ESSAYS ON REPUTATION

A Dissertation

by

JUNG HUN CHO

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2006

Major Subject: Economics

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ABSTRACT

Essays on Reputation. (August 2006)

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This dissertation examines reputation, the belief of the decision maker about types of advisors, in incomplete information games with multiple advisors. The decision maker believes that an advisor can be one of two types – an advisor who is biased towards suggesting any particular advice (bad advisor) or an advisor who has the same preferences as the decision maker (good advisor). I explain why it is not always beneficial for the decision maker to seek advice from two advisors simultaneously compared to seeking advice from a single advisor. It is shown that a strong concern for one's reputation not to be perceived as a bad advisor can make the good advisor sometimes give wrong advice. Also, if each type of advisor considers his future important, the decision maker is better off having a single advisor. Then I show that, when dealing with two advisors, it is better for the decision maker to seek advice simultaneously since the possibility of obtaining information is lower in sequential cheap talk. I also examine how an individual's perception of what he thinks of himself (self-reputation) and what others think of him regarding his ability to resist temptation (perception of reputation) affect his actions. It is shown that higher self-reputation and higher perception of reputation help in making resolutions and keeping up with them both in the short and the long run. However, this result requires that individuals find it relatively easy to resist temptation. Also, even those who find it hard to resist temptation can sustain their resolution after telling friends about the resolution in the short run if they value the future more than the present.

To my family

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CHAPTER I

INTRODUCTION

Many decisions are made after obtaining advice from others. Before going to a movie or buying a new computer, we usually seek advice from our friends each of whom has more private information than we do. In many situations, there are some advisors who are biased towards suggesting a particular choice when the information is conveyed by cheap talk. Before going to the college, we ask advice from many people in order to choose the proper field or the proper college for us. Some advisors are biased towards suggesting a particular field or college because they believe that their suggested field or college can increase the chances of obtaining a job in the future regardless of either our interests or market conditions.

As a more specific example, consider a market expert who is not biased towards suggesting any particular pricing policy and the manager of a firm who is also not biased towards using a particular policy. If the manager of the firm has to choose an action under uncertainty, he seeks the advice from an expert. Because of imperfect information about the type of expert, the manager of the firm may believe that an expert is biased towards suggesting the aggressive pricing policy. Call such an expert as aggressive market expert. Consider the case where the expert concludes that an aggressive pricing policy, a price cut, is needed. If a strong concern by the market expert (who has the same preferences as the manager of the firm) for his reputation makes him suggest a defensive strategy, the manager of the firm loses information. The incentive to tell a lie exists because the expert wants the manager to believe what he will suggest next time. In contrast to the result of informative cheap talk,

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The journal model is *IEEE Transactions on Automatic Control.*

even if the market expert has the same preferences as the manager of the firm, the strong reputational concern of the expert leads to the loss of information. The single advisor's model is examined by Morris [15]. If the manager of the firm believes that he may not obtain the correct information from a single market expert, he may try to seek advice from an additional expert under the belief that the presence of the other expert may change the message of the first expert, and vice-versa. In the first chapter, I extend Morris' model [15] by adding one more advisor to the decision maker and examine the effect of the presence of the other advisor to the message of each advisor. I show that to have two advisors each of whom knows the type of the other advisor and sends the message simultaneously to the decision maker is not always beneficial to the decision maker if each advisor considers his future payoff sufficiently more important. By changing some aspects of the previous model, the second chapter examines the possibility of removing the bad reputation effect of the decision maker. Also, by comparing the conditions of the simultaneous cheap talk which guarantees both the good and the bad reputation effect, I determine whether the simultaneous advice or the sequential advice is preferred by the decision maker.

The first question is whether the existence of the other expert changes the message of one expert when the types of experts are mutually known. If so, I can examine whether the existence of the other expert affects the possibility that the first expert will tell a lie or the truth. Although the manager of the firm obtains additional information when he has two experts compared to the case where he has only one expert, the likelihood of telling a lie may increase because of the existence of the other expert. The next question is whether the manager of the firm can benefit if he takes advice from an additional expert when each expert knows the type of the other expert and both experts send their message simultaneously.

I consider a two period cheap talk model with two experts (or advisors) to the decision maker. The advisors can be one of two types - good or bad. Good advisor is assumed to have exactly the same preferences as the decision maker, while the bad advisor has a payoff bias towards one of the two actions available to the decision maker. Each advisor observes a private signal regarding the state of the world, 0 or 1, and then sends a message (0 or 1) to the decision maker who does not have any prior information regarding the state of the world. The decision maker then chooses an action which can affect all players' payoffs. The real state of the world is revealed after the decision maker takes an action. The decision maker then updates his belief about the type of each advisor taking into account the message sent by each advisor and the real state of the world. The same stage game is repeated in the next period with the decision maker again consulting the same advisors. In the example, the expert who has the same preferences as the manager of the firm is referred to the good expert while the expert who is biased towards suggesting the aggressive pricing policy is termed the bad expert. After obtaining the messages from both experts, the manager of the firm determines the price as an action.

The term "good reputation effect" refers to the bad advisor sending a truthful message after receiving a signal that the state of the world is the one he is not biased towards. The term "bad reputation effect" involves the good advisor sending an untrue message after receiving a signal that the state of the world is the one the bad advisor is biased towards. It is important to note that the both of these effects arise from the reputational concern of the advisor to be perceived as a good advisor by the decision maker.

If each advisor has perfect information about the state of the world, I cannot determine the existence of a bad reputation effect. The first chapter shows that both advisors have a reputational incentive to send the message the bad advisor is not biased towards if each advisor has imperfect private information. This is because the decision maker knows the advisors have imperfect information regarding the state of the world, and suggesting the message the bad advisor is not biased towards is the way for good advisor to distinguish himself from bad advisor.

If the bad advisor considers his second period as sufficiently more important, he tells the truth even if he experiences a loss in current payoff when the signal is the one he is not biased towards. When the signal is the one the bad advisor is biased towards, the good advisor tells a lie if he considers his second period as sufficiently more important. There is a greater (lesser) incentive for the good advisor to tell a lie when the other advisor is bad (good). By using numerical examples, it is shown that the existence of the other advisor reinforces the bad reputation effect. By comparing the expected payoff of the decision maker when he has two advisors with that of the decision maker when he has a single advisor, I find that it is better for the decision maker only to have a single advisor if each type of advisor considers his second period as sufficiently more important. If the decision maker is so skeptical and so believes that each type of advisor considers the second period as sufficiently more important, i.e. if he believes each type of advisor sends the message the bad advisor is not biased towards in the first period, he is better off of asking for advice from a single advisor.

By changing some aspects of the model in the first chapter, the second chapter examines the possibility of obtaining the correct advice to the decision maker. In order to explain the contribution of the second chapter, let's consider the specific example of free consultation of the doctor to the patient. There is a patient who is uncertain about his health condition - either medicine is needed or surgery is needed. So, the patient seeks an advice from one doctor. In the example, there are two uncertainties to the patient. First, the patient is uncertain about his health condition. Secondly, he is also uncertain about the type of the doctor. The patient believes that the doctor can be one of two types - good doctor or bad doctor. The good doctor is the person who has payoff incentive to suggest correct advice to the patient. The bad doctor is the person who is biased towards suggesting surgery.

Let's also consider the case where the patient who needs surgery meets a doctor who is of the good type. If the good doctor suggests surgery, the belief of the patient that the doctor is of the bad type is increased. If the good doctor considers his future payoff sufficiently more important, he may suggest medicine to the patient. In this case, the patient loses information about his health condition. When the patient knows that he may lose information by having a single doctor, he may try to obtain an additional advice from having an additional doctor.

When the patient meets the second doctor, he may or may not tell the advice of the first doctor to the second doctor. If the patient does not tell the advice of the first doctor to the second doctor but only tells the existence of the previous doctor, it is a simultaneous cheap talk. I have studied this situation in the first chapter. In many cases, when the patient meets the second doctor, he also tells the advice sent by the first doctor to the second doctor. Since each doctor works in the same medicine field, each doctor may know the type of the other doctor. Or, each doctor may have a belief about the type of the other doctor. For simplicity, it is assumed that each doctor knows the type of the other doctor and the second advisor knows the advice sent by the first doctor.

In my first chapter, it is shown that the patient sometimes loses information if he seeks advice from two doctors simultaneously. However, that is the case where each doctor considers his reputation to be perceived as the doctor who has payoff incentive to suggest the correct advice. I want to examine that the loss of information is vanished by asking advice sequentially. The first question is if there is an incentive for the second doctor to tell a lie or tell the truth when the second doctor knows the advice sent by the first doctor. If there may be the loss of information in sequential cheap talk model, I examine the welfare of the patient - I examine if it is better for him to seek advice simultaneously or sequentially.

I examine a two period cheap talk model with two advisors. Each advisor observes imperfect private signal about the state of the world (0 or 1). The decision maker tells the existence of the other advisor when he meets the first advisor. After receiving the message from the first advisor, the decision maker tells both the existence of the previous advisor and the message sent by the first advisor to the second advisor. The decision maker chooses the action in each period after obtaining the messages from two advisors. The real state of the world is revealed after the action of the decision maker is determined.

In each period, the message of the first advisor is determined by the type of the advisor. In case of the second advisor, the message is determined by the type of each advisor and the message sent by the first advisor. Given messages from two advisors, the action of the decision maker is determined in each period. Since the decision maker is uncertain about the preferences of each advisor, his action is affected by not only the messages from two advisors but also the type of each advisor. Then, the payoff of each type of the advisor in each period is determined by the action of the decision maker.

By comparing the updated belief of the decision maker about the type of the advisor in each message of the first period, it is shown that the first advisor has an incentive to suggest the message the bad advisor is not biased towards in order to increase his reputation in the first period. Given the message of the first advisor, the second advisor can increase his reputation by sending the message the bad advisor is not biased towards in the first period.

Let me consider the case where the second advisor is of the bad type and observes the signal the bad advisor is not biased towards. If the advisor sends the message he is biased towards, he can increase his current payoff. However, he loses his reputation by doing so. By considering the total payoff of the second advisor who knows the message sent by the first advisor, I find that the bad advisor sometimes tells the truth in the first period regardless of the message of the first advisor and the type of the first advisor. Since the bad advisor tells the truth even if he has a loss in his payoff in current period to increase his reputation, it is called as the good reputation effect. The possibility of the existence of the good reputation effect is greater (or lesser) in sequential cheap talk if the second advisor receives the message the bad advisor is not biased towards (or the bad advisor is biased towards) from the first advisor, compared to the simultaneous cheap talk.

Next is the case where the second advisor is of the good type and observes the signal the bad advisor is biased towards. By sending the message truthfully, he may obtain high payoff in the current period. However, the possibility of being perceived as the advisor who is biased towards suggesting the same message is increased by sending the truthful message. If the second advisor considers his second period sufficiently more important, he tells a lie in the first period regardless of both the type of the previous advisor and the message of the previous advisor. Such as the bad advisor sometimes tells the truth to increase his reputation, the good advisor who knows the message sent by the previous advisor tells a lie to make the decision maker believe what he will suggest in the next period. Compared to the simultaneous cheap talk, the possibility of telling a lie in sequential cheap talk is greater regardless of the message of the first advisor. It is because the advisor does not want to be perceived as bad advisor and wants to separate his type from the bad type. If the decision maker seeks advice from two advisors and believes that each advisor considers his reputation to be perceived as the advisor who sends the message truthfully, it is better for the decision maker to seek the advice simultaneously. It is because the advisor who knows the message of the previous advisor can adjust his message more easily in sequential cheap talk and the possibility of telling a lie in sequential cheap talk is always greater than that in simultaneous cheap talk regardless of both the type and the message of previous advisor.

In cheap talk models, the question is when there exist equilibria where the cheap talk from the sender (the advisor) to the decision maker is informative.[1] To make cheap talk informative, three necessary conditions are needed. The different sender types have different preferences over the actions of the decision maker, the decision maker prefers the different actions depending on the sender's type, and the decision maker's preferences over actions cannot be completely opposed to the sender's (Gibbons [9]). Crawford and Sobel [6] examine the strategic information transmission from one advisor to the decision maker that satisfies these three conditions. By characterizing partially pooling equilibria, they find that more communication is possible when the preferences of the two players are more closely aligned, and the perfect information is conveyed when two players have the same preferences. Compared to the single advisor's cheap talk model, it is also possible to consider the welfare effect of the decision maker in multiple advisors' cheap talk model. Krishna and Morgan [12] examine a one period cheap talk model with two advisors who send messages sequentially. By comparing the payoff of the decision maker when he has a single advisor with that of the decision maker when he has two advisors, they show that it is never beneficial to consult both advisors if both advisors are biased in the same direction.

[1] Battaglini [3],[4] and and Levy and Razin [13] consider multidimensional cheap talk models. In Olszewski [16], the decison maker also has private information regarding the state of the world. Park [18] finds the conditions that make cheap talk informative in infinitely repeated cheap talk game.

However, if the two advisors are biased in opposite directions, it is always beneficial to consult both advisors. Since they examine one period model, each advisor has no reputational concern when he sends the message.

Consultants will most likely be concerned about their reputation if they are engaged in a repeated interaction with the decision maker.[2] Sobel [20] considers a finite cheap talk game where there is a single advisor who can be one of the two types - enemy (an informed advisor with completely opposing interests to the decision maker) or friend (with identical interests to the decision maker), and finds that there is an incentive for an enemy to behave like a friend in order to increase his reputation.[3] Even if he considers the good reputation effect, the enemy is different as the bad advisor because the bad advisor is the person who is biased towards suggesting any particular strategy. Such as the bad advisor sometimes tells the truth to increase his reputation, the good advisor who has the same preferences as the decision maker sometimes tells a lie to increase his reputation in the case where there is one advisor to the decision maker. Morris [15] considers a two period cheap talk model with one advisor having imperfect information regarding the state of the world. In equilibrium condition, an advisor has reputational incentive to send the particular message the bad advisor is not biased towards in order to separate his type from the bad type regardless of the signal. Especially if an advisor considers the second period as sufficiently more important, no information is conveyed in the first period.[4]

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[2]Kreps and Wilson [11] and Milgrom and Roberts [14] examined Selten's chain store game [19] and developed the theoretical model of the reputation where long run player meets a sequence of the short run players.

[3]In Ottaviani and Sorensen [17], the real state of the world is revealed to the decision maker before the decision maker chooses his action. The action of the decision maker is evaluation of the advisor by comparing the message to the real state of the world.

[4]Ely and Valimaki [7] consider a model where long lived mechanic interacts with a sequence of short run motorists. Bad reputation effect can emerge if there is imperfect

The last chapter examines a resolution model. We often make a promise to ourselves (a resolution) and try to keep it. Simple examples include deciding to quit smoking or starting a diet. However, several of us find it very hard to sustain the resolution, or even to make it in the first place because of the strong temptation to keep indulging in the particular mode of behavior. Benabou and Tirole [5] develop a model of resolution and focus on: (a) who will take up a resolution, (b) assuming an agent is somehow constrained to sustain the resolution in the short run, does this external constraint help the agent to stick to the resolution in the long run, and (c) how imperfect recall of past failures in making resolutions affects the chances of making resolutions in the future.[5]

Instead of using external constraint to the decision maker in order to resist impulses in the short run, I make the external constraint endogenous to the model by considering the possibility of telling friends about the resolution as an external commitment.[6] In the last chapter, I consider not only the effect of the belief of an individual that he has strong ability to endure impulses but also the effect of the belief that his friends believe him as a man of strong will. By examining how these effects can change choices of an individual, I try to ask: (a) assuming one has made a resolution, what does it take to sustain it in both the short run and the long run, (b) why is it that people keep making resolutions but (i) often they succeed in keeping up with the resolution in the short run but not in the long run, and (ii) sometimes can't sustain it even in the short run.

information to the motorists regarding the type of the mechanic.

[5]Personal rules have been discussed by Ainslie [1]. Benabou and Tirole [5] developed a model of personal rules based on self-reputation.

[6]This is main difference from other self-control papers of Gul and Pesendorfer [10] and Fudenberg and Levine [8] because I can examine the effect of both self-reputation effect and perception of reputation to the decisions of an individual.

There is one decision maker who wants to be perceived as a man of strong will. A two period model is developed because it is not possible to analyze whether the decision maker succeeds in carrying out his resolution with a single period model. Both periods of the model are divided into three sub-periods. First, whether to follow the temptation or not. Second, whether to tell friends about one's resolution or not. Third, whether to persevere or not. The first and second sub-periods of each period can be thought of as the *Normal* time, and the third as the *Stress* time. The idea behind this distinction is that making resolutions, and telling about them is relatively easier than keeping up with them.

Since there is uncertainty about the type of the decision maker, the decision maker can be one of two types: high type or low type. The high type decision maker has strong ability to endure impulses in the stress time. If the decision maker chooses to follow the temptation (FT) in the first period, it means that he has not made the resolution. Since the focus of the last chapter is to understand what it takes to sustain a resolution, I mainly focus on the case where the decision maker chooses not to follow his temptation (NFT) in the first period. If the decision maker chooses to follow his temptation (FT) at the beginning of the second period, he is considered as low type because he cannot keep the promise with himself in the long run. It is possible that the decision maker chooses not to follow temptation (NFT) at the beginning of the second period but chooses not to persevere (NP) at the end. In this case, the decision maker is also termed as low type. In the last chapter, the high type decision maker is the person who chooses not to follow his temptation (NFT) and to persevere (P) in the second period.

Because of imperfect information regarding the type of the decision maker, I examine two different types of the beliefs of the decision maker: self-reputation and perception of reputation. The decision maker believes that he is high type with some probability. The belief of the decision maker that he is high type is termed as self-reputation. The decision maker also believes that his friends believe him as high type with some probability. The belief of the decision maker that his friends believe him as high type is called as the perception of reputation. These two beliefs are updated by the choices of the decision maker made in the first period.

The chapter starts with the detailed description of the model and the assumptions. By the backward induction, I explain the analysis of the model. The interesting results are

(1) Higher self-reputation, and higher perception of reputation help an agent persevere in the first period, again take up a resolution at the beginning of the second period, and persevere till the end.

(2) Among the low types, there are some who make a resolution again at the beginning of the second period (by choosing not to follow his temptation), but choose not to persevere in the second period. Further, the decision maker who values the future high enough compared to the present, chooses to tell to his friends and to persevere in the first period.

CHAPTER II

MULTIPLE ADVISORS WITH REPUTATION

By adding one more advisor in Morris' model [15] and examining the simultaneous cheap talk, I explain why it is not always beneficial for the decision maker to seek advice from two advisors. I consider a two period cheap talk model where the decision maker has two advisors, each of whom knows the type of the other advisor. It is assumed that the decision maker does not know the state of the world and seeks the advice of both advisors. After receiving a private signal regarding the state of the world, both advisors simultaneously send a costless message to the decision maker.

A. Model

The state of the world in period i is $\omega_i \in W = \{0, 1\}$ for $i = 1, 2$. Each state is equally likely, i.e. $P(\omega_i = 0) = \frac{1}{2} = P(\omega_i = 1)$. The advisor j (for $j = 1, 2$) observes a signal S_i^j regarding the state of the world in period i. The decision maker does not know the type of advisors. The decision maker believes the advisor j is good with probability λ_i^j in each period i. With probability $1 - \lambda_i^j$, the decision maker believes the advisor j is bad. After observing the signal, advisor j sends the message m_i^j to the decision maker. After obtaining messages from both advisors, the decision maker chooses his action $a_i \in R$ which affects all players' payoffs. After the decision maker chooses his action, the state of the world in period i is revealed publicly. The message in the first period plays the additional role of changing the belief of the decision maker about the type of the advisor. After receiving the message from each advisor and the state of the world in the first period, the decision maker updates the belief that the advisor j is good, $\lambda_2^j = \lambda_2^j(\lambda_1^j, m_1^j, \omega_1)$, where λ_1^j is prior belief of the decision maker that advisor j is good.

The utility function of the decision maker who chooses the action and the good advisor is assumed to be $-(a_i - \omega_i)^2$ in each period i. Since it is assumed that the decision maker does not know the state of the world, the decision maker chooses the action a_i which is the probability that the state of the world is 1 given messages from both advisors. The utility function of the bad advisor is assumed to be a_i in each period i. The payoff obtained by the bad advisor in period i is the greatest if the decision maker chooses action 1. Each type of advisor may put different weights on each period. The total utility of advisor j, if he is of the good type, is

$$-x_1^j(a_1 - \omega_1)^2 - x_2^j(a_2 - \omega_2)^2,$$

where x_1^j and x_2^j denote the weights on the payoffs in the first period and in the second period respectively. The total utility of advisor j, if he is of the bad type, is

$$y_1^j a_1 + y_2^j a_2,$$

where y_1^j and y_2^j denote the weights on the payoffs in the first period and in the second period respectively. It is assumed that the sum of the weight in each period is 1.

I will use backward induction to solve the model. I first solve for the action taken by the decision maker after receiving messages from both advisors during the second period. Knowing the decision maker's action for each message in the second period, I am able to determine the value function for each type of the advisor. While sending the message in the first period, the advisors consider not only their payoffs in the first period, but also the expected payoffs in the second period which are determined by the value function.

If each advisor observes a perfect signal regarding the state of the world, each advisor has no reputational incentive to tell a lie in the first period. When the state of the world in the first period is revealed as 1, I cannot guarantee the updated belief of the decision maker that an advisor is good increases if advisor sends the message 0. It is because the decision maker will know that an advisor is a liar and will not believe what he will say in the next period. Each advisor has an incentive to send the message truthfully in the first period if he wants the decision maker to believe what he will say in the next period.

