PROBLEMS OF FLUID FLOW IN A DEFORMABLE RESERVOIR

A Dissertation

by

ILDAR DIYASHEV

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2005

Major Subject: Petroleum Engineering
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Approved by:

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Major Subject: Petroleum Engineering
ABSTRACT

Problems of Fluid Flow in a Deformable Reservoir. (December 2005)

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This research is focused on development and enhancement of the model of fluid flow in a formation with stress-dependent permeability. Several typical axi-symmetrical problems of fluid flow in a multi-layered reservoir with account for wellbore storage and skin have been solved numerically. The permeability was assumed to be a function of the vertical deformation of the reservoir. This deformation is the result of changing stress-strain state in the elastic system, comprised of the reservoir itself and the surrounding rock mass. The change in the stress-strain state of the system is induced by pressure change in the layers of the reservoir.

Numerical results qualitatively agree with observed field behavior. Such behavior includes (1) deviation of an inflow performance curve from the straight-line relationship at pressures above bubble-point pressure, (2) time- and rate-dependence of well-testing derivative, (3) asymmetry of processes of production and of injection, and (4) inconsistent results between drawdown and buildup, or injection and falloff tests.

Based on the results, a procedure to estimate the parameters of the suggested permeability model is proposed.
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I would like to express my gratitude to the members of my committee – I was able to learn a lot from them when taking their classes and from many discussions. I’m especially grateful to Dr. Stephen A. Holditch, as his constant encouragement and support have made it possible for this work to go through.

Many thanks are due to Professors Eduard Skvortsov and Alexander Kosterin of Kazan State University. This research was originated under their guidance and supervision.

My parents and my brother have been very supportive throughout the years. Their love and understanding cannot be overestimated.
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1 INTRODUCTION

1.1 Complications of well test analysis in stress-dependent formations

The analysis of pressure transient data obtained during a conventional single-well test can provide estimates of in-situ permeability, near-wellbore damage or stimulation, and average reservoir pressure. The interpretation is normally based on reservoir, well, and fluid models that best approximate the principal features of fluid flow in porous media. In many cases, the existing fluid-flow models satisfactorily describe the well pressures measured during the test. However, there are situations when the observed pressure data do not “fit” predictions from any of the existing fluid-flow models, but rather exhibit distinctively different trends. For example, an inflow performance curve may deviate from the linear Darcy behavior to a much greater degree than can be explained by the multi-phase flow effects alone. Another example would be that the derivative curve on a diagnostic plot may behave as if the permeability-thickness product ($kh$) were not constant, suggesting the value of $kh$ either increases or decreases with time as the test continues. Such trends in the pressure data can be attributed to a number of causes. One factor that can lead to inconsistencies would be when both the rock and fluid properties are stress dependent.

It is well known from numerous laboratory experiments on cores that rock properties, such as permeability and porosity, measured under various loading conditions, change as the effective stress in a specimen changes. While the relative change of porosity is, generally, fairly small (within several percent), the relative change of permeability may be as large as one or two orders of magnitude. These effects can be more pronounced for naturally fractured reservoirs, where the ability of the rock to transmit fluid is determined by a network

This dissertation follows the style of Journal of Petroleum Technology.
of cracks and fractures. Reduction of pore pressure under constant load may lead to closure of cracks and fissures with a resulting decrease in permeability.

Field experiments also indicate that permeability in most rocks depends upon the effective stress. Oil production above the bubble-point pressure, or gas production above the dew-point pressure, if conducted without pressure support, will cause reservoir pressure to decrease, which will increase the effective stress in the formation. When the effective stress increases, the permeability of the formation will decrease. This decrease in permeability, coupled with multiphase flow, once the pressure drops below the bubble point or dew point, can cause substantial decreases in well productivity in stress-sensitive formations. Derivative analysis of pressure transient tests in such wells suggests that the permeability-thickness product changes with time. Another feature, characteristic of the stress-dependent permeability, is the observed asymmetry of pressure response for drawdown and buildup tests (or injection and falloff tests). Field data also show that there is a degree of interaction of the pressures in individual layers, which may be tens or hundreds of feet apart. The latter indicates that we need to account for the stress changes not only in the reservoir, but also in the surrounding rocks when we are analyzing pressure data in stress-sensitive formations.

Though it is obvious that permeability of any reservoir rock is affected to a greater or lesser degree by the change in effective stress, it can be fairly difficult to recognize the stress-dependent behavior in real data, as many other factors tend to offset such behavior. For example, an increase in the pressure drawdown may lead to closure of the cracks and fractures in the vicinity of a well, which results in decreased near-wellbore permeability and, therefore, a decreased production rate. At the same time, some layers may require such increased drawdown to initiate any production, either due to the non-Newtonian behavior of
the reservoir oil, or because of the smaller magnitude of permeability, or both. Other factors, such as multi-phase flow effects and non-Darcy flow will tend to mask the characteristics of stress-dependent permeability, as many pressure/rate-dependent factors quantitatively work in the same direction of the production rate decrease. Therefore, it can be difficult to recognize and quantify the effect of reduced permeability caused by increased effective stress.

Nevertheless, it is possible to identify and describe the stress-dependent permeability from pressure transient tests on water injection wells, or by observing long-term data from permanent downhole pressure gauges in oil or gas producing wells. However, the tests must be properly designed and analyzed using the best data if one is to quantify the effects of stress-dependent permeability in layered reservoirs.

1.2 Mechanics of stress-dependent permeability

The physics of stress-dependent permeability is based on the deformation of porous rock under changing effective stress. Increasing the effective stress on rock leads to reduction in the size of pores and pore throats which is a consequence of deformation of the matrix. This process can be described with a set of equations that relate changes in effective stress and porosity. Modern geomechanical simulators incorporate such equations to calculate changes in porosity induced by changes in pore pressure and effective stress. One then must assume that changes in porosity cause changes in permeability.

The existing understanding of stress-dependent permeability is based largely on experimental data. In the laboratory, a rock sample is saturated with a fluid, such as water or nitrogen, then the sample is placed in a sleeve so that an overburden stress can be applied to the outside of the sample and the pore pressure inside the rock sample can be set to simulate
the reservoir pressure. The single-phase fluid (water or gas) is flowed through the sample to measure permeability. By reducing the pressure of the fluid flowing through the rock sample, one can measure how the rock permeability changes as the effective stress increases. In most cases, we can establish the relationship between the changes in porosity with changes in permeability as the effective stress increases. The functional form of such relationships varies from a simple power-law dependence of permeability with porosity, such as Kozeny-Carman’s formula, to fairly complicated expressions, which may additionally account for the mechanical constants of the rock, grain size, or other properties, such as surface area. There is no single formula that adequately describes every formation, due to obvious wide variation of lithology, mechanical properties, and pore size distribution of those rocks.

Since it is obvious from the literature that rocks deform under changing effective stress, and the deformation causes changes in values of porosity and permeability, we will not discuss the details concerning the mechanics of the process in this research. Moreover, we will assume that vertical deformation of a reservoir is representative of any of the reasons for permeability decrease – pore and pore throat size reduction, repacking of the matrix, closure of cracks and fissures, etc. Instead of having to describe various mechanical processes, which are impossible to measure under in-situ conditions and, therefore, to verify, we will consider the integral effect of changing effective stress on the two readily available characteristics of a well – flow rate and wellbore pressure. In our work, based on the experimental observations, the permeability is assumed to be a simple exponential function of the vertical deformation only, i.e. \( k = k_0 \exp(-\beta \varepsilon) \). Here \( k_0 \) and \( k \) are the values of permeability at initial and current reservoir pressure, \( \varepsilon \) is the relative vertical deformation, and coefficient \( \beta \) characterizes the sensitivity of permeability to deformation. Relative
vertical deformation is, of course, a function of the effective stress. This approach captures the essence of the stress-dependent behavior of permeability, which allows us to investigate the response of a reservoir to a wide range of producing conditions and is simple enough to allow for identification of the parameters of flow from well test data.

1.3 **Research approach**

To resolve the differences between theoretical predictions of flow rate vs. pressure in a well test, we need to account for physical factors, such as rock deformation. It is known that the weight of the overburden is compensated in the reservoir by stresses in the rock matrix and by the fluid pressure in the pore space. Change in pore pressure will alter the stress-strain state in the reservoir and in the surrounding rocks. The respective deformation induces changes in the structure of the pore space and permeability. Thus, permeability implicitly depends on fluid pressure via deformation, which can help explain non-linear behavior of flow rate vs. pressure that is sometimes observed in field data.

The odd behavior of some well test data can be, to some extent, explained by spatially varying stress state in the reservoir. Such non-uniformity results from two factors. The first factor is due to drilling. Whenever a shale layer is present above and near a producing oil or gas reservoir, drilling allows the shale to be squeezed into the well, which causes relaxation of both the shale layer and the reservoir rock layers near the shale and near the wellbore. Reduced overburden pressure in the zone of relaxation is compensated by the “arch” effect of the overlaying layers. The reservoir beyond the “relaxed” zone remains in an undisturbed condition. The second factor contributing to deformational non-uniformity is associated with fluid pressure redistribution caused by oilfield operations, such as the production of oil and/or gas.
It should be noted that the importance of the described deformational effects depends upon the magnitude of the deformations. Consider, for example, a simple one-dimensional case of an elastic deformation. According to the hypothesis of constant total stress, the magnitude of strain is equal to the ratio of change of pore pressure to Young’s modulus, which in practice is about 0.0001. This means that for a 10 m thick reservoir, the change in thickness will be only 1 mm. Nevertheless, such a tiny change can significantly impact the flow characteristics, especially if the permeability depends on the elements of the structure that are sensitive enough to deform. In naturally fractured formations such elements are the natural fractures.

Let us consider a uniform horizontal layer with a system of vertical fractures\(^1\). The reservoir is characterized by substantial anisotropy of mechanical properties. Young’s modulus in the vertical direction is essentially the same as the formation matrix. However, Young’s modulus in the horizontal direction can be much less than the value in the vertical direction if there are a lot of vertical natural fractures. The vertical natural fractures substantially reduce the effective horizontal stiffness when such fractures are open and conductive. If the fractures are mineral filled or sealed, the stiffness in the horizontal direction will not be reduced as much as when the fractures are open. Assuming that the overburden pressure is constant, the weight of the overburden, \(\Gamma\), is compensated by the stress \(\sigma_z\) in the porous matrix and by the fluid pressure in the pore space as follows:

\[
\Gamma = \phi_1 p_1 + (1 - \phi_1) \phi_2 p_2 - (1 - \phi_1)(1 - \phi_2) \sigma_z
\]

subscripts 1 and 2 denote, respectively, fractures and blocks; \(z\) is vertical coordinate. (Here and henceforth we use engineering, rather than geosciences sign convention for stresses, i.e.}
compression is negative and tension is positive. We denote overburden pressure, $P_{ob}$, as $\Gamma$ to indicate this accepted sign convention.)

Since $\phi_1 \ll 1$, the above expression can be written as

$$\Gamma = -\sigma_z^e + p_2$$

(1.1)

where $\sigma_z^e = (1-\phi_1)(\sigma_z + p_2)$ is the effective stress in blocks. A small change of pressure, $\delta p$, causes a small change of the stress state in the reservoir and in the surrounding formations. According to the hypothesis of constant overburden pressure and from Eq. 1.1, it follows that $\delta \sigma_z^e = \delta p_2$. For the considered conditions the blocks between parallel fractures experience only simple tension (compression), and the respective principal components of strain, $\varepsilon$, are

$$\varepsilon_z = \frac{\delta p_2}{E} \quad \varepsilon_x = \varepsilon_y = -\nu \frac{\delta p_2}{E}$$

where $E$ is Young’s modulus and $\nu$ is Poisson’s ratio.

For this case of linear isotropic behavior of the blocks, the values $\varepsilon_x$ and $\varepsilon_y$ determine the change in the width of the fractures. Compared to the thickness of the overlying formations, the reservoir is of negligible thickness and can be reasonably assumed to deform mainly vertically, so that the sum of the lateral dimension of a block, $a$, and fracture width, $w$, can be considered virtually constant: $\delta a + \delta w = 0$. Hence

$$\delta w = -\delta a = -a \varepsilon_x = -va \frac{p_2 - p_2^0}{E}$$

The pressure of fracture closure $p^*$ is determined from the condition $\delta w = -w^0$ (superscript $0$ denotes initial state, prior to applied $\delta p$), therefore
\[ p^* = p^0_2 - \frac{E w^0}{w a} \]

Since \( p^* \) should be greater than 0, then if \( w^0/a > \nu p^0_2 / E \) the fracture will not close at all.

Let us estimate the change of fracture permeability due to change of fluid pressure in blocks, \( p^0_2 \). For \( E=10^4 \) MPa, \( \nu=0.2 \), and assuming that a fracture transmissibility is proportional to \( w^3 \), a 0.2 m long, 1 \( \mu m \) wide fracture (\( a/w^0 = 2*10^{-4} \)) will change its permeability by a factor of 8 (\( \delta w = 0.5w \)) due to a 1 MPa (145 psi) change in pressure.

A more complicated analysis and similar estimates are available for inclined fractures\(^1\). The estimates illustrate the expected more significant permeability change, which is the greater the closer is the fracture orientation to horizontal.

Even though the above discussion refers to naturally fractured formations, the discussion may also be useful for cases when the formation is not naturally fractured. Indeed, disturbances introduced when drilling a well (including a zone of relaxation around a wellbore) will result in creation of microcracks and fissures in a near wellbore zone. Hydraulic fracturing, when performed, can as well be responsible for creating such microcracks around the fracture. Production or injection will also alter the conditions in the near wellbore zone to some extent, depending on the degree of formation consolidation. It is understood that sand production is sometimes caused by an increase in drawdown pressure which increases the effective stress causing formation failure near the wellbore.

1.4 Objectives and results

The main objective of this research is to investigate the effects of stress-dependent permeability upon transient production/injection behavior of a well. The understanding of
such impact is required in many applications, including reservoir simulation and pressure transient testing. In this research, we build upon a known solution of the coupled problem of elastic deformation and fluid flow in porous media.

Following approach, presented in\textsuperscript{2,1}, the problem of elasticity theory is formulated for an elastic semi-space, which includes the reservoir and the overburden. The solution of the problem results in an operator relationship between the vertical deformation of the reservoir and the pressure distribution in the reservoir. Fluid production or injection alters the stress-strain state in the reservoir and in the overburden, which causes elastic vertical deformation in both the reservoir and the surrounding layers. This, in turn, affects the fluid flow, as the permeability is assumed to be a function of the strain.

In this work, we solve several axisymmetrical problems of fluid flow with typical boundary conditions: constant well rate or wellbore pressure, and with closed or constant pressure outer boundary. The reservoir may consist of an arbitrary number of layers. We investigate the effect of layers’ elastic constants on the flow, as well as how the pressure and deformation fields in layers interact with each other.

A computer program was developed that allows for calculating non-linear problems for flow of a single-phase compressible fluid in a multi-layered circular reservoir with cross-flow between layers. The problems were solved with conventional inner and outer boundary conditions that account for wellbore storage and skin for cases of both production and injection.
2 PREVIOUS WORK

As the petroleum industry moves towards producing oil and gas from increasingly challenging reservoirs, such as low-permeability, naturally fractured or high-pressure, high temperature reservoirs, the problem of understanding and adequately describing such reservoirs also increases. One of the tools used to obtain information about a reservoir is pressure transient testing. Unfortunately, in many cases the recorded pressures can not be matched using any of the existing models. The reasons for such discrepancies are fairly well understood qualitatively and may include, among others, pressure-dependent fluid properties, rate-dependent non-Darcy flow effects, and stress-dependence of formation properties. If quantitative descriptions of the first two factors do not commonly represent a difficulty, a comprehensive description of the third factor is, and, probably always will, remain a problem. There are several reasons why stress-dependent reservoir properties are not widely used in pressure transient analysis solutions. The principal reasons include inability to directly measure the properties under in-situ conditions and limited applicability of laboratory results because of inherent problems with measurements on cores and lithology variations and heterogeneities of the reservoir rocks. Also, many analysts do not recognize the need to take stress-dependent reservoir properties into account during the analysis of the data.

Based on the laboratory results, permeability is the rock property that may undergo significant changes during production. Other properties, such as porosity and rock compressibility, change comparatively less. Efforts of many researchers have been focused on the mechanisms of permeability reduction and incorporation of such mechanisms into flow problems.
Raghavan et al.\textsuperscript{3} defined a pseudo-pressure function, which includes variable reservoir properties, as well as pressure-dependent fluid properties. Raghavan illustrated applicability of his approach in transforming non-linear differential equations to diffusivity-type equations.

Based on the results of measurements on cores, Vairogs et al.\textsuperscript{4} concluded that permeability is a complex function of effective stress. Effective stress is a function of pore pressure, which, in turn, is a function of permeability. The authors investigated rock stresses around a wellbore and developed a model of the effect. They concluded that rock heterogeneities, such as hairline natural fractures and shale streaks, accentuate the permeability reduction due to change of effective stress.

In subsequent work, Vairogs and Rhoades\textsuperscript{5} applied the developed model to simulate drawdown and buildup tests. They applied methods of conventional analysis to simulated data and showed that the results from drawdown tests analyzed with conventional methods are not reliable when pressure dependent permeability exists, but buildup tests if conducted early in the life of the reservoir will give values of $kh$ about 10 percent below true values. Skin factors from either test are not representative of the true condition around the wellbore.

Evers and Soeiinah\textsuperscript{6} used an exponential form of dependence of permeability on effective stress and then related permeability and pressure based on the hypothesis of the constant overburden pressure. They showed how to recognize stress-sensitivity from a series of constant-rate drawdowns, if turbulence and viscosity changes are minimal.

Pedrosa\textsuperscript{7} introduced the concept of permeability modulus – fractional change of permeability with unit change in pressure, which is equivalent to exponential variation of permeability with pressure. Permeability modulus of zero corresponds to the absence of
stress-sensitivity. This approach allowed the author to generate type curves in term of this parameter.

Celis et al.\textsuperscript{8} applied the concept of permeability modulus to develop an analytical model for pressure transient analysis in stress-sensitive, naturally fractured formations. They showed that the variation of fracture properties with pressure can drastically change the estimates of parameters.

Buchsteiner et al.\textsuperscript{9} investigated stress-induced permeability reduction in fissured reservoirs and developed a model to account for directional permeability variation. These same authors\textsuperscript{10} developed an analysis procedure that allows one to measure stress-sensitive permeability of rocks and determine the permeability anisotropy.

Zhang and Ambastha\textsuperscript{11} introduced a stepwise permeability modulus to better describe experimental data and suggested use of long duration pressure transient tests to infer a permeability vs. stress relationship.

When formulating their models, the authors of all of the above cited references considered the reservoir only. Permeability was assumed to be a function of the effective stress and the functional form was normally selected based on the laboratory data\textsuperscript{12}. It was noted that the degree of the permeability reduction is different for homogeneous and fractured/fissured rock, so several authors developed models that allowed them to estimate permeability change vs. change of effective stress for various configurations of pore space – packing of spheres, or cracks, as well as crack orientation\textsuperscript{13,14}.

It also was noted that the properties of the over-, under-, and side-burden (rock mass that surrounds the reservoir) should be also taken into account when examining problems on a field-wide scale\textsuperscript{15,16}. Since most of the solved problems were on a limited scale, both in
time (duration of a drawdown or a buildup test) and in space (area drained by a well), the authors implicitly assumed that the surrounding rocks do not have a significant impact on the flow\textsuperscript{17}.

It is known, however, that substantial deviation of pressure from the initial pressure field in a reservoir is possible during field development. This deviation causes stress-strain state to change not only in the reservoir but also in the surrounding rocks. This, in turn, leads to alteration of the flow characteristics of a reservoir and production indicators of a well.

These and other effects concerned with the influence of the stress-strain state on the fluid flow were investigated in Former Soviet Union by Basniev, Ban, Gorbunov, Yentov, Zazovsky, Zotov, Maximov, Malakhova, Marmorshtein, Nikolayevsky, Ramazanov, and others. They suggested various models to account for interaction of fluid flow and deformation processes.

Yentov and Malakhova\textsuperscript{2} discuss several such models and present the scheme according to which the reservoir within the rocks is modeled as a cut. The approach allowed the authors to investigate the stress-strain state in the rocks adjacent to the reservoir, including the well vicinity. Yentov, et. al.\textsuperscript{18} obtained a qualitative estimate of the change in the well’s productivity index (PI) following step pressure change in adjacent layer. Zazovskii\textsuperscript{19} developed a way to account for shearing stresses in the vicinity of a well.

Nikolayevsky and Ramazanov\textsuperscript{20} used the approach suggested by Yentov, et. al.\textsuperscript{2}, but instead of making assumptions, they calculated the pressure using transient flow equation. The same authors\textsuperscript{21} expanded on their previous method\textsuperscript{20} with account for wellbore radius, which allowed them to estimate the magnitude of the shearing stress on the casing.
Pressure and deformation fields induced by a constant rate well in a thick aquifer were investigated by Lewis, et. al.\textsuperscript{22} using the finite element method. The processes in the aquifer and surrounding rocks are described using equations of consolidation, and the permeability is considered to be vanishingly small. The conclusion was that similar problems should be solved using a system of coupled consolidation equations.

It is known that formation permeability depends on the fluid pressure in the reservoir\textsuperscript{23}. This dependence is a result of formation deformation caused by changes in the stress-strain state in the system “reservoir – surrounding rocks”. The dependence of permeability on the deformation of the skeleton was explained by Lewis, et. al.\textsuperscript{24}. Hence, the interaction of the stress-strain state with the pressure field in the reservoir should be taken into account.

Work of several researchers indicates that permeability of a core sample can be significantly affected by change in confining and pore pressure. This effect can be explained by repacking of the grains and by opening or closure of microcracks. The results show that deformation of low-permeability samples is mainly elastic. In Fig. 2-1 presented are results obtained by Gubanov\textsuperscript{25}. These are permeability change of several core samples under constant confining pressure and changing pore pressure. Each data set has been fitted with a trend of the same color. It has been noted by many workers that the most permeability reduction occurs in low-permeability samples and the measured data can be satisfactorily fitted using exponential function.
Several authors\textsuperscript{24,26} noted that in a number of injection tests increase in injection pressure lead to rapid non-linear increase of injectivity. This observation also is explained by the development or the opening of existing microcracks in the near-wellbore zone.

The effects of permeability reduction as a result of repacking are discussed by Chernikov et al.\textsuperscript{27}. The experiments on specimens indicate that the permeability, when the specimen is affected by the stress caused by the drawdown increase, can reduce significantly – by tens of percent, and for some types of rock in the described conditions the permeability vanishes. These effects can take place within the drawdown ranges that are characteristic for actual field practice. This naturally leads to reduced production rates.
The deviation of an IPR from straight-line behavior can be explained, among other factors, by deformation effects such as closure of microcracks and repacking following the decline of the reservoir pressure. It also has been noted\textsuperscript{28,29} that the observed non-linear behavior effects are not symmetric for production and injection.

Diyashev, et. al.\textsuperscript{1} provide a discussion on permeability reduction in a formation with strong dependence of permeability on effective stress via deformation. The deformation is calculated using model of Yentov et. al.\textsuperscript{2} and incorporating properties of the entire rock mass, i.e. the reservoir itself and the surrounding it rocks. The authors solved several axi-symmetric steady-state and transient problems with constant wellbore pressure and constant pressure outer boundary in a one- and two-layer reservoir and demonstrated the interaction of the deformational and pressure fields in adjacent layers. In the same reference and in the authors have considered linear flow and flow in presence of a circular zone around the wellbore, which may have different properties than the rest of the reservoir.

As follows from the above, it is interesting to evaluate the impact of deformation of the “reservoir – surrounding rocks” system on the pressure behavior and productive indicators of a well that operates in steady-state or transient regimes in a multilayer reservoir.
3 METHODOLOGY

In this section we present the formulation and solution of the elasticity theory problem, followed by the dimensionless equations and boundary conditions, which describe fluid flow in a reservoir with stress-dependent of permeability caused by deformation. We then discuss the additional dimensionless parameters, introduced by accounting for the deformation. Finally, we present the details of numerical solution of the problems of elasticity theory and fluid flow.

3.1 Formulation and solution of the elasticity theory problem

Consider a horizontal, porous, fluid-saturated reservoir of thickness $h$ at depth $H$, penetrated with a well. We will use cylindrical coordinate system $(r, z, \varphi)$, where $z$-axis coincides with the well axis (Fig. 3-1). The plane $z = H$ is perpendicular to the direction of the gravitational force and coincides with the surface. The plane $z = 0$ lies in the middle of the reservoir. The non-uniform pressure distribution in the reservoir is assumed to be axisymmetrical.

For the purpose of the subsequent analysis of fluid flow in an elastic reservoir, interacting with surrounding rocks, we will use the model that was suggested by Yentov and Malakhova. According to this model, the reservoir is considered to be thin ($h \ll H$), so that only its deformations in the vertical direction are important. It is assumed that the reservoir has no impact on the deformations of the surrounding rocks in the radial direction. Thus, uniform elastic semi-space bounded by plane $z = H$ with a cut at $z = 0 \pm 0$ corresponds to the reservoir system.
One could visualize a reservoir in this model as a set of tiny springs separating two elastic domains, which represent the surrounding rock mass (Fig. 3-2). Decrease of reservoir pressure near a well in such visualization is equivalent to compaction of the springs in the vicinity of a well. This compaction leads to deformation of over- and underburden. As the surrounding rocks have certain elastic properties, the deformation will not be local, i.e. limited only to the near-wellbore zone, where the pressure reduction has occurred. The springs further away from the well will also experience compaction, caused by the deformation of the surrounding media (Fig. 3-2b). At the same time, because of the assumption of a thin reservoir, it is reasonable to neglect stresses and strains in all but vertical direction, as illustrated in Fig. 3-3. Of course, when either of the model assumptions do not hold, i.e. the reservoir thickness or extent are comparable to its depth, one needs to properly account for all pertinent stresses and strains.
Figure 3-2. Visualization of the model

Figure 3-3. Schematic of radial stress in (a) thin, and (b) thick reservoir
Because of the assumed axial symmetry of the pressure distribution, the stress-strain state will also be axisymmetrical, so that the components of the stress tensor $\tau_{r\phi} = \tau_{z\phi} = 0$, and the component of the displacement vector $u_\phi = 0$.

Further, we are not interested in finding the absolute values of pressure and displacement, but only the deviations from the values that correspond to a reservoir with constant pressure. In the derivations, which follow, we imply such deviations.

Elastic semi-space is characterized by Poisson’s ratio, $\nu$, and Young modulus, $E$. The displacement vector $\mathbf{u}$ satisfies the equation

$$\text{grad} \ \text{div} \ \mathbf{u} - \frac{1-2\nu}{2(1-\nu)} \text{rot} \ \text{rot} \ \mathbf{u} = 0$$

(3.1)

with the boundary conditions

$$z = H: \quad \sigma_z = \tau_{rz} = 0$$

(3.2)

$$z = 0: \quad [\sigma_z] = [\tau_{rz}] = 0, \quad [u_r] = 0$$

(3.3)

$$[u_z] = \frac{\sigma_z + \eta p(r)}{c}$$

(3.4)

Here the square brackets denote the difference (jump) of a value on the top and bottom borders of the reservoir. $\sigma_z$ and $\tau_{rz}$ are components of the stress tensor. $u_r$ and $u_z$ are components of the displacement vector $\mathbf{u}$. The magnitude of $\eta$ characterizes the degree that the reservoir pressure, $p(r)$, is transmitted to the reservoir top and bottom boundaries; it is close to unity. The rigidity of the vertical deformation, $c$, can be expressed as $c = E/h$, where $E$ is the Young modulus for the reservoir. A linear elastic constitutive law is used.

