ROBUST MODEL-BASED FAULT DIAGNOSIS FOR CHEMICAL PROCESS SYSTEMS

A Dissertation

by

SRINIVASAN RAJARAMAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2006

Major Subject: Chemical Engineering

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Approved by:

Co-Chairs of Committee,	M. Sam Mannan
	Juergen Hahn
Committee Members,	M. El-Halwagi
	Shankar P. Bhattacharyya
Head of Department,	Kenneth R. Hall

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ABSTRACT

Robust Model-Based Fault Diagnosis for Chemical Process Systems. (May 2006)

Srinivasan Rajaraman, B.S., Indian Institute of Technology, Roorkee, India

Co–Chairs of Advisory Committee: Dr. M. Sam Mannan
Dr. Juergen Hahn

Fault detection and diagnosis have gained central importance in the chemical process industries over the past decade. This is due to several reasons, one of them being that copious amount of data is available from a large number of sensors in process plants. Moreover, since industrial processes operate in closed loop with appropriate output feedback to attain certain performance objectives, instrument faults have a direct effect on the overall performance of the automation system. Extracting essential information about the state of the system and processing the measurements for detecting, discriminating, and identifying abnormal readings are important tasks of a fault diagnosis system.

The goal of this dissertation is to develop such fault diagnosis systems, which use limited information about the process model to robustly detect, discriminate, and reconstruct instrumentation faults. Broadly, the proposed method consists of a novel nonlinear state and parameter estimator coupled with a fault detection, discrimination, and reconstruction system.

The first part of this dissertation focuses on designing fault diagnosis systems that not only perform fault detection and isolation but also estimate the shape and size of the unknown instrument faults. This notion is extended to nonlinear processes

whose structure is known but the parameters of the process are *a priori* uncertain and bounded. Since the uncertainty in the process model and instrument fault detection interact with each other, a novel two-time scale procedure is adopted to render overall fault diagnosis. Further, some techniques to enhance the convergence properties of the proposed state and parameter estimator are presented.

The remaining part of the dissertation extends the proposed model-based fault diagnosis methodology to processes for which first principles modeling is either expensive or infeasible. This is achieved by using an empirical model identification technique called subspace identification for state-space characterization of the process.

Finally the proposed methodology for fault diagnosis has been applied in numerical simulations to a non-isothermal CSTR (continuous stirred tank reactor), an industrial melter process, and a debutanizer plant.

To my parents and my sister for all their unalloyed love, support, and encouragement

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CHAPTER I

INTRODUCTION

Early and accurate fault detection and diagnosis is an essential component of operating modern chemical plants in order to reduce downtime and costs, increase safety and product quality, and minimize the impact on the environment. A survey [1] revealed that the US-based petrochemical industry could save up to $10 billion annually if abnormal process behavior could be detected, diagnosed, and appropriately dealt with. Studies suggest [2] that this industry alone loses over $20 billion annually due to inappropriate reaction to abnormal behavior.

The importance of monitoring the variables and interpreting their variations increases with the increase in the level of instrumentation in chemical plants. While most of the variations seen in measurements result from changing operating conditions, some of these can be directly linked to instrument faults. Therefore, gathering essential information about the state of a system and processing data for detecting, discriminating, and reconstructing abnormal readings are important tasks of a fault diagnosis system [3], which entails the following objectives:

(i) **Fault detection**: a Boolean decision about the existence of faults in a system.

(ii) **Fault isolation/discrimination**: determination of the location of a fault, e.g., which sensor or actuator is not operating within normal limits.

(iii) **Fault reconstruction/estimation**: estimation of size and type of a fault.

There exists numerous techniques for fault diagnosis [4]. The majority of these approaches are based upon data from past operations in which statistical measures

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are used to compare current operating data to earlier conditions of the process where the state of the process was known, however, a significant body of literature also exists for fault detection and identification of measurement bias based upon fundamental models [5, 6]. While these techniques are often easy to implement, they do have the drawback that it is not possible to perform fault reconstruction, that a large amount of past data are required, that the method may not be able to detect a fault if operating conditions have changed significantly, or that process exhibiting highly nonlinear behavior may be difficult to diagnose [5].

In order to address above problems, the first part of dissertation presents an approach for fault diagnosis based upon a first principles model of the process with uncertainty in the parameters of the model. Using fundamental models in the procedure allows for accurate diagnosis in spite of a change in operating conditions, while the online estimation of model parameters takes care of the plant-model mismatch. The parameter estimation is performed using an augmented nonlinear observer based on stabilization of an interval family of polynomials using Kharitonov's theorem [7] in order to ensure a certain level of robustness for the designed observer. The fault diagnosis alone uses the computation of residuals (i.e. mismatch between measured output and estimated output using the model) for fault detection [3] and appropriate filters are designed to achieve fault isolation and reconstruction as well. Since it is not possible to simultaneously perform parameter estimation and fault detection, due to the interaction of these two tasks, an approach where these calculations are taking place at different time scales is implemented. It is shown that fault detection, isolation, and reconstruction for nonlinear systems with uncertain parameters can be performed under realistic assumptions with the presented approach.

The second part of this dissertation focuses on enhancing the convergence properties of the aforementioned parameter estimation technique. Finally, the model-based

fault diagnosis technique developed is extended to processes where developing first principles models are either expensive or infeasible by computing subspace models from measured inputs and outputs of the process.

1. Dissertation outline

Chapter II will review existing techniques for model-based fault diagnosis for linear as well as nonlinear systems. The emphasis is on methods that are based on residual generation through observers/estimators, since this is the focus of the thesis. The advantages and limitations associated with observer-based residual generation methods are also described here. In Chapter III, filter design for simultaneously performing detection, isolation, and reconstruction of sensor faults through frequency domain characterization is explained. The concept of frequency domain design for model-based fault diagnosis was first introduced by Vishwanadham et al [8] and was later developed by Ding and Frank [9]. However, the work in this dissertation is the first to extend factorization methods for linear systems to nonlinear systems with parametric uncertainties. Next, the concept of robust residual generation against plant-model mismatch for nonlinear processes is introduced. A detailed background of robust stability of interval family of polynomials [7] is also presented. This serves as the basis for developing a novel nonlinear state and parameter estimation technique required to identify model parameters from process measurements. Further, the synthesis technique of the overall fault diagnosis combining the proposed parameter estimation technique with the filter design using frequency domain approaches for sensor fault detection, isolation, and reconstruction in nonlinear processes with parametric uncertainties is presented. Finally, the procedure is applied to a nonisothermal CSTR in numerical simulations, which results in robust fault diagnosis even in the

presence of parametric uncertainties. Also some techniques on enhancing the speed of convergence of the proposed nonlinear state and parameter estimator is provided in this chapter. This is achieved by placing an upper bound on the real part of the closed-loop eigen values of the estimator over a bounded set of parameter.

In chapter IV, the concept of robust fault diagnosis using the proposed two-time scale approach is extended to processes for which first-principles modeling is either computationally expensive or infeasible. This is achieved by developing empirical state-space models from the actual process measurements through Subspace Model Identification (SMI) [10]. A brief description of SMI followed by design techniques for fault detection filters using subspace models of the process is presented. Finally, the proposed subspace model-based fault diagnosis technique is applied on a non-isothermal CSTR with time-varying parametric uncertainties, data from a debutanizer plant, and data from an industrial melter process through numerical simulations.

CHAPTER II

REVIEW OF MODEL-BASED FAULT DIAGNOSIS

In this chapter a comprehensive review of model-based fault diagnosis techniques in the existing literature is presented. Section 1 starts with definitions, terminologies, and the general scheme used in the field of model-based fault diagnosis. Further motivations for model-based fault diagnosis is also presented in this section. Section 2 introduces model-based fault diagnosis for linear systems, whereas Section 3 reviews model-based fault diagnosis techniques keeping nonlinear systems in focus.

1. Model-based fault diagnosis

Although there are numerous articles and books on fault diagnosis [3, 11, 12, 13], it is still worthwhile to introduce basic terminologies involved in fault diagnosis. Following are the terminologies that resulted through an initiative by The IFAC Technical Committee: SAFEPROCESS (Fault Detection, Supervision and Safety for Technical Processes). Detailed discussions regarding terminology can be found in papers by Isermann and Ballé [14] and Vanschrick [15]

- **Fault**: an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/normal/standard condition.
- **Failure**: a permanent disruption of a system's ability to perform a desired function under specified operating conditions
- **Residual**: a symptom to indicate a fault, based on a deviation between measurements and analytical model-based computations.
- **Fault detection**: a Boolean decision that something has gone wrong or that

everything is fine.

- **Fault isolation**: determination of the location of the fault, for example, which sensor or actuator is not operating within normal limits. Follows fault detection.
- **Fault identification**: determination of the size and time-variant behavior of a fault. Follows fault isolation.
- **Fault diagnosis**: determination of the kind, size, location, and time of detection of a fault. Includes fault detection, isolation, and identification.

1.a. Motivations for model-based fault diagnosis

The need for research in model-based fault diagnosis arises because the most frequently used diagnosis method is to monitor the trend of a particular signal, and take action when it has reached a preset threshold. One of the drawbacks of this method of diagnosis is that false alarms are often triggered due to a change in the input level or in operating conditions. Moreover, a single source of fault can propagate to trigger multiple alarms, and hence root cause analysis becomes very difficult.

Another traditional approach to fault diagnosis in the wider application context is based on "hardware (or physical/parallel) redundancy" [3] techniques which use multiple sensors, actuators, computers and relevant softwares to measure a particular variable. Often, a voting scheme is applied to the redundant sensor system to decide if and when a fault has occurred and its likely location amongst redundant system components. This method, while easy to implement, also suffers certain drawbacks such as additional cost incurred due to redundant equipment installation and maintenance and additional space required to accommodate the equipments.

In view of above points, a mathematical model-based approach which provides

functional relationship between different system signals is required. This is the concept of "analytical (functional) redundancy", which uses redundant analytical (functional) relationships between various measured variables of the process. No redundant hardware is required in this approach, and hence analytical redundancy is potentially more reliable than hardware redundancy [16].

1.b. General scheme for model-based fault diagnosis

As explained in the preceding section, analytical redundancy uses functional relationships to check consistency among different variables to detect fault(s). A residual signal is generated through consistency checking among different variables. The residual signal is designed so that it is zero-valued when the system is normal, and should diverge from zero when a fault occurs in the system. The residual generation is normally achieved through a comparison between a measured signal with its estimation. The estimation is done using the mathematical model of the system being considered.

The major advantage of using model-based approach is that no additional hardware components are required to realize a fault diagnosis algorithm. Furthermore, in many cases, the measurements required for process control are sufficient for performing fault diagnosis [3]. Therefore, only additional storage space and computational power is required for implementing an on-line model-based fault diagnosis algorithm.

1.c. Modeling of faulty systems

Consider a mutiple-input and multiple-output linear dynamic systems as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu_R(t) \\ y_R(t) &= Cx(t) + Du_R(t) \end{aligned} \tag{II.1}$$

where $x \in \mathbb{R}^n$ is a vector of state variables, $u_R \in \mathbb{R}^q$ is a vector of input from the actuators, $y_R \in \mathbb{R}^m$ is a vector of actual outputs, n is the number of states, q refers to the number of input variables and m refers to the number of outputs. A, B, C, and D are matrices of appropriate dimensions. For the purposes of modeling, the open-loop system can be divided into three parts: actuators, system dynamics, and sensors as explained in the Figure 1. When the system has sensor and actuator faults, the system model is described as below:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Bf_a(t) \\ y(t) &= Cx(t) + Du(t) + Df_a(t) + f_s(t) \end{aligned} \tag{II.2}$$

where $f_s(t) \in \mathbb{R}^m$ is the sensor fault vector, $f_a(t) \in \mathbb{R}^q$ is the actuator fault vector, $y(t) = y_R(t) + f_s(t)$ is the actual sensor output and $u(t) = u_R(t) - f_a(t)$ is the actual input to the actuator.

For the sake of convenience, $f_s(t)$ and $f_a(t)$ are lumped together. The state-space model is then rewritten as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + R_1 f(t) \\ y(t) &= Cx(t) + Du(t) + R_2 f(t) \end{aligned} \tag{II.3}$$

The matrices R_1 and R_2 are called the fault entry matrices, and represent the effect of faults on the system. Performing Laplace-transform on eq (II.3), an input-output transfer matrix representation of the above system is then described as:

$$y(s) = G_u(s)u(s) + G_f(s)f(s) \tag{II.4}$$

where

$$G_u(s) = C(sI - A)^{-1}B + D$$
$$G_f(s) = C(sI - A)^{-1}R_1 + R_2 \tag{II.5}$$

The general model for a faulty system described by eq (II.3) in the time domain and eq (II.4) in the frequency domain has been widely accepted in the fault diagnosis literature [17, 18, 19, 20, 21]

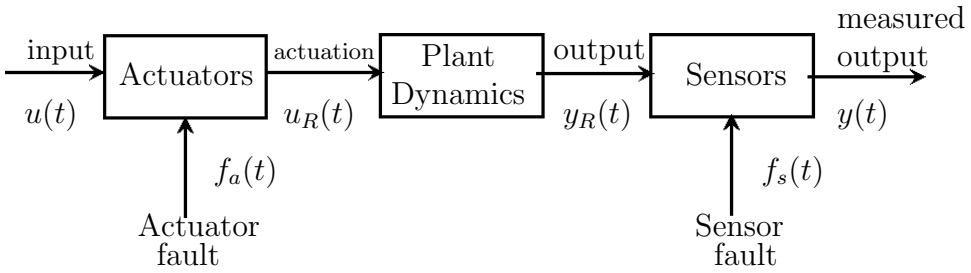

Fig. 1. Open-loop faulty system.

1.d. Structure of residual generation in model-based fault diagnosis in linear systems

Residuals are the quantities that represent the inconsistency between the actual system variables (measured) and their estimated counter part. They are derived using an appropriate mathematical model of the process.

One of the easiest ways for residual generation is to duplicate the system measurements by using an open-loop functional relationship of the model between system inputs and outputs. The major drawback with this method is that the residual generator cannot be guaranteed stable to be stable when the open-loop system is unstable. Therefore, it is mandatory that such a functional relationship is derived between system inputs and outputs so that residual generator is stable. A general residual scheme

for linear dynamic system is shown in Figure 2

Where $F_1(u, y)$ is a stable estimator that generates the redundant signal $z(t)$ to be

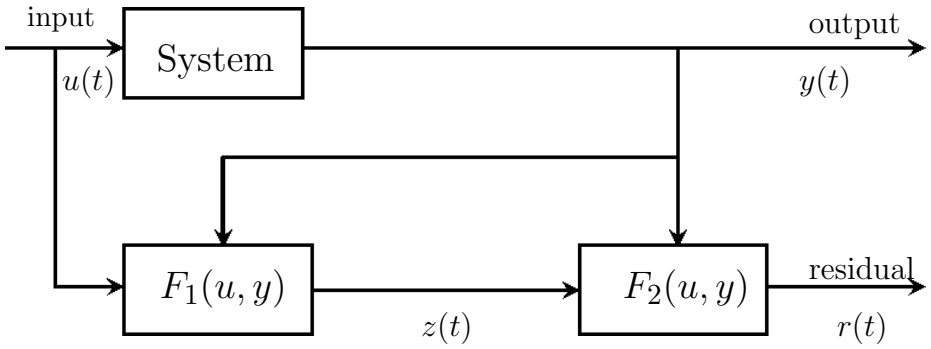

Fig. 2. Residual generation structure.

compared with the actual system output $y(t)$ to generate the residual signal $r(t)$.

2. Model-based fault diagnosis for linear systems

Various methods are available for model-based fault diagnosis [3]. However, this dissertation focuses on developing residuals using the state-space model of the process. Therefore, only observer-based fault diagnosis techniques for linear systems are reviewed in this section.

The basic idea behind observer or estimator-based methods is to design a stable estimator for outputs from available measurements (inputs and outputs) by using either Luenberger observers in deterministic setting [22, 23, 24, 17] or Kalman filters in stochastic settings [25, 26, 24, 27, 28, 29]. In observer-based fault diagnosis one is interested in output estimation through observers, while state estimation is unnecessary. The design of a simple Luenberger observer [30] for residual generation for fault detection and isolation is discussed in the following. It is desired to design an

observer for the dynamic system given by eq (II.3). Consider the following linear, time-invariant dynamic system:

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + L(y - \tilde{y}) + Bu \\ \tilde{y} &= C\tilde{x} + Du \end{aligned} \tag{II.6}$$

Where, $\tilde{x} \in \mathbb{R}^n$ and $\tilde{y} \in \mathbb{R}^m$ are the estimates of x and y, respectively. Assuming the system given by eq (II.3) is observable, then L can be chosen to make the closed-loop observer stable and achieve a desired observer dynamics. Define a term $e(t) = \tilde{x}(t) - x(t)$ and $r(t) = Q(y(t) - \tilde{y}(t))$, Q is a predefined weighting matrix then following is obtained:

$$\begin{aligned} \dot{e}(t) &= (A - LC)e(t) + (LR_2 - R_1)f(t) \\ r(t) &= -QCe(t) + QR_2f(t) \end{aligned} \tag{II.7}$$

One has to keep in mind that the preceding discussion on design of observer-based residual generation is restrictive, since emphasis here is on estimating the states as compared to directly estimating the outputs. A thorough discussion of an observer for linear functional is provided in [31] and its application for residual generation is provided in [3]. The weighting matrix Q is chosen so that, $Q \neq 0$ and residuals are aligned to their respective fault directions. This is the concept of structured residual set generation [20].

It is clear that model-based fault diagnosis uses a mathematical model of the supervised system, however a perfectly accurate model of a physical system is never available. The inaccuracies in the model result from uncertain parameters, unknown disturbances, and noise that cannot be modeled accurately. Such plant-model mismatch can become a source of false and missed alarms which can render the existing fault diagnosis application unusable. To address above concerns, a model-based fault

diagnosis must be made robust against modeling uncertainties. Therefore, the next Section a reviews existing literature in robust fault diagnosis in linear systems.

2.a. Robust fault diagnosis in linear systems

A fault diagnosis system designed to provide satisfactory sensitivity to faults, and with adequate robustness against modeling uncertainties is called as robust fault diagnosis system [19, 3, 11]. To approach this problem, one must start with the mathematical description of the system given by eq (II.3) with all possible sources of modeling uncertainties. Therefore the state-space model is given as follows:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) + R_1 f(t) + E_1 d(t) \\ y(t) &= (C + \Delta C)x(t) + (D + \Delta D)u(t) + R_2 f(t) + E_2 d(t) \end{aligned} \tag{II.8}$$

Here $d(t) \in \mathbb{R}^r$ is an unknown disturbances vector, however the disturbances distribution matrices E_1 and E_2 are assumed to be known. The matrices ΔA, ΔB, ΔC, and ΔD are the errors associated due to parametric uncertainties.

To generate residuals robust against disturbances, disturbance de-coupling designs can be achieved with an unknown-input observer [32, 24, 33, 34, 35] and by assigning the eigenstructure of the observer [36, 37, 11]. The following subsection discusses the design and application of UIO for robust fault diagnosis.

2.a.1. Unknown input observers

Consider a class of linear dynamic systems give by the following equation:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + R_1 f(t) + Ed(t) \\ y(t) &= Cx(t) + R_2 f(t) \end{aligned} \tag{II.9}$$

where $d(t) \in \mathbb{R}^r$ is the unknown input (or disturbance) vector. A,B, and C are as described in eq (II.2) and (II.3), and E is a known matrix with appropriate dimensions.

Definition 1 (Unknown Input Observer (UIO)) *[3] An observer is defined as an unknown input observer for the system described by eq (II.9), if its state estimation error approaches zero asymptotically, regardless of the presence of the unknown input (disturbance) in the system.*

Remarks

(a) The disturbance term may also appear in the output equation(II.9), i.e.,

$$y(t) = Cx(t) + E_y d(t)$$

However this case is not considered here since the disturbance term $E_y d(t)$ in the output equation(II.9) can be eliminated by simply using a transformation of the output signal $y(t)$, i.e.

$$y_E(t) = T_y y(t) = T_y Cx(t) + T_y E_y d(t) = T_y Cx(t)$$

where $T_y E_y = 0$. The problem to the one without output disturbances.

(b) Generally there is a term relating the output $y(t)$ to the control input $u(t)$ in the system output equation (II.9), i.e.

$$y(t) = Cx(t) + Du(t)$$

Since the control input $u(t)$ is known, a new output can be constructed as:

$$\bar{y}(t) = y(t) - Du(t) = Cx(t)$$

If the actual output $y(t)$ is replaced by $\bar{y}(t)$, then the problem is equivalent to the one without the term $Du(t)$ in the output equation (II.9).

Here, observer-based fault diagnosis, using an unknown input obserer [35] is briefly discussed.

Consider the following structure of the unknown input observer:

$$\begin{aligned} \dot{z}(t) &= Nz(t) + My(t) + Gu(t) \\ \hat{w}(t) &= Jy(t) + Fz(t) \end{aligned} \tag{II.10}$$

for estimating $w(t) = Hx(t)$. Where, $z(t) \in \mathbb{R}^p$, and $w(t) \in \mathbb{R}^s$. N,M,G,J, and F are observer parameters of appropriate dimensions to be determined. Then it is said that eq (II.10) observes $Vx(t)$, some nontrivial linear combination of states in the absence of faults ($f(t) = 0$) if and only if

$$\lim_{t\to\infty}[z(t) - Vx(t)] = 0 \qquad \forall x(0), z(0), u(.), d(.) \tag{II.11}$$

with the exponents of convergence in eq (II.11) lie in a desired $\subset \mathbb{C}^-$. Defining $e(t) = z(t) - Vx(t)$. Then for eq (II.11) to be true following holds

$$\begin{aligned} NV - VA + MC &= 0 \\ G &= VB \\ 0 &= VE \end{aligned} \tag{II.12}$$

and $\lambda(N) \subset \mathbb{C}^-$, $\lambda(.)$ denotes the eigen-spectrum. Also, if eq (II.10) estimates $Hx(t)$ asymptotically, then

$$\lim_{t\to\infty}[\hat{w}(t) - w(t)] = 0 \qquad \forall x(0), z(0), u(.), d(.)$$

with its exponents in $\mathbb{C}^-$, if and only if

$$JC + FV = H \tag{II.13}$$

Let $\mathcal{V}$,$\mathcal{C}$, and $\mathcal{H}$ denote the space spanned by the columns of matrices V,C, and H respectively. Then from eqs (II.12) and (II.13) it follows that:

$$\begin{aligned} A^T &\subset \mathcal{C} + \mathcal{V} \\ \mathcal{V} &\subset \mathrm{Ker}E \\ \mathcal{H} &\subset \mathcal{C} + \mathcal{V}. \end{aligned} \tag{II.14}$$

To proceed, an observer is defined to be internally nonredundant if

$$\mathrm{rank}V = p \tag{II.15}$$

and externally nonredundant if

$$\mathrm{rank}\begin{bmatrix} C \\ V \end{bmatrix} = \mathrm{rank}C + \mathrm{rank}V. \tag{II.16}$$

Assuming that eq (II.15) or (II.16) is satisfied, then the observer parameters can be determined by the method described in [35].

