NONLINEAR CONTINUOUS FEEDBACK CONTROLLERS

A Thesis

by

SAI GANESH SITHARAMAN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2004

Major Subject: Computer Science

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Approved as to style and content by:

Dmitri Loguinov (Chair of Committee)	Jyh-Charn (Steve) Liu (Member)
A. L. Narasimha Reddy (Member)	Jennifer Welch (Member)
Valerie Taylor (Head of Department)	

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ABSTRACT

Nonlinear Continuous Feedback Controllers.

(May 2004)

Sai Ganesh Sitharaman, B.E., Regional Engineering College, Trichy, India

Chair of Advisory Committee: Dr. Dmitri Loguinov

Packet-switched communication networks such as today's Internet are built with several interconnected core and distribution packet forwarding routers and several sender and sink transport agents. In order to maintain stability and avoid congestion collapse in the network, the sources control their rate behavior and voluntarily adjust their sending rates to accommodate other sources in the network. In this thesis, we study one class of sender rate control that is modeled using continuous first-order differential equation of the sending rates. In order to adjust the rates appropriately, the network sends continuous packet-loss feedback to the sources. We study a form of closed-loop feedback congestion controllers whose rate adjustments exhibit a nonlinear form.

There are three dimensions to our work in this thesis. First, we study the network optimization problem in which sources choose utilities to maximize their underlying throughput. Each sender maximizes its utility proportional to the throughput achieved. In our model, sources choose a utility function to define their level of satisfaction of the underlying resource usages. The objective of this direction is to establish the properties of source utility functions using inequality constrained bounded sets and study the functional forms of utilities against a chosen rate differential equation.

Second, stability of the network and tolerance to perturbation are two essential factors that keep communication networks operational around the equilibrium point. Our

objective in this part of the thesis is to analytically understand the existence of local asymptotic stability of delayed-feedback systems under homogeneous network delays.

Third, we propose a novel *tangential* controller for a generic maximization function and study its properties using nonlinear optimization techniques. We develop the necessary theoretical background and the properties of our controller to prove that it is a better rate adaptation algorithm for logarithmic utilities compared to the well-studied proportional controllers. We establish the asymptotic local stability of our controller with upper bounds on the increase / decrease gain parameters.

DEDICATION

To my mother Sakunthala and brother Sathya Sai.

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I would like to express my sincere thanks to my advisor, Dr. Dmitri Loguinov, for his constant encouragement through several interesting technical brainstorming sessions. His tireless discussions taught me several ways to approach problems critically. He continuously motivated me to focus on problems narrowly, taught me the need to analyze the solved problems more closely and to present the research in an organized writing style. Without his continuous push to perfection for top-quality research, such an interesting work may not have been possible.

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CHAPTER I

INTRODUCTION

Best-effort Internet prevailing today has evolved from a simple experimentation ARPANET test bed consisting of a fixed set of routers and end-hosts to a complex, fast growing heterogeneous network. Unlike the static circuit-switched telephony network, the Internet has become a highly dynamic network with ever increasing number of hosts and networks connected to the core [1]. In order to maintain scalability in this constantly changing network, early designers devised an open-architecture that would push most of the routing and transport intelligences to the end hosts. This open-architecture that is widely under operation in today's network is the familiar Transmission Control Protocol (TCP) / Internet Protocol (IP) protocol suite [37].

The backbone of the open-architecture prevailing in today's Internet is built with several interconnected core, distribution and edge routers that simply forward the data from the set of incoming interfaces to the corresponding outgoing physical interface. Routers that forward packets do not know the aggregate packet arrival rates in advance owing to the connectionless nature of the underlying Internet Protocol (IP). Thus the traffic patterns at these routers become unpredictable and often bursty. The Internet traffic logs at the service providers' premises show that the aggregations of several traffic streams do not strictly follow the Poisson distribution that was widely used to model the teletraffic networks [2], [3] and [4]. Instead, the IP traffic exhibits a highly bursty nature with spikes in the arrival rates at all time scales (an effect termed as "self-similarity"). Thus, if the incoming rate exceeds the router queue processing overhead, the newly arriving packets are dropped at the bottleneck droptail router owing to the buffer overflow and hence packet losses occur. Packet losses also occur if there is a significant surge in the traffic burstiness at these intermediate routers.

This thesis follows the style and format of *IEEE/ACM Transactions on Networking.*

Assuming link losses are negligible, packet losses at these intermediate routers can be controlled by enforcing flow control schemes that control the sender's rate (or window sizes). Earlier studies on connectionless flow control schemes were based on point-to-point receiver-advertised flow control, but the obvious problem in the protocol is that the resource overhead at the intermediate routers were never considered as a part of the protocol, resulting in a network-wide packet losses. These flow control schemes dictated the point-to-point behavior between the individual sender and the receivers and thus are very selfish in nature. The Internet however, requires global congestion control and avoidance mechanisms collectively among all the senders and the routers in order to efficiently utilize the shared resources along the path from the source to the receiver.

In 1988, Jacobson [5], Chiu and Jain [38] introduced a congestion avoidance scheme in which the network signals of an incipient congestion to the end hosts. Their congestion avoidance scheme is used to adjust the sender's rate dynamically based on the load on the network (implicit packet loss), thus preventing further packet loss in a congested network. Congestion control schemes similar to those introduced by [5] and [38] studies the interaction of sender's rate control with the network feedback signals to maintain stability in the network.

1 Congestion Control Techniques

Network congestion control problem was not studied seriously until Jacobson [5], and Chiu and Jain [38] introduced their congestion avoidance scheme to control the source rate behavior to adapt dynamically to the network conditions. In the same year, Chiu and Jain [38] and Ramakrishnan and Jain [40] viewed congestion control as a resource allocation problem that controls the effective and fair usage of the underlying link bandwidth. In their model, an effective utilization is achieved when resources are utilized to their maximum extent with minimum losses at the underlying link and a fair usage is achieved when the flows of similar nature share the bandwidth equally.

The congestion avoidance mechanism introduced by Jacobson [5] in 1988 became popularly known as TCP-Tahoe and was widely deployed in the Internet since then. The TCP-Tahoe scheme implemented several new algorithms including the TCP slow-start, an automatic self-clocking and a dynamic window-size adjustment scheme. In 1990, TCP Reno proposed modifications to the TCP-Tahoe and performed fast recovery and fast retransmits to improve the sender's throughput. Both the protocol variants used the network-wide packet losses as an implicit signal for congestion avoidance. In 1992, TCP Selective Acknowledgement (SACK) [46] implemented an acknowledgement option to the existing TCP Tahoe that was proved to improve the sender's throughput by maintaining the outstanding packets along the path. In 1994, TCP Vegas was introduced by Brakmo *et al.* [47] as an improvement over TCP Reno. TCP Vegas anticipated the network losses better than Reno and improved the sender's throughput by transmitting the window sizes between the actual and expected windows. The current Internet widely deploys the TCP SACK protocol and is the most popular transport implemented so far.

One of the common features of such congestion control mechanisms is that the source rate adjustment is simply a function of closed-loop implicit or explicit network feedback. Earlier implementations used a binary single-bit feedback scheme [38], [39], [40] that set a congestion indication bit in the packet header to indicate the network load for senders to increase or decrease their rates according to the bit. This single-bit feedback indicated if the packet losses were to be anticipated in the network if all the sources were to continue using the same rate. Such discrete models adjust the sender's rate following the difference equation given by:

$$x(t+1) = x(t) + (1-B)R_i(x(t)) + BR_d(x(t)) . \qquad (1)$$

In the equation (1), the value of the single bit B is used by the sender to adjust the source rate according to the increase function $R_i(x)$ and the decrease function $R_d(x)$, both being functions of current rate $x(t)$.

Additive Increase Multiplicative Decrease (AIMD) is one family of source control scheme that uses an additive increase constant α and a multiplicative decrease constant β according to the equation given by:

AIMD:

$$x(t+1) = \begin{cases} x(t) + \alpha, & \text{additive increase} \\ \beta x(t), & \text{multiplicative decrease} \end{cases} . \quad (2)$$

As long as there are no losses or congestion indication feedback from the receiver or router, the sources increase their rate additively, but once losses occur, the rate is reduced multiplicatively.

The increase and decrease functions $R_i(x)$ and $R_d(x)$ in the generic rate equation (1) dictate several other source control variants including Additive Increase Additive Decrease (AIAD), Multiplicative Increase Additive Decrease (MIAD), and Multiplicative Increase Multiplicative Decrease (MIMD). However, it is shown in literatures [38], [39] that only AIMD with appropriate increase / decrease constants α and β converge towards efficiency and fairness line. However, AIMD is also proved to oscillate around the efficiency and fairness operating points for any given positive constants α and β.

In contrast to the discrete difference equation in (1), continuous controllers model the source rate behavior as a function of differential equations. These controllers model the source rate behavior as a first-order differential equation with the closed-loop continuous feedback loss function explicitly taking into account. Continuous models are thus useful in examining the functional form of the right-hand-side of the first-order rate equation. For instance, the control theoretic frequency response methods can be directly applied to prove the stability of closed loop response system of these first-order systems. Solutions to these set of first-order equations can reveal their properties including existence of stationary points and convergence to fairness. Interestingly, it is possible to model discrete difference equations using the continuous form with discontinuous feedback information.

An ordinary first-order nonlinear differential equation is given by an expression such as (3) below. The equation shows that the source rate changes occurs as a function of the current rate as well as the loss feedback function $p(t)$. Thus, the function f dictates a family of first-order linear or nonlinear differential equation dictated by:

$$\frac{dx(t)}{dt} = f\left(x(t), p(t)\right). \tag{3}$$

In our proposal, we revisit one such form of continuous feedback controllers known as proportional fairness scheme introduced by Kelly [6] that has known useful properties including smoother non-oscillatory convergence (theoretic convergence), fairness towards similar flows and existence of globally stable operating point. Proportional fairness is practically useful in applications like video streaming that require faster convergence than currently existing AIMD models, and be fair to similar flows. If all flows behave (that is, sources are *elastic*), such non-oscillatory rate behavior naturally reduces the congestion in the network and hence prevents packet losses due to congestion. Our underlying motivation is to analytically understand these properties of the variants of the proportional fairness scheme and their applicability to the current Internet.

The general form of Kelly's source rate differential equation studied in [6] can be written as shown in the equation (4) below. For any user k, the rate differential equation has a constant additive component $\kappa^* w_k$ and a negative multiplicative factor $\kappa^* \sum_{j\in k} \mu_j$, in which κ is a constant positive gain factor. The summation represents the link shadow prices or the feedback losses across all links along the path. Intuitively, the link shadow prices are themselves a function of the aggregate rate of all the source agents and hence the rate equation takes a nonlinear form:

$$\frac{dx_k(t)}{dt} = \kappa\left(w_k - x_k(t) \times \sum_{j\varepsilon k} \mu_j(t)\right). \tag{4}$$

Recently, other forms of nonlinear continuous controllers were investigated including Minimum Potential Delay (MPD) [42], [43], and [7]. The objective of MPD is to minimize the time delay for a data transfer and the delay is considered a reciprocal of the allocated rate $x_k(t)$. The functional form of rate differential equation is similar to that of equation (4) but with user rate $x_k(t)$ squared in the second term of the equation. It was proved that this form of MPD controller also achieves fairness among all behaving flows and makes optimum usage of the link bandwidth.

2 Forms of Congestion Feedback

Sources adjust their sending rates based on the congestion feedback signals and these signals may be received in an implicit or in an explicit manner. Typically, such feedback is provided either by the intermediate routers in the network or from the unique end receivers. Schemes that implement network-based indications are commonly referred to as Active Queue Management (AQM) techniques. AQM techniques enforce queuing constraints to the TCP conversations and provide active congestion feedback to the end users. Today's routers implement a wide variety of AQM techniques that monitor the queue sizes and mark / drop packets of the misusing flows. On the other hand, schemes that use end host receiver-based feedback are termed as end-to-end feedback control.

Two of the most widely used AQM techniques are Random Early Detection (RED) and Explicit Congestion Notification (ECN) [44]. RED was introduced in 1993 as a scheme to allow the gateway to detect incipient congestion by monitoring the average queue sizes. As the average queue size exceeds a threshold limit, the incoming packets are randomly marked or dropped and this naturally penalizes flows with higher arriving rates. Sources are thus expected to infer the packet drops as network losses (implicitly) and adjust their rates accordingly. ECN marking is a variant of RED that sets a congestion indication bit in the packet header while still forwarding the packets to the end hosts. ECN-capable transports are expected to explicitly make use of this field in adjusting their window sizes.

Orthogonal to the AQM techniques is the end-to-end feedback from the receivers that acknowledge the received packets. Receiver-based feedback can be ACK-based or NACK-based. Both of these schemes suffer from accurate estimation of available bandwidth of the bottleneck link. This is because the loss ratio feedback by one receiver is alone not sufficient in estimating the bottleneck congestion, as congestion occurs due to the aggregation of all the other flows as well. If the number of flows in a bottleneck router is known, the feedback from one receiver can be scaled up to multiple flows.

Having said this, AQM methods such as RED (and variants of RED) and ECN only provide capabilities to contain and penalize flows that misbehave and use disproportionate share of the link capacity. These techniques by themselves may not be sufficient in implementing a true end-to-end congestion control.

3 Optimal Flow Control

Providing performance incentives to end-to-end congestion control mechanisms can be one of the best ways to encourage deployment of behaving source agents in the

Internet [45]. Recent studies have opened several new avenues for optimizing flow control using a variety of game-theoretic approach [6], [7], and [20]. These methods have spurred a vast interest in deploying such end-to-end congestion schemes in the Internet. There has been a significant shift in the paradigm among the researchers in analyzing congestion control techniques from traditional closed-loop flow control to game-theoretic optimization methods since the latter demonstrate better performance in simulations and implementations. Many of these game-theoretic studies use analytical models to study users' behavior given their utility functions and underlying network costs. One of the recent experiments based on this approach is Caltech's FAST TCP [49] that is expected to provide throughput several times higher than TCP in high bandwidth-delay product networks. FAST TCP uses duality optimization theory to adjust the sender's response based on both the queuing delay and packet loss as cost factors. The success of strategic game theoretic controllers has been demonstrated through analytical study and simulations in several recent literatures as well [7], [49], and [50].