Consider the case where advisor j observes an imperfect signal S_i^j in period i regarding the state of the world ω_i. Let γ denote the probability that the state of the world is the same as the signal received by each advisor, i.e. $\gamma = P(S_i = \omega_i)$. The signal is imperfect but informative, such that $\frac{1}{2} < \gamma < 1$. Since it is assumed that each advisor has the same ability to obtain the signal regarding the state of the world, the same probability that the state of the world is the same as the signal received by an advisor is applied to each advisor.

1. Second Period

There is always a babbling equilibrium in cheap talk models. Since the main question in cheap talk models is under what conditions the cheap talk can be informative, I examine the informative equilibrium in the model. In any informative equilibrium in the second period, since this is the last period each advisor does not consider his reputation. The advisor j, if he is of the good type, sends the message $m_2^j = k$ when his signal in the second period is k for $k = 0$ or 1. If the advisor j is of the bad type, regardless of the signal he sends the message $m_2^j = 1$.

In order to determine the value function in any informative equilibrium, the probability that the state of the world in the second period is 1 given messages from the advisors must be calculated. As discussed before, the decision maker knows that the bad advisor will never send the message 0 in the second period. The probability that the state of the world is 1 given the message 0 from both advisors in the second period is given by

$$P^{2,1}_{0,0} = \frac{(1-\gamma)^2}{\gamma^2 + (1-\gamma)^2}$$

where $P^{i,1}_{m_i^1, m_i^2}$ represents the probability that the state of the world is 1 in period i given the message of the first advisor m_i^1 and the message of the second advisor m_i^2. Since the action of the decision maker is the probability that the state of the world is 1 given messages from both advisors, the decision maker chooses action $P^{2,1}_{0,0}$ when he receives the message 0 from both advisors.

If the decision maker receives $m_2^1 = 0$ and $m_2^2 = 1$ in second period, he will be able to infer that the first advisor is good but the second advisor may be of either good or bad type. The belief of the decision maker that the state of the world is 1 becomes

$$P^{2,1}_{0,1} = \frac{(1-\gamma)(1-\lambda_2^2 + \gamma\lambda_2^2)}{1-(1-2\gamma+2\gamma^2)\lambda_2^2}$$

and the decision maker chooses action $P^{2,1}_{0,1}$ when he receives the message 0 from the first advisor and 1 from the second advisor.

If the decision maker receives the message 1 from the first advisor and the message 0 from the second advisor, the probability that the state of the world in the second period is 1 becomes

$$P^{2,1}_{1,0} = \frac{(1-\gamma)(1-\lambda_2^1 + \gamma\lambda_2^1)}{1-(1-2\gamma+2\gamma^2)\lambda_2^1}$$

and the decision maker chooses action $P^{2,1}_{1,0}$.

When the decision maker receives the message 1 from both advisors, then he believes both advisors may be good, both may be bad, or only one advisor may be good. The probability that the state of the world is 1 will be

$$P^{2,1}_{1,1} = \frac{1 - (1-\gamma)(\lambda^1_2 + \lambda^2_2) + (1-\gamma)^2 \lambda^1_2 \lambda^2_2}{2 - \lambda^1_2 - \lambda^2_2 + (1 - 2\gamma + 2\gamma^2)\lambda^1_2 \lambda^2_2}$$

and the decision maker chooses action $P^{2,1}_{1,1}$ when he receives the message 1 from both advisors.

The value function of the bad advisor is determined using the action of the decision maker given that the message of bad advisor is 1. Since each advisor knows the type of the other advisor, let's start to calculate the value function of the bad advisor when the bad advisor knows the other advisor is of the good type. Suppose that the first advisor is of the bad type. When the bad advisor knows that the other advisor is good, the value function for the bad advisor is

$$v^1_{BG}[\lambda^1_2, \lambda^2_2] = y^1_2 a_2 = \frac{1}{2} y^1_2 (P^{2,1}_{1,0} + P^{2,1}_{1,1}).$$

It is because the bad advisor sends the message 1 regardless of the signal and the good advisor sends the message which is the signal he observes in each state of the world. In each state of the world, each advisor obtains the correct signal with probability γ. When the bad advisor knows that the other advisor is also of the bad type, the value function of the bad advisor is

$$v^1_{BB}[\lambda^1_2, \lambda^2_2] = y^1_2 a_2 = y^1_2 P^{2,1}_{1,1}.$$

It is because both bad advisors send the message 1 regardless of signal.

The value function of the good advisor is determined using the action of the decision maker in each state of the world. Suppose that the first advisor is of the good type. The value function for the good advisor is

$$\begin{aligned}
v^1_{GB}[\lambda^1_2, \lambda^2_2] &= -x^1_2(a_2 - \omega_2)^2 \\
&= -\frac{1}{2}x^1_2[\gamma\{(P^{2,1}_{0,1})^2 + (P^{2,1}_{1,1} - 1)^2\} \\
&\quad +(1-\gamma)\{(P^{2,1}_{1,1})^2 + (P^{2,1}_{0,1} - 1)^2\}]
\end{aligned}$$

when he knows that the other advisor is of the bad type. With probability γ, the good advisor obtains the signal which is the same as the state of the world in each state of the world. Also, the good advisor knows that the other advisor sends the message 1 regardless of the signal. When the good advisor knows that the other advisor is also of the good type, the value function of the good advisor is

$$\begin{aligned}
v^1_{GG}[\lambda^1_2, \lambda^2_2] &= -x^1_2(a_2 - \omega_2)^2 \\
&= -\frac{1}{2}x^1_2[\gamma^2\{(P^{2,1}_{0,0})^2 + (P^{2,1}_{1,1} - 1)^2\} \\
&\quad +\gamma(1-\gamma)\{(P^{2,1}_{0,1})^2 + (P^{2,1}_{1,0})^2 \\
&\quad +(P^{2,1}_{1,0} - 1)^2 + (P^{2,1}_{0,1} - 1)^2\} \\
&\quad +(1-\gamma)^2\{(P^{2,1}_{1,1})^2 + (P^{2,1}_{0,0} - 1)^2\}].
\end{aligned}$$

With probability γ^2, both advisors obtain the signal which is the same as the state of the world in each state of the world. One advisor obtains the signal which is the same as the state of the world and the other advisor is misinformed with probability $\gamma(1-\gamma)$ in each state of the world.

Irrespective of his type and the type of the other advisor, the value function for an advisor is increasing with the updated belief of the decision maker that an advisor is good. If the message in the first period increases the updated belief of the decision maker that the advisor is good, it can also increase the value function in the second period.

2. First Period

In the first period, the payoff of the bad advisor is either

$$y_1^1 a_1 + v_{BG}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)]$$

or

$$y_1^1 a_1 + v_{BB}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)]$$

by following the type of the other advisor. The payoff of the good advisor is expressed as

$$-x_1^1(a_1 - \omega_1)^2 + v_{GB}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)]$$

or

$$-x_1^1(a_1 - \omega_1)^2 + v_{GG}^1[\lambda_2^1(\lambda_1^1, m_1^1, \omega_1), \lambda_2^2(\lambda_1^2, m_1^2, \omega_1)].$$

Suppose that good advisor sometimes tells a lie. The bad advisor also sometimes tells a lie, i.e. he does have a current payoff incentive to tell a lie if he observes the signal 0 and have a reputational incentive to tell a lie if he observes the signal 1. If advisor j is good, he sends the message 0 when his signal is 0, and sends the message 1 with probability z when the signal is 1 because bad advisor has payoff incentive to send the message 1 and good advisor sometimes tells a lie not to be perceived as bad advisor. If advisor j is bad, he sends the message 1 with probability ν when his signal is 0 because he has current payoff incentive to send the message 1. He sends the message 1 with probability ρ when the signal is 1 not to be perceived as bad advisor. It is shown that the probability that good advisor sends the message 0 if his signal is 1 increases with the probability that bad advisor sends the message 1 if his signal is 1. Since this model examines the case where the good advisor can distinguish his type from the bad type by sending the message the bad advisor is not biased towards, let us consider the case where the bad advisor sends the message 1 more often than good advisor, i.e. $\rho \geq z$. By sending the message 1, the likelihood that the advisor is bad increases.

By using Bayes' rule, the decision maker calculates the updated belief about the type of each advisor by using the message and the revealed state of the world. The updated belief of the decision maker that advisor j is good, when the decision maker receives the message 1 and the real state of the world is revealed as 1, is

$$\lambda_2^j(\lambda_1^j, 1, 1) = \frac{\lambda_1^j \gamma z}{\lambda_1^j \gamma z + (1-\lambda_1^j)\{\gamma\rho + (1-\gamma)\nu\}}.$$

If the decision maker receives the message 1 from advisor j, he believes that the advisor j is either of the good type or of the bad type. The real state of the world is revealed as 1. The advisor j obtains the signal 1 with probability γ. The decision maker believes that the advisor j, if advisor j is of the good type, sends the message 1 with probability γz. He also believes that the advisor j, if advisor j is of the bad type, sends the message 1 with probability $\{\gamma\rho + (1-\gamma)\nu\}$. By using the same method, the updated belief of the decision maker for each message and each state of the world is

$$\lambda_2^j(\lambda_1^j, 0, 1) = \frac{\lambda_1^j (1-\gamma z)}{\lambda_1^j (1-\gamma z) + (1-\lambda_1^j)\{1-\gamma\rho - (1-\gamma)\nu\}},$$

$$\lambda_2^j(\lambda_1^j, 0, 0) = \frac{\lambda_1^j \{1-(1-\gamma)z\}}{\lambda_1^j \{1-(1-\gamma)z\} + (1-\lambda_1^j)\{1-(1-\gamma)\rho - \gamma\nu\}}$$

and

$$\lambda_2^j(\lambda_1^j, 1, 0) = \frac{\lambda_1^j (1-\gamma) z}{\lambda_1^j (1-\gamma) z + (1-\lambda_1^j)\{(1-\gamma)\rho + \gamma\nu\}}.$$

Proposition 1 *Regardless of the state of the world in the first period, advisor j has reputational incentive to announce 0 because*

$$\lambda_2^j(\lambda_1^j, 0, 0) > \lambda_1^j > \lambda_2^j(\lambda_1^j, 1, 0)$$

and

$$\lambda_2^j(\lambda_1^j, 0, 1) > \lambda_1^j > \lambda_2^j(\lambda_1^j, 1, 1).$$

Under the condition that bad advisor sends the message 1 more often than good advisor, each type of advisor has reputational incentive to send the message 0 in the first period. Even if real state of the world in the first period is revealed as 1, sending the message 0 is the way to increase the reputation. It is because each advisor knows that there is imperfect signal regarding the state of the world and to send the message 0 is the way to separate his type from the bad type. In the example, each type of market expert has reputational incentive to suggest the defensive pricing policy to the manager of the firm regardless of his signal. It is because the likelihood that the expert is of the aggressive type decreases when the expert suggests the defensive pricing strategy. Proposition 1 will help me consider both good and bad reputation effects.

In order to consider the payoff in the first period, the belief of the decision maker that the state of the world in the first period is 1 given each pair of messages is calculated. If the decision maker receives the message 0 from both advisors, the probability that the state of the world is 1 is

$$P_{0,0}^{1,1} = \frac{Q_{0,0}^{1,1}}{Q_{0,0}^{1,0} + Q_{0,0}^{1,1}}$$

where $Q_{m_1^1, m_1^2}^{1,l}$ represents the conditional probability that the message of the first advisor is m_1^1 and the message of the second advisor is m_1^2 given that the state of the world in the first period is l, and

$$\begin{aligned} Q^{1,0}_{0,0} &= [\lambda^1_1\{1-(1-\gamma)z\}+(1-\lambda^1_1)\{1-(1-\gamma)\rho-\gamma\nu\}]\times \\ &\quad [\lambda^2_1\{1-(1-\gamma)z\}+(1-\lambda^2_1)\{1-(1-\gamma)\rho-\gamma\nu\}] \end{aligned}$$

and

$$\begin{aligned} Q^{1,1}_{0,0} &= [\lambda^1_1(1-\gamma z)+(1-\lambda^1_1)\{1-\gamma\rho-(1-\gamma)\nu\}]\times \\ &\quad [\lambda^2_1(1-\gamma z)+(1-\lambda^2_1)\{1-\gamma\rho-(1-\gamma)\nu\}]. \end{aligned}$$

The probability that both advisors send the message 0 is calculated by considering the 32 possible cases. In each state of the world, the good advisor sends the message 0 if his signal is 0, and sends the message 0 with probability $1-z$ if the signal is 1. The bad advisor sends the message 0 with probability $1-\nu$ if his signal is 0, and sends the message 0 with probability $1-\rho$ if the signal is 1. The decision maker believes that each advisor may be either of the good or of the bad type. The decision maker chooses action $P^{1,1}_{0,0}$ when he receives the message 0 from both advisors in the first period.

The probability that the state of the world in the first period is 1 when the decision maker receives $m^1_1=0$ and $m^2_1=1$ is

$$P^{1,1}_{0,1}=\frac{Q^{1,1}_{0,1}}{Q^{1,0}_{0,1}+Q^{1,1}_{0,1}}$$

where

$$\begin{aligned} Q^{1,0}_{0,1} &= [\lambda^1_1\{1-(1-\gamma)z\}+(1-\lambda^1_1)\{1-(1-\gamma)\rho-\gamma\nu\}]\times \\ &\quad [\lambda^2_1(1-\gamma)z+(1-\lambda^2_1)\{(1-\gamma)\rho+\gamma\nu\}] \end{aligned}$$

and

$$\begin{aligned} Q_{0,1}^{1,1} &= [\lambda_1^1(1-\gamma z) + (1-\lambda_1^1)\{1-\gamma\rho-(1-\gamma)\nu\}] \times \\ & \quad [\lambda_1^2 \gamma z + (1-\lambda_1^2)\{\gamma\rho+(1-\gamma)\nu\}]. \end{aligned}$$

Since the good advisor who observes the signal 0 never sends the message 1, the decision maker believes that the second advisor is either good advisor who observes the signal 1 or bad advisor in each state of the world. The decision maker chooses action $P_{0,1}^{1,1}$ when he receives the message 0 from the first advisor and the message 1 from the second advisor in the first period.

The probability that the state of the world in the first period is 1 if the decision maker receives $m_1^1 = 1$ and $m_1^2 = 0$ is

$$P_{1,0}^{1,1} = \frac{Q_{1,0}^{1,1}}{Q_{1,0}^{1,0} + Q_{1,0}^{1,1}}$$

where

$$\begin{aligned} Q_{1,0}^{1,0} &= [\lambda_1^1(1-\gamma)z + (1-\lambda_1^1)\{(1-\gamma)\rho+\gamma\nu\}] \times \\ & \quad [\lambda_1^2\{1-(1-\gamma)z\} + (1-\lambda_1^2)\{1-(1-\gamma)\rho-\gamma\nu\}] \end{aligned}$$

and

$$\begin{aligned} Q_{1,0}^{1,1} &= [\lambda_1^1 \gamma z + (1-\lambda_1^1)\{\gamma\rho+(1-\gamma)\nu\}] \times \\ & \quad [\lambda_1^2(1-\gamma z) + (1-\lambda_1^2)\{1-\gamma\rho-(1-\gamma)\nu\}]. \end{aligned}$$

When the decision maker receives the message 1 from the first advisor and the message 0 from the second advisor, he chooses action $P_{1,0}^{1,1}$ in the first period.

If both advisors send the message 1, the probability that the state of the world in the first period is 1 is

$$P_{1,1}^{1,1} = \frac{Q_{1,1}^{1,1}}{Q_{1,1}^{1,0} + Q_{1,1}^{1,1}}$$

where

$$\begin{aligned} Q_{1,1}^{1,0} &= [\lambda_1^1(1-\gamma)z + (1-\lambda_1^1)\{(1-\gamma)\rho + \gamma\nu\}] \times \\ &\quad [\lambda_1^2(1-\gamma)z + (1-\lambda_1^2)\{(1-\gamma)\rho + \gamma\nu\}] \end{aligned}$$

and

$$\begin{aligned} Q_{1,1}^{1,1} &= [\lambda_1^1\gamma z + (1-\lambda_1^1)\{\gamma\rho + (1-\gamma)\nu\}] \times \\ &\quad [\lambda_1^2\gamma z + (1-\lambda_1^2)\{\gamma\rho + (1-\gamma)\nu\}] \end{aligned}$$

and the decision maker chooses action $P_{1,1}^{1,1}$ in the first period. The decision maker believes that the advisor who sends the message 1 is either of the good or of the bad type. Since good advisor who observes the signal 0 never sends the message 1, the decision maker infers that the good advisor who sends the message 1 observes the signal 1. He also infers that the bad advisor who observes the signal either 0 or 1 sends the message 1 with some probability in each state of the world.

Under the conditions that each advisor has imperfect information regarding the state of the world and has perfect information regarding the type of the other advisor, I first examine the existence of good reputation effect. Let us consider the case where the first advisor who is of the bad type observes the signal 0 and knows that the second advisor is good. The bad advisor believes that the second advisor sends the message 0 with probability 1 if his signal is 0, or sends the message 1 with probability z if his signal is 1. Since each advisor obtains the correct signal regarding the state of the world with probability γ, the belief of the first advisor that the signal of the second advisor is the same as that of the first advisor is $\frac{1}{2}$.

The bad advisor's total utility of telling the truth ($m_1^1 = 0$) when he observes the signal 0 is

$$\frac{1}{2}[y_1^1\{2P_{0,0}^{1,1} + z(P_{0,1}^{1,1} - P_{0,0}^{1,1})\}$$
$$+\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}\upsilon_{BG}^1[R_\epsilon\lambda_2^1(\lambda_1^1, 0, \epsilon),$$
$$\frac{1}{2}\{R_\epsilon\lambda_2^2(\lambda_1^2, 0, \epsilon) + (1 - R_\epsilon)z_\zeta\lambda_2^2(\lambda_1^2, \zeta, \epsilon)\}]]$$

where $R_0 = \gamma$, $R_1 = 1 - \gamma$, $z_0 = 1 - z$ and $z_1 = z$. The bad advisor who sends the message 0 believes that the other advisor sends the message 0 if the signal of the other advisor is the same as his own signal. He also believes that the other advisor sends the message 0 with probability $1-z$ if the signal of the other advisor is different from his own signal. When the real state of the world is revealed to be 1, i.e. when the bad advisor is misinformed, he needs to consider the cases where the other advisor obtains the correct signal or is also misinformed. Similarly, the bad advisor needs to consider the case where the other advisor obtains the correct signal or is misinformed when the real state of the world is revealed to be 0, i.e. when the bad advisor obtains the correct signal. The total utility to the bad advisor who observes the signal 0 when he tells a lie ($m_1^1 = 1$) is

$$\frac{1}{2}[y_1^1\{2P_{1,0}^{1,1} + z(P_{1,1}^{1,1} - P_{1,0}^{1,1})\}$$
$$+\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}\upsilon_{BG}^1[R_\epsilon\lambda_2^1(\lambda_1^1, 1, \epsilon),$$
$$\frac{1}{2}\{R_\epsilon\lambda_2^2(\lambda_1^2, 0, \epsilon) + (1 - R_\epsilon)z_\zeta\lambda_2^2(\lambda_1^2, \zeta, \epsilon)\}]].$$

If the bad advisor only considers his payoff in the first period, i.e. $y_1^1 = 1$ and $y_2^1 = 0$, then he will send the message 1 after observing the signal 0. Even if the bad advisor weights the two periods equally, i.e. $y_1^1 = \frac{1}{2} = y_2^1$, he will prefer telling a lie. If the bad advisor only considers his second period payoff, i.e. $y_1^1 = 0$ and $y_2^1 = 1$, the payoff when he tells the truth is greater than the payoff when he tells a lie. Truth telling is possible if y_2^1 is sufficiently large. The critical value of y_1^1 which guarantees the existence of the good reputation effect is calculated as a function of parameters.

Proposition 2 *There is good reputation effect for the advisor who observes the signal 0 and knows that the other advisor is good if he considers his second period as sufficiently more important (see Appendix A).*

If the bad advisor who knows that the other advisor is good strongly considers his reputation, then after observing the signal 0 in the first period he sends the message 0. This reputational concern that makes the bad advisor tell the truth is referred to the good reputation effect. In the example of the market expert and the manager of the firm, when the aggressive expert concludes that the defensive pricing policy is needed, the strong reputational concern of the expert not to be perceived as an aggressive expert makes him suggest the defensive pricing strategy to the manager of the firm even if he has loss in the current payoff. This result holds when the aggressive expert (bad expert) knows that the other expert is good expert.

Let us consider a numerical example to examine the relationship between the probability that the bad advisor sends the message truthfully if his signal is 0 (ν) and the belief of the decision maker about the type of each advisor, λ_1^1 and λ_1^2. It is supposed that the first advisor is of the bad type and the second advisor is of the good type. If $\gamma = \frac{2}{3}$, $y_1^1 = \frac{1}{10}$ and $y_2^1 = \frac{9}{10}$, i.e. if the bad advisor considers his second period as more important, the probability that the bad advisor sends the message truthfully if his signal is 0 ($1 - \nu$) is a function of λ_1^1, λ_1^2, ρ and z. In Morris' paper, when the prior belief of the decision maker regarding the type of an advisor is either very low or very high, ν is high, i.e., the probability that the advisor tells a lie after observing the signal 0 is high. However, this chapter shows that the belief of the decision maker about the type of the good advisor (the type of the other advisor) also can change the probability that the bad advisor tells a lie. If $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$, the value of ν lies between 0 and 1. This in turn guarantees that the bad advisor tells the truth with a non-zero probability of $1 - \nu$. Given $\lambda_1^1 = \frac{1}{2}$, the probability that the bad advisor tells the truth increases with λ_1^2, i.e. ν decreases with λ_1^2. Especially if the prior belief of the decision maker about the good advisor (λ_1^2) approaches 1, ν is at its lowest value. The probability that the bad advisor tells the truth in the first period is very high if the decision maker believes the other advisor to be good with a very high probability given that the prior belief of the decision maker that the bad advisor is good is $\frac{1}{2}$.

