

ANALYTICAL FOUNDATIONS OF
PHYSICAL SECURITY SYSTEM ASSESSMENT

A Dissertation

by

GREGORY HOWARD GRAVES

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Industrial Engineering

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ABSTRACT

Analytical Foundations of
Physical Security System Assessment. (August 2006)
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Physical security systems are intended to prevent or mitigate potentially catastrophic loss of property or life. Decisions regarding the selection of one system or configuration of resources over another may be viewed as design decisions within a risk theoretic setting. The problem of revealing a clear preference among design alternatives, using only a partial or inexact delineation of event probabilities, is examined.

In this dissertation, an analytical framework for the assessment of the risk associated with a physical security system is presented. Linear programming is used to determine bounds on the expected utility of an alternative, and conditions for the separation of preferences among alternatives are shown. If distinguishable preferences do not exist, techniques to determine what information may help to separate preferences are presented. The linear programming approach leads to identification of vulnerabilities in a security system through an examination of the solution to the dual problem.

Security of a hypothetical military forward operating base is considered as an illustrative example. For two alternative security schemes, the uncertainty inherent in the scenario is represented using probability assessments consisting of bounds on event probabilities and exact probability assignments. Application of the framework reveals no separation of preferences between the alternatives. Examination of the primal and

dual solutions to the linear programming problems, however, reveals insights into information which, if obtained, could lead to a separation of preferences as well as information on vulnerabilities in one of the alternative security postures.

To Wya, one of my foremost sources of certainty.

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I owe much gratitude to my wife, Wya, for carrying more than her fair share, and to my children, Abigail, Sarah, AnnaBeth, and Hope, for their unwavering love, patience, and understanding throughout the process.

I thank my parents for infusing me with the motivation to learn, to excel, and to balance the important things in life.

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TABLE OF CONTENTS

CHAPTER	Page
I	INTRODUCTION 1
	A. Physical Security Principles 3
	1. Asset Identification 3
	2. Threats 4
	3. Threat Evaluation 4
	4. Risk Mitigation 5
	5. Constraints 5
	6. Evaluation of Alternatives 6
	7. Decide, Implement, and Monitor 6
	B. Objectives and Approach 6
	1. Research Objectives 7
	2. Approach 8
	C. Dissertation Organization 9
II	LITERATURE REVIEW 10
	A. Modeling of Physical Security, Antiterrorism, and Force Protection Systems 10
	B. Decisions in the Absence of a Unique Probability Measure 13
	C. Security Decision Training: Fields and Needs 16
III	RISK ASSESSMENT OF SECURITY SYSTEMS 20
	A. The Expected Utility Theorem 20
	B. Separation of Risk Preferences Using Sets of Probability Laws 23
	C. Physical Security Decisions 25
IV	ASSESSMENT OF PHYSICAL SECURITY SYSTEMS 29
	A. Loss Uncertainty 30
	B. Expected Utility of Alternatives 32
	C. Alternatives Having the Most Preferred Risk 35
	D. Linear Programming Formulation 37
	E. Dual Problem Solutions 40
	1. Bounds on Event Probabilities 41
	2. Bounds on Conditional Probabilities 43

CHAPTER	Page
F. Independence	45
V AN APPLICATION IN A MILITARY SCENARIO	48
A. Background and Scenario Setting	48
B. Intelligence Assessment	51
C. Description of Alternatives	53
D. Determining Sets of Distributions on Loss	53
E. Analysis of Results	57
1. Insight from the Dual Solutions	57
a. Dual Values as Shadow Prices	57
b. Shadow Pricing for Conditional Probabilities	58
2. Consistency and the Primal Solutions	63
VI CONCLUSION	66
A. Future Research Areas	67
1. Problem Description	68
2. Formulation of Bilinear Programs	69
B. Concluding Remarks	71
1. Motivation and Flow of Research	71
2. Further Areas for Application	73
REFERENCES	75
APPENDIX A	84
APPENDIX B	87
APPENDIX C	91
APPENDIX D	92
VITA	100

LIST OF TABLES

TABLE		Page
I	Scenario Attack Types	52
II	Sector Values (in \$ thousands)	54
III	Possible Values of L_i	54
IV	Conditional Probability Constraint Shadow Prices	62
V	Probability of Destruction of Sector 3	64

LIST OF FIGURES

FIGURE		Page
1	Physical Security Design Process	7
2	Perimeter of Forward Operating Base Amaan	50
3	Sectors of FOB Amaan	51
4	Plot of Utility Function	56
5	Boundaries of a Utility Function	69

CHAPTER I

INTRODUCTION

Physical security is certainly not a new concept. The idea of protecting cities through the construction of fortifications dates back thousands of years. Following her excavation of Jericho and analysis of the fortifications and artifacts located there, Kenyon [1] found that the earliest walls and towers of that ancient city dated prior to 6000 B.C. However, a change in the way in which houses were built within the walls indicates that the first occupants may have been conquered. Therefore, some adversary devised a method to defeat the protection offered by those first walls.

The walls of Jericho indicate that as long as mankind has been protecting people and property, threats from adversaries have existed as a motivation to provide protection. As threats change, so must the safeguards. For the citizens of the United States, the events of September 11, 2001 came as a shocking announcement that the threats against the American people had changed. Significant threats were recognized to exist on American soil, and these threats affect civilians, military forces, and law enforcement agencies. Questions regarding the balance of civil liberties with security now arise. Key concerns in this debate are: 1) the cost of security, and 2) the value of inconvenience that people must tolerate. Physical security has emerged as a pressing social concern.

In response to this new emphasis on physical security, government agencies, industries, and businesses are dedicating considerable resources to improving security. The creation of the Department of Homeland Security stands as the largest reorganization within the government of the United States since the creation of the Depart-

The journal model is *IEEE Transactions on Automatic Control*.

ment of Defense in 1947. [2] Transportation security is being overhauled to address requirements and procedures to protect not only passengers and cargo but also terminals and ports. City governments in the United States are spending in excess of \$70 million per week on security. [2]

In addition to the need for security systems, the need for security education and certification is growing. ASIS International provides professional certification in the security industry.¹ While a process for certification by ASIS International as a Certified Protection Professional has existed since 1977, new programs for certification as Professional Certified Investigator and Physical Security Professional were launched in 2002. [3] Prior to 2001, there were no security management or risk management programs in higher education. [4] As of 2006, 21 colleges and universities offer degrees in areas such as security management, risk management, security administration, security systems, and security and loss prevention. [5] Of these, eight are graduate degrees. While these educational programs target the security industry, other sectors have voiced a need for security education. This need will be discussed further in Chapter II.

In addition to the degree programs addressing security, the opportunities to apply operations research methods to security problems are growing. Optimization methods and decision theory lend themselves naturally to application in security resource allocation. It is in this vein that the research reported in this dissertation has been performed.

The research reported here is focused on applying operations research methods to the analysis and evaluation of physical security systems. The remainder of this introductory chapter is divided into three sections. First, we discuss physical security

¹Numerous other organizations offer certification opportunities for information technology security.

principles. Second, we establish research objectives and the approach that is used to accomplish them. Finally, we give the organization of the dissertation.

A. Physical Security Principles

Before introducing an analytical approach for analyzing and evaluating physical security systems, we present an overview of the process of designing physical security systems. This provides the context for the evaluation problem. While these design principles may be applied to security in general, they have been drawn from sources whose focus is on physical security. These sources include Sandia National Laboratories [6], the United States Army [7], and texts of Garcia [8] and Fischer and Green [2].

1. Asset Identification

The primary purpose of a physical security system is the protection of an asset or a set of assets. These assets can include resources, personnel, facilities, homes, locations, or other items of value. The identification of the assets to be protected and their value in turn reveals other items that must be considered such as the environment and threats. Additionally, specificity in identifying assets ensures that the scope of a protective system is not too broad or too narrow. Proper determination of scope seeks to prevent the unnecessary commitment of resources to protection and leaving items vulnerable that require additional protection. The identification of assets establishes the purpose of the protective system.

Included with the identification of assets is the characterization of asset environment. If assets are materials or resources, the environmental characterization may include a description of a facility in which assets are located together with opera-

tional aspects of that facility. Protection of a facility requires an examination of the operating policies and procedures for the facility and its tenants. Garcia [8] provides a discussion of additional considerations which should be addressed when characterizing a facility. A thorough analysis of the environment facilitates the identification of threats which is the next principle.

2. Threats

After identifying assets and environment, threats must be identified. Some considerations used to identify threats are motivations for attacking assets or goals to be achieved through an attack. Information about a potential threat should include the type of threat, capabilities of potential intruders, and tactics commonly used by intruders. Information about a threat should be specific so as to allow for both the assessment of potential damage and the identification of techniques to counter threats.

3. Threat Evaluation

Once threats are identified, the vulnerability of assets can be investigated through the performance of a threat evaluation or assessment. A threat evaluation requires the analysis of the potential threat actions. These threat actions are often characterized in terms of “consequences” and “likelihood.” [9] Consequences of an action must be determined by some measure of value, and likelihood is assessed using probability. Diagrams of these assessments of threat actions, known as “risk maps” (see Scandizzo [10]), can be helpful in identifying threats that pose high risk.² These threats

²In this discussion of physical security principles, we use the word “risk” as defined by Smith, Barrett, and Box [11] to mean “uncertain consequences, and in particular exposure to potentially unfavorable circumstances, or the possibility of incurring nontrivial loss.” This usage is in contrast to the concept of the risk as a distribution function of the reward associated with an alternative as noted in Wortman and Park. [12]

will require attention when identifying procedures to mitigate risk.

4. Risk Mitigation

Once threats are identified, potential countermeasures can be identified to mitigate risk. Garcia [8] categorizes countermeasures or safeguards according to three primary functions: detect, defend, or respond. Detection is the identification of an ongoing or imminent intrusion. Defense can be either the shielding of assets from damage or the delay of an adversary through a physical barrier or obscurity. Response involves action to interdict an intruder. Deterrence, although not a direct countermeasure, can be a by-product of safeguards and may reduce the likelihood of attacks by certain adversaries. This can be addressed when alternative systems are evaluated.³

5. Constraints

Certain constraints will affect the development of alternative physical security systems; principal among them are resource constraints. Such constraints are typically financial and reflect the price that the decision maker is willing to pay for a security system.⁴ Other constraints that may arise are regulatory, legal, and conformance to operational needs.

³Fischer and Green [2] consider the transfer of risk through the purchase of some type of insurance as a way to mitigate risk. Insurance provides for the possible replacement of an item that is lost or damaged. Not only may replacement not be possible for unique or rare valuables, the prevention of loss or damage is the purpose of a physical security system.

⁴In military tactical security operations, this constraint is primarily in terms of the forces available to engage in the operations.

6. Evaluation of Alternatives

After determining the constraints to which alternatives configurations must conform, feasible alternatives can be identified. In the case of existing systems (and the case of no current system), the *status quo* may be a feasible alternative. If a set of feasible alternatives is known, other feasible alternatives may be constructed through the synthesis of two or more alternatives. Once alternatives are identified, they must be evaluated with respect to the protection they provide against the identified threats and how they mitigate risk. This evaluation should provide a criterion by which alternatives may be compared.

7. Decide, Implement, and Monitor

Evaluation of the alternatives provides the basis for a decision to select a security system. The decision process will be discussed further in the following chapters and so is not detailed here. Once a system is selected, it must be implemented in accordance with its design. Additionally, the system must be monitored for performance and reliability, and information concerning new and existing threats must be periodically updated to determine whether modifications to individual safeguards or to the system as a whole are warranted. With this in mind, principles two through seven serve as an assessment cycle that should be included in security operations, not just in system design. This process is illustrated in Figure 1.

B. Objectives and Approach

The goal of this research is to apply operations research techniques to the decision of selecting from alternative physical security systems and, through the use of these techniques, to gain insights into the primary factors affecting the decision.

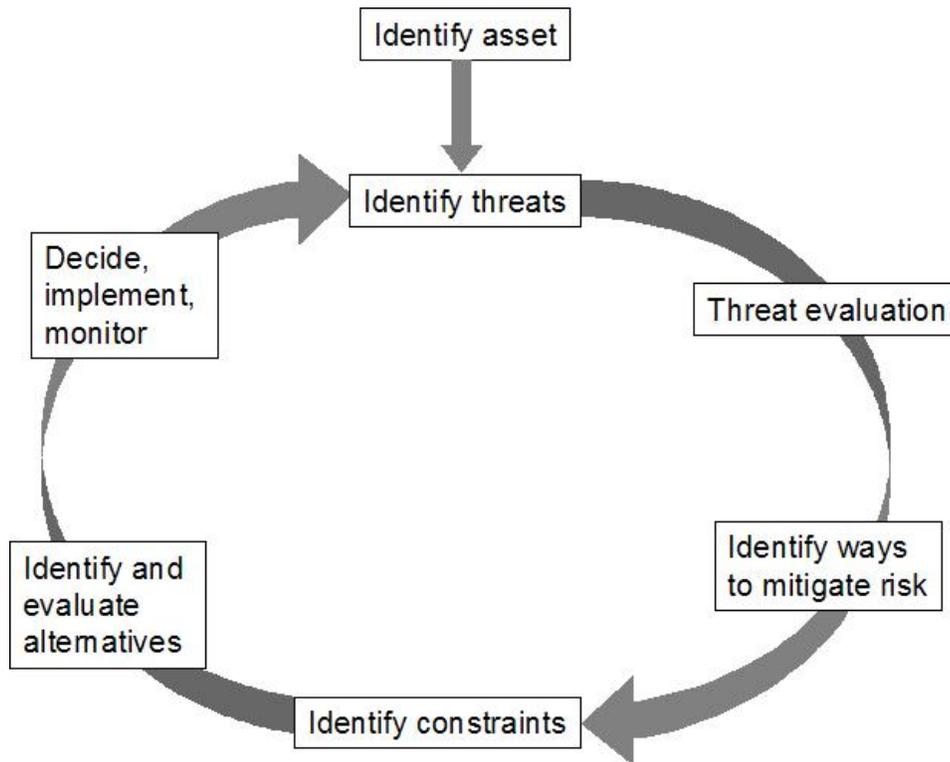


Fig. 1. Physical Security Design Process

Conventional decision problems under conditions of uncertainty require a specified probability measure on the σ -algebra generated by a set of atomic events or outcomes. In this research, we consider the problem where assessments of event probabilities do not lead to a unique probability measure. The research objectives and the approach used to accomplish them are presented here.

1. Research Objectives

We seek to develop an analytical framework for assessment of physical security systems requiring characterization of the risk associated with alternatives. Additionally, the framework must enable the comparison of the risk associated with each alternative design. This approach will not be a prescriptive model dictating how

resources should be configured. Hence, our model is not intended to form the basis of a decision support system. Instead, the model will allow insight into the factors affecting the selection or non-selection of one system over another. It will provide feedback concerning the consistency of the decision maker and the ordering of his preferences. The model is therefore directed toward use in training decision makers.

In developing a model to examine preferences in decisions regarding physical security, we address three primary objectives:

1. Identify an objective function for the decision which orders preferences for the decision maker.
2. Determine conditions for the separation of preferences without complete characterization of probability law.
3. Identify insights available to the decision maker through interpretation of aspects of the model structure.

2. Approach

The assessment of physical security systems requires addressing risk encumbered decisions; hence, risk theory provides a suitable modeling framework. As Wortman and Park [12] assert, risk is defined as “the (cumulative) probability distribution of reward associated with a particular decision alternative.” Thus, to assess the risk associated with an alternative physical security system, our model represents the consequences of threat actions in terms of a random variable representing reward. We characterize atomic events using random variables representing magnitude of loss to the assets and specific types of threat actions, and we present a form of the distribution function on reward in terms of the probabilities of these atomic events. We use this form of the distribution function to develop an expression to calculate expected utility.

We then characterize a set of distribution functions on reward using assessments of probabilities of threat actions and loss. These assessments may be specific probability values, bounds on event probabilities, or other restrictions on the distribution. These assessments are shown to be linear constraints in terms of the probabilities of the atomic events. Using the expression for expected utility as an objective function along with the constraints reflecting the probability assessments, we formulate linear programs to determine bounds on expected utility. Using these bounds, we may compare the risks associated with alternative systems and determine whether a separation of preferences exists. Finally, we show that the primal and dual solutions to the linear programs provide insights regarding consistency, identification of threat actions about which additional information should be pursued, and areas of vulnerability.

C. Dissertation Organization

The remainder of this dissertation is organized in five additional chapters. Chapter II presents a review of literature of physical security. Chapter III provides a discussion of risk theory including the Expected Utility Theorem and the application of risk theory to physical security decisions. Development of our analytical framework is given in Chapter IV. In Chapter V, a hypothetical military scenario is explored. Finally, Chapter VI considers an extension of the model along with concluding remarks.

CHAPTER II

LITERATURE REVIEW

In reviewing literature related to assessment of physical security systems, we consider three primary areas of research. Initially, we examine the design, evaluation, and selection of security systems to include physical security systems, antiterrorism countermeasures, and military force protection efforts. Second, research concerning decisions without unique probability measures on the state space is surveyed. Finally, we review documented areas for security decision training along with identified needs for such training.