Utility-based techniques converges end flow rates to the solution that optimizes a particular objective function. These methods include user response time (faster convergence) [6], [8], [51], [49], [55] or providing better fairness [20], [53], [52] to the users with different utilities. Only a handful of literatures [6], [7] study the relative merits of use of one form of objective function and the rate adaptation against the other. Thus an analytical understanding based on optimization theories and basic calculus would be beneficial to enable the end applications to appropriately choose their utilities regardless of others in the network. From the network perspective, it is equally important to optimally make use of link bandwidth given that users assign dissimilar utilities. In this paper, we analyze the properties of objective functions in choosing an appropriate rate adaptation scheme. We propose a generic maximization function and establish the necessary criteria for convergence to efficiency of our rate adaptation algorithm and its stability. For a given maximization function, we propose a novel *tangential controller* based on familiar tangent vector calculus that proves to be a better choice for rate adaptation scheme. We describe its origins and motivations and prove that it has some interesting properties compared to the widely studied Kelly controller.

In order to study the characteristics of objective functions, it is important to understand the form of network cost factors involved. Researchers have debated over the use of single or several bits of ECN-style feedback versus providing a fine-grained available-bandwidth feedback as in XCP [54]. XCP develops a new congestion control scheme by introducing *precise congestion signal* to provide explicit feedback of the available bandwidth in the packet header. We are partly motivated by the design of XCP that allows end flows to acquire their fair share of bandwidth quicker than TCP. Our model assumes a continuous and uniform loss feedback (all sources receive the same network feedback).

The rest of our thesis is organized as follows: We present the three facets of our research problem, our motivation to investigate them and their corresponding approaches to the solution in chapter II. In chapter II, we present the problems of utility-based flow optimization, stability of delayed feedback systems and our novel tangential controller as our three major objectives of the thesis work. Chapter III describes the related work in our area of research. In chapter IV, we present our network model, the motivation and analysis of utility-based optimized flow control schemes. Specifically, we consider the constrained optimization problem with inequality Kuhn-Tucker constraints and prove the bounds for feasible rate allocations for proportional fairness schemes. Chapter V introduces to the problem of analytical evaluation of the existence of asymptotic stability in delayed feedback environment. Here, we derive the upper bounds for increase/decrease gain parameters of single-flow and N-flow cases using transfer function methods. Chapter VI proposes a novel *tangential controller* using the trajectory-following technique and scaled packet loss penalty. Our tangential controller rate adaptation scheme uses a trajectory following technique to maximize a given objective function with the adjusted penalty. We propose a novel rate control algorithm (for logarithmic utilities) corresponding to our modified pricing scheme that has up to 4 times less packet losses compared to the proportional controllers. We establish the asymptotic stability of our controller using transfer functions. Finally, we analyze and implement a rate-based *tangential TCP* (TTCP) scheme using NS-2 network simulator and show simulation results of the relative behavior with other forms of rate control schemes.

CHAPTER II

RESEARCH PROBLEM AND SOLUTIONS

1 Utility Functions and Network Optimization Problem

Communication networks that use adaptive and elastic transport agents (such as TCP) are highly dynamic in nature and exhibit a non-cooperative (selfish) rational decision making process between the participating sources. Rational game theory-based decision making involves global optimization of individual and system utilities. Although end source agents are unaware of the decisions taken by others, a collective rational decision making requires some form of feedback from the network. In the case of transport protocols like TCP, this information is available in the form of loss rate feedback from the network.

A class of optimization problem that maximizes the system utility globally optimizes the system resources knowing the utilities of the individual usages. Each user chooses its own utility based on its own resource usage. In most problems, the system utility functions are considered to be additive in nature of the individual user utilities.

1.1 Proposed Research

Our proposed research intends to study the properties of utilities within the given inequality constrained set (a set consisting of feasible rate allocations) to establish tighter bounds on the network shadow prices. Primal and dual optimization algorithms studied in, [6], [7] and [8] treat the system maximization problem using equality constraints and solves using traditional Lagrangian techniques (with looser sufficiency conditions). We believe that there are at least 2 problems in this approach. Network problems are always known to operate under the inequality conditions such as available bandwidth. Network queues undergo stochastic fluctuations owing to the arrival and departure of flows in the system and also due to the ON-OFF nature of the flows themselves. Optimization theories suggest that inequality constraints have tighter bounds compared to equality constraints and hence we attempt to take this into account in our problem domain. This will be the fundamental focus of our study of utility-based optimization approach.

Secondly, we intend to study the functional form of the utilities to establish a corresponding rate differential equation. A natural question that arises is whether the rate adaptation algorithm can be uniquely determined given the objective function to be maximized? That is, are there any systematic ways for source k to adapt its rate x_k that "closely follows" the maximization function?

In investigating these two ends, we realize that the problem of choosing the right utility (with appropriate functional properties) is two-fold in nature: to drive the system towards an equilibrium point and to establish the local stability of the chosen rate differential equation. At one side, we establish the necessary and sufficiency criteria of the functional form of utility within the bounded inequality set. At the other side, we struggle to prove the existence of a Liapunov function for this optimization problem and prove its asymptotic stability of the first-order rate equation (using Liapunov first and second stability theorems).

In this section, we give an introduction to the problem. We consider more details of our problem formulation and its solution in the actual thesis work. Consider a network optimization problem that maximizes the system utility for all the k users in the system. We assume our utilities are additive in nature constrained by the link capacity given by:

$$\begin{cases} U(\mathbf{x}) = MAX \ \sum_k U_k(x_k) \\ h(\mathbf{x}) = C - A^T\mathbf{x} \geq 0, x_k \geq 0, \forall k \end{cases}. \tag{5}$$

In equation (5), the bottleneck capacity is represented by positive constant C and $h(\mathbf{x})$ is the constraint to be applied. Matrix A represents a LxN link adjacency matrix of L links of N users that has unity $A_{ik} = 1$ if user i uses link k across its path.

Kelly's dual model for a user r maximizes user objective function given by:

$$MAX \ \sum_r U_r(x_r) - \sum_j \mu_j C_j, \sum_{r\varepsilon L} x_r \leq C . \tag{6}$$

Optimization is done across all the links j of each route set J and μ_j are the link shadow price per rate. The corresponding source rate adjustment of any source k is given by the differential equation:

$$\frac{dx_r(t)}{dt} = \kappa\left(w_r - x_r(t)\sum_{j\varepsilon r}\mu_j(t)\right). \tag{7}$$

Global Liapunov of the systems (6) and (7) with a logarithmic utility is given by:

$$U(x) = \sum_r w_r \log x_r - \sum_{j\varepsilon J} \int p_j(y)dy . \tag{8}$$

2 Stability of Delayed Feedback Systems

Establishing the necessary theoretical criteria for network stability and robustness is a critical factor in understanding the system operation around the equilibrium point. Studying the problem of network stability is orthogonal to studying the optimization of network resources about an efficiency point. While stable systems drive itself back to normal conditions, tolerance to perturbations is considered system robustness. In communication networks, instability primarily arises due to multiple network conditions including stochastic perturbations due to arrival and departure of flows, transmission and queuing delays in the network and resource misuse due to *inelastic* sources. Elastic sources that adjust their rates following a rate differential equation suffer from slower response time owing to the lag in the communication feedback delays in the network. This delay causes delay in convergence to fairness with other similar source agents. Intuitively, the slower the convergence rate is, the longer the transient effects last. These transient effects can lead to unpleasant oscillations due to the delay in learning and adjustment of the total number of competing flows. In this study, we do not consider such transient effects.

Assuming that all sources behave (that is, sources are *elastic*), instability in the network arises solely due to the network delays caused by propagation and queuing delays. Our system thus becomes a closed-loop delayed feedback control system in which the stability depends directly on the end-to-end feedback delay. In order to simplify our study, we consider only a constant propagation delay and ignore the queuing delays. In our study, we investigate the relation between the robustness of the system around the optimal point and the bounds on homogeneous delays of the sources.

2.1 Proposed Research

Our proposed research intends to analytically study the local stability of the chosen rate differential equation under homogeneous delayed feedback conditions. Feedback delays of a source can be constant or stochastic owing to the queuing and noisy factors added to the delays. Stochastic delays with non-zero mean noises are modeled using functional differential equations and are not studied in depth in our study here. Our focus in this thesis is on modeling simple practical delayed feedback systems with constant homogeneous delays and to develop parametric bounds.

We use the control theoretic open-loop transfer function methods to establish the local stability of our first-order rate equation. We linearize our rate equation around the equilibrium point and use the Laplacian methods to transform it to the frequency domain. The transfer function method yields us insights into the system dynamics compared to the simple linearized Jacobian technique. Moreover, feedback delays in the Laplacian domain simply take the form of an exponent factor in the transfer functions and hence are immediately tractable.

In this section, we give an introduction to the problem and consider further details of the problem formulation and solutions in the actual thesis. Consider the simplest form of proportional controller with Kelly's logarithmic utility functions as given.

$$\frac{dr}{dt} = \alpha - \beta rp \tag{9}$$

Our source rate r is increased additively by α and multiplicatively by $\beta * r(t) * p\left(r(t)\right)$. For a single-flow across the bottleneck router, the packet loss function $p(r(t))$ is defined as in:

$$p = \begin{cases} 0, r < C \\ \frac{r - C}{r}, r \geq C \end{cases}, \tag{10}$$

where C is the bottleneck link capacity.

The system transfer function $G(s)$ (output transfer function divided by the input transfer function) of a single-flow constant delayed model is obtained by linearizing (9), followed by transformation to Laplacian domain. The final system open-loop transfer function is as given by:

$$G(s) = \frac{1}{s + r_e + \frac{C\beta}{r_0} e^{-sT}}, \qquad r_e = \frac{\alpha\beta}{C\beta+\alpha},\ r_0 = \frac{C\beta+\alpha}{\beta} \tag{11}$$

where T in our model represents the constant roundtrip delay.

Traditional models [4], [5], [6] study the system stability with known functional forms of utilities under delayed circumstances. These techniques use state-space analysis to prove the existence of negative eigenvalues of the system characteristic equation. Applying the Nyquist stability criteria on the system *return ratio* matrix, they develop the upper bounds on the delay that the system tolerates for given constants α and β. It is proved that such systems become unstable once the delay exceeds the threshold. Our proposal intends to address the stability of the delayed feedback systems as studied earlier. However, we compare the Jacobian approaches and study the pitfalls in the approach and later develop the transform function-based approach to study the dynamics.

3 Tangential Controller

Based on the nonlinear optimization theories developed earlier, we propose a novel tangential controller that has several interesting properties compared to the family of proportional controllers. Specifically, we have the following motivations in developing our analytical model for the tangential controller.

We are motivated by several recent literatures studying decentralized rate adaptation scheme in which the senders adjust the source rate at the same time maximizing the system objective function (utility minus cost). This led us to the following questions: 1) Can the rate adaptation algorithm be uniquely determined given the objective function to be maximized? That is, are there any systematic ways for a source k to adapt its rate x_k that "closely follows" the maximization function and that ultimately maximizes the system throughput of all users at a fair optimum rate x_k^* for all users k? 2) How does choosing of a rate control scheme affect its convergence to efficiency with a minimum packet loss? 3) How can asymptotic and global stability be ensured of such a closed-loop feedback systems under heterogeneous delayed conditions?

Literatures [4], [5], [6] survey several forms of objective maximization functions but they do not necessary explain why the rate control behaves the way it behaves. Specifically, they do not address the following problems:

- What is the correlation between choosing a distributed rate adaptation scheme against maximizing a given objective function?

- Are there other maximization functions that can converge to fair optimum rate and that are Liapunov stable?

- For a given utility, what is the tradeoff between attaining the optimum throughput, the speed of convergence to efficiency and the compromise in stability with delayed feedback?

- Can packet loss scaling be made asymptotically sub-linear with the increasing number of flows across a link?

3.1 Proposed Research

In this section, we use the results from inequality optimizing problem described in earlier sections to develop a novel *tangential controller*. First, we introduce a trajectory-following technique and prove that it satisfies the two Kuhn-Tucker inequality conditions described in previous section. Second, we show the need for additive (positive) packet loss penalty to scale the loss aggressively. Third, we utilize this (positive) packet loss scaling factor and prove that our pricing scheme with additional loss scaling factor results in much smaller error in source rate evolution.

Consider the network rate allocation problem *P* as follows. Our aim now is to develop a network rate allocation scheme with dependence on the following parameters given by:

$$\begin{aligned}&\text{P: } NETWORK(\mathbf{U}(\mathbf{x}), h(\mathbf{x}), p_l(\mathbf{x}), \mathbf{Q}(\mathbf{x}))\\&\mathbf{U}(\mathbf{x}) = \left(U_1(x_1)\ \ U_2(x_2) \ldots U_k(x_k)\right)^T\\&\mathbf{Q}(\mathbf{x}) = \left(Q_{R_1}(\mathbf{x})\ \ Q_{R_2}(\mathbf{x}) \ldots Q_{R_k}(\mathbf{x})\right)^T\end{aligned} \quad , \tag{12}$$

where $NETWORK(\mathbf{U}(\mathbf{x}), h(\mathbf{x}), p_l(\mathbf{x}), \mathbf{Q}(\mathbf{x}))$ is the system that maximizes the overall utility function defined as: $MAX\left(\sum_k U_k(\mathbf{x}) - \sum_l p_l(\mathbf{x}) - Q_{R_k}(\mathbf{x})\right)$. We propose an alternative cost function as follows.

Consider our model with link l with packet loss $p_l(x_l)$, where x_l is the aggregate rate of all flows that pass through link l. The model assumes an additional route-dependent penalty scaling factor $Q_{R_k}(x_l)$ added to the packet loss $p(x_l)$. The scaling factor is path-dependent such that R_k represents the path for user k. Thus the net cost $W_k(x_k)$ paid by the user k in our pricing scheme is given by:

$$W_k(x_k) = \alpha U_k(x_k) - \beta\left(\sum_{l\in R}\int_0^{\sum x_s} p_l(s)ds - \beta_c Q_{R_k}(\mathbf{x})\right), \qquad (13)$$
$$Q_{R_k}(\mathbf{x}) = \max\left(\sum_{s\in l} x_s - C_l\right)^2, \forall l \in R_k$$

In (13), the constant non-negative gain parameters α, β and β_c are used in the rate differential equation for increasing and decreasing the rates.