Remark: Since one is interested in output estimation only for fault detection purposes an unknown input observer for output estimation always exist since $H = C$ provided eq (II.15) or eq (II.16) is satisfied.

In the presence of faults the convergence of eq (II.11) is governed by the following differential equations:

$$\begin{aligned} \dot{e}(t) &= Ne(t) + (MR_1 - VR_2)f(t) \\ r(t) &= Q(y(t) - \hat{y}(t)) = Q(R_2 f(t) - JR_2 f(t) - JFe(t)) \end{aligned} \tag{II.17}$$

where Q is a predefined weighting matrix as shown in eq (II.7). Once again, fault isolation filters can be designed using the structured residual set generation approach [20] or by using Beard Fault Detection Filters (BFDF) [22, 38].

The primary requirement for a UIO based residual generation approaches is that the unknown input distribution matrix must be known *a priori*, although the actual unknown input does not need to be known. If the uncertainty in the modeling is due to external disturbance, then UIO can be designed easily do attain robust fault diagnosis. However in cases where model uncertainty does not appear affine in the system equations, e.g., linearization errors, parametric variations, modeling errors, etc, direct application of disturbance decoupling approaches is not possible. Therefore, this problem has prevented the application of these robust fault diagnosis techniques to real industrial systems. There has been a considerable effort by some investigators [39, 40, 41, 42] to come up with an approach in which modeling errors and other uncertain factors are represented approximately as unknown disturbances, with an estimated distribution matrix. Again, this approximation is often poor in systems where uncertain parameters appear exponentially in the dynamic equations, e.g., chemical reaction systems.

2.a.2. Eigenstructure assignment in observer design

Often it is not necessary to decouple external disturbances from the state estimation but only from the diagnostic signal - residual. A direct approach to design disturbance decoupled residuals is thus required. This section introduces briefly the concept behind decoupling residuals from disturbances through eigenstructure assignment of the observer. The residual generator based on full-order observer for the class of linear dynamic systems given by eq (II.9) is described as:

$$\begin{aligned} \dot{\hat{x}}(t) &= (A - KC)\hat{x}(t) + Bu(t) + Ky(t) \\ \hat{y}(t) &= C\hat{x}(t) \\ r(t) &= Q\left[y(t) - \hat{y}(t)\right] \end{aligned} \tag{II.18}$$

Where, A, B,C are matrices as described in eqs (II.3) and (II.4), Q is a residual weighting matrix of appropriate dimension.

Defining the state estimation error $e(t) = x(t) - \hat{x}(t)$, then the residual is governed by the following equations:

$$\begin{aligned} \dot{e}(t) &= (A - KC)e(t) + Ed(t) + R_1 f(t) - KR_2 f(t) \\ r(t) &= QCe(t) + QR_2 f(t) \end{aligned} \tag{II.19}$$

The Laplace transformed residual response to fault and disturbance is thus:

$$\begin{aligned} r(s) = {} & QR_2 f(t) + QC(sI - A + KC)^{-1}(R_1 - KR_2)f(s) \\ & + QC(sI - A + KC)^{-1}Ed(s) \end{aligned} \tag{II.20}$$

The general principle behind disturbance decoupling design is that:

$$QC(sI - A + KC)^{-1}Ed(s) = 0 \quad \forall d(.) \tag{II.21}$$

The sufficient condition for satisfying the disturbance decoupling conditions of eq (II.21) are [3]:

- $QCE = 0$
- All rows of the matrix QC are left eigenvectors of $(A - KC)$ corresponding to any eigenvalues **OR** all columns of matrix E are right eigenvectors of $(A - KC)$ corresponding to any eigenvalues.

The limitations of applying this approach to industrial systems are similar to those of unknown input observers, i.e., that perfect knowledge of disturbance distribution matrix E is unknown *a priori*. In the event when modeling uncertainties cannot be lumped as an external disturbance perfect disturbance decoupling cannot be achieved. One can consider an optimal or approximate decoupling by minimizing a performance index containing a measure of the effects of both disturbance and faults. The next section focuses on formulating robust fault diagnosis using frequency performance criteria.

2.a.3. Frequency domain design for robust fault diagnosis

When complete elimination of disturbance effects may not be possible due to lack of design freedom, an appropriate criterion for robust residual design should take into account the effects of both modeling errors and faults. There is a trade-off between sensitivity to faults and robustness to modeling uncertainties. Hence, residuals have to be generated so that their sensitivities to faults is maximized while minimizing their sensitivities to modeling uncertainties in a prescribed frequency range.

The problem of maximizing fault effects and at the same time minimizing the disturbance effects was studied by [43, 44, 45] in the frequency domain. Consider the

transfer function description of the system given by eq (II.8):

$$y(s) = (G_u(s) + \Delta G_u(s))u(s) + G_d(s)d(s) + G_f(s)f(s)$$

Here $G_d(s)d(s)$ represent the disturbance effect and $G_f(s)f(s)$ the effect of faults on the system.

$$G_d(s) = E_2 + C(sI - A)^{-1}E_1$$

$$G_f(s) = R_2 + C(sI - A)^{-1}R_1$$

The transfer function description of the residual signal of the system is given as:

$$r(s) = H_y(s)G_f(s)f(s) + H_y(s)\Delta G_u(s) + H_y(s)G_d(s)d(s) \tag{II.22}$$

Here, $H_y(s)$ is a stable transfer function matrix such that the residual has maximum sensitivity to faults while minimizing the effect of disturbances on it. One suitable choice of performance index in the frequency domain was proposed by Frank and Ding [44]:

$$J = \frac{\|H_y(j\omega)G_d(j\omega)\|}{\|H_y(j\omega)G_f(j\omega)\|}$$

where $\|.\|$ is the H_∞ norm [46]. By minimizing the performance index J over a specified frequency range an approximate decoupling can be achieved. The main problem with the aforementioned approach is that only those cases can be considered for which the disturbance distribution matrix is known.

2.a.4. Optimization methods for robust fault diagnosis

Consider the following mathematical description of the monitored system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + R_1 f(t) + d(t) \\ y(t) &= Cx(t) + R_2 f(t) \end{aligned} \tag{II.23}$$

Where A,B,C,R_1, and R_2 are matrices of appropriate dimensions. The vector $d(t)$ is the disturbance vector which is used to represent modeling errors such as:

$$d(t) = \Delta A x(t) + \Delta B u(t) \tag{II.24}$$

The residual generator is described as:

$$\begin{aligned} \dot{\hat{x}}(t) &= (A - KC)\hat{x}(t) + Bu(t) + Ky(t) \\ \hat{y}(t) &= C\hat{x}(t) \\ r(t) &= Q[y(t) - \hat{y}(t)] \end{aligned} \tag{II.25}$$

The residual response to faults and disturbances in transfer-function form is thus:

$$\begin{aligned} r(s) = Q\{R_2 + C(sI - A + KC)^{-1}(R_1 - KR_2)\}f(s) \\ + QC(sI - A + KC)^{-1}[d(s) + e(0)] \end{aligned} \tag{II.26}$$

where $e(0)$ is the initial value of the state estimation error.

To reduce faults and missed alarm rates, the effect of faults on the residuals must me maximized while the effect of disturbances should be minimized. Following performance indices are therefore considered [47, 48, 49]:

$$J_1(K, Q) = \inf_{\omega \in [\omega_1, \omega_2]} \quad \underline{\sigma}\{QR_2 + QC(j\omega I - A + KC)^{-1}(R_1 - KR_2)\} \tag{II.27}$$

$$J_2(K,Q) = \sup_{\omega\in[\omega_1,\omega_2]} \quad \bar{\sigma}\{QR_2 + QC(j\omega I - A + KC)^{-1}\} \tag{II.28}$$

where $\underline{\sigma}\{.\}$ and $\bar{\sigma}\{.\}$ denote minimal and maximal singular values respectively. Therefore by maximizing $J_1(K,Q)$ and minimizing $J_2(K,Q)$ over a specified frequency range, one can obtain optimum values of the weighting matrix Q and the observer gain K. Further the effect of input and sensor noise signals on the residuals can also be incorporated in this fashion [49]. However, the drawback with this general approach for robust fault diagnosis is that it cannot be applied straightforwardly to nonlinear systems. Moreover, this method when applied to linearized models often ends up with solving non-convex optimization problem [3].

3. Model-based fault diagnosis for nonlinear systems

Most of observer-based fault diagnosis are built upon linear system models. Therefore, traditionally for nonlinear systems the existing fault diagnosis techniques have been applied to linearized system models of the original nonlinear systems. Further, the inaccuracy due to linearization is handled by applying robust residual generation against nonlinearity and model parameter variations within a small neighborhood of the operating point.

Unfortunately, this technique only applies to systems that closely operate around a chosen operating point and do not have a large mismatch between the linearized and the nonlinear system model. There have been some attempts to use nonlinear observers to solve nonlinear fault diagnosis problems. The first reported research in nonlinear fault diagnosis was done using an identity nonlinear observer [50]. This method was further carried out in [51, 52]. However, this method involves computation of a gain matrix at each time step for observer stability. It was shown in [51]

that for many practical applications a constant gain matrix is sufficient for observer stability. In many ways the identity observer used in [50] is similar to the extended Luenberger observer [53].

The unknown input observer approach was extended to include nonlinear fault diagnosis problems by using nonlinear unknown input observers [54, 55]. The design of nonlinear unknown input observers is based on differential geometric concepts of disturbance decoupling in linear system theory. However, the extension is restricted to a certain class of nonlinear systems. To apply the nonlinear unknown input observers to general nonlinear systems, a transformation is required, which in practice is very restrictive [5]. Even if the existence conditions for the transformation is satisfied, the computation is hampered by solving some higher order partial differential equations.

An adaptive observer based fault diagnosis for handling parametric uncertainties and nonlinearity was proposed in [56, 57]. In this approach a stable adaptive observer for time-varying nonlinear systems [58] is used. The formulation of the above problem is based on the assumption that nonlinearities and slowly time-varying unknown parameters appear as an affine unknown input to the system. However, most chemical processes are nonlinear and exhibit an exponential dependence of unknown parameters in the process model, e.g., the activation energy.

3.a. Nonlinear fault diagnosis using identity observer

This approach to nonlinear fault diagnosis was proposed by Hengy and Frank [50]. Detailed design considerations are described in [51]. Consider a nonlinear dynamic

system as follows:

$$\begin{aligned} \dot{x}(t) &= g(x(t), u(t)) + R_1 f(t) \\ y(t) &= h(x(t), u(t)) + R_2 f(t) \end{aligned} \tag{II.29}$$

where $g(.,.) : \mathbb{R}^n \times \mathbb{R}^q \rightarrow \mathbb{R}^n$ and $h(.,.) : \mathbb{R}^n \times \mathbb{R}^q \rightarrow \mathbb{R}^m$ are differentiable vector fields in a small neighborhood around the operating point. A nonlinear identity observer can be designed as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= g(\hat{x}(t), u(t)) + K(\hat{x}(t), u(t))[y(t) - \hat{y}(t)] \\ \hat{y}(t) &= h(\hat{x}(t), u(t)) \\ r(t) &= y(t) - \hat{y}(t) \end{aligned} \tag{II.30}$$

The residual $r(t)$ and the state estimation error $e(t) = x(t) - \hat{x}(t)$ obey the following differential equations:

$$\begin{aligned} \dot{e}(t) &= F(t)e(t) + O_1(e^2(t), t) + R_1 f(t) - K(\hat{x}(t), u(t))R_2 f(t) \\ r(t) &= H(t)e(t) + O_2(e^2(t), t) + R_2 f(t) \end{aligned} \tag{II.31}$$

where $O_1(e^2(t), t)$ and $O_2(e^2(t), t)$ represent the second and higher order terms pertaining to $e(t)$ and

$$\begin{aligned} F(t) &= \frac{\partial g(\hat{x}(t), u(t))}{\partial \hat{x}(t)} - K(\hat{x}(t), u(t))H(t) \\ H(t) &= \frac{\partial h(\hat{x}(t), u(t))}{\partial \hat{x}(t)} \end{aligned} \tag{II.32}$$

The remaining problem is to synthesize the observer gain $K(\hat{x}(t), u(t))$ (usually time-varying) such that $e(t) = 0$ is an asymptotically stable solution of the differential equation II.31.

CHAPTER III

ROBUST MODEL-BASED FAULT DIAGNOSIS FOR NONLINEAR SYSTEMS WITH PARAMETRIC UNCERTAINTIES

Chapter II provided a comprehensive overview of work that has been done so far to achieve robustness against modeling uncertainties for both linear and nonlinear systems. However the design of such robust fault diagnosis are either restrictive in their application to chemical processes or are computationally intensive. Moreover, one important aspect in fault diagnosis is fault reconstruction/estimation which can be helpful in reconfiguring closed-loop industrial processes to attain certain desired performance objectives even under the presence of faults. This aspect of fault diagnosis has not been explored for uncertain nonlinear systems and therefore it is the purpose of this chapter to explain techniques for fault detection, isolation, and identification for nonlinear systems with parametric uncertainties.

1. Preliminaries

In Section 1.a, observer-based fault diagnosis for LTI (linear, time-invariant) systems is reviewed. Required background information about the concept of stability of an interval family of polynomials is presented in Section 1.b. This serves as a foundation for the presentation of the new technique in Section 2. Section 3 discusses a case study to demonstrate the application of the proposed fault diagnosis methodology on a non-isothermal CSTR operation through numerical simulations.

1.a. Review of fault diagnosis for LTI systems

Consider a linear time-invariant system with inputs

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + f_s(t) \end{aligned} \tag{III.1}$$

where $x \in \mathbb{R}^n$ is a vector of state variables, $u \in \mathbb{R}^q$ is a vector of input variables, $y \in \mathbb{R}^m$ is a vector of output variables, f_s is the vector of sensor faults, n is the number of states, q refers to the number of input variables and m refers to the number of outputs. A, B, and C are matrices of appropriate dimensions. Assuming that the above system is observable, a Luenberger observer [30] can be designed for the system

$$\begin{aligned} \dot{\tilde{x}}(t) &= A\tilde{x}(t) + L(y(t) - \tilde{y}(t)) + Bu(t) \\ \tilde{y}(t) &= C\tilde{x}(t) \end{aligned} \tag{III.2}$$

where L is chosen to make the closed-loop observer stable and achieve a desired observer dynamics. $\tilde{x} \in \mathbb{R}^n$ and $\tilde{y} \in \mathbb{R}^m$ are the estimates of x and y, respectively. Further, define a residual [3]

$$r(t) = \int_0^t Q(t-\tau)[y(\tau) - \tilde{y}(\tau)]d\tau \tag{III.3}$$

which represents the difference between the observer output and the actual output passed through a filter $Q(t)$. Taking the Laplace transform of eqs (III.1)-(III.3) results in

$$r(s) = Q(s)\{I - C[sI - (A - LC)]^{-1}L\}f_s(s) \tag{III.4}$$

where $Q(t)$ is chosen such that $Q(s)$ is a RH_∞ matrix [46]. It can be shown that

(1) $r(t) = 0$ if $f_s(t) = 0$

(2) $r(t) \neq 0$ if $f_s(t) \neq 0$

indicating that the value of $r(t)$ predicts the existence of a fault in the system [9]. In addition, if one uses the dedicated observer scheme as shown for a system with two outputs in Figure 3, then the fault detection system can also determine the location of the fault:

(3) $r_i(t) = 0$ if $f_{s,i}(t) = 0, i = 1, 2, 3, \ldots, m$

(4) $r_i(t) \neq 0$ if $f_{s,i}(t) \neq 0, i = 1, 2, 3, \ldots, m$,

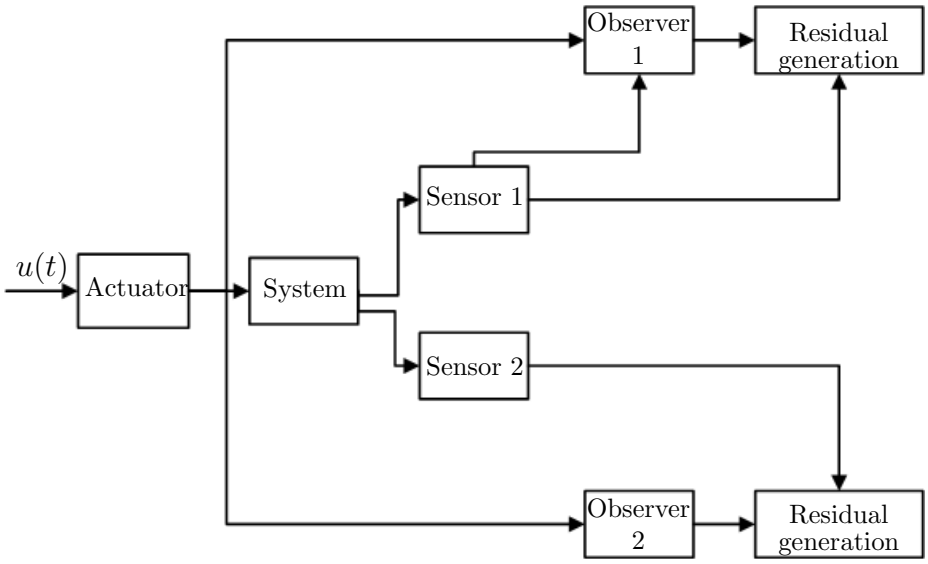

Fig. 3. Schematic of a dedicated observer scheme for a system with two measurements.

where i represents the i_{th} measurement. A fault detection system that satisfies all of the above conditions is called a Fault detection and isolation filter (FDIF). A Fault detection and isolation filter becomes a fault identification filter (FIDF) if additionally the following condition is satisfied [9].

(5) $\lim_{t\to\infty}[r_i(t) - f_{s,i}(t)] = 0, i = 1, 2, 3, \ldots, m$

To meet above conditions, the following restrictions on the choice of $Q(s)$ are imposed:

(a) $Q(s) \neq 0, \forall\ s \in \mathbb{C}$

(b) $Q(s) = \{I - C[sI - (A - LC)]^{-1}L\}^{-1} = [C(sI - A)^{-1}L + I]$.

Linear, observer-based fault detection, isolation, and identification schemes work well if an accurate model exists for the process over the whole operating region and if appropriate choices are made for L and Q.

1.b. Robust stability of an interval polynomial

Consider a set $\delta(s)$ of real polynomials of degree n of the form

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \cdots + \delta_n s^n$$

where the coefficients lie within the given ranges:

$$\delta_0 \in [\delta_0^-, \delta_0^+],\ \delta_1 \in [\delta_1^-, \delta_1^+],\ \cdots,\ \delta_n \in [\delta_n^-, \delta_n^+]$$

Denote that

$$\delta := [\delta_0, \delta_1, \ldots, \delta_n]$$

and define a polynomial $\delta(s)$ by its coefficient vector δ. Furthermore, define a hyper-rectangle of coefficients as follows:

$$\Omega := \{\delta : \delta \in \mathbb{R}^{n+1}, \delta_i^- \leq \delta_i \leq \delta_i^+, i = 0, 1, 2, \ldots, n\}$$

A set of polynomials with above properties is called an interval polynomial family [7]. Kharitonov's theorem provides a necessary and sufficient condition for Hurwitz stability of all members of this family.

Theorem 1 (Kharitonov's Theorem) *Every polynomial in the family $\delta(s)$ is Hurwitz if and only if the following four extreme polynomials are Hurwitz [59].*

$$\begin{aligned}
\delta^{--}(s) =& \delta_0^- + \delta_1^- s + \delta_2^+ s^2 + \delta_3^+ s^3 + \delta_4^- s^4 + \delta_5^- s^5 + \delta_6^+ s^6 + \cdots, \\
\delta^{-+}(s) =& \delta_0^- + \delta_1^+ s + \delta_2^+ s^2 + \delta_3^- s^3 + \delta_4^- s^4 + \delta_5^+ s^5 + \delta_6^+ s^6 + \cdots, \\
\delta^{+-}(s) =& \delta_0^+ + \delta_1^- s + \delta_2^- s^2 + \delta_3^+ s^3 + \delta_4^+ s^4 + \delta_5^- s^5 + \delta_6^- s^6 + \cdots, \\
\delta^{++}(s) =& \delta_0^+ + \delta_1^+ s + \delta_2^- s^2 + \delta_3^- s^3 + \delta_4^+ s^4 + \delta_5^+ s^5 + \delta_6^- s^6 + \cdots,
\end{aligned} \tag{III.5}$$

While this theorem has been extensively used in parametric approaches to robust control, this work will make use of Kharitonov's theorem for developing robust observer designs that can handle parametric uncertainties in the model.

2. Robust fault detection, isolation and identification

2.a. Problem formulation

Consider a nonlinear system with possibly multiple outputs of the following form:

$$\begin{aligned}
\dot{x}(t) &= f(x(t), \theta(t)) \\
y(t) &= h(x(t), \theta(t)) + f_s(t)
\end{aligned} \tag{III.6}$$

where $x \in \mathbb{R}^n$ is a vector of state variables and $y \in \mathbb{R}^m$ is a vector of output variables. It is assumed that $f(x, \theta)$ is a is smooth analytic vector field on $\mathbb{R}^n$ and $h(x, \theta)$ is a smooth analytic vector field on $\mathbb{R}^m$. Let $\theta \in \mathbb{R}^p$ be the parameters assumed to be constant with time but a priori uncertain, and f_s is the sensor fault of unknown nature with the same dimensions as the output. The goal of this paper is to estimate the state vector with limited information about the parameters describing the process model and under the influence of output disturbances such that $\lim_{t\to\infty}(x(t) - \tilde{x}(t)) = 0$,

where $\tilde{x}$ is the estimate of the state vector, x, and to design a set of filters $Q(t)$ so that the residuals, given by the expression $r(t) = \int_0^t Q(t-\tau)[y(\tau) - \tilde{y}(\tau)]d\tau$, have all the five properties discussed in section 1.a.

One of the main challenges of this research is that both faults and plant-model mismatch will have an effect on the fault identification. In order to perform accurate state and parameter estimation, it is desired to have reliable measurements, while at the same time an accurate model of the process is required to identify the fault. This will be taken into account by performing the parameter estimation and the fault detection at different time scales. Each time the parameters are estimated, it is assumed that the fault is not changing at that instance, while the values of the parameters are not adjusted during each individual fault detection. A variety of different techniques exist for designing nonlinear closed-loop observers for state and parameter estimation [53, 58, 60]. However, since the class of problems under investigation includes parametric uncertainty it would be natural to address these issues through a parametric approach instead of the often used extended Kalman filter or extended Luenberger observer. The procedure for designing nonlinear observers under the influence of parametric uncertainty is outlined in the next subsections, which is followed by a description of the fault detection, isolation, and identification algorithm.