A. Modeling of Physical Security, Antiterrorism, and Force Protection Systems

While physical security systems have received renewed interest since 2001, this area is mature. Garcia [8] gives an integrated approach to designing physical security systems. Of particular note are the chapters on evaluation and analysis of protective systems as well as risk assessment. A cost-effectiveness approach is presented, and the measure of effectiveness employed for a physical protection system is the probability of interruption which is defined as “the cumulative probability of detection from the start of an adversary path to the point determined by the time available for response.”

Hicks et al. [13] present a cost and performance analysis for physical protection systems at the design stage. Their system-level performance measure is risk which they define as follows.

$$\text{Risk} = P(A) \times [1 - P(E)] \times C$$

where, $P(A)$ is Probability of Attack

$P(E)$, Probability of System Effectiveness,

$$= P(I) \times P(N),$$

$P(I)$ is Probability of Interruption,

$P(N)$ is Probability of Neutralization,

C is Consequence.

Their discussion of the cost-performance tradeoff is limited and heavily weighted toward cost as a driver in the decision.

Doyon [14] presents a probabilistic network model for a system consisting of guards, sensors, and barriers. He determines analytic representations for determining probabilities of intruder apprehension in different zones between site entry and a target object. Fischer and Green [2] present a very subjective risk analysis approach to ranking threats using a probability/criticality/vulnerability matrix. Cost-effectiveness is discussed as a possible measure of system evaluation.

Schneider and Grassie [15] and Grassie, Johnson, and Schneider [16] present a methodology in which countermeasures are developed in response to asset-specific vulnerabilities. They discuss issues relating to cost-effectiveness tradeoffs individual countermeasures, but fail to give an overall security system evaluation scheme. They do allow for a “system level impression of overall cost and effectiveness” created by considering the interaction of the selected countermeasures.

A small subset of the literature examined presented operations research techniques applied to analysis of physical security systems. Kobza and Jacobson [17] and Jacobson et al. [18] have presented probability models for access security systems with particular applications to aviation security. They are particularly concerned with false clear and false alarm signals. They formulate an optimization problem to determine the minimum false alarm rate for a system with a pre-specified false clear standard.

In light of recent world events, much emphasis has been given to modeling secu-

rity systems for antiterrorism. Bier and Abhichandani [19] examine the problem of defending simple series or parallel systems of components against an intelligent adversary. They present an approach based on game theory and consider the cases where the defender has resource constraints or is unconstrained. In considering series systems, they also differentiate between cases where the attacker has perfect knowledge of the system's defenses or no prior knowledge of the defensive configuration.

In developing a terrorism vulnerability assessment tool, Sher and Guryan [20] present a network approach to site security. They consider a set of resources with fixed locations with the objective of determining the maximum probability of an intruder reaching or damaging potential targets within the site. They transform the problem to a shortest path problem and apply proven methods to solve it.

Wagner[21] relates the physical security design methodology developed by Sandia National Laboratories to a United States Port of Entry in an effort to enhance border security. Hinman and Hammond [22] examine the bombing of the Alfred P. Murrah federal office building in Oklahoma City and present defensive design principles for new and existing structures.

Finally, few examples exist in the literature of analytical models of force protection scenarios or systems. Peck [23] [24] and Peck and Lacombe [25] have explored unattended ground sensors with regard to their employment as part of an intrusion detection system in a force protection role for base camps. They examine environmental effects on system performance and are currently developing a decision aid for sensor selection based on environmental conditions.

Cowdale and Lithgow [26] discuss combining the employment of simulation and geographic information systems (GIS) in the development of force protection planning aids. They describe several tools which may be used by analysts in support of planning decisions for future force protection operations.

To summarize the state of the literature in this area, much effort has been placed on analyzing security needs and countermeasures. Analytical methods have been developed for security decision making and for designing and evaluating security systems. However, techniques given for making physical security decisions have primarily been incomplete or subjective. Moreover, risk (correctly defined) has not been used as a performance measure for physical security systems and has not been incorporated into an expected utility approach for physical security decisions.

B. Decisions in the Absence of a Unique Probability Measure

To address the aspect of the problem where one does not wish to or is unable to assign a unique probability measure to the underlying measurable space, we consider literature dealing with making decisions in situations where the assessment of probabilities does not yield a unique probability distribution on the sample space of possible outcomes. We use the rational decision criterion of maximizing expected utility in the tradition of Bernoulli, de Finetti, and von Neumann and Morgenstern [27] and subsequently expounded upon by Savage [28], Good [29][30], Smith [31], and Fishburn [32] among others.

Although the determination of expected utility depends on a probability measure over the sample space of possible outcomes, the issue of the selection of a unique probability measure by the decision maker which represents his degree of belief regarding the “state of the world” has been discussed and examined by various authors. Good [29] gives the following summary of the issue involving imprecision in decision problems:

Because of a lack of precision in our judgment of probabilities, utilities, expected utilities and “weights of evidence” we may often find that there

is nothing to choose between alternative courses of action, i.e., we may not be able to say which of them has the larger expected utility. Both courses of action may be reasonable and a decision may then be arrived at by the operations known as “making up one’s mind”.

Good further allows that a probability as a degree of belief is usually imprecise and may be viewed as lying in some interval.

Smith [31] discusses the use of upper and lower personal odds and the corresponding upper and lower probabilities. He denotes any value lying in the interval bounded by upper and lower probabilities as a medial probability. Halpern [33] presents a measure theoretic approach using upper and lower probabilities in which the axioms of probability are preserved. He includes discussions of properties of sets of probability measures as well as aspects of decision theory.

Fishburn [34][35] considers the selection of a strategy when the probability distribution on the possible states of nature is imprecise. He provides methods for determining point estimates for the probabilities in the cases where 1) ordering the probabilities is possible, 2) inequalities which relate the probabilities exist, and 3) bounds exist on the probabilities. He also gives conditions for dominance of one strategy over another when utilities of consequences (or “values” in Fishburn’s terminology) can be ordered or bounded and when probabilities can be ordered, bounded, or expressed as linear inequalities in terms of other probabilities. A comparative approach is mentioned by Fishburn et al. [36]

White[37] posits that statements of likelihood or preference are representable by sets of linear inequalities on probabilities or utilities. He discusses their use on an extension of decision analysis known as imprecisely specified multiattribute utility theory.

Yager and Kreinovich [38] provide a method for decisions when intervals for probabilities are given. They use an averaging procedure to determine a single probability measure that is consistent with all given intervals. They do not, however, assert that this method produces optimal decisions in all cases.

Danielson and Ekenberg [39] consider decision situations where statements concerning probabilities and value measures are represented by linear inequalities or intervals. When such restrictions fail to render a unique recommendation for a course of action, they recommend an additional decision criterion based on the strength of one alternative compared to another. They then present conditions under which the resulting quadratic and bilinear programming problems required to select the alternative with the maximal strength can be solved via linear programming algorithms.

An approach to handling the situation when assessment of event probabilities does not lead to the complete characterization of a probability distribution is that of using the maximum entropy distribution which has been applied in the area of decision analysis (see Bickel and Smith [40]). Originally credited to Jaynes [41], the principle of maximum entropy states that “when we make inferences based on incomplete information, we should draw them from that probability distribution that has the maximum entropy permitted by the information we do have.” [42] Entropy is described as a criterion for the “amount of uncertainty” represented by a probability distribution. An entropy method for obtaining joint probability distributions from probability assessments is discussed by Abbas [43]. Good [44] argues that “the principle of maximum entropy is intended only as a heuristic principle” and that the use of entropy involves a “degree of arbitrariness.” We do not use this approach in this dissertation.

Finally, Lowell [45] provides an approach to reducing the number of probability assessments required in a decision analysis scenario by examining the sensitivity of a

decision to the probabilistic dependence between uncertainties in the decision model. He provides a mathematical programming formulation for the problem of determining whether one alternative is preferred over all others when the joint probability distribution over the set of uncertainties is known to lie in a set defined by probability assessments. In his approach, however, the expected utility of the alternatives being considered is computed using a common measurable space for all alternatives. This condition is not required in the approach taken in this dissertation.

C. Security Decision Training: Fields and Needs

The management of security operations or systems requires the ability to decide when a system of countermeasures is adequate or when it should be changed. In this section we discuss areas in which a need for education or expertise in aspects of these types of decisions has been documented or implied in research.

While the focus of this research is physical security, the area of information security has received the most attention in recent years. The principles of physical security apply to computer security as well, and lessons learned from this industry may be applied to physical security in many cases. Similarly, many needs for educating students in computer security can be applied to physical security as well. Barnett [46] addresses the state of computer security education, the skills that should be included in this education, and areas in which industry can facilitate computer security education. He discusses the need for foundational training “to provide the student with the knowledge of tools and techniques to characterize and manage risk.” He also asserts a need for analytical skills and practice in computer security design, evaluation, and engineering. These skills and areas of training would serve professionals in other security fields as well.

Martin [47] and Van Brabant [48] discuss issues in security training for humanitarian aid workers and non-governmental organizations (NGOs). Martin presents the idea of a security triangle consisting of acceptance, protection, and deterrence which must be balanced at the local field office level. The protection portion of the triangle correlates to the concepts of physical security. His decision making rule for security is clearly stated. “It is a matter of identifying what security threats are of the highest probability and greatest consequence to an NGO’s operations and prioritising resources to these threats accordingly.” Van Brabant discusses the need for operators in the field to have good judgement in the area of security management.¹ He puts forth needs which a curriculum in security management should fulfill. The advantage of such a course would be “to develop the analytical, judgmental, and decision-making skills of people with an operational management responsibility for security.”

Tzannatos [50] discusses security requirements in shipping in light of the International Ship and Port Facility Security (ISPS) Code which was enacted in 2003. One of the major requirements of the code is a multi-level security plan for ships and ports.² To enable the development of security plans, he proposes a decision support system (DSS) which uses a risk management methodology. However, he allows that “it usually takes a specialized security expert to determine the vulnerabilities of a threatened ship or port.” At the same time, the DSS he proposes requires that all known vulnerabilities and all existing countermeasures be included in a vulnerability assessment which results in a subjective assessment of residual vulnerability. Additionally, the

¹The development of judgement in decision makers is a key aspect of naturalistic decision making (NDM) which is a descriptive decision theory used by the U.S. Army in decision making education. In describing how to train decision makers under the NDM framework, Klein and Wolf [49] give four strategies: (1) build expertise, rather than teaching generic analytical strategies, (2) support, rather than replace, the strategies people use, (3) make the decision requirements specific to the task context, and (4) model the cognitive processes of subject-matter experts. The last strategy involves attempting to capture the judgement used by experts in decisions so that less experienced decision makers can benefit from it.

²*Multi-level* refers to the ability to handle different security levels, i.e. likelihoods of specific threats attacking. The ISPS defined security levels are normal, elevated, and exceptional.

final step of using the proposed DSS requires the development of multi-level plans to handle the scenarios which pose the most significant risk as determined by the use of the DSS. Both the assessment of vulnerabilities and the development of plans to handle security scenarios would require training or education in security decisions.

A study of security management practices at universities in the United Kingdom by Baron and van Zwanenberg [51] revealed a lack of rational decision making processes by those managers responsible for campus security operations. After studying the indicators used by the managers to indicate the need for security services and computing the correlations that these indicators had with campus crime, the authors made this telling statement:

We have a picture, then, where the inputs to decision making are probably unreliable as influencing factors, and where the outputs of the decisions seem not to be related to the factors which are believed to be shaping them.

Moreover, the authors note a lack of an ability on the part of the security managers to quantify “the value given to the reduction of risk of events or any means of measuring any effects of such risk reduction.” In this sense, educating the university security managers in decision making techniques and concepts would provide the basis for rational decisions regarding the provision of campus security services.

This chapter has presented a brief survey of literature in three areas. First, literature relating to design, analysis, and selection of physical security systems was presented and summarized. The second section of the chapter dealt with decision making when a unique probability measure on the set of possible outcomes is not defined. The techniques and results presented in this dissertation are not replicated in any of the literature surveyed. The research presented here makes a contribution

to the fields searched as detailed in the next chapter. Finally, various areas in which security decision education is needed were examined.

CHAPTER III

RISK ASSESSMENT OF SECURITY SYSTEMS

A rational approach to the selection of a physical security system requires an assessment of the merit of each of the alternatives. Since threat actions are not known with certainty, the value received through the selection of an alternative is also uncertain. Since risk is a distribution function on reward, it provides a logical indicator of the desirability of a security system. Thus, the selection of a physical security system from among alternatives implies the identification of the alternative with the most preferred risk.

Determining preferences among distributions on reward is possible through the calculation of expected utility which requires the existence of a utility function. The Expected Utility Theorem guarantees the existence of a utility function provided certain restrictions (or axioms) governing the alternatives' distributions on reward are satisfied. In this chapter, we first review these axioms and the Expected Utility Theorem. We then explore the application of the theorem to decision making in a discussion of risk theory. Finally, we address physical security decisions using risk theory.

A. The Expected Utility Theorem

The concept of utility has its foundations with Bernoulli's treatment of the St. Petersburg Paradox. Its axiomatization and proof are credited to von Neuman and Morgenstern [27] who gave the characterization of the preferences of an individual which enables their representation by a utility function. Fishburn [52] discusses equivalent formulations of the theorem and its axioms. We use that of Puppe [53] modified to conform to our notation.

Without loss of generality, let $\mathcal{X} = [a, b] \subset \mathbb{R}$ be a compact interval containing 0. The set \mathcal{X} can be viewed as the set of possible amounts of reward where a is the maximum possible loss, 0 represents the *status quo*, and b is an upper bound on reward. Let \mathbb{P} be the set of all distribution functions with support contained in \mathcal{X} . The normative axioms underlying the Expected Utility Theorem are:

Axiom 1 (Weak Order) *The relation \succsim is a weak order, i.e. preferences are complete and transitive.*

The implications of the Weak Order Axiom are twofold. First, one abiding by this axiom can compare any two alternatives and state a preference of one over another. This is a necessary condition for making a choice based on preferences. Second, these preferences must be transitive. This condition prevents a decision maker from being taken advantage of via a money pump situation.

Axiom 2 (Continuity) *For every $F \in \mathbb{P}$ the sets $\{G \in \mathbb{P} : G \succsim F\}$ and $\{G \in \mathbb{P} : F \succsim G\}$ are closed in the topology of weak convergence.*

Suppose that F is a distribution function and that $\mathcal{X}_F \subset \mathcal{X}$ is the set of points of continuity of F . The Continuity Axiom ensures that if there exists a sequence of distribution functions $\{F_n\}_{n=1}^{\infty} \subset \mathbb{P}$ which converge pointwise on \mathcal{X}_F to F , then $F \in \mathbb{P}$. Additionally, if for some $G \in \mathbb{P}$, $F_n \succsim G$ for all n , then $F \succsim G$. Likewise, if $G \succsim F_n$ for all n , then $G \succsim F$.

Axiom 3 (Independence) *For all $F, G, H \in \mathbb{P}$ and all $\alpha \in [0, 1]$, $F \succsim G$ implies*

$$\alpha F + (1 - \alpha)H \succsim \alpha G + (1 - \alpha)H.$$

The Independence Axiom requires that preferences not change if an additional option is added to or mixed with the alternatives that are being compared. In the

statement of the axiom, it is the preference of F over G that is independent of the mixture of H with each distribution in identical proportions.

Accepting the axioms of rational preference allows the separation of preferences with a utility function. This result is formally stated as the Expected Utility Theorem.

Theorem 1 (Expected Utility Theorem) *Let \succsim be a binary relation on \mathbb{P} . There exists a continuous function $u : \mathcal{X} \rightarrow \mathbb{R}$ such that the functional*

$$U(F) = \int_a^b u(x) dF(x), \forall F \in \mathbb{P}$$

represents \succsim if and only if \succsim satisfies axioms 1, 2, and 3. Moreover, the function u is unique up to positive affine transformations.

To use the theorem in the identification of the preferred distribution, let \mathcal{A} be the set of alternatives. For an alternative $\alpha \in \mathcal{A}$, let Ω^α be the sample space and \mathcal{F}^α be a σ -algebra on Ω^α . Let $V^\alpha : \Omega^\alpha \rightarrow \mathcal{X}^\alpha \subset \mathbb{R}$ be a random variable representing the value gained through the realization of an outcome. Let P^α be a probability measure on \mathcal{X}^α . Then the distribution function $F^\alpha(x) = P^\alpha(V^\alpha \leq x)$ completely characterizes P^α . Choose \mathcal{X} such that $\cup_{\alpha \in \mathcal{A}} \mathcal{X}^\alpha \subset \mathcal{X}$. Then $\{F^\alpha : \alpha \in \mathcal{A}\} \subset \mathbb{P}$.