Our novel trajectory-following formulation is as follows. Consider a positive-definite source rate equation $\mathbf{x}(t)$ that evolves according to the first-order differential equation:

$$\frac{d\mathbf{x}}{dt} = \frac{d\phi(\mathbf{x},\eta)}{dt} = f(\mathbf{x}). \qquad (14)$$

The source rate vector $\mathbf{x}(t)$ is adjusted according to trajectory tracking function $f(\mathbf{x})$ that is yet to be determined. Family of functions $\phi(\mathbf{x},\eta)$ are said to be *flows* that are solutions to the rate differential equation (14) such that, every constant η yields an integral curve for (14). If the objective maximization function is $U(\mathbf{x})$, then we theorize that the tangent vector at any point $\mathbf{x}$ (gradient at $\mathbf{x}$) at every step yields the closest possible trajectory towards the unique maximum $\mathbf{x}^*$. In our pricing scheme, the net cost paid by the user k is given by (15). The additional route-dependent penalty scaling factor $Q_{R_k}(\mathbf{x})$ is given by:

$$W_k(x_k) = \alpha U_k(x_k) - \beta\left(\sum_{l\in R}\int_0^{\sum x_s} p_l(x)dx - \beta_c Q_{R_k}(\mathbf{x})\right). \qquad (15)$$

$$Q_{R_k}(\mathbf{x}) = \max\left(\sum_{s\in l} x_s - C_l\right)^2, \forall l \in R_k \qquad (16)$$

We claim that our optimum rate allocation problem P in (12) is solved by the following rate differential equation that follows the cost function closely.

$$\frac{dx_k}{dt} = x_k \left(\alpha U'_k(x_k) - \beta \sum_{l \in R} p_l(\mathbf{x}) \right) - \beta \beta_c \frac{\partial Q_{R_k}}{\partial x_k} \tag{17}$$

We aim to prove the necessary theory behind this allocation problem and establish the stability of this rate controller.

CHAPTER III

RELATED WORK

1 Utility Functions

The concept of utility is borrowed from optimization theory. Utility of a user is a function of commodity consumed against the given prices under the budget set. The user or the agent's objective is to maximize its own utility over the given budget set. Equivalently, in network optimization problem the sender maximizes its own throughput at the cost of other sources in the system and the constrained link capacity (that is, aggregate link rate does not exceed the link capacity). In our model, we assume that the senders' utilities are proportional to their throughput achieved.

In 1998, Kelly [6] introduced a family of game-theoretic utility functions applicable to network optimization and allowed individual users to have their own utility function $U_r(x_r)$ proportional to the current sending rate x_r. The collective set of sources forms the system-wide additive utility which is to be maximized under the link capacity constraint as shown in equation:

$$\begin{aligned} &MAX \quad \textstyle\sum_{r\in R} U_r(x_r) \\ &\textstyle\sum_{l\in L} x_l \leq C, \text{ constraint} \end{aligned} . \tag{18}$$

The constraint indicates that the aggregate rate across link l cannot exceed the link capacity C.

A number of utility functions may be chosen but the important properties are that they have to be strictly concave monotonically increasing functions and are continuously differentiable over positive rate x_r. An implicit assumption made by Kelly *et al.* in [6] is that the utilities increase monotonically (or are concave) as a function of current rate, provided the rate is bounded by the bottleneck capacity. Strict concavity of the constraint ensures a unique global maximum in the constrained bounded set. Such functions are applicable only within the bounded set given by the constraint in the equation (18). Bounded sets are those whose rates are positive values $x_r \geq 0$ and for those whose aggregated rate does not exceed the link capacity C.

Source agents that behave (adjust their rates dynamically) using a certain class of utility functions are said to be elastic. Kelly *et al.* [6] introduced two source rate algorithms, the *primal* and the *dual* forms of optimization problem. These algorithms maximize the given objective function according to a chosen rate differential equation. Optimization problems that maximize an objective function under constraints are treated as primal to begin with, and are later converted to its corresponding dual form. Primal form seeks to maximize the objective function as expressed by:

$$\begin{aligned} &MAX \sum_r U_r(x_r) - \sum_j \mu_j C_j \\ &\sum_{r\varepsilon L} x_r \leq C, \text{ constraint} \end{aligned} \quad . \tag{19}$$

In the context of network flow control, solving the primal form requires a closer coordination between the sources to adjust their individual rates. Figure 1 shows one such model in which sources across routes 1 and 2 are not aware of each other's presence and only the bottleneck router is expected to share the loss ratio information with all sources in the network. In our model, there are two independent sources sets S_1 and S_2 each using utility functions U_r and U_s respectively. Because users do not have any means to share information with other users within the network, any network-wide feedback has to be provided by the network infrastructure itself [45]. In our case, sets S_1 and S_2 ultimately share a single bottleneck resource and hence the bottleneck router sends link usage feedback of one set to the other.

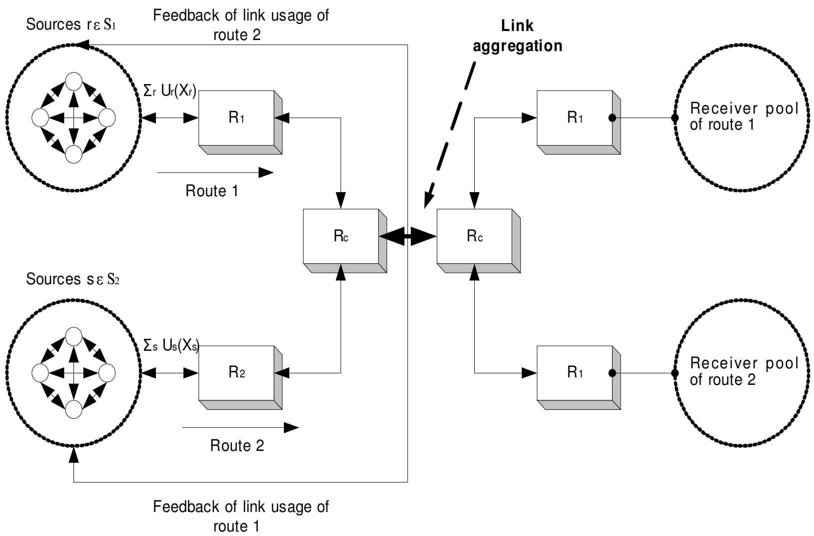

Figure 1 Primal coordinated approach.

The Internet, on the other hand, is vastly distributed and heterogeneous in nature and hence the dual form is more practical to implement. The dual form of network optimization is a distributed approach in which the senders are non-cooperative in nature and are only aware of their own losses in the network. This loss information serves as a global end-to-end congestion feedback to all the senders to adjust their rates. Thus, the whole system functions as a dynamic closed-loop feedback system with network operating as a plant transfer function. We illustrate this in the Figure 2. In the model shown in the Figure 2, we distinguish the sources in two sets S_1 and S_2 based on their utilities. Sources in sets S_1 (r ε S_1) and S_2 (s ε S_2) are distinguished by their utility functions U_r and U_s respectively with S_1 and S_2 taking different route to destinations.

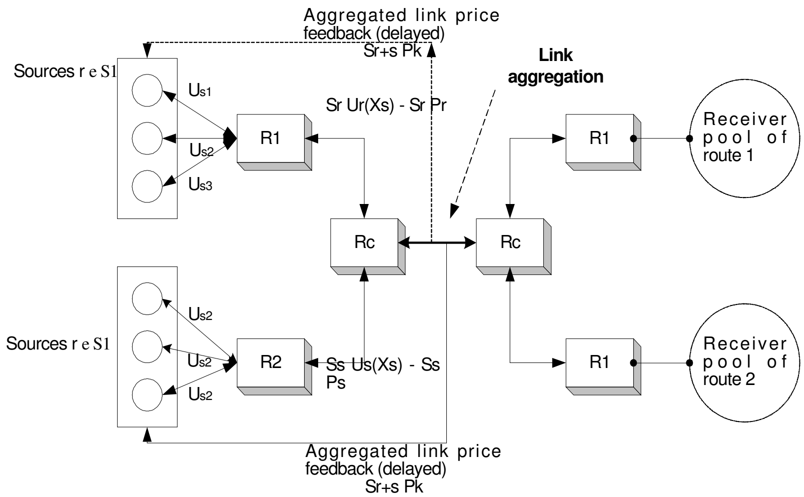

Figure 2 Closed-loop feedback network model.

A decentralized dual form rate algorithm thus solves a first-order rate differential equation with the aggregate loss feedback as given by:

$$\frac{dr_k(t)}{dt} = \kappa * r_k(t) \left(U'_k * w_k - \sum_{j \varepsilon k} \mu_j(t) \right). \tag{20}$$

Rate equation (20) is similar to the Kelly's original (4) for any user *k*, except that (20) is for a generic utility. Kelly's scheme (4) is obtained by substituting logarithmic utility $U(r) = \log r$ in (20).

2 Network Loss Feedback

Several forms of implicit (such as packet loss or drops) and explicit network feedback (such as ECN [44]) are studied in the literature [38], [39], [40], and [44]. In these models, source agents infer the current network load through implicit means such as packet drops, queuing delays, asymptotic increase in queue sizes and variance of the RTT, all of which are nonlinear in nature. ECN marking [44] is a congestion indicator mechanism that that sets a congestion indication bit in the packet header while still forwarding the packets to the end hosts. ECN-capable transports are expected to explicitly make use of this field in adjusting their window sizes. Recently, XCP (eXplicit Control Protocol) [54] develops a new congestion control scheme by introducing *precise congestion signal* to provide explicit feedback of the available bandwidth in the packet header. It is shown by the authors that the design of XCP allows end flows to acquire their fair share of bandwidth quicker than TCP.

Kelly *et al*. [6] propose a rate-adjustment algorithm known as the *dual form* and use the loss feedback as the pricing function. Traditional TCP and earlier utility-based studies , [7], [50], [15], [8] assume loss feedback as the pricing function, but recently La and Ananthram [55] and Alpcan and Basar [49] investigate the use of queuing delays as nonlinear pricing and establish the global stability of such controllers. An important challenge faced in nonlinear rate algorithms is to prove the existence of a unique point towards which all Liapunov trajectories converge.

The success of strategic game theoretic congestion control with network loss rate as feedback has been demonstrated widely through simulations in several recent literatures [7], [49], and [50]. Kunniyur *et al*. [7] simulate agents with three utility functions that simultaneously share the network and show the unfairness in their behavior when congestion occurs. This happens because utilities with significant round-trip delays experience buffer starvation in FIFO queues with the drop-tail mechanism, while other utilities do not aggressively decrease window sizes when congestion is detected. Alpcan and Basar [49] develop a window-based TCP-friendly controller with the linear queuing delay as the pricing and show that the new scheme exhibits smoother convergence and far less-aggressive behavior compared to TCP. Ganesh and Laevens [50] simulate a rate adaptation scheme with a family of utility functions with heterogeneous price estimates and prove that the stability of their controller is not compromised.

In the model studied, we consider an ECN-style explicit network feedback that consists of cumulative packet loss across all links along the path of each user. Cumulative packet loss is *additive* in nature as studied in current literatures [6] and [7]. However in practice, packet marking (or dropping) typically follows a product form probability function $1-\prod_{l\in R}(1-p_l(x_l))$, where $p_l(x_l)$ is the loss rate at link l, x_l is the rate of all flows across link l and R is the set of all links along the route of each user. In the later proposal of tangential controller, we study the additive penalty with a positive route-dependent price that the user pays and prove that this pricing scheme converges to a global optimum.

3 Network Stability

System stability is studied using two well-known approaches: to develop the Liapunov function and prove the first and second asymptotic stability criteria [12] and to develop the control theoretic open-loop stability using transfer methods [56] and [57].

Kelly *et al.* [6], [21], [22] theoretically established that the system maximization function (that is, aggregate sum of logarithmic utility minus the price paid per link) was indeed the Liapunov proving the local stability of the rate equation. Without the stochastic perturbations, the first derivative of the Liapunov function was proved to be a strictly increasing positive definite function with a unique maximizing rate. In their model, Kelly *et al.* established the stability of discrete delayed feedback systems.

Massoulié [11] established an upper bound on the increase / decrease gain parameters with heterogeneous feedback delays for a continuous-time system. His model considered the asymptotic stability of the rate equation with heterogeneous feedback delays using matrix transfer methods. Local stability of the rate equation was verified by proving the existence of Liapunov function, its first and second stability criteria and the existence of strictly concave Liapunov function.

Johari and Tan [9] developed an upper bound on the increase / decrease gain parameters for proving the local stability of a constant-delay feedback system using a discretized characteristic equation. The main difference between their model and Massoulié's [11] is that the upper bound of the gain parameters using a discrete-time equation is tighter as compared to the bound established by a differential-difference equation. One immediate inference is that the practical systems are better analyzed analytically using discrete-time difference equations. Using their model, the authors prove that the same bound exists for non-adaptable users (infinite feedback delay) as well as instantly adaptable users (zero feedback delay).

Vinnicombe [10] developed a transfer function-based methodology to verify the local asymptotic stability of continuous-time systems with heterogeneous delays. His results verified results established by Kelly *et al.*[6]. An important contribution is his explicit accounting of the exponential smoothed pricing information taken into account in the rate adjustment algorithm.

CHAPTER IV

UTILITY FUNCTIONS AND NETWORK OPTIMIZATION

1 Our Network Model

In this section, we present the analytical model for solving the optimization problem under link capacity constraints. Our network model resembles that of Kelly *et al.* [6], [21], [22], but solves the inequality constraint problem using Kuhn-Tucker inequality conditions [13].

The analytical model used in this study considers an underlying network framework with a set of J resources utilized by set of I users. Resources are links (or router queues) that have link capacities C_j and are capable of signaling end users by providing aggregate loss feedback p_l across link l. Since we wish to explicitly evaluate the performance of our model with both end-to-end and AQM methods, we assume both forms of feedback control in our model and experiments.

User $k \in I$ in our model chooses utility function $U_k(x_k)$ which is strictly concave, monotonically increasing and double differentiable over all rates $x_k \geq 0$. For simplicity, we logically group the sources into groups such that the sources in each group attach the same logical meaning to their utilities. Recall that we showed one such example in Figure 2 with groups S_1 and S_2 in the introduction section. The model assumes a distributed approach in which the bottleneck link provides an aggregate feedback to sources in groups S_1 and S_2.

