations and intersection numbers for the first kind of Pati-Salam models, *i. e.*, the Models TI-U-i with i=1, ..., 7, and the Models TI-

Table XII. The exotic particle spectrum in Model TI-U-4 with gauge symmetry $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(10)]_{hidden}$.

Representation	Multiplicity	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$
$(4_a, \bar{2}_d)$	2	1	0	0	-1
$(\bar{4}_a, 2_e)$	3	-1	0	0	0
$(4_a, 10_{O6})$	1	1	0	0	0
$(\bar{2}_b, 2_d)$	1	0	-1	0	1
$(\bar{2}_b, \bar{2}_d)$	5	0	-1	0	-1
$(2_b, 2_e)$	6	0	1	0	0
$(2_c, 2_d)$	2	0	0	1	1
$(\bar{2}_c, 10_{O6})$	2	0	0	-1	0
$(2_d, 2_e)$	6	0	0	0	1

Sp-j with j=1, ..., 4, in Tables XXX-XL in Appendix B and C. In these models, the suitable three-family SM fermion masses and mixings can be generated at stringy tree level, and then the rank one problem for the SM fermion Yukawa matrices can be solved.

The observable gauge symmetry in Models TI-U-i is $U(4)_C \times U(2)_L \times U(2)_R$. Also, the Models TI-U-i with i=1, ..., 5 are on Type IIA $\mathbf{T^6}$ orientifold while the Models TI-U-6 and TI-U-7 are on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold. And only Model TI-U-4 has $U(4)_C$ symmetric representation. Moreover, there are six bidoublet Higgs fields in Models TI-U-1, TI-U-2, TI-U-3 and TI-U-4. There are twelve pairs of vector-like bidoublet Higgs fields from the massless open string states in a $N = 2$ subsector in Model TI-U-5, and six pairs of vector-like bidoublet Higgs fields in Models TI-U-6 and TI-U-7. Especially, the D6-brane configurations in Model TI-U-6 are the same as those in Model I-Z-10 in Ref. [22], and Model TI-U-6 is the only model that the supergravity fluxes do not contribute to the D6-brane RR tadpoles. Also, the D6-brane configurations in the observable sector in Model TI-U-7 are the same as those in Model I-Z-10 in Ref. [22], and there are a lot of exotic particles from extra

Table XIII. The particle spectrum in the observable sector in Model TI-Sp-1 with gauge symmetry $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times[U(2) \times U(1)^4 \times USp(2)]_{hidden}$.

Representation	Multiplicity	$U(1)_a$	$U(1)_b$	$U(1)_c$
$(4_a, \bar{2}_b)$	3	1	-1	0
$(\bar{4}_a, 2_c)$	3	-1	0	1
$(2_b, \bar{2}_c)$	3	0	1	-1
$(4_a, 2_c)$	1	1	0	1
$(\bar{4}_a, \bar{2}_c)$	1	-1	0	-1
Exotic Particles and Hidden Sector Matter				

gauge groups due to the large supergravity fluxes and the RR tadpole cancellation conditions.

The observable gauge symmetry in Models TI-Sp-j is $U(4)_C \times USp(2)_L \times U(2)_R$, and all these models are on Type IIA $\mathbf{T^6}$ orientifold. There are $U(4)_C$ symmetric representations in Models TI-Sp-3 and TI-Sp-4. Also, there are three bidoublet Higgs fields in Models TI-Sp-1, TI-Sp-2 and TI-Sp-3, and three pairs of vector-like bidoublet Higgs fields in Model TI-Sp-4.

We present the complete particle spectrum in Model TI-U-4 with six bidoublet Higgs fields in Tables XI and XII, and the particle spectrum in the observable sector in Model TI-Sp-1 with three bidoublet Higgs fields in Table XIII. The vector-like particles with quantum numbers $(4_a, 2_c)$ and $(\bar{4}_a, \bar{2}_c)$ are the Higgs fields which can break the Pati-Salam gauge symmetry down to the SM gauge symmetry. After suitable D6-brane splittings, only the vector-like particles with quantum numbers $(\mathbf{1}, \mathbf{1}, \mathbf{-1}, \mathbf{1/2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{-1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry from $(4_a, 2_c)$ and $(\bar{4}_a, \bar{2}_c)$ are massless, and they can break the $U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry down to the $U(1)_Y$ gauge symmetry by supersymmetry preserving Higgs mechanism.

b. The Second Kind of Pati-Salam Models

We present the D6-brane configurations and intersection numbers for the second kind of Pati-Salam models, *i. e.*, the Models TII-U-i with i=1, ..., 6, and the Models TII-Sp-j with j=1, ..., 5, in Tables XLI-L in Appendix D and E. In these models, only the SM fermion masses for the third family can be generated at stringy tree level, and we assume that the first two families of the SM fermions may obtain masses from quantum corrections.

Similar to above subsection, the observable gauge symmetry in Models TII-U-i is $U(4)_C \times U(2)_L \times U(2)_R$. And all these models are on Type IIA $\mathbf{T^6}$ orientifold. Moreover, there are two bidoublet Higgs fields in Model TII-U-1, three in Model TII-U-2, and four in Models TII-U-3 and TII-U-4. There are two and four pairs of vector-like bidoublet Higgs fields in Models TII-U-5 and TII-U-6, respectively.

The observable gauge symmetry in Models TII-Sp-j is $U(4)_C \times USp(2)_L \times U(2)_R$. And the Models TII-Sp-i with i=1, ..., 4 are on Type IIA $\mathbf{T^6}$ orientifold while the Model TII-Sp-5 is on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold. Also, there are $U(4)_C$ symmetric representations in Models TII-Sp-1, TII-Sp-3 and TII-Sp-4. Moreover, there are one bidoublet Higgs fields in Model TII-Sp-1, three in Models TII-Sp-2, TII-Sp-3 and TII-Sp-5, and four pairs of vector-like bidoublet Higgs fields in Model TII-Sp-4.

CHAPTER V

FLIPPED $SU(5)$ AND UN-FLIPPED $SU(5)$ GUT MODELS

A. Basic Flipped $SU(5)$ Phenomenology*

In a flipped $SU(5)\times U(1)_X$ [6, 26, 27] unified model, the electric charge generator Q is only partially embedded in $SU(5)$, *i.e.*, $Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$, where Y' is the $U(1)$ internal $SU(5)$ and $\tilde{Y}$ is the external $U(1)_X$ factor. Essentially, this means that the photon is 'shared' between $SU(5)$ and $U(1)_X$. The Standard Model (SM) plus right handed neutrino states reside within the representations $\bar{\mathbf{5}}$, $\mathbf{10}$, and $\mathbf{1}$ of $SU(5)$, which are collectively equivalent to a spinor $\mathbf{16}$ of $SO(10)$. The quark and lepton assignments are flipped by $u^c_L \leftrightarrow d^c_L$ and $\nu^c_L \leftrightarrow e^c_L$ relative to a conventional $SU(5)$ GUT embedding:

$$\bar{f}_{\bar{\mathbf{5}},-\frac{\mathbf{3}}{\mathbf{2}}} = \begin{pmatrix} u^c_1 \\ u^c_2 \\ u^c_3 \\ e \\ \nu_e \end{pmatrix}_L ; \quad F_{\mathbf{10},\frac{\mathbf{1}}{\mathbf{2}}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L d^c_L \;\; \nu^c_L \right) ; \quad l_{\mathbf{1},\frac{\mathbf{5}}{\mathbf{2}}} = e^c_L \tag{5.1}$$

In particular this results in the $\mathbf{10}$ containing a neutral component with the quantum numbers of ν^c_L. We can break spontaneously the GUT symmetry by using a $\mathbf{10}$ and $\overline{\mathbf{10}}$ of superheavy Higgs where the neutral components provide a large vacuum

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expectation value, $\langle\nu^c_H\rangle = \langle\bar{\nu}^c_H\rangle$,

$$H_{\mathbf{10},\frac{1}{2}} = \{Q_H,\ d^c_H,\ \nu^c_H\}\,; \quad \bar{H}_{\overline{\mathbf{10}},-\frac{1}{2}} = \{Q_{\bar{H}},\ d^c_{\bar{H}},\ \nu^c_{\bar{H}}\}\,. \tag{5.2}$$

The electroweak spontaneous breaking is generated by the Higgs doublets H_2 and $\bar{H}_{\bar{2}}$

$$h_{\mathbf{5},\mathbf{-1}} = \{H_2, H_3\}\,; \quad \bar{h}_{\bar{\mathbf{5}},\mathbf{1}} = \left\{\bar{H}_{\bar{2}}, \bar{H}_{\bar{3}}\right\} \tag{5.3}$$

Flipped $SU(5)$ model building has two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet (H_2)-triplet(H_3) splitting problem of the electroweak Higgs pentaplets $h, \bar{h}$ through the trilinear coupling of the Higgs fields: $H_{\mathbf{10}} \cdot H_{\mathbf{10}} \cdot h_{\mathbf{5}} \to \langle\nu^c_H\rangle\, d^c_H H_3$, and (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields ϕ, $F_{\mathbf{10}} \cdot \bar{H}_{\overline{\mathbf{10}}} \cdot \phi \to \left\langle\nu^c_{\bar{H}}\right\rangle \nu^c\phi$.

The generic superpotential W for a flipped $SU(5)$ model will be of the form :

$$\lambda_1 FFh + \lambda_2 F\bar{f}\bar{h} + \lambda_3 \bar{f}l^c h + \lambda_4 F\bar{H}\phi + \lambda_5 HHh + \lambda_6 \bar{H}\bar{H}\bar{h} + \cdots \in W \tag{5.4}$$

the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino mass and the last two terms are responsible for the doublet-triplet splitting mechanism [6].

B. Flipped $SU(5)$ from Type IIA D-branes

Our goal now is to realize a supersymmetric $SU(5) \times U(1)_X$ gauge theory with three generations and a complete GUT and electroweak Higgs sector in the four-dimensional spacetime. We also try to avoid as much extra matter as possible.

1. A First Trial*

We first consider a stack with ten D6-branes to form the desired $U(5)$ group, and then determine additional stacks of two branes which provide $U(1)$ group factors and are compatible with the supersymmetry conditions of the 10-brane stack. To have enough but not too many copies of the antisymmetric and symmetric representation in the first stack a to satisfy the tadpole conditions, it is reasonable to consider the case of no tilted tori ($k = 0$) and we choose a set of proper wrapping numbers to make $\mathcal{M}((\mathrm{A}_a)_L) = 4$ and $\mathcal{M}((\mathrm{A}_a + \mathrm{S}_a)_L) = -2$. Under this setting, one wrapping number is zero and it makes two of the RR-tadpole parameters A, B, C, D zero with the remaining two negative, which forces the structure parameters x_A, x_B, x_C, x_D to be all positive by the SUSY conditions. Then the rest of the 2-brane stacks are chosen in accordance with our requirements.

Because of the combined constraints from RR-tadpole and SUSY conditions, it is harder to get negative values than to get positive values or zero for I_{ab} and $I_{ab'}$ to generate the required bi-fundamental representations. Generally when a negative number is needed, the absolute value cannot be large enough to alone provide three generations of chiral matter. This suggests the consideration of multiple two-brane stacks to share the burden of this task.

Next we turn to the question of the number of stacks we need. Generally speaking a case with three stacks is enough to provide all the required matter to construct a normal $SU(5)$ GUT model. However, as we mentioned we have to ensure that the $U(1)_X$ remains a gauge symmetry after the application of the G-S mechanism. It is clear that at least two more stacks are needed if all the couplings to the four RR

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forms are present.

The pentaplet $\bar{f}$ which contains Standard Model fermions is different from the Higgs pentaplet $\bar{h}$ resulting from the 'flipped' nature of the model as we saw in section 3.1. For example, if we take $U(1)_X$ for $(\mathbf{10},\mathbf{1})$ in both SM and Higgs spectrum as $1/2$, then it is $-3/2$ for $(\bar{\mathbf{5}},\mathbf{1})$ in SM, $5/2$ for $(\mathbf{1},\mathbf{1})$ in SM, $-1/2$ for $(\overline{\mathbf{10}},\mathbf{1})$ in Higgs, 1 or -1 for $(\bar{\mathbf{5}},\mathbf{1})$ and $(\mathbf{5},\mathbf{1})$ in Higgs, and 0 for $(\mathbf{1},\mathbf{1})$ in Higgs. These constrain some coefficients of $U(1)$s from the stacks involving the SM and Higgs spectra, and may require more stacks in addition to the five mentioned above for obtaining the correct $U(1)_X$ charge for all the matter and Higgs representations. Here we first present an example with seven stacks.

However, with seven stacks it was still difficult to find chiral bi-fundamental representations to be identified with the electroweak Higgs pentaplets, $h, \bar{h}$ and at the same time for the $U(1)_X$ group to remain a gauge symmetry. This directed us towards the most natural choice of identifying our Higgs pentaplets as well as some matter representations from intersections which provide non-chiral matter. After all, the Higgs $\mathbf{5}$ and the $\bar{\mathbf{5}}$ construct the vector-like $\mathbf{10}$ representation of $SO(10)$. A zero intersection number between two branes implies additional non-chiral (vector-like) multiplet pairs from $ab+ba$, $ab'+b'a$, and $aa'+a'a$ [63]. This is useful since we can fill the spectrum with this matter without affecting the required global conditions because the total effect of the pairs is zero. For instance in our model, besides the (ae') intersection which provides a vector-like pair of Higgs pentaplets, the intersection (ef') delivers the fermion (singlet under the $SU(5)$ group) $l_{\mathbf{1},\frac{\mathbf{5}}{\mathbf{2}}}$ particles.

In Table XIV we present a consistent model compatible with the constraints we described. Note that this is a (7+1)-stack model, with one stack of two filler branes wrapped along the first orientifold plane and two sets of parallel branes; the latter provide several non-chiral pairs. The gauge symmetry associated with the two filler

Table XIV. Wrapping numbers and the consistent parameters of the model with gauge group $U(5) \times U(1)^6 \times Usp(2)$.

stack	N_a	$(n_1, l_1)(n_2, l_2)(n_3, l_3)$	A	B	C	D	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$
a	$N = 10$	(0,-1) (-1,-1) (-1,-2)	0	0	-2	-1	2	-1	0	0
b	$N = 2$	(-1,-1) (-1, 1) (1, 3)	-1	-3	3	-1	3	1	-1	3
c	$N = 2$	(-1,-1) (-1, 1) (1, 3)	-1	-3	3	-1	3	1	-1	3
d	$N = 2$	(-1, 1) (1, 0) (-1,-2)	-1	0	-2	0	0	-1	0	2
e	$N = 2$	(-1, 1) (1,-1) (0,-1)	0	-1	-1	0	-1	0	0	1
f	$N = 2$	(-1, 1) (1,-1) (0,-1)	0	-1	-1	0	-1	0	0	1
g	$N = 2$	(1,-1) (-4,-1) (-1, 0)	-4	0	0	-1	0	-4	1	0
$O6^{(1)}$	$N^{(1)} = 2$	(1, 0) (1, 0) (1, 0)	-1	0	0	0	0	0	0	0

Table XV. List of intersection numbers of the model in Table XIV. The number in parenthesis indicates the multiplicity of non-chiral pairs.

stk	N	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	$O6^{(1)}$
a	10	2	-2	-2	0(5)	-2	0(5)	0(1)	4	-2	0(1)	-2	0(1)	6	10	2
b	2	24	0	-	-	0(0)	24	2	0(5)	0(2)	0(2)	0(2)	0(2)	30	0(9)	3
c	2	24	0	-	-	-	-	2	0(5)	0(2)	0(2)	0(2)	0(2)	30	0(9)	3
d	2	2	-2	-	-	-	-	-	-	0(1)	-2	0(1)	-2	0(2)	4	0
e	2	0	0	-	-	-	-	-	-	-	-	0(0)	0(4)	0(5)	-6	-1
f	2	0	0	-	-	-	-	-	-	-	-	-	-	0(5)	-6	-1
g	2	-6	6	-	-	-	-	-	-	-	-	-	-	-	-	0

branes is $Usp(2) \cong SU(2)$.

The Result The gauge symmetry of the (7+1)-stack model in table XIV is $U(5) \times U(1)^6 \times Usp(2)$, and the structure parameters of the wrapping space are

$$x_A = 1, \quad x_B = 2, \quad x_C = 8, \quad x_D = 1 \tag{5.5}$$

which means

$$\frac{R_2^1}{R_1^1} = \frac{1}{2}, \quad \frac{R_2^2}{R_1^2} = 2, \quad \frac{R_2^3}{R_1^3} = \frac{1}{4} \tag{5.6}$$

The intersection numbers are listed in Table XV, and the resulting spectrum in

Table XVI. We have a complete Standard Model sector plus right handed neutrinos in three copies, a complete Higgs spectrum, and in addition extra *exotic* matter which includes two $(\overline{\mathbf{15}}, \mathbf{1})$.

The $U(1)_X$ is

$$U(1)_X = \frac{1}{12}\left(3U(1)_a - 20U(1)_b + 45U(1)_d - 15U(1)_e - 15U(1)_f - 20U(1)_g\right) \quad (5.7)$$

while the other two anomaly-free and massless combinations $U(1)_Y$ and $U(1)_Z$ are

$$\begin{aligned} U(1)_Y &= U(1)_b + U(1)_c - 6U(1)_d + 3U(1)_e + 3U(1)_f + 2U(1)_g \\ U(1)_Z &= U(1)_b - U(1)_c + U(1)_e - U(1)_f \end{aligned} \quad (5.8)$$

These two gauge symmetries can be spontaneously broken by assigning vacuum expectation values to singlets from the intersection (bg). Thus, the final gauge symmetry is $SU(5) \times U(1)_X \times Usp(2)$.