The elasticity theory problem Eqs. (3.1)-(3.4) is "external" in relation to the fluid flow in the reservoir and implicitly states the operator relationship of vertical deformations of the
reservoir with the pressure change in it. To obtain the operator explicitly, we need to have an analytical solution of the problem. Such a solution was obtained in by using a stress function. Alternatively, we can use the Papkovich-Neuber representations of components of the stress-strain tensor and displacement vector for the axisymmetrical case via two harmonic functions $\Phi$ and $\varphi$.

$$\sigma_z = 2(1-\nu) \frac{\partial^2 \Phi}{\partial z^2} - \frac{\partial^2 \varphi}{\partial z^2} - z \frac{\partial^2 \Phi}{\partial z^2}$$

$$\tau_{rz} = \frac{\partial}{\partial r} \left[ (1-2\nu)\Phi - \frac{\partial \varphi}{\partial z} - z \frac{\partial \Phi}{\partial z} \right]$$

$$u_z = \frac{1+\nu}{E} \left[ (3-4\nu)\Phi - \frac{\partial \varphi}{\partial z} - z \frac{\partial \Phi}{\partial z} \right]$$

$$u_r = -\frac{1-\nu}{E} \left[ \frac{\partial \varphi}{\partial z} + z \frac{\partial \Phi}{\partial z} \right]$$

We let layer $0 < z < H$ and semi-space $z < 0$ be, respectively, sub-domains 1 and 2. The functions $\Phi_j, \varphi_j,$ and $p$ (here and henceforth $j=1,2$) can then be expressed as Hankel’s integrals

$$0 < z < H : \quad \Phi_1(r, z) = \int_0^\infty \left[ A_1(\xi) e^{\xi r} + B_1(\xi) e^{-\xi r} \right] J_0(\xi z) \xi d\xi$$

$$\varphi_1(r, z) = \int_0^\infty \left[ C_1(\xi) e^{\xi r} + D_1(\xi) e^{-\xi r} \right] J_0(\xi z) \xi d\xi$$

$$z < 0 : \quad \Phi_2(r, z) = \int_0^\infty A_2(\xi) e^{\xi r} J_0(\xi z) \xi d\xi$$

$$\varphi_2(r, z) = \int_0^\infty C_2(\xi) e^{\xi r} J_0(\xi z) \xi d\xi$$

$$p(r) = \int_0^\infty P(\xi) J_0(\xi z) \xi d\xi$$
Satisfying boundary conditions Eqs. (3.2)-(3.4) and accounting for Eq. (3.5), we find $A_j(x)$, $B_j(x)$, $C_j(x)$, and $D_j(x)$. The vertical stress on the borders of the cut $z = 0$ is given by an integral

$$\sigma_z(r,0) = \int_0^\infty \frac{H \xi^2 [1 - f(\xi)] P(\xi) J_0(r \xi)}{a_0 + H \xi - H \xi f(\xi)} d\xi$$

where $a_0 = 4(1 - \nu^2) \frac{E_h H}{\nu \eta}$ and $f(\xi) = e^{-2H\xi}(1 + 2H\xi + 2H^2\xi^2)$, which coincides with the results, obtained in $^2$. Further development is the evaluation of the magnitude of $f(\xi)$ and neglecting the terms that include this small function. This is equivalent to setting $H$ to infinity and, after changing the integration variable, the magnitude of $\sigma_z(r,0)$ does not depend on $H$. Therefore, if $\sigma_z(r,0)$ is needed as a function of $H$, the calculation should be performed using the full integral representation, Eq. (3.6).

Since we are interested in the characteristics of a well, we assume that the reservoir is located at a significant depth ($h/H << 1$), which allows for the above simplification. Simplifying, we obtain

$$[u_z(\rho,0)] = \frac{a}{c} \int_0^\xi \xi p(\xi) A(\xi, \rho) d\xi$$

where $\rho = r/r_w$, where $r_w$ is a characteristic dimension, the wellbore radius

$$A(\xi, \rho) = \int_0^\infty \frac{y J_0(\xi y) J_0(\rho y)}{y + a} dy$$

$$a = 4\eta(1 - \nu^2) \frac{r_w E_1}{hE}$$

The internal integral (3.8) can be written as
\[
A(\xi, \rho) = \int_{0}^{\infty} J_{0}(\xi y)J_{0}(\rho y)dy - a\int_{0}^{\infty} \frac{J_{0}(\xi y)J_{0}(\rho y)}{y + a}dy = I_{1}(\xi, \rho) - aI_{2}(\xi, \rho)
\]

(3.9)

The integral \( I_{1} \) can be written as\(^3\)

\[
I_{1}(\xi, \rho) = \begin{cases} 
(2/\pi \xi)K(\rho / \xi), & \rho < \xi \\
(2/\pi \rho)K(\xi / \rho), & \rho > \xi 
\end{cases}
\]

(3.10)

where \( K(x) \) is full elliptical integral. We can re-write integral \( I_{2} \), which has an oscillating kernel in a form that allows for easy calculations. Let \( \alpha = \rho/\xi < 1 \) and \( b = a\xi \). Then

\[
I_{2}(\alpha, b) = \int_{0}^{\infty} \frac{J_{0}(\alpha y)J_{0}(y)}{y + b}dy
\]

Using the representation of a Bessel function of a complex argument \( u \) with Hankel functions of the third kind\(^3\)

\[
J_{0}(u) = \frac{1}{2}[H_{0}^{(1)}(u) + H_{0}^{(2)}(u)]
\]

consider contour integral

\[
I_{\Gamma_{1}} = \oint_{\Gamma_{1}} \frac{J_{0}(\alpha u)H_{0}^{(1)}(u)}{u + b}du
\]

along a closed contour, which is comprised of the part \((\gamma, M)\) of the real axis \( \text{Im}u = 0 \), quarter of circle \((M, iM)\), part \((i, iy)\) of the imaginary axis \( \text{Re}u = 0 \), and quarter of circle \((iy, i)\) \((\gamma << 1, M >> 1)\). It is known that \( I_{\Gamma_{1}} = 0 \).

Also consider an integral, obtained by replacing \( H_{0}^{(1)}(u) \) with \( H_{0}^{(2)}(u) \) in the integrand and taken along contour \( \Gamma_{2} \), which is symmetric over real axis to \( \Gamma_{1} \). When \( M \to \infty \), the contributions to \( I_{\Gamma_{1}} \) and \( I_{\Gamma_{2}} \) of the quarter circles of infinitely large radii tend to zero. For
\( \gamma \to 0 \) the contributions of the quarter circles of infinitely small radii depend on the behavior of the integrands in the vicinity of zero. It is known that here \( J_0(u) \to 1 \) and functions \( H^{(1)}_0, H^{(2)}_0 \) have integrable logarithmic singularity. Therefore, the contributions of small quarter circles also tend to zero, and the integral \( I_2 \) is equal to the half-difference of the integrals taken along the real axis

\[
I_2(\alpha, b) = \frac{1}{2} \lim_{\gamma \to 0} \left[ -\int_{i\gamma}^{i\gamma} \frac{J_0(\alpha u)H^{(1)}_0(u)}{u + b} du + \int_{-i\gamma}^{-i\gamma} \frac{J_0(\alpha u)H^{(2)}_0(u)}{u + b} du \right]
\]

Using the relations between Bessel functions of integer order and modified Bessel functions \( I_0(x), K_0(x) \):

\[
J_0(\alpha u) = I_0(\alpha \gamma), \quad H^{(1)}_0(u) = \frac{2}{\pi} K_0(\gamma)
\]

with account for equality \( H^{(2)}_0(\tilde{z}) = \overline{H^{(1)}_0(z)} \), obtain

\[
I_2(\xi, \rho) = \frac{2a}{\pi} \int_0^{\xi} \frac{I_0(\rho \gamma)K_0(\xi \gamma)}{\gamma^2 + a^2} d\gamma, \quad \rho < \xi
\] (3.11)

Similarly, we can show that

\[
I_2(\xi, \rho) = \frac{2a}{\pi} \int_0^{\xi} \frac{I_0(\xi \gamma)K_0(\rho \gamma)}{\gamma^2 + a^2} d\gamma, \quad \rho > \xi
\] (3.12)

And the expression \( A(\xi, \rho) \) in Eq. (3.9) is written in the form, which does not contain oscillating functions

\[
A(\xi, \rho) = \begin{cases} 
\frac{2}{\pi} \left[ \frac{1}{\xi} K\left( \frac{\rho}{\xi} \right) - a^2 \int_0^{\infty} \frac{I_0(\rho \gamma)K_0(\xi \gamma)}{\gamma^2 + a^2} d\gamma \right], & \rho < \xi \\
\frac{2}{\pi} \left[ \frac{1}{\rho} K\left( \frac{\xi}{\rho} \right) - a^2 \int_0^{\infty} \frac{I_0(\xi \gamma)K_0(\rho \gamma)}{\gamma^2 + a^2} d\gamma \right], & \rho > \xi
\end{cases}
\] (3.13)
Formulas (3.7), (3.8), (3.13) allow for calculating of the jump $[u_z(\rho,0)]$ for given pressure distribution $p = p(\rho)$, which allows to proceed to analysis of fluid flow in an elastic reservoir, i.e. to formulation and solution of the respective “internal” problem.

We can note that condition (3.4) is physically the rheological equation of one-dimensional flow consolidation of a linearly elastic reservoir. Such interpretation allows for natural generalization to the case of a layered reservoir.

Let the reservoir be comprised of $n$ layers with rigidities $c_j = E_j/h_j$, $j = 1,...n$ and $E_j, h_j$ are Young’s modulus and thickness of layer $j$. The jump of displacement of a layered reservoir is equal to the sum of the jumps of all layers

$$[u_z] = \sum_{j=1}^{n} [u_z^j]$$, \quad where \quad $$[u_z^j] = \frac{\sigma_z + \eta p_j(r)}{c_j}$$

and the condition (3.4) is replaced by

$$[u_z] = \sum_{j=1}^{n} \frac{\sigma_z + \eta p_j(r)}{c_j}$$ \quad (3.14)

Applying the solution procedure to problem (3.1)-(3.3), (3.14), we find

$$[u_z(\rho,0)] = \frac{a}{\rho} \int_0^\infty \xi A(\xi, \rho) \sum_{j=1}^{n} \frac{p_j(\xi)}{c_j} d\xi$$ \quad (3.15)

where $A(\xi, \rho)$ is given by Eq. (3.13) and

$$a = \frac{4\eta \mu_0 (1-\nu^2)}{E \sum_{j=1}^{n} (1/c_j)}$$ \quad (3.16)

The formulas (3.14)-(3.16) allow for effective calculation of the jump $[u_z(\rho,0)]$, and, subsequently, of individual layers’ displacements.
\[
[u_z'] = \left[u_z\right] - \eta \sum_{k=1}^{n} \frac{p_k}{c_k} + \eta \frac{p_j}{c_j} - \frac{1}{c_j} \sum_{k=1}^{n} \frac{1}{c_k} \sum_{j=1}^{n} \frac{1}{c_k}
\]

The obtained expressions allow one to proceed to the solution of the “internal” problem, which is analysis of flow in the reservoir. In simulating fluid flow in the elastic reservoir, we will assume that layer permeability is a function of its vertical deformation, and, for a multi-layer reservoir, dimensionless permeability \( K_j = k_j / k_j^0 = K_j(\varepsilon_j) \), where \( \varepsilon_j = \varepsilon_j(p_1, ..., p_n) = [u_z'] / h_j \), and \( k_j^0 \) are constants.

Further, using several simple models of fluid flow as examples and stating explicitly the functional form of permeability dependence on vertical deformation, we will consider the effect of accounting for interdependent flow and deformations processes on the production indicators of a well in a multi-layered reservoir.

We should again note that in the actual conditions of reservoir development, the reservoir deformations, \( \varepsilon \), are not greater than \( 10^{-3} \). Such small deformations do not usually significantly impact the reservoir permeability; however, in naturally fractured reservoirs, for which resistance to flow is determined by a network of fractures, such small deformations can cause changes in permeability by an order of magnitude or more. Similar response, according to laboratory measurements, can be expected in low-permeability un-fractured reservoirs. In this study we use a formal model of a uniform reservoir that strongly interacts with the surrounding rocks (allowing for large change of \( K \) with change of \( \varepsilon \)). Qualitatively, the model correctly describes actual behavior of a naturally fractured reservoir and other possible situations.
3.2 Transient fluid flow equations

For this problem, we consider radial, Darcy flow of a slightly compressible liquid of constant viscosity to a well located at the center of a circular drainage area. The reservoir is comprised of an arbitrary number of layers of uniform thicknesses with pressure-dependent permeabilities as given by solution of the “external” problem of the elasticity theory. A well is produced at constant total rate, or at constant flowing bottom hole pressure. Following Ehlig-Economides and Joseph\textsuperscript{32}, we introduce dimensionless variables ($n$ is the number of layers)

$$r_D = \frac{r}{r_w}, \quad C_D = \frac{C}{2\pi(\phi h)_i c r_w^2}$$

$$(kh)_i = \sum_{j=1}^{n} (kh)_j$$

$$(\phi h)_i = \sum_{j=1}^{n} (\phi h)_j$$

$$\kappa_j = \frac{(kh)_j}{(kh)_i}$$

$$\omega_j = \frac{(\phi h)_j}{(\phi h)_i}$$

$$p_{jD} = \frac{2\pi(kh)_i}{q_j \mu} p_j$$

$$t_D = \frac{(kh)_i}{(\phi h)_i \mu c r_w^2 t}$$

$$q_{jD} = \frac{q_j}{q_i} = -\kappa_j K_j r_D \frac{\partial p_{jD}}{\partial r_D}$$

The pressure equation for an individual layer in dimensionless variables becomes

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left[ r_D K_j \frac{\partial p_{jD}}{\partial r_D} \right] = \omega_j \frac{\partial p_{jD}}{\kappa j \partial t_D}$$

Initial condition

$$p_{jD}(r_D, 0) = 0$$

Inner boundary condition is formulated using the following expressions
\[ P_{wD} = \left( p_{jD} - s_j r_D K_j \frac{\partial p_{jD}}{\partial r_D} \right)_{r_D = 1} \]

\[ 1 = \left( C_D \frac{dp_{wD}}{dt_D} - \sum_{j=1}^{n} K_j r_D \frac{\partial p_{jD}}{\partial r_D} \right)_{r_D = 1} \]

The outer boundary condition is

\[ (p_{jD})_{r_D = r_o} = 0 \quad \text{or} \quad \left( \frac{\partial p_{jD}}{\partial r_D} \right)_{r_D = r_o} = 0 \]

The problem is solved numerically for \[ K_j = K_j(\varepsilon_j) = \exp(-\beta_j \varepsilon_j) \], where

\[ \varepsilon_j = \varepsilon_j(p_1, \ldots, p_n) = \frac{u_j^\prime}{h_j} \quad \text{and} \quad \left[ u_j^\prime \right] = \frac{[u_z] - \eta \sum_{k=1}^{n} \frac{p_k}{c_k}}{c_j \sum_{k=1}^{n} \frac{1}{c_k}} + \eta \frac{p_j}{c_j} \]

Additionally, we introduce \[ \gamma_j = \frac{E_j}{E} \]. Then, the total deformation in dimensionless variables is

\[ [u_z] = \frac{q_j \mu}{2\pi(kh)} \frac{a}{E} \int_{0}^{\infty} \left( \sum_{k=1}^{n} \frac{h_k}{\gamma_k} p_{kd} \right) y A(y, r_D) dy \quad a = \frac{4\eta r_w (1-\nu^2)}{\sum_{k=1}^{n} (h_k/\gamma_k)} \]

The individual deformation of \[ j \)-th layer is

\[ \left[ u_j^\prime \right] = \frac{q_j \mu}{2\pi(kh)} \frac{\eta}{E} \left( \frac{a}{\eta} \int_{0}^{\infty} \left( \sum_{k=1}^{n} \frac{h_k}{\gamma_k} p_{kd} \right) y A(y, r_D) dy - \sum_{k=1}^{n} \frac{h_k}{\gamma_k} p_{kd} \gamma_j \sum_{k=1}^{n} \frac{h_k}{\gamma_k} + \frac{h_j}{\gamma_j} p_{jD} \right) \]

Denote \[ \alpha_j = \beta_j \frac{\eta}{E} \frac{q_j \mu}{2\pi(kh)} \], then

\[ K_j (r_D) = \exp \left[ -\alpha_j \left( \frac{a}{\eta} \int_{0}^{\infty} \left( \sum_{k=1}^{n} \frac{h_k}{\gamma_k} p_{kd} \right) y A(y, r_D) dy - \sum_{k=1}^{n} \frac{h_k}{\gamma_k} p_{kd} \gamma_j \sum_{k=1}^{n} \frac{h_k}{\gamma_k} + \frac{p_{jD}}{\gamma_j} \right) \right] \]
We can further make use of the radial symmetry of the flow system to simplify the equations by performing the following coordinate transformation

\[
x = \frac{\ln r_D}{\ln r_{cD}} \\
r_D = r_{cD}^x \\
dr_D = x r_{cD}^{x-1} dx
\]

Then, since \(1 \leq r_D \leq r_{cD}\), the new variable \(0 \leq x \leq 1\) and the equation becomes

\[
\frac{\partial}{\partial x} \left[ K_j \frac{\partial p_{jD}}{\partial x} \right] = r_{cD}^x (\ln r_{cD})^2 \frac{\omega_j}{K_j} \frac{\partial p_{jD}}{\partial t_D}
\]

Using the new variable, the dimensionless wellbore pressure is

\[
P_{wD} = \left( p_{jD} - s_j K_j \frac{\partial p_{jD}}{\partial x} \right)_{x=0}
\]

And the rate boundary condition is

\[
1 = \left( C_D \frac{dP_{wD}}{dt_D} - \frac{1}{\ln r_{cD}} \sum_{j=1}^{n} K_j K_j \frac{\partial p_{jD}}{\partial x} \right)_{x=0}
\]

The outer boundary condition is either constant flux (or no-flow), or constant pressure circular boundary.

### 3.3 Parameters of the problem

The problem depends on a set of parameters, which describe various rock and fluid properties, and the geometry of the system. Rock properties are normally determined as a result of core analysis or are derived from open hole well logs and well tests. Fluid properties can be obtained either from PVT analysis of a reservoir fluid sample or estimated using correlations. The geometry of the system can be inferred using data from logs, seismic, well tests, and the geologic understanding of the reservoir. Well-specific parameters are
measured (flow rate, flowing pressure), or estimated/calculated (wellbore storage, skin) from well test and production data.

In addition to conventional dimensionless variables used in well test analysis, our problem depends on the two additional dimensionless groups; which are

\[ a = \frac{4\eta r_w (1 - \nu^2)}{\sum_{k=1}^{n} \left( h_k / \gamma_k \right)} = \frac{4\eta r_w (1 - \nu^2)}{E \sum_{k=1}^{n} \left( h_k / E_k \right)} \]  

(3.17)

that incorporates the elastic properties of rocks, and parameter \( \alpha_j \)

\[ \alpha_j = \beta_j \frac{\eta q_j \mu}{2\pi (kh)} \]  

(3.18)

that characterizes the sensitivity of reservoir rock’s permeability to deformation and also includes the Young’s modulus of the overburden.

Consider parameter \( a \). According to various estimates, Biot’s parameter, \( \eta \), can vary from 0.6 to 1, but is usually close to unity. Poisson’s ratio, \( \nu \), for typical rocks is in the range from 0.15 to 0.25. \( \gamma \), the ratio of Young’s modulus of reservoir rock to that of the surrounding rock mass, may vary within two orders of magnitude, from 0.1 to 10, or more. The ratio of the wellbore radius to the reservoir thickness for all practical purposes is much less than 0.1.

The dimensionless \( \alpha_j \) includes parameter \( \beta_j \), which is the coefficient that characterizes the degree of permeability change with deformation. It follows from the experimental results of permeability change with changes in effective stress that greater permeability reduction with an increase of effective stress occurs in low-permeability rocks. This reduction can be as high as 90% of the initial value. High-permeability rocks also exhibit stress-dependent behavior, though on a much smaller scale.
At the same time, if flow in a reservoir is determined to a major degree by the presence of natural fractures or micro-cracks, the initially high rock permeability (due to micro-cracks) will suffer the most dramatic reduction. According to the estimates, actual relative deformation of a reservoir will not exceed $10^{-3}$. To allow for large permeability variation, parameter $\beta_j$ should be on the order of $10^3$. The magnitude of $\alpha_j$ depends also on the Young’s modulus of the overburden. Let in (3.18) $\alpha_j = 2$ for $E = 10^{10} \text{ Pa}$. If other variables (total flow rate, total permeability-thickness product, and fluid viscosity) are kept the same, then for $E = 10^{11} \text{ Pa}$, the same $\alpha = 2$ is equivalent to a 10-fold increase in $\beta_j$, which is the sensitivity to deformations. It is impossible to narrow down, with certainty, the range of $\beta_j$ for a particular lithology, because its magnitude also will depend on the magnitude of pressure change in the reservoir, stress history, and many other factors such as shale content. We can, however, identify this parameter, $\beta_j$, on a case-by-case basis, as it will be shown later.

For typical values of the parameters and based on the results of laboratory measurements, $\alpha_j$ should be in the range from -3 to 3 (negative values correspond to injection). Within this range of sensitivity values, permeability varies from about 3-fold increase (injection) and to less than 10% retained of the initial value. Clearly, our estimate of the range of parameter $\alpha_j$ reflects only the available data, i.e. measurements on consolidated sandstone samples. For fractured/fissured rocks and/or in presence of shale streaks sensitivity of permeability to deformation may be significantly greater.
It also can be concluded from the experimental data\textsuperscript{25}, that the results can be adequately approximated with an exponential function, which served as justification for selecting permeability dependence in the form $K_j = K_j(e_j) = \exp(-\beta_j e_j)$.

### 3.4 Numerical solution

In solving the problem numerically, we proceed as follows. First, we calculate the vertical deformation of the entire reservoir. This establishes the relationship of vertical deformation with pressure change in the layers. The step is performed only once for the given set of geometric and elastic parameters. Second, we determine the deformation of individual layers. Third, we assign a functional dependence of layers’ permeability on their deformation and calculate the flow characteristics – rate and pressure.

#### 3.4.1 Calculation of deformations

Effective and accurate calculation of the vertical displacement is extremely important to solving this problem. Total deformation is of the following form,

$$U(r_D) = \frac{2\pi(kh)}{q_t, \mu} E[u_z] = a \int_0^\pi \left( \sum_{k=1}^n \frac{h_k}{\gamma_k} p_{kD} \right) yA(y, r_D) dy$$

The integral from zero to infinity is a sum of these three parts

$$U(r_D) = a p_{wD} \left( \sum_{k=1}^n \frac{h_k}{\gamma_k} \int_0^1 yA(y, r_D) dy + a \int_1^r \left( \sum_{k=1}^n \frac{h_k}{\gamma_k} p_{kD} \right) yA(y, r_D) dy + 0 \right)$$

Or, when accounting for the log-transformation of the radial coordinate in the second integral,
\[ U(r_D) = a p_{wD} \sum_{k=1}^{n} \frac{h_k}{\gamma_k} \int_{0}^{1} y A(y, r_D) dy + a \int_{0}^{1} \left( \sum_{k=1}^{n} \frac{h_k}{\gamma_k} P_{kD} \right) y A(y, x) dy = \]

\[ = a p_{wD} \sum_{k=1}^{n} \frac{h_k}{\gamma_k} J_1 + a J_2 \]

Consider, first, integral \( J_2 \) and its kernel
\[
y A(y, x) = \begin{cases} 
2y \left[ \frac{1}{y} K \left( \frac{x}{y} \right) - a^2 \int_{0}^{\pi} I_0(xz) K_0(yz) \frac{dz}{z^2 + a^2} \right], & x < y \\
2y \left[ \frac{1}{x} K \left( \frac{y}{x} \right) - a^2 \int_{0}^{\pi} I_0(yz) K_0(xy) \frac{dz}{z^2 + a^2} \right], & x > y 
\end{cases}
\]

In the integral \( I_1 = a^2 \int_{0}^{\pi} I_0(xz) K_0(yz) \frac{dz}{z^2 + a^2} \) introduce \( t = yz \); \( z^2 = \frac{t^2}{y^2} \); \( dz = \frac{dt}{y} \)
\[
I_1 = a^2 \int_{t}^{\infty} \frac{I_0 \left( \frac{x}{y} \right) K_0(t) dt}{t^2 + a^2} = ya^2 \int_{0}^{\infty} \frac{I_0 \left( \frac{x}{y} \right) K_0(t) dt}{t^2 + y^2 a^2}
\]

Similarly,
\[
I_2 = a^2 \int_{0}^{\infty} \frac{I_0 \left( \frac{y}{x} \right) K_0(t) dt}{t^2 + x^2 a^2} = xa^2 \int_{0}^{\infty} \frac{I_0 \left( \frac{y}{x} \right) K_0(t) dt}{t^2 + x^2 a^2}
\]

And
\[
y A(y, x) = \begin{cases} 
\frac{2}{\pi} \left[ K \left( \frac{x}{y} \right) - y^2 a^2 \int_{0}^{\pi} I_0 \left( \frac{x}{y} \right) K_0(t) \frac{dt}{t^2 + y^2 a^2} \right], & x < y \\
\frac{2}{\pi} \frac{y}{x} \left[ K \left( \frac{y}{x} \right) - yxa^2 \int_{0}^{\pi} I_0 \left( \frac{y}{x} \right) K_0(t) \frac{dt}{t^2 + x^2 a^2} \right], & x > y
\end{cases}
\]

(3.19)
At $y = x$ there is an integrable logarithmic singularity in the full elliptic integral and, for example for $x = 0.5$, the plot of the kernel vs. $y$ is shown in Fig. 3-4.

![Figure 3-4. Kernel vs. integration parameter at x=0.5](image)

To accurately account for the rapidly changing function in the vicinity of $x$, a non-uniform grid is introduced with spacing increasing away from $x$. Both the grid and the values of the kernel for each $x$ are pre-computed and stored, which allows later for fast and accurate calculations of deformation. The internal integral with modified Bessel functions is calculated using the Romberg method.

The double integral
\[
J_1 = \int_0^1 y A(y, r_D) dy = 2 \frac{1}{\pi} \left[ \frac{y}{r_D} K\left( \frac{y}{r_D} \right) - r_D y a^2 \int_0^t \frac{t y}{t^2 + r_D^2 a^2} dt \right] dy
\]

is computed using Romberg integration twice and is also stored.

When calculating deformation for each value of \( x \), the term \( \sum_{k=1}^n \frac{h_k}{\gamma_k} p_{kd} \) is interpolated using cubic spline at the pre-computed grid points. Final integration is performed using the trapezoidal rule. On each stage of the computations extensive numerical testing has been performed to ensure acceptable accuracy.