Given a utility function u and an alternative $\alpha \in \mathcal{A}$ with distribution function F^α , we denote the expected utility of alternative α as $E_\alpha(U)$. We compute the expected utility,

$$E_\alpha(U) = \int_{\mathcal{X}} u(x) dF^\alpha(x).$$

We may thus represent a preference for one distribution over another using expected utility by

$$F^\alpha \succsim F^\beta \Leftrightarrow E_\alpha(U) > E_\beta(U). \quad (3.1)$$

As a result of the theorem, since a utility function exists and represents the underlying preferences, one may use it to separate preferences between alternatives

through the computation of the expected utility for each alternative. Determining which alternative provides the maximum expected utility is equivalent to finding the alternative, α^* , where

$$\alpha^* = \operatorname{argmax}_{\alpha \in \mathcal{A}} E_{\alpha}(U). \quad (3.2)$$

B. Separation of Risk Preferences Using Sets of Probability Laws

Risk is commonly defined in terms of a set of possible outcomes, the consequences associated with the outcomes, and the outcome probabilities. Simply calculating the expected value of the consequences does not sufficiently capture the meaning of risk. Kaplan and Garrick [9] posit that “it is not the mean of the curve, but the curve itself which is the risk.” The curve to which they refer is the risk curve. By examining their construction of the risk curve, it can be seen that the risk curve provides the same information as the distribution function on the value of the consequences. This interpretation agrees with the definition of risk given in Chapter I.

Identification of the preferred risk, then, involves the selection of a distribution on value or reward that is preferred to other possible distributions. A decision is essentially a wager where the one commits an amount of time and resources in return for the selected risk. This selection of risk is reflected in (3.2) by the fact that the only element that varies on the right side of the equation is the distribution function. When a single probability law can be identified to represent the risk for each alternative, an exhaustive search over all alternatives will identify the preferred alternative using (3.2).

However, for a sample space with a realistic number of possible outcomes, assessment of a probability law can be extremely challenging. Additionally, a person’s perception of a situation can change over time which may in turn require adjustments

to a probability law that has been assessed. Kaplan and Garrick [9] admit to the need for a measure of confidence in the level of risk and suggest constructing a probability distribution over a space of risk curves. Rather than entering the philosophical debate concerning the propriety of probabilities of probabilities, we propose the use of a set of probability laws each of which is a potential representation of the beliefs of the decision maker. Such a set can be associated with each alternative.

For an alternative $\alpha \in \mathcal{A}$, a set of potential distribution functions $\mathcal{P}^\alpha \subset \mathbb{P}$ can be defined by adding any number of additional constraints (to include zero) on the allowable assignments of values to probabilities of outcomes or events beyond the requirement that $\int_{\mathcal{X}} dF^\alpha(x) = 1$. Constraints may include the assignment of a specific value to an event probability, placing bounds on the probability of an event, or the inclusion of conditional probabilities with appropriate assessments or bounds. The inclusion of additional constraints serves to reduce the size of \mathcal{P}^α .

Once such a set is identified for each alternative, we may apply optimization methods to compute bounds on the expected utility for each alternative. For each $\alpha \in \mathcal{A}$, let

$$u_{min}^\alpha = \min_{F \in \mathcal{P}^\alpha} \left\{ \int_{\mathbb{R}} u(x) dF(x) \right\} \quad (3.3)$$

and

$$u_{max}^\alpha = \max_{F \in \mathcal{P}^\alpha} \left\{ \int_{\mathbb{R}} u(x) dF(x) \right\}. \quad (3.4)$$

Wortman and Park [12] show that if alternative β has risk belonging to the set of distribution functions \mathcal{P}^β and $u_{max}^\beta < u_{min}^\alpha$ then alternative α is preferred to alternative β . Thus, in a condition similar to but not as strong as (3.1), we have

$$u_{min}^\alpha > u_{max}^\beta \Rightarrow \mathcal{P}^\alpha \succcurlyeq \mathcal{P}^\beta. \quad (3.5)$$

Hence, even if risk cannot be characterized by a unique probability law, (3.5) gives

the condition for the existence of a preference for one alternative over another if sets of probability laws can be identified for each alternative.

C. Physical Security Decisions

The purpose of physical security systems is “to prevent or detect an attack by a malevolent human adversary.” [8] Implicit in this definition is the idea that there exists some asset which one wishes to protect from attacks or threat actions. The asset may be material, human, a location, a facility, or a combination of these items. Regardless of its composition, the asset possesses some value to a person who would select a security system to protect it.

A physical security system is a configuration of various types of safeguards. Safeguards are resources which may either detect, delay, or respond to a threat action. The selection of a security system, then, has costs in terms of the safeguards which comprise the system both in acquiring and operating the safeguards. In order to select the risk associated with an alternative, one commits resources equal to the present value of these costs.

We assume that a person desiring to protect an asset has finite wealth and, hence, a finite set of safeguards from which to configure a system. In order to determine which safeguards to include in a system, the nature of the possible attacks on the asset must be characterized. We assume that the number of classifications of threats actions against an asset is finite. If a threat action involves a person moving through an area, a motion sensor might be a safeguard to be included in a system. Using a risk assessment approach, threat actions should, at a minimum, be characterized in terms of consequences and likelihood. If the safeguard is of significant value, damage or destruction of the safeguard should be included in the consequence analysis. In

the determination of which safeguards to include, the consequences of the possible threat actions are most important.

Once safeguards are identified to counter threat actions against assets, a configuration of the safeguards must be determined. A configuration includes the quantity of each of the different safeguards selected as well as the location and, if applicable, the orientation of each safeguard. For example, two security cameras might be installed at the same location with each camera oriented to cover different areas. Thus, a physical security system is comprised of a set of safeguards as well as their configuration. Note that several alternative systems might be possible through different configurations of the same set of safeguards.¹

Once an alternative system, say alternative α , is identified, the characterization of the risk associated with alternative α may be captured in terms of a set of probability laws. Here, we require a state space S_α of mutually exclusive outcomes representing possible threat actions and the effects of these actions. We also require a σ -algebra Σ_α of subsets of S_α each of which is an event. Finally, we require a probability measure $P_\alpha : \Sigma_\alpha \rightarrow [0, 1]$. Note that the probability space $(S_\alpha, \Sigma_\alpha, P_\alpha)$ is specific to alternative α .

The effects associated with an outcome may be expressed in terms of reward. Reward is expressed in some acceptable common unit of value. We consider reward to be measured in dollars and thus to take on a discrete set of possible values. Since the combined value of an asset and the physical security system protecting it is finite, the set of possible amounts of reward is also finite. Since reward is to be determined by the realization of an outcome, we define a discrete random variable $R : S_\alpha \rightarrow \mathbb{R}$ to represent reward.

¹This situation is commonly found in military defensive scenarios. The FOB scenario in Chapter V is one such situation.

Since the set of possible threat action classifications is finite and set of possible values of reward is finite, we may define a finite set of disjoint events which represent all possible combinations of threat actions and reward. Let N be the number of possible values of reward and M be the number of classifications of threats. We then define

$$\Omega_\alpha = \{\omega_{ij} : i = 1, \dots, N, j = 1, \dots, M\}.$$

Let $\mathcal{F}_\alpha = 2^{\Omega_\alpha}$, the set of all subsets of Ω_α . Then \mathcal{F}_α is a σ -algebra on S_α , and we assume that $\mathcal{F}_\alpha \subset \Sigma_\alpha$. The events $\{\omega_{ij}\}$ are denoted *atomic events* since any event in \mathcal{F}_α may be expressed as a union of these events.

The probability measure may then be used to assess probabilities of events in \mathcal{F}_α . Of particular interest are the events

$$\{R \leq x\} = \{\omega_{ij} : R(\omega_{ij}) \leq x\}$$

since these events are used to characterize the risk, $F_R(x) = P_\alpha\{R \leq x\}$. These probability assessments may be expressed as either a single value in the interval $[0, 1]$ or by placing bounds on such values. Assessments concerning the effects of a threat action will frequently be recorded using conditional probability distributions such as

$$F_{R|A}(x|k) = P_\alpha(R \leq x|A = k)$$

where $A : S_\alpha \rightarrow \{1, \dots, M\}$ is a random variable representing the classification of threat action associated with an outcome. Other characteristics of the distribution function F_R such as the mean or variance may also be assessed. These assessments will result in the formation of a family of distributions on reward, \mathcal{P}^α . This distribution family contains the risk associated with the alternative under consideration.

After determining the distribution family for each alternative, the conditions

given in section B above may be used to determine the existence of a separation of preferences among alternatives. A modeling approach to this assessment methodology is presented in the next chapter.

CHAPTER IV

ASSESSMENT OF PHYSICAL SECURITY SYSTEMS

In this chapter we present an analytical technique to determine whether one alternative is preferred over another when a unique risk distribution is unavailable. We first characterize atomic events in terms of random variables representing loss to assets and classifications of threat actions. Using the probabilities associated with these atomic events, we develop a characterization of the distribution function for loss. After a transformation to a distribution on reward, we show that each alternative may be characterized by the family of distribution functions on reward that conform to a given set of probability assessments. We then present a method using linear programming formulations to identify a most preferred alternative if one exists. An examination of the optimal primal and dual solutions to the linear programs leads to insights with respect to the bounds on expected utility and potential modifications of alternatives. Finally, a discussion of the effects of any assumptions regarding probabilistic independence of events concludes the chapter.

We first summarize the assessment scenario under consideration. Given assets to be protected and a set of alternative configurations of safeguards with which to protect the assets, the alternatives must be assessed to determine which has the most preferred risk. Available information about threat capabilities and modes of operation are used to classify the possible threat actions against the assets. For each alternative configuration, probabilities regarding threat actions and their effects are assessed. Due to the complexity of the situation, time limitations, or other reasons, the probability assessments do not result in a unique probability law on reward. Bounds on expected utility are computed in order to determine the preference ordering of alternatives if separation of preferences is achieved.

A. Loss Uncertainty

Let \mathcal{A} be the set of alternative configurations of safeguards. For each alternative $\alpha \in \mathcal{A}$, the assets to be protected are partitioned into N sectors. Sectors may be determined by location, by function, or by any other means desired. Sectors may consist of safeguards as well as assets. The personnel, equipment, and functions associated with a sector determine its value. Sectors need not be uniform in size, shape, or value. Based on an assessment of available information about the environment, M possible threat classifications are identified. If a sector incurs damage from a malevolent action, a loss (or decrease in value) results.

Let $(\Omega_\alpha, \Sigma_\alpha, P_\alpha)$ be a probability space where Ω_α is the set of all possible threat actions and effects on the assets. We define a random variable $A^\alpha : \Omega_\alpha \rightarrow \{1, \dots, M\}$ to be the classification of threat action. We define the lattice random variables $L_i^\alpha : \Omega_\alpha \rightarrow \mathbb{R}$ to be the loss in sector i due to a threat action for $i = 1, \dots, N$.¹ Since all sector values are considered to be finite, each L_i^α can assume a finite set of values. Note that although these random variables only take on a finite number of real values, they may map a possibly infinite number of outcomes to this finite set of values. Thus, the state space need not be countable.

Define a class \mathcal{C}^α of random variables as

$$\mathcal{C}^\alpha \triangleq \{A^\alpha, L_1^\alpha, \dots, L_N^\alpha\}.$$

Let $\mathcal{F}_\alpha \triangleq \sigma(\mathcal{C}^\alpha)$. Then Ω_α may be partitioned into a set of *atoms* on which each of the random variables in \mathcal{C}^α is constant.² An *atomic event* is an event consisting of

¹A lattice random variable has support on a lattice $L = \{b + h\Delta : b \in \mathbb{R}, h \in \mathbb{Z}, \Delta \in (0, \infty)\}$. We assume that a common lattice contains the supports of the random variables $L_i^\alpha, i = 1, \dots, N$.

²Williams [54] uses the term “Z-atoms” to denote the elements of a partition of a sample space where each element is a set of outcomes on which a discrete random variable Z is constant. We adopt this similar term for the concept described here.

the outcomes corresponding to one atom. Since loss is a decrease in value, each L_i^α takes on non-positive values. Using a vector $\mathbf{n} = (n_1, n_2, \dots, n_N)$ with all $n_i \geq 0$, we can represent the values that each random variable L_i^α possesses for a given atom. Thus, we denote an atomic event as

$$\{\omega_{(\mathbf{n},k)}\} = \{L_1^\alpha = -n_1, \dots, L_N^\alpha = -n_N, A^\alpha = k\}.$$

Each event in \mathcal{F}_α may be represented as a union of atomic events.

Let L^α represent the total loss to the assets. Thus,

$$L^\alpha = \sum_{i=1}^N L_i^\alpha.$$

Using the ℓ^1 -norm defined as $\|\mathbf{n}\|_1 = \sum_{i=1}^N n_i$, we see that for a specified atomic event, we will have

$$\sum_{i=1}^N L_i^\alpha \leq x$$

only when $\|\mathbf{n}\|_1 \geq -x$. For a given threat action of classification k , we can now write

$$\{\omega \in \Omega_\alpha : L^\alpha(\omega) \leq x, A^\alpha(\omega) = k\} = \bigcup_{\mathbf{n} \in \{\|\mathbf{n}\|_1 \geq -x\}} \{\omega_{(\mathbf{n},k)}\}.$$

We assume that given a threat action occurs, no additional loss to the facility will be incurred other than the loss caused by that action. Therefore, if we consider the conditional distribution of L^α , given $A^\alpha = k$, we see that

$$F_{L^\alpha|A^\alpha}(x|k) = P_\alpha(L^\alpha \leq x | A^\alpha = k).$$

Since the classifications of threat actions are mutually exclusive and collectively ex-

haustive, we can express the distribution function for L^α as

$$\begin{aligned}
 F_{L^\alpha}(x) &= \sum_{k=1}^M F_{L^\alpha|A^\alpha}(x|k) P_\alpha\{A^\alpha = k\} \\
 &= \sum_{k=1}^M P_\alpha\{L^\alpha \leq x, A^\alpha = k\} \\
 &= \sum_{(\mathbf{n},k) \in \{\|\mathbf{n}\|_1 \geq -x, 1 \leq k \leq M\}} p_{(\mathbf{n},k)}^\alpha
 \end{aligned}$$

where $p_{(\mathbf{n},k)}^\alpha \triangleq P_\alpha\{\omega_{(\mathbf{n},k)}\}$.

B. Expected Utility of Alternatives

The reward gained through the operation of a physical security system can be expressed using a value measure such as money. The benefits of owning the protected assets as well as the benefits and costs associated with operating the security system are expressed using the same value measure. This value measure is also used to quantify losses to the assets or to the safeguards comprising the security system. Using such a common unit of value permits the association of some amount of reward with each outcome. Thus, the definition of a distribution on reward is reasonable.

We assume that the reward function $v(x)$, which incorporates both the advantage of possessing the assets and a loss of x units of value due to damage from a threat action, is linear in terms of x . The maximum value for reward due to ownership and protection of the assets is $v(0) = v_0$. This quantity incorporates the value of the assets as well as the value of the security system.

We consider the threat actions which the system may face to have effects which may be considered of finite value. Since there are a finite number of threat action types and each type has a finite possible loss, the total possible loss will be finite. We choose a as the bound on loss. The realization of this loss would give a minimum

value for reward of $v(a) = v_0 + a$.