If the prior belief of the decision maker about the bad advisor is very high, i.e. if λ_1^1 is very high, the bad advisor becomes more likely to tell a lie if the prior belief of the decision maker about the type of the good advisor (λ_1^2) approaches 0. The bad advisor becomes more likely to tell the truth if λ_1^2 approaches 1. It is because the reputation of the bad advisor cannot decrease a lot if the prior belief of the decision maker about him (λ_1^1) is very high. The bad advisor will have a greater fear of losing his reputation if the belief of the decision maker about the type of the good advisor (λ_1^2) is very high.

If the prior belief of the decision maker about the bad advisor, λ_1^1, is very low, the incentive to tell the truth increases when λ_1^2 is also very low. The incentive to tell a lie increases if λ_1^2 increases given very low λ_1^1. The bad advisor knows that it is very hard to increase his reputation if the prior belief of the decision maker about his type is very low. However, if the belief of the decision maker about type of both advisors is very low, it is relatively easy for the bad advisor to increase his reputation.

In each case, the presence of the other advisor (especially the belief of the decision maker about the type of the other advisor) plays an important role in determining the choice of the message of one advisor.

Next is the case where the first advisor who is of bad type observes the signal 0, and knows that the second advisor is also bad. Under the belief that the bad advisor sends the message 1 with probability ν if the signal is 0, or sends the message 1 with probability ρ if the signal is 1, the first advisor compares the total utility when he tells the truth with that when he tells a lie. There is good reputation effect for the advisor who knows the other advisor is also bad if he puts a relatively greater weight on the second period payoff (see Appendix B). It is shown that the area which guarantees the existence of good reputation effect is bigger when the bad advisor faces a good advisor rather than a bad advisor. This is because the good advisor sends the message 0 more often than the bad advisor. The strong reputational concern not to be perceived as the bad advisor makes the bad advisor send the message 0 more easily when he knows that the other advisor is of the good type rather than of the bad type.

In order to examine the existence of bad reputation effect, let us consider the case where the first advisor who is of the good type observes the signal 1 and meets the bad advisor. The good advisor knows that the bad advisor sends the message 1 with probability ν if the signal is 0, or sends the message 1 with probability ρ if the signal is 1. The good advisor compares the total utility of sending message truthfully ($m_1^1 = 1$) with the total utility of telling a lie ($m_1^1 = 0$).

The total utility to the good advisor from sending message truthfully ($m_1^1 = 1$)

is

$$
\begin{aligned}
&\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[[\rho_\epsilon\{\gamma^2(P^{1,1}_{1,\epsilon}-1)^2+(1-\gamma)^2(P^{1,1}_{1,\epsilon})^2\}\\
&+\gamma(1-\gamma)\nu_\epsilon\{(P^{1,1}_{1,\epsilon}-1)^2+(P^{1,1}_{1,\epsilon})^2\}](-\frac{1}{2}x^1_1)\\
&+\upsilon^1_{GB}[(1-R_\epsilon)\lambda^1_2(\lambda^1_1,1,\epsilon),\\
&\frac{1}{2}\{(1-R_\epsilon)\rho_\zeta\lambda^2_2(\lambda^2_1,\zeta,\epsilon)+R_\epsilon\nu_\zeta\lambda^2_2(\lambda^2_1,\zeta,\epsilon)\}]]
\end{aligned}
$$

where $R_0=\gamma$, $R_1=1-\gamma$, $\rho_0=1-\rho$, $\rho_1=\rho$, $\nu_0=1-\nu$ and $\nu_1=\nu$. The state of the world is equally likely and the belief of the good advisor that the other advisor obtains the same signal is $\frac{1}{2}$. In the case where the good advisor is misinformed, i.e. the real state of the world is revealed as 0, the good advisor needs to consider the case where the other advisor is also misinformed or obtains the correct signal. The good advisor also needs to consider the case where the other advisor obtains the correct signal or is misinformed when he obtains the correct signal, i.e. when the real state of the world is 1. The total utility to the good advisor from telling a lie ($m^1_1=0$) is

$$
\begin{aligned}
&\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[[\rho_\epsilon\{\gamma^2(P^{1,1}_{0,\epsilon}-1)^2+(1-\gamma)^2(P^{1,1}_{0,\epsilon})^2\}\\
&+\gamma(1-\gamma)\nu_\epsilon\{(P^{1,1}_{0,\epsilon}-1)^2+(P^{1,1}_{0,\epsilon})^2\}](-\frac{1}{2}x^1_1)\\
&+\upsilon^1_{GB}[(1-R_\epsilon)\lambda^1_2(\lambda^1_1,0,\epsilon),\\
&\frac{1}{2}\{(1-R_\epsilon)\rho_\zeta\lambda^2_2(\lambda^2_1,\zeta,\epsilon)+R_\epsilon\nu_\zeta\lambda^2_2(\lambda^2_1,\zeta,\epsilon)\}]].
\end{aligned}
$$

If the good advisor only cares about the first period payoff, i.e. $x^1_1=1$ and $x^1_2=0$, then the good advisor will send message truthfully ($m^1_1=1$) after observing the signal 1. If the good advisor only cares about the second period, i.e. $x^1_1=0$ and $x^1_2=1$, then he tells a lie ($m^1_1=0$) to increase his reputation. The truth telling is possible if x^1_1 is sufficiently large. The critical value of x^1_1 which guarantees the existence of the bad reputation effect is calculated as the function of the parameters. Below the critical point of x_1^1, there is equilibrium where the good advisor sometimes tells a lie.

Proposition 3 *There is bad reputation effect for the advisor who observes the signal 1 and knows that the other advisor is bad if he considers his second period as sufficiently more important (see Appendix C).*

The strong reputational concern of the good advisor who knows that the other advisor is bad makes him send the message 0 in the first period after observing the signal 1. Consider the example where the good market expert who knows that the other expert is of the aggressive type concludes that aggressive pricing policy is needed. If the good expert strongly does not want to be perceived as the aggressive expert, he suggests the defensive pricing strategy to the manager of the firm. It is because he wants the decision maker to believe what he will suggest next time.

If $y_1^2 = \frac{1}{2} = y_2^2$, i.e. if the second advisor weights the two periods equally, it is shown that the bad advisor always sends the message 1 in the first period. The good advisor who observes the signal 1 sends the message 0 if he puts the greater weight on the second period (if $x_1^1 < 0.2923$ for $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$ and $\gamma = \frac{2}{3}$). Since the area which guarantees the existence of bad reputation effect is greater in two advisors' model, Compared to the single advisor's model, the existence of the other advisor reinforces the bad reputation effect.

If the bad advisor sometimes sends the message 0 (this happens for $y_1^2 = \frac{1}{10}$, $y_2^2 = \frac{9}{10}$ and $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$), the area which guarantees the existence of the bad reputation effect is smaller than the area which guarantees the existence of the bad reputation effect when the bad advisor always sends the message 1. It is because the area which guarantees the existence of the bad reputation effect (the critical value) increases with the probability that bad advisor sends the message 1 if his signal is 1 (ρ). The good advisor has greater incentive to tell a lie (i.e. he has greater incentive to send the message 0 after observing the signal 1) in order to increase his reputation if the other bad advisor always sends the message 1.

If the bad advisor always sends the message 0 and the prior belief of the decision maker about the type of each advisor is $\frac{1}{2}$, i.e. if $y_1^2 = 0$, $y_2^2 = 1$ and $\lambda_1^1 = \frac{1}{2} = \lambda_1^2$, the good advisor also sends the message 0 if he considers his second period as sufficiently more important (if $x_1^1 < 0.2692$). This is pooling equilibrium in the first period. In this case, if the real state of the world is revealed as 1 in the first period, the decision maker loses all information regarding the state of the world from having an additional advisor.

When the good advisor observes the signal 1 and knows that the other advisor is also good (and will send the message 0 if his signal is 0, or the message 1 with probability z if the signal is 1), there is bad reputation effect for the advisor if he puts greater weight on the second period payoff (see Appendix D). The area which guarantees the existence of bad reputation effect is bigger when the good advisor faces a bad rather than a good advisor. Since the bad advisor sends the message 1 more often than the good advisor, the good advisor has greater incentive to tell a lie when he meets the other bad advisor in order to separate himself from the bad type.

B. Welfare Effect

The decision maker tries to obtain additional information with an additional advisor. However, bad reputation effect arises if an advisor has strong reputational concern to be perceived as a good advisor. To examine the welfare of the decision maker, I compare the payoff of the decision maker with one advisor and with two advisors.

For simplicity, it is assumed that the decision maker believes that an advisor is good with probability $\frac{1}{2}$ before the first period starts.

In order to calculate the expected payoff of the decision maker when he has a single advisor, the action of the decision maker given each message is calculated in each period. Since the second period is the last period, the good advisor sends the message which is the same as his signal and the bad advisor always sends the message 1. From Morris' paper, the probability that the state of the world is 1 in the second period given each message is

$$P_0^{2,1} = 1 - \gamma$$

and

$$P_1^{2,1} = \frac{1 - \lambda_2^1 + \lambda_2^1 \gamma}{2 - \lambda_2^1}$$

where $P_{m_i^1}^{i,1}$ represents the probability that the state of the world in period i is 1 given the message of one advisor m_i^1.

In the first period, the good advisor sends the message 0 if his signal is 0 in the first period and sends the message 1 with probability z if the signal is 1. The bad advisor sends the message 1 with probability ρ if his signal is 1 and sends the message 1 with probability ν if his signal is 0. The probability that state of the world in the first period is 1 given the message of an advisor is

$$P_0^{1,1} = \frac{\lambda_1^1(1 - \gamma z) + (1 - \lambda_1^1)\{1 - \gamma\rho - (1 - \gamma)\nu\}}{\lambda_1^1(2 - z) + (1 - \lambda_1^1)(2 - \rho - \nu)}$$

and

$$P_1^{1,1} = \frac{\lambda_1^1 \gamma z + (1 - \lambda_1^1)\{\gamma\rho + (1 - \gamma)\nu\}}{\lambda_1^1 z + (1 - \lambda_1^1)(\rho + \nu)}.$$

I first separate each period payoff of the decision maker when he has two advisors or has one advisor, and then compare the total payoff of the decision maker. If each type of the advisor considers his second period as sufficiently more important, each type of advisor sends the message 0 in the first period regardless of his signal (for example, $y_2^2 = 1$ and $x_2^1 = 1$). The payoff of the decision maker when he has two advisors is

$$-\frac{1}{2}\{(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2\}$$

and that when he has a single advisor is

$$-\frac{1}{2}\{(P_0^{1,1})^2 + (P_0^{1,1} - 1)^2\}.$$

The probability that the state of the world in the first period is 0 given that both advisors send the message 0 is greater than the probability that the state of the world in the first period is 0 given that one advisor sends the message 0, i.e. $P_{0,0}^{1,0} > P_0^{1,0}$ or $P_{0,0}^{1,1} < P_0^{1,1}$. If the state of the world is 0, $(P_{0,0}^{1,1})^2$ is less than $(P_0^{1,1})^2$. If the state of the world is 1, $(P_{0,0}^{1,1} - 1)^2$ is greater than $(P_0^{1,1} - 1)^2$. Since the welfare loss of having two advisors if the state of the world is 1 is a lot greater than that of having a single advisor if the state of the world in the first period is 1, compared with the welfare gain of having two advisors if the state of the world is 0, it is better for the decision maker to have a single advisor in the first period.

In the second period, the expected payoff of the decision maker from having two advisors is calculated by considering the cases when both advisors are good, one advisor is good and the other advisor is bad, and both advisors are bad. The expected payoff of the decision maker when he has a single advisor in the second period is also calculated by considering the case when the advisor is good or the advisor is bad (Appendix E). In the second period, the welfare loss from obtaining the wrong signal if the decision maker has two advisors is a lot greater than that if the decision maker has a single advisor, compared with the welfare gain which is from obtaining the correct signal. It is shown that having one good advisor is always better than having two advisors regardless of types of both advisors. To have at least one good advisor if the decision maker has two advisors is better than to have a single bad advisor. However, it is better for the decision maker to have a single bad advisor than two bad advisors. Since it is assumed that each advisor is good with probability $\frac{1}{2}$, the comparison of the total payoff of the decision maker between two advisors case and single advisor case leads to the following proposition.

Proposition 4 *Under the condition that each type of the advisor considers his second period as sufficiently more important, the decision maker cannot benefit of taking advice from an additional advisor.*

If the strong reputational concern makes each type of the advisor send the message 0 in the first period, the decision maker is better off having a single advisor rather than two advisors. If the bad advisor always sends the message 1 in the first period (for example, $y_1^2 = \frac{1}{2} = y_2^2$), and if the good advisor considers his second period as sufficiently more important, it is better for the decision maker to have two advisors. If each type of the advisor sometimes sends the message 1, the decision maker can benefit from taking advice from an additional advisor. Except the case where each type of advisor considers his second period as sufficiently more important, it is better for the decision maker to have two advisors.

CHAPTER III

SEQUENTIAL CHEAP TALK FROM ADVISORS WITH REPUTATION

I consider a two period cheap talk model where the decision maker seeks the advice from two advisors sequentially. I examine the possibility of obtaining the correct advice to the decision maker and explain why it is better for the decision maker to seek advice simultaneously.

A. Model

The first advisor sends the message to the decision maker after obtaining the signal regarding the state of the world. After obtaining the message from the first advisor, the decision maker may seek the advice from the second advisor. Before sending the message to the decision maker, the second advisor knows the message sent by the first advisor in sequential cheap talk. After receiving messages from both advisors sequentially, the decision maker chooses the action which can affect all players' payoffs. Since the method of conveying the message is cheap talk, the message of each advisor cannot enter into the utility function of either the advisors or the decision maker. In doctor-patient example I explained in introduction, the patient is the decision maker who seeks advice about his health condition (the medicine is needed or the surgery is needed) from doctors. After obtaining the advice from the first doctor, the patient meets the second doctor to obtain more information about his health condition. In the meeting of the second doctor, the patient tells what the first doctor said about his health condition. I believe that the second doctor may adjust his advice more easily if the second doctor knows the advice sent by the first doctor.

Assumption 1. Each advisor knows the preferences of the other advisor but the decision maker does not know the type of each advisor.

Before sending the message to the decision maker, each advisor knows both the presence of the other advisor and the type of the other advisor. The decision maker believes that the advisor can be one of two types - good or bad. The good advisor is the person who has the same preferences to the decision maker. The bad advisor is the person who is biased towards suggesting any particular advice. In the example, the good doctor is the person who has payoff incentive to suggest the correct advice to the patient but the bad doctor is the person who has payoff incentive to suggest the surgery regardless of health condition to the patient. In period i (for $i = 1, 2$), the decision maker believes that the advisor j (for $j = 1, 2$) is of the good type with probability λ_i^j. With probability $1 - \lambda_i^j$, the decision maker believes that the advisor j is of the bad type.

The state of the world in period i is $\omega_i \in W = \{0,\, 1\}$. In the example, the state of the world 0 is the case where the patient needs the medicine and the state of the world 1 is the case where the patient needs the surgery.

Assumption 2. Each state of the world is equally likely.

Assumption 3. Each advisor receives imperfect private signal about the state of the world and the signal is imperfect but informative.

In the previous chapter, I show that the bad reputation effect - the good advisor sometimes tells a lie to increase his reputation - is not guaranteed if each advisor receives perfect signal about the state of the world. The advisor j obtains the signal S_i^j in period i which is the same as the state of the world with probability γ, i.e. $P(S_i^j = \omega_i) = \gamma$ and $\frac{1}{2} < \gamma < 1$.

The first advisor who knows the type of the second advisor sends the message m_i^1 to the decision maker in period i by following his type and his signal S_i^1. Since the message of the first advisor m_i^1 is known to the second advisor, the second advisor sends the message m_i^2 by following his type and the signal S_i^2 given the message of the first advisor.

Assumption 4. Each advisor follows his signal if there is a conflict of the signal between two advisors.

In some cases, there is a conflict of the signal between two advisors if each advisor knows the type of the other advisor. I will explain why the assumption 4 is needed for simplicity in sequential cheap talk model later in this chapter.

The decision maker chooses an action $a_i^{DM} \in R$ which can affect all players' payoffs after receiving the messages from both advisors. In cheap talk model, the message of each advisor cannot directly affect the action of the decision maker but can indirectly affect it by changing the belief of the decision maker about the state of the world. After payoffs are determined, the state of the world in period i (ω_i) is revealed publicly. Before starting the second period, the decision maker can update the belief about the type of each advisor by considering the message of the advisor and the realized state of the world.

The preferences of advisors are explained by using the utility function of each type of advisor. Since the good advisor is the person who has the same preferences to the decision maker, the utility function of the decision maker is assumed to be the same as that of the good advisor as $-(a_i - \omega_i)^2$ in period i. Since the decision maker is uncertain about the state of the world, he chooses the action a_i as the belief of the decision maker that the state of the world in period i is 1 given messages from both advisors. Since the bad advisor is the person who is biased towards suggesting any particular message, the utility function of the bad advisor is assumed to be a_i in period i. The total payoff of advisor j, if he is of the good type, is

$$-x_1^j(a_1 - \omega_1)^2 - x_2^j(a_2 - \omega_2)^2,$$

where x_1^j and x_2^j denote the weights on the payoffs in the first period and in the second period respectively. If the advisor is of the bad type, the total payoff of advisor j is

$$y_1^j a_1 + y_2^j a_2,$$

where y_1^j and y_2^j denote the weights on the payoffs in the first period and in the second period respectively.

Assumption 5. $\sum_{i=1}^{2} x_i^j = 1$ and $\sum_{i=1}^{2} y_i^j = 1$ for advisor j in period i.

Since both good and bad reputation effects in equilibrium are determined by the weight on the payoff in the first period, it is assumed that the sum of the weight in each period is 1 for simplicity. I will discuss it later in this chapter.

In each period, the message of each advisor is determined by following the type, the signal and the order of the advisor. Given messages from two advisors, the decision maker's action is determined. Then, I can determine the value function of each type of advisor which is the payoff of the advisor in each period. By comparing total payoff of telling a lie with that of telling the truth, I want to show the existence of both good and bad reputation effect. I will use backward induction to examine the reputation effects in the chapter.

1. Second Period

There is babbling equilibrium where the decision maker does not learn anything about the type of the advisor and the state of the world. In this chapter, the equilibrium where informative message is conveyed in the second period is examined. Since the second period is the last period, there is no reputational concern to each type of advisor.

a. Message of Each Advisor

Let's consider the message of the first advisor. The message of the first advisor is determined by following the type and the signal of the first advisor. If the first advisor is of the good type, he sends the message 0 when he observes the signal 0. When the first advisor observes the signal 1, he sends the message 1 if he is of the good type. Regardless of the signal, the first advisor sends the message 1 if he is of the bad type.

The message of the second advisor is determined by following the type of the second advisor, the signal of the second advisor and the message of the first advisor. Let's consider the case where the second advisor is of the good type and receives the message 0 from the first advisor. The second advisor knows that the first advisor is of the good type and the signal of the first advisor is 0. If the signal of the second advisor is 0, he sends the message 0. But if the second advisor receives the signal 1, the second advisor is confused about the state of the world. The second advisor may have a doubt about his signal because he knows that the first advisor observes the signal 0. In general, he may choose the message 0 or 1 randomly. For simplicity, I apply the assumption that each advisor follows his signal. Thus, the second advisor sends the message 1 by assumption 4.

Next is the case where the second advisor is of the good type and receives the message 1 from the first advisor. The second advisor believes that the first advisor is of the good type and his signal is 1, or believes that the second advisor is of the bad type regardless of the signal. If the signal of the second advisor is 0, he is confused about the state of the world. However, he sends the message 0 by assumption 4. If the second advisor observes the signal 1, he sends the message 1 to the decision maker.

In the case were the second advisor is of the bad type, the second advisor sends the message 1 regardless of the signal. By assumption 4, the second advisor, if he is of the good type, sends the message 0 regardless of the message of the first advisor when the signal of the second advisor is 0. The second advisor sends the message 1 regardless of the message of the first advisor when the signal of the second advisor is 1. Regardless of both the signal and the message of the first advisor, the second advisor sends the message 1 if he is of the bad type.

b. Action of the Decision Maker

In order to determine the action of the decision maker given messages from two advisors sequentially, the conditional probability that the second advisor sends the message 0 or 1 given message of the first advisor as 0 or 1 is calculated. The conditional probability that second advisor sends the message 0 given message of the first advisor as 0 is

$$\begin{aligned} P(m_2^2 &= 0|m_2^1 = 0) \\ &= \frac{\sum_{k=0}^{1} P(m_2^1 = 0 = m_2^2|\omega_2 = k)}{\sum_{k=0}^{1} P(m_2^1 = 0|\omega_2 = k)} \\ &= \frac{\gamma^2 + (1-\gamma)^2}{\lambda_2^1 + (1-\lambda_2^1\lambda_2^2)\{\gamma^2 + (1-\gamma)^2\}}. \end{aligned}$$

The state of the world is equally likely. The denominator is determined given that the message of the second advisor is 0 or 1. The conditional probability that second advisor sends the message 0 given message of the first advisor as 1 is

$$
\begin{aligned}
P(m_2^2 &= 0|m_2^1 = 1) \\
&= \frac{\sum_{k=0}^{1} P(m_2^1 = 0 \text{ and } m_2^2 = 1|\omega_2 = k)}{\sum_{k=0}^{1} P(m_2^1 = 1|\omega_2 = k)} \\
&= \frac{\lambda_2^2[1 - \{\gamma^2 + (1-\gamma)^2\}\lambda_2^1]}{2 - \lambda_2^1}.
\end{aligned}
$$

Since the sum of the conditional probability is 1, the conditional probability that the second advisor sends the message 1 given message of the first advisor is automatically determined.