3.4.2 Numerical solution of set of non-linear equations

The set of non-linear equations, obtained after putting flow equations for individual layers into finite-difference form, was solved numerically using the globally convergent Newton method. Detailed explanation of the method and the code can be found in Ref. 33. The finite-difference equations result in the following system

\[
F(p) = 0
\]

where \( F \) is vector-function, and \( p \) is vector of unknown pressures. On each Newton iteration we calculate the values of vector-function for new vector

\[
p_{\text{new}} = p_{\text{old}} + \delta p
\]

where \( \delta p = -J^{-1} F \), \( J \) is the Jacobian matrix, and the step \( \delta p \) is accepted if it reduces on each iteration the norm

\[
f = \frac{1}{2} F \cdot F.
\]

The Jacobian is computed numerically.
3.4.3 Finite-difference scheme

The non-linear dimensionless pressure equation for an individual layer (omitting dimensionless subscript \(D\) and layer subscript \(j\) for brevity)

\[
\frac{\partial}{\partial x} \left[ K \frac{\partial p}{\partial x} \right] = r_{eD}^2 (\ln r_{eD})^2 \frac{\Omega}{\kappa} \frac{\partial p}{\partial t}
\]

is put in finite-difference form in conventional way. The implicit pressure equation is

\[
\frac{1}{\Delta x} \left( K^n_{i+1/2} p^{n+1}_{i+1} - \frac{p^n_{i+1} - p^n_i}{\Delta x} - K^n_{i-1/2} \frac{p^{n+1}_{i} - p^{n}_{i-1}}{\Delta x} \right) = r_{eD}^2 (\ln r_{eD})^2 \frac{\Omega}{\kappa} \frac{p^{n+1}_{i} - p^n_i}{\Delta t}
\]

here \(K^n_{i+1/2} = \frac{K^n_{i+1} + K^n_{i}}{2}\) and \(i = 2, \ldots, N - 1\). The non-linear coefficients are calculated on the previous time step, \(n\). Denote \(\lambda_i = r_{eD}^2 (\ln r_{eD})^2 \frac{\Omega (\Delta x)²}{\kappa} \frac{1}{\Delta t}\), then

\[
K^n_{i+1/2} p^{n+1}_{i+1} - \left( K^n_{i+1/2} + K^n_{i-1/2} + \lambda_i \right) p^{n+1}_{i} + K^n_{i-1/2} p^{n+1}_{i-1} = -\lambda_i p^n_{i}
\]

The explicit formulation is

\[
K^n_{i+1/2} p^n_{i+1} - \left( K^n_{i+1/2} + K^n_{i-1/2} - \lambda_i \right) p^n_{i} + K^n_{i-1/2} p^n_{i-1} = \lambda_i p^{n+1}_{i}
\]

We then use the weighted average of implicit and explicit formulations

\[
\sigma K^n_{i+1/2} p^{n+1}_{i+1} - \sigma \left( K^n_{i+1/2} + K^n_{i-1/2} + \lambda_i \right) p^{n+1}_{i} + \sigma K^n_{i-1/2} p^{n+1}_{i-1} = -\sigma \lambda_i p^n_{i}
\]

\[
(1 - \sigma) \lambda_i p^{n+1}_{i} = \left[ (1 - \sigma) \left( K^n_{i+1/2} p^n_{i+1} - \left( K^n_{i+1/2} + K^n_{i-1/2} - \lambda_i \right) p^n_{i} + K^n_{i-1/2} p^n_{i-1} \right) \right]
\]

And the layer flow equation at the interior points is approximated with the second order in space and the first order in time (for \(0.5 < \sigma \leq 1\)) by vector-function with components

\[
F_i = \sigma K^n_{i+1/2} p^{n+1}_{i+1} - \left( \sigma K^n_{i+1/2} + \sigma K^n_{i-1/2} + \lambda_i \right) p^{n+1}_{i} + \sigma K^n_{i-1/2} p^{n+1}_{i-1} + \sigma \lambda_i p^n_{i} + \left(1 - \sigma \right) \left[ \left( K^n_{i+1/2} p^n_{i+1} - \left( K^n_{i+1/2} + K^n_{i-1/2} - \lambda_i \right) p^n_{i} + K^n_{i-1/2} p^n_{i-1} \right) \right]
\]
In order to put the rate boundary condition into finite-difference form, we need to approximate pressure derivative at \( x = 0 \). Using Taylor series have

\[
p(\Delta x) = p(0) + \Delta x \frac{\partial p(0)}{\partial x} + \frac{\Delta x^2}{2} \frac{\partial^2 p(0)}{\partial x^2} + O(\Delta x^3)
\]

\[
\frac{\partial p(0)}{\partial x} = \frac{p(\Delta x) - p(0)}{\Delta x} - \frac{\Delta x}{2} \frac{\partial^2 p(0)}{\partial x^2} + O(\Delta x^2)
\]

Expression for the second pressure derivative at \( x = 0 \) follows from the pressure equation

\[
\frac{\partial}{\partial x} \left[ K \frac{\partial p}{\partial x} \right] = \frac{\partial K}{\partial x} \frac{\partial p}{\partial x} + K \frac{\partial^2 p}{\partial x^2} = r_{ed}^2 \left( \ln r_{ed} \right)^2 \frac{\omega}{\kappa} \frac{\partial p}{\partial t} = \left( \ln r_{ed} \right)^2 \frac{\omega}{\kappa} \frac{\partial p}{\partial t}
\]

\[
\frac{\partial p}{\partial x} \left( 1 - \frac{\Delta x}{2} \frac{\partial K}{\partial x} \right) = \frac{p(h) - p(0)}{\Delta x} - \left( \ln r_{ed} \right)^2 \frac{\Delta x \omega}{2K\kappa} \frac{\partial p}{\partial t}
\]

\[
\frac{\partial p}{\partial x} = \frac{1}{1 - \frac{\Delta x}{2} \frac{\partial \ln K}{\partial x}} \left[ \frac{p(h) - p(0)}{\Delta x} - \left( \ln r_{ed} \right)^2 \frac{\Delta x \omega}{2K\kappa} \frac{\partial p}{\partial t} \left( \ln r_{ed} \right)^2 \right]
\]

Since \( p(0) = p_1, p(\Delta x) = p_2, \) etc., obtain

\[
\left. \frac{\partial p}{\partial x} \right|_{x=0} = \frac{2K_1^n}{2 - \ln K_2^n + \ln K_1^n} \left[ \frac{p_2^{n+1} - p_1^n}{\Delta x} - \left( \ln r_{ed} \right)^2 \frac{\Delta x \omega}{2K_1^n \kappa} \frac{p_2^{n+1} - p_1^n}{\Delta t} \right]
\]

Denote

\[
T_0 = \frac{2K_1^n}{2 - \ln K_2^n + \ln K_1^n}; \quad \quad T_1 = \frac{\kappa}{\ln r_{ed}} \frac{T_0}{\Delta x}; \quad \quad T_2 = \frac{\Delta x \omega \ln r_{ed}}{\Delta t} \frac{T_0}{2K_1^n}
\]

Implicit and explicit rate formulations are, respectively,
Recall that dimensionless wellbore pressure for layer $j$ is written as

$$q_{im} = \frac{\kappa K^n_i}{\ln r_{eD}} \frac{\partial p}{\partial x} =$$

$$=-T_1 p_i^{n+1} + T_2 p_2^{n+1} + T_2 p_1^n = -(T_1 + T_2)p_i^{n+1} + T_1 p_2^{n+1} + T_2 p_1^n$$

$$q_{ex} = \frac{\kappa K^n_i}{\ln r_{eD}} \frac{\partial p}{\partial x} =$$

$$= -T_1 p_i^n + T_1 p_1^n - T_2 p_1^{n+1} + T_1 p_2^n = -T_2 p_1^{n+1} - (T_2 - T_1)p_i^n + T_1 p_2^n$$

and individual layer rate boundary condition includes rates from all $m$ layers

$$p_{wD} = \left( p_{jD} - \frac{s_j \kappa_j K_j}{\kappa_j \ln r_{eD}} \frac{\partial p_{jD}}{\partial x} \right)_{x=0}$$

Then, for example, the component of the vector-function, which corresponds to the inner boundary of the first layer is

$$F_{inner}^1 = \frac{C_D}{\Delta t} \left( p_{wD} - p_i^{n+1} + \frac{s_i}{\kappa_1} [\sigma q_{im} + (1-\sigma)q_{ex}] + \sigma q_{im} + (1-\sigma)q_{ex} + \sum_{\text{remaining layers}} [\sigma q_{im} + (1-\sigma)q_{ex}] + 1 \right)$$

If we specify constant wellbore pressure, $p_{wD}$, the boundary condition is

$$F_{inner}^1 = \sigma p_i^{n+1} + (1-\sigma)p_i^n - \frac{s_i}{\kappa_1} [\sigma q_{im} + (1-\sigma)q_{ex}] - p_{wD}$$
Similar expressions are written for the other layers.

Possible cross flow between layers is described by a term that includes pressure difference in adjacent layers and cross flow coefficient. For example, for layers 1 and 2 this term is simply \( X(p_{1,i}^{n+1} - p_{2,i}^{n+1}) \), where \( X \) is the dimensionless cross flow coefficient.

On each time step we first calculate values of the vector-function with the non-linear coefficients, taken from the previous time step. Then, we calculate the Jacobian using forward difference\(^{33}\), and determine the descent direction for the norm of the vector-function. We then move along this direction with steps designed to reduce the norm and to bring the vector-function closer to zero. Once acceptable convergence on the norm has been achieved, we calculate the non-linear coefficients and continue with the next Newton iteration, until the values of the vector-function are sufficiently close to zero. We then proceed to the next time step.

### 3.4.4 Code implementation

In coding of the algorithm, we have applied certain ideas of the object-oriented programming (OOP). The benefits of the object-oriented approach are well-known and discussions of the philosophy behind the OOP can be found elsewhere. Though Visual Basic for Applications (supplied with Microsoft Excel) does not yet have full support for OOP, it was possible to structure the code in such a way that it allows for easy analysis and subsequent modifications. A brief explanation of the code implementation follows (code itself is supplied in the Appendix).

On the initialization stage, we input static parameters of the reservoir (number of layers, extent, temperature, etc.), of layers (porosity, permeability, thickness, skin, etc), of the well (wellbore radius, wellbore storage coefficient), and fluid data. We also specify the
boundary conditions and numerical parameters of the problem (time step, etc.), as well as read-in the values of the kernel, pre-calculated using Eq. (3.19).

Following input of the data, the code creates a number of objects. The first collection of objects is a collection of points. Each point object carries the point-specific grid and values of the kernel at the grid points. The reservoir object includes the collection of points and additionally methods (subroutines and functions) that calculate for given pressure distributions in layers the term $\sum_{k=1}^{n} \frac{h_k}{\gamma_k} p_{klD}$ and total vertical deformation.

Once total deformation has been calculated, its values are passed to layers. The layers are separate objects with different properties (Young’s modulus, thickness, porosity, etc.), but with identical methods to calculate the permeability and fluid non-linear terms. The layer objects are generated using the same template and are assigned respective different values of properties. The non-linear terms, once determined for all layers, allow us to calculate the values of the vector-function, which is also an object. The next two objects perform Newton iterations and iterative solution of a sparse matrix (in non-OOP terms, these modules have subroutines and functions to run these calculations). Along with these principal objects we also generate separate objects, which perform check on the convergence criteria and output.

The result of such an approach is a clearly structured code. On each step of the numerical solution, we are dealing with objects that carry their respective properties and methods. If there is a need to add a property or a method (a subroutine) to a collection of objects, all we have to do is to add the required property to the template. All objects subsequently generated using this template will automatically include the desired features.
3.4.5 Numerical testing

Calculations were performed on a spatial grid with uniform step size and with a time step multiplier. To select these parameters, the finite-difference scheme was tested against Van Everdingen and Hurst constant terminal rate solution in an infinite reservoir. The relative difference between the two solutions was calculated for various combinations of step sizes with $C_0 = 100$ and no sensitivity of reservoir permeability to deformation ($\alpha = 0$).

The testing was performed for 20-, 50-, and 100-point spatial grids and with 1.01, 1.1, 1.2, 1.5 time step multiplier. Since the results have shown that a 20-point grid with time step multiplier of 1.2 (12 steps per log cycle) provides acceptable accuracy (less than 0.1% error in the practical time range) within reasonable calculation times, these grid parameters have been used. The value of the weighting parameter, $\sigma$, in the finite-difference scheme is 0.7.

In the Fig. 3-5, the blue curves are the calculated pressure change and the derivative, respectively, and the red curve is the relative difference with Van Everdingen and Hurst terminal rate solution.
Figure 3-5. Comparison with Van Everdingen and Hurst terminal rate solution
4 SINGLE LAYER CASE

In this chapter we present the results of calculations for a single-layer case. This case is the least complicated and its consideration allows to arrive to certain conclusions, which are helpful in understanding of multi-layer cases. In the first section, we show how the flow principal characteristics – pressure, deformation, and permeability – vary vs. distance and time. Second, we consider several typical transient problems, present the type curves and their sensitivity on the geometric and elastic parameters. And, finally, we consider the steady-state problems and examine the IPRs.

4.1 Sensitivity to elastic and geometrical parameters of the problem

In this section, we investigate the dependence of flow on the elastic and geometric parameters has been investigated. Since the magnitude of permeability change depends also on the direction of flow, we consider separately cases of production and injection.

To develop an understanding of how the magnitude of deformation depends on the elastic and geometric parameters of the problem, we first consider the following example. Let a well be produced at constant flow rate in a circular reservoir with a constant pressure outer boundary located at $r_{oD} = 10,000$. For this case, the permeability is not stress-dependent, i.e. $\alpha = 0$ and $k = 10 \text{ md}$. The other parameters are as follows: $\gamma = E_1/E = 1$ and Young’s modulus of the surrounding rocks $E = 10^{10} \text{ Pa}$; $r_w/h = 10^{-2}$ and $h = 10 \text{ m}$; Poisson’s ratio $\nu = 0.2$, coefficient $\eta = 1$. With these parameters $a = 0.0384$. Pressures vs. distance at several moments in time are presented in Fig 4-1. The pressure distribution curves are approximately one log-cycle apart in time and red curve corresponds to $t_D/C_D \approx 1000$. 


Pressure distributions in time follow an expected pattern. As the duration of the drawdown increases, more of the formation contributes to flow. When pressure finally reaches the constant pressure outer boundary, we observe steady-state flow.

Distributions of deformation at the same moments in time are given in Fig. 4-2. We can see that even though the pressure has not yet reached the boundary, the entire reservoir already “feels” the effect of the reduced pressure closer to the wellbore (red curve in Fig 4-2 corresponds to $t_D/C_D \approx 10,000$). If we assume that permeability is stress-dependent, flow everywhere in the formation will be very quickly affected by the redistribution, caused by well operation, of the stress-strain state in the reservoir and the surrounding rock mass.
It can also be seen that the largest deformation occurs in the vicinity of the wellbore within a distance of about 10 wellbore radii. The mechanical integrity of this zone can be altered fairly significantly by drilling and/or well operation, i.e. it is most prone to fissuring/fracturing. If we assume that permeability is stress-dependent, then most of permeability impairment will occur in the near-wellbore zone.

Let us now consider how the magnitude of the reservoir deformation near the well depends on the elastic parameters of the problem. Presented in Fig. 4-3 is the steady-state normalized deformation at \( r_D = 1 \) for \( E = 10^{10} \) Pa, \( \alpha = 0 \), and different values of \( \gamma \) (blue curve). The deformations are normalized by the value, which corresponds to \( \gamma = 1 \). As one would expect, for the same stiffness of the surrounding rocks, a more pliable or “softer”
reservoir ($\gamma < 1$) deforms to a greater degree. At the same time, the deformations, which are the vertical displacements of the interface reservoir-embedding strata, are limited to some magnitude, which is defined by both of the Young’s moduli. Stiffer reservoir rocks ($\gamma > 1$) deform to a lesser degree. We can conclude that for a softer reservoir its deformation is defined by the properties of the surrounding rocks and the magnitude of deformation does not change significantly for $\gamma < 1$, whereas deformation of a stiffer reservoir depends strongly on its Young’s modulus and not so much on the elastic properties of the surrounding strata.

![Figure 4-3. Steady-state near-well deformation as function of reservoir Young’s modulus for various values of parameter $\alpha$](image)

If formation permeability is stress-dependent, then the magnitude of deformation changes, which reflects the impaired conditions of the flow. Red, green, black, pink, light blue, and light green curves in Fig. 4-3 correspond to normalized as before deformation at
\( r_p = 1 \) for \( \alpha = 1, 2, 3, 4, 5, \) and 6, respectively. It can be seen that for \( \alpha < 4 \) as the sensitivity of permeability to deformation increases, so does the deformation itself. This is a result of an increased pressure drop in the formation in order to offset the decreased permeability, since the well is produced at constant flow rate. Increased pressure drop leads to increase of effective stress in the reservoir. The corresponding normalized steady-state wellbore pressure and permeability curves are given in Figs. 4-4 and 4-5. Though the curves for \( \alpha > 4 \) show less deformation, but coupled with the increased sensitivity of permeability to deformation, permeability is further decreased and for these values of \( \alpha \) the steady-state retained permeability is below 10 percent in a wide range of \( \gamma \).

![Figure 4-4. Steady-state wellbore pressure as function of reservoir Young’s modulus for various values of parameter \( \alpha \).](image-url)
Figure 4-5. Steady-state near-well permeability as function of reservoir Young's modulus for various values of parameter $\alpha$.

Let us now consider sensitivity of deformation to Young’s modulus of the surrounding rocks. The blue curve in Fig. 4-6 corresponds to normalized deformation at $r_p = 1$ for fixed $E_1 = 10^{10} \text{ Pa}$ and $\alpha = 0$. Stiffer surrounding rocks ($\gamma < 1$) do not deform as easily and, respectively, allow for comparatively lesser deformation of the reservoir. A stiffer reservoir ($\gamma > 1$) is less affected by the properties of the surrounding rocks and deforms accordingly to its Young’s modulus. It can be concluded that properties of both the reservoir rock and the surrounding strata have a significant impact on the magnitude of deformations.
Though in reality the variation of the elastic moduli is not that wide, it still should be properly accounted for, especially if permeability is stress-dependent. Consider the red curve in Fig. 4-6. Here we plot similarly normalized deformation at $r_D = 1$ vs. $\gamma$ but now with account for permeability dependence on deformation. The values of the variables in parameter $\alpha$ are such that $\alpha = 1$ (red curve) for $\gamma = 1$. Since $\alpha$ is inversely proportional to $E$, then for other variables, including $\beta$ – sensitivity of reservoir permeability to deformation, kept constant, it follows that $\alpha$ is equal to 100 for $E = 10^8 \ Pa$, and $\alpha$ is equal to 0.01 for $E = 10^{12} \ Pa$, etc. Green, black, and pink curves correspond to $\alpha = 2, 3, 4$. It can be seen that for large $\gamma$ (and large $\alpha$) the deformation is small, as it is limited by the properties of the reservoir. For small $\gamma$ the stiffer surrounding rocks deform to a lesser degree and limit the deformation of the reservoir. The equivalent $\alpha$ is also small, the reservoir deforms as if there were little or no permeability stress-dependence, and blue, red, and green curves almost coincide. On each curve there is a point of maximum, the location of which is determined by the combination of deformation and sensitivity effects.
Let us now consider also negative values of $\alpha$, which correspond to injection. In Fig. 4-7 we present the product $\alpha[u_z]$ vs. $\gamma$ for constant $E=10^{10} \text{ Pa}$. This product combines the elastic properties and sensitivity to deformation and is responsible for the degree of permeability impairment or improvement. Red, green, and black curves correspond to $|\alpha|=2, 4, 6$, respectively. The blue line is for $\alpha[u_z]=0$ and corresponds to the linear case, dashed lines correspond to injection. The respective dimensionless permeability curves, $k/k_0 = \exp(-\alpha[u_z])$, are presented in Fig. 4-8. Here the blue line also corresponds to linear case, i.e. with constant permeability.
Figure 4-7. Steady-state complex $\alpha[u_z]$ as function of Young’s modulus of the surrounding rocks for various values of parameter $\alpha$

We can see that deviation from the linear case of deformation with account for permeability stress-dependence is greater for the case of production (positive $\alpha[u_z]$). Respectively, permeability reduction is much more significant in this case. For large $\gamma$ (greater Young’s modulus of the reservoir than that of the surrounding rocks) both deformation and permeability change to a lesser degree than in a case of a softer reservoir.
In Fig. 4-9, we present the product $\alpha[u_z]$ vs. $\gamma$ for constant $E_1 = 10^{10}$ Pa for cases of production and injection. Here again, as in Fig. 4-6, parameter $\alpha$ changes as $\gamma$ changes, variation of $\gamma$ is due to varying Young’s modulus of the surrounding rocks, $E$, and for each value of $E$ we calculate a value of $\alpha$, which is equivalent to the value at $\gamma = 1$. Red, green, black, and pink curves correspond, respectively, to $|\alpha| = 1, 2, 3, 4$.

It can be seen that deformation of a soft reservoir ($\gamma << 1$) are limited by the elasticity of the surrounding rocks – all curves, inflation and compaction, converge to the linear case (blue line). The degree of compaction of a stiff reservoir ($\gamma >> 1, \alpha[u_z] > 0$) is limited by the...
elasticity of the reservoir itself – all curves converge to the same value. Inflation of a stiff reservoir leads to non-linear increase of its deformation.

Figure 4-9. Steady-state near-well complex $\alpha[u_z]$ as function of Young’s modulus of the surrounding rocks for various values of parameter $\alpha$

The results above were obtained using some typical values for Young’s moduli. Curves in Fig. 4-10 is the product $\alpha[u_z]$ at $r_p = 1$ vs. $\gamma$, calculated for $E_i = 10^9 \text{ Pa}$ (blue curve), $E_i = 10^{10} \text{ Pa}$ (red curve), and $E_i = 10^{11} \text{ Pa}$ (green curve). We can see that the magnitude of deformation depends both on the rigidity of the surrounding rocks and the reservoir stiffness. For example, at $\gamma = 0.01$ deformation of a stiffer reservoir (green curve) is much smaller than deformation of a more pliable reservoir (blue curve). The magnitude of deformation in either case is fairly small as the stiffer surrounding rocks limit the
deformation. As \( \gamma \) increases and the rigidity of the reservoir becomes comparable to the rigidity of the surrounding rocks, so does the deformation. After reaching a certain value of \( \gamma \), the magnitude of deformation does not change significantly. This characteristic value of \( \gamma \) depends on the rigidity of the reservoir. As \( \gamma \) increases, for a weak reservoir (blue curve) the deformation slightly decreases, which indicates less influence of the surrounding rocks.

![Diagram showing deformational complex](image)

**Figure 4-10. Steady-state near-well complex \( \alpha[u]z \) as function of Young’s modulus of the surrounding rocks for various values of reservoir Young’s modulus**

The principal geometric parameter that affects the deformation is the reservoir thickness. Presented in Fig. 4-11 dimensionless permeability curves for \(|\alpha|=1\) and 2 (red and green curves) vs. normalized by the previously used value of reservoir thickness.
(h = 10 m). The blue straight line represents linear case. We can see that a thin reservoir exhibits significant deviation of permeability from the straight line \( k/k_0 = 1 \), more so in case of injection. As the reservoir thickness increases, its relative deformation, \( \varepsilon = [u_z]/h \), decreases, which results in less permeability deviation from the linear case. For very thick reservoirs \( r_w/h > 0.001 \) the deformation is small.

Figure 4-11. Steady-state near-wellbore permeability as function of reservoir thickness for various values of parameter \( \alpha \).

Another geometric parameter that impacts the flow is the radius of the drainage area. Presented in Fig. 4-12 are the dimensionless permeability curves for \( \alpha = -2, -1, 0, 1, 2 \) (red, green, blue, black, and purple curves, respectively) vs. drainage radius, normalized by 1,000
m. It can be seen that the size of the drainage area, compared to other parameters, has only a minor effect for a given magnitude of sensitivity of permeability to deformation.

![Diagram](image)

**Figure 4-12.** Steady-state near-wellbore permeability as function of drainage radius for various values of parameter $\alpha$

### 4.2 Typical drawdown problems

In this section we present the drawdown type curves generated from our model, and examine how the elastic and geometric parameters affect the shape of the pressure/rate and derivative curves. We solve numerically several fluid flow problems with stress-dependent permeability with conventional inner and boundary conditions.
4.2.1 *Constant rate drawdown, infinite reservoir*

First, let us introduce the drawdown base case and list the values of the principal parameters. Calculations for the base case were performed with $a = 0.0384$, $\alpha = 2$, $\gamma = E_i/E = 1$, $E = 10^{10}$ Pa, $r_w/h = 0.001$, $h = 10$ m, $C_D = 100$, $s = 0$. The reservoir is considered to be infinite, which is modeled by setting constant pressure outer boundary at $10^5$ m. Comparison of the base case with the linear case (no account for deformation) is presented in Fig. 4-13.

![Graph showing pressure, derivative vs. dimensionless time for linear (blue) and base (red) case](image)

**Figure 4-13. Drawdown type curves for linear ($\alpha=0$) and base ($\alpha=2$) case**

Linear (blue) and base (red) case dimensionless pressure difference (thick lines) and well-testing derivative (thin lines of same color) are plotted vs. dimensionless time. Let us consider the red curves. The pressure difference is greater than in the linear case because
permeability is stress-dependent and decreases in response to fluid withdrawal and reduction of pressure within an expanding zone around the well. The derivative simply reflects this increase of pressure difference and behaves as if the permeability-thickness product, $kh$, were decreasing with time, or equivalently, if skin were increasing with time. One might also interpret such behavior of the derivative as a closed boundary located fairly close to the well. The possibility of such incorrect interpretation is greater when the duration of the test is short.

![Figure 4-14. Drawdown type curves for various values of $\alpha$](image)

Fig. 4-14 presents the pressure and derivative curves for different values of permeability sensitivity to deformation. For blue, red, green, and black pairs of curves $\alpha = 1, 2, 3, \text{ and } 4$, respectively. The greater is $\alpha$, the greater is the deviation from the linear
case (plotted as filled and empty dots). We can see that for $\alpha=3$ and 4 the derivative first increases, reaches a maximum, and then decreases. It appears as if $kh$ were initially decreasing and then began to increase. To understand such behavior, consider Fig. 4-15, where we plot pressure vs. distance at several moments in time for $\alpha=1$ and 4. Respective permeability distributions are given in Fig. 4-16. The blue curves are for $t_D/C_D \approx 1$, consecutive curves correspond to later times and are approximately two log-cycles in time apart, the last curve is for $t_D/C_D \approx 10^9$

As the pressure transient travels into the formation, the redistribution of the stress-strain state in the entire system causes reduction of permeability everywhere in the reservoir. Since the largest pressure drop occurs within about 5 wellbore radii around the well, this zone suffers the most permeability reduction. Greater sensitivity to deformation leads to a more rapid decrease in permeability. In turn, this zone around the wellbore determines to a large degree the flow into the wellbore. Once the permeability near the wellbore has been significantly reduced, continuing reduction of permeability in the reservoir further away from the well has less significant impact on the flow into the wellbore. The created near the well “deformational choke” is the principal factor that governs subsequent behavior of the well pressure and rate. Once the “choke” is in place, it impedes further decrease of well pressure (increase of drawdown). Well pressure still continues to decrease, but a slower rate, which is reflected by the derivative.
Figure 4-15. Pressure vs. distance for several times
Figure 4-16. Permeability vs. distance for several times
In Fig. 4-17, we present the dimensionless permeability at \( r_D = 1 \) vs. time for the values of \( \alpha = 1, 2, 3, 4, \) and 10. Let us consider the blue and red curves (\( \alpha = 1 \) and 2, respectively). Following the end of wellbore storage distortion (\( t_D / C_D \approx 100 \)), the permeability declines approximately linearly with the logarithm of time. The green and black curves (\( \alpha = 3 \) and 4) also exhibit a linear decline after the end of wellbore storage. There is, however, another period of linear decline at a slower rate, which appears at later times. If we draw straight lines through both of these time periods (dashed lines), they intersect at a time when derivative changes its character (see also Fig. 4-14 – curve \( \alpha = 4 \)).

![Figure 4-17. Near-well permeability vs. time for various values of \( \alpha \)](image)

If the sensitivity of permeability to deformation is substantial, then the permeability at the well drops very quickly, and after that initial rapid decline, the permeability decreases at
a much slower rate (see pink curve for $\alpha=10$). Everywhere else in the reservoir, permeability continues to decline, which shows up as the increasing separation of the pressure and the derivative curves. The increasing separation can be interpreted either as increasing $kh$, or as increasing impedance to flow (skin), which in this case is a better explanation.

As explained previously, the elastic properties of the reservoir and surrounding rocks are of principal importance as these properties determine the magnitude of the reservoir deformation. The effect of the described above near-wellbore deformational “choke” is more pronounced when the surrounding rocks are more pliable (or less stiff) than the formation, i.e. the deformation is not confined significantly by the rigidity of the embedding strata. Less stiff surrounding rocks also mean greater $\alpha$.

Presented in Fig. 4-18 are type curves for constant Young’s modulus of the reservoir, $E_i = 10^{10}$ Pa and $\gamma=100, 10, 1, 0.1, 0.01$ (blue, red, green, black, and purple curves, respectively). We can see that for large $\gamma=100$ (blue curves) the reservoir quickly responds to the drawdown by developing the “choke” – obviously both permeability and pressure on the well drop very quickly (within the time of well unloading) and further production results in expansion of the zone of reduced permeability into the formation. Similar effects, though on a smaller scale, occur for $\gamma=10$ (red curves). Pressure for both pairs of curves rises fast initially and then at a much slower rate. Such behavior might be incorrectly interpreted as a no flow boundary very close to the well.
For $\gamma=1$ (green curves), the behavior of the derivative, when analyzed using conventional methods, might be interpreted as negative skin or constant pressure boundary close to the well. For a reservoir that is stiffer than the surrounding strata ($\gamma=0.1$ and 0.01 – black and purple pairs practically coincide) and with given sensitivity to deformation, the deviation of the type curves from the linear case with no account for deformation is practically undetectable.

Let us now consider sensitivity of the results to the Young’s modulus of the reservoir. In Fig. 4-19, the type curves for $\gamma=0.01$ (red curves), and 100 (green curves) are presented. For the curves $\alpha=2$ and Young’s modulus of the surrounding rocks, $E$, is constant at
$10^{10}$ Pa. The blue curves are linear case pressure and derivative, respectively. It follows from the graph that a softer reservoir (lower $\gamma$) will exhibit more deviation from the linear case.

**Figure 4-19. Drawdown type curves for various values of reservoir Young’s modulus**

**4.2.2 Constant rate drawdown, constant pressure outer boundary**

Let a well be produced at a constant flow rate from a circular reservoir with constant pressure on the outer boundary. In Fig. 4-20, the blue, red, green, and black pairs of curves, respectively, correspond to $r_{oD} = \infty$ (base case); 1,000; 5,000; and 10,000. All other parameters, except for $r_{oD}$, are same as in the base case.
We can see that prior to reaching the boundary, the differences among the pressure and derivative curves, which correspond to various \( r_{eD} \) are very small. The distance to the boundary, if the test is conducted long enough, can be estimated. More discernable results are presented later, when we describe pseudo- and steady-state flow and a drawdown followed by a buildup.

**Figure 4-20. Drawdown type curves for various values of drainage radius and \( \alpha=2 \)**

Sensitivity to parameter \( \alpha \) for \( r_{eD} = 10,000 \) is presented in Fig. 4-21. For blue, red, green, and black pairs of curves \( \alpha = 1, 2, 3, \) and 4, respectively. As the value of \( \alpha \) increases, the effects of deformation and permeability stress-dependence also increase. We can see that if using conventional analysis one could underestimate wellbore storage coefficient and calculate negative skin. It is also interesting to note that for large \( \alpha = 4 \), the boundary
apparently is being “felt” by the derivative somewhat earlier than for smaller values of $\alpha$. This is not the effect of the boundary, this is the described above effect of the deformational “choke”.