The remaining four global $U(1)$s from the Green-Schwarz mechanism are given respectively by

$$\begin{aligned} U(1)_1 &= -10U(1)_a + 2U(1)_b + 2U(1)_c - 2U(1)_d - 8U(1)_g \\ U(1)_2 &= -2U(1)_b - 2U(1)_c + 2U(1)_g \\ U(1)_3 &= 6U(1)_b + 6U(1)_c + 4U(1)_d + 2U(1)_e + 2U(1)_f \\ U(1)_4 &= 20U(1)_a + 6U(1)_b + 6U(1)_c - 2U(1)_e - 2U(1)_f. \end{aligned} \quad (5.9)$$

Table XVI. The spectrum of $U(5)\times U(1)^6\times Usp(2)$, with the four global $U(1)$s from the G-S mechanism. The $\star' d$ representations stem from vector-like non-chiral pairs.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_g$	$12U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_Y$	$U(1)_Z$
$(10, 1)$	3	2	0	0	0	0	0	0	6	-20	0	0	40	0	0
$(\bar{5}_a, 1_e)$	2	-1	0	0	0	1	0	0	-18	10	0	2	-22	3	1
$(\bar{5}_a, 1_f)$	1	-1	0	0	0	0	1	0	-18	10	0	2	-22	3	-1
$(\bar{1}_e, \bar{1}_f)^\star$	3	0	0	0	0	-1	-1	0	30	0	0	-4	4	-6	0
$(10, 1)$	1	2	0	0	0	0	0	0	6	-20	0	0	40	0	0
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	0	-6	20	0	0	-40	0	0
$(5_a, 1_e)^\star$	1	1	0	0	0	1	0	0	-12	-10	0	2	18	3	1
$(\bar{5}_a, \bar{1}_e)^\star$	1	-1	0	0	0	-1	0	0	12	10	0	-2	-18	-3	-1
$(1_b, \bar{1}_g)$	4	0	1	0	0	0	0	-1	0	10	-4	6	6	-1	1
$(\overline{15}, 1)$	2	-2	0	0	0	0	0	0	-6	20	0	0	-40	0	0
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	0	-6	20	0	0	-40	0	0
$(\bar{5}_a, 1_c)$	2	-1	0	1	0	0	0	0	-3	12	-2	6	-14	1	-1
$(5_a, 1_d)$	4	1	0	0	1	0	0	0	48	-12	0	4	20	-6	0
$(\bar{5}_a, 1_b)$	2	-1	1	0	0	0	0	0	-23	12	-2	6	-14	1	1
$(\bar{5}_a, 1_f)$	1	-1	0	0	0	0	1	0	-18	10	0	2	-22	3	-1
$(5_a, \bar{1}_g)$	6	1	0	0	0	0	0	-1	23	-2	-2	0	20	-2	0
$(5_a, 1_g)$	10	1	0	0	0	0	0	1	-17	-18	2	0	20	2	0
$(1_b, 1_c)$	24	0	1	1	0	0	0	0	-20	4	-4	12	12	2	0
$(1_b, \bar{1}_d)$	2	0	1	0	-1	0	0	0	-65	4	-2	2	6	7	1
$(1_b, \bar{1}_g)$	26	0	1	0	0	0	0	-1	0	10	-4	6	6	-1	1
$(1_c, \bar{1}_d)$	2	0	0	1	-1	0	0	0	-45	4	-2	2	6	7	-1
$(1_c, \bar{1}_g)$	30	0	0	1	0	0	0	-1	20	10	-4	6	6	-1	-1
$(\bar{1}_d, \bar{1}_e)$	2	0	0	0	-1	-1	0	0	-30	2	0	-6	2	3	-1
$(\bar{1}_d, \bar{1}_f)$	2	0	0	0	-1	0	-1	0	-30	2	0	-6	2	3	1
$(1_d, 1_g)$	4	0	0	0	1	0	0	1	25	-10	2	4	0	-4	0
$(\bar{1}_e, \bar{1}_g)$	6	0	0	0	0	-1	0	-1	35	8	-2	-2	2	-5	-1
$(\bar{1}_f, \bar{1}_g)$	6	0	0	0	0	0	-1	-1	35	8	-2	-2	2	-5	1
$(\bar{1}, \bar{1})$	2	0	0	0	-2	0	0	0	-90	4	0	-8	0	12	0
$(1, 1)$	6	0	0	0	0	0	0	2	-40	-16	4	0	0	4	0
$(1_e, 1_f)^\star$	4	0	0	0	0	1	1	0	-30	0	0	4	-4	6	0
$(\bar{1}_e, \bar{1}_f)^\star$	1	0	0	0	0	-1	-1	0	30	0	0	-4	4	-6	0
Additional non-chiral Matter															
USp(2) Matter															

Table XVII. Wrapping numbers and the consistent parameters of the model with gauge group $U(5) \times U(1)^5 \times USp(8)$.

stack	N_a	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	B	C	D	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$
a	$N=10$	(0,-1) (-1,-1) (-1,-2)	0	0	-2	-1	2	-1	0	0
b	$N=2$	(-1,-1) (-1, 1) (1, 4)	-1	-4	4	-1	4	1	-1	4
c	$N=2$	(-1,-1) (-1, 1) (1, 4)	-1	-4	4	-1	4	1	-1	4
d	$N=2$	(-1, 1) (1, 0) (-1,-2)	-1	0	-2	0	0	-1	0	2
e	$N=2$	(-1, 1) (1, 0) (-1,-2)	-1	0	-2	0	0	-1	0	2
f	$N=2$	(0, 1) (1, 1) (-1,-2)	0	0	-2	-1	2	-1	0	0
$O6^{(1)}$	$N^{(\Omega R)}=8$	(1, 0) (1, 0) (1, 0)	-1	0	0	0	0	0	0	0

2. A Model without K-theory Anomaly*

Now we present a second model. This model is similar to the previous one, however the K-theory constraints are satisfied. we present an example with 6+1 stacks of branes. The first stack has the same set of wrapping numbers as in our previous model. We also have a stack with $N^{(\Omega R)} = 8$ filler branes which give rise to a $USp(8)$ gauge group. The gauge symmetry of the (6+1)-stack model, whose wrapping numbers are presented in Table XVII, is $U(5) \times U(1)^5 \times USp(8)$, and the structure parameters of the wrapping space are

$$x_A = 1, \quad x_B = 2, \quad x_C = 10, \quad x_D = 1 \tag{5.10}$$

The intersection numbers are listed in Table XVIII, and the resulting spectrum in Table XIX.

The singlet (under the $SU(5)$ symmetry) representation e^c_L, now comes from the bi-fundamentals, namely from the intersection (cf) and we choose the $\mathbf{5}$ and $\overline{\mathbf{5}}$ Higgs

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Table XVIII. List of intersection numbers of the model in Table XVII. The number in parenthesis indicates the multiplicity of non-chiral pairs.

stk	N	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$O6^{(1)}$
a	10	2	-2	-4	0(6)	-4	0(6)	0(1)	4	0(1)	4	0(0)	0(8)	2
b	2	32	0	-	-	0(0)	32	4	0(6)	4	0(6)	4	0(6)	4
c	2	32	0	-	-	-	-	4	0(6)	4	0(6)	4	0(6)	4
d	2	2	-2	-	-	-	-	-	-	0(0)	0(8)	0(1)	4	0
e	2	2	-2	-	-	-	-	-	-	-	-	0(1)	4	0
f	2	2	-2	-	-	-	-	-	-	-	-	-	-	2

pentaplets from a non-chiral intersection (ab'). There is less exotic matter in this model, though we still have two copies of $\overline{\mathbf{15}}$ which is unavoidable since we need $\overline{\mathbf{10}}$ in the Higgs sector. Matter charged under both the $SU(5)\times U(1)_X$ and $USp(8)$ gauge symmetries is also present, as is evident from Table XVIII.

The $U(1)_X$ is

$$U(1)_X = \frac{1}{2}\left(U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d - 5U(1)_e - 5U(1)_f\right) \tag{5.11}$$

while the other anomaly-free and massless combinations $U(1)_Y$ is

$$U(1)_Y = U(1)_b - U(1)_c + U(1)_d - U(1)_e \tag{5.12}$$

The remaining four global $U(1)$s from the Green-Schwarz mechanism are given respectively by

$$\begin{aligned} U(1)_1 &= -10U(1)_a + 2U(1)_b + 2U(1)_c - 2U(1)_d - 2U(1)_e - 2U(1)_f \\ U(1)_2 &= -2U(1)_b - 2U(1)_c \\ U(1)_3 &= 8U(1)_b + 8U(1)_c + 4U(1)_d + 4U(1)_e \\ U(1)_4 &= 20U(1)_a + 8U(1)_b + 8U(1)_c + 4U(1)_f \end{aligned} \tag{5.13}$$

Table XIX. The spectrum of $U(5)\times U(1)^5\times USp(8)$, with the four global $U(1)$s from the G-S mechanism. The $\star' d$ representations stem from vector-like non-chiral pairs.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$2U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_Y$
$(10,1)$	3	2	0	0	0	0	0	2	-20	0	0	40	0
$(\bar{5}_a,1_b)$	3	-1	1	0	0	0	0	-6	12	-2	8	-12	1
$(1_c,\bar{1}_f)$	3	0	0	1	0	0	-1	10	4	-2	8	4	-1
$(10,1)$	1	2	0	0	0	0	0	2	-20	0	0	40	0
$(\overline{10},1)$	1	-2	0	0	0	0	0	-2	20	0	0	-40	0
$(5_a,1_b)^\star$	1	1	1	0	0	0	0	-4	-8	-2	8	28	1
$(\bar{5}_a,\bar{1}_b)^\star$	1	-1	-1	0	0	0	0	4	8	2	-8	-28	-1
$(1_b,1_c)$	4	0	1	1	0	0	0	0	4	-4	16	16	0
$(\overline{15},1)$	2	-2	0	0	0	0	0	-2	20	0	0	-40	0
$(\overline{10},1)$	1	-2	0	0	0	0	0	-2	20	0	0	-40	0
$(\bar{5}_a,1_b)$	1	-1	1	0	0	0	0	-6	12	-2	8	-12	1
$(\bar{5}_a,1_c)$	4	-1	0	1	0	0	0	4	12	-2	8	-12	-1
$(5_a,1_d)$	4	1	0	0	1	0	0	6	-12	0	4	20	1
$(5_a,1_e)$	4	1	0	0	0	1	0	-4	-12	0	4	20	-1
$(1_b,1_c)$	28	0	1	1	0	0	0	0	4	-4	16	16	0
$(1_b,\bar{1}_d)$	4	0	1	0	-1	0	0	-10	4	-2	4	8	0
$(1_b,\bar{1}_e)$	4	0	1	0	0	-1	0	0	4	-2	4	8	2
$(1_b,\bar{1}_f)$	4	0	1	0	0	0	-1	0	4	-2	8	4	1
$(1_c,\bar{1}_d)$	4	0	0	1	-1	0	0	0	4	-2	4	8	-2
$(1_c,\bar{1}_e)$	4	0	0	1	0	-1	0	10	4	-2	4	8	0
$(1_c,\bar{1}_f)$	1	0	0	1	0	0	-1	10	4	-2	8	4	-1
$(1d,1_f)$	4	0	0	0	1	0	1	0	-4	0	4	4	1
$(1e,1_f)$	4	0	0	0	0	1	1	-10	-4	0	4	4	-1
$(\bar{1},\bar{1})$	2	0	0	0	-2	0	0	-10	4	0	-8	0	-2
$(\bar{1},\bar{1})$	2	0	0	0	0	-2	0	10	4	0	-8	0	2
$(\bar{1},\bar{1})$	2	0	0	0	0	0	-2	10	4	0	0	-8	0
$(5_a,1_b)^\star$	5	1	1	0	0	0	0	-4	-8	-2	8	28	1
$(\bar{5}_a,\bar{1}_b)^\star$	5	-1	-1	0	0	0	0	4	8	2	-8	-28	-1
Additional non-chiral Matter													
USp(8) Matter													

Table XX. List of wrapping and intersection numbers for $N_{flux} = 192$. Here $x_A = 62$, $x_B = 1$, $x_C = 1$, and $x_D = 2$. It is obvious that the first K-theory constraint is not satisfied. The gauge symmetry is $U(5) \times U(1)^5 \times USp(6)$.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$D7_2$
a	10	(1, 0)(-1,-1)(-2, 1)	2	-2	-12	24	1	-3	1	-3	0(1)	-2	0(1)	-2	2
b	2	(3,-1)(-5, 1)(4,-1)	332	148	-	-	7	15	7	15	12	16	12	16	12
c	2	(-2, 1)(2, 1)(-1, 0)	0	0	-	-	-	-	0(0)	0(16)	0(0)	0(9)	0(0)	0(9)	2
d	2	(-2, 1)(2, 1)(-1, 0)	0	0	-	-	-	-	-	-	0(0)	0(9)	0(0)	0(9)	2
e	2	(-1, 1)(1, 1)(-1, 0)	0	0	-	-	-	-	-	-	-	-	0(0)	0(4)	1
f	2	(-1, 1)(1, 1)(-1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	1
$O7_2$	6	(0, 1)(1, 0)(0,-1)	-	-	-	-	-	-	-	-	-	-	-	-	-

C. Flipped $SU(5)$ with Type IIB Fluxes*

1. $N_{flux} = 192$

The most ideal situation is to preserve supersymmetry both in the closed string and open string sectors in the spirit of this flux construction. However we found that it is difficult to achieve. An example is shown in Table XX. Although this example is supersymmetric both in the open and closed string sectors, satisfies the conditions for cancellation of RR charges, and yields a three generation flipped $SU(5)$ model with a complete but extended Higgs sector, it does not satisfy the K-theory constraints.

2. $N_{flux} = 128$

We present an example for $N_{flux} = 128$ with four stacks of magnetized D-branes as well as two filler branes presented in Table XXI. Although this particular model does not contain flipped $SU(5)$ symmetry, it is a consistent solution of the RR tadpole conditions and the K-theory constraints, and is supersymmetric both in the open and

Table XXI. $N_{flux} = 128$. The number stacks is only two plus two filler branes, though it has very few exotic particles, we have too few stacks to complete the cancellation of $U(1)_X$ mass. Here $x_A = 27$, $x_B = 1$, $x_C = 1$, and $x_D = 2$.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	$D3$	$D7_2$
a	10	(1, 0)(-1,-1)(-2, 1)	2	-2	-16	24	0(1)	2
b	2	(3,-2)(-3, 1)(4,-1)	374	202	-	-	-2	12
$O3$	4	(1, 0)(1, 0)(1, 0)	-	-	-	-	-	-
$O7_2$	4	(0, 1)(1, 0)(0,-1)	-	-	-	-	-	-

closed string sectors. The gauge symmetry is

$$U(5) \times U(1) \times USp(4) \times USp(4). \tag{5.14}$$

3. $N_{flux} = 1 \times 64$

In this example, we use two sets of parallel D-branes and all conditions are satisfied. No filler brane is needed, and $x_A = 22$, $x_B = 1$, $x_C = 1$, and $x_D = 2$. The complete (n_a^i, m_a^i) and $SU(5)\times U(1)_X$ spectrum are listed in Table XXII and XXIII, and $U(1)_X$ is

$$U(1)_X = \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c - 5U(1)_d + 5U(1)_e - 5U(1)_f). \tag{5.15}$$

The four global $U(1)$s from the Green-Schwarz mechanism are given respectively:

$$\begin{aligned}
U(1)_1 &= 24U(1)_b + 24U(1)_c + 4U(1)_e + 4U(1)_f,\\
U(1)_2 &= 20U(1)_a + 8U(1)_b + 8U(1)_c + 4U(1)_d,\\
U(1)_3 &= -10U(1)_a + 6U(1)_b + 6U(1)_c - 2U(1)_d - 2U(1)_e - 2U(1)_f,\\
U(1)_4 &= -2U(1)_b - 2U(1)_c.
\end{aligned} \tag{5.16}$$