If the decision maker receives the message 0 from the first advisor and then receives the message 0 from the second advisor, he believes that both advisors are of the good type. The decision maker also believes that the second advisor sends the message 0 with probability $P(m_2^2 = 0|m_2^1 = 0)$ because he knows that the second advisor knows the message of the first advisor. The probability that the state of the world is 1 given message 0 from two advisors sequentially is given by

$$P_{0,0}^{2,1} = \frac{1 - 2\gamma + \gamma^2}{1 - 2\gamma + 2\gamma^2}$$

where $P_{m_i^1, m_i^2}^{i,1}$ represents the probability that the state of the world in period i is 1 given the message of the first advisor m_i^1 and the message of the second advisor m_i^2 sequentially. The decision maker chooses action $P_{0,0}^{2,1}$ if he receives the message 0 from two advisors sequentially.

If the decision maker receives the message 0 from the first advisor then receives the message 1 from the second advisor, he is certain that the first advisor is of the good type but is not sure whether the second advisor is of the good type or of the bad type. He will infer that the second advisor sends the message 1 with probability

$P(m_2^2 = 1|m_2^1 = 0)$. The belief of the decision maker that the state of the world in the second period is 1 given message of the first advisor 0 and the message 1 from the second advisor becomes

$$P_{0,1}^{2,1} = \frac{(1-\gamma)\{1-(1-\gamma)\lambda_2^2\}}{1-\{\gamma^2+(1-\gamma)^2\}\lambda_2^2}.$$

If the decision maker receives the message 0 and then receives the message 1, he chooses the action $P_{0,1}^{2,1}$ in the second period.

Similarly, the action of the decision maker in the second period ($P_{1,0}^{2,1}$ or $P_{1,1}^{2,1}$) given messages ($m_2^1 = 1, m_2^2 = 0$ or 1) from two advisors sequentially is determined as

$$P_{1,0}^{2,1} = \frac{(1-\gamma)\{1-(1-\gamma)\lambda_2^1\}}{1-\{\gamma^2+(1-\gamma)^2\}\lambda_2^1}$$

and

$$P_{1,1}^{2,1} = \frac{1-(1-\gamma)(\lambda_2^1+\lambda_2^2)+(1-\gamma)^2\lambda_2^1\lambda_2^2}{2-(\lambda_2^1+\lambda_2^2)+\{\gamma^2+(1-\gamma)^2\}\lambda_2^1\lambda_2^2}.$$

If the action of the decision maker given messages sequentially is determined, the payoff of the second advisor in the second period is determined. This is the value function of the second advisor by following types of both the first advisor and the second advisor. If the value function of the second advisor is increasing with the updated belief of the decision maker that the second advisor is of the good type, the second advisor can adjust his message in the first period to increase his reputation. There are four cases to consider the payoff of the advisor- the second bad advisor meets the other good advisor, meets the other bad advisor, or the second good advisor meets the other bad advisor, or meets the other good advisor.

c. Payoff of the Advisor

Let's consider the case where the second bad advisor meets the other good advisor. Since the second advisor is the advisor who has payoff incentive to suggest the message 1, the value function of the second advisor if the first advisor is of the good type is

$$\upsilon^2_{GB}[\lambda^1_2, \lambda^2_2] = y^2_2 a_2 = \frac{1}{2} y^2_2 (P^{2,1}_{0,1} + P^{2,1}_{1,1}).$$

The first advisor receives the correct signal with probability γ in each period. By assumption 2, the state of the world in the second period is 1 with probability $\frac{1}{2}$. So, the first advisor sends the message 0 with probability $\frac{1}{2}(\gamma + 1 - \gamma) = \frac{1}{2}$.

Next is the case where the second bad advisor knows that the first advisor is also of the bad type. Since both advisors send the message 1 regardless of the signal in the second period, the value function of the second advisor if the first advisor is of the bad type is

$$\upsilon^2_{BB}[\lambda^1_2, \lambda^2_2] = y^2_2 a_2 = y^2_2 P^{2,1}_{1,1}.$$

When the second advisor is of the good type, I need to consider two cases - the first advisor is of the bad type or of the good type. The value function of the second advisor if he knows that the first advisor is of the bad type is

$$\begin{aligned}\upsilon^2_{BG}[\lambda^1_2, \lambda^2_2] &= -x^2_2 (a_2 - \omega_2)^2 \\ &= -\frac{1}{2} x^2_2 \sum_{k=0}^{1} [\gamma_k (P^{2,1}_{1,k})^2 + (1 - \gamma_k)(P^{2,1}_{1,k} - 1)^2]\end{aligned}$$

where $\gamma_0 = \gamma$ and $\sum_{k=0}^{1} \gamma_k = 1$. By assumption 2, the state of the world is equally likely. In each state of the world, the advisor receives the correct signal with probability γ. Since the second advisor has payoff incentive to suggest the correct advice to the decision maker but the first advisor sends the message 1 regardless of his signal, the payoff of the second advisor is determined as $\upsilon^2_{BG}[\lambda^1_2, \lambda^2_2]$.

Similarly, the value function of the second advisor if he knows that the first advisor is also of the good type is

$$\begin{aligned}\upsilon^2_{GG}[\lambda^1_2, \lambda^2_2] &= -x_2^2(a_2-\omega_2)^2 \\ &= -\frac{1}{2}x_2^2\sum_{k=0}^{1}\sum_{i=0}^{1}[\gamma_k\gamma_l(P^{2,1}_{k,l})^2 \\ &\quad +(1-\gamma_k)(1-\gamma_l)(P^{2,1}_{k,l}-1)^2].\end{aligned}$$

In each state of the world, both advisors observe the correct signal with probability γ^2. One of two advisors observes the correct signal with probability $\gamma(1-\gamma)$. In this case, both advisors have payoff incentive to suggest the correct advice to the decision maker. Similarly, the value function of the first advisor is determined by following the type of the second advisor .

So far, I examine 1. the message of each type of the first or the second advisor in the second period, 2. the action of the decision maker in the second period given messages from two advisors sequentially, and 3. the payoff of each type of advisor as a value function in the second period. As an important property in the second period, the value function of the first (second) advisor is increasing with the updated belief of the decision maker that the first (second) advisor is of the good type. If there is the message of each advisor which can increase the updated belief of the decision maker about his type in the first period, the message can also increase the payoff of the advisor in the second period.

2. First Period

In the first period, each advisor sends the message by considering both the first and the second period payoffs. The message in the first period can affect the first period payoff of the advisor by changing the belief of the decision maker about the state of the world. Also, the message in the first period can affect the second period payoff of the advisor by changing the value function of the advisor. If the second advisor is of the bad type, the total payoff of the second advisor is

$$\begin{aligned}
& y_1^2 a_1 + \upsilon_{GB}^2[\lambda_2^1, \lambda_2^2] \\
= \; & y_1^2\{a_1 - \frac{1}{2}(P_{0,1}^{2,1} + P_{1,1}^{2,1})\} + \frac{1}{2}(P_{0,1}^{2,1} + P_{1,1}^{2,1}),
\end{aligned}$$

when the second advisor knows that the other advisor is of the good type. If the second advisor knows that the first advisor is also of the bad type, the value function is changed from $\upsilon_{GB}^2[\lambda_2^1, \lambda_2^2] = \frac{1}{2}y_2^2(P_{0,1}^{2,1} + P_{1,1}^{2,1})$ to $\upsilon_{BB}^2[\lambda_2^1, \lambda_2^2] = y_2^2 P_{1,1}^{2,1} = (1 - y_1^2)P_{1,1}^{2,1}$ in the first case. The total payoff of the second advisor, if he is of the good type, is

$$\begin{aligned}
& -x_1^2(a_1 - \omega_1)^2 + \upsilon_{BG}^2[\lambda_2^1, \lambda_2^2] \\
= \; & x_1^2\{\frac{1}{2}\sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1} - 1)^2] - (a_1 - \omega_1)^2\} \\
& -\frac{1}{2}\sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1} - 1)^2],
\end{aligned}$$

when the second advisor knows that the other advisor is of the bad type. The value function is changed from $\upsilon_{BG}^2[\lambda_2^1, \lambda_2^2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1} - 1)^2]$ to $\upsilon_{GG}^2[\lambda_2^1, \lambda_2^2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}\sum_{i=0}^{1}[\gamma_k\gamma_l(P_{k,l}^{2,1})^2 + (1-\gamma_k)(1-\gamma_l)(P_{k,l}^{2,1} - 1)^2]$ if the second advisor knows that the other advisor is also of the good type. Similarly, the total payoff of each type of the first advisor is determined by following the type of the second advisor.

a. Message of Each Advisor

Suppose that the good advisor sometimes tells a lie in the first period. If the signal is the one the bad advisor is biased towards, the good advisor sometimes sends the message the bad advisor is not biased towards in order not to be perceived as bad advisor. In the model, I suppose that the good advisor sometimes sends the message 0 in the first period if he observes the signal 1. The bad advisor also sometimes tells a lie. If the signal is the one the bad advisor is not biased towards, the bad advisor sometimes sends the message he is biased towards in order to increase his current payoff. Also, the bad advisor may sometimes send the message he is not biased towards if his signal is the one he is biased towards. It is because the bad advisor wants to make the decision maker believe what he will suggest next time. In the model, the bad advisor sometimes sends the message 1 (or 0) if his signal is 0 (or 1).

If the advisor j is of the good type, he sends the message 0 when he observes the signal 0 in the first period. If the signal in the first period is 1, the advisor j sends the message 1 with probability z. In the case where the decision maker believes that the advisor j is of the good type, the decision maker is uncertain about the state of the world if he receives the message 0 from the advisor j. It is because the decision maker believes that the signal of the advisor is 0, or believes that the signal of the advisor is 1 and the advisor sends the message 0 with probability $1-z$. Let's consider the case where the advisor j is of the bad type. If the advisor observes the signal 0, he sends the message 1 with probability ν. The advisor j sends the message 1 with probability ρ if his signal is 1.

Let's consider the message of the first advisor in the first period. The first advisor sends the message 0 or 1 by following his type and the signal he observes. The first advisor sends the message 0 for sure if he is of the good type and his signal is 0. The probability that the first advisor sends the message 0 if he is of the good type and his signal is 1 is $1 - z$. If the first advisor is of the bad type, the probability that he sends the message 0 if his signal is 0 (or 1) is determined as $1 - \nu$ (or $1 - \rho$).

The message of the second advisor is more complicated because the second advisor knows the message sent by the first advisor. So, the message of the second advisor is determined by following the type of the second advisor, the signal of the second advisor, the type of the first advisor and the message sent by the first advisor. Let's consider the case where the second advisor is of the good type and observes the signal 0 in the first period. I need to consider four cases - the first advisor is of the good type or of the bad type, and the first advisor sends the message 0 or 1. As one example, if the second advisor knows that the first advisor is of the good type and sends the message 0, the probability that the second advisor sends the message 0 in the first period is

$$P(m_1^2 = 0 | \text{both advisors are good \& } S_1^2 = 0 \text{ \& } m_1^1 = 0) = 1.$$

In the first period, each state is equally likely. In each state of the world, the second advisor observes the correct signal with probability γ. If the second advisor receives the message 0 from the first good advisor, he is uncertain about the signal of the first advisor. The second advisor infers that the first advisor observes the signal 0 in the first period. Also, the second advisor believes that the first advisor observes the signal 1, but sends the message 0 in order not to be perceived as the bad advisor. By assumption 4, the second advisor follows his signal as 0. So, the second advisor sends the message 0.

By considering different type of the first advisor (good or bad) and the message of the first advisor (0 or 1), what I find here is that the second advisor who is of the good type and observes the signal 0 sends the message 0 regardless of both the type of the first advisor and the message of the first advisor.

Next is the case where the second advisor is of the good type and observes the signal 1 in the first period. The four cases where the first advisor is of the good type or the bad type and the message of the first advisor is 0 or 1 are considered. As one case, if the second advisor knows that the first advisor is of the good type and receives the message 1 from the first advisor, the second advisor is certain that the signal of the first advisor in the first period is 1. So, there is no conflict of the signal between two advisors in this case. Since the advisor who is of the good type and observes the signal 1 sometimes sends the message 0, the probability that the second good advisor sends the message 0 in the first period is

$$P(m_1^2 = 0 | \text{both advisors are good } \& \ S_1^2 = 1 \ \& \ m_1^1 = 1) = 1 - z.$$

In other three cases, there is a conflict of the signal between two advisors. By assumption 4, I find that the second advisor who observes the signal 1 sends the message 0 with probability $1 - z$ regardless of both the type of the first advisor and the message of the first advisor.

The third case is that the second advisor is of the bad type and observes the signal 0 in the first period. By considering different types of the first advisor (good or bad) and the message of the first advisor (0 or 1), the probability that the second advisor sends the message 0 is determined as $1 - \nu$. Finally, by using the same analysis, the second advisor who is of the bad type and observes the signal 1 sends the message 0 with probability $1 - \rho$ regardless of both the type of the first advisor and the message of the first advisor.

b. Updated Belief of the Decision Maker about Type of the Advisor

Since the state of the world is revealed publicly at the end point of the first period, the decision maker can update the belief that the advisor j is of the good type by considering the message of the advisor j and realized state of the world ω_1. Let's first consider the updated belief of the decision maker about the type of the first advisor when the state of the world is revealed as 0. If the decision maker receives the message 0 from the first advisor, the updated belief of the decision maker that the first advisor is of the good type is

$$
\begin{aligned}
& \lambda_2^1(\lambda_1^1, 0, 0) \\
= \quad & \frac{\lambda_1^1(1 - z + \gamma z)}{1 - \rho + \gamma(\rho - \nu) + \lambda_1^1\{\rho - z + \gamma(z - \rho - \nu)\}}
\end{aligned}
$$

where $\lambda_2^j(\lambda_1^j, m_1^j, \omega_1)$ represents the updated belief of the decision maker about the type of the advisor j if the advisor j sends the message m_1^j and the state of the world in the first period is revealed as ω_1. When the first advisor sends the message, he considers his signal as well as the belief of the message of the second advisor. As one example, consider the case where each advisor knows that the other advisor is of the good type. When the first advisor who observes the signal 0 sends the message 0, he knows that the probability that the second advisor who observes the signal 0 and receives the message 0 from the first advisor sends the message 0 (or 1) is 1 (or 0). Similarly, the first advisor knows the probability that the second advisor who observes the signal 1 and receives the message 0 from the first advisor sends the message 0 (or 1) is $1 - z$ (or z). Given these beliefs, the probability that the first advisor who observes the signal 0 sends the message 0 is 1 if both advisors are of the good type. Since the denominator explains all possible cases that the first advisor sends the message 0 if the state of the world is revealed as 0, I need to consider three more cases - 1. the first advisor who is of the good type observes the signal 1 sends the message 0 with probability $1 - z$, 2. the first advisor who is of the bad type observes the signal 0 sends the message 0 with probability $1 - \nu$, and 3. the first advisor sends the message 0 with probability $1 - \rho$ if he is of the bad type and observes the signal 1 in the first period. Among those cases, the numerator explains the possibility of the first advisor who is of the good type sends the message 0.

By using the same method, the updated belief of the decision maker that the first advisor is of the good type if the first advisor sends the message 1 and the real state of the world is revealed as 0 is

$$\begin{aligned} & \lambda_2^1(\lambda_1^1, 1, 0) \\ = \;& \frac{\lambda_1^1(z - \gamma z)}{\rho - \gamma(\rho - \nu) + \lambda_1^1\{z - \rho + \gamma(\rho - z - \nu)\}}. \end{aligned}$$

In this case, the first advisor needs to consider the conditional probability that the second advisor sends the message 0 (or 1) given the type of the first or the second advisor, the signal of the second advisor, and the message 1 from the first advisor.

Next is the case where the state of the world is revealed as 1. The updated belief of the decision maker that the first advisor is of the good type if the first advisor sends the message 0 is

$$\begin{aligned} & \lambda_2^1(\lambda_1^1, 0, 1) \\ = \;& \frac{\lambda_1^1(z - \gamma z)}{1 - \nu + \gamma(\nu - \rho) + \lambda_1^1\{\nu + \gamma(\rho - z - \nu)\}}. \end{aligned}$$

The only difference between the first case and this case is that the state of the world is changed from 0 to 1. If the first advisor is of the good type, he sends the message 0 when he observes wrong signal. The first advisor also sends the message 0 with probability $1 - z$ when he observes the correct signal. When the first advisor sends the message to the decision maker, he also considers the conditional probability that the second advisor sends the message 0 (or 1) given the type of the first or the second advisor, the signal of the second advisor and the message 0 from the first advisor.

Similarly, the updated belief of the decision maker that the first advisor is of the good type if the first advisor sends the message 1 is

$$\begin{aligned} & \lambda_2^1(\lambda_1^1, 1, 1) \\ = \quad & \frac{\lambda_1^1 \gamma z}{\nu + \gamma(\rho - \nu) + \lambda_1^1\{-\nu + \gamma(z + \nu - \rho)\}}. \end{aligned}$$

Proposition 5 *Regardless of the state of the world in the first period, the first advisor has reputational incentive to announce 0 because*

$$\lambda_2^1(\lambda_1^1, 0, 0) > \lambda_1^1 > \lambda_2^1(\lambda_1^1, 1, 0)$$

and

$$\lambda_2^1(\lambda_1^1, 0, 1) > \lambda_1^1 > \lambda_2^1(\lambda_1^1, 1, 1).$$

The first advisor who knows that the second advisor can adjust his message by following his message sends the message 0 regardless of the signal to increase the updated belief of the decision maker about his type. In the example, to suggest the medicine is the way to increase the reputation of the first doctor regardless of health condition of the patient. Especially, even if the patient needs the surgery, the first doctor suggests the medicine if he only considers the increase in his reputation. Since I already show that the payoff of the first advisor in the second period (or value function) is increased with the updated belief of the decision maker about the type of the first advisor, the way to increase the payoff in the second period is to suggest the message the bad advisor is not biased towards.

Now, I want to examine the updated belief of the decision maker about the type of the second advisor. Since the fact that the second advisor knows the message sent by the first advisor before sending his message is common knowledge among players, the conditional probability that the second advisor sends the message 0 (or 1) given message of the first advisor 0 (or 1) is calculated. If the second advisor receives the message 0 from the first advisor, the probability that the second advisor also sends the message 0 in the first period is

$$\begin{aligned} P(m_1^2 \;&=\; 0|m_1^1 \;=\; 0) \\ &= \frac{\sum_{k=0}^{1} Q_{0,0}^{1,k}}{\sum_{i=0}^{1}\sum_{j=0}^{1} Q_{0,i}^{1,j}} \end{aligned}$$

where $Q_{m_1^1,m_1^2}^{1,l}$ represents the conditional probability that the message of the first advisor is m_1^1 and the message of the second advisor is m_1^2 given that the state of the world in the first period is l. The numerator shows the probability that both advisors send the message 0 if the state of the world in the first period is 0 or 1. The denominator explains the conditional probability that only the first advisor sends the message 0 given that the state of the world in the first period is 0 or 1.

Similarly, the conditional probability that the second advisor sends the message 0 given that the first advisor sends the message 1 is

$$\begin{aligned} P(m_1^2 \;&=\; 0|m_1^1 \;=\; 1) \\ &= \frac{\sum_{k=0}^{1} Q_{1,0}^{1,k}}{\sum_{i=0}^{1}\sum_{j=0}^{1} Q_{1,i}^{1,j}}. \end{aligned}$$

The numerator explains the probability that the first advisor sends the message 1 and the second advisor sends the message 0 if the state of the world in the first period is 0 or 1. If the state of the world is 0 or 1, the denominator explains the probability that the first advisor sends the message 1 in the first period. Since the sum of the conditional probability that the advisor sends the message 0 or 1 given the message of the second advisor is 1, the probability that the second advisor sends the message 1 given the message of the first advisor (0 or 1) is determined automatically.

Since the second advisor knows the message of the first advisor, the updated belief of the decision maker that the second advisor is of the good type is calculated given the message of the first advisor. The first case is that the second advisor receives the message 0 from the first advisor in the first period. If the state of the world in the first period is revealed as 0, the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 0 is

$$\begin{aligned} & \lambda_2^2(\lambda_1^2, m_1^2 = 0, 0 | m_1^1 = 0) \\ = \; & \frac{\lambda_1^2\{1-(1-\gamma)z\}}{\lambda_1^2\{1-(1-\gamma)z\} + (1-\lambda_1^2)\{1-(1-\gamma)\rho - \gamma\nu\}}. \end{aligned}$$

The denominator explains all possible cases that the second advisor sends the message 0. If the second advisor is of the good type, he sends the message 0 with probability $P(m_1^2 = 0|m_1^1 = 0)$ when he obtains the correct signal. When the second advisor receives the wrong signal, he sends the message 0 with probability $(1-z) \cdot P(m_1^2 = 0|m_1^1 = 0)$. Let's consider the case where the second advisor is of the bad type. If he obtains the correct signal, he sends the message 0 with probability $(1-\nu) \cdot P(m_1^2 = 0|m_1^1 = 0)$. Similarly, the second advisor sends the message 0 with probability $(1-\rho) \cdot P(m_1^2 = 0|m_1^1 = 0)$ if he receives the wrong signal. Among those cases, the numerator shows the cases that the second advisor sends the message 0 if he is of the good type.

By using the same method, the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 1 if the state of the world is revealed as 0 is

$$\begin{aligned} & \lambda_2^2(\lambda_1^2, m_1^2 = 1, 0 | m_1^1 = 0) \\ = & \frac{\lambda_1^2(1-\gamma)z}{\lambda_1^2(1-\gamma)z + (1-\lambda_1^2)\{(1-\gamma)\rho + \gamma\nu\}}. \end{aligned}$$

In this case, the second advisor sends the message 1 with probability $P(m_1^2 = 1 | m_1^1 = 0)$ by following his type and the signal of the second advisor. It is easily shown that the updated belief of the decision maker that the second advisor sends the message 0 is greater than the updated belief of the decision maker that the second advisor sends the message 1 if the first advisor sends the message 0 in the first period.