![Pressure curves from highest to lowest: $\alpha = 4, 3, 2, 1$](image1)

![Derivative curves from highest to lowest: $\alpha = 4, 3, 2, 1$](image2)

**Figure 4-21.** Drawdown type curves for various values of $\alpha$ and reD = 10,000

### 4.2.3 Constant rate drawdown, no-flow outer boundary

Let us now consider a case of a well put on production with constant rate and located at the center of a closed circular reservoir. The boundary is at $r_{eo} = 10,000$, all other parameters are the same as in the base case. Fig. 4-22 presents type curves for this case (red curves) along with the linear case (blue curves).
We can see that the behavior of the two sets of curves substantially differs from each other. Drawdown from a closed area in the linear case follows a familiar pattern – after the boundary has been reached the pseudo-steady flow takes place. The pressure declines at the same rate everywhere in the reservoir in order to maintain constant production. Both pressure and derivative follow a unit-slope straight line.

![Figure 4-22. Drawdown type curves, closed outer boundary, various $\alpha$](image)

The case with stress-dependent permeability is markedly different. First, the pressure during transient flow is greater than the linear case pressure in order to compensate for ever decreasing permeability. The derivative reflects this trend. Second, as the boundary is reached both pressure and derivative begin to increase, but the increase does not continue indefinitely as in the linear case, rather at some later time pressure stabilizes at a constant
value. The derivative clearly shows it by dropping to zero. And, third, it appears that in the linear case the boundary is reached later.

In order to visualize such behavior let us consider the numerical rate calculation. In its implicit formulation rate is given by

$$\frac{\partial p_D}{\partial x} \bigg|_{x=0} = \frac{2K^n_1}{2 - \ln K^n_2 + \ln K^n_1} \left[ \frac{p^{n+1}_2 - p^{n+1}_1}{\Delta x} - \left( \frac{\Delta x}{2K^n_1} \frac{\omega}{\Delta t} \right) \left( p^{n+1}_1 - p^n_1 \right) \right]$$

$$q_D = C_D \frac{dp_{ed}}{dt_D} - \frac{\kappa}{\ln r_{ed}} \frac{\partial p_D}{\partial x} \bigg|_{x=0} = 1$$

For large times the rate is essentially determined by the product

$$\frac{\kappa}{\ln r_{ed}} \frac{\partial p_D}{\partial x} \bigg|_{x=0} = \frac{\kappa}{\ln r_{ed}} \left( \frac{2K^n_1}{2 - \ln K^n_2 + \ln K^n_1} \right) \left( \frac{p^{n+1}_2 - p^{n+1}_1}{\Delta x} \right)$$

where the two components in brackets change with time as shown in Fig. 4-23. Here blue curve is the rate, red curve is the pressure gradient, multiplied by $\frac{\kappa}{\ln r_{ed}}$, and green curve is the complex with the permeabilities. As time increases the green curve deviates from unity, as permeability near the well declines in response to declining pressure and increasing effective stress. The red curve increases, which corresponds to increasing pressure drop in the well in order to accommodate the permeability decrease and to maintain constant rate production. The product of the curves tends to unity as wellbore storage effects disappear with time.

Following the end of wellbore storage, the red and green curves change uniformly and fairly smoothly until $t_D/C_D \approx 2*10^5$, at which time the boundary is reached and pressure everywhere in the reservoir begins to increase. This rapidly increasing pressure causes respective rapid decrease of permeability, which eventually arrests pressure increase near the
wellbore and subsequently in the entire formation. At \( t_D/C_D \approx 10^7 \) pressure term reaches a maximum and permeability term reaches a minimum and after that both terms change respectively oppositely to each other until stabilization. From the results it appears that production at constant rate from a closed reservoir can be maintained indefinitely, which is impossible.

![Behavior of calculated rate and its components vs. time](image-url)

**Figure 4-23. Behavior of calculated rate and its components vs. time**

In order to visualize the transient processes, let us consider pressure and permeability distributions at several moments in time as given in Figs. 4-24 and 4-25, respectively.
We can see that pressure gradient near the well increases up to $t_D/C_D \approx 10^7$ and then decreases. Pressure in the reservoir finally stabilizes, so that there is not a lot of difference between distributions at $t_D/C_D = 10^{10}$ and $10^{16}$. Near-well permeability is eventually reduced by several orders of magnitude. Permeability everywhere in the formation is less than 10% of the initial value. Clearly, production rate cannot remain constant in such conditions. In fact, it should decrease since pressure gradient near the well decreases and so does permeability. Nevertheless, calculations show that the rate can be maintained constant. The discrepancy is due to numerical evaluation of rate, which is a product of pressure derivative and permeability term. At large times pressure gradient (multiplied by $\frac{\kappa}{\ln r_{ed}}$) is the inverse of permeability term. As a result the dimensionless rate $q_D = 1$. 

Figure 4-24. Pressure vs. distance in closed reservoir at several times and $\alpha=2$
Figure 4-25. Permeability vs. distance in closed reservoir at several times and $\alpha=2$

Let us consider another approach to numerical calculation of rate. Instead of approximating the pressure derivative on the well at $x = 0$, as it was shown in Chapter 3.4, we now will find the derivative on the outer boundary at $x = 1$. Using Taylor series, have

\[ p(1-h) = p(1) - h \frac{\partial p(1)}{\partial x} + \frac{h^2}{2} \frac{\partial^2 p(1)}{\partial x^2} + O(h^3) \]

\[ \frac{\partial p(1)}{\partial x} = \frac{p(1) - p(1-h)}{h} + \frac{h}{2} \frac{\partial^2 p(1)}{\partial x^2} + O(h^2) \]

Expression for the second derivative follows from the pressure equation

\[ \frac{\partial}{\partial x} \left[ K \frac{\partial p}{\partial x} \right] = \frac{\partial K \frac{\partial p}{\partial x}}{\partial x} + K \frac{\partial^2 p}{\partial x^2} = r_{ed}^2 (\ln r_{ed})^2 \frac{\omega \frac{\partial p}{\partial x}}{\kappa \frac{\partial t}{\partial t}} = r_{ed}^2 (\ln r_{ed})^2 \frac{\omega \frac{\partial p}{\partial t}}{\kappa \frac{\partial t}{\partial t}} \]

\[ \frac{\partial^2 p}{\partial x^2} = -\frac{\partial K \frac{\partial p}{\partial x} + (r_{ed} \ln r_{ed})^2 \frac{\omega \frac{\partial p}{\partial x}}{K \frac{\partial t}{\partial t}}}{K \frac{\partial x}{\partial x}} = -\frac{\partial (\ln K) \frac{\partial p}{\partial x} + (r_{ed} \ln r_{ed})^2 \frac{\omega \frac{\partial p}{\partial t}}{K \frac{\partial t}{\partial t}}}{K \frac{\partial x}{\partial x}} \]
\[
\frac{\partial p(1)}{\partial x} = \frac{p(1) - p(1 - h)}{h} + \frac{h}{2} \left[ -\frac{\partial (\ln K)}{\partial x} \frac{\partial p(1)}{\partial x} + \frac{(r_{eoD} \ln r_{eoD})^2}{K} \omega \frac{\partial p(1)}{\partial t} \right]
\]

\[
\frac{\partial p}{\partial x} \left[ 1 + \frac{h}{2} \frac{\partial (\ln K)}{\partial x} \right] = \frac{p_N - p_{N-1}}{h} + \frac{h}{2} \frac{(r_{eoD} \ln r_{eoD})^2}{K} \omega \frac{\partial p}{\partial t} + \frac{1}{1 + \frac{h}{2} \frac{\partial (\ln K)}{\partial x}} \left( \frac{p_N - p_{N-1}}{h} + \frac{h}{2} \frac{(r_{eoD} \ln r_{eoD})^2}{K} \omega \frac{\partial p}{\partial t} \right)
\]

\[
\frac{\partial p}{\partial x} \bigg|_{x=1} = \frac{2}{2 + \ln K_N^n - \ln K_{N-1}^n} \left( \frac{p_N^+ - p_{N-1}^+}{h} + \frac{h}{2} \frac{(r_{eoD} \ln r_{eoD})^2}{K} \omega \left( p_N^+ - p_{N}^+ \right) \right)
\]

which is similar to previous result, except for all variables are taken at \( x = 1 \).

Now we take the pressure equation

\[
\frac{\partial}{\partial x} \left[ K \frac{\partial p}{\partial x} \right] = r_{eoD}^{2x} \left( \ln r_{eoD} \right)^2 \omega \frac{\partial p}{\partial t}
\]

And integrate it formally from 0 to 1

\[
\int_{0}^{1} \frac{\partial}{\partial x} \left[ K \frac{\partial p}{\partial x} \right] dx = \int_{0}^{1} r_{eoD}^{2x} \left( \ln r_{eoD} \right)^2 \omega \frac{\partial p}{\partial t} dx
\]

\[
\left[ K \frac{\partial p}{\partial x} \right]_{x=1} - \left[ K \frac{\partial p}{\partial x} \right]_{x=0} = (\ln r_{eoD})^2 \omega \int_{0}^{1} r_{eoD}^{2x} \frac{\partial p}{\partial t} dx
\]

On the left-hand side we have the difference of pressure derivatives (essentially, rates) at the boundaries and we have an expression to calculate rate at \( x=1 \). We can, therefore, calculate well rate using rate on the outer boundary. To do this we need to consider integral

\[
\int_{0}^{1} r_{eoD}^{2x} \frac{\partial p}{\partial t} dx = \int_{0}^{h} r_{eoD}^{2x} \frac{\partial p}{\partial t} dx + \int_{h}^{2h} r_{eoD}^{2x} \frac{\partial p}{\partial t} dx + \cdots + \int_{1-h}^{1} r_{eoD}^{2x} \frac{\partial p}{\partial t} dx = \sum_{i=1}^{N-1} \int_{x_i}^{x_{i+1}} r_{eoD}^{2x} \frac{\partial p}{\partial t} dx
\]
We now replace the time derivative with its value at the midpoint of the interval

\[
\sum_{i=1}^{N-1} \left( \frac{\partial p}{\partial t} \right)_{i+1/2} = \sum_{i=1}^{N-1} \left( \frac{p_i^{n+1} - p_i^n}{\tau} \right) = \frac{1}{2\tau} \sum_{i=1}^{N-1} \left( p_i^{n+1} + p_{i+1}^{n+1} - p_i^n - p_{i+1}^n \right) \int_{x_i}^{x_{i+1}} \int_{x_{i+1}}^{x_i} r_{ed}^2 \, dx
\]

Integrals individually are

\[
\int_{x_i}^{x_{i+1}} r_{ed}^2 \, dx = \frac{1}{2 \ln r_{ed}} \left( r_{ed}^{2x_i} - r_{ed}^{2x_{i+1}} \right)
\]

Therefore

\[
\int_{0}^{x_{i+1}} \frac{\partial p}{\partial t} \, dx = \frac{1}{4\tau \ln r_{ed}} \sum_{i=1}^{N-1} \left( p_i^{n+1} + p_{i+1}^{n+1} - p_i^n - p_{i+1}^n \right) \left( r_{ed}^{2x_i} - r_{ed}^{2x_{i+1}} \right)
\]

Finally, denoting \( y_i = r_{ed}^{2x_i} - r_{ed}^{2x_{i+1}} \), \( y_1 = r_{ed}^{2x_1} - r_{ed}^{2x_2} \), \( y_N = r_{ed}^{2x_N} - r_{ed}^{2x_1} \), obtain

\[
\left[ K \frac{\partial p}{\partial x} \right]_{x=0}^{x} = \frac{2K_N^n}{2 + \ln K_N^n - \ln K_{N-1}^n} \left( \frac{p_N^{n+1} - p_{N-1}^{n+1}}{h} + \frac{h}{2\tau} \frac{(r_{ed} \ln r_{ed})^2}{K_N^n} \frac{\omega}{\kappa} (p_N^n - p_N^g) \right) - \frac{\omega \ln r_{ed}}{4\tau} \left[ p_1^{n+1} y_1 + p_2^{n+1} y_2 + p_3^{n+1} y_3 + \ldots - p_1^n y_1 - p_2^n y_2 - p_3^n y_3 - \ldots \right]
\]

This expression shows that production rate cannot remain constant but should decline with time. As \( \tau \) increases the numerical value of the expression decreases. Indeed, when this formula was used to calculate rate, the rate steadily declined after pressure reached maximum at \( t_D / C_D \approx 10^7 \).

Type curves for \( \alpha = 0, 1, 3, \) and 5 are presented in Fig. 4-26 (blue, red, green, and black pairs of curves). We can see that the greater is the sensitivity of permeability to deformation, the sooner the above describe effects are observed, i.e. permeability reduction leads to infeasibility of constant rate production.
4.2.4 Constant pressure drawdown, constant pressure outer boundary

The next problem considered is a case of a well put on production with constant wellbore pressure and with constant pressure circular outer boundary. The boundary is at $r_{CD} = 10,000$, all other parameters are the same as in the base case. Rather than observing change of pressure on the well and its derivative we now observe how production rate declines with time. At constant pressure, as suggested by several researchers (see, for example, Ref. 34) a plot of $1/q_o$ vs. time has similar trends as the considered before constant rate case where pressure difference is plotted vs. time.
Figure 4-27. Constant pressure production, linear case

Dimensionless rate (dashed blue curve) as well as $1/q_D$ (thick red curve) and logarithmic derivative $\partial (1/q_D)/\partial (\ln t_D)$ (thin red curve) are presented in Fig. 4-27 for the linear case $\alpha = 0$. The magnitude of dimensionless wellbore flowing pressure is equal to $\ln(r_{eD})$, so that production rate stabilizes at $q_B = 1$. We can see that initially $1/q_D$ increases fairly rapidly (production rate exhibits rapid decline) and after $t_D/C_D \approx 100$ increases at a constant rate, which is indicated by constant derivative. As the boundary is reached at $t_D/C_D \approx 10^6$, both production rate and $1/q_D$ flatten at constant value of 1, respectively the derivative drops to zero.
If permeability is stress-dependent with $\alpha = 2$, then constant pressure production at same as above $p_{wD} = \ln r_{eD}$ results in the following deviation from the linear case. Presented in Fig. 4-28 are the same variables. We can see that at steady-state production rate is lower than in the linear case. The increase of $1/q_D$ is at variable rate at all time, which is reflected by the derivative. Such behavior is due to decrease of permeability in response to caused by production pressure decrease.

![Graph](image)

**Figure 4-28. Constant pressure production, $\alpha=2$**

Distributions of dimensionless pressure, deformation, and permeability at several moments in time are presented in Figs. 4-29, 30, 31. The curves correspond to $t_D/C_D \approx 0.01$, 1, 100, $10^4$, $10^6$, and $10^{13}$ (stabilization), blue dashed straight line in Fig. 4-29 is steady-state linear case pressure distribution.
Let us consider the blue curves. At this early time pressure has not yet moved beyond 3-4 wellbore radii (Fig. 4-29) but the impact of the well pressure drop has been already felt at a significant distance away from the wellbore. The stress-strain state in the system has changed and as a result the formation has deformed far beyond of the zone with altered pressure (Fig. 4-30). This deformation causes reduction of permeability and, for example, at a distance of about 16 wellbore radii the permeability has dropped by 1 percent (Fig. 4-31). The reduced permeability, in turn, affects the flow and the pressure difference, which eventually stabilizes at a lower level than in the linear case.

![Figure 4-29. Pressure vs. distance at several times](image)

The same behavior is observed for later times. At the onset of steady-state flow (red curves) the entire formation has been already significantly affected with respective
magnitudes of deformation and reduced permeability. For this value of $\alpha$, the permeability on the well stabilizes at less than half of the initial value and on the outer boundary drops to about 90 percent of the initial value. The majority of permeability reduction occurs within the distance of about 10 dimensionless wellbore radii. Note that conventional models that account for stress change in the formation only would not predict any permeability change at the outer boundary.

Figure 4-30. Deformation vs. distance at several times
Figure 4-31. Permeability vs. distance at several times

For greater sensitivity of permeability to deformation the deviation from the linear case is, respectively, more significant. The rate type curves for $\alpha = 0, 4,$ and $8$ (blue, red, and green pairs of curves, respectively) are presented in Fig. 4-32. Clearly, the transient process of rate stabilization is affected by the magnitude of $\alpha$. When sufficiently accurate measurements of production rate are available, plot of $1/q_D$ vs. time may provide indication of permeability stress-sensitivity.
Figure 4-32. Rate type curves for various $\alpha$

The steady-state rate vs. $\alpha$ is presented in Fig. 4-33. Increased sensitivity of permeability to deformation expectedly results in a decreased production, so that, for example, for $\alpha = 10$ the rate is only about 30 percent of the rate, calculated with no account for deformation.
Figure 4-33. Steady-state production rate vs. $\alpha$

The causes for such decrease are illustrated in Figs. 4-34 and 4-35 where we plot steady-state pressure and permeability distributions for $\alpha = 0, 4, \text{ and } 8$ (curves colored as in Fig. 4-32). For large $\alpha$ the permeability near the well drops to several percent of the initial value and the pressure difference in the reservoir stabilizes at a lower level. This additional pressure drop is obviously introduced by the decreased permeability and it is plotted (normalized by the total formation pressure drop) in Fig.4-36 at steady state (values of $\alpha$ and curve coloring are as in Fig. 4-32). We can see that even for moderate sensitivity of permeability to deformation the additional resistance to flow is substantial, which is further aggravated by the fact that the entire area of flow is affected.
Figure 4-34. Steady-state pressure distributions for various $\alpha$
Figure 4-35. Steady-state permeability distributions for various $\alpha$
Along with sensitivity of permeability to deformation the constant pressure production is affected by the value of the wellbore pressure, since its magnitude determines the pressure drop in the reservoir and deformation is a function of the effective stress. Presented in Fig.4-37 are rate type curves \(-1/q_D\) and its logarithmic derivative for constant \(p_{wd}=1, 5, \ln(r_D), 20, \text{ and } 50\) (blue, red, green, black, and pink pairs of curves, respectively). Increased drawdown leads to increase of effective stress, which results in larger deformation and for the same for all pairs of curves \(\alpha=2\) – in a larger permeability decrease. This, in
turn, affects the flow and production rate decreases ($1/q_D$ increases). The derivative allows to see more clearly the trend in behavior of $1/q_D$.

Figure 4-37. Rate type curves for various $p_{wD}$

A conventional way to evaluate formation capabilities is examination of an Inflow Performance Relationship (IPR). Several types of IPRs that describe different effects are available. For example, Darcy IPR for oil wells shows that rate changes linearly with drawdown for constant productivity index. It cannot be applied in the entire range of rates and pressures because it assumes single-phase flow and does not account for multiphase flow effects. These effects in oil wells can be accounted for by using either Vogel’s IPR, or by calculating multiphase pseudo-pressure. The latter IPR curves are not straight-line
relationships, as is Darcy’s IPR, and they account for changes in productivity index. Nevertheless, Darcy’s equation and IPR are used routinely to perform quick calculations and to compare the effects of various factors on flow.

In Fig. 4-38 we use Darcy IPR to visualize the effect of permeability stress-dependence on flow rate of a well. Blue straight line is the linear case, calculated with no account for deformation. Curves from red to light green correspond to $\alpha = 1, 2, 3, 4, 5,$ and 6. We can see that the greater is the magnitude of the drawdown and $\alpha$, the greater is the deviation of an IPR from the straight-line behavior. Similar behavior is characteristic of the gas or multiphase flow, which makes it difficult to distinguish between the two. But if these trends on an IPR are observed on an oil well, produced above the bubble-point pressure, then, given that we have enough confidence in the quality of the PVT analysis and rate/pressure measurements, such deviation can be explained by the effects of deformation.
4.2.5 Constant pressure drawdown, no-flow outer boundary

Let us next consider the case of a well put on production with constant wellbore pressure and located in the center of a closed circular drainage area. The boundary is at \( r_{eD} = 10,000 \), all other parameters are the same as in the base case. Next several graphs show how production rate declines with time for different values of the problem parameters.

In Fig. 4-39 pair of blue and red curves correspond to linear case and to the case \( \alpha = 2 \). Both cases were calculated with constant \( p_{nD} = \ln r_{eD} \). As one would expect, production from a closed reservoir eventually results in decrease of rate (increase of \( 1/q_D \)).
Transient flow regime is followed by pseudo-steady state flow, both periods are clearly identifiable.

Figure 4-39. Rate type curves, closed drainage area, various $\alpha$

During transient flow, prior to reaching the outer boundary, the behavior of the curves, calculated with account for deformation, is essentially the same as in the preceding problem. Let us consider distributions of pressure and permeability during transient and pseudo-steady state as presented in Figs. 4-40, 4-41. The blue curves are for $t_D/C_D \approx 0.01$, the red curves are for $t_D/C_D \approx 10^6$ and correspond to the onset of pseudo-steady state flow. All curves are approximately one log-cycle in time apart.
After reaching the boundary pressure decreases everywhere in the reservoir and so does permeability. Note that even though wellbore pressure is constant, permeability on the well drops significantly in response to pressure reduction elsewhere in the reservoir. Curves for late times virtually overlay, which corresponds to rate decrease.

Figure 4-40. Pressure vs. distance at several times
4.3 Buildup problems

In this section we present selected results of calculation of a typical buildup test, following either a constant rate, or a constant pressure drawdown. We have already seen that commencement of production from a stress-sensitive formation causes development of a zone of reduced permeability around a well. As production continues, this zone expands into the formation, affects the fluid flow, and, respectively, the trends in the wellbore flowing pressure. One can expect, therefore, that the duration of production prior to shut-in will significantly affect wellbore pressure behavior during buildup.
Let us consider buildup in an infinite reservoir \( r_{dD} = 10^6 \), following constant rate drawdown as given by the base case with \( a = 0.0384, \alpha = 2, \gamma = E_{t}/E = 1, E = 10^{10} \text{ Pa}, \)
\( r_{w}/h = 0.001, h = 10 \text{ m}, C_D = 100, s = 0 \). Suppose that at \( t_{D}/C_D = 10^{10} \) the well is shut in for an equally extended buildup. These two flow periods are presented in Fig. 4-42. The blue curves represent drawdown, and the red curves represent buildup.

**Figure 4-42. Infinite reservoir, drawdown followed by buildup, \( \alpha=2 \)**

We can see that these two types of tests produce markedly different sets of curves. If interpretation of a drawdown test using conventional methods might have indicated a stimulated well, the conventional buildup interpretation would indicate a significant damage to the well. It would also appear that the magnitude of damage increases, as we are further
into the test. It is interesting to note that if buildup is terminated prior to reaching the outer boundary (which almost always will be the case in low-permeability reservoirs) the estimated reservoir pressure might be higher than the value prior to production. Obviously, conventional interpretation of both tests would provide with incorrect results. If run concurrently, the characteristic behavior as depicted here would clearly indicate stress-sensitive permeability. Such type of asymmetry of response has been observed in several reported field tests.

Let us now investigate how the production time prior to shut-in affects the shape of buildup type curves. For Figs. 4-43-4-45, well is shut-in at $t_D/C_D = 10^3$, $10^5$, and $10^7$, respectively. Duration of buildup is the same as of drawdown, blue and red pairs of curves correspond to drawdown and buildup, respectively. It can be seen that both pressure and derivative curves have the same characteristic features, as described above, but the difference between drawdown and buildup becomes more significant as the duration of the flow period increases.
Figure 4-43. Infinite reservoir, drawdown followed by buildup. Production time $10^3$
Figure 4-44. Infinite reservoir, drawdown followed by buildup. Production time $10^5$
Figure 4-45. Infinite reservoir, drawdown followed by buildup. Production time $10^7$

From consideration of the derivative it appears that the circular zone around the well, affected the most by the drawdown, might be erroneously recognized as drainage area of the well. Another characteristic feature is a larger separation between pressure and derivative curves in case of a longer drawdown and large separation indicates greater damage. The transition period increases with extension of drawdown which also indicates greater damage. It might be possible to identify the sensitivity of permeability to deformation as well as the conventional parameters (permeability, skin, wellbore storage coefficient) from a series of constant rate drawdown tests of increasing duration intermitted by buildup tests. The
duration of buildup tests should be long enough so that the derivative intercepts the derivative, obtained on the preceding drawdown.

It is clear that all these effects will be more pronounced if the properties of the surrounding rocks and of the reservoir allow for large deformation of the reservoir and/or the permeability is substantially sensitive to the deformation. Consider, for example, Fig. 4-46 where we illustrate what effect has the sensitivity of permeability to deformation ($\alpha = 4$) and the distance to the constant pressure reservoir radius ($r_{cD} = 10^4$). Other parameters are the same as in the previous case. As one would expect, during drawdown greater $\alpha$ results in larger deformation and impairment of permeability, small size of drainage area ensures that the entire area experiences a substantial permeability reduction. Shut-in pressure and derivative accordingly respond to these effects by significant deviation from the drawdown curves. The separation between buildup type curves and long transition period show severe formation damage.
We can also note that if a well is shut-in for sufficiently long time, then the pressure builds up to the initial reservoir pressure, which coincides here with the contour constant pressure. It means that in our case of elastic deformations the permeability will fully recover to its initial value. Clearly, if a well produces from a closed reservoir with a no-flow circular boundary for extended period of time and average reservoir drops below its initial value, certain degree of permeability impairment will persist. This effect is shown in Fig. 4-47, with a no-flow boundary is located at $r_{cD} = 10^4$ and other parameters are same as for results in Fig. 4-43. The pressure builds up to a lower value. It is notable that both the separation between the buildup type curves as well as the duration of the transition period increase as a
result of substantial permeability reduction in response to declined pressure everywhere in the reservoir.

![Graph](image)

**Figure 4-47.** No-flow boundary at reD=10^4, drawdown followed by buildup, ϕ = 2

### 4.4 Injection and falloff tests

In the examples above we have investigated how flow characteristics – wellbore pressure and production rate – change during drawdown and buildup if we account for permeability stress-dependence. It is also of interest to consider another type of a well test, injection test, and to examine how permeability stress-dependence influences the transient processes in this case. We have already shown in Section 4.1 that the response of formation
to injection is different from that of to production because of the assumed functional form of permeability dependence to deformation.

Similarly to drawdown test before we introduce base case for injection test. The values of the parameters (elastic, geometric, etc.) are the same as in the drawdown base case, except for $\alpha = -2$. Type curves for linear case (blue curves) and base case (green curves) are presented in Fig. 4-48. Since for the base case permeability increases in response to injection, it requires less pressure difference to maintain constant rate and the pressure curve is lower than the respective curve in linear case. As pressure transient propagates into the formation, it changes the stress-strain state in the system reservoir-surrounding rocks, which results in permeability increase in the reservoir. This behavior can be identified from examination of the derivative – it has a downward trend as if the permeability-thickness product were increasing with increasing duration of injection. Red and black pairs of curves in Fig. 4-48 correspond to $\alpha = -1$ and to -3. Clearly, for greater sensitivity of permeability to deformation these effects become more pronounced.
Similarly to drawdown test transient wellbore pressure response to constant rate injection depends on the elastic and geometric parameters of the system. Though sensitivity of permeability to deformation in case of injection probably varies in a smaller range. There is limited data on the degree of permeability increase due to injection and the available estimates should be taken with caution as in many cases the non-linear increase of permeability could be attributed to breakdown of a rock sample. Field experiments are also difficult to evaluate quantitatively.

It is reasonable to assume that after millions of years of burial and associated creep compaction, permeability of rock would not increase orders of magnitude under injection at pressures below formation parting pressure. Even with the pre-existing network of cracks
and fissures the injected fluid (water) will quickly find a preferential path, which will leave the majority of fissures at their initial sizes. Increased injection pressure might result in involving into flow of a larger number of cracks, but most of the water will still flow along the developed path which will increase the sizes of cracks and fissures mainly along this path. A fairly uniform permeability increase due to injection could be observed in a formation with reservoir pressure lower than its initial value. In such situation permeability increase can be attributed to re-opening of the cracks that were closed because of the increased effective stress due to production.

We will continue using the exponential functional dependence of permeability on deformation, but will limit the magnitude of the parameter $\alpha$. Clearly, for large $\alpha$ we may calculate a substantial non-linear permeability increase, but such increase might be attributed to some other factors, different from the mechanism that we consider in this research. In order to estimate the range of the parameter $\alpha$ for the case of injection we plot in Fig. 4-49 steady-state permeability distributions vs. distance for several values of $\alpha$ and $r_{eb} = 10^4$. Other parameters are as in the base case.
As one would expect, greater sensitivity of permeability to deformation results in higher permeability values in the reservoir and, for example, for $\alpha = -6$ (green curve) permeability increase near the wellbore is 4-fold and at the external contour of the drainage area is about 30%. Though it is fairly easy to visualize a 4-fold permeability increase in the vicinity of a well, a significant increase of permeability further in the formation can look suspicious because of the relative permeability effects, associated with simultaneous flow of water and oil. In this research we consider flow of a single-phase fluid only. Nevertheless, the presented below several injection/falloff problems can provide certain insight into behavior of injection wells. We will also limit the magnitude of $\alpha$ by -4, which corresponds to about 3-fold permeability increase near the well.
In Fig. 4-50 and 4-51 the blue and red pairs curves correspond to injection and falloff, respectively. For both graphs the problem parameters are the same as in the previous case, except for $\alpha = -2$ for Fig. 4-50 and $\alpha = -4$ for Fig. 4-51.