From Table XXIII we found that none of the global $U(1)$s from the G-S anomaly

Table XXII. List of intersection numbers for $N_{flux} = 64$ with gauge group $U(5) \times U(1)^5$. The number in parenthesis indicates the multiplicity of non-chiral pairs.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	10	(1, 0)(-1,-1)(-2, 1)	2	-2	-8	12	-8	12	0(0)	0(8)	0(1)	4	0(1)	4
b	2	(1,-1)(-3, 1)(4,-1)	84	12	-	-	0(0)	96	8	12	4	0(6)	4	0(6)
c	2	(1,-1)(-3, 1)(4,-1)	84	12	-	-	-	-	8	12	4	0(6)	4	0(6)
d	2	(-1, 0)(1, 1)(-2, 1)	2	-2	-	-	-	-	-	-	0(1)	4	0(1)	4
e	2	(1, 1)(1, 0)(2,-1)	2	-2	-	-	-	-	-	-	-	-	0(0)	8
f	2	(1, 1)(1, 0)(2,-1)	2	-2	-	-	-	-	-	-	-	-	-	-

cancellation mechanism provides Yukawa couplings required for generation of mass terms in superpotential (5.4). However, $U(1)_X$ admits these Yukawa couplings, and if we require the other anomaly-free and massless combination $U(1)_Y$ does as well, two conditions can be considered. The first one is to demand all the Yukawa couplings from the assigned intersections, and an example of the $U(1)_Y$ and the corresponding combinations of representations are listed as follows:

$$U(1)^1_Y = 5U(1)_a - 25U(1)_b + 25U(1)_c - 25U(1)_d - 38U(1)_e + 38U(1)_f. \tag{5.17}$$

$$\begin{aligned}
FFh &\rightarrow (\mathbf{10},\mathbf{1})(\mathbf{10},\mathbf{1})(\mathbf{5}_a,\mathbf{1}_d)^\star \\
F\bar{f}\bar{h} &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{5}}_a,\mathbf{1}_b)(\overline{\mathbf{5}}_a,\overline{\mathbf{1}}_d)^\star \\
\bar{f}l^c h &\rightarrow (\overline{\mathbf{5}}_a,\mathbf{1}_b)(\mathbf{1}_c,\overline{\mathbf{1}}_d)(\mathbf{5}_a,\mathbf{1}_d)^\star \\
F\bar{H}\phi &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\mathbf{1}_b,\mathbf{1}_c) \\
HHh &\rightarrow (\mathbf{10},\mathbf{1})(\mathbf{10},\mathbf{1})(\mathbf{5}_a,\mathbf{1}_d)^\star \\
\bar{H}\bar{H}\bar{h} &\rightarrow (\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{5}}_a,\overline{\mathbf{1}}_d)^\star
\end{aligned} \tag{5.18}$$

If we do not require the Higgs pentaplet $\bar{h}'$ coupled with the chiral fermions in the term $F\bar{f}\bar{h}'$ to be the same as the Higgs pentaplet $\bar{h}$ coupled to $\bar{H}$, then we expect a mixture state $\bar{h}_x = c\bar{h}' + s\bar{h}$ of these two different Higgs pentaplets in the Higgs sector, therefore

$$U(1)_Y^2 = U(1)_b - U(1)_c + U(1)_e - U(1)_f. \tag{5.19}$$

$$\begin{aligned} F\bar{f}\bar{h}' &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{5}}_a,\mathbf{1}_b)(\overline{\mathbf{5}}_a,\mathbf{1}_c) \\ \bar{H}\bar{H}\bar{h} &\rightarrow (\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{5}}_a,\overline{\mathbf{1}}_d)^{\star} \end{aligned} \tag{5.20}$$

We should also notice that the superfluous $\bar{\mathbf{5}},\mathbf{5}$, and $\overline{\mathbf{10}}$ representations may be ostracized from the low energy spectrum through trilinear couplings of the generic form $\bar{\mathbf{5}}\cdot\mathbf{5}\cdot\mathbf{1}$ and $\overline{\mathbf{10}}\cdot\mathbf{10}\cdot\mathbf{1}$ satisfying the gauged $U(1)$ symmetries, where the singlets are assumed to acquire string scale *vev*s.

Table XXIII. The spectrum of $U(5) \times U(1)^5$, or $SU(5) \times U(1)_X \times U(1)_Y$, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star'd$ representations indicate vector-like matter. We list the two cases for the $U(1)_Y$.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_Y^1$	$U(1)_Y^2$
$(10, 1)$	3	2	0	0	0	0	0	1	0	40	-20	0	10	0
$(\bar{5}_a, 1_b)$	3	-1	1	0	0	0	0	-3	24	-12	16	-2	-30	1
$(1_c, \bar{1}_d)$	3	0	0	1	-1	0	0	5	24	4	8	-2	50	-1
$(10, 1)$	1	2	0	0	0	0	0	1	0	40	-20	0	10	0
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	-1	0	-40	20	0	-10	0
$(5_a, 1_d)^\star$	1	1	0	0	1	0	0	-2	0	24	-12	0	-20	0
$^1(\bar{5}_a, \bar{1}_d)^\star/^2\bar{h}_x$	1	[1]-1	[1]0	[1]0	[1]-1	[1]0	[1]0	2	[1]0	[1]-24	[1]12	[1]0	[1]20	[2]-1/ 0
$(1_b, 1_c)$	4	0	1	1	0	0	0	0	48	16	12	-4	0	0
$(\overline{15}, 1)$	2	-2	0	0	0	0	0	-1	0	-40	20	0	-10	0
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	-1	0	-40	20	0	-10	0
$(\bar{5}_a, 1_b)$	5	-1	1	0	0	0	0	-3	24	-12	16	-2	-30	1
$(5_a, 1_b)$	12	1	1	0	0	0	0	-2	24	28	-4	-2	-20	1
$(\bar{5}_a, 1_c)$	8	-1	0	1	0	0	0	2	24	-12	16	-2	20	-1
$(5_a, 1_c)$	12	1	0	1	0	0	0	3	24	28	-4	-2	30	-1
$(5_a, 1_e)$	4	1	0	0	0	1	0	3	4	20	-12	0	-33	1
$(5_a, 1_f)$	4	1	0	0	0	0	1	-2	4	20	-12	0	43	-1
$(1_b, 1_c)$	92	0	1	1	0	0	0	0	48	16	12	-4	0	0
$(1_b, \bar{1}_d)$	8	0	1	0	-1	0	0	0	24	4	8	-2	0	1
$(1_b, 1_d)$	12	0	1	0	1	0	0	-5	24	12	4	-2	-50	1
$(1_b, \bar{1}_e)$	4	0	1	0	0	-1	0	-5	20	8	8	-2	13	0
$(1_b, \bar{1}_f)$	4	0	1	0	0	0	-1	0	8	8	-2	0	-63	2
$(1_c, \bar{1}_d)$	5	0	0	1	-1	0	0	5	24	4	8	-2	50	-1
$(1_c, 1_d)$	12	0	0	1	1	0	0	0	24	12	4	-2	0	-1
$(1_c, \bar{1}_e)$	4	0	0	1	0	-1	0	0	20	8	8	-2	63	-2
$(1_c, \bar{1}_f)$	4	0	0	1	0	0	-1	5	20	8	8	-2	-13	0
$(1d, 1_e)$	4	0	0	0	1	1	0	0	4	4	-4	0	-63	1
$(1d, 1_f)$	4	0	0	0	1	0	1	-5	4	4	-4	0	13	-1
$(1, 1)$	12	0	2	0	0	0	0	-5	48	16	12	-4	-50	2
$(1, 1)$	12	0	0	2	0	0	0	5	48	16	12	-4	50	-2
$(\bar{1}, \bar{1})$	2	0	0	0	-2	0	0	5	0	-8	4	0	50	0
$(\bar{1}, \bar{1})$	2	0	0	0	0	-2	0	-5	-8	0	4	0	76	-2
$(\bar{1}, \bar{1})$	2	0	0	0	0	0	-2	5	-8	0	4	0	-76	2
$(5_a, 1_d)^\star$	7	1	0	0	1	0	0	-2	0	24	-12	0	-20	0
$(\bar{5}_a, \bar{1}_d)^\star$	7	-1	0	0	-1	0	0	2	0	-24	12	0	20	0
Additional non-chiral Matter														

Table XXIV. D6-brane wrapping and intersection numbers for the Flipped $SU(5)$ model on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^7 \times USp(16)$.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	$O6$
a	5	(0, 1) (-1,-1) (3, 1)	2	-2	-3	0(3)	0(6)	0(0)	-3	-6	5	4	1	2	0	0	0	0	1
b	1	(-1,-3) (1,-1) (0, 1)	-6	6	-	-	0(3)	3	0	0	-24	0	12	-12	12	-9	-9	12	-3
c	1	(0, 1) (1,-1) (3,-1)	-2	2	-	-	-	-	6	3	-4	-5	-2	-1	0	0	0	0	-1
d	1	(1, 1) (1, 3) (0,-1)	6	-6	-	-	-	-	-	-	-28	52	40	-40	30	-33	-33	30	3
e	1	(1, 3) (9,-1) (1, 0)	0	0	-	-	-	-	-	-	-	-	0	0	-56	7	7	-56	0
f	1	(1,-9) (3, 1) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	14	35	35	14	0
g	1	(0, 1) (7, 1) (-3,-7)	14	-14	-	-	-	-	-	-	-	-	-	-	-	-	0	0	7
h	1	(0, 1) (7,-1) (3,-7)	-14	14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-7
$O6$	8	(1, 0) (2, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

D. Flipped $SU(5)$ with Type IIA Fluxes*

We present the D6-brane configurations and intersection numbers for a Flipped $SU(5)$ model in Table XXIV, and its particle spectrum in the observable sector in Table XXV. The complex structure parameters are $\chi_1 = 1/9, \quad \chi_2 = 6$, and $\chi_3 = 1$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(3q+2)$, $a = 12$, and $m = 2$.

The $U(1)_X$ gauge symmetry is

$$U(1)_X = \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c - 5U(1)_d + 5U(1)_e + 5U(1)_f + 5U(1)_g + 5U(1)_h) \, . \tag{5.21}$$

The four global $U(1)$'s are

$$U(1)_1 \quad = \quad -15U(1)_a + 3U(1)_c + 27U(1)_e - 27U(1)_f - 21U(1)_g + 21U(1)_h \, ,$$

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Table XXV. The particle spectrum in the observable sector with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_g$	$U(1)_h$	$2U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_U$	$U(1)_V$	$U(1)_W$
$(10, 1)$	3	2	0	0	0	0	0	0	0	1	-30	0	0	10	0	-20	70
$(\bar{5}_a, 1_b)$	3	-1	1	0	0	0	0	0	0	-3	15	0	1	-8	0	-20	70
$(1_c, \bar{1}_d)$	3	0	0	1	-1	0	0	0	0	5	3	0	1	-4	0	-50	-7
$(10, 1)$	1	2	0	0	0	0	0	0	0	1	-30	0	0	10	0	-20	70
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	0	0	-1	30	0	0	-10	0	20	-70
$(5_a, \bar{1}_c)^\star$	1	1	0	-1	0	0	0	0	0	-2	-18	0	0	6	0	40	42
$(\bar{5}_a, 1_c)^\star$	1	-1	0	1	0	0	0	0	0	2	18	0	0	-6	0	-40	-42
$(\bar{1}_c, 1_e)$	4	0	0	-1	0	1	0	0	0	0	24	-1	0	1	1	63	7
$(\overline{15}, 1)$	2	-2	0	0	0	0	0	0	0	-1	30	0	0	-10	0	20	-70
$(\overline{10}, 1)$	1	-2	0	0	0	0	0	0	0	-1	30	0	0	-10	0	20	-70
Additional chiral and non-chiral Matter																	

$$
\begin{aligned}
U(1)_2 &= -U(1)_e + U(1)_f \ , \\
U(1)_3 &= U(1)_b - U(1)_d \ , \\
U(1)_4 &= 5U(1)_a - 3U(1)_b - U(1)_c + 3U(1)_d + 7U(1)_g - 7U(1)_h \ . \qquad (5.22)
\end{aligned}
$$

And the other massless $U(1)$'s are:

$$
\begin{aligned}
U(1)_U &= U(1)_e + U(1)_f - U(1)_g - U(1)_h \ , \\
U(1)_V &= -10U(1)_a - 50U(1)_c + 13U(1)_e + 13U(1)_f + 13U(1)_g + 13U(1)_h \ , \\
U(1)_W &= 35U(1)_a - 7U(1)_c - 13U(1)_g + 13U(1)_h \ . \qquad (5.23)
\end{aligned}
$$

Table XXVI. D6-brane configurations and intersection numbers for the $SU(5)$ Model on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^4 \times USp(12) \times USp(8) \times USp(4)$.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	$O6$
a	5	(1, 1) (-1,-1) (-1, 3)	3	0	0(2)	-3	-3	0(6)	3	0(0)	0(3)	0(1)	1	-	-3	-	3
b	1	(0, 2) (1,-3) (1,-3)	-9	9	-	-	-9	0(3)	-3	0(2)	3	0(1)	2	-	0(3)	-	-18
c	1	(1,-1) (1, 3) (2, 0)	0	0	-	-	-	-	0(6)	3	-3	0(3)	-2	-	0(1)	-	0(3)
d	1	(-1, 1) (1,-1) (-1,-3)	-3	0	-	-	-	-	-	-	0(1)	0(3)	-1	-	3	-	-3
e	1	(-1,-1) (0, 2) (1, 3)	3	-3	-	-	-	-	-	-	-	-	0(1)	-	0(3)	-	6
f	6	(2, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-
g	4	(0,-2) (0, 2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-
$O6$	2	(2, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

E. $SU(5)$ with Type IIA Fluxes*

In the previous $SU(5)$ model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold without fluxes, one can easily show that one can not construct the models with three anti-symmetric representations and without symmetric representations [21, 78]. And then, for the models with three anti-symmetric representations and some symmetric representations, the net number of $\bar{\mathbf{5}}$ and $\mathbf{5}$ can not be three due to the non-abelian anomaly free conditions, *i. e.*, one does not have exact three families of the SM fermions [21, 78].

In this Section, we will present $SU(5)$ models with three anti-symmetric $\mathbf{10}$ representations and without symmetric $\mathbf{15}$ representations. Although the net number of $\bar{\mathbf{5}}$ and $\mathbf{5}$ is three due to the non-Abelian anomaly free condition, the initial $\bar{\mathbf{5}}$ number is not three. For a concrete model, we will show that after the additional gauge symmetry breaking via supersymmetry preserving Higgs mechanism, the $\bar{\mathbf{5}}$ and

5 pairs can form the massive vector-like particles with masses around the GUT/string scale. Then we will have exact three $\bar{\mathbf{5}}$ and no **5**. Moreover, we can break the $SU(5)$ gauge symmetry down to the SM gauge symmetry via D6-brane splitting, and solve the doublet-triplet splitting problem. If the extra one pair of Higgs doublets and adjoint particles can obtain GUT/string-scale masses via high-dimensional operators, we only have the MSSM in the observable sector below the GUT scale. And then we can explain the observed low energy gauge couplings. We also briefly comment on two more models where the phenomenological discussions are similar.

We present the D6-brane configurations and intersection numbers for the Model SU(5)-I in Table XXVI, and its particle spectrum in the observable and Higgs sectors in Table XXVII. The complex structure parameters are $\chi_1 = 2\sqrt{3/5}$, $\chi_2 = 2\sqrt{1/15}$, and $\chi_3 = 2\sqrt{15}/9$. To satisfy the RR tadpole cancellation conditions, we choose the fluxes $h_0 = -4(3q+2)$, $a = 8$, and $m = 2$.

The four global $U(1)$'s from the additional $U(1)$ gauge symmetry breaking due to the Green-Schwarz mechanism are

$$\begin{aligned} U(1)_1 &= 5U(1)_a + 2U(1)_b - 2U(1)_c - U(1)_d \ , \\ U(1)_2 &= 5U(1)_a + 6U(1)_c - U(1)_d - 2U(1)_e \ , \\ U(1)_3 &= -15U(1)_a + 3U(1)_d \ , \\ U(1)_4 &= 15U(1)_a - 18U(1)_b - 3U(1)_d + 6U(1)_e \ . \end{aligned} \tag{5.24}$$

And the anomaly-free $U(1)$ is

$$U(1)_{free} = U(1)_a + U(1)_b + U(1)_c + 5U(1)_d + 3U(1)_e \ . \tag{5.25}$$

In this model, we have three **10** representations, thirty $\bar{\mathbf{5}}$ representations, and twenty-seven **5** representations for $SU(5)$. Then, the net number of $\bar{\mathbf{5}}$ and **5** is three.

Table XXVII. The particle spectrum in the observable and Higgs sectors with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_{free}$
$(10,1)$	3	2	0	0	0	0	10	10	-30	30	2
$(\bar{5}_a,\bar{1}_b)$	3	-1	-1	0	0	0	-7	-5	15	-15	-2
$(\bar{1}_b,1_c)$	3	0	-1	1	0	0	-4	6	0	18	0
$(5_a,\bar{1}_b)^\star$	2	1	-1	0	0	0	3	5	-15	33	0
$(\bar{5}_a,1_b)^\star$	2	-1	1	0	0	0	-3	-5	15	-33	0
$(\bar{5}_a,1_c)$	3	-1	0	1	0	0	-7	1	15	-15	0
$(5_a,\bar{1}_d)$	3	1	0	0	-1	0	6	6	-18	18	-4
$(5_a,12_f)$	1	1	0	0	0	0	5	5	-15	15	1
$(\bar{5}_a,8_g)$	3	-1	0	0	0	0	-5	-5	15	-15	-1
$(5_a,4_{O6})$	3	1	0	0	0	0	5	5	-15	15	1
$(1_e,4_{O6})$	6	0	0	0	0	1	0	-2	0	6	3
$(\bar{1}_b,1_c)$	6	0	-1	1	0	0	-4	6	0	18	0
$(\bar{1}_b,1_d)$	3	0	-1	0	1	0	-3	-1	3	15	4
$(1_b,\bar{1}_e)$	3	0	1	0	0	-1	2	2	0	-24	-2
$(1_c,1_d)$	3	0	0	1	1	0	-3	5	3	-3	6
$(\bar{1}_c,1_e)$	3	0	0	-1	0	1	2	-8	0	6	2
$(12_f,8_g)^\star$	4+4	0	0	0	0	0	0	0	0	0	0
$(8_g,4_{O6})^\star$	4+4	0	0	0	0	0	0	0	0	0	0
$(1_c,\bar{1}_d)^\star$	6	0	0	1	-1	0	-1	7	-3	3	-4
$(\bar{1}_c,1_d)^\star$	6	0	0	-1	1	0	1	-7	3	-3	4
$(1_e,12_f)^\star$	1	0	0	0	0	1	0	-2	0	6	3
$(\bar{1}_e,12_f)^\star$	1	0	0	0	0	-1	0	2	0	-6	-3
$(1_e,8_g)^\star$	1	0	0	0	0	1	0	-2	0	6	3
$(\bar{1}_e,8_g)^\star$	1	0	0	0	0	-1	0	2	0	-6	-3
Additional chiral and non-chiral Matter											

So, the key question is whether we can give the GUT/string-scale vector-like masses to twenty-seven pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$.