If we define $R^\alpha = v(L^\alpha) = v_0 + L^\alpha$, then the distribution function on reward, F_{R^α} , is a straightforward transformation using F_{L^α} given by

$$\begin{aligned} F_{R^\alpha}(x) &= P_\alpha\{v(L^\alpha) \leq x\} \\ &= P_\alpha\{L^\alpha \leq x - v_0\} \\ &= F_{L^\alpha}(x - v_0). \end{aligned}$$

Since a joint distribution function of this form induces a probability measure, given a utility function, u , we can compute the expected utility

$$\begin{aligned} E_\alpha(U) &= \int_{\mathbb{R}} u(x) dF_{R^\alpha}(x) \\ &= \int_{v_0+a}^{v_0} u(x) dF_{R^\alpha}(x). \end{aligned}$$

Since F_{R^α} is a monotone function and u is monotone and continuous, we may change measures via integration. Thus

$$E_\alpha(U) = u(x)F_{R^\alpha}(x) \Big|_{v_0+a}^{v_0} - \int_{v_0+a}^{v_0} F_{R^\alpha}(x) du(x).$$

Since $F_{L^\alpha}(a) = 0$, we have $F_{R^\alpha}(v_0 + a) = 0$. By construction, $F_{R^\alpha}(v_0) = 1$. Letting $u_0 = u(v_0)$, we have

$$\begin{aligned} E_\alpha(U) &= u_0 - \int_{v_0+a}^{v_0} F_{R^\alpha}(x) du(x) \\ &= u_0 - \int_{v_0+a}^{v_0} F_{L^\alpha}(x - v_0) du(x). \end{aligned} \tag{4.1}$$

Considering the range on possible loss, since L^α is a lattice random variable, we may discretize the range using sufficiently small subintervals of uniform length, say

Δ , which agrees with the spacing of the lattice containing the support of L^α . Letting

$$H = \frac{a}{\Delta},$$

it follows that

$$\begin{aligned} \int_{v_0+a}^{v_0} F_{L^\alpha}(x - v_0) du(x) &= \sum_{h=1}^H F_{L^\alpha}(a + (h-1)\Delta) [u(a + h\Delta) - u(a + (h-1)\Delta)] \\ &= -u(a)F_{L^\alpha}(a) - \sum_{h=1}^{H-1} u(a + h\Delta) [F_{L^\alpha}(a + h\Delta) \\ &\quad - F_{L^\alpha}(a + (h-1)\Delta)] + u(a + H\Delta)F_{L^\alpha}(a + (H-1)\Delta). \end{aligned}$$

Since $F_{L^\alpha}(a) = 0$, it follows that

$$\begin{aligned} \int_{v_0+a}^{v_0} F_{L^\alpha}(x - v_0) du(x) &= - \sum_{h=1}^{H-1} u(a + h\Delta) P_\alpha(L^\alpha = a + h\Delta) \\ &\quad + u(a + H\Delta) [1 - P_\alpha(L^\alpha = a + H\Delta)] \\ &= u_0 - \sum_{h=1}^H u(a + h\Delta) P_\alpha(L^\alpha = a + h\Delta) \\ &= u_0 - \sum_{h=1}^H u(a + h\Delta) \left(\sum_{(\mathbf{n},k) \in N_h} p_{(\mathbf{n},k)}^\alpha \right) \end{aligned}$$

where

$$N_h = \{(\mathbf{n}, k) : \|\mathbf{n}\|_1 = -a - h\Delta, 1 \leq k \leq M\}.$$

Substituting this result into (4.1), we obtain

$$E_\alpha(U) = \sum_{h=1}^H u(a + h\Delta) \left(\sum_{(\mathbf{n},k) \in N_h} p_{(\mathbf{n},k)}^\alpha \right) \quad (4.2)$$

which gives an expression which may be used to compute the expected utility of each alternative.

C. Alternatives Having the Most Preferred Risk

To determine a preference ordering for a set of alternative physical security systems \mathcal{A} , we require the risk distribution, F_{L^α} , associated with each alternative $\alpha \in \mathcal{A}$. When the unique probability law is obtainable for each alternative, (4.2) gives the value for the expected utility of each alternative. The alternative having the maximum expected utility is the alternative having the most preferred risk.

While methods exist for eliciting distribution functions (see Hampton *et al.* [55] for an analysis of several methods), a complete assessment may not be practical. For instance, in a situation where a large number of event probabilities are required in order to completely characterize the desired distribution function, time constraints may preclude assessing all of the probabilities. Alternatively (and usually), one's perception of uncertainty is limited to information on a set of events which are a subset of those required for delineation of a unique probability law. The result of the assessment of this reduced number of event probabilities is that many distribution functions may agree with this limited perception. The assessed probabilities serve as constraints which characterize this set of distribution functions.

For each alternative $\alpha \in \mathcal{A}$, let \mathcal{P}^α be a set of distribution functions that capture information on uncertainty regarding the protection offered by alternative α . Then the expected utility of alternative α lies in the interval $I^\alpha = [u_{min}^\alpha, u_{max}^\alpha]$ where u_{min}^α and u_{max}^α are defined in (3.3) and (3.4). These intervals may be compared to determine separation of preferences as shown by Wortman and Park [12].

An interval graph $G = (V, E)$ may be constructed using the set $\{I^\alpha : \alpha \in \mathcal{A}\}$ by associating a vertex $v_\alpha \in V$ with each interval I^α and having an edge $e_{\alpha\beta} \in E$ if and only if $I^\alpha \cap I^\beta$ is not empty. Let $\alpha^* = \arg \max_\alpha \{u_{min}^\alpha\}$.³ Then I^{α^*} is the interval

³Note that α^* is not necessarily a unique alternative. Even if it is not unique, Lemma 1 and Corollary

with the greatest lower bound.

Lemma 1 (Wortman and Park [12]) *With $G = (V, E)$ and $\{I^\alpha : \alpha \in \mathcal{A}\}$ defined as above, v_{α^*} belongs to a maximal clique and is adjacent to no other vertices.*

Proof. Let $V_{\alpha^*} \subset V$ be the set of vertices adjacent to v_{α^*} , and let S be the index set of V_{α^*} . Then $\beta = \arg \min_{\gamma \in S} \{u_{max}^\gamma\}$ is well defined, and $\beta > u_{min}^{\alpha^*}$. Hence, $[u_{min}^{\alpha^*}, \beta] \subset I^\gamma, \forall \gamma \in S$. Thus, $\bigcap_{\gamma \in S} I^\gamma \neq \emptyset$, and by the definition of an interval graph, the vertices in V_{α^*} are mutually adjacent and thus belong to a clique. Since v_{α^*} is adjacent to no vertex outside of V_{α^*} , there can be no clique of G that properly contains the clique to which V_{α^*} belongs. Thus, V_{α^*} is the vertex set of a maximal clique. \square

The application of this lemma to the comparison of sets of distribution functions or distribution families is given by the following corollary from Wortman and Park [12].

Corollary 1 (Separation of preferences via distribution families)

1. *Any alternative with risk belonging to the family of distribution functions \mathcal{P}^β with $u_{max}^\beta < u_{min}^{\alpha^*}$ is less preferable than any alternative having risk belonging to the family \mathcal{P}^{α^*} .*
2. *Any alternative β for which $u_{max}^\beta \geq u_{min}^{\alpha^*}$ is indistinguishable from the most preferred alternative.*

As a result of this corollary, a comparison methodology for a set \mathcal{A} of alternative physical security systems to determine the existence of an alternative with the most preferred risk is as follows:

¹ hold for each alternative with $u_{min}^\alpha = u_{min}^{\alpha^*}$. Moreover, a preference between such alternatives cannot be determined as shown in Part 2 of Corollary 1.

1. For each alternative $\alpha \in \mathcal{A}$,
 - a. Determine sectors and threat classifications.
 - b. Assess probabilities reflecting the perception of protection offered by the alternative.
 - c. Compute bounds on expected utility.
2. Determine the alternative α^* with the greatest lower bound on expected utility, $u_{min}^{\alpha^*}$.
3. Compare the upper bounds of all other alternatives with $u_{min}^{\alpha^*}$.

Alternatively, any alternative whose upper bound on expected utility is exceeded by the lower bound of another alternative cannot have the most preferred risk. Using this principle, even if a single most preferred alternative cannot be identified, it is possible to eliminate inferior alternatives and thus cull a set of most preferred alternatives from the original set of alternatives.

D. Linear Programming Formulation

Computing the bounds on expected utility for an alternative may be accomplished via linear programming. Constraints restricting the set of distribution functions \mathcal{P}^α may be specific probability values such as $P_\alpha\{A^\alpha = 2\} = 0.3$. Other constraints may be bounds such as

$$0.2 \leq P_\alpha\{L_1^\alpha \leq 7, A = 3\} \leq 0.4$$

or ordinal relationships such as

$$P_\alpha\{L_2^\alpha = 7, A = 3\} \leq 3P_\alpha\{L_1 \leq 7, A = 3\}.$$

Further constraints might include a specified mean or other characteristic of the distribution.

Since L^α is a discrete random variable, a potential probability mass function can be represented by an assignment of values to each of the $p_{(\mathbf{n},k)}^\alpha$. The sum of all of the atomic event probabilities must equal one, and each $p_{(\mathbf{n},k)}^\alpha$ must lie in the interval $[0, 1]$ in order to describe a probability law. Let D be the number of possible vectors (\mathbf{n}, k) . Then each potential probability mass function can be mapped to a point in the hypercube $[0, 1]^D$ and on the hyperplane defined by

$$\sum_{\mathbf{n},k} p_{(\mathbf{n},k)}^\alpha = 1. \quad (4.3)$$

Additionally, the event probabilities in the constraints on \mathcal{P}^α can be represented as sums of the atomic event probabilities. Therefore, any constraint can be written in the form

$$\sum_{\mathbf{n},k} a_{(\mathbf{n},k)} p_{(\mathbf{n},k)}^\alpha \leq b \quad (4.4)$$

where each $a_{(\mathbf{n},k)}$ is an appropriate real coefficient and b is a constant.

Constraints obtained from bounds on conditional probabilities can also be expressed as sums and will always have a right-hand-side value of zero. Suppose that, for events E and F , the assessment $P_\alpha(E|F) \leq c$ is given for some $c \in [0, 1]$. To create a linear constraint, we must transform the constraint to

$$P_\alpha(E \cap F) - cP_\alpha(F) \leq 0.$$

Since any element in $E \cap F$ is also in F , we rewrite this as

$$(1 - c)P_\alpha(E \cap F) - cP_\alpha(F \setminus E) \leq 0$$

in order to obtain disjoint events. This constraint may now be written as

$$\sum_{(\mathbf{n},k) \in I_1} (1-c)p_{(\mathbf{n},k)}^\alpha + \sum_{(\mathbf{n},k) \in I_2} (-c)p_{(\mathbf{n},k)}^\alpha \leq 0$$

where

$$I_1 = \{(\mathbf{n}, k) : \omega_{(\mathbf{n},k)} \in E \cap F\}$$

and

$$I_2 = \{(\mathbf{n}, k) : \omega_{(\mathbf{n},k)} \in F \setminus E\}.$$

Since each constraint can be written in the form given by (4.4), each constraint represents a halfspace. Let \mathcal{H}^α be the intersection of these halfspaces with the hyperplane given by (4.3). Then \mathcal{H}^α is a polyhedral set and is thus convex. Since (4.3) ensures that this set is bounded, \mathcal{H}^α is a polytope representing all possible probability mass functions corresponding to distribution functions in the set \mathcal{P}^α . The addition of more constraints will cut away additional regions of the polytope making the feasible region smaller.

If the constraints reduce the polytope to a single point, then \mathcal{P}^α is reduced to a unique probability law corresponding to the probability mass function defined by the coordinates of that point. These coordinates are the probabilities of the atomic events. It is also possible that the combination of constraints will create an empty feasible region. In this case, some of the probability assessments are inconsistent. The constraints thus should be altered in order to define a consistent set of constraints.

Since (4.2) is also a sum of the $p_{(\mathbf{n},k)}^\alpha$, we use it as an objective function and hence apply linear programming to determine the maximum and minimum values for expected utility for the set of distribution functions which satisfy the constraints. Let

$$c(\mathbf{p}^\alpha) = \sum_{h=1}^H u(a + h\Delta) \left(\sum_{(\mathbf{n},k) \in N_h} p_{(\mathbf{n},k)}^\alpha \right),$$

and let \mathbf{A}^α and \mathbf{b}^α be the constraint matrix and right-hand-side vector which define the polytope \mathcal{H}^α . Then the lower bound for (4.2) is found by solving

$$\begin{aligned} \underline{z}^\alpha &= \min c(\mathbf{p}^\alpha) \\ \text{s.t. } \mathbf{A}^\alpha \mathbf{p}^\alpha &\geq \mathbf{b}^\alpha. \end{aligned}$$

Similarly, the upper bound for (4.2) is found by solving

$$\begin{aligned} \bar{z}^\alpha &= \max c(\mathbf{p}^\alpha) \\ \text{s.t. } \mathbf{A}^\alpha \mathbf{p}^\alpha &\geq \mathbf{b}^\alpha. \end{aligned}$$

These linear programs are solved for each alternative $\alpha \in \mathcal{A}$. Suppose that we are comparing a set of alternatives $\mathcal{A} = \{\alpha, \beta\}$. Recalling that we are seeking the alternative with the largest value for (4.2), we see that if $\underline{z}^\alpha > \bar{z}^\beta$, then alternative α is preferred to alternative β . Likewise, if $\underline{z}^\beta > \bar{z}^\alpha$, then alternative β is preferred to alternative α . If neither of these conditions exist, then a preference ordering cannot be determined between the two alternatives.

E. Dual Problem Solutions

The solutions to the dual problems of the linear programs provide information regarding the protection offered by a physical security system. Each variable in the dual problems corresponds to a constraint in the primal problems. Similar to the economic interpretation of dual variables as rates of change or marginal returns per additional units of resources, dual variables may be used in security risk assessment to indicate what effects changes in probability assessments for an alternative would have on the bounds of the expected utility of that alternative. In situations where the preference between alternatives cannot be determined, an examination of these

variables can be used to determine what additional information would be useful in order to refine specific probability assessments. This might take the form of either gathering more information about characteristics and capabilities of a specific threat or pursuing more detailed performance characteristics of selected safeguards. Alternatively, an analysis of these variables could suggest a modification to an alternative which would provide tighter bounds on expected utility causing a separation of utility intervals and showing that the modified alternative is preferred to another alternative. Here we consider two types of constraints along with the corresponding information provided by their dual variables.

1. Bounds on Event Probabilities

As noted previously, an event can be represented as a union of the atomic events, and since the atomic events are disjoint, the probability of the event is equal to the sum of the probabilities of the atomic events. A constraint consisting of a bound on an event probability can thus be represented by a traditional inequality constraint in a linear optimization problem.

Suppose that E is an event and $I = \{(\mathbf{n}, k) : \omega_{(\mathbf{n}, k)} \in E\}$. Then if $P_\alpha(E) \leq b$ for some $b \in [0, 1]$, the corresponding constraint in the linear program is

$$\sum_{(\mathbf{n}, k) \in I} p_{(\mathbf{n}, k)}^\alpha \leq b.$$

For constraints of this type, the corresponding dual variables may be examined after solving the linear programs. We denote the objective function value at the optimal solution to a linear program as z^* . Thus, either $z^* = u_{min}^\alpha$ or $z^* = u_{max}^\alpha$, depending on which linear program is under consideration. For nondegenerate solutions, if $\bar{\pi}_b$ is the value of the dual variable corresponding to the constraint above at

the optimal solution, then

$$\frac{\partial z^*}{\partial b} = \bar{\pi}_b.$$

For degenerate solutions, different right-hand and left-hand derivatives may exist for a given constraint. Bazaraa *et al.* [56] give a discussion of shadow pricing in the degenerate case.

By examining the values of the dual variables corresponding to bounded probability assessments, it is possible to determine which assessments will, if changed, have the largest effect on the objective function. Consideration must also be given to the amount by which the assessment may be changed. For example, if a probability is bounded above by 0.05 and the sign of the dual variable indicates that the objective function may be changed in the desired direction by lowering the upper bound, the fact that the probability can be lowered by 0.05 at the most must be considered in concert with the value of the dual variable to determine if attempting to change the bound is reasonable. By focusing effort on the events corresponding to the dual values of the highest magnitude and having potential for adjustment, information gathering efforts may be directed toward those events which have the greatest potential to tighten the bounds on expected utility.

Note that the tightening of a probability constraint cannot increase the size of the feasible region. Any change in the size of the feasible region will be a reduction. Suppose that the constraint $P_\alpha(E) \leq b$ is tightened to $P_\alpha(E) \leq b - \delta$ with $\delta \in (0, b)$. Since $b - \delta < b$, any solution satisfying $P_\alpha(E) \leq b - \delta$ will also satisfy $P_\alpha(E) \leq b$, so we are in effect adding the constraint $P_\alpha(E) \leq b - \delta$ to the original constraint set.

Due to the constraint (4.3), the bound on the probability of the complementary event is also tightened. The original constraint implies that $P_\alpha(E^c) \geq 1 - b$ while the tightened constraint implies $P_\alpha(E^c) \geq 1 - (b - \delta) = 1 - b + \delta$. Since $1 - b + \delta > 1 - b$,

the lower bound on the complementary event probability is increased.

Since the tightening of these probability constraints either does not affect the size of the feasible region or reduces it, u_{min}^α will either remain the same or increase, and u_{max}^α will either remain the same or decrease. Thus, any change in the length of the interval $I^\alpha = [u_{min}^\alpha, u_{max}^\alpha]$ will be a reduction in length.

2. Bounds on Conditional Probabilities

As shown earlier, a constraint obtained from a bound on a conditional probability such as $P_\alpha(E|F) \leq c$ will always have a right-hand-side value of zero with the form

$$\sum_{(\mathbf{n},k) \in I_1} (1-c)p_{(\mathbf{n},k)}^\alpha + \sum_{(\mathbf{n},k) \in I_2} (-c)p_{(\mathbf{n},k)}^\alpha \leq 0.$$

Thus, the dual variable cannot be used with the traditional interpretation. Instead, we want to examine what happens when the value c is changed. Thus, we must consider how the objective function is affected by a change in the coefficient matrix \mathbf{A} . Freund [57] [58] considers the parameterized linear program

$$\begin{aligned} z(\theta) &= \max \mathbf{c}\mathbf{x} \\ \text{s.t. } &(\mathbf{F} + \theta\mathbf{G})\mathbf{x} = \mathbf{b} \\ &\mathbf{x} \geq \mathbf{0}. \end{aligned} \tag{4.5}$$

He shows that the derivative of $z(\theta)$ with respect to θ at the point $\theta = \theta_0$ is

$$z'(\theta_0) = -\bar{\boldsymbol{\pi}}\mathbf{G}\bar{\mathbf{x}}$$

where $\bar{\mathbf{x}}$ is an optimal primal solution at $\theta = \theta_0$ and $\bar{\boldsymbol{\pi}}$ is an optimal solution to the dual problem at the same point.