![Figure 4-50. Injection followed by falloff, $\alpha=-2$](image)

It can be seen that behavior of these tests is completely different from the drawdown-buildup case. Pressure difference during injection period initially increases fairly fast and causes fast increase of permeability near the well. After this initial period, as the deformation and, therefore, permeability become limited either by the elastic properties of the reservoir or those of the surrounding rocks, pressure increases at a much slower rate, which serves as an illustration of importance of the near-wellbore zone in the flow. In fact,
increase of permeability further away in the reservoir has little effect on the pressure, as the injectivity becomes limited by the throughput capability of the near-wellbore zone. The derivative simply reflects this ever diminishing increase of pressure, and its behavior might be interpreted as increasing $kh$. For larger negative $\alpha = -4$ these features are more pronounced.

![Graph](image)

**Figure 4-51. Injection followed by falloff, $\alpha = -4$**

Once an injection well is shut-in for pressure falloff test, the pressure starts to drop. Once the wellbore storage distortion has seized, pressure declines at a fairly constant rate (pressure difference increases steadily) with logarithm of time. The derivative is almost flat on both figures with a tendency to slight increase in the late time region. The derivative response is almost that of a homogeneous layer, because permeability near the well drops.
fairly quickly and not so fast further in the reservoir. Permeability profiles along distance at several moments in time for \( \alpha = -4 \) are given in Fig. 4-52. The thick red curve corresponds to \( t_D/C_D = 100 \), i.e. to the end of the transition period following wellbore storage. We can see that after that time the permeability distributions are approximately uniform throughout the reservoir.

![Figure 4-52. Falloff permeability distributions at several times](image)

Analysis of a falloff test alone using conventional methods would thus provide with an estimate of permeability, which is different from the actual value. In order to illustrate this point we plot in Fig. 4-53 falloff type curves for linear case (blue curves), which is the same as a conventional method, and falloff with \( \alpha = -4 \) (red curves). Additionally, green curves are red curves shifted up and to the right in order to align the flat portion of the
derivative and the wellbore storage region with the type curves. This shift corresponds to the scenario that would be followed when analyzing pressure data using conventional approach.

![Figure 4-53. Conventional analysis applied to falloff data in a stress-sensitive formation](image)

Let us estimate how the shifting of the data performed in order to conform to classical method affects the values of the principal parameters. First, let us recall that the dimensionless pressure and time are defined as

\[
p_D = \frac{kh(p_{sw} - p)}{141.2qB\mu} \quad \text{and} \quad t_D = \frac{0.0002637kt}{\phi\mu\kappa p_w^2}
\]

The upward shift of the curves is equivalent to increase of \( p_D \), the right shift is equivalent to increase of \( t_D \). Both increases can be accommodated only by increase of permeability, since it is the only unknown variable in both equations. Therefore, the value of
permeability if estimated by conventional method will be greater than the actual value. The value of the wellbore storage coefficient should be correct because the same amount of shift is applied in time and pressure directions. We can also see that the value of average reservoir pressure will be determined to be higher than the true value (lower difference between the injection pressure prior to the test and falloff pressure). In order to estimate the effect of the shift on skin, recall than skin can be determined using

$$s = \frac{1}{2} \left[ \frac{\Delta p_r}{(t\Delta p')_r} - \ln \left( \frac{kt}{1688\phi\mu c_r r_w^2} \right) \right]$$

where first term in brackets is the separation between the pressure and the derivative curve. Clearly, shifting does not affect this ratio. The second term includes permeability and since it’s value was incorrectly determined to be higher than the true value, it will result in a lower estimate of skin.

In order to get an understanding of the magnitude of errors introduced by using conventional methods we have used WELLTEST software to analyze falloffs for linear case and for the case with $\alpha = -4$ (Fig. 4-51). The actual permeability and skin were 10 md and 0, respectively. Analysis of the linear case yielded correct values, while application of the conventional approach for the stress-sensitive case resulted in 14 md permeability and skin of -1. The difference between estimates of average reservoir pressure was 1,200 psi, wellbore storage coefficient was essentially the same in both analyses.

It is obvious that if formation has lower sensitivity to deformation the results, obtained using conventional approach, will be fairly close to the correct values. It is also possible to derive the correct values of permeability and skin if there is an independent estimate of the value of average reservoir pressure. In such case we can deduce the magnitudes of pressure and time shift, introduced by application of the conventional analysis
methods, and to come up with true formation permeability and, subsequently, skin. Therefore, falloff tests on injection wells appear to be very attractive practically, as these tests do not normally require long shut-in and can provide useful information even in stress-sensitive formations.
5 TWO LAYER CASE

In this chapter we examine several typical problems and show how the model allows to account for interaction of pressure fields in adjacent layers. We consider sensitivity of the flow to a number of elastic and geometric parameters of the problem. Generally, the layers may have different values of thickness, permeability, porosity, etc. As before, permeability is assumed to be an exponential function of deformation. Having calculated total deformation of the reservoir, we can obtain the values of deformation of individual layers and calculate permeability distributions in layers. Since total deformation is a sum of layers’ deformations and individual deformations depend on pressure distributions in layers, it is clear that the flow in one of the layers will depend on pressure distributions in both layers.

5.1 Constant rate drawdown in an infinite reservoir

Let a well be producing at constant well rate from an reservoir comprised of two layers. A natural check of the validity of the calculations is to first assume that two layers have identical properties. Then the flow behavior of a well should coincide with the previously calculated single layer case of the same total reservoir thickness. Performed calculations indicate that in such case the two layer results are identical to the single layer results presented earlier, which gives confidence in the accuracy of the numerical implementation of the solution.

Let us first consider how the magnitude of sensitivity of permeability to deformation affects the results. We set \( a = 0.0384, \alpha_1 = 0, \alpha_2 = 4, \gamma_j = E_j/E = 1, \quad E = 10^{10} \text{ Pa}, \)
\( r_w/h = 0.001, \quad k_j = 10 \text{ md}, \quad h = 10 \text{ m}, \quad h_j = 5 \text{ m}, \quad C_D = 100, \quad s_j = 0 \) (subscript \( j = 1, 2 \) denotes layer number). Infinite reservoir is modeled by setting constant pressure outer boundary at
$10^3$ m. The results of calculations are presented in Fig. 5-1. Here green pairs of curves correspond to well pressure response for the described two layer case. The blue and red pairs of curves are for the single layer case with $\alpha = 0$ and 4, respectively. We can see that the two layer case deviates only slightly from the linear case with $\alpha = 0$ and in practice may be detected only when running a long-duration drawdown in a well equipped with permanent pressure gauge.

**Figure 5-1. Type curves for two-layer case and two single-layer cases**

Since low-permeability rocks experience more permeability reduction than high-permeability rocks, it is more interesting to consider a case with unequal initial
permeabilities. For results presented in Fig. 5-2 \( k_1 = 100 \text{ md} \) and \( k_2 = 1 \text{ md} \), all other parameters are the same as in the previous case.

\[
\begin{align*}
\text{Figure 5-2. Type curves for two-layer case with equal thicknesses of layers and } \frac{k_1}{k_2}=100
\end{align*}
\]

It can be seen that presence of two layers with significantly different permeability values changes the behavior of the derivative – it appears that the permeability-thickness product increases. This happens because both layers deform under increased effective stress and even though for the high-permeability layer the impact of deformation on permeability is negligible (\( \alpha_1 = 0 \)), compaction of this layer causes some unloading of the low-permeability layer. This unloading leads to increase of permeability in low-permeability layer.
Another even more realistic case is when a relatively thin high-permeability layer is produced simultaneously with a thicker low-permeability one. The results for $h_1 = 1 \text{ m}$, $h_2 = 9 \text{ m}$ and other parameters same as in previous two cases are presented in Fig. 5-3.

![Figure 5-3. Type curves for two-layer case with $h_2/h_1=9$ and $k_1/k_2=100$](image)

We can see that it would be impossible to distinguish this case from a case of a homogeneous reservoir -- the derivative flattens at typical 0.5. It is also clear, that if one were using conventional analysis the determined value of skin would be higher, since permeability reduction manifests itself as increased resistance to fluid flow. The practical implications of such error are obvious. For example, a futile workover with the goal of eliminating damage skin might be performed.
To visualize the transient processes that occur in the latter case, we present results of calculation of pressure, deformation, and permeability distributions in the two layers at several moments in time. In Fig. 5-4, the left and right groups of graphs are the distributions of the above listed parameters at $t_D \approx 0.1$ and at $t_D \approx 1$. The solid blue lines represent the first layer of thickness $h_1 = 1 \text{ m}$ and initial permeability $k_{01} = 100 \text{ md}$. The dashed red lines represent the second thicker low-permeability layer. In Figs. 5-5 through 5-8 we present the same characteristics of flow at later times.

Let us consider $t_D \approx 0.1$ (left group of graphs in Fig. 5-4). Pressure in both layers has traveled a certain distance into the formation in response to constant rate production (first graph). Reduced pressure in the vicinity of the well causes deformation of the reservoir and the deformations of the individual layers are given on the second graph. We can see that the first layer deforms to a greater degree, as one would expect, given that pressure in this layer has dropped to a greater degree and in a larger area around the well. The deformation of the first layer is substantial and causes unloading of the low-permeability second layer – it actually experiences negative (tensile) deformations. Since we have assumed that permeability of the first layer is not sensitive to deformation, its dimensionless permeability remains the same, while dimensionless permeability of the second layer increases in response to unloading (third graph).
Figure 5-4. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 0.1$ (left) and $t_D/C_D \approx 1$ (right group)
Figure 5-5. Distributions of pressure, deformation, and permeability vs. distance at $t_d/C_D \approx 10$ (left) and $t_d/C_D \approx 100$ (right group)
Solid blue line: $h_1=1$ m, $k_1=100$ md  
Dashed red line: $h_2=9$ m, $k_2=1$ md

Figure 5-6. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+3$ (left) and $t_D/C_D \approx 1E+4$ (right group)
Figure 5-7. Distributions of pressure, deformation, and permeability vs. distance at $t_p/C_D \approx 1E+5$ (left) and $t_p/C_D \approx 1E+6$ (right group)
Figure 5-8. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+7$ (left) and $t_D/C_D \approx 1E+8$ (right group)
The same trends are present on subsequent graphs for later times. Compaction of the first layer in response to pressure propagation into the formation leads to unloading of the second layer and its permeability increases. At the same time, the deformation of both layers in the vicinity of the wellbore is positive, i.e. both layers experience compaction. The permeability of the second stress-sensitive layer within a small area around the wellbore (3-5 wellbore radii) decreases, which shows up as increased skin on the diagnostic plot. Further away in the formation, within the zone affected by the well production, the permeability of the low-permeability layer is greater than the initial value. Finally, outside of this zone the reservoir experiences compaction and permeability of the second layer is less than the initial value. The plots illustrate that the deformations of individual layers depend on the magnitude of pressures in the layers, and also on the pressure difference, i.e. on interaction of pressure fields in layers.

5.2 **Constant rate drawdown in a finite reservoir (constant pressure, or no-flow outer boundary)**

Pressure behavior during a constant rate drawdown in a finite reservoir initially follows the trends, described in the previous example. The infinite reservoir in this example was modeled by setting a constant pressure outer boundary at \( r_{eD} = 10^6 \). From the group of graphs on the right side of Fig. 5-8, it is clear that the outer boundary should be reached somewhat later than \( t_D/C_D = 10^8 \). Let us consider what happens at later times. Pressure, deformation and permeability distributions are given in Figs. 5-9 – 5-11.
Figure 5-9. Distributions of pressure, deformation, and permeability vs. distance at $t_d/C_D \approx 1E+9$ (left) and $t_d/C_D \approx 1E+10$ (right group)
Figure 5-10. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+11$ (left) and $t_D/C_D \approx 1E+12$ (right group)
Figure 5-11. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+13$ (left) and $t_D/C_D \approx 1E+14$ (right group)
We can see that once the boundary is reached, the distributions of all characteristics change. Pressure in the low-permeability layer stabilizes at a level which is lower than the typical logarithmic distribution. Both layers experience compaction – the deformation is positive in the entire formation. Permeability of the stress-sensitive layer stabilizes at a fairly uniform level everywhere in the reservoir, except for the near-wellbore zone. In this zone permeability drops from about 70% of the initial value to 10% on the well contour. This distribution is quite different from the single-layer case. In Fig. 5-12 we present stabilized permeability distribution for a 10 m thick single-layer with same values of other elastic and geometric parameters along with the steady-state permeability distributions of the two-layer case (as in Fig. 5-11).

Figure 5-12. Steady-state permeability distributions for one- and two-layer cases
We can see that in comparison with the single-layer case, the presence of a high-permeability layer has a positive effect on the flow in the low-permeability layer as compaction of the former leads to unloading of the latter. Permeability decrease in the near-wellbore zone is still significant and will exacerbate any damage skin, if present.

In the case of a no-flow outer boundary, the above described effect of unloading of the low-permeability layer persists after pressure in the high-permeability layer reaches the outer boundary. As a consequence, the dimensionless permeability of the low-permeability layer increases in the formation by an order of magnitude or more. Near the wellbore, however, it continues to decrease with time. These results for \( r_{eD} = 10^4 \) and other parameters same as in the previous example are presented in Fig. 5-13 (type curves) and Fig. 5-14 – 5-18 (pressure, deformation, and permeability distributions vs. distance at several moments in time). Let us consider Fig. 5-13. Prior to the pressure in the high-permeability thinner layer reaching the closed boundary, the type curves have familiar features. Once the boundary has been reached at approximately \( t_{D}/C_D = 10^4 \), the derivative starts to increase in response to increasing pressure. As the deviation of pressure in the high-permeability layer from the initial pressure increases and the difference between pressures in layers increases, substantial unloading of the thicker low-permeability layer will occur and its permeability away from the well will increase (see left group of graphs in Fig. 5-17), while on the well contour it remains at approximately same low value. Since near-well permeability determines the flow rate, the permeability improvement elsewhere in the reservoir is inconsequential. At about \( t_{D}/C_D = 2*10^5 \) the boundary is reached in the low-permeability layer and for a period of time the pressure curve flattens (derivative drops). Up to and during this time it is still possible to maintain constant rate production, but eventually (at \( t_{D}/C_D \approx 6*10^5 \)) the
pressure drops to such a degree that flow rate declines. The second flattening of the pressure curve occurs at about \( t_D/C_D = 10^7 \) and corresponds to the time when near-well permeability in the low-permeability layer is reduced to about 1-2% of the initial value, therefore, effectively choking this layer, and when the magnitude of drawdown in the high-permeability layer becomes negligible. At this time the flow rate essentially drops to zero.

![Diagram](image)

**Figure 5-13.** Type curves for constant rate drawdown in closed drainage area. Two-layer case with \( h_2/h_1=9 \) and \( k_1/k_2=100 \)

Note again that until the boundary has not been reached the behavior of pressure and derivative curves with time is indistinguishable from a conventional linear case.
Figure 5-14. Closed drainage area. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 0.1$ (left) and $t_D/C_D \approx 1$ (right group)
Figure 5-15. Closed drainage area. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 10$ (left) and $t_D/C_D \approx 100$ (right group)
Figure 5-16. Closed drainage area. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+3$ (left) and $t_D/C_D \approx 1E+4$ (right group)
Figure 5-17. Closed drainage area. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+5$ (left) and $t_D/C_D \approx 1E+6$ (right group)
Solid blue line: $h_1=1$ m, $k_1=100$ md
Dashed red line: $h_2=9$ m, $k_2=1$ md

Figure 5-18. Closed drainage area. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+7$ (left) and $t_D/C_D \approx 3E+7$ (right group)
5.3 Constant pressure drawdown in a finite reservoir with constant pressure outer boundary

The next problem we consider is constant pressure production from a circular reservoir with constant pressure outer boundary. The reservoir is comprised of two layers. We will focus here mainly on the steady-state flow rate, as the transient processes of stabilization have been described earlier and are qualitatively the same in this case.

As in the previous examples, let $a = 0.0384$, $\gamma_j = E_j/E = 1$, $E = 10^{10} Pa$, $r_w/h = 0.001$, $h = 10 m$, $C_D = 100$, $r_{eD} = 10^4$, $s_j = 0$ (subscript $j=1, 2$ denotes layer number). Also, $k_1 = 100 md$, $k_2 = 1 md$, $h_1 = 1 m$, $h_2 = 9 m$, $\alpha_1 = 0$, $\alpha_2 = 4$. The inflow performance curve for this case is presented in Fig. 5-19 along with Darcy straight-line IPR (dotted line). We can see that the difference between the two IPRs is negligible. On the same graph we present as dashed curve the results for a case with $k_1 = 10 md$, $k_2 = 1 md$, $h_1 = 1 m$, $h_2 = 9 m$, $\alpha_1 = 1$, $\alpha_2 = 4$. In the latter case permeability of both layers is sensitive to deformation with the value of parameter $\alpha_i$ in line with the magnitude of the initial layer permeability. We can see that in this case the deviation from the straight line is significant since the magnitudes of individual layers’ flow rates are now comparable and both layers experience permeability reduction. The interaction of pressures in layers results in higher flow rate for the same magnitude of drawdown as compared to a single-layer case with $\alpha = 4$ (see respective curve in Fig. 4-38). The presence of a thin higher-permeability layer causes unloading of low-permeability layer and the total rate increases.
Overall, it is clear that presence of a high-permeability layer, in which permeability does not change noticeably with change of effective stress, masks the stress-sensitive behavior of low-permeability layer when both layers are produced simultaneously. If the permeability contrast between the two layers is not significant then it may be possible to detect the characteristic qualitative response of a stress-sensitive reservoir, given, of course, that we are able to confidently isolate the pressure- and rate-dependent effects (multiphase flow and non-Darcy flow).
5.4 Buildup following constant rate/pressure drawdown in a finite two-layer reservoir with constant pressure outer boundary

Just as it was observed earlier in the single-layer case, a drawdown test followed by a buildup test can provide a much better indication of the stress-sensitive behavior. Depending on the duration of the drawdown and, respectively, the area around a well that was affected, buildup response does reflect the altered permeability in this zone.

Consider the case with \( a = 0.0384 \), \( \gamma_j = E_j/E = 1 \), \( E = 10^{10} \) Pa, \( r_w/h = 0.001 \), \( h = 10 \) m, \( C_D = 100 \), \( r_{ed} = 10^4 \), \( s_j = 0 \), \( k_1 = 10 \) md, \( k_2 = 1 \) md, \( h_1 = 1 \) m, \( h_2 = 9 \) m, \( \alpha_1 = 1 \), \( \alpha_2 = 4 \) (subscript \( j = 1, 2 \) denotes layer number). The type curves are presented in Fig. 5-20 and have all the characteristic features, described earlier. During drawdown the pressure difference increases fairly slowly because of the expanding zone with altered permeability. The derivative reflects the permeability reduction by an upward trend, which can be interpreted as decreasing skin. Since flow rate is proportional to \( kh \) and inversely proportional to skin, it is obvious that for constant rate decrease of \( kh \) should be accompanied by decrease of skin and vice versa.

During a pressure buildup, the permeability in both layers improves in response to decreasing pressure difference in the formation. Such behavior leaves characteristic signature trend on the type curves. The pressure difference in the well increases fairly rapidly and the derivative behaves as if the permeability-thickness product were increasing. As before, a combination of drawdown and buildup test provides a clear qualitative indication of formation stress-sensitivity.
Figure 5-20. Drawdown followed by buildup in a two-layer reservoir

It is of interest to review the transient processes of stabilization of pressure, deformation, and permeability. The distributions of these parameters vs. distance for several moments in time are presented in Figs. 5-21 through 5-25 (drawdown) and Figs. 5-26 through 5-30 (buildup). The dashed curves correspond to the second layer. The series of graphs illustrate the interaction of pressure fields in layers, what impact it has on permeability distributions, and allow to visualize the effects that eventually result in the behavior observed on the type curves.
Figure 5-21. Drawdown. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 0.1$ (left) and $t_D/C_D \approx 1$ (right group)
Figure 5-22. Drawdown. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 10$ (left) and $t_D/C_D \approx 100$ (right group)
Figure 5-23. Drawdown. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+3$ (left) and $t_D/C_D \approx 1E+4$ (right group)
Figure 5-24. Drawdown. Distributions of pressure, deformation, and permeability vs. distance at \( t_D/C_D \approx 1E+5 \) (left) and \( t_D/C_D \approx 1E+6 \) (right group)
Figure 5-25. Drawdown. Distributions of pressure, deformation, and permeability vs. distance at $t_d/C_D \approx 1E+7$ (left) and $t_d/C_D \approx 1E+8$ (right group)
Figure 5-26. Buildup. Distributions of pressure, deformation, and permeability vs. distance at $t_d/C_D \approx 0.1$ (left) and $t_d/C_D \approx 1$ (right group)
Figure 5-27. Buildup. Distributions of pressure, deformation, and permeability vs. distance at $t_d/C_D \approx 10$ (left) and $t_d/C_D \approx 100$ (right group)
Figure 5-28. Buildup. Distributions of pressure, deformation, and permeability vs. distance at $t_d/C_D \approx 1E+3$ (left) and $t_d/C_D \approx 1E+4$ (right group)
Figure 5-29. Buildup. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+5$ (left) and $t_D/C_D \approx 1E+6$ (right group)
Figure 5-30. Buildup. Distributions of pressure, deformation, and permeability vs. distance at $t_D/C_D \approx 1E+7$ (left) and $t_D/C_D \approx 1E+8$ (right group)
5.5 Water injection into a two-layer reservoir at different injection pressures in individual layers

We do not present in detail the cases of injection and falloff tests, as behavior of these tests is consistent with already presented results. It is interesting though to consider the situation that was described in the introduction. According to a series of field tests, injection at increased pressures in one of the layers while maintaining the same injection pressure in the other layer had resulted in a decrease of the volume of water accepted by the layer with constant injection pressure. This could be an indication of interaction of pressure fields in separate layers, which would cause, according to our model, deformation (inflation) of the layer with increased injection pressure and respective compaction of the layer with constant injection pressure. The deformation would cause appropriate change of permeability, specifically its decrease in the layer with constant injection pressure.

Let us consider a two-layer reservoir with the following elastic and geometric parameters: \( a = 0.0384 \), \( \gamma_j = E_j/E = 1 \), \( E = 10^{10} \text{ Pa} \), \( r_w/h = 0.001 \), \( h = 10 \text{ m} \), \( h_j = 5 \text{ m} \), \( C_D = 100 \), constant pressure outer boundary is at \( r_eD = 10^4 \), \( s_j = 0 \), \( k_j = 10 \text{ md} \), \( \alpha_j = -2 \) (subscript \( j=1, 2 \) denotes layer number). Also dimensionless injection pressure in the first layer is \( p_{wD}^1 = 3 \) and let us vary injection pressure in the second layer and observe how the volumes injected in each layer change. In Fig. 5-31 we present the behavior of dimensionless injection rates in each layer vs. dimensionless time. For the blue and red curves \( p_{wD}^2 = 1 \) and for the green and black curves \( p_{wD}^2 = 5 \).
We can see that the amount of fluid a layer accepts depends on the magnitudes of the injection pressures in both layers. At lower $p_{wD}^2 = 1$ the injection rate in the first layer (blue curve) is smaller than in the second layer (red curve). Regardless of the fact that the properties of both layers are identical, the stabilized injection rates differ quite significantly and in non-linear fashion. Higher injection pressure in the second layer causes its inflation and respective compaction of the first layer with lower $p_{wD}^1 = 3$ (green and black curves).

Steady-state injection rates in both layers for constant $p_{wD}^1 = 3$ vs. $p_{wD}^2$ are presented in Fig. 5-32. The solid blue and dashed red curves correspond to the first and second layer, respectively.
Figure 5-32. Layers’ injection rates vs. injection pressure in the second layer for constant injection pressure in the first layer

The results qualitatively agree with the field tests thus suggesting a possible explanation of the observed behavior.
6 THREE LAYER CASE

As stated earlier, we can model a reservoir with an arbitrary number of layers. In this chapter we present several typical problems that have been calculated for a three-layer reservoir. As one would expect, based on examination of the two-layer cases, the increased number of layers should make the characteristic features of the single-layer type curves even less obvious, more so if the layers’ properties are significantly different.

6.1 Constant rate drawdown followed by buildup in a finite reservoir with constant pressure outer boundary

Let us consider a three-layer reservoir with $a = 0.0384$, $\gamma_j = E_j/E = 1$, $E = 10^{10} \, Pa$, $r_w/h = 0.001$, $h = 10 \, m$, $C_D = 100$, constant pressure outer boundary is at $r_{eD} = 10^4$, and $s_j = 0$ (subscript $j$ denotes layer number). The layers have different thicknesses, initial permeabilities, and sensitivities of permeability to deformation. These values are as follows: $h_1 = 1 \, m$, $k_1 = 100 \, md$, $\alpha_1 = 0$; $h_2 = 2 \, m$, $k_2 = 10 \, md$, $\alpha_2 = 1$; $h_3 = 7 \, m$, $k_3 = 1 \, md$, $\alpha_3 = 4$ (the highest-permeability layer having the least thickness and negligible sensitivity of permeability to deformation).

The type curves for this case are presented in Fig. 6-1 (blue solid curves – drawdown, red dashed curves – buildup). We can see that as in the two-layer case, large permeability contrast and respective values of sensitivity of permeability to deformation makes this case virtually indistinguishable from a linear case with no account for deformation.
It has been shown earlier that the results depend also on a layer thickness and thin layers deform to a lesser degree. In Fig. 6-2, we present the results for the case when high-permeability layer is 0.1 m thick, \( h_2 = 1 \text{ m} \), \( h_3 = 8.9 \text{ m} \), and all other parameters are the same as in the previous example. The high-permeability layer deforms less, the lower-permeability layers experience less unloading and the buildup type curves qualitatively resemble more the type curves for a single-layer case.

If permeability contrast among layer is relatively small and, respectively, the layers’ permeabilities have not negligible sensitivity to deformation, then the difference between drawdown and buildup curves becomes larger. This is shown in Fig. 6-3 where we present the results for the case with \( h_1 = 1 \text{ m} \), \( k_1 = 10 \text{ md} \), \( \alpha_1 = 1 \); \( h_2 = 2 \text{ m} \), \( k_2 = 5 \text{ md} \), \( \alpha_2 = 2 \);
\[ h_3 = 7 \, m, \; k_3 = 1 \, md, \; \alpha_3 = 4. \] The rest of the parameters are the same as in previous two examples.

![Graph showing drawdown followed by buildup in a three-layer reservoir](image)

**Figure 6-2. Drawdown followed by buildup in a three-layer reservoir. Thin high permeability layer**

We should note again that it is practically impossible to infer stress-sensitive behavior from drawdown curves alone. This means that for confident identification of the effect under consideration and for its quantitative description one would need an extended flow period, so that a large area around a well is affected by the drawdown, followed by a shut-in period, which is long enough for the downward trend on the derivative to be established. One would also need to have some evidence of layering of the reservoir and some crude estimates of layers’ permeability values.
Figure 6-3. Drawdown followed by buildup in a three-layer reservoir. Low permeability contrast

The type curve behavior of a three-layer system, as evident from the above three examples, is in accordance with the previous results, so we will not consider any other drawdown/buildup problems.

6.2 Water injection into two layers, separated by an impermeable layer, at different injection pressures in individual layers

Let us again consider the case of water injection into two layers at different magnitudes of injection pressure in individual layers. Field tests have shown that the volume of water that any particular layer accepts depends on pressures in both layers. Increase of
injection pressure in one of the layers results in decrease of the volume accepted by the other layer, even though the injection pressure in that layer is kept constant. The layers may be several meters apart.

We consider a three-layer reservoir with $a = 0.0384$, $\gamma_j = E_j/E = 1$, $E = 10^{10}$ Pa, $r_u/h = 0.001$, $h = 10$ m, $C_D = 100$, constant pressure outer boundary is at $r_{DO} = 10^4$, and $s_j = 0$ (subscript $j$ denotes layer number). The two permeable layers with identical properties are separated by a thicker impermeable layer. Let $h_1 = h_3 = 5$ m, $k_1 = k_3 = 10$ md, $\alpha_1 = \alpha_3 = -2$; $k_2 = 10^{-15}$ md, $\alpha_2 = 0$. The injection is at constant $p_{wD}^3 = 3$ in the bottom layer, injection pressure in the first layer varies between 1 and 6. Since the middle layer is impermeable, does not accept any water, and pressure in it does not change, any interaction between two permeable layers is possible by means of deformation of all three layers.