2 Problem Motivation

The motivation in investigating utility-based flow optimization in our study is due to the several recent literatures on decentralized rate adaptation schemes. In these schemes, the senders adjust their rates and at the same time maximize the system objective function, i.e., utility minus cost. This led us to the following questions: 1) Can the rate adaptation algorithm be *uniquely* determined given the objective function to be maximized? That is, is there any systematic way for source k to adapt its rate x_k that "closely follows" the maximization function and that ultimately maximizes the system throughput of all users at a fair optimum rate x_k^* for all users k? 2) How does choosing a rate control scheme affect its convergence to efficiency and the amount of packet loss? 3) How can asymptotic and global stability of such a closed-loop feedback systems be ensured under heterogeneous end user delays?

Literature survey several forms of objective maximization functions, but they often do not address the following problems:

- What is the correlation between choosing a distributed rate adaptation scheme and maximizing a given objective?
- Are there other maximization functions that can converge to a fair optimum rate and are Liapunov stable?
- For a given utility, what are the tradeoffs between attaining the optimum throughput, speed of convergence to efficiency, and stability under delayed feedback?
- Can packet loss be made asymptotically sub-linear with the increasing number of flows through a link?

These questions serve as our primary motivation towards investigating controllers that may yield the above-mentioned properties.

3 Inequality Optimization Problem

Network transport agents are modeled as non-cooperating sender entities that try to maximize their own objective function. Objective functions consist of an unbounded monotonically increasing utility function minus a nonlinear penalty paid for the service. The domain of the objective functions is generally determined by equality or inequality constraints. Typically, inequality or equality constraints are applicable depending on whether the aggregate rate is strictly lesser than the bottleneck link bandwidth or not. Traditional techniques study the optimization problem with equality constraints with the primal and dual form algorithms [6]. The primal and dual optimization algorithms studied in [6], [8] treat the system maximization as equality constrained using the Lagrangian formulation. We believe that there are at least two problems in this approach. First, network flows always are known to operate under inequality conditions such as the bottleneck bandwidth. Second, optimization theories suggest that inequality constraints with application of Kuhn-Tucker conditions [13] establish tighter bounds on shadow prices and hence we attempt to take this into account in our study.

Our inequality problem formulation is as follows. Consider a constrained optimization problem for maximizing a given user objective function $U_k(x_k)$, which is a strictly concave, monotonically increasing function of the user's throughput x_k. The domain of the user maximization function is bounded by the inequality constraint for link l, $h_l(\mathbf{x}) \geq 0$, where $h_l(\mathbf{x})$ is the constraint function and $\mathbf{x}$ is the aggregate rate of all flows passing through link l. Assuming utilities are additive, the system-wide objective function $U(\mathbf{x})$ is a weighted sum of individual user utilities. We further assume that the feasible user allocation rates $x_k \geq 0$ are formed by the inequality constraint $h_l(\mathbf{x}) \geq 0$, which forms a closed and bounded set, i.e., a closed ball D with optimum rate x_k^* as its radius. It is important to consider a closed-ball in order to establish a global optimum for our maximization function and to prevent Kuhn-Tucker conditions from failing at the global optimum. Recall that, Kuhn-Tucker conditions are necessary, but not sufficient for the existence of a global maximum [13]. Furthermore, it is important to realize that the closed-ball radius x_k^* is different for different flows until optimum fairness among all flows is established. However, there exists a global maximum of all flows for a given objective function: $x^* = \max_k x_k^*$.

Consider a network optimization problem that maximizes the system utility for all users in the system. We assume our utilities are additive in nature and constrained by the link capacity:

$$\begin{cases} U(\mathbf{x}) = \max \sum_k U_k(x_k), x_k \geq 0, \forall k \\ \mathbf{h}(\mathbf{x}) = \mathbf{C} - A^T\mathbf{x} \geq 0 \end{cases}, \tag{21}$$

where the inequality constraint vector **h**, the rate vector **x**, the capacity vector **C** and routing matrix *A* are defined as:

$$\begin{aligned} &\mathbf{x} = (x_1, x_2, .., x_l, .., x_n)^T \\ &\mathbf{C} = (C_1, C_2, .., C_l, .., C_n)^T \\ &\mathbf{h}(\mathbf{x}) = (h_1(\mathbf{x}), h_2(\mathbf{x}), .., h_l(\mathbf{x}), ..h_n(\mathbf{x}))^T \\ &A = \begin{cases} A_{kl} = 1, \text{ if user } k \text{ passes through link } l \\ A_{kl} = 0, otherwise \end{cases} \end{aligned} . \tag{22}$$

The vectors defined in (22) solve the optimization problem (21) by defining the Lagrangian. The Lagrangian of the system (21) is defined by: $L(\mathbf{x}, \boldsymbol{\mu}) = U(\mathbf{x}) + \boldsymbol{\mu}^T h(\mathbf{x})$, where vector $\boldsymbol{\mu} = (\mu_1, \mu_2, .., \mu_l, ..\mu_n)^T$ are the Lagrangian multipliers or the shadow prices corresponding to each link. The optimal solution $\mathbf{x}^*$ is determined by applying the inequality theorem [13] to the feasible set of rates under the following conditions:

- Condition 1: $\mu_l \geq 0,\ \mu_l h_l(x_k) = 0,\ h_l(x_k) \geq 0$, for all users k across link l.
- Condition 2: for all users *k*, we must have the following:

$$\begin{cases} U_k'(x_k) + \mu_k \left.\dfrac{\partial h_l(\mathbf{x})}{\partial x_k}\right|_{x_k = x_k^*} = 0 \\ x_k \geq 0 \end{cases} \tag{23}$$

An immediate observation from the above conditions is that the shadow price factor μ_l for link *l* is the same for all users whose route passes through link *l*. Substituting constraint $h_l(\mathbf{x})$ in equation (21) in (23), and simplifying yields:

$$U_k'(x_k^*) = -\mu_l \frac{\partial}{\partial x_k}(C - \sum_m x_m) = \mu_l\,. \tag{24}$$

Lemma 1: Users with the same utilities $U(x_k)$ attain a fair share of the underlying link *l* with shadow price μ_l, and the fair share for all users $\mathbf{x}^*$ is given by:

$$\begin{cases} x_k^* & = & \dfrac{1}{U'(\mu_l)} \\ U'(x_k^*) & \neq & 0 \\ U'(\mu_l) & \neq & 0 \end{cases}\,. \tag{25}$$

Proof: We first note that (23) gives us the first-derivative of utility at optimum x_k^* as $U_k'(x_k^*) = \mu_l$. Assuming that the shadow prices $\boldsymbol{\mu}$ are non-negative, we apply the familiar inverse function theorem to obtain the first-derivative of the utility as a function of shadow price μ_l for link *l* at the optimum point x_k^*. Inverse function theorem is defined as follows. Given a continuously differentiable function $y = f(x)$ and a local optimum value $\mathbf{x}^*$, the inverse function $x = f^{-1}(y)$ exists near $\mathbf{x}^*$ if $f(\mathbf{x}^*) \neq 0$ [58]. In our case the function *f* happens to be $U'(x_k)$. Since the shadow price communicated to all users *k* (whose route lies along link *l*) is the equal, their respective fairness share is the same. Note that the utility function is strictly concave, monotonically increasing function for our proof to hold good. ■

Since the shadow prices μ_l can take any non-negative real value, and the first-derivative of the objective function exists for all μ_l, we conclude that (25) defines the fairness among sources that have routes passing through link *l*. Indeed, in a single-link system, we find that this amounts to an exact fair share among all users provided they use the same utility functions. It is this non-negativity condition on the shadow prices $\mu_l \geq 0$ that enforces fairness among users and establishes tighter feasibility solutions to (21).

We next extend the usefulness of shadow prices and prove that it provides a family of fairness schemes, one of which is *proportional fairness*.

Lemma 2: The optimization problem in (21) with additional fairness constraint $x_k^* - x_k \geq 0$ for all users *k* results in skewed normalized *fairness delay* (reciprocal of the rate) of $\frac{1}{x_i} - \frac{1}{x_j} = \lambda_j - \lambda_i$ for dissimilar users *i* and *j* sharing the same bottleneck link *l*. Proportional fairness is one such family of fairness resulting from $\lambda_i = \lambda_j$.

Proof: The Lagrangian of our new system is determined by the utility minus the penalty which are constrained by additional fairness factor for each user *k*. This results in extra shadow prices $\lambda_l \geq 0$ for each link *l*. The Lagrangian of this system may be written as follows:

$$L(\mathbf{x},\boldsymbol{\mu}) = U(\mathbf{x}) - \sum_l \int p_l(s)ds + \boldsymbol{\mu}^T \mathbf{h}(\mathbf{x}) + \boldsymbol{\lambda}^T(\mathbf{x}^* - \mathbf{x}) \qquad (26)$$

Consider the Lagrangian of a two-user system with packet loss $p(\sum_k x_k) = 1 - \frac{C}{\sum_k x_k}$, where C is the bottleneck link capacity. The Lagrangian of this two-user system for logarithmic utility can be written as:

$$\begin{aligned} &L(x_1, x_2, \mu, \lambda_1, \lambda_2) = (\log x_1 + \log x_2) - \\ &-(x_1 + x_2 + 2C\log(x_1 + x_2)) + \mu(C - x_1 - x_2) + . \\ &+\lambda_1(x_1^* - x_1) + \lambda_2(x_2^* - x_2) \end{aligned} \qquad (27)$$

Taking the partial derivatives of (27) with respect to x_1 and x_2 and equating them to zero yields the normalized skewed fairness delay factor for users i and j that share the same underlying bottleneck link l:

$$\frac{1}{x_i} - \frac{1}{x_j} = \lambda_j - \lambda_i \qquad (28)$$

■

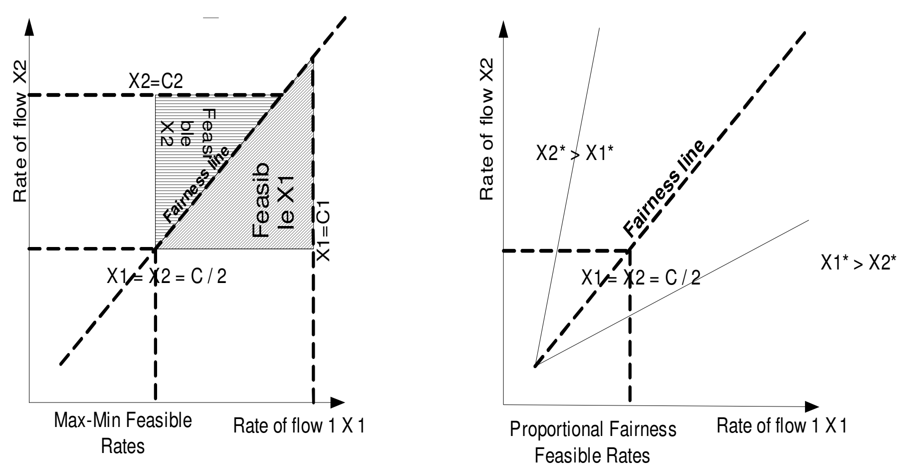

Figure 3 Feasible rates for max-min (left) and proportional fairness (right) for two flows.

We show the results of Lemma 2 in Figure 3 that compares two feasible allocation rate sets for max-min and proportional fairness. Max-min sets can take values in set $C/2 \le x_1 \le C_1$ and $C/2 \le x_2 \le C_2$, bounded by the right-triangles shown in the figure. On the other hand, proportional fairness are defined by $\sum_k \frac{x_k^* - x_k}{x_k} \le 0$ and hence can widely vary between the two lines shown $x_1^* > x_2^*$ and $x_2^* > x_1^*$. The result in Lemma 2 shows precisely this.

CHAPTER V

STABILITY OF DELAYED FEEDBACK SYSTEMS

1 Motivation

Our motivation is to develop control-theoretic transfer function methods to prove the stability of delayed feedback controllers. We develop the bounds on increase / decrease parameters using transfer function methods and show that the methods concurs with the bounds developed by similar literature earlier.

2 Stationarity of Proportional Controllers

In this section, we derive the stationarity of proportional controllers for two cases: a simple single-flow case and a general *N*-flow case. Establishing the stationary steady state rates and packet loss for a general N-flow case is important in linearizing the system around the operating point where the local stability is sought.

2.1 Single-flow Case

Consider a proportionally fair transport agent that adjusts its sending rate using a continuous function of time evolving as per the differential equation:

$$\frac{dx}{dt} = \alpha - \beta x p \,, \tag{29}$$

where α and β are additive and multiplicative constants. For simplicity, consider a single bottleneck link with one flow whose rate evolves as per the differential equation (29). The link loss feedback function $p(x)$ is continuous and is defined as:

$$p(x) = \begin{cases} 0, x < C \\ \dfrac{x - C}{x}, x \geq C \end{cases}. \tag{30}$$

Substituting (30) in (29) for rates $x \geq C$, the stationary steady-state rate of the source is obtained by forcing the time derivate of rate differential equation (29) to zero. That is,

$$\begin{aligned} &\frac{dx}{dt} = 0 \\ &\Rightarrow \alpha - \beta x p(x) = \alpha - \beta x \frac{x - C}{x} = 0 \end{aligned}. \tag{31}$$

We obtain the stationary source rate x^* and the stationary loss feedback $p^*(x^*)$ for the single-flow bottleneck link by simplifying the expression in (31):

$$\begin{cases} x^* = C + \dfrac{\alpha}{\beta} \\ p^* = \dfrac{\alpha}{\beta x^*} = \dfrac{\alpha}{\alpha + C\beta} \end{cases}. \tag{32}$$

2.2 *N*-flow Case

Consider a network model with a simple one bottleneck link with several flows whose path lie along the bottleneck link. The flow rates are proportionally fair and evolve according to (29). We define the rates of all flows passing through the bottleneck link by a vector $\mathbf{x} = (x_1, x_2, .., x_N)^T$. The model is the same as a single-flow case except that the aggregate packet loss function $p(\mathbf{x})$ is a function of the aggregate rate of all flows along the bottleneck link. The individual source rate of user k is defined by function $f_i(x_k)$ that uses the aggregate packet loss $p(\mathbf{x})$:

$$f_k(x_k) = \frac{dx_k}{dt} = \alpha - \beta x_k \frac{(\sum_i x_i - C)}{\sum_i x_i} \;,\;\; k = 1, 2, .., N\,. \tag{33}$$

The stationary rate vector $\mathbf{x}^* = \left(x_1^*, x_2^*, .., x_k^*, .., x_N^*\right)^T$ is obtained by independently maximizing the user rate differential equations and solving the N algebraic equations for the individual stationary rates. At stationary vector $\mathbf{x}^*$, all flows converge to the same equilibrium value $x_1^* = x_2^* = .. = x_k^* = .. = x_N^* = x_c^*$ and is given by:

$$\mathbf{x}^* = \left(x_1^*, x_2^*, .., x_k^*, .., x_N^*\right)^T \;=\; \left(\frac{C}{N} + \frac{\alpha}{\beta}, \frac{C}{N} + \frac{\alpha}{\beta}, ..., \frac{C}{N} + \frac{\alpha}{\beta}\right)^T. \tag{34}$$

Recall that, proportional fairness for single-flow results in the stationary rate exceeding the bottleneck capacity by α/β and hence (34) gives us a general stationary vector for flows $N \geq 1$. Recall that the stationary loss is thus a function of the aggregate rate of all flows passing through the link and this hence is given by:

$$p_c = 1 - \frac{C}{N \times x^*} = \frac{N\alpha}{C\beta + N\alpha}\,. \tag{35}$$

We now prove the asymptotic stability of the generic *N*-flow case using the strong linear stability theorem [12]. Strong linear stability theorem states that the first order differential equation around an equilibrium point is asymptotically stable if the distinct eigenvalues of the Jacobian matrix evaluated at the equilibrium point has strictly negative real part [12]. The theorem however, remains true even if the eigenvalues of the Jacobian matrix are not distinct.