Let us discuss how to decouple the vector-like particles via supersymmetry preserving Higgs mechanism. We have the following superpotential from three-point functions:

$$\begin{aligned} W_3 &= y^A_{ijk}(\bar{\mathbf{5}}_a, \mathbf{1}_c)_i(\mathbf{5}_a, \bar{\mathbf{1}}_d)_j(\bar{\mathbf{1}}_c, \mathbf{1}_d)_k + y^B_{ik}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{12}_f)(\mathbf{12}_f, \mathbf{8}_g)_k \\ &\quad + y^C_{ijk}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{4}_{O6})_j(\mathbf{8}_g, \mathbf{4}_{O6})_k \ . \end{aligned} \tag{5.26}$$

After the Higgs fields $(\bar{\mathbf{1}}_c, \mathbf{1}_d)_k$ obtain vacuum expectation values (VEVs), we can give the vector-like masses to three pairs of $(\bar{\mathbf{5}}_a, \mathbf{1}_c)_i$ and $(\mathbf{5}_a, \bar{\mathbf{1}}_d)_j$ because the $(\bar{\mathbf{5}}_a, \mathbf{1}_c)_i$, $(\mathbf{5}_a, \bar{\mathbf{1}}_d)_j$ and three of six $(\bar{\mathbf{1}}_c, \mathbf{1}_d)_k$ arises from the intersections on the third two-torus. In addition, after the Higgs fields $(\mathbf{12}_f, \mathbf{8}_g)_k$ and $(\mathbf{8}_g, \mathbf{4}_{O6})_k$ obtain VEVs, we can give vector-like masses to eight pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$ in $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$ and $(\mathbf{5}_a, \mathbf{12}_f)$, and to four pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$ in $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$ and $(\mathbf{5}_a, \mathbf{4}_{O6})_j$, respectively.

To further give vector-like masses to additional twelve pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$, we introduce the following superpotential from four-point functions:

$$\begin{aligned} W_3 &= y^D_{ik}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{12}_f)(\mathbf{1}_e, \mathbf{12}_f)(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k + y^{D\prime}_{ik}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{12}_f)(\bar{\mathbf{1}}_e, \mathbf{12}_f)(\mathbf{1}_e, \mathbf{8}_g)_k \\ &\quad + y^E_{ijkl}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{4}_{O6})_j(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k(\mathbf{1}_e, \mathbf{4}_{O6})_l \ . \end{aligned} \tag{5.27}$$

We point out that $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$, $(\mathbf{5}_a, \mathbf{4}_{O6})_j$, $(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k$, and three of six $(\mathbf{1}_e, \mathbf{4}_{O6})_l$ arise from the intersections on the third two-torus. If we give VEVs to the Higgs fields $(\mathbf{1}_e, \mathbf{12}_f)$, $(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k$, $(\bar{\mathbf{1}}_e, \mathbf{12}_f)$, $(\mathbf{1}_e, \mathbf{8}_g)_k$, and $(\mathbf{1}_e, \mathbf{4}_{O6})_l$, we can generate the vector-like masses for the rest twelve pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$ in $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$ and $(\mathbf{5}_a, \mathbf{12}_f)/(\mathbf{5}_a, \mathbf{4}_{O6})_j$. Therefore, we only have three $\bar{\mathbf{5}}$ and do not have $\mathbf{5}$ after the Higgs mechanism at the GUT/string scale. Note that there are three ($U(1)_e$ symmetric) singlets with charge $-\mathbf{2}$ and six

$(\mathbf{1}_e, \mathbf{4}_{O6})_l$ with charge $+\mathbf{1}$ under the $U(1)_e$ gauge symmetry, we obtain that the D-flatness for $U(1)_e$ gauge symmetry can be preserved if we give VEVs to these singlets. And the D-flatness for other broken gauge symmetries can be preserved because all the other relevant Higgs particles are vector-like. Also, it is obvious that we have the F-flatness for above superpotential. Thus, the Higgs mechanism can preserve supersymmetry.

To break the $SU(5)$ gauge symmetry down to the SM gauge symmetry, we split the a stack of D6-branes into a_3 and a_2 stacks with respectively 3 and 2 D6-branes. To have the vector-like MSSM Higgs doublets, we assume that the a_2 and b stacks of D6-branes are parallel and on the top of each other on the third two-torus. Then we obtain two pairs of vector-like Higgs doublets $(\mathbf{2}_{a_2}, \bar{\mathbf{1}}_b)_j$ and $(\bar{\mathbf{2}}_{a_2}, \mathbf{1}_b)_j$ $(j = 1, 2)$ with quantum numbers $(\mathbf{1}, \mathbf{2}, \mathbf{1/2})$ and $(\mathbf{1}, \bar{\mathbf{2}}, -\mathbf{1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. We also assume that the a_3 and b stacks of D6-branes are not on the top of each other on the third two-torus. So, the vector-like triplets will obtain the masses around the string scale, and the doublet-triplet splitting problem is solved. Therefore, below the GUT scale, we have SM gauge symmetry, three families of the SM fermions, two pairs of Higgs doublets, and three adjoint particles for each gauge symmetry in the observable sector.

Suppose that one pair of the Higgs doublets and adjoint particles obtain the GUT/string-scale vector-like masses via high-dimensional operators, we only have the MSSM below the GUT scale. And then, if we choose the suitable grand unified gauge coupling by adjusting the string scale M_S, the observed low energy gauge couplings can be generated via RGE running. Let us discuss the gauge coupling and the string scale. For a generic stack σ of D6-branes, its gauge coupling at the string scale is [14]

$$(g^{\sigma}_{YM})^2 = \frac{\sqrt{8\pi}M_s}{M_{Pl}} \frac{1}{\prod_{i=1}^{3} \sqrt{(n^i_{\sigma})^2 \chi_i^{-1} + (2^{-\beta_i} l^i_{\sigma})^2 \chi_i}} \ . \tag{5.28}$$

So, the $SU(5)$ gauge coupling g^a_{YM} at the string scale is

$$(g^a_{YM})^2 \ = \ \frac{(375\pi^2)^{1/4}}{4} \frac{M_S}{M_{Pl}} \ \simeq \ \frac{2M_S}{M_{Pl}} \ . \tag{5.29}$$

Thus, we can have the suitable grand unified gauge coupling g^a_{YM} by adjusting the string scale. As an example, to have the MSSM unified gauge coupling g_{MSSM} which is about $1/\sqrt{2}$, we choose $M_S \simeq M_{Pl}/4$ which is close to the string scale in the heterotic string theory. We present another $SU(5)$ model in the appendix.

CHAPTER VI

CONCLUSIONS

A physics theory needs to have the ability to explain and even predict the phenomena in the real world, or it is just a mathematical game. String theory though has had great success in unification with its elegant structure, however we still have to face the imperfectness of our universe, at least at the low energy scale. With a structure of extra dimensions and large symmetry groups string theory includes more than what we need in our world, so we have to percolate and identify the substructure of the theory that we can observe and touch, for example, the Standard Model in a four dimensional world. Several methods had been developed to solve this problem, and we found D-branes compactified on $\mathbf{T^6}$ with orientifold has the potential to achieve it by constructing open string models non-perturbatively from the dual of heterotic string [79]. In this theory particles are chiral fermions from D-branes intersecting at angles [7] in a $D = 4$ $N = 1$ world. We will use this property as well as the required constraints to build Standard and GUT models. This method is the basic idea of this dissertation.

In this dissertation, we present in the second chapter some basic knowledge of D-brane theory and the idea to construct a chiral spectrum. By using Kaluza-Klein dimension reduction we introduce D-branes in the dual space, and we realize the concept of duality is the spirit of M-theory and the D-brane construction. Gauge groups arise from the D-branes, and Chan-Paton indices and Wilson lines fertilize their properties. Oribifold and orientifold actions are discussed, and the branes intersect in Type IIA theory is due to the magnetic fluxes introduced in Type IIB picture. We show D6-branes in Type IIA picture can have massless chiral fermions only. In the third chapter, one special orientifold structure $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ is chosen to construct the realistic models and the details are carefully given. This orientifold can have non-rigid-3-cycles so no exceptional/fractional cycles needed to be considered to satisfy the RR-tadpole conditions and the orbifold actions do not fixed the complex moduli too early to satisfy the supersymmetry conditions. Of course we know it is important to fix all the moduli to stabilize the theory, however we leave this task to the non-trivial Type IIA or IIB supergravity fluxes as a string background. We also include the K-theory constraints [40, 66, 67, 68] which are from the discrete anomaly of the orientifold and the Green-Schwarz mechanism for some models which need global gauge groups. Finally, a complete spectrum of the representations of the gauge groups in this intersecting D-brane scenario is given so we can set up the requirements from the realistic models to construct the consistent D-brane models with all conditions satisfied.

In the fourth and fifth chapter we started to discuss examples of the Standard-like Models and GUT models, including Trinification, Pati-Salam, $SU(5)$ and flipped $SU(5)$ models. Our studies start from using D-brane constructions on Type IIA or Type IIB $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold (some Type IIA models with fluxes are on $\mathbf{T^6}$ orientifold), since they are T-dual to each other. Then we introduce Type IIB and Type IIA supergravity fluxes respectively to construct models with the moduli fixed. For Type IIB fluxes, We consider both supersymmetric and non-supersymmetric fluxes in the closed string sector, and we claim that the non-supersymmetric (soft-breaking) models of Pati-Salam and flipped $SU(5)$ we have found are more realistic and consistent with all the constraints of string theory including K-theory and supersymmetry in the open string sector. The non-supersymmetric flux ($N_{flux} = 64$) in this particular flipped $SU(5)$ model breaks supersymmetry in the closed string sector. This leads to a mechanism of soft supersymmetry breaking at a mass scale $M_{soft} \sim \frac{M^2_{string}}{M_{Pl}}$ which implies an intermediate string scale or an inhomogeneous warp factor in the internal space to stabilize the electroweak scale [42, 76, 81]. With this non-supersymmetric flux present, soft supersymmetry breaking terms may be manifested in the effective action of open string fields. Detailed studies in soft-breaking mechanism and some trial investigations into the effective low energy scenario were studied in [76, 80]. Combined with a Yukawa coupling analysis [82], this may provide a clear picture of the low energy physics which we defer for future work.

The four global $U(1)$ symmetries from the G-S anomaly cancellation forbid all the Yukawa couplings necessary for the generation of quark and lepton masses, although if we ignore these global $U(1)$ factors and focus only on the $U(1)_X$ and $U(1)_Y$ symmetries, then we find that all of the required Yukawa couplings in (5.4) are present, as well as those needed for making the extra matter in the model obtain mass $\mathcal{O}(M_{string})$. We need to keep in mind that global $U(1)$ symmetries are valid to all orders in perturbation theory, and can be broken by non-perturbative instanton effects [83]. To solve this problem without these instanton effects, one possibility one may entertain is to use singlets, suitably charged, to trigger spontaneous breaking of global $U(1)$s as well as of the local $U(1)_Y$ at the string scale, while leaving $U(1)_X$ intact. In the case of global $U(1)$s one may hope that we will end up with invisible axion-like bosons. The interested reader may check from Table XIX that such singlets with appropriate charges do exist. Another possibility is that we may need a new D-brane configuration, like Type IIA orientifold with flux compactifications.

On Type IIA orientifolds with flux compactifications in supersymmetric AdS vacua some semi-realistic Pati-salam and GUT models are built, and we for the first time construct the exact three-family $SU(5)$ models. In these models, we have three $\mathbf{10}$ representations, and obtain three $\overline{\mathbf{5}}$ representations after the additional gauge symmetry breaking via supersymmetry preserving Higgs mechanism. So, there are exact three families of the SM fermions, and no chiral exotic particles that are charged under $SU(5)$. In addition, we can break the $SU(5)$ gauge symmetry down to the SM gauge symmetry via D6-brane splitting, and solve the doublet-triplet splitting problem. If the extra one (or several) pair(s) of Higgs doublets and adjoint particles obtain GUT/string scale masses via high-dimensional operators, we only have the MSSM in the observable sector below the GUT scale. Choosing suitable grand unified gauge coupling by adjusting the string scale, we can explain the observed low energy gauge couplings via RGE running. However, how to generate the up-type quark Yukawa couplings, which are forbidden by the global $U(1)$ symmetry, deserves further study.

For the flipped $SU(5)$ models, in order to have at least one pair of Higgs fields $\mathbf{10}$ and $\overline{\mathbf{10}}$, we must have the symmetric representations, and then the net number of $\overline{\mathbf{5}}$ and $\mathbf{5}$ can not be three if the net number of $\mathbf{10}$ and $\overline{\mathbf{10}}$ is three due to the non-abelian anomaly free condition. We constructed the first model with three $\mathbf{10}$ representations, and the first model where all the Yukawa couplings are allowed by the global $U(1)$ symmetries but with a extremely large exotic spectrum.

Despite of these fruitful successes in this D-brane model building we are still in the midway. First so far our best models are built upon AdS vacua with all the moduli fixed, which is still far from our real world on a Minkowski space. But recently a new discussion of D-brane construction [84, 85] including additional non-geometric fluxes [86] provides a way to define (D-brane-)solvable Minkowski vacua. Definitely it is the next topic to search. Secondly we can only construct the gauge group structure of the known models in particle physics but the most important quantities able to be identified from the experiments, particle masses and the gauge coupling constants are still untouched although some trial investigations are taken, for instance in [80]. There are also plenty phenomenology worthy to discuss, such as supersymmetry breaking, Higgs structure and Yukawa couplings. We are getting to an age that both theoretical and experimental physics have exciting development and somehow can communicate and improve with each other. LHC at CERN will start running in 2007 and ILC and other programs are coming in the near future to search physics at a higher energy scale. And WMAP and other observatories are collecting data from the universe for cosmology. Although we do not expect that we should be able to see some direct evidence of string theory soon from the laboratories, we still hope the new found particles can testify the predictions made by some extended theories of strings, like the D-brane configuration discussed in this dissertation, to prove that string theory is correct and all our efforts paid are worthy.

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APPENDIX A

P-S MODELS WITHOUT FLUX*

In this Appendix, we present the spectrum in the $U(4)_C \times U(2)_L \times U(1)' \times U(1)'' \times U(1)_e \times U(1)_f \times USp(4)^2$ model with anomaly free $U(1)_{I_{3R}}$ and $U(1)_X$ gauge symmetries in Tables XXVIII and XXIX.

Table XXVIII. The SM fermions and Higgs fields in the $U(4)_C \times U(2)_L \times U(1)' \times U(1)'' \times U(1)_e \times U(1)_f \times USp(4)^2$ model, with anomaly free $U(1)_{I_{3R}}$ and $U(1)_X$ gauge symmetries.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$2U(1)_{I_{3R}}$	$2U(1)_X$
$(4_a, \bar{2}_b)$	3	1	-1	0	0	0	0	0	2
$(\bar{4}_a, 1_c)$	3	-1	0	1	0	0	0	1	-1
$(\bar{4}_a, 1_d)$	3	-1	0	0	1	0	0	-1	1
$(2_b, \bar{1}_c)$	8	0	1	-1	0	0	0	-1	-1
$(2_b, \bar{1}_d)$	9	0	1	0	-1	0	0	1	-3

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Table XXIX. The extra particles in the $U(4)_C \times U(2)_L \times U(1)' \times U(1)'' \times U(1)_e \times U(1)_f \times USp(4)^2$ model, with anomaly free $U(1)_{I_{3R}}$ and $U(1)_X$ gauge symmetries.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$2U(1)_{I_{3R}}$	$2U(1)_X$
$(4_a, 2_b)$	1	1	1	0	0	0	0	2	0
$(\bar{4}_a, \bar{1}_c)$	1	-1	0	-1	0	0	0	-3	-1
$(\bar{4}_a, \bar{1}_d)$	1	-1	0	0	-1	0	0	-1	-3
$(\bar{4}_a, 1_e)$	3	-1	0	0	0	1	0	-1	-3
$(\bar{4}_a, \bar{1}_e)$	1	-1	0	0	0	-1	0	-1	1
$(4_a, \bar{1}_f)$	1	1	0	0	0	0	-1	1	3
$(4_a, 1_f)$	3	1	0	0	0	0	1	1	-1
$(\bar{2}_b, \bar{1}_d)$	5	0	-1	0	-1	0	0	-1	-1
$(2_b, \bar{1}_e)$	8	0	1	0	0	-1	0	1	1
$(\bar{2}_b, 1_f)$	3	0	-1	0	0	0	1	-1	-1
$(\bar{2}_b, \bar{1}_f)$	1	0	-1	0	0	0	-1	-1	3
$(1_c, \bar{1}_d)$	1	0	0	1	-1	0	0	2	-2
$(1_c, 1_d)$	3	0	0	1	1	0	0	2	2
$(1_c, \bar{1}_f)$	5	0	0	1	0	0	-1	2	2
$(\bar{1}_c, \bar{1}_f)$	9	0	0	-1	0	0	-1	-2	2
$(\bar{1}_d, 1_e)$	1	0	0	0	-1	1	0	0	-4
$(1_d, 1_e)$	3	0	0	0	1	1	0	0	0
$(\bar{1}_d, \bar{1}_f)$	16	0	0	0	-1	0	-1	0	0
$(1_e, \bar{1}_f)$	5	0	0	0	0	1	-1	0	0
$(\bar{1}_e, \bar{1}_f)$	9	0	0	0	0	-1	-1	0	4
1_b	2	0	2	0	0	0	0	2	-2
$\bar{3}_b$	2	0	-2	0	0	0	0	-2	2
1_c	2	0	0	2	0	0	0	4	0
1_d	6	0	0	0	2	0	0	0	4
1_e	2	0	0	0	0	2	0	0	-4
1_f	6	0	0	0	0	0	2	0	-4
Additional non-chiral and $USp(4)$ & $USp(4)$ Matter									

APPENDIX B

FIRST KIND OF $U(4)_C \times U(2)_L \times U(2)_R$ P-S MODELS WITH IIA FLUXES*

We present the D6-brane configurations and intersection numbers for the first kind of Pati-Salam models. Let us explain the convention. Suppose b and c stacks of D6-branes are parallel on a two-torus and the product of intersection numbers on the other two two-tori is i, we denote their intersection number as $0(i)$.