Let us see what effect has the thickness of the impermeable layer. For results presented in Fig. 6-4 $h_2 = 0, 10, 20$ m from top to bottom graph, respectively. Solid blue line corresponds to the third layer with constant injection pressure, dashed red line corresponds to the first layer. For zero thickness of the middle layer the results coincide with the two-layer case, presented earlier. As thickness of the impermeable layer increases, the interaction between permeable layers becomes less. We can see that even though the distance between the producing layers may be substantial they still do affect each other and exhibit the observed in the field experiments trend.
Figure 6-4. Three-layer case. Various thicknesses of impermeable middle layer
7 MULTILAYER CASE

Comparison of single-, two-, and three-layer cases shows that the presence of several layers tends to mask the characteristic behavior of the single-layer type curves, more so when the layers’ properties (such as permeability) vary significantly from layer to layer. In fact, as the heterogeneity increases the results become more similar to the linear case, i.e. calculated without accounting for stress-dependence. The interaction of pressures in layers can be easily understood if we recall the formula for vertical deformation of an individual layer.

\[
\left[u_z\right] = \left[u_z\right] - \eta \sum_{k=1}^{n} \frac{p_k}{c_k} + \eta \frac{p_j}{c_j} \quad (7.1)
\]

At any given point in the reservoir, the first term in the expression above is essentially the same for all layers if the magnitudes of layers Young’s moduli and thicknesses are close. The principal difference among layers is attributed to the last term, \( \eta p_j/c_j \). During drawdown, the radius of investigation in a higher-permeability layer is greater and pressure difference is greater in the high-permeability layer at any point and at any moment in time. According to Eq. 7.1 the deformation of the high-permeability layer is also greater than that of lower-permeability layers. Greater deformation results in a certain degree of “unloading” of lower-permeability layers and an improvement, or at least in a lesser damage, to their permeabilities. The higher-permeability layer, on the other hand, suffers certain permeability reduction. As a result of these deformation effects, the pressure in the lower-permeability layers tends to catch-up with pressure in the higher-permeability layers, as the former experiences permeability improvement and the latter – degradation. The net effect is that pressure disturbance moves into the formation approximately uniformly in all layers. As the
permeability contrast among layers increases and as the thickness of the high-permeability layer increases the effect is more pronounced.

Elsewhere in the reservoir the deformation effects tend to reduce the permeability contrast, but on the well contour, the magnitude of deformation is the same in all layers (if elastic moduli and thicknesses are the same) or at least comparable across layers, and highest- and lowest-permeability layers suffer least and most permeability reduction, respectively. The created deformational choke greatly impedes flow from the low-permeability layer. Even though far-field permeability in it might be improved, the input of this layer into well flow rate is very low.

Let us consider a 6-layer reservoir with the following properties: \( a = 0.0384 \), \( \gamma_j = E_j/E = 1 \), \( E = 10^{10} \) Pa, \( r_w/h = 0.001 \), \( h = 10 \) m, \( C_D = 100 \), constant pressure outer boundary is at \( r_{oD} = 10^4 \), and \( s_j = 0 \). Sensitivity of a layer to deformation depends on the magnitude of the initial permeability. These parameters are shown in Table 7-1.

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</tbody>
</table>

The drawdown and buildup type curves are presented in Fig. 7-1. We can see that it might be difficult to distinguish this case from a single-layer linear case if only short-duration drawdown or buildup data are available. The combination of the two tests of sufficient
duration might allow for interpretation. The individual layers’ parameters is practically impossible to determine.

![Figure 7-1. 6-layer case](image)

Though not discussed in any detail, possible crossflow between adjacent layers can significantly change the processes of stabilization of the major parameters. In the program the crossflow is incorporated as an additional term of the form $CF_{j}^{j+1}(p_D - p_{j+1D})$ in the pressure equation, where $CF_{j}^{j+1}$ is the dimensionless crossflow coefficient between adjacent layers. The greater is this coefficient and the greater is the permeability contrast and, respectively, the contrast of sensitivities of permeability to deformation in adjacent layers, the closer are the type curves to the linear case.
8 FIELD CASE

8.1 Account for pressure-dependent fluid properties and stress-sensitivity

Prior to proceeding to the field case, let us include in calculations the fluid properties and examine how dependence of oil and gas properties on pressure and temperature affect the results. We have incorporated correlations, which allow calculating oil or gas viscosity, formation volume factor, etc. into the code and have performed several runs in order to evaluate how accounting for changing fluid properties in addition to accounting for permeability dependence on deformation affects the results.

The variation of oil viscosity was included into the calculation of the non-linear term in the pressure equation

$$\frac{1}{r_D} \frac{\partial}{\partial r_D} \left[ r_D \frac{K_j \partial p_{jD}}{\mu^*_o(p) \partial r_D} \right] = \frac{\omega_j \partial p_{jD}}{\kappa_j \partial t_D}$$

where $\mu^*_o(p)$ is ratio of oil viscosity at a particular value of pressure to the value at the initial pressure. For simplicity, we did not consider variation of oil or formation compressibility and oil formation volume factor.

The parameters for the example single-layer case are given in Table 8-1.

Table 8-1. Parameters for the case with account for variation of fluid properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial reservoir pressure, psia</td>
<td>4,000</td>
</tr>
<tr>
<td>Formation temperature, oF</td>
<td>200</td>
</tr>
<tr>
<td>Net pay thickness, ft</td>
<td>30.5</td>
</tr>
<tr>
<td>Oil gravity, oAPI</td>
<td>40</td>
</tr>
<tr>
<td>Wellbore radius, ft</td>
<td>0.328</td>
</tr>
<tr>
<td>Gas specific gravity</td>
<td>0.7</td>
</tr>
<tr>
<td>Reservoir radius, ft</td>
<td>3,281</td>
</tr>
<tr>
<td>Solution GOR, scf/bbl</td>
<td>800</td>
</tr>
<tr>
<td>Skin factor</td>
<td>0</td>
</tr>
</tbody>
</table>

Additionally, $a = 0.0384$, $\gamma_j = E_j/E = 1$, $E = 10^{10}$ Pa, $C_D = 100$, no gas impurities. We have also required 10% retained permeability at the largest drawdown, which
corresponds in this case to $\alpha = 1.6$ at $q = 1,600$ BOPD. Since parameter $\alpha$ is proportional to production rate, its magnitude was scaled accordingly to values of rate.

In Fig. 8-1 we present results of calculation of IPRs when accounting only for variation of oil viscosity (dashed blue curve), only for permeability sensitivity to deformation (solid green curve), and for both factors (dotted red curve).

![Graph showing sandface IPRs for the three cases](image)

**Figure 8-1. Sandface IPRs for the three cases**

We can see that permeability stress-dependence results in substantial deviation of an IPR from a conventional case when only variation of oil viscosity is considered. For bottomhole pressures significantly below the bubble-point (3,318 psia) the deviation is larger, as the oil viscosity increases and the magnitude of the non-linear pressure- and deformation-dependent term in the equation of flow further decreases. It should be also
noted that in this case for the conditions of predominantly single-phase formation flow with a small zone of gas liberation around the wellbore, the effect of permeability stress-dependence is much greater than the effect of pressure-dependent fluid properties. For more accurate description of simultaneous flow of oil and gas we need to account also for changing relative permeabilities which would result in additional and approximately the same deviation of all curves in Fig. 8-1.

Properties of gases, on the other hand, normally vary in a much broader range. In order to accurately account for the variations, one should use adjusted pseudo-pressure and pseudo-time. The form of the dimensionless flow equation remains the same and solutions developed for flow of liquid apply. However, when performing analysis one must plot actual data in adjusted variables.

When considering the field case, we assume that deformation is the same as before function of the adjusted pseudo-pressure, since the numerical values of dimensionless pressure in liquid case and of adjusted pseudo-pressure coincide, and permeability depends on deformation.

8.2 Field description

The Opon gas field is located in the La Paz formation in Colombia and produces lean retrograde gas with dew point pressure of about 9,000 psi and 45o API condensate gravity (all information is as of 1995). The area is characterized by high tectonic stresses – 1.3 psi/ft, the reservoir is over pressured at 1 psi/ft. As a result of thrust faulting this single-layer homogeneous reservoir is highly fractured. Common problems in the field include perforation failure, there is also concern about reservoir stability and possible earthquakes.
According to results of core analysis matrix permeability ranges from 0.005 to 0.2 md, core porosity ranges from 5 to 9%. While simulating depletion core permeability is sensitive to increasing stress. Results of pressure transient analysis indicate permeability-thickness product in the range from 1,000 to 2,100 md-ft, which along with the core analysis indicates high density of natural fractures. Some other data is provided in Table 8-2.

**Table 8-2. Opon field data**

<table>
<thead>
<tr>
<th>Field name</th>
<th>Opon field</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir age</td>
<td>Eocene</td>
</tr>
<tr>
<td>Stratigraphic unit</td>
<td>La Paz formation</td>
</tr>
<tr>
<td>Geologic setting</td>
<td>Braided fluvial</td>
</tr>
<tr>
<td>Lithology</td>
<td>Sandstone and shale</td>
</tr>
<tr>
<td>Porosity type</td>
<td>Intergranular, feldspar dissolution</td>
</tr>
<tr>
<td>Trapping mechanism</td>
<td>North-plunging fault bend anticline</td>
</tr>
<tr>
<td>Structural size, acres</td>
<td>7,750</td>
</tr>
<tr>
<td>Gas column, ft</td>
<td>3,600</td>
</tr>
<tr>
<td>Average porosity, %</td>
<td>7.4</td>
</tr>
<tr>
<td>Average Sw, %</td>
<td>35</td>
</tr>
<tr>
<td>Average gross thickness, ft</td>
<td>1,900</td>
</tr>
<tr>
<td>Average pay thickness, ft</td>
<td>775</td>
</tr>
<tr>
<td>Gas gravity</td>
<td>0.725 (recombined)</td>
</tr>
<tr>
<td>Dew point pressure, psia</td>
<td>8,640</td>
</tr>
<tr>
<td>Average well depth, ft</td>
<td>12,500</td>
</tr>
<tr>
<td>Reservoir pressure Opon #3 / Opon #4, psi</td>
<td>11,100 (-9,186’ subsea) / 10,850 (-9,128’)</td>
</tr>
<tr>
<td>Reservoir temperature, oF</td>
<td>210</td>
</tr>
<tr>
<td>Estimated field reserves</td>
<td>2.2 TCF and 94 MMBC</td>
</tr>
<tr>
<td>Production test results Opon #3</td>
<td>45 MMSCFD and 2,000 BCPD</td>
</tr>
<tr>
<td>Production test results Opon #4</td>
<td>58 MMSCFD and 1,900 BCPD</td>
</tr>
</tbody>
</table>
A single isochronal test data set is available (Fig. 8-2). The quality of the data prevents from obtaining a confident type-curve match, though certain qualitative features can be noted.

Figure 8-2. Opon #3. Modified isochronal test

In Fig. 8-3 we present an attempt to match the first drawdown data. Clearly the data is far too noisy (other flow periods are even worse), though one might see an upward trend on the derivative.
Figure 8-3. Type curve analysis of the first flow period

The buildup data is less noisy but lack of early time data prevents from a confident match (see Figs. 8-4 – 8-7). The (arguably) recognizable downward trend on the derivative is in line with prediction of our model. Late data exhibit a more discernable drop, which may be indicative of the end of the zone of altered permeability, created by the first drawdown. Sensitivity of permeability to deformation appears to be fairly high (lowest type curves correspond to the linear case $\alpha = 0$, highest – to $\alpha = 4$).
Figure 8-4. First buildup

Figure 8-5. Second buildup
Figure 8-6. Third buildup

Figure 8-7. Fourth (extended) buildup
Other buildup periods also show the downward trend on the derivative, which gives certain assurance in the validity of the model, but confident analysis is impossible in absence of quality data in the drawdown periods and early-time data in the buildup periods. The data requirements are as follows. We need at least two flow periods followed by two buildups. During the first flow period the flow rate should be fairly low and the duration of the first buildup should be preferably the same as duration of the first drawdown in order for permeability to restore to its initial value. The magnitude of the second flow rate should be higher than that of the first flow period, but preferably low enough to minimize the multi-phase and non-Darcy flow effects. Duration of both flow periods should be long enough in order to observe the data undistorted by the wellbore storage. The second buildup can be of shorter duration and can be terminated as soon the downward trend in the middle-time region is observed.

The data then should plotted using appropriate plotting functions and type curve match should be performed. From the match two values of $\alpha$ will be obtained, which will be in the same proportion as the respective flow rates. Since $\alpha = \beta \frac{\eta}{E} \frac{q}{2\pi(kh)}$, we will be able to determine the sensitivity of permeability to deformation and the permeability-thickness product. Skin factor in its conventional sense of an additional pressure drop on the well contour can not be determined, rather we will calculate permeability impairment in the near-wellbore zone accordingly to the sensitivity of permeability to deformation. The value of average reservoir pressure can be determined fairly accurately for small sensitivity of permeability to deformation if one extrapolates drawdown and respective buildup data until pressure curves intersect. In general, the value of average reservoir pressure will be overestimated.
Once the parameters have been determined, one can make a decision on the optimal magnitude of the drawdown on the well. In some cases it will make economic sense to limit the production rate, as it will result in less permeability reduction and less reservoir drawdown needed to maintain this rate. Fracturing can be very beneficial in the sense that it allows economically produce a well with low drawdown.

We have seen that the majority of permeability reduction occurs in the near-wellbore zone. This reduction is the greater the lower is the permeability of this zone, which makes the problems of introduced near-wellbore damage and of fines migration more acute.
9 CONCLUSIONS

The developed model of fluid flow in a deformable reservoir incorporates elastic properties of the system “reservoir – surrounding rock mass” to calculate deformations in this system due to pore pressure change caused by well operation. We use a formal model of a uniform reservoir with permeability assumed to be a strong function of vertical deformation of the reservoir. This approach is suitable to describe the behavior of a naturally fractured reservoir and some other possible situations. Solution to problems of fluid flow in deformable multi-layer reservoir with account for wellbore storage, skin, and crossflow between layers were obtained by finite-difference methods. We have solved numerically several problems with typical boundary conditions and made the following conclusions.

- The deformation of a reservoir depends on the elastic properties of the entire system comprised of the reservoir itself and the embedding rock mass.
- The behavior of a drawdown (injection) test is inconsistent with a buildup (falloff) test. The respective tests should be analyzed with different sets of type curves.
- The processes of production and injection are asymmetric.
- During drawdown/buildup test, the permeability-thickness product in a stress-sensitive reservoir appears to be decreasing/increasing with time as the test continues.
- The presence of several layers with different properties tends to mask the characteristic drawdown response and to a lesser degree the characteristic buildup response. The greater is the properties contrast of the layers, the more the behavior of both tests is obscured. These effects are due to the interaction of the pressure and deformation fields in individual layers.
• The drawdown response of a highly heterogeneous reservoir with stress-dependent permeability is practically indistinguishable from the response of a homogeneous reservoir with no stress-dependence. The buildup response may still provide a qualitative indication of permeability stress-sensitivity.

• In order to estimate the parameters of a stress-sensitive reservoir at least two drawdown-buildup sequences are required. The magnitudes of flow rates during drawdown periods must be selected so that to minimize the multi-phase and non-Darcy flow effects at the largest possible difference between the two rates.

• The results qualitatively agree with the field data, thus suggesting an explanation to phenomena observed in practice. For quantitative analysis the tests should designed and run accordingly to the recommended here procedure.
REFERENCES


Driver
Option Explicit
Option Base 1
Public nx As Integer, nl As Integer, ntau As Integer, m As Integer
Public tauInit As Double, tau As Double, tauMult As Double, TotalTime As Double, sa() As Double
Public dx As Double, reD As Double, rD() As Double, u() As Double, Eo As Double, CD As Double, pwD As Double, p_mult As Double
Public MaxT As Double, MaxP As Double, ConstantPwD As Double, qq As Double, pwDinit As Double, q As Double, pres As Double, muo As Double
Public bDone As Boolean, bConstantQ As Boolean, bNoFlow As Boolean, bSparse As Boolean
Public Points As Collection, Layers As Collection, Reservoir As cls05Reservoir
Public Initialize As cls01Initialize, FlProp As cls02FlProp, Layer As cls03Layer, Point As cls04Point
Public ExitCriteria As clsExitCriteria, Newton As clsNewton
Public Vector As clsVector, Matrix As clsMatrix, Outp As clsOut
Public VEH As clsVEH, bDrawDown As Boolean, bBuildUp As Boolean, bBoth As Boolean, PrintFrequency As Integer
Sub MainProg()
    Dim timestart As Double, x() As Double, check As Boolean, niter As Integer
    ' On Error GoTo 3
    Set Initialize = New cls01Initialize
    timestart = Timer
    1   Call Initialize.WhatToRun(x)
    While Not bDone
        ntau = ntau + 1
        TotalTime = TotalTime + tau
        If bConstantQ Then
            Call Newton.Run(x, check, niter)
        Else
            While Not ExitCriteria.bConverged(x)
                Call Reservoir.SetLayersPressure(x)
                Reservoir.Pressure
                Call Reservoir.Deformation(0, 0)
                Reservoir.CalculateLayersPermeability
                Call Vector.Progonka(x)
            Wend
            End If
        Reservoir.UpdateLayersPressure
        Outp.Intermediate
        tau = tau * tauMult
        Call ExitCriteria.Evaluate(x)
    Wend
    Sheets("Input").Cells(30, 2) = Timer - timestart
    Exit Sub
    If bBoth Then GoTo 1
    Exit Sub
3   Sheets("Input").Cells(30, 2) = "Error"
End Sub
------------------------------------------------------------------
Initialization
Option Explicit
Option Base 1
Private ms_ftype As String, md_Pb As Double, md_H2S As Double, md_CO2 As Double, md_N2 As Double, md_T As Double
Private md_API As Double, md_SGG As Double, md_Rsb As Double, md_Rs As Double
Private mb_bProd As Boolean, md_LayerParams() As Double, md_kht As Double, md_phiht As Double, md_1ci As Double
Private md_pres As Double, md_qwell As Double
Sub Class_Initialize()
ReadSpreadsheetData
InitializeFileReader
PerformAuxCalculations
InitializeFluidProperties
InitializeLayers
InitializePoints
InitializeReservoir
Set ExitCriteria = New clsExitCriteria
Set Outp = New clsOut
Set Vector = New clsVector
Set Matrix = New clsMatrix
Set Newton = New clsNewton
Set VEH = New clsVEH
End Sub
Private Sub ReadSpreadsheetData()
Dim item As Variant, j As Integer, rw As Double, re As Double
Sheets("Input").Activate
For Each item In ActiveWorkbook.Names
Select Case item.Name
Case Is = "q": md_qwell = Range(item.Name).Value
Case Is = "pres": pres = Range(item.Name).Value
Case Is = "CD": CD = Range(item.Name).Value
Case Is = "pwD": ConstantPwD = Range(item.Name).Value
Case Is = "tau": tauInit = Range(item.Name).Value
Case Is = "taum": tauMult = Range(item.Name).Value
Case Is = "tmax": MaxT = Range(item.Name).Value
Case Is = "pmax": MaxP = Range(item.Name).Value
Case Is = "E": Eo = Range(item.Name).Value
Case Is = "nl": nl = Range(item.Name).Value
Case Is = "brw": bConstantQ = Range(item.Name).Value
Case Is = "bre": bNoFlow = Range(item.Name).Value
Case Is = "both": bBoth = Range(item.Name).Value
Case Is = "rw": rw = Range(item.Name).Value
Case Is = "re": re = Range(item.Name).Value
Case Is = "prod": mb_bProd = Range(item.Name).Value
Case Is = "API": md_API = Range(item.Name).Value
Case Is = "SGG": md_SGG = Range(item.Name).Value
Case Is = "Rsb": md_Rsb = Range(item.Name).Value
Case Is = "yH2S": md_H2S = Range(item.Name).Value
Case Is = "yCO2": md_CO2 = Range(item.Name).Value
Case Is = "yN2": md_N2 = Range(item.Name).Value
Case Is = "Pb": md_Pb = Range(item.Name).Value
Case Is = "T": md_T = Range(item.Name).Value
End Select
Next
End Sub
Case Is = "fltype": ms_ftype = Range(item.Name).Value
Case Else
End Select
Next item
reD = re / rw
md_kht = 0#
md_phiht = 0#
md_1ci = 0#
ReDim md_LayerParams(nl, 7)
Cells(9, 8).Select
For j = 1 To nl
  md_LayerParams(j, 1) = ActiveCell.Offset(j - 1, 0) 'h
  md_LayerParams(j, 2) = ActiveCell.Offset(j - 1, 1) 'phi
  md_LayerParams(j, 3) = ActiveCell.Offset(j - 1, 2) 'k
  md_LayerParams(j, 4) = ActiveCell.Offset(j - 1, 3) 'E
  md_LayerParams(j, 5) = ActiveCell.Offset(j - 1, 4) 's
  md_LayerParams(j, 6) = ActiveCell.Offset(j - 1, 5) 'alfa
  md_LayerParams(j, 7) = ActiveCell.Offset(j - 1, 7) 'xFlow
  md_kht = md_kht + md_LayerParams(j, 3) * md_LayerParams(j, 1)
  md_phiht = md_phiht + md_LayerParams(j, 2) * md_LayerParams(j, 1)
  md_1ci = md_1ci + md_LayerParams(j, 1) / (md_LayerParams(j, 4) / Eo)
Next j
End Sub
Private Sub PerformAuxCalculations()
  Dim a As Double, i As Integer
  a = 1#
  i = 0
  While a < 10#
    i = i + 1
    a = a * tauMult
  Wend
  PrintFrequency = Application.WorksheetFunction.Max(1, CInt(i / 10))
  If mb_bProd Then 'production
    qq = 1#
  ElseIf Not mb_bProd Then 'injection
    qq = -1#
  End If
  If Not bConstantQ Then qq = 0#
  dx = 1# / (nx - 1)
  ReDim rD(nx)
  For i = 1 To nx
    rD(i) = CDbl(i - 1) * dx
  Next i
  m = nx * nl
  bDrawDown = False
  bBuildUp = False
End Sub
Private Sub InitializeLayers()
  Dim j As Integer, lParams() As Double
  Set Layers = New Collection
  For j = 1 To nl
    Set Layer = New cls03Layer
    ReDim lParams(7)
    lParams(1) = md_LayerParams(j, 1): lParams(2) = md_LayerParams(j, 2): lParams(3) = md_LayerParams(j, 3)
lParams(4) = md_LayerParams(j, 4):  lParams(5) = md_LayerParams(j, 5):  lParams(6) = md_LayerParams(j, 6)
  lParams(7) = md_LayerParams(j, 7)
Call Layer.SetConstantParameters(lParams, md_kht, md_phiht)
Layers.Add Layer
Next j
End Sub
Private Sub InitializeReservoir()
  Set Reservoir = New cls05Reservoir
  Reservoir.oneci = md_lci
End Sub
Private Sub InitializeFluidProperties()
  Set FlProp = New cls02FlProp
  Call FlProp.ReadData(md_API, md_SGG, md_Pb, md_Rsb, md_H2S, md_CO2, md_N2, md_T, ms_ftype)
p_mult = (md_qwell * 1.84027) * FlProp.muo(pres) / 2# / Application.WorksheetFunction.Pi() / (md_kht * 0.001 * 100#)
End Sub
Private Sub InitializePoints()
  Dim Point As cls04Point, i As Integer, j As Integer, a1 As Double, a2 As Double
  Set Points = New Collection
  For i = 1 To nx
    Input #1, j, a1, a2
    Set Point = New cls04Point
    Point.number = j
    Point.small = a1
    Point.ints = a2
    Points.Add Point
  Next i
  For Each Point In Points
    With Point
      .SetArrays
        For i = 1 To .number
          Input #1, a1, a2
          .xx(i) = a1
          .yy(i) = a2
        Next i
      End With
    Next
  Close #1
End Sub
Private Sub InitializeFileReader()
  Dim aCalc As Single, aRead As Single, DataPath As String, FoundFile As String
  DataPath = ThisWorkbook.Path & "\Data\red= & CStr(reD)
aCalc = CStr(Cells(21, 8))
FoundFile = Dir(DataPath & "\?????????????????????????????.txt")
Do While FoundFile <> ""
  Open DataPath & "\" & FoundFile For Input As #1
  Input #1, nx, aRead
  If Abs(1# - aCalc / aRead) * 100# < 0.001 Then Exit Sub
  Close #1
  FoundFile = Dir()
Loop
End Sub
Private Sub BuildUp()
tau = tauInit
ntau = 0
TotalTime = 0#
qq = 0#
bDone = False
bDrawDown = False
bBuildUp = True
bConstantQ = True
qq = 0#
pwDinit = pwD
End Sub
Private Sub DrawDown()
    tau = tauInit
    ntau = 0
    TotalTime = 0#
    bDone = False
    bDrawDown = True
    bBuildUp = False
    If bConstantQ Then
        pwDinit = 0#
    Else
        pwDinit = ConstantPwd
    End If
End Sub
Sub WhatToRun(x)
    ReDim x(m)
    If bBoth And Not bDrawDown Then
        DrawDown
    ElseIf (bBoth And bDrawDown) Or Not bBoth Then
        BuildUp
    Call Output.Retrieve(x)
    bBoth = False
End If
pwD = pwDinit
End Sub

Fluid properties
Option Explicit
Option Base 1
Private md_API As Double, md_SGG As Double, md_Rsb As Double, md_H2S As Double, md_CO2 As Double, md_N2 As Double,
md_Tsc As Double, md_Psc As Double, md_T As Double, md_Pb As Double, md_STO As Double
Sub Class_Initialize()
    md_Tsc = 60#
    md_Psc = 14.696
End Sub
Sub ReadData(API, SGG, Pb, Rsb, H2S, CO2, N2, T, ftype)
    md_API = API:   md_SGG = SGG:   md_Rsb = Rsb:   md_H2S = H2S:   md_CO2 = CO2:   md_N2 = N2
    md_Co2 = CO2:
    md_T = T:       md_STO = 141.5 / (md_API + 131.5):
    If Pb = 0# Then
        md_Pb = Pbub
    Else
        md_Pb = Pb
    End If
End Sub
Property Get co(p) As Double
    If p < md_Pb Then
        '        co = Exp(-7.633 - 1.497 * Log(p) + 1.115 * Log(md_T + 459.67) + 0.533 * Log(md_API))
        '        If md_Rsb <> 0 Then co = co * Exp(0.184 * Log(md_Rsb))
        co = Exp(-7.573 - 1.45 * Log(p) - 0.383 * Log(md_Pb) + 1.402 * Log(md_T + 459.67) + 0.256 * Log(md_API))
        If md_Rsb <> 0 Then co = co * Exp(0.449 * Log(md_Rsb))
    Else
        co = (-1433# + 5# * md_Rsb + 17.3 * md_T - 1180# * md_SGG + 12.61 * md_API) * 0.00001 / p
    End If
End Property
Property Get muo(p) As Double
    muo = 10 ^ (10 ^ (1.8653 - 0.025086 * md_API - 0.5644 * Log(md_T) / Log(10#))) - 1#
    muo = (10.715 * (Rso(p) + 100#) ^ -0.515) * muo ^ (5.44 * (Rso(p) + 150#) ^ -0.338)
    If p > md_Pb Then muo = muo * (p / md_Pb) ^ (2.6 * p ^ 1.187 * Exp(-11.513 - 0.0000898 * p))
End Property
Property Get Rso(p) As Double
    Dim alpha As Double, beta As Double, gamma As Double, pr As Double, Rsr As Double
    If p < md_Pb Then
        pr = (p - 14.7) / (md_Pb - 14.7)
        alpha = 0.000000973 * md_SGG ^ 1.672608 * md_API ^ 0.92987 * md_T ^ 0.247235 * (md_Pb - 14.7) ^ 1.056052
        beta = 0.022339 * md_SGG ^ -1.00475 * md_API ^ 0.337711 * md_T ^ 0.132795 * (md_Pb - 14.7) ^ 0.302065
        gamma = 0.725167 * md_SGG ^ -1.48548 * md_API ^ -0.164741 * md_T ^ -0.09133 * (md_Pb - 14.7) ^ 0.047094
        Rsr = alpha * pr ^ beta + (1# - alpha) * pr ^ gamma
        Rso = md_Rsb * Rsr
        If Rso < 0# Then Rso = 0#
    Else
        Rso = md_Rsb
    End If
End Property
Property Get Bo(p) As Double
    Dim DRO As Double
    If p < md_Pb Then
        DRO = rhoo(p)
        Bo = (63.37 * md_STO + 0.01357 * Rso(p) * md_SGG) / DRO
    Else
        DRO = rhoo(md_Pb)
        Bo = (63.37 * md_STO + 0.01357 * Rso(p) * md_SGG) / DRO
        Bo = Bo * Exp(co(p) * (md_Pb - p))
    End If
End Property
Property Get rhoo(p) As Double
    Dim iter As Integer, ddone As Boolean, DA As Double, DN As Double
    iter = 0
    ddone = False
    DN = 52.8 - 0.01 * Rso(p)
    While Not ddone
iter = iter + 1
DA = -49.893 + (85.0149 - (3.70373 - 0.047981 * DN) * DN) * md_SGG + (2.98914 - 0.035688 * DN) * DN
rhoo = (Rso(p) * md_SGG + 4600# * md_STO) / (73.71 + Rso(p) * md_SGG / DA)
If Abs(rhoo - DN) < 0.000001 Then
ddone = True
Else
DN = rhoo
End If
If iter > 100 Then
ddone = True
Debug.Print "WARNING -- DENSITY CALCULATION DID NOT CONVERGE"
End If
Wend
If p < md_Pb Then
rhoo = rhoo + (0.167 + 16.81 * 10 ^ (-0.0425 * rhoo)) * (p / 1000#) _
- 0.01 * (0.299 + 263 * 10 ^ (-0.0603 * rhoo)) * (p / 1000#) ^ 2
Else
rhoo = rhoo + (0.167 + 16.81 * 10 ^ (-0.0425 * rhoo)) * (md_Pb / 1000#) _
- 0.01 * (0.299 + 263# * 10 ^ (-0.0603 * rhoo)) * (md_Pb / 1000#) ^ 2
End If
rhoo = rhoo - (0.00302 + 1.505 * rhoo ^ (-0.951)) * (md_T - md_Tsc) ^ 0.938 _
+ (0.0233 * 10 ^ (-0.0161 * rhoo)) * (md_T - md_Tsc) ^ 0.475
If p > md_Pb Then rhoo = rhoo * Exp(co(p) * (p - md_Pb))
End Property
Property Get Pbub() As Double
Dim VX As Double, VY As Double
VX = 0.013098 * md_T ^ 0.282372 - 0.0000082 * md_API ^ 2.176124
VY = md_Rsb ^ 0.081465 * md_SGG ^ -0.161488 * 10 ^ VX - 0.740152
Pbub = 1091.47 * VY ^ 5.354891
End Property
Property Get dRso_dp(p, Pb, Rsb, API, gasgrav, Tf) As Double
Dim pr As Double
pr = (p - 14.7) / (md_Pb - 14.7)
If pr <= 1# Then
VA = 0.000000973 * md_SGG ^ 1.672608 * md_API ^ 0.92987 * md_T ^ 0.247235 * (md_Pb - 14.7) ^ 1.056052
VB = 0.022339 * md_SGG ^ (-1.00475) * md_API ^ 0.337711 * md_T ^ 0.132795 * (md_Pb - 14.7) ^ 0.302065
VC = 0.725167 * md_SGG ^ (-1.48548) * md_API ^ -0.164741 * md_T ^ (-0.09133) * (md_Pb - 14.7) ^ 0.047094
dRso_dp = md_Rsb / (md_Pb - 14.7) * (VB * VA * pr ^ (VB - 1#) + VC * (1# - VA) * pr ^ (VC - 1#)) _
Else
dRso_dp = 0#
End If
End Property
Property Get dBo_dp(p) As Double
Dim pr As Double
pr = (p - 14.7) / (md_Pb - 14.7)
If pr <= 1# Then
    dBo_dp = 1.2 * (0.00012) * (Sqr(md_SGG / md_STO) * md_Rs + 1.25 * md_T) ^ 0.2 * Sqr(md_SGG / md_STO) * dRso_dp(p)
Else
    dBo_dp = 0#
End If
End Property