Next, I want to consider the case where the state of the world is revealed as 1 and the message of the first advisor is given by 0. By comparing the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 0 ($\lambda_2^2(\lambda_1^2, m_1^2 = 0, 1 | m_1^1 = 0)$) with that when the second advisor sends the message 1 ($\lambda_2^2(\lambda_1^2, m_1^2 = 1, 1 | m_1^1 = 0)$), I conclude that the second advisor can update the belief about his type by sending the message 0 in the first period.

Let's consider the case where the first advisor sends the message 1 and the state of the world is revealed as 1 in the first period. The updated belief of the decision maker that the second advisor is of the good type if the second advisor sends the message 0 is

$$\begin{aligned} & \lambda_2^2(\lambda_1^2, m_1^2 = 0, 1 | m_1^1 = 1) \\ = & \frac{\lambda_1^2(1-\gamma z)}{\lambda_1^2(1-\gamma z) + (1-\lambda_1^2)\{1 - \gamma\rho - (1-\gamma)\nu\}}. \end{aligned}$$

When the second advisor is of the good type, the second advisor sends the message 0 with probability $P(m_1^2 = 0 | m_1^1 = 1)$ if he receives the wrong signal. If the second advisor observes the correct signal, he sends the message 0 with probability $(1-z) \cdot P(m_1^2 = 0|m_1^1 = 1)$. In the case where the second advisor is of the bad type, the second advisor sends the message 0 with probability $(1-\rho) \cdot P(m_1^2 = 0|m_1^1 = 1)$ if his signal is correct. Similarly, the second advisor sends the message 0 with probability $(1-\nu) \cdot P(m_1^2 = 0|m_1^1 = 0)$ if the signal of the second advisor is wrong.

Given that the message of the first advisor is 1, the updated belief of the decision maker that the second advisor is of the good type if the second advisor sends the message 1 is

$$\begin{aligned} & \lambda_2^2(\lambda_1^2, m_1^2 = 1, 1|m_1^1 = 1) \\ = \; & \frac{\lambda_1^2 \gamma z}{\lambda_1^2 \gamma z + (1-\lambda_1^2)\{\gamma\rho + (1-\gamma)\nu\}}. \end{aligned}$$

In order to calculate the updated belief of sending the message 1, the conditional probability that the second advisor sends the message 1 given the message of the first advisor as 1 is needed. Similarly, the updated belief of the decision maker about the type of the second advisor when the second advisor sends the message 0 or 1 if the state of the world is revealed as 0 and the message of the first advisor is 1 is determined.

Proposition 6 *Given the message of the first advisor, the second advisor has reputational incentive to announce 0 regardless of the state of the world because*

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 0|m_1^1 = 0) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 0|m_1^1 = 0),$$

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 1|m_1^1 = 0) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 1|m_1^1 = 0),$$

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 0|m_1^1 = 1) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 0|m_1^1 = 1),$$

and

$$\lambda_2^2(\lambda_1^2, m_1^2 = 0, 1|m_1^1 = 1) > \lambda_1^2 > \lambda_2^2(\lambda_1^2, m_1^2 = 1, 1|m_1^1 = 1).$$

When the message of the first advisor is given as 0, to send the message 0 is the way to increase the reputation of the second advisor regardless of the state of the world. Also, the way to increase the reputation of the second advisor is to send the message 0 when the message of the first advisor is given as 1. In the example, the second doctor can increase his reputation by sending the advice that the medicine is needed even if he knows that the previous doctor recommends the surgery regardless of the health condition of the patient. Since the payoff of the advisor in the second period (or the value function of the advisor) is increased with the reputation of the second advisor, the second advisor sends the message 0 if he wants to increase the second period payoff.

c. Action of the Decision Maker

Given messages from both advisors sequentially in the first period, the decision maker chooses the action which can affect all players' first period payoff. If the decision maker receives the message 0 from the first advisor and then receives the message 0 from the second advisor who knows the message of the first advisor, the probability that the state of the world is 1 in the first period is

$$P_{0,0}^{1,1} = \frac{Q_{0,0}^{1,1}}{\sum\limits_{k=0}^{1} Q_{0,0}^{1,k}}.$$

The denominator shows all possible cases that both advisors send the message 0 given each state of the world. From assumptions, it is known that each state of the world is equally likely and each advisor observes the correct signal with probability γ. In the first period, the advisor who sends the message 0 is either of the good type or of the bad type. If the advisor is of the good type, the advisor sends the message 0 when his signal is 0, and also sends the message 0 with probability $1 - z$ when the signal is 1. Let's think about the message of the bad type advisor. If the signal is 0, the advisor sends the message 0 with probability $1-\nu$ and the advisor sends the message 0 with probability $1-\rho$ if his signal is 1. So, if the first advisor sends the message 0, this is one of four cases in each state of the world. The second advisor who sends the message 0 is also included in these cases. However, I need to apply that the second advisor sends the message 0 with $P(m_1^2 = 0|m_1^1 = 0)$ because the second advisor knows the message sent by the first advisor. Among those cases, the numerator explains the case where both advisors send the message 0 sequentially given that the state of the world in the first period is 1. The decision maker chooses the action $P_{0,0}^{1,1}$ if he receives the message 0 from both advisors sequentially.

If the decision maker receives the message 0 from the first advisor and then receives the message 1 from the second advisor, the probability that the state of the world in the first period is 1 is

$$P_{0,1}^{1,1} = \frac{Q_{0,1}^{1,1}}{\sum_{k=0}^{1} Q_{0,1}^{1,k}}.$$

If the decision maker receives the message 1 from the second advisor, he believes that one of the following cases is possible. The decision maker believes that the second advisor is either of the good type or of the bad type. It is because the good advisor who observes the signal 1 and receives the message 0 from the first advisor sends the message 1 with probability $z \cdot P(m_1^2 = 1|m_1^1 = 0)$. Also, the bad advisor who observes the signal 0 (or 1) and receives the message 0 from the first advisor sends the message 1 with probability $\nu \cdot P(m_1^2 = 1|m_1^1 = 0)$ (or $\rho \cdot P(m_1^2 = 1|m_1^1 = 0)$). The decision maker chooses the action $P_{0,1}^{1,1}$ if he receives the message 0 from the first advisor and then receives the message 1 from the second advisor.

By using the same method, the case where the decision maker receives the message 1 from the first advisor and then receives the message 0 (or 1) from the second advisor is considered. The action of the decision maker if he receives the message 1 and 0 sequentially is

$$P_{1,0}^{1,1} = \frac{Q_{1,0}^{1,1}}{\sum_{k=0}^{1} Q_{1,0}^{1,k}},$$

and that of the decision maker if he receives 1 from both advisors sequentially is

$$P_{1,1}^{1,1} = \frac{Q_{1,1}^{1,1}}{\sum_{k=0}^{1} Q_{1,1}^{1,k}}.$$

The action of the decision maker in the first period can determine the payoffs of all players in the first period. Since the total payoff is determined by considering the action of the decision maker in each period, I want to examine the existence of the good or the bad reputation effect from now on.

B. Reputation Effect

After determining the total payoff of each advisor, it is shown that there is sometimes a conflict between the current payoff and the future payoff of the advisor when the advisor sends the message to the decision maker. In case of the bad advisor, he can increase his current payoff of sending the message 1 but can increase his future payoff of sending the message 0 in the first period. In case of the good advisor, to send the correct message is the way to increase his current payoff but to send the message 0 is only way to increase his future payoff in the first period. By considering the weighted average in each period, I want to determine when there exists the good or the bad reputation effect. The good reputation effect means that the bad advisors sometimes tells the truth to increase the reputation and the bad reputation effect means that the good advisor sometimes tells a lie to increase his reputation. In order to consider the good reputation effect, I examine the case where the bad advisor observes the signal he is not biased towards.

1. Good Reputation Effect

Let's consider the case where the second advisor is of the bad type and observes the signal 0 in the first period. Since each advisor knows the type of the other advisor and the second advisor knows the message sent by the first advisor, there are four cases the second advisor faces- 1. the first advisor is of the good type and sends the message 0 in the first period, 2. the first advisor is of the good type and the message of the first advisor is 1 in the first period, 3. the first advisor is of the bad type and sends the message 0 in the first period, and 4. the first advisor is of the bad type and the message of the first advisor is 1 in the first period. In each case, I first try to find when the second bad advisor tells the truth even if he has loss in current payoff given the message of the first advisor. Then, I examine whether there is an incentive for the first advisor to send the given message I analyzed before to find the equilibrium condition.

If the second advisor meets the other advisor who is of the good type and sends the message 0 in the first period, the total payoff of the second advisor who sends the message 0 in the first period is

$$y_1^2 P_{0,0}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 0)].$$

Since the second advisor receives the message 0 from the first advisor, the payoff of the second advisor in the first period is determined as $P_{0,0}^{1,1}$ if the second advisor also sends the message 0. The value function of the second advisor is determined by both the type of the first advisor and the updated belief of each advisor in each state of the world. The total payoff of the second advisor who sends the message 1 given the message of the first advisor as 0 is

$$y_1^2 P_{0,1}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 0)].$$

If the second advisor only cares about his first period, i.e. $y_1^2 = 1$, he sends the message 1 in the first period. Also, the second advisor who only considers his second period (i.e. $y_1^2 = 0$) sends the message 0 in the first period. So, I can determine the critical value of the weighted average in the first period as a function of parameters to guarantee the existence of the good reputation effect. In order to examine the equilibrium condition, the incentive of sending the message 0 of the first advisor who is of the good type is examined given the area where the second advisor tells the truth or tells a lie. Since the method of considering the equilibrium condition in the good reputation effect is the same as that in the bad reputation effect, I will explain the equilibrium condition in the Appendix in the bad reputation effect. What I find here is that the bad advisor sometimes tells the truth even if he has loss in his current payoff in sequential cheap talk model. After comparing the area which guarantees the existence of the good reputation effect when the second advisor does not know the message sent by the first advisor, I find that the possibility of the existence of the good reputation effect is greater in sequential cheap talk model. It is because the advisor can adjust his message more easily if the advisor knows the message sent by the other advisor. If the bad advisor receives the message he is not biased towards from the previous advisor, he can send the message he is not biased towards easily not to be perceived as the bad advisor.

Next is the case where the first advisor is of the good type and sends the message 1 in the first period. Since the only difference between this case and the first case is that the first advisor sends the message 1, the equations are easily changed. The second advisor who observes the signal 0 sends the message 0 if

$$y_1^2 P_{1,0}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 1)] >$$

$$y_1^2 P_{1,1}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 1)].$$

Since the message of the first advisor is fixed as 1, the first period payoff of the second advisor if the second advisor sends the message 0 (or 1) is determined as $P_{1,0}^{1,1}$ (or $P_{1,1}^{1,1}$). The value function is different from the first case even if the type of each advisor is the same because the updated beliefs of both the first and the second advisor are changed by following the message of the first advisor. By considering the incentive for the first advisor to send the message 1 in the first period given each message of the second advisor, I find that there is still the good reputation effect when the second advisor who knows that the first advisor is of the good type receives the message 1 from the first advisor. However, the possibility of the existence of the good reputation effect is lesser if the second advisor knows that the first advisor sends the message 1 compared to the case where the second advisor does not know the message sent by the first advisor. It is because the possibility of being perceived as the bad advisor to the decision maker is lesser if the second advisor receives the message the bad advisor is biased towards from the first advisor. So, the bad advisor can easily choose to send the message he is biased towards to increase his current payoff.

Now, the case where the first advisor is of the bad type is considered. First is the case where the second bad advisor receives the message 0 from the first advisor. Since each advisor knows that the other advisor is of the bad type, the value function is changed from $\upsilon_{GB}^2[\lambda_2^1, \lambda_2^2] = \frac{1}{2} y_2^2 (P_{0,1}^{2,1} + P_{1,1}^{2,1})$ to $\upsilon_{BB}^2[\lambda_2^1, \lambda_2^2] = y_2^2 P_{1,1}^{2,1}$. The total payoff of the second advisor if he sends the message 0 is

$$y_1^2 P_{0,0}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BB}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 0)],$$

and that of the second advisor if he sends the message 1 is

$$y_1^2 P_{0,1}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{GB}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 0)].$$

The payoff in the first period is the same as the case where the first advisor is of the good type and sends the message 0 in the first period because it is connected with not the type of the advisor but the action of the decision maker who does not know the type of the advisor. By considering equilibrium conditions, the existence of the good reputation effect is easily shown in this case. If the second advisor considers his second period sufficiently more important, the second advisor who knows that the first advisor is of the bad type and sends the message 0 in the first period sends the message 0 in the first period. When the second advisor knows that the first advisor sends the message 0 regardless of the type of the first advisor, the second advisor tells the truth more easily than the case when the second advisor does not know the message sent by the first advisor.

Finally, the case where the first advisor is of the bad type and sends the message 1 is considered. The second advisor sends the message 0 if

$$y_1^2 P_{1,0}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 1)]$$

is greater than

$$y_1^2 P_{1,1}^{1,1} + \frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BB}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 1)].$$

The payoff in the first period is determined given the message of the first advisor as

1. The payoff in the second period is calculated by the updated belief of the decision maker about the type of each advisor. There is the good reputation effect if the second advisor knows that the first advisor is of the bad type and sends the message 1 in the first period. When the second advisor knows that the message of the first advisor is 1 regardless of the type of the advisor, the probability that the second advisor tells the truth is greater in sequential cheap talk compared to the simultaneous cheap talk.

Proposition 7 *There is good reputation effect for the advisor who observes the signal 0 and receives the message 0 from the other advisor if he considers his second period sufficiently more important (See Appendix F).*

Proposition 8 *There is good reputation effect for the advisor who observes the signal 0 and receives the message 1 from the other advisor if he considers his second period sufficiently more important (See Appendix G).*

There is the good reputation effect in sequential cheap talk model. Since the advisor can adjust his message more easily when he knows the message of the other advisor, the advisor can easily tell the truth if he receives the message he is not biased towards. If the advisor receives the message he is biased towards, the probability of telling the truth is lesser compared to the case where the advisor does not know the message sent by the other advisor. It is because the advisor is the person who does not want to be perceived as the bad advisor.

2. Bad Reputation Effect

Since the bad reputation effect means that the good advisor sometimes tells a lie to increase his reputation, the case where the second advisor is of the good type and observes the signal 1 in the first period. Four cases are considered by following the type of the first advisor and the message of the first advisor - 1. the first advisor is of the bad type and sends the message 0 in the first period, 2. the first advisor is of the bad type and the message of the first advisor is 1, 3. the first advisor is of the good type and sends the message 0 in the first period, and 4. the first advisor is of the good type and the message of the first advisor is 1 in the first period.

Let's consider the case where the second advisor who observes the signal 1 knows that the first advisor is of the bad type and knows that the message of the first advisor is 0. The total payoff of the second advisor if he sends the message 0 is

$$-\frac{1}{2}x_1^2\{(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BG}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 0, \xi|m_1^1 = 0)].$$

Since the payoff of the good advisor in each period is affected by the state of the world, the assumption that the state of the world is equally likely is applied. In each state of the world, since both advisors send the message 0 sequentially in the first period, the payoff of the second advisor in the first period is $-\frac{1}{2}x_1^2\{(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2\}$. The payoff of the second advisor in the second period is determined by the updated belief of both first and the second advisor. The updated belief of the decision maker about the type of the first advisor is determined by $\lambda_2^1(\lambda_1^1, 0, \xi)$ given each state of the world and that of the second advisor is determined by the message of each advisor and the state of the world.

The total payoff of the second advisor if he sends the message 1 is

$$-\frac{1}{2}x_1^2\{(P_{0,1}^{1,1})^2 + (P_{0,1}^{1,1} - 1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1}\upsilon_{BG}^{2}[\lambda_2^1(\lambda_1^1,0,\xi),\lambda_2^2(\lambda_1^2,1,\xi|m_1^1=0)].$$

The payoff of the second advisor in the first period is changed from $-\frac{1}{2}x_1^2\{(P_{0,0}^{1,1})^2 +(P_{0,0}^{1,1}-1)^2\}$ to $-\frac{1}{2}x_1^2\{(P_{0,1}^{1,1})^2+(P_{0,1}^{1,1}-1)^2\}$ compared to the previous case because the message of the second advisor is changed from 0 to 1. In case of the payoff of the second advisor in the second period, the updated belief of the decision maker about the type of the second advisor is changed to $\lambda_2^2(\lambda_1^2,1,\xi|m_1^1=0)$ because the message of the second advisor is 1 in this case.

If the second advisor only considers his first period (i.e. $x_1^2=1$), he tells the truth. But if the second advisor only considers his second period (i.e. $x_1^2=0$), he sends the message the bad advisor is not biased towards. By comparing the total payoff of the second advisor in each message of the second advisor, what I find is that the second advisor sends the message 0 in the first period if he considers his second period sufficiently more important. Then, the incentive of the first advisor to send the message 0 is examined given the area where the second advisor sends the message 0 in the first period. I will explain the equilibrium condition in Appendix. There is still the bad reputation effect in sequential cheap talk. Compared to the area which guarantees the existence of the bad reputation effect in the simultaneous cheap talk, the probability that the advisor sends the message 0 is greater if he knows that the first advisor is of the bad type and sends the message 0 in the first period. It is because the advisor who receives the message the bad advisor is not biased towards follows the message of the previous advisor not to be perceived as the bad advisor more easily compared to the case where he does not know the message of the first advisor.

The next is the case where the second advisor knows that the first advisor is of the bad type and sends the message 1 in the first period. The only difference between this case and the previous case is the message of the first advisor. The second advisor who observes the signal 1 sends the message 0 if

$$-\frac{1}{2}x_1^2\{(P_{1,0}^{1,1})^2 + (P_{1,0}^{1,1} - 1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BG}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 1)]$$

is greater than

$$-\frac{1}{2}x_1^2\{(P_{1,1}^{1,1})^2 + (P_{1,1}^{1,1} - 1)^2\}$$

$$+\frac{1}{2}\sum_{\xi=0}^{1} \upsilon_{BG}^2[\lambda_2^1(\lambda_1^1, 1, \xi), \lambda_2^2(\lambda_1^2, 1, \xi | m_1^1 = 1)].$$

The payoff of the second advisor in the first period is determined by the message from each advisor and the state of the world. So, the assumption that the state of the world is equally likely is applied. The payoff of the second advisor in the second advisor is determined by the updated belief of the decision maker about the type of each advisor. The updated belief of the decision maker that the second advisor is of the good type is determined by the message of each advisor and the realized state of the world. By considering the equilibrium condition, it is shown that there is the bad reputation effect if the second advisor considers his second period sufficiently more important. Compared to the simultaneous cheap talk, the possibility of the existence of the bad reputation effect is greater in sequential cheap talk. It is because the advisor wants to separate his type from the bad type more easily if he knows that the message of the other advisor is the one the bad advisor is biased towards.

If the second advisor knows that the first advisor is of the good type, the value function of the second advisor is changed from $\upsilon_{BG}^2[\lambda_2^1, \lambda_2^2]$ to $\upsilon_{GG}^2[\lambda_2^1, \lambda_2^2]$. The first period payoff of the second advisor if he sends the message 0 is

$$-\frac{1}{2}x_1^2\{(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2\}.$$

This is the same as the payoff of the second advisor if he knows that the first advisor is of the bad type and sends the message 0 in the first period. It is because the payoff in the second period is only affected by the message of each advisor. The second period payoff of the second advisor if he sends the message 1 is

$$\frac{1}{2}\sum_{\xi=0}^{1} v_{GG}^2[\lambda_2^1(\lambda_1^1, 0, \xi), \lambda_2^2(\lambda_1^2, 0, \xi | m_1^1 = 0)].$$

The value function of the second advisor is affected by the type of the other advisor. Similarly, the total payoff of the second advisor if he sends the message 1 is calculated. What I find is that the second advisor tells a lie if he considers his second period sufficiently more important. Finally, the case where the first advisor is of the good type and sends the message 1 in the first period. Since the message of the first advisor is changed, the payoff of each period is also changed. By considering the equilibrium condition, I also find that there is the bad reputation effect in this case.

Proposition 9 *There is bad reputation effect for the advisor who observes the signal 1 and receives the message 0 from the other advisor if he considers his second period sufficiently more important (See Appendix H).*

Proposition 10 *There is bad reputation effect for the advisor who observes the signal 1 and receives the message 1 from the other advisor if he considers his second period sufficiently more important (See Appendix I).*

There is the bad reputation effect in sequential cheap talk model regardless of the message or the type of the previous advisor. Compared to the bad reputation effect in simultaneous cheap talk, the advisor tells a lie more easily if he knows the message of previous advisor. If the advisor knows that the message of the previous advisor is the one the bad advisor is not biased towards, he is afraid of telling the truth because he is easily perceived as the bad type by doing so. If the advisor knows that the previous advisor sends the message the bad advisor is biased towards, it is better for him to tell a lie to separate his type from the bad type.

In the point of view of the decision maker, there are some chances for the decision maker to obtain the correct advice by seeking advice sequentially rather than simultaneously. However, the possibility of losing information by asking advice sequentially is greater than that in simultaneous cheap talk in many cases. It is because the second advisor can easily adjust his message in the case where he knows the message of the previous advisor. After obtaining the advice from the first advisor, it is better for the decision maker not to tell the message sent by the first advisor when he meets the second advisor.