2.b. Estimator design: a parametric approach

A nonlinear system of the form given by eq (III.6) can be rewritten by viewing the parameters as augmented states of the system

$$\begin{pmatrix} \dot{x}(t) \\ \dot{\theta}(t) \end{pmatrix} = \begin{pmatrix} f(x(t), \theta(t)) \\ 0 \end{pmatrix} \tag{III.7}$$

$$y(t) = h(x(t), \theta(t)) + f_s(t)$$

and with a change of notation

$$\bar{x} = \begin{pmatrix} x \\ \theta \end{pmatrix}, \bar{f}(x, \theta) = \begin{pmatrix} f(x, \theta) \\ 0 \end{pmatrix} \tag{III.8}$$

this results in the following system:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{f}(\bar{x}(t)) \\ y(t) &= h(\bar{x}(t)) + f_s(t) \end{aligned} \tag{III.9}$$

For the state and parameter estimation step, it is assumed that sensor faults are known, since they are identified at certain "sampling times" and the assumption is made that they remain constant over the time interval between two "sampling points". Furthermore, assume that each component θ_i of the parameter vector

$$\theta := [\theta_0, \theta_1, \theta_2, \ldots, \theta_{p-1}] \tag{III.10}$$

can vary independently of the other components and each θ_i lies within an interval where the upper and lower bounds are known

$$\Pi := \{\theta : \theta_i^- \leq \theta_i \leq \theta_i^+, i = 0, 1, 2, \ldots, p-1\} \tag{III.11}$$

Also, let $\theta = \theta_{ss} \in \Pi$ be a constant vector of a priori uncertain parameters and (x_{ss}, θ_{ss}) an equilibrium point of eq (III.7). The augmented system needs to be observable in order to design an observer, which can also estimate the values of parameters. A sufficient condition for local observability of a nonlinear system is if the observability matrix of the augmented system has rank $n+p$ for $(x, \theta) = (x_{ss}, \theta_{ss})$ [61].

$$W_o(\bar{x}) = \begin{bmatrix} \frac{\partial h(\bar{x})}{\partial \bar{x}} \\ \frac{\partial}{\partial \bar{x}} L_{\bar{f}} h(\bar{x}) \\ \vdots \\ \frac{\partial}{\partial \bar{x}} L_{\bar{f}}^{n+p-1} h(\bar{x}) \end{bmatrix} \tag{III.12}$$

Since the equilibrium points of the system depend upon the values of the parameters which are not known *a priori*, it is required that the rank of $W_o(\bar{x})$ is checked for all $\theta = \theta_{ss} \in \Pi$ and the resulting equilibrium points (x_{ss}, θ_{ss}).

It is assumed that the augmented system is observable over the entire hyperrectangle-like set Π and the equilibrium points corresponding to these parameters values. It is then possible to design an observer for the augmented system

$$\begin{pmatrix} \dot{\tilde{x}}(t) \\ \dot{\tilde{\theta}}(t) \end{pmatrix} = \begin{pmatrix} f(\tilde{x}(t), \tilde{\theta}(t)) \\ 0 \end{pmatrix} + \bar{L}(\tilde{x}(t), \tilde{\theta}(t))(y(t) - \tilde{y}(t)) \tag{III.13}$$

$$\tilde{y}(t) = h(\tilde{x}(t), \tilde{\theta}(t)) + f_s(t)$$

where $\tilde{x}$ is the estimate of x and $\tilde{\theta}$ is the estimate of θ and $\bar{L}(\tilde{x}, \tilde{\theta})$ is a suitably chosen nonlinear observer gain. Also, note that the observer makes use of the assumption that the measurement fault is known from an earlier identification of the fault. When the observer is computed for the first time, it has no knowledge about possible sensors

faults and assumes that no sensor fault was initially present.

2.b.1. Determination of the family of polynomials for observer design

In this section, the result about Hurwitz stability of an interval family of polynomials from section 1.b is utilized to determine a methodology for computing the gain $\bar{L}(\tilde{x}, \tilde{\theta})$ of the nonlinear observer given by eq (III.13). Consider the linearized model of the augmented process model around an equilibrium point (x_{ss}, θ_{ss}).

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}(x_{ss}, \theta_{ss})\bar{x}(t) \\ y(t) &= \bar{C}(x_{ss}, \theta_{ss})\bar{x}(t) + f_s(t) \end{aligned} \tag{III.14}$$

where $\bar{A}(x_{ss}, \theta_{ss})$ is the Jacobian of $\bar{f}(x, \theta)$ at the point (x_{ss}, θ_{ss}) and $\bar{C}(x_{ss}, \theta_{ss}) = \left[\frac{\partial h(x,\theta)}{\partial x}|_{(x_{ss},\theta_{ss})} \quad \frac{\partial h(x,\theta)}{\partial \theta}|_{(x_{ss},\theta_{ss})} \right]$. The characteristic polynomial of the system, which determines its stability, is given by

$$\delta(s) = det[sI - \bar{A}(x_{ss}, \theta_{ss})] = \delta_0(x_{ss}, \theta_{ss}) + \delta_1(x_{ss}, \theta_{ss})s + \cdots + s^n \tag{III.15}$$

It can be seen that the coefficients of the characteristic polynomial are nonlinear functions of the parameter vector x_{ss} and θ_{ss} . Assuming that $f(x_{ss}, \theta_{ss})$ satisfies the conditions of the implicit function theorem, i.e., $|\frac{\partial f(x,\theta)}{\partial x}|_{x_{ss},\theta_{ss}}| \neq 0$, then x_{ss} can be solved for a given θ_{ss}; i.e., $x_{ss} = \phi(\theta_{ss})$, where $\phi : \mathbb{R}^p \rightarrow \mathbb{R}^n$. The characteristic polynomial in s given by eq (III.15) can then be rewritten as

$$\delta(s) = \bar{\delta}_o(\theta_{ss}) + \bar{\delta}_1(\theta_{ss})s + \bar{\delta}_2(\theta_{ss})s^2 + \ldots + s^n \tag{III.16}$$

where $\delta_i[\phi(\theta_{ss}), \theta_{ss}] = \bar{\delta}_i(\theta_{ss}), i = 0, 1, 2, 3, \ldots, n-1$. While it is generally not possible to derive an analytic expression of the coefficients $(\bar{\delta}_o, \bar{\delta}_1, \ldots, \bar{\delta}_{n-1})$ as a function θ_{ss}, x_{ss} can be evaluated by numerically solving the equation $f(x_{ss}, \theta_{ss}) = 0$ for $\theta_{ss} \in \Pi$.

Since $f(x,\theta)$ is assumed to be a smooth vector function, the coefficients of the characteristic polynomial are continuous functions of (x_{ss},θ_{ss}) . Therefore, by discretizing the set Π and evaluating the maximum and minimum values for each coefficient $\bar{\delta}_i(\theta_{ss})$ over all of the points in the set Π, the hyperrectangle of coefficients Ω as described in section 1.b can be obtained. In the case of a multidimensional parameter θ, discretizing the set Π can be computationally expensive; however, advanced nonlinear programming algorithms exist that facilitate the calculation of the required bounds on the coefficients. For the case where θ is a scalar, the range within which the coefficients vary can be determined by plotting $\bar{\delta}_i(\theta_{ss})$ against θ_{ss} $\forall$ $i = 0, 1, 2, \ldots, n-1$. Figure 4 shows the typical plot of the coefficient versus the one-dimensional parameter θ.

To enforce that the estimation error decays asymptotically for the linearized system, the observer gains are chosen to satisfy the condition

$$\lambda[\bar{A}(x_{ss},\theta_{ss}) - \bar{L}(x_{ss},\theta_{ss})\bar{C}(x_{ss},\theta_{ss})] \in \mathbb{C}^{-}, \quad \forall \quad \theta_{ss} \in \Pi \tag{III.17}$$

where $\lambda(.)$ refers to the eigenvalues of the matrix. The following section focuses on computing appropriate gains $\bar{L}$ by making use of Kharitonov's theorem.

2.b.2. Observer gain computation

Since it is assumed that the augmented system given by eq (III.7) is observable over the entire hyperrectangle-like set Π and the equilibrium points corresponding to these parameter values, it is possible to find an invertible transformation $\bar{T}(x_{ss},\theta_{ss})$, s.t. $\forall$ $\theta_{ss} \in \Pi$; the LTI system given by eq (III.14) can be transformed into an observer

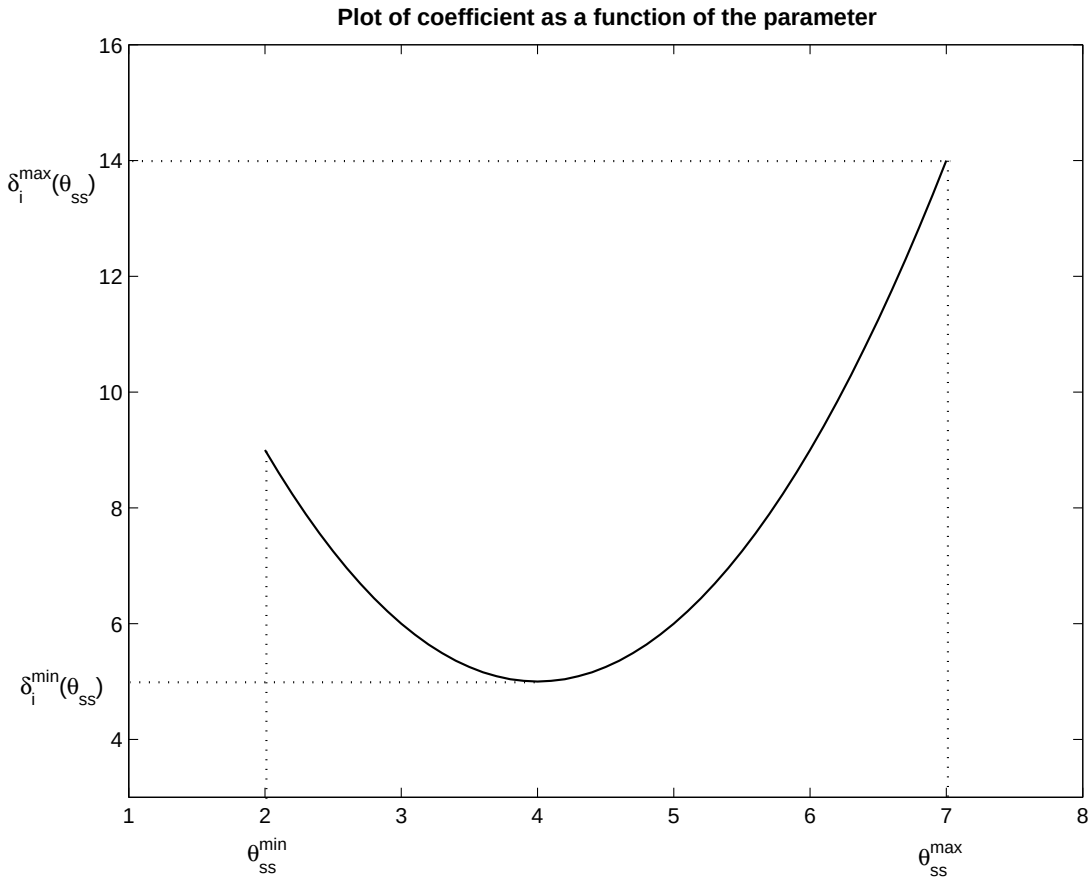

Fig. 4. Sample plot of the coefficient as a function of a scalar parameter.

canonical form [62], considering one output at a time as

$$\begin{aligned} \dot{\bar{z}}(t) =& \breve{A}(x_{ss}, \theta_{ss})\bar{z}(t) \\ y(t) =& \breve{C}\bar{z}(t) \end{aligned} \tag{III.18}$$

where, $\bar{z} = \bar{T}(x_{ss}, \theta_{ss})\bar{x}$, $\breve{A}(x_{ss}, \theta_{ss}) = \bar{T}(x_{ss}, \theta_{ss})\bar{A}(x_{ss}, \theta_{ss})\bar{T}^{-1}(x_{ss}, \theta_{ss})$, $\breve{C} = \bar{C}\bar{T}^{-1}(x_{ss}, \theta_{ss})$ and

$$\breve{A}(x_{ss}, \theta_{ss}) = \begin{bmatrix} -\delta_{n-1}(x_{ss}, \theta_{ss}) & 1 & 0 & \cdots & 0 & 0 \\ -\delta_{n-2}(x_{ss}, \theta_{ss}) & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\delta_2(x_{ss}, \theta_{ss}) & 0 & \cdots & 0 & 1 & 0 \\ -\delta_1(x_{ss}, \theta_{ss}) & 0 & \cdots & 0 & 0 & 1 \\ -\delta_0(x_{ss}, \theta_{ss}) & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \tag{III.19}$$

$$\breve{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}$$

The following analysis provides a method to compute a constant gain vector l for the case of a single-output system such that

$$\lambda(\breve{A}(x_{ss}, \theta_{ss}) - l\breve{C}) \in \mathbb{C}^-, \forall \quad \theta_{ss} \in \Pi$$

Consider the set of n nominal parameters $\{\delta_0^0, \delta_1^0, \delta_2^0, \ldots, \delta_{n-1}^0\}$ together with a set of a priori uncertainty ranges $\Delta\delta_0, \Delta\delta_1, \ldots, \Delta\delta_{n-1}$, which is given by $\Delta\delta_i = \delta_i^+ - \delta_i^-$, $i = 0, 1, \ldots, n-1$. Furthermore, consider the family $\delta(s)$ of polynomials,

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \ldots + \delta_{n-1} s^{n-1} + s^n$$

where the coefficients of the polynomial can vary independently from one another and lie within the given ranges.

$$\left\{\delta : \delta_i^0 - \frac{\Delta\delta_i}{2} \leq \delta_i \leq \delta_i^0 + \frac{\Delta\delta_i}{2}\right\}$$

Further let there be n free parameters $l = (l_0, l_1, l_2, \ldots, l_{n-1})$ to transform the family $\delta(s)$ into the family described by

$$\gamma(s) = (\delta_0 + l_0) + (\delta_1 + l_1)s + \ldots + (\delta_{n-1} + l_{n-1})s^{n-1} + s^n \tag{III.20}$$

The above problem arises, when it is required to suitably place the closed-loop observer poles for a single output system where the system matrices A and C are in observable canonical form and the coefficients of the characteristic polynomial of A are subject to bounded perturbations.

It has been shown in [7] and [63] that it is possible to determine a vector l such that the entire family $\gamma(s)$ is stable. The synthesis of such a vector l is shown in [7]. The result is an observer given by eq (III.13), which estimates the states and parameters of the system given by eq (III.7) by use of an observer gain $\bar{L}(\tilde{x}, \tilde{\theta}) = \bar{T}^{-1}(\tilde{x}, \tilde{\theta})l$. The presented approach yields an analytical expression for the observer gains irrespective of the dimension of the system. This is a significant advantage over other methods, e.g. an extended Luenberger observer where the gains are recomputed after each time step [64]. Therefore, the current methodology is less computationally demanding than other state and parameter estimation techniques while it guarantees local convergence of the error dynamics. It can be seen that there exist an infinite number of vectors l such that the given interval family of polynomials can be transformed into another family $\gamma(s)$ such that $\gamma(s)$ is Hurwitz. The following section discusses how the observer gain computation technique can be extended to also meet certain performance objectives of the closed-loop observer in addition to stability

requirements.

2.b.3. Observer gain computation for desired rate of convergence of estimation error

Often it is desired to choose l such that the real part of the eigenvalues of the family of polynomials $\gamma(s)$ have an upper limit $-\alpha$, where $\alpha \geq 0$ is a predefined real number. This problem arises when it is desired to regulate the rate of convergence of an estimation error for observer design. In order to address this type of problem, apply a coordinate transformation $s = s' - \alpha$ to the interval polynomial family given by eq (III.20), resulting in

$$\begin{aligned} \gamma(s' - \alpha) = (\delta_0 + l_0) + (\delta_1 + l_1)(s' - \alpha) + \dots \\ + (\delta_{n-1} + l_{n-1})(s' - \alpha)^{n-1} + (s' - \alpha)^n \end{aligned} \tag{III.21}$$

This polynomial family can then be rewritten explicitly in terms of s'

$$\bar{\gamma}(s') = (\xi_0 + \beta_0) + (\xi_1 + \beta_1)s' + \dots + (\xi_{n-1} + \beta_{n-1})s'^{n-1} + s'^n \tag{III.22}$$

where, ξ_i is a linear function of $(\delta_0, \delta_0, \cdots, \delta_{n-1})$, $\forall\ i = 0, 1, 2, \cdots, n-1$ and $\beta = (\beta_0, \beta_1, \beta_2, \cdots, \beta_{n-1})$ are n free parameters that can be chosen such that the roots of the polynomial given by eq (III.22) lie to the left of $s' = 0$. Moreover, it can be shown that the following relation exists

$$l_{n-r} = \sum_{k=1}^{r} \binom{n-k}{n-r} \alpha^{r-k} \beta_{n-k}, r = 1, 2, 3, \dots, n \tag{III.23}$$

The above choice of l will ensure that roots of the closed-loop polynomial $\gamma(s)$ always lie to the left of $Re(s) = -\alpha$. Hence, α can be chosen such that the slowest root of the family of closed-loop polynomials is at least 3-5 times larger in magnitude than the slowest root of the open-loop characteristic polynomial. Such a design will ensure

a certain rate of convergence of the observer even under the influence of parametric uncertainty.

2.c. Fault detection

The purpose of fault detection is to determine whether a fault has occurred in the system. It can be seen that $\lim_{t\to\infty}(x - \tilde{x}) \neq 0$ in the presence of sensor faults. To extract information about faults from the system, a residual needs to be defined as $r(t) = \int_0^t Q(t-\tau)[y(\tau) - \tilde{y}(\tau)]d\tau$, where $Q(t)$ is any stable filter. It can be verified that

(i) $r(t) = 0$ if $f_s(t) = 0$

(ii) $r(t) \neq 0$ if $f_s(t) \neq 0$

Additional restrictions on the class of stable filters $Q(t)$ will be imposed in the following sections in order to satisfy the desired objectives.

2.d. Fault isolation

Fault isolation is synonymous with determining the location of a fault, and its computation imposes additional restrictions on the choice of the filter $Q(t)$. To perform fault isolation the augmented system given by eq (III.7) is assumed to be separately locally observable through each of the outputs $y \; \forall \; \theta_{ss} \in \Pi$. It should be noted that this requirement is mandatory for the existence of a fault isolation filter [9] and hence does not pose a stringent condition for using the presented approach.

To achieve fault detection as well as fault isolation, the proposed approach uses a series of dedicated nonlinear observers as shown in Figure 3. In this method as many residuals are generated as there are measurable outputs. It can be verified that

(i) $r_i(t) = 0$ if $f_{s,i}(t) = 0, i = 1, 2, 3, \ldots, m$

(ii) $r_i(t) \neq 0$ if $f_{s,i}(t) \neq 0, i = 1, 2, 3, \ldots, m$,

for an appropriately chosen filter $Q(t)$.

2.e. Fault identification

To estimate the shape and size of the fault, the residuals have also to meet the following objective:

$$\lim_{t\to\infty}[r_i(t) - f_{s,i}(t)] = 0 \quad i = 0, 1, 2, \ldots, m$$

Because a dedicated nonlinear observer scheme is utilized in the presented approach, it remains to choose a suitable filter $Q(t)$ to meet all of the conditions for fault detection, isolation and identification. It was shown in section 1.a that an appropriate choice of $Q(t)$ for a LTI system described by eq (III.1) is given by

$$Q(s) = [C(sI - A)^{-1}L + I]$$

where $Q(s)$ is the Laplace transform of the filter $Q(t)$. Similarly, for the nonlinear system given by eq (III.9), a linear filter

$$Q(s) = \{\bar{C}(x_{ss}, \theta_{ss})[sI - \bar{A}(x_{ss}, \theta_{ss})]^{-1}\bar{L}(x_{ss}, \theta_{ss}) + I\}$$

is locally applicable. Because the equilibrium point is *a priori* unknown, the fault identification filter is modified:

$$Q(s) = \{\bar{C}(\tilde{x}, \tilde{\theta})[sI - \bar{A}(\tilde{x}, \tilde{\theta})]^{-1}\bar{L}(\tilde{x}, \tilde{\theta}) + I\}$$

where $Q(s)$ is the Laplace transform of the filter at any point $(\tilde{x}, \tilde{\theta})$ in the state space. However, since at least as many eigenvalues of $\bar{A}(\tilde{x}, \tilde{\theta})$ are identical to zero as there

are parameters of the original system, the above $Q(t)$ is not stable. To overcome the problem of choosing a stable filter for fault reconstruction, a lower dimensional observer that does not perform the parameter estimation but only estimates the states needs to be considered

$$\begin{aligned} \dot{\hat{x}}(t) &= f(\hat{x}(t), \tilde{\theta}(t)) + L(\hat{x}(t), \tilde{\theta}(t))(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= h(\hat{x}(t), \tilde{\theta}(t)) + f_s(t) \end{aligned} \tag{III.24}$$

where $\hat{x}$ is the estimate of x and

$$L(x_{ss}, \theta_{ss}) \quad s.t. \quad \lambda[A(x_{ss}, \theta_{ss}) - L(x_{ss}, \theta_{ss})C(x_{ss}, \theta_{ss})] \in \mathbb{C}^- \quad \forall \theta_{ss} \in \Pi \tag{III.25}$$

where $A(x_{ss}, \theta_{ss})$ is the Jacobian of $f(x, \theta)$ at the point (x_{ss}, θ_{ss}) and $C(x_{ss}, \theta_{ss}) = \frac{\partial h(x,\theta)}{\partial x}|_{x_{ss}, \theta_{ss}}$.

Lemma 1

The nonlinear system described by eq (III.24) in conjunction with the observer of the augmented system is a locally asymptotic observer to the system given by eq (III.6) if f_s is known [65].

For practical purposes, the original system given by eq (III.6) in the absence of faults is considered to be locally stable around the operating point as the parameters vary in the hyperrectangle as defined by eq (III.11). In other words, it is assumed that the Jacobian $A(x_{ss}, \theta_{ss})$, $\forall\ \theta_{ss} \in \Pi$, is Hurwitz stable.

Using the above assumption, a stable linear fault identification filter $Q(t)$ such that the residual $r(t) = \int_0^t Q(t-\tau)[y(\tau) - \tilde{y}(\tau)]d\tau$, having the property that $\lim_{t\to\infty} r(t) = f_s(t)$, has the following state-space representation:

$$\begin{aligned} \dot{\xi}(t) &= A(\hat{x}, \tilde{\theta})\xi(t) + L(\hat{x}(t), \tilde{\theta}(t))(y(t) - \hat{y}(t)) \\ r(t) &= C\xi(t) + I(y(t) - \hat{y}(t)) \end{aligned} \tag{III.26}$$

$\xi \in \mathbb{R}^n$ is a state with initial condition $\xi(0) = 0$.

2.f. Fault diagnosis framework

The overall fault detection, isolation and identification filter consists of the observers eqs (III.13) and (III.24) and is computed in parallel with eq (III.26) in order to generate residuals.