Property Get Bg(p) As Double
    Bg = (zfucktor(p) * (md_T + 460#) / p) * (md_Psc / (md_Tsc + 460#)) * 1000# / 5.61458333
End Property

Property Get cg(p)
    Dim zj As Double, zk As Double, pTc As Double, pPc As Double, pTr As Double, pPr As Double
    Dim za As Double, zb As Double, zc As Double, zd As Double, dzB_dpr As Double, dz_dpr As Double
    zj = md_H2S * -0.4582 * 672.12 / 1300 + md_CO2 * -0.90348 * 547.58 / 1071 + md_N2 * -0.66026 * 227.16 / 493.1 + 0.11582 + md_SGG * (0.70729 + md_SGG * -0.099397)
    zk = md_H2S * -0.06534 * 672.12 / Sqr(1300#) + md_CO2 * -0.42113 * Sqr(1300#) + md_N2 * -0.91249 * Sqr(1300#) + md_CO2 * -0.42113 * 547.58 / Sqr(1071#) + md_N2 * -0.91249 * 227.16 / Sqr(493.1) + 3.8216 + md_SGG * (17.438 + md_SGG * -3.2191)
    pTc = zk * zk / zj
    pPc = pTc / zj
    pPr = p / pPc
    za = 1.39 * Sqr(pTr - 0.92) - 0.36 * pTr - 0.101
    zb = (0.62 - 0.23 * pTr) * pPr + (0.066 / (pTr - 0.86) - 0.037) * pPr ^ 2 + 0.32 * pPr ^ 6 / 10 ^ (9# * (pTr - 1))
    zc = 0.132 - 0.32 * Application.WorksheetFunction.Log10(pTr)
    zd = Exp(0.3106 + pTr * (-0.49 * pTr * 0.1824))
    dzB_dpr = (0.62 - 0.23 * pTr) + 2# * (0.066 / (pTr - 0.86) - 0.037) * pPr + 6# * 0.32 * pPr ^ 5 / 10 ^ (9# * (pTr - 1#))
    dz_dpr = -(1# - za) * Exp(-zb) * dzB_dpr + zd * zc * pPr ^ (zd - 1#)
    z = za + (1# - za) / Exp(zb) + zc * pPr ^ zd
    cg = 1# / p - dz_dpr / (pPc * z)
End Property

Property Get mug(p) As Double
    Dim Corr_h2s As Double, Corr_co2 As Double, Corr_n2 As Double, Lg10SG As Double
    Dim alpha As Double, beta As Double, gamma As Double, tr As Double, Ma As Double, DG As Double, DG1 As Double
    Lg10SG = Log(md_SGG) / Log(10#)
    Corr_h2s = md_H2S * (3.73 + 8.49 * Lg10SG) / 1000#
    Corr_co2 = md_CO2 * (6.24 + 9.08 * Lg10SG) / 1000#
    Corr_n2 = md_N2 * (9.59 + 8.48 * Lg10SG) / 1000# - 2.062 * md_SGG * md_T / 1000000# + (8.188 - 6.15 * Lg10SG) / 1000# + Corr_h2s + Corr_co2 + Corr_n2
    tr = md_T + 459.67
    Ma = 28.9625 * md_SGG
    DG = p * Ma / zfucktor(p) / 10.32 / tr / 62.368
    alpha = ((9.379 + 0.01607 * Ma) * tr ^ 1.5) / (209.2 + 19.26 * Ma + tr)
    beta = 3.448 + 986.4 / tr + 0.01009 * Ma
    gamma = 2.447 - 0.2224 * beta
    DG1 = 14.696 * Ma / 10.732 / tr / 62.368
mug = mug * Exp(beta * (DG ^ gamma - DG1 ^ gamma))
End Property

Property Get zfucktor(p) As Double
Dim iter As Integer, ddone As Boolean, j As Double, k As Double, pPc As Double, pr As Double, TPC As Double
Dim c0 As Double, c1 As Double, c2 As Double, c3 As Double, c4 As Double
Dim dr As Double, dr0 As Double, dr2 As Double, dr5 As Double, dp As Double, pp As Double
Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, t5 As Double, t6 As Double
Dim tr As Double, tr2 As Double, tr3 As Double, tr4 As Double
j = 0.1158157 + (0.7072878 - 0.0993966 * md_SGG) * md_SGG - 0.2368944 * md_H2S - 0.4619311 * md_CO2 - 0.3041646 * md_N2
k = 3.821599 + (17.43771 - 3.219084 * md_SGG) * md_SGG - 1.218021 * md_H2S - 7.046435 * md_CO2 - 9.334518 * md_N2
TPC = k * k / j
pPc = TPC / j
tr = (md_T + 459.67) / TPC
pr = p / pPc
If tr < 1# Then MsgBox "WARNING -- TR < 1.0"
tr2 = tr * tr
tr3 = tr2 * tr
tr4 = tr2 * tr2
c1 = -0.7361 + 0.1844 / tr
C0 = 0.3265 * tr - 1.07 - 0.5339 / tr2 + 0.01569 / tr3 - 0.05165 / tr4
C2 = 0.5475 * tr + c1
C3 = -0.1056 * c1
C4 = 0.6134 / tr2
dr = 1#
iter = 0
ddone = False
Do While Not ddone
  dr0 = dr
  dr2 = dr * dr
  dr5 = dr2 * dr2 * dr
  t1 = c0 * dr
  t2 = c2 * dr2
  t3 = c3 * dr5
  t4 = c4 * dr2
  t5 = 0.721 * dr2
  t6 = Exp(-t5)
  pp = (tr + t1 + t2 + t3) * dr + t4 * dr * (1# + t5) * t6
  dp = tr + 2# * t1 + 3# * t2 + 6# * t3 + t4 * t6 * (3# + 3# * t5 - 2# * t5 ^ 2)
  dr = dr0 - (pp - 0.27 * pr) / dp
  If dr <= 0# Then
    dr = 0.5 * dr0
  ElseIf dr >= 3# Then
    dr = dr0 + 0.9 * (3# - dr0)
  End If
  If Abs(dr - dr0) < 0.000001 Then ddone = True
  iter = iter + 1
  If iter > 100 Then
    ddone = True
    Debug.Print "WARNING -- DENSITY CALCULATION DID NOT CONVERGE"
  End If
Loop
zfucktor = 0.27 * pr / (dr * tr)
End Property

Layer Class
Option Explicit
Option Base 1
Private md_k As Double, md_phi As Double, md_E As Double, md_h As Double, md_s As Double, md_alpha As Double
Private md_kappa As Double, md_omega As Double, md_lambda As Double, md_gamma As Double, md_hgamma As Double
Private md_q As Double, md_z() As Double, md_p() As Double, md_pnew() As Double, md_u() As Double, md_perm() As Double
Private md_t0 As Double, md_t1 As Double, md_t2 As Double, md_t3 As Double, md_t4 As Double, md_t5 As Double, md_t6 As Double
Private md_a() As Double, md_b() As Double, md_c() As Double, md_d() As Double, md_zz() As Double
Private md_aa As Double, md_sigma As Double, md_sigma1 As Double, md_Xflow As Double
Sub Class_Initialize()
    Dim i As Integer
    ReDim md_p(nx):     ReDim md_pnew(nx):     ReDim md_u(nx):     ReDim md_z(nx):     ReDim md_perm(nx)
    For i = 1 To nx
        md_perm(i) = 1#
        md_z(i) = reD ^ (2# * dx * (i - 1))
    Next i
    ReDim md_zz(nx):    md_zz(1) = md_z(2) - md_z(1):
    For i = 2 To nx - 1
        md_zz(i) = md_z(i + 1) - md_z(i - 1)
    Next i
End Sub
Sub SetConstantParameters(LayerParams, kht, phiht)
    md_h = LayerParams(1)
    md_phi = LayerParams(2)
    md_k = LayerParams(3)
    md_E = LayerParams(4)
    md_s = LayerParams(5)
    md_alpha = LayerParams(6)
    md_Xflow = LayerParams(7)
    md_kappa = md_k * md_h / kht
    md_omega = md_phi * md_h / phiht
    md_lambda = Log(reD) ^ 2 * md_omega / md_kappa * dx ^ 2
    md_gamma = md_E / Eo
    md_hgamma = md_h / md_gamma
    md_q = md_k * md_h / Kht
    md_sigma = 0.7
    md_sigma1 = 1# - md_sigma
End Sub
Property Get hgamma() As Double
    hgamma = md_hgamma
End Property
Property Get q() As Double
    q = md_q
End Property
Property Get pnew(Index As Integer) As Double
pnew = md_pnew(Index)
End Property

Property Get ul(Index As Integer) As Double
    ul = md_u(Index)
End Property

Property Get perm(Index As Integer) As Double
    perm = md_perm(Index)
End Property

Sub Permeability()
    Dim i As Integer, ptrue As Double
    For i = 1 To nx
        md_u(i) = (u(i) + md_pnew(i)) / md_gamma
        ptrue = (pres / 14.696 - md_pnew(i) * p_mult * md_h) * 14.696
        md_perm(i) = Exp(-md_alpha * md_u(i)) / FlProp.muo(ptrue) *
    Next i
    FIPProp.muo(pres)
End Sub

Private Sub CalculateVariableParameters()
    md_t0 = 2# * md_perm(1) / (2# - Log(md_perm(2)) + Log(md_perm(1)))
    md_t1 = md_t0 * md_kappa / Log(reD) / dx
    md_t2 = md_t0 * dx / tau * md_omega * Log(reD) / 2# / md_perm(1)
    md_t3 = md_lambda / tau
End Sub

Sub CalculateArrays()
    CalculateVariableParameters
    GenerateArray
    GenerateInnerBC
    If Not bConstantQ Then tridag
End Sub

Private Sub GenerateInnerBC()
    Dim qImplicit As Double, qExplicit As Double, deriv As Double
    If bConstantQ Then
        qImplicit = -(md_t1 + md_t2) * md_pnew(1) + md_t1 * md_pnew(2) + md_t2 * md_p(1)
        qExplicit = -md_t2 * md_pnew(1) - (md_t1 - md_t2) * md_p(1) + md_t1 * md_p(2)
        md_aa = md_sigma * qImplicit + md_sigma1 * qExplicit
        deriv = CD / tau * (-md_pnew(1) + pwD)
        md_a(1) = deriv + md_aa
    Else
        md_b(1) = 1#
        md_d(1) = ConstantPwD
    End If
End Sub

Sub RetrieveArray(a, aa)
    a = md_a: aa = md_aa
End Sub

Sub pqAndPwd()
    md_q = -(md_t1 + md_t2) * md_pnew(1) + md_t1 * md_pnew(2) + md_t2 * md_p(1)
    If bConstantQ Then
        pwD = md_pnew(1) + md_s / md_kappa * md_q
    Else
        pwD = ConstantPwD
        md_pnew(1) = pwD + md_s / md_kappa * md_q
    End If
    md_p = md_pnew
Private Sub GenerateArray()
Dim i As Integer, am As Double, ap As Double, g As Double, a As Double, b As Double, c As Double, d As Double
ReDim md_a(nx): ReDim md_b(nx): ReDim md_c(nx): ReDim md_d(nx)
    For i = 2 To nx - 1
        g = md_t3 * md_z(i)
        am = (md_perm(i) + md_perm(i - 1)) * 0.5
        ap = (md_perm(i) + md_perm(i + 1)) * 0.5
        a = am * md_sigma
        c = ap * md_sigma
        b = -(a + c) - g
        d = -g * md_p(i) + am * (md_p(i) - md_p(i - 1)) * md_sigma1 - ap * (md_p(i + 1) - md_p(i)) * md_sigma1
        If bConstantQ Then
            md_a(i) = a * md_pnew(i - 1) + b * md_pnew(i) + c * md_pnew(i + 1) - d
        Else
            md_a(i) = a: md_b(i) = b: md_c(i) = c:
            md_d(i) = -d
        End If
    Next i
    If bConstantQ Then
        md_a(nx) = -md_pnew(nx)
        If bNoFlow Then md_a(nx) = md_a(nx) + md_pnew(nx - 1)
    Else
        md_b(nx) = 1#
        If bNoFlow Then md_a(nx) = -1#
    End If
End Sub

Property Let pnew(Index As Integer, ByVal pnewVal As Double)
    md_pnew(Index) = pnewVal
End Property

Sub RetrievePKU(a1, a2, a3)
    a1 = md_pnew: a2 = md_perm: a3 = md_u
End Sub

Sub RetrievePnew(a1)
    a1 = md_pnew
End Sub

Sub tridag()
Dim j As Integer, bet As Double, gam(500) As Double
    If md_b(1) = 0# Then MsgBox "tridag: rewrite equations"
    bet = md_b(1)
    md_pnew(1) = md_d(1) / bet
    For j = 2 To nx - 1
        gam(j) = md_c(j - 1) / bet
        bet = md_b(j) - md_a(j) * gam(j)
        If bet = 0# Then MsgBox "pause 'tridag failed"
        md_pnew(j) = (md_d(j) - md_a(j) * md_pnew(j - 1)) / bet
    Next j
    For j = nx - 1 To 1 Step -1
        md_pnew(j) = md_pnew(j) - gam(j + 1) * md_pnew(j + 1)
    Next j
End Sub
**Point Class**

Option Explicit
Option Base 1
Private mi_number As Integer, md_small As Double, md_ints As Double
Private md_xx() As Double, md_yy() As Double, md-pane() As Double
Property Let number(numberVal As Integer)
    mi_number = numberVal
End Property
Property Get number() As Integer
    number = mi_number
End Property
Property Let small(smallVal As Double)
    md_small = smallVal
End Property
Property Get small() As Double
    small = md_small
End Property
Property Let ints(intsVal As Double)
    md_ints = intsVal
End Property
Property Get ints() As Double
    ints = md_ints
End Property
Sub SetArrays()
    ReDim md_xx(mi_number)
    ReDim md_yy(mi_number)
    ReDim md-pane(mi_number)
End Sub
Property Get xx(Index As Integer) As Double
    xx = md_xx(Index)
End Property
Property Let xx(Index As Integer, ByVal xxVal As Double)
    md_xx(Index) = xxVal
End Property
Property Get yy(Index As Integer) As Double
    yy = md_yy(Index)
End Property
Property Let yy(Index As Integer, ByVal yyVal As Double)
    md_yy(Index) = yyVal
End Property
Property Get pane(Index As Integer) As Double
    pane = md-pane(Index)
End Property
Property Let pane(Index As Integer, ByVal paneVal As Double)
    md-pane(Index) = paneVal
End Property

**Reservoir Class**

Option Explicit
Option Base 1
Private ep As cls04Point, el As cls03Layer
Private md_1ci As Double, md_p() As Double, md_q As Double, md_integral() As Double
Sub Class_Initialize()
    ReDim u(nx)
    ReDim md_integral(nx)
End Sub
Property Let oneci(oneciVal As Double)
    md_lci = oneciVal
End Property
Sub Pressure()
    Dim i As Integer, s As Double
    ReDim md_p(nx)
    For i = 1 To nx
        s = 0#
        For Each el In Layers
            s = s + el.pnew(i) * el.hgamma
        Next
        md_p(i) = s
    Next i
End Sub
Sub Deformation(jjj, ind)
    Dim i As Integer, j As Integer, k As Integer, n As Integer, khi As Integer, klo As Integer
    Dim i1 As Integer, i2 As Integer, jj As Integer
    Dim x1 As Double, x2 As Double, f1 As Double, f2 As Double, f As Double, ff As Double, s As Double, up As Double
    ReDim u(nx)
    i = 0
    If j jj = 0 Then
        For Each ep In Points
            With ep
                i = i + 1
                s = 0#
                n = .number
                x1 = .xx(1)
                klo = 1
                khi = nx / 4
                Call fappar(klo, khi, x1, f)
                f1 = f * .yy(1)
                For j = 2 To n
                    x2 = .xx(j)
                    While x2 > rD(khi)
                        khi = khi + 1
                    Wend
                    While x2 < rD(klo)
                        klo = klo - 1
                    Wend
                    Call fappar(klo, khi, x2, f)
                    f2 = f * .yy(j)
                    ff = (f1 + f2) * Abs(x2 - x1) * 0.5
                    .pane(j) = ff
                    s = s + ff
                    x1 = x2
                    f1 = f2
                Next j
                md_integral(i) = s
                u(i) = (s + md_p(1) * .small - md_p(i)) / md_lci
            End With
        Next
    Else
        If ind = 0 Then
            If j jj = 1 Then ReDim mi_NotEqual(nx, nx, 500)
            jj = Corresponds(jjj)
        End If
    End If
End Sub
For Each ep In Points
    With ep
        i = i + 1
        k = 0
        s = 0#
        n = .number
        x1 = .xx(1)
        klo = 1
        khi = nx / 4
        Call fappar(klo, khi, x1, f)
        f1 = f * .yy(1)
        For j = 2 To n
            x2 = .xx(j)
            While x2 > rD(khi)
                khi = khi + 1
            Wend
            While x2 < rD(klo)
                klo = klo - 1
            Wend
            Call fappar(klo, khi, x2, f)
            f2 = f * .yy(j)
            ff = (f1 + f2) * Abs(x2 - x1) * 0.5
            If ff <> .pane(j) Then
                k = k + 1
                mi_NotEqual(jj, i, k) = j
            End If
            s = s + ff
            x1 = x2
            f1 = f2
        Next j
        mi_NotEqual(jj, i, 500) = k
        u(i) = (s + md_p(1) * .small - md_p(i)) / md_1ci
    End With
Next
Else
    jj = Corresponds(jjj)
    For Each ep In Points
        With ep
            i = i + 1
            s = md_integral(i)
            n = .number
            il = mi_NotEqual(jj, i, 1) - 1
            i2 = mi_NotEqual(jj, i, 500)
            x1 = .xx(il)
            klo = 1
            khi = nx / 4
            Call fappar(klo, khi, x1, f)
            f1 = f * .yy(il)
            For j = il + 1 To i2
                x2 = .xx(j)
                While x2 > rD(khi)
                    khi = khi + 1
                Wend
                While x2 < rD(klo)
                    klo = klo - 1
                Wend
                Call fappar(klo, khi, x2, f)
            Next j
        End With
    Next
End
f2 = f * yy(j)
f1 = f2
ff = (f1 + f2) * Abs(x2 - x1) * 0.5
s = s - pane(j) + ff
x1 = x2
f1 = f2
Next j
u(i) = (s + md_p(l) * small - md_p(i)) / md_1ci
End With
Next
End If
End If
End Sub
Sub CalculateLayersPermeability()
For Each el In Layers
  el.Permeability
Next
End Sub
Function Corresponds(jj)
  Corresponds = jj - Application.WorksheetFunction.RoundDown(jj / nx, 0) * nx
  If Corresponds = 0 Then Corresponds = nx
End Function
Sub SetLayersPressure(x)
Dim i As Integer, j As Integer, k As Integer
j = 0
For Each el In Layers
  j = j + 1
  k = 0
  For i = nx * (j - 1) + 1 To nx * j
    k = k + 1
    el.pnew(k) = x(i)
  Next i
Next
End Sub
Sub UpdateLayersPressure()
Dim s As Double
For Each el In Layers
  el.pqAndPwd
  s = s + el.q
Next
md_q = CD / tau * (Layers(1).pnew(1) - pwD) - s
End Sub
Sub CrossFlow()
Dim i As Integer, j As Integer, k As Integer, dp As Double
If nl > 1 Then
  For j = 1 To nl Step 2
    For i = 1 To nx
      dp = Layer(j).pnew(i) - Layer(j + 1).pnew(i)
      Layer(j).pXflow(i) = -Layer(j).xflow * dp
      Layer(j + 1).pXflow(i) = Layer(j).xflow * dp
    Next i
  Next j
End If
End Sub
Property Get q() As Double
  q = md_q
End Property
Private Sub fappar(klo, khi, z, zz)
Dim i As Integer, k As Integer, a As Double, b As Double, c As Double, f1 As Double, f2 As Double, f3 As Double
Dim x1 As Double, x2 As Double, x3 As Double
If klo = 1 Then
  klo = 2
  If khi <> nx Then khi = khi + 1
End If
For i = klo To khi
  If z = rD(i) Then
    zz = md_p(i)
    GoTo 1
  End If
  If z > rD(i - 1) And z < rD(i) Then
    If i = 2 Then
      k = i - 1
    Else
      k = i - 2
    End If
    x1 = rD(k)
    x2 = rD(k + 1)
    x3 = rD(k + 2)
    f1 = md_p(k)
    f2 = md_p(k + 1)
    f3 = md_p(k + 2)
    b = ((f1 - f3) / (x1 * x1 - x3 * x3) - (f1 - f2) / (x1 * x1 - x2 * x2)) * _
        (x1 + x3) * (x1 + x2) / (x2 - x3)
    a = ((f1 - f3) - b * (x1 - x3)) / (x1 * x1 - x3 * x3)
    c = f1 - a * x1 * x1 - b * x1
    zz = a * z * z + b * z + c
    GoTo 1
  End If
Next i
1   klo = i - 1
   khi = i
End Sub

Exit Criteria
Option Explicit
Option Base 1
Private md_x() As Double
Sub Class_Initialize()
  bDone = False
  ReDim md_x(m)
End Sub
Sub Evaluate(x)
  If TotalTime / CD > MaxT Or pwD > MaxP Then bDone = True
End Sub
Function bConverged(x)
  Dim i As Integer, razn0 As Double, razn As Double
  razn0 = -1000#
  razn = razn0
  bConverged = False
  For i = 1 To m
    If x(i) <> 0# Then razn = Application.WorksheetFunction.Max(razn, Abs(1# - md_x(i) / x(i)) * 100#)
  Next i
End Function
Next i
If razn < 0.00001 And razn <> razn0 Then
    bConverged = True
    ReDim md_x(m)
Else
    md_x = x
End If
End Function