CHAPTER IV

RESOLUTION - PROMISE WITH ONESELF

By adding the possibility of telling to our friends about our resolution in Benabou and Tirole's model [5], I examine how both self reputation and the perception of reputation can change choices of the decision maker. I consider a two period model and each period is divided into three sub-periods. In the first sub-period of each period, the decision maker has to choose whether to follow the temptation or not. If the decision maker chooses to follow the temptation in the first period, it means that he does not make a resolution in the first period. Then, the game of the first period is finished and the decision maker confronts the game in the next period. If the decision maker chooses not to follow his temptation in the first sub-period of each period, the decision maker has to choose whether to tell his friends about his resolution or not in the second sub-period of each period. Regardless of the choice in the second sub-period of each period, the decision maker has to choose whether to follow the temptation or not in the third sub-period of each period. The decision maker can choose a maximum of six decisions.

A. Model

When the decision maker chooses to follow temptation (FT), he can get the immediate benefit A which is explained as a happiness of using a habit in his mind. The benefit from not indulging in the temptation during a period is realized at the end of each period. After choosing not to follow the temptation (NFT), the decision maker has to decide whether to tell his friends (T) or not to tell his friends (NT) about his resolution. This choice for the decision maker is added in the model because many people believe that the best way to quit a bad habit is to tell the friends about the resolution. American Cancer Society [2] mentions the importance of supports of the family to quit smoking. When the decision maker tells his friends about the resolution, he has to consider the perception of the reputation and this perception of the reputation may be helpful to quit the bad habit. It is assumed that there is no immediate cost or benefit from both the choice of NFT and the choice of T or NT.

Regardless of telling his friends or not, the decision maker has to choose to persevere (P) or not to persevere (NP) after choosing not to follow his temptation (NFT). If the decision maker chooses NP, he does not suffer any cost. In each period if the decision maker chooses P, he suffers the cost ($C_P > 0$) of enduring his temptation at the starting point of the third sub-period. When the decision maker chooses to persevere, he obtains the benefit B_K for $K \in \{T, NT\}$ at the end of the third sub-period in each period. Even if the decision maker does not persevere, he gets the benefit b_K for $K \in \{T, NT\}$ at the end of the third sub-period in each period because he at least feels the happiness of making the resolution in the first sub-period and trying to keep the resolution during the second sub-period.

The decision maker discounts the benefits that the choice of NFT would bring at a rate γ. In the third sub-period after choosing NFT, the decision maker has an immediate cost and the delayed benefit. I apply the discount rate β in the third sub-period. Since the decision maker will give up (i.e. choose NP) whenever $\frac{C_P}{\beta} > B_K - b_K$, I define β as the ability to resist impulses in the stress time. Similarly, the discount rate γ is explained as the ability to resist impulses in the normal time because the decision maker chooses to follow the temptation if $\frac{A}{\gamma}$ is greater than the expected payoff of choosing NFT. Next is the assumption regarding the benefits.

Assumption 6.

$$B_T > B_{NT} > b_{NT} > b_T > A.$$

Regardless of telling friends or not, the benefit when the decision maker chooses to persevere (B_K) is greater than that when he chooses not to persevere (b_K). Also, it is better for the decision maker to try to make a resolution than not to try anything even though he cannot succeed to persevere in the third sub-period. Since the decision maker is a person who considers both the self-reputation and the perception of reputation, the benefit of telling his friends is greater than that of not telling his friends if he perseveres. But, the benefit of not telling his friends is greater than that of telling his friends if he does not persevere.

In the first period, the decision maker does not know perfectly whether he has a high or low ability to endure impulses. Also, the decision maker is not sure what his friends believe of his ability to endure impulses. The following assumptions are helpful to formalize these ideas.

Assumption 7.

$$\beta_L < \beta_H < \gamma.$$

The ability to endure impulses in the stress time (β) is lesser than that in the normal time (γ).

Assumption 8.

The decision maker believes that he has high ability to endure impulses in the stress time or is high type in the stress time (β_H) with probability ρ. With probability $1-\rho$, the decision maker is low type in the stress time (β_L). In the first period, the decision maker has a prior belief ρ_1 that he is high type in the stress time. The decision maker updates the belief regarding his own ability from ρ_1 to ρ_2 by the choices made during the first period.

Assumption 9.

The decision maker believes that his friends believe him as a high type individual who has discount factor β_H with probability ξ. With probability $1 - \xi$, the decision maker believes that his friends believe him as a low type individual. In the first period, the decision maker has a prior belief ξ_1. The decision maker updates his belief regarding the perception of reputation from ξ_1 to ξ_2 by the choices made during the first period.

Assumption 10. Self-reputaion (ρ) is positively related with perception of reputation (ξ). Those actions that enhance the self-reputation of the decision maker also enhance his perception of reputation. As an extension of the chapter, if there is a cost of either telling friends about the resolution or not telling friends about the resolution, it is shown that self-reputation and perception of reputation are positively correlated.

B. The Analysis

The main questions in this section are: (a) what does it take to make a resolution, (b) assuming one has made a resolution, what does it take to sustain it in both the short run and the long run, (c) why is it that people keep making resolutions but (i) often they succeed in keeping up with the resolution in the short run but not in the long run, and (ii) sometimes can't sustain it even in the short run. In this chapter, short run refers to the first period of the model.

1. Second Period

Let's start to consider the third sub-period in the second period. Since this is the last period to the decision maker, the decision maker does not need to consider both self-reputation and the perception of the reputation in this period. For simplicity, it is assumed that the high type decision maker in the stress time always chooses to persevere (P) while the decision maker who has weak ability to resist impulses in the stress time chooses not to persevere (NP) in the second period.

Assumption 11.

$$\frac{C_P}{\beta_H} < B_K - b_K < \frac{C_P}{\beta_L}$$

for $K \in \{T,\, NT\}$.

In this chapter, this is the way to distinguish between the high type decision maker and the low type decision maker in the first step. Regardless of telling friends or not, the high type decision maker chooses to persevere (P) and the low type decision maker chooses not to persevere (NP). This assumption implies that the decision maker chooses to persevere (P) with probability ρ_2 and chooses not to persevere (NP) with probability $1 - \rho_2$ where ρ_2 is the updated belief of the decision maker that he has strong ability to endure impulses in the stress time.

Now, I need to determine the relationship between the updated belief of the decision maker that he is high type in the stress time, ρ_2, and the probability that the decision maker tells his resolution to friends, $\phi(\rho_2, \xi_2)$ in the second sub-period of the second period. The decision maker chooses to tell the resolution to his friends (T) if the net expected benefit from telling is greater than that from not telling. After the decision maker chooses not to follow his temptation in the second period, he chooses to tell his friends about the resolution if

$$\rho_2(B_T - B_{NT}) + (1 - \rho_2)(b_T - b_{NT}) \geq 0.$$

If the updated belief of the decision maker that he is high type in the stress time (ρ_2) is greater than the critical value (ρ_2^T), the decision maker chooses to tell his friends about the resolution, i.e. $\phi(\rho_2, \xi_2) = 1$ if $\rho_2 \geq \rho_2^T$ where

$$\rho_2^T = \frac{b_{NT} - b_T}{B_T - B_{NT} + b_{NT} - b_T}.$$

Under assumption 6 and assumption 10, it is easily shown that the probability that the decision maker tells to friends, $\phi(\rho_2, \xi_2)$, is increasing with both the updated belief of the decision maker that he has strong ability to resist impulses in the stress time (ρ_2) and the updated belief of the decision maker that his friends believe him as high type (ξ_2).

In the first sub-period of the second period, the decision maker will choose not to follow his temptation (NFT) if the net expected benefit of NFT is greater than the net expected benefit of following temptation (A). The benefit to the decision maker of following the temptation, i.e. the benefit of not making a resolution is realized immediately and equal to A. When the decision maker chooses not to follow the temptation, then he can tell or not tell, and then persevere or not persevere. The expected benefit of not following the temptation in the second period is

$$\begin{aligned} E_2(NFT) &= \gamma[\phi(\rho_2, \xi_2)\{\rho_2(B_T - C_P) + (1-\rho_2)b_T\} \\ &\quad +(1-\phi(\rho_2, \xi_2))\{\rho_2(B_{NT} - C_P) + (1-\rho_2)b_{NT}\}]. \end{aligned}$$

In the Appendix J, it is shown that the benefit of not following the temptation in the second period increases with the updated belief of the decision maker that he is high type (ρ_2) if both the probability to tell his friends in the second period and the benefit when the decision maker chooses to persevere (P) are sufficiently high. Formally,

$$\frac{\partial E_2(NFT)}{\partial \rho_2} > 0$$

if

$$\phi(\rho_2, \xi_2) > (1-\rho_2)\frac{\partial \phi(\rho_2, \xi_2)}{\partial \rho_2} \text{ and } B_K > C_P + b_K$$

for $K \in \{T,\ NT\}$. Because of this monotonicity, there exists ρ_2^* which satisfies

$E_2(NFT) > A$ if $\rho_2 > \rho_2^*$ where

$$\gamma[\phi(\rho_2^*, \xi_2)\{\rho_2^*(B_T - C_P) + (1-\rho_2^*)b_T\}$$
$$+(1-\phi(\rho_2^*, \xi_2))\{\rho_2^*(B_{NT} - C_P) + (1-\rho_2^*)b_{NT}\}] = A.$$

The choices made by the decision maker in the second period are summarized in the following proposition.

Proposition 11 *Regardless of his type, the decision maker chooses not to follow his temptation (NFT) and to tell his friends (T) in the second period if*

1. $\rho_2 > \rho_2^*$ *where* ρ_2^* *is the critical value which satisfies*

$$E_2(NFT) = A,$$

2. $\phi(\rho_2, \xi_2) \geq (1-\rho_2)\frac{\partial\phi(\rho_2,\xi_2)}{\partial\rho_2}$ *and* $B_K > C_P + b_K$ *for* $K \in \{T,\ NT\}$
3. $\rho_2 \geq \rho_2^T$ *where*

$$\rho_2^T = \frac{b_{NT} - b_T}{B_T - B_{NT} + b_{NT} - b_T}.$$

The decision maker chooses not to follow his temptation (NFT) and not to tell his friends (NT) in the second period regardless of the type of the decision maker if 1. $\rho_2 > \rho_2^*$, 2. $B_K > C_P + b_K$ for $K \in \{T,\ NT\}$ and 3.$\rho_2 < \rho_2^T$.

I define the value function of the decision maker as his net expected benefit from the second period decisions. Since the high type decision maker chooses to persevere in the second period while the low type decision maker chooses not to persevere in the second period, I need to separate the value function for each type. The value function for the decision maker who has strong ability to resist impulses in the stress time is calculated as

$$\begin{aligned} V_2^H(\rho_2, \xi_2) &= \alpha(\rho_2, \xi_2)[\phi(\rho_2, \xi_2)(B_T - C_P) \\ &\quad +(1 - \phi(\rho_2, \xi_2))(B_{NT} - C_P)] + (1 - \alpha(\rho_2, \xi_2))A \end{aligned}$$

where $\alpha(\rho_2, \xi_2)$ is the probability that the decision maker chooses NFT in the second period. Similarly, I can determine the value function of the decision maker who has weak ability to resist impulses in the stress time as

$$V_2^L(\rho_2, \xi_2) = \alpha(\rho_2, \xi_2)[\phi(\rho_2, \xi_2)b_T + (1 - \phi(\rho_2, \xi_2))b_{NT}] + (1 - \alpha(\rho_2, \xi_2))A.$$

For the high type decision maker, the value function increases with both ρ_2 and ξ_2, i.e.

$$\frac{\partial V_2^H(\rho_2, \xi_2)}{\partial \rho_2} > 0 \text{ and } \frac{\partial V_2^H(\rho_2, \xi_2)}{\partial \xi_2} > 0.$$

If the choices of the decision maker in the first period increase both the updated belief of the decision maker that he is high type and that of the decision maker that his friends believe him as high type, the decision maker who has strong ability to endure impulses can increase the payoff from the second period.

The value function of the decision maker who has weak ability to resist impulses is increasing only if the payoff of choosing to follow temptation is sufficiently small. Formally,

$$\frac{\partial V_2^L(\rho_2, \xi_2)}{\partial \rho_2} > 0,$$

and

$$\frac{\partial V_2^L(\rho_2, \xi_2)}{\partial \xi_2} > 0$$

if

$$A < b_{NT} - (b_{NT} - b_T) \cdot \{\alpha(\rho_2, \xi_2)\frac{\partial \phi(\rho_2, \xi_2)}{\partial \alpha(\rho_2, \xi_2)} - \phi(\rho_2, \xi_2)\}.$$

However, when the payoff of choosing to follow the temptation is larger than the critical value which is determined as a function of parameters, the value function of the low type decision maker increases if both the updated belief that he is high type and the updated belief that his friends believe him as high type decrease. Later in this chapter, I will use the numerical example to understand this critical value easily. It is shock for the decision maker who believes himself as high type in normal time to figure out that he may not be the high type in the stress time. However, this shock is weaker if the belief of the decision maker that he is high type in the stress time is lower. It is also because the low type decision maker in the stress time chooses not to persevere more easily than the high type decision maker in the stress time (actually in this chapter, the low type decision maker always chooses not to persevere in the last period). Thus, the low type decision maker in the stress time feels comfortable if he believes that his friends believe him as low type and believes that he is low type.

2. First Period

From this point, I explain how the updated beliefs of the decision maker (ρ_2 and ξ_2) are changed from the prior beliefs of the decision maker (ρ_1 and ξ_1) by the choices in the first period. When the decision maker chooses FT in the first period, then he neither obtains the opportunity to tell his friends nor to test his strength of his will in the stress time. As a result, both ρ and ξ are not changed. The self-reputation (ρ) increases if the decision maker chooses to persevere (P) in the first period regardless of telling his friends or not after choosing NFT. The perception of reputation (ξ) is changed only if the decision maker chooses to tell his friends about his resolution in the first period. It increases if the decision maker tells his friends and then chooses to persevere after choosing NFT, but decreases if he tells his friends and then chooses not to persevere after choosing NFT in the first period. For a given ρ_1 and ξ_1,

$$\rho_2^{T+P} = \rho_2^{NT+P} > \rho_1 > \rho_2^{T+NP} = \rho_2^{NT+NP}$$

and

$$\xi_2^{T+P} > \xi_2^{NT+P} = \xi_1 = \xi_2^{NT+NP} > \xi_2^{T+NP}.$$

a. First Period Decisions of High Type

When the decision maker who has strong ability to endure impulses in the stress time chooses to persevere (P) on the third sub-period in the first period, he can increase both the payoff in the first period and the value function. It is because to choose P can increase his self-reputation and cannot decrease the perception of the reputation (the perception of reputation is not changed if he does not tell his friends but increases if he tells his friends), and the value function of high type decision maker increases with the updated beliefs. Since to choose P is dominant strategy for the high type decision maker, he tells his resolution to his friends with probability 1 in the first period ($B_T > B_{NT}$ in Assumption 6). In the first period, the expected benefit of not following the temptation in the second period is

$$\begin{aligned} E_1(NFT) &= \gamma[\phi(\rho_1, \xi_1)\{\rho_1(B_T - C_P) + (1-\rho_1)b_T\} \\ &\quad +(1-\phi(\rho_1, \xi_1))\{\rho_1(B_{NT} - C_P) + (1-\rho_1)b_{NT}\}]. \end{aligned}$$

I find the following proposition to explain the choices of the high type decision maker in the first period.

Proposition 12 *The high type decision maker in the stress time chooses to tell his resolution to his friends (T) and to persevere (P) after choosing not to follow his* *temptation (NFT) in the first period if* $\rho_1 > \rho_1^*$ *where* ρ_1^* *satisfies*

$$E_1(NFT) = A.$$

The condition, $\phi(\rho_1, \xi_1) \geq (1-\rho_1)\frac{\partial\phi(\rho_1,\xi_1)}{\partial\rho_1}$, is automatically satisfied in the first period because high type decision maker in the stress time chooses to tell his resolution to his friends with probability 1. Thus, to choose NFT, T and P in the first period is dominant strategy for the high type decision maker in the stress time under the condition as $\rho_1 > \rho_1^*$.

b. First Period Decisions of Low Type

Since the value function of the low type decision maker may or may not increase with the updated beliefs of the decision maker by following the payoff of choosing FT, I need to examine two different cases for the low type decision maker.

It is shown that when the payoff of choosing FT (A) is larger than the critical value which is determined as a function of parameters, the value function of the low type decision maker increases if both the updated belief that he is high type and the updated belief that his friends believe him as high type decrease. Among the low type decision maker, this type of the decision maker has relatively strong desire to follow his temptation because the happiness of using bad habit is relatively large. If the low type decision maker chooses not to persevere (NP) in the first period, both the payoff in the first period and that from the value function increase. It is because to choose not to persevere decreases the belief of the decision maker that he is high type and cannot increase the belief of the decision maker that his friends believe him as high type, which leads to the increase in the value function of the low type decision maker. Since to choose NP is dominant strategy in the first period, the low type decision maker who has relatively strong desire to follow the temptation has to choose not to tell his friends about his resolution ($b_{NT} > b_T$ in Assumption 6). Under the condition as $\rho_1 > \rho_1^*$ where ρ_1^* satisfies $E_1(NFT) = A$, the low type decision maker chooses not to follow his temptation (NFT), not to tell his friends (NT) and not to persevere (NP) in the first period.

Let us consider the case where the decision maker believes that the payoff of choosing FT is smaller than the critical value (the low type decision maker who has relatively weak desire to follow his temptation). In this case, the value function of the low type decision maker increases with the updated beliefs of the decision maker. The decision maker who has weak ability to endure impulses in the stress time chooses to persevere (P) on the third sub-period in the first period if the net expected benefit from doing so is greater than the net expected benefit from not persevering. Formally, the low type chooses to persevere (P) if

$$\begin{aligned} & \phi(\rho_1, \xi_1)[y_1(B_T - \frac{C_P}{\beta_L}) + y_2 V_2^L(\rho_2^{T+P}, \xi_2^{T+P})] \\ & +(1-\phi(\rho_1, \xi_1))[y_1(B_{NT} - \frac{C_P}{\beta_L}) + y_2 V_2^L(\rho_2^{NT+P}, \xi_2^{NT+P})] \\ \geq\ & \phi(\rho_1, \xi_1)[y_1 b_T + y_2 V_2^L(\rho_2^{T+NP}, \xi_2^{T+NP})] \\ & +(1-\phi(\rho_1, \xi_1))[y_1 b_{NT} + y_2 V_2^L(\rho_2^{NT+NP}, \xi_2^{NT+NP})] \end{aligned}$$

where $\phi(\rho_1, \xi_1)$ is the probability that the decision maker tells his resolution to his friends and y_K is weighted average to the payoff in the period K.

Let us normalize $y_1 + y_2$ to unity. If the decision maker only cares about the first period payoff, i.e. $y_1 = 1$, he chooses to not to persevere (NP). But, the decision maker chooses to persevere (P) with probability 1 if he only considers his second period payoff, i.e. $y_1 = 0$. The low type decision maker in the stress time chooses to persevere (P) on the third sub-period in the first period if he considers his second period as sufficiently more important (Appendix K).

In this case, if the probability that the decision maker tells his friends in the first period increases, the probability that the decision maker chooses to persevere (P) also increases in the first period, i.e. the area which guarantees the choice of persevering is larger if $\phi(\rho_1, \xi_1)$ increases. The following proposition explains that under what conditions the low type decision maker who has relatively weak desire to follow his temptation chooses to tell his friends (T) and to persevere (P) after choosing not to follow his temptation (NFT) in the first period.

Proposition 13 *Under the conditions as $\rho_1 > \rho_1^*$, the low type decision maker in the stress time chooses not to follow his temptation (NFT) and then chooses to tell his friends (T) and to persevere (P) rather than chooses not to tell (NT) and not to persevere (NP) if he considers his second period as sufficiently more important (See Appendix L).*

The low type decision maker in the stress time who considers his first period as sufficiently more important chooses not to follow his temptation (NFT), not to tell his friends (NT) and not to persevere (NP) in the first period under the condition as $\rho_1 > \rho_1^*$.

If $B_T = 5$, $B_{NT} = 4$, $b_{NT} = 3$, $b_T = 2$ and $C_P = 2$, the decision maker chooses to tell his friends (T) in the second period if $\rho_1 > \frac{1}{2}$. The decision maker chooses not to follow his temptation (NFT) in the second period if $A = \frac{5}{3}$ and $\gamma = \frac{2}{3} = \rho_2^*$. If both the updated belief of the decision maker that he is high type and that of the decision maker that his friends believe him as high type are between $\frac{1}{2}$ and $\frac{2}{3}$, the payoff of choosing to follow the temptation is lesser than the critical value, i.e. $A < 4$. Thus, the decision maker is the low type decision maker who has relatively weak desire to follow his temptation, which means that the value function of the decision maker increases with both beliefs of the decision maker. But if both the updated belief of the decision maker that he is high type and that of the decision maker that his friends believe him as high type are greater than $\frac{2}{3}$, there is a case where the decision maker increases his value function by decreasing beliefs of the decision maker because the payoff of choosing to follow temptation can be greater than the critical value. In the first case, the decision maker chooses not to follow the temptation (NFT), to tell friends (T) and to persevere (P) in the first period if he considers his second period as sufficiently more important ($y_1 \leq 0.2174$ in this example). In the second case, the decision maker chooses not to tell his friends (NT) and not to persevere (NP) in both periods after choosing not to follow his temptation (NFT) because these choices can increase the payoffs in both periods.

Let us explain the choices (tell or not to tell in each period and persevere or not to persevere in each period) of the low type decision maker if the decision maker chooses not to follow his temptation in both periods. For each type of the decision maker, it is shown that the probability to tell his friends about the resolution in the second period is positively related with the updated beliefs of the decision maker that he is high type and that his friends believe him as high type.

1. The low type decision maker who has relatively strong desire to follow his temptation chooses not to tell his friends (NT) and not to persevere (NP) after choosing not to follow his temptation in both periods.

2. The low type decision maker who has relatively weak desire to follow his temptation chooses to tell his friends (T) in both periods but chooses to persevere (P) only in the first period if he considers his second period as sufficiently more important. In this case, the decision maker who succeeds to persevere in the first period believes that to tell his friends is helpful to keep the promise with himself, and then he also chooses to tell his friends in the second period.