Applying this result to the constraint generated by the bound on a conditional

probability, we see that the parameter $\theta = c$ in this case. Here \mathbf{G} is a matrix with entries of -1 in the row corresponding to the constraint in question and in the columns corresponding to the variables included in this constraint and with entries of 0 in all other locations. We denote the value of the dual variable corresponding to the constraint at the optimal solution as $\bar{\pi}_j$, and $\bar{p}_{(\mathbf{n},k)}^\alpha$ is the value of the primal variable $p_{(\mathbf{n},k)}^\alpha$ at the optimal solution. Then we have

$$\frac{\partial z^*}{\partial c_j} = \bar{\pi}_j \sum_{(\mathbf{n},k) \in I_1 \cup I_2} \bar{p}_{(\mathbf{n},k)}^\alpha.$$

Note that

$$\sum_{(\mathbf{n},k) \in I_1 \cup I_2} \bar{p}_{(\mathbf{n},k)}^\alpha = P_\alpha(F)|_{\mathbf{p}=\bar{\mathbf{p}}}$$

where $\bar{\mathbf{p}}$ is the optimal primal solution. We thus have the result that for constraints of the form

$$P_\alpha(E|F) \leq c_j,$$

the partial derivative of the optimal value of the objective function with respect to the right hand side is given by

$$\frac{\partial z^*}{\partial c_j} = \bar{\pi}_j P_\alpha(F)|_{\mathbf{p}=\bar{\mathbf{p}}}. \quad (4.6)$$

Thus the partial derivative is equal to product of the dual variable and the probability of the conditioning event. This has direct implications on attempting to tighten the bounds on expected utility. By considering conditional probability assessments with both a large dual value and a high conditioning event probability, those assessments which will have the most desired effect on the utility bounds can be identified and analyzed.⁴

⁴Incidentally, Army doctrine identifies more desirable alternatives as those which are most likely to succeed against the most probable and most dangerous enemy actions. A typical conditional probability assessment would concern the event that x amount of damage occurs given that an attack of type y takes place.

F. Independence

Until now, independence has not been considered. The introduction of independence has a considerable effect on the structure of the problem since the constraint set is no longer linear.

Suppose events E and F are assumed to be independent where $\{\omega_i\}, i = 1, \dots, D$ are the atomic events and

$$I_1 = \{i : \omega_i \in E\}$$

and

$$I_2 = \{i : \omega_i \in F\}.$$

The independence assumption induces the constraint

$$P_\alpha(E \cap F) - P_\alpha(E)P_\alpha(F) = 0.$$

In terms of the atomic event probabilities, this constraint is

$$\sum_{i \in I_1 \cap I_2} p_i^\alpha - \sum_{\substack{i \in I_1 \\ j \in I_2}} p_i^\alpha p_j^\alpha = 0.$$

This is equivalent to

$$\sum_{i \in I_1 \cap I_2} [p_i^\alpha - (p_i^\alpha)^2] - \sum_{\substack{i \in I_1 \setminus I_2 \\ j \in I_2 \setminus I_1}} p_i^\alpha p_j^\alpha = 0.$$

If two of the random variables representing loss, say L_{11}^α and L_{43}^α , are assumed to be independent, then the σ -algebras generated by those random variables are independent. This condition induces a massive number of nonlinear constraints of the form just shown since each event in the σ -algebra generated by L_{11}^α is independent

By paying particular attention to high probability attacks, the Army takes advantage of this relationship between conditioning events and the objective function.

of each event in the σ -algebra generated by L_{43}^α . Even though we are considering a finite number of atomic events, this complete enumeration of pairs of events from the separate σ -algebras leads to a sizeable number of constraints.

Since independence assumptions introduce nonlinear constraints, the forms of the optimization problems which must be solved to determine the bounds on expected utility change. Instead of linear programming formulations, the problems are expressed in the traditional nonlinear programming form

$$\begin{aligned} \underline{z}^\alpha &= \min c(\mathbf{p}^\alpha) \\ \text{s.t. } g_i^\alpha(\mathbf{p}^\alpha) &\geq 0, \quad i = 1, \dots, m, \end{aligned}$$

and

$$\begin{aligned} \bar{z}^\alpha &= \max c(\mathbf{p}^\alpha) \\ \text{s.t. } g_i^\alpha(\mathbf{p}^\alpha) &\geq 0, \quad i = 1, \dots, m \end{aligned}$$

where all constraints (both linear and nonlinear) are written as $g_i^\alpha(\mathbf{p}^\alpha) \geq 0$. These nonlinear programming problems can be approached using well-known methods.

This chapter has presented an analytical model which may be applied to the assessment of physical security systems in the situation where a unique probability law on reward for each alternative is not available. The model applies expected utility theory and linear programming to determine bounds on expected utility. These bounds may be used to determine the existence of a separation of preferences among alternative systems. We have also shown that elements of sensitivity analysis which are available due to the linear programming formulation can be used to focus efforts to tighten the bounds on expected utility if a separation of preferences is not achieved.

In the next chapter, a military scenario is presented in which an application of the model is illustrated.

CHAPTER V

AN APPLICATION IN A MILITARY SCENARIO

In this chapter, the analytical framework from Chapter IV is applied to the military scenario of protecting a forward operating base (FOB). In this scenario, the risks associated with alternative configurations of soldiers, weapons, and equipment are assessed. An important and interesting aspect of this scenario is the fact that the soldiers, weapons, and equipment are simultaneously the assets being protected and the safeguards used to protect them. After a brief introduction to applicable military concepts, the background for the scenario is presented. An assessment of threats and a description of the alternative configurations are then described, followed by the formulation of the linear programs required to provide bounds on the expected utilities of the alternatives. Results from solving the linear programs are given, and a description of how the dual solutions may be used to gain valuable insights into vulnerabilities of the alternatives is presented. A discussion on using the primal solution to examine the consistency of the probability assessments concludes the chapter.

A. Background and Scenario Setting

Military tactical operations are classified in three categories. *Defensive operations* are conducted primarily to create the environment for offensive operations. Additional purposes for defensive operations are to defeat the enemy, delay the enemy, conserve friendly forces, hold key terrain, or deny enemy access to an area. *Offensive operations* have the primary purpose of destroying or defeating the enemy and are the Army's preferred type of operations since they tend to retain initiative. *Enabling operations* aid in the preparation for or conduct of either offensive or defensive operations. Enabling operations include reconnaissance, troop movement, relief

in place, and security.

The primary types of defensive tactical operations are the retrograde, the mobile defense, and the area defense. A retrograde is a deliberate movement away from the enemy. A mobile defense involves allowing enemy movement into a vulnerable position and then defeating or destroying the enemy with a striking force. An area defense is conducted to hold or deny access to key terrain.

Two specific types of key terrain are bases and installations. These are locations which house specific functions such as logistical support or command and control centers. The term “installation” typically carries the connotation of a location in the United States or in a secure or friendly area. A base is defined as “a locality from which operations are projected or supported.” [59] A base defense is established to deny enemy access to the personnel, equipment, and facilities located within the base. This purpose is essentially the same as the purpose of employing a physical security system. Thus, the assessment of a course of action for a base defense is a special case of a physical security system assessment.

As a specific type of base, a forward operating base is formally defined as “an airfield used to support tactical operations without establishing full support facilities.” [59] In practice, the function of supporting tactical operations is more definitive than the possession of an airfield. In other words, a FOB is a location from which tactical operations are conducted or supported which does not possess full support facilities. FOBs “are normally run by company sized units and have shorter term missions.” [60] However, since Army doctrine dictates that units continually improve their positions as long they occupy them, a FOB may take on certain characteristics of a permanent base or installation. Securing a FOB is a form of a base defense, and this operation is the context for this scenario.

In this scenario, we consider alternative configurations of resources for the phys-

ical security of a hypothetical FOB. An officer has been charged with overseeing the base defense for FOB Amaan. Resources available include soldiers, their weapons and equipment, and construction materials. The perimeter and surroundings of FOB Amaan are shown in Figure 2.

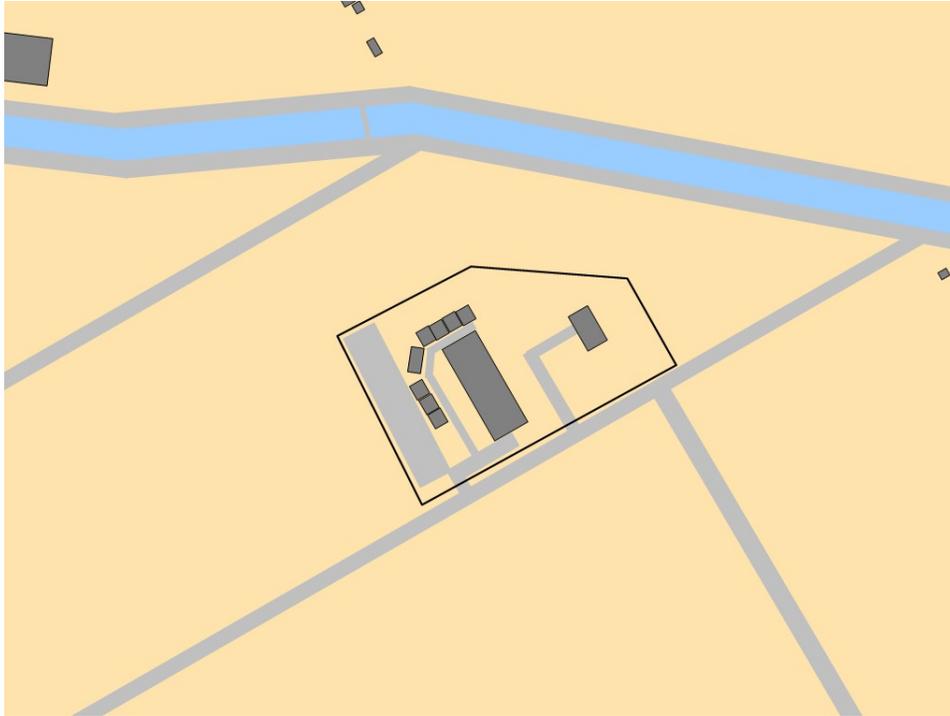


Fig. 2. Perimeter of Forward Operating Base Amaan

FOB Amaan is an established base in that the physical perimeter of the base has been established. A chain link perimeter fence with concertina wire has been erected along with observation towers at the corners. The observation towers are each manned by a crew, and the officer must consider alternatives regarding the sizes of the crews and the types of machine guns that they will use. Additionally, the configurations must specify manning levels and weapons to control vehicle access to the FOB.

FOB Amaan has been partitioned into four sectors according to location and

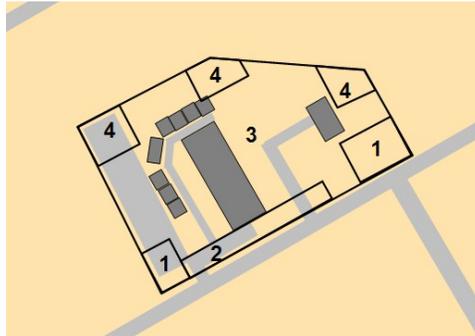


Fig. 3. Sectors of FOB Amaan

function. Sector 1 is comprised of the two southernmost observations towers and the area surrounding each. Sector 2 is the vehicle access gate area. Sector 3 is the living area, headquarters, and vehicle parking area. Sector 4 is the northernmost observation towers and the area surrounding each. A schematic of the sectors is shown in Figure 3.

The available safeguards are twenty soldiers, two heavy machine guns, five light machine guns, five antitank weapons, and 13 rifles to deploy in the defense of the FOB. Additionally, there are 20 soldiers in the living area at any given time, five soldiers in the headquarters area, and five high mobility multipurpose wheeled vehicles (HMMWV) in the parking area.

B. Intelligence Assessment

Based on intelligence about enemy activity in the area, there are four attack types that the base might face. Let $\{A = k\}$ be the event that an attack of type k occurs, where the attack corresponding to k is as shown in Table I. Based on recent intelligence estimates, the probabilities of the different types of attacks are assessed

Table I. Scenario Attack Types

k	Attack description
1	Suicide car bomb via the road from the west
2	Improvised explosive device or suicide bomb at the vehicle gate
3	Suicide car bomb via the road from the south
4	Dismounted attack from the north

as lying in the following ranges:

$$P(A = 1) \leq 0.25 \quad (5.1)$$

$$P(A = 1) \geq 0.15 \quad (5.2)$$

$$P(A = 2) \leq 0.3 \quad (5.3)$$

$$P(A = 3) \leq 0.2 \quad (5.4)$$

$$P(A = 3) \geq 0.05 \quad (5.5)$$

$$P(A = 4) \leq 0.35 \quad (5.6)$$

$$P(A = 4) \geq 0.1 \quad (5.7)$$

Additionally, the certain attacks are believed to be more probable than others.

These beliefs can be described by the following probability statements:

$$P(A = 1) \leq P(A = 2) \quad (5.8)$$

$$P(A = 3) \leq P(A = 2) \quad (5.9)$$

$$P(A = 2) \leq P(A = 4) \quad (5.10)$$

C. Description of Alternatives

Two alternative configurations have been identified. The first, alternative α , is to place three soldiers with one light machine gun, one antitank weapon, and two rifles in each of the five observation towers. The remaining five soldiers, three rifles, and both heavy machine guns would then be placed at the vehicle access gates. All perimeter positions would be reinforced with sandbags.

The second option, alternative β , is to place three soldiers with one light machine gun, one antitank weapon, and two rifles in each of the northern three observation towers. The southern two towers would each have four soldiers, a heavy machine gun, an antitank weapon, and three rifles. The gates would be guarded by three soldiers with two light machine guns and one rifle. Rather than using sandbags in this alternative, all positions are reinforced with cinder blocks and steel roofing.

D. Determining Sets of Distributions on Loss

The value measure used for the determination of loss in each sector is in terms of dollars. The sector value is equal to the sum of equipment value and soldier value. Due to different arrangements of soldiers and equipment in the different alternatives, the sector values may vary from one alternative to another.

Equipment costs are based on current public information from the military and are listed in Appendix C. A value of \$550,000 is used for the life of a soldier based on an average cost of \$50,000 to train a soldier to proficiency [61], the maximum life insurance amount of \$400,000 available from the government [62], and the Fallen Hero Compensation of \$100,000 paid to a soldier's family if the soldier is killed in a combat zone [63].

The total value of the soldiers and equipment in the FOB is \$25.39 million. Sector

values by alternative are shown in Table II.

Table II. Sector Values (in \$ thousands)

Sector	Alternative α	Alternative β
1	3,330	4,446
2	2,790	1,674
3	14,275	14,275
4	4,995	4,995

Using an interval size of $\Delta = \$1,000,000$, the possible values of L_i for the different sectors are shown in Table III. Based on these values, we define the atomic events as all possible combinations of sector losses and attacks. Thus, $\{\omega_{0,0,0,0,1}\}$ corresponds to a suicide car bomb from the west (attack type 1) and no loss in any sector. Similarly, the event $\{\omega_{4,3,15,6,3}\}$ consists of a suicide car bomb from the south and maximal destruction in all sectors (based on values for alternative α).

Table III. Possible Values of L_i

Sector	Alternative α	Alternative β
1	0 to 4	0 to 5
2	0 to 3	0 to 2
3	0 to 15	0 to 15
4	0 to 6	0 to 6

Based on the characteristics of the different alternatives, a series of conditional probability assessments reflects perceptions of the protection of the different sectors

by each alternative against the various attack types. These assessments are located in Appendix A for alternative α and in Appendix B for alternative β .