Matrix Solution
Option Explicit
Option Base 1
Private Const itol As Integer = 1, tiny As Double = 1E-20
Private ija() As Double, indx() As Double
Sub SolveSparse(a, b)
    Dim iter As Integer, err As Double, x() As Double
    ReDim x(m)
    Call sprsin(a, m)
    Call linbcg(m, b, x, itol, 1E-16, 2000, iter, err)
    b = x
End Sub
Private Sub sprsin(a, n)      'convert matrix to sparse format
    Dim i As Integer, j As Integer, k As Integer, nmax As Integer, aij As Double
    ReDim ija(n)
    ija(1) = n + 2
    k = n + 1
    For i = 1 To n
        For j = 1 To n
            aij = a(i, j)
            If Abs(aij) <> 0# Then
                If i <> j Then
                    k = k + 1
                    ReDim Preserve sa(k)
                    ReDim Preserve ija(k)
                    sa(k) = aij
                    ija(k) = j
                End If
            End If
        Next j
        ija(i + 1) = k + 1
    Next i
End Sub
Private Sub dsprsax(x, b, n)                    'product of sparse matrix
    Dim i As Integer, k As Integer
    If ija(1) <> n + 2 Then MsgBox "mismatched vector and matrix in sprsax"
    For i = 1 To n
        b(i) = sa(i) * x(i)
        For k = ija(i) To ija(i + 1) - 1
            b(i) = b(i) + sa(k) * x(ija(k))
        Next k
    Next i
End Sub
Private Sub dsprstx(x, b, n)                    'product of transpose
    Dim i As Integer, k As Integer
    If ija(1) <> n + 2 Then MsgBox "mismatched vector and matrix in sprsax"
    For i = 1 To n
        b(i) = sa(i) * x(i)
        For k = ija(i) To ija(i + 1) - 1
            b(i) = b(i) + sa(k) * x(ija(k))
        Next k
    Next i
End Sub
Dim i As Integer, j As Integer, k As Integer
If ija(l) <> n + 2 Then MsgBox "mismatched vector and matrix in sprstx"
For i = 1 To n
    b(i) = sa(i) * x(i)
Next i
For i = 1 To n
    For k = ija(i) To ija(i + 1) - 1
        j = ija(k)
        b(j) = b(j) + sa(k) * x(i)
    Next k
Next i
End Sub
Private Function snrm(n, sx, itol) 'used by linbcg
for vector norm
Dim i As Integer, isamax As Integer
If itol <= 3 Then
    snrm = 0#
    For i = 1 To n
        snrm = snrm + sx(i) ^ 2
    Next i
    snrm = Sqr(snrm)
Else
    isamax = 1
    For i = 1 To n
        If Abs(sx(i)) > Abs(sx(isamax)) Then isamax = i
    Next i
    snrm = Abs(sx(isamax))
End If
End Function
Private Sub atimes(n, x, r, itrnsp) 'used by linbcg
for sparse multiplication
If itrnsp = 0 Then
    Call dsprsax(x, r, n)
Else
    Call dsprstx(x, r, n)
End If
End Sub
Private Sub asolve(n, b, x, itrnsp) 'used by linbcg
for preconditioner
Dim i As Integer
For i = 1 To n
    x(i) = b(i) / sa(i)
Next i
End Sub
Private Sub linbcg(n, b, x, itol, tol, ITMAX, iter, err)
Dim nmax As Integer, i As Integer, j As Integer
Dim eps As Double
Dim ak As Double, akden As Double, bk As Double, bkden As Double, bknrm As Double, bnrm As Double
Dim dxnrm As Double, xnrm As Double, zmlnrm As Double, znrm As Double, p() As Double, pp() As Double
Dim r() As Double, rr() As Double, z() As Double, zz() As Double
nmax = 1024
eps = 0.00000000000001
iter = 0
Call atimes(n, x, r, 0)
For j = 1 To n
    r(j) = b(j) - r(j)
    rr(j) = r(j)
Next j
If itol = 1 Then
    bnrm = snrm(n, b, itol)
    Call asolve(n, r, z, 0)
ElseIf itol = 2 Then
    Call asolve(n, b, z, 0)
    bnrm = snrm(n, z, itol)
    Call asolve(n, r, z, 0)
ElseIf itol = 3 Or itol = 4 Then
    Call asolve(n, b, z, 0)
    bnrm = snrm(n, z, itol)
    Call asolve(n, r, z, 0)
Else
    MsgBox "illegal itol in linbcg"
End If
100 If iter <= ITMAX Then
    iter = iter + 1
    Call asolve(n, rr, zz, 1)
    bknum = 0#
    For j = 1 To n
        bknum = bknum + z(j) * rr(j)
    Next j
    If iter = 1 Then
        For j = 1 To n
            p(j) = z(j)
            pp(j) = zz(j)
        Next j
        Else
            bk = bknum / bkden
            For j = 1 To n
                p(j) = bk * p(j) + z(j)
                pp(j) = bk * pp(j) + zz(j)
            Next j
        End If
        bkden = bknum
        Call atimes(n, p, z, 0)
        akden = 0#
        For j = 1 To n
            akden = akden + z(j) * pp(j)
        Next j
        ak = bknum / akden
        Call atimes(n, pp, zz, 1)
        For j = 1 To n
            x(j) = x(j) + ak * p(j)
            r(j) = r(j) - ak * z(j)
            rr(j) = rr(j) - ak * zz(j)
        Next j
        Call asolve(n, r, z, 0)
    If itol = 1 Then
        err = snrm(n, r, itol) / bnrm
    ElseIf itol = 2 Then
        err = snrm(n, r, itol) / bnrm
        Call atimes(n, x, r, 0)
        For j = 1 To n
            r(j) = b(j) - r(j)
            rr(j) = r(j)
        Next j
        If itol = 1 Then
            bnrm = snrm(n, b, itol)
            Call asolve(n, r, z, 0)
        ElseIf itol = 2 Then
            Call asolve(n, b, z, 0)
            bnrm = snrm(n, z, itol)
            Call asolve(n, r, z, 0)
        ElseIf itol = 3 Or itol = 4 Then
            Call asolve(n, b, z, 0)
            bnrm = snrm(n, z, itol)
            Call asolve(n, r, z, 0)
        Else
            MsgBox "illegal itol in linbcg"
        End If
        100 If iter <= ITMAX Then
            iter = iter + 1
            Call asolve(n, rr, zz, 1)
            bknum = 0#
            For j = 1 To n
                bknum = bknum + z(j) * rr(j)
            Next j
            If iter = 1 Then
                For j = 1 To n
                    p(j) = z(j)
                    pp(j) = zz(j)
                Next j
                Else
                    bk = bknum / bkden
                    For j = 1 To n
                        p(j) = bk * p(j) + z(j)
                        pp(j) = bk * pp(j) + zz(j)
                    Next j
                End If
                bkden = bknum
                Call atimes(n, p, z, 0)
                akden = 0#
                For j = 1 To n
                    akden = akden + z(j) * pp(j)
                Next j
                ak = bknum / akden
                Call atimes(n, pp, zz, 1)
                For j = 1 To n
                    x(j) = x(j) + ak * p(j)
                    r(j) = r(j) - ak * z(j)
                    rr(j) = rr(j) - ak * zz(j)
                Next j
                Call asolve(n, r, z, 0)
            If itol = 1 Then
                err = snrm(n, r, itol) / bnrm
            ElseIf itol = 2 Then
                err = snrm(n, r, itol) / bnrm
                Call atimes(n, x, r, 0)
err = snrm(n, z, itol) / bnrm
ElseIf itol = 3 Or itol = 4 Then
    zm1nrm = znrm
    znrm = snrm(n, z, itol)
    If Abs(zm1nrm - znrm) > eps * znrm Then
        dxnrm = Abs(ak) * snrm(n, p, itol)
        err = znrm / Abs(zm1nrm - znrm) * dxnrm
    Else
        err = znrm / bnrm
        GoTo 100
    End If
    xnrm = snrm(n, x, itol)
    If err <= 0.5 * xnrm Then
        err = err / xnrm
    Else
        err = znrm / bnrm
        GoTo 100
    End If
End If
End If
If err > tol Then GoTo 100
End If
End Sub
Sub SolveLUD(fjac, p)
Dim d As Double
    ReDim indx(m)
    Call ludcmp(fjac, d)
    Call lubksb(fjac, p)
End Sub
Private Sub ludcmp(a, d)
Dim i As Integer, imax As Integer, j As Integer, k As Integer
    Dim aamax As Double, dum As Double, sum As Double, vv() As Double
    ReDim vv(m)
    d = 1
    For i = 1 To m
        aamax = 0#
        For j = 1 To m
            If Abs(a(i, j)) > aamax Then aamax = Abs(a(i, j))
        Next j
        If aamax = 0 Then MsgBox "singular matrix in ludcmp"
        vv(i) = 1# / aamax
    Next i
    For j = 1 To m
        For i = 1 To j - 1
            sum = a(i, j)
            For k = 1 To i - 1
                sum = sum - a(i, k) * a(k, j)
            Next k
            a(i, j) = sum
        Next i
        aamax = 0#
    For i = j To m
        sum = a(i, j)
        For k = 1 To j - 1
            sum = sum - a(i, k) * a(k, j)
        Next k
        a(i, j) = sum
        dum = vv(i) * Abs(sum)
If dum >= aamax Then
    imax = i
    aamax = dum
End If
Next i
If j <> imax Then
    For k = 1 To m
        dum = a(imax, k)
        a(imax, k) = a(j, k)
        a(j, k) = dum
     Next k
    d = -d
    vv(imax) = vv(j)
End If
indx(j) = imax
If a(j, j) = 0# Then a(j, j) = tiny
If j <> m Then
    dum = 1# / a(j, j)
    For i = j + 1 To m
        a(i, j) = a(i, j) * dum
    Next i
End If
Next j
End Sub
Private Sub lubksb(a, b)
    Dim i As Integer, ii As Integer, j As Integer, ll As Integer
    Dim sum As Double
    ii = 0
    For i = 1 To m
        ll = indx(i)
        sum = b(ll)
        b(ll) = b(i)
        If ii <> 0 Then
            For j = ii To i - 1
                sum = sum - a(i, j) * b(j)
            Next j
        ElseIf sum <> 0# Then
            ii = i
        End If
        b(i) = sum
    Next i
    For i = m To 1 Step -1
        sum = b(i)
        For j = i + 1 To m
            sum = sum - a(i, j) * b(j)
        Next j
        b(i) = sum / a(i, i)
    Next i
End Sub

Newton Method
Option Explicit
Option Base 1
Private fvec() As Double, mi_ind As Integer, sigma As Double
Private Const maxits As Integer = 500, tolf As Double = 0.00001, tolmin As Double = 0.000001, tolx As Double = 0.0000001

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Private Const stpmx As Double = 100#, alf As Double = 0.0001, eps As Double = 0.0001
Sub Class_Initialize()
    mi_ind = 0
End Sub
Sub Run(x, check, niter)
    Dim i As Integer, its As Integer, j As Integer
    Dim d As Double, den As Double, f As Double, fold As Double, stpmax As Double, sum As Double, temp As Double
    Dim test As Double, fjac() As Double, g() As Double, p() As Double, xold() As Double, fjac1() As Double
    ReDim indx(m): ReDim fjac(m, m): ReDim g(m): ReDim p(m):
    f = fmin(x)
    test = 0#
    For i = 1 To m
        If Abs(fvec(i)) > test Then test = Abs(fvec(i))
    Next i
    If test < 0.01 * tolf Then
        check = False
        Exit Sub
    End If
    sum = Application.WorksheetFunction.SumProduct(x, x)
    stpmax = stpmx * Application.WorksheetFunction.Max(Sqr(sum), CDbl(m))
    For its = 1 To maxits
        niter = its
        Call fdjac(x, fvec, fjac)
        For i = 1 To m
            sum = 0#
            For j = 1 To m
                sum = sum + fjac(j, i) * fvec(j)
            Next j
            g(i) = sum
        Next i
        xold = x
        fold = f
        For i = 1 To m
            p(i) = -fvec(i)
        Next i
        If bSparse Then
            Call Matrix.SolveSparse(fjac, p)
        Else
            Call Matrix.SolveLUD(fjac, p)
        End If
        Call lnsrch(xold, fold, g, p, x, f, stpmax, check)
        test = 0#
        For i = 1 To m
            If Abs(fvec(i)) > test Then test = Abs(fvec(i))
        Next i
        If test < tolf Then
            check = False
            Exit Sub
        End If
    End If
    If check Then
        test = 0#
        den = Application.WorksheetFunction.Max(f, 0.5 * m)
        For i = 1 To m

temp = Abs(g(i)) * Application.WorksheetFunction.Max(Abs(x(i)), 1) / den
    If temp > test Then test = temp
Next i
If test < tolmin Then
    check = True
Else
    check = False
End If
Exit If
End If
test = 0
For i = 1 To m
    temp = (Abs(x(i) - xold(i))) / Application.WorksheetFunction.Max(Abs(x(i)), 1)
    If temp > test Then test = temp
Next i
If test < tolx Then Exit Sub
Next its
End Sub
Private Sub lnsrch(xold, fold, g, p, x, f, stpmax, check)
    Dim i As Integer
    Dim temp As Double, test As Double, tmplam As Double
    Dim a As Double, alam As Double, alam2 As Double, alamin As Double, b As Double, disc As Double
    Dim f2 As Double, fold2 As Double, rhs1 As Double, rhs2 As Double, slope As Double, sum As Double
    check = False
    sum = Sqr(Application.WorksheetFunction.SumProduct(p, p))
    If sum > stpmax Then
        For i = 1 To m
            p(i) = p(i) * stpmax / sum
        Next i
    End If
    slope = Application.WorksheetFunction.SumProduct(g, p)
    test = 0
    For i = 1 To m
        temp = Abs(p(i)) / Application.WorksheetFunction.Max(Abs(xold(i)), 1)
        If temp > test Then test = temp
    Next i
    alamin = tolx / test
    alam = 1
    For i = 1 To m
        x(i) = xold(i) + alam * p(i)
    Next i
    f = fmin(x)
    If alam < alamin Then
        x = xold
        check = True
        Exit Sub
    ElseIf f <= fold + alf * alam * slope Then
        Exit Sub
    Else
        If alam = 1 Then
            tmplam = -slope / (2 * (f - fold - slope))
        Else
\[
\text{rhs1} = f - \text{fold} - \text{alam} \times \text{slope} \\
\text{rhs2} = f2 - \text{fold2} - \text{alam2} \times \text{slope} \\
a = (\text{rhs1} / \text{alam} \times 2 - \text{rhs2} / \text{alam2} \times 2) / (\text{alam} - \text{alam2}) \\
b = (-\text{alam2} \times \text{rhs1} / \text{alam} \times 2 + \text{alam} \times \text{rhs2} / \text{alam2} \times 2) / (\text{alam} - \text{alam2})
\]

If \( a = 0 \) Then
\[
\text{tmplam} = -\text{slope} / (2 \times b)
\]
Else
\[
\text{disc} = b \times b - 3 \times a \times \text{slope}
\]
If disc < 0 Then MsgBox "roundoff problem in lnsrch"
\[
\text{tmplam} = (-b + \text{Sqr(disc)}) / (3 \times a)
\]
End If
If tmplam > 0.5 \times \text{alam} Then tmplam = 0.5 \times \text{alam}
End If
\[
\text{alam2} = \text{alam}
\text{f2} = f
\text{fold2} = \text{fold}
\text{alam} = \text{Application.WorksheetFunction.Max(tmplam, 0.1 \times \text{alam})}
\]
\text{GoTo 1}
End Sub

Private Function fdjac(x, fvec, df)
Dim i As Integer, j As Integer, h As Double, temp As Double, f() As Double, dfij As Double, nonzerocount As Long
ReDim f(m)
nonzerocount = 0
ReDim sa(m)
For j = 1 To m
    temp = x(j)
    h = eps * Abs(temp)
    If h = 0 Then h = eps
    x(j) = temp + h
    h = x(j) - temp
    Call funcv(x, f, j)
    x(j) = temp
    For i = 1 To m
        dfij = (f(i) - fvec(i)) / h
        If dfij <> 0 Then nonzerocount = nonzerocount + 1
        If i = j Then sa(j) = dfij
        df(i, j) = dfij
    Next i
Next j
mi_ind = 1
bSparse = False
If nonzerocount / m / m < 0.5 Then bSparse = True
End Function

Private Function fmin(x)
Call funcv(x, fvec, 0)
fmin = 0.5 * Application.WorksheetFunction.SumProduct(fvec, fvec)
End Function

Private Sub funcv(x, ffvec, j)
    Call Reservoir.SetLayersPressure(x)
    Reservoir.Pressure
    Call Reservoir.Deformation(j, mi_ind)
    Reservoir.CalculateLayersPermeability
    Call Vector.Generate(ffvec)
End Sub
Option Explicit
Option Base 1
Private Const md_min As Double = 1E-16
Private mv_rDtD() As Variant, elem As Variant, el As cls03Layer, md_xs() As Variant, md_xxs() As Double, mb_BU As Boolean
Private md_p() As Double, md_k() As Double, md_u() As Double, mi_row As Integer, mi_col As Integer, mi_start As Integer
Sub Class_Initialize()
Dim i As Integer
mv_rDtD = Array("P", "K", "U")
If bBoth Then CleanUp
ReDim md_xxs(nx)
Sheets("ED").Activate
Cells(1, 20).Select
For i = 1 To nx
    md_xxs(i) = reD ^ rD(i)
    ActiveCell.Offset(i - 1, 0) = md_xxs(i)
    ActiveCell.Offset(i - 1, 6) = md_xxs(i)
Next i
ReDim md_xs(1)
For i = 1 To nl
    Call ConcatenateTwoArrays(md_xs, md_xxs)
Next i
For Each elem In mv_rDtD
    Sheets(elem).Activate
    Cells(2, 1).Select
    For i = 1 To m
        ActiveCell.Offset(i - 1, 0) = md_xs(i)
    Next i
Next elem
Sheets("uu").Activate
Cells(2, 1).Select
For i = 1 To nx
    ActiveCell.Offset(i - 1, 0) = md_xxs(i)
Next i
mi_row = 1
mi_col = 2
mb_BU = True
End Sub
Sub Intermediate()
Dim a1() As Double, a2() As Double, a3() As Double, q As Double, rangeX As Range, rangeY1 As Range, rangeY2 As Range
Dim i1 As Integer, i2 As Integer
Application.ScreenUpdating = False
If bBuildUp Then
    mi_start = 9
    mi_row = m + 4
If mb_BU Then
    mi_col = 2
    mb_BU = False
End If
Else
    mi_start = 1
    mi_row = 2
End If
If bConstantQ Then
    i1 = mi_start + 2
    i2 = mi_start + 3
Else
    i1 = mi_start + 3
    i2 = mi_start + 2
End If
ReDim md_k(1): ReDim md_u(1)
q = Reservoir.q
Sheets("tD").Select
Cells(ntau, mi_start) = TotalTime / CD
Cells(ntau, mi_start + 1) = Log(TotalTime / CD)
If Abs(pwDinit - pwD) <> 0# Then
    Cells(ntau, i1) = Abs(pwDinit - pwD)
Else
    Cells(ntau, i1) = md_min
End If
If q <> 0# Then
    Cells(ntau, i2) = 1# / q
Else
    Cells(ntau, i2) = md_min
End If
If ntau > 2 Then
    Set rangeX = Range(Cells(ntau - 2, mi_start + 1), Cells(ntau, mi_start + 1))
    Set rangeY1 = Range(Cells(ntau - 2, mi_start + 2), Cells(ntau, mi_start + 2))
    Set rangeY2 = Range(Cells(ntau - 2, mi_start + 3), Cells(ntau, mi_start + 3))
    Cells(ntau - 1, mi_start + 4) = ___Application.WorksheetFunction.Max(md_min, Application.WorksheetFunction.slope(rangeY1, rangeX))
    Cells(ntau - 1, mi_start + 5) = ___Application.WorksheetFunction.Max(md_min, Application.WorksheetFunction.slope(rangeY2, rangeX))
End If
If TotalTime <> 0# Then
    Cells(ntau, mi_start + 6) = Abs(1# - Abs(pwD - pwDinit) / VEH.Stehfest(TotalTime, 1#)) * 100#
Cells(1, 21).Select
i1 = 0
For Each el In Layers
    i1 = i1 + 1
    Call el.RetrievePKU(a1, a2, a3)
For i2 = 1 To nx
    ActiveCell.Offset(i2 - 1, i1 - 1) = a1(i2)
    ActiveCell.Offset(i2 - 1, i1 - 1 + 6) = a2(i2)
Next i2
If ntau Mod PrintFrequency = 0 Then
    Call ConcatenateTwoArrays(md_p, a1)
    Call ConcatenateTwoArrays(md_k, a2)
    Call ConcatenateTwoArrays(md_u, a3)
End If
Next el
If ntau Mod PrintFrequency = 0 Then
    For Each elem In mv_rDtD
        Sheets(elem).Activate
Cells(mi_row, mi_col).Select
ActiveCell.Offset(-1, 0) = TotalTime / CD
Select Case elem
Case Is = "P"
    For i1 = 1 To m
        ActiveCell.Offset(i1 - 1, 0) = md_p(i1)
    Next i1
Case Is = "K"
    For i1 = 1 To m
        ActiveCell.Offset(i1 - 1, 0) = md_k(i1)
    Next i1
Case Is = "U"
    For i1 = 1 To m
        ActiveCell.Offset(i1 - 1, 0) = md_u(i1)
    Next i1
Case Else
    End Select
Next elem
Next elem
Sheets("uu").Activate
Cells(mi_row, mi_col).Select
ActiveCell.Offset(-1, 0) = TotalTime / CD
For i1 = 1 To nx
    ActiveCell.Offset(i1 - 1, 0) = u(i1)
Next i1
mi_col = mi_col + 1
End If
Sheets("Results").Activate
ActiveSheet.TextBoxes("Progress").Text = "Time = " & Format(TotalTime / CD, "0.00E+00") & Chr(10) & _
    "Error = " & Format(error, "0.00E+00")
If ntau Mod 10 = 0 Then Application.ScreenUpdating = True
End Sub
Private Sub ConcatenateTwoArrays(arr1, arr2)
    Dim i As Integer, n1 As Integer, N2 As Integer
    n1 = UBound(arr1)
    If n1 = 1 Then n1 = 0
    N2 = UBound(arr2)
    ReDim Preserve arr1(n1 + N2)
    For i = 1 To N2
        arr1(i + n1) = arr2(i)
    Next i
End Sub
Private Sub CleanUp()
    Application.ScreenUpdating = False
    For Each elem In mv_rDtD
        Sheets(elem).Activate
        Cells.ClearContents
    Next elem
    Sheets("tD").Activate
    Cells.ClearContents
    Sheets("Input").Activate
    Cells(30, 2).ClearContents
End Sub
Sub Store(x)
    Dim i As Integer
    Application.ScreenUpdating = False
    Sheets("All").Select
Cells(2, 1).Select  
ActiveCell. Offset(-1, 0) = TotalTime / CD  
For i = 1 To m  
    ActiveCell. Offset(i - 1, 0) = x(i)  
Next i  
End Sub  
Sub Retrieve(x)  
Dim i As Integer  
Application. ScreenUpdating = False  
Sheets("All"). Select  
Cells(2, 1). Select  
For i = 1 To m  
    x(i) = ActiveCell. Offset(i - 1, 0)  
Next i  
pwD = x(1)  
Call Reservoir. SetLayersPressure(x)  
Reservoir. UpdateLayersPressure  
End Sub  

Vector-Function Calculation  
Option Explicit  
Option Base 1  
Private el As cls03Layer  
Private Sub ConcatenateTwoArrays(arr1, arr2)  
Dim i As Integer, n1 As Integer, N2 As Integer  
n1 = UBound(arr1)  
If n1 = 1 Then n1 = 0  
N2 = UBound(arr2)  
ReDim Preserve arr1(n1 + N2)  
For i = 1 To N2  
    arr1(i + n1) = arr2(i)  
Next i  
End Sub  
Sub Generate(fvec)  
Dim i As Integer, j As Integer, k As Integer, bi As Double, a() As Double, b() As Double  
ReDim fvec(1)  
ReDim b(1)  
i = 0  
For Each el In Layers  
    i = i + 1  
    ReDim a(nx)  
    el. CalculateArrays  
    Call el. RetrieveArray(a, bi)  
    ReDim Preserve b(i)  
    b(i) = bi  
    Call ConcatenateTwoArrays(fvec, a)  
Next  
k = 0  
For i = 1 To m Step nx  
    k = k + 1  
    For j = 1 To UBound(b)  
        If k <> j Then fvec(i) = fvec(i) + b(j)  
    Next j  
    fvec(i) = fvec(i) + qq  
Next i  
End Sub
Sub Progonka(x)
Dim i As Integer, a() As Double
    ReDim x(1)
    i = 0
    For Each el In Layers
        i = i + 1
        ReDim a(nx)
        el.CalculateArrays
        Call el.RetrievePnew(a)
        Call ConcatenateTwoArrays(x, a)
    Next
End Sub

Van Everdingen and Hurst Solution
Option Explicit
Option Base 1
Function Stehfest(td As Double, rD As Double) As Double
    Dim pwDs As Double, dp_dlntd As Double, dp_dtd As Double, qwd As Double,
        Cumd As Double, v(20) As Double
    Dim v_init As Double, vsum As Double, top As Double, bottom As Double, t1
        As Double, t2 As Double
    Dim b1 As Double, b2 As Double, b3 As Double, b4 As Double, b5 As Double,
        a As Double, s As Double, fs As Double
    Dim k_start As Long, k_end As Long, n As Long, i As Long, k As Long
    n = 10
    With Application
        For i = 1 To n
            v_init = (-1#) ^ (n / 2 + i)
            k_start = Int((i + 1) / 2)
            k_end = .Min(i, n / 2)
            vsum = 0#
            For k = k_start To k_end
                t1 = k ^ (n / 2)
                t2 = .Fact(2 * k):     b1 = .Fact(n / 2 - k):     b2 =
                        .Fact(k)
                b3 = .Fact(k - 1):     b4 = .Fact(i - k):          b5 =
                        .Fact(2 * k - i)
                top = t1 * t2
                bottom = b1 * b2 * b3 * b4 * b5
                vsum = vsum + top / bottom
            Next k
            v(i) = v_init * vsum
        Next i
    End With
    a = Log(2#) / td
    pwDs = 0#
    ' dp_dlntd = 0#: dp_dtd = 0#: qwd = 0#: Cumd = 0#
    For i = 1 To n
        s = a * i
        fs = Lpd(s, rD)
        pwDs = pwDs + v(i) * fs
        ' dp_dtd = dp_dtd + v(i) * s * fs: qwd = qwd + v(i) / (s * s * fs):
        ' Cumd = Cumd + v(i) / (s * s * s * fs)
    Next i
    pwDs = a * pwDs
dp_dtd = a * dp_dtd:  dp_dlntd = dp_dtd * td:  qwd = a * qwd:
Cumd = å * Cumd
Stehfest = pwDs
End Function
Private Function Lpd(s As Double, rD As Double) As Double
Dim bb As Double
bb = BesselK0(Sqr(s))
'    Lpd = (BesselK0(rd * sqr(s))) / (s ^ (3 / 2) * BesselK1(sqr(s)))  
'    Lpd = (BesselK0(rd * Sqr(s))) / s / (1# + CD * s * BesselK0(Sqr(s)))
Lpd = bb / s / (1# + CD * s * bb)
End Function
Private Function BesselK0(arg As Double) As Double
Dim y As Double, z As Double, i As Integer, n As Integer
y = arg / 2#
z = 1# / y
If arg < 2# Then
    BesselK0 = -Log(y) * BesselI0(arg) - 0.57721566 + 0.4227842 * (y ^ 2) 
        + 0.23069756 * (y ^ 4) + 0.0348859 * 
(y ^ 6) _
        + 0.00262698 * (y ^ 8) + 0.0001075 * 
(y ^ 10) _
        + 0.0000074 * (y ^ 12) 
Else
    BesselK0 = 1.25331414 - 0.07832358 * z + 0.02189568 * (z ^ 2) _
        - 0.01602446 * (z ^ 3) + 0.00587872 * (z ^ 4) _
        - 0.0025154 * (z ^ 5) + 0.00053208 * (z ^ 6)
    If arg > 700# Then
        n = Application.WorksheetFunction.RoundDown(arg / 700#, 0)
        BesselK0 = BesselK0 / (arg ^ 0.5) / Exp(arg - 700# * n)
        For i = 1 To n
            BesselK0 = BesselK0 / Exp(arg / 700#)
        Next i
    End If
End If
End Function
Private Function BesselK1(arg As Double) As Double
Dim y As Double, z As Double
y = arg / 2#
z = 1# / y
If arg < 2# Then
    BesselK1 = arg * Log(y) * BesselI1(arg) + 1 + 0.15443144 * (y ^ 2) 
        - 0.67278579 * (y ^ 4) - 0.18156897 * (y ^ 6) 
        - 0.01919402 * (y ^ 8) - 0.00110404 * (y ^ 10) - 
0.00004686 * (y ^ 12)
    BesselK1 = BesselK1 / arg
Else
    BesselK1 = 1.25331414 + 0.23498819 * (z) - 0.0365562 * (z ^ 2) + 
0.01504628 * (z ^ 3) - 0.00780353 * (z ^ 4) 
    + 0.00325614 * (z ^ 5) - 0.00068245 * (z ^ 6)
    BesselK1 = BesselK1 / Exp(arg) / Sqr(arg)
End If
End Function
Private Function Ei(arg As Double) As Double
Dim t1 As Double, t2 As Double, t3 As Double, t4 As Double, t5 As Double
Dim b1 As Double, b2 As Double, b3 As Double, b4 As Double, b5 As Double
If arg < 1# Then
    t1 = -0.57721566 + 0.99999193 * arg
    t2 = -0.24991055 * (arg * arg) + 0.05519968 * (arg * arg * arg)
    t3 = -0.00976004 * (arg * arg * arg * arg) + 0.00107857 * (arg *
        arg * arg * arg * arg) + 0.00107857 * (arg *
ElseIf arg >= 1# And arg <= 50# Then
    t1 = 1 * (arg * arg * arg * arg)
    t2 = 8.5733287401 * (arg * arg * arg)
    t3 = 18.059016973 * (arg * arg)
    t4 = 8.6347608925 * (arg)
    t5 = 0.2677737343
    b1 = 1# * (arg * arg * arg * arg)
    b2 = 9.5733223454 * (arg * arg * arg)
    b3 = 25.6329561486 * (arg * arg)
    b4 = 21.0996530827 * (arg)
    b5 = 3.9584969228
    Ei = (1# / (arg * Exp(arg))) * ((t1 + t2 + t3 + t4 + t5) / (b1 +
        b2 + b3 + b4 + b5))
Else
    Ei = 0#
End If
End Function
Private Function BesselI1(arg As Double) As Double
Dim T As Double
    T = arg / 3.75
    If arg < 3.75 Then
        BesselI1 = 0.5 + 0.87890594 * (T ^ 2) + 0.51498869 * (T ^ 4) +
            0.15084934 * (T ^ 6) + 0.02658733 * (T ^ 8) _
            + 0.00301532 * (T ^ 10) + 0.00032411 * (T ^ 12)
        BesselI1 = arg * BesselI1
    Else
        BesselI1 = 0.39894228 - 0.03988024 / T - 0.00362018 / (T ^ 2) +
            0.00163801 / (T ^ 3) - 0.01303555 / (T ^ 4) _
            + 0.02282967 / (T ^ 5) - 0.02895312 / (T ^ 6) +
            0.01787654 / (T ^ 7) - 0.00420059 / (T ^ 8)
        BesselI1 = BesselI1 * Exp(arg) / Sqr(arg)
    End If
End Function
Private Function BesselI0(arg As Double) As Double
Dim T As Double
    T = arg / 3.75
    If arg < 3.75 Then
        BesselI0 = 1# + 3.5156229 * (T ^ 2) + 3.0899424 * (T ^ 4) +
            1.2067492 * (T ^ 6) + 0.2659732 * (T ^ 8) _
            + 0.0360768 * (T ^ 10) + 0.0045813 * (T ^ 12)
    Else
        BesselI0 = 0.39894228 + 0.01328592 / T + 0.00362018 / (T ^ 2) -
            0.00163801 / (T ^ 3) + 0.00916281 / (T ^ 4) _
            - 0.02282967 / (T ^ 5) + 0.02895312 / (T ^ 6) -
            0.01787654 / (T ^ 7) - 0.00420059 / (T ^ 8)
        BesselI0 = BesselI0 * Exp(arg) / Sqr(arg)
    End If
End Function
VITA

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