In order to achieve this, we first linearize our rate equation (using Taylor series) around the stationary point. Taylor-series linearization requires deriving Jacobian partial derivatives of the set of functions f_k for all k with respect all independent rates x_j. Denoting this by various rows and columns, we evaluate the resulting expression at the stationary vector **x***. The Jacobian yields the following form.

$$\frac{\partial f_k(\mathbf{x})}{dx_j} = \begin{cases} \dfrac{-\beta(N-1)\left(\dfrac{C}{N}+\dfrac{\alpha}{\beta}\right)}{C}, \; if \;\; k \neq j \\ \\ \dfrac{\beta(N-1)\left(\dfrac{C}{N}+\dfrac{\alpha}{\beta}\right)}{C} - \beta, \; if \;\; k = j \end{cases}, \tag{36}$$

Giving generic names for various rows and columns as $a = \dfrac{\partial f_k}{dr_k}; \;\; b = \dfrac{\partial f_k}{dr_j}$, the Jacobian simply becomes an *N*-dimensional Toeplitz matrix for which the eigen-values are to be calculated. The Jacobian Toeplitz matrix is given by the symmetric matrix:

$$-\begin{bmatrix} a & b & b & b \\ b & a & b & b \\ . & . & . & . \\ b & b & b & a \end{bmatrix} = 0. \tag{37}$$

The corresponding eigen-value of the Toeplitz matrix is as given by:

$$-\begin{bmatrix} a-\lambda & b & b & b \\ b & a-\lambda & b & b \\ . & . & . & . \\ b & b & b & a-\lambda \end{bmatrix} = 0\,. \tag{38}$$

Toeplitz matrices are a special form of matrices that has constant elements along the negative diagonals and can be constructed with $2N–1$ unique elements. If all elements of the main diagonal are the same and all other elements (across all other diagonals) are the same, we have a special form of Toeplitz matrix called *circulant matrix*. Circulant matrices are symmetrical with respect to the main diagonal, and each row or column can be formed by circular-rotation of elements in the previous row or column counter-clockwise. An interesting and useful result about an NxN circulant matrix is a generic way to determine its eigenvalues.

Eigenvalues λ_i of a circulant matrix is defined as a series summation and a polynomial of ω_i, which is one of the n^{th} roots of unity. We note that some of the roots may be complex depending on the value of N.

$$\lambda_i = \sum_{j=0}^{n-1} x_j * \omega_i^{\,j}\ . \tag{39}$$

To apply this to our case, we observe that there are only two unique values across the diagonals including a and b defined above. Thus, the polynomial (39) thus reduces to the following:

$$\lambda_i = a + b * \sum_{j=1}^{n-1} \omega_i^{\,j} = \begin{cases} a + (n-1)b, if\, \omega_i = 1 \text{(occurs 1 time)} \\ a - b, if\, \omega_i \neq 1, \text{(occurs } n\text{ - 1 times)} \end{cases}. \tag{40}$$

The eigenvector of our circulant matrix can thus be written in the following form as in equation (38) below. Since it is already known that $a > b > 0$, all eigenvalues λ_i are negative proving that *N*-flow system is also asymptotically stable. Moreover, the eigenvalues of the matrix is:

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ . \\ \lambda_n \end{pmatrix} = \begin{pmatrix} -a-(n-1)b \\ -a+b \\ . \\ -a+b \end{pmatrix}. \tag{41}$$

3 Control-theoretic Model

Recall the generic model depicted in Figure 1 where the sources in sets S_1 and S_2 have an associated route to their unique receivers in the receiver pool. In this model, we assume a constant round-trip delay of *T* for all sources in the same group. Consider a simple model consisting of a single bottleneck link with an arbitrary number of flows across the link. It is possible to extend such a model to include more bottleneck links.

The input rate transfer vector of source rates can be defined as a vector: $\mathbf{X}(s) = (X_1(s), X_2(s),.., X_l(s),..X_N(s))^T$ and the corresponding output vector as: $Y(s) = (Y_1(s), Y_2(s),.., Y_l(s),..Y_N(s))^T$. The open-loop transfer function (output transfer function divided by input transfer function) vector is defined as: $\mathbf{G}(s) = (G_1(s), G_2(s),.., G_l(s),..G_N(s))^T$. The open-loop vector $\mathbf{G}(s)$ is a function of state matrix *A*, the delayed state-matrix A_d, the input matrix *B* and the input-output matrix *C*.

Recall that the open-loop transfer function of a system with multiple-inputs and multiple-outputs with the given vectors can be directly found by [56]:

$$G(s) = C(sI - A - A_d D)^{-1} B \,. \tag{42}$$

The relation between the input transfer $\mathbf{X}(s)$ and the output transfer vector $\mathbf{Y}(s)$ can be defined as below and this is used to write the system transfer function (42).

$$\begin{aligned} sX(s) &= AX + A_d DX + BU \\ Y(s) &= CX \end{aligned} \,. \tag{43}$$

The input matrix B is a unity matrix $B = U_{N\times N}$ and the input-output matrix C is an identity matrix $C = I_{N\times N}$. Additionally we define a delay diagonal matrix $D = diag\{e^{-sT}\}$ of constant delay T of all the sources. The additional state matrix A and delayed-state matrix A_d are dependent on the number of flows in the system and are determined in later sections.

4 Stability of Single-flow

Consider the proportional rate controller as given in equation (29) but with delayed feedback response of delay T. Represent the delayed rate response as $x_d(t)$ and constant delay T, the resulting equation is denoted by:

$$\begin{aligned} &\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}, \mathbf{x}_d) = \alpha - \beta x(t) p(x - T) \\ &\Rightarrow \alpha - x(t) \frac{\beta(x(t-T) - C)}{x(t-T)}, x(t-T) \geq C \end{aligned} \,. \tag{44}$$

The stationary steady state rate of $x(t)$ and x_d=$x(t$-$T)$ for $f(x, x_d)$=0 still remain the same for a single-flow model.

$$
\begin{aligned}
&\frac{dx}{dt} = f(x) = 0 \Rightarrow \beta(x - C) - \alpha = 0 \\
&\Rightarrow r^* = C + \frac{\alpha}{\beta}
\end{aligned}
\quad . \tag{45}
$$

Similarly, the stationary packet loss p^* for this rate also remains the same since the steady rate x_c is the same.

$$
\begin{aligned}
&p^* = x(t-T) \times \left.\frac{(x(t-T) - C)}{x(t-T)}\right|_{x=x^*} \\
&\Rightarrow p^* = \frac{\alpha}{\alpha + C\beta}
\end{aligned}
\quad . \tag{46}
$$

We however treat the independent variable and delayed rate $x_d(t)$ as two significantly different rate equations, since it then becomes easier to linearize. Thus our right side function $f(x, x_d)$ has 2 partial derivatives for each of the rate variables.

$$
\begin{aligned}
&\left.\frac{\partial f(x, x_d)}{\partial x}\right|_{x=x_c} = \frac{-\alpha\beta}{C\beta + \alpha} \\
&\left.\frac{\partial f(x, x_d)}{\partial x_d}\right|_{x_d=x_c} = \frac{-C\beta^2}{C\beta + \alpha}
\end{aligned}
\quad . \tag{47}
$$

Linearizing (44) using the Taylor-series form yields us:

$$\delta \dot{x}(t) = \left.\frac{\partial f}{\partial x}\right|_{x=x_c} \times \delta x(t) - \left.\frac{\partial f}{\partial x_d}\right|_{x_d=x_c} \times \delta x_d = \\ = \frac{-\beta}{C\beta+\alpha}\delta x(t) - \frac{C\beta^2}{C\beta+\alpha}\delta x_d \qquad (48)$$

Taking the Laplace transform of on both sides of (48) and noting that the Laplacian of the delayed rate $x_d(t)$ $L\{\delta x_d\} = e^{-sT}\delta x(s)$, the resulting linearized expression for a single-flow model with constant delay T is given by following expression.

$$\begin{cases} G(s) = \dfrac{C_1}{s + x_e + \dfrac{C\beta}{x_0}e^{-sT}}, \quad C_1 \neq 0 \\ \\ x_e = \dfrac{\alpha\beta}{C\beta+\alpha} \ ; \ x_0 = \dfrac{C\beta+\alpha}{\beta} \end{cases} \qquad (49)$$

5 Stability of *N*-flow Case

Study of stability of general case N-flow system can be done using state-space analysis with positive delays using the above discussed techniques discussed to derive the stability for a single-flow case. Our flows have equal but constant delays represented as T and we use this to construct the state matrix. Recall that, the general case delayed model has the system transfer function $\boldsymbol{G}(s)$ as a function of state-matrix $\boldsymbol{A}$, state-input matrix $\boldsymbol{B}$ and input-output matrix $\boldsymbol{C}$. For convenience and easier matrix manipulation, we have an additional state-matrix $\boldsymbol{A_d}$, called delayed state-matrix and this represents the additional delay exponential elements to the system.

We notice the stationary steady state rate still remains the same as for any no-delay N-flow case.

$$x_i^* = \frac{C}{N} + \frac{\alpha}{\beta} = \frac{C\beta + \alpha}{N\beta} \tag{50}$$

$$p^* = \frac{\sum_i x_i^* - C}{\sum_i x_i^*} = \frac{\alpha N^2}{C\beta + N\alpha}. \tag{51}$$

Our Jacobian matrix for N-flow case is thus extended to a delayed Jacobian (represented as J_d), which involves as much independent variables as the original Jacobian, but with positive delays. Using partial derivatives evaluated for each rate x_i and x_i^d, our Jacobian at the stationary point takes the square matrix form as below:

$$\begin{aligned} J\big|_{stationary} &= \frac{-N\alpha\beta}{C\beta + N\alpha} I_{N\times N} \\ J_d\big|_{stationary} &= \frac{-C\beta^2}{N(C\beta + N\alpha)} U_{N\times N} \end{aligned}. \tag{52}$$

The linearized rate differential is a sum of no-delay and delayed-Jacobian, both evaluated at their corresponding stationary equilibrium points. That is,

$$\dot{\boldsymbol{\delta}\mathbf{x}} = J \times \boldsymbol{\delta}\mathbf{x} + J_d \times \boldsymbol{\delta}\mathbf{x}_\mathbf{d}. \tag{53}$$

For a general case N-flow delayed system, following are the vectors and matrices that are used to derive the system transfer matrix $\boldsymbol{G}(s)$.

$$\delta\dot{\mathbf{X}} = \left(\delta\dot{x}_1, \delta\dot{x}_1, .., \delta\dot{x}_k, ..\delta\dot{x}_N\right)^T$$
$$A = \frac{-N\alpha\beta}{C\beta + N\alpha} I_{N\times N};\ A_d = \frac{-C\beta^2}{N(C\beta + N\alpha)} U_{N\times N}\ . \tag{54}$$
$$B = U_{N\times N};\ \ C = I_{N\times N}$$

Taking the Laplace transforms on both the sides of the input-output matrix yields us the relation between state vector, no-delay and delayed state-matrix and the output. In order to accommodate the delayed state-matrix A_d in the input-state expression, an additional delay matrix D is required. The delay matrix for a general case is a diagonal matrix $D = diag\{e^{-sT}\}$ with series of unequal exponent terms corresponding to the unequal delays of each flow.

With the inclusion of delayed state-matrix A_d and delay-matrix D, the system transfer equation as given in the earlier sections is defined as the following matrix multiplication of state matrices.

$$G(s) = C(sI - A - A_d D)^{-1} B\ . \tag{55}$$

$$A + A_d \times D = \begin{pmatrix} -\alpha' - \beta'' e^{-sT} & -\beta'' e^{-sT} & .. & -\beta'' e^{-sT} \\ -\beta'' e^{-sT} & -\alpha' - \beta'' e^{-sT} & .. & -\beta'' e^{-sT} \\ .. & .. & .. & .. \\ -\beta'' e^{-sT} & -\beta'' e^{-sT} & .. & -\alpha' - \beta'' e^{-sT} \end{pmatrix};\ . \tag{56}$$
$$\alpha' = \frac{N\alpha\beta}{C\beta + N\alpha};\ \ \beta'' = \frac{C\beta^2}{N(C\beta + N\alpha)}$$

The system transfer matrix $G(s)$ reduces to a simpler case for a no-delay N-flow as discussed in the earlier sections. We thus have only two unique elements in the matrix namely $s + \alpha' + \beta' e^{-sT}$ and $\beta'' e^{-sT}$, and using the earlier methods to determine the determinant and inverse, we have the following expression for the system transfer function.