The attack probability assessments (5.1)-(5.10) and the loss probability assessments in the appendices are each then converted to constraints in terms of the atomic events. We use the convention $p_{ijklmk} = P\{\omega_{i,j,l,m,k}\}$. Thus i corresponds to the magnitude of loss in sector 1, j to loss in sector 2, l to loss in sector 3, m to loss in sector 4, and k to the type of attack. Examples of assessments and corresponding constraints are

$$P_{\alpha}(A^{\alpha} = 2) \leq 0.3 \Rightarrow \sum_{ijlm} p_{ijlm2}^{\alpha} \leq 0.3$$

and

$$P_{\beta}(L_1^{\beta} = 0 | A^{\beta} = 1) \leq 0.9 \Rightarrow \sum_{jlm} 0.1 p_{0jlm1}^{\beta} - \sum_{i=1}^5 \sum_{jlm} 0.9 p_{ijlm1}^{\beta} \leq 0.$$

The final constraint for each alternative is the requirement that $P(\Omega) = 1$ which becomes

$$\sum_{ijklmk} p_{ijklmk} = 1. \quad (5.11)$$

The utility function used for the scenario was

$$u(x) = a + be^{-\frac{x}{\rho}}, \quad x \in [0, 28]$$

with $a = 131.43685$, $b = -31.43685$, and $\rho = 18.87391658$. This function is shown plotted over the possible loss values in figure 4. This function returns a utility value of 100 for operation of the FOB with no losses, and a loss of \$27 million, which exceeds the value of the FOB with all its personnel and equipment, returns a utility value of

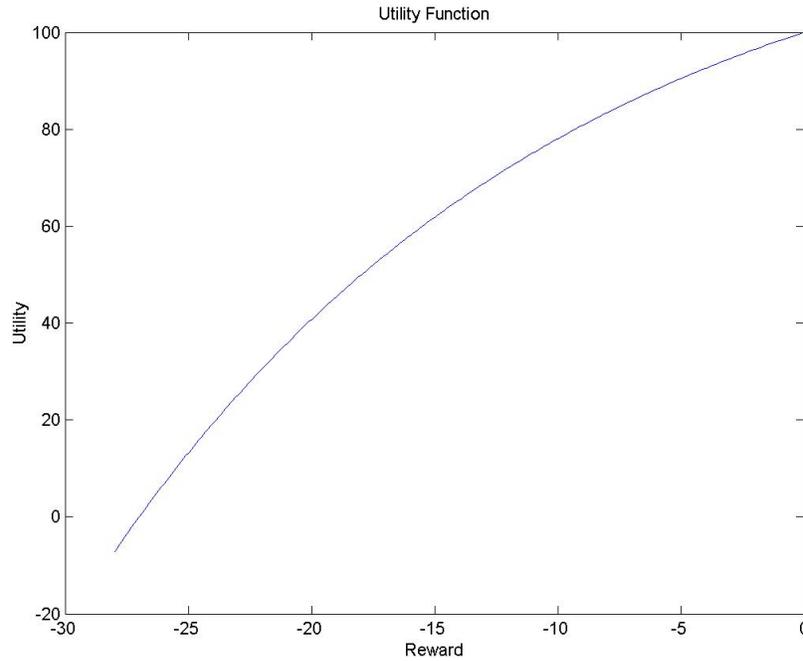


Fig. 4. Plot of Utility Function

zero. The objective function for the linear programs for alternative α thus becomes

$$\begin{aligned} c(\mathbf{p}^\alpha) &= \sum_{ijklmk} [a + b e^{(i+j+l+m)/\rho}] p_{ijklmk}^\alpha \\ &= a + b \sum_{ijklmk} e^{(i+j+l+m)/\rho} p_{ijklmk}^\alpha. \end{aligned}$$

The objective function for alternative β is identical except that the decision variables are given by the vector \mathbf{p}^β .

The linear programs for the two alternatives were each coded using AMPL mathematical programming software and solved using the CPLEX solver.

E. Analysis of Results

As a result of solving each alternative's formulation as both maximization and minimization problems, the expected utility values were determined to be in the interval [93.4667, 99.5059] for alternative α and in the interval [94.4837, 99.4554] for alternative β . Since no separation exists between the intervals, a preference cannot be determined for one alternative over the other with the current state of information. However, examining the linear programming solutions provides insights regarding the situation.

1. Insight from the Dual Solutions

The non-zero values for the dual variables from each of the solutions of the linear programs are shown in Appendix D. Each value π_c corresponds either to constraint c in Appendix A or B (depending on the alternative being considered) or to constraints (5.1)-(5.11). In Appendix D, these values are sorted by magnitude for each of the optimization problems for each alternative.

a. Dual Values as Shadow Prices

The traditional interpretation of the dual variables as shadow prices for the right hand side quantities applies here with respect only to constraints (5.1)-(5.10). The right hand side of constraint (5.11) can never be changed, and the constraints in the appendices are all based on conditional probabilities and will thus have a right hand side value of zero.

For alternative α , the dual variables corresponding to constraints (5.3), (5.4), and (5.6) are all non-zero for both the minimization and maximization problems. Additionally, all have a negative value for the minimization problem and a positive

value for the maximization problem. Since each of these constraints is an upper bound on an attack probability, tightening these upper bounds would potentially tighten both the upper and lower bounds on expected utility. Given the choice of which constraint to focus information gathering efforts on for alternative α , the officer should choose either constraint (5.3) or (5.6) since the magnitudes of their dual values are greater than those of constraint (5.4) in both the minimization and maximization problems.

For alternative β , the dual variables for constraints (5.1), (5.3), and (5.6) are non-zero for the minimization problem, and the dual variables for constraints (5.4), (5.6), and (5.8) are non-zero for the maximization problem. Since the magnitude of $\pi_{5.6}$ is the largest of the three values in the minimization problem and since it is the only constraint with non-zero values for both problems, the officer should focus efforts on tightening constraint (5.6) to tighten the bounds on expected utility for alternative β .

Given that the analysis of the dual variables for the attack probability constraints for both alternatives point to tightening the upper bound on $P(A = 4)$, the officer may want to focus on gathering additional intelligence on the likelihood of a dismounted attack from the north.

b. Shadow Pricing for Conditional Probabilities

Using Equation (4.6) to compute shadow prices for conditional probabilities is not as straightforward as using the traditional interpretation of the dual variables. Several factors must be considered. First, which of the constraints have a dual variable with a large magnitudes? Second, what is the sense of the constraint that corresponds to a large dual value? Is it an equality or an inequality? Third, for the current optimal solution, what is the probability of the conditioning event for the constraint under

consideration? Once a shadow price is able to be computed, we may then turn to the questions of whether the right hand side should be increased or decreased and what must be done to justify such an adjustment in the mind of the officer. To show the implications of each of these questions, we consider the minimization problem of alternative α .

The constraints with the dual variable of the largest magnitude is constraint 52 with $\bar{\pi}_{52} = -59.5041$. Constraint 52 is an equality constraint with a right hand side value of one. In this scenario, constraints of this type serve the purpose of placing an upper bound on the loss in a sector given that an attack occurs and that the loss in the protective sector closest to the attack is not zero. The right hand side of this type of constraint cannot be adjusted and still serve its purpose. In fact, in the probability assessments used for this scenario, all of the equality constraints have a right hand side of one and conditionally either place an upper bound on loss in a sector or fix the loss in a sector at zero. Thus, we should limit the constraints under consideration to those which are inequality constraints.

The inequality constraint with the dual value of highest magnitude is constraint 28 which is

$$P_{\alpha}(L_3^{\alpha} \leq 1 | L_2^{\alpha} > 0, A^{\alpha} = 2) \geq 0.8.$$

To compute the shadow price using Equation (4.6), we must compute the conditioning event probability, $P_{\alpha}(L_2^{\alpha} > 0, A^{\alpha} = 2)$. In terms of the atomic event probabilities, we see that

$$P_{\alpha}(L_2^{\alpha} > 0, A^{\alpha} = 2) = \sum_{j=1}^5 \sum_{ilm} p_{ijlm2} = 0.09.$$

Before applying Equation (4.6) to compute the shadow price, we must consider that this result was obtained from the linear program (4.5) which is a maximization problem while we are working with a minimization problem. Since the conversion from

minimization to maximization is accomplished through negation of the objective function, we may simply change the sign of the right side of Equation (4.6) to apply it to this situation. Thus, we see that

$$\frac{\partial \underline{z}^\alpha}{\partial c_{28}} = -(-47.5062)(0.09) = 4.2756.$$

This value is positive, so an increase in the lower bound on $P_\alpha(L_3^\alpha \leq 1 | L_2^\alpha > 0, A^\alpha = 2)$ would cause an increase in \underline{z}^α , the lower bound on expected utility. Since this would be a tightening of the bounds on expected utility, making such an adjustment would be desirable for trying to separate the intervals containing the two expected utilities.

Examining constraint 27 which has the dual value of next highest magnitude, we see that constraint 27,

$$P_\alpha(L_3^\alpha = 0 | L_2^\alpha = 0, A^\alpha = 2) \geq 0.95,$$

is an inequality constraint, and we compute the conditioning event probability,

$$P_\alpha(L_2^\alpha = 0, A^\alpha = 2) = \sum_{ilm} p_{i0lm2} = 0.3.$$

We find that

$$\frac{\partial \underline{z}^\alpha}{\partial c_{27}} = -(-47.169)(0.3) = 9.9055.$$

Since this value is also positive, increasing the lower bound on constraint 27 would also cause an increase in the lower bound on expected utility.

Since these probability assessments represent the officer's beliefs, however, something must change in relation to the situation or the anticipated consequences of alternative α in order to justify these changes in the bounds on the probabilities. One impetus for a change in the assessment could take the form of additional information about the weapons used by the enemy in carrying out attacks of type 2

(suicide bombings or improvised explosive devices). However, there is no guarantee that such information would cause the probabilities to *increase*. The primary matter that is under the control of the officer is the configuration, placement, and construction of the positions in the sectors. What increasing the lower bound on $P_\alpha(L_3^\alpha = 0 | L_2^\alpha = 0, A^\alpha = 2)$ means, practically, is that the officer is more certain that no damage will occur in sector 3 if an attack of type 2 occurs and no damage occurs in sector 2. Increasing the lower bound on $P_\alpha(L_3^\alpha \leq 1 | L_2^\alpha > 0, A^\alpha = 2)$ would indicate that, if damage in sector 2 occurs due to a type 2 attack, the officer is more certain that a lower magnitude of loss would be incurred in sector 3. Since the location of a type 2 attack is directly adjacent to sector 2, the decision maker can reduce the probability of such an attack affecting sector 3 by increasing the distance between sectors 2 and 3 or by constructing protective barriers between the sectors. Since this adds another safeguard to the current configuration, the officer would in fact be creating a new alternative and should consider whether the previous probability assessments for other events are valid for the new alternative.

Rather than create a new alternative on the basis of the analysis of a limited number of shadow prices, the officer should consider whether the implications of a combination of shadow prices might produce a better alternative. In the problem under consideration, the eight inequality constraints with the largest shadow price values are shown in Table IV. Note that the partial derivatives with respect to the right sides of these assessments are all positive.

Note that five of the eight constraints are assessments that involve loss in sector 3 and that the two highest partial derivative values are included in these five. This is an indication that making the probability of damage to sector 3 as unlikely as possible would increase the lower bound on expected utility. Additionally, since the conditioning events in these assessments involve all of the other sectors in the FOB,

Table IV. Conditional Probability Constraint Shadow Prices

j	Constraint j	$\partial z^\alpha / \partial c_j$
63	$P_\alpha(L_3^\alpha = 0 L_4^\alpha = 0, A^\alpha = 4) \geq 0.8$	12.0206
27	$P_\alpha(L_3^\alpha = 0 L_2^\alpha = 0, A^\alpha = 2) \geq 0.95$	9.9055
17	$P_\alpha(L_2^\alpha = 0 A^\alpha = 2) \geq 0.7$	4.3069
28	$P_\alpha(L_3^\alpha \leq 1 L_2^\alpha > 0, A^\alpha = 2) \geq 0.8$	4.2756
5	$P_\alpha(L_2^\alpha = 0 L_1^\alpha = 0, A^\alpha = 1) \geq 0.95$	3.3602
15	$P_\alpha(L_3^\alpha \leq 4 L_1^\alpha > 0, A^\alpha = 1) \geq 0.99$	3.0048
7	$P_\alpha(L_2^\alpha \leq 1 L_1^\alpha > 0, A^\alpha = 1) \leq 0.9$	2.9980
9	$P_\alpha(L_3^\alpha = 0 L_1^\alpha = 0, A^\alpha = 1) \geq 0.95$	2.8620

this is an indication that the possibility of placing some type of protective barrier between sector 3 and the others should be explored. Such a barrier would practically enclose sector 3. Since sector 3 contains the living and headquarters areas and since those areas contain the most value due to the concentration of soldiers in those areas, construction of hardened buildings or reinforcement of current facilities for these areas should be considered.

This result corresponds to the intuitive notion of using more assets to protect items or areas of higher value. However, this example shows how the analytical technique presented can identify areas of high value which are left vulnerable. This identification of vulnerabilities is of primary importance in the evaluation of security system alternatives. This evaluation is fundamental in the identification of a most preferred alternative.

2. Consistency and the Primal Solutions

Although other methods exist to determine consistency of probability assessments, linear programming provides an automatic check for consistency through the determination of feasibility. Additionally, an examination of the values of the primal variables (that is, the atomic event probabilities) can provide insight into whether the given probability assessment actually reflects one's perception of the situation. The non-zero variables from each of the primal solutions of the linear programs in this scenario are shown in Appendix D. Each value for p_{ijklmk} in these tables represents a probability which corresponds to an atomic event which, according to the probability assessments, may have a non-zero probability and which contributes to either the maximum or minimum value for expected utility for its respective alternative.

One consideration is whether the probabilities do, in fact, represent one's beliefs regarding the possible consequences of the alternative configuration for which they were given. The probabilities in Table V are extracted from the solution for the minimization problem for alternative α . Note that the value corresponding to the index l is 15 for all of these events. Summing these probabilities, we see that for the probability law corresponding to this solution,

$$P_{\alpha}\{L_3^{\alpha} = 15\} = 0.097.$$

This indicates that the officer's assessment of the situation allows for a probability of almost 0.1 that the headquarters, living, and parking areas (sector 3) will be destroyed. If the officer does not believe that the probability may be that high, he should adjust the probability assessments to reflect this belief. If he does believe the probability may be that high, he should consider adjusting the configuration so that those key areas are better protected.

Table V. Probability of Destruction of Sector 3

i	j	l	m	k	p_{ijklmk}
0	0	15	0	1	0.00375
0	0	15	0	4	0.063
0	0	15	5	3	0.001
2	3	15	0	1	0.00075
4	0	15	0	2	0.0105
4	1	15	0	2	0.018

Another consideration for the officer is whether events with non-zero probabilities would actually occur. The first two lines in Table V correspond to events where an attack occurs and none of the protective measures incurs any loss, yet the living area is completely destroyed. The officer must consider whether or not he believes that this could actually happen. If so, then, again, he should consider a modification to the configuration to prevent such a catastrophic occurrence. If not, then he should add constraints to reflect that belief. One possible constraint would be

$$P_{\alpha}(L_3^{\alpha} \leq 7 | A^{\alpha} = 4) = 1$$

which would place a bound on the loss in sector 3 due to attack 4. An assessment of this type will further constrain the feasible region and may also tighten the lower bound on expected utility.

This chapter has presented an application of the analytical framework developed in Chapter IV for identification of a preferred alternative under incomplete characterization of probability law. A military scenario involving the security of a FOB was

considered as a special case of the physical security problem. Linear optimization problems to find the bounds on expected utility for two alternatives were formulated and solved. Results showed that, while neither alternative was preferred over the other, an examination of the primal and dual solutions provided insight into three primary areas:

- information that should be gathered to tighten the bounds on expected utility,
- identification of vulnerabilities in high-value areas, and
- consistency of probability assessments.

CHAPTER VI

CONCLUSION

This dissertation has accomplished the research objectives presented at the outset. The first objective was to identify an objective function for the physical security system decision which orders preferences for the decision maker. In Chapter III, the basis for the use of expected utility as the objective function was presented. In Chapter IV, an objective function was derived which is a computation of expected utility within the context of the problem examined. The second objective was to determine conditions for the separation of preferences without complete characterization of probability law. In Chapter IV, we presented a method to determine upper and lower bounds on expected utility through the use of linear programming. Separation of preferences occurs when the lower bound on utility for one alternative exceeds the upper bound on utility for another. The third objective was to identify insights available to the decision maker through interpretation of aspects of the model structure. Chapter IV discussed sensitivity analysis methods available in linear programming which are applicable to the formulation of the problem studied here. The interpretation of the information provided through sensitivity analysis was illustrated in Chapter V where we provided techniques to give insight both into vulnerabilities that exist within a design and also into which threats merit additional information gathering efforts.

In this final chapter, as an area for further research, we present an extension of physical security system assessment to the case where neither the probability law nor the utility function are uniquely specified. Finding bounds on expected utility in this case can be accomplished via bilinear programming, and we give formulations of these problems. Following the discussion of this research extension, we discuss

the motivation for considering this topic and give possible application areas for the assessment framework.

A. Future Research Areas

In research on educating or training leaders in decision making, Cohen, Freeman, and Thompson [64] present a descriptive model which identifies qualities and skills of proficient decision makers and thus offers specific foci for training. Klein and Wolf [49] include modeling the cognitive processes of subject-matter experts as one of the primary guidelines for training decision makers. Thus, in training people to be better decision makers, efforts are made to describe what characteristics and thought processes a good decision maker possesses.

Applying this approach to training decision makers using an expected utility paradigm, expert decision makers can be considered to have utility functions with specific characteristics. In training people to make good decisions in specific situations, success could be measured by determining whether the utility function used by the student is similar to that of an expert in that situation. One approach would be to identify a range of utility functions that would be acceptable and then to determine if the utility function used to make the decision falls within that range.

The model presented in Chapter IV can be extended to the case where a utility function is not specified but is known to lie within certain boundaries. We show that such an assessment problem may be modeled using bilinear programs which may be solved to determine upper and lower bounds on expected utility for each alternative. A separation of the bounds on expected utility serves to order preferences among alternatives if such a separation exists.