In the presence of unknown sensor faults, the estimate $\tilde{\theta}$ for some $\theta_{ss} \in \Pi$ may diverge from the actual value, and therefore the stability of the overall fault diagnosis system can not be guaranteed. To overcome this problem, parameter estimation and fault reconstruction are performed at different time scales and it is assumed that the algorithm is initialized when no sensor fault occurs until a time t_o such that for some $\epsilon \geq 0$, $\|y - \hat{y}\|_2 \leq \epsilon$, $\forall\ t_o \geq 0$. The sensor fault is of the following form:

$$f_s(t) = f(t)S(t - t_o), \quad S(t - t_o) = \begin{pmatrix} 1: & t \geq 0 \\ \\ 0: & t < 0 \end{pmatrix}$$

The above assumption ensures that the parameter estimate from eq (III.7) converges to its actual value with a desired accuracy

$$\|\theta_{ss} - \tilde{\theta}\|_2 \leq \eta, \quad \eta(\epsilon) > 0 \tag{III.27}$$

before the onset of faults in the original process. Additionally, the parameters are adapted periodically by the augmented observer in order to take process drifts into account. In summary, the presented fault diagnosis system performs parameter estimation and fault reconstruction at different time scales, where the fault identification takes place at a higher frequency than parameter estimation. Figure 5 illustrates this two-time scale behavior, where stages 2 and 3 are repeated alternatively throughout

the operation and the time between the start of each stage is decided by the nature of the process. In general, parameter updating is performed sporadically for short periods of time while faults are diagnosed for the vast majority of time.

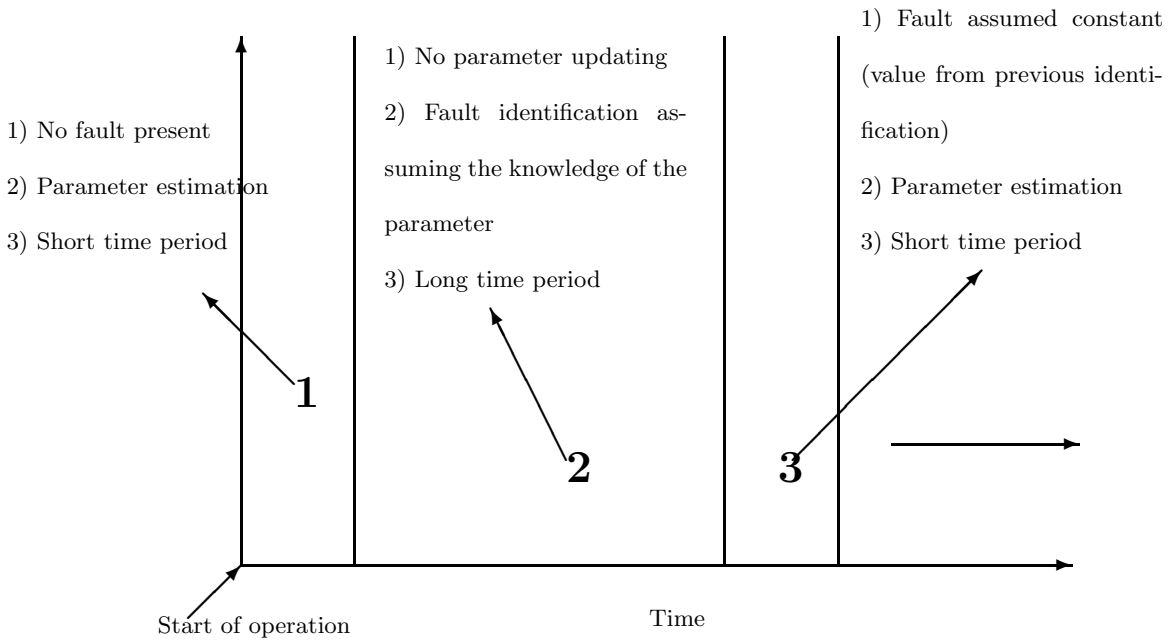

Fig. 5. Schematic fault identification for systems with time-varying parameters.

3. Case study

In this section the performance of the proposed fault diagnosis is demonstrated through taking an example of non-isothermal CSTR with parametric uncertainties.

3.a. Fault diagnosis of CSTR with uncertain parameters

To illustrate the main aspects of the investigated observer-based fault diagnosis scheme, a non-isothermal CSTR is considered with coolant jacket dynamics, where the

following exothermic irreversible reaction between sodium thiosulfate and hydrogen peroxide is taking place [65].

$$2Na_2S_2O_3 + 4H_2O_2 \rightarrow Na_2S_3O_6 + Na_2SO_4 + 4H_2O \tag{III.28}$$

The capital letters A-E are used to denote the chemical compounds $Na_2S_2O_3$, H_2O_2, $Na_2S_3O_6$, Na_2SO_4, and H_2O. The reaction kinetic law is as follows:

$$-r_A = k(T)C_AC_B = (k_o + \Delta k_o)\exp(\frac{-E + \Delta E}{RT})C_AC_B$$

where Δk_o and ΔE represent parametric uncertainties in the model. A mole balance for species A and energy balances for the reactor and the cooling jacket result in the following nonlinear process model:

$$\begin{aligned}
\frac{dC_A}{dt} &= \frac{F}{V}(C_{Ain} - C_A) - 2k(T)C_A^2 \\
\frac{dT}{dt} &= \frac{F}{V}(T_{in} - T) + 2\frac{(-\Delta H)_R + \Delta(-\Delta H)_R}{\rho c_p}k(T)C_A^2 \\
&\quad - \frac{UA + \Delta UA}{V\rho c_p}(T - T_j) \\
\frac{dT_j}{dt} &= \frac{F_w}{V_w}(T_{jin} - T_j) + \frac{UA + \Delta UA}{V_w\rho_w c_{pw}}(T - T_j)
\end{aligned} \tag{III.29}$$

where F is the feed flow rate, F_w is the coolant flow rate, V is the volume of the reactor, C_{Ain} is the inlet feed concentration, T_{in} is the inlet feed temperature, V_w is the volume of the cooling jacket, T_{jin} is the inlet coolant temperature, c_p is the heat capacity of the reacting mixture, c_{pw} is the heat capacity of the coolant, ρ is the density of the reaction mixture, U is the overall heat-transfer coefficient, and A is the area over which the heat is transferred. The process parameter values are listed in Table I. Where ΔK_o, ΔE, $\Delta(\Delta H)$,and ΔUA represent uncertainty in the pre-exponential factor, the activation energy, the heat of the reaction, and the overall heat transfer rate, respectively. When the uncertainties in these kinetic and thermodynamic parameters

Table I. Process parameter values for CSTR operation

process parameters	value
F	120 L/min
$C_{A_{in}}$	1 mol/L
V	100 L
k_o	4.11×10^{13} $L/min.mol$
E	76534.704
T_{in}	275 K
$(-\Delta H)_R$	596619 J/mol
ρ	1000 g/L
c_p	4.2 $J/g.K$
F_w	30 L/min
UA	12×10^5 $J/min.K$
V_w	10 L
ρ_w	1000 g/L
c_{p_w}	4200 $J/kg.K$
$T_{j_{in}}$	250 K

are chosen to be zero, the nominal nonlinear model exhibits multiple steady states, of which the upper steady state (i.e. $C_{Ass} = 0.019$mol/L; $T_{ss} = 384.0\ K$; $T_{js} = 371.3\ K$) is stable and chosen as the point of operation. Since the activation energy appears exponentially in the state space description of the process, therefore the effect of uncertainty in activation energy is expected to be significantly higher than other parameters listed above.

In order to validate the performance of the proposed approach, first of all the results derived from a fault detection scheme based upon a Luenberger observer are used for comparison. The system matrices obtained by linearizing the process model eq (III.29) around the chosen steady state are:

$$\begin{aligned} A &= \begin{bmatrix} -123.75 & -0.0735 & 0 \\ 17408.47 & 6.38 & 2.857 \\ 0 & 28.571 & -31.571 \end{bmatrix} \\ C_1 &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad C_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{aligned} \tag{III.30}$$

with $\lambda(A) = \{-112.94, -1.37, -34.63\}$. For performing fault isolation and identification, it is required to design observers for each of the two measurements as shown in Figure 3, with the eigenvalues of the closed loop observer placed at $\{-6.85, -6.86, -6.87\}$. The observer gain calculated for each of the reaction and coolant temperature

are given as follows:

$$L_1 = \begin{bmatrix} -53.912 \\ 1.55E+3 \\ 5.79E+5 \end{bmatrix}, L_2 = \begin{bmatrix} 1.7E+2 \\ 2.7E+4 \\ 1.55E+3 \end{bmatrix}$$

Both reaction temperature and coolant temperature sensors are induced with an additive fault signal and with zero-mean random noise with normal distribution whose shape and size are shown in Figure 6

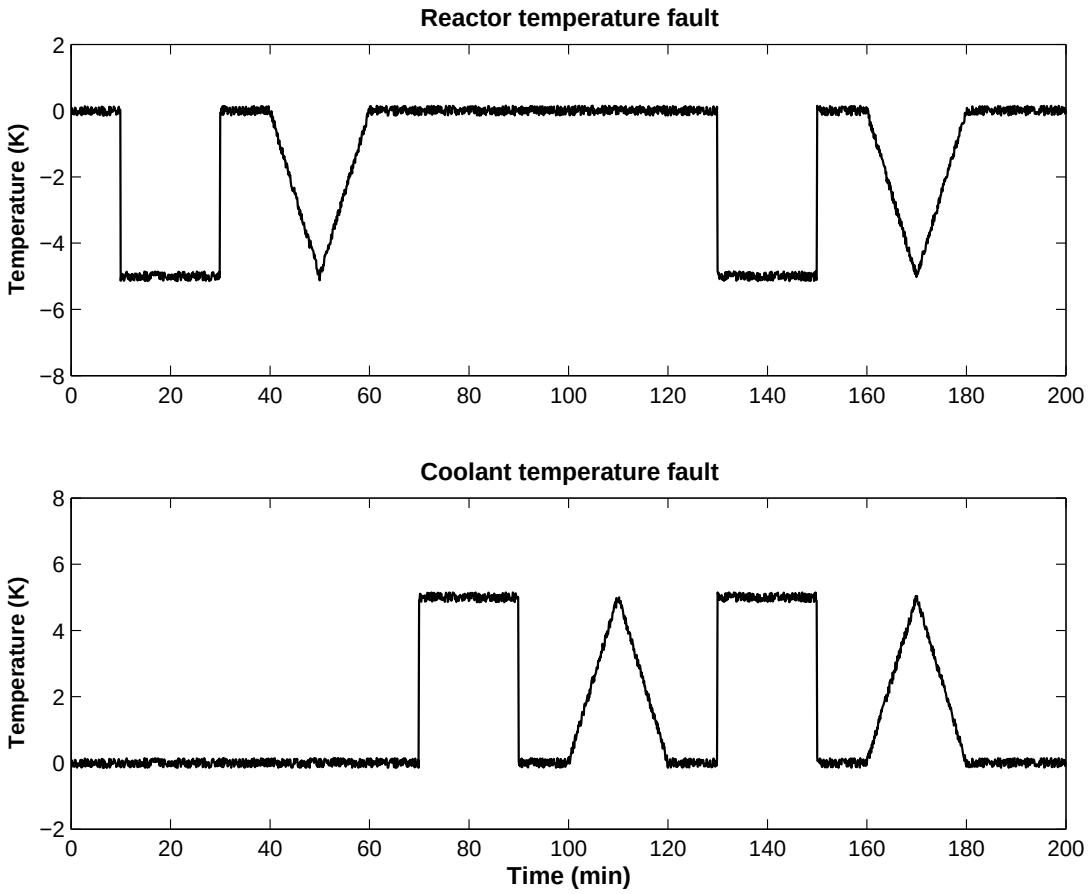

Fig. 6. Reactor and coolant temperature fault signals.

Residuals generated by the technique based upon a Luenberger observer with mismatch in the initial conditions are shown in Figure 7. Comparing Figure 6 and

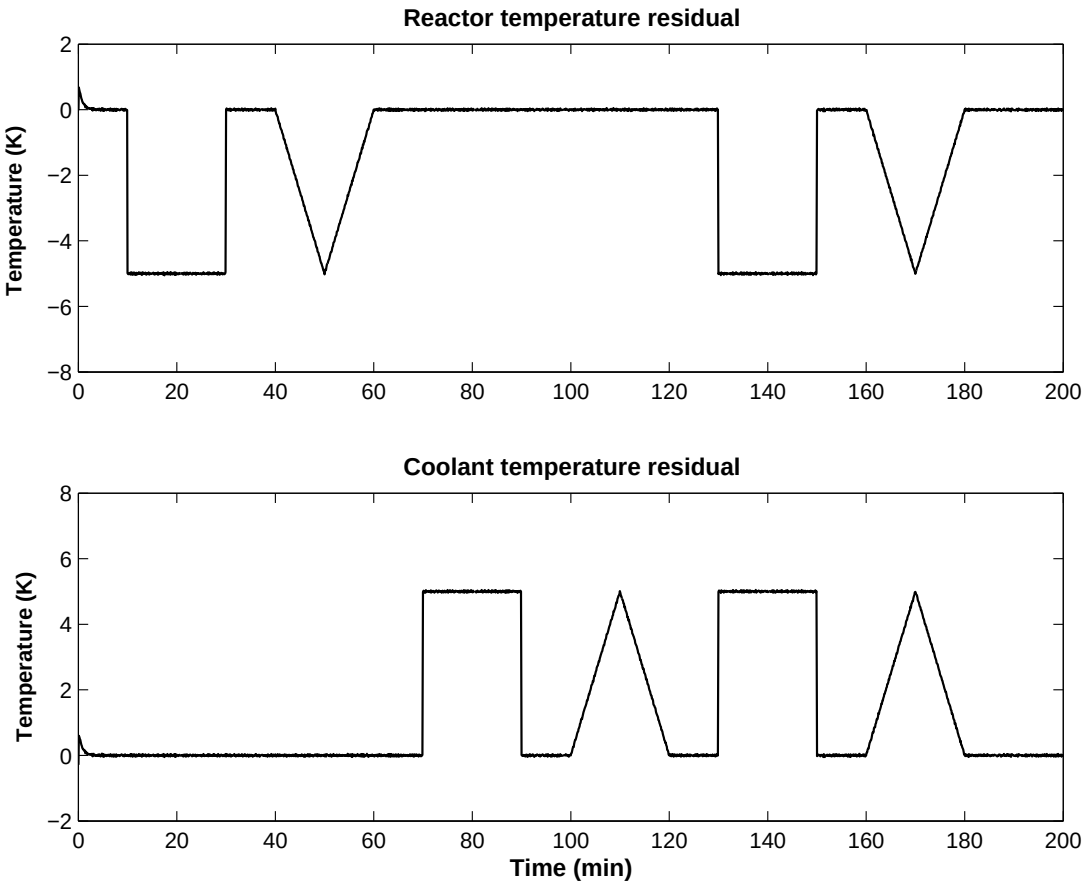

Fig. 7. Reactor and coolant temperature residuals through the Luenberger observer scheme (no model uncertainty).

Figure 7, it can be concluded that the Luenberger observer-based fault diagnosis scheme is able to isolate and identify the approximate nature of the fault in each sensor. Similar simulations have been carried out, where the process model includes uncertainties ($\Delta k_o = 5\% k_o$, $\Delta E = 6\% E$, $\Delta(\Delta H) = 5\% \Delta H$, and $\Delta UA = 5\% UA$). Figure 8 shows the residual generated for the fault signal shown in Figure 6 for one specific case of parametric uncertainty. From Figure 8, it is evident that while the shape of the fault is reproduced almost perfectly, the bias in the residuals results

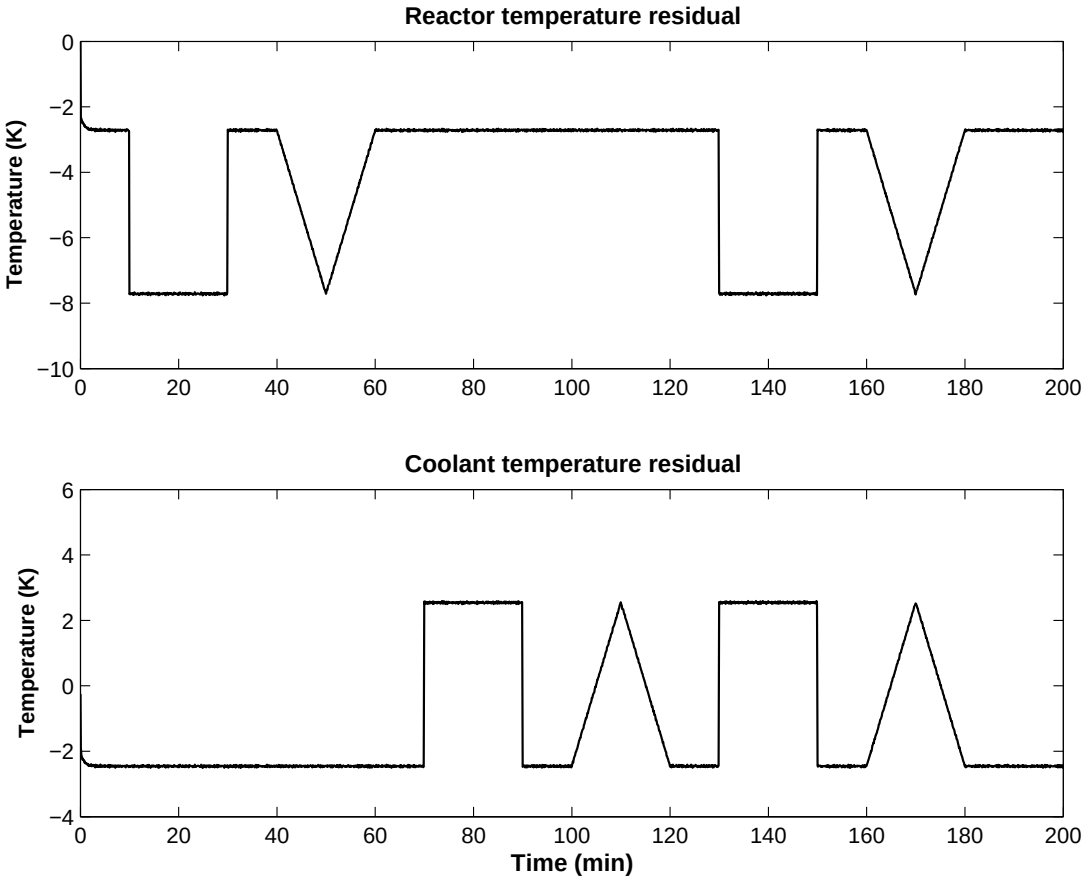

Fig. 8. Reactor and coolant temperature residuals through the Luenberger observer scheme (with model uncertainty).

from modeling uncertainties and can be misinterpreted as a response to a step fault in the sensor. To investigate this point even deeper, simulations of the fault diagnosis scheme based upon the Luenberger observer are performed for a sufficiently large number of scenarios (10,000), which comprises random occurrences of faults in either or both sensors as well as randomly chosen parametric uncertainties within a given bound in order to determine the overall percentage of successfully identifying one or all of the scenarios. The scenarios denoted by 00, 01, 10, and 11 in Tables II and III stand for no faults in both the sensors, no fault in reactor temperature sensor and fault in coolant temperature sensor, fault in reactor temperature sensor and no fault in coolant temperature sensor, and finally fault in both reactor and coolant temperature sensor respectively. Faults of step nature of magnitude 5 K are induced in the sensor and time $t = 0$. Various thresholds are selected to determine whether faults occurred in the sensors, and the fault isolation scheme based upon the Luenberger observer is tested through Monte Carlo simulations with uncertainties chosen randomly with the given intervals.

As an example, Monte Carlo simulations identify the scenario where no faults occur in both the sensors for a preset threshold α if the following condition is satisfied:

(a) if the time average of $|r_T(t)| < \alpha$, where $r_T(t)$ denotes the reactor temperature residual.

(b) if the time average of $|r_C(t)| < \alpha$, where $r_C(t)$ denotes the coolant temperature residual.

The criteria for other (01, 10, and 11) scenarios are chosen similarly. Table II summarizes of how effectively the fault isolation scheme is able to predict the source of faults in the presence of random uncertainties in all the parameters varying within the given uncertainty bound. Table II shows that the parametric uncertainty can

Table II. Monte Carlo simulation (Luenberger observer with model uncertainty)

		threshold			
		1	2	3	4
	00	3.92	9.51	35.80	67.52
scenario	01	24.01	8.82	44.98	16.80
	10	55.38	55.81	44.20	39.04
	11	76.25	58.18	44.36	16.32

have a strong effect on the accuracy of the fault diagnosis scheme and hence requires that can cope with model uncertainty. Therefore, nonlinear fault diagnosis scheme is applied to the same example. Since the effect of uncertainty in activation energy is assumed to be higher than other parameters, only uncertainty in the activation energy $\Pi := \{E := 0.94E_{ss} \leq E \leq 1.06E_{ss}\}$ is considered, with $E_{ss} = 76534.704$J/mol. However, while the design is solely performed based upon uncertainty in this one parameter, the evaluation of the fault diagnosis scheme will consider uncertainty in all of the parameters to compare it to the Luenberger observer scheme. The interval polynomial computed by a step-by-step procedure as discussed in section 2.b.2 is as follows:

(a) The jacobian of the nonlinear dynamic model is symbolically evaluated around

an equilibrium point (x_{ss}, θ_{ss}) as follows:

$$A(x_{ss}, \theta_{ss}) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where the coefficients of the matrix $A(x_{ss}, \theta_{ss})$ are nonlinear functions of (x_{ss}, θ_{ss}).

(b) The characteristic polynomial of the system is determined as shown in eq (III.15).

$$\begin{aligned} \delta(s) = & -a_{11}a_{22}a_{33} + a_{11}a_{23}a_{32} + a_{21}a_{12}a_{23} - a_{21}a_{13}a_{32} \\ & - a_{31}a_{12}a_{23} - a_{31}a_{12}a_{23} + a_{31}a_{13}a_{32} + \dots \\ & + (a_{22}a_{23} - a_{23}a_{32} + a_{11}a_{33} + a_{11}a_{22} - a_{21}a_{12} - a_{31}a_{13})s \\ & + (-a_{11} - a_{22} - a_{33})s^2 + s^3 \end{aligned}$$

The coefficients of the characteristic polynomial are continuous functions of (x_{ss}, θ_{ss}).

(c) x_{ss} is eliminated from the coefficients by using the equation $f(x_{ss}, \theta_{ss}) = 0$ for computation of the upper and lower bounds of the coefficients of the above characteristic polynomial. Not always an analytic expression of the coefficients of the characteristic polynomial as a function of θ_{ss} is possible. To overcome this limitation, x_{ss} is evaluated numerically by solving the equation $f(x_{ss}, \theta_{ss}) = 0$ for θ_{ss}. The maximum and minimum values of each of the coefficients of the characteristic polynomial is computed by varying θ_{ss} in the entire uncertainty set Π.