All elements of the transfer matrix ***G***(s) remain the same as $G_{ii}(s)$ given below.

$$G_{ik}(s) = \\ = \frac{s+\alpha' - \beta'' e^{-sT}}{(s+\alpha' - \beta'' e^{-sT})^2 - (N-2)s + \alpha' - \beta'' e^{-sT} - (N-1)\beta''^2 e^{-2sT}} \tag{57}$$

Thus function represents the relation between any input k and any output i in the multiple flow model and the transfer function remains the same for all input-output combinations.

CHAPTER V

TANGENTIAL CONTROLLER

1 Motivation

In this section, we use the results from inequality optimizing problem described above to develop a novel *tangential controller*. First, we introduce a trajectory-following technique and prove that it satisfies the two Kuhn-Tucker inequality conditions described in previous section. Second, we show the need for additive (positive) packet loss penalty to scale the loss aggressively. Third, we utilize this (positive) packet loss scaling factor and prove that our pricing scheme with additional loss scaling factor results in much smaller error in source rate evolution.

In modeling the controller, we attempt to address some of the motivations described in section II. Specifically, we investigate the correlation between a rate adaptation scheme and a given objective function. We design a novel trajectory-following technique that uniquely maximizes the objective function at the finite optimum rate. Using logarithmic utility, we contrast our scheme with Kelly-style proportional controller. We find that our scheme converges faster and much closer to the bottleneck bandwidth with several times less packet loss. The trajectory following hypothesis is proved using inequality optimization problem in which the user pays an additional (positive) packet loss penalty in addition to the current penalty paid. The additional penalty is supported by our underlying theory that packet loss across all links in the user's route is non-additive in nature and hence an appropriate error scaling factor is required. Indeed, cumulative packet drops across droptail-enabled routers results in a product-form probability given by $1-\prod_{l\in R}(1-p_l(x_l))$, where x_l is the aggregate rate across link l.

While the product-form loss penalty function may be suitable for adjusting user rates, it is applicable to specific topologies and invariably assumes droptail queues across links in user routes. Routes with heterogeneous queuing along the paths may not necessary take the product-form penalty function. We believe that a general-form additive penalty function with suitable *loss scaling* factor (a positive addition to the packet loss) is more appropriate. We have the following motivations to study this form of additive penalty function with suitable a scaling factor:

- What positive error scaling factor is required and how should it be calculated to design a rate adaptation scheme that converges to the optimum?
- How aggressive can the scaling penalty be while maintaining a stable rate control equation?
- Can the scaling factor be utilized to bring the system optimum rate below the bottleneck capacity?
- Does the user rate control equation tolerate negative packet loss penalty?

We justify the addition of an error scaling factor, i.e., the first three points in section B and C below and leave the last point for further study.

2 Trajectory-Following Algorithm

Our trajectory-following formulation is as follows. Consider a positive-definite source rate that evolves according to the first-order differential equation:

$$\frac{d\mathbf{x}}{dt} = \frac{d\phi(\mathbf{x},\eta)}{dt} = f(\mathbf{x}) \tag{58}$$

The source rate vector $\mathbf{x}$ is adjusted according to trajectory tracking function $f(\mathbf{x})$ that is yet to be determined. Functions $\phi(\mathbf{x},\eta)$ are said to be *flows* that are solutions of the rate differential equation such that every constant η yields an integral curve of (58) . We claim that there exists at least one such integral curve of (58) that starts at a non-zero local minimum $\mathbf{x_0}$ (initial source rate) and converges to $\mathbf{x}^*$ along the given objective function curve *f*($\mathbf{x}$).

Lemma 1: Suppose $U(\mathbf{x})$ is the objective maximization function. The tangent vector at any point $\mathbf{x}$ (gradient at $\mathbf{x}$) of the cost function $U(\mathbf{x})$ at every step yields the closest possible trajectory towards unique maximum $U(\mathbf{x}^*)$.

Proof: Consider any feasible allocation vector $\mathbf{x} \in D$, where D is the closed ball of allocation rates. If our incremental rate change is still bounded by ball D, $\mathbf{x} + \Delta\mathbf{x} \in D$, then we can define our objective function around the neighborhood of $U(\mathbf{x})$ using Taylor series expansion:

$$\begin{aligned} U(\mathbf{x}+\Delta\mathbf{x}) &\geq U(\mathbf{x}), && \forall \mathbf{x} \\ U(\mathbf{x}) + \frac{\partial U(\mathbf{x})}{\partial \mathbf{x}}\Delta\mathbf{x} + R(\Delta\mathbf{x}) &\geq U(\mathbf{x}), && \forall \mathbf{x} \\ \Delta\mathbf{x}\left(\frac{\partial U(\mathbf{x})}{\partial \mathbf{x}} + \frac{R(\Delta\mathbf{x})}{\Delta\mathbf{x}}\right) &\geq 0, && \forall \mathbf{x} \end{aligned} \tag{59}$$

Noticing that $\Delta\mathbf{x}$ is positive and $\lim_{\Delta\mathbf{x}\to 0}\frac{\mathrm{R}(\Delta\mathbf{x})}{\Delta\mathbf{x}} = 0$, the gradient of the objective function is positive:

$$\frac{\partial U(\mathbf{x})}{\partial \mathbf{x}} \geq 0 \tag{60}$$

The objective function $U(\mathbf{x})$ grows monotonically with time and because of this property, an integral curve solution for (58) is given by:

$$\frac{d\phi(\mathbf{x},\eta)}{dt} = \left[\frac{\partial U(\mathbf{x})}{\partial \mathbf{x}}\right]^T$$
$$\frac{dU(\mathbf{x})}{dt} = \frac{\partial U(\mathbf{x})}{\partial \mathbf{x}} \frac{d\phi(\mathbf{x},\eta)}{dt} \geq 0 \tag{61}$$

One such proof of the tangent vector and solution (61) is given in [57] in a nonlinear constrained optimization. ■

Since the source rate evolves "closely" according to the trajectory followed along the gradient of the cost optimization function $U(\mathbf{x})$, we term our controller a *tangential controller.*

Lemma 2: The gradient of objective maximization function $U(\mathbf{x})$ is concave and evolves strictly along the *direction* of the gradient $\frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}$ of the inequality constraint function $h(\mathbf{x})$.

Proof: The theorem is proved using Farkas's lemma [57]. The geometric interpretation of Farkas's lemma is that the steepest increase in the gradient $\frac{\partial U(\mathbf{x})}{\partial \mathbf{x}}$ must lie along the direction of the gradient of the constraint function $\frac{\partial h(\mathbf{x})}{\partial \mathbf{x}}$, which is negative in our case. Kuhn-Tucker Condition (23) requires this as a necessary condition and hence the gradient $\frac{\partial U(\mathbf{x})}{\partial \mathbf{x}}$ indeed satisfies Farkas's lemma. ■

Our immediate observation is that the tangent vector scheme is related to the Kuhn-Tucker inequality conditions. We illustrate the trajectory-tracking mechanism using Figure 4. The simple linear constraint $h(\mathbf{x})$ is shown by a thick linearly decreasing function $h(\mathbf{x}) = C - \mathbf{x}$, where C is the bottleneck capacity. As long as the constraint is fulfilled, the objective function monotonically increases along the curve $U(\mathbf{x})$. At the critical rate $\mathbf{x}^*$, the objective function settles at a constant gradient and hence the rate becomes steady. The downward directional vector indicates the negative constraint gradient taking effect at optimum $\mathbf{x}^*$.

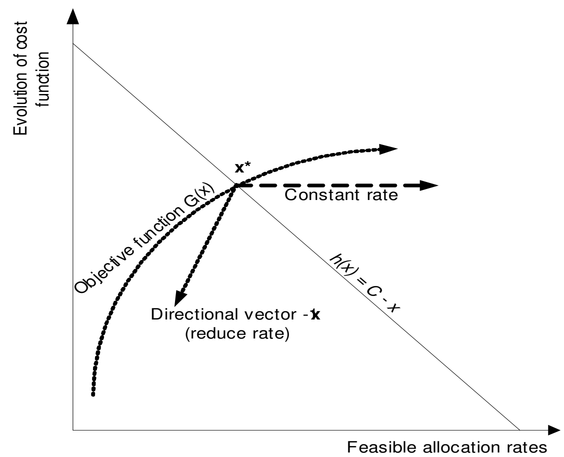

Figure 4 Trajectory-tracking technique and linear constraint.

3 Packet Loss Penalty

Our motivation to reconsider packet loss penalty functions arises from two main sources. Existing literatures [6], [7] consider additive packet losses across the links along the path of the user. Ganesh *et al.* [50] consider an iso-elastic exponentially weighted moving average price estimator with the goal of keeping link utilization close to the bottleneck capacity. A similar gradient-projection price estimator was developed by Low and Lapsley [8] in which link prices are adjusted in the opposite direction to the gradient of the price at every step. Their controller adjusts link prices according to the aggregate price across the bottleneck link. The price gradient is the gradient of the dual objective function, i.e., the Lagrangian itself.

Our penalty adjustment is similar to the controller developed by Low and Lapsley [8], but we develop our motivation from optimization theory. The objective of their price adjustment controller is to solve for the source rate as a function of optimum price. Our tangential controller algorithm solves the dual form introduced by Kelly *et al.* [6]. We use sequential nonlinear programming [57] techniques that add certain penalty functions to the maximization function. In a discretized version of the nonlinear programming technique, successive iterations lead to convergence of the sequence. The goal is to choose an appropriate penalty function that converges to optimum. Using the tangent vector controller developed earlier, we show that our pricing scheme indeed converges to unique optimum.

Consider the link l with packet loss $p_l(x_l)$, where x_l is the aggregate rate of all flows that pass through link l. The model assumes an additional route-dependent penalty scaling factor $Q_{R_k}(x_l)$ added to the packet loss $p(x_l)$. The scaling factor is path-dependent such that R_k represents the path for user k. Thus the net cost $W_k(x_k)$ paid by the user k in our pricing scheme is given by:

$$\begin{aligned} W_k(x_k) &= \alpha U_k(x_k) - \beta\left(\sum_{l \in R} \int_0^{\sum x_s} p_l(s)ds - \beta_c Q_{R_k}(\mathbf{x})\right) \\ Q_{R_k}(\mathbf{x}) &= \max\left(\sum_{s \in l} x_s - C_l\right)^2, \forall l \in R_k \end{aligned} \tag{62}$$

In (62), the constant non-negative gain parameters α, β and β_c are used in the rate differential equation for increasing and decreasing the rates. The original pricing scheme $M_k(x_k)$ as studied by Kelly *et al.* [6] is given by:

$$M_k(x_k) = U_k(x_k) - \beta \sum_{l \in R} \int_0^{\sum x_s} p_l(s)ds \tag{63}$$

Lemma 3: Path-dependent penalty scaling factor $Q_{R_k}(\mathbf{x})$ used in the pricing scheme $W_k(x_k)$ in (62) solves for optimum rate $\mathbf{x}^*$.

Proof: The lemma requires that the scaling factor $Q_{R_k}(\mathbf{x})$ be bounded. The scaling factor adds large penalties initially when the flow starts (i.e., when the rates differs from the constraint widely) and becomes smaller as the aggregate rate stays closer to the bottleneck bandwidth. This proves the convergence of our rate algorithm to an optimum $\mathbf{x}^*$.

■

Notice that, the scaling factor introduced in $W_k(x_k)$ is the square of the constraint $h_k(r_k)$ and this is one form of additional penalty used by sequential nonlinear programming methods.

The penalty function $Q_{R_k}(\mathbf{x})$ is chosen to be the maximum of all the penalties across the user's path.

Corollary: Our pricing vector $\mathbf{W}(\mathbf{x}) = (W_1(x_1), W_2(x_2), .., W_l(x_l), .., W_N(x_N))^T$ in (62) is bounded and it results in a convergence to optimum rate $\mathbf{x}^*$.

In order to demonstrate that our penalty converges to a steady state, we perform experiments that compare our tangential controller with proportional fairness. We perform three experiments with a given three-flow topology below and show its significance.

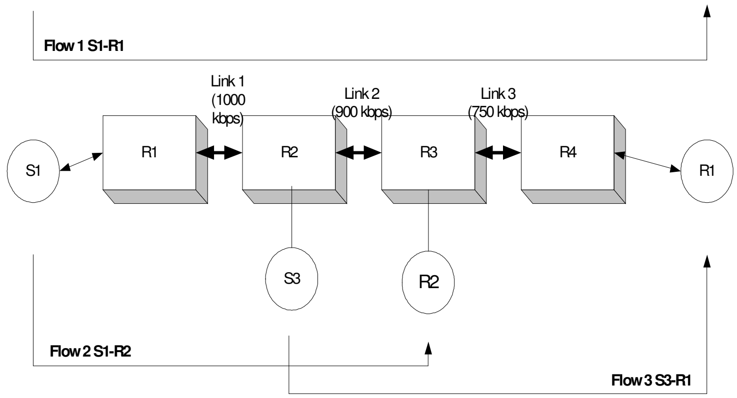

Figure 5 Parking lot topology with three flows.

Figure 5 shows the parking-lot topology for our experiments. We consider two sources, two receivers and three bottleneck links with three flows S_1-R_1, S_1-R_2, and S_3-R_1. Flows either use proportional or tangential controllers.

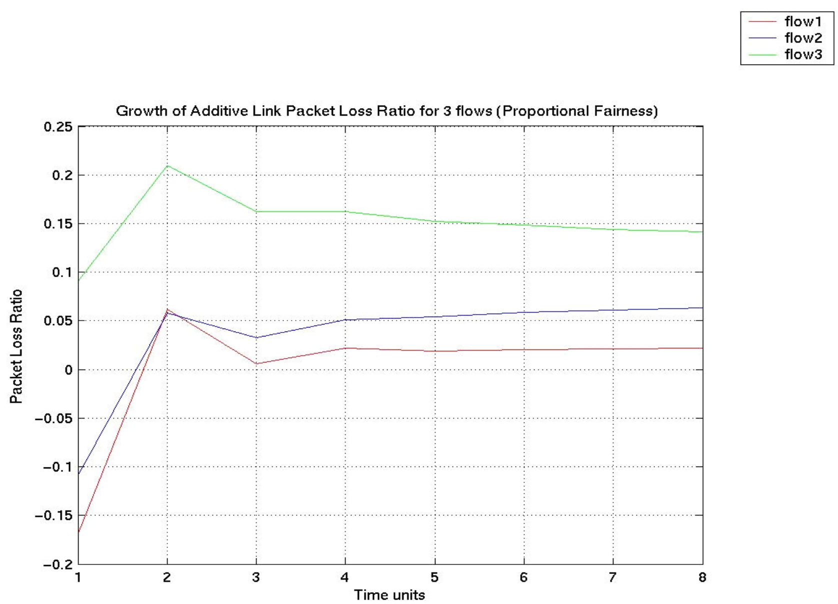

Figure 6 Growth of additive flow loss for proportional controllers.