1. Problem Description

The assessment situation is identical to that in Chapter IV except that the utility function is not known. Instead, the utility function is required to lie between certain boundaries. Some assets are to be protected, and alternative configurations of safeguards have been identified with which to protect the assets. The assets to be protected are partitioned into N sectors, and M possible types of threats have been identified. For each alternative configuration, an assessment is made of the probabilities of the occurrence of the attack types and of consequences of attacks, but these probability assessments do not result in a unique probability law on reward for each alternative.

As shown in (4.2), if a utility function u is given, the calculation of the expected utility for alternative α is

$$E_\alpha(U) = \sum_{h=1}^H u(a + h\Delta) \left(\sum_{(\mathbf{n},k) \in N_h} p_{(\mathbf{n},k)}^\alpha \right)$$

where

$$N_h = \{(\mathbf{n}, k) : \|\mathbf{n}\|_1 = -a - h\Delta, 1 \leq k \leq M\}.$$

However, in the problem under consideration, no utility function is given. We consider two monotone, continuous functions $u_U : \mathbb{R} \rightarrow \mathbb{R}$ and $u_L : \mathbb{R} \rightarrow \mathbb{R}$ as boundaries on the utility function. That is, we require that

$$u_L(x) \leq u(x) \leq u_U(x), \quad x \in \mathbb{R}.$$

An illustration of utility boundary functions is shown in Figure 5 where $[a, b]$ is the support of the risk distribution for alternative α . The plot of an acceptable utility function would fall within the shaded region.

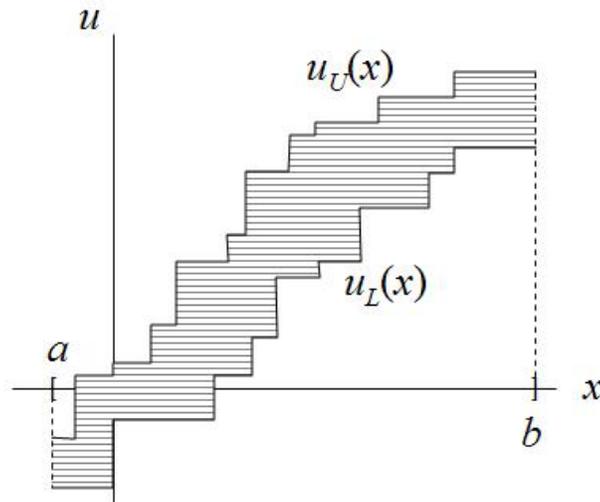


Fig. 5. Boundaries of a Utility Function

2. Formulation of Bilinear Programs

Considering the expression which we have derived for expected utility which we used as an objective function in Chapter IV, we see that H evaluations of the utility function are needed to determine a bound on expected utility. These are

$$u(a + h\Delta), \quad h = 1, \dots, H.$$

Each of these evaluations may be considered to be a decision variable to which a value must be assigned. Let

$$u_h = u(a + h\Delta), \quad h = 1, \dots, H.$$

Then for each u_h , we have the following constraints:

$$u_h \geq u_L(a + h\Delta)$$

$$u_h \leq u_U(a + h\Delta).$$

Additionally, we require that the unknown utility function be monotone. Therefore, we include the constraints

$$u_h \leq u_{h+1}, \quad h = 1, \dots, H - 1.$$

Let $\mathbf{u} = (u_1, \dots, u_H)$ be the vector of utility decision variables. Then the constraints on these decision variables can be expressed as

$$\mathbf{A}_1 \mathbf{u} \geq \mathbf{b}_1.$$

Just as in Chapter IV, we denote the constraints on the probability decision variables for alternative $\alpha \in \mathcal{A}$ as

$$\mathbf{A}_2^\alpha \mathbf{p}^\alpha \geq \mathbf{b}_2^\alpha.$$

The objective function which is simply the adjusted expression for expected utility becomes

$$c(\mathbf{u}, \mathbf{p}^\alpha) = \sum_{h=1}^H \sum_{(\mathbf{n}, k) \in N_h} u_h p_{(\mathbf{n}, k)}^\alpha.$$

Thus we have two bilinear programs for each alternative. The lower bound for expected utility for alternative α is found by solving

$$\begin{aligned} \underline{z}^\alpha &= \min c(\mathbf{u}, \mathbf{p}^\alpha) \\ \text{s.t. } \mathbf{A}_1 \mathbf{u} &\geq \mathbf{b}_1 \\ \mathbf{A}_2^\alpha \mathbf{p}^\alpha &\geq \mathbf{b}_2^\alpha. \end{aligned}$$

Similarly, the upper bound for expected utility for alternative α is found by solving

$$\begin{aligned} \bar{z}^\alpha &= \max c(\mathbf{u}, \mathbf{p}^\alpha) \\ \text{s.t. } \mathbf{A}_1 \mathbf{u} &\geq \mathbf{b}_1 \\ \mathbf{A}_2^\alpha \mathbf{p}^\alpha &\geq \mathbf{b}_2^\alpha. \end{aligned}$$

These bilinear programs both have nonlinear objective functions and linear constraints. Problems of this type have been studied and various solution techniques have been applied with success. Al-Khayyal [65] gives a thorough overview of related problems and a survey of solution methodologies.

For two alternatives $\alpha, \beta \in \mathcal{A}$, a preference for one alternative over another can be determined whenever $\underline{z}^\alpha > \bar{z}^\beta$ or $\underline{z}^\beta > \bar{z}^\alpha$. Thus, in certain instances, it is possible to determine a preference between alternatives when neither the probability law nor the utility function is precisely specified.

B. Concluding Remarks

In this conclusion, we first comment on the motivation and performance of the research presented in this dissertation. We close with a discussion of other potential application areas for our analytical framework.

1. Motivation and Flow of Research

The initial motivation for this research was to develop an analytical framework which could be used to educate and train current and future Army officers in decision making. The focus was on providing an analytical framework which could be used to develop decision scenarios to allow future Army leaders to better understand uncertainty, to discover aspects of their own decision styles and preferences, and to develop a sense of consistency. The need for improved education in decision making for Army officers was recognized by General (Retired) Montgomery Meigs in the spring of 2001, *prior* to the attacks of September 11 and America's entry into the Global War on Terror:

Among staff college students and about-to-be general officers as well, we must foster a better understanding of the uncertainty inherent in operations and the processes by which they can best deal with that uncertainty. Our professional education must engender better decisionmaking by furnishing the intellectual tools that bolster leaders against stress, friction, and fog, and against the pressures of their fears and those of their political masters....In this regard we need a very sophisticated course of hands-on case studies in how decisions are enabled and made, not just the study of staff duties and political science in a military context. [66]

The desire was to provide a tool to enable the training of officers in tough, realistic, ambiguous, combat-related decision scenarios so that making decisions under these conditions would not be as difficult in actual combat situations.

As research into the problem continued, it became evident that the need for decision making education in the security arena was not limited to the Army. Certainly, other branches of the military would benefit from decision education of this sort. However, the challenges facing decision makers in Homeland Security, port security, stadium security, and many other areas would indicate that decision making skills in the security sector are absolutely necessary due to the high price of failure. Thus, the focus of the research broadened, and the military decision scenario was seen to be a special case of the more general physical security decision problem. Since such a decision is a matter of choosing the alternative with the preferred risk, the need to focus on security risk assessment became apparent.

2. Further Areas for Application

The analytical framework that was developed in this research was applied to a military scenario where Army forces are securing a FOB, and a decision maker must decide between alternative configurations of forces. In the scenario presented, the decision maker is concerned with a small area and a limited number of forces. The technique presented, however, has applications across the military services and across the spectrum of force levels. Consider the deployment of a carrier strike group in the U.S. Navy. A carrier strike group consists of an air craft carrier and a mission-dependent combination of guided missile cruisers, guided missile destroyers, attack submarines, and a combat support ship for logistical support. [67] The group commander has the responsibility to protect the aircraft carrier and the other ships in the strike group using the assets that comprise the group. He must consider the threats to the group and array the ships and aircraft coverage to counter the threats appropriately while simultaneously performing the group's mission. While an Army lieutenant or captain might oversee the security of a small FOB, a Navy rear admiral is ultimately responsible for the security of a carrier strike group. Other possible scenarios include the securing of a beachhead by U.S. Marines or the protection of an air base by security forces of the Air Force. The flexibility of the framework in military applications is evident.

Because of this flexibility in the application of the analytical framework, numerous opportunities exist for employing the framework in the education and training of military decision makers. One potential application would be to use decision scenarios to provide feedback on consistency in both decision making and threat assessment. Military leaders must be able to act in accordance with their values and their understanding of the situation at hand. An understanding of consistency and practice in

facing ambiguous decision scenarios will enable leaders to make decisions when facing real life situations in the so-called “fog of war”. Another application is the employment of the model to identify weaknesses in plans and decisions under uncertainty in training scenarios. Using the capability within the framework to identify potential vulnerabilities, officers can learn to develop thorough plans and to see when resource constraints leave missions or facilities exposed to unacceptable levels of loss of life or equipment. Through the development of applications such as these, valuable experience in making hard decisions under combat-related conditions of uncertainty can be gained.

Potential applications for the framework in educating security decision makers exist outside of the military domain as well. Border and port security are prime examples of situations where a fixed asset must be protected by arranging sets of safeguards in response to uncertain threats. Not only are the threats uncertain, they are manifold due to the amount of traffic and cargo that passes through these types of facilities. Officials in these areas must be well versed in the consideration of threat information and the proper response to such information. Moreover, the ability to determine which threats warrant the pursuit of additional information would be of value due to the large number of potential threats faced by border and port security systems. The framework presented here can assist in providing this ability.

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APPENDIX A

PROBABILITY ASSESSMENTS FOR ALTERNATIVE α

The following are the probability assessments for loss under alternative α . The numbers to the right of each line correspond to the constraint numbers and thus dual variable indices for those assessments.

Assessments for Attack Type 1

$$0.1 \leq P_\alpha(L_1^\alpha = 0 | A^\alpha = 1) \leq 0.6 \quad (1, 2)$$

$$0.7 \leq P_\alpha(L_1^\alpha \leq 1 | A^\alpha = 1) \quad (3)$$

$$P_\alpha(L_1^\alpha \leq 2 | A^\alpha = 1) = 1 \quad (4)$$

$$0.95 \leq P_\alpha(L_2^\alpha = 0 | L_1^\alpha = 0, A^\alpha = 1) \quad (5)$$

$$0.5 \leq P_\alpha(L_2^\alpha \leq 1 | L_1^\alpha > 0, A^\alpha = 1) \leq 0.9 \quad (6, 7)$$

$$0.95 \leq P_\alpha(L_2^\alpha \leq 2 | L_1^\alpha > 0, A^\alpha = 1) \quad (8)$$

$$0.95 \leq P_\alpha(L_3^\alpha = 0 | L_1^\alpha = 0, A^\alpha = 1) \quad (9)$$

$$0.5 \leq P_\alpha(L_3^\alpha \leq 1 | L_1^\alpha > 0, A^\alpha = 1) \leq 0.8 \quad (10, 11)$$

$$0.7 \leq P_\alpha(L_3^\alpha \leq 2 | L_1^\alpha > 0, A^\alpha = 1) \leq 0.95 \quad (12, 13)$$

$$0.9 \leq P_\alpha(L_3^\alpha \leq 3 | L_1^\alpha > 0, A^\alpha = 1) \quad (14)$$

$$0.99 \leq P_\alpha(L_3^\alpha \leq 4 | L_1^\alpha > 0, A^\alpha = 1) \quad (15)$$

$$P_\alpha(L_4^\alpha = 0 | A^\alpha = 1) = 1 \quad (16)$$

Assessments for Attack Type 2

$$0.7 \leq P_\alpha(L_2^\alpha = 0 | A^\alpha = 2) \leq 0.9 \quad (17, 18)$$

$$0.85 \leq P_\alpha(L_2^\alpha \leq 1 | A^\alpha = 2) \quad (19)$$

$$0.9 \leq P_\alpha(L_2^\alpha \leq 2 | A^\alpha = 2) \quad (20)$$

$$0.9 \leq P_\alpha(L_1^\alpha = 0 | L_2^\alpha = 0, A^\alpha = 2) \quad (21)$$

$$0.6 \leq P_\alpha(L_1^\alpha \leq 1 | L_2^\alpha > 0, A^\alpha = 2) \leq 0.9 \quad (22, 23)$$

$$0.2 \leq P_\alpha(L_1^\alpha \leq 1 | L_2^\alpha > 1, A^\alpha = 2) \leq 0.7 \quad (24, 25)$$

$$P_\alpha(L_1^\alpha \leq 2 | L_2^\alpha > 1, A^\alpha = 2) = 1 \quad (26)$$

$$0.95 \leq P_\alpha(L_3^\alpha = 0 | L_2^\alpha = 0, A^\alpha = 2) \quad (27)$$

$$0.8 \leq P_\alpha(L_3^\alpha \leq 1 | L_2^\alpha > 0, A^\alpha = 2) \quad (28)$$

$$0.1 \leq P_\alpha(L_3^\alpha \leq 1 | L_2^\alpha > 1, A^\alpha = 2) \leq 0.5 \quad (29, 30)$$

$$0.3 \leq P_\alpha(L_3^\alpha \leq 2 | L_2^\alpha > 1, A^\alpha = 2) \leq 0.7 \quad (31, 32)$$

$$0.6 \leq P_\alpha(L_3^\alpha \leq 3 | L_2^\alpha > 1, A^\alpha = 2) \leq 0.9 \quad (33, 34)$$

$$0.7 \leq P_\alpha(L_3^\alpha \leq 4 | L_2^\alpha > 1, A^\alpha = 2) \leq 0.95 \quad (35, 36)$$

$$0.8 \leq P_\alpha(L_3^\alpha \leq 5 | L_2^\alpha > 1, A^\alpha = 2) \quad (37)$$

$$P_\alpha(L_3^\alpha \leq 6 | L_2^\alpha > 1, A^\alpha = 2) = 1 \quad (38)$$

$$P_\alpha(L_4^\alpha = 0 | A^\alpha = 2) = 1 \quad (39)$$

Assessments for Attack Type 3

$$0.1 \leq P_\alpha(L_1^\alpha = 0|A^\alpha = 3) \leq 0.6 \quad (40, 41)$$

$$0.7 \leq P_\alpha(L_1^\alpha \leq 1|A^\alpha = 3) \quad (42)$$

$$P_\alpha(L_1^\alpha \leq 2|A^\alpha = 3) = 1 \quad (43)$$

$$0.95 \leq P_\alpha(L_4^\alpha = 0|L_1^\alpha = 0, A^\alpha = 3) \quad (44)$$

$$0.7 \leq P_\alpha(L_4^\alpha \leq 1|L_1^\alpha > 0, A^\alpha = 3) \leq 0.95 \quad (45, 46)$$

$$P_\alpha(L_4^\alpha \leq 2|L_1^\alpha > 0, A^\alpha = 3) = 1 \quad (47)$$

$$0.95 \leq P_\alpha(L_3^\alpha = 0|L_1^\alpha = 0, A^\alpha = 3) \quad (48)$$

$$0.85 \leq P_\alpha(L_3^\alpha = 0|L_1^\alpha > 0, A^\alpha = 3) \leq 0.95 \quad (49, 50)$$

$$0.9 \leq P_\alpha(L_3^\alpha \leq 1|L_1^\alpha > 0, A^\alpha = 3) \quad (51)$$

$$P_\alpha(L_3^\alpha \leq 2|L_1^\alpha > 0, A^\alpha = 3) = 1 \quad (52)$$

$$P_\alpha(L_2^\alpha = 0|A^\alpha = 3) = 1 \quad (53)$$

Assessments for Attack Type 4

$$0.7 \leq P_\alpha(L_4^\alpha = 0|A^\alpha = 4) \leq 0.9 \quad (54, 55)$$

$$0.8 \leq P_\alpha(L_4^\alpha \leq 1|A^\alpha = 4) \leq 0.95 \quad (56, 57)$$

$$0.95 \leq P_\alpha(L_4^\alpha \leq 2|A^\alpha = 4) \quad (58)$$

$$0.99 \leq P_\alpha(L_4^\alpha \leq 3|A^\alpha = 4) \quad (59)$$

$$P_\alpha(L_4^\alpha \leq 4|A^\alpha = 4) = 1 \quad (60)$$

$$P_\alpha(L_1^\alpha = 0|A^\alpha = 4) = 1 \quad (61)$$

$$P_\alpha(L_2^\alpha = 0|A^\alpha = 4) = 1 \quad (62)$$

$$0.8 \leq P_\alpha(L_3^\alpha = 0|L_4^\alpha = 0, A^\alpha = 4) \quad (63)$$

$$0.75 \leq P_\alpha(L_3^\alpha = 0|L_4^\alpha > 0, A^\alpha = 4) \leq 0.9 \quad (64, 65)$$