This procedure is used to evaluate the interval polynomial family given by eq (III.31)

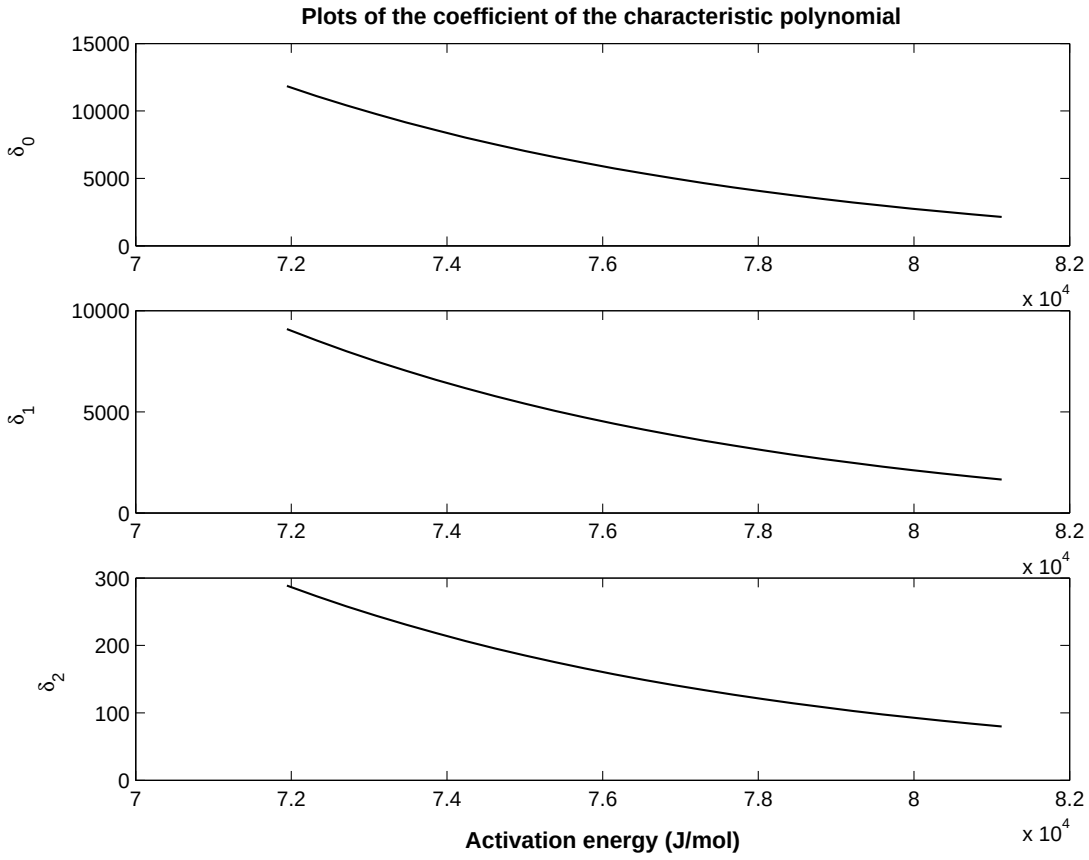

Fig. 9. Plots of the coefficients of the characteristic polynomial as a function of activation energy.

for the nonlinear system given by eq (III.6). Figure 9 shows the plots of coefficients of the characteristic polynomial as the activation energy E varies in the set $\Pi := \{E := 0.94E_{ss} \leq E \leq 1.06E_{ss}\}$, with $E_{ss} = 76534.704\ J/mol$. The interval polynomial thus family takes the following form:

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + s^3 \tag{III.31}$$

where $\delta_0 \in [2143, 11840]$, $\delta_1 \in [1648, 9090]$, $\delta_2 \in [79, 289]$. It can be verified from Theorem 1 that the interval polynomial family given by eq (III.31) is Hurwitz stable, and hence verifying that the nonlinear system given by eq (III.29) is locally stable around the operating point as E varies in the prescribed set $\Pi := \{E := 0.94E_{ss} \leq E \leq 1.06E_{ss}\}$, with $E_{ss} = 76534.704\ J/mol$.

The observer gain computed for the state and parameter estimator from the reactor temperature is

$$\bar{L}(\tilde{x}, \tilde{\theta}) = \bar{T}^{-1}(\tilde{x}, \tilde{\theta}) \begin{bmatrix} -529 \\ -12970 \\ 11347.5 \\ -6113 \end{bmatrix}$$

Similarly the observer gains for the state estimator of the form eq (III.24) to be used for fault isolation are computed to be

$$L_1(\hat{x}, \tilde{\theta}) = T_1^{-1}(\hat{x}, \tilde{\theta}) \begin{bmatrix} -5929 \\ -4143 \\ 878.5 \end{bmatrix}, L_2(\hat{x}, \tilde{\theta}) = T_2^{-1}(\hat{x}, \tilde{\theta}) \begin{bmatrix} -5929 \\ -4143 \\ 878.5 \end{bmatrix}$$

Using the presented technique and applying it to a system with uncertainty in all of

the model parameters, it is found that estimate of the activation energy converges to its true value after 7 min in the absence of sensor faults. The condition that there is no initial sensor fault is a reasonable assumption since one would like to have a certain level of confidence in the measurements before a fault diagnosis procedure is invoked. Figure 10 shows the fault signal $f_s(t)$ that is affecting the senors. The

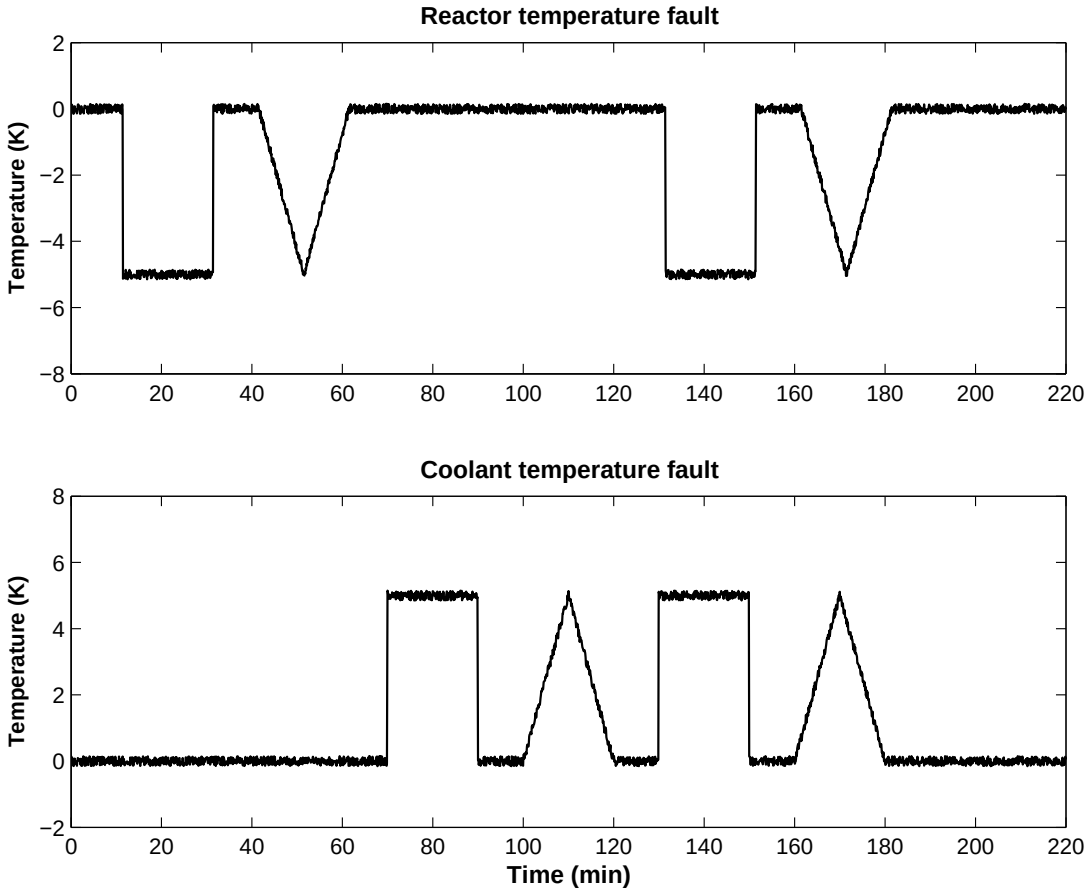

Fig. 10. Reactor and coolant temperature fault signals.

corresponding coolant and reactor temperature residuals generated by the Kharitonov theorem-based fault identification techniques are presented in Figure 11. Apparently the residuals converge to the values of the faults even when uncertainty exists in the model parameters. Additionally, the location, shape, and magnitude of the faults are

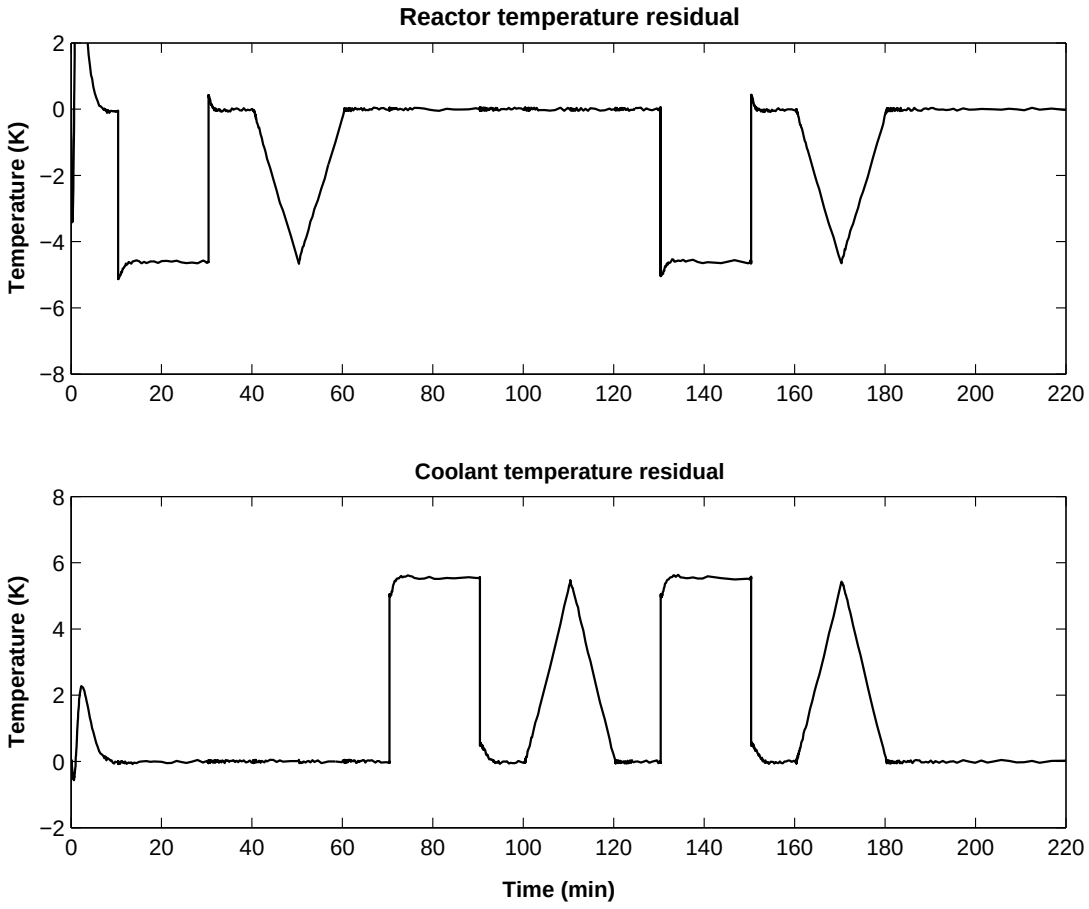

Fig. 11. Reactor and coolant temperature fault residual signals through the presented scheme (with model uncertainty).

correctly reconstructed and sensor noise is filtered.

Since the simulations performed has considered uncertainty in activation energy only, therefore Monte Carlo simulations have obviously a 100% success rate for the scenarios considered in Table II. However, this is not a realistic assumption and in order to compare the proposed fault diagnosis scheme to the Luenberger observer-based method, Monte Carlo simulations are performed by taking the uncertainty in all of the parameters ($\Delta k_o = 5\% k_o, \Delta E = 6\% E, \Delta(\Delta H) = 5\% \Delta H$, and $\Delta UA = 5\% UA$) into account. Table III clearly shows that proposed fault diagnosis scheme performs

Table III. Monte Carlo simulation (presented approach with model uncertainty)

		threshold			
		1	2	3	4
	00	100.00	100.00	100.00	100.00
scenario	01	100.00	100.00	100.00	96.45
	10	89.92	100.00	100.00	100.00
	11	100.00	100.00	100.00	100.00

very well under the influence of uncertainty in all of the model parameters. It can be concluded that assumption that only activation energy has a major impact on the fault diagnosis was a good one since fault identification was only defined for uncertainty in the activation energy; in spite of that reliable fault identification is possible even under the influence of uncertainty in all other parameters. Moreover, in practice it is important to choose an appropriate threshold for detecting faults.

3.b. Fault diagnosis of CSTR with uncertain and time-varying parameters

In this section the performance of the proposed fault diagnosis scheme is evaluated for the previously considered CSTR model but with the kinetic and thermodynamic parameters varying with time. In particular, only activation energy is assumed to be varying with time because it has been determined that uncertainty in the activation energy influences the robustness of a fault diagnosis the most. In practice possible reasons for change in the activation energy is due to catalyst deactivation or coking. Figure 12 shows the plot of the activation energy and its estimate through the

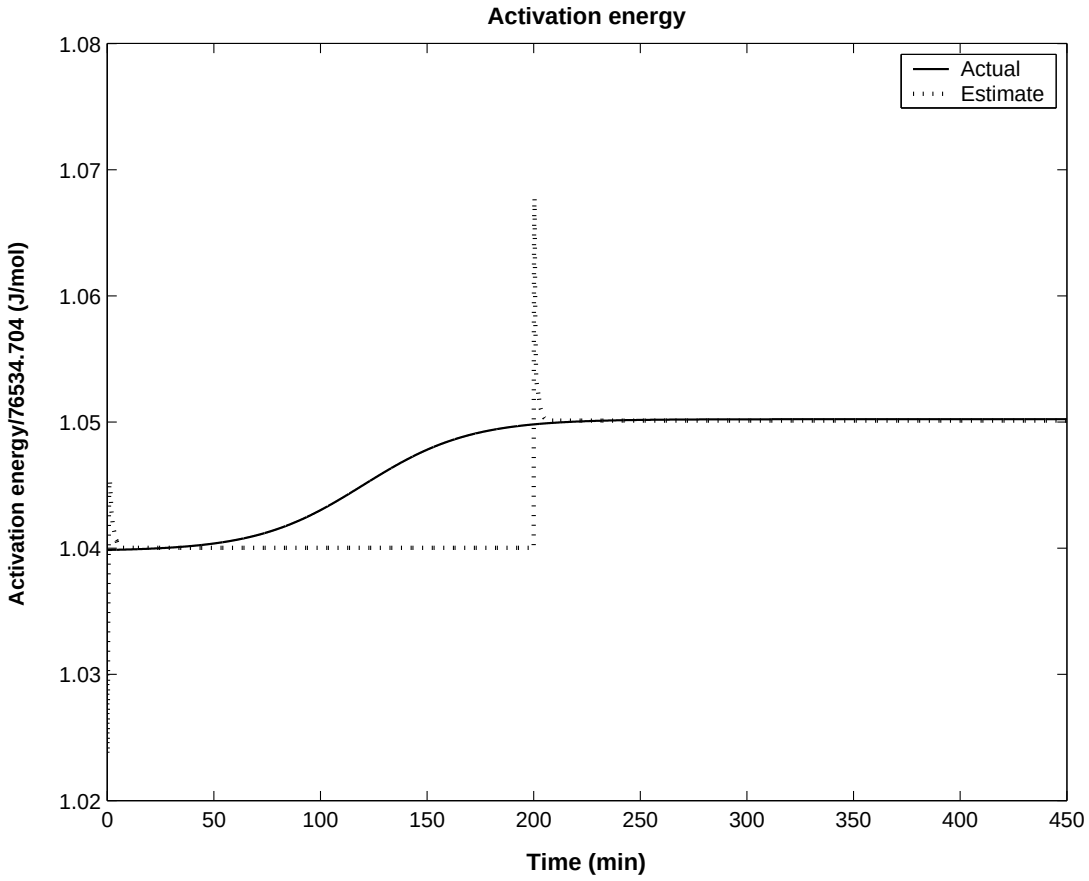

Fig. 12. Activation energy change with time.

proposed nonlinear state and parameter estimator for simulated time span. Figure 13 presents the fault signal $f_s(t)$ being induced in the sensors. The corresponding coolant and reactor temperature residuals generated by Kharitonov theorem-based fault identification are shown in Figure 14. The time period within which the parameter is identified within acceptable limits ranges from $t = 0$ to 10 min. These times were ascertained by comparing the measured and predicted outputs. The first long time period in which fault detection and identification algorithm is invoked ranges from 10 to 200 min. The parameter is updated from $t = 200$ to 210 min. This is followed by another fault detection period ranging from $t = 210$ to 400 min. It can be concluded from Figure 14 that the fault identification scheme is effective even when the activation energy is changing with time. It is apparent from Figure 14 that the fault diagnosis system does not work well if the parameters are not updated periodically.

4. Summary and discussions

In this chapter, a robust observer based nonlinear fault diagnosis was presented. The class of models considered are nonlinear systems whose model structure is known but their parameters are uncertain *a priori*, varying in a bounded set. Since, model identification and fault reconstruction cannot be performed simultaneously, the overall fault diagnosis scheme is designed to periodically identify the model by a nonlinear state and parameter estimator and alternately detect, discriminate, and reconstruct the faults.

The nonlinear state and estimator is designed using a novel parametric approach using Kharitonov's theorem [59]. This approach allows to synthesize a closed-form expression of the nonlinear gain, thereby requiring less computation effort for systems

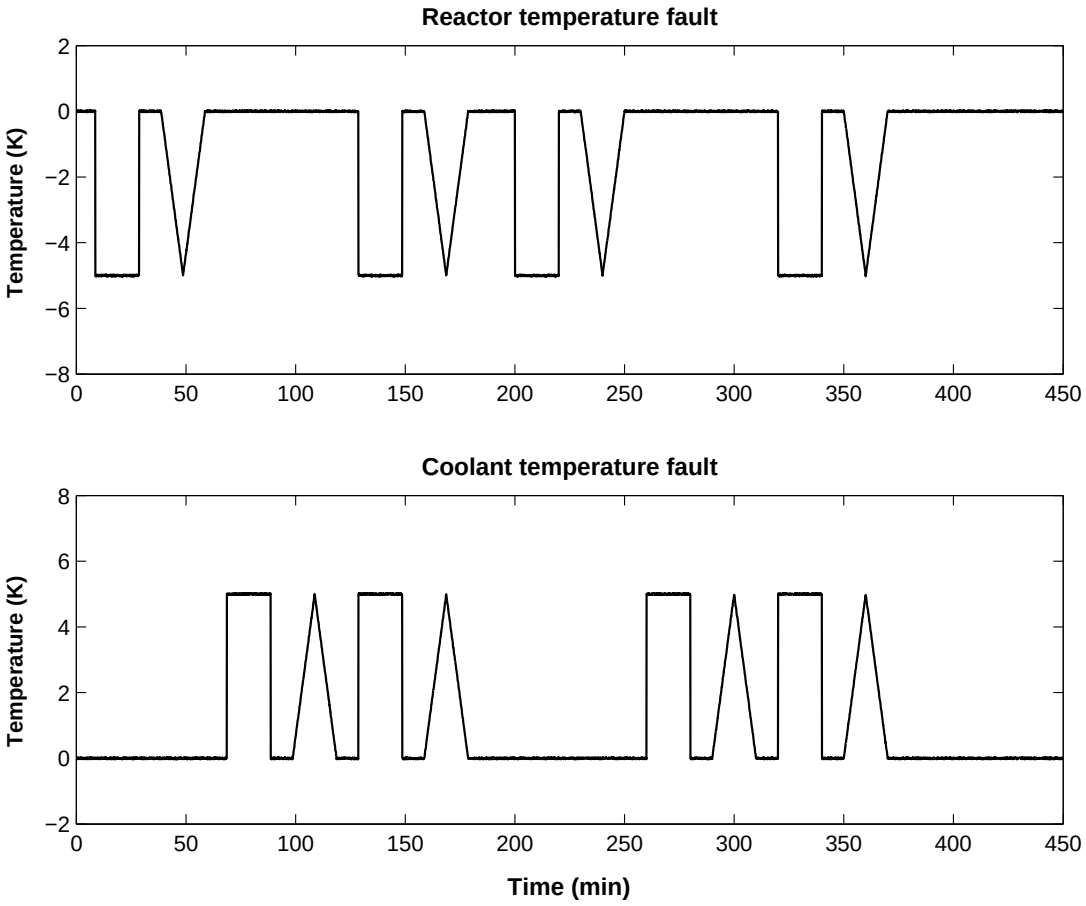

Fig. 13. Reactor and coolant temperature fault signals.

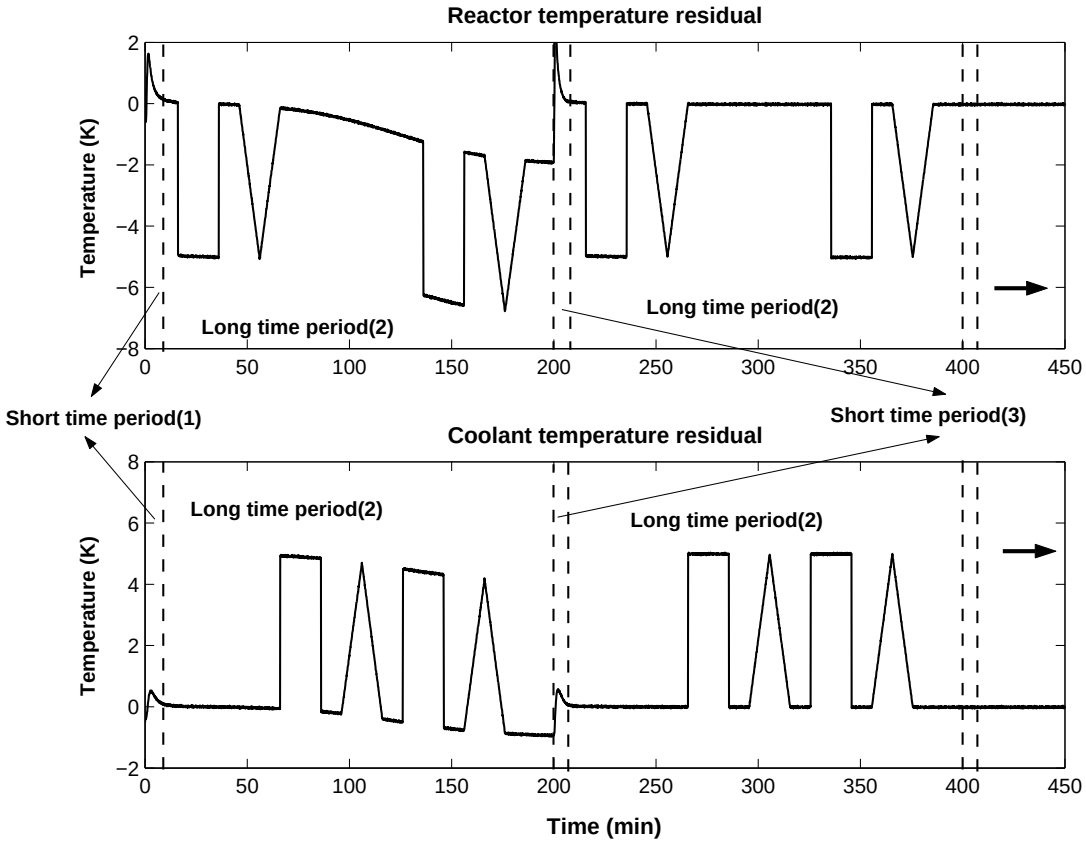

Fig. 14. Reactor and coolant temperature residual signals through the presented scheme (with time-varying parametric uncertainty).

of higher dimensions. The proposed fault diagnosis technique has been applied to two distinct cases of non-isothermal CSTR operation with the model parameters being uncertain *a priori* in one case and slowly time-varying in the other case. The numerical simulations of the fault diagnosis system show the robustness of the presented approach against parametric uncertainties.