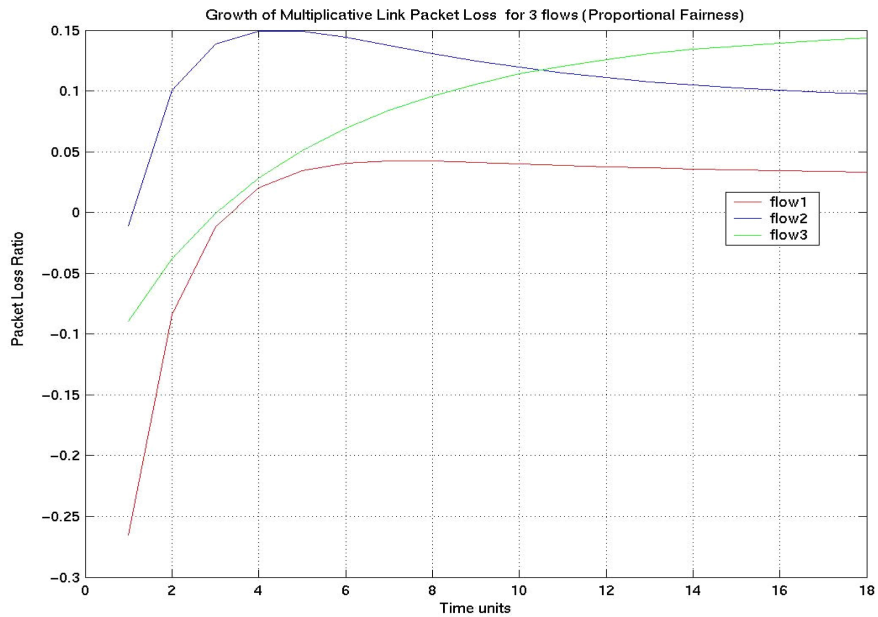

Figure 7 Growth of multiplicative packet loss for proportional controllers.

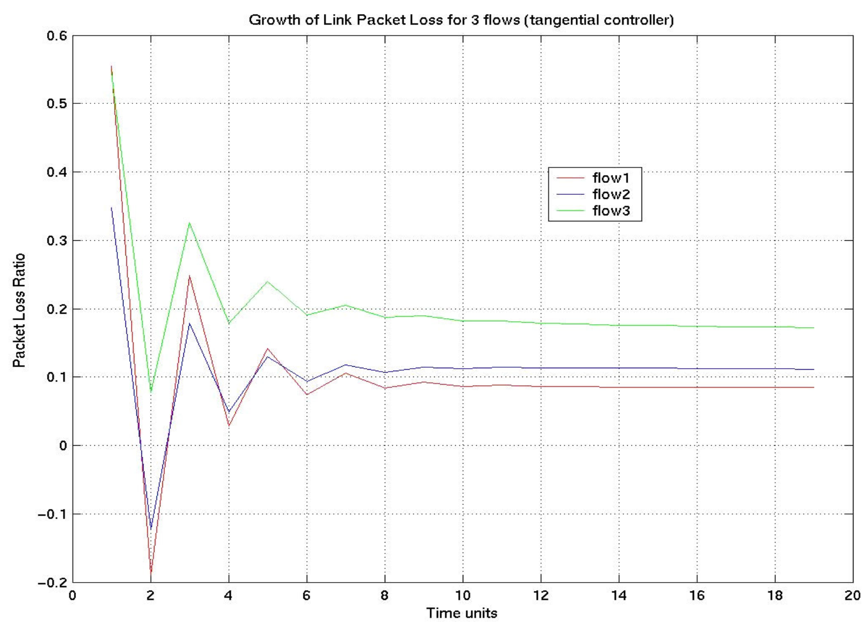

Figure 8 Growth of link packet loss for tangential controller.

In our experiments, we compare the cumulative flow losses of each of the three flows S_1-R_1, S_1-R_2, S_3-R_1 and across links 1, 2 and 3. Figure 6 and Figure 7 show the cumulative additive and multiplicative losses for proportional controllers and Figure 8 shows cumulative losses for our tangential controller. All flows start at initial rate α=10kbps with β=0.5. We find that additive losses were consistently smaller compared to multiplicative for the proportional controller. The cumulative loss of tangential controller is roughly twice that of additive controller suggesting less-aggressive link utilization. On the other hand, the individual link losses are up to 4 times smaller than that of the proportional (not shown above).

4 Pricing Scheme and Rate Adaptation

In this subsection, we demonstrate that our network pricing scheme establishes a unique equilibrium **x*** between the user paid price and the network allocated rates. Our previous subsection describes the necessary motivation for the additional packet loss penalty and we use it to design a pricing scheme in this subsection. We find that all users sharing a common link l pay additional penalty that leads to faster convergence to the optimum point and reduced overall packet loss for all flows through each link. Our aim now is to develop a network rate allocation scheme P with dependence on the following parameters:

$$\begin{aligned} &\text{P: } NETWORK(\mathbf{U}(\mathbf{x}), h(\mathbf{x}), p_l(\mathbf{x}), \mathbf{Q}(\mathbf{x})) \\ &\mathbf{U}(\mathbf{x}) = \left(U_1(x_1) \;\; U_2(x_2) \ldots U_k(x_k)\right)^T \\ &\mathbf{Q}(\mathbf{x}) = \left(Q_{R_1}(\mathbf{x}) \;\; Q_{R_2}(\mathbf{x}) \ldots Q_{R_k}(\mathbf{x})\right)^T \end{aligned} \tag{64}$$

where $NETWORK(\mathbf{U}(\mathbf{x}), h(\mathbf{x}), p_l(\mathbf{x}), \mathbf{Q}(\mathbf{x}))$ is the system that maximizes the overall utility function defined as: $MAX\left(\sum_k U_k(\mathbf{x}) - \sum_l p_l(\mathbf{x}) - Q_{R_k}(\mathbf{x})\right)$.

Utilities are user-dependent, strictly concave, increasing functions of user's throughput x_k constrained by $h(\mathbf{x})$. We now claim that our network problem P in (64) is solved by the pricing scheme (62). We conjecture that our pricing scheme indeed establishes a unique optimum $\mathbf{x}^*$ only if the source rate adaptation uses the tangent-vector algorithm. The gradient vector algorithm simply requires the rate differential equation for user k to vary according to the gradient of the objective function with respect to the rate x_k. The rate algorithm is given by:

$$\frac{dx_k}{dt} = x_k \frac{\partial M_k(x_k)}{\partial x_k} - \beta\beta_c \frac{\partial Q_k(x_k)}{\partial x_k} \tag{65}$$

Expanding the right side of (65), we get the following:

$$\frac{dx_k}{dt} = x_k \left(\alpha U'_k(x_k) - \beta \sum\nolimits_{l \in R} p_l(\mathbf{x})\right) - \beta\beta_c \frac{\partial Q_{R_k}}{\partial x_k} \tag{66}$$

Lemma 4: With a strictly concave increasing utility $U(\mathbf{x})$, the pricing scheme (62) forms a Liapunov function for the source rate control (65).

Proof: Recall that, a Liapunov function requires that the time derivative to monotonically increase for rates below $\mathbf{x}^*$ and negative for rates above the optimum. For a strictly concave increasing utility function $U(\mathbf{x})$, we observe that the rate change of the Liapunov $W_k(x_k)$ as given by:

$$\begin{aligned} \frac{d\mathbf{W}(\mathbf{x})}{dt} &= \sum_k \frac{\partial W_k(x_k)}{\partial x_k} \frac{dx_k}{dt} = \\ &= \sum_k \left(\frac{\partial M_k(x_k)}{\partial x_k} + \beta\beta_c \frac{\partial Q_k(x_k)}{\partial x_k} \right) \frac{dx_k}{dt} \end{aligned} \tag{67}$$

For the given rate adaptation algorithm given in (66), the derivative of $W_k(x_k)$ is positive definite for rates below capacity C of the bottleneck link, as show below:

$$
\begin{aligned}
&\frac{d\mathbf{W}(\mathbf{x})}{dt} = \\
&= \sum_k \left(\begin{aligned} & x_k \left(\frac{\partial M_k(x_k)}{\partial x_k} \right)^2 - \beta \beta_c \left(\frac{\partial Q_k(x_k)}{\partial x_k} \right)^2 + \\ & + (x_k - 1) \frac{\partial M_k(x_k)}{\partial x_k} \frac{\partial Q_k(x_k)}{\partial x_k} \end{aligned} \right)
\end{aligned}
\tag{68}
$$

The time derivative of the cost optimization function for rates below optimum $\mathbf{x}^*$ is positive definite because the partial derivatives of functions $M_k(x_k)$ and $Q_{R_k}(\mathbf{x})$ are strictly positive:

$$
\begin{aligned}
&\frac{d\mathbf{W}(\mathbf{x})}{dt} \geq 0 \\
&\mathbf{Q}\, \frac{\partial M_k(x_k)}{\partial x_k} \geq 0, \forall x_k \geq 0 \\
&\mathbf{Q}\, \frac{\partial Q_k(x_k)}{\partial x_k} \geq 0, \forall x_k \geq 0
\end{aligned}
\tag{69}
$$

Thus, the monotonically increasing Liapunov gradient with respect to time uniquely maximizes the rate evolution to optimum $\mathbf{x}^*$. However, as the aggregate rates exceed $\mathbf{x}^*$, the Liapunov time derivative becomes negative since the term $\frac{\partial M_k(x_k)}{\partial x_k} \leq 0$ becomes negative and dominates in (68). As shown in Figure 3, the constraint $h(\mathbf{x})$ takes effect as the rate approaches the bottleneck bandwidth and it is at this point that the Liapunov reaches its maximum. ■

Lemma 5: Our scaled pricing scheme in (62) introduces a smaller error factor along the rate trajectory compared to the Kelly's pricing scheme (63).

Proof: In order to prove this, we calculate the cumulative area of the error curve along the trajectory starting at the initial rate $\mathbf{x}_0$.

$$E_{W_k}(x_k,\eta)=\int_{x_k^0}^{x_k^*}\left(W_k(x_k)-\eta x_{W_k}\right)dx_k \tag{70}$$

Notice that the error along the trajectory of that of Kelly's cost function in (63) is given by:

$$E_{M_k}(x_k,\eta)=\int_{x_k^0}^{x_k^*}\left(M_k(x_k)-\eta x_{M_k}\right)dx_k \tag{71}$$

We prove that the error resulting from pricing scheme (62) is lesser than that of Kelly's (63). That is, error condition $E_{Wk}<E_{M_k}$ holds. Evaluating integral (70) along the curve and simplifying the expression $E_{Wk}-E_{M_k}$ yields:

$$\begin{aligned}&E_{W_k}-E_{M_k}=\frac{\beta\beta_c}{3}\left(3x_k^*C-\left(x_k^*\right)^2-3C^2\right).\\&E_{W_k}-E_{M_k}\le 0\ \text{ as }\ \lim x_k^*\to C\end{aligned} \tag{72}$$

■

This establishes that for condition $E_{Wk}-E_{M_k}\le 0$, we must have $\beta_c>0$. Notice that if β_c=0, the controller is equivalent to the well-studied proportional controller.

5 Stability Analysis

In this section, we establish local stability of the tangential controller with homogeneous delays using a fluid approximation and the transfer-function method. Using the Jacobian linearization around the equilibrium point, we study the tolerance to perturbation and prove that our controller indeed has a high phase margin. Recall that, controllers possessing high-phase margin are more robust against perturbation against feedback delays. For a given finite time delay, we prove that the open loop transfer function of our plant controller does not encircle negative unity only if the decrease parameters are bounded. That is:

$$\begin{cases} \pi n < \beta(1+\beta_c)Te^{\alpha' T} < \dfrac{(2n+1)}{2}\pi, \quad n \geq 0 \\ \beta_c \geq 0 \end{cases}. \tag{73}$$

where n is a non-negative integer. We thus establish that our controller is delay-tolerant as long as the delay T is finite.

5.1 Conditions for Local Stability

Consider the generic model depicted in Figure 1 where the sources in sets S_1 and S_2 have an associated route to their unique receivers in the receiver pool. We assume constant round-trip delay of T for all sources in this set. Consider a simple model consisting of a single bottleneck link with an arbitrary number of flows across the link. It is possible to extend such a model to include more bottleneck links.