$$0.95 \leq P_\alpha(L_3^\alpha \leq 1|L_4^\alpha > 0, A^\alpha = 4) \leq 0.99 \quad (66, 67)$$

$$0.99 \leq P_\alpha(L_3^\alpha \leq 2|L_4^\alpha > 0, A^\alpha = 4) \quad (68)$$

$$P_\alpha(L_3^\alpha \leq 5|L_4^\alpha > 0, A^\alpha = 4) = 1 \quad (69)$$

APPENDIX B

PROBABILITY ASSESSMENTS FOR ALTERNATIVE β

Assessments for Attack Type 1

$$0.7 \leq P_\beta(L_1^\beta = 0 | A^\beta = 1) \leq 0.9 \quad (1, 2)$$

$$0.85 \leq P_\beta(L_1^\beta \leq 1 | A^\beta = 1) \leq 0.99 \quad (3, 4)$$

$$0.95 \leq P_\beta(L_1^\beta \leq 2 | A^\beta = 1) \quad (5)$$

$$P_\beta(L_1^\beta \leq 3 | A^\beta = 1) = 1 \quad (6)$$

$$0.95 \leq P_\beta(L_2^\beta = 0 | L_1^\beta = 0, A^\beta = 1) \quad (7)$$

$$0.75 \leq P_\beta(L_2^\beta \leq 1 | L_1^\beta > 0, A^\beta = 1) \leq 0.9 \quad (8, 9)$$

$$0.95 \leq P_\beta(L_3^\beta = 0 | L_1^\beta = 0, A^\beta = 1) \quad (10)$$

$$0.75 \leq P_\beta(L_3^\beta \leq 1 | L_1^\beta > 0, A^\beta = 1) \leq 0.9 \quad (11, 12)$$

$$0.85 \leq P_\beta(L_3^\beta \leq 2 | L_1^\beta > 0, A^\beta = 1) \leq 0.95 \quad (13, 14)$$

$$0.95 \leq P_\beta(L_3^\beta \leq 3 | L_1^\beta > 0, A^\beta = 1) \quad (15)$$

$$0.99 \leq P_\beta(L_3^\beta \leq 4 | L_1^\beta > 0, A^\beta = 1) \quad (16)$$

$$P_\beta(L_4^\beta = 0 | A^\beta = 1) = 1 \quad (17)$$

Assessments for Attack Type 2

$$0.5 \leq P_\beta(L_2^\beta = 0 | A^\beta = 2) \leq 0.8 \quad (18, 19)$$

$$0.75 \leq P_\beta(L_2^\beta \leq 1 | A^\beta = 2) \leq 0.95 \quad (20, 21)$$

$$0.95 \leq P_\beta(L_1^\beta = 0 | L_2^\beta = 0, A^\beta = 2) \quad (22)$$

$$0.75 \leq P_\beta(L_1^\beta \leq 1 | L_2^\beta > 0, A^\beta = 2) \leq 0.9 \quad (23, 24)$$

$$0.85 \leq P_\beta(L_1^\beta \leq 2 | L_2^\beta > 0, A^\beta = 2) \leq 0.99 \quad (25, 26)$$

$$P_\beta(L_1^\beta \leq 3 | L_2^\beta > 0, A^\beta = 2) = 1 \quad (27)$$

$$0.95 \leq P_\beta(L_3^\beta = 0 | L_2^\beta = 0, A^\beta = 2) \quad (28)$$

$$0.8 \leq P_\beta(L_3^\beta \leq 1 | L_2^\beta > 0, A^\beta = 2) \leq 0.95 \quad (29, 30)$$

$$0.9 \leq P_\beta(L_3^\beta \leq 2 | L_2^\beta > 0, A^\beta = 2) \quad (31)$$

$$0.95 \leq P_\beta(L_3^\beta \leq 3 | L_2^\beta > 0, A^\beta = 2) \quad (32)$$

$$0.99 \leq P_\beta(L_3^\beta \leq 4 | L_2^\beta > 0, A^\beta = 2) \quad (33)$$

$$0.2 \leq P_\beta(L_3^\beta \leq 1 | L_2^\beta = 2, A^\beta = 2) \leq 0.5 \quad (34, 35)$$

$$0.4 \leq P_\beta(L_3^\beta \leq 2 | L_2^\beta = 2, A^\beta = 2) \leq 0.7 \quad (36, 37)$$

$$0.6 \leq P_\beta(L_3^\beta \leq 3 | L_2^\beta = 2, A^\beta = 2) \leq 0.9 \quad (38, 39)$$

$$0.7 \leq P_\beta(L_3^\beta \leq 4 | L_2^\beta = 2, A^\beta = 2) \leq 0.95 \quad (40, 41)$$

$$0.8 \leq P_\beta(L_3^\beta \leq 5 | L_2^\beta = 2, A^\beta = 2) \quad (42)$$

$$0.9 \leq P_\beta(L_3^\beta \leq 6 | L_2^\beta = 2, A^\beta = 2) \quad (43)$$

$$0.99 \leq P_\beta(L_3^\beta \leq 7 | L_2^\beta = 2, A^\beta = 2) \quad (44)$$

$$P_\beta(L_4^\beta = 0 | A^\beta = 2) = 1 \quad (45)$$

Assessments for Attack Type 3

$$0.7 \leq P_\beta(L_1^\beta = 0 | A^\beta = 3) \leq 0.9 \quad (46, 47)$$

$$0.85 \leq P_\beta(L_1^\beta \leq 1 | A^\beta = 3) \leq 0.99 \quad (48, 49)$$

$$0.95 \leq P_\beta(L_1^\beta \leq 2 | A^\beta = 3) \quad (50)$$

$$P_\beta(L_1^\beta \leq 3 | A^\beta = 3) = 1 \quad (51)$$

$$0.99 \leq P_\beta(L_4^\beta = 0 | L_1^\beta = 0, A^\beta = 3) \quad (52)$$

$$0.7 \leq P_\beta(L_4^\beta \leq 1 | L_1^\beta > 0, A^\beta = 3) \leq 0.95 \quad (53, 54)$$

$$P_\beta(L_4^\beta \leq 2 | L_1^\beta > 0, A^\beta = 3) = 1 \quad (55)$$

$$0.99 \leq P_\beta(L_3^\beta = 0 | L_1^\beta = 0, A^\beta = 3) \quad (56)$$

$$0.9 \leq P_\beta(L_3^\beta = 0 | L_1^\beta > 0, A^\beta = 3) \leq 0.99 \quad (57, 58)$$

$$0.95 \leq P_\beta(L_3^\beta \leq 1 | L_1^\beta > 0, A^\beta = 3) \quad (59)$$

$$P_\beta(L_3^\beta \leq 2 | L_1^\beta > 0, A^\beta = 3) = 1 \quad (60)$$

$$P_\beta(L_2^\beta = 0 | A^\beta = 3) = 1 \quad (61)$$

Assessments for Attack Type 4

$$0.7 \leq P_\beta(L_4^\beta = 0 | A^\beta = 4) \leq 0.9 \quad (62, 63)$$

$$0.8 \leq P_\beta(L_4^\beta \leq 1 | A^\beta = 4) \leq 0.95 \quad (64, 65)$$

$$0.95 \leq P_\beta(L_4^\beta \leq 2 | A^\beta = 4) \quad (66)$$

$$0.99 \leq P_\beta(L_4^\beta \leq 3 | A^\beta = 4) \quad (67)$$

$$P_\beta(L_4^\beta \leq 4 | A^\beta = 4) = 1 \quad (68)$$

$$P_\beta(L_1^\beta = 0 | A^\beta = 4) = 1 \quad (69)$$

$$P_\beta(L_2^\beta = 0 | A^\beta = 4) = 1 \quad (70)$$

$$0.8 \leq P_\beta(L_3^\beta = 0 | L_4^\beta = 0, A^\beta = 4) \quad (71)$$

$$0.75 \leq P_\beta(L_3^\beta = 0 | L_4^\beta > 0, A^\beta = 4) \leq 0.9 \quad (72, 73)$$

$$0.95 \leq P_\beta(L_3^\beta \leq 1 | L_4^\beta > 0, A^\beta = 4) \leq 0.99 \quad (74, 75)$$

$$0.99 \leq P_\beta(L_3^\beta \leq 2 | L_4^\beta > 0, A^\beta = 4) \quad (76)$$

$$P_\beta(L_3^\beta \leq 5 | L_4^\beta > 0, A^\beta = 4) = 1 \quad (77)$$

APPENDIX C

EQUIPMENT VALUES

Equipment	Value [68]
M16 Rifle	\$1,200
M2 .50 Caliber Machine Gun	\$14,000
M60 7.62mm Machine Gun	\$6,000
AT4 Light Anti-Armor Weapon	\$1,500
AN/PVS-4 Individual Weapon Night Sight	\$4,800
AN/TVS-5 Crew Served Weapon Night Sight	\$4,000
High Mobility Multipurpose Wheeled Vehicle (HMMWV)	\$75,000 ¹

¹The value of the M998 variant is \$50,000. This variant includes no armor plating. An additional \$25,000 was added to the value to account for armor plating and communications equipment.

APPENDIX D

LINEAR PROGRAMMING RESULTS

Alternative α Minimization Primal Solution

i	j	l	m	k	p_{ijklmk}^α	i	j	l	m	k	p_{ijklmk}^α
0	0	0	0	1	0.07125	1	0	0	1	3	0.12
0	0	0	0	2	0.189	2	0	1	0	1	0.0075
0	0	0	0	3	0.019	2	0	2	0	1	0.015
0	0	0	0	4	0.252	2	0	3	0	1	0.015
0	0	15	0	1	0.00375	2	2	4	0	1	0.00375
0	0	15	0	4	0.063	2	3	4	0	1	0.003
0	0	0	2	4	0.0175	2	3	15	0	1	0.00075
0	0	0	3	4	0.00875	2	0	0	1	3	0.006
0	0	1	3	4	0.00525	2	0	0	2	3	0.027
0	0	1	4	4	0.00175	2	0	1	2	3	0.009
0	0	2	4	4	0.0014	2	0	2	2	3	0.018
0	0	5	4	4	0.00035	4	0	0	0	2	0.0105
0	0	15	5	3	0.001	4	0	15	0	2	0.0105
1	0	1	0	1	0.03	4	1	1	0	2	0.018
1	1	1	0	2	0.054	4	1	15	0	2	0.018

Alternative α Minimization Dual Solution

c	π_c	c	π_c
52	-59.5041	22	-6.34895
28	-47.5062	43	-4.82928
27	-47.1690	55	-3.43077
69	-46.7866	60	-2.75566
5	-44.8021	5.3	-2.68875
15	-40.0634	5.6	-2.68662
7	-39.9728	8	-2.61346
9	-38.1606	14	-2.22938
48	-38.1606	51	-2.22938
63	-38.1606	66	-2.22938
16	-27.5136	12	-2.11434
39	-27.5136	49	-2.11434
44	-21.1104	59	-2.11434
38	-16.6252	10	-2.00523
61	-16.4295	45	-2.00523
53	-15.6270	64	-2.00523
62	-14.8206	3	-1.90175
17	-14.3562	42	-1.90175
4	-13.2153	58	-1.90175
47	-8.2747	40	-1.69226
30	-8.2391	26	-0.70971
68	-7.4427	5.4	-0.33845
21	-7.4211	5.11	95.28130

Alternative α Maximization Primal Solution

i	j	l	m	k	p_{ijklmk}^α
0	0	0	0	1	0.09
0	0	0	0	2	0.27
0	0	0	0	3	0.12
0	0	0	0	4	0.315
0	1	0	0	2	0.027
0	0	0	1	4	0.014
0	0	1	1	4	0.00315
0	0	2	1	4	0.00035
0	0	0	2	4	0.0175
1	0	0	0	1	0.042
1	0	0	0	3	0.072
1	0	1	0	3	0.004
1	0	2	0	1	0.009
1	0	3	0	1	0.003
1	2	0	0	1	0.006
1	0	0	2	3	0.004
2	1	0	0	2	0.003

Alternative α Maximization Dual Solution

c	π_c
5.11	98.831000
7	3.705360
11	3.705360
23	3.705360
46	3.705360
2	2.922410
18	2.081080
13	2.005230
41	1.985990
55	1.909920
67	1.901750
50	1.803610
57	1.803610
65	1.803610
5.3	0.960856
5.6	0.887791
5.4	0.374569

Alternative β Minimization Primal Solution

i	j	l	m	k	p_{ijklm}^β	i	j	l	m	k	p_{ijklm}^β
0	0	0	0	1	0.16625	2	2	1	0	2	0.006
0	0	0	0	2	0.1425	2	2	2	0	2	0.0015
0	0	0	0	3	0.0693	2	0	0	1	3	0.006
0	0	0	0	4	0.252	2	0	0	2	3	0.004
0	0	15	0	4	0.063	3	1	1	0	1	0.01875
0	2	15	0	1	0.00875	3	2	2	0	1	0.0075
0	0	0	2	4	0.0175	3	2	2	0	2	0.0075
0	0	0	3	4	0.00875	3	2	3	0	1	0.0075
0	0	1	3	4	0.00525	3	2	3	0	2	0.0075
0	0	1	4	4	0.00175	3	2	4	0	1	0.003
0	0	2	4	4	0.0014	3	2	4	0	2	0.006
0	0	5	4	4	0.00035	3	2	7	0	2	0.0012
0	0	15	5	3	0.0007	3	2	15	0	1	0.00075
1	1	1	0	1	0.0375	3	2	15	0	2	0.0003
1	1	1	0	2	0.1125	3	0	0	2	3	0.002
1	0	0	1	3	0.015	3	0	1	2	3	0.0015
2	1	1	0	2	0.0015	3	0	2	2	3	0.0015
2	1	2	0	2	0.006	5	0	15	0	2	0.0075

Alternative β Minimization Dual Solution

c	π_c	c	π_c	c	π_c
60	-61.9277	22	-9.5355	8	-2.3507
33	-59.1201	55	-8.7250	25	-2.3507
41	-50.3952	7	-7.7799	59	-2.3507
28	-49.7356	76	-7.4427	5.1	-2.3223
77	-46.7866	5.6	-5.2626	11	-2.2294
16	-40.0634	1	-5.1670	57	-2.2294
10	-38.1606	51	-5.0921	74	-2.2294
56	-38.1606	18	-4.6989	29	-2.1143
71	-38.1606	3	-4.1196	50	-2.1143
44	-31.3385	46	-3.8635	67	-2.1143
17	-27.5136	63	-3.4308	23	-2.0052
45	-27.5136	5.3	-3.1703	53	-2.0052
52	-21.1104	68	-2.7557	72	-2.0052
69	-21.1104	15	-2.6135	48	-1.9018
6	-13.2153	32	-2.6135	66	-1.9018
27	-13.2153	13	-2.4786	34	-0.1150
61	-10.1397	31	-2.4786	5.11	97.8573
70	-10.1397				

Alternative β Maximization Primal Solution

i	j	l	m	k	p_{ijklm}^β	i	j	l	m	k	p_{ijklm}^β
0	0	0	0	1	0.2025	0	0	0	2	4	0.0175
0	0	0	0	2	0.1125	1	0	0	0	1	0.01575
0	0	0	0	3	0.18	1	0	0	0	3	0.0168
0	0	0	0	4	0.315	1	0	1	0	3	0.0002
0	1	0	0	2	0.07875	1	0	2	0	1	0.001125
0	2	0	0	2	0.01125	1	0	3	0	1	0.001125
0	2	2	0	2	0.0045	1	2	0	0	1	0.00225
0	2	3	0	2	0.0045	1	0	0	2	3	0.001
0	2	4	0	2	0.001125	2	0	0	0	1	0.00225
0	2	5	0	2	0.001125	2	0	0	0	3	0.002
0	0	0	1	4	0.014	2	1	0	0	2	0.010125
0	0	1	1	4	0.00315	3	1	0	0	2	0.001125
0	0	2	1	4	0.00035						

Alternative β Maximization Dual Solution

c	π_c	c	π_c
41	212.46300	26	2.00523
33	210.11300	47	1.91384
5.11	99.10150	63	1.90992
21	15.23750	75	1.90175
35	3.90698	4	1.80361
9	3.70536	49	1.80361
12	3.70536	58	1.80361
24	3.70536	65	1.80361
54	3.70536	73	1.80361
2	2.55187	5.4	0.68907
39	2.22938	5.8	0.62526
37	2.11434	5.6	0.61732
14	2.00523		

VITA

Gregory Howard Graves earned his Bachelor of Science degree in engineering management from the United States Military Academy at West Point, New York in 1988. In 1998, he received his Master of Science degree in industrial engineering from Texas A&M University at College Station, Texas. He returned to Texas A&M University in 2003 and completed his Doctor of Philosophy in industrial engineering in 2006. He is a lieutenant colonel in the United States Army, and his research interests include decision making, probability, and security. LTC Graves may be reached at Department of Mathematical Sciences, United States Military Academy, 646 Swift Road, West Point, New York 10996. His email address is gregory.graves@us.army.mil.