One of practical limitations of observer-based fault diagnosis approach is that fairly accurate first principles dynamic models are required. Fundamental modeling can be expensive for large interconnected, complex chemical processes and at times infeasible [12]. To overcome this limitation an empirical state-space characterization of the process model through subspace identification is utilized. Next chapter focuses on designing fault detection, discrimination, and reconstruction techniques using subspace models.

CHAPTER IV

FAULT DIAGNOSIS USING SUBSPACE MODELS

This chapter focuses on diagnosing sensor faults for systems described by their subspace models. These models are generated from the input and output data collected within the region, where the process is operating.

SMI has become an active area of research and has been successfully applied to identification of multivariate state-space models from input-output data of the process [66, 67, 68]. A variety of different techniques exist for designing fault diagnosis algorithms based on SMI [69, 70, 71, 72]. In [69] the authors develop a subspace model from input and output measurements and use a structured residual approach to generate residuals sensitive to a subset of faults, but insensitive to others. The major restriction with the above approach is that although fault detection and isolation can be performed dynamically, fault reconstruction is only possible at steady state. In [70] the same authors propose a subspace model-based output measurement fault detection, isolation and estimation technique for dynamic processes for four specific types of sensor faults. In [71] a subspace approach towards identification of optimal residual models for process fault detection and isolation is presented. In [72] a SMI technique is used to generate contribution charts to diagnose abnormal behavior by reducing the set of variables to monitor the process performance.

As explained in the previous chapter that fundamental modeling of complex industrial processes in computationally expensive and time consuming, therefore state-space model of the input-output data from subspace model identification techniques is developed [67]. Further, to achieve fault detection, isolation and identification, the subspace model is integrated with the model- based fault diagnosis technique developed in the previous chapter [65]. It is not possible to perform fault reconstruction

and subspace model identification simultaneously due to the interaction of model identification accuracy and faults. Therefore, an approach where these two tasks are taking place at different time scales as shown in Figure 5 is implemented. It is shown that fault detection, isolation and reconstruction for dynamic systems whose normal operational input-output data is known can be performed under realistic assumptions with the presented approach.

1. Preliminaries

In Section 1.a observer-based fault diagnosis for discrete LTI systems is presented. Section 1.b provides necessary background information about subspace model identification (SMI).

1.a. Fault diagnosis for discrete LTI systems

In this section, observer-based fault diagnosis for continuous LTI system developed in section 1.a of chapter III is adapted to discrete linear, time-invariant systems. Consider a discrete linear time-invariant system with inputs

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) + \mathbf{f_s}(k) \end{aligned} \tag{IV.1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is a vector of state variables, $\mathbf{u} \in \mathbb{R}^q$ is a vector of input variables, $\mathbf{y} \in \mathbb{R}^m$ is a vector of output variables, n is the number of states, q refers to the number of input variables, m refers to the number of outputs and $k \in \mathbb{N}$ is the sample index. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are matrices of appropriate dimensions. Assuming that the above system is observable, a closed-loop discrete Luenberger observer can be

designed for the system

$$\begin{aligned} \tilde{\mathbf{x}}(k+1) &= \mathbf{A}\tilde{\mathbf{x}}(k) + \mathbf{L}(\mathbf{y}(k) - \tilde{\mathbf{y}}(k)) + (\mathbf{B} - \mathbf{LD})\mathbf{u}(k) \\ \tilde{\mathbf{y}}(k) &= \mathbf{C}\tilde{\mathbf{x}}(k) \end{aligned} \tag{IV.2}$$

where $\mathbf{L}$ is chosen to make the closed-loop observer stable and achieve a desired observer dynamics. $\tilde{\mathbf{x}} \in \mathbb{R}^n$ and $\tilde{\mathbf{y}} \in \mathbb{R}^m$ are the estimates of $\mathbf{x}$ and $\mathbf{y}$, respectively. Further, define a residual [3]

$$\mathbf{r}(k) = \sum_{\tau=1}^{k} \mathbf{Q}(k-\tau)[\mathbf{y}(\tau) - \tilde{\mathbf{y}}(\tau)] \tag{IV.3}$$

which represents the difference between the actual output and the observer output passed through a filter $\mathbf{Q}(k)$. Taking the z transforms of eqs (IV.1)-(IV.3) results in

$$\mathbf{r}(z) = \mathbf{Q}(z)\{\mathbf{I} - \mathbf{C}[z\mathbf{I} - (\mathbf{A} - \mathbf{LC})]^{-1}\mathbf{L}\}\mathbf{f_s}(z) \tag{IV.4}$$

where $\mathbf{Q}(k)$ is chosen such that $\mathbf{Q}(z)$ is a RH_∞ matrix [46]. It can be shown that

(1) $\mathbf{r}(k) = 0$ if $\quad \mathbf{f_s}(k) = 0$

(2) $\mathbf{r}(k) \neq 0$ if $\quad \mathbf{f_s}(k) \neq 0$

indicating that the value of $\mathbf{r}(k)$ predicts the existence of a fault in the system [9]. In addition, if one uses the dedicated observer scheme as shown for a system with two outputs in Figure 3, then the fault detection system can also determine the location of the fault:

(3) $\mathbf{r_i}(k) = 0$ if $\quad \mathbf{f_{s,i}}(k) = 0, i = 1, 2, 3, \ldots, m$

(4) $\mathbf{r_i}(k) \neq 0$ if $\quad \mathbf{f_{s,i}}(k) \neq 0, i = 1, 2, 3, \ldots, m,$

where i represents the i_{th} measurement. A fault detection system that satisfies all of the above conditions is called a Fault detection and isolation filter (FDIF). A fault

detection and isolation filter becomes a fault identification filter (FIDF) if additionally the following condition is satisfied [9]:

(5) $\lim_{k\to\infty}[\mathbf{r_i}(k) - \mathbf{f_{s,i}}(k)] = 0, i = 1, 2, 3, \ldots, m$

To meet above conditions, the following restrictions on the choice of $\mathbf{Q}(z)$ are imposed [65]:

(a) $\mathbf{Q_i}(z) \neq 0, \forall\ z \in \mathbb{C}$

(b) $\mathbf{Q_i}(z) = \{\mathbf{I}-\mathbf{C_i}[z\mathbf{I}-(\mathbf{A}-\mathbf{L_iC_i})]^{-1}\mathbf{L_i}\}^{-1} = [\mathbf{C_i}(z\mathbf{I}-\mathbf{A})^{-1}\mathbf{L_i}+\mathbf{I}], \quad i = 1, 2, \ldots, m.$

Where, $\mathbf{C_i}$ is the i_{th} row of matrix $\mathbf{C}$ and $\mathbf{L_i}$ is the corresponding observer gain. One of the requirements of the aforementioned approach is that it relies on the fundamental state-space description of the process, which may not be easy to generate for large-scale complex chemical plants. The following section briefly reviews a method to generate empirical state-space models from the input-output data of processes with linear, time-invariant behavior.

1.b. Subspace model identification

It is highlighted in [67] that subspace methods determine the state space matrices $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$ and $\mathbf{D}$. Moreover, the order of the subspace model is directly determined from the input-output measurements.

In this dissertation it is assumed that the dynamic process models are linear, time-invariant, and bounded input bounded output stable over a data horizon where the model is identified. Further, it is assumed that the process noise is Gaussian distributed, uncorrelated and additive to the actual state sequences of the process. In this work, the N4SID algorithm [10] is used and is briefly reviewed below. Since

only application of subspace models in fault diagnosis is the focus of this chapter, interested readers will find a more thorough analysis of SMI algorithms in [67].

The historical process measurements are first scaled to zero mean and unit variance and arranged to form block Hankel matrices $\mathbf{Y_f}$, $\mathbf{Y_p}$, $\mathbf{U_f}$, and $\mathbf{U_p}$. The subscripts "f" and "p" refer to future and past, respectively. The Hankel matrices are then arranged to form a linear least squares regression equation, where the rows of $\mathbf{Y_f}$ are regressed on the matrix $\begin{bmatrix} \mathbf{Y_p}^T & \mathbf{U_f}^T & \mathbf{U_p}^T \end{bmatrix}^T$, producing the regression matrix $\begin{bmatrix} \mathbf{R_{Y_p}} & \mathbf{R_{U_f}} & \mathbf{R_{U_p}} \end{bmatrix}$. By excluding the linear contribution of the $\mathbf{U_f}$, the matrix, $\mathbf{Y_f}$ is predicted as follows:

$$\hat{\mathbf{Y}}_\mathbf{f} = \begin{bmatrix} \mathbf{R_{Y_p}} & \mathbf{R_{U_p}} \end{bmatrix} \begin{bmatrix} \mathbf{Y_p} \\ \mathbf{U_p} \end{bmatrix} \tag{IV.5}$$

It is shown in [10] that the above regression equation can also be formulated as follows:

$$\mathbf{Y_f} = \mathbf{\Gamma X_f} + \mathbf{R_{U_f} U_f} + \mathbf{G_f}, \tag{IV.6}$$

where $\mathbf{\Gamma}$ is the extended observability matrix, $\mathbf{X_f}$ is matrix of state sequences stored as row vectors and $\mathbf{G_f}$ is a residual matrix. Comparing eqs (IV.5) and (IV.6), the matrix product $\mathbf{\Gamma X_f}$ is equivalent to $\hat{\mathbf{Y}}_\mathbf{f}$. By a singular value decomposition of $\hat{\mathbf{Y}}_\mathbf{f}$, $\mathbf{\Gamma}$ and $\mathbf{X_f}$ can be obtained up to a similarity transformation. The number of state variables selected is based on the magnitude of the dominant singular values [10]. The states sequence can be obtained from the estimation of Γ as follows:

$$\mathbf{X_f} = \mathbf{\Xi} \begin{bmatrix} \mathbf{U_p} \\ \mathbf{Y_p} \end{bmatrix} \mathbf{X_f}^{(+1)} = \mathbf{\Xi}^{(+1)} \begin{bmatrix} \mathbf{U_p}^{(+1)} \\ \mathbf{Y_p}^{(+1)} \end{bmatrix}, \tag{IV.7}$$

where $(+1)$ indicates that the elements in $\mathbf{X_f}^{(+1)}$, $\mathbf{U_p}^{(+1)}$ and $\mathbf{Y_p}^{(+1)}$ are forward-

shifted by one instance relative to the elements in $\mathbf{X_f}$, $\mathbf{U_p}$ and $\mathbf{Y_p}$ respectively. This is achieved by rearranging the rows in $\mathbf{Y_f}$, $\mathbf{Y_p}$, $\mathbf{U_f}$ and $\mathbf{U_p}$. Furthermore,

$$\mathbf{\Xi} = \mathbf{\Gamma}^{\dagger} \begin{bmatrix} \mathbf{R_{Y_p}} & \mathbf{R_{U_p}} \end{bmatrix}, \quad \mathbf{\Xi}^{(+1)} = \mathbf{\Gamma}^{(+1)\dagger} \begin{bmatrix} \mathbf{R_{Y_p}}^{(+1)} & \mathbf{R}_{\mathbf{U_p}}^{(+\mathbf{1})} \end{bmatrix},$$

where † represents the generalized inverse.

2. Subspace model-based fault diagnosis scheme

To avoid developing a fundamental state-space model of a process plant, the proposed method uses empirical state-space models derived from input-output data of a plant and subsequently generates residuals for fault diagnosis along the lines as discussed in section 1.a.

2.a. Residual generation using subspace models

Consider a discrete linear, time-invariant system with possibly multiple inputs and outputs:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}x_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{v}_k \end{aligned} \tag{IV.8}$$

where $\mathbf{w}_k$ and $\mathbf{v}_k$ are zero mean, finite dimensional Gaussian white vector sequences. To use the N4SID algorithm developed in [10] measured inputs and outputs are scaled to zero mean and unit variance. The inputs and outputs are scaled as follows:

$$\bar{\mathbf{y}}_k = \mathbf{S_y}^{-\frac{1}{2}}(\mathbf{y}_k - \mathbf{y_s}), \quad \bar{\mathbf{u}}_k = \mathbf{S_u}^{-\frac{1}{2}}(\mathbf{u}_k - \mathbf{u_s}), \tag{IV.9}$$

where $\mathbf{S_y}$ and $\mathbf{S_u}$ are sample covariance matrices of output and input sequences respectively and assumed to be invertible. $\mathbf{y_s}$ and $\mathbf{u_s}$ are sample mean vectors of output

and input sequences, respectively. The estimated state-space model through N4SID algorithm is represented in the following form:

$$\begin{aligned} \mathbf{x}_{k+1} &= \hat{\mathbf{A}} x_k + \hat{\mathbf{B}} \bar{\mathbf{u}}_k \\ \bar{\mathbf{y}}_k &= \hat{\mathbf{C}} \mathbf{x}_k + \hat{\mathbf{D}} \bar{\mathbf{u}}_k \end{aligned} \tag{IV.10}$$

Here $(\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{C}}, \hat{\mathbf{D}})$ are the estimated state-space parameters. It is assumed that the state-space model given by eq (IV.10) captures the scaled input-output response of the actual system given by eq (IV.8) accurately. Further, consider the original discrete linear time-invariant system with an output measurement fault $\mathbf{f}_s \in \mathbb{R}^m$.

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k^f &= \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{v}_k + \mathbf{f}_{sk} \end{aligned} \tag{IV.11}$$

Here, $\mathbf{y}_k^{\mathbf{f}}$ is the faulty output measurement, no faults in the input measurement is considered although it is an equally important problem. To use the fault detection, isolation and reconstruction algorithm presented in section 1.a, the output and input measurements of the system given by eq (IV.11) are scaled according to eq (IV.9). This provides a common metric to compare the predicted and actual system measurements.The following notation is used for the rest of the paper:

$$\bar{\mathbf{y}}_k^{\mathbf{f}} = \bar{\mathbf{y}}_k + \bar{\mathbf{f}}_s, \quad \bar{\mathbf{f}}_{\mathbf{s}} = {\mathbf{S}_{\mathbf{y}}}^{-\frac{1}{2}} \mathbf{f}_{\mathbf{s}} \tag{IV.12}$$

A closed-loop observer is developed using the subspace model from eq (IV.10) to estimate the actual process outputs. It has been shown in [3] that using a closed loop estimator increases the rate of decay of the estimation error and moreover reduces the effect of plant-model mismatch and measurement noise on the estimated outputs.

The estimator equations take the following form:

$$\begin{aligned} \tilde{\mathbf{x}}_{k+1} &= \hat{\mathbf{A}}\tilde{\mathbf{x}}_k + \mathbf{L}(\bar{\mathbf{y}}_k^f - \tilde{\bar{\mathbf{y}}}_k) + (\hat{\mathbf{B}} - \mathbf{L}\hat{\mathbf{D}})\bar{\mathbf{u}}_k \\ \tilde{\bar{\mathbf{y}}}_k &= \hat{\mathbf{C}}\tilde{\mathbf{x}}_k \end{aligned} \tag{IV.13}$$

where, $\tilde{\mathbf{x}}_k \in \mathbb{R}^n$ and $\tilde{\bar{\mathbf{y}}}_k \in \mathbb{R}^m$ are the estimates of $\mathbf{x}_k$ and $\bar{\mathbf{y}}_k$ respectively, $\mathbf{L}$ is the Kalman gain for the closed-loop filter. Also, note that the Kalman filter makes use of the assumption that the measurement fault is known from an earlier identification of the fault. When the subspace model is identified for the first time, it has no knowledge about possible sensor faults and assumes that no sensor fault was initially present. Defining a residual as in Section 1.a, the following is obtained:

$$\mathbf{r}_k = \sum_{\tau=1}^{k} \mathbf{Q}_{k-\tau}[\bar{\mathbf{y}^{\mathbf{f}}}_\tau - \tilde{\bar{\mathbf{y}}}_\tau] \tag{IV.14}$$

Taking z transforms of eqs (IV.11)-(IV.14) results in:

$$\mathbf{r}(z) = \mathbf{Q}(z)\{\mathbf{I} - \hat{\mathbf{C}}[z\mathbf{I} - (\hat{\mathbf{A}} - \mathbf{L}\hat{\mathbf{C}})]^{-1}\mathbf{L}\}\bar{\mathbf{f}}_s(z) \tag{IV.15}$$

2.b. Fault detection

The main purpose of fault detection is to determine whether there exists faults in the measurements. It can be seen from eq (IV.15) that:

(1) $\mathbf{r}_k = 0$ if $\quad \bar{\mathbf{f}}_s = 0, \quad$ since $\mathbf{S_y}^{-\frac{1}{2}}$ is an invertible matrix, $\Rightarrow \mathbf{f_s} = 0$

(2) $\mathbf{r}_k \neq 0$ if $\quad \bar{\mathbf{f}}_s \neq 0, \quad \Rightarrow \mathbf{f}_s \neq 0$

indicating the that the value of $\mathbf{r_k}$ predicts the existence of the fault in the system.

2.c. Fault isolation

To perform fault isolation, the state-space system given by eq (IV.10) is assumed to be observable through each of the outputs $\mathbf{y}$. It was shown that this requirement is mandatory for the existence of a fault isolation filter [9]. However, this limitation can be overcome by identifying a minimal order state-space model through each of the outputs separately.

To achieve fault isolation as well as detection, the proposed approach uses a series of dedicated observers as shown in Figure 3. In this method, as many residuals are generated as the number of measurable outputs. It can be verified that:

(3) $\mathbf{r}_{k_i} = 0$ if $\quad \bar{\mathbf{f}}_{s_i} = 0, i = 1, 2, 3, \ldots, m$

(4) $\mathbf{r}_{k_i} \neq 0$ if $\quad \bar{\mathbf{f}}_{s_i} \neq 0, i = 1, 2, 3, \ldots, m,$

Moreover, since $\mathbf{S_y}^{-\frac{1}{2}}$ is an invertible matrix, it is ensured that if $\mathbf{f}_s \in \mathbb{R}^m$ then $\bar{\mathbf{f}}_s \in \mathbb{R}^m$ and therefore the mutual independence of components of $\mathbf{f_s}$ is preserved in $\bar{\mathbf{f}}_s$.

2.d. Fault identification

Fault identification entails the job of reconstructing the shape and size of the fault signal. To estimate the shape and size of the fault, the residuals have to meet the following objective [9]:

$$\lim_{k \to \infty} [\mathbf{r}_{k_i} - \bar{\mathbf{f}}_{s_i}] = 0, i = 1, 2, 3, \ldots, m$$

In other words, residuals should asymptotically converge to the actual fault signal. Since a dedicated observer scheme is utilized in the proposed approach, it remains to choose a suitable filter $\mathbf{Q}_k$ to meet all the conditions for fault detection, isolation,

and identification. The following choice of filter satisfies the requirement for fault detection, isolation, and identification:

(a) $\mathbf{Q_i}_z \neq 0, \forall\ z \in \mathbb{C}$

(b) $\mathbf{Q_i}_z = \{\mathbf{I} - \hat{\mathbf{C}}_\mathbf{i}[z\mathbf{I} - (\hat{\mathbf{A}} - \mathbf{L_i}\hat{\mathbf{C}}_\mathbf{i})]^{-1}\mathbf{L_i}\}^{-1} = [\hat{\mathbf{C}}_\mathbf{i}(z\mathbf{I} - \hat{\mathbf{A}})^{-1}\mathbf{L_i} + \mathbf{I}], \quad i = 1, 2, \ldots, m.$

Where, $\hat{\mathbf{C}}_\mathbf{i}$ is the i_{th} row of matrix $\hat{\mathbf{C}}$ and $\mathbf{L_i}$ is the corresponding Kalman gain. Finally, the unscaled fault is reconstructed through the following matrix operation:

$$\mathbf{f}_s = \mathbf{S_y}^{\frac{1}{2}}\bar{\mathbf{f}}_s$$

3. Case study

In this section, the performance of the subspace model-based fault diagnosis is evaluated on a discrete LTI system, a nonlinear chemical process with time-varying parameter, a debutanizer plant, and an industrial melter process.

3.a. Application to discrete time LTI system

In this section, the output is generated through numerical simulation of a discrete LTI system shown below:

$$\begin{bmatrix} \mathbf{x_1}(k+1) \\ \mathbf{x_2}(k+1) \\ \mathbf{x_3}(k+1) \end{bmatrix} = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & -0.31 & 0 \\ 0 & 0 & -0.32 \end{bmatrix} \begin{bmatrix} \mathbf{x_1}(k) \\ \mathbf{x_2}(k) \\ \mathbf{x_3}(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \mathbf{u}(k) + \mathbf{W}(k)$$

$$\begin{bmatrix} \mathbf{y_1}(k) \\ \mathbf{y_2}(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x_1}(k) \\ \mathbf{x_2}(k) \\ \mathbf{x_3}(k) \end{bmatrix} + \mathbf{V}(k), \quad \mathbf{x}(0) = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \tag{IV.16}$$

Here $\mathbf{u}(k) = 0.2 \quad$ for all $k \in \mathbb{N}$. $\mathbf{W}(k) \sim N(0, \mathbf{Q})$ is the process noise and $\mathbf{V}(k) \sim N(0, \mathbf{R})$ is the output noise with zero mean, white noise characteristics and follows a normal distribution. It is further assumed that the process noise $\mathbf{W}(k)$ arises due to an additive noise in the input signal. Therefore the actual input signal is redefined as $\mathbf{u_{ac}}(k) = \mathbf{u}(k) + \mathbf{w}(k)$, where $\mathbf{w}(k) = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \mathbf{W}(k)$. After scaling the outputs and input measurements to zero mean and unit covariance, the N4SID algorithm is used to obtain the following model:

$$\mathbf{x}(k+1) = \hat{\mathbf{A}}\mathbf{x}(k) + \hat{\mathbf{B}}\mathbf{u}(\bar{k})$$

$$\bar{\mathbf{y}}(k) = \hat{\mathbf{C}}\mathbf{x}(k)$$

The identified process model matrices are:

$$\hat{\mathbf{A}} = \begin{bmatrix} -0.29786 & -0.68581 & -0.0572 \\ 0.59282 & -0.17853 & -0.67979 \\ 0.092206 & 0.61815 & -0.4536 \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} -0.021503 \\ -0.012004 \\ -0.022702 \end{bmatrix}$$

$$\hat{\mathbf{C}} = \begin{bmatrix} -60.813 & -9.5603 & 69.491 \\ -81.081 & -12.748 & 92.656 \end{bmatrix}$$

The eigenvalues of matrix $\hat{\mathbf{A}}$ are $\{-0.3 \quad -0.31 \quad -0.32\}$. A closed-loop observer is used to estimate the output measurements and generate residuals. Further, a dedicated observer scheme as shown in Figure 3 is used to facilitate fault isolation. Both the outputs are induced with an additive fault in the simulations, whose shape and size are shown in Figure 15. Residuals generated by the technique based upon a subspace model of the process and Kalman filter are shown in Figure 16. When Figures 15 and 16 are compared, it is concluded that the SMI based fault diagnosis scheme is able to correctly isolate and identify the approximate nature of the fault in each output.