1. Model TI-U-1

   D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times USp(2)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $3\chi_1 = \chi_2 = \chi_3 = \sqrt{2}$. To satisfy the RR tadpole conditions, $h_0 = -2(3q+4)$, $a = 4$, and $m = 2$.

Table XXX. D6-brane configurations and intersection numbers for Model TI-U-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times USp(2)^2]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$O6$
a	4	(1, 0) (-1,-1) (-1, 1)	0	0	3	0(3)	-3	0(3)	0(3)	-3	0(3)	3	0(1)	-	0(1)
b	2	(1,-3) (1,-1) (0, 2)	-6	6	-	-	6	0(1)	-6	0(18)	-9	-3	0(3)	-	-6
c	2	(-1,-3) (0, 2) (1, 1)	6	-6	-	-	-	-	9	3	6	0(18)	0(3)	-	6
d	2	(2,-3) (1, 1) (2, 0)	0	0	-	-	-	-	-	-	12	0(1)	-6	-	0(3)
e	2	(2, 3) (2, 0) (1,-1)	0	0	-	-	-	-	-	-	-	-	6	-	0(3)
f	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	0(4)
$O6$	1	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-

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2. Model TI-U-2

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $6\chi_1 = \chi_2 = \chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

Table XXXI. D6-brane configurations and intersection numbers for Model TI-U-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2)^2]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(1, 0) (-1,-1) (-1, 1)	0	0	3	0(3)	-3	0(3)
b	2	(-1, 3) (-2, 0) (1, 1)	0	0	-	-	6	0(1)
c	2	(1, 3) (-1, 1) (-2, 0)	0	0	-	-	-	-
d	1	(1, 1) (2, 0) (3,-1)	0	0	-	-	-	-
e	1	(1,-1) (3, 1) (2, 0)	0	0	-	-	-	-
f	1	(1,-3) (1,-1) (0, 2)	-6	6	-	-	-	-
g	1	(1, 3) (0,-2) (1, 1)	6	-6	-	-	-	-
h	1	(0,-1) (2, 0) (0, 2)	0	0	-	-	-	-
i	1	(0,-1) (0, 2) (2, 0)	0	0	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'
a	4	(1, 0) (-1,-1) (-1, 1)	-1	2	1	-2	3	0(3)	-3	0(3)	1	-	-1	-
b	2	(-1, 3) (-2, 0) (1, 1)	0(8)	0(2)	-2	4	0(1)	-6	0(12)	0(2)	0(1)	-	2	-
c	2	(1, 3) (-1, 1) (-2, 0)	2	-4	0(8)	0(2)	0(12)	0(2)	0(1)	6	-2	-	0(1)	-
d	1	(1, 1) (2, 0) (3,-1)	-	-	-2	0(1)	12	-6	-8	8	0(3)	-	-2	-
e	1	(1,-1) (3, 1) (2, 0)	-	-	-	-	8	-8	-12	6	2	-	0(3)	-
f	1	(1,-3) (1,-1) (0, 2)	-	-	-	-	-	-	6	0(1)	0(1)	-	2	-
g	1	(1, 3) (0,-2) (1, 1)	-	-	-	-	-	-	-	-	-2	-	0(1)	-
h	1	(0,-1) (2, 0) (0, 2)	-	-	-	-	-	-	-	-	-	-	0(4)	-
i	1	(0,-1) (0, 2) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-

3. Model TI-U-3

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times U(1)^2 \times USp(2)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the second two-torus, and the complex structure parameters are $\chi_1 = 3\chi_2 = 2\chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

Table XXXII. D6-brane configurations and intersection numbers for Model TI-U-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times U(1)^2 \times USp(2)^2]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(1, 1) (1,-3) (1, 0)	0	0	3	0(1)	-3	0(3)
b	2	(2, 0) (1, 3) (1,-1)	0	0	-	-	6	0(3)
c	2	(1,-1) (2, 0) (1, 1)	0	0	-	-	-	-
d	2	(1, 1) (-1,-3) (0, 1)	3	-3	-	-	-	-
e	2	(3,-1) (1, 1) (1, 0)	0	0	-	-	-	-
f	1	(0, 2) (-1, 3) (-1, 1)	-6	6	-	-	-	-
g	1	(2, 0) (1,-1) (3, 1)	0	0	-	-	-	-
h	1	(2, 0) (0,-2) (0, 1)	0	0	-	-	-	-
i	1	(0,-2) (0, 2) (1, 0)	0	0	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'
a	4	(1, 1) (1,-3) (1, 0)	0(3)	0(1)	0(4)	0(1)	0(1)	-3	-1	2	1	-	0(1)	-
b	2	(2, 0) (1, 3) (1,-1)	0(1)	3	1	-2	0(6)	0(4)	0(8)	0(2)	0(1)	-	-2	-
c	2	(1,-1) (2, 0) (1, 1)	-3	0(3)	-1	2	6	0(3)	2	-4	-2	-	2	-
d	2	(1, 1) (-1,-3) (0, 1)	-	-	2	2	-3	0(1)	6	3	0(1)	-	-1	-
e	2	(3,-1) (1, 1) (1, 0)	-	-	-	-	6	-3	-1	0(1)	-1	-	0(3)	-
f	1	(0, 2) (-1, 3) (-1, 1)	-	-	-	-	-	-	-8	-8	2	-	0(1)	-
g	1	(2, 0) (1,-1) (3, 1)	-	-	-	-	-	-	-	-	0(3)	-	2	-
h	1	(2, 0) (0,-2) (0, 1)	-	-	-	-	-	-	-	-	-	-	0(2)	-
i	1	(0,-2) (0, 2) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-

4. Model TI-U-4

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(10)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the third two-torus, and the complex structure parameters are $6\chi_1 = 2\chi_2 = \chi_3 = 2\sqrt{6}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(q+2)$, $m = 2$, and $a = 8$.

Table XXXIII. D6-brane configurations and intersection numbers for Model TI-U-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(10)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	$O6$
a	4	(0,-1) (1, 1) (3, 1)	1	-1	3	0(1)	-3	0(3)	2	0(2)	-3	-	1
b	2	(-1,-1) (2, 0) (-3, 1)	0	0	-	-	6	0(3)	-1	-5	6	-	0(1)
c	2	(1,-1) (-1, 1) (0,-2)	-2	2	-	-	-	-	0(10)	2	0(1)	-	-2
d	2	(2, 3) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	6	-	0(3)
e	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	0(4)
$O6$	5	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-

5. Model TI-U-5

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(8)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $6\chi_1 = \chi_2 = \chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

Table XXXIV. D6-brane configurations and intersection numbers for Model TI-U-5 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(8)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$O6$
a	4	(1, 0) (-1,-1) (-1, 1)	0	0	3	0(3)	-3	0(3)	0(1)	0(1)	2	2	-2	-2	0(1)
b	2	(1,-3) (1,-1) (0, 2)	-6	6	-	-	0(12)	0(2)	0(1)	1	2	-1	6	3	-6
c	2	(1, 3) (-1, 1) (-2, 0)	0	0	-	-	-	-	0(1)	-1	-6	-3	-2	1	0(3)
d	2	(0, 1) (1,-1) (1,-1)	-1	1	-	-	-	-	-	-	0(4)	0(1)	0(4)	0(1)	-1
e	1	(0,-1) (3, 1) (1, 3)	3	-3	-	-	-	-	-	-	-	-	0(16)	0(25)	3
f	1	(0,-1) (1, 3) (3, 1)	3	-3	-	-	-	-	-	-	-	-	-	-	3
$O6$	4	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-

6. Model TI-U-6

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [USp(2)^4]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $2\chi_1 = 6\chi_2 = 3\chi_3 = 6$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = \pm 3a$, and $m = \mp q$ so that the supergravity fluxes do not contribute to the D6-brane RR tadpoles.

Table XXXV. D6-brane configurations and intersection numbers for Model TI-U-6 on Type IIA $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [USp(2)^4]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	$O6^1$	$O6^2$	$O6^3$	$O6^4$
a	8	(0,-1) (1, 1) (1, 1)	0	0	3	0(3)	-3	0(3)	1	-1	0	0
b	4	(3, 1) (1, 0) (1,-1)	-2	2	-	-	0(6)	0(1)	0	1	0	-3
c	4	(3,-1) (0, 1) (1,-1)	2	-2	-	-	-	-	-1	0	3	0
$O6^1$	2	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-
$O6^2$	2	(1, 0) (0,-1) (0, 2)	-	-	-	-	-	-	-	-	-	-
$O6^3$	2	(0, -1) (1, 0) (0, 2)	-	-	-	-	-	-	-	-	-	-
$O6^4$	2	(0, -1) (0, 1) (2, 0)	-	-	-	-	-	-	-	-	-	-

7. Model TI-U-7

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T}^6/\mathbb{Z}_2\times\mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C\times U(2)_L\times U(2)_R]_{observable}\times[U(1)^6\times USp(4)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $2\chi_1=6\chi_2=3\chi_3=6$. To satisfy the RR tadpole cancellation conditions, we choose $h_0=-6(q+8)$, $m=8$, and $a=16$.

Table XXXVI. D6-brane configurations and intersection numbers for Model TI-U-7 on Type IIA $\mathbf{T}^6/\mathbb{Z}_2\times\mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C\times U(2)_L\times U(2)_R]_{observable}\times[U(1)^6\times USp(4)^2]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	8	(0,-1) (1, 1) (1, 1)	0	0	3	0(3)	-3	0(3)
b	4	(3, 1) (1, 0) (1,-1)	-2	2	-	-	0(6)	0(1)
c	4	(3,-1) (0, 1) (1,-1)	2	-2	-	-	-	-
d	2	(9,-1) (-3,-1) (-2, 0)	12	-12	-	-	-	-
e	2	(3, 4) (1,-4) (2, 0)	-16	16	-	-	-	-
f	2	(9, 1) (-1,-3) (0, 2)	-12	12	-	-	-	-
g	2	(3,-4) (-4, 1) (0,-2)	16	-16	-	-	-	-
h	2	(1, 0) (5, 3) (5,-3)	0	0	-	-	-	-
i	2	(1, 0) (3,-5) (3, 5)	0	0	-	-	-	-
$O6^1$	4	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-
$O6^2$	4	(1, 0) (0,-1) (0, 2)	-	-	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	$O6^1$	$O6^2$
a	8	(0,-1) (1, 1) (1, 1)	18	36	15	-9	-18	-36	-15	9	8	8	-8	-8	1	-1
b	4	(3, 1) (1, 0) (1,-1)	-12	6	-36	-60	18	36	15	-9	-3	12	20	5	0	1
c	4	(3,-1) (0, 1) (1,-1)	-18	-36	-15	9	12	-6	36	60	-5	-20	-12	3	-1	0
d	2	(9,-1) (-3,-1) (-2, 0)	-	-	0	0	-288	0	462	78	-12	-42	-90	-60	0	-6
e	2	(3, 4) (1,-4) (2, 0)	-	-	-	-	462	-78	-720	0	276	-204	-140	340	0	8
f	2	(9, 1) (-1,-3) (0, 2)	-	-	-	-	-	-	0	0	60	90	42	12	6	0
g	2	(3,-4) (-4, 1) (0,-2)	-	-	-	-	-	-	-	-	-340	140	204	-276	-8	0
h	2	(1, 0) (5, 3) (5,-3)	-	-	-	-	-	-	-	-	-	-	0	0	0	0
i	2	(1, 0) (3,-5) (3, 5)	-	-	-	-	-	-	-	-	-	-	-	-	0	0
$O6^1$	4	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$O6^2$	4	(1, 0) (0,-1) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-

APPENDIX C

FIRST KIND OF $U(4)_C \times USp(2)_L \times U(2)_R$ P-S MODELS WITH IIA FLUXES*

We present the D6-brane configurations and intersection numbers for the first kind of Pati-Salam models. Let us explain the convention. Suppose b and c stacks of D6-branes are parallel on a two-torus and the product of intersection numbers on the other two two-tori is i, we denote their intersection number as $0(i)$.

1. Model TI-Sp-1

   D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the second torus, and the complex structure parameters are $14\chi_1 = 7\chi_2 = \chi_3 = 2\sqrt{7}$. To satisfy the RR tadpole conditions, $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

Table XXXVII. D6-brane configurations and intersection numbers for Model TI-Sp-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(2)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'
a	4	(0,-1) (1, 3) (3, 1)	3	-3	3	-	-3	0(2)	2	1	15	6	-3	0(1)	0(1)	-
b_{O6}	1	(1, 0) (2, 0) (2, 0)	-	-	-	-	3	-	0(1)	-	0(1)	-	6	-	0(2)	-
c	2	(1,-1) (-1, 3) (-1,-1)	-6	0	-	-	-	-	-2	0(2)	-24	0(9)	4	4	1	-
d	2	(1, 1) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	0(1)	-2	4	-2	0(1)	-
e	1	(1, 1) (2, 0) (7,-1)	0	0	-	-	-	-	-	-	-	-	16	-20	-2	-
f	1	(1,-3) (0, 2) (3,-1)	-6	6	-	-	-	-	-	-	-	-	-	-	0(1)	-
g	1	(0,-1) (0, 2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-

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2. Model TI-Sp-2

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^4 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the third two-torus, and the complex structure parameters are $2\chi_1 = 6\chi_2 = \chi_3 = 2\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+2)$, $m = 2$, and $a = 4$.

Table XXXVIII. D6-brane configurations and intersection numbers for Model TI-Sp-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^4 \times USp(2)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(1, 0) (-1,-1) (-3, 1)	0	0	3	-	-3	0(1)
b	1	(0,-1) (1, 0) (0, 2)	0	0	-	-	3	-
c	2	(-1,-1) (0, 1) (3, 1)	2	-2	-	-	-	-
d	2	(-3,-1) (-1, 1) (2, 0)	0	0	-	-	-	-
e	1	(3,-1) (0, 1) (1,-1)	-2	2	-	-	-	-
f	1	(0,-1) (1,-1) (-1, 1)	-2	2	-	-	-	-
g	1	(1,-1) (1, 0) (1, 1)	0	0	-	-	-	-
h	1	(1, 0) (-1,-3) (-1, 1)	0	0	-	-	-	-
$O6$	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	$O6$
a	4	(1, 0) (-1,-1) (-3, 1)	-2	0(1)	1	-2	2	0(2)	2	1	0(2)	0(8)	0(1)
b	1	(0,-1) (1, 0) (0, 2)	6	-	-3	-	0(1)	-	0(1)	-	-3	-	0(2)
c	2	(-1,-1) (0, 1) (3, 1)	2	4	0(8)	0(2)	-2	-1	-2	0(2)	2	-1	1
d	2	(-3,-1) (-1, 1) (2, 0)	-	-	6	0(1)	0(3)	-6	-4	-2	4	2	0(1)
e	1	(3,-1) (0, 1) (1,-1)	-	-	-	-	0(3)	3	2	0(4)	0(1)	-1	-1
f	1	(0,-1) (1,-1) (-1, 1)	-	-	-	-	-	-	-1	0(1)	0(4)	2	-1
g	1	(1,-1) (1, 0) (1, 1)	-	-	-	-	-	-	-	-	-3	0(3)	0(1)
h	1	(1, 0) (-1,-3) (-1, 1)	-	-	-	-	-	-	-	-	-	-	0(3)
$O6$	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-

3. Model TI-Sp-3

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(4)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the third two-torus, and the complex structure parameters are $4\chi_1 = 2\chi_2 = 3\chi_3 = 2\sqrt{2}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+4)$, $m = 2$, and $a = 8$.