3. The low type decision maker who has relatively weak desire to follow his temptation chooses not to tell his friends (NT) and not to persevere (NP) in both periods if he considers his first period as sufficiently more important. The decision maker who fails to persevere in the first period may think that the failure of persevering is from NT. However, the decreasing self-reputation and the perception of reputation makes the decision maker choose not to tell his friends in the second period.

CHAPTER V

CONCLUSIONS

In the first chapter, I characterize the conditions for the existence of both good and bad reputation effects when each advisor knows the type of the other advisor, and the advisors send the message simultaneously. In any informative equilibrium, regardless of the signal both advisors have reputational incentive to send the message the bad advisor is not biased towards. By comparing the total payoff of telling the truth and that of telling a lie, I show that bad advisor sometimes tells the truth to increase his reputation and show that strong reputational concern makes the good advisor sometimes tell a lie regardless of the type of the other advisor. Moreover, bad (good) reputation effect is more likely to emerge when the good (bad) advisor knows that the other advisor is bad (good) rather than good (bad). I then examine whether the decision maker is better off if he obtains information from an additional advisor. The expected payoff of the decision maker is lower with two advisors than with only one advisor if each type of the advisor considers his second period as sufficiently more important.

After receiving the advice from the doctor, the patient may seek more information by asking an advice from the second doctor if he believe that the first doctor may sometimes send the incorrect advice. When the patient meets the second doctor, he may tell the advice of the previous doctor since he believes that the second doctor may adjust the advice which can be beneficial to the patient. By considering two period cheap talk model where advisors send the message sequentially to the decision maker, I try to examine the conditions which guarantee the existence of both good and bad reputation effect in the second chapter. Also, I determine whether simultaneous advice or sequential advice is preferred by the decision maker.

Given each message of the previous advisor, the second advisor who is of the bad type sometimes tells the truth in the first period. Regardless of the type of the first advisor, the possibility of telling the truth of the second advisor is greater (lesser) in sequential cheap talk compared to the simultaneous cheap talk when he receives the message the bad advisor is not biased towards (the bad advisor is biased towards).

There is also bad reputation effect in sequential cheap talk. Given each message of the first advisor, the second advisor who is of the good type tells a lie in the first period if he considers his second period sufficiently more important. This incentive to tell a lie occurs to make the decision maker believe what he will suggest the next time. Regardless of both the message of the first advisor and the type of the first advisor, it is better for the decision maker to consult advice simultaneously. It is because the advisor does not want to be perceived as the bad type and he wants to separate his type from the bad type more easily in sequential cheap talk.

Since the possibility of telling the truth of the second advisor is greater or lesser in sequential cheap talk by following the message of the first advisor and the possibility of telling a lie is always greater in sequential cheap talk, it is better for the decision maker to seek advice simultaneously when he seeks advice from two advisors. The analysis in the chapter is simplified because of the assumption that each advisor follows his signal if there is a conflict between the signal of each advisor. In general, the second advisor chooses 0 or 1 randomly if there is a conflict of the signal with the first advisor. If the second advisor can choose the message randomly, the value function of the second advisor is changed. Without using this assumption, I can extend the analysis of sequential cheap talk in the future.

I can extend the model to analyze the case where each advisor has imperfect information regarding the type of the other advisor. I have shown that the presence of the other advisor can affect the message of an advisor when each advisor knows the type of the other advisor. If each advisor has imperfect information about the type of the other advisor, the strategic choice of an advisor may be changed, which may lead to different results about both good and bad reputation effects.

The last chapter tries to investigate questions related to formation and execution of resolutions by individuals who have imperfect information about their ability to resist temptations. It extends the model of Benabou and Tirole (2004) by examining how an individual's perception of his reputation (i.e., his belief regarding what others think of him), in addition to self-perception, affects his choices.

Each of the two periods in the model consists of three sub-periods. The first requires one to decide whether to make a resolution or not; in the second sub-period the individual has to decide whether to inform others about his resolution or not; and the third requires him to choose between persevering with the resolution or not. The first and second sub-periods of each period are the normal time, and the third sub-period of each period is the stress time.

The decision maker who has strong ability to resist impulses in the stress time (high type) chooses to tell his friends and to persevere in the first period if he makes a resolution at the beginning of the first period. Doing so enhances his self-reputation and his perception of reputation during the second period. If the increase in the two beliefs is high enough then he goes on to make the resolution again and persevere with it during the second period.

In case of low type decision maker, the chapter particularly focuses on the decision maker who makes a resolution in the first period but does not persevere in the second period. By fixing these two choices the other four choices are identified. One of the most important observations regarding the low types is that the utility from not persevering in the second period is not always higher if self-reputation and perception of reputation is higher. It is shown that the utility from not persevering in the second period is higher with both self-reputation and perception of reputation if the payoff of choosing to follow the temptation is small enough.

After making a resolution at the beginning of the first period:

1. The low type decision maker who has a relatively strong desire to follow his temptation chooses not to tell his friends and not to persevere in both periods.

2. If the low type decision maker who has relatively weak desire to follow his temptation values the future high enough compared to the present, he chooses to tell his friends about his resolution and chooses to persevere in the first period. Perseverance in the first period enhances his self-reputation and his perception of reputation for the second period. If the increase in the two beliefs is high enough then he goes on to make the resolution, informs his friends again, but does not succeed in persevering at the end.

3. If the low type decision maker who has relatively weak desire to follow his temptation values the present high enough compared to the future, he chooses not to tell his friends and not to persevere in the first period. This leads to a reduction in self-reputation and his perception of reputation for the second period. However, if the reduction is not substantial he chooses to make the resolution again at the beginning of the second period, informs his friends again, but still fails to persevere.

As an extension of the model, if the possibility of telling or not telling is added after finishing the first period, it may be helpful to understand the effect of telling to the choices of the decision maker directly. If a relationship between the belief of the decision maker that he is a high type and the perception of the reputation is not assumed as in my dissertation, it is possible to explain the case where there is conflict between these two beliefs.

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APPENDIX A

PROOF OF PROPOSITION 2

The value function of the first advisor, if he is of the bad type, is calculated using the probability that the state of the world is 1 given his message is 1. The value function for the bad advisor is

$$\upsilon^1_{BG}[\lambda^1_2, \lambda^2_2] = y^1_2 a_2 = \frac{1}{2} y^1_2 (P^{2,1}_{1,0} + P^{2,1}_{1,1})$$

when the bad advisor meets the good advisor. I first examine the incentive for the first advisor sometimes to send the message 0 in the first period under the belief that the second advisor sometiems tells a lie. Then, I examine the incentive for the second advisor to send the message 0 if his signal is 0 and send the message 1 with probability z if his signal is 1 under the belief that the first advisor sometimes tells the truth to examine the equilibrium condition.

There is good reputation effect of the advisor who observes the signal 0 and meets the other advisor who is of the good type if

$$y^1_1(-\alpha_1) + \beta_1 y^1_2 + \eta_1 y^1_2 > 0$$

where $y^1_1 + y^1_2 = 1$

and $-\alpha_1 = \sum_{\epsilon=0}^{1} (-1)^{\epsilon} \{(P^{1,1}_{\epsilon,0})(1 - \frac{1}{2}z) + z(P^{1,1}_{\epsilon,1})\} < 0$,

$\beta_1 = \frac{1}{4}\{\sum_{\epsilon=0}^{1} (-1)^{\epsilon} \frac{1-\gamma-\lambda^1_2(\lambda^1_1,\epsilon,1)(1-\gamma)^2}{1-\lambda^1_2(\lambda^1_1,\epsilon,1)(1-2\gamma+2\gamma^2)} + f_1(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)\} > 0$ and

$\eta_1 = \frac{1}{4}\{\sum_{\epsilon=0}^{1} (-1)^{\epsilon} \frac{1-\gamma-\lambda^1_2(\lambda^1_1,\epsilon,0)(1-\gamma)^2}{1-\lambda^1_2(\lambda^1_1,\epsilon,0)(1-2\gamma+2\gamma^2)} + f_2(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)\} > 0$.

Here, $-\alpha_1$ explains the difference between the first advisor's first period payoff when he tells the truth and the payoff when he tells a lie. The bad advisor believes that the other advisor obtains the signal 0 with probability $\frac{1}{2}$. He also believes that the good advisor who observes the signal 0 sends the message 0 if his signal is 0 and sends the message 0 with probability $1-z$ if the signal is 1, i.e. with probability $\frac{1}{2}(1-z)$ or $\frac{1}{2}$, the second advisor sends the message 0.

β_1 shows the difference between the first advisor's second period payoff which is determined by the value function of the bad advisor when he tells the truth and the payoff when he tells a lie in the case where the real state of the world is revealed as 1. If the first advisor believes that the second advisor observes the signal 0, i.e. the first advisor believes that the second advisor's signal is the same as his signal, he knows that the updated belief of the decision maker about the type of the second advisor is $\frac{1}{2}(1-\gamma)\lambda_2^2(\lambda_1^2, 0, 1)$. It is because the second advisor is misinformed when the real state of the world is 1. In the case as the first advisor believes that the second advisor observes the signal 1, the updated belief of the decision maker that the second advisor is good is $\frac{1}{2}\gamma\{z\lambda_2^2(\lambda_1^2, 1, 1) + (1-z)\lambda_2^2(\lambda_1^2, 0, 1)\}$ since the good advisor who observes the signal 1 sends the message 0 with probability $1-z$.

η_1 shows the difference of the first advisor's second period payoff between telling the truth and telling a lie in the case where the real state of the world is revealed as 0. Since the expression of β_1 and η_1 is so complicated, I use the functional expression $f_a(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)\}$ for $a = 1$ or 2 to show the rest part of the difference between telling the truth and telling a lie when the state of the world is revealed as 1 or 0 respectively .

Let's consider the incentive of the second advisor to send the message as the first advisor thinks to consider the equilibrium condition. Two cases are examined- the second advisor who observes the signal 0 or the second advisor who observes the signal 1. The second advisor who observes the signal 0 sends the message 0 because

$$
\begin{aligned}
&-\frac{1}{2}x_1^2[\frac{1}{2}\gamma^2\sum_{i=0}^{1}(-1)^i(P_{i,0}^{1,1})^2 \\
&+\frac{1}{2}\gamma(1-\gamma)\{(1-z)\sum_{i=0}^{1}((-1)^i(P_{i,0}^{1,1})^2+(-1)^i(P_{i,0}^{1,1}-1)^2)) \\
&+z\sum_{i=0}^{1}((-1)^i(P_{i,1}^{1,1})^2+(-1)^i(P_{i,1}^{1,1}-1)^2))\} \\
&+\frac{1}{2}(1-\gamma)^2\sum_{i=0}^{1}(-1)^i(P_{i,0}^{1,1}-1)^2]+\upsilon_{BG}^2[\lambda_2^1,\lambda_2^2]
\end{aligned}
$$

is greater than 0. Similarly, I determine the incentive of the second advisor sometimes to send the message 0 if his signal is 1.

If the bad advisor who meets the other good advisor considers his second period as sufficiently more important, he sends the message he is not biased towards, i.e. if

$$y_1^1 < \frac{\beta_1+\eta_1}{\alpha_1+\beta_1+\eta_1} = F_B(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu) < \frac{1}{2},$$

the bad advisor who observes the signal 0 sends the message 0.

APPENDIX B

PROOF (PAGE 29)

The expressions in Appendix B are very similar as those in Appendix A. The value function of the bad advisor, if the first advisor is of the bad type, is

$$\upsilon_{BB}^{1}[\lambda_2^1, \lambda_2^2] = y_2^1 a_2 = y_2^1 P_{1,1}^{2,1}$$

when the bad advisor meets the bad advisor. If the bad advisor who meets the other bad advisor considers his second period as sufficiently more important, after observing the signal 0 he sends the message 0.

I compare the area which guarantees the existence of the bad reputation effect when the bad advisor meets good advisor with the area which guarantees the existence of the bad reputation effect when the bad advisor meets the other bad advisor and find that the good reputation effect occurs more easily when the bad advisor meets the other good advisor.

APPENDIX C

PROOF OF PROPOSITION 3

If the first advisor is of the good type, the value function for the good advisor is

$$\begin{aligned} v_{GB}^{1}[\lambda_2^1, \lambda_2^2] &= -x_2^1(a_2 - \omega_2)^2 \\ &= -\frac{1}{2}x_2^1[\gamma(P_{0,1}^{2,1})^2 + (1-\gamma)(P_{1,1}^{2,1})^2 \\ &\quad +\gamma(P_{1,1}^{2,1} - 1)^2 + (1-\gamma)(P_{0,1}^{2,1} - 1)^2] \end{aligned}$$

when he knows that the second advisor is of the bad type. I first examine the incentive for the first advisor sometimes to tell a lie under the belief that the second advisor sometimes tells a lie. Then, I examine the incentive for the second advisor to send the message 0 or 1 with some probability in each signal under the belief that the first advisor sometimes tells a lie to examine the equilibrium conditions.

The good advisor tells a lie ($m_1^1 = 0$) to increase his reputation if

$$-x_1^1\alpha_3 + x_2^1\beta_3 + x_2^1\eta_3 > 0$$

where $x_1^1 + x_2^1 = 1$ and

$\alpha_3 = \frac{1}{2}\sum_{\epsilon=0}^{1}(P_{0,\epsilon}^{1,1} - P_{1,\epsilon}^{1,1})[\rho_\epsilon\{-2\gamma^2 + (1-2\gamma+2\gamma^2)(P_{0,\epsilon}^{1,1} + P_{1,\epsilon}^{1,1})\} + \nu_\epsilon(2\gamma^2 - 2\gamma)(P_{0,\epsilon}^{1,1} + P_{1,\epsilon}^{1,1} - 1)] > 0,$

$-\beta_3 = f_3(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu) = f_3[g(\lambda_2^1(\lambda_1^1, 0, 0)) - g(\lambda_2^1(\lambda_1^1, 1, 0))]$ and

$-\eta_3 = f_4(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu) = f_4[g(\lambda_2^1(\lambda_1^1, 0, 1)) - g(\lambda_2^1(\lambda_1^1, 1, 1))]$.

α_3 explains the difference between the first advisor's first period payoff when he tells a lie and the payoff when he tells the truth. The first advisor believes that the second advisor has the same signal as him with probability $\frac{1}{2}$, i.e. $S_1^2 = 1$ with probability $\frac{1}{2}$. He also believes that the bad advisor sends the message 0 with probability

ρ if the signal of the second advisor is 1 and sends the message 0 with probability ν if the signal is 0. Since the payoff of the good advisor is affected by both the action of the decision maker and the real state of the world, I need to separate the first period payoff as the case where the real state of the world is revealed as 0 or 1.

$-\beta_3$ shows the difference between the first advisor's second period payoff which is determined by the value function of the first advisor when he tells a lie and the payoff when he tells the truth in the case where the real state of the world is revealed as 0. When the first advisor believes that the second advisor observes the signal 1, the updated belief of the decision maker that the second advisor is good is $\frac{1}{2}(1-\gamma)\{\rho\lambda_2^2(\lambda_1^2,1,0)+(1-\rho)\lambda_2^2(\lambda_1^2,0,0)\}$. Similarly, if the good advisor believes that the second advisor observes the signal 0, the updated belief of the decision maker that the second advisor is good is $\frac{1}{2}\gamma\{\nu\lambda_2^2(\lambda_1^2,1,0)+(1-\nu)\lambda_2^2(\lambda_1^2,0,0)\}$.

$-\eta_3$ shows the difference between the first advisor's payoff when he tells a lie and the payoff when he tells the truth in the case where real state of the world is revealed as 1. Since those expressions are very complicated, I use functional form f and g for simple explanation.

Let's consider the equilibrium condition by examining the incentive of the second advisor sometimes to send the message 0. Given area which guarantees the existence of the bad reputation effect, the second advisor who is of the bad type and observes the signal 0 sends the message 0 if

$$\begin{aligned}&\frac{1}{2}y_1^2\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}\\&+\frac{1}{2}\sum_{k=0}^{1}\sum_{\epsilon=0}^{1}(-1)^k\upsilon_{BG}^1[\gamma_\epsilon\lambda_2^1(\lambda_1^1,k,\epsilon),\\&\frac{1}{2}\{\gamma_\epsilon\lambda_2^2(\lambda_1^2,0,\epsilon)+(1-\gamma_\epsilon)z_k\lambda_2^2(\lambda_1^2,k,\epsilon)\}]\end{aligned}$$

is greater than 0 where $z_0=1-z$ and $z_1=z$. Each advisor observes the correct signal with probability γ. The second advisor knows that the first advisor sends the message 0 if the signal is 0 and sends the message 0 with probability $1-z$ if the first advisor observes the signal 1. The second advisor who observes the signal 0 sends the message 0 if he considers his second period sufficiently more important.

When the second advisor is of the bad type and observes the signal 1 in the first period, he sends the message 0 if

$$\begin{aligned}&\frac{1}{2}y_1^2\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}\\&+\frac{1}{2}\sum_{k=0}^{1}\sum_{\epsilon=0}^{1}(-1)^k v_{BG}^1[(1-\gamma_\epsilon)\lambda_2^1(\lambda_1^1,k,\epsilon),\\&\frac{1}{2}\{(1-\gamma_\epsilon)\lambda_2^2(\lambda_1^2,0,\epsilon)+\gamma_\epsilon z_k\lambda_2^2(\lambda_1^2,k,\epsilon)\}]\end{aligned}$$

is greater than 0. When the state of the world is revealed as 0, the signal of the second advisor is wrong. The second advisor knows that probability that the first advisor observes the signal 1 is $\frac{1}{2}$. Since the payoff in the first period is not changed by the signal of the second advisor, the payoff in this equation is the same as that in previous case. It is easily shown that the second advisor who observes the signal 1 sometimes sends the message 0 in the first period.

If the good advisor who meets the other bad advisor considers his second period as sufficiently more important, he sends the message the bad advisor is not biased towards, i.e. if

$$x_1^1<\frac{\beta_3+\eta_3}{\alpha_3+\beta_3+\eta_3}=F_G(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)<\frac{1}{2},$$

the good advisor who observes the signal 1 sends the message 0.

The area which guarantees the existence of the bad reputation effect increases with the probability that bad advisor sends the message 1 if his signal is 1 because of $\frac{\partial F_G(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)}{\partial\rho}>0$, which means that z decreases if ρ increases.

APPENDIX D

PROOF (PAGE 32)

The expressions in Appendix D is very similar as those in Appendix C. When the first advisor knows that the second advisor is also of the good type, the value function of the good advisor is

$$\begin{aligned}
\upsilon^1_{GG}[\lambda^1_2, \lambda^2_2] &= -x^1_2(a_2 - \omega_2)^2 \\
&= \frac{1}{2}x^1_2[\gamma^2\{(P^{2,1}_{0,0})^2 + (P^{2,1}_{1,1} - 1)^2\} \\
&\quad +\gamma(1-\gamma)\{(P^{2,1}_{0,1})^2 + (P^{2,1}_{1,0})^2 \\
&\quad +(P^{2,1}_{1,0} - 1)^2 + (P^{2,1}_{0,1} - 1)^2\} \\
&\quad +(1-\gamma)^2\{(P^{2,1}_{1,1})^2 + (P^{2,1}_{0,0} - 1)^2\}].
\end{aligned}$$

If the good advisor who meets the other good advisor considers his second period as sufficiently more important, after observing the signal 1 he sends the message 0.

By comparing the area which guarantees the existence of bad reputation effect in Appendix C with that in Appendix D, it is shown that the area which guarantees the existence of bad reputation effect when the good advisor meets the other bad advisor is bigger than the area which guarantees the existence of bad reputation effect when the good advisor meets the other good advisor.

APPENDIX E

PROOF (PAGE 34)

Each type of advisor sends the message 0 in the first period because of the strong reputational concern to be perceived as good advisor. In the second period, if both advisors are good, the expected payoff of the decision maker is

$$E_{GG}^{DM} = -\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[R_\epsilon R_\zeta (P_{\epsilon,\zeta}^{2,1})^2 + (1-R_\epsilon)(1-R_\zeta)(P_{\epsilon,\zeta}^{2,1}-1)^2]$$

where $E_{T_1T_2}^{DM}$ represents the expected payoff of the decision maker if the first advisor is of the type T_1 and the second advisor is of the type T_2, and $R_0 = \gamma$ and $R_1 = 1-\gamma$. It is because each good advisor sends the message which is the same as his signal and each advisor obtains the correct signal with probability γ. If the first advisor is good and the second advisor is bad, the expected payoff of the decision maker is

$$E_{GB}^{DM} = -\frac{1}{2}\sum_{\epsilon=0}^{1}\sum_{\zeta=0}^{1}[R_\epsilon R_\zeta (P_{\epsilon,1}^{2,1})^2 + (1-R_\epsilon)(1-R_\zeta)(P_{\epsilon,1}^{2,1}-1)^2].$$

It is because bad advisor always sends the message 1 regardless of his signal. If both advisors are bad, the payoff of the decision maker is

$$E_{BB}^{DM} = -\frac{1}{2}[(P_{1,1}^{2,1})^2 + (P_{1,1}^{2,1}-1)^2].$$

Similarly, the payoff of the decision maker in the second period when he has a single advisor is calculated. If the decision maker has good advisor, his expected payoff is

$$E_{G}^{DM} = -\frac{1}{2}\sum_{\epsilon=0}^{1}[R_\epsilon (P_{\epsilon}^{2,1})^2 + (1-R_\epsilon)(P_{\epsilon}^{2,1}-1)^2]$$

where $E_{T_1}^{DM}$ represents the expected payoff of the decision maker if advisor is of the type T_1. The payoff of the decision maker when he has bad advisor is

$$E_B^{DM} = -\frac{1}{2}[(P_1^{2,1})^2 + (P_1^{2,1} - 1)^2].$$

Since the decision maker believes both advisors are good, one advisor is good and the other advisor is bad, or both advisors are bad with the same probability, the payoff of the decision maker when he has two advisors is

$$-\frac{1}{2}[(P_{0,0}^{1,1})^2 + (P_{0,0}^{1,1} - 1)^2] + \frac{1}{3}[E_{GG}^{DM} + E_{GB}^{DM} + E_{BB}^{DM}].$$

The payoff of the decision maker when he has a single advisor is

$$\frac{1}{2}[-(P_0^{1,1})^2 - (P_0^{1,1} - 1)^2 + E_G^{DM} + E_B^{DM}].$$

Under the case where each advisor considers his second period as sufficiently more important, since the expected payoff of the decision maker when he has a single advisor is greater than that when he has two advisors, it is better for the decision maker to consult only a single advisor.