Consider the input rate transfer vector $\mathbf{X}(s) = (X_1(s), X_2(s),.., X_l(s),..X_N(s))^T$ of source rates whose open-loop system NxN matrix is $G(s)$. The open-loop vector is a function of state matrix A, delayed state-matrix A_d, input matrix B and input-output matrix C. These matrices are defined as follows:

$$
\begin{aligned}
A &= \frac{-N\alpha\beta}{C\beta + N\alpha} I_{N\times N} \\
A_d &= \frac{-C\beta^2}{N(C\beta + N\alpha)} U_{N\times N} \\
B &= U_{N\times N} \\
C &= I_{N\times N}
\end{aligned} \quad . \tag{74}
$$

Additionally, we define a delay matrix $D = diag\{e^{-sT}\}$ of equal delay T for all sources. The open-loop transfer function of our system with multiple-inputs and multiple-outputs is given by:

$$
G(s) = C(sI - A - A_d D)^{-1} B \, . \tag{75}
$$

The open-loop transfer matrix **G**(s) is constant having the same elements across all rows and columns because the matrix $M(s) = (sI - A - A_d D)^{-1}$ is symmetric and circulant. The elements in the constant matrix $G(s)$ is given by:

$$
\begin{aligned}
G_{ik}(s) = &\frac{s + \alpha' - \beta'' e^{-sT}}{(s + \alpha' + (N-3)\beta'' e^{-sT})} \times \\
&\times \frac{1}{(s + \alpha' + \beta'' e^{-sT})^{N-1}}, \forall i, k
\end{aligned} \quad . \tag{76}
$$

where the constants α' are β'' are defined as below:

$$
\begin{aligned}
\alpha' &= \frac{N\alpha\beta(1+\beta_c)}{C\beta(1+\beta_c) + N\alpha} \\
\beta'' &= \frac{C\beta^2(1+\beta_c)^2}{N(C\beta(1+\beta_c) + N\alpha)}
\end{aligned} \quad . \tag{77}
$$

Lemma 6: Consider a closed-loop feedback system with transfer function in (75) for a network with single bottleneck link *l* consisting of *N* flows. Assume that each flow (or the user) *k* has a non-zero positive delay *T*. Then, the system is locally asymptotically stable if the following bound holds:

$$0 < \beta(1+\beta_c)Te^{\alpha' T} < \frac{\pi}{2}. \tag{78}$$

Proof: We notice that the characteristic polynomial of our open-loop system transfer function is the determinant of circulant matrix *M* given by the following characteristic polynomial for $N > 1$ flows:

$$\begin{aligned}&(s+\alpha'+(N-3)\beta''(1+\beta_c)e^{-sT})\times\\&\times(s+\alpha'+\beta''(1+\beta_c)e^{-sT})^{N-1}=0\end{aligned}. \tag{79}$$

Nyquist stability criterion requires that the roots of (79) be lesser than one. In our case, we prove that our polynomial term $s+\alpha'+\beta''(1+\beta_c)e^{-sT}$ has negative real roots resulting in the stability our controller (66). The roots of this polynomial are given by Lambert's *W* function [59] and the only negative range of values for which our polynomial holds is given by (78).

■

We plot the frequency response of the open-loop transfer polynomial in Bode diagram in Figure 9 for three different values of delay *T*.

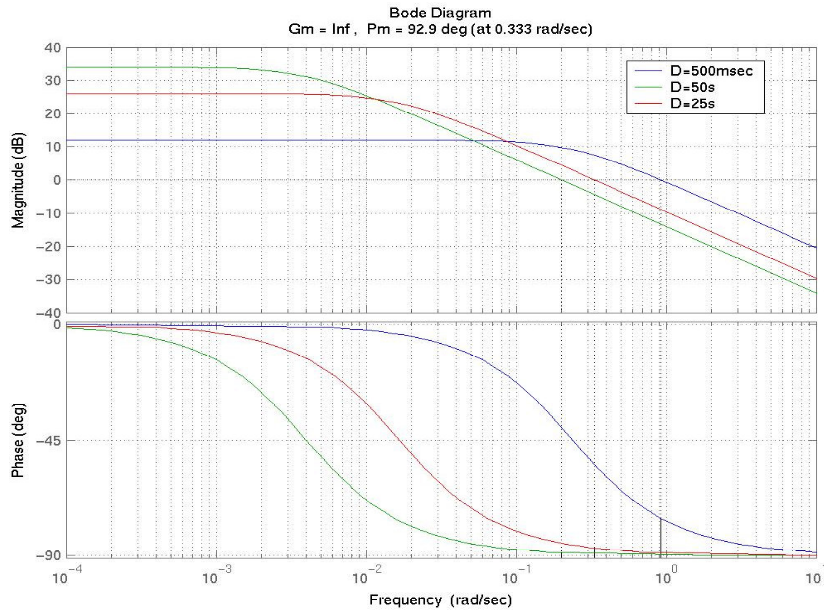

Figure 9 Bode plot of open-loop transfer function for various delays.

CHAPTER VI

SIMULATIONS

1 Introduction

In this section, we perform ns2 simulations to verify our theoretical results. We develop discretized delay-tolerant source and sink agents that emulate the behavior of the various controllers using end-to-end and explicit AQM loss feedback. We evaluate the stationarity and convergence properties of the proportional controller with additive penalty. We perform experiments using three explicit loss feedback AQM schemes, which include max-min, proportional fairness, and tangential loss adjustments. Our simulation results show that proportional controller suffers from overestimation of aggregate rates at the bottleneck links. Moreover, a sliding-window average rate calculation requires estimation of sliding loss. Our observation is that extrapolating the aggregate rate or the averaged loss estimation leads to AIMD-type large oscillations. Since one of our motivations is keep our steady-state oscillations closer to the bottleneck bandwidth, we investigate on developing AQM-based schemes.

2 Simulation Setup

We use an AQM-based loss calculation scheme in our simulations. Routers calculate aggregate link losses at only a specific AQM interval and the sources respond only once during this interval. We consider a standard parking-lot topology with three flows and two intermediate bottleneck links with link capacity 500kbps. All sources start at the same time with an initial rate of 20kbps. We set the increase/decrease constants to α=20kbps, β=0.5 and $\beta_c=1/C$, where C is the bottleneck capacity. Max-min fairness results when our AQM scheme updates the packet header with the largest packet loss of the most congested link across the path from the source. Similarly, proportional fairness is achieved by adding across all the links along the path. In addition to the proportional controller loss, the tangential controller requires a loss scaling factor that is calculated and inserted by the AQM scheme in the packet header.

3 Max-min and Proportional AQM Feedback

Figure 10 shows the rate evolution of three flows of max-min and proportional controllers. The figure shows the convergence of three flows with bottleneck bandwidth of 500kbps. Flow 2 starts 10 time units after flow 1 and flow 3 starts 20 units after flow 1. The initial rates of these flows were set to 20kbps, 250kbps and 500kbps respectively. We observe that max-min converges at 256kbps whereas proportional converges slower to 210kbps, but with less link loss.

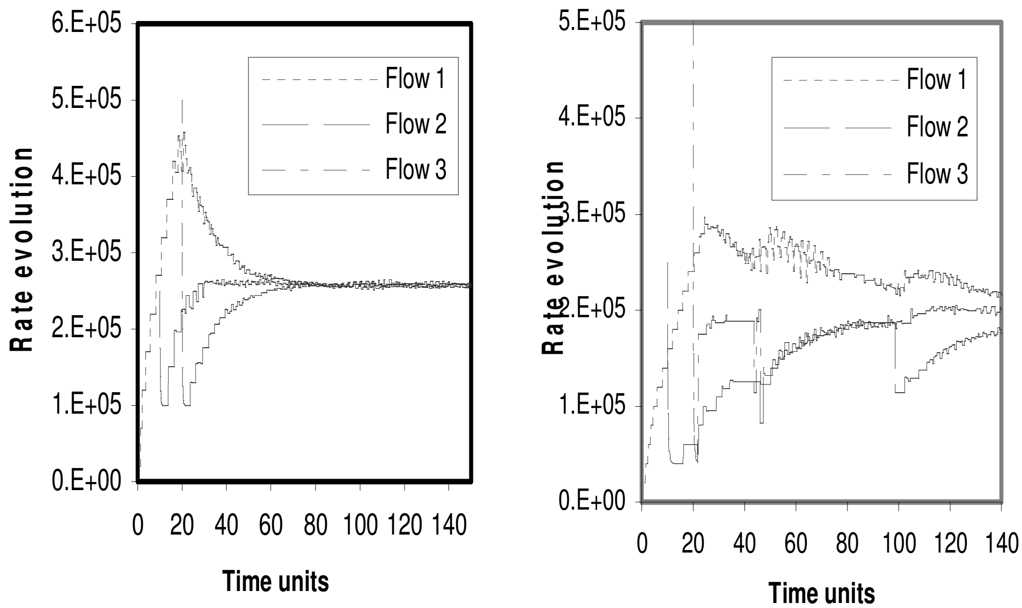

Figure 10 Rate evolution of max-min (left) and proportional controllers (right).

4 Proportional and Tangential AQM Feedback

We use the same agents to adjust the AQM feedback for either proportional (additive penalty) or tangential controllers. Figure 11 shows that the tangential controller is capable of achieving convergence to fairness much closer to the link bottleneck with much lesser packet loss. While proportional controller converges at 256kbps, our fair convergence occurs at around 200kbps for all three flows.

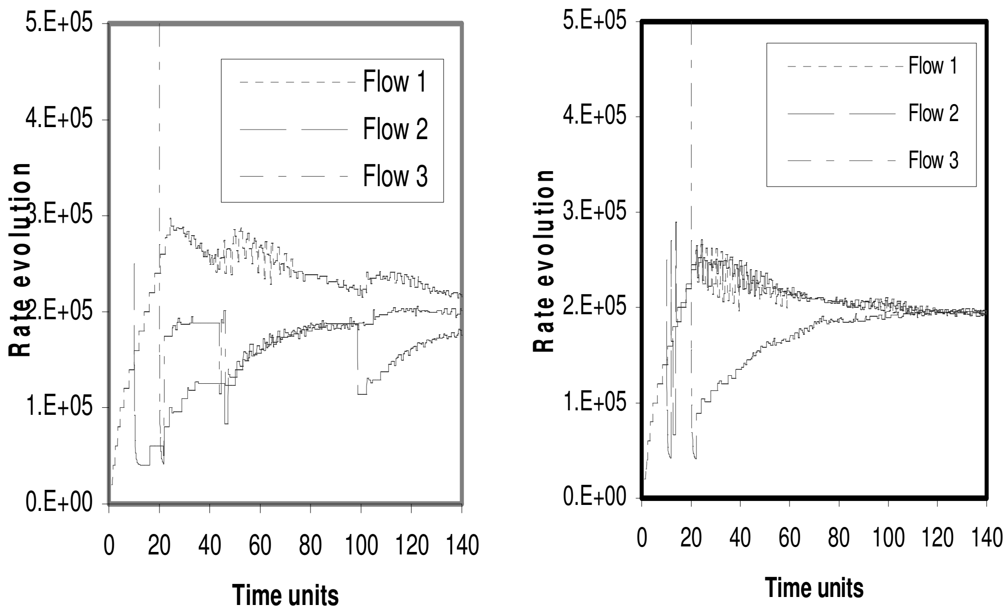

Figure 11 Rate evolution of proportional (left) and tangential controllers (right).

Figure 12 demonstrates the loss across the two bottleneck links for proportional and tangential controllers. The aggregate positive scaling factor results in a much stable and smaller flow losses compared to the proportional controller.

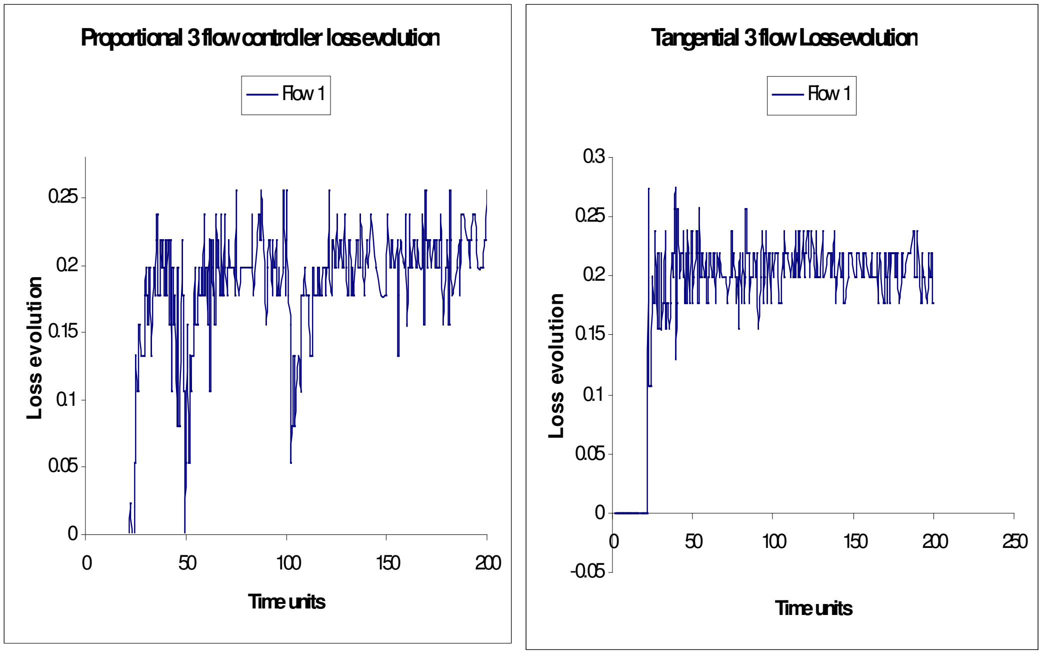

Figure 12 Flow loss evolution of proportional (left) and tangential controllers (right).

CHAPTER VII

CONCLUSIONS

In this thesis, we considered a family of nonlinear continuous feedback controllers based on utility functions, cost penalty and applied optimization theory to the problem. Specifically, we studied the stationarity and stability properties of logarithmic proportional controllers and compared it against our novel tangential controller. We derive our motivation from sequential nonlinear programming methods that allow additional penalty to objective function based on the square of linear constraint. Here, we showed that this additional penalty has immediate application to adjusting our loss penalty function and the network cost factor. We developed a novel tangential source rate controller whose trajectory followed closely that of source's own cost function and proved that the controller indeed minimized the aggregate losses. Using simulations, we also established its convergence and existence of stationary optimal rate. Finally, we established the asymptotic stability of the tangential controller and derived the upper bounds on the increase and decrease parameters α, β, and β_c.

Recollecting some of the motivation in the earlier sections, we see that our scheme well-defined the rate adjustment algorithm for the given cost function. The significance of our work is in its improvement of the speed of convergence and consistent reduction in packet loss compared to the proportional controller. Our tangential controller is thus suited for high bandwidth-delay product networks. In the future, we intend to study the aggressiveness of the loss scaling factor and whether such penalty may be applicable for general form of utility functions.

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VITA

Sai Ganesh Sitharaman was born in Hyderabad, India on July 03, 1974. He received his Bachelor of Engineering in computer science and engineering from Regional Engineering College (currently known as the National Institute of Technology), Trichy, India in May 1995. Since May 1995, he has held various positions as software designer, consultant and senior analyst with Wipro Technologies in India and Nortel Networks in the USA. He last held the position of member of technical staff with Advent Networks, USA during the year of 2001. From August 2001, he has attended the graduate school at Texas A&M University pursuing a Master of Science in computer science. His permanent address in India is: 12, Sai Kripa, Ayyanar Nagar, Madurai, India-625020.

The typist of this thesis was Sai Ganesh Sitharaman.