Table XXXIX. D6-brane configurations and intersection numbers for Model TI-Sp-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(4)^2]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(1, 1) (-1,-1) (-1, 3)	12	0	3	-	-3	0(9)
b_{O6}	1	(1, 0) (2, 0) (1, 0)	-	-	-	-	3	-
c	2	(0,-1) (1,-1) (-2, 3)	-6	6	-	-	-	-
d	1	(2, 1) (0, 2) (-1,-3)	12	-12	-	-	-	-
e	1	(1, 5) (1,-5) (1, 0)	0	0	-	-	-	-
f	1	(1,-1) (1, 1) (1, 0)	0	0	-	-	-	-
g	1	(3,-1) (2, 0) (4, 1)	0	0	-	-	-	-
h	2	(0,-1) (2, 0) (0, 1)	0	0	-	-	-	-
i	2	(0,-1) (0, 2) (1, 0)	0	0	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'
a	4	(1, 1) (-1,-1) (-1, 3)	6	0(3)	-36	-36	0(2)	0(3)	52	22	1	-	-3	-
b_{O6}	1	(1, 0) (2, 0) (1, 0)	-6	-	0(25)	-	0(1)	-	0(1)	-	0(1)	-	0(2)	-
c	2	(0,-1) (1,-1) (-2, 3)	18	6	6	-9	-3	0(3)	-42	-30	0(2)	-	0(3)	-
d	1	(2, 1) (0, 2) (-1,-3)	-	-	-27	33	9	-3	110	26	-4	-	0(6)	-
e	1	(1, 5) (1,-5) (1, 0)	-	-	-	-	0(18)	0(8)	-80	70	-5	-	0(1)	-
f	1	(1,-1) (1, 1) (1, 0)	-	-	-	-	-	-	-2	4	1	-	0(1)	-
g	1	(3,-1) (2, 0) (4, 1)	-	-	-	-	-	-	-	-	0(12)	-	6	-
h	2	(0,-1) (2, 0) (0, 1)	-	-	-	-	-	-	-	-	-	-	0(2)	-
i	2	(0,-1) (0, 2) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-

4. TI-Sp-4

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^2 \times USp(4)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the second two-torus, and the complex structure parameters are $12\chi_1 = 3\chi_2 = \chi_3 = 2\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

Table XL. D6-brane configurations and intersection numbers for Model TI-Sp-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^2 \times USp(4)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	4	(0,-1) (1, 3) (3, 1)	3	-3	3	-	-3	0(2)	12	15	3	-6	0(1)	-
b_{O6}	1	(1, 0) (2, 0) (2, 0)	-	-	-	-	0(3)	-	0(2)	-	-12	-	0(2)	-
c	2	(1, 0) (1,-3) (1, 1)	0	0	-	-	-	-	10	-8	-12	-6	1	-
d	1	(3,-2) (3, 1) (2, 0)	0	0	-	-	-	-	-	-	-40	64	0(9)	-
e	1	(1, 6) (1, 1) (0,-2)	12	-12	-	-	-	-	-	-	-	-	-2	-
f	2	(0,-1) (0, 2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-

APPENDIX D

SECOND KIND OF $U(4)_C \times U(2)_L \times U(2)_R$ P-S MODELS WITH IIA FLUXES*

We present the D6-brane configurations and intersection numbers for the second kind of $U(4)_C \times U(2)_L \times U(2)_R$ Pati-Salam models with IIA fluxes.

1. Model TII-U-1

   D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different tori, and the complex structure parameters are $\chi_1 = 3\chi_2 = 3\chi_3 = \sqrt{2}$. To satisfy the RR tadpole conditions, we choose $h_0 = -2(3q+2)$, $m = 2$, and $a = 4$.

Table XLI. D6-brane configurations and intersection numbers for Model TII-U-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'
a	4	(1, 0) (-1,-3) (-1, 3)	0	0	3	0(1)	-3	0(1)	0(1)	3	6	3	0(3)	-9	-6	3	0(1)	-
b	2	(1,-1) (1,-3) (0, 2)	-6	6	-	-	2	0(1)	0(6)	0(0)	0(2)	0(8)	-6	0(6)	-14	-20	0(1)	-
c	2	(-1,-1) (0, 2) (1, 3)	6	-6	-	-	-	-	0(1)	-2	-4	-2	9	3	15	21	0(1)	-
d	1	(1, 1) (1, 3) (0,-2)	6	-6	-	-	-	-	-	-	0(8)	0(2)	0(6)	6	20	14	0(1)	-
e	1	(1,-3) (-1, 1) (0,-2)	-6	6	-	-	-	-	-	-	-	-	-20	14	0(19)	-34	0(3)	-
f	1	(2,-1) (1, 3) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	0(16)	0(4)	-2	-
g	1	(6, 1) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	2	-
h	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-

2. Model TII-U-2

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(3) \times U(1)^3 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = \chi_2 = \chi_3 = 6$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -36(q+4)$, $m = 2$, and $a = 24$.

Table XLII. D6-brane configurations and intersection numbers for Model TII-U-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(3) \times U(1)^3 \times USp(2)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	$O6$
a	4	(2, 0) (3, 1) (3,-1)	0	0	3	0(1)	-3	0(1)	2	1	0(1)	0(4)	0(6)	0(6)	-9	0(3)	0(1)
b	2	(3,-1) (2, 0) (3, 1)	0	0	-	-	3	0(1)	0(1)	0(4)	-2	-1	9	0(3)	0(6)	0(6)	0(1)
c	2	(3, 1) (3,-1) (2, 0)	0	0	-	-	-	-	-2	-1	2	1	0(3)	-9	0(3)	9	0(1)
d	3	(1,-1) (2, 0) (1, 1)	0	0	-	-	-	-	-	-	-1	0(1)	2	1	4	-4	0(1)
e	1	(2, 0) (1, 1) (1,-1)	0	0	-	-	-	-	-	-	-	-	-4	4	-2	-1	0(1)
f	1	(0, 2) (3,-1) (3,-1)	-1	1	-	-	-	-	-	-	-	-	-	-	27	0(9)	-2
g	1	(3, 1) (0,-2) (3, 1)	1	-1	-	-	-	-	-	-	-	-	-	-	-	-	2
$O6$	1	(2, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

3. Model TII-U-3

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2) \times USp(4) \times USp(6)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = 2\chi_2 = 2\chi_3 = 2\sqrt{6}/3$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+4)$, $m = 2$, and $a = 8$.

Table XLIII. D6-brane configurations and intersection numbers for Model TII-U-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2) \times USp(4) \times USp(6)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(1, 0) (-1,-3) (-1, 3)	0	0	3	0(1)	-3	0(1)
b	2	(2,-1) (1,-3) (0, 2)	-6	6	-	-	4	0(1)
c	2	(-2,-1) (0, 2) (1, 3)	6	-6	-	-	-	-
d	1	(2, 3) (1, 1) (0,-2)	6	-6	-	-	-	-
e	1	(2,-3) (1,-1) (0, 2)	-6	6	-	-	-	-
f	1	(4,-3) (1, 3) (2, 0)	0	0	-	-	-	-
g	1	(4, 1) (1,-1) (2, 0)	0	0	-	-	-	-
h	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-
i	2	(0,-1) (2, 0) (0, 2)	0	0	-	-	-	-
$O6$	3	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	$O6$
a	4	(1, 0) (-1,-3) (-1, 3)	3	6	6	3	0(9)	-27	-6	3	0(1)	-	3	-	0(9)
b	2	(2,-1) (1,-3) (0, 2)	0(16)	0(4)	0(4)	0(16)	12	0(20)	-12	-8	0(1)	-	0(6)	-	-6
c	2	(-2,-1) (0, 2) (1, 3)	-4	-8	-8	-4	30	-6	6	18	0(1)	-	-4	-	6
d	1	(2, 3) (1, 1) (0,-2)	-	-	0(12)	0(0)	-36	24	20	0(28)	0(3)	-	0(2)	-	6
e	1	(2,-3) (1,-1) (0, 2)	-	-	-	-	-24	36	0(28)	-20	0(3)	-	0(2)	-	-6
f	1	(4,-3) (1, 3) (2, 0)	-	-	-	-	-	-	0(32)	0(8)	-6	-	24	-	0(9)
g	1	(4, 1) (1,-1) (2, 0)	-	-	-	-	-	-	-	-	2	-	-8	-	0(1)
h	1	(1, 0) (0,-2) (0, 2)	-	-	-	-	-	-	-	-	-	-	0(2)	-	0(4)
i	2	(0,-1) (2, 0) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	0(2)
$O6$	3	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-

4. Model TII-U-4

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times U(1)^2 \times USp(8)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = \chi_2 = 3\chi_3 = 2/\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

Table XLIV. D6-brane configurations and intersection numbers for Model TII-U-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times U(1)^2 \times USp(8)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(1, 0) (-1,-1) (-1, 3)	0	0	3	0(3)	-3	0(2)
b	2	(2,-3) (1,-1) (0, 2)	-6	6	-	-	4	2
c	2	(0,-1) (1, 3) (1, 3)	9	-9	-	-	-	-
d	2	(2, 1) (1, 3) (0,-2)	6	-6	-	-	-	-
e	2	(-2,-1) (2, 0) (-1, 3)	0	0	-	-	-	-
f	1	(2,-3) (0,-2) (-1, 3)	-18	18	-	-	-	-
g	1	(6,-1) (2, 0) (1, 1)	0	0	-	-	-	-
h	4	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'
a	4	(1, 0) (-1,-1) (-1, 3)	-1	2	0(1)	3	0(3)	-9	2	-1	0(1)	-
b	2	(2,-3) (1,-1) (0, 2)	0(16)	0(4)	-8	-4	0(1)	12	-16	-20	0(3)	-
c	2	(0,-1) (1, 3) (1, 3)	0(2)	-6	18	0(6)	-6	0(2)	18	36	-1	-
d	2	(2, 1) (1, 3) (0,-2)	-	-	0(3)	12	-8	-4	24	12	0(1)	-
e	2	(-2,-1) (2, 0) (-1, 3)	-	-	-	-	0(16)	-24	0(16)	0(4)	2	-
f	1	(2,-3) (0,-2) (-1, 3)	-	-	-	-	-	-	-64	-40	0(3)	-
g	1	(6,-1) (2, 0) (1, 1)	-	-	-	-	-	-	-	-	-2	-
h	4	(1, 0) (0,-2) (0, 2)	-	-	-	-	-	-	-	-	-	-

5. Model TII-U-5

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^2 \times USp(4)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $2\chi_1 = 3\chi_2 = 2\chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -2(3q+4)$, $m = 2$, and $a = 4$.

Table XLV. D6-brane configurations and intersection numbers for Model TII-U-5 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^2 \times USp(4)^2]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'
a	4	(1, 0) (-1,-3) (-1, 1)	0	0	3	0(1)	-3	0(1)	-12	-6	-6	12	0(1)	-	3	-
b	2	(1,-1) (1,-3) (0, 1)	-6	6	-	-	0(2)	0(3)	12	6	-36	-18	0(1)	-	0(3)	-
c	2	(1, 1) (-1, 3) (-1, 0)	0	0	-	-	-	-	-2	4	6	-12	1	-	-3	-
d	1	(1, 3) (0,-2) (3, 1)	12	-12	-	-	-	-	-	-	96	0(20)	0(9)	-	-6	-
e	1	(3, 1) (2, 0) (3,-1)	0	0	-	-	-	-	-	-	-	-	6	-	0(9)	-
f	2	(1, 0) (0,-2) (0, 1)	0	0	-	-	-	-	-	-	-	-	-	-	0(2)	-
g	2	(0,-1) (2, 0) (0, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-

6. Model TII-U-6

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(4)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $4\chi_1 = \chi_2 = \chi_3 = 2\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(q+2)$, $m = 2$, and $a = 8$.

Table XLVI. D6-brane configurations and intersection numbers for Model TII-U-6 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(4)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	4	(1, 0) (3, 1) (3,-1)	0	0	3	0(2)	-3	0(2)	-2	-4	0(9)	-	3	-
b	2	(0, 1) (3,-1) (1,-1)	-1	1	-	-	0(4)	0(1)	-2	1	3	-	0(1)	-
c	2	(0,-1) (1, 1) (3, 1)	1	-1	-	-	-	-	0(1)	1	-3	-	0(3)	-
d	2	(1,-2) (1, 1) (2, 0)	0	0	-	-	-	-	-	-	-4	-	2	-
e	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	0(2)	-
f	2	(0,-1) (2, 0) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-

APPENDIX E

SECOND KIND OF $U(4)_C \times USp(2)_L \times U(2)_R$ P-S MODELS WITH FLUXES*

We present the D6-brane configurations and intersection numbers for the second kind of $U(4)_C \times USp(2)_L \times U(2)_R$ Pati-Salam models with IIA fluxes.

1. Model TII-Sp-1

   D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(4) \times USp(8)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $2\chi_1 = \chi_2 = 2\chi_3 = 2\sqrt{6}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(3q+4)$, $m = 2$, and $a = 24$.

Table XLVII. D6-brane configurations and intersection numbers for Model TII-Sp-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(4) \times USp(8)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	$O6$
a	4	(1, 0) (3, 1) (3,-1)	0	0	3	-	-3	0(1)	0(2)	0(8)	0(3)	9	3	0(5)	-3	-	0(1)
b	1	(0,-1) (2, 0) (0, 1)	0	0	-	-	3	-	-1	-	0(2)	-	0(2)	-	0(2)	-	0(1)
c	2	(3, 1) (3,-1) (1, 0)	0	0	-	-	-	-	2	1	-3	0(5)	0(3)	-9	0(9)	-	0(1)
d	2	(1, 0) (1, 1) (1,-1)	0	0	-	-	-	-	-	-	1	2	-2	3	-1	-	0(1)
e	1	(2, 1) (3, 1) (0,-1)	2	-2	-	-	-	-	-	-	-	-	-12	0(4)	-6	-	1
f	1	(0, 1) (3,-1) (2,-1)	-2	2	-	-	-	-	-	-	-	-	-	-	0(3)	-	-1
g	2	(0,-1) (0, 2) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	0(2)
$O6$	4	(1, 0) (2, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

2. Model TII-Sp-2

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2)^3 \times U(1)^2 \times USp(6) \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = 3\chi_2 = \chi_3/4 = \sqrt{3/2}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+4)$, $m = 2$, and $a = 4$.

Table XLVIII. D6-brane configurations and intersection numbers for Model TII-Sp-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2)^3 \times U(1)^2 \times USp(6) \times USp(2)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(0,-1) (1, 3) (3, 1)	6	-6	3	-	-3	0(1)
b_{O6}	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	1	-
c	2	(1,-1) (0, 1) (3,-1)	-2	2	-	-	-	-
d	2	(1, 0) (1, 6) (1,-1)	0	0	-	-	-	-
e	2	(1, 0) (1, 2) (3,-1)	0	0	-	-	-	-
f	2	(0,-1) (1, 1) (1, 1)	2	-2	-	-	-	-
g	1	(6, 1) (2,-1) (2, 0)	0	0	-	-	-	-
h	1	(3, 1) (1,-1) (2, 0)	0	0	-	-	-	-
i	3	(1, 0) (0,-1) (0, 2)	0	0	-	-	-	-
j	1	(0, -1) (1, 0) (0, 2)	0	0	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	j	j'
a	4	(0,-1) (1, 3) (3, 1)	-6	-9	3	0(5)	0(2)	0(8)	42	30	12	6	-3	-	0(9)	-
b_{O6}	1	(1, 0) (1, 0) (2, 0)	0(6)	-	0(2)	-	-1	-	0(1)	-	0(1)	-	0(2)	-	0(2)	-
c	2	(1,-1) (0, 1) (3,-1)	1	-2	0(1)	-3	2	1	-14	-10	-4	-2	0(3)	-	3	-
d	2	(1, 0) (1, 6) (1,-1)	-	-	0(4)	0(16)	5	0(7)	-13	11	-7	5	0(1)	-	6	-
e	2	(1, 0) (1, 2) (3,-1)	-	-	-	-	2	3	-5	3	-3	1	0(3)	-	6	-
f	2	(0,-1) (1, 1) (1, 1)	-	-	-	-	-	-	18	6	6	0(3)	-1	-	0(1)	-
g	1	(6, 1) (2,-1) (2, 0)	-	-	-	-	-	-	-	-	0(3)	0(27)	4	-	-12	-
h	1	(3, 1) (1,-1) (2, 0)	-	-	-	-	-	-	-	-	-	-	2	-	-6	-
i	3	(1, 0) (0,-1) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-
j	1	(0, -1) (1, 0) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-

3. Model TII-Sp-3

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^6 \times USp(4)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $12\chi_1 = 12\chi_2 = \chi_3 = 2\sqrt{6}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+2)$, $m = 2$, and $a = 4$.