APPENDIX F

PROOF OF PROPOSITION 7

Let's consider the case where the first advisor is of the good type and sends the message 0 to the decision maker. Since the second advisor knows both the type of the first advisor and the message sent by the first advisor, the total payoff of the second advisor is determined by his message in the first period. The total payoff of the second advisor, if he is of the bad type, is

$$\begin{aligned} & y_1^2 a_1 + v_{GB}^2[\lambda_2^1, \lambda_2^2] \\ = \quad & y_1^2 a_1 + \frac{1}{2} y_2^2 (P_{0,1}^{2,1} + P_{1,1}^{2,1}). \end{aligned}$$

When the second advisor is of the bad type and receives the signal 0 in the first period, the second advisor sends the message 0 (i.e., there is the good reputation effect) if

$$y_1^2\{P_{0,0}^{1,1} - P_{0,1}^{1,1} + \frac{1}{2}(\beta_1 - \alpha_1)\} > \frac{1}{2}(\beta_1 - \alpha_1)$$

where $\alpha_1 = \frac{1}{2}[\sum_{\epsilon=0}^{1} \frac{1-\gamma-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)} + f_1(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)]$

$> \beta_1 = \frac{1}{2}[\sum_{\epsilon=0}^{1} \frac{1-\gamma-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)} + f_2(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)]$.

α_1 explains the value function of the second advisor when he meets the other good advisor if the second advisor sends the message 0 in the first period given the message of the first advisor as 0. The first part shows the probability that the state of the world in the second period is 1 given that the message of the first advisor is 0 and then the message of the second advisor is 1. The next expression $f_1(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)$ shows the conditional probability that the state of the world in the second period is 1 given message 0 from both advisors sequentially. Here, the updated belief of the decision maker about the type of the first advisor is $\lambda_2^1(\lambda_1^1, 0, 0)$ or $\lambda_2^1(\lambda_1^1, 0, 1)$ by following the state of the world. The updated belief of the decision maker that the second advisor is of the good type is $\lambda_2^2(\lambda_1^2, 0, 0|m_1^1 = 0)$ or $\lambda_2^2(\lambda_1^2, 0, 1|m_1^1 = 0)$ by following the state of the world.

Only different thing in β_1 is that the second advisor sends the message 1 in the first period. So, the updated belief of the decision maker about the type of the first advisor is the same as that in α_1. But, the updated belief of the decision maker that the second advisor is of the good type is changed to $\lambda_2^2(\lambda_1^2, 1, 0|m_1^1 = 0)$ or $\lambda_2^2(\lambda_1^2, 1, 1|m_1^1 = 0)$ by following the state of the world. The expression $f_2(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)$ shows the conditional probability that the state of the world in the second period is 1 given that the message of the first advisor is 0 and the message of the second advisor is 1 in the first period.

If the bad advisor who meets the other good advisor and receives the message 0 from previous advisor considers his second period sufficiently more important, he sends the message he is not biased towards, i.e. if

$$y_1^2 < \frac{\frac{1}{2}(\alpha_1 - \beta_1)}{P_{0,1}^{1,1} - P_{0,0}^{1,1} + \frac{1}{2}(\alpha_1 - \beta_1)} = F_B^1(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu) < \frac{1}{2},$$

the bad advisor who observes the signal 0 sends the message 0 in the first period.

By using the same method, the case where the first advisor is of the bad type and sends the message 0 is considered. Since the payoff in the first period is only affected by the action of the decision maker who does not know the type of each advisor, the payoff in the first period is the same as the previous case. However, the type of the first advisor is changed, the value function of the second advisor is changed as $\upsilon_{BB}^2[\lambda_2^1, \lambda_2^2] = y_2^2 P_{1,1}^{2,1}$.

When the second advisor is of the bad type and receives the signal 0 in the first period, the second advisor sends the message 0 if

$$y_1^2(P_{0,0}^{1,1} - P_{0,1}^{1,1} + \beta_2 - \alpha_2) > \beta_2 - \alpha_2$$

where $\alpha_2 = \frac{1}{2}\sum_{\epsilon=0}^{1} \frac{1-(1-\gamma)(\lambda_2^1(\lambda_1^1,0,\epsilon)+\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0))+(1-\gamma)^2\lambda_2^1\lambda_2^2}{2-\lambda_2^1(\lambda_1^1,0,\epsilon)-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)+(1-2\gamma+2\gamma^2)\lambda_2^1\lambda_2^2}$
$> \beta_2 = \frac{1}{2}\sum_{\epsilon=0}^{1} \frac{1-(1-\gamma)(\lambda_2^1(\lambda_1^1,0,\epsilon)+\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0))+(1-\gamma)^2\lambda_2^1\lambda_2^2}{2-\lambda_2^1(\lambda_1^1,0,\epsilon)-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)+(1-2\gamma+2\gamma^2)\lambda_2^1\lambda_2^2}$.

α_2 (or β_2) explains the value function of the second advisor if the first advisor is of the bad type when the second advisor sends the message 0 (or 1) in the first period. The updated belief of the decision maker about the type of the first advisor is calculated in each period given the message of the first advisor as 0. The updated belief of the decision maker that the second advisor is of the good type is determined by the message of both the first and the second advisor in each state of the world.

If the second advisor considers his future payoff sufficiently more important, the advisor who observes the signal 0 sends the message 0 in the first period, i.e., if

$$y_1^1 < \frac{\frac{1}{2}(\alpha_2 - \beta_2)}{P_{0,1}^{1,1} - P_{0,0}^{1,1} + \frac{1}{2}(\alpha_2 - \beta_2)} = F_B^2(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu) < \frac{1}{2},$$

there is the good reputation effect.

APPENDIX G

PROOF OF PROPOSITION 8

Let's first consider the case where the first advisor is of the good type and sends the message 1 in the first period. The payoff of the second advisor in the first period is changed as $P^{1,1}_{1,0}$ (or $P^{1,1}_{1,1}$) by following the message of the second advisor. If the second advisor considers his second period sufficiently more important, i.e. if

$$y_1^2 < \frac{\frac{1}{2}(\alpha_3 - \beta_3)}{P^{1,1}_{1,1} - P^{1,1}_{1,0} + \frac{1}{2}(\alpha_3 - \beta_3)} = F^3_B(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu) < \frac{1}{2},$$

where

$\alpha_3 = \frac{1}{2}[\sum_{\epsilon=0}^{1} \frac{1-\gamma-\lambda^2_2(\lambda^2_1,0,\epsilon|m^1_1=1)(1-\gamma)^2}{1-\lambda^2_2(\lambda^2_1,0,\epsilon|m^1_1=1)(1-2\gamma+2\gamma^2)} + f_3(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)]$

$> \beta_3 = \frac{1}{2}[\sum_{\epsilon=0}^{1} \frac{1-\gamma-\lambda^2_2(\lambda^2_1,1,\epsilon|m^1_1=1)(1-\gamma)^2}{1-\lambda^2_2(\lambda^2_1,1,\epsilon|m^1_1=1)(1-2\gamma+2\gamma^2)} + f_4(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)]$, the second advisor sends the message 0 in the first period.

The first expression in α_3 is different to the first case in Appendix A because the updated belief of the decision maker about the type of the second advisor is changed by the message sent by the first advisor. $f_3(\lambda^1_1, \lambda^2_1, \gamma, z, \rho, \nu)$ shows the conditional probability that the state of the world in the second period is 1 given that the message of the first advisor is 1 and that of the second advisor is 0. The expression β_3 explains the value function of the second advisor if he sends the message 1 in the first period.

Similarly, the existence of the good reputation effect is easily shown in the case where the first advisor is of the bad type and sends the message 1 in the first period.

APPENDIX H

PROOF OF PROPOSITION 9

Let's consider the case where the first advisor is of the bad type and sends the message 0 in the first period. The total payoff of the second advisor, if he is of the good type, is

$$\begin{aligned} & -x_1^2(a_1-\omega_1)^2+\upsilon_{BG}^2[\lambda_2^1,\lambda_2^2] \\ = \quad & -x_1^2(a_1-\omega_1)^2-\frac{1}{2}x_2^2\sum_{k=0}^{1}[\gamma_k(P_{1,k}^{2,1})^2+(1-\gamma_k)(P_{1,k}^{2,1}-1)^2]. \end{aligned}$$

The second advisor who observes the signal 1 sends the message 0 in the first period if

$$-\frac{1}{2}x_1^2\sum_{\epsilon=0}^{1}\{(P_{0,\epsilon}^{1,1})^2+(P_{0,\epsilon}^{1,1}-1)^2\}-\frac{1}{2}(1-x_1^2)(\alpha_4-\beta_4)>0$$

where $\alpha_4=\frac{1}{2}[\sum\limits_{\epsilon=0}^{1}\gamma_\epsilon\{(\frac{1-\gamma-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,0,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)}-\epsilon)^2+(f_5(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)-1+\epsilon)^2\}]$

and $\beta_4=\frac{1}{2}[\sum\limits_{\epsilon=0}^{1}\gamma_\epsilon\{(\frac{1-\gamma-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-\gamma)^2}{1-\lambda_2^2(\lambda_1^2,1,\epsilon|m_1^1=0)(1-2\gamma+2\gamma^2)}-\epsilon)^2+(f_6(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)-1+\epsilon)^2\}]$.

Since the first period payoff is affected by both the action of the decision maker and the state of the world in the first period, the assumption that the state of the world is equally likely is applied. In each state of the world, α_4 explains the value function of the second advisor if both advisors send the message 0 in the first period. β_4 explains the value function of the second advisor if the first advisor sends the message 0 and the message of the second advisor is 1 in the first period in each state of the world. $f_5(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)$ and $f_5(\lambda_1^1,\lambda_1^2,\gamma,z,\rho,\nu)$ show the probability that the state of the world in the second period is 1 given message 1 from both advisors if the updated belief of the decision maker about the type of the second advisor is $\lambda_2^2(\lambda_1^2,0,\epsilon)$ and $\lambda_2^2(\lambda_1^2,1,\epsilon)$ respectively.

Let's consider the equilibrium condition by examining the incentive of the first advisor to send the message 0. Given area which guarantees the existence of the bad reputation effect, the first advisor who is of the bad type and observes the signal 0 sends the message 0 if

$$\begin{aligned}
&\frac{1}{2}y_1^1\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}\\
&+\frac{1}{2}\sum_{k=0}^{1}\sum_{\epsilon=0}^{1}(-1)^k\upsilon_{BG}^1[\gamma_\epsilon\lambda_2^1(\lambda_1^1,k,\epsilon),\\
&\frac{1}{2}\{\gamma_\epsilon\lambda_2^2(\lambda_1^2,0,\epsilon)+(1-\gamma_\epsilon)z_k\lambda_2^2(\lambda_1^2,k,\epsilon)\}]
\end{aligned}$$

is greater than 0 where $z_0 = 1 - z$ and $z_1 = z$. By using Bayes' rule, the probability that the second advisor observes the same signal to the first advisor is $\frac{1}{2}$. Each advisor observes the correct signal with probability γ. The first advisor knows that the second advisor sends the message 0 if the signal is 0 and sends the message 0 with probability $1 - z$ if the second advisor observes the signal 1. The first advisor who observes the signal 0 sends the message 0 if he considers his second period sufficiently more important, i.e. if $y_1^1 < y_1^* = f_7(\lambda_1^1, \lambda_1^2, \gamma, z, \rho, \nu)$, the first advisor sends the message 0 in the first period.

When the first advisor is of the bad type and observes the signal 1 in the first period, he sends the message 0 if

$$\begin{aligned}
&\frac{1}{2}y_1^1\{(2-z)(P_{0,0}^{1,1}-P_{1,0}^{1,1})+z(P_{0,1}^{1,1}-P_{1,1}^{1,1})\}\\
&+\frac{1}{2}\sum_{k=0}^{1}\sum_{\epsilon=0}^{1}(-1)^k\upsilon_{BG}^1[(1-\gamma_\epsilon)\lambda_2^1(\lambda_1^1,k,\epsilon),\\
&\frac{1}{2}\{(1-\gamma_\epsilon)\lambda_2^2(\lambda_1^2,0,\epsilon)+\gamma_\epsilon z_k\lambda_2^2(\lambda_1^2,k,\epsilon)\}]
\end{aligned}$$

is greater than 0. When the state of the world is revealed as 0, the signal of the first advisor is wrong. The first advisor knows that probability that the second advisor observes the signal 1 is $\frac{1}{2}$. Since the payoff in the first period is not changed by the signal of the first advisor, the payoff in this equation is the same as that in previous case. It is easily shown that the first advisor who observes the signal 1 sometimes sends the message 0 in the first period.

In equilibrium, the second advisor who observes the signal 1 and receives the message 0 from the first advisor sends tells a lie if he considers his second period payoff sufficiently more important, i.e. if

$$x_1^2 < \frac{\alpha_4 - \beta_4}{(P_{0,\epsilon}^{1,1})^2 + (P_{0,\epsilon}^{1,1} - 1)^2 + \alpha_4 - \beta_4},$$

the second advisor sends the message 0 in the first period.

If the second advisor knows that the first advisor is of the good type and receives the message 0 in the first period, the payoff of the second advisor in the first period is the same as previous case. However, the payoff of the second advisor in the second period is changed from $\upsilon_{BG}^2[\lambda_2^1, \lambda_2^2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}[\gamma_k (P_{1,k}^{2,1})^2 + (1-\gamma_k)(P_{1,k}^{2,1} - 1)^2]$ to $\upsilon_{GG}^2[\lambda_2^1, \lambda_2^2] = -\frac{1}{2}x_2^2 \sum_{k=0}^{1}\sum_{i=0}^{1}[\gamma_k\gamma_l (P_{k,l}^{2,1})^2 + (1-\gamma_k)(1-\gamma_l)(P_{k,l}^{2,1} - 1)^2]$. By comparing the total payoff of the second advisor of each message, it is shown that the second advisor who observes the signal 1 sends the message 0 if he considers his second period sufficiently more important. Let's consider the incentive of the first advisor to send the message 0 in the condition where the second advisor sometimes tells a lie in the first period. Two cases are examined- the first advisor who observes the signal 0 sends the message 0 if the first advisor considers his second period sufficiently more important or the first advisor who observes the signal 1 sends the message 0 if the second period is sufficiently more important to the first advisor. The first advisor who observes the signal 0 sends the message 0 if

$$
\begin{aligned}
&-\frac{1}{2}x_1^1[\frac{1}{2}\gamma^2\sum_{i=0}^{1}(-1)^i(P_{i,0}^{1,1})^2 \\
&+\frac{1}{2}\gamma(1-\gamma)\{(1-z)\sum_{i=0}^{1}((-1)^i(P_{i,0}^{1,1})^2+(-1)^i(P_{i,0}^{1,1}-1)^2)) \\
&+z\sum_{i=0}^{1}((-1)^i(P_{i,1}^{1,1})^2+(-1)^i(P_{i,1}^{1,1}-1)^2))\} \\
&+\frac{1}{2}(1-\gamma)^2\sum_{i=0}^{1}(-1)^i(P_{i,0}^{1,1}-1)^2]+\upsilon_{GG}^2[\lambda_2^1,\lambda_2^2]
\end{aligned}
$$

is greater than 0. The payoff of the first period in the second period is determined by the updated belief of the decision maker in each message.

APPENDIX I

PROOF OF PROPOSITION 10

If the second advisor who observes the signal 1 meets the other advisor who is of the bad type and receives the message 1 from the first advisor in the first period, he sends the message 0 if

$$
\begin{aligned}
&-\frac{1}{2}x_1^2\sum_{\epsilon=0}^{1}\{(P_{1,\epsilon}^{1,1})^2+(P_{1,\epsilon}^{1,1}-1)^2\}\\
&+\frac{1}{2}\sum_{\epsilon=0}^{1}\upsilon_{BG}^2[\lambda_2^1(\lambda_1^1,1,\epsilon),\lambda_2^2(\lambda_1^2,0,\epsilon)]\\
&-\frac{1}{2}\sum_{\epsilon=0}^{1}\upsilon_{BG}^2[\lambda_2^1(\lambda_1^1,1,\epsilon),\lambda_2^2(\lambda_1^2,1,\epsilon)]
\end{aligned}
$$

is greater than 0. By considering the incentive of the first advisor to send the message 1 in the first period, it is shown that the second advisor tells a lie if he considers his second period sufficiently more important. In the case where the first advisor is of the good type and sends the message 1 in the first period, the value function of the second advisor is changed from $\upsilon_{BG}^2[\lambda_2^1,\lambda_2^2]$ to $\upsilon_{GG}^2[\lambda_2^1,\lambda_2^2]$.

APPENDIX J

PROOF (PAGE 75)

The expected payoff of choosing not to follow his temptation (NFT) in the second period is

$$\begin{aligned} E_2(NFT) &= \gamma[\phi(\rho_2, \xi_2)\{\rho_2(B_T - C_P) + (1-\rho_2)b_T\} + \\ &\quad (1-\phi(\rho_2, \xi_2))\{\rho_2(B_{NT} - C_P) + (1-\rho_2)b_{NT}\}]. \end{aligned}$$

From the partial derivative of the expected payoff of not following the temptation to ρ_2, I can find that

$$\begin{aligned} \frac{\partial E_2(NFT)}{\partial \rho_2} &= \gamma[\frac{\partial \phi(\rho_2, \xi_2)}{\partial \rho_2}\{(b_T - b_{NT}) + \\ &\quad \rho_2\{(B_T - B_{NT}) - (b_T - b_{NT})\}\} + \\ &\quad (B_{NT} - C_P - b_{NT}) + \\ &\quad \phi(\rho_2, \xi_2)\{(B_T - B_{NT}) - (b_T - b_{NT})\}] \end{aligned}$$

is positive if $\phi(\rho_2, \xi_2)$ is greater than $(1-\rho_2)\frac{\partial \phi(\rho_2, \xi_2)}{\partial \rho_2}$ and $B_K > C_P + b_K$.

APPENDIX K

PROOF (PAGE 82)

The low type decision maker in the stress time chooses to persevere (P) on the third sub-period in the first period if he considers his second period as sufficiently more important, i.e. if

$$y_1 \leq y_1^* = \frac{f_1(\rho_1, \xi_1, V_2^L(\rho_2, \xi_2))}{f_2(\rho_1, \xi_1, V_2^L(\rho_2, \xi_2), B_K, b_K, C_P)} < \frac{1}{2}$$

for $K \in \{T, NT\}$ where $f_1(\rho_1, \xi_1, V_2^L(\rho_2, \xi_2))$ represents the difference of the value function between the choice of perseverance and the choice of not persevering when the decision maker chooses to tell or not, and $f_2(\rho_1, \xi_1, V_2^L(\rho_2, \xi_2),\ B_K, b_k, C_P)$ represents the sum between $f_1(\rho_1, \xi_1, V_2^L(\rho_2, \xi_2))$ and the payoff difference of choosing P or NP.

APPENDIX L

PROOF OF PROPOSITION 13

For the low type decision maker in the stress time, the decision maker chooses to persevere if

$$y_1(B_T - \frac{C_P}{\beta_L}) + y_2 V_2^L(\rho_2^{T+P}, \xi_2^{T+P}) \geq y_1 b_T + y_2 V_2^L(\rho_2^{T+NP}, \xi_2^{T+NP})$$

when the decision maker tells the resolution to his friends. Similarly when the decision maker does not tell his friends, he chooses to persevere if

$$y_1(B_{NT} - \frac{C_P}{\beta_L}) + y_2 V_2^L(\rho_2^{NT+P}, \xi_2^{NT+P}) \geq y_1 b_{NT} + y_2 V_2^L(\rho_2^{NT+NP}, \xi_2^{NT+NP}).$$

Since the probability that the decision maker tells to his friends, $\phi(\rho_1, \xi_1)$, is increasing with the probability that the decision maker chooses to persevere (P) in the first period, I can find the total payoff of the decision maker when he chooses to tell his friends and to persevere as $y_1(B_T - \frac{C_P}{\beta_L}) + y_2 V_2^L(\rho_2^{T+P}, \xi_2^{T+P})$, and that of the decision maker when he chooses not to tell and not to persevere as $y_1 b_{NT} + y_2 V_2^L(\rho_2^{NT+NP}, \xi_2^{NT+NP})$. Thus, the decision maker chooses to tell his friends and to persevere if

$$y_1 \leq y_1^{**} = \frac{f_3(V_2^L(\rho_2, \xi_2))}{f_4(V_2^L(\rho_2, \xi_2) B_K, b_K, C_P)} < \frac{1}{2}$$

where $f_3(V_2^L(\rho_2, \xi_2))$ represents the difference between the value function when he tells his friends and chooses to persevere and that when he does not tell his friends and chooses not to persevere in the first period, and $f_4(V_2^L(\rho_2, \xi_2) B_K, b_K, C_P)$ represents the sum between $f_1(\rho_1, \xi_1, V_2^L(\rho_2, \xi_2))$ and the payoff difference of choosing either P and T, or NP and NT.

VITA

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