3.b. Application to a chemical process with time-varying parameters

In this section, a non-isothermal continuous stirred tank reactor (CSTR) in the presence of variation of activation energy with time as in section 3.b of chapter III is considered. The model equations and the corresponding process parameters are same as in section 3.a of chapter III. In this case study only the activation energy is varying with time and all other parameters are assumed to be constant and known. The activation energy varies with time as shown in Figure 17. Since it is not possible to perform fault reconstruction and subspace identification simultaneously, the two-

time scale approach explained in section 2.f of chapter III, where these two tasks are taking place at different time scales is implemented. In general subspace model identification requires relatively short periods of time, so that the fault can be identified for the vast majority of the time. In this example, $u(t)$, the input of the process is manipulated by the reactant feed flow rate F. It is further assumed that there is an additive zero mean Gaussian white noise to the input variable. There are two output measurements from this process, namely the reaction temperature and the coolant temperature, each of which has an additive zero mean Gaussian white noise. The continuous input and output measurements are sampled with a sampling interval of 0.1 min. After scaling input and output measurements to zero mean and unit variance, the N4SID algorithm is used to obtain the subspace model for the period starting from $t = 0$ min to 100 min. Figure 18 shows the fault induced additively in each of the output measurements through simulations. The residuals generated using the subspace model generated from the input-output data are shown in Figure 19. In the absence of sensor faults for $t = 0$ min to 100 min, the residuals converge to zero value. Moreover, for this particular case study, the linear subspace model adequately captures the nonlinear behavior of the actual process at steady state. Further, for the time period starting from t=100 min to 200 min, the residuals match their corresponding fault signals accurately. As shown in Figure 17 the parameter drifts from its original value at time 200 min and therefore the subspace model generated from the sampled input-output data from t=0 to 100 min is not valid after t=200 min. This observation is supported by the Figure 19, where both the residuals drift from the fault signals after t=200 min. Since the proposed fault diagnosis technique is based on the subspace model, the subspace model identification accuracy and faults reconstruction influence each other. To overcome this problem, the two-time scale scheme shown in Figure 5 is used. The subspace model is re-identified from the input-output

data from t=400 min to 500 min, where no faults exist in the measurements as shown in Figure 18. Again the residuals generated from the new subspace model converge to zero for the time period t=400 min to 500 min, which proves that the subspace model has been adequately adapted to the changing process conditions. The residuals generated from the re-identified model accurately reconstructs the true fault signals after t=400 min. In practice, one can adjust the time for model identification and fault reconstruction according to the nature of the process.

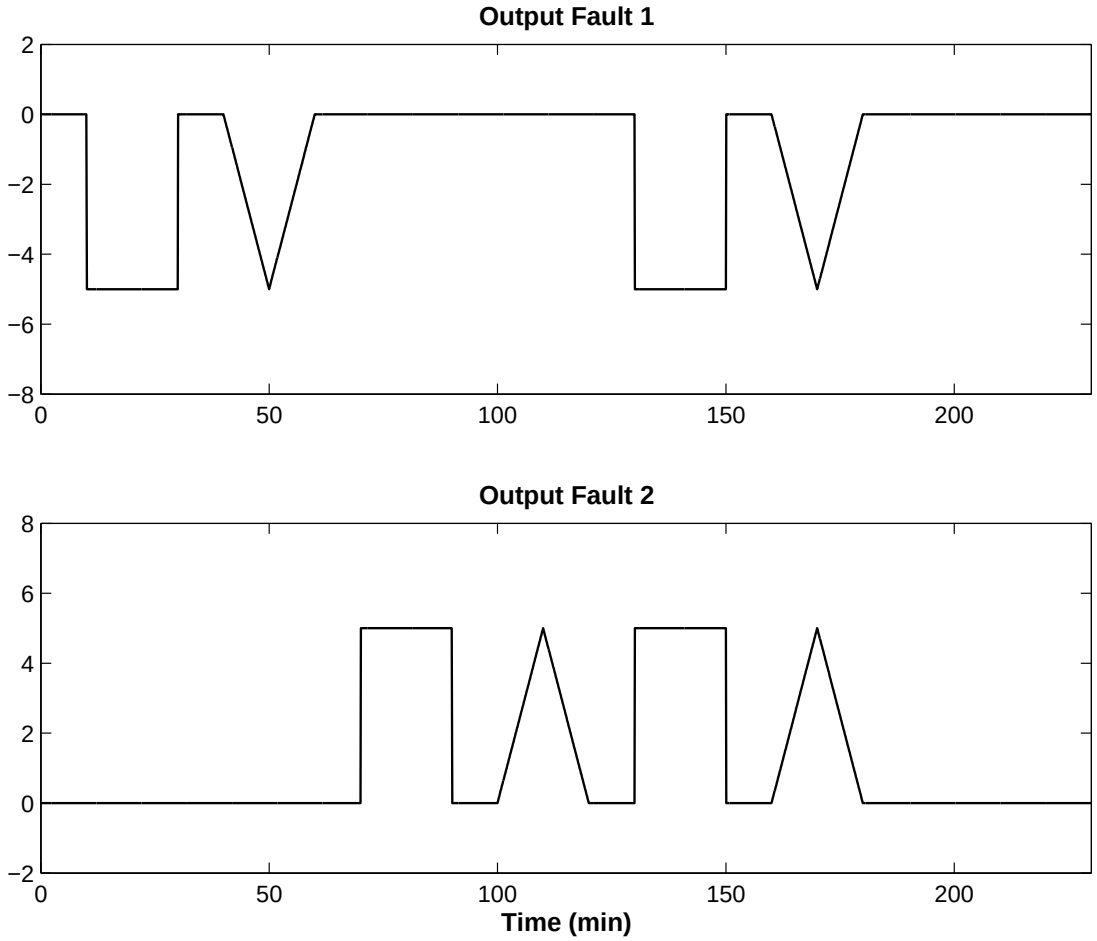

Fig. 15. Output fault signals.

3.c. An industrial distillation process

This case study demonstrates the application of the linear subspace model-based fault diagnosis to a debutanizer plant, which is designed to purify Butane from fresh feed comprising of a mixture of hydrocarbons, mainly Butane and Hexane and impurities of Propane. This process has twelve outputs and four inputs as shown in Table IV. The

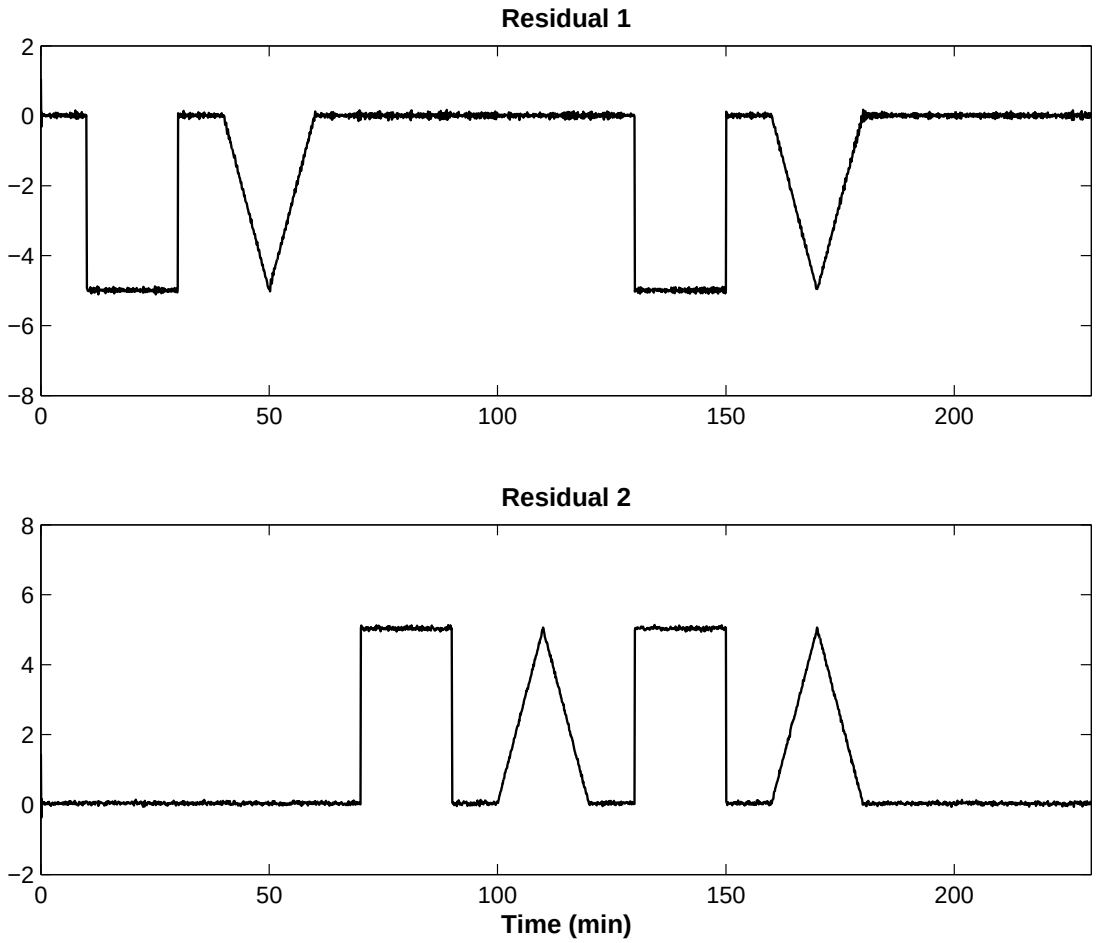

Fig. 16. Output residuals using SMI and Kalman filter.

distillation tower includes 33 trays with which the separation is achieved. A purified Butane stream leaves the distillation process as the top product and consequently, the Hexane and impurities leave the distillation process with the bottom draw. The

input-output data is sampled at an interval of 30 sec and the entire process runs for 5000 min. It is learnt from the operation that the operational data is fault-free. The data of the process comes from a real debutanizer plant, but the faults that will be added in this section were imposed to show the validity of the presented approach. For sake of simplicity the fault detection, isolation, and identification problem for only the first two outputs is considered. Figure 20 illustrates the nature of the fault signal added to two of the sensors of the system, no faults are added to the actuators of the system. The residuals generated by using the proposed fault diagnosis scheme using the subspace model generated from the input-output plant data is illustrated in Figure 21. Comparing Figures 20 and 21 it is concluded that the subspace model-based fault diagnosis performs well.

3.d. An industrial melter process

The last of the case studies is another application of the linear subspace model-based fault diagnosis technique on an operational data of an industrial melter process. The process has 8 outputs representing temperatures at various zones and 6 inputs which represent the power in the induction coil and/or voltage. The process is sampled at an interval of 5 min for approximately 5245 minutes. Neither the dynamic behavior nor the nature of faults in outputs are known *a priori* for this process. However, it is learnt that the thermocouple corresponding to output 6 fails towards the end of the process, thereafter no further measurements were recorded. Linear subspace model-based fault diagnosis is performed on the operational data of this process. Figure 22 presents the output residuals generated for all the outputs. Moreover, from the output residual for temperature 6, it is deduced that the thermocouple fails around 5240 minutes. This observation exactly matches what was determined for the melter

process.

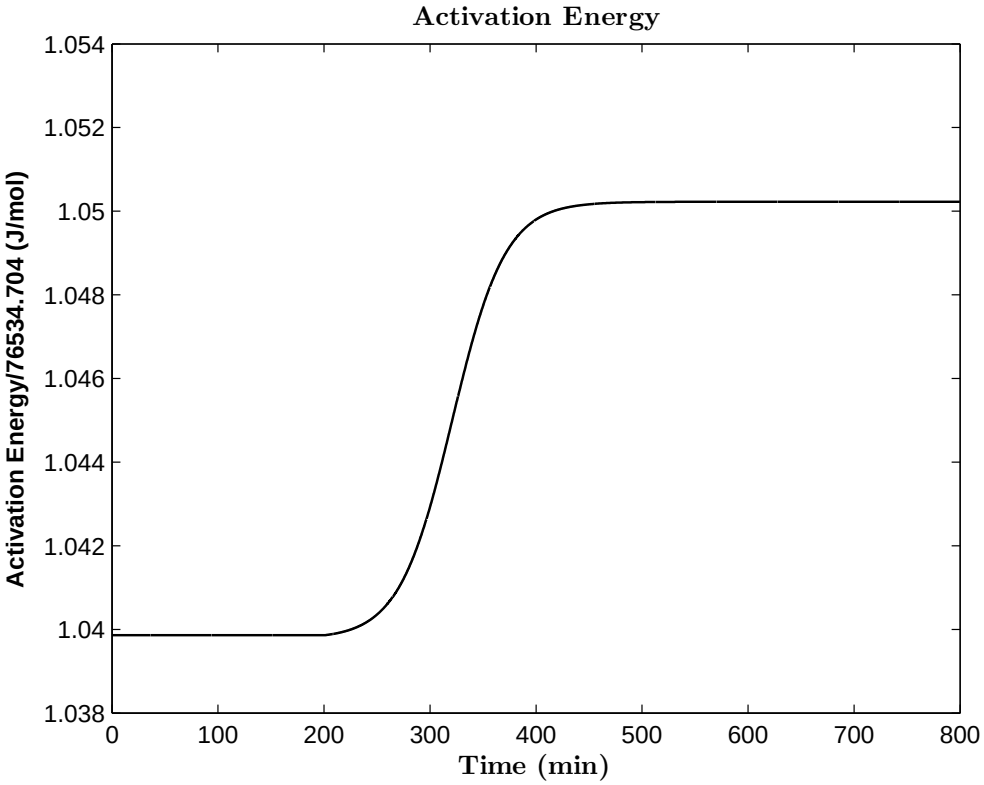

Fig. 17. Activation energy change with time.

4. Summary and discussions

A new data-driven fault diagnosis scheme for dynamic systems was presented. This approach revolves around two main components: the design of a subspace model from normal process data, and the choice of an appropriate fault isolation and identification filter for reconstructing the location and nature of the fault. The subspace model was developed using the N4SID algorithm after scaling the input-output variables to zero mean and unit variance. The fault isolation and identification filter was designed based upon the computed subspace model.

Since it is not possible to simultaneously perform model identification and fault reconstruction, these two tasks were implemented at different time scales. The model was identified at periodic intervals where no faults were assumed to be present or were assumed to be known and constant, while the fault diagnosis scheme was invoked during rest of the time. The performance of the proposed fault diagnosis scheme was evaluated using a numerical example of LTI system, non-isothermal CSTR with time-varying parameter, a debutanizer operation, and an industrial melter process. Through simulations it is concluded that the faults were reconstructed correctly in all the examples.

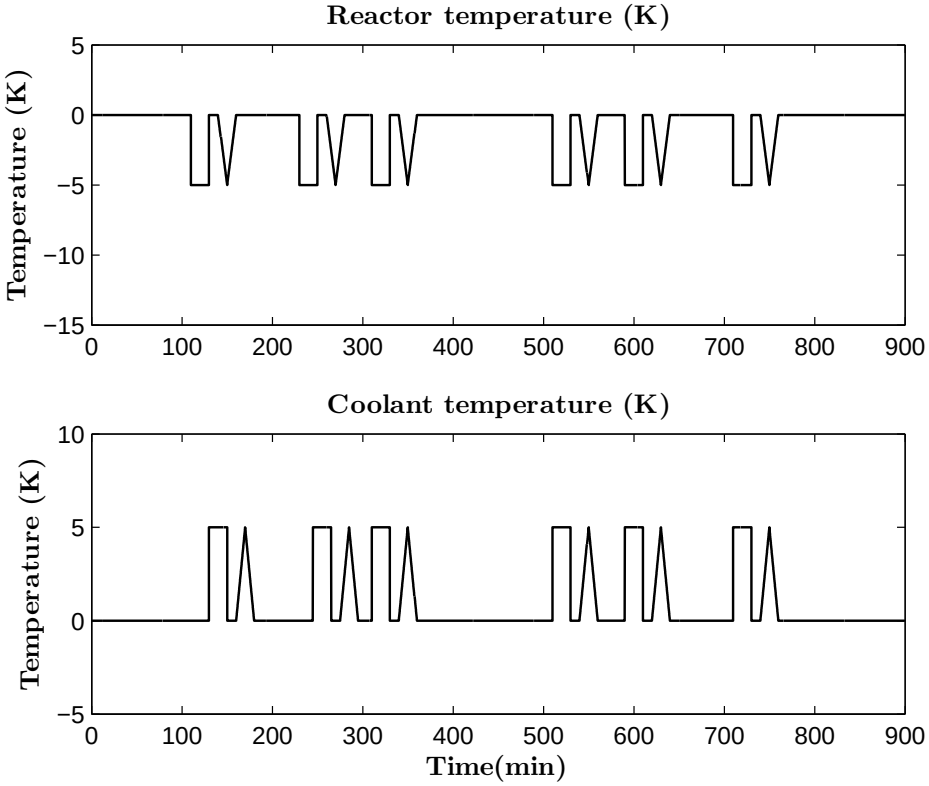

Fig. 18. Reactor and coolant temperature fault signals.

Table IV. Considered process variables for the industrial distillation process

VARIABLE NO.		**DESCRIPTION**
Predictor Variables	1	Feed flow rate
	2	Temperature of fresh feed
	3	Reflux flow
	4	Reboiler steam flow
Response Variables	5	Tray 14 temperature
	6	Column overhead pressure
	7	Tray 2 temperature
	8	Reflux vessel level
	9	Butane product flow (top draw)
	10	Percentage of C3 in C4
	11	Percentage of C5 in C4
	12	Tray 31 temperature
	13	Reboiler vessel level
	14	Bottom draw
	15	Percentage of C4 in C5
	16	Reboiler temperature

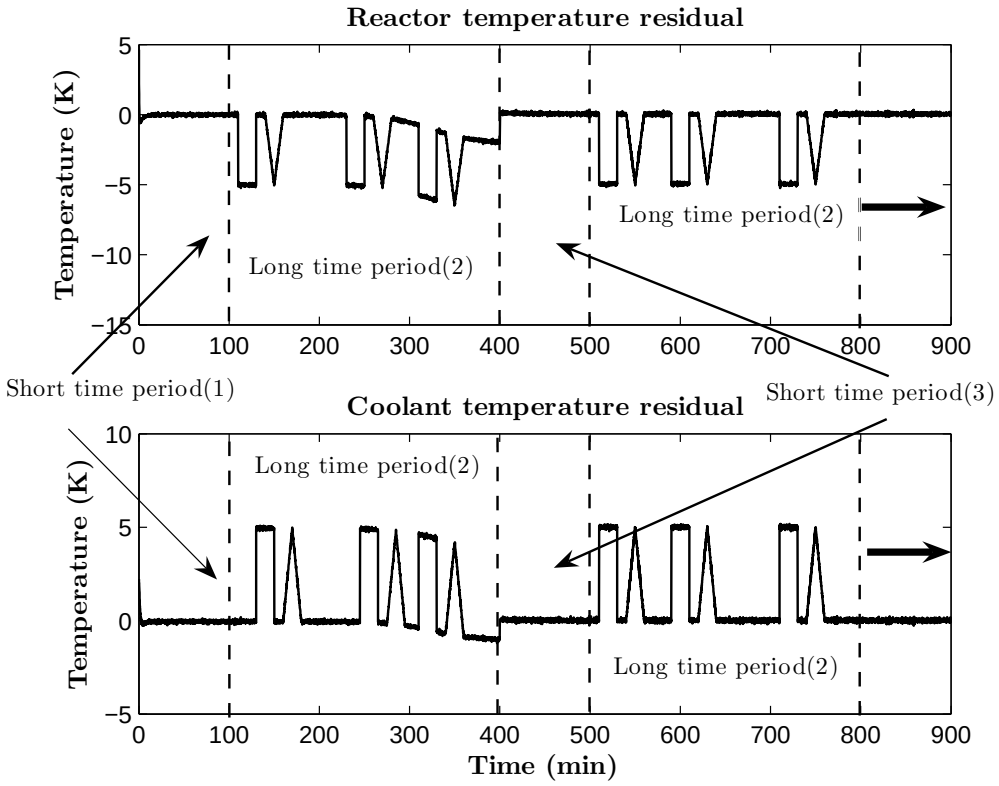

Fig. 19. Reactor and coolant temperature residual signals through proposed approach (with time-varying parametric uncertainty).

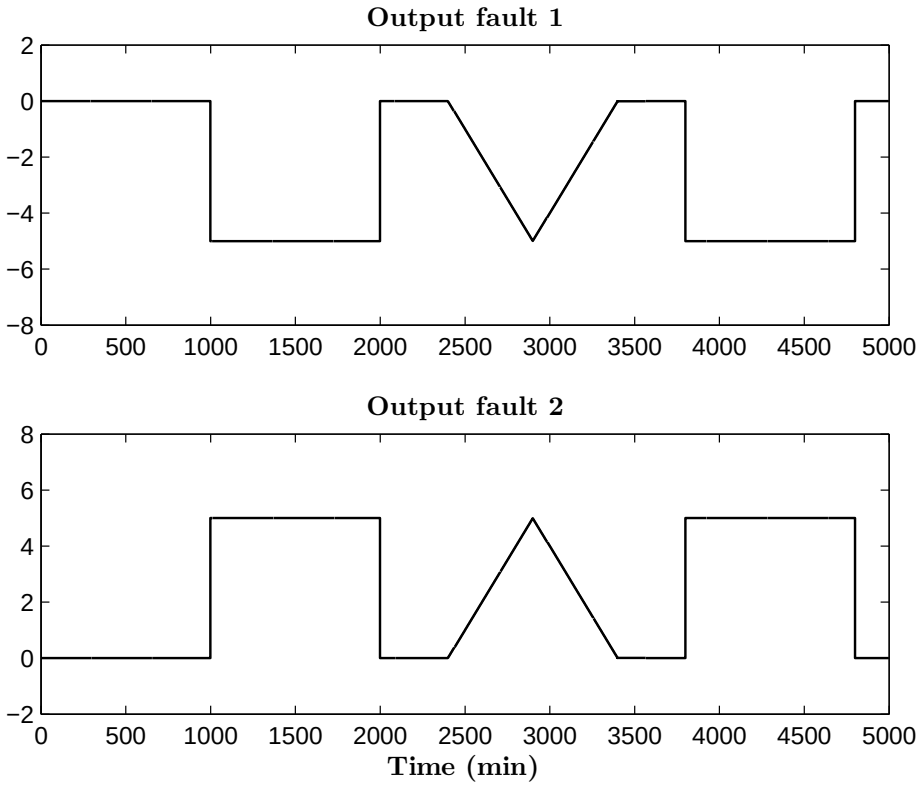

Fig. 20. True output fault signals.