Table XLIX. D6-brane configurations and intersection numbers for Model TII-Sp-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^6 \times USp(4)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(0,-1) (1, 3) (3, 1)	6	-6	3	-	-3	0(1)
b_{O6}	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	3	-
c	2	(1,-3) (0, 1) (3,-1)	-6	6	-	-	-	-
d	1	(1,-1) (0, 1) (1,-1)	-2	2	-	-	-	-
e	1	(3, 1) (0, 1) (-1,-3)	6	-6	-	-	-	-
f	1	(3, 2) (3,-2) (2, 0)	0	0	-	-	-	-
g	1	(1, 2) (1,-2) (2, 0)	0	0	-	-	-	-
h	1	(1,-2) (-1, 0) (-3,-1)	0	0	-	-	-	-
i	1	(1, 2) (1, 0) (3,-1)	0	0	-	-	-	-
j	2	(0,-1) (1, 0) (0, 2)	0	0	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	j	j'
a	4	(0,-1) (1, 3) (3, 1)	-2	-1	-12	-15	33	21	5	1	0(3)	9	9	0(3)	0(9)	-
b_{O6}	1	(1, 0) (1, 0) (2, 0)	1	-	-3	-	0(4)	-	0(4)	-	0(2)	-	0(2)	-	0(2)	-
c	2	(1,-3) (0, 1) (3,-1)	0(2)	0(8)	0(50)	0(32)	-33	-21	-5	-1	-3	0(5)	0(5)	-3	3	-
d	1	(1,-1) (0, 1) (1,-1)	-	-	0(8)	0(2)	-15	-3	-3	1	2	-3	-3	2	1	-
e	1	(3, 1) (0, 1) (-1,-3)	-	-	-	-	-27	81	-15	21	28	-25	-25	28	-3	-
f	1	(3, 2) (3,-2) (2, 0)	-	-	-	-	-	-	0(16)	0(64)	-16	-8	-8	-16	-12	-
g	1	(1, 2) (1,-2) (2, 0)	-	-	-	-	-	-	-	-	-8	0(2)	0(2)	-8	-4	-
h	1	(1,-2) (-1, 0) (-3,-1)	-	-	-	-	-	-	-	-	-	-	0(12)	0(0)	0(3)	-
i	1	(1, 2) (1, 0) (3,-1)	-	-	-	-	-	-	-	-	-	-	-	-	0(3)	-
j	2	(0,-1) (1, 0) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-

4. Model TII-Sp-4

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^3 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $12\chi_1 = 24\chi_2 = \chi_3 = 4\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -8(3q+2)$, $m = 2$, and $a = 16$.

Table L. D6-brane configurations and intersection numbers for Model TII-Sp-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^3 \times USp(2)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	4	(0, 1) (-1,-3) (3, 1)	6	-6	3	-	-3	0(7)
b_{O6}	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	0(4)	-
c	2	(-1, 0) (1, 4) (-3, 1)	0	0	-	-	-	-
d	2	(1, 1) (1,-2) (2, 0)	0	0	-	-	-	-
e	1	(-1, 2) (-1, 3) (0, 2)	-24	24	-	-	-	-
f	1	(2, 5) (-1, 5) (-2, 0)	0	0	-	-	-	-
g	1	(4, 1) (2,-1) (2, 0)	0	0	-	-	-	-
h	1	(1, 0) (0,-1) (0, 2)	0	0	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'
a	4	(0, 1) (-1,-3) (3, 1)	5	1	-18	0(3)	16	-4	28	20	-3	-
b_{O6}	1	(1, 0) (1, 0) (2, 0)	0(2)	-	12	-	0(25)	-	0(1)	-	0(2)	-
c	2	(-1, 0) (1, 4) (-3, 1)	-6	2	42	6	-45	-5	-9	7	0(3)	-
d	2	(1, 1) (1,-2) (2, 0)	-	-	6	-10	0(9)	0(49)	0(9)	0(25)	2	-
e	1	(-1, 2) (-1, 3) (0, 2)	-	-	-	-	36	16	-90	-98	0(2)	-
f	1	(2, 5) (-1, 5) (-2, 0)	-	-	-	-	-	-	0(162)	0(242)	10	-
g	1	(4, 1) (2,-1) (2, 0)	-	-	-	-	-	-	-	-	4	-
h	1	(1, 0) (0,-1) (0, 2)	-	-	-	-	-	-	-	-	-	-

5. Model TII-Sp-5

   $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^8 \times USp(6) \times USp(4)]_{hidden}$ on Type IIA $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. The SM fermions and Higgs fields arise from the intersections on different two-tori, and $\chi_1 = \chi_2/2 = \chi_3 = \sqrt{2}$. To satisfy the RR tadpole conditions, we choose $h_0 = -4(3q+8)$, $m = 8$, and $a = 32$.

Table LI. D6-brane configurations and intersection numbers for Model TII-Sp-5 on Type IIA $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^8 \times USp(6) \times USp(4)]_{hidden}$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	8	(1, 0) (3, 1) (3,-1)	0	0	3	-	-3	0
b_{O6^3}	2	(0,-1) (2, 0) (0, 1)	-	-	-	-	3	-
c	4	(3, 1) (3,-1) (1, 0)	0	0	-	-	-	-
d	2	(1,-1) (-1,-1) (-1, 0)	0	0	-	-	-	-
e	2	(1, 3) (1,-3) (1, 0)	0	0	-	-	-	-
f	2	(2,-3) (-2, 0) (-2,-3)	0	0	-	-	-	-
g	2	(1,-3) (-2, 0) (-1,-3)	0	0	-	-	-	-
h	2	(4,-1) (0, 2) (1,-2)	4	-4	-	-	-	-
i	2	(3,-1) (0, 2) (2,-3)	6	-6	-	-	-	-
j	2	(2,-1) (1,-1) (0, 1)	1	-1	-	-	-	-
k	2	(0, 1) (-1,-3) (6, 1)	-3	3	-	-	-	-
$O6^1$	6	(1, 0) (2, 0) (1, 0)	-	-	-	-	-	-
$O6^4$	4	(0,-1) (0, 2) (1, 0)	-	-	-	-	-	-

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	j	j'	k	k'	$O6^1$	$O6^4$
a	8	(1, 0) (3, 1) (3,-1)	-1	-2	-15	-12	33	21	30	24	15	-21	21	-33	6	-2	-36	-15	0	-3
b_{O6^3}	2	(0,-1) (2, 0) (0, 1)	-1	-	3	-	0	-	0	-	-8	-	-12	-	0	-	0	-	0	0
c	4	(3, 1) (3,-1) (1, 0)	0	0	0	0	-33	-21	-30	-24	42	6	54	0	5	-2	-15	12	0	0
d	2	(1,-1) (-1,-1) (-1, 0)	-	-	0	0	3	15	6	12	-6	-10	-6	-12	-1	0	-1	2	0	0
e	2	(1, 3) (1,-3) (1, 0)	-	-	-	-	-81	27	-54	0	26	22	30	24	-7	-16	-3	0	0	0
f	2	(2,-3) (-2, 0) (-2,-3)	-	-	-	-	-	-	0	0	-140	-28	-168	0	-8	-16	96	120	0	12
g	2	(1,-3) (-2, 0) (-1,-3)	-	-	-	-	-	-	-	-	-110	26	-144	60	-5	-7	51	57	0	6
h	2	(4,-1) (0, 2) (1,-2)	-	-	-	-	-	-	-	-	-	-	0	0	2	6	52	-44	-4	0
i	2	(3,-1) (0, 2) (2,-3)	-	-	-	-	-	-	-	-	-	-	-	-	2	10	60	-48	-6	0
j	2	(2,-1) (1,-1) (0, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	24	12	-1	2
k	2	(0, 1) (-1,-3) (6, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	0

APPENDIX F

FLIPPED $SU(5)$ MODELS WITH TYPE IIA FLUXES*

We present the D6-brane configurations and intersection numbers for flipped $SU(5)$ models with IIA fluxes.

1. Model FSU(5)F-I

Flipped $U(5) \times U(1)^7 \times USp(6) \times USp(2) \times USp(4)$ on Type IIA $\mathbf{T^6}$ orientifold. The complex structure parameters are $\chi_1 = 1/\sqrt{3}$, $\chi_2 = 2/\sqrt{3}$, and $\chi_3 = 2/\sqrt{3}$. To satisfy the RR tadpole conditions, $h_0 = -12(3q+2)$, $a = 16$, and $m = 2$.

The $U(1)_X$ in flipped $SU(5) \times U(1)_X$ gauge symmetry is

$$
\begin{aligned}
U(1)_X &= \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d - 5U(1)_e \\
&\quad +5U(1)_f + 5U(1)_g + 5U(1)_h) \ .
\end{aligned} \tag{F.1}
$$

The other massless $U(1)$'s are:

$$
\begin{aligned}
U(1)_U &= 5U(1)_a - 25U(1)_b + 25U(1)_c + 25U(1)_d + 107U(1)_e \\
&\quad +25U(1)_f - 19U(1)_g + 25U(1)_h \ , \\
U(1)_V &= U(1)_c - 2U(1)_d + U(1)_e + U(1)_f + U(1)_h \ , \\
U(1)_W &= 4U(1)_b - 6U(1)_d - 10U(1)_e - U(1)_f + 2U(1)_g - U(1)_h \ .
\end{aligned} \tag{F.2}
$$

And the four global $U(1)$'s are

$$
U(1)_1 = -5U(1)_a + 2U(1)_c + U(1)_d - 2U(1)_e + 2U(1)_f - 6U(1)_g \ ,
$$

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Table LII. D6-brane configurations and intersection numbers for the Model FSU(5)F-I on Type IIA $\mathbf{T^6}$ orientifold.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	5	(0, 1)(-1,-1)(1, 3)	3	-3	-3	0(1)	3	0	0	0	0	3	2	1
b	1	(1,-1)(0, 2)(1, 3)	-6	6	-	-	-6	0	-1	2	0	-6	-4	0
c	1	(1, 1)(1,-1)(2, 0)	0	0	-	-	-	-	1	-2	0	0	0	-2
d	1	(0, 1)(1,-3)(1,-1)	-3	3	-	-	-	-	-	-	-2	-1	0	-3
e	1	(1,-1)(1, 1)(2, 0)	0	0	-	-	-	-	-	-	-	-	2	0
f	1	(1, 1)(2, 0)(1,-1)	0	0	-	-	-	-	-	-	-	-	-	-
g	1	(3,-1)(3, 1)(2, 0)	0	0	-	-	-	-	-	-	-	-	-	-
h	1	(-1, 1)(-1, 3)(0, 2)	-6	6	-	-	-	-	-	-	-	-	-	-
i	3	(1, 0)(0,-2)(0, 2)	0	0	-	-	-	-	-	-	-	-	-	-
j	1	(0,-1)(2, 0)(0, 2)	0	0	-	-	-	-	-	-	-	-	-	-
k	2	(0,-1)(0, 2)(2, 0)	0	0	-	-	-	-	-	-	-	-	-	-

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	g	g'	h	h'	i	i'	j	j'	k	k'
a	5	(0, 1)(-1,-1)(1, 3)	3	-3	9	18	-2	-1	-1	-	0	-	0	-
b	1	(1,-1)(0, 2)(1, 3)	-6	6	-18	-36	0	2	0	-	2	-	0	-
c	1	(1, 1)(1,-1)(2, 0)	0	0	0	0	4	0	2	-	-2	-	0	-
d	1	(0, 1)(1,-3)(1,-1)	-3	3	-15	-12	0	3	1	-	0	-	0	-
e	1	(1,-1)(1, 1)(2, 0)	0	0	0	0	0	-4	-2	-	2	-	0	-
f	1	(1, 1)(2, 0)(1,-1)	0	0	-2	1	3	0	2	-	0	-	-2	-
g	1	(3,-1)(3, 1)(2, 0)	0	0	-	-	20	-32	-6	-	6	-	0	-
h	1	(-1, 1)(-1, 3)(0, 2)	-6	6	-	-	-	-	0	-	0	-	2	-

$$\begin{aligned} U(1)_2 &= 2U(1)_b - 2U(1)_c + 2U(1)_e + 6U(1)_g \ , \\ U(1)_3 &= -2U(1)_f + 2U(1)_h \ , \\ U(1)_4 &= 15U(1)_a - 6U(1)_b - 3U(1)_d - 6U(1)_h \ . \end{aligned} \tag{F.3}$$

Table LIII. The particle spectrum in the observable sector of Model FSU(5)F-I with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

Rep.	$\mathcal{M}$	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_g$	$U(1)_h$	$2U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_U$	$U(1)_V$	$U(1)_W$
$(10,1)$	3	2	0	0	0	0	0	0	0	1	-10	0	0	30	10	0	0
$(\bar{5}_a, 1_b)$	3	-1	1	0	0	0	0	0	0	-3	5	2	0	-21	-30	0	4
$(1_d, 1_h)$	3	0	0	0	1	0	0	0	1	5	1	0	2	-9	50	-1	-7
$(10,1)$	1	2	0	0	0	0	0	0	0	1	-10	0	0	30	10	0	0
$(\overline{10},1)$	1	-2	0	0	0	0	0	0	0	-1	10	0	0	-30	-10	0	0
$(5_a, 1_b)^\star$	1	1	1	0	0	0	0	0	0	-2	-5	2	0	9	-20	0	4
$(\bar{5}_a, \bar{1}_b)^\star$	1	-1	-1	0	0	0	0	0	0	2	5	-2	0	-9	20	0	-4
$(1_c, \bar{1}_h)$	4	0	0	1	0	0	0	0	-1	0	2	-2	0	6	0	0	1
$(\overline{15},1)$	3	-2	0	0	0	0	0	0	0	-1	10	0	0	-30	-10	0	0
$(10,1)$	2	2	0	0	0	0	0	0	0	1	-10	0	0	30	10	0	0
$(\overline{10},1)$	2	-2	0	0	0	0	0	0	0	-1	10	0	0	-30	-10	0	0
Additional chiral and non-chiral Matter																	

2. Model FSU(5)F-II

We construct the Model FSU(5)F-II on Type IIA $\mathbf{T^6}$ orientifold in which unlike the previous flipped $SU(5)$ model building [25, 43, 44], all the Yukawa couplings are allowed by the global $U(1)$ symmetries. The D6-brane configurations and intersection numbers for the Model FSU(5)-II are given in Tables LIV and LV, and its particle spectrum in the observable sector is given in Table LVI. The complete gauge symmetry is $U(5) \times U(1)^{11} \times USp(16)$, and the complex structure parameters are $\chi_1 = \sqrt{3}/27$, $\chi_2 = 2\sqrt{3}$, and $\chi_3 = \sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+2)$, $a = 24$, and $m = 12$.

The $U(1)_X$ gauge symmetry is

$$U(1)_X = \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d + 5U(1)_e + 5U(1)_f + 5U(1)_g + 5U(1)_h + 5U(1)_i - 5U(1)_j - 5U(1)_k - 5U(1)_l) \,. \quad \text{(F.4)}$$

Table LIV. D6-brane configurations and intersection numbers (Part 1) for the Model FSU(5)-II on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^{11} \times USp(16)$.

stk	N	(n_1,l_1) (n_2,l_2) (n_3,l_3)	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	5	(0, 1) (-1,-1) (3, 1)	2	-2	-3	-6	0(1014)	0(864)	0(242)	0(392)	0(6)	0(0)	-3	0(3)
b	1	(1, 3) (1, 3) (0,-1)	18	-18	-	-	-114	111	-200	-425	-6	3	0(24)	0(6)
c	1	(0, 1) (25,-1) (3,-25)	-50	50	-	-	-	-	0(197192)	0(193442)	0(864)	(1014)	36	39
d	1	(0, 1) (-3,-25) (25, 1)	50	-50	-	-	-	-	-	-	0(392)	0(242)	-250	275
e	1	(0, 1) (1,-1) (3, 1)	-2	2	-	-	-	-	-	-	-	-	0	3
f	1	(1,-9) (1,-1) (0, 1)	-18	18	-	-	-	-	-	-	-	-	-	-
g	1	(1, 0) (3,-1) (3, 1)	0	0	-	-	-	-	-	-	-	-	-	-
h	1	(1, 0) (3, 1) (3,-1)	0	0	-	-	-	-	-	-	-	-	-	-
i	1	(1, 1) (1, 9) (0,-1)	18	-18	-	-	-	-	-	-	-	-	-	-
j	1	(1,-1) (1,-9) (0, 1)	-18	18	-	-	-	-	-	-	-	-	-	-
k	1	(1,-1) (27, 1) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-
l	1	(1, 1) (27,-1) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-
$O6$	8	(1, 0) (2, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-

The four global $U(1)$'s are:

$$
\begin{aligned}
U(1)_1 &= -15U(1)_a + 75U(1)_c - 75U(1)_d + 3U(1)_e - 27U(1)_k + 27U(1)_l \ , \\
U(1)_2 &= -3U(1)_g + 3U(1)_h + U(1)_k - U(1)_l \ , \\
U(1)_3 &= -U(1)_b + U(1)_f + 3U(1)_g - 3U(1)_h - U(1)_i + U(1)_j \ , \\
U(1)_4 &= 5U(1)_a + 9U(1)_b - 25U(1)_c + 25U(1)_d - U(1)_e \\
&\quad -9U(1)_f + 9U(1)_i - 9U(1)_j \ . \qquad \text{(F.5)}
\end{aligned}
$$

There are seven other massless $U(1)$'s. As an example, we present two of them:

$$
\begin{aligned}
U(1)_V &= U(1)_b - U(1)_f + 2U(1)_g + 2U(1)_h - 2U(1)_i \ , \\
U(1)_W &= -36U(1)_b - 27U(1)_c + 36U(1)_f + 4U(1)_g \\
&\quad +29U(1)_h - 3U(1)_i + 75U(1)_l \ . \qquad \text{(F.6)}
\end{aligned}
$$

Table LV. D6-brane configurations and intersection numbers (Part 2) for the Model FSU(5)-II on Type IIA $\mathbf{T^6}$ orientifold.

stk	N	(n_1,l_1) (n_2,l_2) (n_3,l_3)	g	g'	h	h'	i	i'	j	j'	k	k'	l	l'	$O6$
a	5	(0, 1) (-1,-1) (3, 1)	0	6	6	0	-12	-15	-15	-12	13	14	14	13	1
b	1	(1, 3) (1, 3) (0,-1)	45	36	36	45	0	0	0	0	160	82	82	160	9
c	1	(0, 1) (25,-1) (3,-25)	858	-1008	-1008	858	339	336	0	0	160	82	82	160	-25
d	1	(0, 1) (-3,-25) (25, 1)	-858	1008	1008	-858	-25	-650	-650	-25	336	339	339	336	25
e	1	(0, 1) (1,-1) (3, 1)	-6	0	0	-6	15	12	12	15	-14	-13	-13	-14	-1
f	1	(1,-9) (1,-1) (0, 1)	-27	-54	-54	-27	0	0	0	0	-112	-130	-130	-112	-9
g	1	(1, 0) (3,-1) (3, 1)	-	-	0	0	-42	39	39	-42	15	-12	-12	15	0
h	1	(1, 0) (3, 1) (3,-1)	-	-	-	-	-39	42	42	-39	12	-15	-15	12	0
i	1	(1, 1) (1, 9) (0,-1)	-	-	-	-	-	-	0	0	242	0	0	242	0
j	1	(1,-1) (1,-9) (0, 1)	-	-	-	-	-	-	-	-	0	-242	-242	0	-9
k	1	(1,-1) (27, 1) (1, 0)	-	-	-	-	-	-	-	-	-	-	0	0	0
l	1	(1, 1) (27,-1) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	0
$O6$	8	(1, 0) (2, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-

This is the first trial flipped $SU(5)$ model where all the Yukawa couplings in superpotential in (5.4) are allowed by the global $U(1)$'s from the Green-Schwarz mechanism. To make the terms like FFh or HHh to be neutral under the global $U(1)$ symmetries, we need to set the Higgs pentaplet h from the intersection between the $N=5$ stack and a stack with large wrapping numbers (by a factor of 25 due to the flipped $SU(5)$ structure) and therefore we can not avoid extremely large exotic matter in the spectrum. In this model the Yukawa terms are:

$$
\begin{aligned}
FFh &\rightarrow (10,1)(10,1)(5_a,\bar{1}_d)\ , \\
F\bar{f}\bar{h}' &\rightarrow (10,1)(\bar{5}_a,1_b)(\bar{5}_a,1_f)\ , \\
\bar{f}l^c h &\rightarrow (\bar{5}_a,1_b)(\bar{1}_b,1_d)(5_a,\bar{1}_d)\ , \\
F\bar{H}\phi &\rightarrow (10,1)(\overline{10},1)(1_b,1_f)\ ,
\end{aligned}
$$

Table LVI. The particle spectrum in the observable sector in the Model FSU(5)-II, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

Rep.	Multi.	$U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_V$	$U(1)_W$	$\cdots$
$(10,1)$	3	1	-30	0	0	10	0	0	$\cdots$
$(\bar{5}_a, 1_b)$	3	-3	15	0	-1	4	1	-36	$\cdots$
$(\bar{1}_b, 1_d)$	3	5	-75	0	1	16	-1	36	$\cdots$
$(10,1)$	1	1	-30	0	0	10	0	0	$\cdots$
$(\overline{10},1)$	1	-1	30	0	0	-10	0	0	$\cdots$
$(5_a, \bar{1}_d)^\star$	1	-2	60	0	0	-20	0	0	$\cdots$
$\bar{h}_x$ $((\bar{5}_a, 1_d)^\star/(\bar{5}_a, 1_f)^\star)$	1	2	-60/15	0	0/1	20/-14	0/-1	0/36	$\cdots$
$(1_b, 1_f)$	4	0	0	0	0	0	0	0	$\cdots$
$(\overline{15},1)$	2	-1	30	0	0	-10	0	0	$\cdots$
$(\overline{10},1)$	1	-1	30	0	0	-10	0	0	$\cdots$
Additional chiral and non-chiral Matter									

$$HHh \rightarrow (10,1)(10,1)(5_a, \bar{1}_d) ,$$

$$\bar{H}\bar{H}\bar{h} \rightarrow (\overline{10},1)(\overline{10},1)(\bar{5}_a, 1_d) . \qquad \text{(F.7)}$$

Because of the structure of Green-Schwarz mechanism in D-brane construction, to cancel the global $U(1)$'s charges for all the Yukawa couplings we expect a mixture state of Higgs pentaplet $\bar{h}_x = c\bar{h}' + s\bar{h}$ where $\bar{h}'$ is from $F\bar{f}\bar{h}'$ and $\bar{h}$ is from $\bar{H}\bar{H}\bar{h}$. However, we may reintroduce the doublet-triplet splitting problem.

3. Model FSU(5)-III

   We present the D6-brane configurations and intersection numbers for the Model FSU(5)-III on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold in Tables LVII and LVIII, and its particle spectrum in the observable sector in Table LIX. The complete gauge symmetry is $U(5) \times U(1)^{10} \times USp(10) \times USp(8) \times USp(2)$, and the complex structure parameters are $\chi_1 = 2/3, \quad \chi_2 = 1$, and $\chi_3 = 1$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(3q+2)$, $a = 16$, and $m = 2$.

Table LVII. D6-brane configurations and intersection numbers (Part 1) for the Model FSU(5)-III on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^{10} \times USp(10) \times USp(8) \times USp(2)$.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'
a	10	(1, 3) (1, 1) (0,-1)	2	-2	-3	0(1)	0(2)	0(3)	0	0	24	24	12	-6	6	-3
b	2	(1,-3) (0,-1) (-1, 1)	-2	2	-	-	-3	0(1)	2	-1	-15	-60	-12	6	12	12
c	2	(1, 3) (-1, 1) (-1, 0)	2	-2	-	-	-	-	4	4	0	0	18	36	-18	-9
d	2	(1, 1) (-1,-3) (0, 1)	2	-2	-	-	-	-	-	-	84	-36	6	0	15	-12
e	2	(-5, 9) (-5,-3) (1, 0)	-30	30	-	-	-	-	-	-	-	-	-486	-324	162	243
f	2	(2, 0) (-1, 3) (-1,-3)	0	0	-	-	-	-	-	-	-	-	-	-	0	-162
g	2	(1,-9) (-1, 0) (-1,-3)	-6	6	-	-	-	-	-	-	-	-	-	-	-	-
h	2	(1,-7) (0, 1) (7,-3)	0	0	-	-	-	-	-	-	-	-	-	-	-	-
i	2	(0, 2) (4,-3) (3,-4)	0	0	-	-	-	-	-	-	-	-	-	-	-	-
j	2	(1,-3) (-1, 0) (-1,-1)	-2	2	-	-	-	-	-	-	-	-	-	-	-	-
k	2	(0, 2) (-3,-1) (1, 3)	0	0	-	-	-	-	-	-	-	-	-	-	-	-
$O6^1$	10	(2, 0) (1, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$O6^3$	8	(0,-2) (1, 0) (0, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$O6^4$	2	(0,-2) (0, 1) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-

The $U(1)_X$ gauge symmetry is

$$
\begin{aligned}
U(1)_X \quad = \quad & \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d + 5U(1)_e + 5U(1)_f - 5U(1)_g \\
& -5U(1)_h + 5U(1)_i - 5U(1)_j - 5U(1)_k) \ . \qquad \text{(F.8)}
\end{aligned}
$$

Table LVIII. D6-brane configurations and intersection numbers (Part 2) for the Model FSU(5)-III on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	h	h'	i	i'	j	j'	k	k'	$O6^1$	$O6^3$	$O6^4$
a	10	(1, 3) (1, 1) (0,-1)	-35	-14	-21	3	3	0	2	-4	3	0	-1
b	2	(1,-3) (0,-1) (-1, 1)	0	0	4	28	0	0	12	6	-3	1	-3
c	2	(1, 3) (-1, 1) (-1, 0)	15	-6	-4	-28	-3	0	-12	-6	0	-1	0
d	2	(1, 1) (-1,-3) (0, 1)	-28	-21	-45	27	6	-3	8	-10	3	3	-1
e	2	(-5, 9) (-5,-3) (1, 0)	195	-330	540	-60	9	-36	60	210	0	15	0
f	2	(2, 0) (-1, 3) (-1,-3)	168	126	-234	150	18	-36	0	-96	0	-6	6
g	2	(1,-9) (-1, 0) (-1,-3)	-24	144	39	15	0	0	0	6	0	0	3
h	2	(1,-7) (0, 1) (7,-3)	-	-	76	0	-20	20	72	54	-21	7	0
i	2	(0, 2) (4,-3) (3,-4)	-	-	-	-	-21	-3	0	0	-24	0	0
j	2	(1,-3) (-1, 0) (-1,-1)	-	-	-	-	-	-	-2	4	0	0	1
k	2	(0, 2) (-3,-1) (1, 3)	-	-	-	-	-	-	-	-	6	0	0
$O6^1$	10	(2, 0) (1, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-
$O6^3$	8	(0,-2) (1, 0) (0, 1)	-	-	-	-	-	-	-	-	-	-	-
$O6^4$	2	(0,-2) (0, 1) (1, 0)	-	-	-	-	-	-	-	-	-	-	-

And the four global $U(1)$'s are

$$\begin{aligned} U(1)_1 &= 6U(1)_c - 90U(1)_e - 18U(1)_g + 48U(1)_i - 6U(1)_j - 12U(1)_k \ , \\ U(1)_2 &= 2U(1)_b - 2U(1)_c + 30U(1)_e - 12U(1)_f + 14U(1)_h \ , \\ U(1)_3 &= -10U(1)_a - 2U(1)_d + 12U(1)_f + 6U(1)_g + 2U(1)_j \ , \\ U(1)_4 &= 30U(1)_a - 6U(1)_b + 6U(1)_d - 42U(1)_h - 48U(1)_i + 12U(1)_k \ . \end{aligned} \tag{F.9}$$

There are six other massless $U(1)$'s. As an example, we present two of them:

$$\begin{aligned} U(1)_U &= -10U(1)_b + U(1)_c - U(1)_e - 2U(1)_f + 4U(1)_g + 2U(1)_h + 2U(1)_k \ , \\ U(1)_V &= 125U(1)_b - 80U(1)_c + 26U(1)_e - 85U(1)_h + 47U(1)_i - 47U(1)_k \end{aligned} \tag{F.10}$$

Table LIX. The particle spectrum in the observable sector in the Model FSU(5)-III, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

Rep.	Multi.	$U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_U$	$U(1)_V$	$\cdots$
$(10, 1)$	3	1	0	0	-20	60	0	0	$\cdots$
$(\bar{5}_a, 1_b)$	3	-3	0	2	10	-36	-10	125	$\cdots$
$(\bar{1}_b, 1_c)$	3	5	6	-4	0	6	11	-205	$\cdots$
$(10, 1)$	1	1	0	0	-20	60	0	0	$\cdots$
$(\overline{10}, 1)$	1	-1	0	0	20	-60	0	0	$\cdots$
$(5_a, 1_b)^\star$	1	-2	0	2	-10	24	-10	125	$\cdots$
$\bar{5}_a, \bar{1}_b$	1	2	0	-2	10	-24	10	-125	$\cdots$
$(1_c, \bar{1}_d)$	4	0	6	-2	2	-6	1	-80	$\cdots$
$(\overline{15}, 1)$	2	-1	0	0	20	-60	0	0	$\cdots$
$(\overline{10}, 1)$	1	-1	0	0	20	-60	0	0	$\cdots$
Additional chiral and non-chiral Matter									

APPENDIX G

$SU(5)$ MODELS WITH TYPE IIA FLUXES*

We present the D6-brane configurations and intersection numbers for the Model SU(5)-II on Type IIA $\mathbf{T^6}$ orientifold and the Model SU(5)-III on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold in Tables LX and LXI, respectively. Similar to the Model SU(5)-I, we have three 10 representations, and three $\bar{5}$ representations after the additional gauge symmetry breaking by the supersymmetry preserving Higgs mechanism. With suitable fine-tuning, we can have the MSSM below the GUT scale, and generate the correct low energy gauge couplings via renormalization group equation running.

1. Model SU(5)-II

   We present the D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^6 \times USp(8) \times USp(8) \times USp(4)$, and the complex structure parameters are $\chi_1 = 6/\sqrt{7}$, $\chi_2 = 2/\sqrt{7}$, and $\chi_3 = 2\sqrt{7}/3$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(3q+2)$, $a = 24$, and $m = 2$.

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Table LX. D6-brane configurations and intersection numbers for the Model SU(5)-II on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^6 \times USp(8) \times USp(8) \times USp(4)$.

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'
a	5	(1, 1) (-1,-1) (-1, 3)	3	0	-3	0(6)	-2	-2
b	1	(1,-1) (1, 3) (2, 0)	0	0	-	-	3	0(1)
c	1	(0, 2) (1,-3) (1,-1)	-3	3	-	-	-	-
d	1	(-3, 1) (1,-1) (-1,-1)	-2	-1	-	-	-	-
e	1	(1,-1) (3,-7) (0, 2)	-7	7	-	-	-	-
f	1	(2, 0) (-1, 1) (-7,-3)	0	0	-	-	-	-
g	1	(1,-3) (0, 2) (3, 1)	3	-3	-	-	-	-
h	4	(2, 0) (0,-2) (0, 2)	0	0	-	-	-	-
i	4	(0,-2) (0, 2) (2, 0)	0	0	-	-	-	-
$O6$	2	(2, 0) (2, 0) (2, 0)	-	-	-	-	-	-

stk	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	$O6$
a	5	(1, 1) (-1,-1) (-1, 3)	4	0(1)	5	0(2)	12	0(9)	-5	-8	1	-	-3	-	3
b	1	(1,-1) (1, 3) (2, 0)	-2	2	0(16)	2	-6	3	-2	1	-2	-	0(1)	-	0(3)
c	1	(0, 2) (1,-3) (1,-1)	-3	0(6)	-1	8	-10	-8	2	-1	2	-	0(1)	-	-6
d	1	(-3, 1) (1,-1) (-1,-1)	-	-	2	-10	0(2)	-5	5	8	-2	-	3	-	-1
e	1	(1,-1) (3,-7) (0, 2)	-	-	-	-	-14	-35	18	9	0(3)	-	6	-	-14
f	1	(2, 0) (-1, 1) (-7,-3)	-	-	-	-	-	-	3	24	0(7)	-	6	-	0(3)
g	1	(1,-3) (0, 2) (3, 1)	-	-	-	-	-	-	-	-	0(9)	-	0(1)	-	6
h	4	(2, 0) (0,-2) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-
i	4	(0,-2) (0, 2) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-
$O6$	2	(2, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-

2. Model SU(5)-III

D6-brane configurations and intersection numbers on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^{10} \times USp(8)$, and the complex structure parameters are $\chi_1 = 2/\sqrt{7}$, $\chi_2 = 2/\sqrt{7}$, and $\chi_3 = 2\sqrt{7}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q + 8)$, $a = 32$, and $m = 8$.

Table LXI. D6-brane configurations and intersection numbers for the Model SU(5)-III on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^{10} \times USp(8)$.

stk	N	(n_1,l_1) (n_2,l_2) (n_3,l_3)	A	S	b	b'	c	c'
a	10	(1, 3) (1, 1) (1,-1)	3	0	-3	0(2)	-2	-2
b	2	(1,-3) (1, 3) (2, 0)	0	0	-	-	3	0(1)
c	2	(0, 2) (1,-3) (3,-1)	0	0	-	-	-	-
d	2	(1,-3) (1,-1) (1, 1)	-3	0	-	-	-	-
e	2	(-3,11) (-3,-11) (2, 0)	0	0	-	-	-	-
f	2	(1,-3) (-2, 0) (-7,-3)	-9	9	-	-	-	-
g	2	(2, 0) (1,-3) (7, 3)	-9	9	-	-	-	-
h	2	(-3,-7) (-3, 7) (2, 0)	0	0	-	-	-	-
i	2	(0, 2) (-3,-7) (7, 3)	0	0	-	-	-	-
j	2	(-1, 7) (-1, 1) (0, 2)	-3	3	-	-	-	-
k	2	(1, 1) (1, 7) (0,-2)	-3	3	-	-	-	-
$O6^2$	8	(2, 0) (0,-2) (0, 2)	-	-	-	-	-	-

stk	N	(n_1,l_1) (n_2,l_2) (n_3,l_3)	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	j	j'	k	k'	$O6^2$
a	10	(1, 3) (1, 1) (1,-1)	3	0	-40	-7	15	0	30	-6	5	-16	-10	-10	5	6	3	8	3
b	2	(1,-3) (1, 3) (2, 0)	0	3	0	0	0	27	-27	0	0	0	3	24	8	10	-8	-10	-6
c	2	(0, 2) (1,-3) (3,-1)	-2	2	-40	3	-24	3	0	6	-3	-24	0	0	-3	6	15	6	6
d	2	(1,-3) (1,-1) (1, 1)	-	-	7	40	0	-15	6	-30	16	-5	10	10	0	-5	-8	-3	-3
e	2	(-3,11) (-3,-11) (2, 0)	-	-	-	-	-33	330	330	33	0	0	54	243	70	-35	-70	-128	-66
f	2	(1,-3) (-2, 0) (-7,-3)	-	-	-	-	-	-	0	189	168	-21	0	147	14	-35	-98	-49	-42
g	2	(2, 0) (1,-3) (7, 3)	-	-	-	-	-	-	-	-	-21	168	0	-42	-49	-98	-35	24	0
h	2	(-3,-7) (-3, 7) (2, 0)	-	-	-	-	-	-	-	-	-	-	189	0	-56	-70	56	70	42
i	2	(0, 2) (-3,-7) (7, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-35	24	-49	-98	-42
j	2	(-1, 7) (-1, 1) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0
k	2	(1, 1) (1, 7) (0,-2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0
$O6^2$	8	(2, 0) (0,-2) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

VITA

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