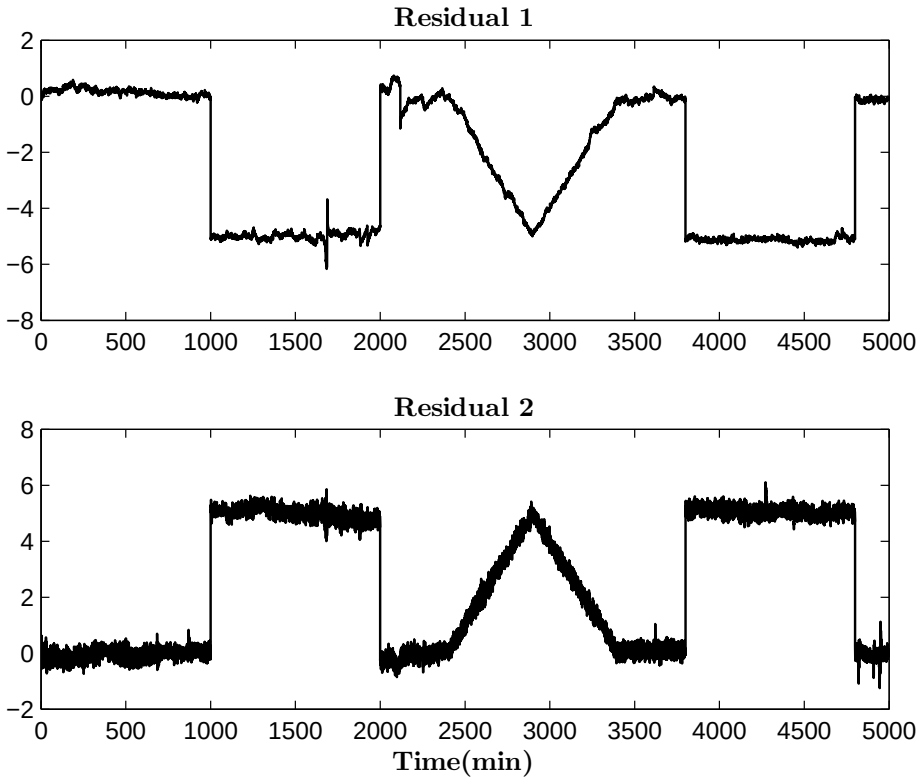

Fig. 21. Output residuals through the proposed scheme.

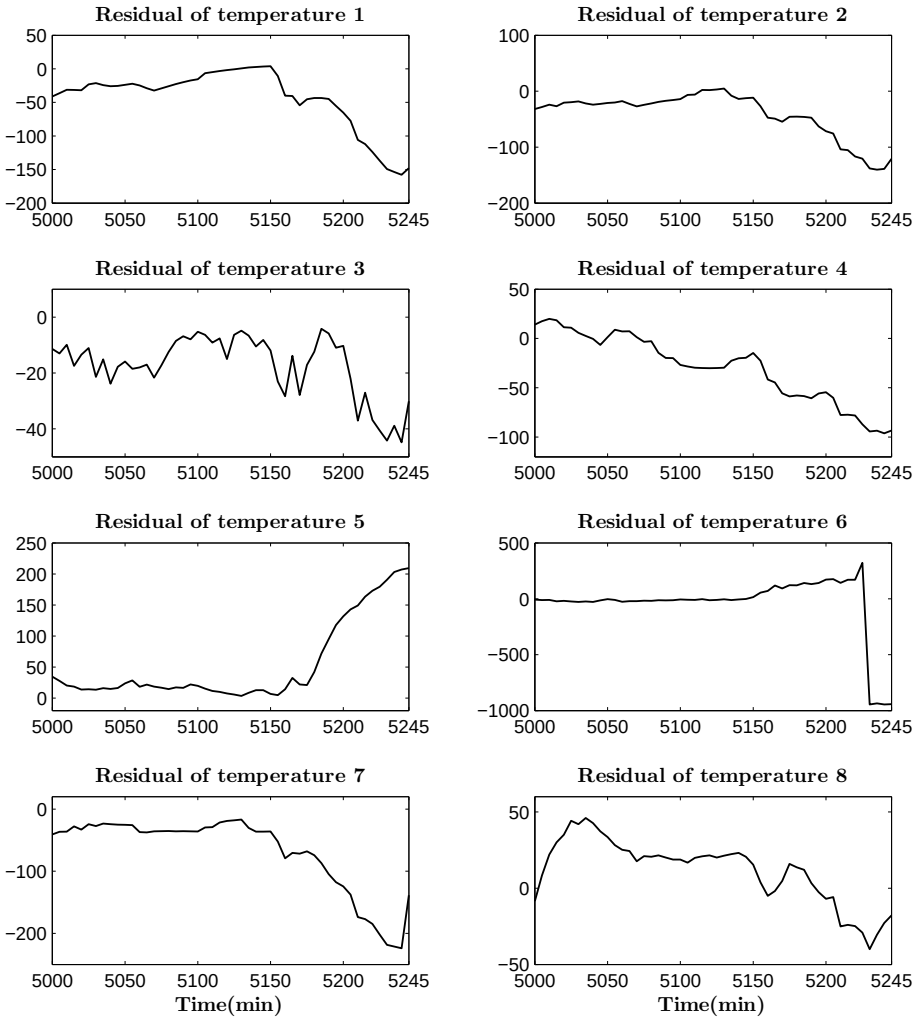

Fig. 22. Output residuals for melter data through the proposed scheme.

CHAPTER V

SUMMARY AND CONCLUSIONS

This dissertation presented a novel model-based fault diagnosis scheme for nonlinear dynamic systems with parametric uncertainties. The entire research focuses on two problems, one in which the structure of the process model is known but is poorly characterized by the parameters and second in which no knowledge of the process model is known but only input-output data is available.

For the first problem the fault diagnosis scheme is centered around two components: the design of the nonlinear observer, which includes uncertain parameters are augmented states, and the choice of an appropriate fault isolation and identification filter for reconstructing the nature and the location of the fault. The observer design was performed based upon Kharitonov's theorem but takes into account the effect that changes in the parameters have on the steady of the system. This resulted in a nonlinear, augmented observer, which has the property that it is locally stable for a parametric uncertainty within a specified range. The fault isolation and identification filter was designed based upon the linearization of the nonlinear model at each time step. Repeatedly computing the linearization of the model does not pose a problem in practice since it is computationally inexpensive.

As it is not possible to perform parameter estimation and fault detection, these two tasks were implemented in different time scales. The parameters were estimated in periodic intervals where the fault was assumed to be zero or known and constant, whereas the fault detection scheme was invoked at all times with the exception of short periods used for the parameter estimation. The performance of the proposed fault diagnosis method was evaluated using a numerical example of an exothermic CSTR and by performing Monte-Carlo simulations on a bounded set of parametric

uncertainties for a series of faults in both of the available measurements. The faults were reconstructed correctly even in the presence of severe uncertainties in the model parameters and measurement noise.

For the second problem the design approach revolves around two main components: the design of a subspace model from normal process data, and the choice of an appropriate fault isolation and identification filter for reconstructing the location and nature of the fault. The subspace model was developed using the N4SID algorithm after scaling the input-output variables to zero mean and unit variance. The fault isolation and identification filter was designed based upon the computed subspace model.

Since it is not possible to simultaneously perform model identification and fault reconstruction, these two tasks were implemented at different time scales. The model was identified at periodic intervals where no faults were assumed to be present or were assumed to be known and constant, while the fault diagnosis scheme was invoked during rest of the time. The performance of the proposed fault diagnosis scheme was evaluated using a numerical example of LTI system, nonisothermal CSTR with time-varying parameter, a debutanizer operation, and an industrial melter process. Through simulations it is concluded that the faults were reconstructed correctly in all the examples.

REFERENCES

[1] I. Nimmo, "Adequately address abnormal operations," *Chemical Engineering Progress*, vol. 91, pp. 36–45, 1995.

[2] H. Vedam and V. Venkatasubramanian, "PCA-SDG based process monitoring and fault detection," *Control Engineering Practice*, vol. 7, pp. 903–917, 1999.

[3] J. Chen and R. Patton, *Robust Model Based Fault Diagnosis For Dynamic Systems*. London: Kluwer Academic Publishers, 1999.

[4] A. Garcia and P. M. Frank, "Deterministic nonlinear observer-based approaches to fault diagnosis: A survey," *Control Engineering Practice*, vol. 5, p. 663, May 1997.

[5] P. M. Frank and X. Ding, "Survey of robust residual generation and evaluation methods in observer-based fault detection systems," *Journal of Process Control*, vol. 7, p. 403, 1997.

[6] T. Soderstrom, D. M. Himmelblau, and T. F. Edgar, "The Extension of Mixed integer Optimization approach to simultaneous data reconciliation and bias identification." in *Foundations of Computer-Aided Process Operations*, Boca Raton, FL, 2003.

[7] S. P. Bhattacharyya, H. Chapellat, and L. H. Keel, *Robust Control: The Parametric Approach*. Upper Saddle River, NJ: Prentice Hall PTR, 1995.

[8] N. Vishwanadham, J. Taylor, and E. Luce, "A frequency-domain approach to failure detection and isolation with application to ge-21 turbine engine control systems," *Control - Theory and Advanced Technology*, vol. 3(1), p. 45, 1987.

[9] X. Ding and P. M. Frank, "Fault detection via factorization approach," *Systems & Control Letters*, vol. 14, p. 431, 1990.

[10] P. V. Van Overschee and B. D. de Moor, "System identification for the identification of combined deterministic-stochastic systems," *Automatica*, vol. 30, no. 1, pp. 75–93, 1994.

[11] R. J. Patton, P. M. Frank, and R. N. Clark, Eds., *Fault Diagnosis in Dynamic Systems, Theory and Application*, ser. Control Engineering Series. New York: Prentice Hall, 1989.

[12] E. L. Russell, L. H. Chiang, and R. Braatz, *Data-Driven Techniques for Fault Detection and Diagnosis in Chemical Processes*. New York: Springer-Verlag, 2000.

[13] S. Simani, F. Cesare, and R. J. Patton, *Model-based Fault Diagnosis in Dynamic Systems Using Identification Techniques*. New York, London: Springer, 2003.

[14] R. Isermann and P. Ballé, "Trends in the application of model-based fault detection and diagnosis of technical process," *Control Engineering Practice*, vol. 5(5), p. 709, 1997.

[15] D. van Schrick, "Remarks on terminology in the field of supervision, fault detection and diagnosis," in *Proc. IFAC Sympo. on Fault Detection, Supervision and Safety for Technical Processes: SAFEPROCESS'97*, Pergamon, Univ. of Hull, UK, 1997, p. 959.

[16] ——, "Investigations of reliability for instrument fault detection state-estimator schemes," *European J. of Diagnosis and Safety in Automation (Revue européene Diagnostic et sûreté de fonctionnement)*, vol. 1(1), p. 63, 1991.

[17] P. M. Frank, "Fault diagnosis in dynamic system using analytical and knowledge based redundancy - a survey and some new results," *Automatica*, vol. 26, no. 3, p. 459, 1990.

[18] ——, "Enhancement of robustness in observer-based fault detection," in *Preprints of IFAC/IMACS Sympo. SAFEPROCESS'91*, Baden-Baden, 1991, pp. 275–287.

[19] R. J. Patton, "A review of parity space approaches to fault diagnosis," in *Preprints of IFAC/IMACS Sympo. SAFEPROCESS'91*, Baden-Baden, 1991, p. 239.

[20] J. Gertler, "Analytical redundancy methods in failure detection and isolation," in *Preprints of IFAC/IMACS Sympo. SAFEPROCESS'91*, Baden-Baden, 1991, p. 9.

[21] P. M. Frank, "Advances in observer-based fault diagnosis," in *Proc. of Int. Conf. on Fault Diagnosis: TOOLDIAG'93*, Toulouse, 1993, pp. 817–836.

[22] R. V. Beard, "Failure accomodation in linear systems through self reorganization," Ph.D. dissertation, Massachussets Institute of Technology, Cambridge, 1971.

[23] R. N. Clark, D. C. Fosth, and V. M. Walton, "Detecting instrument malfunctions in control systems," *IEEE Trans. Aero. & Electron. Syst.*, vol. AES-11, pp. 465–473, 1975.

[24] P. M. Frank and J. Wunnenberg, "Sensor fault detection via robust observers," in *System Fault Diagnosis, Reliability and Related Knowledge-based Approaches*,

S. G. Tzafestas, M. G. Singh, and G. Schmidt, Eds. Dordrecht: D.Reidel Press, 1987, vol. 1, p. 147.

[25] R. K. Mehra and J. Peschon, "An innovations approach to fault detection and dynamics in dynamic systems," *Automatica*, vol. 7, pp. 637–640, 1971.

[26] A. S. Willsky, "A survey design methods for failure detection in dynamic systems," *Automatica*, vol. 12(6), pp. 601–611, 1976.

[27] M. Basseville, "Detecting changes in signals and systems - a survey," *Automatica*, vol. 3, p. 309, 1988.

[28] S. G. Tzafestas and K. Watanabe, "Modern approaches to system/sensor fault detection and diagnosis," *Journal A*, vol. 31(4), pp. 42–57, 1990.

[29] R. Da and C. F. Lin, "Sensitivity analysis algorithm for the state chi-square test," *J. of Guidance, Contr. & Dynamics*, vol. 19(1), pp. 219–222, 1996.

[30] D. G. Luenberger, "Observing the state of a linear system," *IEEE Trans. Milit. Electron.*, vol. 8, p. 74, 1963.

[31] J. O'Reilly, *Observers for Linear Systems*. London, New York: Academic Press, 1983.

[32] K. Watanabe and D. M. Himmelblau, "Instrument fault detection in systems with uncertainties," *Int. J. Sys. Sci.*, vol. 13(2), pp. 137–158, 1982.

[33] Y. Xiong and M. Saif, "Robust fault detection and isolation via a diagnostic observer," *International Journal of Robust Nonlinear Control*, vol. 10, p. 1175, 2000.

[34] Y. Guan and M. Saif, "A novel approach to design of unknown input observers," *IEEE Trans. Automat. Contr.*, vol. AC-36(5), pp. 632–635, 1993.

[35] S. P. Bhattacharyya, "Observer design for linear systems with unknown inputs," *IEEE Trans. Automat. Contr.*, vol. AC-23, pp. 483–484, 1978.

[36] R. J. Patton and S. W. Willcox, "Fault detection in dynamic system using a robust output zeroing design method," in *System Fault Diagnosis, Reliability & Related Knowledge-based Approaches.* Dordrecht: D. Reidel Press, 1987, vol. 1, pp. 177–192.

[37] R. J. Patton, "Robust fault detection using eigenstructure assignment," in *Proc. 12th IMACS World Congess on Scientific Computation*, vol. 2, Paris, 1988, pp. 431–434.

[38] M. A. Massoumia, "A geometric approach to the synthesis of failure detection filters," *IEEE Transactions on Automatic Control*, vol. 31(9), p. 839, 1986.

[39] R. J. Patton and J. Chen, "Optimal design of unknown input distribution matrix in the design of robust observers for fault diagnosis," *Automatica*, vol. 29(4), pp. 837–841, 1991.

[40] R. J. Patton, J. Chen, and H. Y. Zhang, "Modeling methods for improving robustness in fault diagnosis of jet engine systems," in *Proc. of the 31st IEEE Conf. on Control & Decision*, Tuscon, Arizona, 1992, pp. 2330–2335.

[41] J. Gertler and M. K. Kunwer, "Optimal residual decoupling for robust fault diagnosis," *Int. J. Contr.*, vol. 61(2), pp. 395–421, 1991.

[42] J. Gertler, "Modeling errors as unknown inputs," in *Preprints of IFAC Sympo.*

on Fault Detection, Supervision and Safety for Technical Processes: SAFEPROCESS'94, vol. 1, Espoo, Finland, 1994, pp. 266–271.

[43] X. Ding and P. M. Frank, "Fault detection via optimally robust detection filters," in *Proc. of 28th IEEE Conf. on Decision & Control*, Tampa, FL, 1989, pp. 1767–1772.

[44] ——, "Frequency domain approach and threshold selector for robust model-based fault detection and isolation," in *Preprints of IFAC/IMACS Symp. SAFEPROCESS'91*, vol. 13, Baden-Baden, 1991, p. 307.

[45] P. M. Frank and X. Ding, "Frequency domain approach to optimally robust residual generation and evaluation for model-based fault diagnosis," *Automatica*, vol. 30, no. 4, p. 789, 1994.

[46] B. A. Francis, *A Course in H_∞ Control Theory.* Berlin, New York: Springer Verlag, 1987.

[47] J. Chen, R. J. Patton, and G. P. Liu, "Design of optimal residuals for detecting sensor faults using multi-object optimization and genetic algorithms," in *Proc. of 1994 AIAA Guidance, Navigation, and Control Conf.*, Scottsdale, AZ, 1994, pp. 349–357.

[48] ——, "Detecting incipient sensor faults in flight control systems," in *Proc. of The Third IEEE Conf. on Control Applcations*, Glasgow, Scotland, 1994, pp. 871–876.

[49] ——, "Optimal residual design for fault diagnosis using multiobjective optimization and genetic algorithms," *Int. J. Sys. Sci.*, vol. 27(6), pp. 567–576, 1996.

[50] D. Hengy and P. M. Frank, "Component failure detection via nonlinear state observers," in *Proc. of IFAC Workshop on Fault Detection and Safety in Chemical Plants*, Kyoto, Japan, 1986, pp. 153–157.

[51] P. M. Frank, "Fault diagnosis in dynamic system via state estimation - a survey," in *System Fault Diagnosis, Reliability and Related Knowledge-based Approaches*, S. G. Tzafestas, M. G. Singh, and G. Schmidt, Eds. Dordrecht: D.Reidel Press, 1987, vol. 1, pp. 35–98.

[52] K. Adjallah, D. Maquin, and J. Ragot, "Nonlinear observer-based fault detection," in *Proc. of The Third IEEE Conf. on Control Applications*, Glasgow, Scotland, 1994, pp. 1115–1120.

[53] M. Zeitz, "The extended luenberger observer for nonlinear systems," *Systems & Control Letters*, vol. 9, p. 149, 1987.

[54] J. Wünnenberg, "Observer-based fault detection in dynamic systems," Ph.D. dissertation, Univ. of Duisburg, Germany, 1990.

[55] R. Seliger and P. M. Frank, "Robust component fault detection and isolation in nonlinear dynamical system using nonlinear unknown input observers." in *Preprints of IFAC/IMACS Symp. SAFEPROCESS'91*, vol. 1, Baden-Baden, 1991, p. 313.

[56] X. Ding and P. M. Frank, "On-line fault detection in uncertain systems using adaptive observers," *European Journal of Diagnosis and Safety in Automation*, vol. 3, p. 9, 1993.

[57] X. Ding, P. M. Frank, and L. Guo, "An adaptive observer based fault detection system for uncertain nonlinear systems," in *Preprints of The 12th IFAC World*

Congress, vol. 7, Australia, 1993, pp. 307–310.

[58] G. Bastin and M. R. Gevers, "Stable adaptive observers for nonlinear time-varying systems," *IEEE Transactions on Automatic Control*, vol. 7, p. 650, 1988.

[59] V. L. Kharitonov, "Asymptotic stability of an equilibrium position of family of systems of linear differential equations," *Differential Uraven*, vol. 14, p. 2086, 1978.

[60] C. Kravaris, V. Sotiropoulos, C. Georgiou, N. Kazantzis, M. Xiao, and A. J. Krener, "Nonlinear design for state and disturbation estimation," in *Proceedings of the 2004th American Control Conference.*, Boston, MA, 2004, p. 2931.

[61] R. Hermann and A. J. Krener, "Nonlinear controllability and observability," *IEEE Transactions on Automatic Control*, vol. AC-22, p. 728, 1977.

[62] F. W. Fairman, *Linear Control Theory*. New York: John Wiley and Sons, 1998.

[63] H. Chapellat and S. P. Bhattacharyya, "Robust stability and stabilization of interval plants," in *Robustness in Identification and control*, M. Milanese, R. Tempo, and A. Vicino, Eds. New York: Plenum Press, 1989, p. 207.

[64] M. J. Kurtz and M. A. Henson, "State and disturbance estimation for nonlinear system affine in the unmeasurable variables," *Computers & Chemical Engineering*, vol. 22, no. 10, p. 1441, 1998.

[65] S. Rajaraman, J. Hahn, and M. S. Mannan, "A methodology for fault detection, isolation and identification for nonlinear processes with parametric uncertainties," *Industrial & Engineering Chemistry Research*, vol. 43, no. 21, pp. 6774–6786, 2004.

[66] M. Moonen, B. D. de Moor, L. Vandenberghe, and J. Vandewalle, "On and off-line identification of linear state-space models," *International Journal of Control*, vol. 49, pp. 219–232, 1989.

[67] P. V. Van Overschee and B. D. de Moor, *Subspace Identification For Linear Systems - Theory, Implementation and Applications*. Boston: Kluwer Academic Publishers, 1996.

[68] M. Viberg, "Subspace-methods for the identification of linear time-invariant systems," *Automatica*, vol. 31, pp. 1835–1851, 1995.

[69] S. J. Qin and W. Li, "Detection, identification, and reconstruction of faulty sensors with maximized sensitivity," *AIChE Journal*, vol. 45, p. 1963, 1999.

[70] ——, "Detection, identification of faulty sensors in dynamic processes," *AIChE Journal*, vol. 47, pp. 1581–1593, 2001.

[71] W. Li, H. Raghavan, and S. Shah, "Subspace identification of continuous time models for process fault detection and isolation," *Journal of Process Control*, vol. 13, pp. 407–421, 2003.

[72] R. J. Treasure, U. Kruger, and J. E. Cooper, "Dynamic multivariate statistical process control using subspace identification," *Journal of Process Control*, vol. 14, pp. 279–292, 2004.

VITA

Srinivasan Rajaraman was born on January 3, 1979, in Kumbakonam, India. He completed his Bachelor of Science degree in chemical engineering from the Indian Institute of Technology Roorkee, Roorkee, India in May 2000. He joined the chemical engineering department at Texas A & M University and received his Ph.D. degree in May 2006. His permanent address is S.D-130 Tower Apartments, Pitampura, Delhi-110034, India.

The typist for this thesis was Srinivasan Rajaraman.